

# Estimating the Parameter Risk of a Loss Ratio Distribution

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CARE Seminar  
Boston, MA  
June 7-8, 2004





# Motivation For Study

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- Stop Loss Reinsurance Contracts
- Terms for Quota Share Contracts
- DFA Analysis



# Goals of Method

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- Relatively simple to understand
- Uses actual data to determine load
- Can be used in real time
- Provides a reasonably good estimate of parameter risk



# Key Concept of Method

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- This method determines sets of parameters that could have calculated the actual data, and the relative probability of each parameter set.
- This will be compared to “Best Fit” for pricing.

# Base Case Data

## Loss Ratio Distribution

	Actual	Actual	75.0%	78.0%	81.0%
Year	LR	Ln(LR)	Loss	Loss	Loss
1	73.4%	-0.309339	0.0%	0.0%	0.0%
2	71.7%	-0.332105	0.0%	0.0%	0.0%
3	77.9%	-0.249513	2.9%	0.0%	0.0%
4	83.6%	-0.178645	3.0%	3.0%	2.6%
5	64.1%	-0.444367	0.0%	0.0%	0.0%
6	76.2%	-0.27227	1.2%	0.0%	0.0%
7	69.5%	-0.364527	0.0%	0.0%	0.0%
8	69.1%	-0.369625	0.0%	0.0%	0.0%
9	73.3%	-0.310132	0.0%	0.0%	0.0%
10	74.2%	-0.29802	0.0%	0.0%	0.0%
Average	73.3%	(0.3129)	0.708%	0.300%	0.264%
Stdev	0.0534	0.0726	0.0124	0.0095	0.0083
Skew	0.2861				
Expected Loss Ratio		73.3%			
Actual Data Exp Loss On Line			23.609%	10.000%	8.801%
Fitted Exp Loss			0.812%	0.384%	0.151%
Fitted Exp Loss On Line			27.061%	12.811%	5.039%

# Parameter Sets and Relative Probabilities



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- How to determine relative probability of each parameter set?
  - Simulate
  - Likelihood function



# Bootstrap with Simulations

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- Simulate 10,000 ten-year blocks of loss ratios for each parameter set
- Simulated ten-year blocks that have a mean, standard deviation and skewness close to the actual data are considered viable



# Simulated at Best Fit

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- Use MLE Parameter Set with
  - Mu equal to  $-0.3129$
  - Sigma equal to  $0.0726$
- 114 Simulated ten-year blocks had a mean, standard deviation and skewness close to the actual data





# Relative Probabilities

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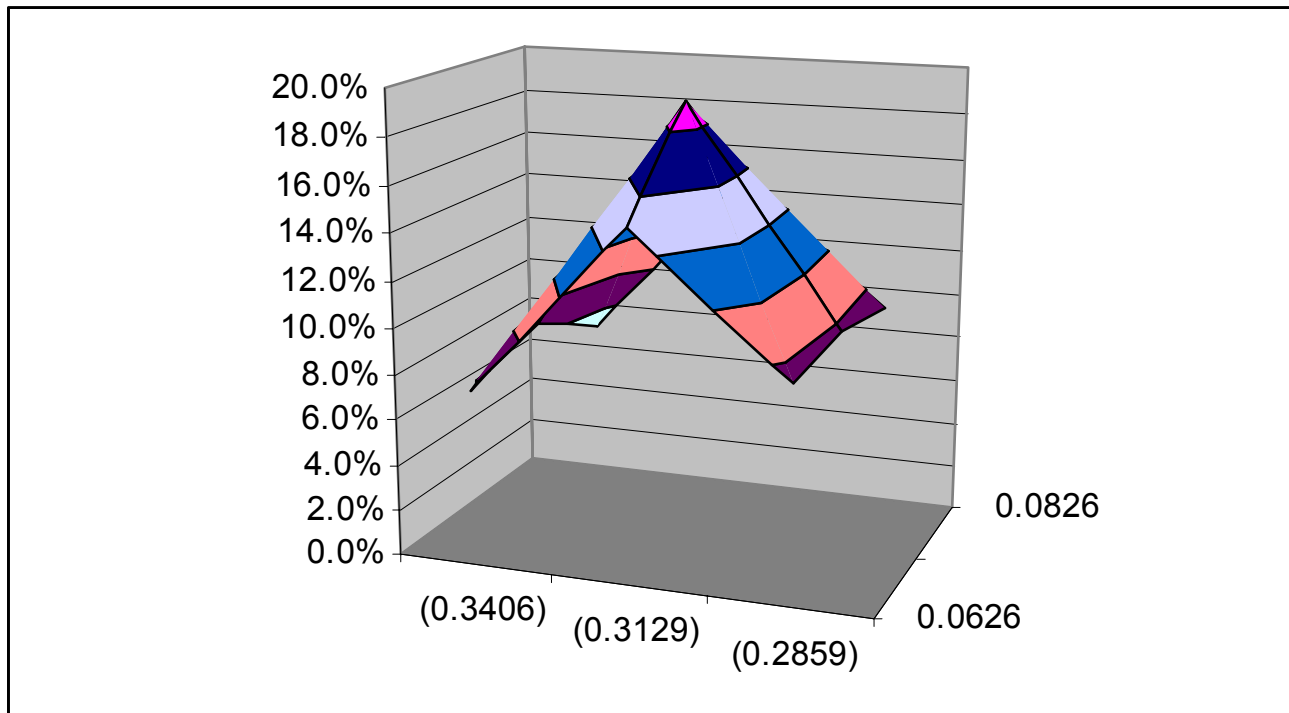
## Alternative Parameter Sets

