CHAPTER 7

REINSURANCE

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INTRODUCTION

What is Reinsurance?

Reinsurance is a form of insurance. A reinsurance contract is legally an insurance contract. The *reinsurer* agrees to indemnify the *cedant* insurer for a specified share of specified types of insurance claims paid by the cedant for a single insurance policy or for a specified set of policies. The terminology used is that the reinsurer *assumes* the liability *ceded* on the *subject policies*. The *cession*, or share of claims to be paid by the reinsurer, may be defined on a *proportional share* basis (a specified percentage of each claim) or on an *excess basis* (the part of each claim, or aggregation of claims, above some specified dollar amount).

The nature and purpose of insurance is to reduce the financial cost to individuals, corporations, and other entities arising from the potential occurrence of specified contingent events. An insurance company sells insurance policies guarantying that the insurer will indemnify the policyholders for part of the financial losses stemming from these contingent events. The pooling of liabilities by the insurer makes the total losses more predictable than is the case for each individual insured, thereby reducing the risk relative to the whole. Insurance enables individuals, corporations and other entities to perform riskier operations. This increases innovation, competition, and efficiency in a capitalistic marketplace.

The nature and purpose of reinsurance is to reduce the financial cost to insurance companies arising from the potential occurrence of specified insurance claims, thus further enhancing innovation, competition, and efficiency in the marketplace. The cession of shares of liability spreads risk further throughout the insurance system. Just as an individual or company purchases an insurance policy from an insurer, an insurance company may purchase fairly comprehensive reinsurance from one or more reinsurers. A reinsurer may also reduce its assumed reinsurance risk by purchasing reinsurance coverage from other reinsurers, both domestic and international; such a cession is called a *retrocession*.

Reinsurance companies are of two basic types: *direct writers*, which have their own employed account executives who produce business, and *broker companies* or *brokers*, which receive business through *reinsurance intermediaries*. Some direct writers do receive a part of their business through brokers, and likewise, some broker reinsurers assume some business directly from the ceding companies. It is estimated that more than half of U.S. reinsurance is placed via intermediaries.

The form and wording of reinsurance contracts are not as closely regulated as are insurance contracts, and there is no rate regulation of reinsurance between private companies. A reinsurance contract is often a manuscript contract setting forth the unique agreement between the two parties. Because of the many special cases and exceptions, it is difficult to make correct generalizations about reinsurance. Consequently, as you read this chapter, you should often supply for yourself the phrases "It is generally true that..." and "Usually..." whenever they are not explicitly stated.

This heterogeneity of contract wordings also means that whenever you are accumulating, analyzing, and comparing various reinsurance data, you must be careful that the reinsurance coverages producing the data are reasonably similar. We will be encountering this problem throughout this chapter.

The Functions of Reinsurance

Reinsurance does not change the basic nature of an insurance coverage. On a long-term basis, it cannot be expected to make bad business good. But it does provide the following direct assistance to the cedant.

Capacity

Having reinsurance coverage, a cedant can write higher policy limits while maintaining a manageable risk level. By ceding shares of all policies or just larger policies, the net retained loss exposure per individual policy or in total can be kept in line with the cedant's surplus. Thus smaller insurers can compete with larger insurers, and policies beyond the capacity of any single insurer can be written.

The word "capacity" is sometimes also used in relation to aggregate volume of business. This aspect of capacity is best considered below in the general category of financial results management.

Stabilization

Reinsurance can help stabilize the cedant's underwriting and financial results over time and help protect the cedant's surplus against shocks from large, unpredictable losses. Reinsurance is usually written so that the cedant retains the smaller, predictable claims, but shares the larger, infrequent claims. It can also be written to provide protection against a larger than predicted accumulation of claims, either from one catastrophic event or from many. Thus the underwriting and financial effects of large claims or large accumulations of claims can be spread out over many years. This decreases the cedant's probability of financial ruin.

Financial Results Management

Reinsurance can alter the timing of income, enhance statutory and/or GAAP surplus, and improve various financial ratios by which insurers are judged. An insurance company with a growing book of business whose growth is stressing their surplus can cede part of their liability to a reinsurer to make use of the reinsurer's surplus. This is essentially a loan of surplus from the reinsurer to the cedant until the cedant's surplus is large enough to support the new business. We will see other ways that reinsurance can be used to alter a cedant's financial numbers. As you might expect in a free market, this aspect of reinsurance has led to some abuses in its use. As we discuss the various forms of reinsurance coverage, we will note their financial effects.

Management Advice

Many professional reinsurers have the knowledge and ability to provide an informal consulting service for their cedants. This service can include advice and assistance on underwriting, marketing, pricing, loss prevention, claims handling, reserving, actuarial, investment, and personnel issues. Enlightened self-interest induces the reinsurer to critically review the cedant's operation, and thus be in a position to offer advice. The reinsurer typically has more experience in the pricing of high limits policies and in the handling of large and rare claims. Also, through contact with many similar cedant companies, the reinsurer may be able to provide an overview of general issues and trends. Reinsurance intermediaries may also provide some of these same services for their clients.

The Forms of Reinsurance

Facultative Certificates

A facultative certificate reinsures just one primary policy. Its main function is to provide additional capacity. It is used to cover part of specified large, especially hazardous or unusual exposures to limit their potential impact upon the cedant's net results or to protect the cedant's ongoing ceded treaty results in order to keep treaty costs down. The reinsurer underwrites and accepts each certificate individually; the situation is very similar to primary insurance individual risk underwriting. Because facultative reinsurance usually covers the more hazardous or unusual exposures, the reinsurer must be aware of the potential for antiselection within and among classes of insureds.

Property certificate coverage is sometimes written on a proportional basis; the reinsurer reimburses a fixed percentage of each claim on the subject policy. Most casualty certificate coverage is written on an excess basis; the reinsurer reimburses a share (up to some specified dollar limit) of the part of each claim on the subject policy that lies above some fixed dollar attachment point (net retention).

Facultative Automatic Agreements or Programs

A facultative automatic agreement reinsures many primary policies of a specified type. These policies are usually very similar, so the exposure is very homogeneous. Its main function is to provide additional capacity, but since it covers many policies, it also provides some degree of stabilization. It may be thought of as a collection of facultative certificates underwritten simultaneously. It may cover on either a proportional or excess basis. It is usually written to cover new or special programs marketed by the cedant, and the reinsurer may work closely with the cedant to design the primary underwriting and pricing guidelines. For example, a facultative automatic agreement may cover a 90% share of the cedant's personal umbrella business, in which case the reinsurer will almost certainly provide expert advice and will monitor the cedant's underwriting and pricing very closely.

Facultative automatic agreements are usually written on a fixed cost basis, without the retrospective premium adjustments or variable ceding commissions sometimes used for treaties (as we shall see below).

There are also *non-obligatory* agreements where either the cedant may not be required to cede or the reinsurer may not be required to assume every single policy of the specified type.

Treaties

A treaty reinsures a specified part of the loss exposure for a set of insurance policies for a specified coverage period. For ongoing treaty coverage, the claims covered may be either those occurring during the treaty term or those occurring on policies written during the term. In the case of claims-made coverage, the word "occurring" means those claims made to the ceding company during the term. The premium *subject* to the treaty corresponds to the types of claims covered: it is earned premium arising from policies of the specified type either in force or written during the term of the treaty. The subject exposure is usually defined by Annual Statement line of business or some variant or subsets thereof. Because an ongoing treaty relationship involves a close sharing of much of the insurance exposure, it can create a close working partnership between the parties; the expertise and services of the reinsurer or broker are available to the cedant. This is especially true for treaties written by a direct writer or where there is a strong reinsurer leading a brokered treaty.

Treaty Proportional Covers

A *quota-share* treaty reinsures a fixed percentage of each subject policy. Its main function is financial results management, although it also provides some capacity. The reinsurer usually receives the same share of premium as claims, and pays the cedant a *ceding commission* commensurate with the primary production and handling costs (underwriting, claims, etc.). Quota-share treaties usually assume in-force exposure at inception. The cedant's financial results are managed because the ceding commission on the ceded unearned premium reserve transfers statutory surplus from the reinsurer to the cedant. (We shall see this later.) The cession of premium also reduces the cedant's net-premium-to-surplus ratio.

The ceding commission on quota-share treaties is often defined to vary within some range inversely to the loss ratio. This allows the cedant to retain better-than-expected profits, but protects the reinsurer somewhat from adverse claims experience.

The term quota-share is sometimes (mis-)used when the coverage is a percentage share of an excess layer; we will more properly treat this kind of coverage as being excess.

A *surplus-share* treaty also reinsures a fixed percentage of each subject policy, but the percentage varies by policy according to the relationship between the policy limit and the treaty's specified *net line* retention. Its main function is capacity, but it also provides some stabilization. A surplus-share treaty may also assume in-force exposure at inception, which together with a ceding commission provides some management of financial results. This is typically a property cover; it is rarely used for casualty business.

Treaty Excess Covers

An excess treaty reinsures, up to a limit, a share of the part of each claim that is in excess of some specified *attachment point* (cedant's *retention*). Its main functions are capacity and stabilization. An excess treaty typically covers exposure earned during its term on either a losses-occurring or claims-made basis, but run-off exposure may be added in. The definition of "subject loss" is important.

For a *per-risk excess* treaty, a subject loss is defined to be the sum of all claims arising from one covered loss event or occurrence for a single subject policy. Per-risk excess is mainly used for property exposures. It often provides protection net of facultative coverage, and sometimes also net of proportional treaties. It is used for casualty less often than per-occurrence coverage.

For a *per-occurrence excess* treaty, a subject loss is defined to be the sum of all claims arising from one covered loss event or occurrence for all subject policies. Per-occurrence excess is used for casualty exposures to provide protection all the way up from working cover layers through clash layers.

A *working cover* excess treaty reinsures an excess layer for which claims activity is expected each year. The significant expected claims frequency creates some stability of the aggregate reinsured loss. So working covers are often *retrospectively rated*, with the final reinsurance premium partially determined by the treaty's loss experience.

A *higher exposed layer* excess treaty attaches above the working cover(s), but within policy limits. Thus there is direct single-policy exposure to the treaty.

A *clash treaty* is a casualty treaty that attaches above all policy limits. Thus it may be only exposed by:

1. extra-contractual-obligations (i.e., bad faith claims)
2. excess-of-policy-limit damages (an obligation on the part of the insurer to cover losses above an insurance contract's stated policy limit)
3. catastrophic workers compensation accidents
4. the "clash" of claims arising from one or more loss events involving multiple coverages or policies.

Both higher exposed layers and clash are almost always priced on a fixed cost basis, with no variable commission or additional premium provision.

Catastrophe Covers

A catastrophe cover is a per-occurrence treaty used for property exposure. It is used to protect the net position of the cedant against the accumulation of claims arising from one or more large events. It is usually stipulated that two or more insureds must be involved before coverage attaches. The coverage is typically of the form of a 90% or 95% share of one or more layers (separate treaties) in excess of the maximum retention within which the cedant can comfortably absorb a loss, or for which the cedant can afford the reinsurance prices.

Aggregate Excess, or Stop Loss Covers

For an *aggregate excess* treaty, also sometimes called a *stop loss* cover, a loss is the accumulation of all subject losses during a specified time period, usually one year. It usually covers all or part of the net retention of the cedant and protects net results, providing very strong stabilization. Claims arising from natural catastrophes are often excluded, or there may be a per-occurrence maximum limit.

Finite, or Nontraditional, Reinsurance Covers

Over the past few years, there has been a growing use of reinsurance, especially treaties, whose only or main function is to manage financial results. The word "finite" means that the reinsurer's assumed risk is significantly reduced by various contractual conditions, sometimes called "structure." Of course, the reinsurer's expected margin (expense and profit) is also reduced to reflect this reduced risk transfer. Sometimes these covers are structured to induce a favorable tax treatment for the cedant. Often they are based on the ability of offshore reinsurers to book claims on a discounted basis in anticipation of the future investment income that will be earned from the premium income received before the claims are settled. The reinsurance premium reflects this discounting, thus giving the cedant a statutory and GAAP accounting benefit.

There have been cases where the risk transfer was nonexistent or negligible. In order to stop accounting abuses through reinsurance, the Financial Accounting Standards Board issued FAS 113 in 1992. FAS 113 requires a measurable and significant transfer of risk before a contract can receive the benefit of reinsurance accounting. Although the standard is somewhat ambiguous, it has largely stopped abusive reinsurance practices.

There continues to be debate in the reinsurance community about a workable distinction between the categories: traditional and finite reinsurance. Other than the rather ambiguous FAS 113, there is no clear boundary between traditional reinsurance and finite reinsurance; there is a continuum of risk transfer possibility between full risk transfer and no transfer. Virtually any reinsurance contract can be structured in a way to reduce the risk transfer and become "finite." We shall see this in the following discussion of typical forms for finite reinsurance. Throughout this chapter, we assume that any reinsurance contract under discussion has sufficient risk transfer to pass FAS 113 requirements.

The first typical form for finite reinsurance is a *financial proportional cover*. As noted above, proportional treaties quite often have a ceding commission that varies inversely with the losses; this limits the risk transfer. The degree of variation can be increased to further limit the risk transfer. Also, the loss share may be defined to decrease somewhat if the losses exceed some maximum. Quite often, these treaties may also have some kind of funding mechanism, wherein the aggregate limit of coverage is based upon the fund (net cash position less the reinsurer's margin) together with some remote risk layer. Whatever the risk-limiting structure, the contract must be checked with the cedant's accountants to assure that they will approve the risk transfer for FAS 113 guidelines.

A *loss portfolio transfer* is also a very prevalent form for finite reinsurance. This is a retrospective cover, a cession of part of the cedant's loss liabilities as of a specified accounting date. It may be a cession of the total liability or, more often, a cession of some aggregate excess layer of the liability. An aggregate excess cover attaching at the cedant's carried loss reserve is often called an *adverse development cover*. It is clear that a loss portfolio transfer could be a pure risk non-finite cover. To make the risk transfer finite, it has an aggregate limit and may have sublimits for various types of claims, and it is priced to be essentially a present-value funding of liabilities with a substantial upfront provisional margin for the reinsurer. Part of this margin will be paid back to the cedant in the form of a profit commission if the loss experience is favorable.

A *funded aggregate excess cover* is, as you might expect, an aggregate excess treaty in which the premium is high enough to fund the loss payments except in extraordinary circumstances. It is analogous to a funded loss portfolio transfer except that it covers future occurring claims. In addition to financial results management, it may provide strong stabilization of the cedant's net results.

A Typical Reinsurance Program

There is no such thing as a typical reinsurance program. Every insurance company is in a unique situation with regard to loss exposure, financial solidity, management culture, future plans, and marketing opportunities. Thus each company needs a unique reinsurance program, a combination of ceded reinsurance covers tailor-made for that company.

Nevertheless, Table 7.1 displays what we might regard as a "typical" reinsurance program for a medium-sized insurance company.

If the company writes surety, fidelity, marine, medical malpractice, or other special business, other similar reinsurance covers would be purchased. If the company were entering a new market (e.g., a new territory or a new type of business), it might purchase a quota-share treaty to lessen the risk of the new business and the financial impact of the new premium volume, and to obtain the reinsurer's assistance. Or it might purchase a proportional facultative automatic agreement for an even closer working relationship with a reinsurer. If the company were exiting a market, it might purchase a loss portfolio transfer, especially an adverse development cover, to cover part of the run-off claims payments.

The Cost of Reinsurance to the Cedant

The Reinsurer's Margin

In pricing a reinsurance cover, the reinsurer charges a margin over and above the ceded loss expectation, commission, and brokerage fee (if any). The margin is usually stated as a percentage of the reinsurance premium. It is theoretically based upon the reinsurer's expenses, the degree of risk transfer, and the magni-

TABLE 7.1

A REINSURANCE PROGRAM
FOR A MEDIUM-SIZED INSURANCE COMPANY

Line of Business	Type of Reinsurance
Fire and Allied Lines HO Section I SMP Section II	1. Proportional and excess facultative certificates to bring each individual policy's net exposure down to $2M 2. Surplus share of four lines not to exceed $1.6M; maximum cedant retention of $400,000 3. Per-risk excess working cover of $200k excess of $200k 4. Catastrophe covers: 4.1. 95% of $5M excess of $5M 4.2. 95% of $10M excess of $10M 4.3. 95% of $10M excess of $20M 4.4. 95% of $10M excess of $30M
Casualty Lines excluding Umbrella	1. Facultative certificates for primary per policy coverage excess of $2M 2. Working cover $500k excess of $500k 3. Higher exposed layer $1M excess of $1M 4. Clash layers: 4.1. $3M excess of $2M 4.2. $5M excess of $5M 4.3. $10M excess of $10M
Personal Umbrellas	1. 90% share facultative automatic program

tude of capacity and financial support, but it is practically influenced by competition in the reinsurance market. Of course, as with most insurance, the actual resulting margin will differ from that anticipated because of the stochasticity of the loss liability and cash flow transferred.

Brokerage Fee

A reinsurance broker charges a brokerage fee for placing the reinsurance coverage and for any other services performed on behalf of the cedant. This fee is incorporated into the reinsurance premium and is paid by the reinsurer. Offsetting this cost is the fact that broker reinsurers usually have lower internal expenses because they don't maintain separate marketing staffs. The brokerage fee is usually a fixed percentage of the reinsurance premium, but on occasion may be defined as either a fixed dollar or as some other variable amount.

Lost Investment Income

For most reinsurance contracts, the premium funds (net of ceding commission) are paid to the broker, if any, who then passes them on (also net of brokerage fee) to the reinsurer. The cedant thus loses the use of those funds, and the reinsurer gains the investment income earned on those funds until returned as loss payments, ceding commission adjustments or other premium adjustments. The price of the reinsurance cover accounts for this investment income.

Some contracts incorporate a *funds withheld* provision, where the cedant pays only a specified margin to the reinsurer, from which the broker, if any, deducts the brokerage fee. The remaining reinsurance premium is "withheld" by the cedant. The cedant then pays reinsurance losses out of the funds withheld until they are exhausted, at which time payments are made directly by the reinsurer. The reinsurance contract may define a mechanism for crediting investment income to the funds withheld. The reinsurer will want a higher profit margin for a funds withheld contract because of the added risk (the credit worthiness of the cedant) and the lost investment income.

Additional Cedant Expenses

The cedant incurs various expenses for ceding reinsurance. These include the cost of negotiation, the cost of a financial analysis of the reinsurer, accounting, and reporting costs, etc. If a broker is involved, the brokerage fee covers some of these services to the cedant. In general, facultative coverage is more expensive than treaty because of individual policy negotiation, accounting, and loss cessions.

Reciprocity

In some cases, in order to cede reinsurance, the cedant may be required to assume some reinsurance from the reinsurer, in this case usually another primary company. If this reciprocal reinsurance assumption is unprofitable, the loss should be considered as part of the cost of reinsurance. Reciprocity is not prevalent in the United States.

Balancing Costs and Benefits

In balancing the costs and benefits of a reinsurance cover or of a whole reinsurance program, the cedant should consider more than just the direct costs versus the benefits of the loss coverage and reinsurance functions discussed previously. A major consideration should be the reinsurer's financial solidity—Will the reinsurer be able to quickly pay claims arising from a natural catastrophe? Will the reinsurer be around to pay late-settled claims many years from now? Also important is the reinsurer's reputation: does the reinsurer pay reasonably presented claims in a reasonable time? Another consideration may be the reinsurer's or broker's services, including possibly underwriting, marketing, pricing, loss prevention, claims handling, reserving, actuarial, investment, and personnel advice and assistance.

Reinsurance Introduction: Final Comments

This introduction is only a brief review of basic reinsurance concepts and terminology. The interested reader will find more extensive discussions in the general reinsurance texts listed in the references.

REINSURANCE PRICING

General Considerations

In general, reinsurance pricing is more uncertain than primary pricing. Coverage terms can be highly individualized, especially for treaties. These terms determine the coverage period, definition of "loss," commission arrangements, premium and loss payment timing, etc. It is often difficult and sometimes impossible to get credible loss experience directly pertaining to the cover being evaluated. Often the claims and exposure data are not as they first seem. So you must continually ask questions in order to discover their true nature. Because of these problems of coverage definition and interpretation of loss and exposure statistics, the degree of risk relative to premium volume is usually much greater for reinsurance than for primary insurance.

Additional risk arises from the low claim frequency and high severity nature of many reinsurance coverages, from the lengthy time delays between the occurrence, reporting, and settlement of many covered loss events, and also from the leveraged effect of inflation upon excess claims. In general, the lower the expected claims frequency, the higher the variance of results relative to expectation, and thus the higher the risk level. In addition, *IBNR* claims emergence and case reserve development are severe problems for casualty excess business. Claims development beyond ten years can be large, highly variant, and extremely difficult to evaluate, as we shall discuss in the Loss Reserving section. Because of this long tail and extreme variability of loss payments, the matching of assets with liabilities is more difficult. Future predictability is also decreased by the greater uncertainty about claims severity inflation above excess cover attachment points. All these elements create a situation where the variance (and higher moments) of the loss process and its estimation are much more important relative to the expected value than is usually the case for primary coverage. For some reinsurance covers, the higher moments (or at least the underwriter/actuary's beliefs regarding uncertainty and fluctuation potential) determine the technical rate.

Reinsurance Pricing Methods

There are many ways to price reinsurance covers. For any given situation, there is no single right way. In this section, we will discuss a few actuarially sound methods. In general, the exposition of pricing methods will begin simply and become more complex as the situation demands and as we ask more questions. In many real situations, you might want to get a quick first evaluation via the simplest methods. Indeed, if you judge the situation to be either fairly predictable by these methods, or if you judge the risk to be small, you may decide to stop there. If not, you may want to pursue your analysis and pricing along the lines presented here. As in most actuarial work, you should try as many reasonable methods as time permits (and also reconcile the answers, if possible).

In this spirit, please note that the simple flat rate pricing formula and the use of the Pareto and Gamma distributions in this chapter are for illustrative purposes. The pricing formula a reinsurance actuary would use depends upon the reinsurer's pricing philosophy, information availability, and complexity of the coverage. The probability models should be selected to describe the actual situation as best as possible given all the real statistical and analytical cost constraints preventing you from obtaining more information. Klugman, Panjer, and Willmot [12], Hogg and Klugman [11], and Patrik [18] all discuss model selection and parameter estimation.

A Flat Rate Reinsurance Pricing Formula

As we get into the formulas, there will be a lot of notation. For clarity, we will preface variables with *PC* for "primary company" and *R* for "reinsurer" or "reinsurance." *PV* will be used in the traditional sense to mean "present value."

A discussion of the pricing formula to be used in this chapter will illustrate certain differences from primary pricing. We will often use the word "technical" to distinguish the actuarially calculated premium, rate, etc. from the actual final premium, rate, etc., agreed to by the cedant and reinsurer. Formula 7.2 calculates the technical reinsurance premium in terms of reinsurance loss cost, external expenses, internal expenses, and target economic return in the simple case where there are no later commission or premium adjustments based upon the actual loss experience. You can see that 7.2 is an "actuarial" formula, based explicitly upon costs.[1]

Formula 7.2: A Flat Rate Reinsurance Pricing Formula

$$RP = \frac{PVRELC}{(1 - RCR - RBF) \times (1 - RIXL) \times (1 - RTER)}$$

where

RP = reinsurance premium

$PVRELC$ = PV of $RELC$

= $RDF \times RELC$

$RELC$ = reinsurer's estimate of the reinsurance expected loss cost, $E[RL]$

RL = reinsurance loss

$E[RL]$ = reinsurance aggregate loss expectation

RDF = reinsurance loss payment discount factor

RCR = reinsurance ceding commission rate (as a percent of RP)

RBF = reinsurance brokerage fee (as a percent of RP)

$RIXL$ = reinsurer's internal expense loading (as a percent of RP net of RCR and RBF)

$RTER$ = reinsurer's target economic return (as a percent of reinsurance pure premium, RP net of RCR, RBF and $RIXL$)

[1]Formulas traditionally used by reinsurance underwriters have more often been of the form: undiscounted loss estimate divided by a judgmental loading factor such as 0.85. Of course the problem with this type of formula is that all the information about the expenses, discounting and profit loading are buried in one impenetrable number.

Let's break down this somewhat complicated-looking formula. First, as an actuarial technical pricing formula, we build up the premium starting with the loss cost. The reinsurance pure premium (*RPP*) can be written as follows.

Formula 7.3: Reinsurance Pure Premium

$$RPP = \frac{PVRELC}{1 - RTER}$$

This is the real "risk" premium for the risk transfer. The reinsurer's target economic return, *RTER*, is the reinsurer's charge for profit and risk. It is properly related to *PVRELC* for the contract. By writing the formula this way, the reinsurer's expected profit (present value) is $RTER \times RPP$. In this flat rated case, the reinsurer's expected profit is a simple linear function of the present value of the reinsurance expected loss cost, $RTER/(1 - RTER) \times PVRELC$. A discussion of how to select *RTER*'s for various contracts or lines of business is well beyond the scope of this chapter. The modern actuarial and economic literature derives these profit margins from an overall corporate target return on equity (RoE) and the relative risk level of each particular contract. There is extensive actuarial literature on this topic; see especially Bühlmann [9], Daykin, Pentikäinen and Pesonen [8], and, generally, *The ASTIN Bulletin, The Proceedings of the Casualty Actuarial Society* and any publications connected with the CAS Casualty Actuaries in Reinsurance.

