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FOREWORD

Actuarial science originated in England in 1792 in the early days of life insurance. Because of the technical nature of the business, the first actuaries were mathematicians. Eventually, their numerical growth resulted in the formation of the Institute of Actuaries in England in 1848. Eight years later, in Scotland, the Faculty of Actuaries was formed. In the United States, the Actuarial Society of America was formed in 1889 and the American Institute of Actuaries in 1909. These two American organizations merged in 1949 to become the Society of Actuaries.

In the early years of the 20th Century in the United States, problems requiring actuarial treatment were emerging in sickness, disability, and casualty insurance—particularly in workers compensation, which was introduced in 1911. The differences between the new problems and those of traditional life insurance led to the organization of the Casualty Actuarial and Statistical Society of America in 1914. Dr. I. M. Rubinow, who was responsible for the Society's formation, became its first president. At the time of its formation, the Casualty Actuarial and Statistical Society of America had 97 charter members of the grade of Fellow. The Society adopted its present name, the Casualty Actuarial Society, on May 14, 1921.

The purpose of the Society is to advance the body of knowledge of actuarial science in applications other than life insurance, to establish and maintain standards of qualification for membership, to promote and maintain high standards of conduct and competence for the members, and to increase the awareness of actuarial science. The Society's activities in support of this purpose include communication with those affected by insurance, presentation and discussion of papers, attendance at seminars and workshops, collection of a library, research, and other means.

Since the problems of workers compensation were the most urgent at the time of the Society's formation, many of the Society's original members played a leading part in developing the scientific basis for that line of insurance. From the beginning, however, the Society has grown constantly, not only in membership, but also in range of interest and in scientific and related contributions to all lines of insurance other than life, including automobile, liability other than automobile, fire, homeowners, commercial multiple peril, and others. These contributions are found principally in original papers prepared by members of the Society and published annually in the *Proceedings of the Casualty Actuarial Society.* The presidential addresses, also published in the *Proceedings*, have called attention to the most pressing actuarial problems, some of them still unsolved, that have faced the industry over the years.

The membership of the Society includes actuaries employed by insurance companies, industry advisory organizations, national brokers, accounting firms, educational institutions, state insurance departments, and the federal government. It also includes independent consultants. The Society has two classes of members, Fellows and Associates. Both classes require successful completion of examinations, held in February, and in the spring and fall of each year in various cities of the United States, Canada, Bermuda, and selected overseas sites. In addition, Associateship requires completion of the CAS Course on Professionalism.

The publications of the Society and their respective prices are listed in the Society's *Yearbook*. The *Syllabus of Examinations* outlines the course of study recommended for the examinations. Both the *Yearbook*, at a charge of $40 (U.S. funds), and the *Syllabus of Examinations*, without charge, may be obtained from the Casualty Actuarial Society, 1100 North Glebe Road, Suite 600, Arlington, Virginia 22201.

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NOTICE

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HOMEOWNERS RATEMAKING REVISITED (USE OF COMPUTER MODELS TO ESTIMATE CATASTROPHE LOSS COSTS)

MICHAEL A. WALTERS AND FRANÇOIS MORIN

Abstract

Recent improvements in computer technology and easy access to large quantities of data have eliminated some traditional limitations on insurance ratemaking. The emergence of catastrophe simulation using computer modeling has helped actuaries develop new methods of measuring catastrophe risk and providing for it in insurance rates. This paper addresses these new methods and illustrates the features and benefits of computer modeling for catastrophe ratemaking. Hurricane loss costs as part of homeowners coverage are treated in the main body of the paper; modeling for other catastrophic perils is reviewed in the Appendix.

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1. WHY MODELING?

According to the CAS Principles of Ratemaking, a rate is "an estimate of the expected value of future costs, provides for all costs associated with the transfer of risk, and provides for the costs associated with an individual risk transfer."

Traditionally, ratemaking has been regarded as the art of extrapolating valid conclusions about the future from scientifically measured past experience. However, for lines of business with catastrophe potential, questions always arise as to how much past insurance experience is needed to represent possible future outcomes and how much weight should be assigned to each year's experience. For instance, if a 1954 hurricane was the last severe event in a given state, may one assume that the return period for an event of the same severity is 43 years? What if historical records show that more severe storms occurred in the 1930s, before the advent of homeowners coverage? If the same storm struck in 1997, would it affect the same properties? What level of damage would occur, given that the distribution of insureds has shifted to coastal communities and that the insured values at risk have trended at a pace that has exceeded inflation?

For these rare calamities, reliance on actual insured experience does not allow accurate measurement of future expected loss. Therefore, one must use a much longer experience period, especially for event frequency. Computer simulation of events to obtain current insured losses has replaced traditional methods based exclusively on reported loss experience. These new methods can now be used not only to measure expected losses, but also to develop risk loadings to compensate for the variance in outcomes, compared to lower-risk insurance products.

The need for catastrophe modeling to aid in reinsurance purchase decisions and in insurance ratemaking has existed for some time. However, computer limitations on the amount of data that could be manipulated to develop a catastrophe model rendered the concept impractical in the past. In recent years, computer capacity has improved dramatically, making catastrophe simulation feasible. Increased computer capability has also enabled scientists to expand their research and produce better simulations through a better understanding of catastrophic events.

2. WHAT TO MODEL

A state's most recent historical losses may not be indicative of its true catastrophe potential because what happens in a given year is only a sample of what could have happened. The goal is to build a model to simulate what could realistically occur, based on information relevant to that state and to all refined geographic areas within the state.

Building a computer model requires that the estimation process be separated between frequency and severity. For the frequency of hurricanes, there is a long history (more than 100 years) of recorded information to help gauge the relative likelihood of landfall in a given state. Even so, there may be a need to supplement that history with geologic information dating back several thousand years to measure the relative frequency of Category 5 hurricanes. Such investigations are now feasible. Scientists believe that they can determine the return periods of very severe events by examining tempestites—ocean floor and coastal lagoon samples, where catastrophic events have left telltale signs in the sand.

For severity of hurricanes, however, older storms over the past hundred years do not offer any useful insured loss information. Even for storms in the 1950s and 1960s, the extent of loss if that same storm occurred again would depend on *today's* insured values, deductibles and level of windstorm-resistant structures. However, a computer simulation model for the hurricane peril can take the characteristics of a storm and replicate the wind speeds over its course after landfall. The damage to buildings and contents and the resulting effect on insured values are based on the wind field created by the modeled storm. Validation of the model examines actual loss experience obtained from storms that have occurred over the recent past. This is an ongoing process as new catastrophes occur.

Because storm simulation by computer was the initial breakthrough, we start with it as the basis for modeling the severity component in estimating hurricane loss costs.

3. HOW TO MODEL FOR SEVERITY

The severity component of catastrophe modeling generally comprises three distinct modules requiring three separate skills:

- event simulation (science)
- damageability of insured properties (engineering)
- loss effect on exposures (insurance).

The event simulation module is designed to reproduce natural phenomena. For a hurricane model, wind physics is now understood well enough to predict wind speeds at every location over the course of a single storm. A model would use such key inputs as central pressure, radius of maximum wind, and forward speed of storm. For practical purposes, each risk can be viewed as being at the geographic centroid of the ZIP code in which it is located. This is generally the finest level of detail currently coded by insurers for their risks. However, greater availability of exposure information at the street level (especially for personal lines) will eventually allow models with even finer levels of detail.

The damageability module estimates the damage sustained by a given property exposed to the simulated event. The damage functions used in a catastrophe model are generally developed by engineers familiar with structural vulnerabilities who test the resistance of various materials to high wind speeds. (The results of these studies are also used to develop new materials and to implement new building codes to limit the damage from catastrophes.)

The insured loss effect module incorporates the results of the first two modules and adjusts for such factors as deductibles, coinsurance, insurance to value, and reinsurance. The loss effect is generally the only company-specific module because it includes all the factors that describe an insurer's in-force book of business. It is also the one used for risk analysis (probable maximum loss) for an individual insurer.

The severity component of catastrophe modeling is usually deterministic, calculating the impact of a predetermined event with known characteristics. The computer, in effect, simulates that event today, with the resulting losses to insured exposures. Of course, even for a particular set of parameters (e.g., wind speed or landfall), the actual distribution of losses will be stochastic. However, the use of a damage factor curve, with validation over a number of storms, can adequately represent the average loss results. This is especially true when a large number of events are simulated. Appendix A provides a detailed description of the process of developing and validating the severity component of a catastrophe model.

4. HOW TO MODEL FOR FREQUENCY

Deterministic catastrophe models were the first ones created, calibrated and validated. They helped to approximate probable maximum loss calculations for risk analysis, by postulating possible storms in different locations to estimate insured losses from adverse events. This deterministic method, however, is not appropriate for ratemaking, which needs to incorporate relative frequency or the probabilities of each type of storm.

To add a frequency component to the hurricane model, one must analyze long-term meteorological records of hurricanes by landfall area, supplemented with informed judgment obtained from professionals in the field of meteorology. One can obtain the historical data from National Oceanic and Atmospheric Administration (NOAA) publications. The past data are then fitted to derive probability distributions of the key input parameters, such as radius of maximum wind, forward speed and pressure differential at the eye of the storm. For example, an analysis of the radius of maximum winds of historical events in South Florida yields a conclusion that they are normally distributed ($N(\mu,\sigma)$), with parameters of 16.840 and 10.567 nautical miles.

Sampling techniques (Monte Carlo, stratified, or a combination of both) can randomly select the parameters from each distribution. Monte Carlo sampling generally assigns an equal probability to all sampled items from the entire population, which makes it easy to use and explain to a nonstatistical audience. One of its drawbacks, however, is a lack of precision in estimating unlikely events. This can be overcome by generating a very large sample size. However, in certain situations, the sample size may become enormous and create problems of efficiency, even with today's computers. An alternative is stratified random sampling.

By dividing the entire population into smaller groups (or strata), stratified sampling allows a more accurate estimation of their distribution, considering homogeneity. These estimates can then be combined into a precise estimate of the overall population with a smaller sample size than with Monte Carlo sampling.[1] Another benefit of stratified random sampling is the ability to sample a larger number of events in each strata than their relative probability in the overall population. This makes the estimation of extremely unlikely events possible, such as a Category 5 hurricane in Maine. This is important because the potential damage associated with such an event, even though only remotely

[1]Refer to Cochran [2, p. 87] for additional information on the benefits of stratified sampling.

conceivable, may be of significance for certain insurers for risk analysis or for ratemaking. When this approach is utilized, the relative probability of each sampled storm must be adjusted to reflect its overall probability in the distribution.

In conjunction with storm intensity distributions, one must also develop the storm path and landfall location for each modeled storm. The selected parameters are based on actual historical events over the last hundred years and on other available sources of information.

After selecting the storm intensity parameters and deriving their respective conditional probabilities, the results are combined. The probabilities are conditional because they refer to the likelihood of a hurricane of a certain size, once a hurricane makes landfall. By definition, the sum of the probabilities will add up to one. The end result is the probabilistic library, which comprises a large enough number of events (in excess of 5,000) to represent all likely scenarios, each with an associated probability. While there is no minimum set of events or sample size required, it is important that it be large enough to ensure that every ZIP code exposed to hurricane force winds will be subjected to a significant number of events. By using stratified sampling techniques, it will be typical for a given ZIP code to be affected by over 1,000 events, rendering the loss estimates fully credible.

5. BASIC OUTPUT OF MODEL

A probabilistic database is the key to calculating expected loss costs. Because the basic premise is that all possible events have been identified along with their probabilities, one can calculate expected loss costs directly for the base class risk in a geographic locale. Simply run the entire event library against a base class house at $100,000 of Coverage A at the centroid of each ZIP code. The resulting expected losses can be divided by the amount of insurance in thousands to produce an expected loss cost per $1,000 of insurance for each ZIP code.

The reason the ZIP code is used as the basic building block is that virtually all insurers are capturing this value. If insurers were geo-coding risks (i.e., by street address mapped to latitude and longitude), the model could also produce loss costs at that level of detail. However, the ultimate rating territories for hurricane are likely to include multiple ZIP codes, so the results can be initially produced by ZIP code.

To ensure that all coverages are handled appropriately in the simulation for a homeowners policy (HO-3), one would assign an additional 10% of the Coverage A (building) amount for Coverage B (appurtenant structures), 50% for Coverage C (contents), and 20% for Coverage D (additional living expense; i.e., loss of use).

Annual expected loss costs for a given ZIP code are obtained by multiplying the sum of the probability-weighted simulated results across all storms by an annual hurricane frequency. The average annual frequency of hurricanes making landfall in the U.S. has been approximately 1.3 for storms with central pressure under 982 millibars.

For a given line of business, the expected losses by ZIP code are then:

$$EL_{\text{ZIP}} = F \times \sum_{\text{storm}} (P_{\text{storm}} \times E_{\text{ZIP}} \times DF_{\text{storm}}),$$

where

EL_{ZIP} = Expected losses for ZIP code for base class

F = Annual hurricane frequency

P_{storm} = Probability of storm

E_{ZIP} = Total exposure amount (Base class constant for all ZIP codes)

DF_{storm} = Damage factor for base class by ZIP code by storm.

These expected losses represent insured losses for a base class amount of insurance, construction type and deductible. These may be selected as frame building with a \$250 deductible, \$100,000 of Coverage A, \$10,000 of Coverage B, \$50,000 of Coverage C and \$20,000 of Coverage D. Because loss adjustment expenses for catastrophes are generally related to the overall level of losses, it is appropriate to include them in the expected losses as a percentage of total losses.

To convert this to a loss cost expressed as a rate per \$1,000 of Coverage A, divide by the exposure base times 1,000.

$$ELC_{\mathrm{ZIP}} = \frac{EL_{\mathrm{ZIP}}}{COVA_{\mathrm{ZIP}}} \times 1{,}000,$$

where

ELC_{ZIP} = Expected loss cost for ZIP code

$COVA_{\mathrm{ZIP}}$ = Base class Coverage A amount in ZIP code.

Independence from Company Experience

A major feature of this calculation is its independence from an individual company's actual loss experience and exposure distribution. Being independent of individual company data, it is, in fact, appropriate for each insurer.

What would happen if an insurer tried to use its own exposure distribution to estimate base class loss costs? First, it would have to run the model in complete class and ZIP code detail over its latest exposure distribution, which would produce expected losses in dollars for the insurer by ZIP code. However, dividing by the total exposures by ZIP code would only yield average loss costs by ZIP code. What if the insurer had a disproportionate number of high-risk exposures in that ZIP code? The insurer would have to divide by the average class relativity in each ZIP code to get the average base class loss cost.

Furthermore, the class relativities to divide out should, in theory, be the indicated class relativities, not the current relativities. Section 6 will deal with how to calculate indicated class relativities using a model. Doing all this using company exposures would then only produce the same answer as using the base class exposure method described above.

In traditional loss ratio methods of ratemaking, with actual loss experience determining loss costs, it is important to use the insurer's actual losses and exposures. However, in catastrophe ratemaking using computer modeling, large volumes of industry loss experience have been used over the last ten years to calibrate the average severity, and meteorologic data over a hundred years have been used to calibrate frequency.

Hence, the value of an individual insurer's actual loss experience is very limited. First, it may not be relevant to know that, for hurricane, a house was insured by Company A versus Company B. Second, an individual insurer may be such a small subset of the total industry loss experience that it has little credibility, especially if the insurer has less than a 5% market share. The example here is for such an insurer, for whom the hurricane model represents the best estimate of future expected costs.

Combining ZIP Codes Into Territories

The next step is to use the insurer's actual exposure distribution by ZIP code to get the base class loss costs for the territory structure it selects after reviewing the indicated hurricane loss costs by ZIP code. The use of geographic mapping is especially useful in this selection process because the ZIP codes can be grouped in ranges and then printed on color-coded maps to help visualize the boundaries of possible territories. For the early years of ratemaking via catastrophe models, broad groups of ZIP codes are likely, such as those with loss costs in ranges of \$.25 per \$1,000 of Coverage A. Once the ZIP code groupings are selected, the loss costs for the new territories can be calculated by the following formula:

$$ELC_{\text{terr}} = \frac{\sum_{\text{ZIP}} (ELC_{\text{ZIP}} \times COVA_{\text{ZIP}})}{\sum_{\text{ZIP}} COVA_{\text{ZIP}}},$$

where

$$ELC_{\text{terr}} = \text{Expected Loss Cost for territory, and}$$

$$COVA_{\text{ZIP}} = \text{Coverage A amount for territory.}$$

In Exhibit 1, the ZIP code loss costs per $1,000 of Coverage A for homeowners are averaged for a given territory structure to derive the territorial loss costs for hurricane coverage. It is likely that the more appropriate territory structure for hurricane will differ from regular homeowners territories. Because the latter evolved over time to respond to homogeneity considerations in setting rates for the perils of fire and theft, there is a need to create new territories to reflect differences in hurricane loss potential.

6. ATTRIBUTES OF LOSS COSTS VIA COMPUTER MODELING

Credibility

Through computer simulation and stratified sampling, the individual ZIP codes are fully credible in the traditional sense because the inputs have theoretically accounted for all the useful information (from industry-validated damage factors to more than 100 years of storm frequency experience). One would not want to assign the complement of credibility to an insurer's actual results on a statewide basis over the past few years, because the recent insurer results add no useful new information and, in fact, could bias the answer because of too much randomness. The idea of the model is to substitute the random variation of low-frequency *actual* storms with the use of a reasonable set of *possible* storms, with their probabilities. (It is understood that even the past 100 years of hurricane history do not contain the set of all possible storms and their inherent likelihood.)

While theoretical full credibility can be assigned in refined cell detail from the computer simulation, this only means that random statistical variation can be resolved to minimize the process risk from a ratemaking standpoint. However, there is still parameter risk in the selection of the key variables because the event frequencies of the past 100 years may not be representative of the next 100 years. (This is especially true in earthquake simulation, where return periods may be in the hundreds or even thousands of years. Also, the understanding of the physics of shake intensity is still evolving among earthquake experts.)

Overcoming parameter risk is the goal of scientific research in the future. As geologic findings help measure the return periods of large hurricanes by region, better estimates of frequency will be developed. This is really no different from the basic ratemaking paradigm that the recent past history will repeat itself, and that the five-year experience period of loss ratio reviews is assumed to be predictive of the next few years. In the case of hurricane modeling, the pure premium method actually calculates the long-term frequencies separately from the more recent average severities, so the existence of parameter risk is highlighted, especially in the frequency calculation. Also, the answer to parameter risk is not to abandon modeling as a method, but to continually strive for better input parameters.

The pure premium method also allows the calculation of loss costs in refined detail directly, using the model's frequency and severity features. For traditional loss ratio ratemaking, the actual insured loss experience from the recent past is used, beginning with statewide totals. Each refinement of statewide data to territory or class carries with it a reduction in credibility because of much smaller experience volumes. This stems from the experience loss ratio method used to derive the result—actual insured experience that is a sample taken from what is expected to occur over time. In contrast, hurricane loss costs are derived from an estimated set of all possible events as constructed in the computer model.

Frequency of Review

Hurricane loss costs derived from modeling do not need frequent updates for two reasons. First, with more than 100 years of event characteristics shaping the model design, another year of actual results is unlikely to change the model parameters much. However, in the early years of model usage, the potential exists to update some of the damage factors. Also, when new class variables are developed, one can refine initial estimates with the loss experience of subsequent actual storms. For example, one could test new kinds of shutters and incorporate the results in the model. For estimating territory loss costs in the early years of model implementation, ZIP code distributions could change, as insureds and insurers react to high loss costs in certain coastal areas.

Second, once adequate rate levels are achieved, annual updates are not critical because the exposure base ($1,000 of Coverage A) is inflation sensitive. The accompanying premium trend can usually offset modest amounts of loss trend from partial losses. This makes for an easier validation of the damage factors using storm results over the past ten years. If there is any residual trend in hurricane loss costs, it may ultimately be difficult to measure directly, because of the relatively low frequency of hurricanes.

Risk Variations

Non-hurricane homeowners loss costs vary significantly by fire protection class, reflecting the large portion of the coverage represented by the fire peril. Yet, the hurricane peril is obviously independent of protection class.

Policy form relativities increase as additional perils are covered. In Forms 1 and 2, the perils are specified, while Form 3 gives essentially all-risk coverage on the building, but not on contents. Form 5 provides all-risk coverage on contents. Yet, the wind coverage is identical in all the homeowners policy forms. Hence, if the hurricane loss costs are a material portion of total homeowners costs, the policy form relativities would have to vary substantially by territory, if applied to an indivisible homeowners premium.

For construction class, a frame house can be almost as hurricane resistant as one made of brick or stone. For large hurricanes, the key is to protect the envelope of the building from penetration—i.e., the windows and the roof. Hence, the relative fire resistance of the construction is essentially irrelevant for the hurricane peril.

The hurricane peril ultimately needs a separate class plan because of different risk variation from the traditional covers. For example, new rating factors will likely emerge for shuttering and for roof type (e.g., gable versus hip roof). Local enforcement of building codes is another rating distinction that is implementable. Redoing all the traditional homeowners class relativities to meld with the new hurricane classes would be very cumbersome. Perhaps the traditional homeowners territories could be retained, with a separate set of territory definitions for the hurricane rate.

A possible class plan with sample surcharges and discounts is shown in Table 1.

TABLE 1

POSSIBLE HURRICANE RELATED SURCHARGES AND DISCOUNTS

Category	Criteria	Sample Factor
Hurricane Shutters	None	+0.20
	Add-On	−0.20
	Built-In	−0.40
Roof Type	Hip	−0.25
	Gable	+0.30
Location	Shielded by buildings	−0.20
	Subject to projectiles	+0.20
	Beach front or subject to surge	+0.10
Town Building Code	Not enforced	+0.15
	Enforced; not inspected	−0.10
	House inspected; within code	−0.25

Table 1 is just an illustration of possible risk variation. In reality, some of the criteria would interact. For example, a house with excellent shuttering protection would not be as susceptible to debris and projectiles penetrating the envelope of the building. Hence, the relativities may not be uniformly multiplicative or additive.

To calculate the indicated classification factors, one would run the model on a single house in each ZIP code, and vary the house based on different resistance characteristics. Next, using geographic mapping features, one would derive the relationships to the base class in ranges of relativities; e.g., .8 to .9, .9 to 1.0, 1.0 to 1.1. Because the ultimate selected relativities are usually expressed in a table used by the marketing force as well as by underwriters and regulators, one would select average relativities that form the dominant pattern from the map illustrations. If, within one state, the masonry house discount averaged 5%, but varied from 3% to 7% by territory, one could conceivably have several zones statewide for construction relativities. Alternatively, if the insurer printed all the rates by territory, instead of just the base class rates, then more flexibility could be allowed in the relativities.

7. FORM OF RATING

If the hurricane peril does not vary by class the same way non-hurricane perils do, should the hurricane rate be split out from the heretofore indivisible premium for homeowners? Should it have its own class plan? The answer to both questions is yes.

Basically, one can have the best of both worlds. The indivisible premium concept was originally introduced almost 50 years ago to simplify the review of loss experience and the rating of the homeowners policy. It also lowered the cost of the monoline coverages, because all the major perils were essentially compulsory.

With catastrophe modeling available today, virtually all of the advantages of the indivisible premium can be retained while still making the hurricane coverage mandatory. Ironically, it is the very difficulty of an overall loss experience review that suggests the unbundling of coverages for ratemaking—using the pure premium method for hurricane ratemaking and allowing a loss ratio approach for the other perils.

Computer modeling could also be used for other catastrophe perils within homeowners (e.g., tornado and winter storm), while the remaining non-catastrophe perils in homeowners would use the more traditional methods of ratemaking. Computer modeling of catastrophe perils actually makes ratemaking for the other perils much easier, because of results that fluctuate less. With loss costs supplied by modeling and with a separate rate for each catastrophe peril, the actual catastrophe losses only need to be removed from the experience period, and nothing need be loaded back to the normal homeowners losses. This means that catastrophe serial numbers ought to be retained for loss coding—to subtract catastrophe losses for the regular loss ratio ratemaking, to supply catastrophe losses to calibrate the models in the future, and, of course, to report to the reinsurers for recovery.

Thus, the overwhelming advantages of separate catastrophe rates are the simplification of the normal coverage rating and ratemaking, as well as the better class and territory rating of the catastrophe coverages.

This does mean an extra rating step for the catastrophe coverages, but there already are so many endorsements in homeowners that this should not be much of a burden. Furthermore, if hurricane loss costs are left in the indivisible premium, the homeowners classes will become much more complicated to rate. The class relativities will have to vary greatly by hurricane zone, and the actuarial calculation of relativity indications will also be much more complex.

Another simplification achieved through separate hurricane rating is the elimination of a complicated set of statewide indications including hurricane. Instead, the indications can be produced, and actual rates selected, separately. Ostensibly, this creates a problem in rate filings, where tradition has called for a combined statewide average *indicated* rate change as well as a *filed* rate level change. However, this is mere custom, and not strictly required by the rating laws—which usually call for *rates* to be filed, not *rate changes*. In other words, statutory requirements are for *rates* to be reasonable, not excessive, inadequate or unfairly discriminatory. Filed measures of *rate changes* have merely been a convenient way for regulators to monitor reasonableness.

This is not to suggest that a rate filing should repress the estimate of statewide rate change. However, given the different ways of calculating the appropriate rates (via a pure premium approach for catastrophes and a loss ratio method for other perils), the statewide indication does not as readily come out of the ratemaking method as, for example, it does for auto insurance. Hence, other reasonable ways of estimating changes will need to be developed, instead of directly from the ratemaking method. A sample indicated rate change calculation appears in Appendix C.

8. EXPENSE LOAD CONSIDERATIONS

If the hurricane peril is reinsured in a reasonable fashion, then the primary insurer ought to be able to pass those costs through to the policyholder. The reinsurance premium can be expressed as a function of the primary layer and added to the equation. Some portion of catastrophe treaty reinstatement premium should also be considered part of the reinsurance cost. If the reinsurance period does not coincide with the ratemaking period, then reasonable estimates of prospective reinsurance premiums might be considered.

The total expected hurricane loss costs need to be adjusted to exclude the reinsured portion by having the hurricane computer model simulate the reinsurance layer. This is done by running all probabilistic storms against the insurer's exposure base by ZIP code and line of business. Each storm's losses in the reinsurance layer are then allocated to line and ZIP code in proportion to total losses for that storm. Then each storm's probability is multiplied by the losses in the layer and accumulated. This produces the expected losses in the reinsurance layer.

$$L_{XS} = \text{MIN}\left(\text{MAX}\left(\left(\sum_{\text{ZIP}} E_{\text{ZIP}} \times DF_{\text{storm}}\right) - RET, 0\right), LIM\right), \tag{8.1}$$

where

L_{XS} = Total losses in layer for each storm,

RET = Reinsurance retention, and

LIM = Reinsurance layer size.

$$L_{XS,ZIP} = L_{TOT,ZIP} \times L_{XS} \div L_{TOT}, \tag{8.2}$$

where

$L_{XS,ZIP}$ = Excess losses by ZIP code for each storm,

L_{TOT} = Total ground-up losses for each storm, and

$L_{TOT,ZIP}$ = Ground-up losses by ZIP code for each storm.

$$EL_{XS,ZIP} = F \times \sum_{\text{storm}} P_{\text{storm}} \times L_{XS,ZIP}, \tag{8.3}$$

where

$EL_{XS,ZIP}$ = Expected losses in layer by Zip code,

F = Annual hurricane frequency, and

P_{storm} = Probability of storm.

The reinsurance premium can then be allocated to line of business and ZIP code in proportion to the expected excess losses in the reinsurance layer. Those premiums are then ratioed to the primary premium by line and ZIP code to get a factor to add to the indicated rate by line and ZIP code.

The remaining expected loss costs outside the reinsurance layer (above and below) would then be loaded for risk margin and expenses. The reinsurance pass-through would already have included the expenses and risk margin of the reinsurer.

9. RISK LOAD CONSIDERATIONS

Splitting the homeowners premium into a catastrophe and non-catastrophe component also allows for a separate calculation of a risk margin. As a result, the non-catastrophe component becomes easier to price, with less variability and a lower margin needed for profit. This makes it closer to a line of business like automobile physical damage in its target total rate of return and total target operating margin needed, which can be expressed as a percentage of premium.

Once a target margin is selected for the non-catastrophe component, the margin for the catastrophe piece can be calculated as a multiple of the non-catastrophe component, using some basic assumptions. One assumption is that profit should be proportional to the standard deviation of the losses. (Some actuarial theorists argue that risk load should be proportional to variance. It is important to note that these arguments apply to individual risks. The assumption that the required risk load for an entire portfolio is related to the standard deviation is not inconsistent with a variance-based risk margin for individual risks. In addition, the high correlation of losses exposed to the risk of a catastrophe, as well as the large contribution of parameter risk to the total risk load requirement, provides additional arguments in favor of a standard deviation basis for risk load.)

The calculation of the risk load should be performed on a basis net of reinsurance because the reinsurance premium is being built back into the rates separately. However, calculating the risk load both gross and net of reinsurance may be an important exercise for an insurer analyzing retention levels. By doing so, the insurer may be able to evaluate its reinsurance protection by considering the total risk load required.

In Table 2, a homeowners non-catastrophe pretax operating profit margin of 3% is assumed. At a 2.5 to 1 premium to surplus ratio, this is equivalent to about a 9.4% aftertax return on surplus

$(((2.5 \times 3 + 7) \times .65) = 9.4)$, assuming surplus can be invested at 7% pretax.

Next, assume that the total pure premium can be split into 80%/20% proportions for the non-catastrophe and catastrophe components, respectively. (This split is expected to be state-specific, since the hurricane loss cost in hurricane-prone states will represent a greater proportion of the total loss cost.) Based on direct homeowners industry data adjusted to eliminate catastrophes, the coefficient of variation of non-catastrophe loss ratios has been about 8% over the past 40 years. The corresponding coefficient of variation for hurricane losses, based on computer models, might be 350%, for example. This implies that the standard deviation of hurricane catastrophe losses would be 10.94 times the standard deviation of non-catastrophe losses.

If a 3% operating margin for non-catastrophe homeowners produces a $2.40 operating profit on an $80 pure premium, then the operating profit for the hurricane pure premium should be 10.94 times that, or $26.25. Expressed as a percentage of the pure premium, this would result in a risk margin of 131% on top of the expected hurricane loss costs. (These operating margins would include investment income from policyholder-supplied funds, and therefore that quantity must be subtracted to derive an underwriting profit margin to be applied to loss costs.)

TABLE 2

CALCULATION OF THE HURRICANE RISK MARGIN AS A FUNCTION OF THE NON-CATASTROPHE RISK MARGIN

(1)	% of Loss (2)	Coefficient of Variation (3)	Standard Deviation (4) = (2) × (3)	Relativity (5)	Risk Margin (% of Mean) (6)	Dollar Return (7)
Non-Catastrophe	80%	0.08	0.064	1.00	3%	0.0240
Hurricane	20%	3.50	0.700	10.94	131%	0.2625

These calculations assume that all policies are issued for one-year terms. If the duration of policies changes to include multi-year policies, then the lower variance of actual results should ultimately result in a lower risk margin to be included in the rates.

One can actually convert the risk margin to be a direct function of the ratio of CVs , as the risk margin incorporates the ratio of the dollar profit to the mean:

$$\text{Risk Margin}_{\text{CAT}} = \text{Risk Margin}_{\text{NON-CAT}} \times \text{CV}_{\text{CAT}} \div \text{CV}_{\text{NON-CAT}}.$$

10. DERIVING HURRICANE BASE RATES

Once the hurricane loss costs by ZIP code have been averaged to territory, expenses and profit margins must be included to derive base class rates. Exhibit 1 shows the derivation of a base class loss cost of $1.545 for Territory B. Using the following values of expenses and profits:

Commission (C): 5% of Premium,

General Expenses (GE): 10% of Premium,

Taxes, Licenses and Fees (T): 3% of Premium,

Investment Income Offset (I): 3% of Premium, and

Profit and Contingencies (P): 131% of Losses,

the base class rate (BCR) for Territory B would be equal to:

$$\begin{aligned} BCR_{\text{terr}} &= \frac{ELC_{\text{terr}} \times (1 + P)}{(1 - C - GE - T + I)} \\ &= 1.545 \times \frac{2.31}{0.85} \\ &= 1.545 \times 2.718 \\ &= 4.199 \text{ per \$1,000 of Coverage A.} \end{aligned}$$

If the insurer decides to pass through the cost of catastrophe reinsurance, then both the loss cost and the profit provision must be adjusted accordingly. Table 3 shows the total territory loss costs and those outside the catastrophe reinsurance layer (refer to Section 8 for more details):

TABLE 3

TERRITORY LOSS COSTS

	Expected Loss Cost	
Territory	Without Reinsurance	Excluding Reinsurance Layer
A	.401	.309
B	1.545	1.113
C	2.806	1.824
D	3.937	2.362
Statewide	2.464	1.646

From the allocation of the catastrophe treaty cost to ZIP code and line of business, one derives a cost of \$2.015 per \$1,000 of Coverage A for Territory B. Also, the required risk load for the losses retained by the company drops from 131% to 65%. Hence, the following rate calculation results:

$$BCR_{\text{terr}} = \frac{ELC_{\text{terr}} \times (1 + P) + R}{(1 - C - GE - T + I)}$$

$$= \frac{1.113 \times 1.65 + 2.015}{0.85}$$

$$= 4.531 \text{ per \$1,000 of Coverage A,}$$

where R = Catastrophe reinsurance cost per \$1,000 of Coverage A.

This indicates that the cost of the reinsurance treaty has a slightly higher embedded risk load than the overall indicated company risk load.

Another advantage of separating the hurricane rate from the heretofore indivisible premium is in the treatment of expenses. For example, a company may wish to implement a different commission structure for its hurricane coverage than for its non-hurricane coverage.

Since the hurricane coverage is intended to be part of the homeowners policy, fixed expenses that are part of the non-hurricane policy must not be double-counted. An easy way to achieve this is to include only variable expenses in the hurricane rates and to incorporate all fixed expenses in the non-hurricane rates.

Once the base class hurricane rates are calculated, they can be filed, along with the table of relativities for hurricane described above. As part of the filing, non-hurricane base rates (which are generally expressed as a dollar amount for the base class amount of insurance in each territory) will also be submitted. We have not demonstrated the calculation of non-hurricane rates in this paper because the topic has been covered extensively in other actuarial literature.

11. RATE FILING ISSUES

The approval of computer models as the source of expected catastrophe loss and risk margin can be a lengthy process because it changes the way regulators can verify the calculations. Under traditional filings, basic data are included with the filing, and the underlying source data are often part of statistical plan information that has been implicitly approved by the regulators in the past.

With catastrophe modeling, the frequency of events is often taken from published information tracking 100 or more years of event history. For the key simulation of a catastrophe event (e.g., hurricane or earthquake), the source is usually a scientific paper describing the ability of various equations to simulate the event. For the probabilistic model generating expected losses, often thousands of events are used, each with a specific probability derived from past distributions of input parameters.

Computer modeling presents a dimensionally different approach to the regulatory approval process. A separate evaluation of each independent modeler is necessary—to clear each model before an actual rate filing is made utilizing that model's calculation of expected loss costs. This pre-clearing process can take several months' time, depending on the level of due diligence needed and on the amount of rate level increase implied by the use of models to replace the old ratemaking system.

Once the independent modelers have been approved, the resulting set of indicated loss costs can provide a range of reasonable answers with which to evaluate specific company filings if the insurer has built its own model. If that company-specific model has loss costs within the pre-cleared range, that is usually *prima facie* evidence of the overall reasonableness of the company model. Even if the insurer model has some results outside the range, that should not necessarily disqualify the result. It merely places an additional burden on the insurer to prove the result is reasonable, based on its own assumptions and judgments.

The following steps can be considered in the regulatory approval process (the details of which are included in Appendix D):

- review general design of the model
- examine event simulation module
- test ability of module to simulate known past events
- check distributions of key input variables
- perform sensitivity checks on most important inputs
- verify damage and insurance relationship functions
- test output for hypothetical new events

- compare different modelers' results for loss costs
- conduct on-site due diligence and review of actual assumptions.

For independent modelers, and even for insurer-specific models, it is important to preserve trade secret information during the approval process and afterwards. The knowledge that research and development investments can be protected will encourage future innovations.

The on-site due diligence of regulators should keep the inner workings of the models confidential, as long as the examining process is documented by the regulator, much in the same way a financial examination of an insurance company keeps key information confidential.

Even after the approval of a model, the regulator can preserve the confidentiality of indicated loss costs by ZIP code by not publishing the ranges that it plans to use in reviewing other company filings. First, it is better policy not to disclose the high end of the range lest some insurers be tempted to file that answer rather than using a rigorous model. Second, publishing the rate may be tantamount to the regulator setting the rate instead of approving reasonable filed rates. Finally, the regulator would not be receiving the direct public attention on why the rates are so high in certain areas.

12. FINAL PERSPECTIVE

In summary, computer models are now capable of simulating catastrophic events and creating probabilistic models of reality that can be used to generate expected loss costs for catastrophe perils. These same models also provide a means of including the reinsurance premiums in the primary pricing process and can help quantify the needed risk load in relation to profit margins required for the non-catastrophe perils.

The same model can also be used for insurer or corporate risk analysis, including reinsurance purchase decisions, and for insurer marketing and underwriting strategies. These analyses are beyond the scope of this paper.

Use of computer models for ratemaking involves a different approach from the customary one, in that it is a pure premium method in contrast to the usual loss ratio method involving past insured loss experience. That carries advantages as well as challenges, because it attempts to deal with the true underlying probabilities of loss, not just with what appears in the last few years of actual insured loss experience—which is merely a sample of what could have occurred. The computer models attempt to simulate the entire spectrum of what could have occurred.

Thus, the models rely heavily on computer simulations and new technical methods made possible by the vast improvement in personal computer potential. This also requires a heavy investment in research and design as well as in resources to have the model evaluated and accepted by regulators and others.

But it is worth the process, not only for the practical results in insurer ratemaking and planning, but also for the insights gained on these catastrophic events and the reduction in uncertainty for society in dealing with them.

Furthermore, the techniques developed in producing these computer models might ultimately be applied to other perils as well. After all, the essence of actuarial work is modeling reality to assess the present financial impact of future contingent events.

REFERENCES

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[3] Walters, Michael A., "Homeowners Insurance Ratemaking," *PCAS* LXI, 1974, pp. 15–61.

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EXHIBIT 1

SAMPLE INSURANCE COMPANY
STATE XYZ

Expected Hurricane Loss Cost
Per $1,000 of Homeowners Coverage A
Base Class: Frame
Base Deductible: $250

Zip Code Loss Costs

Base Territory	Zip Code	Exposure in Coverage A Amount	Expected Loss Cost
(1)	(2)	(3)	(4)
A	02001	3,227,000	0.351
	02002	12,495,000	0.342
	02003	8,113,000	0.421
	02004	9,204,000	0.482
B	02005	1,198,000	1.232
	02006	3,254,000	1.425
	02007	6,681,000	1.647
	02008	11,341,000	1.552
C	02009	7,295,000	2.565
	02010	6,400,000	2.752
	02011	8,508,000	2.832
	02012	9,212,000	3.011
D	02013	17,346,000	3.742
	02014	15,212,000	3.953
	02015	13,900,000	4.032
	02016	6,573,000	4.211
Total		139,959,000	2.464

Territory Loss Costs

Base Territory	Exposure in Coverage A Amount	Expected Loss Cost
(1)	(2)	(3)
A	33,039,000	0.401
B	22,474,000	1.545
C	31,415,000	2.806
D	53,031,000	3.937
Total	139,959,000	2.464

Notes:
In-force Coverage A amounts are as of June 30, 1995.
Expected Loss Costs are derived from probabilistic hurricane modeling.

EXHIBIT 2

SAMPLE INSURANCE COMPANY
STATE XYZ

Calculation of Statewide Rate Level Change
Homeowners

(1) Total premiums on current rate level		$4,544,326
(2) Current amount of insurance years (000's)		$872,589
(3) Current average rate per $1,000	(1)/(2)	$5.21
(4) Catastrophe factor from last approved filing		1.327
(5) Portion of rate from catastrophes	1 – [1/(4)]	24.6%
(6) Portion of catastrophe from hurricane (est.)		80.0%
(7) Portion of rate from hurricane	(5) × (6)	19.7%
(8) Current average hurricane rate per $1,000	(3) × (7)	$1.03
(9) Current average non-hurricane rate per $1,000	(3) – (8)	$4.18
(10) Indicated average non-hurricane rate per $1,000		$4.02
(11) Indicated average hurricane rate per $1,000		$4.53
(12) Indicated total rate per $1,000	(10) + (11)	$8.55
(13) Indicated rate level change—non-hurricane	(10)/(9) – 1	–3.8%
(14) Indicated rate level change—hurricane	(11)/(8) – 1	339.8%
(15) Indicated total rate level change	(12)/(3) – 1	64.1%
(16) Filed average non-hurricane rate per $1,000		$4.02
(17) Filed average hurricane rate per $1,000		$4.25
(18) Filed average total rate per $1,000	(16) + (17)	$8.27
(19) Filed average non-hurricane rate level change	(16)/(9) – 1	–3.8%
(20) Filed average hurricane rate change	(17)/(8) – 1	312.6%
(21) Filed average total rate change	(18)/(3) – 1	58.7%

Rate Change Status for Future On-Level Calculations

(22) Approved average non-hurricane rate per $1,000		$4.02
(23) Approved average hurricane rate per $1,000		$3.75
(24) Approved average total rate per $1,000	(22) + (23)	$7.77
(25) Approved average total rate level change	(24)/(3) – 1	49.1%
(26) Premium level change for non-hurricane coverage	(22)/(3) – 1	–22.8%

APPENDIX A

How to Construct a Model

The severity component of a catastrophe model generally contains three modules built separately and later integrated. These modules are:

- event simulation (science)
- damageability of properties (engineering)
- loss effect on exposures (insurance).

Before it can be used for ratemaking purposes, a catastrophe model must undergo a high level of research and testing.

Science Module

As a first step, the modeler must incorporate the physics of the natural phenomena in a module (also called the event generator module) that simulates as closely as possible the actual event. Examples of input for a hurricane model include the radius of maximum winds, pressure differential at the eye of the storm (ambient pressure minus central pressure), forward speed, angle of incidence, landfall location and directional path. For an earthquake model, such factors as magnitude, location of the epicenter, soil conditions, liquefaction potential and distance from the fault rupture are used to estimate the shaking intensity of the ground at a given location.

The event generator module must be tested before its use to reproduce historical events and simulate hypothetical or probabilistic events. As a first step for a hurricane model, actual wind speed records for recent events should be compared to modeled results. Such organizations as the National Hurricane Center can provide records for the historical events.

Next, the hurricane model should be tested for reasonableness by predicting wind speeds for hypothetical events along the

Atlantic and Gulf coasts. Because one of the key drivers of a hurricane model is the roughness parameter, this testing will help evaluate the sensitivity of the model to this terrain factor and will allow necessary refinements to the initial assumptions.

The model's predictive accuracy is limited by the fact that data are not currently captured for some site-specific factors that affect an individual property (e.g., topographic peculiarities that influence wind speeds or liquefaction propensity at a given location for earthquakes). Therefore, one should not expect a model to exactly reproduce a single past event, but rather verify that it can adequately simulate hypothetical events with a given set of parameters. Over a range of input parameters, the model should generate intensity levels that are consistent and reasonable. Thus, actual future events with other site differences do not require major modifications to the model, but rather provide additional information to further refine it.

Engineering Module

Once the event generator has been developed, damageability functions are needed to estimate the damage to a property subject to an event of a given intensity. Input from various fields of the engineering profession, such as wind engineering and structural engineering, must be gathered to develop these functions. For damage by hurricane wind speeds, numerous studies have been performed that estimate these relationships. The functions should vary by line of business, region, construction, and coverage (building versus contents).

As was the case for the event generator module, accuracy of the damage functions is improved by analyzing actual past events. Actual loss experience of insurance companies should be compared to modeled losses in the most refined level of detail available. Whereas only aggregate loss amounts by catastrophe used to be collected by insurers, it is now generally possible to see loss data by line of business and county (or even ZIP code).

Next, on-site visits to the locations of catastrophes can help assess the damageability of exposed structures. While not imperative, these visits provide additional insight to the modeler, especially in identifying future classification distinctions.

The refinement of the damage functions is an ongoing process that is dependent on input generally provided by the engineering community. Engineering studies and loss mitigation reports are constantly being published, and their conclusions should be adapted and incorporated into the damage functions being used in the catastrophe model.

Insurance Module

Once the science and engineering modules have been developed, they must be integrated with the insurance module to determine the resulting insured loss from a given event. For risk analysis, Kozlowski and Mathewson [4] stress the importance of developing and maintaining a database of in-force exposures that captures the relevant factors that can be used in assessing the damage to a given risk. This database will not only include such factors as location, construction type, number of stories, age of building and coverage limits, but also replacement cost provisions, deductibles, co-insurance and reinsurance (both proportional and non-proportional).

Integration of Modules

Table 4 presents a sample calculation of the loss estimate generated by the model for a sample hurricane after integrating the three modules.

The example assumes that there is one single-family dwelling in each ZIP code, each with a different deductible. Based on the parameters of the storm simulated, the event generator module calculates the average wind speed sustained by all structures within the ZIP code. In this case, the wind speeds decrease as the ZIP codes are further away from the coast.

TABLE 4

SAMPLE CALCULATION OF HURRICANE LOSSES

ZIP Code	Exposure Amount	Deductible	Windspeed (mph)	Corresponding Damage Factor	Gross Resulting Loss	Net Resulting Loss
2001	$180,000	$250	100	.15	$27,000	$26,750
2002	180,000	$500	90	.08	14,400	13,900
2003	180,000	2%	80	.05	9,000	5,400

The damageability module then predicts the damage sustained by each structure as a function of the windspeed. The damage factors generally vary based on factors such as construction type (e.g., frame versus wind-resistive), age of building and number of stories. The gross resulting loss is then calculated by multiplying the exposure amount by the damage factor. The estimate is then adjusted for insurance features, such as deductibles and reinsurance. In this example, the gross loss is reduced by the deductible to derive the net resulting loss.

How to Validate

The final task in developing a catastrophe model lies in validating the simulated results. While intermediate levels of calibration are performed for each module, the modeler must verify how they interact by completing an overall analysis of the results.

Because the model is designed to simulate reality, actual incurred loss experience is the obvious candidate to be used in testing modeled losses. Of course, all comparisons are dependent on the quality of the data captured from the loss records of insurers. As described above, the modeler should gain access to various sets of insured loss data and verify that all relevant factors are reflected in the model. These would include line of business, classification, coverage (e.g., building versus contents), and loss adjustment expense (LAE) as a percentage of loss.

One issue often raised when validating a catastrophe model is demand surge (or "price gouging"). Because this phenomenon is dependent on the time, size and location of the event, it should not be incorporated in the damage functions, except to the extent it is "expected." For example, most models underestimated the actual losses from Hurricane Andrew. If the models were adjusted to exactly reproduce Andrew's losses, they would effectively include a provision for factors specific to Andrew and not expected in the long run, such as:

- inflation in reconstruction costs due to the excess of demand over supply
- excess claim settlements that occurred because adjuster resources were overwhelmed by the volume of claims.

While these factors can be included separately in the reproduction of a single storm, they should not be part of the base model because they would inappropriately increase the expected level of future losses.

Another issue is storm surge from a hurricane. While a flood loss is not officially covered by a homeowners policy, some adjusters of losses on houses affected will construe coverage from wind damage prior to the house being flooded. This can be handled with a small additional factor on those locales in low areas most susceptible to surge. However, from a ratemaking and rate filing standpoint, it is difficult to support much of an increase from a coverage that does not officially apply to homeowners.

APPENDIX B

HOW OTHER PERILS ARE MODELED

Earthquake

The library of historical earthquake events producing significant insured losses is scant compared to that of historical hurricane events. Hence, the precision level of computerized earthquake models will not reach that of hurricane models. Nevertheless, numerous models have been developed and a great amount of research done to define the various factors and relationships.

In the science module, the model begins with simulating the magnitude of an earthquake, generally expressed as a unit on the Richter scale. This implies a rupture length on a fault. Using other factors, such as distance to the rupture, soil conditions and the liquefaction potential of the areas affected, the model estimates the shaking intensity for each ZIP code. For the engineering module, resulting shaking intensities are usually converted to the Modified Mercalli Intensity (MMI) scale, because most models use the ATC-13 damage functions as a starting point. These functions were developed by a group of 13 engineers and scientists commissioned by the Applied Technology Council (ATC) in 1982 to estimate the damage to California properties from earthquake.

The insurance module for an earthquake model is generally similar to a hurricane model. However, the use of percentage deductibles (which is not common on a standard homeowners policy) and separate coverage deductibles present a new twist. Hence, the model must have the capability of handling various deductible combinations. For instance, some earthquake policies apply a building deductible different from the contents deductible and the additional living expense deductible. The deductible credit applies separately for each coverage.

The insured loss data available to validate an earthquake model are more limited than for hurricanes. Also limiting is the fact that earthquakes are not all similar. For instance, most major faults in California have been of the strike-slip type. These faults generally run in a north-south direction, with energy being released when western blocks of crust move north past the eastern block. This causes ground displacements that are mostly horizontal.

Yet the 1994 Northridge quake was a "blind" thrust-fault earthquake. In this type of event, sections of rock overriding others at an angle are displaced. The movements are generally upward and sideways, which creates strong shaking that is generally more damaging. In the case of Northridge, the fault did not reach the surface. Hence the term "blind" fault.

These two types of earthquakes are by their nature very different, and the event generator module will vary to reflect the different types of shaking intensities.

Once the deterministic earthquake model has been developed, a probabilistic version must be generated. For earthquake modeling, a set of known faults is generally used as a starting point in building the library of events. Events of various strengths and locations are simulated for each fault. A probability is then assigned to each event in the library. These probabilities are generally expressed in a return time format such as 1 in 400 years. They can be obtained from geological sources, such as the United States Geological Survey.

The Northridge event highlighted the fact that serious damage could be caused by earthquakes not located on known fault systems. This has implications for earthquake ratemaking because the frequency of these events is very much unknown at this time, and inclusion of this type of event could increase the expected loss costs substantially. However, the modeler needs to take care that the long-run frequency of earthquakes remains reasonable.

Tornado and Hail

The actual loss experience of tornadoes and hailstorms is more readily available than for any other type of natural catastrophe. Given that there are roughly 1,000 tornadoes in the U.S. each year, the traditional way of developing a tornado catastrophe loading in states with exposure to these perils has been to smooth the actual loss experience over a number of years. However, this methodology does not capture the essence of why catastrophe modeling is the preferred approach, which is to estimate the loss potential of a company given its current distribution of exposures. Also implicit in any modeling approach is the simulation of events that have not occurred much in some areas but are reasonably foreseeable given the historical database of events.

Tornadoes and hailstorms are typically generated by inland storms when moist, warm air masses collide with cooler, drier air masses. Such conditions are often present in the southcentral United States (e.g., northern Texas and Oklahoma) and the plains states (e.g., Iowa and Kansas) where the Gulf of Mexico provides a continuous source of warm, moist air, and the Rocky Mountains create a source of cooler drier air as weather systems move over them. Tornadoes do, however, occur in all 50 states.

An inland storm capable of generating tornadoes may create dozens of individual funnels over a widely dispersed area. A single funnel will produce damage over the portion of its track making contact with the earth. The length of that ground contact track can range from a few hundred feet to a hundred miles. The width of the track funnel can range from ten feet to a mile. In order to model the loss effects of a single funnel, it is therefore necessary to consider the small scale (nine-digit ZIP code) location of exposures relative to the funnel path.

Because tornadoes and hailstorms are more sudden and unpredictable than hurricanes, most historical information has been the result of human observation. Current tornado databases generally consist of date and time, initial observed location, path width, path length and storm intensity for each event. Tornado intensity is generally measured on the basis of the Fujita scale, which translates an expected degree of damage to a range of windspeeds. For example, a tornado with a Fujita-scale intensity of F2 will be expected to tear roofs from frame houses. Engineering studies indicate that damage of this intensity can be generated by windspeeds between 113 and 157 miles per hour.

Tornadoes do not behave like hurricanes. The spinning funnel-shaped updraft of a mature tornado is the most damaging windstorm produced by nature. The damage relationships at a given windspeed for a tornado are quite different from those of a hurricane. The results of engineering and damage studies specific to tornadoes must be collected to develop a representative model.

The development of a hail model resembles that of a tornado model. However, difficulties lie in the definition of what is considered a hailstorm and which hailstorms are already included in a tornado database. The interpretation of the data present in the databases therefore has a significant impact on the overall frequency assumptions used in both models.

The validation of a tornado and/or a hail model against actual loss experience is dependent on the availability of loss data and on how much differentiation between the two perils is possible. (If this cannot be obtained, the modeler may have to calibrate the models on a combined basis. As a result, this would make the development and justification of territorial loss costs for all severe local storm perils easier.)

Winter Storm

Winter storm and freeze activity has been quite severe over the last few years. As a result, the need for better risk measurement and expected loss calculations has increased. Also, some of the same characteristics as hurricanes prompt the use of a catastrophe model to simulate winter storm losses—changes in exposure and longer return periods than in an individual insurer's data base.

However, contrary to the other catastrophe perils, winter storms do not have a specific unit of measure that describes the intensity of a given event, and individual temperature is not the only factor that can describe these events. For example, wide temperature swings and absolute highs and lows over consecutive days have been identified as some of the factors that affect the intensity and duration of these events.

The damage functions associated with winter storms are also very different from those of the other perils. Because little of the damage is structural, damage functions are less severe than those of hurricanes, for example.

Similar to a hurricane model, the creation of a probabilistic database requires simulation of multiple events. While the parameters are different, each event is defined by a location (or landfall), size, intensity and duration.

Because individual winter storms have not been as surplus threatening as hurricanes or earthquakes, the motivation to develop computer models has not been as high for risk analysis and development of PMLs. However, for ratemaking, this peril is equally as compelling as hurricane toward the use of computer modeling. Not only does it yield better expected loss estimates, but it allows the exclusion of past catastrophes from the normal homeowners ratemaking database for better stability in rate level indications.

APPENDIX C

ESTIMATING STATEWIDE RATE LEVEL CHANGES FOR HOMEOWNERS USING HURRICANE-MODELED LOSS COSTS

In the initial year of implementing hurricane ratemaking using a model, it may be necessary to split the current homeowners rates into the estimated portion due to hurricane and non-hurricane. (See Exhibit 2 for the calculations.) The next year's rate level review for non-hurricane can then use the non-hurricane rate as the basis for review using a traditional loss ratio method. However, until the actual written premiums can be coded into hurricane and non-hurricane, the on-level premium calculations will need to consider the separation of the rate into the two components. This can be done by treating the separation of the premium as a premium level reduction. In the example on Exhibit 2, the premium reduction statewide is 22.8% for non-hurricane coverage versus the heretofore total coverage. Thus, future experience reviews containing unbundled premiums must separate out the non-hurricane portion with this factor. When all the premiums are recorded separately for non-hurricane and hurricane, this on-level method is not necessary.

The accuracy of the split may not be critical to the outcome of the rate review, especially if the credibility of the insurer's experience is high. If credibility is 100%, then it matters little what the current rate level is, because the loss experience will completely determine the indicated premium level. Of course, the amount of the quoted rate level change may vary, but the indicated rates are the key to any filing, unless the amount of the change is very large, in which case there may be some regulatory objections to the size of the change.

For the hurricane coverage, the actual premium change is irrelevant to the calculation of next year's indicated rates because the model produces those on a pure premium basis. However, there may be a continuing need to use the average rates charged to keep the regulator informed of the size of the changes for the current customer base.

APPENDIX D

METHODS TO REVIEW CATASTROPHE MODELS IN REGULATORY PROCESS

1. **Review general design of model**
    - Examine the credentials of the modeler.
    - What is the scientific basis for the key event simulation?
    - What is the engineering support for the damage factors produced by each event severity?
    - Are the insurance limitation features reasonable; e.g., deductibles, coinsurance and reinsurance calculations?
2. **Examine event simulation module**
    - What are the credentials of the scientists who specified it?
    - Has their work been published and/or peer reviewed?
    - What special insights are they offering on the particular event to be simulated?
3. **Test event generator's ability to simulate known past events**
    - Use published information from some critical events, such as Hurricanes Andrew and Hugo, the Loma Prieta earthquake (1989) or even the 1906 San Francisco earthquake.
    - Input some key parameters, such as central pressure, landfall, speed and radius of maximum wind, and examine the output wind field at various locations compared to published information on wind speeds. This can be done for any event, even if no current estimates of insured losses are available, as a test of the event simulation accuracy.
4. **Conduct sensitivity checks**
    - Use a few sample events.
    - Promulgate a sample exposure base statewide (e.g., 25 risks).

- Vary the parameters one at a time, or perhaps a few in pairs.
- Observe changes in output (insured losses) for incremental changes in input.
- The goal is a rough measurement of the effect of changing inputs (e.g., central pressure, radius of maximum winds, forward speed).

5. **Check key input distributions**

   Compare the distributions of key input values among the different modelers, to see if there is any disparity in the key drivers of results. For hurricanes, a possible approach could be to look at the:

   - Distributions of central pressure at ten millibar intervals: 900–909, 910–919, etc.,
   - Distributions of radius of maximum winds in five nautical mile ranges, and forward speeds in five knot ranges, and
   - Probabilities of landfall for all storms affecting the state (direct hit and nearby landfalls).

6. **Verify damage and insurance relationship functions**

   - Examine the credentials of the engineers.
   - Has the analysis been published and/or peer reviewed?
   - Analyze the damage curves (functions of increasing damage for increasing event intensity) separately for types of exposure, class and coverage.
   - Review the insurance module for effects by deductible and reinsurance or coinsurance.
   - Review the validation of the two components (damage and insurance effects) via multiple events over the past few years for multiple insurers; each event does not have to be replicated, but the components should average out over all events and all insurers.

7. **Test output for hypothetical new events**
   - Select some new events defined by key parameters.
   - Use a sample database of exposures by ZIP code.
   - Compare results for different modelers and ask outside experts for their opinions on the reasonableness of these results.
8. **Compare indicated loss costs for different modelers**
   - Select sample ZIP codes throughout the state.
   - Have modelers run all events with probabilities for those ZIP codes.
   - Use several base classes and coverages:
     - homeowners, $100,000 frame house, $250 deductible,
     - tenants, $30,000 contents, masonry, $250 deductible,
     - businessowners, $200,000, masonry, $1,000 deductible.
   - Compare modelers' loss costs per $1,000 of coverage by ZIP code.
   - Ask outliers to explain large differences from average.
9. **Conduct on-site due diligence and review of key assumptions**
   - View a live running of the model, with actual input data.
   - Review input data sources—published and non-published:
     - all key input parameters,
     - frequency of events by location,
     - key damage factors and sources.
   - Review output, including color-coded maps showing ranges of expected loss costs.

DISCUSSION OF PAPER PUBLISHED IN VOLUME LXXX

SURPLUS—CONCEPTS, MEASURES OF RETURN, AND DETERMINATION

RUSSELL E. BINGHAM

DISCUSSION BY ROBERT K. BENDER

1. INTRODUCTION

If the purpose of policyholder surplus is to provide a cushion against possible errors in the estimation of balance sheet assets and liabilities for an insurance company, then surplus is required wherever estimation errors might exist, regardless of their source. In particular, the balance sheet contains estimates of liabilities due to the runoff of previously written policies as well as due to current business. For that reason, the required or *benchmark* surplus that appears on a given balance sheet should be allocated to the exposure period (e.g., policy year, accident year, contract year, etc.) that gives rise to the uncertainty.

Russell Bingham advocates such a decomposition of balance sheet surplus and income statement flows into the contributing accident years. Because a given exposure period frequently impacts many annual statements, this decomposition results in the formation of historical supporting surplus triangles that are analogous to the loss development triangles used in the analysis of reserve level adequacy. Once the supporting surplus and income flows for each exposure period are known, the overall return on the supporting surplus can be determined. When evaluating the return earned by a particular product line, it is this long term investment of surplus that must be considered.

This is in sharp contrast to calendar year measures in which it is assumed that all of the company surplus supports the currently written exposure. The long-term commitment of supporting surplus to each accident year and the corresponding reflection of that commitment is the major idea presented in Bingham's paper.

To illustrate the segregation of surplus and income flows and the formation of insurance company balance sheet triangles on a present value basis, Bingham presents a simplified example. Some aspects of the example are more complicated than need be to illustrate the basic concepts (e.g., the explicit consideration of federal income tax), whereas other aspects are deceptively simple (e.g., the adoption of a constant leverage ratio). Several issues are left unresolved if the single example is to be used as the springboard to a more comprehensive return on equity (i.e., return on benchmark supporting surplus) model. In the course of this discussion, a more transparent illustrative model for the determination of the return on equity is described. Additional levels of complexity are introduced to the model as the previously unresolved issues are considered.

By means of the more transparent example, the essential features of Bingham's methodology are summarized and the invariant nature of Bingham's present value ratio of total return to supporting surplus is demonstrated. Two refinements to the model are then introduced. The first refinement involves changing the basis for determining the benchmark surplus from nominal loss reserves to discounted loss reserves. This allows for a reflection of both ultimate loss *amount* risk and payout *timing* risk. The second refinement involves replacing the constant reserve-to-surplus ratio with a variable leverage ratio. Both of these refinements are compatible with the agreement inherent in Bingham's scheme for releasing operating gain (i.e., internal rate of return = average annual return on supporting surplus = Bingham's present value ratio).

An examination of the behavior of Bingham's methodology in two extreme pricing situations (severe rate inadequacy and severe rate redundancy) discloses that the simplified model does not produce reasonable results under these extreme conditions unless the leverage ratio is a function of the expected retained operating gain. Determining a functional relationship while maintaining the advantages of Bingham's release scheme is shown to be a non-trivial exercise.

Bingham's example assumes that, at each point in time, events which were *expected* to have occurred previously actually *did* occur. Because of that, the earned investment income and retained operating gain at any given time are exactly what they were expected to be when the supporting surplus requirement for that time was originally determined. This discussion considers whether or not supporting surplus to be carried during the runoff should be modified if the actual history is not what was expected a priori. Resolution of this issue affects both the prospective and retrospective determination of the return on equity for an insurance product.

Two appendices serve to flesh out the discussion. The first appendix provides a rigorous proof that Bingham's timing of the release of the insurance operating earnings always leads to agreement among the internal rate of return (IRR), average annual return on equity (ROE), and present value ratio, regardless of the level of sophistication introduced into the insurance model (e.g., the reflection of federal income tax, policyholder dividend payment, etc.) or the nature of the reserve-to-surplus leverage ratio (e.g., dependence upon the number of open claims, the expected retained operating gain, etc.). It is this proof that allows simplified models to be used to illustrate the methodology. The second appendix provides evidence that, contrary to common wisdom, a *decreasing* leverage ratio may be appropriate even for a line such as workers compensation with lifetime pension cases.

2. OVERVIEW OF BINGHAM'S METHODOLOGY

The world can be divided into three parts. These are the insurance product itself, shareholder funds, and everything that is external to the other two parts.

The *insurance product* can be narrowly defined as a single contract (e.g., a primary company policy or single reinsurance treaty, etc.), or the definition can be broadened to include a portfolio of similar contracts. In the extreme case, a portfolio could encompass all of the writings of a company.

Bingham refers to the second division as *shareholder funds*. While this designation works well for stock companies, the more general name, *surplus account*, allows us to extend the discourse to encompass mutual companies. The surplus account consists of two types of surplus, the surplus that is required to support the particular insurance operation (Bingham's benchmark surplus) and free surplus or *surplus* surplus. Surplus surplus is available to pay stockholder dividends, back new insurance operations, or simply remain idle with a return equal to that of the company's investment portfolio.

The third division includes everything external to the insurance company such as the policyholders, the stock market, and the Internal Revenue Service. Elements of the third division are relevant only to the extent that their existence results in cash flows either into or out of the other two divisions.

Having (implicitly) assumed this division, Bingham states that the purpose of supporting surplus is to act as a buffer to ensure an acceptably low probability of ruin. The buffer must be made available because of uncertainty that gives rise to both investment risk and underwriting risk. He, therefore, concludes that *supporting surplus must be allocated to the insurance product as long as any uncertainty exists.* A logical corollary to this is that supporting surplus can be released to the surplus surplus portion of the surplus account only as uncertainties are resolved and the corresponding probability of ruin decreases. This implies that supporting surplus must be a function of all of the stochastic asset and liability variables, not simply a function of one year's written premium as many surplus allocation formulae dictate. At any point in time, surplus is required to support not only the uncertainty associated with the current exposure period (accident year, policy year, contract year, etc.), but also the uncertainty associated with the runoff of prior exposure periods as well.

With that in mind, Bingham turns his attention to a single accident year and its contribution to each subsequent balance sheet. Bingham goes on to observe that *the timing of the release of the supporting surplus and operating gain*[1] *from an insurance product to the surplus account affects the measured return on equity for that accident year.* This observation is the second major point raised in Bingham's paper. While reserves and supporting surplus are clearly identified as "belonging" to the insurance product, the time at which other funds that arise from the insurance product are released to the surplus account is somewhat arbitrary. As a result of the sensitivity of the ROE to the arbitrary identification of these funds as insurance funds vs. surplus funds, the calculated return on supporting surplus can be manipulated by users of these models to produce a wide range of values purporting to be *the* ROE. Bingham proposes a timing scheme which, while still arbitrary, has a logical foundation.

By means of a simple example, Bingham illustrates the consequences of releasing supporting surplus and operating earnings as uncertainty is resolved and releasing investment income on supporting surplus as it is earned. A significant observation is that, under this release scheme, the annual return on supporting

[1]Operating gain is usually thought of as a calendar year concept. In this context, the operating gain associated with a particular exposure period is the amount by which the present value of the premium income exceeds the present value of the loss payments and other expenses (including federal income tax and policyholder dividends). It reflects both the underwriting result and investment income on underwriting funds. In contrast to a calendar year concept, operating gain in this context applies to the entire history of a particular exposure period.

surplus, the internal rate of return of the flows to and from the surplus account, and the present value of the flows to the surplus account divided by the present value of the supporting surplus (over the life of the product) are all equal.

The aesthetically pleasing agreement of the three measures of return on equity that results from Bingham's scheme for the release of surplus, investment income, and operating earnings is a strong argument in favor of following Bingham's lead. An additional argument in support of this scheme is that not only is the required amount of supporting surplus kept available to act as a buffer against insolvency risk, but a portion of the operating gain is also retained to serve as an additional buffer against the possibility of ruin. Both the supporting surplus and the retained operating gain are released as the uncertainties regarding occurrences that could lead to ruin (insolvency) are resolved.

Another strong argument supporting the resulting measure of the ROE is that Bingham's present value ratio measure is an *invariant* measure of the rate of return corresponding to an entire class of models, regardless of when and how the operating earnings are actually released to the surplus account.

In order to illustrate the relationship between the various measures of the return on surplus, Bingham presents a simplified example. As simple as the example is, more details concerning the workings of the insurance product were described than were necessary. In particular, only the operating profit associated with the insurance product and the time at which reserves (with the associated uncertainties) are present need to be known in order to determine the return on surplus. This is not to say that such issues as expenses and federal income tax timing are not important; rather, these aspects of the insurance product can be left inside the "black box" that determines the operating profit and establishes reserves. Leaving them out of the example serves to make the illustration more transparent. To that end, an even more simplified illustrative example is presented in this discussion.

On day one of this example (denoted as the last day of year zero in all of the exhibits), $400 of premium is collected. Payments of $264, $96, $32, $8, and $4 are made at the ends of years one through five, respectively. The series of payments may be thought of as either claim payments or as the aggregation of claim, expense, and tax payments. Only the magnitude and timing of the payments, together with the establishment of a liability in recognition of future payments, are germane to this discussion. So as not to obscure the basic concepts with the unnecessary details concerning how federal tax law would apply to the hypothetical situation, it will be assumed that there are no federal income taxes or other expenses. The payments, therefore, may be thought of as claim (loss) payments. Any reference to *losses* or *loss ratio* is equally valid for *losses*, *expenses*, and *taxes* together with the corresponding *combined ratio*. Investment income is assumed to be earned at a 5% annual effective rate.

Ruin occurs whenever there are insufficient funds available with which to make payments (loss, expense, and tax) as they become due. Sources of available funds include policyholder premium, investment income on underwriting funds, and supporting surplus. It is usually assumed that premiums and investment income on underwriting funds provide sufficient funds to cover all of the *expected* payments as they become due. Unexpected events such as unexpected loss payments (both with respect to amount and timing) are a major source of potential ruin. Supporting surplus (surplus allocated to the insurance product) provides additional funds to cushion against possible unexpected events. The more supporting surplus that is allocated to the insurance product, the more extreme the unexpected event would have to be in order to cause ruin. Assume that, for the insurance product under consideration, the probability of ruin can be kept to an acceptable level (e.g., less than 2%) by supporting each dol-

TABLE 1

Year End	Paid to Date Loss	Nominal[2] O/S Loss	Discounted[3] O/S Loss	Supporting Surplus
0	$0.00	$404.00	$375.86	$202.00
1	264.00	140.00	130.66	70.00
2	360.00	44.00	41.19	22.00
3	392.00	12.00	11.25	6.00
4	400.00	4.00	3.81	2.00
5	404.00	0.00	0.00	0.00

lar of outstanding loss reserve with $0.50 of surplus (i.e., a 2 : 1 reserve to surplus leverage ratio). This assumption presupposes that a rigorous determination of the appropriate leverage ratio has been conducted and that the result was the 2 : 1 ratio. As was the case in Bingham's paper, the details of this determination fall beyond the scope of this discussion.

The payment and leverage ratio assumptions are almost identical to the situation presented by the NAIC as an illustration of an IRR model [2]. Table 1 shows the loss and supporting surplus under the assumption of a constant reserve-to-surplus leverage ratio and payment pattern.

Exhibit 1 displays the essential features of the situation. The insurance product has an operating gain equal to $24.14 (present value of the premium on day one less the present value of the loss payments on day one). In this particular example, the entire operating gain was allowed to accrue interest as part of the insurance product account for a year before it was released to the

[2]"Nominal O/S" is the (estimated) sum of all future claim payments whether or not the claims have been reported to the carrier at the time that the estimate is made. This outstanding amount includes carried reserves and bulk reserves such as true IBNR and the less restrictive IBNE (Incurred But Not Enough).

[3]"Discounted O/S" is the present value of the expected flows that make up the nominal outstanding amount. As such, it may include a provision for the present value of claim payments that are expected to be made on claims that have not yet been reported. Discounted outstanding is more inclusive than the present value of the carried reserves.

surplus account. The accrued value of $24.14 after one year is $25.34.

Because the reserves along with their associated uncertainty remained constant during the first year, the supporting surplus was held constant at $202.00 for the entire year. The total return on this supporting surplus during the first year consisted of the $25.34 accrued operating gain from the insurance product and the $10.10 of investment income earned by the supporting surplus. The total, $35.44, represents a 17.55% return on the $202.00 surplus investment.

During subsequent years there was no contribution to the total return on supporting surplus arising from the insurance product. Investment income that was earned on underwriting funds during each calendar year was exactly sufficient to establish the year-end discounted loss reserve after all of the calendar year loss payments were made. In this respect, the insurance product did not participate in any further fund transfers between its own account and the surplus account after the end of the first year. Regardless of this, the fact that there was uncertainty regarding the ultimate loss outcome during each subsequent year led to the requirement that some surplus had to be allocated to the insurance product. During these subsequent years there was a 5% annual return on the supporting surplus. This is the same return as would have been earned had the surplus been idle (i.e., not supporting an insurance product). This surplus was, however, committed to supporting uncertainties during the runoff and was *not* available to support *new* writings. The average annual return on supporting surplus [Σ(released operating gain plus investment income on the supporting surplus)/Σ(supporting surplus), where the summation is over all years] was 13.39%.

It would not be correct to consider only the first year and to report a 17.55% return on equity for the product. Doing so would ignore the commitment of surplus during the subsequent four years. To see that this is precisely what is done when calendar year earnings are compared to average calendar year surplus amounts, consider what the calendar year return would be if the carrier wrote only a single contract during its lifetime. The first year-end balance sheet would indicate an average surplus amount equal to $202.00, while the statement of income would reflect $35.44, giving us a 17.55% return on surplus for the calendar year.

It can be argued that a carrier's calendar year return on equity would be equal to the average annual return on equity if the book of business were to be repeatedly renewed until a steady state had been achieved. Mathematically, this is a true statement. In the case of the single contract described above, repeated renewal for four or more years would result in an annual commitment of $302 of surplus ($202 for the most recently renewed contract, $70 for the contract written one year before, etc.) and annual income equal to $40.44 ($35.44 for the most recently renewed contract, $3.50 for the contract written one year before, etc.), which would yield a 13.39% return on supporting surplus for the calendar year.

Conceptually, the two measures are very different. While the average annual return measure relates to a single contract, the calendar year measure requires identical contracts to be written year after year. If the mix of business changes from year to year or if all of the company surplus is not being used to support the runoff of previously written contracts, then the equality no longer holds.

The conceptual difference between calendar year and average annual return on surplus is similar to the one that exists between accident year (or policy year) loss ratio and calendar year loss ratio. Here, too, the two measures are numerically equal once a steady state situation has been achieved. Each age to age development that is observed in the accident year (or policy year) triangle would be contributed to the calendar year experience by different accident year contracts in the corresponding stages of development. Just as one would not rely upon this equality when estimating the ultimate loss ratio for a single accident year, Bingham advocates determining the return on surplus associated with each underwriting period separately. Since the activity associated with a single underwriting period often spans several calendar years, balance sheet triangles arise in a manner that is analogous with loss and premium development triangles.

A second measure of the rate of return is the IRR implied by the flows to and from the surplus account. For this purpose, the flows of invested supporting surplus to and from the surplus surplus (surplus that is not supporting an insurance product) account must be reflected as well as the release of the operating gain and investment income on the supporting surplus. The total flows (initial supporting surplus investment, return of supporting surplus as it is released, investment income on the supporting surplus as it is earned, and the accrued operating gain as it is released) are displayed in the IRR column in Exhibit 1. For this example, the IRR is 13.87%.

A third measure of the return on equity is the ratio of the present value of the flows to surplus (the released operating gains and the investment income on the supporting surplus) to the present value of the *year-end* supporting surplus, 13.58% in this example. This measure is similar to the average return except that the present values of the numerator and denominator have been taken prior to forming the ratio. Of the three measures of return, only the present value ratio appears to lack an intuitively satisfying context. Taking the present value of a year-end surplus amount, which does not represent a discrete cash flow at year-end, contributes to the initial uneasiness with this measure.

The last three columns in Exhibit 1 are for reference. They display the *retained earnings*, the *investment balance*, and insurance product *overfund*. The retained earnings represent the accrued underwriting gain or loss at each year-end. In a way, the retained earnings reflect the impact of statutory accounting requirements on the surplus account. The investment balance at any point is the amount of insurance product funds that are available for investment (accrued premium less paid losses and released operating gain). The overfund is the amount by which the investment balance exceeds the discounted outstanding loss reserve. The overfund represents the portion of the operating gain that has been retained to act as an additional buffer against insolvency risk. No model is acceptable that does not come to an end with exactly zero overfund and zero investment balance. Any alternative to closing out the insurance product account after the last claim has been paid would result in allocating a portion of surplus to the insurance product (or floating it a loan) long after all claims had closed and all uncertainties had been resolved.

Exhibit 2 represents the same situation but with a withdrawal of operating gain at the opposite extreme from that which was depicted in Exhibit 1. In Exhibit 2, the operating gain is retained within the insurance product until all claims have been paid. As long as the operating gain is retained within the insurance product account, all interest accrued on it will be attributed to the insurance product. At the end of the fifth year, when the accrued operating gain is finally released to the surplus account, it carries with it \$6.67 of accrued interest, all of which is considered as part of the total return on supporting surplus.

Once funds are released to the surplus surplus account, subsequent investment income earned by them is not attributed to the insurance product. Because the operating gain was released to the surplus account later than in Exhibit 1, more of the investment income earned on these funds was attributed to the insurance product (\$6.67 vs. \$1.20). As a result of the difference between these two arbitrary segregations of funds—between the insurance product account and surplus account—the average annual return on supporting surplus increases from 13.39% as displayed in Exhibit 1 to 15.20% under the operating gain release timing of Exhibit 2. Whereas reflecting more dollars of investment income causes the average return on surplus to increase, it has the opposite effect on the IRR. The IRR corresponding to 13.87% of Exhibit 1 is 11.84% in Exhibit 2.

If the insurance company had set a 13.5% target for its return on equity and used these measures of return to evaluate this product, it would have found the product to fall short of the target if it had adopted the average return measure under the Exhibit 1 scenario, but to be acceptable under the Exhibit 2 scenario. Just the opposite conclusions would be reached if the IRR measures were used. Now, simply earmarking funds as belonging to a particular company account (insurance product account or surplus account) does not affect the overall well-being of the company.[4] There must be something misleading about a model that produces different results for different earmarkings. While the present value measure of the return on equity remained equal to 13.58% for both alternatives, invariance alone does not provide sufficient support for it to be adopted as the true measure of the return on equity.

Exhibit 3 provides that support. This example begins by specifying how the operating gain is to be released to the surplus account. In this alternative, the operating gain is released Bingham's way, as uncertainty is resolved (i.e., under the same criteria that the supporting surplus is released). This timing results in releasing the operating gain in such a way that the ratio of released dollars to the invested surplus remains constant. In symbolic form, if $S(j)$ is the invested surplus during the jth year, and $O(j)$ is the accrued operating gain that is released at the end of the jth year, then the set $\{O(j)\}$ must satisfy two conditions:

1. $PV[\{O(j)\}]$ = the operating gain, and
2. $O(j)/S(j)$ = constant for all years, independent of j.

The bottom of Exhibit 3 displays the detailed calculation of the set $\{O(j)\}$ corresponding to this example.

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[4] While actions taken as a result of this earmarking, such as the declaration of stockholder dividends, can affect the overall well-being of the company, the *act* of earmarking funds cannot affect the company's well-being.

As promised by Bingham, the three measures of the return on equity are equal when his release of operating gain is adopted. This is more than a coincidence. Appendix A presents a general proof that Condition 2 is sufficient to force the three measurements into agreement.

The invariant measure is equal to the average annual return on equity and to the internal rate of return corresponding to the case in which operating earnings are released in the same manner as supporting surplus, as uncertainty is resolved. The invariant measure does have a context.

It should be emphasized that the only feature of the insurance product cash flow that is explicitly reflected in the determination of the return on surplus is the present value of the operating gain. Increasing the degree of sophistication of the insurance product model (e.g., reflecting federal income tax, other expenses, policyholder dividends, etc.) almost certainly will change the numerical value of the operating gain but will not alter any of the concepts that have been discussed. Once the operating gain is determined, the manner in which it is released remains unchanged (i.e., according to the two conditions), and the agreement among the IRR, average annual return, and the model invariant continues to hold.

3. DOES THE BINGHAM METHODOLOGY LEAD TO REASONABLE RESULTS?

All of the exhibits thus far have been based upon a situation that generates an operating profit. Furthermore, while the Bingham invariant ratio, average annual return on equity, and internal rate of return produce different measures of the return on equity, they do not differ significantly in the absence of the Bingham release scheme. For the purpose of a reasonableness check, a new example will be presented. The longer payout period accentuates the differences between the three measures of ROE when the

TABLE 2

ELEMENTS COMMON TO EXHIBITS 4–6

Year End	Paid to Date Loss	Nominal O/S Loss	Discounted O/S Loss	Supporting Surplus	Required Funds
0	$0.00	$2,000.00	$961.38	$1,000.00	$1,961.38
1	0.00	2,000.00	1,009.45	1,000.00	2,009.45
2	0.00	2,000.00	1,059.92	1,000.00	2,059.92
3	6.00	1,994.00	1,106.92	997.00	2,103.92
4	34.00	1,966.00	1,134.26	983.00	2,117.26
5	120.00	1,880.00	1,104.98	940.00	2,044.98
6	184.00	1,816.00	1,096.23	908.00	2,004.23
7	258.00	1,742.00	1,077.04	871.00	1,948.04
8	332.00	1,668.00	1,056.89	834.00	1,890.89
9	404.00	1,596.00	1,037.73	798.00	1,835.73
10	474.00	1,526.00	1,019.62	763.00	1,782.62
11	526.00	1,474.00	1,018.60	737.00	1,755.60
12	574.00	1,426.00	1,021.53	713.00	1,734.53
13	618.00	1,382.00	1,028.61	691.00	1,719.61
14	660.00	1,340.00	1,038.04	670.00	1,708.04
15	693.00	1,304.00	1,053.94	652.00	1,705.94
16	730.00	1,270.00	1,072.64	635.00	1,707.64
17	760.00	1,240.00	1,096.27	620.00	1,716.27
18	788.00	1,212.00	1,123.08	606.00	1,729.08
19	1,312.00	688.00	655.24	344.00	999.24
20	2,000.00	0.00	0.00	0.00	0.00

release of operating gain does not follow the resolution of uncertainty.

The longer payout period of this example is similar to that of high attachment point workers compensation excess of loss reinsurance. By the end of the 18th year, less than 40% of the ultimate loss is expected to have been paid. Table 2 displays the elements that are common to Exhibits 4 through 6.

The Required Funds column consists of the funds that must be allocated to the insurance product, an amount equal to the discounted outstanding loss plus the supporting surplus. Any additional funds may be released to the surplus account at any time.

The reasonableness check begins with a simple observation. If the operating gain associated with the insurance product is exactly zero (i.e., the premium is just sufficient to fund the discounted loss reserves), then there can be no net flow from the insurance product to or from the surplus account. Supporting surplus must be allocated, but all that can be earned on the supporting surplus is the 5% return that could be earned on idle surplus. No release of the operating gain (i.e., a set of non-zero flows that have a present value equal to zero) that results in a return on equity other than 5% is reasonable.

Exhibits 4A and 4B present just such a zero operating gain situation. With $961.38 of premium and $2,000 of expected loss, the underwriting loss would be $1,038.62 and the incurred loss ratio would be 208%. A premium equal to $961.38, paid on day one, exactly funds the discounted outstanding loss reserve. With no funds to spare, the operating gain is exactly zero. While supporting surplus is required during the 20 year runoff, its return will be exactly the same as if the insurance product had not been written, 5%. No measurement of the return on surplus other than 5% would be reasonable for this situation.

A quick glance at Exhibit 4A discloses that Bingham's invariant ratio passes the test, whereas the average annual return, at 8.1%, clearly fails the test.

The rather peculiar looking release of operating gain, $\{O(j)\}$, mimics the requirements of statutory accounting (SAP). Under SAP, the carrier must fund the nominal reserves rather than the discounted reserves. As a result of this requirement, the $961.38 premium falls short by $1,038.62. Consistent with the SAP requirement, $1,038.62 must be transferred *from* surplus surplus to the product on day one. The equivalent year-end transfer is displayed on Exhibit 4A. The $990.55 transfer can be thought of as the day one transfer of $1,038.62 plus interest (totaling $51.93) less the interest earned on the $2,000 nominal reserve (a total of $100.00).

While this set of operating gain flows is allowable (their present value is zero and they produce a zero investment balance by the end of the 20 year runoff), the corresponding 8.1% average annual return on supporting surplus is, clearly, unreasonable. This paradoxical result is an example of the type of manipulation that Bingham's release scheme is designed to prevent.

This manipulation was previously encountered in the first example for which the operating gain was greater than zero and for which all flows were positive. In that example, it was noted that once a flow is released to the surplus account, no further investment income earned on this money is credited to the insurance product. The longer the operating gain is retained as part of the insurance product, the more of its earned investment income is credited to the insurance product. Interest earned on surplus surplus is ignored, regardless of its source.

Likewise, when some of the flows are negative, the interest that is not earned (lost) by the surplus surplus is ignored. The insurance product, rather than surplus surplus, receives credit for the earned investment income. The $485.65 of nominal gain (sum of the stream of $O(j)$ flows) that appears to have been generated by the insurance product was at the (unrecognized) expense of the surplus surplus account.

If the average annual return on surplus is viewed as being the calendar year return once a steady state situation has been achieved, then the identification of the source of the additional $485.65 return is somewhat different. Under a steady state interpretation, the flows from year-ends 1 through 20 represent the contribution of previously written policies to the current calendar year. Under this interpretation, the policies in runoff do provide sufficient funds to establish the initial reserve on newly written policies and provide the missing 3.1% return on the steady state supporting surplus. What is missing in this interpretation is the cost of establishing the steady state (transferring funds from surplus to establish the first twenty years of writings). The additional 3.1% return is exactly equal to the investment income being lost by the surplus surplus account as a result of funding the underwriting loss for the first 20 years.

The final columns of Exhibit 4A display the calendar year return on equity during each year, if the SAP release scheme were to be followed. During the first 20 years, the runoff from successively more accident years is reflected in each annual statement. Eventually by year 20, the statement ROE reflects one year-end ROE for each of the 20 accident years. Growth from year to year affects the relative amount of each maturity that is reflected in the year-end ROE. Only when the exposure growth rate is 5% does the calendar year ROE approach the reasonable 5% figure.

Exhibit 4B looks at the same situation from the insurance carrier's perspective rather than from the perspective of a stockholder who is focused on the surplus account. In this representation, no distinction is made between supporting surplus and discounted loss reserves. All of the funds belong to the insurance carrier. Funds are released to the general surplus account as soon as they are not required to support the insurance product.

There is an initial investment of $1,000 from general surplus which, together with the premium, leaves $1,961.38 to be invested at 5% per year. The required funds are also equal to $1,961.38. At the end of a year, the invested funds will have accrued to $2,059.45 (there having been no loss payments). Only $2,009.45 is required by the insurance carrier to fund the discounted outstanding loss amount and supporting surplus, so the $50.00 difference can be released to surplus. Continuing in this fashion results in cash flows to the surplus account that have an internal rate of return equal to 5%.

With both the Bingham invariant ratio and the insurance carrier perspective treatment having passed the first reasonableness test, a new situation (depicted in Exhibits 5A and 5B) is considered. In these exhibits, less premium is collected. This results in a net operating loss for the product. Clearly, the supporting surplus must earn less than if it were not supporting this product.

As can be seen in Exhibit 5A, Bingham's invariant ratio represents a reasonable measure of the return on surplus; it is less than that of idle surplus. The statutory accounting model, again, fails the test because its ROE is greater than that of idle surplus.

From the insurance carrier's perspective (Exhibit 5B), \$1,961.38 is required to support the product, but only \$600.00 is received in the form of premium. The additional \$1,361.38 must be supplied from the surplus account. While not producing the same ROE as Bingham's scheme does, this measure is reasonable.

Both the Bingham scheme and the insurance carrier perspective agree that surplus would increase faster if this product, with its 333% loss ratio, were not written. The statutory accounting model does not agree.

A reasonable model should report an ROE that is greater than that of idle surplus if there is an operating gain produced by the insurance product. The purpose of supporting surplus is to cushion against uncertainty. If the premium is sufficient to fund the discounted loss reserve for the expected losses and to provide the required cushion against uncertainty, then no contribution of supporting surplus should be required. As the premium approaches this "no risk to the carrier" amount, the ROE should increase without bound. This expectation provides another test of a model's behavior.

Exhibit 6A displays the first portion of the reasonableness test when there is a net operating profit. With \$1,700 of premium, there is a \$738.62 operating gain. All three measures of ROE are greater than that of idle surplus (5%).

Both Bingham and the statutory model allocate the same amount of supporting surplus that they did in the other two cases. From the insurance carrier perspective (Exhibit 6B), only \$261.38 of surplus is needed in addition to the premium in order to fully fund the discounted outstanding loss and supply the required amount of cushion against uncertainty.

TABLE 3

RETURN ON EQUITY FOR EACH MEASURE

Premium	Operating Gain	Loss Ratio	SAP Measure	Bingham Invariant Measure	Insurance Carrier IRR Measure
$ 500.00	$ −461.38	400.0%	5.0%	0.6%	1.6%
600.00	−361.38	333.3%	5.7%	1.5%	2.2%
700.00	−261.38	285.7%	6.3%	2.5%	2.9%
800.00	−161.38	250.0%	7.0%	3.4%	3.6%
900.00	−61.38	222.2%	7.7%	4.4%	4.4%
961.38	0.00	208.0%	8.1%	5.0%	5.0%
1,000.00	38.62	200.0%	8.3%	5.4%	5.4%
1,100.00	138.62	181.8%	9.0%	6.3%	6.5%
1,200.00	238.62	166.7%	9.7%	7.3%	7.8%
1,300.00	338.62	153.8%	10.3%	8.3%	9.4%
1,400.00	438.62	142.9%	11.0%	9.2%	11.4%
1,500.00	538.62	133.3%	11.7%	10.2%	14.1%
1,600.00	638.62	125.0%	12.3%	11.1%	18.0%
1,700.00	738.62	117.6%	13.0%	12.1%	24.4%
1,800.00	838.62	111.1%	13.7%	13.1%	37.5%
1,900.00	938.62	105.3%	14.3%	14.0%	88.3%
1,920.00	958.62	104.2%	14.5%	14.2%	126.6%
1,940.00	978.62	103.1%	14.6%	14.4%	237.3%
1,950.00	988.62	102.6%	14.7%	14.5%	441.1%
1,960.00	998.62	102.0%	14.7%	14.6%	3,620.8%

For the second part of the reasonableness test, allow the premium to increase until it is sufficient to fund the $961.38 discounted loss reserve and to supply the required $1,000 cushion against adversity. At that point, zero surplus is required and the return on equity should become undefined. Table 3 displays the resulting returns on equity for each of the measures (average annual return under a release dictated by statutory accounting, Bingham's present value ratio, and the insurance carrier's IRR) as the zero risk extreme is approached.

The shaded sections of the table indicate regions in which the model fails a reasonableness test. The Bingham invariant ratio appears to fail the test at the high operating profit extreme because the required supporting surplus does not reflect the fact that the retained operating funds provide an additional (unquantified) cushion against uncertainty.

If the required surplus were to be reduced in recognition of the retained operating gain, with the sum of the supporting surplus and retained operating gain providing the required cushion (at a 2 : 1 reserve to cushion ratio), then insurance products with a larger expected operating gain would require less surplus. As the expected operating gain approached the required cushion amount, the required surplus would approach zero, and the resulting return on surplus would increase without bound as the expected operating gain approached this no risk situation.

Were it not for the Bingham requirement that the operating gain be released so as to maintain a constant return on the supporting surplus, reducing the surplus in recognition of the retained operating gain would be a trivial exercise. Difficulty arises because the set of release flows, $\{O(j)\}$, depends upon the year-end surplus amounts, $\{S(j)\}$, which in turn depend upon the set of retained operating gains, $\{R(j)\}$. These retained gains depend upon what has been previously released, the set $\{O(j)\}$.

Attempting to find a set of flows and surplus amounts that satisfy the two relations,

$$O(j)/S(j-1) = k, \quad \text{independent of } j, \text{ and}$$

$$R(j) + S(j) = \text{Reserves at year-end } j \text{ divided by the reserves-to-cushion ratio}$$

is not a trivial matter.

Solving this linked set of equations in closed form requires solving a polynomial of degree 20 for a product with a 20 year runoff.[5] Attempting to solve the system by an iterative technique

[5]The polynomial arises as the result of an attempt to determine the operating gain to be released at each year-end. As demonstrated in Appendix A, in the absence of modifying the supporting surplus to reflect the cushioning effect of the retained operating gain, a set

requires the imposition of additional conditions that are not specified by Bingham.[6]

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of linear equations in k, the constant annual return on equity resulting from the release of accrued operating gain,

$$O(j) = k*S(j-1),$$

must be solved. Because the $\{S(j)\}$ are independent of the $\{O(j)\}$, the set of n equations in k is linear.

When the supporting surplus is a function of the retained operating gain, as it is when the amount of supporting surplus is reduced in recognition of the operating gain that has been retained, $S(j-1)$ becomes a function of the previously released operating gain. Each $O(j)$ is, itself, a linear function of k. As a result,

$$O(j) = k*F(\{O(n)\}), \quad \text{where } n \text{ ranges from zero to } j-1.$$

It is this functional dependence of $O(j)$ upon the $O(n)$ that introduces increasingly higher powers of k as j increases.

To be more concrete, let

OP be the expected operating gain for the product (i.e., the present value at time zero),
$\{C(j)\}$ be the required amount of cushion at year-end j,
$\{R(j)\}$ be the retained operating gain at year-end j, and
i be the investment income rate.

During the first year, the required cushion, $C(0)$, consists of the sum of the operating gain, OP, and a contribution from surplus, $S(0)$. At the end of the year, $O(1)$ will be released such that

$$O(1)/S(0) = k.$$

With the exception of the fact that $S(0)$ is not equal to $C(0)$, this equation is identical to the first equation in the set of linear equations.

During the second year, the required cushion is $C(1)$. This is supplied by the retained operating gain, $(1+i)*OP - O(1)$ together with a contribution from surplus, $S(1)$, where

$$\begin{aligned} S(1) &= C(1) - (1+i)*OP - O(1) \\ &= C(1) - (1+i)*OP - k*S(0) \\ &= C(1) - (1+i)*OP - k*[C(0) - OP]. \end{aligned}$$

The condition that

$$O(2)/S(1) = k$$

becomes

$$O(2)/[C(1) - (1+i)*OP - k*[C(0) - OP]] = k$$

which is quadratic in k. Each additional year that is reflected introduces another power of k into the polynomial.

[6]The iterative solution begins with an initial solution that sets

$$S(j)_0 = C(j).$$

4. REFINEMENTS TO THE BINGHAM METHODOLOGY I (NOMINAL VS. DISCOUNTED RESERVES)

While several sources of uncertainty are enumerated in Bingham's paper, his example deals with only one of these sources, the uncertainty associated with the ultimate loss *amount*. As a result, his supporting surplus is a function of the nominal outstanding loss reserve. Bingham does not describe how the risk associated with the *timing* of loss payments would influence the amount of supporting surplus, nor does he discuss the effect of investment risk on the amount of supporting surplus that would be required.

A minor change is required to reflect not only the uncertainty in the ultimate *amount* but also *timing risk* and a portion of the *investment rate* uncertainty as well. The change involves applying the leverage ratio to the discounted reserves rather than to the nominal reserves. The variance of the expected discounted reserves can be modeled to reflect the uncertainty in the ultimate

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For this solution, a set of $O(j)_0$ are determined. Using these $O(j)_0$, the set of retained operating gains, $\{R(j)_0\}$, can be determined at each year-end.

The next iteration begins by setting

$$S(j)_1 = C(j) - R(j)_0$$

and completing another cycle.

The iteration is said to converge if, for all n greater than a fixed N, $S(j)_n - S(j)_N$ is not material.

When applied to the 20 year payout example, the iterative procedure ran into problems (failed to converge) when the premium was sufficient to cause

$$S(j)_m = C(j) - R(j)_{m-1} < 0 \qquad \text{for some } j \text{, on the } m\text{th iteration.}$$

A logical additional condition to impose upon $S(j)$ is that it be greater than or equal to zero.

At even larger premium amounts (above $1,800), multiple $S(j)$s "zeroed out." Again, the iteration failed to converge to a single accumulation point, as $S(j)$s that were previously equal to zero became positive at the next iteration.

A determination of the conditions that must be imposed upon the iteration in order to make it converge for all premium amounts is beyond the scope of this discussion. It is very interesting to note that, when the procedure did converge, the indicated rate of return on surplus was numerically equal to the IRR produced by looking at the process from the insurance carrier perspective. Finding a logical set of constraints that would insure (proven rigorously) this equality at all premium levels would be a significant contribution to the literature.

amount, uncertainty in the cash flow timing, and uncertainty in the investment income rate as well. A description of how one would determine the leverage ratio that would cushion against variation of the expected discounted reserve around its mean is beyond the scope of this discussion.

When the role of the cushion is restricted to covering the *ultimate amount* at risk, it is still appropriate to apply a leverage ratio to discounted reserves. Even if the actual future loss payments are greater than expected, only the present value of the unexpected payments needs to be available now. It will accrue to the required amount by the time it must be used.

Returning to the original example, the nominal loss reserve is $44.00 at the end of year two. The 2 : 1 reserve to surplus ratio[7] implies that if $66.00 is made available to pay losses ($44.00 of loss reserve and $22.00 of supporting surplus), then the probability of ruin can be kept below some pre-established amount (e.g., less than 0.02). If there is no uncertainty regarding the timing of the future payments (i.e., the percentages of the actual ultimate loss to be paid by each year end are exactly those which were expected), then each future loss payment will be 50% higher than expected in this worst case scenario. If the $41.19 discounted loss reserve accrues to pay the expected future losses, then an additional $20.60 (50% of the discounted outstanding loss amount) should be sufficient to make the unexpected payments if they become due.

Differences between the expected and actual timing of loss payments have no impact upon the nominal loss reserves that should be carried but do affect the amount of discounted loss reserve that should be carried at any point in time. It is logical to cushion against the timing uncertainty that increases the

[7] It has been assumed that the original 2 : 1 leverage ratio does not reflect any implicit discounting for interest.

variance in the discounted loss reserve by adopting the discounted reserve as the surplus allocation base.

5. REFINEMENTS TO THE BINGHAM METHODOLOGY II (A DECREASING LEVERAGE RATIO)

Bingham assumes that a constant reserve-to-surplus leverage ratio results when supporting surplus is established to maintain a constant probability of ruin. While this assumption is consistent with the other simplifications that he adopted for illustrative purposes, it must be emphasized that it is neither required to achieve an invariant ratio, nor is it realistic. Many models that allocate surplus over the life of a product assume that a constant leverage ratio is appropriate. Some models even allow the leverage ratio to increase over time. For many circumstances, the leverage ratio must decrease over the long run if a constant probability of ruin is to be maintained. This is not to say that a short term increase in the ratio of reserves to surplus is impossible, but that such a short term increase will be followed by a long term decrease as the runoff becomes increasingly more volatile.

For illustrative purposes, consider the hypothetical case of excess of loss casualty reinsurance with a very high attachment point. Because of the high attachment point, assume that small claims will be eliminated. Assume, further, that those claims that remain can be modeled by one of the more common distributions (e.g., the lognormal or Pareto distribution). A suitably high attachment point assures us that all of the possible claims will fall in the relatively flat tail of the severity distribution. This means that the likelihood of any particular claim size is almost equal to that of any other size claim. If each claim closes with a single payment and this payment does not depend upon how long the claim remained open before being settled, then the ultimate closing amount on each open claim can be represented by a stochastic variable where the same underlying distribution applies to all of the open claims.

When there are exactly N independent open claims,[8] the best estimate of the outstanding loss is

$$\text{Nominal outstanding loss reserve} = Ns,$$

where s is the mean severity from the single claim severity distribution.[9] Likewise, the variance of the possible loss outcomes for the group of N claims is given by

$$\text{Variance of the aggregate ultimate loss around the expected} = N\sigma^2,$$

where σ is the standard deviation of the single claim severity distribution. If N is sufficiently large, the aggregate loss distribution will be approximately normal. The ultimate loss outcome will be less than

$$\text{98th percentile ultimate loss} = Ns + 2.06\sqrt{N}\sigma$$

98% of the time. If, for every Ns of expected loss, $2.06\sqrt{N}\sigma$ of supporting surplus is allocated, then the probability of ruin can be maintained at 2%. Here ruin means that more funds are required than are available. If only a single contract is being considered, ruin may be less catastrophic than company insolvency. The corresponding leverage ratio to cushion against this single contract ruin is given by

$$Ns : 2.06\sqrt{N}\sigma$$

or

$$\sqrt{N}s/2.06\sigma : 1.$$

[8]Here, N may reflect not only the known open claim count but also an estimate of the IBNR claim count as well.

[9]The claim severity distribution is that which describes losses in the layer of reinsurance. For excess of loss reinsurance, this would not be the same distribution as the ground up severity distribution.

TABLE 4

Year End	Paid to Date Loss	Nominal O/S Loss	Discounted O/S Loss	Leverage Ratio	Supporting Surplus
0	$0.00	$404.00	$375.86	2.00 : 1.00	$187.93
1	264.00	140.00	130.66	1.18 : 1.00	110.97
2	360.00	44.00	41.19	0.66 : 1.00	62.40
3	392.00	12.00	11.25	0.34 : 1.00	32.63
4	400.00	4.00	3.81	0.20 : 1.00	19.14
5	404.00	0.00	0.00	N/A	0.00

As claims close, N, the number of open claims, decreases. As shown above, the leverage decreases in proportion to the square root of N.

If there are insufficient open claims to warrant the normal approximation, then the 98th percentile would have to be determined by means of some other aggregate loss modeling technique. The important point is that as the number of open claims decreases, the *relative* uncertainty increases as a function of the expected loss amount. In other words, the *absolute* amount of surplus may decrease, but the *relative* amount increases.

If claims are closed with a single payment and the same severity distribution can represent each claim, then the percentage of ultimate loss that is paid at any point in time is a measure of the number of claims that have been paid. Assuming that a 2 : 1 reserve-to-surplus ratio is appropriate at time zero, when none of the claims are closed, then the appropriate reserve-to-surplus ratio would become $2\sqrt{.75}$: 1 when 25% of the claims have closed. Returning to the original example with a five year runoff, and introducing both the modified leverage ratio and the discounted outstanding loss reserve as the base, the supporting surplus amounts shown in Table 4 are required at each year-end.

The *initial* supporting surplus, $187.93, is less than the $202.00 of supporting surplus for the unmodified case. This quickly changes as the leverage ratio decreases (i.e., more surplus is required to support a dollar of loss reserve as the number of open claims decreases and the proportionate volatility increases) and the offsetting loss discount unwinds.

Exhibit 7 displays the correspondingly modified Bingham model. Notice that the average return on surplus and internal rate of return remain equal to the Bingham invariant ratio after the modification. Because the modification involves changing the amount of supporting surplus, the invariant ratio is not equal to the corresponding invariant ratio displayed on the other exhibits. Such agreement would not be expected.

Exhibits 8A and 8B apply the modifications to the second example. While the invariant ratio is numerically equal to the internal rate of return from the perspective of the insurance carrier, this is simply a coincidence produced by rounding errors. Table 5 provides a comparison of the three models under the modified surplus determination.

6. OTHER ISSUES

There are a number of issues that fall outside the scope of this discussion paper. They are briefly mentioned in the hope that they may encourage further discussion.

1. How can the other sources of insurance product uncertainty be reflected?
2. How can the appropriate leverage ratios for a selected probability of ruin be determined empirically?
3. As presented, the model produces a *point estimate* of the return on equity. Expected loss amounts and payout timing are all that have been reflected in the determination of the return on equity. If $\{L_t\}$ represents the actual loss payments at times $\{t\}$, then the return on equity that has been determined is $\text{ROE}(\{\langle L_t\rangle\})$ rather than

TABLE 5

RESULTS UNDER MODIFIED SURPLUS DETERMINATION

Premium	Operating Gain	Loss Ratio	Statutory Measure	Invariant Measure	Insurance Carrier IRR Measure
$500	$ -461.38	400.0%	5.0%	-2.1%	0.4%
600	-361.38	333.3%	6.0%	-0.6%	1.1%
700	-261.38	285.7%	7.0%	1.0%	1.9%
800	-161.38	250.0%	8.0%	2.5%	2.9%
900	-61.38	222.2%	9.1%	4.1%	4.1%
961	0.00	208.0%	9.7%	5.0%	5.0%
1,000	38.62	200.0%	10.1%	5.6%	5.6%
1,100	138.62	181.8%	11.1%	7.1%	7.6%
1,200	238.62	166.7%	12.1%	8.7%	10.6%
1,300	338.62	153.8%	13.1%	10.2%	15.8%
1,400	438.62	142.9%	14.1%	11.7%	32.6%
1,410	448.62	141.8%	14.2%	11.9%	37.7%
1,420	458.62	140.8%	14.3%	12.0%	45.6%
1,430	468.62	139.9%	14.4%	12.2%	60.7%
1,440	478.62	138.9%	14.5%	12.4%	123.9%
1,442	480.62	138.7%	14.5%	12.4%	381.8%

$\langle \text{ROE}(\{L_t\}) \rangle$, where $\langle \cdots \rangle$ denotes taking the expected value of the quantity that is enclosed. If the model is not a linear function of the $\{L_t\}$ then the two averages need not be equal. There are many possible sets of loss payments that *may* be made. Out of this population, only one set of payments *will* occur. Prior to their occurrence, the best estimate of what *will* occur is $\{\langle L_t \rangle\}$. Each of the possible $\{L_t\}$ will result in a different return on supporting surplus. There is no guarantee that the expected return is equal to the return corresponding to the expected loss payments. Even for our simple example, whether or not the two averages are equal depends upon how the next issue is resolved.

4. At a particular point in time there is an expected outstanding loss reserve. A corresponding amount of sur-

plus will be allocated in such a way that the probability of ruin is less than some predetermined amount. The estimate of future payments will, undoubtedly, change over time. After several years have elapsed and the first few years of actual payments have been made, as details concerning the actual open claims become known, and as IBNR emerges, expectations regarding payments yet to be made will probably not be the same as they were in the beginning. The question is whether or not these changed expectations of future loss payments should result in a modification of the supporting surplus during future periods.

In the first example, the a priori expected reserve at the end of year two is $44.00. Based upon this expectation, $22.00 of supporting surplus is considered to be an adequate cushion against ruin. Together, there will be enough funds available to cover $66.00 of future loss payments. But $44.00 is the a priori (at time zero) expected loss to be paid after the end of year two. What if the best estimate of the future loss payout is $60.00 when the end of year two actually arrives? Certainly, the reserve would be changed to reflect this additional information. Should the cushion at year-end two and subsequent periods be adjusted accordingly?

There appear to be three alternative ways in which to determine the required supporting surplus for future periods under this scenario.

- Assume that $16.00 of the $22.00 safety margin has been used to establish the originally unanticipated additional outstanding loss reserve. The remaining $6.00 of surplus continues to provide an adequate safety net. This approach assumes that the a priori outstanding amount defines the distribution of possible outcomes, and that the safety margin is always measured against the a priori expectation. Regardless of what the actual estimate is, the supporting surplus cushions against the a priori estimate of the worst case scenario. Under this alternative, all differences between the expected values and the actual values are attributed to process variance. There is no cushion provided for parameter errors contained in the a priori expectations.

In a sense, this method is analogous to a loss ratio reserving methodology in which IBNR reserves are established equal to the difference between the a priori loss ratio and the reported loss ratio. Only if the difference becomes negative (i.e., reported amounts exceed expected amounts) is the a priori assumption questioned. A negative difference means that ruin has occurred.

- Assume that the a priori outstanding loss amount defines the size of the uncertainty, $22.00. Even when year two ends and the outstanding loss estimate (and it is still just an estimate as of year-end two) is $60.00 rather than the expected $44.00, $22.00 of surplus provides the necessary safety margin.

  This alternative is analogous to the Bornhuetter/Ferguson loss reserving methodology. Future development (and uncertainty) depends upon an a priori assumption which is not modified to reflect current information.

- Assume that the $60.00 estimate contains the same percentage of uncertainty as did the $44.00 a priori estimate. In this case, the supporting surplus must be increased from $22.00 to $30.00. Intuitively, this approach is less than satisfying because it appears to imply that the claim department's opinion at the end of year two not only does nothing to decrease the uncertainty over the a priori estimate that was available at the beginning of year zero but actually increases the dollar amount of uncertainty. This alternative assumes that the a priori estimate was based upon so much parameter error as to be worthless once additional information becomes available.

This alternative is analogous to the chain ladder reserving methodology which is 100% responsive to the current information.

A resolution of how to deal with *actual estimates* vs. *a priori expectations* will be necessary in order to determine whether or not the point estimate, ROE($\{\langle L_t \rangle\}$), will be equal to the ensemble average (i.e., ROE($\{L_t\}$) run for each of the $\{L_t\}$ and then weighted by the probability of occurrence), $\langle$ROE($\{L_t\}$)$\rangle$. If the two estimates are not equal, then even a prospective evaluation of the rate of return must be performed on an ensemble of possible insurance product outcomes rather than a single expected value outcome.

For our simple example, the second alternative results in a linear model whereas the other two do not. This can easily be demonstrated by running several possible loss outcomes through the Bingham model. The expected loss for the example is $404.00. Without changing the payout pattern (i.e., the percentage of ultimate loss paid at any particular point in time), consider Table 6, the possible loss outcomes and their corresponding probabilities of occurrence.

Note that Alternatives 1 and 3 produce deviations from the point estimate that are in opposite directions. The more volatile the loss distribution (the larger the variance), the more pronounced the deviation between the ensemble and point estimates will be for non-linear models.

While linearity makes the calculations easier, computational difficulty should not be the only criterion that is used in the selection of an alternative.

TABLE 6

OUTCOMES AND THEIR PROBABILITIES

Probability of Occurrence	$\sum_{t=0}^{5} L_t$	Alternative 1	Alternative 2	Alternative 3
0.35	$380.00	19.76%	21.51%	22.56%
0.14	392.00	16.84%	17.55%	17.93%
0.02	404.00	13.58%	13.58%	13.58%
0.14	416.00	9.90%	9.61%	9.48%
0.35	428.00	5.73%	5.64%	5.61%
Ensemble Average	404.00	12.94%	13.58%	13.97%
Point Average	404.00	13.58%	13.58%	13.58%

5. Closely related to the ensemble vs. point estimate discussion is the appropriate allocation of surplus when a policy year is analyzed *retrospectively* to determine the actual return on surplus. Since the a priori expectations are rarely realized, how much supporting surplus should be reflected? When actual results deviate from expected results, actual outstanding loss reserves will deviate from those that were expected. At what point in the retrospective determination of the return on surplus should the actual reserves be reflected? Should it reflect *carried* reserves or what *should have been carried* at any point in time?

7. SUMMARY AND CONCLUSIONS

Russell Bingham made a significant contribution to the literature concerning the allocation of surplus and determination of the rate of return on that surplus. His advocacy of keeping the results of each exposure period separate so that the long-term commitment of surplus can be appropriately reflected is right on target.

In the process of taking the present values of the insurance flows and supporting surplus, Bingham has produced an invariant measure of the return on surplus.

The difference between the commonly used *calendar* year determination of the return on surplus and Bingham's accident year approach can be illustrated by the following two descriptions of the same investment:

- Calendar Year Approach: A carrier invests $1,000 of surplus and receives a $400 return, so the return on surplus is 40%;
- Accident Year Approach: A carrier invests $1,000 of surplus *for 10 years* and receives an *average annual return* equal to 3.4% on its investment.

The second approach takes into account the time over which the surplus funds are invested (until all of the uncertainties are resolved). This time horizon is well beyond the time that premiums are in force for most insurance products.

If a given probability of ruin is to be maintained by cushioning funds, then there must be some recognition of the cushion afforded by the premium provision for expected operating profit. Otherwise, the probability of ruin will vary with premium in a manner that is difficult to rationalize. This would appear to imply that ruin occurs whenever the expected operating profit is not achieved rather than whenever there are insufficient funds to meet the unexpected losses and expenses. The latter definition seems to be a more logical way to define ruin. This is an area that warrants further investigation.

Two modifications that can be made to enhance Bingham's model have been proposed. In actual practice, the leverage ratio will vary, but not in such a simple manner as suggested by the square root rule. A more detailed investigation of the characteristics of a particular line must be undertaken in order to establish actual leverage ratios for the runoff of a maturing policy year.

While not exhaustive, a list of additional considerations provides issues that must be addressed. In particular, the idea of averaging the returns over an ensemble of possible loss outcomes forces us to refine our ideas concerning the role of surplus as it cushions against ruin.

REFERENCES

[1] Heckman, Philip E. and Glenn G. Meyers, "The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions," *PCAS* LXX, 1983, pp. 22–71.

[2] N.A.I.C. Study of Investment Income, Supplement to the Proceedings, National Association of Insurance Commissioners, 1984, Vol. II, pp. 504–507.

[3] The Workers Compensation Rating and Inspection Bureau of Massachusetts, Filing for rates to become effective January 1, 1988, p. 701.

EXHIBIT 1

A NOT COMPLETELY BINGHAM MODEL

	INSURANCE PRODUCT				SURPLUS ACCOUNT					IRR	INSURANCE PRODUCT		
End of Year	Written Premium	Paid Loss	Operating Gain	$O(n) \Rightarrow$ Funds Released	Supporting Surplus S	Funds Released to Surplus	Idle Surplus Investment Income	Idle Surplus Investment Income	Total Return on Supporting Surplus	Total Surplus Flows	Retained Earnings	Investment Balance	Overfund
0	$400.00	$0.00								$-202.00	$-4.00	$400.00	$24.14
1		264.00		$25.34	$202.00	12.55%	$10.10	5.0%	17.55%	167.44	-9.34	130.66	0.00
2		96.00		0.00	70.00	0.00%	3.50	5.0%	5.00%	51.50	-2.81	41.19	0.00
3		32.00		0.00	22.00	0.00%	1.10	5.0%	5.00%	17.10	-0.75	11.25	0.00
4		8.00		0.00	6.00	0.00%	0.30	5.0%	5.00%	4.30	-0.19	3.81	0.00
5		4.00		0.00	2.00	0.00%	0.10	5.0%	5.00%	2.10	0.00	0.00	0.00
NPV	400.00	375.86	$24.14	24.14	281.38	8.58%	14.07	5.0%	13.58%				
									Average = 13.39%	IRR = 13.87%			

EXHIBIT 2

ANOTHER NOT COMPLETELY BINGHAM MODEL

	INSURANCE PRODUCT				SURPLUS ACCOUNT					IRR	INSURANCE PRODUCT		
End of Year	Written Premium	Paid Loss	Operating Gain	$O(n) \Rightarrow$ Funds Released	Supporting Surplus S	Funds Released to Surplus	Idle Surplus Investment Income	Idle Surplus Investment Income	Total Return on Supporting Surplus	Total Surplus Flows	Retained Earnings	Investment Balance	Overfund
0	$400.00	$0.00								$-202.00	$-4.00	$400.00	$24.14
1		264.00		$0.00	$202.00	0.00%	$10.10	5.0%	5.00%	142.10	16.00	156.00	25.34
2		96.00		0.00	70.00	0.00%	3.50	5.0%	5.00%	51.50	23.80	67.80	26.61
3		32.00		0.00	22.00	0.00%	1.10	5.0%	5.00%	17.10	27.19	39.19	27.94
4		8.00		0.00	6.00	0.00%	0.30	5.0%	5.00%	4.30	29.15	33.15	29.34
5		4.00		30.81	2.00	1,540.35%	0.10	5.0%	1,545.35%	32.91	0.00	0.00	0.00
NPV	400.00	375.86	$24.14	24.14	281.38	8.58%	14.07	5.0%	13.58%				
									Average = 15.20%	IRR = 11.84%			

EXHIBIT 3

THE BINGHAM MODEL

	INSURANCE PRODUCT				SURPLUS ACCOUNT					IRR	INSURANCE PRODUCT		
End of Year	Written Premium	Paid Loss	Operating Gain	$O(n) \Rightarrow$ Funds Released	Supporting Surplus S	Funds Released to Surplus	Idle Surplus Investment Income	Idle Surplus Investment Income	Total Return on Supporting Surplus	Total Surplus Flows	Retained Earnings	Investment Balance	Overfund
0	$400.00	$0.00								$-202.00	$-4.00	$400.00	$24.14
1		264.00		$17.33	$202.00	8.58%	$10.10	5.0%	13.58%	159.43	−1.33	138.67	8.02
2		96.00		6.00	70.00	8.58%	3.50	5.0%	13.58%	57.50	−0.40	43.60	2.41
3		32.00		1.89	22.00	8.58%	1.10	5.0%	13.58%	18.99	−0.11	11.89	0.65
4		8.00		0.51	6.00	8.58%	0.30	5.0%	13.58%	4.81	−0.03	3.97	0.16
5		4.00		0.17	2.00	8.58%	0.10	5.0%	13.58%	2.27	0.00	0.00	0.00
NPV	400.00	375.86	$24.14	24.14	281.38	8.58%	14.07	5.0%	13.58%				
								Average =	13.58%	IRR = 13.58%			

Determination of the $\{O(n)\}$

1. Constant annual ROE $\Rightarrow O(n)/S(n) = k$, or $O(n) = k * S(n)$
2. $\text{NPV}(\{O(n)\}) = k * \text{NPV}(\{S(n)\}) \Rightarrow k = \text{NPV}(\{O(n)\})/\text{NPV}(\{S(n)\})$

n	$S(n)$		$O(n)$
1	$S(1) =$	$202.00	$17.33
2	$S(2) =$	70.00	6.00
3	$S(3) =$	22.00	1.89
4	$S(4) =$	6.00	0.51
5	$S(5) =$	2.00	0.17
	$\text{NPV}(\{S(n)\}) =$	$281.38	
	$\text{NPV}(\{O(n)\}) =$	$ 24.14	$24.14
	$k =$	0.085784	

EXHIBIT 4A

Statutory Accounting Model—Zero Operating Gain

	INSURANCE PRODUCT				SURPLUS ACCOUNT							ROE Under Statutory Accounting With a Growth Rate equal to		
End of Year	Written Premium	Paid Loss	Operating Gain	$O(n) \Rightarrow$ Funds Released	Supporting Surplus S	Funds Released to Surplus	Idle Surplus Investment Income	Idle Surplus Investment Income	Total Return on Supporting Surplus	Insurance Investment Balance	Year	0.0%	5.0%	10.0%
0	$961.38	$0.00								$961				
1		0.00		$ − 990.55	$1,000.00	−99.1%	$50.00	5.0%	−94.1%	2,000	1	−94.1%	−94.1%	−94.1%
2		0.00		100.00	1,000.00	10.0%	50.00	5.0%	15.0%	2,000	2	−39.5%	−40.9%	−42.1%
3		6.00		100.00	1,000.00	10.0%	50.00	5.0%	15.0%	1,994	3	−21.4%	−23.1%	−24.9%
4		28.00		99.70	997.00	10.0%	49.85	5.0%	15.0%	1,966	4	−12.3%	−14.3%	−16.3%
5		86.00		98.30	983.00	10.0%	49.15	5.0%	15.0%	1,880	5	−6.9%	−9.1%	−11.2%
6		64.00		94.00	940.00	10.0%	47.00	5.0%	15.0%	1,816	6	−3.4%	−5.7%	−8.0%
7		74.00		90.80	908.00	10.0%	45.40	5.0%	15.0%	1,742	7	−1.0%	−3.3%	−5.8%
8		74.00		87.10	871.00	10.0%	43.55	5.0%	15.0%	1,668	8	0.8%	−1.6%	−4.1%
9		72.00		83.40	834.00	10.0%	41.70	5.0%	15.0%	1,596	9	2.2%	−0.3%	−2.9%
10		70.00		79.80	798.00	10.0%	39.90	5.0%	15.0%	1,526	10	3.3%	0.7%	−2.0%
11		52.00		76.30	763.00	10.0%	38.15	5.0%	15.0%	1,474	11	4.2%	1.6%	−1.2%
12		48.00		73.70	737.00	10.0%	36.85	5.0%	15.0%	1,426	12	4.9%	2.2%	−0.6%
13		44.00		71.30	713.00	10.0%	35.65	5.0%	15.0%	1,382	13	5.6%	2.8%	−0.1%
14		42.00		69.10	691.00	10.0%	34.55	5.0%	15.0%	1,340	14	6.1%	3.3%	0.3%
15		36.00		67.00	670.00	10.0%	33.50	5.0%	15.0%	1,304	15	6.5%	3.7%	0.6%
16		34.00		65.20	652.00	10.0%	32.60	5.0%	15.0%	1,270	16	7.0%	4.0%	0.9%
17		30.00		63.50	635.00	10.0%	31.75	5.0%	15.0%	1,240	17	7.3%	4.4%	1.2%
18		28.00		62.00	620.00	10.0%	31.00	5.0%	15.0%	1,212	18	7.6%	4.6%	1.4%
19		524.00		60.60	606.00	10.0%	30.30	5.0%	15.0%	688	19	7.9%	4.9%	1.6%
20		688.00		34.40	344.00	10.0%	17.20	5.0%	15.0%	0	20	8.1%	5.0%	1.6%
											21	8.1%	5.0%	1.6%
Total	961.38	2,000.00		485.65	15,762.00	3.1%	788.10	5.0%	8.1%		etc.	8.1%	5.0%	1.6%
PV	961.38	961.38	$0.00	0.00	10,386.00	0.0%	519.31	5.0%	5.0%					

EXHIBIT 4B

ZERO OPERATING GAIN FROM THE INSURANCE CARRIER PERSPECTIVE

				Funds Released as they Become Available			
End of Year	Invested Surplus	Invested Premium	Paid Loss	Required Funds	Invested Funds	Released to Surplus	Flows for IRR
0	$1,000.00	$961.38	$0.00	$1,961.38	$1,961.38	$0.00	$ – 1,000.00
1			0.00	2,009.45	2,059.45	50.00	50.00
2			0.00	2,059.92	2,109.92	50.00	50.00
3			6.00	2,103.92	2,156.92	53.00	53.00
4			28.00	2,117.26	2,181.12	63.86	63.86
5			86.00	2,044.98	2,137.12	92.14	92.14
6			64.00	2,004.23	2,083.23	79.00	79.00
7			74.00	1,948.04	2,030.44	82.40	82.40
8			74.00	1,890.89	1,971.44	80.55	80.55
9			72.00	1,835.73	1,913.43	77.70	77.70
10			70.00	1,782.62	1,857.52	74.90	74.90
11			52.00	1,755.60	1,819.75	64.15	64.15
12			48.00	1,734.53	1,795.38	60.85	60.85
13			44.00	1,719.61	1,777.26	57.65	57.65
14			42.00	1,708.04	1,763.59	55.55	55.55
15			36.00	1,705.94	1,757.44	51.50	51.50
16			34.00	1,707.64	1,757.24	49.60	49.60
17			30.00	1,716.27	1,763.02	46.75	46.75
18			28.00	1,729.08	1,774.08	45.00	45.00
19			524.00	999.24	1,291.53	292.29	292.29
20			688.00	0.00	361.20	361.20	361.20
							IRR = 5.0%

EXHIBIT 5A

STATUTORY ACCOUNTING MODEL—NEGATIVE OPERATING GAIN (I.E., A LOSS)

	INSURANCE PRODUCT				SURPLUS ACCOUNT							ROE Under Statutory Accounting With a Growth Rate equal to		
End of Year	Written Premium	Paid Loss	Operating Gain	$O(n) \Rightarrow$ Funds Released	Supporting Surplus S	Funds Released to Surplus	Idle Surplus Investment Income	Idle Surplus Investment Income	Total Return on Supporting Surplus	Insurance Investment Balance	Year	0.0%	5.0%	10.0%
0	$600.00	$0.00								$600				
1		0.00		$ − 1,370.00	$1,000.00	−137.0%	$50.00	5.0%	−132.0%	2,000	1	−132.0%	−132.0%	−132.0%
2		0.00		100.00	1,000.00	10.0%	50.00	5.0%	15.0%	2,000	2	−58.5%	−60.3%	−62.0%
3		6.00		100.00	1,000.00	10.0%	50.00	5.0%	15.0%	1,994	3	−34.0%	−36.4%	−38.7%
4		28.00		99.70	997.00	10.0%	49.85	5.0%	15.0%	1,966	4	−21.8%	−24.5%	−27.2%
5		86.00		98.30	983.00	10.0%	49.15	5.0%	15.0%	1,880	5	−14.5%	−17.5%	−20.4%
6		64.00		94.00	940.00	10.0%	47.00	5.0%	15.0%	1,816	6	−9.8%	−12.9%	−16.0%
7		74.00		90.80	908.00	10.0%	45.40	5.0%	15.0%	1,742	7	−6.5%	−9.7%	−13.0%
8		74.00		87.10	871.00	10.0%	43.55	5.0%	15.0%	1,668	8	−4.1%	−7.4%	−10.8%
9		72.00		83.40	834.00	10.0%	41.70	5.0%	15.0%	1,596	9	−2.2%	−5.6%	−9.1%
10		70.00		79.80	798.00	10.0%	39.90	5.0%	15.0%	1,526	10	−0.8%	−4.2%	−7.9%
11		52.00		76.30	763.00	10.0%	38.15	5.0%	15.0%	1,474	11	0.4%	−3.1%	−6.9%
12		48.00		73.70	737.00	10.0%	36.85	5.0%	15.0%	1,426	12	1.4%	−2.2%	−6.1%
13		44.00		71.30	713.00	10.0%	35.65	5.0%	15.0%	1,382	13	2.3%	−1.4%	−5.4%
14		42.00		69.10	691.00	10.0%	34.55	5.0%	15.0%	1,340	14	3.0%	−0.8%	−4.8%
15		36.00		67.00	670.00	10.0%	33.50	5.0%	15.0%	1,304	15	3.6%	−0.2%	−4.4%
16		34.00		65.20	652.00	10.0%	32.60	5.0%	15.0%	1,270	16	4.2%	0.2%	−4.0%
17		30.00		63.50	635.00	10.0%	31.75	5.0%	15.0%	1,240	17	4.6%	0.7%	−3.7%
18		28.00		62.00	620.00	10.0%	31.00	5.0%	15.0%	1,212	18	5.1%	1.0%	−3.4%
19		524.00		60.60	606.00	10.0%	30.30	5.0%	15.0%	688	19	5.5%	1.4%	−3.1%
20		688.00		34.40	344.00	10.0%	17.20	5.0%	15.0%	0	20	5.7%	1.5%	−3.0%
											21	5.7%	1.5%	−3.0%
Total	600.00	2,000.00		106.20	15,762.00	0.0	788.10	5.0%	5.7%		etc.	5.7%	1.5%	−3.0%
PV	600.00	961.38	$ − 361.38	− 361.38	10,386.19	−3.5%	519.31	5.0%	1.5%					

EXHIBIT 5B

NEGATIVE OPERATING GAIN FROM THE INSURANCE CARRIER PERSPECTIVE

				Funds Released as they Become Available			
End of Year	Invested Surplus	Invested Premium	Paid Loss	Required Funds	Invested Funds	Released to Surplus	Flows for IRR
0	$1,361.38	$600.00	$0.00	$1,961.38	$1,961.38	$0.00	$ – 1,361.38
1			0.00	2,009.45	2,059.45	50.00	50.00
2			0.00	2,059.92	2,109.92	50.00	50.00
3			6.00	2,103.92	2,156.92	53.00	53.00
4			28.00	2,117.26	2,181.12	63.86	63.86
5			86.00	2,044.98	2,137.12	92.14	92.14
6			64.00	2,004.23	2,083.23	79.00	79.00
7			74.00	1,948.04	2,030.44	82.40	82.40
8			74.00	1,890.89	1,971.44	80.55	80.55
9			72.00	1,835.73	1,913.43	77.70	77.70
10			70.00	1,782.62	1,857.52	74.90	74.90
11			52.00	1,755.60	1,819.75	64.15	64.15
12			48.00	1,734.53	1,795.38	60.85	60.85
13			44.00	1,719.61	1,777.26	57.65	57.65
14			42.00	1,708.04	1,763.59	55.55	55.55
15			36.00	1,705.94	1,757.44	51.50	51.50
16			34.00	1,707.64	1,757.24	49.60	49.60
17			30.00	1,716.27	1,763.02	46.75	46.75
18			28.00	1,729.08	1,774.08	45.00	45.00
19			524.00	999.24	1,291.53	292.29	292.29
20			688.00	0.00	361.20	361.20	361.20
							IRR = 2.2%

EXHIBIT 6A

STATUTORY ACCOUNTING MODEL—POSITIVE OPERATING GAIN

	INSURANCE PRODUCT			SURPLUS ACCOUNT								ROE Under Statutory Accounting With a Growth Rate equal to		
End of Year	Written Premium	Paid Loss	Operating Gain	$O(n) \Rightarrow$ Funds Released	Supporting Surplus S	Funds Released to Surplus	Idle Surplus Investment Income	Idle Surplus Investment Income	Total Return on Supporting Surplus	Insurance Investment Balance	Year	0.0%	5.0%	10.0%
0	$1,700.00	$0.00								$1,700				
1		0.00		$ – 215.00	$1,000.00	–21.5%	$50.00	5.0%	–16.5%	2,000	1	–16.5%	–16.5%	–16.5%
2		0.00		100.00	1,000.00	10.0%	50.00	5.0%	15.0%	2,000	2	–0.8%	–1.1%	–1.5%
3		6.00		100.00	1,000.00	10.0%	50.00	5.0%	15.0%	1,994	3	4.5%	4.0%	3.5%
4		28.00		99.70	997.00	10.0%	49.85	5.0%	15.0%	1,966	4	7.1%	6.5%	6.0%
5		86.00		98.30	983.00	10.0%	49.15	5.0%	15.0%	1,880	5	8.7%	8.0%	7.4%
6		64.00		94.00	940.00	10.0%	47.00	5.0%	15.0%	1,816	6	9.7%	9.0%	8.4%
7		74.00		90.80	908.00	10.0%	45.40	5.0%	15.0%	1,742	7	10.4%	9.7%	9.0%
8		74.00		87.10	871.00	10.0%	43.55	5.0%	15.0%	1,668	8	10.9%	10.2%	9.5%
9		72.00		83.40	834.00	10.0%	41.70	5.0%	15.0%	1,596	9	11.3%	10.6%	9.8%
10		70.00		79.80	798.00	10.0%	39.90	5.0%	15.0%	1,526	10	11.6%	10.9%	10.1%
11		52.00		76.30	763.00	10.0%	38.15	5.0%	15.0%	1,474	11	11.9%	11.1%	10.3%
12		48.00		73.70	737.00	10.0%	36.85	5.0%	15.0%	1,426	12	12.1%	11.3%	10.5%
13		44.00		71.30	713.00	10.0%	35.65	5.0%	15.0%	1,382	13	12.3%	11.5%	10.6%
14		42.00		69.10	691.00	10.0%	34.55	5.0%	15.0%	1,340	14	12.4%	11.6%	10.7%
15		36.00		67.00	670.00	10.0%	33.50	5.0%	15.0%	1,304	15	12.6%	11.7%	10.8%
16		34.00		65.20	652.00	10.0%	32.60	5.0%	15.0%	1,270	16	12.7%	11.8%	10.9%
17		30.00		63.50	635.00	10.0%	31.75	5.0%	15.0%	1,240	17	12.8%	11.9%	11.0%
18		28.00		62.00	620.00	10.0%	31.00	5.0%	15.0%	1,212	18	12.9%	12.0%	11.1%
19		524.00		60.60	606.00	10.0%	30.30	5.0%	15.0%	688	19	13.0%	12.1%	11.1%
20		688.00		34.40	344.00	10.0%	17.20	5.0%	15.0%	0	20	13.0%	12.1%	11.1%
											21	13.0%	12.1%	11.1%
Total	1,700.00	2,000.00		1,261.20	15,762.00	8.0%	788.10	5.0%	13.0%		etc.	13.0%	12.1%	11.1%
PV	1,700.00	961.38	$738.62	738.62	10,386.19	7.1%	519.31	5.0%	12.1%					

EXHIBIT 6B

POSITIVE OPERATING GAIN FROM THE INSURANCE CARRIER PERSPECTIVE

				Funds Released as they Become Available			
End of Year	Invested Surplus	Invested Premium	Paid Loss	Required Funds	Invested Funds	Released to Surplus	Flows for IRR
0	$261.38	$1,700.00	$0.00	$1,961.38	$1,961.38	$0.00	$ – 261.38
1			0.00	2,009.45	2,059.45	50.00	50.00
2			0.00	2,059.92	2,109.92	50.00	50.00
3			6.00	2,103.92	2,156.92	53.00	53.00
4			28.00	2,117.26	2,181.12	63.86	63.86
5			86.00	2,044.98	2,137.12	92.14	92.14
6			64.00	2,004.23	2,083.23	79.00	79.00
7			74.00	1,948.04	2,030.44	82.40	82.40
8			74.00	1,890.89	1,971.44	80.55	80.55
9			72.00	1,835.73	1,913.43	77.70	77.70
10			70.00	1,782.62	1,857.52	74.90	74.90
11			52.00	1,755.60	1,819.75	64.15	64.15
12			48.00	1,734.53	1,795.38	60.85	60.85
13			44.00	1,719.61	1,777.26	57.65	57.65
14			42.00	1,708.04	1,763.59	55.55	55.55
15			36.00	1,705.94	1,757.44	51.50	51.50
16			34.00	1,707.64	1,757.24	49.60	49.60
17			30.00	1,716.27	1,763.02	46.75	46.75
18			28.00	1,729.08	1,774.08	45.00	45.00
19			524.00	999.24	1,291.53	292.29	292.29
20			688.00	0.00	361.20	361.20	361.20
							IRR = 24.4%

EXHIBIT 7

THE MODIFIED BINGHAM MODEL

	INSURANCE PRODUCT				SURPLUS ACCOUNT					IRR	INSURANCE PRODUCT		
End of Year	Written Premium	Paid Loss	Operating Gain	$O(n) \Rightarrow$ Funds Released	Supporting Surplus S	Funds Released to Surplus	Idle Surplus Investment Income	Idle Surplus Investment Income	Total Return on Supporting Surplus	Total Surplus Flows	Retained Earnings	Investment Balance	Overfund
0	$400.00	$0.00								$-187.93	$-4.00	$400.00	$24.14
1		264.00		$12.08	$187.93	6.43%	$9.40	5.0%	11.43%	98.44	3.92	143.92	13.26
2		96.00		7.14	110.97	6.43%	5.55	5.0%	11.43%	61.25	3.98	47.98	6.79
3		32.00		4.01	62.40	6.43%	3.12	5.0%	11.43%	36.90	2.36	14.36	3.11
4		8.00		2.10	32.63	6.43%	1.63	5.0%	11.43%	17.22	0.98	4.98	1.17
5		4.00		1.23	19.14	6.43%	0.96	5.0%	11.43%	21.33	0.00	0.00	0.00
NPV	400.00	375.86	$24.14	24.14	375.38	6.43%	18.77	5.0%	11.43%				
								Average =	11.43%	IRR = 11.43%			

Determination of the $\{O(n)\}$

1. Constant annual ROE $\Rightarrow O(n)/S(n) = k$, or $O(n) = k * S(n)$
2. $\text{NPV}(\{O(n)\}) = k * \text{NPV}(\{S(n)\}) \Rightarrow k = \text{NPV}(\{O(n)\})/\text{NPV}(\{S(n)\})$

n	$S(n)$		$O(n)$
1	$S(1) =$	$187.93	$12.08
2	$S(2) =$	110.97	7.14
3	$S(3) =$	62.40	4.01
4	$S(4) =$	32.63	2.10
5	$S(5) =$	19.14	1.23
	$\text{NPV}(\{S(n)\}) =$	$375.38	
	$\text{NPV}(\{O(n)\}) =$	$ 24.14	$24.14
	$k =$	0.064303	

EXHIBIT 8A

STATUTORY ACCOUNTING MODEL WITH VARIABLE R : S LEVERAGE RATIO

	INSURANCE PRODUCT				SURPLUS ACCOUNT							ROE Under Statutory Accounting With a Growth Rate equal to		
End of Year	Written Premium	Paid Loss	Operating Gain	$O(n) \Rightarrow$ Funds Released	Supporting Surplus S	Funds Released to Surplus	Idle Surplus Investment Income	Idle Surplus Investment Income	Total Return on Supporting Surplus	Insurance Investment Balance	Year	0.0%	5.0%	10.0%
0	$1,000.00	$0.00								$1,000				
1		0.00		$ − 950.00	$480.69	−2.0%	$24.03	0.0%	−1.9%	2,000	1	−1.9%	−1.9%	−1.9%
2		0.00		100.00	504.73	0.2%	25.24	0.1%	0.2%	2,000	2	−0.8%	−0.8%	−0.9%
3		6.00		100.00	529.96	0.2%	26.50	0.1%	0.2%	1,994	3	−0.4%	−0.5%	−0.5%
4		28.00		99.70	553.46	0.2%	27.67	0.0%	0.2%	1,966	4	−0.3%	−0.3%	−0.3%
5		86.00		98.30	567.13	0.2%	28.36	0.1%	0.2%	1,880	5	−0.2%	−0.2%	−0.2%
6		64.00		94.00	552.49	0.2%	27.62	0.0%	0.2%	1,816	6	−0.1%	−0.1%	−0.2%
7		74.00		90.80	548.12	0.2%	27.41	0.1%	0.2%	1,742	7	−0.0%	−0.1%	−0.1%
8		74.00		87.10	538.52	0.2%	26.93	0.1%	0.2%	1,668	8	−0.0%	−0.1%	−0.1%
9		72.00		83.40	528.45	0.2%	26.42	0.0%	0.2%	1,596	9	0.0%	−0.0%	−0.1%
10		70.00		79.80	518.87	0.2%	25.94	0.0%	0.2%	1,526	10	0.0%	−0.0%	−0.1%
11		52.00		76.30	509.81	0.1%	25.49	0.0%	0.2%	1,474	11	0.0%	−0.0%	−0.1%
12		48.00		73.70	509.30	0.1%	25.47	0.1%	0.2%	1,426	12	0.1%	0.0%	−0.0%
13		44.00		71.30	510.77	0.1%	25.54	0.1%	0.2%	1,382	13	0.1%	0.0%	−0.0%
14		42.00		69.10	514.31	0.1%	25.72	0.1%	0.2%	1,340	14	0.1%	0.0%	−0.0%
15		36.00		67.00	519.02	0.1%	25.95	0.0%	0.2%	1,304	15	0.1%	0.0%	−0.0%
16		34.00		65.20	526.97	0.1%	26.35	0.1%	0.2%	1,270	16	0.1%	0.0%	−0.0%
17		30.00		63.50	536.32	0.1%	26.82	0.1%	0.2%	1,240	17	0.1%	0.0%	−0.0%
18		28.00		62.00	548.14	0.1%	27.41	0.1%	0.2%	1,212	18	0.1%	0.1%	−0.0%
19		524.00		60.60	561.54	0.1%	28.08	0.1%	0.2%	688	19	0.1%	0.1%	0.0%
20		688.00		34.40	327.62	0.1%	16.38	0.0%	0.2%	0	20	0.1%	0.1%	0.0%
											21	0.1%	0.1%	0.0%
Total	1,000.00	2,000.00		526.20	10,386.22	0.1%	519.31	0.0%	0.1%		etc.	0.1%	0.1%	0.0%
PV	1,000.00	961.38	$38.62	38.62	6,509.42	0.1%	325.47	0.0%	0.1%					

EXHIBIT 8B

INSURANCE CARRIER PERSPECTIVE WITH A VARIABLE R : S LEVERAGE RATIO

			Funds Released as they Become Available				
End of Year	Invested Surplus	Invested Premium	Paid Loss	Required Funds	Invested Funds	Released to Surplus	Flows for IRR
0	$442.07	$1,000.00	$0.00	$1,442.07	$1,442.07	$0.00	$ – 442.07
1			0.00	1,514.18	1,514.17	–0.01	– 0.01
2			0.00	1,589.88	1,589.89	0.01	0.01
3			6.00	1,660.38	1,663.37	2.99	2.99
4			28.00	1,701.39	1,715.40	14.01	14.01
5			86.00	1,657.47	1,700.46	42.99	42.99
6			64.00	1,644.35	1,676.34	31.99	31.99
7			74.00	1,615.56	1,652.57	37.01	37.01
8			74.00	1,585.34	1,622.34	37.00	37.00
9			72.00	1,556.60	1,592.61	36.01	36.01
10			70.00	1,529.43	1,564.43	35.00	35.00
11			52.00	1,527.90	1,553.90	26.00	26.00
12			48.00	1,532.30	1,556.30	24.00	24.00
13			44.00	1,542.92	1,564.92	22.00	22.00
14			42.00	1,557.06	1,578.07	21.01	21.01
15			36.00	1,580.91	1,598.91	18.00	18.00
16			34.00	1,608.96	1,625.96	17.00	17.00
17			30.00	1,644.41	1,659.41	15.00	15.00
18			28.00	1,684.62	1,698.63	14.01	14.01
19			524.00	982.86	1,244.85	261.99	261.99
20			688.00	0.00	344.00	344.00	344.00
							IRR = 0.1%

APPENDIX A

PROOF OF THE CONSISTENCY OF THE THREE RATE OF RETURN MEASURES

In Bingham's paper and in this discussion, it is demonstrated that releasing both surplus and operating gain as uncertainty regarding the outstanding loss amounts is resolved results in agreement among the three measures of return on equity. This appendix presents a rigorous proof that when this release scheme is adopted, the internal rate of return, annual return on surplus, and the invariant ratio are equal.

Begin the proof with the following variable designations. Let:

$S(j)$ be the supporting surplus at year-end j,
$I(j)$ be the investment income earned on the supporting surplus during the jth year,
$O(j)$ be the operating return that is released at year-end j,
i be the annual effective investment income rate on invested assets, and
$v_i = 1/(1+i)$, the discounting factor at interest rate i.

With these definitions, the investment income earned by the supporting surplus during the jth year can be expressed as

$$I(j) = i * S(j-1), \qquad \text{(A.1)}$$

where it is assumed that the supporting surplus remains unchanged during the course of a year. This is consistent with Bingham's assumption that losses are paid at year-end.

Bingham's release scheme dictates that accrued operating earnings be released as uncertainty is resolved (i.e., as losses are paid). More specifically, the ratio of the released accrued operating gain at any year-end j to the supporting surplus during the jth year must be a constant independent of the particular year. Symbolically,

$$O(j)/S(j-1) = k, \quad \text{a constant } \forall j, \quad 1 \le j \le \omega, \qquad \text{(A.2)}$$

where ω denotes the end of the last year during which there are open (or IBNR) claims. Note that a second requirement placed upon the set of released operating gains is that the present value of the $\{O(j)\}$ equals the present value of the operating gain. While this requirement insures releasing an amount exactly equal to the accrued operating gain, it is not a necessary condition for agreement of the three measures of ROE.

Upon solving Equation A.2 for $O(j)$, we obtain the released operating earnings at the end of the jth year in terms of the supporting surplus that was allocated at the end of the previous year,

$$O(j) = k * S(j - 1). \tag{A.3}$$

During the jth year, the supporting surplus is $S(j - 1)$ and the return on that surplus is the investment income on that surplus, $I(j)$, plus the released operating gain, $O(j)$, or

$$I(j) + O(j) = (i + k) * S(j - 1). \tag{A.4}$$

Dividing by the invested surplus, $S(j - 1)$, gives the *average return on surplus during the jth year*, $(i + k)$. This expression is independent of j, making it a constant for all years.

Taking the present value of the total return on supporting surplus using *any* interest rate gives

$$\text{NPV}[I(j) + O(j)] = (i + k) * \text{NPV}[S(j - 1)]. \tag{A.5}$$

Divide the present value of the total return by the present value of the year-end supporting surplus, to obtain Bingham's present value ratio,

$$\text{NPV}[I(j) + O(j)]/\text{NPV}[S(j - 1)] = (i + k). \tag{A.6}$$

Note that the present value ratio is equal to the average return on surplus.

To show that the IRR of the surplus flows is also equal to the present value ratio and average return on surplus, begin by observing that, for a set of cash flows $\{CF_j\}$, the internal rate of return is defined as the interest rate that satisfies the equation

$$\mathrm{NPV}[\{CF_j\}] = \sum_{j=0}^{\omega} v^j * CF_j = 0. \tag{A.7}$$

In addition to the total return on the supporting surplus during the jth year ($j \geq 1$), the surplus flow also includes the return of supporting surplus as it is released,

$$\Delta S(j) = S(j-1) - S(j). \tag{A.8}$$

$S(\omega)$, the supporting surplus at the *end* of the last year in which there are any carried reserves, is zero.

Combining Equations A.4 and A.8 and remembering that at the end of year zero, $S(0)$ is transferred *out of* the surplus account,

$$CF_j = \begin{cases} -S(0), & \text{if} \quad j = 0, \\ (1+i+k)*S(j-1) - S(j), & \text{if} \quad j \neq 0. \end{cases} \tag{A.9}$$

At the internal rate of return,

$$\mathrm{NPV}_{\mathrm{IRR}}[\{CF_j\}] = -S(0) + (1+i+k)\sum_{j=1}^{\omega+1} v_{\mathrm{IRR}}^{l} * S(j-1) - \sum_{j=1}^{\omega+1} v_{\mathrm{IRR}}^{l} \cdot S(j) = 0. \tag{A.10}$$

To test the average annual return as a possible solution, substitute

$$v_{\mathrm{IRR}} = \frac{1}{(1+i+k)}, \tag{A.11}$$

then

$$\mathrm{NPV}[\{CF_j\}] = -S(0) + \sum_{j=1}^{\omega+1} v^{j-1} * S(j-1) - \sum_{j=1}^{\omega+1} v^j * S(j). \tag{A.12}$$

With a change of the dummy variable in the first sum, (A.12) becomes

$$\mathrm{NPV}[\{CF_j\}] = -S(0) + \sum_{j=0}^{\omega} v^j * S(j) - \sum_{j=1}^{\omega+1} v^j * S(j) = -S(\omega + 1), \tag{A.13}$$

but $S(\omega + 1)$ is zero because all uncertainties will have been resolved by the end of the last year, ω. Therefore,

$$\mathrm{NPV}[\{CF_j\}] = 0, \tag{A.14}$$

which proves that $(i + k)$ is an internal rate of return for the surplus flows.

It has been proven that, as a result of the release of operating gain scheme, the average total return on invested supporting surplus, the internal rate of return of the surplus flows, and the ratio of the present value of the total returns to the present value of the supporting surplus are all equal.

Nothing in this proof depends upon the specific relationship between the supporting surplus and the insurance product. In fact, $\{S(j)\}$ could be been selected at random (as long as all of the $S(j)$s are equal to zero after all of the uncertainty is resolved). None of the details leading to a determination of the insurance product operating gain, in fact not even its numerical value, enter into the proof. The conclusion that can be drawn from this is that no additional level of sophistication in the determination of the operating gain (e.g., the reflection of federal income tax, expenses, and policyholder dividends) or refinement in the selection of a reserve to surplus leverage ratio will invalidate the conclusions that have been proven in this appendix. The Bingham release scheme automatically insures the equality of the three measures of return.

APPENDIX B

EVIDENCE FOR A DECREASING LEVERAGE RATIO

Workers compensation is often cited as a line of business in which the uncertainty in the outstanding loss reserve decreases rapidly because of the highly predictable nature of lifetime pension cases. The conventional wisdom is that once the more volatile minor cases have been resolved, all that remains are claimants with lifetime benefits. As soon as the open claims consist only of lifetime pension cases, supporting surplus can be released rapidly. In particular, as a result of a decision of the hearing officer during the workers compensation rating hearing for rates to become effective January 1, 1988 in Massachusetts, the leverage ratio *increases* uniformly from the end of the fifth quarter until all claims are closed [3]. The pension case argument has been used to support the accelerated release of surplus.

Workers compensation claims probably arise from several underlying distributions. Clearly, minor cuts and bruises cannot be described by the same severity distribution that would apply to more serious injuries of the type that can lead to long term disability. As groups of claims close, the remaining open claims may be of a more homogeneous nature. This, in itself, may decrease the relative uncertainty in the open claim reserves. Initially, at least, as certain classes of claims close, an increase in the leverage ratio may be possible.

Once the population of open claims consists of nothing but lifetime pension cases, the long term behavior of the leverage ratio manifests itself. For a reasonable example, it can be demonstrated that the leverage ratio must decrease over time. This is not a rigorous proof that the leverage ratio for workers compensation coverage *must always* decrease but, rather, it is evidence that one cannot assume that once pension cases dominate the open claim reserves, the leverage ratio *will always* increase. This appendix serves as a counter example, disproving the common contention.

It is an indication that even in the case of workers compensation runoff, additional research is necessary.

For the example, assume that there are exactly 100 open claims at a given point in time. To simplify matters, assume that each of these claims involves a 40 year old claimant who is receiving a $5,000 annual amount paid in weekly installments. There are no cost of living adjustments. Benefits terminate upon death of the claimant.

Further assume that the 1979–1981 U.S. Decennial Life Mortality table for the Total Population (that adopted by the National Council on Compensation Insurance for Unit Statistical Plan reporting) reflects the life expectancies of these claimants. The aggregate nominal outstanding loss reserve for these claimants would be $18,392,500 (100 claimants * $5,000 per year per claimant * 36.785 years per claimant on the average). Exhibit B-1 displays a section of the mortality table and the life expectancy calculation. The $18,392,500 reserve is only a point estimate. The actual amount paid to these claimants could be significantly more or less than this amount.

The mortality table shows a 0.014 probability that a claimant could die within five years rather than living the expected 36.785 years. Likewise, there is approximately a .02 probability that the claimant could live 58 more years rather than the expected number of years. If each claim were reserved to a 98% confidence level, the leverage ratio would be approximately 1.73 : 1.00; for every $1.00 of reserves, $0.58 of surplus would have to be allocated (i.e., of the 58 years that must be provided for, 36.785 would be provided for in the form of loss reserves with the remaining 21.215 coming from supporting surplus). Alternatively, a dollar of surplus can support $1.73 of reserves.

Of course, 100 times the individual claim supporting surplus is not necessary to maintain a 98% confidence level in the aggregate. The single claimant loss distribution was input into an aggregate loss model, such as the one described by Heckman and Meyers [1], to determine that the appropriate reserve-to-surplus ratio for a 2% probability of ruin would be 14.015 : 1.000 (i.e., the 98th percentile occurs at 1.07135 times the expected mean, so $0.07135 of surplus is required to support every $1.00 of reserves). Exhibit B-2 displays the cumulative probability corresponding to various aggregate loss amounts where the entry ratio is the ratio of the selected aggregate loss to the mean aggregate loss.

Ten years later, if everything has gone as was expected, there will be 96 open claims (consisting of lifetime pension cases for 50 year olds). At that time, there will be approximately a 2% probability of living at least 48 more years (almost no difference between the probability of a 40 year old living to 98, 0.0230, and the probability of a 50 year old living to 98, 0.0239). With a 27.939 year life expectancy, the individual claim leverage ratio for the 50 year old claimants is 1.39 : 1.00, which represents a decrease from 1.73. The 96 claim aggregate leverage ratio is 11.521 : 1.000, also a decrease from the 14.015 leverage ratio. Exhibit B-2 displays the aggregate loss distribution for 96.418 claims (96 being the result of rounding to whole numbers for the sake of the narrative).

By the time the claimants are 60 years of age, the individual claimant leverage ratio will have fallen to 1.11 : 1.00 (with a 20.019 year life expectancy and approximately a 2% probability of living to 98 years of age or longer, almost equal amounts of surplus and reserves are required). Of the original 100 claimants, 88 (88.2 claims were used in the aggregate loss model) are expected to reach age 60. The aggregate leverage ratio for these 88 living 60 year old claimants would be 9.25 : 1.00.

Unless there is another group of claims that both increases the variability of the open claim reserves in total and closes rapidly enough to more than offset the increasing variability of the pension claims, the leverage ratio for open workers compensation claims must *decrease* in the long run. The preceding example does not constitute a proof that the leverage ratio decreases; rather, it makes the conventional wisdom less obvious. The appropriate leverage ratio for any line of business must be the result of an investigation of the underlying volatility of its open claims at any point in time.