	Loss Ratio	71.1%	73.1%	75.1%
	Mu	(0.3406)	(0.3129)	(0.2859)
Sigma a	0.0626	46	90	56
	0.0726	53	114	58
	0.0826	42	86	54

## Relative Probabilities

	Loss Ratio	71.1%	73.1%	75.1%
	Mu	(0.3406)	(0.3129)	(0.2859)
Sigma a	0.0626	7.7%	15.0%	9.3%
	0.0726	8.8%	19.0%	9.7%
	0.0826	7.0%	14.4%	9.0%

# Relative Probability Graph





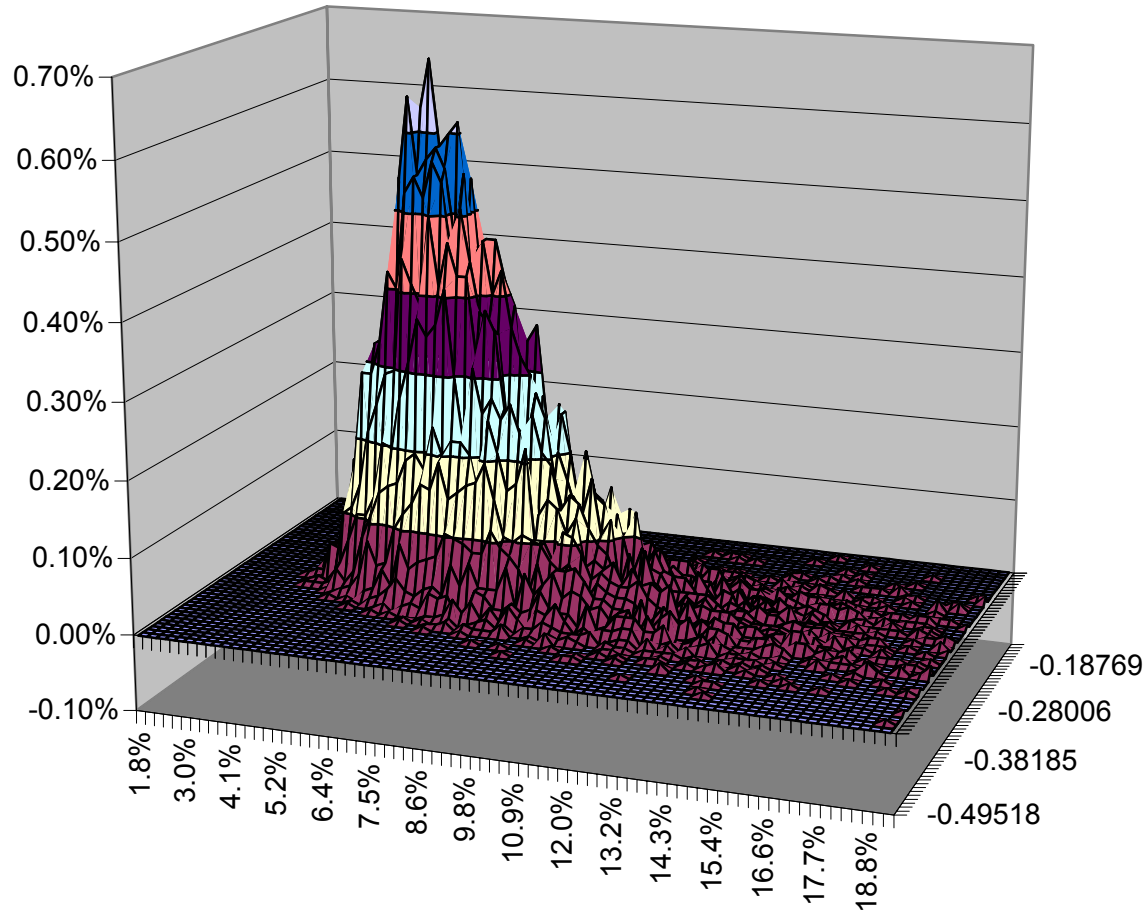
# Expand Concept

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- Develop ranges for Mu and Sigma
- Develop macro to step through ranges
- Determine which parameter sets are viable
- Determine relative probabilities for each viable parameter set

