Don’t put all your AALs in One Basket

Casualty Actuarial Society RPM
Boston, MA

Risk Concentration Pricing for Primary Insurers

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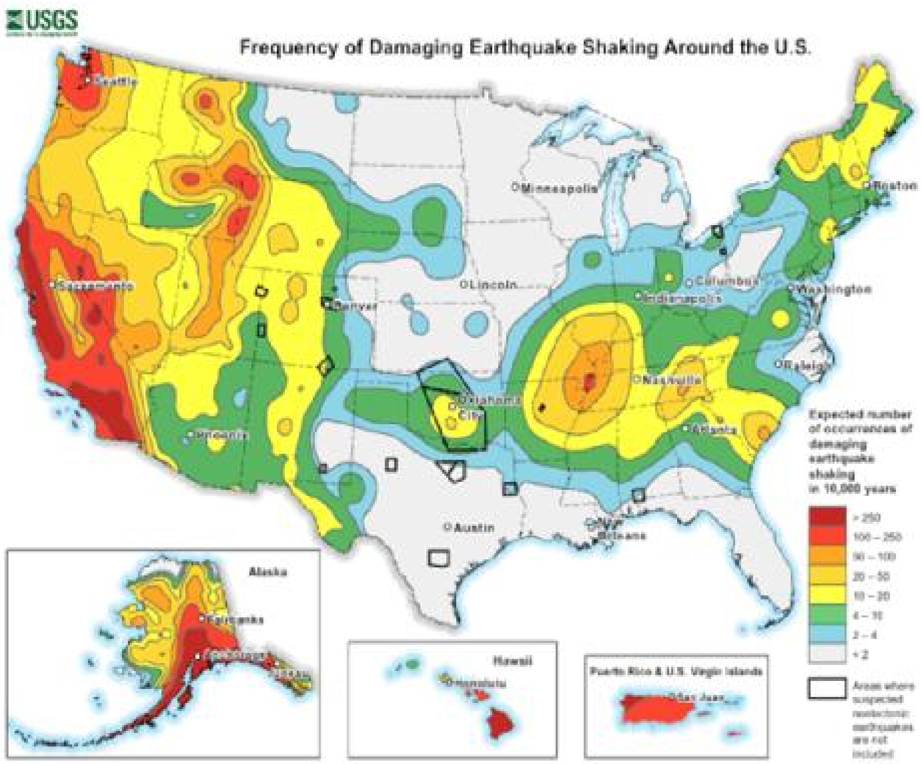
CATs are Dependent!



My book is mostly in Minneapolis

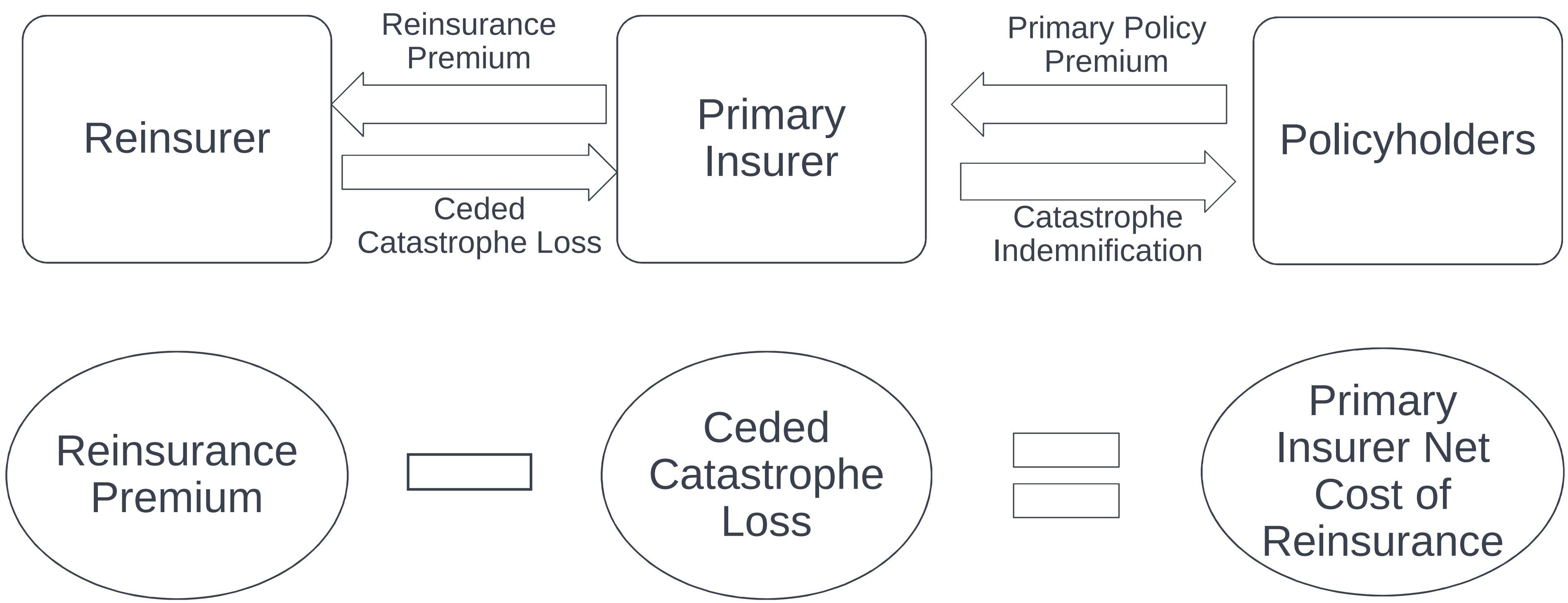
My book is mostly in Sacramento

Primary Insurer

I considered concentration risk!

