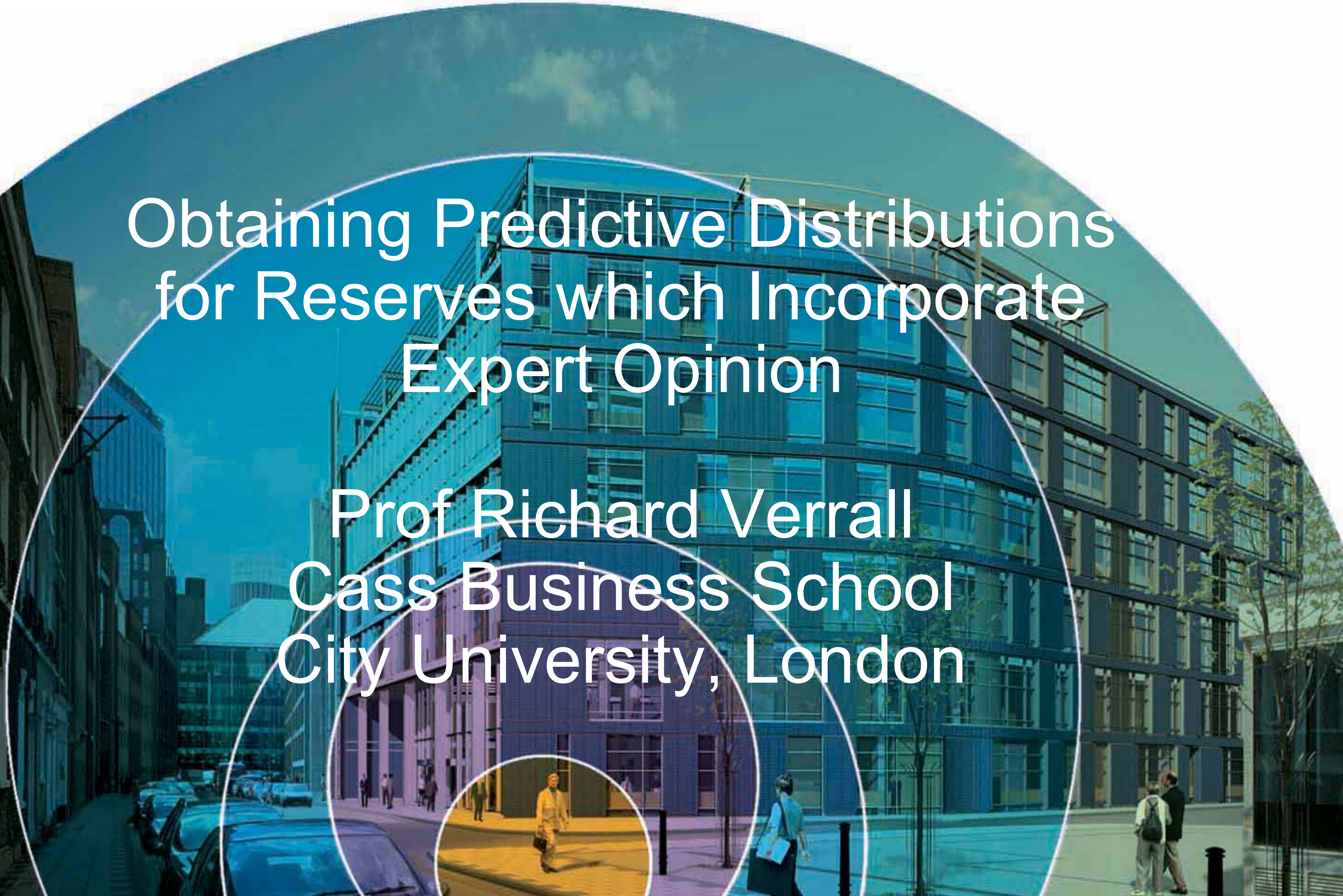

Obtaining Predictive Distributions for Reserves which Incorporate Expert Opinion

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Data

357848	766940	610542	482940	527326	574398	146342	139950	227229	67948	0
352118	884021	933894	1183289	445745	320996	527804	266172	425046		94,634
290507	1001799	926219	1016654	750816	146923	495992	280405			469,511
310608	1108250	776189	1562400	272482	352053	206286				709,638
443160	693190	991983	769488	504851	470639					984,889
396132	937085	847498	805037	705960						1,419,459
440832	847631	1131398	1063269							2,177,641
359480	1061648	1443370								3,920,301
376686	986608									4,278,972
344014										4,625,811
										18,680,856
3.491	1.747	1.457	1.174	1.104	1.086	1.054	1.077	1.018	1.000	