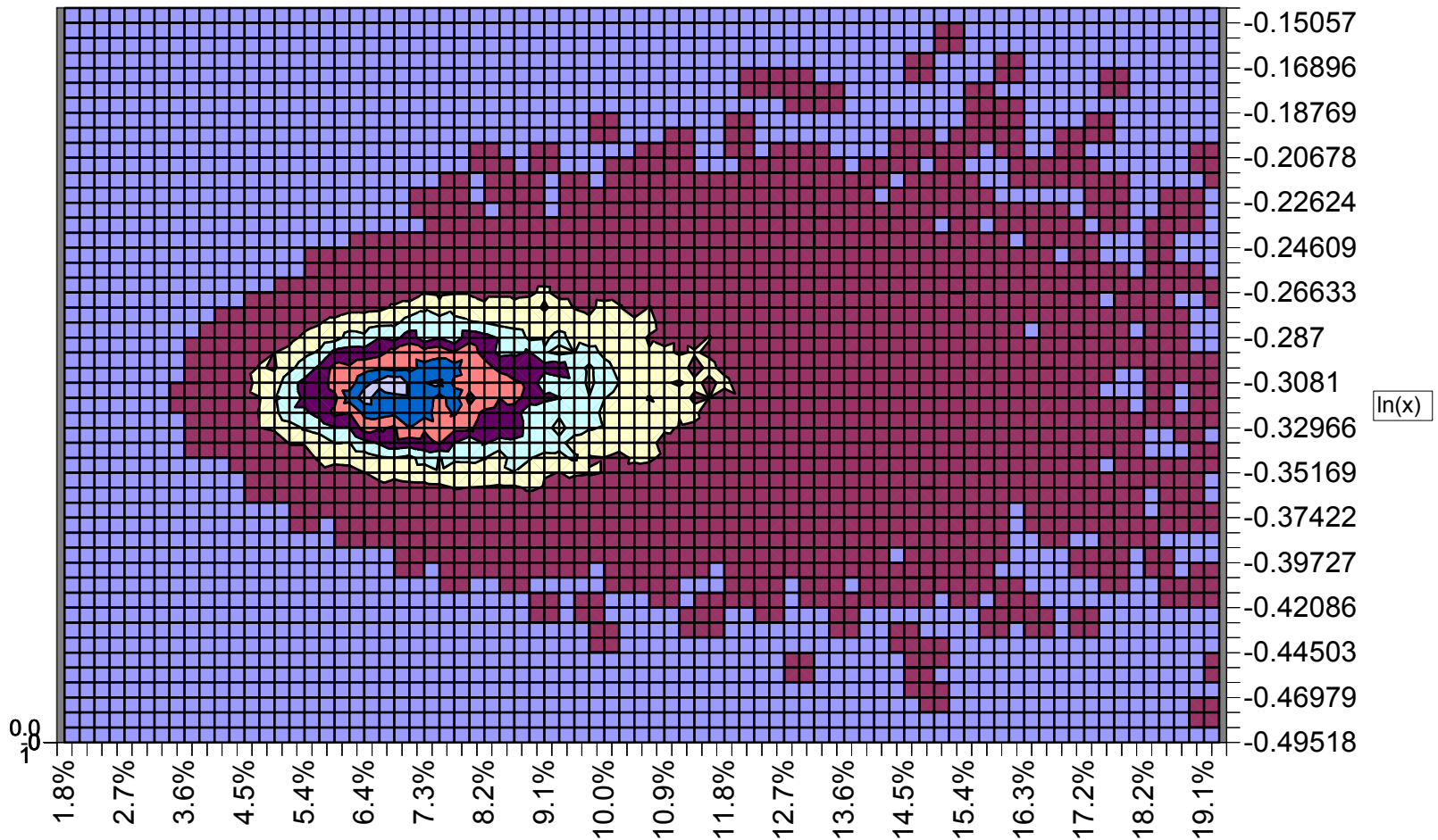
# Relative Probability Graph Side View

Sum of Adj Z



# Relative Probability Graph Top View

Sum of Adj Z





# Parameter Set

## Expected Losses by Layer

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### Expected Loss

		Actual	Best Fit	Simulation
75.00%	78.00%	0.708%	0.812%	0.905%
78.00%	81.00%	0.300%	0.384%	0.510%
81.00%	84.00%	0.264%	0.151%	0.269%

### Difference to Actual Data

		Actual	Best Fit	Simulation
75.00%	78.00%	0.0%	14.6%	27.8%
78.00%	81.00%	0.0%	28.1%	70.1%
81.00%	84.00%	0.0%	-42.7%	1.8%



# Method Meet Goals?

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- Has an intuitive feel
- Uses actual data
- Takes hours to run
- Not sure how accurate



# Simulation Considerations

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- Using simulations - not exact
- Only a sample of possible parameter sets





# Likelihood Function

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- Instead of simulating use likelihood function
- Multiplicative of PDF for all actual loss ratios
- Use same parameter sets as simulation for comparison