EXHIBIT B-1

PART 1

79/81 U.S. DECENNIAL LIFE MORTALITY TABLE

		$x = 40$				$x = 50$				$x = 60$			
Age, x	$l(x)$	n	$p(n)$	$np(n)$	Sum[$p(n)$]	n	$p(n)$	$np(n)$	Sum[$p(n)$]	n	$p(n)$	$np(n)$	Sum[$p(n)$]
38	95,317												
39	95,129												
40	94,926	0.5	0.00232	0.001	0.002								
41	94,706	1.5	0.00254	0.004	0.005								
42	94,465	2.5	0.00278	0.007	0.008								
43	94,201	3.5	0.00303	0.011	0.011								
44	93,913	4.5	0.00331	0.015	0.014								
45	93,599	5.5	0.00361	0.020	0.018								
46	93,256	6.5	0.00394	0.026	0.022								
47	92,882	7.5	0.00432	0.032	0.026								
48	92,472	8.5	0.00475	0.040	0.031								
49	92,021	9.5	0.00521	0.050	0.036								
50	91,526	10.5	0.00569	0.060	0.042	0.5	0.00590	0.003	0.006				
51	90,986	11.5	0.00615	0.071	0.048	1.5	0.00638	0.010	0.012				
52	90,402	12.5	0.00665	0.083	0.054	2.5	0.00689	0.017	0.019				
53	89,771	13.5	0.00721	0.097	0.062	3.5	0.00747	0.026	0.027				
54	89,087	14.5	0.00779	0.113	0.069	4.5	0.00807	0.036	0.035				
55	88,348	15.5	0.00840	0.130	0.078	5.5	0.00871	0.048	0.043				
56	87,551	16.5	0.00902	0.149	0.087	6.5	0.00935	0.061	0.053				
57	86,695	17.5	0.00968	0.169	0.096	7.5	0.01004	0.075	0.063				
58	85,776	18.5	0.01040	0.192	0.107	8.5	0.01078	0.092	0.074				
59	84,789	19.5	0.01120	0.218	0.118	9.5	0.01161	0.110	0.085				
60	83,726	20.5	0.01206	0.247	0.130	10.5	0.01251	0.131	0.098	0.5	0.01368	0.007	0.014
61	82,581	21.5	0.01299	0.279	0.143	11.5	0.01347	0.155	0.111	1.5	0.01473	0.022	0.028

EXHIBIT B-1

PART 2

79/81 U.S. DECENNIAL LIFE MORTALITY TABLE

		x = 40				x = 50				x = 60			
Age, x	l(x)	n	p(n)	np(n)	Sum[p(n)]	n	p(n)	np(n)	Sum[p(n)]	n	p(n)	np(n)	Sum[p(n)]
62	81,348	22.5	0.01395	0.314	0.157	12.5	0.01447	0.181	0.126	2.5	0.01581	0.040	0.044
63	80,024	23.5	0.01491	0.350	0.172	13.5	0.01546	0.209	0.141	3.5	0.01690	0.059	0.061
64	78,609	24.5	0.01582	0.388	0.188	14.5	0.01641	0.238	0.158	4.5	0.01794	0.081	0.079
65	77,107	25.5	0.01672	0.426	0.204	15.5	0.01734	0.269	0.175	5.5	0.01895	0.104	0.098
66	75,520	26.5	0.01763	0.467	0.222	16.5	0.01829	0.302	0.193	6.5	0.01999	0.130	0.118
67	73,846	27.5	0.01858	0.511	0.241	17.5	0.01927	0.337	0.212	7.5	0.02107	0.158	0.139
68	72,082	28.5	0.01964	0.560	0.260	18.5	0.02037	0.377	0.233	8.5	0.02226	0.189	0.161
69	70,218	29.5	0.02075	0.612	0.281	19.5	0.02152	0.420	0.254	9.5	0.02353	0.224	0.185
70	68,248	30.5	0.02194	0.669	0.303	20.5	0.02276	0.467	0.277	10.5	0.02488	0.261	0.210
71	66,165	31.5	0.02310	0.728	0.326	21.5	0.02396	0.515	0.301	11.5	0.02619	0.301	0.236
72	63,972	32.5	0.02422	0.787	0.350	22.5	0.02512	0.565	0.326	12.5	0.02746	0.343	0.263
73	61,673	33.5	0.02522	0.845	0.376	23.5	0.02616	0.615	0.352	13.5	0.02859	0.386	0.292
74	59,279	34.5	0.02613	0.901	0.402	24.5	0.02710	0.664	0.379	14.5	0.02962	0.429	0.322
75	56,799	35.5	0.02697	0.957	0.429	25.5	0.02797	0.713	0.407	15.5	0.03058	0.474	0.352
76	54,239	36.5	0.02781	1.015	0.456	26.5	0.02884	0.764	0.436	16.5	0.03153	0.520	0.384
77	51,599	37.5	0.02866	1.075	0.485	27.5	0.02973	0.818	0.466	17.5	0.03250	0.569	0.416
78	48,878	38.5	0.02957	1.138	0.515	28.5	0.03067	0.874	0.497	18.5	0.03353	0.620	0.450
79	46,071	39.5	0.03046	1.203	0.545	29.5	0.03159	0.932	0.528	19.5	0.03453	0.673	0.484
80	43,180	40.5	0.03131	1.268	0.576	30.5	0.03247	0.990	0.561	20.5	0.03550	0.728	0.520
81	40,208	41.5	0.03198	1.327	0.608	31.5	0.03317	1.045	0.594	21.5	0.03626	0.780	0.556
82	37,172	42.5	0.03241	1.378	0.641	32.5	0.03362	1.093	0.627	22.5	0.03675	0.827	0.593
83	34,095	43.5	0.03248	1.413	0.673	33.5	0.03368	1.128	0.661	23.5	0.03682	0.865	0.630
84	31,012	44.5	0.03215	1.431	0.705	34.5	0.03335	1.150	0.695	24.5	0.03645	0.893	0.666
85	27,960	45.5	0.03159	1.437	0.737	35.5	0.03277	1.163	0.727	25.5	0.03582	0.913	0.702
86	24,961	46.5	0.03079	1.432	0.768	36.5	0.03194	1.166	0.759	26.5	0.03491	0.925	0.737
87	22,038	47.5	0.02953	1.403	0.797	37.5	0.03063	1.148	0.790	27.5	0.03348	0.921	0.770
88	19,235	48.5	0.02778	1.347	0.825	38.5	0.02881	1.109	0.819	28.5	0.03150	0.898	0.802
89	16,598	49.5	0.02575	1.274	0.851	39.5	0.02670	1.055	0.845	29.5	0.02919	0.861	0.831

EXHIBIT B-1

PART 3

79/81 U.S. DECENNIAL LIFE MORTALITY TABLE

		x = 40				x = 50				x = 60			
Age, x	l(x)	n	p(n)	np(n)	Sum[p(n)]	n	p(n)	np(n)	Sum[p(n)]	n	p(n)	np(n)	Sum[p(n)]
90	14,154	50.5	0.02366	1.195	0.875	40.5	0.02454	0.994	0.870	30.5	0.02683	0.818	0.858
91	11,908	51.5	0.02154	1.109	0.896	41.5	0.02234	0.927	0.892	31.5	0.02442	0.769	0.882
92	9,863	52.5	0.01929	1.013	0.915	42.5	0.02001	0.850	0.912	32.5	0.02187	0.711	0.904
93	8,032	53.5	0.01694	0.906	0.932	43.5	0.01757	0.764	0.930	33.5	0.01921	0.643	0.923
94	6,424	54.5	0.01455	0.793	0.947	44.5	0.01509	0.671	0.945	34.5	0.01649	0.569	0.940
95	5,043	55.5	0.01221	0.678	0.959	45.5	0.01266	0.576	0.958	35.5	0.01384	0.491	0.954
96	3,884	56.5	0.00996	0.562	0.969	46.5	0.01032	0.480	0.968	36.5	0.01129	0.412	0.965
97	2,939	57.5	0.00794	0.457	0.977	47.5	0.00824	0.391	0.976	37.5	0.00901	0.338	0.974
98	2,185	58.5	0.00618	0.362	0.983	48.5	0.00641	0.311	0.983	38.5	0.00701	0.270	0.981
99	1,598	59.5	0.00472	0.281	0.988	49.5	0.00489	0.242	0.987	39.5	0.00535	0.211	0.986
100	1,150	60.5	0.00353	0.214	0.991	50.5	0.00366	0.185	0.991	40.5	0.00400	0.162	0.990
101	815	61.5	0.00258	0.159	0.994	51.5	0.00268	0.138	0.994	41.5	0.00293	0.121	0.993
102	570	62.5	0.00186	0.117	0.996	52.5	0.00193	0.102	0.996	42.5	0.00211	0.090	0.995
103	393	63.5	0.00133	0.084	0.997	53.5	0.00138	0.074	0.997	43.5	0.00150	0.065	0.997
104	267	64.5	0.00093	0.060	0.998	54.5	0.00096	0.052	0.998	44.5	0.00105	0.047	0.998
105	179	65.5	0.00063	0.041	0.999	55.5	0.00066	0.036	0.999	45.5	0.00072	0.033	0.999
106	119	66.5	0.00043	0.029	0.999	56.5	0.00045	0.025	0.999	46.5	0.00049	0.023	0.999
107	78	67.5	0.00028	0.019	0.999	57.5	0.00029	0.017	0.999	47.5	0.00032	0.015	0.999
108	51	68.5	0.00019	0.013	1.000	58.5	0.00020	0.012	1.000	48.5	0.00021	0.010	1.000
109	33	69.5	0.00035	0.024	1.000	59.5	0.00036	0.021	1.000	49.5	0.00039	0.020	1.000
110	0	70.5	0.00000	0.000	1.000	60.5	0.00000	0.000	1.000	50.5	0.00000	0.000	1.000
		Total	1.00000	36.785			1.00000	27.939			1.00000	20.019	

EXHIBIT B-2

PART 1

AGGREGATE LOSS DISTRIBUTION

100 Forty Year Old Claimants			96 Fifty Year Old Claimants			88 Sixty Year Old Claimants		
Entry Ratio	Aggregate Loss	Cumulative Probability	Entry Ratio	Aggregate Loss	Cumulative Probability	Entry Ratio	Aggregate Loss	Cumulative Probability
1.00000	18,393,470	0.4967	1.0750	14,480,151	0.9617	1.0800	9,535,527	0.9356
1.00625	18,508,429	0.5673	1.0800	14,547,501	0.9706	1.0900	9,623,818	0.9563
1.01250	18,623,388	0.6359	1.0860	14,628,320	0.9790	1.1000	9,712,110	0.9713
1.01875	18,738,347	0.7006	1.0861	14,629,667	0.9791	1.1010	9,720,940	0.9725
1.02500	18,853,306	0.7597	1.0862	14,631,014	0.9792	1.1020	9,729,769	0.9737
1.03125	18,968,266	0.8120	1.0863	14,632,361	0.9794	1.1030	9,738,598	0.9748
1.03750	19,083,225	0.8567	1.0864	14,633,708	0.9795	1.1040	9,747,427	0.9759
1.04375	19,198,184	0.8937	1.0865	14,635,055	0.9796	1.1050	9,756,256	0.9770
1.05000	19,313,143	0.9234	1.0866	14,636,402	0.9797	1.1060	9,765,086	0.9780
1.05625	19,428,102	0.9464	1.0867	14,637,749	0.9798	1.1061	9,765,968	0.9781
1.06250	19,543,062	0.9635	1.0868	14,639,096	0.9800	1.1062	9,766,851	0.9782
1.06860	19,655,262	0.9757	1.0869	14,640,443	0.9801	1.1063	9,767,734	0.9783
1.06870	19,657,101	0.9759	1.0870	14,641,790	0.9802	1.1064	9,768,617	0.9784
1.06880	19,658,940	0.9761	1.0871	14,643,137	0.9803	1.1065	9,769,500	0.9785
1.06890	19,660,780	0.9762	1.0872	14,644,484	0.9804	1.1066	9,770,383	0.9786
1.06900	19,662,619	0.9764	1.0873	14,645,831	0.9805	1.1067	9,771,266	0.9787
1.06910	19,664,458	0.9765	1.0874	14,647,178	0.9806	1.1068	9,772,149	0.9788
1.06920	19,666,298	0.9767	1.0875	14,648,525	0.9808	1.1069	9,773,032	0.9789
1.06930	19,668,137	0.9769	1.0876	14,649,872	0.9809	1.1070	9,773,915	0.9790
1.06940	19,669,976	0.9770	1.0877	14,651,219	0.9810	1.1071	9,774,798	0.9791
1.06950	19,671,816	0.9772	1.0878	14,652,566	0.9811	1.1072	9,775,681	0.9792

EXHIBIT B-2

PART 2

AGGREGATE LOSS DISTRIBUTION

100 Forty Year Old Claimants			96 Fifty Year Old Claimants			88 Sixty Year Old Claimants		
Entry Ratio	Aggregate Loss	Cumulative Probability	Entry Ratio	Aggregate Loss	Cumulative Probability	Entry Ratio	Aggregate Loss	Cumulative Probability
1.06960	19,673,655	0.9773	1.0879	14,653,913	0.9812	1.1073	9,776,563	0.9793
1.06970	19,675,494	0.9775	1.0880	14,655,260	0.9813	1.1074	9,777,446	0.9794
1.06980	19,677,334	0.9777	1.0881	14,656,607	0.9814	1.1075	9,778,329	0.9795
1.06990	19,679,173	0.9778	1.0882	14,657,954	0.9815	1.1076	9,779,212	0.9796
1.07000	19,681,013	0.9780	1.0883	14,659,301	0.9816	1.1077	9,780,095	0.9797
1.07010	19,682,852	0.9781	1.0884	14,660,648	0.9817	1.1078	9,780,978	0.9798
1.07020	19,684,691	0.9783	1.0885	14,661,995	0.9819	1.1079	9,781,861	0.9799
1.07030	19,686,531	0.9784	1.0886	14,663,342	0.9820	1.1080	9,782,744	0.9799
1.07040	19,688,370	0.9786	1.0887	14,664,689	0.9821	1.1081	9,783,627	0.9800
1.07050	19,690,209	0.9787	1.0888	14,666,036	0.9822	1.1082	9,784,510	0.9801
1.07060	19,692,049	0.9789	1.0889	14,667,383	0.9823	1.1083	9,785,393	0.9802
1.07070	19,693,888	0.9790	1.0890	14,668,730	0.9824	1.1084	9,786,276	0.9803
1.07080	19,695,727	0.9792	1.0891	14,670,077	0.9825	1.1085	9,787,158	0.9804
1.07090	19,697,567	0.9793	1.0892	14,671,424	0.9826	1.1086	9,788,041	0.9805
1.07100	19,699,406	0.9795	1.0893	14,672,771	0.9827	1.1087	9,788,924	0.9806
1.07110	19,701,245	0.9796	1.0894	14,674,118	0.9828	1.1088	9,789,807	0.9807
1.07120	19,703,085	0.9798	1.0895	14,675,465	0.9829	1.1089	9,790,690	0.9808
1.07130	19,704,924	0.9799	1.0896	14,676,812	0.9830	1.1090	9,791,573	0.9809
1.07135	19,705,844	0.9800	1.0897	14,678,159	0.9831	1.1091	9,792,456	0.9809
1.07140	19,706,763	0.9801	1.0898	14,679,506	0.9832	1.1092	9,793,339	0.9810
1.07150	19,708,603	0.9802	1.0899	14,680,853	0.9833	1.1093	9,794,222	0.9811

EXHIBIT B-2

PART 3

AGGREGATE LOSS DISTRIBUTION

100 Forty Year Old Claimants			96 Fifty Year Old Claimants			88 Sixty Year Old Claimants		
Entry Ratio	Aggregate Loss	Cumulative Probability	Entry Ratio	Aggregate Loss	Cumulative Probability	Entry Ratio	Aggregate Loss	Cumulative Probability
1.07160	19,710,442	0.9803	1.0900	14,682,200	0.9834	1.1094	9,795,105	0.9812
1.07170	19,712,281	0.9805	1.0901	14,683,547	0.9835	1.1095	9,795,988	0.9813
1.07180	19,714,121	0.9806	1.0902	14,684,894	0.9836	1.1096	9,796,871	0.9814
1.07190	19,715,960	0.9808	1.0903	14,686,241	0.9837	1.1097	9,797,754	0.9815
1.07200	19,717,799	0.9809	1.0904	14,687,588	0.9838	1.1098	9,798,636	0.9816
1.07210	19,719,639	0.9810	1.0905	14,688,935	0.9839	1.1099	9,799,519	0.9816
1.07220	19,721,478	0.9812	1.0906	14,690,282	0.9840	1.1100	9,800,402	0.9817
1.07230	19,723,318	0.9813	1.0907	14,691,629	0.9841	1.1200	9,888,694	0.9887
1.07240	19,725,157	0.9814	1.0908	14,692,976	0.9842	1.1300	9,976,986	0.9933
1.07250	19,726,996	0.9816	1.0909	14,694,323	0.9843	1.1400	10,065,278	0.9961
1.07260	19,728,836	0.9817	1.0910	14,695,670	0.9844	1.1500	10,153,570	0.9978
1.07270	19,730,675	0.9818	1.0911	14,697,017	0.9845	1.1600	10,241,862	0.9988
1.07280	19,732,514	0.9820	1.0912	14,698,364	0.9846	1.1700	10,330,154	0.9994
1.07290	19,734,354	0.9821	1.0913	14,699,711	0.9846	1.1800	10,418,446	0.9997
1.07300	19,736,193	0.9822	1.0914	14,701,058	0.9847	1.1900	10,506,738	0.9999

ADDRESS TO NEW MEMBERS—MAY 19, 1997

CHARLES A. BRYAN

Good morning. Thank you, Bob, for inviting me to make a presentation to this class of 18 new Fellows. In 1974 I was a new Fellow and sitting where you are sitting, expectantly waiting to participate in my first CAS meeting as a full member. It is a great honor to address you here in San Antonio where I spent many happy years with USAA. I didn't realize then, that earning the Fellowship would open so many doors.

This is a very diverse class—there are 15 different organizations represented. This class of Fellows illustrates how strong our profession is. We have new members from traditional companies like SAFECO, Allstate, and State Farm; from companies rapidly expanding overseas such as AIG-Europe; from reinsurance; and from brokers. All of us are proud to welcome you to full membership.

By passing your exams and earning your Fellowship, you have joined a select few within the business world. A number of years ago, for some of you more than 10 years ago, you chose to follow the path to becoming an actuary. At that time, you saw with apprehension a formidable set of examinations and knew that only a small percentage of those people who began that path ever arrived at their destination.

Some people try several exams and never pass any. Others pass several examinations and then decide that the amounts of time and effort required are too great. Still others reach the Associateship, and after that major achievement decide not to pursue the remaining exams. But a few, a select few, persevere and pass all the required exams and become Fellows of the Casualty Actuarial Society. By obtaining the designation, you have linked yourself to a collection of over 1,500 people, all of whom are distinguished in their careers by being actuaries. You will find life-long friends among other people who share the exam experience. I can tell you from my own career that some of your most valued friends will be in the actuarial profession.

Each and every one of you has displayed two characteristics that will serve you well in the future. First, you have demonstrated self-discipline. It takes tremendous self-discipline to spend the three hundred plus hours of study time per examination, and to plow through so many complicated scientific works to gather the knowledge needed to answer the questions in the examination. Second, you have demonstrated perseverance. There are very few individuals that have passed every examination they took the first time. You have decided to persevere over the discouragement of not passing an exam every time, and over the discouragement of thinking you knew material and having to restudy that material. Those two characteristics, self-discipline and perseverance, will distinguish you throughout your careers.

You have credentials, very valuable credentials. The FCAS is the most well respected educational credential in the insurance world today. You clearly have knowledge, you know the risk transfer business better than almost anybody else in the country and better than anybody else in the insurance industry; and you have a reputation you inherit, because the reputation of the FCAS has been built by a series of people like you who have passed exams and then gone on to bring honor to this profession.

In all honesty, however, receiving your Fellowship is merely the end of the beginning of your careers. Now at this end of the beginning, your most important decision is what your next goal will be. My experience has taught me that there are two general paths down which an FCAS can go. Both paths are satisfying, and both paths are worthy of your best efforts.

The first path is the path of technical actuary. There are many great individuals that you can follow down this path. For example, today when I think of a technical actuary, I think of Sholom Feldblum, Gary Venter, Steve Philbrick, Bob Conger, Glenn Meyers and others. These people have honored our profession by the use of mathematics and science to make otherwise un-understandable issues, tractable and understandable. If you choose this path, then you must accumulate more mathematical, computer, and statistical skills and integrate those skills with your proven actuarial skills. You must write papers and make presentations to add to our store of knowledge. On you, we will build the intellectual foundation of our profession.

Or you may follow the second path, a general management actuary. Here also, you can follow some great actuaries. Some examples include Bob Anker, our current CAS President and the CEO of American States Insurance; Jay Brown, who led the reorganization of what was Crum & Foster and is now Talegen; Steve Groot, who has led the phenomenal success of Allstate Indemnity; Charlie Rinehart, the chairman of Home Savings of America; Ron Ferguson and Ron Bornhuetter, and many others.

If you choose the path of being a general management actuary, then you also must gain more skills, but these will now be in the area of speaking, writing, and motivational skills. If you go down this path, you should join an organization such as Toastmasters to perfect your speaking skills. You should make sure that you are publishing articles in general insurance periodicals, and you should measure and benchmark yourself by whether you have published in magazines like *Best's*, or *National Underwriter* within the next 24 months. You should become experts on the business of insurance and familiar with other disciplines, particularly claims and underwriting.

So there are two paths, one of which you should choose. The most important thing to you now is choosing that path. Both paths are honorable, both paths are interesting, both are open to you now that you have your credentials. Once you have chosen your path, if you apply the same self-discipline and perseverance you applied in becoming a Fellow, you will surely be successful. Some of you may be able to go down both paths, but I urge you to focus your energies on the one or the other.

Congratulations to you for what you have accomplished. Congratulations to your spouses, to your friends, to your parents, to your co-workers, and to your employers for the role that they have played in allowing you to achieve this designation, and on this day, the end of the beginning, and the beginning of the next phase of your career. Congratulations for what you will accomplished in your oh so bright future.

MINUTES OF THE 1997 SPRING MEETING

May 18–21, 1997

SAN ANTONIO RIVERCENTER, SAN ANTONIO, TEXAS

Sunday, May 18, 1997

The Board of Directors held their regular quarterly meeting from noon to 5:00 p.m.

Registration was held from 4:00 p.m. to 6:00 p.m.

From 5:30 p.m. to 6:30 p.m., there was a special presentation to new Associates and their guests. All 1997 CAS Executive Council members briefly discussed their roles in the Society to the new members. In addition, Michael L. Toothman, who is a past president of the CAS, briefly discussed his role with the American Academy of Actuaries' Casualty Practice Council.

A welcome reception for all members and guests was held from 6:30 p.m. to 7:30 p.m.

Monday, May 19, 1997

Registration continued from 7:30 a.m. to 8:30 a.m.

CAS President Robert A. Anker opened the Business Session at 8:30 a.m. and recognized past presidents of the CAS who were in attendance at the meeting including: Irene K. Bass (1994), Phillip N. Ben-Zvi (1985), Ronald L. Bornhuetter (1975), Charles A. Bryan (1990), Michael Fusco (1989), Allan M. Kaufman (1995), C. K. Stan Khury (1984), W. James MacGinnitie (1979), Kevin M. Ryan (1988), Jerome A. Scheibl (1980), Michael L. Toothman (1991), and Michael A. Walters (1986).

Mr. Anker also recognized special guests in the audience: Neville S. Henderson, President of the Canadian Institute of Actuaries, and Wilson Wyatt, Executive Director of the American Academy of Actuaries.

Paul Braithwaite, Kevin B. Thompson, Susan T. Szkoda, and Robert S. Miccolis announced the 117 new Associates and Mavis A. Walters announced 16 new Fellows. The names of these individuals follow.

NEW FELLOWS

Timothy Atwill*
Margaret A. Brinkmann*
Andrew J. Doll
Eric J. Gesick
Alessandrea Corinne Handley
James M. MacPhee*
Mark Joseph Moitoso*
Marlene D. Orr
Kathleen M. Pechan
Dale S. Porfilio
Robert Emmett Quane III
Jean-Denis Roy
Mark L. Thompson
James F. Tygh
Steven Boyce White
Floyd M. Yager

*Admitted as new Fellow and Associate

NEW ASSOCIATES

Ethan David Allen
Mark B. Anderson
Timothy Atwill
Wayne F. Berner
Jonathan Everett Blake
Edmund L. Bouchie
David John Braza
Cary J. Breese
Margaret A. Brinkmann
Hugh E. Burgess
Christopher J. Burkhalter
Stephanie T. Carlson
Sharon C. Carroll
Richard Joseph Castillo
Richard M. Chiarini
Theresa Anne Christian
Alfred Denard Commodore
Margaret Eleanor Conroy
Kenneth S. Dailey
John D. Deacon
Sharon C. Dubin
Denis Dubois
Rachel Dutil
Wayne W. Edwards
Jennifer R. Ehrenfeld
Kristine Marie Esposito
Joseph G. Evleth
Benedick Fidlow
Tracy Marie Fleck
John E. Gaines
David Evan Gansberg
Jay C. Gotelaere
Allen Jay Gould
John W. Gradwell
David Thomas Groff
Alexander Archibold Hammett
Daniel J. Henderson
David E. Heppen
William N. Herr, Jr.
Thomas Edward Hinds
Christopher Todd Hochhausler
Luke Delaney Hodge
Amy L. Hoffman
Dave R. Holmes
Jane W. Hughes
Jason Israel

Paul Ivanovskis
Jeremy M. Jump
Scott Andrew Kelly
David Neal Kightlinger
Deborah M. King
George A. Kish
Karen Lee Krainz
Robin M. LaPrete
Jean-Sebastien Lagarde
Yin Lawn
Kevin A. Lee
Neal M. Leibowitz
Bradley H. Lemons
Michael Victor Leybov
Janet G. Lindstrom
Christina Link
Michelle Luneau
James M. MacPhee
Andrea Wynne Malyon
Jason Noah Masch
William J. Mazurek
Phillip E. McKneely
Allison Michelle McManus
Paul D. Miotke
Mark Joseph Moitoso
Benoit Morissette
Janice C. Moskowitz
Michael James Moss
Vinay Nadkarni
Darci Z. Noonan
Michael A. Nori
Mihaela Luminita S. O'Leary
Christopher Edward Olson
Rebecca Ruth Orsi
Harry Todd Pearce
John S. Peters
Amy Ann Pitruzzello
Jennifer K. Price
Richard Bronislaus Puchalski
Patricia Ann Pyle
Kara Lee Raiguel
Rebecca J. Richard
John R. Rohe
Sandra L. Ross
Joanne Emily Russell
Lisa M. Scorzetti
Marc Shamula
Michael Shane
Bret Charles Shroyer
Katherine R. S. Smith
G. Dennis Sparks
Alan M. Speert
Nathan R. Stein
Lisa M. Sukow
C. Steven Swalley
Adam Marshall Swartz
Christopher C. Swetonic
Elizabeth Susan Tankersley
Patricia Therrien
Jeffrey S. Trichon
Kimberly S. Troyer
Timothy J. Ungashick
Martin Vezina
Karen E. Watson
Mark Steven Wenger
Miroslaw (Mirek) Wieczorek
Jerelyn S. Williams
Wendy Lynn Witmer
Simon Kai-Yip Wong
Jeffrey F. Woodcock
Edward J. Zonenberg

Mr. Anker then introduced Charles A. Bryan, a past president of the Society, who presented the Address to New Members.

Patrick J. Grannan, CAS Vice President—Programs and Communications, spoke to the meeting participants about the highlights of this meeting and what was planned in the program.

Gary R. Josephson, chairperson of the CAS Committee on Review of Papers, announced that one *Proceedings* paper would be presented at this meeting. In addition, one discussion of a *Pro-*

ceedings paper that was published in the 1993 *Proceedings of the Casualty Actuarial Society* would be presented at this meeting.

Mark E. Fiebrink, chairperson of the Michelbacher Award Committee, gave a brief description of this year's Call Paper Program on Health Care Issues for Property/Casualty Insurers. He announced that three of the four call papers would be presented at this meeting, and all four call papers were bound in the 1997 CAS *Discussion Paper Program*.

Mr. Anker then began the presentation of awards. He explained that the CAS Harold W. Schloss Memorial Scholarship Fund benefits deserving and academically outstanding students in the actuarial program of the Department of Statistics and Actuarial Science at the University of Iowa. The student recipient is selected by the Trustees of the CAS Trust, based on the recommendation of the department chair at the University of Iowa. Mr. Anker announced that Ranee Thiagarajah is the recipient of the 1997 CAS Harold W. Schloss Memorial Scholarship Fund. She will be presented with a $500 scholarship.

Mr. Anker also announced that Theresa W. Bourdon, Keith Passwater, and Mark Priven are the recipients of the 1997 CAS Michelbacher Award for their paper, "An Introduction to Capitation and Health Care Provider Excess Insurance." Mr. Anker explained that this award commemorates the work of Gustav F. Michelbacher and honors the authors of the best paper submitted in response to a call for discussion papers. The papers are judged by a specifically appointed committee on the basis of originality, research, readability, and completeness.

Mr. Anker then concluded the business session of the Spring Meeting by calling for a review of *Proceedings* papers.

After a refreshment break, Mr. Anker introduced the featured speaker, Lee Sherman Dreyfus, Ph.D., who is President of Lee Sherman Dreyfus, Inc. and a weekly columnist for the *Waukesha Freeman*, a Milwaukee area daily newspaper. Dr. Dreyfuss was formerly governor of Wisconsin, President of Sentry Insurance Corporation, and Chancellor of the University of Wisconsin at Stevens Point.

The first General Session was held from 11:00 a.m. to 12:30 p.m.

"Distribution Systems in the 21st Century"

Moderator: Cecily A. Gallagher
Consulting Actuary
Tillinghast-Towers Perrin

Panelists: Charles A. Bryan
Chief Operating Officer
Director Response Corporation

Nancy Carini
Assistant Vice President
Conning & Company

Brig. Gen. Wilson C. (Bill) Cooney, USAF Ret.
President
USAA Property and Casualty Insurance Group

After a luncheon, the afternoon was devoted to presentations of concurrent sessions and discussion papers. The call papers presented were:

1. "An Introduction to Capitation and Health Care Provider Excess Insurance"

   Authors: Theresa W. Bourdon
   Vice President and Consulting Actuary
   Aon Risk Management Services

   Keith Passwater
   Actuary
   Aon Managed Care

   Mark Priven
   Vice President
   Aon Risk Services

2. "Integration of Managed Care in Workers Compensation"

   Authors: Brian Z. Brown
   Consulting Actuary
   Milliman & Robertson, Inc.

   Michael C. Schmitz
   Associate Actuary
   Milliman & Robertson, Inc.

3. "Identifying and Pricing Managed Care Errors and Omissions"

   Authors: Michael Sapnar
   Vice President
   Transatlantic Reinsurance Company

   Elizabeth A. Wellington
   Vice President and Associate Actuary
   Transatlantic Reinsurance Company

The concurrent sessions presented from 1:30 p.m. to 3:00 p.m. were:

1. Questions and Answers with the CAS Board of Directors

   Moderator: Mavis A. Walters
   CAS President-Elect
   Executive Vice President
   Insurance Services Office, Inc.

   Panelists: Alice H. Gannon
   Vice President
   United Services Automobile Association

   David N. Hafling
   Senior Vice President and Actuary
   American States Insurance Companies

   Richard J. Roth, Jr.
   Chief Property/Casualty Actuary
   California Department of Insurance

2. The Future of Rating Bureaus

Moderator: Philip O. Presley
Chief Actuary
Texas Department of Insurance

Panelists: Michael Camilleri
President
Insurance Data Resources, Inc.

William D. Hager
President and CEO
National Council on Compensation Insurance, Inc.

Kevin M. Ryan
President
U.S. Rating Bureau

3. Actuaries in Non-Traditional Roles

Moderator: Sanford R. Squires
Vice President
ISI Systems, Inc.

Panelists: David Koegel
Senior Vice President
Gill & Roeser, Inc.

Eileen M. Sweeney
President
ZC Healthcare

4. Pricing Decisions for Marketing Reasons

Presenter: Charles L. McClenahan
Principal
William M. Mercer, Inc.

After a refreshment break from 3:00 p.m. to 3:30 p.m., presentations of call papers, concurrent sessions, and *Proceedings* papers continued. Certain call papers and concurrent sessions presented earlier were repeated. Additional concurrent sessions presented from 3:30 p.m. to 5:00 p.m. were:

1. Dynamic Financial Analysis—What Does It Look Like?

   Moderator/ Joseph A. Herbers
   Panelist: Principal and Consulting Actuary
   Miller, Rapp, Herbers & Terry, Inc.

   Panelist: Susan E. Witcraft
   Consulting Actuary
   Milliman & Robertson, Inc.

2. Introduction to the Examination Committee

   Moderator: David L. Menning
   Chairperson, CAS Examination Committee
   Senior Associate Actuary
   State Farm Mutual Automobile Insurance Company

   Panelists: J. Thomas Downey
   Manager, Admissions
   Casualty Actuarial Society

   Thomas G. Myers
   Vice President
   Prudential Property & Casualty

Proceedings papers presented during this time were:

1. "Homeowners Ratemaking Revisited (Use of Computer Models to Estimate Catastrophe Loss Costs)"

   Authors: Michael A. Walters
   Consulting Actuary
   Tillinghast-Towers Perrin

   François Morin
   Consulting Actuary
   Tillinghast-Towers Perrin

2. Discussion of "Surplus—Concepts, Measures of Return, and Determination"

   (by Russell E. Bingham, *PCAS* LXXX, 1993, p. 110)

   Discussion by: Robert K. Bender
   Associate Actuary
   Kemper Reinsurance Company

A reception for new Fellows and guests was held from 5:30 p.m. to 6:30 p.m., and the general reception for all members and their guests was held from 6:30 p.m. to 7:30 p.m.

Tuesday, May 20, 1997

Certain discussion papers and concurrent sessions that had been presented earlier during the meeting were repeated this morning from 8:30 a.m. to 10:00 a.m. Additional concurrent sessions presented were:

1. General Principles of Actuarial Science

   Moderator/ Panelist: Michael A. Walters
   Consulting Actuary
   Tillinghast-Towers Perrin

   Panelist: Linda L. Bell
   Senior Vice President and Chief Actuary
   The Hartford

   Michael A. McMurry
   Consulting Actuary
   Milliman & Robertson, Inc.

2. Texas Issues in Current Legislative Debate

   Moderator: Alice H. Gannon
   Vice President
   United Services Automobile Association

   Panelists: Philip O. Presley
   Chief Actuary
   Texas Department of Insurance

   Fred C. Bosse
   Vice President
   United Services Automobile Association

   Brian L. Mibus
   Division Underwriting Manager
   Liberty Mutual Insurance Group

3. Catastrophe Reserves—Alternatives and Issues

Moderator: Phillip N. Ben-Zvi
Principal-in-Charge
Coopers & Lybrand, L.L.P.

Panelists: Ross J. Davidson, Jr.
Vice President
United Services Automobile Association

Vincent L. Laurenzano
Assistant Deputy Superintendent and Chief Examiner
New York State Insurance Department

Wayne Upton
Project Manager
Financial Accounting Standards Boards

4. ABCD and Qualification Standards

Moderator: Henry K. Knowlton
Chairperson
Actuarial Board for Counseling and Discipline

Panelists: Walter J. Fitzgibbon, Jr.
Vice Chairperson
Actuarial Board for Counseling and Discipline

Charles L. McClenahan
Vice Chairperson
Committee on Qualifications

Jerome A. Scheibl
Member
Actuarial Board for Counseling and Discipline

After a refreshment break, a General Session was held from 10:30 a.m. to noon. The General Session presented was:

"Insurance Company CEOs' Perspectives on Future Industry Challenges"

Moderator: Robert A. Anker
CAS President
Chairman and CEO
American States Insurance Companies

Panelists: Ramani Ayer
Chairman and CEO
The Hartford

Ronald L. Bornhuetter
Chairman, President and CEO
NAC Re Corporation

Brian Duperreault
Chairman, President and CEO
A.C.E. Insurance Company, Ltd.

General Robert T. Herres, USAF Ret.
Chairman and CEO
United Services Automobile Association

Various CAS committees met from 1:00 p.m. to 5:00 p.m. In addition, three new concurrent sessions were held from 1:30 p.m. to 3:00 p.m.:

1. Actuaries Online

Panelists: J. Michael Boa
Communications and Research Coordinator
Casualty Actuarial Society

Stephen P. Lowe
Consulting Actuary
Tillinghast—Towers Perrin

2. Neural Networks

Moderator/ Frank M. Zizzamia
Panelist: Assistant Vice President
Travelers Property Casualty Corporation

Panelists: Cheng-Scheng (Peter) Wu
Manager
Deloitte & Touche LLP

Todd W. Gutschow
Vice President
HNC Software

3. Casualty Practice Council

Facilitator: Michael L. Toothman
1997 Vice President

All members and guests enjoyed a buffet dinner at the Institute of Texan Cultures from 6:30 p.m. to 9:30 p.m.

Wednesday, May 21, 1997

Certain concurrent sessions that had been presented earlier during the meeting were repeated this morning from 8:30 a.m. to 10:00 a.m. Additional concurrent sessions presented were:

1. Employment Practices Liability Insurance

Moderator: George M. Levine
Manager
KPMG Peat Marwick LLP

Panelists: Bernard R. Horovitz
Actuary
Executive Risk, Inc.

Brian Z. Brown
Consulting Actuary
Milliman & Robertson, Inc.

Mark W. Larsen
Consultant, D & O Survey
Watson Wyatt Worldwide

2. Automobile Safety Features and Their Impact on Insurance Costs

Moderator: Kathleen M. Pechan
Actuary
State Farm Mutual Automobile Insurance Company

Panelists:	Adrian Lund, Ph.D. Senior Vice President of Research Insurance Institute for Highway Safety
	Steven G. Lehmann Consulting Actuary Miller, Rapp, Herbers & Terry, Inc.
	John Werner Assistant Director of Research State Farm Mutual Automobile Insurance Company

The final General Session was held from 10:30 a.m. to noon after a 30-minute refreshment break:

"Exploring the Turnaround in the Workers Compensation Market"

Moderator/ Panelist:	Ronald C. Retterath Actuarial Consultant
Panelists:	Richard W. Palyczynski Senior Vice President Travelers Group
	Richard I. Fein Principal Coopers & Lybrand, L.L.P.

Robert A. Anker officially adjourned the 1997 CAS Spring Meeting at noon after closing remarks and an announcement of future CAS meetings.

Attendees of the 1997 CAS Spring Meeting

The 1997 CAS Spring Meeting was attended by 220 Fellows, 188 Associates, and 158 Guests. The names of the Fellows and Associates in attendance follow:

FELLOWS

Mark A. Addiego
Martin Adler
Rhonda K. Aikens
Terry J. Alfuth
Richard B. Amundson
Charles M. Angell
Robert A. Anker
Nolan E. Asch
William M. Atkinson
Timothy Atwill
Karen F. Ayres
Anthony J. Balchunas
Timothy J. Banick
D. Lee Barclay
Donald T. Bashline
Irene K. Bass
Todd R. Bault
Gregory S. Beaulieu
Linda L. Bell
Phillip N. Ben-Zvi
Robert K. Bender, Ph.D.
Regina M. Berens
James E. Biller
Gavin C. Blair
Ralph S. Blanchard III
William H. Bland
Cara M. Blank
Ronald L. Bornhuetter
Charles H. Boucek
Theresa W. Bourdon
Christopher K. Bozman
John G. Bradshaw, Jr.
Paul Braithwaite
Robert S. Briere
Margaret Ann Brinkmann
Ward M. Brooks
Brian Z. Brown
Charles A. Bryan
James E. Buck
Jeanne H. Camp
Michael J. Cascio
Galina M. Center
Francis D. Cerasoli
Janet L. Chaffee
Scott K. Charbonneau
David R. Chernick
Kasing Leonard Chung
Robert F. Conger
Alan C. Curry
Thomas J. DeFalco
Curtis Gary Dean
Jerome A. Degerness
George T. Dodd
Michael C. Dolan
Andrew Joseph Doll
Michael C. Dubin
M. L. Dye
Richard D. Easton
Grover M. Edie
Paul E. Ericksen
James G. Evans
Matthew G. Fay
Richard I. Fein
Judith M. Feldmeier
Mark E. Fiebrink
Robert J. Finger
Walter J. Fitzgibbon, Jr.
Kenneth R. Frohlich
Michael Fusco
Cecily A. Gallagher
Alice H. Gannon
Eric J. Gesick
John F. Gibson
Bryan C. Gillespie
Bradley J. Gleason
Steven F. Goldberg
James F. Golz
Karen Pachyn Gorvett
Leon R. Gottlieb
Patrick J. Grannan
Gary Grant
Gregory T. Graves
Anthony J. Grippa
Carleton R. Grose
Kyleen Knilans Hale
Robert C. Hallstrom
Alessandrea Corinne Handley
E. LeRoy Heer
Agnes H. Heersink
James S. Higgins
Kathleen A. Hinds
Paul E. Hough
Marvin A. Johnson
Stephen H. Kantor
Allan M. Kaufman
Eric R. Keen
Tony J. Kellner

C. K. Stan Khury
Frederick O. Kist
Douglas F. Kline
Rodney E. Kreps
David J. Kretsch
Jeffrey L. Kucera
Paul E. Lacko
Dean K. Lamb
John A. Lamb
Nicholas J. Lannutti
Robert H. Lee
Merlin R. Lehman
Steven G. Lehmann
Aaron S. Levine
George M. Levine
Richard W. Lo
Janet G. Lockwood
Andre Loisel
Stephen P. Lowe
Aileen C. Lyle
W. James MacGinnitie
James M. MacPhee
Howard C. Mahler
Mark J. Mahon
Donald E. Manis
Burton F. Marlowe
Steven E. Math
Robert W. Matthews
Kevin C. McAllister
Michael G. McCarter
Charles L. McClenahan
Michael A. McMurray
Dennis T. McNeese
David L. Menning
Stephen V. Merkey
Robert S. Miccolis
David L. Miller
Michael J. Miller
Neil B. Miner
Charles B. Mitzel
Frederic James Mohl
Mark Joseph Moitoso
Richard B. Moncher
François Morin
Evelyn Toni Mulder
Todd B. Munson
Donna S. Munt
Nancy R. Myers
Thomas G. Myers
Richard T. Newell, Jr.
John Nissenbaum
Ray E. Niswander, Jr.
Layne M. Onufer
Marlene D. Orr
Richard W. Palczynski
Kathleen M. Pechan
Joseph W. Pitts
Dale Steven Porfilio
Joseph J. Pratt
Philip O. Presley
Mark Priven
John M. Purple
Robert Emmett Quane III
Timothy P. Quinn
Jeffrey C. Raguse
Jerry W. Rapp
Ronald C. Retterath
James F. Richardson
Donald A. Riggins
Diane R. Rohn
William P. Roland
A. Scott Romito
Deborah M. Rosenberg
Richard J. Roth, Jr.
Jean-Denis Roy
David A. Russell
Kevin M. Ryan
Pierre A. Samson
Jerome A. Scheibl
Kim A. Scott
Mark Robert Shapland
Bonnie C. Shek
Edward C. Shoop
Rial R. Simons
Raleigh R. Skaggs, Jr.
David Skurnick
David A. Smith
Richard H. Snader
Daniel L. Splitt
Sanford R. Squires
Thomas N. Stanford
Elton A. Stephenson
James P. Streff
Eileen M. Sweeney
John A. Swift
Susan T. Szkoda
Catherine Harwood Taylor
Kevin B. Thompson
Mark L. Thompson
Michael L. Toothman
Michel Trudeau
Everett J. Truttmann
Warren B. Tucker
James F. Tygh
Peter S. Valentine

William R. Van Ark
Trent R. Vaughn
Joseph L. Volponi
Gregory M. Wacker
Robert H. Wainscott
Mavis A. Walters
Michael A. Walters
Dominic A. Weber
Debra L. Werland
Steven Boyce White
William D. White
Gregory S. Wilson
Martha A. Winslow
Timothy L. Wisecarver
Susan E. Witcraft
Cheng-Sheng P. Wu
Floyd M. Yager
Ronald J. Zaleski

ASSOCIATES

Rimma Abian
Jonathan D. Adkisson
Ethan David Allen
Mark B. Anderson
James A. Andler
Martin S. Arnold
Martha E. Ashman
William P. Ayres
Wayne F. Berner
Jonathan Everett Blake
Edmund L. Bouchie
Kimberly Bowen
Cary J. Breese
Linda M. Brockmeier
Lisa A. Brown
Kirsten R. Brumley
Hugh Eric Burgess
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Stephanie T. Carlson
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Brenda L. Reddick
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Martin Vezina
Cynthia L. Vidal
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Geoffrey T. Werner
Miroslaw Wieczorek

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Mary E. Wills	Calvin Wolcott	Mark L. Woods
Kirby W. Wisian	Brandon L. Wolf	Michele N. Yeagley

Volume LXXXIV, Part 2 **No. 161**

PROCEEDINGS
November 9, 10, 11, 12, 1997

FUNDING FOR RETAINED WORKERS COMPENSATION EXPOSURES

BRIAN Z. BROWN AND MICHAEL D. PRICE

Abstract

The number of firms retaining part of their workers compensation exposure has grown dramatically over the last 5 to 10 years. It is important that firms fund and reserve for their retained exposure so that their balance sheet and income statements are accurate. This paper outlines several methods that can be used to establish funding levels for self-insured employers. Additionally, we outline several considerations which employers face in deciding whether or not to self-insure and some of the factors which affect the structure of a self-insured program.

1. INTRODUCTION

The self-insured workers compensation market grew dramatically between 1986 and 1991. Table 1 displays the growth in

TABLE 1

WORKERS COMPENSATION
PERCENTAGE OF MARKET SELF-INSURED

Calendar Year	Self-Insured Percentage
1986	20.1%
1987	21.2
1988	22.3
1989	25.5
1990	25.9
1991	29.0

the percentage of the total market that is self-insured (based on premiums and premium equivalents).[1]

In this paper we will outline various methods that can be used to estimate the self-insured employers' liability for their retained exposures. Although a more rigorous definition will be provided later, the "funding level" can be thought of as the contributions needed to:

- pay the expected amount of claims and related costs in the "upcoming year," and
- establish an appropriate accrual as of the end of the year.

Establishing funding levels for entities that self-insure their workers compensation exposure is a complex process. This paper defines the term "funding level" and describes methods that can be used to estimate the funding level.

The paper is divided into seven sections. The first section is the introduction. The second section discusses some of the benefit and cost considerations involved in deciding whether to commercially insure or retain some of the exposure in-house.

[1] See Johnson & Higgins [1]. The term self-insurance denotes any program employing risk retention as the primary method for funding expected losses. This definition includes self-insured programs deemed "qualified" under state laws, but does not include self-insured retentions or deductibles in conventional insurance programs.

The third section describes some of the significant requirements that states impose on firms that self-insure their workers compensation exposure. In the fourth section, the funding level is defined.

The fifth section provides two detailed funding level calculations. The first calculation presented is for an employer that has been self-insured for a number of years and has substantial historical loss and exposure information. The second calculation is for an employer that has been self-insured for only a short time period and has limited loss and exposure information.

The sixth section of the paper discusses several additional items that an entity may want to consider in structuring and funding a workers compensation self-insurance program:

- confidence levels,
- discounting, and
- excess insurance.

The final section of the paper is the conclusion.

2. BENEFITS AND COSTS OF SELF-INSURANCE

An employer faces costs and benefits when evaluating the decision to retain or self-insure part of its workers compensation exposure. Each organization will perceive the overall value of self-insuring differently.

A. *Benefits of Self-Insuring Workers Compensation Exposures*

The potential benefits of self-insuring workers compensation exposures result from:

- cost savings to employers,
- enhanced awareness and control of loss costs, and
- other considerations.

A.1. Cost Savings to Employers

Lower cost is often considered to be the most important benefit of self-insurance. However, cost should not be considered in isolation. The cost of self-insuring must be considered in relation to the cost of purchasing insurance from the commercial marketplace and the increased risk assumed by the self-insured employer.

Premiums charged by commercial insurers contain several distinct components: expected loss costs (including allocated loss adjustment expenses), operating expenses, expected profit (excluding risk load), and risk load.[2] The self-insured entity can potentially achieve cost savings in three of these four premium components. The entity cannot avoid the risk load "cost."

The expected loss costs underlying commercial premiums generally reflect the insurance company's estimate of the average loss cost for a group of similar insureds. To the extent that the entity considering self-insurance has lower expected loss costs than the "average" entity in the group, the difference between the average loss costs and the entity's loss costs is expected to be realized as cost savings by the self-insurer. That is, the self-insurer reaps the full benefit of better-than-expected loss experience. This is not to say that commercial insurer pricing is inaccurate. Rather, an entity may have recently changed its risk management and/or loss control policies and these changes have not yet been reflected in data which is measurable. Therefore, by self-insuring, the entity is "betting" that its changes are more favorable than measured by the commercial insurance market.

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[2] We are using the term "profit" to include both underwriting results and investment returns. One way to measure this profit is to compute the discounted (present value) of the net cashflows (premium less expenses and losses) at the insurer's projected yield rate. We believe it is important to consider investment returns in the profit calculation since the self-insured losses will be paid over an extended period of time whereas the commercial insurance premium is paid at policy inception. To focus solely on underwriting income (and ignore investment results) would ignore the fact that the self-insured can invest the funds it would have paid for commercial insurance.

Furthermore, the self-insurer benefits directly and immediately from any reduction in expected loss costs that results from the successful implementation of loss control or loss prevention strategies. This incentive to self-insure has not escaped the attention of the commercial marketplace. There are numerous mechanisms used by the commercial insurer wishing to compete for the business of the better-than-average risk, including experience rating, retrospective rating, prospective rating (e.g., schedule rating), and dividend plans. However, in most cases, these options either dilute or delay (or both) the full benefit of reduced loss potential. For example, under a dividend plan, a $1 reduction in loss experience does not usually translate into a $1 dividend; furthermore, the dividend payment is made many months after the close of the policy period.

The operating expense component of commercial premiums may include a provision for such costs and services as claims handling, underwriting, taxes, dividends, assigned risk assessment, administrative costs, marketing, acquisition costs, and overhead. Self-insurance may potentially eliminate or reduce the need for several components of operating expense, thus resulting in cost savings to the self-insured entity. Self-insured entities will not incur expenses for underwriting, marketing, dividends, or acquisition of business (commissions). Also, subject to various state regulations, self-insured entities may be exempt from assigned risk assessments and premium taxes. Self-insurers can further achieve cost savings by retaining the provision for expected profit in the rates.

We believe that the self-insurer cannot avoid the uncertainty of outcomes associated with retaining its exposure to loss. This cost will be borne by the self-insurer either through the opportunity cost of funds, in excess of the expected value, set aside for possible adverse claim results, or the need to "borrow" from other parts of the organization (or an outside source) during those years with poor loss experience. Commercial insurers often include a provision in their rates, known as a risk load, to compensate for this uncertainty. More discussion on this component will follow in a later section.

A.2. Enhanced Awareness and Control of Loss Costs

As a consequence of the decision to self-insure workers compensation exposures, the employer becomes responsible for many aspects of the risk management and financing processes that may otherwise be addressed by the commercial insurer. Claims handling, database management, loss prevention, and loss control functions are often moved in-house or purchased from a third-party provider.

Oftentimes this may provide the self-insurer with a firsthand opportunity to witness the magnitude of the financial and human costs associated with workplace accidents. Self-insuring may provide a more direct link between employer actions, such as loss control or loss prevention, and the company's bottom line. This greater awareness may often lead to measures enacted with the intention of reducing costs and providing a safer workplace.

A.3. Other Considerations

Through the mechanism of self-insurance, the employer is able to provide workers compensation benefits to its employees (subject to regulatory approval). While all employers are able to obtain workers compensation coverage from the residual market, if not from the voluntary market, many employers wish to avoid the stigma of being considered a substandard risk when they are forced to obtain coverage from an assigned risk mechanism. Furthermore, while coverage availability is guaranteed, there is no guarantee that an insured can place its business with the company of its choice.

By means of potential cost savings and enhancement of employee morale, the employer is given a direct incentive to aggressively rehabilitate injured workers. This may result not only in cost savings for the employer, but also in a societal benefit associated with restoring an individual to a state of health and productivity. Furthermore, overall employee loyalty may be enhanced. The self-insurer retains more control over the claims handling process, and thus has more authority over decisions to deny claims or investigate fraud.

Finally, the self-insurer retains authority over its investment portfolio; that is, it controls the assets that back the liabilities incurred by self-funding. This freedom allows the company to seek potentially higher rates of return than are reflected in commercial premiums.

B. Costs of Self-Insuring for Workers Compensation Exposures

The costs of self-insuring for workers compensation exposures result from:

- increased cost to employers,
- increased variability of insurance related costs,
- additional staffing costs, and
- other considerations.

B.1. Increased Cost to Employers

To the extent that the entity considering self-insurance has higher-than-expected loss costs, this difference is realized as an additional cost when self-insuring. Additionally, many states will require a letter of credit (LOC) or other collateral to be posted by self-insured entities. The fee for obtaining this collateral is an additional cost.

B.2. Increased Variability of Insurance-Related Costs

While the expected value of costs under a self-funding arrangement may be equal to or lower than the cost of purchasing commercial insurance, the variability of these costs is potentially much greater. This result follows from consideration of the Law of Large Numbers. That is, the variance associated with the sample mean is less than or equal to the variance associated with a single observation [2].

Premiums charged by commercial insurers and funding levels established by self-insurers may contain a provision for contingencies referred to as a risk load. The relative magnitude of the risk load is usually dependent on the variance of possible losses relative to the expected amount of losses associated with insured exposures. Additionally, there may be greater uncertainty when the self-insurer estimates its ultimate future costs than when an insurance company develops average rates. The uncertainty involved in estimating cost or rate parameters is referred to as "parameter risk." Estimates of claim frequency and severity that are derived from large credible databases, such as those available to most large commercial insurers, are more statistically reliable than estimates developed from smaller, less credible databases, such as those maintained by self-insurers.

An insurance company can provide coverage for a large number of employers, who are diverse both economically and geographically, while a self-insurer is limited to providing coverage for its own exposures. Thus, the self-insurer requires a proportionately larger loading than the insurance company does for the risk that losses will, in the aggregate, exceed their expected value by some percentage. This differential represents a cost of self-insurance.

Furthermore, the amount of funding required to pay insurance claims is less certain and more variable for a self-insured employer. Although estimates are made and funding levels may include a risk load, the actual cost of self-insuring may not be known for many years. This increased uncertainty can complicate the financial planning process of the employer. This complication can be viewed as a cost of self-insurance.

B.3. Additional Staffing Costs

The employer that decides to self-insure must provide or purchase many services otherwise provided by the commercial insurer, including claims handling, database management, and loss control/prevention services. Other services required by a self-insurer include audit, actuarial, and investment management services.

These services are essential to the successful management and financing of workers compensation exposures. Therefore, the self-insurer must either purchase these services from an outside party, or move the functions in-house. Often, especially at first, the self-insurer cannot undertake these operations as cost-effectively as the commercial insurer.

Generally, additions to staff will be required to perform or monitor these functions, as well as handle other administrative tasks associated with managing a self-insurance program. Skilled risk management personnel will be required to supervise these functions as well as address the technical needs of the program (e.g., what excess limits of coverage to purchase). Often, a company must purchase computer hardware and software to establish a risk management database required for monitoring and analyzing exposure to loss. Actuarial, audit, and investment management services can be purchased from professional firms specializing in these areas.

It should also be noted that the commercial insurer, due to economies of scale, may provide better service and/or provide the service at a lower overall cost than the self-insured entity.

B.4. Other Considerations

One additional cost associated with the decision to self-insure is the potential adverse impact on the employer's relationship with its employees. If the employer chooses to move the claims adjusting process in-house, the employer and the employee can be thrust into an adversarial relationship under certain circumstances. Consider the decision to deny claims. If the employer denies an employee's claim, the employer may be viewed as unsympathetic by the injured person's friends and co-workers. This can have a damaging effect on the firm's relationships and reputation. Similar difficulties arise if the employer takes a hard line on investigating and eliminating fraudulent claims. For these reasons many firms that self-insure their exposure choose to contract for claims management services with a third party administrator

(TPA). The TPA is often viewed as an objective decision maker, balancing the goals of the employer against the needs and rights of injured workers.

Another potential cost pertains to excess insurance. Many self-insured entities will want (or be required) to purchase excess insurance, and this subjects these companies to:

- the uncertainty regarding market conditions, and the effect upon the availability and affordability of the coverage; and
- the payment risk due to insolvency associated with future excess insurance recoveries.

It should be noted that, although federal income tax considerations are outside the scope of this paper, they may be significant. Typically a self-insured employer can deduct losses only as they are paid, whereas commercial insurance premiums are fully deductible. Also, many states require self-insured entities to meet various administrative requirements. These requirements may involve substantial time and cost.

3. SELF-INSURANCE REGULATORY REQUIREMENTS

Most states have established requirements to provide funds for injured workers in the case of a self-insured entity's bankruptcy. In addition, states have attempted to limit the "availability" of self-insurance to financially strong firms. This section discusses several common self-insurance requirements imposed by the various states. The requirements are divided into initial filing requirements and additional requirements.

Self-insurance initial filing requirements often include:[3]

1. a parental guarantee (if applicable),

[3]"The Self-Insurance Manual" [3] summarizes each state's statute related to workers compensation self-insurer requirements.

2. the most recent audited financial statement of the entity considering self-insurance, and
3. loss experience and payroll information.

The parental guarantee is a promise by the parent corporation to "guarantee" the workers compensation payments of a subsidiary. This requirement will decrease the credit risk associated with the self-insured entity's exposure by committing not only the subsidiary's assets but also the parent's assets to guarantee the self-insurer's workers compensation payments.[4]

The second requirement, a recent audited financial statement, allows the state to evaluate the potential (or current) self-insured employer in order to determine if the employer is financially strong enough to self-insure. This procedure should reduce the number of financially weak self-insured employers.

The last requirement, loss and payroll information, allows the Insurance Department to determine the reasonableness of the collateral (which is discussed later).

As a note, some states have established additional and more specific requirements. For example, the Vermont regulations require that the applicant must meet target ratios in six categories.[5]

If a self-insured employer meets the initial filing requirements and the state is satisfied with the entity's financial condition, then two additional requirements may be imposed [3]:

- excess insurance, and
- security or bonding.

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[4]Credit risk is the possibility that one entity will suffer a financial loss due to the inability of a second entity to satisfy its obligations. For example, if a self-insured employer went bankrupt, other employers in the state may be required to pay claimants' bills. Credit risk is discussed in more detail in Brown [4].

[5]There are minimum target ratios for: cash flow, liquidity, working capital, net worth, profitability, and turnover [3].

One reason to require excess insurance is to increase the predictability of the self-insured employer's retained loss experience. The purchase of excess insurance may make the loss experience more predictable from year to year and may reduce the probability of an insolvency (of the self-insured entity) due to poor loss experience in one particular year. States will usually require excess insurance if the self-insured employer has some financial shortcomings. The importance of excess insurance and its relationship to the funding level will be discussed in Section 6.

The security or collateral requirement is the mechanism that states have established to compensate claimants in the event of a self-insured employer's bankruptcy. Most states do not have prefunded guarantee funds covering the obligations of self-insured employers. Therefore many states require self-insured employers to provide the state with a letter of credit (LOC) or surety bond. These funds would then be available in the case of a self-insured employer's bankruptcy. States use various methods to establish the security requirement. In reviewing the various state regulations, it appears that many states use one (or more) of the following three methods to determine the amount of security:

- a minimum flat dollar amount,
- a factor times case reserves, or
- a formula approach based on the recent loss experience of the insured.

A few states require an actuarial analysis to assist in determining the amount of collateral. It should be noted that, in general, states do not require security for municipalities and political subdivisions that self-insure. This may be due to the fact that these entities typically have taxing authority and therefore are unlikely to be unable to meet claim obligations.

This section has discussed some of the more common self-insurance requirements. However, the reader is cautioned that specific requirements vary significantly from state to state.

4. FUNDING LEVEL

For illustrative purposes, the discussion of the funding level in this section assumes that the self-insured entity is utilizing a risk financing technique for its retained exposure that involves earmarking assets.[6] A partial list of the most commonly used risk financing techniques for retained exposures includes:

- current expensing of losses,
- an unfunded reserve,
- a funded reserve (i.e., earmarking assets),
- use of borrowed funds, and
- retention through an affiliated ("captive") insurer.

There are advantages and disadvantages associated with each of the above mentioned techniques. Some of the advantages of using a funded reserve as a risk financing technique include the following.

1. It may be more likely that liquid assets will be available to pay for retained losses. If an entity earmarks assets for retained exposures, oftentimes a cash flow (or duration) analysis will be performed on the retained exposure.
2. Accounting considerations may require the entity to accrue a liability for its retained exposure. The applicable standard board statements are Financial Accounting Standards Board (FASB)-5 for private companies and Governmental Accounting Standards Board (GASB)-10 for public entities.[7] An appropriate (i.e., reasonable)

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[6]A risk financing option involving earmarking assets has several advantages from a financial planning standpoint, as the text discusses. The gross liability to the employer is similar regardless of the risk financing option. The risk financing options affect only the distribution of assets.

[7]It should be noted that these accounting obligations could be met through an unfunded reserve.

funded reserve would probably satisfy these requirements.

3. Regulators may prefer that firms formally establish a funded reserve. In fact, some states have allowed, in essence, a formally structured funded reserve (escrow account) to meet the collateral requirements established by the state.[8]

Two potential disadvantages of a funded reserve as a risk financing technique are:

1. The entity may have better use of its funds than merely to invest in financial instruments in anticipation of paying future losses. The firm may be able to generate a better return by devoting funds to regular productive activities.
2. The funded reserve may appear as idle funds and be redeployed for other corporate purposes.

We define the required "fund" as the amount of assets needed to satisfy all past years' retained insurance obligations plus insurance obligations for the upcoming self-insurance year. This is analogous to (but not identical to) an insurance company's

- liabilities as of year-end, plus
- next year's premium.

The required fund for a self-insured employer consists of the following elements:

- Liabilities as of year-end—
  - Claim liabilities (including a provision for allocated loss adjustment expenses [ALAE])

[8]An escrow account is a written agreement entered into among three parties. Funds are deposited for safekeeping with the third party as custodian. The custodian or depository is obliged to follow strictly the terms of the agreement agreed upon by the other parties.

  - Other loss adjustment expense liabilities
  - Any potential loss sensitive premium related obligations prior to self-insuring (e.g., additional retrospective rating plan premium)
  - Expected additional excess insurance premium payments for prior years' exposure (due to a positive payroll audit)
  - Second injury fund assessments, taxes payable, etc.
  - Other (general) expense liabilities
  - A provision for uncollectible excess insurance
- Funding obligations for the upcoming self-insurance year—
  - Claim costs including ALAE
  - Unallocated loss adjustment expense (ULAE) costs
  - Marketing/sales costs (for a group self-insurer)
  - Excess insurance costs
  - Second injury fund assessment, taxes etc.
  - Risk charge (this is discussed under loss probability levels in Section 6)
  - Other expense (expected to be incurred in the upcoming self-insurance year)

As a note, the above mentioned claim costs refer to the retained (after the application of excess insurance) exposure. We are assuming that a self-insurance year will provide coverage for all claims occurring during the year.

The "funding level" for the upcoming calendar year is then equal to:

- the prior years' liabilities, plus
- the funding obligations for the future accident year, minus

- the amount of assets earmarked to pay for the obligations.

If investment income is intended to remain in the fund, then the assets should include the investment income earned on the earmarked assets.

We have not defined claim costs with regard to whether the amount is discounted or undiscounted or whether the amount is an expected value or established at some confidence level amount. Section 6 will cover these concepts.

There are probably other ways to define funding levels. However, it appears that many self-insured entities use the definitions discussed in this section.

5. FUNDING LEVEL EXAMPLES

In this part of the paper, we will outline approaches that can be used to estimate the funding level of a self-insured employer, the claim related liabilities as of year-end, and the expected claim costs for the upcoming year. We will assume that the self-insured employer is able to estimate the amount of non-claim related items (e.g., excess insurance costs). In addition, we will provide funding level calculations for two scenarios:

- Scenario One—The self-insured employer has adequate data to utilize several commonly accepted actuarial projection methods.
- Scenario Two—The self-insured employer does not have sufficient data to utilize commonly used actuarial projection techniques and therefore some creative but necessary techniques are required.

A. *Adequate Data Example*

For scenario one, the employer has been self-insured for ten years. The employer purchases specific excess coverage above

$500,000 per claim. The employees are in two classes (based on National Council on Compensation Insurance [NCCI] class codes).

We will first discuss a procedure to project gross losses, although it may not be necessary to project gross losses to estimate net losses. However, we will discuss the projection of gross losses for the following two reasons:

- a projection of net losses could involve subtracting projected excess losses from gross losses, and
- if any excess carriers are insolvent or financially troubled, a projection of gross losses is needed to estimate an uncollectible excess insurance provision.

We will use the term "loss" to include both losses and ALAE.

The following data is available by self-insured year and development year:

- Exhibit 1 displays the employer's paid loss experience,
- Exhibit 2 displays the employer's incurred loss experience,
- Exhibit 3 displays the corresponding claim count data (for lost time claims), and
- Exhibit 4 displays the employer's average incurred severity.

Additionally, Exhibit 5 displays the self-insured employer's workers compensation payroll by self-insured year and class.

A.1. Projection of Gross Losses

Based on the above-mentioned data items, we can use several methods to estimate ultimate losses by self-insured year. The unpaid claim liability can be computed as the ultimate losses less the losses paid to date. The following generally accepted projection methods are used to project ultimate losses by self-insured year:

- paid loss development (Exhibit 6),
- incurred loss development (Exhibit 7),
- a count times average method (Exhibit 8),
- an expected loss method (Exhibit 9),
- a trended pure premium approach (Exhibit 10), and
- a Bornhuetter–Ferguson method (Exhibit 11).

We will not provide the details on these methods in the text as they are well documented in the actuarial literature. The exhibits should be self-explanatory.

Note that if more refined data are available, several enhancements could be made to the projection methods outlined on Exhibits 6 through 11. For example, the projection methods outlined on Exhibits 6 through 11 could be performed separately:

1. by class,
2. by type of loss (medical, indemnity, and expense), or
3. a combination of 1 and 2 above (e.g., by class for medical costs versus by class for indemnity costs).

Further breakdown of the data may reveal trends not apparent by viewing the data more globally. However, this will involve less data and hence introduce credibility concerns.

It should also be noted that while we have not explicitly introduced credibility into the loss projection methods, we have used various projection methods. Presumably the analyst will be in a position to assign credibility to the various projection methods in selecting ultimate losses.

The above mentioned data items and hence the above estimates are gross (i.e., before the application of the entity's excess insurance program). In the gross loss projections we have assumed that there were no unusually large losses that would distort the projections. If there are unusually large losses, they should be treated separately.[9]

A.2. Projection of Net Losses

Several methods can be used to estimate the retained losses for the entity. We will discuss two. The first set derives the retained losses by repeating the projection techniques performed for gross losses. However, retained losses are used in lieu of gross losses in constructing the triangles. Therefore, individual losses will be limited at the per claim retentions. With regard to aggregate recoveries, it may be more reasonable to construct "triangles" gross of aggregate retentions and limit the projected losses at the aggregate retention. As a note, both the Bornhuetter–Ferguson method and the expected loss method will require an independent estimate of the ultimate retained losses. These retained losses can be calculated based on:

- an estimate of unlimited losses, and
- excess ratios published by the NCCI.

The second technique is a Bornhuetter–Ferguson method for the excess layer and involves subtracting estimated excess losses from gross losses. The *a priori* estimate of ultimate excess losses is based on the selected gross losses and an estimate of the percentage of losses which will exceed a specific amount. For discussion purposes, we relied on excess ratios from Gillam [5].

These excess ratios will vary by state and hazard group. A discussion of the procedures necessary to calculate excess ratios is beyond the scope of this paper.

[9]For example, the large losses can be removed from the projection methodology and evaluated independently.

Several sources can be used to estimate the required excess reporting patterns. A partial list includes:

- data published by the Reinsurance Association of America (RAA),
- data from A. M. Best for reinsurance companies, and
- data from the individual entity (if the entity is large enough).

It should be noted that both the RAA and A. M. Best data have several limitations, including:

- a mixture of attachment points and retention levels,
- a mixture of different types of risks, and
- varying company reporting requirements and reserving philosophies.

Exhibit 12 displays the calculation of the *a priori* excess losses. Exhibit 13 displays the Bornhuetter–Ferguson calculation for excess losses.

The retained losses are then calculated by subtracting the estimated excess losses from the estimated gross losses. Exhibit 14 displays our selected gross losses, excess losses, retained losses, and retained unpaid claim liability.

The expected value of losses for the upcoming year (1994) can be determined based on an expected loss method and a trended pure premium approach. The required fund (on an expected value basis) is then equal to the sum of:

- the net unpaid claim liabilities, plus
- the expected retained claim costs for the upcoming year.

Exhibit 15 summarizes the estimates and displays the calculation.

B. Limited Data Example

The XYZ Manufacturing Company has self-insured its workers compensation exposures for the past six years. While the firm has paid over $7,000,000 in claims during that time period, it has only recently begun to establish case reserves for individual claims. Aggregate loss payments are available by calendar year, but individual claim detail is not available. The paid loss data is available for medical versus indemnity payments.

The company has recently established a database capturing information on all open and newly reported claims as of January 1, 1993. The accident date and the current reserve amount are captured; however, prior payments and prior reserve levels on claims are not known. Reserves are available separately for medical versus indemnity losses. The company has not captured exposure information by class code.

The absence of a complete set of cumulative data triangles for paid and incurred losses poses a problem for estimating the unpaid claim liabilities of the company. Traditional actuarial methodologies cannot be employed without modification. The first step is to estimate the reserve accrual for the company from inception of the self-insured period as of year-end 1993 (i.e., self-insured years 1988–1993 valued as of 12/31/93).

Three nonstandard actuarial techniques will be employed to estimate the reserve accrual of the XYZ Manufacturing Company:

- case reserve development method,
- calendar year incremental payment method, and
- a de-trended Bornhuetter–Ferguson projection method.

For reference, Exhibit 16 displays the available loss experience of the company.

B.1. Case Reserve Development Method

The case reserve development method is similar to the paid and incurred loss development methods and is predicated on the assumption that case reserves have been established in a manner consistent with industry standards. Unusually large losses may distort the development projection and therefore should be treated separately.

A set of multiplicative factors, which vary according to the maturity of a given accident year, are applied to the known case reserves for each accident year as of a common evaluation date. The factors are referred to as case development factors. For a given year, the product of the case development factor and the case reserve amount yields an estimate of the total unpaid losses (including incurred but not reported losses [IBNR]) for that accident year.

This method may be well suited for application to workers compensation losses since most of the development beyond 24 months is attributable to supplemental development on known case reserves. Case development factors can be derived from cumulative paid and incurred loss development factors. Define the following notation:

P_t = Paid loss development factor from t months to ultimate,

I_t = Incurred loss development factor from t months to ultimate,

P = Paid losses at t months of development,

I = Incurred losses at t months of development, and

U = Ultimate losses.

Then, on an expected value basis:

$$(P) \times (P_t) = U \text{ implies } P = (U)/(P_t), \qquad \text{and}$$

$$(I) \times (I_t) = U \text{ implies } I = (U)/(I_t).$$

We desire a factor, k, such that (on an expected value basis):

$$(I - P) \times (k) = (U - P);$$

that is, case reserves at t months, $(I - P)$, multiplied by the factor k yields total unpaid losses, $(U - P)$. Therefore, on an expected value basis:

$$(U/I_t - U/P_t) \times (k) = U - U/P_t;$$

$$(U) \times (1/I_t - 1/P_t) \times (k) = (U) \times (1 - 1/P_t); \qquad \text{and}$$

$$(1/I_t - 1/P_t) \times (k) = (1 - 1/P_t).$$

Thus, $k = (1 - 1/P_t)/(1/I_t - 1/P_t)$.

In the example, no credible development history exists from which to select paid and incurred development factors. Therefore, external data sources will be used to derive development patterns. Exhibit 17 displays paid and incurred development factors based on our interpretation of data published by the NCCI in a specific state, for medical and indemnity losses, as well as the calculation of case development factors according to the formula derived above.

Exhibits 18 and 19 depict the application of the case development factors to the case reserves of the company and the resulting estimates of unpaid losses.

B.2. Calendar Year Incremental Payment Method

The calendar year incremental payment method is based on an assumed loss payout pattern, a loss trend, and a constant exposure (payroll) trend to derive a factor that can be applied to calendar year paid losses to produce an estimate of unpaid losses for all accident years. This method is based on the following assumptions:

- there is no change in the payment pattern by accident year (e.g., no speed up in claim settlements),

- the loss trend is constant and does not vary by accident year or calendar year, and
- there have been no usually large claim payments.

The payout pattern employed is derived from the development pattern we used in the case development method. Exhibit 20 displays the selected payment patterns. For this example, we assume that medical losses (pure premiums) will increase at a rate of 10% annually and indemnity losses will increase by 3% annually.[10] As a note, these trends are in excess of payroll growth. We assume that the company's exposures have increased by approximately 4% per year (including payroll growth).

Let AY_0 denote accident year 0, and let P_0^t represent the incremental percentage of ultimate losses paid in year t for AY_0.

Then, given the amount paid in calendar year t on AY_0 losses, unpaid losses at time t on AY_0 exposures can be estimated by multiplying calendar year payments by the following factor:

$$\frac{\left(1 - \sum_{i=0}^{t} P_0^i\right)}{P_0^t},$$

which is the ratio of the percentage of ultimate losses yet to be paid at time t, to the percentage paid in year t.

Allowing for the effect of trend in accident year loss costs and exposures, the factor to estimate unpaid losses on AY_k exposures is given by:

$$\frac{(1+r)^k - \sum_{i=0}^{(t-k)} P_k^i (1+r)^k}{P_k^{t-k}(1+r)^k}.$$

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[10] A good starting place in seeking trend factors would be a bureau filing. For example, NCCI provides separate medical and indemnity loss ratio trends in most states.

As a note, the trend factor is the product of the loss and exposure trend. Notice that the trend factor $(1 + r)$ could be factored out of this expression, yielding the result that trend is irrelevant to the calculation of the reserve factor for a single accident year. However, as will be seen below, trend is important when multiple accident years are combined.

Now suppose that the calendar year losses resulting from z accident years are known, but their breakdown by accident year is unknown. An expression can be developed which, when applied to the calendar year payments at time t, yields an estimate of unpaid losses for all accident years at time t.

Conceptually, this expression should reflect the sum of all future payments for each of the z accident years (z is the number of years self-insured), divided by the sum of the calendar year t payments for the z accident years (based on an assumed payment pattern). The expression is:

$$\frac{\sum_{k=0}^{z}\left[(1+r)^k - \sum_{i=0}^{(t-k)} P_k^i (1+r)^k\right]}{\sum_{k=0}^{z} P_k^{t-k}(1+r)^k}.$$

This expression can be seen to be the ratio of the sum of the numerators for each of the z accident year factors to the sum of the denominators for each of the z accident year factors. Notice that the trend factor cannot be factored out of this expression. The trend factor affects the relative weights given to each accident year factor.

Exhibits 21 and 22 display the mechanics of the methodology as well as the resulting estimate of unpaid indemnity and medical losses for the XYZ Manufacturing Company.

As a note, this model can also be used to vary the future trend from historical averages. For example if XYZ entered into a long-term contract with a particular hospital that would reduce expected future medical costs by 1% per year (and almost all of the injured workers were treated at this hospital), then this 1% reduction could be factored into the model.

The future projected medical payments would be reduced by 1% annually or multiplied by a factor of $(.99)^x$ (where x is the number of years from the date the long-term contract began to the date the projected payment is made).

B.3. De-Trended Bornhuetter–Ferguson Method

The last method discussed is a De-Trended Bornhuetter–Ferguson [6] projection method. This method can be used to estimate the unpaid claim liability as well as provide an estimate of the upcoming year's expected losses. For this method the following elements are required:

- an estimate of ultimate losses for the most recent year,
- an assumed reporting pattern for losses,
- an assumed loss trend, and
- an assumed exposure trend.

For XYZ, the ultimate losses for 1993 are estimated based on incurred and paid loss projection methods. The ultimate losses for prior accident years are then estimated based on the combined loss and exposure trend. For example, the ultimate losses for self-insured year 1990 are equal to 1993 ultimate losses divided by $(1 + r)^3$. A Bornhuetter–Ferguson method can then be used to estimate the total reserves by year. Exhibit 23 displays the calculation.

The upcoming year's expected losses are estimated by multiplying the results of the incurred projection method by the selected trend factor of $(1 + r)$. Exhibit 24 displays this calculation. Exhibit 25 displays the selected unpaid claim liability at 12/31/93 along with expected 1994 claim costs. The funding for 1994 is equal to the required fund less the amount of assets set aside to pay claim liabilities.

6. ADDITIONAL CONSIDERATIONS

This section will discuss factors other than cost estimates that an entity may want to consider in structuring a self-insured program (and determining a funding level):

- the variability associated with cost estimates,
- the time value of money, and
- issues related to excess insurance.

Loss Probability Levels

The estimates described in Section 5 are expected values. Therefore, a significant percentage of the time the actual losses will exceed the estimates derived in Section 5. The attached Exhibit 26 displays a hypothetical example of a distribution of projected losses for the upcoming self-insurance year for a risk with $500,000 of expected losses.

As this graph displays, for a risk with expected losses of $500,000, there is a 9.6% probability that actual losses will exceed $1,000,000 in the upcoming self-insurance year. The self-insured entity will want to consider this information in determining funding levels. Exhibit 27 displays some of the key figures underlying the graph.

In determining the probability level at which to fund, the employer may also want to consider:

- How easy would it be to obtain additional funds if loss experience is worse than expected?
- Would bonds have to be liquidated at a loss to fund for adverse insurance results?

- What are the insurance costs relative to the net worth, sales, and net income of the entity?
- What is the entity's philosophy with regard to assuming risk?

These factors, along with the variability of losses, should be used by the entity to determine the funding level.

In deriving losses associated with probability levels, we are interested in the distribution of the funding level. The assets as of year-end are fixed (ignoring credit risk); therefore; the probability level is a function of the combined distribution of:

- next year's claim costs, and
- the future loss payments associated with the unpaid claim liabilities for prior years as of year-end.

While a discussion of the combined aggregate loss distribution is outside the scope of this paper, we would point the interested reader to "Hospital Self-Insurance Funding: A Monte Carlo Approach" by David Bickerstaff [7]. This is one of the few papers that attempts to estimate the aggregate loss distribution of the combination of:

- the run-off of the fund's prior years' losses, plus
- the prospective year's losses.

Discounting

Another item that the self-insured entity may wish to consider is the time value of money. Exhibit 28 displays how $100 of workers compensation losses are projected to be paid out over time. If the entity invested funds and received interest payments equal to 6% of the invested funds annually, then less than $100 could be invested at the beginning of the period to cover the expected loss payments. This is due to the fact that the interest earnings will be available to satisfy future loss payments. In this example, approximately $90 invested at the beginning of the period, along with projected interest earnings (at 6%) are anticipated to be sufficient to cover the expected loss payments shown on Exhibit 28.

In determining discounted unpaid claim liabilities, the Actuarial Standards Board has outlined several issues and considerations that an actuary should take into account [8]. A partial list of issues and considerations includes:

- the timing of future payments and potentially a range of payment timing estimates,
- the interest rate selected for discounting, and
- risk margins associated with the discounted loss reserves (as the discounting process introduces additional uncertainties).

The entity may also want to consider the interaction of the loss payment stream and the probability level of the undiscounted losses. For example, if the entity suffers an unusual number of large claims (resulting in a relatively high probability level) it may be more likely that the payment pattern will be extended. Large lifetime workers compensation claims are typically paid out over an extended period. This consideration has resulted in some analysts assuming that the discounted losses associated with various probability levels (the present value of the losses associated with the probability level) are simply equal to the undiscounted amounts multiplied by the best estimate of the present value factor (based on the premise that this assumption is conservative). Given this assumption, the discounted probability level amounts could be computed by multiplying the undiscounted amounts by a uniform factor of .90 (see Exhibit 28).

Excess Insurance Issues

It appears that the most common types of excess insurance for workers compensation are per occurrence coverage and aggregate coverage. Per occurrence coverage provides coverage in excess of a dollar threshold per occurrence. Aggregate coverage limits the entity's exposure in total for a self-insured year. It provides coverage in excess of a dollar threshold for all claims occurring in a self-insured year.

Excess insurance reduces the variability associated with the retained claim liabilities. The per occurrence coverage limits individual claim amounts that are retained; therefore, for a large claim only the first $\$x$ will be retained. The aggregate coverage limits the retained losses for any one self-insured year and therefore provides an upper limit to the retained exposure (ignoring credit risk and policy limits being exhausted).

Exhibit 29 displays the effect of the per occurrence excess insurance on the distribution of costs for the upcoming self-insurance year. The exhibit displays the probability level amounts for a risk with $500,000 of expected unlimited losses, both with and without a $50,000 per occurrence loss limit. For the latter, we have added a provision for the cost of excess insurance. For illustrative purposes, we have assumed that the excess insurer would include a 25% loading of the undiscounted expected value to determine premium.[11]

If the employer does not purchase per occurrence excess insurance, the actual claim payments are projected to exceed $980,000 one year in every ten or 10% of the time. However if the employer purchases excess insurance, the corresponding probability for approximately $980,000 of insurance costs is 5%, or one year in every twenty. Exhibit 30 graphically displays the distribution of loss outcomes assuming the employer purchased per occurrence excess insurance. In comparing Exhibit 30 and Exhibit 26 it should be noted that:

[11]While the 25% on its face appears low (for expenses, profit, and a risk margin), it should be noted that excess workers compensation payments are made over an extended period. Therefore, if the excess insurer reflects the time value of money, the discounted expected losses will be significantly less than the undiscounted amounts.

- the distribution of insurance costs is less dispersed for the employer that purchases excess insurance, and
- the employer is forgoing the possibility of very favorable insurance costs (with the purchase of excess insurance) for reducing the possibility of adverse loss experience.

7. CONCLUSION

This paper has outlined several methods that can be used to establish funding levels for an entity that retains its workers compensation exposure. In addition we have discussed:

- benefit and cost considerations involved in self-insuring,
- regulatory requirements associated with self-insuring, and
- funding level considerations.

We believe that the concepts outlined in this paper can assist an entity in:

- structuring a self-insurance program (or deciding whether to self-insure), and
- funding for a self-insurance program.

REFERENCES

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[3] Lilly, C. C. and H. G. Boggs, "The Self Insurance Manual," NILS Publishing Company.

[4] Brown, Brian Z., "Pricing for Credit Exposure," *PCAS* LXXIX, 1992, pp. 186–214.

[5] Gillam, William R., "Retrospective Rating: Excess Loss Factors," *PCAS* LXXVIII, 1991, pp. 1–40.

[6] Bornhuetter, R. L., and R. E. Ferguson, "The Actuary and IBNR," *PCAS* LIX, 1972, pp. 181–195.

[7] Bickerstaff, David, "Hospital Self-Insurance Funding: A Monte-Carlo Approach," *Casualty Actuarial Society Forum*, Spring 1989, pp. 89–138.

[8] Actuarial Standards Board, Actuarial Standard of Practice No. 20, "Discounting of Property and Casualty Loss and Loss Adjustment Expense Reserves," American Academy of Actuaries, April 1992.

EXHIBIT 1

ABC COMPANY PAID LOSSES*—MEDICAL AND INDEMNITY COMBINED ($000'S)

Self Insured	Months of Development									
Year	12	24	36	48	60	72	84	96	108	120
1984	145	711	900	1,001	1,100	1,113	1,124	1,130	1,130	1,130
1985	201	845	1,011	1,101	1,151	1,170	1,170	1,170	1,170	
1986	290	1,011	1,294	1,412	1,480	1,500	1,513	1,519		
1987	359	1,210	1,421	1,513	1,570	1,590	1,600			
1988	450	1,445	1,551	1,701	1,851	1,940				
1989	680	1,599	1,819	2,001	2,100					
1990	750	2,150	2,445	2,550						
1991	980	2,050	2,500							
1992	1,325	2,700								
1993	1,522									

*Including ALAE.

Development Factors

Self Insured Year	12–24	24–36	36–48	48–60	60–72	72–84	84–96	96–108	108–120	
					Months of Development					
1984	4.903	1.266	1.112	1.099	1.012	1.010	1.005	1.000	1.000	
1985	4.204	1.196	1.089	1.045	1.017	1.000	1.000	1.000		
1986	3.486	1.280	1.091	1.048	1.014	1.009	1.004			
1987	3.370	1.174	1.065	1.038	1.013	1.006				
1988	3.211	1.073	1.097	1.088	1.048					
1989	2.351	1.138	1.100	1.049						
1990	2.867	1.137	1.043							
1991	2.092	1.220								
1992	2.038									
Average	3.169	1.186	1.085	1.061	1.021	1.006	1.003	1.000	1.000	
Column Sum	2.649	1.174	1.080	1.060	1.023	1.006	1.003	1.000	1.000	
Selected Age to Age Factor	2.200	1.174	1.080	1.060	1.023	1.011	1.005	1.002	1.001	
Selected Cumulative Factor	3.113	1.415	1.205	1.116	1.053	1.029	1.018	1.013	1.011	1.010 Tail

Note: In selecting factors, we would suggest reviewing ABC Company data as well as development factors published by the NCCI for State X.
Note: The most recent diagonal has been brought to year end based on data through September 30.

EXHIBIT 2

ABC COMPANY INCURRED LOSSES*—MEDICAL AND INDEMNITY COMBINED ($000'S)

Self Insured	Months of Development									
Year	12	24	36	48	60	72	84	96	108	120
1984	400	800	990	1,111	1,115	1,125	1,130	1,130	1,130	1,130
1985	510	902	1,096	1,151	1,160	1,170	1,170	1,190	1,190	
1986	790	1,180	1,396	1,500	1,540	1,560	1,500	1,519		
1987	901	1,391	1,501	1,559	1,570	1,590	1,690			
1988	1,120	1,460	1,661	1,842	1,950	2,000				
1989	1,401	1,701	1,900	2,011	2,110					
1990	1,761	2,340	2,465	2,550						
1991	1,700	2,316	2,675							
1992	2,400	2,995								
1993	2,600									

*Including ALAE.

Development Factors

Self Insured Year	Months of Development									
	12–24	24–36	36–48	48–60	60–72	72–84	84–96	96–108	108–120	
1984	2.000	1.238	1.122	1.004	1.009	1.004	1.000	1.000	1.000	
1985	1.769	1.215	1.050	1.008	1.009	1.000	1.017	1.000		
1986	1.494	1.183	1.074	1.027	1.013	0.962	1.013			
1987	1.544	1.079	1.039	1.007	1.013	1.063				
1988	1.304	1.138	1.109	1.059	1.026					
1989	1.214	1.117	1.058	1.049						
1990	1.329	1.053	1.034							
1991	1.362	1.155								
1992	1.248									
Average	1.474	1.147	1.070	1.026	1.014	1.007	1.010	1.000	1.000	
Column Sum	1.373	1.132	1.065	1.030	1.015	1.008	1.010	1.000	1.000	
Selected Age to Age Factor	1.373	1.132	1.065	1.030	1.015	1.008	1.005	1.000	1.000	
Selected Cumulative Factor	1.753	1.277	1.128	1.059	1.028	1.013	1.005	1.000	1.000	1.000 Tail

Note: In selecting factors, we would suggest reviewing ABC Company data as well as development factors published by the NCCI for State X.
Note: The most recent diagonal has been brought to year end based on data through September 30.

EXHIBIT 3

ABC COMPANY INDEMNITY INCURRED CLAIM COUNTS*

Self Insured Year	Months of Development										Ultimate Claim Counts	Ultimate Frequency Per $Million of Payroll**
	12	24	36	48	60	72	84	96	108	120		
1984	382	400	409	409	409	409	409	409	409	409	409	2.525
1985	400	412	418	418	418	418	418	418	418		418	2.416
1986	444	462	480	480	480	480	480	480			480	2.619
1987	469	487	500	501	502	502	502				502	2.619
1988	523	548	566	580	584	584					584	2.925
1989	559	580	590	591	591						591	2.947
1990	600	613	620	622							623	2.937
1991	657	680	688								693	3.124
1992	700	725									745	3.200
1993	761										811	3.303

*Claims that either have closed with an indemnity payment or have an indemnity reserve.
**These frequencies imply an exponential trend of 3.7% per year.

Development Factors

Self Insured Year	Months of Development 12–24	24–36	36–48	48–60	60–72	72–84	84–96	96–108	108–120	
1984	1.047	1.023	1.000	1.000	1.000	1.000	1.000	1.000	1.000	
1985	1.030	1.015	1.000	1.000	1.000	1.000	1.000	1.000		
1986	1.041	1.039	1.000	1.000	1.000	1.000	1.000			
1987	1.038	1.027	1.002	1.002	1.000	1.000				
1988	1.048	1.033	1.025	1.007	1.000					
1989	1.038	1.017	1.002	1.000						
1990	1.022	1.011	1.003							
1991	1.035	1.012								
1992	1.036									
Average	1.037	1.022	1.005	1.001	1.000	1.000	1.000	1.000	1.000	
Column Sum	1.037	1.021	1.005	1.002	1.000	1.000	1.000	1.000	1.000	
Selected Age to Age Factor	1.037	1.021	1.005	1.002	1.000	1.000	1.000	1.000	1.000	
Selected Cumulative Factor	1.066	1.028	1.007	1.002	1.000	1.000	1.000	1.000	1.000	1.000 Tail

EXHIBIT 4

ABC COMPANY INCURRED LOSS SEVERITY TRIANGLE

Self Insured	Months of Development										Ultimate
Year	12	24	36	48	60	72	84	96	108	120	Severity*
1984	1,047	2,000	2,421	2,716	2,726	2,751	2,763	2,763	2,763	2,763	2,763
1985	1,275	2,189	2,622	2,754	2,775	2,799	2,799	2,847	2,847		2,847
1986	1,779	2,554	2,908	3,125	3,208	3,250	3,125	3,165			3,165
1987	1,921	2,856	3,002	3,112	3,127	3,167	3,367				3,400
1988	2,141	2,664	2,935	3,176	3,339	3,425					3,483
1989	2,506	2,933	3,220	3,403	3,570						3,681
1990	2,935	3,817	3,976	4,100							4,333
1991	2,588	3,406	3,888								4,366
1992	3,429	4,131									5,168
1993	3,417										5,784

*Based on an exponential trend, we selected an annual trend factor for severity of 8.3%.

Development Factors

Self Insured Year	Months of Development									
	12–24	24–36	36–48	48–60	60–72	72–84	84–96	96–108	108–120	
1984	1.910	1.210	1.122	1.004	1.009	1.004	1.000	1.000	1.000	
1985	1.717	1.198	1.050	1.008	1.009	1.000	1.017	1.000		
1986	1.435	1.139	1.074	1.027	1.013	0.962	1.013			
1987	1.487	1.051	1.037	1.005	1.013	1.063				
1988	1.244	1.101	1.082	1.051	1.026					
1989	1.170	1.098	1.057	1.049						
1990	1.301	1.042	1.031							
1991	1.316	1.142								
1992	1.205									
Average	1.421	1.123	1.065	1.024	1.014	1.007	1.010	1.000	1.000	
Column Sum	1.353	1.114	1.062	1.025	1.014	1.007	1.010	1.000	1.000	
Selected Age to Age Factor	1.353	1.114	1.062	1.025	1.014	1.007	1.010	1.000	1.000	
Selected Cumulative Factor	1.693	1.251	1.123	1.057	1.031	1.017	1.010	1.000	1.000	1.000 Tail

EXHIBIT 5

ABC COMPANY PAYROLL BY CLASS CODE ($000'S)

Self Insured Year	Class Code A	Class Code B	Total
1984	131,004	31,004	162,008
1985	140,001	33,001	173,002
1986	147,762	35,492	183,254
1987	154,672	37,001	191,673
1988	159,843	39,836	199,679
1989	160,510	40,001	200,511
1990	169,452	42,671	212,123
1991	177,001	44,806	221,807
1992	185,811	47,001	232,812
1993	196,152	49,398	245,550
1994*	203,998	51,374	255,372

*Based on 1993 payroll trended 4%.

EXHIBIT 6

ABC COMPANY PROJECTION OF ULTIMATE LOSSES PAID LOSS PROJECTION ($000'S)

Self Insured Year	Paid Loss	Cumulative Development Factor	Projected Ultimate Losses
1984	1,130	1.010	1,141
1985	1,170	1.011	1,183
1986	1,519	1.013	1,539
1987	1,600	1.018	1,629
1988	1,940	1.029	1,996
1989	2,100	1.053	2,211
1990	2,550	1.116	2,846
1991	2,500	1.205	3,013
1992	2,700	1.415	3,821
1993	1,522	3.113	4,738
Total	18,731		24,117

EXHIBIT 7

ABC COMPANY PROJECTION OF ULTIMATE LOSSES INCURRED LOSS PROJECTION ($000'S)

Self Insured Year	Incurred Loss	Cumulative Development Factor	Projected Ultimate Losses
1984	1,130	1.000	1,130
1985	1,190	1.000	1,190
1986	1,519	1.000	1,519
1987	1,690	1.005	1,698
1988	2,000	1.013	2,026
1989	2,110	1.028	2,169
1990	2,550	1.059	2,700
1991	2,675	1.128	3,017
1992	2,995	1.277	3,825
1993	2,600	1.753	4,558
Total	20,459		23,833

EXHIBIT 8

ABC COMPANY PROJECTION OF ULTIMATE LOSSES AVERAGE SEVERITY PROJECTION

Self Insured Year	Projected Ultimate Severity	Projected Ultimate Incurred Claims	Projected Ultimate Loss ($000's)
1984	2,763	409	1,130
1985	2,847	418	1,190
1986	3,165	480	1,519
1987	3,400	502	1,707
1988	3,483	584	2,034
1989	3,681	591	2,175
1990	4,333	623	2,699
1991	4,366	693	3,026
1992	5,168	745	3,850
1993	5,784	811	4,691
Total			24,021

EXHIBIT 9

ABC COMPANY PROJECTION OF ULTIMATE LOSSES BASED ON NCCI LOSTS COSTS

	Class Code = A			Class Code = B			
Self Insured Year	Class Payroll ($000's)	Loss Cost*	Expected Losses ($000's)**	Class Payroll ($000's)	Loss Cost*	Expected Losses ($000's)**	Total Expected Losses ($000's)
1990	169,452	1.23	2,081	42,671	2.08	889	2,970
1991	177,001	1.31	2,326	44,806	2.23	998	3,324
1992	185,811	1.41	2,613	47,001	2.38	1,121	3,734
1993	196,152	1.50	2,951	49,398	2.55	1,260	4,211
1994***	203,998	1.61	3,284	51,374	2.73	1,403	4,687

*The expense components of the rates have been stripped out.
**Subject to rounding error.
***Based on 1993 payroll trended at 4%.
Note: The loss costs for the prior years have been de-trended based on the NCCI trend factor.

EXHIBIT 10

ABC COMPANY PROJECTION OF ULTIMATE LOSSES
TRENDED PURE PREMIUM APPROACH
SELF-INSURED YEAR 1992–1994

Self Insured Year	Total Payroll (000's)	Selected Ultimate Loss* ($000's)	Pure Premium Per $100 Payroll	Pure Premium Trended to 1992**	Selected Pure Premium	Selected Ultimate Loss ($000's)
1988	199,679	2,011	1.007	1.370		
1989	200,511	2,190	1.092	1.376		
1990	212,123	2,773	1.307	1.524		
1991	221,807	3,015	1.359	1.468		
1992	232,812				1.435***	3,341
1993	245,550				1.550	3,806
1994	255,372				1.673****	4,272

*Based on an average of the paid and incurred projections.
**Selected Trend Factor of 8.00% based on analyzing industry data.
***1.435 = {(1.37 + 1.376 + 1.524 + 1.468)/4}.
****1.673 = (1.435)*(1.08)^2.

EXHIBIT 11

ABC COMPANY SELECTION OF ULTIMATE LOSSES BORNHUETTER–FERGUSON PROJECTION METHOD ($000'S)

Self Insured Year	Preliminary Selected Ultimate Loss*	Expected** Percentage Unreported	Expected IBNR	Incurred Loss	Indicated Ultimate
1992	3,734	21.69%	810	2,995	3,805
1993	4,211	42.96%	1,809	2,600	4,409

*Based on the expected loss method from Exhibit 9.
**Selected from Exhibit 2. The expected percentage unreported = (1 – (1/LDF)).

EXHIBIT 12

ABC COMPANY PROJECTION OF ULTIMATE LOSSES EXCESS OF 500,000 PER CLAIM ($000'S)

Self Insured Year	Expected* Unlimited Losses	Excess** Ratio	Projected Excess Losses
1990	2,970	0.030	89
1991	3,324	0.032	106
1992	3,734	0.034	127
1993	4,211	0.037	156
1994	4,687	0.039	183

*From Exhibit 9.
**From Exhibit 2 of Gillam [5]. As a note, we have assumed that the factors are appropriate for the 1990 year and adjusted the excess ratio by adjusting the loss limit for inflationary factors for the more recent years. For example, a $500,000 loss limit in 1990 is equivalent to a $450,000 loss limit in 1992.

EXHIBIT 13

ABC COMPANY PROJECTION OF EXCESS LOSSES BORNHUETTER–FERGUSON METHOD ($000'S)

At September 30, 1993

Self Insured Year	Projected Excess Losses*	Expected Percentage of Excess Losses Unreported	Estimated IBNR Reserves	Reported Case Incurred	Projected Ultimate Excess Losses
1990	89	55%	49	0	49
1991	106	70%	74	300	374
1992	127	80%	102	0	102
1993	156	95%	148	0	148
1994	183	100%	183	0	183

*From Exhibit 12.

Note: For purposes of this paper, it is assumed that the entity will not have any excess claims for self-insured years 1989 and prior.

EXHIBIT 14

ABC COMPANY SELECTION OF ULTIMATE LOSSES ($000'S)

	Indicated Ultimate Gross Loss Based on:						(A)	(B)	(C)	(A)–(B)–(C)
Self Insured Year	Paid Loss Projection	Incurred Loss Projection	Average Severity Projection	Expected Loss Method	Trended Pure Prem Approach	Bornhuetter-Ferguson Projection	Selected Ultimate Gross Loss	Projected Excess Recoveries	Retained Paid Losses	Total Retained Reserves
1984	1,141	1,130	1,130	xxxx	xxxx	xxxx	1,136	0	1,130	6
1985	1,183	1,190	1,190	xxxx	xxxx	xxxx	1,187	0	1,170	17
1986	1,539	1,519	1,519	xxxx	xxxx	xxxx	1,529	0	1,519	10
1987	1,629	1,698	1,707	xxxx	xxxx	xxxx	1,664	0	1,600	64
1988	1,996	2,026	2,034	xxxx	xxxx	xxxx	2,011	0	1,940	71
1989	2,211	2,169	2,175	xxxx	xxxx	xxxx	2,190	0	2,100	90
1990	2,846	2,700	2,699	2,970	xxxx	xxxx	2,804	49	2,550	205
1991	3,013	3,017	3,026	3,324	xxxx	xxxx	3,451	374	2,500	577
1992	3,821	3,825	3,850	3,734	3,341	3,805	3,807	102	2,700	1,005
1993	4,738	4,558	4,691	4,211	3,806	4,409	4,521	148	1,522	2,851
Total	24,117	23,833	24,021				24,300	673	18,731	4,896

EXHIBIT 15

ABC COMPANY PROJECTED ULTIMATE LOSSES FOR SELF INSURED YEAR 1994 ($000'S)

(1) Self Insured Year	(2) Expected Loss Method	(3) Trended Pure Premium Method	(4) Selected Gross Losses	(5) Projected Excess Losses	(6) Projected Retained Losses
1994	4,687	4,272	4,480	183	4,297
		Unpaid Claim Liability @ 12/31/93*			4,896
				Required Fund	9,193

Col. 2: From Exhibit 9.
Col. 3: From Exhibit 10.
Col. 5: From Exhibit 12.
*From Exhibit 14.

EXHIBIT 16

XYZ MANUFACTURING COMPANY RETAINED WORKERS COMPENSATION LOSS EXPERIENCE

Accident Year	Medical Paid	Medical Reserves as of 12/31/93	Indemnity Paid	Indemnity Reserves as of 12/31/93	Total Paid	Total Reserves as of 12/31/93
1988	N/A	$ 311,429	N/A	$ 467,143	N/A	$ 778,572
1989	N/A	80,355	N/A	120,533	N/A	200,888
1990	N/A	128,002	N/A	192,003	N/A	320,005
1991	N/A	180,331	N/A	270,497	N/A	450,828
1992	N/A	460,633	N/A	690,949	N/A	1,151,582
1993	593,137	470,377	400,991	875,066	994,128	1,345,443
Total	$593,137	$1,631,127	$400,991	$2,616,191	$994,128	$4,247,318

Note: Values have been projected through year-end based on data through September 30.

Calendar Year	Paid Medical Losses	Paid Indemnity Losses	Total Paid Losses
1988	$ 200,663	$ 209,649	$ 410,312
1989	500,794	359,415	860,209
1990	670,651	490,477	1,161,128
1991	700,133	600,702	1,300,835
1992	790,143	800,853	1,590,996
1993	950,949	1,100,759	2,051,708
Total	$3,813,333	$3,561,855	$7,375,188

EXHIBIT 17

DERIVATION OF CASE DEVELOPMENT FACTORS BASED ON NCCI DATA FOR A SPECIFIC STATE

	Cumulative Medical Development Factors			Cumulative Indemnity Development Factors		
Age	Paid	Incurred	Case	Paid	Incurred	Case
72	1.177	1.069	1.752	1.218	1.043	1.304
60	1.203	1.070	1.633	1.288	1.058	1.325
48	1.237	1.076	1.584	1.416	1.069	1.282
36	1.299	1.074	1.427	1.659	1.092	1.269
24	1.463	1.103	1.419	2.197	1.170	1.364
12	2.611	1.346	1.714	4.297	1.517	1.799

EXHIBIT 18

XYZ MANUFACTURING COMPANY CASE DEVELOPMENT METHOD

Accident Year	Medical Reserves as of 12/31/93	Medical Case Development Factor	Indicated Total Unpaid Medical Loss as of 12/31/93
1988	$ 311,429	1.752	$ 545,615
1989	80,355	1.633	131,232
1990	128,002	1.584	202,746
1991	180,331	1.427	257,373
1992	460,633	1.419	653,445
1993	470,377	1.714	806,299
Total	$1,631,127		$2,596,710

EXHIBIT 19

XYZ MANUFACTURING COMPANY CASE DEVELOPMENT METHOD

Accident Year	Indemnity Reserves as of 12/31/93	Indemnity Case Development Factor	Indicated Total Unpaid Indemnity Loss as of 12/31/93
1988	$ 467,143	1.304	$ 609,038
1989	120,533	1.325	159,682
1990	192,003	1.282	246,065
1991	270,497	1.269	343,311
1992	690,949	1.364	942,227
1993	875,066	1.799	1,574,347
Total	$2,616,191		$3,874,670

EXHIBIT 20

SELECTED PAYMENT PATTERNS BASED ON NCCI DATA FOR A SPECIFIC STATE

	Paid Losses as a Percent of Ultimate Losses			
	Medical		Indemnity	
Age	Cumulative	Incremental	Cumulative	Incremental
72	0.850	$0.018 = P^5$	0.823	$0.047 = P^5$
60	0.831	$0.023 = P^4$	0.776	$0.070 = P^4$
48	0.808	$0.039 = P^3$	0.706	$0.103 = P^3$
36	0.770	$0.086 = P^2$	0.603	$0.148 = P^2$
24	0.684	$0.301 = P^1$	0.455	$0.222 = P^1$
12	0.383	$0.383 = P^0$	0.233	$0.233 = P^0$

EXHIBIT 21

XYZ MANUFACTURING COMPANY CALENDAR YEAR INCREMENTAL PAYMENT METHOD
MEDICAL LOSSES

Accident Year	Trend (in Years)	Calendar Year Incremental Payments			
		1991	1992	1993	1994 & Subsequent
1988 AY0	0	0.039	0.023	0.018	0.150
1989 AY1	1	0.099	0.044	0.026	0.193
1990 AY2	2	0.393	0.113	0.050	0.251
1991 AY3	3	0.573	0.450	0.129	0.345
1992 AY4	4		0.656	0.515	0.542
1993 AY5	5			0.750	1.209
Total		1.104	1.286	1.489	2.690

	Indication 1	Indication 2	Indication 3	Selected
Calendar Year Unpaid Loss Factor:	2.436*	2.092	1.806	
Calendar Year Paid Losses:	700,133	790,143	950,949	
Indicated Unpaid Medical Losses @ 12/31/93:	1,705,762	1,652,842	1,717,392	1,691,999

Loss Trend:	10.0%
Exposure Trend:	4.0%
r =	14.4%

*2.436 = 2.690/1.104 or the sum of all future payments (1994 and subsequent) for accident years 1988–1993 divided by calendar year 1991 payments on accident years 1988–1991.

EXHIBIT 22

XYZ MANUFACTURING COMPANY CALENDAR YEAR INCREMENTAL PAYMENT METHOD INDEMNITY LOSSES

Accident Year	Trend (in Years)	Calendar Year Incremental Payments			
		1991	1992	1993	1994 & Subsequent
1988 AY0	0	0.103	0.070	0.047	0.177
1989 AY1	1	0.158	0.111	0.075	0.240
1990 AY2	2	0.255	0.169	0.119	0.337
1991 AY3	3	0.286	0.273	0.181	0.488
1992 AY4	4		0.306	0.293	0.717
1993 AY5	5			0.328	1.082
Total		0.803	0.930	1.043	3.041

	Indication 1	Indication 2	Indication 3	Selected
Calendar Year Unpaid Loss Factor:	3.788*	3.270	2.916	
Calendar Year Paid Losses:	600,702	800,853	1,100,759	
Indicated Unpaid Indemnity Losses @ 12/31/93:	2,275,596	2,618,481	3,209,583	2,701,220
Indicated Unpaid Medical Losses @ 12/31/93:	1,705,762	1,652,842	1,717,392	
Indicated Total Unpaid Losses @ 12/31/93:	3,981,358	4,271,323	4,926,975	

Loss Trend: 3.0%
Exposure Trend: 4.0%
r = 7.1%

*3.788 = 3.041/.803 or the sum of all future payments (1994 and subsequent) for accident years 1988–1993 divided by calendar year 1991 payments on accident years 1988–1991.

EXHIBIT 23

XYZ MANUFACTURING COMPANY DE-TRENDED BORNHUETTER-FERGUSON METHOD

	Indemnity			Medical					
Accident Year	Selected Ultimates*	% Unreported	Estimated IBNR	Selected Ultimates**	% Unreported	Estimated IBNR	Estimated IBNR	Case Reserves	Unpaid Claim Liability
1993	1,800,000	34.08%	613,448	1,500,000	25.71%	385,587	999,035	1,345,443	2,344,478
1992	1,680,672	14.53%	244,200	1,311,189	9.34%	122,441	366,641	1,151,582	1,518,223
1991	1,569,255	8.42%	132,208	1,146,144	6.89%	78,971	211,179	450,828	662,007
1990	1,465,224	6.45%	94,575	1,001,874	7.06%	70,764	165,339	320,005	485,344
1989	1,368,090	5.48%	74,999	857,764	6.54%	57,293	132,292	200,888	333,180
1988	1,277,395	4.12%	52,663	765,528	6.45%	49,412	102,075	778,572	880,647
Total			1,212,094			764,468			6,223,880

* Indemnity Trend Factor: 7.1% ** Medical Trend Factor: 14.4%

Ultimate Loss Projection
Accident Year 1993

Indemnity	Amount	LDF	Ultimate	Medical	Amount	LDF	Ultimate
Paid	400,991	4.297	1,723,058	Paid	593,137	2.611	1,548,681
Incurred	1,276,057	1.517	1,935,778	Incurred	1,063,514	1.346	1,431,490
Selected			1,800,000	Selected			1,500,000

EXHIBIT 24

XYZ MANUFACTURING COMPANY PROJECTED ULTIMATE LOSSES FOR SELF-INSURED YEAR 1994

	Indemnity	Medical	Total
Selected 1993 Ultimate Loss	1,800,000	1,500,000	3,300,000
Selected Annual Trend Factor	1.03	1.10	
Anticipated Exposure Growth	1.04	1.04	
Ultimate Losses Self Insured Year 1994	1,928,160	1,716,000	3,644,160

EXHIBIT 25

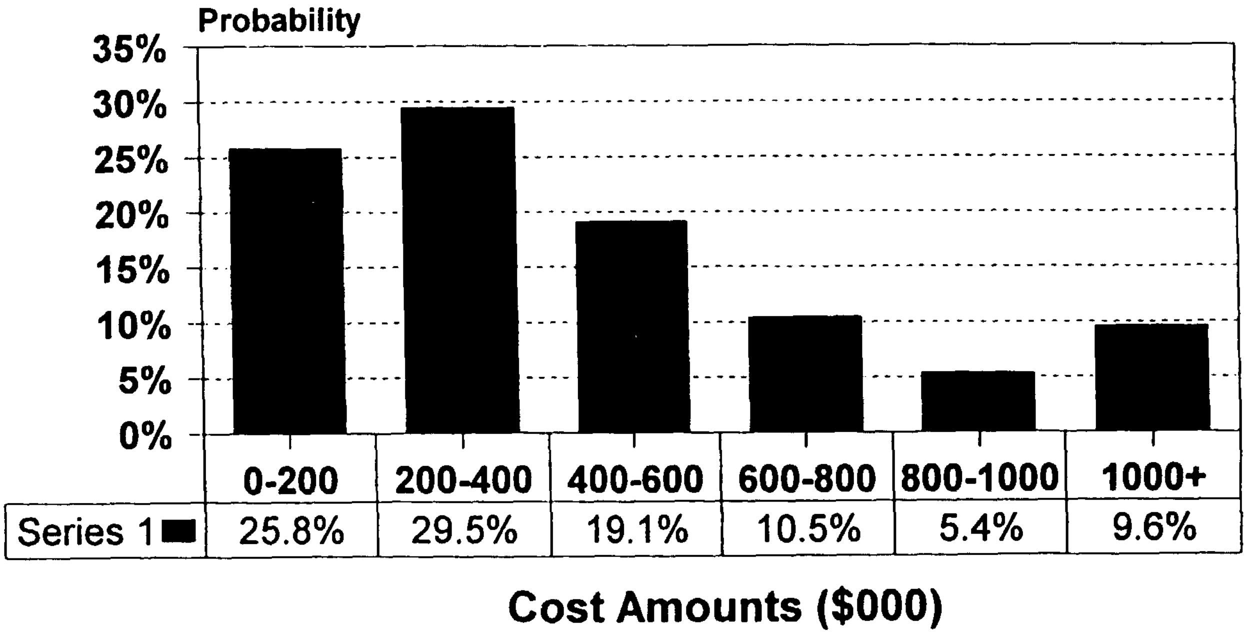
XYZ MANUFACTURING COMPANY SELECTED FUND AT 12/31/93 ($000'S)

1)	Estimated Unpaid Claim Liability— Case Development Method	6,471
2)	Estimated Unpaid Claim Liability— Incremental Payment Method	4,393
3)	Estimated Unpaid Claim Liability— De-Trended Bornhuetter–Ferguson Method	6,224
4)	Selected Unpaid Claim Liability as of December 31, 1993 {Average[(1) + (2) + (3)]}	5,696
5)	Selected Claim Costs for 1994	3,644
6)	Required Fund at 12/31/93 (4) + (5)	9,340

EXHIBIT 26

PROBABILITY DISTRIBUTION OF LOSSES
EXPECTED UNLIMITED LOSSES = $500,000

No Per Occurrence Loss Limitation



For Illustrative Purposes Only

EXHIBIT 27

PROBABILITY DISTRIBUTION OF LOSSES
EXPECTED UNLIMITED LOSSES = $500,000

No Per Occurrence Loss Limitation

Probability Level	Loss Amount	Relativity to Expected Values
Exp value	$ 500,000	1.00
75%	605,000	1.21
90%	980,000	1.96
95%	1,425,000	2.85

EXHIBIT 28

ABC COMPANY WORKERS COMPENSATION PROJECTED PAYOUT PATTERN

Number of Years From Inception of the Exposure	Cumulative Loss Payments	Incremental Loss Payments	Discounted Incremental Loss Payments
1	32	32	31
2	71	39	35
3	83	12	11
4	90	7	5
5	95	5	4
6	97	2	2
7	98	1	1
8	99	0	0
9	99	0	0
10	99	0	0
11	99	0	0
12	100	0	0
13	100	0	0
Total		100	90

Discount @ 6.0%

Discount Factor 0.90

EXHIBIT 29

CONFIDENCE LEVEL ANALYSIS

Expected Losses = 500,000
No Per Occurrence Loss Limitation

Probability Level	Loss Amount	Relativity to Expected Value
Expected Value	$ 500,000	1.00
75%	605,000	1.21
90%	980,000	1.96
95%	1,425,000	2.85

Expected Ultimate Losses = 500,000
Per Occurrence Loss Limitation = 50,000

Probability Level	Loss* Amount	Expected Excess	Total Insurance Costs	Relativity To Expected Value
Expected Value	$321,000	223,750	544,750	1.00
75%	398,040	223,750	621,790	1.14
90%	587,430	223,750	811,180	1.49
95%	747,930	223,750	971,680	1.78

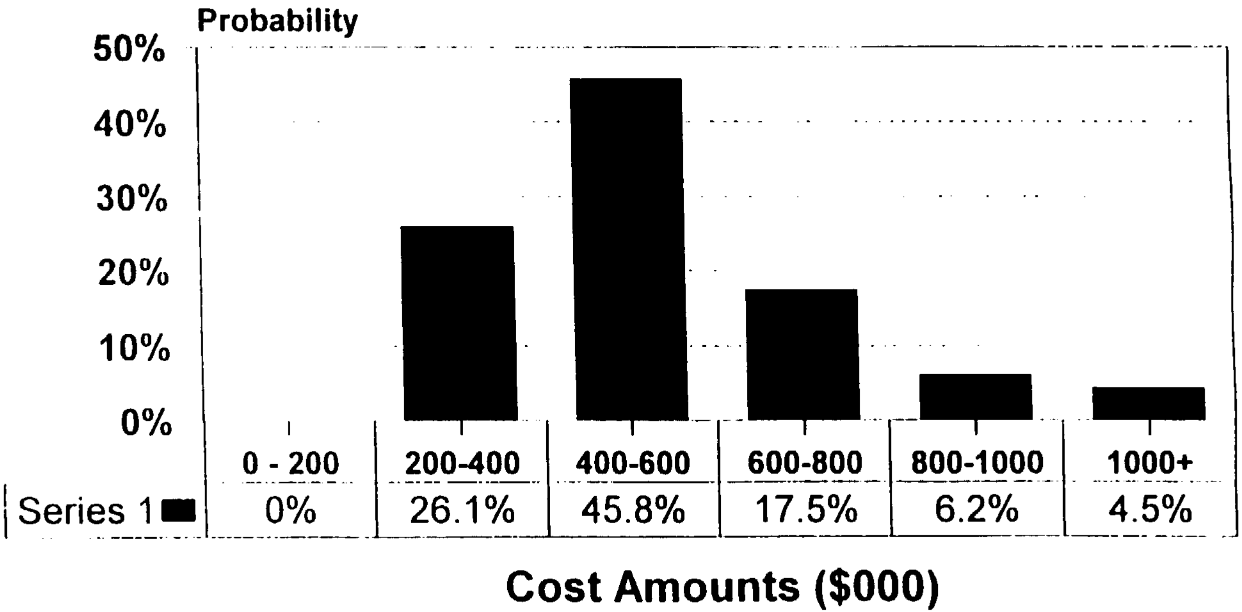
*Excludes 179,000 of expected excess losses which based on a 25% loading results in an excess premium amount of 223,750.

For Illustrative Purposes Only

EXHIBIT 30

PROBABILITY DISTRIBUTION OF LOSSES
EXPECTED UNLIMITED LOSSES = $500,000

Per Occurrence Retention of $50,000



For Illustrative Purposes Only

MEASUREMENT OF ASBESTOS BODILY INJURY LIABILITIES

SUSAN L. CROSS AND JOHN P. DOUCETTE

Abstract

This paper presents a model for projecting an insurer's or reinsurer's potential asbestos bodily injury (BI) liabilities through an analysis of exposed policy limits. The model projects the ground-up aggregate liabilities of individual insureds, allocates those liabilities to policy years, and carves out the portion of the liabilities falling in the layers of coverage written by the insurer or reinsurer. That is, the underlying process of claim filings against the insureds is modeled and then compared to the insurer's or reinsurer's identified policy exposures.

1. INTRODUCTION

This paper presents a methodology for estimating an insurer's or reinsurer's potential liabilities from asbestos-related bodily injury (BI) claims associated with notified exposures. Property damage (PD) claims resulting from asbestos are not considered in this model. The approach is a policy limits analysis on a sample group of insureds.

The first step in developing the methodology is obtaining an understanding of the nature of the potential liabilities. Thus, our paper begins with a brief discussion of the significant historical developments relating to the emergence of asbestos-related BI claims. Section 2 presents historical uses of asbestos, problems arising from asbestos use, legal issues related to the asbestos problem, and insurance issues emerging from asbestos litigation. This information is important in understanding how these claims differ from traditional products and general liability BI claims and, therefore, why traditional actuarial projection techniques are not directly applicable. Section 3 describes the asbestos diseases: mesothelioma, lung and other cancers, asbestosis, and pleural plaques.

Knowledge of the unique characteristics of these diseases is necessary to understand the legal issues surrounding asbestos BI insurance coverage litigation. Although this paper provides an overview of relevant legal issues, it is by no means a comprehensive review of such issues. Individuals involved in handling asbestos claims and analyzing asbestos liabilities should seek legal advice as necessary.

Section 4 explains the motivation for the model presented in this paper as well as the requirements of any methodology that projects asbestos BI liabilities. Section 5 presents details on the steps in the asbestos BI model. The steps may be grouped into the following categories: 1) determine the sample group and collect data; 2) adjust the sample group data; 3) use the model to estimate the insurance or reinsurance company's liabilities for the sample group; 4) conduct sensitivity testing of model assumptions; and 5) extrapolate the model results to all insureds. To facilitate the discussion, we run a fictitious reinsurer, ABC Re, through each of the steps of the model. Finally, Section 6 discusses strengths and weaknesses of the model and identifies areas related to asbestos liability projections requiring further research.

2. BACKGROUND

Asbestos and Its Uses

What is asbestos? It is a generic term referring to a variety of naturally occurring minerals which share similar properties. There are six major recognized species of asbestos: chrysotile (white asbestos), amosite (brown asbestos), crocidolite (blue asbestos), anthophyllite, tremolite, and actinolite. These six species of asbestos come in two general forms: chrysotile comes in the serpentine form, and the other five come in the amphibole form [5, p. I-1-1]. Chrysotile represents over 95% of all asbestos used in buildings [13]. Though each variety of asbestos has unique characteristics, in general the asbestos minerals form fibers which are incombustible, flexible, durable, strong, and resistant to heat, corrosion, and wear. Because of these properties, asbestos was targeted for use in an estimated 3,000 commercial, public, and industrial applications [5, p. I-1-2]. Examples include building insulation, pipe coverings, wire coatings, brake linings, roofing products, and flooring products. By the year 1900, asbestos was in use in the building construction industry. Asbestos was also used extensively in World War II ship building. Following the war, there was significant expansion of the use of asbestos products in construction and manufacturing. Exhibit 1 provides details on the uses and composition of asbestos-containing building products as of the mid-1980s. "Friable" means that the material can be reduced to powder by hand pressure. Other commonly cited products in asbestos litigation include industrial ceramic furnace products, ceiling tiles, and heat protection equipment (e.g., gloves, blankets, jackets).

Problems Arising from Asbestos Use

The virtually indestructible nature of asbestos fibers, which makes it so attractive in commercial applications, causes asbestos to be a health risk to humans. When airborne asbestos fibers are inhaled into the lungs, they tend to persist indefinitely. Thus, exposure to asbestos dust has been the cause of such diseases as mesothelioma, lung cancer, asbestosis, and pleural plaques. Historically, the population with the greatest exposure to asbestos dust was workers involved in the production or installation of asbestos [12, pp. 21–52]. However, significant numbers of claims relate to other workers and bystanders in proximity to the asbestos products or operations.

The United States government did not take action to limit workers' exposure to asbestos until the early 1970s. Today, the permissible exposure limit for workers exposed to asbestos set forth in the Occupational Safety and Health Administration's (OSHA) Asbestos Regulations is less than one one-hundredth of the average exposure level of an insulation worker prior to 1970 [11; 12, pp. 99–120]. Table 1 shows the exposure standards over the past 20 years. In 1989, the Environmental Protection Agency (EPA) issued a ban on the manufacture, importation, processing, and distribution in commerce of asbestos in almost all products [4]. The legality of the ban is currently being addressed in court.

Legal Issues Related to the Asbestos Problem

Prior to the asbestos litigation onslaught during the 1970s and 1980s, asbestos-related occupational diseases were traditionally compensated through workers compensation insurance. Claims have been filed under workers compensation since the 1950s for asbestos-related disease; the first significant liability lawsuit against asbestos manufacturers was not filed until 1970.

The first significant asbestos-related lawsuit, Borel v. Fibreboard, filed in 1970 [1] and decided in 1973, was a landmark case in asbestos litigation. The decision held that a defendant manufacturer of insulation materials containing asbestos could be found strictly liable when: 1) an individual's disease was caused

TABLE 1

OSHA EXPOSURE STANDARDS

Year Enacted	Permissible Fibers/ Cubic Centimeter 8 Hour Average
1972	5 f/cc
1976	2 f/cc
1983	.5 f/cc
1988	.2 f/cc
1994	.1 f/cc

Source: OSHA

by exposure to the defendant's product, and 2) despite the defendant's knowledge of the risk, the defendant failed to provide adequate warning to the individual. In reaching its decision, the court found that asbestos was defective and "unreasonably dangerous" under the law. The court also stated that all asbestos manufacturers found liable would be "jointly and severally" liable for the entire injury if they are unable to demonstrate divisible harm. The burden of demonstrating divisible harm was placed on the manufacturer. The Borel decision opened the door for further actions against manufacturers. Since Borel, there has been an expansion of the theories of liability applied in asbestos litigation.

As additional claims were filed in the late 1970s, defendants pursued coverage for these claims under their products liability insurance policies. The long latency period of asbestos-related diseases (i.e., an asbestos-related disease may not manifest itself for 40 or more years after first exposure [12, pp. 104–106]) required legal decisions regarding the date of occurrence of asbestos-related BI in order to determine which insurance policies were triggered. Consequently, beginning in 1980, insurance coverage decisions were handed down by the courts. The decisions have generally followed either a continuous trigger (or injury-in-fact trigger interpreted similarly to a continuous trigger) or, in some cases, an exposure trigger. There has been one case decided on a manifestation trigger basis [3] and one case based on a combination of exposure and manifestation triggers [16]. Under the *continuous trigger* theory, injury is deemed to occur continuously from the first inhalation of the asbestos fibers through the manifestation of the disease. Thus, any and all policies in effect during this time period can be triggered and called upon to pay the claim. Under the *exposure trigger* theory, injury is assumed to occur only during the period of exposure to asbestos. Thus, the exposure theory triggers a subset of the policies triggered by the continuous theory. Under the *manifestation trigger* theory, no bodily injury occurs, and thus no insurance coverage is triggered, until the asbestos-related disease becomes reasonably capable of medical diagnosis. Thus, manifestation theory triggers policies in a single year [2, pp. 25–38].

Since the early 1980s, asbestos litigation has grown at a staggering rate. As of June 1991, there had been over 71,000 cases filed nationwide in federal courts. As of June 1992, there were over 120,000 additional lawsuits pending in state courts. Despite defendants' attempts to settle lawsuits, many still face tens of thousands of pending suits. Note that these are numbers of lawsuits, not numbers of plaintiffs. The number of plaintiffs is even higher, because some lawsuits are consolidations of hundreds or thousands of plaintiffs.

A plaintiff typically names several defendants in a suit, even dozens, so adding the reported number of claims for all defendants would overstate the total number of claims. Many defendants are being named in thousands of new cases each month. The asbestos litigation problem is not going away and cannot be ignored by potential defendants or their insurers [7, 15].

Insurance Coverage Issues

In practice, the method of handling claims and allocating loss and expense dollars to policies or self-insured periods is negotiated between the insured and its group of insurers. These negotiations are consistent with the applicable trigger theory. With the total filed claim count exceeding 200,000 for some defendants, such agreements are necessary for the efficient processing of claims. For purposes of this paper, we define the defendant's insurance coverage block as the years of agreed-upon coverage. That is, through negotiations and/or litigation, insureds generally reach agreement with some or all of their carriers as to which policy years will be triggered by asbestos claims. This block of policy years is referred to as the insured's coverage block. Some of the policy years may relate to periods of self-insurance for which an insured may be responsible. The coverage block forms a starting point for allocation of claim dollars to insurance coverage by defining the end points, i.e., the earliest and latest dates to be used in the allocation. Once the coverage block is agreed upon, a simplified procedure for sharing the costs (referred to as cost sharing agreements) may be negotiated by the responsible parties, or each individual claim may be allocated based on the particulars of the claim.

Given the predominant trigger theories, coverage blocks generally begin with commencement of asbestos product manufacture or distribution and end with either: 1) the end of the product's commercial use (often early to mid-1970s), or 2) the last year of products liability coverage without an asbestos exclusion (generally late 1970s or early to mid-1980s). However, it should be noted that negotiating an ending date for the coverage block is likely to be problematic as insurers seek to include years where policies contain asbestos exclusions or other protective underwriting measures such as per claim deductibles or SIRs. Such inclusion would result in a greater allocation to the insured, which the insured would no doubt resist. In most cases, the coverage block will span 15 or more years.

It is interesting to note that unlike the absolute pollution exclusion introduced into the Insurance Services Office's (ISO) Comprehensive General Liability (CGL) policy in 1986, an asbestos exclusion was not consistently incorporated into policies during a certain year. Rather, various forms of asbestos exclusions were phased in during the 1970s (generally *late* 1970s) and early 1980s, first for primary manufacturers and later for secondary manufacturers and distributors. Even today, many insurers do not routinely incorporate an asbestos exclusion in all CGL policies. This complicates the determination of the end of the coverage block for each insured.

Today there continues to be considerable unresolved insurance coverage litigation. This litigation tends to revolve around three issues: 1) existence and terms of lost policies, 2) interpretation of asbestos exclusion wordings, and 3) applicability of the known loss exclusion [2, pp. 25–110]. In addition to these cited issues, a significant amount of litigation and negotiation centers upon issues of number of occurrences as relates to policy limits and SIR/deductible application, duty to defend, horizontal versus vertical exhaustion of limits, and contributions for uninsured periods. Although unresolved issues may hinder analysis of an insurer's potential liabilities for a particular insured related to specific years of coverage, case law is sufficiently established to permit the estimation of a range of total potential liabilities for the known asbestos defendant group.

The trend in asbestos litigation of an increasing universe of defendants must be understood before quantifying liabilities for a particular group of insureds. Early in the asbestos litigation process, only major manufacturers and distributors of asbestos were named as defendants in the suits. However, the asbestos defendant group has expanded considerably over time. This is due in large part to the bankruptcy of major asbestos defendants such as Johns–Manville and UNR Industries as well as the search by plaintiff attorneys for other sources of compensation. In addition, significant expansion occurred around 1989 when defendant Owens Corning Fiberglas drew a large number of companies into the asbestos litigation via third-party actions [9]. Companies first identified as defendants subsequent to 1989 are generally companies with more limited asbestos exposures due to the encapsulation of asbestos in their products or their involvement only as a local or regional distributor. However, these companies and their insurers are still facing potentially substantial indemnification and defense costs. A further expansion of the defendant group may yet occur. In this paper we do not address quantification of an IBNR provision associated with as yet unidentified defendants. Such a provision could be estimated by extrapolating from historical emergence activity.

Another insurance issue requiring discussion is the type of coverage under which asbestos BI defendants are filing and the implications of limits under that coverage. Since the asbestos litigation explosion, insurers' asbestos-related costs under workers compensation have been limited because employees have sued the manufacturers and distributors of asbestos products rather than file workers compensation claims against employers. Asbestos BI claims have historically been filed by defendants as products and completed operations claims under general liability (GL) policies. The majority of such policies include an aggregate limit applicable to products claims. As thousands of claims are allocated across an insured's coverage block, the portion of the claims allocated to each policy accumulates to exhaust that policy's aggregate limit.

In situations where no aggregate limit is included in the policy, the asbestos claims are applied against the occurrence limits and a determination of the number of occurrences must be made. Court decisions have been mixed on whether the decision to manufacture asbestos products constitutes a single occurrence, whether each claim is a separate occurrence, or whether some other definition of occurrence should apply. Thus, policies without aggregate limits may end up paying multiples of the occurrence limits.

In the mid-1980s, several defendants and insurers formed the Asbestos Claims Facility (ACF) to deal with the enormous number of asbestos claims. Participants in the ACF addressed the treatment of policies without aggregate limits, as well as other coverage issues, in the Wellington Agreement signed by insureds and insurers [2, pp. 100–109]. The Wellington Agreement specified an aggregate limit as a multiple of the per occurrence limit, with the multiple varying with the magnitude of the per occurrence limit. Although the ACF was dissolved in 1988, the provisions of the Wellington Agreement remain. Thus, most products liability coverage is subject to aggregate limits for indemnity.

A number of asbestos defendants owned subsidiaries that installed asbestos products as well as manufactured and/or distributed the products. As these defendants are exhausting their products liability coverage, they are seeking premises and operations coverage for claims related to the installation subsidiary. Since general liability policies did not generally contain aggregate limits for premises and operations claims, significant additional coverage could be available to defendants if they are successful in obtaining coverage on this basis. Also, the expansion of the defendant group to include premises owners and operators, as discussed in a later section, has resulted in additional premises and operations claim filings.

3. ASBESTOS DISEASES

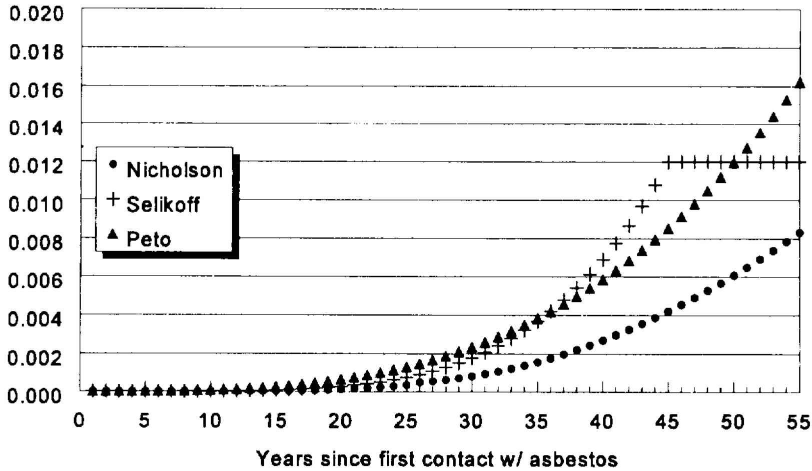
Life-threatening or disabling diseases can be caused by exposure to airborne asbestos, particularly at the high exposure levels in occupational settings during the first 70 years of this century. Diseases associated with asbestos exposure include mesothelioma, lung and other cancers such as gastrointestinal, asbestosis, and pleural plaques. Mesothelioma has been strongly associated with asbestos exposure. Lung cancer and other cancers have been associated with asbestos exposure at occupational levels. Asbestosis has been observed mainly after high occupational exposure to asbestos [6].

According to the *Journal of the National Cancer Institute*, "asbestos is the only known risk factor for mesothelioma, a tumor of the membranes lining the chest or abdominal cavities" [8]. It should be noted that cases of mesothelioma have been diagnosed in individuals without known asbestos exposure. However, if individuals can demonstrate exposure to asbestos, the courts appear to universally accept that mesothelioma was caused by such exposure.

Mesothelioma generally manifests itself 15 to 50 years from first exposure to asbestos and is almost always fatal within one

FIGURE 1

PROBABILITY OF DEATH DUE TO MESOTHELIOMA



Sources: Nicholson [20]. Adopted by Dunbar [21].
Selikoff [22]. Adopted by Tillinghast [23] and Peterson [24].
Peto [25]. Adopted by Walker [26].

to two years of diagnosis. Figure 1 shows three functions derived from epidemiological studies and used to project future mesothelioma incidence rates for an insulation worker with cumulative asbestos exposure of 250 fiber-years/ml [12, pp. 101–106]. Cumulative exposure is calculated as the sum over all years of the annual averages of the average exposure levels of an individual measured in fibers per milliliter (i.e., measured on a basis consistent with the OSHA standards presented in Table 1). For example, an individual exposed to an average of 10 fibers/ml for 25 years would have a cumulative exposure of 250 fiber-years/ml. This would be the same as an exposure of 25 fibers/ml for 10 years.

The graph demonstrates the relationship between mesothelioma incidence rates and time since first exposure (i.e., the latency period). This helps explain why workers exposed in the 1950s and 1960s are just now filing claims and why, when incorporating exposures from the 1970s, claim reportings are expected to continue well into the next century.

Epidemiological studies have demonstrated an increased risk of lung and other cancers among workers exposed to asbestos. For insulation workers with cumulative exposure of 250 fiber-years/ml, the risk of lung cancer is two to seven times the normal risk. Following a minimum latency period of 8 to 10 years from date of first exposure, the relative risk (i.e., the risk for an asbestos-exposed population versus an unexposed population) of developing lung cancer increases linearly until 35 to 40 years past first exposure and then begins to decrease [14].

Another asbestos-related disease is asbestosis. Asbestosis is a fibrotic or scarring process within the lung tissue, potentially causing an inflammatory response and fluid collection resulting in various levels of disability from respiratory problems. Severe cases of asbestosis are generally associated with heavy occupational exposure such as that of insulators or shipyard workers. While it is generally acknowledged that the relative incidence of asbestosis has declined in recent years, we are not aware of any evidence showing a similar decrease in asbestosis claim filings.

The mildest of the asbestos related diseases is pleural plaques. Pleural plaques is a benign condition of the lungs which is generally not debilitating. However, pleural plaques is associated with asbestos exposure and claims are being filed by individuals with this condition. Some jurisdictions do not recognize pleural plaques alone as a compensable injury.

Plaintiffs with mesothelioma generally receive the highest indemnity payments, averaging well over five hundred thousand dollars (though some individual awards total several million dollars).

While certain lung cancer plaintiffs without contributing factors such as smoking receive average indemnity payments comparable to mesothelioma, the overall average indemnity for lung cancer plaintiffs is approximately 50% of the average mesothelioma payment. Non-fatal asbestosis plaintiffs receive payments averaging approximately 10% to 15% of mesothelioma payments [10].

4. PROJECTION CONSIDERATIONS

One thing is clear with regard to projecting ultimate asbestos liabilities: traditional loss development techniques which rely on historical accident year loss development to derive development factors cannot be used. Traditional methodology is inappropriate for asbestos loss development because: 1) historical asbestos loss development is not representative of expected future development; 2) asbestos loss development is not a function of the age of the accident or policy year; 3) diseases caused by asbestos are latent for long periods of time; and 4) asbestos claims are allocated over many years based on the courts' decisions on occurrence of injury.

Any loss development patterns used in projecting asbestos liabilities should reflect what is happening at the underlying insured level as well as the insurance or reinsurance company's exposure. It will be shown in Section 5 that asbestos loss development for insurers and reinsurers does not relate to the age of the policy, but to factors such as the underlying claim allocation procedure and the attachment points and limits of the exposed policies.

Any methodology for projecting an insurer's or reinsurer's potential liabilities for asbestos BI claims must reflect the following elements of the company's exposure:

- years and volume of general liability business underwritten,
- use and wording of asbestos exclusions,
- type of insureds underwritten,

TABLE 2

ASBESTOS BI RISK ASSESSMENT

GL Book of Business Characteristic	Low Risk	Medium Risk	High Risk
Policy Years	1986 and subsequent	1976–1985	1975 and prior
Premium Volume (GL Market Share)	< 0.5%	0.5%–1.5%	1.5%+
Asbestos Exclusion	Consistent use of comprehensive exclusion by early-1970s	Consistent use of comprehensive exclusion by late 1970s	Asbestosis exclusion and inconsistent use until mid 1980s
Type of Insureds	Small/Local Businesses	Regional Companies	Fortune 1000 Manufacturing/ Construction
Layers Written	Very High Excess (> $20 million)	High Excess (> $5 million)	Primary/Umbrella/ Low Excess
Aggregate Limits	No Exceptions	Few Exceptions	Many Exceptions
Expense Treatment	Indemnity Only	Expense included in limit	Expense in addition to limit

- layers of liability underwritten and retained,
- use of aggregate limits, and
- expense treatment in policies.

Table 2 is useful in doing a preliminary assessment of the level of an insurance or reinsurance company's potential asbestos BI liabilities. It gives several characteristics relating to the general liability book of business. For each characteristic there is a typical answer for low risk, medium risk, and high risk. Low risk means the insurer or reinsurer is not likely to have significant potential asbestos liability. High risk means the insurer or reinsurer is likely to have significant potential asbestos liability. This is not a comprehensive list of factors to consider. Obviously, the number of asbestos claims for insureds, average indemnity for insureds, and similar information are required before the potential liability for an insurer or reinsurer can be quantified.

Of course, these factors need to be considered in total, but insurers or reinsurers falling in the low risk category for all factors (unlikely, as small businesses purchasing coverage above $20 million are rare) and limited claim activity to date are most likely not facing significant liabilities. Likewise, insurance or reinsurance companies consistently rated high risk should carefully review their potentially significant liabilities.

To do a more detailed and rigorous analysis of an insurance or reinsurance company's liability, a projection methodology must be selected based on its appropriateness for the line of business being reviewed. Given the unique characteristics of asbestos losses, such as development being unrelated to age of policy or accident year, a policy limits analysis is a strong candidate for a methodology that can incorporate all of the necessary factors in an ultimate loss estimate. A policy limits analysis will be presented in the next section.

5. POLICY LIMITS ANALYSIS

Our model differs from most traditional actuarial loss development methods by explicitly quantifying the impact of each policy's limits when estimating the insurance or reinsurance company's liability. In our model, ground-up losses for each insured are calculated using a frequency and severity approach. For each policy for each insured, the losses in the insurance layer are calculated based on the policy's limits and the ground-up losses. Other actuarial projection methods, such as the incurred loss development method, are assumed to implicitly take into account the insured's policy limits in the selection of loss development factors.

Our approach is more appropriate for asbestos losses because of the extremely long latency of asbestos diseases and the allocation of an asbestos claim across several policy years. If a court ruled that an asbestos-related injury had been caused by exposure spanning 30 years, all 30 years of insurance policies could be triggered. Typically over such a long period the defendant's policy limits have grown. A primary policy written in 1948 may have been at $50,000 limits, while a primary policy written in 1977 may have been at $1 million limits. This change in limits needs to be reflected, at least in the aggregate.

A policy limits analysis of a sample group of defendant companies can be supplemented with individual case estimates for defendants with unusual exposures to provide an assessment for all known asbestos defendants. Unusual exposures could include policies without aggregate limits or those with significant outstanding coverage issues.

In the remainder of this section, we discuss our asbestos BI model, from the initial stages involving the sample group determination to extrapolation of the model results. The steps of the policy limit analysis are as follows:

I. *Determine the sample group and collect data.*

1. Determine the desired group of insured defendants to be included in the detailed analysis;

2. Collect information on each defendant's claim experience and the company's exposure to the defendant's asbestos claims; and

3. Re-evaluate which insureds to include in the sample group based on the compiled information.

II. *Adjust the policy exposure data.*

4. Adjust the sample group's policy information to restate it on a ground-up basis.

III. *Use the model to estimate the insurance or reinsurance company's liability for the sample group.*

5. Project future aggregate ground-up costs for each sample group defendant;

6. Allocate the aggregate ground-up costs to years within the defendant's coverage block;

7. Determine the amount of the ground-up loss and expense in each year falling in the layers of coverage provided by the insurer or reinsurer; and

8. Sum the losses in the insurance layer across all sample group defendants.

IV. *Conduct sensitivity testing of the model's parameters and make adjustments.*

9. Test alternative scenarios regarding future claim activity and alternative claim allocation procedures; and

10. Develop a range of outcomes for the sample group based on the sensitivity analysis.

V. *Extrapolate model results from the sample group to all insureds.*

11. Use the model results to develop assumptions applicable to the remaining group of insured defendants; and

12. Incorporate individual case estimates for unusual exposures.

In the following sections, we discuss each of these steps.

Determine the Sample Group and Collect Data

The use of a sample group in estimating liabilities for a large group of insureds is sometimes desirable. For large insurers or reinsurers, it may not be feasible to model the future claim activity for all insured asbestos defendants. For these companies, the number of insureds who may have filed precautionary notices related to potential asbestos claim activity could easily total five hundred or even one thousand. Information may be limited on certain defendants, including a large number of defendants whose exposure to asbestos claims is small, due to a small market share or the use of encapsulated asbestos only. The sample group must be representative of the total exposures of the company so that an extrapolation of the model results to the remaining exposures can be done.

To facilitate selection of a sample group and extrapolation of model results for insurance and reinsurance companies, categorize all potential defendants in the asbestos universe into five tiers. Each tier rating is based upon the nature and extent of potential asbestos liabilities of the defendant. Thus, the first step in determining the appropriate sample group for an insurer or reinsurer is to apply a tier rating to each of the insureds.

The first tier includes defendants who have been involved in asbestos litigation since its inception and who were the primary manufacturers, suppliers, or miners of raw asbestos or producers of asbestos products throughout North America. Each defendant in this category is estimated to face ground-up ultimate aggregate liabilities of $1 billion or more. Considering that across the industry fewer than 20 companies fall into this category and the required information on these defendants is generally available through the claim department and/or public sources, all of these defendants should be reviewed for inclusion in the sample group for detailed model or individual analysis. Since most Tier 1 insureds are expected to exhaust available products liability coverage, individual review may be substituted for detailed modeling. In such cases, individual analysis may involve simply verifying that reserves have been established to policy limits.

Our second tier includes defendants who have also been involved in asbestos litigation almost since inception, but due to lower market shares or more limited-use products, their estimated ground-up ultimate liabilities are in the $100 million to $1 billion range. Tier 2 would include manufacturers of asbestos-containing products such as those used in the construction, petrochemical, and shipbuilding industries as well as smaller mining concerns. The distinction between Tiers 1 and 2 is subject to some judgment. Based on our current estimates, there are approximately 50 Tier 2 defendants, with any one insurer having exposure to a subset of this group of defendants. A majority of a company's exposure to Tier 2 defendants should be included in the sample group.

The third and fourth tiers include the remaining hundreds of non-railroad defendants that have been enjoined as third party defendants brought into the asbestos litigation as Tier 1 and Tier 2 defendants have filed for bankruptcy protection. Tier 3 includes those defendants whose exposure relates to encapsulated and similar low exposure asbestos products (e.g., friction and protective products) and local or regional suppliers and distributors of asbestos products. It should be noted that some manufacturers of encapsulated asbestos products with extensive national distribution were targeted early by the plaintiffs' bar and should be categorized as Tier 2. Many Tier 3 defendants face substantial numbers of claims, high defense costs, and relatively low indemnity payments (in comparison to Tiers 1 and 2). In total, their potential liabilities are significant, though well below the Tier 2 level. There are also numerous Tier 3 defendants facing very small liabilities, e.g., in situations where exposure to a company's products will be difficult to establish by plaintiffs.

Tier 4 defendants are those who never manufactured or distributed asbestos products, but rather owned or operated property where asbestos products were used. A Tier 4 defendant's liability is thus related to contractors or third parties, other than employees, who were exposed to asbestos on the defendant's premises. Claims are filed as premises/operations liability rather than products liability generally with per occurrence rather than products aggregate limits applicable. An example of a Tier 4 defendant is a utility or oil company.

The sample group should contain Tier 3 and 4 defendants for which the necessary claim statistics are available. In selecting the sample defendants from these tiers, policies providing coverage in various layers representing the type of coverage provided to insureds in Tiers 3 and 4 should be included.

Tier 5 has been reserved for railroads facing liabilities from exposed workers under the Federal Employers Liability Act. The claim reporting pattern of railroads is expected to be faster than that of most other types of defendants. This results from the fact that heavy asbestos exposure of railroad workers is tied to steam engines which were replaced by diesel engines in the early 1960s. Also, attorneys and unions have been active in identifying exposed workers and facilitating claim filings. Many railroads have reached settlement agreements with their insurers related to asbestos claims. To the extent that an insurance company has exposure to railroads not subject to a settlement agreement, a sampling of the railroad insureds should be included in the model analysis.

The intent is for the sample group to be representative of the insurer's or reinsurer's total exposure to asbestos liability from its insureds known to have asbestos exposure. If a defendant has an unusual exposure, or a coverage dispute, which is not representative of the other insureds in the tier, a separate analysis or adjustments to the defendant's policy information may be necessary.

Once the sample group has been selected, data for each defendant in the sample group must be collected for input into the asbestos BI model. The following data elements should be compiled for each defendant:

1. number of claims filed, disposed, and pending;
2. cumulative paid and reported indemnity;
3. expense-to-indemnity ratio;
4. dates of coverage block;
5. details of all products liability coverage (or premises/operations liability coverage, if applicable) provided by the insurer or reinsurer within the coverage block including—
    a) policy term;
    b) attachment point relative to the first dollar of loss;
    c) aggregate (or per occurrence) limit of liability;
    d) participation percentage or percentage share in the layer of liability;
    e) expense treatment under the policy;
    f) asbestos exclusions;
    g) erosion of limits by non-asbestos products claims; and
    h) (for reinsurers only) ceding company's policy information, i.e., (5a) through (5g) for the ceding company's policy.
6. details of negotiated settlement agreements; and
7. details of pending coverage disputes.

Note that these data do not completely describe every aspect of all insurance policies in the sample group. This is particularly true for reinsurance policies. However, the data collected does allow for a good estimate of the insurance or reinsurance company's asbestos exposure from each policy in the sample group.

The claim counts, indemnity payments, and expense ratio information are required at the defendant level in order to project the defendant's ground-up aggregate liabilities. Details regarding negotiated settlement agreements and pending coverage disputes are useful in determining whether an insured defendant should be included in the sample group (with or without adjustments to reflect the uncertainty presented by pending coverage disputes) or if case reserves established by the claim department reflecting agreements/disputes should be relied upon instead. Of course, case reserve estimates should be relied upon only if the reserve contemplates future claim reporting to an ultimate basis, which could happen if the insurer's policy limits are exhausted.

Several potential sources for the required data exist, including the claims department of the insurance company, annual reports of the various defendants, insurance company attorneys, and court documents. While some of the required data is relatively easy to obtain, certain information is difficult to get directly. Data for some potential candidates may not be available at all. It may be necessary to estimate missing information and test the sensitivity of the model results to alternative assumptions, or leave some insureds out of the sample group entirely. Ultimately, the decision to include an insured should be based on whether inclusion of that insured will help make the sample group representative and whether there is enough data on that insured for use in the model.

The policy information (attachment point, company's percentage share in the layer, and limit of liability) on a first dollar of loss (ground-up) basis may be difficult to collect. This data should be readily available from the policy files for primary companies. For excess writers and reinsurers, however, this information can be particularly difficult to obtain. For assumed reinsurance business, additional information is required on the ceding company's policies in order to identify the ground-up loss required to penetrate the reinsurer's layer. In other words, we need to restate the reinsurer's limit, percentage share, and attachment point relative to the first dollar of loss in order to determine when the policy is expected to be hit by the aggregate asbestos claims generated by the model.

Adjust the Policy Exposure Data

The calculations shown in this paper assume that all limits apply on an aggregate basis. Policies without aggregate limits can be handled in a number of ways. First, if the policy is governed by the Agreement Concerning Asbestos-Related Claims entered into by various insurers and asbestos producers, referred to as the Wellington Agreement, the occurrence limits could be restated on an aggregate basis reflecting the multipliers in the agreement. Simplifying assumptions could also be made as to the number of occurrences applicable and the relative magnitude of each occurrence. This would facilitate either a restatement of the limits to apply to aggregate claims or a breaking down of the aggregate claims into the separate occurrences and a comparison of the per occurrence amounts to the per occurrence limits.

To effectively reflect the insurer's or reinsurer's exposure to asbestos loss on a policy, the policy information must be stated on a first dollar of loss, or ground-up, basis. This is necessary for the stated attachment point, percentage share, and policy limit. A first dollar policy does not require adjustment. For a direct excess policy, it may only be necessary to adjust the attachment point by adding the underlying primary limit to the stated attachment point. For an assumed reinsurance policy, especially treaty reinsurance, all three parameters might require a restatement to a first dollar of loss basis. Facultative reinsurance policy information may already be stated on a first dollar of loss basis for stated policy limit and participation share, thereby requiring only an attachment point adjustment similar to that mentioned for direct excess policies.

If the ceding company information is known, the reinsurance policy parameters can be restated to a first dollar basis using the formulas described below. If the ceding company's policy information is incomplete, estimates can be made of the appropriate adjustments based on an analysis of a sample of policies. To illustrate the adjustments necessary for reinsurance policies, we examine some policies of a reinsurer, ABC Re, with ceding insurer XYZ which wrote policies for two insureds.

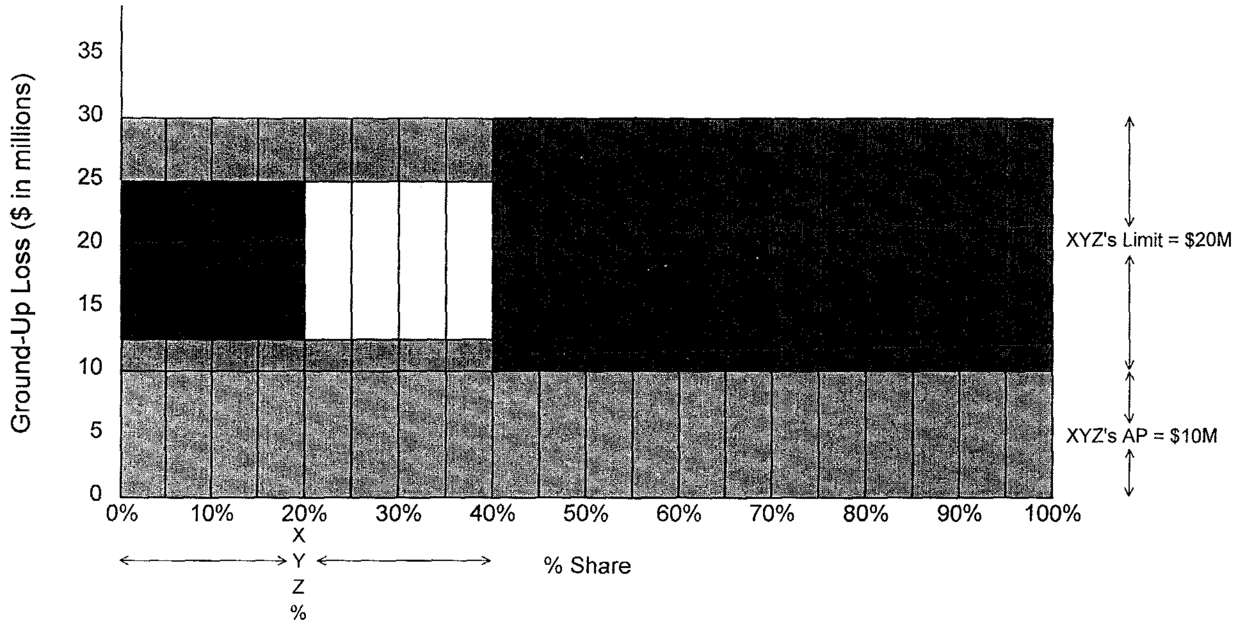
Exhibit 2 shows sample adjustment calculations. The exhibit shows three sets of policy information: cedent XYZ's direct policy information in Columns 3 through 5, ABC Re's stated reinsurance policy information in Columns 6 through 8, and the calculated ground-up reinsurance policy information for ABC Re in Columns 9 through 11. Columns 3, 6, and 9 are the percentage shares. Columns 4, 7, and 10 are the attachment points. Columns 5, 8, and 11 are the policy limits. Expenses are ignored in Exhibit 2 for simplicity.

Definitions of the three restated policy parameters in the context of this paper are in order. All three are adjusted reinsurance policy parameters which express the ground-up exposure to loss for the reinsurer. The restated reinsurance percentage share is the amount that, when multiplied by the restated reinsurance policy limit, equals the reinsurer's maximum dollar share of the ground-up losses. The restated reinsurance attachment point equals the amount of ground-up losses which must be incurred before the reinsurance layer is penetrated. The restated reinsurance limit is the amount that, when added to the restated reinsurance attachment point, equals the amount of ground-up losses necessary to exhaust the reinsurance policy.

Figures 2, 3, and 4 graphically illustrate the need to make the adjustment to ABC Re's policies shown in Exhibit 2. Note that for some policies, the reinsurer has no exposure to loss, even though the ceding company does. Again, expenses have been ignored in this example for simplicity.

FIGURE 2

ABC RE'S RESTATED POLICY TERMS FOR POLICY 3 FROM EXHIBIT 1 CAPPED BY UPPER CONSTRAINT 1



a) XYZ Attachment Point = $10M

b) Other direct writers = 60% of $20M xs $10M

c) Retained by XYZ = 40% of $2.5M xs $10M (for its reinsurance AP), 40% of $5M xs $25M (above its reinsurance layer)

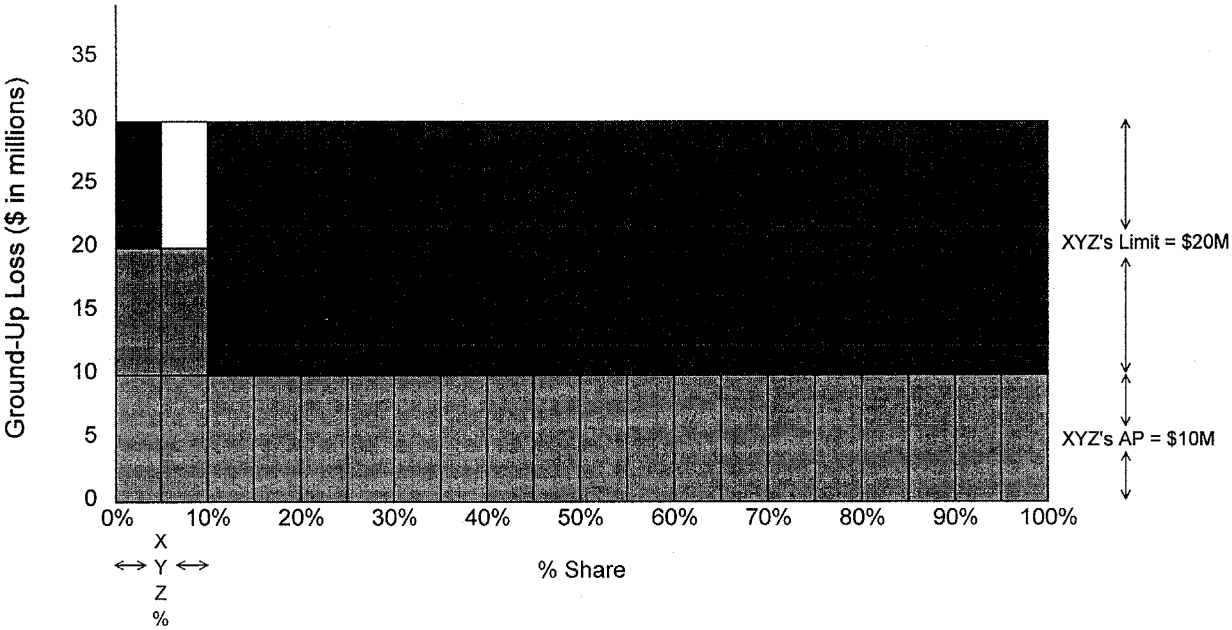
d) XYZ ceded to other reinsurers = 20% of $12.5M xs $12.5M

e) XYZ ceded to ABC = 20% of $12.5M xs $12.5M

(Assume XYZ purchased 1 layer of reinsurance, ABC is one writer of layer. Assume no expenses for simplicity.)

FIGURE 3

ABC RE'S RESTATED POLICY TERMS FOR POLICY 4 FROM EXHIBIT 1 CAPPED BY UPPER CONSTRAINT 2



a) XYZ Attachment Point = $10M

b) Other direct writers = 90% of $20M xs $10M

c) Retained by XYZ = 10% of $10M xs $10M (for its reinsurance AP)

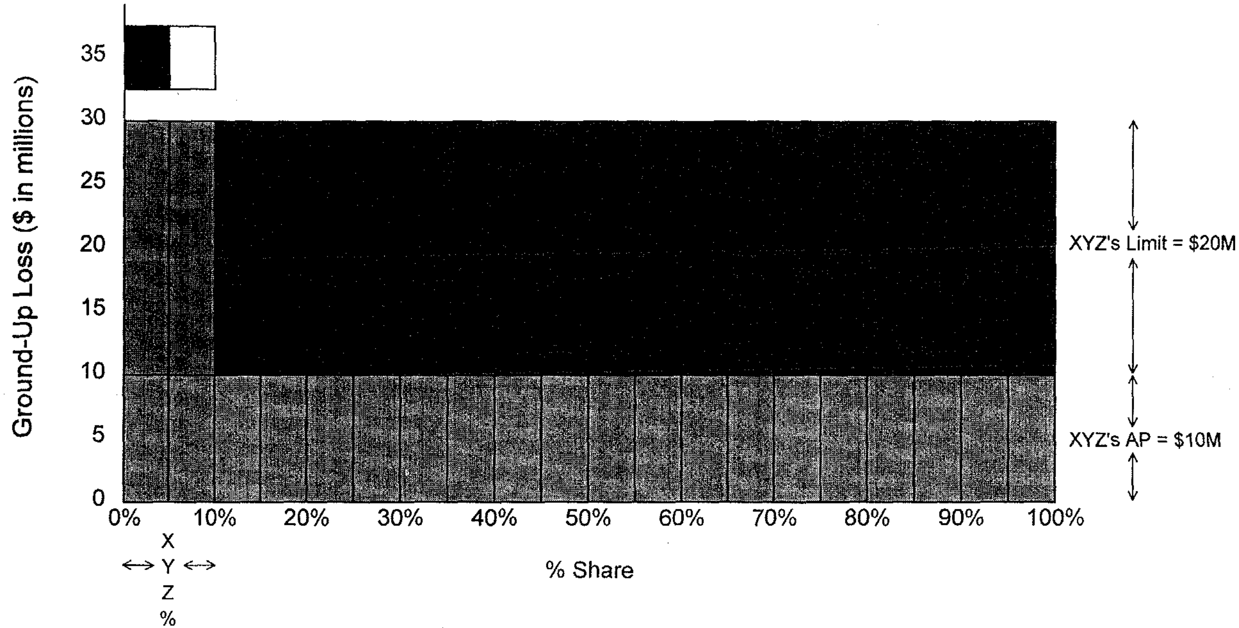
d) XYZ ceded to other reinsurers = 5% of $10M xs $20M

e) XYZ ceded to ABC = 5% of $10M xs $20M

(Assume XYZ purchased 1 layer of reinsurance, ABC is one writer of layer. Assume no expenses for simplicity.)

FIGURE 4

ABC Re's Restated Policy Terms for Policy 5 from Exhibit 1 Capped by Lower Constraint 1



a) XYZ Attachment Point = $10M

b) Other direct writers = 90% of $20M xs $10M

c) Retained by XYZ = 10% of $22.5M (capped at $20M) xs $10M (for its reinsurance AP)

d) XYZ ceded to other reinsurers = $0, attaches at $32.5M

e) XYZ ceded to ABC = $0, attaches at $32.5M

(Assume XYZ purchased 1 layer of reinsurance, ABC is one writer of layer. Assume no expenses for simplicity.)

The calculation of the restated reinsurance percentage share in Column 9 is straightforward. Ignoring expenses and extra-contractual situations, the ceding company is limited to the percentage share stated in the policy. ABC Re's percentage share is a portion of the cedent's share of the insurance layer. Hence the restated percentage share relative to first dollar of loss must be the product of the two percentages, or Column 3 × Column 6.

The restated reinsurance attachment point in Column 10 follows similar logic. The ceding company's layer of liability begins at the attachment point in the primary policy. In order for the cedent to incur any losses, the ground-up losses must be greater than the attachment point in the ceding company's policy. Likewise, ABC Re's layer of liability begins at the attachment point on the reinsurance policy. Only when the cedent's losses have reached the reinsurance attachment point will ABC Re's layer be penetrated. If the cedent's percentage share was 100%, ABC Re's layer could be penetrated only if the ground-up losses exceeded the sum of the two attachment points. However, in cases where the cedent's percentage share is less than 100%, the reinsurance attachment point must be divided by the primary policy percentage share and then added to the primary attachment point to calculate the restated ground-up attachment point, or ([(7)/(3)] + (4)). The division by the primary percentage share is required because for every dollar of loss incurred by the cedent, the insured must have incurred the reciprocal of the primary percentage share.

The logic for the restated ground-up attachment point and percentage share must be kept in mind to determine the appropriate calculation for the restated reinsurance limit in Column 11. We look at the interaction of the direct policy with the reinsurance policy to understand the calculation. The formula for Column 11 reflects two upper constraints, a lower constraint, and an adjustment for the direct policy's percentage share.

First, we examine the intuitive upper constraint of the Column 11 formula. Ignoring expenses and again assuming the cedent's percentage share is 100%, the maximum restated reinsurance limit relative to the first dollar of loss equals the reinsurance limit, or Column 8. Note that this is just the limit of the reinsurance policy; the maximum dollar share of the reinsurance layer would be the reinsurance limit times the reinsurance percentage share. Here we are concerned only with the calculation of the limit. If the ceding company participation share is less than 100%, then this maximum for the restated limit needs to be divided by the cedent's participation share, or (8)/(3), for the same reason this adjustment was made in calculating the restated attachment point.

The second upper constraint for the restated reinsurance limit is the maximum imposed by the ceding company's dollar share of the layer (i.e., cedent's percentage share times cedent's limit, or (3) × (5)) less the cedent's retention (i.e., the reinsurer's unadjusted attachment point, or Column 7), all divided by the cedent's percentage share, or Column 3. Once the reinsurance attachment point is exhausted and the reinsurance layer has been penetrated, every dollar that consumes the reinsurance limit is due to ground-up losses equal to the reciprocal of the cedent's percentage share, or $1/(3). Stated another way, the restated reinsurance limit cannot exceed the cedent's limit minus the quantity of the reinsurance attachment point divided by the cedent's percentage share, ((5) – [(7)/(3)]), equal to the second upper constraint. Remember, in calculating the restated reinsurance limit, we are trying to determine the amount of ground-up dollars that, when added to the restated reinsurance attachment point, will exhaust the reinsurance policy limits.

By including a lower constraint, we complete the formula for the restated reinsurance limit in Column 11. The lower constraint of the formula is zero; the restated reinsurance limit cannot be negative. Combining all the pieces of the restated reinsurance limit, we now have the formula used to derive Column 11,

$MAX[0, MIN\{(8)/(3), (5) - ((7)/(3))\}]$. Thus, if we know the cedent's policy information, we can adjust the reinsurance policy information to restate it on a first dollar of loss basis.

The two upper constraints discussed above contribute to what we refer to as "underlap." That is, the interaction of the cedent's policy terms with the reinsurer's policy terms may reduce the reinsurer's stated exposure. Exhibit 2 shows the calculation of the underlap for each of the policies presented and the underlap factor of 54.5% calculated in total for all policies related to Insureds 1 and 2.

Once the ground-up policy information for each of the sample defendants' products liability policies has been determined and other required information is obtained, the data preparation for the sample group is complete and the model can be used.

Use the Model to Estimate the Insurance or Reinsurance Company's Liability for the Sample Group

The asbestos BI model presented in this paper uses a frequency and severity approach to calculate ground-up losses and applies a policy limits analysis to the ground-up losses. It calculates an estimate of an insurance or reinsurance company's asbestos liability for a sample group of representative underlying insureds. This sample can later be used to estimate the total asbestos liability for the insurer or reinsurer. Whether we are analyzing liabilities for an insurer or a reinsurer, the underlying insureds are the manufacturers, installers, and distributors of asbestos products, and not the reinsured insurance companies. For simplicity of presentation, reinsurer ABC Re will be used in this section of the paper to demonstrate the model for both insurance and reinsurance companies.

For each underlying insured in ABC Re's selected sample group, the model projects by calendar year ground-up reported claim counts, ground-up average severity, and thus ground-up aggregate indemnity costs. Expenses are loaded based on the historical expense-to-indemnity ratios of the particular insured. The projected costs are spread over the policy years in the insured's coverage block. Having projected ground-up indemnity and expense costs for each calendar year by policy year, the model can then carve out ABC Re's liability from the ground-up costs for each policy of each insured in the sample group. Summing ABC Re's liability for all insureds gives ABC Re's estimated liability for the entire sample group.

Exhibit 3 presents a partial list of ABC Re's insureds with a known potential for asbestos loss. Insureds 1 through 15 are included in the sample group; the remaining insureds are not. Exhibits 4 through 9 demonstrate the use of the asbestos BI model to calculate ABC Re's estimated asbestos liability for one insured company in the sample group, Insured 3. Exhibit 4 presents the required model policy input assumptions for Insured 3; Exhibit 5 presents the required model claim input assumptions for Insured 3. Exhibits 5.1 through 9.1 show the baseline scenario with selected severity trend of 5% and 15 year coverage block. Exhibits 5.2 through 9.2 have 0% trend and 15 years selected. Exhibits 6.3 through 9.3 have 5% trend and 25 years selected. Exhibits 6.4 through 9.4 have 0% trend and 25 years selected. Exhibit 10 shows the aggregate results of all insured defendants in ABC Re's sample group. ABC Re's percentage shares, limits, and attachment points for Insured 3, presented in Exhibit 4, have already been restated on a first dollar of loss basis.

