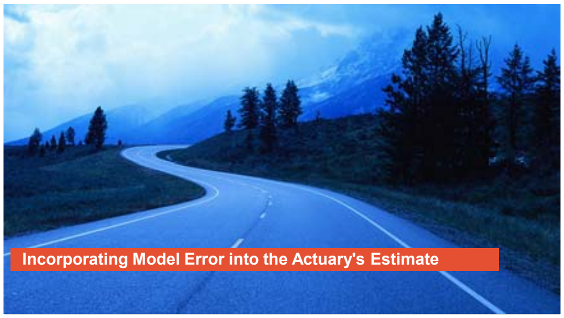


Incorporating Model Error into the Actuary's Estimate

Paul Bailey and Jamie Mackay

CAS Loss Reserving Seminar

September 9-11, 2015

TOWERS WATSON

Session Description

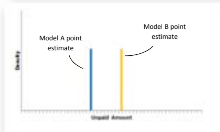
- Within the property/casualty insurance industry, increased interest is being placed on understanding the variability inherent in a point estimate of unpaid claims

- (!) The session will begin with a dilemma that confronts actuaries when relying upon a *single* model to measure the variability around a central estimate based on *multiple* models
- We will then provide an overview of the basic building blocks to estimating reserve variability and will then address a component of reserve variability that is often overlooked: model uncertainty
- This session will present practical methodologies for incorporating model uncertainty into the actuary's estimate of uncertainty and will use a case study to demonstrate their use

Dilemma

Dilemma

- Consider a situation where we have two models, Model A & Model B, that each produce a point estimate:

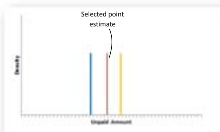


Dilemma

- Consider a situation where we have two models, Model A & Model B, that each produce a point estimate:
- Assume the actuary selects the central estimate to be the average of the point estimates from the two models:

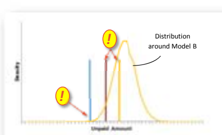
$$\mathit{Central\ Estimate} = \frac{(\mathit{Model\ A} + \mathit{Model\ B})}{2}$$

- How do we estimate the uncertainty in our central estimate?



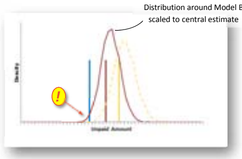
Dilemma

- One way might be to estimate uncertainty using one of our underlying models as the basis
  - Using Model B as the basis for estimating uncertainty:
- This raises two issues:
  - ! Central estimate (red) is not "central" within distribution
  - ! Model A point (blue) estimate appears unlikely yet given 50% weight



Dilemma

- The first issue can be resolved by scaling:



$$Additive\ Scaling = \hat{y}'_p = \hat{y}_p + [central\ estimate - \bar{y}]$$

$$Multiplicative\ Scaling = \hat{y}'_p = \hat{y}_p \frac{[central\ estimate]}{\bar{y}}$$

- The central estimate (red) is now “central” with distribution
- However, the second issue remains:
  - Model A point estimate (blue) still appears unlikely yet given 50% weight

Dilemma

- It is common to estimate unpaid claims using more than one model
- It is rare for different models to produce point estimates that are equivalent
- Current approaches to estimating uncertainty tend to derive variability within the context of a single model
- Central estimate is often not equivalent to any single model.

How do we derive a suitable distribution of variability?

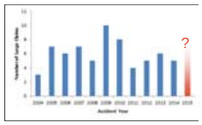
Introduction

Types of Uncertainty

A Basic Forecasting Problem

- Suppose you are an established auto insurer and have written the same number of policies for the last 11 years
- You have had the following number of 'large' claims:

Accident Year	No. of Large Claims
2004	3
2005	7
2006	6
2007	7
2008	5
2009	10
2010	8
2011	4
2012	5
2013	6
2014	5
2015	?



- How many large claims do you expect next year?
- What is the uncertainty in your estimate?
- What is the uncertainty in the outcome?

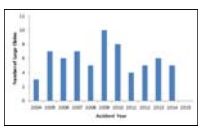
A Basic Forecasting Problem Defining the model

- The first step is to develop a model upon which to base our prediction

Accident Year	No. of Large Claims
2004	3
2005	7
2006	6
2007	7
2008	5
2009	10
2010	8
2011	4
2012	5
2013	6
2014	5
2015	?

Model:	

2015	?



