Verisk Insurance Solutions

Optimization Applications in Insurance

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THE SCIENCE OF **RISK**℠

Optimization Applications

- Generalized Linear Models
  - View this essential tool as an optimization problem
- Deriving factor-based rates from existing non-factor-based rates
  - Straightforward optimization problem significantly reduced time spent and improved results
- Price Optimization
  - What is the optimization problem behind “Price Optimization”?
- Geo-spatial smoothing of GLM-based geographic models
  - Controlling changes across a boundary

Steps in Formulation of a Model

- Determination of Decision Variables
  - What does the model seek to determine?
  - Decision variables should completely describe the decisions to be made. $(x_1, x_2, \ldots, x_n)$
- Determination of Objective Function
  - What is measure of performance (Profit, Time, Speed, ...) $(3x_1 + x_2 - x_3)$
  - What is the goal of the problem (usually minimization or maximization)?
- Determination of Constraints
  - What are the resources limiting the values of the decision variables? $(2x_1 + x_2 + x_3 \leq 5)$
  - Are there “laws” or continuity relationships that limit the solution?

Generalized Linear Models

- What are the Decision Variables?
  A. The data variables (the X's)
  B. The parameters (the β's)
  C. The mean of the distribution (e.g. Gamma, Poisson)

- What is the Objective Function?
  A. Minimize sum of squared errors to the data
  B. Maximize the likelihood of the data
  C. Minimize the deviance function

- What are the constraints on the Decision Variables?
  A. Non-negativity
  B. Monotonicity (i.e., non-decreasing, no reversals)
  C. None

Other questions about GLMs

- Why don't GLMs allow constraints?
  – Different type of optimzation algorithms to solve constrained problems

- Why are GLMs limited to exponential distributions?
  – Simplifies the structure of the objective function
  – Allows efficient optimization algorithms

Deriving factor-based rates from existing rates

- Problem:
  - Existing rates are essentially “rates in a table”. No factor based relationships among them.
  - Difficult to manage rates and ensure consistency during rate changes
  - Problem existed countrywide in multiple sub-lines of business.

- Goal:
  - Move all states and sub-lines to factor-based rating
  - Try to minimize rate disruption
  - Required factors to be derived for hundreds of tables

- Initial Solution – Not Optimization Oriented
  - Trial-and-error - extremely time consuming

Microsoft Office Excel Worksheet

Deriving factor-based rates from existing rates

- Advantages of optimization approach
  - Ability to automate the decisioning
    - Reduced many hours of guesswork per state/sub-line down to a few minutes.
    - Can be programmed in Excel (with upgraded solver due to problem size), SAS, Matlab, or R.
  - Ability to build constraints into the optimization
    - Eliminated reversals
  - Assurance of the “optimal solution”

Price Optimization

- Problem:
  - Set prices to maximize a measure of customer value
  - Consider retention/conversion effects and price elasticity
  - Subject to certain business objectives

- Focus on how the optimization problem can be formulated

- Many choices in model formulation
  - Choices should be dictated by intended use of the solution

Thanks to Reuven Shnaps, VP of Professional Services
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Price Optimization – Decision Variables

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Individual vs. Rating Factor Optimization

Individual optimization

- PO at most granular level available in the data (individuals)
- Not necessarily aligned with segmentation used for risk pricing
- Applicable in regions with lighter regulation,
- Easier to apply with real-time online pricing
- Usually integrated online or via reverse engineering of prices to IT- system dictated structure

Rating factor optimization

- Direct optimisation of factor variables within factor tables of a rating structure
- Typically aligned with segments used for risk calculations
- Applied where individual optimization is impractical due to restriction in regulation or IT systems
- Properties:
  - Simultaneous optimization of factor parameters
  - Constraints on the allowable changes
  - "Monotonicty" constraints to preserve indicated/observed risk trends
  - Global business constraints

Rating Factor Optimization – what is optimized?

Problem: In order to change the final price of the policy we need to change several factor parameters. But each factor change influences more than one risk profile.

Requirements:

- Need to optimize the factor values directly rather than the final prices, within a reasonable run-time
- Need to constrain the change for each factor (e.g. not more than +/-2% relative to current value)

Current premium = Base premium x

Age	Factor
<21	2.8
21-35	1.9
35-49	1.2
50-59	1
60+	1.4

x

Gender	Factor
Male	1.25
Female	1

x

Car age	Factor
<1	2.7
1-2	1.8
2-4	1
4-7	0.8
7+	0.5

x …

Optimizer

Optimized premium = Base premium x

Age	Factor
<21	2.7
21-35	1.8
35-49	1.3
50-59	1.1
60+	1.5

x

Gender	Factor
Male	1.2
Female	1

x

Car age	Factor
<1	2.7
1-2	1.8
2-4	0.9
4-7	0.9
7+	0.6

x …

Limitations of “Reverse Engineering” an Individual Price Optimization

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- Hard/impossible to find a solution
- Time consuming:
- Can only be performed by experts
- Loss of global constraints
- Loss of “Order” constraints

Price Optimization – Objective Function

- Choice of several objectives
  - Maximize profit
  - Maximize lifetime customer value
  - Over what time period?
  - On new business or renewal business?

