INCREASED LIMITS RATEMAKING FOR LIABILITY INSURANCE

July 2006

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1. INTRODUCTION

Increased limits ratemaking focuses on the development of appropriate charges for various limits of liability coverages. Common liability lines of insurance include Personal Automobile Liability, Commercial Automobile Liability, General Liability, and Medical Professional Liability. However, as society continues to develop and grow, new sources of potential liability for individuals and businesses may arise --- such as liability for E-Commerce.

Increased limits ratemaking requires specialized techniques. Quite often, the actuary is confronted with only limited available data when attempting to develop charges for high limits of liability coverages --- which may represent very significant potential loss exposures for an insurance company.

Techniques for evaluating appropriate charges for various limits of liability insurance have evolved over the years. The current approach involves calculating a series of factors --- increased limits factors or ILFs --- which are applied to a rate or "loss cost" for a lower limit of liability (generally referred to as the "basic limit").

2. LIABILITY INSURANCE INCREASED LIMITS FACTORS

Why are increased limits factors used? Why not just calculate rates or loss costs at every desired limit of insurance? An important reason is credibility. A larger volume of data generally leads to more reliable loss cost estimates. There usually is not enough data at higher loss sizes to calculate higher limit loss costs in a fine level of detail --- separately by limit, class, state and territory, for example. The standard actuarial answer to this dilemma has been to split the analysis of expected costs for liability insurance into two parts.

First, the larger volume of data on claims of relatively smaller sizes is used to calculate basic limit loss costs in full class, state and territory detail. (For example, for General Liability and Commercial Automobile Liability, Insurances Services Office (ISO) uses a basic limit of $100,000.) Then, increased limits factors --- which represent the ratio of expected costs at higher limits of liability to expected costs at the basic limit --- are calculated using a broader combination of experience, such as class group, state group or countrywide experience. These increased limits factors are applied to the class, state and territory specific basic limit loss costs to produce higher limit loss costs that reflect individual class, state and territory differences.

The expected costs of liability losses generally include several items: indemnity costs (actual losses paid to plaintiffs); allocated loss adjustment expense (or ALAE, most notably legal defense costs, plus other expenses that are associated with an individual claim); and unallocated loss adjustment expense (general overhead expenses associated with the claim settlement process). In some cases a quantity called risk load, which is a function of the estimated variability of expected losses at each liability limit, is also included as a cost.

An Increased Limit Factor (ILF) at limit L relative to basic limit B can be defined as:

$$ILF(L) = \frac{\text{Expected Indemnity Cost}(L) + ALAE(L) + ULAE(L) + RL(L)}{\text{Expected Indemnity Cost}(B) + ALAE(B) + ULAE(B) + RL(B)}$$

where ALAE(X) = the Allocated Loss Adjustment Expense provision at each limit,

ULAE(X) = the Unallocated Loss Adjustment Expense provision at each limit, and

RL(X) = the Risk Load provision at each limit.

Increased limits factors are generally developed on a per-claim or per-occurrence basis. A per-claim limit is a limit on the amount that will be paid to a *single plaintiff* for losses arising from a single incident. A per-occurrence limit is a limit on the total amount that will be paid to *all plaintiffs* for losses arising from a single incident. Other types of liability limits encountered will be discussed in a later section.

Of the four general quantities listed above, the most important component is the expected indemnity cost. At times, ALAE and risk load may also be significant components of overall costs. In most analyses, ALAE, ULAE and RL are all expressed as some function of the expected indemnity cost.

For illustrative purposes, let us examine the "indemnity-only" ILF:

$$ILF(L) = \frac{\text{Expected Indemnity Cost}(L)}{\text{Expected Indemnity Cost}(B)}$$

An important simplifying assumption is generally used when working with increased limits factors. Underlying frequency is assumed to be independent of severity. In other words, the size distribution of each loss is assumed to be independent of the number of losses. The above formula can then be expressed as:

$$\text{ILF(L)} = \frac{\text{Expected Frequency (L) x Expected Severity (L)}}{\text{Expected Frequency (B) x Expected Severity (B)}}$$

Secondly, it is generally assumed that the frequency is independent of the policy limit purchased, or in other words, that Expected Frequency (L) = Expected Frequency (B). Thus, for example, insureds who purchase policies with a $1,000,000 limit of liability are assumed to have the same expected claim frequency as insureds who purchase policies with a $100,000 limit of liability. With this assumption, both the numerator and denominator of the above formula can be divided by the same expected frequency to give:

$$\text{ILF(L)} = \frac{\text{Expected Severity (L)}}{\text{Expected Severity (B)}}$$

Thus, the indemnity-only increased limits factors can be entirely developed by examining the expected severities at various limits. Here we are speaking of the most basic forms of policy limits, per-claim and per-occurrence limits. Other types of limits do require consideration of claim frequency. Also, if either of the two simplifying assumptions stated above is violated, frequency would need to be considered.

The expected severity at a given limit of liability is known as the Limited Average Severity (LAS). Stated simply, the limited average severity is the average size of loss when all losses have been capped at the given policy limit. Various methods of developing this quantity will be discussed in this paper. The limited average severity is also referred to as the limited expected value, a term which the reader will encounter in other actuarial texts and papers.

As noted earlier, while the indemnity cost is the most significant component of increased limits factors, there are also provisions for ALAE, ULAE and Risk Load. It is worth restating our more general formula as:

$$\text{ILF(L)} = \frac{\text{LAS(L)} + \text{ALAE(L)} + \text{ULAE(L)} + \text{RL(L)}}{\text{LAS(B)} + \text{ALAE(B)} + \text{ULAE(B)} + \text{RL(B)}}$$

Here all quantities are evaluated on a per-claim or per-occurrence basis.

Before turning to a discussion of the development of the indemnity cost component of ILFs, several comments about the types of liability limits generally encountered in practice are in order.

3. TYPES OF LIMITS OF LIABILITY

Limits of liability can be defined relative to several different loss measurements. The first limit to be discussed is a per-claim limit. As noted earlier, a per-claim limit is a limit on the amount that will be paid to a *single plaintiff* for losses arising from a single incident (accident or occurrence). The second limit is a per-occurrence (or per-accident) limit. A per-occurrence limit is a limit on the total amount that will be paid to *all plaintiffs* for losses arising from a single incident. Increased limits factors for both per-claim and per-occurrence limits can be calculated in the same way. In one case, records of per-claim loss amounts are needed; in the other, records of per-occurrence amounts are used.

