Reserving and Pricing Excess Insurance: Taming the Wild Burning Cost

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Taming the Wild Burning Cost

- In excess insurance reserving, sometimes see reserving done by applying ground up LDFs to all the individual losses.
  - Losses that then develop above the attachment point are the excess claims to include in excess reserves
  - Excess reserves for those claims are then developed cost-attachment point

Taming the Wild Burning Cost

- Also sometimes used for large deductible policies
- Also sometimes used to estimate reinsurance recoveries to compute net from gross reserves

Taming the Wild Burning Cost

- Genesis of that approach appears to be “burning cost” pricing methodology from excess pricing
  1. List the claims that are “large enough” to adjust beyond the attachment point.
  2. Multiply each individual claim by ground up LDF and trend to midpoint of contract being priced
  3. For each claim, subtract the attachment point from the results of 2.
  4. Add up the total of 3. for each prior policy, AY, etc. coverage period
  5. Adjust the results of 4 for exposure changes, underlying rate level changes, to get estimates for the excess policy being priced.
  6. Combine the results of 5. for various coverage groups (policies, accident years, etc.) to make single estimate of either excess loss cost, or , if divided by exposures, loss cost rate.
  7. Add in expenses and profit.

Taming the Wild Burning Cost

- Prominent statement in Gary Patrik's discussion of "An Actuarial Approach to Loss Rating" by Ferguson (PCAS 1978)
- **"Don't Use Burning Cost"**

Taming the Wild Burning Cost

- Why not use burning cost?
  1. Credibility problem: Only a few losses will develop into the excess layer.
  2. Bias problem #1: loss development problem #1-Shouldn't the claims that eventually get very big have more loss development?
  3. Bias problem #2: loss development problem #2- Should you really apply LDFS to closed claims?
  4. Bias problem #3: loss development problem #3- Isn't the development due to IBNR really something of different character than development on claims that are already reported?
  5. Can you reliably adjust for differences in exposure, especially if exposure is premium written?

Taming the Wild Burning Cost

- Will start with problem 2
- Shouldn't the claims that eventually get very big have more loss development?
  - Core of the problem: There's a distribution of possible final costs around each case reserve, not a single value.
  - Some claims with given case will develop to be very large, some will develop to zero.
  - Precedent: 1997 Robin Gillam/Jose Couret paper 'Retrospective Rating: 1997 Excess Loss Factors'

Taming the Wild Burning Cost

- Basic approach of this paper- Generate "Adjustment Distribution", common to all claims, for ratio of ultimate value of each claim to the values of those claims at some maturity m
  - Same distribution of possible LDFs for all claims
  - Distribution applies to each claim independently,
  - Large claims then more prone to be excess claims than small claims, but not all become excess claims

Taming the Wild Burning Cost

- Computing Adjustment Distribution
  - Set adjustment distribution so that distribution generated by multiplying the samples from the claim size distribution at m months by independent samples from the adjustment distribution exactly (or closely in the upper claim sizes) simulates the ultimate loss distribution.

Taming the Wild Burning Cost

- Differences from Gillam/Couret
  - Single distribution for all claims sizes-
    - G/C has separate development distribution for each claim size
    - High complexity/data issues preparing various distributions
  - Less pinpoint accuracy than G/C
    - But still produces unbiased estimates
    - Only need compute a single distribution
    - Special utility (discussed later) for IBNR claims

Taming the Wild Burning Cost

- Properties of Simulation Using Adjustment Distribution
  - Produces unbiased (across all possible groups of maturity “m”, claims, since they average to severity distribution at m months) estimate of ultimate/excess losses
    - Takes bias out of burning cost

Taming the Wild Burning Cost

- "Tamed" burning Cost pre-exposure, etc. adjustment is (following Mahler discussion of Gillam/Couret article)

$$\sum_{claims} \sum_{"i"} X_i \int_{L/X_i}^{\infty} a(r)\left(r - \frac{L}{X_i}\right) dr$$

X = reported claim amount at m months,
L =attachment point,
R = random sample (R=ratio) from adjustment distribution,
a(r) = mass function of adjustment distribution

Taming the Wild Burning Cost

- Mitigation of problems listed for burning cost
  1. Credibility problem : “Tamed” method uses a large volume of claims, each times a probability it becomes “large”.
  2. Shouldn’t the claims that eventually get very big have more loss development (bias problem 1)?: Under the “Tamed” method they do.
  3. Bias problems 2 and 3: One may see that since the expected outcome of the “Tamed” distribution across all possible sets of claims reported to-date (at some maturity) is the ultimate loss distribution, the tamed method is unbiased.
  4. Exposure adjustment item-beyond the scope of this study.

Taming the Wild Burning Cost

- Some Quibbles and Questions Follow

Taming the Wild Burning Cost

- Well, Joe, applying development to closed claims and claims that are mostly paid off might produce a nice estimate of the excess costs for my reserves or pricing, but clients will howl that it's unreasonable. What do you say to that?

Taming the Wild Burning Cost

- Eliminating development on closed and partially paid claims
  - Alternately, could development an adjustment distribution a(r) that takes distribution of case reserves at , say 36 months, and converts it into distribution of true unpaid claims cost sizes (including IBNR claims) at 36 months
  - Unbiasedness of the result depends on the distribution of possible percentage increases between case and payout being independent of what is paid-to-date.

