Aggregate Reserve Distributions from Detailed Process Models for Individual Claims

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Data Requirements

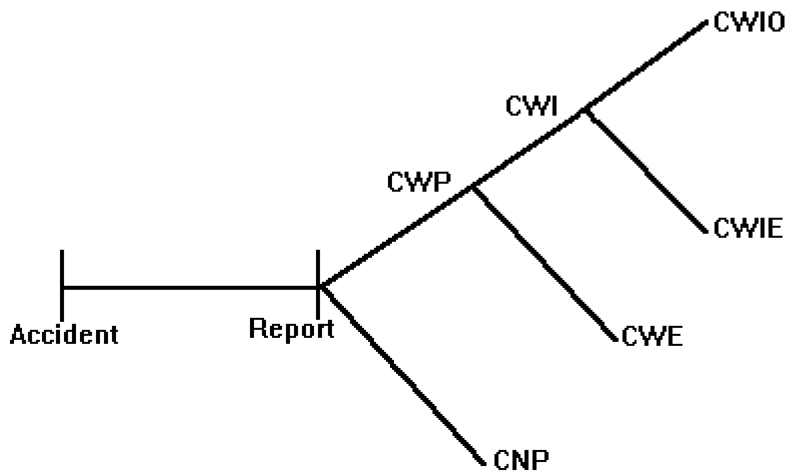
Current model uses only the most recent claim status file.

Minimal:
- Claim identifier
- Valuation date
- Status: open/closed
- Claim dates
  - Accident
  - Report
  - Closing
- Indemnity paid
- Loss adjustment expense paid
- Policy limit.

Optional:
- Policy dates
  - Inception
  - Retroactive
  - Expiration
- Indemnity reserve
- Loss adjustment expense reserve
- Deductible information
- Reopen indicator
- Reopen date
- Legal status
  - Attorney involvement
  - Trial indicator
  - Jury indicator.
- Reinsurance information
  - Treaty year
  - Treaty type
  - Layers.

Process Diagram

Figure 1: Claim Process Decision Tree



Default Model Hypotheses

Variable	Conditionals	Distribution	Remarks
Report lag	Accident date	Weibull	Right truncated Spike at zero
Payment lag	Report date	Weibull	Right censored
Indicators CWP CWI|CWP CWE|CWI	 Accident date Report lag Payment lag	 Logistic	 Details difficult to verify.
Expense on CNI	Accident date Report lag Payment lag Payment lag	Lognormal	Logs of lag variables usually better.
Indemnity	Accident date Report lag Payment lag	Lognormal	Right censored at policy limits.
Expense on CWI	Accident date Report lag Payment lag Indemnity (log)	Lognormal	Full list of conditionals usually not needed.

Random Variables

X_T = I(CWP| t_S, r, s) ·
{ I(CWI|CWP, t_S, r, s) · [X_I| t_S, r ,s +
I(CWE|CWI, t_S, r, s) · $X_E|X_I$, t_S, r, s] +
I(CNI|CWP, t_S, r, s) · $X_E|X_I = 0$, t_S, r, s } ,

where

X_T = total payments for indemnity and expense;

I(·) = indicator functions, modeled as Bernoulli variables with linear logits;

X_I = indemnity payments conditional on accident date, lag variables, censored at policy limit U
= min[U, exp($a_I + b_I \cdot t_S + c_I \cdot \ln(r) + d_I \cdot \ln(s) + N(0, s_I^2)$)] ;

$X_E|X_I$ = expense payments on CWI's
= exp($a_{EI} + b_{EI} \cdot t_S + c_{EI} \cdot \ln(r) + d_{EI} \cdot \ln(s) + e_{EI} \cdot \ln(X_I) + N(0, s_{EI}^2)$);

$X_E|X_I = 0$ = expense payments on CNI's
= exp($a_E + b_E \cdot t_S + c_E \cdot \ln(r) + d_E \cdot \ln(s) + N(0, s_E^2)$) ;

t_S = settlement date minus valuation date in days;

r = report lag from accident date ~ W(a_r , b_r), a Weibull variable truncated on the right at the valuation lag;

s = payment lag from report date ~ W(a_s , b_s), a Weibull variable censored on the right at the valuation lag.