Compound limits combine at least two types of loss limitations. One type of compound limit is a "split limit" claim/accident limit that is commonly encountered in Personal and Commercial Automobile bodily injury liability coverage. For example, a 100/300 BI liability policy will pay up to $100,000 per claimant per accident, but no more than $300,000 in total to all claimants involved in any one accident, regardless of the number of claimants.

Another common type of compound limit is an occurrence/annual aggregate limit. Such limits are common in many lines of insurance, including General Liability and Professional Liability. For example, a General Liability policy with a $1,000,000/$2,000,000 policy limit will pay up to $1,000,000 in total to all claimants for a single occurrence, but will not pay more than $2,000,000 in total for all occurrences that occur within a one-year policy period.

Compound limit factors are more difficult to calculate than per-claim or per-occurrence factors. They require consideration of the frequency distribution of claims and/or occurrences. As noted earlier, under common assumptions, frequency is not considered in the calculation of simple per-claim or per-occurrence increased limits factors. Often, compound increased limits factors are calculated in a two-step process. The first step is the calculation of per-occurrence or per-claim increased limits factors. As a second step, a simulation is run to evaluate the effects of multiple claimant or multiple occurrence situations.

This paper will only cover calculating increased limits factors for per-claim and per-occurrence limits.

4. INDEMNITY COST

As stated earlier, two basic assumptions are frequently made when calculating increased limits factors:

- The underlying claim frequency is independent of the underlying claim severity.
- The choice of limit purchased is independent of claim frequency.

As noted above, these two assumptions allow us to ignore frequency (and hence to ignore premium and exposure data) and to only consider size of loss data in our calculation of per-claim and per-occurrence increased limits factors.

The methods illustrated in this paper rely on these two assumptions. The theoretical framework for the calculation of the indemnity cost components of per-claim and per-occurrence increased limits factors under these assumptions was developed by Miccolis [5].

In order to evaluate an appropriate provision for indemnity costs at various limits of liability, we need to develop the limited average severity (LAS) at various limits of liability. Recall that the limited average severity is simply the average size of loss when all losses have been capped at the given limit.

This definition leads directly to a simple means of developing the LAS by using empirical loss information. Assuming we have available a body of individual loss experience, we proceed to examine each loss. Each individual loss is capped (if necessary) at the desired limits for our analysis.

For illustration, consider the development of an indemnity-only ILF for a $1,000,000 limit, using a $100,000 limit as the basic limit. Using a small sample of losses, we first cap the losses at each limit:

Loss Amount	**Loss at $100,000 Limit**	**Loss at $1,000,000 Limit**
50,000	50,000	50,000
75,000	75,000	75,000
150,000	100,000	150,000
250,000	100,000	250,000
1,250,000	100,000	1,000,000
Total	425,000	1,525,000
Limited Average Severity	85,000	305,000

This leads directly to an indemnity-only ILF of:

3.588 = 305,000 / 85,000

In practice, this analysis should be based on a large number of losses. As noted in the introductory section, at higher limits of liability, there generally is less loss experience available for analysis. Thus, increased limits analyses should be based on a broad body of loss data.

At times, individual loss experience may be cumbersome to obtain and difficult to work with. There is also a straightforward technique for working with summarized loss experience. This technique requires a compilation of loss experience to be developed for various size of loss intervals. At a minimum, we will need to examine loss intervals that correspond to the limits at which we wish to develop limited average severities. (Finer intervals may be used, but it is necessary to compile the loss experience in such a manner that each desired limit corresponds to an endpoint of an interval.)

For example, consider the following empirical loss data:

Lower($)	**Upper($)**	**Losses($)**	**Occurrences**
1	100,000	25,000,000	1000
100,001	250,000	75,000,000	500
250,001	500,000	60,000,000	200
500,001	1,000,000	30,000,000	50
1,000,001	-	15,000,000	10

In this table, a loss is slotted into the interval corresponding to its total loss size. Thus, a loss of $40,000 would be included in the first interval, while a loss of $150,000 would be included in the second interval.

In order to develop a limited average severity at each desired limit, we need to consider two parts of this size of loss data. To evaluate an LAS at a $100,000 limit, we need to include all loss dollars from losses of $100,000 or less, *plus* the first $100,000 of each loss that is in excess of $100,000.

Thus:

LAS($100,000) = ($25,000,000 + 760 × $100,000) / 1760 = $57,386

Similarly:

LAS($500,000) = ($160,000,000 + 60 × $500,000) / 1760 = $107,955
LAS($1,000,000) = ($190,000,000 + 10 × $1,000,000) / 1760 = $113,636

The indemnity-only ILFs are:

ILF($500,000) = 107,955 / 57,386 = 1.88
ILF($1,000,000) = 113,636 / 57,386 = 1.98

The losses used in this example are assumed to be the total losses that actually occurred. In other words, none of these losses were limited, or "censored," by the insured's policy limit. This is a very significant issue when working with empirical data. In actual practice, many losses are censored by the insured's policy limit. Our concern is how to use occurrences (or claims) whose actual size has been censored by the policy limit purchased. For example, we may see that a $500,000 loss amount has been reported to us for a policy with a $500,000 limit. We know that the underlying loss size is *at least* $500,000. But, we do not know if it is $500,000, $501,000, $900,000 or $20,000,000. If our reported data contains many occurrences where the underlying reported losses have already been capped at various policy limits, this can potentially result in an underestimate of the LAS for higher limits of liability.

In order to work with censored losses, we will need to group the data by the policy limits at which the policies were written, and retain the information on the various intervals of losses. For illustration, let us assume that that the loss experience we used in the above example was actually the total experience generated from policies written proportionally (20:30:50) at $100,000, $500,000 and $1,000,000 limits. To facilitate comparison, we will split each interval of actual losses and occurrences, using these proportions, into three parts.

