THE DIRECT DETERMINATION of RISK-ADJUSTED DISCOUNT RATES and LIABILITY BETA

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Abstract

The development of a complete financial structure including balance sheet, income and cash flow statements, coupled with conventional accounting and economic valuation rules, provides the foundation from which risk-adjusted discount rates and liability betas can be determined. Since liability betas cannot be measured directly, a shift in focus is proposed to one based on measures more readily available and better understood, such as cost of capital, equity beta, leverage, etc. The risk-adjusted discount rate is shown as a function of these variables based on the developed financial structure and valuation framework.

The liability beta is then shown to follow as a consequence, also to be calculated as a function of these same variables. The risk-adjusted discount rates that result are less than the risk-free rate and the liability betas are negative to a greater degree than often suggested.

Several relationships are demonstrated including: risk / return versus leverage, equity beta versus liability beta, and underwriting profit margin related in turn to loss payout, investment yield, market risk premium, and leverage.

1. Summary

The original Myers-Cohn "model" [11] presented basic principles of discounted cash flow, with losses risk-adjusted, for use in the determination of a "fair" premium in ratemaking. Determination of the risk-adjustment to be used in discounting, a critical model parameter, was based on the liability beta. Unfortunately, determination of liability beta has proven to be both elusive and controversial, since data does not exist to support its direct measurement. As a consequence, arguments in rate hearings regarding the value of underwriting beta have become influenced more by subjective matters such as one's philosophical view of the role of insurance in society than by concrete facts. The ratemaking focus must be brought back to one based on analytics and supported by financially based, quantifiable assumptions and data. In the end, some means must be established for more rigorously incorporating underwriting risk and variability in the ratemaking process.

While elegant in many respects, what Myers-Cohn first presented was more conceptual than substantive, and it lacked many elements needed to permit its use in a ratemaking environment. Successful implementation of these concepts in a ratemaking context requires the development of a more complete and sophisticated financial model structure. At a minimum, the means to determine the rate of return implied by a particular rate must be provided. In addition, the present overly subjective practice by which liability beta is selected in Massachusetts, must give way to a more rigorous and quantifiable one.

The purpose of this paper is to first recap the essential changes which need to be made to the Myers-Cohn model, presented in detail in [3], to round it into a complete financial model containing the key components of total return. Secondly, the importance of using after-tax discount rates and the equivalency of net present value rates of return and internal rates of return that follow as a consequence is reviewed, again discussed in detail in [1], [2] and [3]. This foundation provides the critical model structure and valuation framework from which risk-adjusted discount rates and liability beta can be determined.

An important principle is introduced – that being that the risk-adjusted total rate of return must equal the risk-free rate. This fundamental principle provides a stepping stone from which a direct estimate of the liability beta becomes possible within the total return framework. Liability betas are shown in relationship to the total return to shareholders and the linkage with equity betas demonstrated. The sensitivity of the underwriting profit margin to variations in loss payout, investment yield, market risk premium and leverage is demonstrated and discussed.

Liability betas cannot be directly measured, and Cummins [6] and Fairley [9] presented approaches to estimate them. Kozik [10] discussed the many problematic aspects of CAPM and liability beta theory, demonstrating why any estimate of liability beta is likely to be subject to much debate. It is important to keep in mind, however, that the development of a liability beta is a secondary objective to that of determining the appropriate risk-adjusted discount rate. This paper proposes a shift in focus from liability beta, to one based on measures more readily available and better understood, such as cost of capital, equity beta, leverage, etc. The risk-adjusted discount rate will be shown as a function of these variables. While not essential to this ratemaking process, the liability beta which MUST follow as a consequence can be calculated as a function of these same variables, if one desires to do so.

The shift to a total return focus supported by equity betas and indicated cost of capital requirements, gives rise to the discussion of another important principle - the need to maintain consistency in financial leverage and equity beta due to the influence of leverage on the magnitude and volatility in shareholder returns.

2. Total Return Model

Practitioners recognize that a more rigorous financial model framework is necessary to implement the basic Myers-Cohn principles. See [3] and [7]. A brief overview of Myers-Cohn and the "fair" premium determination is recapped in the Appendix. In addition to adding the missing elements needed to provide the complete total return model framework necessary to support ratemaking, some of the more critical "shortcomings" of the original Myers-Cohn presentation which must be addressed include:

1. A single period focus, utilizing the rather simplistic premium to surplus relationship, which avoids dealing with more involved issues that follow from the need to link surplus flows to policyholder liability flows over a multi-period timeframe.
2. The simplified view in which only losses are risky (i.e. require use of a risk-adjusted discount rate). Other uncertain underwriting cash flows, and variables such as underwriting income tax and surplus, which are dependent on losses, also require risk-adjustment.
3. The reliance on a liability beta, needed within the CAPM framework to develop an estimate of the required risk adjustment, for which no direct measurement or actual data exists.

As discussed in detail in [3], several changes are required in order to convert the Myers-Cohn model into a total rate of return model as listed below:

1. Introduce surplus flows into the model, including related investment income.
2. Separate and clearly delineate income from (1) underwriting, (2) investment of policyholder funds, and (3) investment of shareholder surplus.
3. Construct balance sheets and income statements, valued on both a nominal and a present value basis,

given the respective cash flows. The present value of liabilities and surplus are of particular importance.

4. Discount all flows using after-tax rates, whether risk-free or risk-adjusted rates.
5. Develop rate of return measures from the net present value income components (underwriting, operating income, and total income) by forming a ratio to the relevant balance sheet liability item. Display net present value calculations both with and without risk-adjustment.
6. Discount surplus and underwriting taxes also on a risk-adjusted basis to the degree they are influenced by losses. Surplus is determined by use of a leverage ratio relative to liabilities inclusive of loss. Therefore, both surplus and underwriting taxes, which are both affected by loss, must also be risk-adjusted for the portion so affected. As in the case of losses, display net present value calculations both with and without risk-adjustment.
7. Control surplus flows through a linkage with liabilities, both with respect to amount and timing.
8. Distribute operating earnings in proportion to the liability exposure over the period for which exposures exist. Essentially this rule distributes operating earnings in proportion to the loss reserve over time.

The above changes are merely those which permit Myers-Cohn entry into the discounted cash flow / net present value family of models. The first six represent change with respect to model structure and analytics. The last two represent rules which specify the pattern of surplus flows and earnings realization based on relationships between risk and return. The appendix provides a recap of these steps, converting Myers-Cohn into a net present value total return model. D'Arcy and Dyer in [8] review many important principles with respect to discounted cash flow and other models in a broad economic context.

The introduction of surplus, via the leverage ratio, is necessary if a total rate of return is to be calculated. This provides an indication as to whether the cost of capital is being met, along with insurance costs, as specified in the actuarial ratemaking principles.

As a result of these steps, equivalency is achieved in rates of return as determined on either a net present value, internal rate of return, or shareholder return basis. This is reviewed in the Appendix and discussed in detail in [2] and [3]. An important element in this reconciliation is the proper reflection of taxes with respect to discounting and the time value of money. This area is worthy of review.