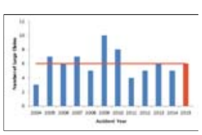
A Basic Forecasting Problem Defining the model

- The first step is to develop a model upon which to base our prediction
- In this case, we can use a simple average of historically observed values:

Accident Year	No. of Large Claims
2004	3
2005	7
2006	6
2007	7
2008	5
2009	10
2010	8
2011	4
2012	5
2013	6
2014	5

Model:	Mean

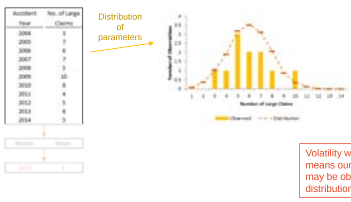
2015	6



- This answers our initial question related to our prediction
- Now, we need address the question of uncertainty around:
  - our estimate
  - the outcome

A Basic Forecasting Problem Parameter Uncertainty

- Firstly, our given model contains parameters obtained from a sample of data
- But are they the 'true' parameters?



Volatility within our underlying data means our observed parameters may be observations from a distribution of potential outcomes

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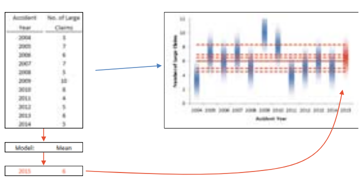
A Basic Forecasting Problem Parameter Uncertainty

- If we believe that our observations come from a wider distribution of possible outcomes, then the results of our model are susceptible to the variance of those parameters



Parameter or *Estimation Uncertainty*

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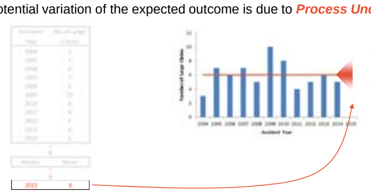
A Basic Forecasting Problem Process Uncertainty

- Even if we were to know the correct parameters of our model…
- …our forecast is susceptible to future variation from our expectation
- The potential variation of the expected outcome is due to ***Process Uncertainty***



Process Uncertainty

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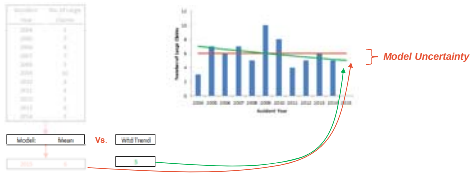
A Basic Forecasting Problem **Model Uncertainty**

- The range of outcomes produced this far share one key assumption…
- …which is that the model itself is correct
- But what if we are not 100% sure that our model is correct?

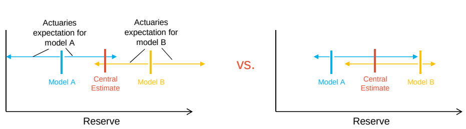


A Basic Forecasting Problem **Summary**

- To summarize, we have two potential sources of uncertainty related to our selected model(s)…
- …and uncertainty associated with the selection of the model itself:

Model vs User Error

- When multiple models are used, there is an important distinction to be made between knowing a model is incorrect and not knowing which model is correct
- An important note to make about model error is the resulting bias on the actuary's prediction, if any, should be unknown



Model error

Known bias

Types of Uncertainty Available models

Mack
Alternative Methods
Sensitivity Testing
Parametric Bootstrap
Bootstrap
MCMC
Practical Stochastic
Scenario Testing

- There are a number of methods and models available to the actuary when analyzing, or reporting on the uncertainty around our selected reserve
- Although we do not have time to cover each in depth, understanding the uncertainty components the output of these approaches includes is important…

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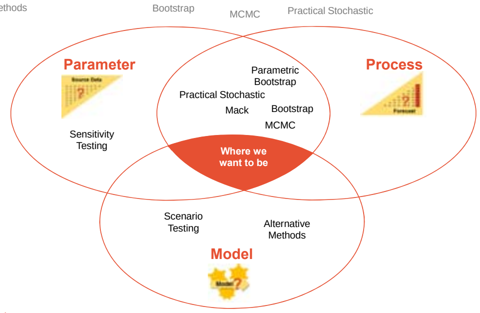
Types of Uncertainty Available models

Mack
Alternative Methods
Sensitivity Testing
Parametric Bootstrap
Bootstrap
MCMC
Practical Stochastic
Scenario Testing



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Incorporating Model Uncertainty

Overview of Approach

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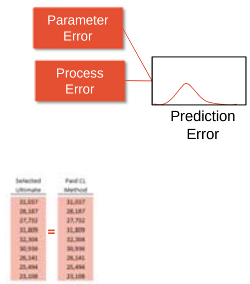
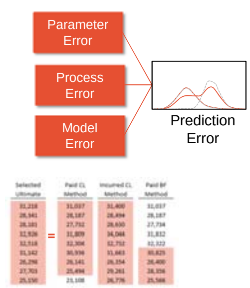
Uncertainty in an Actuarial Central Estimate

- Measuring uncertainty is a challenge in our profession because the unpaid claim process is unknown and the output from this process is not a repeatable exercise
- Many approaches exist to estimating the uncertainty in an unpaid claim estimate
  - Mack, Bootstrapping, MCMC, practical stochastic simulation
- Two common themes in these approaches:
  - Prediction error is comprised of parameter error and process error
  - A **single** model is assumed to be representative of the unpaid claim process