Now, in our illustration, the following experience will be seen:

$100,000 Policy Limit Losses (prior to limiting at the policy limit)

Lower($)	Upper($)	Losses($)	Occurrences
1	100,000	5,000,000	200
100,001	250,000	15,000,000	100
250,001	500,000	12,000,000	40
500,001	1,000,000	6,000,000	10
1,000,001	-	3,000,000	2

$500,000 Policy Limit Losses (prior to limiting at the policy limit)

Lower($)	Upper($)	Losses($)	Occurrences
1	100,000	7,500,000	300
100,001	250,000	22,500,000	150
250,001	500,000	18,000,000	60
500,001	1,000,000	9,000,000	15
1,000,001	-	4,500,000	3

$1,000,000 Policy Limit Losses (prior to limiting at the policy limit)

Lower($)	Upper($)	Losses($)	Occurrences
1	100,000	12,500,000	500
100,001	250,000	37,500,000	250
250,001	500,000	30,000,000	100
500,001	1,000,000	15,000,000	25
1,000,001	-	7,500,000	5

When the effect of each policy limit is considered, we have:

$100,000 Policy Limit Losses

Lower($)	Upper($)	Losses($)	Occurrences
1	100,000	20,200,000	352
100,001	250,000	-	-
250,001	500,000	-	-
500,001	1,000,000	-	-
1,000,001	-	-	-

$500,000 Policy Limit Losses

Lower($)	Upper($)	Losses($)	Occurrences
1	100,000	7,500,000	300
100,001	250,000	22,500,000	150
250,001	500,000	27,000,000	78
500,001	1,000,000	-	-
1,000,001	-	-	-

$1,000,000 Policy Limit Losses

Lower($)	Upper($)	Losses($)	Occurrences
1	100,000	12,500,000	500
100,001	250,000	37,500,000	250
250,001	500,000	30,000,000	100
500,001	1,000,000	20,000,000	30
1,000,001	-	-	-

In these examples, when a loss exceeds the policy limit, it has been capped at that limit. Thus, for the \$100,000 policy limit example, 152 losses have been capped at \$100,000 and added to the \$1-\$100,000 interval.

When calculating the limited average severities at each limit of liability, we will use the largest amount of data possible, yet avoid any bias caused by policy limit censorship.

First, let us calculate LAS(\$100,000):

$$
\begin{aligned}
\text{LAS}(\$100{,}000) = {} & [\ (\ \$20{,}200{,}000\ ) \\
& + (\ \$7{,}500{,}000 + 228 \times \$100{,}000\ ) \\
& + (\ \$12{,}500{,}000 + 380 \times \$100{,}000\ )\ \ ] \\
& /\ \ [\ 352 + 528 + 880\ ] \\
& = \$57{,}386
\end{aligned}
$$

Here we were able to use the information from all three policy limits in our calculation. Note that this agrees with the LAS(\$100,000) that we developed previously.

We will now develop a layer average severity for the \$100,000 to \$500,000 layer. This will then be combined with the LAS at \$100,000 to produce the LAS at \$500,000.

From \$500,000 Limit Policies:

Lower(\$)	Upper(\$)	Losses(\$)	Occurrences	\$100,000 to \$500,000 Layer Losses
1	100,000	7,500,000	300	-
100,001	250,000	22,500,000	150	7,500,000
250,001	500,000	27,000,000	78	19,200,000

From \$1,000,000 Limit Policies:

Lower(\$)	Upper(\$)	Losses(\$)	Occurrences	\$100,000 to \$500,000 Layer Losses
1	100,000	12,500,000	500	-
100,001	250,000	37,500,000	250	12,500,000
250,001	500,000	30,000,000	100	20,000,000
500,001	1,000,000	20,000,000	30	12,000,000

In the above tables, we have calculated the losses in the \$100,000 to \$500,000 layer. This is a simple calculation. For loss intervals between \$100,000 and \$500,000, we start with the first-dollar (or "ground-up") losses from the previous tables, and remove the first

$100,000 of each loss. Thus, for the losses from policies written at a $500,000 limit, we have:

$$\$22{,}500{,}000 - 150 \times \$100{,}000 = \$7{,}500{,}000$$
$$\$27{,}000{,}000 - 78 \times \$100{,}000 = \$19{,}200{,}000$$

Similar calculations are performed for the losses between $100,000 and $500,000 from policies written at a $1,000,000 limit:

$$\$37{,}500{,}000 - 250 \times \$100{,}000 = \$12{,}500{,}000$$
$$\$30{,}000{,}000 - 100 \times \$100{,}000 = \$20{,}000{,}000$$

We also must consider the $100,000 to $500,000 layer from losses in excess of $500,000. Thus, we include $400,000 for each of those 30 losses, or:

$$30 \times \$400{,}000 = \$12{,}000{,}000$$

Now, the total losses in the $100,000 to $500,000 layer equal:

$$\$7{,}500{,}000 + \$19{,}200{,}000 + \$12{,}500{,}000 + \$20{,}000{,}000 + \$12{,}000{,}000$$

$$= \$71{,}200{,}000$$

Dividing by the total occurrences in the layer --- 608 --- yields an average loss for the layer of $117,105.

Note that this is the average loss severity for the layer *given that a loss has entered the layer.*

For the purpose of developing LAS($500,000), we need to adjust for the fact that this higher layer severity was based on a smaller body of losses --- 608 --- than our LAS($100,000), which was based on 1760 losses. In order to make this adjustment we will calculate the probability that a loss is greater than $100,000 given that a loss has occurred. Our first inclination might be to divide 608 by 1760 to calculate this probability, but this would not be correct. Many of those 1760 occurrences were written at a policy limit of $100,000. Therefore, they could not have entered the $100,000 to $500,000 layer.

When we exclude the 352 loss occurrences from policies written at a limit of $100,000, we find that 1408 loss occurrences remain. Therefore, we calculate the probability that a loss is greater than $100,000 given that a loss has occurred as:

$$608 / 1408 = 0.431818$$

And our estimate of the loss severity in the layer given that a loss *of any size* has occurred is:

$$\$117{,}105 \times 0.431818 = \$50{,}568$$

Note that this estimate of the average loss in the layer can also be developed directly, by dividing the total losses in the layer --- $71,200,000 --- by the total number of occurrences from the $500,000 and $1,000,000 limit policies --- 1408:

$$\$71{,}200{,}000 / 1408 = \$50{,}568$$

We now calculate LAS($500,000) as:

$$\$57{,}386 + \$50{,}568 = \$107{,}954$$

We will proceed similarly to develop the cost of the $500,000 to $1,000,000 layer. We now will only use losses written at a policy limit of $1,000,000.

From $1,000,000 Limit Policies:

Lower($)	**Upper($)**	**Losses($)**	**Occurrences**	**$500,000 to $1,000,000 Layer Losses**
1	100,000	12,500,000	500	-
100,001	250,000	37,500,000	250	-
250,001	500,000	30,000,000	100	-
500,001	1,000,000	20,000,000	30	5,000,000

Here, the losses in the layer are developed as:

$$\$20{,}000{,}000 - 30 \times \$500{,}000 = \$5{,}000{,}000$$

Dividing by the total number of occurrences from $1,000,000 limit policies --- 880 --- yields an average loss for the layer of $5,682:

$$\$5{,}000{,}000 / 880 = \$5{,}682$$

This allows us to calculate LAS($1,000,000) as:

$$\$57{,}386 + \$50{,}568 + \$5{,}682 = \$113{,}636$$

Note the exact agreement between the LAS values we have calculated here for \$500,000 and \$1,000,000 and the earlier values calculated for the sample of uncensored loss data. (The one dollar difference in LAS(\$500,000) is simply due to rounding.) This agreement arises because we have purposely constructed this example by overlaying a policy limit distribution on our earlier sample of uncensored loss data to illustrate this technique.