The first step of the asbestos model is to calculate the future aggregate ground-up indemnity and expense costs for each sample insured. For ABC Re's Insured 3, this is done in Exhibit 5. Several inputs are necessary to estimate the future aggregate indemnity and expense costs: a claim count reporting pattern, an average severity, a severity trend, and future expense-to-indemnity ratios.

First, a claim count reporting pattern must be calculated for the insured companies in ABC Re's sample group to be used as input in Exhibit 5. This pattern is not ABC Re's claim reporting pattern but rather that of the underlying insureds. The selected pattern for Insured 3 is shown in Exhibits 5.1 and 5.2. Actual calculation of the reporting pattern is beyond the scope of this paper.

Ideally, the necessary claim count reporting pattern is derived from claim count projections developed by researchers expert in both the asbestos-exposed population and the mathematical models which tie claim incidences to such factors as exposure levels and latency period. Such studies are available through bankruptcy courts, which have overseen the formation of liability trust funds for companies undergoing restructuring, and in academic literature. For example, the Manville and National Gypsum bankruptcies and related hearings addressed projections of future claim filings by disease. Judgmental extrapolation of historical claim reporting patterns can alternatively be made, particularly if a shorter time horizon, such as ten years, rather than an ultimate run-off, is selected for the review. If sufficient information is available, claim count patterns by tier should be calculated. However, this may be difficult, particularly due to the limited available research on Tier 3 and Tier 4 companies.

The second required input on Exhibit 5 is the selected average severity. Dividing total indemnity paid by total closed claims gives a historical paid severity. Dividing indemnity paid in each recent year by its related number of closed claims gives a starting point for the selection of an average reported indemnity to be used for the projection of future costs. The most recent year's average reported severity should also be examined before making the selection.

The third input for Exhibit 5 is the selected severity trend. A 5% severity trend is chosen for Insured 3. Exhibits 5.1 through 10.1, and Exhibits 6.3 through 10.3 use this assumption. To show the impact of different severity trend selections, Exhibits 5.2 through 10.2 and Exhibits 6.4 through 10.4 use a 0% inflation rate.

The severity trend can be based on a review of historical average claim amounts, but should also consider expected future changes. For example, Tier 3 insureds may be expected to experience greater severity trends and consequently a larger share of the total cost, due to the bankruptcy of Tier 1 and 2 insureds and the impact of courts imposing joint-and-several liability. Changes in the mix of claims by disease type could also affect future trends. A decrease in severe asbestosis cases coupled with an increase in claims filed for pleural plaques would be expected to reduce future claim trends as plaintiffs with pleural plaques may receive little or no compensation. Given these potential impacts on future average severities, alternative claim trend assumptions should be tested to derive a range of estimated liabilities.

The fourth input required for Exhibit 5 is the selected expense-to-indemnity ratio for each calendar year. A 50% expense-to-indemnity ratio is selected for Insured 3 as shown on Exhibits 5.1 and 5.2 for all future calendar years.

The expense-to-indemnity ratio for each insured in the sample should be based on several factors. The historical expense-to-indemnity ratio for the particular insured is a good starting point. However, other factors must also be considered. The existence of legal precedents for many once hotly debated legal issues relating to asbestos personal injury liability suggests a declining trend in defense costs. The likelihood of out of court settlements must also be considered. A systematic approach by the underlying insured defendant to settlement of asbestos cases, such as a matrix of specific dollar ranges for each disease, would suggest that more cases would settle than go to court, lowering defense costs. The Manville Personal Injury Settlement Trust utilizes such a matrix-type approach to settlement with specified dollar amounts by disease. However, a Tier 3 or Tier 4 company increasingly being named in suits might start aggressively defending suits, thus raising defense costs. Each underlying insured must be examined carefully to determine reasonable expense-to-indemnity ratios for each projected calendar year.

The second step of the model is to allocate the projected aggregate ground-up indemnity and expense costs to policy years within the insured's coverage block. If an insured's actual coverage block is known, it should be used. Exhibit 6 presents the projected calendar year ground-up indemnity costs from Exhibit 5 spread across Insured 3's coverage block. Exhibit 7 differs from Exhibit 6 by including both indemnity and expense costs, calculated by applying the selected expense-to-indemnity ratios from Exhibit 5. Insured 3's coverage block includes 1960 through 1974. There is a chance that Insured 3 will pursue a coverage block of 1960–1984 to get more insurance coverage. Exhibits 6.1 through 10.1 and Exhibits 6.2 through 10.2 use the 15 year coverage block. To demonstrate the impact of a different coverage block selection, Exhibits 6.3 through 10.3 and Exhibits 6.4 through 10.4 use a coverage block selection of 25 years, 1960 through 1984.

As mentioned previously, allocation of claims within a coverage block will depend on the applicable trigger theory and the outcome of negotiations on this issue. If known, an insured's actual procedure for allocating costs to years within its coverage block should be used; otherwise the allocation should be based on a logical procedure. Possible default allocation methods include:

- an even allocation to each year in the coverage block,
- an allocation to year which reflects the proportion of total coverage written in the year, and
- an allocation which reflects the expected aggregate distribution of claims based on dates of exposure and manifestation.

An even allocation to year is a reasonable approximation when the coverage block is relatively short in length. An allocation in proportion to coverage is a reasonable approximation primarily because the typical increase in limits purchased over time tends to follow a typical allocation pattern based on a continuous trigger. An allocation pattern based on the expected distribution of claims is likely to be the closest to actual, but requires more data and analysis to develop. For simplicity in our example, each year in Insured 3's coverage block receives an equal allocation (or weighting) in Exhibits 6 and 7.

The third step in the model is to calculate for each policy year the ground-up indemnity and expense dollars which fall into the insurance or reinsurance company's layers of coverage. ABC Re's liability for Insured 3 is calculated by carving out Insured 3's projected ground-up indemnity and expense dollars that hit ABC Re's layers of insurance as shown in Exhibit 8. ABC Re's 1958 policy for Insured 3 is not included because policy year 1958 is outside Insured 3's coverage block, 1960 through 1974 for Exhibits 8.1 and 8.2, and 1960 through 1984 for Exhibits 8.3 and 8.4. As long as 1958 is outside Insured 3's coverage block, ABC Re's 1958 policy with Insured 3 is not exposed to potential asbestos losses. Seven ABC Re policies are within Insured 3's coverage block (both the 15 and 25 year scenarios). For simplicity of presentation, each of the policies in the example is in a distinct policy year. If ABC Re had multiple layers of insurance coverage for Insured 3 in the same policy year, a simple adjustment to Exhibit 8 could be made: each policy's appropriate layer would be carved out of the total indemnity and expense costs allocated to that particular policy year.

To demonstrate the effects of different expense treatments on policies, Exhibit 8 shows examples of each of the three most common expense treatments: indemnity only, expenses included in the limit, and expenses in addition to limits. The attachment point, percentage share in the layer, and total limit of liability also vary in these seven policies to show the effects of each. Typically, for a given layer of insurance for a particular company, the expense treatment would be more consistent; expense treatment is varied here for illustrative purposes only. The determination of whether loss and expense hit a layer can be calculated in two ways for policies with expenses included in the limit: either add expenses before applying the attachment point or add expenses once indemnity is in the layer. Both methods should be tested in the real world because the lower layer policies' expense treatment determines the appropriate method.

The projected loss and expense in ABC Re's layers shown on Exhibits 8.1 through 8.4 is calculated by carving out the appropriate ground-up loss and expense from Exhibits 5, 6, and 7. The method of carving out the loss and expense varies based on whether the policy for which the liability is being calculated has expense treatment of indemnity only, expenses included in the limit, or expenses in addition to the limit. For all three types of policies, the general methodology to calculate Exhibit 8's cumulative reported liability in the layer is: the prior calendar year's liability in the layer for the policy year (the number to its left on Exhibit 8) added to the incremental increase in indemnity and expense (where appropriate), taking into account attachment point, limit, and percentage share. To illustrate this, the calculation of the calendar year 2003 numbers for policy years 1971, 1969, and 1968 from Exhibit 8.1 will be shown.

The 1971 policy is an indemnity only policy with a projected reported liability of $1,629 ($ in 000's). The $1,629 equals $1,455 from the prior calendar year added to $174. The $174 is 100% (the policy percentage share in 1971) times ($3,629 – $3,455), the incremental increase in indemnity shown on Exhibit 6.1. Development on this policy year continues until calendar year 2006 when the policy is projected to exhaust its 100% share of the $2 million limit.

The 1969 policy is an ultimate net loss, or expenses included in the limit, policy. As the footnote on Exhibit 8.1 indicates, the process of calculating when losses and expenses hit this layer varies depending on underlying policies. For all policies of this type in Exhibit 8.1, expenses are added to indemnity before applying the attachment point and limits. The $1,944 for policy year 1969 as of calendar year 2003 equals $1,683 from the prior calendar year plus \$261. \$261 is calculated as 100% (1969 policy's percentage share) times (\$5,444 – \$5,183), the incremental indemnity and expense during calendar year 2003 from Exhibit 7.1. Note that the 1969 policy is penetrated much earlier than the 1968 policy, one that is identical to the 1969 policy except for its expense treatment. Also note that the 1969 policy's ultimate liability is \$4,000,000, equaling 100% of \$4 million.

The 1968 policy is an expense in addition to limit policy. In calendar year 2003, its reported liability is \$194. Because this is the first calendar year in which the policy is penetrated, the calculation needs to take into account the attachment point of the policy. Therefore the calculation is \$0 added to 100% times (\$5,444-\$5,183), incremental indemnity and expense during calendar year 2003 from Exhibit 7.1, times (\$3,629 – \$3,500)/(\$3,629 – \$3,455), the portion of indemnity that penetrated the 1968 policy layer of \$4 million excess \$3.5 million. These indemnity amounts come from Exhibit 6.1. Note that ultimately its liability is \$5,163, greater than the 1969 liability of \$4,000, because expenses are in addition to the limit on this 1968 policy. Furthermore, the 1970 policy is identical to the 1968 policy except that its percentage share is 25 percent. At every calendar year, the 1970 policy's reported liability is 25 percent of the 1968 policy's liability.

Contrasting the development of ground-up costs in Exhibits 6.1 and 7.1 with the development of costs in the insurance layers in Exhibit 8.1 provides much insight. As expected, Insured 3 has projected reported ground-up losses (in Exhibits 6.1 and 7.1) several years before ABC Re has reported losses in its layer. However ABC Re's loss reporting pattern is not necessarily faster or slower than Insured 3's. In Exhibit 9.1, ABC Re's pattern is ultimately faster because Insured 3 will exhaust some or all of ABC Re's retained layers and yet will continue to incur losses for several years. This is due primarily to ABC Re's attachment points (its ground-up attachment points are low relative to the total amount of ground-up losses) and the size of ABC Re's limits (its ground-up limits are small relative to total ground-up losses). Exhibit 9.2 demonstrates the reverse. If ABC Re's layers attached at a very high point relative to the total amount of ground-up losses, as is the case for some underlying sample insureds in Exhibit 3, ABC Re's pattern might be slower than the underlying insureds and policies might incur little or no loss, as seen in Exhibit 10. This relationship between attachment point, limit, and asbestos loss development is a point to be considered by both the underlying insureds and insurers in evaluating asbestos insurance coverage issues.

The comparison of the development of costs across policies in Exhibit 8.1 provides further insight. As would be expected, reported development is a function of the magnitude of the attachment point and total limits, while total liability is a function of the percentage share and total limits of the layer. Each of the policy years for Insured 3 were allocated the same ground-up cost. However, the different expense treatment in the 1965 and 1967 reinsurance policies (see Exhibit 8.1) causes the 1967 policy year to report over 200% more liability than the 1965 policy year in calendar year 2000. Furthermore, the 1965 policy year has $0.6 million more reported liability in calendar year 2000 than does the 1968 policy year, even though the 1968 policy has a larger total limit and the policies have the same expense treatment; this is because the higher attachment point on the 1968 policy causes less of the total ground-up indemnity and expenses to hit the layer in that year.

A comparison of the 1968 and 1970 policies in Exhibit 8.1 illustrates the effect of the percentage share. Each has the same attachment point and the same total limit, but the insurer's participation in 1968 was 100% while in 1970 it was 25%. Thus, for every dollar that penetrates these layers of $4.0 million excess $3.5 million, $1 hits the 1968 policy and only $.25 hits the 1970 policy.

The most important point illustrated on Exhibit 8.1 is that development for asbestos losses is not a function of the age of the accident or policy year. The least mature policy for ABC Re for Insured 3 is 1971. The 1971 policy year develops to ultimate faster than all but one other policy year, 1967. This pattern of development is not unusual because of the long latency of asbestos-related diseases and the allocation to policy year. Therefore, historical asbestos accident or policy year loss development is not representative of future development.

Exhibit 9 gives a comparison of Insured 3's allocation of costs on a ground-up basis versus ABC Re's liability in the layer. This exhibit demonstrates the differences in development for policy year 1968 versus all policy years in the coverage block, both in dollars and as a percentage of ultimate.

The fourth step of the asbestos BI model is to sum the losses in the insurance layers across all sample group defendants. The steps performed in Exhibits 5 through 8 for Insured 3 under the four scenarios are repeated for all other insureds in ABC Re's sample group. The sum of these calculations for all insureds in the sample group is shown on Exhibit 10. The totals from Exhibit 10 represent the estimate of ABC Re's liability under the various scenarios for the sample group.

ABC Re's loss reporting pattern for each insured and for the entire sample group can be derived from Exhibit 10. The sum of the asbestos liabilities for all companies in the sample group gives an overall loss reporting pattern for ABC Re. If enough companies from each tier are included in the sample group to give credible results by tier, ABC Re's reporting pattern by tier can also be calculated from Exhibit 10. Using ABC Re's estimated reported losses in the insurance layers for each calendar year, overall loss development factors for ABC Re can be calculated.

Conduct Sensitivity Testing of Model

Due to the inherent uncertainty in asbestos litigation, different scenarios should be examined to: 1) test the model's sensitivity to certain parameters or estimates, and 2) compute a range of estimates of liability for the sample group. The two parameters in the model with the most uncertainty are the future severity trend and the insureds' coverage blocks. Therefore, variations in the assumptions for each of these should be examined, as was done with the four scenarios included in Exhibits 5–10. Other parameters, such as the projected expense-to-indemnity ratio, should be considered to determine if sensitivity testing is necessary.

Exhibit 10 also shows ABC Re's aggregate exposure to each underlying insured in the sample group. Given an aggregate exposure for each insured and ABC Re's estimated ultimate loss for each insured, a projected percentage of exposure eroded by claims for each insured can be calculated as well as subtotaled by tier. This can be helpful in extrapolating the model results to all of ABC Re's underlying insureds.

Using the results of the different scenarios, a range of estimates can be derived for the sample group's liability. Weights applied to each scenario should be based on the expected likelihood of the scenario. Exhibit 11 calculates the average ABC Re asbestos liability for its sample group insureds using the results from Exhibits 10.1–10.4. The size of the indicated range in Exhibit 11, about $50 million, is large on both a percentage and a dollar basis. However, note that approximately $20 million of the range comes solely from the selection of the severity trend. This emphasizes the need to do sensitivity testing when working with projections so far into the future. We have shown a selected range based on averages of the two 25-year coverage block projections and the two 15-year coverage block projections. Thus, we are averaging the 0% and 5% severity trend indications. Note that this gives a different indication than simply selecting a 2.5% severity trend assumption, due to the interaction of the ground-up losses and the policy layers.

Our overall selected estimate is based on a 75%/25% weighting of the 15-year and 25-year coverage block indications. These weights have been selected for illustrative purposes only. Actual weights could vary by insured and should reflect such factors as court decisions on trigger issues in the applicable state and the nature of the insured's involvement with asbestos. Given the objective of the insured to maximize total coverage and the tendency of courts to further this objective, the nature and extent of insurance coverage available to the insured under alternative trigger scenarios should also be considered in selecting appropriate coverage block scenarios and their weights. It is important to note that maximizing total coverage to the insured may or may not be consistent with a worst case scenario for a given insurer or reinsurer. The impact of changing the length of an insured's coverage block on a given insurer depends on the attachment points, limits, and placement within the coverage block of all policies issued by the insurer to the insured.

Before extrapolating the model results of the sample group to all insureds, the model results should be reviewed for reasonableness. Alternative assumptions should be tested as necessary to gain a better understanding of the factors affecting the indications and the sensitivity of the results to changes in those factors.

In reviewing the reasonableness of the results, it should be noted that the loss reporting pattern produced by the model will likely be faster than that experienced by the insurance or reinsurance company, because of the inherent lag in reporting between the insured, the insurer, and the reinsurer. That is, the reporting pattern produced by the model is developed from each underlying insured's expected claim reporting pattern and does not reflect delays in the insurance reporting and reserving process. Likewise, if the insurance or reinsurance company establishes case reserves that incorporate a provision for IBNR claims (as may be the case when it is apparent that, with continued claim reporting, policy limits will be exhausted) then the model-produced pattern may be too slow. Both of these possibilities need to be considered.

Extrapolation of Model Results

With the model results for the sample group quantified, the estimated ultimate asbestos liabilities for all of ABC Re's underlying insureds can now be calculated. There are several ways to extrapolate the sample group model results to reflect ABC Re's total expected liabilities. The appropriateness of a particular method depends on the nature of the company's exposures as well as its claims handling and reserving procedures. Potential methods are: 1) percentage of layer exhausted by tier, 2) development factor by tier, 3) percentage of exposed limits exhausted by tier, 4) average ultimate loss by tier times number of insureds, and 5) extrapolation from Tiers 1 and 2.

Method One

The first extrapolation method is a percentage of layer exhausted method. By tier, one can develop estimates of the percentage of layers expected to be exhausted by asbestos BI claims. That is, the sample group Tier 2 insureds could be run through the model with the company's policy limits and attachment points overwritten by the following layers:

— primary $500,000;

— $500,000 xs $500,000;

— $4 million xs $1 million;

— $5 million xs $5 million;

— $15 million xs $10 million;

— $25 million xs $25 million; and

— $50 million xs $50 million.

The model output would provide an estimate of the percentage of these layers expected to be exhausted (or burned) by BI claims. We refer to these percentages as burn factors. Thus, exposures for non-sample Tier 2 insureds could be arrayed by layer and the selected percentages applied to derive estimates of the company's ultimate liabilities associated with all Tier 2 insureds. This could then be repeated for other tier categories. Burn factors could be calculated in total for each tier or with further refinement by policy year.

Exhibit 12 provides an example of one part of this analysis, the calculation of ABC Re's liability for Insured 3 in the $5 million excess $5 million layer. To do this, the model is used for Insured 3 policies, with the policies' width, attachment points, and percentage shares overridden by $5 million, $5 million, and 100%, respectively. This is done for all Insured 3 policies.

Exhibit 13 shows a grid which would ultimately be completed for use in this extrapolation method. In calculating the burn factors or percentage eroded by layer by tier, all insureds in the sample group would be run through the model using the desired policy layers in place of the actual policy exposures. The exposures from the insureds not in the sample group would be arrayed in a similar matrix as they are in Exhibit 13, by layer by tier. The matrix of exposures would be multiplied by each corresponding cell in the percentage eroded matrix to determine the ultimate liability of the non-sample group. For example, assume ABC Re's exposure in the $5 million excess $5 million layer was $100 million for Tier 2 non-sample group companies. $100 million times 42% from Exhibit 13 gives projected ultimate liability of $42 million for the Tier 2, $5 million excess $5 million layer. This calculation would be repeated for each tier and layer combination and the results would be summed. It would then be necessary to combine this estimate for the non-sample group with the selected estimate of $153 million (Exhibit 11) for the sample group to produce an estimate of ABC Re's total liabilities.

This approach is likely better than the other approaches outlined below, particularly when differences by policy year are recognized. However, it is also the most cumbersome as it requires attachment point and limits information on all exposures. The likelihood of asbestos exclusions applying in certain years or policies falling outside the insureds' coverage blocks should be considered.

Method Two

The second method is performed by determining the development factor to ultimate by tier implied by the model output relative to the reported case incurred loss and expense held by the company for the sample group. The development factors are then applied to the total incurred loss and expense for each tier category. This approach assumes consistent case reserving for sample group insureds versus other insureds. Grouping the insureds by tier is expected to result in more homogeneous groupings with respect to case reserving and layers exposed, but differences between the sample and non-sample group should be explored in the extrapolation procedure. For example, if the information available for insureds in the sample group were more complete than the non-sample group, then an extrapolation might result in an understatement of total liability because too small a development factor would be applied to the less developed losses. Likewise, if the company wrote policies with a wide range of attachment points and the sample group represented insureds with lower layer policies, case reserving might not be as adequate on the non-sample group with higher layer policies. Thus, the development factors may be expected to differ for the two groups due to the different layers exposed.

The reported case incurred loss and expense development factors by tier by scenario are found on Exhibit 10. The selection of development factors based on all four scenarios is shown on Exhibit 14. These factors by tier would be multiplied by the non-sample group reported loss and expense by tier to calculate an ultimate loss and expense for non-sample group insureds. For example, assuming ABC Re's non-sample group Tier 1 insureds have reported loss and expense of $20 million dollars, the calculated non-sample group Tier 1 ultimate liability would be $20 million times 1.935 from Exhibit 14, or $39 million. This calculation would be repeated for each tier and summed. Adding to this sum the ultimate liability of the sample group, $153 million from Exhibit 11, would yield ABC Re's total asbestos BI liability based on this extrapolation method.

It should be noted that, as in this example, the development factors are generally relatively large (close to 2 in our example and potentially much greater). Thus, the presence or absence of a large reported loss could significantly impact the projection.

Method Three

The third extrapolation method is to calculate by tier the percentage of exposed policy limits ultimately exhausted by the asbestos BI claims, as projected in the model, and apply these percentages to the total exposed policy limits by tier. Differences in exposed limits by attachment point for the sample versus non-sample group should be considered in applying this procedure.

The ultimate loss and expense as a percentage of exposure can be found on Exhibit 10. The selection of percentage of exposure factors based on all four scenarios is shown on Exhibit 15. These factors by tier would be multiplied by the non-sample group exposure by tier to calculate the estimated liability for the non-sample group. For example, assuming ABC Re's non-sample group Tier 2 insureds have exposure of $50 million for all layers, the estimated Tier 2 liability would be $50 million times 30.7%, or $15 million. This calculation would be repeated for each tier and summed. Note that the non-sample group exposure by tier is the sum of each tier's non-sample group exposure by layer which was used in the first extrapolation method. Adding the sample group's ultimate liability of $153 million from Exhibit 11 to the summed estimated ultimate liability for the non-sample group yields ABC Re's total asbestos BI liability based on this extrapolation method.

Method Four

The fourth method is a frequency times ultimate severity method. By tier, one could calculate an average ultimate loss and expense amount per insured in the sample group and multiply by the total number of insureds. This approach assumes that the sample group represents a typical distribution of limits written per insured and that the sample group and non-sample group are composed of insureds with similar exposure distributions. For example, the sample group should not be selected from the set of claims and the average results applied to the set of precautionary notices. However, extrapolation of the precautionary notice group could be accomplished by estimating the percentage of notices expected to become claims in the future. This could be done by reviewing the magnitude of movement from the notice to the claim category over the past several years.

Exhibit 16 shows the average ultimate loss and expense by tier for each of the four scenarios. From these an average ultimate loss and expense by tier is selected, based on a 75% weight to the 15-year coverage block scenarios and a 25% weight to the 25-year coverage block scenarios. This selected average amount by tier would be multiplied by the number of non-sample group insureds by tier. For example, if ABC Re had 50 Tier 3 insureds, then ABC Re's projected liability for non-sample group Tier 3 companies would be 50 times $794,000, or $40 million. The $794,000 is from Exhibit 16. This calculation would be repeated for each tier and summed. The sum, equal to the estimated liability for all non-sample group insureds, would be added to $153 million, ABC Re's estimated sample group liability, to derive the estimate of ABC Re's overall liability based on this extrapolation method.

Method Five

The fifth method is an extrapolation of Tiers 1 and 2. It is accomplished by using one of the above methods for the Tier 1 and 2 exposures and then extrapolating from the Tier 1 and 2

TABLE 3

	Average Ground-Up Liabilities (in Millions)	Percentage of Exposed Limits Exhausted
Tier 1	3,000	100%–110%
Tier 2	700	25%–35%
Tier 3	50	6%–10%

results to the remaining tiers. For example, given the following information for Tiers 1 and 2 versus Tier 3, an extrapolation of the percentage of exposed limits exhausted may indicate a range of 6% to 10% for Tier 3 insureds. The selected percentage could then be applied to the aggregate of exposed policy limits for Tier 3 insureds. The assumptions used in this method are presented in Table 3.

A subjective extrapolation could also be carried out using the expected percentage reported by tier. For example, if Tier 1 insureds are 55% reported and Tier 2 30% reported, we might estimate that Tier 3 insureds are 15% to 20% reported.

In extrapolating the model results to reflect the company's total liabilities, insureds presenting an unusual type or degree of exposure to the company should be considered separately. For example, an unusual degree of exposure would exist when a vast majority of the company's products liability policies were written with aggregate limits but one old policy without an aggregate has surfaced with a Tier 1 named insured. Similarly, if the company generally insured risks categorized as "main street," but a Tier 1 or Tier 2 company was insured for a number of years on a first or second excess of loss layer, the magnitude of the potential asbestos BI liabilities could be substantial relative to other insureds. In addition, a pending dispute regarding significant amounts of potential coverage for a Tier 1 or Tier 2 insured or an applicable settlement agreement would warrant separate consideration. Such cases require discussions with claims department personnel and a review of assumptions underlying case reserves. Estimates for these unusual exposures should be derived on a case-by-case basis and included in the total ultimate loss estimates for the company.

6. SUMMARY AND CONCLUSIONS

This paper demonstrates a methodology for modeling asbestos BI liabilities. While this policy limits methodology was designed specifically for modeling asbestos BI liability, there may be potential for application to other insurance situations where traditional actuarial techniques do not apply well. There are two clear strengths of this model: 1) its flexibility, and 2) enhanced documentation.

With the model's flexibility, any parameter can be changed for sensitivity analysis. As noted earlier, the average severity trend can be adjusted to test the impact of various inflation assumptions. The claim count reporting pattern for the sample group can be sped up or lagged. If evidence suggests that certain insureds' expenses are declining relative to indemnity (particularly now that the courts have resolved many legal issues), the expense-to-indemnity ratio can be adjusted on a year-by-year basis. Finally, if the coverage block of the insured is unknown or changed in a court ruling, the number of years and the weighting of each year in the coverage block can be varied.

Enhanced documentation for modeling asbestos BI liability is another strength of the model and a benefit for claims professionals handling asbestos BI claims. These professionals are often requested to provide input into the process of estimating IBNR claim liabilities on known insureds or are specifically assigned the responsibility of establishing case reserves incorporating unreported claim activity for the foreseeable future. They are likely to follow an approach similar to that used in our model with insureds for which sufficient policy information is known. Benefits of a more formalized model analysis include: 1) an automated process which permits the testing of alternative scenarios and facilitates future updates as additional information emerges; 2) an aggregate view of the company's estimated liabilities to help analyze cash flow requirements or produce benchmarks when historical claims data is not available; and 3) enhanced documentation to support aggregate reserve levels to outside auditors and regulators.

Possible weaknesses of the model as presented include: 1) it is a deterministic rather than a stochastic approach to the estimation of asbestos BI liabilities; and 2) it is dependent on reasonably accurate selection of model parameters. Both of these disadvantages can be minimized through sensitivity analysis. Several scenarios should be run through the model to estimate the range of potential liabilities and to minimize errors due to parameter mis-estimation. Also, with additional programming or use of appropriate computer software, model parameters can be varied stochastically.

Possible enhancements to the model or additional areas requiring research in projecting asbestos liabilities include: 1) the inclusion of extra parameters to more comprehensively describe the insurance or reinsurance policy and the potential asbestos exposure associated with the policy; 2) a provision for IBNR associated with insureds who have not yet notified their insurance carriers and are not yet identified by the company; 3) a methodology for estimating liabilities associated with premises and operations claims not subject to policy aggregates; and 4) a methodology for estimating property damage claims related to asbestos.

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EXHIBIT 1

PART 1

LOCATION, COMPOSITION, AND DATES OF USE OF ASBESTOS-CONTAINING BUILDING PRODUCTS

Product	Location	Percent Asbestos	Dates of Use	Binder	Friable/ Nonfriable	How Fibers Can Be Released
Roofing and Siding						
Roofing felts	Flat, built-up roofs	10–15	1910–present	Asphalt	Nonfriable	Replacing, repairing, demolishing
Roof felt shingles	Roofs	1	1971–1974	Asphalt	Friable	Replacing, demolishing
Roofing Shingles	Roofs	20–32	1930–present	Portland cement	Nonfriable	Replacing, repairing, demolishing
Siding Shingles	Siding	12–14	?–present	Portland cement	Nonfriable	Replacing, repairing, demolishing
Clapboards	Siding	12–15	1944–1945	Portland cement	Nonfriable	Replacing, repairing, demolishing
Walls and Ceilings						
Sprayed coating	Ceilings, walls, and steelwork	1–95	1935–1978	Portland cement, sodium silicate, organic binders	Friable	Water damage, deterioration, impact
Troweled coating	Ceilings, walls	1–95	1935–1978	Portland cement, sodium silicate	Friable	Water damage, deterioration, impact
Asbestos-cement sheet	Near heat sources such as fireplaces, boilers	20–50	1930–present	Portland cement	Nonfriable	Cutting, sanding, scraping
Spackle	Walls, ceilings	3–5	1930–1978	Starch, casein, synthetic resins	Friable	Cutting, sanding, scraping

EXHIBIT 1

PART 2

LOCATION, COMPOSITION, AND DATES OF USE OF ASBESTOS-CONTAINING BUILDING PRODUCTS

Product	Location	Percent Asbestos	Dates of Use	Binder	Friable/ Nonfriable	How Fibers Can Be Released
Joint compound	Walls, ceilings	3–5	1945–1977	Asphalt	Friable	Cutting, sanding, scraping
Textured paints	Walls, ceilings	4–15	?–1978		Friable	Cutting, sanding, scraping
Millboard, rollboard	Walls, commercial buildings	80–85	1925–?	Starch, lime, clay	Friable	Cutting, demolition
Vinyl wallpaper	Walls	6–8	?		Nonfriable	Removal, sanding, dryscraping, cutting
Insulation board	Walls	30	?	Silicates	Friable	Removal, sanding, dryscraping
<u>**Floors**</u>						
Vinyl-asbestos tile	Floors	21	1950–1980?	Poly(vinyl) chloride	Nonfriable	Removal, sanding, dryscraping, cutting
Asphalt-asbestos tile	Floors	26–33	1920–1980?	Asphalt	Nonfriable	Removal, sanding dryscraping, cutting
Resilient sheet flooring	Floors	30	1950–1980?	Dry oils	Nonfriable	Removal, sanding, dryscraping, cutting
Mastic adhesive	Sheet and tile backing	5–25	1945–1980?	Asphalt	Friable	Removal, sanding, dryscraping, cutting

EXHIBIT 1

PART 3

LOCATION, COMPOSITION, AND DATES OF USE OF ASBESTOS-CONTAINING BUILDING PRODUCTS

Product	Location	Percent Asbestos	Dates of Use	Binder	Friable/ Nonfriable	How Fibers Can Be Released
Pipes and boilers						
Cement and pipe and fittings	Water and sewer	20–?	1935–present	Portland cement	Nonfriable	Demolition, cutting, removing
Block insulation	Boilers	6–15	1890–1978	Magnesium carbonate, calcium silicate	Friable	Damage, cutting, deterioration
Preformed pipe wrap	Pipes	50	1926–1975	Magnesium carbonate, calcium silicate	Friable	Damage, cutting, deterioration
Corrugated asbestos paper	Pipes	high temp. 90 mod. temp. 35–70	1935–1980? 1910–1980?	Sodium silicate, starch	Friable	Damage, cutting, deterioration
Paper tape	Furnaces, steam valves, flanges, electrical wiring	80	1901–1980?	Polymers, starches, silicates	Friable	Tearing, deterioration
Putty (Mudding)	Plumbing joints	20–100	1900–1973	Clay	Friable	Water damage, deterioration

Source: U.S. Environmental Protection Agency

EXHIBIT 2

ADJUSTMENT TO ABC REINSURANCE COMPANY'S POLICY LIMITS FOR POLICIES ASSUMED FROM XYZ INSURANCE COMPANY—INDEMNITY ONLY*

($ in Millions)

		XYZ Direct Policy Information			ABC Re's Stated Policy Information			ABC Re's Restated Policy Information					
ABC Re Policy Number (1)	Insured Company (2)	Percentage Share (3)	Attachment Point (4)	Limit (5)	Percentage Share (6)	Attachment Point (7)	Limit (8)	Percentage Share (9)	Attachment Point (10)	Limit (11)	ABC Re's Stated Dollar Share (12)	ABC Re's Restated Dollar Share (13)	Underlap Amount (14)
1	Insured 1	100.00%	60.00	10.00	7.25%	5.00	5.00	7.25%	65.00	5.00	0.36	0.36	0.00
2	Insured 1	100.00%	5.00	20.00	30.00%	5.00	10.00	30.00%	10.00	10.00	3.00	3.00	0.00
3	Insured 2	40.00%	10.00	20.00	50.00%	1.00	5.00	20.00%	12.50	12.50	2.50	2.50	0.00
4	Insured 2	10.00%	10.00	20.00	50.00%	1.00	5.00	5.00%	20.00	10.00	2.50	0.50	2.00
5	Insured 2	10.00%	10.00	20.00	50.00%	2.25	5.00	5.00%	32.50	0.00	2.50	0.00	2.50
6	Insured 2	50.00%	7.00	25.00	100.00%	5.00	15.00	50.00%	17.00	15.00	15.00	7.50	7.50
7	Insured 2	32.00%	7.00	10.00	100.00%	2.00	2.00	32.00%	13.25	3.75	2.00	1.20	0.80
8	Insured 2	100.00%	7.00	5.00	20.00%	5.00	5.00	20.00%	12.00	0.00	1.00	0.00	1.00
9	Insured 2	100.00%	7.00	5.00	20.00%	2.00	3.00	20.00%	9.00	3.00	0.60	0.60	0.00
10	Insured 2	65.00%	6.00	20.00	20.00%	10.00	5.00	13.00%	21.38	4.62	1.00	0.60	0.40
11	Insured 2	65.00%	11.00	20.00	20.00%	5.00	10.00	13.00%	18.69	12.31	2.00	1.60	0.40
12	Insured 2	10.00%	11.00	50.00	40.00%	4.00	5.00	4.00%	51.00	10.00	2.00	0.40	1.60
13	Insured 2	10.00%	11.00	50.00	40.00%	1.00	5.00	4.00%	21.00	40.00	2.00	1.60	0.40
											36.46	19.86	
									(15) Underlap Factor			54.5%	

Notes:

(3)–(5) Direct policy information. Given.

(6)–(8) Stated reinsurance policy information. Given.

(9) = (3) × (6).

(10) = [(7)/(3)] + (4).

(11) = Max{0, Min{(8)/(3), {(5) – ((7)/(3))}}}.

(12) = (6) × (8).

(13) = (9) × (11).

(14) = (12) – (13).

(15) = Total of (13)/Total of (12).

* Expenses are ignored for simplicity of presentation.

EXHIBIT 3

PARTIAL LIST OF ABC RE'S KNOWN ASBESTOS DEFENDANTS

($ in Millions)

Name of Company	Tier	Ceding Company Policy Information	ABC Re's Policy Information	Included in Sample Group
Insured 1	4	Known	Known	Yes
Insured 2	4	Known	Known	Yes
Insured 3	2	Known	Known	Yes
Insured 4	1	Known	Known	Yes
Insured 5	1	Known	Known	Yes
Insured 6	1	Known	Known	Yes
Insured 7	2	Known	Known	Yes
Insured 8	2	Known	Known	Yes
Insured 9	2	Known	Known	Yes
Insured 10	3	Known	Known	Yes
Insured 11	2	Known	Known	Yes
Insured 12	3	Known	Known	Yes
Insured 13	3	Unknown	Known	Yes
Insured 14	3	Unknown	Known	Yes
Insured 15	3	Unknown	Known	Yes
Insured 16	3	Unknown	Unknown	No
Insured 17	3	Unknown	Unknown	No
Insured 18	3	Unknown	Unknown	No
Insured 19	3	Unknown	Unknown	No
Insured 20	3	Unknown	Unknown	No
Insured 21	3	Unknown	Unknown	No
Insured 22	3	Unknown	Unknown	No
Insured 23	2	Unknown	Unknown	No

EXHIBIT 4

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
POLICY INFORMATION FOR UNDERLYING INSURED 3, A TIER 2 COMPANY

Coverage Block under Baseline Scenario: 1960–1974
Coverage Block under Alternative Scenario: 1960–1984

25 Year Cov. Block	15 Year Cov. Block	Policy Year	ABC Re Policy w/Insured 3	Restated Percentage Share	Restated Attachment Point	Restated Limits	Expense Treatment
		1958	Yes	100.00%	3,500,000	4,000,000	Pro Rata in Addition to Limit
		1959	None				
1	1	1960	None				
2	2	1961	None				
3	3	1962	None				
4	4	1963	None				
5	5	1964	None				
6	6	1965	Yes	100.00%	2,700,000	2,000,000	Pro Rata in Addition to Limit
7	7	1966	Yes	100.00%	2,700,000	2,000,000	Pro Rata in Addition to Limit
8	8	1967	Yes	100.00%	2,700,000	2,000,000	Expenses included within Limit
9	9	1968	Yes	100.00%	3,500,000	4,000,000	Pro Rata in Addition to Limit
10	10	1969	Yes	100.00%	3,500,000	4,000,000	Expenses included within Limit
11	11	1970	Yes	25.00%	3,500,000	4,000,000	Pro Rata in Addition to Limit
12	12	1971	Yes	100.00%	2,000,000	2,000,000	Indemnity Only
13	13	1972	None				
14	14	1973	None				
15	15	1974	None				
16		1975	None				
17		1976	None				
18		1977	None				
19		1978	None				
20		1979	None				
21		1980	None				
22		1981	None				
23		1982	None				
24		1983	None				
25		1984	None				

EXHIBIT 5.1

PART 1

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
PROJECTION OF FUTURE AGGREGATE GROUND-UP INDEMNITY AND EXPENSES, ANNUAL INFLATION = 5.0%

Inputs into Model	1991	1992	1993
1) Cumulative Reported Claims to Date	35,000	37,500	40,000
2) Cumulative Reported Indemnity	23,349,294	25,730,246	28,230,246
3) Historical Exp-to-Indem Ratio	0.5	0.5	0.5
4) Cumulative Reported Indem & Expense	35,023,941	38,595,369	42,345,369
5) Claims Closed in Year	1,600	1,800	2,000
6) Indemnity and Expense Paid in Year	1,312,000	1,530,000	1,800,000
7) Average Pd Indemnity & Expense in Year	820	850	900
8) Selected Average Reported Claim Severity			1,000

	Calendar Year						
	1994	1995	1996	1997	1998	1999	2000
9) Projected Incremental Reported Claims	2,500	2,200	2,200	2,200	2,100	2,000	1,900
10) Selected Annual Severity Trend	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
11) Trended Severity	1,050	1,103	1,158	1,216	1,276	1,340	1,407
12) Projected Incremental Indemnity Costs	2,625,000	2,425,500	2,546,775	2,674,114	2,680,191	2,680,191	2,673,491
13) Selected Expense-to-Indemnity Ratio	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%
14) Projected Incremental Indemnity & Expense Costs	3,937,500	3,638,250	3,820,163	4,011,171	4,020,287	4,020,287	4,010,236
15) Projected Cumulative Indemnity Costs	30,855,246	33,280,746	35,827,521	38,501,635	41,181,826	43,862,018	46,535,508
16) Projected Cumulative Indemnity & Expense Costs	46,282,869	49,921,119	53,741,282	57,752,453	61,772,739	65,793,026	69,803,263

EXHIBIT 5.1

PART 2

	Calendar Year						
	2001	2002	2003	2004	2005	2006	2007
9) Projected Incremental Reported Claims	1,800	1,700	1,600	1,500	1,400	1,300	1,200
10) Selected Annual Severity Trend	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
11) Trended Severity	1,477	1,551	1,629	1,710	1,796	1,886	1,980
12) Projected Incremental Indemnity Costs	2,659,420	2,637,258	2,606,231	2,565,509	2,514,199	2,451,344	2,375,918
13) Selected Expense-to-Indemnity Ratio	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%
14) Projected Incremental Indemnity & Expense Costs	3,989,130	3,955,887	3,909,347	3,848,264	3,771,298	3,677,016	3,563,877
15) Projected Cumulative Indemnity Costs	49,194,928	51,832,186	54,438,418	57,003,927	59,518,125	61,969,469	64,345,387
16) Projected Cumulative Indemnity & Expense Costs	73,792,392	77,748,279	81,657,626	85,505,890	89,277,188	92,954,204	96,518,081

	Calendar Year						Projected
	2008	2009	2010	2011	2012	2013	Ultimate*
9) Projected Incremental Reported Claims	1,100	1,000	900	800	700	600	
10) Selected Annual Severity Trend	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	
11) Trended Severity	2,079	2,183	2,292	2,407	2,527	2,653	
12) Projected Incremental Indemnity Costs	2,286,821	2,182,875	2,062,816	1,925,295	1,768,865	1,591,979	
13) Selected Expense-to-Indemnity Ratio	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	
14) Projected Incremental Indemnity & Expense Costs	3,430,231	3,274,312	3,094,225	2,887,943	2,653,298	2,387,968	
15) Projected Cumulative Indemnity Costs	66,632,208	68,815,083	70,877,899	72,803,195	74,572,060	76,164,038	104,131,118
16) Projected Cumulative Indemnity & Expense Costs	99,948,312	103,222,624	106,316,849	109,204,792	111,858,090	114,246,058	156,196,678

Notes:

(1)–(6) From Insured 3's claim experience.

(7) = (6)/(5).

(8), (10) Selected based on historical and anticipated claim severity trends.

(9) See paper for discussion of calculation of reporting pattern.

(11) = Prior (11) × (1.0 + Current (10)).

*Ultimate value is calculated by continuation of patterns beyond years shown.

(12) = (9) × (11).

(13) Selected based on historical and anticipated claim expense to indemnity ratios.

(14) = (12) × (1.0 + (13)).

(15) = Cumulative (12).

(16) = Cumulative (14).

EXHIBIT 5.2

PART 1

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
PROJECTION OF FUTURE AGGREGATE GROUND-UP INDEMNITY AND EXPENSES, ANNUAL INFLATION = 0.0%

Inputs into Model	1991	1992	1993
1) Cumulative Reported Claims to Date	35,000	37,500	40,000
2) Cumulative Reported Indemnity	23,349,294	25,730,246	28,230,246
3) Historical Exp-to-Indem Ratio	0.5	0.5	0.5
4) Cumulative Reported Indem & Expense	35,023,941	38,595,369	42,345,369
5) Claims Closed in Year	1,600	1,800	2,000
6) Indemnity and Expense Paid in Year	1,312,000	1,530,000	1,800,000
7) Average Pd Indemnity & Expense in Year	820	850	900
8) Selected Average Reported Claim Severity			1,000

	Calendar Year						
	1994	1995	1996	1997	1998	1999	2000
9) Projected Incremental Reported Claims	2,500	2,200	2,200	2,200	2,100	2,000	1,900
10) Selected Annual Severity Trend	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
11) Trended Severity	1,000	1,000	1,000	1,000	1,000	1,000	1,000
12) Projected Incremental Indemnity Costs	2,500,000	2,200,000	2,200,000	2,200,000	2,100,000	2,000,000	1,900,000
13) Selected Expense-to-Indemnity Ratio	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%
14) Projected Incremental Indemnity & Expense Costs	3,750,000	3,300,000	3,300,000	3,300,000	3,150,000	3,000,000	2,850,000
15) Projected Cumulative Indemnity Costs	30,730,246	32,930,246	35,130,246	37,330,246	39,430,246	41,430,246	43,330,246
16) Projected Cumulative Indemnity & Expense Costs	46,095,369	49,395,369	52,695,369	55,995,369	59,145,369	62,145,369	64,995,369

EXHIBIT 5.2

PART 2

	Calendar Year						
	2001	2002	2003	2004	2005	2006	2007
9) Projected Incremental Reported Claims	1,800	1,700	1,600	1,500	1,400	1,300	1,200
10) Selected Annual Severity Trend	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
11) Trended Severity	1,000	1,000	1,000	1,000	1,000	1,000	1,000
12) Projected Incremental Indemnity Costs	1,800,000	1,700,000	1,600,000	1,500,000	1,400,000	1,300,000	1,200,000
13) Selected Expense-to-Indemnity Ratio	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%
14) Projected Incremental Indemnity & Expense Costs	2,700,000	2,550,000	2,400,000	2,250,000	2,100,000	1,950,000	1,800,000
15) Projected Cumulative Indemnity Costs	45,130,246	46,830,246	48,430,246	49,930,246	51,330,246	52,630,246	53,830,246
16) Projected Cumulative Indemnity & Expense Costs	67,695,369	70,245,369	72,645,369	74,895,369	76,995,369	78,945,369	80,745,369

	Calendar Year						Projected
	2008	2009	2010	2011	2012	2013	Ultimate*
9) Projected Incremental Reported Claims	1,100	1,000	900	800	700	600	
10) Selected Annual Severity Trend	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	
11) Trended Severity	1,000	1,000	1,000	1,000	1,000	1,000	
12) Projected Incremental Indemnity Costs	1,100,000	1,000,000	900,000	800,000	700,000	600,000	
13) Selected Expense-to-Indemnity Ratio	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	
14) Projected Incremental Indemnity & Expense Costs	1,650,000	1,500,000	1,350,000	1,200,000	1,050,000	900,000	
15) Projected Cumulative Indemnity Costs	54,930,246	55,930,246	56,830,246	57,630,246	58,330,246	58,930,246	65,755,246
16) Projected Cumulative Indemnity & Expense Costs	82,395,369	83,895,369	85,245,369	86,445,369	87,495,369	88,395,369	98,632,869

Notes:
(1)–(6) From Insured 3's claim experience.
(7) = (6)/(5).
(8), (10) Selected based on historical and anticipated claim severity trends.
(9) See paper for discussion of calculation of reporting pattern.
(11) = Prior (11) × (1.0 + Current (10)).
(12) = (9) × (11).
(13) Selected based on historical and anticipated claim expense to indemnity ratios.
(14) = (12) × (1.0 + (13)).
(15) = Cumulative (12).
(16) = Cumulative (14).

*Ultimate value is calculated by continuation of patterns beyond years shown.

EXHIBIT 6.1

PART 1

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
INSURER 3'S CUMULATIVE GROUND-UP LOSSES, INDEMNITY ONLY,
ANNUAL INFLATION = 5.0%/COVERAGE BLOCK = 15 YEARS
($000's)

Policy	Selected	Calendar Year									
Year	Weights	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
1960	6.67%	2,057	2,219	2,389	2,567	2,745	2,924	3,102	3,280	3,455	3,629
1961	6.67%	2,057	2,219	2,389	2,567	2,745	2,924	3,102	3,280	3,455	3,629
1962	6.67%	2,057	2,219	2,389	2,567	2,745	2,924	3,102	3,280	3,455	3,629
1963	6.67%	2,057	2,219	2,389	2,567	2,745	2,924	3,102	3,280	3,455	3,629
1964	6.67%	2,057	2,219	2,389	2,567	2,745	2,924	3,102	3,280	3,455	3,629
1965	6.67%	2,057	2,219	2,389	2,567	2,745	2,924	3,102	3,280	3,455	3,629
1966	6.67%	2,057	2,219	2,389	2,567	2,745	2,924	3,102	3,280	3,455	3,629
1967	6.67%	2,057	2,219	2,389	2,567	2,745	2,924	3,102	3,280	3,455	3,629
1968	6.67%	2,057	2,219	2,389	2,567	2,745	2,924	3,102	3,280	3,455	3,629
1969	6.67%	2,057	2,219	2,389	2,567	2,745	2,924	3,102	3,280	3,455	3,629
1970	6.67%	2,057	2,219	2,389	2,567	2,745	2,924	3,102	3,280	3,455	3,629
1971	6.67%	2,057	2,219	2,389	2,567	2,745	2,924	3,102	3,280	3,455	3,629
1972	6.67%	2,057	2,219	2,389	2,567	2,745	2,924	3,102	3,280	3,455	3,629
1973	6.67%	2,057	2,219	2,389	2,567	2,745	2,924	3,102	3,280	3,455	3,629
1974	6.67%	2,057	2,219	2,389	2,567	2,745	2,924	3,102	3,280	3,455	3,629
1975–84	0.00%	0	0	0	0	0	0	0	0	0	0
Total	100.00%	30,855	33,281	35,828	38,502	41,182	43,862	46,536	49,195	51,832	54,438

EXHIBIT 6.1

PART 2

Policy Year	Selected Weights	Calendar Year 2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	Ultimate
1960	6.67%	3,800	3,968	4,131	4,290	4,442	4,588	4,725	4,854	4,971	5,078	6,942
1961	6.67%	3,800	3,968	4,131	4,290	4,442	4,588	4,725	4,854	4,971	5,078	6,942
1962	6.67%	3,800	3,968	4,131	4,290	4,442	4,588	4,725	4,854	4,971	5,078	6,942
1963	6.67%	3,800	3,968	4,131	4,290	4,442	4,588	4,725	4,854	4,971	5,078	6,942
1964	6.67%	3,800	3,968	4,131	4,290	4,442	4,588	4,725	4,854	4,971	5,078	6,942
1965	6.67%	3,800	3,968	4,131	4,290	4,442	4,588	4,725	4,854	4,971	5,078	6,942
1966	6.67%	3,800	3,968	4,131	4,290	4,442	4,588	4,725	4,854	4,971	5,078	6,942
1967	6.67%	3,800	3,968	4,131	4,290	4,442	4,588	4,725	4,854	4,971	5,078	6,942
1968	6.67%	3,800	3,968	4,131	4,290	4,442	4,588	4,725	4,854	4,971	5,078	6,942
1969	6.67%	3,800	3,968	4,131	4,290	4,442	4,588	4,725	4,854	4,971	5,078	6,942
1970	6.67%	3,800	3,968	4,131	4,290	4,442	4,588	4,725	4,854	4,971	5,078	6,942
1971	6.67%	3,800	3,968	4,131	4,290	4,442	4,588	4,725	4,854	4,971	5,078	6,942
1972	6.67%	3,800	3,968	4,131	4,290	4,442	4,588	4,725	4,854	4,971	5,078	6,942
1973	6.67%	3,800	3,968	4,131	4,290	4,442	4,588	4,725	4,854	4,971	5,078	6,942
1974	6.67%	3,800	3,968	4,131	4,290	4,442	4,588	4,725	4,854	4,971	5,078	6,942
1975–84	0.00%	0	0	0	0	0	0	0	0	0	0	0
Total	100.00%	57,004	59,518	61,969	64,345	66,632	68,815	70,878	72,803	74,572	76,164	104,131

Notes:

— Cumulative projected calendar year ground-up indemnity costs from Exhibit 5.1, Item 15.
— Allocation method of calendar year losses to policy year is by equal weighting to each year.
— Ultimate value is calculated by continuation of patterns beyond months shown.

EXHIBIT 6.2

PART 1

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
INSURER 3'S CUMULATIVE GROUND-UP LOSSES, INDEMNITY ONLY, ANNUAL INFLATION = 0.0%/COVERAGE BLOCK = 15 YEARS
($000's)

Policy	Selected	Calendar Year									
Year	Weights	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
1960	6.67%	2,049	2,195	2,342	2,489	2,629	2,762	2,889	3,009	3,122	3,229
1961	6.67%	2,049	2,195	2,342	2,489	2,629	2,762	2,889	3,009	3,122	3,229
1962	6.67%	2,049	2,195	2,342	2,489	2,629	2,762	2,889	3,009	3,122	3,229
1963	6.67%	2,049	2,195	2,342	2,489	2,629	2,762	2,889	3,009	3,122	3,229
1964	6.67%	2,049	2,195	2,342	2,489	2,629	2,762	2,889	3,009	3,122	3,229
1965	6.67%	2,049	2,195	2,342	2,489	2,629	2,762	2,889	3,009	3,122	3,229
1966	6.67%	2,049	2,195	2,342	2,489	2,629	2,762	2,889	3,009	3,122	3,229
1967	6.67%	2,049	2,195	2,342	2,489	2,629	2,762	2,889	3,009	3,122	3,229
1968	6.67%	2,049	2,195	2,342	2,489	2,629	2,762	2,889	3,009	3,122	3,229
1969	6.67%	2,049	2,195	2,342	2,489	2,629	2,762	2,889	3,009	3,122	3,229
1970	6.67%	2,049	2,195	2,342	2,489	2,629	2,762	2,889	3,009	3,122	3,229
1971	6.67%	2,049	2,195	2,342	2,489	2,629	2,762	2,889	3,009	3,122	3,229
1972	6.67%	2,049	2,195	2,342	2,489	2,629	2,762	2,889	3,009	3,122	3,229
1973	6.67%	2,049	2,195	2,342	2,489	2,629	2,762	2,889	3,009	3,122	3,229
1974	6.67%	2,049	2,195	2,342	2,489	2,629	2,762	2,889	3,009	3,122	3,229
1975–84	0.00%	0	0	0	0	0	0	0	0	0	0
Total	100.00%	30,730	32,930	35,130	37,330	39,430	41,430	43,330	45,130	46,830	48,430

EXHIBIT 6.2

PART 2

Policy Year	Selected Weights	Calendar Year 2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	Ultimate
1960	6.67%	3,329	3,422	3,509	3,589	3,662	3,729	3,789	3,842	3,889	3,929	4,384
1961	6.67%	3,329	3,422	3,509	3,589	3,662	3,729	3,789	3,842	3,889	3,929	4,384
1962	6.67%	3,329	3,422	3,509	3,589	3,662	3,729	3,789	3,842	3,889	3,929	4,384
1963	6.67%	3,329	3,422	3,509	3,589	3,662	3,729	3,789	3,842	3,889	3,929	4,384
1964	6.67%	3,329	3,422	3,509	3,589	3,662	3,729	3,789	3,842	3,889	3,929	4,384
1965	6.67%	3,329	3,422	3,509	3,589	3,662	3,729	3,789	3,842	3,889	3,929	4,384
1966	6.67%	3,329	3,422	3,509	3,589	3,662	3,729	3,789	3,842	3,889	3,929	4,384
1967	6.67%	3,329	3,422	3,509	3,589	3,662	3,729	3,789	3,842	3,889	3,929	4,384
1968	6.67%	3,329	3,422	3,509	3,589	3,662	3,729	3,789	3,842	3,889	3,929	4,384
1969	6.67%	3,329	3,422	3,509	3,589	3,662	3,729	3,789	3,842	3,889	3,929	4,384
1970	6.67%	3,329	3,422	3,509	3,589	3,662	3,729	3,789	3,842	3,889	3,929	4,384
1971	6.67%	3,329	3,422	3,509	3,589	3,662	3,729	3,789	3,842	3,889	3,929	4,384
1972	6.67%	3,329	3,422	3,509	3,589	3,662	3,729	3,789	3,842	3,889	3,929	4,384
1973	6.67%	3,329	3,422	3,509	3,589	3,662	3,729	3,789	3,842	3,889	3,929	4,384
1974	6.67%	3,329	3,422	3,509	3,589	3,662	3,729	3,789	3,842	3,889	3,929	4,384
1975–84	0.00%	0	0	0	0	0	0	0	0	0	0	
Total	100.00%	49,930	51,330	52,630	53,830	54,930	55,930	56,830	57,630	58,330	58,930	65,755

Notes:

— Cumulative projected calendar year ground-up indemnity costs from Exhibit 5.2, Item 15.
— Allocation method of calendar year losses to policy year is by equal weighting to each year.
— Ultimate value is calculated by continuation of patterns beyond months shown.

EXHIBIT 6.3

PART 1

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
INSURER 3'S CUMULATIVE GROUND-UP LOSSES, INDEMNITY ONLY,
ANNUAL INFLATION = 5.0%/COVERAGE BLOCK = 25 YEARS
($000's)

Policy	Selected	Calendar Year									
Year	Weights	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
1960	4.00%	1,234	1,331	1,433	1,540	1,647	1,754	1,861	1,968	2,073	2,178
1961	4.00%	1,234	1,331	1,433	1,540	1,647	1,754	1,861	1,968	2,073	2,178
1962	4.00%	1,234	1,331	1,433	1,540	1,647	1,754	1,861	1,968	2,073	2,178
1963	4.00%	1,234	1,331	1,433	1,540	1,647	1,754	1,861	1,968	2,073	2,178
1964	4.00%	1,234	1,331	1,433	1,540	1,647	1,754	1,861	1,968	2,073	2,178
1965	4.00%	1,234	1,331	1,433	1,540	1,647	1,754	1,861	1,968	2,073	2,178
1966	4.00%	1,234	1,331	1,433	1,540	1,647	1,754	1,861	1,968	2,073	2,178
1967	4.00%	1,234	1,331	1,433	1,540	1,647	1,754	1,861	1,968	2,073	2,178
1968	4.00%	1,234	1,331	1,433	1,540	1,647	1,754	1,861	1,968	2,073	2,178
1969	4.00%	1,234	1,331	1,433	1,540	1,647	1,754	1,861	1,968	2,073	2,178
1970	4.00%	1,234	1,331	1,433	1,540	1,647	1,754	1,861	1,968	2,073	2,178
1971	4.00%	1,234	1,331	1,433	1,540	1,647	1,754	1,861	1,968	2,073	2,178
1972	4.00%	1,234	1,331	1,433	1,540	1,647	1,754	1,861	1,968	2,073	2,178
1973	4.00%	1,234	1,331	1,433	1,540	1,647	1,754	1,861	1,968	2,073	2,178
1974	4.00%	1,234	1,331	1,433	1,540	1,647	1,754	1,861	1,968	2,073	2,178
1975–84	40.00%	12,342	13,312	14,331	15,401	16,473	17,545	18,614	19,678	20,733	21,775
Total	100.00%	30,855	33,280	35,828	38,502	41,182	43,862	46,535	49,195	51,832	54,438

EXHIBIT 6.3

PART 2

Policy Year	Selected Weights	Calendar Year 2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	Ultimate
1960	4.00%	2,280	2,381	2,479	2,574	2,665	2,753	2,835	2,912	2,983	3,047	4,165
1961	4.00%	2,280	2,381	2,479	2,574	2,665	2,753	2,835	2,912	2,983	3,047	4,165
1962	4.00%	2,280	2,381	2,479	2,574	2,665	2,753	2,835	2,912	2,983	3,047	4,165
1963	4.00%	2,280	2,381	2,479	2,574	2,665	2,753	2,835	2,912	2,983	3,047	4,165
1964	4.00%	2,280	2,381	2,479	2,574	2,665	2,753	2,835	2,912	2,983	3,047	4,165
1965	4.00%	2,280	2,381	2,479	2,574	2,665	2,753	2,835	2,912	2,983	3,047	4,165
1966	4.00%	2,280	2,381	2,479	2,574	2,665	2,753	2,835	2,912	2,983	3,047	4,165
1967	4.00%	2,280	2,381	2,479	2,574	2,665	2,753	2,835	2,912	2,983	3,047	4,165
1968	4.00%	2,280	2,381	2,479	2,574	2,665	2,753	2,835	2,912	2,983	3,047	4,165
1969	4.00%	2,280	2,381	2,479	2,574	2,665	2,753	2,835	2,912	2,983	3,047	4,165
1970	4.00%	2,280	2,381	2,479	2,574	2,665	2,753	2,835	2,912	2,983	3,047	4,165
1971	4.00%	2,280	2,381	2,479	2,574	2,665	2,753	2,835	2,912	2,983	3,047	4,165
1972	4.00%	2,280	2,381	2,479	2,574	2,665	2,753	2,835	2,912	2,983	3,047	4,165
1973	4.00%	2,280	2,381	2,479	2,574	2,665	2,753	2,835	2,912	2,983	3,047	4,165
1974	4.00%	2,280	2,381	2,479	2,574	2,665	2,753	2,835	2,912	2,983	3,047	4,165
1975–84	40.00%	22,802	23,807	24,788	25,738	26,653	27,526	28,351	29,121	29,829	30,466	41,652
Total	100.00%	57,004	59,518	61,970	64,345	66,632	68,815	70,878	72,803	74,572	76,164	104,131

Notes:

— Cumulative projected calendar year ground-up indemnity costs from Exhibit 5.1, Item 15.

— Allocation method of calendar year losses to policy year is by equal weighting to each year.

— Ultimate value is calculated by continuation of patterns beyond months shown.

EXHIBIT 6.4

PART 1

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
INSURER 3'S CUMULATIVE GROUND-UP LOSSES, INDEMNITY ONLY, ANNUAL INFLATION = 0.0%/COVERAGE BLOCK = 25 YEARS
($000's)

Policy Year	Selected Weights	Calendar Year 1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
1960	4.00%	1,229	1,317	1,405	1,493	1,577	1,657	1,733	1,805	1,873	1,937
1961	4.00%	1,229	1,317	1,405	1,493	1,577	1,657	1,733	1,805	1,873	1,937
1962	4.00%	1,229	1,317	1,405	1,493	1,577	1,657	1,733	1,805	1,873	1,937
1963	4.00%	1,229	1,317	1,405	1,493	1,577	1,657	1,733	1,805	1,873	1,937
1964	4.00%	1,229	1,317	1,405	1,493	1,577	1,657	1,733	1,805	1,873	1,937
1965	4.00%	1,229	1,317	1,405	1,493	1,577	1,657	1,733	1,805	1,873	1,937
1966	4.00%	1,229	1,317	1,405	1,493	1,577	1,657	1,733	1,805	1,873	1,937
1967	4.00%	1,229	1,317	1,405	1,493	1,577	1,657	1,733	1,805	1,873	1,937
1968	4.00%	1,229	1,317	1,405	1,493	1,577	1,657	1,733	1,805	1,873	1,937
1969	4.00%	1,229	1,317	1,405	1,493	1,577	1,657	1,733	1,805	1,873	1,937
1970	4.00%	1,229	1,317	1,405	1,493	1,577	1,657	1,733	1,805	1,873	1,937
1971	4.00%	1,229	1,317	1,405	1,493	1,577	1,657	1,733	1,805	1,873	1,937
1972	4.00%	1,229	1,317	1,405	1,493	1,577	1,657	1,733	1,805	1,873	1,937
1973	4.00%	1,229	1,317	1,405	1,493	1,577	1,657	1,733	1,805	1,873	1,937
1974	4.00%	1,229	1,317	1,405	1,493	1,577	1,657	1,733	1,805	1,873	1,937
1975–84	40.00%	12,292	13,172	14,052	14,932	15,772	16,572	17,332	18,052	18,732	19,372
Total	100.00%	30,730	32,930	35,130	37,330	39,430	41,430	43,330	45,130	46,830	48,430

EXHIBIT 6.4

PART 2

Policy Year	Selected Weights	Calendar Year 2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	Ultimate
1960	4.00%	1,997	2,053	2,105	2,153	2,197	2,237	2,273	2,305	2,333	2,357	2,630
1961	4.00%	1,997	2,053	2,105	2,153	2,197	2,237	2,273	2,305	2,333	2,357	2,630
1962	4.00%	1,997	2,053	2,105	2,153	2,197	2,237	2,273	2,305	2,333	2,357	2,630
1963	4.00%	1,997	2,053	2,105	2,153	2,197	2,237	2,273	2,305	2,333	2,357	2,630
1964	4.00%	1,997	2,053	2,105	2,153	2,197	2,237	2,273	2,305	2,333	2,357	2,630
1965	4.00%	1,997	2,053	2,105	2,153	2,197	2,237	2,273	2,305	2,333	2,357	2,630
1966	4.00%	1,997	2,053	2,105	2,153	2,197	2,237	2,273	2,305	2,333	2,357	2,630
1967	4.00%	1,997	2,053	2,105	2,153	2,197	2,237	2,273	2,305	2,333	2,357	2,630
1968	4.00%	1,997	2,053	2,105	2,153	2,197	2,237	2,273	2,305	2,333	2,357	2,630
1969	4.00%	1,997	2,053	2,105	2,153	2,197	2,237	2,273	2,305	2,333	2,357	2,630
1970	4.00%	1,997	2,053	2,105	2,153	2,197	2,237	2,273	2,305	2,333	2,357	2,630
1971	4.00%	1,997	2,053	2,105	2,153	2,197	2,237	2,273	2,305	2,333	2,357	2,630
1972	4.00%	1,997	2,053	2,105	2,153	2,197	2,237	2,273	2,305	2,333	2,357	2,630
1973	4.00%	1,997	2,053	2,105	2,153	2,197	2,237	2,273	2,305	2,333	2,357	2,630
1974	4.00%	1,997	2,053	2,105	2,153	2,197	2,237	2,273	2,305	2,333	2,357	2,630
1975–84	40.00%	19,972	20,532	21,052	21,532	21,972	22,372	22,732	23,052	23,332	23,572	26,302
Total	100.00%	49,930	51,330	52,630	53,830	54,930	55,930	56,830	57,630	58,330	58,930	65,755

Notes:

— Cumulative projected calendar year ground-up indemnity costs from Exhibit 5.2, Item 15.
— Allocation method of calendar year losses to policy year is by equal weighting to each year.
— Ultimate value is calculated by continuation of patterns beyond months shown.

EXHIBIT 7.1

PART 1

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
INSURER 3'S CUMULATIVE GROUND-UP LOSSES, INDEMNITY AND EXPENSES, ANNUAL INFLATION = 5.0%/COVERAGE BLOCK = 15 YEARS
($000's)

Policy Year	Selected Weights	Calendar Year 1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
1960	6.67%	3,086	3,328	3,583	3,850	4,118	4,386	4,654	4,919	5,183	5,444
1961	6.67%	3,086	3,328	3,583	3,850	4,118	4,386	4,654	4,919	5,183	5,444
1962	6.67%	3,086	3,328	3,583	3,850	4,118	4,386	4,654	4,919	5,183	5,444
1963	6.67%	3,086	3,328	3,583	3,850	4,118	4,386	4,654	4,919	5,183	5,444
1964	6.67%	3,086	3,328	3,583	3,850	4,118	4,386	4,654	4,919	5,183	5,444
1965	6.67%	3,086	3,328	3,583	3,850	4,118	4,386	4,654	4,919	5,183	5,444
1966	6.67%	3,086	3,328	3,583	3,850	4,118	4,386	4,654	4,919	5,183	5,444
1967	6.67%	3,086	3,328	3,583	3,850	4,118	4,386	4,654	4,919	5,183	5,444
1968	6.67%	3,086	3,328	3,583	3,850	4,118	4,386	4,654	4,919	5,183	5,444
1969	6.67%	3,086	3,328	3,583	3,850	4,118	4,386	4,654	4,919	5,183	5,444
1970	6.67%	3,086	3,328	3,583	3,850	4,118	4,386	4,654	4,919	5,183	5,444
1971	6.67%	3,086	3,328	3,583	3,850	4,118	4,386	4,654	4,919	5,183	5,444
1972	6.67%	3,086	3,328	3,583	3,850	4,118	4,386	4,654	4,919	5,183	5,444
1973	6.67%	3,086	3,328	3,583	3,850	4,118	4,386	4,654	4,919	5,183	5,444
1974	6.67%	3,086	3,328	3,583	3,850	4,118	4,386	4,654	4,919	5,183	5,444
1975–84	0.00%	0	0	0	0	0	0	0	0	0	0
Total	100.00%	46,283	49,921	53,741	57,752	61,773	65,793	69,803	73,792	77,748	81,658

EXHIBIT 7.1

PART 2

Policy Year	Selected Weights	Calendar Year 2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	Ultimate
1960	6.67%	5,700	5,952	6,197	6,435	6,663	6,882	7,088	7,280	7,457	7,616	10,413
1961	6.67%	5,700	5,952	6,197	6,435	6,663	6,882	7,088	7,280	7,457	7,616	10,413
1962	6.67%	5,700	5,952	6,197	6,435	6,663	6,882	7,088	7,280	7,457	7,616	10,413
1963	6.67%	5,700	5,952	6,197	6,435	6,663	6,882	7,088	7,280	7,457	7,616	10,413
1964	6.67%	5,700	5,952	6,197	6,435	6,663	6,882	7,088	7,280	7,457	7,616	10,413
1965	6.67%	5,700	5,952	6,197	6,435	6,663	6,882	7,088	7,280	7,457	7,616	10,413
1966	6.67%	5,700	5,952	6,197	6,435	6,663	6,882	7,088	7,280	7,457	7,616	10,413
1967	6.67%	5,700	5,952	6,197	6,435	6,663	6,882	7,088	7,280	7,457	7,616	10,413
1968	6.67%	5,700	5,952	6,197	6,435	6,663	6,882	7,088	7,280	7,457	7,616	10,413
1969	6.67%	5,700	5,952	6,197	6,435	6,663	6,882	7,088	7,280	7,457	7,616	10,413
1970	6.67%	5,700	5,952	6,197	6,435	6,663	6,882	7,088	7,280	7,457	7,616	10,413
1971	6.67%	5,700	5,952	6,197	6,435	6,663	6,882	7,088	7,280	7,457	7,616	10,413
1972	6.67%	5,700	5,952	6,197	6,435	6,663	6,882	7,088	7,280	7,457	7,616	10,413
1973	6.67%	5,700	5,952	6,197	6,435	6,663	6,882	7,088	7,280	7,457	7,616	10,413
1974	6.67%	5,700	5,952	6,197	6,435	6,663	6,882	7,088	7,280	7,457	7,616	10,413
1975–84	0.00%	0	0	0	0	0	0	0	0	0	0	0
Total	100.00%	85,506	89,277	92,954	96,518	99,948	103,223	106,317	109,205	111,858	114,246	156,197

Notes:
- — Cumulative projected calendar year ground-up indemnity and expense costs from Exhibit 5.1, Item 16.
- — Allocation method of calendar year losses to policy year is by equal weighting to each year.
- — Ultimate value is calculated by continuation of patterns beyond months shown.

EXHIBIT 7.2

PART 1

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
INSURER 3'S CUMULATIVE GROUND-UP LOSSES, INDEMNITY AND EXPENSES, ANNUAL INFLATION = 0.0%/COVERAGE BLOCK = 15 YEARS
($000's)

Policy	Selected	Calendar Year									
Year	Weights	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
1960	6.67%	3,073	3,293	3,513	3,733	3,943	4,143	4,333	4,513	4,683	4,843
1961	6.67%	3,073	3,293	3,513	3,733	3,943	4,143	4,333	4,513	4,683	4,843
1962	6.67%	3,073	3,293	3,513	3,733	3,943	4,143	4,333	4,513	4,683	4,843
1963	6.67%	3,073	3,293	3,513	3,733	3,943	4,143	4,333	4,513	4,683	4,843
1964	6.67%	3,073	3,293	3,513	3,733	3,943	4,143	4,333	4,513	4,683	4,843
1965	6.67%	3,073	3,293	3,513	3,733	3,943	4,143	4,333	4,513	4,683	4,843
1966	6.67%	3,073	3,293	3,513	3,733	3,943	4,143	4,333	4,513	4,683	4,843
1967	6.67%	3,073	3,293	3,513	3,733	3,943	4,143	4,333	4,513	4,683	4,843
1968	6.67%	3,073	3,293	3,513	3,733	3,943	4,143	4,333	4,513	4,683	4,843
1969	6.67%	3,073	3,293	3,513	3,733	3,943	4,143	4,333	4,513	4,683	4,843
1970	6.67%	3,073	3,293	3,513	3,733	3,943	4,143	4,333	4,513	4,683	4,843
1971	6.67%	3,073	3,293	3,513	3,733	3,943	4,143	4,333	4,513	4,683	4,843
1972	6.67%	3,073	3,293	3,513	3,733	3,943	4,143	4,333	4,513	4,683	4,843
1973	6.67%	3,073	3,293	3,513	3,733	3,943	4,143	4,333	4,513	4,683	4,843
1974	6.67%	3,073	3,293	3,513	3,733	3,943	4,143	4,333	4,513	4,683	4,843
1975–84	0.00%	0	0	0	0	0	0	0	0	0	0
Total	100.00%	46,095	49,395	52,695	55,995	59,145	62,145	64,995	67,695	70,245	72,645

EXHIBIT 7.2

PART 2

Policy Year	Selected Weights	Calendar Year 2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	Ultimate
1960	6.67%	4,993	5,133	5,263	5,383	5,493	5,593	5,683	5,763	5,833	5,893	6,576
1961	6.67%	4,993	5,133	5,263	5,383	5,493	5,593	5,683	5,763	5,833	5,893	6,576
1962	6.67%	4,993	5,133	5,263	5,383	5,493	5,593	5,683	5,763	5,833	5,893	6,576
1963	6.67%	4,993	5,133	5,263	5,383	5,493	5,593	5,683	5,763	5,833	5,893	6,576
1964	6.67%	4,993	5,133	5,263	5,383	5,493	5,593	5,683	5,763	5,833	5,893	6,576
1965	6.67%	4,993	5,133	5,263	5,383	5,493	5,593	5,683	5,763	5,833	5,893	6,576
1966	6.67%	4,993	5,133	5,263	5,383	5,493	5,593	5,683	5,763	5,833	5,893	6,576
1967	6.67%	4,993	5,133	5,263	5,383	5,493	5,593	5,683	5,763	5,833	5,893	6,576
1968	6.67%	4,993	5,133	5,263	5,383	5,493	5,593	5,683	5,763	5,833	5,893	6,576
1969	6.67%	4,993	5,133	5,263	5,383	5,493	5,593	5,683	5,763	5,833	5,893	6,576
1970	6.67%	4,993	5,133	5,263	5,383	5,493	5,593	5,683	5,763	5,833	5,893	6,576
1971	6.67%	4,993	5,133	5,263	5,383	5,493	5,593	5,683	5,763	5,833	5,893	6,576
1972	6.67%	4,993	5,133	5,263	5,383	5,493	5,593	5,683	5,763	5,833	5,893	6,576
1973	6.67%	4,993	5,133	5,263	5,383	5,493	5,593	5,683	5,763	5,833	5,893	6,576
1974	6.67%	4,993	5,133	5,263	5,383	5,493	5,593	5,683	5,763	5,833	5,893	6,576
1975–84	0.00%	0	0	0	0	0	0	0	0	0	0	0
Total	100.00%	74,895	76,995	78,945	80,745	82,395	83,895	85,245	86,445	87,495	88,395	98,633

Notes:

— Cumulative projected calendar year ground-up indemnity and expense costs from Exhibit 5.2, Item 16.

— Allocation method of calendar year losses to policy year is by equal weighting to each year.

— Ultimate value is calculated by continuation of patterns beyond months shown.

EXHIBIT 7.3

PART 1

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
INSURER 3'S CUMULATIVE GROUND-UP LOSSES, INDEMNITY AND EXPENSES, ANNUAL INFLATION = 5.0%/COVERAGE BLOCK = 25 YEARS
($000's)

Policy	Selected	Calendar Year									
Year	Weights	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
1960	4.00%	1,851	1,997	2,150	2,310	2,471	2,632	2,792	2,952	3,110	3,266
1961	4.00%	1,851	1,997	2,150	2,310	2,471	2,632	2,792	2,952	3,110	3,266
1962	4.00%	1,851	1,997	2,150	2,310	2,471	2,632	2,792	2,952	3,110	3,266
1963	4.00%	1,851	1,997	2,150	2,310	2,471	2,632	2,792	2,952	3,110	3,266
1964	4.00%	1,851	1,997	2,150	2,310	2,471	2,632	2,792	2,952	3,110	3,266
1965	4.00%	1,851	1,997	2,150	2,310	2,471	2,632	2,792	2,952	3,110	3,266
1966	4.00%	1,851	1,997	2,150	2,310	2,471	2,632	2,792	2,952	3,110	3,266
1967	4.00%	1,851	1,997	2,150	2,310	2,471	2,632	2,792	2,952	3,110	3,266
1968	4.00%	1,851	1,997	2,150	2,310	2,471	2,632	2,792	2,952	3,110	3,266
1969	4.00%	1,851	1,997	2,150	2,310	2,471	2,632	2,792	2,952	3,110	3,266
1970	4.00%	1,851	1,997	2,150	2,310	2,471	2,632	2,792	2,952	3,110	3,266
1971	4.00%	1,851	1,997	2,150	2,310	2,471	2,632	2,792	2,952	3,110	3,266
1972	4.00%	1,851	1,997	2,150	2,310	2,471	2,632	2,792	2,952	3,110	3,266
1973	4.00%	1,851	1,997	2,150	2,310	2,471	2,632	2,792	2,952	3,110	3,266
1974	4.00%	1,851	1,997	2,150	2,310	2,471	2,632	2,792	2,952	3,110	3,266
1975–84	40.00%	18,513	19,968	21,497	23,101	24,709	26,317	27,921	29,517	31,099	32,663
Total	100.00%	46,283	49,921	53,742	57,752	61,773	65,793	69,803	73,792	77,748	81,658

EXHIBIT 7.3

PART 2

Policy Year	Selected Weights	Calendar Year 2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	Ultimate
1960	4.00%	3,420	3,571	3,718	3,861	3,998	4,129	4,253	4,368	4,474	4,570	6,248
1961	4.00%	3,420	3,571	3,718	3,861	3,998	4,129	4,253	4,368	4,474	4,570	6,248
1962	4.00%	3,420	3,571	3,718	3,861	3,998	4,129	4,253	4,368	4,474	4,570	6,248
1963	4.00%	3,420	3,571	3,718	3,861	3,998	4,129	4,253	4,368	4,474	4,570	6,248
1964	4.00%	3,420	3,571	3,718	3,861	3,998	4,129	4,253	4,368	4,474	4,570	6,248
1965	4.00%	3,420	3,571	3,718	3,861	3,998	4,129	4,253	4,368	4,474	4,570	6,248
1966	4.00%	3,420	3,571	3,718	3,861	3,998	4,129	4,253	4,368	4,474	4,570	6,248
1967	4.00%	3,420	3,571	3,718	3,861	3,998	4,129	4,253	4,368	4,474	4,570	6,248
1968	4.00%	3,420	3,571	3,718	3,861	3,998	4,129	4,253	4,368	4,474	4,570	6,248
1969	4.00%	3,420	3,571	3,718	3,861	3,998	4,129	4,253	4,368	4,474	4,570	6,248
1970	4.00%	3,420	3,571	3,718	3,861	3,998	4,129	4,253	4,368	4,474	4,570	6,248
1971	4.00%	3,420	3,571	3,718	3,861	3,998	4,129	4,253	4,368	4,474	4,570	6,248
1972	4.00%	3,420	3,571	3,718	3,861	3,998	4,129	4,253	4,368	4,474	4,570	6,248
1973	4.00%	3,420	3,571	3,718	3,861	3,998	4,129	4,253	4,368	4,474	4,570	6,248
1974	4.00%	3,420	3,571	3,718	3,861	3,998	4,129	4,253	4,368	4,474	4,570	6,248
1975–84	40.00%	34,202	35,711	37,182	38,607	39,979	41,289	42,527	43,682	44,743	45,698	62,479
Total	100.00%	85,506	89,277	92,955	96,518	99,948	103,223	106,317	109,205	111,858	114,246	156,197

Notes:

— Cumulative projected calendar year ground-up indemnity and expense costs from Exhibit 5.1, Item 16.
— Allocation method of calendar year losses to policy year is by equal weighting to each year.
— Ultimate value is calculated by continuation of patterns beyond months shown.

EXHIBIT 7.4

PART 1

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
INSURER 3'S CUMULATIVE GROUND-UP LOSSES, INDEMNITY AND EXPENSES, ANNUAL INFLATION = 0.0%/COVERAGE BLOCK = 25 YEARS
($000's)

Policy Year	Selected Weights	Calendar Year 1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
1960	4.00%	1,844	1,976	2,108	2,240	2,366	2,486	2,600	2,708	2,810	2,906
1961	4.00%	1,844	1,976	2,108	2,240	2,366	2,486	2,600	2,708	2,810	2,906
1962	4.00%	1,844	1,976	2,108	2,240	2,366	2,486	2,600	2,708	2,810	2,906
1963	4.00%	1,844	1,976	2,108	2,240	2,366	2,486	2,600	2,708	2,810	2,906
1964	4.00%	1,844	1,976	2,108	2,240	2,366	2,486	2,600	2,708	2,810	2,906
1965	4.00%	1,844	1,976	2,108	2,240	2,366	2,486	2,600	2,708	2,810	2,906
1966	4.00%	1,844	1,976	2,108	2,240	2,366	2,486	2,600	2,708	2,810	2,906
1967	4.00%	1,844	1,976	2,108	2,240	2,366	2,486	2,600	2,708	2,810	2,906
1968	4.00%	1,844	1,976	2,108	2,240	2,366	2,486	2,600	2,708	2,810	2,906
1969	4.00%	1,844	1,976	2,108	2,240	2,366	2,486	2,600	2,708	2,810	2,906
1970	4.00%	1,844	1,976	2,108	2,240	2,366	2,486	2,600	2,708	2,810	2,906
1971	4.00%	1,844	1,976	2,108	2,240	2,366	2,486	2,600	2,708	2,810	2,906
1972	4.00%	1,844	1,976	2,108	2,240	2,366	2,486	2,600	2,708	2,810	2,906
1973	4.00%	1,844	1,976	2,108	2,240	2,366	2,486	2,600	2,708	2,810	2,906
1974	4.00%	1,844	1,976	2,108	2,240	2,366	2,486	2,600	2,708	2,810	2,906
1975–84	40.00%	18,438	19,758	21,078	22,398	23,658	24,858	25,998	27,078	28,098	29,058
Total	100.00%	46,095	49,395	52,695	55,995	59,145	62,145	64,995	67,695	70,245	72,645

EXHIBIT 7.4

PART 2

Policy Year	Selected Weights	Calendar Year 2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	Ultimate
1960	4.00%	2,996	3,080	3,158	3,230	3,296	3,356	3,410	3,458	3,500	3,536	3,945
1961	4.00%	2,996	3,080	3,158	3,230	3,296	3,356	3,410	3,458	3,500	3,536	3,945
1962	4.00%	2,996	3,080	3,158	3,230	3,296	3,356	3,410	3,458	3,500	3,536	3,945
1963	4.00%	2,996	3,080	3,158	3,230	3,296	3,356	3,410	3,458	3,500	3,536	3,945
1964	4.00%	2,996	3,080	3,158	3,230	3,296	3,356	3,410	3,458	3,500	3,536	3,945
1965	4.00%	2,996	3,080	3,158	3,230	3,296	3,356	3,410	3,458	3,500	3,536	3,945
1966	4.00%	2,996	3,080	3,158	3,230	3,296	3,356	3,410	3,458	3,500	3,536	3,945
1967	4.00%	2,996	3,080	3,158	3,230	3,296	3,356	3,410	3,458	3,500	3,536	3,945
1968	4.00%	2,996	3,080	3,158	3,230	3,296	3,356	3,410	3,458	3,500	3,536	3,945
1969	4.00%	2,996	3,080	3,158	3,230	3,296	3,356	3,410	3,458	3,500	3,536	3,945
1970	4.00%	2,996	3,080	3,158	3,230	3,296	3,356	3,410	3,458	3,500	3,536	3,945
1971	4.00%	2,996	3,080	3,158	3,230	3,296	3,356	3,410	3,458	3,500	3,536	3,945
1972	4.00%	2,996	3,080	3,158	3,230	3,296	3,356	3,410	3,458	3,500	3,536	3,945
1973	4.00%	2,996	3,080	3,158	3,230	3,296	3,356	3,410	3,458	3,500	3,536	3,945
1974	4.00%	2,996	3,080	3,158	3,230	3,296	3,356	3,410	3,458	3,500	3,536	3,945
1975–84	40.00%	29,958	30,798	31,578	32,298	32,958	33,558	34,098	34,578	34,998	35,358	39,453
Total	100.00%	74,895	76,995	78,945	80,745	82,395	83,895	85,245	86,445	87,495	88,395	98,633

Notes:

— Cumulative projected calendar year ground-up indemnity and expense costs from Exhibit 5.2, Item 16.
— Allocation method of calendar year losses to policy year is by equal weighting to each year.
— Ultimate value is calculated by continuation of patterns beyond months shown.

EXHIBIT 8.1

PART 1

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
INSURER 3'S LOSSES IN ABC RE'S REINSURANCE LAYER, INDEMNITY AND EXPENSES, ANNUAL INFLATION = 5.0%/COVERAGE BLOCK = 15 YEARS
($000's)

Policy Year	Width/Attch Pt/ % Share/Expenses ($ in millions)	Calendar Year 1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
1960	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1961	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1962	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1963	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1964	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1965	2.0/2.7/100.0%/Pro Rata	0	0	0	0	68	336	604	869	1,133	1,394
1966	2.0/2.7/100.0%/Pro Rata	0	0	0	0	68	336	604	869	1,133	1,394
1967	2.0/2.7/100.0%/Included in Limit	386	628	883	1,150	1,418	1,686	1,954	2,000	2,000	2,000
1968	4.0/3.5/100.0%/Pro Rata	0	0	0	0	0	0	0	0	0	194
1969	4.0/3.5/100.0%/Included in Limit	0	0	83	350	618	886	1,154	1,419	1,683	1,944
1970	4.0/3.5/25.0%/Pro Rata	0	0	0	0	0	0	0	0	0	48
1971	2.0/2.0/100.0%/Indem Only	57	219	389	567	745	924	1,102	1,280	1,455	1,629
1972	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1973	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1974	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1975–84	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
Total		443	847	1,354	2,067	2,918	4,169	5,417	6,438	7,405	8,603

EXHIBIT 8.1

PART 2

Policy Year	Width/Attch Pt/ % Share/Expenses ($ in millions)	Calendar Year 2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	Ultimate
1960	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1961	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1962	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1963	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1964	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1965	2.0/2.7/100.0%/Pro Rata	1,650	1,902	2,147	2,385	2,613	2,832	3,000	3,000	3,000	3,000	3,000
1966	2.0/2.7/100.0%/Pro Rata	1,650	1,902	2,147	2,385	2,613	2,832	3,000	3,000	3,000	3,000	3,000
1967	2.0/2.7/100.0%/Included in Limit	2,000	2,000	2,000	2,000	2,000	2,000	2,000	2,000	2,000	2,000	2,000
1968	4.0/3.5/100.0%/Pro Rata	450	702	947	1,185	1,413	1,632	1,838	2,030	2,207	2,366	5,163
1969	4.0/3.5/100.0%/Included in Limit	2,200	2,452	2,697	2,935	3,163	3,382	3,588	3,780	3,957	4,000	4,000
1970	4.0/3.5/25.0%/Pro Rata	113	175	237	296	353	408	459	508	552	592	1,291
1971	2.0/2.0/100.0%/Indem Only	1,800	1,968	2,000	2,000	2,000	2,000	2,000	2,000	2,000	2,000	2,000
1972	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1973	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1974	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1975–84	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
Total		9,864	11,101	12,175	13,184	14,156	15,084	15,885	16,318	16,716	16,958	20,454

Notes:

— Policy information from Exhibit 4. Only policies in Insured 3's coverage block for this scenario, 1960 through 1974, are included.

— Losses in layer are calculated by using the policy information to carve out losses and expenses from Exhibits 5.1, 6.1, and 7.1.

— Expenses are added to indemnity before applying attachment point and limits for expenses included in limits policies (Policy Years 1967 and 1969). When all lower layer policies are indemnity only or pro rata, this would not be true. In this case, indemnity only should be used to determine if the attachment point is reached. In the real world the true answer is somewhere between adding expenses to indemnity or just indemnity in determining satisfaction of the attachment point. Both scenarios should be examined.

— Ultimate value is calculated by continuation of patterns beyond months shown.

EXHIBIT 8.2

PART 1

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
INSURER 3'S LOSSES IN ABC RE'S REINSURANCE LAYER, INDEMNITY AND EXPENSES, ANNUAL INFLATION = 0.0%/COVERAGE BLOCK = 15 YEARS
($000's)

Policy Year	Width/Attch Pt/ % Share/Expenses ($ in millions)	Calendar Year 1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
1960	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1961	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1962	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1963	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1964	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1965	2.0/2.7/100.0%/Pro Rata	0	0	0	0	0	93	283	463	633	793
1966	2.0/2.7/100.0%/Pro Rata	0	0	0	0	0	93	283	463	633	793
1967	2.0/2.7/100.0%/Included in Limit	373	593	813	1,033	1,243	1,443	1,633	1,813	1,983	2,000
1968	4.0/3.5/100.0%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1969	4.0/3.5/100.0%/Included in Limit	0	0	13	233	443	643	833	1,013	1,183	1,343
1970	4.0/3.5/25.0%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1971	2.0/2.0/100.0%/Indem Only	49	195	342	489	629	762	889	1,009	1,122	1,229
1972	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1973	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1974	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1975–84	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
Total		422	788	1,168	1,755	2,315	3,034	3,921	4,761	5,554	6,158

EXHIBIT 8.2

PART 2

Policy Year	Width/Attch Pt/ % Share/Expenses ($ in millions)	Calendar Year 2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	Ultimate
1960	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1961	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1962	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1963	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1964	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1965	2.0/2.7/100.0%/Pro Rata	943	1,083	1,213	1,333	1,443	1,543	1,633	1,713	1,783	1,843	2,526
1966	2.0/2.7/100.0%/Pro Rata	943	1,083	1,213	1,333	1,443	1,543	1,633	1,713	1,783	1,843	2,526
1967	2.0/2.7/100.0%/Included in Limit	2,000	2,000	2,000	2,000	2,000	2,000	2,000	2,000	2,000	2,000	2,000
1968	4.0/3.5/100.0%/Pro Rata	0	0	13	133	243	343	433	513	583	643	1,326
1969	4.0/3.5/100.0%/Included in Limit	1,493	1,633	1,763	1,883	1,993	2,093	2,183	2,263	2,333	2,393	3,076
1970	4.0/3.5/25.0%/Pro Rata	0	0	3	33	61	86	108	128	146	161	331
1971	2.0/2.0/100.0%/Indem Only	1,329	1,422	1,509	1,589	1,662	1,729	1,789	1,842	1,889	1,929	2,000
1972	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1973	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1974	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1975–84	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
Total		6,708	7,221	7,714	8,304	8,845	9,337	9,779	10,172	10,517	10,812	13,783

Notes:

— Policy information from Exhibit 4. Only policies in Insured 3's coverage block for this scenario, 1960 through 1974, are included.

— Losses in layer are calculated by using the policy information to carve out losses and expenses from Exhibits 5.2, 6.2, and 7.2.

— Expenses are added to indemnity before applying attachment point and limits for expenses included in limits policies (Policy Years 1967 and 1969). When all lower layer policies are indemnity only or pro rata, this would not be true. In this case, indemnity only should be used to determine if the attachment point is reached. In the real world the true answer is somewhere between adding expenses to indemnity or just indemnity in determining satisfaction of the attachment point. Both scenarios should be examined.

— Ultimate value is calculated by continuation of patterns beyond months shown.

EXHIBIT 8.3

PART 1

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
INSURER 3'S LOSSES IN ABC RE'S REINSURANCE LAYER, INDEMNITY AND EXPENSES, ANNUAL INFLATION = 5.0%/COVERAGE BLOCK = 25 YEARS
($000's)

Policy Year	Width/Attch Pt/ % Share/Expenses ($ in millions)	Calendar Year 1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
1960	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1961	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1962	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1963	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1964	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1965	2.0/2.7/100.0%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1966	2.0/2.7/100.0%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1967	2.0/2.7/100.0%/Included in Limit	0	0	0	0	0	0	92	252	410	566
1968	4.0/3.5/100.0%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1969	4.0/3.5/100.0%/Included in Limit	0	0	0	0	0	0	0	0	0	0
1970	4.0/3.5/25.0%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1971	2.0/2.0/100.0%/Indem Only	0	0	0	0	0	0	0	0	73	178
1972	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1973	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1974	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1975–84	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
Total		0	0	0	0	0	0	92	252	483	744

EXHIBIT 8.3

PART 2

Policy Year	Width/Attch Pt/ % Share/Expenses ($ in millions)	Calendar Year 2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	Ultimate
1960	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1961	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1962	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1963	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1964	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1965	2.0/2.7/100.0%/Pro Rata	0	0	0	0	0	79	203	318	424	520	2,198
1966	2.0/2.7/100.0%/Pro Rata	0	0	0	0	0	79	203	318	424	520	2,198
1967	2.0/2.7/100.0%/Included in Limit	720	871	1,018	1,161	1,298	1,429	1,553	1,668	1,774	1,870	2,000
1968	4.0/3.5/100.0%/Pro Rata	0	0	0	0	0	0	0	0	0	0	998
1969	4.0/3.5/100.0%/Included in Limit	0	71	218	361	498	629	753	868	974	1,070	2,748
1970	4.0/3.5/25.0%/Pro Rata	0	0	0	0	0	0	0	0	0	0	249
1971	2.0/2.0/100.0%/Indem Only	280	381	479	574	665	753	835	912	983	1,047	2,000
1972	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1973	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1974	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1975–84	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
Total		1,000	1,323	1,715	2,095	2,461	2,968	3,546	4,085	4,580	5,026	12,391

Notes:

— Policy information from Exhibit 4. Only policies in Insured 3's coverage block for this scenario, 1960 through 1984, are included.

— Losses in layer are calculated by using the policy information to carve out losses and expenses from Exhibits 5.1, 6.3, and 7.3.

— Expenses are added to indemnity before applying attachment point and limits for expenses included in limits policies (Policy Years 1967 and 1969). When all lower layer policies are indemnity only or pro rata, this would not be true. In this case, indemnity only should be used to determine if the attachment point is reached. In the real world the true answer is somewhere between adding expenses to indemnity or just indemnity in determining satisfaction of the attachment point. Both scenarios should be examined.

— Ultimate value is calculated by continuation of patterns beyond months shown.

EXHIBIT 8.4

PART 1

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
INSURER 3'S LOSSES IN ABC RE'S REINSURANCE LAYER, INDEMNITY AND EXPENSES, ANNUAL INFLATION = 0.0%/COVERAGE BLOCK = 25 YEARS
($000's)

Policy Year	Width/Attch Pt/ % Share/Expenses ($ in millions)	Calendar Year 1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
1960	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1961	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1962	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1963	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1964	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1965	2.0/2.7/100.0%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1966	2.0/2.7/100.0%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1967	2.0/2.7/100.0%/Included in Limit	0	0	0	0	0	0	0	8	110	206
1968	4.0/3.5/100.0%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1969	4.0/3.5/100.0%/Included in Limit	0	0	0	0	0	0	0	0	0	0
1970	4.0/3.5/25.0%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1971	2.0/2.0/100.0%/Indem Only	0	0	0	0	0	0	0	0	0	0
1972	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1973	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1974	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1975–84	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
Total		0	0	0	0	0	0	0	8	110	206

EXHIBIT 8.4

PART 2

Policy Year	Width/Attch Pt/ % Share/Expenses ($ in millions)	Calendar Year 2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	Ultimate
1960	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1961	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1962	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1963	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1964	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1965	2.0/2.7/100.0%/Pro Rata	0	0	0	0	0	0	0	0	0	0	0
1966	2.0/2.7/100.0%/Pro Rata	0	0	0	0	0	0	0	0	0	0	0
1967	2.0/2.7/100.0%/Included in Limit	296	380	458	530	596	656	710	758	800	836	1,245
1968	4.0/3.5/100.0%/Pro Rata	0	0	0	0	0	0	0	0	0	0	0
1969	4.0/3.5/100.0%/Included in Limit	0	0	0	0	0	0	0	0	0	36	445
1970	4.0/3.5/25.0%/Pro Rata	0	0	0	0	0	0	0	0	0	0	0
1971	2.0/2.0/100.0%/Indem Only	0	53	105	153	197	237	273	305	333	357	630
1972	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1973	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1974	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1975–84	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
Total		296	433	563	683	793	893	983	1,063	1,133	1,229	2,321

Notes:

— Policy information from Exhibit 4. Only policies in Insured 3's coverage block for this scenario, 1960 through 1984, are included.

— Losses in layer are calculated by using the policy information to carve out losses and expenses from Exhibits 5.2, 6.4, and 7.4.

— Expenses are added to indemnity before applying attachment point and limits for expenses included in limits policies (Policy Years 1967 and 1969). When all lower layer policies are indemnity only or pro rata, this would not be true. In this case, indemnity only should be used to determine if the attachment point is reached. In the real world the true answer is somewhere between adding expenses to indemnity or just indemnity in determining satisfaction of the attachment point. Both scenarios should be examined.

— Ultimate value is calculated by continuation of patterns beyond months shown.

EXHIBIT 9.1

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
COMPARISON OF GROUND-UP INDEMNITY & EXPENSE VS. INDEMNITY & EXPENSE IN LAYER
ANNUAL INFLATION = 5.0%/COVERAGE BLOCK = 15 YEARS
($000's)

	Insured 3's 1968 Policy Year Cumulative Indemnity and Expense				All Policy Years for Insured 3 in its Coverage Block Cumulative Indemnity and Expense			
Calendar Year (1)	On a Ground-Up $ Basis (2)	Implied Ground-Up Reporting Pattern (3)	In ABC Re's Reinsurance Layer (4)	ABC Re's Implied Reporting Pattern (5)	On a Ground-Up $ Basis (6)	Implied Ground-Up Reporting Pattern (7)	In ABC Re's Reinsurance Layer (8)	ABC Re's Implied Reporting Pattern (9)
1994	3,086	29.63%	0	0.00%	46,283	29.63%	443	2.16%
1995	3,328	31.96%	0	0.00%	49,921	31.96%	847	4.14%
1996	3,583	34.41%	0	0.00%	53,741	34.41%	1,354	6.62%
1997	3,850	36.97%	0	0.00%	57,752	36.97%	2,067	10.11%
1998	4,118	39.55%	0	0.00%	61,773	39.55%	2,918	14.27%
1999	4,386	42.12%	0	0.00%	65,793	42.12%	4,169	20.38%
2000	4,654	44.69%	0	0.00%	69,803	44.69%	5,417	26.48%
2001	4,919	47.24%	0	0.00%	73,792	47.24%	6,438	31.48%
2002	5,183	49.78%	0	0.00%	77,748	49.78%	7,405	36.20%
2003	5,444	52.28%	194	3.75%	81,658	52.28%	8,603	42.06%
2004	5,700	54.74%	450	8.72%	85,506	54.74%	9,864	48.23%
2005	5,952	57.16%	702	13.59%	89,277	57.16%	11,101	54.27%
2006	6,197	59.51%	947	18.34%	92,954	59.51%	12,175	59.52%
2007	6,435	61.79%	1,185	22.94%	96,518	61.79%	13,184	64.46%
2008	6,663	63.99%	1,413	27.37%	99,948	63.99%	14,156	69.21%
2009	6,882	66.09%	1,632	31.60%	103,223	66.09%	15,084	73.75%
2010	7,088	68.07%	1,838	35.59%	106,317	68.07%	15,885	77.66%
2011	7,280	69.91%	2,030	39.32%	109,205	69.91%	16,318	79.78%
2012	7,457	71.61%	2,207	42.75%	111,858	71.61%	16,716	81.73%
2013	7,616	73.14%	2,366	45.83%	114,246	73.14%	16,958	82.91%
Ultimate	10,413	100.00%	5,163	100.00%	156,197	100.00%	20,454	100.00%

Notes:
(2), (6) From Exhibit 7.1.
(3) = (2)/(2) at Ultimate.
(4), (8) From Exhibit 8.1.
(5) = (4)/(4) at Ultimate.
(7) = (6)/(6) at Ultimate.
(9) = (8)/(8) at Ultimate.

EXHIBIT 9.2

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
COMPARISON OF GROUND-UP INDEMNITY & EXPENSE VS. INDEMNITY & EXPENSE IN LAYER
ANNUAL INFLATION = 0.0%/COVERAGE BLOCK = 15 YEARS
($000's)

	Insured 3's 1968 Policy Year Cumulative Indemnity and Expense				All Policy Years for Insured 3 in its Coverage Block Cumulative Indemnity and Expense			
Calendar Year (1)	On a Ground-Up $ Basis (2)	Implied Ground-Up Reporting Pattern (3)	In ABC Re's Reinsurance Layer (4)	ABC Re's Implied Reporting Pattern (5)	On a Ground-Up $ Basis (6)	Implied Ground-Up Reporting Pattern (7)	In ABC Re's Reinsurance Layer (8)	ABC Re's Implied Reporting Pattern (9)
1994	3,073	46.73%	0	0.00%	46,095	46.73%	422	3.06%
1995	3,293	50.08%	0	0.00%	49,395	50.08%	788	5.72%
1996	3,513	53.43%	0	0.00%	52,695	53.43%	1,168	8.47%
1997	3,733	56.77%	0	0.00%	55,995	56.77%	1,755	12.73%
1998	3,943	59.97%	0	0.00%	59,145	59.97%	2,315	16.79%
1999	4,143	63.01%	0	0.00%	62,145	63.01%	3,034	22.01%
2000	4,333	65.90%	0	0.00%	64,995	65.90%	3,921	28.45%
2001	4,513	68.63%	0	0.00%	67,695	68.63%	4,761	34.54%
2002	4,683	71.22%	0	0.00%	70,245	71.22%	5,554	40.30%
2003	4,843	73.65%	0	0.00%	72,645	73.65%	6,158	44.67%
2004	4,993	75.93%	0	0.00%	74,895	75.93%	6,708	48.67%
2005	5,133	78.06%	0	0.00%	76,995	78.06%	7,221	52.39%
2006	5,263	80.04%	13	0.98%	78,945	80.04%	7,714	55.97%
2007	5,383	81.86%	133	10.04%	80,745	81.86%	8,304	60.25%
2008	5,493	83.54%	243	18.33%	82,395	83.54%	8,845	64.17%
2009	5,593	85.06%	343	25.88%	83,895	85.06%	9,337	67.74%
2010	5,683	86.43%	433	32.67%	85,245	86.43%	9,779	70.95%
2011	5,763	87.64%	513	38.70%	86,445	87.64%	10,172	73.80%
2012	5,833	88.71%	583	43.98%	87,495	88.71%	10,517	76.30%
2013	5,893	89.62%	643	48.51%	88,395	89.62%	10,812	78.44%
Ultimate	6,576	100.00%	1,326	100.00%	98,633	100.00%	13,783	100.00%

Notes: (2), (6) From Exhibit 7.2. (3) = (2)/(2) at Ultimate. (4), (8) From Exhibit 8.2. (5) = (4)/(4) at Ultimate. (7) = (6)/(6) at Ultimate. (9) = (8)/(8) at Ultimate.

EXHIBIT 9.3

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
COMPARISON OF GROUND-UP INDEMNITY & EXPENSE VS. INDEMNITY & EXPENSE IN LAYER
ANNUAL INFLATION = 5.0%/COVERAGE BLOCK = 25 YEARS
($000's)

	Insured 3's 1968 Policy Year Cumulative Indemnity and Expense				All Policy Years for Insured 3 in its Coverage Block Cumulative Indemnity and Expense			
Calendar Year (1)	On a Ground-Up $ Basis (2)	Implied Ground-Up Reporting Pattern (3)	In ABC Re's Reinsurance Layer (4)	ABC Re's Implied Reporting Pattern (5)	On a Ground-Up $ Basis (6)	Implied Ground-Up Reporting Pattern (7)	In ABC Re's Reinsurance Layer (8)	ABC Re's Implied Reporting Pattern (9)
1994	1,851	29.63%	0	0.00%	46,283	29.63%	0	0.00%
1995	1,997	31.96%	0	0.00%	49,921	31.96%	0	0.00%
1996	2,150	34.41%	0	0.00%	53,742	34.41%	0	0.00%
1997	2,310	36.97%	0	0.00%	57,752	36.97%	0	0.00%
1998	2,471	39.55%	0	0.00%	61,773	39.55%	0	0.00%
1999	2,632	42.12%	0	0.00%	65,793	42.12%	0	0.00%
2000	2,792	44.69%	0	0.00%	69,803	44.69%	92	0.74%
2001	2,952	47.24%	0	0.00%	73,792	47.24%	252	2.03%
2002	3,110	49.78%	0	0.00%	77,748	49.78%	483	3.90%
2003	3,266	52.28%	0	0.00%	81,658	52.28%	744	6.00%
2004	3,420	54.74%	0	0.00%	85,506	54.74%	1,000	8.07%
2005	3,571	57.16%	0	0.00%	89,277	57.16%	1,323	10.68%
2006	3,718	59.51%	0	0.00%	92,955	59.51%	1,715	13.84%
2007	3,861	61.79%	0	0.00%	96,518	61.79%	2,095	16.91%
2008	3,998	63.99%	0	0.00%	99,948	63.99%	2,461	19.86%
2009	4,129	66.09%	0	0.00%	103,223	66.08%	2,968	23.95%
2010	4,253	68.07%	0	0.00%	106,317	68.07%	3,546	28.62%
2011	4,368	69.91%	0	0.00%	109,205	69.91%	4,085	32.97%
2012	4,474	71.61%	0	0.00%	111,858	71.61%	4,580	36.96%
2013	4,570	73.14%	0	0.00%	114,246	73.14%	5,026	40.56%
Ultimate	6,248	100.00%	998	100.00%	156,197	100.00%	12,391	100.00%

Notes:
(2), (6) From Exhibit 7.3.
(3) = (2)/(2) at Ultimate.
(4), (8) From Exhibit 8.3.
(5) = (4)/(4) at Ultimate.
(7) = (6)/(6) at Ultimate.
(9) = (8)/(8) at Ultimate.

EXHIBIT 9.4

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
COMPARISON OF GROUND-UP INDEMNITY & EXPENSE VS. INDEMNITY & EXPENSE IN LAYER
ANNUAL INFLATION = 0.0%/COVERAGE BLOCK = 25 YEARS
($000's)

	Insured 3's 1968 Policy Year Cumulative Indemnity and Expense				All Policy Years for Insured 3 in its Coverage Block Cumulative Indemnity and Expense			
Calendar Year (1)	On a Ground-Up $ Basis (2)	Implied Ground-Up Reporting Pattern (3)	In ABC Re's Reinsurance Layer (4)	ABC Re's Implied Reporting Pattern (5)	On a Ground-Up $ Basis (6)	Implied Ground-Up Reporting Pattern (7)	In ABC Re's Reinsurance Layer (8)	ABC Re's Implied Reporting Pattern (9)
1994	1,844	46.73%	0	NA	46,095	46.73%	0	0.00%
1995	1,976	50.08%	0	NA	49,395	50.08%	0	0.00%
1996	2,108	53.43%	0	NA	52,695	53.43%	0	0.00%
1997	2,240	56.77%	0	NA	55,995	56.77%	0	0.00%
1998	2,366	59.97%	0	NA	59,145	59.97%	0	0.00%
1999	2,486	63.01%	0	NA	62,145	63.01%	0	0.00%
2000	2,600	65.90%	0	NA	64,995	65.90%	0	0.00%
2001	2,708	68.63%	0	NA	67,695	68.63%	8	0.34%
2002	2,810	71.22%	0	NA	70,245	71.22%	110	4.73%
2003	2,906	73.65%	0	NA	72,645	73.65%	206	8.87%
2004	2,996	75.93%	0	NA	74,895	75.93%	296	12.75%
2005	3,080	78.06%	0	NA	76,995	78.06%	433	18.66%
2006	3,158	80.04%	0	NA	78,945	80.04%	563	24.26%
2007	3,230	81.86%	0	NA	80,745	81.86%	683	29.43%
2008	3,296	83.54%	0	NA	82,395	83.54%	793	34.17%
2009	3,356	85.06%	0	NA	83,895	85.06%	893	38.48%
2010	3,410	86.43%	0	NA	85,245	86.43%	983	42.36%
2011	3,458	87.64%	0	NA	86,445	87.64%	1,063	45.80%
2012	3,500	88.71%	0	NA	87,495	88.71%	1,133	48.82%
2013	3,536	89.62%	0	NA	88,395	89.62%	1,229	52.95%
Ultimate	3,945	100.00%	0	NA	98,633	100.00%	2,321	100.00%

Notes: (2), (6) From Exhibit 7.4. (4), (8) From Exhibit 8.4. (7) = (6)/(6) at Ultimate.
(3) = (2)/(2) at Ultimate. (5) = (4)/(4) at Ultimate. (9) = (8)/(8) at Ultimate.

EXHIBIT 10.1

PART 1

ASBESTOS BI MODEL FOR ABC RE'S SAMPLE GROUP
INDEMNITY AND EXPENSES WITH ABC RE'S LAYER OF COVERAGE FOR ALL SAMPLE INSUREDS, ANNUAL INFLATION = 5.0%/COVERAGE BLOCK = 15 YEARS
($000's)

Sample Insureds	Tier	Average Ground-Up Attachment Pt	Total Exposure	ABC Re's Reported Loss & Exp	Projected Losses and Expenses from All Policies with Insured in Calendar Year 1994	1995	1996	1997	1998	1999
Insured 1	4	37,500	3,363	0	0	0	0	0	0	0
Insured 2	4	1,994	13,960	20	143	158	173	188	203	218
Insured 3	2	2,943	17,000	2,300	443	847	1,354	2,067	2,918	4,169
Insured 4	1	48,750	38,480	21,500	44,301	46,334	46,334	46,334	46,334	46,334
Insured 5	1	50,357	30,280	19,300	30,212	30,344	30,344	30,344	30,344	30,344
Insured 6	1	48,333	40,680	22,450	44,059	45,224	46,371	47,233	47,233	47,233
Insured 7	2	37,813	13,581	1,500	1,500	1,500	1,500	1,556	1,668	1,777
Insured 8	2	40,000	14,290	300	300	300	300	300	300	529
Insured 9	2	40,313	10,233	300	300	300	300	300	457	673
Insured 10	3	17,143	6,000	150	186	190	193	197	279	391
Insured 11	2	37,813	31,940	200	281	300	300	300	300	300
Insured 12	3	26,429	16,300	0	0	0	0	0	0	0
Insured 13	3	25,938	24,800	15	0	0	0	0	0	0
Insured 14	3	21,111	9,500	15	0	0	0	0	0	42
Insured 15	3	25,313	6,400	200	236	253	270	312	415	533
	Subtotal Tier 1		109,440	63,250						
	Subtotal Tier 2		87,045	4,600						
	Subtotal Tier 3		63,000	380						
	Subtotal Tier 4		17,323	20						
	Total		276,807	68,250	121,961	125,750	127,439	129,132	130,452	132,544
	% of Ultimate				70.48%	72.67%	73.65%	74.62%	75.39%	76.60%

EXHIBIT 10.1

PART 2

Sample Insured	Tier	Projected Losses and Expenses from All Policies with Insured in Calendar Year 2000	2001	2002	2003	2004	2005	2006	2007	2008
Insured 1	4	0	0	0	0	0	0	0	0	0
Insured 2	4	233	248	263	278	292	306	320	334	346
Insured 3	2	5,417	6,438	7,405	8,603	9,864	11,101	12,175	13,184	14,156
Insured 4	1	46,334	46,334	46,334	46,334	46,334	46,334	46,334	46,334	46,334
Insured 5	1	30,344	30,344	30,344	30,344	30,344	30,344	30,344	30,344	30,344
Insured 6	1	47,233	47,233	47,233	47,233	47,233	47,233	47,233	47,233	47,233
Insured 7	2	2,394	3,473	4,462	5,008	5,258	5,503	5,741	5,972	6,195
Insured 8	2	869	1,198	1,317	1,423	1,527	1,629	1,729	1,825	1,918
Insured 9	2	858	937	1,016	1,093	1,169	1,243	1,316	1,387	1,454
Insured 10	3	488	531	574	616	658	698	738	777	831
Insured 11	2	300	300	300	300	300	300	300	300	300
Insured 12	3	0	0	0	0	0	0	0	0	0
Insured 13	3	7	47	87	127	166	200	200	200	200
Insured 14	3	86	129	172	200	200	200	200	200	200
Insured 15	3	644	714	750	786	821	856	889	922	962
	Subtotal Tier 1									
	Subtotal Tier 2									
	Subtotal Tier 3									
	Subtotal Tier 4									
	Total	135,207	137,927	140,257	142,344	144,166	145,947	147,519	149,011	150,474
	% of Ultimate	78.13%	79.71%	81.05%	82.26%	83.31%	84.34%	85.25%	86.11%	86.96%

EXHIBIT 10.1

PART 3

Sample Insured	Tier	Projected Losses and Expenses from All Policies with Insured in Calendar Year 2009	2010	2011	2012	2013	Ultimate	Ultimate as % of Exposure	Case Inc'd Loss Devel. Factor
Insured 1	4	0	0	0	0	0	0	0.0%	0.000
Insured 2	4	359	371	383	395	403	411	2.9%	20.529
Insured 3	2	15,084	15,885	16,318	16,716	16,958	20,454	120.3%	8.893
Insured 4	1	46,334	46,334	46,334	46,334	46,334	46,334	120.4%	2.155
Insured 5	1	30,344	30,344	30,344	30,344	30,344	30,344	100.2%	1.572
Insured 6	1	47,233	47,233	47,233	47,233	47,233	47,233	116.1%	2.104
Insured 7	2	6,407	6,619	6,830	7,039	7,246	7,449	54.8%	4.966
Insured 8	2	2,007	2,095	2,183	2,270	2,357	5,475	38.3%	18.250
Insured 9	2	1,519	1,584	1,648	1,691	1,709	3,314	32.4%	11.045
Insured 10	3	892	953	1,013	1,063	1,099	1,928	32.1%	12.853
Insured 11	2	300	313	1,027	1,735	2,435	4,290	13.4%	21.450
Insured 12	3	0	0	0	0	0	586	3.6%	0.000
Insured 13	3	200	200	200	200	200	2,057	8.3%	137.164
Insured 14	3	200	200	200	200	200	1,595	16.8%	106.351
Insured 15	3	1,005	1,047	1,090	1,126	1,152	1,575	24.6%	7.873
	Subtotal Tier 1						123,911	113.2%	1.959
	Subtotal Tier 2						40,981	47.1%	8.909
	Subtotal Tier 3						7,741	12.3%	20.372
	Subtotal Tier 4						411	2.4%	20.127
	Total	151,883	153,179	154,804	156,348	157,670	173,044	62.5%	2.535
	% of Ultimate	87.77%	88.52%	89.46%	90.35%	91.12%	100.00%		

Notes:

— This exhibit is a compilation of Exhibit 8.1 for each insured in the sample group.

— Average ground-up attachment point and total exposure from insured policy information are given.

— ABC Re's reported loss & expense from ABC Re's claim files are given. The amount could be lower than implied by model because of reporting lags to ABC Re or higher because of additional reserves.

EXHIBIT 10.2

PART 1

ASBESTOS BI MODEL FOR ABC RE'S SAMPLE GROUP
INDEMNITY AND EXPENSES WITH ABC RE'S LAYER OF COVERAGE FOR ALL SAMPLE INSUREDS, ANNUAL INFLATION = 0.0%/COVERAGE BLOCK = 15 YEARS
($000's)

Sample Insureds	Tier	Average Ground-Up Attachment Pt	Total Exposure	ABC Re's Reported Loss & Exp	Projected Losses and Expenses from All Policies with Insured in Calendar Year 1994	1995	1996	1997	1998	1999
Insured 1	4	37,500	3,363	0	0	0	0	0	0	0
Insured 2	4	1,994	13,960	20	141	154	166	178	190	200
Insured 3	2	2,943	17,000	2,300	422	788	1,168	1,755	2,315	3,034
Insured 4	1	48,750	38,480	21,500	43,967	45,878	46,318	46,318	46,318	46,318
Insured 5	1	50,357	30,280	19,300	30,115	30,344	30,344	30,344	30,344	30,344
Insured 6	1	48,333	40,680	22,450	43,890	44,901	45,845	46,728	47,200	47,200
Insured 7	2	37,813	13,581	1,500	1,500	1,500	1,500	1,500	1,564	1,642
Insured 8	2	40,000	14,290	300	300	300	300	300	300	300
Insured 9	2	40,313	10,233	300	300	300	300	300	300	401
Insured 10	3	17,143	6,000	150	185	189	192	195	197	250
Insured 11	2	37,813	31,940	200	269	300	300	300	300	300
Insured 12	3	26,429	16,300	0	0	0	0	0	0	0
Insured 13	3	25,938	24,800	15	0	0	0	0	0	0
Insured 14	3	21,111	9,500	15	0	0	0	0	0	0
Insured 15	3	25,313	6,400	200	234	248	262	276	318	388
	Subtotal Tier 1		109,440	63,250						
	Subtotal Tier 2		87,045	4,600						
	Subtotal Tier 3		63,000	380						
	Subtotal Tier 4		17,323	20						
	Total		276,807	68,250	121,323	124,903	126,695	128,193	129,346	130,378
	% of Ultimate				81.33%	83.73%	84.93%	85.94%	86.71%	87.40%

EXHIBIT 10.2

PART 2

Sample Insured	Tier	Projected Losses and Expenses from All Policies with Insured in Calendar Year 2000	2001	2002	2003	2004	2005	2006	2007	2008
Insured 1	4	0	0	0	0	0	0	0	0	0
Insured 2	4	210	220	229	238	246	253	260	267	273
Insured 3	2	3,921	4,761	5,554	6,158	6,708	7,221	7,714	8,304	8,845
Insured 4	1	46,318	46,318	46,318	46,318	46,318	46,318	46,318	46,318	46,318
Insured 5	1	30,344	30,344	30,344	30,344	30,344	30,344	30,344	30,344	30,344
Insured 6	1	47,200	47,200	47,200	47,200	47,200	47,200	47,200	47,200	47,200
Insured 7	2	1,714	1,781	1,943	2,574	3,161	3,661	4,126	4,555	4,873
Insured 8	2	320	532	733	922	1,099	1,231	1,281	1,328	1,370
Insured 9	2	543	674	799	871	914	953	990	1,024	1,055
Insured 10	3	324	392	457	495	518	540	560	578	595
Insured 11	2	300	300	300	300	300	300	300	300	300
Insured 12	3	0	0	0	0	0	0	0	0	0
Insured 13	3	0	0	0	18	40	60	79	96	112
Insured 14	3	19	47	73	98	122	143	164	182	200
Insured 15	3	467	541	611	665	705	723	740	756	770
	Subtotal Tier 1									
	Subtotal Tier 2									
	Subtotal Tier 3									
	Subtotal Tier 4									
	Total	131,680	133,111	134,560	136,202	137,674	138,949	140,077	141,253	142,255
	% of Ultimate	88.27%	89.23%	90.20%	91.30%	92.29%	93.15%	93.90%	94.69%	95.36%

EXHIBIT 10.2

PART 3

Sample Insured	Tier	Projected Losses and Expenses from All Policies with Insured in Calendar Year 2009	2010	2011	2012	2013	Ultimate	Ultimate as % of Exposure	Case Inc'd Loss Devel. Factor
Insured 1	4	0	0	0	0	0	0	0.0%	0.000
Insured 2	4	278	283	288	292	297	301	2.2%	15.034
Insured 3	2	9,337	9,779	10,172	10,517	10,812	13,783	81.1%	5.993
Insured 4	1	46,318	46,318	46,318	46,318	46,318	46,318	120.4%	2.154
Insured 5	1	30,344	30,344	30,344	30,344	30,344	30,344	100.2%	1.572
Insured 6	1	47,200	47,200	47,200	47,200	47,200	47,200	116.0%	2.102
Insured 7	2	4,966	5,054	5,137	5,216	5,290	5,359	39.5%	3.573
Insured 8	2	1,409	1,446	1,481	1,514	1,544	1,958	13.7%	6.528
Insured 9	2	1,083	1,110	1,135	1,159	1,182	1,484	14.5%	4.946
Insured 10	3	611	626	640	653	665	817	13.6%	5.447
Insured 11	2	300	300	300	300	300	300	0.9%	1.500
Insured 12	3	0	0	0	0	0	0	0.0%	0.000
Insured 13	3	127	141	154	166	177	200	0.8%	13.333
Insured 14	3	200	200	200	200	200	200	2.1%	13.333
Insured 15	3	783	796	808	819	829	909	14.2%	4.546
	Subtotal Tier 1						123,862	113.2%	1.958
	Subtotal Tier 2						22,885	26.3%	4.975
	Subtotal Tier 3						2,126	3.4%	5.595
	Subtotal Tier 4						301	1.7%	14.739
	Total	142,956	143,596	144,176	144,697	145,158	149,174	53.9%	2.186
	% of Ultimate	95.83%	96.26%	96.65%	97.00%	97.31%	100.00%		

Notes:

— This exhibit is a compilation of Exhibit 8.2 for each insured in the sample group.

— Average ground-up attachment point and total exposure from insured policy information are given.

— ABC Re's reported loss & expense from ABC Re's claim files are given. The amount could be lower than implied by model because of reporting lags to ABC Re or higher because of additional reserves.

EXHIBIT 10.3

PART 1

ASBESTOS BI MODEL FOR ABC RE'S SAMPLE GROUP
INDEMNITY AND EXPENSES WITH ABC RE'S LAYER OF COVERAGE FOR ALL SAMPLE INSUREDS, ANNUAL INFLATION = 5.0%/COVERAGE BLOCK = 25 YEARS
($000's)

Sample Insureds	Tier	Average Ground-Up Attachment Pt	Total Exposure	ABC Re's Reported Loss & Exp	Projected Losses and Expenses from All Policies with Insured in Calendar Year 1994	1995	1996	1997	1998	1999
Insured 1	4	37,500	3,363	0	0	0	0	0	0	0
Insured 2	4	1,994	13,960	20	40	46	53	60	67	74
Insured 3	2	2,943	17,000	2,300	0	0	0	0	0	0
Insured 4	1	48,750	38,480	21,500	21,011	22,026	23,025	24,586	26,127	27,780
Insured 5	1	50,357	30,280	19,300	19,628	20,344	20,344	20,778	21,365	22,253
Insured 6	1	48,333	40,680	22,450	22,484	24,860	26,048	27,015	28,367	29,988
Insured 7	2	37,813	13,581	1,500	0	0	333	675	1,011	1,339
Insured 8	2	40,000	14,290	300	0	62	135	207	277	300
Insured 9	2	40,313	10,233	300	52	129	205	279	300	300
Insured 10	3	17,143	6,000	150	36	76	116	155	167	168
Insured 11	2	37,813	31,940	200	0	0	0	0	0	0
Insured 12	3	26,429	16,300	0	0	0	0	0	0	0
Insured 13	3	25,938	24,800	15	0	0	0	0	0	0
Insured 14	3	21,111	9,500	15	0	0	0	0	0	0
Insured 15	3	25,313	6,400	200	58	84	111	137	150	158
Subtotal Tier 1			109,440	63,250						
Subtotal Tier 2			87,045	4,600						
Subtotal Tier 3			63,000	380						
Subtotal Tier 4			17,323	20						
Total			276,807	68,250	63,309	67,627	70,370	73,892	77,830	82,360
% of Ultimate					45.36%	48.45%	50.41%	52.94%	55.76%	59.00%

EXHIBIT 10.3

PART 2

Sample		Projected Losses and Expenses from All Policies with Insured in Calendar Year								
Insured	Tier	2000	2001	2002	2003	2004	2005	2006	2007	2008
Insured 1	4	0	0	0	0	0	0	0	0	0
Insured 2	4	83	92	101	110	119	128	136	144	152
Insured 3	2	92	252	483	744	1,000	1,323	1,715	2,095	2,461
Insured 4	1	29,616	31,398	33,166	34,913	36,633	38,318	39,961	41,554	42,774
Insured 5	1	23,185	24,091	24,990	25,878	26,752	27,608	28,443	29,252	29,769
Insured 6	1	31,567	33,101	34,623	36,127	37,607	39,058	40,472	41,843	42,948
Insured 7	2	1,500	1,500	1,500	1,500	1,500	1,500	1,500	1,500	1,500
Insured 8	2	300	300	300	300	300	300	300	300	300
Insured 9	2	300	300	300	300	300	300	300	300	300
Insured 10	3	171	173	175	178	180	182	184	186	188
Insured 11	2	0	0	0	11	56	100	143	184	224
Insured 12	3	0	0	0	0	0	0	0	0	0
Insured 13	3	0	0	0	0	0	0	0	0	0
Insured 14	3	0	0	0	0	0	0	0	0	0
Insured 15	3	168	178	189	199	209	219	228	237	246
	Subtotal Tier 1									
	Subtotal Tier 2									
	Subtotal Tier 3									
	Subtotal Tier 4									
	Total	86,982	91,386	95,827	100,259	104,655	109,035	113,383	117,596	120,862
	% of Ultimate	62.32%	65.47%	68.65%	71.83%	74.98%	78.12%	81.23%	84.25%	86.59%

EXHIBIT 10.3

PART 3

Sample Insured	Tier	Projected Losses and Expenses from All Policies with Insured in Calendar Year 2009	2010	2011	2012	2013	Ultimate	Ultimate as % of Exposure	Case Inc'd Loss Devel. Factor
Insured 1	4	0	0	0	0	0	0	0.0%	0.000
Insured 2	4	159	167	174	181	188	195	1.4%	9.770
Insured 3	2	2,968	3,546	4,085	4,580	5,026	12,391	72.9%	5.387
Insured 4	1	43,683	43,975	44,182	44,182	44,182	44,182	114.8%	2.055
Insured 5	1	30,066	30,344	30,344	30,344	30,344	30,344	100.2%	1.572
Insured 6	1	43,754	44,312	44,812	45,307	45,548	45,548	112.0%	2.029
Insured 7	2	1,500	1,500	1,500	1,502	1,552	1,601	11.8%	1.067
Insured 8	2	300	300	300	300	300	1,848	12.9%	6.161
Insured 9	2	300	300	300	300	300	1,403	13.7%	4.678
Insured 10	3	190	192	193	195	197	751	12.5%	5.004
Insured 11	2	263	300	300	300	300	300	0.9%	1.500
Insured 12	3	0	0	0	0	0	0	0.0%	0.000
Insured 13	3	0	0	0	0	0	200	0.8%	13.333
Insured 14	3	0	0	0	0	0	200	2.1%	13.333
Insured 15	3	254	262	271	282	313	618	9.7%	3.092
	Subtotal Tier 1						120,074	109.7%	1.898
	Subtotal Tier 2						17,543	20.2%	3.814
	Subtotal Tier 3						1,769	2.8%	4.655
	Subtotal Tier 4						195	1.1%	9.578
	Total	123,438	125,197	126,460	127,474	128,250	139,581	50.4%	2.045
	% of Ultimate	88.43%	89.69%	90.60%	91.33%	91.88%	100.00%		

Notes:

— This exhibit is a compilation of Exhibit 8.3 for each insured in the sample group.

— Average ground-up attachment point and total exposure from insured policy information are given.

— ABC Re's reported loss & expense from ABC Re's claim files are given. The amount could be lower than implied by model because of reporting lags to ABC Re or higher because of additional reserves.

EXHIBIT 10.4

PART 1

ASBESTOS BI MODEL FOR ABC RE'S SAMPLE GROUP
INDEMNITY AND EXPENSES WITH ABC RE'S LAYER OF COVERAGE FOR ALL SAMPLE INSUREDS, ANNUAL INFLATION = 0.0%/COVERAGE BLOCK = 25 YEARS
($000's)

Sample Insureds	Tier	Average Ground-Up Attachment Pt	Total Exposure	ABC Re's Reported Loss & Exp	Projected Losses and Expenses from All Policies with Insured in Calendar Year 1994	1995	1996	1997	1998	1999
Insured 1	4	37,500	3,363	0	0	0	0	0	0	0
Insured 2	4	1,994	13,960	20	39	45	50	55	61	65
Insured 3	2	2,943	17,000	2,300	0	0	0	0	0	0
Insured 4	1	48,750	38,480	21,500	20,868	21,744	22,567	23,512	24,662	25,732
Insured 5	1	50,357	30,280	19,300	19,395	20,344	20,344	20,369	20,807	21,215
Insured 6	1	48,333	40,680	22,450	22,149	24,201	25,732	26,262	27,077	27,953
Insured 7	2	37,813	13,581	1,500	0	0	173	442	692	925
Insured 8	2	40,000	14,290	300	0	42	102	158	210	259
Insured 9	2	40,313	10,233	300	41	107	170	228	283	300
Insured 10	3	17,143	6,000	150	30	65	97	128	156	166
Insured 11	2	37,813	31,940	200	0	0	0	0	0	0
Insured 12	3	26,429	16,300	0	0	0	0	0	0	0
Insured 13	3	25,938	24,800	15	0	0	0	0	0	0
Insured 14	3	21,111	9,500	15	0	0	0	0	0	0
Insured 15	3	25,313	6,400	200	54	77	99	119	139	149
	Subtotal Tier 1		109,440	63,250						
	Subtotal Tier 2		87,045	4,600						
	Subtotal Tier 3		63,000	380						
	Subtotal Tier 4		17,323	20						
	Total		276,807	68,250	62,577	66,625	69,334	71,273	74,086	76,764
	% of Ultimate				51.44%	54.77%	57.00%	58.59%	60.91%	63.11%

EXHIBIT 10.4

PART 2

Sample Insured	Tier	Projected Losses and Expenses from All Policies with Insured in Calendar Year 2000	2001	2002	2003	2004	2005	2006	2007	2008
Insured 1	4	0	0	0	0	0	0	0	0	0
Insured 2	4	70	75	80	85	90	95	99	103	107
Insured 3	2	0	8	110	206	296	433	563	683	793
Insured 4	1	26,726	27,796	28,881	29,903	30,860	31,754	32,584	33,350	34,052
Insured 5	1	21,677	22,261	22,812	23,331	23,818	24,272	24,694	25,083	25,440
Insured 6	1	29,012	30,001	30,935	31,814	32,638	33,408	34,122	34,781	35,386
Insured 7	2	1,142	1,342	1,500	1,500	1,500	1,500	1,500	1,500	1,500
Insured 8	2	300	300	300	300	300	300	300	300	300
Insured 9	2	300	300	300	300	300	300	300	300	300
Insured 10	3	168	169	170	171	173	174	175	176	177
Insured 11	2	0	0	0	0	0	0	0	0	0
Insured 12	3	0	0	0	0	0	0	0	0	0
Insured 13	3	0	0	0	0	0	0	0	0	0
Insured 14	3	0	0	0	0	0	0	0	0	0
Insured 15	3	154	159	165	171	177	182	187	191	195
	Subtotal Tier 1									
	Subtotal Tier 2									
	Subtotal Tier 3									
	Subtotal Tier 4									
	Total	79,547	82,409	85,253	87,782	90,152	92,417	94,523	96,468	98,250
	% of Ultimate	65.39%	67.75%	70.09%	72.16%	74.11%	75.97%	77.71%	79.30%	80.77%

EXHIBIT 10.4

PART 3

Sample Insured	Tier	Projected Losses and Expenses from All Policies with Insured in Calendar Year 2009	2010	2011	2012	2013	Ultimate	Ultimate as % of Exposure	Case Inc'd Loss Devel. Factor
Insured 1	4	0	0	0	0	0	0	0.0%	0.000
Insured 2	4	110	113	116	119	122	124	0.9%	6.206
Insured 3	2	893	983	1,063	1,133	1,229	2,321	13.7%	1.009
Insured 4	1	34,691	35,297	35,872	36,414	36,925	43,240	112.4%	2.011
Insured 5	1	25,764	26,073	26,365	26,640	26,900	29,904	98.8%	1.549
Insured 6	1	35,935	36,457	36,952	37,419	37,858	43,315	106.5%	1.929
Insured 7	2	1,500	1,500	1,500	1,500	1,500	1,500	11.0%	1.000
Insured 8	2	300	300	300	300	300	300	2.1%	1.000
Insured 9	2	300	300	300	300	300	300	2.9%	1.000
Insured 10	3	178	178	179	180	180	181	3.0%	1.207
Insured 11	2	5	21	36	50	64	242	0.8%	1.209
Insured 12	3	0	0	0	0	0	0	0.0%	0.000
Insured 13	3	0	0	0	0	0	0	0.0%	0.000
Insured 14	3	0	0	0	0	0	0	0.0%	0.000
Insured 15	3	199	202	206	209	212	215	3.4%	1.073
	Subtotal Tier 1						116,459	106.4%	1.841
	Subtotal Tier 2						4,663	5.4%	1.014
	Subtotal Tier 3						396	0.6%	1.041
	Subtotal Tier 4						124	0.7%	6.085
	Total	99,875	101,425	102,888	104,264	105,590	121,642	43.9%	1.782
	% of Ultimate	82.11%	83.38%	84.58%	85.71%	86.80%	100.00%		

Notes:

— This exhibit is a compilation of Exhibit 8.4 for each insured in the sample group.

— Average ground-up attachment point and total exposure from insured policy information are given.

— ABC Re's reported loss & expense from ABC Re's claim files are given. The amount could be lower than implied by model because of reporting lags to ABC Re or higher because of additional reserves.

EXHIBIT 11

ASBESTOS BI MODEL FOR ABC RE'S SAMPLE GROUP CALCULATION OF RANGE OF ESTIMATES OF ABC RE'S LIABILITIES FOR THE SAMPLE GROUP

Estimated Ultimate Loss & Expense for Sample Group of ABC Re's Policies			
Inflation = 5.0% 15 Yr Cov Blck Baseline Scenario (1)	Inflation = 0.0% 15 Yr Cov Blck Scenario (2)	Inflation = 5.0% 25 Yr Cov Blck Scenario (3)	Inflation = 0.0% 25 Yr Cov Blck Scenario (4)
$173,044	$149,174	$139,581	$121,642
	(5) Selected Low End of Range		$130,612
	(6) Selected High End of Range		$161,109
	(7) Selected Best Estimate		$153,485

Notes:
(1) From Exhibit 10.1.
(2) From Exhibit 10.2.
(3) From Exhibit 10.3.
(4) From Exhibit 10.4.
(5) Average of Columns (3) and (4).
(6) Average of Columns (1) and (2).
(7) Weighted average of Items (5) and (6). The weights are 25% and 75% respectively. The weights were selected based on likelihood of each scenario.

EXHIBIT 12.1

PART 1

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
INSURED 3'S LOSSES IN $5M XS $5M LAYER, INDEMNITY AND EXPENSES, ANNUAL INFLATION = 5.0%/COVERAGE BLOCK = 15 YEARS
($000's)

Policy Year	Width/Attch Pt/ % Share/Expenses ($ in millions)	Calendar Year 1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
1960	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1961	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1962	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1963	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1964	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1965	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1966	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1967	5/5/100%/Included in Limit	0	0	0	0	0	0	0	0	183	444
1968	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1969	5/5/100%/Included in Limit	0	0	0	0	0	0	0	0	183	444
1970	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1971	5/5/100%/Indem Only	0	0	0	0	0	0	0	0	0	0
1972	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1973	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1974	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1975–84	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
Total		0	0	0	0	0	0	0	0	366	888

EXHIBIT 12.1

PART 2

Policy Year	Width/Attch Pt/ % Share/Expenses ($ in millions)	Calendar Year 2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	Ultimate
1960	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1961	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1962	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1963	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1964	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1965	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	116	2,913
1966	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	116	2,913
1967	5/5/100%/Included in Limit	700	952	1,197	1,435	1,663	1,882	2,088	2,280	2,457	2,616	5,000
1968	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	116	2,913
1969	5/5/100%/Included in Limit	700	952	1,197	1,435	1,663	1,882	2,088	2,280	2,457	2,616	5,000
1970	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	116	2,913
1971	5/5/100%/Indem Only	0	0	0	0	0	0	0	0	0	78	1,942
1972	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1973	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1974	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1975–84	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
Total		1,401	1,904	2,394	2,869	3,326	3,763	4,176	4,561	4,914	5,776	23,595

Notes:

— $5M XS $5M layer for all policies. Only policies in Insured 3's coverage block for this scenario, 1960 through 1974, are included.

— Losses in layer are calculated by using $5M XS $5M to carve out losses and expenses from Exhibits 5.1, 6.1, and 7.1.

— Expenses are added to indemnity before applying attachment point and limits for expenses included in limits policies (Policy Years 1967 and 1969). When all lower layer policies are indemnity only or pro rata, this would not be true. In this case, indemnity only should be used to determine if the attachment point is reached. In the real world the true answer is somewhere between adding expenses to indemnity or just indemnity in determining satisfaction of the attachment point. Both scenarios should be examined.

— Ultimate value is calculated by continuation of patterns beyond months shown.

EXHIBIT 12.2

PART 1

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
INSURED 3'S LOSSES IN $5M XS $5M LAYER, INDEMNITY AND EXPENSES, ANNUAL INFLATION = 0.0%/COVERAGE BLOCK = 15 YEARS
($000's)

Policy Year	Width/Attch Pt/ % Share/Expenses ($ in millions)	Calendar Year 1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
1960	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1961	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1962	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1963	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1964	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1965	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1966	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1967	5/5/100%/Included in Limit	0	0	0	0	0	0	0	0	0	0
1968	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1969	5/5/100%/Included in Limit	0	0	0	0	0	0	0	0	0	0
1970	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1971	5/5/100%/Indem Only	0	0	0	0	0	0	0	0	0	0
1972	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1973	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1974	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1975–84	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
Total		0	0	0	0	0	0	0	0	0	0

EXHIBIT 12.2

PART 2

Policy Year	Width/Attch Pt/ % Share/Expenses ($ in millions)	Calendar Year 2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	Ultimate
1960	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1961	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1962	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1963	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1964	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1965	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0	0
1966	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0	0
1967	5/5/100%/Included in Limit	0	133	263	383	493	593	683	763	833	893	1,576
1968	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0	0
1969	5/5/100%/Included in Limit	0	133	263	383	493	593	683	763	833	893	1,576
1970	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0	0
1971	5/5/100%/Indem Only	0	0	0	0	0	0	0	0	0	0	0
1972	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1973	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1974	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1975–84	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
Total		0	266	526	766	986	1,186	1,366	1,526	1,666	1,786	3,151

Notes:

— $5M XS $5M layer for all policies. Only policies in Insured 3's coverage block for this scenario, 1960 through 1974, are included.

— Losses in layer are calculated by using $5M XS $5M to carve out losses and expenses from Exhibits 5.2, 6.2, and 7.2.

— Expenses are added to indemnity before applying attachment point and limits for expenses included in limits policies (Policy Years 1967 and 1969). When all lower layer policies are indemnity only or pro rata, this would not be true. In this case, indemnity only should be used to determine if the attachment point is reached. In the real world the true answer is somewhere between adding expenses to indemnity or just indemnity in determining satisfaction of the attachment point. Both scenarios should be examined.

— Ultimate value is calculated by continuation of patterns beyond months shown.

EXHIBIT 12.3

PART 1

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
INSURED 3'S LOSSES IN $5M XS $5M LAYER, INDEMNITY AND EXPENSES, ANNUAL INFLATION = 5.0%/COVERAGE BLOCK = 25 YEARS
($000's)

Policy Year	Width/Attch Pt/ % Share/Expenses ($ in millions)	Calendar Year 1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
1960	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1961	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1962	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1963	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1964	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1965	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1966	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1967	5/5/100%/Included in Limit	0	0	0	0	0	0	0	0	0	0
1968	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1969	5/5/100%/Included in Limit	0	0	0	0	0	0	0	0	0	0
1970	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1971	5/5/100%/Indem Only	0	0	0	0	0	0	0	0	0	0
1972	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1973	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1974	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1975–84	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
Total		0	0	0	0	0	0	0	0	0	0

EXHIBIT 12.3

PART 2

Policy Year	Width/Attch Pt/ % Share/Expenses ($ in millions)	Calendar Year 2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	Ultimate
1960	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1961	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1962	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1963	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1964	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1965	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0	0
1966	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0	0
1967	5/5/100%/Included in Limit	0	0	0	0	0	0	0	0	0	0	1,248
1968	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0	0
1969	5/5/100%/Included in Limit	0	0	0	0	0	0	0	0	0	0	1,248
1970	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0	0
1971	5/5/100%/Indem Only	0	0	0	0	0	0	0	0	0	0	0
1972	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1973	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1974	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1975–84	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
Total		0	0	0	0	0	0	0	0	0	0	2,496

Notes:

— $5M XS $5M layer for all policies. Only policies in Insured 3's coverage block for this scenario, 1960 through 1984, are included.

— Losses in layer are calculated by using $5M XS $5M to carve out losses and expenses from Exhibits 5.1, 6.3, and 7.3.

— Expenses are added to indemnity before applying attachment point and limits for expenses included in limits policies (Policy Years 1967 and 1969). When all lower layer policies are indemnity only or pro rata, this would not be true. In this case, indemnity only should be used to determine if the attachment point is reached. In the real world the true answer is somewhere between adding expenses to indemnity or just indemnity in determining satisfaction of the attachment point. Both scenarios should be examined.

— Ultimate value is calculated by continuation of patterns beyond months shown.

EXHIBIT 12.4

PART 1

ASBESTOS BI MODEL FOR ABC RE'S INSURED 3
INSURED 3'S LOSSES IN $5M XS $5M LAYER, INDEMNITY AND EXPENSES,
ANNUAL INFLATION = 0.0%/COVERAGE BLOCK = 25 YEARS
($000's)

Policy Year	Width/Attch Pt/ % Share/Expenses ($ in millions)	Calendar Year 1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
1960	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1961	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1962	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1963	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1964	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1965	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1966	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1967	5/5/100%/Included in Limit	0	0	0	0	0	0	0	0	0	0
1968	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1969	5/5/100%/Included in Limit	0	0	0	0	0	0	0	0	0	0
1970	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0
1971	5/5/100%/Indem Only	0	0	0	0	0	0	0	0	0	0
1972	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1973	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1974	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
1975–84	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0
Total		0	0	0	0	0	0	0	0	0	0

EXHIBIT 12.4

PART 2

Policy Year	Width/Attch Pt/ % Share/Expenses ($ in millions)	Calendar Year 2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	Ultimate
1960	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1961	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1962	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1963	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1964	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1965	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0	0
1966	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0	0
1967	5/5/100%/Included in Limit	0	0	0	0	0	0	0	0	0	0	0
1968	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0	0
1969	5/5/100%/Included in Limit	0	0	0	0	0	0	0	0	0	0	0
1970	5/5/100%/Pro Rata	0	0	0	0	0	0	0	0	0	0	0
1971	5/5/100%/Indem Only	0	0	0	0	0	0	0	0	0	0	0
1972	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1973	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1974	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
1975–84	No ABC Re Policy	0	0	0	0	0	0	0	0	0	0	0
Total		0	0	0	0	0	0	0	0	0	0	0

Notes:

— $5M XS $5M layer for all policies. Only policies in Insured 3's coverage block for this scenario, 1960 through 1984, are included.

— Losses in layer are calculated by using $5M XS $5M to carve out losses and expenses from Exhibits 5.2, 6.4, and 7.4.

— Expenses are added to indemnity before applying attachment point and limits for expenses included in limits policies (Policy Years 1967 and 1969). When all lower layer policies are indemnity only or pro rata, this would not be true. In this case, indemnity only should be used to determine if the attachment point is reached. In the real world the true answer is somewhere between adding expenses to indemnity or just indemnity in determining satisfaction of the attachment point. Both scenarios should be examined.

— Ultimate value is calculated by continuation of patterns beyond months shown.

EXHIBIT 13

EXTRAPOLATION METHOD 1 USING ABC RE'S SAMPLE GROUP
CALCULATION OF PERCENTAGE OF EXPOSURE ERODED BY LAYER BY TIER

Example Calculation of Matrix Box for Tier 2, $5M XS $5M

			Projected Ultimate Loss and Expense from BI Model in the Layer Assuming Each ABC Re Policy is $5M XS $5M							
Name	Tier	Exposure Assuming each Policy $5M XS $5M	5% Infltn 15 Yr Spread Scenario	0% Infltn 15 Yr Spread Scenario	Average of 15 Yr Spread Scenarios	5% Infltn 25 Yr Spread Scenario	0% Infltn 25 Yr Spread Scenario	Average of 25 Yr Spread Scenarios	Wtd 75% 15 Yr Wtd 25% 25 Yr Average	Percentage of $5M XS $5M Layer Eroded
Insured Co 3	2	35.0	23.6	3.2	13.4	2.5	0.0	1.3	10.4	30%
Insured Co 7	2	40.0	33.6	7.8	20.7	6.0	0.0	3.0	16.3	41%
Insured Co 8	2	40.0	37.9	10.9	24.4	8.5	0.0	4.3	19.4	48%
Insured Co 9	2	40.0	35.7	9.4	22.6	7.2	0.0	3.6	17.8	45%
Insured Co 11	2	40.0	35.7	9.4	22.6	7.2	0.0	3.6	17.8	45%
		195.0	166.5	40.7	103.6	31.4	0.0	15.7	81.6	42%

Selected Percentage of Layer Eroded

	Layer						
Tier	.5M XS 0	.5M XS .5M	4M XS 1M	5M XS 5M	15M XS 10M	25M XS 25M	50M XS 50M
1							
2				42%			
3							
4							

Notes:
— The exposure for an insured here is the number of policies with the insured times the $5M layer.
— Ultimate loss and expense from Exhibit 12 for each Tier 2 insured in the sample group.
— Average ultimate loss and expense judgmentally selected based upon weighted average of four scenarios.

EXHIBIT 14

EXTRAPOLATION METHOD 2 USING ABC RE'S SAMPLE GROUP CALCULATION OF CASE INCURRED LOSS DEVELOPMENT FACTORS

	Case Incurred Loss and Expense Development Factor by Tier for			
Tier	5% Infltn 15 Yr Spread Scenario	0% Infltn 15 Yr Spread Scenario	5% Infltn 25 Yr Spread Scenario	0% Infltn 25 Yr Spread Scenario
Tier 1	1.959	1.958	1.898	1.841
Tier 2	8.909	4.975	3.814	1.014
Tier 3	20.372	5.595	4.655	1.041
Tier 4	20.127	14.739	9.578	6.085

	Case Incurred Loss and Expense Percentage Reported by Tier for							
Tier	5% Infltn 15 Yr Spread Scenario	0% Infltn 15 Yr Spread Scenario	Average of 15 Yr Spread Scenarios	5% Infltn 25 Yr Spread Scenario	0% Infltn 25 Yr Spread Scenario	Average of 25 Yr Spread Scenarios	Wtd 75% 15 Yr Wtd 25% 25 Yr Average % Reported by Tier	Selected Development Factor by Tier
Tier 1	51.05%	51.07%	51.06%	52.69%	54.32%	53.50%	51.67%	1.935
Tier 2	11.22%	20.10%	15.66%	26.22%	98.62%	62.42%	27.35%	3.656
Tier 3	4.91%	17.87%	11.39%	21.48%	96.06%	58.77%	23.24%	4.304
Tier 4	4.97%	6.78%	5.88%	10.44%	16.43%	13.44%	7.77%	12.875

Notes:

— Development factors from Exhibit 10.
— Percentage reported equals reciprocal of appropriate development factor.
— Weighted average of percentage reported for the four scenarios judgmentally selected.
— Selected development factor equals reciprocal of weighted average percentage reported.

EXHIBIT 15

EXTRAPOLATION METHOD 3 USING ABC RE'S SAMPLE GROUP
CALCULATION OF PERCENTAGE OF EXPOSURE EXHAUSTED BY TIER

	Ultimate Loss & Expense as a Percentage of Exposure for						Wtd 75% 15 Yr Wtd 25% 25 Yr Average
Tier	5% Infltn 15 Yr Spread Scenario	0% Infltn 15 Yr Spread Scenario	Average of 15 Yr Spread Scenarios	5% Infltn 25 Yr Spread Scenario	0% Infltn 25 Yr Spread Scenario	Average of 25 Yr Spread Scenarios	Percentage of Exposure Exhausted by Tier
Tier 1	113.2%	113.2%	113.2%	109.7%	106.4%	108.1%	111.9%
Tier 2	47.1%	26.3%	36.7%	20.2%	5.4%	12.8%	30.7%
Tier 3	12.3%	3.4%	7.9%	2.8%	0.6%	1.7%	6.3%
Tier 4	1.8%	1.3%	1.6%	0.8%	0.5%	0.7%	1.3%

Notes:

— Percentage of exposure factors from Exhibit 10.

— Weighted average of four scenarios judgmentally selected.

— Some percentage of exposure factors bigger than 100% because of policies with pro rata expense treatment.

EXHIBIT 16

EXTRAPOLATION METHOD 4 USING ABC RE'S SAMPLE GROUP CALCULATION OF AVERAGE ULTIMATE LOSS AND EXPENSE BY TIER

	Ultimate Loss & Expense by Scenario by Tier				
Tier	5% Infltn 15 Yr Spread Scenario	0% Infltn 15 Yr Spread Scenario	5% Infltn 25 Yr Spread Scenario	0% Infltn 25 Yr Spread Scenario	Number of Sample Group Insureds by Tier
Tier 1	123,911	123,862	120,074	116,459	3
Tier 2	40,981	22,885	17,543	4,663	5
Tier 3	7,741	2,126	1,769	396	5
Tier 4	411	301	195	124	2

	Average Ultimate Loss & Expense by Scenario by Tier						
Tier	5% Infltn 15 Yr Spread Scenario	0% Infltn 15 Yr Spread Scenario	Average of 15 Yr Spread Scenarios	5% Infltn 25 Yr Spread Scenario	0% Infltn 25 Yr Spread Scenario	Average of 25 Yr Spread Scenarios	Wtd 75% 15 Yr Wtd 25% 25 Yr Average Ultimate Loss & Expense
Tier 1	41,304	41,287	41,296	40,025	38,820	39,422	40,827
Tier 2	8,196	4,577	6,387	3,509	933	2,221	5,345
Tier 3	1,548	425	987	354	79	217	794
Tier 4	206	151	178	98	62	80	153

Notes:

— Ultimate loss and expense from Exhibit 10.

— Number of sample group insureds by Tier from Exhibit 10.

— Weighted average of four scenarios judgmentally selected.

RATEMAKING: A FINANCIAL ECONOMICS APPROACH

STEPHEN P. D'ARCY AND MICHAEL A. DYER

Abstract

Financial pricing models are replacing traditional ratemaking techniques for property-liability insurers. This paper provides an introduction to the target total rate of return approach, the capital asset pricing model, the discounted cash flow technique, and the option pricing model, all in an insurance context. Examples of each method, along with discussions of their advantages and weaknesses, are provided.

ACKNOWLDGEMENTS

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1. INTRODUCTION TO FINANCIAL ECONOMICS

Financial economics deals with the acquisition, issuance, valuation, and investment of securities in capital markets. Much of the early work in financial economics dealt with determining the appropriate value of stocks. Models were developed to predict the value of a stock, which was compared with its actual price. The strategy of buying underpriced stocks and selling overpriced stocks was expected to produce returns above the general market performance. Benjamin Graham and David Dodd were major proponents of this approach [18]. However, valuation of individual stocks proved to be difficult, and to this day no consensus exists among financial economists about what the price of a given stock should be.

In 1952, Harry Markowitz directed the focus away from individual stock picking with his work entitled "Portfolio Selection" [24]. Markowitz calculated the variance, which was used as a measure of risk of returns, and demonstrated the effect on portfolio risk of the addition and subtraction of stocks to and from a group of stocks. He showed that a portfolio of stocks could generate a higher return at a lower level of risk than individual stocks held alone. This concept, known as portfolio diversification, reduced the emphasis on individual stock picking.

However, investors were still interested in the returns of individual stocks. Building upon Markowitz's work, William Sharpe [29] published an article in 1964 that explained the expected return of individual securities in a well-diversified portfolio. In this model, termed the Capital Asset Pricing Model (CAPM), the investor is compensated only for bearing *systematic risk*, which cannot be diversified away by adding more stocks to a portfolio. *Unsystematic risk*, which can be diversified away, is the second component of total risk of a portfolio. Markowitz-like portfolio-diversified investors do not need to be compensated for unsystematic risk. The expected return of a security, thus, is the rate of return on a risk-free asset, plus the security's beta multiplied by the market risk premium:

$$\mathrm{E}(R_i) = R_f + \beta_i[\mathrm{E}(R_m) - R_f], \tag{1.1}$$

where

$$\begin{aligned}
\mathrm{E}(R_i) &= \text{expected return for security } i, \\
R_f &= \text{risk-free rate}, \\
\mathrm{E}(R_m) - R_f &= \text{market risk premium, and} \\
\beta_i &= \text{beta of security } i \\
&= \mathrm{Cov}(R_i, R_m)/\mathrm{Var}(R_m).
\end{aligned}$$

The market risk premium is the amount by which a portfolio of stocks diversified against all unsystematic risk is expected to exceed the risk-free rate. This value is frequently determined based on historical experience.

Initially, empirical tests of the CAPM tended to support the model. However, some studies, notably Roll [28], have stated that the tests were essentially joint tests of the model and the market proxy. The market portfolio should include such assets as bonds, real estate, collectibles, and even human capital, but valid measures of the total value of these assets on a regular basis are not available. Since empirical tests of the model tend to use a stock market portfolio, which is readily available, as the market portfolio, the model has not been, and actually cannot be, fully tested. More recently, Fama and French [16] report the results of an extensive test of the CAPM on stock market data from 1941 to 1990 and conclude that size and the ratio of book-to-market value are more important than beta in explaining returns. Over the entire period, the relationship between beta and average returns is insignificant. Despite this damaging evidence, the Capital Asset Pricing Model's risk and return relationship is still considered important today and is the foundation of several financial models that have been applied to insurance ratemaking.

Merton Miller, in his 1958 work with Franco Modigliani entitled "The Cost of Capital, Corporation Finance, and the Theory of Investment," laid the groundwork for corporate financial theory [25]. This work examined the impact that the use of debt and dividends had upon the value of the firm. Miller found that the value of the firm is independent of the level of debt and the dividend payout level chosen by the firm. This conclusion, derived from strict assumptions including no taxes, was controversial, but led to the understanding of optimal capital structure and dividend policy used by corporations today.

One other important development in financial economics is the option pricing theory developed by Black and Scholes [4] in the early 1970s. Options are derivative securities, meaning that they derive their value from their relationship with another security. Options give the holder the right to buy, in the case of a *call* option, or to sell, in the case of a *put* option, an asset at a specified price. Options exist on stocks, bonds, futures, commodities, stock indices, and even insurance catastrophe losses. For example, an investor may own a call option on IBM stock to buy IBM for $100 a share. If the price of IBM is greater than $100, the investor can exercise the option to buy IBM at $100 and then sell the stock at the higher price, thereby earning a profit. Many assets and claims exist that can be thought of as options, or contingent claims. For example, stockholders of a corporation can be thought of as holding an option on the company's assets being greater than its liabilities. If the assets are less than the liabilities, stockholders receive nothing; if greater, stockholders receive the entire difference.

Option pricing models such as the Black–Scholes model have been fairly successful at valuing options. The Black–Scholes model was so successful that its model prices were used by option traders as actual market prices in the early 1970s when organized option exchanges were opened.

The option concept along with the use of option pricing models can also be applied to the claims of insurers. The claims of policyholders, stockholders, and tax authorities against the insurer can be thought of as options. These applications will be discussed in detail later.

The contributions of Markowitz, Sharpe, Miller, Modigliani, Black, and Scholes have led to the development of financial models that have been applied to investment and corporate finance. These models have also been applied to ratemaking in the insurance industry. The formulation of these financial models and their application to insurance will be explained in this paper.

Even a cursory review of insurance profitability demonstrates that, at least since the 1970s, the industry has not achieved the target underwriting profit margin of five percent based on the 1921 National Convention of Insurance Commissioners profit formula. This result could be due to an inability to achieve the appropriate rate of return because of unexpected inflation, disasters and other insured catastrophes, social changes that raised costs in an unpredictable manner, or other unforeseen developments. However, the persistency of the shortfall, the fact that insurance markets have remained attractive enough to continue to draw new entrants and investment capital, and the fact that bankruptcies and failures among insurers have not risen to especially unusual levels suggest that a more appropriate explanation is that the model for determining the profit margin is at fault, and insurers have not been trying to obtain a 5 percent underwriting profit margin. The search for an alternative pricing model has yet to be concluded. For a description of the early regulatory decisions repudiating the 5 percent underwriting profit margin and a summary of alternative models, see Derrig [11].

2. TARGET TOTAL RATE OF RETURN MODEL

Early alternative pricing models were proposed by Bailey [2], Ferrari [17], and Cooper [6]. In these models, the total return of an insurer, the sum of underwriting and investment results, was recognized as the key measure of profitability. When investment income increases, as it did in the 1960s due to longer-tailed claim payments and higher interest rates, the underwriting income can be expected to decline, depending on the required total rate of return. An example of this approach is the Target Total Rate of Return Model.

The target total rate of return combines the two sources of income for an insurer: investment income and underwriting income. In this approach, a target total rate of return is set equal to the total return from investments plus the total rate of return from underwriting. Once the investment income is projected, the required underwriting profit margin can be calculated. The formula for the target total rate of return for insurers can be written as:

$$TRR = (IA/S)(IR) + (P/S)(UPM), \qquad (2.1)$$

where

$$TRR = \text{target total rate of return,}$$
$$IA = \text{investable assets,}$$
$$S = \text{owners' equity in the insurer,}$$
$$IR = \text{investment return,}$$
$$P = \text{premium, and}$$
$$UPM = \text{underwriting profit margin.}$$

In Equation 2.1, investment income and underwriting income are expressed as a percentage of equity.

In order to use this technique, the appropriate target total rate of return must be determined. Various procedures could be used to determine the total rate of return, such as an industry average return on equity, an arbitrary target such as 15 percent, a variable value tied to an alternative investment such as 5 percent over long-term Treasury bonds, or some appropriate rate of return for the investor based on the riskiness of the firm. The latter procedure of providing investors with an appropriate rate of return to compensate for the risk that they undertake is used in public utility rate regulation. The Capital Asset Pricing Model, discussed in detail in the next section, is often used in utility rate regulation to determine the appropriate risk-adjusted return that stockholders should expect to receive.

To apply the target total rate of return model, $E(R_e)$ from the CAPM in Equation 1.1 is set equal to the target total rate of return TRR in Equation 2.1. Solving for the underwriting profit margin UPM leads to the following equation:

$$UPM = (S/P)[R_f + \beta_e(E(R_m) - R_f) - (IA/S)(IR)]. \quad (2.2)$$

To use Equation 2.2 for a stock insurer, current company data for the ratios of investable assets to equity and premium are used along with a forecast of the insurer's investment rate of return.

The insurer's beta and the market risk premium can be gathered through historical estimates, and the current one year Treasury bill rate can be used as the risk-free rate in this single period model.

