Boston, Massachusetts
June 7–8, 2004

Seminar on Reinsurance

June 7-8, 2004

Advanced Exposure Rating
Beyond the Basics

Steve White, FCAS MAAA, Guy Carpenter
Kari Mrazek, FCAS MAAA, Employers Reinsurance Corporation

Advanced Exposure Rating
Topics

- History of Casualty Loss Curves
- ISO's Truncated Pareto
- Casualty Exposure Rating
- Using ILFs for Exposure Rating
- Working with Personal Auto Split Limits
- Property Exposure Rating – Using First Loss Scales
- Property Exposure Rating – Using PSOLD
- Stacking and Participation
- Miscellaneous Topics

Section 1

History of Casualty Loss Curves

And Concerns with some of them

ISO Casualty Loss Curves
History

- Truncated Pareto ("5-parameter Pareto") – prior to 1994
- Full Mixed Pareto (Pareto Soup) – 1994 thru 1998/1999
  - Single Mixed Pareto approximation
  - Truncated Pareto approximation
- Mixed Exponential (ME) – introduced in 1998/1999
  - Single Mixed Pareto approximation
  - Truncated Pareto approximation

ISO Casualty Loss Curves
Advantages/Disadvantages

- Pareto Soup
  - Preferred by many reinsurers because of thicker tail
  - (Not updated, new tables not available)
  - (Many parameters needed)
- Truncated Pareto Approximation to the Pareto Soup
  - Same as Pareto Soup
  - Fewer parameters (5)
  - (Difficult to use below truncation point)
- Mixed Pareto Approximation to the Pareto Soup
  - Same as Pareto Soup
  - Fewer parameters (5)

ISO Casualty Loss Curves
Advantages/Disadvantages

- Mixed Exponential (ME)
  - Current ISO Methodology
  - Source of Latest Information (by-State Prem/Ops and Auto Groups)
  - Better fit than Mixed Pareto over wide range of loss sizes
  - Simpler, fewer parameters than Pareto Soup, more flexible
  - (Many Reinsurers believe these are too thin in the tail)
- Truncated Pareto Approximation to the Mixed Exponential
  - Same as Mixed Exponential
  - (Difficult to use below truncation point)
- Mixed Pareto Approximation to the Mixed Exponential
  - Same as Mixed Exponential

ISO Casualty Loss Curves
Formulas

Mixed Exponential

$$CDF\_ME(x;\overline{\mu},\overline{w}) = \sum_{i=1}^{8} w_i \times \left(1 - e^{-\frac{x}{\mu_i}}\right)$$

Truncated Pareto (w/ Split Uniform)

$$CDF\_TP(x;b,q,p,s,t) = \begin{cases} \dfrac{x \times p \times (t-s)}{s \times t} & s \geq x \\ \dfrac{p \times (t-s)}{t} + \dfrac{(x-s) \times p \times s}{t \times (t-s)} & t \geq x > s \\ p + (1-p)\left[1 - \left(\dfrac{b+T}{b+x}\right)^{q}\right] & x > t \end{cases}$$

Single Mixed Pareto

$$CDF\_MP(x;b_1,q_1,p,b_2,q_2) = (1-p)\left(1 - \left(1 + \frac{x}{b_1}\right)^{-q_1}\right) + p\left(1 - \left(1 + \frac{x}{b_2}\right)^{-q_2}\right)$$

Full Mixed Pareto (Pareto Soup)

$$CDF\_PS(x;\overline{b}_1,q_1,\overline{p},\overline{b}_2,q_2,\overline{w}) = \sum_{i=1}^{7} w_i CDF\_MP(x;b_{1,i},q_1,p_i,b_{2,i},q_2)$$

Section 2

ISO’s Truncated Pareto

Estimating Losses below the Truncation Point T

Truncated Pareto Curve
Estimating below Truncation Point

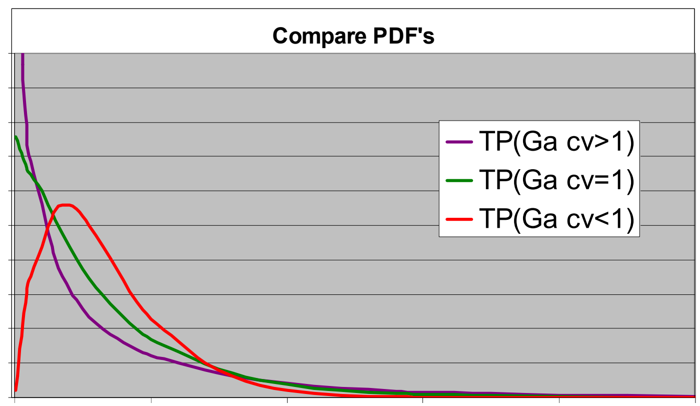
- Standard Truncated Pareto parameters
  - B, Scale Parameter of the Ballasted Pareto
  - Q, Shape Parameter of the Ballasted Pareto
  - T, Truncation point
  - P, Probability of being less that T
  - S, Mean of the Losses smaller than T
- For losses greater than T, the Curve is a Truncated Pareto
- For losses less than T, the Curve is undefined (other than the mean)

Truncated Pareto Curve
Uniform Option

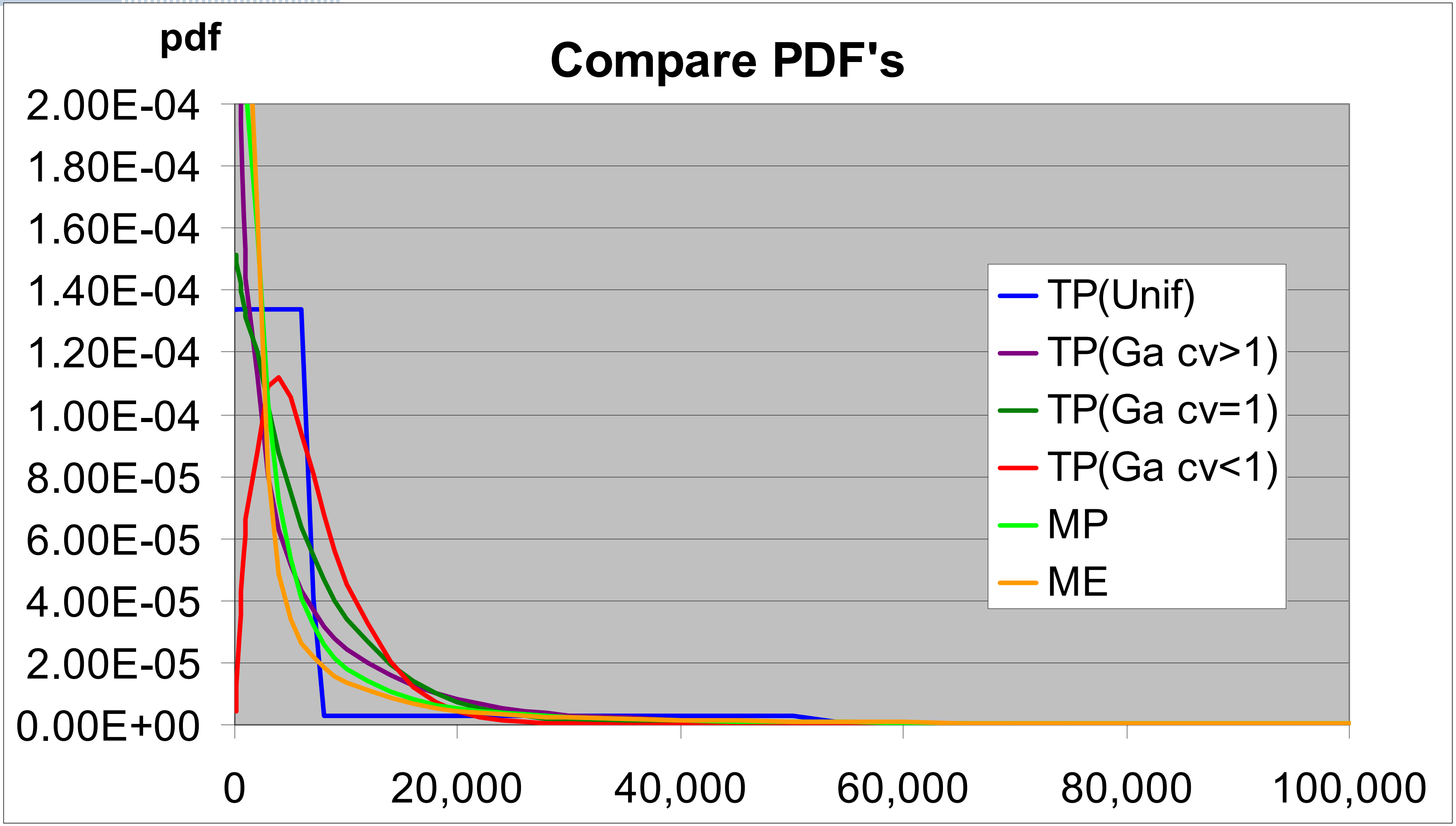
- One option is to split the curve below T into two Uniform Distributions
  - For $0 < X \leq S$, $f(x) = P \times (T - S) / (S \times T)$
  - For $S < X \leq T$, $f(x) = P \times S / (T \times (T - S))$
  - For $T < X$, $f(x)$ = Ballasted Pareto with weight $(1-P)$

Truncated Pareto Curve
Gamma Option

- Another option is to choose another Distribution (e.g. Gamma) to model the losses less than T
- Use solver for the Gamma CV and Theta so that the PDFs at T match and so that the mean of the truncated gamma is S.
  - You can solve for a CV which will match the two PDFs
  - You can solve for Theta which will match the means
  - Or solve both at the same time
- The Gamma can take many shapes, the CV determines the shape below T
  - CV < 1, then you get “Log Normal-ish”
  - CV = 1, Exponential Shape
  - CV > 1, “Hyper Exponential”