Parameter Variability

- All parameters transformed to real line, no constraints, and estimated by Maximum Likelihood.
- Parameters as variables modeled as multivariate normal using covariance from ML fitting.
- For model *m*, *j*th parameter,

$$\tilde{P}_{mj} = \hat{P}_{mj} + f_m \sum_k E_{mk} V_{mjk} e_{mk} + s_{mj}^{2} \ln(Mix)$$

  - *E*, *V* are eigenvalues and eigenvectors of the parameter covariance.
  - *e* is a N(0, 1) random variate drawn once per case.
  - *f* is a fudge factor. Too large a value generates bizarre results.
  - The last term is an *ad hoc* device describing parameter variability between models. The mixing variable (unit mean, adjustable variance) is drawn once per case.

- This approach is improvable. Much remains to be done.

IBNR Simulation

$$\Pr(C = c) = p(1 - p)^c ,$$

$$E[C] = (1 - p)/p,$$

p = probability for reporting before valuation date.

Inverse Cumulative:

$$C = \text{floor}[\ \ln U / \ln(1 - p)];$$

Report Lag Statistics (sample)

Test_GL		LOB =	GL
Model Type:	Report Lags: Weibull		
Parameter Summary	6177 Cases		73 Iterations

Parameter		Estimate	Standard Error
1	lnExponent	-0.72143	0.010272
2	lnScale	-3.57164	0.03195
3	Prompt Logit	-2.00377	0.038933

Correlation Matrix

	1	2	3
1	1	-0.18105	0.012009
2	-0.18105	1	3.13E-05
3	0.012009	3.13E-05	1

- ln Likelihood =	27447.28	Akaike IC =	54900.56
Exponent =	0.486059		
Scale =	35.57486		
Mean =	75.06592		
Pr(Same Day) =	0.118808		

CWP Statistics (sample)

Test_GL		LOB =	GL
Model Type:	Pr(CWP)		
Parameter Summary	5356 Cases	610 Iterations	

Parameter		Estimate	Standard Error	T Value
1	Intercept	-1.0198	0.13284	-7.6768
2	lnReport Lag	0.00672	0.01942	0.34595
3	lnStl Lag	0.45459	0.02685	16.9298
4	Trend	6.9E-05	4.8E-05	1.4403
5	In Suit	-1.34672	0.12541	-10.738
6	In Trial	-2.531	0.311	-8.1382

Correlation Matrix

	1	2	3	4	5	6
1	1	-0.154	-0.83165	0.40718	0.231	0.06882
2	-0.15437	1	-0.04912	0.15041	-0.5	-0.1218
3	-0.83165	-0.049	1	0.01882	-0.3	-0.1497
4	0.407176	0.1504	0.01882	1	-0.21	-0.1414
5	0.231255	-0.505	-0.30388	-0.2118	1	0.00607
6	0.068819	-0.122	-0.14974	-0.1414	0.006	1

- ln Likelihood	3121.839	Akaike IC	6255.68

Indemnity Statistics (sample)

