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SURPLUS—CONCEPTS, MEASURES OF RETURN, AND DETERMINATION

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1. INTRODUCTION

Dr. Bender has made the results obtained in Mr. Bingham's paper more accessible by focusing on the essential elements that influence measurement of return, and by providing a variety of detailed examples. In addition, Dr. Bender has extended the work in several directions. Several of the results obtained in Dr. Bender's discussion paper are fundamental to the study of surplus and return on equity (ROE). In particular, Dr. Bender describes two basic tests of reasonableness that can be applied to any rate-of-return model in order to check the model's soundness. Because of their universal applicability, these tests are a major contribution.

Two major results presented in the discussion are: 1) the three measures of return discussed in the paper are equal to each other under a specific earnings release pattern, and 2) one of the measures (the NPV ratio) is constant with respect to the earnings release pattern. As will be discussed below, there are some accounting issues that must be dealt with in order to make use of these results, and they do not generally hold true for a model that does not include reserve margin (or some equivalent mechanism). Despite this caveat, Dr. Bender's paper contains other

important findings, and represents a substantial contribution to the actuarial literature on surplus and profitability measurement.

2. CALENDAR YEAR MEASURES

In his introduction, Dr. Bender states: "When evaluating the return earned by a particular product line, it is this long-term investment of surplus that must be considered. This is in sharp contrast to calendar year measures in which it is assumed that all of the company surplus supports the currently written exposure."

Dr. Bender correctly points out that surplus supports exposures from all accident years that have not yet been closed, as well as current writings. In particular, it should not be assumed that surplus supports only the current year's written premium. Although this is a common interpretation of calendar year profitability measures, no such assumption necessarily exists, even when a premium-to-surplus ratio is used in profitability measurement. Such calculations use premium as a measure of the volume of business (including prior years' exposures), while the surplus serves as a measure of internal capitalization.

The premium-to-surplus ratio measure must be used with caution because the current year's written premium is a very imperfect measure of the volume of the business. A hidden assumption is that the ratio of outstanding liabilities plus the expected future liabilities arising from the current writings to the current year's written premium is a constant. This is generally not true because current writings will fluctuate according to many factors such as entry into and exit from lines of business, pricing adequacy, market conditions, etc. Additionally, it is affected through changes on the liability side such as loss payout characteristics, inflation, etc. It can be shown that in a steady-state situation (with no underlying price, exposure, or loss characteristic changes) that the current year's written premium is an accurate measure of the volume of liabilities of the business. Although the premium-to-surplus ratio measure has problems, it is a convenient way to

allocate capital in a model. If a premium-to-surplus ratio measure is used, it must reflect current premiums and past liabilities and the volatility inherent in both.

For example, suppose a company plans to write $1,000,000 of premium during a given calendar year and has $2,500,000 of loss reserves at the beginning of the year. (For simplicity, unearned premium reserves will be omitted from this example.) Also suppose that the company performs a comprehensive analysis of risk for its portfolio, determining that $250,000 of surplus should be allocated to support the expected future liabilities from the writing of the premium and $550,000 of surplus should be allocated to supporting the outstanding loss reserves. The total surplus commitment is $800,000, which can be construed to produce a written premium to surplus ratio of 1.25. This does not imply that $100 of surplus supports each $125 of premium written.

3. PRODUCT ACCOUNT AND SURPLUS ACCOUNT

Dr. Bender discusses a useful perspective that was developed in the Bingham paper. In his overview of Bingham's methodology, Dr. Bender writes, "The world can be divided into three parts … the insurance product, shareholder funds [surplus], and everything that is external to the other two parts." The conceptual distinction between product account and surplus account can either be directly incorporated into a model or at least kept in mind by an actuary while developing and testing a model.