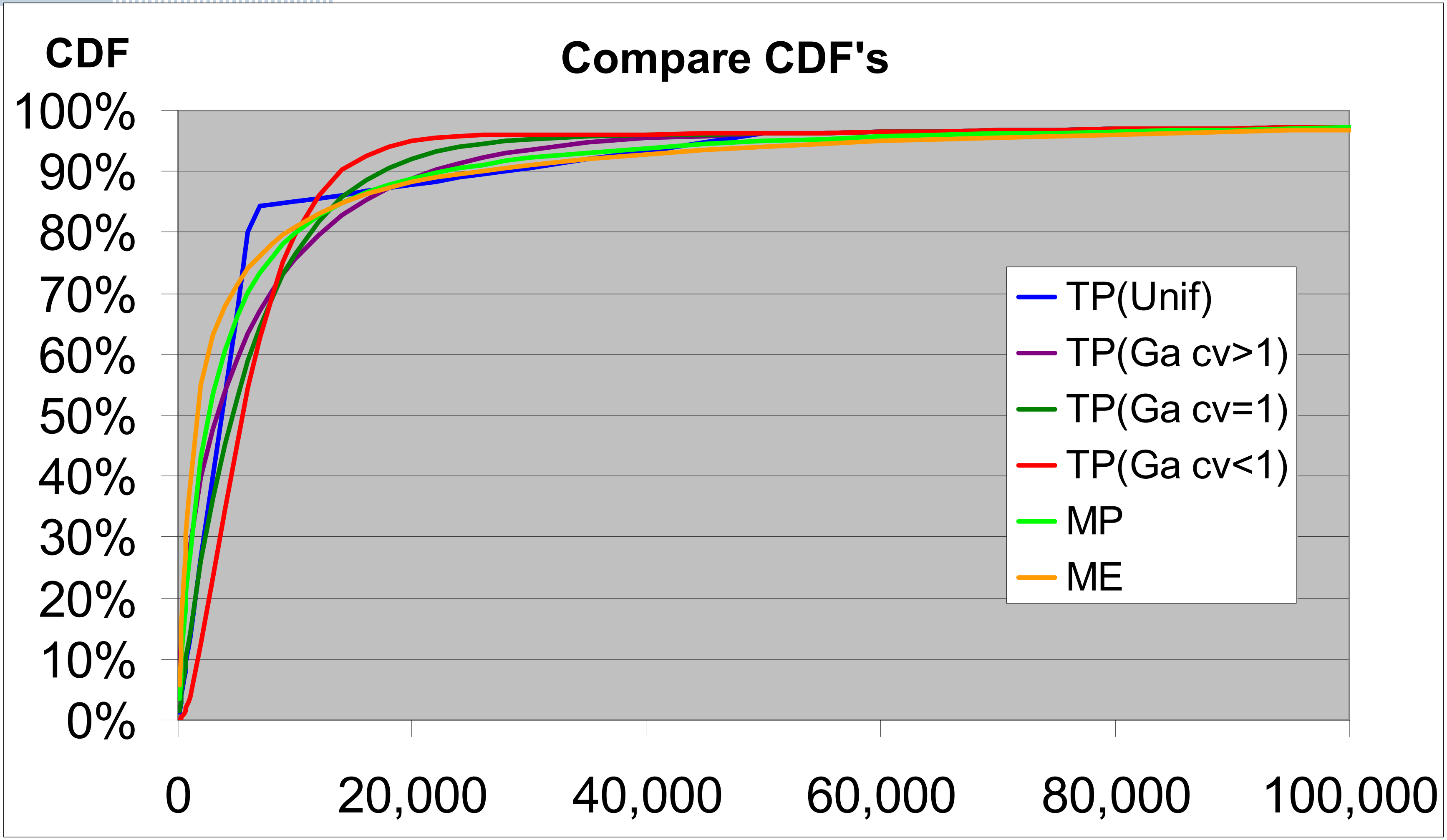


Truncated Pareto Curve
PDF Comparison

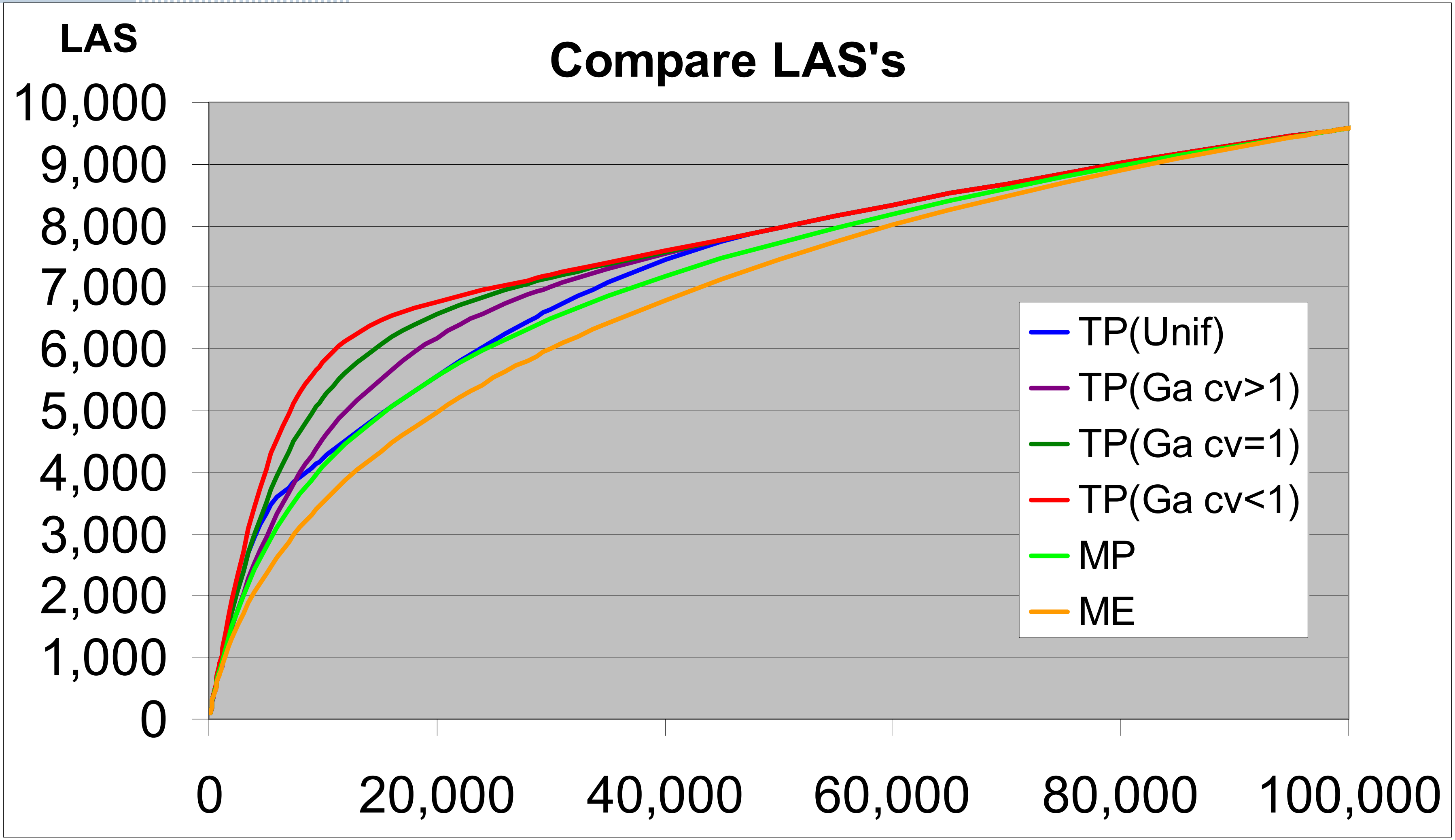


Truncated Pareto Curve

CDF Comparison for a representative table



Truncated Pareto Curve
LAS Comparison



Section 3

Casualty Exposure Rating

Working with ISO's Mixed Exponential

The Mixed Exponential
Issues – Data Limitations

Issues

- Many Reinsurers feel that the tail of the Mixed Exponential is too thin
- ISO states that there are limitations to the data and recommended usage of the Mixed Exponential

Line of Business	Truncation	Tempering	Max Filed	Max Limit
Commercial Auto-Light/Medium/All	2,000,000	Not Tempered	10,000,000	10,000,000
Commercial Auto-Heavy/XHeavy/Zone	1,000,000	Not Tempered	10,000,000	10,000,000
Prem/Ops	5,000,000	Excluded**	10,000,000	10,000,000
Products	4,000,000	Excluded**	10,000,000	10,000,000
Hospital/Physicians/Surgeons	5,000,000	1,000,000	3,000,000	10,000,000
Dentists/Allied Healthcare/Nursing	2,000,000	1,000,000	3,000,000	10,000,000
Veterinarians	200,000	Not Tempered	3,000,000	10,000,000

- Unfortunately knowing this doesn't help us exposure rate high excess layers

The Mixed Exponential
Some Common Solutions

- Use the Mixed Exponential and assume everything is fine
- Use the Truncated Pareto Approximation
- Use the Pareto Soup since many reinsurers feel more comfortable with the tail of the Pareto Soup compared to the Mixed Exponential

The Mixed Exponential

Common Beliefs, Urban Legends and Myths #1

Pareto Soup is thicker tailed than the Mixed Exponential

- Generally True But……
- For the lighter exposures, many times the Pareto Soup is lighter than the Mixed Exponential in the high layers (ie 5M xs 5M)
  - PremOps/Products 1
    - Prem1 – Pareto Soup is much lighter than Mixed Exponential
    - Prod A – Pareto Soup and Mixed Exponential are about the same
  - Commercial Auto
    - Light/Medium – Pareto Soup is lighter than the Mixed Exponential in most state groups
    - All Other – Pareto Soup is lighter than the Mixed Exponential in some state groups
  - Medical
    - Pareto Soup thicker than Mixed Exponential in all cases
- For heavier exposures, the Pareto Soup is typically heavier tailed than the Mixed Exponential

The Mixed Exponential
Common Beliefs, Urban Legends and Myths #2

Truncated Pareto is thicker tailed than the Mixed Exponential

- Many times the Truncated Pareto is lighter than the Mixed Exponential in the high layers (ie 5M xs 5M)
  - PremOps/Products
    - Prem1,2,3 – Truncated Pareto is lighter than Mixed Exponential
    - ProdA – Truncated Pareto is lighter than Mixed Exponential
  - Commercial Auto
    - Light/Medium – Truncated Pareto is lighter than the Mixed Exponential in most state groups
    - Other Tables – Varied by Table and Group
- For Attachment points less than 100K, Truncated Pareto can be significantly Higher than the Mixed Exponential
  - Especially true when Truncation point, T, is low (Comm Auto)
  - Overstated LAS for low limits, implies an overstated credit for deductible policies

The Mixed Exponential
Advantages of Using the Latest Mixed Exponential

- Mixed Exponential includes the most recent available information
  - State Specific Prem/Ops
  - Updated Class Code definitions associated with the Mixed Exponential
  - New State Groupings for Commercial Auto
  - Variable ALAE factors in MILD were not available prior to the Mixed Exponential
- Latest Pareto Soup parameters are from 1999 and are not getting any younger

“Adjusted” Mixed Exponential
Compromise between Mixed Exponential and Pareto Soup

A proposed method that blends the most recent information from the latest Mixed Exponential methods with a distribution that has a thicker tail than the Mixed Exponential

- Assume there is uncertainty on the mean parameters of the Mixed Exponential
  - Inclusion of uncertainty will thicken the tail of the distribution
  - If you assume the uncertainty is modeled by an Inverse Gamma distribution, the resulting distribution is a Ballasted Pareto
  - Therefore the Mixed Exponential becomes a “Mixed Pareto”
  - This “Mixed Pareto” has the same number of Ballasted Paretos as the Mixed Exponential has Exponentials (not to be confused with the Mixed Pareto which is one of the approximations to the Mixed Exponential published by ISO)

Adjusted Mixed Exponential

Using a Mixing Distribution on a Mixed Exponential to get a Mixed Pareto

$$h(y;\phi,\theta) = \int f(y;\phi,\psi)g(\psi;\theta)d\psi$$

$$h(y;\phi,\theta) = f(y;\phi,\hat{\psi})\psi\, g(\psi;\theta)$$

$f(y;\phi,\psi)$ - structural loss distribution with independent parameter(s) ϕ and dependent parameter(s) ψ

$g(\psi;\theta)$ - mixing distribution on parameter(s) ψ with it's own parameter(s) ϕ

$h(y;\phi,\theta)$ - mixed distribution with parameters ϕ and $\boldsymbol{\theta}$

Adjusted Mixed Exponential
A Simple Example using Theoretical Mixing

Ballasted Pareto can result from an Exponential distribution mixed with an Inverse Gamma distribution

$$BP(y;\theta,\alpha) = Exp(y;\mu)\,\hat{\mu}\,InvGA(\mu;\theta,\alpha)$$

Choosing Parameters for the distributions

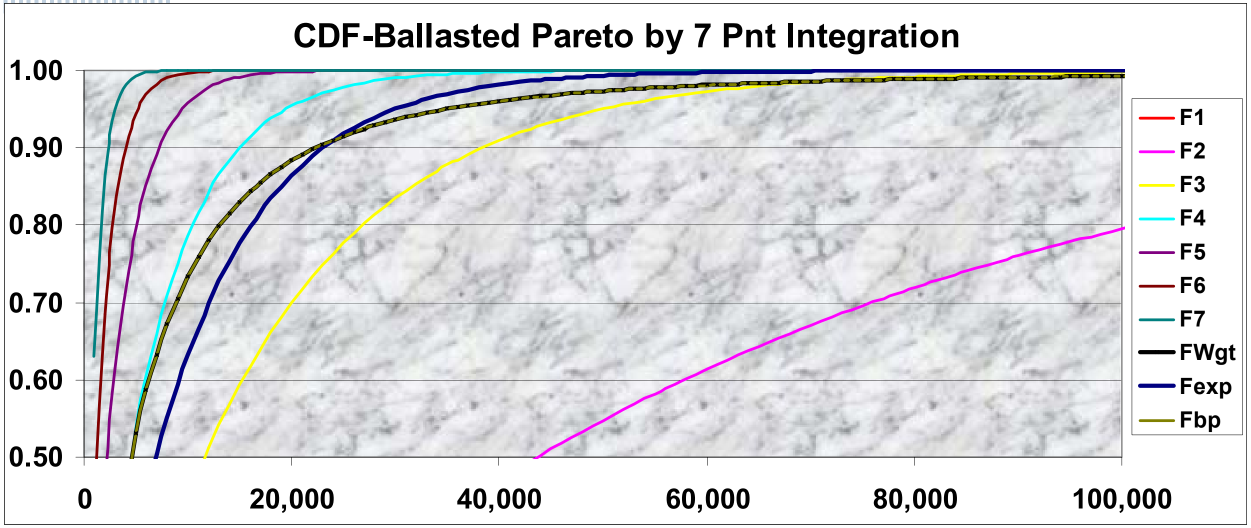
The Exponential has an assumed mean, ***μ***

