Easy Tree-sy
Everyday Applications of Decision Trees

CAS Ratemaking and Product Management

San Diego, CA

March 2017

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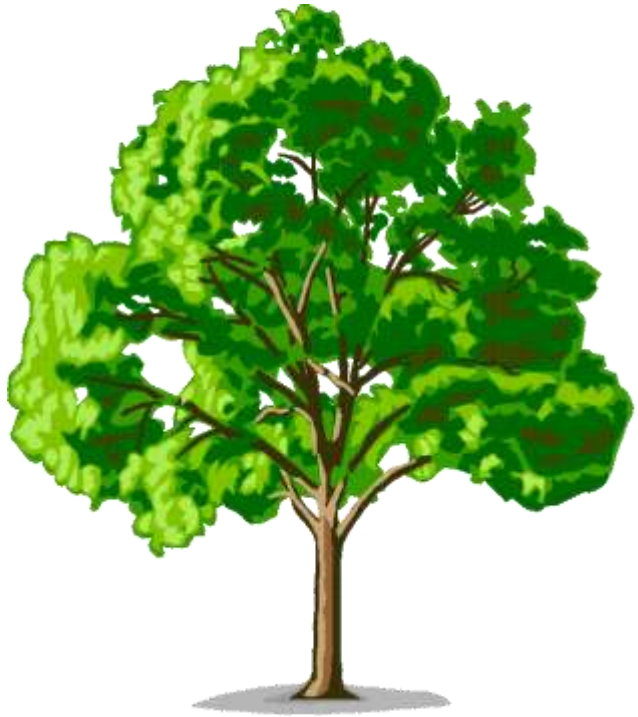
Introductions and Agenda

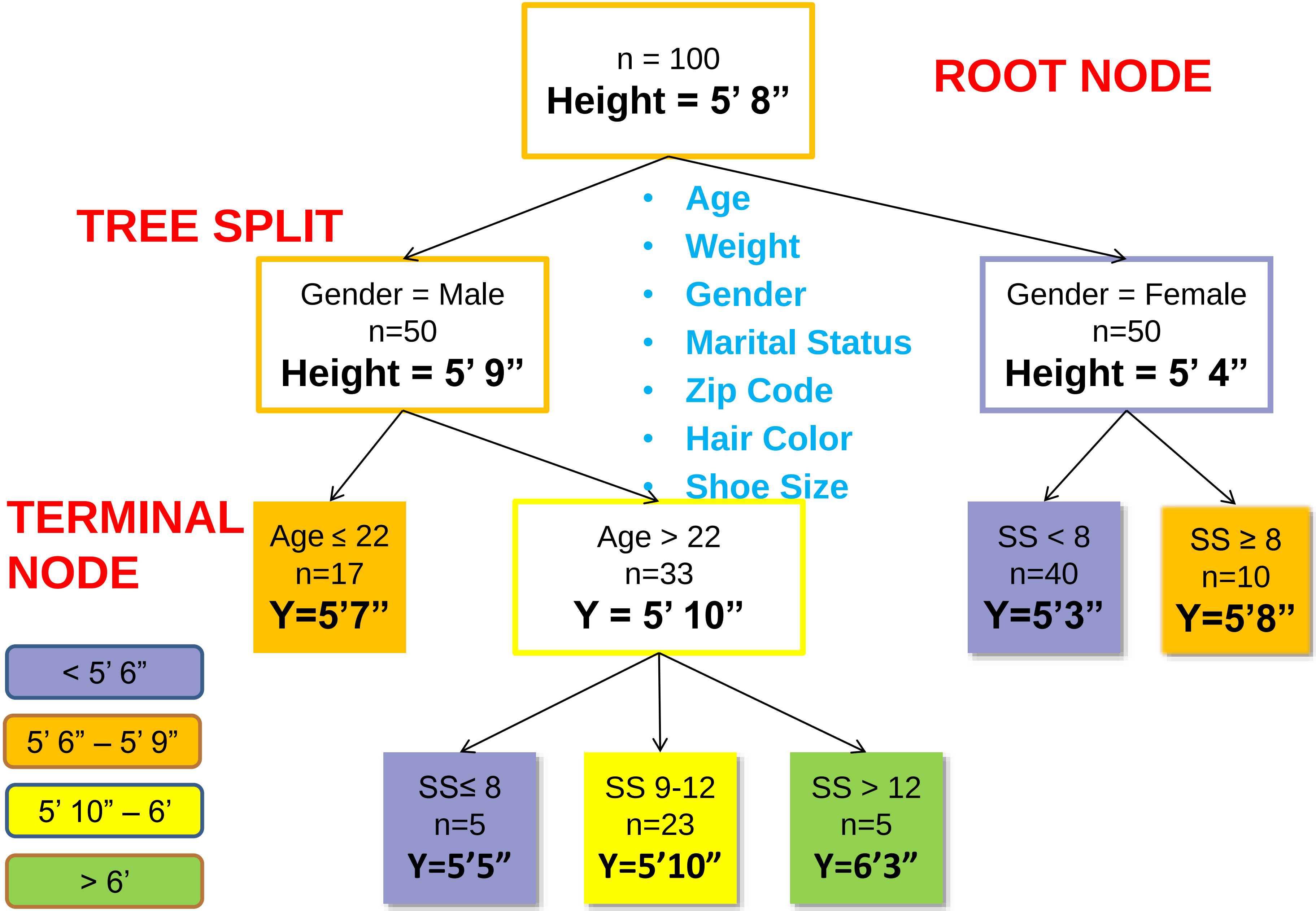
- Decision Tree Basics
  - An Example
  - Terminology
  - Objectives/Theory
- Applications of Decision Trees
- Case Study using Free Software
- Customization

