GLM: The Predictive Modeling Context With Minimum Bias, GLMs and Credibility

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Outline

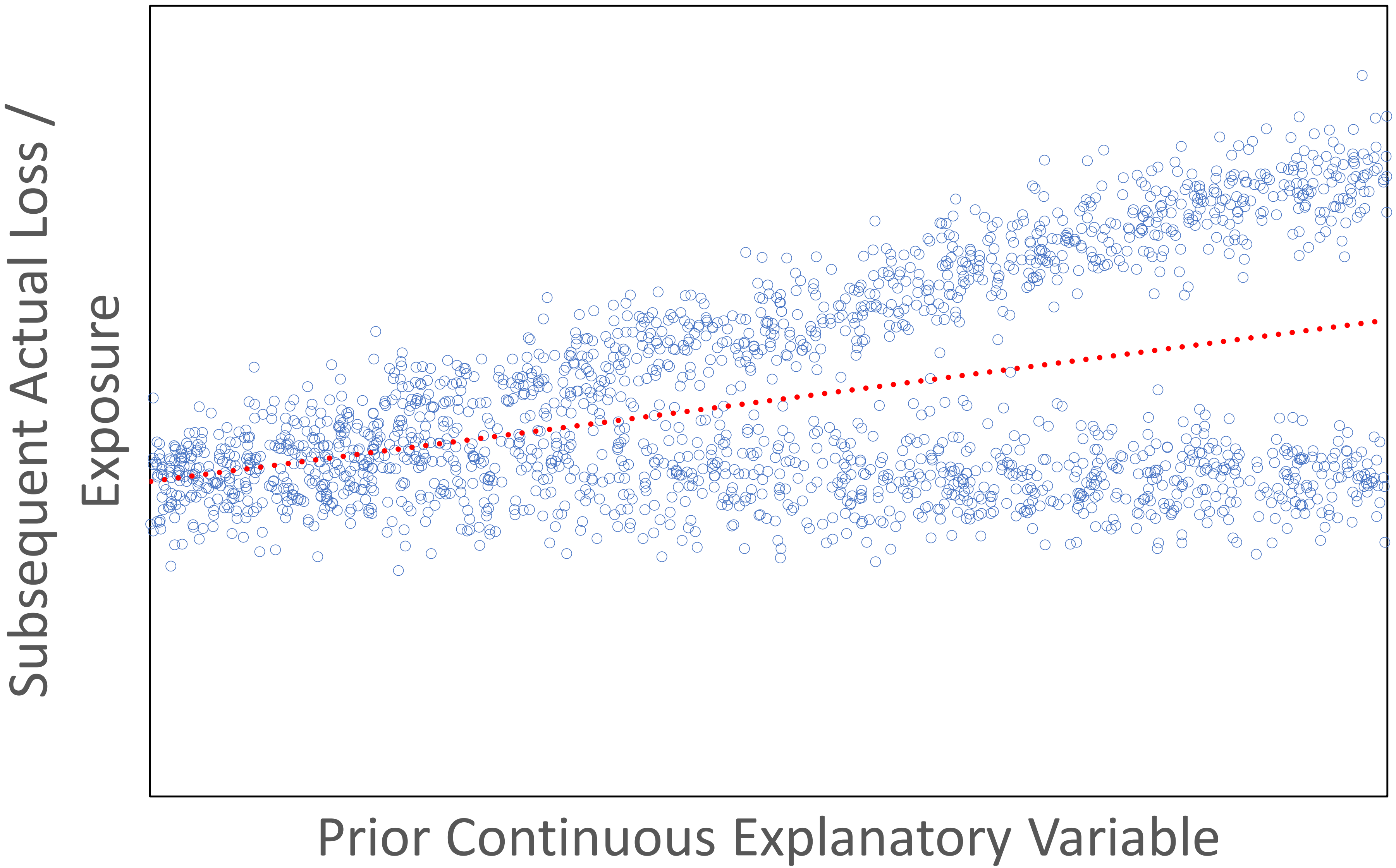
- Simple example of traditional assumption tested model
- Simple example of predictively tested model
- Multiplicative minimum bias models
- Generalized Linear Models (GLM)
- Comparison of multiplicative minimum bias and GLM
- Incorporating credibility into minimum bias versus GLM
- Case study from paper by Gross and Evans, "Minimum Bias, GLMs, and Credibility in the Context of Predictive Modeling," *CAS E-Forum* Winter 2017, available at www.casact.org

Simple Example Of Traditional Assumption Tested Model

Simple Linear Regression Model

- $Y = m X + b + \xi$
- $\xi \sim \text{Normal}(0, \sigma^2)$
- σ^2 is constant
- ξ is independent of Y and X
- ξ is sequentially auto-independent
- Least Squares = Maximum Likelihood Estimation (MLE), used for fitting

2 Years Data


Subsequent Actual Loss / Exposure
Prior Continuous Explanatory Variable

Simple Example Of Traditional Assumption Tested Model

Invalidity of Assumptions on 2 Years Data:

- ξ is clearly not Normal.
- σ^2 is not constant.
- ξ is dependent on X.

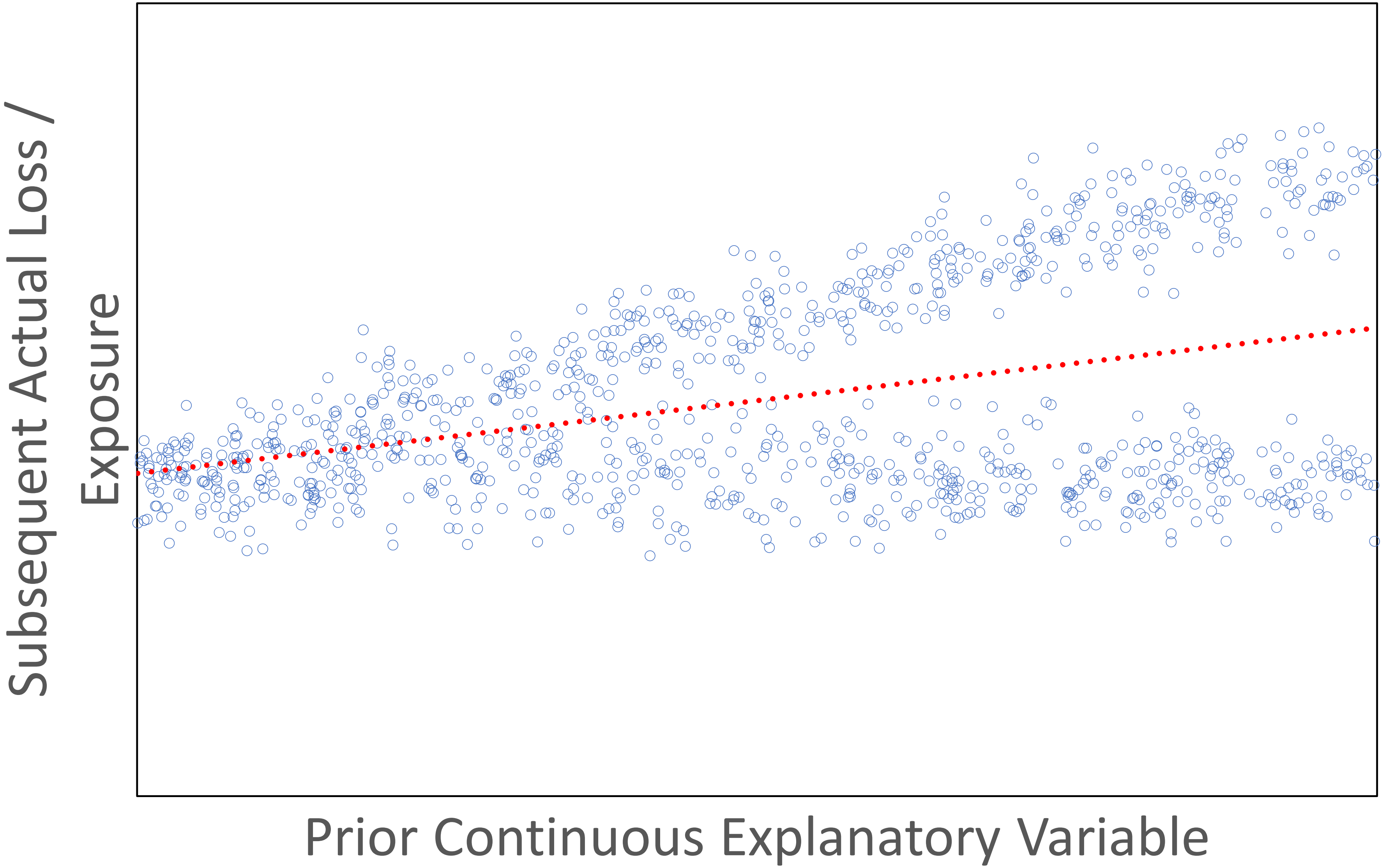
Game Over! The model is discarded! Sad!