Empirical loss data can be viewed as a sample of loss data from an underlying continuous size of loss distribution. If we fit a continuous distribution to empirical loss data, we can then calculate limited average severities using this continuous distribution.

Recall that the expected value of a continuous random variable can be calculated as

$E(x) = \int_0^\infty x f(x)dx$, where $f(x)$ is the probability density function of that random variable.

Recall that the cumulative distribution function (CDF) of any random variable, X, evaluated at x, is defined as $F(x) = \Pr(X \le x)$. The Survival Function of X, evaluated at x, is defined as $S(x) = 1 - F(x) = \Pr(X > x)$.

The limited average severity (LAS), at a limit of k, in the case of a continuous distribution, is:

$$\text{LAS}(k) = \int_0^k x f(x)dx + k[1 - F(x)]$$

In this equation, the first term includes all losses less than or equal to k, while the second term includes the first k dollars of losses greater than k.

The above expression for the limited average severity at a limit of k can be equivalently written as:

$$\text{LAS}(k) = \int_0^k [1 - F(x)]dx = \int_0^k S(x)dx$$

This relationship can be established via integration by parts.

An excellent graphical presentation of this relationship, and other aspects of increased limits factors, may be found in Lee [4].

5. EFFECT OF TREND ON VARIOUS LAYERS OF LOSS --- "THE LEVERAGED EFFECT OF INFLATION"

Before proceeding any further, a few comments about loss trend, and its effect on layers of size of loss distributions are in order.

Briefly stated, assuming a constant percentage trend acting on all sizes of loss, basic limit losses will trend at a lower rate than losses limited at higher limits of liability, which in turn will trend at a lower rate than excess layers. This phenomenon is commonly referred to as the "leveraged effect of inflation."

This relationship does require that losses are steadily increasing over time. In the example below, we use a positive trend of +10%. The relationships we will discuss will always hold for an environment of steadily increasing losses. Note that a *mirror-image relationship* will occur with steadily *decreasing* losses. As we examine pre-trend and post-trend losses in the following example, the reader may visualize the effects of a negative trend by imagining that the pre-trend and post-trend loss columns are reversed.

Please note, however, that when working with loss *severity* trends, steadily increasing losses --- a positive trend --- are generally to be expected. This is simply due to the prevalence of upward cost inflation. Periods of negative trend are somewhat rare, and generally indicate some underlying change in the experience that is counteracting the effects of cost inflation.

Let us examine the effects of a constant percentage trend on several loss amounts, in situations involving several limits. We will assume a +10% underlying trend, acting on all sizes of loss. We will examine loss amounts of $50,000, $250,000, $490,000, $750,000, $925,000, and $1,825,000. On an unlimited basis, these losses will grow to $55,000, $275,000, $539,000, $825,000, $1,017,500 and $2,007,500, respectively. We will evaluate the effects of trend on losses limited at $100,000, losses limited at $500,000, and losses limited at $1,000,000. The following table illustrates the results:

Effects of +10% Trend on Various Loss Limits

Loss Amount ($)	$100,000 Limit		$500,000 Limit		$1,000,000 Limit	
	Pre Trend($)	Post Trend($)	Pre Trend($)	Post Trend($)	Pre Trend($)	Post Trend($)
50,000	50,000	55,000	50,000	55,000	50,000	55,000
250,000	100,000	100,000	250,000	275,000	250,000	275,000
490,000	100,000	100,000	490,000	500,000	490,000	539,000
750,000	100,000	100,000	500,000	500,000	750,000	825,000
925,000	100,000	100,000	500,000	500,000	925,000	1,000,000
1,825,000	100,000	100,000	500,000	500,000	1,000,000	1,000,000
Total	550,000	555,000	2,290,000	2,330,000	3,465,000	3,694,000
Realized Trend	+0.9%		+1.7%		+6.6%	

This example illustrates the effects of various liability limits on loss trends. The basic underlying cause of these varying realized trends is that while the *loss amounts* are trending upward, the *limits of liability* are not. Thus, when a loss, in the pre-trend period, has already been capped at a given limit, it can grow no further. The lower the limit, the lower the effective trend.

In the example above, the relatively low $100,000 limit of liability reduces the effect of the unlimited +10% trend to +0.9%. At a $500,000 limit of liability, the unlimited +10% trend is reduced to +1.7% --- which is greater than the effective trend at $100,000 but still significantly below the unlimited trend. At a $1,000,000 limit of liability, the effective trend is now +6.6%.

It is worth noting that, were the limits of liability to be *increased at the same rate* as the loss trend used, then the +10% trend would have been fully realized in each case. For example, if the $100,000 limit were increased to $110,000 in the post-trend period, the total post-trend losses would have been $605,000 --- exactly a 10% increase over the pre-trend total.

On the excess layer side, there are two factors to consider. First, the portions of losses below the layer are removed from both the pre-trend and post-trend loss amounts. This is a smaller percentage of the post-trend loss, which produces a "leveraging" effect. However, some losses may be capped by the upper limit of the layer, mitigating the effect. The following table provides an illustration:

Effects of +10% Trend on Various Excess Loss Layers

Loss Amount ($)	$250,000 excess of $250,000 layer		$500,000 excess of $500,000 layer		$1,000,000 excess of $1,000,000 layer	
	Pre Trend($)	Post Trend($)	Pre Trend($)	Post Trend($)	Pre Trend($)	Post Trend($)
50,000	-	-	-	-	-	-
250,000	-	25,000	-	-	-	-
490,000	240,000	250,000	-	39,000	-	-
750,000	250,000	250,000	250,000	325,000	-	-
925,000	250,000	250,000	425,000	500,000	-	17,500
1,825,000	250,000	250,000	500,000	500,000	825,000	1,000,000
Total	990,000	1,025,000	1,175,000	1,364,000	825,000	1,017,500
Realized Trend	+3.5%		+16.1%		+23.3%	

At the $250,000 excess of $250,000 layer, note two losses in particular: The $250,000 loss grows to $275,000 and now enters the excess layer in the post-trend period. The $490,000 loss grows to $539,000, and is now capped by the upper limit of the layer. The effective trend is +3.5%, well below the unlimited trend of +10%.