3. After-Tax Discounting

The economic value that can be realized over time by holding onto an asset is determined through the process of discounting. The reasonably risk-free, pre-tax rate at which an asset can be invested, net of the tax payable on such implied investment income, is the rate that fully reflects the economics involved.

While it is common to see models that use pre-tax discounting (and some of these introduce taxes as a last step), this is incorrect in principle. Insurance companies are tax paying entities, obligated to pay taxes on income, including investment, as earned. Thus insurers only realize an after-tax economic return on their investments. Just as bottom-line net income from underwriting is top-line premium less underwriting expense and tax, bottom-line net income from investment is top-line pre-tax investment income less investment expense and tax. Simply put, taxes are a significant expense that cannot be ignored.

To illustrate this point, consider a $1000 asset to be held for one year, with risk-free government yields available of 6%. At the end of a year $60 of pre-tax investment income will be realized, and be subject to tax. At a 35% tax rate, only $39 will remain, the net economic value generated from this asset. The effective earnings rate is thus 3.9%, or 6% taxed.

Now suppose a claim for $1000 is to be paid in one year. If one assumes that the present value can be based on a pre-tax rate of 6% then only $943 dollars need be set aside to cover it. ($1,000 discounted with a factor of 1.06). The $943 will grow at 6% to $1,000. However, tax will have to be paid on the $57 dollars of income, leaving the company short of the $1,000. The necessary amount that must be set aside to cover the claim is $962 ($1,000 discounted with a factor of 1.039). The $962 will earn $58 dollars, less a tax of $20, leaving the company with the necessary $1,000 to pay the claim. Thus the economic value associated with the $1,000 loss payable in a year is $38 and the discounted loss reserve is $962 at the beginning of the year.

While models that apply taxes to calculate the final answer in a last step may be reasonably close and simpler to construct, this is akin to assuming a life insurance-like inside buildup, and the degree of error will increase as the holding period extends beyond a single year.

4. Derivation of Risk-Adjusted Discount Rate and Liability Beta

The model framework supporting the calculation of a rate of return, both with and without risk-adjustment, with taxes fully reflected in the discount rate, provides the key to being able to directly estimate liability beta. The following principles will be utilized in conjunction with the rate of return model:

(i) If no adjustment is made for risk in the discount rate, then the total calculated rate of return must equal the required cost of equity, whereas,

(ii) if all risk is taken into account in the discount process, then the total calculated rate of return must equal the risk-free rate.

These principles state simply that rates of return should normally equal the cost of equity when no adjustment is made for risk in the discount rate, but that they should equal the risk-free rate in the absence of risk, as occurs when risk-adjusted discount rates are used. The first principle is simply a statement that total return should be the cost of capital.

The second principle is at the core of the risk adjustment process with respect to rate of return. The purpose of risk adjustment is to mathematically adjust for risk such that the result becomes comparable to other such risk adjusted rates of return. Usually this process targets the adjusted result to a common reference point represented by the risk free rate of return. In the case of discounted cash flow calculations, this involves an economically based formula which reflects the time value of money. The important point is that the risk-adjustment to the discount rate has the effect of mathematically accounting for (i.e. eliminating) risk so that the risk-adjusted total return that then results is the risk-free rate. If this were not the result, then by definition further risk would remain and the risk adjustment process would not have been complete.

The rate of return model formulation, both with and without risk-adjustment will be used to demonstrate, by way of simple examples, how the required risk adjustment and liability beta can be determined directly. For simplification in the examples presented here, expenses will be assumed to be 0, premium to be fully collected at policy inception (i.e. at time 0), taxes paid without delay, and losses 100% fully paid on a single date. Only losses will be assumed to require risk adjustment. The formulae used below for calculating net present value rates of return were presented in detail in [3], [4] and [5], and are reviewed in the Appendix.

First, given loss (L), tax (T), before-tax interest rate (R_b), loss payment date (N), liability / surplus leverage factor (F), equity beta (β_e) and the market risk premium (R_p), a Premium (P) is determined which generates a total return, without risk-adjustment, equal to the CAPM-based cost of equity of $R_b + \beta_e R_p$:

$$(1) \qquad \frac{(P-L)(1-T) + L\{1 - 1/(1+R)^N\}}{L\{1 - 1/(1+R)^N\}/R} F + R = R_b + \beta_e R_p$$

Second, this premium (P) is used to determine the after-tax risk adjustment (R_L) which produces a risk-adjusted total return equal to the risk-free rate:

$$(2) \qquad \frac{(P-L)(1-T) + L\{1 - 1/(1+R+R_L)^N\}}{L\{1 - 1/(1+R+R_L)^N\}/(R+R_L)} \; F + R = R_b$$

Finally, the implied liability beta is determined using the relationship:

$$\beta_L = R_{Lb} / R_p = \{ R_L / (1-T) \} / R_p$$

Required Assumptions:	R_b: Interest Rate, Before-Tax	R: Interest Rate, After-Tax
	L: Loss	F: Liability/Surplus Leverage Factor
	R_p: Market Risk Premium	β_e: Equity Beta

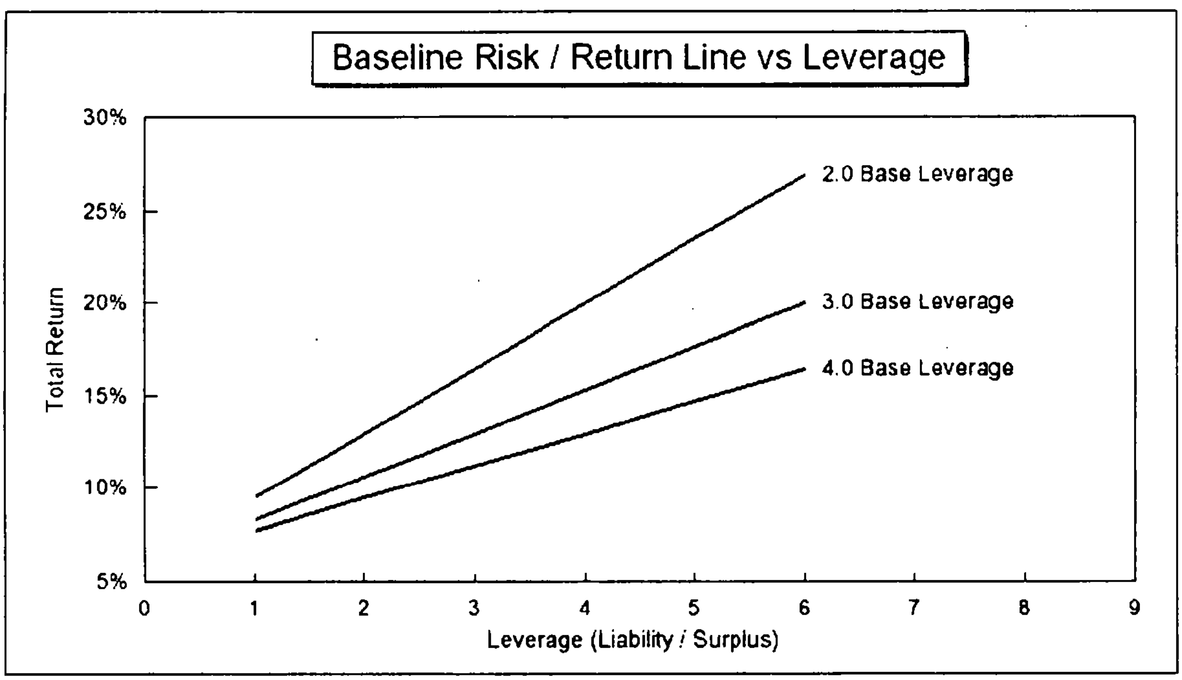
Derived by Formula:	P: Premium	R_L: Risk Discount Adjustment, After-Tax
	β_L: Liability Beta	R_{Lb}: Risk Discount Adjustment, Before-Tax