Uncertainty in an Actuarial Central Estimate

- However, actuaries rarely rely upon one model in selecting a central estimate; instead, actuaries commonly consider the estimates from multiple models
- Therefore we need a way to reflect the additional uncertainty that results from the selection between multiple models

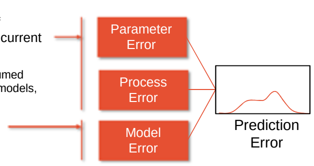


Our Approach

Generate a distribution comprised of simulations about each model using current approaches:

- Bootstrapping; simulation from an assumed distribution; simulation from analytical models, simulating and scaling; etc.

Weighted sample



Our Approach

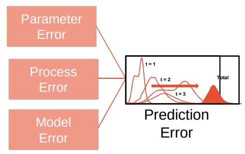
Generate a distribution comprised of simulations about each model using current approaches:

- Bootstrapping; simulation from an assumed distribution; simulation from analytical models; simulating and scaling, etc.

Weighted sample

Aggregating results across multiple years requires additional rigor:

- **Rank Tying** and **Model Tying** approaches are available to generate aggregate distributions

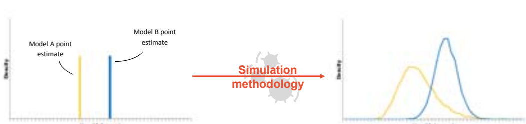


Weighted Sampling

Single Years

Sampling of methods

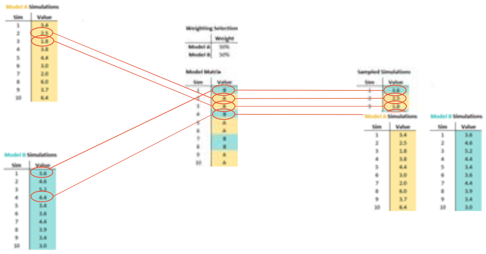
- Start by creating simulated distributions for each of Model A and B:



Model A Simulations: Sim	Model A Simulations: Value	Model B Simulations: Sim	Model B Simulations: Value
1	3.4	1	3.6
2	2.5	2	4.6
3	1.8	3	5.2
4	3.8	4	4.4
5	4.4	5	3.4
6	3.0	6	3.6
7	2.0	7	4.4
8	6.0	8	3.9
9	3.7	9	3.4
10	6.4	10	3.0

Sampling of methods

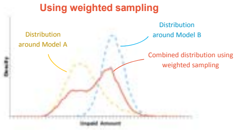
- Create a ‘Model Matrix’ based on selected weighting
- In this case, we will use 50-50 weighting between Model A and B
- Simulations are pulled from each model based on this ‘Model Matrix’



Sampling of methods

- Comparison of results from Weighted Sampling between Model A and Model B:

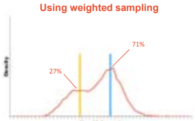
Using weighted sampling



Sampling of methods

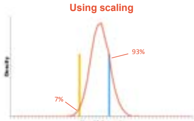
- Comparison of results from Weighted Sampling vs. Scaling:

Using weighted sampling



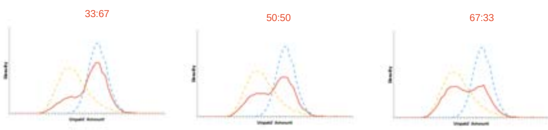
vs.

Using scaling



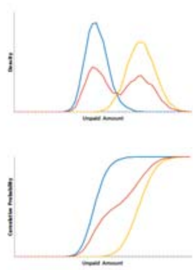
Sampling of methods

- Adjusting our underlying weights will shift the resulting distribution accordingly:



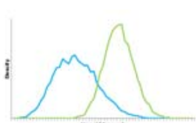
Sampling of methods Multi-modal distributions

- Weighted Sampling may produce 'lumpy', or multi-modal, probability density distributions
- However, the probabilities across a range of outcomes may be more easily interpreted using the associated cumulative probabilities graph
- Further adjustments could be made to the simulated results if such an outcome was deemed problematic



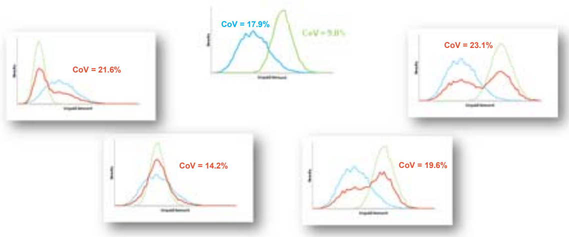
Sampling of methods Impact on Variability

- The effect that weighted sampling will have on the overall distribution is dependent on two factors:
  - The dispersion in the means of the underlying models
  - The variance of the distribution of the underlying models