Simple Example Of Predictively Tested Model

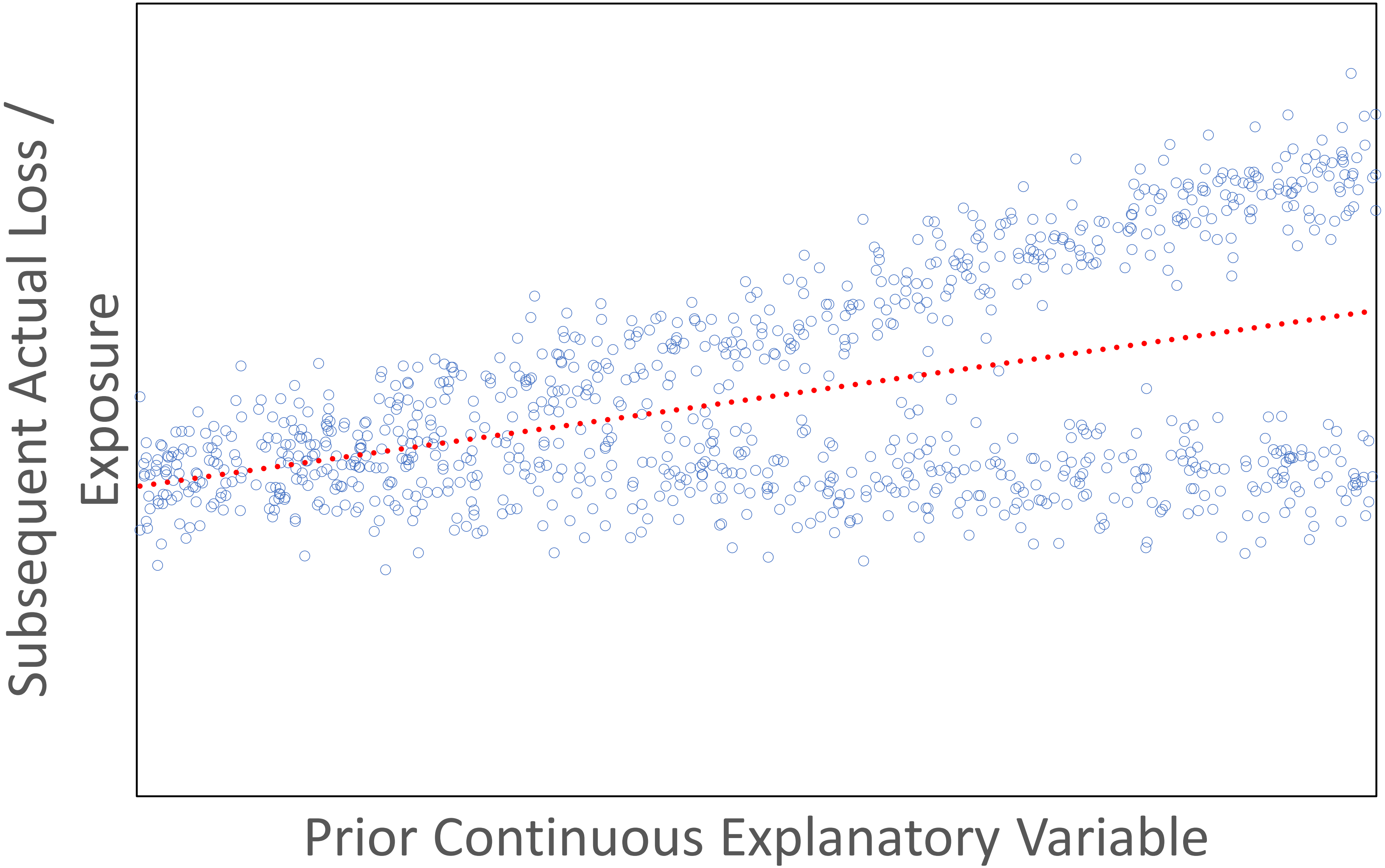
"Simplified" Simple Linear Model

- $E[Y] = m X + b$
- Typically least squares would still be used to fit this model, but the likelihood, needed for MLE, cannot even be defined.
- However, we will split out the two years of data and use Year 1 to predict Year 2.