An application of this paradigm occurs later in the paper, when Dr. Bender observes that the ROE must equal the investment rate of return if the insurance product account generates an operating gain of zero. If one imagines the product account generating no outflow or inflow of funds, and the surplus account generating the investment rate, then the result becomes readily apparent without the need for calculations. This test can be employed by an actuary to check the soundness of a return

model being considered for use. Dr. Bender's conclusion that the calendar year steady-state model fails the test, and is therefore inherently inaccurate, appears correct. Both the reasonableness test and this conclusion are noteworthy contributions.

4. SURPLUS ACCOUNTING AND RETURN MEASUREMENT

An accounting problem arises when Dr. Bender discusses income that is generated from funds in the product account (generally known as "income from insurance operations"). This includes earned premium and investment income on underwriting funds (but not investment income on surplus) minus incurred losses and expenses. Dr. Bender writes, "While reserves and supporting surplus are clearly identified as 'belonging' to the insurance product, the time at which other funds that arise from the insurance product are released to the surplus account is somewhat arbitrary."

The problem is that there is no such action as "releasing funds to the surplus account." Surplus by definition is the amount of assets in excess of liabilities and is thus the balancing item on the balance sheet. Assuming that liabilities are consistently stated without bias (which is generally assumed in models of this kind), the only way surplus can be deliberately increased or decreased is through transactions with external shareholders. Operating gain cannot remain in the insurance product account, even if generated by funds in the product account: as soon as any such gain is recognized, it immediately and automatically becomes surplus, by the definitions of income and surplus.

The model shown in Dr. Bender's exhibits allows income to accumulate as "retained earnings" in the product account, rather than as an increase in surplus. But these "retained earnings" are actually additional surplus and must either be distributed to shareholders or counted as surplus in the denominator of ROE. Either way, the actual surplus levels and flows differ from those shown in Dr. Bender's exhibits. Although his demonstration and

proof of the equality of the three return measures is mathematically sound, this equality is not a true representation of ROE because the surplus is inaccurately stated.

That said, Dr. Bender's analysis and results are valid when reserve margin is included in the model. Reserve margin is the amount by which a reserve (the stated value of a liability) exceeds the unbiased estimate of the liability's value. Reserve margins have an important, legitimate use that has been documented in the literature [1].

Reserve margin neatly fills the role of "retained earnings" in the paper's exhibits. Since reserve margin is part of total reserves, it is in the product account. A reserve margin can be viewed as an asset or "operating gain" that has not yet been recognized as an increase to surplus, which is exactly what "retained earnings" are. Dr. Bender notes that retained earnings act as "… an additional buffer against insolvency risk." A positive reserve margin does act as an additional buffer, absorbing the impact of adverse results before surplus is affected. Finally, the level of reserve margin can be selected to increase or decrease the surplus level, providing a mechanism for releasing funds to the surplus account.

If we substitute the label "Reserve Margin" for "Retained Earnings" in the paper's exhibits, all of the paper's results hold. The only question is whether it is reasonable to include reserve margin in a return model. This is a question to be decided by the individual model designer, based in part on the particular application for which the model is being developed.

A minor remaining problem is that the paper's exhibits often show a negative value for retained earnings. Negative reserve margin implies inadequate nominal reserves, which would inflate the calculated return. A negative reserve margin condition may not be acceptable in some return modeling applications.

5. NOMINAL VS. DISCOUNTED RESERVES

Dr. Bender makes an important point: if a company calculates required supporting surplus based on nominal unpaid losses so that a performance criterion (e.g., probability of ruin less than 2%) is met, then the result is a surplus requirement for the future (when the loss payments are to be made). A lesser amount of surplus is sufficient at the time of the evaluation, since the surplus can accumulate investment income during the interim. The question that Dr. Bender then addresses is how much surplus is required at the time of evaluation to meet the performance criterion.

Dr. Bender advocates calculating the surplus requirement based on discounted loss reserves. His method is to apply a leverage ratio to the discounted reserves. The leverage ratio is calculated from the probability distribution of discounted future payments, so that timing risk and investment return risk are accounted for in the distribution. The resulting surplus meets the performance criterion with respect to the discounted reserves at the time of evaluation.