Next, the reinsurer's internal expenses are loaded onto the reinsurance pure premium by dividing it by $1 - RIXL$. Thinking of this from the opposite direction (top down), it may also be thought of as a loading on reinsurance premium less external expenses. It is convenient to think of the loading for the reinsurer's internal expenses this latter way for at least three reasons:

1. This is the reinsurer's actual cash income from the cedant (unless there are funds withheld).
2. It is relatively easy to account for external expenses by reinsurance line of business. Within a line, the reinsurer's

underwriting and claims handling effort, and thus internal expenses, should be similar for each contract, varying only by claims and "risk" volume.

3. Internal expenses by contract should be independent (or almost independent) of commissions or brokerage expenses. Thus the loading should be independent of these expenses.

Together, *RTER* and *RIXL* determine the reinsurer's desired pricing margin for internal expenses and profit (economic return).

Finally, the reinsurance ceding commission rate and brokerage fee are specified in each particular contract; they are almost always stated as percentages of the total reinsurance premium *RP*. There is often no ceding commission on excess coverage; this is very different from primary coverage where commissions almost always exist. Of course, the existence of a reinsurance brokerage fee also depends upon the existence of a reinsurance intermediary for the contract.

An example will help clarify this.

Example 7.4:

$PVRELC$ = \$100,000 (calculated by actuarial analysis and formulas)

$RTER$ = 20% (The reinsurer believes this is appropriate to compensate for the uncertainty and risk level of the coverage.)

$RIXL$ = 10% (The reinsurer's allocation for this type of business.)

RCR = 25% (specified in the contract)

RBF = 5% (specified in the contract)

Then

$$RPP = \frac{\$100{,}000}{0.8} = \$125{,}000$$

$$RP = \frac{\$125{,}000}{(1-.10)\times(1-.25-.05)}$$

$$= \$198{,}413$$

Please note that the reinsurance premium less external expenses is

$$\frac{RPP}{1-RIXL} = \frac{\$125{,}000}{0.9}$$

$$= \$138{,}889$$

$$= 0.7\times\$198{,}413$$

$$= (1-RCR-RBF)\times RP$$

Also, the reinsurer's desired margin for internal expenses and profit is

$$\$138{,}889 - \$100{,}000 = \$38{,}889$$

$$= \$13{,}889 + \$25{,}000.$$

Very often the reinsurance premium is not a fixed dollar amount, but is calculated as a rate times a rating basis, quite often *PCP, the primary company (subject) premium*. In our example, if *PCP* was expected to be \$5,000,000, the reinsurance rate would most likely be rounded to 0.04 or 4%. Then the expected reinsurance premium would be \$200,000 and the expected reinsurance premium less external expenses would be $0.7\times\$200{,}000 = \$140{,}000$, and the expected reinsurance margin would be $\$140{,}000 - \$100{,}000 = \$40{,}000$, greater than the desired margin. So, if the reinsurer's internal expenses were still \$13,889, then the reinsurer's expected economic return (profit) would be $\$40{,}000 - \$13{,}889 = \$26{,}111$, greater than the target of \$25,000.

Later we will see cases where the commission or premium is dependent upon the actual loss experience on the contract. This leads to a more complicated formula, where in order to obtain a proper actuarial technical premium or rate, you must consider the reinsurer's profit as a more complicated function of the loss experience. But let us start simply.

Thirteen-step Program to Reinsurance Pricing Happiness

In general, when pricing reinsurance, it is desirable to perform both an *exposure rating* and an *experience rating*. An exposure rate is akin to a primary manual rate, using general rating factors independent of the cedant's particular claims experience. An experience rate is akin to a primary loss rate, completely dependent upon the cedant's particular claims experience. The final technical rate will be a weighing together of both of these rates.

The steps in the rating procedure may be abstracted as follows.

1. Gather and reconcile primary exposure, expense and rate information segregated by major rating class groups.
2. Calculate an exposure expected loss cost, *PVRELC*, and, if desirable, a loss cost rate, *PVRELC*/*PCP*.
3. Gather and reconcile primary claims data segregated by major rating class groups.
4. Filter the major catastrophe claims out of the claims data.
5. Trend the claims data to rating period.
6. Develop the claims data to settlement values.
7. Estimate the catastrophe loss potential.
8. Adjust the historical exposures to the rating period.

9. Estimate an experience expected loss cost, *PVRELC*, and, if desirable, a loss cost rate, *PVRELC*/*PCP*.
10. Estimate a “credibility” loss cost or loss cost rate from the exposure and experience loss costs or loss cost rates.
11. Estimate the probability distribution of the aggregate reinsurance loss, if desirable, and perhaps other distributions, such as for claims payment timing.
12. Specify values for *RCR*, *RIXL*, and *RTER*.
13. Negotiate, reconcile opinions and estimates, alter terms, and finalize.

Steps 1, 2, and 12 may be considered to be exposure rating, steps 3–9 and 12 to be experience rating, and steps 10–13 to be rate finalization. Step 11 is usually performed only for more complex contracts. We will try to use this same sequence of steps whenever possible. But sometimes the order of the steps will differ depending upon the particular situation. Let us start with the simplest case.

Facultative Certificates

Since a facultative certificate covers a share of a single insurance policy or set of policies covering a single insured, the individual insured can be underwritten and priced. The exposure of the individual insured can be evaluated and manual rates and rating factors can be used to calculate an exposure rate. However, since most facultative certificates are written on larger or more hazardous exposures, manual rates and rating factors may not exist or must often be modified. Thus, the analysis of individual exposure and loss experience, together with a great deal of underwriting judgment, is important.

In contemplating any form of facultative coverage, the underwriter first evaluates the exposure to decide if the risk is acceptable, and may then evaluate the rate used by the cedant to estimate its degree of adequacy. The underwriter also determines if the ceding commission fairly covers the cedant's expenses, but does not put the cedant into a situation significantly more advantageous than that of the reinsurer. That is, except in unusual circumstances, the cedant should not make a significant profit in those circumstances where the reinsurer is losing money, and vice versa.

The Actuary's Role

Historically, actuaries have seldom been directly involved in facultative certificate pricing. But they can be useful in the following ways by:

1. Being sure that the facultative underwriters are provided with, and know how to use, the most current and accurate manual rates and rating factors, e.g., increased limits factors, loss development factors, trend factors, actuarial opinions on rate adequacy by exposure type and by territory (state), etc.
2. Working with the underwriters to design and maintain good pricing methodologies, almost always in the form of interactive computer programs.
3. Working with the underwriters to design and maintain good portfolio monitoring systems for meaningful categories of their business, for relative price level and for the monitoring of claims, underwriting, and bottomline profit experience.
4. Working with the underwriters to evaluate and determine which lines of business and which exposure layers to concentrate upon as market conditions change.

Property Certificates

The evaluation and pricing of property certificate coverage on a proportional share basis usually needs little further actuarial assistance. However, the actuary should be involved in the evaluation of the accumulation of catastrophe exposure, and also in the interpretation of long-term results from a corporate portfolio perspective.

The evaluation and pricing of property certificate coverage on an excess basis is more difficult. Many tables used by underwriters calculate the excess rate as a factor times the primary rate.[2] Instead of relating the excess rate directly to the primary rate, a more actuarially sound method for determining property per-risk excess rating factors expresses the expected excess loss cost for coverage above an attachment point as a percentage of the total expected loss cost. This allows us to use Formula 7.2 to treat expected loss cost, expenses, and profit separately. The curves underlying these factors depend upon the class of business (its claim severity potential) as determined by the most important rating variables: amount insured, *MPL* (maximum possible loss), *PML* (probable maximum loss), construction, occupancy, and protection.

The *MPL*, sometimes called the "amount subject," is a very conservative estimate by the individual underwriter of the maximum loss possible on the policy. For example, it includes the maximum full value of contiguous buildings together with contents, and also reflects maximum time element (e.g., business interruption) coverage. The *PML* is a less conservative estimate of the largest loss, assuming for example, that the sprinkler system works, that the contents are normal, etc. The difference between *MPL* and *PML* is illustrated by considering an office building:

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[2]Some underwriters use so-called Lloyd's Scales. Underwriters bring these tables with them from job to job; the parentage of the Lloyd's Scales floating around the industry seems to be highly questionable. So, be careful.

Some underwriters also use tables of factors from a 1960 *PCAS* paper by Ruth Salzmann. But these factors were developed for homeowners business. So even if they were adjusted for inflation over the last 40 years, they are of questionable use for the typical commercial property facultative exposure. A paper by Stephen Ludwig [13] uses commercial risk experience to estimate loss curves. ISO and various reinsurance companies have been doing research and developing property loss curves. But so far there are no published, actuarially sound claims severity curves or tables of factors for rating property per-risk excess coverage.

the *MPL* is the total value; the definition of *PML* varies from underwriter to underwriter, but is usually thought to be three to five floors. The *MPL* and *PML* affect the shape of the loss cost curve because you expect, for example, very different loss severity distributions for an insured with a $100,000 *MPL* and *PML* versus an insured with a $10,000,000 *MPL* and $5,000,000 *PML*.

This is illustrated by the accompanying Graph 7.5. An actuary might think of the *MPL* as being essentially the 100^{th} percentile of the probabilistic loss distribution, and the *PML* as being somewhere around the 95^{th} to 99^{th} percentiles. Note that the $10,000,000 *MPL* property has a smaller probability of a total loss. In fact, at every loss level, its graph lies above the graph for the $100,000 *MPL* property, thus having a smaller probability of exceeding each percentage loss level.

Appropriate *RTER*'s and *RIXL*'s could be incorporated into the table or could be recommended as additional loading factors.

An appropriate pricing formula for an excess cover could use Formula 7.2 with (dropping the *PV* for short-tailed property coverage) *RELC* calculated as follows.

Formula 7.6: Reinsurance Expected Loss Cost

$$RELC = ELCF \times PCP \times PCPLR \times RCF$$

where

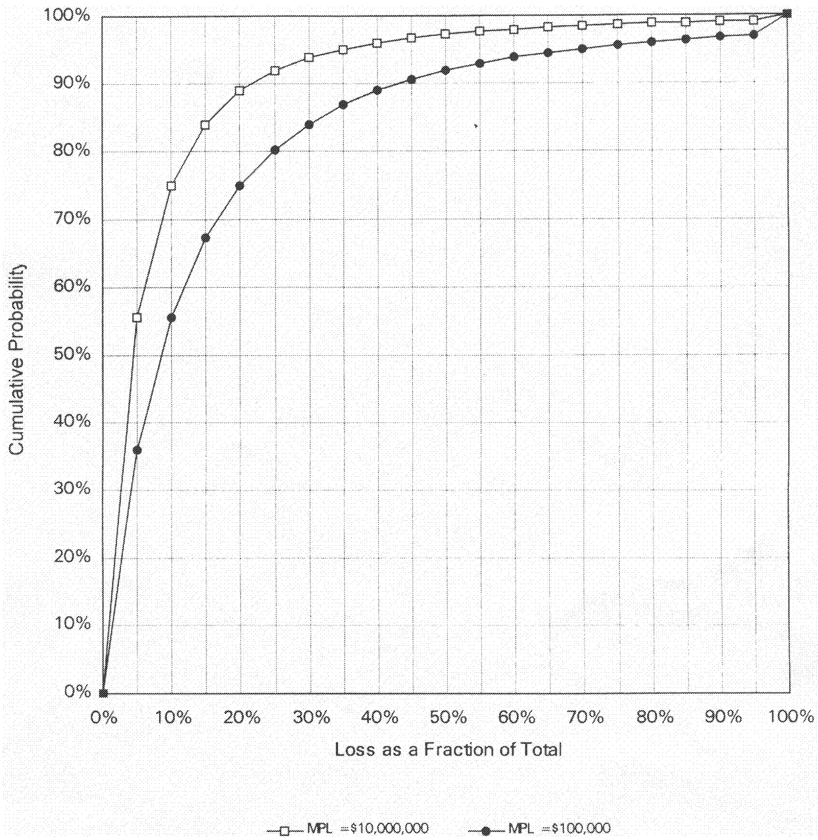
$ELCF$ = excess loss cost factor
(from the table; as a percent of total loss cost)

PCP = primary company (subject) premium

$PCPLR$ = primary company permissible loss ratio
(including any loss adjustment expenses covered as part of loss)

GRAPH 7.5

EXAMPLES OF PROPERTY CLAIM SEVERITY
CUMULATIVE DISTRIBUTION FUNCTIONS



RCF = rate correction factor
(reinsurer's adjustment for the estimated
(in)adequacy of the primary rate)

Again, the reinsurance premium can be translated into a reinsurance rate by dividing it by the primary company premium *PCP*. An example will help clarify this.

Example 7.7:

$$\begin{aligned}
PCP &= \$100{,}000\\
PCPLR &= 65\%\\
RCF &= 1.05 \text{ (estimated 5\% inadequacy)}\\
RCR &= 30\%\\
RBF &= 0\% \text{ (no broker)}\\
RIXL &= 15\%\\
MPL &= PML = \$10{,}000{,}000.\\
Attpt &= \text{Attachment Point} = \$1{,}000{,}000.\\
RLim &= \text{Reinsurance Limit} = \$4{,}000{,}000\\
RTER &= 10\%
\end{aligned}$$

Thus the reinsurer believes the total expected loss cost is as follows.

Formula 7.8: Total Expected Loss Cost

$$\begin{aligned}
PCP \times PCPLR \times RCF &= \$100{,}000 \times (.65) \times (1.05)\\
&= \$68{,}250
\end{aligned}$$

Now assume that we believe that the claim severity, including loss adjustment expense, for this class of business and this *MPL*, is given by a censored (at *MPL*) Pareto distribution of the following form.

Formula 7.9: Censored Pareto Model

$$1 - F(x) = \text{Prob}[X > x] = \begin{cases} \dfrac{b}{(b+x)^q} & \text{for} \quad x < 1 \\ 0 & \text{for} \quad x \geq 1 \end{cases}$$

where the claim size X is expressed as a percent of *MPL*.

(Properties of the Pareto distribution are outlined in Appendix A.)

Suppose that the parameters are given by $b = 0.1$ and $q = 2$. We can now verify the following facts.

Formula 7.10: Expected Claim Severity (as a fraction of MPL)

$$E[X;1] = \left\{\frac{b}{q-1}\right\} \times \left\{1 - \left(\frac{b}{b+1}\right)^{q-1}\right\} \quad \text{(Appendix A)}$$
$$= \left\{\frac{0.1}{2-1}\right\} \times \left\{1 - \left(\frac{0.1}{1.1}\right)^{1}\right\}$$
$$= \{0.1\} \times \{1 - 0.91\}$$
$$= 0.0909$$

Thus, if a loss occurs, our estimate of its average size is $0.0909 \times \$10{,}000{,}000 = \$909{,}000$.

We can also calculate the expected claim count.

Formula 7.11: Expected Claim Count (ground-up)

$$\text{Expected claim count} = \frac{\text{Total expected loss cost}}{\text{Expected loss severity}}$$
$$= \frac{\$68{,}250}{\$909{,}000} = 0.075$$

Formula 7.12: Expected Claim Severity in the Layer [\$0, \$1,000,000]

$$E[X;0.1] = \left\{\frac{b}{q-1}\right\} \times \left\{1 - \left(\frac{b}{b+0.1}\right)^{q-1}\right\}$$
$$= \left\{\frac{0.1}{2-1}\right\} \times \left\{1 - \left(\frac{0.1}{0.1+0.1}\right)^{1}\right\}$$
$$= \{0.1\} \times \{1 - 0.5\}$$
$$= 0.05$$

Thus, if a loss occurs, its average size in the layer $[\$0, \$1{,}000{,}000]$ is $0.05 \times \$10{,}000{,}000 = \$500{,}000$.

Formula 7.13: Expected Claim Severity in the Layer [$0, $5,000,000]

$$E[X;0.5] = \left\{\frac{b}{q-1}\right\} \times \left\{1 - \left(\frac{b}{b+0.5}\right)^{q-1}\right\}$$
$$= \left\{\frac{0.1}{2-1}\right\} \times \left\{1 - \left(\frac{0.1}{0.1+0.5}\right)^{1}\right\}$$
$$= \{0.1\} \times \{1 - 0.167\}$$
$$= 0.0833$$

Thus, if a loss occurs, its average size in the layer [$0, $5,000,000] is $0.0833 \times \$10,000,000 = \$833,000$. Therefore, the *ELCF* can be calculated as follows.

Formula 7.14: ELCF (Excess Loss Cost Factor)[3]

$$ELCF = \frac{E[X;0.5] - E[X;0.1]}{E[X;1]}$$
$$= \frac{.0833 - 0.5}{0.0909}$$
$$= 0.367$$

We can now calculate the Reinsurance Expected Loss Cost (*RELC*).

Formula 7.15: Reinsurance Expected Loss Cost

$$RELC = \text{est. } E[RL] = ELCF \times \{\text{Total expected loss cost}\}$$
$$= 0.367 \times \$68,250$$
$$= \$25,028$$

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[3]The factors in a Lloyd's-type table, mentioned in a previous footnote, would technically be of the following form:

$$\text{Expected loss cost fraction at limit } c\% \text{ of total amount insured}$$
$$= \frac{E[X;c]}{E[X;1]} \quad \text{for} \quad 0 \le c \le 1$$

Thus the *ELCF* is simply a difference of two of these factors: at the attachment point and at the attachment point plus the limit.

Then we can calculate the technical reinsurance pure premium as

$$\begin{aligned} RPP &= \frac{RELC}{1-RTER} \\ &= \frac{\$25{,}028}{0.9} \\ &= \$27{,}808 \end{aligned}$$

and the technical reinsurance premium as

$$\begin{aligned} RP &= \frac{RPP}{(1-RIXL)\times(1-RCR-RBF)} \\ &= \frac{\$27{,}808}{(0.85)\times(0.7)} \\ &= \$46{,}737. \end{aligned}$$

To squeeze out more information about the reinsurance coverage in order to measure the risk transfer, as we will discuss later, we can also estimate the reinsurance expected claim count and severity.

Formula 7.16: Expected Reinsurance Claim Count

$$\begin{aligned} &\text{Expected excess claim count} \\ &\quad = \text{Expected claim count (ground-up)} \times \text{Excess probability} \\ &\quad = 0.075 \times \left(\frac{b}{b+0.1}\right)^{q} \\ &\quad = 0.075 \times \left(\frac{0.1}{0.1+0.1}\right)^{2} \\ &\quad = 0.075 \times 0.25 \\ &\quad = 0.0188 \end{aligned}$$

We can then calculate the excess expected claim severity.

Formula 7.17: Expected Claim Severity in the Layer [\$1,000,000, \$5,000,000]

$$\begin{aligned}\text{Excess expected claim severity}\\ &= \frac{RELC}{\text{expected reinsurance claim count}}\\ &= \frac{\$25{,}028}{0.0188}\\ &= \$1{,}333{,}333.\end{aligned}$$

This is the average size (in the layer) of a claim entering the layer. Note that it is one third of the reinsurance limit. Average intra-layer claim sizes for excess layers are typically (but not always) between one quarter to one half of the layer limit.

Further Property Certificate Pricing Considerations

Quite often the pricing situation is much more complicated, with multiple locations and coverages. The underwriter/pricer generally determines a price for each location and each coverage, and then adds them to obtain the total premium.

Instead of working directly with an estimated underlying loss severity distribution like this Pareto, the *ELCF* in Formula 7.14 might be obtained from a Lloyd's-type table. Better yet, a complete pricing procedure such as this can be programmed into an interactive computer package for the underwriters. The package would contain all the appropriate rates and rating factors or underlying loss severity models and parameters to be called upon by the user. It would ask most of the relevant questions of the user and would document the decision trail for each submission seriously contemplated by the underwriter. If desirable, the reinsurer's values for *RIXL*'s and *RTER*'s could be built into the system or as part of the *ELCF*'s.

For facultative certificate property coverage as with any reinsurance business segment, the pricing cycle is very severe. This is mainly due to naive capital flowing into the market because of easy access, but also due to the short-term nature of most peoples' memories. Thus it is very important to monitor the results closely. Renewal pricing and rate competition in the marketplace should be watched monthly; perhaps summaries derived from the aforementioned pricing system would be appropriate. Quarterly updates of underwriting results by accident year in appropriate business segment detail are very important.

Casualty Certificates

The evaluation and pricing of facultative certificate casualty coverage is even trickier than property coverage, due mainly to the additional uncertainty arising from delayed claims reporting and settlement. Because of this increased uncertainty, the actuary's role can be more important in the pricing and in the monitoring and interpretation of results.

As with property excess, a cover may be exposure rated via manual rates and increased limits factors, together with exposure evaluation and underwriting judgement. The same Formula 7.6 may be used to determine *RELC*, except that the *ELCF* will be based upon increased limit loss cost tables, based upon claim severity curves, and the *RCF* may be determined both by facts and by judgments regarding the cedant's basic limit rate level and increased limit factors.

Since most companies use Insurance Services Office (ISO) increased limit factors for third party liability pricing (especially for commercial lines), it is very important that the actuaries very closely monitor ISO factors and understand their meaning. Likewise, it is important to monitor and understand information from the National Council on Compensation Insurance (NCCI) regarding workers compensation claim severity. However, you should not use the published excess loss factors (*ELF*'s) for pricing excess loss coverage, since they underestimate per-occurrence excess loss potential.

Allocated Loss Adjustment Expense (ALAE)

ALAE per claim is usually covered on an excess basis either:

- In proportion to the indemnity loss share of the excess cover vis-a-vis the total, or
- by adding the *ALAE* to the indemnity loss before applying the attachment point and limit.

For example, assume the layer \$500,000 excess of \$500,000 is reinsured. The following claims induce the indicated reinsurance reimbursements.

TABLE 7.18

REINSURANCE PAYMENTS BY *ALAE* SHARE

(1) Indemnity Payment	(2) *ALAE*	(3) Reinsurer's payment if *ALAE* pro rata	(4) Reinsurer's payment if *ALAE* added
\$ 500,000	\$ 50,000	\$0	\$ 50,000
750,000	75,000	275,000	325,000
1,000,000	100,000	550,000	500,000
2,000,000	150,000	537,500	500,000

Increased limits factors published by ISO have no provision for *ALAE* outside of the basic limit. Thus *ELCF*'s based upon these increased limit factors must be adjusted to cover the reinsurer's share of *ALAE*.

Pricing Methods

Since policies subject to facultative coverage are larger than usual, experience rating often comes into play. One method is to first experience rate a lower layer with more credible experience. Then the experience-based loss cost on the lower layer may be used together with the reinsurer's *ELCF* table to extrapolate up to the intended layer of coverage.

For a buffer layer of coverage where the likelihood of loss penetration is significant, it might also be possible to obtain a loss cost estimate directly from a careful analysis of the large loss experience of the insured.

An example should clarify this.

Example 7.19:

- Policy period priced = 2001
- Exposure = General liability premises/operations
- Policy limit = $2,000,000, no aggregate
- *PCP* = $550,000 (estimated for 2001)
- *BLP* (basic limit premium @ $100,000 limit) = $200,000 (estimated for 2001)
- *PCPLR* = 70% (excluding *un*allocated loss adjustment expense from the loss cost)
- *Attpt* (attachment point) = $500,000
- *RLim* (reinsurance limit) = $1,500,000
- *ALAE* is covered pro rata.
- *RCR* (reinsurance commission) = 25%
- *RBF* (reinsurance brokerage fee) = 5%
- *RIXL* (reinsurer's internal expense loading) = 15%
- *RTER* (reinsurer's target economic return) = 20%
- Have exposure and loss experience for policy years 1995 through 1999, consisting of exposures, basic and total limits premiums, current evaluation of basic limits losses and a detailed history for each known claim larger than $100,000

Suppose that the cedant's basic limit premium was determined from a standard experience and schedule rating plan that we believe to be adequate. Also suppose that the cedant uses the appropriate ISO increased limit factors, which we believe also to be adequate, and which include the ISO risk loading but no *ALAE* provision for the layer. Suppose the ISO increased limit factors for this exposure are as follows.

TABLE 7.20

(FICTITIOUS) ISO INCREASED LIMIT FACTORS

(1) Policy limit	(2) Published *ILF*	(3) *ILF* without risk load
$ 100,000	1.00	1.0000
500,000	2.10	1.9077
2,000,000	2.75	2.4264

Suppose that the cedant is offering a manual difference excess premium of $130,000 calculated as follows.

Formula 7.21: Manual Difference Excess Premium

$$\text{Manual difference excess premium} = \$200{,}000 \times (2.75 - 2.10)$$
$$= \$130{,}000$$

This is the simplest technical price determination possible. Some reinsurers, after multiplying by a judgment factor to adjust for rate (in)adequacy for this insured or for this type of insured, stop here. However, because there is too much information hidden in this calculation, let us continue onward to calculate an actuarial, technical price based upon the estimated loss cost and the other rating parameters.

Suppose that, based upon a study of the relationship of *ALAE* to claim size for this type of exposure, we believe that an appropriate loading for pro rata *ALAE* is 10% of indemnity loss cost for this layer. Also suppose that we have an estimated expected loss payment pattern for this type of exposure and this layer, and suppose that the corresponding discount factor, using current U.S. Treasury rates (risk-free) timed to the loss payment pattern and reflecting the implications of the current tax law, is .80. Then if we believe the ISO increased limit factors to be adequate for this exposure, the present value reinsurance expected loss cost could be calculated as follows.

