Generalized Linear Models II: Applying GLMs in Practice

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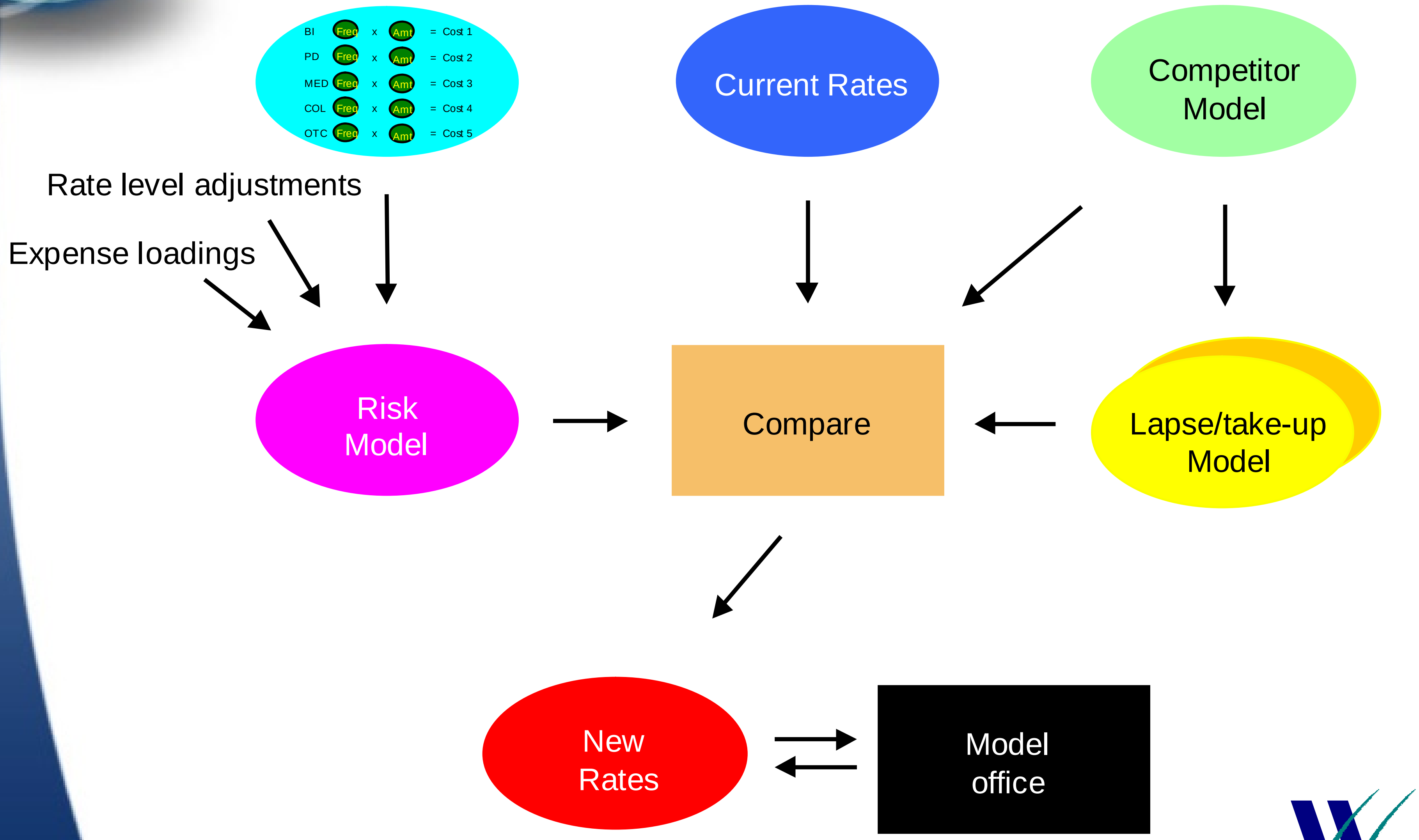
Agenda

- Introduction / recap
- Model forms and model validation
- Aliasing and "near aliasing"
- Interactions
- Smoothing, Combining models, Restrictions
- Tweedie GLMs
- Applications and interpreting the results

The premium rating process



BI Freq x Amt = Cost 1
PD Freq x Amt = Cost 2
MED Freq x Amt = Cost 3
COL Freq x Amt = Cost 4
OTC Freq x Amt = Cost 5

Current Rates

Competitor Model

Rate level adjustments

Expense loadings

Risk Model

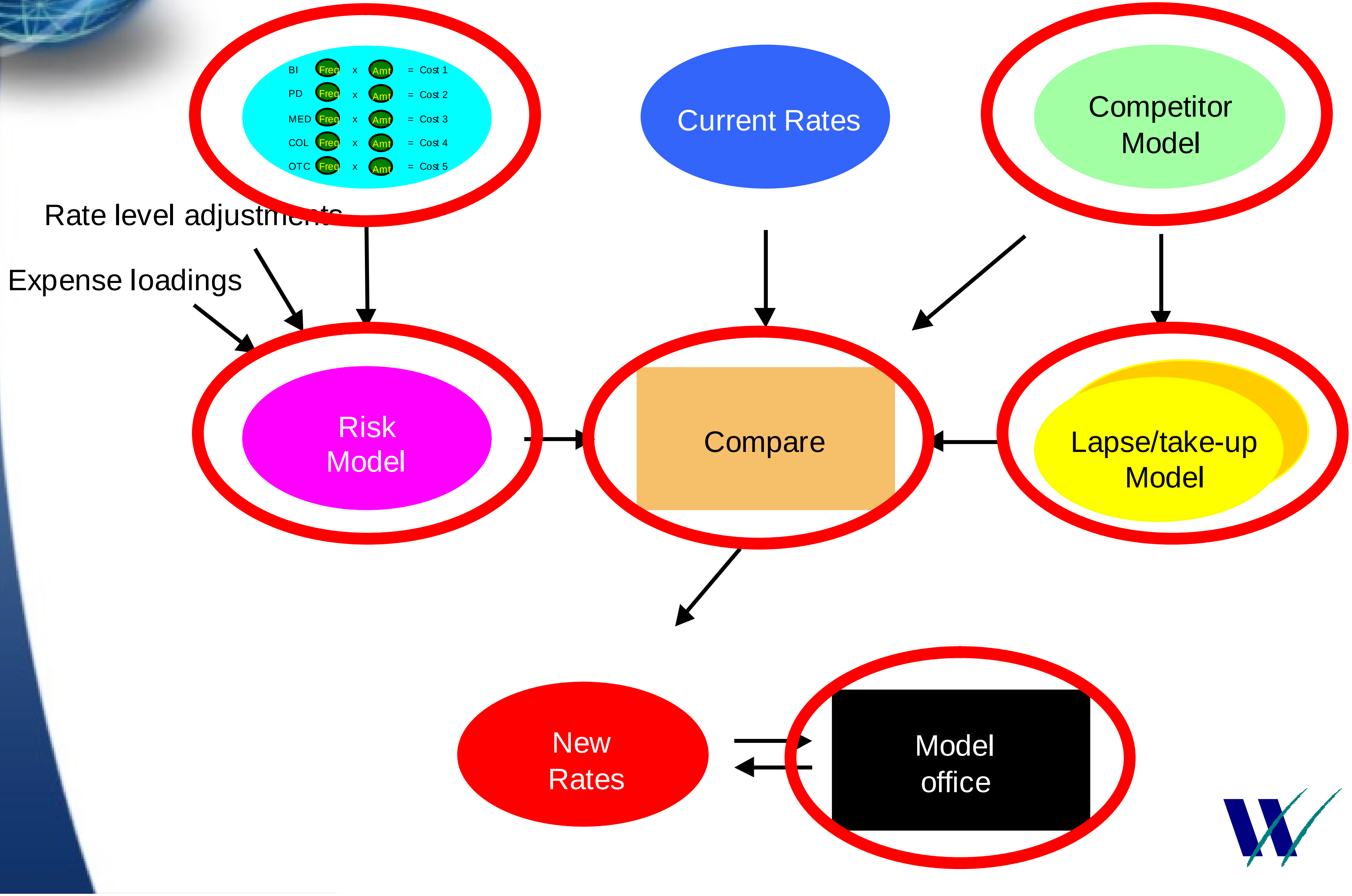
Compare

Lapse/take-up Model

New Rates

Model office

The premium rating process



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Current Rates

Competitor Model

Rate level adjustments

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Lapse/take-up Model

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Model office

Generalized linear models

$$E[Y_i] = \mu_i = g^{-1}(\Sigma X_{ij}\beta_j + \xi_i)$$

$$Var[Y_i] = \phi.V(\mu_i)/\omega_i$$

- Consider all factors simultaneously
- Allow for nature of random process
- Robust and transparent
- Increasingly a global industry standard

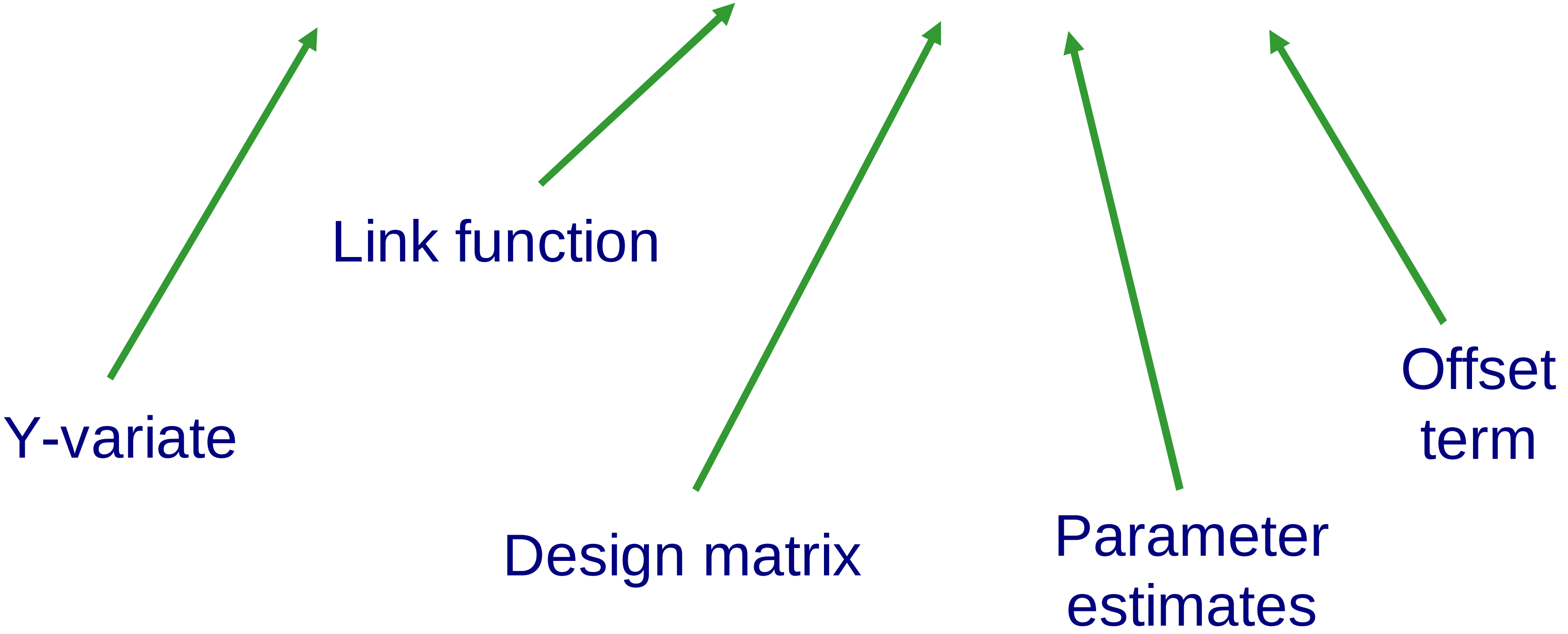
Generalized linear models

$$E[\underline{Y}] = \underline{\mu} = g^{-1}(\mathbf{X}.\underline{\beta} + \underline{\xi})$$

$$\mathrm{Var}[\underline{Y}] = \phi.V(\underline{\mu})/\underline{\omega}$$

Generalized linear models

$$E[\underline{Y}] = \underline{\mu} = g^{-1}(\mathbf{X}.\underline{\beta} + \underline{\xi})$$



Link function

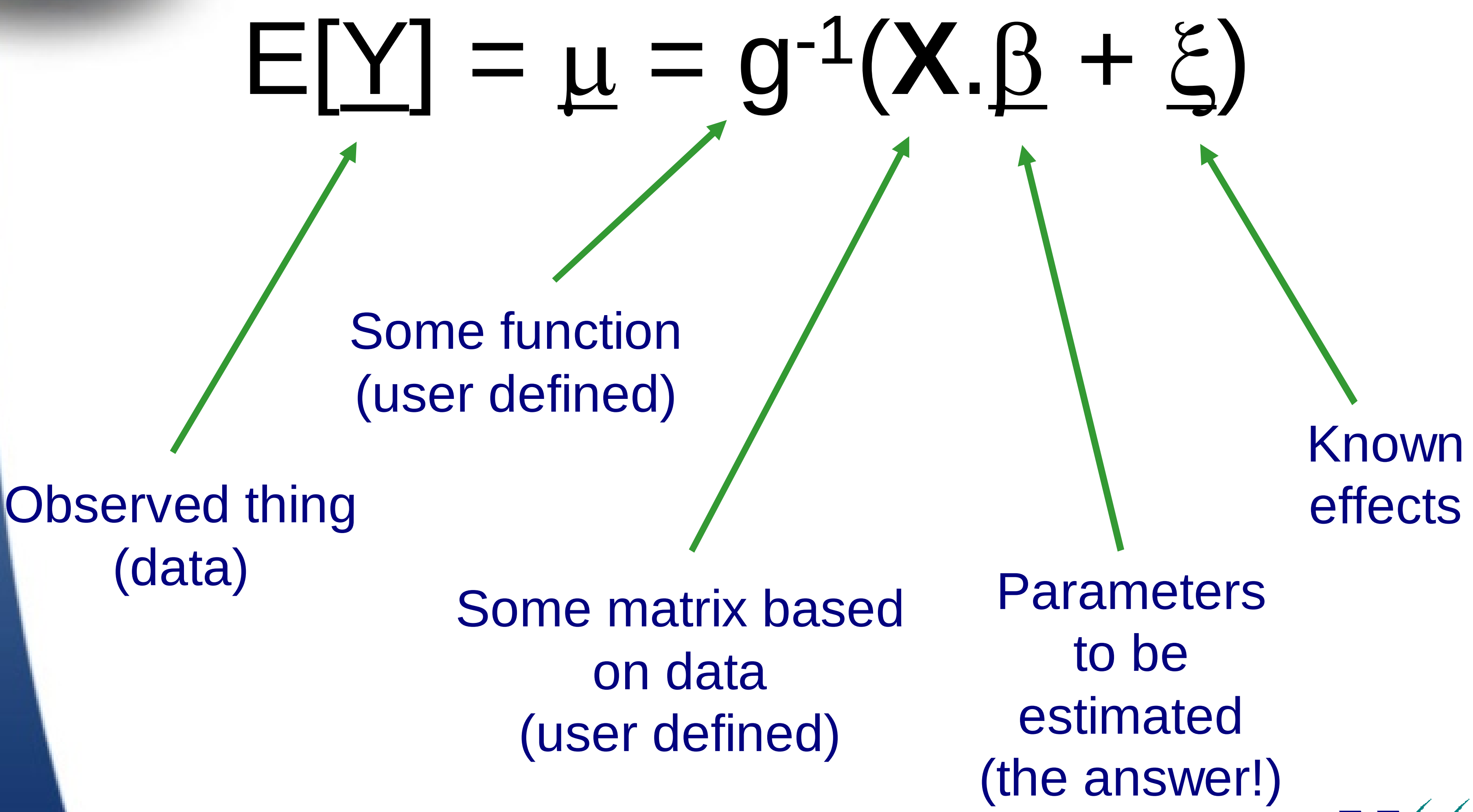
Y-variate

Design matrix

Parameter estimates

Offset term

Generalized linear models



$$E[\underline{Y}] = \underline{\mu} = g^{-1}(\mathbf{X}.\underline{\beta} + \underline{\xi})$$

Observed thing
(data)

Some function
(user defined)

Some matrix based
on data
(user defined)

Parameters
to be
estimated
(the answer!)

Known
effects

Generalized linear models

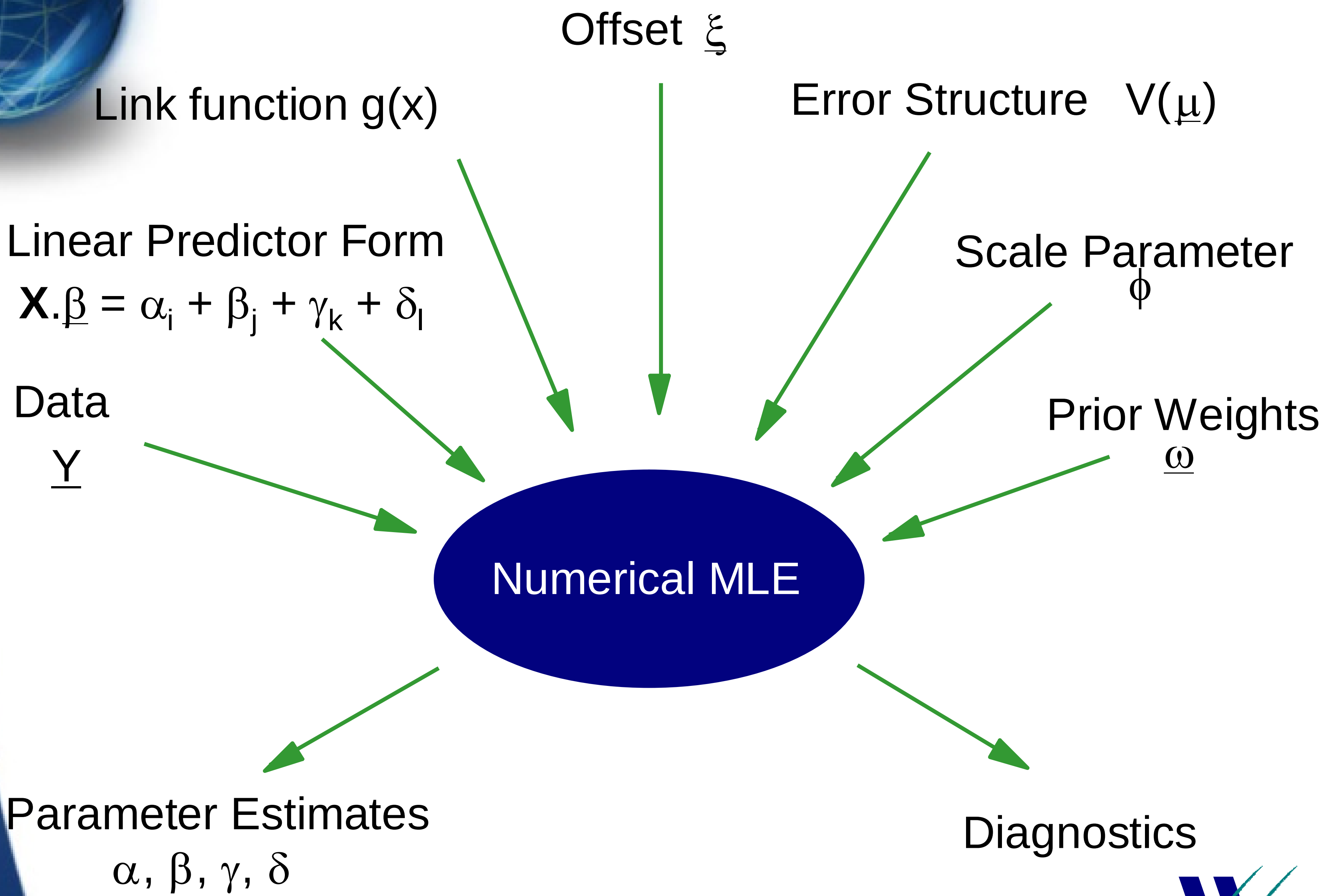
$$\mathrm{Var}[\underline{Y}] = \phi.V(\underline{\mu})/\underline{\omega}$$

Scale parameter

Variance function

Prior weights

- Usually assume exponential family, eg
- $\phi = \sigma^2$ (estimated), $V(x) = 1$ $\Rightarrow \mathrm{Var}[Y_i] = \sigma^2$ Normal
- $\phi = 1$ (specifed), $V(x) = x$ $\Rightarrow \mathrm{Var}[Y_i] = \mu_i$ Poisson
- $\phi = k$ (estimated), $V(x) = x^2$ $\Rightarrow \mathrm{Var}[Y_i] = k\mu_i^2$ Gamma



Offset $\underline{\xi}$

Link function g(x)

Error Structure $V(\underline{\mu})$

Linear Predictor Form

$\mathbf{X}.\underline{\beta} = \alpha_i + \beta_j + \gamma_k + \delta_l$

Scale Parameter

ϕ

Data

$\underline{Y}$

Prior Weights

$\underline{\omega}$

Numerical MLE

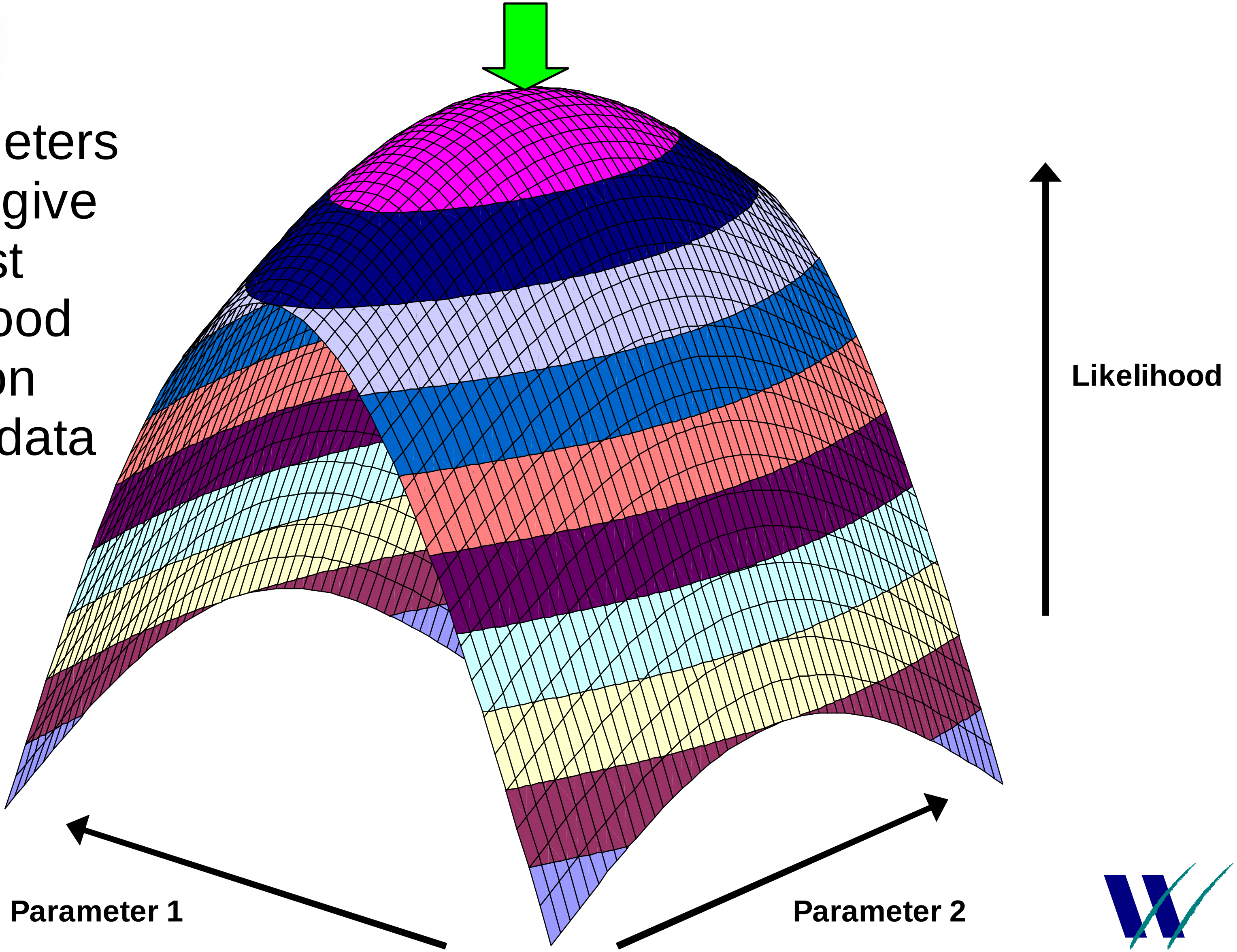
Parameter Estimates

$\alpha, \beta, \gamma, \delta$

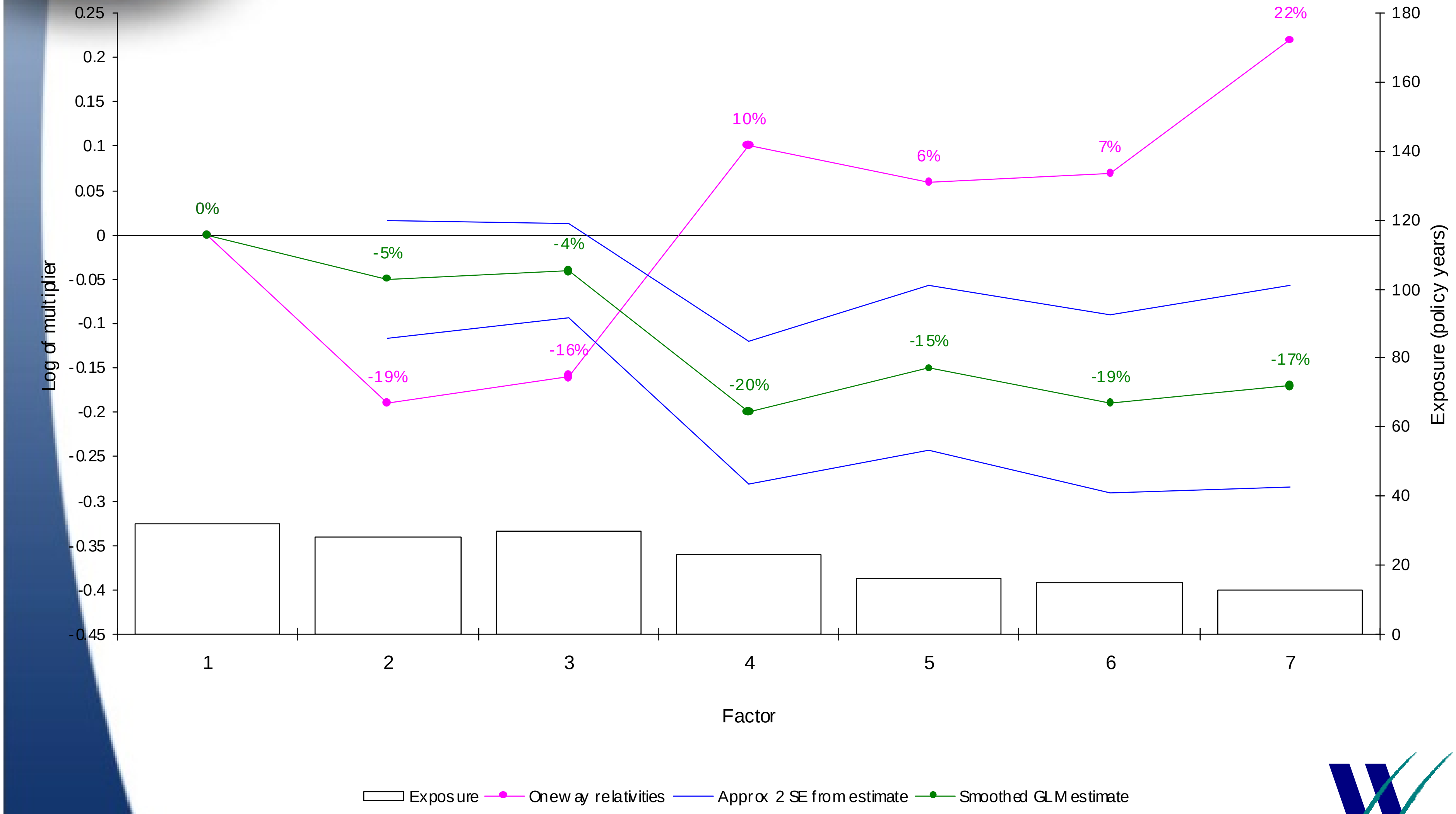
Diagnostics

Maximum likelihood estimation

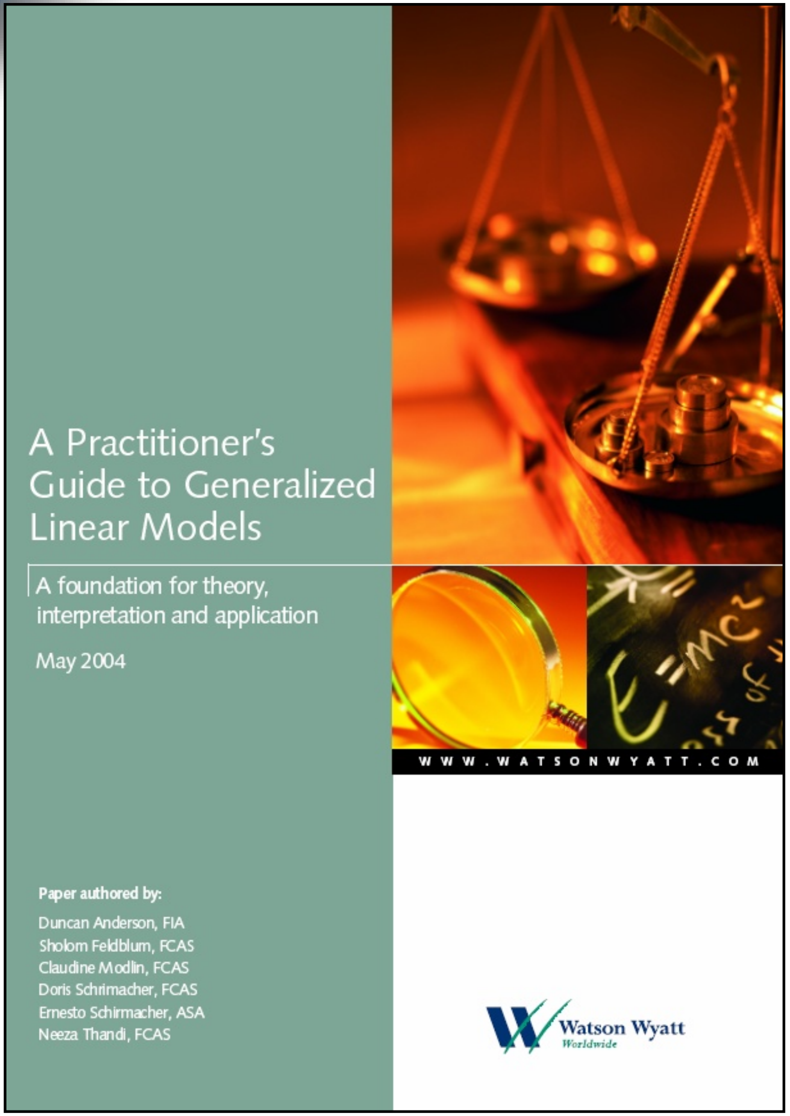
- Seek parameters which give highest likelihood function given data



Example of GLM output (real UK data)



"A Practitioner's Guide to Generalized Linear Models"



A Practitioner's Guide to Generalized Linear Models

A foundation for theory, interpretation and application

May 2004

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- Copies available here

  and at

  www.watsonwyatt.com/glm

  and

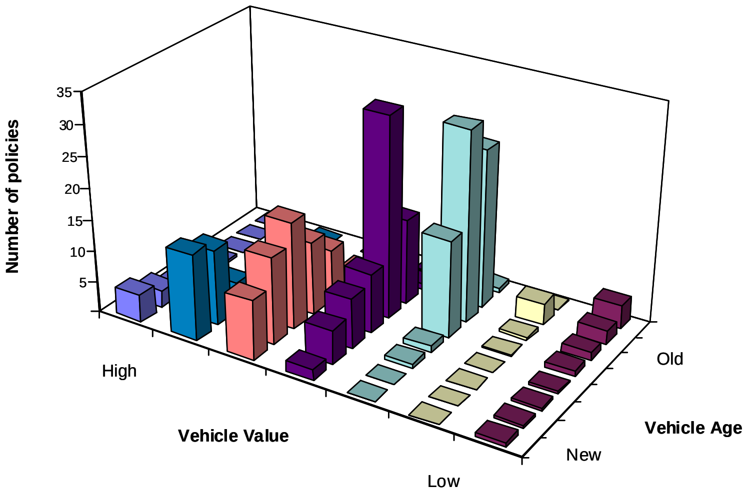
  www.casact.org

Data required

- Minimum 100,000 earned exposures
- Model separately by claim type
- Key fields for each risk
  - period of earned exposure
  - rating factors applicable at time (start of period)
  - other "dummy" factors (area, time)
  - incurred claim count (by type)
  - incurred loss amount (by type), based on most recent estimates
  - (optionally) earned premium on current basis

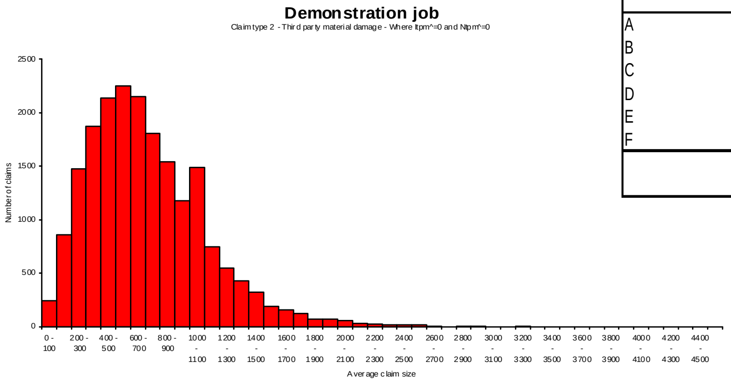
Preliminary analyses

- One-way analyses
- Two-way analyses
- Correlation analyses
- Distribution analyses



Claim type 1 - Third party property damage
Vehicle type (Type)

Level	Number of records	Exposure	Premium	Number of claims	Incurred losses	Claim frequency	Average cost per claim	Pure premium	Loss ratio
A	27,661	24,757	10,584,626	1,807	8,457,208	7.3%	4,681	342	79.9%
B	22,089	19,777	9,623,698	1,598	6,957,135	8.1%	4,354	352	72.3%
C	13,768	12,334	6,305,906	1,011	4,245,902	8.2%	4,200	344	67.3%
D	19,662	17,592	9,382,767	1,584	6,070,943	9.0%	3,832	345	64.7%
E	11,235	10,076	5,676,363	982	3,262,384	9.7%	3,321	324	57.5%
F	5,607	5,037	3,118,064	550	1,858,753	10.9%	3,379	369	59.6%
	100,022	**89,572**	**44,691,424**	**7,532**	**30,852,324**	**8.4%**	**4,096**	**344.44**	**69.0%**



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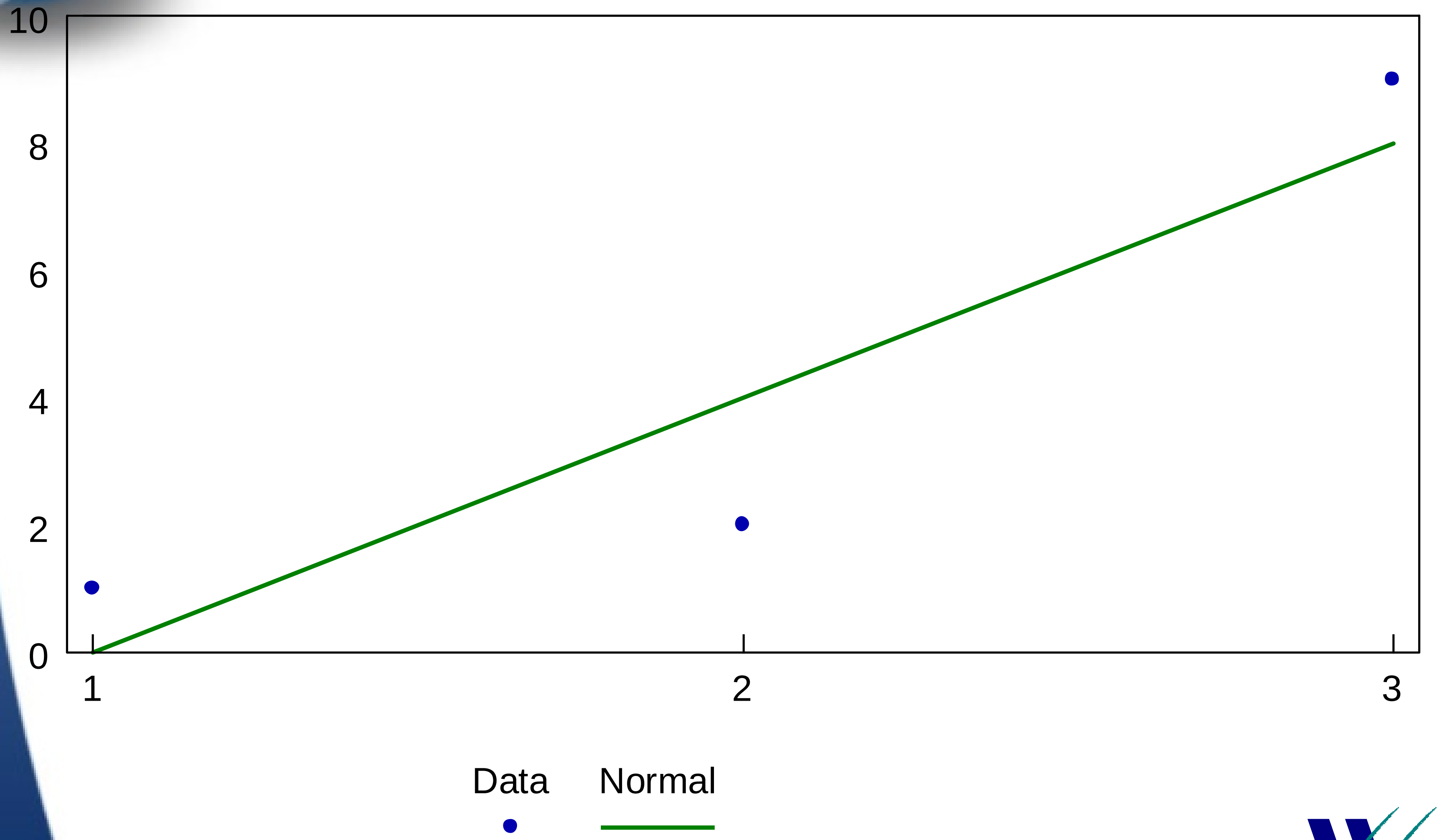
Interesting properties