At the $500,000 excess of $500,000 layer, we see similar effects. The $490,000 loss grows to $539,000 and now enters the excess layer in the post-trend period. The $925,000 pre-trend loss grows to $1,017,500, and is now capped by the upper limit of the layer. The effective trend is +16.1%, which is *greater* than the unlimited trend of +10%.

When we examine the $1,000,000 excess of $1,000,000 layer, we find that the effective trend is now +23.3%.

If the upper and lower boundaries of a layer of losses are increased at the same rate as the loss trend used, then the trend would be fully realized. For example, consider the above example for the $250,000 excess of $250,000 layer, with the post-trend layer covering from $275,000 to $550,000. The post-trend total now becomes $1,089,000 --- exactly a 10% increase over the pre-trend total.

Note that this is an illustrative example, and not intended to necessarily reflect current severity trends. For a more detailed discussion of the effect of trend on excess layers, see Keatinge [1]. For an elegant visual representation of the leveraged effect of inflation, see Lee [4].

6. ISSUES WITH EMPIRICAL DATA

There are several issues involved in using empirical data to construct increased limits factor tables. First, as we have discussed above, is the issue of policy limit censorship. The size of the total underlying loss faced by the insured is not available if the loss is greater than the policy limit. Instead, the actuary will know only the size of the payment made by the insurer.

A second, related issue is that some reported data may be from policies written with a deductible, or may be from excess or umbrella policies. Because the deductible eliminates losses below the deductible, experience from policies written with a deductible is missing any information on the layers of loss below the deductible. Similarly, no information on losses below the attachment point of an excess or umbrella policy will be available.

Third, losses that occurred in the past cannot be used to model future claim size distributions without adjusting for trends in claim costs. For liability losses, trends may, of course, be due to general cost inflation. Other societal changes, such as changes in the patterns of jury awards, or legislative actions, may impose additional effects on liability losses. Also, when developing loss trends from empirical data, it is important to consider the credibility of the experience. The presence or absence of large losses can dramatically impact trend fits, particularly at higher layers of loss.

Fourth, recent claims may not be settled at the time of their evaluation, and hence only incomplete information about their true size will be known. If we wish to use information from recent losses we must adjust for the effect of loss development on loss size distributions. This is a more complex problem than it may appear initially. For basic limits ratemaking, a loss development factor is generally applied to a body of losses to project the future growth in this body of losses as the experience matures. Here, we are concerned with overall average growth in a body of losses *in aggregate.*

For increased limits analyses, we are concerned with the growth of a *distribution* of losses. Thus, in addition to average growth over time, we also need to address changes in the shape of the loss distribution. For example, a loss initially reserved at a 15-month evaluation for $50,000 may be settled for $50,000, $55,000, $100,000 or a much higher amount. Alternatively, the defense of the insured may be successful, and the loss may be reduced to $0, with only ALAE resulting for the claim. This issue is complicated by the fact that more serious potential losses will generally be more involved, and take longer to move through the stages of the claims process, due to later reporting, longer discovery and investigation, possibly lengthy settlement discussions, and a possible jury trial. All of these issues serve to make loss development a non-trivial exercise for increased limits analyses.

Finally, empirical data is subject to random fluctuations, especially in higher layers where data is sparse. If information on various loss layers is sparse, the presence or absence of larger losses in the experience period can result in estimates of ILFs that may be significantly too high or too low. Thus, it is advisable to use loss data from many accident years when conducting an analysis of increased limits factors.

7. CONTINUOUS DISTRIBUTIONS

A common approach used in the analysis of increased limits factors is to fit a continuous distribution to the empirical data. A fitted distribution serves to smooth random fluctuations in the empirical data, enabling appropriate ILFs to be determined for any desired limit.

The following continuous distributions have been commonly used in various analyses of increased limits factors:

Lognormal
Pareto
Truncated Pareto
Mixed Exponential

Various actuarial references, such as Klugman, Panjer, and Wilmot [3], offer a detailed discussion of many distributions. Several general comments are provided here.

The lognormal distribution is a positively skewed distribution. This is consistent with the general behavior of size of loss distributions, where there are many occurrences of relatively smaller size, and fewer occurrences of larger size. Yet, these less frequent occurrences of larger size represent a sizable proportion of the total dollars of loss in the distribution.

The Pareto distribution is also a positively skewed distribution. Notably, this distribution will generally have a heavier tail (i.e., higher probabilities of larger loss sizes) than the lognormal distribution.

A modification of the Pareto distribution, the truncated Pareto distribution has been commonly used for many years in analyses of increased limits factors. The truncated Pareto distribution employs a Pareto above a truncation point t. Losses below t are modeled by a bi-level uniform distribution, with parameters based on the empirical data. Note that the resulting increased limits factors are not affected by the behavior of the distribution below t, as t is always selected to be below the basic limit.

The probability density function, $f(x)$, and cumulative distribution function, $F(x)$, of this distribution are as follows:

$$f(x) = \frac{p(t-s)}{st} \qquad \text{for } 0 < x \le s$$

$$f(x) = \frac{ps}{t(t-s)} \qquad \text{for } s < x \le t$$

$$f(x) = \frac{(1-p)q(t+b)^q}{(x+b)^{q+1}} \qquad \text{for } x > t$$

$$F(x) = \frac{p(t-s)}{st}x \qquad \text{for } 0 < x \le s$$

$$F(x) = \frac{p(t-s)}{t} + \frac{ps(x-s)}{t(t-s)} \qquad \text{for } s < x \le t$$

$$F(x) = 1-(1-p)\left(\frac{(t+b)}{(x+b)}\right)^q \qquad \text{for } x > t$$

where x = size of the occurrence
t = truncation point
p = probability that the size of an occurrence is less than or equal to t
s = average size of occurrences that are less than or equal to t
b = Pareto distribution parameter that determines the scale of the curve
q = Pareto distribution parameter that determines the shape of the curve

The truncated Pareto distribution may be fit to the empirical data above the truncation point using the method of maximum likelihood, taking care to adjust the likelihood function for any censoring and truncation in the data. The truncation is used because adequate fits often cannot be obtained if the fit encompasses all sizes of loss down to zero. The parameters p and s are selected to match the empirical data.

The fitted parameters are then used to define the per-occurrence loss severity distribution. For a truncated Pareto distribution, the limited average severity at a policy limit k is:

$$ps + \frac{(1-p)}{(q-1)}\{(qt+b)-(k+b)[\frac{(t+b)}{(k+b)}]^q\}$$

The current ISO increased limits methodology uses the mixed exponential distribution. The mixed exponential distribution provides an excellent fit to many types of liability occurrence data over a wide range of loss sizes. The mixed exponential distribution is simple, and is more flexible than most other distributions. The details of the methodology are described in the following section.