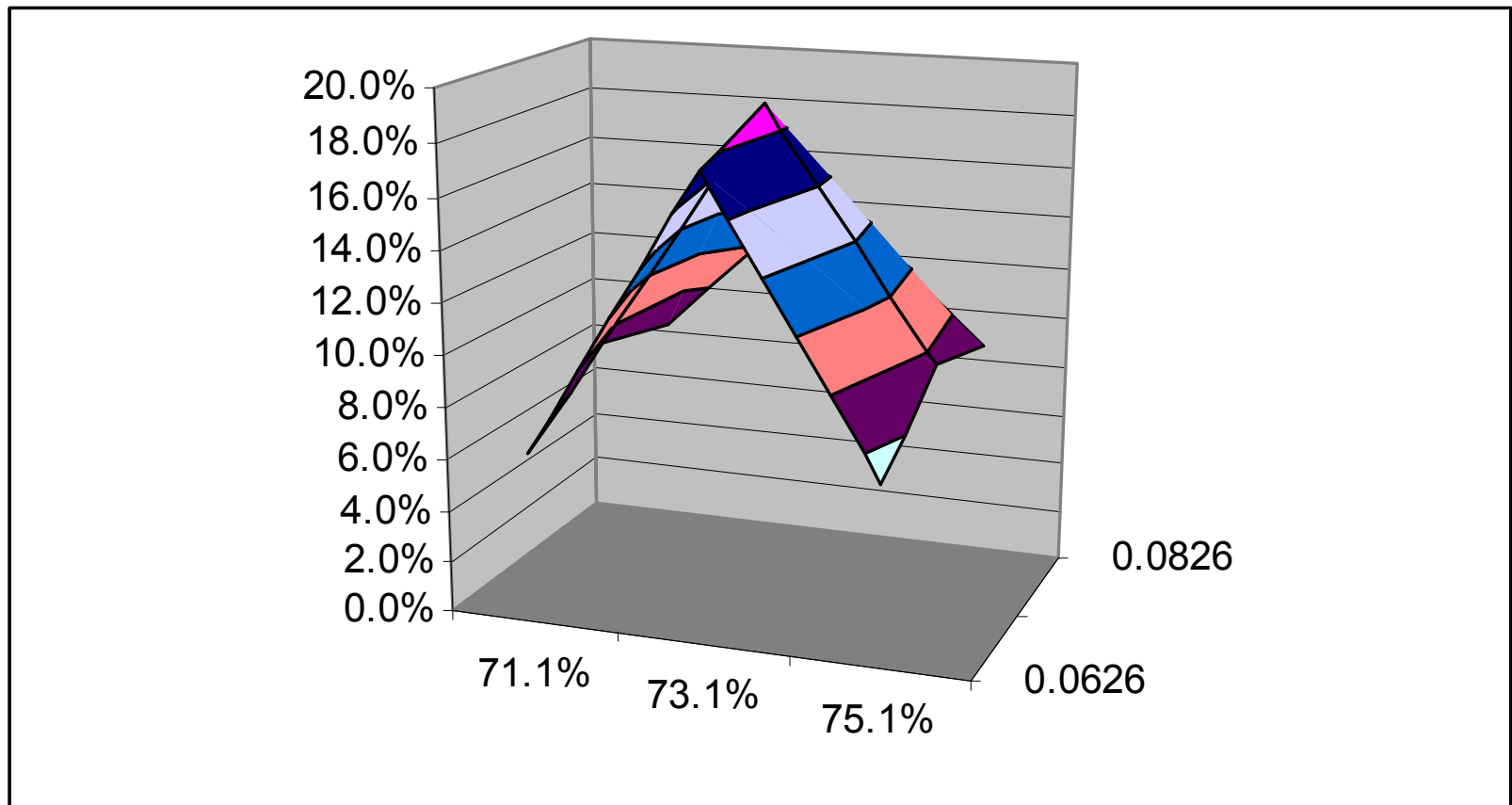
# Relative Probabilities

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## Relative Probabilities