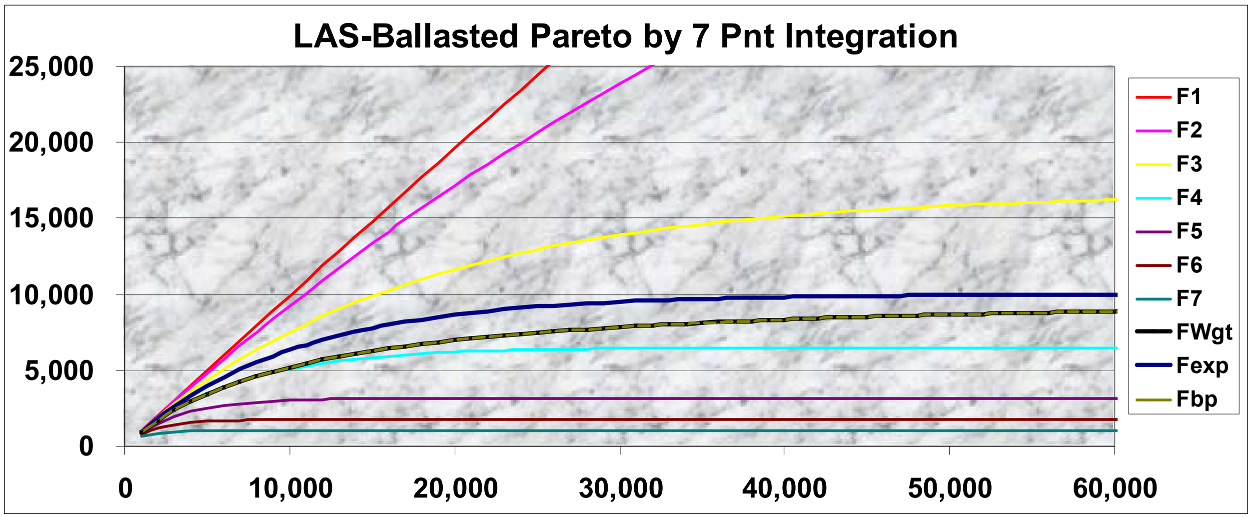
- Select parameters for the Inverse Gamma Distribution
- You can assume an Inverse Gamma with a mean, μ, and an assumed CV
  - $\alpha = 2 + 1/CV^2$, $\theta = \mu^*(\alpha - 1)$
  - Nice intuitive approach, 2nd moment exists for Ballasted Pareto
  - For a thicker tailed Ballasted Pareto, you can select an $\alpha < 2$. While this a valid approach, it lacks some of the intuitive appeal.

$$pdf_{BP}(x;\theta,\alpha) = \frac{\alpha}{\theta}\frac{1}{\left(1+\left(x/\theta\right)\right)^{(\alpha+1)}} \qquad pdf_{Exp}(x;\mu) = \frac{1}{\mu}e^{-(x/\mu)} \qquad pdf_{IG}(\mu;\theta,\alpha) = \frac{1}{\theta}\frac{\left(\theta/\mu\right)^{\alpha+1}}{\Gamma(\alpha)}e^{-\theta/\mu}$$

$$CDF_{BP}(x;\theta,\alpha) = 1 - \frac{1}{\left(1+\left(x/\theta\right)\right)^{\alpha}} \qquad CDF_{Exp}(x;\mu) = 1 - e^{-(x/\mu)}$$

Adjusted Mixed Exponential
A Simple Example using Theoretical Mixing





Adjusted Mixed Exponential

A “Mixed” Mixed Exponential becomes a “Mixed” Pareto

A single Ballasted Pareto can result from an single Exponential distribution mixed with an Inverse Gamma distribution

$$BP(y;\theta_i,\alpha) = Exp(y;\mu_i) \bigwedge_{\mu_i} InvGA(\mu_i;\theta_i,\alpha)$$

$$\theta_i = \mu_i \times (\alpha - 1); \alpha = 2 + 1/CV^2$$

The same mixing idea can be extended to a weighted average of Exponentials (Mixed Exponential) to created a weighted average of Ballasted Paretos

$$\sum_i w_i \times BP(y;\theta_i,\alpha) = \sum_i w_i \times Exp(y;\mu_i) \bigwedge_{\mu_i} InvGA(\mu_i;\theta_i,\alpha)$$

Section 4

Using ILFs for Exposure Rating

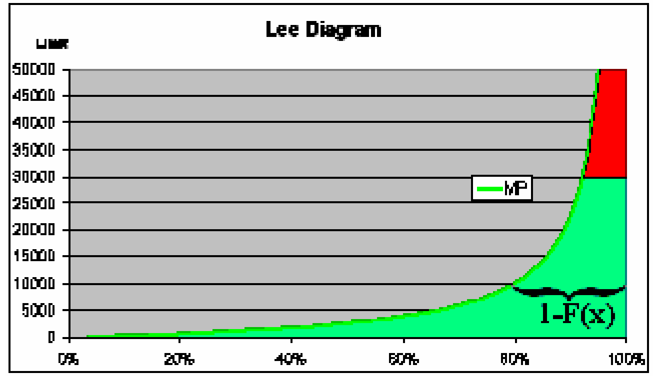
Using ILFs
Considerations

Sometimes the ceding company (or other company) ILFs are the only information available

- If the deal is sessions rated, you may be required to use a company ILF, is the resulting rate fair
- Does ILF include risk and expense loads
- Is ILF sufficiently detailed or is interpolation required

Using Ceding Company ILFs
A Quick Review of ILFs

$$ILF(X) = \frac{LAS(X)}{LAS(B)} = \frac{\int_0^X [1-F(x)]dx}{\int_0^B [1-F(x)]dx} = \frac{\int_0^X [1-F(x)]dx}{LAS(B)}$$



The ILF is non-decreasing (non-negative 1st derivative)

$$\frac{d\ ILF(X)}{dX} = \frac{d\left\{\frac{1}{LAS(B)}\int_0^X [1-F(x)]dx\right\}}{dX} = \frac{[1-F(X)]}{LAS(B)} \geq 0$$

The ILF is non-decreasing at a decreasing rate (non-positive 2nd derivative)

$$\frac{d^2\ ILF(X)}{dX^2} = \frac{-f(X)}{LAS(B)} \leq 0$$

LAS(B), the limited average severity at the base limit is constant with respect to X

Using Ceding Company ILFs
A few quick tests of ILFs

First Order test, ILFs should be strictly non-decreasing

$$\frac{d\ ILF(X)}{dX} \geq 0$$

•Fortunately, I think every ILF (excluding typos) I have seen have passed this test

Second Order Test, ILFs should be non-decreasing at an decreasing rate

$$\frac{d^2\ ILF(X)}{dX^2} \leq 0$$

- I have frequently seen this test violated, even when the ILFs were supposedly based on actual data
- Test=0 implies f(x)=0 which means there is zero probability of loss in the range where the second derivative is equal to zero
- Test > 0 implies f(x)<0 which means there is negative probability of loss in the range where the second derivative is positive

Using Ceding Company ILFs

Good ILFs

Limit	ILF	F.O.	S.O.
1,000,000	1.000	2.3E-07	-7.2E-14
2,000,000	1.231	1.6E-07	-3.4E-14
3,000,000	1.390	1.3E-07	-2.0E-14
4,000,000	1.516	1.0E-07	-2.1E-14
5,000,000	1.621	8.4E-08	-7.1E-15
7,500,000	1.830	6.6E-08	
10,000,000	1.995		

$$\frac{(1.231-1.000)}{(2M-1M)}$$

$$\frac{(8.4\times10^{-08}-1.0\times10^{-07})}{(5M-4M)}$$

should be positive and decreasing

should be negative

Example ILF is of the form

$$ILF(X)=\left(\frac{X}{1M}\right)^{.3};\frac{d\,ILF(X)}{dX}=(.3)\left(\frac{X}{1M}\right)^{-.7};\frac{d^2\,ILF(X)}{dX^2}=(-.7)(.3)\left(\frac{X}{1M}\right)^{-1.7}$$

This is a very common functional form for severe ILFs

Using Ceding Company ILFs

Bad ILFs

Limit	ILF	F.O.	S.O.
1,000,000	1.000	2.3E-07	0.0E+00
2,000,000	1.231	2.3E-07	-1.8E-13
3,000,000	1.462	5.3E-08	5.2E-14
4,000,000	1.516	1.0E-07	-6.3E-14
5,000,000	1.621	4.2E-08	0.0E+00
7,500,000	1.726	4.2E-08	
10,000,000	1.831		

Probability of exceeding 2M same as exceed 1M, therefore no probability of being in the range 1M-2M

Probability of exceeding 4M is greater than the probability of exceeding 3M, therefore there is a negative probability of being in the range 3M-4M

If your ILF fails these tests, then it is an invalid ILF and you likely will not be able to get a good results from the ILF

Using Ceding Company ILFs
Interpolating between ILFs

Same rules apply to the interpolation routine as apply to the ILFs

- First Order test $\frac{d\, ILF(X)}{dX} \geq 0$
- Second Order Test $\frac{d^2\, ILF(X)}{dX^2} \leq 0$

Common Interpolation Routines

- Linear
- Log-Linear
- Log-Log-Linear
- Power
- Fitted CDF (Single Parameter Pareto Example)

Let's test these routines assuming you have a two point Table of ILFs

X_{LO}	$ILF(X_{LO})$	where	$X_{HI} > X_{LO}$
X_{HI}	$ILF(X_{HI})$	and	$ILF(X_{HI}) \geq ILF(X_{LO})$

Using Ceding Company ILFs
Testing Linear Interpolation

$$ILF(x) = ILF(X_{Low}) + \frac{(x - X_{Low})\left(ILF(X_{Hi}) - ILF(X_{Low})\right)}{\left(X_{Hi} - X_{Low}\right)}$$

$$\frac{d\,ILF(X)}{dX} = \frac{\left(ILF(X_{Hi}) - ILF(X_{Low})\right)}{\left(X_{Hi} - X_{Low}\right)} \geq 0$$ Passes Test 1

$$\frac{d^2\,ILF(X)}{dX^2} = 0$$ Fails Test 2

- Most people acknowledge that linear interpolation is bad
- Since it is so easy to do, many can’t seem to resist using it

Using Ceding Company ILFs

Testing Log_y-Linear Interpolation

$$ILF(x) = Exp\left(Ln(ILF(X_{Low})) + \frac{(x - X_{Low})(Ln(ILF(X_{Hi})) - Ln(ILF(X_{Low})))}{(X_{Hi} - X_{Low})} \right)$$

$$\frac{d\,ILF(X)}{dX} = ILF(x)\left(\frac{\ln(ILF(X_{Hi})) - \ln(ILF(X_{Low}))}{X_{Hi} - X_{Low}} \right) \geq 0$$

Passes Test 1

$$\frac{d\,ILF(X)}{dX} = ILF(x)\left(\frac{\ln(ILF(X_{Hi})) - \ln(ILF(X_{Low}))}{X_{Hi} - X_{Low}} \right)^2 \geq 0$$

Fails Test 2

This one is worst than Linear Interpolation which will be shown in an example that follows

Using Ceding Company ILFs
Testing Log_x-Linear Interpolation

$$ILF(x) = ILF(X_{Low}) + \frac{(\ln(x) - \ln(X_{Low}))(ILF(X_{Hi}) - ILF(X_{Low}))}{(\ln(X_{Hi}) - \ln(X_{Low}))}$$

$$\frac{d\, ILF(X)}{dX} = \frac{1}{x} \times \frac{(ILF(X_{Hi}) - ILF(X_{Low}))}{(\ln(X_{Hi}) - \ln(X_{Low}))} \geq 0$$

Passes Test 1

$$\frac{d^2\, ILF(X)}{dX^2} = -\frac{1}{x^2} \frac{(ILF(X_{Hi}) - ILF(X_{Low}))}{(\ln(X_{Hi}) - \ln(X_{Low}))} \geq 0$$

Passes Test 2

- Passes both tests
- Performs poorly when extrapolation below the lowest ILF
- Ln(0) does not exist

Using Ceding Company ILFs
Testing Log-Log-Linear Interpolation

$$ILF(x) = Exp(Ln(ILF(X_{Low})) + \frac{(Ln(x) - Ln(X_{Low}))(Ln(ILF(X_{Hi})) - Ln(ILF(X_{Low})))}{(Ln(X_{Hi}) - Ln(X_{Low}))}$$

$$\frac{d\, ILF(X)}{dX} = \frac{ILF(x)}{x} \times \frac{\ln(ILF(X_{Hi})) - \ln(ILF(X_{Low}))}{\ln(X_{Hi}) - \ln(X_{Low})} \geq 0$$

Passes Test 1

$$\frac{d^2\, ILF(X)}{dX^2} = \frac{-ILF(x)}{x^2} \times \left[\frac{(\ln(ILF(X_{Hi})) - \ln(ILF(X_{Low})))}{(\ln(X_{Hi}) - \ln(X_{Low}))} \right] +$$

$$\frac{ILF(x)}{x^2} \times \left[\frac{(\ln(ILF(X_{Hi})) - \ln(ILF(X_{Low})))}{(\ln(X_{Hi}) - \ln(X_{Low}))} \right]^2 \leq 0$$

Passes Test 2

Using Ceding Company ILFs
Power Interpolation

$$ILF(x) = ILF(X_{Low})\left(\frac{ILF(X_{Hi})}{ILF(X_{Low})}\right)^{\frac{(x-X_{Low})}{(X_{Hi}-X_{Low})}}$$

$$ILF(x) = Exp\left(Ln\left(ILF(X_{Low})\left(\frac{ILF(X_{Hi})}{ILF(X_{Low})}\right)^{\frac{(x-X_{Low})}{(X_{Hi}-X_{Low})}}\right)\right)$$

$$ILF(x) = Exp(Ln(ILF(X_{Low})) + \frac{(x-X_{Low})(Ln(ILF(X_{Hi})) - Ln(ILF(X_{Low})))}{(X_{Hi}-X_{Low})}$$

