Machine Learning, Regression Models, and Prediction of Claims Reserves

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Mathias Lindholm

CLRS, September, 2020

The majority of the presented material is based on recent and ongoing joint work with

- Richard Verrall
- Felix Wahl
- Henning Zakrisson

in particular the manuscript

"Machine Learning, Regression Models, and Prediction of Claims Reserves"

which is accepted for publication in CAS E-Forum, see Lindholm et al. (2020)

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(i.e. prepare for some abuse of notation, for more details see Lindholm et al. (2020))

Outline of the talk:

► Regression models: Double Chain-Ladder (DCL) type models

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- Parametrisations and machine learning estimation
- Numerical illustrations: Gradient Boosting Machines and Neural Networks
- Closing remarks

 Aggregated payment data used by e.g. the std chain-ladder technique is not sufficient in order to produce separate IBNR / RBNS reserves

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 \Rightarrow "Double Chain-Ladder" (DCL) type models

- DCL type models are similar to standard chain-ladder technique models, that
 - can produce separate RBNS / IBNR reserves
 - have parameters with constructive interpretations
 - are easy to bootstrap

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We will not present the standard DCL model from Miranda et al. (2012), but rather the so-called Collective Reserving Model (CRM) from Wahl et al. (2019) which is an over-dispersed Poisson model

Model 1

The CRM based on $X_{i,j}$ and $N_{i,j}$ data from Wahl et al. (2019) can be written on the form

$$X_{i,j} \mid \mathcal{N} \sim \mathsf{ODP}\left(\sum_{k=0}^{j \wedge d} \psi_{i,j-k,k} \mathsf{N}_{i,j-k}, \varphi\right),$$

and

$$N_{i,j} \sim \mathsf{ODP}(\nu_{i,j}, \phi),$$

that is,

$$\mathsf{E}[X_{i,j} \mid \mathcal{N}] = \sum_{k=0}^{j \wedge d} \psi_{i,j-k,k} \mathsf{N}_{i,j-k} = \mathsf{Var}(X_{i,j} \mid \mathcal{N}) / \varphi,$$

and

$$\mathsf{E}[N_{i,j}] = \nu_{i,j} = \mathsf{Var}(N_{i,j})/\phi.$$

All $X_{i,j}$ are assumed to be conditionally independent, given N, and all $N_{i,j}$ are assumed to be independent.

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- (ii) The N_{ij} s act as exposures in the X_{ij} part of the model, and the N_{ij} s are modelled following a standard ODP cross-classified Chain-Ladder model
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- (iv) The RBNS reserves are obtained by conditioning on knowledge of N_{ij} s, and the IBNR reserves are obtained by also predicting future N_{ij} s, i.e. (for combinations of (i, j) corresponding to the future, with allowed values of k)

RBNS contribution: $E[X_{i,j} | \mathcal{N}] = \sum_k \psi_{i,j-k,k} N_{i,j-k}$

IBNR contribution: $E[X_{i,j}] = \sum_k \psi_{i,j-k,k} E[N_{i,j-k}]$

Note that (iii) discusses (i,j,k)-information, which is something needed in the derivation of the CRM (and DCL):

▶ Let X_{ijk} denote the total amount of payments from accident year *i*, that come from claims that are reported with *j* years delay, and that are paid *k* years after reporting

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- ▶ Let X_{ijk} denote the total amount of payments from accident year *i*, that come from claims that are reported with *j* years delay, and that are paid *k* years after reporting...
- ...but when using X_{i,j,k} data the N_{i,j}s are not needed to produce separate RBNS and IBNR reserves, since the outstanding claim payments

$$R_i^{\mathcal{R}} := \sum_{j=0}^{m-i} \sum_{k>m-(i+j)} X_{i,j,k},$$

and

$$R_i^{\mathcal{I}} := \sum_{j=m-i+1}^{m-1} \sum_k X_{i,j,k}.$$

This suggests two additional models:

Model 2

The CRM based on $X_{i,j,k}$ and $N_{i,j}$ data from Wahl et al. (2019) can be written on the form

$$X_{i,j,k} \mid \mathcal{N} \sim \mathsf{ODP}\left(\psi_{i,j,k} \mathsf{N}_{i,j}, \varphi\right),$$

and

 $N_{i,j} \sim \text{ODP}(\nu_{i,j}, \phi),$

and in particular,

 $\mathsf{E}[X_{i,j,k} \mid \mathcal{N}] = \psi_{i,j,k} \mathsf{N}_{i,j},$

All $X_{i,j,k}$ are assumed to be conditionally independent, given N, and all $N_{i,j}$ are assumed to be independent.