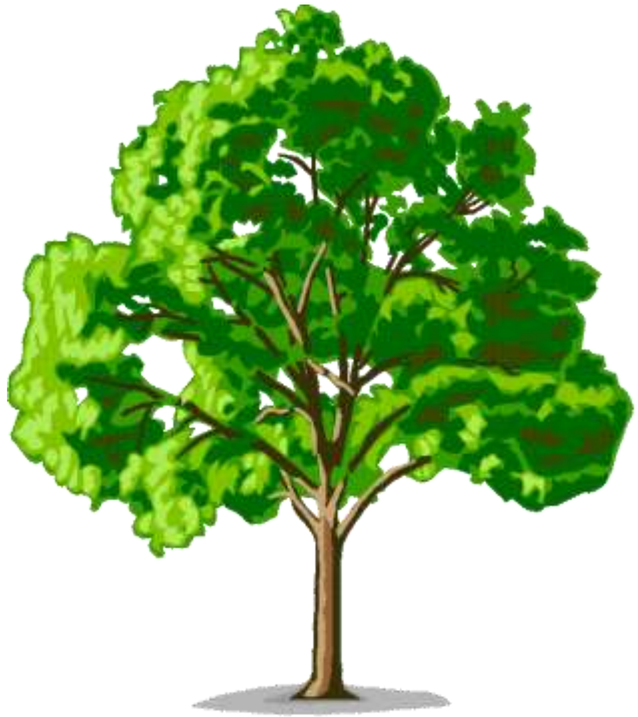


An Example

Estimate the height of an adult, given the following information:

- Age
- Weight
- Gender
- Marital Status
- Zip Code
- Hair Color
- Shoe Size


n = 100
Height = 5’ 8”
ROOT NODE
TREE SPLIT
Age
Weight
Gender
Marital Status
Zip Code
Hair Color
Shoe Size
Gender = Male
n=50
Height = 5’ 9”
Gender = Female
n=50
Height = 5’ 4”
TERMINAL NODE
Age ≤ 22
n=17
Y=5’7”
Age > 22
n=33
Y = 5’ 10”
SS < 8
n=40
Y=5’3”
SS ≥ 8
n=10
Y=5’8”
< 5’ 6”
5’ 6” – 5’ 9”
5’ 10” – 6’
> 6’
SS≤ 8
n=5
Y=5’5”
SS 9-12
n=23
Y=5’10”
SS > 12
n=5
Y=6’3”



