Pricing and Reserving Adverse Development Cover

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Modelling the past

· Information from the past is an essential component of our predictions of the future.

·To reserve or price excess-type reinsurance must have a good model for past trends, and ***distributions*** about those trends. (must adequately describe upper tail)

Modelling Framework

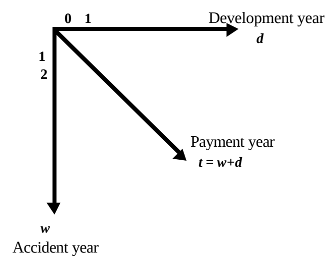
Hence a model has two essential components:

- A model for the trends

- A model for the distribution of each amount around the underlying trend

Modelling Framework

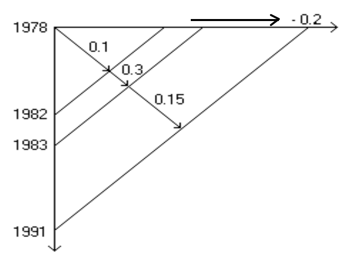
Trends occur in three directions:

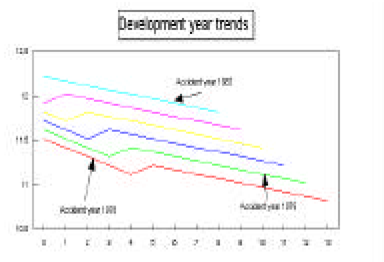


Trend properties of loss development arrays

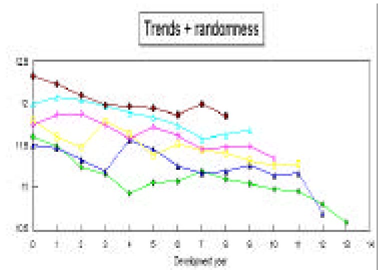
- Trends in payment year direction project onto the other two directions and vice versa
- Changing trends can be hard to pick up in the presence of noise, unless main trends are removed first (regression as a form of adjustment)
- Modeling a changing trend as a single trend will result in pattern in the residual plots

Underlying Trends in the Data





Projection of trends onto other directions

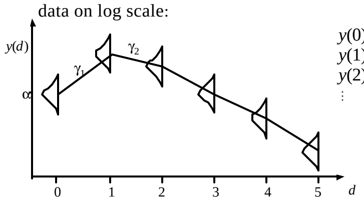


Changing trends hard to pick without removing main development and payment year trend.

Distribution of data about those trends

(Data = Trends + Random Fluctuations)

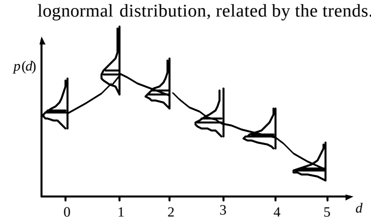
Development trend for single accident year, data on log scale:



$y(0) = \alpha + \varepsilon_0$
$y(1) = \alpha + \gamma_1 + \varepsilon_1$
$y(2) = \alpha + \gamma_1 + \gamma_2 + \varepsilon_2$
⋮

$$log(p_d) = y_d = \alpha + \sum_{i=1}^{d} \gamma_i + \varepsilon_d$$

On the original (dollar) scale, each payment has a lognormal distribution, related by the trends.



All years - trends in 3 directions

$$log(p_{w,d}) = y_{w,d} = \alpha_w + \sum_{i=1}^{d} \gamma_i + \sum_{j=1}^{w+d} \iota_j + \varepsilon_{w,d}$$

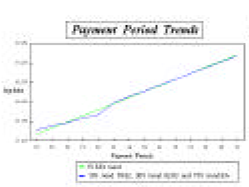
Different levels for accident years

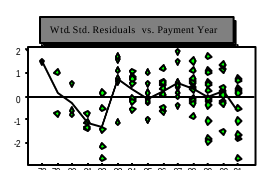
Payment year trends

You would never use all these parameters at the same time - parsimony is as important as flexibility.

A model with too many parameters gives *terrible* forecasts.

Fitting a single trend to changing trends





- estimates as an average trend
- changes show up in the residuals

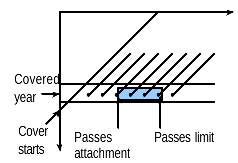
Checking the modeling framework

• if the modeling framework "works", it should be hard to differentiate between real data and data simulated from an identified model

• if you create (simulate) data, you should be able to identify the (known) changing trends in the data; mean forecasts should usually be within about 2 standard errors of the true mean

Adverse Development Cover

For illustrative purposes, consider cover for a single year:



Dots represent future payments on original line of business.

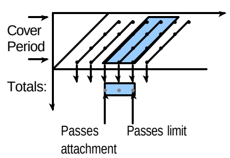
Generate payment stream from model, including parameter uncertainty and covariances.