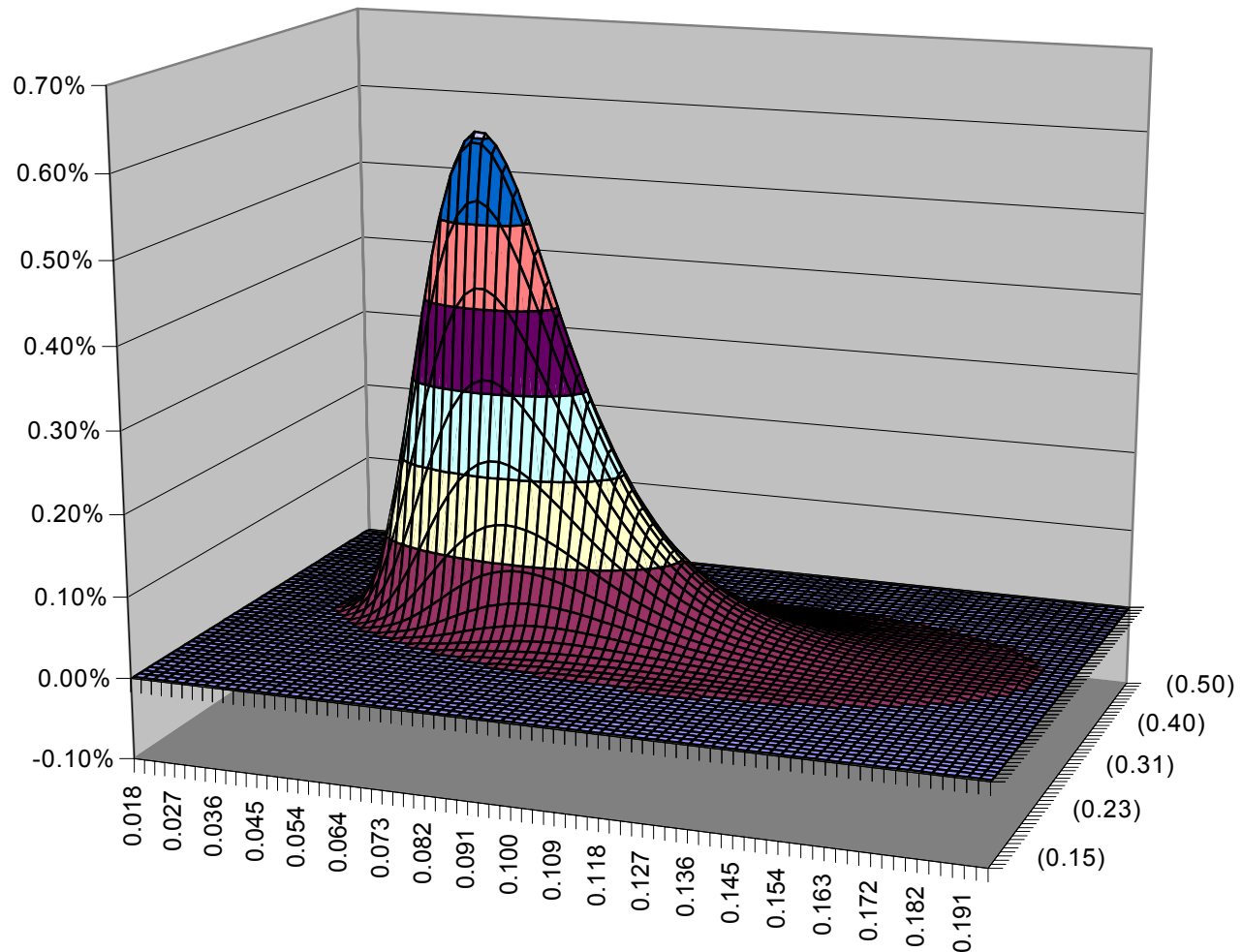
	Loss Ratio	71.1%	73.1%	75.1%
	Mu	(0.3406)	(0.3129)	(0.2859)
Sigma	0.0626	6.6%	17.6%	7.0%
	0.0726	9.1%	18.9%	9.5%
	0.0826	8.3%	14.5%	8.5%