Year 1 Data

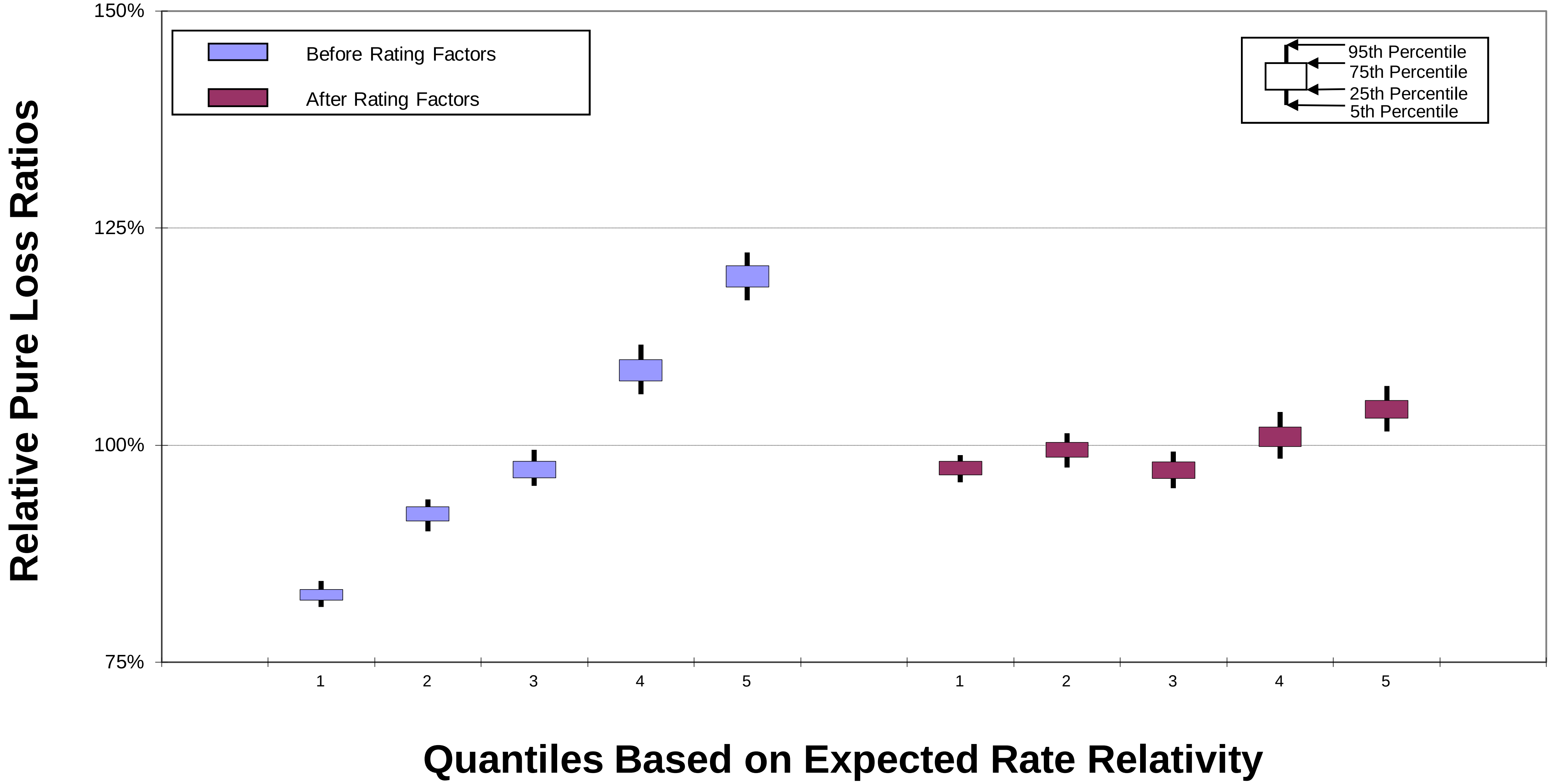

Subsequent Actual Loss / Exposure
Prior Continuous Explanatory Variable

Year 2 Data



Subsequent Actual Loss / Exposure
Prior Continuous Explanatory Variable

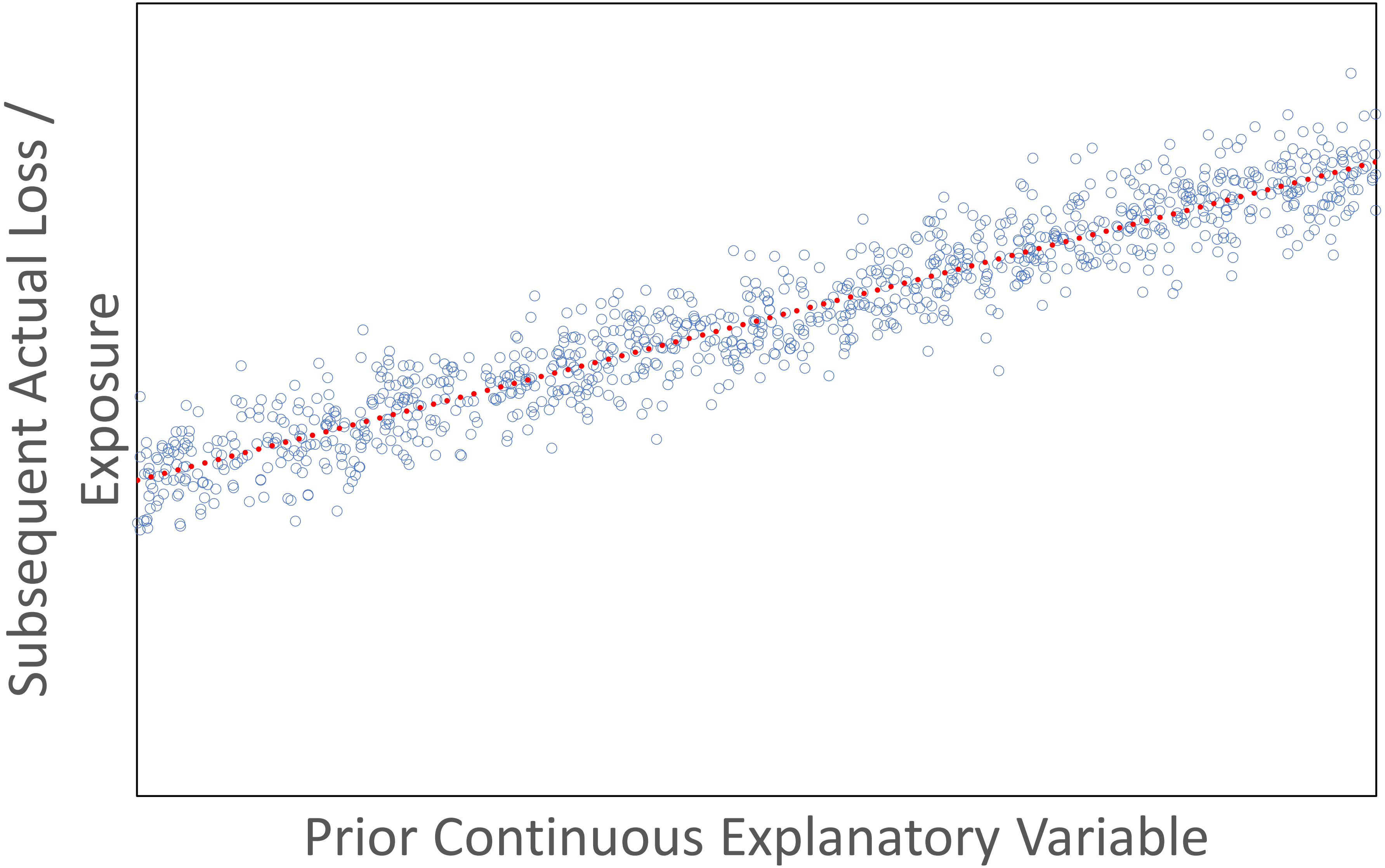
Bootstrap Quantile Test (Year 2 Predicted By Year 1)



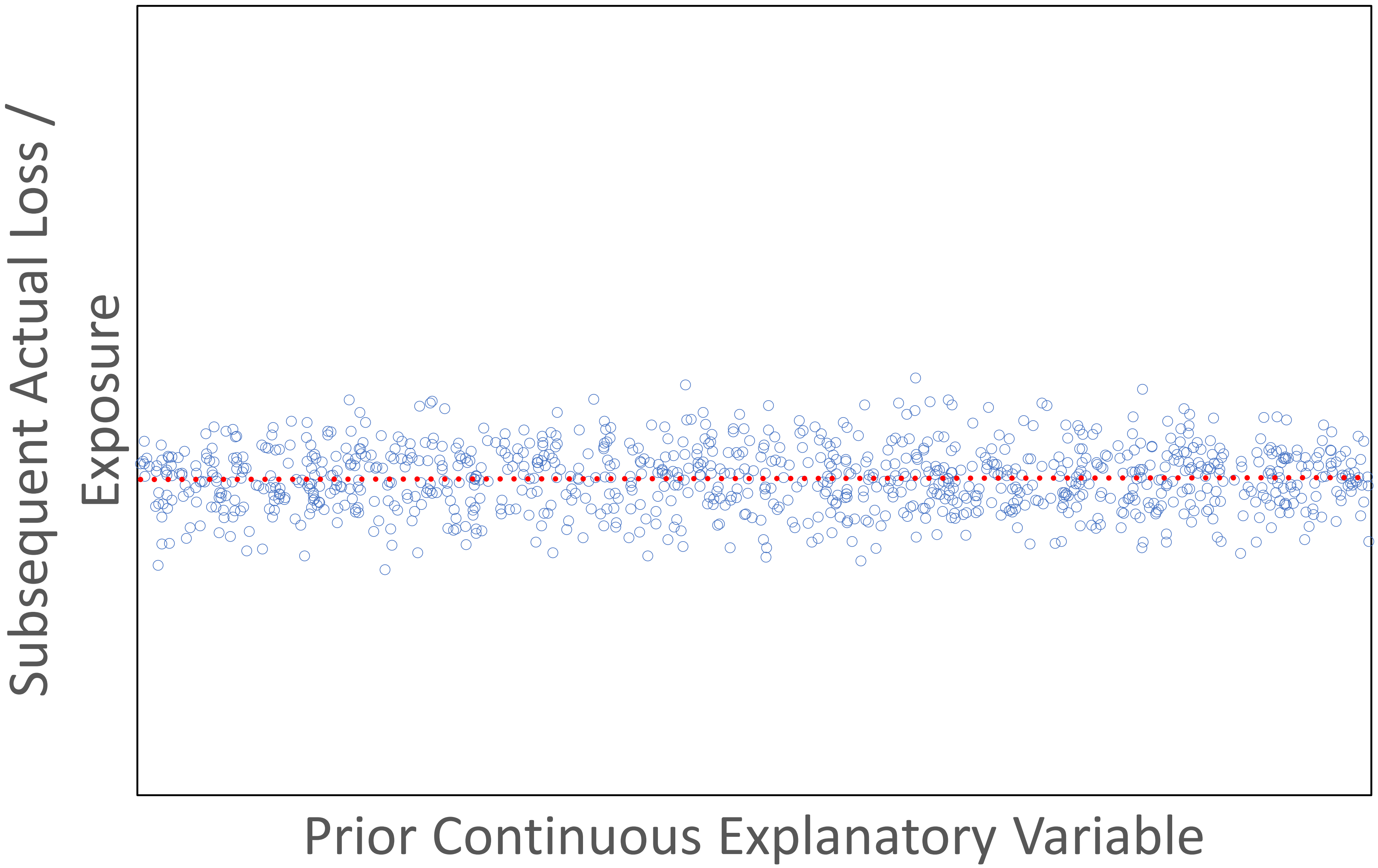
Simple Example Of Predictively Tested Model

- The linear fit is very predictive even though the assumptions of a standard simple linear regression model were severely violated.
- Now we will do this again in a situation where the data from Year 1 and Year 2 are very different.