Formula 7.22: PVRELC

$$\begin{aligned}
PVRELC &= RDF \times RELC \\
&= RDF \times ELCF \times BLP \times PCPLR \times RCF \\
&= RDF \times ELCF \times \{\text{basic limit expected loss cost}\} \\
&= (0.8) \times \{(2.4264 - 1.9077) \times (1.10)\} \\
&\quad \times \{\$200{,}000 \times (0.7) \times (1.00)\} \\
&= (0.8) \times \{(0.5187) \times (1.10)\} \times \{\$140{,}000\} \\
&= 0.8 \times 0.5706 \times \$140{,}000 \\
&= \$63{,}904
\end{aligned}$$

Then the reinsurance premium can be calculated via Formula 7.2.

Formula 7.23: Reinsurance Premium (RP)

$$\begin{aligned}
RP &= \frac{PVRELC}{(1 - RCR - RBF) \times (1 - RIXL) \times (1 - RTER)} \\
&= \frac{\$63{,}904}{(1 - 0.25 - 0.05) \times (1 - 0.15) \times (1 - 0.2)} \\
&= \frac{\$63{,}904}{(0.7) \times (0.85) \times (0.8)} \\
&= \frac{\$63{,}904}{0.476} \\
&= \$134{,}252
\end{aligned}$$

Please note that the assumption that ISO increased limit factors appropriately describe the claim severity potential for this insured is a very crucial assumption. Since facultatively reinsured exposures are often more hazardous, the actuary or underwriter may believe that often the claim severity distribution is more dangerous. However, the actuary or underwriter designing a facultative certificate pricing procedure may believe that the increased severity hazard is already reflected in the first $100,000 basic limit price for these insureds. Otherwise, you may wish to adjust ISO claim severity curves accordingly, or allow the certificate pricer to do so on a case-by-case basis.

In this case, the offered $130,000 premium looks reasonably close to our technical premium. So the pricing endeavor usually stops here. But what about the large loss experience? Suppose that for accident years 1995–1999, there are some claims known as of June 30, 2000 whose indemnity values are greater than $100,000. Can any of this insured's large loss information be used to help price the cover, or at least to help form an opinion as to the adequacy of the exposure rate premium?

A recommended rating procedure in this case is to experience rate the layer $400,000 excess of $100,000, and then use the non-risk-loaded increased limit factors to extrapolate upward to a reinsurance loss cost for the layer $1,500,000 excess of $500,000. We will describe a common reinsurance experience rating methodology later in the section on pricing excess treaties.

As with property excess, it is clear that the exposure rating methods can be programmed into an interactive computer package for underwriters. Also, as with property coverage, it is very important to monitor relative rate level and results in appropriate business segment detail. The actuarial evaluations and opinions regarding future case reserve development and *IBNR* emergence should be very important to the underwriters.

Facultative Automatic Programs

These large multi-insured programs are very similar to treaties. One difference, however, is that the reinsurance premium for a facultative automatic excess cover is usually computed on a policy-by-policy basis using agreed-upon excess rates, instead of as a rate times total subject premium, as is usually the case for treaties. Thus the reinsurance premium is more responsive to the individual exposures ceded to the reinsurer. Nevertheless, the risk of anti-selection against the reinsurer on a nonobligatory contract should be evaluated by the underwriter.

The pricing of these agreements is the same or similar to the pricing of excess treaties, discussed below.

Reinsurance Treaties in General

In this discussion the word "treaty" is used interchangeably for both treaties and automatic facultative agreements.

Since a treaty covers a share of a portfolio of insurance policies, insureds are rarely individually underwritten and priced by the reinsurer; many of the policies to be covered may not be written by the cedant until well into the reinsurance coverage period. Instead, the reinsurance underwriter/pricer considers the whole portfolio of potentially subject policies. To do this, the reinsurer evaluates first the management of the potential cedant. What is their management philosophy and ability? Are they honest, fair-dealing? Do they know what they are doing? Is the company financially solid? What are their business plans? Why do they want reinsurance? Why do they need reinsurance?

Once the reinsurer is satisfied that this is a company and these are people we would like to deal with on a long-term basis, we can then evaluate their underwriting, primary pricing, marketing, and claims handling ability. Since individual insureds are not usually underwritten by the reinsurer, we must be generally satisfied with the cedant's underwriting and pricing expertise for the exposure we may assume. For any treaty, we must understand the cedant's insurance exposures, rate level, and policy limits sold. Many reinsurers will send a team of marketing and underwriting people to perform a pre-quote audit, and will also send claims people to review the company's claims handling and reserving practices.

The reinsurer (or the lead reinsurer on a multi-reinsurer brokered treaty) usually reviews the structure of the cedant's reinsurance program to understand how all the reinsurance contracts, facultative and treaty, fit together to provide benefits to the cedant. Lastly, the reinsurer evaluates the particular reinsurance treaties and suggested rates if offered, or creates a program and rates to offer to the cedant company.

Actuaries can provide extremely useful, and often necessary, technical support for treaty business. They can perform the four functions listed at the beginning of the section on facultative certificate pricing. They can also get involved in the technical evaluation and pricing of individual large and/or difficult treaties through many or all of the rating steps. Experience rating is much more important for treaties than for facultative certificates. Consequently, the actuarial tools of data analysis and loss modeling can be critical to a reinsurer's ability to write difficult exposures, especially casualty exposures where long-tail loss development is a significant factor.

Property Proportional Treaties

Proportional treaties are usually priced by evaluating the amount of the commission to be paid ultimately to the cedant. The ultimate commission is comprised of a provisional commission paid at the time of the cession, plus any later adjustments specified by the terms of the treaty.

Property Quota-Share Treaties

A traditional quota-share treaty covers a share of the cedant's net retention after facultative covers. To evaluate the loss exposure, we follow the rating steps outlined before. To facilitate the discussion, we will consider an example using our favorite primary insurer, da Ponte Insurance Company.

Example 7.24:

- Rating period = 2001
- 25% quota share on various property lines
- Risks attaching coverage (primary policy year; reinsurers generally call this "*underwriting year*")
- Estimated PCP (written premium) = \$10,000,000
- Estimated unearned premium reserve @ 12/31/01 = \$3,000,000
- Per occurrence limit of \$7,500,000 for reinsurance losses
- Proposed $RCR = 35\%$
- $RBF = 0\%$ (no broker)

Step 1: Gather and reconcile primary exposure, expense and rate information segregated by major rating class groups.

The grouping may be by Annual Statement line of business, or could be a finer decomposition if the proposed reinsurance coverage is more restricted or if there are any important exposures you may want to evaluate separately. What is meant by the word "exposure" for reinsurance purposes is usually primary subject premiums. The reconciliation should be to publish financial records as much as possible. If there is significant catastrophe potential, we would want the exposure by zip code in order to perform the seventh rating step.

Let us suppose that the proposed treaty is to cover the property exposures in Annual Statement lines 1–5 and 12, net of facultative reinsurance; the liability parts of lines 3–5 are excluded. In this example, the reinsurer would ask for gross written premium by line by year for 1995 through 6/30/00, together with estimates for 2000 and 2001. The expense information could be from the cedant's Insurance Expense Exhibit. The rate information would be contained in the cedant's underwriting line guide. We also want information on average rate deviations.

Step 2: Calculate an exposure expected loss cost, PVRELC, and, if desirable, a loss cost rate, PVRELC/PCP.

For proportional coverage, an exposure loss cost rating is simply an evaluation of the adequacy of the cedant's rates for the exposures to be covered, leading to an estimate of the expected loss ratio. An underwriting review can compare the cedant's rates to those of other primary insurers or to the reinsurer's own database of adequate primary rates.

Many people in the reinsurance business would say that you cannot calculate an exposure rate for proportional coverage, or that you cannot rate the coverage at all; you can only evaluate the ceding commission. The ceding commission should fairly reimburse the cedant's expenses, but should not put the cedant into a position significantly more advantageous than that of the reinsurer. Except in unusual circumstances, the cedant should not make a significant profit while the reinsurer is losing money, and vice versa. The point here is to evaluate the (in)adequacy of the cedant's rates in order to evaluate whether or not the proposed reinsurance commission will work for the reinsurer *and* for the cedant.

Let us suppose that the review indicates that overall, the expected loss ratio for policy year 2001 is 65% including all catastrophe losses. Suppose we estimate that a per occurrence catastrophe limit will only reduce this by a few percent, but less than the amount of the reinsurer's expense and profit loadings, *RIXL* and *RTER*. Thus, it looks as if the reinsurer cannot afford a 35% ceding commission. We will deal with the ceding commission issue further in steps 10–12.

Step 3: Gather and reconcile primary claims data segregated by major rating class groups.

We want the claims data segregated as the exposure data. We usually want *ALAE* separately from indemnity losses. For proportional coverage, the data are usually historical aggregate claims data for the past five to ten policy years, plus individual large claims. We also want some history of claims values (an historical policy year/development year triangle) for step five. The data should be adjusted so that they are approximately on the same basis as our coverage with respect to any other *inuring* reinsurance, that is, reinsurance that applies to the cedant's loss before our coverage.

Suppose we have net aggregate loss development triangles by line for policy years 1995–2000 at evaluation dates 12/31/95, 12/31/96,...,12/31/99, 6/30/00, plus concurrent evaluations by line of all numbered catastrophes occurring during this time period. These are the catastrophes designated by Property Claims Service (PCS). Also suppose that there haven't been significant changes in da Ponte's insurance portfolio or in their reinsurance program during this time. So the data are consistent with the expected 2001 loss exposure.

Step 4: Filter the major catastrophic claims out of the claims data.

This is straightforward. Subtract the numbered catastrophe values by line at each evaluation from the loss development triangles.

Step 5: Trend the claims data to the rating period.

We want to be sure that the historical claims data are adjusted in a manner that makes them reasonably relevant to the rating period. The trending should be for inflation and for other changes in exposure (such as higher policy limits) that might affect the loss potential. For proportional coverage, we may skip this step, and simply look for any apparent trend in the historical loss ratios in step nine.

Step 6: Develop the claims data to settlement values.

Claims development is usually not much of a problem for filtered primary property claims. If we have historical policy year/development year triangles, we can use standard methods to estimate loss development factors and apply these to the filtered data. If we don't have historical triangles, we may be able to use Annual Statement Schedule P accident year data, if they are reasonably reflective of the proposed coverage and if we can filter out major catastrophes, to estimate policy year loss development factors. We also want to compare the development patterns estimated from the cedant's data to what we expect based upon our own historical data for comparable coverages to check for reasonableness.

If the reinsurer believes that the development patterns should be reasonably similar for the various covered lines, we usually estimate the total development from the combined data instead of by line.

Step 7: Estimate the catastrophic loss potential.

This deserves a whole section by itself. The problem is that the historical data, even developed and trended, may not indicate the true potential for catastrophic losses. Reinsurers who have relied entirely upon five to ten years of historical claims to estimate catastrophe potential have often blundered into coverages where the catastrophic events occurring during their coverage period have more than wiped out the "normal" non-catastrophic profits. For example, da Ponte's 1995–1999 experience period does not include such major catastrophes as Hurricane Andrew (1992) and the Northridge earthquake (1994). It also does not include such possibilities as a New Madrid fault earthquake, a Long Island hurricane, etc. If da Ponte's exposures include any potential catastrophe areas, then we must estimate the corresponding catastrophe potential.

We will discuss various methods for this evaluation in the later section on catastrophe covers. If we collected gross written premiums by zip code, we can use these data to parameterize one of the available catastrophe computer models.

Let us assume that our evaluation of this portfolio indicates an expected catastrophic loss ratio to *PCP* of 12% with respect to a $7,500,000 per occurrence limit on reinsurance losses.

Step 8: Adjust the historical exposures to the rating period.

We want to be sure that the historical exposure (premium) data are adjusted in a manner that makes them reasonably relevant to the rating period. The trending should be for primary rate and underwriting changes and for other changes in exposure that might affect the loss potential. For proportional coverage, we only adjust the premiums for significant rate level changes, so that the historical loss ratios in step nine are all consistent with 2001 rate level. The rate adjustments are accomplished using the standard methods.

Step 9: Estimate an experience expected loss cost, PVRELC, and, if desirable, a loss cost rate, PVRELC/PCP.

Suppose that our data and estimates so far are as displayed in Table 7.25.

TABLE 7.25

DA PONTE INSURANCE COMPANY PROPERTY BUSINESS (IN $1,000'S)

(1) Policy Year	(2) Onlevel *PCP*	(3) Subject Loss	(4) Cat. Loss	(5) Filtered Loss	(6) Loss Devel. Factor	(7) Devel'd Loss (5) × (6)	(8) Loss Ratio (7) ÷ (2)
1995	$ 7,000	$ 3,472	$ 512	$ 2,960	1.00	$ 2,960	42%
1996	7,500	4,116	403	3,713	1.00	3,713	50%
1997	8,000	4,772	188	4,584	1.01	4,630	58%
1998	8,500	4,855	1,286	3,569	1.05	3,747	44%
1999	9,000	4,144	622	3,522	1.20	4,227	47%
2000	4,750	1,000	75	925	n.a.	n.a.	n.a.
Total w/o 2000	$40,000	$21,359	$3,011	$18,348	n.a.	$19,277	48%

Note: Columns 3–5 evaluated @ 6/30/00.

The five-year average filtered loss ratio is 48%. There seems to be no particular trend, and there are no significant rate revisions planned. So we simply take the average to be our estimate of the 2001 loss ratio. According to Step 7, we must add on 12% for catastrophe losses. We thus have a total expected loss ratio of 60%.

Step 10: Estimate a "credibility" loss cost or loss cost rate from the exposure and experience loss costs or loss cost rates.

We must reconcile the experience rating estimate of a 60% loss ratio with our exposure rating estimate of a 65% loss ratio. Remember that the exposure estimate includes unlimited catastrophe losses. The reconciliation is a process of asking questions and judging the relative credibility of the two loss ratio estimates. In the later discussion of treaty working cover excess pricing, we will list some of the questions we should ask.

Let us suppose that our "credibility" estimate of the 2001 expected loss ratio is 62%.

Steps 11–13 are very intertwined. Normally, we would want to perform Step 11 before Step 12, since Step 11 helps us quantify the risk transfer on the contract with respect to the particular contract terms. But sometimes, we have preliminary estimates of *RCR*, *RIXL* and *RTER* which may later be modified. The negotiations with the cedant will often send us back to Step 11 or 12, or even to earlier steps. We will present Steps 11 and 12 simultaneously.

Step 11: Estimate the probability distribution of the aggregate reinsurance loss if desirable, and perhaps other distributions such as for claims payment timing.

Step 12: Specify values for RCR, RIXL, and RTER

Let us start with preliminary values for Step 12. Suppose that a review of the cedant's expenses for the past few years indicates an expense ratio of 33% for this business. Suppose that the cedant will accept a ceding commission of 33%. Does this leave the reinsurer with enough profit?

With a 33% ceding commission, the expected reinsurance premium net of commission is $(1 - RCR) \times RP = 0.67 \times \$2{,}500{,}000 = \$1{,}675{,}000$. Suppose we determine that the reinsurer's costs for underwriting and accounting for this treaty, plus a general overhead allocation, will be about $50,000. This translates into an $RIXL$ of $50,000/$1,675,000 = 3.0% (approx.). The reinsurance risk pure premium with respect to a 33% ceding commission and a 3.0% reinsurer's internal expense loading is as follows.

Formula 7.26: Reinsurance Risk Pure Premium

$$\begin{aligned} RPP &= RP \times (1 - RCR) \times (1 - RIXL) \\ &= \$2{,}500{,}000 \times 0.67 \times 0.97 \\ &= \$1{,}675{,}000 \times 0.97 \\ &= \$1{,}624{,}750 \end{aligned}$$

Since the reinsurer's expected loss ratio is 62%, the expected loss is $0.62 \times \$2{,}500{,}000 = \$1{,}550{,}000$. This leaves a profit of $\$1{,}624{,}750 - \$1{,}550{,}000 = \$74{,}750$. Is this enough?

Suppose that the reinsurer generally wants $RTER$ = 8% for the risk transfer on this type of exposure. With RPP = $1,624,750, the usual 8% profit margin is expected to be $0.08 \times \$1{,}624{,}750 = \$129{,}980$. In addition, since there will be a statutory surplus loan equal to the ceding commission on the unearned premium reserve at year-end, the reinsurer should add some margin for this. Suppose the historical premium data indicate that the average year-end unearned premium reserve is about 40% of written premium. With a 33% ceding commission, the reinsurer will be giving da Ponte an estimated $0.33 \times 0.4 \times 0.25 \times \$10{,}000{,}000 = \$330{,}000$ of surplus relief at the end of 2001 for the 25% quota share. Suppose that an appropriate loan rate would be 7%. Thus,

$0.07 \times \$330{,}000 = \$23{,}100$ should be added to the reinsurer's profit margin. Adding $23,100 to the profit margin produces an overall profit margin of $\$129{,}980 + \$23{,}100 = \$153{,}080$. This is $\$153{,}080 - \$74{,}750 = \$78{,}330$ more than the expected profit.

It is time for us to sharpen our pencil. Do we want to accept this risk below our desired margin? Is this cedant that valuable to us? Do we already have other business with da Ponte whose profit makes up for this reduced margin, or will they cede such business to us? Do we already have or can we build a long-term profitable relationship with da Ponte and a close personal relationship with their president, Yakema Canutt? We also remember that our estimates may have a large random error component. What is the competitive situation?

Perhaps we can interest da Ponte in a more sophisticated contractual alternative involving a variable commission rate, or a loss and profit carryforward to successive years if we believe that primary rates, and thus loss ratios, will improve in the next few years. To evaluate a variable commission rate or a carryforward provision properly, we need an estimate of the ceded aggregate loss distribution. So now we return to Step 11.

There are many ways to estimate the probability distribution of the aggregate reinsurance loss. In a situation like this, where we are not dealing with aggregate deductibles or aggregate limits, it is best to use a simple method. The simplest method is to estimate the first two moments of the aggregate loss distribution from the annual historical aggregate filtered loss data plus a component for the catastrophe exposure. The obvious problem with this is that in this case we have only five data points for the filtered losses, a very small sample.

Considering the filtered loss ratios in column 8 of Table 7.25, the mean of the policy years 1995–1999 is 48% and the standard deviation is 6.1%. We use loss ratios instead of dollars to normalize the distribution, so it can be used for policy year 2001. Since the 6.1% estimate is derived from a small sample, we ask whether or not it is comparable with other standard deviation estimates from similar exposure situations. Let us assume it is.

Suppose our evaluation of the catastrophe loss potential yields the estimated expectation of 12% mentioned previously, and an estimated standard deviation of 13.9% (details in Appendix B). It is appropriate to assume that the filtered loss and catastrophic loss are independent. Under this assumption, their variances are additive. Let us further adjust the expectation of the filtered loss to 50% in order that the expectations add to 62%. Assuming that the reinsurance premium *RP* is constant, we have the following estimates.

Formula 7.27: First Two Central Moments of the Loss Ratio Distribution

$$\frac{RELC}{RP} = 62\%$$

$$\begin{aligned}\mathrm{Var}\left[\frac{RL}{RP}\right] &= \mathrm{Var}\left[\frac{RL_F}{RP}\right] + \mathrm{Var}\left[\frac{RL_C}{RP}\right]\\ &= \{(6.1\%)^2 + (13.9\%)^2\} \text{ estimate}\\ &= 2.29\%\end{aligned}$$

where RL_F = filtered reinsurance loss

RL_C = catastrophic reinsurance loss

We thus have an estimate of the standard deviation, $SD[RL/RP] = 15.13\%$. We approximate the loss ratio distribution with a Gamma distribution. This is described and justified in Appendix C, and also in Papush, Patrik and Podgaits [17]. We will later discuss more sophisticated models for approximating the aggregate loss or loss ratio distribution. In our simple case, it is enough to assume that the distribution of L can be represented by a Gamma distribution whose parameters can be estimated by the Method of Moments. We can now evaluate the ceding commission terms with this distribution, and thus estimate the reinsurer's profit.

Let us now finalize Step 12 before negotiations. We will use the values for *RIXL* and *RTER* already obtained. Let us now evaluate a *sliding scale commission* arrangement of the following form in order to specify a formula for *RCR*. Proportional treaties often have sliding-scale or *contingent commissions*. In these cases, the reinsurer pays the cedant a *provisional commission* on the reinsurance gross written premium as it is transferred to the reinsurer. At suitable dates (often quarterly), the cumulative experience on the treaty (usually from the beginning if there is a *deficit carryforward*, or over some period such as three years) is reviewed. If it is profitable, the reinsurer pays the cedant an additional commission; if it is unprofitable, the cedant returns some of the provisional commission to the reinsurer. An example should clarify this.

Formula 7.28: Sliding Scale Commission Arrangement

$$RCR = PRCR - SF \times \left\{ \frac{RL}{RP} - (1 - PRCR - RM) \right\}$$

Subject to $\min RCR \leq RCR \leq \text{Max}\, RCR$

where $PRCR$ = provisional reinsurance commission rate

= 33%

SF = slide factor = 50%

RM = reinsurer's margin = 5%

in RCR = minimum RCR = 25%

$\text{Max}\, RCR$ = maximum RCR = 35%

This ceding commission formula varies the commission one half point with each loss ratio point, with a minimum of 25% and a maximum of 35%. Loss development is an issue to be aware of in these types of formulas. If it is not accounted for in the loss evaluation that determines the loss ratio, the formula will compute too high a commission. Thus, the reinsurer will normally be in a position of paying some commission that will eventually be returned. This may usually be a minor point, but it does effect the reinsurer's cash flow, and thus economic profit, on the contract.

If we use the Gamma distribution with the first and second moments estimated in formula 7.27, then the expected reinsurance profit becomes \$108,512, as shown in Table 7.29. This is still less than the desired profit margin of \$153,080. Many reinsurers who think that the da Ponte Insurance Company is or will be a good client, and thus want to deal with them in the future, will accept these terms. Or the reinsurer may request a change in the commission terms: perhaps a reduction in *PRCR* together with an increase in *RM*.

Please note that the table treats the Gamma as being discreet, and displays probabilities for only selected loss ratios; the computation of the average profit uses the more complete distribution shown in Appendix C. Also note the nonsymmetry of the probabilities in column 2; the skewed right-hand tail is typical of property and casualty aggregate loss distributions.

A more sophisticated evaluation would account for cash flow timing, e.g., when commission adjustments would be made. On the basis of the distribution of the reinsurer's profit displayed in columns 2 and 4, we may specify a different target *RTER*. Suppose this profit distribution is more compact than usual for contracts of this type. This would indicate that there is less risk than usual, thus indicating that a lesser *RTER* value is appropriate. If the profit distribution is more spread than usual, thereby indicating more risk, then a greater value is appropriate.

Step 13: Negotiate, reconcile opinions and estimates, alter terms and finalize

We now have contract terms that meet our underwriting standards and largely meet our profit guidelines. We also believe these terms will be acceptable to da Ponte Insurance Company. Then we meet with them and explain our analysis and evaluation,

TABLE 7.29

AGGREGATE LOSS DISTRIBUTION AND CALCULATION OF REINSURER'S PROFIT

(1) Loss Ratio (selected)	(2) Probability Density (as percent)	(3) Reinsurance Commission	(4) Reinsurer's Profit
20%	0.4%	35.0%	$1,074,750
25%	3.5%	35.0%	949,750
30%	16.1%	35.0%	824,750
35%	47.2%	35.0%	699,750
40%	100.3%	35.0%	574,750
45%	166.2%	35.0%	449,750
50%	226.5%	35.0%	324,750
55%	263.3%	35.0%	199,750
60%	268.5%	34.0%	99,750
65%	245.4%	31.5%	37,250
70%	204.2%	29.0%	(25,250)
75%	156.8%	26.5%	(87,750)
80%	112.2%	25.0%	(175,250)
85%	75.5%	25.0%	(300,250)
90%	48.1%	25.0%	(425,250)
95%	29.2%	25.0%	(550,250)
100%	16.9%	25.0%	(675,250)
105%	9.5%	25.0%	(800,250)
110%	5.1%	25.0%	(925,250)
115%	2.7%	25.0%	(1,050,250)
120%	1.3%	25.0%	(1,175,250)
125%	0.7%	25.0%	(1,300,250)
130%	0.3%	25.0%	(1,425,250)
135%	0.1%	25.0%	(1,550,250)
140%	0.1%	25.0%	(1,675,250)
145%	0.0%	25.0%	(1,800,250)
150%	0.0%	25.0%	(1,925,250)
			$108,512

answer their questions and listen to their counter-arguments and alternative interpretations of information and data. If necessary, we redo the previous steps until we reach a contract structure that satisfies both parties, if possible.

Evaluating an Ongoing Treaty

A reinsurer is in a similar evaluation situation when interpreting the experience on an ongoing treaty. But in addition to evaluating the year 2001 profit potential, we must also evaluate our cumulative experience on the treaty and compare the cumulative profit to our guideline *RTER* (including the charge for year-end statutory surplus loans, if any). The advantage is that we now have our own experience data on the contract for some years, and we also know the cedant and their people better. The questions are similar to the previous pencil-sharpening exercise. How much money do we think we have made, or lost? Do we have other business with this cedant? What is the total account bottomline profit? Is this a good relationship to continue? How credible is our answer?

Property Surplus Share Treaties

A property surplus-share treaty is somewhat more difficult to evaluate. Since the reinsurer does not provide coverage for small insureds, and covers larger insureds in proportion to their size above some fixed retention, the reinsurer must be more concerned with the cedant's pricing of larger insureds. An example should clarify this.