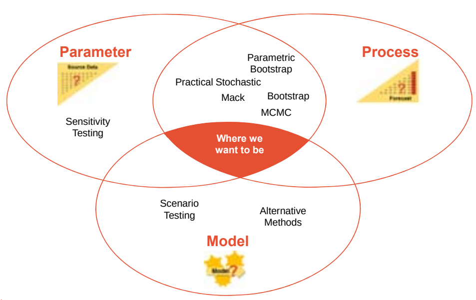


Sampling of methods Impact on Variability

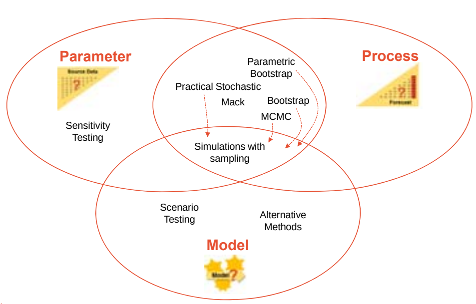
- Take the following example
- Model A (blue) and model B (green) have the same CoV in each example, however, we can see the impact of shifting the mean (multiplicatively) and re-sampling



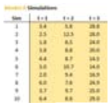
Sampling of methods

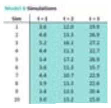


Sampling of methods



Sampling of methods





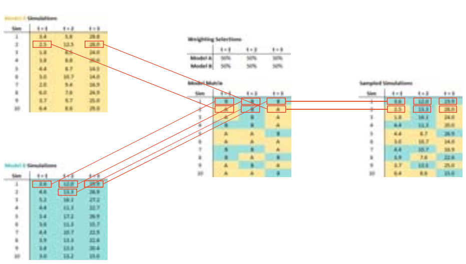
- So far, we have considered a scenario with just a single set of simulations
- What if we have multiple sets of predictions?
  - Multiple accident years, for example

Weighted Sampling

Multiple Years

Sampling of methods **Multiple Year Aggregations**

- Again, for each time period, we can create a 'Model Matrix' based on the selected weighting



Sampling of methods Multiple Year Aggregations

A note on simulation tying

- Typically, the methods that are used to generate the simulations around each of the underlying models do not treat each accident year in isolation, but rather produce year-by-year results that are intrinsically related to each other
- This is reflected in each and every simulation, which we can think of as 'strings'
- This means that we are able to calculate the total unpaid amount for each simulation by simply summing across each row

Sim	t = 1	t = 2	t = 3	Total
#1	3.4	5.8	28.8	37.9
#2	2.5	12.5	28.0	43.0

- In this manner, any accident year correlation that is inherent to the model can be maintained

Sampling of methods Multiple Year Aggregations

A note on simulation tying: Another Dilemma !

- What happens when we mix samples from different models in simulation 'strings'
- Where a break occurs in a 'string', we destroy any correlation that may have been included in our model

Sim	t = 1	t = 2	t = 3	Total
#1	3.4	5.8	19.9	37.9
#2	4.6	12.5	28.0	43.0

Sampling of methods Multiple Year Aggregations

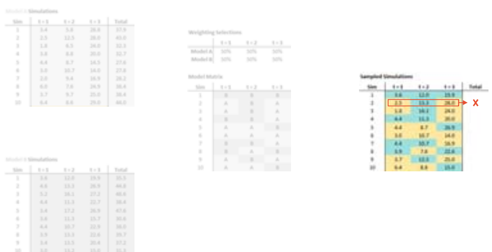
A note on simulation tying: Another Dilemma !

- What happens when we mix samples from different models in simulation 'strings'
- Where a break occurs in a 'string', we destroy any correlation that may have been included in our model
- If we simply randomly-arrange our samples across simulations, we essentially destroy any year-by-year correlation in our results and we are no longer able to sum across the rows to get the total (unless this is desired)

Sim	t = 1	t = 2	t = 3	Total
#1	3.4	5.8	19.9	37.9
#2	4.6	12.5	28.0	43.0

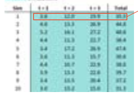
Sampling of methods Multiple Year Aggregations

- Going back to our sampled simulations - because we sampled independently for each time period, we have broken the links intrinsic to the underlying model(s)



Sampling of methods Multiple Year Aggregations

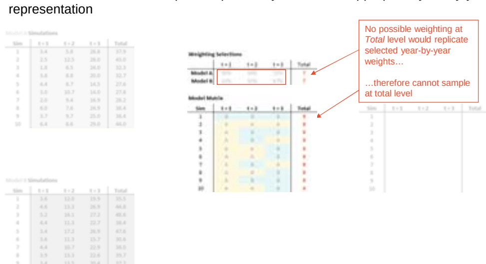
- With this example (equal weighting for each accident period), we can get around the problem by sampling just one time and ensuring that we pick the same simulation for every time period