For example, suppose nominal loss reserves are $10,000 and discounted reserves are $8,000. Suppose also that ultimate paid losses will be less than $15,000 with 98% probability, and that the distribution of discounted unpaid losses has its 98th percentile at $9,600 (considering all possible interest rate and payout scenarios). To meet the performance criterion of P(ruin) $<$ 2% using nominal loss reserves, the supporting surplus would be $\$15{,}000 - \$10{,}000 = \$5{,}000$, which corresponds to a 2.00 reserves-to-surplus leverage ratio. Using discounted reserves, the surplus required would be $\$9{,}600 - \$8{,}000 = \$1{,}600$ for a 5.00 leverage ratio. Although the 5.00 leverage ratio seems high, there is a 98% probability that the $9,600 fund will accumulate sufficient investment income to pay all claims as they come due.

This method meets the performance criterion on discounted reserves at the date of evaluation and simultaneously provides

proper funding to meet the performance criterion at the future payment dates. It is a mathematically correct answer to the question that was posed.

There are two notable objections to using Dr. Bender's discounted reserves approach: 1) it is presently impossible to accurately quantify the probability distributions of future interest rate levels and claims payment patterns, both of which are fundamental elements for determining the distribution of discounted unpaid losses; 2) if claims develop adversely as of a later evaluation, more surplus may have to be obtained to continue to meet the performance criterion. If additional surplus is available at each evaluation point (as could be the case for an insurance company within a holding company group), this is not a problem. If not (as could be the case for a small stand-alone company), there is no margin for such a contingency.

Both of these objections are addressed by using nominal loss reserves. The only distribution to be considered is the aggregate loss distribution, which can usually be estimated reasonably. If additional surplus should be required at a later evaluation, a portion of the investment income earned on surplus can be retained, rather than released as earnings.

Future developments in financial analysis may eventually provide solutions to the first objection. The second objection could be addressed by setting the surplus level a little higher, so as to provide a prescribed cushion on top of the surplus level that is dictated by the performance criterion. The amount of cushion would thus be selected more precisely than the somewhat arbitrary investment income cushion provided by using nominal loss reserves.

Dr. Bender did raise the possibility of adverse loss development and the consequent need for additional surplus. He treated this issue in Section 6 of his paper, using the following example: expected nominal losses of $44 are initially allocated $22 of surplus (using a 2 : 1 rule), for a total funding requirement

of $66. Two years later, the losses are re-evaluated, and the best estimate is $60. Dr. Bender offered three possible solutions:

1. Allow the surplus level to drop as a result of the adverse loss development. In the example, the additional $16 of adverse development would be absorbed by the original surplus allocation, and the new surplus level would be $6. The total funding requirement is still $66.

2. Restore surplus to its original level. For the example, this would mean increasing the surplus level to $22, for a total funding requirement of $82.

3. Increase the surplus level, following the original surplus rule. In this example, the rule was a 2 : 1 ratio, so the new surplus level would be $30, and total funds would be $90.

Which of these alternatives is used may depend on the application. For example, the first approach is often implicit in a pricing model, where surplus is set with the knowledge that worse or better results will be achieved over the sample space of lines and years. In fact, a total exhaustion of the surplus ("ruin") is actually expected to occur a certain percentage of the time, if a probability of ruin method is used to set surplus.

None of these three alternatives corresponds to the surplus calculation method that Dr. Bender proposes. The new information that produced the higher reserve valuation should be incorporated into the leverage ratio. We propose a fourth alternative: calculate a new leverage ratio in the same way that the original 2 : 1 ratio was calculated, perhaps based on variability of outstanding losses (nominal or discounted). Apply the leverage ratio to the current valuation of outstanding losses to determine current surplus requirements. This alternative resembles the third approach, but is more consistent with the surplus calculation ideas that Dr. Bender puts forth.