Reinsurer

Calculating Net Cost of Reinsurance



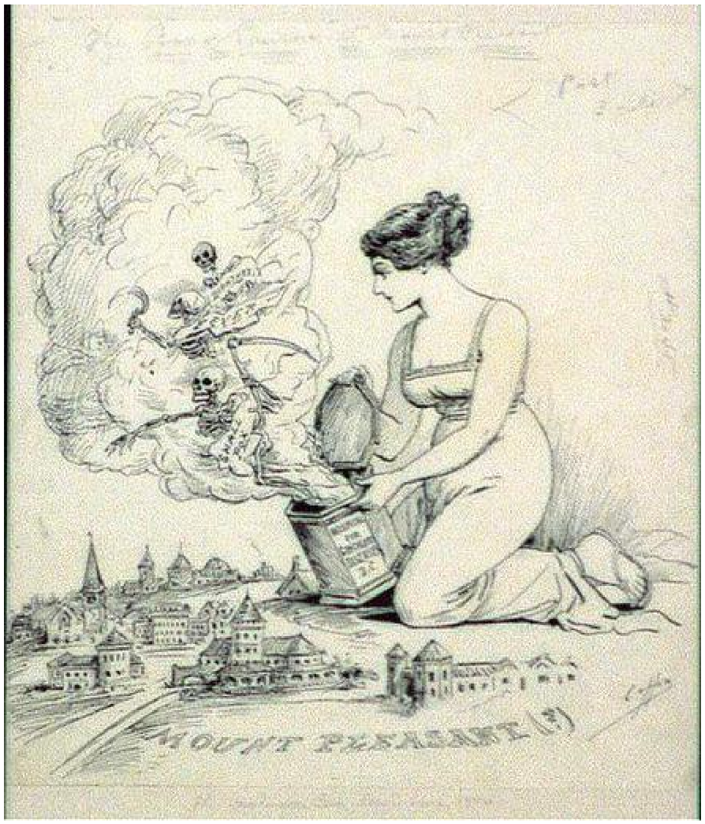
Primary Insurers' Traditional Treatment of NCOR

Flat Load in Base Rate

Allocated NCOR

Actual Cost of Risk Transfer

Catastrophe Model Components – Stochastic Event Module



- Includes a database of stochastic events
- Defined by:
    - Strength
    - Location
    - Probability of occurrence

Catastrophe Model Components – Hazard Module

§ Generates event information

§ Examples (for Hurricane):

- q Central pressure
- q Radius of maximum wind
- q Translational speed

Catastrophe Model Components – Vulnerability Module

- Estimates relationship between event parameters and damage to property
- Calculates:
  - Mean damage
  - Uncertainty around the mean

Catastrophe Model Components – Financial Module

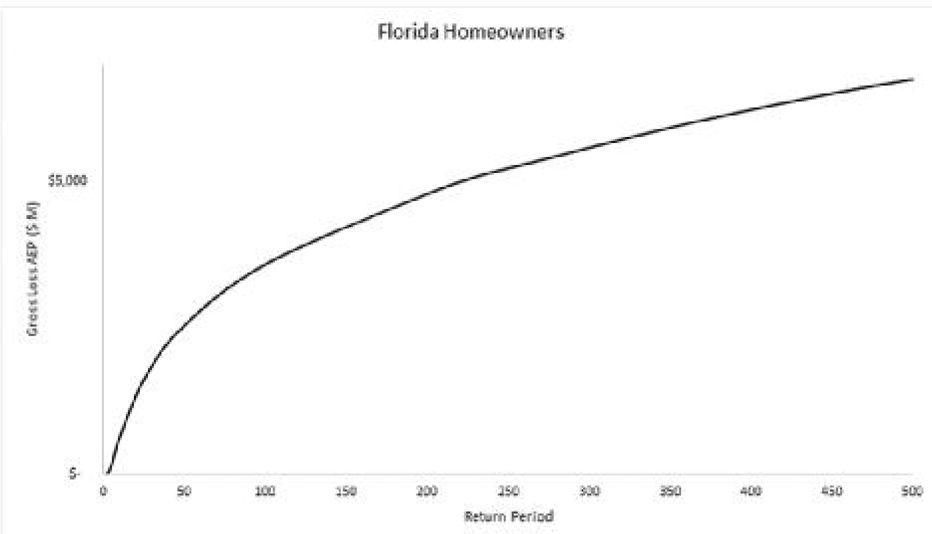
- Applies insurance contract terms to the modeled damage
- Translates the physical loss into a financial loss figure
- Calculate losses incurred to insured, primary insurer, and reinsurer

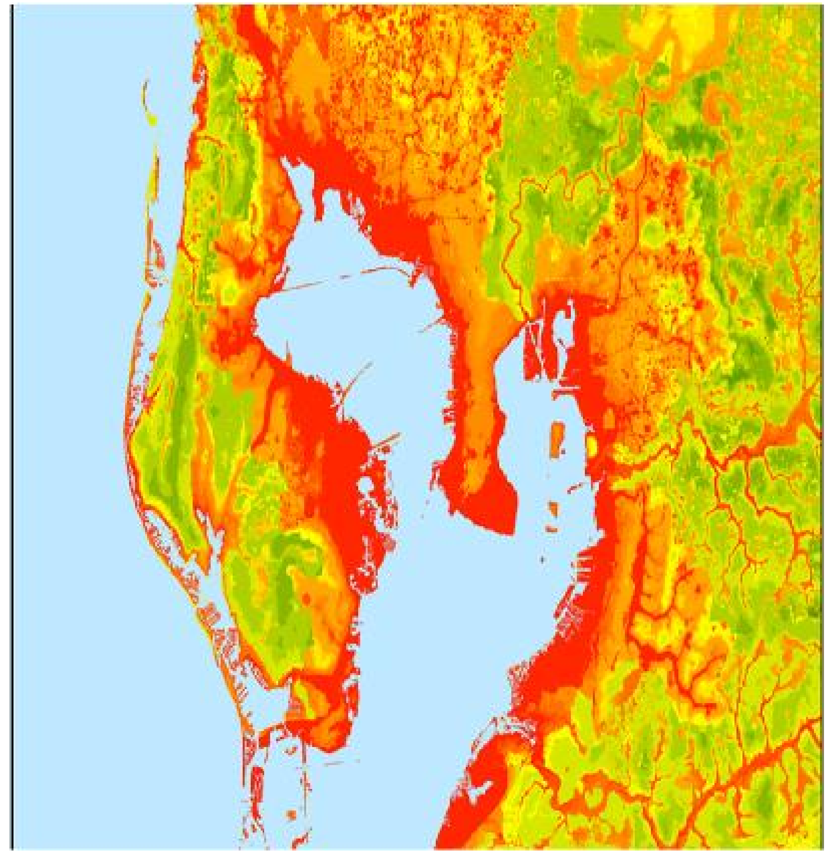
Insurer

Insured

Catastrophe Model - Output

- Event Level vs Region/Risk Level
- By Reinsurance Treaty Layer
- Probable Maximum Loss Curves





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Common Application in Pricing