Example 7.30:

- Four line first surplus not to exceed $800,000
- Maximum cedant retention = $200,000

  Then the following statements are true:

- Maximum reinsurance limit per policy = $800,000
- For a policy with limit < $200,000, the reinsurer receives no premium and pays no losses.

- For a policy with limit = $500,000, the reinsurer receives 60% of the policy's premium less ceding commission and brokerage fee, and pays 60% of the policy's losses.
- For a policy with limit = $1,000,000, the reinsurer receives 80% of the policy's premium less ceding commission and brokerage fee, and pays 80% of the policy's losses.
- For a policy with limit = $2,000,000, the reinsurer receives 40% of the policy's premium less ceding commission and brokerage fee, and pays 40% of the policy's losses.

It is easy to see that, given this complicated proportional structure depending upon the limit of each policy, the premium and loss accounting for a surplus-share treaty is somewhat complex. Despite this, surplus-share treaties are popular, because they provide more large loss protection than a quota-share, and are much easier for the reinsurer to evaluate and price (usually only the ceding commission and slide is the subject of negotiations) than an excess treaty.

A surplus-share treaty is generally riskier relative to ceded premium volume than is a simple quota-share. So the reinsurer will charge a correspondingly higher margin for risk assumption.

Casualty Quota-Share Treaties

The pricing or evaluation of a true ground-up net retention quota-share on casualty exposure would be similar to the pricing of a property cover except that the reinsurer would have to be very careful about loss development. The historical catastrophe exposure would be asbestos and pollution clean-up, plus any other mass tort situation that significantly distorts the cedant's claims data patterns. The future catastrophe exposure would be ________ (Fill in the blank: Tobacco? Medical implant devices? Workers compensation claims arising from a major earthquake during working hours? Etc.). The additional uncertainty arising from the longer-tail claims development and trend estimation would add risk to the rate determination, thus tending to increase the reinsurer's *RTER*. But the final evaluation of risk and necessary *RTER* also depends upon the relative catastrophe exposure.

Working Cover Excess Treaties

A working cover is an excess layer where losses are expected. An excess cover is usually riskier than a proportional cover. So the reinsurer will be more mindful of prediction error and fluctuation potential, and will charge a higher *RTER* for assuming this risk. If losses are covered per-occurrence, then the reinsurer is exposed by policy limits below the attachment point because of the "clash" of losses on different policies or coverages arising from the same occurrence.

The reinsurance premium is usually calculated via a reinsurance rate times subject premium. However, for liability coverage, it may be on an increased limits premium collected basis; this is often the premium calculation method used for facultative automatic programs. Here the total reinsurance premium is the sum of the individually calculated reinsurance premiums for each policy.

Working cover treaties are often large enough so that many of the risk parameters can be estimated either directly from the exposure and claims history or by a credibility weighting of the history with more general information. Ideally, the reinsurance pricing consists of both an exposure rating and an experience rating, together with a reconciliation of the two rates. We will illustrate the differences from pricing either facultative certificates or proportional treaties as we deal with an example. We will use a casualty example. Pricing a property per-risk excess working cover would be similar.

Example 7.31:

- da Ponte Insurance Company wants a proposal for a three-year retrospective-rated treaty incepting Jan. 1, 2001
- All liability and workers compensation exposure

- Losses occurring coverage (accident year basis)
- Per-occurrence coverage
- *ALAE* added to indemnity for each claim
- Proposed attachment point *AP* = $300,000
- Proposed reinsurance limit *RLim* = $700,000
- *RCR* = 0%
- *RBF* = 0%
- Estimated 2001 subject premium *PCP* = $100,000,000
- Possible reinsurance premium range up to $10,000,000

We will follow the same rating steps we did for Example 7.24.

Step 1: Gather and reconcile primary exposure, expense, and rate information segregated by major rating class groups.

Da Ponte's casualty exposure is as follows.

TABLE 7.32

CASUALTY EXPOSURE CATEGORIES

• Private passenger automobile
• Commercial automobile
• Premises/operations
• Homeowners Section II
• Special Multi-Peril Section II
• Workers compensation
• Personal umbrella

These categories should be further broken down to split the underlying exposure at least according to the most significant applicable increased limit tables and policy limits, or, in the case of workers compensation, by major states and hazard groups. If we cannot get data from da Ponte to do this, we can, as a last resort, use default distributions of premiums by increased limit tables and policy limits for this type of company, that we have estimated from general information.

We should want to start with the information listed in Step 1 for Example 7.24. In addition, we assume that our underwriters have visited da Ponte and have performed an underwriting review. We want to know about deviations from bureau manual rates and average schedule and experience credits. We want historical premiums for at least the last five-and-one-half years 1995 through June 30, 2000 plus predictions for 2001–2003. We also want the names of contact people at da Ponte to talk with, in particular, their pricing actuary.

Step 2: Calculate an exposure expected loss cost, PVRELC, and, if desirable, a loss cost rate, PVRELC/PCP.

Da Ponte writes limits up to $10,000,000, but purchases facultative reinsurance for coverage in excess of $2,000,000. They also purchase facultative coverage above $300,000 for any difficult exposures on the reinsurer's exclusion list, and a 90% facultative automatic cover for their umbrella programs.

Treaty excess exposure rating differs from the facultative certificate excess pricing in Example 7.19 in that the reinsurer deals with broad classes of business by increased limit table or state/hazard group instead of individual insureds. We consider manual rate relativities to bureau rates and/or to other companies writing the same exposure, and evaluate da Ponte's experience and schedule rating plans and pricing abilities. The increased limit factors used by the cedant for liability coverages are especially important. The same Formulas 7.2 and 7.6 can be used. Since the coverage is per-occurrence, we must load the manual difference rate for the clash exposure.

Since *ALAE* is added to each claim in order to determine the reinsurer's excess share, claims from some policy limits below the attachment point will bleed into the excess layer. We may have our own data describing the bivariate distribution of indemnity and *ALAE*, or we may obtain such information from ISO. Using these data, we can construct increased limits tables where *ALAE* is added to loss instead of residing entirely in the basic limits coverage.

Alternatively, and more simplistically, we can adjust the manual increased limits factors to account for the addition of *ALAE* to loss. A simplistic way of doing this is to assume that the *ALAE* for each claim is a deterministic function of indemnity amount for the claim, adding exactly $\gamma\%$ to each claim value for the range of claim sizes that are near the layer of interest. Note that this γ factor is smaller than the overall ratio of *ALAE* to ground-up indemnity loss, because much of the total *ALAE* applies to small claims or claims closed with no indemnity. In our example then, we hypothesize that when *ALAE* is added to loss, every claim with indemnity greater than $\$300{,}000/(1+\gamma)$ penetrates the layer \$700,000 excess of \$300,000, and that the loss amount in the layer reaches \$700,000 when the ground-up indemnity reaches $\$1{,}000{,}000/(1+\gamma)$. From this you can see how to modify standard increased limits factors to account for *ALAE* added to loss. In this liability context, Formula 7.6 can be reinterpreted with *PCP* as the basic limit premium and *PCPLR* as the primary company permissible basic limits loss ratio. Given the clash exposure, suppose we believe that for this type of coverage, an overall loss loading of $\delta\%$ is sufficient to adjust the loss cost for this layer estimated from the stand-alone policies. Then *ELCF* calculates the excess loss in the layer \$700,000 excess of \$300,000 arising from each policy limit (plus its contribution to clash losses) as a percent of basic limits loss arising from the same policy limit. The formula for *ELCF* evaluated at limit, Lim, is displayed in the following formula.

Formula 7.33: Liability ELCF for ALAE Added To Indemnity Loss

$$ELCF(Lim) = 0$$

$$\text{if} \quad Lim \leq \frac{AP}{1+\gamma}$$

$$ELCF(Lim) = (1+\delta)\times(1+\gamma)\times\left\{ILF(Lim) - ILF\left(\frac{AP}{1+\gamma}\right)\right\}$$

$$\text{if}\quad \frac{AP}{1+\gamma} < Lim \leq \frac{AP+RLim}{1+\gamma}$$

$$ELCF(Lim) = (1+\delta)\times(1+\gamma)$$
$$\times\left\{ILF\left(\frac{AP+RLim}{1+\gamma}\right) - ILF\left(\frac{AP}{1+\gamma}\right)\right\}$$

$$\text{if}\quad \frac{AP+RLim}{1+\gamma} < Lim$$

where AP = attachment point = \$300,000

$RLim$ = reinsurance limit = \$700,000

δ = clash loading = 5%

γ = excess $ALAE$ loading = 20%

Table 7.34 displays this simplistic method for a part of da Ponte's *GL* exposure using hypothetical increased limits factors (excluding both *ALAE* and risk load) to calculate excess loss cost factors. In Table 7.34, we see that policies with limits \$300,000, \$500,000 and \$1,000,000 and above expose the excess layer.

TABLE 7.34

EXCESS LOSS COST FACTORS WITH *ALAE* ADDED TO INDEMNITY LOSS @ 20% ADD-ON AND A CLASH LOADING OF 5%

(1) Policy Limit	(2) *ILF* w/o risk load and w/o *ALAE*	(3) *ELCF*
\$ 100,000	1.0000	0
250,000	1.2386	0
300,000	1.2842	0.0575
500,000	1.4084	0.2139
833,333	1.5271	0.3635
1,000,000 or more	1.5681	0.3635

Using Formula 7.33, we calculate $ELCF(\$300{,}000) = 1.05 \times 1.20 \times (1.2842 - 1.2386) = 0.0575$, and $ELCF(\$1{,}000{,}000) = 1.05 \times 1.20 \times (1.5271 - 1.2386) = 0.3635$. Back up for Table 7.34 is contained in Appendix D.

Suppose da Ponte's permissible basic limit loss ratio for this exposure is $PCPLR = 65\%$. Suppose our evaluation indicates that their rates and offsets are adequate, so $RCF = 1.00$. Then we can translate Table 7.34 into an exposure rate *RELC*, the reinsurer's estimate of loss cost (undiscounted) in the excess layer as shown in Table 7.35.

TABLE 7.35

RELC (REINSURANCE EXPECTED LOSS COST (UNDISCOUNTED))

(1) Policy Limit	(2) Estimated Subject Premium Year 2000	(3) Manual *ILF*	(4) Estimated Basic Limit Loss Cost $0.65 \times (2)/(3)$	(5) *ELCF* from 6.37	(6) *RELC* $(4) \times (5)$
Below \$ 300,000	\$1,000,000	1.10 (avg.)	\$ 590,909	0	\$0
300,000	1,000,000	1.35	481,482	0.0575	27,664
500,000	1,000,000	1.50	433,333	0.2139	92,711
1,000,000 or more	2,000,000	1.75 (avg.)	724,857	0.3635	270,036
Total	\$5,000,000	n.a.	\$2,248,810	n.a.	\$390,411

The estimation of *RELC* for other categories would be similar. For liability exposure categories where increased limit factors are not published, use judgment to assign appropriate factors. For workers compensation, excess loss cost factor differences would be weighted by estimated subject premium by hazard group, by major state grouping, or the underlying claim severity distributions could be estimated by state, by hazard group. We would then combine these *RELC* estimates with our estimates of loss discount factors for each exposure in order to calculate *PVRELC*.

A better way of estimating an exposure loss cost is to work directly from probability models of the claim size distributions. This directly gives us claim count and claim severity information to use in our simple risk theoretic model for aggregate loss. Suppose we know that the indemnity loss distribution underlying Table 7.34 is Pareto with $b = 5{,}000$ and $q = 1.1$. Then our simple model of adding 20% *ALAE* to indemnity per claim (per-occurrence) changes the indemnity Pareto distribution to another Pareto with $b = 5{,}000 \times 1.20 = 6{,}000$ and $q = 1.1$. Please note that these parameters are selected simply to make the computations easier for you to check. We then multiply by $1 + \delta = 1.05$ to adjust the layer severity for clash.[4] We can then calculate excess expected claim sizes from each policy limit. Dividing by the *RELC* for each limit yields estimates of expected claim count. This is done in Table 7.36.

TABLE 7.36

EXCESS EXPECTED LOSS, CLAIM SEVERITY, AND COUNT

(1) Policy Limit	(2) *RELC*	(3) Expected Claim Size (App. E)	(4) Expected Claim Count (2)/(3)
$ 300,000	$ 27,664	$ 57,016	0.485
500,000	92,711	212,210	0.437
1,000,000 or more	270,036	360,517	0.749
Total	$390,411	$233,624	1.671

Dividing *RELC* by the estimate of the total expected claim count, we can back into an estimate of the total excess expected claim size of $233,624 for this exposure. Assuming indepen-

[4]One may argue that clash affects the excess claim frequency, not the excess claim severity. The truth is that it affects both. Here, for simplicity, we only adjust the excess claim severity.

dence of claim events across all exposures, we can also add to obtain estimates for the expected claim (occurrence) count and overall excess expected claim (occurrence) size.

Now we turn to experience rating.

Step 3: Gather and reconcile primary claims data segregated by major rating class groups.

As in Example 7.24, we want the claims data segregated as the exposure data, and we want some history of individual large claims. We usually receive information on all claims greater than one-half the proposed attachment point, but, the more data, the better. Suppose a claims review has been performed. In our example, suppose we receive a detailed history for each known claim larger than \$100,000 occurring 1990–2000, evaluated $12/31/90, 12/31/91, \ldots, 12/31/99, 6/30/00$.

Step 4: Filter the major catastrophic claims out of the claims data.

In our example, we want to identify clash claims, if possible, and any significant mass tort claims. By separating out the clash claims, we can estimate their frequency and size relative to non-clash claims, and compare these statistics to values we know from other cedants, thus enabling us to get a better estimate for our δ loading. It should be obvious that the mass tort claims need special treatment.

Step 5: Trend the claims data to the rating period.

As in Example 7.24, the trending should be for inflation and for other changes in exposure (such as higher policy limits) that might affect the loss potential. Unlike proportional coverage, we may not skip this step. The reason is the leveraged effect of inflation upon excess claims: a constant inflation rate increases the aggregate loss above any attachment point faster than the aggregate loss below, because claims grow into the excess layer, while their value below is stopped at the attachment point. We trend each ground-up claim value at each evaluation, including

ALAE, from year of occurrence to 2001. For example, consider the treatment of a 1993 claim in Table 7.37.

TABLE 7.37

TRENDING AN ACCIDENT YEAR 1993 CLAIM

(1) Evaluation Date	(2) Value at Evaluation	(3) Trend Factor	(4) 2001 Level Value	(5) Excess Amount
12/31/93	$0	1.59	$0	$0
12/31/94	0	1.59	0	0
12/31/95	125,000	1.59	198,750	0
12/31/96	125,000	1.59	198,750	0
12/31/97	150,000	1.59	238,500	0
12/31/98	200,000	1.59	318,000	18,000
12/31/99	200,000	1.59	318,000	18,000
6/30/00	200,000	1.59	318,000	18,000

Note the use of a single trend factor. The reasoning here is that the trend affects claim values according to accident date (or year), not by evaluation date. Of course a more sophisticated model for claims inflation could be used. A delicate issue is the trending of policy limits. If a 1993 claim on a policy with limit less than $250,000 inflates to above $300,000 (including *ALAE*), would the policy limit sold in 2001 be greater than $250,000, so to allow this excess claim (including *ALAE* @ 20%)? The da Ponte underwriter and your own marketing people may argue that the policy limit does not change. But, over long time periods, it would appear that the answer is that policy limits do change with inflation. If possible, information on the da Ponte's policy limit distributions over time should be obtained. If this is a real issue, you can try some sensitivity testing on the extreme alternatives:

1. The historical policy limits change with claims inflation.
2. The historical policy limits remain constant.

Then reach some judgmental compromise.

Step 6: Develop the claims data to settlement values.

We want to construct historical accident year/development year triangles from the data produced in column 5 of Table 7.37 for each type of large claim. We would usually combine together all claims by major line of business. Then we can use standard methods to estimate loss development factors and apply these to the excess claims data. We also want to compare the development patterns estimated from da Ponte's data to what we expect based upon our own historical data for comparable coverages, in order to check for reasonableness. This may be a problem, even for filtered claims, if the historical data has insufficient credibility. Consider the claim in Table 7.37: only $18,000 exceeds the attachment point, and only at the fifth development point. Suppose our triangle looks like Table 7.38.

TABLE 7.38

TRENDED HISTORICAL CLAIMS
IN THE LAYER $700,000 EXCESS OF $300,000
(IN $1,000'S)

	Development Year						
Acc. Year	Age 1	Age 2	Age 3	…	Age 9	Age 10	Age 10.5
1990	$0	$80	$254	…	$259	$321	$321
1991	0	0	148	…	743	788	
⋮	⋮	⋮	⋮				
1998	57	117	236				
1999	0	0					
2000	0						
ATA	4.236	1.573	1.076	…	1.239	n.a.	n.a.
ATU	15.026	3.547	2.255	…	1.301	1.050	= tail
Smoothed Lags	10.9%	28.7%	45.7%	…	93.1%	95.3%	96.1%

where *ATA* = Age-To-Age development factor

ATU = Age-To-Ultimate development factor

Lag(t) = percentage of loss reported at time t

The tail factor of 1.05 is selected based upon general information about development beyond ten years for this type of exposure.

By switching the point of view from age-to-ultimate factors to their inverse, time lags of claim dollar reporting, as we do later in the Loss Reserving section, we transform the loss reporting view to that of a cumulative distribution function (cdf) whose domain is $[0,\infty)$. This gives us a better view of the loss development pattern. It allows us to consider and measure the average (expected) lag and other moments, which can then be compared to the moments of the loss development patterns from other exposures.

Since excess claims development is almost always extremely chaotic, it is a good idea to employ some kind of smoothing technique, as discussed in the later loss reserving section. If properly estimated, the smoothed factors should yield more credible loss development estimates. They also allow us to unambiguously evaluate the function Lag( ) at any positive time. This is handy, since our latest data evaluation point is 6/30/00, and we want to use these data. The smoothing introduced in the last row of Table 7.38 is based upon a Gamma distribution with mean 4 (years) and standard deviation 3. This model is based upon the data, plus our judgment regarding the general pattern we should expect for loss development, the shape of the cdf. A more sophisticated approach could estimate parameters for the Gamma distribution directly from the development data via the method of Maximum Likelihood Estimation by treating the data for each accident year as a sample truncated at the last age.

Frequently, it is also useful to analyze large claim paid data, if they are available, both to estimate excess claims payment patterns and to supplement the ultimate estimates based only upon the reported (incurred with no *IBNR*) claims used above. It is sometimes true that the paid claims development, although it is of longer duration, is more stable than the reported claims development.

Occasionally, only aggregate excess claims data are available. This would be an historical accident year/development year \$700,000 excess of \$300,000 aggregate loss triangle. Pricing in a situation like this, with no specific information about large claims, is very risky, but it is sometimes done. Technically, in such a situation, the *RTER* should be higher because of the added risk because of greater (mis-)information uncertainty. But often, the opposite occurs because of marketing pressures.

Step 7: Estimate the catastrophic loss potential.

It should be obvious that mass tort claims need special treatment. Asbestos and pollution clean-up claims, if any, distort the historical data. As is done for property coverage, an analysis of da Ponte's exposures can allow us to guess some suitable loading for future mass tort claim potential.

As we said in Step 4, we want to identify clash claims, if possible. By separating out the clash claims, for each claim, we add its various parts together to apply properly the attachment point and the reinsurance limit to the occurrence loss amount. If we cannot identify the clash claims, then our experience estimate of *RELC* must include a clash loading based upon judgment for the general type of exposure.

Step 8: Adjust the historical exposures to the rating period.

As we did in Example 7.24, we want to be sure that the historical exposure (premium) data are adjusted in a manner that makes them reasonably relevant to the rating period. The trending should be for primary rate and underwriting changes and for other changes in exposure that might affect the loss potential. Table 7.39 shows such a calculation.

Step 9: Estimate an experience expected loss cost, PVRELC, and, if desirable, a loss cost rate, PVRELC/PCP.

Suppose we now have trended and developed excess losses for all classes of da Ponte's casualty exposure. The usual practice is

TABLE 7.39

GL PREMIUM ADJUSTED TO 2001 LEVEL (IN $1,000'S)

(1) Accident Year	(2) *PCP*	(3) Rate Change	(4) Deviation from Manual	(5) Exposure Inflation Growth	(6) Written Premium Growth	(7) Earned Premium Growth	(8) Cumulative On-level Factor	(9) Adjusted 2001-level *PCP*
1990	$ 5,829	3.2%	4.6%	2.9%	101.3%	n.a.	n.a.	n.a.
1991	6,045	2.5%	7.3%	2.8%	103.7%	n.a.	n.a.	n.a.
1992	6,095	1.9%	7.6%	1.9%	107.4%	103.9%	69.8%	$ 8,733
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1996	14,484	3.7%	8.3%	1.7%	137.6%	129.4%	86.9%	16,661
1997	16,114	2.6%	9.1%	1.6%	142.2%	137.1%	92.1%	17,488
1998	16,458	1.3%	10.2%	1.4%	144.3%	141.9%	95.3%	17,265
1999	16,810	0.5%	11.4%	1.3%	144.9%	144.1%	96.8%	17,362
2000 (@ 6/30)	8,346	1.0%	11.0%	1.0%	148.5%	145.3%	97.6%	8,549
2001	n.a.	2.0%	10.0%	1.0%	154.7%	148.8%	100.0%	n.a.

Notes:

$$\text{Col. (6)}[199N] = (1-(4)) \times \{(1+(3)[1990] \times (1+(5)[1990]) + (1+(3)[1991] \times (1+(5)[1991]) + \cdots$$
$$+ (1+(3)[199(N-1)]) \times (1+(5)[199(N-1)]) + (1+(3)[199N] \times (1+(5)[199N])\}$$

$$\text{Col. (7)}[199N] = 0.125 \times \{(6)[199N] + (6)[199(N-2)] + 0.75 \times (6)[199(N-1)]$$

$$\text{Col. (8)}[199N] = \frac{(7)[199N]}{(7)[2001]}$$

$$\text{Col. (9)} = \frac{(2)}{(8)}$$

to add up the pieces to assemble an exhibit that looks like Table 7.40.

TABLE 7.40

DA PONTE INSURANCE COMPANY CASUALTY BUSINESS (IN $1,000'S)

(1) Accident Year	(2) Onlevel *PCP*	(3) Trended and Developed Loss Excess Loss (Estimated *RELC*)	(4) Estimated Cost Rate (3) ÷ (2)
1992	$ 85,847	$ 3,357	3.91%
1993	87,953	4,644	5.28%
1994	89,076	6,761	7.59%
1995	92,947	5,410	5.82%
1996	94,172	4,567	4.85%
1997	95,674	3,329	3.48%
1998	98,561	4,268	4.33%
1999	99,226	6,420	6.47%
2000	49,750	1,413	2.84%
Total	$793,206	$40,168	5.06%
Total w/o 2000	$743,456	$38,755	5.21%

The eight-year average loss cost rate, eliminating 2000 as being too green (undeveloped), is 5.21%. There seems to be no particular trend from year to year. If there were, we would want to see what its extrapolation to 2001 might be. The standard deviation is 1.37%. These estimates seem to be quite stable. This is not always the case.

Alternative Estimation Methods and Some Potential Problems

Problem: Chaotic excess development

Sometimes the loss development data for the layer $700,000 excess $300,000 for some of the categories of exposure are more chaotic than we have seen in this example, so that we can't estimate model parameters with enough confidence. We have alternatives:

1. We can use our general loss development patterns for these types of exposure, this attachment point.
2. We can experience rate a lower layer, where we may have more stable, more credible development patterns.

For alternative 2, for example, perhaps we would drop down to experience rate the layer \$800,000 excess of \$200,000, or \$300,000 excess of \$200,000. We would then calculate the *RELC* for \$700,000 excess of \$300,000 for each exposure category by an extrapolation employing excess layer relativities derived from claim severity curves.

Problem: Incomplete excess claims sample

If the cumulative trend factor for some exposure for some historical accident year exceeds the ratio of the attachment point divided by the lower bound of the large claims, then we will have an incomplete excess sample extrapolated to the year 2001. Let us illustrate this point with the 1993 accident year. Suppose we were to drop down to experience rate the layer \$150,000 excess of \$150,000 for 2001. Returning to the claim trending in Table 7.38, since our 1993 trend factor is 1.59, the 2001 values of all the 1993 claims greater than \$100,000 will lie above \$159,000 = $1.59 \times$ \$100,000. So we are missing claims whose 2001 values would be in the interval between \$150,000 and \$159,000. So our trended claims data from 1993 are representative of the layer \$141,000 excess of \$159,000, not the layer \$150,000 excess of \$150,000.

Alternate Estimation Method

With the individual large claims data we have, a more sophisticated methodology would have us look at excess claim count development and excess claim size development separately. If the data give us reasonably stable development indications, we readily obtain claim count and claim size estimates useful for the simple risk theoretic model for excess aggregate loss. With a sufficient number of large claims, we can even estimate excess claim size distribution parameters directly from the data. These can then be compared to general parameters we have to reach credible models for excess claim size.