- This approach achieves the objective in that each individual accident period reflects the desired weighting *and* we maintain the correlation inherent to each individual simulation
- *However…*

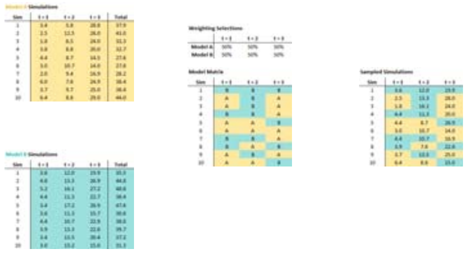
Sampling of methods Multiple Year Aggregations

- …what if our selected weightings vary for each origin year?
- In this case, we need to sample independently to maintain appropriate year-by-year representation



Sampling of methods Multiple Year Aggregations

- We require some manner of rearranging our simulations to reflect underlying correlations
- Going back to our earlier example, sampling individually by years, we suggest 2 ways in which to achieve this….



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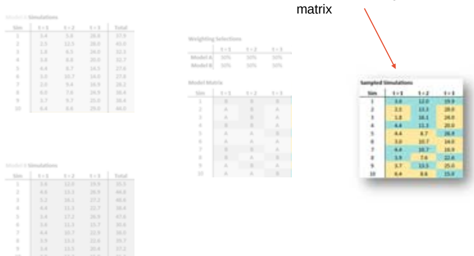
Sampling of methods Multiple Year Aggregations

1) Rank Tying:

Rearrange the sampled simulations themselves using a 'borrowed' correlation matrix



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Sampling of methods Multiple Year Aggregations

2) Model Tying:

Rearranging the Weighted Samples 'Model Matrix' prior to pulling through the reserves from the underlying model

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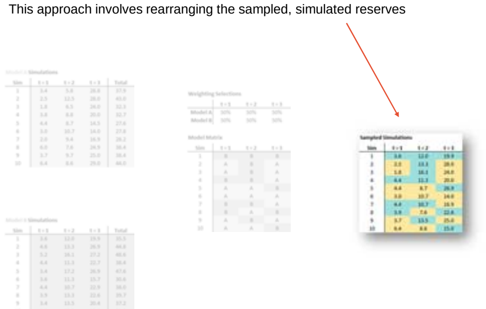
Aggregating Results

Rank Tying

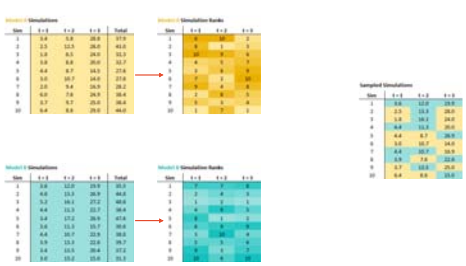
Aggregating Results Rank Tying

- This approach involves rearranging the sampled, simulated reserves



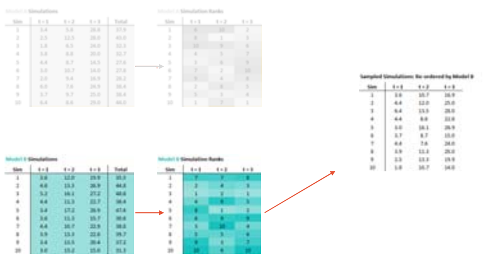
Aggregating Results Rank Tying

- We can 'borrow' a rank correlation matrix from one of the underlying models
- We do this by calculating the reserve ranks for each year for the underlying models



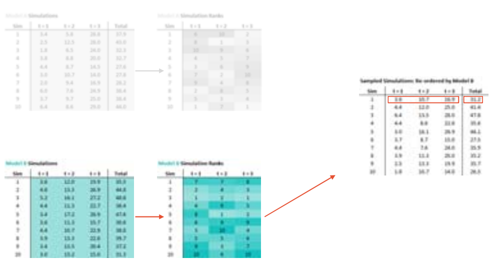
Aggregating Results Rank Tying

- We then select which model to use as the basis for our rank-tying (in this case, Model B)…
- …and reorder the sampled simulations accordingly on a year-by-year basis

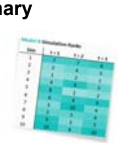


Aggregating Results Rank Tying

- Once rearranged, we can then sum across the rows to calculate a total reserve for each simulation