For example, assume the risk-free rate is 7 percent, the insurer's beta is 1.0, the market risk premium is 8 percent, the insurer's ratio of investable assets to equity is 3 to 1, the insurer's investment return is 7 percent, and the ratio of premiums to equity is 2 to 1. The target total rate of return is given by the CAPM in Equation 1.1 as follows:

$$TRR = 7\% + 1.0(8\%) = 15\%.$$

The underwriting profit margin is given by Equation 2.2:

$$UPM = (1/2)[15\% - 2(7\%)] = 0.5\%.$$

The investment return on equity of 14 percent is subtracted from the total rate of return of 15 percent yielding an underwriting return on equity of 1 percent, which translates into an underwriting profit margin of 0.5 percent.

The target underwriting profit margin for an insurer with equity of \$500,000, premiums of \$1,250,000, investable assets of \$2,000,000, investment return of 7.5 percent, and beta of 1.15, when the risk-free rate is 7 percent, and the market risk premium is 9 percent is determined as follows:

$$\begin{aligned} UPM &= (S/P)[R_f + \beta_e(\mathrm{E}(R_m) - R_f) - (IA/S)(IRR)] \\ &= (500{,}000/1{,}250{,}000) \\ &\quad \times [7\% + 1.15(9\%) - (2{,}000{,}000/500{,}000) \times (7.5\%)] \\ &= -5.06\%. \end{aligned}$$

In addition to the difficulty in determining the target total rate of return for this model, measuring the owners' equity in the insurer is another complex issue. This value should represent the current investment in the company, the amount that could be deployed elsewhere if the owners decided not to continue to write insurance. For a stockholder-owned insurer, this can be estimated in total by the market value of the company. However, insurers do not set rates in aggregate, but on a by-line by-state basis. Estimating the owners' equity in, for example, Kansas private passenger automobile, is far more difficult.

Equity is not statutory surplus. If statutory surplus, instead of the insurer's actual equity, is used in the target total rate of return method, the required underwriting profit margin derived from the model will be distorted. The statutory surplus figure is lower than most insurers' actual equity levels, since statutory surplus ignores the time value of money in loss reserves, excludes the value of tangible assets and non-admitted reinsurance, and values bonds and real estate at other than market values. Thus, the target total rate of return calculation based on statutory surplus will generate a lower underwriting profit margin than if the true equity figure were used. If this lower underwriting profit margin were forced upon insurers, they might react by investing in more risky assets to boost their investment rate of return in order to compensate for the lower underwriting profit margin. The reaction of increased risk taking by insurers could lead to an increase in insolvency among insurance companies.

A statutory surplus figure higher than actual equity levels, which could occur in times of increasing interest rates, would indicate a higher than necessary underwriting profit margin. This would cause excessive premiums to be charged to customers.

By itself, the target total rate of return approach lacks any theoretical justification for a proper rate of return. A model well supported by theory will be discussed next.

3. CAPITAL ASSET PRICING MODEL

The CAPM, developed in the 1960s, is one of the most powerful tools of finance and one of the foundations of most current financial theories. The CAPM has been applied to many financial issues: estimating stock returns and prices, determining appropriate corporate capital budgeting rates of return, establishing allowable rates of return for utilities, and pricing insurance. Insurance applications of the CAPM include estimating underwriting profit margins for insurance pricing purposes and determining the appropriate rate for discounting loss reserves.

The CAPM is based on several straightforward investment principles—asset allocation, portfolio return and risk, efficient portfolios, and portfolio diversification—that will be described and explained in this section.

Asset allocation involves dividing capital among broad asset categories. These asset categories include stocks, bonds, real estate, bank deposits, certificates of deposit (CDs), and Treasury bills. Most of the former asset categories can be described as risky assets, meaning they have an uncertain return. Some of the latter categories, such as Treasury bills and, to a lesser extent, bank deposits and CDs, are considered to be risk-free assets, meaning they have a virtually guaranteed return. The following simplified example will describe and illustrate asset allocation.

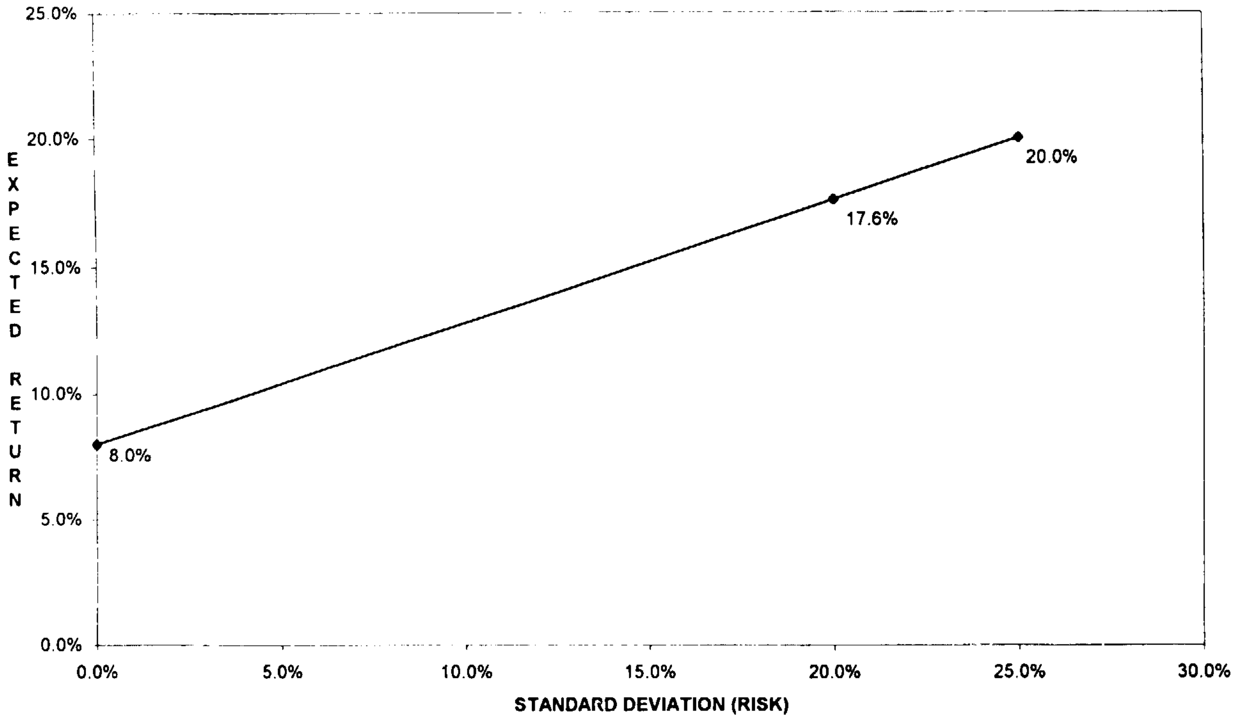
Two Asset Allocation Case

Assume that there are only two assets available to investors, one risk-free asset and one risky asset. The risk-free asset has a rate of return of 8 percent and has no risk. The risky asset has an expected rate of return of 20 percent and a standard deviation of 25 percent. The standard deviation measures the total variability of returns over time and is frequently used as a risk measure. This standard deviation of return will be our risk measure initially.

The two assets' expected return and risk are known, and an investor wants to allocate money between these two assets. Figure 1 illustrates this asset allocation choice. The y-axis intercept

FIGURE 1

Two Asset Allocation



represents one possible asset allocation choice: investing all of the money in the risk-free asset, giving the investor a guaranteed return of 8 percent. Another possibility is the endpoint of the line plotted in Figure 1, which represents investing all of the money in the risky asset. This choice has an expected return of 20 percent and a standard deviation of 25 percent. The line between these two points, termed the Capital Allocation Line (CAL), represents the expected return and standard deviation of different combinations of the two assets. The expected return of each portfolio is simply a weighted average of each asset's expected return and is given by the following general formula:

$$\mathrm{E}(R_p) = (1 - W)R_f + W\mathrm{E}(R_k), \tag{3.1}$$

where

$\mathrm{E}(R_p)$ = the expected return of the combination portfolio,

R_f = the risk-free rate,

W = the proportion of money invested in the risky asset, and

$\mathrm{E}(R_k)$ = the expected return of the risky asset.

In this example, with R_f = 8 percent, and $\mathrm{E}(R_k)$ = 20 percent, the formula for the expected combination portfolio return is:

$$\mathrm{E}(R_p) = (1 - W)(8\%) + W(20\%). \tag{3.2}$$

The riskiness of the portfolio is given by the portfolio's standard deviation, which depends on the standard deviation of each asset, the proportion invested in each asset, and the covariance between the two assets' returns. The general formula for a two asset portfolio's standard deviation is:

$$\sigma_p = [(1 - W)^2\sigma_1^2 + 2(1 - W)(W)\mathrm{Cov}(R_1,R_2) + W^2\sigma_2^2]^{(1/2)}, \tag{3.3}$$

where

σ_i = the standard deviation of returns for asset i,

W = the proportion invested in asset 2, and

$\mathrm{Cov}(R_1, R_2)$ = the covariance of returns between asset 1 and asset 2.

The covariance of returns equals the product of the standard deviation of each of the two assets and the correlation coefficient between the two assets. In this example, the first asset is a risk-free asset, meaning it has a standard deviation of zero; therefore, the covariance between the risk-free asset and the risky asset is also zero, which yields the following simple formula for the standard deviation of our example's portfolio:

$$\sigma_p = [W^2 \sigma_k^2]^{1/2} = W\sigma_k. \tag{3.4}$$

Thus, in this example, the portfolio's standard deviation is the proportion of money invested in the risky asset multiplied by the standard deviation of the risky portfolio, which is 25 percent.

For illustrative purposes, consider a sample portfolio to verify a point on the Capital Allocation Line in Figure 1. Assume 20 percent of an investor's money is invested in the risk-free asset and the remaining 80 percent in the risky asset. The expected return for this portfolio would be:

$$\mathrm{E}(R_p) = (0.2)(8\%) + (0.8)(20\%) = 17.6\%.$$

The standard deviation of the portfolio would be:

$$\mathrm{SD}(\mathrm{E}(R_p)) = (0.8)(25\%) = 20\%.$$

This point is shown on the Capital Allocation Line in Figure 1.

To apply this technique, an individual investor would choose the desired level of risk and/or return and would solve for the appropriate proportion to invest in the risky asset from the risk and return Equations 3.2 and 3.4 above. The investor selects the point along the Capital Allocation Line that indicates the expected return and risk level of the investor's choice. For example, if an investor wanted an expected rate of return of 12 percent, he or she could use the expected return formula given in Equation 3.1 to solve for the risky asset proportion that would yield the 12 percent expected return. Solving Equation 3.1, the appropriate risky asset proportion would be:

$$W = [E(R_p) - R_f]/[E(R_k) - R_f]. \tag{3.5}$$

In this example Equation 3.5 would be written as the following:

$$W = [12\% - 8\%]/[20\% - 8\%] = 0.33 \text{ or } 33\%.$$

If the investor wanted an expected return of 12 percent, based on Equation 3.5, the investor would have to invest 33 percent of the portfolio in the risky asset and the remaining 67 percent in the risk-free asset. According to Equation 3.4, this portfolio would have a standard deviation of (0.33)(25%) or 8.25 percent.

An investor could also establish the portfolio according to the amount of risk desired. For example, if an investor could tolerate a risk level of only a 10 percent standard deviation in the expected return, he or she could use the portfolio standard deviation equation given in Equation 3.4 to solve for the risky portfolio proportion that would yield the 10 percent combination portfolio standard deviation. Solving Equation 3.4, the appropriate risky proportion would be:

$$W = \sigma_p/\sigma_k. \tag{3.6}$$

From the above example, the appropriate risky asset proportion would be:

$$W = 10\%/25\% = .40 \text{ or } 40\%.$$

Therefore, to achieve the desired risk level of 10 percent, an investor would have to invest 40 percent of the portfolio in the risky asset and 60 percent in the risk-free asset. From Equation 3.2, this would give the investor an expected portfolio return of

(.6)(8%) + (.4)(20%), or 12.8 percent for the desired 10 percent risk level.

What if an investor wanted an expected return greater than the expected return of the risky asset of 20 percent? In this case, the investor would have to invest more than 100 percent in the risky asset by borrowing. For simplicity, financial models often assume that investors can borrow and lend at the same interest rate. Figure 2 illustrates borrowing at the risk-free rate. The extension of the Capital Allocation Line beyond the horizontal line at an expected return of 20 percent represents a negative investment at the risk-free rate (borrowing), giving the investor the necessary funds to invest more than 100 percent in the risky asset. For example, if an investor wanted an expected return of 26 percent, the investor would solve for W, the proportion invested in the risky asset, from Equation 3.5. In this example, Equation 3.5 is written as the following:

$$W = [26\% - 8\%]/[20\% - 8\%] = 1.5 \text{ or } 150\%.$$

To achieve an expected return of 26 percent, the investor would have to invest 150 percent of the value of the portfolio in the risky portfolio and borrow an amount equal to 50 percent of the portfolio at the risk-free rate. This portfolio would have a standard deviation of (1.5)(25%), or 37.5 percent.

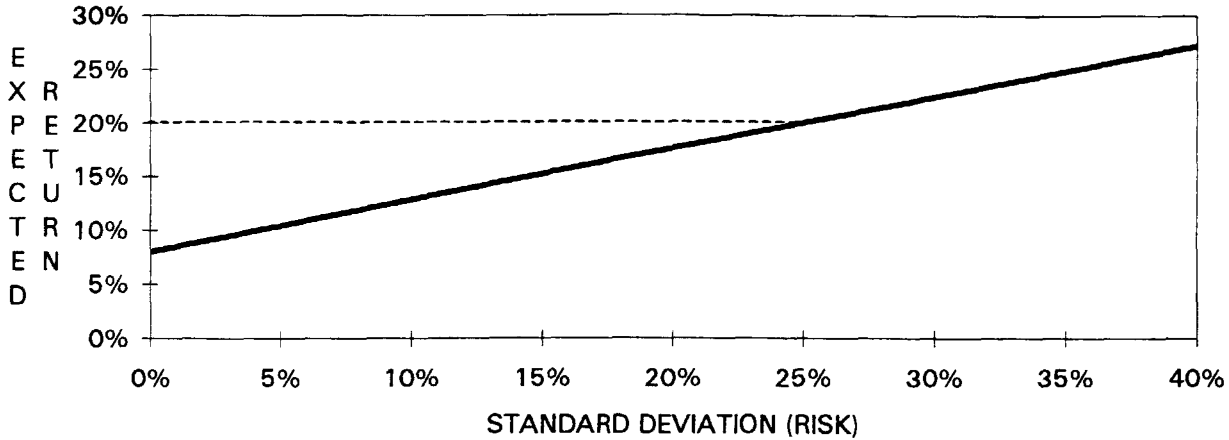
This concludes the discussion of the simple two asset allocation case. Next, the case that is more realistic, with a myriad of assets available for investment, is introduced. It will then be shown that this "entire universe" case can be simplified to a two asset allocation choice, leading to the concept of the Capital Asset Pricing Model.

Multiple Asset Allocation Case

Assume that any or all risky assets in the world are available for investment, all investors know the expected return and standard deviation of each asset and the covariance of returns

FIGURE 2

2 ASSET ALLOCATION WITH BORROWING AT THE RISK-FREE RATE



among different assets, and everyone has the same expectations regarding these returns and standard deviations. All the risk and return information could be used to form millions of different portfolios of these assets, and the expected return and standard deviation of each portfolio could be calculated. The risk and return calculation would be performed using a more general form of Equations 3.1 and 3.3 as given by Equations 3.7 and 3.8:

$$E(R_p) = \Sigma_i(W_i E(R_i)), \qquad \text{and} \tag{3.7}$$

$$\sigma_p^2 = \Sigma_i W_i \sigma_i^2 + \Sigma_j \Sigma_i W_i W_j \sigma_{ij}, \tag{3.8}$$

where

j does not equal i, and

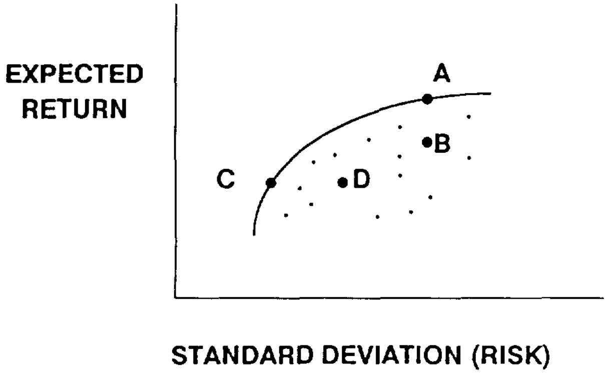
σ_{ij} = covariance between stocks i and j.

A sample of these portfolios is plotted on Figure 3. The next step is to decide which portfolios investors might select. Assume investors are rational risk averse investors. This means these investors prefer to maximize return for the same level of risk and to minimize risk for the same level of return. First, look at portfolios A and B in Figure 3. Notice that both portfolios have the same standard deviation, but portfolio A has a higher expected return. Rational risk averse investors would prefer portfolio A to portfolio B because it has a higher level of expected return for the same level of risk. Portfolio A is said to dominate portfolio B and any other portfolio below portfolio A on the graph with the same level of risk but a lower expected return.

Now, examine portfolios C and D in Figure 3. Both portfolios have the same expected return, but portfolio C has a lower standard deviation than portfolio D. Again, rational risk averse investors would rather invest in portfolio C because it has a lower level of risk for the same level of return when compared to portfolio D. Portfolio C is said to dominate portfolio D and any other portfolio to the right of portfolio C in the graph with the same expected return but a higher standard deviation.

FIGURE 3

EFFICIENT FRONTIER

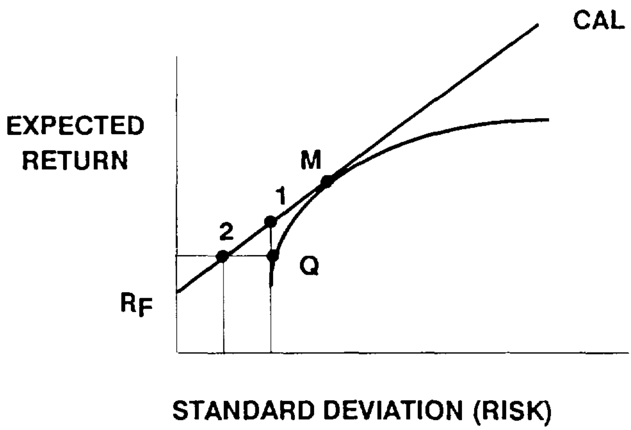


Portfolios A and C are called efficient portfolios because they have the highest level of return for a given level of risk and the lowest level of risk for a given level of return. All the portfolios on the curve in Figure 3 are efficient portfolios and investors would want to invest only in this set of efficient portfolios because their risk and return characteristics dominate all portfolios under the curve. The curve representing the efficient portfolio set is called the efficient frontier.

An investor could select a portfolio on this efficient frontier according to the investor's desired risk and return level. However, there is a better way to choose a portfolio which coincides with our earlier two asset allocation example.

Assume the risk-free asset still exists and a line can be drawn from the risk-free asset to the efficient frontier. The line could intersect the efficient frontier at any point on the curve, but the line from the risk-free rate that is tangent to the curve is the most desirable line from the standpoint of the investor. This line, included in Figure 4, is exactly the same as the capital allocation line discussed earlier in the two asset example. In this case, the

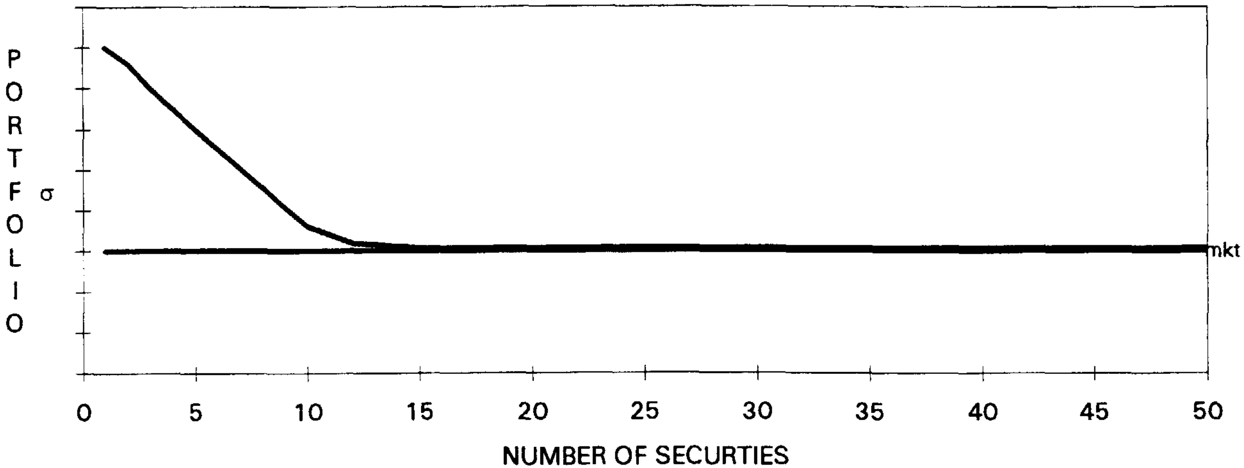
FIGURE 4



risky asset is the one portfolio, M, that is common to both the line and the efficient frontier. Since every investor has the same (homogenous) expectations about asset risk and return, portfolio M in Figure 4 would be the risky portfolio that all investors would want to own. In this case, the investor's portfolio choice is the same as the earlier two asset case of a combination between the risk-free asset and the efficient risky asset portfolio M. The investor's portfolio would be divided between the two assets according to the level of risk and/or return desired. If the investor wanted the level of risk and return that portfolio M offered, then 100 percent of the money would be placed in portfolio M. An investor wanting a level of risk lower than portfolio M would prefer to invest a portion of the total portfolio in the risk-free asset and the rest in portfolio M according to Equation 3.6 instead of investing in a portfolio on the efficient frontier like Q in Figure 4. A portfolio on the CAL has either a higher return for the same level of risk, such as point 1, or a lower level of risk for the same level of return, such as point 2, when compared to portfolio Q. In other words, the CAL is more efficient than the efficient frontier at every point except M; that is why this expanded asset world example collapses to the two asset allocation choice.

Measurement of Expected Returns on Individual Securities

At this point, the investor holds a well-diversified portfolio that consists of the risk-free asset and the optimal risky portfolio M which contains many assets (theoretically all assets available in the world). To define a well-diversified portfolio, we must first discuss the two types of risk of a portfolio. As in insurance, where an insurer can reduce risk by writing more policies, an investor can reduce risk by adding more assets to a portfolio. Figure 5 illustrates this point where the standard deviation, or total risk, of a portfolio is denoted on the y-axis and the number of assets on the x-axis. Generally, by choosing assets at random and adding them to the portfolio, the investor can reduce the overall risk of a portfolio. However, eventually the investor reaches a saturation point where more assets added to the portfolio do not significantly reduce the total risk of the portfolio. At this saturation point, the investor still has risk remaining in the portfolio. This remaining risk has three different names in finance, but all mean the same thing: nondiversifiable, systematic, or market risk. This market risk is risk that cannot be diversified away by adding more assets to a portfolio and is the inherent risk associated with the market portfolio of all risky assets. However, if asset returns were uncorrelated, then this residual risk would disappear in the same way that the law of large numbers applies to insurance. Risk associated with individual assets that can be diversified away is called diversifiable, unsystematic, or company-specific risk. Therefore, total risk is equal to company-specific risk plus market risk. This means that the investor should be concerned with both risks, the total risk, if a portfolio contains only a few different assets, and with just market risk if the portfolio is well-diversified. Returning to the case discussed earlier where the investor holds a portfolio that consists of a combination of the risk-free asset and the "market" portfolio M in Figure


PORTFOLIO
σ
mkt
0
5
10
15
20
25
30
35
40
45
50
NUMBER OF SECURTIES

4, this investor holds a well-diversified portfolio and should be concerned only with market risk rather than total risk.

How can this market risk be measured? Consider an investor who holds a well-diversified portfolio and is thinking about adding a new asset to the portfolio. The investor should be concerned with how the new asset's returns vary with the market portfolio's returns. The following simple linear regression can be calculated to measure the new asset's market risk:

$$R_{it} = a_i + b_i R_{mt} + e_{it}, \qquad (3.9)$$

where

a_i and b_i = the regression intercept and slope coefficient,

R_i = the return of asset i,

R_m = the return of the market portfolio,

e_{it} = the residual error at time t, and

t = time.

The regression line slope coefficient, $b_i = \text{Cov}(R_i, R_m)/\text{Var}(R_m)$, measures the time series variation between the asset's return and the market portfolio's return and can be used as a measure of the asset's market risk. Let's now call b_i, beta or β. The market portfolio has a β of one. If an asset's β is greater than one, it means the asset's return tends to go up more than the market when the market rises and decline more when the market return drops.

Returning to the asset allocation choice between the risk-free asset and the market portfolio M, the formula for the expected return of this portfolio is given by rewriting Equation 3.1 as:

$$\text{E}(R_p) = (1 - W)R_f + (W)\text{E}(R_m). \qquad (3.10)$$

Equation 3.10 can be rewritten as

$$\text{E}(R_p) = R_f + W(\text{E}(R_m) - R_f). \qquad (3.11)$$

The second half of Equation 3.11 can be called a market risk premium, which is the return an investor expects to receive above the risk-free rate for investing in the market portfolio. If W is 1 in Equation 3.11, the expected portfolio return equals the expected market return, and the portfolio risk equals the standard deviation of the market portfolio. Figure 4 shows this relationship but still uses total risk as a risk measure although the investor should only be interested in market risk, as measured by β, when looking at adding a new asset to a well-diversified portfolio.

β measures an individual asset's sensitivity to movements in the market portfolio, and $E(R_m) - R_f$ is the excess return demanded on the market portfolio, or market risk premium. The excess return demanded on an individual asset added to a well-diversified portfolio should be $\beta[E(R_m) - R_f]$ which is the asset risk premium. From this relationship, the following formula for the expected or required return for an individual asset in a well-diversified portfolio can be developed:

$$E(R_i) - R_f = \beta_i[E(R_m) - R_f], \tag{3.12}$$

which is the formula for the asset risk premium just explained above. Rewriting Equation 3.12 gives a formula for the expected return on asset i,

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f]. \tag{3.13}$$

Equation 3.13, shown previously as Equation 1.1, is known as the Capital Asset Pricing Model, and it is a single period linear relationship between market risk as denoted by β and expected return. The CAPM was developed by Sharpe [29], Lintner [21], and Mossin [26] independently in the 1960s. The assumptions of the model are:

1. Investors are risk averse diversifiers who try to maximize expected return and minimize risk.

2. Investors are price takers, in that they act as if their trades have no effect on asset prices.

3. Investors have homogeneous or identical expectations about asset expected returns and standard deviations.
4. Investors have no transaction costs or taxes.
5. Investors can borrow or invest at the risk-free rate without any limit.
6. Assets are infinitely divisible.

Equation 3.13 forms a line in Figure 6 known as the Security Market Line (SML). The y-axis in Figure 6 is the expected return and the x-axis is beta. The slope of the SML is the market risk premium, $E(R_m) - R_f$, and the market portfolio has a beta of 1.

In the example in Figure 6, the risk-free rate is 8 percent, and the market risk premium return is 9 percent. This leads to an expected market return, where beta equals 1, of 17 percent, as depicted by the horizontal line at 17 percent. From this graphical relationship, we can find the expected return of any asset as long as we know its beta. For example, assume stock A has a beta of 1.2. From Equation 3.13, stock A's expected return would be:

$$E(R_A) = 8\% + 1.2[9\%] = 18.8\%.$$

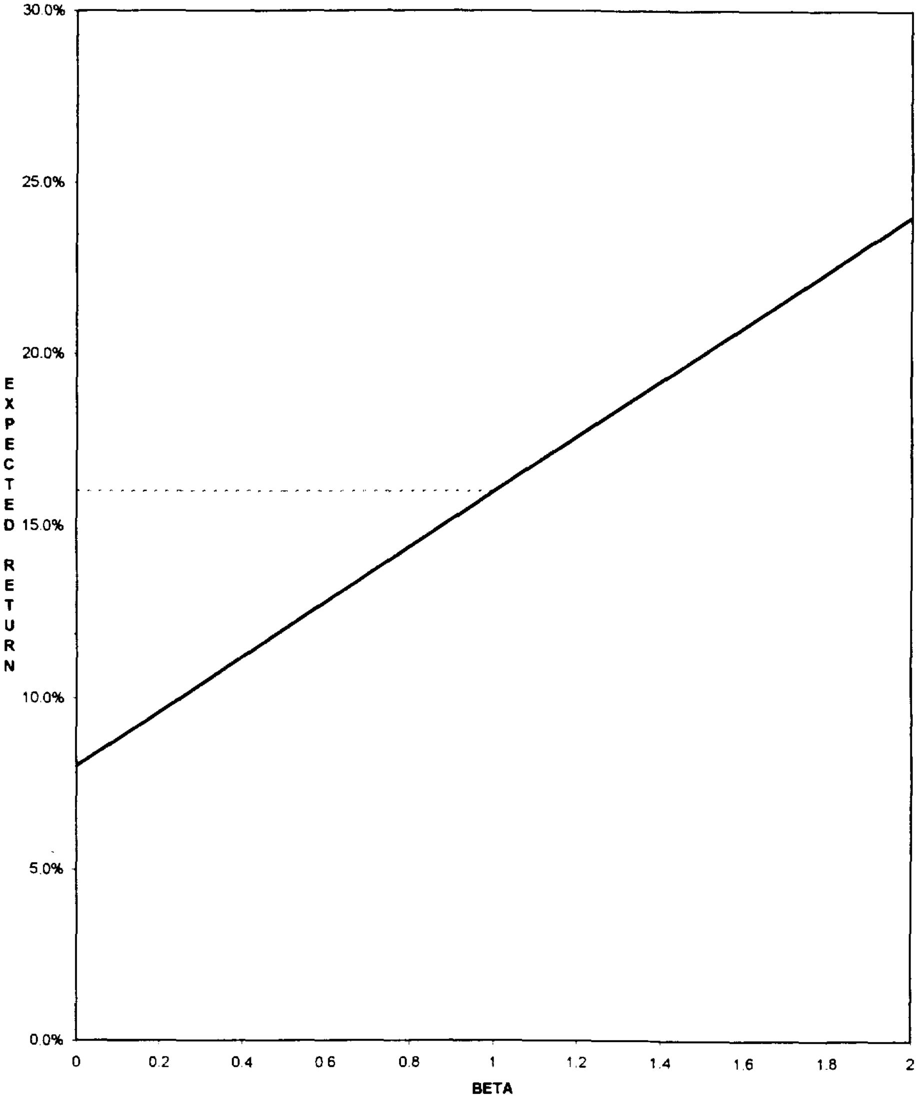
This point is shown on the SML in Figure 6. A stock with a beta of 0.6 would have an expected return of (8% + 0.6[9%]), or 13.4 percent.

An asset with a negative beta is assumed to have returns that move in the opposite direction of the return of the market portfolio. Examples of assets that may have negative betas are gold and gold mining stocks, which tend to have increased returns when the market falls. Continuing with the previous examples, a stock with a beta of −0.4 would have an expected return of (8% + (−0.4)[9%]), or 4.4 percent.

The implication of the CAPM for asset prices is that when an asset price is in equilibrium, its actual expected return equals its expected return as given by the CAPM. If an asset, such as

FIGURE 6

CAPITAL ASSET PRICING MODEL



stock A with a beta of 1.2, has an expected return of 20 percent, higher than its equilibrium expected return of 18.8 percent, then its return should decline to 18.8 percent. In order for stock A's return to go down, its price must go up. Investors, noticing that stock A's return lies above the SML, will put buying pressure on the stock until the price rises to the equilibrium point where its expected return equals 18.8 percent. The opposite would happen if the actual expected return were below the expected return given on the SML.

Some cautions about using the CAPM must be mentioned. The model requires the use of the market risk premium and past market portfolio returns and individual asset returns to arrive at beta estimates for individual assets. This assumes that such relationships are stable, when in fact they are likely to change over time. There has also been much debate in finance literature about what the market portfolio actually is. Most of the debate centers around the fact that the market portfolio in theory consists of all the assets in the world, and its return has never been measured. To use the CAPM in practice, a proxy for the market portfolio is used. Typical market proxies are stock market indices such as the Standard & Poor's 500 stock index, the New York Stock Exchange Composite stock index, the Wilshire 5000 stock index, the Value Line Investment Survey 1700 stock index, the American Stock Exchange Index, or combinations of some of these indices along with bond and real estate indices. Despite the problems with choosing an appropriate market proxy, the CAPM is a model that can easily be applied to many applications and has been applied to insurance.

4. APPLICATION OF THE CAPITAL ASSET PRICING MODEL TO INSURANCE

The Capital Asset Pricing Model as defined in Equation 3.13 has been used to determine insurance underwriting profit margins by Fairley [15], Hill [19], and Hill and Modigliani [20] among others. The basic form of Fairley's CAPM model is given by the following equation:

$$UPM = -kR_f + \beta_u[\mathrm{E}(R_m) - R_f], \tag{4.1}$$

where

k = the funds generating coefficient, and

β_u = the underwriting beta.

In this form of the model, the appropriate underwriting profit margin is equal to the insurer's systematic underwriting risk premium, $\beta_u[\mathrm{E}(R_m) - R_f]$, which is offset by the investment inflow rate of return, $-kR_f$. The underwriting beta is determined by the historical movements of underwriting returns in relation to the market portfolio returns, and can be applied to individual lines of business. The investment inflow rate of return arises because of the time lag between the receipt of premiums by the insurer and the payment of losses and expenses. The funds generating coefficient, k, represents the average time the insurer holds premiums. This model ignores actual insurance company investment performance but assumes insurers will earn the risk-free rate of return. The insurer bears the risk and incurs the gain or loss on any risky investment.

Use of the Fairley CAPM requires an estimate of the underwriting beta and the funds generating coefficient for the company as a whole or for the line of business under consideration. The underwriting beta is frequently estimated by running a simple linear regression of historical underwriting returns against the returns of the market portfolio as described in Equation 3.9. The beta coefficient, the estimate for β_u, from this regression is equal to $\mathrm{Cov}(R_u, R_m)/\mathrm{Var}(R_m)$.

Cummins and Harrington [9] used quarterly underwriting results for insurers to arrive at an empirical β_u estimate that was insignificantly different from zero. Other empirical studies have estimated the beta of insurer liabilities, which is then converted to an underwriting beta. Hill found a liability beta of −0.23 through a regression approach. Fairley used an indirect estimation approach depending on the insurer's financial leverage, the systematic risk of investable assets (the asset beta), and the funds generating coefficient, from which an estimate of −0.21 was found for the liability beta. Fairley then used the relationship of $\beta_u = -k\beta_L$ which yielded a positive underwriting beta of approximately 0.2.

The funds generating coefficient estimate, k, can be given by the insurer's projection of the loss and expense payment pattern expected from the insurer's current exposures. The estimate k would be the weighted average of the length of time expected between the receipt of premium and the payment of losses and expenses among these different exposures. A value of 1 for k would mean an expected time lag of one period between the receipt of premium and payment of losses and expenses, and a value of 0 would mean that losses and expenses are paid as soon as the premiums are received. Fairley found empirical estimates of k for various lines of insurance that ranged from 0.31 and 0.35 for auto property damage and homeowners to 1.60 for both auto bodily injury and workers compensation to a high of 3.74 for medical malpractice.

To illustrate the use of the model, consider the following example. Assume an insurer wants to determine the minimum annual underwriting profit margin to factor into the upcoming year's premiums for homeowners insurance. The company has determined that homeowners coverages consist of three distinct payment pattern groups. Group 1, which represents 30 percent of the homeowners premium, has an expected loss payment pattern of three months or 0.25 years after receipt of premiums. Group 2, which represents 40 percent of the line's premium, has an expected loss payment pattern of 0.5 years after receipt of premiums. Group 3, which represents 30 percent of the line's premium, has an expected payment pattern of 0.75 years after receipt of premiums. The estimate of the funds generating coefficient, k, for the line of business would be:

$$k = .3(0.25) + .4(0.5) + .3(0.75) = 0.5.$$

The insurer has determined the homeowners underwriting beta to be 0.2 based on historical information. The risk-free rate is 6 percent and the market risk premium is 8 percent. The appropriate homeowners insurance underwriting profit margin according to the model in Equation 4.1 would be:

$$UPM = -0.5(6\%) + 0.2[8\%] = -1.4\%.$$

In this example, the underwriting risk premium, 0.2[8%] = 1.6%, is offset by the interest received from the investment of premiums at 6 percent for one-half year to yield a negative underwriting profit margin of 1.4 percent.

Now, assume a lower risk-free rate of 4 percent and a higher underwriting beta of .50. The underwriting profit margin in the above example would be:

$$UPM = -0.5(4\%) + 0.5[8\%] = 2.0\%.$$

In this second example, the lower risk-free rate results in a lower investment rate of return and the higher beta produces a larger underwriting risk premium, yielding a higher indicated underwriting profit margin.

The insurance CAPM described in Equation 4.1 does not include the effects of taxation. The Hill and Modigliani tax version of the insurance CAPM takes into account the corporate taxation of underwriting income and differential tax rates for the assets in an insurer's investment portfolio of tax-exempt bonds, capital gains on stocks and bonds, and corporate dividend income from other non-controlled corporations. The tax version insurance CAPM can be written as the following equation:

$$UPM = -kR_f(1 - T_A)/(1 - T) + \beta_u[E(R_m) - R_f] + (S/P)R_f[T_A/(1 - T)], \tag{4.2}$$

where

$$T_A = \text{the tax rate on investment income,}$$
$$T = \text{the tax rate on underwriting income, and}$$
$$S/P = \text{the insurer's equity to premium ratio.}$$

In the above equation, T_A is the weighted average of the different tax rates on the insurer's investment portfolio.

The first term in Equation 4.2 is the after-tax adjusted risk-free return on the insurer's investment portfolio during the time lag between receipt of premiums and payment of losses. The second term is the underwriting risk premium.

To illustrate the use of the tax version insurance CAPM, consider the following example: the risk-free rate = 6%; the market risk premium = 8%; the underwriting beta = 0.2; the funds generating coefficient = 0.5; the corporate tax rate = 35%; the equity to premium ratio = 1.0; and the insurer invests 30 percent of its investment portfolio in tax exempt bonds, 20 percent in corporate dividend income stocks which are taxed at 30 percent of the corporate tax rate, and 50 percent in investments that are taxed as ordinary taxable income. The investment income tax rate is the weighted average of the tax rates of each investment category and is given by the following:

$$T_A = .3(0\%) + .2(.3)(35\%) + .5(35\%) = 19.6\% \text{ or } .196.$$

The first term above is for the tax exempt bonds, the second term for the corporate dividend income which has an effective tax rate of .3(35%) = 10.5%, and the third term for ordinary income. Given the investment income tax rate, the tax version insurance CAPM yields the following underwriting profit margin:

$$UPM = -0.5(6\%)(1 - .196)/(1 - .35) + 0.2[8\%]$$
$$+ 1.0(6\%)(.196/(1 - .35)] = -0.30\%.$$

Notice this is the same as the first insurance CAPM example but with the addition of taxes; the effect of taxes is to generate a higher underwriting profit margin.

The models that apply the CAPM to insurance have been criticized for ignoring risk unique to insurance that is not systematic with investment risk. Ang and Lai [1] determine that insurance premiums should be based on both systematic insurance risk and systematic investment risk. Turner [30] indicates that the insurance market cannot simply be appended to the stock market. Both studies conclude that CAPM insurance pricing models would underprice insurance. These conclusions are supported in D'Arcy and Garven [10] by the finding that actual underwriting profit margins significantly exceeded the CAPM indications over most of the period from 1926 to 1985. Thus, while it is important to understand the mechanics of both the CAPM and its applications to insurance, this method is not necessarily the appropriate pricing technique.

For example, consider the following situation in which the CAPM is not likely to produce the correct indication. An insurer is pricing earthquake insurance and assumes that the underwriting beta for this coverage is zero, the funds generating coefficient is .4, the risk-free rate is 5 percent, and the market risk premium is 7 percent. The insurer operates at a 2 to 1 premium to equity ratio, has a 35 percent tax rate on underwriting income, and a 15 percent tax rate on investment income. The indicated underwriting profit margin based on the tax version insurance CAPM is:

$$
\begin{aligned}
UPM &= -.4(5\%)(1-.15)/(1-.35) + 0(7\%) \\
&\quad + (1/2)(5\%)(.15/(1-.35)) \\
&= -2.0\%.
\end{aligned}
$$

What factors are not reflected in this calculation that would affect the appropriate underwriting profit margin? The CAPM provides a risk premium only for risk that is systematic with market returns, ignoring catastrophe risk. Also, the CAPM ignores bankruptcy costs. Insurers must be concerned with insurance-specific risk and with bankruptcy. Thus, the CAPM indicated underwriting pricing margin is likely to be too low.

5. DISCOUNTED CASH FLOW ANALYSIS

Discounted cash flow analysis is another foundation of most financial theories and models. Discounted cash flow (DCF) analysis converts cash flows from different times to a common point based on the time value of money so that cash inflows and outflows can be more easily compared. Discounted cash flow analysis is used to value bonds, stocks, and corporate investments in capital projects. DCF can also be useful in insurance where differences in timing between receipt of premiums and payment of losses are common.

The typical DCF analysis is a straightforward calculation that finds the present value of expected future cash flows by discounting these cash flows at the appropriate discount rate. The present values are then summed to determine the value of the investment. The basic concept behind the time value of money is that a dollar in the future is worth less than a dollar today. A dollar today can be invested and earn interest so that more than a dollar will be available in the future, or can be used for current consumption, which is assumed to be worth more than a similar amount of consumption in the future.

To illustrate this approach, consider the following example: an insurer sells a one-year policy for a premium of $1,200 that has $200 of expenses paid concurrently with the receipt of premium and an expected loss of $1,050 that is paid at the end of the year, and the insurer can invest the premium less expenses at 7 percent. The insurer would like to know the gross profit from both underwriting and investment on this policy in today's dollars. This problem can be approached in two ways.

First, the end of period gross profit could be determined by finding the future value of the net premium investment and then subtracting the end of period expected loss. The \$1,000 net premium is invested at 7 percent and the future, or end of period, value is given by the following:

$$FV = PV(1 + r)$$
$$= \$1{,}000(1 + .07) = \$1{,}070, \qquad (5.1)$$

where

FV = the future value,

PV = the present value (which is the premium in this case), and

r = the interest rate.

The end of period gross profit would be \$1,070 – \$1,050 = \$20, but the insurer wants to know what this is worth in today's dollars. This means the present value of the \$20 end of period gross profit must be found. Assuming the discount rate is equal to the insurer's investment rate of 7 percent, the present value can be found by solving for PV in Equation 5.1:

$$PV = FV/(1 + r). \qquad (5.2)$$

The present value of the future profit of \$20 is:

$$PV = \$20/(1 + .07) = \$18.69.$$

A second and more direct approach to find the present value of the policy's gross profit is to subtract the present value of the expected loss from the premium:

$$PV\text{ (gross profit)} = \$1{,}000 - \$1{,}050/(1 + .07)$$
$$= \$1{,}000 - \$981.31 = \$18.69.$$

Again, the present value of the policy's gross profit is found to be \$18.69.

More general versions for the future value and present value equations with a time span of more than one year are given in Equations 5.3 and 5.4:

$$FV_t = PV(1 + r)^t, \tag{5.3}$$

$$PV = FV_t/(1 + r)^t, \tag{5.4}$$

where

FV_t = the future value at time t,

PV = the present value at time 0, and

r = the interest (discount) rate.

To illustrate the use of the above formulae, consider the following examples.

EXAMPLE 1 An investor places \$500 today in an account paying 8 percent annually for three years. How much would the investor have in the account at the end of three years?

$$FV_3 = \$500(1.08)^3 = \$629.86.$$

The investor will have \$629.86 in the account at the end of three years.

EXAMPLE 2 An insurer expects to make a loss payment of \$5,000 five years from now and has a discount rate of 9 percent; the insurer wants to know the present value of this future payment:

$$PV = \$5{,}000/(1.09)^5 = \$3{,}249.66.$$

The present value of the expected \$5,000 payment in five years is \$3,249.66.

DCF analysis can also be used to find the present value of multiple cash flows, as illustrated by examples valuing bonds, corporate investment projects, and stocks.

For example, assume a bond matures in two years and pays annual interest of \$100 per year, with the next payment occurring in one year and the last payment occurring in two years. In addition to the interest payments, the bond has a maturity value of \$1,000 also payable in two years. If the appropriate rate of return on this bond is 8 percent, the value of the bond can be determined according to the following formula:

$$V = \Sigma_t\{CF_t/(1+r)^t\}, \tag{5.5}$$

where

$$V = \text{the value of the investment,}$$

$$CF_t = \text{the cash flow at time } t, \text{ and}$$

$$r = \text{the discount rate.}$$

For the above bond example, Equation 5.5 is as follows:

$$V = \$100/(1.08) + \$100/(1.08)^2 + \$1000/(1.08)^2$$

$$= \$92.59 + \$85.73 + \$857.34 = \$1,035.66.$$

The present value of the cash flows discounted at 8 percent from the bond is \$1,035.66. A use for this technique is to determine the appropriate price to pay for an investment. If the investor requires an 8 percent return to make the above investment desirable, then the maximum purchase price that would be paid to obtain these cash flows is \$1,035.66. Any higher price would generate a return less than 8 percent.

A variation of the formula in Equation 5.5, which includes a cash flow at time zero, can be used to help corporate managers determine whether to invest in a given project

$$NPV = CF_0 + \Sigma_t\{CF_t/(1+r)^t\}. \tag{5.6}$$

Equation 5.6 used in this corporate capital budgeting environment is called the net present value (*NPV*) of the project. To calculate the *NPV* of an investment, the manager simply needs the estimates of the future cash flows from the investment, the estimated cost of the investment, and the required return or discount rate demanded by the firm on this type of investment. If the *NPV* is positive, the present value of the expected cash inflows is greater than the expected cost of the investment, and the project would be profitable to invest in, assuming the projected cash inflows turned out to be correct. A negative *NPV* means that the estimated costs of the investment exceed the present value of the expected cash inflows, and such a project would be considered unacceptable for investment purposes.

Consider the following example for net present value analysis.

NPV ANALYSIS : $r = 15\%$

Period	Cash Flow
0	−$10,000
1	$4,000
2	$5,000
3	$4,000
4	$2,000
5	$1,000

The *NPV* from Equation 5.6 for this sample would be written as:

$$NPV = -\$10{,}000 + \$4{,}000/(1.15) + \$5{,}000/(1.15)^2$$

$$+ \$4{,}000/(1.15)^3 + \$2{,}000/(1.15)^4 + \$1{,}000/(1.15)^5$$

$$= -\$10{,}000 + \$3{,}478 + \$3{,}781 + \$2{,}630 + \$1{,}144 + \$497$$

$$= \$1{,}530.$$

The project in this example has a positive *NPV* of $1,530, which means it is an acceptable project for investment purposes.

Another application of the discounted cash flow analysis on the same project evaluation is the internal rate of return (*IRR*) method. The *IRR* is simply the discount rate that gives the project a net present value of zero. Equation 5.7 is the formula for the internal rate of return:

$$NPV = CF_0 + \Sigma_t\{CF_t/(1 + IRR)^t\} = 0. \qquad (5.7)$$

The *IRR* is found by trial and error or by computer programs that iterate to find the appropriate rate. The decision rule for the *IRR* approach is as follows: if the *IRR* is greater than the required rate of return for the project, then accept the project; if the *IRR* is less than the required rate of return, the project is rejected. The *IRR* for the previous example is 22.63 percent. The *NPV* approach with a discount rate of 22.63 percent is used below to prove the *IRR* result

$$\begin{aligned} NPV = & -10{,}000 + 4{,}000/(1.2263) + 5{,}000/(1.2263)^2 \\ & + 4{,}000/(1.2263)^3 + 2{,}000/(1.2263)^4 + 1{,}000/(1.2263)^5 \\ = & -10{,}000 + 3{,}262 + 3{,}324 + 2{,}169 + 884 + 361 \\ = & \ 0. \end{aligned}$$

For typical projects, the *NPV* and *IRR* will always provide the same decision.

However, problems exist with the *IRR* method. If the cash flows change sign (positive to negative or vice-versa) more than once, then multiple *IRR*s can occur. For example, a project may require an initial investment (negative cash flow), then generate a positive cash flow, and then negative cash flows. In this case, one *IRR* may be negative and another one very high, but usually only one *IRR* appears reasonable. For example, consider a project with the following cash flows:

Period	Cash Flow
0	−\$5,000
1	\$5,000
2	\$4,000
3	−\$3,000
4	\$2,000
5	−\$1,000

This project has a *NPV* at 15 percent of \$1,046; however, the project has two *IRR* values: −46.9 percent and 36.4 percent. Since the project has a positive *NPV* at 15 percent, the latter *IRR* of 36.4 percent must be the reasonable value for the internal rate of return.

Stock Valuation

Another application of discounted cash flow analysis is stock valuation. The value of a stock can be thought of as the present value of its future cash flows, similar to the earlier bond valuation example. The relevant cash flows for a stock are its expected future cash dividends. The valuation is a little more difficult for stocks because they have no maturity value. One stock valuation model is the Gordon growth model. It assumes the following present value of expected dividends model:

$$V = \Sigma_t \{D_t/(1 + r_s)^t\}, \qquad (5.8)$$

where

D_t = the dividend expected at time t, and

r_s = the required return on stock.

The Gordon model is a specialized version of the model in Equation 5.5 that assumes a constant annual growth rate in dividends, which causes Equation 5.5 to reduce to the following:

$$V = D_0(1 + g)/(r_s - g), \qquad (5.9)$$

where

D_0 = the current dividend paid,

g = the constant dividend growth rate, and

r_s = the required return on stock.

The model in Equation 5.9 cannot be used if the growth rate is greater than or equal to the required stock return rate.

To illustrate the use of Gordon's stock valuation model, consider the following example. Stock A currently pays a dividend of \$3 per share and has a required rate of return of 17 percent. Stock A's dividend is expected to grow at a constant rate of 9 percent annually. Equation 5.9 can be used in this case and would be written as follows:

$$V = \$3(1.09)/(.17 - .09) = \$40.875.$$

The value of \$40.875 given by the model for Stock A is called the stock's intrinsic value. An investor could compare the model price to the actual market price for Stock A and decide whether to buy or sell the stock.

A word of caution about using stock valuation models. The model is only as good as the estimates used in it. It is quite difficult to arrive at accurate estimates of future growth rates in dividends.

6. DISCOUNTED CASH FLOW MODELS APPLIED TO INSURANCE

Two basic methods of applying discounted cash flow analysis to insurance have developed. One, termed the Risk Adjusted Discount Technique, analyzes the cash flows from the point of view of the policyholder, and was first applied at the 1982 Massachusetts automobile rate hearings [27]. The other approach, used by the NCCI, is an internal rate of return calculation. Cummins [7] explains and compares these two approaches. Derrig [13] explains how the Risk Adjusted Discount Technique has been used in Massachusetts to set automobile and workers compensation rates and discusses the key issues in selecting parameter values. In retrospect, as cited by Derrig, the factor most responsible for underpricing these coverages has been the underestimation of losses and expenses, rather than the choice of financial model or value. The material presented here does not attempt to duplicate the specific approach in Massachusetts, but does apply the same general technique. Although the calculations seek to determine an appropriate premium rather than an underwriting profit margin, the underwriting profit margin can be calculated in the conventional manner after the premium is determined.

The basic premise of the Risk Adjusted Discount Technique is that, on a risk adjusted basis, the present value of the premium equals the present value of all the cash flows resulting from writing an insurance policy. Specifically, the present value of the premium equals the sum of the present values of the losses, expenses, and taxes on both underwriting and investment income, generated by the contract. For an explanation of the importance of considering taxes post-Tax Reform Act of 1986, see Derrig [12]. The term "risk adjusted" means that the interest rate selected to discount cash flows varies to account for the degree of risk inherent in the cash flow: a risky cash flow will be discounted at a different rate than a certain cash flow.

To illustrate this concept, assume that an insurer is trying to set a premium level for a one year policy. The premiums will be collected when the policy is effective. Expenses on the policy are $20 and will be paid when the policy is written. Losses on the policy are expected to be $80, and will be paid at the end of the year. (Assume, for example, that the average loss will occur halfway through the coverage period and there will be a six month lag in paying the claim.) The insurer will incur taxes on the underwriting profit (or a tax reduction on an underwriting loss) at the 35 percent level. The insurer will earn investment income on the premium less the expenses paid, and on the surplus, or equity, devoted to this policy. The insurer will assign $50 of equity to support writing this policy. In this case, the insurer pays the same 35 percent tax rate on investment income as on underwriting income. All taxes will be paid at the end of the year. In this first example, risk will be ignored and all cash flows will be discounted at the same interest rate of 7 percent.

The general format of the discounting approach is quantified as follows:

$$PV(P) = PV(L) + PV(E) + PV(TUW) + PV(TII), \qquad (6.1)$$

where

PV = present value operator,

P = premiums,

L = losses and loss adjustment expenses,

E = underwriting expenses,

TUW = taxes on underwriting profit or loss,

TII = taxes on investment income, and

UPM = underwriting profit margin.

For the first example, the calculation becomes:

$$P = \frac{80}{1.07} + 20 + \frac{(P - 20 - 80)(.35)}{1.07} + \frac{(50 + P - 20)(.07)(.35)}{1.07}$$

$$P = 74.766 + 20 + .327P - 6.542 - 26.168 + .687 + .023P$$

$$.65P = 62.743$$

$$P = \$96.53$$

$$UPM = 1 - \frac{80}{96.53} - \frac{20}{96.53} = -3.59\%.$$

In the first case, the premium is $96.53 for an underwriting profit margin of negative 3.59 percent. This represents the

TABLE 1

SUMMARY OF NOMINAL AND DISCOUNTED VALUES
EXAMPLE 1

	Nominal Values	Discounted Values
Losses	$ 80.00	$74.77
Expenses	20.00	20.00
Taxes on Underwriting	– 1.21	– 1.14
Taxes on Investments	3.10	2.90
Total	$101.89	$96.53*

*Premium = Sum of the Discounted Value of Losses, Expenses, and Taxes

present value of the losses ($80/1.07), the expenses ($20), the tax reduction on the underwriting loss ([P – 100][.35]/1.07), and the tax on the equity and premiums, less expenses, invested at interest for one year ([50 + P – 20][.07][.35]/1.07). The nominal and discounted values from Example 1 are shown on Table 1. Note that an underwriting loss occurs and the tax on this underwriting loss is negative, representing a cash inflow or an offset to other taxes. Since investment income is positive, the tax on investment income is also positive, raising the required premium. This calculation demonstrates discounting, and the various cash flows generated by writing an insurance policy. It does not represent risk adjusted discounting, though, which will be introduced in the next example.

Example 2 will recognize that some of the cash flows from the insurance contract are risky. Specifically, losses will vary around the expected value. Since risk is involved, it is not reasonable to discount them at what was, in Example 1, a risk-free rate. However, the premium income is certain once the policy is written and the underwriting expenses can be assumed to be known. Taxes emanating from these certain cash flows can also be assumed to be risk-free. However, a critical problem rests with how to determine an appropriate risk adjusted discount rate. One approach is outlined below.

The insurance company is assuming the risk of guaranteeing to pay losses for the insured. The insurer should not be expected to place its capital at risk without compensation. In Example 1, where all cash flows were discounted at the risk-free rate, the insurer would be better off investing the equity directly in financial markets and not assuming the risk involved in paying claims. Thus, discounting the risky cash flows at an interest rate below the risk-free rate represents a form of compensation to the insurer for placing its capital at risk in the insurance contract.

Conversely, the policyholder in an insurance contract is receiving a guarantee from the insurer to pay claims. The guarantee represents a value to the policyholder. Thus, much in the manner that a life insurance policyholder is willing to accept a guaranteed interest rate below the market interest rate, a property/liability insurance policyholder is willing to accept a lower interest rate on the risky cash flows relating to that insurance policy. Another way to view this issue is on a CAPM basis. The insurance policy represents an asset with a negative beta because it has value when the policyholder's tangible assets are reduced in value. The required return on a negative beta asset is below the risk-free rate. The problem becomes, though, the determination of an appropriate risk adjusted discount rate.

For Example 2 we will sidestep that thorny issue and select a discount rate of 4 percent for the risky loss payment cash flow, but maintain the 7 percent discount rate for the risk-free cash flows. The calculation for Example 2 becomes:

$$P = \frac{80}{1.04} + 20 + \frac{(P-20)(.35)}{1.07} - \frac{80(.35)}{1.04} + \frac{(50+P-20)(.07)(.35)}{1.07}$$

$$P = 76.923 + 20 + .327P - 6.542 - 26.923 + .687 + .023P$$

$$P = 64.145/.65 = \$98.68$$

$$UPM = 1 - \frac{80}{98.68} - \frac{20}{98.68} = -1.34\%.$$

The effect of discounting loss payments at a risk adjusted rate is to increase the appropriate premium level and reduce the underwriting loss. The increase in the value of discounted losses (76.923 versus 74.767) is partially offset by the increased reduction in taxes generated by the losses (26.923 versus 26.168). The higher the tax rate, the less the overall effect of a lower risk adjusted discount rate would be.

Reflecting a more realistic loss payment pattern makes the determination a bit more complex. For Example 3, assume that the losses will still total \$80, but half will be paid after one year and the other half after two years. Now we have to address the issue of how long equity should be allocated to a given policy. Conventional insurance accounting deals with premium to surplus ratios as if surplus were necessary only to support writing policies. However, it is not the writing of policies that requires a surplus, but the assumption of the obligation to pay claims. Surplus, or equity, is required in the event that claims exceed the expected values so that the insurer can absorb the excess without defaulting on the commitment to pay claims. Thus, equity should not be released as soon as the premium is written, or even earned, but more realistically should continue to be allocated to a given policy until the obligation to pay claims is extinguished, that is, when all losses are settled. In Example 3, the equity devoted to this policy will be released in proportion to the payment of losses. Thus, the full \$50 of equity will be invested for the first year of the policy, but only \$25 will be invested during the second year because one-half of the losses have already been settled. Similarly, the full premium, less expenses, is available for investment the first year, but for the second year the premium less expenses and losses paid in the first year is available to invest.

Another complication is the calculation of the taxes on underwriting income. The Tax Reform Act of 1986 requires discounting of loss reserves based on a five year moving average of mid-maturity U.S. government obligations. Insurers use either industry or company loss payment patterns to discount outstanding reserves. For this example, the company pattern will be used. The interest rate required for discounting bears no relationship to rates actually earned by the insurer and, since the required rate is based on a five year moving average, the required rate may not even be available to the insurer. When interest rates have been rising, the required discount rate may be below the current risk-free rate. At other times the required rate will exceed the risk-free rate. Since the mid-maturity rate is based on three to nine year maturities for U.S. government obligations, in normal times this rate will be slightly above the rate for short term U.S. bonds on which the risk-free rate is frequently based. Thus, in this example, the outstanding reserve will be discounted at a rate of 1 percent above the risk-free rate, or at 8 percent. In determining the tax on underwriting income, the incurred losses in the first year are reduced to reflect the discount at the mid-maturity interest rate. In the second year, the incurred losses equal the difference between the paid losses and the initial, discounted, loss reserve. The Tax Reform Act of 1986 also reduces the unearned premium reserve deduction by 20 percent to reflect the timing difference between earning premiums and paying expenses. This adjustment does not affect these examples, as the premium is considered fully earned at the end of the year.

The calculation for Example 3 is:

$$P = \frac{40}{1.04} + \frac{40}{(1.04)^2} + 20$$

$$+ \frac{(P-20)(.35)}{1.07} - \frac{(40+40/1.08)(.35)}{1.04}$$

$$- \frac{(40-40/1.08)(.35)}{(1.04)^2} + \frac{(50+P-20)(.07)(.35)}{1.07}$$

$$+ \frac{(50(.5)+P-20-40)(.07)(.35)}{(1.07)^2}$$

$$P = 38.462 + 36.982 + 20 + .327P - 6.542 - 25.926 - .959$$
$$+ .687 + .023P - .749 + .021P$$

$$P = 61.955/.629 = \$98.50$$

$$UPM = 1 - \frac{80}{98.50} - \frac{20}{98.50} = -1.52\%.$$

In this case, the delay in claim payments decreases the premium level and increases the underwriting loss. The present value of the loss payments declines, but this decline is partly offset by an increase in taxes on investment income.

The prior examples assumed that the expenses were paid when the premium was received, which is a common assumption in insurance ratemaking. Realistically, however, many expenses are incurred well before the premium is collected. The work involved in setting premium levels is done years before the premium is actually collected. Computer systems, underwriting guidelines, contract language, advertising, and many other aspects of an insurance transaction are developed well before a given policy is written. The expenses associated with training staff are incurred before the work for which they are trained is actually performed. Although some expenses are contemporaneous with the receipt of premium, primarily commissions, premium taxes, underwriting inspection reports, and clerical policy insurance expenses, other expenses are paid before the policy is written. To reflect the prepayment of some expenses, Example 4 is calculated on the basis that \$10 of expenses was paid two years before the premium was collected and \$10 was paid when the policy was written. For simplicity it will be assumed that the insurer is content to earn the risk-free rate on the prepaid expenses, although a higher rate may be more reasonably expected, as some prepaid expenses may not be recovered by future policy writings. This calculation is:

$$P = \frac{40}{1.04} + \frac{40}{(1.04)^2} + 10(1.07)^2 + 10$$

$$+ \frac{(P - 10(1.07^2) - 10)(.35)}{1.07} - \frac{(40 + 40/1.08)(.35)}{1.04}$$

$$- \frac{(40 - 40/1.08)(.35)}{(1.04)^2} + \frac{(50 + P - 20)(.07)(.35)}{1.07}$$

$$+ \frac{(50(.5) + P - 20 - 40)(.07)(.35)}{(1.07)^2}$$

$$P = 38.462 + 36.982 + 11.449 + 10 + .327P - 7.016$$

$$- 25.926 - .959 + .687 + .023P - .749 + .021P$$

$$P = 62.930/.629 = \$100.05$$

$$UPM = 1 - \frac{80}{100.05} - \frac{20}{100.05} = 0.05\%.$$

The prepayment of expenses increases the indicated premium level. Failure to reflect the fact that many expenses are actually expended before the premium is received leads to an understating of the premiums determined by the risk adjusted discounted cash flow models.

The risk adjusted discounted cash flow models are often adjusted to reflect the fact that premiums are not received at the inception of the policy term. The method developed by the Insurance Services Office, termed the ISO State X calculation, includes this adjustment. These delays may be several months, especially if an agent is given a certain amount of time before being expected to submit the premiums. However, if a representative of the company, such as an agent, has collected the premiums but not remitted them to the insurer, it is incorrect to reflect this delay by discounting the premiums for this lag. This delay reflects a form of agent compensation and should be reflected as an expense rather than as a discounted premium. If the policyholder has paid the premiums, then the insurance rates should not be increased because the insurer has not invested the funds.

As most insurance policies include grace periods, though, it is not unusual for premiums to be submitted after the coverage is in effect. Thus, reflecting the lag in collecting premiums is proper in these circumstances. To illustrate this effect, Example 5 assumes that premiums are paid, either to an agent or the company, one month after policy inception. The calculation becomes:

$$\frac{P}{(1.07)^{1/12}} = \frac{40}{1.04} + \frac{40}{(1.04)^2} + 10(1.07)^2 + 10$$

$$+ \frac{(P - 10(1.07^2) - 10)(.35)}{1.07} - \frac{(40 + 40/1.08)(.35)}{1.04}$$

$$- \frac{(40 - 40/1.08)(.35)}{(1.04)^2} + \frac{(50 + P - 20)(.07)(.35)}{1.07}$$

$$+ \frac{(50(.5) + P - 20 - 40)(.07)(.35)}{(1.07)^2}$$

$$.994P = 38.462 + 36.982 + 11.449 + 10 + .327P - 7.016$$

$$- 25.926 - .959 + .687 + .023P - .749 + .021P$$

$$P = 62.930/.623 = \$101.01$$

$$UPM = 1 - \frac{80}{101.01} - \frac{20}{101.01} = 1.00\%.$$

The effect of assuming a one month delay in the policyholders' payment of premiums is to increase the indicated premium level by one percentage point. In this example, the delay of premium payment generates a positive underwriting profit margin for the insurer. This is not, in itself, a more favorable financial position for the insurer than the prior underwriting loss or breakeven indications. In all cases, premiums simply equal the risk adjusted cash flows emanating from writing the policy. The underwriting profit margin is irrelevant to this method and is shown here only as a frame of reference with traditional insurance accounting conventions.

The general formula for the Risk Adjusted Discount Technique, as illustrated by the above examples, can be written as:

$$P\sum_{i=0}^{N}\frac{a_i}{(1+R_f)^i} = L\sum_{i=0}^{N}\frac{b_i}{(1+R_L)^i} + E\sum_{i=-M}^{N}\frac{c_i}{(1+R_f)^i}$$
$$+\frac{\left(P - E\sum_{i=-M}^{N}\frac{c_i}{(1+R_f)^i}\right)t}{1+R_f}$$
$$-Lt\left(\frac{\sum_{i=1}^{N}\frac{b_i}{(1+R_T)^{i-1}}}{1+R_L} + \sum_{j=2}^{N}\frac{\sum_{i=j}^{N}\frac{R_T b_i}{(1+R_T)^{i-j+1}}}{(1+R_L)^j}\right)$$
$$+R_f t\left(\sum_{j=1}^{N}\left[\frac{S\left(\sum_{i=j}^{N} b_i\right) + P - E - L\sum_{i=0}^{j-1} b_i}{(1+R_f)^j}\right]\right), \tag{6.2}$$

where

a_i = fraction of premium received in time period i,

b_i = fraction of losses paid in time period i,

c_i = fraction of expenses paid in time period i,

S = owners' equity in insurer,

P = premiums,

L = losses and loss adjustment expenses,

E = underwriting expenses,

t = tax rate,

R_T = discount rate required for tax purposes,

R_f = risk-free rate,

R_L = risk adjusted rate for losses,

M = number of time periods before policy effective date that the first prepaid expenses are paid, and

N = number of time periods after policy effective date that the last loss payment is made.

Equation 6.2 applies the same methodology as Equation 6.1. The present value of premiums is set equal to the sum of the present values of losses, expenses, and taxes. In this formulation, expenses are allowed to be paid before the policy is written. Taxes on underwriting income are based on the provisions of the Tax Reform Act of 1986, in which outstanding reserves each year are discounted at a mandated rate and losses are incurred for tax purposes each year reflecting the loss payments compared to the discounted reserves and the fact that as time elapses, outstanding reserves are discounted for shorter periods of time. The tax on investment income reflects an equity allocation based on the percent of losses that are still unpaid.

In this example, expenses are listed as known values that do not depend on premiums. Some, but not all, expenses could more properly be stated as a percentage of premiums. Commissions and premium taxes tend to be percentages of premiums, and could be shown accordingly in Equation 6.2. Other expenses, such as administration, systems development, employee training, underwriting, and overhead may not depend on the premium level and should be treated as given values in the same way that losses and equity are independent of the premium level. For example, if an increase in taxes results in a higher premium loading, it is not appropriate to increase the full expense loading proportionally, as many expenses will not change.

This formula has other shortcomings as well. The present value of premiums is determined based on the risk-free rate. However, the lag in premium collection is not equivalent to an investment in a risk-free security. Some premiums are never paid, and the insurer is forced to cancel coverage. Other premiums are paid within the grace period, but only after losses have occurred; those that do not have losses simply do not pay the premiums, in essence obtaining free insurance protection for which the insurer must pass along the cost to other insureds.

The shortcomings described above relating to expenses proportional to premiums and a risk adjustment for premium delays could be accounted for by revising the formula to reflect these items. However, more serious drawbacks to the Risk Adjusted Discount Technique exist that cannot be so easily corrected. One major problem is the proper determination of the risk adjusted discount rate. In the examples, this rate was set at 4 percent. However, no widely accepted approach for setting this rate has yet been determined. In the original development of this technique, Myers and Cohn use the Capital Asset Pricing Model to determine the appropriate rate. Recent research suggests that the CAPM does not provide a large enough risk margin for insurance transactions. Also, research in finance has raised serious questions about the validity of the CAPM to investment returns in general. Not knowing how to select the appropriate risk adjusted rate is a serious flaw in this technique.

Another major problem relates to the allocation of equity to policies. Since the taxes incurred on investment income allocated to equity supporting a given line are included in the premium determination, knowing how much and how long equity is allocated are of critical importance. The traditional consideration of premium to surplus measures is inappropriate because, as described earlier, surplus is needed to protect against losses exceeding expected values. Thus, at least some equity must continue to be committed to a policy until all the losses are paid. In the approach outlined above, equity is released proportionately with loss payments. As expense payments have very little risk of exceeding expected values, only loss payments are considered in releasing equity.

A key decision in the Risk Adjusted Discount Technique is how much equity should be allocated to a given policy. The allocation may consider such items as the degree of variability in losses, the length of time loss payments will be made, covariability among different lines of insurance, or other factors. Perhaps an insurer should be required to maintain a higher level of equity for property coverages during tornado and hurricane seasons than at other times. A stop loss reinsurance contract would reduce the need for equity for a covered line. Liability lines may need additional equity in times of judicial instability, more than when the doctrine of *stare decisis* is likely to be applied. A consensus on the proper equity determination has not yet been reached. The approach applied in the examples, in which equity is predetermined, perhaps in proportion to expected losses but not as a function of premiums, is reasonable, but is not the only approach.

Another serious drawback to the Risk Adjusted Discount Technique is the fact that it considers only one policy term. The profitability of insurance policies depends on how many renewal cycles the policy has been through. New business tends to be unprofitable, but long term business becomes increasingly profitable. This tendency is termed the aging phenomenon and appears to occur for all insurers and for all lines of business. Thus, in determining a proper premium level, the aging phenomenon should be recognized. The cash flows emanating not only from the current policy but also from future renewals of the policy should be considered. This would be a multidimensional risk adjusted discounting approach that has not yet been developed.

In summary, the Risk Adjusted Discount Technique establishes the premium level for a policy by equating the present value of premiums to the present value of losses, expenses, and taxes on both underwriting and investment income. If appropriate values for the cash flows and the discount rates could be determined, this approach should generate valid premium levels. The technique is illustrated by simplistic examples. More realistic examples become increasingly complex, but still follow the same logic. The major difficulties in applying the Risk Adjusted Discount Technique revolve around selecting the appropriate discount rate and the equity allocation. Unless these values are correct, the premium levels resulting from this approach will not be valid.

7. OPTION PRICING

Option Mechanics

An option is termed a derivative security, one that derives its value based on the price of another asset. Typical options are traded on stocks, bonds, commodities, and stock indices. The owner of an option has the right to trade the underlying asset at a specified price by or on a given date. However, the owner does not have to exercise this right. Two types of options exist. A call option gives the owner the right to buy the asset at the specified price, which is called the strike or exercise price. A put option gives the owner the right to sell the asset. The seller of the option, called the writer of the option, has the obligation to sell (in the case of a call option) or to buy (in the case of a put option) the underlying asset at the exercise price if the buyer elects to exercise the option.

Options are also classified according to when they can be exercised. A European option can only be exercised on the expiration date. An American option can be exercised at any time up until expiration.

To illustrate how options work, let us examine the decision of the owners of European options on the expiration date. Assume an investor owns a call option on ABC stock with an exercise price of \$50, and on the expiration date ABC stock sells for \$60. The owner has to decide whether the call option should be exercised. If the option is exercised, the investor would pay \$50 per share for ABC stock which currently sells for \$60 per share. Once the investor exercises the call option for \$50 per share, he or she can sell the stock for \$60 a share, thereby making a profit of \$10 per share. Alternatively, the investor can simply keep the stock that was purchased for a bargain price. If the investor did not exercise the call option, the option would expire worthless, and the investor would receive nothing. Obviously, the call option owner should exercise the option in this case.

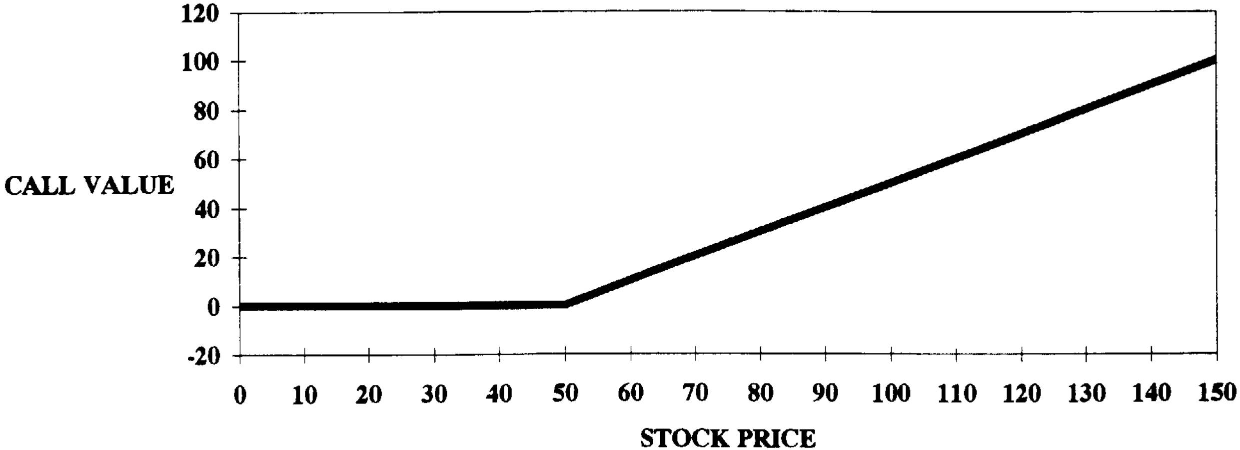
Now, using the same \$50 exercise price call option example, assume that the price of ABC stock is \$40 per share on the expiration date. If the call option owner exercised the option in this case, the owner would pay \$50 per share for a stock that could be purchased for only \$40 per share. If the call holder wanted to own ABC stock, the holder should let the option expire worthless and buy the stock for \$40 per share. The value at expiration of the call option in this situation would be zero.

From these two simple call option examples, a pattern emerges. The owner of a European call option should exercise the option if the underlying stock or asset price is greater than the exercise price at expiration. The value at expiration of the call option is the higher of the stock price minus the exercise price or zero. This payoff can be seen in Figure 7 for our ABC stock example and expressed generally in the following formula:

$$C = \max[S - X, 0], \tag{7.1}$$

FIGURE 7

CALL OPTION PAYOFF AT EXPIRATION



where

C = the value of the call option at expiration,

S = the price of the underlying asset, and

X = the exercise price of the option.

For the first example where the stock price was \$60 and the exercise price was \$50, Equation 7.1 would be:

$$C = \max[60 - 50, 0] = \$10.$$

Equation 7.1 for the second example where the stock price was \$40 and the exercise price was \$50 would be:

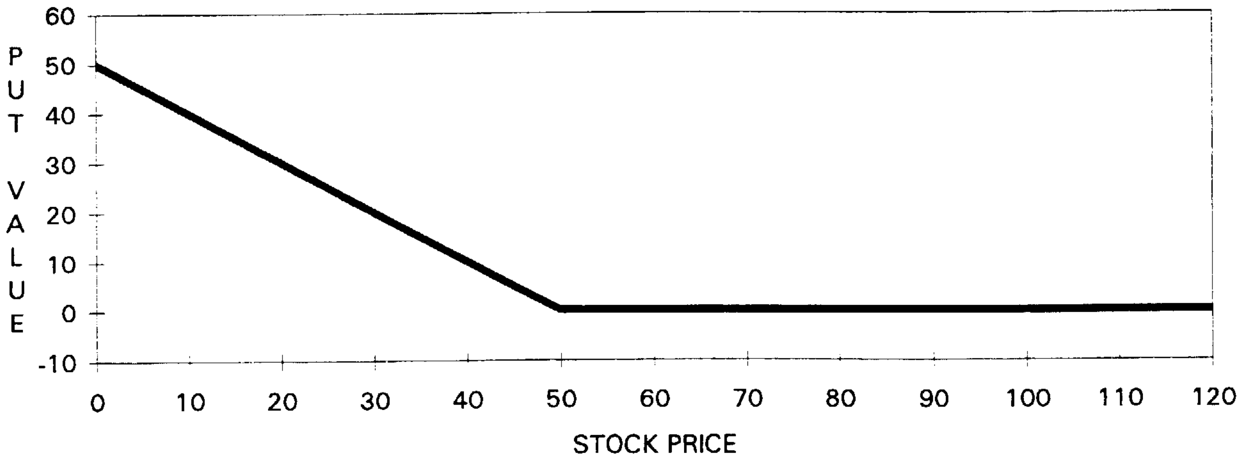
$$C = \max[40 - 50, 0] = 0.$$

Now consider the owner of a put option on ABC stock with an exercise price of \$50. At expiration, the price of ABC stock is \$35, for example. If the owner of the put option exercises the option, the put owner must first acquire the stock in order to sell the stock to the writer of the put option. So, the put owner buys ABC stock for \$35 per share in the market and completes the transaction by exercising the option to sell the stock for \$50 per share, which nets the option holder \$15 per share. Alternatively, if the put holder already owned ABC stock, the stock could be sold to the put writer for the higher price of \$50 per share, rather than the market price of \$35. However, if the price of the stock were \$55 per share at expiration, the put option owner would not exercise the option to sell the stock for \$50 per share even if the holder already owned the stock. Therefore, the decision rule for an owner of a put option is to exercise the option at expiration only if the stock price is less than the exercise price. The payoff at expiration for a put option is the larger of the exercise price minus the price of the underlying asset or zero. The payoff for our put option example is represented graphically in Figure 8 and given by the following general formula:

$$P = \max[X - S, 0], \tag{7.2}$$

FIGURE 8

PUT OPTION PAYOFF AT EXPIRATION



where

P = the value of a put option at expiration,

X = the exercise price of the option, and

S = the price of the underlying asset.

Verifying the first put option example where the stock price was \$35 and the exercise price was \$50, Equation 7.2 would be:

$$P = \max[50 - 35, 0] = \$15.$$

The second example where the stock price was \$55 and the exercise price was \$50 yields the following for Equation 7.2:

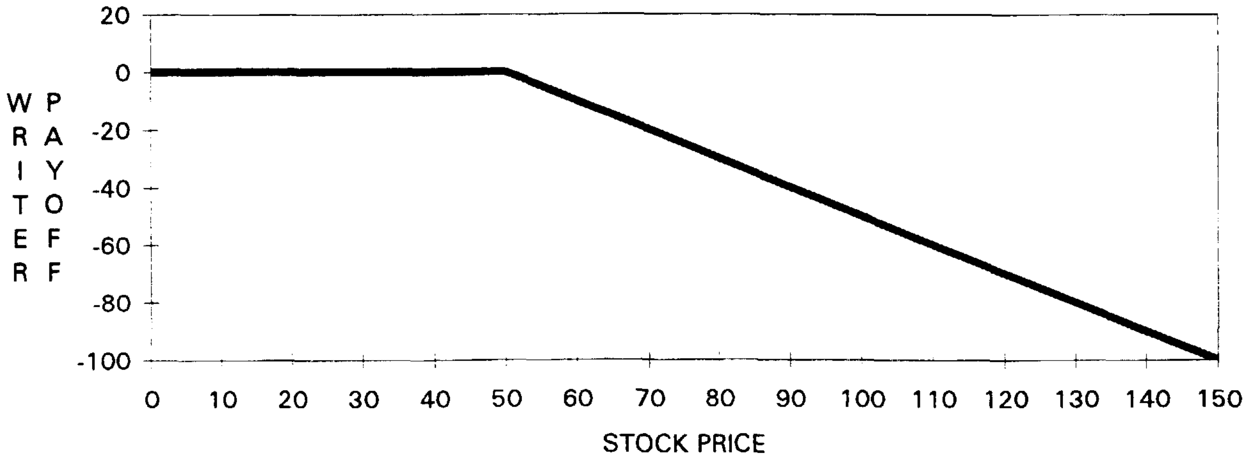
$$P = \max[50 - 55, 0] = 0.$$

One may wonder what happens to the writer of these options while all this action occurs at expiration. Going back to the first call option example, where the stock price was \$60 and the exercise price was \$50, the writer of this call option has to sell the stock to the owner of the call option who is exercising the option to buy ABC stock for \$50 per share. This means the writer of the call has to buy the stock if he or she does not own the stock already. In this situation the call writer has to buy the stock for \$60 and then sell it to the owner of the call for \$50 per share, incurring a loss of \$10 per share. In the second call option example, where the price of the stock was \$40 and the exercise price was \$50, the holder of the call would not exercise the option, which means the writer of the call would have a payoff of zero at expiration. In option terminology, even though the writer's expiration value is zero or negative, this expiration value is still called a payoff. The payoff to the writer of a call option at expiration can be expressed by the following equations and graphically in Figure 9:

$$W_c = \min[-(S - X), 0] \tag{7.3}$$

FIGURE 9

CALL WRITER PAYOFF AT EXPIRATION



or

$$W_c = \min[X - S, 0], \tag{7.4}$$

where

W_c = payoff at expiration to call option writer,

S = the underlying stock or asset price, and

X = the exercise price of the option.

Using Equation 7.4 to verify the previous call option examples yields the following in the first case where the stock price was \$60 and the exercise price was \$50:

$$W_c = \min[50 - 60, 0] = -\$10.$$

For the second call example where the stock price was \$40 and the exercise price was \$50, Equation 7.4 yields the following:

$$W_c = \min[50 - 40, 0] = 0.$$

The writer of a put option has to buy the stock at the exercise price if the option is exercised at expiration. Returning to the first put option example, where the stock price is \$35 and the exercise price is \$50, the owner of the put would exercise the option to sell the stock to the put writer for \$50 per share. The put option writer would have to raise \$50 per share to buy the stock that sells for \$35 per share in the market. If the put writer then sells the stock in the market for \$35 per share, a loss of \$15 per share is realized immediately. In the second put option example, where the stock price is \$55 and the exercise price is \$50, the owner of the put would not exercise the option, and the writer of the put will have a payoff of zero at expiration. In the put option case, as in the call option case, the put option writer's payoff at expiration is a loss equal to the put option owner's gain if the option is exercised and zero if the option is not exercised. The put option writer's payoff at expiration is given in the following equations:

$$W_p = \min[-(X - S), 0], \qquad \text{or} \tag{7.5}$$

$$W_p = \min[S - X, 0]. \tag{7.6}$$

For put option Example 1, $S = \$35$ and $X = \$50$, so Equation 7.6 yields:

$$W_p = \min[35 - 50, 0] = -\$15,$$

and for Example 2, $S = \$55$ and $X = \$50$, so:

$$W_p = \min[55 - 50, 0] = 0.$$

The above examples can also be seen graphically in Figure 10.

Given the maximum payoff of zero for writers of options in the previous examples, why would anyone want to write an option? The answer lies in the one important variable omitted from the discussion thus far: the original selling price of the option. This selling price will now be integrated into the discussion.

Let C_0 be the original selling price of a call option and P_0 be the original selling price of a put option. The writer of the option initially receives either C_0 or P_0 from the buyer of the option depending on the type of option sold. Given this fact, Equations 7.1 through 7.6 can be rewritten for the total payoff at expiration for the owners and writers of options as the following equations:

$$C = \max[S - X - C_0, -C_0], \tag{7.7}$$

$$P = \max[X - S - P_0, -P_0], \tag{7.8}$$

$$W_c = \min[C_0 - (S - X), C_0] \quad \text{or} \tag{7.9}$$

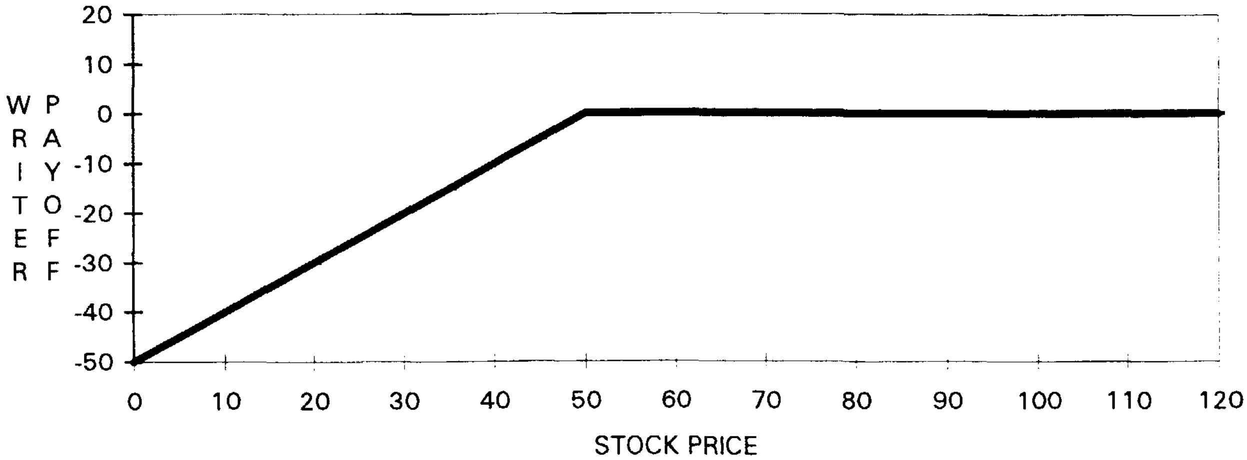
$$= \min[C_0 + X - S, C_0], \tag{7.10}$$

$$W_p = \min[P_0 - (X - S), P_0] \quad \text{or} \tag{7.11}$$

$$= \min[P_0 + S - X, P_0]. \tag{7.12}$$

FIGURE 10

PUT WRITER PAYOFF AT EXPIRATION



For ease of terminology, the time value of money concept of the price for the option being paid at the beginning of the period and the expiration payoff coming at the end of the period will be ignored for now. Equations 7.7 and 7.8 are the payoffs at expiration for the owner of a call option and put option, respectively, and Equations 7.9 and 7.11 are the payoffs at expiration for the writers of call and put options, respectively. With the integration of the option selling price and Equations 7.7 through 7.12 in the discussion, the motives of both the writers and buyers of options become more apparent. The buyer of a call option buys the option with the hope or belief that the price of the underlying asset will go up, and the writer sells the call option with the belief that the price of the underlying asset will go down. If the writer of the call option is correct and the stock or asset price is lower than the exercise price at expiration, the option will not be exercised, and the call option writer will pocket the original selling price of the call option.

To illustrate this point consider the case where a person decides to write a call option on INS stock that expires in three months with a current stock price of \$30 and an exercise price of \$30 and sells this call option to a buyer for \$3 per option. Assume three months later INS's stock price is \$25. The owner of the call option will not exercise the option; therefore, the call option owner's net payoff is negative \$3, the cost of the option. This value can be verified by Equation 7.7 and seen graphically in Figure 11:

$$\begin{aligned} C &= \max[S - X - C_0, -C_0] \\ &= \max[25 - 30 - 3, -3] = -\$3. \end{aligned}$$

The call option writer's net profit (or loss) is given by Equation 7.9 and graphically in Figure 12:

$$\begin{aligned} W_c &= \min[C_0 - (S - X), C_0] \\ &= \min[3 - (25 - 30), 3] = \$3, \end{aligned}$$

FIGURE 11

CALL OWNER NET PAYOFF AT EXPIRATION

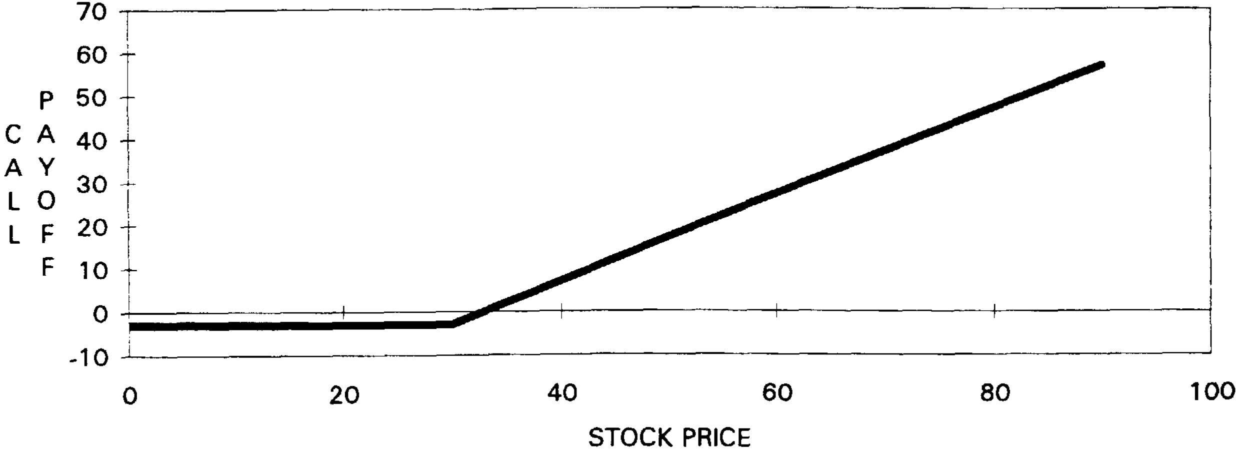
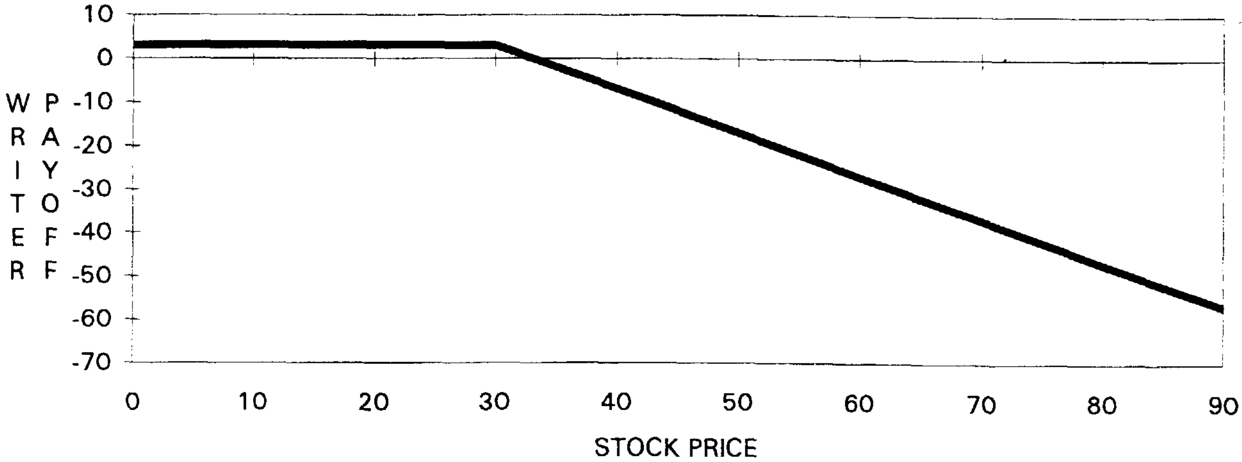


FIGURE 12

CALL WRITER NET PAYOFF AT EXPIRATION



or by Equation 7.10,

$$W_c = \min[C_0 + X - S, C_0]$$
$$= \min[3 + 30 - 25, 3] = \$3.$$

Now suppose the stock price at the end of three months is \$32. Will the owner of this call option exercise the option? Consider the previous example, where the stock price was \$25, and the call option owner did not exercise the option and had a net loss of \$3 per option. As Equation 7.7 shows, the most the owner of a call option can lose is the amount paid for the option, which is the case when the owner does not exercise the option. Using this equation for the new example where the stock price is now \$32 and the exercise price is \$30, gives the following payoff to the owner of the call option:

$$C = \max[32 - 30 - 3, -3] = -\$1.$$

This example shows that the owner of the call option should exercise the option, even though the payoff is negative, because exercising the option results in a smaller net loss than letting the option expire worthless. The example also verifies the decision rule stated earlier of always exercising a call option if the stock price is greater than the exercise price. The writer of the call option in this case would have a payoff of \$1, as verified by Equation 7.10:

$$W_c = \min[3 + 30 - 32, 3] = \$1.$$

A writer of a put option sells the put in the belief that the stock price will go up, and the buyer of a put option buys the option in the belief the stock price will go down. Consider the following example: a person writes a put option that expires in three months with an exercise price of \$40 and a current stock price of \$40 and sells this option for \$5. Three months later at expiration the stock price is \$48. Since the stock price is greater than the exercise price, the owner of this put option will not exercise the option to sell the stock for the exercise price of \$40.

The put owner's payoff, or net profit (or loss), is equal to the amount paid for the option as given by Equation 7.8 below:

$$P = \max[X - S - P_0, -P_0]$$
$$= \max[40 - 48 - 5, -5] = -\$5.$$

The writer of the put option will keep the amount paid for the option in this case because the put option is not exercised. This put writer's payoff is expressed by Equation 7.12 below:

$$W_p = \min[P_0 + S - X, P_0]$$
$$= \min[5 + 48 - 40, 5] = \$5.$$

Now assume the stock price is \$37 instead of \$48 on the expiration date. The owner of the put would exercise the option because the exercise price is greater than the stock price. This would give the owner of the put option a net loss of \$2 as given below by Equation 7.8, but this \$2 loss is better than the \$5 loss derived in the previous example where the put option was not exercised:

$$P = \max[40 - 37 - 5, -5] = -\$2.$$

The writer of the put in this case would buy the stock for \$40 per share and then sell the stock for \$37 in the market resulting in a net gain of \$2 per option, as shown in Equation 7.12 below:

$$W_p = \min[5 + 37 - 40, 5] = \$2.$$

See Figures 13 and 14 for a graphical representation of the payoffs to the owner and writer of the put option in the above example.

The owner of a call option has the potential for an unlimited gain as there is no theoretical upper limit to the price of a stock. The most an owner of a call option can lose is the price paid for the option. Therefore, a call option owner has unlimited upside potential and limited downside potential. The writer of a call option has no limit on the amount of the potential loss, but the gain is limited to the selling price of the option..

FIGURE 13

PUT OWNER NET PAYOFF AT EXPIRATION

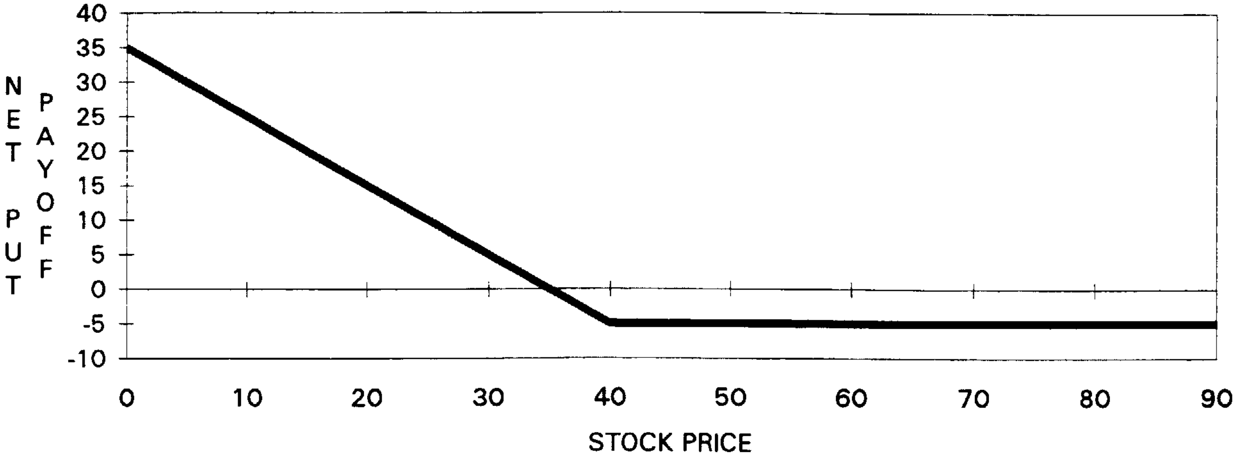
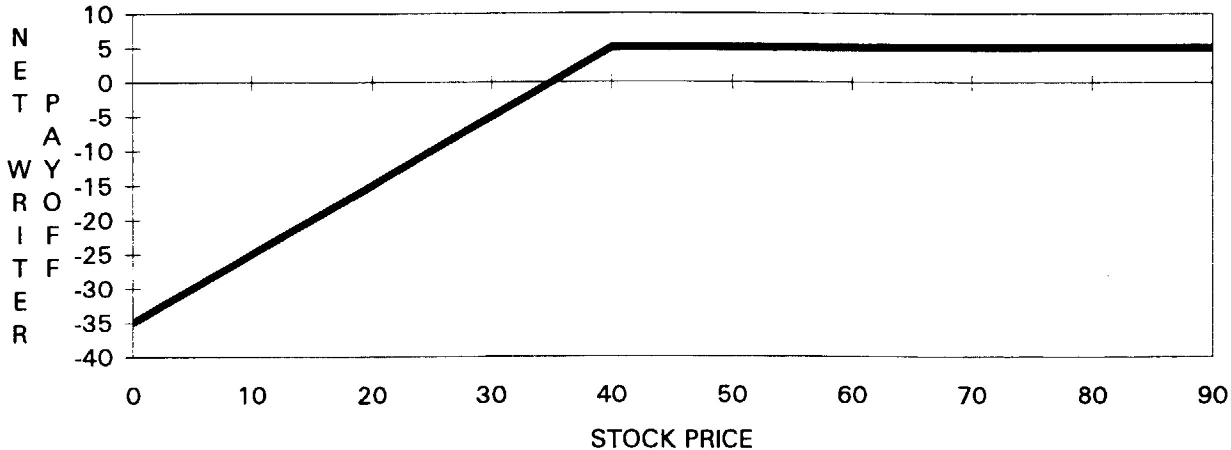


FIGURE 14

PUT WRITER NET PAYOFF AT EXPIRATION



The best possible outcome for the owner of a put option is that the underlying asset becomes worthless at expiration. This means the put option owner's maximum possible gain is the exercise price less the price paid for the option. The most a put option owner can lose is the price paid for the option. The writer of a put option has a maximum possible loss equal to the original selling price minus the exercise price of the option, and the gain of the put writer is limited to the selling price of the option.

This concludes the discussion of option mechanics at expiration. This discussion is important because the payoff at expiration for calls and puts given by Equations 7.1 and 7.2 represents the absolute lowest price of an option at any time and shows that the price for an option is directly related to the option's underlying asset price and exercise price.

Uses of Options

Investors trade options primarily for two reasons. One reason is a speculative motive in that an investor can profit (or lose) from the price movements of the underlying asset for a fraction of the cost of buying the asset itself. An investor who thinks the price of a \$50 stock will increase could buy the stock at \$50 per share or could buy a call option on that stock with an exercise price of \$50 for a much lower price. Buying call options allows the investor to control the price appreciation potential of more shares of stock and still receive at least the same dollar gain in the option price that occurs in the stock price. For example, if the options were valued at \$5, for \$500 an investor could buy call options on 100 shares with an exercise price of \$50 for the stock described above or 10 shares of the actual stock. If the stock price rose to \$60 before option expiration, the price of the call option should rise from \$5 to at least \$10. If the investor bought options on 100 shares of stock at \$5 per option, he or she could sell the options now for \$10 each and receive a gain of \$500 on the original \$500 option investment. The owner of

10 shares of stock could sell the stock for $60 per share for a gain of $10 per share which results in a total gain of $100 on the 10 shares. The investor in options in this example was able to receive a much higher return in comparison to the investor in the underlying stock, highlighting the speculative motive for buying options.

However, a down side to this strategy exists. Assume in the above example that the stock price remains stable. At expiration, the call options will be worthless, resulting in a loss of $500 to the call option holder. At the same time, the owner of the stock will still have 10 shares of stock worth $50 per share, resulting in no loss of value to this investor.

The second reason for buying options is to hedge a position taken in the underlying asset. For example, an investor buys a stock in the hope that the price of the stock will rise above the original price paid for the stock. However, the investor may be concerned about the price of the stock falling below the original purchase price and want to minimize this possible loss, while still receiving a gain if the stock price rises. To hedge the stock position, the investor could buy a put option on the underlying stock. The put option increases in value when the price of the underlying stock decreases. For example, assume the investor bought 100 shares of XYZ stock for $50 per share and bought put options with an exercise price of $50 on 100 shares of XYZ stock for $3 per option. If the price of XYZ stock is $40 when the options expire, the investor will exercise the put options to sell the stock for $50 per share. Alternatively, if XYZ stock sells for $60, the investor would let the put options expire worthless and either sell XYZ for $60 or hold the stock. The expiration value of this hedged stock position is equal to:

$$S - X + \max[X - S - P_0, -P_0], \tag{7.13}$$

which is

$$S - X + X - S - P_0 = -P_0, \tag{7.14}$$

if S is less than X, and

$$S - X - P_0, \tag{7.15}$$

if S is greater than X, where

S = the stock price at expiration,

X = the exercise price of the option and the original purchase price of the stock, and

P_0 = the cost of the put option.

In the first example with XYZ stock selling for \$40 on the option expiration date, Equation 7.13 becomes:

$$40 - 50 + \max[50 - 40 - 3 = 7, -3] = 40 - 50 + 7 = -\$3,$$

$$\text{or} \qquad 40 - 50 + 50 - 40 - 3 = -\$3.$$

In the second example with XYZ stock selling for \$60 on the option expiration date, Equation 7.13 becomes:

$$60 - 50 + \max[50 - 60 - 3 = -13, -3] = 60 - 50 - 3 = \$7.$$

This hedging strategy limits the investor's loss to the cost of the put option in the first case and reduces the gain by the cost of the put option in the latter case.

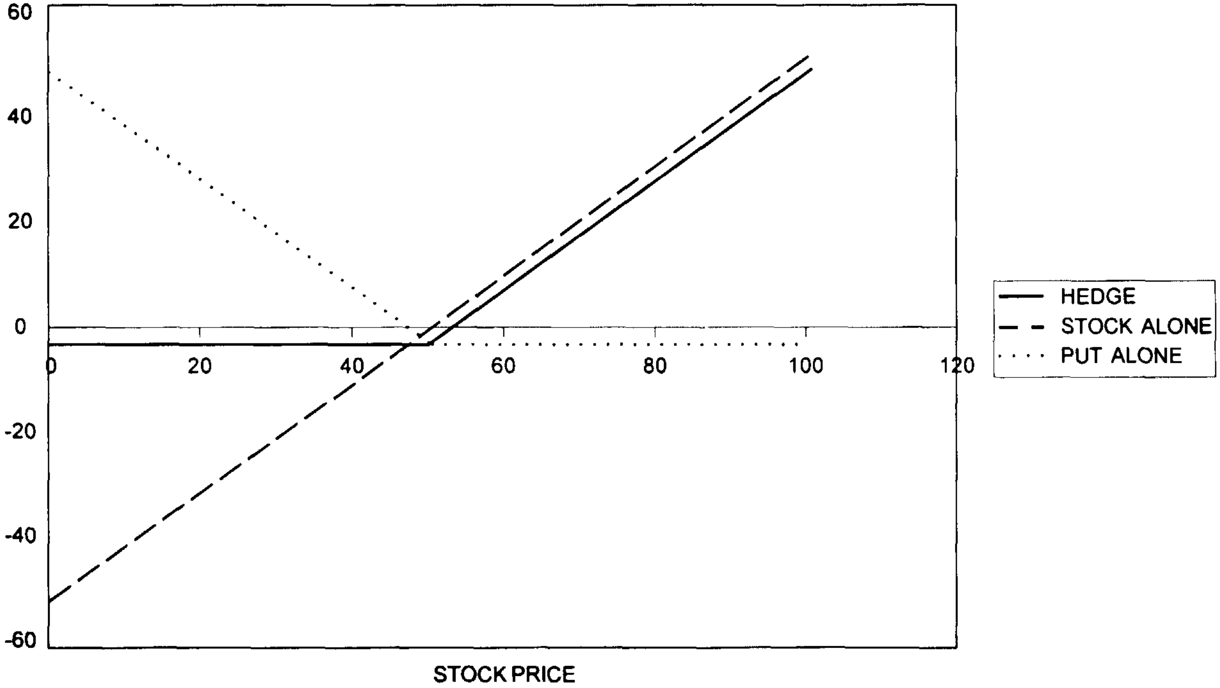
The net value of the hedged stock-put position is illustrated in Figure 15. The diagram of the hedged stock position looks exactly like the payoff of owning a call option in Figure 11 in shape and direction. The net value of the hedged position of owning a stock and a put option has the same characteristics as owning a call option, and in essence a call option has been created as a result of this hedged position. This can be seen by combining Equations 7.14 and 7.15 into one equation:

$$\text{Hedge payoff} = \max[S - X - P_0, -P_0]. \tag{7.16}$$

A put option can be created in a similar fashion. The creation of a put involves buying a call option to hedge a short stock

FIGURE 15

PAYOFF OF HEDGED STOCK-PUT POSITION



position. A short stock position is when an investor borrows shares of stock from someone else and sells them at the current price and then agrees to buy back the stock later in order to return the stock to the lender. A person in a short stock position profits if the stock price falls below the short sale price and loses if the stock price rises after the sale. Therefore, the short stock investor can hedge the downside potential of a rising stock price by buying a call option with an exercise price equal to the sale price of the stock, which increases in value when the price of the underlying stock rises. This position can be called a short hedge. A short hedge has a loss limited to the cost of the call option if the stock price rises above the original selling price of the stock, because the investor would exercise the call option of buying the underlying stock at the same price at which the stock was originally sold. If the stock price is below the original sale price of the stock at expiration, the call option will be allowed to expire worthless and the investor's payoff is the original stock sale price or exercise price minus the expiration stock price minus the cost of the call option. This short hedge net position can be expressed in the following formula:

$$SH = X - S + \max[S - X - C_0, -C_0], \qquad (7.17)$$

which is

$$SH = X - S + S - X - C_0 = -C_0, \qquad (7.18)$$

if the stock price is greater than the exercise price and

$$SH = X - S - C_0, \qquad (7.19)$$

if the stock price is less than the exercise price. Rewriting Equation 7.17 by combining 7.18 and 7.19 leads to a payoff structure identical to owning a put option

$$SH = \max[X - S - C_0, -C_0].$$

This hedging idea was used in the development of one of the most popular option pricing models, the Black–Scholes option pricing model developed in 1973. To highlight the importance and popularity of the Black–Scholes model, secondary option markets were organized after the development of the Black–Scholes model, and traders used the model to set market option prices in the early stages of the secondary stock option markets.

The Black–Scholes option pricing model takes into account five variables that affect option prices:

1. the underlying stock price,
2. the exercise price,
3. the time to expiration of the option,
4. the volatility of price movements in the underlying stock, and
5. the risk-free rate of interest.

The derivation of the model is based on the idea that, if an investor is able to continuously maintain a perfect hedge using an option on the underlying stock or asset, and borrow or lend at the risk-free rate (borrowing at the risk-free rate to raise the money to buy the underlying asset or lending the funds generated from the short sale of an asset at the risk-free rate), then this hedging portfolio must yield the risk-free rate of return to the investor. The Black–Scholes model uses continuous time compounding of interest and a lognormal distribution of asset or stock prices. The lognormal distribution of asset prices is used because an asset cannot sell for a price less than zero, and a lognormal distribution model of asset price satisfies the reality of non-negative asset prices.

The formula for the Black–Scholes option pricing model is as follows:

$$C = S \times N(d_1) - Xe^{-R_f t} \times N(d_2), \qquad (7.20)$$

where

C = the model price for a European call option,

S = the price of the underlying stock or asset,

X = the exercise price of the option,

$N(*)$ = the normal distribution function evaluated at $*$,

R_f = the risk-free rate of return,

t = the time to expiration,

$$d_1 = \frac{\ln(S/X) + (R_f + .5\sigma^2)t}{\sigma t^{1/2}},$$

$d_2 = d_1 - \sigma t^{1/2}$, and

σ = the standard deviation of the continuously compounded returns of the underlying asset.

The Black–Scholes model in Equation 7.20, although appearing quite complex, is fairly easy to use. All variables except the standard deviation are readily observable. The standard deviation can be estimated from historical asset return data, or derived by setting the current market call option price equal to Equation 7.20 and solving for the standard deviation.

Consider this example. Find the Black–Scholes call value for a call option with an exercise price of \$70, stock price of \$90, risk-free rate of .08 per year, time to expiration of 0.5 years, and standard deviation of the stock price returns of .25. The first step is to find d_1 and d_2:

$$d_1 = [\ln(90/70) + (.08 + .5(.25)^2).5]/[.25(.5)^{1/2}] = 1.7363$$

$$d_2 = 1.7363 - .25(.5)^{1/2} = 1.5595.$$

The values of d_1 and d_2 are substituted into Equation 7.20 which yields the following:

$$\begin{aligned} C &= 90 \times N(1.7363) - 70e^{-.08(.5)}N(1.5595) \\ &= 90(.9588) - 70(.9608)(.9405) = 23.04. \end{aligned}$$

The values of the normal distribution functions at d_1 and d_2 are interpolated from the normal z table reproduced in the appendix.

From observation of the model, relationships between the variables and call option prices can be described. A positive relationship exists between the stock price and the call price. A negative relationship exists between the exercise price and the call price. Also, positive relationships exist between the call price and the remaining variables: time to expiration, the risk-free rate, and standard deviation.

Other Applications of Options

Besides options on stocks, bonds, and other financial assets, other financial instruments and insurance have characteristics of options and can be priced by option pricing models. For example, the value of a corporation has the characteristics of a European call option.

The value of a corporation's equity, E, is equal to the value of its assets, A, less the value of its debts or liabilities, D. Assume at the end of the period the corporation will liquidate, and the equityholders will receive the difference between the corporation's assets and liabilities if assets are greater than the liabilities or nothing if assets are less than liabilities. This relationship can be expressed by the following equation:

$$E = \max[A - D, 0]. \tag{7.21}$$

This end of period value of equity relationship in Equation 7.21 is the same as the payoff of a European call option at expiration where the value of the assets is the stock or asset price and the value of liabilities is the exercise price. The debtholders receive the full face value of their claims, D, or the value of the assets, A, if the corporation's assets are less than its liabilities at the end of the period. The end of period value of the debtholders' claims, V_D, can be written as the following:

$$V_D = \min[D, A]. \tag{7.22}$$

The debtholders have, in effect, written a put option whose maximum value is the face value of their claims, D, if the value of the corporation's assets, A, is greater than or equal to D, and whose minimum value is 0 if the corporation's assets are worthless at the end of the period.

An insurance contract is another example of a financial asset that has option characteristics. Assume an insurance company writes a single period policy with a premium, P, a deductible amount, B, and an unknown loss amount, L. Ignoring the time value of money for simplicity, the insurer's end of period policy value (V_p) would be written as the following:

$$V_p = \min[P, P-(L-B)] \qquad \text{or} \qquad \min[P, P-L+B]. \tag{7.23}$$

The insurer would net the premium if no loss occurred or if the loss did not exceed the deductible. If the loss were greater than the deductible, the insurer's income would be reduced by the difference between the loss and the deductible. This expression in Equation 7.23 is very similar to the payoff at expiration to the writer of a European call option. The insurer has in effect written a European call option with an exercise price of the deductible amount. In this case, the policyholder can be thought of as owning a European call option with an exercise price of the deductible amount. The value of the policyholder's claim (V_h) can be written as the following:

$$V_h = \max[L-B-P, -P]. \tag{7.24}$$

These straightforward examples of options have been used to determine the fair rate of return in pricing insurance with an option pricing framework. This application of option pricing to insurance will be discussed in the following section.

8. APPLICATION OF OPTION PRICING MODELS TO PRICING INSURANCE

The application of option pricing to insurance pricing was developed by Doherty and Garven [14]. Their formulation assumes a single-period insurer with initial equity of S_0 and premiums collected (net of expenses) of P_0. The aim of the model is to find the premium that gives the insurer an adequate or "fair" rate of return on equity. This is done by setting the present value of the expected end of period market value of equity equal to the beginning of period amount of equity.

The sum of initial equity and premiums represents the insurer's initial cash flow or asset portfolio of Y_0:

$$Y_0 = S_0 + P_0. \tag{8.1}$$

The insurer has this initial asset portfolio available to invest at rate R. All of the equity can be invested for the entire period and the premiums can be invested for a portion of the period because of the time lag between receipt of premiums and payment of losses. The time lag between receipt of premiums and payment of losses is called the funds generating coefficient and will be denoted by k. The end of period asset portfolio available to the insurer is the initial asset portfolio Y_0 plus the income generated from the investment of the initial asset portfolio at rate R which is written as the following:

$$Y_1 = S_0 + P_0 + (S_0 + kP_0)R. \tag{8.2}$$

The insurer has this end of period asset portfolio available to pay claimholders. The claimholders include policyholders who expect to have their losses paid and the government that expects taxes to be paid.

The policyholders hope the insurer has an adequate end of period asset value to pay their losses of amount L. If the insurer's end of period asset value is greater than or equal to L, the policyholders receive L. If the insurer does not have adequate asset value to cover the losses, the policyholders receive the amount of the insurer's end of period asset value Y_1. The policyholders' end of period claim, H_1, is represented by the following:

$$H_1 = \max\{\min[L, Y_1], 0\}. \tag{8.3}$$

This is equivalent to the expiration payoff to the owner of a European call option with an exercise price of L.

The government holds a similar type of call option based on whether the insurer makes a payoff. If the insurer has positive taxable income, the government receives taxes from the insurer based on the amount of the insurer's profits. If the insurer does not make a profit, the government receives no tax revenue. The value of the government's end of period tax claim, T_1, can be written as the following:

$$T_1 = \max\{t[i(Y_1 - Y_0) + P_0 - L], 0\}, \tag{8.4}$$

where

t = the insurer's corporate tax rate, and

i = the portion of investment income that is taxable.

The term $Y_1 - Y_0$ represents the insurer's investment income which may come from investments such as tax exempt bonds, corporate dividend income, and capital gains which may have differential tax rates from ordinary income.

Any portion of the asset portfolio remaining after the policyholders and taxes are paid reverts to the equityholders. Therefore, the end of period value of equity, V_e, is:

$$V_e = Y_1 - H_1 - T_1. \tag{8.5}$$

However, the end of period values in the previous equation are not known with certainty at the beginning of the period. The present value of the expected equity value must be found to begin the process of deriving a premium value that yields a "fair" rate of return on equity.

The present value of the policyholders' claim and the government's claim can be written as the following:

$$H_0 = V(Y_1) - C[Y_0; \mathrm{E}(L)], \tag{8.6}$$

$$T_0 = tC[i(Y_1 - Y_0) + P_0; \mathrm{E}(L)], \tag{8.7}$$

where

$V(Y_1)$ = the market value of the insurer's asset portfolio,

$C[A;B]$ = the current value of a European call option with exercise price B written on an asset with a value of A, and

$\mathrm{E}(L)$ = the expected losses and loss adjustment expenses during the period.

The market value of equity can now be written as:

$$\begin{aligned} V_e &= V(Y_1) - H_0 - T_0 \\ &= C[Y_0; \mathrm{E}(L)] - tC[i(Y_i - Y_0) + P_0; \mathrm{E}(L)] \\ &= C_1 - tC_2. \end{aligned} \tag{8.8}$$

For example, assume that an insurer is in the following situation (figures are in millions):

Initial Equity	\$100
Premiums Written	200
Expenses	40
Expected Losses	150
Standard Deviation of Investment Returns	0.5
Standard Deviation of Losses	0.0
Risk-Free Interest Rate	4.0%
Funds Generating Coefficient	1.0

In the context of the Doherty–Garven option pricing notation, S_0 is \$100, P_0 is \$160 (\$200 in premium less \$40 in expenses), E(L) is \$150 and there is no uncertainty about the value of these losses, and k is 1 year, at which time the insurer will pay the losses out of available assets.

Assuming no taxes initially, the value of the stockholders' interest in this insurer is, based on Equation 8.8 and the Black–Scholes option pricing model:

$$C[Y_0; \mathrm{E}(L)] = C[100 + 200 - 40; 150] = C[260; 150]$$

$$d_1 = \frac{\ln\left(\frac{260}{150}\right) + (.04 + .5(.5)^2)1}{.5(1)^{1/2}}$$

$$= 1.43$$

$$d_2 = 1.43 - .5(1)^{1/2} = .93$$

$$C = 260N(1.43) - 150e^{-.04(1)}N(.93)$$

$$= 260(.9236) - 150(.9608)(.8238)$$

$$= 121.41.$$

Thus, the value of this insurer, based on the option pricing methodology, is \$121.41 million if taxes are ignored. This is higher than would be anticipated if the only consideration were given to the initial equity of \$100 million and the underwriting profit of \$10 million (premiums of \$200 less expenses of \$40 and losses of \$150). Adding the initial equity to the underwriting profit totals \$110 million. The reason for the much higher value based on the option pricing methodology is that the model considers the default option. If the end of period assets are less than \$150 million, the policyholders bear the loss, but the stockholders incur all the gains over that level.

Now taxes will be added to the calculation. Assume that all investment income is fully taxable, so the i in Equations 8.7 and 8.8 is 1.0. The insurer's tax rate, t, is 35 percent. The end of period asset portfolio is calculated based on Equation 8.2:

$$Y_1 = 100 + 160 + (100 + 1.0(160))(.04) = 270.4.$$

The value of the government's tax claim on the insurer is:

$$\begin{aligned} T_0 &= .35C[(270.4 - 260) + 160; 150] \\ &= .35C[170.4; 150] \\ d_1 &= \frac{\ln\left(\frac{170.4}{150}\right) + (.04 + .5(.5)^2)1}{.5(1)^{1/2}} \\ &= .5850 \\ d_2 &= .5850 - .5(1)^{1/2} = .0850 \\ C &= 170.4N(.5850) - 150e^{-.04(1)}N(.0850) \\ &= 170.4(.7207) - 150(.9608)(.5339) \\ &= 45.86 \\ T_0 &= .35C = 16.05. \end{aligned}$$

This value of the government's tax claim of \$16.05 million also may seem high, given that the insurer has investment income, based on the risk-free rate, of \$10.4 million and an underwriting profit of \$10 million. However, taxes are asymmetric, with the government collecting 35 percent of any gains, but not sharing in any losses. (In this model, tax loss carryforwards and carrybacks are ignored. In reality, taxes are much more complicated than the model provides.)

Considering taxes in determining the stockholders' value of the insurer described in this example yields the following, based on Equation 8.8:

$$V_e = 121.41 - 16.05 = 105.36.$$

Since V_e exceeds the initial equity value of \$100, the insurer gains value by writing insurance at this premium level. In this example, the expected losses are assumed to have no uncertainty.

When losses are allowed to vary, then, in essence, the exercise prices of the options for the stockholders and government are random variables. This variation can be accounted for, but complicates the calculation.

Doherty and Garven next use this methodology, including allowing losses to vary, to find the appropriate premium the insurer should charge. The premium should be set so that the market value of equity is equal to the initial equity amount of S_0 and yields a "fair" rate of return to shareholders. The values of Y_0 and Y_1 are functions of the "fair" premium of P^* as are the call options in Equation 8.8, rewritten as:

$$\begin{aligned} V_e &= C[Y_1(P^*);\mathrm{E}(L)] - tC[i(Y_1(P^*) - Y_0(P^*) + P^*;\mathrm{E}(L)] \\ &= C_1^* - tC_2^* \\ &= S_0. \end{aligned} \tag{8.9}$$

The insurer's fair underwriting profit margin is given by:

$$UPM = [P^* - \mathrm{E}(L)]/P^*. \tag{8.10}$$

The call option values are found by an option pricing model based on the Black–Scholes model. Doherty and Garven use two different option pricing models to price the options in Equation 8.9. These two models are arrived at by different assumptions about investor risk preferences and asset price distributions. One model is based on constant absolute risk aversion (CARA) and a normal distribution of asset prices, and the other assumes constant relative risk aversion (CRRA) and a lognormal distribution of asset prices similar to the Black–Scholes model. Since the models do not have closed form solutions, P^* is found by trial and error from properly parameterized versions of the models. Parameter estimates needed for the models are the initial equity level, standard deviation of claim costs and investment returns, and the correlation between claim costs and investment returns. The general results of that research indicate that the appropriate underwriting profit margins are higher under the option pricing model than under the CAPM.

This option pricing approach to pricing insurance is more complex than the CAPM or Discounted Cash Flow approaches, but it avoids many of the problems, such as estimating betas and market risk premiums, of the CAPM-based models. Also, the option model is different in that it uses the total risk of the insurer's investment portfolio and underwriting operations, rather than systematic risk.

The option pricing model has also been applied to insurance solvency considerations. Cummins [8] calculates the appropriate guaranty fund charge by using a diffusion process for assets and liabilities similar to the Black–Scholes option pricing model. Through the use of realistic parameters, Cummins is able to obtain a guaranty fund premium in line with past experience. Boyle and Kemna [5] use the option pricing model to examine incentives for cooperating behavior and for assuming excessive risk under the risk sharing arrangement inherent in guaranty funds.

One problem in applying the Black–Scholes option pricing model to insurance cases is the documented tendency of this model to underprice options in which the stock price is well above the exercise price (see [22], [23]). These options, termed in-the-money options, are exactly the type of option that is used in applications of the option pricing model to insurance, since the expected terminal value of the insurer's assets is generally much higher than expected losses. Thus, although the option pricing model has significant advantages over other valuation models, the bias inherent in the model needs to be taken into consideration.

9. CONCLUSION

The insurance industry, including regulators, insurers, and researchers, has grappled with the issue of a profit provision for over 70 years. The issue is as yet unresolved. The easy rules of thumb are based on invalid techniques. The valid techniques require input values that may not be possible to measure accurately.

Efforts to refine the techniques and advance the use of appropriate methods are continuing. Some results are quite promising, but additional work is necessary.

An analogy to loss reserving is appropriate. No actuary uses one method to set loss reserves, as no single method is applicable in all cases. The paid loss development method is very accurate, but takes a long time to reflect changes. Incurred loss methods reflect changes more quickly, but are sensitive to changes in case reserve adequacy. Data availability problems sometimes require reliance on less robust techniques. As discussed in Berquist and Sherman [3], the proper approach to establishing loss reserves is to use a variety of techniques, analyze the distribution of indications, try to determine if outliers are caused by errors or reflect early warnings of shifts in development patterns, and then use actuarial judgment to arrive at the best figure.

Ratemaking should be no different. Actuaries should apply a variety of ratemaking techniques to see what the various indications turn out to be. Some methods rely on parameters that are difficult to measure. Other methods are not responsive to changes in risk, interest rates, or other economic conditions. The techniques described in this paper can provide useful information about rate levels, but they should not be expected to determine, under all circumstances, the correct rate level. Within a portfolio of ratemaking techniques, each can contribute some value.

Actuaries have played, and will continue to play, a key role in the effort to determine appropriate profit provisions. However, since the mid-1970s, the playing field for investigation of these issues has shifted into relatively unfamiliar terrain for actuaries, the field of financial economics. Actuaries need to master this area in order to continue to play an influential role. The addition of finance to the Casualty Actuarial Society *Syllabus* is a useful step in developing this expertise. Hopefully, this paper will also be useful as an educational, or reference, tool.

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APPENDIX

NORMAL DISTRIBUTION FUNCTION (FOR NEGATIVE X, $N(X)$ = 1 – VALUE LISTED)

X	.00	.01	.02	.03	.04	.05	.06	07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8079	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986

THE INSURANCE EXPENSE EXHIBIT AND THE ALLOCATION OF INVESTMENT INCOME

SHOLOM FELDBLUM

Abstract

The Insurance Expense Exhibit (IEE) provides a statutory allocation of investment income to lines of business, thereby measuring the underlying profitability of the insurance operations. Casualty actuaries must understand the allocation procedure, both for completing the Insurance Expense Exhibit of their own companies and for interpreting the results of their peer companies.

Although the allocation procedure is strictly prescribed by the NAIC, the method is not shown on the IEE itself and the formulas are difficult to decipher from the IEE Instructions. This paper explains the philosophy underlying the allocation procedure, the adjustments made to various components, and the formulas that are used to determine the amount of investment income assigned to each line of business. In addition, the paper provides an illustration, enabling the reader to trace the steps of the allocation procedure, from the initial data elements to the final allocated amounts.

The allocation of investment income to lines of business is a contentious issue, for which various methods are currently being used. The paper concludes with a comprehensive analysis of the pros and cons of the statutory allocation procedure prescribed in the IEE.

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1. INTRODUCTION

The statutory Annual Statement enables state regulators to monitor the profitability and financial strength of insurance enterprises. Most revenues and expenditures that relate to particular policies, such as premiums and losses, are shown by line of business. Revenues and expenditures that cannot be directly associated with particular policies, such as investment income and general expenses, are shown only in the aggregate.

The primary focus of state regulation is on the ability of the insurance company to meet its obligations to policyholders and claimants. Profitability is an important consideration, since a consistently unprofitable insurer may soon find itself in financial distress. But the focus is on overall profitability, not on the profitability of each business segment.

Aggregate information does not suffice for all users. Rate regulators, for instance, must determine if premium rates by line of business are inadequate or excessive. Investors must determine if the capital used to support a given block of business is earning a satisfactory return. The insurer's management must determine which segments of the company are meeting desired profit levels.

The Insurance Expense Exhibit (IEE), filed by April 1 as a supplement to the statutory Annual Statement, provides the needed additional information. All revenues and expenditures, whether or not they are associated with particular policies, are allocated to lines of business. Various sets of operating returns are calculated, so that profitability by line of business may be measured.

Expense allocation may be complicated, but it is not conceptually difficult. Investment income allocation, however, particularly when used to measure the total return by line of business, requires subjective assumptions: "How should surplus be allocated to lines of business?" "Should the investment returns on policyholder supplied funds differ from those on capital and surplus funds?" "How should policyholder supplied funds be defined?"

These are not idle questions. They have been debated for years by actuaries and regulators, and their answers form the framework of the new investment income allocation procedure in the IEE. This paper reviews this allocation procedure and the resultant measures of profitability by line of business in the NAIC financial statements.

Casualty actuaries are often asked to complete the investment income columns in their companies' Insurance Expense Exhibits.[1] In addition, they are often asked to evaluate the IEE profitability measures: to tell their managements whether the operating returns shown in the IEE accurately reflect the performance of each line of business. Careful study of the investment income allocation procedures in the IEE is needed to respond to such questions.

The Structure of the Insurance Expense Exhibit

The structure of the IEE is as follows:

- Part I—Allocation to Expense Groups

[1]The statutory procedures for completing the IEE are documented in the *NAIC Proceedings*, 1992, Volume IA, pages 338–341, "Summary of Changes to the Proposal of the Insurance Expense Exhibit Working Group to the Blanks (EX4) Task Force," as well as in the NAIC instructions to the IEE.

- Part II—Allocation to Lines of Business Net of Reinsurance
- Part III—Allocation to Lines of Direct Business Written

Part I of the IEE, like Part 4 of the "Underwriting and Investment Exhibit," divides expenses along two dimensions:

a. *Expense classification*, such as advertising, rent, salaries, or equipment, and

b. *Expense groups*, which are loss adjustment expenses, other underwriting expenses, and investment expenses.

The IEE has a more refined division of "other underwriting expenses" into

- Acquisition, field supervision and collection expenses,
- General expenses, and
- Taxes, licenses and fees.

Part II of the IEE shows the allocation of all revenues and expenditures to lines of business, where the figures are net of reinsurance. Part III shows a similar allocation for direct business, except that investment income is not included in Part III.

In Parts II and III, lines of business are shown along the vertical axis (i.e., they are rows), and revenue and expenditure categories are shown along the horizontal axis (i.e., they are columns). A decimal point in an IEE line of business indicates that a finer breakdown is being used than is shown in the Underwriting and Investment Exhibit. Automobile liability provides a good illustration. The pre-1995 Underwriting and Investment Exhibit in the Annual Statement showed a single Line 19: Auto liability.[2] The IEE shows

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[2]In 1995, automobile liability was split in the Underwriting and Expense Exhibit as well as into personal and commercial auto liability, following the IEE split. In the Underwriting and Investment Exhibit, 19.1 is now private passenger auto liability and 19.2 is now commercial auto liability.

Lines 19.1, 19.2: Private passenger auto liability and
Lines 19.3, 19.4: Commercial auto liability.

The exhibits of premiums and losses by state (Page 15 of the Annual Statement) show all four components separately:

Line 19.1: Private passenger auto no-fault (personal injury protection)
Line 19.2: Other private passenger auto liability
Line 19.3: Commercial auto no-fault (personal injury protection)
Line 19.4: Other commercial auto liability

Personal and commercial auto often have different expense characteristics (e.g., agents' contracts may provide a higher commission rate on personal auto), so this subdivision is appropriate for the IEE.

This paper concentrates on the investment income allocation procedures used for completing Part II of the IEE, Columns 18 and 20. There are only passing references to Parts I and III of the IEE; in particular, there is no discussion of the expense classifications in Part I of the IEE. Moreover, the first 16 columns of Part II of the IEE, which contain the data needed for the investment income allocation procedure, are described in Appendix A, not in the body of the paper. The text of the paper deals with the computations needed to determine the entries for Columns 18 and 20, and it provides an arithmetic example of the procedure.

2. IEE PART II: ALLOCATION TO LINES OF BUSINESS NET OF REINSURANCE

> The purpose of Part II is to allocate elements of total profit (or loss) net of reinsurance to lines of business.
>
> —*NAIC Proceedings*, 1992, Volume IA, page 339

The completion procedures for the first 16 columns of Part II of the Insurance Expense Exhibit are documented in detail in Appendix A. Readers who are preparing to complete an actual IEE will find the information in Appendix A to be essential. The focus of this paper is on the allocation of investment income in the IEE, so we begin with Column 17 of Part II.

Allocation of Investment Income by Line of Business

The allocation of investment income by line of business in the 1992 and subsequent Insurance Expense Exhibits differs from the corresponding allocation in previous years. However, the allocation in the IEE is now the same as the allocation in the NAIC "Profitability by Line by State" reports.

Before 1992, the allocation procedure was documented in the footnotes to the IEE. Now the allocation procedure appears in the instructions to the IEE. The allocation procedure is also described in the *Proceedings of the NAIC*, 1992, Volume 1A, pages 339–341.[3]

This paper examines the allocation procedure on three levels:

- *Conceptual*: the philosophy underlying the allocation procedure,
- *Components*: the insurance elements comprising the allocation formula, as well as the adjustments made to several of these elements, and
- *Data*: the data sources for the elements of the allocation formula (primarily the previous columns of Part II of the IEE).

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[3]The allocation procedure is strictly prescribed by the NAIC: "Although various methodologies might result in reasonable allocations of investment income to lines of business, the following formulae for allocating investment gain must be used in completing the allocation for Column 18, Investment Gain on Funds Attributable to Insurance Transactions and the allocation for Column 20, Investment Gain Attributable to Capital and Surplus" (*Proceedings*, page 339).

The NAIC instructions to the IEE show the arithmetic formula, with little or no explanation of the allocation philosophy or the rationale for the adjustments. This paper describes the concepts and formulas of the allocation procedure, and it provides a detailed example to assist the reader in understanding the method.

Conceptual Level

The allocation of investment income to line of business in the IEE rests upon three principles:

1. Investment income is allocated to each line of business in proportion to the investable funds associated with each line of business. Investable funds consist of (i) funds attributable to insurance transactions and (ii) funds attributable to capital and surplus.
2. Funds attributable to insurance transactions are loss reserves plus unearned premium reserves minus prepaid expenses and minus uncollected premiums. (The adjustments to the unearned premium reserves for prepaid expenses and uncollected premiums occur in some parts of the allocation procedure, not in all parts. See *The Allocation*, below.)
3. Capital and surplus are allocated to lines of business in proportion to total reserves plus earned premium for the year.

Component Level

The allocation procedure uses the following principles to derive the items in the "conceptual level":

1. For balance sheet items, the *averages* of the current year-end values and the prior year-end values are used. These balance sheet items are
   - Net loss and loss adjustment expense reserves,

- Net unearned premium reserves,
- Net agents' balances, and
- Policyholders' surplus.

The allocation procedure refers to these as "mean surplus," "mean net agents' balances," and so forth. (For example, mean surplus is the average of policyholders' surplus at December 31 of the current year and policyholders' surplus at December 31 of the prior year.)

2. Prepaid expenses, or "acquisition expenses," are

   Commission and brokerage expenses incurred

   + Taxes, licenses, and fees incurred
   + Other acquisition, field supervision, and collection expenses incurred
   + One half ($\frac{1}{2}$) of general expenses incurred.

3. Net investment gain or loss is composed of net investment income earned and net realized capital gains or losses. It does *not* include unrealized capital gains or losses.

The Allocation

The allocation procedure works as follows:

A. Allocate the company's mean surplus to line of business in proportion to

   Mean net loss and loss adjustment expense reserves

   + Mean net unearned premium reserves
   + Earned premium for the year.

   Unearned premium reserves are not adjusted for agents' balances or for prepaid expenses, in this part of the allocation procedure. The unearned premium reserves represent the amount the insurer is required to hold, not the amount of investable funds derived from premiums.

B. Determine the company's overall "investment gain ratio" as

Net investment gain

÷ (Mean net loss and loss adjustment expense reserves

+ Mean net unearned premium reserves

− Mean net agents' balances

+ Mean policyholders' surplus).

"Net investment gain (or loss)" is composed of net investment income earned and net realized capital gains or losses. It does *not* include unrealized capital gains or losses.

Agents' balances are a component of written premium and therefore of the unearned premium reserve. But agents' balances are not an investable asset, so they are subtracted from the unearned premium reserve in determining the investment gain ratio.

In statutory accounting, prepaid expenses are an expenditure, not an asset. Prepaid expenses reduce policyholders' surplus, so they are already "subtracted" from the investable assets in the denominator of the "investment gain ratio." (In contrast, the agents' balances considered here are admitted assets, so they do not reduce policyholders' surplus.[4])

In this part of the formula, the reserves, agents' balances, and surplus are for all lines combined.

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[4]Non-admitted agents' balances do not appear on the balance sheet, since they are already deducted from policyholders' surplus.

C. For each line of business, the "investment gain on funds attributable to insurance transactions" (Column 18) is the company's investment gain ratio times the funds attributable to insurance transactions for that line of business. This latter item is determined as

Funds attributable to insurance transactions

= Mean net loss and loss adjustment expense reserves

+ {Mean net unearned premium reserves

× [1 − (prepaid expenses ÷ written premiums)]}

− Mean net agents' balances.

Prepaid expense are funded from surplus, not from insurance transactions, since the full (gross) unearned premium reserve must be held as a liability. The ratio of prepaid expenses to written premiums shows the percentage of each premium dollar that must be funded from surplus. The mean net unearned premium reserves are therefore multiplied by the complement of this ratio.

D. For each line of business, the "investment attributable to capital and surplus" (Column 20) is the total investment gain for that line of business minus the "investment gain on funds attributable to insurance transactions." The total investment gain for that line of business is the company's investment gain ratio times the investable funds associated with that line of business. The investable funds associated with that line of business equal that line's

Mean net loss and loss adjustment expense reserves

+ Mean net unearned premium reserves

− Mean net agents' balances

+ Allocated policyholders' surplus.

Since policyholders' surplus is already reduced by prepaid expenses in statutory accounting, there is no need to reduce the unearned premium reserves by these expenses.

This completes the allocation procedure for investment income. The section below shows the data sources for each element of the procedure.

Data Level

All the data elements for the allocation of investment income to line of business are taken from the Annual Statement or from prior columns of the IEE. The following abbreviations clarify the formulas:[5]

LR_{lob}	Mean net loss and loss adjustment expense reserves by line of business
LR_{tot}	Mean net loss and loss adjustment expense reserves for all lines combined
$UEPR_{lob}$	Mean net unearned premium reserves by lines of business
$UEPR_{tot}$	Mean net unearned premium reserves for all lines combined
PPE_{lob}	Net prepaid expenses, or net acquisition expenses, by line of business
AB_{lob}	Mean net agents' balances by line of business
AB_{tot}	Mean net agents' balances for all lines combined
WP_{lob}	Net written premium by line of business for the current year
EP_{lob}	Net earned premium by line of business for the current year
EP_{tot}	Net earned premium for all lines combined for the current year

[5]The NAIC instructions use different abbreviations: A1 for LR_{lob}, A2 for LR_{tot}, B1 for $UEPR_{lob}$, and so forth, through L for IG_{it} and M for IG_{cs}. Actuaries familiar with ancient BASIC variable naming conventions should have no difficulty with the NAIC abbreviations.

PHS_{tot}	Mean policyholders' surplus for all lines combined
PHS_{lob}	Policyholders' surplus allocated to the line of business
PHS_{rat}	Policyholder surplus ratio
IG	Net investment gain
IGR	Investment gain ratio
IG_{it}	Investment gain by line of business on funds attributable to insurance transactions
IG_{cs}	Investment gain by line of business attributable to capital and surplus

1. *Net loss and loss adjustment expense reserves* are taken from page 11 of the Annual Statement, "Underwriting and Investment Exhibit," Part 3A, Column 5, "net losses unpaid excluding loss adjustment expenses," plus Column 6, "unpaid loss adjustment expenses." The "mean" value is determined by averaging the amounts in the current and prior Annual Statements.

2. *Net unearned premium reserves* are taken from Page 9 of the Annual Statement, "Underwriting and Investment Exhibit," Part 2A, Column 5, "total reserve for unearned premium." The "mean" value is determined by averaging the amounts in the current and prior Annual Statements.

3. *Net prepaid expenses* are determined from the prior columns in Part II of the IEE, as

$$\text{Net prepaid expenses} = (\text{Column } 12 + \text{Column } 13 + \text{Column } 14 + \tfrac{1}{2} \text{ Column } 15).$$

4. *Net agents' balances* for all lines combined is taken from Page 2 of the Annual Statement, Line 10.1 plus Line

10.2. Agents' balances by line of business are taken from Column 11 of Part II of the IEE. The "mean" values are determined by averaging the amounts in the current and prior Annual Statements and Insurance Expense Exhibits.

5. *Written and earned premium*: Net written premium is taken from Column 1 of Part II of the IEE, and net earned premium is taken from Column 2.

6. *Mean policyholders' surplus* for all lines combined is the average of Columns 1 and 2 on Line 26 of Page 3 of the Annual Statement.

7. The *policyholders' surplus ratio* is defined as the ratio of policyholders' surplus to the sum of loss reserves, unearned premium reserves, and annual earned premium, or

$$PHS_{rat} = PHS_{tot} \div (LR_{tot} + UEPR_{tot} + EP_{tot}).$$

8. The *policyholders' surplus allocated to each line of business* is determined as the product of the policyholders' surplus ratio and the sum of loss reserves, unearned premium reserves, and annual earned premium for that line of business, or

$$PHS_{lob} = PHS_{rat} \times (LR_{lob} + UEPR_{lob} + EP_{lob}).$$

9. The *net investment gain* is taken from the Annual Statement, Page 4, "Statement of Income," Line 9A, "net investment gain or loss." Line 9A of Page 4 is the sum of Line 8 ("net investment income earned," or interest, dividends, and rent) and Line 9 ("realized capital gains or losses"). Unrealized capital gains and losses, which appear on Line 19 of Page 4, are not included in Line 9A.

10. The *investment gain ratio* is defined as the investment gain divided by investable assets, or

$$IGR = IG \div (LR_{tot} + UEPR_{tot} + PHS_{tot} - AB_{tot}).^{6}$$

11. The *investment gain by line of business on funds attributable to insurance transactions* is determined as

$$IG_{it} = IGR \times \{LR_{lob} + UEPR_{lob} \times [1 - (PPE_{lob} \div WP_{lob})] - AB_{lob}\}.$$

This is the entry for Column 18.

12. The *investment gain by line of business attributable to capital and surplus* is determined as

$$IG_{cs} = [IGR \times (LR_{lob} + UEPR_{lob} + PHS_{lob} - AB_{lob})] - IG_{it}.$$

This is the entry for Column 20.

The 1992 Revisions

The major differences introduced in the 1992 IEE regarding the allocation of investment income are as follows:

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[6]In theory, one might make other adjustments to investable assets, such as for "bills receivable, taken for premiums" (Line 11 of Page 2 of the Annual Statement). Most of these other adjustments are minor, and would not materially affect the allocation procedures.

David Eley has pointed out to me that the "investment gain ratio" is applied to the investable assets by line of business. It would be extremely difficult, if at all practical, to make these adjustments by line of business. To properly allocate investment income, the investable assets by line of business should sum to the total investable assets used in the allocation procedure. Moreover, although the investment gain ratio without these adjustments may be slightly inaccurate in any one year, over a period of several years the ratio works well.

Mr. Eley is correct. These practical considerations overwhelm any theoretical advantages from additional adjustments.

- Before 1992, there was a separate "capital and surplus" account, similar to a line of business. The investment income attributable to capital and surplus was not allocated to lines of business. In 1992, the separate "capital and surplus" account was removed, and the investment income attributable to capital and surplus is allocated to lines of business.
- Before 1992, the investment income allocated to lines of business reflected primarily bond returns, not common stock dividends or capital gains.[7] Thus, the investment yield on funds attributable to capital and surplus differed from the investment yield on funds attributable to insurance transactions. In 1992, stock dividends and realized capital gains are treated as other investment income, so the difference in investment yields has been eliminated.[8]

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[7]In the 1991 IEE, the "adjusted investment income" that is allocated to lines of business is defined as "Annual Statement, Page 6, Part 1, Column 8, Lines 10–11–12–2.1–2.11–2.2–2.21" (see step "B" of footnote "D" in the 1991 IEE). Part 1 of Page 6 shows "interest, dividends, and real estate income," not capital gains. Column 8 shows the amount earned during the year. Line 10 shows the total (gross) investment income. Column 11 shows the investment expenses incurred, and Column 12 shows the real estate depreciation. Lines 2.1, 2.11, 2.2, and 2.21 show dividends on (i) unaffiliated preferred stock, (ii) affiliated preferred stock, (iii) unaffiliated common stock, and (iv) affiliated common stock, respectively.

The investable assets to which the "adjusted investment income" was compared also excluded common stocks. The "investment income ratio" used for the allocation of investment income to line of business therefore reflected primarily bond returns, not stock returns.

Step "J" of footnote D to the 1991 IEE defines "investment income attributable to the capital and surplus accounts" as Annual Statement Page 4, Line 8, less the investment income allocated to lines of business. Page 4, Line 8, equals Page 6, Column 8, Lines 10–11–12–13. Line 13 is "aggregate write-ins for deductions from investment income," and it is usually a small amount.

A major portion of net investment income attributed to the capital and surplus account reflected the difference between stock and bond returns. Step "K" of footnote D to the 1991 IEE says "Realized capital gains attributable to capital and surplus accounts = Annual Statement, Page 4, Line 9. Page 4, Line 9, comprises all realized capital gains, as shown on Page 6, Part 1A, Line 11.

This separation of stock dividends and realized capital gains from other investment income was no longer considered appropriate. In 1992, the division between investment income attributable to insurance transactions and that attributable to capital and surplus relates to the earnings base (i.e., the amount of funds in each section), not to the type of investments "associated" with each section.

[8]Compare the *NAIC Proceedings*, 1991 Volume IIA, "Insurance Expense Exhibit Working Group of the Blanks (EX4) Task Force," March 22, 1991, Attachment Four-B, page

- More funds are attributable to insurance transactions in the 1992 and subsequent Insurance Expense Exhibits than were attributed to policyholders in the pre-1992 IEE.

Profit or Loss

Part II of the IEE shows three columns of profit or loss:

- Column 17: Pre-tax profit or loss excluding all investment gain,
- Column 19: Profit or loss excluding investment gain attributable to capital and surplus, and
- Column 21: Total profit or loss.

All three columns are pre-federal income tax, though the "pre-tax" caption appears only in Column 17.

The profit or loss equals revenues minus expenditures, on an accrual (not paid) basis. Thus

- Column 1, "premium written," is on a paid basis. Column 2, "premium earned," is on an accrual basis. Earned premium (Column 2) is used in the profit and loss calculation, not written premium (Column 1).
- Columns 7 through 10, the loss reserves, loss adjustment expense reserves, and unearned premium reserves, are liabilities, not expenditures. Column 11, "agents' balances," is an asset, not a revenue item. Columns 7 through 11 do not enter the profit or loss calculation.

450: "The separate treatment of realized capital gains was eliminated with the effect of relating the same rate of return to capital and surplus that is related to insurance transaction funds." Compare also the letter from David F. Eley to Dan Atkinson of February 22, 1991, "Formula for Allocating Investment Income to Lines of Business" in the *NAIC Proceedings*, 1991 Volume IIA, page 454: "A second change is that all investment gain, including realized capital gain or loss, is allocated equally. There is no longer any disparity between the rate of return earned on funds derived from the insurance transaction and the rate of return earned on capital and surplus."

- Column 16, "other income" is a revenue item. Columns 3 through 6 (policy benefits, or losses incurred, loss adjustment expenses incurred, and policyholder dividends) and Columns 12 through 15 (expenses) are expenditure items, so they enter the profit or loss calculation.

The formula for Column 17 is therefore

$$\text{Column } 17 = \text{Columns } 2 + 16 - 3 - 4 - 5 - 6 - 12 - 13 - 14 - 15.$$

Investment income is a revenue item. Thus Column 19 equals Column 17 + Column 18, and Column 21 equals Column 19 + Column 20. This completes Part II of the IEE.

3. ALLOCATION PROCEDURES: AN ILLUSTRATION

The discussion above is abstract; an illustration should clarify the procedures. The example below reviews the various steps in the allocation of investment income:

- Allocating surplus to lines of business,
- Calculating the investment gain ratio,
- Calculating the prepaid ("acquisition") expense ratio,
- Determining the investment gain on funds attributable to insurance transactions, and
- Determining the investment gain attributable to capital and surplus.

In the illustration, we are completing the investment gain columns in Part II of the 1996 Insurance Expense Exhibit, using data from the 1995 and 1996 statutory financial statements. The IEE is for a commercial lines insurer that writes only two lines of business: workers compensation and other liability. All amounts in the illustration are in millions of dollars.

Allocation of Surplus to Lines of Business

We must first allocate policyholders' surplus to lines of business. Line 26 of Page 3 of the 1996 Annual Statement shows statutory surplus of $500 million at December 31, 1995, and of $700 million at December 31, 1996. The earned premiums, unpaid losses, unpaid allocated loss adjustment expenses, unpaid unallocated loss adjustment expenses, and unearned premium reserves for workers compensation and other liability shown in the table below are taken from the 1995 and 1996 Insurance Expense Exhibits, Columns 2, 7, 8, 9, and 10, for Rows 16 and 17. Alternatively, these figures may be taken from the Underwriting and Investment Exhibits in the 1995 and 1996 Annual Statements: earned premiums from Part 2 (Page 7), Column 4; unearned premium reserves from Part 2A (Page 8), Column 5; unpaid losses from Part 3A (Page 10), Column 5; and unpaid loss adjustment expenses from Part 3A (Page 11), Column 6.

(Figures in millions of dollars)	Workers Compensation		Other Liability	
	1995	1996	1995	1996
Earned premium, year ending 12/31/9_	350	450	200	200
Loss and LAE reserves, 12/31/9_	1,400	1,700	600	600
Unearned premium reserves, 12/31/9_	75	125	100	100

The IEE investment income allocation procedure requires that we allocate the company's *mean* surplus to line of business in proportion to

Mean net loss and loss adjustment expense reserves

+ Mean net unearned premium reserves

+ Earned premium for the year.

In this allocation, there is no adjustment of the unearned premium reserves for agents' balances or for prepaid expenses. Mean surplus is the average of the December 31, 1995, surplus and the December 31, 1996, surplus, or ($500 million + $700 million) ÷ 2 = $600 million. Mean surplus is used because investment income is earned over the course of the year.

Mean reserves are used, both for loss and loss adjustment expenses and for unearned premium. The 1996 earned premium is used, not the average 1995 and 1996 earned premiums.

- For workers compensation, the sum of mean reserves and annual earned premium is

$$[(1{,}400 + 1{,}700) \div 2] + [(75 + 125) \div 2] + 450$$
$$= \$2{,}100 \text{ million.}$$

- For other liability, the sum of mean reserves and annual earned premium is

$$[(600 + 600) \div 2] + [(100 + 100) \div 2] + 200$$
$$= \$900 \text{ million.}$$

- The mean surplus allocated to workers compensation is

$$(600) \times [2{,}100 \div (2{,}100 + 900)]$$
$$= \$420 \text{ million.}$$

- The mean surplus allocated to other liability is

$$(600) \times [900 \div (2{,}100 + 900)]$$
$$= \$180 \text{ million.}$$

Investment Gain Ratio

We proceed to determine the "investment gain ratio." The workers compensation and other liability figures are reproduced below.

(Figures in millions of dollars)	Workers Compensation		Other Liability	
	1995	1996	1995	1996
Agents' balances, 12/31/9_	35	45	10	10
Earned premium, year ending 12/31/9_	350	450	200	200
Loss and LAE reserves, 12/31/9_	1,400	1,700	600	600
Unearned premium reserves, 12/31/9_	75	125	100	100

In addition, we take the following investment income and capital gains figures from the 1995 and 1996 Annual Statements, from the following exhibits:

- Net investment income: Page 4, Line 8 = Underwriting and Investment Exhibit, Page 6, Part 1, Item 15.
- Realized capital gains: Page 4, Line 9 = Underwriting and Investment Exhibit, Page 6, Part 1A, Item 11.
- Unrealized capital gains: Page 4, Line 19 = Underwriting and Investment Exhibit, Page 6, Part 1A, Item 12.

Policyholders' surplus was $500 million at December 31, 1995, and $700 million at December 31, 1996, as shown on Page 3, Line 26.

Investment Income and Policyholders' Surplus ($000,000)

	1995	1996
Net investment income, year ending 12/31/9_	250	250
Realized capital gains, year ending 12/31/9_	100	50
Unrealized capital gains, year ending 12/31/9_	100	150
Policyholders' surplus, year ending 12/31/9_	500	700

The company's overall "investment gain ratio" is defined as

Net investment gain

$\div$ (Mean net loss and loss adjustment expense reserves

+ Mean net unearned premium reserves

− Mean net agents' balances

+ Mean policyholders' surplus).

"Net investment gain" for 1996 is used, not the average of the 1995 and 1996 values. It consists of net investment income earned (Line 8 of Page 4) and net realized capital gains or losses (Line 9 of Page 4). It does *not* include unrealized capital gains or losses (Line 19 of Page 4).

In this example, "net investment gain," or Line 9A of Page 4 of the Annual Statement, equals

$$\$250 \text{ million} + \$50 \text{ million} = \$300 \text{ million}.$$

The reserves, agents' balances, and surplus figures are needed for the company as a whole, not for each line of business. In this example, the figures are:

- Mean net loss and loss adjustment expense reserves are

$$(\$1{,}400 \text{ M} + \$1{,}700 \text{ M} + \$600 \text{ M} + \$600 \text{ M}) \div 2 = \$2{,}150 \text{ million};$$

- Mean net unearned premium reserves are

$$(\$75 \text{ M} + \$125 \text{ M} + \$100 \text{ M} + \$100 \text{ M}) \div 2 = \$200 \text{ million};$$

- Mean net agents' balances are

$$(\$35 \text{ M} + \$45 \text{ M} + \$10 \text{ M} + \$10 \text{ M}) \div 2 = \$50 \text{ million};$$

- Mean policyholders' surplus is

$$(\$500\text{ M} + \$700\text{ M}) \div 2 = \$600\text{ million};$$

- The "investment gain ratio" is

$$[\$300\text{ M} \div (\$2{,}150\text{ M} + \$200\text{ M} - \$50\text{ M} + \$600\text{ M})] = 10.34\%.$$

Prepaid ("Acquisition") Expenses

We now proceed to determine the prepaid expenses by line of business. We take the following data from the 1995 and 1996 Insurance Expense Exhibits:

(Figures in millions of dollars)	Workers Compensation		Other Liability	
	1995	1996	1995	1996
Written premium, year ending 12/31/9_	400	500	200	200
Commission & brokerage, year ending 12/31/9_	40	50	25	30
Taxes, licenses & fees, year ending 12/31/9_	8	10	5	5
Other acquisition expenses, year ending 12/31/9_	8	10	5	5
General expenses, year ending 12/31/9_	40	60	20	20

Prepaid expenses, or "acquisition expenses," are defined as

Commission and brokerage expenses incurred
+ Taxes, licenses, and fees incurred
+ Other acquisition, field supervision, and collection expenses incurred
+ One half ($\frac{1}{2}$) of general expenses incurred.

For prepaid expenses, we use the 1996 figures, not the average of the 1995 and 1996 figures.

- For workers compensation, prepaid expenses are

$$[\$50\text{ M} + \$10\text{ M} + \$10\text{ M} + (0.5)(\$60\text{ M})]$$
$$= \$100\text{ million.}$$

- For other liability, prepaid expenses are

$$[\$30\text{ M} + \$5\text{ M} + \$5\text{ M} = (0.5)(\$20)] = \$50\text{ million.}$$

The prepaid expense ratio is prepaid expenses divided by written premium, not earned premium (see the calculations below). Acquisition expenses, underwriting expenses, and premium taxes all relate to written premiums (or written exposures), not to earned premiums.

Investment Gain on Funds Attributable to Insurance Transactions

Column 18 of the Insurance Expense Exhibit asks for the "investment gain on funds attributable to insurance transactions." We now determine the appropriate Column 18 entries for workers compensation and other liability, using the accounting information from the company's 1995 and 1996 financial statements, as shown above.

For each line of business, the "investment gain on funds attributable to insurance transactions" is the company's investment gain ratio times the funds attributable to insurance transactions for that line of business.

In this example, the investment gain ratio is 10.34%. The funds attributable to insurance transactions are defined as

Funds attributable to insurance transactions

= Mean net loss and loss adjustment expense reserves

+ {Mean net unearned premium reserves

× [1 − (prepaid expenses ÷ written premiums)]}

− Mean net agents' balances.

Prepaid expenses were determined as $100 million for workers compensation and $50 million for other liability. The 1996 written premium is $500 million for workers compensation and $200 million for other liability, so the factor of

$$1 - (\text{prepaid expenses} \div \text{written premiums})$$

is 80% for workers compensation and 75% for other liability.

The mean values for reserves and agents' balances were determined above. Using these values, the funds attributable to insurance transactions are as follows:

- For workers compensation:

$$[(1{,}400 + 1{,}700) \div 2] + \{[(75 + 125) \div 2] \times 80\%\}$$
$$- [(35 + 45) \div 2] = \$1{,}590 \text{ million.}$$

- For other liability:

$$[(600 + 600) \div 2] + \{[(100 + 100) \div 2] \times 75\%\}$$
$$- [(10 + 10) \div 2] = \$665 \text{ million.}$$

The "investment gain on funds attributable to insurance transactions" is therefore 10.34% × $1,590 million = $165 million for workers compensation and 10.34% × $665 million = $69 million for other liability.

Investment Gain Attributable to Capital and Surplus

Column 20 of the Insurance Expense Exhibit asks for the "investment gain attributable to capital and surplus." We now determine the appropriate Column 20 entries for workers compensation and other liability, using the accounting information from the company's 1995 and 1996 financial statements.

For each line of business, the "investment gain attributable to capital and surplus" (Column 20) is the total investment gain for that line of business minus the "investment gain on funds attributable to insurance transactions."

- The "investment gain on funds attributable to insurance transactions" for workers compensation and other liability were determined above.

- The total investment gain for the line of business is the company's investment gain ratio times the investable funds associated with the line of business. The investable funds associated with the line of business equal the line's

  Mean net loss and loss adjustment expense reserves

  + Mean net unearned premium reserves

  − Mean net agents' balances

  + Allocated policyholders' surplus.

Note carefully the distinction between "investable funds attributable to insurance operations" and "investable funds associated with the line of business." The former has an adjustment for prepaid ("acquisition") expenses. The latter includes policyholders' surplus allocated to lines of business. As noted above, prepaid expenses are already deducted from surplus. So if surplus enters the formula, there is no deduction of prepaid expenses from the unearned premium reserves.

The mean values for reserves and agents' balances were determined above, as was the allocation of policyholders' surplus to lines of business. Using these values, the investable funds associated with the lines of business are as follows:

- For workers compensation:

$$[(1{,}400 + 1{,}700) \div 2] + [(75 + 125) \div 2] - [(35 + 45) \div 2] + 420 = \$2{,}030 \text{ million.}$$

The total investment gain = 10.34% of $2,030 million = $210 million. The investment gain attributable to funds from insurance operations is $165 million, so the investment gain attributable to capital and surplus is $45 million.

- For other liability:

$$[(600 + 600) \div 2] + [(100 + 100) \div 2] - [(10 + 10) \div 2] + 180 = \$870 \text{ million.}$$

The total investment gain = 10.34% of $890 million = $90 million. The investment gain attributable to funds from insurance operations is $69 million, so the investment gain attributable to capital and surplus is $21 million.

4. PART III—ALLOCATION TO LINES OF DIRECT BUSINESS WRITTEN

> The purpose of Part III is to allocate elements of profit (or loss) on a direct basis to lines of business. Part III simulates what the results were without reflecting the effect of reinsurance.
>
> —*NAIC Proceedings*, 1992, Volume IA, page 340

Part III, "Allocation to Lines of Direct Business Written," is similar to Part II, except that Part III shows direct experience whereas Part II shows net experience. Two other differences result from this:

- Because most Annual Statement exhibits show net experience, not direct experience, there are few direct cross-checks from Part III of the IEE to the Annual Statement.
- Because investment income relates to net experience, not to direct experience, there are no investment income columns in Part III of the IEE.

Profit or Loss

Part III of the IEE shows only underwriting gain or loss, in Column 17: "Pre-tax profit or loss excluding all investment gain." Column 17 of Part III is calculated in the same fashion as Column 17 of Part II: revenues minus expenditures, on an accrual basis.

Part III has no allocation of investment income. Investment income is earned on assets actually held by the company: that is, on assets net of reinsurance. Investment income on direct business is a theoretical amount. In 1991, the IEE Working Group of the NAIC debated whether to show a theoretical investment income figure for direct business. In April 1991, the Insurance Expense Exhibit Working Group of the Blanks (EX4) Task Force voted to show such a figure in Part III:

> The working group then discussed the proposal to calculate investment income on a direct basis. Members of the advisory committee expressed concerns that the proposal creates assumptions on what would exist on a direct basis; that the numbers go beyond the financial accounting data historically included in annual statement data; that companies would be projecting income that they do not have. Members of the working group indicated that it would assist a state in seeing the impact of the state's premium dollar without excluding the reinsured portion of the premium dollar. Further, the information would be qualified using italics and footnotes in order to caution users of the nature of the data. It was moved and second that investment income on funds attributable to insurance transactions be calculated on a direct basis using italics to qualify the data. Voted to adopt with California opposed.
>
> —*NAIC Proceedings*, 1991 Volume IIA, "Insurance Expense Exhibit Working Group of the Blanks (EX4) Task Force," April 13, 1991, Attachment Four-A, page 448.

