CAPITAL ALLOCATION
AND
THE COST OF CAPITAL

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papers

Neil Doherty, 1991, “The Design of Insurance Contracts when Liability Rules are Unstable” Journal of Risk and Insurance, LVIII,- 227-246

Robert Merton and Andre Perold, 1993, “Theory of Risk Capital in Financial Firms, Journal of Applied Corporate Finance, 6, 16-32

Stewart Myers and James Read, 2001, “Surplus Allocation for Insurance Companies” fothcoming, Journal of Risk and Insurance.

BASIC FUNCTION OF INSURER

COST OF RISK ARISES FROM

Increases probability of ruin

Increases conflicts - owners v policyholders

Jeopardizes financing of future projects

Provides noise to earnings

Tax convexity

MANAGING RISK

RISK MANAGEMENT
hedge - contingent capital - debt conversion/mutual reinsurance

CAPITAL MANAGEMENT
leverage - contingent capital - debt conversion/mutual reinsurance

CAPITAL MANAGEMENT

Proper measurement of risk capital

How much capital

What type of capital

What is hurdle rate of return

How to allocate capital (why allocate capital)

RISK CAPITAL(Merton/Perold)

Risk capital is capital that "insurers" net assets of firm relative to riskless investment of those net assets.

Risk capital can be:

shareholder guarantee
shareholders capital at risk to pay losses

cost of reinsurance

cost of asset insurance

loss of revenue from sale of risky policies
default sensitive premiums

RISK CAPITAL - INSURANCE EXAMPLES

ADAPTED FROM MERTON/PEROLD

Example 1

Insurer issues policies with premium of 100. Insurer provides 12 in equity. Buys default free bonds with r/r of 10% giving end of year value of 100(1.1)=110. Policy liabilities as follows. Reinsurance bought for premium of 12

Loss	Reinsurance	Loss-reinsurance	Assets	Equity
90	0	90	110	20
150	40	110	110	0
200	90	110	110	0

ACCOUNTING BALANCE SHEET

ASSETS		LIABILITIES	
bonds	100	policies	100
reinsurance	12	equity	12

RISK CAPITAL BALANCE SHEET

ASSETS		LIABILITIES	
bonds	100	policies	100
reinsurance	12	equity (risk cap)	12

Example 2

Insurer issues risky policies with premium of 88 (discounted by a risk premium of 12). Insurer provides 12 in equity. Insurer buys default free bonds with r/r of 10% giving end of year value of 100(1.1)=110. Policy liabilities as follows. No reinsurance purchased

Loss	Assets	Default Implicit reinsurance	Equity
90	110	0	20
150	110	40	0
200	110	90	0

ACCOUNTING BALANCE SHEET

ASSETS		LIABILITIES	
bonds	100	policies	88
		equity	12

RISK CAPITAL BALANCE SHEET

ASSETS		LIABILITIES	
bonds	100	policies (riskless)	100
implicit reinsurance	12	equity (risk cap)	12

***Example 3A*. This case has risky assets and liabilities. Insurer tries to use as much asset insurance and reinsurance as possible to avoid all default risk on policies. The reinsurance premium is 12. Insurer buys risky bonds for a price of 98 and with a payout of 110 if there is no default and 67 if default. However, the asset risk is insured up to a value of 90 (the insurance will pay 23 if there is default) for a premium of 4. It can be seen that, without dual interconnected triggers, all policy risk cannot be removed. Thus, the policies sell for a discount of 2 (i.e., 100-2=98). To help the insurer pay the premia for asset insurance and reinsurance, shareholders provide cash of 12+4=16**

Assets		Loss	Reins-urance	Asset Insurance	Loss- reins - asset ins.	Pol default	Equity
No default	110	90	0	0	90	0	20
	110	150	40	0	110	0	0
	110	200	90	0	110	0	0
Default	67	90	0	23	67	0	0
	67	150	40	23	87	20	0
	67	200	90	23	87	20	0

ACCOUNTING BALANCE SHEET

ASSETS		LIABILITIES	
bonds	98	policies	98
reinsurance	12	equity	16
asset ins	4		

RISK CAPITAL BALANCE SHEET

ASSETS		LIABILITIES	
bonds	98	policies	100
reinsurance	12	equity cash	(2)
asset ins	4	equity (risk cap)	18
implicit reins	2		

The negative cash equity can be explained as follows. The owner does not have to provide all risk capital. Policyholders pay 2 for the implicit reinsurance.