Same as Log_y-Linear

Passes Test 1

Fails Test 2

Using Ceding Company ILFs

Single Parameter (Simple) Pareto Interpolation

$$F(x) = 1 - \left(\frac{c}{x}\right)^{\alpha} ; x > c$$

$$LAS(x) = \int_0^x (1 - F(t))dt = \int_0^x \left(\frac{c}{t}\right)^{\alpha} dt = \int_0^x \left(\frac{t}{c}\right)^{-\alpha} dt$$

$$LAS(x) = \frac{c}{(1-\alpha)}\left(\frac{x}{c}\right)^{1-\alpha} = \frac{c}{(1-\alpha)}\left(\frac{c}{x}\right)^{\alpha-1}$$

$$\frac{LAS(X_{Hi})}{LAS(X_{Low})} = \frac{\frac{c}{(1-\alpha)}\left(\frac{c}{X_{Hi}}\right)^{\alpha-1}}{\frac{c}{(1-\alpha)}\left(\frac{c}{X_{Low}}\right)^{\alpha-1}} = \left(\frac{X_{Hi}}{X_{Low}}\right)^{1-\alpha}$$

$$\tilde{\alpha} = 1 - \frac{Ln(LAS(X_{Hi})) - Ln(LAS(X_{Low}))}{Ln(X_{Hi}) - Ln(X_{Low})}$$

Using Ceding Company ILFs
Single Parameter (Simple) Pareto Interpolation (cont.)

$$ILF(x) = \frac{LAS(x)}{LAS(X_{Low})} = \left(\frac{x}{X_{Low}}\right)^{1-\tilde{\alpha}}$$

$$\frac{d\,ILF(x)}{dx} = \left(\frac{1-\tilde{\alpha}}{X_{Low}}\right)\left(\frac{x}{X_{Low}}\right)^{-\tilde{\alpha}} > 0$$

Passes Test 1
If $\alpha < 1$

$$\frac{d^2\,ILF(x)}{dx^2} = \frac{(-\tilde{\alpha})(1-\alpha)}{c^2}\left(\frac{x}{c}\right)^{-\tilde{\alpha}-1} < 0$$

Passes Test 2
If $\alpha > 0$

- Any valid distribution function is a candidate for an interpolation routine
- SPP is an easy to use function that many find intuitively appealing
- Questionable when less than the original truncation point (Extrapolation)
- Better at Extrapolating than many other methods
- Essentially the same as the Log-Log method

Using Ceding Company ILFs

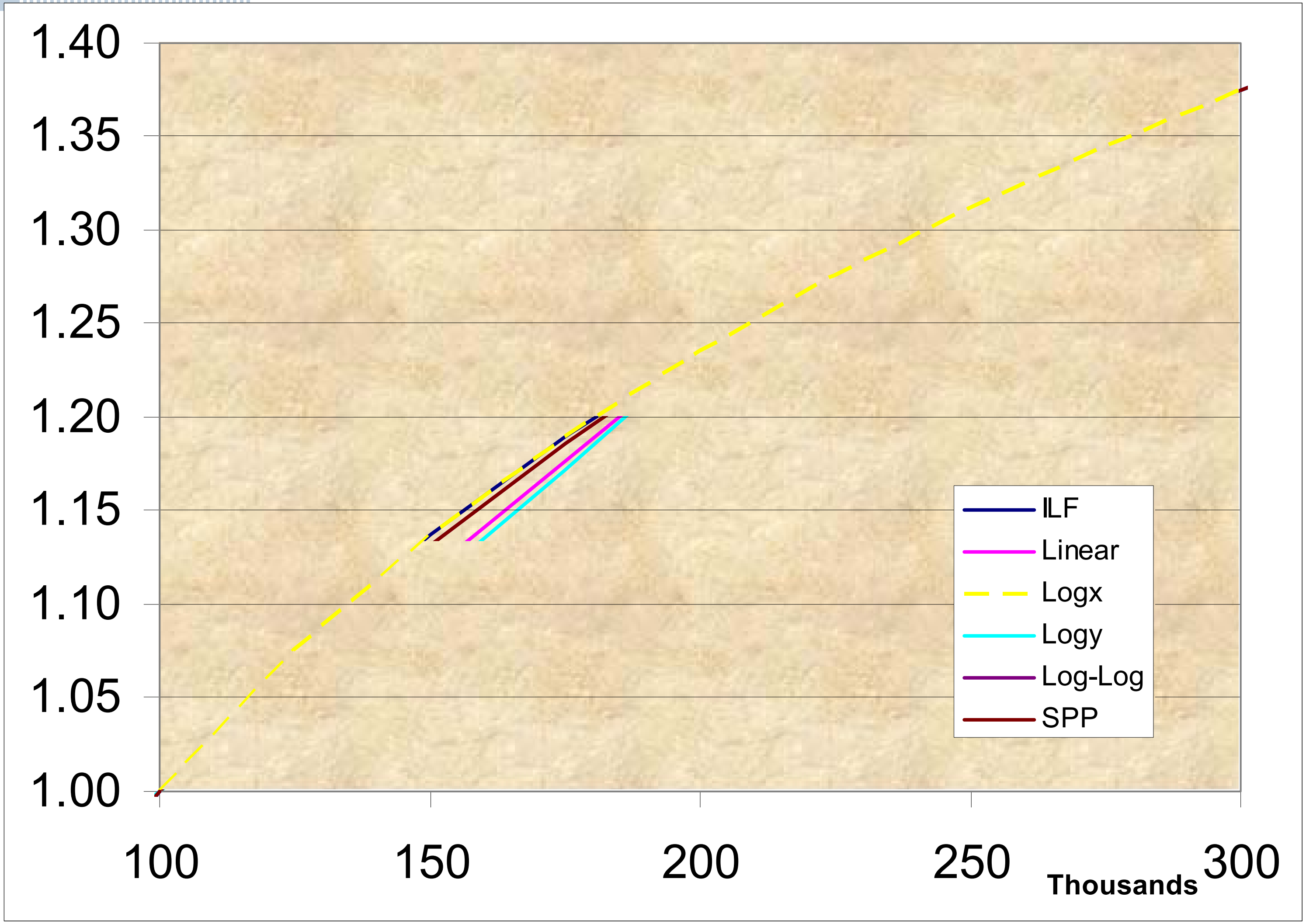
Comparing the Interpolation Methods

Comparing Various Interpolation Routines						
Limit	ILF	Linear	Log_x	Log_y	Log-Log	SPP
	1.000	1.000	1.000	1.000	1.000	1.000
	1.076	1.059	1.076	1.054	1.070	1.070
	1.138	1.118	1.138	1.112	1.132	1.132
	1.190	1.177	1.190	1.172	1.186	1.186
	1.236	1.236	1.236	1.236	1.236	1.236
	1.276	1.270	1.276	1.269	1.275	1.275
	1.312	1.305	1.312	1.303	1.310	1.312
275,000	1.345	1.340	1.345	1.339	1.344	1.345
300,000	1.375	1.375	1.375	1.375	1.375	1.375
	1.403	1.400	1.403	1.399	1.402	1.402
	1.429	1.425	1.429	1.424	1.428	1.428
	1.453	1.450	1.453	1.449	1.452	1.453
	1.475	1.475	1.475	1.475	1.475	1.475
	1.496	1.494	1.496	1.494	1.495	1.495
	1.515	1.513	1.515	1.513	1.515	1.515
	1.534	1.532	1.534	1.532	1.533	1.534
	1.551	1.551	1.551	1.551	1.551	1.551

Blue -Actual Values, Brown – Interpolated Values

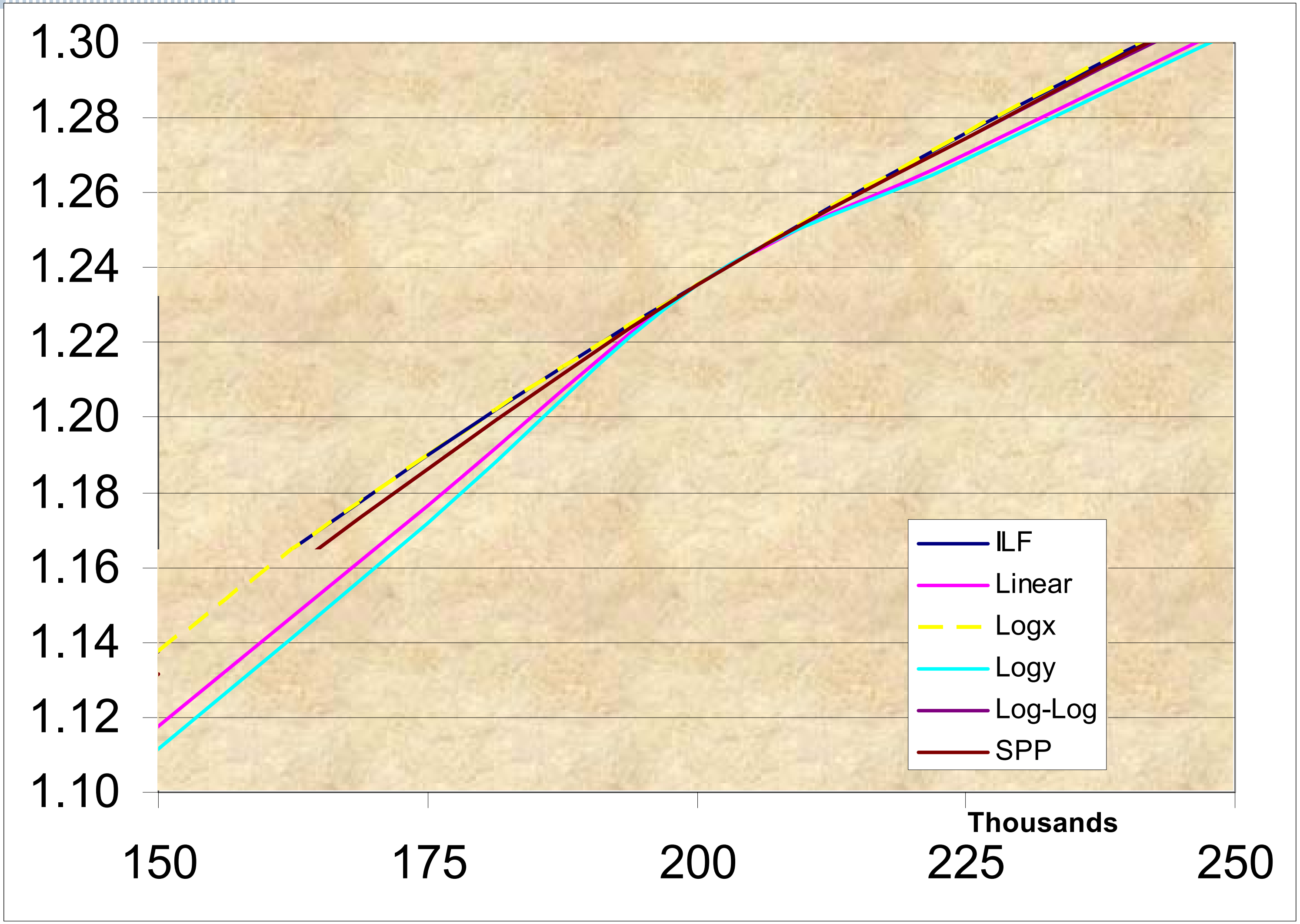
Using Ceding Company ILFs

Comparing the Interpolation Methods



Using Ceding Company ILFs

Comparing the Interpolation Methods



Using Ceding Company ILFs
Comparing the Extrapolation Methods

Comparing Various Interpolation Routines

Limit	Retention	ILF	Linear	Log_x	Log_y	Log-Log	SPP
25,000	**100,000**	4.89%	3.80%	4.89%	**3.50%**	4.54%	4.54%
25,000	**125,000**	3.99% -	3.80% **=**	3.99% -	**3.69% +**	3.95% -	3.95% -
25,000	**150,000**	3.37% -	3.80% **=**	3.38% -	**3.89% +**	3.51% -	3.51% -
25,000	**175,000**	2.93% -	3.80% **=**	2.92% -	**4.10% +**	3.18% -	3.18% -
25,000	**200,000**	2.60% -	2.25%	2.61% -	**2.16%**	2.52% -	2.52% -
25,000	**225,000**	2.33% -	2.25% **=**	2.34% -	**2.22% +**	2.32% -	2.40% -
25,000	**250,000**	2.12% -	2.25% **=**	2.12% -	**2.28% +**	2.15% -	2.17% -
25,000	**275,000**	1.95% -	2.25% **=**	1.93% -	**2.34% +**	2.01% -	1.91% -
25,000	**300,000**	1.79% -	1.61%	1.79% -	**1.57%**	1.75% -	1.75% -
25,000	**325,000**	1.66% -	1.61% **=**	1.66% -	**1.60% +**	1.65% -	1.70% -
25,000	**350,000**	1.54% -	1.61% **=**	1.54% -	**1.62% +**	1.56% -	1.57% -
25,000	**375,000**	1.44% -	1.61% **=**	1.44% -	**1.65% +**	1.48% -	1.42% -
25,000	**400,000**	1.35% -	1.23%	1.34% -	**1.21%**	1.31% -	1.31% -
25,000	**425,000**	1.26% -	1.23% **=**	1.26% -	**1.22% +**	1.25% -	1.29% -
25,000	**450,000**	1.19% -	1.23% **=**	1.19% -	**1.24% +**	1.20% -	1.21% -
25,000	**475,000**	1.12% -	1.23% **=**	1.13% -	**1.25% +**	1.15% -	1.11% -