The Working Group subsequently decided not to include such figures in Part III:

> Columns 18 and 19 on Part III, Allocation to Lines of Direct Business Written will be deleted. Column 18 developed an implicit investment gain on funds attributable to insurance transactions. Column 19 developed an implicit profit or loss excluding investment gain attributable to capital and surplus.
>
> —*NAIC Proceedings*, 1992, Volume IA, page 338.

5. THE MEASUREMENT OF PROFITABILITY

The previous sections of this paper have dealt with the statutory procedures for the allocation of policyholders' surplus and of investment income to lines of business in the Insurance Expense Exhibit. These statutory procedures, when combined with premium, loss, and expense data, enable regulators and companies to quantify the total return earned on each line of business.

The procedures embodied in the Insurance Expense Exhibit are one of many potential techniques for measuring total returns. The profitability of insurance operations is a widely debated public concern, and casualty actuaries have repeatedly been called upon to testify on behalf of various positions. It is important that actuaries understand the pros and cons of the major procedures, so that they may be better able to judge the appropriateness of each of them.

The issues in the measurement of insurance profitability may be grouped into the following categories:

- Prospective versus retrospective measurement of profitability,
- The allocation of policyholders' surplus,
- Run-off of past business versus writing of new business, and
- Insurance returns versus investment returns.

This section deals only with methods of measuring insurance profitability. It does not touch upon how much profit, or what rate of return, is appropriate. Other actuarial papers have dealt with this last issue, and the interested reader is referred to them.[9]

Prospective versus Retrospective

Much of the actuarial literature on profitability measurement deals with the pricing of insurance products. Pricing is fundamentally a prospective task. The Insurance Expense Exhibit, in contrast, is a retrospective measure of insurance profitability.

This difference pervades each of the other issues dealt with below. Actuarial procedures for prospective pricing are not necessarily appropriate for retrospective profitability measurement. The prospective versus retrospective dichotomy runs through many of the comments below.

Allocation of Surplus

The allocation of policyholders' surplus is the first step in the IEE allocation of investment income. The allocation of surplus is also an essential component of financial pricing models for insurance products, such as discounted cash flow models and internal rate of return models.[10] But the meaning of this phrase, the "allocation of surplus," differs radically in these two contexts.

The Insurance Expense Exhibit is allocating the company's actual policyholders' surplus to lines of business. If a company has more surplus than its peers, more surplus is allocated to each line. Conversely, a "capital-poor" company would have less surplus allocated to each line.

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[9]See, for instance, Feldblum [8], which discusses five commonly used methods of setting profit targets by line of business, and the discussion by Bault, which compares the methods in Feldblum's paper to those used by other actuaries.

[10]For a full discussion of the allocation of surplus in insurance pricing models, see Derrig [3] and Feldblum [6], and the references cited therein.

The pricing actuary using an internal rate of return model or a discounted cash flow model does *not* allocate the company's actual surplus to line of business. In fact, the pricing actuary may never even ask how much surplus the company has. Rather, the pricing actuary uses various "surplus assumptions." For instance, the pricing actuary may *assume* that each $1,000 of business that is written is "supported" by $500 of surplus.

The surplus *assumptions* used in pricing models may be compared with the *allocation procedures* in profitability measures. For instance, the most common surplus assumptions in pricing models are leverage ratios to premiums or to reserves. Similarly, the most common surplus allocation procedures in profitability measures are based on the premiums or reserves associated with each line of business. Let us examine more closely the relationship between the surplus assumptions and the allocation procedures.

The *retrospective surplus allocation* procedure begins with the company's actual surplus and proceeds to subdivide it by line of business. One of two methods is used for this allocation:

A. Allocation by leverage ratios, such as "premium to surplus ratios" or "reserves to surplus ratios," or

B. Allocation by the relative risk of each line of business, where risk may be quantified by the volatility of each line's loss ratio.[11]

The IEE uses leverage ratios, both premium to surplus and reserves to surplus. Some analysts have opined that reserve leverage ratios might serve as a proxy for risk. That is, the slow-paying lines, such as Products Liability and Medical Malpractice, are also the more risky lines. These more risky lines of business therefore have higher reserves to surplus ratios. Thus,

[11]Compare Feldblum [8], and the reviews by Philbrick and Todd, as well as Meyers [13].

an allocation of capital by reserves to surplus ratios is a method of allocating capital according to relative risk.

This reasoning is specious. Products Liability and Medical Malpractice are high risk lines and are also slow paying lines. But there are high risk lines which are fast paying lines, and there are slow paying lines that are low risk lines. Property insurance in regions prone to natural catastrophes, such as Homeowners insurance in the Gulf Coast states or earthquake insurance in California, are high risk lines, but their loss payout is rapid. Conversely, annuity payments, such as long-term disability coverage or workers' compensation pension claims, have slow payouts, but their risk is relatively small.[12]

The *prospective surplus assumptions* used for pricing purposes generally proceeds in one of two manners:

A. The needed surplus is determined for each line of business independently of the surplus required for other lines of business or of the overall surplus needs of the insurance enterprise. This needed surplus is calculated by consideration of the line's volatility in conjunction with selected calibration yardsticks, such as a "probability of ruin" yardstick or an "expected policyholder deficit" yardstick.[13]

B. The insurance industry as a whole is assumed to be neither over-capitalized nor under-capitalized. This assump-

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[12]For a more complete discussion of reserve duration, pricing risk, and reserving risk by line of business, see Feldblum [reply to Philbrick, 8]. Hodes, Feldblum, and Blumsohn [10] provide a detailed analysis of workers' compensation reserve volatility. Although compensation reserves have a long average duration, the steady payment pattern, which results from the mandated (statutory) benefits, causes the volatility of the reserves to be extremely low (on a discounted basis).

[13]On the "probability of ruin" yardstick, see Pentikäinen, Bonsdorff, Pesonen Rantala, and Ruohonen [15] or Daykin, Pentikäinen, and Pesonen [2]. On the "expected policyholder deficit" yardstick, see Butsic [1] or Hodes, Feldblum, and Blumsohn [10], Appendix B. Compare also the NAIC's risk-based capital formula, which determines capital requirements to guard against the underwriting risks in each line of business (see Feldblum [7]). Although the NAIC explicitly counsels against use of the risk-based capital results for pricing purposes, there is no theoretical reason why they could not be used for this purpose.

tion is justified by the efficiency of capital markets and the competitiveness of the insurance product markets. If the insurance industry were overcapitalized, returns on capital would be insufficient, and capital would leave the industry. Conversely, if the insurance industry were undercapitalized, returns on capital would be excessive, and additional capital would enter the industry.[14]

The overall industry capital would be allocated to lines of business, by means of leverage ratios or relative risk measures. This procedure differs from the former one in that the leverage ratios or the relative risk measures would be calibrated to achieve the existing industry surplus for all lines of business combined.

Once the appropriate leverage ratio is determined for any given line of business, any particular company's needed capital is determined from this leverage ratio. These are surplus assumptions. For any particular company, of course, the assumed surplus requirements for all lines of business combined will not equal its actual (held) surplus. (As mentioned earlier, this differentiates the prospective surplus assumptions from the retrospective surplus allocation procedure.)

In sum, the prospective surplus assumptions and the retrospective surplus allocation procedures often look similar. However, they serve different functions, and a procedure that is appropriate for one function may not applicable to the other function.

Reserve Run-Off versus New Business

Actuarial pricing is concerned with setting premium rates for new business. To accurately set rates, the pricing actuary must

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[14]See, however, Joskow [11], who objects to this reasoning, arguing that cartelization of the property-casualty insurance industry by means of rating bureaus has led to excessive prices along with overcapitalization, resulting in "normal" returns on capital and therefore equilibrium in the capital and product markets.

estimate the amount of investment income to be earned for each dollar of new business.

Investment income is earned on assets supporting reserves (both unearned premium reserves and loss reserves), as well as on the capital and surplus funds supporting the policy. But the "reserves" considered by the pricing actuary are not the reserves held by the company. Rather, they are the anticipated reserves that will be held in the future for each dollar of new business. This is the essence of prospective ratemaking.

The Insurance Expense Exhibit, in contrast, has a retrospective measurement of profitability. The investment income that is allocated is the investment income that is actually earned on the assets supporting the held reserves in each line of business.

The difference between the two approaches is clearest when the company grows or declines in a line of business.[15] Suppose a company is setting rates for workers compensation insurance, and the pricing actuary expects that losses will be paid out on average about four years after the accident date. If the actuary assumes an expected loss ratio of 75%, then (in a steady state) there will be about three dollars of reserves for each dollar of annual premium.

Similarly, for a steady state company, the Insurance Expense Exhibit will show about three dollars of reserves for each dollar of workers compensation premium. For the steady state company, the IEE information can be used in rate setting.

Suppose, however, that the company first began writing workers compensation in the current calendar year. To the pricing actuary, the past history of the company is irrelevant. The pricing actuary still assumes that there will be three "dollar-years" of

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[15]For a complete discussion of the effects of business expansion on statutory measures of total return, see Feldblum [6].

reserves for each dollar of premium earned during the year, and the future rates are determined accordingly.[16]

The actual held reserves of the company at the end of the year for each dollar of annual premium earned is probably only about 65¢. (There are 75¢ of incurred loss for each dollar of earned premium, and some of the losses have already been paid out by the end of the year.)

In other words, the IEE shows very little investment income earned on the workers compensation line of business. One is tempted to say that the IEE and the pricing actuary are addressing different questions and therefore they come up with different answers. The pricing actuary wants to ascertain the expected profitability of a new policy, so he or she considers the expected investment income on the assets supporting the future reserves of this policy. The IEE seeks to measure the retrospective profitability of a given line of business, so it considers the investment income earned during the past year on the held reserves.

This explanation is incorrect. The IEE aims to compute the "total profit or loss" in each line of business. In theory, one should compute this figure by using discounted reserves. For prospective ratemaking, one would use anticipated losses discounted at an expected interest rate or investment yield. For retrospective profitability measurement, one would use actual losses discounted at market interest rates or current investment yields.

The Insurance Expense Exhibit is wedded to statutory accounting. Accordingly, it uses undiscounted loss reserves, not discounted reserves, for computing the underwriting profit margin. This figure, shown in Column 17 of Part II, "Pre-tax profit

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[16]Note carefully the units of each insurance element. Reserves are a "stock," or a balance sheet item existing at a given valuation date. Earned premium is a "flow," or an income statement items, whose magnitude depends on the length of time in the valuation period. When pricing actuaries compare these two elements, they generally assume a one-year time period. In other words, they are comparing *reserve-years* and *annual earned premium*. Because this convention is so common, it is rarely stated explicitly, and one generally reads of a comparison of reserves with earned premium.

or loss excluding all investment gain," uses the current calendar year earned premiums, incurred losses, and incurred expenses.

In theory, Column 19 of Part II, "Profit or loss excluding investment gain attributable to capital and surplus," should be computed by using discounted loss costs. In practice, the IEE uses "investment income on funds attributable to insurance transactions" as a proxy for the amount of the discount. This procedure is reasonable for companies in a steady state. It is misleading when a company grows or declines significantly in a particular line of business.

Insurance Returns and Investment Returns

A traditional insurance industry trade practice is to divide a company's operational results into "underwriting income" and "investment income." Underwriting income is defined as earned premiums minus incurred losses minus incurred expenses. Investment income consists of interest, dividends, and rents earned on the company's invested assets. Capital gains, either realized capital gains or all capital gains, are generally included in investment income as well.

The insurance trade press often says that "underwriting operations" were not profitable, because underwriting income was negative, and that the insurance industry was "saved" only by its investment income. Such a view, of course, is primarily for public consumption. Underwriting income that takes no account of the time value of money does not properly measure the profitability of insurance operations.

The Insurance Expense Exhibit rectifies this problem by allocating the investment income earned by the company to lines of business. In doing so, it must consider what investment income to allocate.

There are three interlocking components of this issue.

1. "What portion of the company's investment income should be considered when measuring the return on insurance operations?" The IEE procedure addresses this question by considering separately (a) the investment income on funds attributable to insurance transactions and (b) the investment income attributable to capital and surplus. One may take either of the two common views—all investment income or only investment income attributable to insurance transactions—and find the appropriate figures in the IEE.[17]

2. "Which investable assets should be associated with funds attributable to insurance transactions and which investable assets should be associated with capital and surplus?" There is a common view that loss reserves and unearned premium reserves should be supported by fixed income securities, such as bonds and mortgages, because of the relative safety of these instruments. Capital and surplus, however, may be supported by common stock and other equities (such as real estate), because of the higher yields afforded by these financial instruments (compare Noris [14]).

   The pre-1992 Insurance Expense Exhibit differentiated between the returns on funds attributable to insurance transactions and the returns on capital and surplus. Bond coupon payments, for instance, were more likely to be associated with the former, whereas common stock dividends were more likely to be associated with the latter.

   The current IEE did away with this differentiation. The "common view" mentioned above is but one investment strategy among many, and it is not necessarily the optimal one. It is not the place of the IEE to implicitly

[17]"Investment income attributable to insurance transactions" is also called "investment income on the insurance cash flow" or "investment income on policyholder funds."

prescribe or even to presume the investment strategies of individual companies.

3. "What investment returns should be allocated to the insurance operations?" That is, "What investment income should be considered a part of insurance operations, and what investment income should be considered separately, either as unanticipated gains or losses or as attributable to the superior or inferior skills of the investment department?"

   To clarify this question, suppose that the insurance company's investment portfolio consists of Treasury bonds yielding 8% per annum, investment grade corporate bonds yielding 10% per annum, lower grade corporate bonds, some of which are yielding between 12% and 15% per annum and some of which have defaulted, common stocks with various dividend yields, some realized capital gains, and some unrealized capital losses. What parts of this investment income should be allocated to lines of business?

One may answer this question in several ways.

A. *Allocate all investment income*: One view says that the regulator should not decide what investment returns are normal and what returns are extraordinary. Roth [17] champions this view, arguing that all investment income should be taken into account.

B. *Differentiate by type of investment income*: The IEE allocates net investment income earned (i.e., interest, dividends, and rents) and realized capital gains and losses to the lines of business (i.e., to the insurance operations). Unrealized capital gains and losses are not included in the IEE allocation of investment income.

The theoretical justification for the distinction between realized and unrealized capital gains is that unrealized capital gains represent unanticipated and random market movements that do not reflect the company's investment strategy. Moreover, unrealized capital gains are often reversed as the market turns, unlike the steady receipt of interest, dividends, and rents. For these reasons, unrealized capital gains and losses should not be included in the company's investment income.

Accounting conventions, unless theoretically justified, are a hindrance to proper measurement of profitability. The justification above is particularly dubious. The realization of capital gains is often driven by federal income tax considerations or by short-term needs for cash. In fact, the inclusion of only realized capital gains in investment income often distorts profitability measurement. Two examples should clarify this:

a. *Federal Income Taxes*: Suppose that companies ABC and XYZ have the same investment portfolios, each having a large common stock component with substantial unrealized capital gains. Company ABC has an underwriting gain during the year. To avoid incurring additional income tax liabilities, it leaves the capital gains unrealized. Company XYZ has a large underwriting loss during the year. To compensate for the operating loss, it sells stocks and realizes the capital gains.

   In truth, the investment returns of the two companies are identical, and they should be treated in identical fashions for the purpose of profitability measurement. Tax considerations determined whether the capital gains would be realized. This should not be allowed to distort the measurement of profitability.

b. *Cash Needs*: Suppose that companies ABC and XYZ differ mainly in their need for cash. Company ABC is "cash poor," so it invests primarily in short- and medium-term Treasury securities and mortgage backed securities with consistent coupon payments. Company XYZ has no immediate need for cash, so it invests heavily in a diversified portfolio of aggressive, growth stocks, with low dividend payments but high expected capital gains. Its investment strategy calls for keeping the stocks for the long-term.

In this example, company ABC is trading expected long-term return for immediate cash. Yet the IEE sees the opposite: it shows higher investment returns for company ABC than for company XYZ.

The reason for the exclusion of unrealized capital gains from the allocation of investment income in the IEE is that unrealized capital gains and losses are a direct credit or charge to surplus (see Feldblum [5]). They do not flow through the statutory income statement, just as they do not flow through the GAAP income statement and they are not included in taxable income. Unfortunately, this accounting attribute of unrealized capital gains distorts the actuarial measurement of profitability.

C. *Allocate "risk-free" investment income*: An approach that is gaining significant acceptance in the actuarial community is that only a "risk-free" investment return should be ascribed to underwriting operations.

The remaining investment income—that is, the difference between the risk-free return and the actual return—is the reward either for the assumption of investment risk by the company or for the superior (or inferior) expertise of the investment department (see, for example Woll [19] or Lowe [12]). In this approach, the investment income allocated to lines of business does not depend on the type of assets owned by the company or on the investment performance of the company's securities. Rather, all investable assets would be assigned a risk-free rate of return for the purpose of allocating investment income to lines of business. The remaining investment income stems from the performance of the investment department; it has nothing to do with the total return associated with the insurance operations.

6. PERSPECTIVES ON THE IEE PROCEDURES

The previous section discussed the theoretical underpinnings of the IEE procedure for the measurement of profitability, though with few normative comments on the general appropriateness of this procedure. The primary purpose of this paper is to describe the IEE procedure and to place it within the broader context of profitability measurement procedures. It is not the purpose of this paper to defend or to criticize the statutory procedures.

The method used in the Insurance Expense Exhibit and discussed in this paper may be viewed as an "official" NAIC method. Casualty actuaries must understand this method well, both for completing the statutory financial statements and for evaluating the reasonableness of the statutory figures.

However, the IEE allocation procedures must be treated with caution: they are useful for some purposes but not for others. The following comments by two actuaries who have worked extensively in state regulation (and particularly with the measurement of insurance profitability) should make this clear.

Mr. Martin Rosenberg, formerly with the New Jersey Insurance Department, writes [16]:

> The allocation of surplus to the various lines of business [in the Insurance Expense Exhibit] can (and will) cause much confusion because the allocation is arbitrary.
>
> ... both from the regulator's point of view as well as the insurance company's point of view, the financial results shown in the IEE for the various lines of business can not and should not be used to measure whether the premium rates are adequate or excessive. Nor should the IEE figures be used to determine if the capital used to support a line of business is earning a satisfactory return ...
>
> ... a regulated enterprise has a right to the opportunity to earn an adequate rate of return. However, the right to an adequate rate of return does not extend to all individual services provided by the regulated entity but rather applies to the enterprise as a whole ...
>
> ... This principle was applied to a 1992 case in which an insurer wanted to increase personal auto rates to recoup assessments to support the personal auto residual market. An Administrative Law Judge in New Jersey decided in that case that a multi line insurer's right to a fair rate of return pertains to the enterprise as a whole and does not extend to each line of insurance. Thus, the relevant measure of the insurer's rate of return was the rate of return of all lines of business combined and not just personal auto insurance.
>
> Insurance companies often price lines of business such as homeowners and personal auto in tandem. For ex-

> ample, typically one consideration in deciding whether to sell personal auto at a discount is whether the policyholder also has a homeowners policy with the same company ...
>
> The point is that from the company's point of view, surplus is not allocated on a line by line basis. An independent measure of the return of personal auto and homeowners is not useful to the company because the financial results of personal auto and homeowners are dependent on each other.
>
> The rate of return for the entire enterprise is the appropriate consideration from both the regulator's and company's point of view in many important applications. Therefore, it must be recognized that an allocation of surplus to the various lines of business may be arbitrary.

Mr. Richard Roth, Assistant Commissioner in the California Department of Insurance, made a tongue-in-cheek observation regarding the IEE allocation procedures during a recent panel presentation [18]:

> ... according to the new IEE, since the underwriting and investment income is allocated based on national surplus, the loss of surplus caused by Hurricane Andrew will cause the profitability of automobile insurance in Massachusetts to improve.

These comments underscore the need for casualty actuaries to carefully analyze the profitability results that may be inferred from the Insurance Expense Exhibit.

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APPENDIX A

THE PART II ENTRIES

Appendix A describes, column by column, the entries in Part II of the Insurance Expense Exhibit. The phrase in italics at the start of each subsection gives the column number and the column caption. The description notes the cross-checks to other statutory exhibits, the type of allocation to line of business, and sundry differences between the entries in the IEE and those in the Annual Statement.

Readers interested only in the theoretical aspects of the allocation of investment income to line of business do not need the information in this appendix, though they may find it a useful reference. Readers who must complete an actual IEE will find this information essential.

Premiums

1. *Premiums written*: The written premium entries correspond by line of business to the "Underwriting and Investment Exhibit," Part 2B (Page 9), "Premiums Written," Column 4, "Net premiums written."[18]

2. *Premiums earned*: These entries correspond by line of business to Page 7, "Underwriting and Investment Exhibit," Part 2, "Premiums Earned," Column 4, "Premiums earned during year."[19]

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[18]These Annual Statement entries are carried to Part 2 of the "Underwriting and Investment Exhibit" (Page 7), "Premiums Earned," Column 1, "Net premiums written." The total for all lines combined is also carried to Page 14, "Exhibit 2—Reconciliation of Ledger Assets," Line 1, "Net premiums written." This figure should agree with Line 32 of the IEE.

[19]In addition, the aggregate amount for all lines combined on Line 32 of the IEE ("Total") should correspond to the entry on Page 4, "Statement of Income," Line 1, "Premiums earned."

Accrued retrospective premiums are reported in two ways in the Annual Statement:

- They may be reported as a separate asset and not as an offset to the unearned premium reserve. This is the treatment on the statutory balance sheet, where an asset for accrued retrospective premiums is shown on Line 10.3. The unearned premium reserve on Line 9 of Page 3 does *not* have an offset for accrued retrospective premiums, since it is taken from Part 2A of the "Underwriting and Investment Exhibit," Page 8, Column 5, Line 34, not Line 32. (The line of business offsets in Column 4, Lines 1 through 31, are removed in Column 5, Line 33.)
- They may be reported as an offset to the unearned premium reserve and thereby included in earned premiums. This is the treatment in Part 2 of the Underwriting and Investment Exhibit (Page 7) and in the earnings statement (Page 4).

An illustration should help clarify this. Suppose an insurer has the following accounting entries for written premium, unearned premium reserves, and accrued retrospective premium reserves:

- Written premium during the year = $20,000,000.
- Unearned premium reserve (liability):
    - Beginning of year = $6,000,000.
    - End of year = $8,000,000.
- Accrued retrospective premium reserve (asset):
    - Beginning of year = $1,000,000.
    - End of year = $2,000,000.

To determine earned premiums, the Underwriting and Investment Exhibit, on Pages 7 and 8 of the Annual Statement, treats accrued retrospective premiums as an offset to unearned premiums. In this example, the net unearned premium reserve at the beginning of the year is $5 million and at the end of the year it is $6 million. Earned premium for the year is $19 million (= written premium minus the change in reserve).

For the balance sheet, the full unearned premium reserve of $8 million is shown on Page 3, Line 9, Column 1. The $2 million of accrued retrospective premiums are carried to Page 2, Line 10.3, Column 2, a non-admitted portion is deducted in Column 3, and the net admitted portion,is shown in Column 4.

The IEE uses the accounting procedure in the Underwriting and Investment Exhibit. Accrued retrospective premiums are reflected in the unearned premium reserves and in premium earned (Columns 2 and 10), not in agents' balances (Column 11). See also the discussion below of Column 11.

The earned premium entries should also equal the figures in Schedule P, Part 1, Column 4, Line 11: "net earned premiums in the current year," according to the Schedule P subdivision of lines of insurance. In most instances, Schedule P does not have as fine a breakdown by line of business as the IEE has. For example, Schedule P combines "Fire," "Allied lines," "Inland Marine," "Earthquake," "Glass," and "Burglary and Theft" into a single "Special Property" category, though these are separate lines of business in the IEE. In a few instances, however, both Schedule P and the IEE have a finer breakdown by line of business than other Annual Statement exhibits have.

10. *Unearned Premium Reserves*: The unearned premium reserves correspond by line of business to Page 7, "Underwriting and Investment Exhibit," Part 2, "Premiums Earned," Column 4, "Unearned Premiums." These unearned premium reserves reflect accrued retrospective premiums; see also the discussions of Column 2 and of Column 11.

Dividends

3. *Dividends to policyholders*: Dividends to policyholders on *net* business is reported in aggregate (all lines combined) on Page

4, "Statement of Income," Line 14a, "Dividends to policyholders." The "allocation" to lines of business in the IEE is a direct allocation, not a formula allocation. That is, the insurer knows which policies received the dividends and therefore to which lines of business they should be allocated.[20]

Dividends to policyholders on *direct* business are reported

- by line of business in the IEE, Part III;
- by state in the Annual Statement, Schedule T, "Exhibit of Premiums Written," Column 4, "Dividends paid or credited to policyholders on direct business;" and
- by line and by state on Page 15, Column 4, of the Annual Statement, "Dividends paid or credited to policyholders on direct business."[21]

Paid dividends to policyholders are shown on Page 14 of the Annual Statement, "Exhibit 2—Reconciliation of Ledger Assets," Line 16, "Dividends to policyholder on direct business less $____ dividends on reinsurance assumed or ceded (net)." Paid dividends may be reconciled to incurred dividends by adding the change in reserves:

Paid dividends – beginning of year reserve + end of year reserve

= incurred dividends.

The required reserve figures are shown on Page 3 of the Annual Statement, "Liabilities, Surplus and Other Funds," Line 10(b): "Dividends to policyholders declared and unpaid," Column 1

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[20]In some cases, the policy form does not correspond to Annual Statement lines of business. For instance, a policy may cover both "Other Liability" and "Products Liability," and the dividend may not differentiate between them. In such instances, the insurer must make a formula allocation of the dividend.

[21]Dividends to policyholders is more closely related to direct business than to net business. Most reinsurance arrangements reimburse the primary insurer for losses paid to policyholders, not for dividends paid to policyholders.

(current year) and Column 2 (previous year). In other words

Page 4, Line 14a (incurred dividends)

= Page 14, Line 16 (paid dividends)

− Page 3, Line 10(b), Column 2 (beginning of year reserve)

+ Page 3, Line 10(b), Column 1 (end of year reserve).

Note that *statutory* accounting requires reserves only for declared dividends to policyholders, not for projected (but undeclared) dividends to policyholders. *GAAP* requires dividend reserves for projected dividends as well. For instance, suppose that on each March 1 the insurer's board of directors declares dividends to policyholders based on the previous calendar year's experience. For GAAP financial statements, the company must project expected dividends relating to the experience of the current accounting period and book these as a liability, even though the company will have no legal obligation to policyholders until the declaration by the board of directors on March 1. For statutory financial statements, no estimate need be made and no reserve need be booked for undeclared dividends.[22]

Losses and Loss Adjustment Expenses

4. *Incurred Loss*: The incurred losses correspond by line of business to Page 10 of the Annual Statement, "Underwriting and Investment Exhibit," Part 3—Losses paid and incurred," Column 7, "Losses incurred, current year."

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[22]Compare AICPA's *Audits of Property and Liability Insurance Companies* (New York: American Institute of Certified Public Accountants, 1993): "GAAP requires policyholder dividends that are undeclared as of the balance sheet date to be estimated and accrued. Under SAP, however, policyholder dividends are not recorded as liabilities until declared." See also David L. Holman and Chris C. Stroup, "Generally Accepted Accounting Principles," in Insurance Accounting and Systems Association, Inc., *Property-Liability Insurance Accounting*, Sixth Edition (Durham, NC: 1994), page 14-7: "Under SAP, dividends to policyholders generally are not recorded as liabilities until they are declared by the company's board of directors. GAAP requires that all undeclared policyholder dividends be accrued at the balance sheet date, using an estimate of the amount to be paid."

These are *calendar year* incurred losses. The incurred losses in Schedule P are *accident year* incurred losses. The supporting exhibits in Schedule P (Parts 2, 3, and 5) show losses combined with allocated loss adjustment expenses. Losses are shown separately from allocated loss adjustment expenses only in Part 1 of Schedule P. To determine calendar year incurred losses from Schedule P, one must use Annual Statements of successive years and subtract the incurred losses (for all accident years combined) in the previous statement from the corresponding incurred losses in the current statement.[23]

Paid losses by line of business are shown on Page 10 of the Annual Statement, "Underwriting and Investment Exhibit," Part 3, Column 4, "net losses paid." Net loss reserves by line of business are shown in Column 5 for the current year and Column 6 for the previous year. Incurred losses are therefore Columns 4 + 5 − 6, or

$$\text{Paid losses} - \text{beginning of year reserve} \\ + \text{end of year reserve} = \text{incurred losses.}$$

5, 6, 8, and 9: *Loss adjustment expenses*: Unpaid loss adjustment expenses are shown by line of business separately for allocated and unallocated expenses in Columns 8 and 9 in the IEE. Total (i.e., allocated plus unallocated) unpaid loss adjustment expenses by line of business are shown on Page 11 of the Annual Statement, "Underwriting and Investment Exhibit," Part 3A, Column 6, "Unpaid loss adjustment expenses." Thus, the sum of Columns 8 and 9 in the IEE should equal Column 6 of Page 11 of the Annual Statement.

Incurred loss adjustment expenses are shown by line of business separately for allocated and unallocated expenses in Columns 5 and 6 of the IEE. Calendar year incurred loss adjust-

[23]Care must be taken in the treatment of the "prior years" lines in Schedule P. See the discussion below regarding loss adjustment expenses.

ment expenses are *not* shown by line of business in the Annual Statement. The aggregate loss adjustment expenses incurred for all lines combined is shown on Page 4, "Statement of Income," Line 3, "Loss expenses incurred," and on Page 12, "Underwriting and Investment Exhibit," Part 4, "Expenses," Line 22, Column 1, "Total loss adjustment expenses incurred."

Schedule P shows cumulative paid loss adjustment expenses by line of business and by accident year in Part 1, Columns 7 and 8 for allocated expenses and in Column 10 for unallocated expenses. The loss adjustment expenses paid in the current calendar year can be derived from successive Annual Statements. For instance, the unallocated loss adjustment expenses paid in the current calendar year equals

- Part 1, Column 10, Line 12 ("total") of the current year's Schedule P
- Part 1, Column 10, Line 12 – Line 2 – Line 1 (= "total" – "oldest accident year" – "prior years") of the previous year's Schedule P.[24]

The previous year's unpaid loss adjustment expense is found on Page 11, "Underwriting and Investment Exhibit," Part 3A, Column 6, of the previous year's Annual Statement. As is true for losses (see above), the current calendar year's incurred loss adjustment expenses, as reported in the IEE, equals the current calendar year's payments plus the change in reserve.

7. *Unpaid losses*: Unpaid losses by line of business should agree with the entries on Page 11 of the Annual Statement, "Underwriting and Investment Exhibit," Part 3A, Column 5, "Net

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[24]The "oldest accident year" in the previous year's Schedule P is no longer separately recorded in the current year's Schedule P, so it is removed from the calculation. The "prior years" line in Part 1 of Schedule P shows the paid amount in the current calendar year, not a cumulative paid amount. Since one wants the amount paid in the current calendar year for this cross-check, one wants the current statement's figure for the "prior years" line, not the change from last year's figure to this year's figure.

losses unpaid excluding loss adjustment expenses." The aggregate figure for all lines combined is also shown on Page 3, "Liabilities, Surplus and Other Funds," Line 1, "Losses," Column 1 (current year).

Agents' Balances

11. *Agents' balances*: The aggregate total for all lines combined should equal the sum of

- Line 10.1, "Premiums and agents' balances in the course of collection," and
- Line 10.2, "Premium, agents' balances and installments booked but deferred and not yet due."

Line 10.3, "Accrued retrospective premiums," is *not* included in the IEE definition of agents' balances, since they are already deducted from unearned premium reserves. On Page 8 of the Annual Statement, "Underwriting and Investment Exhibit," Part 2A, "Recapitulation of all Premiums," accrued retrospective premiums are entered as negative amounts in Column 4, "Reserve for rate credits and retrospective adjustments based on experience." The "total reserve for unearned premiums" in Column 5 is the sum of Columns 1 through 4, where Columns 1 through 3 are

- Column 1: Amount unearned, running one year or less from date of policy,
- Column 2: Amount unearned, running more than one year from date of policy, and
- Column 3: Advance premiums.

Earned premium is defined as written premium minus the change in the unearned premium reserve, or

Earned premium

= Written premium

+ Beginning of year unearned premium reserve

– End of year unearned premium reserve.

A decrease in the end of year unearned premium reserve causes a corresponding increase in the year's earned premium. The accrued retrospective premium asset decreases the end of year unearned premium reserve on Page 8, so it increases the earned premium on Page 7, Column 4, of the Annual Statement. The "profit or loss" in Column 17 of the IEE begins with the earned premium in Column 2. Thus, accrued retrospective premiums are already included in the "profit or loss" figure, and they need not be entered again in "agents' balances" (Column 10).[25]

In most cases, the allocation of agents' balances to line of business is a direct allocation, not a formula allocation. The allocation shown in Column 10, as well as the allocation for the previous year end, is used in the allocation of investment income by line of business (see below).

Underwriting Expenses

12, 13, 14, and 15. *Expenses*: The expense items for all lines of business combined should equal the corresponding amounts in Part I of the IEE, as follows:

- IEE, Part II, Column 12, "Commission and brokerage expenses incurred," Line 32 (total) should equal IEE, Part I, Column 2, "Acquisition, field supervision and collection expenses," Line

[25]Compare the *NAIC Proceedings*, 1991 Volume IIA, "Insurance Expense Exhibit Working Group of the Blanks (EX4) Task Force," March 22, 1991, Attachment Four-B, Page 450: "Unearned premium reserves will be net of retrospective premiums, therefore line 9.3 will no longer be subtracted from reserves." (The 1991 Line 9.3 is the current Line 10.3.)

2h, "Net commission and brokerage." The allocation to line of business is generally a direct allocation.

- IEE, Part II, Column 13, "Taxes, licenses and fees incurred," Line 32 (total) should equal IEE, Part I, Column 4, "Taxes, licenses and fees," Line 22, "Total." The allocation to line of business is a combination of direct allocation and formula allocation.

- IEE, Part II, Column 14, "Other acquisition, field supervision and collection expenses incurred," Line 32 (total) should equal IEE, Part I, Column 2, "Acquisition, field supervision and collection expenses," Line 22, "Total," minus Line 2h, "Net commission and brokerage." The allocation to line of business is generally a formula allocation (see New York Regulation 30).

- IEE, Part II, Column 15, "General expenses incurred" Line 32 (total) should equal IEE, Part I, Column 3, "General expenses," Line 22, "Total." The allocation to line of business is generally a formula allocation (see New York Regulation 30).

16. *Other Income less Other Expenses*: The aggregate amount for all lines of business combined in this column should equal Page 4 of the Annual Statement, Line 13 minus Line 5. Page 4, Line 13 is "total other income," and it may be a positive or negative amount. Page 4, Line 5 is "aggregate write-ins for underwriting deductions," and it is generally a positive amount.

Do not confuse the "other expenses" in Column 16 of the IEE, Part II, with "other underwriting expenses" on Page 4, Line 4, of the Annual Statement. The "other underwriting expenses" on Page 4, Line 4, equals the sum of Columns 12, 13, 14, and 15 in Part II of the IEE.

Also, note that the "net gain or loss from agents' or premium balances charged off," which appears on Line 10 of Page 4 of the Annual Statement, shows up on Part II of the IEE in Column 16, "other income less other expenses," not in Column 11, "agents' balances." Column 11 shows the currently admitted portion of agents' balances. Recoveries of amounts previously not admitted, as well as charge-offs of amounts previously admitted, show up in Column 16.

APPENDIX B

THE PART III ENTRIES

Appendix B describes, column by column, the entries in Part III of the Insurance Expense Exhibit. The phrase in italics at the start of each subsection gives the column number and the column caption. The description notes the cross-checks to other statutory exhibits, the type of allocation to line of business, and sundry differences between the entries in the IEE and those in the Annual Statement.

Most readers will not need the information in Appendix B, since there is no allocation of investment income in Part III of the IEE. Readers who must complete an actual IEE, however, will find this information essential.

1. *Premiums written*: Direct premiums written by line of business are shown in the Underwriting and Investment Exhibit, Page 9, Part 2B, "Premiums written," Column 1, "direct business."

2. *Premiums earned* and 3. *Dividends to policyholders*: Direct premiums earned and dividends to policyholders on direct business are shown in the Annual Statement in Schedule T by state (Columns 3 and 4) and on Page 15 by line of business and by state (Columns 3 and 4). The column headings in the IEE note the cross-check to Schedule T, not to Page 15. The cross-check to Schedule T applies to the all lines combined row, not to the individual line of business amounts.

4. *Incurred loss* and 7. *Unpaid losses*: Direct unpaid losses are shown in the Annual Statement by line of business in the Underwriting and Investment Exhibit, Page 11, Part 3A, "Unpaid losses and loss adjustment expenses," Column 1a, "Adjusted or in process of adjustment: direct," plus Column 4a, "Incurred but not reported: direct;" in Schedule T by state (Column 7); and on Page 15 by line of business and by state (Column 8). The

column headings in the IEE note the cross-check to Schedule T, not to the Underwriting and Investment Exhibit or to Page 15. The cross-check to Schedule T applies to the all lines combined row, not to the individual line of business amounts.

Direct losses incurred are shown in Schedule T and on Page 15, but not in the Underwriting and Investment Exhibit. Direct paid losses are shown in all three places. Direct incurred losses by line of business can be derived from the Underwriting and Investment Exhibits of successive Annual Statements, since incurred losses equal paid losses plus the change in reserves. In any case, the column headings in the IEE note the cross-check to Schedule T, not to Page 15 or to the Underwriting and Investment Exhibits of successive Annual Statements. The cross-check to Schedule T applies to the all lines combined row, not to the individual line of business amounts.

5, 6, 8, and 9. *Loss adjustment expenses*: Loss adjustment expenses are not reported in Schedule T, and direct loss adjustment expenses are not shown in the Underwriting and Investment Exhibit. The only cross-check listed in the IEE instructions or the NAIC *Proceedings* says:

> IEE Part III, Columns 5, 6, 8 and 9 must agree with IEE Part II, Columns 5, 6, 8 and 9, respectively, excluding expense relating to reinsurance assumed and ceded.

However, direct allocated loss adjustment expenses incurred and unpaid are shown on Page 15 by line of business and by state (Columns 10 and 11), so a cross-check is available to Columns 5 and 8 of Part III of the IEE.

10. *Unearned premium reserves*: Unearned premium reserves are not shown in Schedule T, and direct unearned premium reserves are not shown in the Underwriting and Investment Exhibit. For this column, however, the IEE instructions do reference the

cross-check to Page 15:

> Column 10 must agree with the sum of Page 15, Column 5 totals for all states plus any alien business.

11. *Agents' balances,* 14. *Other acquisition, field supervision, and collection expenses incurred,* 15. *General expenses incurred,* and 16. *Other income less other expenses*: There are no direct cross checks to any of these columns. The IEE instructions say that these figures should agree with the Part II entries after exclusion of balances or expenses related to reinsurance assumed or ceded.[26]

12. *Commissions and brokerage expenses incurred*: In Part I of the IEE, as well as on Part 4 of the Underwriting and Investment Exhibit in the Annual Statement, commissions and brokerage expenses are divided into seven categories:

2a. Direct excluding contingent,

2b. Reinsurance assumed excluding contingent,

2c. Reinsurance ceded excluding contingent,

2d. Contingent—direct,

2e. Contingent—reinsurance assumed,

2f. Contingent—reinsurance ceded, and

2g. Policy and membership fees.

Commission and brokerage expenses should appear in Column 2 of Part I: "Acquisition, field supervision and collection expenses."

The column heading in Part III of the IEE notes that the total for Column 12 for all lines of business combined should equal

[26]Agents' balances related to reinsurance ceded are disclosed on Page 2, Lines 10.1 and 10.2 (in the parenthetical phrase in the line label), though there is no corresponding disclosure for amounts related to reinsurance assumed.

the sum of Rows 2a and 2d, Column 2, from Part I. Commissions and brokerage expenses were added to the Page 15 exhibits in 1992 (Column 12), so a cross-check by line of business is now available as well.

13. *Taxes, licenses and fees incurred*: The IEE instructions list no explicit cross-check. Taxes, licenses, and fees were also added to the Page 15 exhibits in 1992, so a cross-check by line of business is available.

RETROSPECTIVE RATING: 1997 EXCESS LOSS FACTORS

WILLIAM R. GILLAM AND JOSE R. COURET

"Fly me to the moon, and let me swing among the stars ... "

—Bart Howard

Abstract

The NCCI methodology for deriving Excess Loss Factors (ELFs), based largely on research performed in 1986, is documented in "Retrospective Rating: Excess Loss Factors" by William R. Gillam [2]. This paper updates that 1991 paper. The changes in the way ELFs are produced have been significant, if not extensive. The work done to support those changes was extensive.

In the Fall of 1992, after an intense but focused study, NCCI updated the parametric size-of-loss distributions described in Gillam's paper. The associated changes were in production for most 1993 filings.

A much more in-depth review of the ELF model was completed in 1995. In this report, we detail some of the investigations made in that review and the features of the resulting model.

The researchers checked to see that the existing groupings of claim types were optimal, or at least superior to any other obvious groupings. They also determined that the groupings of states by benefit type (escalating, non-escalating, and limited) was not justified.

Loss distributions by claim group have again been updated, this time using a new method to model fifth-to-ultimate loss development, overcoming the lack of individual loss information after that report. (The Workers Compensation Statistical Plan ends at fifth report.)

The risk loadings for parameter risk and contagion were also updated to be more appropriate in an open competition rating environment.

1. MODELING OF LOSSES BY INJURY TYPE

Under the Workers Compensation Statistical Plan (WCSP), an *injury type* code is reported for each claim—corresponding to the carrier's belief at the valuation date as to the ultimate injury type of the claim. The injury types are: Fatal, Permanent Total (PT), Permanent Partial (PP), Temporary Total (TT), Medical-Only, and Contract Medical. For ratemaking, NCCI makes the distinction between a *Major* and *Minor* Permanent Partial claim according to whether its indemnity component is above or below a state-specific *critical value.* This results in seven injury types being coded into NCCI's databases.

As described in the 1991 paper by Gillam, Excess Loss Factors (ELFs) were based on weighted excess ratios for each of three injury groups. In the 1995 study, we tried to determine the ideal grouping of injury types.

Description of NCCI Approach

In the 1986 study, curves were fit to data from each of a sample of states. Combining data for various states prior to curve fitting was not done, apparently due to concern over differences in scale between the states. Consequently, one problem the researchers encountered was the scarcity of data within each state for PT claims. Their solution was to combine PT claims with Major PP claims, yielding the composite injury type *PT/Major.*

Similarly, *TT/Minor* is a combination of TT and Minor PP claims.

In the 1992 study, NCCI developed a procedure for combining multiple states' data. At each report, losses for each state were grouped into three categories: Fatal, PT/Major, and TT/Minor. Next, differences in scale by state were removed through *normalization*; for each claim group, this was done by dividing each claim by the average cost per case for the appropriate state-injury-type combination. Then claim sizes would be calibrated by "entry ratio" to the average cost per case. For example, a Florida claim in the PT/Major category would be divided by the average cost per case for Florida PT/Major claims. For Fatal and PT/Major, claims were combined respective of benefit type (escalating, limited, and non-escalating). This normalization is consistent with the methodology for production of ELFs, wherein excess ratios are calibrated for entry ratios.

Statistical distributions were then fit to the normalized empirical distributions using maximum likelihood.

Since by definition, no Major PP claim can be less than the critical value and it is unlikely that a permanent total claim would be, it made sense to fit a shifted distribution to the normalized fifth report PT/Major claims; that is, all normalized PT/Major claims were reduced by some flat amount prior to curve fitting.

The average state critical value for the claims in the database was roughly a fourth of the average cost per case for PT/Major claims. Consequently, a shift parameter of .25 or 25% of the average cost per case was reasonable. The actual dollar value would of course vary by state and year.

Performance Testing Injury Groups

Exhibit 1 summarizes the testing used to gauge the effectiveness of these three ways of grouping claims: 1) PT and PP modeled separately, 2) PT and Major PP combined, TT and Minor PP combined, 3) a single distribution combining PT and all PP, leaving TT by itself, and 4) a control, the simple use of last year's raw data. The testing attempted to determine which approach best predicted the relative magnitude of the empirical fifth report excess ratio at given loss limits. We tested using the following loss limits: 12,500, 50,000, 250,000, and 500,000.

We first considered the option of separating out PT and consolidating Major and Minor PP claims, then modeling claims according to the normally reported injury type. This would be the common-sense approach. In recent studies, we observed considerable variation in the proportion of PT/Major corresponding to PT. In some states, PT is nearly 40% of the PT/Major loss dollars; whereas, in others it is only 5%. This variation seemed to argue against a model which combined PT and Major PP.

To review the rationale behind the current option, labeled Option 2 in Exhibit 1, it is apparent that the practical distinction between PT and Major PP varies from state to state and year to year. A claim classified as PT in one state might well be considered a PP claim in another. This blurring of PT and Major PP would not be a factor if PT and Major PP were combined prior to curve fitting. The combination of Minor PP with TT is made for ease of computation and has little impact on the final factors.

As a third option for grouping claims, we considered the use of a single ground-up distribution combining PT and all of PP, calling this the "Permanent Claims."

Excess ratio tables were calculated at fifth report for each of the groupings of claims. Each of the three models above were used to calculate ELFs. For fixed loss limits, the relative magnitude of the modeled excess ratios by state should roughly track the empirical ratios. If a model predicts a higher excess ratio in, say, Georgia than in Florida, the empirical fifth report excess ratio for Georgia should be higher than that for Florida. The accuracy of the tracking can be quantified using R^2. The models used as inputs the average cost per case and injury weight values corresponding to the target data.

The model using the current grouping of claims produced the best estimates, as measured by R^2. That is, the current injury groupings did the best job of predicting which states would have high or low empirical excess ratios. It may be that PT average costs per case and injury weights, which are based on relatively small samples, are too volatile, leading to unstable partial excess ratios when PT is modeled separately.

The selected model is based on the injury groupings: Fatal, PT/Major, and TT/Minor.

2. THE GROUPING OF STATES BY BENEFIT TYPE

Description of NCCI Approach

In the 1992 study described above, differences in scale by state were removed by dividing each claim by the average cost per case for the appropriate state/claim group combination. Once the differences in scale were removed for claims of each injury group, the states were combined according to state benefit type. There were five groupings: 1) Escalating Fatal, 2) Non-escalating or Limited Fatal, 3) Escalating or Limited PT, 4) Non-escalating PT, and 5) TT/Minor. States would be in either 1) or 2), 3) or 4), and all states would be in 5).

Performance Testing State Groupings

We have tried to determine whether there exists a systemic relationship between the shape of the distribution (after removing the effects of scale) and the state benefit type. Three injury types were tested: Fatal, PT, and PP. These have by far the most weight in the calculation of excess ratios. Fatal and PT are the ones that could be logically impacted by escalation, non-escalation, or limitation, but we also tested PP for completeness.

We first examined the variance and skewness statistics of the normalized fifth report losses for each of the three injury types—Fatal, PT, and PP.

For Fatal claims, neither the variance (Exhibit 2-A) nor the skewness (Exhibit 2-B) of the normalized loss seem to have any significant relationship to state benefit type. Similarly, no useful relationship could be deduced for PT claims (Exhibits 3-A and 3-B) or PP claims (Exhibit 4-A and 4-B).

Treating benefit type as a categorical variable, we performed ANOVA testing and calculated coefficients of determination (R^2)

for each of the comparisons. The categorical variable *state benefit type* appears to be of little use in predicting fifth report normalized loss skewness or variance for Fatal or PT losses.

Analysis based on the likelihood ratio test further supports this argument. For both Fatal and PT claims, we compared normalized loss distributions for claims in states grouped by benefit type with those for all other states combined. Differences for these groupings were statistically insignificant.

We have decided that states should not be grouped by benefit classification, based on the large variation by state in higher moments of the distribution and on the fact that these are not correlated with benefit type.

As another possible change from the prior procedure, we considered eliminating the countrywide distributions and using distributions for each state. For most states we found the statistical significance of the difference between the state and countrywide normalized size-of-loss distributions for Fatal and PT is questionable—as indicated by the likelihood ratio test (Exhibits 5-A and 5-B). The enhanced credibility and utility of using countrywide distributions, on the other hand, are of clear value.

3. MODELING LOSS DEVELOPMENT

The impact of loss development on individual claims is not uniform since claims obviously have unique development patterns. Some settle for less than originally estimated, some for more. Accordingly, as losses mature, the dispersion among losses increases and so we expect the shape of the size-of-loss distribution at an ultimate report to be very different from that at a fifth report. We would expect the former to be more heavy-tailed than the latter.

For our purposes, it is the shape of the ultimate-report normalized loss distribution that we wish to model for each injury type. Unfortunately, Workers Compensation Statistical Plan data is available only through a fifth report. We are able to account for average claim size development on open serious claims using financial data. What is needed is a procedure which can account for the distortion of the shape of the size-of-loss distribution due to post-fifth-report loss development. In this section, we describe just such a procedure—the Random Development Divisor algorithm.

In the Appendix, we discuss the Black–Scholes model used by stock traders to price securities options, noting the similarity between the mathematics of pricing an option in the financial arena and that of excess of loss pricing in insurance. The Random Development Divisor algorithm described below bears more than coincidental resemblance to the Black–Scholes model.

The Random Development Divisor algorithm was designed to account for the post-fifth-report development in the shape of the severity distribution. The process is to 1) organize the partially developed fifth report loss distribution into a series of uniform distributions derived from empirical grouped data, 2) model loss development using a gamma distributed divisor, whose parameters are determined by matching the moments of the loss development factors for individual claims, and 3) compound the uniform and gamma distributions to derive an ultimate report distribution. The use of the piecewise linear approximation to a continuous distribution is a standard technique.

The basic building blocks of the model are a prior uniform distribution representing open or closed claims in each layer of fifth report loss range, and a corresponding gamma distribution quantifying development for such losses in the layer.

Empirical Fifth Report Severity Distribution

We construct n intervals of the grouped empirical claim distribution F_y for the fifth report size-of-loss random variable

Y. Each interval may contain several claims. These $n + 1$ points,

$$(a_1 = 0, F_y(a_1) = 0), (a_2, F_y(a_2)), \ldots, (a_n, F_y(a_n)),$$
$$(a_{n+1}, F_y(a_{n+1}) = 1),$$

divide the probability space of Y into n intervals. Let p_k represent the probability associated with the kth interval:

$$p_k = F_y(a_{k+1}) - F_y(a_k), \qquad k = 1, 2, \ldots, n.$$

p_k is the number of empirical claims in the kth interval divided by the total number of claims.

The following discussion is in terms of a basic building block. However, it should be kept in mind that the complete model would involve an application of the method to each subinterval of fifth report size-of-loss. Compounding the posterior distributions for all layers is a task made easy by the computer.

Gamma Distributed Fifth-to-Ultimate Development Divisors

Let Z denote the random variable representing the reciprocal of the fifth-to-ultimate loss development factor. Our a priori assumption is that such loss development is dependent on the size of the fifth report losses and whether they are open or not. Of course, the proportion of open claims varies by layer, so we were able to model loss development using two gamma distributions, one for open, one for closed.

Modeling development using a divisor rather than a multiplier facilitates the derivation of closed form formulas for the cumulative distribution function and excess ratio functions.

Constructive Model of Ultimate Losses

A heuristic description of the process for generating ultimate losses ($X = Y/Z$) is as follows:

STEP 1 Select one of the n fifth report loss intervals. The probability of selecting a given interval equals the amount of probability in the interval (p_k).

STEP 2 Assume that losses are uniformly distributed within each selected interval. Randomly select a fifth report loss (Y), which may be open or closed, from the uniform distribution chosen in Step 1.

STEP 3 The result of Step 1 determines which gamma distribution will be used to select a loss development divisor (Z). Randomly choose Z from the respective gamma distribution with parameters (α_o, β_o) or (α_c, β_c), where o is open and c is closed.

STEP 4 Divide Y by Z. The result is X (the ultimate report loss).

The Relationship Between the Conditional Distribution Functions of X and Y

$$\begin{aligned} F_x(x \mid z) &= \Pr(X \leq x \mid z) \\ &= \Pr(Y/Z \leq x \mid z) \\ &= \Pr(Y \leq zx) \\ F_x(x \mid z) &= F_y(zx). \end{aligned} \tag{1}$$

Derivation of Distribution Function of X

We treat each y-interval as a separate random variable. Let $u_k(z)$ denote the probability density function (p.d.f.) for Z. Note that there are n such conditional distributions—one for each y-loss interval.

Then, using Equation 1 in

$$F_x(x) = \int_0^\infty F_x(x \mid z) u_k(z)\, dz,$$

we have

$$F_x(x) = \int_0^\infty F_y(zx)u_k(z)\,dz. \tag{2}$$

For each interval $(a_k, a_{k+1}]$, we assume that fifth report losses are uniformly distributed. Then,

$$F_y(y) = \begin{cases} 0 & \text{for} \quad y \le a_k \\ \dfrac{y - a_k}{a_{k+1} - a_k} & \text{for} \quad a_k < y \le a_{k+1} \\ 1 & \text{for} \quad y > a_{k+1} \end{cases}.$$

For (fixed) $x > 0$, $k = 1, 2, \ldots, n$

$$\begin{aligned} & a_k < y \le a_{k+1} \\ \Leftrightarrow\ & a_k < xz \le a_{k+1} \\ \Leftrightarrow\ & \frac{a_k}{x} < z \le \frac{a_{k+1}}{x}. \end{aligned}$$

Thus,

$$F_y(zx) = \begin{cases} 0 & \text{for} \quad z \le a_k/x \\ \dfrac{y - a_k}{a_{k+1} - a_k} & \text{for} \quad a_k/x < z \le a_{k+1}/x \\ 1 & \text{for} \quad z > a_{k+1}/x \end{cases}.$$

Using the above in Equation 2, we can calculate $F_x(x)$:

$$F_x(x) = \int_{a_k/x}^{a_{k+1}/x} \frac{zx - a_k}{a_{k+1} - a_k} u_k(z)\,dz + \int_{a_{k+1}/x}^{\infty} u_k(z)\,dz. \tag{3}$$

The above applies to the kth interval ($k = 1, 2, \ldots, n$) treated in isolation. To calculate $F_x(x)$ over all intervals, we take a probability-weighted average. $F_x(x)$ is the fully developed sample to which we fit the final parametrized distributions leading to the excess ratio table used in production of ELFs.

4. DERIVATION OF EXPECTED EXCESS LOSS FUNCTION FOR EACH INTERVAL

Let x denote the loss limit. Then for a random loss Y/Z, the excess of Y/Z over x is: $(Y/Z - x)$ for $Y/Z > x$, and 0 otherwise.

Thus to calculate the expected excess loss, we need to integrate over the set of (y,z) for which y/z is greater than x.

Let $f(y,z)$ denote the joint probability density function for Y and Z. Since Y and Z are independent,

$$f(y,z) = f(y)f(z).$$

Now, recall that Y is uniformly distributed in $(a_k, a_{k+1}]$ which means that $f(y,z)$ is zero whenever $y < a_k$ or $y > a_{k+1}$. This reduces the area over which we must integrate to a trapezoidal region.

This trapezoidal region consists of the rectangular "AREA A" and the triangular "AREA B" in Figure 1. The expected excess loss can then be calculated as

$$\text{Excess}_x = \int_0^{a_k/x} \int_{a_k}^{a_{k+1}} \left(\frac{y}{z} - x\right) f(y,z)\,dy\,dz + \int_{a_k/x}^{a_{k+1}/x} \int_{xz}^{a_{k+1}} \left(\frac{y}{z} - x\right) f(y,z)\,dy\,dz. \tag{4}$$

DEFINITION *If Z is gamma distributed with parameters (α,β), then the cumulative distribution function (c.d.f.) of Z is*

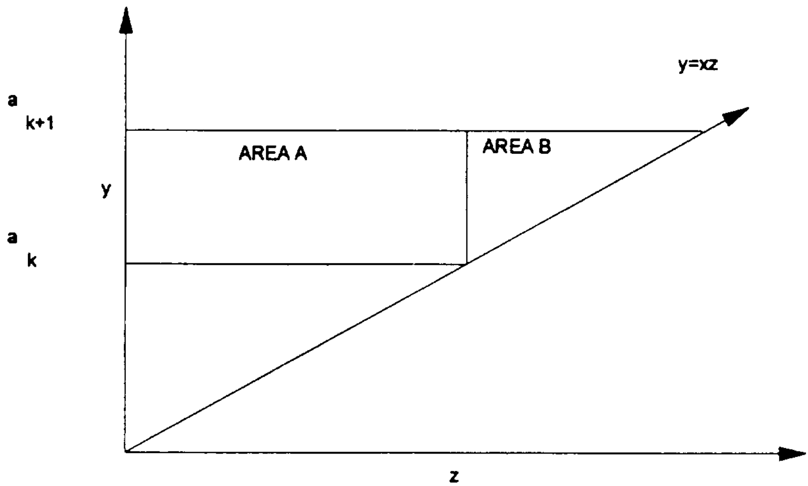
$$F_z(z) = \Gamma(\alpha;\beta z) = \frac{\int_0^{\beta z} u^{\alpha-1} e^{-u}\,du}{\Gamma(\alpha)}. \tag{5}$$

For this distribution, non-central moments can be calculated as follows:

$$\mathrm{E}[Z^n] = \frac{\Gamma(\alpha + n)}{\beta^n \Gamma(\alpha)}. \tag{6}$$

FIGURE 1

AREA OVER WHICH EXCESS LOSS INTEGRAL IS EVALUATED



THEOREM 1 *Let the fifth report size-of-loss random variable Y ($Y > 0$) be uniformly distributed on the interval $(a_k, a_{k+1}]$, and let the loss development divisor random variable Z ($z > 0$) be gamma distributed with parameters (α, β). Let Y and Z be independent. Then the ultimate report size-of-loss random variable X, equal to the ratio of Y to Z ($X = Y/Z$), has cumulative distribution function*

$$F_x(x) = \frac{\alpha x}{\beta(a_{k+1} - a_k)}\left[\Gamma\left(\alpha + 1; \frac{a_{k+1}\beta}{x}\right) - \Gamma\left(\alpha + 1; \frac{a_k\beta}{x}\right)\right] - \frac{a_k}{a_{k+1} - a_k}\left[\Gamma\left(\alpha; \frac{a_{k+1}\beta}{x}\right) - \Gamma\left(\alpha; \frac{a_k\beta}{x}\right)\right] + 1 - \Gamma\left(\alpha; \frac{a_{k+1}\beta}{x}\right).$$

THEOREM 2 *Let the random variable X be as defined in Theorem 1. Then for any x ($x > 0$) the expected portion of loss in* *excess of x for a randomly selected claim is equal to*

$$\begin{aligned}
\text{Excess}_x &= \frac{\beta(a_k + a_{k+1})}{2(\alpha-1)}\Gamma\left(\alpha-1;\frac{a_k\beta}{x}\right) - x\Gamma\left(\alpha;\frac{a_k\beta}{x}\right)\\
&+ \frac{a_{k+1}^2\beta}{2(\alpha-1)(a_{k+1}-a_k)}\left\{\Gamma\left(\alpha-1;\frac{a_{k+1}\beta}{x}\right) - \Gamma\left(\alpha-1;\frac{a_k\beta}{x}\right)\right\}\\
&+ \frac{\alpha x^2}{2\beta(a_{k+1}-a_k)}\left\{\Gamma\left(\alpha+1;\frac{a_{k+1}\beta}{x}\right) - \Gamma\left(\alpha+1;\frac{a_k\beta}{x}\right)\right\}\\
&- \frac{x a_{k+1}}{a_{k+1}-a_k}\left\{\Gamma\left(\alpha;\frac{a_{k+1}\beta}{x}\right) - \Gamma\left(\alpha;\frac{a_k\beta}{x}\right)\right\}.
\end{aligned}$$

Excess_x *is the numerator of the excess ratio, whose denominator is*

$$\text{E}[X] = \beta\frac{(\alpha_{k+1} + \alpha_k)}{2(\alpha-1)}.$$

Illustration #1: Estimation of Z-parameters

For fifth report losses in the interval (20,000, 30,000) suppose we have observed that the first moment (mean) of the fifth-to-ultimate loss development factor distribution is $m_1 = 1.00$ and the second moment is $m_2 = 1.81$.

Estimate the α and β parameters of random development division that correspond to the observed moments.

Solution:

Using Equation 6, set m_1 equal to $\text{E}[1/Z]$ and m_2 equal to $\text{E}[1/Z^2]$

$$\text{E}[1/Z] = \beta/(\alpha-1) = m_1 = 1.00,$$

and

$$\text{E}[1/Z^2] = \beta^2/\{(\alpha-1)(\alpha-2)\} = m_2 = 1.81.$$

Solving the two equations simultaneously gives $\alpha = 3.2346$ and $\beta = 2.2346$.

Illustration #2: Calculation of $F_x(x)$

Given the scenario in Illustration 1, estimate the probability that a fully developed claim will not exceed \$35,000.

Solution:

We apply Theorem 1 with the following parameter values: $a_k = 20{,}000$, $a_{k+1} = 30{,}000$, $\alpha = 3.2346$, $\beta = 2.2346$, and $x = 35{,}000$

$$F_x(35{,}000) = .825178.$$

Illustration #3: Calculation of Excess Ratio

Given the assumptions in Illustration 1, estimate the expected proportion of loss dollars in excess of 35,000.

Solution:

We apply Theorem 2 with the following parameter values: $a_k = 20{,}000$, $a_{k+1} = 30{,}000$, $\alpha = 3.2346$, $\beta = 2.2346$, and $x = 35{,}000$

$$\text{Excess}_x(35{,}000) = 4092.57$$

$$\text{E}[X] = 25{,}000 \times 1.00 = 25{,}000.$$

The ratio is $4097.57/25000 = 0.1639$.

5. THE ISSUE OF RISK LOAD

The Flat Loading

The flat loading, which accounts for parameter risk and anti-selection, was added to the ELF which is a ratio to premium that includes expenses; it was .005, subject to a maximum of half the ELF. In the selected procedure, we have chosen to remove ourselves from the expense arena by instead applying the .005 flat loading to the pure excess ratio and limiting it to half of that.

Prior Load for Contagion

The loss distributions underlying the prior ELF procedure correspond to individual claims by injury type; however, ELFs apply on a per occurrence basis (a single occurrence may contain multiple claims). The adjustment used to account for the per occurrence basis of the coverage was to inflate the average cost per case for each injury type by a factor of 1.1. So, for example, if the average Fatal claim for a given state and hazard group was projected to be $100,000, an average value of $110,000 was assumed in the ELF calculations. In other words, the Fatal occurrence size distribution was scaled to an average value of $110,000—from which the Fatal contribution to the ELF (the partial excess ratio) is calculated. This was done for all claim types.

Selected Contagion Load

As stated in Section 1, removing the differences in scale by state made it possible to combine experience from more than one state. For each injury type, normalized claims had an average size of unity. Parametrized statistical distributions were then fit to the sample distributions by maximum likelihood. The scale parameters of the fitted distributions did not necessarily result in a mean of unity but had to be adjusted once again to normalize the result.

We are sampling from highly skewed distributions for PT/Major and Fatal. Consider the distributions of the sample means. In theory, these distributions approach normality as the sample sizes approach infinity; but is this the case in practice? These sampling distributions of the means at an ultimate report, based on finite state sample sizes, are likely still skewed. This means that in more cases than not, the sample means will be less than the true means.

The empirical cumulative distribution function (c.d.f.) is based on a sample, and a sample contains a largest observed claim.

A theoretical distribution such as a transformed beta would not have a maximum possible claim; some probability (albeit small) would be assigned to claims greater than the largest claim observed in the sample. For this reason, the mean of the fitted distribution (the maximum likelihood estimator) may be greater than that observed in the data (the method of moments estimator).

The choice of an adequate statistical model produces a fitted statistical distribution whose cdf very closely matches that observed for the data, except at very high entry ratios. For example, if 25% of the observed normalized claims are below entry ratio 1.00, we expect the theoretical model's cdf to be very close to .25 at input value 1.00. As stated above, the prior approach was to re-scale the distribution fitted to combined data to a mean of unity. A consequence of this re-scaling is that the cumulative distribution values do not match the empirical. In 1995, we chose not to re-scale the fitted distributions, thereby providing, in effect, a natural contagion load. We are using distributions that closely match the observed empirical distribution values, but assign small probabilities to large unobserved claim values. As in the prior procedure, the small probabilities assigned to the tail of each distribution are determined by the fitting procedure. The difference is that the means of the models are greater than unity. By allowing the means to float, our models more closely match the observed claim distributions and at the same time provide some risk load.

The way the Fatal and PT/Major claim data is fit enhances the impact of the above strategy. The model accounts for these occurrences by fitting a distribution to the claim data censored from above; heuristically, the observed values correspond to single-claim occurrences and the censored portion of the distribution corresponds to multiple claim occurrences. The result is an occurrence size-of-loss distribution, with entry ratios to the average cost per claim. This is described in more detail below.

6. DEVELOPMENT OF PRODUCTION MODEL

The above sections cover the major issues addressed by our research and decisions made on these issues. Following is the application of these decisions in creating a new model.

Construction of Normalized Database

The countrywide claims database comes from Workers Compensation Statistical Plan data. This database contains fifth report claims for NCCI states along with an open/closed claim indicator. Each claim is identified by injury type.

Fifth-to-ultimate development factors (from a separate database used for class ratemaking) by state and injury type were used to develop these open claims. The development factor for open claims was such that the overall development (on open and closed claims) averaged to the loss development factors in our class ratemaking database.

Claims were then normalized (scaled to unity) by state and injury group, retaining the open/closed indicator. At this point, states can be combined, and the distributions can be grouped into n uniform claim size intervals.

In the procedure described thus far, no adjustment has been made for dispersion in the development by claim, other than the application of a flat factor by state to open claims only.

Application of Random Development Divisor (RDD) Algorithm

As in the 1992 study, we assume that only the distributions for Fatal and PT/Major claims change shape beyond a fifth report.

We introduced development uncertainty via the Random Development Divisor (RDD) algorithm. Based officially on judgment, but unofficially on an analysis of confidential data, we used a coefficient of variation (cv) of .9 for open claims and .1 for closed claims. Section 4 explains how we developed the open claim sub-intervals using a cv of .9. A similar procedure was used for the intervals of closed claims. We weighted together all $2n$ resulting distributions to form the sample for the next step.

Curve Fitting

Using maximum likelihood, we fit parametric distributions to the developed sample claim distributions. Actual fifth report data was used without further adjustment to fit the TT/Minor model.

Fit to Fatal Claims

The fatal loss size distribution encompasses two distinct types of claim—those with and those without survivor. Survivor benefits range over a lot of possible values, generally large to larger. Without a survivor, there is still a range of values depending on medical care, but a cluster of smallish values for claims in which medical care is minimal. Looking at the actual data, we concluded this could not be easily modeled by a single parametrized distribution.

A linear mixture of three distributions is used to model Fatal losses. Let R represent the "entry ratio" random variable. $F(r)$ is the cumulative distribution function of R:

$$F(r) = w_1 F_1(r) + w_2 F_2(r) + w_3 F_3(r),$$

where the w's represent the weights given to each of the three pieces.

For $R < 1$, the distribution of R is modeled using a *censored* Weibull distribution. The censoring parameter, c, is 1. This distribution, $F_1(r)$, received the largest weight (w_1) of 0.608.

For $R > 1$, we model, $R - 1$, the *excess* above entry ratio 1, with a transformed beta distribution. Each normalized occurrence in this interval can be thought of as unity plus a transformed beta deviate. This distribution received the next largest weight (w_2).

To eliminate clustering and improve the fit of the model, a small portion of the claims in the interval $(.75, 1)$ were modeled separately using a conditional (truncated and censored) Weibull. This distribution received a weight (w_3). The parameters are the same as those for the Weibull used in the $(0, 1)$ interval, except for the truncation point. Following is a comparison of the composite fitted distribution, $F(r)$, and the sample distribution generated by the RDD model, $F_n^*(r)$.

COMPARISON OF FATAL DISTRIBUTIONS

r	$F_n^*(r)$	$F(r)$
0.10	0.176340	0.176835
0.50	0.413960	0.419246
1.00	0.626340	0.626340
5.00	0.987300	0.986424
10.00	0.997330	0.996420
50.00	0.999980	0.999779

The severity distribution for Fatal has a mean of 1.039.

Fit to PT/Major

Following is a comparison of PT/Major cdfs.

The empirical ultimate report cdf for normalized claims prior to application of the RDD algorithm (but after development of open claims) is $F_n(r)$; after RDD it is $F_n^*(r)$. To account for the per occurrence basis of the coverage, a conditional distribution $F(r \mid r \leq 90)$ was fit via maximum likelihood to $F_n^*(r)$, also censored at 90. The corresponding uncensored distribution $F(r)$ is used to model occurrences.

The RDD algorithm causes most claims to develop downward but at the same time makes the tail of the distribution thicker, as can be noted from a comparison of $F_n(r)$ and $F_n^*(r)$.

The fitted conditional distribution ($F(r) \leq 90$) fits the post-RDD cdf ($F_n^*(r)$) well.

r	$F_n(r)$	$F_n^*(r)$	$F(r \mid r \leq 90)$	$F(r)$
0.10	0.00030	0.00101	0.00123	0.00123
0.50	0.37237	0.41468	0.41221	0.41214
1.00	0.71205	0.74970	0.74759	0.74745
5.00	0.98692	0.98053	0.97958	0.97940
10.00	0.99741	0.99472	0.99350	0.99332
50.00	0.99986	0.99967	0.99970	0.99951

The severity distribution for PT/Major has a mean of 1.066.

Fit to TT/Minor Claims

A Transformed Beta was fit to TT/Minor claims.

r	$F(r)$	$F_n(r)$
1.00	0.69826	0.68660
5.00	0.97017	0.96897
10.00	0.99635	0.99731

In other words, finis.

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[3] Heckman, P. E., and G. G. Meyers, "The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions," *PCAS* LXX, 1983, pp. 22–61.

[4] Hogg, R. V., and S. A. Klugman, *Loss Distributions*, 1984, John Wiley & Sons, Somerset, NJ.

EXHIBIT 1

COMPARISON OF INJURY GROUPING SCHEMES

PERFORMANCE TESTING SUMMARY
THEORETICAL STATE EXCESS RATIOS REGRESSED ON EMPIRICAL 5TH REPORT EXCESS RATIOS

35 States

Loss Limit	Option 1 R-squared	Option 2 R-squared	Option 3 R-squared	Option 4 R-squared
$12,500	0.969	0.960	0.970	0.903
$50,000	0.853	0.953	0.843	0.791
$250,000	0.406	0.408	0.281	0.404
$500,000	0.290	0.213	0.157	0.147

NCCI States At Least 5,000 Serious Claims
(17 States)

Loss Limit	Option 1 R-squared	Option 2 R-squared	Option 3 R-squared	Option 4 R-squared
$12,500	0.958	0.952	0.960	0.936
$50,000	0.843	0.965	0.837	0.921
$250,000	0.618	0.696	0.534	0.553
$500,000	0.361	0.371	0.296	0.215

Option 1: PT and PP modeled separately.
Option 2: PT and Major PP together, TT and Minor PP together.
Option 3: PT and all PP modeled together, TT by itself.
(Control) Option 4: Excess Ratio predicted using previous year's observed values.

EXHIBIT 2-A

FATAL VARIANCE

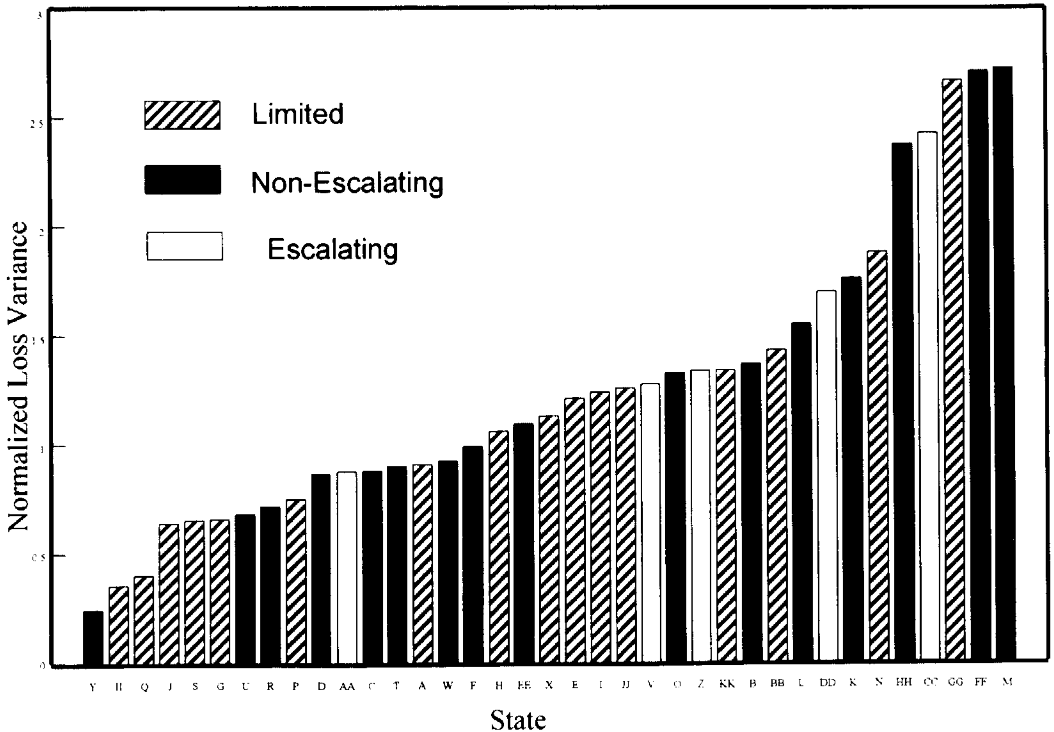

EXHIBIT 2-B

FATAL SKEWNESS

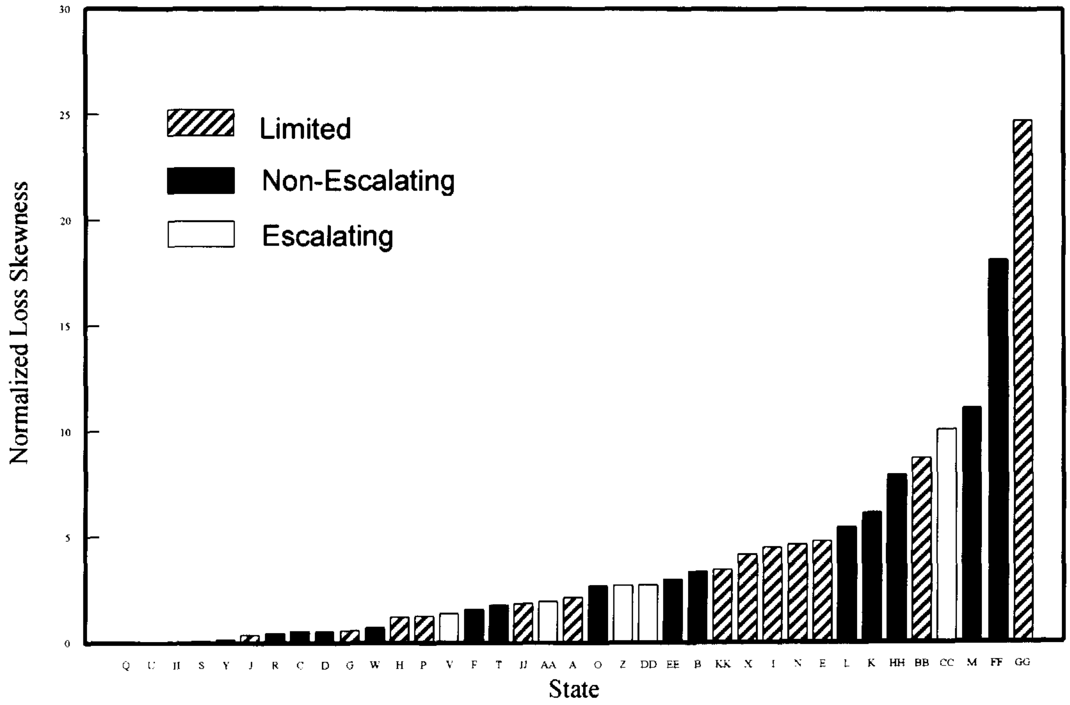

EXHIBIT 3-A

PERMANENT TOTAL VARIANCE

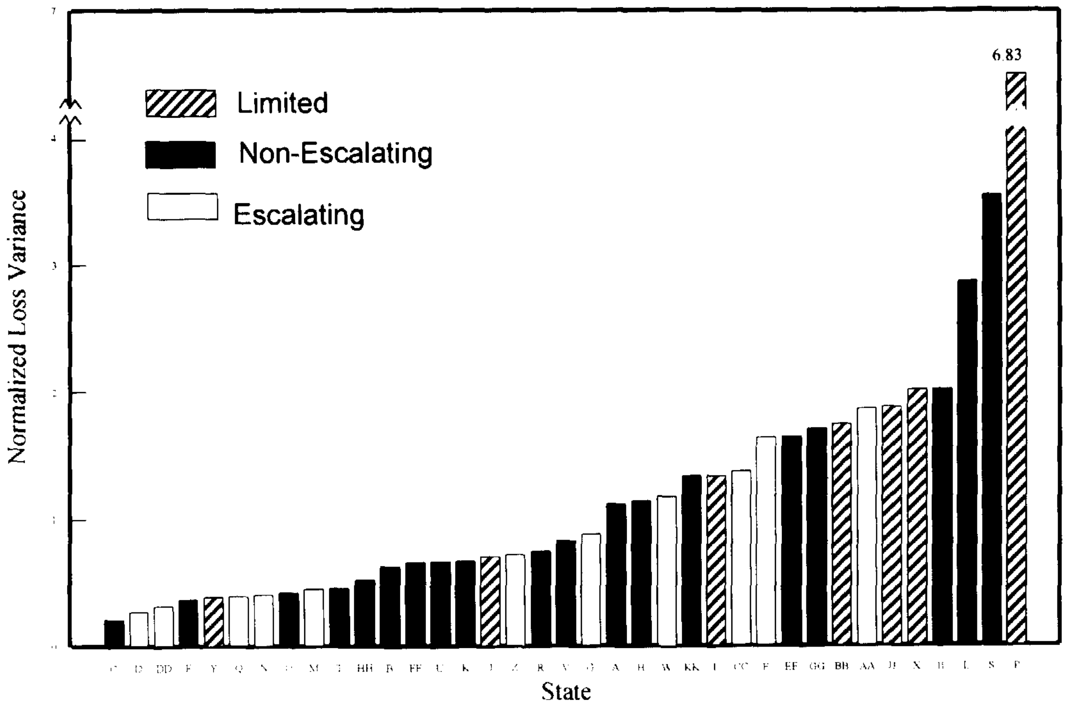

EXHIBIT 3-B

PERMANENT TOTAL SKEWNESS

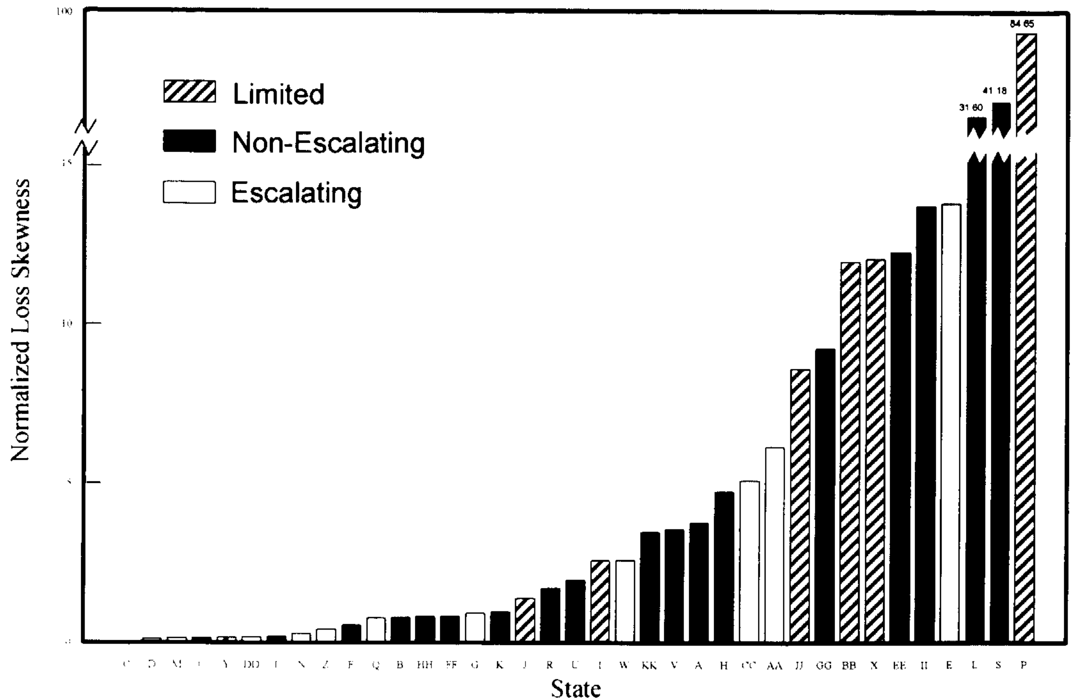

EXHIBIT 4-A

PERMANENT PARTIAL VARIANCE

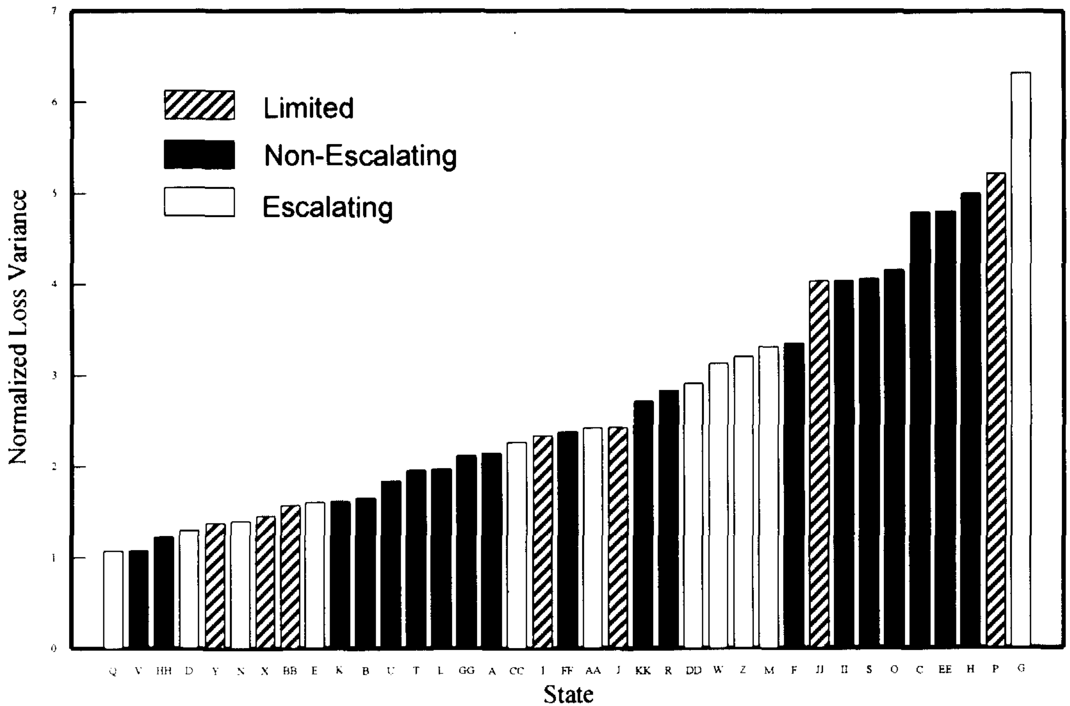

EXHIBIT 4-B

PERMANENT PARTIAL SKEWNESS

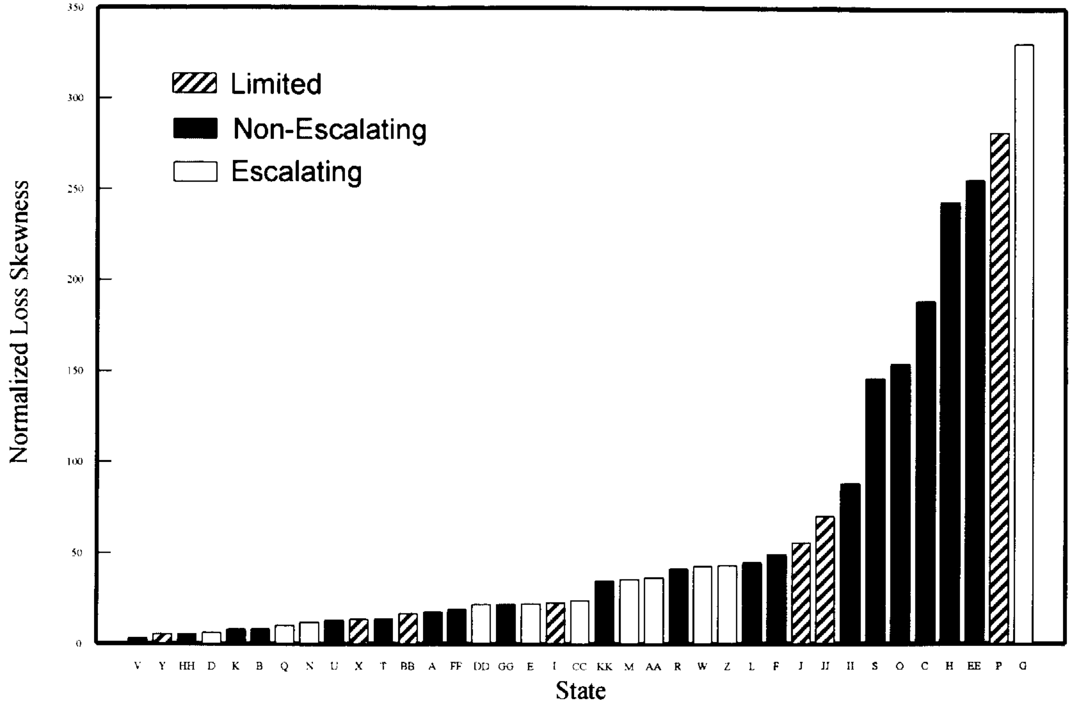

EXHIBIT 5-A

SUMMARY
STATE FATAL CLAIMS DISTRIBUTION COMPARED TO COUNTRYWIDE

State	Claims	Degrees of Freedom	Likelihood Ratio Test Statistic	p-value
A	77	3	0.780	0.854
B	36	3	0.435	0.933
C	40	2	1.279	0.528
D	69	2	1.587	0.452
E	187	3	10.567	0.014
F	14	2	0.680	0.712
G	22	2	0.182	0.913
H	168	2	0.767	0.681
I	103	2	4.518	0.104
J	56	2	4.193	0.123
K	51	2	2.188	0.335
L	134	3	5.542	0.136
M	50	3	19.877	0.000
N	15	2	5.653	0.059
O	118	3	0.882	0.830
P	75	3	0.049	0.997
Q	27	1	0.560	0.454
R	28	2	0.382	0.826
S	27	1	0.560	0.454
T	78	2	2.434	0.296
U	79	1	2.968	0.085
V	7	2	2.622	0.270
W	14	2	0.680	0.712
X	84	2	3.253	0.197
Y	4	1	0.451	0.502
Z	10	2	1.825	0.402
AA	66	3	0.208	0.976
BB	61	2	6.313	0.043
CC	59	3	9.003	0.029
DD	13	1	1.860	0.173
EE	132	3	2.510	0.473
FF	47	3	5.757	0.124
GG	131	3	9.280	0.026
HH	10	2	1.825	0.402
II	99	1	4.429	0.035
JJ	91	3	0.389	0.943
KK	78	3	1.392	0.707
All States	2,360	118	117.88	0.486

EXHIBIT 5-B

SUMMARY
STATE PT CLAIMS DISTRIBUTION COMPARED TO COUNTRYWIDE

State	Claims	Degrees of Freedom	Likelihood Ratio Test Statistic	*p*-value
A	86	3	1.016	0.797
B	29	2	2.076	0.354
C	21	1	1.040	0.308
D	358	2	55.105	0.000
E	738	3	8.843	0.031
F	64	2	0.195	0.907
G	5	1	0.134	0.714
H	144	3	0.133	0.988
I	20	2	2.148	0.342
J	47	2	0.264	0.876
K	98	2	1.887	0.389
L	156	3	7.160	0.067
M	36	1	2.342	0.126
N	29	2	3.464	0.177
O	33	1	3.753	0.053
P	32	3	10.665	0.014
Q	82	2	0.635	0.728
R	13	2	0.619	0.734
S	45	2	3.346	0.188
T	78	1	4.402	0.036
U	109	3	0.511	0.916
V	80	2	2.031	0.362
W	11	2	2.961	0.228
X	56	2	2.422	0.298
Y	14	1	0.693	0.405
Z	5	1	0.134	0.714
AA	75	3	26.425	0.000
BB	61	3	0.246	0.970
CC	48	3	3.016	0.389
DD	7	1	0.043	0.836
EE	130	3	2.049	0.562
FF	32	2	3.041	0.219
GG	102	3	2.864	0.413
HH	28	2	0.765	0.682
II	52	3	1.726	0.631
JJ	51	2	2.197	0.333
KK	52	3	2.117	0.548
All States	3,027	115	162.468	0.002
Excl. D	2,669	113	107.363	0.632

APPENDIX

Black–Scholes Model

Let us think of the future price of a share of stock, S, as the current share price, S_0, times a random "development factor." The development factor random variable is customarily modeled by a lognormal distribution. Then the future price—equal to a lognormally distributed random factor times a constant (the current price) is also lognormally distributed.

Let the indexed random variable S_t represent the unknown future price of a share of stock at time t ($t > 0$). Suppose S_t is lognormally distributed with parameters $(\mu_t, \sigma^2 t)$.

At time zero, we wish to price a call option exercisable at time t, at exercise price d. A call gives the holder the option of buying a share of stock at the exercise price at some future date. Let r_f represent the force of interest at the risk free rate. Then:

$$PV(CALL) = e^{-r_f t} \int_d^{\infty} (s-d) f(s) ds$$

$$= e^{-r_f t}\{\mathrm{E}(S) - \mathrm{E}(S;d)\}$$

$$= e^{-r_f t}\left(e^{\mu_t + \sigma^2 t/2} - e^{\mu_t + \sigma^2 t/2}\Phi\left(\frac{\ln(d) - \mu_t - \sigma^2 t}{\sigma\sqrt{t}}\right)\right.$$

$$\left. -d\left\{1 - \Phi\left(\frac{\ln(d) - \mu_t}{\sigma\sqrt{t}}\right)\right\}\right)$$

$$= e^{-r_f t}\left(e^{\mu_t + \sigma^2 t/2}\left\{1 - \Phi\left(\frac{\ln(d) - \mu_t - \sigma^2 t}{\sigma\sqrt{t}}\right)\right\}\right.$$

$$\left. -d\left\{1 - \Phi\left(\frac{\ln(d) - \mu_t}{\sigma\sqrt{t}}\right)\right\}\right)$$

$$= e^{-r_f t}\left[e^{\mu_t+\sigma^2 t}\Phi\left(\frac{-\ln(d)+\mu_t+\sigma^2 t}{\sigma\sqrt{t}}\right)\right.$$

$$\left. -d\Phi\left(\frac{-\ln(d)+\mu_t}{\sigma\sqrt{t}}\right)\right].$$

The Black–Scholes form for the above is arrived at by equating the mean of the distribution to the current ($t = 0$) share price times the discount factor:

$$e^{\mu_t+\sigma^2 t/2} = S_0 e^{r_f t}.$$

The above is highly suggestive. First, the idea of creating Black–Scholes analogs based on distributions other than the lognormal may come to mind. We could, for example, assume that the stock "development factors" follow the gamma distribution instead of the lognormal; the share price itself would also then be gamma distributed. Not surprisingly, the use of such Black–Scholes analogs is not unknown in the world of finance.

The second item which may come to mind is the resemblance between the mathematics of pricing an option and the reserving of excess of loss coverage. Of major significance is the following: if a fifth report open claim is currently valued below a given retention, it does not follow that the expected contribution of the claim to the excess layer is zero, just as the value of a call for a stock currently priced below the exercise price is not zero.

BALANCING DEVELOPMENT AND TREND IN LOSS RESERVE ANALYSIS

SPENCER M. GLUCK

Abstract

The most common loss reserving procedures emphasize development-based projections, with implied trends examined for reasonableness and considered on an ad-hoc basis. This paper presents relatively simple methods for reflecting development and trend simultaneously, with weights that reasonably reflect the relative accuracy of the two types of projections. The Stanard–Bühlmann or "Cape Cod" method is a special case of these methods, which are denoted "Generalized Cape Cod" methods. The Appendices present underlying variance structures under which the weights used in the Generalized Cape Cod methods are optimal.

1. INTRODUCTION AND OVERVIEW

Commonly applied actuarial procedures involve projections in two "directions" of the traditional loss development triangle:

1. The development direction

We use the term "development" to refer to the emergence of information for a single year of origin. Development projections involve the measurement, selection, and application of development patterns. While the measurement and selection of development patterns often involve data from multiple years of origin, the application of the development pattern is made to each year of origin independently.

2. *The trend direction*

We use the term "trend" to refer to projections made using relationships among amounts for different years of origin. Trends, as used herein, refer to expected changes in the ratio of a projected amount to an exposure base. For simplicity, we will generally refer to the projected amount as "losses," the exposure base as "exposures" and the ratio of losses to exposures as "pure premiums."

In fact, the methods presented are more general, and have much wider potential application. For example, if the projected amount is losses, potential exposure bases include ultimate claim counts and premiums, in which case the quantity denoted "pure premium" herein would really be severity or loss ratio, respectively. Other examples of potential combinations of projected amounts and exposure bases are listed in Section 9.

In reserving methodology, primary emphasis is often given to development projections, with implied trends perhaps examined for reasonableness and ad hoc modifications sometimes made to development projections, particularly for recent years of origin.

The Bornhuetter–Ferguson method is commonly used to blend a development projection with an "a priori" result. While that a priori result may well be based on a trend projection of some kind, the basis for the a priori result is, in general, unspecified [1]. The Stanard–Bühlmann or "Cape Cod" method is an application of the Bornhuetter–Ferguson method in which the a priori result comes from a specified, trend-based calculation.[1] Regression-based or dynamic stochastic models can be used to reflect the development and trend directions simultaneously, but these models are not in widespread use.[2]

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[1]See Stanard [2], Bühlmann [3] and Patrik [4].

[2]See, for example, DeJong and Zehnwirth [5], Taylor [6] and Wright [7].

The goal of this paper is to present methods that simultaneously reflect development and trend in a unified approach, reasonably reflecting the relative accuracy of the two types of projections. Additional goals are that the methods be practical, accessible to most practicing actuaries, and easily integrated with the most common actuarial procedures and the types of data that are readily available.

The methods discussed all estimate ultimate losses in a two step process; first, the expected losses for each year of origin are estimated based on a weighted average of the results from all years, developed and trended as appropriate; then the expected losses and actual losses for each year of origin are weighted together using the Bornhuetter–Ferguson (or similar) method. The Stanard–Bühlmann or Cape Cod method is a special case of this general approach, and we refer to the broader family of methods as Generalized Cape Cod Methods.

After introducing preliminary notation in Section 2, the paper proceeds as follows:

Section 3: The Bornhuetter–Ferguson method is presented along with a statistical justification for the Bornhuetter–Ferguson weights.

Section 4: The general framework for calculating expected losses as a weighted average of all years' results is presented, along with a discussion of variance relationships that should be reflected in the weights.

Section 5: The Stanard–Bühlmann or Cape Cod Method is presented and is shown to fit the general framework of Section 4. Two potentially significant shortcomings of the method are identified.

Sections 6 and 7: Two generalizations of the Cape Cod method are presented, designed to overcome the shortcomings identified in Section 5.

Section 8: The Bornhuetter–Ferguson calculation is revisited in conjunction with the expected loss estimates presented in Section 5 through 7.

Section 9: A number of potential applications of the methodology are listed.

Section 10: Conclusion

A glossary of notation is provided at the end of the paper including preliminary notation from Section 2, plus additional notation introduced in other Sections and Appendices.

The Appendices provide additional details of simple variance models consistent with the methods presented. Using these models, calculations are presented that use the data triangle to assist in the selection of parameters for Section 6 and Section 7 models.

The paper is organized with mathematics of any length or complexity consigned to the Appendices, and it is intended that the body of the paper can be read without the Appendices. Furthermore, while the Appendices provide calculations for estimating certain model parameters, the procedures of the paper will provide reasonable and useful results with judgmentally selected values for these parameters.

2. AVAILABLE DATA AND NOTATION

The following are presumed to be available (with $i = 1 \ldots N$):

	Notation
• The current evaluation of losses for each year of origin i	LTD_i
• Cumulative development factors appropriate to project losses to their ultimate value (note that the subscript refers to year of origin rather than maturity)	DF_i

- A measurement of the relative exposure per year of origin — E_i
- Trend factors to adjust for the change in expected losses per exposure from year of origin i to year of origin j — TF_{ij}

Additional Notation:

- Ultimate losses for year of origin i — ULT_i
- Thus, $LTD_i \times DF_i$ is an estimate of ULT_i
- $ULT_i \div E_i$ (i.e. pure premium) — PP_i
- Expected Value — E( )
- Variance — Var( )

The carat (^) is used to denote estimation; i.e., a quantity with a hat over it is an estimate of the quantity beneath the hat.

The derivation of the factors DF_i and TF_{ij} is not addressed in this paper. It is presumed that the actuary has applied appropriate calculations, adjustments, and judgments in selecting the factors so that $LTD_i \times DF_i$ is the best available development estimate of ULT_i, and TF_{ij} is the best available estimate of $E(PP_j) \div E(PP_i)$.[3]

For the most part, the above information is presumed to constitute all of the available information. In addition, calculations are presented in the Appendices that use the underlying data triangle to estimate certain model parameters.

Additional notation is introduced at later points in the paper. For convenience, a glossary containing all notation is included at the end of the paper.

3. THE BORNHUETTER–FERGUSON METHOD

The Bornhuetter–Ferguson method is the most commonly used approach to blending development and trend projections if

[3]Some of the types of adjustments that may be necessary are discussed in Berquist and Sherman [8].

trended values from other years of origin are the basis for the estimate of expected ultimate losses. In the Bornhuetter–Ferguson method, ultimate losses are estimated as follows:

$$\hat{ULT}_i = LTD_i + \left(1 - \frac{1}{DF_i}\right) \times \hat{\mathrm{E}}(ULT_i) \qquad (3.1)$$

where the source of the estimate $\hat{\mathrm{E}}(ULT_i)$ is unspecified. Expanding the first term, we have:

$$\hat{ULT}_i = \left(\frac{1}{DF_i}\right) \times LTD_i \times DF_i + \left(1 - \frac{1}{DF_i}\right) \times \hat{\mathrm{E}}(ULT_i) \qquad (3.2)$$

and the Bornhuetter–Ferguson estimate is seen to be a weighted average of the development based estimate of ULT_i and $\hat{\mathrm{E}}(ULT_i)$. The weights are optimal[4] under the following constraints:

1. Expected losses are known (i.e. $\hat{\mathrm{E}}(ULT_i) = \mathrm{E}(ULT_i)$);
2. Unemerged losses are independent from the emerged losses;
3. The DF_is are known; and
4. For a given year of origin i, the variance of the development-based estimate of ultimate losses (i.e. $LTD_i \times DF_i$) is proportional to the development factor DF_i.

Proof of the above statement is provided in Appendix A.

In practice, Constraint 1 is obviously not met; the majority of this paper concentrates on producing the best possible estimate of $\mathrm{E}(ULT_i)$ using all of the available information per Section 2. Section 8 and Appendix E deal with the implications of eliminating this constraint, and it is demonstrated that the same weights

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[4]Optimal weights are defined as those that produce the best (i.e. the minimum variance) linear unbiased estimate, given that the individual estimates being weighted together are themselves considered to be unbiased.

remain optimal if the estimate $\hat{E}(ULT_i)$ is determined using the techniques of this paper.

Constraint 2 is assumed to hold in both the Bornhuetter–Ferguson method and in underlying variance models developed in Appendix B. The independence assumption is modified for the model in Section 7 and Appendix C.

Constraint 3 is assumed to hold throughout.[5] Given the imperfection of this assumption, results described as optimal should be considered only approximately optimal.

Constraint 4 will subsequently be denoted as the "Cape Cod variance assumption." This assumption, along with several other assumptions, and the "Cape Cod variance model" is presented in Appendix B. Relaxation of this constraint and the use of an alternative variance model is addressed in Section 7 and Appendix C.

4. A FRAMEWORK FOR ESTIMATING EXPECTED LOSSES FOR A GIVEN YEAR

Using the available information and notation per Section 2, the expected pure premium for year i can be estimated based on the data from year j as follows:

$$ {}_j\hat{E}(PP_i) = \frac{LTD_j \times DF_j}{E_j} \times TF_{ji} \qquad (4.1) $$

where the subscript on the left denotes that the estimate is based on data from year of origin j. Although we usually think of trend factors moving forward in time, note that j can also equal i or be greater than i.

Thus, the data from each year of origin j provide a different estimate ${}_j\hat{E}(PP_i)$. If these estimates were independent, then

[5]Constraint 3 may be violated in practice. The DF_is are usually themselves random variables, which makes the mathematical properties of development estimates less than ideal. Stanard concluded that development estimates are not generally unbiased. See Appendix A of [2]. On the other hand, in Mack's model, development estimates can be unbiased [10].

the optimal estimate of $\hat{E}(PP_i)$ would be a weighted average of the estimates ${}_j\hat{E}(PP_i)$ with the weights inversely proportional to the variances of the estimation errors.[6,7] If the DF_is were known, such an independence assumption would be plausible. Given the methods normally used to estimate the DF_is, independence is unlikely. Nevertheless, we will attempt to develop weights roughly in inverse proportion to the relative variances of the estimation errors associated with the individual projections.

Differences among the variances of the estimation errors associated with the estimates ${}_j\hat{E}(PP_i)$ are generally related to the volume of the data and to the development and trending calculations as follows:

1. <u>*Volume*</u>

   All other things being equal, we normally expect that a larger volume of data produces a lower variance estimate of pure premium than a smaller volume of data. If we consider the loss data itself as the result of a random sample of size E_j, then the variance of the pure premium projection would be inversely proportional to E_j, and the indicated weight directly proportional to E_j. All models discussed herein assume that variance is inversely proportional to E_j and all weighting systems include E_j as an element of the weights.

2. <u>*Development*</u>

   All other things being equal, we normally expect that less mature data will produce higher variance estimates

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[6] A common statistics result. See Rohatgi [9, p. 352].

[7] When the amount being estimated is an expected value, the variance of the estimation errors equals the variance of the estimate, and the two terms may be used interchangeably. When the amount being estimated is an actual value (i.e., the realization of a random variable), then the distinction between the variance of the estimation error and the variance of the estimate is important.

than more mature data. Thus, in a reasonable weighting system, the relative weight will increase with the maturity of the data.

3. *Trending*

Given the imperfections in exposure measurement and trend estimation, the use of one year's data to estimate pure premium for another year would be expected to increase the variance of the estimation error as compared to using the data from the year itself. The relative variance would be expected to increase as the length of time between the years increases. This effect, which could be described as the deterioration in the value of information with time, is dealt with in many areas of actuarial practice.

While the general variance relationships discussed above will usually hold, they should not be considered absolute nor is the list necessarily exhaustive. There may be specific cases when one or more of the above relationships do not hold. Furthermore, the variances associated with the estimates ${}_j\hat{E}(PP_i)$ may come from sources that are not reflected in the above relationships, with some complex interactions among them. Limited to the practical goal of a reasonable and useful weighting system, this paper presents simple, practical models of the variance structure that reasonably account for the variance relationships listed above.

5. THE STANARD–BÜHLMANN OR CAPE COD METHOD

The Stanard–Bühlmann or Cape Cod method compared favorably with other loss reserving techniques in a study by Stanard [2]. Stanard also cites unpublished work by Bühlmann [3], who coined the name "Cape Cod." Patrik presents the method as a reinsurance reserving technique in the Foundations of Casualty Actuarial Science textbook, using the name Stanard–Bühlmann [4, pp. 352–354].

Stanard's presentation of the method assumed that exposures were equal for all years. The presentation below allows for varying levels of exposure, using the notation from Section 2. For clarity of presentation, we will omit the trend factor from the formulas in the remainder of the paper; it is assumed that losses and/or exposures have been adjusted for trend so that the pure premiums are expected to be equal for all years.

The expected pure premium is estimated as follows:

$$\hat{\mathrm{E}}(PP) = \frac{\sum_i LTD_i}{\sum_i [E_i/DF_i]}. \tag{5.1}$$

Note that $\hat{\mathrm{E}}(PP)$ is written without subscript, since the value is presumed to be equal for all years of origin. The expected pure premium thus calculated using the data from all available years is then used to calculate a priori expected losses in the Bornhuetter–Ferguson procedure.

Table 1 includes the trend adjustment and displays the calculation of the expected pure premium.

In Table 2, the expected pure premium is used in the Bornhuetter–Ferguson calculation.

It is instructive to expand Equation 5.1. Rewriting the numerator, we have:

$$\hat{\mathrm{E}}(PP) = \frac{\sum_i [(LTD_i \times DF_i/E_i) \times (E_i/DF_i)]}{\sum_i [E_i/DF_i]}. \tag{5.2}$$

In this form, the value of $\hat{\mathrm{E}}(PP)$ can be seen to be the weighted average of the developed projected pure premiums for each year ($LTD_i \times DF_i \div E_i$) with the weights equal to the values E_i/DF_i. Thus, the method falls into the general framework discussed in Section 4.

TABLE 1

COMPANY XYZ
WORKERS COMPENSATION STUDY
DATA AS OF 12/31/92

CAPE COD METHOD
CALCULATION OF EXPECTED PURE PREMIUM

(1) Accident Year	(2) E_i Exposures	(3) LTD_i Paid Losses @ 12/31/92 (000's)	(4) $TF_{i,1992}$ Trend Factor to 1992 @ 11%	(5) Trended Paid Losses @ 12/31/92 (3) × (4) (000's)	(6) DF_i Cumulative Paid Loss Development Factor	(7) E_i/DF_i (2)/(6)
1979	914	491	3.8833	1,907	1.1200	816
1980	1,203	385	3.4985	1,347	1.1312	1,063
1981	1,264	949	3.1518	2,991	1.1538	1,096
1982	1,372	769	2.8394	2,184	1.1769	1,166
1983	1,422	944	2.5580	2,415	1.2122	1,173
1984	1,502	909	2.3045	2,095	1.2624	1,190
1985	2,090	1,345	2.0762	2,792	1.3239	1,579
1986	2,338	1,298	1.8704	2,428	1.4175	1,649
1987	2,456	1,375	1.6851	2,317	1.5531	1,581
1988	2,617	2,086	1.5181	3,167	1.7053	1,535
1989	2,774	2,153	1.3676	2,945	1.9171	1,447
1990	3,021	2,265	1.2321	2,791	2.4865	1,215
1991	3,067	2,345	1.1100	2,603	3.4906	879
1992	3,428	1,186	1.0000	1,186	6.6569	515
Total	29,468	18,500		33,166		16,903

(8) Expected Pure Premium (at accident year 1992 level): 1.9621
(8) = (total Col. 5)/(total Col. 7)

The Cape Cod weights reflect two of the three variance relationships identified in Section 4. Volume is reflected by having the weights proportional to E_i. Development is reflected by having the weights inversely proportional to DF_i. Variance related to trending is not reflected.

TABLE 2

COMPANY XYZ
WORKERS COMPENSATION STUDY
DATA AS OF 12/31/92

CAPE COD METHOD
ESTIMATION OF ULTIMATE LOSSES
(BORNHUETTER–FERGUSON METHOD)

(1) Accident Year	(9) Expected Pure Premium (8)/(4)	(10) Expected Ultimate Losses/1,000 (2) × (9) (000's)	(11) Expected Unpaid Losses [1 − 1/(6)] ×(10) (000's)	(12) Estimated Ultimate Losses (3) + (11) (000's)
1979	0.5053	462	49	540
1980	0.5608	675	78	463
1981	0.6225	787	105	1,054
1982	0.6910	948	143	912
1983	0.7670	1,091	191	1,135
1984	0.8514	1,279	266	1,175
1985	0.9451	1,975	483	1,828
1986	1.0490	2,453	722	2,020
1987	1.1644	2,860	1,018	2,393
1988	1.2925	3,382	1,399	3,485
1989	1.4347	3,980	1,904	4,057
1990	1.5925	4,811	2,876	5,141
1991	1.7677	5,421	3,868	6,213
1992	1.9621	6,726	5,716	6,902
Total		36,849	18,819	37,319

Using weights inversely proportional to the DF_is makes the implicit assumption that the relative variance of the development-based pure premium estimates are proportional to the DF_is[8] (the Cape Cod variance assumption). This same assumption was previously listed as underlying the Bornhuetter–Ferguson method.

[8]This result is developed in Appendix B of [2].

This very simple variance model is often adequate to account for the decreasing reliability of projections as development factors increase—but not always. For example, incurred loss development factors will often approach unity well before all the losses are settled and all variance is eliminated. Incurred loss development factors less than unity provide an example where the Cape Cod variance assumption is clearly unreasonable.[9] Note that the Bornhuetter–Ferguson method also produces unreasonable results in this case.[10] This potential problem is addressed in Section 7.

The failure to reflect variance related to trending can be a serious shortcoming in practice. Practitioners have sometimes found that the Cape Cod method gives excessive weight to out of date results. The problem can be severe when a very long data base is used, as is often the case in reinsurance applications. The problem can be addressed to some degree by limiting the number of years entering the Cape Cod calculation. A less arbitrary approach is to specifically account for the relationship between variance and trending in the weighting scheme, as is presented in the following section.

6. ACCOUNTING FOR YEAR-TO-YEAR VARIANCE

We present two approaches for accounting for variance related to trending. The first uses an exponential decay factor, which is simple to apply and has proven practical in applications, although it is not based directly on a mathematical model. The decay factor approach is the one cited in most other sections of this paper. The second approach, using an additive "adaptive variance" term, is based on a specific mathematical model and is directly analogous to techniques used in dynamic stochastic modeling. With suitably

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[9]The implication would be that the immature projected pure premium is more reliable (i.e., has lower variance) than the actual ultimate pure premium.

[10]Used with a development factor less than unity, the Bornhuetter–Ferguson method produces a projection outside of the range of the development result and the expected result.

chosen parameters, the two approaches produce similar results. The adaptive variance section can be skipped without substantial loss of continuity. The mathematical model underlying the adaptive variance approach is used in Appendix D to develop indicated values of the adaptive variance and of the approximately equivalent decay factor.

The Decay Factor Approach[11]

We account for the variance related to trending by introducing an exponential decay factor to the original Cape Cod weighting scheme. Equation 5.2 becomes:

$$\hat{\mathrm{E}}(PP_i) = \frac{\sum_j \left[\left(\frac{LTD_j \times DF_j}{E_j} \right) \times \left(\frac{E_j}{DF_j} \right) \times D^{|i-j|} \right]}{\sum_j \left[\left(\frac{E_j}{DF_j} \right) \times D^{|i-j|} \right]};$$

$$\text{where} \quad 0 \le D \le 1. \qquad (6.1)$$

The weights $(E_j/DF_j) \times D^{|i-j|}$ now reflect volume (via E_j), development using the Cape Cod variance assumption (via $1/DF_j$), and trending via the exponentially decaying weight $D^{|i-j|}$. The exponentially decaying weight has the required property that the relative weight decreases as the length of the trending period, $|i-j|$, increases. Note that the value $\hat{\mathrm{E}}(PP_i)$ now contains a subscript denoting the year of origin, since the weights will now shift for each year of origin, causing the values of $\hat{\mathrm{E}}(PP_i)$ to "drift."[12]

In Table 3, the example from Table 1 is re-worked with an annual exponential decay factor of 0.75. The calculation of

[11]Used for many years in various consulting reports [11].

[12]The drifting value of $\hat{\mathrm{E}}(PP_i)$ is roughly analogous to the techniques of dynamic stochastic modeling, where the various model parameters may be allowed to drift over time. See, for example DeJong and Zehnwirth [5] and Wright [7].

TABLE 3

COMPANY XYZ
WORKERS COMPENSATION STUDY
DATA AS OF 12/31/92

GENERALIZED CAPE COD METHOD WITH DECAY
CALCULATION OF EXPECTED PURE PREMIUM FOR ACCIDENT YEAR 1990
(USING A DECAY RATE OF 75%)

(1)	(2) E_i	(3) LTD_i	(4) $TF_{i,1992}$	(5) DF_i	(6)	(7) $D^{\|i-1990\|}$	(8)
Accident Year	Expo-sures	Paid Losses @ 12/31/92 (000's)	Trend Factor to 1992 @ 11%	Cumu-lative Paid Loss Devel-opment Factor	Indicated Ultimate Trended Pure Premium (3) × (4) × (5)/(2)	Decay @ 75%	Total Weight Assigned to Indicated Ult. Pure Premium (2) × (7)/(5)
1979	914	491	3.8833	1.1200	2.3364	0.0422	34.467
1980	1,203	385	3.4985	1.1312	1.2665	0.0563	59.888
1981	1,264	949	3.1518	1.1538	2.7303	0.0751	82.256
1982	1,372	769	2.8394	1.1769	1.8730	0.1001	116.709
1983	1,422	944	2.5580	1.2122	2.0585	0.1335	156.586
1984	1,502	909	2.3045	1.2624	1.7607	0.1780	211.758
1985	2,090	1,345	2.0762	1.3239	1.7689	0.2373	374.626
1986	2,338	1,298	1.8704	1.4175	1.4719	0.3164	521.875
1987	2,456	1,375	1.6851	1.5531	1.4652	0.4219	667.133
1988	2,617	2,086	1.5181	1.7053	2.0635	0.5625	863.228
1989	2,774	2,153	1.3676	1.9171	2.0349	0.7500	1,085.233
1990	3,021	2,265	1.2321	2.4865	2.2970	1.0000	1,214.961
1991	3,067	2,345	1.1100	3.4906	2.9625	0.7500	658.984
1992	3,428	1,186	1.0000	6.6569	2.3031	0.5625	289.662
Total	29,468	18,500					6,337.366

(10) Expected Pure Premium for Accident Year 1990 (at 1992 AY level): 2.0675
(10) = Average of Col. 6 weighted by Col. 8

TABLE 4

COMPANY XYZ
WORKERS COMPENSATION COMPANY
DATA AS OF 12/31/92

GENERALIZED CAPE COD METHOD WITH DECAY
ESTIMATION OF ULTIMATE LOSSES
(BORNHUETTER–FERGUSON METHOD)

(1) Accident Year	(11) Expected Pure Premium @ AY 1992 Level	(12) Expected Pure Premium (11)/(4)	(13) Expected Ultimate Losses (2) × (12) (000's)	(14) Expected Unpaid Losses [1 − 1/(5)] × (13) (000's)	(15) Estimated Ultimate Losses (3) + (14) (000's)
1979	1.9586	0.5044	461	49	540
1980	1.9246	0.5501	662	77	462
1981	1.9676	0.6243	789	105	1,054
1982	1.9290	0.6794	932	140	909
1983	1.9019	0.7435	1,057	185	1,129
1984	1.8644	0.8090	1,215	253	1,162
1985	1.8397	0.8861	1,852	453	1,798
1986	1.8246	0.9755	2,281	672	1,970
1987	1.8511	1.0985	2,698	961	2,336
1988	1.9250	1.2680	3,318	1,372	3,458
1989	1.9915	1.4562	4,039	1,932	4,085
1990	2.0675	1.6781	5,069	3,031	5,296
1991	2.1399	1.9278	5,913	4,219	6,564
1992	2.1486	2.1486	7,365	6,259	7,445
Total			37,652	19,708	38,208

the expected pure premium for accident year 1990 is shown in Table 3.

The analogous calculation is then performed for other accident years, and the results are recorded in Column 11 of Table 4. Note the drifting values of the expected pure premium in Column 11. (The single value 1.9617 was used for all years in the Table 1 and Table 2 calculation).

This structure, using a decay factor between zero and unity, conveniently collapses to the original Cape Cod method when $D = 1$ and to the development method when $D = 0$. Thus, adding the decay factor produces a compromise between the Cape Cod method and the development method, with the degree of compromise controlled by the decay factor. The value of the decay factor should be a function of the variance associated with development projections compared with the variance associated with trend projections. In general, lower decay factors are appropriate for large data bases exhibiting stable development with higher decay factors for smaller data bases with more erratic development.

Given that using the decay factor produces a compromise between the Cape Cod and development methods, and that both the Cape Cod and development methods represent documented methodology, use of the decay factor will fall within the framework of documented methodology with any value of the decay factor between zero and unity, and it is reasonable that the decay factor may be judgmentally selected. Alternatively, the relative variances in the development and trend directions can be measured from the data triangle and used to aid in the selection of the decay factor.

Appendix B presents a variance model for data in a development triangle array, consistent with the Cape Cod variance assumption. Using that model, a method for using the data triangle to determine the indicated decay factor is developed in Appendix D.

In judgmentally selecting decay factors over many years in practice, we have generally used values ranging from 50% to 100%, with 75% as a "default" value. Estimates using the Appendix D methodology appear to confirm that this range is reasonable.

The Adaptive Variance Approach

This approach is justified by assuming that the unknown values $\mathrm{E}(PP_i)$ observe a simple random walk, i.e.,

$$\mathrm{E}(PP_i) = \mathrm{E}(PP_{i-1}) + d \qquad (6.2)$$

where d is a random "disturbance" with mean zero and variance ${}_d\sigma^2$. We refer to the value of ${}_d\sigma^2$ as the adaptive variance.

Denote the variance of the development-based estimate of PP_j as σ_j^2, i.e.

$$\mathrm{Var}(LTD_j \times DF_j \div E_j) = \sigma_j^2.$$

Then, it can be shown that the variance of the estimation error associated with using the development-based estimate of PP_j as an estimate of $\exp(PP_i)$ is as follows:

$$\mathrm{Var}(\mathrm{E}(PP_i) - LTD_j \times DF_j \div E_j) = \sigma_j^2 + {}_d\sigma^2 \times |i-j|. \qquad (6.3)$$

The Cape Cod weights in Equation 5.2 assume that σ_j^2 is directly proportional to DF_j and inversely proportional to E_j, i.e.

$$\sigma_j^2 = k \times DF_j / E_j \qquad (6.4)$$

for some proportionality constant k. Substituting in Equation 6.3, we have:

$$\mathrm{Var}(\mathrm{E}(PP_i) - LTD_j \times DF_j \div E_j) = \frac{k \times DF_j}{E_j} + {}_d\sigma^2 |i-j| \qquad (6.5)$$

and the indicated weights would be in inverse proportion to the variances in Equation 6.5. To calculate these weights requires estimates of both the adaptive variance ${}_d\sigma^2$ and the proportionality constant k.

The adaptive variance approach collapses to the original Cape Cod method when ${}_d\sigma^2 = 0$ and approaches the development method as ${}_d\sigma^2$ approaches infinity. Methods for estimating k and ${}_d\sigma^2$ are provided in Appendices B and D.

Selecting an Approach

Although the adaptive variance approach is more directly tied to a mathematical model, we generally prefer the decay factor approach since:

- the two approaches produce similar results;
- it is simpler to apply;
- it directly reflects the degree of compromise between the Cape Cod and development methods; and
- it is unitless, and thus is more amenable to judgmental selection, evaluation of reasonableness, and comparisons among different data bases.

7. GENERALIZING THE DEVELOPMENT VARIANCE ASSUMPTION

In each method presented thus far, the relative variances arising from development have been modeled using the Cape Cod variance assumption. While this simple assumption is often adequate, it is rather crude and is sometimes sufficiently inaccurate that the methods (including the Bornhuetter–Ferguson method in general) are unusable or of limited effectiveness.

Rather than specify a relationship between the development-related variance and the development factors, we address the issue more generally by introducing an additional "variance factor," VF_i, defined as follows:

$$VF_i = \frac{\mathrm{Var}[LTD_i \times DF_i]}{\mathrm{Var}(ULT_i)} \qquad (7.1)$$

The Cape Cod variance assumption is the special case when $VF_i = DF_i$.

In each previously presented formula for estimating expected pure premiums, the VF_is replace the DF_is as an element of the weights. Thus, the original Cape Cod weighting scheme (Equation 5.2) becomes:

$$\hat{\mathrm{E}}(PP) = \frac{\sum_i \left[\left(\frac{LTD_i \times DF_i}{E_i} \right) \times \left(\frac{E_i}{VF_i} \right) \right]}{\sum_i \left[\left(\frac{E_i}{VF_i} \right) \right]}. \qquad (7.2)$$

Using the decay factor, Equation 6.1 becomes:

$$\hat{\mathrm{E}}(PP_i) = \frac{\sum_j \left[\left(\frac{LTD_j \times DF_j}{E_j} \right) \times \left(\frac{E_j}{VF_j} \right) \times D^{|i-j|} \right]}{\sum_j \left[\left(\frac{E_j}{VF_j} \right) \times D^{|i-j|} \right]}. \qquad (7.3)$$

After the expected pure premium is estimated, the final step of the reserving procedure has been the application of the Bornhuetter–Ferguson method; however, with the alternative assumption, an "alternative Bornhuetter–Ferguson" calculation is indicated. We modify Equation 3.2, replacing the weights based on DF_i with weights based on VF_i, as follows:

$$ULT_i = (1/VF_i) \times LTD_i \times DF_i + (1 - 1/VF_i) \times \hat{\mathrm{E}}(ULT_i). \qquad (7.4)$$

In Table 5 the expected pure premium for accident year 1990 is calculated using a decay factor of 0.75 and variance factors in Column 8 different from the development factors in Column 5.

After performing the analogous calculation for other accident years, the remainder of the methodology is shown in Table 6.

Appendix C presents an alternative variance model consistent with using variance factors different from the development

TABLE 5

COMPANY XYZ
WORKERS COMPENSATION STUDY
DATA AS OF 12/31/92

GENERALIZED CAPE COD METHOD WITH DECAY AND ALTERNATIVE VARIANCE FACTORS CALCULATION OF EXPECTED PURE PREMIUM FOR ACCIDENT YEAR 1990 (USING A DECAY RATE OF 75%)

(1)	(2) E_i	(3) LTD_i	(4) $TF_{i,1992}$	(5) DF_i	(6)	(7) $D^{\|i-1990\|}$	(8) VF_i	(9)
Accident Year	Exposures	Incurred Losses @ 12/31/92 (000's)	Trend Factor to 1992 @ 11%	Cumulative Incurred Loss Development Factor	Indicated Ultimate Trended Pure Premium (3) × (4) × (5)/(2)	Decay Rate @ 75%	Variance Factors	Total Weight Assigned to Indicated Ult. Pure Premium (2) × (7)/(8)
1979	914	684	3.8833	1.0000	2.9061	0.0422	1.1200	34.467
1980	1,203	490	3.4985	1.0050	1.4321	0.0563	1.1312	59.888
1981	1,264	1,068	3.1518	1.0100	2.6897	0.0751	1.1538	82.256
1982	1,372	817	2.8394	1.0151	1.7164	0.1001	1.1769	116.709
1983	1,422	1,022	2.5580	1.0252	1.8848	0.1335	1.2122	156.586
1984	1,502	913	2.3045	1.0406	1.4577	0.1780	1.2624	211.758
1985	2,090	1,597	2.0762	1.0614	1.6838	0.2373	1.3239	374.626
1986	2,338	1,485	1.8704	1.0880	1.2926	0.3164	1.4175	521.875
1987	2,456	1,554	1.6851	1.1206	1.1948	0.4219	1.5531	667.133
1988	2,617	2,538	1.5181	1.1830	1.7417	0.5625	1.7053	863.228
1989	2,774	2,705	1.3676	1.2715	1.6957	0.7500	1.9171	1,085.233
1990	3,021	3,181	1.2321	1.4253	1.8491	1.0000	2.4865	1,214.961
1991	3,067	3,345	1.1100	1.7462	2.1140	0.7500	3.4906	658.984
1992	3,428	2,109	1.0000	2.2026	1.3551	0.5625	6.6569	289.662
Total	29,468	23,508						6,337.366

(11) Expected Pure Premium for Accident Year 1990 (at AY 1992 level): 1.6868
(11) = Average of Col. 6 weighted by Col. 9

TABLE 6

COMPANY XYZ
WORKERS COMPENSATION STUDY
DATA AS OF 12/31/92

GENERALIZED CAPE COD METHOD
WITH DECAY AND ALTERNATIVE VARIANCE FACTORS
ESTIMATION OF ULTIMATE LOSSES

(1) Accident Year	(11) Expected Pure Premium @ AY 1992 Level	(12) Expected Pure Premium (11)/(4)	(13) Expected Ultimate Losses (2) × (12) (000's)	(14) Development Basis Ultimate Losses (3) × (5) (000's)	(15) Estimated Ultimate Losses (13) × [1 − 1/(8)]+ (14)/(8) (000's)
1979	1.9586	0.5044	461	684	660
1980	1.9025	0.5438	654	492	511
1981	1.8916	0.6002	759	1,079	1,036
1982	1.8072	0.6365	873	829	836
1983	1.7450	0.6822	970	1,048	1,034
1984	1.6784	0.7283	1,094	950	980
1985	1.6377	0.7888	1,649	1,695	1,684
1986	1.5946	0.8525	1,993	1,616	1,727
1987	1.5873	0.9420	2,314	1,741	1,945
1988	1.6261	1.0712	2,803	3,002	2,920
1989	1.6557	1.2106	3,358	3,439	3,401
1990	1.6868	1.3690	4,136	4,534	4,296
1991	1.7071	1.5380	4,717	5,841	5,039
1992	1.6883	1.6883	5,787	4,645	5,616
Total			31,568	31,597	31,685

factors. Using that model, Equation 7.4 is demonstrated to be the indicated alternative to the Bornhuetter–Ferguson method.

It is beyond the scope of this paper to develop specific models of the relationship between variance and development. In practice, any reasonable variance factors will produce reasonable weights.

Measurement of the variance factors based on the actual data triangle is possible, but the available data may frequently be too limited to parameterize a model of any complexity. As a practical alternative, a "reference pattern" can be used, or a simple modification to a reference pattern can be made.

For the reference pattern to be useful, the values should be greater than unity as long as there is any significant remaining uncertainty in the development projection of ultimate losses. For example, if the Cape Cod variance assumption has been rejected for incurred development because the development factors decrease to unity (or less) faster than the uncertainty is eliminated, the paid development factors for the same business may provide a logical reference pattern (in the example of Table 5, the alternative variance factors are the paid development factors for the same business). A compromise between the paid and incurred development factors is another possible choice.

8. USING EXPECTED VALUE ESTIMATES IN THE BORNHUETTER–FERGUSON CALCULATIONS

The methodologies described in this paper estimate ultimate losses with a two step process: first, estimating expected ultimate losses by optimally combining information from all years; then using the estimated expected ultimate losses in the Bornhuetter–Ferguson or alternative Bornhuetter–Ferguson calculation. Proofs are provided in Appendices A and C that the Bornhuetter–Ferguson and alternative Bornhuetter–Ferguson weights are optimal, but the proofs are dependent on the constraint that the expected ultimate losses are assumed known.

In fact, the expected ultimate losses are not known. Rather, we are using an estimate of the expected ultimate losses. Furthermore, that estimate is not independent from the development result, since the development result from each year is part of the expected ultimate loss estimate.

In each of the estimates of expected ultimate losses presented in this paper, the expected ultimate loss estimate can be expressed as a weighted average of the development estimate from the year itself and other estimates independent of the data from the year itself (i.e., data from other years).

Thus:

$$\hat{E}(ULT) = W' \times LTD \times DF + (1 - W') \times (Other)$$

where *Other* is an estimate of E(*ULT*), independent of *LTD* and *ULT*.

Additionally, the weights W' and $(1 - W')$ are inversely proportional to the variances of the estimates $LTD \times DF$ and *Other*, under the assumed variance models.

Appendix E addresses the issue of optimal Bornhuetter–Ferguson or alternative Bornhuetter–Ferguson weights, replacing the original assumption that E(*ULT*) is known with an assumption that the estimate $\hat{E}(ULT)$ has the properties listed above. The result is that the exact same weights continue to be optimal.

9. APPLICATION

For convenience, we have referred to the quantity being projected by development methods as "losses," the exposure base as "exposures" and the ratio of the two as "pure premiums." However, there are many other potential applications. The methods described herein are useful any time we make a development-based projection and compare the result to some other predictive quantity. The following chart gives some examples:

QUANTITY BEING PROJECTED	"EXPOSURE" BASE	TREND ADJUSTMENT
Losses	Ratemaking Exposures	Pure Premium Trend
Losses	Ultimate Claim Counts	Severity Trend
Losses	Earned Premiums	Loss Ratio Index, or equivalently, Rate Adequacy Index
Claim Counts	Ratemaking Exposures	Frequency Trend
ALAE	Ultimate Losses	Expected Trend in ALAE/Loss Ratio (if any)
Salvage	Ultimate Losses	Expected Trend in Salvage/Loss (if any)
Excess Loss	Ultimate Limited Losses	Expected Trend in Excess/ Limited Losses (if any)

10. CONCLUSION

The techniques of this paper are useful in a wide variety of applications, and provide an alternative to the somewhat arbitrary judgments that are required when trend projections are incorporated only through reasonableness tests and ad hoc modifications to development projections.

The weighting methods presented herein are based on simplified variance structures designed to reasonably reflect the variance relationships that we typically expect to see. There is undoubtedly a good deal of room for improvement in this area, and the development of more rigorous variance models is an interesting and useful area for further research. However, the difference between reasonably good weights and optimal weights is often not significant, and the use of these techniques need not wait for improved variance models.

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GLOSSARY OF NOTATION

	Notation	Definition	Statistical Conception[13]
Section 2	N	Number of years of origin	
	LTD_i	Cumulative losses for year of origin i at current evaluation	Random variable
	DF_i	Cumulative development factor to ultimate applicable to losses for year of origin i at current evaluation	Treated as a known constant
	E_i	Measurement of relative exposure for year of origin i	Known constant
	TF_{ij}	Pure premium trend factor from year of origin i to year of origin j	Treated as a known constant
	ULT_i	Ultimate losses for year of origin i	Random variable
	PP_i	$ULT_i \div E_i$	Random variable
	E( )	Expectation	Operator
	Var( )	Variance	Operator
	$\hat{}$	Denotes estimation; i.e. the value is an estimate of the value under the "*hat*"	
Section 4	${}_j\hat{\mathrm{E}}(PP_i)$	Estimate of $\mathrm{E}(PP_i)$ based only on data from year of origin j. Defined in Equation 4.1	Estimated parameter, therefore, random variable
Section 5	PP	Single value of PP assumed to apply to all years of origin in the Cape Cod model	Random variable

[13] A number of random variables result from summarized data, and may be conceived of as sample sums or sample means (which are still random variables).

	Notation	Definition	Statistical Conception
Section 6	D	Usually used in the form $D^{\|i-j\|}$. First introduced in Equation 6.1. An exponential decay factor between zero and one, used to decrease relative weight as the length of the trend period increases	Unknown parameter, mostly treated as a known constant. If estimated, it is then an estimated parameter, therefore a random variable
	d	Random disturbance term, used to define a random walk in the values E(PP_i). First introduced in Equation 6.2	Random variable
	${}_d\sigma^2$	Variance of d, called the "adaptive variance"	Unknown parameter
	k	A proportionality constant. First introduced in Equation 6.4. Also refer to Equation B.1.	Unknown parameter
Section 7	VF_i	"Variance factor" to reflect relative variances of development-based ultimate losses for different years of origin. Defined in Equation 7.1.	Treated as a known constant
Section 8	W'	Weight assigned to $LTD \times DF$ in an estimate of E(ULT)	
	Other	Estimate of E(ULT) that is independent of the values LTD and ULT (normally from other years of origin)	
Appendix A	V^2	Var(ULT)	Unknown parameter
	W	Weight assigned to $LTD \times DF$ in Bornhuetter–Ferguson estimate of ULT	
	$\propto$	"Is proportional to"	
Appendix B	X_{ij}	Cumulative losses for year of origin i through development period j	Random variable

	Notation	Definition	Statistical Conception
	x_{ij}	Non-cumulative losses for year of origin i in development period j	Random variable
	P_j	Cumulative development pattern through period j	Treated as a known constant
	p_j	Non-cumulative development pattern in period j	Treated as a known constant
	n	Number of points of data	
	$\widehat{PP}_i$	Development based estimate of PP_i. Also serves as development based estimate of $E(PP_i)$	Estimated parameter, therefore, random variable
	$\widehat{PP}$	Cape Cod estimate of PP. Also serves as Cape Cod estimate of $E(PP)$. Defined in Equation B.5.	Estimated parameter therefore, random variable
	w_{ij}	Weight given to the value $x_{ij}/E_i p_j$ in calculating $\widehat{PP}$. Defined in Equation B.6.	
	${}_D\widehat{PP}_i$	Estimate of $E(PP_i)$ using the Cape Cod with decay model, using decay factor D.	Estimated parameter therefore, random variable
		Note a small inconsistency in the use of the "^" in that the value is an estimate of $E(PP_i)$ rather than PP_i	
	${}_Dw_{ij}$	The weight applied to the value $x_{ij}/E_i p_j$ in calculating ${}_D\widehat{PP}_i$. Defined in Equation B.8.	
	$\hat{k}_D$	Estimate of k using decay factor D	Estimated parameter, therefore, random variable
Appendix C	LTD'	$LTD \times DF/VF$. Transformed value of LTD for use in the alternative variance model	Random variable

	Notation	Definition	Statistical Conception
	V_j	Cumulative "variance pattern" through development period j	Treated as known constant
	v_j	Non-cumulative variance pattern in development period $j (= V_j - V_{j-1})$	Treated as known constant
	X'_{ij}	$X_{ij} \times V_j / P_j$. Transformed value of X_{ij}	Random variable
	x'_{ij}	Non-cumulative transformed value $(= X'_{ij} - X'_{i,j-1})$	Random variable
Appendix D	ε_i	Random error related to estimate ${}_0\widehat{PP}_i$. Introduced in Equation D.1.	Random variable
	Δ_i	${}_0\widehat{PP}_i - {}_0\widehat{PP}_{i-1}$	Random variable
	Σ	Variance-covariance matrix of the vector Δ	Unknown parameter
	$\bar{\sigma}_i^2$	Variance of ${}_0\widehat{PP}_i$ for year of origin with average variance. Defined in Equation D.5.	

APPENDIX A

OPTIMALITY OF THE BORNHUETTER–FERGUSON CALCULATION

This Appendix provides a proof that the Bornhuetter–Ferguson weights are optimal (i.e., they produce the minimum variance estimate) under the constraints listed in Section 3. We will use the notation from Section 2, dropping the subscript denoting year of origin.

In addition, let $V^2 = \text{Var}(ULT)$.

By Constraint 2, LTD is independent from $ULT - LTD$.

Therefore, $\text{Cov}(ULT, LTD) = \text{Var}(LTD)$.

By Constraint 4, $\text{Var}(LTD \times DF) \propto DF$.

Noting that $DF = 1$ when $LTD = ULT$, the proportionality constant is $\text{Var}(ULT)$, or V^2:

$$\text{Var}(LTD \times DF) = V^2 \times DF$$

$$\text{Var}(LTD) = V^2/DF$$

$$\begin{aligned}\text{Var}(ULT - LTD) &= \text{Var}(ULT) + \text{Var}(ULT) - 2\,\text{Cov}(ULT, LTD)\\ &= \text{Var}(ULT) - \text{Var}(LTD)\\ &= V^2(1 - 1/DF).\end{aligned}$$

The Bornhuetter–Ferguson method estimates ultimate losses as a weighted average of a development-based estimate of ultimate losses and expected ultimate losses, i.e.:

$$\widehat{ULT} = W \times LTD \times DF + (1 - W) \times \text{E}(ULT); \qquad 0 \le W \le 1$$

Since the two estimates, $LTD \times DF$ and $\text{E}(ULT)$, are clearly independent, optimal weights are inversely proportional to the variances of the estimation errors [9, p. 352]. Starting with the estimate $LTD \times DF$:

$$\begin{aligned} \text{Var}(ULT - LTD \times DF) &= \text{Var}(ULT) + \text{Var}(LTD \times DF) \\ &\quad - 2\,\text{Cov}(ULT, LTD \times DF) \\ &= V^2 + V^2 \times DF - 2 \times DF \times \text{Var}(LTD) \\ &= V^2 + V^2 \times DF - 2V^2 \\ &= V^2[DF - 1]. \end{aligned}$$

The variance of the estimation error associated with using $\text{E}(ULT)$ as an estimate of ULT:

$$\text{Var}[ULT - \text{E}(ULT)] = \text{Var}(ULT) = V^2.$$

Calculating W in inverse proportion to the variances:

$$\begin{aligned} &= \frac{V^2}{V^2[DF - 1] + V^2} = \frac{1}{DF - 1 + 1} \\ &= 1/DF \end{aligned}$$

which is the weight used in the Bornhuetter–Ferguson method.

APPENDIX B

THE CAPE COD VARIANCE MODEL

This Appendix provides a simple model for the variance structure of the development triangle consistent with the Cape Cod variance assumption.

We introduce additional notation to deal with the full data triangle. Also, note that whereas the DF_is in the prior notation carry a subscript denoting year of origin, the development patterns included in this Appendix carry subscripts denoting development period. Capital letters are used to denote cumulative values with lower case letters denoting the corresponding non-cumulative values.

All losses are presumed to have been trended to a common level.

<u>Notation</u>:

Cumulative Triangle Values: X_{ij}, $1 \leq i \leq N$, $1 \leq j \leq N$
Non-Cumulative Triangle Values: x_{ij} ($= X_{ij}$ for $j = 1$; $= X_{ij} - X_{i,j-1}$ for $j > 1$)
Cumulative Development Pattern: P_j, ($= 1/DF_{N-j+1}$)
Non-Cumulative Development Pattern: p_j ($= P_j$ for $j = 1$; $= P_j - P_{j-1}$ for $j > 1$)

For year i, the number of points in X_{ij} is $N - i + 1$. Denote the total number of points in X_{ij} as $n = N(N + 1)/2$.

Additional Assumptions

- The values x_{ij} are assumed to be mutually independent. Within a given year of origin, this is somewhat more restrictive than (although clearly consistent with) the previous assumption of independence between the emerged and unemerged losses. Independence among values from different years of origin is an additional assumption briefly touched on in Section 4.

- The variance of PP_i is inversely proportional to E_i, i.e.

$$\mathrm{Var}(PP_i) = \frac{k}{E_i}. \tag{B.1}$$

This assumption was previously discussed in Section 4.

These additional assumptions are sufficient to determine that:

$$\mathrm{Var}(x_{ij}) = k \times E_i \times p_j. \tag{B.2}$$

Proof The Cape Cod variance assumption (using notation of this Appendix):

$$\mathrm{Var}\left(\frac{X_{ij}}{P_j}\right) \propto \frac{1}{P_j}.$$

Noting that $P_j = 1$ when $ULT_i = X_{ij}$, we have:

$$\begin{aligned}
\mathrm{Var}\left(\frac{X_{ij}}{P_j}\right) &= \frac{\mathrm{Var}(ULT_i)}{P_j} \\
\mathrm{Var}(X_{ij}) &= P_j \times \mathrm{Var}(ULT_i) \\
\mathrm{Var}(X_{i,j-1} + x_{ij}) &= \mathrm{Var}(X_{i,j-1}) + \mathrm{Var}(x_{ij}) \\
\mathrm{Var}(x_{ij}) &= \mathrm{Var}(X_{ij}) - \mathrm{Var}(X_{i,j-1}) \\
\mathrm{Var}(x_{ij}) &= P_j \times \mathrm{Var}(ULT_i) - P_{j-1} \times \mathrm{Var}(ULT_i) \\
\mathrm{Var}(x_{ij}) &= (P_j - P_{j-1}) \times \mathrm{Var}(ULT_i) \\
&= p_j \times \mathrm{Var}(ULT_i).
\end{aligned} \tag{B.3}$$

Noting that $ULT_i = E_i \times PP_i$ and using Equation B.1, we have:

$$\mathrm{Var}(ULT_i) = E_i^2 \times \mathrm{Var}(PP_i) = k \times E_i. \tag{B.4}$$

Substituting (B.4) in (B.3) produces Equation B.2.

The value k may be interpreted as the variance associated with one unit of exposure, when losses are fully developed. Each point, x_{ij} provides an independent estimate of $E(PP_i)$, as $x_{ij}/E_i p_j$, with variance $k/E_i p_j$.

Optimality of the Development Estimates

Weighting all estimates of $E(PP_i)$ from a given year of origin in inverse proportion to variances:

$$\hat{E}(PP_i) = \frac{\sum_{j=1}^{N-i+1} (x_{ij}/E_i p_j) \times E_i p_j}{\sum_{j=1}^{N-i+1} E_i p_j}$$

$$= X_{i,N-i+1}/E_i P_{N-i+1}$$

which is the development estimate; call it $\widehat{PP}_i$.

Optimality of the Cape Code Estimate

We next assume that $E(PP_i)$ is the same for all years i (we will write it as $E(PP)$). Weighting all estimates of $E(PP)$ in inverse proportion to variances:

$$\hat{E}(PP) = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N-i+1} (x_{ij}/E_i p_j) \times E_i p_j}{\sum_{i=1}^{N} \sum_{j=1}^{N-i+1} E_i p_j}$$

$$= \frac{\sum_{i=1}^{N} X_{i,N-i+1}}{\sum_{i=1}^{N} E_i \times P_{N-i+1}} \qquad \text{(B.5)}$$

which is the estimate of the original Cape Cod method; call it $\widehat{PP}$.

Estimating the Proportionality Constant k

Estimates of the proportionality constant k are used in quantifying the decay factor (in Appendix D). The remainder of this Appendix deals with making these estimates. Noting that the estimate $\widehat{PP}$ is a weighted average of the individual estimates $(x_{ij}/E_i \times p_j)$, the value k can be estimated by averaging the sample variance estimates at each point:

$$\frac{\hat{k}}{E_i P_j} = \left(\widehat{PP} - \frac{x_{ij}}{E_i p_j}\right)^2$$

$$\hat{k} = \frac{(\widehat{PP} \times E_i p_j - x_{ij})^2}{E_i p_j}.$$

The sample variance estimates are biased low due to degrees of freedom of the estimate $\widehat{PP}$.[15] In a weighted average, the bias is different at each point. If a given point has weight w_{ij} such that $\sum w_{ij} = 1$, then the bias correction at that point is $1/(1 - w_{ij})$.

Define:

$$w_{ij} = \frac{E_i p_j}{\sum_{l=1}^{N} \sum_{j=1}^{N-l+1} E_l p_j} = \frac{E_i p_j}{\sum_{l=1}^{N} E_l P_{N-l+1}}. \qquad \text{(B.6)}$$

Then the individual estimates of k, corrected for bias are:

$$\hat{k} = \frac{(\widehat{PP} \times E_i p_j - x_{ij})^2}{E_i p_j \times (1 - w_{ij})}.$$

---

[15]In keeping with our previously stated simplifying assumptions, we are treating the development pattern p_j and any trend factors used as known values. Since in practice these values are likely to be estimated from the x_{ij}, the measurement of variances is improved if the number of parameters in the development pattern is kept to the minimum necessary; thus, using a fitted curve for the development pattern is recommended.

Averaging all available estimates of k:

$$\hat{k} = \frac{\displaystyle\sum_{i=1}^{N}\sum_{j=1}^{N-i+1} \frac{(\widehat{PP} \times E_i p_j - x_{ij})^2}{E_i p_j (1 - w_{ij})}}{N}. \tag{B.7}$$

Estimating k Under the Cape Cod with Decay Model

The Cape Cod with Decay model allows that the value $E(PP_i)$ may vary among the years. Let ${}_D\widehat{PP}_i$ represent the estimate of $E(PP_i)$ using decay factor D. Using the notation of this section:

$${}_D\widehat{PP}_i = \frac{\displaystyle\sum_{l=1}^{N}\sum_{j=1}^{N-l+1} (x_{lj}/E_l p_j) \times E_l p_j \times D^{|i-l|}}{\displaystyle\sum_{l=1}^{N}\sum_{j=1}^{N-l+1} E_l \times p_j \times D^{|i-l|}}$$

$$= \frac{\displaystyle\sum_{l=1}^{N} X_{l,N-l+1} \times D^{|i-l|}}{\displaystyle\sum_{l=1}^{N} E_l \times P_{N-l+1} \times D^{|i-l|}}$$

Note that ${}_1\widehat{PP}_i = \widehat{PP}$ for all i, and that ${}_0\widehat{PP}_i$ represents the development estimate for year i ($\widehat{PP}_i$).

Two modifications to Equation B.7 are indicated. First, the value $\widehat{PP}$ is replaced with the individual year values ${}_D\widehat{PP}_i$. Second, the degree of freedom correction is changed due to the change in weighting system. Denote the weight given to the point x_{ij} in calculating the value ${}_D\widehat{PP}_i$ as ${}_Dw_{ij}$. Then,

$${}_Dw_{ij} = \frac{E_i p_j}{\displaystyle\sum_{l=1}^{N}\sum_{j=1}^{NJ-l+1} E_l \times p_j \times D^{|i-l|}} = \frac{E_i p_j}{\displaystyle\sum_{l=1}^{N} E_l \times P_{N-l+1} \times D^{|i-l|}}. \tag{B.8}$$

Note that if $D < 1$, the values ${}_Dw_{ij}$ are strictly greater than the values w_{ij}. Thus, the full set of values ${}_Dw_{ij}$ does not make a single set of weights. This is because when you change the subscript i, ${}_Dw_{ij}$ now refers to a weight in a different weighted average.

The formula for k reflecting decay factors less than one is as follows:

$$\hat{k}_D = \frac{\displaystyle\sum_{i=1}^{N}\sum_{j=1}^{N-i+1} \frac{({}_D\widehat{PP}_i \times E_i p_j - x_{ij})^2}{E_i p_j(1 - {}_Dw_{ij})}}{N}. \tag{B.9}$$

Note that in the special case when $D = 0$, $\hat{k}_0$ is based on observed within-year variance only. Also note that there will be no variance estimate available at the point $x_{N,1}$ (there will be no degrees of freedom). Thus, in the case of $D = 0$, Equation B.9 is modified by ending the first summation at $i = N - 1$ and changing the denominator to $N - 1$.

APPENDIX C

THE ALTERNATIVE VARIANCE MODEL

This Appendix provides an alternative variance model corresponding to the use of variance factors different from the development factors.

Justification for the Alternative Model

Given that our method is to weight together individual year development estimates that are assumed to be mutually independent, and that relative variances of those estimates are those assumed in Section 7, the weights in Section 7 follow directly.

However, in developing a consistent underlying model, we encounter the following difficulty: for the overall estimate to be optimal, each year's development estimate must be the optimal estimate based on the data from that year alone. In fact, it can be proven that the following three assumptions are irreconcilable:

1. optimality of the individual development estimates;
2. independence of the emerged and unemerged losses; and
3. variance factors different from the development factors.

We address the difficulty by changing the independence assumption. We will demonstrate that with the alternative assumption, the "alternative Bornhuetter–Ferguson" calculation (Equation 7.4) is indicated, that the development estimate is optimal for each individual year, and that the Cape Cod method with alternate variance factors is optimal using data from all years.

Note, however, that the alternative model was selected for mathematical convenience only. We will conclude this section with a brief discussion of whether the alternative model is intuitively reasonable.

Changing the Independence Assumption

We accomplish the alternative independence assumption by defining a transformation of the data. Independence assumptions are then assumed to hold for the transformed data, rather than the original data.

First, we use the cumulative notation of Appendix A.

Define $LTD' = LTD \times DF/VF$.

Thus $LTD' \times VF = LTD \times DF$.

We assume that LTD' is independent from $ULT - LTD'$.

Next, we present the alternative model in full triangle detail, using the notation of Appendix B.

In addition, we introduce the following notation:

Cumulative Variance Pattern: V_j $(= 1/VF_{N-j+1})$

Non-Cumulative Variance Pattern: v_j $(= V_j$ for $j = 1;$ $= V_j - V_{j-1}$ for $j > 1)$

Let

$$X'_{ij} = X_{ij} \times V_j/P_j$$

$$x'_{ij} = X'_{ij} \qquad \text{for} \quad j = 1;$$

$$= X'_{ij} - X'_{i,j-1} \qquad \text{for} \quad j > 1$$

The values x'_{ij} are now assumed to be mutually independent.

Revisiting the Bornhuetter–Ferguson Calculation

We perform calculations analogous to those of Appendix A, with some changes to the underlying constraints. As in Appendix A, we drop the subscript denoting year of origin. Reviewing the underlying constraints:

1. Expected losses are known (i.e. $\hat{\mathrm{E}}(ULT) = \mathrm{E}(ULT)$).
2. Originally, unemerged losses $(ULT - LTD)$ were assumed to be independent from emerged losses (LTD). As defined previously, we now assume that $(ULT - LTD')$ is independent from (LTD').
3. Both development factors (DF) and variance factors (VF) are now assumed to be known.
4. The variance of $LTD \times DF$ is now assumed to be proportional to VF.

Let $V^2 = \mathrm{Var}(ULT)$. Then $\mathrm{Var}(LTD \times DF) = \mathrm{Var}(LTD' \times VF)$ $= V^2 \times VF$

$$\mathrm{Var}(LTD') = V^2/VF$$

$$\mathrm{Var}(ULT - LTD') = V^2(1 - 1/VF).$$

Expressing the estimate of ULT as a weighted average of the development result and the expected ultimate losses:

$$\begin{aligned}\widehat{ULT} &= W \times LTD \times DF + (1 - W)\mathrm{E}(ULT)\\ &= W \times LTD' \times VF + (1 - W)\mathrm{E}(ULT).\end{aligned}$$

The remaining calculations, which are not shown here, exactly parallel those of Appendix A, except that LTD is replaced with LTD' and DF is replaced with VF. The indicated value of W is $1/VF$, which is the weight used in the alternative Bornhuetter–Ferguson calculation.

Optimality of the Development Estimates

We have previously defined mutually independent values x'_{ij}. An exactly analogous proof to that performed in Appendix B establishes that the variance of x'_{ij} is $k \times E_i \times v_j$.

Each value x'_{ij} now produces an independent estimate of $E(PP_i)$, as $x'_{ij}/E_i v_j$ with variance $k/E_i v_j$.[16] Weighting all estimates for a given year of origin in inverse proportion to variances:

$$\hat{E}(PP_i) = \frac{\sum_{j=1}^{N-i+1} (x'_{ij}/E_i v_j) \times E_i v_j}{\sum_{j=1}^{N-i+1} E_i v_j} = X'_{i,N-i+1}/E_i V_{N-i+1} = X_{i,N-i+1}/E_i P_{N-i+1}$$

or the development estimate.

Optimality of the Cape Cod Estimates with Alternate Variance Factors

If $E(PP_i)$ is assumed to be equal for all years i, then the weighted average of all estimates of $E(PP)$ from all years of origin is as follows:

$$\hat{E}(PP) = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N-i+1} (x'_{ij}/E_i v_j) \times E_i v_j}{\sum_{i=1}^{N} \sum_{j=1}^{N-i+1} E_i v_j}$$

$$= \frac{\sum_{i=1}^{N} X'_{i,N-i+1}}{\sum_{i=1}^{N} E_i V_{N-i+1}} = \frac{\sum_{i=1}^{N} (X_{i,N-i+1}/P_{N-i+1}) \times V_{N-i+1}}{\sum_{i=1}^{N} E_i V_{N-i+1}}$$

[16]A proof that x'_{ij}/v_j is an estimate of ULT_i is provided later in this Appendix.

which is the Cape Cod estimate with the alternate variance factors.

Estimating the Proportionality Constant k

The estimate of the proportionality constant k exactly parallels the calculations presented in Appendix B, except that X'_{ij} and x'_{ij} replace X_{ij} and x_{ij}, and V_j and v_j replace P_j and p_j.

Thus Equations B.6 through B.9 become:

$$w_{ij} = \frac{E_i v_j}{\sum_{l=1}^{N} E_l V_{N-l+1}} \tag{C.1}$$

$$\hat{k} = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N-i+1} \frac{(\widehat{PP} \times E_i v_j - x'_{ij})^2}{E_i v_j \times (1 - w_{ij})}}{n} \tag{C.2}$$

$$_D w_{ij} = \frac{E_i v_j}{\sum_{l=1}^{N} E_l V_{N-l+1} D^{|i-1|}} \tag{C.3}$$

$$\hat{k}_D = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N-i+1} \frac{({}_D\widehat{PP}_i \times E_i v_j - x'_{ij})^2}{E_i v_j (1 - {}_D w_{ij})}}{N}. \tag{C.4}$$

Note the discussion in Appendix B regarding modifying Equation C.4 for the special case when $D = 0$.

We now provide a proof that x'_{ij}/v_j is an unbiased estimate of ULT_i, assuming that each development result is an unbiased estimate of ULT_i, and that the values V_j are known (i.e. not random variables).

Assume, for all X_{ij}:

$$ULT_i = \mathrm{E}(X_{ij}/P_j) = \mathrm{E}(X'_{ij}/V_j).$$

Then,

$$\mathrm{E}\left(\frac{x'_{ij}}{v_j}\right) = \frac{\mathrm{E}(x'_{ij})}{v_j} = \frac{\mathrm{E}(X'_{ij}) - \mathrm{E}(X'_{i,j-1})}{v_j}$$

$$= \frac{V_j(ULT_i) - V_{j-1}(ULT_i)}{v_j}$$

$$= \frac{(V_j - V_{j-1})(ULT_i)}{v_j} = ULT_i.$$

Plausibility of the Alternative Model

Having replaced the assumption of independence between the emerged and unemerged losses, the alternative model implies dependence. It can be proven that when the variance factor is larger than the development factor, the alternative model implies negative correlation between emerged and unemerged losses. In this section, we discuss the plausibility of that result under two scenarios.

The first scenario is an incurred development projection, which is the most common situation in which variance factors different from the development factors will be needed. In this situation, the inclusion of the case reserves in the data may lead to variance factors higher than the development factors and a presumed negative correlation between emerged and unemerged losses. In this case, the sign of the dependence is logical: relative over-reserving of cases for a particular year of origin will lead to a high error on the emerged losses and a low error on the unemerged losses, and vice versa. Of course, we have not addressed whether the amount of dependence predicted by the alternative model is reasonable.

A paid loss development scenario provides a counter-example. Assume that there are no partial payments, that average claim size tends to grow with the lag to settlement, and that the coefficient of variation of the claim size distribution is constant.

These assumptions imply that the independence model is appropriate, and yet the variance factors will be different from the development factors. In this case the alternative model appears inappropriate. If the variance factors are correct, the generalized Cape Cod weights still produce the optimal combination of the individual year development projections; however, the individual year development projections do not represent the optimal combination of data from a particular year of origin.

APPENDIX D

ADAPTIVE VARIANCES AND DECAY FACTORS

This Appendix provides a method for calculating the indicated decay factor D. The approach first calculates the indicated adaptive variance under the random walk model, and then calculates the approximately equivalent decay factor.

Calculating the Adaptive Variance

This approach for calculating the adaptive variance was used by Wright [7].

Recalling that the development-based pure premium estimate for year i is denoted ${}_0\widehat{PP}_i$,

$$ {}_0\widehat{PP}_i = \mathrm{E}(PP_i) + \varepsilon_i. \tag{D.1} $$

The random walk model connecting the values $\mathrm{E}(PP_i)$ is:

$$ \mathrm{E}(PP_i) = \mathrm{E}(PP_{i-1}) + d \qquad \text{for} \quad i = 2,3,\ldots \tag{D.2} $$

The error term ε_i and the random disturbance term d are presumed to have variances σ_i^2 and ${}_d\sigma^2$, respectively. For the purposes of this calculation, we will also assume that ε_i and d are normally distributed.

The adaptive variance ${}_d\sigma^2$ is the variance of the differences between the true (unknown) parameters $\mathrm{E}(PP_i)$, not the estimates ${}_0\widehat{PP}_i$. We measure the adaptive variance by observing the differences in the estimates ${}_0\widehat{PP}_i$, and correcting for the estimation errors ε_i.

Let

$$ \Delta_i = {}_0\widehat{PP}_i - {}_0\widehat{PP}_{i-1} \qquad \text{for} \quad i = 2,3,\ldots $$

Then,

$$ \Delta_i = (\mathrm{E}(PP_i) + \varepsilon_i) - (\mathrm{E}(PP_{i-1}) + \varepsilon_{i-1}) = d + \varepsilon_i - \varepsilon_{i-1}. $$

The variance-covariance matrix, Σ, of the vector Δ is given by:

$$\Sigma = \begin{bmatrix} {}_d\sigma^2 + \sigma_1^2 + \sigma_2^2 & -\sigma_2^2 & 0 & 0 \\ -\sigma_2^2 & {}_d\sigma^2 + \sigma_2^2 + \sigma_3^2 & -\sigma_3^2 & 0 \\ 0 & -\sigma_3^2 & {}_d\sigma^2 + \sigma_3^2 + \sigma_4^2 & -\sigma_4^2 \\ 0 & 0 & -\sigma_4^2 & \text{etc.} \end{bmatrix}$$

Δ is normally distributed, and the value $\Delta^T \times \Sigma^{-1} \times \Delta$ is chi-squared distributed with $N - 1$ degrees of freedom ($N - 1$ is the length of the vector Δ).

The expected value of the chi-squared distribution is $N - 1$, so an estimate of σ_d^2 is given by solving the equation:

$$\Delta^T \times \Sigma^{-1} \times \Delta = N - 1 \qquad \text{for} \quad {}_d\sigma^2, \tag{D.3}$$

which can be solved numerically, given that estimates of σ_i^2 are available. It is possible that the variances σ_i^2 may be large enough compared to the differences Δ_i that $\Delta^T \times \Sigma^{-1} \times \Delta < N - 1$ for all ${}_d\sigma^2 > 0$. In this case, there is no demonstrable random walk and we set ${}_d\sigma^2$ to zero (and the decay factor D to unity).

Under the alternative variance model of Appendix C (which includes the Cape Cod variance model as a special case),

$$\sigma_i^2 = \frac{k}{E_i V_{N-i+1}}. \tag{D.4}$$

Given an estimate of k, Equation D.4 can be applied and then Equation D.3 solved for ${}_d\sigma^2$.