- Poisson multiplicative
  - parameter estimates unchanged if group by unique combination of rating factor
  - invariant to measures of time
- Gamma multiplicative
  - parameter estimates unchanged by grouping but standard errors are not
  - generally do not group except for multiple claims on a risk in a policy period
  - invariant to measures of currency
- Logistic (binomial / logit)
  - maps (0,1) to (-∞,∞)
  - invariant to whether measuring of success / failure (eg same if model lapse / renew)
  - more appropriate for retention/conversion analyses, but harder to communicate

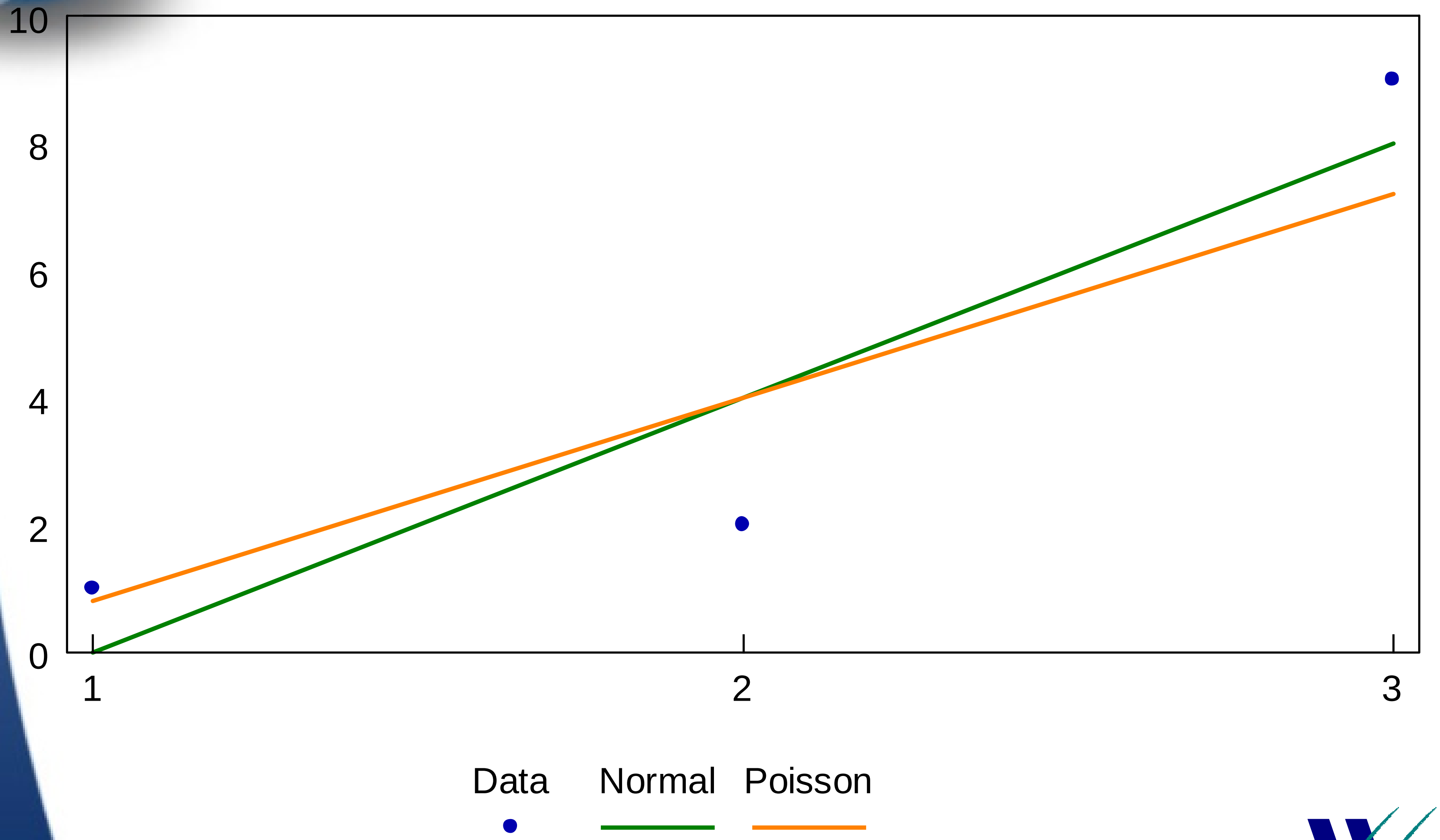
Typical model forms

Y	Claim frequency	Claim number	Average claim amount	Probability (eg lapses)
g(x)	ln(x)	ln(x)	ln(x)	ln(x/(1-x))
Error	Poisson	Poisson	Gamma	Binomial
ϕ **V(x)**	1 x	1 x	estimate x^2	1 x(1-x)
ω	exposure	1	# claims	1
ξ	0	ln(exposure)	0	0

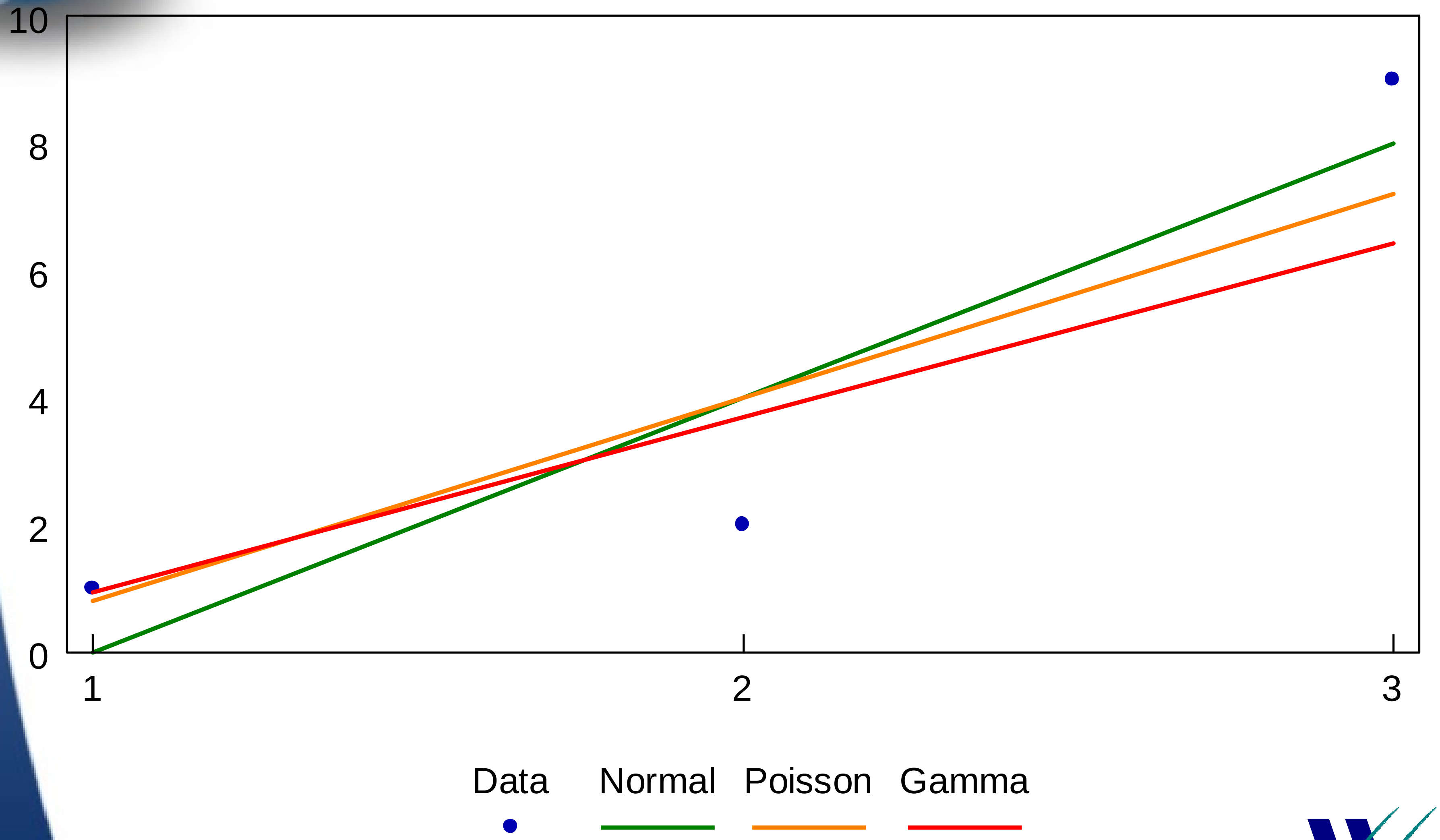
Example of effect of changing assumed error - 1



Example of effect of changing assumed error - 1



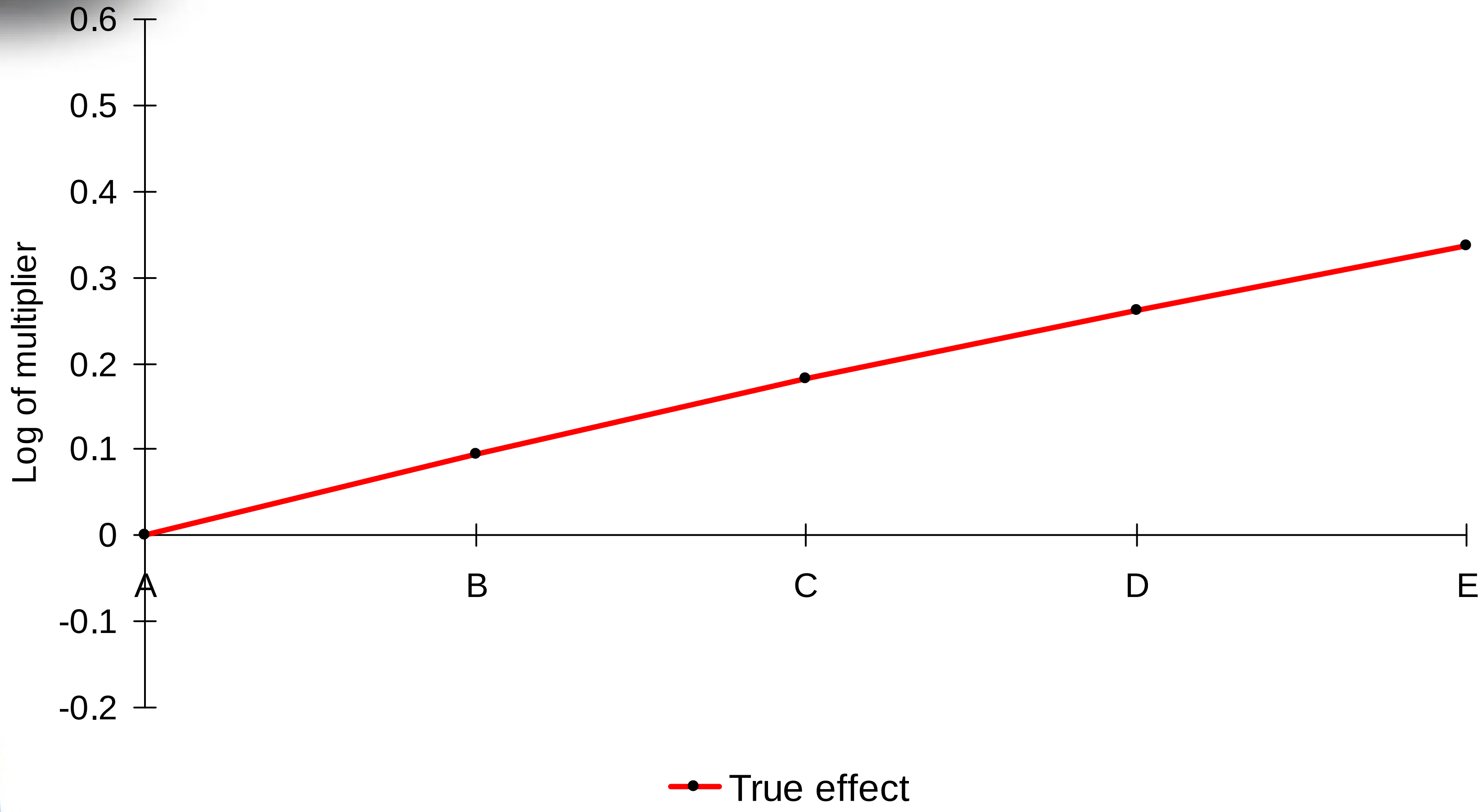
Example of effect of changing assumed error - 1



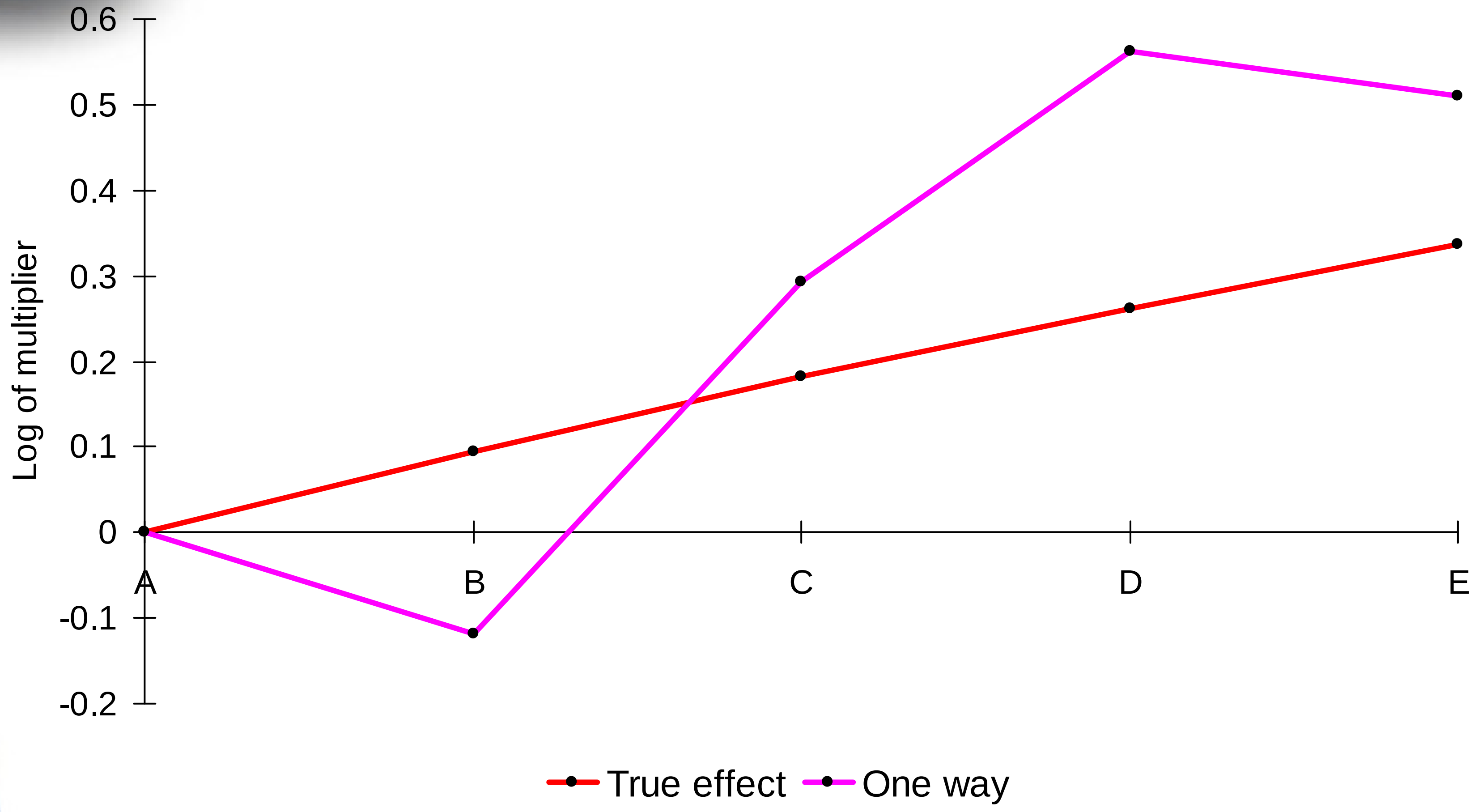
Example of effect of changing assumed error - 2

- Example portfolio with five rating factors, each with five levels A, B, C, D, E
- Typical correlations between those rating factors
- Assumed true effect of factors
- Claims randomly generated (with Gamma)
- Random experience analysed by three models

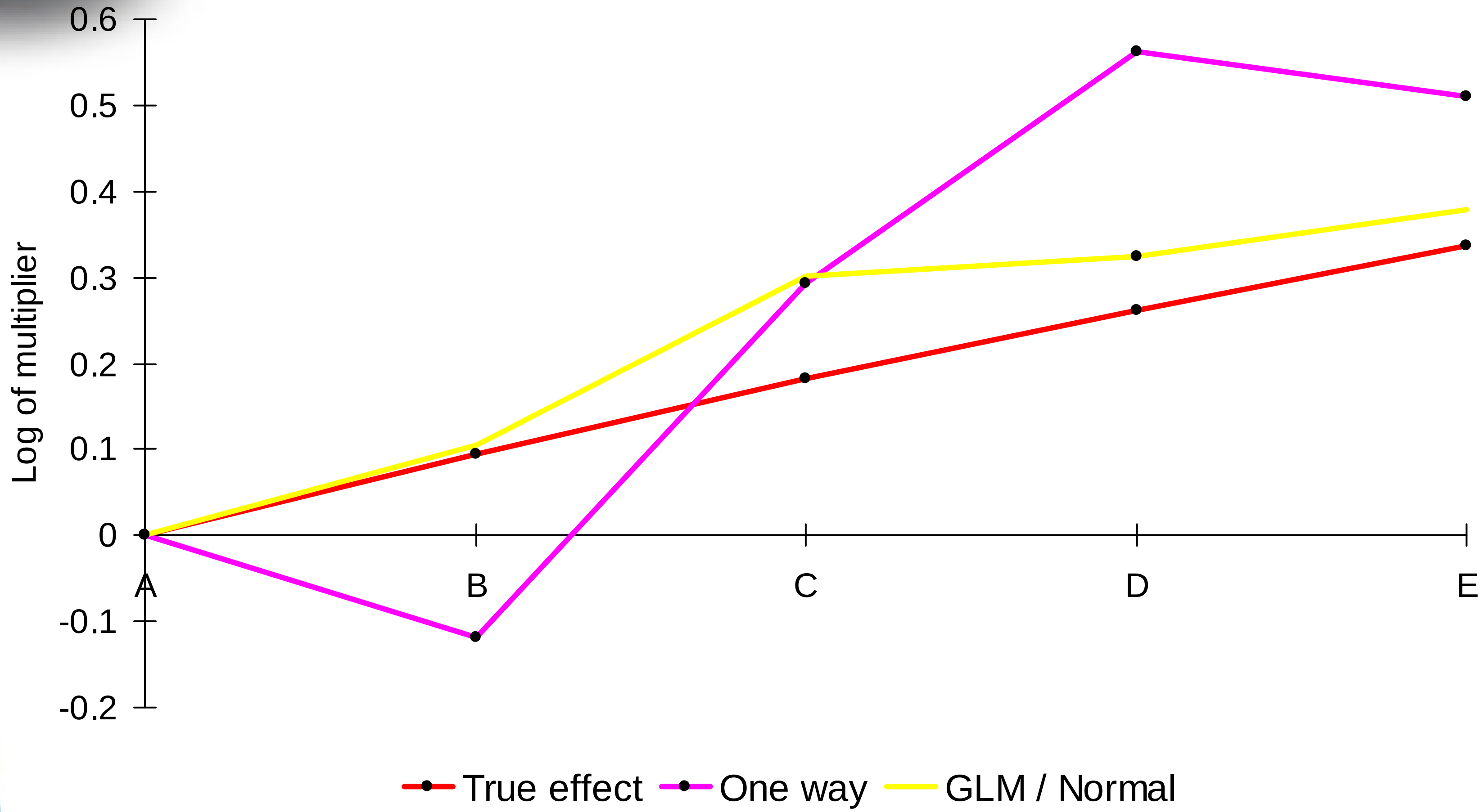
Example of effect of changing assumed error - 2



Example of effect of changing assumed error - 2



Example of effect of changing assumed error - 2



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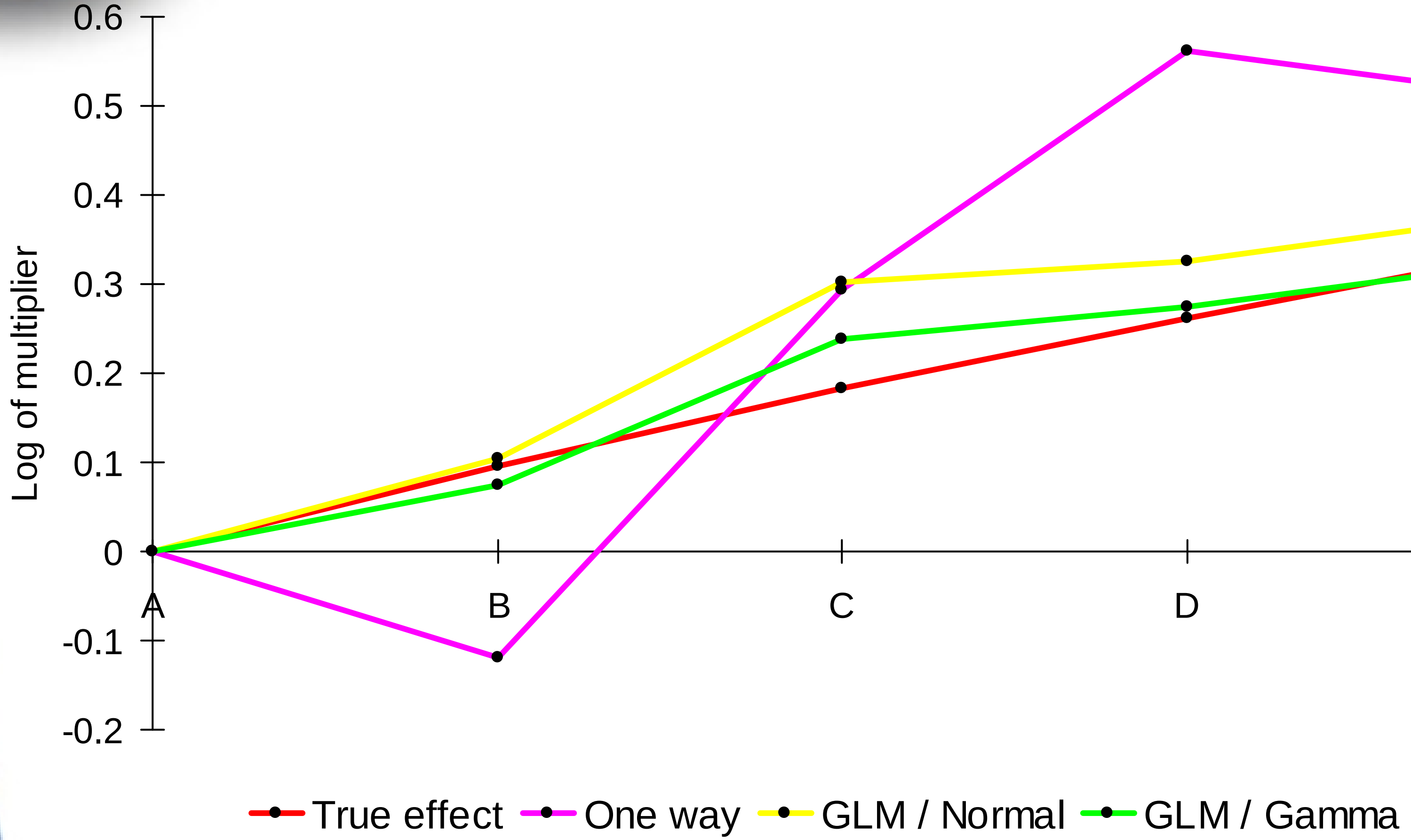

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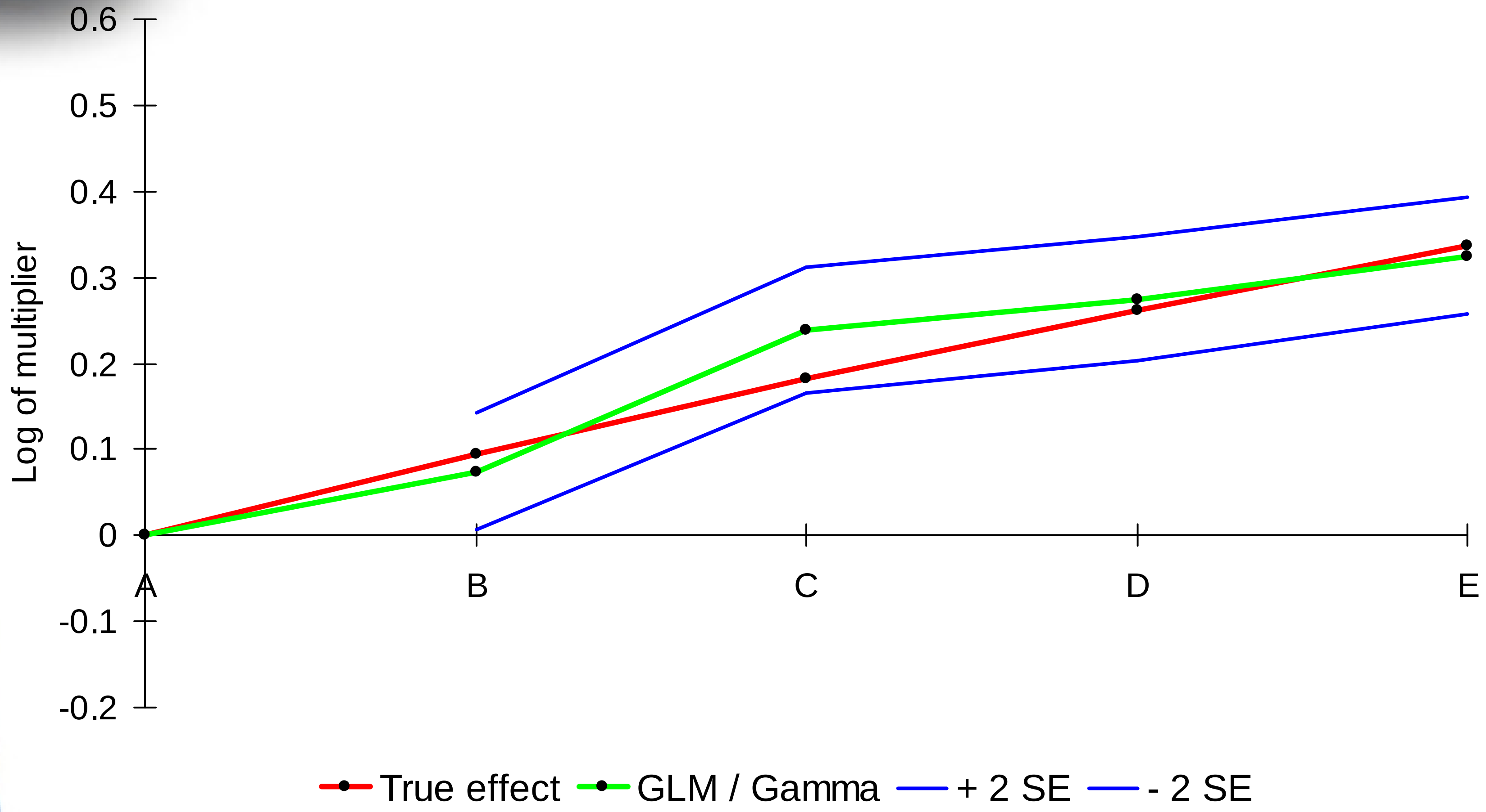
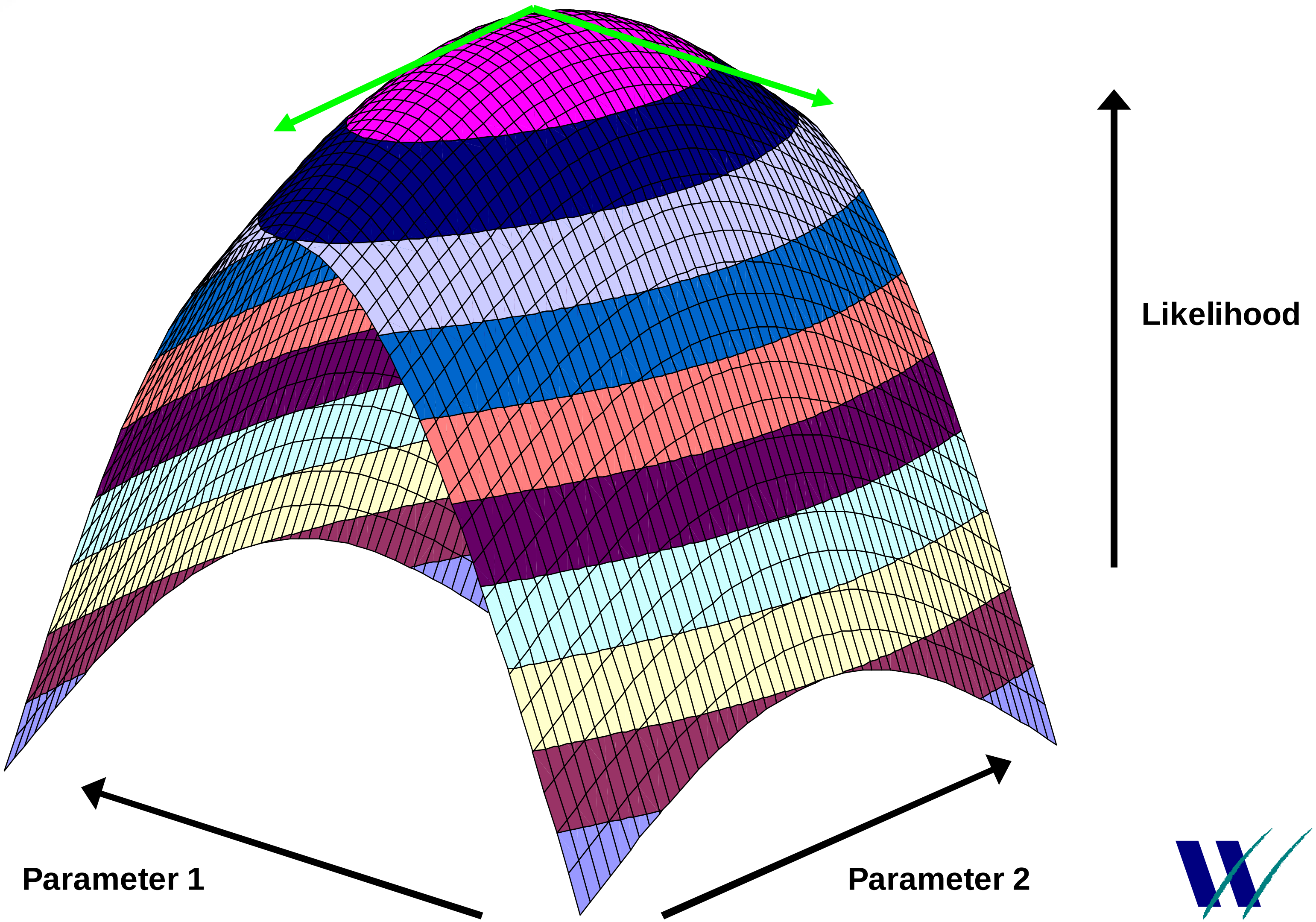


Model testing

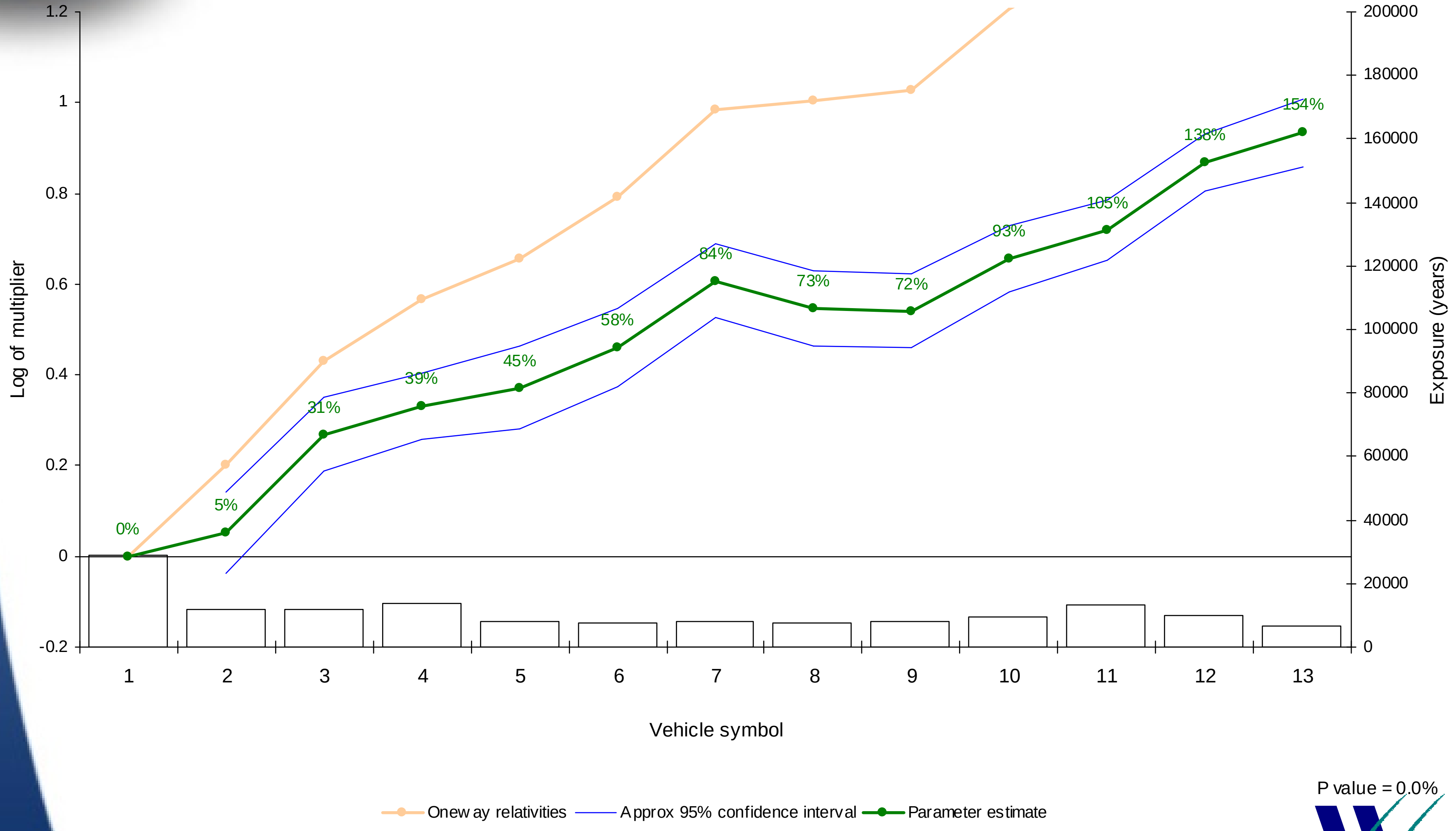
- Use only those factors which are predictive
  - standard errors of parameter estimates
  - F tests / χ^2 tests on deviances
  - stepwise approach (helpful if used with care)
  - consistency over time
  - human intuition
- Make sure the model is reasonable
  - residual plots (histograms / Q-Q / residual vs fitted value etc)
  - leverage / Cook's distance
  - Box-Cox

Standard errors

- Roughly speaking, for a parameter p: $SE = -1 / (\partial^2/ \partial p^2 \text{ Likelihood})$