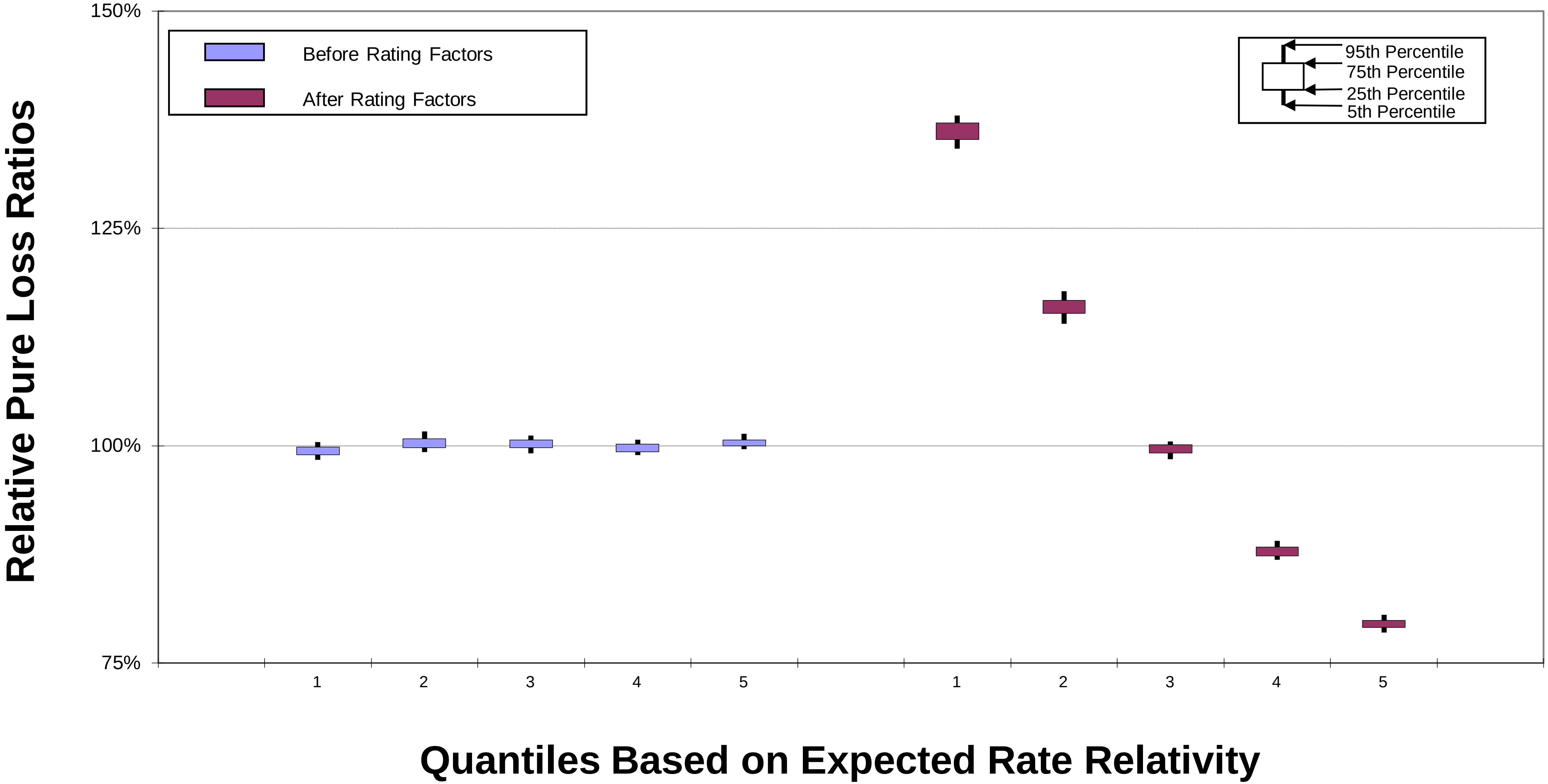
Alternate Year 1 Data



Alternate Year 2 Data



Bootstrap Quantile Test (Alt. Year 2 Predicted By Alt. Year 1)



Simple Example Of Predictively Tested Model

- In this alternate situation the linear model is very unpredictive, actually doing damage to estimates.

- These two situations together demonstrate the value of predictive performance testing over testing model assumptions.

Direct Measurement

$$y = x_1 x_2 x_3 \dots x_n + error$$

$$\widehat{x_1} = \frac{\sum y}{\sum x_2 x_3 \dots x_n}$$

$$\widehat{x_2} = \frac{\sum y}{\sum x_1 x_3 \dots x_n}$$

$$\widehat{x_3} = \frac{\sum y}{\sum x_1 x_3 \dots x_n}$$

etc.

Multiplicative Minimum Bias Model

Total losses $L_{i_1,\ldots,i_n} \geq 0$ for a particular combination of rating characteristics $i_1, \ldots, i_n$ is equal to a product of factors X_{j,i_j} times total cell exposure $P_{i_1,\ldots,i_n} \geq 0$ plus a bias (residual error) $B_{i_1,\ldots,i_n}$ so that:

$$L_{i_1,\ldots,i_n} = B_{i_1,\ldots,i_n} + P_{i_1,\ldots,i_n} \prod_{j=1,\ldots,n_j} X_{j,i_j}$$

The factors are estimated through repeated iteration:

$$X_{j,k,1} = 1 \qquad X_{j,k,t+1} = \frac{\sum_{i_j=k} L_{i_1,\ldots,i_n}}{\sum_{i_j=k} P_{i_1,\ldots,i_n} \prod_{l \neq j} X_{l,i_j,t}}$$

Generalized Linear Models (GLM)

- Losses L_i for each risk follow a distribution from the exponential family (Poisson, Normal, etc.).
- $E[L_i] = g^{-1}(X_i\beta)$
- $g(x)$ is a strictly monotonic function, called the link function.
- X_i is the vector of explanatory variables (usually dummy variables for categorical values of rating characteristics) for the particular risk.
- β is a vector of parameters estimated, along with variance parameters of the distribution, through maximum likelihood as $\hat{\beta}$.

Comparison Of Multiplicative Minimum Bias And GLM

- GLM completely specifies probability density.
- Multiplicative minimum bias only specifies expected value.
- Numerical values that solve (are a fixed point) the multiplicative minimum bias iteration equations will also produce the MLE for a corresponding GLM based on a logarithmic link function and Poisson distribution.

The Argument Against Multiplicative BMB

- Multiplicative Bailey Minimum Bias formulas are replicated by GLM with a log-link function and a Poisson error distribution
- A GLM that includes this potential specification subsumes the multiplicative BMB approach.
- The GLM with log-link/Poisson is considered and the errors are not described will by the assumption.
- Therefore multiplicative BMB must be a bad choice.

A Similar Line of Reasoning

- Measure the average temperature in January in Toronto.
  - Daily temperatures gathered and the mean is calculated
- The MLE of the mean of a Poisson distribution is equal to the mean of the observed data.
- But some of the observed temperatures are **negative**. Cleary the Poisson is a poor choice.
- The observed mean therefore is a bad measurement of the true underlying mean.

The Reasoning Flaw

- A true statement **does not** imply its converse.

True Statement	Converse Statement
X ~Poisson(λ) => Best estimate of λ = $\bar{X}$	Best estimate of λ = $\bar{X}$ => X ~Poisson(λ) Wrong
GLM with log link and Poisson distribution => Multiplicative BMB formulas	Multiplicative BMB formulas => GLM with log link and Poisson distribution Wrong

The strength of Multiplicative BMB

- The strength of the Multiplicative BMB formulas lies **not** in the quality of assumptions of an implied GLM and maximizing likelihood.
- In fact, the associated assumptions in a GLM will often be invalidated.
- The strength instead lies in the **direct measurement** of the items of interest adjusting for other variables.

Direct Measurement

$$y = x_1 x_2 x_3 \dots x_n + error$$

$$\widehat{x_1} = \frac{\sum y}{\sum x_2 x_3 \dots x_n}$$

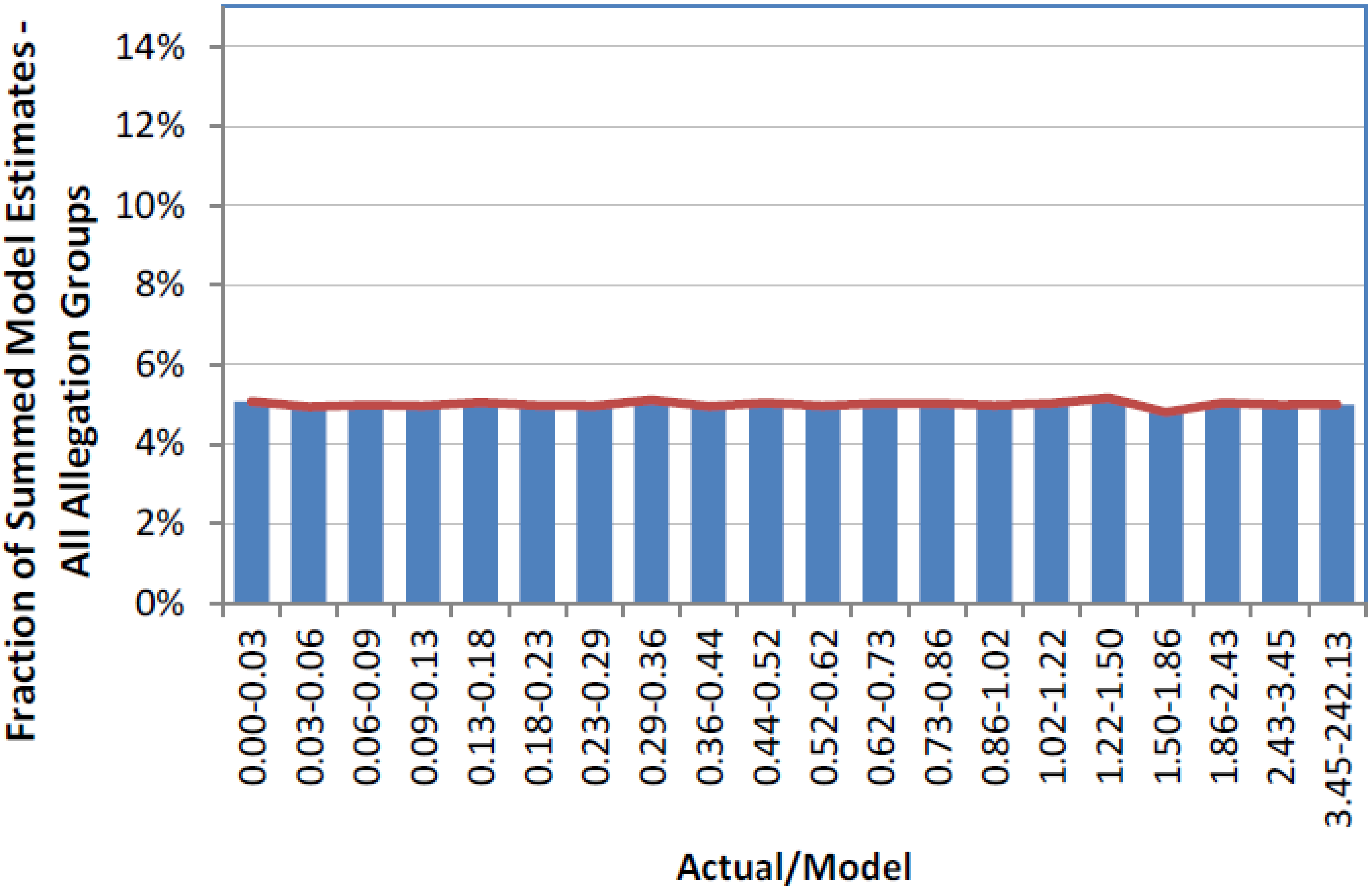
$$\widehat{x_2} = \frac{\sum y}{\sum x_1 x_3 \dots x_n}$$

