A Generic Claims Reserving Model

A Fundamental Risk Analysis

Graciela Vera

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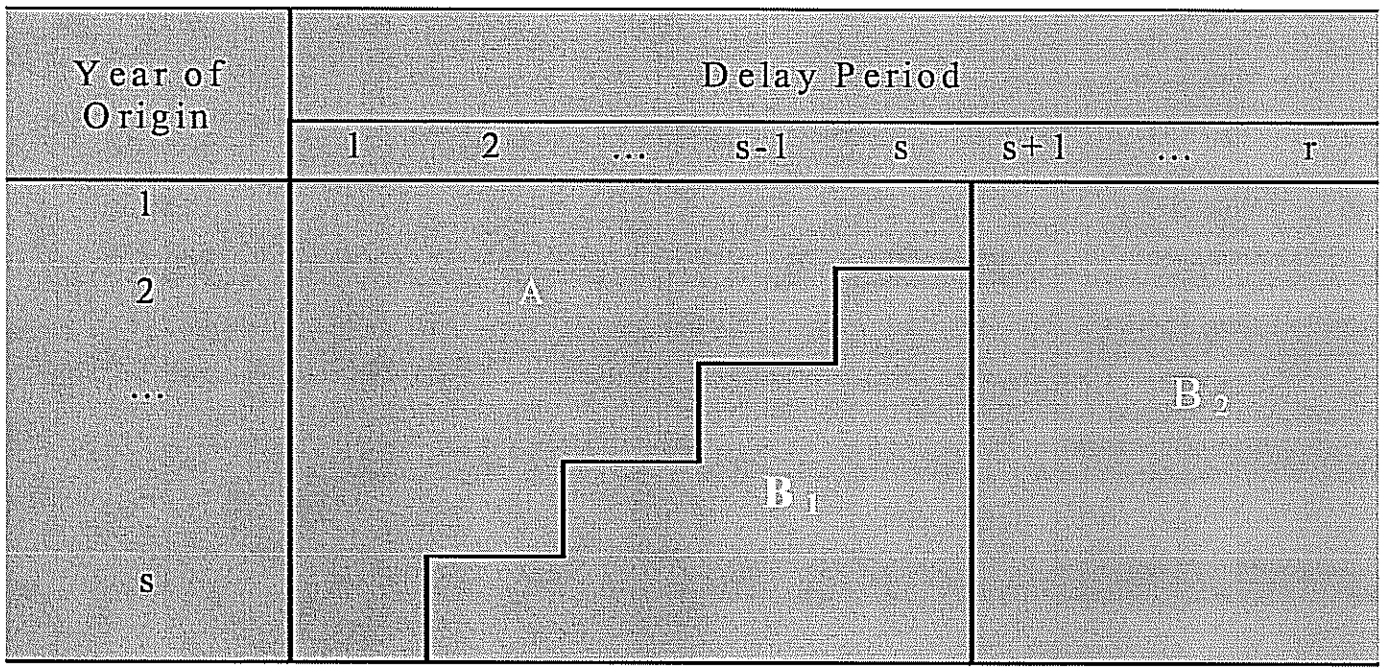
Outline

- Overview of the chain ladder's assumptions
- Generic claims reserving model
- Actuarial risk characteristics
- Advantages of the generic model
- More complex models
- Predicted mean square errors

The chain ladder

- Predictive Scope
- Assumptions (Kremer (1982))
- Conclusions

Chain ladder
Predictive Scope limited to B_1



Chain ladder - Assumptions

(Kremer (1982))

$$y^{**}_{w,j} = \ln y^{*}_{w,j} = \mu + \alpha_w + \beta_j + e_{w,j}$$

$$e_{w,j} \sim N\left(0, \sigma^2\right)$$

$$\sum_{w=1}^{s} \alpha_w = \sum_{j=1}^{s} \beta_j = 0$$

Chain ladder - Assumptions

(Alternative expressions for Kremer (1982))

If

$$\begin{cases} \hat{c}^*_{w,s} \equiv \exp(\mu + \alpha_w)\left(\sum_{k=1}^{s} \exp(\beta_k)\right) \\ \hat{p}^*_j \equiv \begin{cases} \exp(\beta_j)\left(\sum_{k=1}^{s} \exp(\beta_k)\right)^{-1} & j \le s \\ 0 & j > s \end{cases} \end{cases}$$