= - The previous range has the same percent of losses exposed, not realistic

\+ - The previous range has a lower percent of losses exposed, not possible

Using Ceding Company ILFs

Comparing the Extrapolation Methods

Comparing Various Extrapolation Routines						
Limit	**ILF**	**Linear**	**Log_x**	**Log_y**	**Log-Log**	**SPP**
0	0.078	0.764	-6.041	0.809	0.002	0.002
1,000	0.078	0.767	-0.565	0.811	0.245	0.245
2,000	0.133	0.769	-0.329	0.813	0.303	0.303
3,000	0.177	0.772	-0.191	0.815	0.343	0.343
5,000	0.244	0.776	-0.018	0.818	0.401	0.401
10,000	0.364	0.788	0.218	0.827	0.495	0.495
15,000	0.451	0.800	0.355	0.835	0.561	0.561
20,000	0.518	0.812	0.453	0.844	0.612	0.612
25,000	0.574	0.823	0.529	0.853	0.655	0.655
50,000	0.771	0.882	0.764	0.900	0.809	0.809
75,000	0.903	0.941	0.902	0.949	0.916	0.916
100,000	1.000	1.000	1.000	1.000	1.000	1.000

Using the values at 100K and 200K to extrapolate below 100K

None of the methods are reliable. At least the Log-Log/SPP gives feasible answers

Blue -Actual Values, Red – Extrapolated Values

Using Ceding Company ILFs
Other Ideas

Fit a curve directly to the ILF
- If using CDF, need estimate of either E(x) or LAS(B)
- May get improved estimates for values less than C in SPP

Compare ILF to known ILFs
- Calculate SSE comparing ILF to Collections of known ILFs
- Consider inclusion of ALAE and Risk Load
- Possible Method for backing out Risk Load

Section 5

Working with Personal Auto Split Limits

Private Passenger Auto (PPA) Split Limits

Conversion to CSL

- Many PPA Loss Curves are Occurrence-based distributions, for use with Combined Single Limits (CSL) profiles.

- But, most PPA business is written on a Split Limits basis. How do you convert these limits profiles for use with CSL curves?

Private Passenger Auto (PPA) Split Limits
Split Limit to One CSL

Option 1:

- Assume a distribution of number of claimants and convert into each Split Limit into a CSL.
- For example, assume 60% 1 claimant, 30% 2 claimants, 10% 3 claimants.
  - 100k/300k/50k Split Limits would be converted to a 200k CSL, assuming a full 50k PD limit (100k * .6 + 200k * .3 + 300k * .1 + 50k)
- Using this method will not give any exposure greater than 200k, even though there is really a possibility of a 350k loss.

Private Passenger Auto (PPA) Split Limits
Split Limit to Multiple CSLs

Option 2:

- Assume a distribution of number of claimants and create a new CSL profile.
- For example, assume 60% 1 claimant, 30% 2 claimants, 10% 3 claimants.
  - Split total premium using these percentages
  - Allocate premium to each per person combination

PPA Split Limit Conversion
Option 2 Example

Limits		
Per Person	Per Occurrence	Premium
25,000	50,000	100,000
50,000	100,000	100,000
100,000	200,000	100,000
100,000	300,000	100,000
250,000	500,000	100,000
300,000	300,000	100,000
500,000	1,000,000	100,000
1,000,000	1,000,000	100,000
Total		800,000

Single Limit	Premium
25,000	60,000
50,000	40,000
50,000	60,000
100,000	40,000
100,000	60,000
200,000	40,000
100,000	60,000
200,000	30,000
300,000	10,000
250,000	60,000
500,000	40,000
300,000	100,000
500,000	60,000
1,000,000	40,000
1,000,000	100,000
Total	800,000

PPA Split Limit Conversion

Option 2 Example

Single Limit	Premium
25,000	60,000
50,000	40,000
50,000	60,000
100,000	40,000
100,000	60,000
200,000	40,000
100,000	60,000
200,000	30,000
300,000	10,000
250,000	60,000
500,000	40,000
300,000	100,000
500,000	60,000
1,000,000	40,000
1,000,000	100,000
Total	800,000

	Data for Exposure Rating	
Single Limit	Limit Incl PD	Premium
25,000	35,000	60,000
50,000	60,000	100,000
100,000	110,000	160,000
200,000	210,000	70,000
250,000	260,000	60,000
300,000	310,000	110,000
500,000	510,000	100,000
1,000,000	1,010,000	140,000
Total		800,000

This example assumes a flat $10k PD limit. May want to use actual PD limit or some other assumption.

Private Passenger Auto (PPA) Split Limits
Questions

- If PPA curves are built using primarily Split Limits data, is there an inherent assumption of a claimant distribution underlying the curve?
  - If so, when we apply a claimant distribution assumption again in converting split limits profiles, are we underestimating the resulting CSL? Or should we simply use the per occurrence limit?
- How are curves loaded for other liability coverages?

Private Passenger Auto (PPA) Split Limits
Loading for other coverages

- Important to understand what your curves include.
  - Some curves may include only BI and PD losses.
  - UM usually mirrors BI limits and does not occur at the same time as BI – probably no adjustment needed
  - Medical Payments are usually very small, so should have minor impact.
  - If PIP is not included, could have significant impact in some states.
- Subject Premium Base should include premium from all liability coverages

Section 6

Property Exposure Rating

Working with First Loss Scales

Property Loss Curves
Example

- Curve generally are expressed as percentage of policy limit or TIV.
  - When curves are expressed as a percentage of policy limits or TIV, impact of trend is not as significant – may not need to update as often.

Example of Percentage curve:

Assume: Coverage A limit = 100k, Layer = 100k xs 50k

Given the curve below, what is the % of losses to the layer?

% of Cvg A Limit	**% of Loss Below**					
0%	0.00%		P(L < X) =	F(X / covg A)		
50%	61.70%		P(L < ret) =	F(50k / 100k) =	F(50%) =	61.70%
100%	84.20%		P(L < limit + ret) =	F(150k / 100k) =	F(150%) =	96.50%
150%	96.50%		% of Loss to Layer =	96.50% - 61.70% =		34.80%
200%	100.00%					

Property Loss Curves
History

- Lloyds
- Salzmann (1960 INA Homeowners data)
- Reinsurer Curves (Swiss Re, Munich Re, etc)
- Ludwig (1984-1988 Homeowners and Small Commercial data)
- ISO's PSOLD (Recent Commercial data)

Property Loss Curves
Advantages/Disadvantages

- Lloyds Curves
  - (Very old data)
  - (Does not vary by amount of insurance or occupancy class)
  - (Underlying data is largely unknown (marine losses?))
- Salzmann (Personal Property)
  - Based on actual Homeowners data
  - Varies by Construction/Protection Class
  - (Very old data – from 1960)
  - (Does not vary by amount of insurance)
  - (Building losses only and Fire losses only)
- Swiss Re Curves
  - Documented study on personal & commercial reinsurance business
  - (Old data)
  - (Does not vary by amount of insurance or occupancy class)

Property Loss Curves
Advantages/Disadvantages

- Ludwig Curves (Personal and Commercial)
  - Based on actual Homeowners and Commercial data, (but uses Hartford small commercial property book – may not be good for large national accts)
  - Varies by Construction/Protection Class for HO and Occupancy Class for Commercial
  - Includes all property coverages and perils
  - (Old data: 1984 - 1988)
- ISO's PSOLD
  - Recent Data – updated every 2 years
  - Varies by amount of insurance, occupancy class, state, coverage, and peril
  - (Based on ISO data only)

First Loss Scales
A Quick Review of FLSs

$$FLS(X) = \frac{LAS(X)}{E(X)} = \frac{\int_0^X [1-F(x)]dx}{\int_0^\infty [1-F(x)]dx} = \frac{\int_0^X [1-F(x)]dx}{E(X)}$$

The FLS is also non-decreasing (non-negative 1st derivative), similar to ILF

$$\frac{d\,ILF(X)}{dX} = \frac{d\left\{\frac{1}{E(X)}\int_0^X [1-F(x)]dx\right\}}{dX} = \frac{[1-F(X)]}{E(X)} \geq 0$$

The FLS is also non-decreasing at a decreasing rate (non-positive 2nd derivative)

$$\frac{d^2\,ILF(X)}{dX^2} = \frac{-f(X)}{E(X)} \leq 0$$

E(X), the unlimited average severity of X

First Loss Scales
Consistency Tests and Interpolation routines

Consistency tests are exactly the same for Increased Limits Factors and for First Loss Scales

Issues regarding valid interpolation routines are also the same for Increased Limits Factors and First Loss Scales

Section 7

Property Exposure Rating

Working with PSOLD

PSOLD Methodology
Adjustments to the PSOLD Methodology

Many users of PSOLD use the model as a source for the underlying parameters for the mixed exponentials, but then use their own model to do the calculations based on the PSOLD curves

If you are going to make your own model, you should consider making improvements to the methodology

- Limited Average Severities over a Range of Value within a Single AOI Group
- Weighting between AOI groups
- Weighting between Occupancy Classes
- Stacking and Participation
- Exposure Above Policy Limit and Stacking
- Exposure Above Policy Limit and Margin Clauses

PSOLD Methodology
Notation

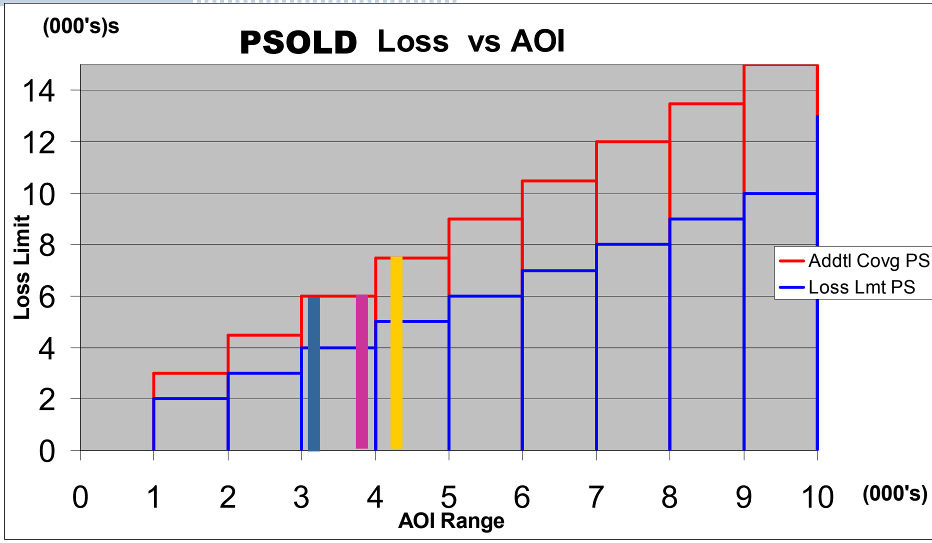
$$LAS_{Exp}(x) = \mu \times \left(1 - e^{-\frac{x}{\mu}}\right)$$

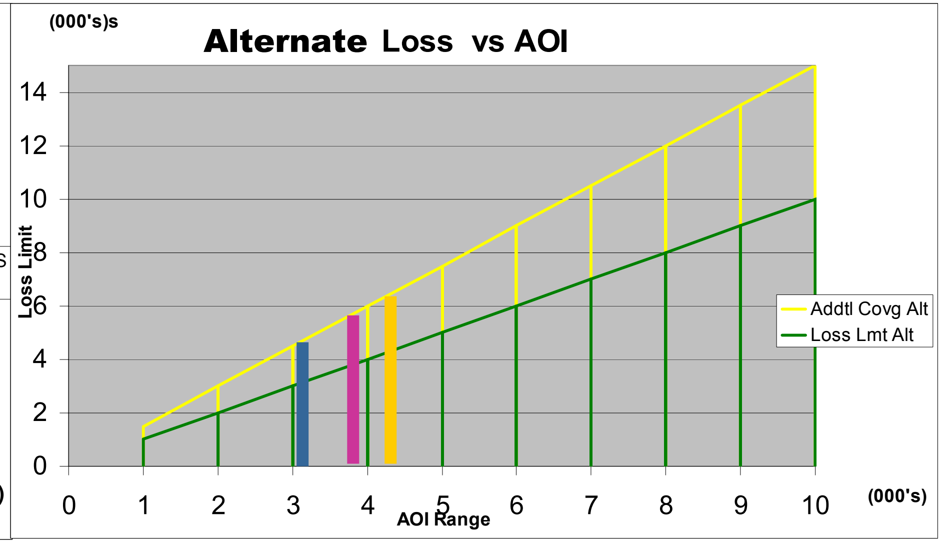
$$LAS_{ME}(x) = \sum_{i=1}^{\#Lags} w_i \mu_i \times \left(1 - e^{-\frac{x}{\mu_i}}\right)$$

$$\mu_{i,psold} = 10^{.5 \times (i+1)}$$

w_i varies by:
- Coverage (B, C, B+C, B+C+I)
- Peril (BG1, BG2, Special Causes, All)
- Net of Deductible vs Ground Up
- Occupancy Class
- State

PSOLD Methodology
Interpreting a single policy LAS in an AOI Ranges in PSOLD





- Is the movement from one AOI range to the next a step Function or a smooth progression?
- Consider three policies, two within a single AOI range and the third in the next highest AOI range but close in value to the second policy
- Should two different policy limits within a single AOI range have the same LAS or should the difference in policy limits be reflected?
- PSOLD currently calculates the LAS at a single point, the minimum of the loss limit and 1.5x(upper bound of the AOI range) for all policies in the range.