$$\widehat{x_3} = \frac{\sum y}{\sum x_1 x_3 \dots x_n}$$

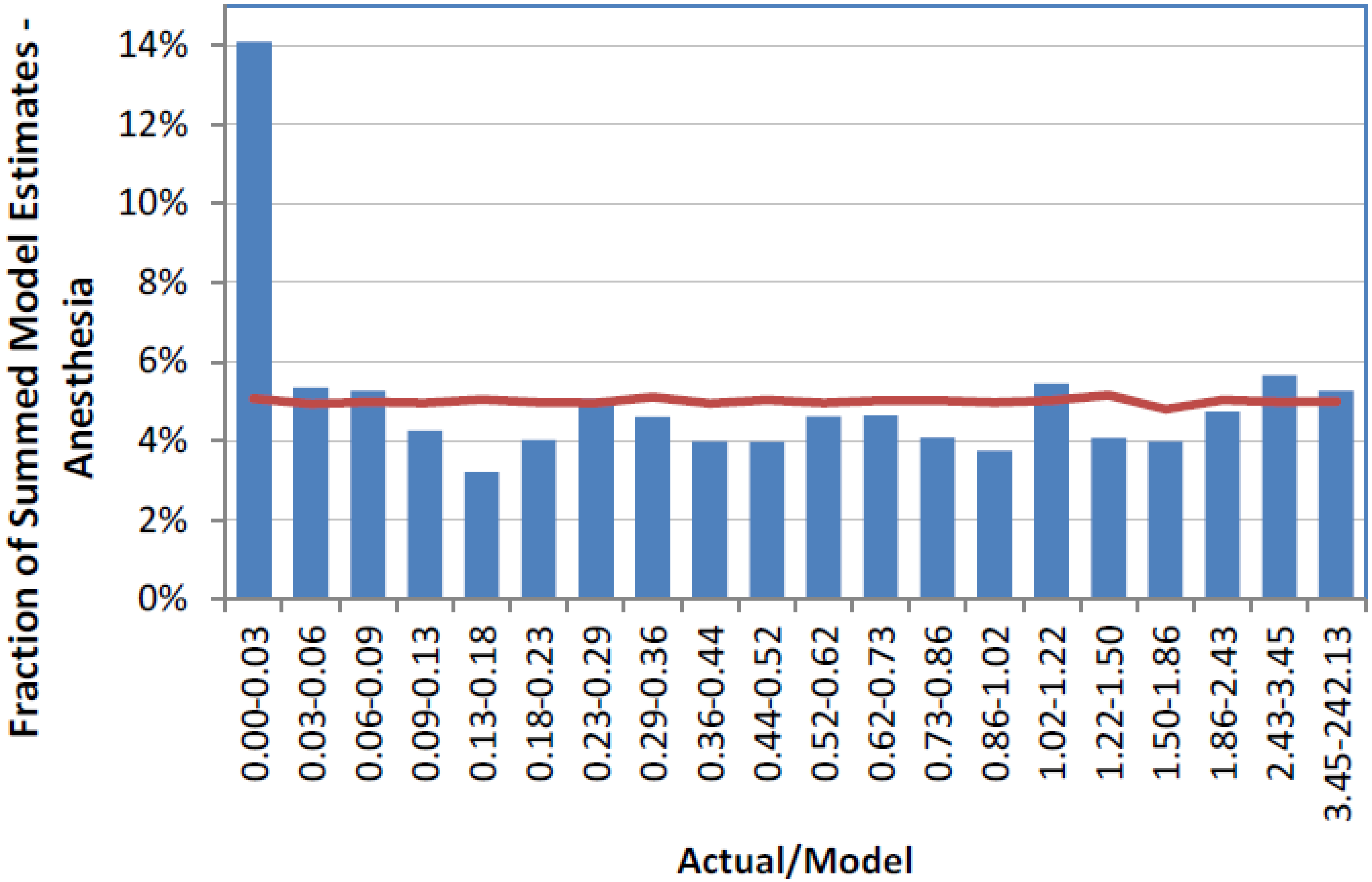
etc.

Identical Distribution?

- One of the common assumptions made in a GLM is that all distributional errors are identically distributed.
- Taken with a log link (to build a multiplicative model), the implication is that variance and higher moments scale with the mean.
- Is this true?
- What is the impact of the assumption?



Fraction of Summed Model Estimates - All Allegation Groups
14%
12%
10%
8%
6%
4%
2%
0%
0.00-0.03
0.03-0.06
0.06-0.09
0.09-0.13
0.13-0.18
0.18-0.23
0.23-0.29
0.29-0.36
0.36-0.44
0.44-0.52
0.52-0.62
0.62-0.73
0.73-0.86
0.86-1.02
1.02-1.22
1.22-1.50
1.50-1.86
1.86-2.43
2.43-3.45
3.45-242.13
Actual/Model


Fraction of Summed Model Estimates - Anesthesia
14%
12%
10%
8%
6%
4%
2%
0%
0.00-0.03
0.03-0.06
0.06-0.09
0.09-0.13
0.13-0.18
0.18-0.23
0.23-0.29
0.29-0.36
0.36-0.44
0.44-0.52
0.52-0.62
0.62-0.73
0.73-0.86
0.86-1.02
1.02-1.22
1.22-1.50
1.50-1.86
1.86-2.43
2.43-3.45
3.45-242.13
Actual/Model

(Actual-Modeled)/Modeled on Test Data (Bootstrapped)

	Multiplicative Minimum Bias			Log-Gaussian		
	Mean	5th %	95th %	Mean	5th %	95th %
Diagnosis	1.0%	0.1%	2.0%	1.3%	0.4%	2.3%
Anesthesia	4.3%	0.0%	9.5%	7.1%	2.5%	11.9%
Surgery	0.8%	-0.3%	2.1%	1.1%	-0.2%	2.5%
Medication	0.9%	-2.2%	4.0%	2.2%	-0.6%	5.4%
IV & Blood Products	3.0%	-11.3%	20.5%	3.6%	-6.8%	15.9%
Obstetrics	0.1%	-2.4%	2.8%	-0.4%	-2.3%	1.8%
Treatment	-0.5%	-2.0%	1.1%	-2.5%	-4.0%	-1.0%
Monitoriing	0.2%	-5.1%	6.2%	0.9%	-4.3%	5.7%
Equipment/Product	-3.4%	-11.0%	5.4%	0.0%	-9.3%	8.7%
Other	-11.1%	-15.8%	-5.7%	-14.3%	-19.6%	-8.9%
Behavioral Health	11.9%	-6.5%	34.4%	13.2%	-10.4%	40.9%
Blank	-17.0%	-38.8%	5.5%	-20.7%	-37.7%	-0.6%

The results were generally better using the Multiplicative Minimum Bias formulas

This despite “better” assumptions on the GLM using Log-Gaussian instead of Log-Poisson

One of the reasons for this is that the identical distribution assumption simply isn’t true

Incorporating Credibility Into Minimum Bias Versus GLM

For multiplicative minimum bias just insert credibility values

0≤ $Z_{j,k} \leq 1$, that can be determined any way you like, directly into the iterative equations:

$$X_{j,k,1} = 1 \qquad X_{j,k,t+1} = \left(1 - Z_{j,k}\right) + Z_{j,k} \frac{\sum_{i_j=k} L_{i_1,\ldots,i_n}}{\sum_{i_j=k} P_{i_1,\ldots,i_n} \prod_{l \neq j} X_{l,i_j,t}}$$

Note: Including a base rate (not shown above) may be desirable to preserve overall balance of expected and actual losses.

Incorporating Credibility Into Minimum Bias Versus GLM

For GLM inserting credibility is generally much more complicated with the primary approaches being:

- Adding random effects to produced a mixed effects model, a somewhat awkward process.
- Constructing a Bayesian Network model for Gibbs Sampling (MCMC), a straight forward but complex process.