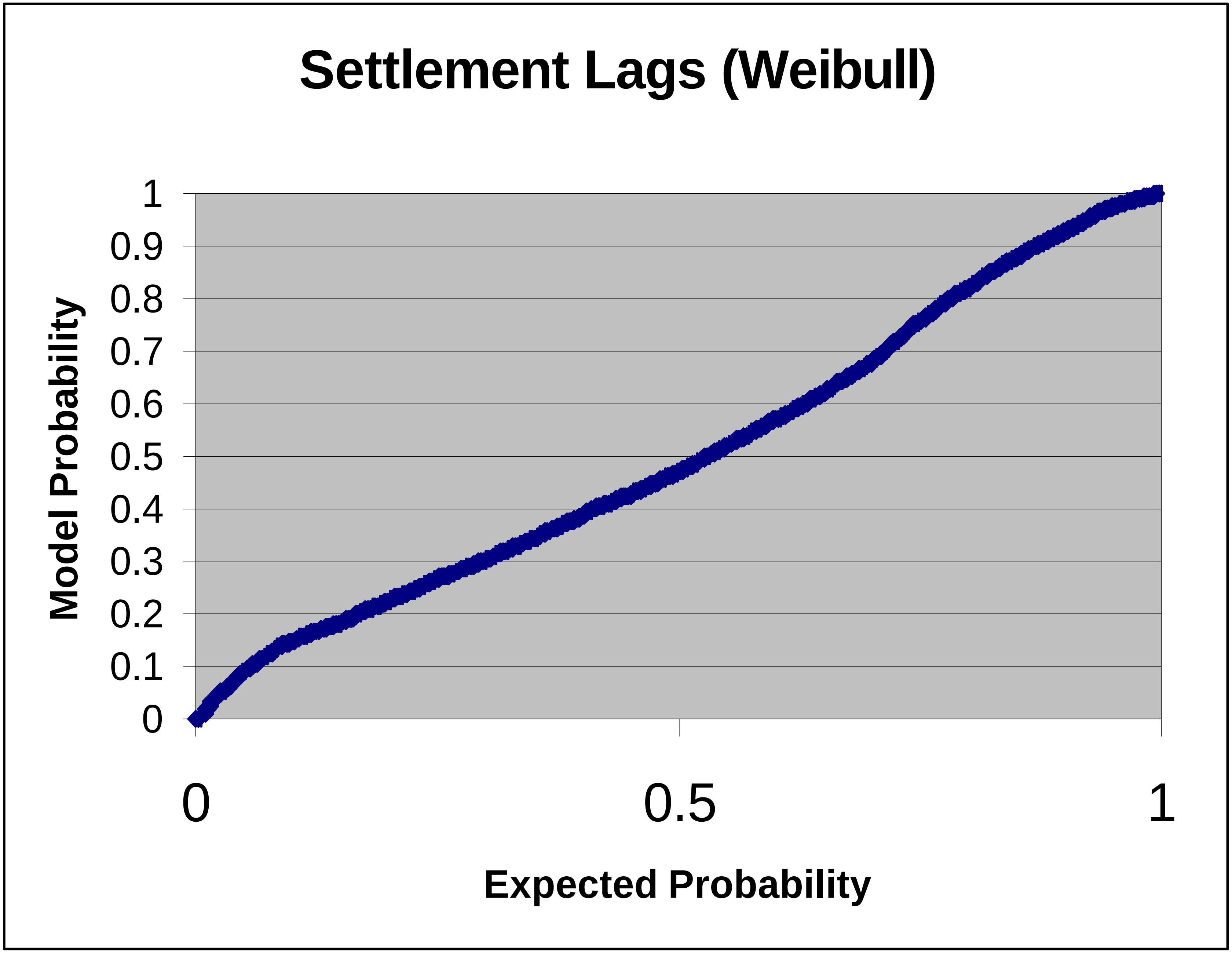
Test_GL LOB = GL
Model Type: Paid Indemnity
Parameter Summary 2119 Cases 2752 Iterations