PSOLD Methodology
Evaluating LAS functions over a range

PSOLD evaluates the function at a fixed point, consider the LAS

$$LAS(x) = \sum_{i=1}^{\#Lags} w_{i,AOI}\mu_i \times \left(1 - e^{-\frac{x}{\mu_i}}\right) = LAS_{ME}(x)$$

Consider evaluating this over a range of values

$$LAS(x \mid X_U \geq x > X_L) = \frac{\int_{X_L}^{X_U} g(x) LAS(x) dx}{\int_{X_L}^{X_U} g(x) dx}$$

Consider two forms for g(x)

1. g(x) is Uniform on the range ($X_U \geq x > X_L$)
2. g(x) follows same distribution as losses conditional on ($X_U \geq x > X_L$)

PSOLD Methodology

Evaluating LAS functions over a range – Uniform Distribution

Will use a single exponential in the example for simplicity.
This is easily generalized to a mixed exponential

$$g(x) = \frac{1}{X_U - X_L} \Rightarrow LAS(x \mid X_U \geq x > X_L) = \int_{X_L}^{X_U} \frac{1}{X_U - X_L} LAS(x)dx$$

$$LAS(x \mid X_U \geq x > X_L) = \frac{\mu}{X_U - X_L} \int_{X_L}^{X_U} \left(1 - e^{-\frac{x}{\mu}}\right) dx$$

$$LAS(x \mid X_U \geq x > X_L) = \frac{\mu(X_U - X_L)}{(X_U - X_L)} - \frac{\mu}{(X_U - X_L)} \int_{X_L}^{X_U} e^{-\frac{x}{\mu}} dx$$

$$LAS(x \mid X_U \geq x > X_L) = \mu - \frac{\mu\left(LAS_{Exp}(X_U) - LAS_{Exp}(X_L)\right)}{(X_U - X_L)}$$

Not difficult to calculate

PSOLD Methodology

Evaluating LAS functions over a range – Exponential Distribution

Will use a single exponential in the example for simplicity. This exponential has the same μ as the loss distribution but it is conditional on being within the range (X_L, X_H)

$$g(x) = \frac{1}{\mu} e^{-\frac{x}{\mu}} \Rightarrow LAS(x \mid X_U \geq x > X_L) = \left(e^{-\frac{X_L}{\mu}} - e^{-\frac{X_U}{\mu}} \right)^{-1} \int_{X_L}^{X_U} \frac{1}{\mu} e^{-\frac{x}{\mu}} \times \mu \left(1 - e^{-\frac{x}{\mu}} \right) dx$$

$$LAS(x \mid X_U \geq x > X_L) = \left(e^{-\frac{X_L}{\mu}} - e^{-\frac{X_U}{\mu}} \right)^{-1} \int_{X_L}^{X_U} \left(e^{-\frac{x}{\mu}} - e^{-\frac{x}{\mu}} \times e^{-\frac{x}{\mu}} \right) dx$$

$$LAS(x \mid X_U \geq x > X_L) = \left(e^{-\frac{X_L}{\mu}} - e^{-\frac{X_U}{\mu}} \right)^{-1} \int_{X_L}^{X_U} \left(e^{-\frac{x}{\mu}} - e^{-\frac{2x}{\mu}} \right) dx$$

$$LAS(x \mid X_U \geq x > X_L) = \left(e^{-\frac{X_L}{\mu}} - e^{-\frac{X_U}{\mu}} \right)^{-1} \int_{X_L}^{X_U} \left(e^{-\frac{x}{\mu}} - e^{-\frac{x}{(.5\times\mu)}} \right) dx$$

PSOLD Methodology

Evaluating LAS functions over a range – Exponential Distribution

$$LAS(x \mid X_U \geq x > X_L) = \frac{\int_{X_L}^{X_U} e^{-\frac{x}{\mu}}dx - \int_{X_L}^{X_U} e^{-\frac{x}{(.5\times\mu)}}dx}{\left(e^{-\frac{X_L}{\mu}} - e^{-\frac{X_U}{\mu}}\right)}$$

$$LAS(x \mid X_U \geq x > X_L) = \frac{\left[\mu\left(e^{-\frac{X_L}{\mu}} - e^{-\frac{X_U}{\mu}}\right) - .5\times\mu\left(e^{-\frac{X_L}{(.5\times\mu)}} - e^{-\frac{X_U}{(.5\times\mu)}}\right)\right]}{\left(e^{-\frac{X_L}{\mu}} - e^{-\frac{X_U}{\mu}}\right)}$$

$$LAS(x \mid X_U \geq x > X_L) \approx \mu\left(1 - \frac{e^{-\frac{X_U}{\mu}} + e^{-\frac{X_L}{\mu}}}{2}\right)$$

This relationship is by observation and not rigorous proof. Intuitively it is around what I would like the result to be.
The average of the LAS(X_{HI}) and LAS(X_{LO})

PSOLD Methodology

PSOLD LAS Calculations over Single AOI Range

LAS for an Mixed Exponential

$$LAS_{ME}(x) = \sum_{i=1}^{\#Lags} w_i \mu_i \times \left(1 - e^{-\frac{x}{\mu_i}}\right)$$

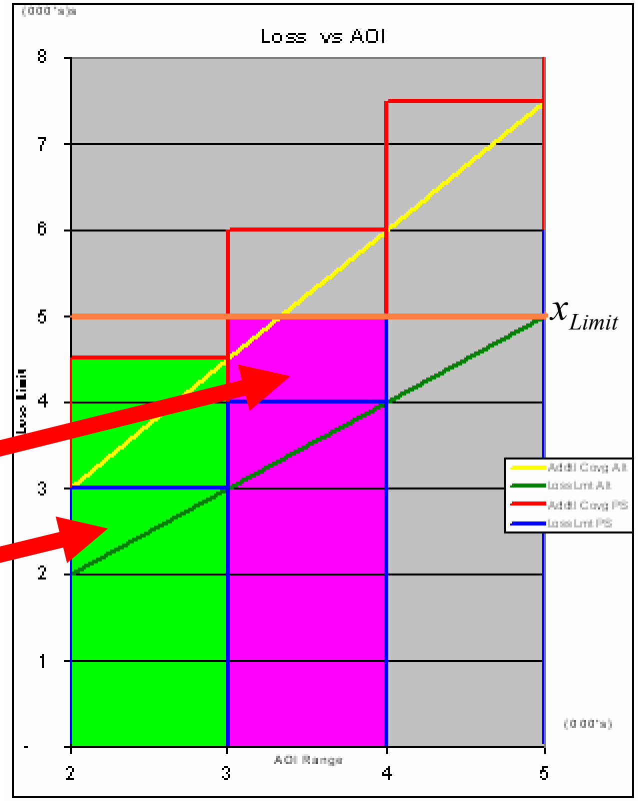
For Coverages B, C and B+C PSOLD constrains the LAS Calculation

PSOLD has two Ranges of Interest

$$x_{Limit} < AOI^*_{Upper} : LAS_{PSOLD}(x_{Limit}) = LAS_{ME}(x_{Limit})$$

$$x_{Limit} \geq AOI^*_{Upper} : LAS_{PSOLD}(x_{Limit}) = LAS_{ME}\left(AOI^*_{Upper}\right)$$

$$where \ \ AOI^*_{Upper} = AOI_{Upper} \times (1 + Additional \ \ Exposure\%)$$



PSOLD Methodology

Alternate LAS Calculations over a Continuous AOI Range

Calculating the LAS over a continuous range adds one more degree of complexity

$$x_{Limit} < AOI^*_{Lower} : LAS_{ALT}(x_{Limit}) = LAS_{ME}(x_{Limit})$$

$$x_{Limit} \geq AOI^*_{Upper} : LAS_{ALT}(x_{Limit}) = \frac{LAS_{ME}\left(AOI^*_{Upper}\right) + LAS_{ME}\left(AOI^*_{Lower}\right)}{2}$$

$$AOI^*_{Upper} \geq x_{Limit} \geq AOI^*_{Lower} :$$

$$LAS_{ALT}(x_{Limit}) = \left(\frac{LAS_{ME}\left(AOI^*_{Lower}\right) + LAS_{ME}\left(x_{Limit}\right)}{2}\right)\left(\frac{x_{Limit} - AOI^*_{Lower}}{AOI^*_{Upper} - AOI^*_{Lower}}\right) +$$

$$LAS_{ME}\left(x_{Limit}\right)\left(\frac{AOI^*_{Upper} - x_{Limit}}{AOI^*_{Upper} - AOI^*_{Lower}}\right)$$

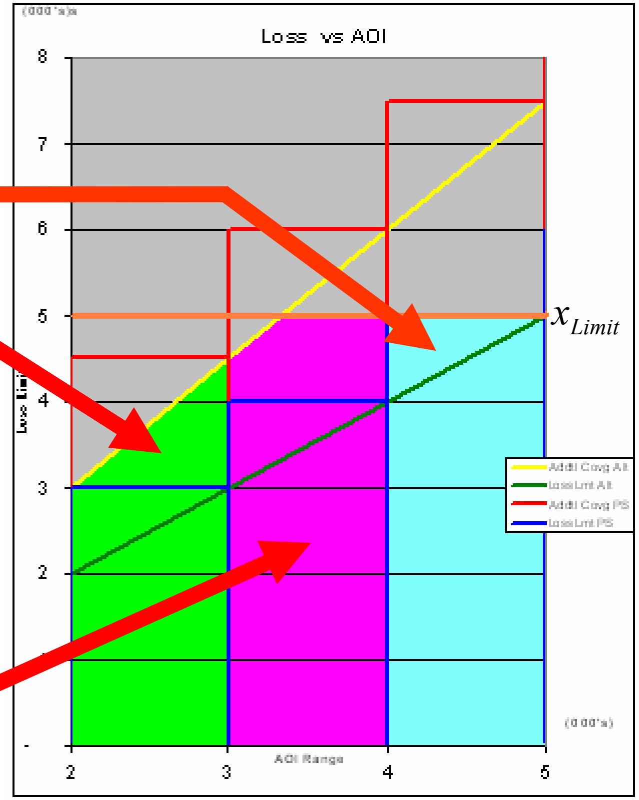
Which simplifies to

$$LAS_{ALT}(x_{Limit}) = LAS_{ME}\left(AOI^*_{Lower}\right)\left(\frac{x_{Limit} - AOI^*_{Lower}}{2\left(AOI^*_{Upper} - AOI^*_{Lower}\right)}\right) +$$

$$LAS_{ME}\left(x_{Upper}\right)\left(1 - \frac{x_{Limit} - AOI^*_{Lower}}{2\left(AOI^*_{Upper} - AOI^*_{Lower}\right)}\right)$$

$$where\ \ AOI^*_{Upper} = AOI_{Upper} \times (1 + Additional\ \ Exposure\%)$$

$$where\ \ AOI^*_{Lower} = AOI_{Lower} \times (1 + Additional\ Exposure\%)$$



PSOLD Methodology
Advantages of the Alternate LAS Calculations

- Different policy limits within the same AOI range will get different LAS
- Smoother transition as you move from one AOI range to the next
- Since this impacts the unlimited average severity for the policy, it will change the allocation of losses to the layer for any exposed policy

- An additional enhancement would be to adjust the w_i's as you move within an AOI range

PSOLD Methodology
Weighting between AOI Ranges

If a range of Insured values spans more than one AOI Range. You need to combine the results of the Individual AOI ranges

- In PSOLD any AOI group included within the range will be given full weight
- An improvement would be to only Include an AOI range in proportion To the percentage that the range is covered

PSOLD Methodology
Weighting between Occupancy Classes

In PSOLD, when using more than one Occupancy class on a single policy group, the relative weight assigned to each occupancy class is based on the occupancy counts in the underlying industry data base.