Incorporating Credibility Into Minimum Bias Versus GLM

The Bayesian Network model (that ultimately "failed" for reasons as yet unknown) for the case study:

$$U_{1,j} = 0 \quad j = 1,2,3$$

$$U_{1,4} = \text{Uniform}(0, 20)$$

$$U_{i,1} \sim \text{Normal}(-\sigma_1^{\,2}/2,\ \sigma_1^{\,2}) \quad i = 2,\ldots,83$$

$$U_{i,2} \sim \text{Normal}(-\sigma_1^{\,2}/2,\ \sigma_1^{\,2}) \quad i = 2,\ldots,12$$

$$U_{i,3} \sim \text{Normal}(-\sigma_1^{\,2}/2,\ \sigma_1^{\,2}) \quad i = 2,\ldots,9$$

$$\sigma_1^{\,2} \sim \text{Lognormal}(0,10)$$

$$\sigma_2^{\,2} \sim \text{Lognormal}(0,10)$$

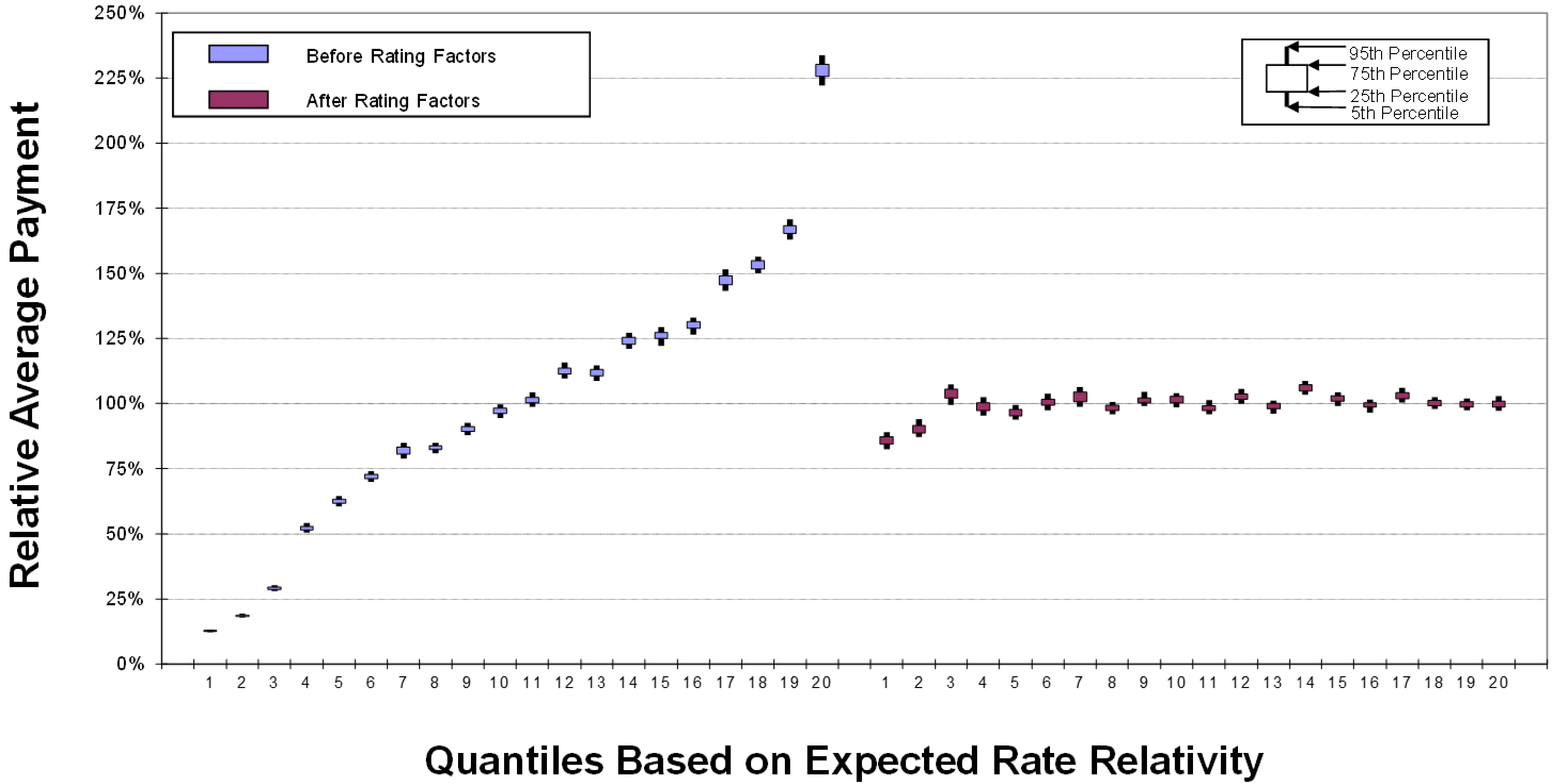
$$\delta_k \sim \text{Normal}(-\sigma_2^{\,2}/2,\ \sigma_2^{\,2}) \quad k = 1,\ldots,n$$

$$Y_k \sim \text{Poisson}(\text{Exp}(\delta_k + U_{1,4} + U_{i_{1,k},1} + U_{i_{2,k},2} + U_{i_{3,k},1})) \quad k = 1,\ldots,n$$

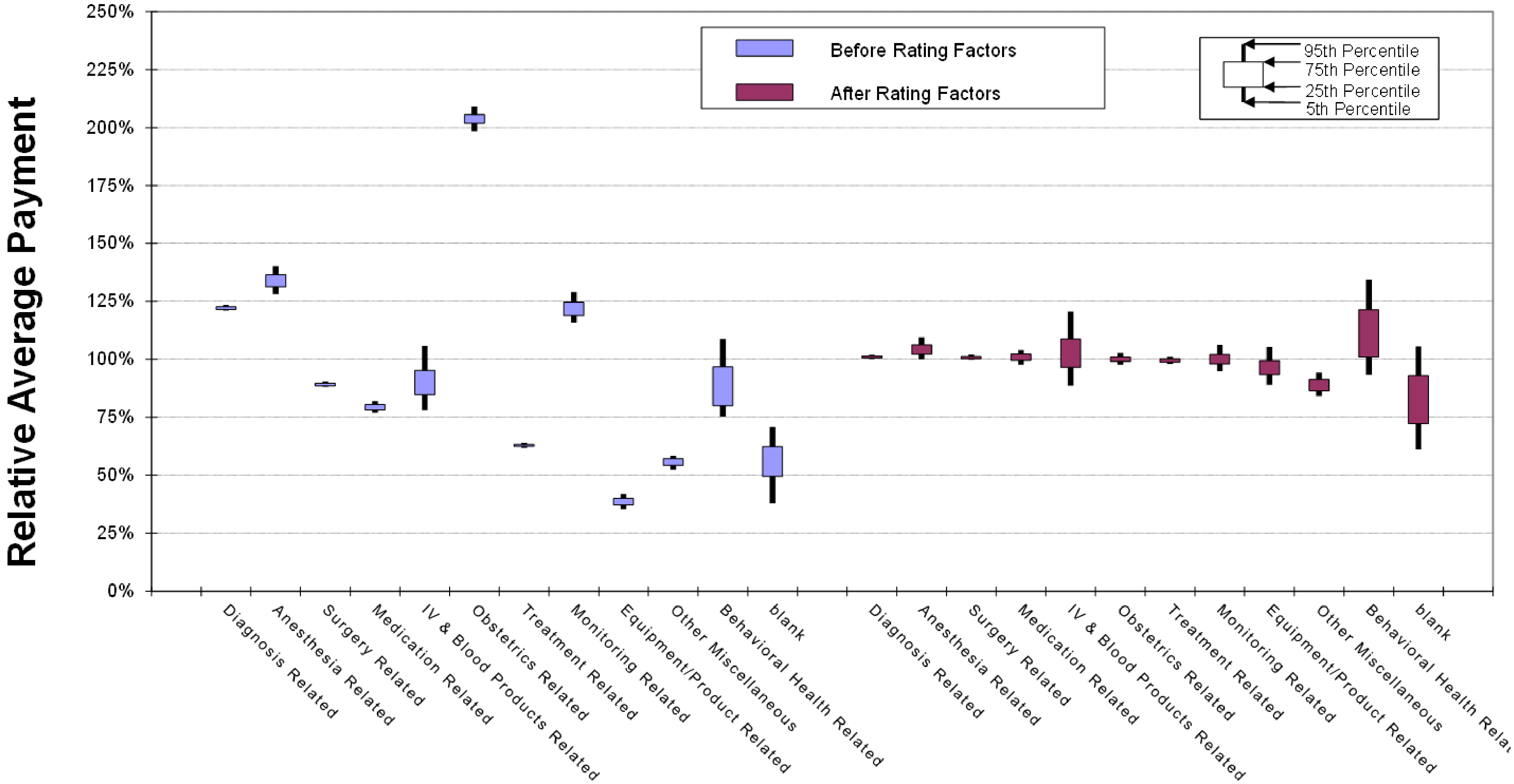
Case Study From Paper By Gross And Evans

- 371,123 records of medical malpractice payments from the National Practitioner Data Bank
- Three explanatory variables were used for modeling payment amounts: *Original Year*, *Allegation Group* and *License Field*
- Records randomly split into two sets for model fitting and validation, respectively
- A random sets of only 5,000 records for fitting models incorporating credibility