# Relative Probability Graph



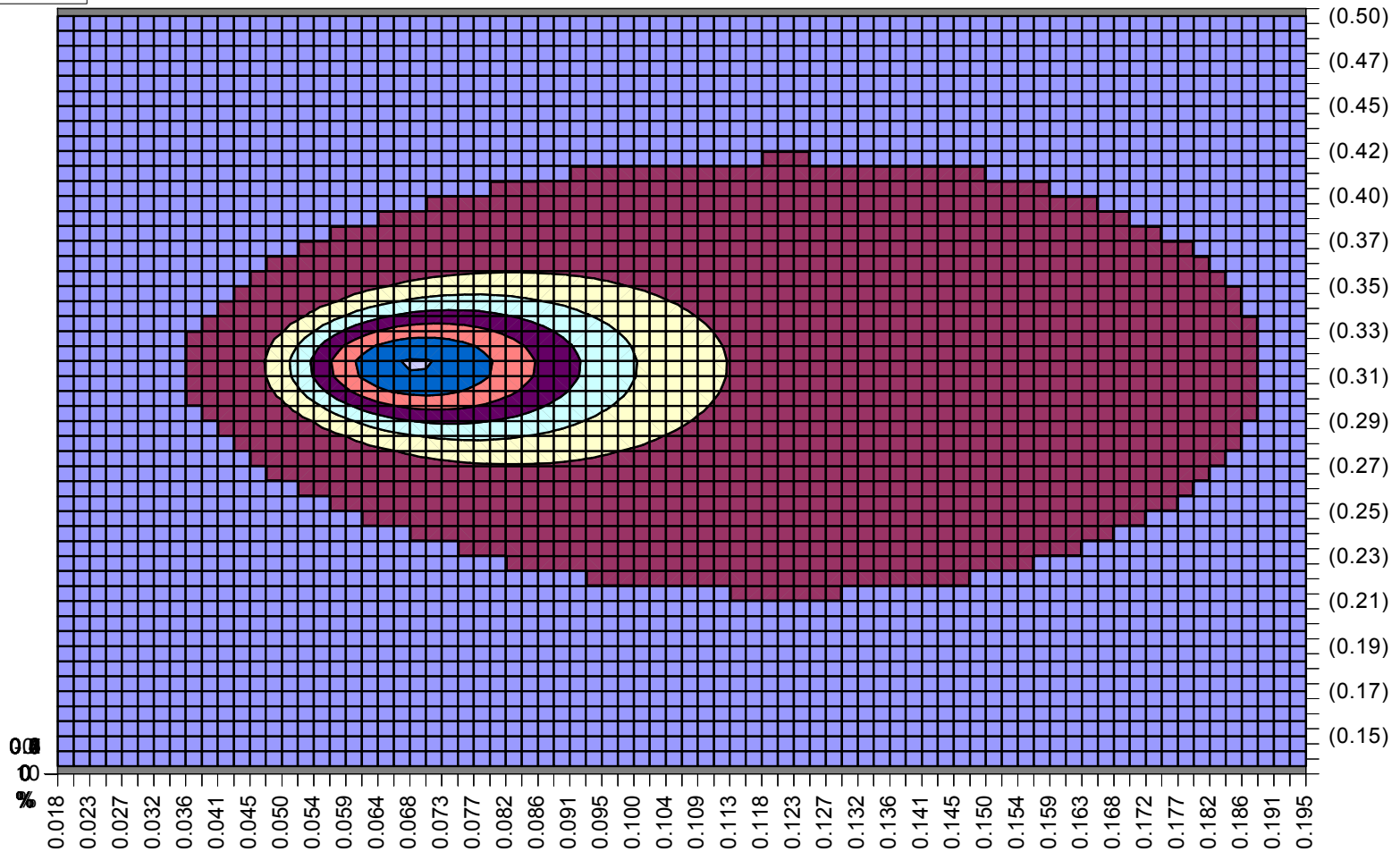
# Relative Probability Graph Side View

Sum of Prob



# Relative Probability Graph Top View

Sum of Prob



Sigma



# Comparison of Methods

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## Expected Loss

		Actual	Best Fit	Simulation	Likelihood
75.00%	78.00%	0.708%	0.812%	0.905%	0.900%
78.00%	81.00%	0.300%	0.384%	0.510%	0.505%
81.00%	84.00%	0.264%	0.151%	0.269%	0.264%