- Complex function, considering many things
  - Expected profit at the specified price
  - Probability of retention/conversion at the specified price

- Can result in highly non-linear function, requiring more complex or specialized optimization algorithms

Price Optimization – Constraints

- Can specify several forms of constraints to reflect business strategies
  - Limits on volume of business to be written
  - Limits on desired retention rate

- Rate Factor Optimization formulation allows additional constraints
  - Monotonicity of rate factors
  - Maximum change from current rates – by factor

Price Optimization

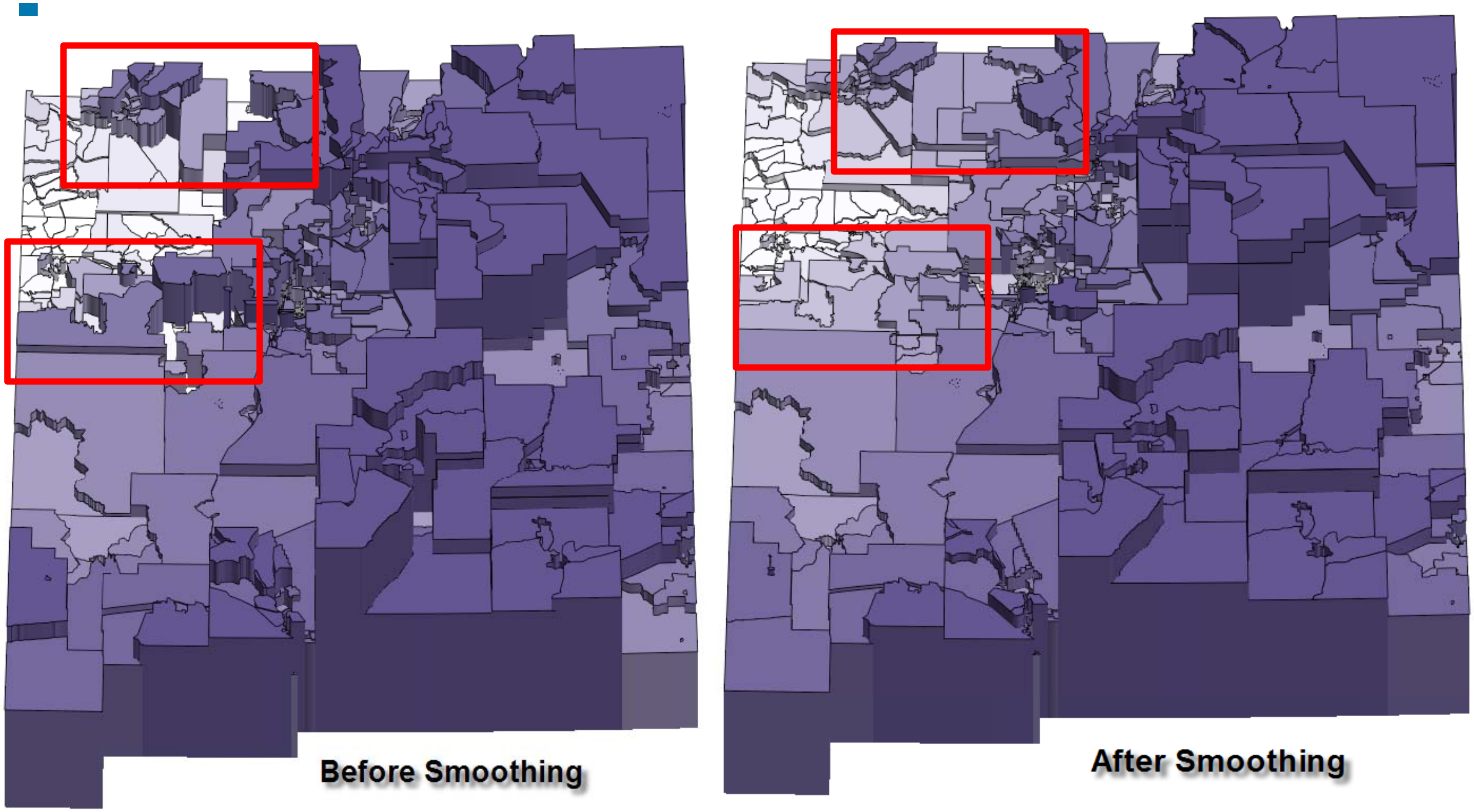
- Many choices
  - Decision Variables
  - Objective Function
  - Constraints
- So choose wisely