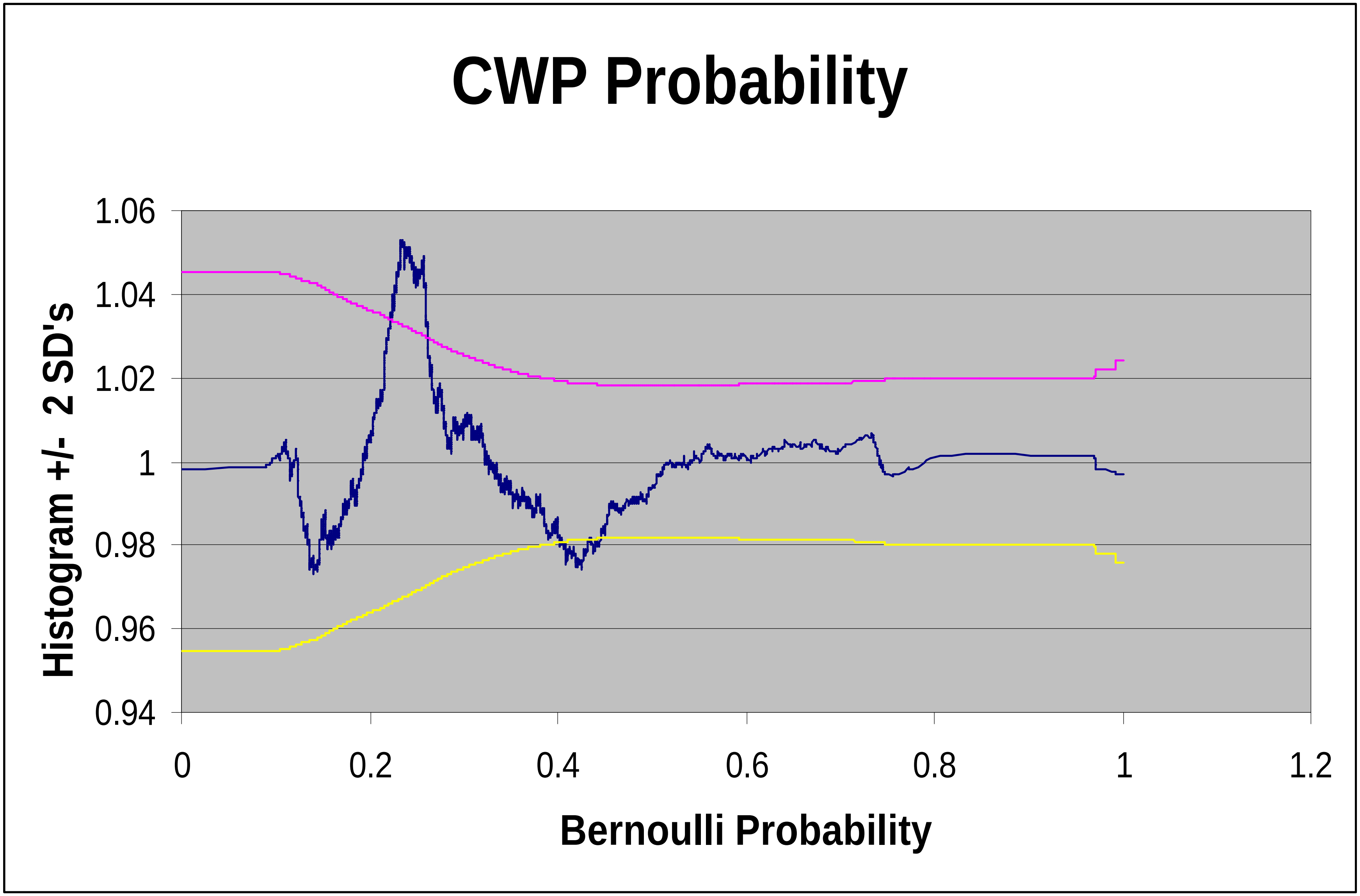
Parameter		Estimate	Standard Error	T Value
1	lnStd Dev	0.68719	0.03196	21.502
2	Intercept	2.98375	0.23356	12.775
3	lnReport Lag	0.19619	0.02582	7.5975
4	lnStl Lag	0.67157	0.03941	17.039
5	Trend	3.4E-05	7.6E-05	0.4395
6	In Suit	0.83753	0.16996	4.9278
7	In Trial	0.82781	0.50186	1.6495