Model 3

Let $X_{i,j,k}$ be defined as an over-dispersed model according to

$$X_{i,j,k} \sim \mathsf{ODP}(\psi_{i,j,k}, \varphi),$$

with

$$\Xi[X_{i,j,k}] = \psi_{i,j,k} = \mathsf{Var}(X_{i,j,k})/\varphi,$$

where all $X_{i,j,k}$ are assumed to be independent.

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Problem: overfitting

Parametrisations and machine learning estimation – Calibration and overfitting

 GBMs and NNs are estimated using iterative numerical procedures

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Parametrisations and machine learning estimation – Calibration and overfitting

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- Problem: In a claims reserving context we don't have access to fully developed triangles that are representative for the future

Following Gabrielli et al. (2019) we instead start from the *claims database* and construct

- (i) one set of *historical* triangles to be used as "in-sample" data for the purpose of estimation
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Next: numerical illustration using GBMs and NNs

The models to be considered are (primarily)

- The (homogeneous) CRM: ψ_{ijk} = γ_k (special case of Model 1)
- The Generalized CRM (GCRM): ψ_{ijk} = α_iβ_jγ_k (special case of Model 2)
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- All implementations are done using standard R-packages (keras and gbm)

Table: True reserve compared to CL, the CRM and GRWNN reserves. Relative bias (error) of the predictions in the parentheses. The data is the same as that used in Gabrielli et al. (2019).

	LoB 1	LoB 2	LoB 3	LoB 4	LoB 5	LoB 6
True reserves	39,689	37,037	16,878	71,630	72,548	31,117
CL reserves	38,569	35,460	15,692	67,574	70,166	29,409
	(-2.82)	(-4.26)	(-7.02)	(-5.66)	(-3.28)	(-5.49)
CRM reserves	32,485	29,901	13,040	55,782	59,390	24,403
	(-18.15)	(-19.27)	(-22.74)	(-22.12)	(-18.14)	(-21.58)
GCRM reserves	38,293	35,117	15,448	66,961	69,397	29,104
	(-3.52)	(-5.18)	(-8.47)	(-6.52)	(-4.34)	(-6.47)
GRWNN	39,233	35,899	15,815	70,219	70,936	30,671
	(-1.15)	(-3.07)	(-6.30)	(-1.97)	(-2.22)	(-1.43)

The relative bias is calculated based on that we know the true simulated payment outcome

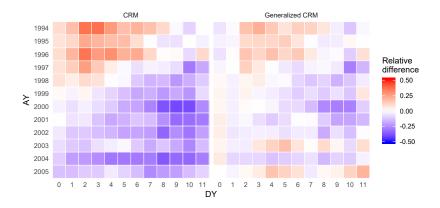


Figure: Heatmap for the relative biases (errors) of the total payment prediction within a specific accident year and development year combination for LoB 1 using the CRM and GCRM.

So there seems to be room for improvement for the RBNS / IBNR split models!

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- Loss function used is the unscaled Poisson deviance

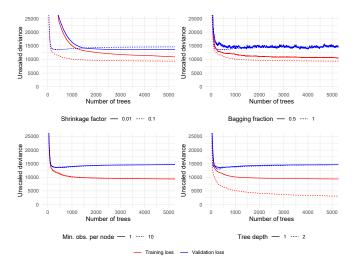


Figure: Training and validation loss for our tuning of the GBM for the payment part of Model 2 when varying the shrinkage factor (upper left), the bagging fraction (upper right), the minimum number of observations per node (lower left), and the tree depth (lower right).

 In the remainder we use shrinkage 0.1, bagging fraction 1, min. observation per node 1, and tree depth 1 or 2, depending on LoB

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- ► We use an analogous procedure for the NN models, see Gabrielli et al. (2019); Lindholm et al. (2020)
- ► The best performing model turns out to be Model 2 which is illustrated next using 100 simulated datasets, see Lindholm et al. (2020) for more details

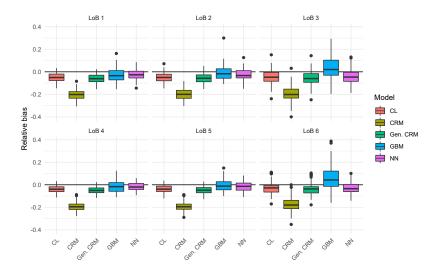


Figure: Boxplots of relative biases for total reserves from 100 simulations. Here GBM and NN refers to Model 2.