Formula (1) expresses the sum of after-tax underwriting income and the present value of investment income on loss reserves in ratio to the present value of balance sheet loss reserve liabilities. This is the operating return, and it is multiplied by leverage and the investment return on surplus added to produce the total return. Formula (2) is differs only by the introduction of the risk discount adjustment. These formulas are simplified due to the assumptions that premium is collected at policy inception, expenses are zero, there is no delay in tax payments, and that losses are paid in a single payment. Formulas (1) and (2) are reviewed in more detail in the Appendix.

These basic relationships were used to produce Figures 1-7 which demonstrate various relationships among the variables. Figure 1 first establishes a base point of reference, by demonstrating the relationship between leverage and return, at three given leverage levels. In practice, the measured equity beta and CAPM target cost of capital are at some "typical" leverage. If leverage were to be higher, then the required return would be higher, and if leverage were to be lower, then the required return would be lower. Presumably this would affect measured equity betas. This interplay between leverage, return and risk should be considered when solving for the target premium.

The actual leverage level is an extremely important, yet often overlooked aspect of risk and return. Leverage simultaneously and similarly impacts both return and risk, as measured by the variability in return. All else being equal, higher leverage should produce greater returns (i.e. higher cost of capital) and greater variability in returns. Although one might expect higher betas to be produced when leverage is higher, this aspect is seldom considered when they are calculated and published. Given the significant impact of leverage, and since industry leverage has been declining steadily over the past several years, three specific values were selected to represent this range and to reflect this dynamic in the following discussion. The fact that insurance industry beta's today are around 1.0 is consistent with the effect that large amounts of surplus and low leverage have in suppressing variability in return, and in making insurer returns align more closely to overall market returns. Both the cost of capital and equity beta are expected to flex with leverage changes over time

Figure 1



3 Average Loss Payout (Years) 1.0 Equity Beta 6% Investment Yield Before-Tax 7% Market Risk Premium

Note: Each line will produce a 13% total return when the actual leverage is the same as the base leverage was when the equity beta was determined to be 1.0.

Three base leverage levels (2.0, 3.0 and 4.0) have been assumed. This represents three possible levels of leverage in existence at the point in time when the equity beta was determined to be 1.0. Actual leverage may subsequently vary from these respective base points as shown by the three lines on the chart. Each of the lines, however, must produce a total return of 13% when the actual leverage matches the base level corresponding to the original calculation of the equity beta. This is the CAPM framework in which the cost of capital (13%) is equal to the risk-free rate (6%) plus the market risk premium (7%) times the equity beta (1.0). These lines are used in the analysis to adjust the total return target up or down if actual leverage increases or decreases from the respective base.

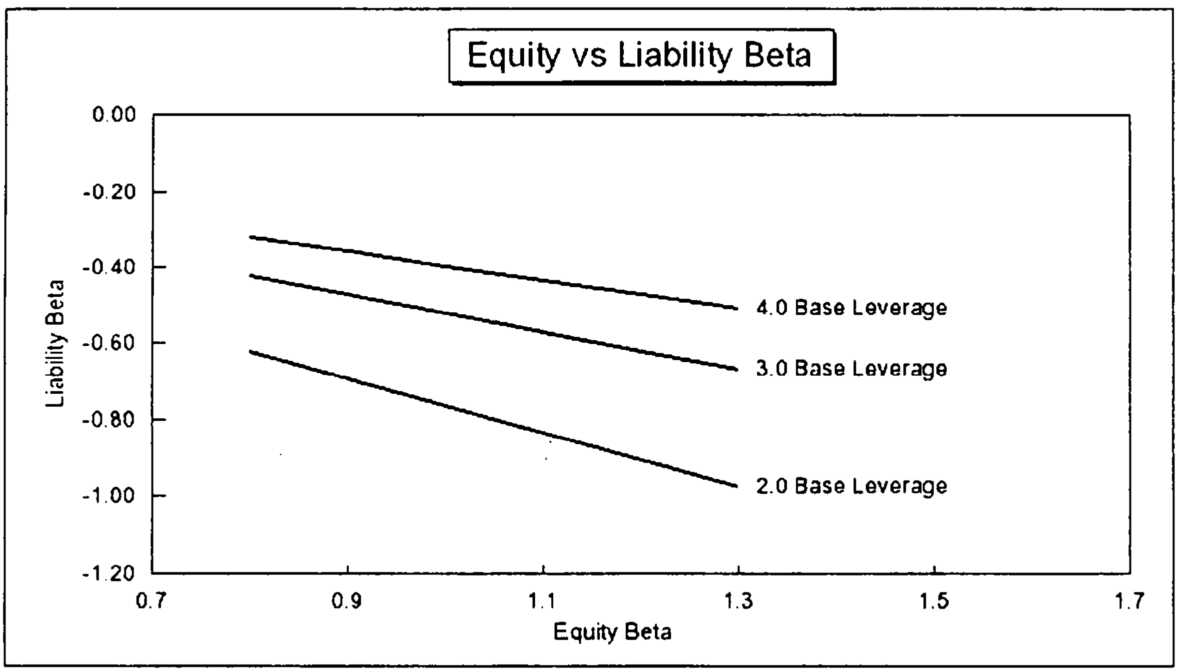
5. Liability Beta

Following the steps discussed above, the liability betas determined by formula are shown in Figure 2 in relationship to equity betas. The example shown is for a 3 year loss payout, 7% market risk premium, and 6% risk-free yield. ***The liability beta is negative in all cases.*** The magnitude shown here is substantially more negative than most of the literature has indicated. This is likely due to the fact that more sources of risk (i.e. variability) exist than may have been recognized by previous measures which have assumed that losses alone are risky. This narrow assumption excludes sources of risk from the variability in the timing of loss payout and the variability in the amount and timing of all other cash flows, including premium, expense, tax and investment.