Terminology

Target Response, Predicted Outcome, Dependent Variable

Y: Height

Explanatory, Predictor, Independent Variables

X_i: Age, Gender, Marital Status
Zip Code, Hair Color, Shoe Size

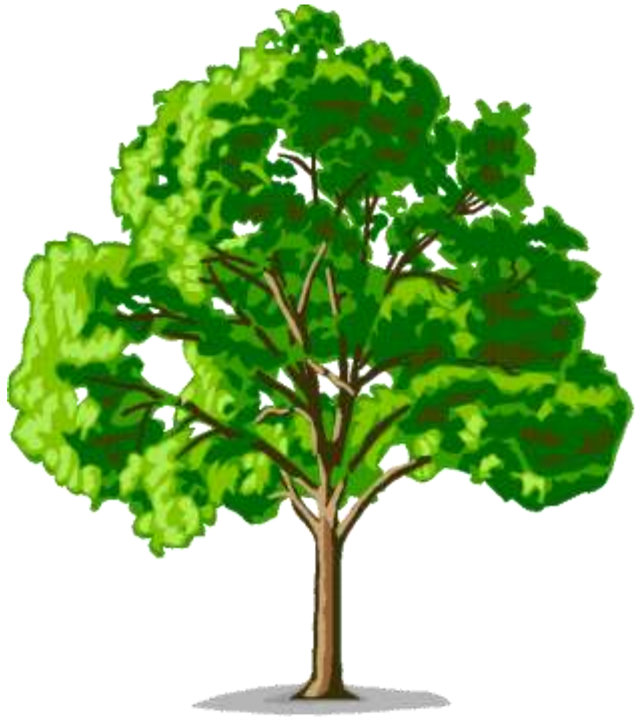
If the Target Variable is:

Categorical

➔ **Classification Tree**

Continuous

➔ **Regression Tree**



Objectives/Theory

Two Objectives:

Purity

➔ **Measure of variation**

Parsimony

➔ **Desire for simple**

The Process

- **Splitting Procedure**

  The domain space of explanatory variables $X_1,\ldots X_{\mathbf{n}}$ is split into two subsets where observed values in X_j belong to one of the subsets