- Event-level AALs for projecting loss costs in rate filings
- Portfolio risk/PML for negotiating reinsurance contracts
- Inforce portfolio commonly used for the simulation runs
    - Often assumes results are fully credible
    - However, may be a weakness if the underlying portfolio is limited in size or has systematic biases
- Model results typically treated as if they were at ultimate
    - Development assumptions implicit in model validation procedures
    - Some aspects of loss trends captured by changing exposure attributes

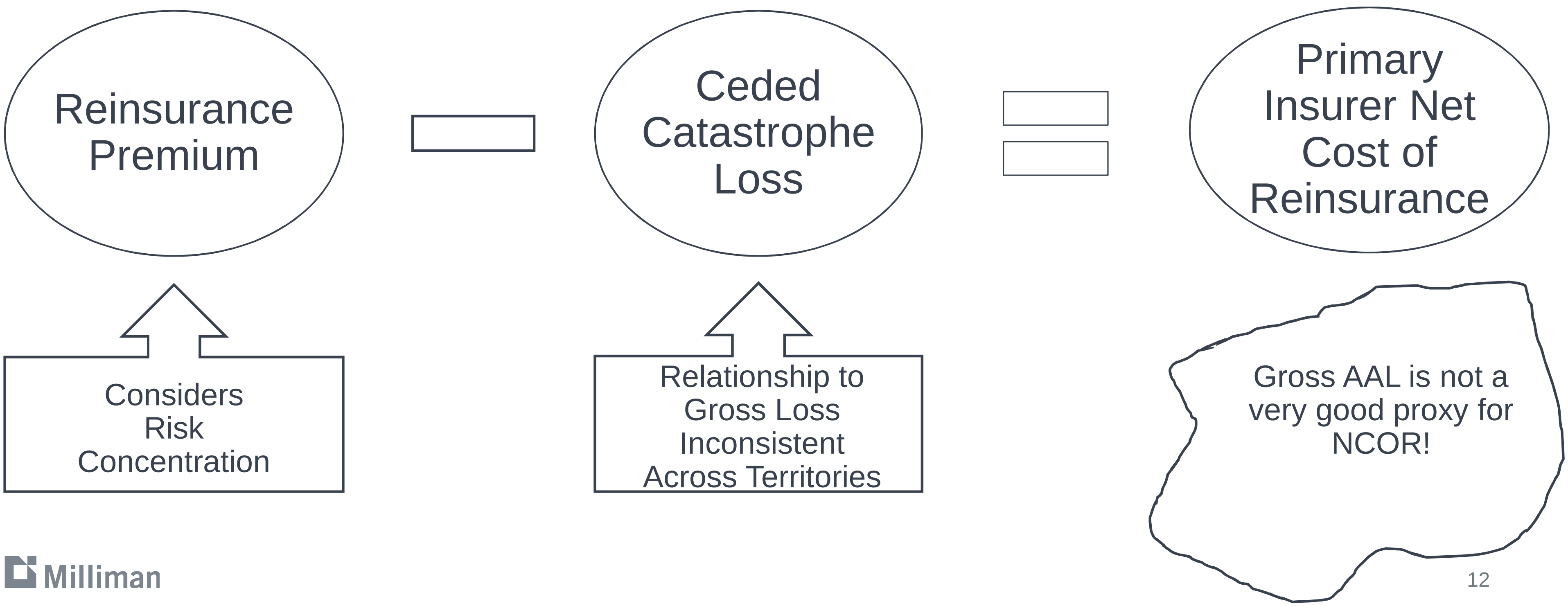
Regulatory Concerns in Concentration Pricing

- Model Versions and Specifications
  - Primary insurers more regulated than reinsurers
  - Inconsistent versioning/specifications between contract and filing
  - Model reviews may be outside of regulators' area of expertise
  - Reinsurance program subject to change
  - Extreme discontinuities for proposed rates

- Resulting rates not equitable from market's perspective
  - Cost of risk transfer depends on other risks in the insurer's portfolio
  - May be difficult for regulators to justify carriers charging different rates for the same risk

Primary Insurers' Traditional Treatment of NCOR

Proportional to Gross AAL

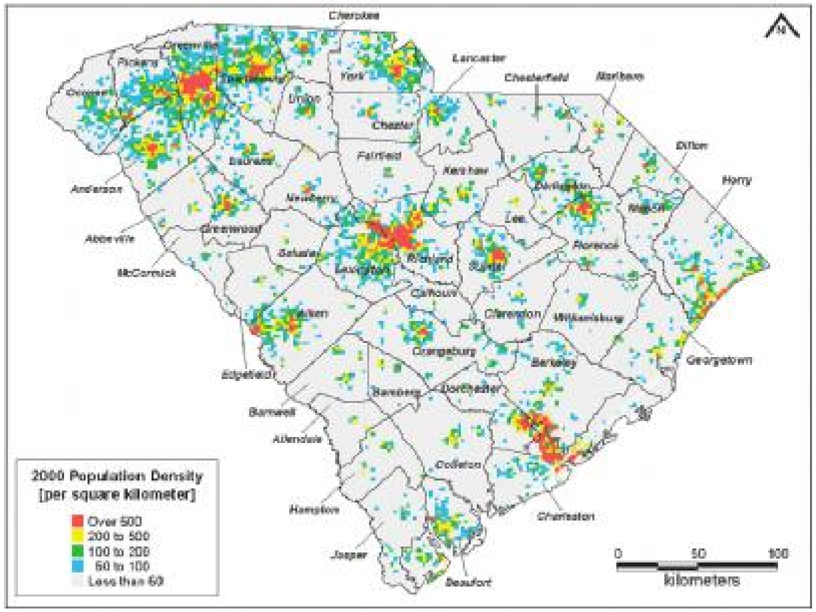


Alternative Options for NCOR Allocation

- Proportional Method Based on VaR/TVaR
- Incremental and Co-Measures Methods
- Shapley Value and Covariance Share Methods
- Event-Based Allocation by Reinsurance Layer Method