Aggregating Results Rank Tying: Summary



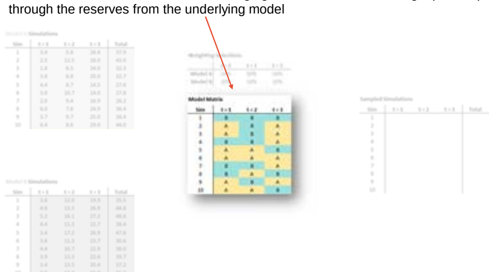
- Rank Tying is a means of combining simulations across origin periods while maintaining the same parameter variance dependency structure associated with one of the underlying projection models
- In essence, this approach assumes that the introduction of model uncertainty does not produce any additional dependency across origin periods
- Rank Tying dependencies across accident years:
  - Process Error = None
  - Parameter Error = Select a single model for source
  - Model Error = None
- *Should* there be correlation among accident years for model uncertainty?
  - It may be argued that should a model turn-out to overestimate or underestimate then it will likely have a similar tendency across all origin periods

Aggregating Results

Model Tying

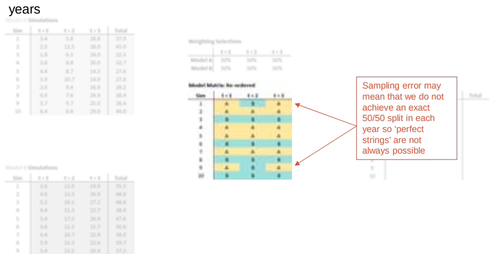
Aggregating Results Model Tying

- This method also involves reordering the simulations
- However, in this case, we will be rearranging at the 'Model Matrix' stage, prior to pulling through the reserves from the underlying model



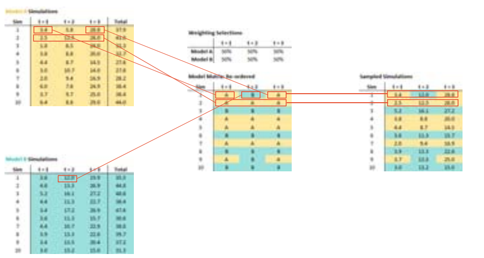
Aggregating Results Model Tying

- We wish to reorder the 'Model Matrix' to maximize the degree to which 'A's in one year are grouped with 'A's in other years, and the degree to which 'B's are grouped with 'B's
- We do this to maximize the correlation of the <u>method selected</u> in each of the accident years



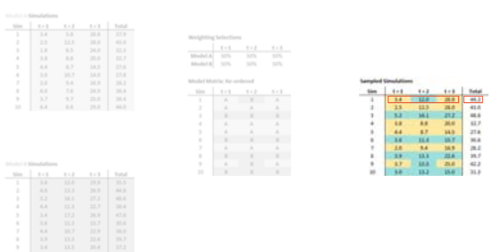
Aggregating Results Model Tying

- We can now select the samples from our underlying methods using the sampling 'Model Matrix' reordered such that we maximize the 'model correlation'…



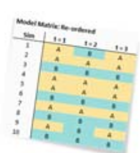
Aggregating Results Model Tying

- …allowing us to simply sum across the sampled simulation 'strings' to derive our set of total simulated reserves



Aggregating Results Model Tying: Summary

- Using the Method Tying approach ensures that, where possible, the original 'strings' of simulations through each year are kept intact, thereby inherently including the dependencies implied by the underlying models
- However, where perfect 'strings' aren't possible due to changing weights, we are essentially breaking origin period correlation caused by parameter error within a model, as we are combining simulations from different models randomly.

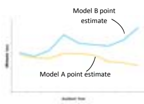


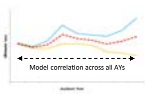
Aggregating Results Model Tying: Summary

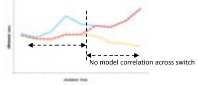
- This may be a desirable effect

Example

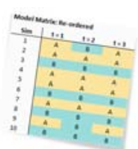
Underlying Models | Equal weights across all AYs | Weight switching from Model A to Model B







Aggregating Results Model Tying: Summary

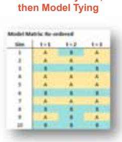


- Using the Method Tying approach ensures that, where possible, the original 'strings' of simulations through each year are kept intact, thereby inherently including the dependencies implied by the underlying models
- However, where perfect 'strings' aren't possible due to changing weights, we are essentially breaking origin period correlation caused by parameter error within a model, as we are combining simulations from different models randomly.
  - This may be a desirable effect
  - Pre-sorting the original sets of simulations (prior to sampling) imposes a proxy dependency between models
- Model Tying dependencies across accident years:
  - Process Error = None
  - Parameter Error = Yes, to the extent selection weights between models implies it should exist
  - Model Error = Yes

Aggregating Results Model Tying

- In a situation where equal weights are applied to each accident year, this approach will yield very similar results to the method suggested earlier – i.e. sampling just once and ensuring that the same simulation is picked for each time period:

Weighted sampling at individual years, then Model Tying ≈ **Weighted sampling at total**