  i.e. **< *s* or >= *s*** *OR* ***s_1=male s_2=female***

- **Improvement Value**

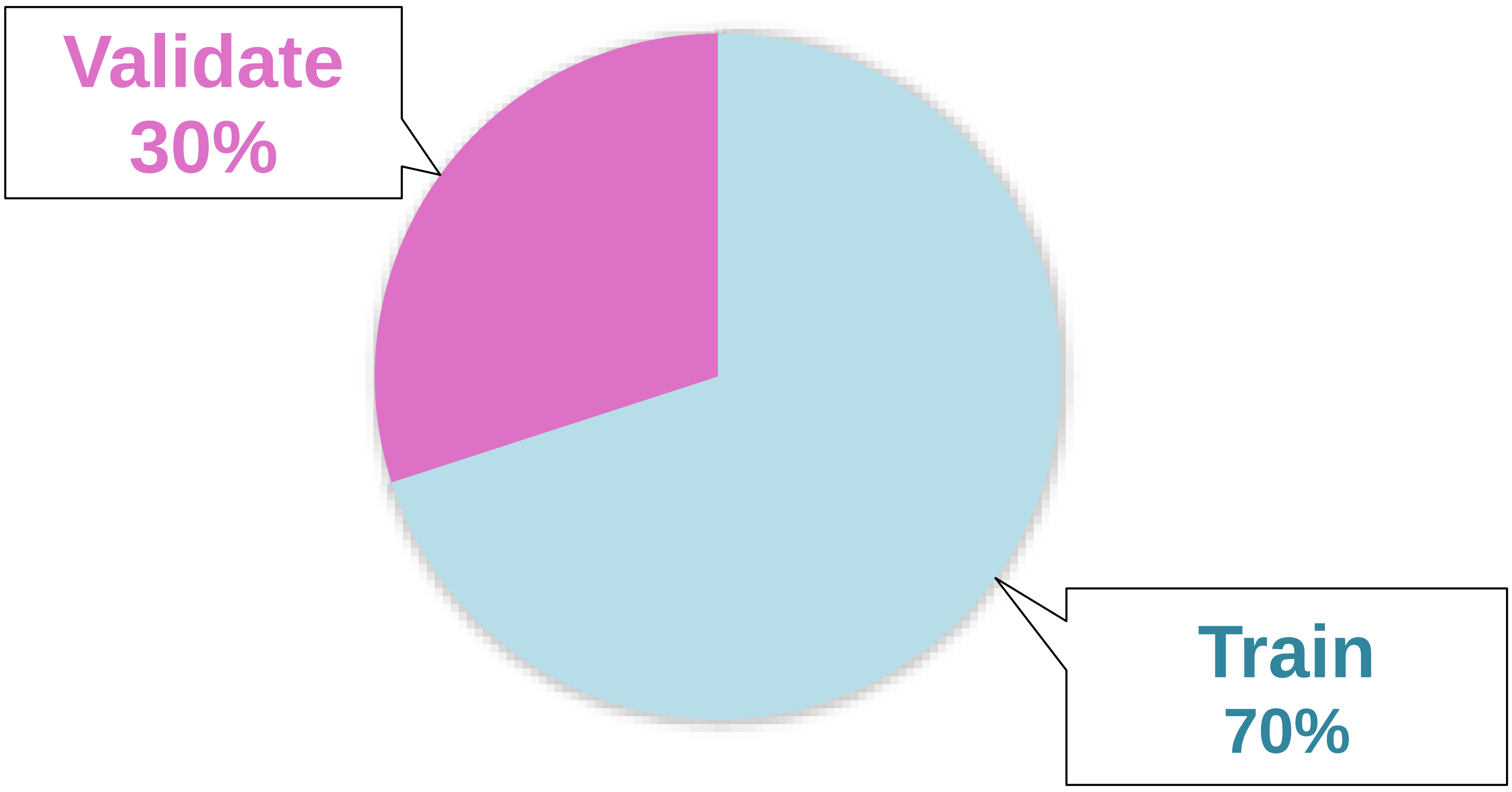
  The dimensions ***j*** and ***s*** above are chosen to minimize the error in the prediction among all such binary (two-leveled) trees. Process is iterated.