To convert an estimate of ${}_d\sigma^2$ to the approximately equivalent decay factor, we have used the following formula:[17]

$$D = \frac{\overline{\sigma}_i^2}{\overline{\sigma}_i^2 + {}_d\sigma^2}, \qquad \text{where} \quad \overline{\sigma}_i^2 = \frac{Nk}{\sum_{i=1}^{N} E_i V_{N-i+1}}. \tag{D.5}$$

[17]The above formula equates the decay factor and adaptive variance approaches at a lag of one year, for a year of origin with average variance.

To estimate k, we use the formulas of Appendix B or C; however, the derivation of $\hat{k}$ is dependent on an estimate of D. For the first iteration, we assume $D = 0$ and use Equation B.9 or C.4 to calculate $\hat{k}_0$.[18] We then apply Equations D.4, D.3, and D.5 above to estimate D. Equation B.9 or C.4 can then be used to estimate $\hat{k}_D$, and D can be re-estimated. This process can be applied repeatedly.

[18] $D = 0$ is a logical starting point since $\hat{k}_0$ is based entirely on within-year variance and provides an unbiased estimate of k regardless of the appropriate value of D. Given that $D > 0$, $\hat{k}_D$ is a superior (i.e. lower variance) estimate of k.

APPENDIX E

OPTIMALITY OF BORNHÜETTER–FERGUSON CALCULATION WITH RELAXED CONSTRAINTS

Appendices A and C provide proofs that the Bornhuetter–Ferguson weights are optimal based on the Cape Cod variance model and alternative variance model, respectively, with the additional constraint that the expected ultimate losses are known (i.e., $\hat{\mathrm{E}}(ULT) = \mathrm{E}(ULT)$).

The Bornhuetter–Ferguson and alternative Bornhuetter–Ferguson weights remain optimal using an estimate, $\hat{\mathrm{E}}(ULT)$, if:

$$\hat{\mathrm{E}}(ULT) = W' \times LTD \times DF + (1 - W')(Other) \qquad \text{(E.1)}$$

where *Other* is an estimate of $\mathrm{E}(ULT)$, independent of *LTD* and *ULT*, and

$$\frac{\mathrm{Var}(Other)}{\mathrm{Var}(LTD \times DF)} = \frac{W'}{(1 - W')} \qquad \text{(E.2)}$$

i.e., $LTD \times DF$ and *Other* are weighted in inverse proportion to the variances of the estimates. All of the estimates of $\mathrm{E}(ULT)$ described in this paper meet these conditions under the assumed variance models.

We provide the proof using the alternative variance model of Appendix C, which includes as a special case the original Cape Cod variance model.

Let $V^2 = \mathrm{Var}(ULT)$.

$$\mathrm{Var}(LTD \times DF) = V^2 \times VF \qquad \text{(E.3)}$$

$$\widehat{ULT} = W \times LTD \times DF + (1 - W)\hat{\mathrm{E}}(ULT) \qquad \text{(E.4)}$$

Substituting Equation E.1 in Equation E.4:

$$\widehat{ULT} = W \times LTD \times DF + (1 - W)(W' \times LTD \times DF + (1 - W')(Other))$$

$$= (W + (1 - W)W') \times LTD \times DF + (1 - W)(1 - W')(Other)$$

$$= W^* \times LTD \times DF + (1 - W^*)(Other),$$

where $W^* = W + (1 - W)W'$. (E.5)

This is a weighted average of two independent estimates of *ULT*. The variance of the estimation error associated with the first estimate,

$$\text{Var}(ULT - LTD \times DF) = V^2(VF - 1),$$

is a result developed in Appendices A and C.

For the second estimate,

$$\text{Var}(ULT - Other)$$

$$= \text{Var}(ULT) + \text{Var}(Other)$$

$$= V^2 + \text{Var}(LTD \times DF)(W'/1 - W'), \quad \text{using E.2}$$

$$= V^2 + V^2 \times VF(W'/1 - W'), \quad \text{using E.3}$$

$$= V^2[1 + VF(W'/1 - W')].$$

Calculating $1 - W^*$ in inverse proportion to variances:

$$1 - W^* = \frac{V^2(VF - 1)}{V^2(VF - 1) + V^2(1 + VF(W'/1 - W'))}$$

$$= \frac{VF - 1}{VF - 1 + 1 + VF(W'/1 - W')} = \frac{VF - 1}{VF(1 + W'(1 - W')}$$

$$= \frac{VF - 1}{VF(1/1 - W')} = \frac{(VF - 1)(1 - W')}{VF}. \quad \text{(E.6)}$$

Substituting $1 - W^* = (1 - W)(1 - W')$ (see E.5)

$$(1 - W)(1 - W') = \frac{(VF - 1)(1 - W')}{VF}$$

$$= 1 - \frac{VF - 1}{VF} = \frac{1}{VF}$$

which is the weight used in the alternative Bornhuetter–Ferguson calculation.

ON APPROXIMATIONS IN LIMITED FLUCTUATION CREDIBILITY THEORY

VINCENT GOULET

Abstract

Different approximation methods to find a full credibility level in limited fluctuation credibility are studied, and it is concluded that, in most cases, there is no significant difference between the various results. Since Venter [9] presented an opposite conclusion, it is emphasized that his approach to the problem is different and that the formula he derives should be used only in his given context.

ACKNOWLEDGEMENT

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1. INTRODUCTION

"Limited fluctuation" credibility is the oldest branch of credibility theory, the other branch being "greatest accuracy" credibility. Also sometimes called American credibility, limited fluctuation credibility originates from the beginning of this century with Mowbray's paper "How Extensive a Payroll Exposure is Necessary to Give a Dependable Pure Premium?" [7]. The title is self-explanatory: Mowbray was interested in finding a level of payroll in workers compensation insurance for which the pure premium of a given risk would be considered fully credible.

The theory has not evolved much since then. The answer to Mowbray's question—which is Mowbray's answer, as a matter of fact—has remained basically the same (see Section 2). With the emergence of risk theory methods, though, the original problem has been formulated in a more general way, and new techniques have been used to find the full credibility level. This paper will first investigate if more powerful and sophisticated approximation methods are more worthwhile than the straightforward normal approximation. Then, because our conclusion will differ from that of Venter [9], the paper will show that Venter's full credibility requirement systematically exceeds that given by the normal approximation.

2. THE MODEL

Let

S = random variable of the total claim amount of a risk over a given period of time (usually 1 year);

X_j = random variable of the amount of the jth claim;

N = random variable of the claim count of the risk over the given period.

Then,

$$S = X_1 + X_2 + \cdots + X_N,$$

where $X_1, X_2, \ldots, X_n$ are independent, identically distributed (i.i.d.) random variables mutually independent of N.[1] This is the classical collective model of risk theory. Most of the situations usually encountered in limited fluctuation credibility can be described by an application of this model. It is also well known (see Gerber [3]) that

$$\mathrm{E}[S] = \mathrm{E}[N]\mathrm{E}[X_j],$$

$$\mathrm{Var}[S] = \mathrm{E}[N]\mathrm{Var}[X_j] + \mathrm{Var}[N]\mathrm{E}[X_j]^2.$$

[1] In reality, the losses may be only conditionally independent given some parameters, such as the inflation rate, to which the losses will all be exposed jointly.

The fundamental problem of limited fluctuation credibility, only slightly adapted from Mowbray's original idea, is: What are the parameters of the distribution of S such that the Equation

$$\Pr[(1-k)\mathrm{E}[S] \leq S \leq (1+k)\mathrm{E}[S]] \geq p \tag{2.1}$$

is verified? Using distribution functions, Equation 2.1 can also be written

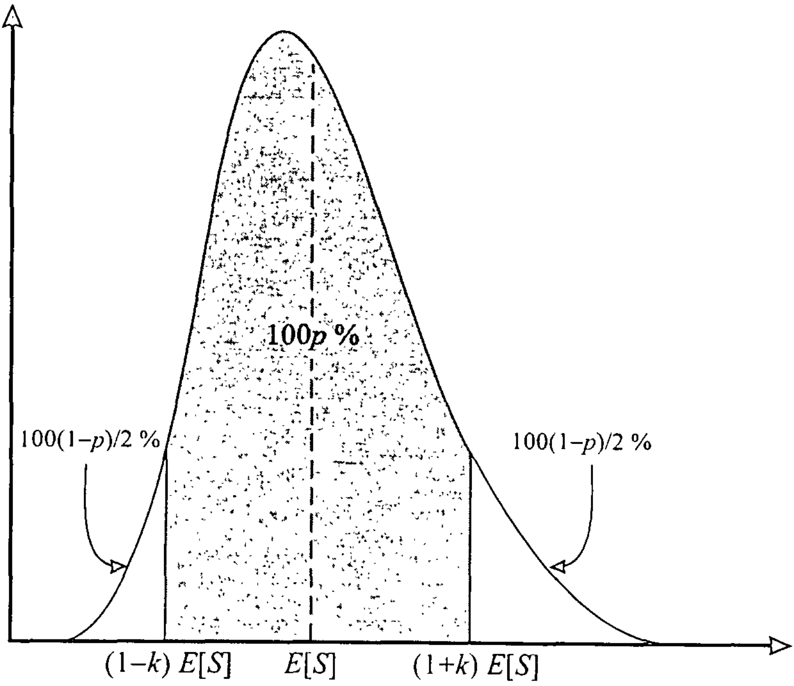
$$F_S((1+k)\mathrm{E}[S]) - F_S((1-k)\mathrm{E}[S]) \geq p. \tag{2.2}$$

This requires that with probability $100p\%$, the total claim amount of a risk stays within $100k\%$ of its expected value (see Figure 1). When a risk meets these requirements, we say that it deserves a full credibility of order (k,p). That is, the risk is charged a pure premium based solely on its own experience. After m periods of time, that premium would simply be the empirical mean $\bar{S} = (S_1 + S_2 + \cdots + S_m)/m$, where each S_i $(i = 1,2,\ldots,m)$ is distributed as S.

In a usual limited fluctuation credibility situation, the parameter k will be quite small, e.g., 5–10%, while the parameter p will be large, often above 90%. Equation 2.1 thus requires the distribution of S to be relatively concentrated around its expected value. Since S is a (random) sum of i.i.d. random variables, one way to achieve such a kind of distribution is to sum a "large" number of those random variables—provided their second moment is finite. The distribution of the sum will then tend towards a normal distribution more relatively concentrated around its mean (that is, the ratio of the standard deviation to the expected value decreases) as the number of terms in the sum increases. Accordingly, the natural way to verify Equation 2.1 is to base the criteria for full credibility on the expected number of claims. (Note that the severity still enters the calculation through the X_js, as it should.) The level of full credibility will then usually be expressed in terms of the expected value of N, which could

FIGURE 1

MOWBRAY'S TWO-SIDED FULL CREDIBILITY CRITERION REQUIRES $100p\%$ OF THE PROBABILITY OF S TO BE CONCENTRATED WITHIN $100k\%$ OF ITS EXPECTED VALUE



represent, for example, the number of claims, the number of employees, or the total payroll. Besides, it is intuitively preferable to base the criterion on some kind of exposure base rather than on the individual amount of the claims.

At this point, most of the theory of limited fluctuation credibility has been covered. What follows are the calculations needed to satisfy Equation 2.1. However, these calculations are more relevant to general risk theory than to credibility theory.

Before going further, we define the skewness of a random variable X as

$$\gamma_1(X) = \mathrm{E}\left[\left(\frac{X - \mathrm{E}[X]}{\sqrt{\mathrm{Var}[X]}}\right)^3\right]. \tag{2.3}$$

Obviously, a symmetrical random variable has $\gamma_1(X) = 0$.

3. THE COMPOUND POISSON CASE

The compound Poisson is a distribution frequently used for S. It is said that the distribution of S is compound Poisson of parameters λ and G when the random variable N follows a Poisson distribution of parameter λ and the random variables X_j $(j = 1,\ldots,n)$ have distribution function G. Let $P_k = \mathrm{E}[X_j^k]$. Then (see, e.g., Gerber [3]):

$$\mathrm{E}[S] = \lambda P_1, \tag{3.1}$$

$$\mathrm{Var}[S] = \lambda P_2, \tag{3.2}$$

$$\gamma_1(S) = \frac{P_3}{\sqrt{\lambda} P_2^{3/2}}. \tag{3.3}$$

Equation 3.1 says that the expected value of the total claim amount is simply the product of the expected values of the number of claims and the amount per claim. In Equation 3.2, we see that the variance of the total claim amount is given by the second moment (P_2) of the claim amount times the expected number (λ) of claims. Finally, Equation 3.3 shows that the skewness of S decreases as the expected number of claims increases.

Further on, we will refer to this model simply as the "compound Poisson case."

4. THREE APPROXIMATION METHODS

In theory, the exact solution of the limited fluctuation credibility problem would be obtained by calculating the exact distribution of S with the convolution formula (see, for example, Gerber

[3]). However, since the expected number of claims is usually quite large, that calculation would represent too long and laborious a task and would, in general, first require transforming the continuous distribution into a discrete one (a procedure known as "discretization"). In fact, nobody ever really intended to calculate the convolutions to solve the problem under study, approximations instead always being used. We present here three common approximation methods that could be used to estimate the distribution of S and then find the parameters that satisfy Equation 2.1.

The first approximation method we present is the most widely used in limited fluctuation credibility: the classic normal approximation. In general, the distribution of S is not symmetrical, even if that of X_j is. However, the limited fluctuation criteria will require the number of claims to be large, thus yielding an almost symmetrical distribution for S. By the version of the Central Limit Theorem applicable to random sums (Feller [2], p. 258), it is reasonable to approximate the distribution of $(S - \mathrm{E}[S])/\sqrt{\mathrm{Var}[S]}$ by a standard normal distribution. Equation 2.1 may then be rewritten

$$\begin{aligned}\Pr\left[-\frac{k\mathrm{E}[S]}{\sqrt{\mathrm{Var}[S]}} \le \frac{S-\mathrm{E}[S]}{\sqrt{\mathrm{Var}[S]}} \le \frac{k\mathrm{E}[S]}{\sqrt{\mathrm{Var}[S]}}\right] &\approx \Phi\left(\frac{k\mathrm{E}[S]}{\sqrt{\mathrm{Var}[S]}}\right) - \Phi\left(-\frac{k\mathrm{E}[S]}{\sqrt{\mathrm{Var}[S]}}\right) \\ &= 2\Phi\left(\frac{k\mathrm{E}[S]}{\sqrt{\mathrm{Var}[S]}}\right) - 1 \ge p. \end{aligned} \tag{4.1}$$

Thus,

$$\mathrm{E}[S] \ge \left(\frac{z_{1-\varepsilon/2}}{k}\right)\sqrt{\mathrm{Var}[S]}, \tag{4.2}$$

where $\varepsilon = 1 - p$ and z_α is the αth percentile of a standard normal distribution. In the compound Poisson case, one finds (see, for example, Perryman [8])

$$\lambda \geq \left(\frac{z_{1-\varepsilon/2}}{k}\right)^2 \left(\frac{P_2}{P_1^2}\right). \tag{4.3}$$

The first ratio represents the normality assumption, while the second accounts for the variability of the claim amounts. Indeed, the full credibility level increases with the square of the coefficient of variation of the random variable X_j. The choice $k = 5\%$, $p = 90\%$, and X_j degenerated at 1 (that is, taking value one with probability one) leads to the famous λ value of 1,082.

The popularity of this approximation, aside from its good precision in the limited fluctuation context when the expected value of N is large, comes from the fact that $F(x) = 1 - F(-x)$. This greatly simplifies the calculations, as it may be seen in Equation 4.1. However, even at the price of heavier calculations, one might be interested to take into account the skewness of S by using more refined approximations.

Two approximations that take the skewness of S into account and are generally considered precise and relatively simple to use will be studied here: the normal power II approximation (using the first three moments; simply called normal power hereafter) and the Esscher approximation. The general formula of the normal power approximation as found in Beard et al. [1] is:

Let

$$y = \frac{x - \mathrm{E}[S]}{\sqrt{\mathrm{Var}[S]}} \quad \text{and} \quad y_0 = -\sqrt{7/4},$$

then

$$F_S(x) \approx \begin{cases} \Phi\left(-\dfrac{3}{\gamma_1(S)} + \sqrt{1 + \dfrac{9}{\gamma_1^2(S)} + \dfrac{6}{\gamma_1(S)}y}\right), & y \geq 1 \\ \Phi\left(y - \dfrac{\gamma_1(S)}{6}(y^2 - 1) + \dfrac{\gamma_1^2(S)}{36}(4y^3 - 7y)\delta(y_0 - y)\right), & y < 1, \end{cases} \tag{4.4}$$

where $\delta(y) = 0$ if $y = 0$ and 1 otherwise. Note that for $y = 1$, both formulae produce $\Phi(1)$.

To use the Esscher approximation, the moment generating function (m.g.f.) of S must exist (preferably in a known form, to simplify the calculations). If the distribution of S is compound Poisson with parameters λ and G, then the Esscher approximation for the distribution function of S is

$$1 - F_S(x) \approx e^{\lambda[m(h)-1]-hx}\left[E_0(u) - \frac{m'''(h)}{6\lambda^{1/2}(m''(h))^{3/2}}E_3(u)\right]. \tag{4.5}$$

where $m(\cdot)$ is the m.g.f. of G,h the solution of $\lambda m'(h) = x$, and $u = h\sqrt{\lambda m''(h)}$. The functions $E_k(\cdot)$ $(k = 0,1,2,\ldots)$ are the Esscher functions:

$$\begin{aligned} E_0(u) &= e^{u^2/2}[1 - \Phi(u)] \\ E_3(u) &= \frac{1-u^2}{\sqrt{2\pi}} + u^3 E_0(u). \end{aligned} \tag{4.6}$$

A more complete description of the Esscher approximation may be found in Gerber [3].

Quite obviously, it is not possible to simplify Equation 2.1 in a form like Equation 4.1 when using the normal power or Esscher approximations. The search for parameters such that Equation 2.1 is satisfied must then be made iteratively. For example, if S is compound Poisson, one must find the smallest value of λ such that $F_S((1+k)\mathrm{E}[S];\lambda) - F_S((1-k)\mathrm{E}[S];\lambda) \geq p$. If the probability obtained with a particular value of λ is smaller than p, then the value of λ must be raised—and vice-versa—until convergence to a unique minimal value is achieved. Note that if the distribution of S remains right-skewed once the full credibility level has been reached, then there will be more probability mass in the right tail than in the left one.

Now, the question is: Are these more complicated and time consuming approximations better (more precise) than the usual normal approximation, still in the context of limited fluctuation credibility? To study this, we made some tests where the distribution of S was held compound Poisson and the distribution of the individual claim amount changed. The parameters of the latter were chosen such that its expected value remained constant at 5,000, but its variance, and especially its skewness, varied. The idea was to make X_j very skewed and then check if the values of λ given by the three approximations would be significantly different, and what would be the resulting skewness of S. Gamma and lognormal distributions were used for X_j, but as the m.g.f. of the latter does not exist, the Esscher approximation was not calculated. The inversion of characteristic functions (ICF) method has also been used to cross-check the results in the gamma cases. This numerical method is used to calculate distribution functions, and its precision is as high as the user desires (see, for example, the Heckman-Meyers algorithm in [4]). It then appeared that the normal power and Esscher approximations can be considered as almost exact in the present application. Table 1 summarizes the results.

From the results of Table 1, we must conclude that it is not necessary to complicate the estimation of the full credibility level by using more sophisticated approximation methods. Indeed, the differences between the various methods are minor—often less than 0.5%. These results and the conclusion drawn from them should not be very surprising since, as stated in Section 2, it is a requirement of the limited fluctuation problem that most of the probability mass be concentrated around the expected value of S. Thus, for k and p constant, the more X_j is skewed, the more the number of claims has to be large to make S a "concentrated" distribution. Intuitively, such a distribution can not be very skewed, thus leading to a good normal approximation. Besides, a quick look at the last column of Table 1 shows that whatever the skewness of X_j, the value of λ will be sufficiently large to result in a quite symmetrical distribution for S.

There remains a peculiar case to be discussed in Table 1: the first lognormal case, where the difference between the normal

TABLE 1

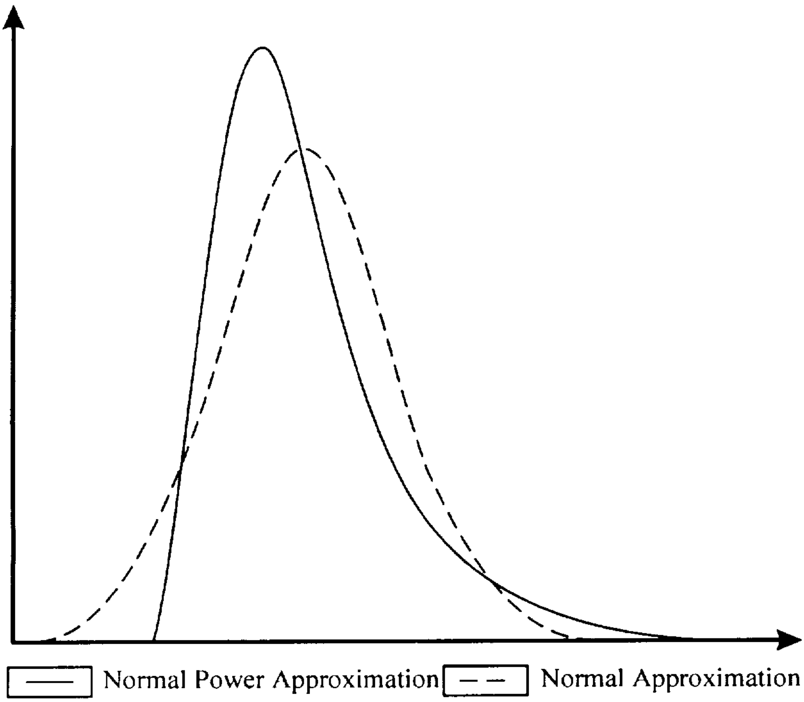
FULL CREDIBILITY LEVELS OBTAINED WITH THREE DIFFERENT APPROXIMATION METHODS IN COMPOUND POISSON CASES

Distribution of X_j	$\gamma_1(X_j)$	k	p	Value of $E[N] = \lambda$: Normal Approximation	Normal Power	Esscher	Largest Difference (%)	$\gamma_1(S)$
Gamma								
$\alpha = 0.01$	20.00	0.050	0.90	109,323	109,258	109,234	0.08	0.06
$\alpha = 0.05$	8.94	0.050	0.95	32,269	32,256	32,257	0.04	0.05
$\alpha = 0.20$	4.47	0.100	0.90	1,624	1,621	1,620	0.23	0.11
$\alpha = 1.10$	1.91	0.025	0.90	8,266	8,264	8,264	0.03	0.02
$\alpha = 5.00$	0.89	0.100	0.95	461	461	461	0.06	0.06
Lognormal								
$\sigma^2 = \ln 50$	364.00	0.050	0.90	54,121	49,232	—	9.03	1.52
$\sigma^2 = 2.00$	23.73	0.050	0.95	11,354	11,301	—	0.47	0.19
$\sigma^2 = 1.50$	12.09	0.100	0.90	1,213	1,203	—	0.77	0.27
$\sigma^2 = 0.75$	4.35	0.025	0.90	9,166	9,163	—	0.03	0.03
$\sigma^2 = 0.65$	3.75	0.100	0.95	736	735	—	0.14	0.10

and the normal power approximations reaches 9%. Clearly, the skewness of 1.52 for S is not insignificant in that case; we would not find that there is precisely a probability of 0.05 both above and below 5% of the mean (in fact, we get 0.089 above and 0.011 below). The normal power estimation of the full credibility level is thus slightly more precise than the normal approximation in that case. The interesting point, though, is that the normal approximation is the higher, or more conservative, of the two. We can thus also conclude from Table 1 that taking the skewness of S into account does not yield higher full credibility levels. In fact, the normal approximation is, in all cases studied, the most conservative one. This can be explained by, for the same expected number of terms in the sum, the normal approximation imputing more probability mass in the left tail than the normal power gains with heavier right tail (see Figure 2).

FIGURE 2

FOR THE SAME EXPECTED NUMBER OF TERMS IN THE SUM, THE NORMAL APPROXIMATION IMPUTES MORE PROBABILITY MASS IN THE LEFT TAIL THAN THE NORMAL POWER GAINS WITH ITS HEAVIER RIGHT TAIL



The above conclusions also mean that the skewness of X_j is not a big issue on the level of full credibility. There is still another interesting way to see that point with the normal power approximation in the compound Poisson case. Since the normal power approximation is only calculated at the points $(1 \pm k)\text{E}[S]$, it is easily seen from Equation 4.4 and Equations 3.1 to 3.3 that all the information one needs about the distribution of X_j to calculate the approximation are the ratios $r_1 \equiv P_1/P_2^{1/2}$ and $r_2 \equiv P_3/P_2^{3/2}$. Assuming that $X_j > 0$, it is easily shown with Jensen's inequality that $r_1 \in [0,1]$ and $r_2 \geq 1$. The left end of the interval for r_1 is not interesting, though, since it represents a zero expected value. The right end represents a zero variance and is thus the frequently used—rightly or wrongly—degenerated case. The ratio r_1 is also the only one needed to calculate the normal approximation and, as such, fully determines in that case the full credibility level—given k and p, of course. Entering in the calculation of $\gamma_1(S)$, the ratio r_2 thus brings the skewness of X_j into the normal power approximation.

Table 2 presents full credibility levels of order (0.05,0.90) for various combinations of the above ratios. For illustration purposes, we have included the $\lambda = 1{,}082$ level, obtained with the combination $r_1 = r_2 = 1$. It should be noted that the entries in the upper left and lower right corners of the table are most unlikely. For the most common distributions (e.g., gamma, log-normal, Pareto), a small r_1 comes with a large r_2, and vice versa. Then, in the really interesting area of the table, we clearly see that the effect of a rather small variation in the value of the ratio r_1 is much more important than a large variation in the value of the ratio r_2. This could also be interpreted as r_1 determining most of the final value of the full credibility level, while r_2 causes only a small, and in most cases negligible, correction to that value.

5. A WORD OF CAUTION

The book *Foundations of Casualty Actuarial Science* published by the Casualty Actuarial Society presents, as the title suggests, different subjects central to casualty insurance practice. The chapter on Credibility Theory—Chapter 7—was written by Gary G. Venter [9]. In the section on limited fluctuation credibility, it is demonstrated by an example (Example 3.1) that the normal power approximation gives a much different estimation of the full credibility level than the one obtained with the nor-

TABLE 2

FULL CREDIBILITY LEVELS OF ORDER (0.05,0.90) IN THE COMPOUND POISSON CASE CALCULATED WITH THE NORMAL POWER APPROXIMATION

	$r_2 = P_3/P_2^{3/2}$		
$r_1 = P_1/P_2^{1/2}$	1	10	300
0.1	108,222	108,210	102,458
0.2	27,055	27,044	24,377
0.3	12,025	12,013	11,172
0.4	6,764	6,753	6,947
0.5	4,329	4,318	4,857
0.6	3,006	2,995	3,652
0.7	2,208	2,198	2,884
0.8	1,691	1,681	2,359
0.9	1,336	1,326	1,981
1.0	1,082	—	—

mal approximation. Naturally, the former is considered the better. This contradiction with the results of the previous section is due to the fact that Venter is not considering exactly the same limited fluctuation problem as above; therefore both normal power approximations can not be directly compared.

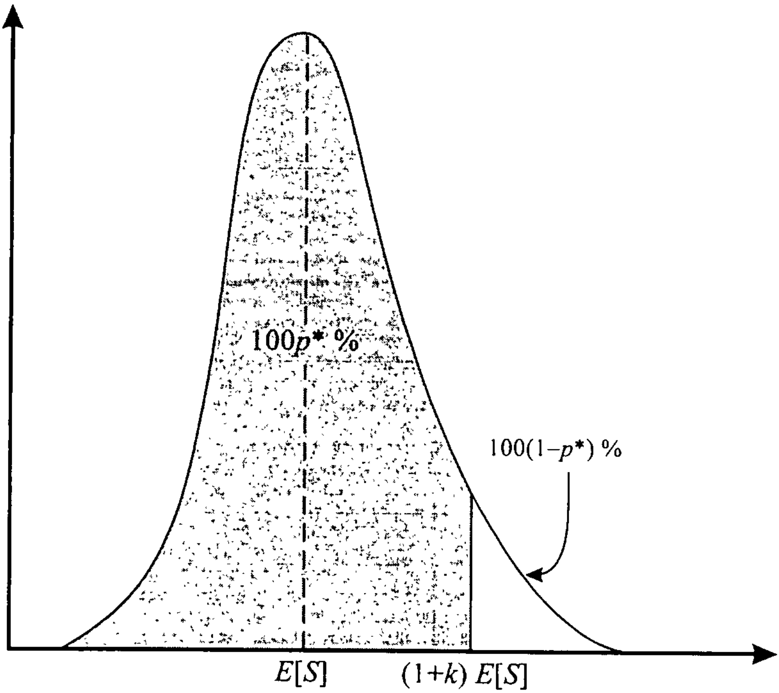
As said before, the normal approximation leads to simple formulae because

$$F_S((1+k)\mathrm{E}[S]) \approx \Phi\left(\frac{k\mathrm{E}[S]}{\sqrt{\mathrm{Var}[S]}}\right) = 1 - \Phi\left(-\frac{k\mathrm{E}[S]}{\sqrt{\mathrm{Var}[S]}}\right) \approx 1 - F_S((1-k)\mathrm{E}[S]). \tag{5.1}$$

Those equalities are not found in the normal power approximation. A simple look at Equation 4.4 is enough to be convinced that $F_S((1+k)\mathrm{E}[S]) \neq 1 - F_S((1-k)\mathrm{E}[S])$. Now, Mr. Venter's approach to the problem is slightly different, as he introduces a simplifying hypothesis right at the beginning. Instead of considering Equation 2.1, he considers a one-sided requirement for full

FIGURE 3

VENTER'S ONE-SIDED FULL CREDIBILITY CRITERION REQUIRES $100p^*\%$ OF THE PROBABILITY OF S TO BE UNDER $100(1 + k)\%$ OF ITS EXPECTED VALUE



credibility, namely:

$$\Pr[S \leq (1 + k)\mathrm{E}[S]] \geq (1 + p)/2 \equiv p^*. \tag{5.2}$$

Thus, instead of requiring that there is a probability of p that S does not deviate from its expected value by more than $100k\%$, it is only required that there is a probability of p^* that S does not exceed its expected value by more than $100k\%$. Therefore, while Equation 2.1 looks at both left and right tails of S, Equation 5.2 looks only at the right tail (see Figure 3).

This different definition of the problem has no effect on the normal approximation since the distribution of S is in any case approximated by a symmetrical distribution. However, when considering the normal power approximation, the necessary iterative calculation can be avoided by applying the simplification assumed in Equation 5.2 to derive a formula of the same form as Equation 4.2. Indeed, Venter [9] defines

$$m_2 = \frac{\mathrm{Var}[N]}{\mathrm{E}[N]} + CV(X)^2$$
$$m_3 = \gamma_1(X)CV(X)^3 + 3\frac{\mathrm{Var}[N]}{\mathrm{E}[N]}CV(X)^2 + \frac{\mathrm{E}[(N-\mathrm{E}[N])^3]}{\mathrm{E}[N]}. \quad (5.3)$$

(where $CV(X)$ is the coefficient of variation of the random variable X) and then the following condition is obtained:[2]

$$\mathrm{E}[N] \geq \frac{1}{4k^2}\left[z_{1-\varepsilon/2}\sqrt{m_2} + \sqrt{z_{1-\varepsilon/2}^2 m_2 + \frac{2}{3}\frac{m_3}{m_2}k(z_{1-\varepsilon/2}^2 - 1)}\right]^2. \quad (5.4)$$

In Example 3.1 of [9], S is a compound Poisson distribution. The distribution of the individual claim amount is lognormal with expected value 5,000 and coefficient of variation equal to 7, which amounts to parameter σ^2 equal to ln 50. The full credibility level is defined by $p = 0.90$ ($p^* = 0.95$) and $k = 0.05$. The normal approximation for λ is then correctly given as 54,120. As can be seen in Table 1, the "usual" two-sided normal power approximation would in that case be 53,927, while the result obtained with Equation 5.4 is 80,030. Full credibility levels have also been calculated with Venter's formula for every other case of Table 1. They are compared with previous results in Table 3. In the last column of Table 3 are also displayed the "true" values

[2] There is a misprint in *Foundations of Casualty Actuarial Science*: the square root sign in Equation 3.6 should be longer and end just before the rightmost parenthesis.

TABLE 3

COMPARISON OF FULL CREDIBILITY RESULTS OBTAINED WITH THE NORMAL AND NORMAL POWER APPROXIMATIONS AND WITH VENTER'S FORMULA

			Value of $E[N] = \lambda$			
Distribution of X_j	k	p	Normal Approximation	Normal Power	Venter's Formula	p for Venter's Results
Gamma						
$\alpha = 0.01$	0.050	0.99	109,323	109,258	111,598	0.9036
$\alpha = 0.05$	0.050	0.95	32,269	32,256	33,042	0.9527
$\alpha = 0.20$	0.100	0.90	1,624	1,621	1,686	0.9065
$\alpha = 1.10$	0.025	0.90	8,266	8,264	8,330	0.9013
$\alpha = 5.00$	0.100	0.95	461	461	474	0.9532
Lognormal						
$\sigma^2 = \ln 50$	0.050	0.90	54,121	49,232	80,029	0.9500
$\sigma^2 = 2.00$	0.050	0.95	11,354	11,301	12,367	0.9596
$\sigma^2 = 1.50$	0.100	0.90	1,213	1,203	1,325	0.9157
$\sigma^2 = 0.75$	0.025	0.90	9,166	9,163	9,268	0.9020
$\sigma^2 = 0.65$	0.100	0.95	736	735	770	0.9552

of p induced by Venter's results and calculated with the normal power approximation.

Venter's one-sided full credibility levels are consistently higher than the two-sided ones calculated with both the normal and normal power approximations. Since the distributions of S are usually positively skewed in the fields where limited fluctuation credibility is applied, this is indeed a direct consequence of the formulation of the problem in the form of Equation 5.2 coupled with the use of the normal power approximation to take the third moment of S into account.

The rationale of the author for adopting a one-sided criterion is not very clear. It is first suggested in [9] that, for most distributions of interest, Mowbray's two-sided criteria will be satisfied if the one-sided is. This can be verified in Table 3. But the main idea was probably to use the normal power approximation to obtain more refined—more accurate—full credibility levels. The task is then facilitated by the one-sided criterion as it leads to the easy to use, closed-form credibility formula, Equation 5.4. The problem with this formula is that it sometimes unnecessarily overstates the full credibility levels, a fact it appears Venter was aware of, as he summarizes Dale Nelson (*PCAS*, 1969):

> ... although the *NP* [normal power approximation] gives useful approximations of the higher percentiles, it may overstate the volume needed for full credibility relative to given standards.

Once the desired degree of conservatism has been fixed through the parameters k and p, there exists a "true" full credibility level satisfying Equation 2.1. We said earlier that our normal power approximation almost gives the true levels[3] and that the normal approximation is sufficiently close to these levels. Now, the normal approximation levels satisfying Equation 5.2 will be the same and as such should be satisfactory. Equation 5.4 may thus be simpler than our application of the normal power approximation, but as it yields higher results than the even simpler normal approximation, its usefulness becomes questionable.

Finally, it is not clearly stated in [9] that Equation 5.4 yields higher—and sometimes much higher, as Table 3 shows—full credibility levels as a solution to a problem defined in the form of Equation 5.2. This could lead to the perception that using the third moment in any full credibility level estimation will necessarily increase these levels. We have concluded earlier that this is not the case. An eventual user of Equation 5.4 should thus be aware of its implications and ensure it is used in conjunction with the one-sided definition of the limited fluctuation credibility problem.

[3] At least in the gamma cases.

6. CONCLUSION

The conclusion of the first part of this paper is drawn from the tests summarized in Table 1. Although giving very accurate results, more sophisticated approximation methods like normal power or Esscher are not worth the added complexity and calculation time as compared to the normal approximation to estimate full credibility levels. It has been shown that when staying with Mowbray's original definition of the limited fluctuation problem, the differences between the various approximations are hardly significant. In more peculiar situations, the normal approximation yields the more conservative result, so we stay on the safe side.

While not necessary in limited fluctuation credibility, we nonetheless emphasize that the normal power and Esscher approximations remain very useful tools in general risk theory because of their good estimation of the percentiles of an aggregate claim distribution.

The paper then discussed the apparently different conclusions put forward by Venter [9]. We mainly argue that it should be stated more clearly in [9] that the definition of the limited fluctuation problem differs from Mowbray's traditional one. Moreover, the formula based on the normal power approximation used in the paper and the conclusions drawn with it pertain only to the problem studied and should not be carried over to general limited fluctuation credibility.

The reader should note that when the limited fluctuation problem is treated as in this paper (that is, with Mowbray's definition), it is not possible to derive a simple, explicit formula for the expected value of N (which usually gives the full credibility level in limited fluctuation credibility) while using the normal power approximation.

Mayerson et al. [6] also obtained significantly higher (from 3% to 10%) full credibility levels when using the third moment of the distribution of S, but this is also due to their conservative approach to the problem. When worked out with the normal power approximation, the compound Poisson examples of Mayerson et al. lead to full credibility levels almost equal to the normal approximation. The most important idea of that paper, though, was that the full credibility level should be based on the pure premium (namely the distribution of S) rather than on only the number of claims (the distribution of N). This should still be stressed today.

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PARAMETER UNCERTAINTY IN (LOG)NORMAL DISTRIBUTIONS

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Abstract

The modeling of parameter uncertainty due to sample size in normal and lognormal distributions with diffuse Bayesian priors is solved exactly and compared to the large-sample approximation. Large-scale simulation results are presented. The results suggest that (1) the large-sample approximation is not very good in this case; and (2) estimates of reserve uncertainty may be considerably understated. A consequence is that intrinsic risk loads and reinsurance premiums may also be considerably understated. An example is given from Best's Homeowners paid data, where the mean estimate of IBNR hardly changes: it is $9.96B without parameter uncertainty and $10.01B with it, but the corresponding distribution standard deviations are 6.9% and 24.9% of the respective means.

1. INTRODUCTION

One of the most ubiquitous sources of parameter uncertainty is the fact that samples in real life are never infinite. Thus, when using a sample to estimate parameters of a presumed underlying distribution, the size of the sample must play a role in the uncertainty in the derived values of the parameters. In general, this uncertainty goes to zero as the sample size gets large. The converse, that the uncertainty can be large and even infinite when the sample size is small, is generally unappreciated.

For large samples the parameter distributions can be approximated by normal distributions, using the inverse of the matrix of second derivatives of the negative log-likelihood as the covariance matrix[1]. This is what is usually done for all sample sizes. What is not often understood is how wrong this approximation can be for small samples, say less than 10 data points.

The present paper is an attempt to give both an exact theoretical underpinning[2] and the practical cumulative distribution functions for use with these distributions. Section 2 is the theory; Section 3 is a numerical description of the actual distributions; and Section 4 is a reserving application to stable paid data. Of course, these results apply to any use of normal or lognormal distributions on empirical data. Claim severity distributions would be one example, and especially for reinsurance data the claim volume can be very small.

The general approach here will be to assume that we know the form of the distribution, thus ignoring what is in practice a very real source of parameter uncertainty. What is treated here is only the effect of finite sample size. What is desired is the probability of the parameters, given the observed sample. Given that, the predictive distribution of the variable itself may be obtained by summing over different parameter probabilities. In the present case, this is done using simulation.

The method of treatment is to use a Bayesian approach. The likelihood function gives the probability of the sample actually seen, given the parameters of the underlying distribution. Bayes' theorem says that the desired parameter probability distribution is, up to a normalization, the product of the likelihood function and an assumed prior distribution of the parameters. The assumed prior is here taken to be "diffuse," meaning that it contains as little information as possible in some sense.

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[1]This results essentially from taking just a second-order Taylor expansion of the negative log-likelihood in the neighborhood of the minimum, as will be done in the special case below. See [1, Section 18.26, page 675].

[2]This particular case is simple enough that it must have been solved many times. However, I am not aware of an actuarial application, and the derivation is instructive.

2. THEORY

We will do the lognormal case, as the normal case is essentially the same with the substitution of x for $\ln(x)$. We are given a sample of data x_i with $i = 1,2,\ldots,n$. The probability density function is

$$f(x) = \frac{1}{\sqrt{2\pi}x\sigma} \exp\left\{ -\frac{(\ln(x)-\mu)^2}{2\sigma^2} \right\}. \tag{2.1}$$

The corresponding negative log-likelihood (NLL) is, up to constant terms,

$$NLL = \frac{1}{2}\sum_{i=1}^{n} \frac{(\ln(x_i)-\mu)^2}{\sigma^2} + \sum_{i=1}^{n} \ln(x_i) + n\ln(\sigma) + cst. \tag{2.2}$$

The analysis begins by constructing the partial derivatives

$$\frac{\partial NLL}{\partial \mu} = \frac{1}{\sigma^2}\sum_{i=1}^{n}(\mu - \ln(x_i)) = \frac{1}{\sigma^2}\left\{ n\mu - \sum_{i=1}^{n}\ln(x_i) \right\} \tag{2.3}$$

and

$$\frac{\partial NLL}{\partial \sigma} = -\frac{1}{\sigma^3}\sum_{i=1}^{n}(\mu - \ln(x_i))^2 + \frac{n}{\sigma}. \tag{2.4}$$

The maximum likelihood estimators are obtained by finding μ_0 and σ_0 such that these partial derivatives are both zero:

$$\mu_0 = \frac{1}{n}\sum_{i=1}^{n}\ln(x_i), \qquad \text{and} \tag{2.5}$$

$$\sigma_0 = \sqrt{\frac{1}{n}\sum_{i=1}^{n}[\ln(x_i) - \mu_0]^2}. \tag{2.6}$$

The usual large sample approximation continues by creating the second partial derivatives:

$$\frac{\partial^2 NLL}{\partial \mu^2} = \frac{n}{\sigma^2}, \tag{2.7}$$

$$\frac{\partial^2 NLL}{\partial \mu \partial \sigma} = \frac{-2}{\sigma^3}\left\{n\mu - \sum_{i=1}^{n} \ln(x_i)\right\} \tag{2.8}$$

$$= \frac{-2n}{\sigma^3}(\mu - \mu_0), \qquad \text{and} \tag{2.9}$$

$$\frac{\partial^2 NLL}{\partial \sigma^2} = \frac{3}{\sigma^4}\sum_{i=1}^{n}(\mu - \ln(x_i))^2 - \frac{n}{\sigma^2} \tag{2.10}$$

$$= \frac{3n}{\sigma^4}\{(\mu - \mu_0)^2 + \sigma_0^2\} - \frac{n}{\sigma^2}. \tag{2.11}$$

Evaluating them at the maximum likelihood (minimum of the NLL),

$$\frac{\partial^2 NLL}{\partial \mu^2}(\mu_0, \sigma_0) = \frac{n}{\sigma_0^2}, \tag{2.12}$$

$$\frac{\partial^2 NLL}{\partial \mu \partial \sigma}(\mu_0, \sigma_0) = 0, \qquad \text{and} \tag{2.13}$$

$$\frac{\partial^2 NLL}{\partial \sigma^2}(\mu_0, \sigma_0) = \frac{2n}{\sigma_0^2}. \tag{2.14}$$

We note in passing that the mixed partial derivative is zero only on the line $\mu = \mu_0$. This means (as will shortly be made explicit) that in general the variables μ and σ are correlated.

The matrix of second-order partial derivatives evaluated at the minimum is

$$\begin{Bmatrix} \frac{\partial^2 NLL}{\partial \mu^2} & \frac{\partial^2 NLL}{\partial \mu \partial \sigma} \\ \frac{\partial^2 NLL}{\partial \mu \partial \sigma} & \frac{\partial^2 NLL}{\partial \sigma^2} \end{Bmatrix} = \frac{n}{\sigma_0^2}\begin{Bmatrix} 1 & 0 \\ 0 & 2 \end{Bmatrix}. \tag{2.15}$$

The inverse of this matrix is the covariance matrix for μ and σ around the minimum when they are expressed as a bivariate normal distribution:

$$\begin{Bmatrix} \mathrm{var}(\mu) & \mathrm{cov}(\mu,\sigma) \\ \mathrm{cov}(\mu,\sigma) & \mathrm{var}(\sigma) \end{Bmatrix} = \frac{\sigma_0^2}{n} \begin{Bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{Bmatrix}. \tag{2.16}$$

A simulation consists of drawing three deviates z, z_1, and z_2 from a standard normal distribution and setting

$$\ln(x) = \mu + \sigma z \tag{2.17}$$

with

$$\mu = \mu_0 + z_1 \frac{\sigma_0}{\sqrt{n}} \tag{2.18}$$

and

$$\sigma = \sigma_0 + z_2 \frac{\sigma_0}{\sqrt{2n}}. \tag{2.19}$$

Equivalently,

$$\ln(x) = \mu_0 + \sigma_0 z_{\mathrm{app}} \tag{2.20}$$

where the effective z in the large sample approximation, z_{app}, is given by

$$z_{\mathrm{app}} = \frac{z_1}{\sqrt{n}} + z\left(1 + \frac{z_2}{\sqrt{2n}}\right). \tag{2.21}$$

We note that the distribution for z_{app} is symmetric about the origin, which implies a mean of zero, and that the variance is given by

$$\mathrm{var}(z_{\mathrm{app}}) = 1 + \frac{3}{2n}. \tag{2.22}$$

It has been pointed out to the author[3] that another approach to a large sample approximation is to use $\ln(\sigma)$ as a variable in place of σ in the *NLL*. Following the same procedure through,[4]

[3]By the reviewer, to whom thanks are given for this remark.

[4]Although the derivation is straightforward, it is somewhat tedious and not relevant for the rest of the paper. Interested readers are invited to contact the author.

Equation 2.20 remains the same but Equations 2.21 and 2.22 become

$$z_{\text{app}} = \frac{z_1}{\sqrt{n}} + z \exp\left(\frac{z_2}{\sqrt{2n}}\right) \tag{2.23}$$

and

$$\text{var}(z_{\text{app}}) = \frac{1}{n} + \exp\left(\frac{1}{n}\right). \tag{2.24}$$

The variable z_{app} has zero mean, but is no longer symmetric. The lack of symmetry is disturbing to the author. However, the variance is larger than before [Equation 2.22] at any n. The increased dispersion of this large sample approximation will be closer to reality.

The underlying technique for the large sample approximation is to approximate the NLL by its Taylor series to second order around the minimum and to take the Bayesian prior to be one (i.e., not dependent on the parameters). However, the resulting simple quadratic form for the NLL is exactly what one gets from a normal (Gaussian) distribution. Hence the remark that, for large samples, the parameter distribution is taken to be normal. The hope is that by the time the NLL deviates significantly from the approximation, its value is sufficiently large that it represents a very small probability.[5]

However, in the present instance this hope is not fulfilled. Returning to the exact problem, the NLL may be rewritten as

$$NLL = n\left\{\frac{\sigma_0^2 + (\mu - \mu_0)^2}{2\sigma^2} + \mu_0 + \ln(\sigma)\right\}. \tag{2.25}$$

Rescale the problem by defining normalized variables v and y such that

$$\mu = \mu_0 + v\sigma_0 \tag{2.26}$$

and

$$\sigma = y\sigma_0. \tag{2.27}$$

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[5]The justification for this technique is essentially the same as for the central limit theorem. For a heuristic approach, see the discussion after Equation 2.28.

Then the NLL becomes

$$NLL = n\left\{\frac{1+v^2}{2y^2} + \ln(y) + \mu_0 + \ln(\sigma_0)\right\}. \qquad (2.28)$$

The range of v and y is the same as that for μ and σ: $-\infty < v < \infty$ and $0 \le y < \infty$. It is perhaps not quite obvious, but easy to prove, that the minimum NLL is at $v = 0$ and $y = 1$.

Although the NLL is exactly quadratic in v, it is not so in y. In fact, it is the rather extreme asymmetry in y around the minimum which results in the inadequacies of the large sample approximation. The large sample approximation results from noticing that, as n gets large, only values of v and y which get nearer to the minimum will give NLL values near its minimum. Specifically, one could take NLL of, say, 20 plus the minimum to be the largest value of interest. This corresponds to assuming a probability for the parameters involved of exp(−20) to be effectively zero. Then as n gets larger the values of v and y which give NLL = minimum + 20 get closer and closer to their minimum values, approximately inversely with the square root of n. This approximation gets better as n increases. In this approximation, terms in the Taylor series expansion of order higher than the second all have contributions to the NLL which decrease as n increases, and the NLL is better and better represented by just the second order term.

We take a Bayesian approach and use diffuse prior distributions for v and y. Since v runs along the full axis from minus infinity to infinity, the prior used is just 1. Since y runs along the semi-axis, the suggested prior is proportional to $1/y^\theta$ where θ is either 0 or 1, depending on one's preference[6]. The choice $\theta = 1$ emphasizes small values of y and corresponds to the assumption that the prior distribution of ln(y) is flat; the choice $\theta = 0$ assumes that the prior distribution of y is flat. Venter[7] has

[6] [1, Section 8.28 p. 304]. A reference is made to an article by Jeffries, advocating $\theta = 1$.
[7] Gary Venter, private communication. He points out that on a semi-axis a flat prior corresponds to assuming that it is as likely for the variable to lie between a million and

emphasized that any choice of prior has strong implications. Ideally, the nature of the data being fitted would give some clues as to proper priors.

The joint distribution of v and y is, up to normalization factors (we use the symbol $\sim$), given by the product of the Bayesian priors and the likelihood:

$$f(v,y) \sim \frac{\exp\left\{-n\left(\frac{1+v^2}{2y^2}\right)\right\}}{y^{n+\theta}}. \tag{2.29}$$

We now change variables from y to w by

$$y = \sqrt{\frac{n(1+v^2)}{w}} \tag{2.30}$$

so that

$$\frac{\partial y}{\partial w} = -\frac{1}{2}\sqrt{\frac{n(1+v^2)}{w^3}}, \tag{2.31}$$

and for the variables v and w, the joint distribution behaves as

$$f(v,w) \sim \left\{w^{(n+\theta-3)/2}\exp\left(-\frac{w}{2}\right)\right\}\left\{(1+v^2)^{-(n+\theta-1)/2}\right\}. \tag{2.32}$$

This transformation does several nice things. First, since the joint distribution is a product, the variables are independent (and therefore uncorrelated) and may be simulated separately. A corollary of this is that v and y, and hence μ and σ, are correlated. Second, we can recognize the variable distributions as well known.

The variable w is chi-squared distributed[8] with parameter $(n+\theta-1)$. Equivalently, $w/2$ is gamma distributed [2, p. 104] with parameter $(n+\theta-1)/2$. Both of the inverse functions exist

a million and one as it is for the variable to lie between zero and one, and that it is infinitely more likely to be excess of any finite amount than to be less than that amount.

[8] Almost any text on statistics has the chi-squared and t-distributions, e.g., [2, p. 107].

in Excel,[9] and can be used in simulations. The mean value of w is $(n+\theta-1)$, and its variance is $2(n+\theta-1)$. Thus w/n has a mean of $(1+(\theta-1)/n)$ and a standard deviation of $\sqrt{2(n+\theta-1)}/n$. As n, the sample size, becomes large these go respectively to 1 and 0.

The variable $v\sqrt{n+\theta-2}$ is t-distributed [2, p. 145] with parameter $(n+\theta-2)$. Therefore the mean value of v is zero, and its standard deviation[10] is $1/\sqrt{n+\theta-4}$. The standard deviation does not exist if $n+\theta \leq 4$, but goes to zero as the sample size increases.

In simulation situations if the underlying distribution does not have a finite variance then the mean of the simulation will not converge, because the mean of the simulation itself will have an infinite standard deviation. In practice, this shows up as occasional large jumps in the mean, even with millions of simulations (in fact, no matter how many simulations are done). If the simulation is being done in a situation where the upper end is limited—for example in a ceded layer of reinsurance—then the variance will always be finite. However, "finite" does not mean the same as "of reasonable size." In some numerical modeling the author has come across cases where a distribution with finite variance and a theoretical mean of a million dollars was producing an occasional value of a trillion dollars. Clearly, very many millions of simulations would be necessary to get a reasonable amount of convergence. It is recommended that actuaries should try to avoid small sample sizes and/or at least work with lognormal distributions which are truncated at the upper end.

Equation 2.32 shows that as far as v and w are concerned taking $\theta = 1$ is the same as assuming that there is one more data point than actually exists and taking $\theta = 0$. The results in Sections 3 and 4 and Appendix A are all done with $\theta = 0$. If one can convince oneself that an appropriate value of θ is 4, then all

[9]Microsoft Excel 5.0. These functions may also be found elsewhere.

[10]The variance of the Student's t distribution with parameter n is $n/(n-2)$.

worries of convergence are over and as little as one data value can be used. Trying to justify this may take some doing—not to mention getting both a mean and standard deviation from one value!

Another representation for v can be obtained by changing to

$$u = \frac{v^2}{1 + v^2}. \tag{2.33}$$

Clearly, the support of this variable runs from 0 to 1, rather than from $-\infty$ to ∞, but

$$v = \pm\sqrt{\frac{u}{1-u}} \tag{2.34}$$

can be obtained from a u deviate by another random choice to get the sign. Since

$$\frac{dv}{du} = \frac{1}{2\sqrt{u(1-u)^3}} \tag{2.35}$$

then

$$f(u) \sim u^{-1/2}(1-u)^{(n+\theta-2)/2} \tag{2.36}$$

which is recognizable as the beta distribution with parameters 1/2 and $(n + \theta - 2)/2$. Random deviates for the beta distribution can be obtained either from the inverse function in Excel or as a ratio of gamma deviates. Specifically, a beta(α, β) deviate can be obtained [2, p. 139] as $x/(x + y)$ where x is gamma distributed with parameter α and y is gamma distributed with parameter β.

Returning to the simulation methodology, if we let z be a deviate from the standard normal distribution, then in parallel with Equations 2.17, 2.18, and 2.19 for the large sample approximation we have the exact results

$$\ln(x) = \mu + \sigma z \tag{2.37}$$

with

$$\mu = \mu_0 + v\sigma_0, \tag{2.38}$$

and

$$\sigma = \sigma_0 \sqrt{\frac{n(1+v^2)}{w}}. \tag{2.39}$$

Combining Equations 2.37, 2.38, and 2.39

$$\ln(x) = \mu_0 + \sigma_0 z_{\text{eff}} \tag{2.40}$$

where the effective deviate z_{eff}, is given by

$$z_{\text{eff}} = v + z\sqrt{\frac{n(1+v^2)}{w}}. \tag{2.41}$$

Equation 2.41 for z_{eff} is the exact result for which z_{app} of Equation 2.21 is an approximation. Like z_{app}, z_{eff} is symmetric about the origin and has mean zero. This effective deviate generally has a much broader tail than the large sample approximation. However, in the limit of large n (as mentioned earlier) v goes to zero and w goes to n, so that z_{eff} goes to z. In fact, z_{eff} goes to z_{app} to order $1/n$ and they both go to z.

In order to get the variance of z_{eff}, the expectation of $1/w$ is needed. To obtain this, use the fact that for any variable x which is gamma distributed with parameter α, the expectation of any power p of x is

$$\text{E}(x^p) = \frac{\Gamma(\alpha + p)}{\Gamma(\alpha)} \tag{2.42}$$

so

$$\text{E}\left(\frac{1}{x}\right) = \frac{\Gamma(\alpha - 1)}{\Gamma(\alpha)} = \frac{1}{\alpha - 1}. \tag{2.43}$$

Since $w/2$ is gamma distributed with parameter $(n + \theta - 1)/2$,

$$\text{E}\left(\frac{2}{w}\right) = \frac{2}{n + \theta - 3}. \tag{2.44}$$

Since the mean of z_{eff} is zero, its variance is just the expectation of its square

$$\text{var}(z_{\text{eff}}) = \text{E}([z_{\text{eff}}]^2). \tag{2.45}$$

Because of the independence of the variables, this implies

$$\text{var}(z_{\text{eff}}) = \text{var}(v) + \text{var}(z)n\text{E}\left(\frac{1}{w}\right)[1 + \text{var}(v)] \tag{2.46}$$

$$= \frac{1}{n+\theta-4} + \frac{n}{n+\theta-3}\left(1 + \frac{1}{n+\theta-4}\right) \tag{2.47}$$

$$= \frac{n+1}{n+\theta-4}. \tag{2.48}$$

In the end, this is a remarkably simple result. Although this variance clearly goes to 1 as n becomes large, for $n = 5$ and $\theta = 0$ its value is 6! Of course, for $n + \theta \leq 4$ it is infinite. This formula also tempts one to choose $\theta = 5$ so that $\text{var}(z_{\text{eff}}) = 1$ for all n.

3. PRACTICE

All of the results for z_{eff} were done using Equation 2.41 with $\theta = 0$ and different values of n. The tables and graphs are useful for getting a feel for how the distributions change with n. If one is uncomfortable with the diffuse prior used, then it is recommended to generate one's own values. It may in the course of simulations be faster to look up values in tables rather than generate them on the fly, but as a matter of general preference the author would rather generate than look up, especially in someone else's tables.

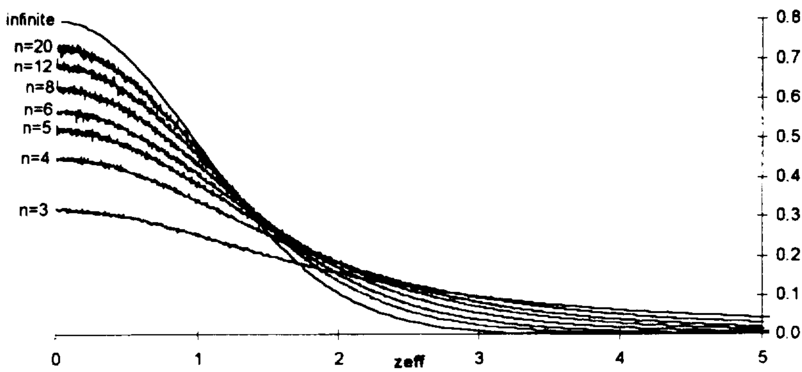
For various values of n, the density function of z_{eff} was simulated in two stages. In the first, 10,000,000 simulations were run to get the range from 50% to 90% on the cumulative distribution function (CDF). Then for values of $n < 10$, 50,000,000 simulations were run to get 5,000,000 simulations of values greater than the 90% level[11] in order to get the tails of the distributions.

Let us look first at the general shape of the density functions. As usual, the effect of parameter uncertainty is to push probability away from the mean out into the tail, and the effect is more

[11]This was not done in a spreadsheet, but in a C++ program.

FIGURE 1

PROBABILITY DENSITY FUNCTIONS FOR DIFFERENT SAMPLE SIZES



pronounced with increasing parameter uncertainty (i.e., decreasing sample size). See Figure 1.

The differences begin to show up dramatically when we look at the Cumulative Distribution Function (CDF) for various sample sizes. Because of the symmetry, only the portion from 50% to 100% is shown in Figure 2.

The extension to even larger z_{eff} is shown in Figure 3. The conclusion from these graphs is at least that the effect of sample size can be substantial even for what might be thought to be relatively large samples.

It is also of interest to compare for a fixed sample size the normal distribution (infinite sample size, no parameter variation), the large sample approximation, and the exact result. Figure 4 displays this comparison for sample size $N = 3$.

Clearly, the large sample approximation is not very good. On the other hand, we didn't expect it to be. However, sample size $N = 8$ shows a similar pattern. See Figure 5.

FIGURE 2

CUMULATIVE DISTRIBUTION FUNCTION FOR VARIOUS SAMPLE SIZES

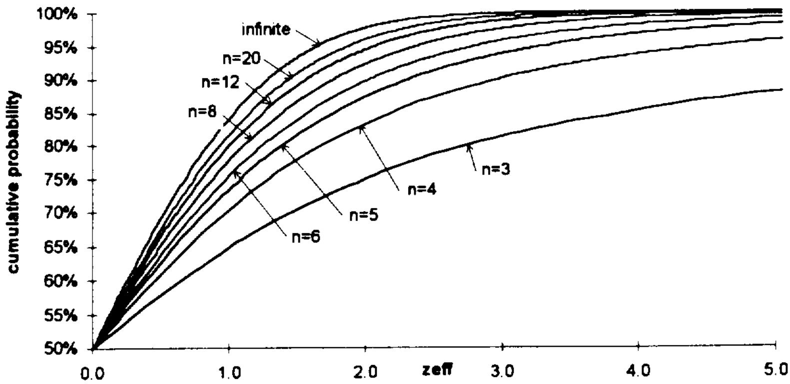


FIGURE 3

CUMULATIVE DISTRIBUTION FUNCTION FOR VARIOUS SAMPLE SIZES

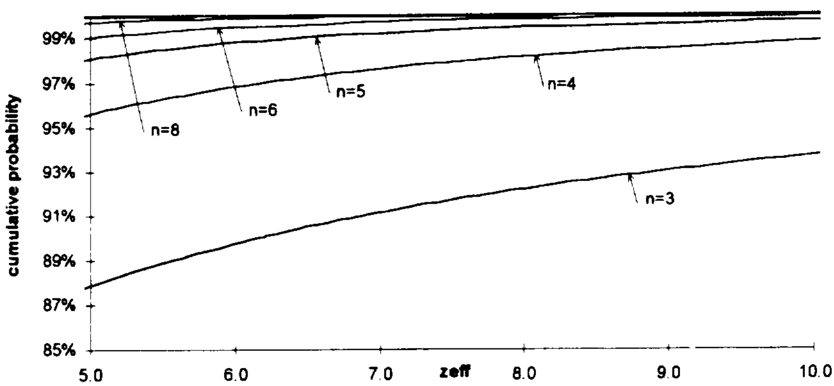


FIGURE 4

$n = 3$ CUMULATIVE DISTRIBUTION FUNCTION FOR VARIOUS TECHNIQUES

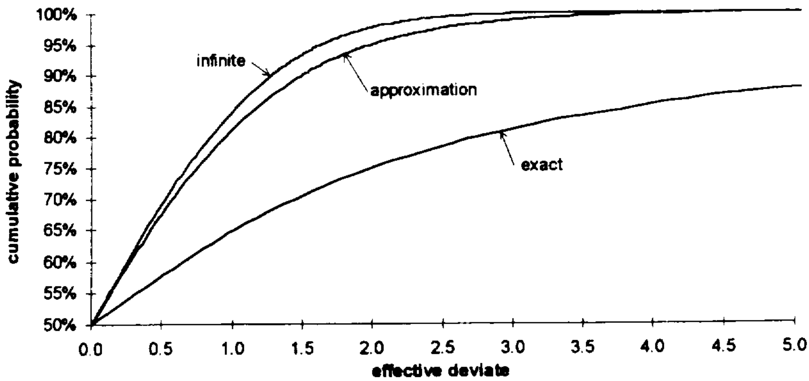


FIGURE 5

$n = 8$ CUMULATIVE DISTRIBUTION FUNCTION FOR VARIOUS TECHNIQUES

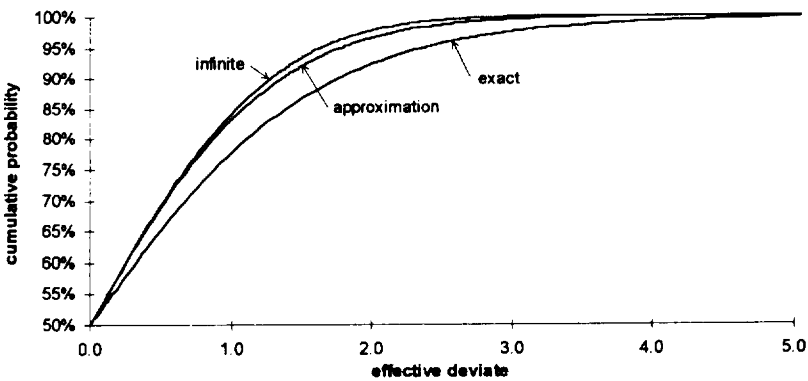


TABLE 1

EFFECTIVE z BY SAMPLE SIZE FOR SOME KEY CDF VALUES

Sample Size	3	4	5	6	8	12	20	Infinite
CDF								
50%	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
60%	0.650	0.456	0.391	0.358	0.325	0.297	0.278	0.253
70%	1.454	0.976	0.826	0.752	0.677	0.618	0.576	0.524
80%	2.752	1.677	1.384	1.245	1.109	1.002	0.931	0.842
90%	6.159	2.981	2.315	2.028	1.762	1.564	1.436	1.282
95.0%	12.62	4.617	3.327	2.819	2.380	2.067	1.873	1.645
97.5%	25.41	6.802	4.501	3.672	2.998	2.541	2.269	1.960
98.0%	31.81	7.666	4.923	3.965	3.200	2.691	2.391	2.054
99.0%	63.71	11.02	6.422	4.956	3.850	3.151	2.757	2.326
99.5%	127.5	15.71	8.260	6.089	4.542	3.614	3.108	2.576
99.9%	**639.3**	**35.35**	**14.47**	**9.526**	**6.392**	**4.726**	**3.897**	**3.090**
99.95%	1,308	50.13	18.46	11.49	7.328	5.247	4.232	3.290
99.99%	6,476	130.4	32.58	17.53	9.822	6.513	5.023	3.719
99.995%	12,470	164.1	42.64	20.75	11.18	7.108	5.371	3.891
99.999%	57,550	345.4	67.41	31.11	14.73	9.353	6.158	4.265

Even here, the large sample approximation is much closer to the pure normal than it is to the exact result, especially in the region of high cumulative probability. The approximation has essentially the same tail behavior as a normal, while the exact result has a much fatter tail. This suggests that the approximation does not hold well for these sample sizes, which are, unfortunately, typical of those usable in chain-ladder reserving.

A complete set of appropriate effective deviates for various CDF values and various sample sizes all at $\theta = 0$ is given in Appendix A. That set is intended for use in simulations if the reader does not want to generate directly the underlying distributions. A subset for some key values of the CDF is given in Table 1.

If we look, for example, at the 99.9% level (in **bold** type), then for n infinite we recognize $z_{\text{eff}} = 3.090$ as a familiar friend from the normal distribution. As the sample size decreases, the location of the 99.9% level increases from 3.09. For $n = 8$ it has more than doubled to 6.4; for $n = 5$ it has almost quintupled to 14.5; and for $n = 3$ it is up to 639! In general, in order to reach any CDF level one must go to increasingly higher multiples of the sigma estimator as the sample size decreases, and the effect is more pronounced as the CDF level increases.

All of the above indicates that the tails are much fatter than one might have thought when using either the large sample approximation to the parameter uncertainty or no parameter uncertainty at all.

4. RESERVING

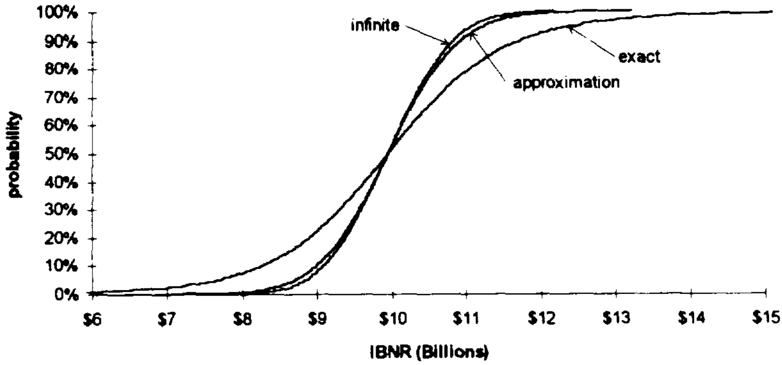
Typically in chain-ladder reserving, the age-to-age factors are implicitly or explicitly taken to be normal or lognormal. For example, the not atypical procedure which we will use here starts by taking the most recent five calendar years of data and averages the logs of the appropriate age-to-age factors in the data to get the log of the projected age-to-age factor. This gives point estimates of the age-to-age factors, which generate the age-to-ultimate factors, which give the IBNR.

Five years is chosen as an intuitive compromise between wanting to stabilize the results by having lots of data and wanting to use only data which is close enough to the current business to be relevant. Clearly there will always be judgment calls of some sort.

In order to go beyond a point estimate of IBNR, the next step is to explicitly assume that the age-to-age factors are lognormally distributed independently at each age. Then we have a sample of five for each age-to-age factor and can calculate the maximum likelihood estimators for both μ and σ. Since the product of lognormal variables is also lognormal, the age-to-ultimate factors are lognormal and their parameters can be easily calculated. This allows the representation of IBNR as a distribution, rather than just a single value.

FIGURE 6

CDF FOR HOMEOWNERS PAID IBNR



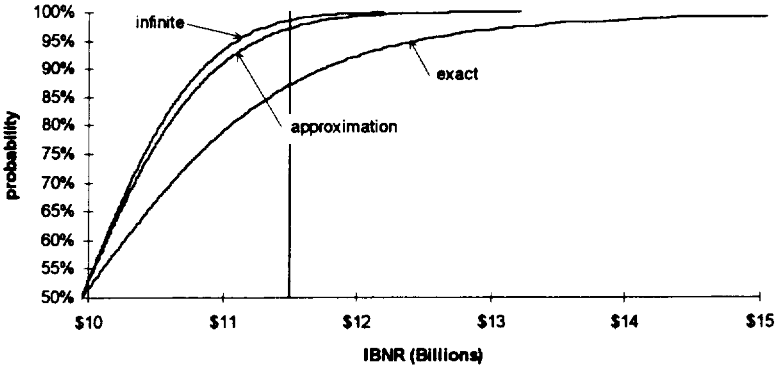
However, this procedure corresponds to using the infinite sample size approximation for the parameter variation—i.e., assuming that there isn't any. Given the above discussion, it will be no surprise that we recommend that the z_{eff} for $n = 5$ be used. It does mean that the distributions for the age-to-age factors must be numerically rather than analytically generated, but this is a relatively minor difficulty.

For a concrete example, we use industry data from Best's 1995 Aggregates and Averages. The original data is Homeowners-Farmowners Schedule P paid data from accident years 1985 to 1994 inclusive, which is displayed in Appendix B. The CDFs are shown in Figure 6, and the labels "infinite," "approximation," and "exact" refer as before to the situations with no parameter variation (infinite sample), the large sample approximation, and the exact result.

An expansion of the dangerous half of the distribution is shown in Figure 7. A line has been put in at $11.5 billion to guide the eye. The probability of exceeding that value is 1.39% for the "infinite" calculation, which would seem a conservative

FIGURE 7

CDF FOR HOMEOWNERS PAID IBNR



reserving level. However, for the approximation the probability is 2.81%, and for the exact result it is 12.78%. To get to the exact 1.39% level, it is necessary to reserve $14.1 billion! These differences are clearly important for a reinsurer. Even for an insurer who reserves at the mean value, the unexpectedly large variability will show up either as an increased risk load cost—probably as cost of liquidity—or as a nasty surprise.

The main simulation results[12] are summarized in Table 2.

It should be noted that even these results are somewhat optimistic (in the sense of providing a small coefficient of variation) in that all factors were taken to have $n = 5$ and in reality the tail of the triangle did not have that much data.

Since this is industry data on a relatively stable line, the 24.9% coefficient of variation for the exact result may be indicative of the minimum reserve variation to be expected.

[12]For 1,000,000 simulations in each case. Run times were 10 minutes, 20 minutes, and 40 minutes.

TABLE 2

SIMULATION RESULTS

CDF	Infinite	Approximation	Exact
20%	$9,377,999	$9,323,378	$8,889,821
40%	$9,775,408	$9,762,350	$9,638,914
60%	$10,121,909	$10,137,497	$10,267,019
80%	$10,530,213	$10,586,319	$11,050,725
90%	$10,839,277	$10,944,285	$11,743,068
95%	$11,097,636	$11,257,818	$12,453,300
98%	$11,393,344	$11,637,681	$13,550,822
99.0%	$11,590,893	$11,912,637	$14,599,413
99.5%	$11,769,344	$12,172,122	$16,014,574
99.9%	$12,144,913	$12,745,661	$21,581,916
mean	$9,956,034	$9,959,629	$10,007,938
standard deviation	$685,580	$782,023	$2,489,269
coefficient of variation	**6.9%**	**7.9%**	**24.9%**

REFERENCES

[1] Stuart, Alan and J. Keith Ord, *Kendall's Advanced Theory of Statistics*, Fifth Edition, Volume 2, 1991, Wiley and Sons.

[2] Hogg, Robert V. and Allen T. Craig, *Introduction to Mathematical Statistics*, Fourth Edition, 1978, Macmillan, New York.

APPENDIX A

TABLE OF EFFECTIVE z FOR $\theta = 0$ BY CDF VALUE BY SAMPLE SIZE

Size	3	4	5	6	7	8	10	12	20	Infinite
CDF										
50%	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
51%	0.0628	0.0448	0.0384	0.0354	0.0335	0.0320	0.0304	0.0292	0.0274	0.0251
52%	0.1259	0.0895	0.0769	0.0706	0.0668	0.0641	0.0608	0.0585	0.0550	0.0502
53%	0.1892	0.1344	0.1155	0.1060	0.1002	0.0962	0.0911	0.0879	0.0825	0.0753
54%	0.2531	0.1795	0.1542	0.1414	0.1336	0.1283	0.1216	0.1175	0.1100	0.1004
55%	0.3173	0.2247	0.1931	0.1770	0.1673	0.1606	0.1523	0.1470	0.1376	0.1257
56%	0.3821	0.2702	0.2322	0.2129	0.2010	0.1929	0.1830	0.1766	0.1652	0.1510
57%	0.4476	0.3161	0.2715	0.2488	0.2349	0.2255	0.2138	0.2062	0.1931	0.1764
58%	0.5143	0.3624	0.3109	0.2848	0.2689	0.2583	0.2448	0.2361	0.2210	0.2019
59%	0.5816	0.4089	0.3507	0.3212	0.3032	0.2913	0.2760	0.2662	0.2492	0.2275
60%	0.6503	0.4560	0.3909	0.3579	0.3378	0.3245	0.3074	0.2966	0.2776	0.2533
61%	0.7207	0.5038	0.4315	0.3950	0.3727	0.3580	0.3391	0.3271	0.3061	0.2793
62%	0.7925	0.5525	0.4728	0.4324	0.4079	0.3918	0.3711	0.3579	0.3348	0.3055
63%	0.8665	0.6018	0.5143	0.4703	0.4435	0.4259	0.4034	0.3891	0.3637	0.3319
64%	0.9422	0.6519	0.5566	0.5087	0.4794	0.4603	0.4360	0.4203	0.3929	0.3585
65%	1.0198	0.7030	0.5996	0.5477	0.5161	0.4952	0.4691	0.4522	0.4226	0.3853
66%	1.1003	0.7549	0.6433	0.5873	0.5532	0.5307	0.5023	0.4844	0.4525	0.4125
67%	1.1835	0.8079	0.6876	0.6274	0.5908	0.5664	0.5360	0.5170	0.4829	0.4399
68%	1.2703	0.8623	0.7328	0.6683	0.6290	0.6028	0.5703	0.5501	0.5136	0.4677
69%	1.3598	0.9182	0.7791	0.7100	0.6678	0.6399	0.6053	0.5837	0.5446	0.4958
70%	1.4535	0.9755	0.8262	0.7524	0.7074	0.6775	0.6407	0.6176	0.5762	0.5244
71%	1.5518	1.0345	0.8747	0.7959	0.7476	0.7159	0.6768	0.6522	0.6082	0.5534
72%	1.6547	1.0953	0.9244	0.8402	0.7889	0.7553	0.7135	0.6875	0.6409	0.5828
73%	1.7634	1.1579	0.9754	0.8855	0.8310	0.7953	0.7510	0.7235	0.6742	0.6128
74%	1.8784	1.2229	1.0280	0.9321	0.8743	0.8365	0.7893	0.7602	0.7082	0.6433
75%	2.0002	1.2902	1.0821	0.9800	0.9186	0.8787	0.8285	0.7980	0.7431	0.6745
76%	2.1291	1.3607	1.1379	1.0292	0.9642	0.9221	0.8689	0.8365	0.7788	0.7063
77%	2.2681	1.4341	1.1957	1.0804	1.0112	0.9666	0.9103	0.8762	0.8152	0.7388
78%	2.4171	1.5110	1.2557	1.1331	1.0598	1.0125	0.9530	0.9169	0.8526	0.7722
79%	2.5778	1.5918	1.3184	1.1878	1.1101	1.0601	0.9971	0.9588	0.8911	0.8064
80%	2.7521	1.6767	1.3838	1.2448	1.1623	1.1092	1.0429	1.0021	0.9306	0.8416
81%	2.9420	1.7666	1.4524	1.3042	1.2165	1.1603	1.0901	1.0470	0.9716	0.8779
82%	3.1509	1.8623	1.5245	1.3667	1.2734	1.2136	1.1390	1.0937	1.0140	0.9154
83%	3.3808	1.9642	1.6004	1.4318	1.3329	1.2691	1.1900	1.1422	1.0580	0.9542
84%	3.6372	2.0735	1.6809	1.5008	1.3954	1.3275	1.2433	1.1929	1.1039	0.9945
85%	3.9250	2.1914	1.7669	1.5736	1.4611	1.3888	1.2993	1.2461	1.1519	1.0364
86%	4.2498	2.3191	1.8592	1.6515	1.5313	1.4537	1.3586	1.3021	1.2023	1.0803
87%	4.6222	2.4593	1.9582	1.7343	1.6056	1.5228	1.4210	1.3613	1.2556	1.1264
88%	5.0524	2.6146	2.0665	1.8240	1.6853	1.5967	1.4881	1.4243	1.3121	1.1750
89%	5.5561	2.7866	2.1844	1.9217	1.7717	1.6764	1.5597	1.4915	1.3721	1.2265
90%	6.1588	2.9813	2.3153	2.0283	1.8658	1.7624	1.6376	1.5640	1.4362	1.2816

TABLE OF EFFECTIVE z FOR $\theta = 0$ BY CDF VALUE BY SAMPLE SIZE (*Continued*)

Size	3	4	5	6	7	8	10	12	20	Infinite
CDF										
90.1%	6.2199	3.0024	2.3297	2.0389	1.8757	1.7727	1.6456	1.5720	1.4435	1.2873
90.2%	6.2876	3.0234	2.3437	2.0503	1.8858	1.7819	1.6538	1.5797	1.4503	1.2930
90.3%	6.3568	3.0448	2.3579	2.0618	1.8959	1.7911	1.6621	1.5875	1.4571	1.2988
90.4%	6.4270	3.0665	2.3722	2.0734	1.9061	1.8005	1.6704	1.5953	1.4639	1.3047
90.5%	6.4988	3.0885	2.3867	2.0852	1.9164	1.8099	1.6788	1.6032	1.4709	1.3106
90.6%	6.5723	3.1108	2.4014	2.0970	1.9269	1.8194	1.6873	1.6111	1.4779	1.3165
90.7%	6.6473	3.1336	2.4163	2.1090	1.9374	1.8291	1.6959	1.6191	1.4849	1.3225
90.8%	6.7235	3.1567	2.4313	2.1212	1.9481	1.8388	1.7046	1.6272	1.4920	1.3285
90.9%	6.8017	3.1800	2.4466	2.1335	1.9588	1.8486	1.7134	1.6354	1.4992	1.3346
91.0%	6.8814	3.2037	2.4621	2.1458	1.9696	1.8585	1.7222	1.6436	1.5065	1.3408
91.1%	6.9627	3.2277	2.4777	2.1583	1.9806	1.8686	1.7311	1.6520	1.5138	1.3469
91.2%	7.0456	3.2520	2.4935	2.1710	1.9918	1.8787	1.7401	1.6604	1.5212	1.3532
91.3%	7.1312	3.2768	2.5096	2.1838	2.0031	1.8889	1.7492	1.6688	1.5286	1.3595
91.4%	7.2189	3.3018	2.5258	2.1969	2.0144	1.8994	1.7584	1.6774	1.5361	1.3658
91.5%	7.3084	3.3272	2.5423	2.2101	2.0259	1.9098	1.7678	1.6861	1.5436	1.3722
91.6%	7.3998	3.3531	2.5590	2.2235	2.0375	1.9204	1.7772	1.6948	1.5513	1.3787
91.7%	7.4930	3.3795	2.5759	2.2369	2.0493	1.9311	1.7867	1.7037	1.5590	1.3852
91.8%	7.5892	3.4063	2.5931	2.2506	2.0613	1.9419	1.7964	1.7126	1.5669	1.3917
91.9%	7.6873	3.4335	2.6106	2.2644	2.0734	1.9528	1.8061	1.7217	1.5749	1.3984
92.0%	7.7878	3.4612	2.6284	2.2785	2.0856	1.9639	1.8159	1.7307	1.5828	1.4051
92.1%	7.8913	3.4895	2.6464	2.2926	2.0980	1.9752	1.8258	1.7399	1.5908	1.4118
92.2%	7.9968	3.5182	2.6646	2.3071	2.1106	1.9865	1.8359	1.7492	1.5990	1.4187
92.3%	8.1049	3.5476	2.6831	2.3216	2.1233	1.9980	1.8460	1.7587	1.6072	1.4255
92.4%	8.2158	3.5772	2.7020	2.3364	2.1360	2.0096	1.8563	1.7682	1.6155	1.4325
92.5%	8.3294	3.6076	2.7211	2.3514	2.1490	2.0213	1.8667	1.7778	1.6240	1.4395
92.6%	8.4467	3.6385	2.7404	2.3666	2.1621	2.0331	1.8773	1.7876	1.6325	1.4466
92.7%	8.5661	3.6700	2.7600	2.3821	2.1754	2.0452	1.8879	1.7975	1.6411	1.4538
92.8%	8.6892	3.7020	2.7799	2.3978	2.1888	2.0575	1.8987	1.8075	1.6498	1.4611
92.9%	8.8153	3.7347	2.8003	2.4137	2.2026	2.0699	1.9096	1.8176	1.6586	1.4684
93.0%	8.9453	3.7680	2.8210	2.4299	2.2164	2.0825	1.9207	1.8279	1.6675	1.4758
93.1%	9.0797	3.8019	2.8420	2.4463	2.2304	2.0952	1.9320	1.8384	1.6764	1.4833
93.2%	9.2163	3.8365	2.8634	2.4630	2.2447	2.1082	1.9434	1.8489	1.6855	1.4909
93.3%	9.3579	3.8720	2.8852	2.4800	2.2593	2.1214	1.9549	1.8596	1.6948	1.4985
93.4%	9.5033	3.9081	2.9072	2.4972	2.2740	2.1347	1.9666	1.8704	1.7041	1.5063
93.5%	9.6537	3.9449	2.9295	2.5148	2.2890	2.1482	1.9784	1.8813	1.7136	1.5141
93.6%	9.8088	3.9825	2.9524	2.5326	2.3043	2.1619	1.9904	1.8925	1.7232	1.5220
93.7%	9.9690	4.0211	2.9759	2.5506	2.3198	2.1759	2.0026	1.9037	1.7329	1.5301
93.8%	10.134	4.0605	2.9997	2.5690	2.3356	2.1901	2.0150	1.9152	1.7427	1.5382
93.9%	10.305	4.1007	3.0240	2.5877	2.3516	2.2045	2.0275	1.9267	1.7527	1.5464
94.0%	10.481	4.1420	3.0488	2.6068	2.3678	2.2191	2.0402	1.9386	1.7628	1.5548
94.1%	10.663	4.1843	3.0741	2.6262	2.3842	2.2340	2.0532	1.9504	1.7731	1.5632
94.2%	10.852	4.2276	3.0999	2.6460	2.4009	2.2490	2.0664	1.9624	1.7835	1.5718
94.3%	11.047	4.2721	3.1262	2.6661	2.4179	2.2643	2.0797	1.9748	1.7940	1.5805

TABLE OF EFFECTIVE z FOR $\theta = 0$ BY CDF VALUE BY SAMPLE SIZE (*Continued*)

Size	3	4	5	6	7	8	10	12	20	Infinite
CDF										
94.4%	11.247	4.3174	3.1530	2.6867	2.4354	2.2800	2.0933	1.9872	1.8048	1.5893
94.5%	11.455	4.3643	3.1803	2.7076	2.4533	2.2960	2.1071	2.0000	1.8158	1.5982
94.6%	11.670	4.4121	3.2085	2.7289	2.4713	2.3123	2.1212	2.0129	1.8269	1.6072
94.7%	11.895	4.4614	3.2372	2.7507	2.4898	2.3288	2.1355	2.0260	1.8381	1.6164
94.8%	12.127	4.5119	3.2667	2.7729	2.5087	2.3456	2.1501	2.0394	1.8495	1.6258
94.9%	12.369	4.5636	3.2966	2.7957	2.5279	2.3627	2.1650	2.0530	1.8610	1.6352
95.0%	12.621	4.6168	3.3275	2.8189	2.5477	2.3802	2.1800	2.0668	1.8728	1.6449
95.1%	12.883	4.6718	3.3593	2.8428	2.5684	2.3981	2.1955	2.0809	1.8849	1.6546
95.2%	13.155	4.7282	3.3915	2.8671	2.5889	2.4163	2.2112	2.0953	1.8972	1.6646
95.3%	13.439	4.7863	3.4247	2.8920	2.6098	2.4348	2.2272	2.1100	1.9096	1.6747
95.4%	13.736	4.8458	3.4587	2.9176	2.6313	2.4539	2.2436	2.1250	1.9224	1.6849
95.5%	14.045	4.9072	3.4938	2.9437	2.6532	2.4734	2.2603	2.1402	1.9354	1.6954
95.6%	14.371	4.9711	3.5297	2.9706	2.6758	2.4933	2.2774	2.1558	1.9486	1.7060
95.7%	14.708	5.0372	3.5669	2.9980	2.6988	2.5136	2.2949	2.1717	1.9619	1.7169
95.8%	15.064	5.1054	3.6047	3.0263	2.7224	2.5343	2.3128	2.1880	1.9756	1.7279
95.9%	15.438	5.1759	3.6438	3.0554	2.7464	2.5556	2.3310	2.2046	1.9897	1.7392
96.0%	16.830	5.2490	3.6843	3.0851	2.7710	2.5775	2.3497	2.2217	2.0040	1.7507
96.1%	16.241	5.3246	3.7260	3.1158	2.7964	2.6000	2.3689	2.2390	2.0188	1.7624
96.2%	16.673	5.4032	3.7688	3.1474	2.8226	2.6229	2.3886	2.2570	2.0337	1.7744
96.3%	17.131	5.4848	3.8130	3.1800	2.8494	2.6465	2.4088	2.2752	2.0489	1.7866
96.4%	17.611	5.5698	3.8589	3.2134	2.8771	2.6709	2.4295	2.2940	2.0647	1.7991
96.5%	18.117	5.6577	3.9065	3.2480	2.9055	2.6960	2.4507	2.3133	2.0807	1.8119
96.6%	18.652	5.7498	3.9556	3.2836	2.9348	2.7219	2.4726	2.3332	2.0971	1.8250
96.7%	19.219	5.8455	4.0063	3.3205	2.9652	2.7484	2.4951	2.3536	2.1140	1.8384
96.8%	19.824	5.9463	4.0595	3.3588	2.9967	2.7760	2.5183	2.3744	2.1315	1.8522
96.9%	20.472	6.0513	4.1146	3.3983	3.0290	2.8044	2.5422	2.3958	2.1495	1.8663
97.0%	21.160	6.1607	4.1723	3.4394	3.0626	2.8338	2.5669	2.4180	2.1679	1.8808
97.1%	21.892	6.2756	4.2322	3.4822	3.0974	2.8645	2.5924	2.4409	2.1868	1.8957
97.2%	22.674	6.3965	4.2945	3.5268	3.1336	2.8960	2.6187	2.4647	2.2064	1.9110
97.3%	23.518	6.5244	4.3603	3.5732	3.1711	2.9286	2.6461	2.4892	2.2263	1.9268
97.4%	24.431	6.6590	4.4286	3.6216	3.2101	2.9624	2.6743	2.5147	2.2474	1.9431
97.5%	25.414	6.8024	4.5006	3.6721	3.2507	2.9977	2.7037	2.5410	2.2691	1.9600
97.6%	26.481	6.9538	4.5760	3.7251	3.2934	3.0345	2.7343	2.5686	2.2916	1.9774
97.7%	27.640	7.1139	4.6558	3.7805	3.3379	3.0730	2.7661	2.5972	2.3149	1.9954
97.8%	28.902	7.2850	4.7399	3.8388	3.3848	3.1136	2.7997	2.6272	2.3394	2.0141
97.9%	30.284	7.4690	4.8287	3.9003	3.4342	3.1560	2.8347	2.6583	2.3648	2.0335
98.0%	31.809	7.6656	4.9230	3.9655	3.4863	3.2004	2.8714	2.6908	2.3912	2.0537
98.1%	33.482	7.8775	5.0239	4.0344	3.5411	3.2472	2.9102	2.7251	2.4191	2.0748
98.2%	35.355	8.1065	5.1315	4.1081	3.5996	3.2969	2.9507	2.7612	2.4482	2.0969
98.3%	37.446	8.3553	5.2479	4.1868	3.6613	3.3497	2.9936	2.7993	2.4788	2.1201
98.4%	39.785	8.6267	5.3734	4.2710	3.7274	3.4059	3.0392	2.8396	2.5113	2.1444
98.5%	42.451	8.9249	5.5087	4.3615	3.7983	3.4660	3.0881	2.8826	2.5454	2.1701

TABLE OF EFFECTIVE z FOR $\theta = 0$ BY CDF VALUE BY SAMPLE SIZE (*Continued*)

Size	3	4	5	6	7	8	10	12	20	Infinite
CDF										
98.6%	45.492	9.2531	5.6560	4.4599	3.8742	3.5306	3.1400	2.9282	2.5816	2.1973
98.7%	48.998	9.6177	5.8178	4.5669	3.9573	3.6007	3.1960	2.9771	2.6209	2.2262
98.8%	53.097	10.027	5.9985	4.6837	4.0473	3.6765	3.2571	3.0302	2.6626	2.2571
98.9%	57.907	10.489	6.1984	4.8125	4.1458	3.7592	3.3235	3.0877	2.7077	2.2904
99.0%	63.707	11.019	6.4222	4.9558	4.2560	3.8501	3.3963	3.1507	2.7573	2.3263
99.1%	70.795	11.635	6.6779	5.1181	4.3794	3.9518	3.4770	3.2210	2.8114	2.3656
99.2%	79.707	12.356	6.9732	5.3036	4.5194	4.0677	3.5679	3.2993	2.8716	2.4089
99.3%	91.036	13.230	7.3215	5.5182	4.6799	4.2011	3.6714	3.3882	2.9391	2.4573
99.4%	106.30	14.315	7.7411	5.7747	4.8698	4.3566	3.7915	3.4911	3.0164	2.5121
99.5%	127.54	15.709	8.2600	6.0891	5.1006	4.5425	3.9348	3.6137	3.1085	2.5758
99.6%	159.43	17.595	8.9434	6.4912	5.3913	4.7766	4.1131	3.7633	3.2206	2.6521
99.7%	212.56	20.367	9.8927	7.0398	5.7817	5.0854	4.3460	3.9576	3.3629	2.7478
99.8%	318.87	25.001	11.384	7.8785	6.3601	5.5437	4.6817	4.2374	3.5614	2.8782
99.9%	639.32	35.346	14.466	9.5264	7.4639	6.3924	5.2836	4.7261	3.8973	3.0902
99.91%	710.10	37.358	15.013	9.7723	7.6388	6.5205	5.3910	4.8041	3.9464	3.1214
99.92%	802.48	39.704	15.607	10.091	7.8452	6.6830	5.5026	4.8898	4.0060	3.1560
99.93%	924.73	42.935	16.388	10.453	8.0855	6.8630	5.6142	4.9882	4.0662	3.1947
99.94%	1071.4	46.165	17.200	10.871	8.3560	7.0571	5.7557	5.0962	4.1476	3.2390
99.95%	1308.2	50.132	18.463	11.487	8.6942	7.3285	5.9347	5.2472	4.2324	3.2905
99.96%	1603.7	55.886	19.756	12.104	9.2022	7.6146	6.1392	5.4053	4.3366	3.3528
99.97%	2129.8	64.578	21.719	13.037	9.7135	8.0344	6.4159	5.6194	4.4780	3.4319
99.98%	3195.1	79.327	24.905	14.473	10.582	8.6579	6.8130	5.9433	4.6819	3.5402
99.99%	6476.5	130.35	32.577	17.533	12.262	9.8218	7.5466	6.5133	5.0235	3.7195
99.991%	7155.0	137.10	34.590	17.863	12.547	10.020	7.6674	6.6017	5.0765	3.7462
99.992%	8105.2	143.86	36.603	18.424	13.066	10.219	7.7953	6.7262	5.1366	3.7742
99.993%	9463.0	150.61	38.617	19.038	13.677	10.539	7.9589	6.8534	5.1956	3.8091
99.994%	10820.	157.36	40.630	19.774	14.288	10.861	8.1225	6.9806	5.2681	3.8464
99.995%	12470.	164.12	42.644	20.754	14.899	11.182	8.3533	7.1077	5.3707	3.8906
99.996%	15513.	177.26	44.657	22.133	15.510	11.558	8.6325	7.3289	5.4865	3.9442
99.997%	20567.	204.65	47.133	23.752	16.122	12.166	8.9968	7.5696	5.6387	4.0140
99.998%	30128.	249.52	53.817	26.086	17.333	13.066	9.5083	8.0240	5.8325	4.1071
99.999%	57549.	345.39	67.405	31.115	20.156	14.730	11.495	9.3525	6.1575	4.2655
99.9991%	123124	528.79	89.391	38.011	23.330	16.773	13.385	11.975	6.5736	4.2841
99.9992%	139934	558.35	93.151	39.277	23.970	17.150	13.557	12.113	6.6238	4.3213
99.9993%	152587	606.41	99.113	40.437	24.688	17.528	13.729	12.330	6.6901	4.3400
99.9994%	174499	652.78	104.17	42.105	25.339	17.931	13.901	12.396	6.8068	4.3772
99.9995%	200226	703.32	109.14	43.939	26.214	18.529	14.072	12.651	6.9205	4.4145
99.9996%	237470	771.56	118.18	46.238	27.656	19.410	14.244	12.824	7.0653	4.4703
99.9997%	281976	856.72	128.12	49.570	28.945	20.215	14.416	13.351	7.2843	4.5449
99.9998%	354906	1079.4	145.55	56.462	31.573	21.907	14.588	13.818	7.5316	4.6194
99.9999%	566663	1407.4	176.46	67.100	36.220	24.042	15.525	14.616	8.0696	4.7684
100%	4870750	8260.6	958.00	156.91	121.20	57.244	27.447	19.958	10.705	#NUM!

The last line of the table may seem surprising, as the values should all be infinite, as indicated in the last column. However, in doing simulations it is necessary to have some way of creating very large values. The best way is simply to generate deviates as one needs them. If one is going to use a table such as the above, then a theoretically correct possibility is to create a tail distribution, and simulate off that. A possibility which also works is to have explored the high end in enough detail and to include a value for 100%, in order to interpolate. The values shown here are the largest obtained during the 50,000,000 simulations. Here, the table is reasonably accurate to the one chance in a million level at the high end. If this is not good enough for the problem at hand, then other procedures must be used. This could happen, for example, if many million simulations are to be used, or if results are sensitive to the very high end of the distribution.

APPENDIX B

SCHEDULE P PART 3 HOMEOWNERS-FARMOWNERS PAID DATA FROM BEST'S 1995 AGGREGATES AND AVERAGES

Years in Which Losses Were Incurred	1 12 Months	2 24 Months	3 36 Months	4 48 Months	5 60 Months
1. Prior	0	961,195	1,539,215	1.853,854	2,162,283
2. 1985	7,122,424	9,387,076	9,733,306	9,975,586	10,142,891
3. 1986	6,540,125	8,549,792	8,959,180	9,210,201	9,363,385
4. 1987	6,549,833	9,431,522	9,348,973	9,606,804	9,757,094
5. 1988	7,387,876	9,934,924	10,367,041	10,614,036	10,736,491
6. 1989	9,159,289	12,691,762	13,200,544	13,558,787	13,670,011
7. 1990	9,204,653	12,321,906	12,859,522	13,155,938	13,337,299
8. 1991	10,631,838	13,987,066	14,667,645	15,022,004	
9. 1992	17,421,697	22,112,982	22,871,006		
10. 1993	11,304,871	14,537,267			
11. 1994	13,181,700				

Years in Which Losses Were Incurred	6 72 Months	7 84 Months	8 96 Months	9 108 Months	10 120 Months
1. Prior	2,275,182	2,340,769	2,390,115	2,415,395	2,432,657
2. 1985	10,226,434	10,270,069	10,301,410	10,327,519	10,339,393
3. 1986	9,456,400	9,505,716	9,530,693	9,546,517	
4. 1987	9,858,142	9,914,405	9,943,700		
5. 1988	10,832,847	10,889,518			
6. 1989	13,778,348				
7. 1990					
8. 1991					
9. 1992					
10. 1993					
11. 1994					

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SCHEDULE P PART 3 HOMEOWNERS-FARMOWNERS PAID DATA FROM BEST'S 1995 AGGREGATES AND AVERAGES (*Continued*)

Years in Which Losses Were Incurred	LN (Age-to-Age Factors) 1–2	2–3	3–4	4–5	5–6
2. 1985	0.27608573	0.03621976	0.02458710	0.01663236	0.00820287
3. 1986	0.26795068	0.04677175	0.02763297	0.01649520	0.00988489
4. 1987	0.36461793	−0.0087910	0.02720510	0.01552301	0.01030310
5. 1988	0.29621595	0.04257541	0.02354564	0.01147104	0.00893459
6. 1989	0.32618457	0.03930492	0.02677678	0.00816963	0.00789392
7. 1990	0.29166954	0.04270589	0.02278867	0.01369133	
8. 1991	0.27427996	0.04751100	0.02387201		
9. 1992	0.23844847	0.03370514			
10. 1993	0.25148180				

Years in Which Losses Were Incurred	LN (Age-to-Age Factors) 6–7	7–8	8–9	9–10
2. 1985	0.00425781	0.00304704	0.00253130	0.00114908
3. 1986	0.00520154	0.00262413	0.00165894	
4. 1987	0.00569104	0.00295043		
5. 1988	0.00521777			
6. 1989				
7. 1990				
8. 1991				
9. 1992				
10. 1993				

Taking the last five calendar years, which are shaded in the previous table, the results for the maximum likelihood estimators are:

Period	μ_0	σ_0
1 to 2	0.27641	0.03091
2 to 3	0.04116	0.00456
3 to 4	0.02484	0.00180
4 to 5	0.01307	0.00299
5 to 6	0.00904	0.00093
6 to 7	0.00509	0.00052
7 to 8	0.00287	0.00018
8 to 9	0.00210	0.00044
9 to ultimate	0.00115	0.00100

The sigma estimator for 9 to ultimate is, of course, a guess. In the actual calculation, all estimators were taken to have come from a sample of size five calendar years, whereas the last four really have less than that. In reserving practice, since there is always judgment involved in the tail factor and its standard deviation, it seems a good idea to use only estimators which are from at least five calendar years. At least this way the assumptions are made explicit, rather than hidden in factors whose standard deviation is actually infinite due to parameter variation.

A MARKOV CHAIN MODEL OF SHIFTING RISK PARAMETERS

HOWARD C. MAHLER

Abstract

In this paper, a practical and flexible model involving simple Markov chains is developed that incorporates the phenomenon of shifting risk parameters. One can view this model as a generalization of the gamma-Poisson, beta-binomial, and similar models.

The model is applied to a variety of examples in order to illustrate its possible uses:

- *dice,*
- *a mixture of four Poissons,*
- *California driving data (modeled by a gamma-Poisson), and*
- *baseball data (modeled by a mixture of binomials).*

The model is sufficiently flexible to be applied to other situations.

In each case, the Markov chain model is used to explore the effects of shifting risk parameters over time. A formula is developed and used to calculate covariances. Based on the Markov chain model, when shifting risk parameters over time are significant, the logs of the covariances between years of data are expected to decline linearly as the separation between years increases.

A formula is developed and used to calculate credibilities from the variances and covariances. When shifting risk parameters are significant, older years receive less credibility and as more and more years of data are added, the sum of the credibilities goes to a limit less than one.

1. INTRODUCTION

The phenomenon of shifting risk parameters over time has been explored in past *Proceedings* papers by Venezian [14, 15] and Mahler [7, 9, 10]. It has been shown that this phenomenon can significantly impact the relative value of data for use in predicting the future. Specifically, it can significantly affect the credibility assigned to data to be used for experience rating.

In this paper, a practical model involving simple Markov chains is developed that incorporates the phenomenon of shifting risk parameters. One can view this model as a generalization of the gamma-Poisson, beta-binomial, and similar models. The model is applied to a variety of examples in order to illustrate its possible uses.

Bühlmann credibility[1] is discussed, for example, in Mayerson [11], Hewitt [4, 5], Philbrick [12], and Herzog [3]. Bühlmann derived, under certain assumptions, the linear least squares estimator; a similar derivation is performed for the more general situation in this paper in Appendix C. In order to apply Bühlmann credibility, the Bühlmann credibility parameter is calculated as

$$K = \frac{\text{expected value of process variance}}{\text{variance of hypothetical means}},$$

where the expected value of process variance and the variance of hypothetical means are each calculated for a single observation of the risk process. Then for N observations, the Bühlmann credibility is $Z = N/(N + K)$.

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[1]Bühlmann credibility is also referred to as Bayesian credibility or least squares credibility.

2. SIMPLE EXAMPLE INVOLVING DICE

2.1. *Bühlmann Credibility*

Assume Joe selects N dice of the same type and rolls them. Assume Joe selected either four-sided, six-sided, or eight-sided dice, with a priori probabilities of 25%, 50%, and 25%, respectively. Joe tells you how many dice he rolled and the resulting sum, but you do not know the type of dice Joe selected. Joe will roll the same dice again.

You can use Bühlmann credibility to predict the sum of that next roll. The expected value of the process variance[2] (for one die) is 3.08. The variance of the hypothetical means[3] is .500. Therefore K = expected value of process variance/variance of hypothetical means = 6.16. The credibility assigned to the observation is $Z = N/(N+K) = N/(N+6.16)$. Thus for example, if Joe rolls 3 dice which sum to 14, then $Z = 33\%$ and the credibility estimate of the sum of the next roll of three dice[4] is $(14)(33\%) + (10.5)(67\%) = 11.7$.

The credibility can also be written as:

$$Z = \frac{.5N^2}{.5N^2 + 3.08N} \tag{2.1}$$

$$Z = \frac{\text{variance of hypothetical means for the sum of } N \text{ dice}}{\begin{array}{c}(\text{variance of hypothetical means for the sum of } N \text{ dice} \\ + \text{ expected value of the process variance for} \\ \text{the sum of } N \text{ dice})\end{array}}$$

$$= \frac{\text{variance of hypothetical means for the sum of } N \text{ dice}}{\text{total variance for the sum of } N \text{ dice}},$$

---

[2]The process variances for 4, 6, and 8-sided dice are, respectively, 1.25, 2.92, and 5.25.

[3]The means for 4, 6, and 8-sided dice are, respectively, 2.5, 3.5, and 4.5.

[4]The complement of credibility of $1 - .33 = .67$ is assigned to the overall a priori mean of 3.5 per die times 3 dice.

where we have used the fact that the total variance is the sum of the expected value of process variance and variance of hypothetical means. Also note that the variance of the hypothetical means for the sum of N identical dice is simply N^2 times the variance of hypothetical means for one die since each of the means is multiplied by N. This is a special case of the general result for any random variable Y, $\text{Var}[NY] = N^2 \text{Var}[Y]$. In this case, Y is the hypothetical mean for a single roll of each type of die. In contrast, the expected value of the process variance for the sum of N identical dice is just N times the expected value of process variance for a single die. This is a special case of the general result, $\text{Var}[X_1 + X_2 + \cdots + X_N] = N\,\text{Var}[X]$ for X_i independent and identically distributed.

This simple example has so far been a review of basic[5] Bühlmann credibility. Next we will complicate the risk process by adding shifting risk parameters over time.

2.2. *Dice Example, Shifting Parameters Over Time*

Let's introduce a somewhat different risk process. Joe selects a die and rolls it. Then prior to the next trial, Beth may at random replace that die with another die. Assume Beth's replacement process works such that:

1. A four-sided die will be replaced 20% of the time by a six-sided die.[6]
2. A six-sided die will be replaced 10% of the time by a four-sided die and 15% of the time by an eight-sided die.
3. An eight-sided die will be replaced 30% of the time by a six-sided die.

Then the process repeats: Joe rolls a die and Beth (possibly) replaces the die. Beth's actions will eventually scramble the

[5]This material is currently included on the Part 4B Exam syllabus.

[6]The remaining 80% of the time the die is left alone.

information one could obtain in her absence by summing the results of many trials. However, if one uses the most recent trial's result, it is unlikely that Beth will have affected the situation. Thus more recent trials provide more valuable information for predicting the future. Therefore, more recent trials of data should be given more credibility than less recent trials of data.

This is generally the case when one has shifting risk parameters over time. We will determine how to calculate the credibilities for this example as well as in more general situations applicable to insurance.

2.3. *Markov Chains*

Beth's risk process is a simple example of a Markov chain.[7] See Appendix A for a discussion of Markov chains. There are three "states": 4-sided die, 6-sided die, and 8-sided die. For each trial there is a new, possibly different, state. The probability of being in a state depends only on the state for the previous trial. Beth's Markov chain was completely described by the "transition probabilities" between the states.

Label the states 1, 2, and 3 corresponding to 4-sided, 6-sided, and 8-sided die. Then let P_{21} = the probability of being in state 1 given that the previous trial was in state 2. This is the probability of Beth replacing a 6-sided die with a 4-sided die, or 10%. Thus $P_{21} = 10\%$. Similarly, P_{23} = the chance of Beth replacing a 6-sided die with an 8-sided die = 15%. P_{22} = the chance of Beth leaving a 6-sided die alone = 75%. Note that $P_{21} + P_{22} + P_{23} = 10\% + 75\% + 15\% = 100\%$. The probabilities of all the things Beth can do to a 6-sided die add up to 100%.

Generally, the transition probabilities for a Markov chain are arranged in a matrix $\boldsymbol{P}$. For Beth's "risk process," the matrix of

[7] Feller [2], Resnick [13].

transition probabilities is:

$$\begin{pmatrix} .80 & .20 & 0 \\ .10 & .75 & .15 \\ 0 & .30 & .70 \end{pmatrix}$$

where we have previously discussed the second row. Note we have assumed no chance of Beth's "risk process" replacing an eight-sided die with a four-sided die so that $P_{31} = 0$.

We note that each of the rows of the matrix sums to unity. As discussed previously, this is a general property of transition matrices. In addition, this matrix was chosen to have a special property.

We have assumed Joe's probability of initially picking each of three types of dice is 25%, 50%, and 25%. Thus the initial probability vector is $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{4}$. The Markov chains we will be dealing with will, in the limit, go to a so-called stationary distribution. For the chosen transition probabilities, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{4}$ is that stationary distribution.[8] We expect this initial distribution to, on average, continue over time.

We can see this by thinking of the expected number of each type of die Beth adds or subtracts.

1. There is a $\frac{1}{4}$ chance that Joe picks a 4-sided die. There is a $\frac{1}{4} \times 20\% = 5\%$ chance that Beth adds a 6-sided die and subtracts a 4-sided die.

2. There is a $\frac{1}{2}$ chance that Joe picks a 6-sided die. There is a $10\%/2 = 5\%$ chance that Beth adds a 4-sided die and subtracts a 6-sided die and a $15\%/2 = 7\frac{1}{2}\%$ chance Beth adds an 8-sided die and subtracts a 6-sided die.

[8] The transition probabilities were chosen so that the initial state would be a stationary distribution. See Appendix D for a discussion of how such a transition matrix can be constructed from a given stationary distribution.

3. There is a $\frac{1}{4}$ chance that Joe picks an 8-sided die. There is a $30\%/4 = 7\frac{1}{2}\%$ chance that Beth adds a 6-sided die and subtracts an 8-sided die.

In summary, the change in the probability of a 4-sided die is expected to be $5\% - 5\% = 0$. The change in the probability of a 6-sided die is expected to be $5\% - 5\% + 7\frac{1}{2}\% - 7\frac{1}{2}\% = 0$. The change in the probability of an 8-sided die is expected to be $7\frac{1}{2}\% - 7\frac{1}{2}\% = 0$. Thus we indeed have a stable situation on an expected basis.

Let α be the vector of a priori probabilities. All we have done is verify the matrix equation that $\alpha \boldsymbol{P} = \alpha$. This is the definition of a stationary distribution.

In general, if $\boldsymbol{\beta}$ is a vector of the initial probabilities of being in each state, then the matrix product of $\boldsymbol{\beta}$ and the transition matrix $\boldsymbol{P}$, $\boldsymbol{\beta P}$, is the vector of probabilities after one trial.

One last important point is how we would calculate, for example, the probability, if Joe initially picked a 4-sided die, of Beth after 2 trials replacing this 4-sided die with an 8-sided die. This would be the product of the probabilities of replacing a 4-sided die with a 6-sided die after the first trial and then replacing the 6-sided die with an 8-sided die after the second trial. In this case, that probability is $P_{12}P_{23} = (.20)(.15) = 3\%$.

If Joe initially picked a 4-sided die, what is the probability of having a 4-sided die after two trials? Either Beth did not replace the die at both trials or she replaced the 4-sided die at trial one with a 6-sided die and then at trial two replaced the 6-sided die with a 4-sided die. These probabilities are $P_{11}P_{11} + P_{12}P_{21} = (.80)(.80) + (.20)(.10) = .66$.

Similarly, if Joe initially picked a 4-sided die, the chance of having a 6-sided die after two trials is $P_{11}P_{12} + P_{12}P_{22} = (.80)(.20) + (.20)(.75) = .31$. Note that given that Joe picked a 4-sided die, the probabilities of the three possible situations after two trials add up to unity: .66 + .31 + .03 = 1.00. One can also verify that these are the entries of the first row of $\boldsymbol{P}^2$.

In general, one could easily compute such probabilities by taking matrix products of $\boldsymbol{P}$. $\boldsymbol{P}^2 = \boldsymbol{P} \times \boldsymbol{P}$ contains the transition probabilities for two trials. $\boldsymbol{P}^3 = \boldsymbol{P} \times \boldsymbol{P} \times \boldsymbol{P}$ contains the transition probabilities for three trials, etc. Thus if β is a vector of the initial probabilities of being in each state, then $\beta \boldsymbol{P}^N$ is the vector of probabilities after N trials.

2.4. *Eigenvectors and Eigenvalues*

In order to more easily compute credibilities as well as gain a better understanding of the behavior in specific examples, eigenvectors and eigenvalues are useful. An eigenvector $\boldsymbol{v}_i$ and related eigenvalue λ_i of a matrix $\boldsymbol{M}$ are such that

$$\boldsymbol{M}\boldsymbol{v}_i = \lambda_i \boldsymbol{v}_i.$$

Appendix B contains a brief discussion of eigenvectors and eigenvalues. If the transpose of $\boldsymbol{P}$ has an eigenvector $\boldsymbol{v}_i$, then

$$\boldsymbol{P}^T \boldsymbol{v}_i = \lambda_i \boldsymbol{v}_i \quad \text{or} \quad \boldsymbol{v}_i \boldsymbol{P} = \lambda_i \boldsymbol{v}_i.$$

Recall Beth's transition matrix:

$$\begin{pmatrix} .80 & .20 & 0 \\ .10 & .75 & .15 \\ 0 & .30 & .70 \end{pmatrix}.$$

Its transpose has eigenvalues[9] of 1, .769, and .481. It has corresponding eigenvectors[10] of (1,2,1), (1,−.314,−.686), and (1,−3.186,2.186). The eigenvalue 1 corresponds to the stationary distribution; its corresponding eigenvector (1,2,1) is proportional to the stationary distribution.[11] By the definition of an eigenvector with an eigenvalue of 1: $\boldsymbol{v}_1 \boldsymbol{P} = 1\boldsymbol{v}_1 = \boldsymbol{v}_1$.

---

[9] We have chosen to list the eigenvalues starting with unity for the sake of convenience.
[10] Note, the eigenvectors may each be multiplied by any constant and remain eigenvectors.
[11] Recall that in this example, the stationary distribution was $(\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$.

Let λ be the vector of eigenvalues of P^T. Let V be the matrix of corresponding eigenvectors, with each row being an eigenvector. In this case $\lambda = (1, .769, .481)$, while

$$V = \begin{pmatrix} 1 & 2 & 1 \\ 1 & -.314 & -.686 \\ 1 & -3.186 & 2.186 \end{pmatrix} \quad \text{and}$$

$$V^{-1} = \begin{pmatrix} .250 & .658 & .092 \\ .250 & -.103 & -.147 \\ .250 & -.451 & .201 \end{pmatrix}.$$

The elements of the first column of V^{-1} are all equal, and are the proportionality constant to convert the first eigenvector (the elements of the first row of V) into the stationary distribution α. In our example, the first column of V^{-1} is (.25,.25,.25) where each element is the inverse of the sum of the first eigenvector[12] (1,2,1). The sum of any eigenvector but the first is zero.

We have the following result of multiplying matrices[13]:

$$VPV^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & .769 & 0 \\ 0 & 0 & .481 \end{pmatrix}.$$

So the matrix of eigenvectors of P^T can be used to convert P to a diagonal matrix whose elements are the eigenvalues of P^T. Let this diagonal matrix be Λ.

2.5. *Limits*

$$VPV^{-1} = \Lambda. \tag{2.2}$$

---

[12]We could have just as easily chosen the first eigenvector as $(\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$, in which case since it sums to unity, it is the stationary distribution.

[13]This follows from the matrix equation $VP = \Lambda V$, which taking each row in turn says $v_i P = \lambda_i v_i$.

In general, for any matrix $\boldsymbol{P}$ and any invertible matrix $\boldsymbol{V}$,

$$\boldsymbol{P}^2 = \boldsymbol{V}^{-1}(\boldsymbol{VPV}^{-1})(\boldsymbol{VPV}^{-1})\boldsymbol{V} = \boldsymbol{V}^{-1}(\boldsymbol{VPV}^{-1})^2\boldsymbol{V}.$$

This result extends similarly to higher powers:

$$\boldsymbol{P}^g = \boldsymbol{V}^{-1}(\boldsymbol{VPV}^{-1})^g\boldsymbol{V}.$$

Substituting the particular expression for Λ from Equation 2.2, one obtains

$$\boldsymbol{P}^g = \boldsymbol{V}^{-1}\Lambda^g\boldsymbol{V}. \tag{2.3}$$

So taking powers of the transition matrix corresponds to taking powers of the diagonal matrix Λ. We use the matrix of eigenvectors $\boldsymbol{V}$ to translate back and forth. Λ^g is diagonal with elements λ_i^g. As $g \to \infty$, $\lambda_i^g \to 0$ for $|\lambda_i| < 1$. Since $|\lambda_i| < 1$ for $i > 1$, Λ^g approaches a matrix, all but one of whose elements is zero, and element $(\Lambda^g)_{11} = 1^g = 1$.

As discussed in Appendix A, $\boldsymbol{P}^g \to \boldsymbol{A}$, as $g \to \infty$, where $\boldsymbol{A}$ is a matrix each of whose rows is proportional to the first eigenvector; each row of $\boldsymbol{A}$ is the stationary distribution.

For any initial distribution $\boldsymbol{\beta}$, $\lim_{g\to\infty}\boldsymbol{\beta P}^g = \boldsymbol{\beta A} = \boldsymbol{\alpha}$ since the sum of the elements of $\boldsymbol{\beta}$ is unity and since the rows of $\boldsymbol{A}$ are each the stationary distribution $\boldsymbol{\alpha}$. Thus for any initial distribution, after enough time passes, we approach the stationary distribution:

$$\boldsymbol{\beta P}^g \to \boldsymbol{\alpha}, \qquad \boldsymbol{P}^g \to \boldsymbol{A}, \qquad \text{and} \qquad \Lambda^g \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{2.4}$$

The speed with which this convergence takes place is dependent on $|\lambda_i|$ for $i \neq 1$. The smaller $|\lambda_i|$ for $i \neq 1$, the larger the effect of shifting parameters over time. In the current example, $\lambda_2 = .769$ and $\lambda_3 = .481$, so convergence takes a while.

For $N = 5$,

$$\Lambda^5 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & .269 & 0 \\ 0 & 0 & .026 \end{pmatrix}, \quad \text{and}$$

$$P^5 = V^{-1}\Lambda^5 V = \begin{pmatrix} .429 & .437 & .134 \\ .219 & .521 & .261 \\ .134 & .521 & .344 \end{pmatrix}.$$

For $N = 20$,

$$\Lambda^{20} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & .005 & 0 \\ 0 & 0 & 4 \times 10^{-7} \end{pmatrix}, \quad \text{and}$$

$$P^{20} = V^{-1}\Lambda^{20} V = \begin{pmatrix} .253 & .499 & .248 \\ .249 & .500 & .250 \\ .248 & .501 & .252 \end{pmatrix}.$$

Thus after five trials we expect to have retained a small amount of information about Joe's initial pick. For example, if Joe initially picked a 4-sided die, after five trials there is .429 chance of a 4-sided die, a .437 chance of a 6-sided die, and a .134 chance of an 8-sided die. After 20 trials, for all practical purposes the probabilities are independent of Joe's initial pick. Beth's process has scrambled things sufficiently in order to remove any trace of the initial pick.

We conclude that the outcome of the first trial would provide no useful information for the prediction of the 21st trial. On the other hand, the outcome of the 16th trial would provide some small amount of useful information for the prediction of the 21st trial, being only five trials apart. Thus, we would expect to give the 16th trial some small credibility and the first trial virtually zero credibility when predicting the outcome of the 21st trial.

2.6. *Covariances*

In insurance applications, a year of data takes the place of a trial in the example involving dice. In order to calculate credibilities, we need to calculate the variances as well as the covariances between different years of data. As developed in Appendix E,

let ζ be the vector such that

$$\zeta_i = ((\boldsymbol{\mu} \times \boldsymbol{\alpha})^T \boldsymbol{V}^{-1})_i (\boldsymbol{V}\boldsymbol{\mu})_i. \tag{2.5}$$

Then, for $g > 0$, the covariance of two years of data separated by g years is given by

$$\text{Cov}[X,U] = \sum_{i>1} \zeta_i \lambda_i^g. \tag{2.6}$$

Note that λ_i and ζ_i which determine the behavior of the covariances are each directly and easily calculable[14] from the assumed transition matrix and the means of the states. The steps developed in Appendix E are:

1. Assume[15] a transition matrix $\boldsymbol{P}$ corresponding to the assumed states with means given by the vector $\boldsymbol{\mu}$.
2. Calculate the eigenvalues and eigenvectors of the transpose of the transition matrix $\boldsymbol{P}^T$.
3. Arrange the eigenvalues in descending order with the first one unity; this is the vector $\boldsymbol{\lambda}$.
4. $\boldsymbol{V}$ is the matrix whose rows are the eigenvectors corresponding (in the same order) to the eigenvalues λ_i.
5. The stationary distribution $\boldsymbol{\alpha}$ is proportional to the eigenvector corresponding to the eigenvalue of unity; the elements of $\boldsymbol{\alpha}$ should sum to unity since it is a probability distribution.

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[14] Assuming the calculations will be performed on a computer.

[15] In many of the examples, we will assume a stationary distribution $\boldsymbol{\alpha}$ and then construct a transition matrix $\boldsymbol{P}$ such that $\boldsymbol{\alpha P} = \boldsymbol{\alpha}$, using the method in Appendix D.

6. $(\mu \times \alpha)$ is the vector whose ith element is $\mu_i \alpha_i$.

7. V^{-1} is the matrix inverse of V.

8. ζ is the vector whose ith element is the product of the ith element of the vector $(\mu \times \alpha)^T V^{-1}$ and the ith element of the vector $V\mu$.

9. For X and U separated by g years, $g > 0$:

$$\text{Cov}[X, U] = \sum_{i>1} \zeta_i \lambda_i^g.$$

The vector ζ is defined in Equation 2.5 in terms of μ, α, and V. μ, the vector of means for each state, and α, the distribution of probabilities for the states, are not dependent on the rate of shifting parameters.

V is a matrix whose rows are the eigenvectors of P^T. The eigenvectors of $(P^g)^T = (P^T)^g$ are the same as those of P^T. By raising P to a power, one can alter the rate at which parameters shift over time without changing the eigenvectors. Therefore, since V does not depend on the power to which P is raised, it does not reflect the speed of shifting risk parameters.

Therefore, ζ, which is calculated from μ, α, and V, reflects the "structure" of the Markov chain rather than the rate of shifting risk parameters. In contrast, the eigenvalues λ_i *do* reflect the rate at which risk parameters shift. If P is raised to the power g, so are the eigenvalues.

Thus writing the covariance between two years of data in terms of ζ and λ as in Equation 2.6 isolates the effect of the rate of shifting parameters into λ_i^g.

2.7. *The Variance-Covariance Matrix*

For a single year, the variance of X (the covariance of X with itself) is not affected directly by shifting risk parameters over time.[16] With stationary probabilities of being in the different states, the variance of X can be calculated ignoring shifting risk parameters.

The variance of X is computed in the usual way as the sum of the variance of hypothetical means and the expected value of process variance. As discussed previously in Section 2.1, for Joe rolling a single die, the variance of hypothetical means is .50 and the expected value of process variance is 3.08. Therefore, the total variance of X is 3.58.

For this example, as calculated in Appendix E, the covariances for trials separated by given amounts are:

Separation	Covariance[17]	Covariance ÷ Variance of Hypothetical Means
0	3.5833[18]	
1	.3750	.750
2	.2837	.568
3	.2159	.432
4	.1649	.330
5	.1263	.253
6	.0968	.194
7	.0743	.149
8	.0570	.114
9	.0438	.088
10	.0337	.067
20	.0024	.0048
30	.0002	.0004

In this case for $g > 0$, the covariances are $(.468)(.769^g) + (.032)(.418^g)$. Therefore, the variance of hypothetical means is .5, we expect the covariance ÷ variance of hypothetical means to be approximately $.77^g$.

[16] To the extent the probabilities of being in different states is not stationary, the expected value of the process variance may move over time. That is not the situation here.

[17] Based on exact calculation with no intermediate rounding.

[18] Total variance = sum of the variance of hypothetical means of .50 and the process variance of 3.08.

Note that setting $g = 0$ in the formula for Cov[X,U] gives .468 + .032 = .500, the variance of hypothetical means. In general, one can calculate the variance of X by getting the variance of hypothetical means in this manner and adding the expected value of process variance. The process variance will depend, among other things, on the particular type of process, e.g., binomial, Poisson, negative binomial, rolling a die, spinning a spinner, etc.

In summary, as shown in Appendix E, the variance-covariance matrix between years of data is given by

$$\mathrm{Cov}[X,U] = (.468)(.769^g) + (.032)(.481^g) + 3.08 \quad (\text{if } g = 0).$$

In general, for years of data X_i and X_j:

$$\mathrm{Cov}[X_i,X_j] = \sum_{k>1} \zeta_k \lambda_k^{|i-j|} + \delta_{ij} \quad (\mathrm{EPV}) \tag{2.7}$$

where

$$\mathrm{EPV} = \text{expected value of the process variance}$$

$$\delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}.$$

2.8. *Credibilities*

Assume we have data from years $1,2,\ldots,Y$ and we wish to predict the outcome in year $Y + \Delta$. Then, as shown in Appendix C, the least squares credibilities are given by solving the Y linear equations in Y unknowns:[19]

$$\sum_{j=1}^{Y} \mathrm{Cov}[X_i,X_j]Z_j = \mathrm{Cov}[X_i,X_{Y+\Delta}] \qquad i = 1,2,\ldots,Y. \tag{2.8}$$

[19]The equations are those in Mahler [10].

Given values for the variances and covariances, one can solve for the credibilities to assign to each year by simple matrix techniques.

In our example, assume we use the outcome of one trial to predict the outcome of the next trial. Then we get one equation:

$$\mathrm{Cov}[X_1,X_1]Z_1 = \mathrm{Cov}[X_1,X_2], \qquad \text{and}$$

$$Z_1 = \frac{\mathrm{Cov}[X_1,X_2]}{\mathrm{Var}[X]} = .3750/3.5833 = 10.5\%.$$

Note that this is lower than the credibility for a single trial in the absence of shifting parameters, which is .50/3.58 = 14.0%.[20]

If we use two years of data to predict the subsequent year, then we get two equations in two unknowns:

$$Z_1\,\mathrm{Cov}[X_1,X_1] + Z_2\,\mathrm{Cov}[X_1,X_2] = \mathrm{Cov}[X_1,X_3], \qquad \text{and}$$

$$Z_1\,\mathrm{Cov}[X_1,X_2] + Z_2\,\mathrm{Cov}[X_2,X_2] = \mathrm{Cov}[X_2,X_3].$$

For this example,

$$3.5833Z_1 + .3750Z_2 = .2837, \qquad \text{and}$$

$$.3750Z_1 + 3.5833Z_2 = .3750.$$

The solution is

$$\begin{pmatrix} 3.5833 & .3750 \\ .3750 & 3.5833 \end{pmatrix}^{-1} \begin{pmatrix} .2837 \\ .3750 \end{pmatrix} = \begin{pmatrix} .069 \\ .097 \end{pmatrix}.$$

We would give 9.7% credibility to the most recent year of data, 6.9% to the second most recent year, and the complement of credibility, 83.4%, to the overall a priori mean of 3.5.

[20]The ratio of credibilities is .75, the ratio of the covariance (with shifting parameters) to the variance of hypothetical means.

Similarly for three years of data, the equations are:

$$3.5833Z_1 + .3750Z_2 + .2837Z_3 = .2159,$$

$$.3750Z_1 + 3.5833Z_2 + .3750Z_3 = .2837, \qquad \text{and}$$

$$.2837Z_1 + .3750Z_2 + 3.5833Z_3 = .3750,$$

which has the solution $Z_1 = 4.6\%$, $Z_2 = 6.4\%$, $Z_3 = 9.4\%$.

If, instead of using years 1 through 3 to predict year 4, we were using them to predict year 5, the right hand sides of the equations would be instead .1649, .2159, and .2837. This would instead result in a solution $Z_1 = 3.5\%$, $Z_2 = 4.9\%$, $Z_3 = 7.1\%$. The additional year of delay in the availability of data has resulted in lower credibilities.[21]

2.9. *Varying the Rate at which Parameters Shift*

One can easily modify this example to either slow down or speed up the rate at which parameters shift over time. For example, the transition probabilities could be revised so it is one-fifth as likely for Beth to switch the type of die after each trial. Such a revised transition matrix

$$\begin{pmatrix} .96 & .04 & 0 \\ .02 & .95 & .03 \\ 0 & .06 & .94 \end{pmatrix}$$

has the same stationary distribution .25, .5, .25, but the parameters shift about one-fifth as fast.

One can speed up the rate at which parameters shift by raising the transition matrix to a power. For example, squaring the given transition matrix yields a new transition matrix in which the parameters shift exactly twice as fast.[22]

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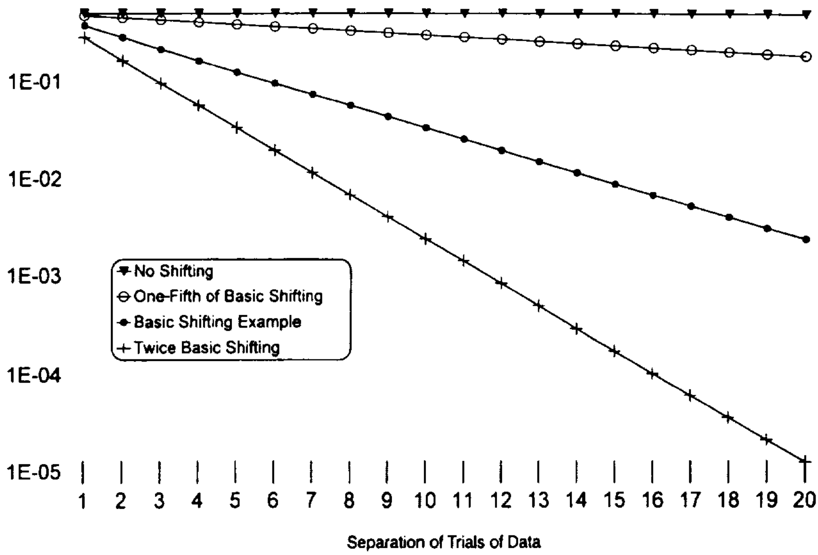
[21] Note the Equations 2.8 are sufficiently general to accommodate gaps between the years of data as well as a gap between the last year of data and the year being predicted.

[22] If $\alpha P = \alpha$, we have $\alpha P^2 = (\alpha P)P = \alpha P = \alpha$. Therefore, if α is a stationary distribution for P, it is a stationary distribution for P^2 (or P^3, or P^4, etc.).

FIGURE 1

COVARIANCES BETWEEN DATA SEPARATED BY GIVEN NUMBER OF TRIALS, EXAMPLES WITH DICE, SHIFTING PARAMETERS OVER TIME



Y-Axis on a Log Scale. One die rolled per trial.

The Markov chain that corresponds to the transition matrix $\boldsymbol{P}^2$ results in covariances of data X_i' that follow from those for data X_i from the Markov chain corresponding to $\boldsymbol{P}$:

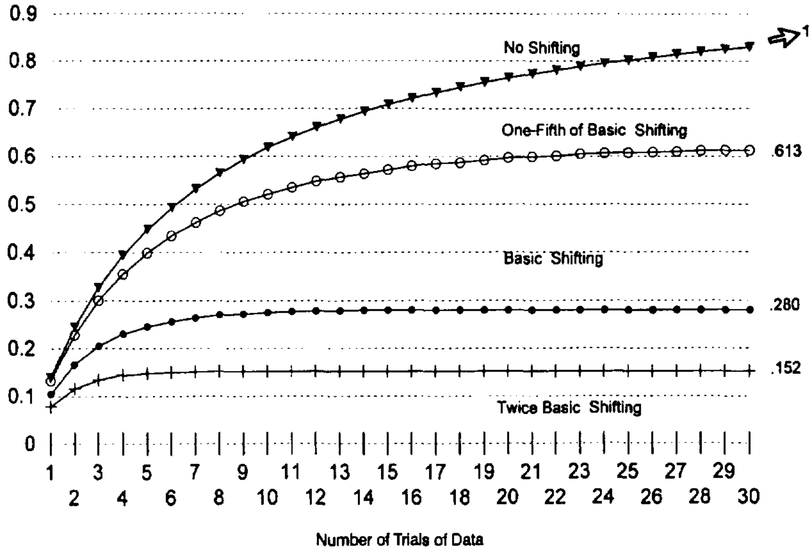
$$\mathrm{Cov}[X_1', X_{1+g}'] = \mathrm{Cov}[X_1, X_{1+2g}].$$

The covariance for a separation of ten years with transition matrix $\boldsymbol{P}^2$ is the same as that for a separation of 20 years with transition matrix $\boldsymbol{P}$.

Figure 1 compares the covariance structure for the basic example to one with no shifting, one-fifth the amount of shifting, and twice the shifting. The vertical axis is on a logarithmic scale. As expected on this logarithmic scale, the covariances decline ap-

FIGURE 2

SUM OF CREDIBILITIES, EXAMPLES WITH DICE, SHIFTING PARAMETERS OVER TIME, VARYING RATES



No gap between data and trial to be estimated. One die rolled per trial.

proximately linearly, with the slope of the decline approximately proportional to the amount of shifting. In the absence of shifting risk parameters over time, the covariances do not decline, rather they are the same regardless of the number of years of separation.[23]

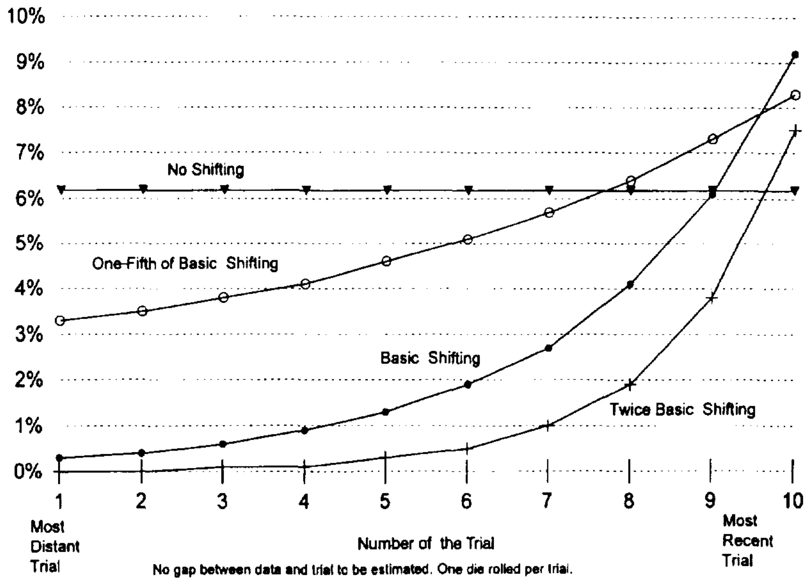
Figure 2 compares the sum of the credibilities one would assign to individual years of data for different amounts of shifting. In the case of no shifting, the sum of the credibilities approaches unity as the number of years approaches infinity.[24] The greater

[23]In many practical applications, the decline will be so small over the time periods of interest that it makes sense to ignore the decline. Thus while the case of no decline *forever* is not realistic, it is a very good approximation for many practical applications.

[24]For this example with no shifting and Y years of data, each year is assigned $1/(Y + 6.1666)$ credibility. The sum is $Y/(Y + 6.1666)$.

FIGURE 3

CREDIBILITIES ASSIGNED TO EACH OF TEN TRIALS OF DATA EXAMPLES WITH DICE, VARIOUS RATES OF SHIFTING PARAMETERS OVER TIME



the shifting, the smaller the sums of the credibilities. The sum of credibilities approaches a value less than unity as the number of years of data increases. The greater the shifting, the lower the limit and the faster it is reached.

Figure 3 compares the credibilities that would be assigned to individual years of data when using ten years of data. In the absence of shifting, each year is assigned equal credibility.[25] The greater the shifting, the greater the difference in credibilities as-

[25]For $Y = 10$, each year is assigned credibility of $1/(10 + 6.1666) = 6.2\%$.

signed to the different years of data. When risk parameters shift rapidly over time, the value of recent information is greater relative to older information.

We note that the most recent year of data is assigned more credibility for the basic example than it is in the absence of shifting. This reflects the fact that in the former situation the *relative* value of the most recent year's data is large compared to the data available from other years. When using ten years of data in the absence of shifting, the total value of the available information is higher, as is the value of the most recent year. However, the value of the most recent year's data relative to all the information available is lower without shifting than with shifting. In contrast, as was shown previously, when using only one year of data, the credibility is lower in the presence of shifting.

Finally, Figure 4 compares the effects of delays in gathering the data. For the basic example, we see how the credibilities decrease as the delay increases. When risk parameters are changing quickly over time, the effect of any delay in collecting data can be very substantial.

2.10. *Size of Risk and Shifting Risk Parameters*

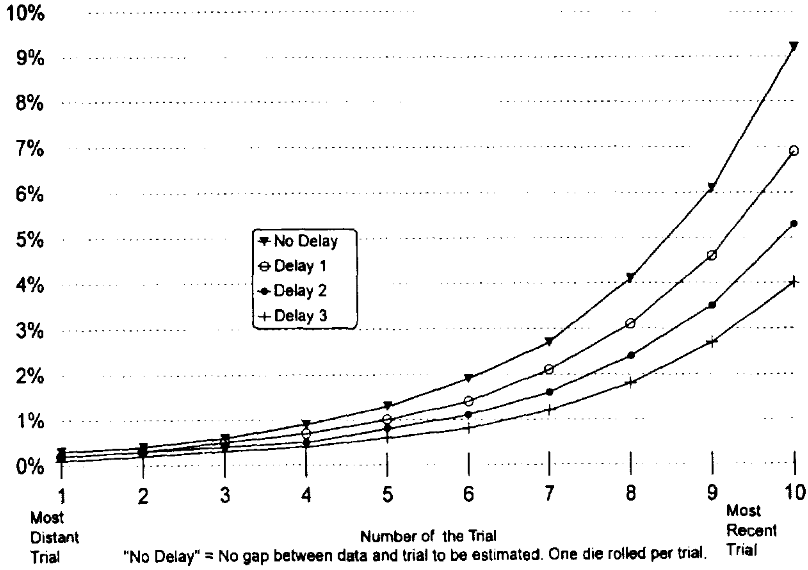
Assume that Joe selects N dice (of the same kind) and rolls them. The resulting sum is the result of one trial or year. After each trial, Beth (possibly) changes the type of dice with transition matrix $\boldsymbol{P}$. (We assume Beth either changes the type of all N dice or leaves them all alone.)

Since we are just adding the results of rolling N identical dice in each year, the covariance between two separate years is given by N^2 times what it was for the case with a single die:

$$\mathrm{Cov}[X_1, X_{1+g}] = N^2 \sum_{i>1} \zeta_i \lambda_i^g.$$

FIGURE 4

CREDIBILITIES ASSIGNED TO EACH OF TEN TRIALS OF DATA, EXAMPLES WITH DICE, EFFECTS OF VARIOUS DELAYS, SHIFTING PARAMETERS OVER TIME



The variance for a year is given by N (expected value of process variance for a single die) + N^2 (variance of hypothetical means for a single die).

As before, given Y years of data, we can solve Y equations in Y unknowns[26] for the credibilities assigned to each year. As in the case of standard Bühlmann credibility, since the expected value of process variance increases with N rather than N^2, as N increases, so does the credibility.

[26]The same Equations 2.8 apply, but the actual values of the variance and covariances depend on N, the number of dice.

FIGURE 5

SUM OF CREDIBILITIES, SHIFTING PARAMETERS OVER TIME, VARIOUS NUMBERS OF DICE PER TRIAL

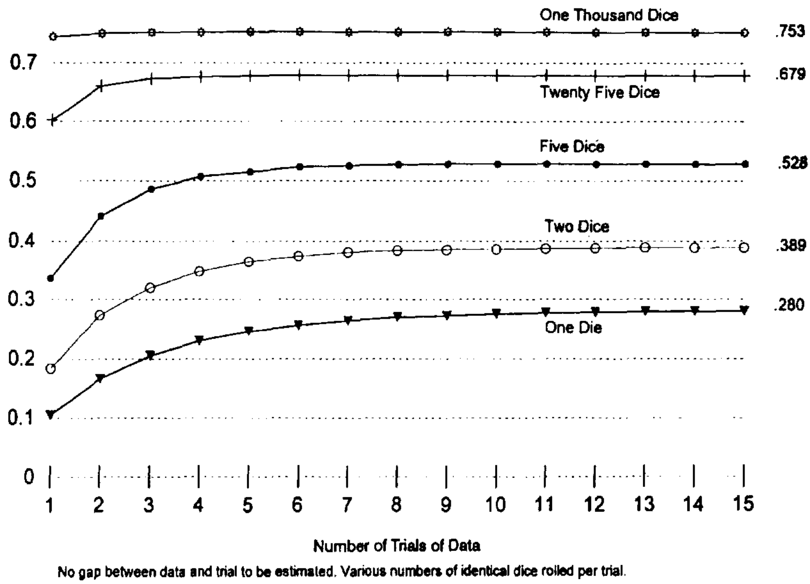


Figure 5 compares the sum of credibilities assigned to Y years for different numbers of dice, N, for the transition matrix discussed previously:

$$\begin{pmatrix} .80 & .20 & 0 \\ .10 & .75 & .15 \\ 0 & .30 & .70 \end{pmatrix}.$$

As expected, the more dice used per roll, the higher the credibility. Also, the more dice, the quicker the limit is approached as the number of years of data increases. For a fixed amount of shifting, for larger risks, the more recent years are relatively more valuable compared to older years than is the case for smaller risks. For larger risks, the random noise in the observation of a single year is less, so one can rely on fewer years of information.

As the number of dice approaches infinity, one relies almost solely on the most recent year of data. In this case, the sum of credibilities approaches 75.6%, with about 74% credibility being assigned to the most recent year.

In general, as the number of dice approaches infinity, the sum of the credibility approaches a number less than unity in the presence of shifting risk parameters. Having Joe roll more dice per trial does not get rid of the effect of Beth (possibly) shifting all the dice between trials. Increasing the size of the risk will not eliminate the uncertainty caused by shifting risk parameters over time.[27]

3. SIMPLE POISSON EXAMPLE

3.1. Bühlmann Credibility

To take a simplified insurance example, assume that for individual insureds the claim frequency in each year is given by a Poisson distribution.[28] Assume that there are four types of insureds with different frequencies:

Type of Insured	A Priori Probability	Mean Frequency
Excellent	40%	.25
Good	30%	.50
Bad	20%	.75
Ugly	10%	1.00

Then the overall mean is .50. The variance of hypothetical means is 1/16. The expected value of process variance is the

[27]Note in the model used here, the rate of shifting was assumed to be independent of the size of risk. This was a simplifying assumption which may or may not be a reasonable approximation to a particular real world application.

[28]For parameter θ, $f(n) = e^{-\theta}\theta^n/n!$. The mean and variance of the Poisson are each equal to θ.

expected value of hypothetical means[29] which is the overall mean of .50.

The Bühlmann credibility parameter is $K = .50/(\frac{1}{16}) = 8$. Therefore, the credibility of Y years of data from an individual insured is:[30]

$$Z = \frac{Y}{Y+8}. \tag{3.1}$$

For example, one year of data would be assigned a credibility of about 11%. Note that the total variance is $.5 + \frac{1}{16} = .5625$. The credibility of a single year is the variance of hypothetical means divided by the total variance $= .0625/.5625 = \frac{1}{9}$.

3.2. *Shifting Risk Parameters, Simple Poisson Example*

Assume that in the previous example, the individual insured has a chance of shifting states each year. For example, an excellent insured might have an 18% chance of switching to a good insured the following year,[31] and an 82% chance of remaining an excellent insured. Assume the following transition matrix for illustrative purposes:

$$\begin{pmatrix} .820 & .180 & 0 & 0 \\ .240 & .592 & .168 & 0 \\ 0 & .252 & .608 & .140 \\ 0 & 0 & .280 & .720 \end{pmatrix}.$$

This transition matrix has the selected initial distribution (.4,.3,.2,.1) as a stationary distribution.[32]

---

[29]Since for the Poisson, the mean is equal to the variance.

[30]We assume that we do not know what type of risk the individual is and that the complement of credibility is to be assigned to the overall mean.

[31]Note that we are referring to presumed changes in the unobservable expected claim frequency rather than observed changes in the actual number of claims from year to year.

[32]It can be easily verified that $(.4,.3,.2,.1)\boldsymbol{P} = (.4,.3,.2,.1)$. See Appendix D for how this transition matrix was constructed.

As with the dice example, one can compute the variance-covariance matrix and thus the credibilities.

The expected value of process variance for a single year in this example is .50. Note that this depends on the fact that for each insured for each year we have assumed a Poisson process.

The transpose of the transition matrix has eigenvalues of

$$\lambda = (1, .855, .580, .305).$$

The eigenvectors are the rows of:

$$V = \begin{pmatrix} 1 & .75 & .5 & .25 \\ 1 & .1456 & -.5623 & -.5833 \\ 1 & -1 & -.6667 & .6667 \\ 1 & -2.1456 & 1.729 & -.5833 \end{pmatrix} \quad \text{and}$$

$$V^{-1} = \begin{pmatrix} .4 & .3309 & .2 & .0691 \\ .4 & .0643 & -.2667 & -.1976 \\ .4 & -.3722 & -.2666 & .2388 \\ .4 & -.7722 & .5333 & -.1612 \end{pmatrix}$$

$$\begin{aligned}
\mu \quad &= (.25,\ .50,\ .75,\ 1.00) = \text{the assumed means} \\
\alpha \quad &= (.40,\ .30,\ .20,\ .10) = \text{the stationary distribution} \\
(\mu \times \alpha) \quad &= (.10,\ .15,\ .15,\ .10) \\
(\mu \times \alpha)V^{-1} \quad &= (.2,\ -.0903,\ -.0067,\ -.0030) \\
V\mu \quad &= (1.25,\ -.6822,\ -.0833,\ -.1094)
\end{aligned}$$

ζ has as its ith element the product of the ith element of the above two vectors, as shown in Equation 2.5; therefore,

$$\zeta = (.25, .0616, .0006, .0003).$$

Therefore, for $g > 0$, the covariance of two different years is given by Equation 2.6:

$$\text{Cov}[X_1, X_{1+g}] = \sum_{i>1} \zeta_i \lambda_i^g$$

$$\text{Cov}[X_1, X_{1+g}] = (.0616)(.855^g) + (.0006)(.580^g)$$

$$+ (.0003)(.305^g) \qquad \text{for} \quad g > 0. \qquad (3.2)$$

For a single year, we set $g = 0$ and add the expected value of process variance:[33]

$$\text{Var}(X) = .0625 + .5 = .5625. \qquad (3.3)$$

One can use this variance-covariance structure in the Equations 2.8 for the credibilities. For example, if using three years of data X_1, X_2, X_3 to estimate the next year, X_4, then the three equations in three unknowns are:[34]

$$.5625Z_1 + .0531Z_2 + .0453Z_3 = .0386,$$

$$.0531Z_1 + .5625Z_2 + .0531Z_3 = .0453, \qquad \text{and}$$

$$.0453Z_1 + .0531Z_2 + .5625Z_3 = .0531.$$

The solution is $Z_1 = 5.6\%$, $Z_2 = 6.7\%$, and $Z_3 = 8.4\%$. Table 1 displays the solutions for various numbers of years of data.

Figure 6 shows the sum of the credibilities both in the presence of shifting risk parameters and in the absence of shifting risk parameters.[35] Also shown are credibilities corresponding to twice the original rate of shifting[36] and to five times the original rate of shifting.[37] As was seen before, the presence of shifting risk parameters lowers the credibilities. The more rapid the shifting, the greater the effect on the credibilities.

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[33]This matches the result prior to considering shifting parameters over time, as it should.

[34]$\text{Cov}[X_1,X_2] = .0531$. $\text{Cov}[X_1,X_3] = .0453$. $\text{Cov}[X_1,X_4] = .0386$. $\text{Var}[X] = .5625$.

[35]As was seen in the previous section, in the absence of shifting risk parameters, $1/(Y + 8)$ credibility is assigned to each of Y years for a total of $Y/(Y + 8)$.

[36]Based on using the square of the original transition matrix.

[37]Based on using the fifth power of the original transition matrix.

TABLE 1

CREDIBILITY
SIMPLE POISSON EXAMPLE WITH SHIFTING RISK PARAMETERS
(No Delay in Receiving Data)

Years Between Data and Estimate	Number of Years of Data Used					
	1	2	3	4	5	10
1 (Most Recent)	9.4%	8.8%	8.4%	8.1%	8.0%	7.8%
2		7.2%	6.7%	6.4%	6.3%	6.0%
3			5.6%	5.2%	5.0%	4.7%
4				4.3%	4.0%	3.7%
5					3.3%	2.9%
6						2.2%
7						1.8%
8						1.4%
8						1.1%
10						0.9%
Total Credibility	9.4%	16.0%	20.7%	24.0%	26.6%	32.5%

With shifting risk parameters, as the number of years of data approaches infinity, the sum of the credibilities approaches a limit less than unity. For faster shifting, this limit is lower and it is approached more rapidly.

Since the first term in the covariance in Equation 3.2 dominates, the variance-covariance structure in Equations 3.2 and 3.3 can be approximated by:

$$\text{Cov}[X_i, X_j] = (.0625)(.85^{|i-j|}) + .5\delta_{ij} \tag{3.4}$$

where $\delta_{ij} = 0$ for $i \neq j$ and 1 for $i = j$.

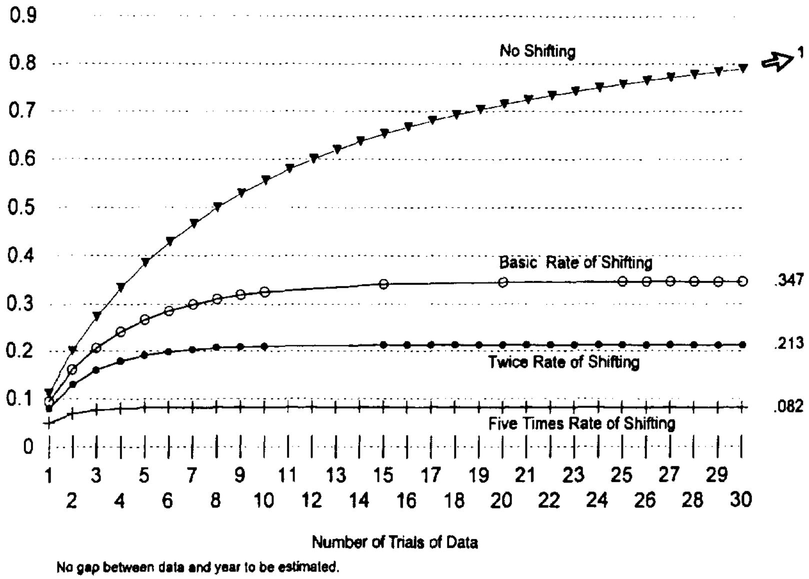
In general,

$$\text{Cov}[X_i, X_j] = \tau^2 \lambda^{|i-j|} + \delta_{ij}\eta^2 \tag{3.5}$$

where η^2 is the expected value of process variance, τ^2 is the variance of hypothetical means and λ is the dominant eigen-

FIGURE 6

SUM OF CREDIBILITIES, SIMPLE POISSON EXAMPLE, VARIOUS RATES OF SHIFTING PARAMETERS OVER TIME



value (other than unity) of the transpose of the transition matrix of the Markov chain.

For one year of data, predicting year $1 + \Delta$, the credibility is obtained by solving Equation 2.8:

$$Z(\tau^2 + \eta^2) = \tau^2 \lambda^\Delta \qquad Z = \lambda^\Delta/(1 + K) \tag{3.6}$$

where $K = \eta^2/\tau^2$ = Bühlmann credibility parameter.

As shown in Mahler [9], when one has years 1 to Y predicting year $Y + \Delta$, the sum of the credibilities is approximately:[38]

[38] As discussed on pages 162–164 of Mahler [9], this approximation underestimates the credibilities. However, here we have also approximated the covariances, therefore the approximation can go in either direction.

$$\sum_{i=1}^{Y} Z_i \approx \frac{\lambda^{\Delta}\left(\sum_{i=1}^{Y}\lambda^{i-1}\right)}{\sum_{i=1}^{Y}\lambda^{i-1} + K}. \tag{3.7}$$

In the absence of shifting risk parameters, $\lambda = 1$ and the sum of the credibilities given by Equation 3.7 becomes the familiar $Y/(Y+K)$.

In the current example, $\lambda = .855$, $\eta^2 = .5$, $\tau^2 = .0625$, and $K = \eta^2/\tau^2 = 8$. Thus Equation 3.7 becomes for $\Delta = 1$

$$\sum_{i=1}^{Y} Z_i \approx \frac{(.855)\left(\sum_{i=1}^{Y}(.855)^{i-1}\right)}{\left(\sum_{i=1}^{Y}(.855)^{i-1}\right) + 8}. \tag{3.8}$$

For $Y = 3$, Equation 3.8 gives $\sum Z_i \approx 20.9\%$. As seen above, the exact solution gives $Z_1 + Z_2 + Z_3 = 5.6\% + 6.7\% + 8.4\% = 20.7\%$, which happens to be somewhat lower in this case.

As the number of years of data increases in Equation 3.7, the approximate sum of the credibilities approaches:

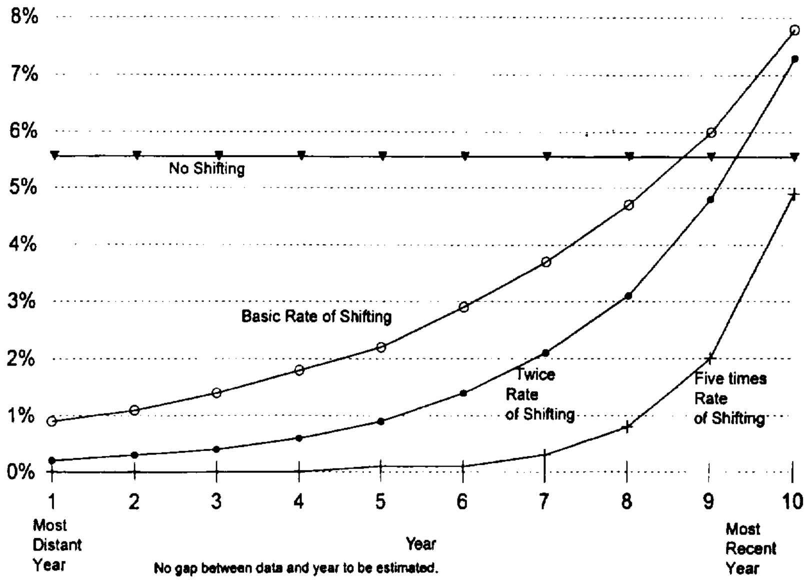
$$\frac{\lambda^{\Delta}\left(\frac{1}{1-\lambda}\right)}{\left(\frac{1}{1-\lambda}\right) + K} = \frac{\lambda^{\Delta}}{1 + K(1-\lambda)}.$$

In this example, for $\Delta = 1$, $\lambda = .855$ and $K = 8$, the sum of the credibilities approaches approximately 39.6%. As seen in Figure 6, the sum of the credibilities actually approaches 34.7%. Thus while this approximation is conceptually useful, one should be cautious in using it for precise numerical results.

Figure 7 displays the credibilities that would be assigned to each of ten years of data. In the absence of shifting risk parameters, each year of data is assigned equal credibility. With shifting

FIGURE 7

CREDIBILITIES ASSIGNED TO EACH OF TEN YEARS OF DATA, SIMPLE POISSON EXAMPLE, VARIOUS RATES OF SHIFTING PARAMETERS OVER TIME



risk parameters, more recent years of data are given more weight than older years of data. The faster the shifting, the less weight is given to the older years of data.

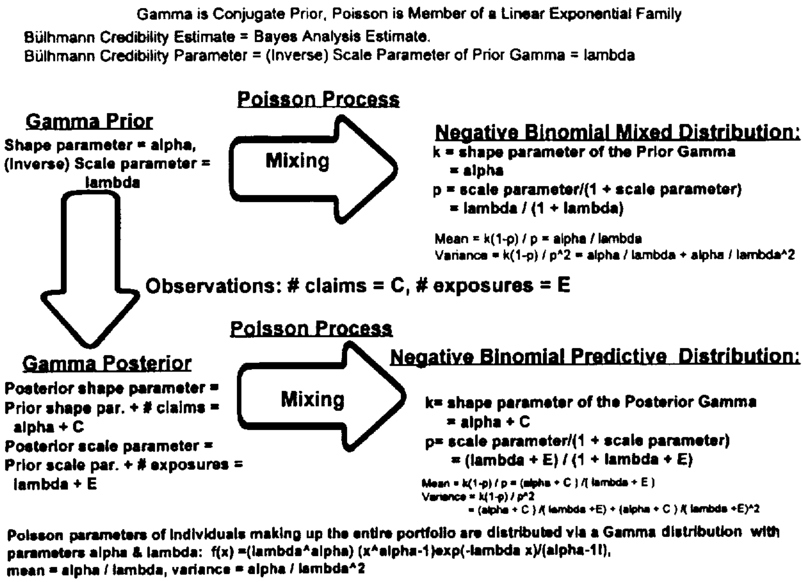
4. CALIFORNIA DRIVING DATA

Mahler [7] examined California driving data. Two sets of data were examined: male and female drivers. The latter set showed more significant evidence of shifting parameters over time. The Markov chain model will be used to model the data for female drivers.[39]

[39]The techniques could be applied in a similar manner to the male drivers.

FIGURE 8

GAMMA-POISSON FREQUENCY PROCESS



For 23,872 female drivers over a period of nine years, there were 7,988 accidents, for an annual accident frequency of .0372. The average variance of a year of data was .0386.

4.1. *Gamma-Poisson*

Such data can be commonly fit with a "gamma-Poisson" in which each insured's frequency is a Poisson process and the Poisson parameters vary over the portfolio via a gamma distribution.[40] Key features of the gamma-Poisson are displayed in Figure 8. The frequency distribution for the portfolio is negative

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[40] See for example, Mayerson [11], Dropkin [1], Herzog [3], and Hossack, Pollard, and Zehnwirth [6].

TABLE 2

NUMBER OF DRIVERS WITH VARYING NUMBERS OF CLAIMS OVER NINE YEARS

Number of Claims	California Female Drivers	Maximum Likelihood Negative Binomial*	Markov Chain Simulation
0	17,649	17,654	17,695
1	4,829	4,822	4,852
2	1,106	1,101	1,029
3	229	235	239
4	44	48	49
5	9	10	6
6	4	2	2
7	1	0	0
8	1	0	0
9+	0	0	0
	23,872	23,872	23,872

*Negative binomial distribution with parameters $p = .8164$, $k = 1.4876$. The California data has a total of 7,988 accidents while the simulated data has a total of 7,865.

binomial. As shown in Table 2, a negative binomial is a reasonable fit to this data.[41] The overall mean is the mean of the gamma distribution. The total variance minus the mean is the variance of the gamma distribution. Thus one can use the method of moments to determine the parameters of the gamma distribution; for this data, the mean of the gamma would be .0372 and the variance of the gamma would be $.0386 - .0372 = .0014$.

For a gamma distribution with shape parameter α and (inverse) scale parameter λ, this would lead to two equations:

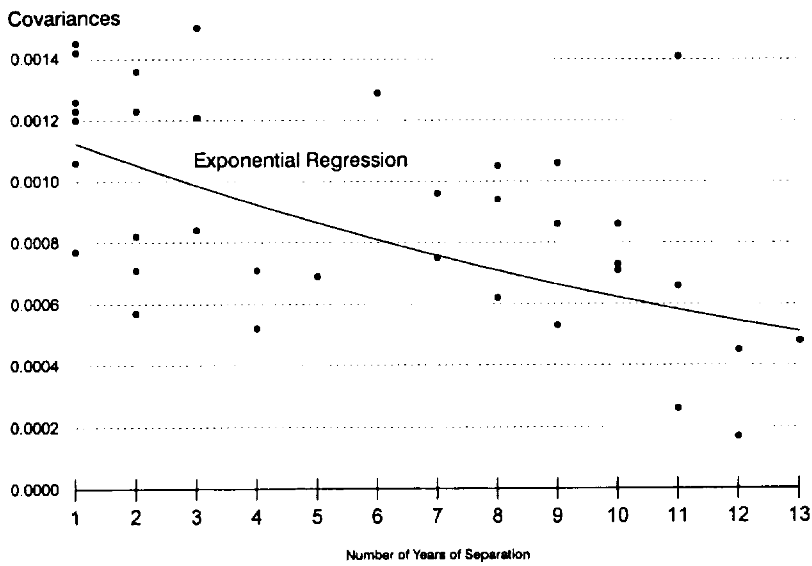
$$\alpha/\lambda = 0.372, \qquad \text{and}$$

$$\alpha/\lambda^2 = .0014.$$

[41] As discussed in reviews of Dropkin [1], this does not imply that the gamma-Poisson model is appropriate.