Problem: Free cover

Also, an analysis that deals with individual claims may uncover problems buried in the simpler excess aggregate loss estimation. One problem may be that of so-called "free cover." This arises when, in our example, the maximum trended historical claim value is \$794,826, for instance. Then the experience rate loss cost estimate for the layer \$700,000 excess of \$300,000 is the same as the experience rate loss cost estimate for the layer \$500,000 excess of \$300,000. So we would be charging \$0 for the layer \$200,000 excess of \$800,000. It may be the case that there are many claims whose 2001 values are close to \$800,000, and it is simply by chance that none of them exceeded \$800,000. In this case, you may simply let the experience rate loss cost estimate stand. If this is not the case, then it looks like an insufficient rate is being calculated for coverage excess of \$800,000, and consequently a different tactic is called for. We can estimate the loss cost for the layer \$200,000 excess of \$800,000 via a relative exposure rate extrapolation from the layer \$500,000 excess of \$300,000. Judgment is important here, together with a review of da Ponte's higher policy limits exposure and a review of the types of historical large claims that might pierce the layer excess of \$800,000.

RELC and PVRELC and PVRELC/PCP

Let us complete our estimation of the reinsurance expected loss cost, discounted reinsurance expected loss cost, and the reinsurance loss cost rate. Table 7.40 gives us an experience-based estimate, $RELC/PCP = 5.21\%$. However, this must be loaded for whatever mass tort exposure exists, and also loaded for clash claims if we judge that we had insufficient information on clash claims in the claims data. A more sophisticated approach would add in the catastrophe loss exposure rate, like we did in Example 7.24 for the property catastrophe loss.

Step 10: Estimate a "credibility" loss cost or loss cost rate from the exposure and experience loss costs or loss cost rates.

We must also weigh the experience loss cost rate against the exposure loss cost rate we calculated. If we have more than one answer, and the various answers differ but cannot be further reconciled, final answers for $700,000 excess of $300,000 claim count and severity can be based upon a credibility balancing of the separate estimates. However, all the differences should not be ignored, but should indeed be included in your estimates of parameter (and model) uncertainty, thus giving rise to more realistic measures of variances, etc., and of risk.

Suppose we are in the simple situation, where we are only weighing together the exposure loss cost estimate and the experience loss cost estimate. In Table 7.41 we list six considerations for deciding how much weight to give to the exposure loss cost estimate. You can see that the credibility of the exposure loss cost estimate is decreased if there are problems with any of the six items.

Likewise, in Table 7.42 we list six considerations for deciding how much weight to give to the experience loss cost estimate. You can see that the credibility of the experience loss cost estimate is lessened by problems with any of the six items.

Appendix F has a more detailed discussion of the items in Tables 7.41 and 7.42.

Let us assume that our credibility loss cost rate is $RELC/PCP = 5.73\%$.

For each exposure category, we estimate a loss discount factor. This is based upon the expected loss payment pattern for the

TABLE 7.41

Items to Consider in Determining the Credibility of the Exposure Loss Cost Estimate

- The accuracy of the estimate of *RCF*, the primary rate correction factor, and thus the accuracy of the primary expected loss cost or loss ratio
- The accuracy of the predicted distribution of subject premium by line of business
- For excess coverage, the accuracy of the predicted distribution of subject premium by increased limits table for liability, by state for workers compensation, or by type of insured for property, within a line of business
- For excess coverage, the accuracy of the predicted distribution of subject premium by policy limit within increased limits table for liability, by hazard group for workers compensation, by amount insured for property
- For excess coverage, the accuracy of the excess loss cost factors for coverage above the attachment point
- For excess coverage, the degree of potential exposure not contemplated by the excess loss cost factors

TABLE 7.42

Items to Consider in Determining the Credibility of the Experience Loss Cost Estimate

- The accuracy of the estimates of claims cost inflation
- The accuracy of the estimates of loss development
- The accuracy of the subject premium on-level factors
- The stability of the loss cost, or loss cost rate, over time
- The possibility of changes in the underlying exposure over time
- For excess coverage, the possibility of changes in the distribution of policy limits over time

exposure in the layer $700,000 excess of $300,000, and upon a selected investment yield. Most actuaries advocate using a risk-free yield, usually U.S. Treasuries for U.S. business, for a maturity approximating the average claim payment lag. Discounting is only significant for longer tail business. So for simplicity, on a practical basis, it's better to use a single, constant fixed rate for a bond maturity between five to ten years. But of course this selection is entirely up to the reinsurer's pricing philosophy.

Let us suppose that the overall discount factor for our loss cost rate of 5.73% is $RDF = 75\%$, giving $PVRELC/PCP = RDF \times RELC/PCP = 0.75 \times 5.73\% = 4.30\%$, or $PVRELC = 4.3\% \times \$100{,}000{,}000 = \$4{,}300{,}000$.

Note that we will reverse steps 11 and 12.

Step 12: Specify values for RCR, RIXL, and RTER

Suppose our standard guidelines for this type and size of contract and this type of exposure specify $RIXL = 5\%$ and $RTER = 15\%$. We can then calculate the reinsurance pure premium of $RPP = PVRELC/(1 - RTER) = \$4{,}300{,}000/0.85 = \$5{,}058{,}823$, with an expected profit of $RPP - PVRELC = \$5{,}058{,}823 - \$4{,}300{,}000 = \$758{,}823$ for the risk transfer. Since $RCR = 0\%$, we have an indicated technical reinsurance premium of $RP = RPP/(1 - RIXL) = \$5{,}058{,}823/0.95 = \$5{,}325{,}077$. This technical premium is above the maximum of $5,000,000 specified by da Ponte. Assuming that there is nothing wrong with our technical calculations, the reinsurer has at least two options:

1. We can accept an expected reinsurance premium of $5,000,000 at a rate of 5%, with a reduced expected profit (assuming our internal expenses stay at $5,325,077 – $5,058,823 = $266,254) of $5,000,000 – $4,300,000 – $266,254 = $434,746.

2. We can offer a variable rate contract, where the reinsurance rate varies according to the reinsurance loss experience, in this case a retrospectively rated contract.

Let us select the more interesting second option, especially since, in this example da Ponte is asking for a retrospectively rated contract. In order to construct a balanced and fair rating plan, we need once again to estimate the distribution of the reinsurance aggregate loss. So we proceed with step 11.

Step 11: Estimate the probability distribution of the aggregate reinsurance loss if desirable, and perhaps other distributions such as for claims payment timing.

We will again use a Gamma distribution approximation. But this time, in this lower (excess) claim frequency situation, we will obtain a better approximation of the distribution of aggregate reinsurance loss using the standard risk theoretic model for aggregate losses together with the first two moments of the claim count and claim severity distributions.

The Standard Risk Theoretic Model for the Distribution of Aggregate Loss

The standard model writes the aggregate loss naturally as the sum of the individual claims (or events) as follows.

Formula 7.43: Aggregate Loss

$$L = X_1 + X_2 + \cdots + X_N$$

where $L = rv$ (random variable) for aggregate loss

$N = rv$ for number of claims (occurrences, events)

$X_i = rv$ for the dollar size of the i^{th} claim

Here N and X_i refer to the excess number of claims and the amount of the i^{th} claim respectively. The standard risk theoretic model relates the distributions of L, N and the X_i's, as shown in Appendix G. Under the assumption that the X_i's are independent and identically distributed and also independent of N (reasonable independence assumptions), the k^{th} moment of L is completely determined by the first k moments of N and the X_i's. In particular, we have the following relationships.

Formula 7.44: First Two Central Moments of the Distribution of Aggregate Loss under the Standard Risk Theoretic Model

$$E[L] = E[N] \times E[X]$$

$$\text{Var}[L] = E[N] \times E[X^2] + (\text{Var}[N] - E[N]) \times E[X]^2$$

We start by assuming that $E[L] = RELC = 5.73\% \times$ \$100,000,000 = \$5,730,000 (undiscounted). We simplistically assume that the excess claim sizes are independent and identically distributed and are independent of the excess claim (occurrence) count. In most cases this is a reasonable assumption. For our layer \$700,000 excess of \$300,000, our modeling assumptions and results are shown in Formula 7.45, using the notation in Appendices A and G.

Formula 7.45: da Ponte \$700,000 Excess of \$300,000 Aggregate Loss Modeling Assumptions and Results

$$
\begin{aligned}
N(300) &\sim \text{Negative Binomial with} \\
E[N(300)] &= 24.64 \\
\text{Var}[N(300)] &= 2 \times E[N(300)] \\
X(300) &\sim \text{Pareto}(350, 2) \\
E[X(300); 700] &= \$232{,}543 \text{ censored at } 700 \\
E[X(300)^2; 700] &= 105.308 \times 10^6 \\
E[RL] &= E[N(300)] \times E[X(300); 700] \\
&= 24.64 \times \$232{,}543 \\
&= \$5{,}729{,}860 \\
\text{Var}[RL] &= E[N(300)] \times E[X(300)^2; 700] \\
&\quad + \{\text{Var}[N(300)] - E[N(300)]\} \times E[X(300); 700]^2 \\
&= E[N(300)] \times \{E[X(300)^2; 700] + E[X(300); 700]^2\} \\
&= 24.64 \times \{105.308 \times 10^6 + 232.543 \times 10^6\} \\
&= 24.64 \times \{159.384 \times 10^6\} \\
&= 3.9272 \times 10^9 \\
SD[RL] &= \$1{,}981{,}724 \\
\frac{SD[RL]}{E[RL]} &= 0.346
\end{aligned}
$$

More sophisticated modeling would more explicitly take into account modeling and parameter risks. A good general mathematical reference for "collective risk theory" is Bühlmann [9]. We will continue with our simple model here.

Let us now set up the following retrospective rating plan.

Formula 7.46: Retrospective Rate Plan

$$RP = RC + LF \times RL$$

$$\text{Subject to} \quad \min RP \leq RP \leq \text{Max}\, RP$$

$$\begin{aligned}
\text{where} \quad RP &= \text{final reinsurance premium} \\
PRP &= \text{provisional reinsurance premium} \\
&= \$4{,}000{,}000 \\
RC &= \text{reinsurance charge} = \$500{,}000 \\
LF &= \text{loss factor} = 1.00 \\
\min RP &= \text{minimum reinsurance premium} \\
&= \$2{,}000{,}000 \\
\text{Max}\, RP &= \text{maximum reinsurance premium} \\
&= \$8{,}000{,}000
\end{aligned}$$

RP, PRP, RC, $\min RP$, and $\text{Max}\, RP$ can also be stated as rates with respect to PCP. Sometimes a loss factor of 1.05 or 1.10 is used. You can see that these are all basic parameters you can play with in order to structure a balanced rating plan.

If we use the Gamma distribution obtained from $E[RL]$ and $\text{Var}[RL]$ in formula 7.45, then the expected reinsurance profit becomes \$745,075, as shown in Table 7.47. This is close enough to the desired profit margin of \$758,823 that any reinsurer will accept these terms.

TABLE 7.47

AGGREGATE LOSS DISTRIBUTION AND CALCULATION OF REINSURER'S PROFIT

(1) Loss Cost Rate (selected)	(2) Probability Density (as percent)	(3) Reinsurance Premium Rate	(4) Reinsurer's Profit
0.0%	0.0%	2.00%	$ 2,984
1.0%	5.2%	2.05%	2,234
2.0%	198.9%	3.10%	1,484
3.0%	914.1%	4.15%	1,109
4.0%	1765.6%	5.20%	1,109
5.0%	2120.8%	6.25%	1,109
6.0%	1886.3%	7.30%	1,109
7.0%	1363.7%	8.00%	734
8.0%	847.0%	8.00%	(16)
9.0%	468.5%	8.00%	(766)
10.0%	236.5%	8.00%	(1,516)
11.0%	110.9%	8.00%	(2,266)
12.0%	48.9%	8.00%	(3,016)
13.0%	20.5%	8.00%	(3,766)
14.0%	8.2%	8.00%	(4,516)
15.0%	3.2%	8.00%	(5,266)
16.0%	1.2%	8.00%	(6,016)
17.0%	0.4%	8.00%	(6,766)
18.0%	0.2%	8.00%	(7,516)
19.0%	0.1%	8.00%	(8,266)
20.0%	0.0%	8.00%	(9,016)
Expected		5.75%	$745,075

The simplified profit formula used in Table 7.47 is as follows.

Formula 7.48: Simplified Profit Formula

$$\text{Reinsurer's profit} = PRP - RIXL - RDF \times \{PRP - (RP - RL)\}$$

$$\text{where} \quad RIXL = \$266{,}000 \text{ (in dollars)}$$

This simple formula assumes that the single reinsurance claims payment, RL, and reinsurance premium adjustment, RP –

PRP, are simultaneous. A more sophisticated evaluation would account for cash flow timing, e.g., when premium adjustments would be made according to the premium formula timing in the contract.

Step 13: Negotiate, reconcile opinions and estimates, alter terms and finalize

We now have contract terms that meet our underwriting standards and also largely meet our profit guidelines. We also believe these terms will be acceptable to da Ponte Insurance Company. We then meet with da Ponte and explain our analysis and evaluation, answer their questions and listen to their counter-arguments and alternative interpretations of information and data. If necessary, we repeat the previous steps until we reach a contract structure that satisfies both parties, if possible.

If we cannot reach an agreement with da Ponte Insurance Company on the pricing of the layer \$700,000 excess of \$300,000, then the best bet is to recommend that the attachment point be increased to \$350,000 or \$400,000. Attachment points should naturally increase over time in an inflationary environment. In this example, the expected excess claim count of 24.64 ($E[N(300)]$) is fairly high for an excess working layer. The standard deviation of about 7 ($SD[N(300)]$) is fairly low relative to the expectation. Perhaps now is the time for an increase so that we and da Ponte aren't simply trading dollars for the more predictable claims just above \$300,000.

As with facultative covers, it is clear that much of the above can and should be programmed into an interactive computer package for the underwriters and actuaries. And it is also extremely important to monitor the results of large treaties and groups of treaties. The monitoring of the pricing experience and the monitoring of *IBNR* emergence and the reconciliation of both is important to the reinsurer.

Higher Exposed Excess Layer Treaties

Since there is policy limits exposure on these contracts, an exposure rate may be determined with the same general methodology as for a working cover. The higher the layer, the greater the relative significance of the workers compensation exposure, if any, and the greater the importance of clash and other multiple limit claims' exposure. The clash load δ in Example 7.31, Formula 7.33 must be larger. Since losses are not "expected" for these layers, historical loss data are sparse. And yet the layers have loss potential, or else cedants wouldn't buy reinsurance. An experience loss cost rate on a contract may be calculated by experience rating a working layer below, and using the working cover loss cost estimate as a basis for estimating the higher exposed layer rate via extrapolation using claim size probability distributions or non-risk-loaded increased limit factors.

Since claim frequency is lower than it is for a working layer, the risk transfer is greater relative to premium volume. Thus, *RTER*'s are greater.

Clash Treaties

Since there is no policy limit exposure, the usual exposure rating methodology does not work, except for evaluating workers compensation exposure. Prices for clash covers are usually set by market conditions. The actuarial prices are largely determined by very high *RTER*'s, and may or may not be close to the market-determined prices.

For clash layer pricing, the reinsurer should keep experience statistics on clash covers to see, in general, how often and to what degree these covers are penetrated, and to see if the historical market-determined rates have been reasonable overall, and also the degree to which rating cycles exist. The rates for various clash layers should bear reasonable relationships to one another, depending both upon the underlying exposures and upon the distances of the attachment points from the policy limits sold.

Underwriters sometimes view clash rates with regard to a notion of payback–the premium should cover one loss every m years for some selected m. These kinds of underwriting-judgment rates could be translated into judgmental expected loss cost rates plus *RIXL*'s and *RTER*'s to put them into a consistent actuarial context.

Property Catastrophe Treaties

The price for a catastrophe treaty should depend upon the attachment point, upon the cedant's accumulation of property exposure in localities prone to natural catastrophes, and upon the cedant's net position on each policy after all other reinsurance. Changes in the cedant's noncatastrophe net retentions may have a great effect upon the catastrophe exposure excess of a given attachment point. That is, a reinsurance program can be tricky: a small change here can have a big effect there.

The recent evolution of commercially available, large-scale computer simulation models for natural catastrophe exposures, especially hurricane and earthquake, has greatly increased the accuracy with which these exposures can be evaluated. Of course these models demand much more input information, such as property premium by zip code and information about construction, etc., than was the case for the old underwriter rules-of-thumb. The models are not yet perfect—it seems that after every major catastrophe, the model parameters are re-adjusted to reflect a revised opinion regarding event frequency and severity potential. But they are a significant step forward.

The reinsurer's actuaries should be knowledgeable about and be involved in the use of these models for both pricing reinsurance contracts and for the measurement of the reinsurer's aggregate accumulation of catastrophe exposure. This aggregate exposure, if not properly understood and measured, can have the potential to blow the roof off a reinsurance company, as was demonstrated by Hurricane Andrew in 1992. The reinsurance exposure from every large property contract or portfolio of smaller contracts must be estimated; this would be based upon the cedants' exposed policy limits, locality, and the reinsurance coverage. Then the reinsurer can see where too much catastrophe potential is accumulated, and can then better structure our own catastrophe retrocessional program.

Aggregate Excess, or Stop Loss, Treaties

Stop loss treaties may be used to protect a cedant's net loss ratio. In a sense, this is the ultimate reinsurance cover for protecting a net result. Because of the magnitude of the risk transfer, these covers are quite expensive, and often are not available unless written on a nontraditional basis.

For example, suppose we return briefly to the quota-share Example 7.24. Table 7.29 displays part of the estimated probability distribution for the loss ratio. As an alternative to the quota-share, perhaps da Ponte Insurance Company might be interested in the following stop loss cover.

Formula 7.49: Stop Loss Cover on da Ponte Property Net Retained Exposure

$$PCP = \text{primary company premium} = \$10{,}000{,}000$$

$$AP = \text{attachment point} = 70\% \text{ loss ratio} = \$7{,}000{,}000$$

$$RLim = \text{reinsurance limit} = 30\% \text{ loss ratio} = \$3{,}000{,}000$$

$$RP = \text{reinsurance premium} = \$550{,}000$$

Suppose that the reinsurer's expected internal expense for this cover is about \$50,000. Assuming that we use the same Gamma distribution as before to represent the loss ratio distribution, the profit distribution for the reinsurer is displayed in Table 7.50.

The reinsurer's expected profit of \$199,704 looks high in relation to the reinsurer's expected loss of $E[RL]$ = \$295,230. But the standard deviation of the reinsurer's loss, $SD[RL]$ = \$651,000, is extremely high compared to the expectation, and the probability

TABLE 7.50

STOP LOSS COVER ON DA PONTE PROPERTY EXPOSURE

(1) Primary Co. Loss Ratio	(2) Reinsurer's Loss Cost	(3) Probability	(4) Reinsurer's Profit
70% or less	$0	72.17%	$ 500,000
71%	100,000	1.95%	400,000
72%	200,000	1.85%	300,000
73%	300,000	1.76%	200,000
74%	400,000	1.66%	100,000
75%	500,000	1.57%	(0)
76%	600,000	1.47%	(100,000)
77%	700,000	1.38%	(200,000)
78%	800,000	1.29%	(300,000)
79%	900,000	1.21%	(400,000)
80%	1,000,000	1.12%	(500,000)
81%	1,100,000	1.04%	(600,000)
82%	1,200,000	0.96%	(700,000)
83%	1,300,000	0.89%	(800,000)
84%	1,400,000	0.82%	(900,000)
85%	1,500,000	0.75%	(1,000,000)
86%	1,600,000	0.69%	(1,100,000)
87%	1,700,000	0.63%	(1,200,000)
88%	1,800,000	0.58%	(1,300,000)
89%	1,900,000	0.53%	(1,400,000)
90%	2,000,000	0.48%	(1,500,000)
91%	2,100,000	0.44%	(1,600,000)
92%	2,200,000	0.40%	(1,700,000)
93%	2,300,000	0.36%	(1,800,000)
94%	2,400,000	0.32%	(1,900,000)
95%	2,500,000	0.29%	(2,000,000)
96%	2,600,000	0.26%	(2,100,000)
97%	2,700,000	0.24%	(2,200,000)
98%	2,800,000	0.21%	(2,300,000)
99%	2,900,000	0.19%	(2,400,000)
100% or more	3,000,000	1.37%	(2,500,000)
Total	$ 295,230	100%	$ 199,704

that the reinsurer loses money is 19%. This is quite common for stop loss covers. It indicates a very high risk transfer; the reinsurer is getting a relatively small premium to cover part of the more unpredictable tail of da Ponte's aggregate property loss distribution. In relation to the total premium of \$10,000,000, the reinsurer's expected profit is only about 2%. Note the shape of the reinsurer's distribution of aggregate loss. It is definitely non-Gamma. It has a positive probability of zero loss, Prob[RL = \$0] = 72.17%, and it is stopped (censored) above at 30%, or \$3,000,000, with Prob[RL = \$3,000,000] = 1.37%.

If we were seriously pricing an aggregate stop loss like this, we would construct our aggregate excess loss model more carefully than we did with an overall Gamma distribution. We might use a Gamma model for the filtered claims, but build up the catastrophe component by modeling event frequency and severity. We would then carefully put the two (or more) pieces together, most likely via simulation. We would also put a lot of effort into sensitivity testing our assumptions and estimates in order to see how wrong we could be and still not get hurt too badly. For example, we would find that if the primary expected loss ratio were 68% instead of 62%, but otherwise with the same standard deviation and also represented by a Gamma distribution, then the reinsurer's expected profit is about \$0, or breakeven.

This is a simplified example. More often, to protect an aggregate stop loss from catastrophe shocks, it would either cover only noncatastrophe claims, or there would be a per-occurrence limit.

Aggregate Excess Cover on An Excess Layer

Another form of aggregate excess treaty provides coverage over an aggregate deductible on a per-risk or per-occurrence excess layer. Let us return to Example 7.31, covering da Ponte's casualty exposure \$700,000 excess of \$300,000. Our previous aggregate loss modeling assumptions for this layer are listed in Formula 7.45. Again, we will simplistically use the Gamma model to price an aggregate deductible and limit. As discussed above, if we were seriously pricing this coverage, we would more carefully construct our aggregate loss model. We would also be very careful to account for our model and parameter uncertainty in order to get a proper spread for the aggregate loss distribution. And we would also perform a lot of sensitivity testing of our modeling assumptions and parameters. In this excess case, with a low claim frequency, the aggregate loss calculations would be most likely performed via simulation or via Panjer recursion as described in Appendix I.

Remember that for the layer \$700,000 excess of \$300,000 we have the following.

Formula 7.51: \$700,000 Excess of \$300,000 for da Ponte's Casualty Exposure at a Flat Rate of 5% (Expected RP = \$5,000,000)

$$\begin{aligned}
E[RL] &= \$5{,}729{,}860 \\
SD[RL] &= \$1{,}981{,}724 \\
RDF &= \text{reinsurer's loss payment discount factor}=75\% \\
RIXL &= 5\% \text{ (or } \$266{,}254) \\
\text{Reinsurer's expected profit} &= \$434{,}746 \\
RTER &= \text{Reinsurer's target economic return (profit)} \\
&= 15\% \text{(or } \$758{,}823)
\end{aligned}$$

Suppose that da Ponte Insurance Company is interested in a cover of the following form.

Formula 7.52: Aggregate Excess Cover on the Layer \$700,000 Excess of \$300,000 of da Ponte's Casualty Exposure

$$\begin{aligned}
AP &= \text{aggregate attachment point} = \$5{,}000{,}000 \\
RLim &= \text{aggregate limit} = \$5{,}000{,}000 \\
RP &= \text{reinsurance premium} = \$1{,}200{,}000
\end{aligned}$$

Since the loss expectation for the excess layer is \$5,729,860, with a \$5,000,000 aggregate deductible, da Ponte avoids trading dollars with the reinsurer for fairly predictable loss payments. Keeping the premium for the deductible, da Ponte also keeps the investment income.

Suppose that the reinsurer's expected internal expense for this cover is about \$75,000. Also, since the reinsurer is now covering only the tail of the excess claims payments after the first \$5,000,000, let us suppose that the reinsurer's loss discount factor decreases from 75% to 60%. Then the reinsurer's profit distribution is displayed in Table 7.53.

Again, the reinsurer's expected profit of \$445,041 may look high in relation to the reinsurer's expected loss of $E[RL] =$ \$1,133,265. But the standard deviation of the reinsurer's loss, $SD[RL] =$ \$1,420,620, is again very high compared to the expectation, indicating a high risk transfer. Again, the reinsurer is getting a relatively small premium to cover part of the tail of da Ponte's casualty aggregate excess loss distribution. The reinsurer's expected profit is about the same as the \$434,746 for the previous flat-rated whole excess cover, and significantly less than the RTER the reinsurer had wanted for that cover. The reinsurer calculates that there is about a 3% probability that the aggregate excess loss will exceed $AP + RLim =$ \$10,000,000; thus a 3% probability that da Ponte will have to pay claims in this excess tail beyond the reinsurer's coverage. The aggregate limit of \$5,000,000 may be acceptable to da Ponte because they believe an excess aggregate loss of \$10,000,000 is impossible or, at least, highly improbable.