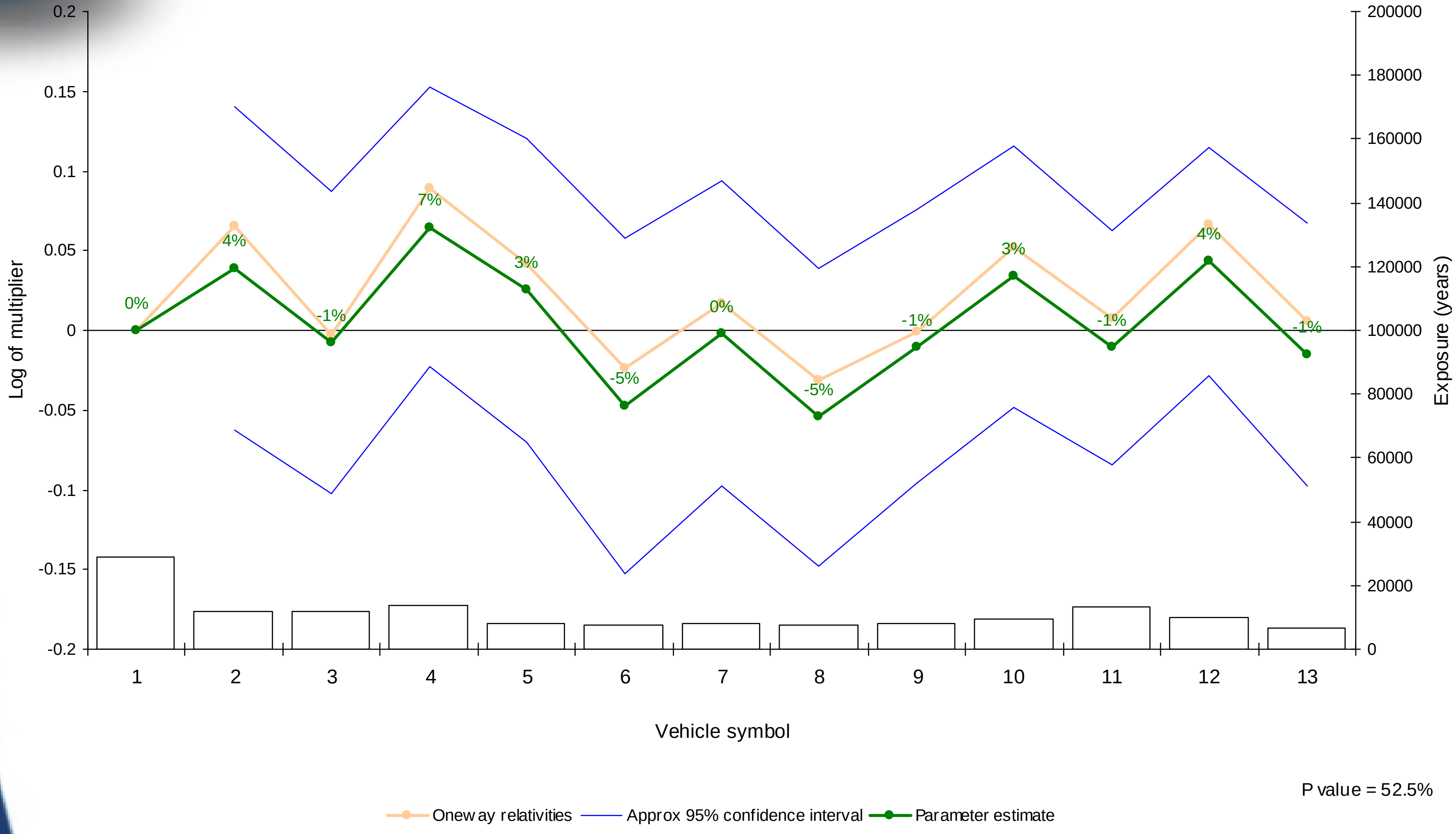


GLM output (significant factor)

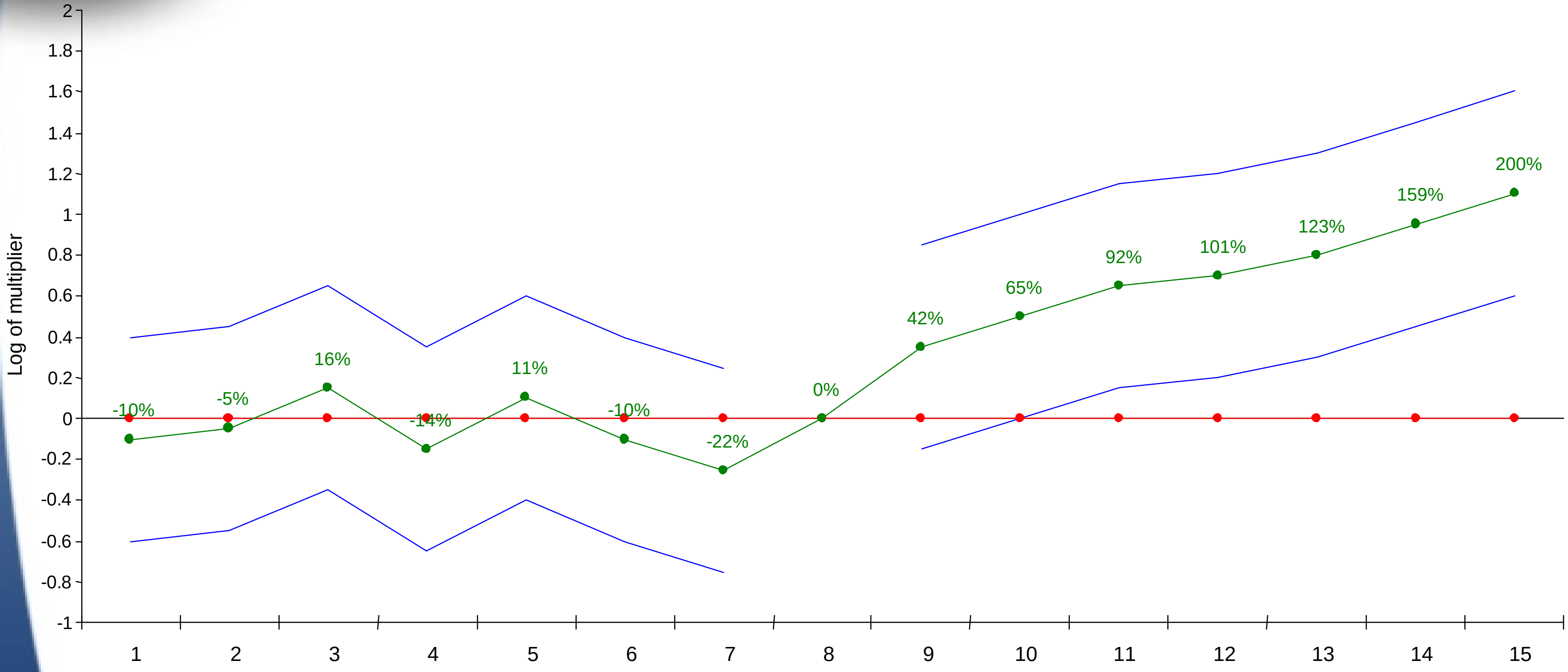


P value = 0.0%

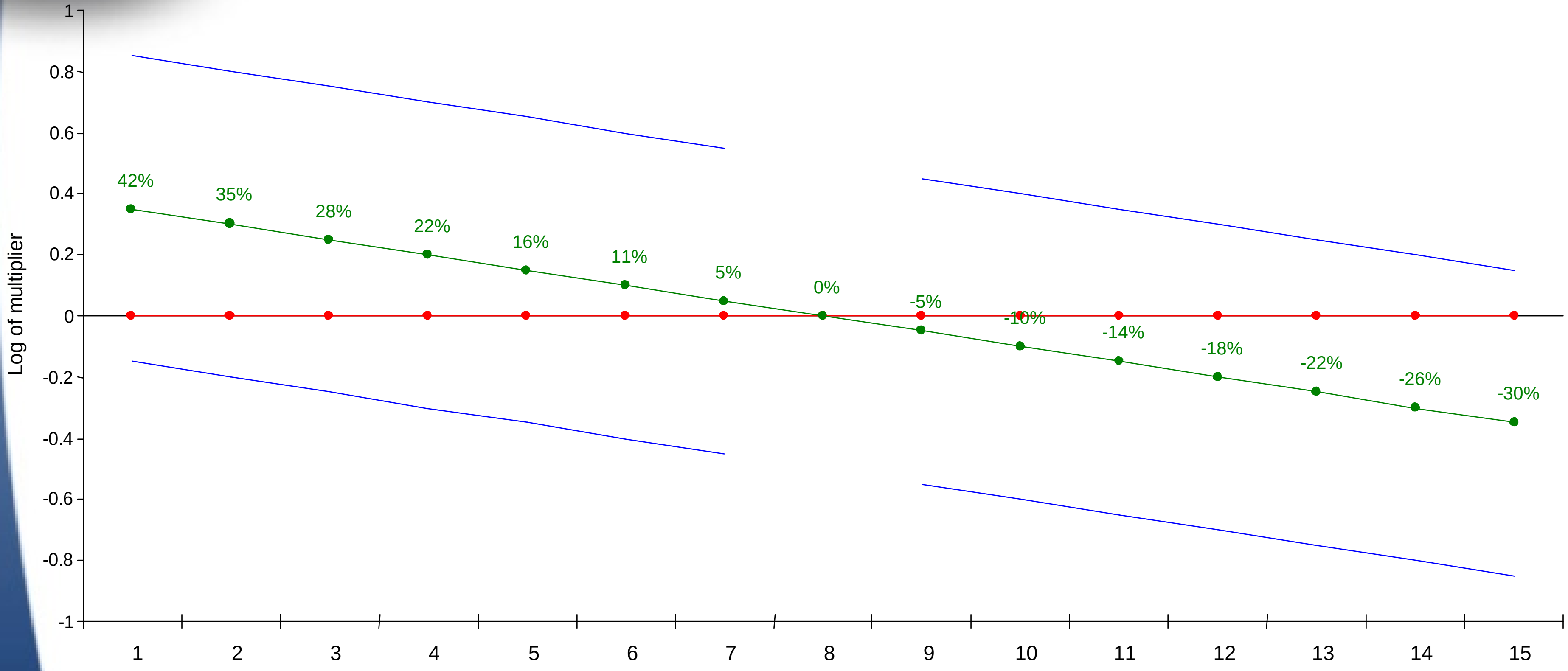
GLM output (insignificant factor)



Awkward cases

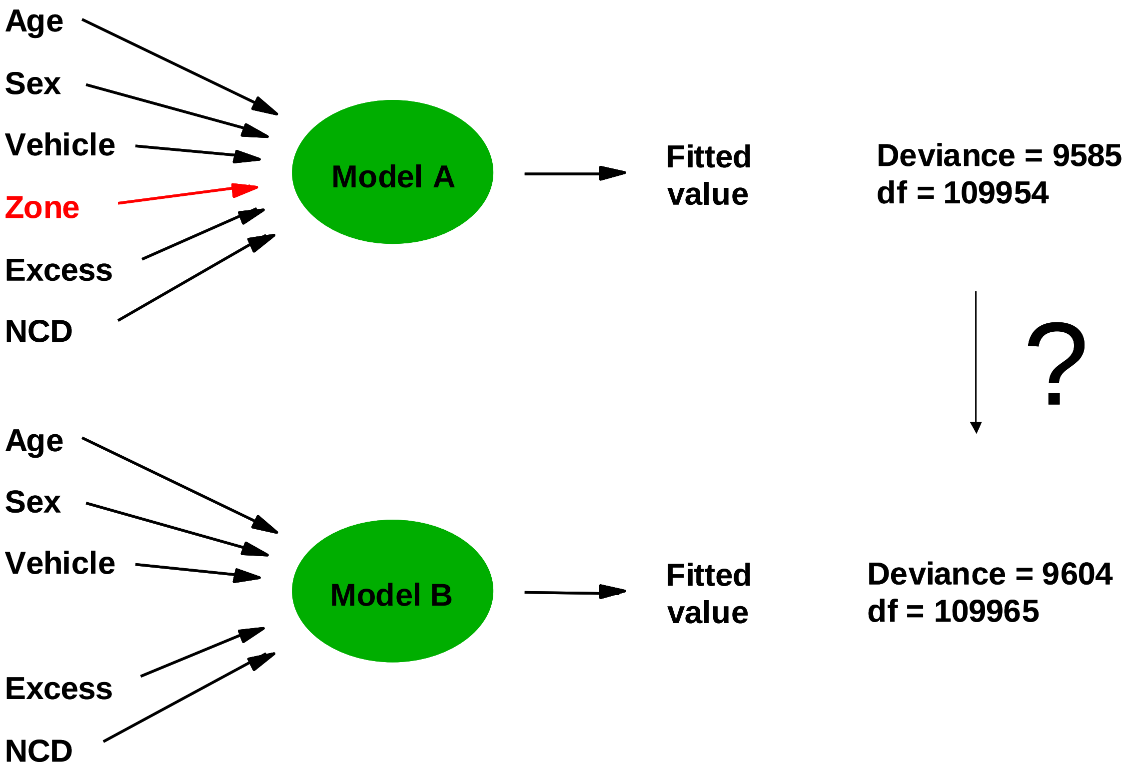


Awkward cases



Deviance type III tests

- Single figure measure of goodness of fit
- Try model with & without a factor
- Statistical tests show the theoretical significance given the extra parameters



Age
Sex
Vehicle
Zone
Excess
NCD

Model A

Fitted value

Deviance = 9585
df = 109954

?

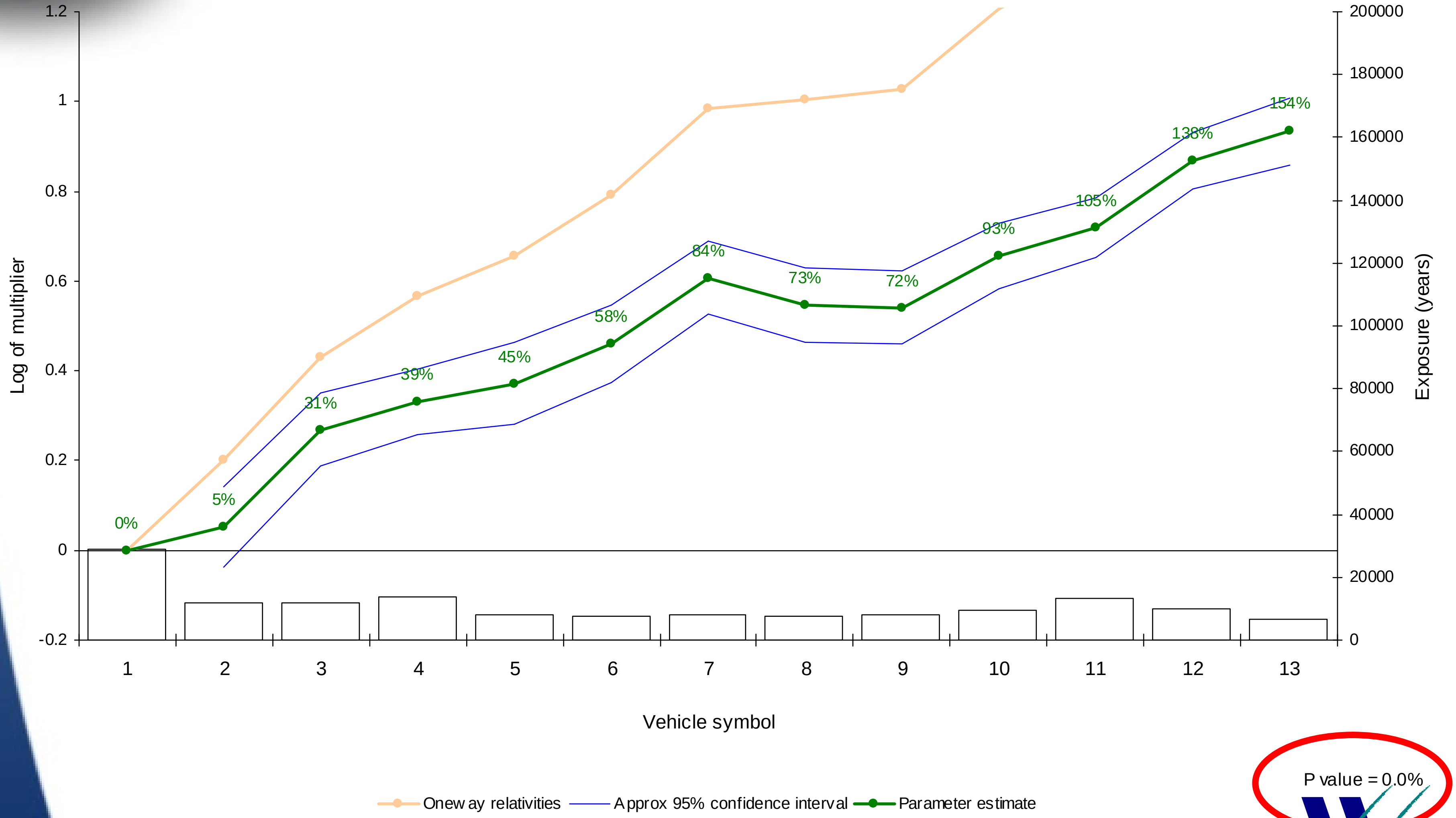
Age
Sex
Vehicle
Excess
NCD

Model B

Fitted value

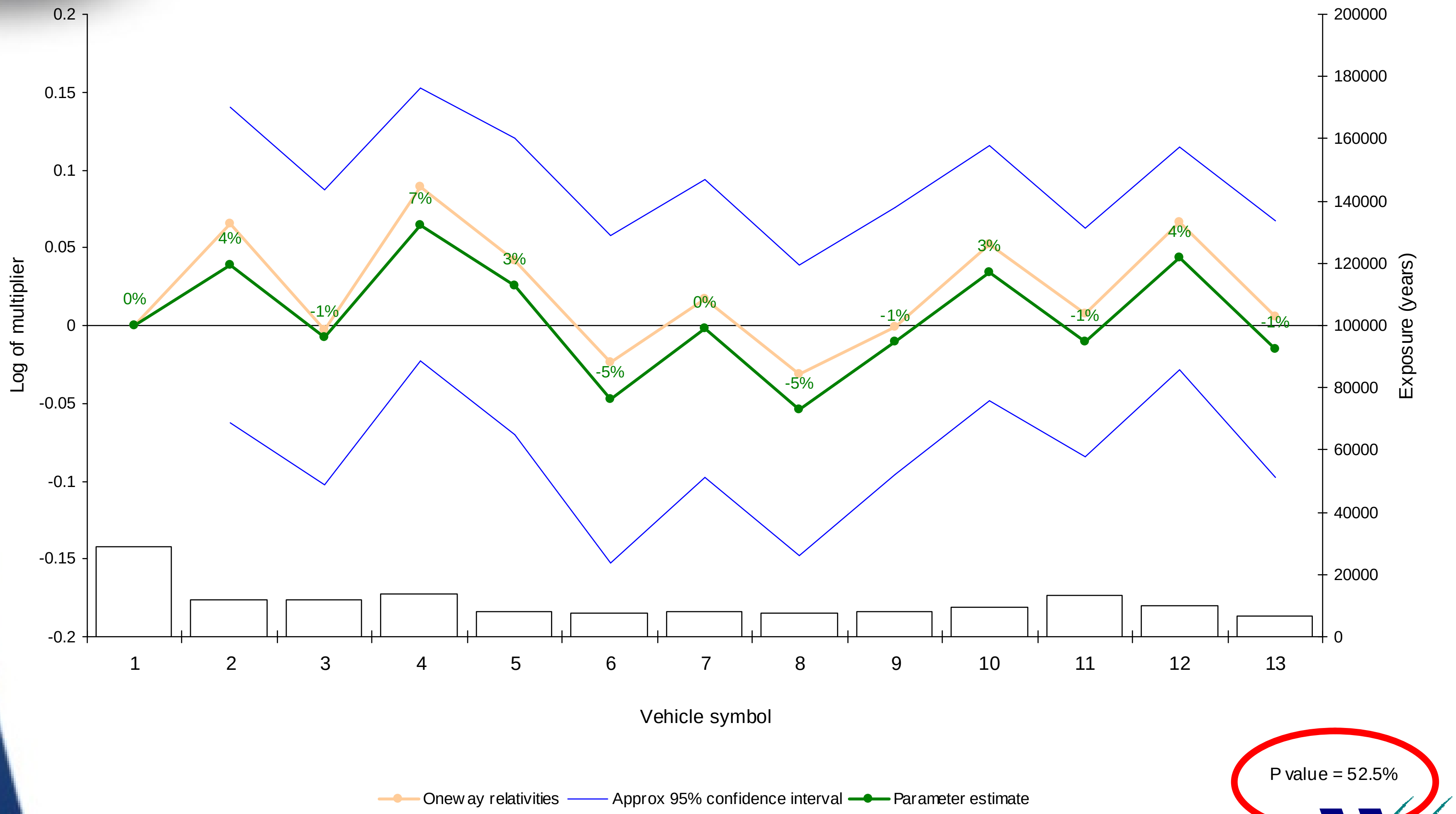
Deviance = 9604
df = 109965

GLM output (significant factor)



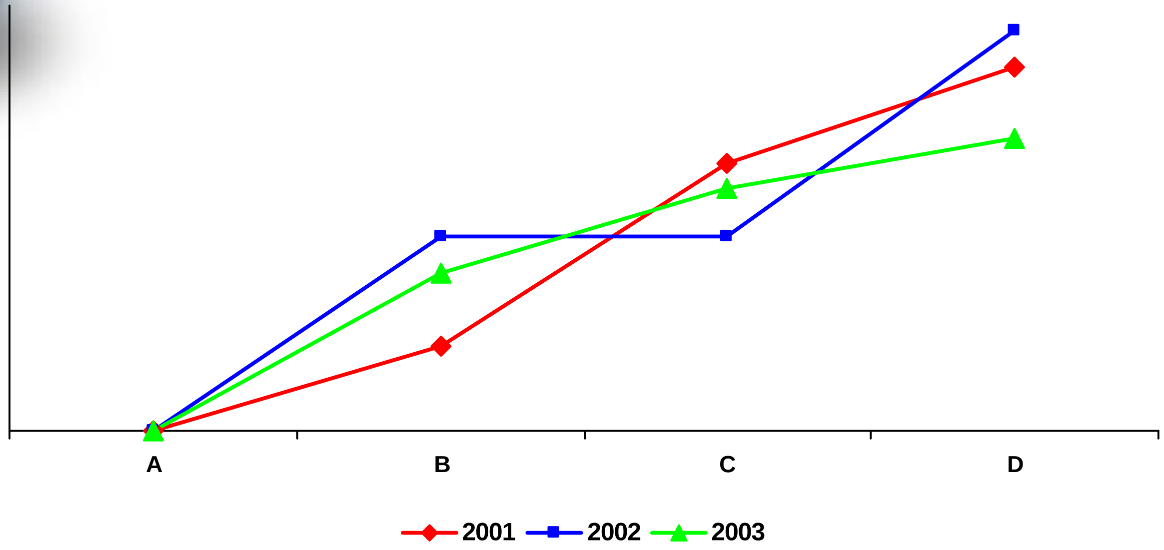
P value = 0.0%

GLM output (insignificant factor)



P value = 52.5%

Consistency over time



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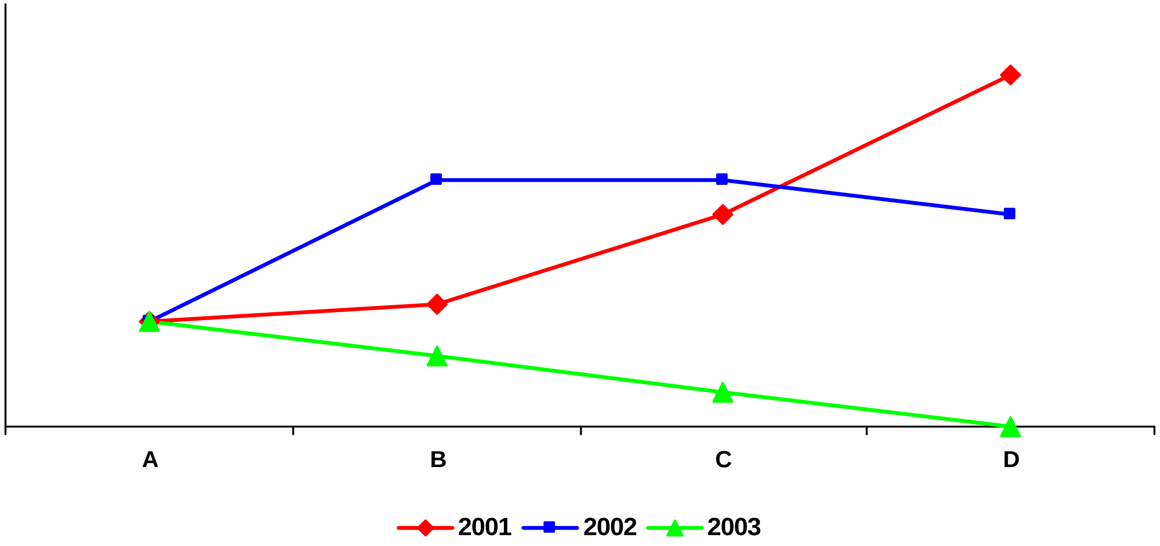
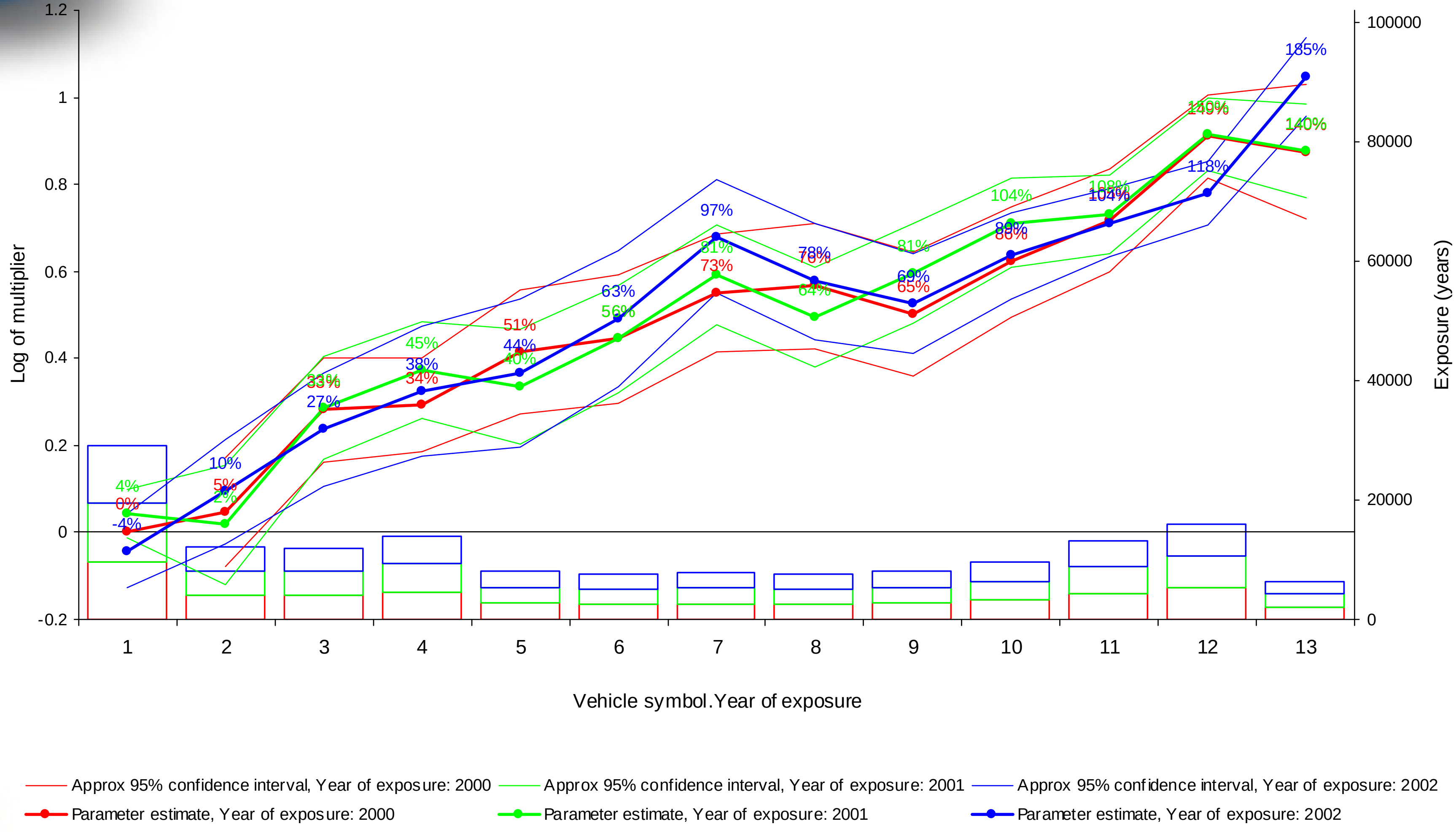
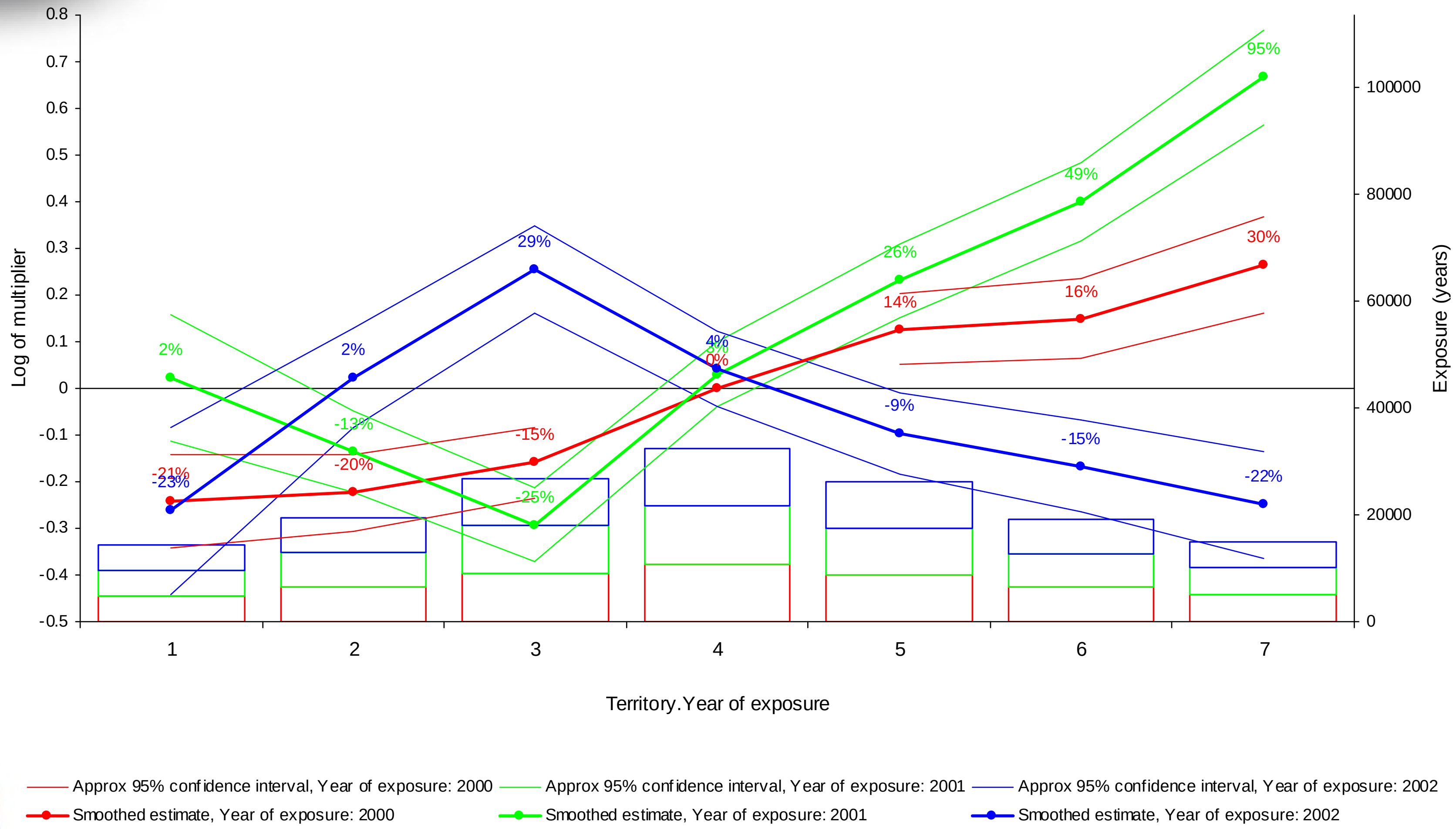


✗

Consistency over time



Consistency over time



Intuition

- Are ordered categorical variables well behaved?
- Can you believe it, given correlations with other factors?
- Can the underwriters believe it?
- How different is it to the one-way?
- What does this factor do in other frequency/amounts models and for other claim types?