$$\left\{C_{ij} : j = 1,\ldots,n-i+1; i = 1,\ldots,n\right\}$$

$$\begin{array}{llll} C_{11} & C_{12} & \ldots & C_{1n} \\ C_{21} & \ldots & C_{2,n-1} & \\ \vdots & & & \\ C_{n1} & & & \end{array}$$

$$D_{ij} = \sum_{k=1}^{j} C_{ik}$$

$C_{ij} \mid x, y, \varphi \sim$ independent over-dispersed Poisson,

with mean $x_i y_j$

and $\sum_{k=1}^{n} y_k = 1$

$$Y = \varphi X$$

Y has mean and variance $\varphi\mu$ and $\varphi^2\mu$, where $\mu = E[X]$

Variability in Claims Reserves

Variability of a forecast includes estimation error and process error

$$\text{prediction error} = (\text{process variance} + \text{estimation variance})^{1/2}$$

Problem reduces to estimating two components

Generalised Linear Models

$$C_{ij} \sim IPoi(\mu_{ij})$$

$$\mathrm{E}(C_{ij}) = \mu_{ij}$$

$$\mathrm{g}\left(\mu_{ij}\right) = \eta_{ij}$$

Prediction Variance

Individual Cell

$$MSE = \phi\mu_{ij} + \mu_{ij}^{2} Var(\eta_{ij})$$

Row/Overall Total

$$MSE = \sum \phi\mu_{ij} + \sum \mu_{ij}^{2} Var(\eta_{ij}) + 2\sum Cov(\eta_{ij}, \eta_{ik})\mu_{ij}\mu_{ik}$$

Predictive Distribution

$$f\left(C_{kl} \mid C_{ij}, i=1,\ldots,n, j=1,\ldots,n-i+1\right)$$

$$=\iint f\left(C_{kl} \mid x,y,\varphi\right) f\left(x,y \mid C_{ij}, i=1,\ldots,n, j=1,\ldots,n-i+1,\varphi\right) dxdy$$

$$=\int\left(\int f\left(C_{kl} \mid x,y,\varphi\right) f\left(x \mid C_{ij}, i=1,\ldots,n, j=1,\ldots,n-i+1,y,\varphi\right) dx\right) f\left(y \mid C_{ij}, i=1,\ldots,n, j=1,\ldots,n-i+1,\varphi\right) dy$$

Consider

$$\int f\left(C_{kl} \mid x,y,\varphi\right) f\left(x \mid C_{ij}, i=1,\ldots,n, j=1,\ldots,n-i+1,y,\varphi\right) dx$$

$$\int f\left(C_{kl} \mid x, y, \varphi\right) f\left(x \mid C_{ij}, i=1,\ldots,n, j=1,\ldots,n-i+1, y, \varphi\right) dx$$

is an over-dispersed negative binomial distribution with parameters

$$k = D_{i,n-i+1} \qquad \text{and} \qquad p = \frac{1}{\lambda_{n-i+2}}$$

The mean is $D_{i,n-i+1} \dfrac{\left(1-\dfrac{1}{\lambda_{n-i+2}}\right)}{\left(\dfrac{1}{\lambda_{n-i+2}}\right)} = D_{i,n-i+1}\left(\lambda_{n-i+2}-1\right)$

Cumulative Claims:

$$\int f\left(D_{i,n-i+2} \mid x, y, \varphi\right) f\left(x \mid C_{ij}, i=1,\ldots,n, j=1,\ldots,n-i+1, y, \varphi\right) dx$$

$$D_{i,n-i+1}\lambda_{n-i+2}$$

Bayes Reserving

Specify prior distributions for the column parameters, y or λ .

The order matters. This method will leave the row parameters unchanged. In other words, we will always be projecting from the latest cumulative. Putting prior distributions on all the parameters at the same time will have a different effect.

Implementation

This is implemented in winBUGS. It makes the stochastic version of the chain-ladder technique more flexible because it allows (easily) the practitioner to intervene. This can be done by:

Using only part of the data.

Assuming that the run-off pattern changes after a particular row.