As mentioned above, these kind of covers are highly sensitive to the assumptions and estimates. The aggregate excess cover in Formula 7.52 becomes unprofitable for the reinsurer if the expected excess loss were really 18% more than calculated, or if the claims were paid out much faster than anticipated. When such a cover is in place, it is important to monitor carefully the develop-

TABLE 7.53

AGGREGATE EXCESS COVER ON THE LAYER $700,000 EXCESS OF $300,000 OF DA PONTE INSURANCE COMPANY'S CASUALTY EXPOSURE

(1) Primary Co. Aggregate Loss	(2) Reinsurer's Aggregate Loss	(3) Probability	(4) Reinsurer's Profit
$5,000,000 or less	$0	39.42%	$1,125,000
5,200,000	200,000	4.23%	1,005,000
5,400,000	400,000	4.17%	885,000
5,600,000	600,000	4.07%	765,000
5,800,000	800,000	3.94%	645,000
6,000,000	1,000,000	3.77%	525,000
6,200,000	1,200,000	3.59%	405,000
6,400,000	1,400,000	3.38%	285,000
6,600,000	1,600,000	3.17%	165,000
6,800,000	1,800,000	2.95%	45,000
7,000,000	2,000,000	2.73%	(75,000)
7,200,000	2,200,000	2.51%	(195,000)
7,400,000	2,400,000	2.29%	(315,000)
7,600,000	2,600,000	2.08%	(435,000)
7,800,000	2,800,000	1.88%	(555,000)
8,000,000	3,000,000	1.69%	(675,000)
8,200,000	3,200,000	1.52%	(795,000)
8,400,000	3,400,000	1.35%	(915,000)
8,600,000	3,600,000	1.20%	(1,035,000)
8,800,000	3,800,000	1.06%	(1,155,000)
9,000,000	4,000,000	0.94%	(1,275,000)
9,200,000	4,200,000	0.82%	(1,395,000)
9,400,000	4,400,000	0.72%	(1,515,000)
9,600,000	4,600,000	0.63%	(1,635,000)
9,800,000	4,800,000	0.55%	(1,755,000)
10,000,000 or more	5,000,000	2.98%	(1,875,000)
Total	$1,133,265	100%	$ 445,041

ment of claims payments below the aggregate excess attachment point to see how the cover is doing. Some reinsurers have been surprised when an aggregate cover has suddenly blown up when claims payments reached the attachment point after many years of no loss to the reinsurer.

Finite, or Nontraditional, Reinsurance Covers

We start with the simplest contract form.

Financial Quota-Share Treaties

The simplest example of a reinsurance cover that might be classified as nontraditional is a financial quota-share. The Example 7.24 quota share could be modified in various ways to emphasize the financial aspects of the cover and decrease the risk transfer, thus decreasing the reinsurer's margin. For example, in Formula 7.28, increasing the slide factor from 50% to 100% and decreasing the minimum commission from 25% to 10%, say, increases the reinsurer's expected profit from $109,000 to $182,000 and decreases the reinsurer's probability of losing money from 34% to 6%. Then the reinsurer can offer a significant profit sharing arrangement for those coverage years that run well. Introducing a loss carryforward from year to year also makes the distribution of the multi-year loss ratio relatively more compact, thus decreasing the risk transfer. Thus the reinsurer can take a lower ultimate margin (after all adjustments).

Da Ponte Insurance Company still gets surplus relief from the provisional commission on the ceded unearned premium reserve, and still decreases their premium-to-surplus ratio. If casualty exposure were covered, the reinsurer would credit da Ponte with some (most) of the investment income earned on the contract's cash balance according to some specified formula. As long as the contract is in a profitable position, this would be returned to da Ponte as an additional commission upon commutation or sooner. On the other hand, there might be a penalty negative commission if da Ponte were to cancel when the ongoing contract were in a deficit position.

Loss Portfolio Transfers

When most insurance people think of nontraditional reinsurance, they think of loss portfolio transfers. A cedant may cede all or part of its liability as of a specified accounting date; this may be for a line of business or territory no longer written, for an impending sale of the company, or for other reasons. Usually, the reinsurance premium is essentially the present value of the transferred estimated liability, plus reinsurer's expense, surplus use, and risk charges. And the cedant can take reinsurance credit for the liability ceded, thus offsetting all or part of the loss reserve previously set up. For a U.S. cedant, this induces a surplus benefit with respect to statutory accounting.

An example may clarify this. Suppose the da Ponte Insurance Company has been told by its domiciliary insurance department that it should increase loss reserves as of December 31, 2000 by 10%. With insurance department approval, da Ponte wishes to purchase a loss portfolio cover for this additional liability. Suppose they would like to minimize the adverse statutory surplus effect as much as possible. Suppose we have the following situation.

Example 7.54: da Ponte Loss Reserves @ 12/31/00

Carried loss reserve 12/31/00 = \$300,000,000

Required additional reserve = \$30,000,000

Suppose that, based upon a thorough analysis of the da Ponte's financial reports, historical exposure, historical reinsurance program, net loss development, and claim payment distributions by line and in aggregate, we determine that the additional \$30,000,000 could easily be funded by a \$15,000,000 payment. To get to this point, besides evaluating the adequacy of their loss reserves, we would pay careful attention to the historical claim payment patterns and their fluctuations. Has the recent exposure changed in such a way to cause a significant change in future claims payments? Have there been major changes in da Ponte's claims department or claims processing? A common analytical technique is to study ratios of cumulative loss payment for each accident year divided by ultimate estimates for each category of exposure. A simplified example is displayed in Tables 7.55 and 7.56.

TABLE 7.55

Claim Payment Development for Net GL Business: Cumulative Paid Loss as a Ratio of Ultimate Loss

		Evaluation Year (End of):					
Accident Year	Estimated Ultimate Loss (in $1,000's)	1	2	3	…	9	10
1990	$60,000	0.150	0.300	0.500	…	0.984	1.000
1991	65,000	0.050	0.370	0.650	…	0.990	
⋮	⋮	⋮	⋮	⋮	⋮		
1997	80,000	0.100	0.380	0.550			
1998	85,000	0.170	0.450				
1999	90,000	0.120					
	1. Weighted Average	0.140	0.390	0.570	…	0.987	1.000
	2. 3-yr Wtd Average	0.130	0.410	0.520	…	0.987	1.000
	3. Maximum	0.170	0.450	0.650	…	0.990	1.000
	4. Minimum	0.150	0.300	0.450	…	0.984	1.000
	5. Trimmed Average	0.124	0.367	0.589	…	0.987	1.000
	6. Selected "Mean"	0.124	0.367	0.589	…	0.987	1.000
	7. Selected Extreme	0.170	0.450	0.650	…	1.000	1.000

The ultimate loss in the second column of Table 7.55 is our best estimate obtained by our evaluation. In our case, the trimmed average (weighted average of each column excluding the maximum and minimum) is selected as our "mean" estimate of the claim payment distribution over time. We also select a probable extreme value to use in our sensitivity testing.

The second column of Table 7.56, the "Estimated Liability as % of Total," is simply 1 – "Mean" values from Table 7.55; this is the *expected* tail left to pay for each accident year at 12/31/00. The calendar year columns display the *percent* of the second column expected to be paid in each year; these are stated as percents of the current reserve. For

TABLE 7.56

CUMULATIVE PAID LOSS AS A RATIO OF NET GL ULTIMATE LOSS: MEAN PAYMENT DISTRIBUTION

Acc. Year	Estimated Liability as % of Total	Percent of Liability To Be Paid in Year: 2000	2001	…	2007	2008	2009
1990	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%
1991	1.3%	100.0%	0.0%	…	0.0%	0.0%	0.0%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1997	41.1%	38.9%	25.6%	…	0.0%	0.0%	0.0%
1998	63.3%	35.1%	25.3%	…	2.1%	0.0%	0.0%
1999	87.6%	27.7%	25.3%	…	1.4%	1.5%	0.0%

Acc. Year	Estimated Liability	Amount To Be Paid in Year (in $1,000's): 2000	2001	…	2007	2008	2009
1990	$0	$0	$0	…	$0	$0	$0
1991	650	650	0	…	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1997	36,000	14,004	9,216	…	0	0	0
1998	46,750	16,409	11,828	…	982	0	0
1999	79,200	21,938	20,038	…	1,109	1,188	0
Total	$202,548	$70,257	$51,666	…	$2,091	$1,188	$0

example, 27.7% = (87.6% – 63.3%)/87.6%, 25.3% = (63.3% – 41.1%)/87.6%, 1.5% = 1.3%/87.6%, 35.1% = (63.3% – 41.1%) /63.3%, etc. In the lower part of Table 7.56, these percentages are applied to our best estimate loss reserve in the second column to get the expected claim payments in the calendar year columns. The best estimate loss reserves are $650 = (1.000 – 0.990) × $650,000, $32,880 = (1.000 – 0.550) × $80,000, etc. Then, $12,790 = 0.277 × $32,880, etc. The Total line displays our expected claim payments by calendar year according to our "mean" claim payment distribution. The totals include payments from years not displayed.

We would also produce a Table 7.56 using the "extreme" claim payment distribution from Table 7.55.

The claim payment predictions from all the covered liabilities would be assembled. If a lower risk, lower margin treaty were contemplated, greater care would be taken with the loss discounting: the reinsurer would probably price the claim payment stream via the use of an immunizing asset portfolio. The bond maturities would be selected to allow adequate margin for a possible speed-up of the claim payments.

As mentioned earlier, suppose we determined that a premium payment of $15,000,000 could easily fund an additional $30,000,000 of claims payments, since these claims payments are out in the tail beyond the first $300,000,000 of payments. But we wish to be more clever. To zero out the statutory surplus effect on da Ponte, we would look for an attachment point where the premium payment for the loss portfolio transfer would approximately match the resulting loss reserve takedown. For example, suppose a reinsurance premium of $50,000,000 is sufficient for a cover of $80,000,000 excess of $250,000,000. This transaction would not change da Ponte's beginning statutory surplus (before reserving the additional $30,000,000). Thus, it would have zero initial statutory effect,[5] and da Ponte would be covered for the additional $30,000,000 loss reserve.

To make the loss portfolio transfer into a finite cover, the premium would be more than necessary for a full risk cover, but there would be substantial profit sharing if the cover ran off favorably, and the reinsurer would expect to keep a lower profit margin. A virtual cash fund, sometimes called an *experience account balance*, would keep track of a fund balance, including investment income, usually according to some prescribed calculations. If this experience account balance ended in a positive position, or were in a positive position when the parties agreed

[5]The $50,000,000 surplus benefit to da Ponte induced by this treaty would be part of surplus under statutory accounting, but would be designated "restricted surplus."

to commute the treaty some years hence, all or most of it would be returned to da Ponte as profit sharing.

Funded Aggregate Excess Covers

Another example of a nontraditional reinsurance treaty is a funded aggregate excess cover. It is clear that the aggregate excess cover of Formula 7.52 could be transformed into such a cover by increasing the premium and introducing profit sharing. One possible structure for a funded cover would be that the reinsurer takes an initial premium of $2,000,000 instead of $1,200,000. This increases the reinsurer's expected profit by $800,000 from $445,041 to $1,245,041 and decreases the reinsurer's probability of losing money from 26% to 11%. The reinsurer would deduct an expense and profit margin (*RIXL* and *RTER* combined) of perhaps only 5%, instead of the 43% previously, and allocate perhaps 90% of the calculated investment income to the experience account balance. Further, the aggregate limit at any point in time might be equal to the experience account balance plus $1,000,000, up to a maximum of $5,000,000. Loss payments might be made only annually, to allow the fund to grow as large as possible. As with the financial quota-share above, there probably would be a penalty negative commission if da Ponte cancelled in a deficit position.

Double Triggers and Other Such Animals

With the recent advent of financial derivatives and the concept of marketable catastrophe bonds, the area of finite reinsurance is changing rapidly. One concept is a reinsurance coverage that applies only if the primary insurance claims penetrate the reinsurance layer, and also simultaneously some financial index is above or below some designated value. Thus the name "double trigger." The finite reinsurance people and the investment people are busy inventing new forms of reinsurance designed to have very specific financial attributes. This promises to be a real growth area.

Reinsurance Pricing: Final Comments

We have seen some examples of how standard actuarial methods and some not-so-standard actuarial methods apply to reinsurance pricing. We must remember that there is no one right way to price reinsurance. But there are many wrong ways. Common actuarial methods should be used only to the extent they make sense. To avoid major blunders, an underwriter/actuary must always understand as well as possible the underlying primary insurance exposure and must always be aware of the differences between the reinsurance cover contemplated and that primary exposure. The differences usually involve much less specificity of information, longer claim report and settlement timing delays, and often much lower claim frequency together with much larger claim severity, all inducing a distinctly higher risk situation. But with this goes a glorious opportunity for actuaries and other technically sophisticated people to use fully their theoretical mathematical and stochastic modeling abilities and their statistical data analytical abilities.

In the next section, we will see how reinsurance loss reserving differs from primary insurance loss reserving, and we will discuss some simple methods for dealing with these differences.

REINSURANCE LOSS RESERVING

General Considerations

For a reinsurance company, the loss reserve is usually the largest uncertain number in the statement of the company's financial condition. To estimate a loss reserve properly, we must study the run-off of the past business of the company. As a result of this process, we should not only be able to estimate a loss reserve as of a certain point in time. We should also be able to estimate historical loss ratios, loss reporting patterns, and loss settlement patterns by year, by line, and by type of business in enough detail to know whether or not a particular contract or business segment is unprofitable, and if so, when. This information should also be applicable to future pricing and decision-making. The goal is to deliver good management information regarding the company's historical contract portfolio, and also deliver some indications of where the company may be going.

Reinsurance loss reserving has many of the same problems as primary insurance loss reserving, and many of the same methods can be used. But there are also various technical problems that make reinsurance loss reserving somewhat more difficult. First, we will survey some of these problems, and then examine various methods for handling them.

Reinsurance Loss Reserving Problems

There seem to be seven major technical problems that make reinsurance loss reserving somewhat more difficult than loss reserving for a primary company. These technical problems are as follows.

Problem 1: Claim report lags to reinsurers are generally longer, especially for casualty excess losses.

The claim report lag, the time from date of accident until first report to the reinsurer, is exacerbated by the lengthy reporting pipeline. A claim reported to the cedant must first be perceived as being reportable to the reinsurer, then must filter through the cedant's report system to its reinsurance accounting department, then may journey through an intermediary to the reinsurer, then must be booked and finally appear in the reinsurer's claim system. The report lag may also be lengthened by an undervaluation of serious claims by the cedant—for a long time an ultimately serious claim may be valued below the reinsurance reporting threshold (usually one half of an excess contract attachment point). This is not an indictment of primary company claims staffs, but simply an observation that a claims person, faced with insufficient and possibly conflicting information about a potentially serious claim, may tend to reserve to "expectation," which is most likely interpreted by the claims person as the mode of the probability distribution. While this modal reserving may be sufficent for most claims with a certain probable fact pattern, it is those few which blow up above this modal average which will ultimately be covered by the excess reinsurer. Thus these larger claims generally are reported later to the reinsurer than are the smaller claims the cedant carries net.

Also, certain kinds of mass tort claims, such as for asbestosis-related injuries, may have really extreme delays in discovery or in reporting to the cedant, and may have dates of loss specified finally by a court. The extreme report lags of these claims may have a big impact on the reinsurer's experience. Just as we saw these time delays adding greatly to the uncertainty in reinsurance pricing, they also add greatly to the uncertainty in reinsurance loss reserving.

Problem 2: There is a persistent upward development of most claim reserves.

Economic and social inflation cause this development. It may also be caused by a tendency of claims people to reserve at modal values, as noted in Problem 1. Also, there seems to be a tendency to underreserve allocated loss adjustment expenses. Thus, early on, the available information may indicate that a claim will pierce the reinsurance retention, but not yet indicate the ultimate severity.

Problem 3: Claims reporting patterns differ greatly by reinsurance line, by type of contract and specific contract terms, by cedant, and possibly by intermediary.

The exposure assumed by a reinsurance company can be extremely heterogeneous. This is a problem because most loss reserving methods require the existence of large, homogeneous bodies of data. The estimation methods depend upon the working of the so-called law of large numbers; that is, future development *en masse* will duplicate past development because of the sheer volume of data with similar underlying exposure. Reinsurers do not have this theoretical luxury, since many reinsurance contracts are unique. And even when there exist larger aggregates of similar exposure, claim frequency may be so low and report lags so long that there is extreme fluctuation in historical loss data. Thus, normal actuarial loss development methods may not work very well.

As we discussed in the pricing section, a reinsurer knows much less about the specific exposures being covered than does a primary carrier. Also, the heterogeneity of reinsurance coverages and specific contract terms creates a situation where the actuary never has enough information and finds it difficult to comprehend what is being covered and what is the true exposure to loss. This is especially true for a reinsurer writing small shares of brokered business.

Problem 4: Because of the heterogeneity stated in Problem 3, industry statistics are not very useful.

Every two years, the Reinsurance Association of America (RAA) publishes a summary of casualty excess reinsurance loss development statistics in the biannual *Historical Loss Development Study* [34]. These statistics give a very concrete demonstration of the long report and development lags encountered by reinsurers. However, as is noted by the RAA, the heterogeneity of the exposure and reporting differences by company must be considered when using the statistics for particular loss reserving situations.

For any two reinsurers, the Annual Statement Schedule P primary line of business exposure and loss development data are essentially incomparable. The reason for this is that Annual Statement lines of business do not provide a good breakdown of reinsurance exposure into reasonably homogeneous exposure categories useful for loss reserving. And also, most reinsurers' loss reserves are aggregated in one line of business, 30B, excess casualty.

Proper categorization follows the pricing categories we have already seen, and will vary by reinsurance company according to the types of business in which the company specializes. This is a problem because many people who are not expert in reinsurance insist upon evaluating a reinsurer's loss reserves according to Schedule P statistics. For an actuary examining a reinsurer for the purpose of loss reserving, an appropriate exposure categorization for the particular company may not be as apparent or as easily accomplished as for a primary company.

Likewise, ISO loss development statistics by line are not directly applicable to reinsurance loss development without significant adjustments that may greatly increase the indicated growth. This is so because for excess coverage, the lag in reserving or reporting claims grows with the attachment point (see Pinto and Gogol [32]), and also because primary company direct statistics do not reflect the additional delays noted in Problem 1 above. See the Reinsurance Association of America Study [33] for a comparison of reinsurance and primary claims reporting distributions. The RAA Study also has a comparison of loss development patterns by excess attachment point.

Problem 5: The reports the reinsurer receives may be lacking some important information.

Most proportional covers require only summary claims information. Often the data are not even split by accident year or by coverage year, but are reported by calendar year or by underwriting year. An underwriting year is the same as a policy year—all premiums and claims for a contract are assigned to the effective or renewal date of each contract. Calendar year or underwriting year statistics are not sufficient for evaluating loss liabilities by accident year, so various interpretations and adjustments must be made.

Even when there is individual claims reporting, as on excess covers, there often is insufficient information for the reinsurer's claims people to properly evaluate each claim without exerting great effort in pursuing information from the cedant. This is why it is desirable to have a professional reinsurance claims staff even though the cedant is handling the claims. Also, reinsurance claims people are more accustomed to handling large claims with catastrophic injuries. Thus they are able to advise the cedant's staff (especially in the rehabilitation of seriously injured parties), and sometimes reduce the ultimate payments.

For loss reserving, it is useful to have an exposure measure against which to compare loss estimates. One possible measure is reinsurance premium by year by primary line of business. On most contracts, losses may be coded correctly by primary line, but very often the reinsurance premium is assigned to line according to a percentage breakdown estimate made at the beginning of the contract and based upon the distribution of subject premium by line. To the degree that these percentages do not accurately reflect the reinsurer's loss exposure by primary line, any comparisons of premiums and losses by line may be distorted. This adds to the difficulties noted in Problem 4.

For most treaties, premiums and losses are reported quarterly in arrears; they may not be reported (and paid) until some time in the following quarter. Thus there is an added *IBNR* exposure for both premiums and losses. The actuary must remember that, at year-end, the latest year premiums may be incomplete, so they may not be a complete measure of latest year exposure.

Problem 6: Because of the heterogeneity in coverage and reporting requirements, reinsurers often have data coding and IT systems problems.

All reinsurers have management information systems problems. The business has grown in size and complexity faster, and expectations regarding the necessary level of data detail have also grown faster, than the ability of reinsurers' data systems to handle and produce the reports requested by marketing, underwriting, claims, accounting, and actuarial staffs. This problem may be endemic to the insurance business, but it is even more true for reinsurance.

Problem 7: The size of an adequate loss reserve compared to surplus is greater for a reinsurer.

This is not a purely technical problem. It is more a management problem, and many reinsurance companies have stumbled over it. Problems 1–6 act to increase the size of an adequate loss reserve and also make it more uncertain. Thus, it is difficult for the actuary to overcome the disbelief on the part of management and marketing people, and convince them to allocate adequate resources for loss liabilities. Eventually, claims emerging on old exposure overwhelms this disbelief, at least for those who listen. A cynic might say that many reinsurance managers change jobs often enough to stay ahead of their *IBNR*. Start-up operations in particular have this problem—if there is no concrete claims runoff experience to point to, why believe a "doom-saying" actuary?

So What?

These seven problems imply that uncertainty in measurement and its accompanying financial risk are large factors in reinsurance loss reserving. This became even more important after the U.S. Tax Reform Act of 1986 required the discounting of loss reserves for income tax purposes. This discounting eliminated the implicit margin for adverse deviation that had been built into previous insurance loss reserves simply by not discounting. Insurers lost this implicit risk buffer. Since this buffer then flowed into profits and thus was taxed sooner, assets decreased. This clearly increased insurance companies' risk level. The effect upon reinsurers was even greater.

Components of a Reinsurer's Loss Reserve

The six general components of a reinsurer's statutory loss reserve are as follows.

Component 1: Case reserves reported by the ceding companies

These may be individual case reports or may be reported in bulk, depending upon the loss reporting requirements of each individual contract. Most excess contracts require individual case reports, while most proportional contracts allow summary loss reporting.

Component 2: Reinsurer additional reserves on individual claims

The reinsurer's claims department usually reviews individual case reserve reports and specifies *additional case reserves* (*ACR*) on individual claims as necessary. Additional case reserves may vary considerably by contract and by cedant.

Component 3: Actuarial estimate of future development on Components (1) and (2)

The future development on known case reserves in total is sometimes known is *IBNER*, Incurred (and reported) But Not Enough Reserved.

Component 4: Actuarial estimate of pure IBNR

Most actuaries would prefer that separate estimates be made for Components (3) and (4), the estimate of pure *IBNR*, Incurred But Not Reported. However, because of limitations in their data systems, in practice most reinsurers combine the estimates of Components (3) and (4). Depending upon the reinsurer's mix of business, these together may amount to more than half the total loss reserve.

Unless otherwise noted, the term *IBNR* in this chapter stands for the sum of *IBNER* and pure *IBNR*.

Component 5: Discount for future investment income

Insurance companies are allowed to take statutory accounting credit for future investment income on the assets supporting workers compensation permanent total cases, automobile PIP annuity claims and medical professional liability claims. Some companies do discount these claims reserves, and some don't. And, of course, as mentioned above, the U.S. Tax Reform Act of 1986 requires discounting of loss reserves for income tax purposes.

Component 6: Risk load

The last component of a loss reserve should be the risk loading or adverse deviation loading necessary to keep the reserve at a suitably conservative level, so as not to allow very uncertain income to flow into profits too quickly. Some loss reserving professionals prefer to build this into the reserve implicitly by employing conservative assumptions and methodologies. However, many actuaries would prefer to see it estimated and accounted for explicitly. Because of the long-tailed nature of much of their exposure and its heterogeneity and the uncertainty of their statistics, this component is theoretically more important for reinsurers.

A General Procedure

The four steps involved in a reinsurance loss reserving methodology are as follows.

Step 1: Partition the reinsurance portfolio into reasonably homogeneous exposure groups that are relatively consistent over time with respect to mix of business (exposures).

It is obviously important to segregate the contracts and loss exposure into categories of business on the basis of loss development potential. Combining loss data from nonhomogeneous exposures whose mix has changed over time can increase measurement error rather than decrease it.

Reasonably homogeneous exposure categories for reinsurance loss reserving have been discussed in the actuarial literature and follow closely the categories used for pricing. Table 7.57 lists various important variables for partitioning a reinsurance portfolio. All affect the pattern of claim report lags to the reinsurer and the development of individual case amounts. The listing is meant to be in approximate priority order.

TABLE 7.57

IMPORTANT VARIABLES FOR PARTITIONING A REINSURANCE PORTFOLIO INTO REASONABLY HOMOGENEOUS EXPOSURE CATEGORIES

- Line of business: property, casualty, bonding, ocean marine, etc.
- Type of contract: facultative, treaty, finite (or "financial")
- Type of reinsurance cover: quota share, surplus share, excess per-risk, excess per-occurrence, aggregate excess, catastrophe, loss portfolio transfer, etc.
- Primary line of business—for casualty
- Attachment point—for casualty
- Contract terms: flat-rated, retro-rated, sunset clause, share of loss adjustment expense, claims-made or occurrence coverage, etc.
- Type of cedant: small, large, or E&S company
- Intermediary

Obviously, a partition by all eight variables would split a contract portfolio into numerous pieces, many with too little credibility. However, after partitioning by the first five variables, it may be desirable to recognize the effects of some of the other variables on certain classes of business. For example, whether or not a casualty contract covers on an occurrence or claims-made basis is obviously important. The RAA now requests loss development data for its biannual study to be segregated by specified attachment point ranges. These ranges are a useful guide to any reinsurer wishing to so classify their own claims data and exposure.

Each reinsurer's portfolio is unique and extremely heterogeneous. In order to determine a suitable partition of exposure for reserving and results analysis, we must depend greatly upon the knowledge and expertise of the people writing and underwriting the exposures, the people examining individual claim reports, and the people processing data from the cedants. Their knowledge, together with elementary data analysis (look at simple loss development statistics), help the actuary understand which of the variables are most important.