Concentration Pricing: Actuarial Considerations

- ***Cost of Risk Transfer*[1]** *– A rate provides for all costs associated with the transfer of risk.*
- **Coherence[2]** – Desirable properties for allocation method to “make sense”.
    - *Positive Homogeneity – “The risk of a multiple is equal to the multiple of a risk.” (applies to perfectly independent risks only)*
    - *Subadditivity – The risk of the sum of individual risks is less than or equal to the sum of the individual risks.*
    - *Translation Invariance – Adding a riskless constant to a risky position does not increase the risk of the position.*
    - *Monotonicity – Measures for less risky positions are smaller than for riskier positions.*
- **Order Dependence / Renewal Additivity – Desirable?**
    - *Risk’s contribution to portfolio risk may vary based on order in which it was added to portfolio, or order in which it was analyzed.*
    - *“For a given portfolio of accounts, a risk load method is renewal additive if the sum of the renewal risk loads calculated for each account equals the risk load calculated when the entire portfolio is treated as a single account.”[3]*



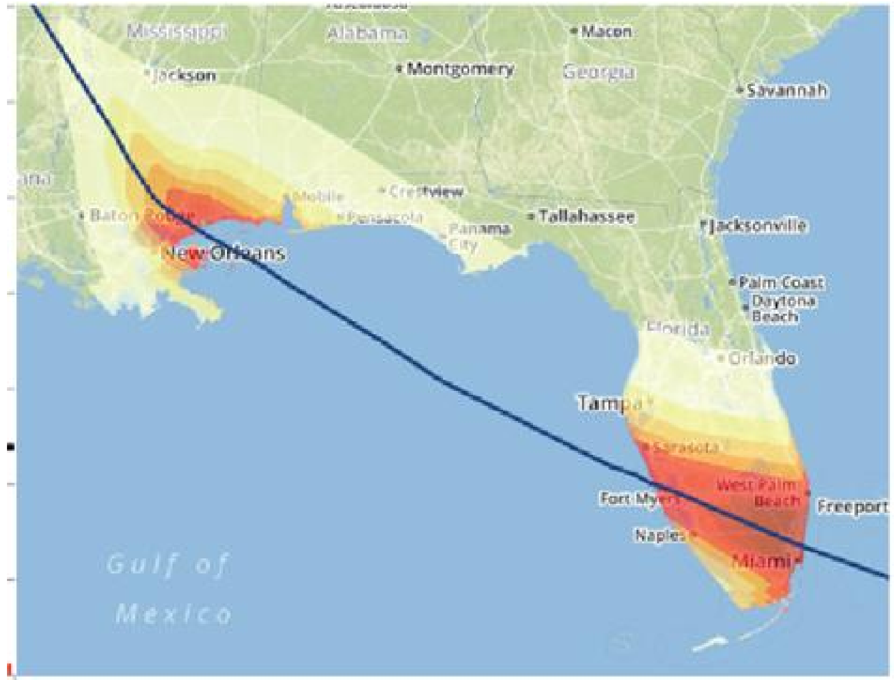
1.Statement of Principles, Property and Casualty Insurance Ratemaking
Source (right): https://pubs.usgs.gov/circ/1306/pdf/c1306_ch2_a.pdf
2.FROM GAME THEORY TO SOLVENCY QUANTILE CALCULATION: CAPITAL ALLOCATION WITH USE IN NONLIFE INSURANCE Nicolas ZEC1
3. AN APPLICATION OF GAME THEORY: PROPERTY CATASTROPHE RISK LOAD by Donald F. Mango

Concentration Pricing: Practical Considerations

- ***Recognition of Extreme Scenarios*** – Extreme scenarios can pose existential threats to insurers. Should risks implicated in these events pay more?

- ***Correlations and Aggregate Measures*** - Writing more risks independent of an insurer's existing portfolio could reduce the overall riskiness of the portfolio.

- ***Analytical Practicality*** – What are the costs and benefits associated with analysis and implementation?

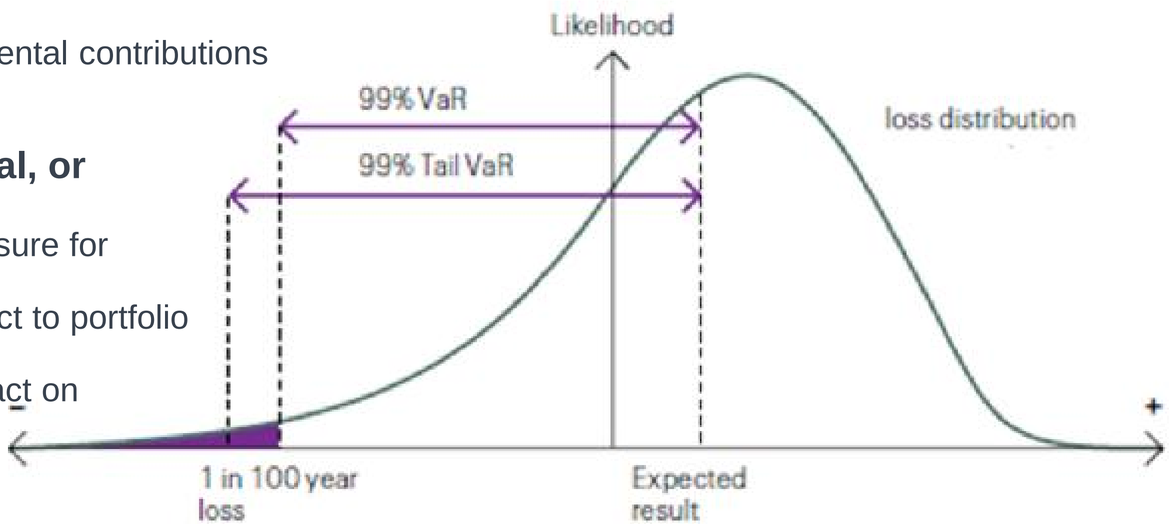


- Hypothetical storm track from "https://www.cnbc.com/2015/04/14/100-year-hurricane-could-cause-more-than-250b-losses-in-florida.html"

Methods Based on VaR/TVaR

- **Risk Measures**
  - Value at Risk (VaR) – Expected outcome given some arbitrary percentile.
  - Tail Value at Risk (TVaR) – Average expected outcome above the arbitrary percentile.
  - Can be calculated for individual policies, territories, portfolios
  - Methods can be *proportional*, *incremental*, or *marginal*.
  -
- **Example: Incremental Allocation Method based on Portfolio VaR/TVaR:**
  - Calculate risk measure at portfolio level
  - Remove one territory and calculate risk measure
  - Difference is the incremental contribution for that territory
  - Repeat for all territories
  - Allocate NCOR proportionally to the incremental contributions

- **Methods can be proportional, incremental, or marginal**.
  - Proportional – Allocate in proportion to measure for territory
  - Incremental – Allocate in proportion to impact to portfolio of removing territory
  - Marginal – Allocate in proportion to the impact on portfolio measure of next dollar or policy.