Practical model iteration

- Start with all factors if possible
- In theory reject one at a time
- In practice be more brutal if one-ways are similar to GLM parameter estimates
- Re-check excluded factors at end
- Interactions
- Stepwise algorithms can be useful if used with care
- This is what takes time

Model testing

- Use only those factors which are predictive
  - standard errors of parameter estimates
  - F tests / χ^2 tests on deviances
  - stepwise approach (helpful if used with care)
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- Make sure the model is reasonable
  - residual plots (histograms / Q-Q / residual vs fitted value etc)
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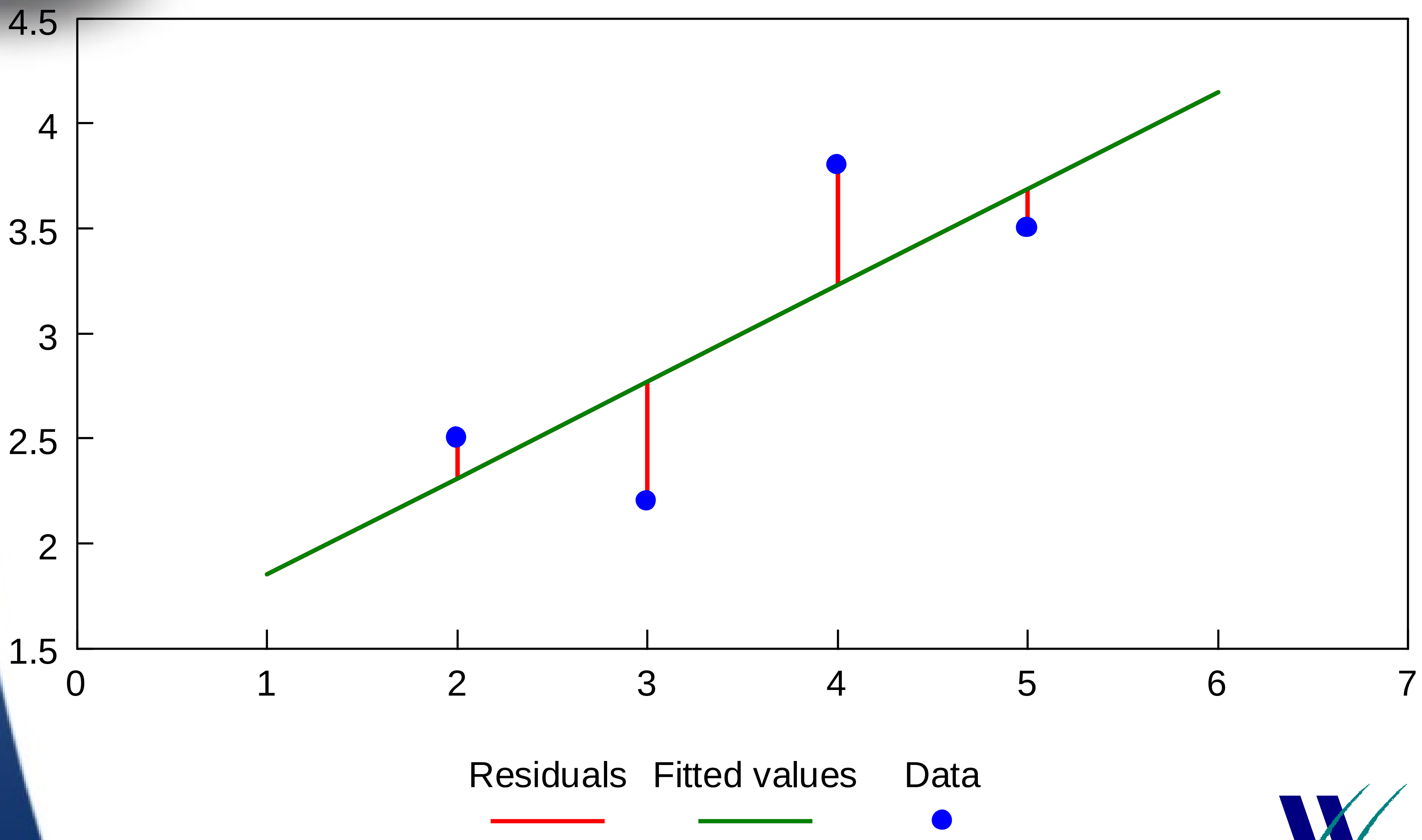
Residuals



Residuals

- Several forms, eg
  - standardized deviance

$$\mathrm{sign}\,(Y_u-\mu_u)\,/\,(\,\phi\,(1-h_u)\,)^{½}\sqrt{2\,\omega_u\int_{\mu_u}^{Y_u}(\,Y_u-\zeta\,)\,/\,V(\zeta)\,d\zeta}$$

  - standardized Pearson

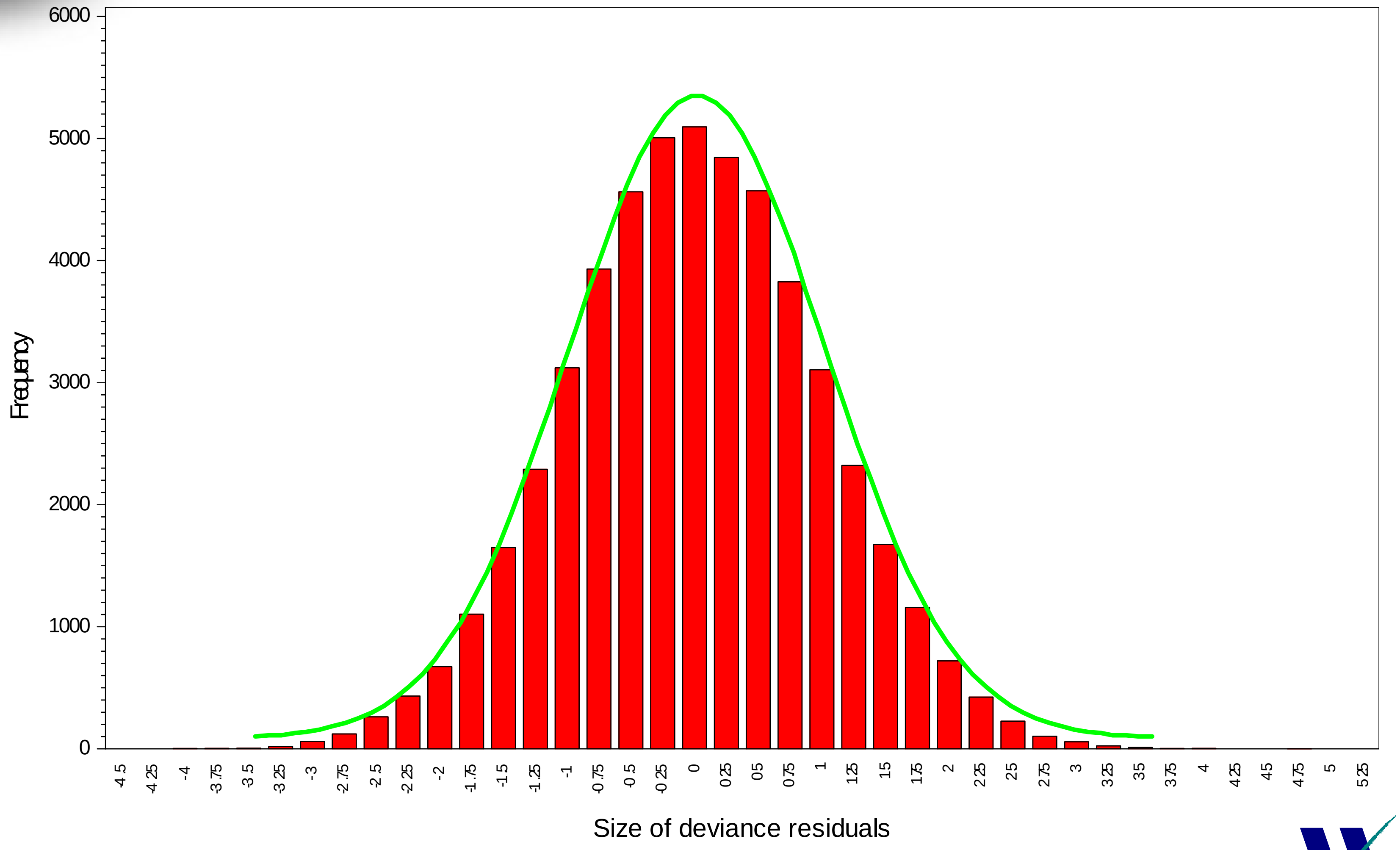
$$\frac{Y_u\ -\ \mu_u}{(\,\phi.V(\mu_u).(1-h_u)\,/\ \omega_u\,)^{½}}$$

- Standardized deviance - Normal (0,1)
- Numbers/frequency residuals problematical

Residuals

Histogram of Deviance Residuals

Run 12 (Final models with analysis) Model 8 (AD amounts)



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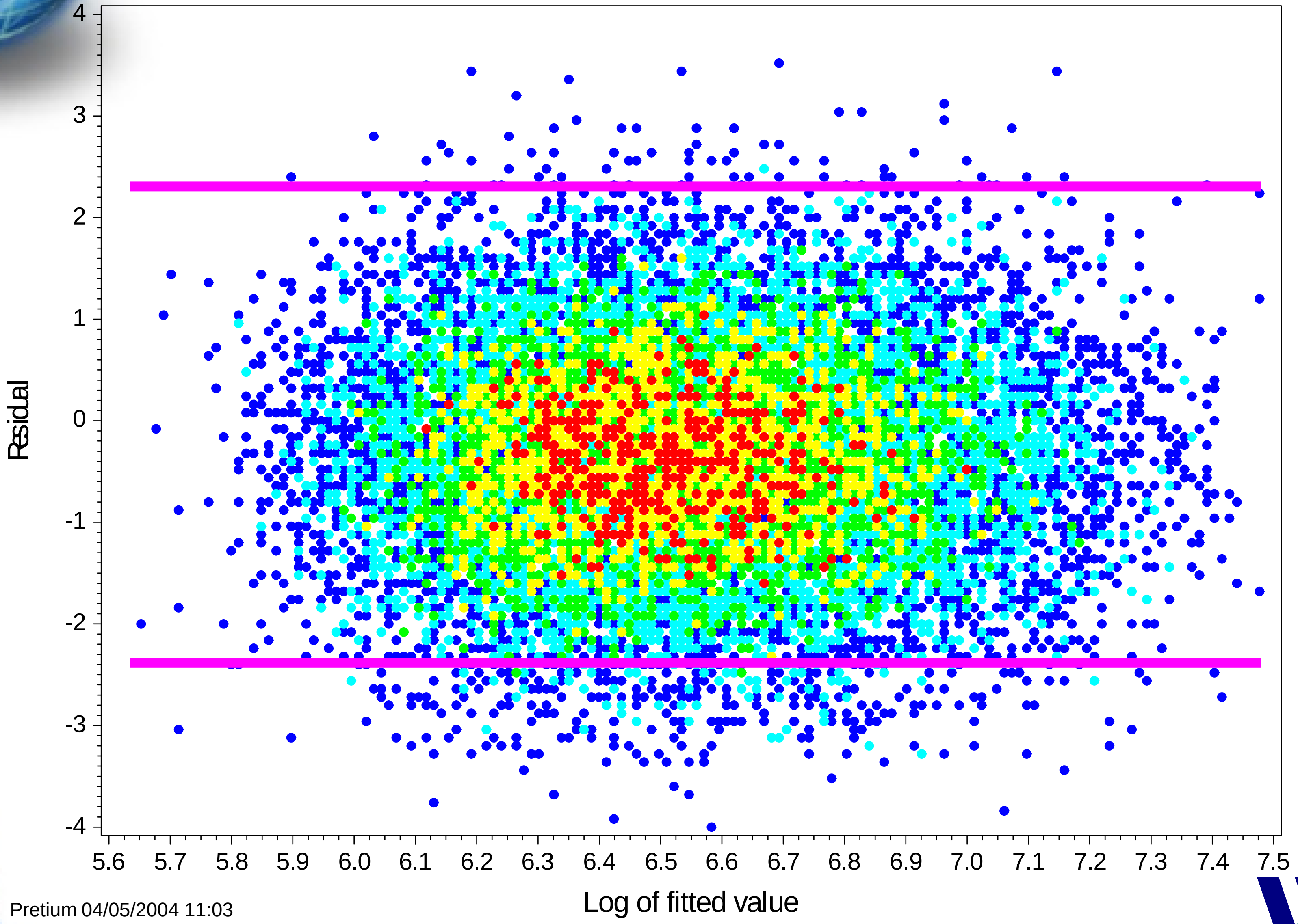
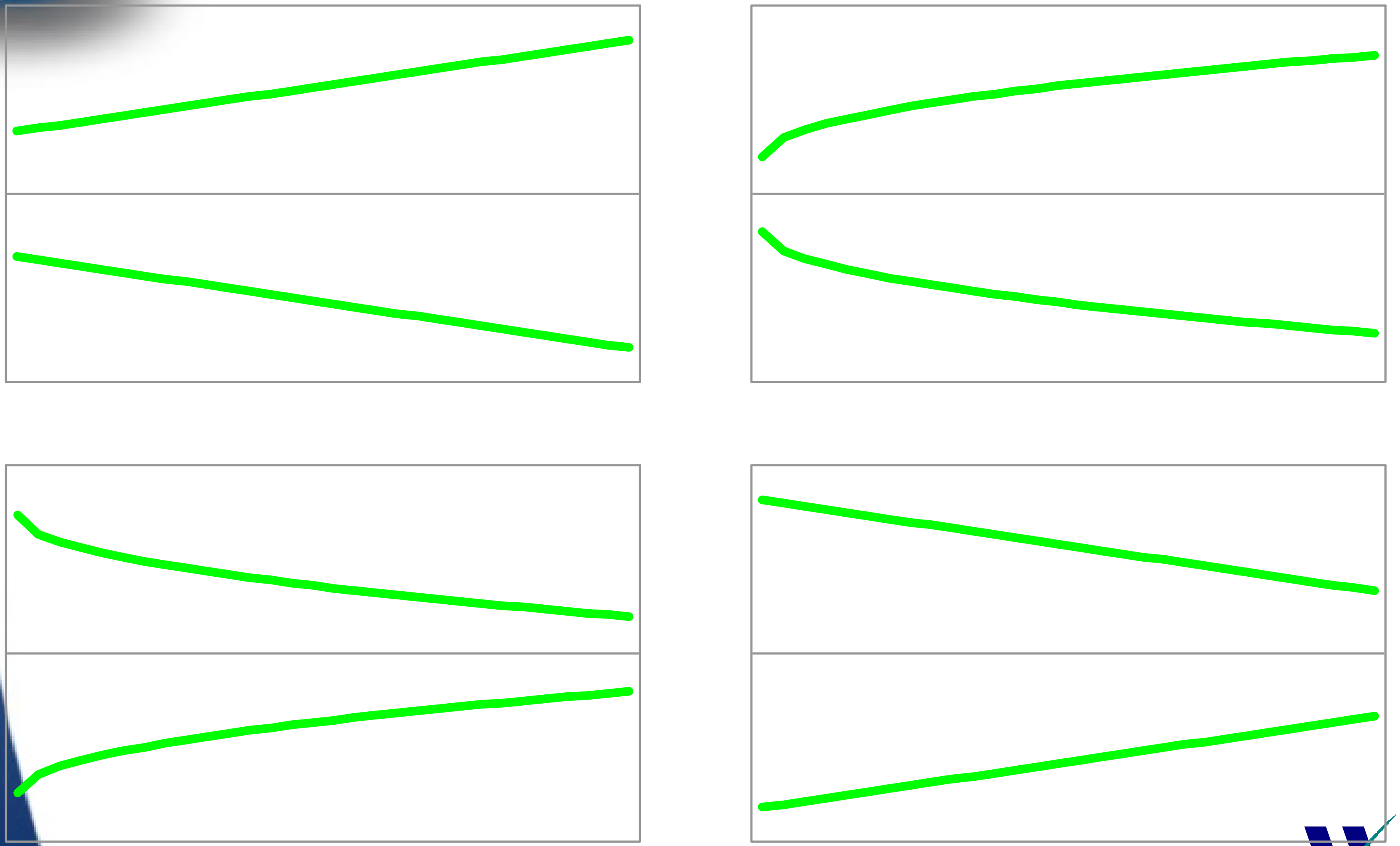
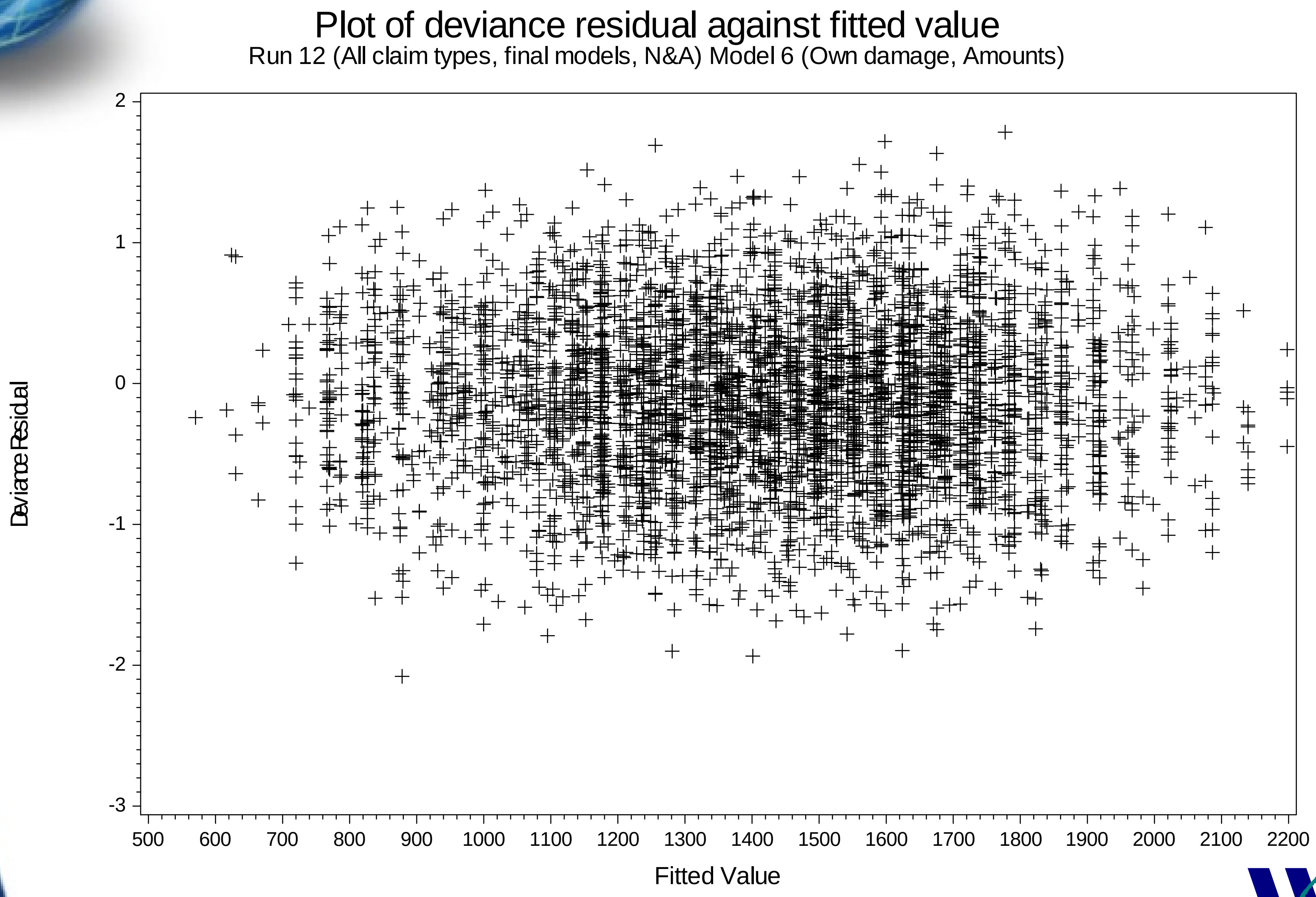
Residuals


4
3
2
1
0
-1
-2
-3
-4
Residual
5.6
5.7
5.8
5.9
6.0
6.1
6.2
6.3
6.4
6.5
6.6
6.7
6.8
6.9
7.0
7.1
7.2
7.3
7.4
7.5
Log of fitted value

Residuals



Gamma data, Gamma error



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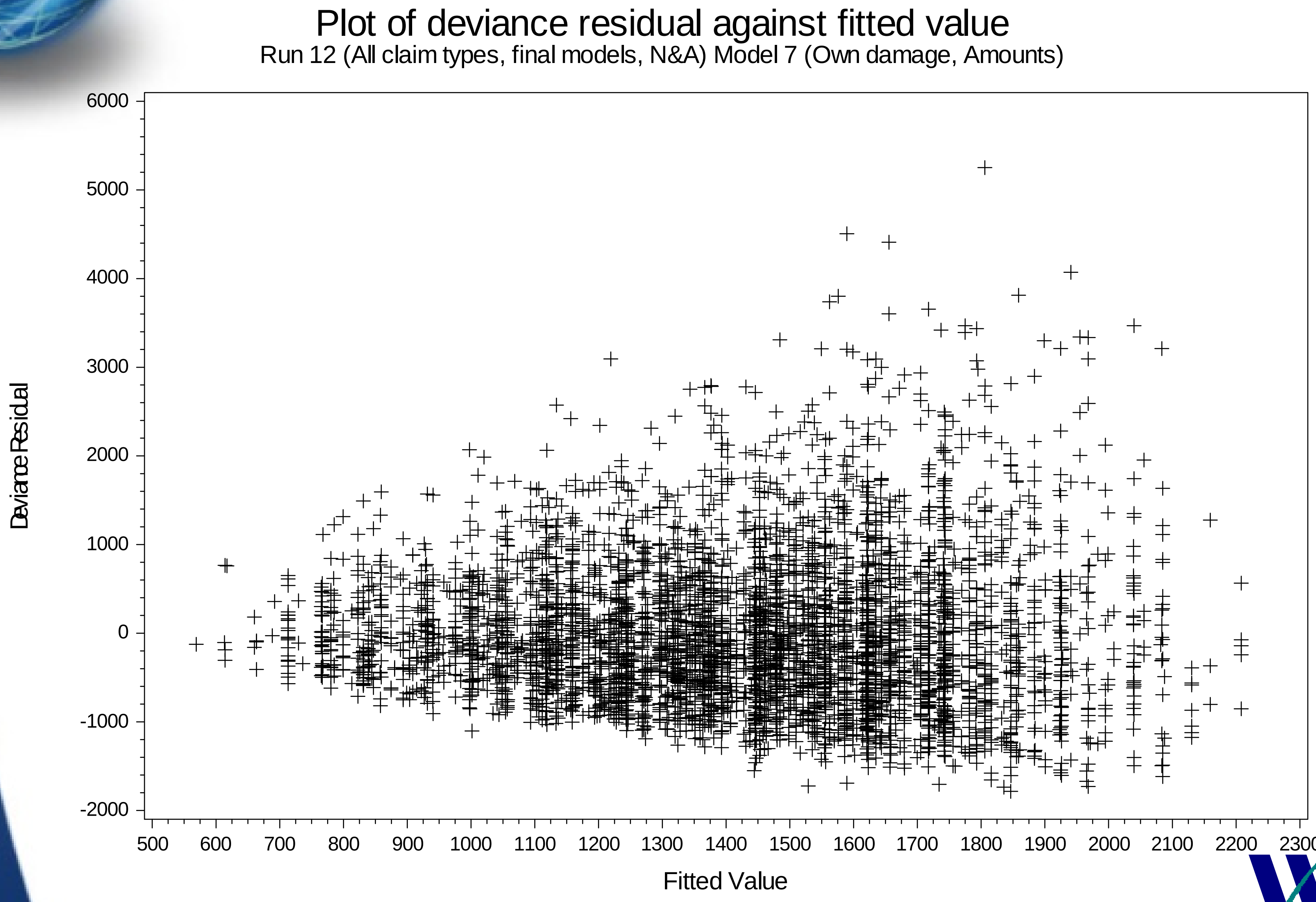
Gamma data, Normal error



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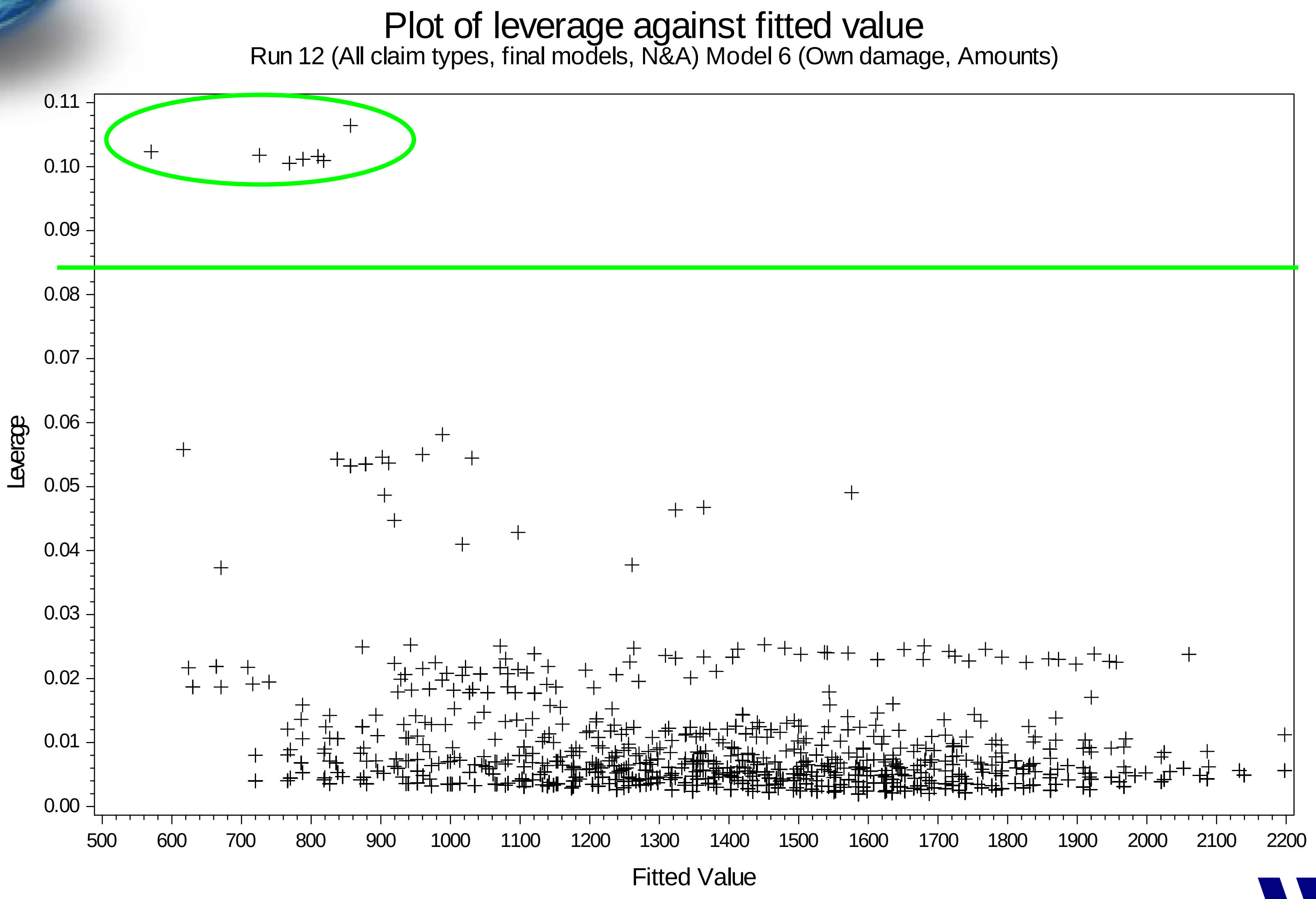
Leverage



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Box-Cox link function investigation

- GLM structure is

  $E[\underline{Y}] = \underline{\mu} = g^{-1}(\mathbf{X}.\underline{\beta} + \underline{\xi}) \qquad Var[\underline{Y}] = \phi.V(\underline{\mu}) / \underline{\omega}$

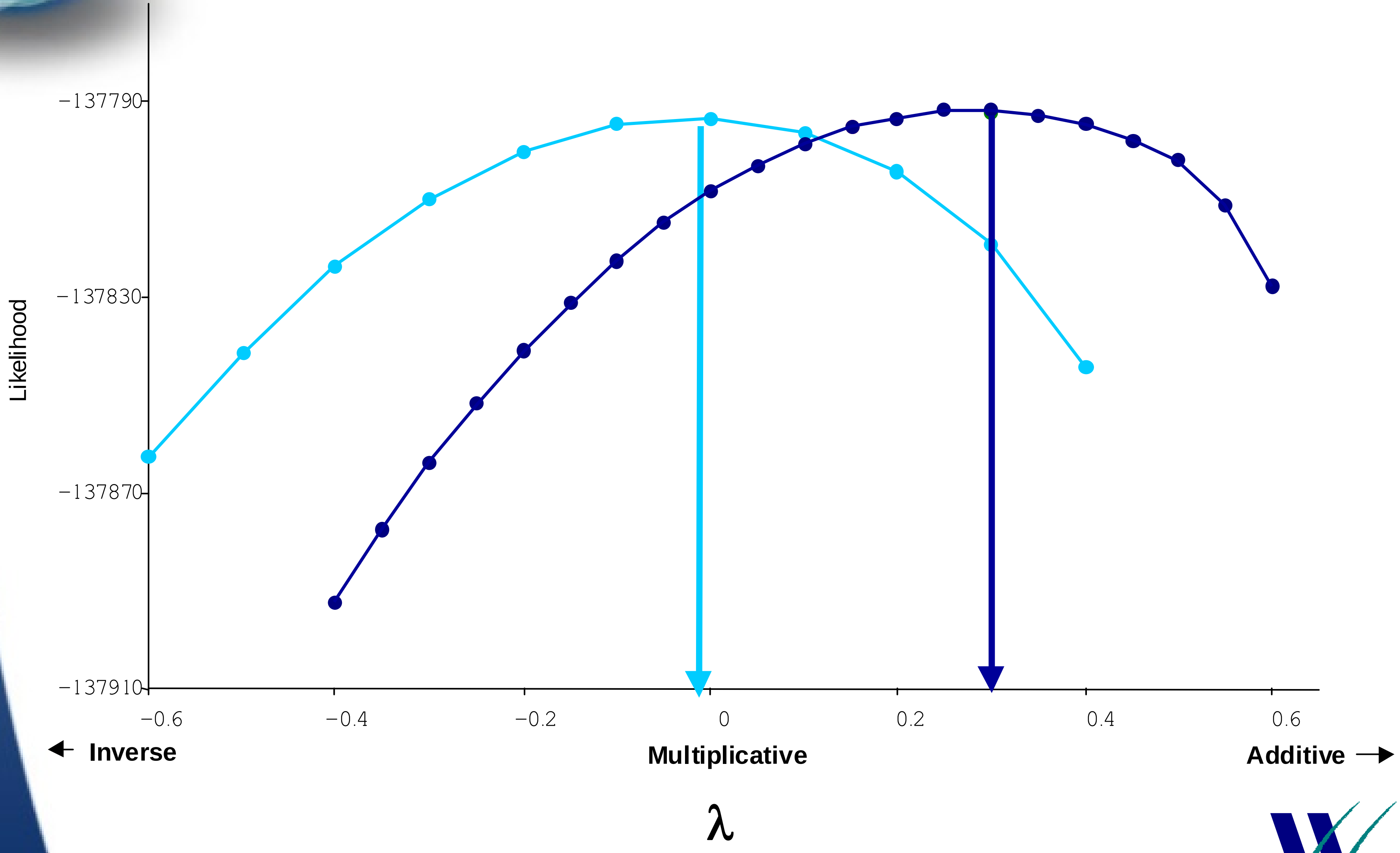
- Box Cox transforms defines

  $g(x) = (x^{\lambda} - 1) / \lambda$ for $\lambda \neq 0$, $\ln(x)$ for $\lambda = 0$

- $\lambda = 1 \Rightarrow g(x) = x - 1 \Rightarrow$ additive (with base level shift)
- $\lambda \to 0 \Rightarrow g(x) \to \ln(x) \Rightarrow$ multiplicative (via maths)
- $\lambda = -1 \Rightarrow g(x) = 1 - 1/x \Rightarrow$ inverse (with base level shift)
- Try different values of λ and measure goodness of fit to see which fits experience best

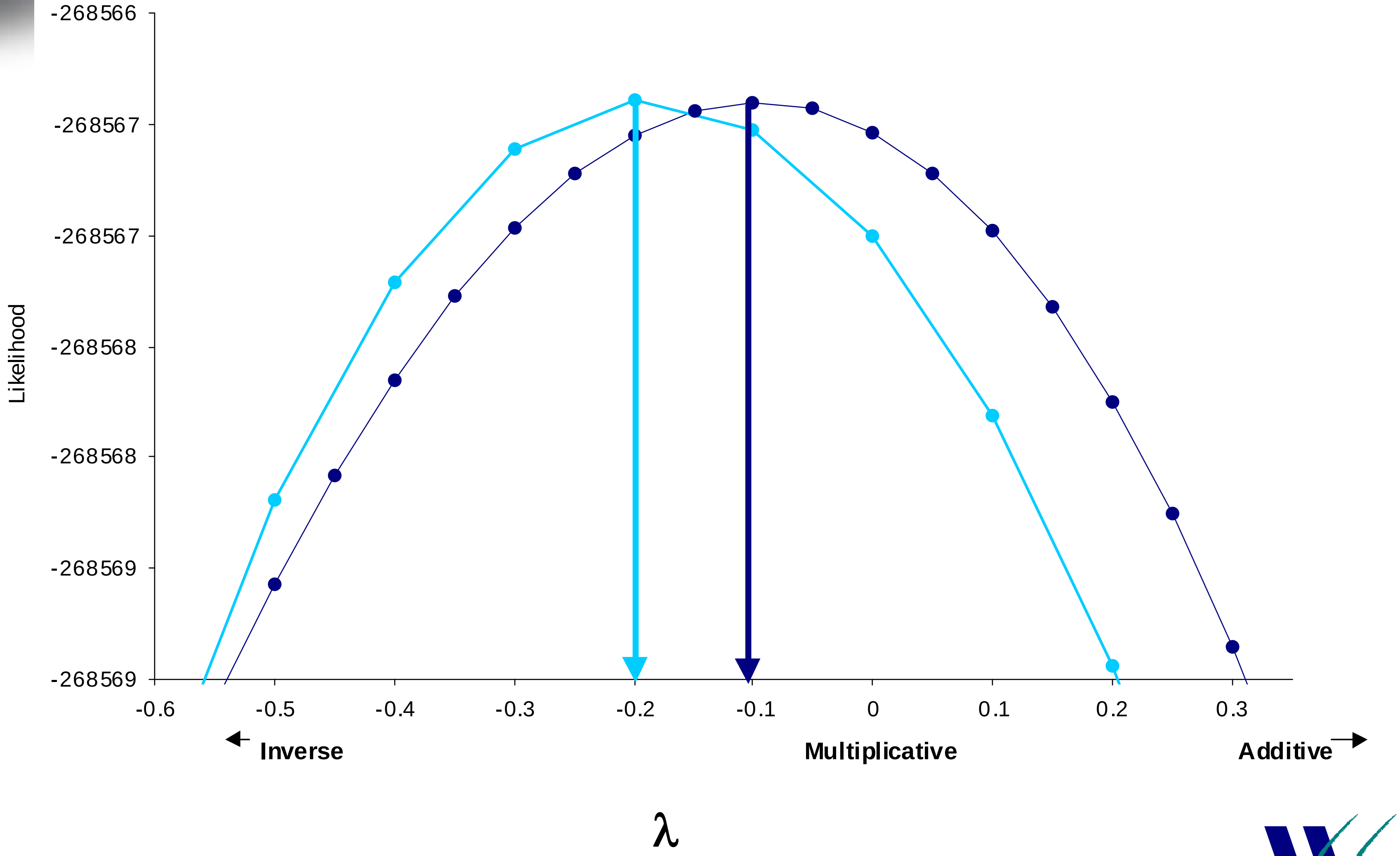
Box-Cox link function investigation

Motor third party property frequencies



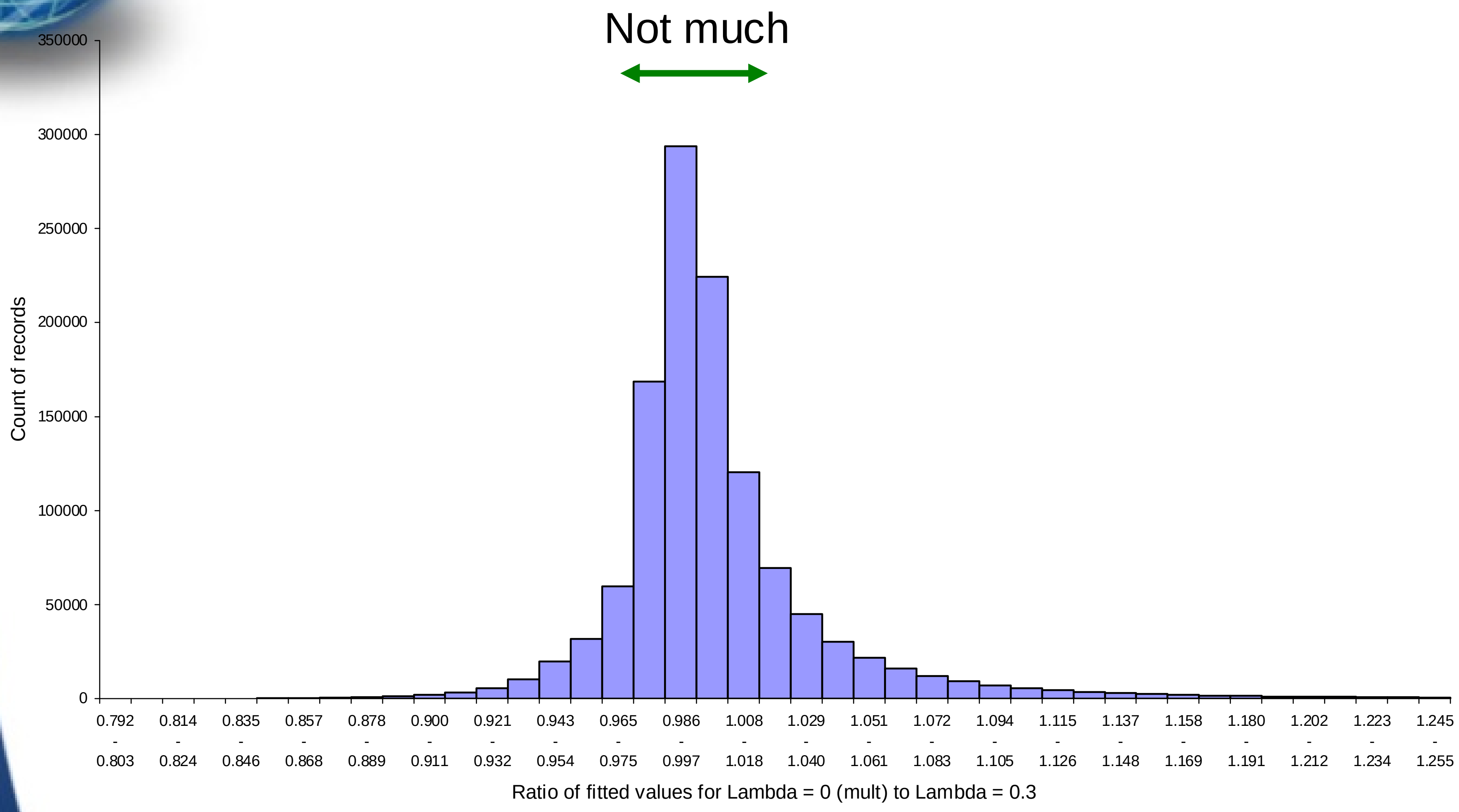
Box-Cox link function investigation

Motor third party property average amounts



Box-Cox link function investigation

Comparing fitted values of different link functions



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Aliasing and "near aliasing"

- Aliasing
  - the removal of unwanted redundant parameters
- Intrinsic aliasing
  - occurs by the design of the model
- Extrinsic aliasing
  - occurs "accidentally" as a result of the data

Example

- Suppose we wanted a model of the form:

$$\underline{\mu} = \alpha + \beta_1 \text{ if } \underline{\text{age}} < 30$$

$$+ \beta_2 \text{ if } \underline{\text{age}} \text{ 30 - 40}$$

$$+ \beta_3 \text{ if } \underline{\text{age}} > 40$$

$$+ \gamma_1 \text{ if } \underline{\text{sex}} \text{ male}$$

$$+ \gamma_2 \text{ if } \underline{\text{sex}} \text{ female}$$

Form of $X.\underline{\beta}$ in this case

$$
\begin{array}{c}
\begin{array}{c|cccccc}
 & & \multicolumn{3}{c}{\text{Age}} & \multicolumn{2}{c}{\text{Sex}} \\
 & & <30 & 30\text{-}40 & >40 & \text{M} & \text{F} \\
\hline
1 & 1 & 0 & 1 & 0 & 1 & 0 \\
2 & 1 & 1 & 0 & 0 & 1 & 0 \\
3 & 1 & 1 & 0 & 0 & 0 & 1 \\
4 & 1 & 0 & 0 & 1 & 1 & 0 \\
5 & 1 & 0 & 1 & 0 & 0 & 1 \\
 & \multicolumn{6}{c}{\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots} \\
 & \multicolumn{6}{c}{\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots}
\end{array}
\end{array}
\; . \;
\begin{pmatrix} \alpha \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \gamma_1 \\ \gamma_2 \end{pmatrix}
$$

Example

- Suppose we wanted a model of the form:

$\underline{\mu} = \alpha + \beta_1$ if $\underline{\text{age}} < 30$

~~$+ \beta_2$ if $\underline{\text{age}}$ 30 - 40~~

"Base levels" $+ \beta_3$ if $\underline{\text{age}} > 40$

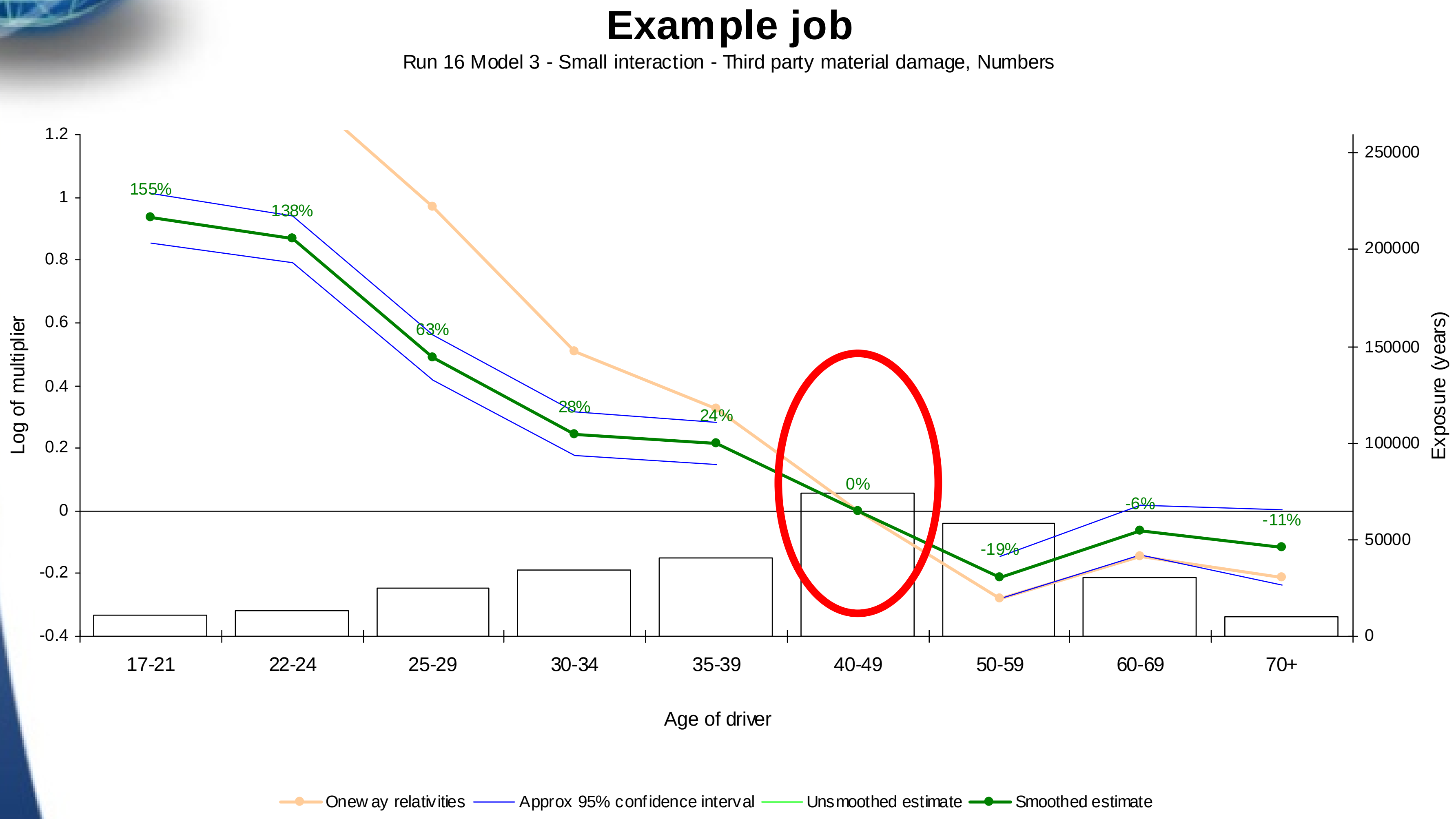
~~$+ \gamma_1$ if $\underline{\text{sex}}$ male~~

$+ \gamma_2$ if $\underline{\text{sex}}$ female

X.$\underline{\beta}$ having adjusted for base levels

$$
\begin{array}{c}
\\
\\
1 \\
2 \\
3 \\
4 \\
5 \\
\\
\\
\end{array}
\left(
\begin{array}{ccccccc}
 & & & \text{Age} & & & \text{Sex} \\
 & <30 & & 30\text{-}40 & >40 & \text{M} & \text{F} \\
1 & 0 & & \cancel{1} & 0 & \cancel{1} & 0 \\
1 & 1 & & \cancel{0} & 0 & \cancel{1} & 0 \\
1 & 1 & & \cancel{0} & 0 & \cancel{0} & 1 \\
1 & 0 & & \cancel{0} & 1 & \cancel{1} & 0 \\
1 & 0 & & \cancel{1} & 0 & \cancel{0} & 1 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{array}
\right)
.
\left(
\begin{array}{c}
\alpha \\
\beta_1 \\
\cancel{\beta_2} \\
\beta_3 \\
\cancel{\gamma_1} \\
\gamma_2
\end{array}
\right)
$$

Intrinsic aliasing



Extrinsic aliasing

- If a perfect correlation exists, one factor can alias levels of another
- Eg if doors declared first:

Exposure: # Doors → Colour ↓	2	3	Selected base ~~4~~	5	Unknown
Selected base ~~Red~~	13,234	12,343	13,432	13,432	0
Green	4,543	4,543	13,243	2,345	0
Blue	6,544	5,443	15,654	4,565	0
Black	4,643	1,235	14,565	4,545	0
Further aliasing ~~Unknown~~	0	0	0	0	3,242

- This is the only reason the order of declaration can matter (fitted values are unaffected)

Extrinsic aliasing

"Near aliasing"

- If two factors are almost perfectly, but not quite aliased, convergence problems can result and/or results can become hard to interpret

Exposure: # Doors →	2	3	Selected base ~~4~~	5	Unknown
Colour ↓					
Selected base ~~Red~~	13,234	12,343	13,432	13,432	0
Green	4,543	4,543	13,243	2,345	0
Blue	6,544	5,443	15,654	4,565	0
Black	4,643	1,235	14,565	4,545	2
Unknown	0	0	0	0	3,242

- Eg if the 2 black, unknown doors policies had no claims, GLM would try to estimate a very large negative number for unknown doors, and a very large positive number for unknown colour

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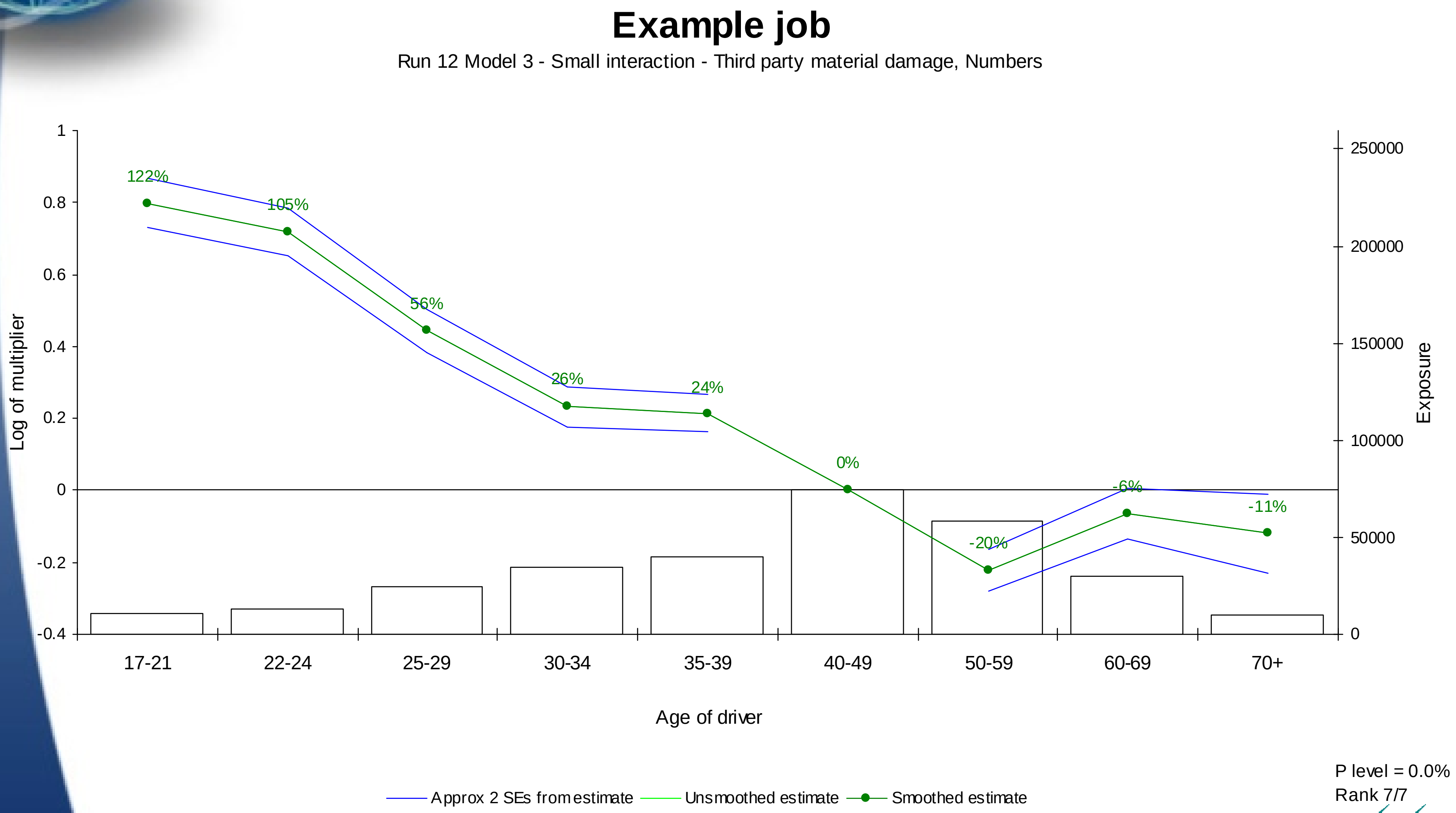
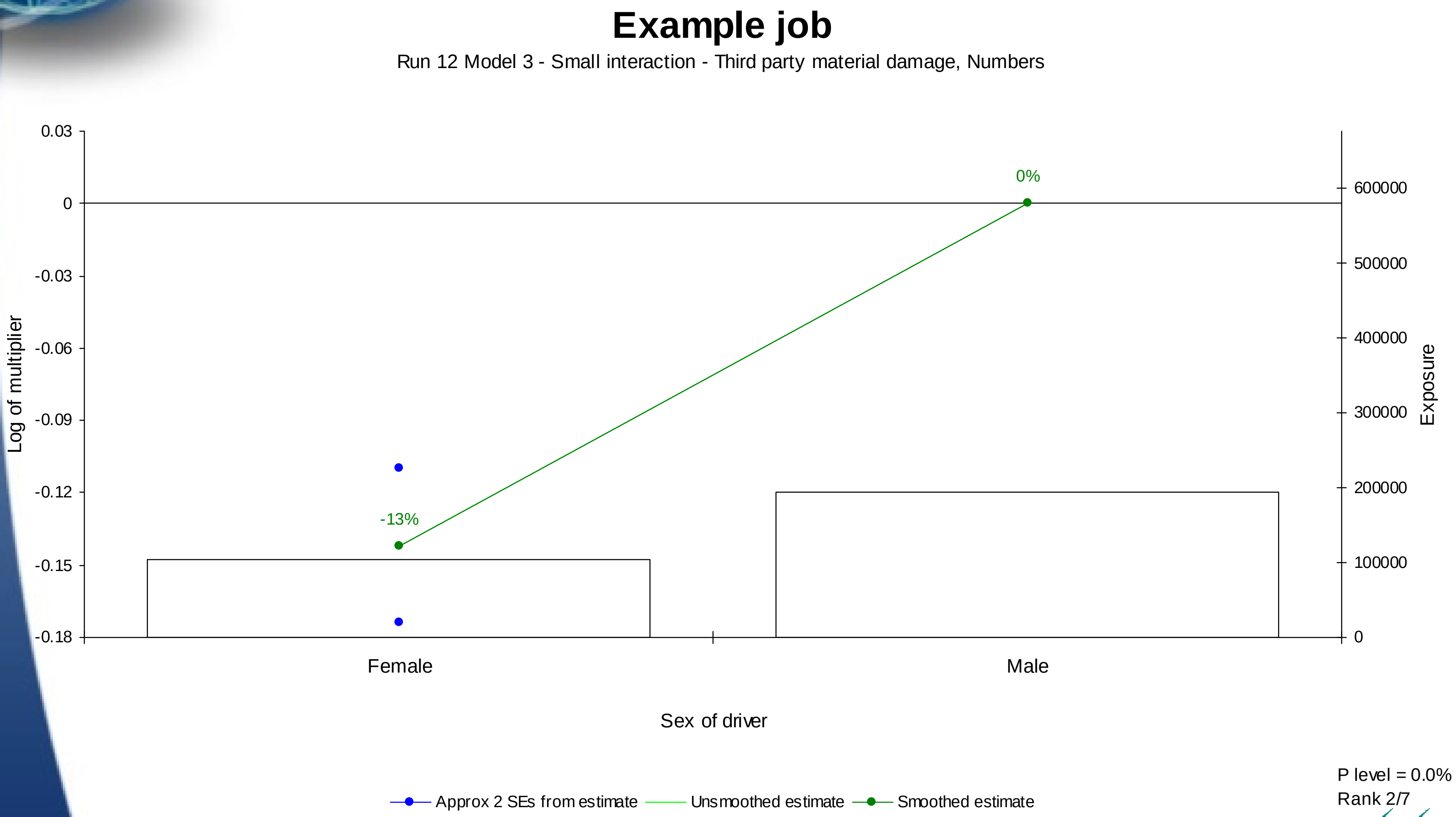
No interaction - age



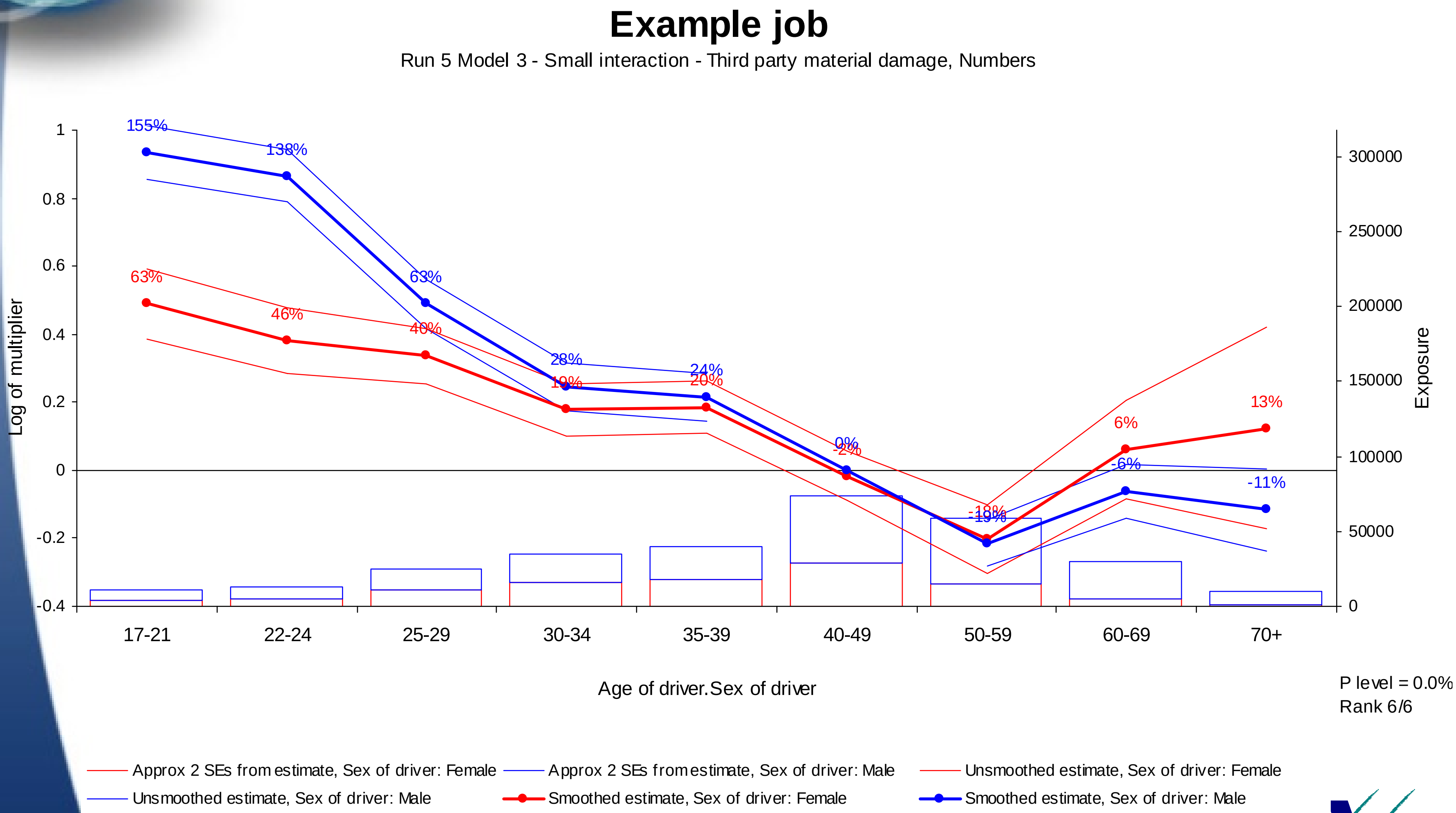
P level = 0.0%
Rank 7/7

No interaction - sex



P level = 0.0%
Rank 2/7

Age - sex interaction



Interactions

No interaction

		A	B	C	D
		x	-	x	x
W	x				
X	-				
Y	x				
Z	x				

Full interaction

	A	B	C	D
W	x	x	x	x
X	x	-	x	x
Y	x	x	x	x
Z	x	x	x	x

Marginal interaction

		A	B	C	D
		x	-	x	x
W	x	x	-	x	x
X	-	-	-	-	-
Y	x	x	-	x	x
Z	x	x	-	x	x

Interactions

Full interaction

	Factor 1:	A	B	C	D
Factor 2:	W	0.72	0.80	0.88	0.96
	X	0.90	1.00	1.10	1.20
	Y	0.97	1.20	1.45	1.66
	Z	1.26	1.40	1.85	2.10

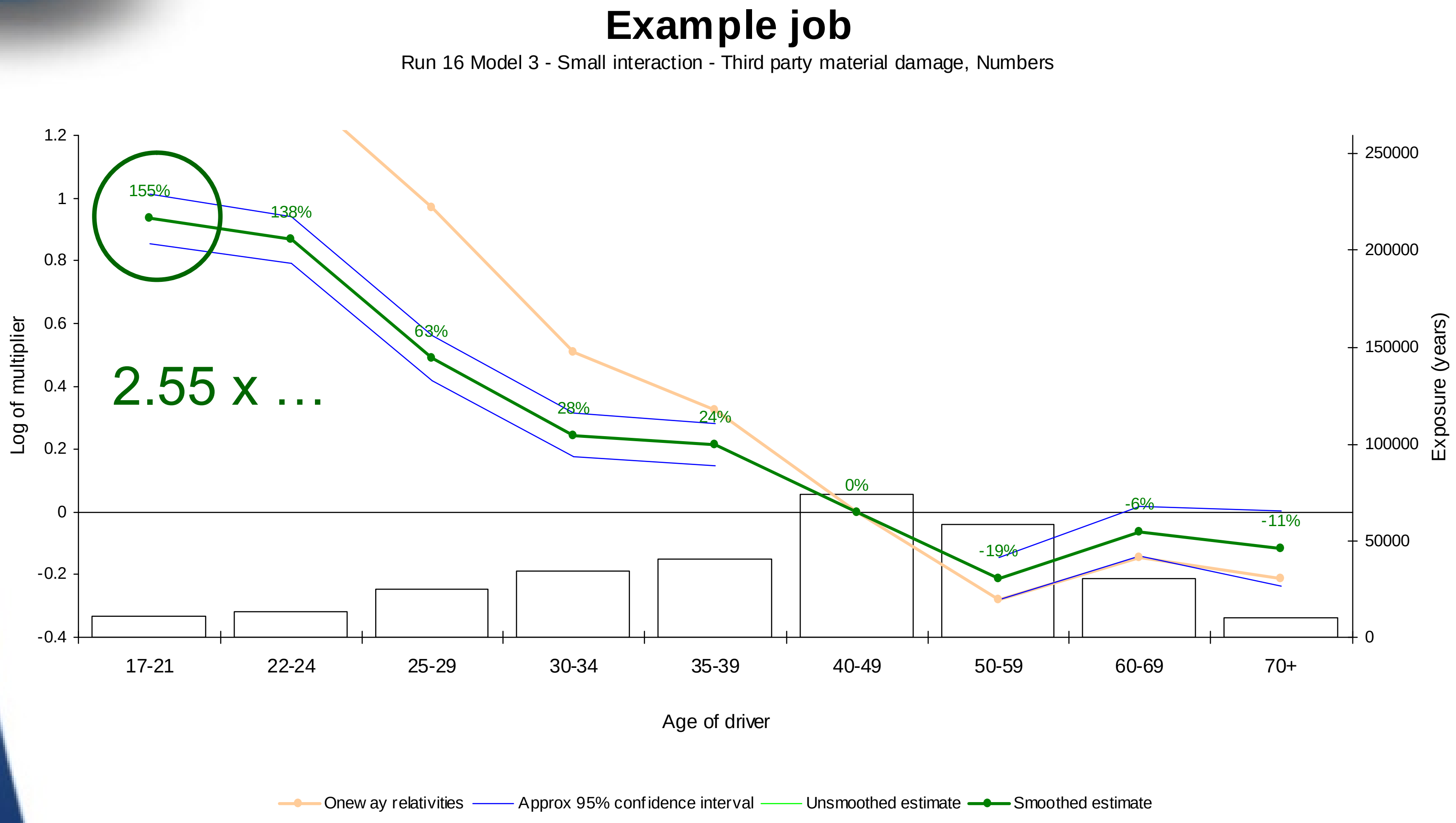
Marginal interaction

	Factor 1:		A	B	C	D
			0.90	-	1.10	1.20
Factor 2:	W	0.80	1	-	1	1
	X	-	-	-	-	-
	Y	1.20	0.9	-	1.1	1.15
	Z	1.40	1	-	1.2	1.25

Consider D/Z: 2.10 = 1.20 * 1.40 * 1.25

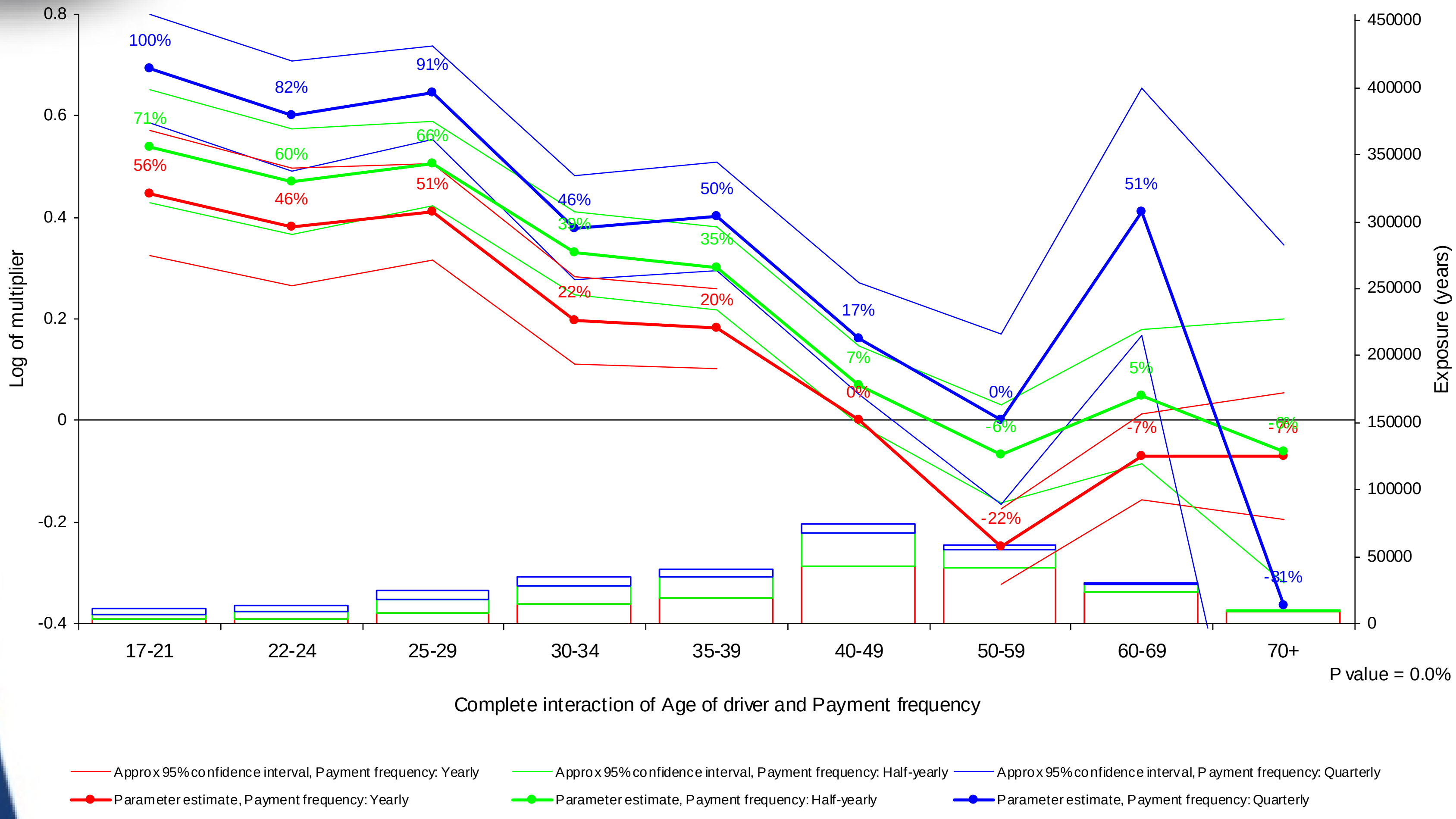
Age - sex interaction

Marginal interaction: Age effect

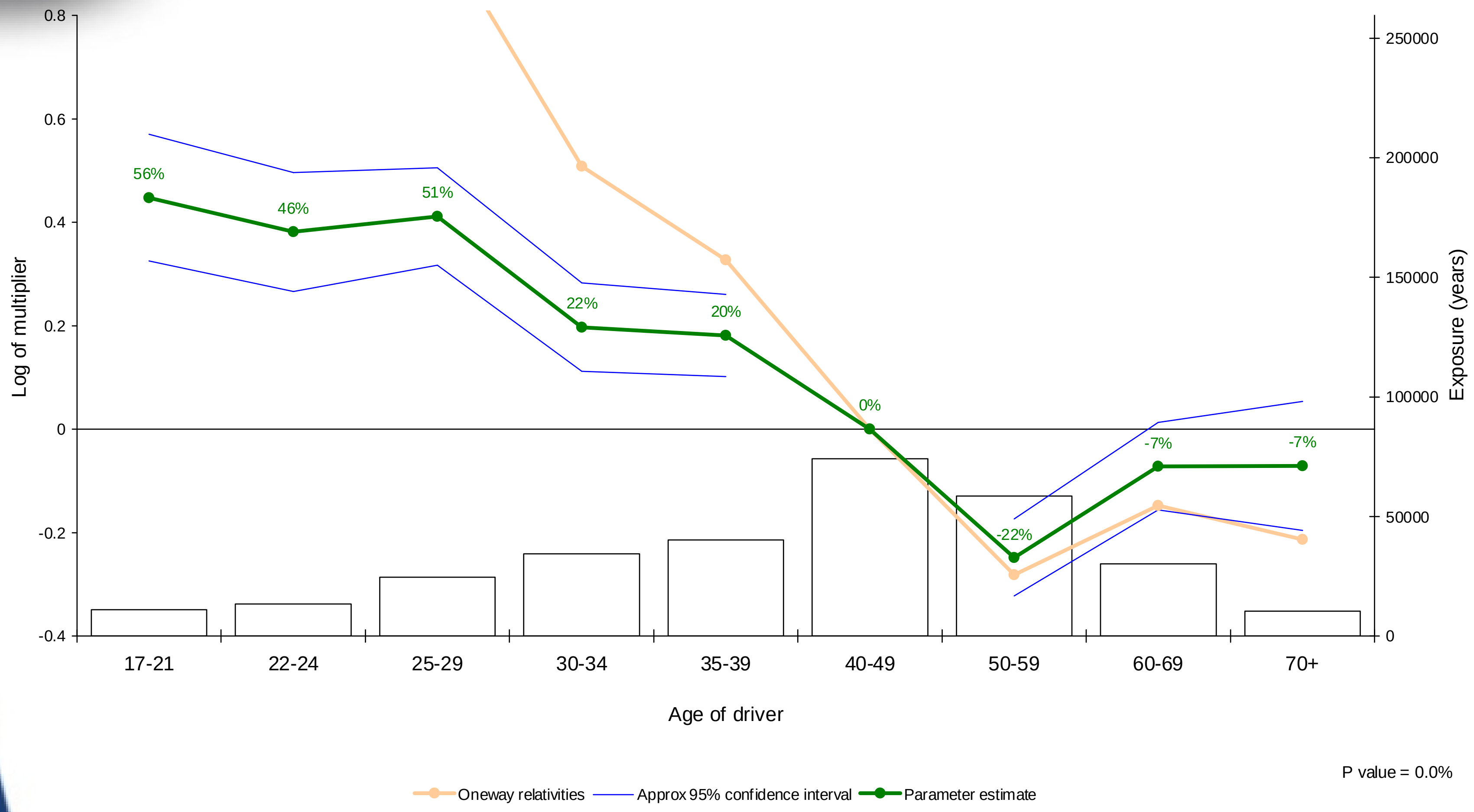


Marginal interaction: Age.Sex (ie additional female multipliers)

An example of no interaction

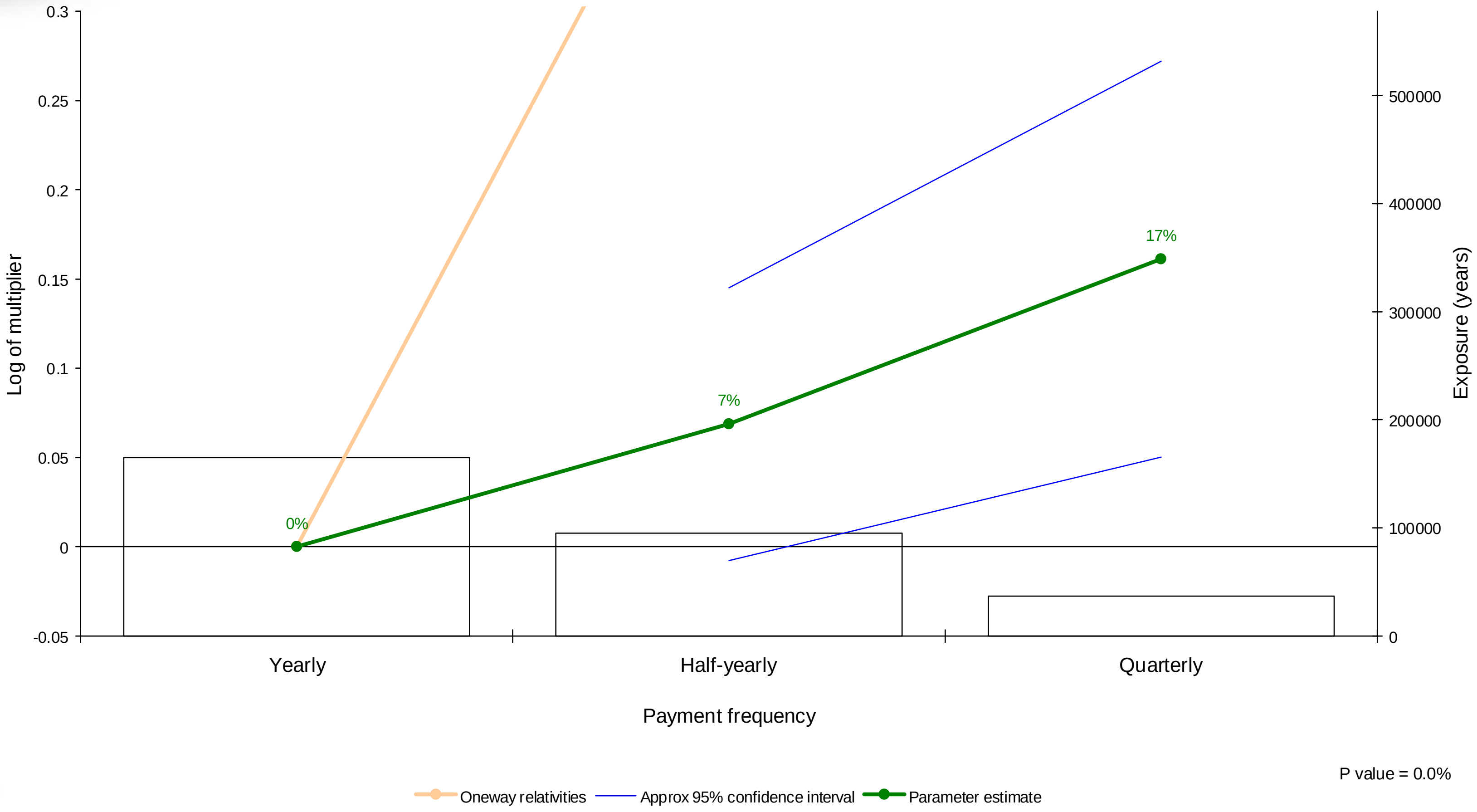


An example of no interaction

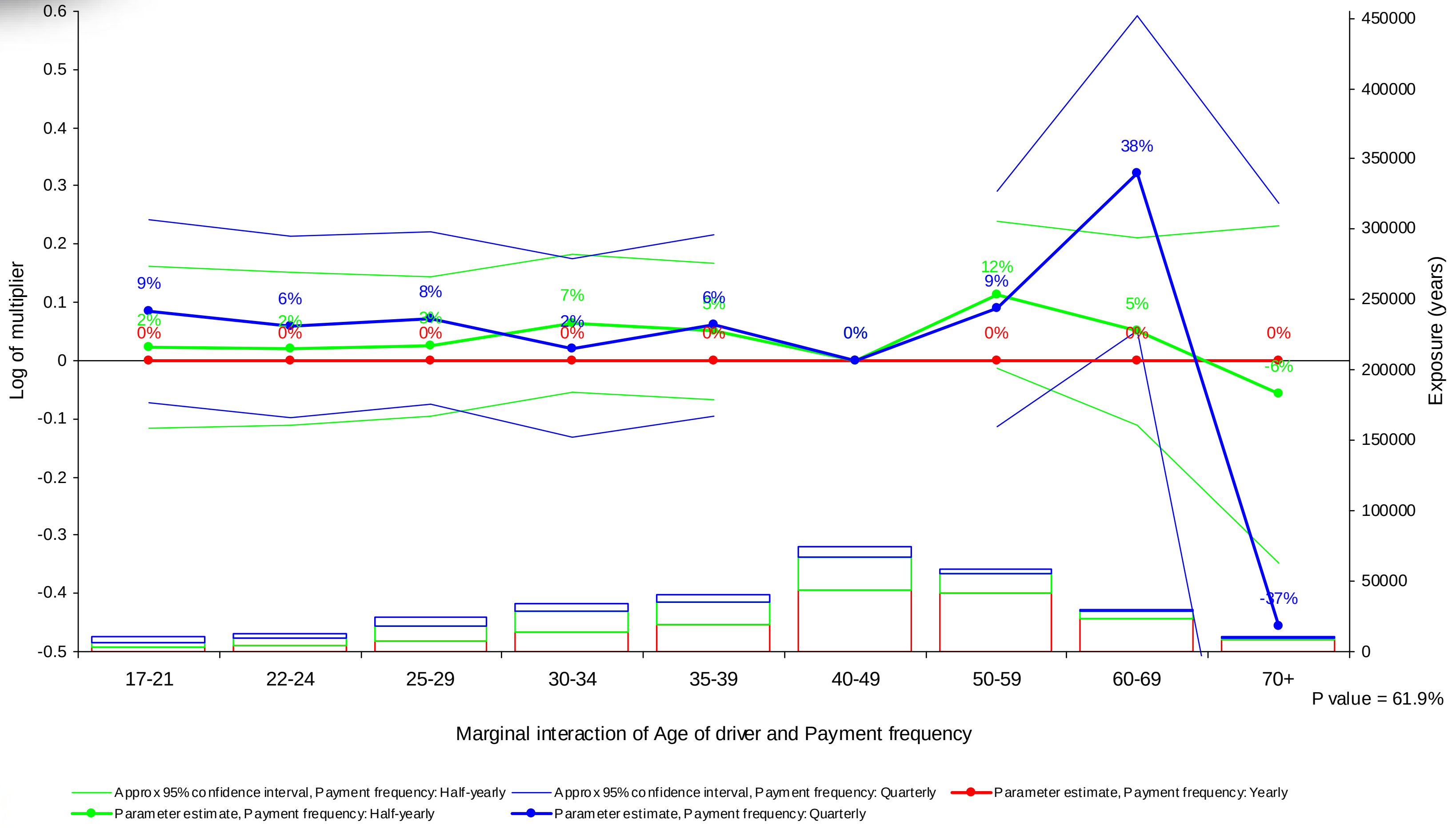


P value = 0.0%

An example of no interaction



An example of no interaction



Why marginal interactions can be hard to interpret

Exposure (000's)

	A	B	C	D
A	980	1,310	1,420	870
B	1,300	1,840	1,340	**10**
C	2,620	3,580	2,110	2,430
D	4,800	4,760	2,030	1,680

With low frequency

Marginal interaction parameter estimates

		A	B	C	D
		0.07	-	-0.05	**-99.00**
A	0.05	0.04	-	-0.01	**99.10**
B	-	-	-	-	-
C	-0.10	-0.20	-	0.10	**99.20**
D	-0.20	0.20	-	0.20	**98.10**

Interactions

Group	1	2	3	4	5	6	7	8	9	10	11	12	13
Factor	0.54	0.65	0.73	0.85	0.92	0.96	1.00	1.08	1.19	1.26	1.36	1.43	1.56

Age	Factor
17	2.52
18	2.05
19	1.97
20	1.85
21-23	1.75
24-26	1.54
27-30	1.42
31-35	1.20
36-40	1.00
41-45	0.93
46-50	0.84
51-60	0.76
60+	0.78