One would hope that liability betas would be estimable within a more narrow range than that shown in Figure 2. Since total returns are affected by leverage, it would seem logical to expect that equity betas would flex to some degree as leverage changed, whereas liability betas should be relatively unaffected and more stable. Increasingly more negative liability betas occur when moving from the upper, higher leverage line to the lower, less leveraged line in Figure 2. More negative betas are what should be expected given the historical trends in declining industry leverage and the likely delay in market response in forcing equity betas down proportionally in line with this.

The fact that the liability beta must be negative is intuitively obvious. Suppose that a $1,000 loss payable in one year is to be reinsured (100%), with risk-free rates at 6%. If the amount and timing are both absolutely certain, it should be possible to find a reinsurer who would agree to assume the loss obligation for a premium of $962 ($1,000 / 1.039). If, on the other hand, losses are uncertain, the additional risk transfer that occurs from the insurer to the reinsurer requires that the reinsurer receive additional compensation. The reinsurer will require a premium ***greater*** than $962. In other words, ***the risk-adjusted discount rate must be less than the risk-free rate***, and liability beta must be negative.

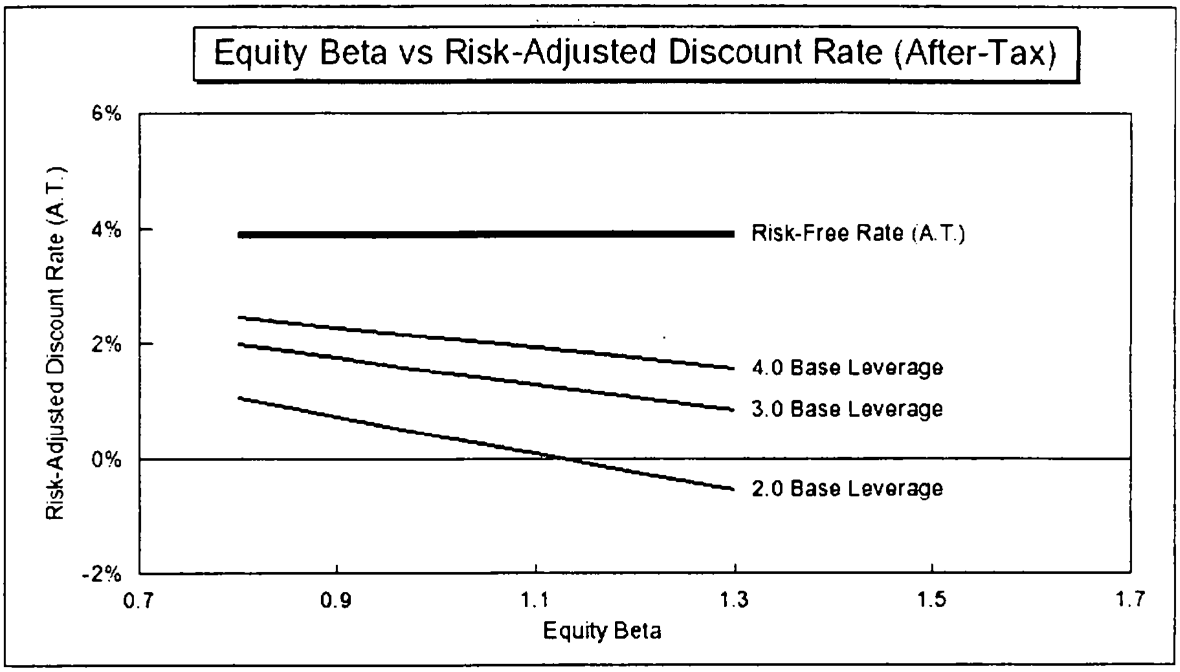
Figure 2



3 Average Loss Payout (Years) 6% Investment Yield Before-Tax 7% Market Risk Premium

The degree to which the risk-adjusted discount rate must be less than the risk-free rate is shown in Figure 3, for the same example, and also in relation to the equity beta.

Figure 3



3 Average Loss Payout (Years)   6% Investment Yield Before-Tax   7% Market Risk Premium

Although low leverage would not generally be associated with a large equity beta, this extreme (lower right, bottom line above) would result in a negative risk-adjusted discount rate. In other words, the discounted liability would be greater than the nominal liability. A sufficiently large surplus base, without corresponding reductions in the equity beta and the cost of capital, impose an unrealistic burden on insurance pricing. This is the true essence of "surplus-surplus" as discussed at times in the ratemaking context. This will be explored further with respect to the underwriting profit margin below.

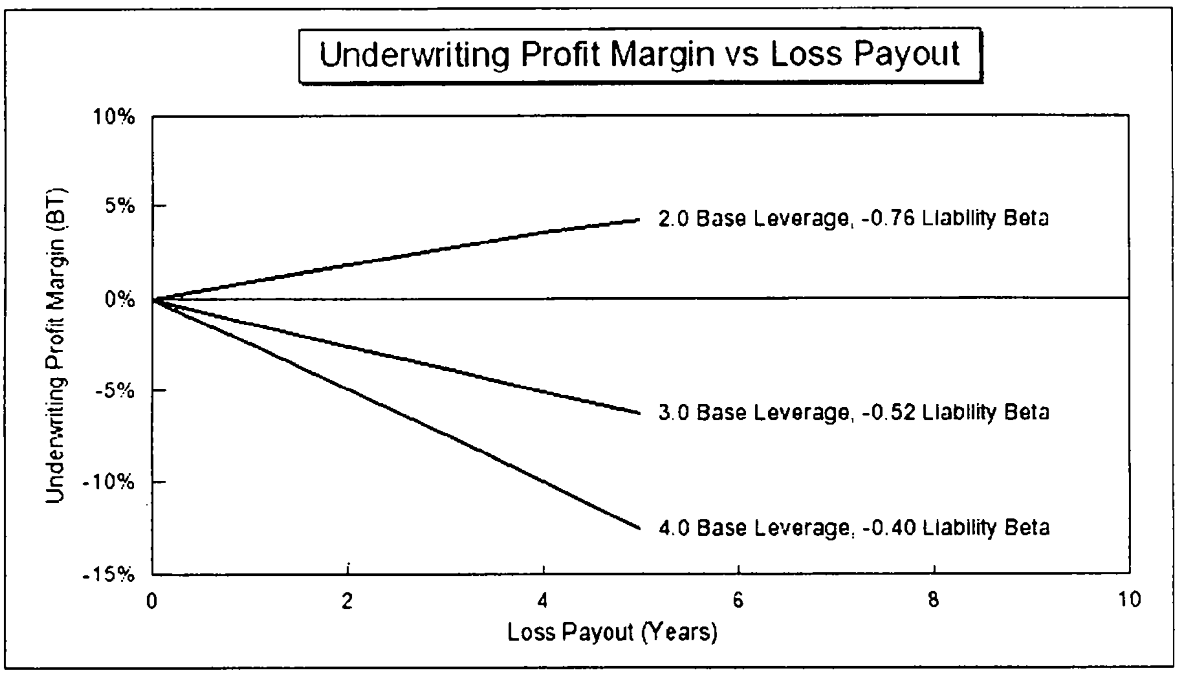
6. Underwriting Profit Margin

The ultimate goal in ratemaking is to determine the premium, given assumptions on all costs and financial conditions, that in some way is "fair" to policyholders and owners of the company alike. Figures 4-7 present the premium rate as an underwriting profit margin as a function of loss payout, investment yield, market risk premium, and leverage, respectively.