Stopping Criterion

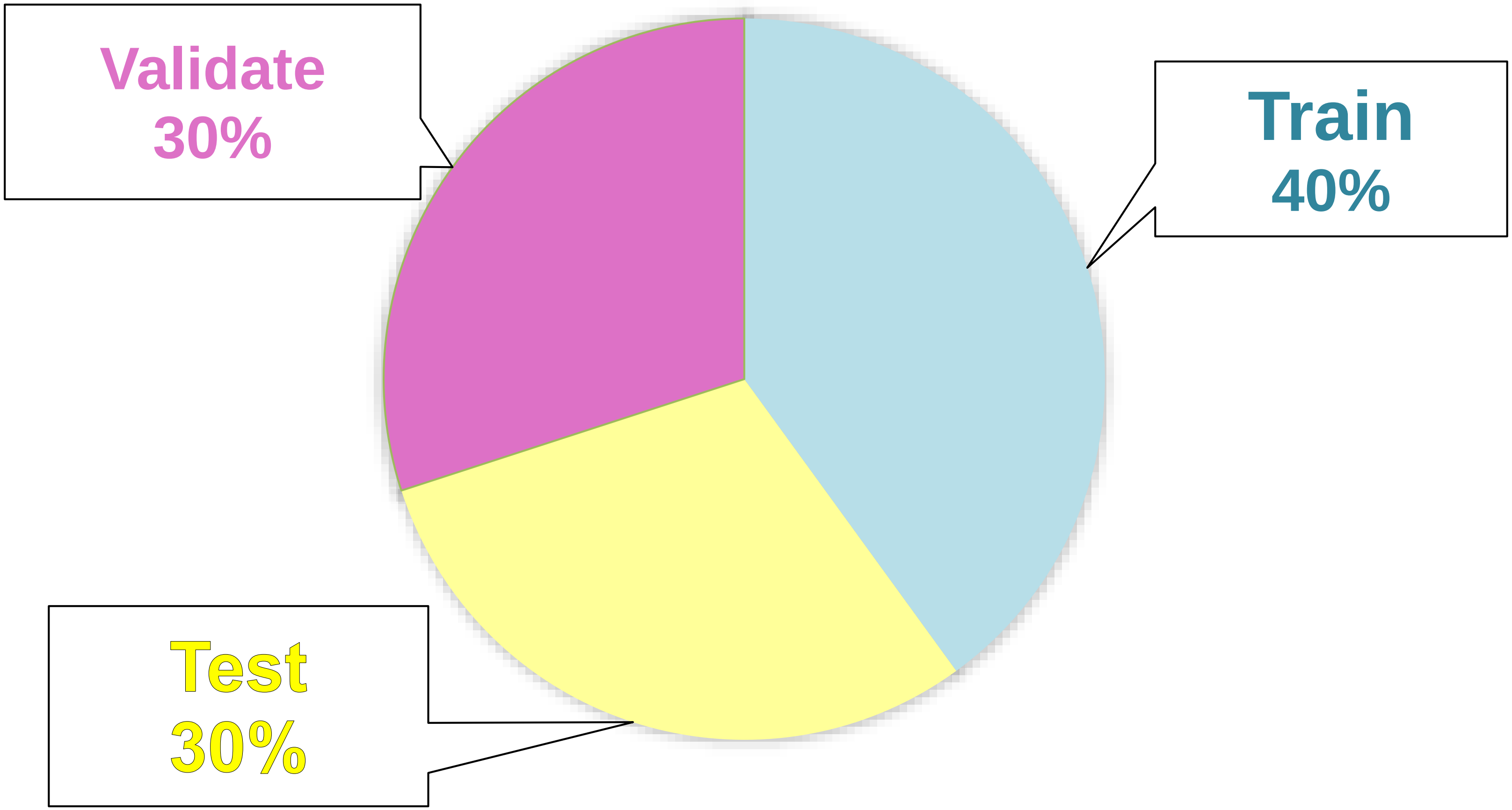
- No stopping criterion
- Minimum leaf (node) size
- Maximum number of levels or splits
- Let data determine the stopping criterion (see Appendix)

Validating Results - Avoiding Over Fit

The validation dataset ensures a way to accurately measure your model's performance.