One possible first-cut partition of assumed reinsurance exposure is shown in Table 7.58. Remember that there is no such thing as a "typical" reinsurance company.

TABLE 7.58

EXAMPLE OF MAJOR EXPOSURE CATEGORIES FOR A REINSURANCE COMPANY

- Treaty casualty excess
- Treaty casualty proportional
- Treaty property excess
- Treaty property proportional
- Treaty property catastrophe
- Facultative casualty
- Facultative property
- Surety
- Fidelity
- Ocean marine
- Inland marine
- Construction risks
- Aviation
- Finite, or nontraditional, reinsurance
- Miscellaneous special contracts, pools and associations
- Asbestos, pollution, and other health hazard or mass tort claims

The last item in Table 7.58 is not really an exposure category. But anyone in their right mind would advocate that these types of claims should be treated separately for any insurance company, especially a reinsurance company.

Within the major categories, the exposure should be further refined into treaty and facultative, if not already specified. Also, all significant excess exposure should be further segregated by type of retention (per-occurrence excess vs. aggregate excess).

Treaty casualty excess exposure should be further segregated by attachment point range and by primary line (automobile liability, general liability, workers compensation, medical professional liability). Each of these categories would be expected to have distinctly different lags for claims reported to the reinsurer.

Categories for treaty casualty proportional business would be similar. As we have discussed, some contracts classified as proportional are not shares of first dollar primary layers, but rather shares of higher excess layers. Thus, whether the exposure is ground-up or excess is an important classification variable.

Loss reserving categories for facultative casualty would certainly separate out automatic primary programs (pro rata share of ground-up exposure) and automatic nonprimary programs (excess). Certificate exposure should be split by attachment point range, if possible, but at least into buffer versus umbrella layers, and then further by primary line.

Likewise for property and other exposures, the loss reserving categories should correspond closely to the pricing categories.

It will be convenient to discuss steps 2 and 3 together.

Step 2: Analyze the historical development patterns. If possible, consider individual case reserve development and the emergence of IBNR claims separately.

Step 3: Estimate the future development. If possible, estimate the bulk reserves for IBNER and pure IBNR separately.

In our discussion, we will only deal with aggregate claim dollar development of both *IBNER* and pure *IBNR* combined. For certain exposures, especially longer-tail lines, it is often a good idea to deal with them separately. Some techniques for doing so are discussed in the chapter on loss reserving.

For suitably homogeneous categories of exposure, *expected* reinsurance claim development patterns are very stable. However, because of the extreme variability in the year-to-year data, these patterns should be studied using claims data over long time periods, as long as the *expected* patterns are reasonably stable from year-to-year. Usually the longer the time period, the better, in order to obtain accurate estimates of the underlying pattern. This is usually a large, time-consuming analytical job. Because of year-end time pressures, it is almost always a good idea to perform the in-depth analysis of claim development patterns during the third or fourth quarter. The idea is to construct our models and estimate most of the parameters before we get to the year-end crunch. Then, at year-end, and at the end of each quarter, we simply apply our parameterized models to the year-end or quarter-end claims and exposure data to estimate our *IBNR* (remember, here we mean both *IBNER* and pure *IBNR*).

Claim Report and Payment Lags

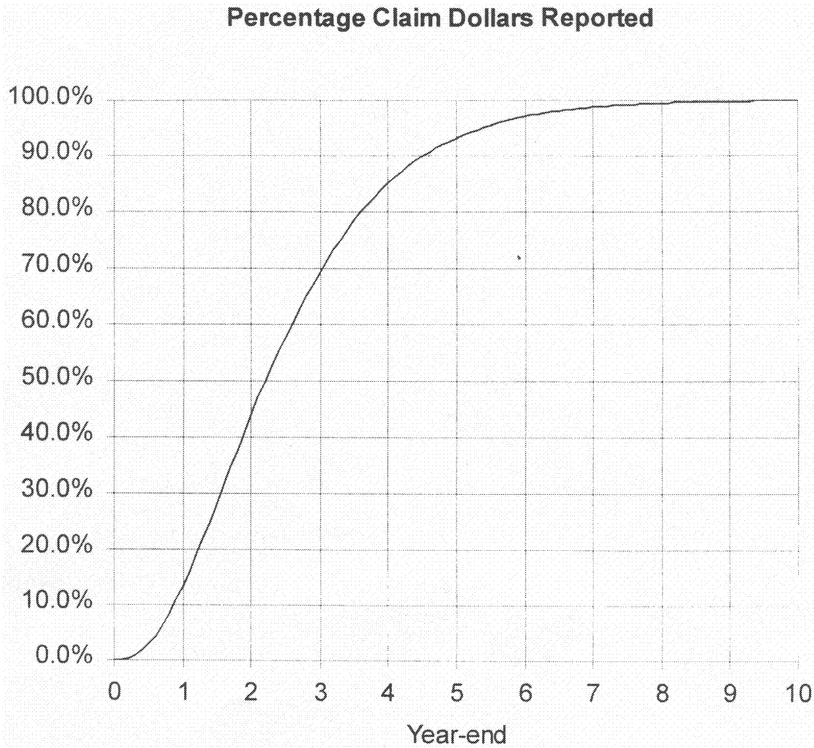
For analyzing and understanding reinsurance claims development patterns, it is useful to consider the inverse of the usual chainladder age-to-ultimate development factors—we call the factor inverses "*lags*." This view produces a time lag curve, $y = Rlag(t)$, where t measures time, like that shown in Graph 7.59.

As the age goes from 0 years to 10 years, the lag goes from $Rlag(0) = 0\%$ to $Rlag(10) = 99.9\%$. The graph looks like a probability cumulative distribution function (cdf), and, with a bit of imagination, can be interpreted as one. $Rlag(t)$ can be read as the probability that any particular claims dollar will be reported to the reinsurer by time t. This view of the claims reporting process allows us to compute statistics, such as the expected value (in years), by which we can compare one claim report pattern with another. This view also helps us fit smooth curves to the often chaotic tails of claims development data, and compute values at intermediate points. The claims report lag in Graph 7.59 is a Gamma distribution with mean 2.5 years and standard deviation 1.5 years.

The same lag idea applies to claim payment patterns.

GRAPH 7.59

CLAIM REPORT LAG GRAPH



It is convenient to discuss the methods to be used for the historical analyses and the estimation methods for the different exposure categories according to the lengths of the average claim dollar report lags.

Methods for Short-tailed Exposure Categories

As is generally true, the best methodologies to use are those that provide reasonable accuracy for least effort and cost. For short-tailed lines of business, such as most property coverage exposure, losses are reported and settled quickly, so loss liabilities are relatively small and run off very quickly. Thus, elaborate loss development estimation machinery is unnecessary.

Reinsurance categories of business that are usually short-tailed are listed in Table 7.60. But, as with any statement about reinsurance, be careful of exceptions.

TABLE 7.60

REINSURANCE CATEGORIES THAT ARE USUALLY SHORT-TAILED (WITH RESPECT TO CLAIM REPORTING AND DEVELOPMENT)

Category	Comments*
Treaty property proportional	Be wary of recent catastrophes
Treaty property catastrophe	Be wary of recent catastrophes
Treaty property excess	Possibly exclude high layers, and be wary of recent catastrophes
Facultative property	Exclude construction risks, and be wary of recent catastrophes
Fidelity proportional	

*Exclude all international exposure, if possible, since there may be significant reporting delays.

Estimation Methods

Many reinsurers reserve property business by setting *IBNR* equal to some percentage of the latest-year earned premium. This is sometimes a reasonable method for non-major-catastrophe "filtered" claims, as defined in the pricing section. It is a good idea to consider major storms and other major catastrophes separately. A recent catastrophe will cause real *IBNR* liability to far exceed the normal reserve. Claims from major catastrophes may not be fully reported and finalized for years, even on proportional covers.

Another simple method used for short-tailed exposure is to reserve up to a selected loss ratio for new lines of business or for other situations where the reinsurer has few or no loss statistics. For short-tailed exposure, provided the selected loss ratio bears some reasonable relationship to past years' experience and provided that it is larger than that computed from already-reported claims, this is a reasonable method for filtered (noncatastrophic) claims.

For some proportional treaties, summary loss reporting may assign claims by underwriting year, according to inception or renewal date of the reinsurance treaty, instead of by accident year. If the reinsurer's claims accounting staff records the reported claims likewise, the loss statistics for each false "accident" year may show great development because of future occurring accidents. In this situation, a more accurate loss development picture and estimation of *IBNR* can be obtained by assigning these "accident" year losses to approximate true accident year by percentage estimates based upon the underwriting year inception date and the general report lag for the type of exposure. Summary claims reported on a calendar (accounting) year basis can likewise be assigned to accident year by percentage estimates, if necessary. For short-tailed lines for which the *IBNR* is estimated as a percentage of premium or reserved up to a selected loss ratio, these re-assignments are unnecessary.

Methods for Medium-Tailed Exposure Categories

Let us consider any exposure for which claims are almost completely settled within five years and with average aggregate dollar claim report lag of one to two years to be medium-tailed for this discussion. Reinsurance categories of business that are usually medium-tailed are listed in Table 7.61.

Even if a property claim is known almost immediately, its ultimate value may not be. Thus it may take longer to penetrate a higher per-risk excess attachment point. This happens more often if time element coverage is involved. The discovery period for construction risk covers may extend years beyond the contract (loss occurrence) period. So for both these exposures, claim report lags may be significantly longer than normal for property business.

TABLE 7.61

REINSURANCE CATEGORIES THAT ARE USUALLY MEDIUM-TAILED (WITH RESPECT TO CLAIM REPORTING AND DEVELOPMENT)

Category	Comments
Treaty property excess higher layers	If it is possible to separate these from working layers
Construction risks	If it is possible to separate these from other property exposure
Surety	
Fidelity excess	
Ocean marine	
Inland marine	
International property	
Non-casualty aggregate excess	Lags are longer than for the underlying exposure

For surety exposure, it is usually a good idea to consider losses gross of salvage and, separately, salvage recoveries. The gross losses are reported fairly quickly, but the salvage recoveries have a longer tail. It is instructive to consider the ratio of salvage to gross loss for mature years. This ratio is fairly stable and may help explain predictions for recent coverage years as long as the underwriters can predict how the salvage ratio may have slowly changed over time.

Estimation Methods

A useful *IBNR* estimation method for medium-tailed lines of business is to use the standard American casualty actuarial *Chain-ladder (CL) Method* of age-to-age factors calculated from cumulative aggregate incurred loss triangles, with or without *ACR*s. This is described fully in the chapter on loss reserving. If accident year data exist, this is good methodology. An advantage of this method is that it strongly correlates future development both with an overall lag pattern and with the claims reported for each accident year. A major disadvantage is simply that the *IBNR* is so heavily correlated with reported claims that, at least for longer-tailed lines, the reported, very random nose wags the extremely large tail estimate for recent, immature years.

It is sometimes true that paid loss development is more stable than reported loss development. If so, then a chainladder estimate of ultimate loss by accident year may be obtained using paid lags. Of course, the problem is that the estimation error may be even greater for immature, recent accident years than it is for reported loss chainladder estimation.

Methods for Long-Tailed Exposure Categories

Just as for pricing, the real problem in loss reserving is long-tailed exposure, especially excess casualty reinsurance. These are the exposures for which the average aggregate claims dollar report lag is over two years and whose claims are not settled for many years. Reinsurance categories of business that are usually long-tailed are listed in Table 7.62.

TABLE 7.62

REINSURANCE CATEGORIES THAT ARE USUALLY LONG-TAILED (WITH RESPECT TO CLAIM REPORTING AND DEVELOPMENT)

Category	Comments
Treaty casualty excess	Includes the longest lags except for the *APH* claims listed below
Treaty casualty proportional	Some of this exposure may possibly be medium-tailed
Facultative casualty	
Casualty aggregate excess	Lags are longer than for the underlying exposure
Asbestos, pollution and other health hazard and mass tort claims	May be the longest of the long tails

For most reinsurance companies, this is where most of the loss reserve lies, and almost all the *IBNR*. So it is important for us to spend most of our analytical and estimation effort here. The first step is to separate these exposures into finer, more homogeneous categories. This is, of course, an iterative process. We depend upon our company's marketing, underwriting, claims, and accounting personnel for the first-stage categorization. Further refinements will then depend upon our hypothesis testing and upon our investigation of various comments from the marketing and underwriting people as they receive from us the estimated *IBNR* by major contract or category based upon the latest categorization. Some of the larger, more important contracts are looked upon at least partially on a standalone basis.

Asbestos, Pollution, Other Health Hazard and Other Mass Tort Exposures

We separate out claims arising from asbestosis, pollution clean-up, other health hazard (sometimes collectively known as "*APH*") and other mass tort situations for special consideration. Because of the catastrophic significance of these types of claims (nothing for many years, then suddenly, gigantic totals), they would drastically distort the development statistics if left in the remaining filtered (noncatastrophic) claims data. Also, it is unlikely that the usual actuarial loss development techniques, if used blindly, would yield reasonable answers for these types of claims.

In the past few years, various models have been developed for estimating asbestos and pollution clean-up liabilities. The large actuarial consulting firms have been in the forefront of this development. The models are extremely complex and require very detailed input regarding coverages exposed to these types of claims. Unless we, the reinsurer, have a very large, long, stable and fairly complete claims development history, we judge that it is better that we and our *APH* claims specialists work with one of the actuarial consultants to develop our liability estimates.

Beyond asbestos and pollution clean-up, the question of which exposures or claims should be treated separately or specially is difficult. We should discuss this thoroughly with our claims and underwriting staff. For example, it is desirable to treat claims-made coverages separately from occurrence coverages, if possible. Also, it should be clear that claims from commuted contracts should be excluded from the usual triangles, since their development is artificially truncated (cut off), and thus would distort the noncommuted complete development patterns.

Estimation Methods

The standard *CL* Method is sometimes used for long-tail exposures. But, the problem is that for very long-tailed lags, the resulting *IBNR* estimate for recent, green years is extremely variable, depending upon the few reported or paid claims to date.

An alternative estimation method is the *Bornheutter–Ferguson (BF) Method* (Bornheutter and Ferguson [25]), which is discussed in the chapter on loss reserving. This method uses a selected loss ratio for each coverage year and an aggregate dollar report lag pattern specifying the percentage of ultimate aggregate loss expected to be reported as of any evaluation date. An advantage of this method is that it correlates future development for each year with an exposure measure equal to the reinsurance premium multiplied by a selected loss ratio. Disadvantages with the *BF IBNR* estimate are:

1. It is very dependent upon the selected loss ratio.
2. The estimate for each accident year does not reflect the particular to-date reported losses for that year, unless the selected loss ratio is chosen with this in mind.

Since the loss ratio for a given accident year is strongly correlated with the place of that year in the reinsurance profitability cycle, so is the to-date reported loss. It would seem to be desirable to use this fact in the *IBNR* estimate. As noted before, the reinsurance profitability cycles are more extreme than primary insurance cycles. Thus, when using the *BF* Method, one must select the accident year loss ratios carefully.

Stanard–Bühlmann (Cape Cod) Method

An estimation method that overcomes some of the problems with the *CL* and *BF* Methods was independently derived by James Stanard (described in Patrik [31] and Weissner [36]) and by Hans Bühlmann (internal Swiss Re publications). This *Stanard–Bühlmann (SB) Method* is known to European actuaries as the *Cape Cod Method* because Hans Bühlmann first proposed it at a meeting on Cape Cod. As with the *CL* and *BF* Methods, this method uses an aggregate reported claim dollar lag pattern, which may or may not be estimated via the *CL* Method or via some other method. The key innovation of the *SB* Method is that the ultimate expected loss ratio for all years combined is estimated from the overall reported claims experience, instead of being selected judgmentally, as in the *BF* Method. A problem with the *SB* Method is that the *IBNR* by year is highly dependent upon the rate-level adjusted premium by year. The user must adjust each year's premium to reflect the rate-level cycle on a relative basis. But this is also a problem with the *BF* Method.

A simple example will help explain the *SB* Method.

For a given exposure category, assume that the yearly earned risk pure premiums (net of reinsurance commissions, brokerage fees and internal expenses) can be adjusted to remove any suspected rate-level differences by year. Thus, we believe that each adjusted year has the same expected loss ratio. In primary insurance terminology, assume that the risk pure premiums have been put on-level. This adjustment is difficult and uncertain, but must be done if we are to have a reasonably consistent exposure base. Netting out the commissions and brokerage fees is usually easy, since these are separately coded by contract in our accounting database. Netting out the internal expenses is more difficult, since these are strictly internal numbers, not usually coded to contract. If it is too difficult or impossible to net out the internal expense, we need not bother, since all we need is a relative exposure base, not an absolute one. We study primary business rate levels by considering industrywide Schedule P line loss ratios by year, and work with our underwriters to adjust these rate relativities for our own reinsurance exposure categories. Let *ELR* represent the unknown expected loss ratio to adjusted earned risk pure premium.

Suppose that Table 7.63 displays the current experience for this category. For clarity, to deal with an example with only five years, we use a report lag that reaches 100% at the end of the sixth year.

TABLE 7.63

STANARD–BÜHLMANN (CAPE COD) METHOD
DATA AS OF 12/31/00 (IN 1,000'S)

(1) Calendar/ Accident Year	(2) Earned Risk Pure Premium	(3) Adjusted Premium	(4) Aggregate Reported Loss	(5) Aggregate Loss Report Lag	(6) "Used-Up" Premium (3) × (5)
1996	$ 6,000	$ 8,000	$ 7,000	95%	$ 7,600
1997	7,000	7,000	5,000	85%	5,590
1998	8,000	6,000	3,000	70%	4,200
1999	9,000	7,000	2,000	50%	3,500
2000	10,000	10,000	4,000	30%	3,000
Total	$40,000	$38,000	$21,000	n.a.	$24,250

We will explain column 6 of Table 7.63 in the following discussion.

The *SB ELR* and *IBNR* estimates are given by Formula 7.64.

Formula 7.64: Stanard–Bühlmann (Cape Cod) ELR and IBNR Estimates

$$SBELR = \frac{\Sigma\{RRL(k)\}}{\Sigma\{ARPP(k) \times Rlag(k)\}}$$

$$SBIBNR(k) = SBELR \times ARPP(k) \times (1 - Rlag(k))$$

where $SBELR$ = Stanard–Bühlmann estimate of the *ELR*

$SBIBNR(k)$ = *SB IBNR* estimate, year k

$$RRL(k) = \text{reported reinsurance loss, year } k$$

$$ARPP(k) = \text{adjusted risk pure premium, year } k$$

$$Rlag(k) = \text{aggregate claim dollar report lag, year } k$$

Some *SB* practitioners call the term $ARPP(k) \times Rlag(k)$, the *used-up premium* for year k. You can see that it is that fraction of premium corresponding to the percent of ultimate claims expected to be reported as of the evaluation date. It is the premium earned according to the expected reported claims. Thus *SBELR* is the loss ratio of reported loss divided by used-up premium. So how do we get to these equations?

First of all, simply define *SBIBNR(k)* to be $SBELR \times ARPP(k) \times (1 - Rlag(k))$. This definition is the same as in the *BF* Method. The only difference is that the *SB* Method will tell us how to estimate *SBELR*. Since the total *IBNR* is the sum of the *IBNR* from each year, then *SBIBNR* can be written as follows.

$$\begin{aligned} SBIBNR &= \Sigma SBIBNR(k) \\ &= \Sigma\{SBELR \times ARPP(k) \times (1 - Rlag(k))\} \text{ by definition} \\ &= SBELR \times \Sigma\{ARPP(k) \times (1 - Rlag(k))\} \end{aligned}$$

Then, since a loss ratio is simply losses divided by premium, we may write *SBELR* as follows.

$$\begin{aligned} SBELR &= \frac{RRL + SBIBNR}{ARPP} \\ &= \frac{RRL + SBELR \times \Sigma\{ARPP(k) \times (1 - Rlag(k))\}}{ARPP} \end{aligned}$$

Or

$$\begin{aligned} SBELR \times ARPP &= RRL + SBELR \times \Sigma\{ARPP(k) \times (1 - Rlag(k))\} \\ &= RRL + SBELR \times ARPP \\ &\quad - SBELR \times \Sigma\{ARPP(k) \times Rlag(k)\} \end{aligned}$$

Or, subtracting $SBELR \times ARPP$ from both sides,

$$SBELR \times \Sigma\{ARPP(k) \times Rlag(k)\} = RRL$$

Or

$$\begin{aligned} SBELR &= \frac{RRL}{\Sigma\{ARPP(k) \times Rlag(k)\}} \\ &= \$21{,}000/(\$8{,}000 \times 0.95 + \$7{,}000 \times 0.85 \\ &\qquad + \$6{,}000 \times 0.70 + \$7{,}000 \times 0.50 \\ &\qquad + \$10{,}000 \times 0.30) \\ &= \frac{\$21{,}000}{\$24{,}250} \\ &= 0.866 \end{aligned}$$

Table 7.65 compares *IBNR* and estimated ultimate loss ratios for the *CL* and *SB* Methods, using the information in Table 7.63 (remember to use the adjusted risk pure premium, *ARPP*, not the original premium, in the *SB* calculation). The *BF* Method cannot be compared, since the *BF* loss ratios are not estimated by formula.

TABLE 7.65

COMPARISON OF CHAINLADDER AND STANARD–BÜHLMANN METHODS

(1) Cal/Acc. Year	(2) Earned Risk Pure Premium	(3) *CL IBNR*	(4) *CL* Loss Ratio	(5) *SB IBNR*	(6) *SB* Loss Ratio
1996	$ 6,000	$ 368	123%	$ 346	122%
1997	7,000	882	84%	909	84%
1998	8,000	1,286	54%	1,589	57%
1999	9,000	2,000	44%	3,031	56%
2000	10,000	9,333	133%	6,062	100%
Total	$40,000	$13,870	87%	$11,908	82%

Look at the differences in the 1999 and 2000 loss ratios. As long as the rate relativity adjustments to yearly earned risk pure premium are reasonably accurate, the estimates for recent, immature years and the overall answer are more accurate with the *SB* Method. It is easy to see that the above example would be even more vivid if a real longer-tailed exposure were used.

Credibility IBNR Estimates

For situations where we don't have complete confidence in the year-to-year rate-level premium adjustments, we can calculate a "credibility" *IBNR* estimate by weighing together the *CL* and *SB* estimates. Intuitively, we would want to give more weight to the *SB* estimate for recent, green years, where the *CL* estimate has a high level of variance, and more weight to the *CL* estimate for older years, where we trust the cumulative rate-level adjustments less. One way of doing this is to use some simple, monotonic function of the report lag as the credibility weight for the *CL* estimate. A simple linear function that yields such a credibility estmate as follows.

Formula 7.66: A Simple "Credibility" IBNR Estimate

$$\text{Cred}\,IBNR(k) = Z(k) \times CLIBNR(k) + (1 - Z(k)) \times SBIBNR(k)$$

$$\text{where} \quad Z(k) = CF \times Rlag(k)$$

$$\text{and} \quad 0 \leq CF = \text{credibility factor} \leq 1$$

$$Rlag(k) = \text{report lag for year } k$$

$$\text{Cred}\,IBNR(k) = \text{credibility } IBNR \text{ for year } k$$

$$CLIBNR(k) = CL\ IBNR \text{ for year } k$$

$$SBIBNR(k) = SB\ IBNR \text{ for year } k$$

Using the information in Tables 7.63 and 7.65, we can calculate a credibility *IBNR*, with $CF = 0.5$ for example, in Table 7.67.

TABLE 7.67

CALCULATION OF "CREDIBILITY" *IBNR*

(1) Cal/Acc. Year	(2) Earned Risk Pure Premium	(3) Claim Report Lag	(4) *CL IBNR*	(5) *SB IBNR*	(6) "Credibility" *IBNR*
1996	$ 6,000	95%	$ 368	$ 346	$ 356
1997	7,000	85%	882	909	898
1998	8,000	70%	1,286	1,589	1,483
1999	9,000	50%	2,000	3,031	2,773
2000	10,000	30%	9,333	6,062	6,553
Total	$40,000	n.a.	$13,870	$11,908	$12,063

For example, in Table 7.67, $356 = (0.5) × (0.95) × $368 + (1 – (0.5) × (0.95)) × $346.

If we were dealing with a real live long-tail example, we would see many more accident years worth of data. Depending upon the length of the claim report and payment lags, we would probably want to be looking at 25 or more years of claims data. We would talk with our marketing people, underwriters, and claims people to see if there are any special contracts, exposures, or types of claims that should be treated separately. We would want to know of any particularly large individual claims which should be censored or otherwise dealt with so as not to have an undue random impact upon either the estimation of the claims report and payment lags or upon the *IBNR* estimation.

Other "Credibility" Procedures

Some reinsurance actuaries also weigh together the *IBNR* estimates obtained separately from reported claims data and paid claims data. A problem with reported claims data is that they include case reserves set up according to the judgments of many claims people, perhaps varying over time. Thus they may lack consistency over time. If we have paid claims data over enough accident and payment years to capture the long tail, and we believe the data to have a fairly consistent *expected* payment pattern over the accident years, these data may exhibit more stability than the reported claims data. Thus they may also be useful for estimating *IBNR* liabilities. The weighing can again be intuitively simplistic. Perhaps we might decide to use the relative claim report and payment lags for each year as weights.