In each of the examples the base leverage cases (2.0, 3.0 and 4.0) require liability betas of approximately

-0.8, -0.5, and -0.4, respectively. Profit margins typically become more negative (i.e. higher combined ratios) as loss payouts lengthen as shown in Figure 4, due to the greater investment income that will be generated prior to loss payment. This is shown in the lower two lines on the chart. However, as noted above, when leverage levels become so low as to create burdensome amounts of surplus the opposite can happen, if cost of capital and equity betas are not adjusted. This is the case in the upper line in Figure 4 below in which the cost of equity and the equity beta have not been altered to reflect the lesser risk implied by the lower leverage. If the equity beta were to decline to at least 0.8, (and the capital target return decline to 11.6% from 13.0%) in this example, this effect is avoided, with the resulting expected downward sloping line.

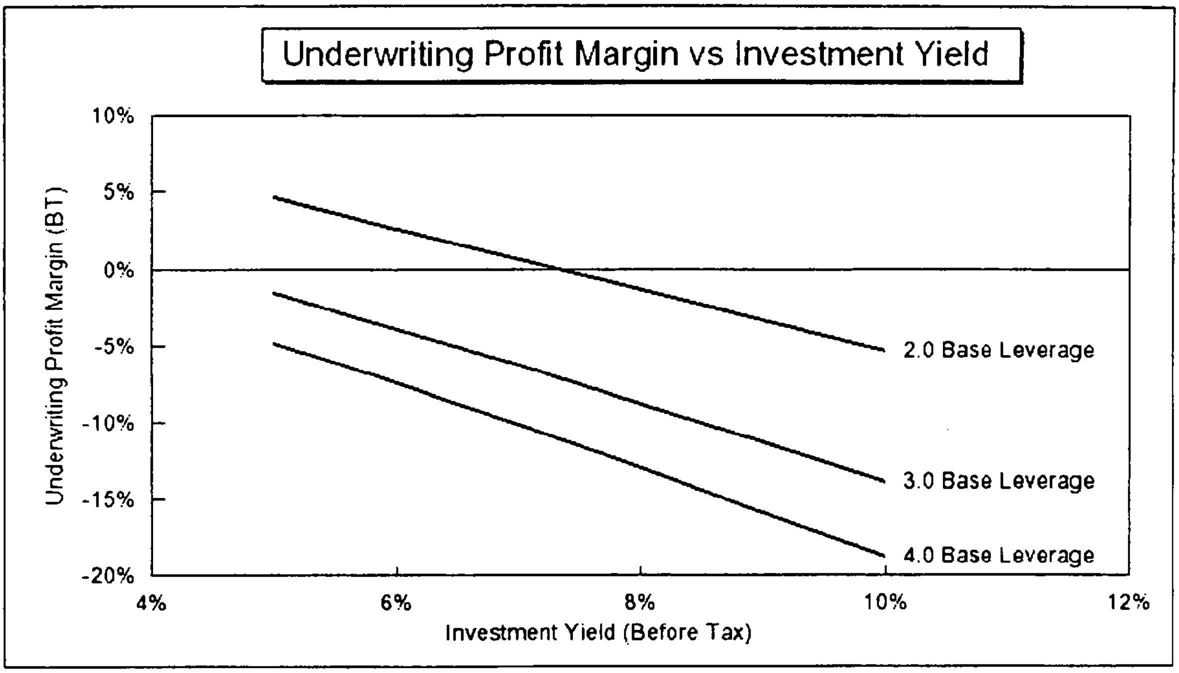
Figure 4



6% Investment Yield Before-Tax 7% Market Risk Premium 1.0 Equity Beta

As investment yields increase, profit margins deteriorate as shown in Figure 5. While this sounds a bit like cash flow underwriting, if premium rates are to fully reflect the benefit of higher yields, this is the result. It is important, however, that individual policy or accident periods be self-sustaining, and that historical portfolio yields not be used to subsidize new writings, truly the negative sort of cash flow underwriting that has occurred in the past in the industry.

Figure 5



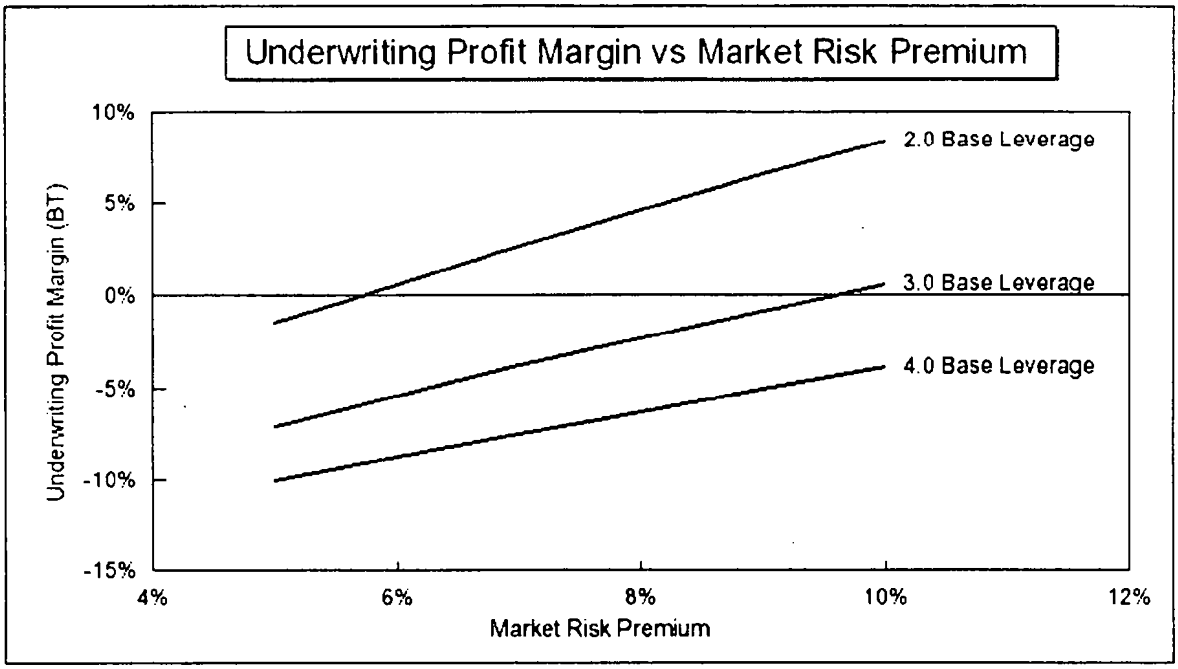
3 Average Loss Payout (Years) 1.0 Equity Beta 7% Market Risk Premium

As in the previous example, the burden on underwriting when leverage is low, is shown on the top line, upper left. Here again, the equity beta and cost of capital have not responded to the lower leverage induced risk.