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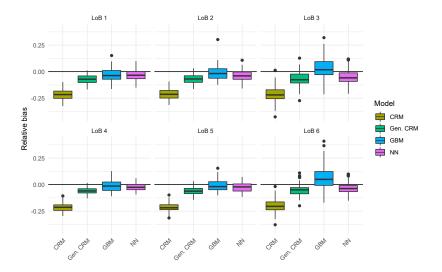


Figure: Boxplots of relative biases for RBNS reserves from 100 simulations. Here GBM and NN refers to Model 2.

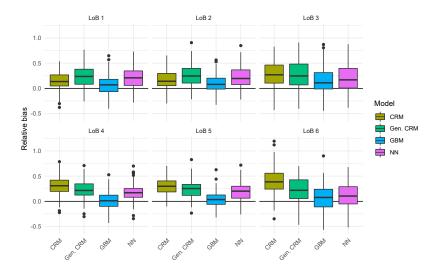


Figure: Boxplots of relative biases for IBNR reserves from 100 simulations. Here GBM and NN refers to Model 2.

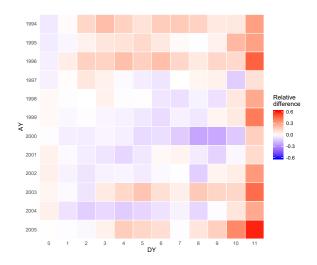


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- The numerical illustrations show how the use of machine learning techniques may improve the predictive performance
- All implementations are done using standard publicly available software packages (in R)

For more details about the regression modelling aspects, see e.g. Wahl et al. (2019); Lindholm and Verrall (2020), and for a more detailed account of the presentation (including MSEP), see Lindholm et al. (2020)

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Appendix

Table: True reserve compared to benchmark models and the GBMs and NNs. Relative biases of the reserve predictions in the parentheses. The data is the same as that used in Gabrielli et al. (2019).

	LoB 1	LoB 2	LoB 3	LoB 4	LoB 5	LoB 6
True reserves	39,689	37,037	16,878	71,630	72,548	31,117
<u>(</u>]	38,569	35,460	15,692	67,574	70,166	29,409
CL reserves	(-2.82)	(-4.26)	(-7.02)	(-5.66)	(-3.28)	(-5.49)
CDM	32,485	29,901	13,040	55,782	59,390	24,403
CRM reserves	(-18.15)	(-19.27)	(-22.74)	(-22.12)	(-18.14)	(-21.58)
GCRM reserves	38,293	35,117	15,448	66,961	69,397	29,104
GCRIVI reserves	(-3.52)	(-5.18)	(-8.47)	(-6.52)	(-4.34)	(-6.47)
GRWNN	39,233	35,899	15,815	70,219	70,936	30,671
GRWINN	(-1.15)	(-3.07)	(-6.30)	(-1.97)	(-2.22)	(-1.43)
CDM (M + I+1 2)	39,697	37,253	16,508	72,679	71,828	31,941
GBM (Model 2)	(0.02)	(0.58)	(-2.19)	(1.46)	(-0.99)	(2.65)
CRM (Madel 2) without inflation	38,324	37,053	16,327	73,386	70,486	32,100
GBM (Model 2) without inflation	(-3.44)	(0.04)	(-3.26)	(2.45)	(-2.84)	(3.16)
GBM (Model 3)	40,114	35,729	15,761	69,448	72,418	30,061
	(1.07)	(-3.53)	(-6.62)	(-3.05)	(-0.18)	(-3.39)
NN (Model 2)	41,587	37,587	15,680	71,155	71,309	28,984
	(4.78)	(1.48)	(-7.10)	(-0.66)	(-1.71)	(-6.86)
NN (Model 3)	39,757	38,719	16,245	70,916	74,600	28,943
Nin (Wodel 3)	(0.17)	(4.54)	(-3.75)	(-1.00)	(2.83)	(-6.99)