***Example 3B*. This example is basically the same as 3A, but now we will allow the insurer's owners to provide more capital to avoid the default risk to policyholders. An additional equity of 20/(1+r) is provided and is invested risk free at the riskless interest rate "r". This will yield and additional 20 at year end. The remaining money is invested in a risky asset as shown in example 3A. This means that policyholders are not asked to provide implicit reinsurance on their own policies and will pay the full 100 premium for the policy. Thus, the total cash available is 100 in premium plus 14 in equity, (the 16 equity in example 3A can be reduced to 14 because the policyholders now pay 100 instead of 98 for the policies), plus 20/(1+r). The total value of assets is now 98 + 20/(1+r). The reinsurance and asset insurance as kept as before.**

Assets		Loss	Reins-urance	Asset Insurance	Loss- reins - asset ins.	Pol default	Equity
No default	130	90	0	0	90	0	40
	130	150	40	0	110	0	20
	130	200	90	0	110	0	20
Default	87	90	0	23	67	0	20
	87	150	40	23	87	0	0
	87	200	90	23	87	0	0

ACCOUNTING BALANCE SHEET

ASSETS		LIABILITIES	
bonds	98 + 20/(1+r)	policies	100
reinsurance	12	equity	14 + 20/(1+r)
asset ins	4		

RISK CAPITAL BALANCE SHEET

ASSETS		LIABILITIES	
bonds	**98 + 20/(1+r)**	**policies**	**100**
INSURANCE			
reinsurance	**12**	**equity cash**	**20/(1+r) -2**
asset ins	**4**	**equity risk cap**	**18**
equity residual	**2**		

COST OF CAPITAL

METHODS FROM CAPITAL MARKET

$R_f + \beta(E(R_m - R_f)$ **single factor model**

$R_f + \sum\beta_i (E(R_{m\,i} - R_f)$ **multi factor model**

problem **: fails to account for firm specific costs of risk**

RAROC (RISK ADJUSTED RETURN ON CAPITAL

R* = F(FIRM RISK)

problem **: fails to reflect the price of risk in capital markets; i.e., β's**

COMPOSITE METHODS (Doherty 1991; Froot 1998)

1. **Activities that are risky increase frictional costs**
   - **Costs of insolvency**
   - **Distort incentives**
   - **Jeopardize funding of new projects**

   THE RETURN MUST BE HIGH ENOUGH TO ACCOUNT FOR THESE COSTS

2. **Earnings which display high SYSTEMATIC risk are valued less in the capital market**

   RETURN ON INVESTORS MUST COMPENSATE FOR THE "BETA" RISK

$$VALUE\ OF\ (C) = \frac{E(C)}{1 + R}$$

$$R = \{1 + R_f + \sum\beta_i (E(R_{m\,i} - R_f)\} \{1 + R^*\}$$

CAPITAL BUDGETING PROBLEM

Value the capital market places NOW on an income stream "C_1" expected in one period's time

$$V(C_1) = E(C_1) - \frac{E(R_m) - R_f}{\sigma^2(R_m)} COV(C_1, R_m)$$

Firm ADDS PROJECT with random income of P in one period and capital cost K(P) .

what is value of firm? ***$V(C_1 ; P; K(P))$***

First recognize frictional costs of risk *WITHIN* firm

$E(C_1 ; P) = E(C_1) + E(P) - f\{ \Delta \text{ RISK} \} \equiv E(C_1) + E(C_2) \equiv C$

$E(C_1 ; P) = E(C_1) + E(P) - f\{ COV (P; C_1) \} \equiv E(C_1) + E(C_2) \equiv C$

<u>Addition of P increase firm risk and thus frictional costs BY f{..}</u>

SO THE VALUE INCLUDING THE PROJECT IS:

$$V(C) = E(C_1) - \left(E(C_2) - K(P)\right) - \frac{E(R_m) - R_f}{\sigma^2(R_m)} COV(C, R_m)$$

OR, SUBSTITUTING FOR C_2,

$$V(C) = E(C_1) + (E(P) - K(P)) - \lambda[COV(P; C_1)] - \frac{E(R_m) - R_f}{\sigma^2(R_m)} COV(C, R_m)$$

VALUE OF FIRM INCLUDING NEW PROJECT IS:

$$V(C) = E(C_1) + (E(P) - K(P)) - \lambda[COV(P; C_1)] - \frac{E(R_m) - R_f}{\sigma^2(R_m)} COV(C, R_m)$$