Dr. Bender indicates that using discounted reserves to calculate required surplus allows one to account for timing risk and investment return risk. A caution is in order: simply applying a leverage ratio to discounted reserves to calculate required surplus does not account for either timing risk or investment return risk. Both of these risks are higher for long payment patterns, but discounted reserves are lower for longer patterns. Applying a fixed leverage ratio to discounted reserves would result in less surplus being assigned to a longer pattern, but the increased timing and investment risks would warrant more surplus (all else being equal). If a leverage ratio is used with discounted reserves, then the ratio must be explicitly calculated based on the variability of the discounted future payments, as Dr. Bender advises.

6. INACCURACY OF THE CALENDAR YEAR RETURN MEASURE

Dr. Bender provides excellent explanations and exhibits to show that calendar year accounting distorts the measurement of return. For Dr. Bender's first "reasonableness test," the insurance product is priced at break-even so that the total return should equal the investment rate obtained on surplus. In the paper's example, the calendar year return (under statutory accounting) is 8.1%, much higher than the 5.0% investment rate. We constructed our own model and independently verified the accuracy of this result, assuming the surplus levels presented in the paper's exhibit.

Dr. Bender continues with a discussion of the calendar year distortion, explaining the result from several perspectives. His lucid explanations make it possible for readers to understand how the calendar year measure fails to produce the proper result. Dr. Bender then notes that the exposure growth rate assumption influences the calendar year return, so that if the growth rate is assumed to be equal to the investment rate, the calendar year return will then produce the correct result. Finally, another example is given in which the insurance product clearly loses money, but

the calendar year return is erroneously higher than the investment rate.

The case that is made against calendar year return is so compelling that the unavoidable conclusion seems to be that calendar year return is (in general) an inaccurate measure of actual return. But what if calendar year return is used to measure a company's performance, either by internal management or external parties? An actuary who is building a return model for, say, pricing purposes will probably still have to include calendar year return in the model (perhaps alongside another return measure). The actuary also will have to consider the calendar year return in the decision-making process, while at the same time recognizing that the calendar year result does not accurately depict profitability.

The fact is that calendar year ROE is currently a prevalent method of calculating return. Dr. Bender's findings should motivate us to conduct research into alternative return measures.

7. SELF-SUPPORTING PREMIUM AND INFINITE RETURN

Dr. Bender's second "reasonableness test" considers the situation where premium is large enough to produce its own supporting surplus as it earns. Surplus allocation formulas often allocate surplus to a policy or line before any premium is earned, on the theory that risk is related to the unearned premium and is present from the time a policy is written. Another perspective is that losses are incurred as premium earns, so the surplus associated with a portion of premium is not needed until the moment that premium is earned, because that's the time when the insurer is actually exposed to loss (not before). After the premium earns, some of the surplus then remains associated with the corresponding loss reserves and runs off accordingly.

Both perspectives are useful. A surplus allocation formula can be used to budget needed surplus for a line of business at annual intervals, based on upward variability of losses from the

expected level. The earning perspective can then be used to reduce the amount of budgeted surplus by the profit that the line is expected to generate as the premium earns. This expected profit will accrue to surplus if actually realized, so it is "future surplus:" not available at the time of budgeting but also not needed until realized and available. If losses are greater than expected, the impact will first be a reduction in this "future surplus," before budgeted surplus is impacted.

As Dr. Bender states, if premium is high enough, the budgeted surplus requirement becomes zero, because the entire surplus need is met by the earning of the premium. Therefore, no investment is required up-front, and the return (under the expected losses scenario) is infinite. Dr. Bender then compares the three return measures as the premium rises to the infinite-return value and observes that only the internal rate of return (IRR) measure yields the correct result. The other two measures produce finite values for return, even when the premium is high enough to generate its own supporting surplus "on-the-fly."

The problem again is that surplus is not being calculated according to the correct formula. The liabilities are discounted at the investment rate, and there is no recognition of the unearned premium reserve liability at the beginning. (In earlier exhibits, it appears that the concepts of "invested capital" and "surplus" are being confused with each other.) In spite of this, the IRR results that are presented in the paper can be reproduced under correct accounting by setting assets equal to Dr. Bender's funding requirement at each point in time.