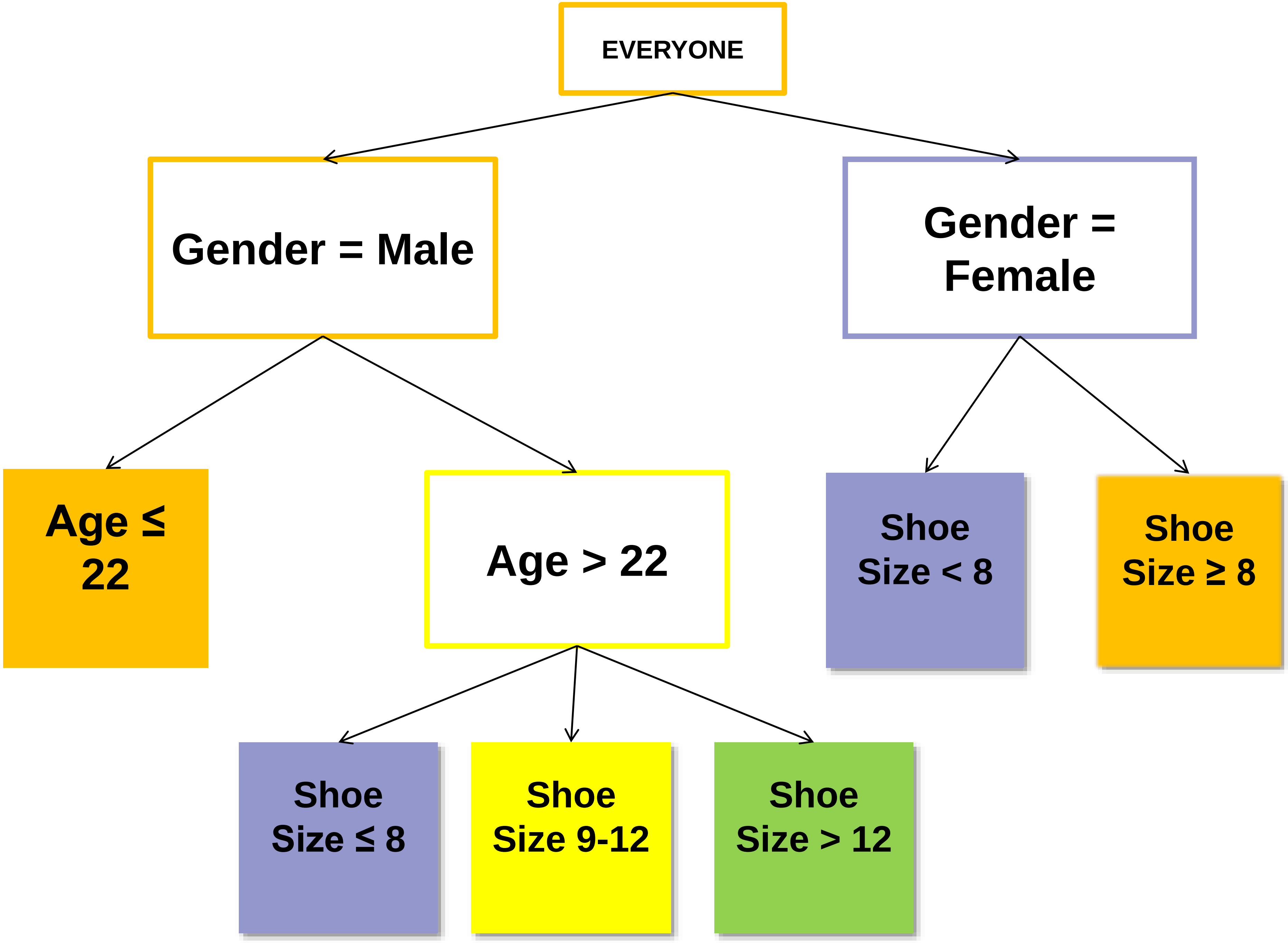


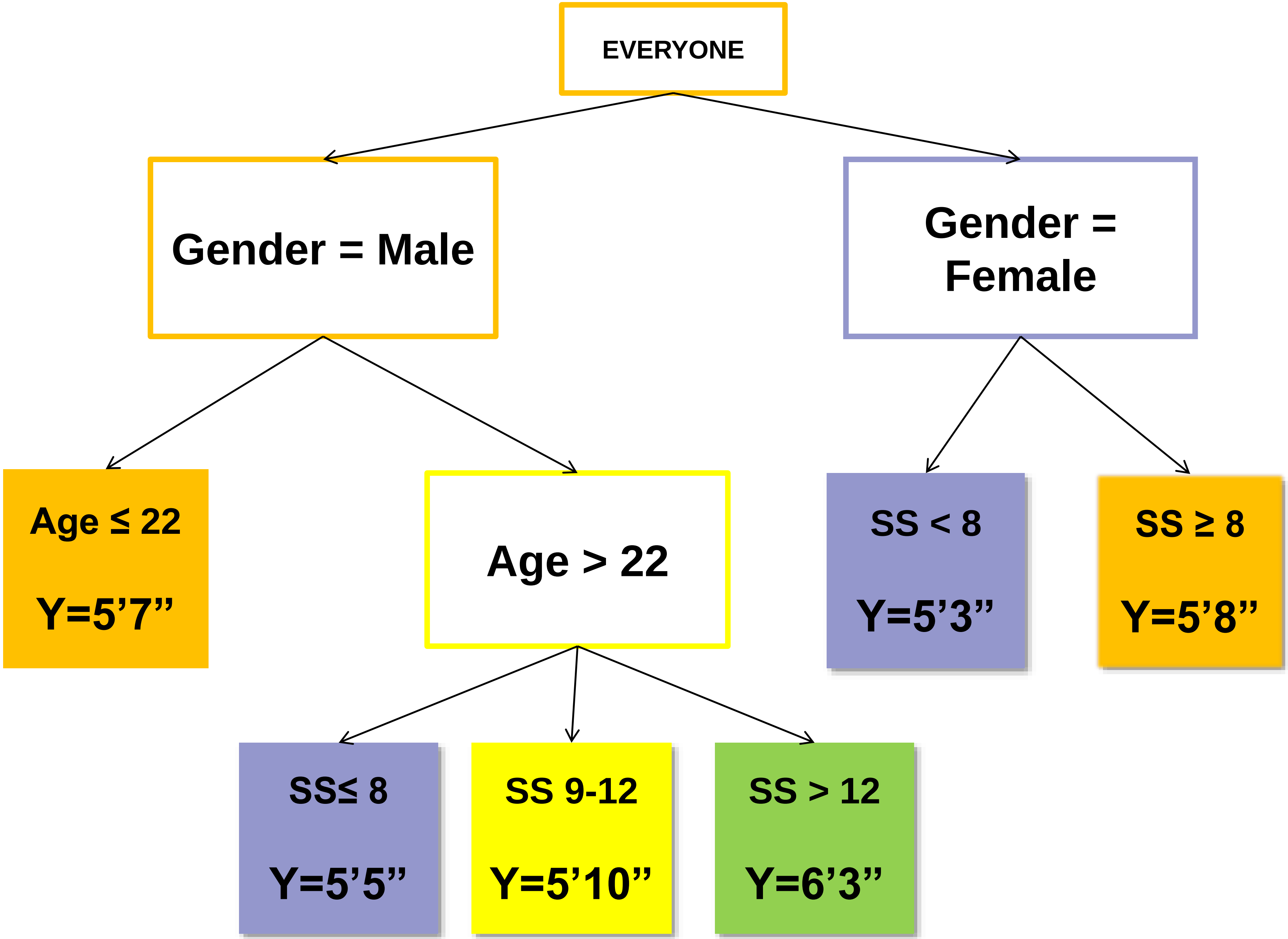
Validating Results - Avoiding Over Fit



Large datasets can be split into 3 unique subsets.

If there is time…


EVERYONE
Gender = Male
Gender = Female
Age ≤ 22
Age > 22
Shoe Size < 8
Shoe Size ≥ 8
Shoe Size ≤ 8
Shoe Size 9-12
Shoe Size > 12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SS≤ 8
Y=5’5”
SS 9-12
Y=5’10”
SS > 12
Y=6’3”

Appendix

Stopping Criterion – Regression Trees

- To begin, we need to define an error function $E()$ on any leaf of a tree. Think of $E()$ as a measure of how far the predicted are from observed
- Then, for a fixed $\alpha > 0$, find that tree T that minimizes

$$C_\alpha(|T|) \quad = \quad \sum_{k=1}^{|T|} E(L_k) + \alpha|T|$$

- $E(L_k)$ is the error contributed by the kth leaf and α is a parameter that rewards parsimony
- One can see that minimizing the cost complexity criterion $C_\alpha()$ requires a balance between predictive power and parsimony to be struck

Stopping Criterion – Regression Trees (cont.)

- Define
  1. $|L_k| = \sum_{\substack{i=1 \\ \mathbf{x}_i \in L_k}}^{K} w_i$
  2. $\bar{y}_k = \frac{1}{|L_k|} \sum_{\substack{i=1 \\ \mathbf{x}_i \in L_k}}^{K} w_i y_i$
- A standard choice for $E()$ is

$$E(L_k) = \sum_{\mathbf{x}_i \in L_k} w_i (y_i - \bar{y}_k)^2$$

- There are other standard functions for $E()$, for example
  1. $E(L_k) = \sum_{\mathbf{x}_i \in L_k} w_i |y_i - \bar{y}_k|$
  2. $E(L_k) = \sum_{\mathbf{x}_i \in L_k} w_i |y_i - \bar{y}_k|^p$ for $1 < p < 2$
- User may have choice on what functional form $E()$ may take depending on the software

Bibliography

- Hastie, T. et al. (2011) *The Elements of Statistical Learning: Data Mining, Inference, and Prediction (2nd Edition)*, Springer, New York.