Allocating Correlation: Shapley Value And Covariance Share

- ***Shapley Value Method[4]:***
  - Calculate the variance within each territory
  - Calculate all covariances between pairs of territories
  - Calculate the variance at the portfolio level
  - For each territory, add its variance with any covariance that involves it
  - Divide by portfolio level variance
  - Allocate NCOR proportional to this ratio

- ***Covariance Share Method[4]:***
  - Same first 3 steps as Shapley Value Method
  - Calculate AAL proportions for all pairs of territories
  - Allocate two times the covariances to each territory based on variance proportions
  - For each territory, add its variance with its allocated covariance share
  - Divide by portfolio level variance
  - Allocate NCOR proportional to this ratio

L= Existing Account
N=New Account
Marginal Variance = Var(n)+2Cov(L,n)
Shapley Value = Var(n)+Cov(L,n)
Covariance Share -> Allocates the value Cov(L,n) in proportion to the variance of the individual territory or account

4. AN APPLICATION OF GAME THEORY: PROPERTY CATASTROPHE RISK LOAD by Donald F. Mango

Using Simulation: Co-Measures Method

Co-Measures Methods[5]:

- Obtain portfolio level loss distribution via simulation
- Calculate contribution of territory or account within those simulations
- Advantages:
  - Allocated contributions add up to total contribution
  - Efficient – Only one simulation run necessary in contrast with incremental or marginal approaches which require many
- Disadvantages
  - May still rely on arbitrary threshold
  - Must have ability to run simulation

Sorted Scenario	Market	Reserves	Line A	Line B	Total
1	779,323	12,180,298	3,188,429	4,994,583	21,142,632
2	494,425	8,169,822	3,734,913	8,695,665	21,094,825
3	-3,407,081	13,140,377	7,607,985	788,471	18,129,751
4	-779,922	2,587,705	5,675,660	10,386,216	17,869,658
5	-1,311,004	-1,203,142	3,238,333	16,924,158	17,648,345
6	-1,392,828	5,488,457	6,646,703	6,799,820	17,542,152
7	-255,475	4,812,487	4,018,249	7,904,885	16,480,145
8	-10,210	6,710,721	2,273,968	7,472,474	16,446,953
9	-1,896,169	4,433,724	1,652,542	12,169,231	16,359,328
10	758,494	3,132,459	2,330,630	10,003,805	16,225,388
11	-1,291,494	8,133,807	5,475,393	3,899,206	16,216,912
12	1,523,399	8,164,027	1,320,562	4,996,263	16,004,250
13	-1,507,026	8,701,922	4,941,913	3,358,494	15,495,303
14	-418,192	-390,473	1,172,596	15,112,222	15,476,153
15	348,569	4,904,846	4,173,982	6,001,026	15,428,423
:	:	:	:	:	:
490	-470,761	3,622,090	-148,615	4,519,262	7,521,976
491	-980,559	3,630,412	1,980,834	2,889,533	7,520,220
492	-2,921,510	2,906,628	-200,015	7,730,833	7,515,936
493	-1,179,044	3,552,559	2,343,631	2,794,807	7,511,953
494	-2,744,202	2,173,409	4,717,356	3,364,141	7,510,703
495	127,947	1,318,389	4,749,312	1,308,659	7,504,307
496	42,016	1,663,231	1,653,643	4,143,005	7,501,894
497	-1,062,298	2,170,695	6,366,285	27,183	7,501,865
498	-901,735	4,579,393	-124,816	3,947,145	7,499,986
499	-2,782,565	972,163	1,896,786	7,411,779	7,498,163
500	-2,959,845	6,146,281	863,894	3,441,193	7,491,523
Co-CTE	-908,399	3,715,533	2,279,319	4,549,138	9,635,591

5.Risk-Adjusted Performance Measurement for P&C Insurers Richard Goldfarb

Event-Based Allocation by Reinsurance Layer Method

Steps

- Obtain event-account level AALs from cat model
- Apply reinsurance treaties to get layer AALs
- Use reinsurance premium multiples to allocate portfolio reinsurance premium to event-account level
- Calculate NCOR at event-account level
- Pick a geographical unit as basis (census block group, census tract, etc.)
- Summarize total AAL and NCOR at the geographical unit level
- Calculate geographical unit level cost ratio to Gross AAL

$$= 1 + \frac{NCOR}{Gross\ AAL}$$

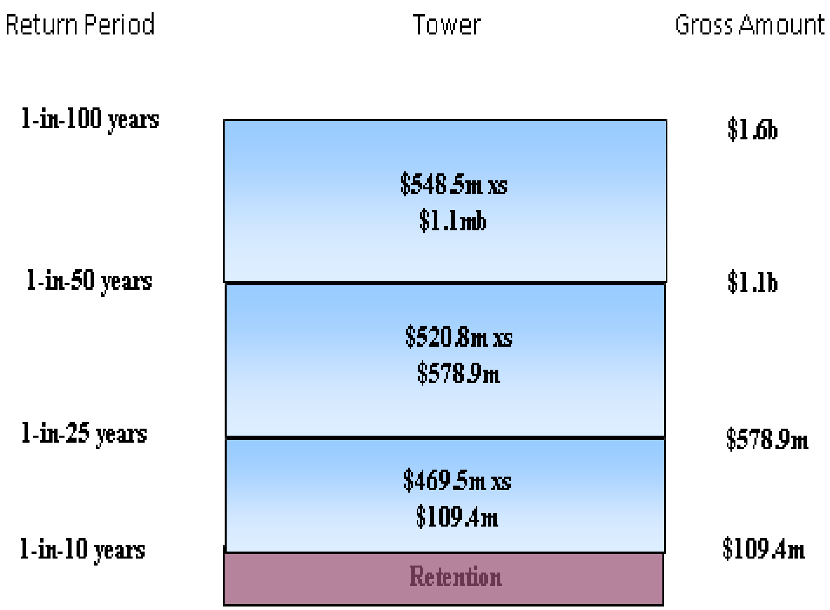
- Apply this multiple to AAL relativities