Geospatial Smoothing of GLM Output

- Problem:
  - Predictive models built using geographic data elements at the census block group level.
  - The scores on adjacent block groups can be very different, which is not desired
  - Adjusting one pair of scores may create a ripple effect creating large adjacency differences in other places
  - Potential adjustments needed in every state for every model

- Goal:
  - Adjust scores for areas with large adjacency differences
  - Stay close to the original model scores
  - Try to preserve rank-ordering among adjacent scores

Smoothing Example



Formulation of Smoothing Problem

- Decision Variables: Final Scores after smoothing adjustments

- Constraint I: Composite adjacency constraint
  - For larger scores, we constrain the score ratio between 0.8 and 1.25
  - For smaller scores, (below the 25th percentile of score values), we constrain the absolute difference between scores. This allows fewer changes when the impact is less material.

Formulation of Feasible Set

- Constraint II: Relaxed adjacent order preserving
- "Relaxed" means that we allow the order violation but we want to control the violation and make it as small as possible
- Original order preserving constraint is:

$$x_a - x_b \le 0, \quad \text{if block group } a \text{ and } b \text{ are in the same territory AND } \frac{s_a}{s_b} \le 1$$

$$-x_a + x_b \le 0, \quad \text{if block group } a \text{ and } b \text{ are in the same territory AND } \frac{s_a}{s_b} > 1$$

- Relaxed order preserving constraint is: (y is a slack variable)

$$x_a - x_b - y_j \le 0, \quad \text{if block group } a \text{ and } b \text{ are in the same territory AND } \frac{s_a}{s_b} \le 1$$

$$-x_a + x_b - y_j \le 0, \quad \text{if block group } a \text{ and } b \text{ are in the same territory AND } \frac{s_a}{s_b} > 1$$

Formulation of Target Function

- In Quadratic Programming (QP) form
  - Put higher penalty on large score changes, so the smoothing algorithm will try to keep the magnitude of changes as small as possible
  - Convex function (U-shape): the local optimum is guaranteed to be the "global" optimum
- Let x=unknown smoothed score, s=original score, w=weight for block group
- Minimize the weighted sum squared score difference

$$MINIMIZE\ f(x) = \left[\sum_{i=1}^{N} w_i * (x_i - s_i)^2 + \sum_{j=1}^{K} c * y_j^2\right]$$

- Minimize the order violation (c is a small constant, let c=0.0001)

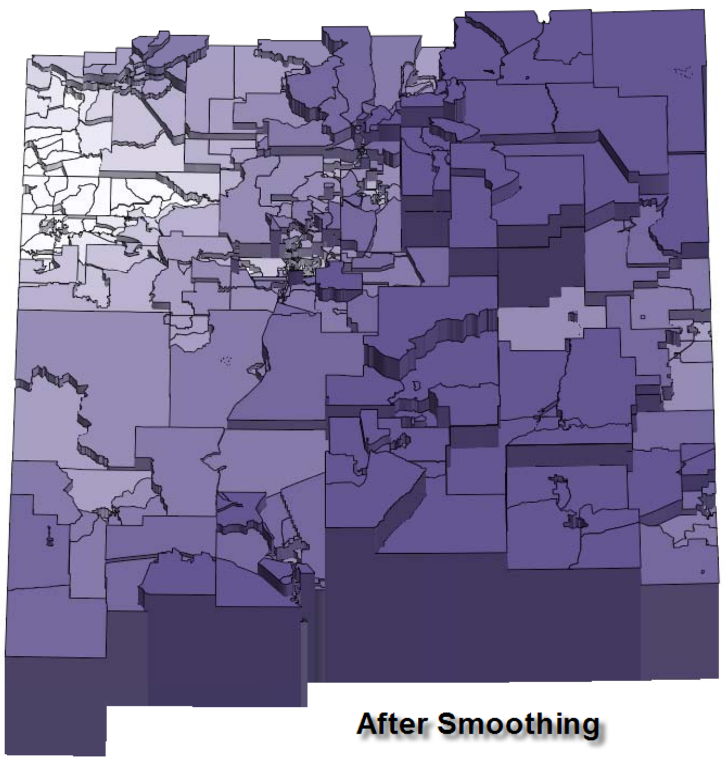
Other optimization problems in insurance

- Portfolio Risk Management Applications
  - Seeking optimal reinsurance structures
  - Managing catastrophe exposure subject to capacity constraints

- Applications in underwriting or claims operations
  - Facility placement
  - Staffing and Service time management

Optimization problems in insurance

- Optimization is a powerful tool to have available in your analytical toolkit.



After Smoothing