Interactions

Group >	1	2	3	4	5	6	7	8	9	10	11	12	13
Age v													
17	1.36	1.64	1.79	2.09	2.27	2.42	2.56	2.65	3.27	3.71	4.08	4.36	4.84
18	1.12	1.31	1.47	1.76	1.84	2.00	2.11	2.19	2.43	2.97	3.29	3.55	3.90
19	1.08	1.30	1.46	1.63	1.82	1.91	2.02	2.11	2.53	2.88	3.30	3.35	3.63
20	0.98	1.18	1.36	1.54	1.68	1.79	1.83	1.97	2.19	2.66	3.02	3.20	3.38
21-23	0.96	1.13	1.24	1.51	1.65	1.64	1.80	1.85	2.04	2.26	2.55	2.53	2.89
24-26	0.82	0.99	1.10	1.31	1.43	1.52	1.51	1.64	1.81	1.93	2.13	2.22	2.47
27-30	0.78	0.90	1.07	1.19	1.32	1.39	1.41	1.51	1.65	1.77	1.91	2.01	2.24
31-35	0.63	0.78	0.86	0.99	1.09	1.17	1.22	1.32	1.42	1.54	1.66	1.71	1.88
36-40	0.55	0.64	0.71	0.85	0.91	0.93	0.99	1.07	1.18	1.29	1.40	1.41	1.53
41-45	0.51	0.61	0.66	0.79	0.88	0.88	0.94	0.99	1.09	1.15	1.29	1.31	1.42
46-50	0.46	0.55	0.61	0.70	0.76	0.81	0.84	0.92	1.02	1.07	1.12	1.18	1.31
51-60	0.40	0.49	0.56	0.64	0.68	0.71	0.78	0.82	0.90	0.99	1.02	1.12	1.20
60+	0.43	0.52	0.55	0.67	0.72	0.73	0.78	0.83	0.93	0.98	1.04	1.11	1.25

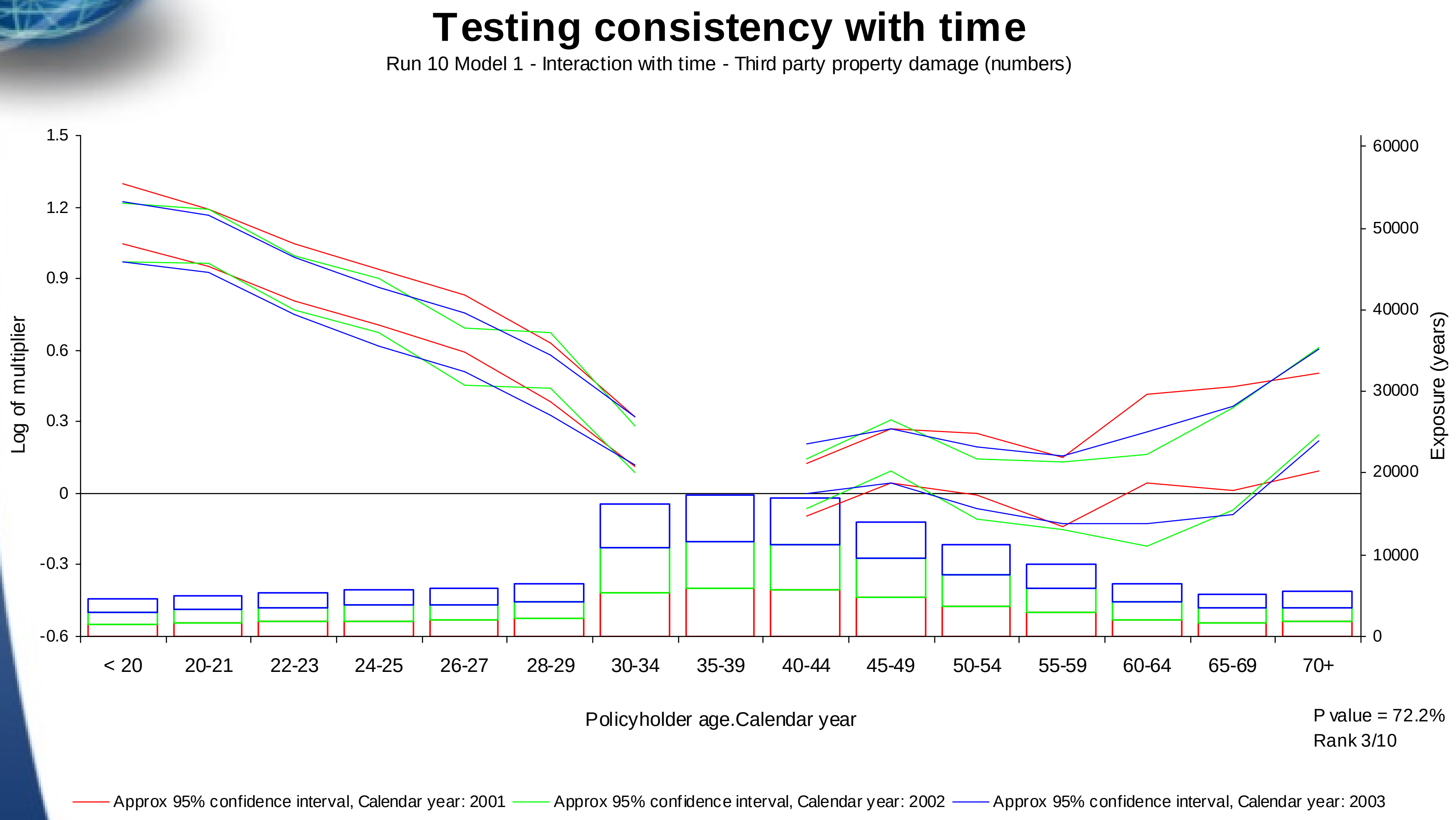
Interactions

Group	1	2	3	4	5	6	7	8	9	10	11	12	13
Factor	0.54	0.65	0.73	0.85	0.92	0.96	1.00	1.08	1.19	1.26	1.36	1.43	1.56

Age	Factor
17	2.52
18	2.05
19	1.97
20	1.85
21-23	1.75
24-26	1.54
27-30	1.42
31-35	1.20
36-40	1.00
41-45	0.93
46-50	0.84
51-60	0.76
60+	0.78

1.00	1.17
1.00	1.00

Interactions - testing consistency with time



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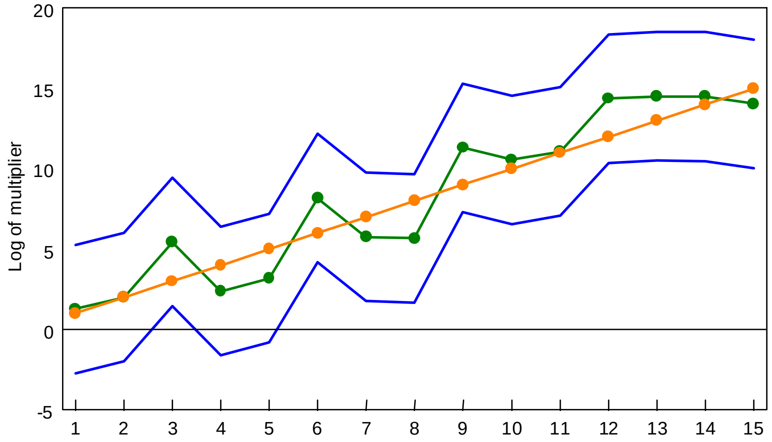
Smoothing



- Models can often be improved by smoothing raw statistical parameter estimates where factors have a natural order
- Best done at frequency / severity level
- Artificial constraints and commercial smoothing come later

Combining claim elements - I

BI Freq x Amt = Cost 1

PD Freq x Amt = Cost 2

MED Freq x Amt = Cost 3

COL Freq x Amt = Cost 4

OTC Freq x Amt = Cost 5

- Multiply factors for frequencies and amounts
- Calculate risk premium as sum of claim elements

Combining claim elements - II

BI	Freq	x	Amt	= Cost 1
PD	Freq	x	Amt	= Cost 2
MED	Freq	x	Amt	= Cost 3
COL	Freq	x	Amt	= Cost 4
OTC	Freq	x	Amt	= Cost 5

- Consider current exposure
- Calculate expected frequency and amount for each claim type for each record
- Combine to give expected total cost of claims for each record
- Fit model to this expected value

Calculation of risk premium

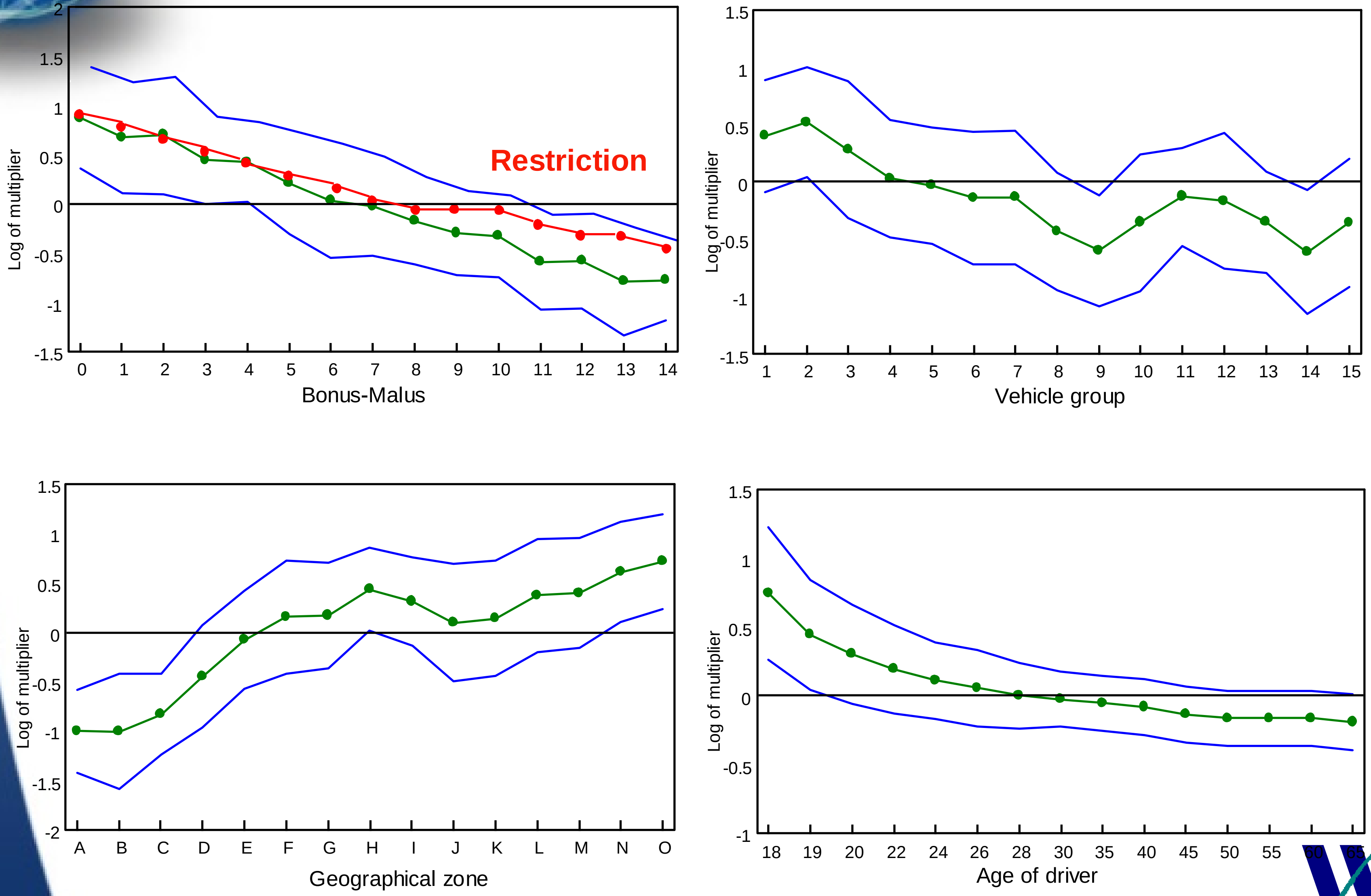
		TPPD Numbers	TPPD Amounts	TPBI Numbers	TPBI Amounts
Intercept		32%	£1000	12%	£4860
Sex	Male	1.000	1.000	1.000	1.000
	Female	0.750	1.200	0.667	0.900
Area	Town	1.000	1.000	1.000	1.000
	Country	1.250	0.700	0.750	0.833

Policy	**Sex**	**Area**	**WWNUM1**	**WWAMT1**	**WWNUM2**	**WWAMT2**	**WWCC1**	**WWCC2**	**WWRSKPRM**
…	…	…	…	…	…	…	…	…	…
82155654	M	T	32%	1000	12%	4860	320	583.20	903.20
82168746	F	T	24%	1200	8%	4374	288	349.92	637.92
82179481	M	C	40%	700	9%	4050	280	364.50	644.50
82186845	F	C	30%	840	6%	3645	252	218.70	470.70
…	…	…	…	…	…	…	…	…	…

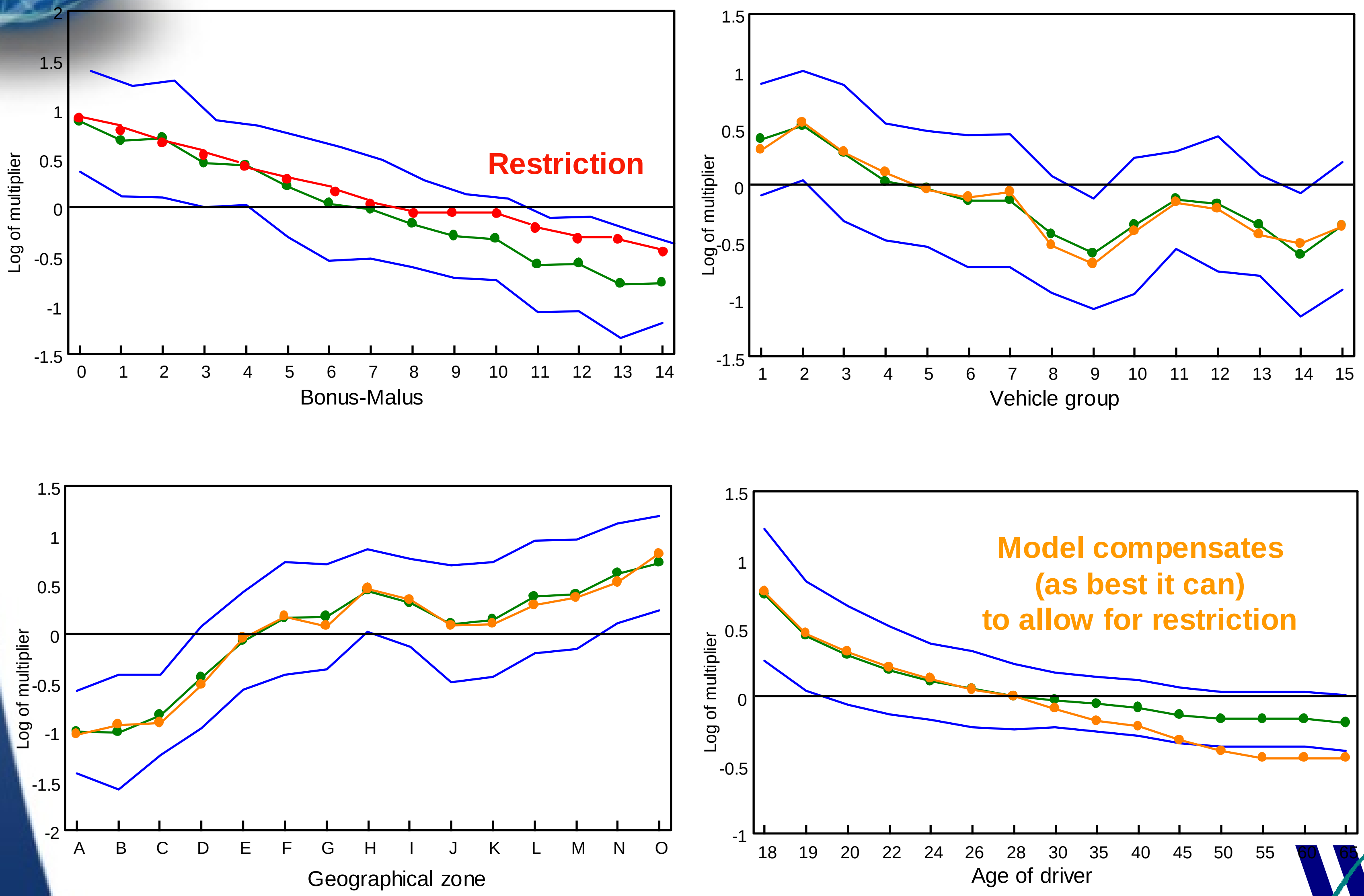
Risk premium standard errors

- Risk premium model standard errors are small owing to the smoothness of the expected value
- It is possible to approximate standard error of risk premium parameter estimates based on standard errors of parameter estimates in underlying models
- Care needed in interpretting such approximations since they do not reflect model error, eg deciding to exclude a marginal factor

Restricted models



Restricted models



Restriction

Model compensates
(as best it can)
to allow for restriction

Restricted models

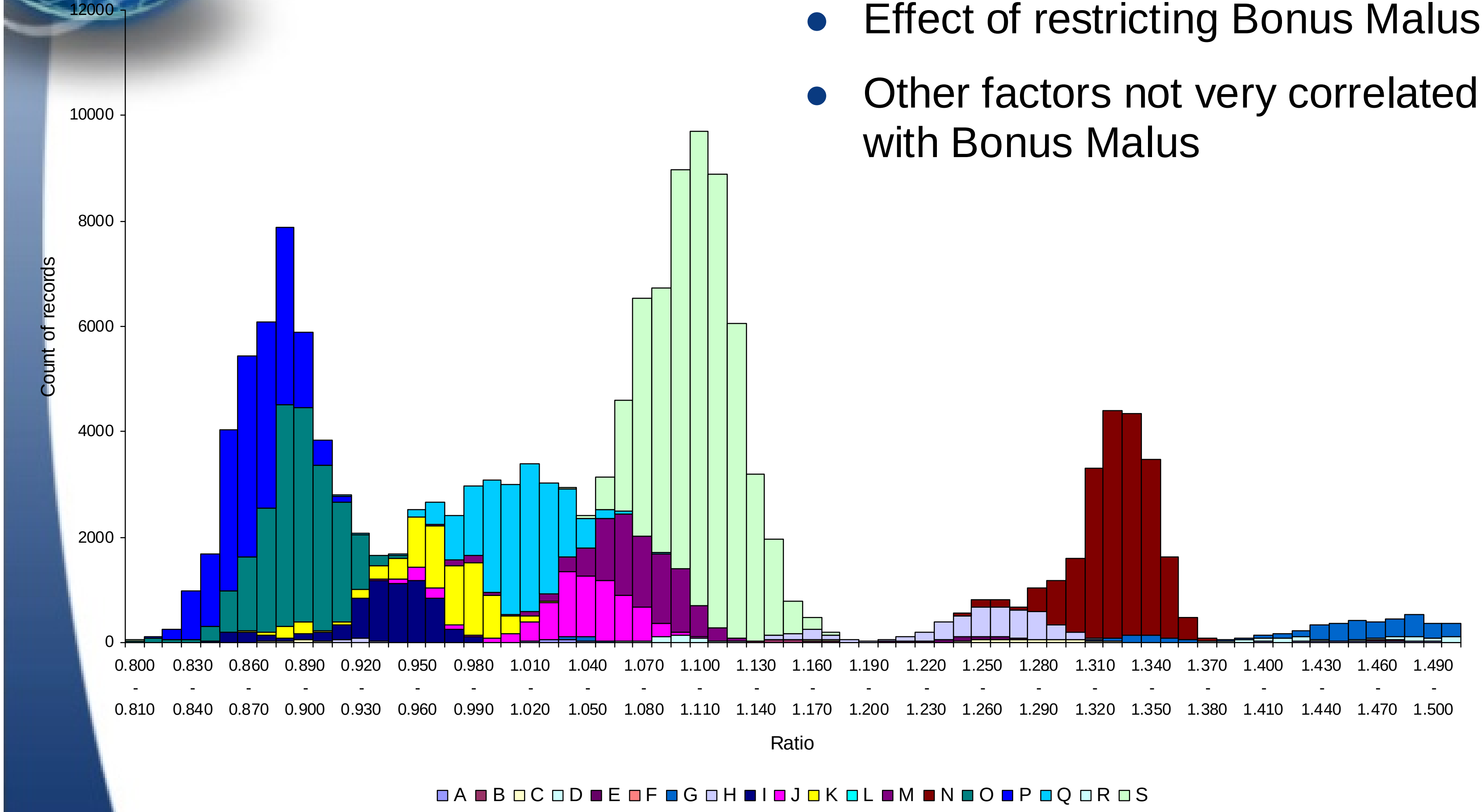
$$E[\underline{Y}] = \underline{\mu} = g^{-1}(\mathbf{X}.\underline{\beta} + \underline{\xi})$$

Offset

- Offset term used for known effects, eg exposure in a numbers model
- Can also be used to constrain model (eg claim free years / payment frequency / amount of cover)
- Other factors adjusted to compensate

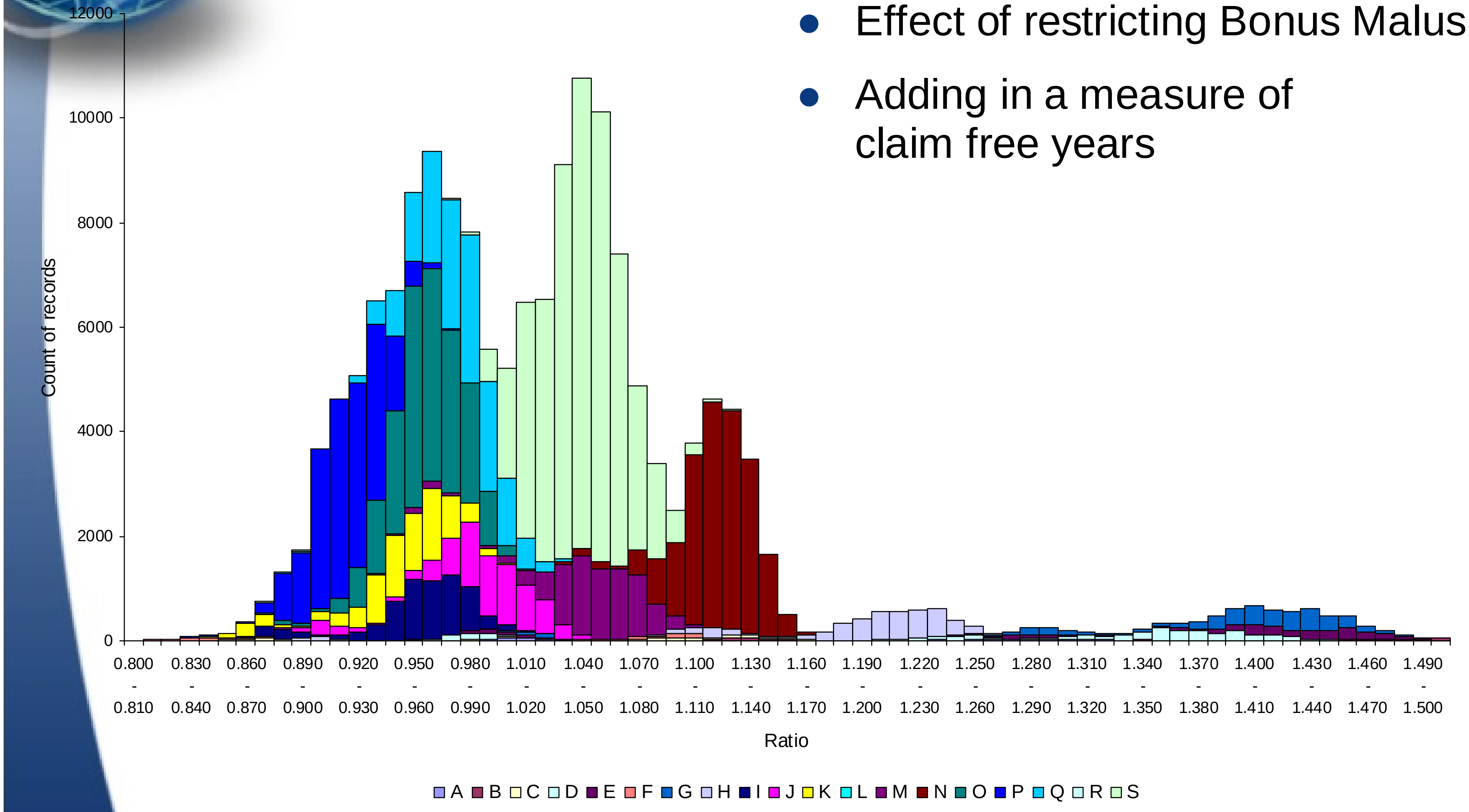
Testing the effectiveness of restrictions

- Effect of restricting Bonus Malus
- Other factors not very correlated with Bonus Malus



Testing the effectiveness of restrictions

- Effect of restricting Bonus Malus
- Adding in a measure of claim free years



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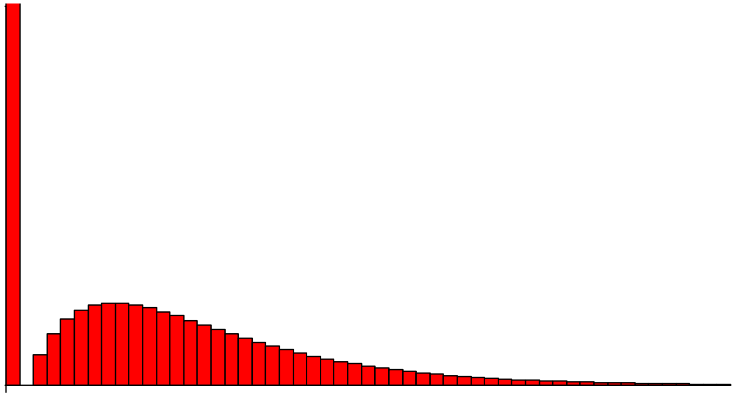
Tweedie distributions

- Incurred losses have a point mass at zero and then a continuous distribution
- Poisson and gamma not suited to this



- Tweedie distribution has point mass and parameters which can alter the shape to be like Poisson and gamma above zero

$$f_Y(y;\theta,\lambda,\alpha)=\sum_{n=1}^{\infty}\frac{\left\{(\lambda\omega)^{1-\alpha}\kappa_\alpha(-1/y)\right\}^n}{\Gamma(-n\alpha)n!\,y}.\exp\left\{\lambda\omega[\theta_0 y-\kappa_\alpha(\theta_0)]\right\}\quad \text{for } y>0$$

$$p(Y=0)=\exp\left\{-\lambda\omega\kappa_\alpha(\theta_0)\right\}$$

Tweedie distributions

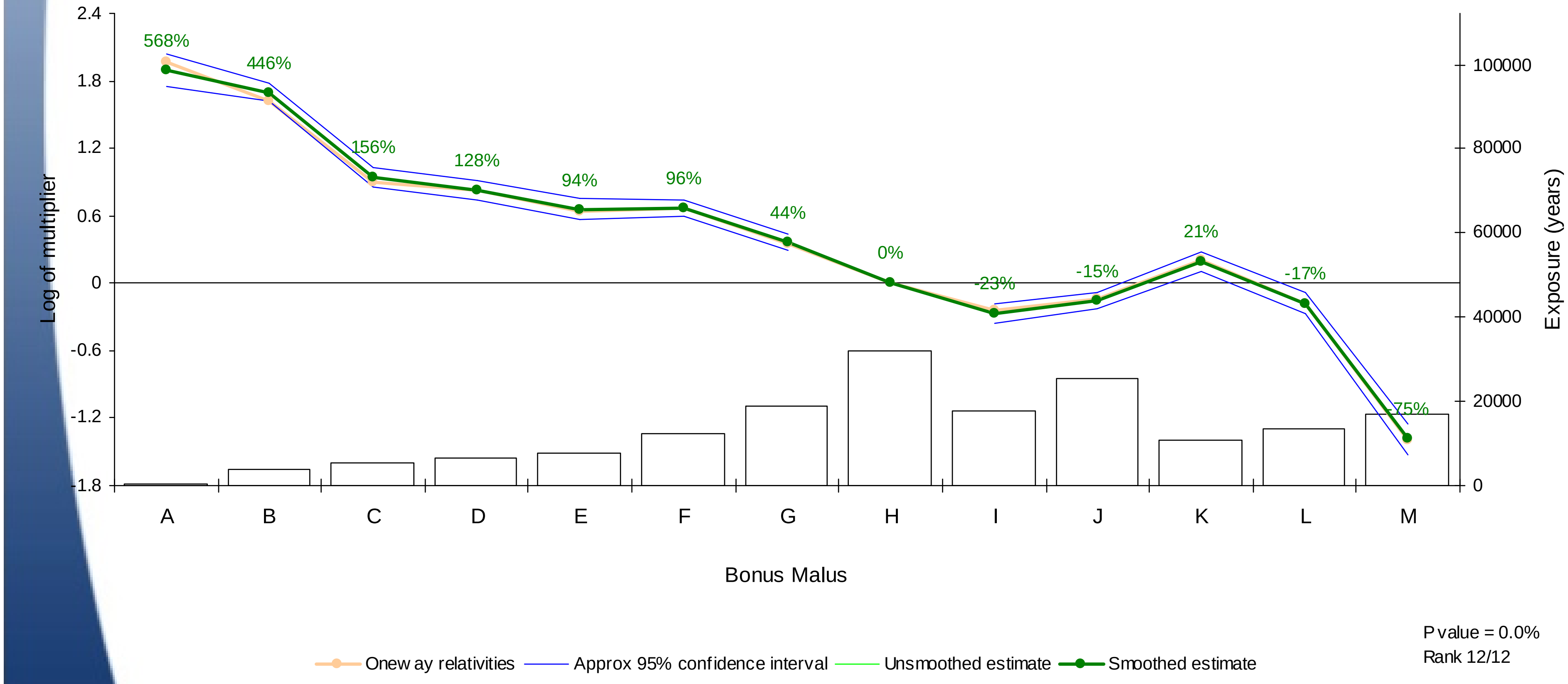
$$\text{Tweedie: } \phi = k,\ V(x) = x^p \Rightarrow \text{Var}[\underline{Y}] = k\underline{\mu}^p$$

- p=1 corresponds to Poisson, p=2 to gamma
- Defines a valid distribution for p<0, 1<p<2, p>2
- Can be considered as Poisson/gamma process for 1<p<2
- Need to estimate both k and p when fitting models - often estimate a where p = (2-a)/(1-a)
- Typical values of p for insurance incurred claims around, or just under, 1.5