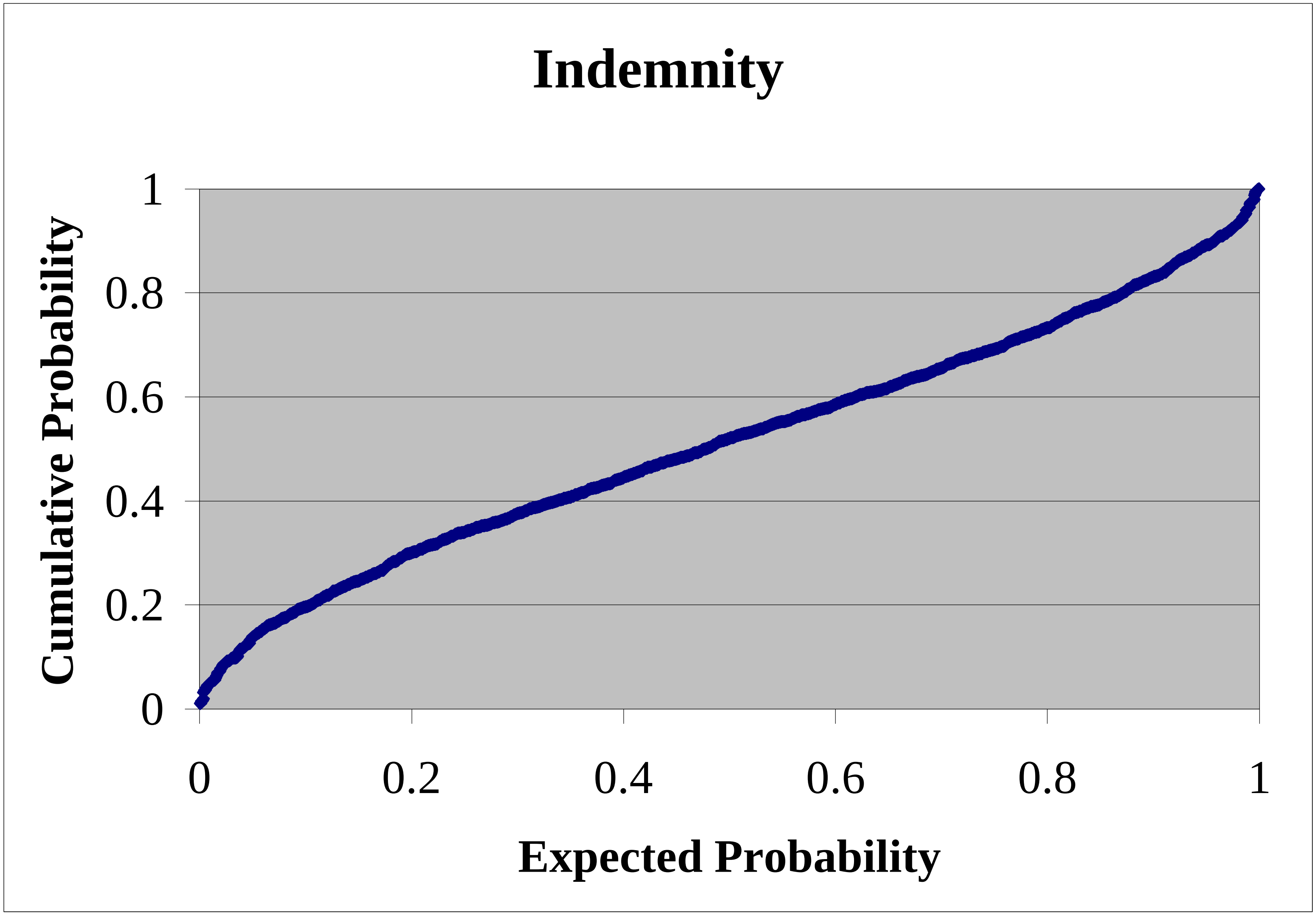
Correlation Matrix

	1	2	3	4	5	6	7
1	1	0.4517	0.12761	-0.207	0.733	-0.11	-0.04
2	0.4517	1	-0.0667	-0.8746	0.619	0.1421	0.015
3	0.1276	-0.067	1	-0.1043	0.173	-0.51	-0.05
4	-0.207	-0.875	-0.1043	1	-0.29	-0.189	-0.06
5	0.7332	0.6191	0.17328	-0.286	1	-0.148	-0.05
6	-0.11	0.1421	-0.5098	-0.1895	-0.15	1	0.006
7	-0.04	0.0149	-0.047	-0.0593	-0.05	0.0062	1

- ln Likelihood = 3205.1
Akaike IC = 6424.12


Settlement Lags (Weibull)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Model Probability
0
0.5
1
Expected Probability


CWP Probability
Histogram +/- 2 SD's
1.06
1.04
1.02
1
0.98
0.96
0.94
0
0.2
0.4
0.6
0.8
1
1.2
Bernoulli Probability


Indemnity
Cumulative Probability
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
Expected Probability

Simulation Algorithm

Enter simulation specifications
Retrieve parameter values
Retrieve layer specifications
Summarize paids
For each simulation case
  Simulate stochastic parameter values
  Initialize accumulators
  For each known claim
    Simulate IBNR Count
    For each associated IBNR claim
      Simulate CWP (Y/N)
      If CWP
        Simulate CWI (Y/N)
        IF CWI
          Simulate Indemnity amount
          Simulate CWE (Y/N)
          If CWE simulate Expense with Indemnity
        Else simulate Expense only
      Else zero Indemnity and Expense
      Accumulate and layer IBNR
    Next IBNR claim
    Simulate CWP
    If simulated CWP or actual payments then CWP
    If CWP
      Simulate CWI
      If simulated CWI or actual indemnity then CWI
      If CWI
        Simulate tail Indemnity
        Simulate CWE
        If simulated CWE or actual expense then CWE
        If CWE simulate tail Expense with Indemnity
      Else simulate tail Expense only

Simulation Algorithm (continued)

Else zero Indemnity and Expense
Accumulate and layer known claim outcomes
Next known claim
Apply aggregate layering
Output case results
Next simulation case
End algorithm

The State of the Art

- Need greater modeling flexibility, robustness.
- Treatment of trend needs improvement.
- Treatment of parameter uncertainty is still primitive.
- Excel VBA OK for prototyping, not for prime time.
- Introduce Bayesian priors into the estimation
  - Ensure plausibility
  - Make use of experience