NOTICE THERE ARE TWO ADJUSTMENTS FOR RISK

one relating to internal frictional costs of risk

$$\lambda[COV(P; C_1)]$$

one relating to cost of risk in capital market

$$\frac{E(R_m) - R_f}{\sigma^2(R_m)} COV(C, R_m)$$

TECHNICAL DERIVATION OF "DOUBLE RISK" PREMIUM
ADAPTED FROM DOHERTY 1991

Value the capital market places on an income stream "C"

$$V(C) = E(C) - \frac{E(R_m) - R_f}{\sigma^2(R_m)} COV(C, R_m)$$

However, if C is generated from risky cash flows in a firm then its value will already have been reduced by the various frictional risk costs. Think of an investment "I" generated a value "P" in one period time if there were no risk costs. However, given the risk costs the expected value will be "C" as follows:

where $E(C) = E(P)\left[\frac{1}{1 + f(\sigma^2(P))}\right]$

where f(.) Can be thought of as the "discount" to reflect the various costs of risk (costs of distress, incentive conflicts, jeopardy of new projects) etc.

Now return on investment is $\frac{C - I}{I} = R_i$.

Therefore, $COV(R_i, R_m) = COV\left(\frac{C-I}{I} : R_m\right) = \frac{1}{I} COV(C ; R_m)$

Now assuming $\sigma^2(P) = \sigma^2(C)$ and COV ($P; R_m$) = COV ($C; R_m$) (which does not seem too unreasonable),

$$R_i = R_f + \beta\left(E(R_m) - R_f\right) = R_f + \frac{COV\left(\frac{C-I}{I} ; R_m\right)}{\sigma^2(R_m)}\left(E(R_m) - R_f\right)$$

$$V(P) = \frac{E(P)\left[\frac{1}{1 + f(\sigma^2(P))}\right]}{1 + R_i} = \frac{E(P)}{\left[1 + R_f + \beta\left(E(R_m) - R_f\right)\right]\left[1 + f(\sigma^2(P))\right]}$$

CAPITAL ALLOCATION

WHY DO WE WISH TO ALLOCATE CAPITAL?

VALID REASONS

CAPITAL BUDGETING: TO ENSURE PROPER MACRO CAPITAL BUDGETING DECISIONS;

EACH BUSINESS UNIT SECURES ADEQUATE RATE OF RETURN ON THE CAPITAL THAT IS NEEDED TO KEEP THAT UNIT WITHIN THE CORPORATE UMBRELLA

PRICING: TO ENSURE THAT THE CAPITAL EMBEDDED IN THE SUPPLY OF EACH PRODUCT IS PROPERLY PRICED

EACH POLICY IS PRICED ACCORDING TO THE CAPITAL REQUIRED TO DELIVER THAT POLICY

INVALID REASON

CAPITAL MUST BE "PARKED" SOMEWHERE.

THE TROUBLE WITH "PARKING" IS THAT IS AFFECTS INCENTIVES;

parked capital tends to be used rather than be returned to owners

MARGINAL CAPITAL ALLOCATION APPROACHES

MERTON/ PEROLD

INSURERS CAN DIVERSIFY RISK.

AS MORE POLICIES ARE ADDED TO A PORTFOLIO, RISK INCREASES AT A SLOWER RATE THAN THE INCREASE IN POLICIES. AVERAGE RISK PER POLICY FALLS WITH “n”

A CONSEQUENCE IS AS FOLLOWS

SUPPOSE THE FIRM WISHES TO KEEP THE RISK PER POLICY (EXPECTED VALUE OF DEFAULT PER POLICY) CONSTANT. THEN IT CAN MAINTAIN THIS TARGET BY INCREASING EQUITY AT A LOWER RATE THAN THE INCREASE IN POLICIES

SO WE CAN THINK OF THE CAPITAL “ATTRIBUTED” TO EACH LINE OF BUSINESS AS THE ADDITIONAL CAPITAL THAT IS NEEDED TO MAINTAIN THE DEFAULT RISK WHEN THAT LINE IS ADDED TO ALL PRE-EXISTING LINES OF BUSINESS.