FIGURE 9

COVARIANCES VERSUS YEARS OF SEPARATION
CALIFORNIA FEMALE DRIVER DATA



This would give values of $\lambda = 26.6$ and $\alpha = .988$. If the shape parameter $\alpha = 1$, one would get an exponential distribution. As a first approximation, assume the frequencies are given by an exponential density function with parameter 26.9: $f(\theta) = 26.9e^{-26.9\theta}$ with mean $1/26.9 = .03717$.

4.2. *Shifting Parameters*

As stated above, the data for California female drivers shows evidence of shifting risk parameters over time. The covariances between years of data with given separations is shown in Figure 9. The covariances appear to decrease for larger separations. The observed covariances were fit to an exponential regression: $\text{Cov}[X_i, X_{i+g}] = .00120e^{-.066g}$, where g is the years of separation.

This is the same general type of behavior one would expect from a Markov chain model of shifting parameters over time. In order for a Markov chain model to fit the observed covariances, the variance of hypothetical means should be about .0012, since setting $g = 0$ in the Markov model gives the variance of hypothetical means.[42] The factor in the exponent, $-.066$, should approximate the log of the dominant eigenvalue (other than unity) of the transition matrix, since this is the approximate rate of decline of the log of the covariances in the model. Thus, in order to match the observed decline, the dominant eigenvalue(s) (other than unity) must be about $e^{-.066} \approx .94$.

4.3. Markov Chain Model

In order to apply the Markov chain model, one has to convert the assumed continuous distribution of frequency parameters into a discrete approximation. For example, take mean frequencies of:

$$\theta_i = .0025, .0075, .0125, \ldots, .3975 \qquad i = 1, 2, \ldots, 80.$$

Take the (initial) probabilities of being in each of these 80 states as α_i proportional to $e^{-26.9\theta_i}$, such that the sum of the α_i is unity.[43] Then, as shown in Appendix D, one can construct an (80×80) transition matrix that has these α as a stationary distribution. For illustrative purposes, assume about $\frac{2}{3}$ chance of shifting up or down a state per year.[44] For this transition matrix,

---

[42]Given the random fluctuation in the data, this estimate of .0012 is not inconsistent with the previous estimate of .0014.

[43]This is a discrete approximation to the selected exponential distribution. The technique will work exactly the same for a gamma distribution with a shape parameter other than unity (which is an exponential).

[44]In order to match the observed covariance structure, this transition matrix will be taken to an appropriate power.

the first ten elements of ζ and λ are:[45]

i	ζ_i	λ_i
1	.00139	1
2	.00059	.9980
3	.00042	.9964
4	.00019	.9939
5	.00008	.9903
6	.00004	.9857
7	.00002	.9801
8	.00001	.9735
9	.00001	.9659
10	.00000	.9574

with all the remaining elements of $\zeta < .0001$. Since only the first few terms contribute significantly to the sum that calculates the model covariances,

$$\text{Cov}[X_1, X_{1+g}] = \sum_{i>1} \zeta_i \lambda_i^g \approx .0013(.997^g) \qquad g > 0.$$

As discussed previously, in order to approximate the observed covariance structure for California female drivers, one would want a decline in the log covariances of about $-.066g$. The above transition matrix has a decline of the log covariances of about $-.003g$. Raising the above transition matrix to the 20th power[46] will multiply the decline in log covariances by about a factor of 20, producing a decline of about $-.060g$, and so should roughly approximate the observed decline.

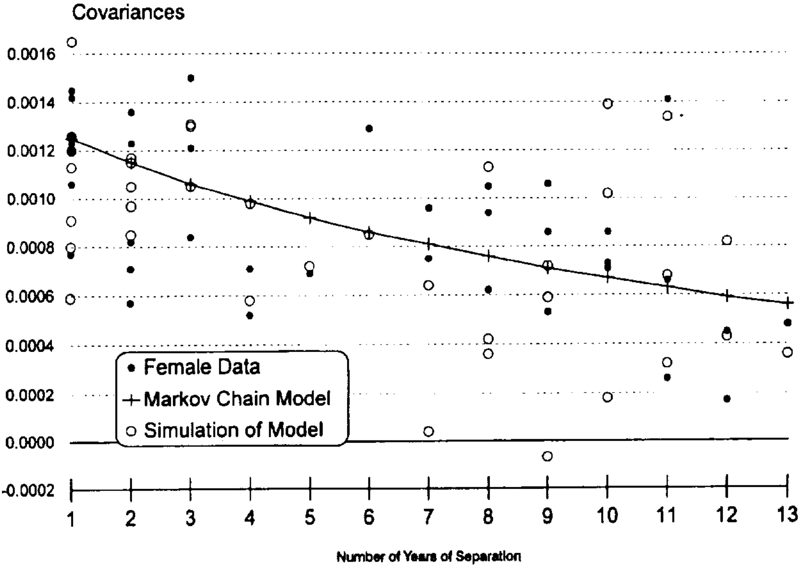
The model covariances for such a transition matrix are shown in Figure 10. The model covariances are a reasonable fit to the

[45]See the previous discussion and Equation 2.5 for the definition of ζ. λ is the set of eigenvalues of the transpose of the transition matrix.

[46]Taking the transition matrix to the 20th power gives a matrix whose eigenvalues are all taken to the 20th power. Thus the logs of all the eigenvalues are multiplied by 20. Since the log covariances decline approximately proportionally to the log of the dominant eigenvalue (other than one), they will decline about 20 times as fast for the new transition matrix as for the original.

FIGURE 10

COVARIANCES VERSUS YEARS OF SEPARATION, FEMALE DRIVER DATA VS. SIMULATED DATA, SHIFTING PARAMETERS OVER TIME



observed data.[47] While the observations extend out to a 13 year separation, one can calculate the model covariances for any number of years of separation.[48]

4.4. *Simulation*

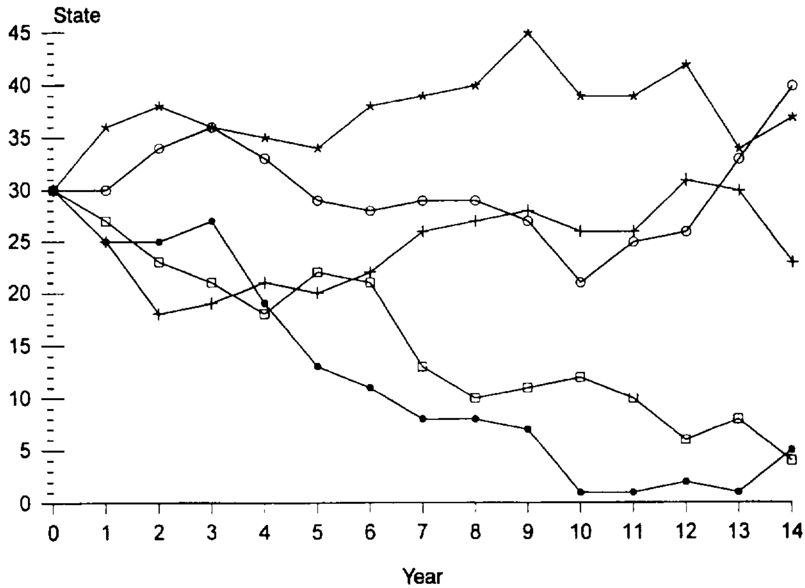
A simulation of this Markov chain model was performed. The first step is to simulate the movement of the Poisson parameters

[47] The limited amount of data would allow other models to fit reasonably well. The observed fit indicates that the form of the proposed model might be useful. It falls well short of demonstrating that it is superior to some other form of model. However, it is clearly superior to a static model without shifting risk parameters.

[48] Beyond 13 years, the model covariances follow a power curve, declining slowly towards zero.

FIGURE 11

MARKOV CHAIN
ILLUSTRATIVE EXAMPLES OF MOVEMENT FOR FIVE RISKS



from year to year, using the probabilities in the transition matrix.[49] Figure 11 shows the result for five risks, each of which started out in State 30, in Year 0. Over the course of 14 years, these risks randomly moved up and down from state to state, with corresponding changes in their assumed expected claim frequency.[50]

The initial configuration of 23,872 drivers by state in Year 0 was chosen to match the selected probability distribution. Then

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[49]The selected 80 × 80 transition matrix was the constructed transition matrix to the 20th power. The constructed transition matrix had an average chance of shifting of about $\frac{2}{3}$ per year and had the selected discrete exponential distribution as a stationary distribution.
[50]State 30 corresponds to a Poisson frequency of .1475.

each of the risks moved randomly each year from state to state via the Markov chain. The five risks shown in Figure 11 for illustrative purposes ended up in vastly different states at the end of the 14 year period. They each started with the same assumed annual Poisson frequency of 14.75% in Year 0. Over the course of the 14 year period, they had Poisson parameters ranging from .25% to 22.25%.

There were 61 risks initially in State 30. Over the course of time, their expected claim frequency declined towards the average of 3.7% for the portfolio:

Risks in State 30 in Year 0

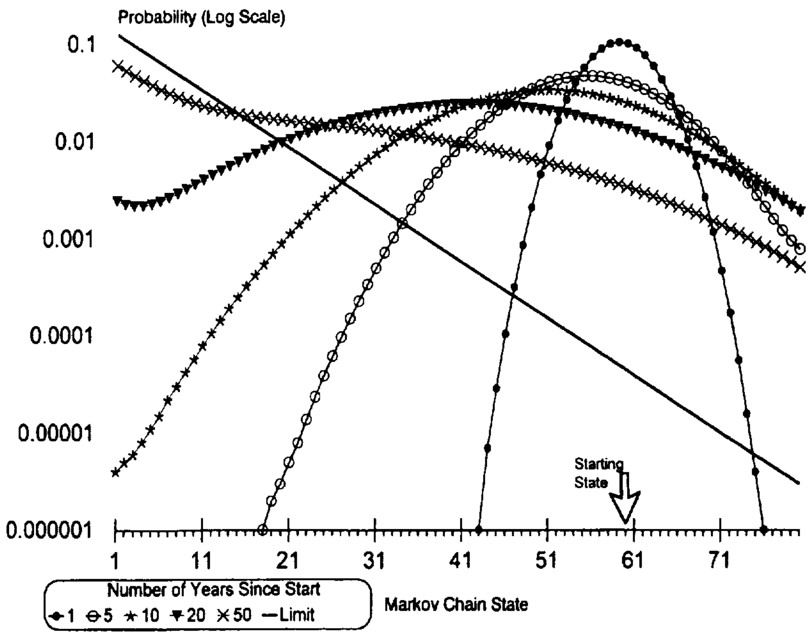
Year	Average Poisson Parameter
0	14.75
1	13.87
2	13.29
3	13.27
4	12.89
5	12.84
6	12.24
7	11.98
8	11.66
9	11.33
10	11.76
11	10.39
12	9.84
13	9.25
14	9.15

After 14 years, the average frequency for these risks moved reasonably towards the overall average.[51] Given enough time, the

[51] The speed at which this occurred was dependent on the particular speed of the shifting parameters over time selected for this example.

FIGURE 12

HAVING STARTED IN STATE 60, CHANCE OF BEING IN A CERTAIN STATE MARKOV CHAIN MODEL OF CALIFORNIA FEMALE DRIVERS

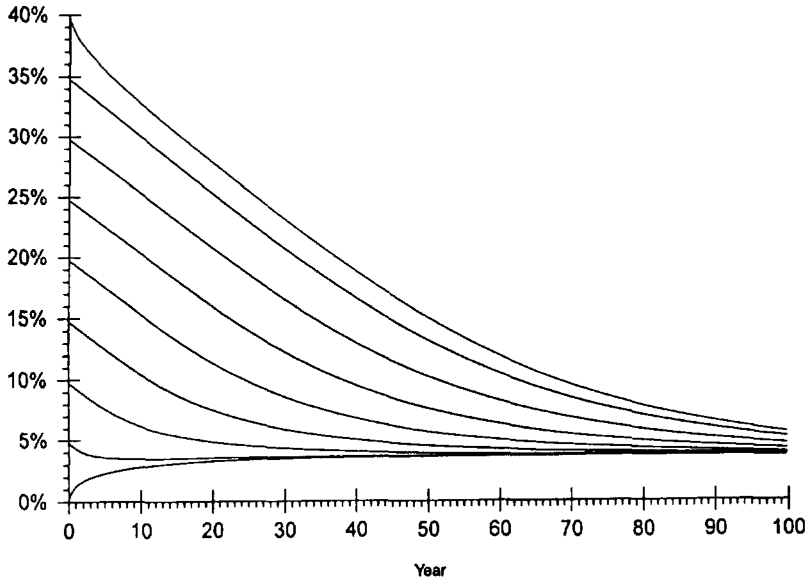


average frequency would have become virtually indistinguishable from the overall average. Figure 12 shows how the distribution evolves over time for risks that start in State 60, (with an initial expected claim frequency of 29.75%). Over time, the distribution approaches the assumed stationary exponential distribution.

This illustrates a general feature of Markov chains: initial information fades over time. For each risk, we have *not* modeled a very long term expected risk propensity that is different than

FIGURE 13

AVERAGE CLAIM FREQUENCY OVER TIME OF DRIVERS STARTING IN VARIOUS STATES
MARKOV CHAIN MODEL OF CALIFORNIA FEMALE DRIVERS



average. Rather, the very long term expected risk propensity is the same for each risk. This is again illustrated in Figure 13, which shows how the expected claim frequency approaches the overall average of 3.7% regardless of which Markov chain state the insured started in. In some applications, this may prevent the model from being useful.

Looking at all 23,782 risks, the simulation resulted in a generally similar mix of Poisson parameters each year. So while individual risks' Poisson parameters changed, the portfolio as a whole was approximately "stationary" over time. For example, the mean frequencies and variances of the portfolio for this simulation were:

Year	Mean Poisson Parameter	Variance of Poisson Parameters	Number of Risks in State 30
1	.03724	.001384	72
2	.03715	.001390	72
3	.03697	.001393	63
4	.03701	.001388	69
5	.03707	.001386	62
6	.03706	.001386	64
7	.03691	.001376	58
8	.03712	.001391	60
9	.03703	.001371	65
10	.03707	.001366	51
11	.03684	.001361	47
12	.03687	.001356	51
13	.03684	.001360	53
14	.03689	.001364	47

Also shown for illustrative purposes is the number of risks in State 30 in each (simulated) year. It fluctuates considerably around its expected value of 61. When looked at in this level of detail, the simulation of Poisson parameters results in some differences in the portfolio composition from year to year. In this case, the states are only .5% apart in annual claim frequency, so exactly how many risks are in any single state is of no practical importance, as well as being unobservable in the real world.

The covariances between the Poisson parameters for the simulation decrease approximately in the manner expected by the model (see Table 3). Thus the simulation of the Poisson parameters in this example does not introduce much random fluctuation into the covariances between years.[52]

[52]With a different number of drivers or different transition matrix, the result could differ.

TABLE 3

COVARIANCES (.00001)

Years of Separation	Model	Simulated Poisson Parameters
1	125	126,126,127,126,126,125,126,125,124,123,123,123,123,124
2	115	116,116,116,116,115,116,115,115,114,113,113,113
3	106	107,107,107,107,106,106,106,105,105,105
4	99	100,100,99,100,99,99,98,98,98,98
5	92	93,92,93,92,92,92,92,92,91
6	86	86,87,86,87,86,86,86,86
7	81	81,81,81,81,81,81,81
8	76	75,76,76,76,76
9	71	71,71,71,71,72
10	67	67,67,67,67
11	63	63,63,63
12	59	60,60
13	56	56

Unfortunately, the second step of the simulation does introduce considerable fluctuation into this example. Once one has a set of Poisson parameters (one for each driver), one can simulate the number of accidents that each driver had in a year. In the particular example, since the annual accident frequencies are so low, there is a lot of noise relative to the information. Any one simulation of a year of accident data does not provide much information. In particular, the covariances between simulated years of accident data are subject to considerable random fluctuation.

For example, for two years of Poisson parameters[53] with a covariance of .00086 between the two years, the covariances between years for seven simulated sets of accident data were:

.00115, .00091, .00113, .00040, .00121, .00118, and .00050.

[53]Of 23,872 drivers distributed as per the model of female drivers in California.

This large amount of random fluctuation implies that one should not draw very precise conclusions from the limited available data.

Figure 10 compares the observed covariances for the California female driving data and those for a set of data simulated using the Markov chain model. Within the context of the large amount of random fluctuation, the actual and simulated data sets look generally similar.

Table 2 compares the numbers of insureds with various numbers of accidents over nine years. The simulated data seems to have a somewhat lighter tail than the observed data, although the overall fit is not unreasonable.[54] One could revise the particular inputs used here to attempt to get a somewhat heavier tail. One could increase the variance of hypothetical means[55] and/or have relatively less shifting over time for high frequency drivers.[56] However, these details are beyond the scope of this paper.

One should note that adding shifting risk parameters in the manner done here reduces the probability of an extremely large number of accidents for an insured over an extended period, since the Poisson parameter for an insured tends towards the overall average over time. The most likely insureds to have extremely large numbers of accidents are those whose Poisson parameters are high for all the observed years.

Overall, the Markov chain model presented here does a reasonable job of fitting the female driver data from California. On the other hand, due to the limited amount of data, one should be cautious in drawing any definitive conclusions. There are un-

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[54]The negative binomial fit to the data also seems to have a slightly light tail compared to the data, indicating that perhaps a gamma-Poisson model might be improved upon.

[55]For the gamma distribution, the variance of hypothetical means is the overall mean divided by the shape parameter of the gamma. Thus for a fixed overall mean, the smaller the shape parameter, the larger the variance of hypothetical means.

[56]The particular transition matrix (which was raised to the 20th power) assumed approximately $\frac{2}{3}$ chance of shifting per year regardless of the state. One could have had the amount of shifting depend on the accident frequency.

TABLE 4

CREDIBILITY
FEMALE CALIFORNIA ACCIDENT DATA
MARKOV CHAIN MODEL
(No Delay in Receiving Data)

Years Between Data and Estimate	Number of Years of Data Used					
	1	2	3	4	5	10
1 (Most Recent)	3.2%	3.1%	3.1%	3.0%	3.0%	2.8%
2		2.9%	2.8%	2.7%	2.7%	2.5%
3			2.6%	2.5%	2.4%	2.3%
4				2.3%	2.2%	2.1%
5					2.1%	1.9%
6						1.7%
7						1.6%
8						1.5%
8						1.4%
10						1.3%
Total Credibility	3.2%	6.0%	8.5%	10.5%	12.4%	19.1%

doubtedly refinements that would allow a somewhat better fit to the observed data.

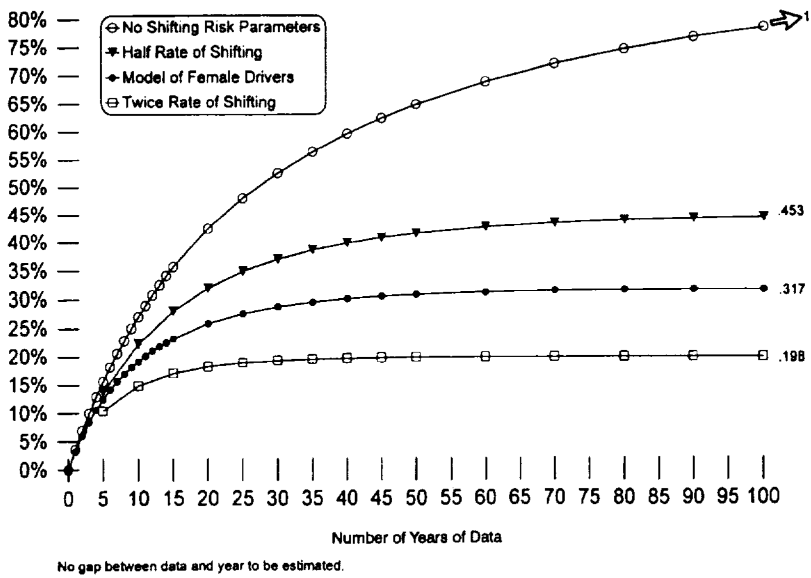
4.5. *Credibilities*

The covariance-variance structure for the Markov chain model fit to the data for the female drivers from California can be used together with Equations 2.8 to solve for the credibilities of different numbers of years of data. These credibilities have the same pattern as in Mahler [7] although the magnitudes are different. The latter appears to be due to a mistake in Mahler [7] in computing the credibilities.[57] In any case, note that the current method has the advantage that it does not require the dividing of the

[57] Unfortunately, it appears that a mistake was made in Mahler [7] in adopting the work in Mahler [10]. The step in Mahler [10] of dividing the variance into three pieces: within

FIGURE 14

SUM OF CREDIBILITIES
MARKOV CHAIN MODEL OF CALIFORNIA FEMALE DRIVER DATA



covariances (or variances) into separate pieces, some of which must be inferred rather than observed. The current method relies on the observable total variances and covariances.[58]

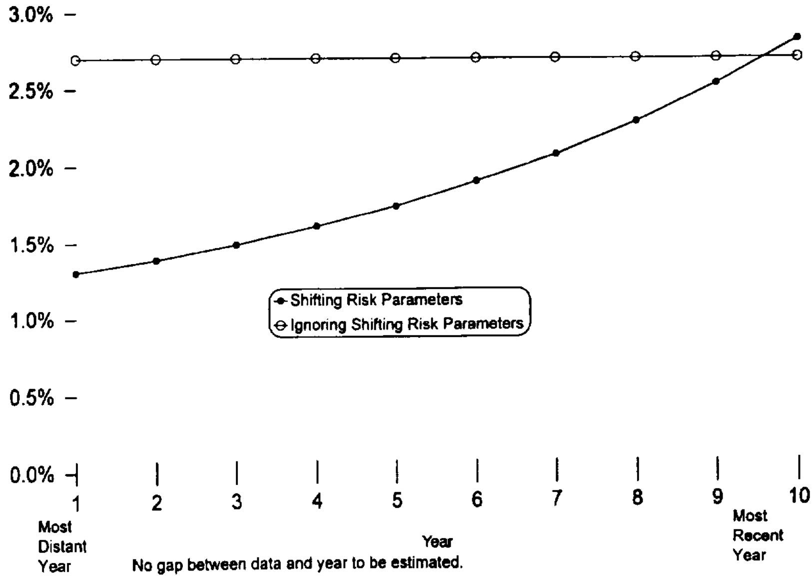
Table 4 displays the credibilities assigned to individual years as well as the sum of the credibilities. Figure 14 compares the sum of the credibilities for the Markov chain model to those

variance, between variance, and the variance due to shifting parameters over time, was not performed in Mahler [7]. This led to an inappropriate total covariance between years being used in the equations for credibility; these covariances were too big by an amount equal to the between variance.

[58]This difference from Mahler [10] is, to a large extent, a matter of presentation and emphasis. (See for example, *PCAS* LXXVII 1990, p. 297.)

FIGURE 15

CREDIBILITIES ASSIGNED TO EACH OF TEN YEARS OF DATA
MARKOV CHAIN MODEL OF CALIFORNIA FEMALE DRIVER DATA



that would result from ignoring shifting risk parameters.[59] With shifting risk parameters, the credibilities are lower.[60] As the number of years approaches infinity, the sum of the credibilities approaches 31.7% rather than 100%.[61] Also shown are credibil-

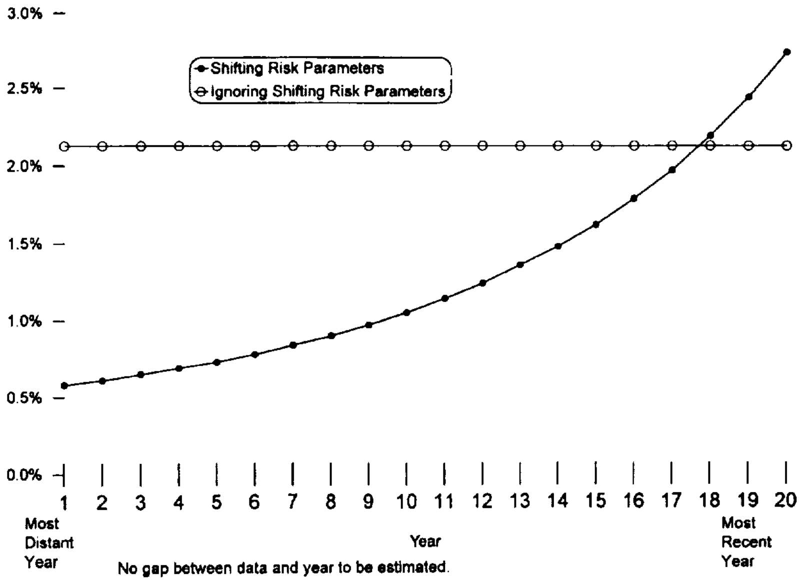
[59]In the absence of shifting risk parameters over time, one has the gamma-Poisson situation summarized in Exhibit 9. The credibility assigned to each of Y years is $1/(Y+\lambda)$ where λ is the scale parameter of the gamma distribution. In the present example λ was taken equal to 26.9. However, the discrete approximation in Section 4.1 produces an expected value of process variance of .037222, and variance of hypothetical mean frequencies of .0013765. Their ratio is a credibility parameter of 27.04. Therefore, in the absence of shifting risk parameters, each of Y years of data would be given a credibility of $1/(Y+27.04)$ for a sum of credibilities of $Y/(Y+27.04)$.

[60]The effect of shifting risk parameters in this case starts to have a significant impact after 10 or 15 years.

[61]While the model can be run for more than 50 years of data, it is unclear what the connection to reality is in this case.

FIGURE 16

CREDIBILITIES ASSIGNED TO EACH OF TWENTY YEARS OF DATA
MARKOV CHAIN MODEL OF CALIFORNIA FEMALE DRIVER DATA



ities for half this rate of shifting as well as twice this rate of shifting.

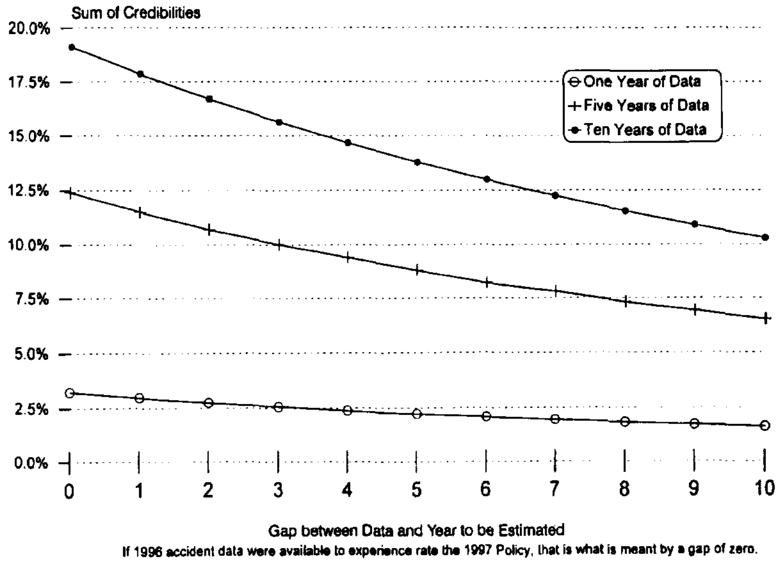
Figure 15 displays the individual credibilities for ten years of data. Figure 16 is similar, but for 20 years of data. In each case, the credibilities assigned to older years of data are significantly lower than those for more recent years of data. While the total credibility is less than in the absence of shifting risk parameters, the most recent year actually receives more credibility.[62]

Figure 17 displays the effect of delays in receiving data. Even in this situation with a relatively slow shifting of risk parameters, the effects of delays are noticeable.

[62]This is the same pattern as was displayed in the simple Poisson example.

FIGURE 17

EFFECTS OF DELAYS IN COLLECTING DATA ON SUM OF CREDIBILITIES, MARKOV CHAIN MODEL OF CALIFORNIA FEMALE DRIVER DATA



5. BASEBALL DATA

Mahler [10] examines the won-lost records of baseball teams. The Markov chain model developed here can be fit to this data.

There are two data sets, American League (AL) and National League (NL), each covering a 60 year period. As in Mahler [10], we will assume for simplicity 150 games per team per year, and convert the losing percentages to numbers of games lost. Table 5 displays the covariances between years of data separated by different amounts.[63] It is evident that the covariances decline as

[63]The separate observations of covariances were averaged. For example, there are 59 pairs of years separated by one year. There is considerable random fluctuation. For example, the covariances for the 59 pairs of years separated by one year for the AL data average to 138.7 with a standard deviation of 78.7.

TABLE 5

COVARIANCES VERSUS YEARS OF SEPARATION, BASEBALL DATA*

Number of Years Separation	American League	National League	Number of Years Separation	American League	National League
0	213.6	205.2	20	45.8	33.2
1	138.7	139.3	21	33.4	26.9
2	109.8	106.2	22	27.4	19.1
3	92.8	99.4	23	14.1	19.9
4	77.7	86.2	24	3.2	15.7
5	55.0	70.5	25	−2.7	4.7
6	45.3	65.2	26	4.0	1.2
7	33.5	53.1	27	3.6	−12.9
8	23.5	40.7	28	0.4	−18.4
9	12.1	30.7	29	−5.4	−11.4
10	15.4	23.8	30	3.4	−3.7
11	12.1	20.9	31	5.5	−6.8
12	9.9	25.2	32	9.4	−3.1
13	18.4	34.7	33	9.7	−8.9
14	17.6	37.1	34	28.3	−12.4
15	26.0	42.9	35	37.7	−2.3
16	36.1	52.2	36	32.6	−8.6
17	34.5	57.4	37	40.8	−16.6
18	42.9	47.2	38	53.4	−16.4
19	43.5	40.6	39	33.2	−7.9
			40	21.4	−33.2

*Covariances between number of games lost per team, based on observed losing percentage and assuming 150 games per team per year.

TABLE 6

CORRELATIONS VERSUS YEARS OF SEPARATION, BASEBALL DATA

Number of Years Separation	American League	National League	Number of Years Separation	American League	National League
0	1.000	1.000	20	0.225	0.136
1	0.633	0.651	21	0.159	0.090
2	0.513	0.498	22	0.125	0.065
3	0.438	0.448	23	0.093	0.055
4	0.360	0.386	24	0.048	0.004
5	0.265	0.312	25	0.006	−0.024
6	0.228	0.269	26	0.010	−0.028
7	0.157	0.221	27	−0.002	−0.095
8	0.124	0.190	28	−0.013	−0.128
9	0.078	0.135	29	−0.032	−0.107
10	0.090	0.100	30	0.006	−0.062
11	0.058	0.083	31	−0.019	−0.061
12	0.063	0.103	32	0.027	−0.028
13	0.101	0.154	33	0.002	−0.015
14	0.104	0.176	34	0.088	0.017
15	0.141	0.180	35	0.143	0.038
16	0.178	0.246	36	0.156	−0.014
17	0.166	0.278	37	0.214	−0.024
18	0.198	0.219	38	0.238	−0.012
19	0.219	0.176	39	0.138	−0.017
			40	0.093	−0.095

the years get further apart. As discussed in Mahler [10], these data display a relatively large impact of shifting risk parameters over time. Table 6 shows the similar pattern for the correlations.

Fitting an exponential regression to the covariances for separations of one to ten years, one obtains:

$$\text{NL}: \quad \text{Cov}[X_1, X_{1+g}] = \exp(5.156 - .185g),$$

$$\text{AL}: \quad \text{Cov}[X_1, X_{1+g}] = \exp(5.317 - .272g).$$

5.1. Markov Chain Model

To fit a Markov chain model to this data, one would want the log of the covariances to decline at a slope of about .23.

The first step in modeling the baseball data is to assume for simplicity that each team's number of games lost in a year is approximately binomial with parameters p and 150. The mean number of games lost, $p150$, will be assumed to have the following discrete distribution:[64]

Expected Number of Games Lost (μ)										
50	55	60	65	70	75	80	85	90	95	100
Probability (α)										
4%	6%	10%	11%	12%	14%	12%	11%	10%	6%	4%

Then using the technique of Appendix D, one can construct an 11×11 transition matrix that has the above α as a stationary distribution:[65]

[64]This simple distribution was chosen for illustrative purposes and is intended to approximate the observed spread of results. The chosen distribution has the desired mean of 75 and together with a binomial risk process would produce a total variance of about 207 compared to the observed total variance of about 209.

[65]The particular matrix was constructed in order to have about a $\frac{1}{2}$ chance of shifting either up or down one state per year.

$$\begin{pmatrix}
.7000 & .3000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
.2000 & .4875 & .3125 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & .1875 & .5506 & .2619 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & .2381 & .5010 & .2609 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & .2391 & .4916 & .2692 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & .2308 & .5385 & .2308 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & .2692 & .4916 & .2391 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & .2609 & .5010 & .2381 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & .2619 & .5506 & .1875 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .3125 & .4875 & .2000 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .3000 & .7000
\end{pmatrix}$$

The eigenvalues and ζ vector are:

i	ζ_i	λ_i	i	ζ_i	λ_i
1	5625	1	7	0	.4292
2	169.6	.9670	8	.004	.2846
3	0	.9034	9	0	.1881
4	1.36	.8119	10	.007	.0966
5	0	.7154	11	0	.0330
6	.069	.5708			

$$\mathrm{Cov}[X_1, X_{1+g}] = \sum_{i>1} \zeta_i \lambda_i^g \approx 170(.967^g).$$

Therefore, the log of the covariances would decline at a slope of about .033. To match the baseball data, we desire a decline at a slope of about .23 or about 6 or 7 times as much. Therefore, this transition matrix raised to the 6th power should roughly match the baseball data.[66]

[66]This will correspond to about a 1 in 2,000 chance of a team moving up or down 6 states (±.2 in expected losing percentage) in a single year.

FIGURE 18

COVARIANCES VERSUS YEARS OF SEPARATION, BASEBALL DATA VS. MARKOV SHIFTING PARAMETERS OVER TIME

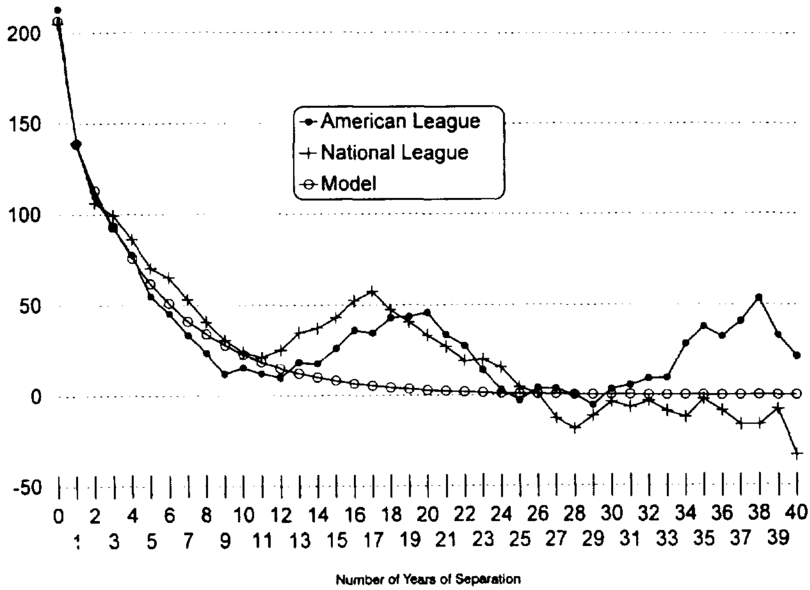


Figure 18 compares the covariances observed for the baseball data and those for the Markov chain model. There is an overall reasonable fit. There are higher covariances than would be predicted by the model for separations of about 15 to 23 years. This may be due to some long term cycle in the data, but in any case is beyond the scope of this paper.[67]

[67] Some factors which remained relatively stable over this 60 year period of time might lead to a tendency for an individual team's expected losing percentage to revert to a long term average different than the overall average of .5. The Markov chain model does not capture any such behavior. Rather, it assumes that given sufficiently long time periods, the average for each risk will be the same. Yet in Section 4.1 of Mahler [10], it is demonstrated that over the 60 year data period, the teams are significantly different. Thus while, as will be shown below, the estimated credibilities are reasonable, the Markov chain model is far from a complete description of the risk process that produced this baseball data.

TABLE 7

CREDIBILITY
BASEBALL DATA MARKOV CHAIN MODEL
(No Delay in Receiving Data)

Years Between Data and Estimate	Number of Years of Data Used					
	1	2	3	4	5	10
1 (Most Recent)	67.0%	55.1%	54.3%	54.2%	54.2%	54.2%
2		17.7%	15.0%	14.8%	14.8%	14.8%
3			4.9%	4.2%	4.1%	4.1%
4				1.4%	1.2%	1.2%
5					.4%	.3%
6						.1%
7						0
8						0
8						0
10						0
Total Credibility	67.0%	72.8%	74.2%	74.6%	74.7%	74.7%

5.2. *Credibilities*

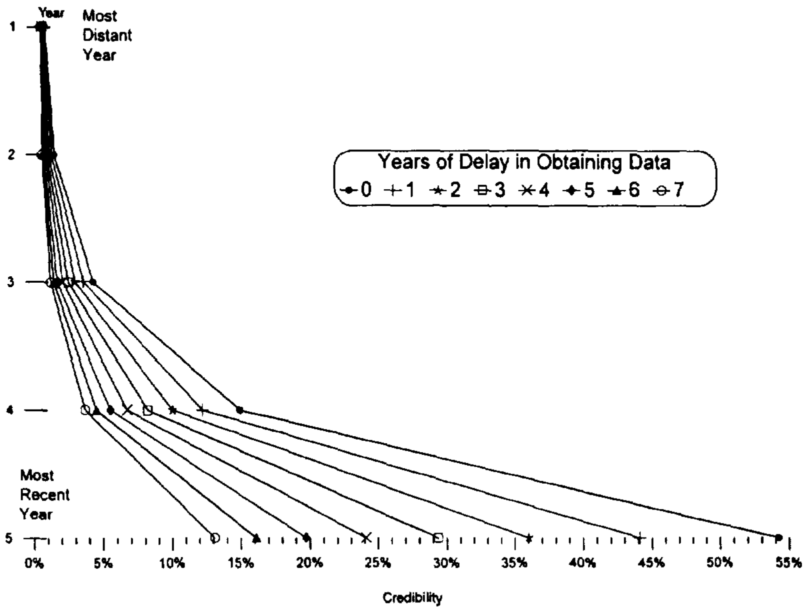
The covariances calculated in the Markov chain model can be used to calculate the credibilities to be assigned to individual years of data. Table 7 displays these credibilities, assuming no delay in receiving the information.

The sum of the credibilities[68] quickly reaches a limit of 74.7% as the number of years of data is increased. Due to the quickly shifting risk parameters over time, the amount of credibility assigned to more distant years of data is small. The credibilities in Table 6 are generally similar to those in Table 16 of Mahler [10]. However, due to the structure imposed by the Markov chain model, the credibilities in Table 6 have a more reasonable pattern when looked at in detail. The credibilities are all between 0 and

[68]The complement of credibility is given to the grand mean.

FIGURE 19

CREDIBILITIES ASSIGNED TO EACH OF FIVE YEARS OF DATA
MARKOV CHAIN MODEL OF BASEBALL DATA



1. They decrease for years more distant in time. The credibility assigned to any individual year of data declines as more years are added. The sum of the credibilities increases smoothly as years of data are added.

Figure 19 displays the credibilities assigned to five separate years of data for various delays in obtaining information. Due to the quickly changing risk parameters, the effect of any delay in obtaining data is significant. As the delay increases, the credibility assigned to any individual year decreases. The smooth pattern shown in Figure 19 demonstrates the effect of the structure imposed by the Markov chain model.

FIGURE 20

CREDIBILITIES ASSIGNED TO EACH OF TEN YEARS OF DATA VARYING THE RATE OF SHIFTING PARAMETERS IN THE MARKOV CHAIN MODEL OF BASEBALL DATA

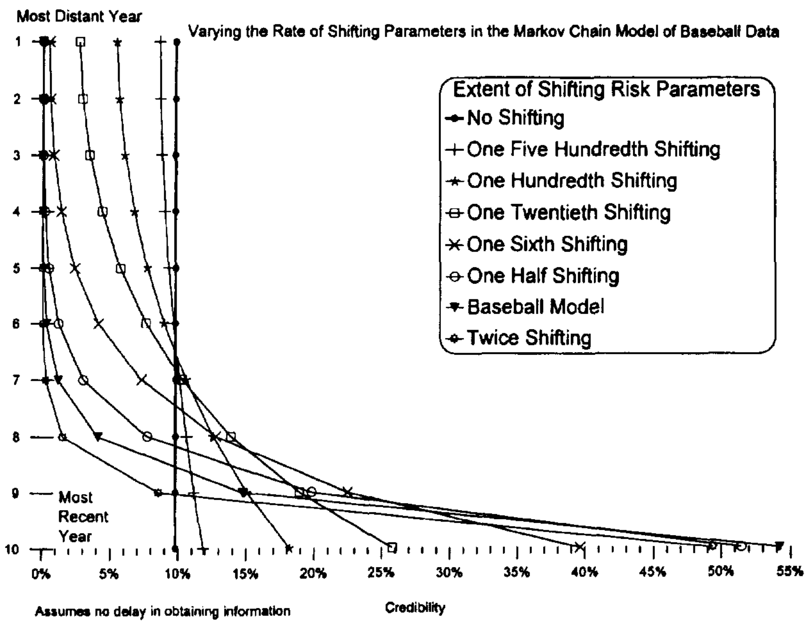
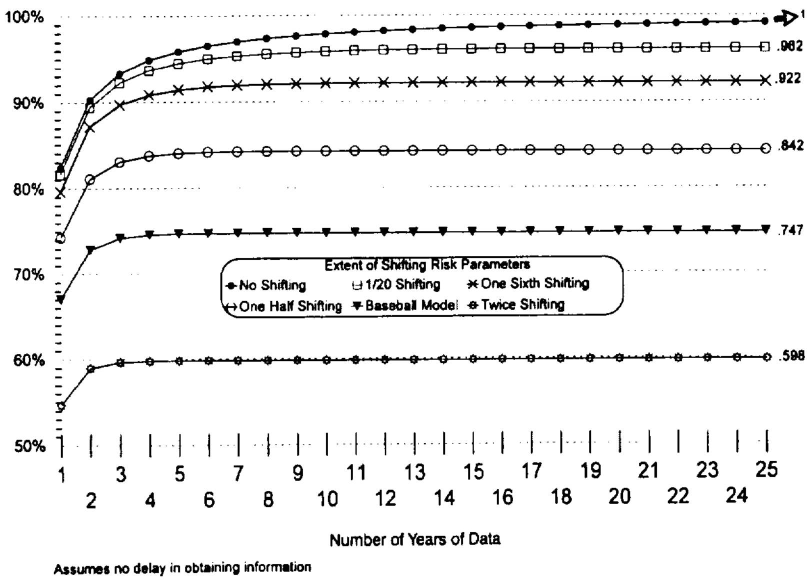


Figure 20 compares the credibilities one would assign to ten individual years of data with either more or less quickly shifting risk parameters than in the baseball data. If a major change in circumstances leads one to believe there has been a significant change in the rate at which parameters shift,[69] then the Markov chain model can be easily adjusted to incorporate one's estimate of the rate at which parameters will shift in the future.

[69]In the baseball example, many changes have occurred since 1960, the last year used to calibrate the model. For example, free agency might allow a more frequent movement of players between teams leading to a somewhat quicker rate of shifting of risk parameters.

FIGURE 21

SUM OF CREDIBILITIES, VARYING THE RATE OF SHIFTING PARAMETERS IN THE MARKOV CHAIN MODEL OF BASEBALL DATA



In the absence of shifting risk parameters, the same credibility is assigned to each of the ten years of data. As more and more shifting is introduced, the credibilities for the older years decline. (The curve on Figure 20 gets further and further from a vertical line.) This illustrates the effect of fine tuning the rate of shifting in the Markov chain model.

Figure 21 compares the sums of the credibilities for various numbers of years of data for various amounts of shifting. In the absence of shifting risk parameters, the sum of the credibilities approaches unity as the number of years increases. As the amount of shifting increases, the limit of the sum of the credibilities decreases.

If parameters are shifted at one-hundredth of the rate at which parameters shifted in the baseball example, the maximum sum of the credibilities is 98.4%. For the baseball example, it is 74.7%. For twice the shifting, it is 59.8%. The greater the rate of shifting of risk parameters, the lower the limit and the faster the convergence.

6. AREAS FOR POSSIBLE FUTURE REFINEMENTS

The model presented here was applied to claim frequency situations. It would probably be valuable to extend this to situations involving claim severity or pure premiums.

The model presented here did not fully explore the impact of size of risk. In order to properly explore the impact of size of risk on insurance situations, one would probably have to incorporate the effects of parameter uncertainty and risk heterogeneity as well as shifting risk parameters over time.[70]

The model presented here does not allow for an expected long term difference between risks. Averaged over a sufficiently long period of time, every risk's average frequency is the same. This is undoubtedly a poor model of certain situations.

There is no specific treatment of the entry of new insureds or the exit of current insureds from the database. Venezian [15] specifically models the change in accident propensity of new drivers entering the system as they gain experience and get older. The model as presented here would not accommodate this phenomenon.

Thus while the model presented here is practical and flexible, it would require further work to adapt it to many situations of potential interest.

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[70] See for example, Mahler [8].

7. SUMMARY

The effects of shifts over time in the risk process of an insured can be quantified in the covariances between years of data. For this Markov chain model, in most cases the covariances can be approximated by[71]:

$$\mathrm{Cov}[X_i, X_j] = \tau^2\lambda^{|i-j|} + \delta_{ij}\eta^2$$
$$\text{where} \quad \delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i \neq j \end{cases} \tag{7.1}$$

η^2 is the expected value of the process variance,

τ^2 is the variance of the hypothetical means,

and λ is the dominant eigenvalue (other than unity) of the transpose of the transition matrix of the Markov chain.

One has $\mathrm{Var}[X] = \mathrm{Cov}[X,X] = \tau^2 + \eta^2$. This is the usual relationship that the total variance can be split into the expected value of the process variance and the variance of the hypothetical means.

As the separation between years of data increases, the (expected) covariance and correlation between years decline.

It is not vital to understand the precise derivation of λ; rather it is important to understand that λ quantifies the rate at which the parameters shift. The smaller λ is, the faster the parameters shift. The closer λ is to unity, the slower the parameters shift. In the limit for $\lambda = 1$, there is no shifting of parameters.

Four examples have been considered, involving dice, a mixture of four Poisson distributions, California driving data (modeled by a gamma-Poisson), and baseball data (modeled by a mixture of binomials). The Markov chain model was applied to each of these situations. The resulting values of λ

[71] This is Equation 3.5.

were:

Example	λ	"Half-Life"
Dice[72]	.769	2.6 trials
Mixture of 4 Poissons[73]	.855	4.4 years
Female California Drivers[74]	.961	17.3 years
Baseball Team Results[75]	.818	3.4 years

where the "half-life" is the length of time for the correlations between years to decline by a factor of one-half:

$$\lambda^{\text{half-life}} = .5$$

$$\text{half-life} = \frac{\ln .5}{\ln \lambda} = \frac{-.693}{\ln \lambda}. \tag{7.2}$$

The longer the half-life, the slower the rate of shifting parameters over time. Thus, the impact of shifting parameters was most significant in the dice example, followed by the baseball data and the mixture of four Poissons example.[76] The female California drivers data with a half-life of about 17 years has much less impact from shifting risk parameters.[77]

If the Markov chain model holds, the correlations between different years of data should decline approximately exponentially. For $i \neq j$, Equation 7.1 gives $\text{Cov}[X_i, X_j] = \tau^2 \lambda^{|i-j|}$.

[72]See Section 2.7.

[73]See Section 3.2.

[74]The dominant eigenvalue shown in Section 4.3 is .998. This transition matrix is then taken to the 20th power, therefore so are the eigenvalues. $(.998)^{20} = .961$.

[75]The dominant eigenvalue shown in Section 5.1 is .967. However, this transition matrix is taken to the 6th power, therefore so are all the eigenvalues. $(.967)^6 = .818$.

[76]The dice example and mixture of four Poissons example were specifically designed to have a significant effect of shifting risk parameters for illustrative purposes. One of the reasons the baseball data was selected for presentation was because it showed a significant impact.

[77]The male driving data displayed even less impact from shifting risk parameters than female driving data. See Mahler [7].

Also, $\mathrm{Var}[X_i] = \mathrm{Var}[X_j] = \eta^2 + \tau^2$. Therefore,

$$\begin{aligned} \mathrm{Corr}[X_i, X_j] &= \left(\frac{\tau^2}{\tau^2 + \eta^2}\right) \lambda^{|i-j|} \\ \ln \mathrm{Corr}[X_i, X_j] &= \ln\left(\frac{\tau^2}{\tau^2 + \eta^2}\right) + |i-j| \ln \lambda \qquad i \neq j. \end{aligned} \tag{7.3}$$

Therefore, if the Markov chain model holds, the log-correlations for years separated by a given amount should decline approximately linearly. The slope of this line is (approximately) $\ln \lambda$. The intercept is approximately $\ln(\tau^2/\tau^2 + \eta^2)$. Note that $\tau^2/(\tau^2 + \eta^2)$ = VHM/total variance = credibility in the absence of shifting risk parameters.

Thus given a data set, one can determine whether this (simple) Markov chain model might be appropriate. One determines whether the log correlations as a function of the separation between years (not including zero separation) can be approximated by a straight line.[78] Then one can estimate the parameter λ and the ratio $\tau^2/(\tau^2 + \eta^2)$ from the slope and intercept of the fitted straight line.

These estimates can be used in turn to estimate credibilities. If one has data from years $1, 2, \ldots, Y$ and is estimating year $Y + \Delta$, then the least squares credibilities are given by solving the Y linear equations in Y unknowns:[79]

$$\sum_{j=1}^{Y} \mathrm{Cov}[X_i, X_j] Z_i = \mathrm{Cov}[X_i, X_{Y+\Delta}], \qquad i = 1, 2, \ldots, Y. \tag{7.4}$$

[78]In many cases there is a large amount of random fluctuation so even if the expected log correlations are precisely along a straight line, the log correlations estimated from the data will vary widely around a straight line. See Figure 10.

[79]See Equations 2.8.

8. CONCLUSIONS

A Markov chain model has been developed and applied to a number of different examples in which risk parameters shift over time. The model is sufficiently flexible to be applied to other situations.

In each case, the Markov chain model was used to explore the effects of shifting risk parameters over time. Covariances are calculated. Based on the Markov chain model, when shifting risk parameters over time are significant, the logs of the covariances between years of data are expected to decline linearly as the separation between years increases.

Credibilities are calculated from the variances and covariances. When shifting risk parameters are significant, older years receive less credibility and as more years of data are added, the sum of the credibilities goes to a limit less than one. The longer the delay in collecting data, the lower the credibilities.

The Markov chain model can be used to simulate the claims process when there are shifting risk parameters over time in the same manner as the gamma-Poisson and similar models can be used in the absence of shifting parameters. The Markov chain model should aid the actuary's understanding of situations in which shifting risk parameters are significant. It is both practical and sufficiently flexible to be applied in a wide variety of circumstances.

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APPENDIX A

MARKOV CHAINS[80]

Assume each year[81] an individual is in a "state." In this paper, each state corresponds to a different average claim frequency. In this paper, there are a finite number of different states.

Assume with each new year that an individual in state i has a chance P_{ij} of going to state j. This chance is independent of which individual we have picked, what his past history was, or what year it is. The transition probability from state i to state j, P_{ij}, is dependent on only the two states, i and j.

Arrange these transition probabilities P_{ij} into a matrix $\boldsymbol{P}$. This transition matrix $\boldsymbol{P}$, together with the definition of the states, defines a (finite dimensional) Markov chain.

If an individual is in state i, P_{ii} is the probability that he remains in state i. $1 - P_{ii}$ is the probability that he changes his state.

$\sum_{j=1}^{n} P_{ij}$ is the sum of the probability of this individual changing to each of the possible states (including remaining in state i). Since all the possibilities are exhausted, $\sum_{j=1}^{n} P_{ij} = 1$. Each of the rows of the transition matrix $\boldsymbol{P}$ for a Markov chain must sum to unity.

A vector containing the probability of finding an individual in each of the possible states is called a "distribution." If the distribution in year 1 is β, then the expected distribution in year 2 is $\beta\boldsymbol{P}$, where $\beta\boldsymbol{P}$ is the matrix product of the (row vector) distribution β and the transition matrix $\boldsymbol{P}$. The expected distribution in year 3 is $(\beta\boldsymbol{P})\boldsymbol{P} = \beta(\boldsymbol{PP}) = \beta\boldsymbol{P}^2$. The expected distribution in year $1 + g$ is $\beta\boldsymbol{P}^g$.

A stationary distribution is a vector α such that $\alpha\boldsymbol{P} = \alpha$. On an expected basis, the portfolio of risks stays in the stationary

[80] See Feller [2] and Resnick [13].

[81] Although in this paper the time interval is a year, in general, it can be anything.

distribution over time. Note that, from its definition, a stationary distribution (if it exists) is an eigenvector corresponding to an eigenvalue of unity. When it exists, the quickest way to compute α, the stationary distribution of $\boldsymbol{P}$, is via:

$$\alpha = (1, 1, \ldots, 1)(\boldsymbol{I} - \boldsymbol{P} + \mathbf{ONE})^{-1}$$

where $\boldsymbol{I}$ is the identity matrix and **ONE** is the n by n matrix, all of whose entries are one.[82]

All of the Markov chains in this paper have been specifically constructed to have a stationary distribution using the techniques in Appendix D.

For a finite dimensional Markov chain such that each state can be reached from every other state and such that no states are periodic,[83] a unique stationary distribution α exists and for any initial distribution β, $\beta \boldsymbol{P}^g \to \alpha$ as $g \to \infty$. Thus eventually the distribution of risks in the portfolio is α (for all practical purposes) regardless of the initial distribution β.

Taking $\beta = (1, 0, 0 \cdots)$, $\beta = (0, 1, 0 \cdots)$, etc., implies that the rows of $\boldsymbol{P}^g \to \alpha$ as $g \to \infty$. If A is a matrix all of whose rows are the stationary distribution α, $\boldsymbol{P}^g \to A$ as $g \to \infty$.

Let $\boldsymbol{P}^T$ be the matrix transpose of $\boldsymbol{P}$. Let Λ be the diagonal matrix with entries equal to the eigenvalues of $\boldsymbol{P}^T$. Let $\boldsymbol{V}^T$ be the matrix, each of whose columns are the eigenvectors of $\boldsymbol{P}^T$. ($\boldsymbol{V}$ has as its rows the eigenvectors of $\boldsymbol{P}^T$.) Then, as stated in Appendix B, $(\boldsymbol{V}^T)^{-1}\boldsymbol{P}^T\boldsymbol{V}^T = \Lambda$. Taking the transpose of both sides of this equation and noting that $\Lambda^T = \Lambda$, since Λ is symmetric: $\boldsymbol{VPV}^{-1} = \Lambda$. So the matrix $\boldsymbol{V}$ can be used to diagonalize the transition matrix $\boldsymbol{P}$:

$$\boldsymbol{V}^{-1}\Lambda^2\boldsymbol{V} = \boldsymbol{V}^{-1}(\boldsymbol{VPV}^{-1})^2\boldsymbol{V} = \boldsymbol{V}^{-1}\boldsymbol{VPV}^{-1}\boldsymbol{VPV}^{-1}\boldsymbol{V} = \boldsymbol{P}^2.$$

[82]See Section 2.14 of Resnick [13].
[83]See Section 2.13 of Resnick [13].

In general, $P^g = V^{-1}(VPV^{-1})^g V = V^{-1}\Lambda^g V$. So powers of P can be computed by taking powers of the diagonal matrix Λ and using the eigenvector matrix V to transform back. The elements of the diagonal matrix Λ^g are λ_i^g.

If A is matrix whose rows are the stationary distribution α, then:

$$P^g \to A.$$

$$\therefore \quad \Lambda^g = VP^g V^{-1} \to VAV^{-1}.$$

But Λ^g has diagonal element λ_i^g. These only converge to a limit as $g \to \infty$ if $\lambda_i = 1$ or $|\lambda_1| < 1$. Let $\lambda_1 = 1$, since the order of eigenvalues is arbitrary. Then $|\lambda_1| < 1$ for $i > 1$ (ignoring the very unusual situation where $\lambda = 1$ is a multiple root of the characteristic equation).

Let the limit of Λ^g as $g \to \infty$ be denoted by Λ^∞. Then $(\Lambda^\infty)_{ij} = 0$ for $i \neq 1$ or $j \neq 1$, and $(\Lambda^\infty)_{1,1} = 1$.

Therefore $V^{-1}\Lambda^\infty V = \lim_{g\to\infty} P^g = A$.

Thus $(V^{-1})_{i1} V_{1j} = A_{ij} = \alpha(j)$, since the rows of A are the stationary distribution, and $(\Lambda^\infty)_{ij} = 0$ for $i \neq 1$ or $j \neq 1$.

Thus $(V^{-1})_{i1} = \alpha(j)/V_{1j}$.

Note that the left hand side is independent of j, while the right hand is independent of i. Since the equation holds for all i and j, both sides must be independent of i and j. Therefore, the elements of the first column of V^{-1} are all equal. The elements of the first row of V are proportional to the stationary distribution α.

Since $VV^{-1} = I$, where I is the identity matrix, the product of the first column of V^{-1} with any row of V other than the first is zero. But the product of the first column of V^{-1} with any row is proportional to the sum of that row since all the elements of the first column of V^{-1} are equal. Therefore, the sum of any row of V other than the first is zero. Therefore, the sum of any eigenvector other than the first is zero, since the rows of V are the eigenvectors of P^T.

APPENDIX B

EIGENVALUES AND EIGENVECTORS

Given a square matrix $\boldsymbol{P}$, if for a vector, $\boldsymbol{v}$, $\boldsymbol{Pv} = \lambda \boldsymbol{v}$, then $\boldsymbol{v}$ is called an eigenvector of $\boldsymbol{P}$ with eigenvalue λ. Note that if $\boldsymbol{v}$ is an eigenvector of $\boldsymbol{P}$, so is $\boldsymbol{v}$ times any non-zero constant. So eigenvectors can be determined only up to a proportionality constant.

One can find the eigenvalues and thus the corresponding eigenvectors by solving the characteristic equation:[84]

Determinant $(\boldsymbol{P} - \lambda \boldsymbol{I}) = 0$, where $\boldsymbol{I}$ is the identity matrix.

If V is a matrix whose columns are the eigenvectors of $\boldsymbol{P}$, then $V^{-1}\boldsymbol{P}V$ is a diagonal matrix Λ whose elements are the eigenvalues of $\boldsymbol{P}$. ($V^{-1}\boldsymbol{P}V = \Lambda$ follows from the matrix equation $\boldsymbol{P}V = V\Lambda$, which when we take each column reduces to the eigenvalue equation: $\boldsymbol{P}\boldsymbol{v}_i = \boldsymbol{v}_i\lambda_i$.)

If $\boldsymbol{v}$ is an eigenvector of $\boldsymbol{P}$ with eigenvalue λ, then $\boldsymbol{Pv} = \lambda\boldsymbol{v}$. Therefore,

$$\boldsymbol{P}^2\boldsymbol{v} = \boldsymbol{P}(\boldsymbol{Pv}) = \boldsymbol{P}(\lambda \mathbf{v}) = \lambda\boldsymbol{Pv} = \lambda^2\boldsymbol{v}.$$

Thus $\boldsymbol{v}$ is also an eigenvector of $\boldsymbol{P}^2$ with eigenvalue λ^2. In general, $\boldsymbol{v}$ is an eigenvector of $\boldsymbol{P}^g$ with eigenvalue λ^g. Raising a matrix to a power does not alter the eigenvectors and raises the eigenvalues to the same power.

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[84]Eigenvalues and eigenvectors are calculated by many computer software packages. The author used the APL program EIG provided by Manugistics (formerly STSC).

APPENDIX C

MATRIX EQUATIONS FOR LEAST SQUARES CREDIBILITY[85]

In this Appendix, Equations 2.8 in the main text are derived by minimizing the squared error. The result is N linear equations for the credibilities to be assigned to each of N years of data. Thus, the credibilities can be solved for in terms of the covariance structure.

Let

$$C_{ij} = \text{Cov}[X_i, X_j]$$
$$= \text{Covariance of year } X_i \text{ and year } X_j, \qquad \text{and}$$
$$C_{ii} = \text{Variance of year } X_i.$$

Let Z_i be the credibility assigned to year X_i. We wish to predict year $X_{N+\Delta}$ using N years of data $X_1, X_2, \ldots, X_N$ and the grand mean M. Let $Z_0 = 1 - \sum_{i=1}^{N} Z_i$ = complement of credibility.

Then the estimate is:

$$F = \sum_{i=1}^{N} Z_i X_i + Z_0 M.$$

Let

$$X_0 = M, \qquad \text{then} \quad F = \sum_{i=0}^{N} Z_i X_i$$

$$F - X_{N+\Delta} = \left(\sum_{i=0}^{N} Z_i X_i \right) - X_{N+\Delta} = \sum_{i=0}^{N} Z_i (X_i - X_{N+\Delta}),$$

since $\sum_{i=0}^{N} Z_i = 1$.

[85]The derivation is adopted from that in Mahler [10].

Therefore,

$$(F - X_{N+\Delta})^2 = \left(\sum_{i=0}^{N} Z_i(X_i - X_{N+\Delta})\right)\left(\sum_{j=0}^{N} Z_j(X_j - X_{N+\Delta})\right)$$

$$= \sum_{i=0}^{N}\sum_{j=0}^{N} Z_i Z_j (X_i - X_{N+\Delta})(X_j - X_{N+\Delta}).$$

Then the expected value of the squared difference between the estimate F and $X_{N+\Delta}$ is, as a function of the credibilities Z:

$$V(Z) = \mathrm{E}[(F - X_{N+\Delta})^2]$$

$$= \sum_{i=0}^{N}\sum_{j=0}^{N} Z_i Z_j \mathrm{E}[(X_i - X_{N+\Delta})(X_j - X_{N+\Delta})].$$

Now

$$\mathrm{E}[(X_i - X_{N+\Delta})(X_j - X_{N+\Delta})] = \mathrm{E}[X_i X_j] - \mathrm{E}[X_i X_{N+\Delta}]$$

$$- \mathrm{E}[X_j X_{N+\Delta}] - \mathrm{E}[X_{N+\Delta}^2]$$

$$\mathrm{E}[X_i X_j] = \mathrm{Cov}[X_i, X_j] + \mathrm{E}[X_i]\mathrm{E}[X_j]$$

$$= C_{ij} + M^2,$$

where

$$C_{0j} = \mathrm{Cov}[M, X_j] = 0.$$

Thus

$$\mathrm{E}[(X_i - X_{N+\Delta})(X_j - X_{N+\Delta})]$$

$$= C_{ij} - C_{i,N+\Delta} - C_{j,N+\Delta} + C_{N+\Delta,N+\Delta}, \qquad \text{and}$$

$$V(Z) = \sum_{i=0}^{N}\sum_{j=0}^{N} Z_i Z_j \{C_{ij} - C_{i,N+\Delta} - C_{j,N+\Delta} + C_{N+\Delta,N+\Delta}\}.$$

$$V(Z) = \sum_{i=0}^{N}\sum_{j=0}^{N} Z_i Z_j C_{ij} - \left(\sum_{i=0}^{N} C_{i,N+\Delta} Z_i\right)\left(\sum_{j=0}^{N} Z_j\right)$$

$$- \left(\sum_{j=0}^{N} C_{j,N+\Delta} Z_j\right)\left(\sum_{i=0}^{N} Z_i\right)$$

$$+ C_{N+\Delta,N+\Delta}\left(\sum_{i=0}^{N} Z_i\right)\left(\sum_{j=0}^{N} Z_j\right).$$

The last three terms all simplify, since

$$\sum_{i=0}^{N} Z_i = Z_0 + \sum_{i=1}^{N} Z_i$$

$$= 1 - \sum_{i=1}^{N} Z_i + \sum_{i=1}^{N} Z_i = 1.$$

Therefore,

$$V(Z) = \sum_{i=0}^{N}\sum_{j=0}^{N} Z_i Z_j C_{ij} - \sum_{i=0}^{N} C_{i,N+\Delta} Z_i$$

$$- \sum_{j=0}^{N} C_{j,N+\Delta} Z_j + C_{N+\Delta,N+\Delta}.$$

Also, since $C_{0j} = 0 = C_{i0}$, the elements involving $i = 0$ or $j = 0$ drop out, leaving

$$V(Z) = \sum_{i=1}^{N}\sum_{j=1}^{N} Z_i Z_j C_{ij} - 2\sum_{i=1}^{N} C_{i,N+\Delta} Z_i + C_{N+\Delta,N+\Delta}.$$

Taking the partial derivative of $V(Z)$ with respect to Z_k and setting it equal to zero:

$$2\sum_{i=1}^{N} Z_i C_{ik} - 2C_{k,N+\Delta} = 0$$

$$\sum_{i=1}^{N} Z_i C_{ik} = C_{k,N+\Delta}.$$

This results in N equations in N unknowns for $k = 1,2,\ldots,N$. These are Equations 2.8 in the main text.

APPENDIX D

CONSTRUCTING A TRANSITION MATRIX

Assume we are given a set of probabilities corresponding to a set of n states:

$$\alpha_i, \qquad i = 1,2,\ldots,n \qquad \sum \alpha_i = 1.$$

There are many transition matrixes, $\boldsymbol{P}$, such that α is a stationary state, $\alpha\boldsymbol{P} = \alpha$. A method of constructing one such matrix from α will be shown.

The constructed transition matrix will be such that most of its elements are zero. The only non-zero elements will be on the main diagonal, just above the main diagonal or just below the main diagonal. Such a matrix is sometimes referred to as "tri-diagonal."

Such a transition matrix corresponds to each year, an insured either staying in the same state or possibly moving up or down by a single state in a single year.[86]

As a concrete example, take the simple Poisson example in the main text, with four states and $\alpha = (.4, .3, .2, .1)$.

The equation $\alpha\boldsymbol{P} = \alpha$ becomes

$$(.4 \quad .3 \quad .2 \quad .1)\begin{pmatrix} P_{11} & P_{12} & 0 & 0 \\ P_{21} & P_{22} & P_{23} & 0 \\ 0 & P_{32} & P_{33} & P_{34} \\ 0 & 0 & P_{43} & P_{44} \end{pmatrix} = (.4 \quad .3 \quad .2 \quad .1).$$

[86] If this transition matrix were raised to a power, then one could move more than a single state per year. Also, the overall speed of parameter shifting would be increased.

So,

$$.4P_{11} + .3P_{21} = .4$$
$$.4P_{12} + .3P_{22} + .2P_{32} = .3$$
$$.3P_{23} + .2P_{33} + .1P_{43} = .2$$
$$.2P_{34} + .1P_{44} = .1.$$

In addition, each row of any transition matrix sums to unity. (Every risk ends up in some state.)

$$P_{11} + P_{12} = 1$$
$$P_{21} + P_{22} + P_{23} = 1$$
$$P_{32} + P_{33} + P_{34} = 1$$
$$P_{43} + P_{44} = 1.$$

Thus

$$P_{21} = \frac{.4(1 - P_{11})}{.3} = \frac{4}{3}P_{12}.$$

Similarly

$$\begin{aligned} P_{32} &= \frac{.3(1 - P_{22}) - .4P_{12}}{.2} \\ &= \tfrac{3}{2}(P_{21} - P_{23}) - 2P_{12} \\ &= \tfrac{3}{2}P_{23} + 2P_{12} - 2P_{12} = \tfrac{3}{2}P_{23}. \end{aligned}$$

Similarly, one gets

$$P_{43} = (\tfrac{2}{1})P_{34}.$$

In general, we need:

$$\alpha_i P_{i,i+1} = \alpha_{i+1} P_{i+1,i}.$$

The left hand side of this equation is the probability of being in state i times the probability of going from state i to state $i+1$. Thus, this is the expected number of transitions from state i to state $i+1$. Similarly, the right hand side is the expected number of transitions from state $i+1$ to state i. In this case, these expected numbers of transitions will cancel and on average will result in no net change.

There are still arbitrary scale factors. (Within large bounds one can pick P_{12} and then P_{21} follows.) For purposes of illustration,[87] let $P_{i,i+1} + P_{i+1,i} = v < 1$ for all i, where v is a parameter that controls the amount of shifting. It represents the approximate probability of shifting either up or down one state; $1 - v$ is the approximate probability of remaining in the same state.

Then once $0 < v < 1$ is chosen, one constructs the transition matrix:

$$P_{i,i+1} = \frac{\alpha_{i+1}}{\alpha_i + \alpha_{i+1}} v,$$

$$P_{i+1,i} = \frac{\alpha_i}{\alpha_i + \alpha_{i+1}} v,$$

$$P_{ii} = 1 - P_{i,i-1} - P_{i,i+1}, \qquad \text{and}$$

$$P_{ij} = 0 \qquad \text{for} \quad |i - j| > 1.$$

This results in a transition matrix with the given α as a stationary distribution and with about a $(1 - v)$ chance of remaining in the same state per year.

This construction algorithm is relatively simple and easily programmable.

In the particular example with $\alpha = (.4, .3, .2, .1)$, taking $v = .42$, the algorithm produces a transition matrix of:

$$\begin{pmatrix} .820 & .180 & 0 & 0 \\ .240 & .592 & .168 & 0 \\ 0 & .252 & .608 & .140 \\ 0 & 0 & .280 & .720 \end{pmatrix}.$$

This is the transition matrix shown in the main text, which has a stationary distribution of (.4, .3, .2, .1).

[87]For certain applications, one may choose to vary the probability of remaining in a state among the different states.

APPENDIX E

COVARIANCES

Let μ_i be the mean for state i. Let X and U be two different years of data separated by g years, $g > 0$. Let X have probability vector $\boldsymbol{\beta}$ for the probability of being in a given state. Then $\boldsymbol{\beta P^g}$ is the probability vector for year U. Then

$$\mathrm{E}[XU] = \sum_{i,j} \Pr(X \text{ in state } i \text{ and } U \text{ in state } j) \times \mathrm{E}[XU \mid X \text{ in state } i \text{ and } U \text{ in state } j].$$

If X is in state i and U is in state j, then

$$\mathrm{E}[XU] = \mathrm{E}[X \mid X \text{ in state } i]\mathrm{E}[U \mid U \text{ in state } j] = \mu_i \mu_j,$$

since the die rolls in year X and U are independent.

$$\begin{aligned}
&\Pr(X \text{ in state } i \text{ and } U \text{ in state } j) \\
&\quad = \Pr(X \text{ in state } i)\Pr(U \text{ in state } j \mid X \text{ in state } i) \\
&\quad = \beta_i (P^g)_{ij},
\end{aligned}$$

since the transition matrix from X to U which are g years apart is $\boldsymbol{P^g}$. Thus

$$\begin{aligned}
\mathrm{E}[XU] &= \sum_{ij} \mu_i \mu_j \beta_i (P^g)_{ij} \\
&= (\mu \times \boldsymbol{\beta})^T \boldsymbol{P^g} \mu,
\end{aligned}$$

where $\mu \times \boldsymbol{\beta}$ is the vector whose ith element is $\mu_i \beta_i$ and we have taken the matrix product of the transpose of this vector with the matrix $\boldsymbol{P^g}$ and then with the (column) vector μ, so,

$$\boldsymbol{P^g} = \boldsymbol{V}^{-1} \Lambda^g \boldsymbol{V}, \qquad \text{and}$$

$$\therefore \; \mathrm{E}[XU] = (\mu \times \boldsymbol{\beta})^T \boldsymbol{V}^{-1} \Lambda^g \boldsymbol{V} \mu.$$

Now assume that we are in the stationary distribution α; i.e., either the process has been going on long enough that the initial state no longer has any practical importance or the initial distribution was chosen to be equal to α. If $\beta = \alpha$, then

$$\mathrm{E}[XU] = (\mu \times \alpha)^T V^{-1} \Lambda^g V \mu.$$

Let C be the vector given by $(\mu \times \alpha)^T V^{-1}$ and let D be the vector given by $V\mu$, then since g is diagonal with $\Lambda_{ii}^g = \lambda_i^g$; $\mathrm{E}[XU] = \sum_k C_k D_k \lambda_{kk}^g$.

Thus we have written $\mathrm{E}[XU]$ as a sum of coefficients (independent of g) times the eigenvalues raised to the power g.

In the dice example in Section 2:

$\mu = (2.5, 3.5, 4.5)$ = means

$\alpha = (.25, .50, .25)$ = stationary distribution

$$V = \begin{pmatrix} 1 & 2 & 1 \\ 1 & -.314 & -.686 \\ 1 & -3.186 & 2.186 \end{pmatrix} = \begin{matrix}\text{matrix whose rows are} \\ \text{eigenvectors of the} \\ \text{transpose of the transition} \\ \text{matrix}\end{matrix}$$

$$V^{-1} = \begin{pmatrix} .250 & .658 & .092 \\ .250 & -.103 & -.147 \\ .250 & -.451 & .201 \end{pmatrix}$$

$$C = (2.5 \times .25, 3.5 \times .5, 4.5 \times .25) \begin{pmatrix} .250 & .658 & .092 \\ .250 & -.103 & -.147 \\ .250 & -.451 & .201 \end{pmatrix}$$

$$= (.875, -.277, .027)$$

$$D = \begin{pmatrix} 1 & 2 & 1 \\ 1 & -.314 & -.686 \\ 1 & -3.186 & 2.186 \end{pmatrix} \begin{pmatrix} 2.5 \\ 3.5 \\ 4.5 \end{pmatrix} = \begin{pmatrix} 14.000 \\ -1.686 \\ 1.186 \end{pmatrix}$$

$C \times D = (12.25, .468, .032)$.

Thus

$$\begin{aligned} \mathrm{E}[XU] &= 12.25(1^g) + .468(.769^g) + (.032)(.481^g) \\ &= 12.25 + (.468)(.769^g) + (.032)(.481^g) \\ \mathrm{E}[X] &= \mathrm{E}[U] = \mathrm{Sum}(\alpha \times \mu) = 3.5 \\ \therefore\ \mathrm{E}[X]\mathrm{E}[U] &= 3.5^2 = 12.25 \\ \mathrm{Cov}[X,U] &= \mathrm{E}[XU] - \mathrm{E}[X]\mathrm{E}[U] \\ &= (.468)(.769^g) + (.032)(.481^g). \end{aligned}$$

Note how the first term of $\mathrm{E}[XU]$ cancels with $\mathrm{E}[X]\mathrm{E}[U]$; this will happen if the eigenvalue of unity is placed first. In general, the covariance of X and U is a sum of coefficients times eigenvalues (other than unity) raised to the power g.

Since $|\lambda_i| < 1$ for $i > 1$, the covariance will converge to zero as $g \to \infty$, because it is limited by a constant times the largest λ_i in magnitude (other than unity) raised to the power g.

Let ζ be the vector such that:

$$\zeta_i = C_i D_i = ((\mu \times \alpha)^T V^{-1})_i (V\mu)_i.$$

This is Equation 2.5 in the main text.

Then we have, for $g > 0$, Equation 2.6 in the main text:

$$\mathrm{Cov}[X,U] = \sum_{i>1} \zeta_i \lambda_i^g.$$

Note that λ_i and ζ_i which determine the behavior of the covariance are each directly and easily calculable[88] from the assumed transition matrix and the means of the states.

[88] Assuming the calculations will be performed on a computer.

WORKERS COMPENSATION AND ECONOMIC CYCLES: A LONGITUDINAL APPROACH

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Abstract

This paper uses cross-sectional time series techniques in an econometric framework to model workers compensation frequency, severity, and loss ratios over the course of the business cycle. Empirical evidence from 37 states over the 1979–1993 period strongly suggests that frequency is strongly pro-cyclical, tending to increase during periods of economic expansion and fall during periods of economic decline or sluggishness. Similarly, the analyses reported in this study indicate that the economic determinants of indemnity and medical severity and loss ratios can be characterized by large pro-cyclical and small counter-cyclical components. The latter finding is contrary to conventionally held beliefs concerning this topic.

At the time this study was performed, the authors were employed by the National Council on Compensation Insurance: Robert Hartwig as Senior Economist; Ronald Retterath as Senior Vice President and Chief Actuary; Tanya Restrepo as Economist; and William Kahley as Assistant Vice President for Economic Research.

1. INTRODUCTION

The explosion in workers compensation costs during the 1980s and early 1990s continues to be the subject of much debate among actuaries, economists, and regulators even though the crisis began to show signs of abating in 1993 and has continued to improve since then. This ongoing dialogue has been constructive inasmuch as it has identified many of the cost drivers responsible for the tumult in the industry over the past fifteen years.

Much has been written about some important cost drivers—such as medical inflation, medical cost shifting, attorney involvement, and fraud—from many angles, including premium avoidance, employee malingering, and medical/legal workers compensation mills. The popular media have even sensationalized some fraud-related activities in print and television.

In contrast, important relationships between workers compensation costs and changes in the economic environment generally have been ignored or discussed only anecdotally in the literature. The objective of this longitudinal study is to demonstrate empirically that workers compensation costs across states are fundamentally dependent on the economic environment. Specifically, empirical evidence suggests that frequency is strongly pro-cyclical, tending to increase during periods of economic expansion and fall during periods of economic decline or sluggishness. Similarly, the analyses reported in this study indicate that the economic determinants of indemnity and medical severity and loss ratios can be characterized by large pro-cyclical and small counter-cyclical components. The latter finding is contrary to conventionally held beliefs concerning this topic.

The remainder of this paper is structured as follows. Section 1 concludes with some background information and a brief literature review on the economic factors affecting workers compensation results. Section 2 contains definitions of the economic variables used in the study and their hypothesized impact on the workers compensation market. Section 3 describes the modeling methodology and defines the dependent variables. In Section 4, empirical results from analyses of the impact of business cycle effects on workers compensation frequency, severity, and loss ratios are presented. Section 5 concludes the paper with a summary of results presented in this study and directions for future research.

The term "business cycle," as it is typically used by business analysts, describes the periodic, but irregular, ups and downs of the real economy over time.[1] The cumulative processes at work during business cycles ensure that at some point every industry and virtually every firm will be either directly or indirectly affected. The workers compensation insurance industry is no exception. In almost all states, workers compensation insurance (or its equivalent) is compulsory, and policies are purchased by businesses in all industries. The fact that workers compensation premiums change with companies' payrolls ensures that fluctuations in economic activity will have direct impacts on the workers compensation industry. Losses, of course, will be affected by shifting levels of exposure during the different phases of the cycle, and also by the changing claim filing incentives facing workers in a dynamic economic environment.

Much of the economic literature in workers compensation insurance has focused on the factors that motivate individual workers to use the workers compensation system, and contrasted employee incentives with the incentives employers have to maintain a safer workplace to hold down workers compensation costs. The preponderance of evidence indicates that higher benefits, particularly indemnity benefits, precipitate increased claim filing by workers. Other research suggests that during an expansion, em-

[1] It is important to distinguish between the meaning of the term "cycle," as it is frequently applied in insurance, versus the economic meaning intended here. The insurance usage of the term "cycle" generally refers to the so-called "underwriting cycle," where "hard markets," characterized by high prices and profits and limited availability of coverage, are followed by "soft markets," when prices and profits are low and the availability of coverage increases.

ployers' incentives to increase revenue may outweigh incentives to contain increases in workers compensation costs through investments in safety. The influence of economic incentives on workers and employers is explored in Moore and Viscusi [20], Butler and Appel [6], and Worrall and Butler [30, 31].[2]

Research into the costs and determinants of workers compensation medical expenditures has burgeoned in recent years in response to deteriorating market conditions through much of the 1980s and early 1990s. The Clinton administration's effort in 1993 and 1994 to enact national health care reform also provided a forum and impetus for such research. Some research findings suggested that price discrimination by medical care providers was at the root of workers compensation medical cost inflation [2]. Others have argued that higher costs for workers compensation cases (relative to non-occupational injuries) are due to the different mix and intensity of treatments necessary to hasten return to work [11]. The range and effectiveness of various medical cost containment strategies such as fee schedules, provider choice, bill/utilization review, and anti-fraud initiatives have also been analyzed.

The potential influence of the macroeconomy on workers compensation frequency and severity was recognized in a 1991 study performed by the Insurance Services Office (ISO). Using policy year data for thirty states, the ISO study found that higher interest rates and real growth in gross national product (GNP) are associated with higher claim frequency and that medical claim severity increased faster than an ISO-modified consumer price index [17].[3] Similarly, a 1996 National Council on Compensation Insurance (NCCI) study found a strong, positive association

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[2]A more detailed literature survey of the relationship between workers compensation frequency, severity, and benefit structures is found in Butler [5]. One early effort to model workers compensation losses and premiums econometrically is given in Lommele and Sturgis [19].

[3]The results of the ISO study were derived using data for just seven policy years (1980–1986). In contrast, the sample period used in this study runs from 1979 to 1993. The ISO study therefore fails to include observations over the full course of a business cycle.

between workers compensation claim frequency, total claim costs per worker, and real growth in gross domestic product (GDP) over the 1980 to 1995 period [14].

Apart from the ISO and NCCI analyses, the implications of studies dealing with economic factors generally are confined to the microeconomic influences on claim frequency and severity. In other words, they focus on the incentives facing individual workers, employers, medical service providers, and insurers. Current research, reported here, complements these efforts through empirical analysis of the important *macroeconomic* factors that can affect frequency, severity, and loss ratios in the workers compensation line. For this reason, the economic explanatory variables used in the present analysis are confined to overall state- and national-level measures of economic performance.

Workers Compensation and the Economic Environment

The complex relationship between the workers compensation industry (measured in this study by frequency, severity, and loss ratios) and the economic environment can be crudely decomposed into the several broad categories listed below. Each of the first three categories is based on our prior expectations of rising frequency, severity, and loss ratios during the expansionary phase of the business cycle. Other forces, as suggested in the final category, may work in the opposite direction. Empirical evidence, however, suggests that these countervailing effects are relatively small. The various effects are discussed briefly here, and then again in more detail in subsequent sections of this study as they specifically apply to the empirical results.

- Workers compensation is influenced by both the level and rate of growth of production. Higher levels and/or growth rates of production tend to be associated with an increase in losses that is in excess of wage increases.
- Periods of rising capacity utilization are associated with adverse impacts on workers compensation. This is in part due

to pressure to increase the speed and volume of production, which may lead to a decreased emphasis on workplace safety. Moreover, worker fatigue due to increased overtime also contributes to a general worsening in the line, as does the reuse of older, less efficient, machinery.

- Changes in the composition of employment also play an important role in workers compensation insurance. During the initial phases of economic expansions, for example, employment in many hazardous industries, such as construction, manufacturing, and trucking, expands rapidly.
- The incentives facing workers and employers shift over the course of the business cycle. Workers, for example, are less likely to file workers compensation claims during an economic expansion when the opportunity cost of being out of work is relatively high.[4]

It is important to recognize that the four broad relationships discussed in this section are themselves influenced by innumerable other socio-economic, demographic, and regulatory factors. The result is that there can be a great deal of variability between the strength and speed with which economic influences are transmitted to the workers compensation insurance line over time and across states. Nevertheless, certain important fundamental relationships that are the subject of this study are useful when discussing the impact of economic conditions on workers compensation frequency, severity, and loss ratios.

2. BUSINESS CYCLE EFFECTS ON WORKERS COMPENSATION

The econometric models used to determine business cycle effects on the workers compensation market typically include a measure of overall economic activity (employment), a measure

[4]The concept of *opportunity cost* is used by economists to denote the foregone value of the next best alternative which is *not* chosen. Hence, it is more costly for a worker to file a workers compensation claim during an economic expansion when overtime work is more available and wage increases are greater due to tightening labor market conditions.

of the health of the labor market (the unemployment rate), and possibly a measure of crisis labor market conditions (business failures). The waiting period and a measure of cost containment are also included in some cases to control for non-business cycle effects specifically related to workers compensation. The medical severity and medical loss ratio models include hospital cost per stay as a measure of medical costs. Sources and definitions of the explanatory variables are listed in Table 1.

The hypothesized effects of the explanatory variables on the workers compensation market are discussed below. In some cases, such as with the unemployment rate, there are countervailing effects, and the dominant impact is determined empirically. The expected sign of some explanatory variables may change depending on the model's dependent variable: either frequency, severity, or the loss ratio.

The employment variable is included in the models as a broad-based measure of economic activity within the state. Gross state product (GSP) is the broadest measure of economic activity at the state level. However, federal statistics on GSP from the Bureau of Economic Analysis are available only with a significant lag (through 1991 at the time of this analysis). Thus, employment is used as a proxy for GSP or the measure of production.[5] The expected sign on the estimated coefficient of the employment variable is positive. In other words, economic activity (as proxied by employment growth) and frequency, severity, and the loss ratio are expected to move in the same direction. Such an expectation is consistent with both previous research and economic theory. For example, during an expansion when employment rises, overtime increases, less-experienced workers are hired, and employment in hazardous industries increases. All these factors could be expected to lead to increased frequency, severity, and loss ratios.

[5]Historically, the correlation between GSP and employment has been extremely high—roughly 98 percent.

TABLE 1

EXPLANATORY VARIABLE DEFINITIONS AND SOURCES

Variable Name	Definition	Source
NAGEMP	Annual average nonagricultural employment measured by persons on establishment payrolls. It excludes proprietors, the self-employed, unpaid volunteer or family workers, farm workers, and domestic workers. Persons who worked in more than one establishment during the reporting period are counted each time their names appear on payrolls [24].	U.S. Department of Labor, Bureau of Labor Statistics, Current Employment Statistics, Survey of Establishments
UNRATE	Annual average unemployment rate measured by the number of unemployed persons as a percent of the labor force. Unemployed persons are those who had no employment during the reference week of the household survey, were available for work, and had made specific efforts to find employment some time during the 4-week period ending with the reference week. The labor force includes all unemployed persons and employed persons. In the household survey, a person is considered employed if they did any work at all (at least 1 hour) as a paid employee, worked in their own business, profession, or on their own farm, or worked 15 hours or more as unpaid workers in an enterprise operated by a member of the family. Each person employed is counted only once, even if he or she holds more than one job [24].	U.S. Department of Labor, Bureau of Labor Statistics, Current Population Survey of households

BUSFAIL	Total business failures measured as businesses that ceased operations and were involved in court proceedings or voluntary actions involving losses to creditors. This does not include business discontinuances which are defined as businesses that cease operations for reasons such as loss of capital, inadequate profits, ill health, retirement, etc., if creditors are paid in full. Although business failures represent only a percentage of total closings, they have the most severe impact upon the economy [10].	The Dun & Bradstreet Corporation, *Business Failure Record*
WAITPER	State-mandated waiting period defined as the time that must elapse during which income benefits are not payable [23].	U.S. Chamber of Commerce, *Workers Compensation Laws*
COSTCON	Dummy variable proxy for cost containment initiatives, equal to unity for years 1991 through 1993 in all states and zero otherwise. See Appendix A for a discussion of dummy variables.	NCCI
PERSTAY	The product of the average daily hospital charge and the average duration of a hospital stay. The average daily charge is expenses incurred for inpatient care divided by inpatient days. Average length of stay is the average stay of inpatients derived by dividing the number of inpatient days by the number of admissions [1].	American Hospital Association, *Hospital Statistics*

The unemployment rate is included in the models as a gauge of the health or tightness of the state's labor market. While there is some small degree of correlation between employment and the unemployment rate, the latter is a better indicator of general labor market conditions.[6] Employment, on the other hand, is an indicator or proxy of the health of the overall economy. For example, during the early stages of an economic recovery, the unemployment rate often increases despite strong gains in employment. This is because the number of new entrants to the labor force exceeds the number of jobs created.

It is difficult to determine the impact of the unemployment rate on the workers compensation market a priori. Various effects are present, and the models will determine the dominant impact. For instance, an increase in the unemployment rate could have an inverse impact on the workers compensation market because inexperienced workers, who often have higher accident rates, are laid off first and workers may defer filing claims during a recession for fear of losing their jobs. Conversely, a decrease in the unemployment rate could increase frequency or severity since those inexperienced workers are rehired and workers who deferred filing a claim may file at the first sign of economic recovery.

On the other hand, the unemployment rate could have a direct impact on the workers compensation market. There are several hypotheses suggesting that frequency, severity, and the loss ratio should rise as the unemployment rate increases during a recession. These include increased duration and frequency due to diminished job opportunities, increased claim-filing incentives due to layoffs (i.e., workers substitute relatively generous workers compensation benefits for unemployment insurance benefits), jumps in the number of claims filed following business layoffs and failures, and increased incentives for workers to take time off to heal nagging injuries. Conversely, as the unemployment

[6]The correlation coefficient between employment and the unemployment rate over the entire 37-jurisdiction, 15-year sample is 0.035.

rate decreases in an economic expansion, the opportunity cost of filing a claim rises, reducing frequency, severity, and the loss ratio. The dominant impact of the unemployment rate on workers compensation loss ratios will be determined empirically.

The business failure variable is sometimes included in the models to measure crisis labor market conditions. The a priori expectation is that the sign on this variable will be positive. The preponderance of anecdotal evidence from industry executives, business owners, risk managers, and others suggests that business failures and plant closings provide a direct and special motivation for employees to file workers compensation claims because the benefits generally are larger and paid over a longer period than unemployment benefits. Moreover, workers compensation benefits are non-taxable. By one estimate, approximately forty to fifty percent of laid-off workers will file workers compensation claims against their employers within six months of termination [3, 4]. An increase in business failures is also expected to lead to an increase in severity because the employee's objective is to obtain a total workers compensation benefit that exceeds the expected unemployment benefit. Severity may also be higher for these claims as a result of the type of injury. Some chronic injuries may be concealed for extended periods of time, only to be revealed upon layoff.

Two variables that are independent of the business cycle, but are included in the models, are waiting period and cost containment. The underlying rationale for including the waiting period variable in the frequency model is straightforward. Longer waiting periods are a barrier and disincentive for workers to file claims, especially for minor injuries. Hence, an inverse relationship between frequency and the waiting period is expected. However, the expected sign on the waiting period coefficient in the indemnity severity regression is positive. Because longer waiting periods are a disincentive to file minor claims, the average severity of the remaining claims will be higher in states with longer waiting periods (holding all other factors constant), particularly if after a retroactive period the employee collects back to the date of injury.

An increase in the waiting period should have no statistically significant impact on the loss ratio. This is because the effects on losses and premiums associated with the waiting period increase should cancel out after on-leveling. However, on-level adjustments do not take into consideration the fact that law changes alter the incentives for workers to use the system. The sign on this variable in the loss ratio equation will depend on whether workers respond to the altered incentives and is discussed in more detail in Section 4.

The cost containment dummy variable is included as a proxy for the vigorous efforts adopted by many states and insurers in the early 1990s to attack rapidly rising workers compensation costs. The logical expectation is that such efforts reduce system costs. Thus, a negative sign on the cost containment coefficient is anticipated. Commonly employed reform initiatives, such as medical fee schedules, anti-fraud campaigns, managed care, and utilization review, have met with widely varying degrees of success across the states. Moreover, because the savings generated through legislative reform or insurer cost containment initiatives are not always readily observable, the dummy variable approach is a practical alternative way to quantifying these initiatives in dollar terms.[7]

Hospital cost per stay is included in the medical severity and medical loss ratio models as a measure of medical costs. The relationship between this variable and medical severity and the medical loss ratio is expected to be direct or positive. Ideally, the medical models would include a physician cost variable. Unfortunately, these data are not available at the state level.

Indemnity severity is included in the medical severity equation and the indemnity loss ratio is in the medical loss ratio equation

[7]A detailed discussion of dummy variables is contained in Appendix A.

to account for some of the variables that indirectly affect medical losses. NCCI believes that medical costs are driven, in part, by workers' demands to file indemnity claims. Beyond the obvious observation that an increase in overall claim severity drives up both indemnity and medical severity, explanations for this expectation include the possible longer observed durations of claims for higher wage earners, greater expectations from medical care by the high earners, and the tendency of these high earners to be in more urban areas where access to medical specialists and state-of-the-art technologies are more prevalent [22].

3. MODELING METHODOLOGY AND DESCRIPTION OF THE DATA

Modeling Methodology

The specifications of the frequency, severity, and loss ratio econometric models used in this study were determined after evaluating and testing families of models containing alternative specifications. These families of models used alternative variables and lag structures to measure the production and labor market effects discussed in the previous section. During this testing phase of the analysis, candidate explanatory variables were selected and evaluated.[8] Variables were rejected when they either failed to achieve statistical significance, behaved erratically over time, or were inconsistent with economic theory. The models themselves were rejected when they did not meet specific goodness-of-fit criteria based on the adjusted-R^2 statistic.

In the cross-sectional time series study reported here, parameters of the model were estimated using ordinary least squares (OLS) multiple regression analysis. All variables were in natural logarithmic form. Where appropriate, estimates were corrected

[8]The testing of many models may overstate at times the statistical significance of the chosen models since, for example, out of every 100 invalid models tested, one would likely pass significance tests at the one percent level. However, in the case of this study, the various families of models were similar and measured the same effects, but with different variables.

for autocorrelation and heteroscedasticity.[9] State dummy variables were included to control for omitted state-specific effects such as fraud, administrative variations, or other differences such as the propensity to litigate claims.[10]

NCCI maintains an extensive data base containing insurance industry, economic, demographic, and regulatory data in order to support its modeling efforts. Initial choices of explanatory variables were determined by researching the historic and current economic conditions nationally and in individual states. The data base information on economic conditions was obtained from four sources: federal and state government agencies, state universities, and private forecast and database concerns, such as Dun & Bradstreet. This information has enabled NCCI to account for certain state-specific cost drivers as well as more regional and national developments that have affected workers compensation frequency, severity, and loss ratios.

There is no compelling practical or theoretical reason to expect that the final specification of any of the models will be identical. For example, the model that best estimates indemnity severity generally will not result in the best estimates of indemnity loss ratios, even though both dependent variables are statistically correlated.

Structural stability of the estimated models is often an issue in econometric analyses. In this study, Chow tests were performed to measure the stability of the models across states and over time [12]. When performing Chow tests, the regression is rerun

[9]Durbin–Watson statistical tests were performed to check for the presence of autocorrelation. Autocorrelation arises when regression error terms are correlated through time. The consequences of failing to account for the presence of autocorrelation include inefficient ordinary least squares estimates of the regression model parameters (i.e., standard errors are inflated) and misleading hypothesis tests.

Heteroscedasticity is the situation where the model error terms do not have the same variance. When heteroscedasticity is present and the problem is not corrected, hypothesis tests will also be misleading. See Appendix B for a discussion of the method used to correct for autocorrelation and heteroscedasticity.

[10]See Appendix A for a detailed discussion of the role and interpretation of dummy variables.

on subsamples of the overall sample, and the estimated models are compared to determine if they are significantly different. In this analysis the sample was split by state randomly and based on population. The sample was also divided in half by year. The results from these various tests were mixed. In order to control for the differences among states and years, state dummy variables and the cost containment dummy variable discussed above were added to the initial models.

Dependent Variable Definitions

The dependent variables used in the analyses were computed using statewide (voluntary and residual market) premium, loss, and claim count data collected from workers compensation carriers through financial data calls. The loss and claim count data used are on an accident year basis, while accident year premium data are derived from a weighted average of two policy years. In the models, the accident year insurance data are paired with the economic data for the corresponding calendar year. Data are for 37 jurisdictions over the 15-year period 1979–1993.[11]

All data used in this study were thoroughly validated. Severities and frequencies by state were compared to Unit Statistical Plan data, and the analysis excluded all companies with suspect claim counts. This is the same data used in NCCI's rate level analyses.

The claim count data and indemnity and medical losses were developed to ultimate. Development factors varied by state based on a by-state analysis to determine which method would produce the most accurate estimate of ultimate losses. In general, the de-

[11]The jurisdictions included in the study are: Alabama, Alaska, Arizona, Arkansas, Colorado, Connecticut, Florida, Georgia, Hawaii, Idaho, Illinois, Indiana, Iowa, Kansas, Kentucky, Louisiana, Maine, Maryland, Michigan, Mississippi, Missouri, Montana, Nebraska, New Hampshire, New Mexico, North Carolina, Oklahoma, Oregon, Rhode Island, South Carolina, South Dakota, Tennessee, Utah, Vermont, Virginia, Wisconsin, and the District of Columbia.

velopment methodology is comparable to that selected in each state's rate filing. The data were also brought on-level. Indemnity and medical losses were brought to current benefit levels, and the measure of premium used in this study, designated statistical reporting (DSR) level standard premium, was brought to current bureau loss cost level.[12] All loss data used in the calculation of the dependent variables exclude all loss adjustment expenses.

The following describes how the frequency variable used in this analysis was calculated from the data elements of the financial call:

$$\text{Frequency} = \frac{\text{indemnity claims developed to ultimate}}{\text{workers (in hundred thousands)}}.$$

The number of workers (in hundred thousands) is estimated by year by state from premium as follows:

$$\left(\frac{\text{on-level DSR premium}}{\text{DSR average rate}} \times 100\right) \div (\text{average weekly wage} \times 52 \times 100{,}000).$$

The DSR average rate is the weighted average of the DSR rate by class multiplied by the Unit Statistical Plan (USP) payroll by class for the state. The average weekly wage is from the Current Population Survey performed by the Bureau of Labor Statistics, adjusted to exclude businesses not generally covered by workers compensation.

The severity dependent variables are defined as follows:

Indemnity severity

$$= \frac{\text{real indemnity losses on-level and developed to ultimate}}{\text{indemnity claims developed to ultimate}},$$

[12] DSR standard premium essentially represents the premium that would have been charged before adjustments such as company deviations, loss cost multipliers, premium discounts, retrospective rating, or schedule rating.

and

$$\text{Medical severity} = \frac{\text{real medical losses (excluding medical-only) on-level and developed to ultimate}}{\text{indemnity claims developed to ultimate}}.$$

In this study, the nominal or current (unadjusted) indemnity severity was converted to real terms using an average weekly wage index constructed for each state. Medical severity was deflated using the medical component of the Consumer Price Index (CPI) produced by the United States Bureau of Labor Statistics. For the medical severity dependent variable, a factor derived from USP data was applied to total medical losses to remove all medical-only dollars. Since claim counts in the financial calls exclude medical-only claims, this was necessary to achieve consistency between the numerator and denominator of medical severity.

The loss ratio dependent variables are defined as follows:

$$\text{Indemnity loss ratio} = \frac{\text{indemnity losses on-level and developed to ultimate}}{\text{loss cost portion of on-level DSR premium}},$$

and

$$\text{Medical loss ratio} = \frac{\text{medical losses on-level and developed to ultimate}}{\text{loss cost portion of on-level DSR premium}}.$$

In some states DSR premium is loss costs, while in others it is rates. In this analysis, however, it was adjusted to loss costs for all states. While the loss ratio typically uses full premium in the denominator, in this analysis the term "loss ratio" is used to describe the ratios defined above.

To summarize, the data base constructed for this study permits the analysis of a significant number of potential cost drivers affecting frequency, medical and indemnity severity, and medical and indemnity loss ratios. The ultimate goal has been to ensure that the final model is statistically sound, logically consistent in terms of economic theory, and that it is the best among alternative specifications.

4. RESULTS AND INTERPRETATION

Claim Frequency and the Business Cycle

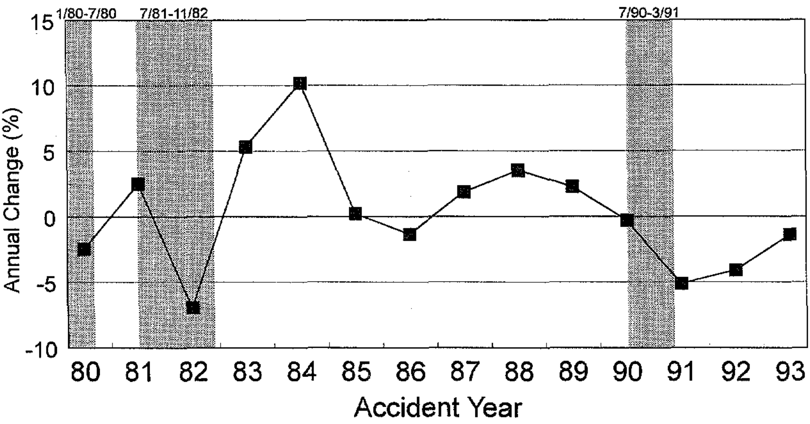
Numerous hypotheses suggest a relationship between workers compensation claim frequency and the economic environment, as discussed in Section 2. The a priori expectation in this study is that claim frequency will tend to rise during periods of economic expansion and fall during contractions.

Figure 1 shows the average annual percentage change in work-related claim frequency for the 37 jurisdictions in our sample.[13] The frequency decreases shown for 1980, 1982, 1990, and 1991 are coincident with national economic recessions (shaded regions) during those years. Frequency increased sharply during the economic recovery that began in 1983 before reaching a plateau in 1985. Modest further increases in frequency were recorded as the economic recovery matured during the late 1980s. The period beginning in 1983 was the longest sustained recovery in the post-World War II era, but eventually gave way to recession during the second half of 1990 and into 1991. The modest decrease in claim frequency in 1992 and 1993 is consistent with the most recent recovery's unusually slow rate of job creation during the first two years of expansion. Job creation and

[13] A simple average of the claim frequency data for the 37 jurisdictions was calculated for each year, 1979 to 1993. The annual percentage changes included in Figure 1 were calculated based on these simple averages. A simple average was used instead of a weighted average to more closely match the data in the study since it is cross-sectional time series rather than total national.

FIGURE 1

ANNUAL CHANGES IN FREQUENCY (%)



an expected upswing in frequency were limited because 18 of the 37 jurisdictions in the sample remained mired in recession even after the national recession ended in March 1991. Frequency increases were also limited because of the workers compensation system reforms that were initiated in these years in several states.

Table 2 summarizes the results from two econometric estimates of claim frequency. The absolute *t*-statistic is shown in italics below the estimated coefficient.[14]

The dependent variable in both regressions, *l*FREQPW, is the natural logarithm of frequency. The "*l*" preceding NAGEMP, UNRATE, and WAITPER indicates that these variables have also been converted to natural logarithms.

The empirical results for both models shown in Table 2 are consistent with prior expectations. The coefficient on the employment variable is positive, while the coefficients on the waiting

[14]The *t*-statistics are used to test the null hypothesis that the respective coefficient is equal to zero.

TABLE 2

WORKERS COMPENSATION FREQUENCY MODELS
COEFFICIENTS AND *t*-STATISTICS
Dependent Variable = *l*FREQPW

Independent Variable/Constant	Models (1)	(2)
Constant	3.64 *5.59*	3.78 *5.81*
*l*NAGEMP	0.58 *7.16**	0.59 *7.31**
*l*UNRATE	−0.09 *3.18**	−0.09 *3.19**
COSTCON	−0.06 *3.48**	−0.05 *3.25**
*l*WAITPER	—	−0.16 *2.54**
State Dummies	Yes	Yes
Adjusted-R^2:	0.907	0.909
N:	555	555

*Significant at the 1% level.

period and cost containment variable are negative. As discussed in Section 2, it is difficult to determine a priori the expected sign on the unemployment rate variable. The sign on the unemployment rate coefficient in the frequency regression is negative. That is, a *decrease* in the unemployment rate is associated with an increase in frequency. The interpretation is that the marginal hires are more likely to be injured [28]. This effect dominates any possible countervailing effects as discussed in Section 2.

It is worth noting that the nonagricultural employment and unemployment rate variables tend to move in opposite directions. For this reason, the opposite signs on the two coefficients reinforce each other, resulting in an amplification effect. Hence, claim frequency is expected to increase during economic expansions and decline during contractions or periods of sluggish growth. Changes in the cost containment and waiting period variables are independent of the business cycle.

Because the models were estimated in logarithms rather than in levels, the variable coefficients can be interpreted as elasticities or sensitivities. For example, the coefficient on the *l*NAGEMP variable in model (1) indicates that a 10 percent increase in nonagricultural employment leads (approximately) to a 5.8 percent increase in claim frequency. Likewise, a 10 percent decrease in the unemployment rate is associated with a 0.9 percent increase in frequency.

This example can be made more realistic by using actual 1995 national forecast values from Regional Financial Associates (RFA), an econometric forecasting organization.[15] Using Model 1, RFA's forecast for 2.5 percent employment growth nationally is expected to lead to a 1.45 percent increase in frequency. Similarly, the projected 8.2 percent decline in the unemployment rate (from 6.1 percent to 5.6 percent) is associated with a frequency increase of 0.74 percent.

Based on the results from the above models, Table 3 presents specific examples of how cyclical economic factors can influence the workers compensation line. Because the arguments presented here generally are symmetric with respect to the phase of the business cycle, discussion is limited to the case of economic expansion. While the cited factors should not be construed as an exhaustive list, they are representative of some of the more important developments that affect frequency when employment and the unemployment rate change.

An increase in employment and decrease in the unemployment rate during an economic expansion is expected to increase

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[15] *Précis*, Vol. 3, No. 6, Regional Financial Associates, West Chester, PA, June 1995.

TABLE 3

ECONOMIC RATIONALE FOR CHANGES IN FREQUENCY DURING ECONOMIC EXPANSIONS

Employment Increases (increases frequency)	Unemployment Rate Decreases (increases frequency)
• Employment increases in hazardous industries	• Overtime increases, leading to increased worker fatique
• Overtime increases, leading to increased worker fatigue	• Inexperienced workers, more prone to injuries, are hired or rehired
• Less machine maintenance diminishes job safety	• Workers who deferred claims in a recession for fear of losing job may file now
• Older, less safe and less efficient equipment may be reused	
• Inexperienced workers, more prone to injuries, are hired or rehired	
• Workers who deferred claims in recession for fear of losing job may file now	

claim frequency for several reasons, some of which are listed in Table 3 and discussed here.

First, employers must hire new workers to meet increased demand during an economic expansion. In general, these workers tend to be younger and less experienced, resulting in more frequent injuries. In this case, it is the level of economic activity that compels employers to expand employment.

Increased demand for goods and services also leads to higher workers compensation costs indirectly. For example, during an economic expansion, producer shipments will rise and business-related vehicle traffic increases. Increasing vehicle travel will lead to an increase in claim frequency. Moreover, motor vehicle accidents are the leading cause of on-the-job fatalities [4].

Another reason why an economic expansion may lead to higher claim frequency is linked to employment growth in hazardous or risky industries, such as in the highly cyclical construction, heavy manufacturing, and mining industries. In this case, it is the shifting mix of employment that is the key transmittal mechanism from the economy to the workers compensation system.

Increased utilization of the system by workers also contributes to an increase in frequency during an economic expansion. Employees who feared they would lose their jobs if they filed a claim during the recession may file once the economy recovers. A 1993 NCCI survey of twenty of the largest workers compensation carriers found that workers' concerns over job security can override the incentive to file a workers compensation claim. The same observation has been made by some in the risk management community [26].

Finally, as employment rises and the pace of economic activity quickens, machine usage increases, and less maintenance and overall safety may accompany the higher capacity [25]. Also, workers' overtime hours increase, leading to fatigue and an increase in accidents. During a 1994 strike, workers at General Motors Corporation cited excessive overtime as the primary reason for their walkout. GM ended the dispute by agreeing to hire hundreds of new workers [27].

The association between higher claim frequency in general and economic expansion has been documented most recently in California. The California Workers Compensation Institute (CWCI) recently reported a 21 percent increase in indemnity claims frequency and an 11 percent increase in medical-only claims frequency; it attributes these increases to the state's recovering economy [8].

The converse of the above arguments is also generally true. In other words, the economic factors that contribute to increasing claim frequency during an economic expansion work to reduce frequency during a contraction.

Claim Severity and the Business Cycle

Both indemnity and medical severities increased rapidly during the early 1980s. Over the 1980–1982 period, the average annual increase in indemnity severity was 10.2 percent and 15.0 percent in medical severity. It is important, however, to recognize that the early 1980s was an inflationary period and that the behavioral and economic decisions made by workers, employers, insurers, and others are responses to real (inflation adjusted) changes in benefits and costs. Moreover, the true impacts of the explanatory variables on severity cannot be observed without first controlling for inflation. For these reasons, it is more informative to use inflation adjusted or real severity data in this analysis.[16]

Figure 2A and Figure 2B compare annual changes in nominal and real indemnity and medical severity, respectively. Indemnity severity increases recorded in 1981, 1983, 1984, 1992, and 1993 were actually declines when measured in real terms. The interpretation is that the average cost of an indemnity claim fell relative to the mean increase in average weekly wages during those years. These years approximately correspond to the recession induced frequency declines discussed above.[17] The decreases in severity are consistent with the decline in hazardous industry employment and the previously discussed incentive for workers to defer filing claims during recessionary periods out of fear of losing their jobs.

[16]As discussed in Section 3, nominal or current (unadjusted) indemnity severity was converted to real terms using an average weekly wage index constructed for each state, while medical severity was deflated using the medical component of the Consumer Price Index (CPI).

[17]Recession induced severity declines, as shown in Figure 2A and Figure 2B, are not precisely coincident with national economic recessions because the experience of our 37-jurisdiction sample differs from the nation's experience. For example, 27 states in the sample were in recession for some part of 1983, even though the national recession ended in November 1982.

FIGURE 2A

ANNUAL CHANGES IN INDEMNITY SEVERITY (%)

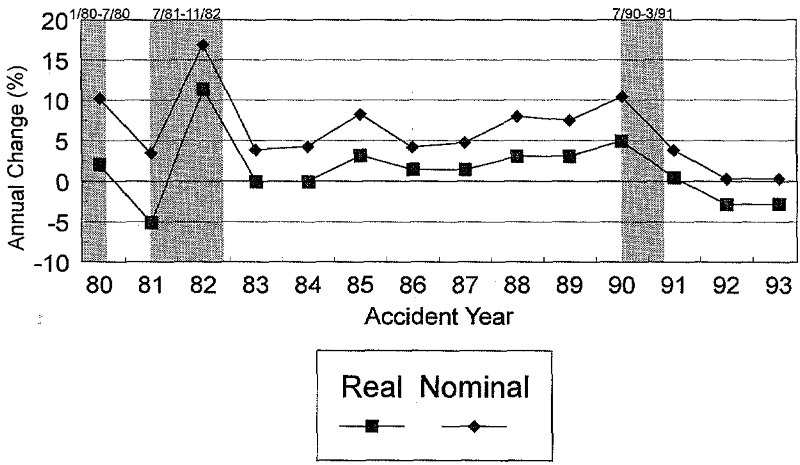
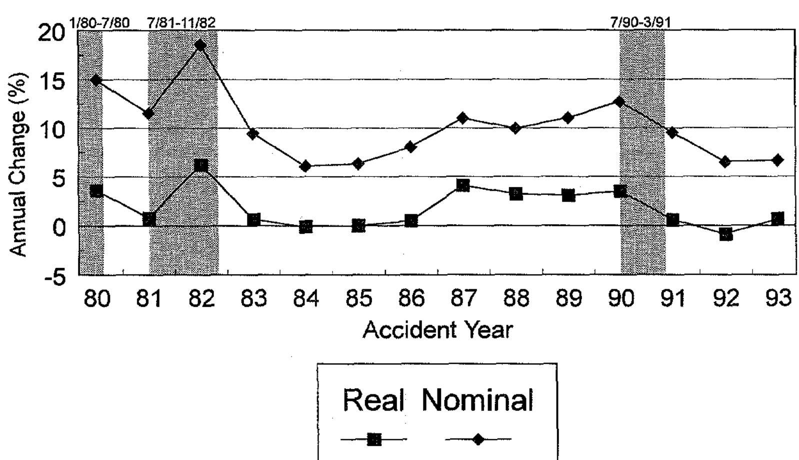


FIGURE 2B

ANNUAL CHANGES IN MEDICAL SEVERITY (%)



Changes in real medical severity, on the other hand, have remained positive for all years over the sample period except 1984 and 1992. Increases have occurred despite adjusting for inflation using the medical component of the CPI, which is significantly higher than the overall CPI. Thus, workers compensation medical severity not only grew faster than the general rate of inflation (as measured by the CPI), but also faster than medical inflation nationwide.

The hypothesis that real indemnity and medical severity vary over the business cycle can be tested econometrically. Table 4 summarizes the results from the econometric estimate of indemnity and medical claim severity. The absolute *t*-statistic is shown in italics below the estimated coefficient.

All variables have been converted to natural logarithms. As in the frequency regressions, estimation of the severity models in logarithmic form implies that the variable coefficients can be interpreted as elasticities. For example, the coefficient on the *l*NAGEMP variable in the indemnity model implies that a 10 percent increase in nonagricultural employment leads to a 6.8 percent increase in claim severity. Similarly, a 10 percent increase in hospital cost per stay is associated with a 2.5 percent increase in medical claim severity.

Unlike the frequency regressions, the unemployment rate variable in the severity regressions has a positive sign, indicating that a direct effect is the dominant impact. The signs on employment, business failures, waiting period, and hospital cost per stay variables are consistent with prior expectations discussed in Section 2.

As discussed previously, employment and the unemployment rate generally move in opposite directions. The positive sign on the coefficients of both variables means that a decrease in the unemployment rate during an economic expansion, for example, tends to dampen the severity increasing influence of expanding employment. Empirical evidence indicates that the employment

TABLE 4

WORKERS COMPENSATION REAL SEVERITY MODELS COEFFICIENTS AND t-STATISTICS Dependent Variable = lINDSEV, lMEDSEV

Independent Variable/Constant	Models: Indemnity	Models: Medical
Constant	2.11 *1.54*	2.07 *4.77*
lNAGEMP	0.68 *3.86**	—
lUNRATE	0.13 *2.25***	—
lBUSFAIL	0.04 *2.59**	—
lWAITPER	0.35 *4.16**	—
lPERSTAY	—	0.25 *5.74**
lINDSEV	—	0.44 *8.51**
State Dummies	Yes	Yes
Adjusted-R^2:	0.908	0.906
N:†	370	555

*Significant at the 1% level.
**Significant at the 5% level.
†Because business failures data were available beginning only in 1984, the number of observations in the indemnity model is less than for the medical model, which spans the entire 1979–1993 sample period.

effect is dominant. To illustrate this point, RFA's 1995 forecast for 2.5 percent employment growth would be expected to lead to a 1.7 percent increase in indemnity severity, while the projected 8.2 percent decrease in unemployment would be associated with a 1.1 percent *decrease* in severity. In combination, there is an expected 0.6 percent increase in severity, holding all else constant.

TABLE 5

ECONOMIC RATIONALE FOR CHANGES IN INDEMNITY CLAIM SEVERITY DURING ECONOMIC EXPANSIONS

Employment Increases (increases severity)	Unemployment Rate Decreases (decreases severity)
• Employment increases in hazardous industries	• Opportunity cost of claim rises, reducing duration
• Truck shipments increase	
• Overtime increases, leading to increased worker fatigue	
• Less maintenance; older machinery reused	
• Unfamiliarity of new employees with machinery	

Some of the important economic factors that contribute to increasing indemnity claim severity during an economic expansion are shown in Table 5. The converse is generally true during economic contractions.

Indemnity and Medical Loss Ratios and the Business Cycle

The statistical evidence presented thus far in this paper has documented that econometric techniques are an important tool that can be used to quantify the relationship between the economic environment and workers compensation frequency and severity. In this section, the econometric methodologies developed in the previous two sections are extended and modified to model indemnity and medical loss ratios over the course of the business cycle.

Loss ratios provide a more complete picture of the economy's impacts on the workers compensation industry because they incorporate information on both losses and premiums. Unlike other lines of insurance, workers compensation premiums are assessed as a proportion of total payroll. Payroll at the firm level often is subject to considerable variability over the course of the business cycle, thereby affecting premium collections. Variability in losses is due to changes in both the corresponding exposure base and the complex interactions between economic variables as they impact frequency and severity.

In general, the factors that best explain changes in medical and indemnity loss ratios will be different from those that best explain frequency and medical and indemnity severity. This is because modeling loss ratios incorporates the interactions between frequency and severity, while modeling frequency and severity separately does not incorporate these effects.

Table 6 summarizes the results of the estimated indemnity and medical loss ratio models.

The positive sign on the employment variable, *l*NAGEMP, in the indemnity model is consistent with a priori expectations and with the frequency and severity findings presented above. As employment rises and the pace of economic activity quickens, the indemnity loss ratio tends to deteriorate. This is partly because the accelerated rate and higher level of production cause machine usage to increase and older, less safe machinery to be brought back on line as capacity constraints are approached. Increased worker fatigue due to a faster pace of production and abundant overtime opportunities is likely to contribute to higher injury severities as well as increased claim frequency.

As in the severity model, the sign on the unemployment rate variable in the loss ratio regression is positive. A decline in the unemployment rate during an expansion is associated with a probable slight decline in the loss ratio. The decline is principally the result of the increasing opportunity cost to the employee (and employer) of a workers compensation claim. Rising wages and overtime opportunities diminish the incentive of the worker to file or stay out on a claim. Concomitantly, the cost to the

TABLE 6

WORKERS COMPENSATION LOSS RATIO MODELS
COEFFICIENTS AND t-STATISTICS
Dependent Variable = lINDLRAT, lMEDLRAT

Independent Variable/Constant	Models: Indemnity	Models: Medical
Constant	−12.32 *6.42*	−8.02 *29.90*
lNAGEMP	1.44 *5.98**	—
lUNRATE	0.08 *1.34*****	—
lWAITPER	0.22 *2.26**	—
COSTCON	−0.06 *1.56****	—
lPERSTAY	—	0.91 *29.41**
lINDLRAT	—	0.53 *16.05**
State Dummies	Yes	Yes
Adjusted-R^2:	0.757	0.898
N:	555	555

*Significant at the 1% level.
**Significant at the 5% level.
***Significant at the 10% level.
****Significant at the 20% level.

employer of losing a worker increases during periods of strong demand. Hence, the increasing opportunity cost of labor income and the rising value of the worker to the employer contribute to a decline in the loss ratio during an economic expansion.

As discussed in Section 2, an increase in the waiting period should have no statistically significant impact on the loss ratio since the effects on losses and premiums associated with the waiting period increase should cancel out after on-leveling. However, the coefficient on this variable in the above equation is positive and significant.

One reason for this result is that on-level adjustments do not take into consideration the fact that law changes alter the incentives for workers to use the system. Specifically, some proportion of workers will be sufficiently motivated to stretch the duration of their claim to meet the new waiting period, thereby increasing average claim severity. Importantly, the new (longer) waiting period will be several days closer to the retroactive date (assuming the retroactive period is not also increased), at which point benefits are paid retroactively to the injury date. Because the time interval between the waiting period and retroactive period has been reduced, the cost to the worker of extending the claim to the retroactive period is also reduced. Ironically, the net result of an increase in the waiting period may be to increase average severity and total system costs.[18]

Finally, the negative sign on the cost containment coefficient indicates that cost containment initiatives are successful in reducing system costs.

Overall, however, the employment effect is dominant, leading to the conclusion that the indemnity loss ratio will increase during an economic expansion and decrease during an economic contraction.[19]

The numerical interpretation of the results is analogous to the frequency and severity models discussed previously. Estima-

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[18]Some industry observers believe that the on-level (law amendment) factors were generally overestimated. Systematic overestimation of these factors would lead to a perceived increase in severity. We thank an anonymous referee for bringing this possibility to our attention.

[19]Previous research examining the incentive effects contributing to workers compensation loss ratios includes Butler and Worrall [7]. Butler [5] also contains a comprehensive review of incentive effect studies, including several using NCCI data.

tion of the models in logarithmic form permits interpretation of the estimated coefficients as elasticities. The 1995 employment growth forecast of 2.5 percent implies a 3.6 percent increase in the indemnity loss ratio. Similarly, the projected 8.2 percent decrease in the unemployment rate would be expected to lead to a 0.7 percent decline in the indemnity loss ratio. In combination, they would lead to an expected 2.9 percent increase in the indemnity loss ratio, holding all else constant.

The medical loss ratio model is analogous to the medical severity model. The 2.9 percent increase in the indemnity loss ratio discussed above implies a 1.5 percent increase in the medical loss ratio.

The coefficients on the economic variables displayed in Table 6 indicate how loss ratios are expected to change over the course of the business cycle—rising during expansions and falling during contractions. Failure to consider these factors may result in systematic and cyclical bias when actuarial trend indications are used.

5. SUMMARY AND CONCLUSIONS

The empirical results presented in this study provide significant statistical evidence that econometric modeling is a powerful explanatory and diagnostic tool for explaining variability in the workers compensation line over the course of the business cycle. Specifically, this study demonstrates that cross-sectional time series techniques can be used to estimate the relationship between the economic environment and workers compensation claim frequency, indemnity and medical severity, and indemnity and medical loss ratios over the 15-year, 37-state, sample period. The estimated frequency, severity, and medical loss ratio models explain ninety percent or more of the variability in the respective dependent variable. More than seventy-five percent of the variability in the indemnity loss ratio is explained.

This study's major empirical finding is that there is a strong association between economic growth and rising frequency, severity, and loss ratios.[20] The converse is also true. This finding displaces the conventional wisdom on this subject, which has held that the workers compensation line generally improves during economic expansions and worsens in recessions. A possible consequence of not accounting for the impacts of economic factors on loss ratios is rate inadequacy. During a prolonged economic expansion, the inadequacy problem is compounded. Conversely, during recessionary periods, rates may become redundant.

Extensions and Directions for Future Research

One logical application of this study's findings is to employ econometric time series techniques at the individual state level to generate econometric estimates of trend. Actuarial methodologies traditionally have relied on curve fitting techniques that depend on historical values of the loss ratios and time to estimate trend. Econometric trend models, in contrast, draw upon a rich base of economic, demographic, and insurance variables.[21] Along with their intuitive appeal, econometric models can incorporate predicted future changes in the economy that may affect the direction and growth of losses and premiums. Moreover, econometric forecasts use all available data points to capture the impacts of the economy on loss ratios over the full course of the business cycle.

Econometric trend estimates have been filed by NCCI in several states, using models similar to those explored in this study. The models typically include a measure of overall economic activity, a measure of the health of the labor market, and possibly

[20]The conclusion is demonstrated for the business cycles during the data period used in this analysis, but may not follow over different periods of time.

[21]For a more complete discussion of econometric trending methodologies, see Hartwig, Kahley, and Restrepo [15].

variables relating to crisis labor market conditions or the share of employment in risky industries in the state. As in this study, logarithmic regression models are estimated separately for indemnity and medical loss ratios. Forecast values of the independent variables are then used to forecast expected loss ratios (the dependent variable). Applying these techniques to workers compensation claim frequency and severity at the state level is another logical extension that NCCI plans to explore.

NCCI plans to continue its study of the relationship between the business cycle and the workers compensation industry. To date, the results of these analyses are promising. Research and methodological refinements are ongoing, and the knowledge gained from modeling a larger number of states across a broad spectrum of economic and workers compensation experience should prove to be invaluable.

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APPENDIX A

DUMMY VARIABLES IN ECONOMETRIC ANALYSIS

Variables used in regression analysis are usually, but not always, continuous over some range. There are situations, however, where explanatory or independent variables are qualitative in nature, taking on two or more distinct values. For example, individuals are male or female, and gender may explain certain behaviors or outcomes. Similarly, investment and consumption decisions may be influenced by whether a country is at war or peace. In such instances, a proxy variable, referred to as a "dummy variable" in econometrics, must be constructed. The dummy variable takes on the value unity whenever the qualitative phenomenon it represents occurs, and zero otherwise. For estimation purposes, the dummy variable is treated no differently than any other explanatory variable, and no modifications to the chosen estimation technique are necessary.[22]

Fixed Effects Models

Qualitative variables are frequently not dichotomous and may assume more than two distinct values (female/male). In the context of the present study, the possibility that the intercept varies across the $N = 37$ cross-sectional units (states) is recognized and incorporated in the frequency, severity, and loss ratio models by constructing dummy variables for $(N - 1)$ states in the sample.[23] One state is omitted to avoid perfect multicollinearity. With perfect multicollinearity, the regression matrix is singular and, therefore, coefficients for the explanatory variables cannot be estimated. The individual contribution of the omitted state is given by the value of the constant term in the regression equation. Such observation specific dummy variables are equal to unity

[22] A more detailed discussion on dummy variables is given in Kennedy [18] and most other econometric textbooks.

[23] State dummy variables in this study are generally statistically significant at the one percent level.

for a specific observation (e.g., Alabama) and zero for all other observations (i.e., every state except Alabama).

The fact that the 37-state sample is observed over a 15-year period suggests another possible application of dummy variables. Dummy variables could be constructed for $(T - 1)$ years, under the hypothesis that the intercept varies over time, as well as across states (for a total of $(N - 1) + (T - 1)$ dummies). Time dummies would assume the value of one for a specific year and zero for all other years. This hypothesis was tested and rejected using the data in this study.

The use of cross-sectional and/or time dummy variables in a longitudinal (or panel) data context is commonplace in econometrics. This method of analyzing longitudinal data is known as *fixed effects* modeling.[24] Essentially, the dummy variable coefficients reflect ignorance. In other words, dummy variables are introduced for the purpose of measuring shifts in the regression line arising from unknown variables. Of course, other explanatory variables, with hypothesized functional relationships to the dependent variable, generally are included in such models as well.

Non-Observable and Difficult-to-Quantify Variables

Many variables believed to influence the level and growth rate of workers compensation frequency, severity, and loss ratios are difficult to quantify and/or cannot be observed directly or in a practical fashion. For example, governments, insurance companies, and employers embarked upon a myriad of legislative reforms, cost containment initiatives, and loss control programs across the country during the early 1990s. From a practical standpoint, it is impossible to survey and quantify all of these changes in state workers compensation systems. Indeed, insurers are applying their acquired knowledge and experience to states

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[24] A detailed and rigorous discussion of fixed effects modeling is found in Hsiao [16].

where no major legislative reforms have been approved. Moreover, from an econometric perspective, modeling such a large variety of reforms and initiatives would require far more degrees of freedom than are available in this study sample of 37 states over 15 years.

A practical solution to modeling the many and diverse cost containment initiatives across states individually is to model them collectively using a dummy variable as a proxy. In this study, a single dummy variable is constructed, *COSTCON*; it is equal to one for the years 1991–1993 in every state and zero otherwise.[25] The coefficient on the dummy variable can be interpreted as an average estimate of the net impact of legislative reform and other initiatives on workers compensation frequency and the indemnity loss ratio (the variable was not statistically significant in the severity regressions).

[25] Anecdotal evidence suggested that the "turning point" for workers compensation came in the early 1990s. The choice of 1991 as the beginning point is based on the statistical strength of *COSTCON* in that year relative to other years. The ending point is 1993 because that is the last year of data in the sample used in this study.

APPENDIX B

AUTOCORRELATION AND HETEROSCEDASTICITY CORRECTION

When correcting for autocorrelation and heteroscedasticity, Regression Analysis of Time Series (RATS), the programming software used in this study, computes the regression using least squares, but then computes a consistent estimate of the covariance matrix allowing for heteroscedasticity and serial correlation up to a first-order moving average.

Ordinary least squares provides a consistent estimate for β in the regression model $\mathbf{Y} = \mathbf{X}\beta + \mathbf{u}$ in a large number of settings where the standard assumption that the residuals satisfy the equation $\mathbf{V} = \mathbf{E}(\mathbf{uu}') = \sigma^2\mathbf{I}$ is violated. Although least squares may provide consistent estimates of the coefficients, $s^2\mathbf{X}'\mathbf{X}^{-1}$ is not a consistent estimate of the variance of the coefficient estimates. Therefore, tests based on the regression output will be incorrect.

To correct the problem, RATS computes consistent estimators for the covariance matrix of estimators using a procedure that imposes little structure on the matrix $\mathbf{V}$. The estimators for least squares are $(\mathbf{X}'\mathbf{X})^{-1}\text{mcov}(\mathbf{X},\mathbf{u})(\mathbf{X}'\mathbf{X})^{-1}$ where $\text{mcov}(\mathbf{X},\mathbf{u})$ refers to the following matrix

$$\sum_{k=-L}^{L} \sum_{t} u_t X_t' X_{t-k} u_{t-k},$$

and u_t is the residual at time t. Serial correlation is handled by making L non-zero. This corrects the covariance matrix for serial correlation in the form of a moving average of order L [9].[26]

[26]Additional detail regarding the correction can be found in Hansen [13], Newey and West [21], and White [29].

APPLICATION OF THE OPTION MARKET PARADIGM TO THE SOLUTION OF INSURANCE PROBLEMS

MICHAEL G. WACEK

Abstract

The Black–Scholes option pricing formula from finance theory is consistent with the assumption that the market price of the underlying asset at any future date is lognormally distributed with time-dependent parameters and can be shown to be a special case of both a more general option model and a familiar actuarial function used in excess of loss applications. This insight leads to an understanding of the similarity between options and certain insurance concepts. Because insurance and finance have developed separately, different paradigms are used by the practitioners in each field. When these paradigms are shared, a new perspective on risk management, product development, and pricing, especially of insurance and reinsurance, emerges.

1. RELATIONSHIP OF THE BLACK–SCHOLES FORMULA AND THE ACTUARIAL EXCESS OF LOSS FUNCTION

In 1973, Fischer Black and Myron Scholes published their now classic paper entitled "The Pricing of Options and Corporate Liabilities," [1] in which they derived the option pricing formula that bears their name. Gerber and Shiu [2] described that paper as "perhaps the most important development in the theory of financial economics in the past two decades." The advent of the modern derivatives market is generally traced back to the introduction of exchange-traded equity options in the U.S. (1973) and the development of the Black–Scholes model [3].

Black and Scholes showed that under certain conditions the current pure premium,[1] $c_t(S)$, for a "call option" to buy a particular asset for price S, at, and only at, time t (where t is the time to expiry) is

$$c_t(S) = P_0 \cdot N(d_1) - Se^{-rt} \cdot N(d_2), \qquad \text{where}$$

$$d_1 = \frac{\ln(P_0/S) + (r + 0.5\sigma^2)t}{\sigma\sqrt{t}}, \tag{1.1}$$

$$d_2 = \frac{\ln(P_0/S) + (r - 0.5\sigma^2)t}{\sigma\sqrt{t}},$$

and where P_0 is the current market price, r is the risk-free force of interest, σ is a measure of annualized price volatility, and N is the cumulative distribution function of the standard normal distribution.

This is a daunting formula, and in this form it provides little insight into the underlying options pricing problem. One of the key points of this paper is that Formula 1.1, the Black–Scholes formula, is actually a special case of a familiar actuarial function written in an unfamiliar form. This will lead us to some important insights about both options and insurance.

Consider that the pure premium of a call option exercisable only on the expiry date (a "European" option) depends on the market's current opinion about the probability distribution of the market price of the underlying asset on the expiry date. If the option exercise price is S, the option will only be exercised in the event the market price at expiry exceeds S. Its value in these circumstances will be the amount by which the market price exceeds S. In other circumstances, the optionholder will let the

[1] Financial economists use the term "price" or "premium." However, to make clear to actuarial readers that there is no embedded charge for risk or expenses in the Black–Scholes valuation, we shall use the actuarial term "pure premium."

option expire unexercised and, if he wants to own the asset, buy the asset at the market price. The option to buy the asset at a higher than market price will be worthless. The value of the option at expiry is the probability-weighted average of all possible expiry scenarios.

Suppose the probability distribution of market prices at expiry is represented by the random variate x. Then the expected value of the option at expiry is

$$\text{Future Value } [c_t(S)] = \int_S^{\infty} (x - S) \cdot f(x)dx. \qquad (1.2)$$

The expression on the right hand side of Formula 1.2 is the *future value* of the option pure premium, since x is defined for the expiry date, which is in the future. Its present value, discounted at the risk-free interest rate,[2] is

$$c_t(S) = e^{-rt} \int_S^{\infty} (x - S) \cdot f(x)dx. \qquad (1.3)$$

Now compare Formulas 1.1 and 1.3. Formula 1.1, the Black–Scholes formula, depends on the assumption that market prices are lognormally distributed. Formula 1.3 is more general and has no embedded distributional assumption. However, if the variate x in Formula 1.3 is assumed to be lognormal and the correct

[2]This is justified on the basis that using any other rate would open the door to risk-free arbitrage profits. It is possible to create a riskless portfolio by hedging a long position in the underlying asset by selling short an appropriate number of call options on the underlying asset. Because it is riskless, this hedged portfolio must earn the risk-free rate of return. However, for this to be true (and it must be true to avoid risk-free arbitrage profit opportunities), it turns out that the interest rate for discounting the expected value of the call option at expiry must also be the risk-free rate. The finance literature refers to this phenomenon as "risk-neutral valuation" and it applies to valuation of all financial derivatives of assets where suitable conditions for hedging exist. For further discussion of risk-neutral valuation and risk-free discounting, see Hull [7].

In actuarial applications involving insurance claims (where hedging is not possible), it is sometimes implicitly recognized that the risk-free rate is not appropriate by discounting at the risk-free rate, and then adding a risk charge to the discounted result. This is equivalent to discounting at a rate less than the risk-free rate. We have deliberately chosen to characterize $c_t(S)$ as a "pure premium" to leave the door open to an additional risk charge where appropriate.

distribution parameters are chosen,[3] Formula 1.1 can be derived from Formula 1.3. In other words, the Black–Scholes formula is a special case of Formula 1.3. The proof of this is in Appendix A.

Formula 1.2, which differs from Formula 1.3 only by a present value factor, also defines a familiar actuarial function seen frequently in excess of loss insurance applications. For example, if x is a random variate representing the aggregate value of losses occurring during an annual period, then Formula 1.2 defines the expected value of losses in excess of an aggregate loss amount of S. This function is an important tool in pricing aggregate excess or stop-loss reinsurance covers.

A second example relates to the more common type of excess of loss coverage, where the excess attachment point S is defined in terms of individual losses, rather than in the aggregate over a period. In this context, if x is a random variate representing the loss severity distribution with mean M, then Formula 1.2 defines the expected portion of M attributable to losses in excess of S. If the result of Formula 1.2 equals C, then C/M is the excess pure premium factor. If N is the expected number of losses, then NC is the expected value of excess losses.

Let us summarize what we have established. Formula 1.2 defines an important element of excess of loss pricing. It differs only by a present value factor from Formula 1.3, which defines a general formula for European call option pricing. Formula 1.1, the Black–Scholes formula, is a special case of Formula 1.3.

The implication of this is that excess of loss insurance and call options are essentially the same concepts. The one deals with insurance claims and the other deals with asset prices, but the pricing mathematics is basically the same.

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[3]Formulas 1.1 and 1.3 produce the same result if x is a lognormal variate with parameters $(\ln P_0 + rt - 0.5\sigma^2 t, \sigma\sqrt{t})$, where this characterization follows Hogg and Klugman [4], who define a lognormal distribution by reference to the μ and σ of the related normal distribution. See Appendix A for the proof of this.

This insight is potentially tremendously powerful. If excess of loss insurance and call options are essentially the same concept in different contexts, then it must be possible to translate ideas from one context into the other context. In the remainder of this paper, we will explore some of the potential applications of the options market paradigm to insurance problems.

2. IMPLICATIONS OF THE EQUIVALENCE OF OPTION AND ACTUARIAL EXCESS OF LOSS MODELS

The mathematical equivalence of finance theory's Black–Scholes formula and an important actuarial function used in excess of loss insurance applications has a number of important implications for the convergence of insurance and finance. In this paper we will explore a few of them.

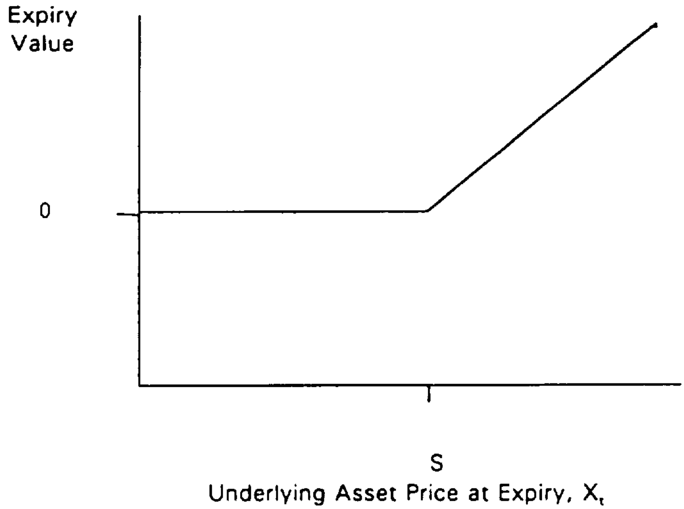
- Option market paradigms can be used to think about insurance problems; and this may well lead to new insurance or, perhaps more likely, reinsurance products.
- The more general actuarial excess of loss paradigm, which encompasses and frequently uses distributions other than the lognormal, can be used to think about the pricing of options on assets for which market prices are not lognormally distributed.
- Taking the two previous points together, it is possible to move beyond existing options and actuarial paradigms to spawn a new one that encompasses both. This, in turn, may lead to new product opportunities for insurers, investors, or both.

3. THE OPTION MARKET PARADIGM

The financial markets have been tremendously creative in devising products and techniques for managing financial risk. Most of this activity has occurred in what is loosely called the "derivatives market." Options are at the core of this market, and it is on

FIGURE 1

EXPIRY VALUE PROFILE: CALL OPTION, $c_t(S)$



this part of the derivatives market that we will focus our attention. Many derivative products are built around option features.

Basic Options

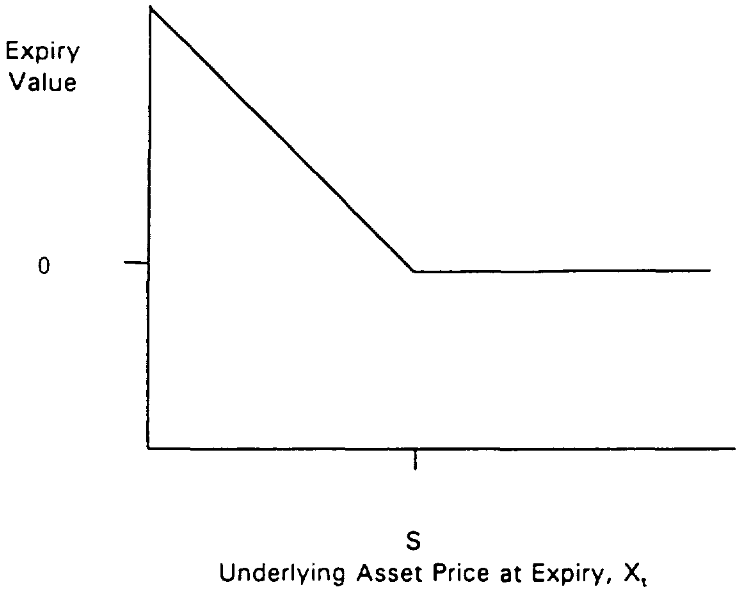
A "European" *call* option, $c_t(S)$, represents the right but not the obligation to buy the underlying asset at, and only at, time t at a price of S. Formula 1.3 describes the price of such a call option. Figure 1 shows its expiry value profile.

An "American" call option incorporates the right to exercise at any time up to and including time t. The Black–Scholes formula applies to the pricing of European calls. In this discussion our references will be to European-style options unless otherwise specified.

A "European" *put* option, $p_t(S)$, represents the right but not the obligation to sell the underlying asset at, and only at, time t

FIGURE 2

EXPIRY VALUE PROFILE: PUT OPTION, $p_t(S)$



at a price of S. Figure 2 shows the expiry value profile of a put option.

The general formula for the price of a put, $p_t(S)$ is

$$p_t(S) = e^{-rt} \int_0^S (S - x) \cdot f(x) dx. \qquad (3.1)$$

Spreads

The combination of two call options, one bought and one sold; e.g.,

$$c_t(S, T) = c_t(S) - c_t(T), \qquad \text{with} \quad T > S \qquad (3.2)$$

is known as a *call option spread.*

In insurance parlance, $c_t(S,T)$ refers to an *excess layer*. $c_t(S,T)$ is the pure premium for the layer of $T-S$ excess of S.

Put option spreads can be defined in a similar way to call option spreads.[4]

Implications for Insurance Applications

Once we recognize that a call spread is the same thing as an excess layer, a new world opens up. In theory, every option and related derivative product must have an insurance analogue! Since the derivative markets have been enormously creative in developing new product ideas, it should be possible to mine that trove of ideas for potentially innovative insurance and reinsurance product concepts.

As an example of how this can be done, we will analyze the derivatives concept of a *cylinder*. Then we will reconstruct it as a reinsurance product.

In its extreme form, a zero cost cylinder is created by the simultaneous purchase of a call and sale of a put (or vice versa) of equal value, usually at different out-of-the-money exercise prices but having the same expiration date.[5] If the cylinder involves a long call (i.e., the purchase of a call) and a short put (i.e., the sale of a put), its value increases when the value of the underlying asset increases and decreases when the asset value decreases. This is a "bullish" position. If the cylinder involves a short call and a long put, its value increases when the value of the underlying asset decreases and declines when the underlying asset value increases. This is a "bearish" position.

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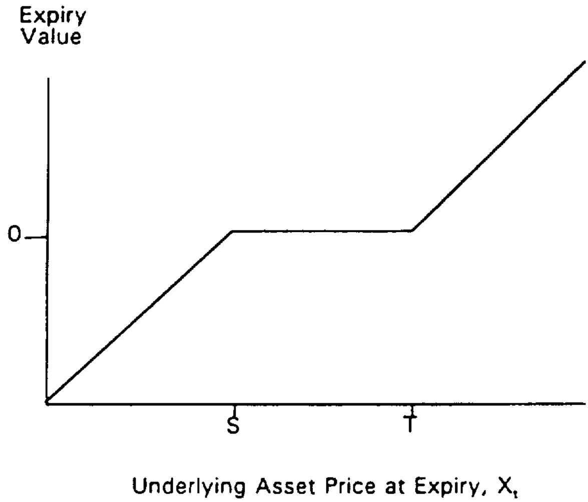
[4]For a detailed discussion of the mathematics of call, put, and cylinder spreads, see Appendix B. There are also a number of good reference books on financial derivatives, including Redhead [5] and Hull [7], that provide more comprehensive treatment of the subject. There is also a British paper, Kemp [8], which examines the subject from a more actuarial perspective, although it is not particularly oriented toward non-life issues.

[5]This is the extreme form. Note that a cylinder need not be "zero cost." For further discussion of cylinders and other option combinations, see [5].

FIGURE 3A

EXPIRY VALUE PROFILE: BULL CYLINDER OPTION $\text{cyl}_t(S,T)$



Bull and bear cylinders are defined as follows:

$$\text{cyl}_t(S,T) = c_t(T) - p_t(S), \qquad T > P_0 > S \quad \text{(bull)}$$

$$-\text{cyl}_t(S,T) = p_t(S) - c_t(T), \qquad T > P_0 > S \quad \text{(bear)}$$

and their expiry value profiles are shown in Figures 3A and 3B.

For an owner of the underlying asset, establishing a *bear* cylinder position partially hedges his asset position and reduces its volatility. Since in the case of a zero cost cylinder the values of the short call and long put are exactly offsetting, no money changes hands at inception of this position. At expiration, if the value of the underlying asset is X_t, the value of the cylinder position is

$$\begin{array}{ll} -(X_t - T), & X_t \geq T; \\ 0, & T > X_t > S; \\ S - X_t, & S \geq X_t. \end{array}$$

FIGURE 3B

EXPIRY VALUE PROFILE: BEAR CYLINDER OPTION $cyl_t(S,T)$

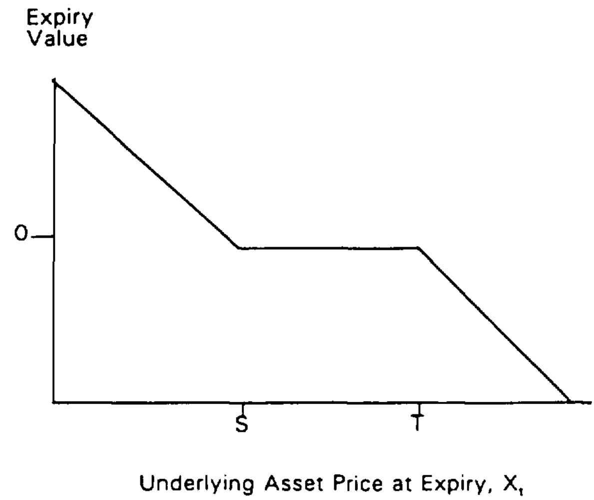


TABLE 1

Expiry Asset Price	Value of Cylinder	Expiry Value Asset + Cylinder
$X_t \geq T$	$-(X_t - T)$	T
$T > X_t > S$	0	X_t
$S \geq X_t$	$S - X_t$	S

The holder of this position gains $S - X_t$ for small values of X_t and loses $X_t - T$ for large values of X_t. For middle values of X_t he gains or loses nothing. His hedged position at expiry of the cylinder is summarized in Table 1.

In words, this implies that the hedged position yields the returns of the underlying asset (i.e., $X_t - P_0$), but subject to a maximum loss of $P_0 - S$ and a maximum gain of $T - P_0$.

If, rather than owning the underlying asset, an investor has a short position in it (i.e., it is a liability), he can partially hedge that position with a *bull* cylinder.

Suppose the underlying asset is the right to recover insurance claims. To an insured, this is an asset (a "long" position). To an insurer, it is a liability (a "short" position). Therefore, an insurer could use a bull cylinder to partially hedge his exposure.

If there were an established derivatives market trading options on insurance claims, as there is for a number of other financial assets, an insurer would be able to hedge its exposure by buying any of a variety of products; e.g., call options, call spreads, bull cylinders, or bull cylinder spreads. At present, there is only a limited derivatives market for options on insurance claims (namely, the excess of loss reinsurance market) and, broadly speaking, it offers only one product: the call spread.[6] One of the key themes of this paper is that conceptually there is no reason why the reinsurance market could not offer similar products to those found in the broader derivatives market.

Now let us consider how the cylinder concept, which has the advantage of lower initial cost to the buyer compared to a simple call option, might be translated into a reinsurance product. To illustrate one way this might work, first imagine a high level excess of loss layer with a retention of T_1 and a limit of $T_2 - T_1$. The market premium, ignoring all expenses, for conventional coverage is $c_t(T_1, T_2)$.

To create the cylinder type structure, we need to introduce a feature equivalent to the sale of a put. Consider a second, unreinsured, layer of $S_1 - S_2$ excess of S_2 *within* the company's reinsurance retention, which will form the basis of the required put spread. Let $p_t(S_1, S_2)$ denote the value of this put spread.

[6]At the time this paper was written, the Chicago Board of Trade's efforts to create a market for options on U.S. catastrophe losses had not yet produced significant capacity.

A reinsurance cylinder spread can be created by the purchase by a ceding company of the high level excess of loss layer at a cost of $c_t(T_1,T_2)$ and the equivalent of the sale of a put spread on the lower layer at a price of $p_t(S_1,S_2)$. (This is not necessarily a zero cost cylinder.) The premium outlay of the ceding company at the beginning of the contract would then be $c_t(T_1,T_2) - p_t(S_1,S_2)$. Since the reinsurer may require a minimum initial premium of $M \geq 0$, it may be necessary to allow the ratio of puts to calls to be different from one. If this ratio is represented by Q, the initial premium is given by

$$M = c_t(T_1,T_2) - Qp_t(S_1,S_2).$$

Under this structure, the premium of $c_t(T_1,T_2)$ buys exactly the same excess protection against large claims as the conventional reinsurance provides. The premium credit of $Qp_t(S_1,S_2)$ embedded in the initial premium represents the sale of a put spread on the lower layer by the ceding company to the reinsurer, the final value of which will be settled as an additional premium of $\min(Q(S_1 - X_t), Q(S_1 - S_2))$ when claim experience is known.

Let us now put some numbers to it. Let

$$\begin{aligned} c_t(T_1,T_2) &= \$2{,}500{,}000, \\ p_t(S_1,S_2) &= \$3{,}889{,}000, \\ Q &= 45\%, \\ S_1 &= \$15{,}000{,}000, \qquad \text{and} \\ S_1 - S_2 &= \$5{,}000{,}000. \end{aligned}$$

Then the initial premium is calculated as follows:

$$\begin{aligned} M &= c_t(T_1,T_2) - Q \cdot p_t(S_1,S_2) \\ &= \$2{,}500{,}000 - (.45)(\$3{,}889{,}000) \\ &= \$750{,}000. \end{aligned}$$

TABLE 2

Claims X_t	Initial Premium	Additional Premium	Total Premium
$X_t < S_2$	\$750	\$2,250	\$3,000
$S_2 \le X_t \le S_1$	\$750	(45%)(\$15,000 − X_t)	Slides \$750 to \$3,000
$S_1 < X_t$	\$750	0	\$750

Note: Premium figures in thousands.

At expiry of the contract (or at such time as agreed), an additional premium, A, equal to the expiry value of the "put spread" is due:

$$A = \min[Q(S_1 - X_t), Q(S_1 - S_2)]$$
$$= \text{lesser of: } (.45)(\$15{,}000{,}000 - X_t) \text{ and } (.45)(\$5{,}000{,}000).$$

The total premium under a "cylinder" reinsurance structure depends on the final cost of claims, X_t, as shown in Table 2.

This compares to the fixed premium of \$2,500,000 under the conventional contract and is shown graphically on Figure 4. In the cylinder structure, the ceding company pays a higher premium for its coverage of $T_2 - T_1$ excess of T_1 when the claim experience in the retained sublayer of $S_1 - S_2$ excess of S_2 is good (up to \$3,000,000 versus \$2,500,000). It pays a lower premium when claim experience in that layer is bad (\$750,000 versus \$2,500,000). In other words, the company pays more when its net claims experience is relatively good and it can afford higher reinsurance premiums, and less when its net is poor and it can least afford the burden of even normal reinsurance premiums. This is illustrated graphically in Figure 5 in terms of the effect on underwriting profit. This premium structure is more effective in reducing the volatility of a ceding company's net underwriting result than the conventional structure. Because of this stability, it might appeal to reinsurance buyers who use excess of loss coverage to reduce underwriting volatility.

FIGURE 4

ILLUSTRATION OF "CYLINDER" REINSURANCE PREMIUM STRUCTURE

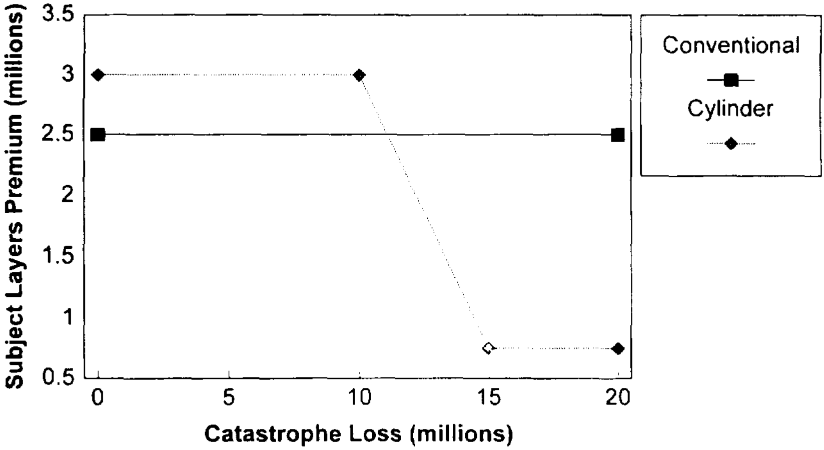
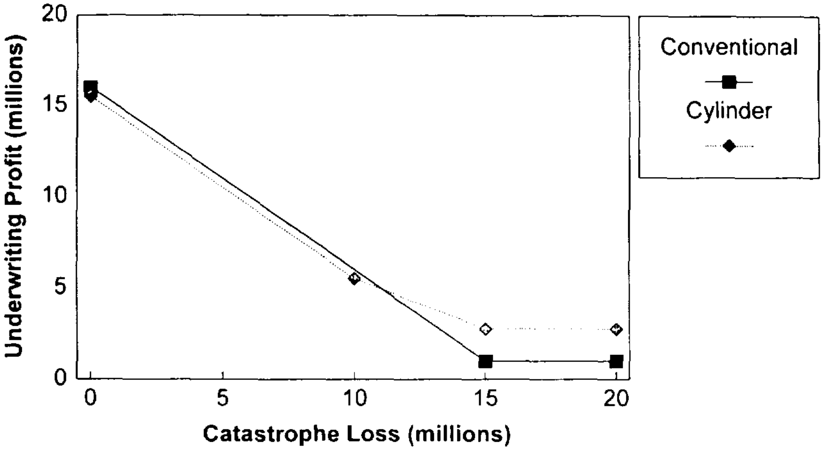


FIGURE 5

ILLUSTRATION OF "CYLINDER" REINSURANCE EFFECT ON UNDERWRITING PROFIT



The flip side of this is that the reinsurer's volatility is increased. Why would a reinsurer be willing to offer such a structure, which reduces premiums when claims are higher? The answer is that, in the context of a reinsurer's diversified portfolio, the incremental volatility will be small, while the extra benefit to the reinsurer's customer may well strengthen the overall reinsurance relationship. The reinsurance market has sometimes been criticized for selling "off the shelf" products that it wants to sell, rather than what ceding companies actually want to buy. In classes of reinsurance where reinsurers can sell as much off-the-shelf product as they want, there exists little or no pressure for them to introduce innovative structures like the foregoing example. However, to the extent some reinsurers want to pursue a more customer-focused strategy or simply feel competitive pressure, product innovation will increasingly begin to emerge. Indeed, the author is aware of at least one major reinsurer that has developed a product that has features similar to this example.

The cylinder is only one example. There are undoubtedly many other practical insurance and reinsurance products waiting to be discovered by exploring the derivatives product paradigm.

4. PRICING OPTIONS WHEN FUTURE PRICES ARE NOT LOGNORMAL

The Black–Scholes model relies on the assumption that market price changes over any finite time interval (expressed by the ratio P_n/P_{n-1}) are lognormally distributed. Since the product of lognormal variates is also lognormal, this assumption leads to the convenient conclusion that future market prices are also so distributed with predictable time-dependent parameters. The beauty of this is that the same framework can be used to determine the pure premium price for a one month, six month, or one year option, or one for any other time period.

Other stochastic price movement models have been described by others [2]. Like Black–Scholes, they support the pricing of options of any maturity. However, for assets subject to sudden or extreme price movements, or which are highly illiquid, a realistic stochastic price movement model may not exist. (Indeed, some analysts (e.g., Peters [6]) argue that *all* such models are flawed since they rely on too many assumptions that market experience has shown to be unrealistic.) This does not mean that options cannot be priced for such assets, but we need a different model.[7]

To price a call option exercisable at time t, we need an estimate of the probability distribution of the underlying asset price at time t as viewed from the vantage point of today. If it is possible to estimate this price distribution, it is possible to price an option. Pricing options of different maturities consistently is more difficult without a price movement model, because it requires separate estimates of the price distribution for each exercise date; but it can be done.

Formula 1.3, without the requirement that x be lognormal, can be used to price any option in this way. Of course, if the asset price at time t is not lognormal, the call option pure premium derived using Formula 1.3 is not equivalent to Black–Scholes. As with the estimation of loss distributions, determination of the price distribution of an asset may be made difficult by sparseness of data.

5. COMBINING THE OPTION AND ACTUARIAL PARADIGMS

Section 1 established that option pricing is analogous to excess of loss insurance pricing. Section 3 showed how new insurance innovations can be developed using the option market product paradigm. Section 4 discussed how to price options out-

[7]Even for the pricing of options on equities, for which Black–Scholes is widely used, traders recognize its imperfections. Fischer Black even wrote a paper entitled "How to Use the Holes in Black–Scholes," reprinted in [3]!

side the Black–Scholes framework. This section will illustrate how the synthesis of these ideas can lead to new product concepts outside the current scope of anything widely offered in either the financial or insurance market today.

Options on Reinsurance Premiums

Consider the following. A reinsurance contract can be thought of as an asset, namely the right to recover the monetary value of qualifying insurance claims from a reinsurer.

The price of a reinsurance contract is normally negotiated in the two or three months prior to the inception or anniversary of the contract. Sometimes there is significant uncertainty about the final price until the completion of the negotiations between the ceding company and reinsurers. Under certain circumstances, it might be valuable to a ceding company to fix the cost of its reinsurance coverage at an earlier date, or at least establish an upper bound. Using the option pricing paradigm, it is possible to establish a way to price such a cap.

Since the reinsurance premium, $prem_t$, for coverage incepting at time $t > 0$ (where time 0 would be today) is not known with certainty today, it is a random variable. The pure premium of a call option on $prem_t$ can therefore be calculated using Formula 1.3! Let us use an example to illustrate this.

Suppose the rate on line (i.e., the premium divided by the limit) of a catastrophe reinsurance contract currently in force is 20%. It is six months into the year and there has been a total loss to the layer. There was also a total loss three years ago.

In light of this experience, the premium for renewal will probably be increased, reflecting an upward reassessment by reinsurers of the exposure to loss. The ceding company will also probably be willing to pay a somewhat increased rate to begin to "pay back" reinsurers. However, the new rate will not be established until closer to the renewal date. In the meantime, for the next several months the premium the cedant faces for renewal is unknown and uncertain.

Suppose the market rate on line for renewal, viewed from the point six months prior to renewal, has a mean of 30% and is lognormally distributed with parameters (−1.20,.125). This implies that a rate increase of some size is nearly certain. It also implies about a 10% chance of a price of 35% or greater and about a 1% chance of a renewal price over 40%.

Formula 1.3 can be used to determine the pure premium of a call option to buy the reinsurance at renewal at a 30% rate on line (or any other price). If $r = 5\%$ and $t = .5$ (= 6 months), Formula 1.3 implies an option pure premium of (.975)(1.5%) = 1.46% rate on line, or 4.9% of the strike price of 30% rate on line.

If the ceding company were to buy this call option, it would be certain that the total cost of renewal would be no more than 31.46% rate on line (30% + 1.46%), and it might be less, since if the reinsurance market quotes less than 30%, the cedant would let the option expire unexercised.