Aggregating Results

Summary

Aggregating Results Summary

- We have outlined three ways in which yearly reserve uncertainty estimates can be aggregated to determine the variability around the **total** (i.e. all year) unpaid loss estimates:
  - Weighted sampling at a **total** level
  - Weighted sampling and re-arranging **sampled simulations** with **Rank Tying**
  - Weighted sampling and re-arranging the **Model Matrix** with **Model Tying**
- It is not always easy to predict how the approaches will compare as it depends on the weightings employed and the results of the respective models across accident years

Sampling the total

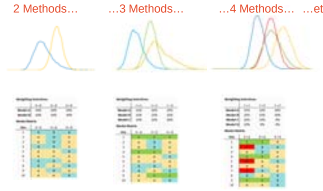
Rank Tying

Model Tying

Aggregating Results Summary

- All three approaches are scalable to allow for the incorporation of multiple models and multiple accident years in the estimate of reserve uncertainty
- Furthermore, the Rank Tying and Model Tying approaches involve sampling at the individual year level and therefore also support the ability to apply weights specific to each accident year

2 Methods… …3 Methods… …4 Methods… …etc.…

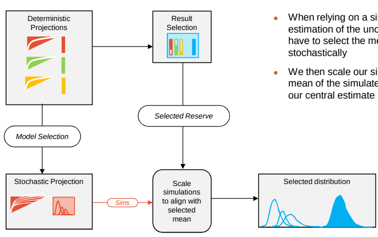


- This allows actuaries to reflect the same weighting philosophy in their uncertainty estimate as employed in their selection of the central estimate

Approach

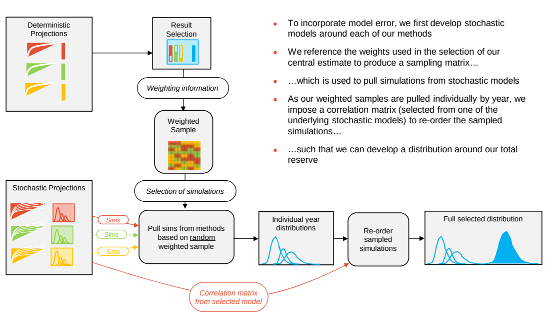
Modus Operandi

Approach Using Scaling



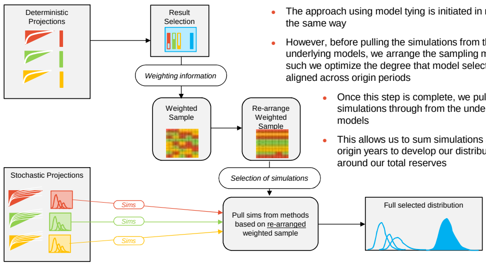
- When relying on a single model as the basis for our estimation of the uncertainty in our prediction, we first have to select the method that we wish to model stochastically
- We then scale our simulated output, such that the mean of the simulated distribution is equal to that of our central estimate

Approach Incorporating Model Error Rank Tying



- To incorporate model error, we first develop stochastic models around each of our methods
- We reference the weights used in the selection of our central estimate to produce a sampling matrix…
- …which is used to pull simulations from stochastic models
- As our weighted samples are pulled individually by year, we impose a correlation matrix (selected from one of the underlying stochastic models) to re-order the sampled simulations…
- …such that we can develop a distribution around our total reserve

Approach Incorporating Model Error Model Tying



- The approach using model tying is initiated in much the same way
- However, before pulling the simulations from the underlying models, we arrange the sampling matrix such we optimize the degree that model selections are aligned across origin periods
  - Once this step is complete, we pull the simulations through from the underlying models
  - This allows us to sum simulations across origin years to develop our distribution around our total reserves

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Case Study

Application of Approach

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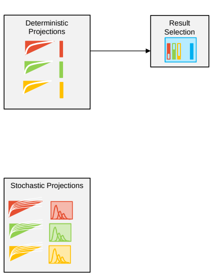
Case Study Introduction



- Three models are investigated
- For the central estimate, each model is given equal weight (for each accident year)
- Traditional methods are used to produce predictive distribution around each model (based on Bootstrap approach)
- We are now faced with the challenge of deriving an estimate of the uncertainty around our prediction, reflecting each model used
- We can employ alternative methods for deriving the uncertainty for individual accident years:
  - Using scaling
  - Using weighted sampling
- The latter approach may employ either Rank Tying or Model Tying approaches in order to assess the variability around the *total* reserve