A LOGICAL CONSEQUENCE OF DIVERSIFICATION IS

SUM OF CAPITAL SO ALLOCATED TO THE LINES IS LESS THAN THE TOTAL CAPITAL OF THE FIRM

UNALLOCATED CAPITAL

ILLUSTRATIONS OF UNALLOCATED CAPITAL

Firm has “n” business units each with gross assets having value of 1000 and the standard deviation of the ratio of assets to liabilities is 0.375. In example 1 the correlation coefficient between any two business units is 0.1; in example 2 it is 0. The risk capital was calculated by Meron/Perold’s approximation formula. Risk Capital = $0.4\sigma\sqrt{T}$

TABLE 1 - ALL BUSINESS UNITS HAVE CORRELATION OF 0.1

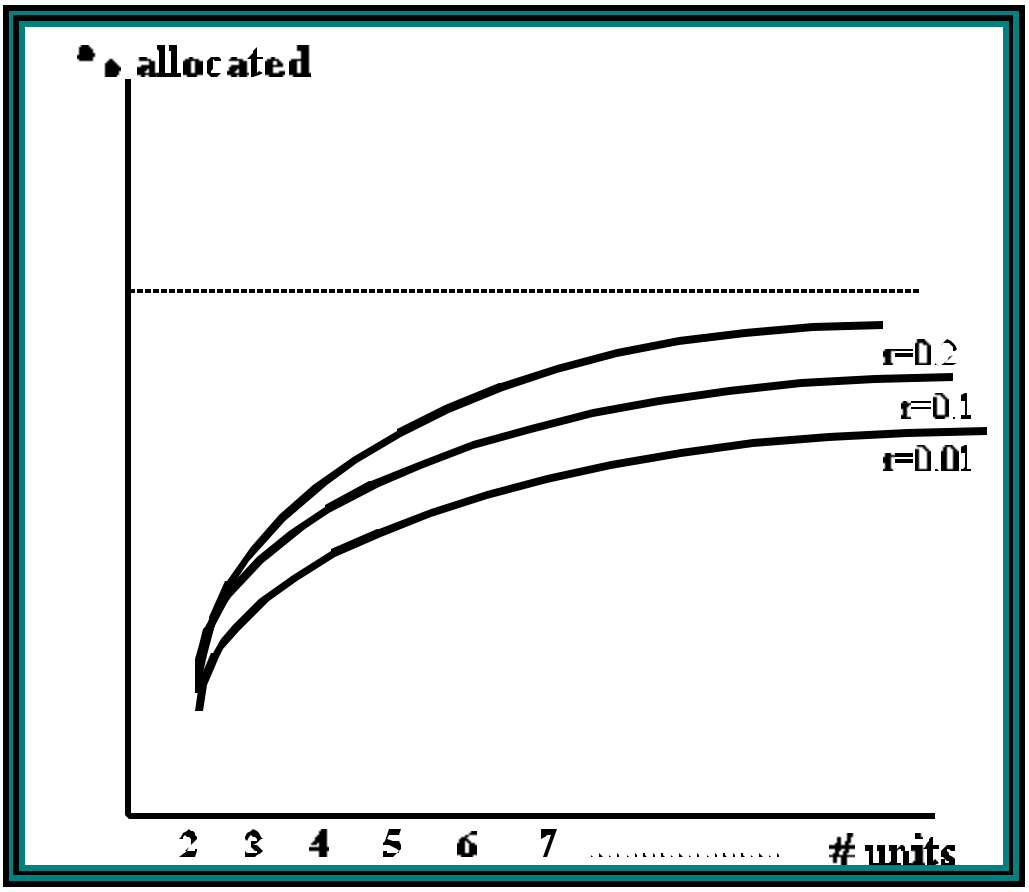
	gross assets	capital	marginal capital	total allocated	percent allocated
1	1000	150			
2	2000	222	72	144	64.8
3	3000	285	63	189	66.3
4	4000	342	57	228	66.6
5	5000	397	55	275	69.3
6	6000	450	53	318	70.6
7	7000	502	52	364	72.5
100					95.9
1000					99.6

TABLE 2. ALL BUSINESS UNITS ARE INDEPENDENT

	gross assets	capital	marginal capital	total allocated	percent allocated
1	1000	150			
2	2000	212	62	124	58.5
3	3000	260	48	141	55.4
4	4000	300	41	164	54.6
5	5000	335	35	175	52.8
6	6000	367	32	192	52.3
7	7000	397	30	210	52.8
100					50.2
1001					29.1

TABLE 3 - ALL BUSINESS UNITS HAVE CORRELATION OF 0.01

	gross assets	capital	marginal capital	total allocated	percent allocated
1	1000	150			
2	2000	213.2	63.2	126.4	59.3
3	3000	262.4	49.2	147.6	56.2
4	4000	304.5	42.07	168.3	55.3
5	5000	342.05	37.55	187.76	54.89
6	6000	376.50	34.45	206.67	54.89
7	7000	408.59	32.09	224.66	54.98
100					75.23
1000					96.82



Limit undefined at r=0

MYERS/ READ

SPECIAL CASE OF MERTON - PEROLD

IMAGINE THERE IS A ***VERY*** LARGE NUMBER OF ACTIVITIES IN THE FIRM AND WE WISH TO ALLOCATE CAPITAL TO EACH MICRO ACTIVITY.