Bootstrap 20 Quantiles Test Validation of Minimum Bias Rating Factors



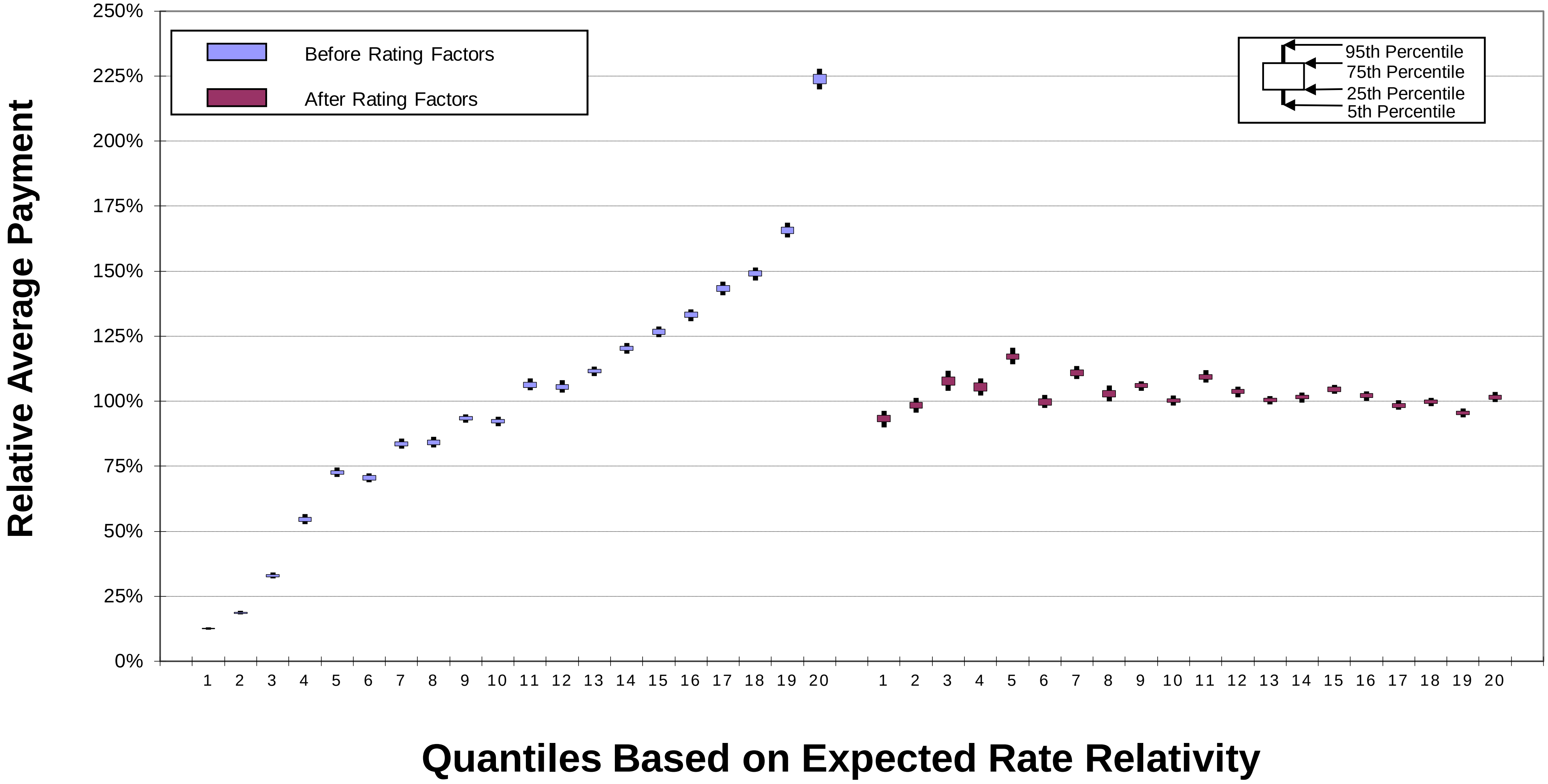
Allegation Nature - Bootstrap Test Validation of Minimum Bias Rating Factors



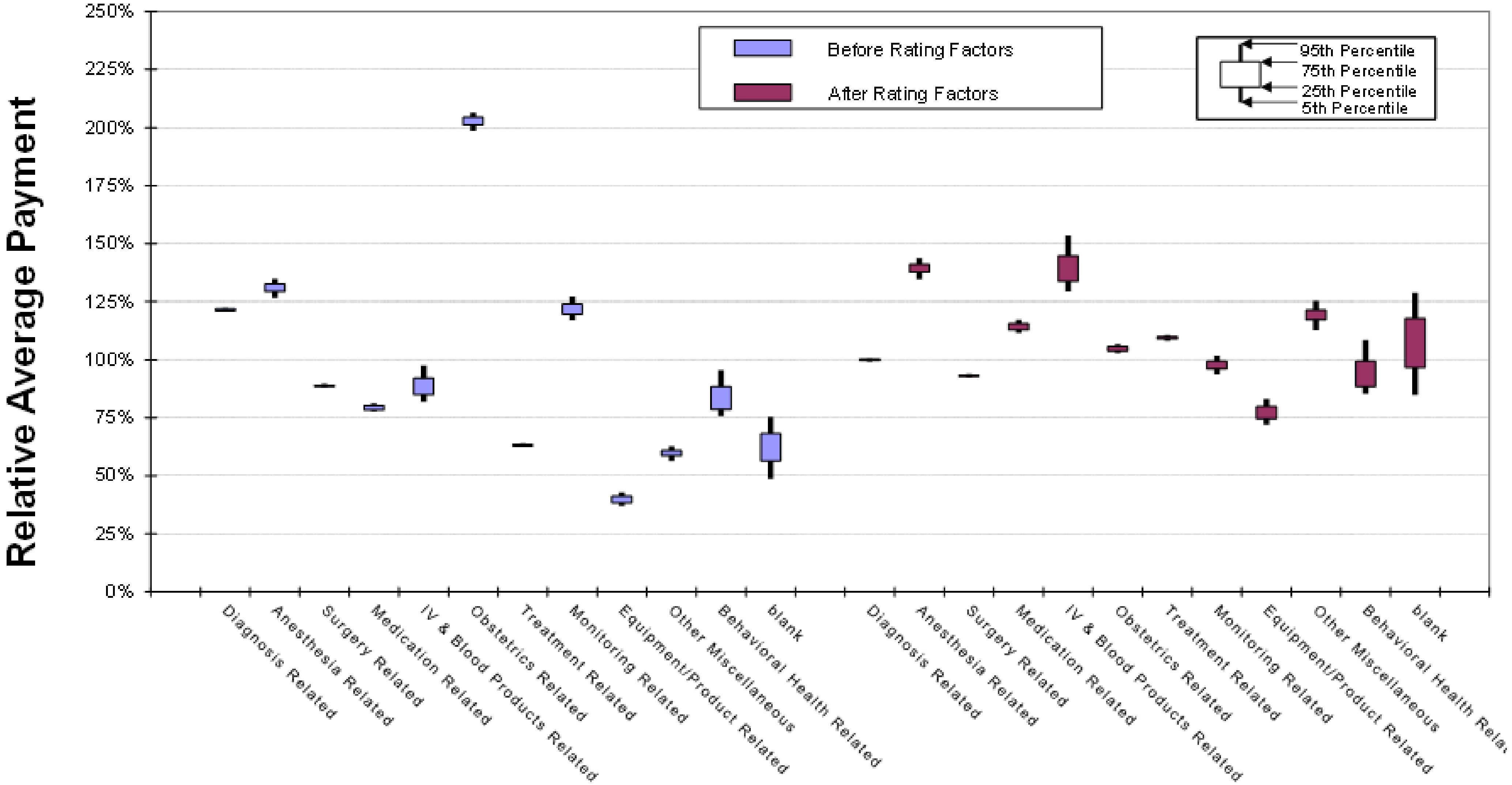
Predictive Performance Statistics for Various Models

	20 Quantiles		Allegation Nature	
	Old Statistic	New Statistic	Old Statistic	New Statistic
Mult. Minimum Bias	0.007	0.512	0.023	0.425
GLMs				
Log-Gaussian	0.010	0.511	0.041	0.422
Log-Poisson	0.007	0.512	0.023	0.425
Log-Gamma	0.009	0.511	0.033	0.422
Log-InverseGaussian	Failed to Converge		Failed to Converge	
Traditional	0.135	0.470	0.089	0.408

Full Test of Smaller Sample Bootstrap 20 Quantiles Test Validation of Minimum Bias (Credibility K = 10) Rating Factors



Full Test of Smaller Sample Allegation Nature - Bootstrap Test Validation of Minimum Bias (Credibility K = 10) Rating Factors