8. THE ISO MIXED EXPONENTIAL METHODOLOGY

The methodology described here was designed by ISO to address the issues involved is working with empirical data and to provide a close fit to the underlying data. The steps of the methodology are:

1. Application of trend

2. Calculating empirical survival distributions by payment lag

3. Combining the empirical survival distributions from each payment lag to produce an overall empirical survival distribution

4. Smoothing the tail of the lag-weighted empirical survival distribution

5. Fitting a mixed exponential distribution to the lag-weighted empirical survival distribution

The methodology uses *only* paid (settled) occurrences. An occurrence is considered to be settled if it has no outstanding reserve. This has both an advantage and a disadvantage relative to a model that relies on incurred data. Incurred data includes information contained in the judgments made by claims adjusters in setting case reserves, and so incorporates the latest available information. However, this information may be subject to the potential distorting effects of changes in reserving practices. Conversely, paid data does not depend on claims reserve estimates, but also does not contain potentially valuable information from these reserves.

Application of Trend

The first step of the methodology is to bring all settled occurrences from the experience period to the same cost level by trending each occurrence to the midpoint of the period during which the resulting increased limits factors will be applied. In selecting a trend factor, an estimate of the *unlimited* trend is generally used.

Calculating Empirical Survival Distributions by Payment Lag

Empirical survival distributions are then constructed using a method based on the Kaplan-Meier product-limit estimator described in Klugman, Panjer, and Wilmot [3]. First, trended, settled occurrences are organized by payment lag. The payment lag is the length of time between when an accident occurs and when the associated indemnity is paid. For purposes of the model, the payment date is considered to be the dollar-weighted average of the dates of the indemnity payments. Payment lag is calculated based on the year in which an accident occurs and the year in which the occurrence is paid:

$$\text{Payment Lag} = (\text{Payment Year} - \text{Accident Year}) + 1$$

Payment lags can vary considerably by line of business and by type of claim. While most property claims are paid quickly, liability claims generally take longer to settle, particularly those involving protracted litigation. Among liability claims, there is considerable variation in payment lag.

Generally, occurrences with longer payment lags involve larger loss sizes. For example, the average loss size for occurrences paid in lag 4 will tend to be considerably larger than the average loss size for those paid in lag 1. Payment lags beyond a certain point (usually four to six years depending on the line of business) generally have similar loss sizes and are often combined to increase credibility. Lags before this are treated individually. The data used in an increased limits analysis often consists of settled occurrences with payments that fall in the most recent five calendar years. Thus, data from different lags generally includes different accident years, which likely have different exposure amounts.

Next, an empirical survival distribution is constructed for each payment lag using discrete loss size intervals. Typically, at least 50 intervals are used. The probability that an occurrence exceeds the upper bound of an interval given that it exceeds the lower bound of the interval is known as the conditional survival probability (CSP). The survival distribution is generated by multiplying successive CSPs together. This procedure allows for the inclusion of censored occurrences as well as occurrences with deductibles and occurrences from excess and umbrella policies. Two conditions must be met in order for a particular occurrence to be used in the calculation of the CSP in a particular interval. These conditions are:

- The policy limit (plus attachment point or deductible) must be greater than the upper bound of the interval. This avoids a downward severity bias by excluding loss occurrences that are precluded by their policy limit from penetrating the upper bound of the interval.

- The attachment point or deductible must be less than or equal to the lower bound of the interval. This avoids an upward severity bias because loss information below the attachment point or deductible is not known.

A simple example should aid in the conceptual understanding of this construction. Assume we have twelve occurrences, all from a single payment lag.

We will calculate the empirical survival distribution using combinations of conditional survival probabilities. The interval boundaries are \$10,000, \$20,000, and \$40,000. The following tables display the calculations:

Sample Data

Occurrence Number	Occurrence Size	Attachment Point	Policy Limit	Comment
1	5,000	0	15,000	
2	5,000	0	15,000	
3	15,000	0	15,000	Censored Data
4	5,000	7,500	15,000	Deductible Data
5	5,000	0	30,000	
6	15,000	0	30,000	
7	25,000	0	30,000	
8	10,000	15,000	30,000	Excess Data
9	15,000	0	100,000	
10	25,000	0	100,000	
11	30,000	0	100,000	
12	50,000	15,000	100,000	Excess Data

Summary of Conditions

	Condition:
CSP (10,000| 0) = P(X > 10,000| X > 0)	PL + AP > 10,000 AP = 0
CSP (20,000| 10,000) = P(X > 20,000| X > 10,000)	PL + AP > 20,000 AP ≤ 10,000
CSP (40,000| 20,000) P(X > 40,000| X > 20,000)	PL + AP > 40,000 AP ≤ 20,000
Where AP = Attachment Point, PL = Policy Limit, X= Occurrence Size	

Calculation of Conditional Survival Probability at \$10,000

$CSP\ (10{,}000 \mid 0) = P(X > 10{,}000 \mid X > 0) =$

$$\frac{\text{Number of Occurrences with: } X + AP > 10{,}000,\ PL + AP > 10{,}000, \text{ and } AP = 0}{\text{Number of Occurrences with: } X + AP > 0,\ PL + AP > 10{,}000, \text{ and } AP = 0}$$

$$= \frac{6 \text{ (occurrences 3, 6, 7, 9, 10, 11)}}{9 \text{ (occurrences 1, 2, 3, 5, 6, 7, 9, 10, 11)}}$$

Only occurrences with policy limit plus attachment point greater than 10,000 are used.
Only occurrences with attachment point equal to zero are used.

Calculation of Conditional Survival Probability at \$20,000

$CSP\ (20{,}000 \mid 10{,}000) = P(X > 20{,}000 \mid X > 10{,}000) =$

$$\frac{\text{Number of Occurrences with: } X + AP > 20{,}000,\ PL + AP > 20{,}000, \text{ and } AP \le 10{,}000}{\text{Number of Occurrences with: } X + AP > 10{,}000,\ PL + AP > 20{,}000, \text{ and } AP \le 10{,}000}$$

$$= \frac{3 \text{ (occurrences 7, 10, 11)}}{6 \text{ (occurrences 4, 6, 7, 9, 10, 11)}}$$

Only occurrences with policy limit plus attachment point greater than 20,000 are used.
Only occurrences with attachment point less than or equal to 10,000 are used.