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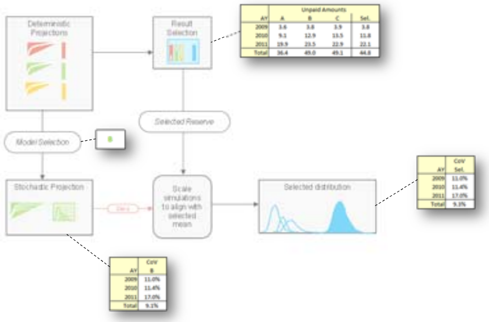
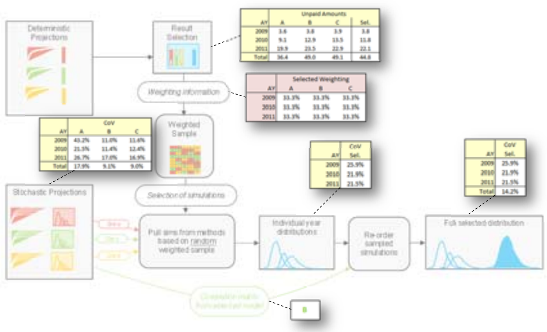
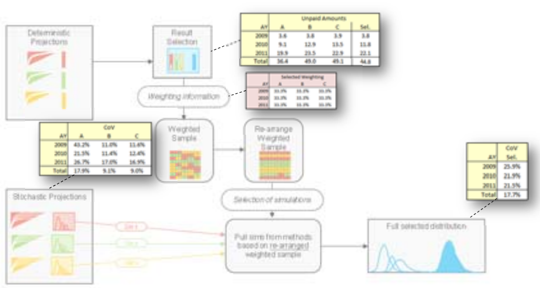
Case Study Using Scaling



Case Study Using Weighted sampling and <u>Rank</u> Tying



Case Study Using Weighted sampling and <u>Model</u> Tying

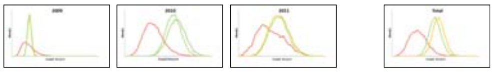


Case Study Comparison

- The table below summarize the reserves and coefficients of variation produced by each different underlying model and each approach to reflecting the total uncertainty

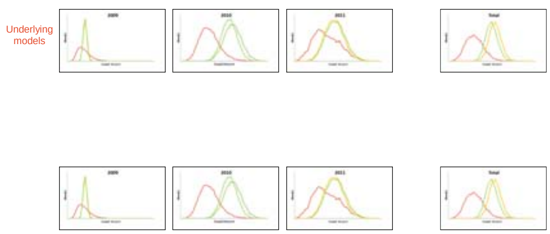
	Unpaid Amounts				Coefficient of Variation					
AY	Model A	Model B	Model C	Sel.	A	B	C	Scaled	Wtd Rank	Wtd Model
2009	3.6	3.8	3.9	3.8	43.2%	11.0%	11.6%	11.0%	25.9%	25.9%
2010	9.1	12.9	13.5	11.8	21.5%	11.4%	12.4%	11.4%	21.9%	21.9%
2011	19.9	23.5	22.9	22.1	26.7%	17.0%	16.9%	17.0%	21.5%	21.5%
Total	36.4	49.0	49.1	44.8	17.9%	9.1%	9.0%	9.1%	14.2%	17.7%

- We can also view the results of our underlying models graphically:

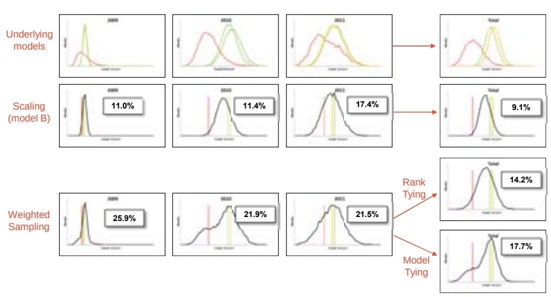


- We can also now compare the results of each of the different approaches to aligning our uncertainty analysis with our selected reserve

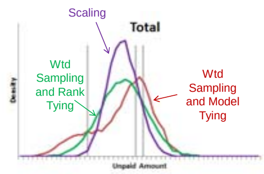
Case Study Comparison



Case Study Comparison



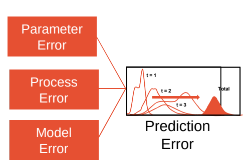
Case Study Comparison



Incorporating Model Error into Actuary's Estimate of Uncertainty

Summary

Summary



- The uncertainty of a prediction is comprised of three components:
- A number of commonly-employed approaches compute uncertainty under the assumption that a single model is representative of the phenomenon
- **Model error** is evident when the actuary places reliance on multiple models as being instructive of their central estimate of unpaid amounts
- **Weighted sampling** is an approach that can be used to incorporate model uncertainty around a central prediction
- **Rank Tying** and **Model Tying** are practical approaches that can be used to incorporate model uncertainty into an aggregation of multiple predictions (e.g. multiple accident years)
- What we produce is a predictive distribution (or a range around our predictions)
- Such approaches allow the actuary to tackle their analysis of uncertainty in an intuitively similar manner to how they derive their central estimate – i.e. with the use of multiple models and application of weights