The effects of the market risk premium on rates and the profit margin are shown in Figure 6. Clearly, higher required risk premiums lead to higher required total return, and in the premium to achieve it. Once again, the importance of achieving consistency in the leverage and the measured equity beta and the resultant cost of capital target is evident by the wide spread between the three lines on the chart.

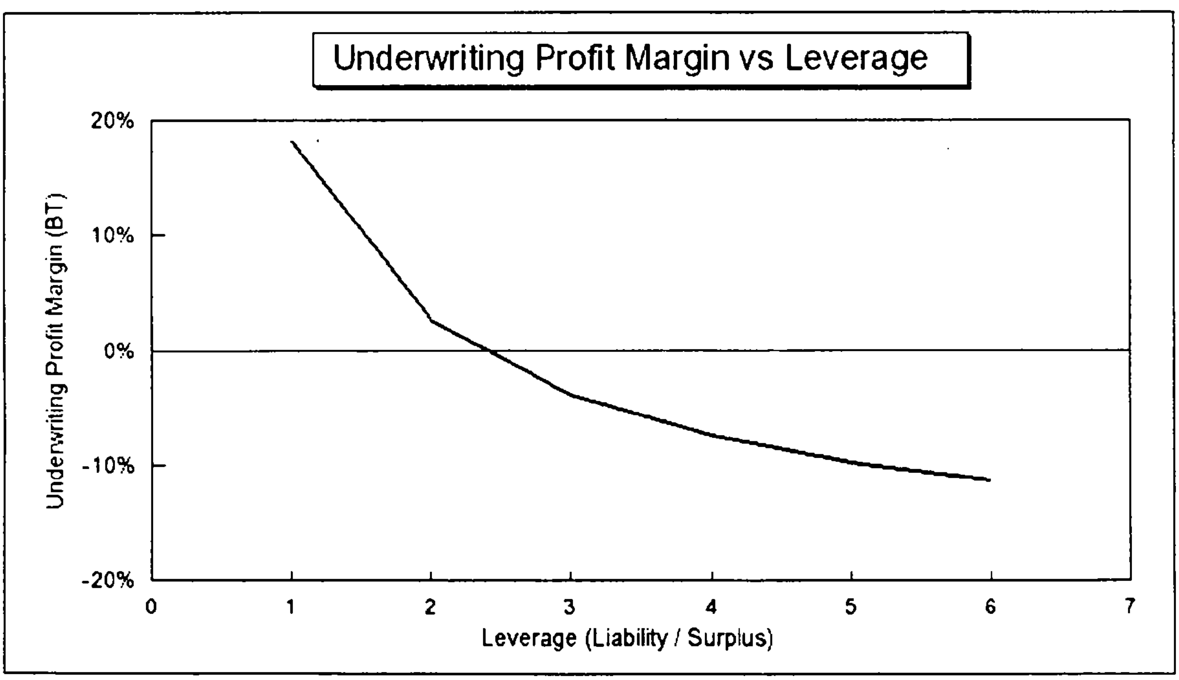
The relationship between leverage and the profit margin is shown in Figure 7. Note the severe impact caused when leverage is very low. If target returns are to be achieved when leverage declines to very low levels, significant increases in premium are required. Once again one has to question at what point surplus levels become "excessive" in relation to current writings, and whether it is reasonable to require target rates of return on the full amount of surplus beyond this point. Perhaps the current low levels of industry leverage are now creating just such a dilemma in which it is becoming increasingly more difficult to generate adequate returns on the entire amount of surplus available.

Figure 6



3 Average Loss Payout (Years) 1.0 Equity Beta 6% Investment Yield Before-Tax

Figure 7



3 Average Loss Payout (Years) 1.0 Equity Beta 6% Investment Yield Before-Tax 7% Market Risk Premium

7. Conclusion

This article has presented a methodology for the direct determination of risk-adjusted discount rates and liability betas. It involves the utilization of a "complete" total rate of return model (albeit in simplified form) in which rates of return can be determined both with and without risk-adjustment. The total return without risk-adjustment must equal the target cost of capital-based return. The total return with risk-adjustment must equal the risk-free rate. Within this formulation it is important that taxes be reflected by utilizing discount rates on an after-tax basis.

This formulation provides the capability to directly determine the required risk-adjusted discount rate and liability beta, given standard underwriting financials, leverage factor, market risk premium and equity beta. The risk-adjusted discount rates that result are less than the risk-free rate and the liability betas are negative, to a much greater degree than are often suggested. In no instance are they positive.

The important influence of leverage, and the need for consistency with the cost of capital and equity beta measurements, is noted. Subsequent changes in leverage require adjustment to these critical CAPM parameters. While the estimation and application of equity beta and the cost of capital are not without debate, at least there is a wide body of comparative data available to help judge the reasonableness of the results. This is not the case with respect to liability beta.

Hopefully, in the future the conceptual dialogue over risk-adjustment and liability betas can be made more meaningful by combining clearly specified parameter assumptions into a concrete total return model framework, such as has been presented in this paper.

References

1. Bingham, Russell E., "Surplus - Concepts, Measures of Return, and its Determination." Casualty Actuarial Society Annual Proceedings, 1993
2. Bingham, Russell E., "Total Return - Policyholder, Company, and Shareholder Perspectives." Casualty Actuarial Society Annual Proceedings, 1993.
3. Bingham, Russell E., "Cash Flow Models in Ratemaking: A Reformulation of Myers-Cohn NPV and IRR Models for Equivalency," Actuarial Considerations Regarding Risk and Return in Porperty-Casualty Insurance Pricing, Chapter 4, Casualty Actuarial Society, 1999.
4. Butsic, Robert P., "Determining the Proper Interest Rate for Loss Reserve Discounting : An Economic approach," Evaluating Insurance Company Liabilities, Casualty Actuarial Society Discussion Paper Program, May 1988.
5. Copeland, Tom., Koller, Tim., and Murrin, Jack., Valuation - Measuring and Managing the Value of Companies, Second Edition, John Wiley & Sons, Inc., 1994.
6. Cummins, J. David, and Scott E. Harrington, "Property-Liability Insurance Rate Regulation: Estimation of Underwriting Betas Using Quarterly Profit Data," Journal of Risk and Insurance, March 1985.
7. Cummins, J. David, "Multi-Period Discounted Cash Flow Models in Property-Liability Insurance," Journal of Risk and Insurance, March, 1990.
8. D'Arcy, Stephen P. and Dyer, Michael A., "Ratemaking: A Financial Economics Approach," Casualty Actuarial Society Annual Proceedings, 1997.
9. Fairley, William, "Investment Income and Profit Margins in Property-Liability Insurance: Theory and Empirical Results," The Bell Journal of Economics, Spring 1979, reprinted in J. David Cummins and Scott E. Harrington (eds.), Fair Rate of Return in Property-Liability Insurance, Kluwer-Nijhoff, 1987.
10. Kozik, Thomas J., "Underwriting Betas – The Shadow of Ghosts," Casualty Actuarial Society

Annual Proceedings, 1996.