Intervention in a particular parameter.

$$\int f\left(C_{kl} \mid x, y, \varphi\right) f\left(x \mid C_{ij}, i=1,\ldots,n, j=1,\ldots,n-i+1, y, \varphi\right) dx$$

is an over-dispersed negative binomial distribution with mean

$$D_{i,n-i+1}\left(\lambda_{n-i+2}-1\right)$$

Specify $f\left(\lambda \mid C_{ij}, i=1,\ldots,n, j=1,\ldots,n-i+1, \varphi\right)$

$$D_{i,n-i+1}\left(\lambda_{i,n-i+2}-1\right)$$

(1) the intervention in a development factor in a particular row, and
(2) the choice of how many years of data to use in the estimation.

Bornhuetter-Ferguson

1. Obtain an initial estimate of ultimate claims, D_{in} , for each accident year, i.
2. Estimate the proportion of ultimate claims that are outstanding for each accident year, using, for example, the chain-ladder technique.
3. Apply the proportion from 2 to the initial estimate of ultimate claims from 1, to obtain the estimate of outstanding claims.

Let the initial estimate of ultimate claims for accident year *i* be M_i, for example taken from the premium calculation.

The estimate of outstanding claims for accident year *i* is

$$M_i\left(1-\frac{1}{\lambda_{n-i+2}\lambda_{n-i+3}\ldots\lambda_n}\right)$$

$$=M_i\frac{1}{\lambda_{n-i+2}\lambda_{n-i+3}\ldots\lambda_n}\left(\lambda_{n-i+2}\lambda_{n-i+3}\ldots\lambda_n-1\right)$$

Bayes Bornhuetter-Ferguson

$$x_i \mid \alpha_i, \beta_i \sim \text{ independent } \quad \Gamma\left(\alpha_i, \beta_i\right)$$

$$\int f\left(C_{kl} \mid x, y, \varphi\right) f\left(x \mid C_{ij}, i = 1, \ldots, n, j = 1, \ldots, n-i+1, y, \varphi\right) dx$$

The mean of this distribution is

$$\left(Z_{ij} \frac{D_{i,j-1}}{S_{j-1}} + \left(1 - Z_{ij}\right) \frac{\alpha_i}{\beta_i}\right) y_j \qquad \text{where} \qquad Z_{ij} = \frac{S_{j-1}}{\beta_i \varphi + S_{j-1}}$$

The mean can also be written as

$$\left(Z_{ij}D_{i,j-1}+\left(1-Z_{ij}\right)\frac{\alpha_i}{\beta_i}\frac{1}{\lambda_j\lambda_{j+1}\ldots\lambda_n}\right)\left(\lambda_j-1\right)$$

Again, the order of estimation makes a difference. If all the parameters are estimated together, then the prior distributions on the row parameters will affect the posterior distributions of the column parameters. This can give undesirable results, and it is not what a practitioner would do. They would simply use the chain-ladder estimates of the column parameters.

We therefore need to estimate the column parameters first.

$$C_{ij} \mid x', y', \varphi \sim \text{independent over-dispersed Poisson}$$

mean $x_i' y_j'$ $\qquad \sum_{k=1}^{n} x_k' = 1$

$$C_{ij} \mid C_{1,j}, C_{2,j}, \ldots, C_{i-1,j}, x', \varphi \sim \text{over-dispersed negative}$$

binomial, with parameters $\frac{1}{\gamma_i}$ and $\sum_{m=1}^{i-1} C_{mj}$

The mean of this distribution is $(\gamma_i - 1)\sum_{m=1}^{i-1} C_{mj}$

$$\gamma_i = \frac{x_i \left(1 - \dfrac{1}{\prod_{k=n-i+2}^{n} \lambda_k}\right)}{\sum_{k=n-i+2}^{n} \left[\left(\prod_{l=1}^{i+k-n-1} \gamma_l \right) \sum_{m=1}^{n-k+1} C_{m,k} \right]} + 1$$

$$\gamma_1 = 1$$

Apply prior distributions to x or reserves.

Implementation

This can also be implemented in winBUGS.

Conclusions

Bayesian methods are flexible and as easy to apply as bootstrapping

Expert opinion can be used without losing the stochastic framework

The Bornhuetter-Ferguson method can be applied as a stochastic model

If you email me I will send you the winBUGS programmes so that you can cut and paste them

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