Then

$$E\left(y^*_{w,j}\right) = \hat{c}^*_{w,s}\hat{p}^*_j$$

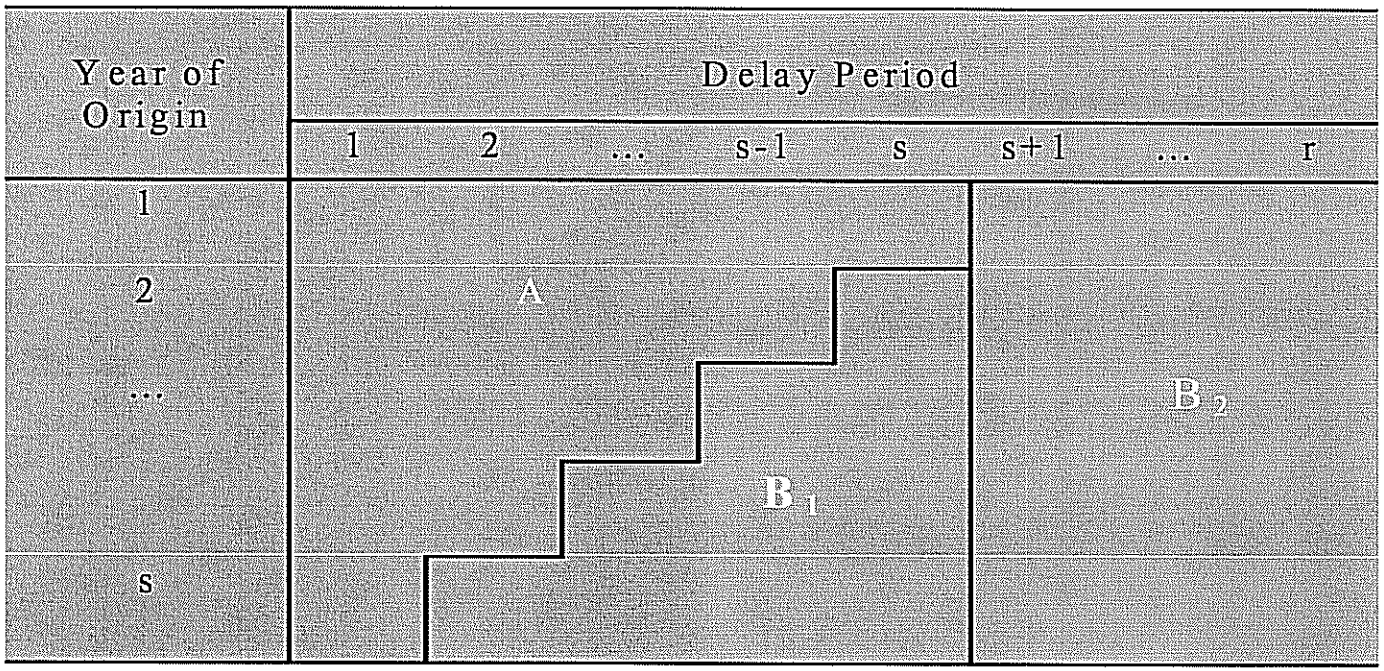
The chain ladder - Conclusion

- Each term of the claims array can be expressed in terms of the marginal estimates.
- To address the limitations of the chain ladder and related methods
  - The row estimates $\hat{c}^*_{w,s}$ should be independent of s.
  - $\hat{p}^*_j$ should be defined for all values of j

Generic claims reserving model

- Predictive Scope
- Assumptions
- Conclusions

Generic claims reserving model
Predictive Scope: B_1 and B_2



Generic model - Assumptions

- Probability density function $\pi(w,t)$
- Percentage cash flow and related functions
- Definition of $\pi(w,t)$
- Mean response and IBNR

Generic model - Assumptions

Probability density function $\pi(w,t)$.