11. Myers, Stewart C. and Cohn, Richard A., "A Discounted Cash Flow Approach to Property Liability Insurance Rate Regulation," J. David Cummins and Scott E. harrington (eds.) Fair Rate of Return in Property-Liability Insurance, Kluwer-Nijhoff, 1987.

APPENDIX

The following example provides high level balance sheet, income and cash flow statements. These are used to demonstrate various rate of return calculations and to show the resulting equivalency between conventionally reported rates of return and net present value rates of return, assuming certain rules are followed to control the flow of surplus and to distribute profits. The net present value rate of return is shown with and without risk-adjustment. Following this, the Myers-Cohn fair premium approach is briefly recapped, as modified to use after-tax discounting, shown in relation to this same example.

The following financial assumptions form the basis for the example presented:

103.85 Combined Ratio

$9,629 Premium, collected without delay

$10,000 Loss, single payment at end of year 3

$0 Expense

35% Income Tax Rate, no delay in payment

6.0% Investment Interest Rate before-tax, 3.9% after-tax

No Loss Discount Tax

3.0 Liability / Surplus Ratio

This example corresponds to the following:

1.0 Equity Beta

7.0% Market Risk Premium

3.65% Liability Risk Discount Adjustment, Before-Tax

-0.521 Liability Beta

Simplified balance sheet, income and cash flow statements are shown for this example in Exhibit I. The rules governing the flow of surplus are as follows: (1) the level of surplus is maintained at a 1/3 ratio with loss reserves, (2) investment income on surplus is paid to the shareholder as earned, and (3) operating earnings are distributed in proportion to the level of insurance exposures in each year, measured by loss reserve levels, relative to the total exposure. Since loss reserves are equal at $10,000 in each of the three years, operating earnings are distributed to the shareholder equally in each year.

Three "levels" of return exist within an insurance company. The first is the underwriting rate of return, which reflects what the company earns on pure underwriting cash flows, before reflecting investment income on the float. This is a "cost of funds" to the company. The second, operating return, reflects what the company earns on underwriting, when investment income on the float is included. This is the "risk charge" to the policyholder for the transfer of risk to the company. Finally, the total return is the net result of underwriting and investment income from operations together with investment income on surplus.

These rates of return can be determined by either a cash flow-based internal rate of return (IRR) calculation or by relating income earned to the amount invested. With regards to the shareholder total return perspective, the internal rate of return (IRR) based on cash flows from and to the shareholder indicates a 13.0% return over the three year period. The income versus investment approach (i.e. ROE) relates the income over the full three year aggregate financial life of the business to the shareholder's investment over this same period. This is shown in both nominal (i.e. undiscounted) and in present value (discounted, but without risk adjustment) dollars to produce a 13.0% rate of return on investment.

Furthermore, the return realized by the shareholder via dividends is also an identical 13.0% in each year. (This attribute follows from the rules used to control the flow of surplus). When risk-adjusted, the total net present valued rate of return in 6.0%, which is identical to the risk-free rate.

The Operating return, inclusive of underwriting and investment income, is shown to generate a cash flow-based internal rate of return of 3.0%. Equivalently, the operating income of $910 is a 3.0% return on the "investment equivalent" of $30,000, the total balance sheet policyholder supplied float upon which these earnings were generated. Also, $843 of present valued income is 3.0% on the present valued liability of $27,804.

The formulas which can be used to directly determine the net present value based rates of return, both without and with risk adjustment, are shown in Exhibits II and III, respectively.

Exhibit I
Three Period Demonstration Example

	PERIOD 0	1	2	3	Total	Present Value Not Risk Adjusted	Present Value Risk Adjusted
BALANCE SHEET							
Invested Assets	13092	13170	13250	0	39512	37072	38808
Loss Reserves	10000	10000	10000	0	30000	27804	29106
Retained Earnings	-65	-44	-22	0	-132	0	0
Surplus Contributed	3333	3333	3333	0	10000	9268	9702
Liabilities/Surplus	3.0	3.0	3.0	0	3.0	3.0	3.0
INCOME AFTER-TAX							
Underwriting	-241	0	0	0	-241	-241	-241
Inv Inc Retained Earnings		-9	-6	-3	-19		
Inv Inc Loss Reserves		390	390	390	1170	1084	445
Total Operating	-241	381	384	387	910	843	204
Surplus		130	130	130	390	361	378
Total Net Income	-241	511	514	517	1300	1205	582
Return On Beginning Contributed Surplus	-7.2%	15.3%	15.4%	15.5%	**13.0%**	**13.0%**	**6.0%**
CASH FLOW							
Operating Cash Flows							
Premium Receipt	9629	0	0	0	9629	9629	9629
Loss Payment	0	0	0	-10000	-10000	-8916	-9555
Tax Paid	130	0	0	0	130	130	130
Ret Earns "Funding"	241	-87	-87	-87	-19	0	0
Total Underwriting	10000	-87	-87	-10087	-260		
Underwriting Return			**Underwriting IRR =**		**-0.9%**		
Investment Receipts (After-Tax)		390	390	390	1170		
Total Operating	10000	303	303	-9697	910	843	204
Operating Return			**Operating IRR =**		**3.0%**		
Surplus Cash Flows							
Contributed Surplus	3333	0	0	3333	0		
Dividend							
Surplus Inv Inc		-130	-130	-130	-390		
Operating Earnings		-303	-303	-303	-910		
Net Shareholder	3333	-433	-433	-3767	-1300		
Shareholder Rate of Return			**Shareholder IRR =**		**13.0%**		
Shareholder "Dividend" Yield		**13.0%**	**13.0%**	**13.0%**	**13.0%**		

RATE OF RETURN

	IRR	(Income / Investment) Nominal	Net Present Value Basis: Not Risk Adjusted	Net Present Value Basis: Risk Adjusted
Underwriting Return	-0.9%	-0.9% = -260 / 30000	-0.9% = -241 / 27804	-0.8% = -241 / 29106
Operating Return	3.0%	3.0% = 910 / 30000	3.0% = 843 / 27804	0.7% = 204 / 29106
Shareholder Total Return	13.0%	13.0% = 1300 / 10000	13.0% = 1205 / 9268	6.0% = 582 / 9702

Exhibit II
Net Present Value Income, Balance Sheet and Rate of Return
Definitions, Formulas and Calculations WITHOUT Risk Adjustment