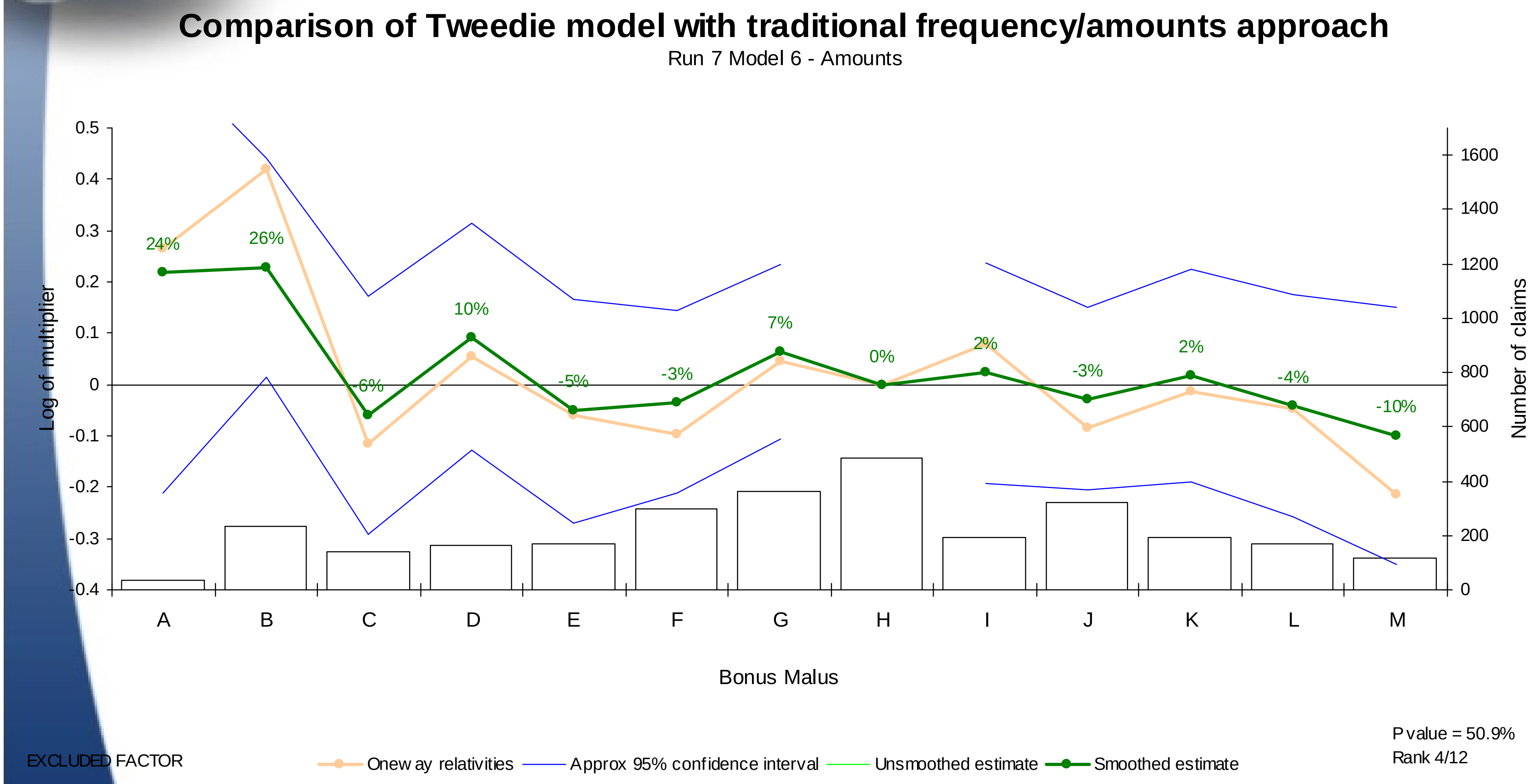
Example 1: frequency

Comparison of Tweedie model with traditional frequency/amounts approach

Run 7 Model 2 - Frequency



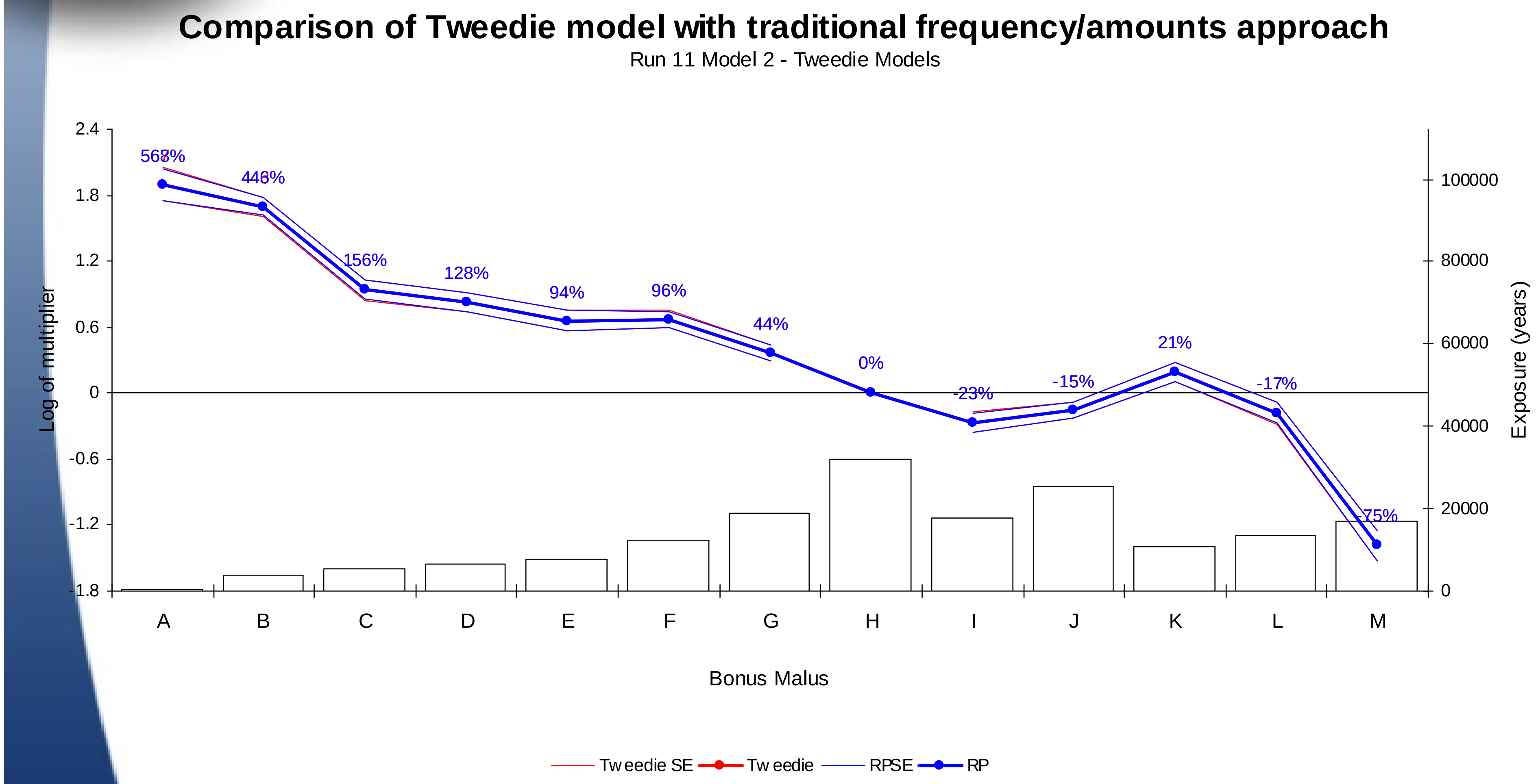
Example 1: amounts



Comparison of Tweedie model with traditional frequency/amounts approach

Run 7 Model 6 - Amounts

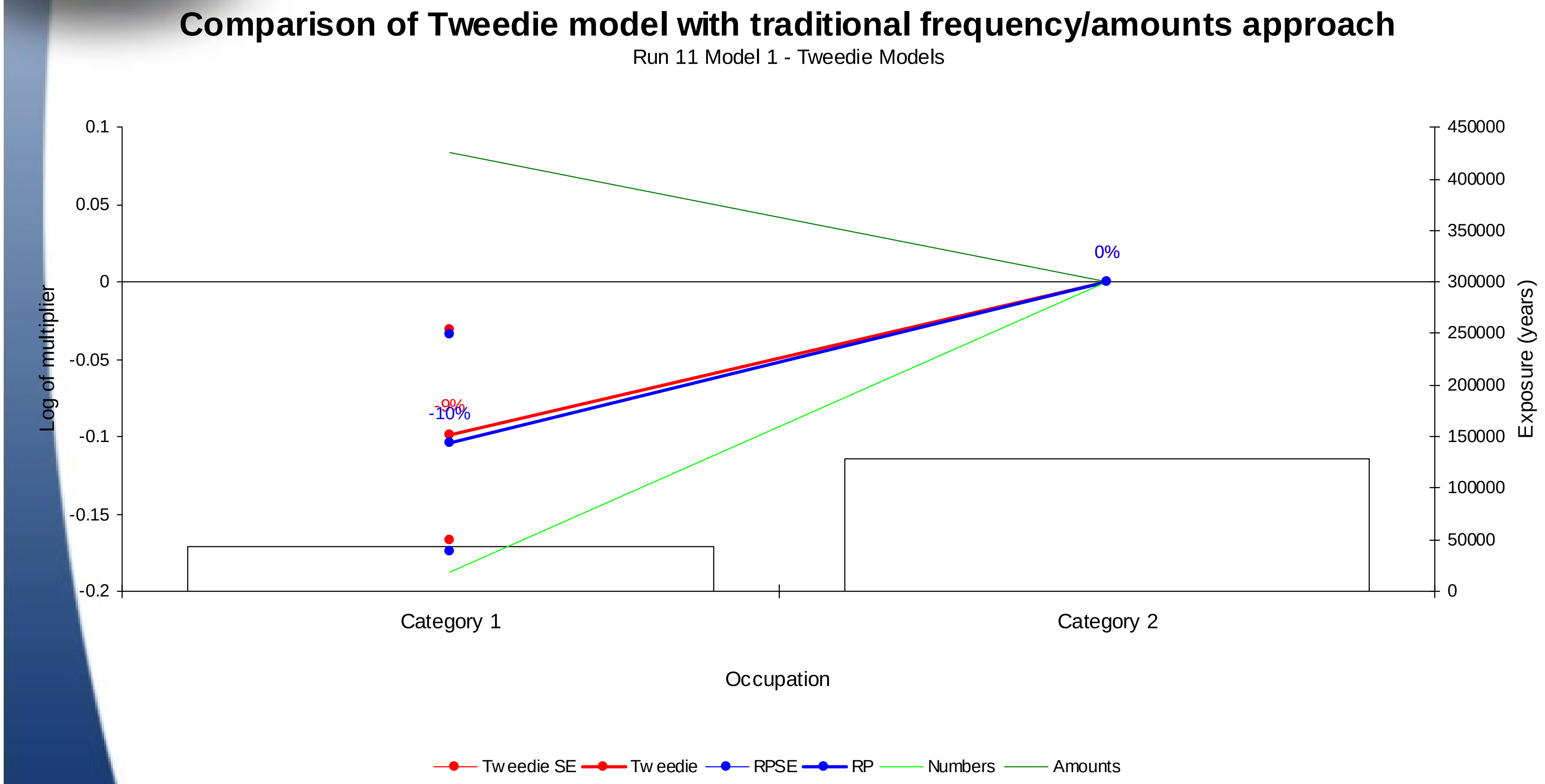
Example 1: traditional RP vs Tweedie



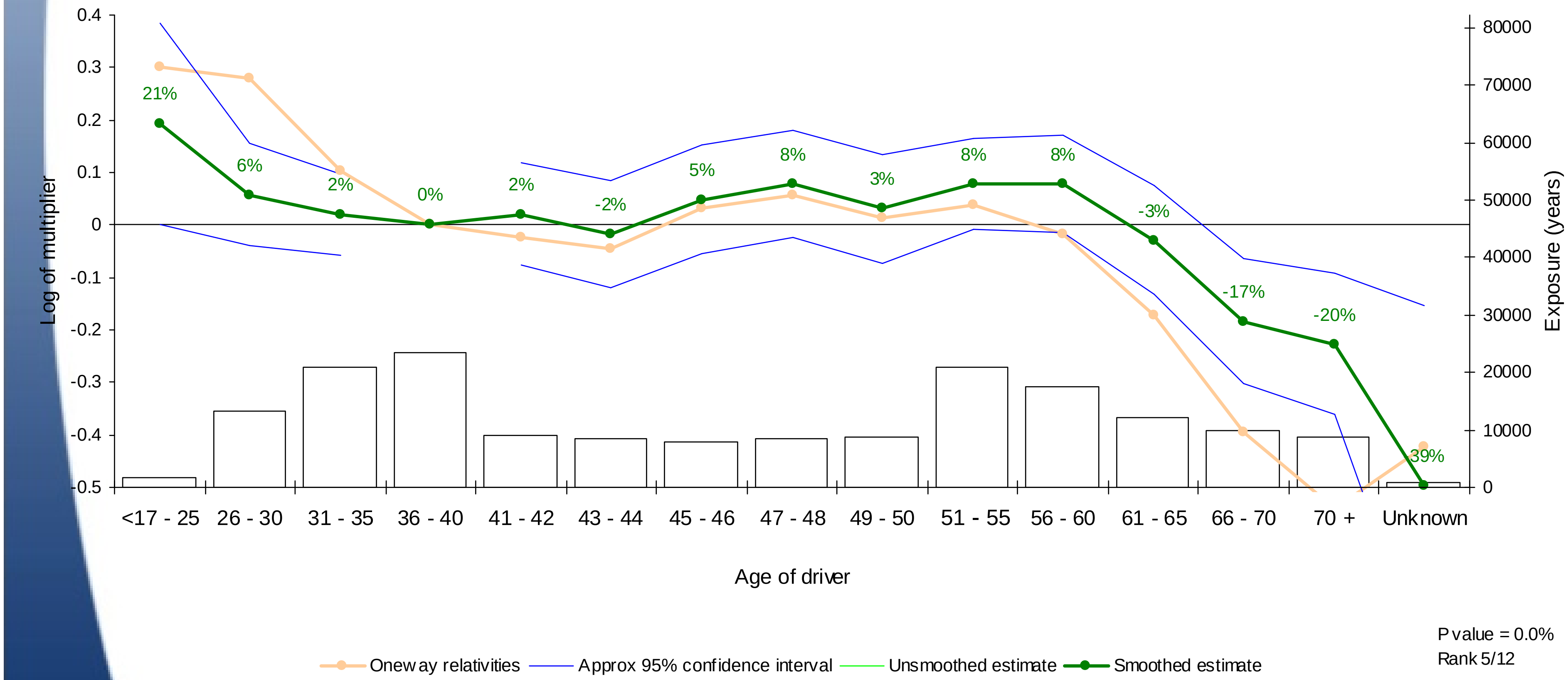
Comparison of Tweedie model with traditional frequency/amounts approach

Run 11 Model 2 - Tweedie Models

Example 2: traditional RP vs Tweedie



Example 3: frequency



Comparison of Tweedie model with traditional frequency/amounts approach

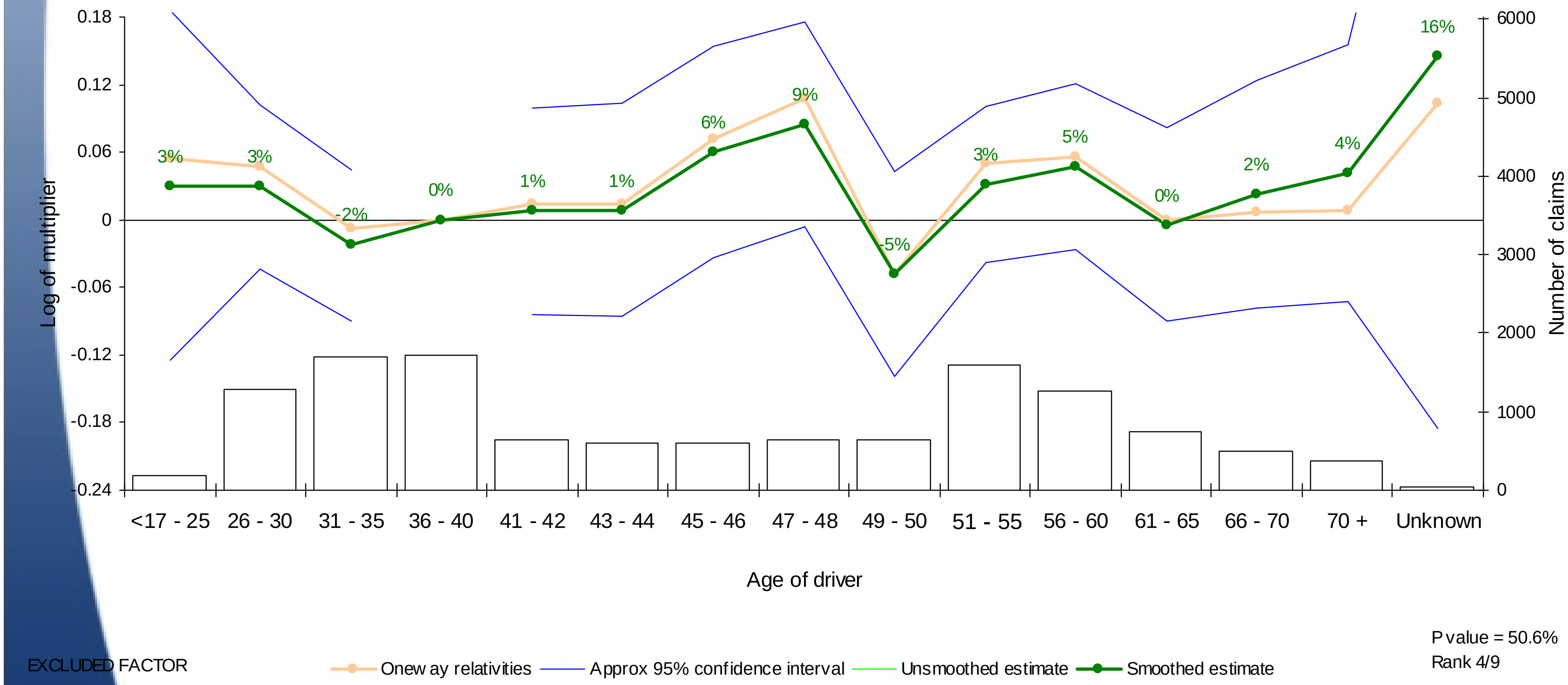
Run 7 Model 1 - Frequency

P value = 0.0%
Rank 5/12

Example 3: amounts

Comparison of Tweedie model with traditional frequency/amounts approach

Run 7 Model 5 - Amounts

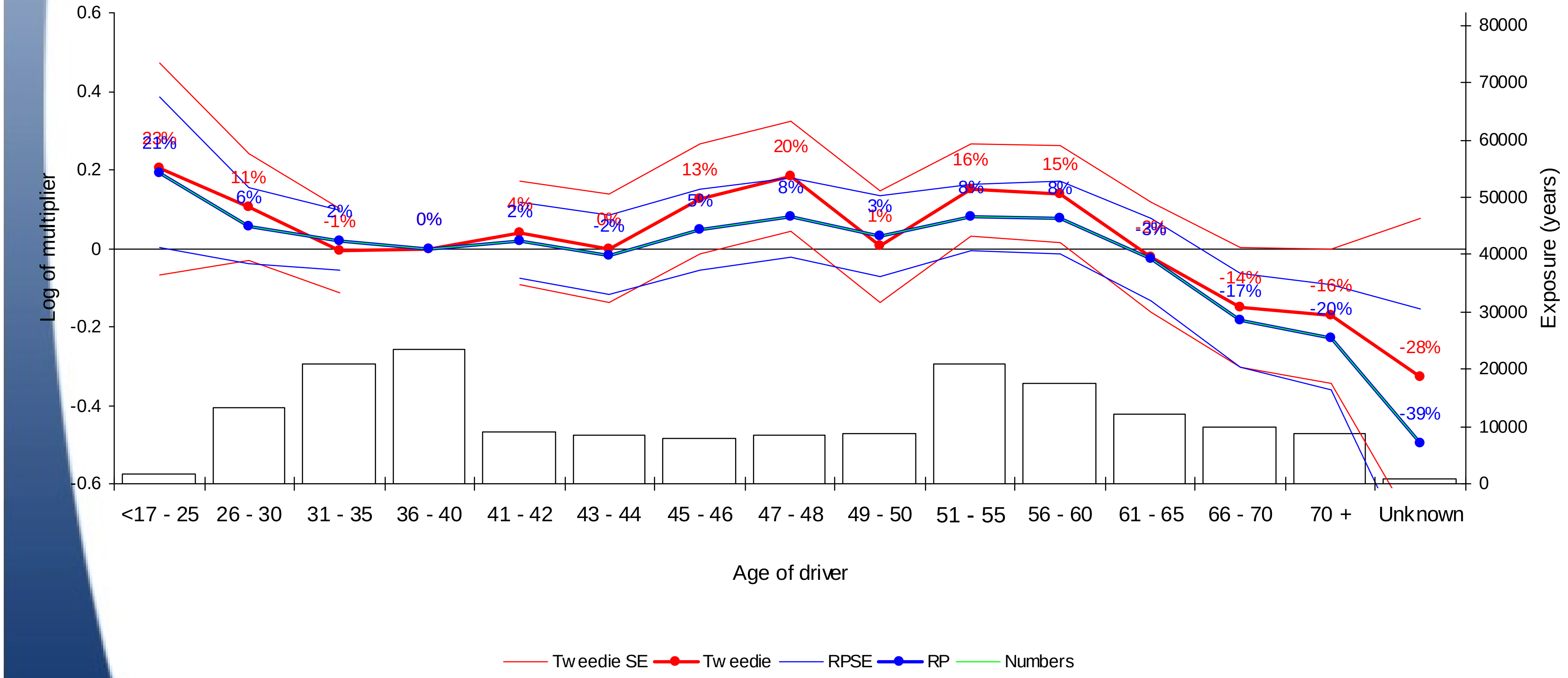


P value = 50.6%
Rank 4/9

Example 3: traditional RP vs Tweedie

Comparison of Tweedie model with traditional frequency/amounts approach

Run 11 Model 1 - Tweedie Models



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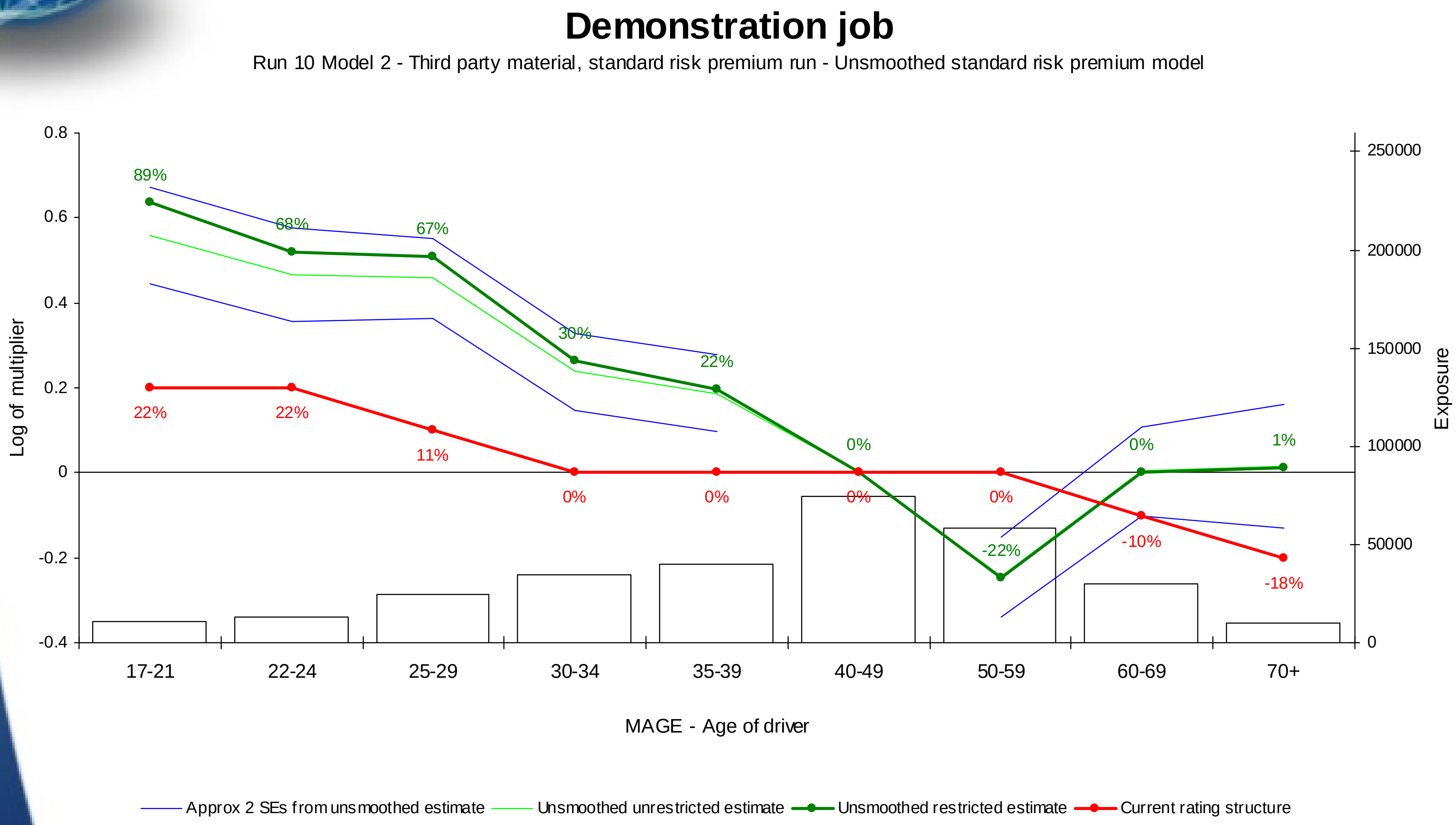
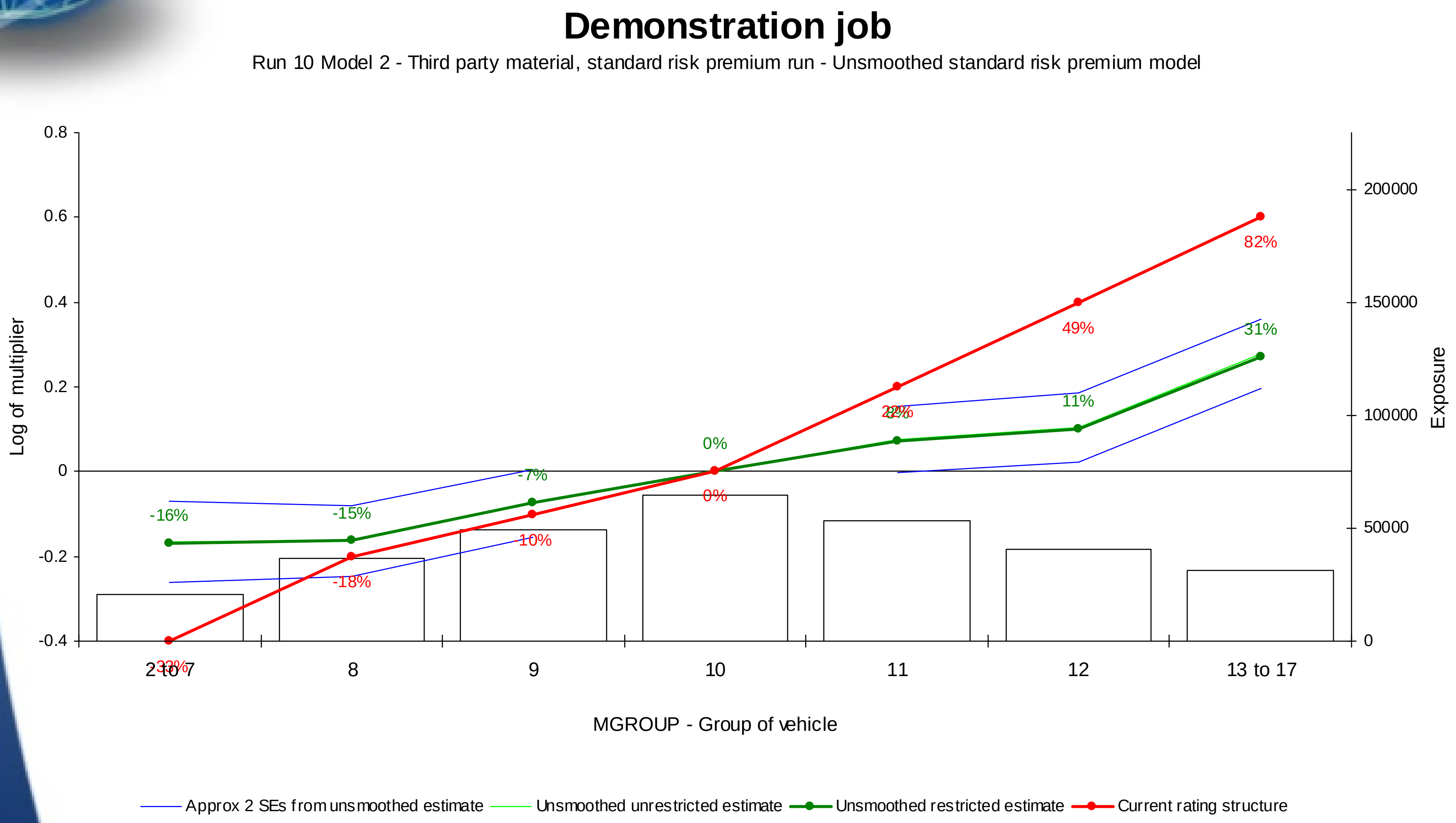
Comparison with actual rates

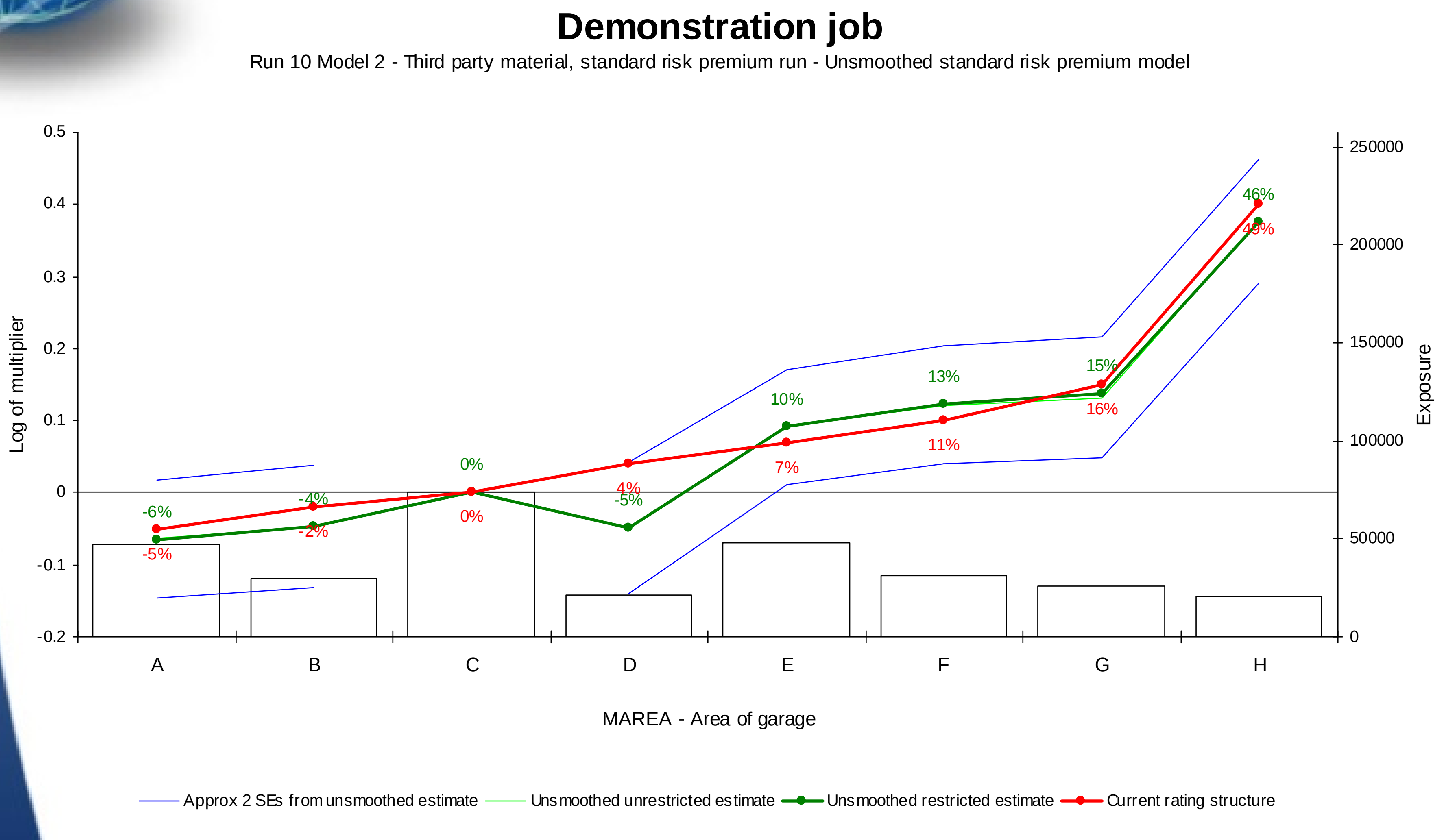


Comparison with actual rates

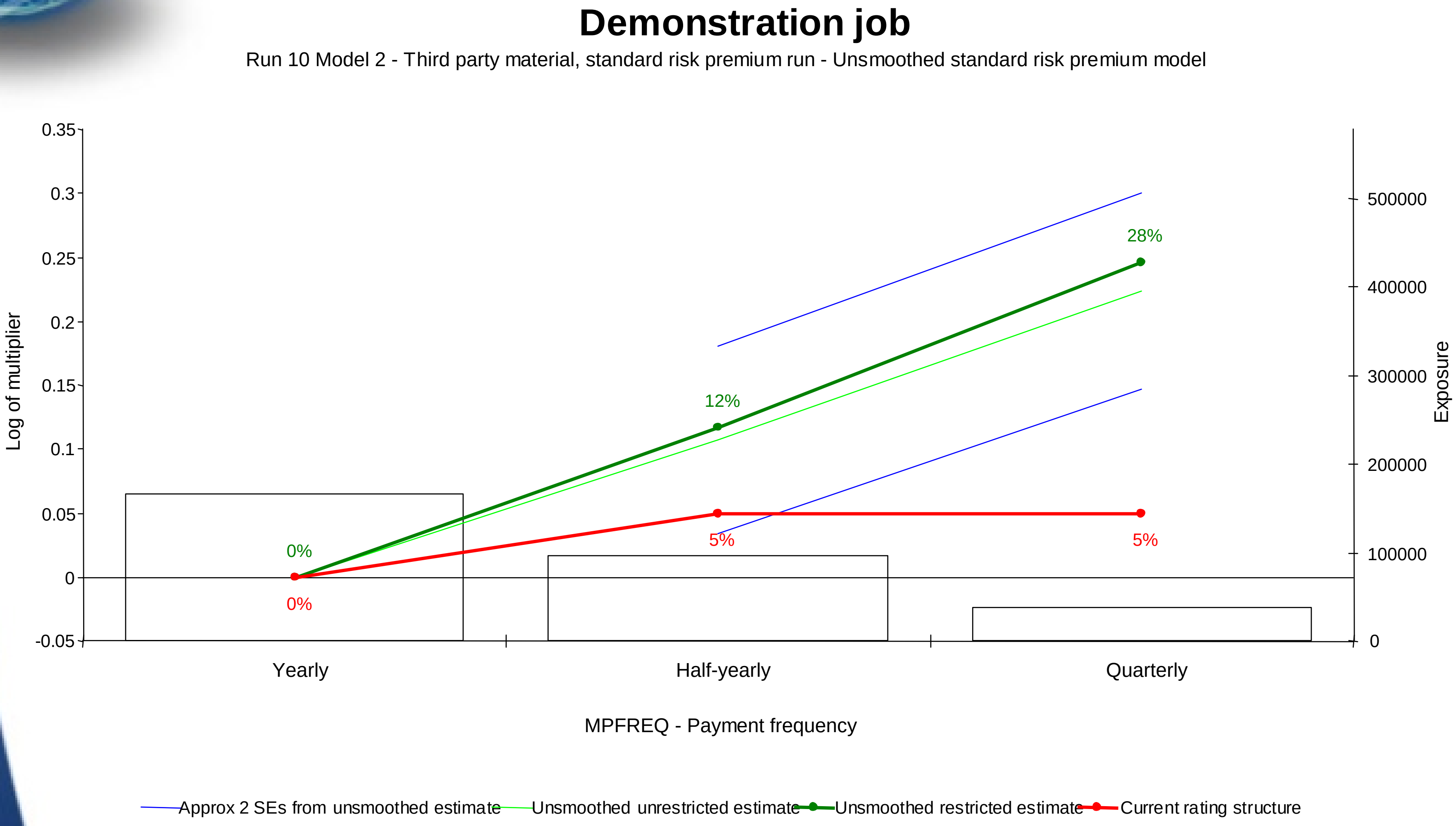

Demonstration job

Run 10 Model 2 - Third party material, standard risk premium run - Unsmoothed standard risk premium model