An improvement would be to allow the user to define the weights between the occupancy classes so that you can more accurately reflect the individual ceding companies exposure

PSOLD Methodology
Additional Exposure Percentage

PSOLD uses the following additional exposure percentage

- Building Only – 50%
- Contents Only – 50%
- Building+Contents Only – 50%
- Building+Contents+Business Interruption – Unlimited

You may want to select a different percentage due to any of the following

- Stacking of Excess Policies – you do not want the policies to overlap
- Margin Clause – contractually limits exposure greater than the limit
- Company Experience
- Judgement

PSOLD Methodology
Stacking and Participation

Additional consideration when dealing with stacking an participation

- The selected AOI group should be based on a full value on the insured risk (same AOI group as if the risk was fully covered by a single policy
- All stacked policies should have the same AOI group
- When stacking, assume additional coverage % is zero or the policies will overlap

Section 8

Stacking and Participation

Stacking and Participation
Participation

- Participation allows you to correctly model the situation where a contract only covers a proportional share of the underlying loss.
- It is most common in a subscription type market like Lloyds, but it is also useful for modeling some facultative business.

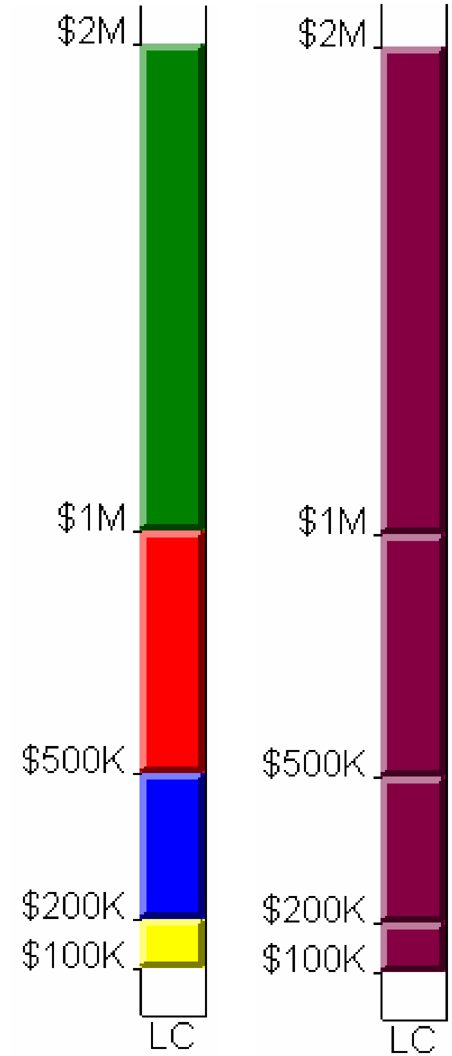
Example

- Assume the following:
  - Write 25% participation on a $1M Contract.
  - You reinsure a 200K xs 200K layer
- In order to get a loss that will expose the Reinsurance Cover
  - You must have a loss to the primary contract greater than 800K (200K / 25%)
  - The largest loss you can have exposing the layer is 250K (25% of 1M) or 50K to the layer
  - Actually, you would take 25% of losses ceded to an 800K xs 800K reinsurance layer. But since the primary policy is $1M, it is effectively 25% of 200k xs 800k.

Stacking and Participation
Stacking

Stacking is where an insurer issues multiple excess contracts covering the same underlying risk

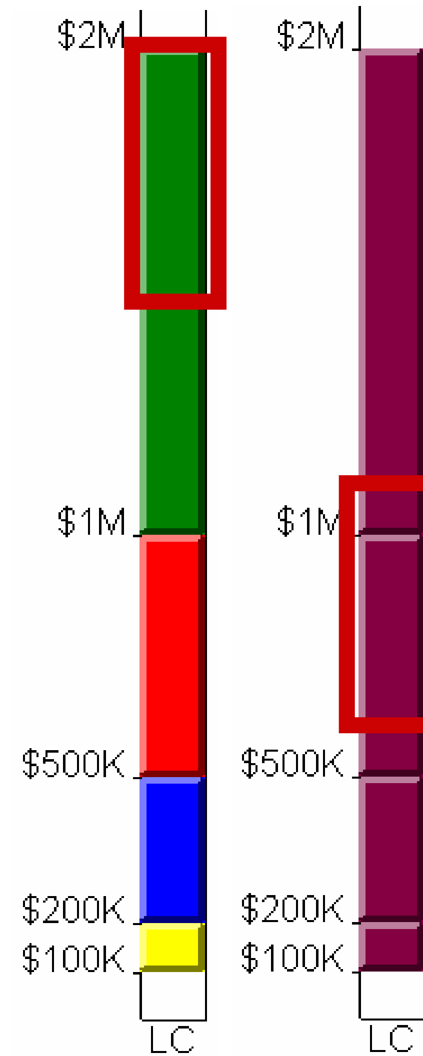
- Assume someone writes a series of policies covering the same risk, 100K x 100K (Yellow), 300K x 200K (Blue), 500K x 500K (Red) and 1M x 1M (Green)
- If all are written at the same level of participation then effectively it is the same as a single 1.9M xs 100K (Purple) policy with the given participation
- In practice, not all contracts are at the same participation and not all contract are written (can be thought of as participation=0%, this is sometimes called ventilation)



Stacking and Participation
Stacking

Now Assume there is a 500K x 500K reinsurance contract covering these contracts

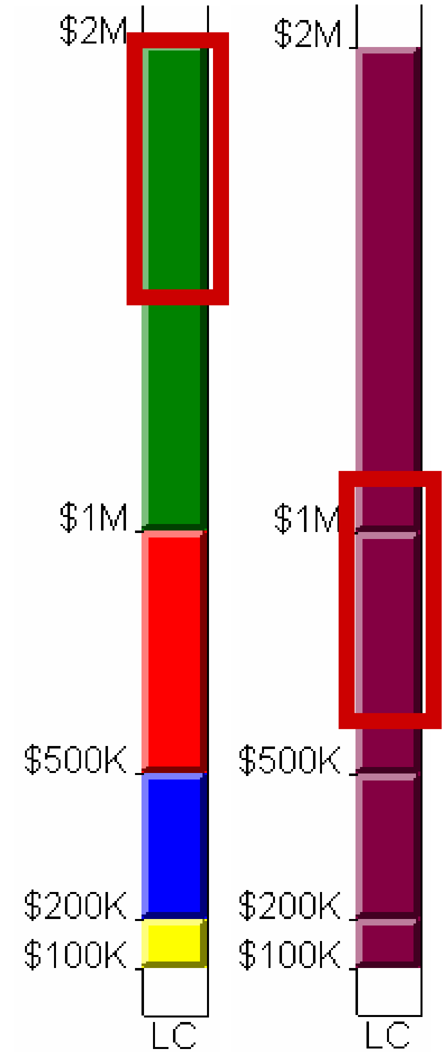
- If the contracts are assumed to be independent, then you would only cover the 500K x 500K layer on the 1M x 1M policy. No other policy would expose.
- If the contracts are assumed to be stacked, then you would cover the 500K x 500K layer on the 1.9M x 100K policy.
- There can be significantly greater exposure to the Reinsurance Contract under the stacked assumption



Stacking and Participation
Stacking

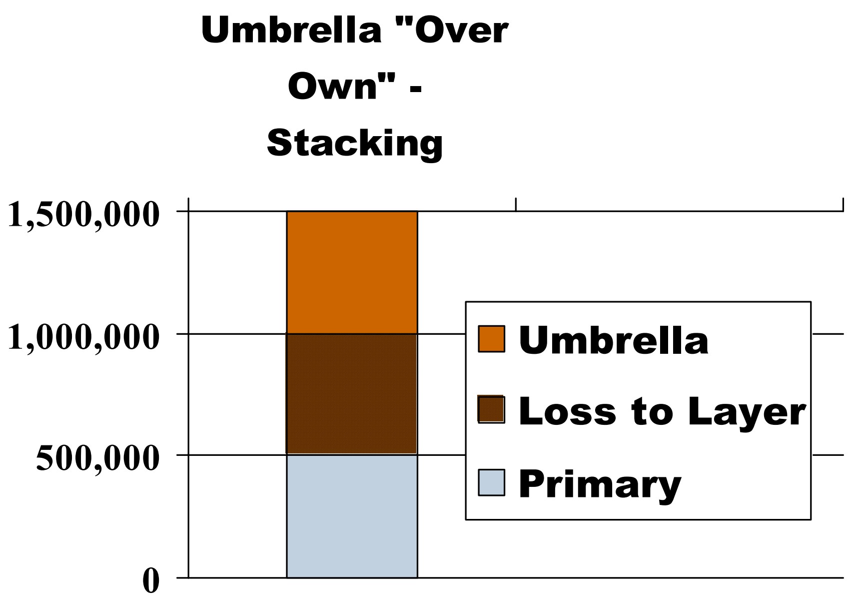
Stacking is Generally thought of as an International Issue, but…

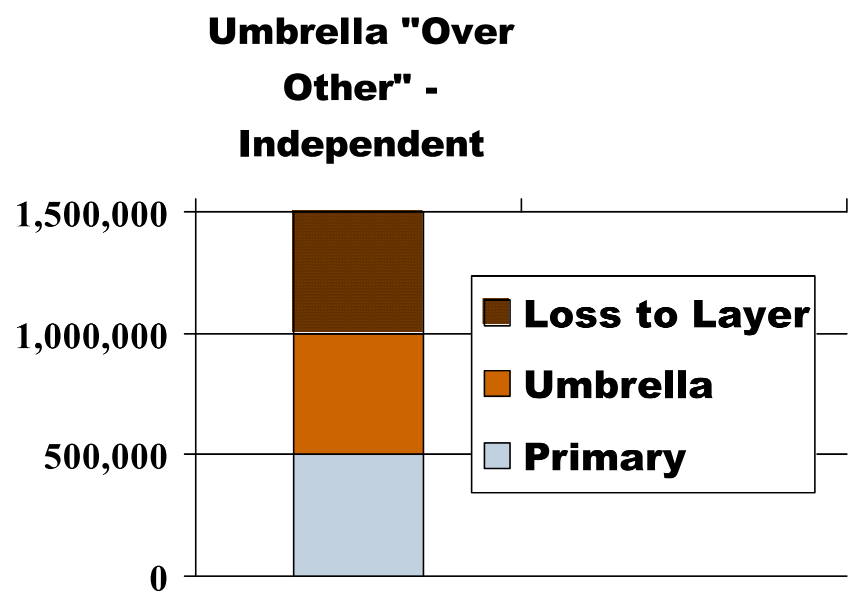
- Stacking can be used in the Facultative Markets
- Stacking can be used to model Umbrella written over a company's own underlying policies
- Stacking is commonly used in combination with participation in a subscription market like Lloyds



Stacking and Participation
Umbrella Pricing

- Umbrella Comparison - Assume: Layer 500k xs 500k, Umbrella Limit = 1M, Underlying Limit = 500k





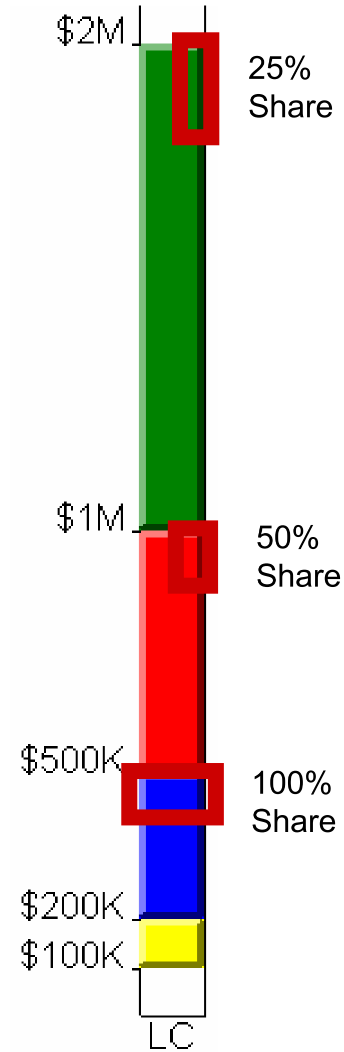
Stacking and Participation
Partial Participation without Stacking

Layer: 300k xs 200k - no stacking

Limits Profile

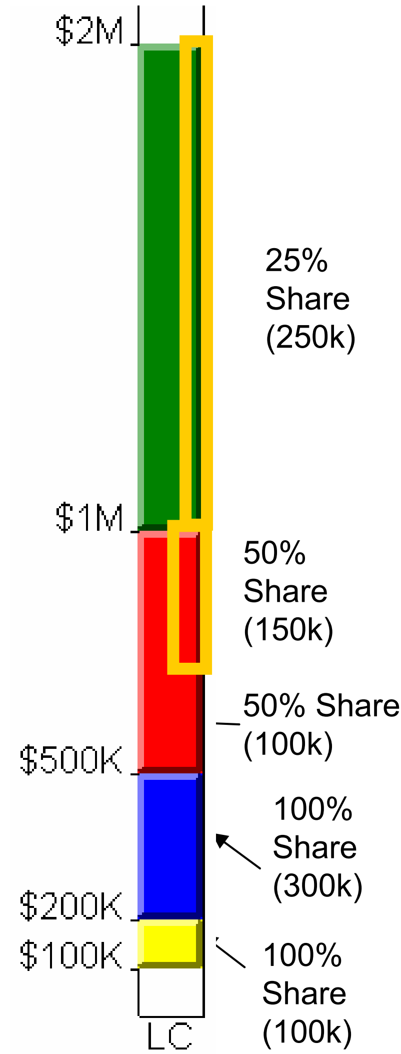
Policy Limit	SIR/ Retention	Participation	Rescaled Treaty Limit (Capped)	Rescaled Treaty Retention
100,000	100,000	100.0%	0	200,000
300,000	200,000	100.0%	100,000	200,000
500,000	500,000	50.0%	100,000	400,000
1,000,000	1,000,000	25.0%	200,000	800,000