Record amounts paid under cover, and (if appropriate) discount back to current values.

Represents a single simulation of payment under the cover.

Adverse Development Cover

Cover for multiple years, works in a similar way.



Need to take account of all covariances.

Once generated, just need payment year totals.

Perform multiple simulations to obtain complete distrib-ution of future cost to cover

Adverse Development Cover

Taking account of covariances:

Forecasts are correlated with this modeling framework because the parameter estimates from which the forecasts are built are correlated.

The variance-covariance matrix of parameters is available as a result of the estimation process.
e.g. it is a standard output from regression packages.

From that variance-covariance matrix, covariances of forecasts on the log scale can be computed, and so we may generate from the appropriate multivariate distribution of log-forecasts.

Adverse Development Cover

From that variance-covariance matrix, covariances of forecasts on the log scale can be computed, and so we may generate from the appropriate multivariate distribution of log-forecasts in any of a number of standard ways.

e.g. One way for Multivariate normals: y = Lz
For *z* vector of independent normals
and L the *Choleski decomposition* of V, where
V is the variance-covariance matrix of y.

Finally, the forecasts are exponentiated back to the dollar scale.

Adverse Development Cover

Once the data are generated, just need payment year totals, P_1, P_2, ...

$P_t = \Sigma_i\, p_{i,t-i} = \Sigma_i \exp(y_{i,t-i})$. Let $C_j = \Sigma_t P_t$

Take attachment point *l*.
Let the reinsurance pay up to a limit of *u-l*.
Take cover to be 100% in that layer.

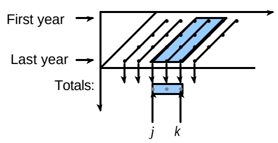
So cumulate P's until you pass *l*; say this happens at time j ($C_j > l$).

Payment ends when $C_k \geq u$.

Adverse Development Cover

Payment stream from reinsurance is:

time	1	2	...	*j*	*j*+1	*j*+2 ...	*k*–1	*k*	*k*+1 ...		
pmt	0	0		$C_j - l$	P_{j+1}	P_{j+2} ...	P_{k-1}	$C_k - u$	0 ...		



Adverse Development Cover

If discounting appropriate these amounts are discounted back to current values before adding together

Gives present value of cost to the layer (discount rate often zero when reserving - the case for many countries).

This is *one* simulation iteration.

Adverse Development Cover

Repeat many times (e.g. 10,000, or to whatever level of accuracy is necessary)

Usually pointless to simulate millions of times
- uncertainty due to simulations usually much smaller than other uncertainties (e.g. parameter uncertainty).

Adverse Development Cover

So (PV of) cost to cover from *n* simulations:

$T_1, T_2, ..., T_n$

These are simulations from *distribution* of cost to cover.

- Can work out mean, median, 75th percentile, s.d., etc.

- Hence mean reserves and risk margins easily obtained (at desired level of security).

- Similarly, for pricing.

Example

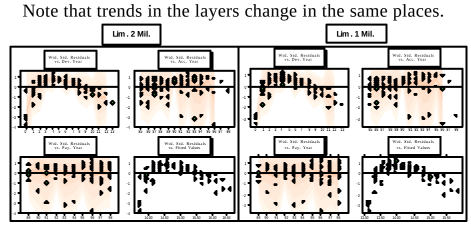
Company M

Paid losses on long tail line of business. The insurer (Company M) has data on several excess of loss layers:

A) Limited to \$2 million,
B) Limited to \$1 million,
C) \$1 million excess of \$1 million (B+C = A)

Example

Note that trends in the layers change in the same places.



Plots are residuals vs 3 directions after removing average trend in development year and payment year directions, to we can see trend changes. Excess layer also same trends.

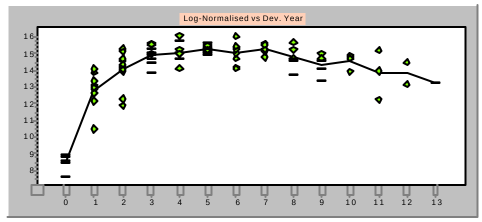
Example

At end of 1998, insurer (who retains B) is concerned about recent experience.

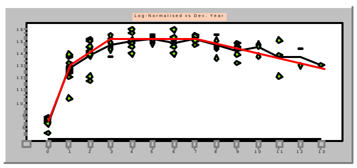
Takes adverse development cover in respect of all accident years of 250M in excess of 500M. The provider of the adverse development cover now wishes to reserve it.