INCOME ITEMS	FORMULAS
Underwriting Income	$(P-L)(1-T)$ $(9629-10000)(1-0.35) = -241$
Operating Income	$PV(P)-PV(L)-PV(UWPT) = P-L/(1+R)^N-T(P-L)$ $9629-10000/(1+0.039)^3-(0.35)(9629-10000)$ $= (P-L)-T(P-L)/(1+R)^{N}{}_{T} + L[(1-1/(1+R)^{N}]$ $(9629-10000)-(0.35)(9629-10000)/(1+039)^0$ $+ 1000[1-1/(1+0.065)^2]$ Underwriting Income + Investment Income Credit on Policyholder Liabilities -241 + 1084 = 843
Surplus Investment Income	R (Surplus) (0.039)(9268) = 361
Total Income	= Operating Income + Investment Income on Surplus 843 + 361 = 1205
BALANCE SHEET ITEMS	
Policyholder Liabilities	$L(1-1/(1+R)^N)/R$ $10000(1-1/(1+0.039)^3)/0.039 = 27804$
Surplus	$S(1-1/(1+R)^N)/R$ $3333(1-1/(1+0.039)^3)/0.039 = 9268$
RATES OF RETURN	
Underwriting Return on Liabilities (Cost of Policyholder Supplied Funds)	Underwriting Income / Policyholder Liabilities -241 / 27804 = -0.9%
Operating Return on Liabilities (Risk Charge to Policyholder)	Operating Income / Policyholder Liabilities 843 / 27804 = 3.0%
Total Return on Surplus (ROS) (Shareholder Return)	Total Income / Surplus 1205 / 9268 = 13.0 % = (Operating Return on Liabilities) (Liability / Surplus) + R 3.0 % (3) + 3.9 % = 13.0 %

Exhibit III
Net Present Value Income, Balance Sheet and Rate of Return
Definitions, Formulas and Calculations WITH Risk Adjustment

INCOME ITEMS	**FORMULAS**
Underwriting Income	$(P-L)(1-T)$ $(9629-10000)(1-0.35) = -241$
Operating Income	$PV(P)-PV(L)-PV(UWPT) = P-L/(1+R-R_L)^N-T(P-L)$ $9629-10000/(1+0.039-0.024)^3-(0.35)(9629-10000)$ $= (P-L)-T(P-L)/(1+R)^{N_T} + L[(1-1/(1+R-R_L)^N]$ $(9629-1000)-(0.35)(9629-1000)/(1+0.039)^0$ $+ 10000[1-1/(1+0.039-0.024)^3]$ = Underwriting Income + Investment Income Credit on Policyholder Liabilities $-241 + 445 = 204$
Surplus Investment Income	R (Surplus) $(0.039)(9702) = 378$
Total Income	Operating Income + Investment Income on Surplus $204 + 378 = 582$
BALANCE SHEET ITEMS	
Policyholder Liabilities	$L(1-1/(1+R-R_L)^N)/(R-R_L)$ $10000(1-1/(1+0.039-0.024)^3)/(0.039-0.024) = 29106$
Surplus	$S(1-1/(1+R-R_L)^N)/(R-R_L)$ $3333(1-1/(1+0.039-0.024)^3)/(0.039-0.024) = 9702$
RATES OF RETURN	
Underwriting Return on Liabilities (Cost of Policyholder Supplied Funds)	Underwriting Income / Policyholder Liabilities -241 / 29106 = -0.8%
Operating Return on Liabilities (Risk Charge to Policyholder)	Operating Income / Policyholder Liabilities 204 / 29106 = 0.7%
Total Return on Surplus (ROS) (Shareholder Return)	Total Income / Surplus 582 / 9702 = 6.0 % = (Operating Return on Liabilities) (Liability / Surplus) + R 0.7 % (3) + 3.9 % = 6.0 %

Myers-Cohn Fair Premium With After-Tax Discounting

The $9629 premium shown in the example can be derived using the traditional Myers-Cohn format, as long as all discounting is on an after-tax basis, and given a liability beta that is "consistent" with the equity beta. The traditional MC model format as shown in reference [1] is as follows:

$$P = PV(L) + PV(UWPT) + PV(IBT).$$

This states that the fair premium, P, is equal to the sum of the present value of the losses, L, the tax on underwriting profit, UWPT, and the tax on investment income derived from the investable balance, IBT. The investable balance includes all policyholder liabilities (net of premium, loss and expense) and surplus. Note that underwriting expense is combined with loss as total liabilities in the example in the cited reference. It is suggested that the discount rates be adjusted for risk (i.e. uncertainty), particularly the rate applicable to losses. No mention is made as to whether discount rates are on a before-tax or after-tax basis.

The example presented previously differs from that in the reference to some degree, first by extending from one to three periods and then by assuming that taxes on underwriting and investment are paid without delay. In the original reference presentation underwriting taxes were assumed to have a one year delay in their payment. The tax loss discount (TRA 86) was excluded for simplification in both instances. In the reference S was set equal to P for the single period example presented. In the example presented, surplus was set at each point in time to an amount equal to L/F, where F is the liability/surplus leverage factor. Since surplus is set as a function of liabilities, surplus is implicitly risk-adjusted as well.

Exhibit IV presents the derivation of the "fair" premium that results when the Myers-Cohn approach is reformulated to use after-tax discounting and to control surplus via a linkage to liabilities over the multi-period timeframe. In this example the interest rate is 6%, the tax rate is 35%, and a risk adjustment of 3.65%, before-tax (i.e. 2.4% after-tax) is made when discounting. This is the risk adjustment that results from a liability beta of (0.521). A liability/surplus ratio of 3 to 1 is used to determine the level of surplus. The fair premium that results is \$9626. As stated previously, premiums and taxes are assumed to have no delay in their receipt or payment.

Exhibit IV

Derivation of "Fair" Premium with After-Tax Discounting

P = PV(L)	9555	$L / (1+R-R_L)^N$ $10000 / (1 + 0.039 - 0.024)^3$
+ PV(UWPT)	-130	$T [P / (1+R)^{N_T} - L / (1+R-R_L)^{N_T}]$
		$(0.35) [(9629)/(1+0.039)^0 - 1000/(1+0.039-0.024)^0]$
+ PV(IBT)	204	$T R_b S [(1 - 1 / (1+R-R_L)^N) / (R-R_L)]$
	_____	$(0.35)(0.06)(3333)[(1-1/(1+0.039-0.024)^3/(0.039-0.024)]$
"Fair" Premium	**9629**	

P: Premium
L: Loss
N: Loss Payment Date
T: Tax Rate
N_T: Underwriting Tax Payment Delay
F: Liability / Surplus Leverage Factor

R_b: Interest Rate, Before -Tax
R: Interest Rate, After -Tax
R_L; Risk Discount Adjustment, After -Tax
UWPT: Underwriting Profit Tax
IBT: Investable Balance Investment Income Tax
S: Initial Surplus Contribution (L / F)

Notes: Due to After-Tax Discounting PV(IBT) reduces to simply tax on investment income derived from the investable surplus balance.

Liability/Surplus Relationship implies Surplus level affected by risk adjustment.