Comparison with actual rates

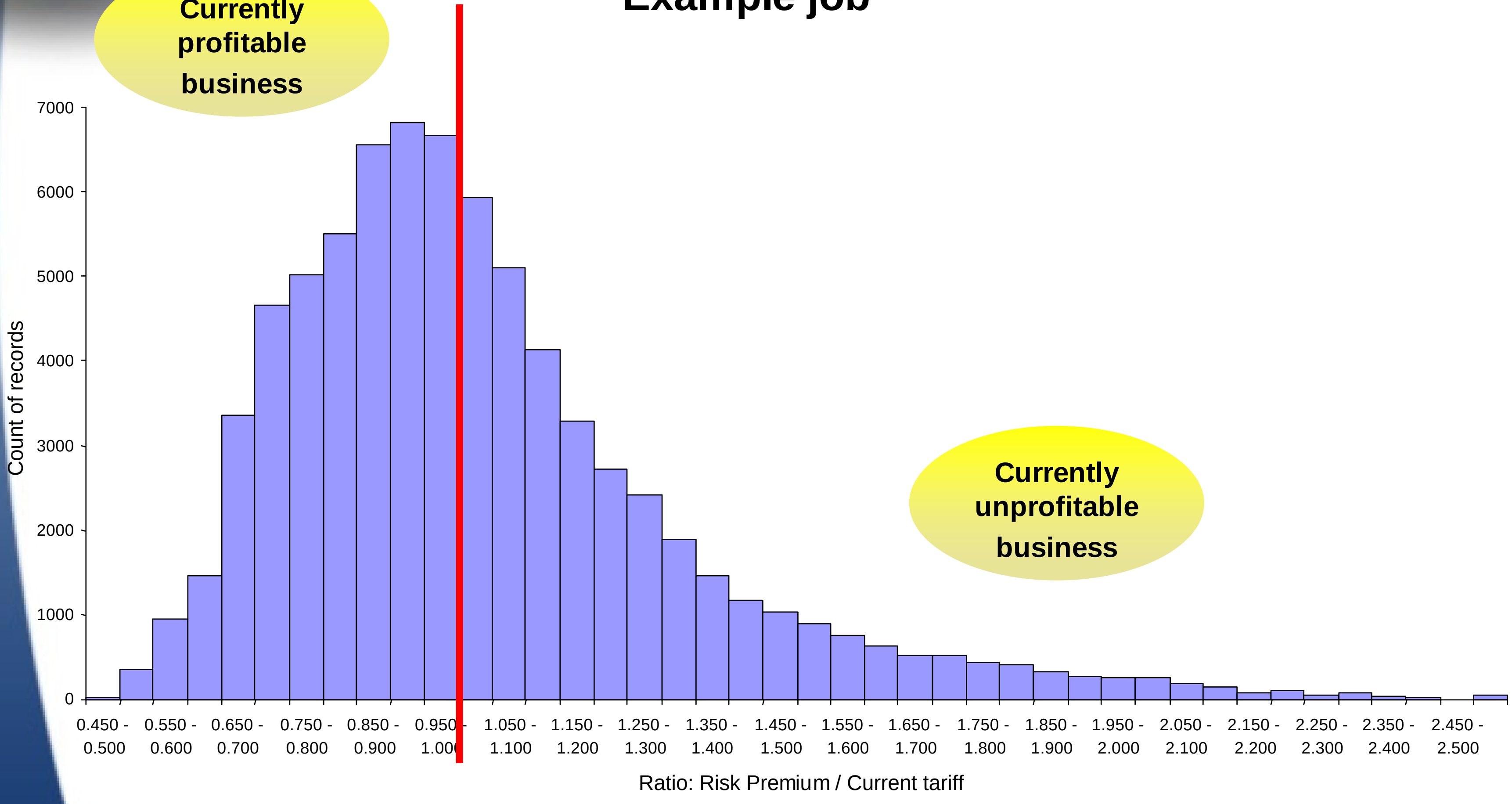


Comparison with actual rates



Impact analysis

Example job

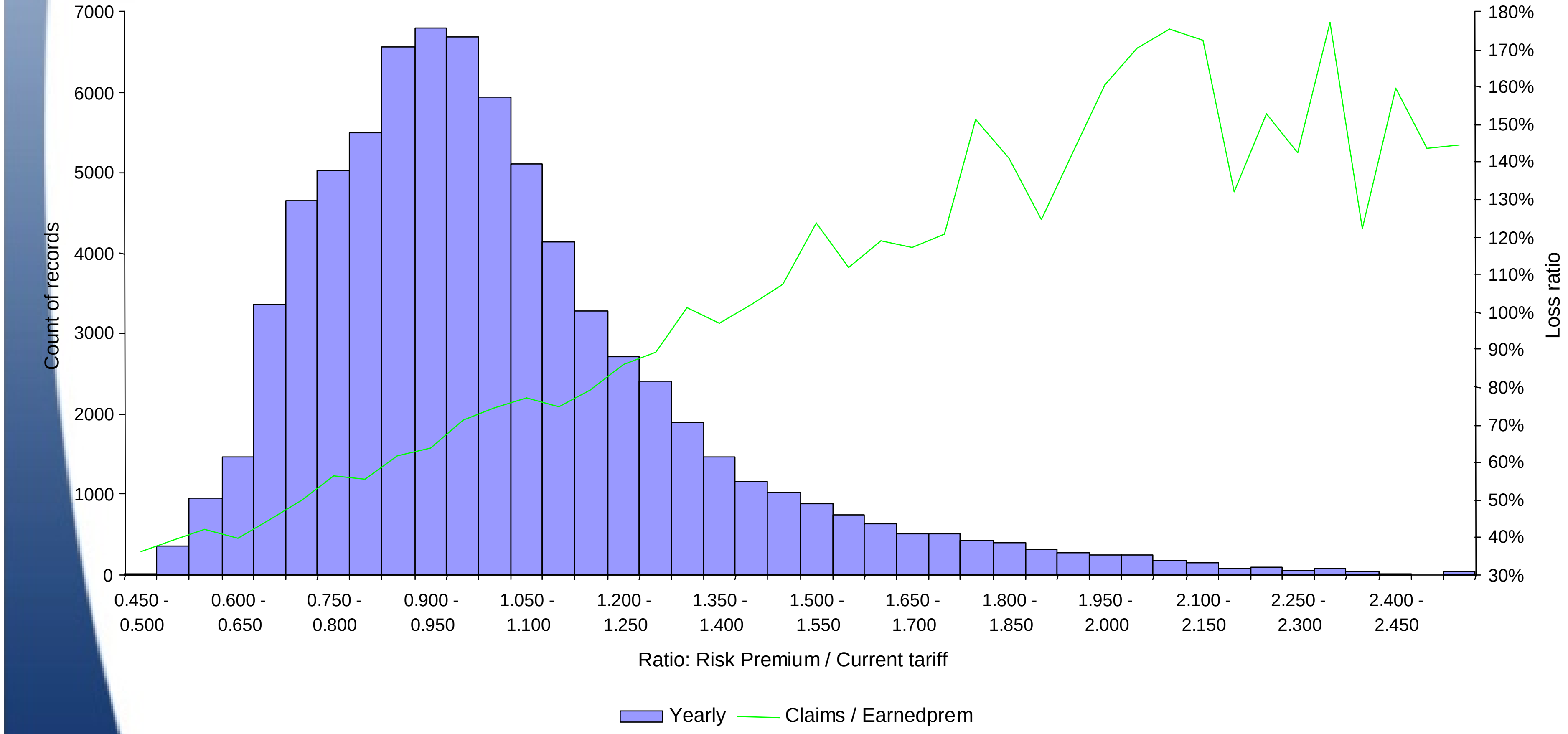


Currently profitable business

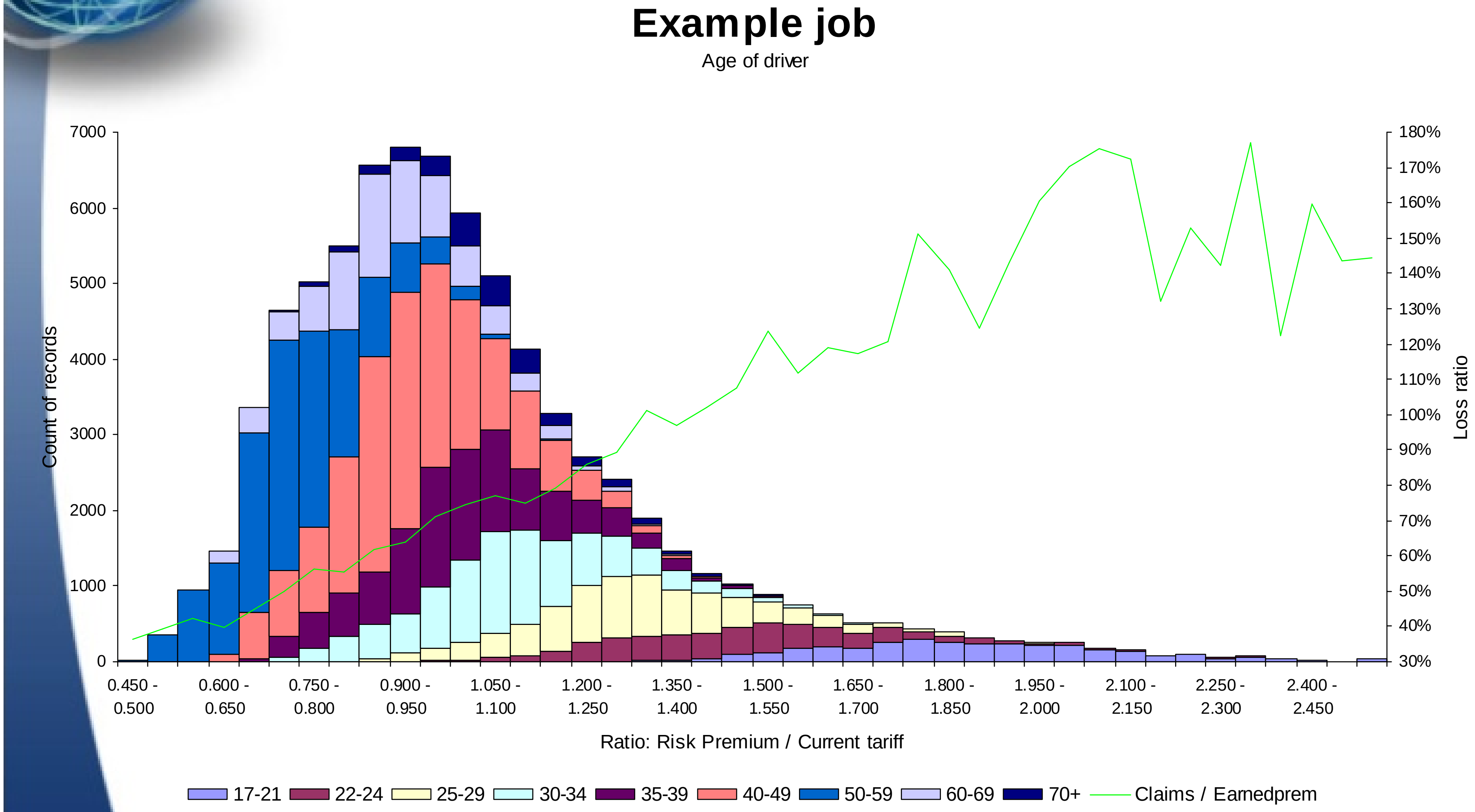
Currently unprofitable business

Impact analysis

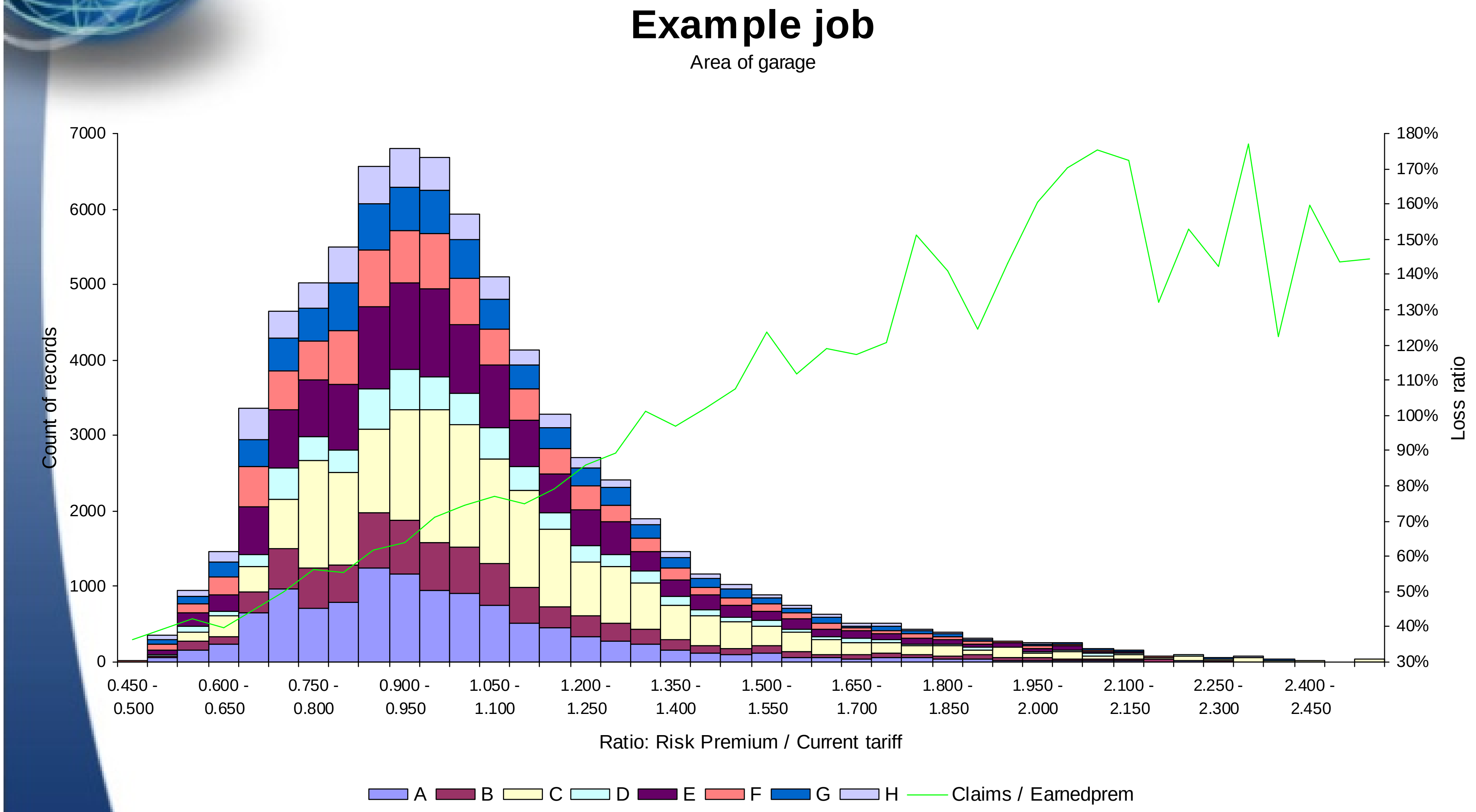
Example job



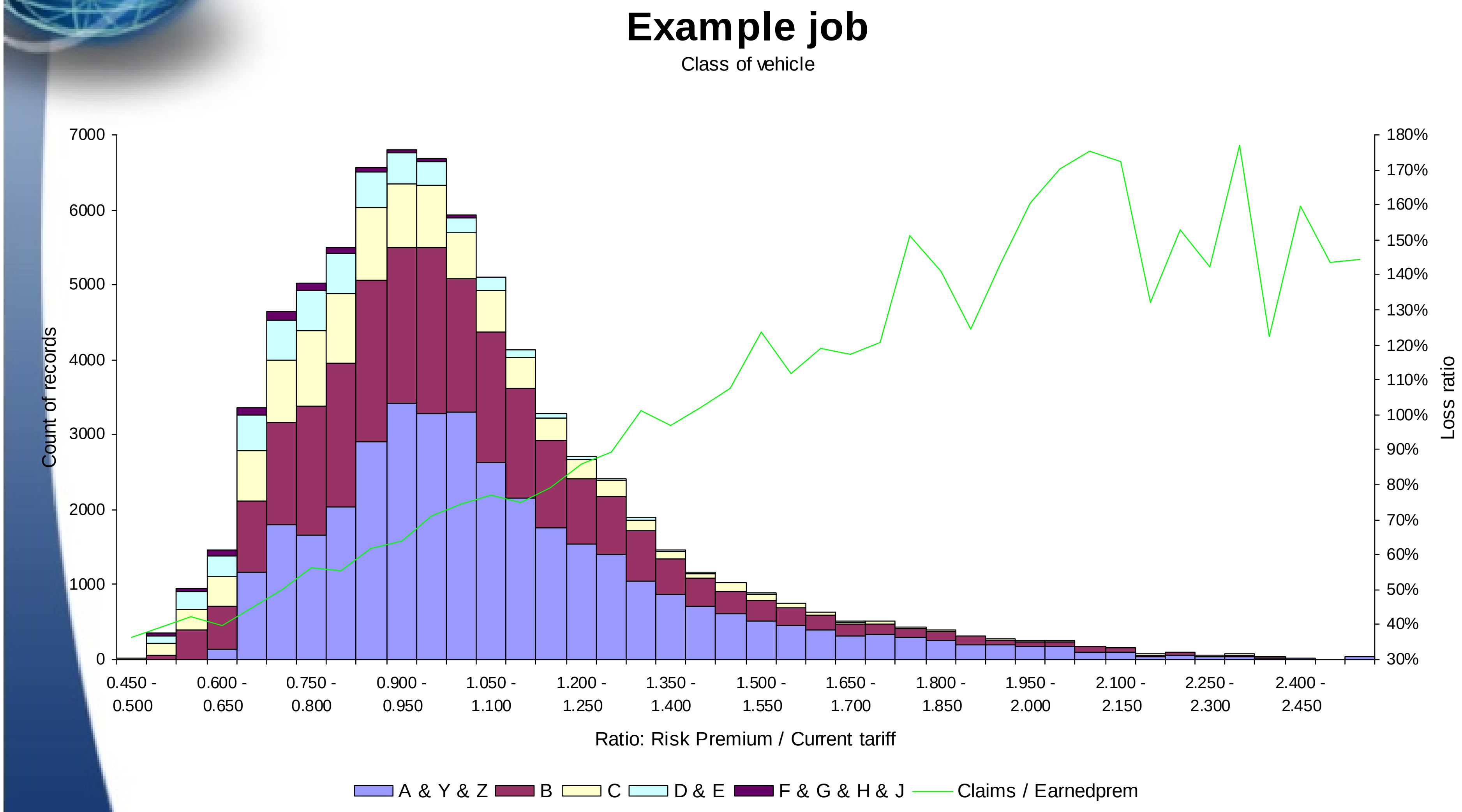
Impact analysis



Impact analysis



Impact analysis



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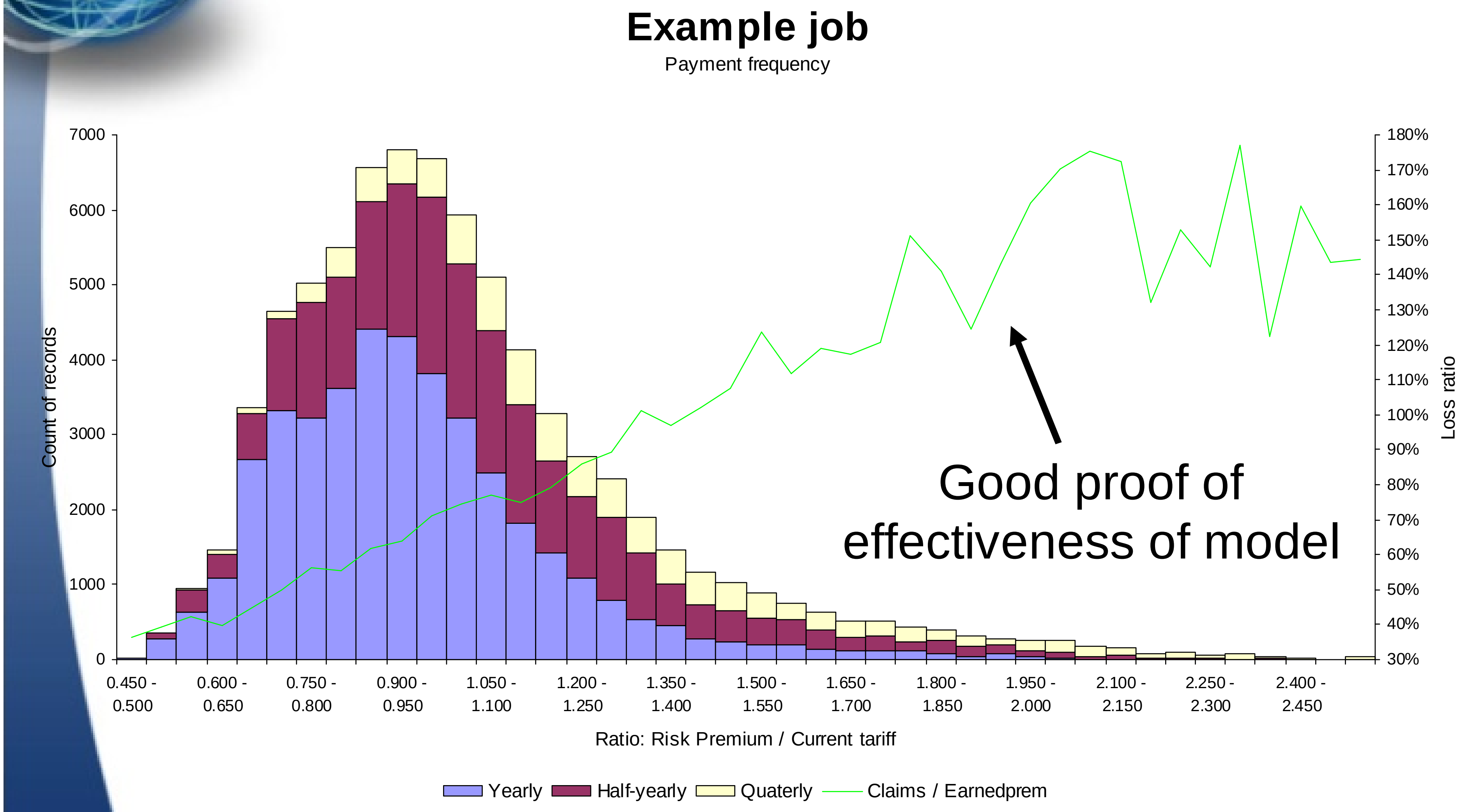
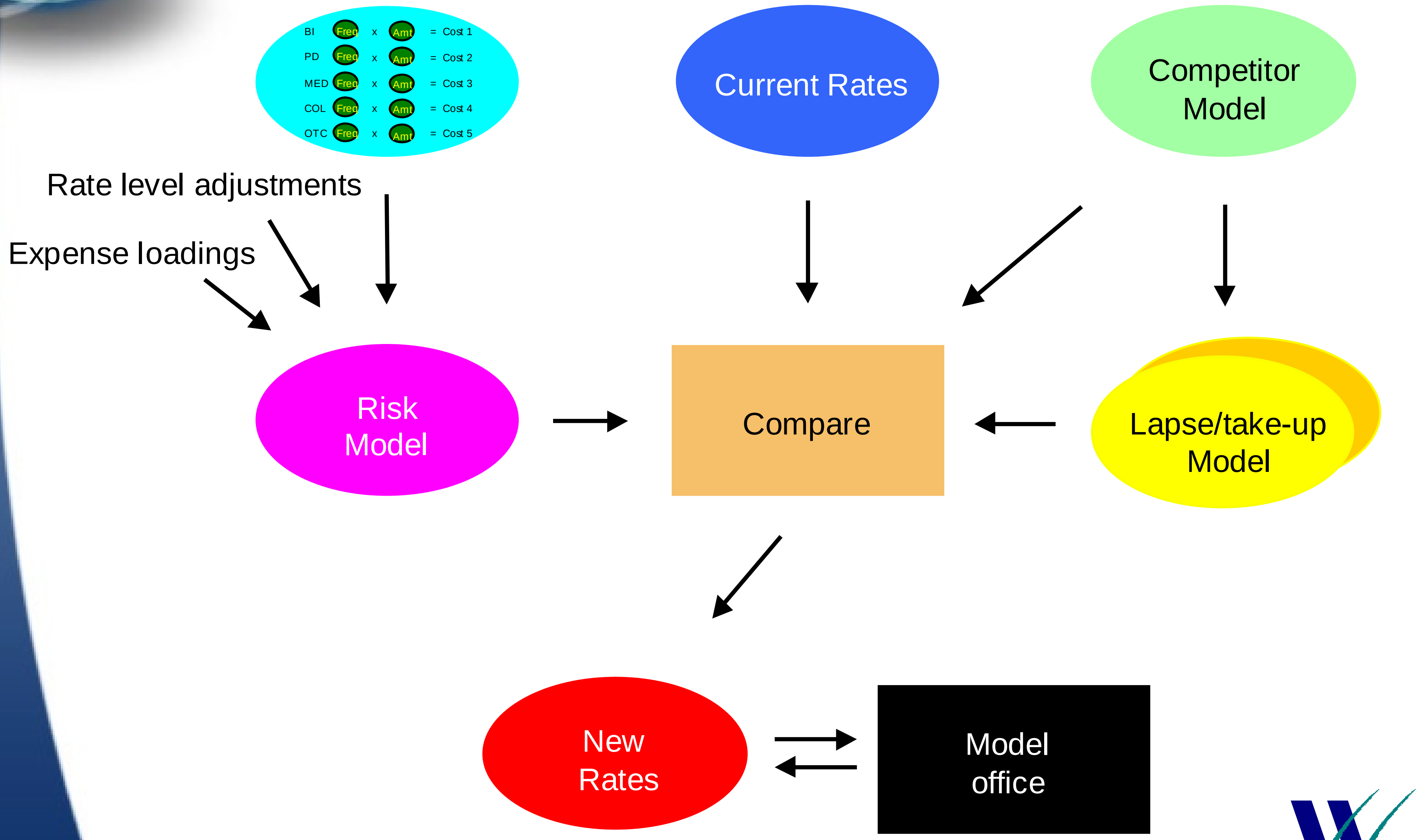


Example job

Payment frequency

Good proof of effectiveness of model

The premium rating process



BI Freq x Amt = Cost 1
PD Freq x Amt = Cost 2
MED Freq x Amt = Cost 3
COL Freq x Amt = Cost 4
OTC Freq x Amt = Cost 5

Current Rates

Competitor Model

Rate level adjustments

Expense loadings

Risk Model

Compare

Lapse/take-up Model

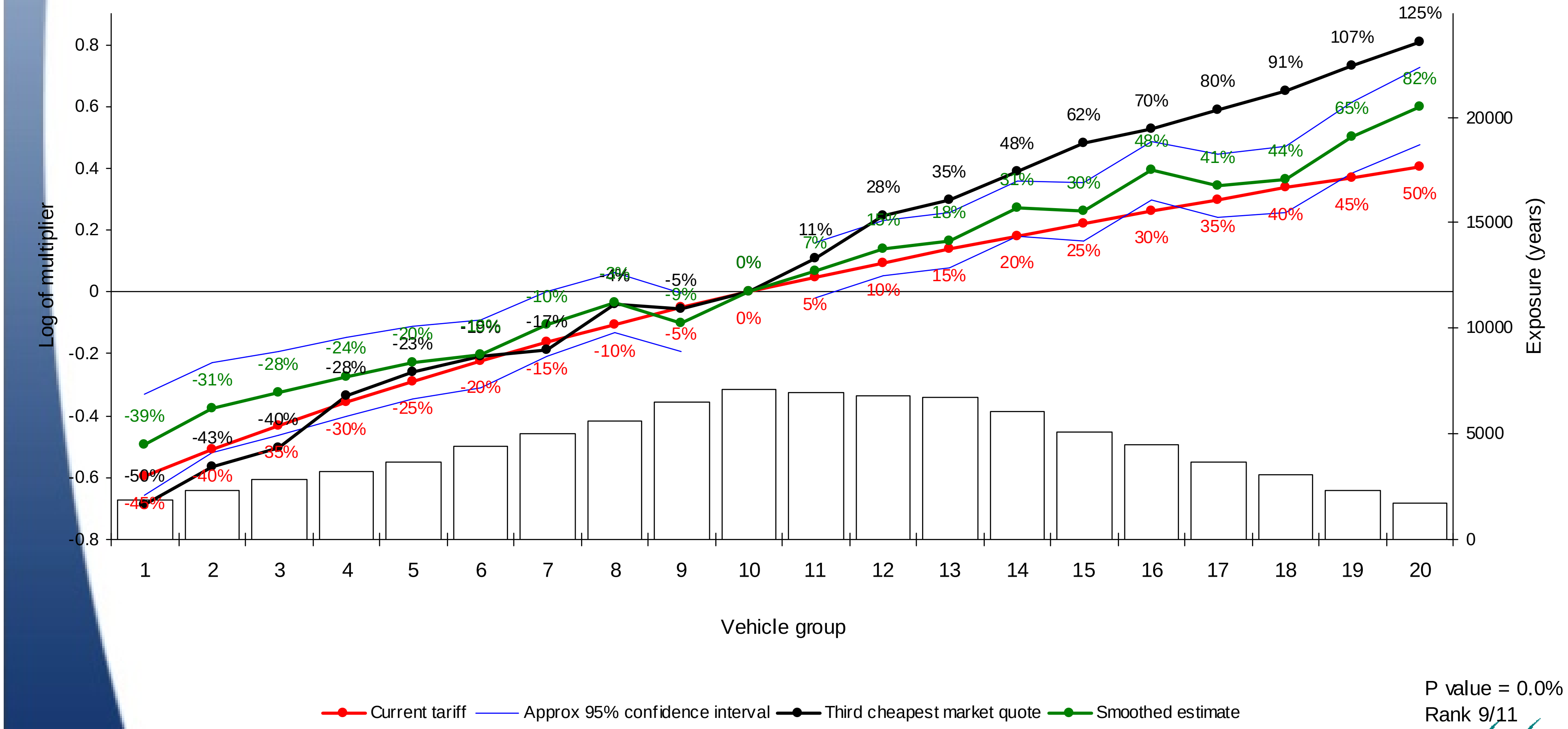
New Rates

Model office

Considering the competitive position

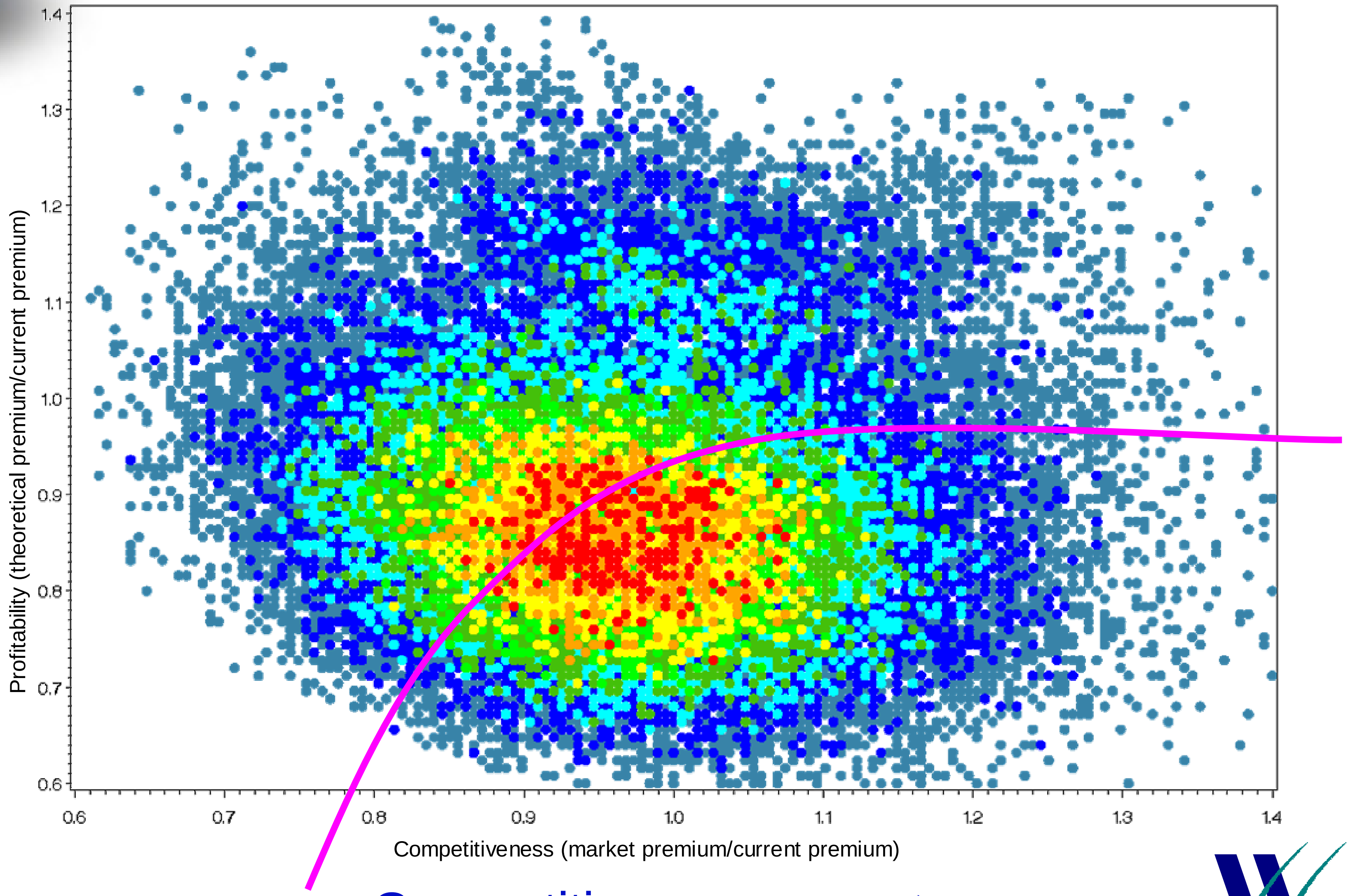
Example of competitor analysis

Third party cover



P value = 0.0%
Rank 9/11

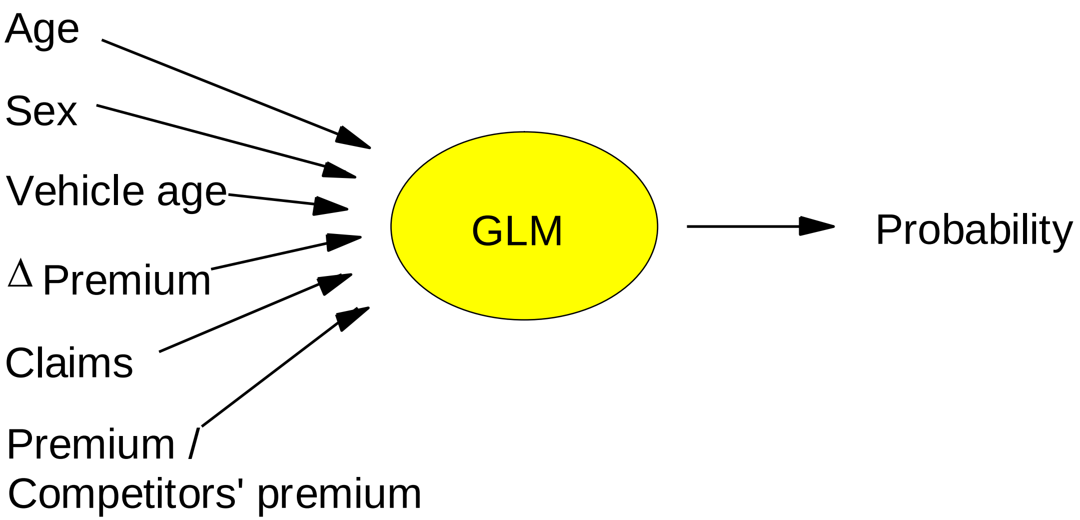
Considering the competitive position



Profitable ↓

Competitive →

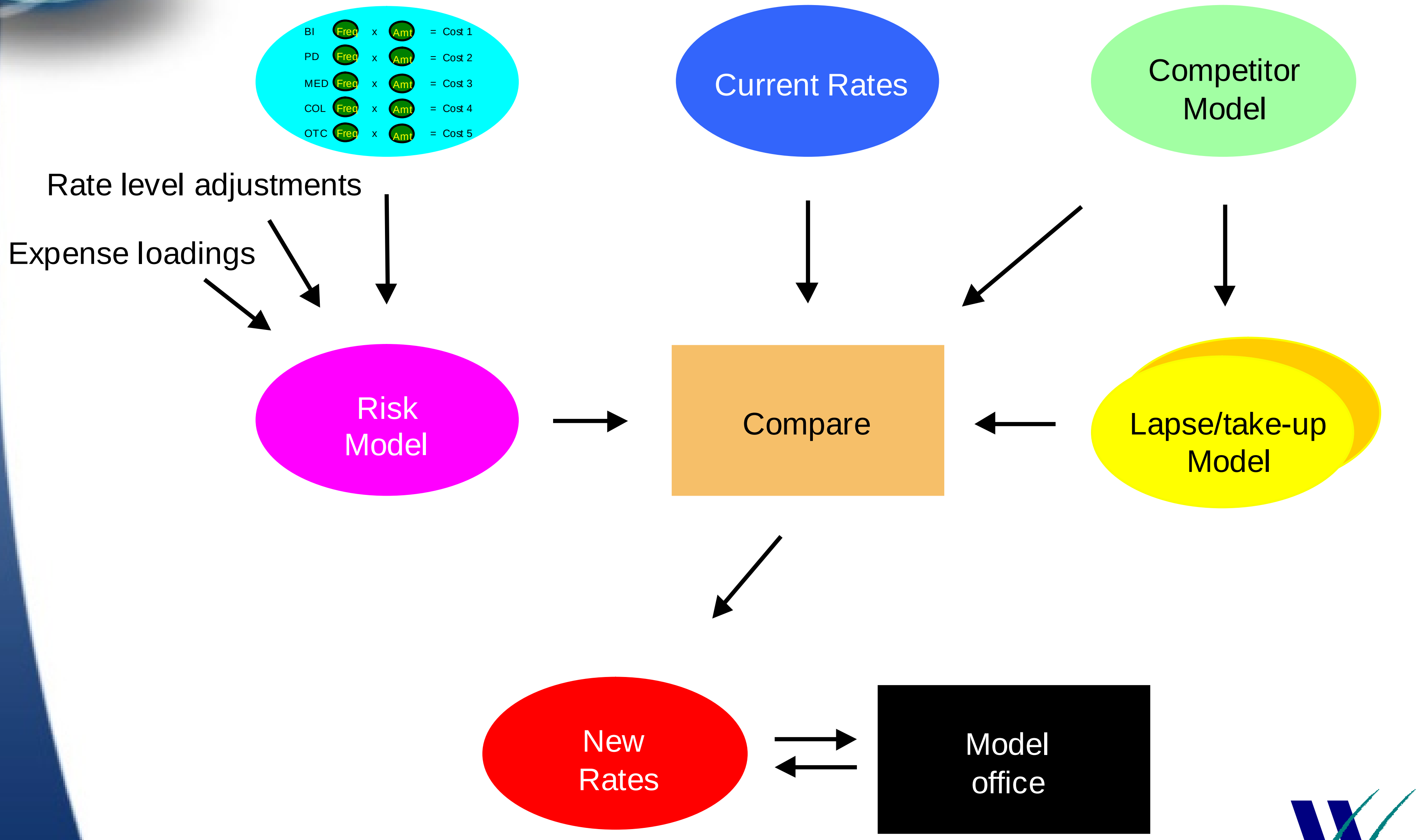
Modeling retention / conversion



- Model

- normal factors	- other products held
- payment method	- change in cover
- discount expectation	*plus…*
- source	- change in premium
- claims history	- competitiveness

The premium rating process



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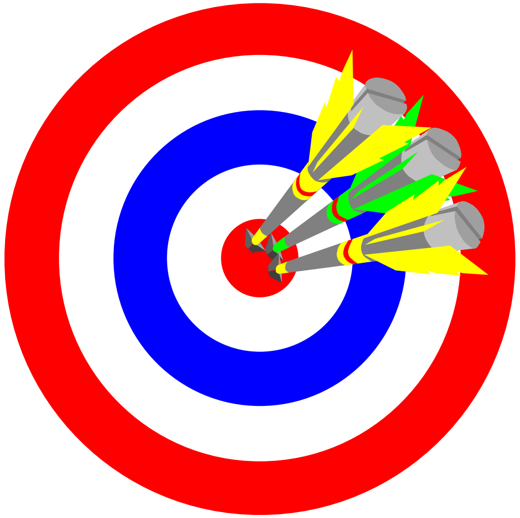
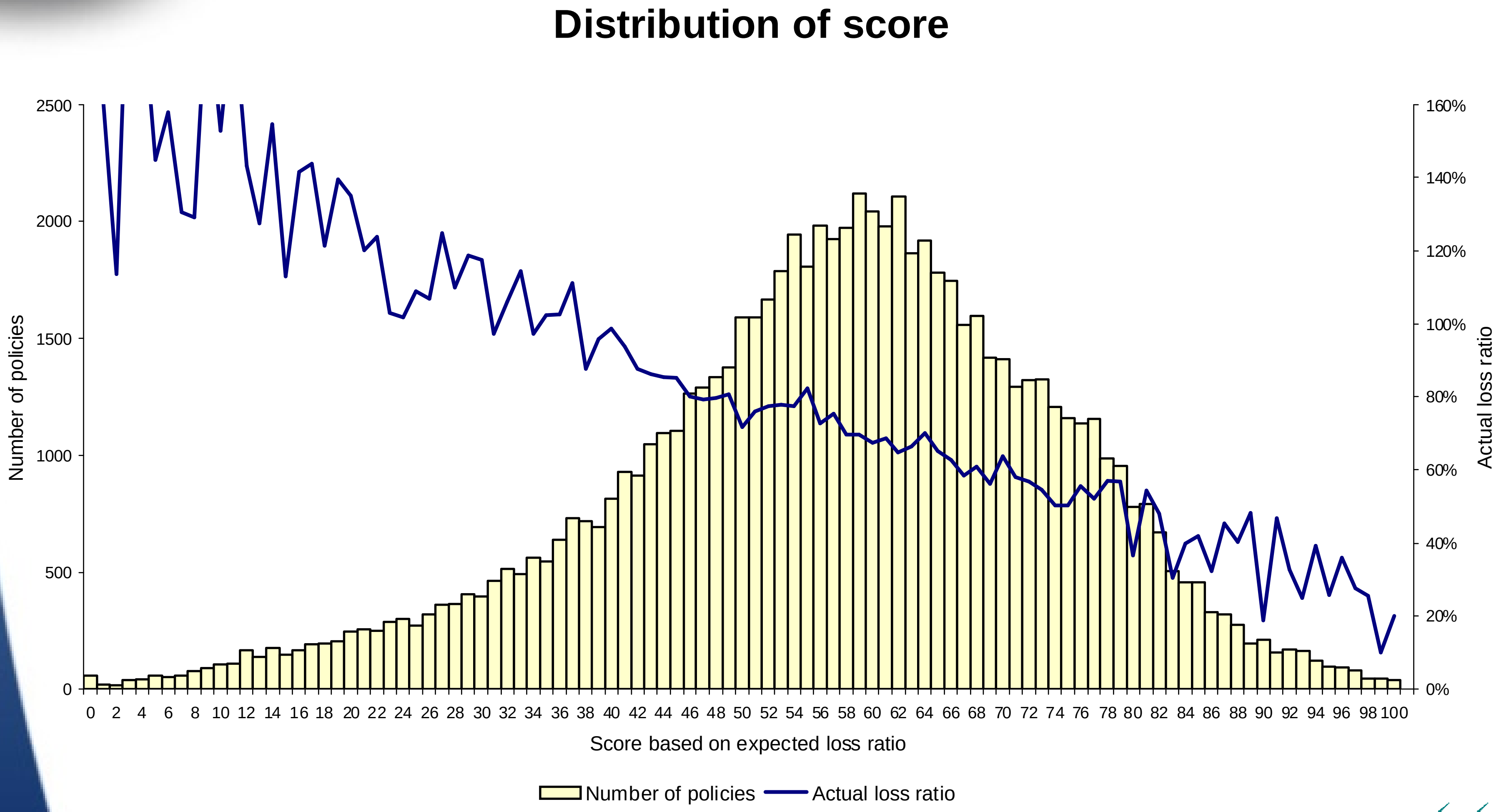
Profitability scoring



- Construct profitability score based on expected loss ratio
- Profitability score can then be used to target sections of a portfolio
- Expected loss ratio can be modeled using a risk premium model offset by current premium rates
- Expected loss ratio can be banded into discrete bands if desired

Profitability scoring



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