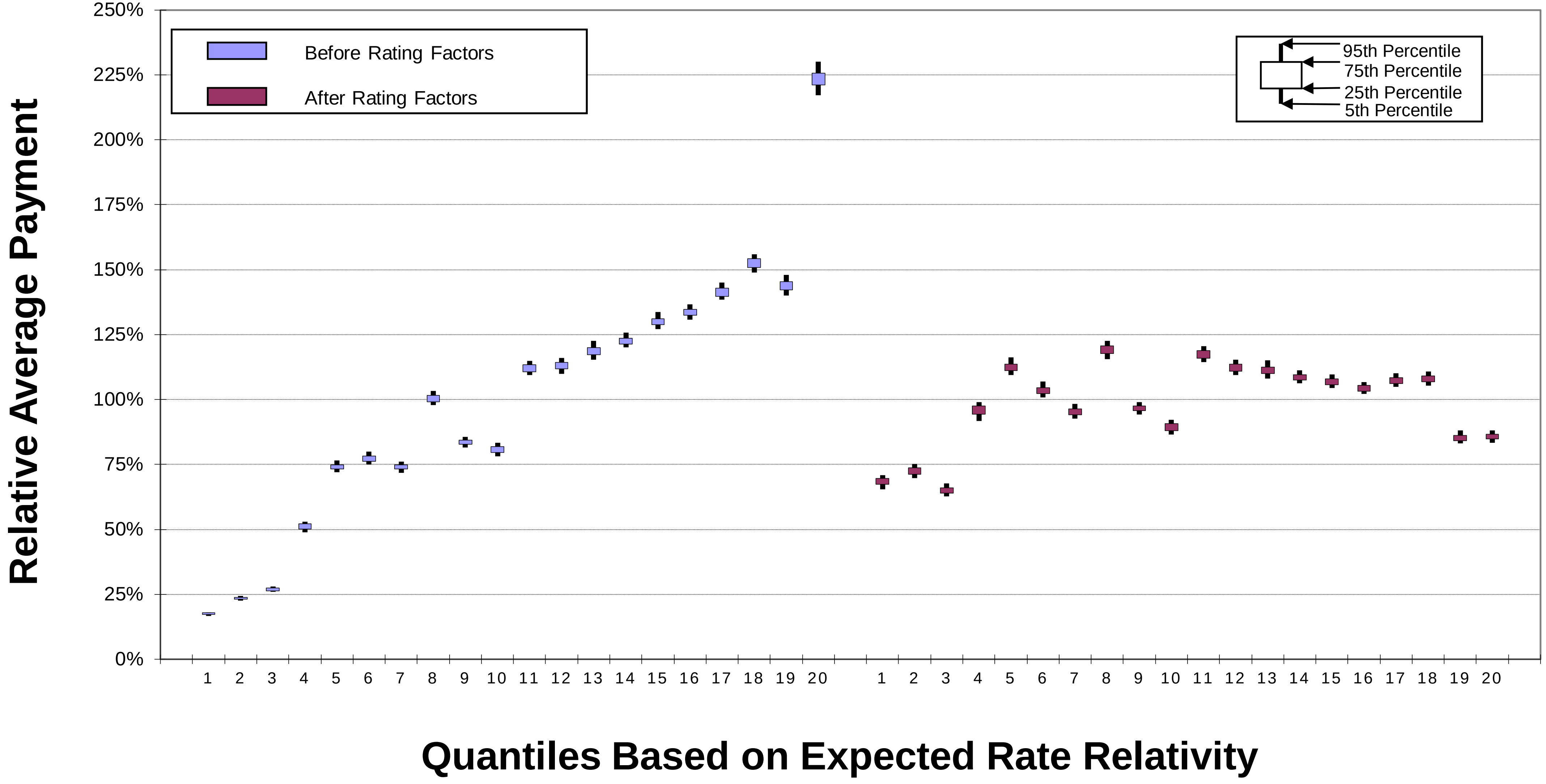
Full Test of Smaller Sample Predictive Performance Statistics for Various Models

	20 Quantiles		Allegation Nature	
	Old Statistic	New Statistic	Old Statistic	New Statistic
Mult. Minimum Bias	0.031	0.488	1.906	-0.403
GLMs				
Log-Gaussian	0.038	0.482	2.673	-0.556
Log-Poisson	0.031	0.488	1.906	-0.403
Log-Gamma	0.072	0.474	3.256	-0.653
Log-InverseGaussian	Failed to Converge		Failed to Converge	
Traditional	0.489	0.350	2.158	-0.471

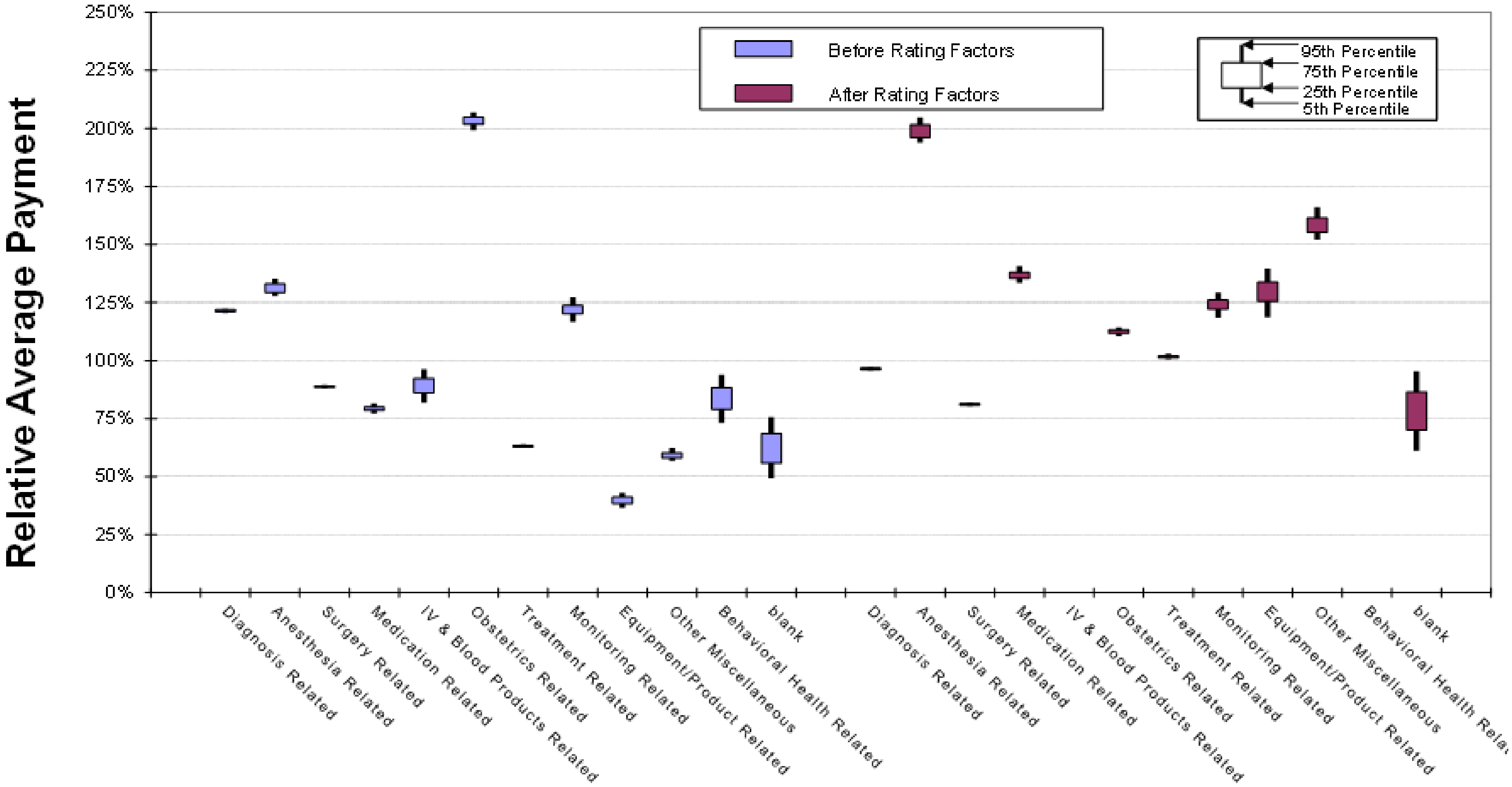
Full Test of Smaller Sample Predictive Performance Statistics for Credibility Adjusted Multiplicative Minimum Bias

	20 Quantiles		Allegation Nature	
	Old Statistic	New Statistic	Old Statistic	New Statistic
Mult. Minimum Bias				
K = 0	0.031	0.488	1.906	-0.403
K = 1	0.020	0.492	0.835	0.139
K = 10	0.012	0.494	0.169	0.380
K = 25	0.013	0.493	0.187	0.379
K = 50	0.026	0.489	0.215	0.372
K = 100	0.063	0.479	0.246	0.364
K = 200	0.117	0.460	0.289	0.355
K = 700	0.300	0.399	0.427	0.317

Full Test of Smaller Sample Bootstrap 20 Quantiles Test
Validation of Gibbs Sampled Rating Factors with Shrinkage



Full Test of Smaller Sample - Allegation Nature - Bootstrap Test Validation of Gibbs Sampled Rating Factors with Shrinkage



Test Statistics for Gibbs Sampled Rating Factors

	Quantiles		Allegation Nature	
	Old Statistic	New Statistic	Old Statistic	New Statistic
Large Split (20 Quantiles)				
w/o overdispersion	0.007	0.512	0.023	0.425
w overdispersion	0.102	0.463	0.219	0.376
Smaller Sample (6 Quantiles)				
w/o overdispersion	0.021	0.463	2.216	-0.683
w overdispersion	0.101	0.403	3.616	-0.943
Full Test Smaller Sample (20 Quantiles)				
w/o overdispersion	0.031	0.488	1.906	-0.403
w overdispersion	0.098	0.448	4.723	-0.818

Conclusions

- A model may perform very well predictively even when its assumptions are clearly and severely violated by data.
- Predictive performance is usually the relevant goal, not correct model assumptions.
- Predictive performance validation, rather than testing model assumptions, allows for simpler incompletely specified models such as multiplicative minimum bias.
- Incorporating credibility into minimum bias is much simpler, more flexible, and more transparent than incorporating credibility into the more complicated and completely specified GLMs.