Example

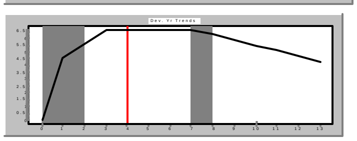
Logs of paid data against development year



Example Model

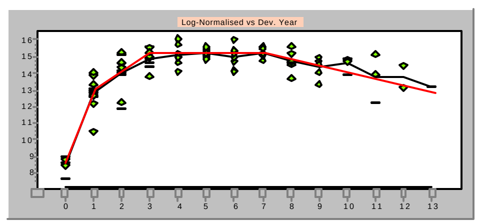


Log-data vs Devel. Year



Fitted Trends

Example Model

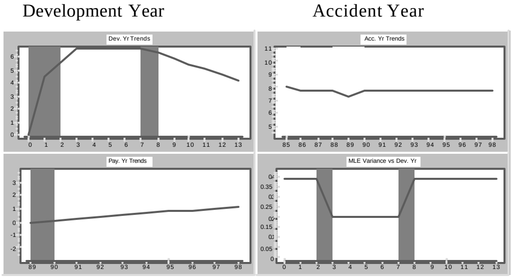


Fit looks good, but remember this apparent fit doesn't include the trends in all three directions.

What else is in there?

Example Model

Development Year Accident Year



Payment Year Variance vs Dev. Year

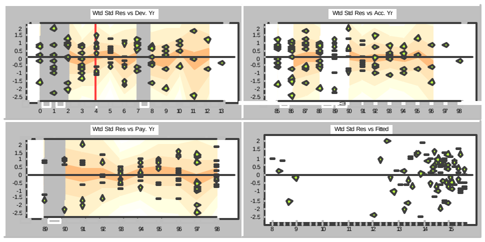
Example Model

Parameter Estimates

	Dev. Yr	**Param.**	**S.E.**	**t-Ratio**	**Diff**	**S.E.**	**t-Ratio**
Gamma	**0- 1**	**4.5240**	**0.3438**	**13.16**			
	1- 3	**1.0762**	**0.1116**	**9.64**	**-3.4479**	**0.4072**	**-8.47**
	3- 7	**0.0000**	**0.0000**	**0.00**	**-1.0762**	**0.1116**	**-9.64**
	7-13	**-0.4015**	**0.0736**	**-5.46**	**-0.4015**	**0.0736**	**-5.46**
	Acc. Year		**Alpha**				
Alpha	**1985**	**7.9985**	**0.3541**	**22.59**			
	1986-88	**7.7557**	**0.3229**	**24.02**	**-0.2428**	**0.1935**	**-1.25**
	1989	**7.3677**	**0.3578**	**20.59**	**-0.3880**	**0.1926**	**-2.01**
	1990-98	**7.7847**	**0.3455**	**22.53**	**0.4170**	**0.2068**	**2.02**
	Pay. Year		**Iota**				
Iota	**1989-98**	**0.1276**	**0.0350**	**3.65**			

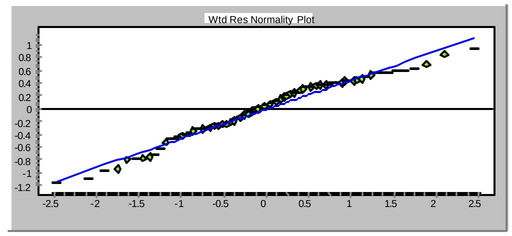
Example Assessing the model for adequacy

residual plots vs 3 directions and predicted



Example Assessing the model for adequacy

normality of residuals



Example Simulated payment year totals:

Sim. Payment by Periods

Sim.	1999	2000	2001	2002	2003	2004	2005	2006	...
1	55.113	48.154	65.497	46.604	45.610	47.702	40.441	19.670	...
2	52.850	80.862	61.967	103.294	102.779	58.783	53.515	93.926	...
3	71.007	72.874	96.228	79.684	62.854	51.276	74.017	25.004	...
4	44.779	56.091	46.382	65.043	71.465	64.843	65.789	53.510	...
5	49.402	53.315	69.434	62.067	49.357	50.242	58.644	49.584	...
6	65.011	87.631	94.196	73.828	84.815	73.969	42.364	52.092	...
7	48.895	58.711	67.086	68.539	62.162	56.206	97.492	47.998	...
⋮									

Example Simulated incremental undiscounted cost to ADC:

1999	2000	2001	2002	2003	2004	2005	2006	2007	
0	0	0	0	10.978	47.702	40.441	19.670	22.363	...
0	0	0	48.973	102.779	58.783	53.515	93.926	47.927	...
0	0	0	69.794	62.854	51.276	74.017	25.004	34.585	...
0	0	0	0	33.761	64.843	65.789	53.510	23.663	...
0	0	0	0	33.574	50.242	58.644	49.584	30.526	...
0	0	0	70.667	84.815	73.969	42.364	52.092	32.849	...
0	0	0	0	55.393	56.206	97.492	47.998	37.683	...
0	0	18.578	125.210	131.911	95.167	56.339	72.794	0	...
⋮									

Example Summary of Simulated distribution

Discounted at $(1.045)^{-1}$

Percentile	Value
50%	62,013,412
75%	132,817,212
95%	183,369,017
Mean	75,261,130