Calculation of Conditional Survival Probability at \$40,000

$CSP\ (40{,}000 \mid 20{,}000) = P(X > 40{,}000 \mid X > 20{,}000) =$

$$\frac{\text{Number of Occurrences with: } X + AP > 40{,}000,\ PL + AP > 40{,}000, \text{ and } AP \le 20{,}000}{\text{Number of Occurrences with: } X + AP > 20{,}000,\ PL + AP > 40{,}000, \text{ and } AP \le 20{,}000}$$

$$= \frac{1 \text{ (occurrence 12)}}{4 \text{ (occurrences 8, 10, 11, 12)}}$$

Only occurrences with policy limit plus attachment point greater than 40,000 are used.
Only occurrences with attachment point less than or equal to 20,000 are used.

Calculation of Empirical Survival Distribution

The CSPs generate the following empirical survival probabilities:

S (10,000) = 6/9

S (20,000) = 6/9 × 3/6 = 1/3

S (40,000) = 6/9 × 3/6 × 1/4 = 1/12

Combining the Empirical Survival Distributions

We now estimate a set of payment lag parameters that produces weights representing the proportion of occurrences paid in each lag. We will use these to weight together the empirical survival distributions for each payment lag to produce an overall empirical survival distribution.

The payment lag model uses three parameters (R1, R2, R3) to generate the weights given to the empirical survival distribution associated with each payment lag. The parameters are defined as follows:

$$R1 = \frac{\text{expected percentage of occurrences paid in lag 2}}{\text{expected percentage of occurrences paid in lag 1}}$$

$$R2 = \frac{\text{expected percentage of occurrences paid in lag 3}}{\text{expected percentage of occurrences paid in lag 2}}$$

$$R3 = \frac{\text{expected percentage of occurrences paid in lag (n + 1)}}{\text{expected percentage of occurrences paid in lag (n)}}, \; n \geq 3$$

For this example, we assume that occurrence size data from payment lags beyond 5 has been combined with that from lag 5.

The weights for each lag are then determined as follows:

Lag 1 weight =		$1 / k$
Lag 2 weight =		$R1 / k$
Lag 3 weight =		$R1\times R2 / k$
Lag 4 weight =		$R1\times R2\times R3 / k$
Lag 5 weight =		$[R1\times R2\times R3^2 / (1-R3)] / k$
	where	$k = 1 + R1 + R1\times R2 / (1-R3)$

Note that the lag 5 weight includes lag 5 and all subsequent lags. In these equations, k is simply a normalizing factor, which ensures that the weights will balance to unity. The equation for the Lag 5 weight is based on the formula for the sum of an infinite geometric series $[a/(1-r)]$.

The payment lag weights represent the percentage of ground-up occurrences in each lag. Therefore, occurrences from policies with deductibles and occurrences from excess and umbrella policies are not included. The payment lag parameters (R1, R2, and R3) are estimated via maximum likelihood. It would be possible to include additional R parameters in the model, but three parameters are generally sufficient to provide a good fit to the data.

The following results were obtained for one recent increased limits analysis:

$$\begin{aligned} R1 &= 0.2551 \\ R2 &= 0.1798 \\ R3 &= 0.4107 \\ k = 1 + R1 + R1\times R2 / (1-R3) &= 1.3329 \end{aligned}$$

Leading to:

$$\begin{aligned} \text{Lag 1 weight} &= 1/k = 0.7502 \\ \text{Lag 2 weight} &= R1/k = 0.1914 \\ \text{Lag 3 weight} &= R1\times R2/k = 0.0344 \\ \text{Lag 4 weight} &= R1\times R2\times R3/k = 0.0141 \\ \text{Lag 5 weight} &= [R1\times R2\times R3^2/(1-R3)]/k = 0.0098 \end{aligned}$$

Smoothing the Tail

At large occurrence sizes, the empirical data is sparse. To limit random fluctuations between consecutive analyses at the larger occurrence sizes, a procedure is used to smooth the tail of the lag-weighted empirical survival distribution. A truncation point is selected above which the empirical survival distribution is not sufficiently stable (generally between $600,000 and $1,500,000, depending upon the nature of the distribution). Then a parametric distribution (very often a truncated Pareto) is selected to model the empirical survival distribution from the truncation point through the larger occurrence sizes. Percentile matching is used to determine the parameters, and the resulting curve is used to replace the empirical survival distribution above the truncation point. The empirical survival distribution below the truncation point is unaffected by this procedure.

Fitting a Mixed Exponential Distribution

The final step is to fit a mixed exponential distribution to the lag-weighted empirical survival distribution with smoothed tail.

The mixed exponential distribution is a weighted average of exponential distributions. The probability density function and cumulative distribution function of the mixed exponential distribution are as follows:

$$f(x) = \sum_i w_i (1/\mu_i) e^{(-x/\mu_i)}, \sum w_i = 1$$

$$F(x) = 1 - \sum_i w_i e^{(-x/\mu_i)}, \sum w_i = 1$$

The number of exponential distributions needed to produce an optimal fit may vary in different analyses and is allowed to be as large as necessary. In most practical situations, the optimal distribution can be found using less than ten distributions in the mixture. For a detailed discussion of the mixed exponential distribution, see Keatinge [2].

We use the fitted mixed exponential distribution to define the per-occurrence loss severity distribution. The limited average severity at policy limit *k* is:

$$LAS(k) = \sum_i w_i \mu_i [1 - e^{(-k/\mu_i)}], \sum w_i = 1$$

9. OTHER COMPONENTS OF INCREASED LIMITS FACTORS

As noted earlier, an increased limit factor may be generally calculated as:

$$\text{ILF(L)} = \frac{\text{LAS(L) + ALAE(L) + ULAE(L) + RL(L)}}{\text{LAS(B) + ALAE(B) + ULAE(B) + RL(B)}}$$

While the majority of this paper focuses on the evaluation of the limited average severity, this section offers several comments on the other components of ILFs.

Loss Adjustment Expense

Loss adjustment expenses are expenses incurred to settle claims. Traditionally they have been classified as allocated loss adjustment expense (ALAE) and unallocated loss adjustment expense (ULAE). ALAE consists of loss adjustment expenses that can be identified with and allocated to a specific loss. For liability coverage, ALAE is largely comprised of legal defense expenses. ULAE consists of all other loss adjustment expenses. These include salaries for in-house claims adjusters and other "overhead" loss adjustment expenses.

Generally, liability policies provide for *unlimited* legal defense of the insured, provided by the insurer, and regardless of the policy limit purchased. This defense is provided *in addition to* any indemnity amounts. At times, a successful defense of a policyholder may result in a paid indemnity loss of zero dollars, yet a certain amount of ALAE will be incurred for that claim. Consistent with this duty to defend, an assumption is often made that the same ALAE provision should be included at all policy limits.