"Our share" of the layer would be Participation x Capped Treaty Limit



Stacking and Participation
Partial Participation with Stacking

- Assume someone writes a series of policies covering the same risk, 100K x 100K (Yellow), 300K x 200K (Blue), 500K x 500K (Red) and 1M x 1M (Green).
  - Your participation on each is: 100K xs 100K (100%), 300K xs 200K (100%), 500K xs 500K (50%), 1M xs 1M (25%)
  - These policies are stacked
  - You reinsure a 500K xs 500K layer
- In order to get a loss that will expose the Reinsurance Cover
  - You must have a loss to the excess contracts greater than 600K (100K / 100% + 300K / 100% + 100K / 50%)
  - The largest loss you can have exposing the layer is 900K (100K * 100% + 300K * 100% + 500K * 50% + 1M * 25%) or 400K to the layer



Section 9

Miscellaneous Topics

Miscellaneous Topics

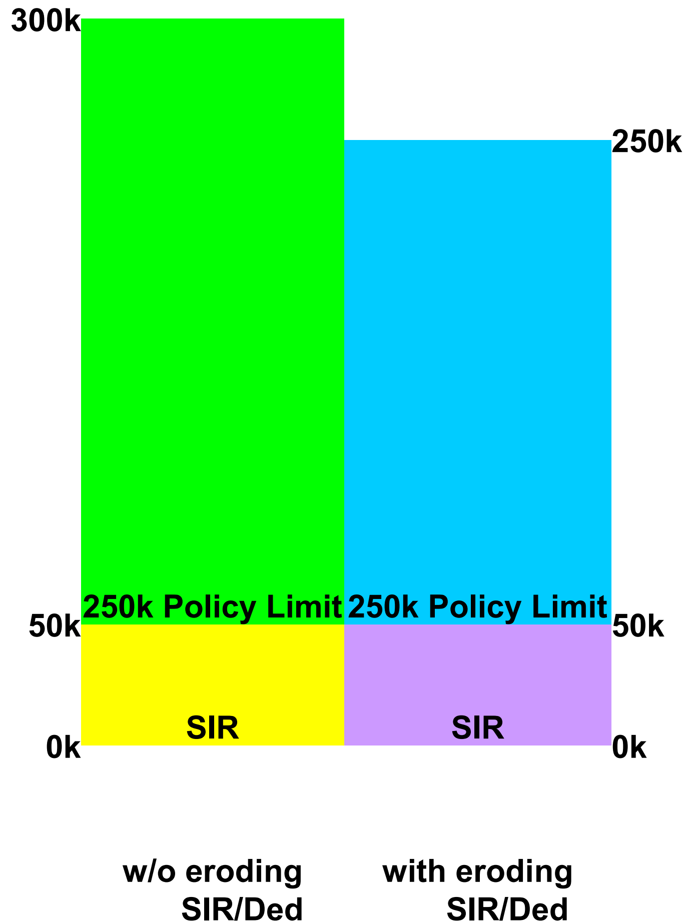
- SIR/Deductibles
- ALAE options
- Policy Count vs Premium

Miscellaneous Topics
Treatment of Self-Insured Retentions (SIRs) and Deductibles

- Deductible/SIR Retains Policy Limit
  - Limit floats on top of the Deductible/SIR

- Deductible /SIR Reduces Policy Limit
  - Limit stays fixed relative to ground-up and the Deductible /SIR effectively reduces or erodes the policy limit

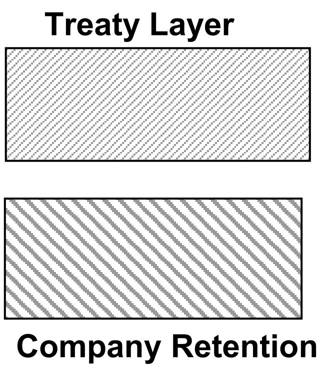
Miscellaneous Topics
Treatment of Self-Insured Retentions (SIRs) and Deductibles

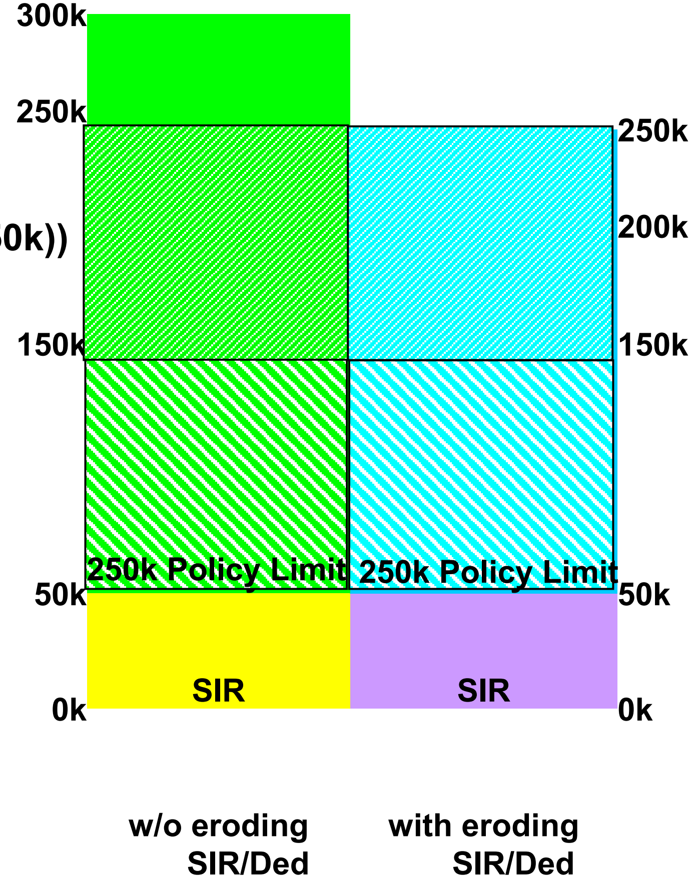


Miscellaneous Topics
Treatment of Self-Insured Retentions (SIRs) and Deductibles

- Assume: Layer 100k xs 100k , Policy Limit = 250k, SIR/Ded = 50k

% Exposed = (LAS(250k) – LAS(150k) / (LAS(300k) – LAS(50k))





% Exposed = (LAS(250k) – LAS(150k) / (LAS(250k) - LAS(50k))

Miscellaneous Topics
ALAE Options

- ALAE Excluded
- ALAE Prorata in Addition to Loss
- ALAE Included outside Policy Limit
- ALAE Included within Policy Limit

Miscellaneous Topics
Burning Cost Formulas

ALAE Excluded

$$BC = LR_{PureLoss} \times \frac{LAS(Min(LayLmt, PolLmt) + PolDed) - LAS(Min(Lay\,\mathrm{Re}\,t, PolLmt) + PolDed)}{LAS(PolLmt + PolDed) - LAS(PolDed)}$$

$$BC_{SIR\_Erode} = LR_{PureLoss} \times \frac{LAS(Min(LayLmt + PolDed, PolLmt)) - LAS(Min(Lay\,\mathrm{Re}\,t + PolDed, PolLmt))}{LAS(PolLmt) - LAS(PolDed)}$$

ALAE Prorata

$$BC = LR_{Loss+ALAE} \times \frac{LAS(Min(LayLmt, PolLmt) + PolDed) - LAS(Min(Lay\,\mathrm{Re}\,t, PolLmt) + PolDed)}{LAS(PolLmt + PolDed) - LAS(PolDed)}$$

$$BC_{SIR\_Erode} = LR_{Loss+ALAE} \times \frac{LAS(Min(LayLmt + PolDed, PolLmt)) - LAS(Min(Lay\,\mathrm{Re}\,t + PolDed, PolLmt))}{LAS(PolLmt) - LAS(PolDed)}$$

The only difference between Excluded and Prorata is the Loss Ratio assumption

Miscellaneous Topics
Burning Cost Formulas

ALAE Included Within Policy Limit

$$BC = LR_{Loss+ALAE} \times \frac{LAS\left(\frac{Min(LayLmt, PolLmt) + PolDed}{A}\right) - LAS\left(\frac{Min(Lay\,Ret, PolLmt) + PolDed}{A}\right)}{LAS\left(\frac{PolLmt + PolDed}{A}\right) - LAS\left(\frac{PolDed}{A}\right)}$$

$$BC_{SIR\_Erode} = LR_{Loss+ALAE} \times \frac{LAS\left(\frac{Min(LayLmt + PolDed, PolLmt)}{A}\right) - LAS\left(\frac{Min(Lay\,Ret + PolDed, PolLmt)}{A}\right)}{LAS\left(\frac{PolLmt}{A}\right) - LAS\left(\frac{PolDed}{A}\right)}$$

ALAE Included Outside Limit

$$BC = LR_{Loss+ALAE} \times \frac{LAS\left(Min\left(\frac{LayLmt}{A}, PolLmt\right) + \frac{PolDed}{A}\right) - LAS\left(Min\left(\frac{Lay\,Ret}{A}, PolLmt\right) + \frac{PolDed}{A}\right)}{LAS\left(PolLmt + \frac{PolDed}{A}\right) - LAS\left(\frac{PolDed}{A}\right)}$$

$$BC_{SIR\_Erode} = LR_{Loss+ALAE} \times \frac{LAS\left(Min\left(\frac{(LayLmt + PolDed)}{A}, PolLmt\right)\right) - LAS\left(Min\left(\frac{(Lay\,Ret + PolDed)}{A}, PolLmt\right)\right)}{LAS(PolLmt) - LAS\left(\frac{PolDed}{A}\right)}$$

ALAE Included vs Prorata is based on the Reinsurance Contract.

ALAE Included Inside the Limit versus Outside the Limit depends on underlying policy.

Miscellaneous Topics
Premium vs Policy Count

- When exposure rating, it is best to use a policy limits profile by premium.
- If you only have a policy count profile, you can estimate a premium distribution
  - Property – multiply policy count by policy limit to estimate TIV
    - Multiply policy count by the Limited Average Severity for the policy when using size of loss distributions (PSOLD) rather than first loss scales
  - Casualty – multiply policy count by ILF at corresponding policy limit

Miscellaneous Topics
Premium vs Policy Count – Property Example

Multiply policy count by policy limit to estimate TIV

Average Policy Limit	Policy Count	Policy Count Distribution	Estimated Premium (TIV)	Premium Distribution
75,000	2,755	60.31%	206,625,000	38.31%
150,000	1,439	31.50%	215,850,000	40.02%
250,000	268	5.87%	67,000,000	12.42%
350,000	52	1.14%	18,200,000	3.37%
450,000	28	0.61%	12,600,000	2.34%
550,000	12	0.26%	6,600,000	1.22%
650,000	3	0.07%	1,950,000	0.36%
750,000	3	0.07%	2,250,000	0.42%
850,000	4	0.09%	3,400,000	0.63%
950,000	2	0.04%	1,900,000	0.35%
1,250,000	1	0.02%	1,250,000	0.23%
1,750,000	1	0.02%	1,750,000	0.32%
Total	4,568		539,375,000	

Miscellaneous Topics
Premium vs Policy Count – Property LAS Example

Multiply policy count by LAS

Average Policy Limit	Policy Count	Policy Count Distribution	LAS	LAS * Count	Premium Distribution
75,000	2,755	60.31%	8,000	22,040,000	48.41%
150,000	1,439	31.50%	12,000	17,268,000	37.93%
250,000	268	5.87%	15,000	4,020,000	8.83%
350,000	52	1.14%	17,000	884,000	1.94%
450,000	28	0.61%	21,000	588,000	1.29%
550,000	12	0.26%	22,000	264,000	0.58%
650,000	3	0.07%	25,000	75,000	0.16%
750,000	3	0.07%	30,000	90,000	0.20%
850,000	4	0.09%	35,000	140,000	0.31%
950,000	2	0.04%	37,000	74,000	0.16%
1,250,000	1	0.02%	39,000	39,000	0.09%
1,750,000	1	0.02%	45,000	45,000	0.10%
Total	4,568			45,527,000	

Miscellaneous Topics
Premium vs Policy Count – Casualty Example

Multiply policy count by ILF at corresponding policy limit

Policy Limit	Policy Count	Policy Count Distribution	ILF	Estimated "Premium"	Premium Distribution
25,000	14	0.24%	1.000	14.00	0.13%
50,000	50	0.87%	1.200	60.00	0.54%
100,000	636	11.10%	1.500	954.00	8.57%
300,000	1,817	31.70%	1.800	3,270.60	29.39%
500,000	1,221	21.30%	2.000	2,442.00	21.95%
1,000,000	1,994	34.79%	2.200	4,386.80	39.42%
Total	5,732			11,127.40	