Taming the Wild Burning Cost

- Okay, Joe, but what about those pesky IBNR claims? How do you develop an IBNR claim from nothing?

Taming the Wild Burning Cost

- IBNR claims
  - Note that the adjustment distribution is not intended to be a perfect depiction of how each claim might develop. It's just supposed to be an unbiased predictor, and it (in the average) generates the ultimate severity distribution.
  - To include IBNR claims you just multiply the results of the base analysis by the ratio of the ultimate claim count to the reported claim count at the given maturity (36 months in the example)

Taming the Wild Burning Cost

- Okay, Joe, so an adjustment distribution could get me unbiased answers. But, even if I do know what ultimate loss severity distribution is and my “m” months of maturity reported severity distribution, how on earth could I derive the adjustment distribution?

Taming the Wild Burning Cost

- Computing the adjustment distribution
  - To start with, you could express the problem as

$$X \times R = Y$$

  - Where “X” is a random variable representing a claims loss severity at months, “R” is the random adjustment factor, and “Y” is the random loss severity at ultimate.

Taming the Wild Burning Cost

- Computing the adjustment distribution
  - Taking natural logarithms gives you

$$\ln(X) + \ln(R) = \ln(Y)$$

  - Now's it's an additive problem
    - In fact, a matrix problem, given "n" values of ln(x) and ln(Y), and their associated probabilities, when you seek "m" values of the probabilities of ln(R).

Taming the Wild Burning Cost

- Computing the adjustment distribution via matrix equation
- Example – if values logs take are specified to be 0,1, 2, 3, up to either m (ln(R))or n (ln(X) and ln(Y), then since the probability of ln(Y) is the sum of the compound probabilities of all ways ln(X) and ln(R) sum to ln(Y)

$$P(\ln(Y)=i)=\sum_{j=0}^{\min(i,m)}P(\ln(X)=(i-j))\times P(\ln(R)=j)$$

Taming the Wild Burning Cost

- Computing the adjustment distribution via matrix equation
  - Since they are logs of the multipliers, might actually have values below zero
  - Might have to sort of spread the probability from, say "X", among the "ln(X)" cells
  - Matrix equation is solvable in Excel
  - Might generate spurious values outside the primary range
    - Suggest minimizing this by using m = n/2.
    - In overdetermined system $C \times D = E$, solve $C^T \times C \times D = C^T \times E$ for best estimate.

Taming the Wild Burning Cost

- Calculation issues
  - Making “R” distribution may be messy
    - Matrix method worked but where ln(R)’s had low probability massive errors were present
      - Had to trim range in results during testing
    - May have to extrapolate if very unusual development is relevant
    - May be poor choice for very immature claims such as “formula” claims
    - Best strategy may be to just show clients the results
      - Maybe show validation with Monte Carlo

Taming the Wild Burning Cost

- Alright, so I have the tools to compute the a(r)'s given the severity distributions at some maturity and at ultimate. But, Mr. Joe (Dr. Joe?) what do I use for the at maturity m and at ultimate loss distributions, especially when my data is immature or of insufficient volume to understand the upper tail of the ultimate loss distributions?

Taming the Wild Burning Cost

- Sources of ultimate severity distributions
  - ISO
    - Immature severity available too
  - NCCI
    - Immature severity available too
  - Another large program with similar characteristics
    - Immature data of other program may be different
  - Combine data from multiple programs
    - Immature data of other program may be different

Taming the Wild Burning Cost

- Hold on Dr. Joe, you obviously do not understand my problem. I'm a consultant and I handle lots of small-to-medium sized programs. Almost all of them have different TPAs and no two programs use the same claim philosophy. How can I properly compute an adjustment distribution given this loss development fruit salad?

Taming the Wild Burning Cost

- Data at immature periods may need to be adjusted to common/industry distribution characteristics
  - First, presence or absence of a few large claims may indicate mean and variance of TPA data that is not reflective of their true reserve adequacy and reserve variance
  - Suggest comparing mean and variance of data limited by claim to benchmark data, and make adjustment factors for mean and variance of TPA data
  - Target overall mean = adj. factor times present mean, similarly for variance
  - Mean adjustment (use straight multiplier of M)-affects mean and standard deviation equally

Taming the Wild Burning Cost

- Adjusting data (each TPA?) at immature periods to common/industry distribution characteristics
  - Mean adjustment (use straight multiplier of M)- affects mean and standard deviation equally

Taming the Wild Burning Cost

- Adjusting data (each TPA?) at immature periods to common/industry distribution characteristics
  - Dispersion – try mapping $X \to (X / \mu^*)^\alpha \mu^* = X^*$
    - Doesn't send any values to zero
    - μ^* = geometric mean = exp(E[ln(X)])
    - α= scale factor
  - Target CV of distribution you seek
    - Under lognormal, α of $\sqrt{\frac{\ln\left(CV^2(desired)+1\right)}{\ln\left(CV^2(present)+1\right)}}$

      will convert to target CV
    - Since data not lognormal, may need multiple iterations

Taming the Wild Burning Cost

- Summary
  - Method that removes development bias in burning cost
  - Characteristics of R distribution
  - Computing R distribution
  - Finding and developing data for computing R distribution

Taming the Wild Burning Cost

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