Under this assumption ALAE(L) does not vary with L, the policy limit, and is a constant expected dollar provision included in the loss cost for each limit. Under this assumption increases in ALAE will lead to increases in the basic limit loss cost, and to the loss cost at every higher limit. However, as the same amount is included in both the numerator and denominator of the ILF calculation, an increase in the ALAE provision will actually lead to a *decrease* in increased limits factors.

Fully developed historical experience is generally used to calculate an average ratio of ALAE to indemnity loss, which is then applied to the average limited average severity (averaged over an historical policy limit distribution) to produce an ALAE estimate. This ALAE provision is then included in the ILF calculation.

Allocated loss adjustment expense can be a very substantial part of the underlying cost of liability insurance. This is especially true for some commercial lines, such as General Liability and Medical Professional Liability. In some jurisdictions, the ratio of ALAE to indemnity losses for these lines is greater than fifty percent.

In some specialized lines of business, ALAE may be included *within* the policy limit. This special policy provision is generally considered in situations where legal defense costs may be large. For example, this policy provision is often included in Directors & Officers Liability insurance and Lawyers Professional Liability insurance.

At times, an analysis of historical ALAE in relation to various policy limits may be conducted, in order to develop a more refined treatment of these expenses. For example, when a reinsurer and a ceding company are developing a provision for sharing the cost of ALAE, a detailed estimate of expected ALAE for the reinsurance contract may be prepared.

As noted earlier, unallocated loss adjustment expense (ULAE) reflects the general overhead expenses of the claim settlement process. A provision for ULAE is generally applied as a percentage loading to expected indemnity losses and ALAE. This same percentage is generally used for all policy limits.

Risk Load

A primary function of insurance is the reduction of the potential for catastrophic financial loss to the insured. The insured pays the insurer a premium to assume a portion of the potential financial consequences of covered events that may or may not occur. The insurer attempts to limit its aggregate exposure to the risks that it assumes through diversification of these risks. Providing insurance at high limits tends to result in less diversification and greater expected variation in the insurer's financial results. For example, the sum of the expected losses of two $100,000 limit policies may equal the expected loss of one $1,000,000 policy, but the variation in losses of the single $1,000,000 policy is likely to be much higher than for the two smaller policies. One way of compensating the insurer for this is by reflecting a cost for risk in the increased limits factors that increases with policy limit.

Risk load is often viewed as consisting of two types, process risk load and parameter risk load. Process risk is customarily defined as the inherent variability of the insurance process, reflected in the difference between actual losses and expected losses. Parameter risk refers to the inherent variability of the estimation process, reflected in the difference between the theoretical (true but unknown) expected losses and the estimated expected losses. Charges for both of these types of risk vary by limit.

There are many measures of variation and risk and new ones continue to be developed. As a result the actuarial literature is filled with discussion about this subject.

10. SOME PRACTICAL CONSIDERATIONS

Before concluding this discussion, it is worth emphasizing several practical issues involved with increased limits ratemaking. While some detailed mathematical techniques have been presented, it is always necessary to keep in mind the need to closely review and understand the underlying data with which you are working.

All data will present unique characteristics, which must be thought about and addressed. In this paper, we have previously discussed the effects of policy limits, deductibles, trend, and loss development. Yet, there may be other, less obvious, underlying features of the data which may not be immediately apparent. There is always a need to review and examine the underlying loss experience from several perspectives. These include: 1) Does the data appear to be accurately reported? 2) Does the data appear to be reasonable? 3) Are there any external factors or developments that need to be considered?

Two examples may clarify the need to remain alert to the characteristics of the underlying data.

1) Assume that you are analyzing loss experience obtained from a book of Medical Professional Liability policies in State X. You note that after two years in which the number of claims averaged about 100 per month, there is a sudden rise to 300 claims in the month of June in the latest year of experience. From July to December, the number of claims drops again, to about 75 per month.

Initially you are puzzled. As you examine these 300 claims, you see no particularly unusual patterns in the data. In all respects, they appear to be like any other claims reported that year.

After some discussions with your colleagues, you learn that, in State X, there has been recent legislation passed which limits awards for non-economic damages in Medical Professional Liability cases to $250,000. You also learn that this law goes into effect for all suits brought after July 1st of the latest year for which you have data. It does appear that many claims were filed in June ahead of the effective date of the cap on non-economic damages.

In our hypothetical example, the data is exhibiting unusual patterns because of a change in the underlying legal and social environment in State X. You now are faced with a host of questions. What will be the effect of this law change? How will my prospective loss experience differ from that of the past? How do I need to adjust my past experience to help with an analysis of future claim costs? What if the "tort reform" legislation is ultimately overturned by the courts in the state?

2) As a second example, assume you are developing increased limits factors for Commercial Automobile Liability. These factors will be developed on a combined single limits (CSL) basis --- that is, a single limit will apply to *both* bodily injury liability (BI) and property damage liability (PD) claims arising from a single occurrence. However, the claims data reported to you separately identifies BI and PD claims. As you analyze the experience, you will need to be aware of the fact that you are actually dealing with two different types of claims. PD claims tend to be relatively quickly settled, as they generally involve damage to another driver's vehicle. The cost of the claim is relatively easy for all parties to agree on. However, BI claims may involve serious injuries to claimants, which may result in lawsuits with high damage awards or settlements.

During the 1990s, numerous safety features became standard equipment on vehicles. Air bags, in particular, became required standard equipment on cars during this period. The introduction of air bags had very different effects on BI claims, as contrasted with PD claims. Air bags had the effect of reducing or eliminating many BI claims. Simultaneously, the cost of "recharging" or replacing the air bag on damaged vehicles added an often significant cost to the PD claims in which the airbag fired. So, the potential exists for data from this period to exhibit changes in the shape of the combined BI and PD loss distribution over time, and care needs to be taken when working with the data. However, somewhat softening this change is the fact that it takes years for a new safety feature that becomes standard equipment to work its way fully into the population of vehicles being driven --- as older vehicles are replaced with new vehicles.

As these examples illustrate, it is always important to review and understand the underlying data in any increased limits factor analysis --- particularly with regard to any changes that might influence future cost projections.

11. CONCLUSION

Techniques for developing appropriate charges for higher limits of liability coverage have been evolving and developing over the years. This paper has presented some of the issues frequently encountered when conducting an analysis of increased limits factors. These techniques will be refined, and additional techniques will be developed, as exposures to potential liability losses continue to grow and evolve.

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