## Difference to Actual Data

		Actual	Best Fit	Simulation	Likelihood
75.00%	78.00%	0.0%	14.6%	27.8%	27.1%
78.00%	81.00%	0.0%	28.1%	70.1%	68.3%
81.00%	84.00%	0.0%	-42.7%	1.8%	-0.1%



# Method Meet Goals?

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- Has an intuitive feel
- Uses actual data
- Takes seconds to run
- Not sure how accurate, but consistent with simulations



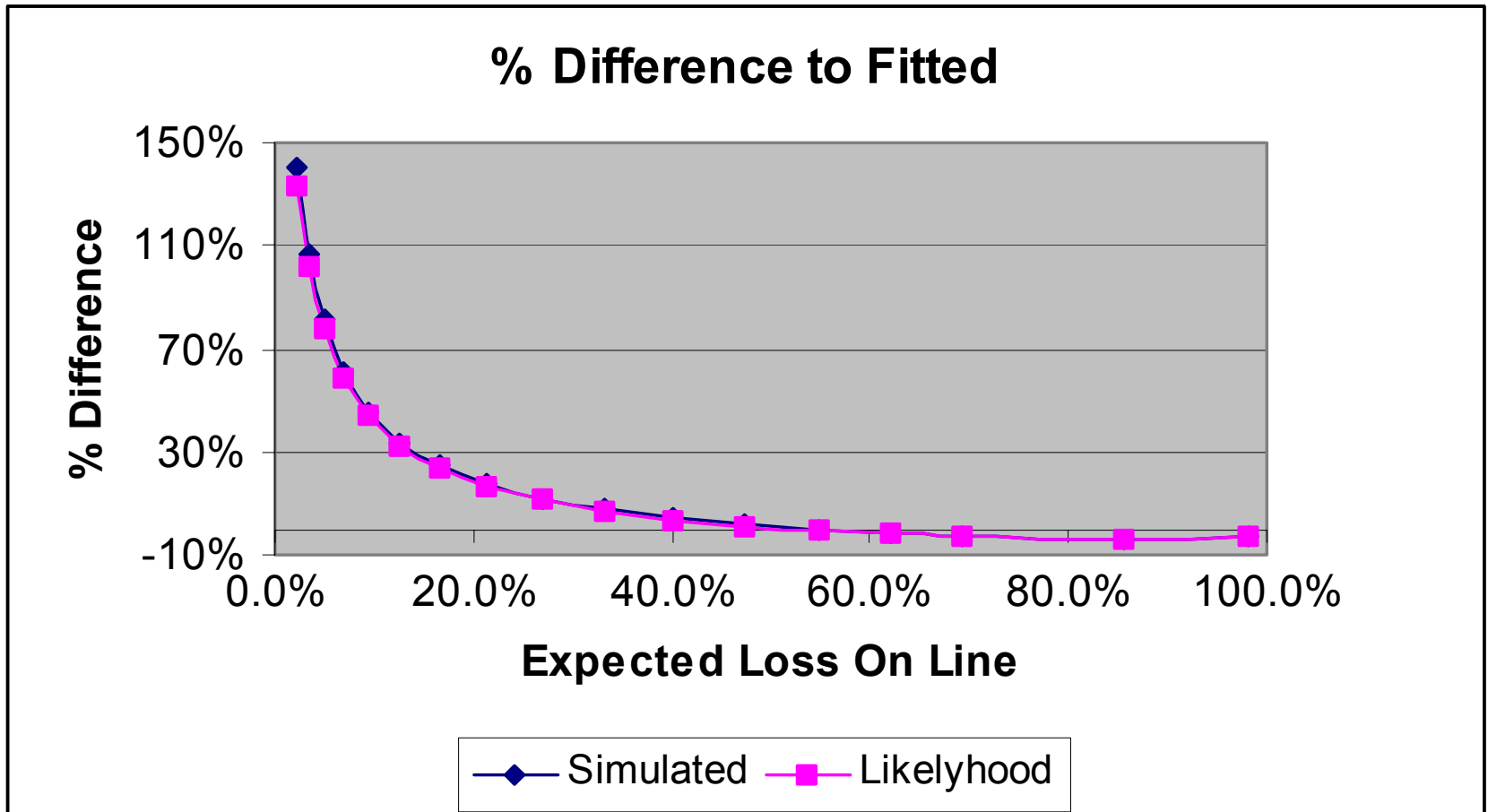


# Likelihood Considerations

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- Using likelihood function – answers do not change
- Only a sample of possible parameter sets

# % Differences By Expected Loss On Line





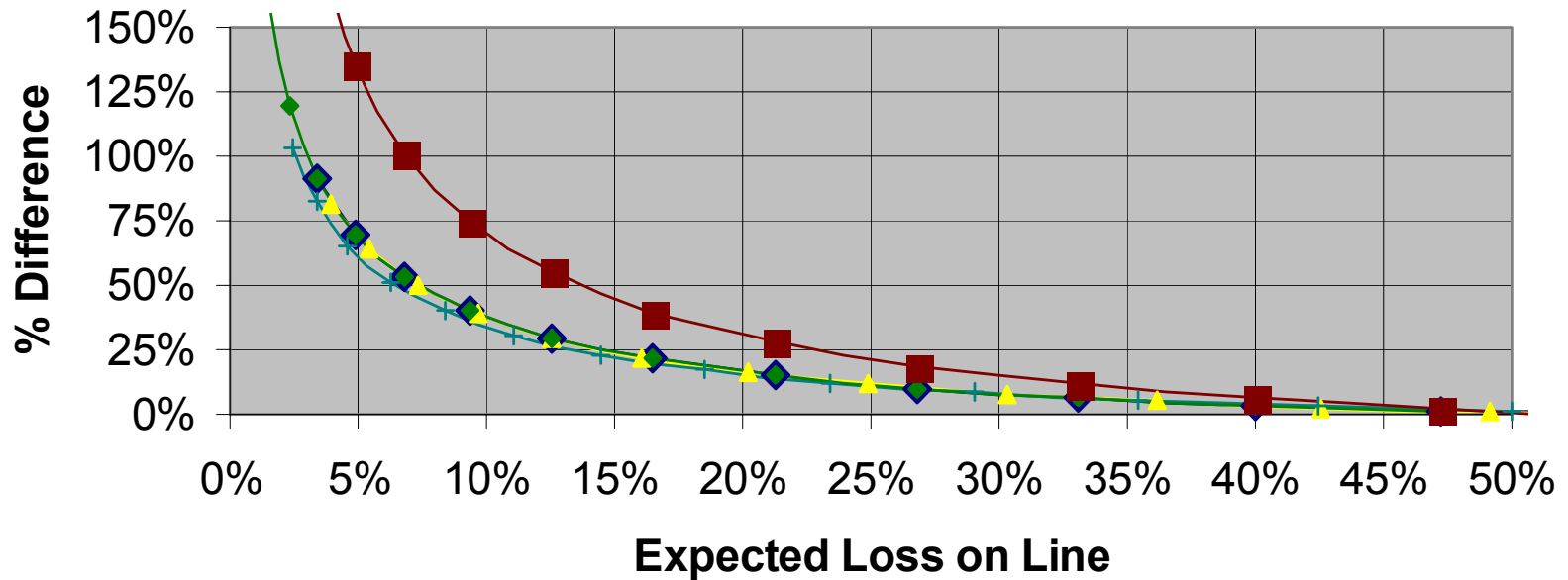
# Sensitivity Testing

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- Increase the mean
- Increase the standard deviation
- Increase the skew
- Fewer years of data

# % Difference By Expected Loss on Line

## Differences by Expected Loss on Line



◆ Base    ▲ Higher Mean    + Higher Std    ◆ Higher Skew    ■ 5 Yrs Data



# Comments on Determining Parameter Sets

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- Step size through the parameter set ranges
- Size of parameter set ranges



# Practical Considerations

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- Using CDF Vs PDF
- Using a floor on relative minimum probabilities



# Other Considerations

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- Only a sample of possible parameter sets
- Cat exposures should be removed
- Requires some judgment
- Exposures not present in historical data are not taken into account
- Process Risk still present and not accounted for in this methodology