In any case, the other two return measures (Calendar Year ROE and net present value (NPV) Ratio) will not produce values that approach infinity, no matter how high the premium is. This is because both of these measures are ratios, with total surplus in the denominator. Calendar Year ROE equals Total Income/Total Surplus, and NPV Ratio equals NPV(Total Income)/NPV(Total Surplus). The only way either ratio could be infinite is if the surplus level is kept at zero for the entire period, which would

not make any sense since some supporting surplus must be held until losses are completely paid. Dr. Bender states that the NPV Ratio measure would approach infinity if surplus requirements were reduced "in recognition of the retained operating gain," but again this "retained operating gain" is actually surplus. The NPV Ratio simply cannot produce the infinite return result.

Exhibit 1 shows a simple example that compares the three return measures. The premium has been set to a high level, so that the policy generates its own surplus (and then some) as premium earns. As the exhibit shows, IRR is infinite because there is zero initial investment and all the cash flows to the investors are positive. The other two return measures produce values that are finite, though large.

The IRR measure produces an infinite return in this example because it is focused on the flows between the company and the shareholders (or the "surplus surplus" account, to use Dr. Bender's terminology), rather than on the company's internal surplus. The other measures implicitly identify the company's internal surplus as invested funds, and measure the return against those funds. Ironically, the Internal Rate of Return (IRR) is distinguished here by its reliance on the company's external transactions with shareholders, versus the alternative return measures, which are based on internal company surplus.

8. CONCLUSION

In summary, Dr. Bender has written a discussion paper that stands on its own. All of Dr. Bender's findings discussed above are essential to a complete understanding of return measurement, and many of them can be directly incorporated into return modeling applications.

REFERENCES

[1] Balcarek, Rafal J., "Effect of Loss Reserve Margins in Calendar Year Results," *PCAS* LIII, 1966, pp. 1–17.

EXHIBIT 1

A SELF-SUPPORTING LINE

Premium = $2,000
Loss = $1,000 paid 2 years after inception
Surplus = 50% of Nominal Loss Reserves
Investment Income = 5% per year
Taxes are omitted

Underwriting Quantities

Time, yrs	Written Premium	Earned Premium	Incurred Loss	Paid Loss	Unearned Premium Reserve	Loss Reserve
Inception	2,000	0	0	0	2,000	0
1	0	2,000	1,000	0	0	1,000
2	0	0	0	1,000	0	0
Total	2,000	2,000	1,000	1,000		

Assets, Liabilities, and Surplus

Time, yrs	UEP Reserve	Loss Reserve	Total Liabilities	Surplus	Total Assets
Inception	2,000	0	2,000	0	2,000
1	0	1,000	1,000	500	1,500
2	0	0	0	0	0

Investment Income Calculation

Time, yrs	Total Assets	Assets Not Investable	Investable Assets	5.00% Investment Income
Inception	2,000	0	2,000	0
1	1,500	0	1,500	100
2	0	0	0	75
Total				175

EXHIBIT 1

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A SELF-SUPPORTING LINE

Calculation of Total Income

Time, yrs	Earned Premium	Incurred Loss	Net U/W Income	Investment Income	Total Income
Inception	0	0	0	0	0
1	2,000	1,000	1,000	100	1,100
2	0	0	0	75	75
Total	2,000	1,000	1,000	175	1,175

Calculation of Flows to Shareholder

Time, yrs	Surplus	Change in Surplus	Total Income	Flows To/(From) Shareholder
Inception	0	0	0	0
1	500	500	1,100	600
2	0	−500	75	575
Total	500		1,175	
NPV	476		1,171	

NPV(Income)/NPV(Surplus) = 1,171/476 = 246%
Calendar Year Average Return = 1,175/500 = 235%
IRR = Infinity