Some reinsurance actuaries also want to use the information inherent in the pricing of the exposure. If actuarial pricing techniques similar to those discussed in the pricing section are used, we automatically have estimates of *ELR* with respect to risk pure premium for each contract. We may monitor their accuracy and average them over suitable exposure categories. We may call these average pricing *ELR*'s our *a priori ELR* estimates and use them instead of or in conjunction with the *SB ELR*'s. We can use them as our *BF ELR* estimates and calculate *BF IBNR* with them. We can then weigh this *a priori IBNR* against the *CL IBNR* to obtain our final "credibility" *IBNR* estimates. You will find an interesting discussion of this credibility method in Benktander [23] and Mack [30].

You can see there are many possibilities, and no single right method. Any good actuary will want to use as many legitimate methods for which reasonably good information and time is available, and compare and contrast the estimates from these methods. As with pricing, it is often informative to see the spread of estimates derived from different approaches. This helps us understand better the range and distribution of possibilities, and may give us some idea of the sensitivity of our answers to varying assumptions and varying estimation methodologies. A distribution-free method for calculating the variance of chain-ladder loss reserve estimates is described in Mack [29].

Alternative Estimation Methodologies

Suppose, as above, we have only the summary development triangles of reported or paid claims. We can obtain maximum likelihood estimates of the parameters for a continuous lag model (such as the Gamma used above) by treating the increments for each development interval as grouped data, exactly as discussed by Hogg and Klugman [11] for claim severity. The reported claim dollars for each accident year can be considered to be a sample from a truncated model (unknown tail). A slight practical problem here may be negative increments. But for the estimation, the time intervals for the grouping of the data need not necessarily all be one-year periods, so the intervals can always be adjusted to avoid negative increments.

Various statistical and reasonableness tests can then help us decide which lag model best describes the data, and which we believe will best predict future claims development. This model with the fitted parameters can then be used to estimate the probability distribution for the *IBNR*, calculate various statistics such as the expectation and standard deviation, and predict *IBNR* claim emergence.

More advanced models, as discussed in the chapter on loss reserving, could use claim counts and amounts and construct a stochastic model for the whole claims development process. These models are useful for very long-tail exposures. They can give us more information and provide insight we can't get from the simpler models discussed above. But they are very much more complicated, and look to many actuaries like black boxes. A problem with increasingly sophisticated methodologies is that the answers may become less intuitive and may be much more difficult for the actuary to understand and explain to management.

An advantage of using a claim count/claim severity model is that we can contemplate intuitively satisfying models for various lag distributions, such as the time from loss event occurrence until first report and the lag from report until settlement. We can then connect these lags with appropriate models for the dollar reserving and payments on individual claims up through settlement. For surveys of some advanced methodologies, see the chapter on Loss Reserving and various Advanced Techniques sessions in Casualty Loss Reserve Seminar transcripts [26].

Monitoring and Testing Predictions

A loss reserve or an *IBNR* reserve is derived from a hypothesis about future claims settlements for past events. In order to validate our methodology, we must test our predictions against actual future experience. Monitoring and testing quarterly claims run-off against predictions may provide early warning of problems.

For short-tailed and medium-tailed lines, this can be fairly simple. As long as current accident year claims can be reasonably separated from past accident year run-off, the run-off can be compared with the previous year-end reported open and *IBNR* reserves. For long-tailed lines, slightly more sophisticated comparisons are necessary.

Perhaps the best way to describe a simple *IBNR* monitoring methodology is through the use of an example. We will consider *GL* exposure for treaty casualty excess contracts with attachment points in RAA range 4. RAA range 4 currently has attachment points between $1,251,000 to $3,500,000. We, of course, deflate this range back via estimated *GL* ground-up claims severity trend, so that we include the appropriate contracts from earlier years in our database.

Table 7.68 is one possible format. Columns 4 and 5 are a Gamma distribution with mean 5 years and standard deviation 3 years. Column 5 is column 4 adjusted by one half year.

Interpreting Table 7.68 is difficult. Column 8 tells us there was $809,000 more claims emergence in the first half of 2001 than the $2,437,000 = $63,240,000 – $60,803,000 expected. This is about 33% more than expected. Is this purely random? Or does it indicate that the beginning *IBNR* reserve was too small, or the lags too short? We will want to watch these claims emergence

TABLE 7.68

Treaty Casualty Excess Working Cover Claims Emergence Test Monitoring 12/31/00 *IBNR* as of 6/30/01 (in $1,000's)

(1) Acc. Year	(2) Claims @ 12/31/00	(3) *IBNR* @ 12/31/00	(4) Claim Report Lag@ 12/31/00	(5) Claim Report Lag@ 6/30/01	(6) Predicted Claims @ 6/30/01	(7) Actual Claims @ 6/30/01	(8) Difference (7) – (6)
1981	$ 3,927	$ 3	99.92%	99.94%	$ 3,928	$ 3,927	$ –1
1982	2,743	3	99.88%	99.90%	2,744	2,743	–1
1983	1,052	2	99.80%	99.84%	1,052	1,152	100
1984	5,721	18	99.69%	99.75%	5,725	5,641	–84
1985	2,325	12	99.50%	99.61%	2,327	2,325	–2
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1993	6,167	1,075	85.15%	87.74%	6,354	6,285	–69
1994	2,364	647	78.52%	82.10%	2,472	3,549	1,077
1995	4,642	2,031	69.56%	74.35%	4,962	4,752	–210
1996	2,086	1,512	57.98%	64.11%	2,306	2,201	–105
1997	1,148	1,468	43.89%	51.22%	1,340	1,245	–95
1998	2,359	5,997	28.23%	36.15%	3,021	2,859	–162
1999	542	3,533	13.30%	20.47%	834	1,478	644
2000	0	5,182	2.86%	7.25%	234	0	–234
Total	$60,803	$22,635	5.00 years average	5.00 years average	$63,240	$64,049	$ 809

Notes:

Col. 3 = (Col. 2) × (1.00 – Col. 4)/(Col. 4) except for 2000. For simplicity here, the Col. 3 *IBNR* is straight chainladder; in practice, it would be our credibility *IBNR*.

Col. 6 = Col. 2 + (Col. 3) × (Col. 5 – Col. 4)/(1.00 – Col. 4) tests each quarter to see if a positive difference persists. This might lead us to decide to lengthen the lags for this exposure.

Reinsurance Loss Reserving: Final Comments

We have seen some examples of how standard actuarial methods and some not-so-standard actuarial methods apply to reinsurance loss reserving. We must remember that there is no one right way to estimate reinsurance loss reserves. But there are many wrong ways. Common actuarial methods should be used only to the extent they make sense. To avoid major blunders, the actuary must always understand as well as possible the types of reinsurance exposure in the reinsurance company's portfolio. The differences from primary company loss reserving mainly involve much less specificity of information, longer report and settlement timing delays, and often much smaller claim frequency together with much larger severity, all inducing a distinctly higher risk situation. But with this goes a glorious opportunity for actuaries to use fully their theoretical mathematical and stochastic modeling abilities and their statistical data analytical abilities.

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APPENDIX A

PARETO DISTRIBUTION

1. Support: $X > 0$

2. Parameters: $b > 0,\ q > 0$

3. C.d.f.:

$$F(x \mid b,q) = 1 - \left(\frac{b}{b+x}\right)^q$$

4. P.d.f.:

$$f(x \mid b,q) = \frac{qb^q}{(b+x)^{q+1}}$$

5. Moments:

$$E[X^k \mid b,q] = \frac{b^k(k!)}{(q-1)(q-2)(q-k)} \qquad \text{for} \quad k < q$$

6.

$$E[X \mid b,q] = \frac{b}{q-1}$$

7.

$$\mathrm{Var}[X \mid b,q] = \frac{qb^2}{(q-2)(q-1)^2}$$

8. Censored (from above) c.d.f.: (general definition)

$$F(X;c) = \begin{cases} F(x) & \text{for} \quad x < c \\ 1 & \text{otherwise} \end{cases}$$

9. Censored moments: If $q-k$ is not an integer, then

$$
\begin{aligned}
E[X^k; c \mid b, q] = {} & \frac{b^k(k!)}{(q-1)(q-2)\ldots(q-k)} - q\left(\frac{b}{(b+c)}\right)^q \\
& \times \left\{ \frac{(b+c)^k}{q-k} + \cdots + (-1)^i \left(\frac{k!}{(i!)(n-i)!}\right) \right. \\
& \quad \times \left(\frac{b^i(b+c)^{k-i}}{q-k+i}\right) + \cdots \\
& \quad \left. + (-1)^k \left(\frac{b^k}{q}\right) - \left(\frac{c^k}{q}\right) \right\}
\end{aligned}
$$

10. Censored expectation:

$$E[X; c \mid b, q] = E[X]\left\{1 - \left(\frac{b}{b+c}\right)^{q-1}\right\} \qquad \text{for} \quad q > 1$$

11. Conditional probability:

$$\text{Prob}[X > y \mid X > x] = \left(\frac{b+x}{b+y}\right)^q$$

12. Truncated (from below-conditional) distribution:

Definition: $X(d) = X - d \qquad \text{for} \quad X > d$

Then $X(d)$ is Pareto with parameters $b+d, q$:

$$
\begin{aligned}
F_{X(d)}(x) &= \text{Prob}[X(d) \le x] \\
&= 1 - \text{Prob}[X(d) > x] \\
&= 1 - \text{Prob}[X > x + d \mid X > d] \\
&= 1 - \left(\frac{b+d}{b+(x+d)}\right)^q \\
&= 1 - \left(\frac{b+d}{(b+d)+x}\right)^q \\
&= F(x \mid b+d, q)
\end{aligned}
$$

13. Trended distribution:

Definition: $Y = tX \qquad t > 0$

Then Y is Pareto with parameters tb, q:

$$\begin{aligned} F_Y(y) &= \text{Prob}[Y \leq y] \\ &= \text{Prob}\left[X \leq \frac{y}{t}\right] \\ &= 1 - \left(\frac{b}{b + \left(\frac{y}{t}\right)}\right)^q \\ &= 1 - \left(\frac{tb}{tb + y}\right)^q \\ &= F(y \mid tb, q) \end{aligned}$$

APPENDIX B

CATASTROPHE LOSS MODEL: EXAMPLE 7.24

For simplicity, the catastrophe distribution for Example 7.24, property quota share pricing, is modeled with the following assumptions.

1. N = cat. event count $\sim$ Poisson[3]

   So,

   $$E[N] = 3 = \text{Var}[N]$$

2. X = cat. event severity (as loss ratio) $\sim$ Pareto[8%, 3]

   (see Appendix A)

   So,

   $$E[X] = \frac{8\%}{3-1} = 4\%$$

   $$E[X^2] = \frac{2(0.08)^2}{(3-1)(3-2)} = 0.0064$$

3. L = annual aggregate cat. loss

   Using the standard risk theoretic model described in Appendix G, we have:

   $$E[L] = E[N]E[X]$$
   $$= 3(4\%) = 12\%$$
   $$\text{Var}[L] = E[N]E[X^2]$$
   $$= 3(0.0064) = 0.0192$$
   $$SD[L] = (0.0192)^{1/2} = 13.86\%$$

APPENDIX C

GAMMA DISTRIBUTION

1. Support: $X > 0$
2. Parameters: $\alpha > 0,\ \beta > 0$
3. P.d.f.:

$$f(x \mid \alpha, \beta) = [\beta^{\alpha}\Gamma(\alpha)]^{-1} x^{\alpha-1} \exp\left(\frac{-x}{\beta}\right)$$

$$\text{where} \quad \Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx = (\alpha - 1)\Gamma(\alpha - 1)$$

4. $E[X \mid \alpha, \beta] = \alpha\beta$
5. $\text{Var}[X \mid \alpha, \beta] = \alpha\beta^2$
6. Mode $= \beta(\alpha - 1)$ if $\alpha \geq 1$
7. If $\alpha = 1$, then Exponential distribution.
8. The Gamma distribution is used in the text to model the distribution of aggregate claims. Recent research (Papush, Patrik and Podgaits [17]) has shown that the simple 2-parameter Gamma distribution is a more accurate representation of the aggregate claims distribution induced by the standard risk theoretic model (Appendix G) than is the more commonly used Lognormal distribution. Also, see Venter [22].

APPENDIX D

ELCF CALCULATION: TABLE 7.34

For simplicity, the claim indemnity severity model underlying Table 7.34 is assumed to be Pareto, with parameters $b = 5{,}000$ and $q = 1.1$. Then Formula 7.33 is used to adjust for *ALAE* added to indemnity loss, with $\gamma = 20\%$, and for a clash loading, $\delta = 5\%$. The following table shows the calculation of the increased limits factors using Appendix A, #10 to compute the censored expectation at each limit.

Pareto[5,000, 1.1] Increased Limits Factors

(1) Policy Limit	(2) Pareto $E[X; PLim \mid 5{,}000, 1.1]$	(3) *ILF* w/o risk load and w/o *ALAE* (2) ÷ (2[$100,000])
$100,000	$13,124	1.0000
250,000	16,255	1.2386
300,000	16,854	1.2842
500,000	18,484	1.4084
833,333	20,041	1.5271
1,000,000 or more	20,579	1.5681

APPENDIX E

EXCESS EXPECTED CLAIM SIZE: TABLE 7.36

For simplicity, the claim indemnity severity model underlying Table 7.36 is the Table 7.34 Pareto adjusted to add *ALAE* to indemnity by altering the Pareto b parameter by the $\gamma = 20\%$ *ALAE* loading (new $b = 1.2 \times 5{,}000 = 6{,}000$) and leaving $q = 1.1$. Since the indemnity loss is censored by policy limits, the 20% add-on for *ALAE* is simply modeled by adjusting the combined indemnity-plus-*ALAE* limits upward by 20%. Using Appendix A, #12 (truncated below at \$300,000) and #13 (adding 20% *ALAE* is like trending by 20%), we get the following result.

1. Expected excess claim severity, over attachment point d and subject to reinsurance limit *RLim*, for policy limit λ:

$$(1+\delta)E[X(d);\ \lambda(1+\gamma)-d \mid (1+\gamma)b+d,q]$$

$$\text{for}\quad d(1+\gamma)^{-1} \le \lambda \le (RLim+d)(1+\gamma)^{-1}$$

$$= (1.05)E[X;(1.2)\lambda - 300{,}000 \mid 306{,}000, 1.1]$$

$$\text{for}\quad 250{,}000 \le \lambda \le 833{,}333$$

$$= (1.05)(306{,}000/0.1)$$

$$\times \left\{1 - \left(\frac{306{,}000}{6.000 + (1.2)\lambda}\right)^{0.1}\right\}$$

$$= (3{,}213{,}000)\left\{1 - \left(\frac{306{,}000}{6.000 + (1.2)\lambda}\right)^{0.1}\right\}$$

2. If $\lambda = 300{,}000$, we computed

$$= (3{,}213{,}000)\left\{1 - \left(\frac{306{,}000}{366{,}000}\right)^{0.1}\right\}$$

$$= (3{,}213{,}000)(1 - 0.9823)$$

$$= (3{,}213{,}000)(0.0177)$$

$$= 57{,}016$$

APPENDIX F

CREDIBILITY CONSIDERATIONS FOR EXPOSURE RATING AND EXPERIENCE RATING

We further discuss the items in Tables 7.41 and 7.42.

TABLE 7.41

ITEMS TO CONSIDER IN DETERMINING THE CREDIBILITY OF THE EXPOSURE LOSS COST ESTIMATE

- *The accuracy of the estimate of RCF, the primary rate correction factor, and thus the accuracy of the primary expected loss cost or loss ratio*

The accuracy of the estimate of *RCF* for the rating year, for each line of business, determines the accuracy of the estimate of the primary expected loss cost or loss ratio, $RCF \times PCPLR$. And this estimate is critical because the exposure rating estimate of the excess loss cost for each line of business is proportional to the loss ratio estimate.

- *The accuracy of the predicted distribution of subject premium by line of business*

If this prediction of the mix of business is accurate, and the estimates of loss cost for each line are accurate, then the estimate of the overall loss cost should be accurate. If the cedant has a fairly stable mix of business, and no plans for substantive changes, a fairly accurate prediction can be made of the mix of business for the rating year.

- *For excess coverage, the accuracy of the predicted distribution of subject premium by increased limits table for liability, by state for workers compensation, or by type of insured for property, within a line of business*

Within many liability lines, there are sublines with different increased limits tables, and thus different increased limits factors.

When rating excess coverage, it is clear that the more accurate our estimate of the distribution of primary premium volume by table, the more accurate our exposure rating estimate of loss cost. Since many primary companies do not keep accurate statistics on these distributions, we often use industry statistics, which may not be very accurate for the particular company.

Likewise, for workers compensation, there are large differences in excess exposure by state.

For property, clearly the type of insured (occupancy, construction, protection) are also important in determining excess exposure.

- *For excess coverage, the accuracy of the predicted distribution of subject premium by policy limit within increased limits table for liability, by hazard group for workers compensation, by amount insured for property*

For excess coverage, the policy limit, hazard group or amount insured (or *MPL* or *PML*) clearly affect the amount of exposure to excess claims. Thus, the more accurate these estimates are, the more accurate the overall estimate of excess exposure to loss. Again, since many primary companies do not keep accurate statistics on these distributions, we often use industry statistics, which may not be very accurate for the particular company. If there is a substantial change in the limits written during the rating year, the accuracy of the estimated excess loss cost is reduced.

- *For excess coverage, the accuracy of the excess loss cost factors for coverage above the attachment point*

This is obviously important. If your excess loss cost factors are based upon ISO's increased limits factors or claims severity data, you must recognize that, since ISO's increased limits data are mainly from policies with limits at or below $1,000,000, their increased limits factors may not be very accurate for limits above $1,000,000. Therefore, the higher the attachment point, the less accurate the exposure rating estimate of loss cost. Estimates of property or casualty claims severity above $1,000,000 are subject to great uncertainty.

- *For excess coverage, the degree of potential exposure not contemplated by the excess loss cost factors*

Neither the excess exposure arising from the clash of separate policies or coverages, nor from stacking of limits, are contemplated by bureau increased limits factors. Thus, some adjustment must be made, by using a clash loading factor, to price these additional exposures. Obviously, a great deal of judgment is used. Another part of this problem is the pricing of excess exposure for which there are no bureau increased limits factors, such as umbrella liability, farm owner's liability, or various professional liability lines.

For property coverage, most excess loss cost factors are derived from fire claims severity statistics. There is then the assumption that roughly the same claim severity applies to other perils.

If these other exposures are known to be a minor part of the overall exposure, the same loss cost rates estimated for similar exposures may be used to estimate the loss costs for these minor exposures. This is obviously subject to judgment and uncertainty.

TABLE 7.42

ITEMS TO CONSIDER IN DETERMINING THE CREDIBILITY OF THE EXPERIENCE LOSS COST ESTIMATE

- *The accuracy of the estimates of claims cost inflation*

This has an obvious effect upon the estimates of ground-up or excess loss cost. Claims cost inflation is usually estimated from broad insurance industry data, because most companies and many individual lines don't have enough claims data in order to make accurate estimates. Historical inflation trends should be modified for anticipated economic and societal changes. There is a lot of judgment involved in these estimates and in the use for particular companies and lines.

- *The accuracy of the estimates of loss development*

Historical claims development statistics for a particular line or company are often chaotic, especially for excess coverage. Thus, we often select claims development lags based upon broader data. These lags may not accurately reflect the development potential of the claims for the exposure being evaluated. And all historical data may not accurately reflect the future claims development potential.

- *The accuracy of the subject premium on-level factors*

As with the estimate of *RCF* for exposure rating, the accuracy of the estimates of the premium on-level factors has a direct effect upon the accuracy of the experience rating estimate of the loss cost. Although most companies have records of their manual rate changes, most do not keep good statistics on their rating plan deviations from manual rates. Therefore, broader insurance data must be used. Also, further rate deviations occur during soft markets when, for competitive reasons, some exposures may be undercounted or more optimistic rate credits are given to retain business.

- *The stability of the loss cost, or loss cost rate, over time*

It should be obvious that the greater the fluctuation in the historical loss cost or loss cost rate from year to year, the greater the sample error in the estimate of the mean, and also the greater the difficulty in measuring any possible trend in the data. This is especially the case for excess experience rating, where there may be few historical claims, so that the so-called process risk generates a high coefficient of variation (see Appendix G).

- *The possibility of changes in the underlying exposure over time*

Experience rating depends upon our ability to adjust past claims and exposure data to future cost level. If there have been significant changes in the book of business of the cedant, such as writing a new line, the experience rate will reflect these changes. Thus it must be adjusted, and this is, of course, subject to error. The less certain you are about the stability of the mix of business over time, the less certain you should be about the experience rate.

- *For excess coverage, the possibility of changes in the distribution of policy limits over time*

It is usually true that policy limits increase over time to keep pace with the inflation in the values being insured, thus with the increase in claim severity. To the extent this is not true, the claim severity data must be adjusted for the slower or faster change in limits.

APPENDIX G

THE STANDARD RISK THEORETIC MODEL

We further discuss Formulas 7.43 and 7.44.

1. Aggregate loss: $L = X_1 + X_2 + \cdots + X_N$

   where

   $L = rv$ (random variable) for aggregate loss

   $N = rv$ for number of claims (occurrences, events)

   $X_i = rv$ for the dollar size of the i^{th} claim

   This formula is true no matter what form the probability models have.

2. Assumptions:

   2.1. The X_i are independent and identically distributed,

   with c.d.f. $F_X(x) = \text{Prob}[X \leq x]$

   2.2. N is independent of the X_i's, and has p.d.f. $p_N(n) = \text{Prob}[N = n]$.

3. Then, the c.d.f. for L has the following form:

$$F_L(x) = \Sigma_N p_N(n) F_X^{*n}(x)$$

where $F_X^{*n}(x) = \text{Prob}[X_1 + X_2 + \cdots + X_n \leq x]$ convolution

4. Thus, each noncentral moment of L can be written:

$$E[L^k] = \int x^k dF_L(x)$$
$$= \int x^k d\{\Sigma_N p_N(n) F_X^{*n}(x)\}$$

$$= \Sigma_N\{p_N(n)\int x^k dF_X^{*n}(x)\}$$
$$= \Sigma_N\{p_N(n)E[(X_1 + X_2 + \cdots + X_n)^k]\}$$

5. So, in particular:

$$\begin{aligned}
E[L] &= \Sigma_N p_N(n)E[X_1 + X_2 + \cdots + X_n] \\
&= \Sigma_N p_N(n)\{nE[X]\} \\
&= E[X]\{\Sigma_N p_N(n)n\} \\
&= E[N]E[X] \\
E[L^2] &= \Sigma_N p_N(n)\{E[(X_1 + X_2 + \cdots + X_n)^2]\} \\
&= \Sigma_N p_N(n)\{E[\Sigma_{i,j}(X_iX_j)]\} \\
&= \Sigma_N p_N(n)\{\Sigma_{i,j}(E[X_iX_j])\} \\
&= \Sigma_N p_N(n)\{\Sigma_i E[X_i^2] + \Sigma_{i\neq,j}(E[X_iX_j])\} \\
&= \Sigma_N p_N(n)\{nE[X^2] + \Sigma_{i\neq,j}(E[X_i]E[X_j])\} \\
&= \Sigma_N p_N(n)\{nE[X^2] + (n-1)nE[X]^2\} \\
&= \{\Sigma_N p_N(n)n\}E[X^2] + \{\Sigma_N p_N(n)n^2\}E[X]^2 \\
&\quad - \{\Sigma_N p_N(n)n\}E[X]^2 \\
&= E[N]\mathrm{Var}[X] + E[N^2]E[X]^2
\end{aligned}$$

6. Thus, $\mathrm{Var}[L] = E[N]E[X^2] + (\mathrm{Var}[N] - E[N])E[X]^2$

7. With patience, you can also show that:

$$\begin{aligned}
E[(L - E[L])^3] = E[N]E[(X - E[X])^3] \\
+ E[(N - E[N])^3]E[X] \\
+ 3\,\mathrm{Var}[N]E[X]\mathrm{Var}[X]
\end{aligned}$$

8. If N is Poisson, then:

$$\mathrm{Var}[L] = E[N]E[X^2]$$
$$E[(L - E[L])^3] = E[N]E[X^3]$$

APPENDIX H

ABBREVIATIONS

- *ACR* — additional case reserve
- *ALAE* — allocated loss adjustment expense
- *BF* Method — Bornheutter–Ferguson method of loss development
- *CL* Method — chainladder method of loss development
- *ELCF* — excess loss cost factor
- *ELR* — expected loss ratio
- $E[X;c]$ — expected loss cost up to censor c
- *IBNER* — incurred by not enough reserve
- L — random variable for aggregate loss
- *MPL* — maximum possible loss
- N — random variable for number of claims
- *PML* — probable maximum loss
- *PCP* — primary company premium
- *PCPLR* — primary company permissible loss ratio
- *PV* — present value
- *RBF* — reinsurance brokerage fee
- *RCF* — rate correction factor
- *RCR* — reinsurance ceding commission rate
- *RDF* — reinsurer's loss discount factor
- *REP* — reinsurer earned premium
- *RELC* — reinsurance expected loss cost
- *RIXL* — reinsurer's internal expense loading
- *RLim* — reinsurance limit
- *RP* — reinsurance premium
- *SB* Method — Stanard–Bühlmann (Cape Cod) method of loss development
- *TER* — target economic return
- X — random variable for size of claim
- $X(d)$ — random variable X excess of truncation point d