$$\pi(w,t) \geq 0 \quad \forall t$$

$$\Pi(w,t) = \int_{z=0}^{t} \pi(w,z)dz = \Pr[z \leq t]$$

$$\int_{z=0}^{\infty} \pi(w,z)dz = 1$$

Generic model

Percentage cash flow and related functions

$$P(w,t_j) = \int_{z=0}^{t_j} \pi(w,z)dz$$

Cumulative Percentage Cash Flow

$$S(w,t_j) = \int_{z=t_j}^{t_e} \pi(w,z)dz = 1 - P(w,t_j)$$

$$p(w,j) = \int_{z=t_{j-1}}^{t_j} \pi(w,z)dz$$

Incremental Percentage Cash Flow

Generic model

Definition of $\pi(w,t)$

$$\text{If}\begin{cases} \pi(w,z) = g_w(.)G(w,\lambda_w,z) \\ g_w^{-1}(.) = \int_{z=0}^{\infty} G(w,\lambda_w,z)dz \\ f(j,\lambda_w) = \int_{z=j-1}^{z=j} G(w,\lambda_w,z)dz \end{cases}$$

$$\text{Then}\begin{cases} C_w = \exp(\alpha_w)g_w^{-1}(.) \\ p(w,j) = f(j,\lambda_w)g_w(.) \end{cases}$$

Generic model - Assumptions

Mean response and IBNR

Incremental model:

$$E\left(y\left(w,j\right)\right)=\exp\left(\alpha_w\right)f\left(j,\lambda_w\right)$$

Cumulative model:

$$E\left(Y\left(w,t\right)\right)=\exp\left(\alpha_w\right)\int_{z=0}^{t}G(w,\lambda_w,z)dz$$

IBNR:

$$IBNR_{(w,s-w+1)}=\exp\left(\alpha_w\right)\int_{z=t}^{\infty}G(w,\lambda_w,z)dz$$

Actuarial risk characteristics

- Depending on the moments of $\pi(w,t)$
  - Skewness
  - Kurtosis
- Hazard rate

$$h(w,t) = \frac{\left(\frac{\partial \Pi(w,z)}{\partial z}\right)_{z=t}}{1-\Pi(w,t)} = -\left(\frac{\partial\left(\log\left(1-\Pi(w,z)\right)\right)}{\partial z}\right)_{z=t}$$

Advantages of the generic model

- Predictive scope
- Basis of innumerable reserving models
- Delay for a given IBNR and inverse function Π^{-} can be obtained

$$t = \Pi^{-}\left(1 - \frac{IBNR_{(w,t)}}{C_w}\right) = \Pi^{-}\left(P(w,t)\right)$$

- Extracts characteristics of actuarial risk
- A platform for various important portfolio studies

More complex models

If $$E\left(Y(w,j)\right) = \sum_{k=1}^{n} C_{kw} P_k(w,j) = C_w P(w,j)$$

then
$$\begin{cases} C_w = \sum_{k=1}^{n} C_{kw} \\ P(w,j) = \sum_{k=1}^{n} \vee_{w_k} P_k(w,j) \\ h(w,j) = \sum_{k=1}^{n} \wedge_{w_k} h_k(w,j) \end{cases}$$
where
$$\begin{cases} \vee_{w_i} = \dfrac{C_{iw}}{\sum_{k=1}^{n} C_{kw}} \\ \wedge_{w_i} \dfrac{IBNR_{(w,j)_i}}{\sum_{k=1}^{n} IBNR_{(w,j)_i}} \end{cases}$$

Predicted mean square errors

Let $z(w,j)$ be the unknown future incremental claims in development year j. Mean square errors are derived for payment year totals and for

$$E\left(z\left(w,j\right)\right) = \exp\left(\hat{\lambda}_w\right) \int_{j-1}^{j} G(w,\beta_w,t)dt$$

$$E\left(\sum_{j=s-w+2}^{\infty} z\left(w,j\right)\right) = IBNR_{(w,s-w+1)}$$

$$E\left(\sum_{w=1}^{u} \sum_{j=s-w+2}^{\infty} z\left(w,j\right)\right) = IBNR_s$$

Predicted mean square errors

Comments

The derived mean square errors are intended for

- incremental claims reserving models
- models with simple variance/covariance structures that can be easily extracted from the model

The formulation of the mean response drastically simplifies the mean square error calculations

Bayesian Markov chain Mote Carlo methods offer a simpler alternative approach