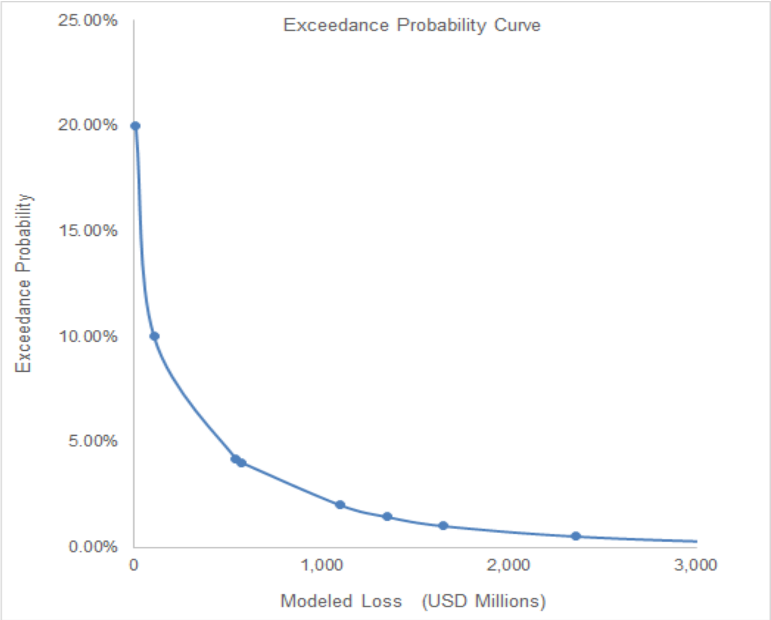
Event-Based Allocation by Reinsurance Layer Method

Data and Tool Requirements

- Data Requirements:
  - Risk characteristics required to run catastrophe simulation models
  - Reinsurance structure (limits, retention, premium)
  - Geographical information
  - Latitude and longitude
  - Census information
  - Catastrophe simulation model outputs for all sub-perils
  - Event level gross and ceded AALs by layer
  - Event-policy level gross AALs
- Tool Requirements:
  - Catastrophe simulation models
  - Geographical clustering techniques (optional)