WE CAN NOW USE CONTINUOUS MATHEMATICS TO APPROXIMATE THE RESULT.

MARGINAL CAPITAL :

> SUPPOSE WE WISH TO ADD ONE MORE POLICY TO OUR EXISTING PORTFOLIO, BUT DID NOT WISH TO INCREASE THE DEFAULT RISK OF THE FIRM. HOW MUCH ADDITIONAL CAPITAL WOULD WE NEED TO WRITE THIS POLICY?

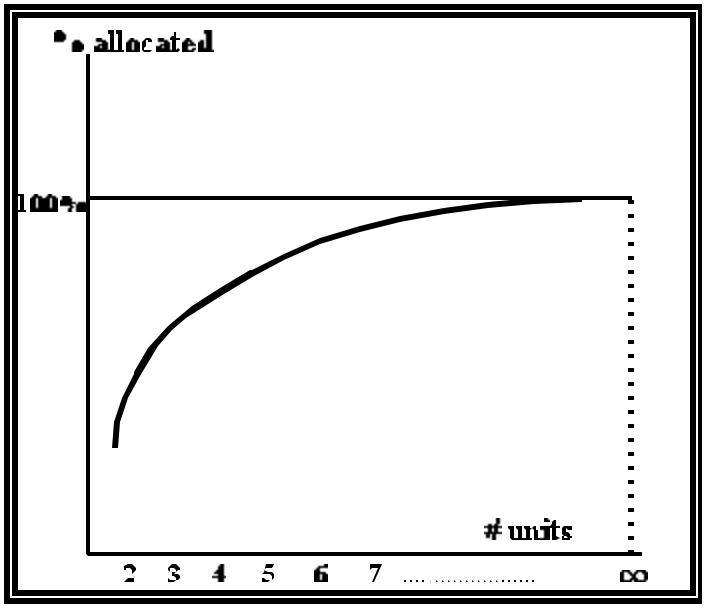
THE MYERS/READ CAPITAL ALLOCATION METHOD

> ALLOCATE CAPITAL TO EACH ACTIVITY SUCH THAT THE MARGINAL CONTRIBUTION TO THE DEFAULT VALUE IS EQUALIZED ACROSS ACTIVITIES
>
> ***SEDUCTIVE BEAUTY*** IS THAT THE MARGINAL CAPITAL ALLOCATIONS ***ADD UP TO THE TOTAL CAPITAL OF THE FIRM***

NO UNALLOCATED CAPITAL

GOOD FOR: PRICING POLICIES WHERE VERY LARGE 3 OF POLICIES

NOT GOOD FOR: CAPITAL BUDGETING WHERE SMALL # OF UNITS



Myers and Read truly marginal approach is the limit of Merton Perold as $n \to \infty$

DANGER OF ALLOCATING ALL CAPITAL

TWO INCUMBENT DIVISION A AND B

DIVISION	EXPECTED LIABILITIES	CASH FLOW	CAPITAL CONSTANT DEFAULT	ROE
A + B	200	8+8=16	100	16%
C	100	4		
A+B+C	300	8+8+4=20	125	16%

ALLOCATE ALL CAPITAL

ALL DIVISIONS ARE IDENTICAL IN RISK AND CROSS CORRELATIONS - THEREFORE ALL WOULD HAVE SIMILAR CAPITAL ALLOCATIONS OF 125/3 = 41.667 EACH

CAPITAL BUDGETING DECISION WITH FULL ALLOCATION

CAPITAL ALLOCATED TO C	41.667		
CASH FLOW	4		
ROE	0.96	< 12.5	REJECT

MERTON PEROLD

MARGINAL CAPITAL OF C IS 125 -100 = 25

CAPITAL ALLOCATED TO C	25.00		
CASH FLOW	4		
ROE	0.16	> 12.5	ACCEPT