Demonstration – South Carolina Hypothetical Insurer

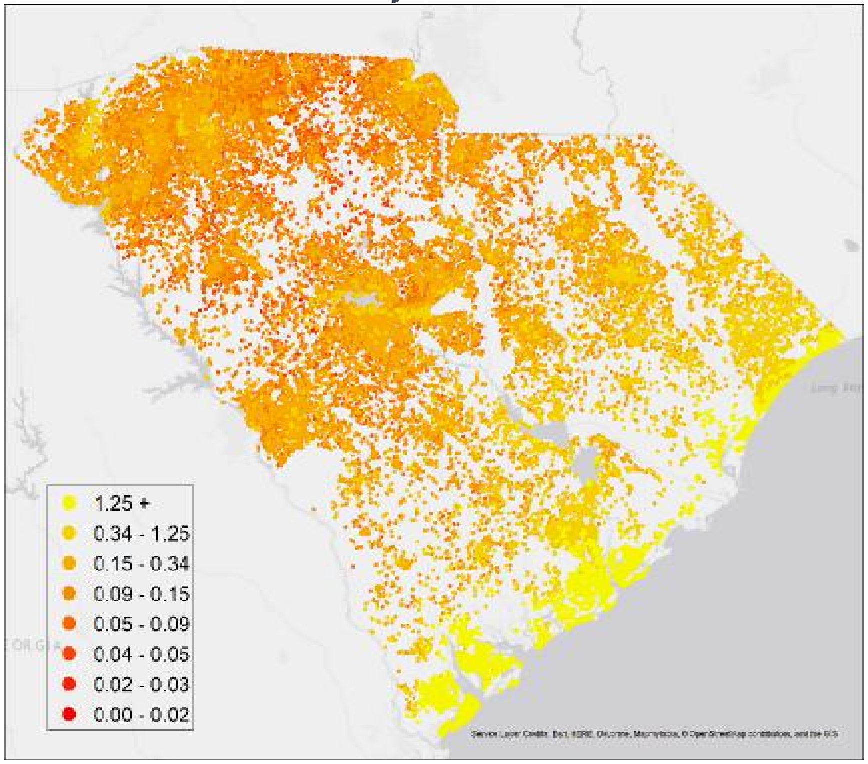




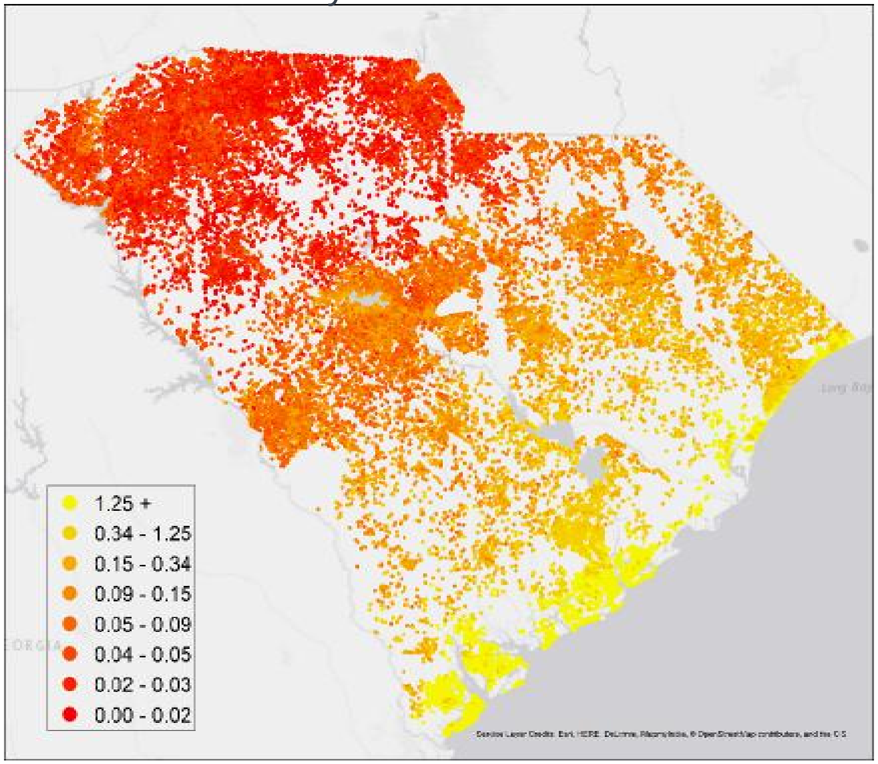
Demonstration – South Carolina Insurer

Relativity Point Maps – AAL vs Reinsurance Adjusted

Relativity Based on AAL

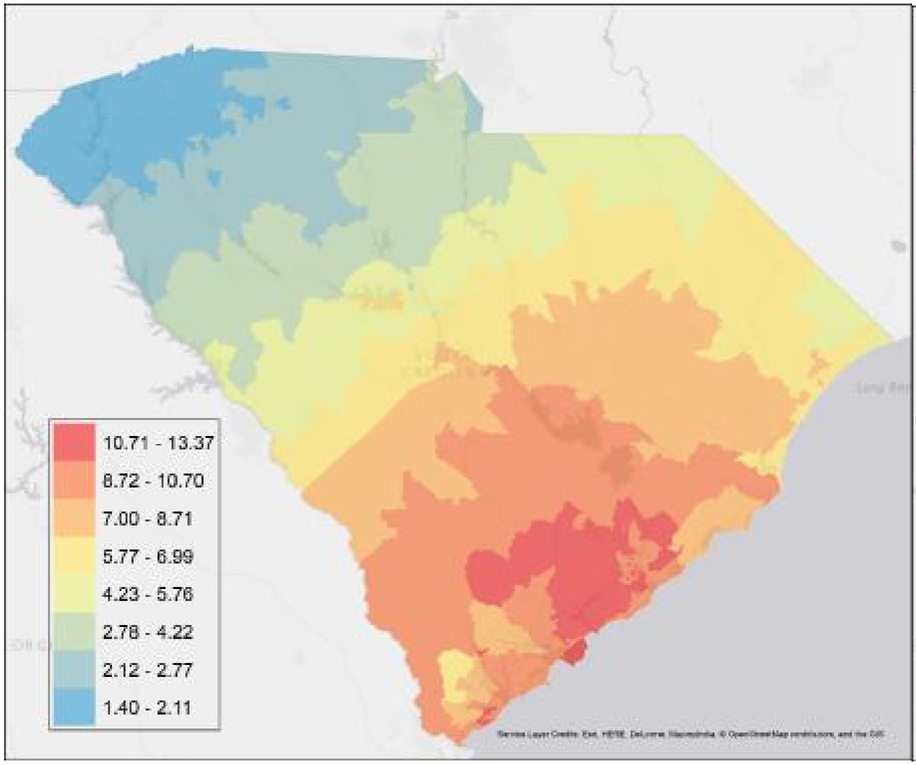


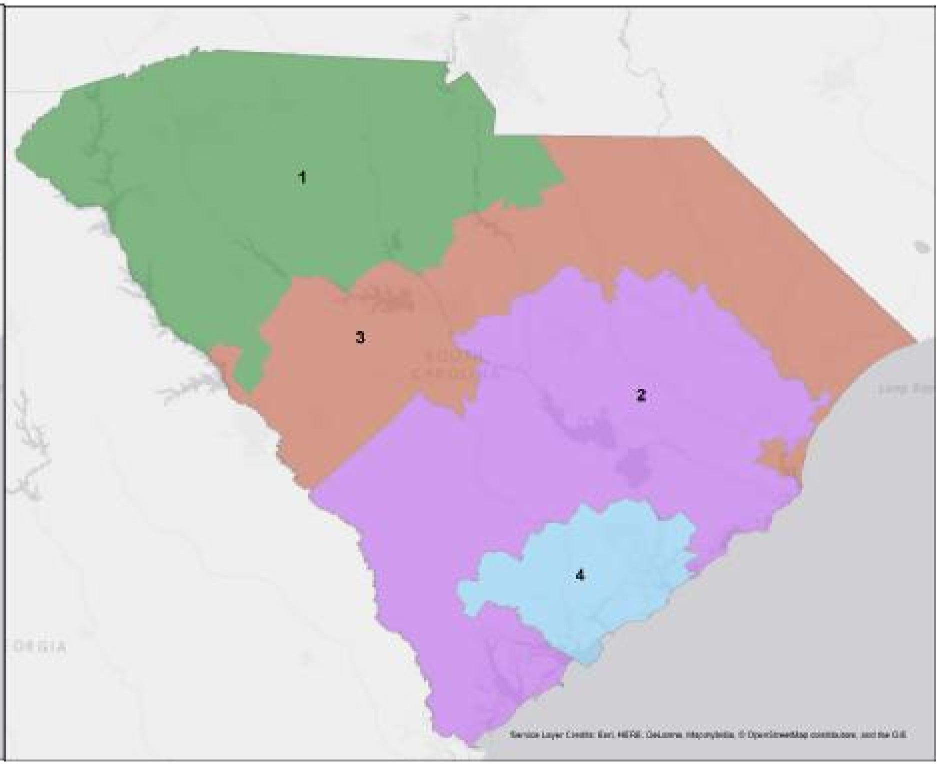
Relativity Based on Total Cost



Demonstration – South Carolina Hypothetical Insurer

Clustering Based on NCOR Multiples – Market Basket Sample Data

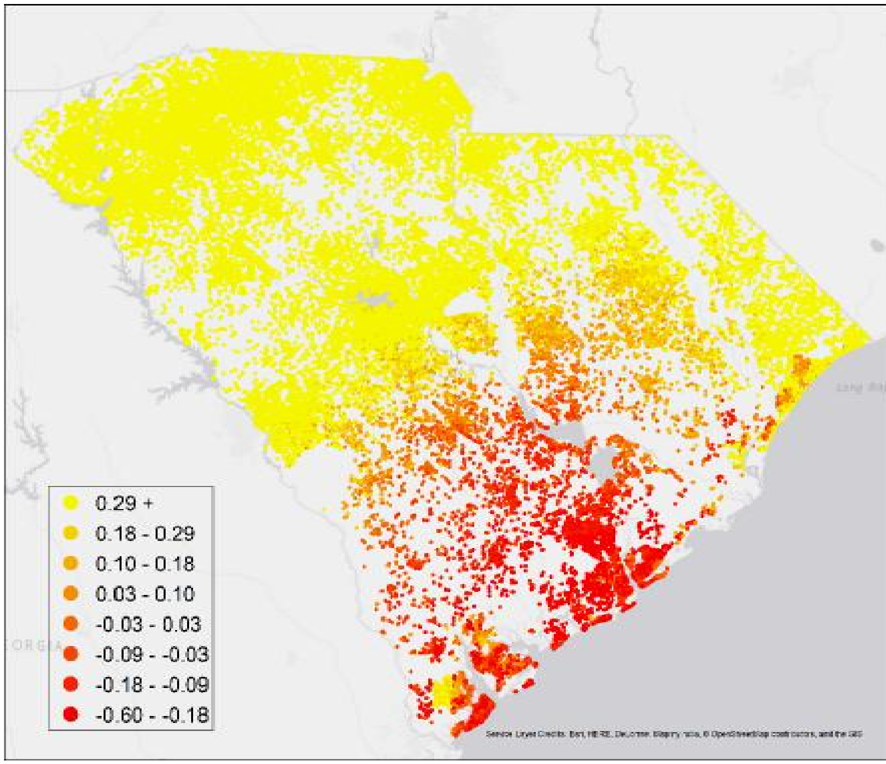




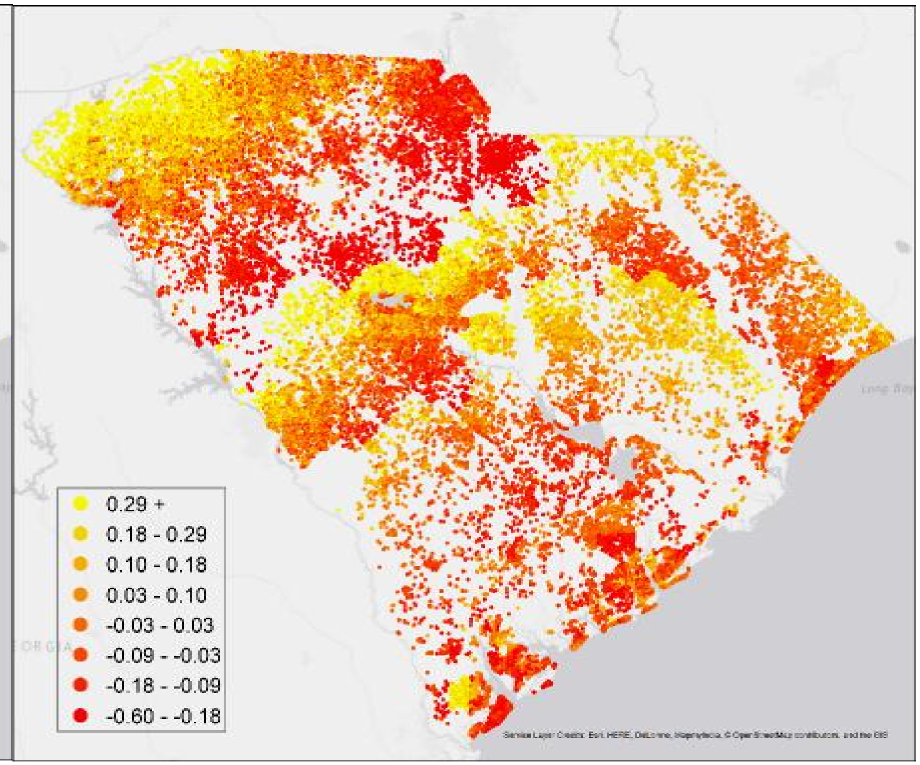
Demonstration – South Carolina Hypothetical Insurer

Deviance Point Maps – AAL vs Reinsurance Adjusted

Cost Deviance Based AAL

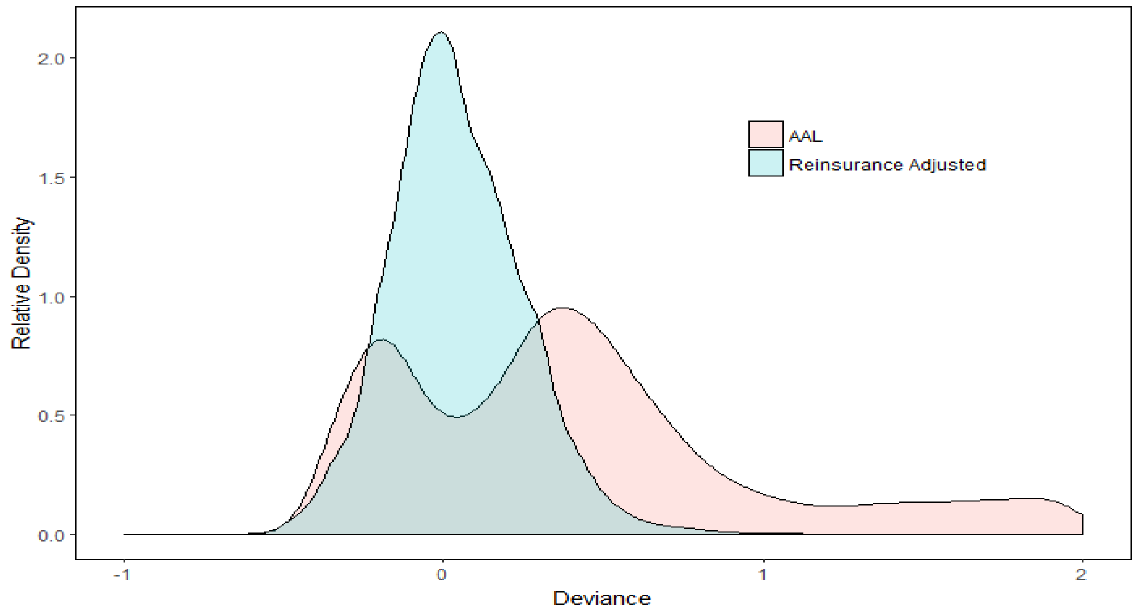


Cost Deviance after Clustering Procedure



Demonstration – South Carolina Hypothetical Insurer

Deviance Density Plot – AAL vs Reinsurance Adjusted



	Allocation Basis	
Statistic	AAL	Reinsurance Adjusted
Average	1.26	0.04
Standard Deviation	1.41	0.21

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Actuarial Allocation Technique Comparison Chart

	Concentration Risk	Extreme Scenario	Matching Rate/Cost	Order Dependence	Practicality
Proportional - AAL	û	û	û	û	ü
Proportional - VaR / TVaR	®	®	®	û	ü
Incremental/Marginal	®	®	®	û	û
Co-Measures	ü	®	ü	®	ü
Shapley Value	ü	®	ü	ü	û
Covariance Share	ü	ü	ü	ü	û
Event-Based by Reins. Layer	ü	ü	ü	®	®

Thank you

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