Is this reinsurance premium call option a financial derivative or a reinsurance premium? The answer is, it could be either. In the way it was described above, it has the form of a derivatives market instrument. But the concept can also easily be incorporated into a reinsurance contract. Let us assume the renewal date is January 1. The option to buy the 12 months coverage incepting next January 1 can be embedded in a reinsurance contract with a premium payment warranty. If a certain required premium payment is not received before inception, the contract does not come into force.

In periods of significant reinsurance pricing uncertainty, purchasing a premium option will reduce that uncertainty and facilitate a ceding company's reinsurance planning and budgeting process. The specialist reinsurance market for this type of coverage historically has been largely found in London.

Rate Guarantees

The option paradigm can also be used to think properly about multi-year rate guarantees in the primary insurance market. Insureds sometimes seek to negotiate a fixed rate for several years or a limit on future rate increases. In these cases the insured is seeking, in effect, to secure a call option, or series of options, on future rate levels.

Suppose the insured wants a three-year rate guarantee for coverage that would normally be subject to an annual rate review. The current rate (which is guaranteed) is denoted by R_0. The market rates for coverage renewing one year and two years from now, respectively, are random variables R_1 and R_2. If the distributions of R_1 and R_2 can be estimated, it is possible to price the call options the insured is seeking. Then the insured can be charged for the options. Alternatively, the insurer may decide not to charge for the options, and merely use the options pricing exercise to determine the effective rate decrease the three-year guarantee represents.

If the options cannot be priced because the distributions of R_1 and R_2 cannot be estimated with sufficient confidence, perhaps it would be unwise for the insurer to agree to the rate guarantee!

At the time this paper was being prepared, multi-year contracts were beginning to appear in the reinsurance market as well. Obviously the same thought process applies to both insurance and reinsurance.

6. CONCLUSION

This paper has sought to demonstrate the value of the options market paradigm in thinking about and developing new insurance solutions. As the relationship between Formulas 1.1 and 1.3 makes clear, the underlying mathematics of insurance and the broader financial markets is the same. Apart from potential regulatory constraints, there is no logical reason why we should not see a convergence of insurance and other financial services in the coming years. This is especially likely at the wholesale level (e.g., reinsurance), where the relative importance of distribution systems and customer interface recedes and the importance of pure risk characteristics increases.

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APPENDIX A

DERIVATION OF THE BLACK–SCHOLES OPTION PRICING FORMULA FROM A LOGNORMAL ASSET PRICE ASSUMPTION

Let

P_0 = the current market price of the security underlying the option,
t = time (in years) to option expiry,
r = the risk-free interest rate used for continuous compounding (i.e., the force of interest),
x = a random variable for the future market price of the security underlying the option, at time t (expiry).

Assume x is lognormally distributed with parameters $\ln P_0 + rt - 0.5\sigma^2 t$ and $\sigma\sqrt{t}$, and mean $E(x) = P_t = \exp(\ln P_0 + rt)$. This implies $P_t = P_0 \cdot e^{rt}$.

X_t = the actual future market price of the security underlying the option, at expiry.
$c_t(S)$ = the current pure premium (i.e., ignoring transaction costs and risk) for an option to buy the underlying security at a price of S at time t. This is known as a "call option with a strike price of S." Because of its feature of exercise at only one date, it is known as a European option.

The call option $c_t(S)$ will have no intrinsic value at expiry if the market price, X_t, of the security is below the strike price, S. In that case, it is cheaper to buy the security directly at price X_t than to exercise to option to buy at expiry price S. No rational investor would pay a non-zero premium for such an option; hence its nil value.

$c_t(S)$ will have intrinsic value of $X_t - S$ at expiry if the market price X_t exceeds the strike price S. An investor would be indifferent to buying the security directly at price X_t and buying the call option $c_t(S)$ at a price of $X_t - S$ for immediate exercise at price S.

The pure premium of $c_t(S)$ is the probability weighted mean of all possible intrinsic values at expiry, discounted to reflect present value.[8]

If the correct interest rate for discounting is the risk-free rate, the pure premium is expressed as:

$$c_t(S) = e^{-rt} \cdot \int_S^{\infty} (x - S) \cdot f(x)\,dx \tag{A.1}$$

$$= e^{-rt} \cdot \left(\int_S^{\infty} x \cdot f(x)\,dx - S \int_S^{\infty} f(x)\,dx \right) \tag{A.2}$$

$$= e^{-rt} \cdot \left(\int_0^{\infty} x \cdot f(x)\,dx - \int_0^{S} x \cdot f(x)\,dx - S\left(1 - \int_0^{S} f(x)\,dx\right)\right). \tag{A.3}$$

In general, the first moment distribution

$$\frac{\int_0^A x \cdot f(x)\,dx}{E(x)}$$

of a lognormal variate x with parameters (μ, σ) is also lognormal with parameters $(\mu + \sigma^2, \sigma)$.

In the present case, x is lognormal $(\ln P_0 + rt - 0.5\sigma^2 t, \sigma\sqrt{t})$ and its first moment distribution has parameters $(\ln P_0 + rt + 0.5\sigma^2 t, \sigma\sqrt{t})$. Accordingly, the second term within the main

[8]The justification for use of the risk-free rate is described in footnote 2 in the body of the paper.

brackets of Formula A.3 can be restated as follows:

$$\int_0^S x \cdot f(x)dx = \mathrm{E}(x) \cdot N\left(\frac{\ln S - (\ln P_0 + rt + 0.5\sigma^2 t)}{\sigma\sqrt{t}}\right)$$
$$= P_t \cdot N\left(\frac{\ln S - (\ln P_0 + rt + 0.5\sigma^2 t)}{\sigma\sqrt{t}}\right),$$

where N is the cumulative distribution function of the standard normal distribution.

Evaluation of the other terms of Formula A.3 is straightforward, and this formula can now be rewritten as:

$$c_t(S) = P_t e^{-rt} \cdot \left(1 - N\left(\frac{\ln S - (\ln P_0 + rt + 0.5\sigma^2 t)}{\sigma\sqrt{t}}\right)\right)$$
$$- Se^{-rt} \cdot \left(1 - N\left(\frac{\ln S - (\ln P_0 + rt - 0.5\sigma^2 t)}{\sigma\sqrt{t}}\right)\right)$$
$$= P_0\left(1 - N\left(\frac{\ln S - \ln P_0 - (r + 0.5\sigma^2)t}{\sigma\sqrt{t}}\right)\right)$$
$$- Se^{-rt} \cdot \left(1 - N\left(\frac{\ln S - \ln P_0 - (r - 0.5\sigma^2)t}{\sigma\sqrt{t}}\right)\right)$$
$$= P_0\left(1 - N\left(\frac{\ln(S/P_0) - (r + 0.5\sigma^2)t}{\sigma\sqrt{t}}\right)\right)$$
$$- Se^{-rt} \cdot \left(1 - N\left(\frac{\ln(S/P_0) - (r - 0.5\sigma^2)t}{\sigma\sqrt{t}}\right)\right); \quad \text{(A.4)}$$

and, since $1 - N(z) = N(-z)$,

$$c_t(S) = P_0 \cdot N\left(\frac{\ln(P_0/S) + (r + 0.5\sigma^2)t}{\sigma\sqrt{t}}\right)$$
$$- Se^{-rt} \cdot N\left(\frac{\ln(P_0/S) + (r - 0.5\sigma^2)t}{\sigma\sqrt{t}}\right). \quad \text{(A.5)}$$

Let

$$d_1 = \frac{\ln(P_0/S) + (r + 0.5\sigma^2)t}{\sigma\sqrt{t}},$$

and

$$d_2 = \frac{\ln(P_0/S) + (r - 0.5\sigma^2)t}{\sigma\sqrt{t}}.$$

Then Formula A.5 can be restated as

$$c_t(S) = P_0 \cdot N(d_1) - Se^{-rt} \cdot N(d_2). \qquad \text{(A.6)}$$

This is the Black–Scholes option pricing formula.

APPENDIX B

VALUATION OF CALL, PUT, AND CYLINDER SPREADS

Call Spreads

The value of a *call spread* $c_t(T_1,T_2)$ with $T_2 > T_1$ and time t to expiry is given by

$$\begin{aligned}
c_t(T_1,T_2) &= c_t(T_1) - c_t(T_2) \\
&= e^{-rt}\left[\int_{T_1}^{\infty}(x-T_1)\cdot f(x)dx - \int_{T_2}^{\infty}(x-T_2)\cdot f(x)dx\right] \\
&= e^{-rt}\left[\int_{T_1}^{T_2}(x-T_1)\cdot f(x)dx + \int_{T_2}^{\infty}(x-T_1)\cdot f(x)dx\right. \\
&\qquad \left. - \int_{T_2}^{\infty}(x-T_2)\cdot f(x)dx\right] \\
&= e^{-rt}\left[\int_{T_1}^{T_2}(x-T_1)\cdot f(x)dx + \int_{T_2}^{\infty}(T_2-T_1)\cdot f(x)dx\right].
\end{aligned} \tag{B.1}$$

Note the similarity to the formulas used to work with excess layers in insurance applications.

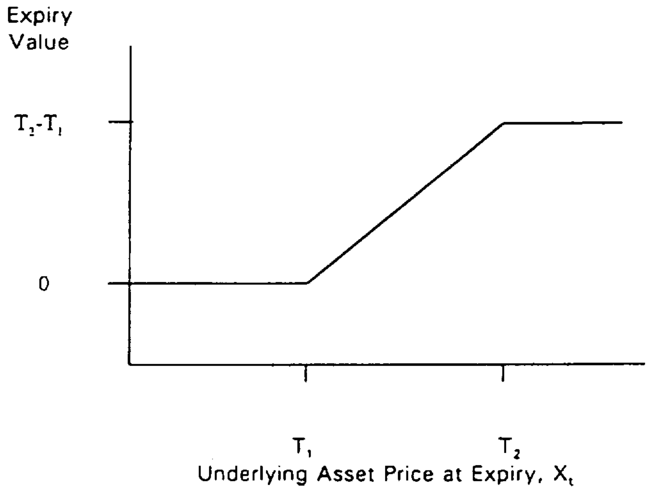
If the actual price of the underlying asset at expiry of the option is X_t, the value of the long call spread position at expiry is given by

$$\begin{aligned}
&T_2 - T_1, && X_t \geq T_2; \\
&X_t - T_1, && T_2 > X_t > T_1; \\
&0, && T_1 \geq X_t.
\end{aligned}$$

This is shown graphically in Figure B-1.

FIGURE B-1

EXPIRY VALUE PROFILE: CALL OPTION SPREAD $c_t(T_1, T_2)$



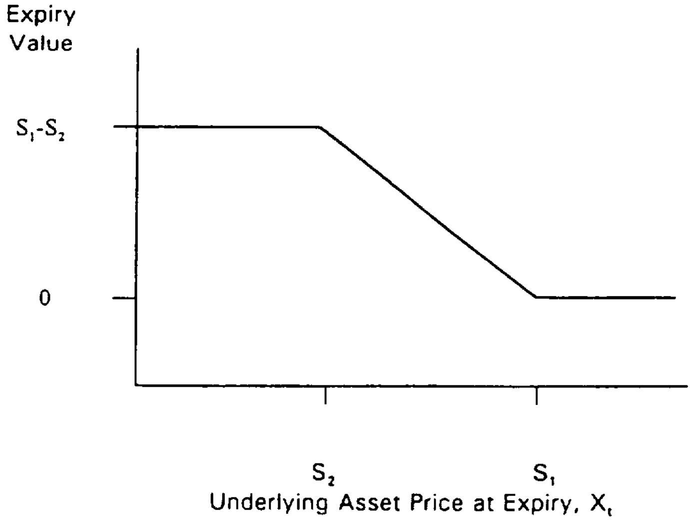
Put Spreads

The value of a *put spread* $p_t(S_1, S_2)$ with $S_1 > S_2$ and time t to expiry is given by

$$\begin{aligned} p_t(S_1, S_2) &= p_t(S_1) - p_t(S_2) \\ &= e^{-rt}\left[\int_0^{S_1}(S_1 - x)\cdot f(x)dx - \int_0^{S_2}(S_2 - x)\cdot f(x)dx\right] \\ &= e^{-rt}\left[\int_0^{S_2}(S_1 - x)\cdot f(x)dx + \int_{S_2}^{S_1}(S_1 - x)\cdot f(x)dx\right. \\ &\qquad \left. - \int_0^{S_2}(S_2 - x)\cdot f(x)dx\right] \\ &= e^{-rt}\left[\int_{S_2}^{S_1}(S_t - x)\cdot f(x)dx + \int_0^{S_2}(S_1 - S_2)\cdot f(x)dx\right]. \end{aligned} \tag{B.2}$$

FIGURE B-2

EXPIRY VALUE PROFILE: PUT OPTION SPREAD $p_t(S_1, S_2)$



The value of the long put spread position at expiry is given by

$$\begin{array}{ll} 0, & X_t \geq S_1; \\ S_1 - X_t, & S_1 > X_t > S_2; \\ S_1 - S_2, & S_2 \geq X_t. \end{array}$$

This is shown graphically in Figure B-2.

Put-Call Parity

There is an important relationship between the value of calls and puts known as "put-call parity." Consider two portfolios. The first consists of an asset with a value of P_0 and a related put option worth $p_t(T_1)$. The second consists of a T-bill valued at $T_1 \cdot e^{-rt}$ and a call option on the asset in the first portfolio, valued at $c_t(T_1)$.

These two portfolios have identical expiry value profiles (namely, $\max(T_1, P_t)$), so unless there are obstacles to arbitrage trading, they must have equal market values for any $T_1 \geq 0$:

$$P_0 + p_t(T_1) = T_1 e^{-rt} + c_t(T_1). \tag{B.3}$$

We can use put-call parity to derive the analogous relationship between put and call spreads:

Since

$$T_1 e^{-rt} = P_0 + p_t(T_1) - c_t(T_1)$$

and

$$T_2 e^{-rt} = P_0 + p_t(T_2) - c_t(T_2),$$

then

$$\begin{aligned}(T_2 - T_1)e^{-rt} &= p_t(T_2) - c_t(T_2) - p_t(T_1) + c_t(T_1) \\ &= c_t(T_1, T_2) + p_t(T_2, T_1). \end{aligned} \tag{B.3a}$$

A brief analysis of Formula B.3a shows that it is consistent with using the risk-free rate for discounting European option pure premiums. If we restate Formula B.3a in terms of integrals and treat the interest rate to be used for discounting the right side of the equation as an unknown, i, we obtain:

$$\begin{aligned}&(T_2 - T_1)e^{-rt} \\ &\quad = e^{-it}\left(\int_{T_1}^{T_2}(x - T_1)\cdot f(x)dx \right.\\ &\qquad + \int_{T_2}^{\infty}(T_2 - T_1)\cdot f(x)dx + \int_0^{T_1}(T_2 - x)\cdot f(x)dx \\ &\qquad \left. + \int_{T_1}^{T_2}(T_2 - x)\cdot f(x)dx - \int_0^{T_1}(T_1 - x)\cdot f(x)dx\right)\end{aligned}$$

TABLE 3

Expiry Price	Long Call Value	Short Put Value	Cash Value	Short Put+ Cash Value
$X_t \geq T_2$	$T_2 - T_1$	0	$T_2 - T_1$	$T_2 - T_1$
$T_2 > X_t > T_1$	$X_t - T_1$	$-(T_2 - X_t)$	$T_2 - T_1$	$X_t - T_1$
$T_1 \geq X_t$	0	$-(T_2 - T_1)$	$T_2 - T_1$	0

$$= e^{-it}\left(\int_0^{T_2}(x - T_1)\cdot f(x)dx \right.$$

$$\left. + \int_{T_2}^{\infty}(T_2 - T_1)\cdot f(x)dx + \int_0^{T_2}(T_2 - x)\cdot f(x)dx\right)$$

$$= e^{-it}\left(\int_0^{T_2}(T_2 - T_1)\cdot f(x)dx + \int_{T_2}^{\infty}(T_2 - T_1)\cdot f(x)dx\right)$$

$$= e^{-it}\int_0^{\infty}(T_2 - T_1)\cdot f(x)dx$$

$$= e^{-it}(T_2 - T_1),$$

which implies $i = r$.

Formula B.3a also implies a definition for a call spread in terms of a put spread and T-bills:[9]

$$C_t(T_1,T_2) = (T_2 - T_1)e^{-rt} - p_t(T_2,T_1). \qquad \text{(B.3b)}$$

This means that it is possible to achieve a synthetic call spread position using put spreads and vice versa. In particular, Formula B.3b says that selling a put spread, $p_t(T_2,T_1)$, and holding the present value of $T_2 - T_1$ in T-bills is equivalent to buying a call spread, $c_t(T_1,T_2)$. To see this, Table 3 compares the expiry values of these two positions.

[9]Note that formulas B.3a and B.3b imply a put-call parity relationship for spreads that, unlike the ordinary put-call parity formula, has no reference to P_0.

TABLE 4

Expiry Price	Long Put Value	Short Call Value	Cash Value	Short Call+ Cash Value
$X_t \geq T_2$	0	$-(T_2 - T_1)$	$T_2 - T_1$	0
$T_2 > X_t > T_1$	$T_2 - X_t$	$-(X_t - T_1)$	$T_2 - T_1$	$T_2 - X_t$
$T_1 \geq X_t$	$T_t - T_1$	0	$T_2 - T_1$	$T_2 - T_1$

Alternatively, since

$$p_t(T_2,T_1) = (T_2 - T_1)e^{-rt} - c_t(T_1,T_2),$$

buying a put spread $p_t(T_2,T_1)$ is equivalent to selling a call spread $c_t(T_1,T_2)$ and holding the present value of $T_2 - T_1$ in T-bills, as shown in Table 4.

Cylinder Spreads

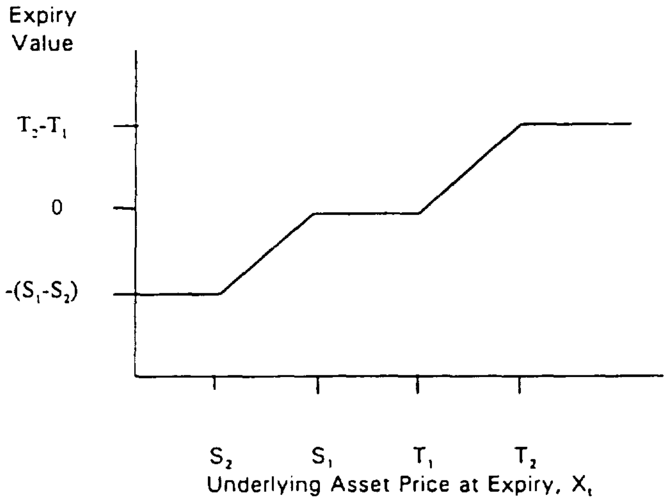
The *bull cylinder spread*, $\text{cyl}_t(S_1,S_2;T_1,T_2)$, created from the call and put spreads defined above, where $T_2 > T_1 > S_1 > S_2$, has the following value:

$$\begin{aligned}\text{cyl}(S_1,S_2;T_1,T_2) &= c_t(T_1,T_2) - p_t(S_1,S_2) \\ &= e^{-rt}\left[\int_{T_1}^{T_2}(x - T_1)\cdot f(x)dx + \int_{T_2}^{\infty}(T_2 - T_1)\cdot f(x)dx \right. \\ &\quad - \int_{S_2}^{S_1}(S_1 - x)\cdot f(x)dx \\ &\quad \left. - \int_0^{S_2}(S_1 - S_2)\cdot f(x)dx\right]. \qquad \text{(B.4)}\end{aligned}$$

The value of $\text{cyl}_t(S_1,S_2;T_1,T_2)$ depends on the choices of S_1, S_2, T_1 and T_2. These parameters can be chosen to create a cylinder structure that produces the desired cylinder value at time t to

FIGURE B-3

EXPIRY VALUE PROFILE: BULL CYLINDER OPTION SPREAD
$\text{cyl}_t(S_1, S_2, T_1, T_2)$



expiry. Additional flexibility can be introduced in the cylinder structure by relaxing the requirement that the same number of call and put spreads are used. If Q is defined as the ratio of the number of puts to the number of calls, then the value of $\text{cyl}_t(S_1, S_2; T_1, T_2)$ is given by

$$\begin{aligned}
&\text{cyl}(S_1, S_2; T_1, T_2) \\
&= e^{-rt} \left[\int_{T_1}^{T_2} (x - T_1) \cdot f(x)\,dx + \int_{T_2}^{\infty} (T_2 - T_1) \cdot f(x)\,dx \right. \\
&\qquad - Q \cdot \left(\int_{S_2}^{S_1} (S_1 - x) \cdot f(x)\,dx \right. \\
&\qquad\qquad \left. \left. + \int_0^{S_2} (S_1 - S_2) \cdot f(x)\,dx \right) \right].
\end{aligned}$$

At expiry the value of the bull cylinder spread position is given by

$$
\begin{array}{ll}
T_2 - T_1, & X_t \geq T_2; \\
X_t - T_1, & T_2 > X_t \geq T_1; \\
0, & T_1 > X_t > S_1; \\
-Q \cdot (S_1 - X_t), & S_1 \geq X_t > S_2; \\
-Q \cdot (S_1 - S_2), & S_2 \geq X_t.
\end{array}
$$

This is illustrated for $Q = 1$ in Figure B-3.

ADJUSTING INDICATED INSURANCE RATES: FUZZY RULES THAT CONSIDER BOTH EXPERIENCE AND AUXILIARY DATA

VIRGINIA R. YOUNG

Abstract

This paper describes how an actuary can use fuzzy logic to adjust insurance rates by considering both claim experience data and supplementary information. This supplementary data may be financial or marketing data or statements that reflect the philosophy of the actuary's company or client. The paper shows how to build and fine-tune a rate-making model by using workers compensation insurance data from an insurance company.

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1. INTRODUCTION

Through the education programs of the Society of Actuaries and the Casualty Actuarial Society, actuaries are equipped with statistical tools to analyze experience data and to determine necessary rate changes for their insurance products. Students are often surprised to learn that those rate changes are frequently not accepted "as is" by company management. Actuaries work with sales, marketing, and underwriting personnel to develop rates that will be competitive and adequate.

Actuaries frequently consider statistical data specific to rates, such as the results of experience studies. In setting premiums, actuaries also consider constraints that supplement experience data. These constraints may reflect company philosophy, such as "We wish to increase our market share *moderately* from year to year." They may also include financial data, such as "Raise the rates if we experience *high* loss ratios or *low* profit margins."

The theory of fuzzy sets provides a natural setting in which to handle such statements. Through fuzzy sets, one can account for vague notions whose boundaries are not clearly defined, such as "*large* amount of business." Fuzzy logic provides a uniform way to handle such factors that influence the indicated rate change (Zadeh [20]). A fuzzy logic system is a type of expert system. An advantage of using a fuzzy logic system is that it provides a systematic way to develop mathematical rules from linguistic ones. This paper describes step-by-step how an actuary can adjust rates by beginning with linguistic rules that consider both experience data and supplementary information.

Fuzzy sets have only recently been applied to problems in actuarial science. DeWit [5] and Lemaire [13] show how to apply fuzzy sets in individual underwriting, and Young [16] indicates how to use fuzzy sets in group health underwriting. Ostaszewski [15] suggests several areas in actuarial science in which fuzzy sets may prove useful. Cummins and Derrig [2] apply a form of fuzzy logic to calculate fuzzy trends in property-liability insurance. Derrig and Ostaszewski [4] employ fuzzy clustering in risk classification and provide an example in automobile insurance. Cummins and Derrig [3] use fuzzy arithmetic in pricing property-liability insurance. In an earlier paper [18], I show how to develop a fuzzy logic model with which actuaries can adjust insurance rates by considering only constraints or information that are ancillary to experience data.

Section 2 introduces fuzzy sets and defines operators corresponding to the linguistic connectors *and* and *or* and the modifier *not*. It also describes a simple fuzzy inference system. References for fuzzy sets include Dubois and Prade [7], Kosko [12], and Zadeh [19]. Some references for fuzzy logic and fuzzy inference are Bellman and Zadeh [1], Driankov et al. [6], Kandel and Langholz [9], Klir and Folger [11], Mamdani [14], Zadeh [20], and Zimmermann [21].

Section 3 describes how to construct and fine-tune a pricing model using fuzzy inference. Section 4 shows how to build a pricing model using workers compensation insurance data from an insurance company. Finally, Section 5 summarizes the paper's key points.

2. FUZZY INFERENCE

Fuzzy sets describe concepts that are vague (Zadeh [19]). The fuzziness of a set arises from the lack of well-defined boundaries. This lack is due to the imprecise nature of language; that is, objects can possess an attribute to various degrees. A fuzzy set corresponding to a given characteristic assigns a value to an object, the degree to which the object possesses the attribute.

Examples of fuzzy sets encountered in insurance pricing are *stable* rates, *large* profits, and *small* amounts of business renewed or written. Indeed, rates can be *stable* to different degrees depending on the relative or absolute changes in the premium rate. Also, profits can be *large* to different degrees depending on the relative or absolute amount of profits.

Fuzzy sets generalize nonfuzzy, or crisp, sets. A crisp set, C, is given by a characteristic function:

$$\chi_C : X \rightarrow \{0,1\},$$

in which $\chi_C(x) = 1$ if x is in C; otherwise, $\chi_C(x) = 0$. Fuzzy sets recognize that objects can belong to a given set to different degrees. They essentially expand the notion of set to allow partial membership in a set.

DEFINITION 2.1 *A fuzzy set, A, in a universe of discourse, X, is a function m_A on X that takes values in the unit interval* [0, 1]*:*

$$m_A : X \to [0,1].$$

The function m_A is called the membership function of A, and for any x in X, $m_A(x)$ in [0, 1] *represents the grade of membership of x in A.*

EXAMPLE 2.1 One may define *stable* rates by the following hypothetical fuzzy set:

$$m_{stable}(r) = \begin{cases} 0, & \text{if} \quad r < -0.10, \\ \dfrac{r+0.10}{0.05}, & \text{if} \quad -0.10 \le r < -0.05, \\ 1, & \text{if} \quad -0.05 \le r < 0.05, \\ \dfrac{0.10-r}{0.05}, & \text{if} \quad 0.05 \le r < 0.10, \\ 0, & \text{if} \quad r \ge 0.10, \end{cases}$$

in which r is the relative rate change. For example, the degree to which a rate increase of 8% is *stable* is 0.40. It does *not* mean, however, that one will view an 8% rate increase as stable 40% of the time and unstable the rest of the time. See Figure 1 for the graph of this fuzzy set. The points ± 0.05 and ± 0.10 depend on the line of business. Also, one may want to use a fuzzy set that is not necessarily piecewise linear.

We now define three basic operations on fuzzy sets.

DEFINITION 2.2 *The union, $A \cup B$, of two fuzzy sets, A and B, is given by*

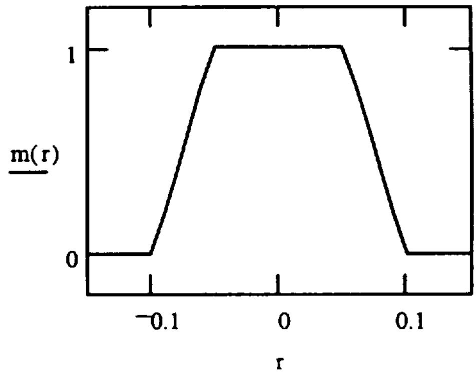
$$m_{A \cup B}(x) \equiv \max[m_A(x), m_B(x)], \qquad x \in X,$$

and the intersection, $A \cap B$, is given by

$$m_{A \cap B}(x) \equiv \min[m_A(x), m_B(x)], \qquad x \in X.$$

FIGURE 1

GRAPH OF FUZZY SET OF STABLE RATES, EXAMPLE 2.1



The complement, −A, of fuzzy set A is given by

$$m_{-A}(x) \equiv 1 - m_A(x), \qquad x \in X.$$

The union operation acts as an *or* operator, the intersection operation acts as *and*, and the complement operation acts as *not*. Thus, for example, $m_{A \cap B}(x)$ represents the degree to which x is a member of both A and B. The given definitions are not the only acceptable ones for these operations. Klir and Folger [11] specify axioms that union, intersection, and complement satisfy. Also, Dubois and Prade [7] and Young [16] discuss alternative operators. One in particular is the intersection operator called the algebraic product. The algebraic product of two fuzzy sets A and B is given by

$$m_{AB}(x) = m_A(x) \cdot m_B(x).$$

The algebraic product allows the fuzzy sets to interact in the intersection. That is, both fuzzy sets contribute to the value of the intersection, as opposed to the min operator in which the minimum of the two values determines the value of the intersection.

We will consider this intersection operator in some of the examples below. Unless otherwise stated, however, the intersection operator is the min operator.

EXAMPLE 2.2 Suppose we want to intersect the fuzzy set of *stable* rates from Example 2.1 with the fuzzy set of *low* actual-to-expected ratios given by

$$m_{low}(x) = \begin{cases} 1, & \text{if } x < 0.90, \\ \dfrac{1.10 - x}{0.20}, & \text{if } 0.90 \leq x < 1.10, \\ 0, & \text{if } 1.10 \leq x, \end{cases}$$

in which x is the ratio of actual claims to expected claims (A/E ratio).[1] We first imbed these fuzzy sets in the product space of pairs $\{(r,x) : r \geq -1.00,\ x \geq 0\}$, as follows

$$m_{stable}(r,x) = m_{stable}(r)$$
$$m_{low}(r,x) = m_{low}(x).$$

See Figure 2 for the graph of the intersection of these two fuzzy sets using the min operator and Figure 3 for the graph of the intersection of these two fuzzy sets using the algebraic product.

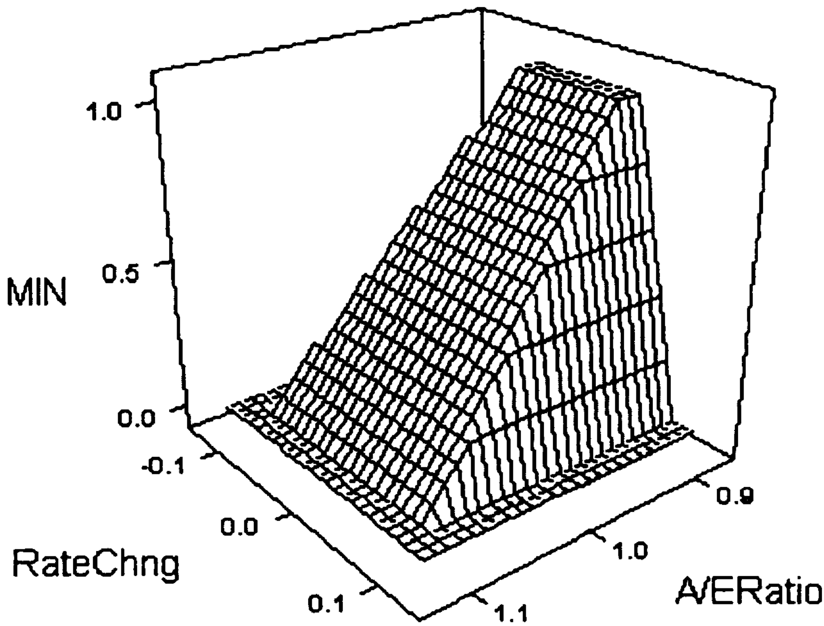
Note that the algebraic product operator allows the two fuzzy sets to interact more than does the min operator. For example, suppose the rate decrease is 6% and the A/E ratio is 0.95. Then, the degree to which the rate change is *stable* is 0.80, and the degree to which the A/E ratio is *low* is 0.75. The degree to which the rate change is stable *and* the A/E ratio is low is min(0.80,0.75) = 0.75 if we use the min operator to intersect the

---

[1]One type of experience study is called an actual-to-expected study. In this study, one compares the actual (incurred) claims relative to the expected claims built into the premium. If this study is performed before the claims have run out, then one develops the actual claims to an ultimate basis to estimate actual incurred claims. One result of this study is the ratio of actual claims to expected claims, called the *actual-to-expected ratio*, or briefly A/E ratio. A high A/E ratio indicates that the allowance for claims in the premium is too low.

FIGURE 2

INTERSECTION OF STABLE RATES AND LOW ACTUAL-TO-EXPECTED RATIO (USING THE MIN OPERATOR)

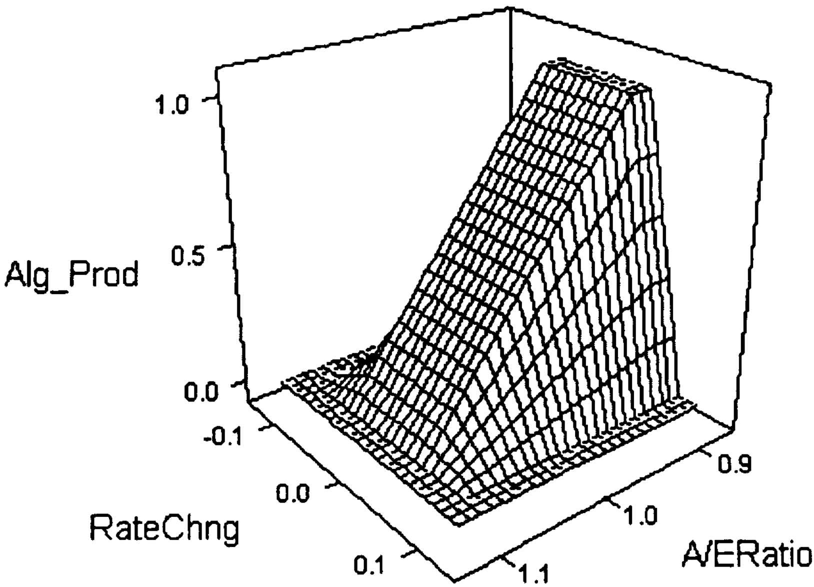


two fuzzy sets, and is (0.80)(0.75) = 0.60 if we use the algebraic product to intersect them.

A few years after fuzzy sets were introduced, Bellman and Zadeh [1] developed the first fuzzy logic model in which goals and constraints were defined as fuzzy sets and their intersection was the fuzzy set of the decision. Cummins and Derrig [2] use the method of Bellman and Zadeh to calculate a trend factor in property-liability insurance. They calculate several possible trends using accepted statistical procedures. For each trend, they determine the degree to which the estimate is *good* by intersecting several fuzzy goals. They suggest that one may choose the trend

FIGURE 3

INTERSECTION OF STABLE RATES AND LOW ACTUAL-TO-EXPECTED RATIO (USING THE ALGEBRAIC PRODUCT)



that has the highest degree of *goodness*. Cummins and Derrig also propose that one may calculate a trend that accounts for all the trends by forming a weighted average of these trends using the membership degrees as weights. It is this latter method that more closely relates to the technique proposed below.

This paper shows how actuaries may incorporate supplementary information in their pricing models, for example, amount of business written or profit earned. Instead of using the method designed by Bellman and Zadeh [1], we follow Zadeh [20] by applying fuzzy inference. In particular, we use a simple form of fuzzy inference proposed by Mamdani [14], who has been a pioneer in applying fuzzy logic in industry. We describe this fuzzy inference after the following example.[2]

EXAMPLE 2.3

(a) If the A/E ratio is *high* and the amount of business is *large*, then raise the rates.

(b) If the A/E ratio is *moderate* and the amount of business is *moderate*, then do not change the rates.

(c) If the A/E ratio is *low* and the amount of business is *small*, then lower the rates.

An actuary can only apply a crisp rate change, not a fuzzy expression such as "*raise* the rates." We therefore set the phrase "raise the rates" equal to the largest rate increase we are willing to administer; similarly, "lower the rates" is replaced by the largest rate decrease we are willing to administer. The reason for doing so will become evident as we proceed below.

In general, our fuzzy system is a collection of n fuzzy rules:

$$\text{If } x \text{ is } A_1, \text{ then } y \text{ is } y_1.$$
$$\text{If } x \text{ is } A_2, \text{ then } y \text{ is } y_2.$$
$$\dots$$
$$\text{If } x \text{ is } A_n, \text{ then } y \text{ is } y_n.$$

If we are given specific input, or explanatory, data $\tilde{x}$ (possibly multi-dimensional if the A_i are compound hypotheses, as in Example 2.3), then measure the degree to which $\tilde{x}$ satisfies the hypothesis A_i in rule i, $i = 1,\dots,n$, namely, $m_{A_i}(\tilde{x})$. To calculate

[2]Throughout this paper, by default, assume that if none of the hypotheses is satisfied to a positive degree, then do nothing. In the following example, this would mean "do not change the rates." This convention is consistent with the weighting scheme defined below in Equation 2.1 if one sets 0/0 equal to 0.

the output $\tilde{y}$, form the weighted average

$$\tilde{y} = \frac{\sum_{i=1}^{n} y_i m_{A_i}(\tilde{x})}{\sum_{i=1}^{n} m_{A_i}(\tilde{x})}. \tag{2.1}$$

A fuzzy hypothesis A may be a compound statement, such as "our company has been writing a *great deal* of business and earning a *small* amount of profit." In this case, we intersect the fuzzy sets corresponding to a *great deal* of business and a *small* amount of profit with the min operator, as in Definition 2.2. Alternatively, one may use the algebraic product operator to intersect the fuzzy sets, as in Example 2.2. Also, if a compound hypothesis involves the connector *or* and modifier *not*, then use the max and negative operators, respectively, to combine the individual fuzzy sets. In Section 3, we describe how to obtain a specific output y_i, $i = 1,\ldots,n$, if the conclusion is expressed as a fuzzy statement, such as "raise the rates a *great deal*."

EXAMPLE 2.4 To continue with Example 2.3, suppose that we have determined the following values of y_i that correspond to the conclusions in the fuzzy rules that we state in that example:

(a) If the A/E ratio is *high* and the amount of business is *large*, then raise the rates 15%.

(b) If the A/E ratio is *moderate* and the amount of business is *moderate*, then do not change the rates.

(c) If the A/E ratio is *low* and the amount of business is *small*, then lower the rates 10%.

Again, if none of the hypotheses is satisfied, then do not change the rates. We are given that the actual-to-expected (A/E) ratio is 1.05, and the amount of business is 3.0 (on some appropriate scale). Given fuzzy sets for the components of the hypotheses, the next step is to calculate the degree to which the input satisfies each hypothesis. Evaluate the degree of membership of the A/E ratio, 1.05, in the fuzzy sets for *high*, *moderate*, and *low*. Hypothetically, suppose that the A/E ratio is *high* to degree 0.75, *moderate* to degree 0.25, and *low* to degree 0.0. Similarly, evaluate the degree of membership of the amount of business, 3.0, in the fuzzy sets for *large*, *moderate*, and *small*. Suppose that the amount of business is *large* to degree 0.50, *moderate* to degree 0.50, and *small* to degree 0.0. The hypothesis of the first rule is, thus, satisfied to degree min(0.75,0.50) = 0.50; the second rule, min(0.25,0.50) = 0.25; and the third rule, min(0.0,0.0) = 0.0. Our rate change is, therefore,

$$\tilde{y} = \frac{0.50(0.15) + 0.25(0.00) + 0.0(-0.10)}{0.50 + 0.25 + 0.0} = 0.10,$$

or increase the rates 10%. Compare the expression for $\tilde{y}$ with Equation 2.1. If, instead of the min operator, we were to use the algebraic product operator for intersection, the rate change would be

$$\tilde{y} = \frac{0.375(0.15) + 0.125(0.00) + 0.0(-0.10)}{0.375 + 0.125 + 0.0} = 0.1125.$$

In Examples 2.3 and 2.4, we incorporate experience data, the actual-to-expected ratio, in the hypotheses of our fuzzy rules. One may also include experience data in the conclusion, as in the following example.

EXAMPLE 2.5 The following fuzzy rules may more accurately reflect the philosophy of the company:

(a) If the amount of business is increasing *greatly* and the profit margin is decreasing *greatly*, then raise the rates *more* than indicated by the A/E ratio.

(b) If the amount of business is *stable* and the profit margin is *stable*, then change the rates as indicated by the A/E ratio.

(c) If the amount of business is decreasing *greatly* and the profit margin is increasing *greatly*, then *lower* the rates *more* than indicated by the A/E ratio.

3. BUILDING A FUZZY INFERENCE MODEL

The previous section describes how to obtain a crisp output $\tilde{y}$ given a fuzzy inference model and crisp input $\tilde{x}$. This section explains how to construct and fine-tune a fuzzy logic model. Young [18] presents steps that may be followed to build a fuzzy logic model. They are repeated here so that this work is self-contained. Section 4 shows how to follow these steps in creating and fine-tuning a fuzzy logic model. Because the following procedure formalizes the discussion in Section 2, the casual or first-time reader may wish to skip to Section 4.

1. *Verbally state linguistic rules.* These rules may reflect current or desired company philosophy. They may arise from the business sense of actuaries. They may result from the combined input of several functions in the insurance company.

2. *Create the fuzzy sets corresponding to the hypotheses.* Assume that the linguistic variables used are naturally ordered. For example, the linguistic variable of amount of business is naturally ordered because *large* amounts of business correspond with large numbers that measure the amount of business, and similarly for *small* amounts of business.

   (a) To create the fuzzy sets for the jth dimension of the input, partition the input space $X_j = [x_{j,1}, x_{j,n(j)}]$ into $n(j) - 1$ disjoint subintervals, one fewer than the number, $n(j)$, of fuzzy sets defined on X_j. Write the boundary points of the subintervals:

$$x_{j,1} \leq x_{j,2} \leq \cdots \leq x_{j,n(j)}.$$

Even though the input space X_j may be infinitely long, the example below describes how to determine $x_{j,1}$ and $x_{j,n(j)}$ so that we can effectively limit X_j to the finite interval $[x_{j,1}, x_{j,n(j)}]$.

(b) The graph of the leftmost fuzzy set $A_{j,1}$ is defined to be the line segment joining the points $(x_{j,1}, 1)$ and $(x_{j,2}, 0)$ and 0 elsewhere. The graph of each of the middle $n(j) - 2$ sets $A_{j,k(j)}$ is the triangular fuzzy set that connects the points $(x_{j,k(j)-1}, 0)$, $(x_{j,k(j)}, 1)$, and $(x_{j,k(j)+1}, 0)$ and 0 elsewhere, $k(j) = 2, \ldots, n(j) - 1$. Finally, the $n(j)$-th fuzzy set $A_{j,n(j)}$ is the line segment joining the points $(x_{j,n(j)-1}, 0)$ and $(x_{j,n(j)}, 1)$ and 0 elsewhere. Note that for any input value of x_j, the sum (over $k(j)$) of its membership values in the sets $A_{j,k(j)}$ is 1; thus, we say that the $A_{j,k(j)}$ form a *fuzzy partition* of X_j.

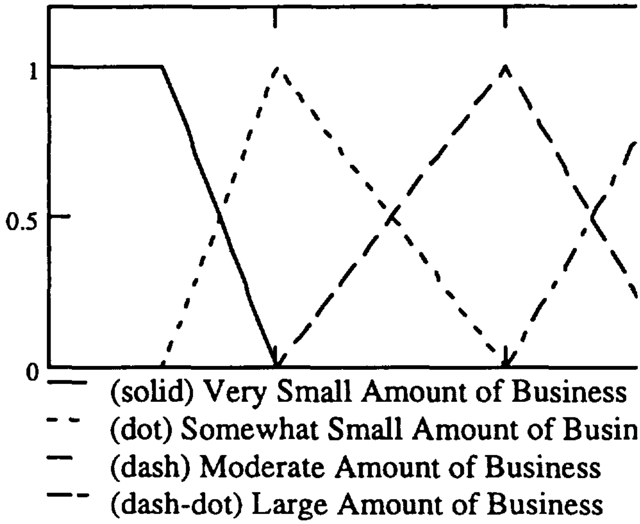
See Figure 4 for an illustration of a partition of the variable of amount of business into four fuzzy sets. Other forms of fuzzy sets may be used to partition a variable, but triangular fuzzy sets are easy to compute and are completely determined by the points in the partition of X_j.

(c) Combine the fuzzy sets that comprise each hypothesis into one fuzzy set using the operators min, max, and negative, corresponding to the linguistic connectors *and* and *or* and modifier *not*, respectively.

3. *Determine the output values $\{y_i\}$ for the conclusions.* Set the output value y_i to the desired output if the hypothesis of rule i is met to degree 1.0.

FIGURE 4

A FUZZY PARTITION OF AMOUNT OF BUSINESS



4. *Fine-tune the fuzzy rules, if applicable.* If learning data is available, either historical data that is still relevant or hypothetical data from experts, then use that data to modify the values $x_{j,k(j)}$ and the values y_i. This is done to optimize any one of a number of objectives. In this work, we minimize a squared-error loss function.

Given data of the form $\{(x_l^*, y_l^*) : l = 1, \ldots, L\}$, pairs of input and output values, either from prior rating periods or from experts' opinions, the model may be fine-tuned using the following simple method: Perturb the parameters $\{x_{j,k(j)}\}$ and $\{y_i\}$ to minimize the squared error

$$\sum_{l=1}^{L} (y_l^* - \tilde{y}(x_l^*))^2,$$

in which $\tilde{y}(x_l^*)$ is the output of the fuzzy logic model, given the input x_l^*. These errors may also be weighted to reflect the relative importance of each ordered pair. In the next section, we minimize such a weighted sum of squared errors:

$$\sum_{l=1}^{L} w_l(y_l^* - \tilde{y}(x_l^*))^2, \qquad (3.1)$$

in which w_l, $l = 1,\ldots,L$, is the weight for the pair (x_l^*, y_l^*). The data, $\{(x_l^*, y_l^*) : l = 1,\ldots,L\}$, is called *learning data* because one "trains" the fuzzy logic system to follow the data to the degree measured by Equation 3.1.

The interested reader may wish to explore other methods for fine-tuning a fuzzy logic model. Glorennec [8], Katayama et al. [10], and Driankov et al. [6] describe several methods for adjusting the parameters to fit learning data. Also, Young [17] proposes using a measure of implication derived from fuzzy subsethood to fine-tune fuzzy logic models. This measure of implication measures the degree to which the input implies the output. To fine-tune a given model, therefore, perturb the parameters of the model to maximize this measure of implication.

4. WORKERS COMPENSATION EXAMPLE

Here is an example of building and fine-tuning fuzzy logic models, using workers compensation insurance data from an insurance company for four consecutive rating periods. Call the insurance company Workers Compensation Insurer (WCI). To protect the interests of this insurance company, the data has been masked by linearly transforming it and by relabeling the geographic regions and the dates involved.

There is a distinction between prescriptive and descriptive modeling. The first part of this section briefly explains the decision process that WCI works through every six months, and proposes and builds fuzzy logic systems that model that process. That is, fuzzy models are built based on the expert opinions of the actuaries and other managers at WCI. This is prescriptive modeling, and it corresponds to Steps 1 through 3 in Section 3. The second part of this section fits three fuzzy logic models based on the data that WCI provides, using Step 4 in Section 3. That is, we seek to find fuzzy models that describe what WCI has actually done in the past.

WCI files rates for its workers compensation insurance line in various states. Every six months, WCI determines the adequacy of those filed rates. WCI represents that adequacy by an *indicated target*. For example, an indicated target of +5% in a state means that WCI requires premiums equal to 105% of its filed rates to reach a specified return on surplus. Similarly, an indicated target of −7% means that WCI requires premium equal to 93% of its filed rates.

In the fuzzy models, the indicated target is based on the experience data. WCI calculates it by comparing the filed rates in a state with the sum of the experience loss ratio and expense ratio in that state, among other items. Based on the indicated target and supplementary (financial and marketing) data, WCI then chooses a selected target for each state. (See Section 4.1.1 for more about how WCI selects a target.) Financial data include competitively-driven rate departures with respect to previous selected targets. For example, a rate departure of −1% means that actual premium was 99% of (filed rates)∗(1 + selected target). Marketing data include retention ratios and actual versus planned initial premium.

4.1. Prescriptive Modeling

4.1.1 *Verbally state linguistic rules.* To develop linguistic rules for a prescriptive model, the pricing actuaries and product developers at WCI provide information about how an ideal "target selector" would use the data for choosing a target. As a rule of thumb, if the indicated target increases over the previous six months, then the selected target increases, and vice versa. However, this rate change is tempered by how well the region met its previous targets and by how much business is written in the region. For example, if the region had a positive rate departure recently, then WCI might consider increasing the selected target. Also, if the amount of business is low relative to planned, then WCI might consider decreasing the selected target in order to stimulate growth. On the other hand, a large amount of initial business (relative to planned initial business) may not be desirable because of the legal or competitive climate in a given state.

In view of the opinions of the experts at WCI, the following linguistic rules are developed on which to base a prescriptive fuzzy logic model:

(a) If the change in indicated target from time $t-1$ to time t is *positive*, and if the recent rate departure is *positive*, and if the amount of business is *good*, then the change in selected target from time $t-1$ to time t is *positive*.

(b) If the change in indicated target from time $t-1$ to time t is *zero*, and if the recent rate departure is *zero*, and if the amount of business is *moderate*, then the change in selected target from time $t-1$ to time t is *zero*.

(c) If the change in indicated target from time $t-1$ to time t is *negative*, and if the recent rate departure is *negative*, and if the amount of business is *bad*, then the change in selected target from time $t-1$ to time t is *negative*.

Methods for measuring the amount of business include premium, number of accounts, retention ratio, close ratio (percentage of new business written to new business quoted), and premium for new business. This paper measures amount of business by the sum of the retention ratio and the minimum of the ratio of actual initial premium to planned initial premium and the inverse of that ratio, that is, min(actual/planned, planned/actual). This minimum lies between 0.0 and 1.0, and it takes into account that writing a great deal of business (relative to the planned initial premium) is not necessarily a profitable goal. The closer the minimum is to 1.0, the better the region has met its target. If the ratio actual/planned is very small or very large, then min(actual/planned, planned/actual) is close to 0.0. Therefore, a *good* amount of business is measured relative to a maximum of 2.0, after expressing the retention ratio as a decimal.

4.1.2 *Create the fuzzy sets corresponding to the hypotheses and determine the output values for the conclusions.* In the above linguistic rules, each hypothesis is a compound statement that combines three fuzzy sets with the connector *and*. Denote the space of change in indicated target by X_1, the space of rate departures (RD) by X_2, and the space of amount of business by X_3. On each of these spaces, define three fuzzy sets—one for each fuzzy rule.

To get the endpoints of each of these spaces and the intermediate boundary points, work backwards as follows: Determine the maximum and minimum changes in the selected target from time $t-1$ to time t. For example, suppose that the maximum allowable change in selected target is +10%, and the minimum is −10%. Then, determine the changes in indicated target, the rate departures, and the sum of retention ratio and min(A/P, P/A) that would lead to those maximum and minimum changes. Suppose that the selected target would be increased 10% if the indicated target increased by at least 15%, if the rate departure were at least +3%, and if the measure of the amount of business were greater than or equal to 1.8. Also, suppose that the selected target would be decreased by 10% if the indicated target decreased by at least 10%, if the rate departure were at least −5%, and if the measure of the amount of business were less than or equal to 1.0.

Then, the space of change in indicated target is effectively $X_1 = [-10\%, 15\%]$, the space of rate departures is $X_2 = [-5\%, 3\%]$, and the space of amount of business is $X_3 = [1.0, 1.8]$. In this representation, all rate departures less than -5% are identified with -5% because the change in selected target resulting from any rate departure less than -5% is the same as the change in selected target if the rate departure were identically -5%. Observed values outside the ranges selected for other variables are treated similarly.

To get the intermediate points at which no change in selected target occurs, decide what values of change in indicated target, rate departure, and amount of business would lead to no change. Suppose that these values are 0%, 0%, and 1.6, respectively. The defining equations of the fuzzy sets for *positive*, *zero*, and *negative* changes in indicated target (*chind*) are, respectively,

$$m_{positive}(chind) = \max\left[0, \min\left(\frac{chind - 0}{15 - 0}, 1\right)\right]$$

$$m_{zero}(chind) = \max\left[0, \min\left(\frac{15 - chind}{15 - 0}, \frac{chind + 10}{0 + 10}\right)\right]$$

$$m_{negative}(chind) = \max\left[0, \min\left(1, \frac{0 - chind}{0 + 10}\right)\right].$$

Similarly, the defining equations of the fuzzy sets for *positive*, *zero*, and *negative* rate departures are, respectively,

$$m_{positive}(rd) = \max\left[0, \min\left(\frac{rd - 0}{3 - 0}, 1\right)\right]$$

$$m_{zero}(rd) = \max\left[0, \min\left(\frac{3 - rd}{3 - 0}, \frac{rd + 5}{0 + 5}\right)\right]$$

$$m_{negative}(rd) = \max\left[0, \min\left(1, \frac{0 - rd}{0 + 5}\right)\right],$$

and the defining equations of the fuzzy sets for *good*, *moderate*, and *bad* amounts of business are, respectively,

$$m_{good}(bus) = \max\left[0, \min\left(\frac{bus - 1.6}{1.8 - 1.6}, 1\right)\right]$$

$$m_{moderate}(bus) = \max\left[0, \min\left(\frac{1.8 - bus}{1.8 - 1.6}, \frac{bus - 1.0}{1.6 - 1.0}\right)\right]$$

$$m_{bad}(bus) = \max\left[0, \min\left(1, \frac{1.6 - bus}{1.6 - 1.0}\right)\right].$$

Finally, the change in selected target is given by

$$\begin{aligned}&[m_{A_1}(chind, rd, bus) \cdot 10 + m_{A_2}(chind, rd, bus) \cdot 0 \\ &\quad + m_{A_3}(chind, rd, bus) \cdot (-10)] \\ &\div [m_{A_1}(chind, rd, bus) + m_{A_2}(chind, rd, bus) \\ &\quad + m_{A_3}(chind, rd, bus)],\end{aligned}$$

(see Equation 2.1) in which

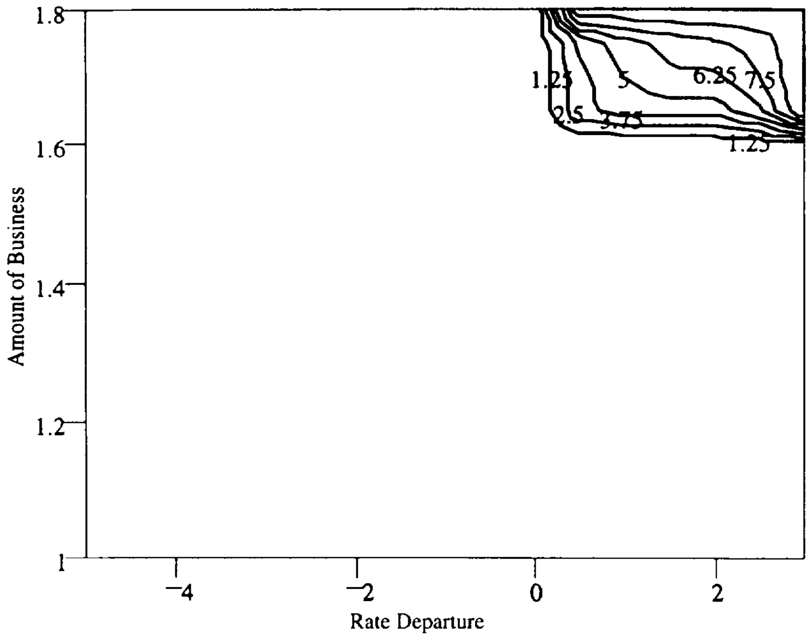
$$\begin{aligned}&m_{A_1}(chind, rd, bus) \\ &\quad = \min[m_{positive}(chind), m_{positive}(rd), m_{good}(bus)] \\ &m_{A_2}(chind, rd, bus) \\ &\quad = \min[m_{zero}(chind), m_{zero}(rd), m_{moderate}(bus)] \\ &m_{A_3}(chind, rd, bus) \\ &\quad = \min[m_{negative}(chind), m_{negative}(rd), m_{bad}(bus)].\end{aligned} \tag{4.1}$$

Figure 5 plots contours of the change in selected target against rate departure and amount of business while fixing the change in indicated target at +10%. Amount of business is along the vertical and rate departure lies along the horizontal. Note that the region for "no change" is relatively large.

If the three variables—change in indicated target, rate departure, and amount of business—may interact when connected by

FIGURE 5

CONTOURS OF CHANGE IN SELECTED TARGET AGAINST AMOUNT OF BUSINESS AND RD (USING AND IN ALL THREE RULES)



and, then consider replacing the min operator with the algebraic product. Also other intersection operators may be used, including those that form weighted averages of the values of the membership functions. There are many ways to formulate the fuzzy rules, but an actuary should, at a minimum, check contour plots to see which formulation coincides with the philosophy or practices of the company. For example, in Figure 5, it should be verified that such a large area of no change is consistent with the company's pricing philosophy when the change in the indicated target is +10%.

TABLE 1

Variable 1	Variable 2	Weighted Correlations
Current Indicated	Current Selected	0.952
Change in Indicated	Change in Selected	0.812
Previous Selected	Current Selected	0.909
Previous RD	Change in Selected	0.281
Current RD	Change in Selected	0.151
Previous Retention	Current Selected	−0.412
Previous Retention	Change in Selected	0.236
Current Retention	Current Selected	−0.429
Current Retention	Change in Selected	−0.035
Actual/Planned Initial	Current Selected	−0.115
Actual/Planned Initial	Change in Selected	−0.149
min(Act/Plan,Plan/Act)	Current Selected	−0.270
min(Act/Plan,Plan/Act)	Change in Selected	0.005

4.2. Descriptive Modeling

Turning to the descriptive portion of fuzzy modeling, fuzzy models are fit to the data that WCI provided. In selecting a target, the actuaries consider the relative amount of business in each state. For this reason, the fuzzy models were fine-tuned by minimizing a weighted sum of squared errors, as in Equation 3.1. The data for each period were weighted according to the premium in each state, after normalizing the weights so that they add to 1.00. Then, each six-month period was weighted equally. That is, a weighted sum of squared errors was calculated for each six months, then those four numbers were added together to get a total sum of squared errors. As a benchmark for the fuzzy models, linear functions were fitted via weighted least squares regression.

Weighted correlations were calculated between variables of interest. See Table 1 for those correlations. The weights used were the same as those used in fine-tuning the fuzzy logic models and in calculating the linear regressions. Note that the correlations between the current indicated and current selected, between

TABLE 2

WEIGHTED CORRELATIONS BETWEEN ERRORS AND REMAINING VARIABLES

	Model (1), Exhibit 1	Model (1), Exhibit 3	Model (2), Exhibit 3
Previous RD	0.025	−0.075	−0.074
Current RD	0.085	0.027	−0.058
Previous Retention	−0.124	−0.059	−0.079
Current Retention	0.160	−0.063	−0.132
Actual/Planned Initial	0.064	−0.007	0.006
min(Actual/Plan,Plan/Actual)	−0.066	0.081	0.027

the change in indicated and change in selected, and between the previous and current selected targets are fairly high.

An actuary may begin by considering simple fuzzy logic models involving one or two explanatory variables that correlate highly with the change in the selected target or the target itself. Starting from the simple models that fit most closely to the data, an actuary may then expand them to include more complicated models with two or more explanatory variables. Those more complicated models may not be substantially more accurate than the simple ones. In this case, the correlations between the errors from the simple models and remaining variables of interest (see Table 2) are fairly small, thus confirming that more complicated models may not add accuracy to the description of WCI's target selecting practices.

In general, models with two rules are nearly as accurate as those with three or more rules. For this reason, only the results of fine-tuning models with two rules (plus the default rule of no action if none of the hypotheses is satisfied) are presented here. Exhibits 1 through 3 display the results obtained using three simple models. To fine-tune these fuzzy logic models, Excel's Solver was used to minimize the weighted sum of squared errors from the four six-month periods, given the starting values, as listed in Exhibits 1 through 3. Solver uses a gradient-descent method (beginning with the starting values) to optimize an objective function subject to constraints. The starting values are the endpoints of the input spaces and the changes in the selected (or the selected target itself) that correspond to the conclusions. Each model described in the exhibits involves only two fuzzy rules, plus the default rule of no action, so only the boundary points of the input spaces and the two output values (either the selected target or changes in the selected) were specified. In the optimization, the left-hand endpoint was constrained to be less than or equal to the right-hand endpoints. In general, the solution obtained by Excel's Solver yields a local minimum. Although the global minimum may not have been reached, the solution may be desirable because, in some sense, it is close to the initial system.

Exhibit 1 considers rules that depend on the value of the change in the indicated target from the previous selection period. To compare with a standard model, the weighted least squares regression line that uses the same explanatory variable, namely, the change in indicated target, is included. The fuzzy model fits only slightly better, as measured by the sum of squared errors, than does the linear regression. In the fuzzy model in Exhibit 1, the starting values of -10.00 and 10.00 for the change in the indicated target from the previous period imply that the space of indicated targets is partitioned into the two fuzzy sets graphed in Figure 6. The set $\{-10.00, 10.00\}$ associated with the change in the selected target means that if the change in the indicated target were -10.00 or less, then the change in the selected would be -10.00. Similarly, if the change in the indicated target were 10.00 or more, then the change in the selected would be 10.00. Figure 7 graphs the change in the selected target as a function of the change in the indicated, before fine-tuning using Excel's Solver.

To minimize the squared-error loss in Equation 3.1, both the endpoints of the interval for the change in the indicated tar-

FIGURE 6

PARTITION OF CHANGE IN THE INDICATED TARGET BEFORE SOLVER SOLUTION

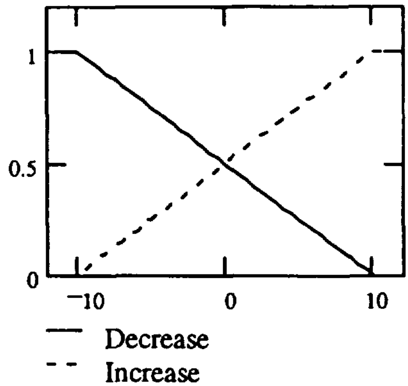


FIGURE 7

CHANGE IN SELECTED TARGET AS A FUNCTION OF CHANGE IN INDICATED TARGET BEFORE SOLVER SOLUTION

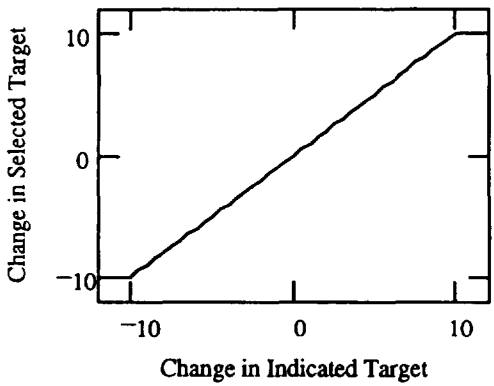
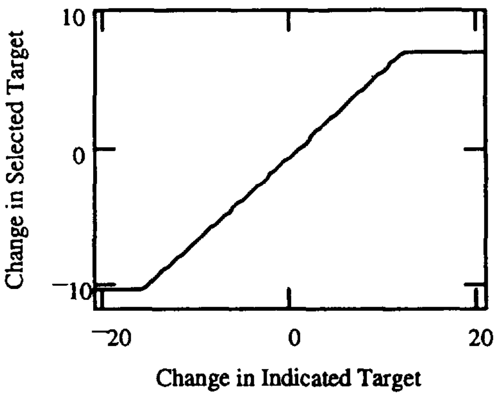


FIGURE 8

CHANGE IN SELECTED TARGET AS A FUNCTION OF CHANGE IN INDICATED TARGET AFTER SOLVER SOLUTION



get, -10.00 and 10.00, and the changes in the selected target, -10.00 and 10.00, are varied. The interval for change in the indicated target becomes the interval $[-15.73, 12.34]$, and the interval for changes in selected target becomes $[-10.52, 6.92]$. Thus, the maximum decrease in selected target is -10.52 and the maximum increase is 6.92. See Figure 8 for a graph of the change in the selected target as a function of the change in the indicated target, after fine-tuning using Excel's Solver.

The form of the presentation and the results are similar in the following two exhibits. Exhibit 2 expands on the model in Exhibit 1 by considering the most recent and the previous rate departures. Exhibit 3 calculates the selected target itself as a function of the current indicated target and the previous selected target. Two fuzzy models were fitted—one that joins the phrases in the hypotheses with *or* (the max operator), and another that uses *and* (the min operator). The former model provides the better fit of the two models, and it has an average error 0.35% smaller than that of linear regression.

5. SUMMARY AND CONCLUSIONS

This paper demonstrates how to build and fine-tune a fuzzy logic system from linguistic rules to finished model, while distinguishing between the prescriptive phase and the descriptive phase. It emphasizes models that combine experience data with supplementary data. It compares those fuzzy models with linear regressions to judge their performance.

Even though a given fuzzy logic model may fit only slightly better than a standard linear regression model, the main advantage of fuzzy logic is that an actuary can begin with verbal rules and create a mathematical model that follows those rules. Fuzzy logic allows linguistic rules to be handled in a consistent manner; it allows possibly conflicting goals and constraints to be combined. By fine-tuning a model using historical data, an actuary can judge whether his or her company has followed those rules. A model can also be fine-tuned based on information from several (possibly conflicting) experts.

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EXHIBIT 1

CHANGE IN SELECTED TARGET AS A FUNCTION OF THE CHANGE IN INDICATED TARGET

(1) **Fuzzy model:**

(a) If the indicated target decreases from time $t-1$ to time t, then decrease the selected target from time $t-1$ to time t.

(b) If the indicated target increases from time $t-1$ to time t, then increase the selected target from time $t-1$ to time t.

	Starting Values	Solver Solution
Indicated Change	$[-10.00, 10.00]$	$[-15.73, 12.34]$
Selected Change	$\{-10.00, 10.00\}$	$\{-10.52, 6.92\}$

Sum of squared errors	34.98
Average error	$\sqrt{34.98/4} = 2.96$

(2) **Linear regression:**

Change in selected = $-0.84 + 0.48 *$(change in indicated)

Sum of squared errors	39.62
Average error	$\sqrt{39.62/4} = 3.15$

EXHIBIT 2

CHANGE IN SELECTED TARGET AS A FUNCTION OF THE CHANGE IN INDICATED TARGET AND OF THE RATE DEPARTURE

(1) **Fuzzy model using the most recent rate departure:**

(a) If the indicated target decreases from time $t-1$ to time t and if the recent rate departure (RD_t) is negative, then decrease the selected target from $t-1$ to time t.

(b) If the indicated target increases from time $t-1$ to time t and if the recent rate departure (RD_t) is positive, then increase the selected target from time $t-1$ to time t.

Note: By default, if both hypotheses have zero weight, then do not change the selected target.

	Starting Values	Solver Solution
Indicated Change	[−10.00, 10.00]	[−15.73, 12.34]
RD_t	[−10.00, 10.00]	[−37.87, 23.20]
Selected Change	{−10.00, 10.00}	{−11.09, 7.32}

Sum of squared errors 32.88

Average error $\sqrt{32.88/4} = 2.87$

(2) **Fuzzy model using the previous rate departure:**

(a) If the indicated target decreases from time $t-1$ to time t and if the previous rate departure (RD_{t-1}) is negative, then decrease the selected target from time $t-1$ to time t.

(b) If the indicated target increases from time $t-1$ to time t and if the previous rate departure (RD_{t-1}) is positive, then increase the selected target from time $t-1$ to time t.

	Starting Values	Solver Solution
Indicated Change	[−10.00, 10.00]	[−13.31, 11.85]
RD_{t-1}	[−10.00, 10.00]	[−23.06, 12.67]
Selected Change	{−10.00, 10.00}	{−11.25, 7.09}

Sum of squared errors 32.74

Average error $\sqrt{32.74/4} = 2.86$

(3) **Linear regression using the most recent rate departure:**

Change in selected $= -0.75 + 0.48 *$(change in indicated)$ + 0.22 * RD_t$

Sum of squared errors 38.28

Average error $\sqrt{38.28/4} = 3.09$

(4) **Linear regression using the previous rate departure:**

Change in selected $= -0.34 + 0.47 *$(change in indicated)$ + 0.42 * RD_{t-1}$

Sum of squared errors 37.26

Average error $\sqrt{37.26/4} = 3.05$

EXHIBIT 3

SELECTED TARGET AS A FUNCTION OF THE INDICATED TARGET AND OF THE PREVIOUS SELECTED TARGET

(1) **Fuzzy model using *or*:**

(a) If the current indicated target or the previous selected target is low, then the current selected target is low.

(b) If the current indicated target or the previous selected target is high, then the current selected target is high.

	Starting Values	Solver Solution
Indicated_t	[−20.00, 20.00]	[−131.38, 62.67]
Selected_{t-1}	[−20.00, 20.00]	[−143.55, 48.75]
Selected_t	{−20.00, 20.00}	{−124.99, 48.03}

Sum of squared errors 27.09

Average error $\sqrt{27.09/4} = 2.60$

(2) **Fuzzy model using *and*:**

(a) If the current indicated target and the previous selected target are low, then the current selected target is low.

(b) If the current indicated target and the previous selected target are high, then the current selected target is high.

	Starting Values	Solver Solution
Indicated_t	[−20.00, 20.00]	[−20.27, 48.18]
Selected_{t-1}	[−20.00, 20.00]	[−31.18, 36.64]
Selected_t	{−20.00, 20.00}	{−21.55, 28.53}

Sum of squared errors 34.99

Average error $\sqrt{34.99/4} = 2.96$

(3) **Linear regression:**

$\text{Selected}_t = -3.84 + 0.45 * \text{Indicated}_t + 0.38 * \text{Selected}_{t-1}$

Sum of squared errors 34.90

Average error $\sqrt{34.90/4} = 2.95$

DISCUSSION OF PAPER PUBLISHED IN VOLUME LXXIX

WORKERS COMPENSATION EXPERIENCE RATING: WHAT EVERY ACTUARY SHOULD KNOW

WILLIAM R. GILLAM

DISCUSSION BY THE AUTHOR

"Welcome to the working week.
I know it don't thrill ya', I hope it won't kill ya'."
—Elvis Costello

Abstract

The calculation of plan parameters and rating values, which is described in Sections 5 and 6 of the original paper, has been significantly improved.[1] *The calculation of expected loss rates (ELRs) has been improved by breaking down published rates into (partial) pure premium*[2] *components before adjusting for trend, development, and amendment factors that vary by component. Coupled with other refinements, this better reflects the distribution of injuries by class, and improves on the accuracy of the ELRs. The calculation of D-ratios has been improved, using data more closely matching that of the experience period. Finally, changes have been made in the plan parameters to reduce swing in the experience modifications of small risks; in the course of implementation, this was referred to as graduating the plan.*

[1]The National Council on Compensation Insurance (NCCI) is always working to improve its products, and this discussion documents several improvements made to the Experience Rating Plan since the time of the original article. To the credit of the regulatory actuaries involved in the NAIC Examination of NCCI conducted in 1992, many of the changes stemmed from the useful recommendations made at that time.

[2]In this paper we refer to the partial pure premiums that underlie workers compensation rates simply as pure premiums, consistent with internal production staff usage.

The following discussion essentially replaces the part of the original paper beginning with Section 5D.

1. CALCULATION OF PLAN PARAMETERS AND EXPERIENCE RATING VALUES

A. Calculation of Plan Parameters

A.1. State Reference Point (SRP)

This calculation is unchanged.

$$SRP = 250 \times SACC, \quad \text{rounded to the nearest 5,000,} \qquad \text{and}$$

$$G = SRP/250{,}000, \quad \text{rounded to the nearest 0.05,}$$

where SACC is the state average cost per claim, at a maturity consistent with that of the experience rating period.

For the State C example in the new exhibits, $SRP = 1{,}400{,}000$, and $G = 5.6$. Individual losses in experience rating are limited to 10% of the SRP, a value called the state accident limit (SAL) on ratable losses.

A.2. B *and* W *Values*

The primary credibility ballast B is calculated using the same formula as in the original paper, but subject to a new indexed minimum, rather than the previously used $7,500.

$$B = E(0.1E + 2{,}570G)/(E + 700G),$$

$$\text{subject to a minimum of } 2{,}500G.$$

Similarly, the excess credibility ballast C has a new indexed minimum, rather than the previously used $150,000.

$$C = E(0.75E + 203{,}825G)/(E + 5{,}100G),$$

$$\text{subject to a minimum of } 60{,}000G.$$

Because typical G values in the 1990s average about 4, the new minimum B and C values already tend to be larger than \$7,500, reducing swing (i.e., responsiveness) in the modifications of small risks. Larger ballast values mean lower credibilities.

A.3. Caps On Modifications

To prevent large swings in the modification of small risks, an indexed maximum limits the calculated value as a function of risk size and G:

$$\text{Max} = 1 + 0.00005(E + 2E/G).$$

Thus a risk with \$5,000 of expected loss in a state where $G = 4$ could have a modification no larger than 1.38. This replaces a table of maximum modifications by size range, whose discontinuity occasionally led to surprise changes in the modifications of small insureds when data in preliminary modifications were updated.

B. Calculation of Rating Values

B.1. Expected Loss Rates (ELRs)

In the experience rating plan, payroll (in \$100s) by class is extended by the respective ELRs to obtain expected losses.

The purpose of Exhibit 1 is to calculate Expected Loss Rate Factors, as shown on Line 8 of Exhibit 1, Part 2. The product of these factors and the corresponding partial pure premiums for each class are summed in order to get a provisional Expected Loss Rate for that class.

The principles of the calculation have not changed, but a treatment more focused on the impacts of loss development, loss based expenses, law amendments, and trend has been implemented. Starting with the partial pure premiums underlying the rates (i.e., serious, non-serious and medical), we must back up in time and maturity to the experience period used for modifications. Loss development, law amendment factors, and trend are all calculated by pure premium for each of the policy periods used in ratemaking. So, in Column 3 of Exhibit 1, Part 1, we calculate average serious development factors to ultimate for each of three periods to be used in experience rating—third, second, and first reports. These are weighted to get one average serious development factor. This is also done for non-serious and medical losses. Notice that medical development factors exist for both serious and non-serious medical; these are weighted to be appropriate for the entire medical pure premium.

To account for law changes between the experience period and the effective period, law amendment factors by injury type and year are weighted to calculate average benefit on-level factors for each pure premium. These are in Column 5 of the exhibit. Note that the Unit Statistical Plan (USP) period is not the same as the experience period for prospective modification. (The reader may note the magnitude of the amendment factor for serious claims in State C. This is an example of the reforms implemented in the crisis period of the late 1980s and early 1990s.)

In Column 7, indemnity on-level loss ratio trend is used for the serious and non-serious pure premiums, and medical on-level loss ratio trend is used for the medical pure premium. These trends are taken from the rate level calculations. Trend factors of two years for first report, three years for second report, and four years for third report are weighted by on-level losses from Column 6 to produce an average trend used to unwind each pure premium.

ELRs are calculated for open competition states as well as administered pricing states. Expenses in manual rates are well-documented in the rate filing. An expense left in the component pure premiums that may still need to be unwound is loss adjustment expense, which in the State C example is 12.5% of loss. There may also be a factor for loss-based assessments, which are occasionally a part of component pure premiums.

Part 2 of Exhibit 1 displays important adjustments needed to calculate final ELR factors. In Lines 1 and 2, we adjust for expenses. In Line 3, we calculate the so-called loss ratio adjustment factor, which accounts for the difference between the latest on-level financial data loss ratio used for the rate level and the trended value of the USP loss ratio. This difference stems in part from the large number of adjustments between manual rates and final prices in the workers compensation rating plan. Another part is that the USP compiles data on each policy at 18 months and annually thereafter, while the financial data is compiled from a calendar year of policies once a year, starting with a 24 month evaluation date. Testing as described in Section 2 confirms the value of this adjustment.

On Line 3a of the exhibit, we see the USP indicated change of 0.927 adjusted by a trend factor of 1.092 to be comparable to the proposed rate level change. In State C, the financial data would indicate a lower change in loss costs than the trended USP loss ratio. The final ELR factors must be higher than those that would be obtained by unwinding trend from the prospective rates; as such, the loss ratio adjustment factor is 1.066.

Line 4 is needed because ELR factors apply to the voluntary level pure premiums, which have been offset downward to account for a new assigned risk pricing program. (This huge offset is another sign of the times.) The final ELRs will be at a total market level, higher than the voluntary level.

The adjustment on Line 6 on Part 2 accounts for losses in excess of the SAL. The Excess Loss Adjustment Factors (ELAFs) are calculated using a weighting of excess ratios in the same way as described in Exhibits 5 and 6 in the original paper. Separate ELAFs are calculated to be applied to the serious pure premium, which is indemnity only, and the medical pure premium, which includes a portion for medical associated with all indemnity claims. Since the excess ratios are for ground-up serious claims including medical, there must be an ELAF applied to the medical pure premium, adjusted by the proportion of the medical pure premium that is for serious claims. It is assumed that no adjustment for loss limitation is necessary for non-serious claims.

For State C, the Hazard Group II serious ELAF is $0.855 = 1 - 0.145$, based on an excess ratio of 0.145. Using data from ratemaking, the serious medical portion of the medical pure premium is 0.379. Thus, the Hazard Group II Medical ELAF is $1 - (0.145)(0.379) = 0.945$. The eight resulting ELAFs appear on Line 6 of Exhibit 1, Part 2.

Notice that there is no adjustment to account for the higher average quality of rated risks. Previously, ELRs were divided by 1.01 to account for this phenomenon. It has been decided that it is more appropriate to let the modification seek its own level. Rate adequacy is based on standard premium and will automatically adjust for the impact of this change on the average modification.

The provisional ELR factors to be applied to the partial pure premiums are shown on Line 8 of Exhibit 1, Part 2.

A further step is necessary after summing these components by class. The partial pure premiums include trend, development, and amendment factors, as well as the impact of credibility, limits on change by class, and the test correction factor to reconcile to the proposed rate level change. In the final manual rate or loss cost, a factor for manual/earned premium, which varies by industry group, is applied to the sum of the pure premiums. This rate factor should be maintained in the ELR, so it is applied in the calculation of the final ELR. For example, consider class 4021 in Hazard Group II and the manufacturing industry group in State C. Suppose it has a serious pure premium of \$3, a non-serious pure premium of \$1, and a medical pure premium of \$2. Then its provisional Expected Loss Rate is $(3)(.554) + (1)(1.085) + (2)(.520) = \$3.79$ per \$100 of payroll. If the manual to earned ratio for manufacturing is 0.98, the final ELR would be $(\$3.79)(.98) = \$3.71$. Thus \$100,000 in payroll in this class in State C would contribute (1,000)(3.71) = $3,710 of expected losses for experience rating.

B.2. D-Ratios

In the calculation of the experience modification, expected losses by class are extended by the respective D-ratios in order to compute expected primary losses for use in experience rating.

The calculation of D-ratio factors is shown in Exhibit 2. These are weighted by the partial pure premiums by class to produce final D-ratios.

The calculation of D-ratios has changed in ways alluded to in the original paper. Specifically, partial D-ratios are based on losses in the latest three statistical plan reports, rather than the most recent single policy year. Actual losses are limited by the state accident limit ("ratable" losses), but adjusted for severity trend from the date of available statistical plan reports to the period that will actually be used for experience rating.[3] Consideration of Table 1 in the original paper should lead to the conclusion that this is normally about two years, although sometimes a bit less. Partial D-ratios are computed for each pure premium component of the rate. The serious partial D-ratio represents the ratio of serious indemnity primary losses to total serious indemnity losses. There is a similar non-serious partial D-ratio. The medical partial D-ratio is primary medical losses divided by the medical total.

Since the primary/excess split in experience rating applies to total losses of indemnity plus medical, and partial D-ratios apply to pure premiums that are (serious) indemnity only, (non-serious) indemnity only, and all medical, some care must be taken in the calculation of the partial D-ratios. This entails the separation of indemnity primary and medical primary (Columns 5 and 6 in

[3]Originally, the author wanted to leave the data untrended and deflate the split point, as this would lead to the adjustment of only one number. The concept of deflation was considered too avant-garde for a highly regulated line of business.

Exhibit 2, Part 1) in the serious and non-serious losses, and the addition of medical primary to the medical partial D-ratio (see Column 6 in the medical row of Exhibit 2, Part 1).

The calculation of final D-ratio factors must reflect the change in pure premium weights between the statistical plan period and the prospective rates. This adjustment (in production parlance, a transition from partial D-ratios to D-ratio factors) is calculated in Exhibit 2, Part 2. The partial D-ratios would be applied to the pure premiums from the calculation of classification rates. Unfortunately, these pure premiums are not in the proper proportions to represent the losses expected for experience rating. Specifically, at ultimate, there is a relatively larger proportion of serious and medical loss and a smaller portion of non-serious loss than will be found in statistical plan data at first, second, and third reports. Therefore, an adjustment is necessary or the estimate of the proportion of primary losses will be too low. The partial D-ratios are multiplied by adjustment factors, Exhibit 2, Part 2, Line 2 divided by Line 6. This is the arithmetic equivalent of adjusting the classification partial pure premiums. The results are the final D-ratio factors as shown on Line 7 of Exhibit 2, Part 2. If a particular class rate in State C was based on pure premiums of \$3 for serious, \$1 for non-serious, and \$2 for medical, then the D-ratio would be $\{(\$3)(.056) + (\$1)(.688) + (\$2)(.266)\}/(\$3 + \$1 + \$2) = .23$.

2. EX-ANTE TESTING OF NEW ELR CALCULATION

Exhibit 3 shows some of the testing done at NCCI to support the new ELR calculation. The idea is to calculate sample ELRs for a historical period, using a new method, and then apply them in their respective time period. We looked at what would have happened to average modifications by class group and ELR accuracy by class. We desired as uniform a result as possible; that is, that ELR accuracy and average modification be as close as possible for the different classes or class groups. The new calculation achieves this better than the prior methodology.

Testing this requires considerable investigation of old ratemaking files and a good bit of care in programming modification calculations using hypothetical ELRs.

Consider Exhibit 3, Part 1. It shows average modifications by hazard group, industry group, and overall for four states. The prior calculation produces modifications that tend to be low for the higher severity class groups; i.e. Hazard Groups III and IV or the Contracting industry group. The revised calculation does not completely correct the problem, but decidedly reduces it. This is expected, since the highest severity classes tend to have the most weight in the serious and medical pure premiums. These two pure premiums should have the lowest ELR factors, and the ELRs for high severity classes should bear a correspondingly low relationship to the rate.

Exhibit 3, Part 2 shows statistics pertaining to ELR accuracy by class for the prior and revised ELR calculations.

One problem is that losses less than $2,000 may be (and often are) summarized in statistical plan reporting, and are always summarized in the data used for normal experience rating. We are not able to attribute these losses to class, since carriers are not required to report the class on medical only claims. As such, we leave them out of the accuracy calculation. Hence, the actual/expected (A/E) ratios tend to be 10–20% low for both calculations. We need to normalize to an overall unity A/E ratio for each hypothetical calculation of ELRs; this puts each on a level playing field when sample variance from expected is calculated as described below.

Overall sample weighted squared variation from unity in A/E by class is calculated on a statewide basis by individual class by hazard group and industry group. Performance of the proposed ELR calculation is measured by squared variation. With occasional exception, performance of the revised method is better than the prior method. In particular, the variation between hazard groups and the variation between industry groups are both reduced. Notice that the *within* variation from the class group mean A/E ratio plus the variation *between* class groups equals the total variation. (This is a hip-pocket fact the manager can use to check the unwary student's work.)

3. CONCLUSION

It is a pleasure to report these improvements in the administration of experience rating. It is clear that such improvements will continue to be needed and will continue to be made.

EXHIBIT 1

PART 1

EXPECTED LOSS RATE CALCULATION
DEVELOPMENT, AMENDMENT, AND TREND BY PURE PREMIUM

STATE: C
EFF DATE: 7/1/1994
USP PERIOD 1/89–12/89 Experience Rating Policy Period: 7/90–7/91

(1)	(2) Losses		(3) Dev. Fact. 3rd to Ult		(4) Ultimate Losses		(5) Amend. Factor		(6) On-Level Losses		(7) Trend Factor		(8) Trended Losses
Death	11,685,084	×	1.690	=	19,747,792	×	0.411	=	8,116,343	×	1.134	=	9,203,933
P.T.	4,702,449	×	1.690	=	7,947,139	×	0.174	=	1,382,802	×	1.134	=	1,568,097
Major	236,250,583	×	1.690	=	399,263,485	×	0.711	=	283,876,338	×	1.134	=	321,915,767
Serious:	252,638,116	×	1.690	=	426,958,416	×	0.687	=	293,375,483	×	1.134	=	332,687,797
Minor	48,728,103	×	1.018	=	49,605,209	×	0.710	=	35,219,698	×	1.134	=	39,939,138
T.T.	49,596,614	×	1.018	=	50,489,353	×	0.885	=	44,683,077	×	1.134	=	50,670,609
Non-Serious:	98,324,717	×	1.018	=	100,094,562	×	0.798	=	79,902775	×	1.134	=	90,609,747
Ser. Med.	96,609,465	×	1.765	=	170,515,706	×	0.943	=	160,796,311	×	1.432	=	230,260,317
Non-Ser. Med.	87,721,420	×	1.047	=	91,844,327	×	0.943	=	86,609,200	×	1.432	=	124,024,374
Medical	184,330,885	×	1.423	=	262,360,033	×	0.943	=	247,405,511	×	1.432	=	354,284,691

USP PERIOD 1/90–12/90 SUMMARIZED Experience Rating Policy Period: 7/91–7/92

			2nd to Ult										
Serious:	184,901,837	×	1.929	=	356,675,644	×	0.705	=	251,425,573	×	1.099	=	276,316,705
Non-Serious:	91,859,386	×	1.029	=	94,523,308	×	0.821	=	77,627,553	×	1.099	=	85,312,681
Medical	158,584,459	×	1.456	=	230,904,175	×	0.943	=	217,742,637	×	1.309	=	285,025,112

EXHIBIT 1

PART 1—PAGE 2

(1)	(2) Losses		(3) Dev. Fact. 3rd to Ult		(4) Ultimate Losses		(5) Amend. Factor		(6) On-Level Losses		(7) Trend Factor		(8) Trended Losses
USP PERIOD	1/91–12/91		SUMMARIZED						Experience Rating Policy Period: 7/92–7/93				
					1st to Ult								
Serious:	102,700,971	×	2.617	=	268,768,441	×	0.816	=	219,337,131	×	1.065	=	233,594,044
Non-Serious:	96,880,811	×	0.996	=	96,493,288	×	0.894	=	86,290,577	×	1.065	=	91,899,465
Medical	152,050,518	×	1.524	=	231,692,605	×	0.968	=	224,278,442	×	1.197	=	268,461,295
THREE YEAR WEIGHTED VALUE													PRODUCT
Serious:			1.948			×	0.726			×	1.103	=	1.560
Non-Serious:			1.014			×	0.837			×	1.098	=	0.932
Medical			1.465			×	0.951			×	1.317	=	1.835

EXHIBIT 1

PART 2

Calculation of ELR Factors by Premium and Hazard Group

	Factor	Serious	Non-Serious	Medical
1) Combined report, development, amendment, and trend factor from Part 1				
From Part 1	—	1.560	.932	1.835
LAE Factor	1.125	—	—	—
Product	—	1.755	1.048	2.064
2) Reciprocal of Factor above	—	0.570	0.954	0.484
3) (a) Adjusted USP Experience Change				
0.927 × 1.092 =	1.012			
(b) Financial Data Experience Change	0.9500			
(c) Loss Ratio Adj. Factor (a)/(b)	1.066			
4) Offset for New Assigned Risk Programs	0.937			
5) Average ELR Factor (2) × (3c)/(4)		0.648	1.085	0.550
6) Excess Loss Adjustment Factors				
HG I		0.873	1.0	0.949
HG II		0.855	1.0	0.945
HG III		0.803	1.0	0.898
HG IV		0.729	1.0	0.850
7) Adjustments:	None			
8) ELR Factors (5) × (6) × (7)				
HG I		0.566	1.085	0.522
HG II		0.554	1.085	0.520
HG III		0.520	1.085	0.494
HG IV		0.472	1.085	0.468

EXHIBIT 2

PART 1

STATE C

CALCULATION OF D-RATIO FACTORS
SPLIT VALUE = $5,000

	First, Second and Third Reports Combined							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Trended Ratable	Primary	Ratable Indemnity	Ratable Medical	Estimated Indemnity Primary (2) × ((3)/(1))	Estimated Medical Primary (2)–(5)	3-Yr Total Partial D-Ratio (5)/(3)	3-Yr Total Loss Dist. (3)/Tot (1)
Serious	739,637,237	44,498,824	522,295,420	217,341,817	31,422,880	13,075,944	0.060	0.377
Non-Serious	579,960,300	224,227,305	331,478,713	248,481,587	128,158,045	96,069,260	0.387	0.240
Medical Only	63,838,341	60,718,083	0	63,838,341	0	60,718,083		
Total	1,383,435,878	329,444,212	853,774,133	529,661,745				
					Medical	169,863,287	0.321*	0.383**

NOTE
THIS REPORT IS FOR STATEWIDE DATA

Proposed Effective Date: 07-01-94
10% of State Reference Point = 140,000
Severity Trend = 1.147

*Medical D-Ratio Factor (6)/(4)
**Loss Distribution (4)/(1)

EXHIBIT 2

PART 2

CALCULATION OF D-RATIO FACTORS

Adjustment For Use With Ultimate Pure Premiums

	(A) Serious	(B) Non-Serious	(C) Medical	(D) Total
1. Three Year Partial D-ratios, Part 1, Col. 7	0.060	0.387	0.321	XXX
2. Three Year Loss Distribution, Part 1, Col. 8	0.377	0.240	0.383	1.000
3. Ultimate USP Experience On-Level	950,281,470	319,380,590	1,009,590,996	XXX
4. Rate Factors Applied by Parts	0.980	0.979	1.060	
5. Experience Underlying Final Rates (3) × (4)	931,275,841	312,673,598	1,070,166,456	2,314,115,894
6. Experience Distribution (5)/sum(5)	0.402	0.135	0.463	1.000
7. Final D-ratio Factors (1) × (2)/(6)	0.056	0.688	0.266	XXX

EXHIBIT 3

PART 1

REVISION TO ELR CALCULATION

Ex-Ante Test of Effect on Average Mod
Rating Year 1992

State	HAZARD GROUP I			HAZARD GROUP II			HAZARD GROUP III			HAZARD GROUP IV		
	Prior	Revised	% Diff	Prior	Revised	% Diff	Prior	Revised	% Diff	Prior	Revised	% Diff
A	1.011	1.008	−0.3%	1.010	1.024	−1.4%	1.032	1.049	1.6%	0.967	1.001	3.5%
B	1.035	1.016	−1.8%	1.012	1.008	−0.4%	0.980	0.984	0.4%	0.922	0.936	1.5%
C	1.062	1.060	−0.2%	1.102	1.106	0.4%	1.003	1.001	−0.2%	0.945	0.960	1.6%
D	1.025	0.992	−3.2%	1.001	0.997	−0.4%	0.979	0.984	0.5%	0.950	0.970	2.1%
Total	1.035	1.022	−1.3%	1.023	1.024	0.1%	0.996	1.002	0.6%	0.934	0.952	1.9%

State	MANUFACTURING			CONTRACTING			ALL OTHER			ALL RISKS		
	Prior	Revised	% Diff	Prior	Revised	% Diff	Prior	Revised	% Diff	Prior	Revised	% Diff
A	1.003	1.012	0.9%	0.974	1.003	3.0%	1.032	1.046	1.4%	1.006	1.025	1.9%
B	1.013	1.008	−0.5%	0.927	0.941	1.5%	1.001	0.998	−0.3%	0.974	0.978	0.4%
C	1.166	1.167	0.1%	0.949	0.954	0.5%	1.000	1.008	0.8%	1.030	1.036	0.6%
D	1.008	1.006	−0.2%	0.952	0.970	1.9%	0.993	0.989	−0.4%	0.980	0.985	0.5%
Total	1.033	1.032	−0.1	0.940	0.956	1.7%	1.007	1.009	0.2%	0.988	0.996	0.7%

EXHIBIT 3

PART 2

NEW ELR CALCULATION

Ex-Ante Test of Class Accuracy
State B, Rating Year 1992

Hazard Group	Actual Ratable Loss	Observed Actual/Expected Ratio:		Weighted Average Squared Deviation By Class from State Avg.		Weighted Average Squared Deviation By Class from HG Avg.	
		Prior	Revised	Prior	Revised	Prior	Revised
All	1,171,432,952	0.8401	0.8522	0.1999	0.1976	0.1965	0.1958
1	84,552,535	0.8084	0.7704	0.0602	0.0684	0.0588	0.0592
2	404,926,751	0.9024	0.8933	0.5062	0.4865	0.5007	0.4842
3	294,811,475	0.8472	0.8571	0.0807	0.0788	0.0806	0.0788
4	387,142,191	0.7852	0.8279	0.0349	0.0353	0.0306	0.0344
Between Variation		0.0034	0.0018				

Hazard Group	Actual Ratable Loss	Observed Actual/Expected Ratio:		Weighted Average Squared Deviation By Class from State Avg.		Weighted Average Squared Deviation By Class from HG Avg.	
		Prior	Revised	Prior	Revised	Prior	Revised
All	1,171,432,952	0.8401	0.8522	0.1999	0.1976	0.1963	0.1966
Mfg.	284,558,957	0.9109	0.8975	0.2869	0.2744	0.2798	0.2716
Ctg.	443,594,453	0.7845	0.8248	0.0327	0.0324	0.0283	0.0314
AO	443,279,542	0.8582	0.8528	0.3302	0.3217	0.3298	0.3217
Between Variation		0.0035	0.0011				

ADDRESS TO NEW MEMBERS—NOVEMBER 10, 1997

RONALD L. BORNHUETTER

When Mike Fusco addressed the new members last year, he said that we would run out of past presidents quite soon. I guess he was right, as this is my second time around.

Although my brief remarks this morning are directed to the new Fellows and Associates we honor today, I do hope the rest of the audience will glean some insights from them.

Because your time has been consumed by CAS actuarial studies during the past several years, your focus has, of necessity, been inward. Therefore, I would like to briefly turn your attention outward to the insurance world in which we live and the one that you are entering.

Each one of you in this room knows how hard you have worked to attain Associateship or Fellowship. It is a difficult road—one we have all traveled and one that will eventually lead to rewards in the years ahead. I cite, as an example, the recent emergence of casualty actuaries serving in a variety of senior management positions in many property and casualty companies.

To be more specific, at one point in 1996, seven major reinsurers—totaling over sixty-five percent of the United States reinsurance market—employed casualty actuaries as chief executive officers. Perhaps that is why we have such a terrible market!

As an aside, I once asked the chairman of a former employer of mine why the title Actuary was never used within the company. Interestingly, the response was that the title was too limiting. In retrospect, I can now say that I fully understand the intended meaning.

On occasion in the past, I have asked actuarial audiences how many of them spend at least ten percent of their time on the asset side of the balance sheet. Very few responded positively. Even today, I would wager that there are probably fewer than twenty-five in this room who do. I would also wager that very few of you are familiar with AFIR. If you are at all interested in broadening your horizons, I would suggest that you look into the activities of this organization.

An interesting piece of trivia concerns one of the sister organizations, the Institute of Actuaries in the United Kingdom, which is very similar to the two North American organizations giving examinations. Do you realize that over 25% of their members do not work in the insurance industry? Most of that 25% are employed in the securities industry—an interesting commentary, and one that does makes some sense.

I realize that the last thing you want to think about is more examinations; however, I would ask that you look around at your world. Some actuaries are taking CPCU exams, and there is also an achievable designation known as CFA—Chartered Financial Analyst—which is an entry into the securities industry. I guess what I am trying to say is that, for many of you, your actuarial training is a platform that prepares you for entry into many different and enriching areas of our business. Surprising results can occur if you are willing to take some risks.

Let me turn to today's catchword—"global." The comments in my 1987 address are just as valid today as they were ten years ago—perhaps even more so. Some ten years ago, the International Actuarial Association had about 4,600 members, 1,200 of whom came from Canada and the United States—even though there were over 10,000 eligible to join from the two countries. In ten years, the ratio is not much different. Out of a total IAA membership approaching 6,700, fewer than 2,500 members come from the United States and Canada, while some 20,000 are eligible to join. What is different today is that many U.S. domestic insurers and reinsurers—your employers—are branching outward—globally—with or without actuarial help. I might also mention that Europe is becoming much more litigious as we speak. Trial lawyers are very good at exporting their product. I am convinced that European actuaries do not have the background or the knowledge to cope with pricing the insurance product as it emerges in the casualty area. Each of you brings a wealth of talent and information to the expansion; your task is to make it known and used by your overseas managers. To those of you with global interests, I would also recommend that you look into ASTIN—a ready-made global forum for casualty actuaries. More than half of ASTIN's membership is composed of actuaries residing outside of North America.

I would be remiss if I did not comment on the American Academy of Actuaries. This is the outward-looking organization for all actuaries in the United States. Many of the CAS members in this room are involved in the various activities provided by the Academy. The Academy is your spokesperson in the public arena—don't hesitate to become involved.

Somewhat related is the Actuarial Standards Board, which has been in existence for ten years. This organization establishes the actuarial Statements of Principles and Standards of Practice. I would ask that you take heed of these policies, as "Principles" are the "thou shall" and "thou shall not," while "Standards" are the "how to" of our profession.

Ladies and gentlemen, I welcome you to the world of actuaries—it is an exciting, ever-changing, challenging, and very rewarding world, and you should feel very proud today to be a part of it. The CAS is only an organization of individuals and it is those individuals—its members—who make it great. May your careers blossom and prosper in the years ahead—the opportunity is there.

Thank you for your kind attention.

PRESIDENTIAL ADDRESS—NOVEMBER 10, 1997

WE NEED TO SERVE THEM BETTER

ROBERT A. ANKER

I have never really been sure whether the curse was Chinese, Turkish, Italian or Irish. This year it must have been actuarial, for these have indeed been interesting times. It has felt like a lifetime in the CAS leadership chair.

The past twelve months were filled with events, planned and unplanned, which have deepened the underpinnings of our profession and organization and remarkably broadened the dialogue among us over who we are. The publication of the first CAS Strategic Plan late last year became a catalyst for decisions taken by the Board of Directors and a guidepost for the Executive Council. Together we wrestled with a cascade of actions and reactions, definitions and interpretations, perceptions and misunderstandings that ensued. Independence of the CAS and our relationships with other actuarial bodies, key elements of the strategic plan, became hot issues. The passion lurking behind the cerebral facade of every actuary became manifest as the *Actuarial Review* suddenly became the most widely read and written-to actuarial publication on the planet.

Yes, these have been interesting times—and they will continue to be so. However, my purpose today is not to revisit a tumultuous year. Instead I want to remove the cloak of diplomacy that has necessarily guarded my words and actions this past year and share my thoughts with you—for whatever use they may or may not serve.

Before doing so, however, there is a part of these last twelve months I want to acknowledge in some detail—the tremendous personal support I have received. Tim Tinsley, our glittering gem of an Executive Director, was invaluable. I simply would not have made it through the year without him. Thanks, Tim—thanks, thanks, thanks. Thanks also to the entire office staff, without whom Tim might not have made it through the year. It takes an enormous effort to support the large and dynamic organization we are today.

This was a time of particular challenge for our Board. On more than one occasion they were asked to deal with unusually difficult circumstances. They did so with courage and aplomb. Without the knowledge that they were solidly behind me, I may well have turned tail and run both early and often, as they say. But they were there. Thanks.

I am enormously proud of the Executive Council with whom I served. Paul, Kevin, Sue, Pat and Bob all fulfilled their responsibilities effectively and admirably, each along the way helping to propel the CAS to a better future. I owe quintuple thanks to them.

I offer very special thanks to both my predecessor and my successor. This was a time during which I needed to draw on large portions of help and counsel. Al Beer and Mavis Walters were always there, responding with speed and wisdom, making the challenges manageable. Last November I assumed the reins from strong, firm and competent hands. This November I pass them on to strong, firm and competent hands.

Penultimately, I offer thanks to the virtual army of members of this body who have contributed part of their time and effort to making me better. Clearly, the job is incomplete, but that is a failure of neither quality nor quantity of effort. Thanks to all of you.

Ultimately, (I promise no jokes about actuaries developing to ultimate—it has already been done) my deepest thanks go to the only person who has lived my entire actuarial career with me, my wife Pat. She has provided thirty years of support—and distraction—in equal measure and with impeccable timing. I have learned from experience the full meaning of the word soulmate. Thank you, my love.

So ... what are the thoughts I would like to share? They are simply a collection of beliefs about what and who we are, actuaries in general, and what we need to do. By the way, the phrase "actuaries in general" applies in both meanings, generic and European. It is a strange quirk—perhaps a casualty—of the language that we must separate general discussion of actuaries from discussion of general actuaries. And, although for actuaries there is no life in general, that certainly does not mean general actuaries, while often casual, are without life.

All of this is prefatory to the point that we are part of a larger, worldwide actuarial profession. We need always to know and understand that. The very word "actuary" is owned collectively by those of us who share it in use. We must work globally with all with whom we share the title to assure that it ceaselessly retains the respect and professional character we cherish. There is important work here, work in which we, as the world's largest specialty educator of actuaries, need to take a more active role. The CAS is a unique organization on the world's actuarial stage, one whose "distinct identity" gives us great opportunity to teach others as well as to learn new ways to drive our profession and science forward.

Actuarial science, unlike law, medicine, ministry or accountancy, is a very young profession. It is yet to be seen whether, as practitioners of the discipline, we may someday rise to the level of recognition or pervasive application such professions have achieved. The prospects are positive, I believe. Nearly everything happening in the world as it approaches the new millennium plays to our strengths.

Economies are exploding, gathering the prerequisite size and velocity to create the need for actuarial tools to measure and reduce uncertainty, establish and refine classifications and provide a rational basis for that most abstract of actuarial skills, determining what is not unfairly discriminatory.

Information is also exploding, providing grist for our analytic mills and making competition and competitiveness ubiquitous. Every increase in competitive need is an opportunity for us to add value. Every quantum of acceleration in the rate of change increases uncertainty in the future. We *should* prosper. As change itself changes everything, we are well positioned to grow in non-traditional fields, even as our traditional fields expand. The "actuarial control cycle," as defined by the Australians, is a problem-solving algorithm equally at home in any area needing financial valuation of contingent events. If you do not know about the actuarial control cycle, please learn about it—continue your education. Similarly, the catch phrase being used by the Institute of Actuaries in celebrating their 150th anniversary in 1998, "Actuaries make financial sense of the future," does not restrict itself to any particular field.

What is the message?

- We need to be aware of and involved with all developments in the profession and relevant to the profession no matter where they occur.
- We need to continue to adopt the new as rapidly as we adapt the old.
- We need to be prepared to apply our science wherever and whenever it is needed.
- Change happens—and it is our friend.

Many of you know that part of my academic background lies in philosophy. Because of this, before I had ever heard of actuarial science, I encountered a school of philosophy based on probability theory. I determined to be a probabilist in my applied approach to life. Probabilism is the only school of philosophy containing a finite possibility it could be wrong.

This appeals to both my basic indecisiveness and my need to protect my self esteem. Yes, in this school, even if you are wrong, you not only are right, you predicted it. Given my refined aptitude for being wrong, the philosophy has served me well. But it also served me well in making the mathematical adaptation from pencil and wide-ruled paper to pencil and graph paper to the use of the Chemical Rubber Publishing Company's *Standard Mathematical Tables* to electrical calculators to electronic calculators to mainframe computers to generations of personal computers to the worldwide web to whatever follows...(With my luck it will be a variation of the abacus, with which I have no skill whatsoever.)

In my view, this philosophy serves any science well. After all, applied science is always based on the latest theory, even as researchers try to disprove the theory and theoreticians try to supplant it. In our case it is yet more fundamental. In every case, our best answer, developed with the unlimited measure of science and professionalism our clients deserve, is merely an estimate of the "right" answer. We define being close as being right in order to avoid being always wrong—but also because, if the right answer is currently knowable, then the world needs neither actuarial science nor actuaries.

These are all general thoughts, meaning those of a generic nature. What about thoughts that are specifically general, meaning about general contingencies? What about the CAS? What do we need?

First, as our record of growth and success indicates, we are in very good shape. We have been operating under a good estimate of the right answer. We do an enormous number of things well. It would be foolish to try to list them. Conversely, I am not able to identify anything we clearly do wrong. There are, however, areas in which we can do better.

We often continue to view ourselves as a United States organization. It is true that the vast majority of our 3,000 members live and practice in the "States," as those outside its borders refer to this country. On the other hand, many have heard me cite the convoluted statistic that there are today more members of the

CAS practicing outside the US than there were Fellows of the CAS when I became a Fellow. We do not serve them nearly as well as they deserve, and I believe they deserve at least as well as we served our total membership "way back when" when the Society inducted me. True, these members are widely dispersed, giving the task different dimensions. Equally true, the communication and service tools of today are incredibly advanced by comparison to even a decade ago, much less to "way back when." We can serve our non-US members better. We need to serve them better.

Some will note that our friends from Canada are included in my "outsiders" classification. Does that mean I believe we underserve them? Absolutely! We need to serve them better.

Our strategic plan notes that we have no geographic boundaries. We need to behave organizationally in ways that demonstrate that truth. We have members, including Fellows, at this meeting, who have never lived or practiced in North America. We need to serve them better. Even today, twenty-five percent of our regional affiliates are located outside the US. With our members in the British Isles and on the Continent organizing a new affiliate, the percentage will increase. We need to serve them better.

We also, in my opinion, need to beef up our support in research and further development of our science and its applications. There should be no significant research effort related to our field of endeavor in which we are not a participant. The support may be monetary through The Actuarial Foundation, the AERF, the CAS Trust or the CAS directly. It may be cooperative with any other actuarial, academic or related-discipline institution. It may be through the diligent personal efforts of each of us as individuals. But I believe it must happen, because we have clients and we need to serve them better.

There is another area we must improve concurrently. It is an area in which we have seen great improvement already, as demonstrated by the "hernia maker" issue of the *Proceedings* that most of us have just received. We need to do everything actuarially and humanly possible to be certain that the *Proceedings* is the preferred publication in the world for the best work in our field. I would not claim that any other publication occupies the pedestal today, but neither would anything in my experiences this past year with actuaries and others from all over the globe make me believe that the *Proceedings* sits on that pedestal. I believe it must. Why? In part, because we have competition. In part because all practitioners of our discipline need it, and we need to serve them better. But there is a more important and overriding reason—it has to do with vision.

This tumultuous year began with the publication of our strategic plan. I believe in that plan. I believe in the guiding principles it articulates. I believe in the functions it lays out for the CAS. I believe in the strategies it identifies. I believe in the definitions it uses and the direction in which it points. Most of all, I believe in its vision for the CAS: to be the pre-eminent resource for education, knowledge, experience and applied research for those actuaries who specialize in property, casualty and similar risk exposures, including the field known as general insurance.

We *can* achieve the vision, but in order to do so, in my view, we have work to do. Work including improving service to our worldwide membership, expanding research and refining and promoting the *Proceedings*. This work alone will not be sufficient. We will need constant attention to optimizing our relationships with other actuarial organizations of all types. We need to serve them better. We will need to continuously evolve and develop our education processes, keeping focus on adding value to students, members and our clients alike. We need to serve them better. We will need to strive eternally for higher quality as an organization and as professionals—all the more to serve them better. We have a great start, but we are young and it is only a start. We need to keep moving in the right direction, keep the strategic plan alive and adaptive, apply good common sense and, oh, by the way, keep it fun. It will help us serve them better.

Well, that's it. You know and I know there is nothing particularly deep or original here, but such are my thoughts, such is my philosophy. I share it in hope to serve you better.

MINUTES OF THE 1997 CAS ANNUAL MEETING

November 9–12, 1997

MARRIOTT'S DESERT SPRINGS

RESORT & SPA, PALM DESERT, CALIFORNIA

Sunday, November 9, 1997

The Board of Directors held their regular quarterly meeting from noon to 5:00 p.m.

Registration was held from 4:00 p.m. to 6:00 p.m.

From 5:30 p.m. to 6:30 p.m., there was a special presentation to new Associates and their guests. All 1997 CAS Executive Council members briefly discussed their roles in the Society with the new members. In addition, Michael L. Toothman, who is a past president of the CAS, briefly discussed his role with the American Academy of Actuaries' (AAA) Casualty Practice Council.

A welcome reception for all members and guests was held from 6:30 p.m. to 7:30 p.m.

Monday, November 10, 1997

Registration continued from 7:00 a.m. to 8:00 a.m.

CAS President Robert A. Anker opened the business session at 8:00 a.m. and recognized past presidents of the CAS who were in attendance at the meeting, including: Albert J. Beer (1995), Phillip N. Ben-Zvi (1985), Ronald L. Bornhuetter (1975), Michael Fusco (1989), David G. Hartman (1987), Allan M. Kaufman (1994), Thomas E. Murrin (1963-1964), LeRoy J. Simon (1971), Michael L. Toothman (1991), and Michael A. Walters (1986).

Mr. Anker also recognized special guests in the audience: Bruce Palmer, President of the American Risk and Insurance Association; Yoshiro Koike of the Institute of Actuaries of Japan; Allan M. Kaufman, President of the American Academy of Actuaries; Wilson W. Wyatt Jr., Executive Director of the American Academy of Actuaries; Peter F. Moser, President-Elect of the Canadian Institute of Actuaries; Anna M. Rappaport, President of the Society of Actuaries.

Mr. Anker then announced the results of the CAS elections. The next President will be Mavis A. Walters, and the President-Elect will be Steven G. Lehmann. Members of the Executive Council for 1997-1998 will be: Curtis Gary Dean, Vice President—Administration; Kevin B. Thompson, Vice President—Admissions; Susan T. Szkoda, Vice President—Continuing Education; Patrick J. Grannan, Vice President—Programs and Communication; and Robert S. Miccolis, Vice President—Research and Development. New members of the CAS Board of Directors will be: Paul Braithwaite, Jerome A. Degerness, Michael Fusco, and Stephen P. Lowe.

Patrick J. Grannan, Robert S. Miccolis, and Kevin B. Thompson announced the new Associates and Mavis A. Walters announced the new Fellows. The names of these individuals follow.

NEW FELLOWS

Jonathan David Adkisson
Timothy Paul Aman
Larry D. Anderson
Robert Sidney Ballmer II
Andrea C. Bautista
Steven L. Berman
Daniel David Blau
Carol Ann Blomstrom
Pierre Bourassa
George Peter Bradley
Mary Hemerick Bready
Kirsten Rose Brumley
Peter Vincent Burchett
John Frederick Butcher II
J'ne Elizabeth Byckovski
Dennis K. Chan
Rita E. Ciccariello
Jean Cloutier
William Brian Cody
Charles Anthony Dal Corobbo
Guy Rollin Danielson
Jeffrey F. Deigl
Robert G. Downs
Bernard Dupont
Jeffrey Eddinger
Carole M. Ferrero
Ginda Kaplan Fisher
Mary Elizabeth Fleischli
Christian Fournier
Julie Therese Gilbert
Michael Ambrose Ginnelly
Annette J. Goodreau
Mari Louise Gray
Leigh Joseph Halliwell

Paul James Hancock
Bradley Alan Hanson
David S. Harris
Amy Jean Himmelberger
Robert J. Hopper
David Dennis Hudson
Paul Robert Hussian
Hou-wen Jeng
Daniel Keith Johnson
Kurt Jeffrey Johnson
Mark Robert Johnson
Ira Mitchell Kaplan
Lowell J. Keith
Rebecca Anne Kennedy
Joan M. Klucarich
Jason Anthony Kundrot
Edward M. Kuss
Salvatore T. LaDuca
Gregory D. Larcher
Elizabeth Ann Lemaster
Jennifer McCullough Levine
Keith A. Mathre
Robert F. Maton
Richard Timmins McDonald
Daniel Julian Merk
Timothy Messier
Claus S. Metzner
Andrew Wakefield Moody
Robert Joseph Moser
Kimberly Joyce Mullins
Turhan E. Murguz
Aaron West Newhoff
Kevin Jon Olsen
Milary Nadean Olson
Regina Marie Puglisi
Patrice Raby
Raymond J. Reimer
Andrew Scott Ribaudo
Meredith Gay Richardson
Gregory Riemer
Stephen Paul Sauthoff
Christine E. Schindler
Terry Michael Seckel
Huidong Kevin Shang
Robert Daniel Share
Jeffrey Parviz Shirazi
Gary E. Shook
Jill C. Sidney
Lori Ann Snyder
Carl J. Sornson
Victoria Grossack Stachowski
Julia Causbie Stenberg
John A. Stenmark
Deborah L. Stone
Kevin Douglas Strous
Thomas Struppeck
Collin John Suttie
Steven John Symon
Daniel A. Tess
Glenn Allen Tobleman
Linda Kay Torkelson
Philippe Trahan
Theresa Ann Turnacioglu
Robert Ward Van Epps
Erica Lynn Weida
Geoffrey Todd Werner
Jeffrey D. White
Gayle Lynne Wiener
Elizabeth Ruth Wiesner
Tad E. Womack
Roger Allan Yard
Gerald Thomas Yeung
Jeffery Michael Zacek
Doug A. Zearfoss
Alexander Guangjian Zhu

NEW ASSOCIATES

John Scott Alexander
Paul C. Barone
Anna Marie Beaton
Andrew S. Becker
Frank J. Bilotti
Linda Jean Bjork
Michael J. Bluzer
Sherri Lynn Border
Richard Albert Brassington
Rebecca Schafer Bredehoeft
Kevin D. Burns
Joyce Chen
Michael Joseph Christian
Stephen Daniel Clapp
Christopher Paul Coelho
Kathleen T. Cunningham
Michael J. Curcio
Kevin Francis Downs
Michael Edward Doyle
Sophie Dulude
Kristine Marie Firminhac
Chauncey Fleetwood
David Michael Flitman
Sy Foguel
Hugo Fortin
Kevin Jon Fried
Noelle Christine Fries
Micah R. Gentile
Susan I. Gildea
Christopher David Goodwin
Philippe Gosselin
Jacqueline Lewis Gronski
Christopher Gerald Gross
Nasser Hadidi
Kenneth Jay Hammell
Gregory Hansen
Michelle Lynne Harnick
Ia F. Hauck
Cynthia Jane Heyer
Ali Ishaq
Christopher Donald Jacks
Jean-Claude Joseph Jacob
Walter L. Jedziniak
William Rosco Jones
Robert B. Katzman
Brandon Daniel Keller
Linda Kong
Richard Scott Krivo
Sarah Krutov
Alexander Krutov
Kirk L. Kutch
Todd William Lehmann
Charles Letourneau
Marc E. Levine
Daniel Patrick Maguire
David E. Marra
William A. Mendralla
Richard Ernest Meuret
Stephanie J. Michalik
Michael J. Miller
Christopher James Monsour
Robert John Moss
Charles P. Neeson
Helen Patricia Neglia
Tieyan Tina Ni
Christopher Maurice Norman
Steven Brian Oakley
David Anthony Ostrowski
James Alan Partridge
Lisa Michelle Pawlowski
Mark Paykin
Julie Perron
Anthony George Phillips
David John Pochettino
Matthew H. Price
Denise Farnan Rosen
Tracy A. Ryan
Shama S. Sabade
Christy Beth Schreck
Michael Robert Schummer
William Harold Scully III
Halina H. Smosna

Avivya Simon Stohl	Alice M Underwood	Laura Markham Williams
Brian Tohru Suzuki	David M. Vogt	Bruce Philip Williams
Nitin Talwalkar	Nathan Karl Voorhis	Joel F. Witt
Jonathan Garrett Taylor	Claude A. Wagner	Yuhong Yang
	Patricia Cheryl White	

Mr. Anker then introduced Ronald L. Bornhuetter, a past president of the Society, who presented the Address to new members.

Mr. Anker then announced Paul M. Otteson as the recipient of the 1997 CAS Matthew S. Rodermund Service Award. David G. Hartman presented the 1997 CAS Charles A. Hachemeister Prize to Stephen P. Lowe and James N. Stanard for their paper, "An Integrated Dynamic Financial Analysis and Decision Support System for a Property Catastrophe Reinsurer."

Gary R. Josephson, chairperson of the CAS Committee on Review of Papers, announced that seven *Proceedings* papers and one discussion of a *Proceedings* paper would be presented at this meeting. Mr. Josephson also announced that five papers would be published in the 1997 *Proceedings* but would not be presented at this meeting. The papers and the authors are: "A Markov Chain Model of Shifting Risk Parameters" by Howard C. Mahler; "The Insurance Expense Exhibit and the Allocation of Investment Income" by Sholom Feldblum; "On Approximations in Limited Fluctuation Credibility Theory" by Vincent Goulet; "Adjusting Indicated Insurance Rates: Fuzzy Rules that Consider both Experience and Auxiliary Data" by Virginia R. Young; and "Measurement of Asbestos Bodily Injury Liabilities" by Susan L. Cross and John P. Doucette.

Mr. Josephson presented the 1997 Woodward-Fondiller Prize to Leigh J. Halliwell for his paper "Loss Prediction by Generalized Least Squares." Mr. Josephson then presented the 1997 CAS Dorweiler Prize to co-winners Sholom Feldblum for his paper, "Personal Automobile Premiums: An Asset Share Pricing Approach for Property-Casualty Insurance," and Glenn G. Meyers for his paper, "The Competitive Market Equilibrium Risk Load Formula for Catastrophe Ratemaking."

Mr. Josephson introduced Stephen P. D'Arcy who presented the American Risk and Insurance Association (ARIA) Prize to Daniel Zajdenweber for his paper, "Extreme Values in Business Interruption Insurance." Mr. D'Arcy then introduced Bruce Palmer, President of ARIA, who spoke briefly about ARIA.

Mr. Anker then requested a moment of silence in honor of those CAS members who passed away since November 1996. They were: Robert D. Bart, Sr.; Douglas Critchley; Harold E. Curry; Richard C. Ernst; Alfred V. Fairbanks; Daniel J. Lyons; John S. McGuinness; Earl Nicholson; Max J. Schwartz; Byron Wright; and Hubert W. Yount.

Mr. Anker acknowledged a donation of $15,000 from D. W. Simpson & Company to the CAS Trust (CAST) on May 9, 1997.

Mr. Anker then concluded the business session of the Annual Meeting by announcing that the minutes of the Spring Meeting, this meeting, and the report of the Vice-President—Administration would be included in the next *Proceedings*.

After a refreshment break, Mr. Anker introduced the featured speaker, Oren Harari, an author and columnist on business and management issues.

The first general session was held from 10:45 a.m. to 12:15 p.m.

Consolidation in the Insurance Industry—A Forward Perspective

Moderator:	Frederick O. Kist Senior Vice President CNA
Panelists:	Heidi E. Hutter Chairman, President, and Chief Executive Officer Swiss Reinsurance America Corporation

Stephan L. Christiansen
Senior Vice President
Conning & Company

George N. Cochran
Managing Director
Cochran, Caronia & Co.

Following the general session, CAS President Robert A. Anker gave his Presidential Address at the luncheon after which he officially passed the CAS presidential gavel on to new CAS President Mavis A. Walters.

After the luncheon, the afternoon was devoted to concurrent sessions, which included presentations of the Hachemeister and ARIA Prize papers, and *Proceedings* papers. The panel presentations from 1:30 p.m. to 3:00 p.m. covered the following topics:

1. Quality Assurance for the Actuarial Work Product

Moderator: Patrick B. Woods
Assistant Vice President
Insurance Services Office, Inc.

Panelists: David B. Cox
Consulting Actuary

Gregory L. Hayward
Actuary
State Farm Mutual Automobile Insurance Company

Russel L. Sutter
Consulting Actuary
Tillinghast-Towers Perrin

2. Current Claims Issues

Moderator: James Surrago
Vice President
Insurance Services Office, Inc.

Panelists: Richard Boehning
Senior Vice President
American Insurance Services Group, Inc.

Joseph Jensen
Vice President
Computer Sciences Corporation

Barry C. Lipton
Vice President
Fireman's Fund Insurance Companies

3. Building a Dynamic Financial Analysis Model

Speaker: Gerald Kirschner
Senior Consulting Actuary
Ernst & Young LLP

4. Insurance Risk With the Year 2000

Moderator: George Burger
Assistant Vice President
Insurance Services Office, Inc.

Panelists: Hilary N. Rowen
Partner
Thelen, Marrin, Johnson & Bridges, LLP

Elizabeth Sterne
Counsel and Director of Reference Maintenance
Property Loss Research Bureau

David Wampold
Product Development Director
The Hartford

5. Casualty Practice Council

Moderator: Michael L. Toothman
AAA Vice President, Casualty Practice Council
Partner
Arthur Andersen LLP

Panelists: Ralph S. Blanchard III
Chairperson, Task Force on Property and Casualty Risk-Based Capital
Assistant Vice President & Actuary
Travelers Group

Frederick O. Kist
Chairperson, Committee on Property and Liability Issues
Senior Vice President
CNA

Jan A. Lommele
Chairperson, Committee on Property and Liability Financial Reporting
Principal
Deloitte & Touche LLP

The following 1997 CAS Hachemeister Prize Paper was presented:

"An Integrated-Dynamic Financial Analysis and Decision Support System for a Property Catastrophe Reinsurer"

Authors: Stephen P. Lowe
Consulting Actuary
Tillinghast-Towers Perrin

James N. Stanard
Chairman, President and Chief Executive Officer
Renaissance Reinsurance, Ltd.

The following 1997 ARIA Prize Paper was presented:

"Extreme Values in Business Interruption Insurance"

Author: Daniel Zajdenweber, U.F.R. SEGMI
Université Paris X-Nanterre

The following 1997 *Proceedings* paper was presented:

"Ratemaking: A Financial Economics Approach"

Authors: Stephen P. D'Arcy
Professor, Department of Finance
University of Illinois

Michael Dyer
Visiting Assistant Professor
University of Illinois

After a refreshment break from 3:00 p.m. to 3:30 p.m., concurrent sessions continued, and two *Proceedings* authors gave presentations of their papers. Certain concurrent sessions presented earlier were repeated. Additional concurrent sessions presented from 3:30 p.m. to 5:00 p.m. were:

1. Catastrophe Models and Some Applications

Moderator/ Panelist: John L. Tedeschi
Senior Vice President
Guy Carpenter & Company

Panelists: John P. Drennan
Consulting Actuary
JPD & Associates

Eric F. Lemieux
Chief Actuary
CAT, Ltd.

2. What Your Mother Never Told You About Part 10

Moderator: Roger M. Hayne
Chairperson, Committee on Theory of Risk
Consulting Actuary
Milliman & Robertson, Inc.

Panelists: John G. Aquino
Senior Vice President
Aon Re Services

Philip E. Heckman
Vice President and Actuary
Aon Risk Consultants, Inc.

Glenn G. Meyers
Assistant Vice President
Insurance Services Office, Inc.

3. Developing Scenarios as Input Into a DFA Model

Speakers: Stephen P. Lowe
Consulting Actuary
Tillinghast-Towers Perrin

François Morin
Consulting Actuary
Tillinghast-Towers Perrin

4. Introduction to the CAS Examination Committee

Moderator: William F. Murphy
Vice Chairperson, CAS Examination Committee
Consulting Actuary
Milliman & Robertson, Inc.

Panelists: J. Thomas Downey
Manager, Admissions
Casualty Actuarial Society

Donald D. Sandman
Actuary
Sentry Insurance

Richard P. Yocius
Actuary
Allstate Insurance Company

5. ASB Standard of Actuarial Practice on Loss Reserve Opinions

Moderator: Robert S. Miccolis
Chairperson, ASB Task Force on Reserving
Senior Vice President and Actuary
Reliance Reinsurance Corporation

Panelists: Members of the Subcommittee on Reserving of the ASB Casualty Committee

The following *Proceedings* papers were presented:

1. "1997 Retrospective Rating: Excess Loss Factors"

   Authors: William R. Gillam
   Vice President/Actuary
   National Council on Compensation Insurance, Inc.

   Jose Couret
   Senior Actuarial Associate
   Swiss Re America

2. Discussion by the original author of "Workers Compensation Experience Rating: What Every Actuary Should Know"

   Author: William R. Gillam
   Vice President/Actuary
   National Council on Compensation Insurance, Inc.

A reception for new Fellows and guests was held from 5:30 p.m. to 6:30 p.m., and the general reception for all members and their guests was held from 6:30 p.m. to 7:30 p.m.

Tuesday, November 11, 1997

Registration and a continental breakfast took place from 7:00 a.m. to 8:00 a.m.

Two general sessions were held simultaneously from 8:00 a.m. to 9:30 a.m. One was:

Actuarial Work Around the World

Moderator: David G. Hartman
Senior Vice President and Chief Actuary
Chubb Group of Insurance Companies

Panelists: Thomas R. Bayley
Vice President
Aetna International, Inc.

Gail M. Ross
Vice President
AM-RE Consultants, Inc.

Thomas Struppeck
Actuary
Centre Re

The other session, presented simultaneously, was

Putting the Catastrophe Challenge in Perspective

Moderator: Jeffrey C. Warren
Vice President
General Reinsurance Corporation

Panelists: Anthony Michaels
Director–Wrigley Institute for Environmental Studies
University of Southern California

Michael G. McCarter
Vice President, Industry and Regulatory Affairs
American International Group, Inc.

After a refreshment break, concurrent sessions were held from 10:00 a.m. to 11:30 a.m. In addition to concurrent sessions that were presented the previous day, the following three additional concurrent sessions and two additional *Proceedings* papers were presented:

1. Should a Merger or Acquisition be Part of Your Company Strategy?

   Moderator: Elizabeth M. Riczko
   Vice President
   Ohio Casualty Group

Panelists: George E. Councill
Director
Coopers & Lybrand, L.L.P.

2. Catastrophe Securitization

Moderator: Lowell J. Keith
Actuary
Zurich Centre ReSource, Ltd.

Panelists: Bryon G. Ehrhart
Executive Vice President
Alternative Financial Products
AON Worldwide Resources

Andrew J. Kaiser
Head of Insurance Products Group
Goldman, Sachs & Company

3. An Evolving Career Path—Financial Actuary

Moderator/ Panelist: James M. Bartie
Vice President
Chase Securities Inc.

Panelists: Steven J. Johnston
Chief Financial Officer and Treasurer
State Auto Insurance Companies

Robert F. Scott, Jr.
Senior Vice President
Willis Faber North America

The following *Proceedings* papers were presented:

1. "Balancing Development and Trend in Loss Reserve Analyses"

Author: Spencer M. Gluck
Executive Vice President
Swiss Re Services Corporation

2. "Funding for Retained Workers Compensation Exposure"

Authors: Brian Z. Brown
Consulting Actuary
Milliman & Robertson, Inc.

Michael D. Price
President
London Life and Casualty Reinsurance

Various CAS committees met from 1:00 p.m. to 5:00 p.m. In addition, tennis and golf tournaments were held during that time.

All meeting participants and their guests enjoyed a California Beach Barbecue from 6:30 p.m. to 10:00 p.m.

Wednesday, November 12, 1997

A continental breakfast was held from 7:00 a.m. to 8:00 a.m.

In addition to concurrent sessions that had been given previously and which were repeated, three additional concurrent sessions and three additional *Proceedings* papers were presented from 8:00 a.m. to 9:30 a.m. The concurrent session was:

1. Emerging Markets in the Pacific Rim

Moderator/ Panelist:	Herbert G. Desson Chief Actuary Aon Risk Consultants (Europe) Ltd.
Panelist:	Rejean S. Besner Deloitte & Touche LLP ABC Consulting

2. Questions and Answers with the CAS Board of Directors

Moderator:	Mavis A. Walters CAS President-Elect Executive Vice President Insurance Services Office, Inc.
Panelists:	Regina M. Berens Consulting Actuary MBA, Inc.
	David N. Hafling Senior Vice President and Actuary American States Insurance Companies

Steven G. Lehmann
Consulting Actuary
Miller, Rapp, Herbers & Terry, Inc.

3. New Fellows' Perspectives

Moderator: Steven G. Lehmann
Consulting Actuary
Miller, Rapp, Herbers & Terry, Inc.

Panelists: Mari L. Gray
Property and Casualty Actuary
State of Montana

Leigh J. Halliwell
Consulting Actuary
Milliman & Robertson, Inc.

Linda K. Torkelson
Associate Actuary, International
Allstate Insurance Company

Geoffrey T. Werner
Executive Director
United Services Automobile Association

The following *Proceedings* papers were presented:

1. "Parameter Uncertainty in (Log) Normal Distributions"

Author: Rodney E. Kreps
Executive Vice President and Chief Actuary
Sedgwick Re Insurance Strategy, Inc.

2. "Application of the Options Market Paradigm to the Solution of Insurance Problems"

Author: Michael G. Wacek
Managing Director
St. Paul Reinsurance Company, Ltd.

3. "Workers Compensation and Economic Cycles: A Longitudinal Approach"

Authors:	Robert P. Hartwig Swiss Re
	William J. Kahley California Workers Compensation Institute
	Tanya E. Restrepo National Council on Compensation Insurance, Inc.
	Ronald C. Retterath Retired Boca Raton, Florida

The final general session was held from 10:00 a.m. to 11:30 a.m. after a 30-minute refreshment break:

Banks in the Insurance Business

Moderator:	Linda L. Bell Senior Vice President and Chief Actuary The Hartford
Panelists:	David M. Klein Executive Vice President The Hartford International
	Chip Sharkey Senior Vice President and Chief Marketing Officer CNA

After the general session, Robert A. Anker announced future CAS meetings and seminars and officially adjourned the 1997 CAS Annual Meeting at 11:40 a.m.

Attendees of the 1997 CAS Annual Meeting

The 1997 CAS Annual Meeting was attended by 434 Fellows, 226 Associates, and 259 guests. The names of the Fellows and Associates in attendance follow.

FELLOWS

Shawna Ackerman
Jonathan David Adkisson
Jean-Luc E. Allard
Timothy Paul Aman
Larry D. Anderson
Richard R. Anderson
Scott C. Anderson
Robert A. Anker
John G. Aquino
Robert Sidney Ballmer II
William N. Bartlett
Edward J. Baum
Andrea C. Bautista
Thomas R. Bayley
Linda L. Bell
Gary F. Bellinghausen
Phillip N. Ben-Zvi
Regina M. Berens
Steven Louis Berman
James R. Berquist
Lisa M. Besman
David R. Bickerstaff
Richard A. Bill
James E. Biller
Richard S. Biondi
Ralph S. Blanchard III
Daniel David Blau
Carol Ann Blomstrom
Gary Blumsohn
LeRoy A. Boison, Jr.
Joseph A. Boor
Ronald L. Bornhuetter
Charles H. Boucek
George Peter Bradley
Nancy A. Braithwaite
Paul Braithwaite
James F. Brannigan
Yaakov B. Brauner
Mary Hemerick Bready
Paul J. Brehm
Dale L. Brooks
Ward M. Brooks
J. Eric Brosius
Brian Z. Brown
William W. Brown, Jr.
Lisa J. Brubaker
Peter Vincent Burchett
George Burger
Richard F. Burt, Jr.
J'ne Elizabeth Byckovski
Claudette Cantin
John E. Captain
Christopher S. Carlson
Sanders B. Cathcart
Martin Cauchon
Dennis K. Chan
David R. Chernick
James K. Christie
Allan Chuck
Rita E. Ciccariello
Jean Cloutier
Jo Ellen Cockley
William Brian Cody
Jeffrey R. Cole
Robert F. Conger
Eugene C. Connell
Francis X. Corr
David B. Cox
Catherine Cresswell
Richard M. Cundy
Robert J. Curry
Daniel J. Czabaj
Stephen P. D'Arcy
Ronald A. Dahlquist
Charles Anthony Dal Corobbo
Guy Rollin Danielson
Michael L. DeMattei
Curtis Gary Dean
Jerome A. Degerness

Jeffrey F. Deigl
Joseph J. Demelio
Daniel Demers
Howard V. Dempster
Anthony M. DiDonato
Kevin G. Dickson
Behram M. Dinshaw
Robert G. Downs
John P. Drennan
Brian Duffy
Bernard Dupont
Maribeth Ebert
Jeffrey Eddinger
Dale R. Edlefson
Bob D. Effinger, Jr.
Gary J. Egnasko
Valere M. Egnasko
Nancy R. Einck
Douglas D. Eland
Donald J. Eldridge
Thomas J. Ellefson
John W. Ellingrod
David Engles
Martin A. Epstein
Dianne L. Estrada
John S. Ewert
Doreen S. Faga
Madelyn C. Faggella
Dennis D. Fasking
Denise A. Feder
Richard I. Fein
Carole M. Ferrero
Ginda Kaplan Fisher
Russell S. Fisher
Beth E. Fitzgerald
Mary Elizabeth Fleischli
James M. Foote
Christian Fournier
Barry A. Franklin
Bruce F. Friedberg
Michael Fusco
James E. Gant
Andrea Gardner
Robert W. Gardner
Roberta J. Garland
James J. Gebhard
Richard N. Gibson
Julie Therese Gilbert
Bonnie S. Gill
William R. Gillam
Michael Ambrose Ginnelly
Spencer M. Gluck
Daniel C. Goddard
Annette J. Goodreau
Patrick J. Grannan
Mari Louise Gray
Ronald E. Greco
Eric L. Greenhill
Russell H. Greig, Jr.
Linda M. Groh
Denis G. Guenthner
Farrokh Guiahi
David N. Hafling
James W. Haidu
Allen A. Hall
Leigh Joseph Halliwell
Paul James Hancock
Elizabeth E. L. Hansen
Bradley Alan Hanson
Alan J. Hapke
Christopher L. Harris
David S. Harris
David G. Hartman
Marcia C. Hayden
Roger M. Hayne
David H. Hays
Gregory L. Hayward
E. LeRoy Heer
Suzanne E. Henderson
Dennis R. Henry
Kirsten Costello Hernan
Amy Jean Himmelberger
Mark J. Homan
Robert J. Hopper
Paul E. Hough
Ruth A. Howald
George A. Hroziencik
David Dennis Hudson
Heidi E. Hutter
Robert P. Irvan
Richard M. Jaeger
Hou-Wen Jeng
Daniel Keith Johnson
Eric J. Johnson
Kurt Jeffrey Johnson
Mark Robert Johnson
Steven J. Johnston
Jeffrey R. Jordan
Gary R. Josephson
Ira Mitchell Kaplan
Allan M. Kaufman

Lowell J. Keith
Anne E. Kelly
Rebecca Anne Kennedy
Deborah E. Kenyon
Michael B. Kessler
Frederick W. Kilbourne
Joe C. Kim
Gerald S. Kirschner
Frederick O. Kist
David M. Klein
Michael F. Klein
Joel M. Kleinman
Charles D. Kline, Jr.
Joan M. Klucarich
Leon W. Koch
Timothy F. Koester
John J. Kollar
Thomas J. Kozik
Israel Krakowski
Gustave A. Krause
Rodney E. Kreps
Adam J. Kreuser
Jane Jasper Krumrie
John R. Kryczka
Jeffrey L. Kucera
Andrew E. Kudera
Ronald T. Kuehn
Jason Anthony Kundrot
David R. Kunze
Edward M. Kuss
Salvatore T. LaDuca
Blair W. Laddusaw
David A. Lalonde
John A. Lamb
Gregory D. Larcher
James W. Larkin
Pierre Guy Laurin
Paul W. Lavrey
John P. Lebens
Joseph R. Lebens
Nicholas M. Leccese, Jr.
Steven G. Lehmann
Elizabeth Ann Lemaster
Eric F. Lemieux
Jennifer McCullough Levine
Orin M. Linden
Barry C. Lipton
Jan A. Lommele
Stephen P. Lowe
Donald F. Mango
Lawrence F. Marcus
Blaine C. Marles
Steven E. Math
Keith A. Mathre
Robert F. Maton
Robert W. Matthews
Heidi J. McBride
Michael G. McCarter
John W. McCutcheon, Jr.
Richard Timmins McDonald
Liam Michael McFarlane
Dennis C. Mealy
William T. Mech
Daniel Julian Merk
Timothy Messier
Claus S. Metzner
Glenn G. Meyers
Robert S. Miccolis
Jon Wright Michelson
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Mary Frances Miller
Michael J. Miller
Ronald R. Miller
Andrew Wakefield Moody
Brian C. Moore
Russell E. Moore
Kenneth B. Morgan, Jr.
Francois Morin
Jay B. Morrow
Robert Joseph Moser
Thomas G. Moylan
Robert V. Mucci
Evelyn Toni Mulder
Kimberly Joyce Mullins
Turhan E. Murguz
Daniel M. Murphy
William F. Murphy
Thomas E. Murrin
James J. Muza
John C. Narvell
Antoine A. Neghaiwi
Karen L. Nester-Schmitt
Richard T. Newell, Jr.
Aaron West Newhoff
Patrick R. Newlin

William A. Niemczyk
Victor A. Njakou
G. Christopher Nyce
David J. Oakden
Kevin Jon Olsen
Milary Nadean Olson
Paul M. Otteson
Joanne M. Ottone
Richard D. Pagnozzi
Robert G. Palm
Donald D. Palmer
Edward F. Peck
Wende A. Pemrick
Melanie Turvill Pennington
Andre Perez
Julia L. Perrine
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John M. Purple
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Richard A. Quintano
Patrice Raby
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Jerry W. Rapp
Raymond J. Reimer
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Meredith Gay Richardson
Elizabeth M. Riczko
Gregory Riemer
Brad M. Ritter
Tracey S. Ritter
Kevin B. Robbins
Sharon K. Robinson
Steven Carl Rominske
Jay Andrew Rosen
Deborah M. Rosenberg
Gail M. Ross
Randy J. Roth
Richard J. Roth, Jr.
Bradley H. Rowe
James B. Rowland
William J. Rowland
John M. Ruane, Jr.
James V. Russell
Sean W. Russell
Stuart G. Sadwin
Donald D. Sandman
Stephen Paul Sauthoff
Christine Ellen Schindler
Harold N. Schneider
David C. Scholl
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Sarah Krutov
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Nitin Talwalkar
Craig P. Taylor
Jonathan Garrett Taylor
John L. Tedeschi
David M. Terne
Trina C. Terne
Joseph O. Thorne
John P. Thorrick
Robert C. Turner, Jr.
Alice Mary Underwood
Frederick A. Urschel
David M. Vogt
Nathan Karl Voorhis
Claude Alain Wagner
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David L. Whitley
Bruce Philip Williams
Laura Markham Williams
Joel F. Witt
Bonnie S. Wittman
Robert F. Wolf
Yuhong Yang
Vincent F. Yezzi
Edward J. Zonenberg

REPORT OF THE VICE PRESIDENT–ADMINISTRATION

This report provides a summary of CAS activities since the last CAS Annual Meeting. I will first comment on these activities as they relate to the following purposes of the Casualty Actuarial Society as stated in our Constitution:

1. Advance the body of knowledge of actuarial science in applications other than life insurance;
2. Establish and maintain standards of qualification for membership;
3. Promote and maintain high standards of conduct and competence for the members; and
4. Increase the awareness of actuarial science.

I will then provide a summary of other activities that may not relate to a specific purpose, but yet are critical to the ongoing vitality of the CAS. And lastly, I will summarize the current status of our finances and key membership statistics.

In support of Purpose 1, the CAS has devoted significant resources during the past year to a variety of research initiatives, including the following projects as assigned to the appropriate committee:

- Creation of a World Wide Web site containing component variables for use in dynamic financial analysis models (DFA Task Force on Variables).
- Completion of a joint call paper program with the Insurance Data Management Association (Committee on Management Data and Information).
- Completion of 1997 call paper programs on ratemaking (Committee on Ratemaking), reserving (Committee on Reserves), reinsurance (Committee on Reinsurance Research), and dynamic financial analysis (Dynamic Financial Analysis Task Force on Variables).

- Collaboration with the SOA to identify areas where joint statements of principles can be developed (Committee on Principles).
- Progress on commissioned research on advanced collective risk models (Committee on Risk Theory).

New papers published in the *Proceedings*, four volumes of the *Forum*, and the *Discussion Paper Program*, all increase the body of knowledge available to our profession. Publications released in 1997 included papers on ratemaking, DFA, reserving, data management, and health care issues. The 1996 *Proceedings* included eleven new papers on a variety of topics—the most new papers published in a *Proceedings* in recent years.

The Admissions Committees provide the major support for Purpose 2. They make continuous improvements to the *Syllabus* and exam preparation and grading process, while overseeing the administration of the testing of approximately 6,400 registered candidates. Major initiatives in this area this year included:

- Design of a new CAS educational system beginning in 2000, along with the transition rules for gaining credit for the new exams.
- Inclusion of material on dynamic financial analysis in the 1998 *Syllabus* for Part 7.

Purpose 3 is partially achieved through a quality program of continuing education. The CAS provides these opportunities through the publication of actuarial materials and the sponsorship of a number of meetings and seminars. This year's sessions included:

- The Spring and Annual Meetings, held in San Antonio, Texas and Palm Desert, California;
- The 1995 CAS Seminar on Ratemaking, held in Boston, Massachusetts, which had 781 registrants;

- The Casualty Loss Reserve Seminar in Atlanta, Georgia, of which the CAS is a cosponsor with the American Academy of Actuaries, attended by 641;
- The special interest seminar on "Mergers and Acquisitions" in April, attended by 122;
- The special interest seminar on "Dynamic Financial Analysis" in July, attended by 257;
- The special interest seminar on "International Issues" held in October, attended by 90;
- The Reinsurance Seminar in June, attended by 277;
- The CIA/CAS Seminar for the Appointed Actuary, cosponsored by the Canadian Institute of Actuaries and the CAS, attended by 293; and
- Limited attendance seminars on "Reinsurance," "Principles of Finance," "Loss Distributions," and "Managing Asset Risk and Return."

The CAS Regional Affiliates also provide valuable opportunities for the members to participate in educational forums. In addition, the Regional Affiliates are a resource to help increase the awareness of the profession at the local level.

In support of Purpose 3, the External Communications Committee developed a CAS Communication Plan that includes promoting dynamic financial analysis; fostering awareness of casualty actuaries in corporate America, government and the P&C industry; encouraging students to consider a casualty actuary career; supporting AAA activities; and improving communications both internally and worldwide.

The CAS also promoted awareness of the profession through continued financial support of the Forecast 2000 Program. This program seeks to align the actuarial profession with crucial public policy issues and increase visibility of actuaries with the general public.

Also related to the fourth purpose, but generally affecting all purposes, are the CAS's international activities. In addition to the ongoing attendance at various international actuarial society meetings by the CAS leadership, the CAS continued to be an active participant in shaping the role of the International Forum of Actuarial Associations.

The CAS Office continues to provide excellent support and to expand its service and capabilities. Significant service additions have been realized with the establishment of the CAS World Wide Web Site and the offering of credit card services for payment of dues and fees. The web site proved to be a popular service very quickly with 1,000 members registered for the CAS "Members' Only" section during the first year of operations. The office also planned and budgeted for reimbursement of exam graders' travel expenses in fiscal year 1998.

The office coordinated a review of CAS operations by an outside law firm, which served as an update to the 1992 legal review. The 1997 update concluded that the CAS "appears to continue to be well-organized and operated from a legal perspective, with relatively low risk of legal liability."

Another resource of the CAS, and an integral part of its fabric and success, is its committees and many volunteers. Member participation on our committees remains high. The annual Leadership Meeting in March was highlighted by discussion of key strategic planning issues including CAS independence and relationship with the SOA, the CAS international role, and managing the consequences of growth.

In addition to approving the new educational system, the Board of Directors addressed several other aspects of the future of the CAS. With assistance from the Long Range Planning Committee, the Board amplified the 1996 Strategic Plan as it relates to CAS independence, CAS growth, and international issues. A vigorous and constructive dialogue with the Society of

Actuaries concerning CAS/SOA relationships and joint activities took place throughout the year.

New members elected to the Board of Directors for next year include Paul Braithwaite, Jerome A. Degerness, Michael Fusco, and Stephen P. Lowe. The membership elected Steven G. Lehmann to the position of president-elect, while Mavis A. Walters will assume the presidency.

The Executive Council, with primary responsibility for day-to-day operations, met either by teleconference or in person at least once a month during the year. The Board of Directors elected the following Vice Presidents for the coming year.

Vice President–Administration, Curtis Gary Dean

Vice President–Admissions, Kevin B. Thompson

Vice President–Continuing Education, Susan T. Szkoda

Vice President–Programs and Communications, Patrick J. Grannan

Vice President–Research and Development, Robert S. Miccolis

In closing, I will provide a brief status of our membership and financial condition. Our size continued its rapid increase as we added 208 new Associates and 125 new Fellows. Our membership now stands at 2,899.

The CPA firm of Feddeman & Company has been engaged to examine the CAS books for fiscal year 1997 and its findings will be reported by the Audit Committee to the Board of Directors in February 1998. The fiscal year ended with unaudited net income of $369,070, which compares favorably to a budgeted amount of $133,508. Members' equity now stands at $2,476,681. This represents an increase in equity of $394,943 over the amount reported last year.

For 1997–1998, the Board of Directors has approved a budget of approximately $3.7 million. Members' dues for next year will be $270, an increase of $10, while fees for the Subscriber Program will increase by $15 to $335.

In my final year as Vice President–Administration, it has been very gratifying to see the CAS continue to grow and strengthen.

Respectfully submitted,
Paul Braithwaite
Vice President–Administration

FINANCIAL REPORT
FISCAL YEAR ENDED 9/30/97

OPERATING RESULTS BY FUNCTION

FUNCTION	*INCOME*	*EXPENSE*	*DIFFERENCE*
Membership Services	$ 905,695 (a)	$ 959,541	$ (53,846)
Seminars	1,084,652	838,519	246,133
Meetings	542,727	529,566	13,161
Exams	2,395,668 (b)	2,246,261 (b)	149,407
Publications	68,378	39,051	29,327
TOTAL	$ 4,997,120	$ 4,612,938	$ 384,182 (c)

NOTES: (a) Includes income of $15,112 to adjust marketable securities to market value (SFAS 124).

(b) Includes $1,475,850 of Volunteer Services for income and expense (SFAS 116).

(c) Change in CAS Surplus net of $52,000 of interfund transfers ($50,000 to Research Fund and $2,000 to ASTIN Fund).

BALANCE SHEET

ASSETS	*9/30/96*	*9/30/97*	*DIFFERENCE*
Checking Account	$ 149,550	$ 237,098	$ 87,548
T-Bills/Notes	2,595,152	2,922,852	327,700
Accrued Interest	45,728	49,875	4,147
Prepaid Expenses	28,405	31,798	3,393
Prepaid Insurance	8,256	11,467	3,211
Accounts Receivable	8,555	13,782	5,227
Textbook Inventory	0	14,435	14,435
Computers, Furniture	253,266	270,717	17,451
Less: Accumulated Depreciation	(199,649)	(223,531)	(23,882)
TOTAL ASSETS	$ 2,889,263	$ 3,328,493	$ 439,230

LIABILITIES	*9/30/96*	*9/30/97*	*DIFFERENCE*
Exam Fees Deferred	$ 328,948	$ 338,649	$ 9,701
Annual Meeting Fees Deferred	62,675	52,860	(9,815)
Seminar Fees Deferred	67,376	21,106	(46,270)
Accounts Payable and Accrued Expenses	269,427	372,617	103,190
Deferred Rent	33,407	21,744	(11,663)
Accrued Pension	45,692	44,835	(857)
TOTAL LIABILITIES	$ 807,525	$ 851,811	$ 44,286

MEMBERS' EQUITY

Unrestricted	*9/30/96*	*9/30/97*	*DIFFERENCE*
CAS Surplus	$ 1,766,753	$ 2,150,935	$ 384,182
Michelbacher Fund	94,856	98,425	3,569
Dorweiler Fund	4,371	3,591	(780)
CAS Trust	3,641	18,825	15,184
Research Fund	185,404	154,207	(31,197)
ASTIN Fund	6,000	31,550	25,550
Subtotal Unrestricted	2,061,025	2,457,533	396,508
Temporarily Restricted			
Scholarship Fund	7,182	7,042	(140)
Rodermund Fund	13,531	12,106	(1,425)
Subtotal Restricted	20,713	19,148	(1,565)
TOTAL EQUITY	$ 2,081,738	$ 2,476,681	$ 394,943

C. Gary Dean, Vice President–Administration

This is to certify that the assets and accounts shown in the above financial statement have been audited and found to be correct.

CAS Audit Committee: Regina M. Berens, Chairperson;
Anthony J. Grippa, David N. Hafling and William J. Rowland

1997 EXAMINATIONS—SUCCESSFUL CANDIDATES

Examinations for Parts 3B, 4A, 4B, 5A, 5B, 6, 8, 8C (Canadian), and 10 of the Casualty Actuarial Society were held on May 5, 6, 7, 8, and 9, 1997. Examinations for Parts 3B, 4A, 4B, 5A, 5B, 7, 7C (Canadian) and 9 of the Casualty Actuarial Society were held on October 27, 28, 29, and 30, 1997.

Examinations for Parts 1, 2, 3A, and 3C (SOA courses 100, 110, 129, and 135, respectively) are jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries. Parts 1 and 2 were given in February, May, and November 1997, and Parts 3A and 3C were given in May and November of 1997. Candidates who were successful on these examinations were listed in joint releases of the two Societies.

The Casualty Actuarial Society and the Society of Actuaries jointly awarded prizes to the undergraduates ranking the highest on the Part 1 CAS Examination.

For the February 1997 Part 1 CAS Examination, the $200 first prize winners were Damon C. Capehart, University of Texas, and Yuri Zaderman, Rutgers University. The $100 second prize winners were: Michel D. Allain, University of Laval; Tom W. Baehr-Jones, Stuyvesant High School; Alice Y.H. Chan, University of Toronto; Pierre Girard, University of Laval; Michael B. Lewis, Yeshiva University; Hon-Bun E. Li, University of California; Elliot S. Moskowitz, Yeshiva University; and Rachel Yudovich, Massachusetts Institute of Technology.

For the May 1997 Part 1 CAS Examination, the $200 first prize winners were: Wang Han Sheng, Peking University; Tao Shi, Peking University; Junni Zhang, University of Science and Technology of China; Regina G. Dolgoarshinnykh, Central Connecticut State University; and Jie Gao, University of Science and Technology of China.

For the November 1997 Part 1 CAS Examination. The $200 first prize winners were: Wen Zhi Chen and Huilang Xie, both of the University of Science and Technology of China. The $100 second prize winners were: Jiehui Li, Zhongshan University; Zhi Li, Peking University; Jing Ning, University of Science and Technology of China; David Isaac Rudel, Harvey Mudd College; Ma Shuangge, University of Science and Technology of China; and Yu Zhou, Renmin University.

The following candidates were admitted as Fellows and Associates at the 1997 CAS Spring Meeting in May. By passing November 1996 CAS examinations, these candidates successfully fulfilled the Society requirements for Fellowship or Associateship designations.

NEW FELLOWS

Timothy Atwill
Margaret A. Brinkmann
Andrew J. Doll
Eric J. Gesick
Alessandrea Corinne Handley
James M. MacPhee
Mark Joseph Moitoso
Marlene D. Orr
Kathleen M. Pechan
Dale S. Porfilio
Robert Emmett Quane III
Jean-Denis Roy
Mark L. Thompson
James F. Tygh
Steven Boyce White
Floyd M. Yager

NEW ASSOCIATES

Ethan David Allen
Mark B. Anderson
Timothy Atwill
Wayne F. Berner
Jonathan Everett Blake
Edmund L. Bouchie
David John Braza
Cary J. Breese
Margaret A. Brinkmann
Hugh E. Burgess
Christopher J. Burkhalter
Stephanie T. Carlson
Sharon C. Carroll
Richard Joseph Castillo
Richard M. Chiarini
Theresa Anne Christian
Alfred Denard Commodore
Margaret Eleanor Conroy
Kenneth S. Dailey
John D. Deacon
Sharon C. Dubin
Denis Dubois
Rachel Dutil
Wayne W. Edwards
Jennifer R. Ehrenfeld
Kristine Marie Esposito
Joseph G. Evleth
Benedick Fidlow
Tracy Marie Fleck
John E. Gaines
David Evan Gansberg
Jay C. Gotelaere
Allen Jay Gould

John W. Gradwell
David Thomas Groff
Alexander Archibold Hammett
Daniel J. Henderson
David E. Heppen
William N. Herr, Jr.
Thomas Edward Hinds
Christopher Todd Hochhausler
Luke Delaney Hodge
Amy L. Hoffman
Dave R. Holmes
Jane W. Hughes
Jason Israel
Paul Ivanovskis
Jeremy M. Jump
Scott Andrew Kelly
David Neal Kightlinger
Deborah M. King
George A. Kish
Karen Lee Krainz
Jean-Sebastien Lagarde
Robin M. LaPrete
Yin Lawn
Kevin A. Lee
Neal M. Leibowitz
Bradley H. Lemons
Michael Victor Leybov
Janet G. Lindstrom
Christina Link
Michelle Luneau
James M. MacPhee
Andrea Wynne Malyon
Jason Noah Masch
William J. Mazurek
Phillip E. McKneely
Allison Michelle McManus
Paul D. Miotke
Mark Joseph Moitoso
Benoit Morissette
Janice C. Moskowitz
Michael James Moss
Vinay Nadkarni
Darci Z. Noonan
Michael A. Nori
Mihaela Luminita S. O'Leary
Christopher Edward Olson
Rebecca Ruth Orsi
Harry Todd Pearce
John S. Peters
Amy Ann Pitruzzello
Jennifer K. Price
Richard Bronislaus Puchalski
Patricia Ann Pyle
Kara Lee Raiguel
Rebecca J. Richard
John R. Rohe
Sandra L. Ross
Joanne Emily Russell
Lisa M. Scorzetti
Marc Shamula
Michael Shane
Bret Charles Shroyer
Katherine R. S. Smith
G. Dennis Sparks
Alan M. Speert
Nathan R. Stein
Lisa M. Sukow
C. Steven Swalley
Adam Marshall Swartz
Christopher C. Swetonic
Elizabeth Susan Tankersley
Patricia Therrien
Jeffrey S. Trichon
Kimberly S. Troyer
Timothy J. Ungashick
Martin Vezina
Karen E. Watson
Mark Steven Wenger
Miroslaw (Mirek) Wieczorek
Jerelyn S. Williams
Wendy Lynn Witmer
Simon Kai-Yip Wong
Jeffrey F. Woodcock
Edward J. Zonenberg

The following candidates successfully completed the Parts of the Spring 1997 CAS Examinations that were held in April and May.

Part 3B

Mustafa Bin Ahmad
Yassir T. Albaharna
Gwendolyn Lilly Anderson
Brandie J. Andrews
Jonathan L. Ankney
James D. Asahl
Nancy A. Ashmore
Grazyna A. Bajorska
Kevin J. Bakken
John L. Baldan
Mark Belasco
Johanne Belleau
Jody J. Bembenek
Brad D. Birtz
Joan M. Blenker
Michael J. Bluzer
Elaine K. Brunner
Donna L. Burchfield
Sarah Burns
Brian J. Cefola
Christian J. Coleianne
Angela L. Cooley
Christopher William Cooney
Deanna L. Crist
Keith R. Cummings
Dustin W. Curtit
Walter C. Dabrowski
Scott C. Davidson
Patricia A. Dillon
Brian S. Donovan
Lise Duhaime
Jeffrey S. Ernst
Lawrence K. Fink
David Michael Flitman
Sharon L. Fochi
Dustin W. Gary
William J. Gerhardt
Todd B. Glassman
Stacey C. Gotham
Jennifer T. Grimes
Michael B. Gunn
David L. Hanzlik
Jeffery T. Hay
Michael J. Hebenstreit
William S. Hedges
Kristina S. Heer
Terri-Beth Heffernan
Monica L. Herenstein
Marcy R. Hirner
Albert E. Holler III
Sean M. Housley
Long Fong Hsu
Alice H. Hung
Victoria K. Imperato
Jesse T. Jacobs
Philippe Jodin
David B. Johnson
Shantelle A. Johnson
Nancy E. Joyce
Kyewook Kang
Sarah M. Kemp
Stacey M. Kidd
Anne Marie Klein
Thomas F. Klem
Susanlisa Koppelman
Thomas F. Krause
Chingyee Teresa Lam
Sean R. Lawley
Wendy R. Leferson
Dengxing Lin
Andrea A. Lombardi
Eric A. Madia
Kevin B. Mahoney
Joshua N. Mandell
Paul T. Marino
Peter K. Markiewicz
Charles C. Martin
Joseph W. Mawhinney
Stephane McGee
William A. Mendralla
Richard Ernest Meuret
Christopher James Monsour
Kenneth D. Moore
Lori A. Moore
Catherine A. Morse
Craig S. Mosher
Maureen A. Motter

Anita D. Mountain
Bilal Musharraf
Brian C. Neitzel
Kelly T. Nguyen
Tieyan Tina Ni
Loren J. Nickel
Sylvain Nolet
Joshua M. Nyros
James P. O'Donovan
William S. Ober
Livia Oh
Rodrick R. Osborn
Apryle L. Oswald
Richard Matthew Pilotte
Bradley A. Price
Donald S. Priest
Eileen A. Prunty
Rebecca A. Putman
Timothy J. Regan
Arnie W. Rippener
Edward C. Roberts
Denise Farnan Rosen
John W. Rosengren
Marc R. Rothschild
Michael M. Rubin
Mark T. Rutherford
Samuel E. Sackey
Elizabeth A. Sexauer
Larry J. Seymour
Cheryl R. Shen
Lori A. Sheppard
Vadim S. Shulman
Annemarie Sinclair
John J. Skowronski
Douglas E. Smith
Molly A. Stark
Nita J. Stone
Ken P. Streng
Thomas E. Thun
Nelson C. E. Townsend
William D. Van Dyke
Justin M. Van Opdorp
Tara J. Van Wagenen
Lawrence A. Vann
Mark A. Verheyen
John T. Volanski
Claude A. Wagner
Lori A. Welch
Erica H. Wheeler
Arthur S. Whitson
William M. Winnis
Yoke Wai Wong
Walter R. Wulliger
Grace Zakaria
Gene Q. Zhang
Yin Zhang
Yingjie Zhang

Part 4A

Michael B. Adams
William J. Albertson
Brian C. Alvers
Frank Daniel K. Amfo
Ashwin Arora
Michael William Barlow
Paul C. Barone
Jason E. Berkey
Alex V. Bondarev
Michael J. Bradley
Jennifer L. Bramschreiber
John R. Broadrick
Derek D. Burkhalter
David R. Cabana
Jason A. Campbell
Samuel C. Cargnel
Caryn C. Carmean
Patrick J. Charles
Ching-Zen Cheng
Laurel A. Cleary
Paul L. Cohen
Crystal Dawn Danner
Timothy M. Devine
Mary Jane B. Donnelly
Brian S. Donovan
Scott H. Drab
Greg J. Engl
Laura A. Esboldt
Brian A. Fannin
Patrick V. Fasciano
Gina C. Ferst
Shelly A. Fowler
Geoffrey A. Fradkin
Matthew P. Gatsch
Cary W. Ginter
Todd B. Glassman
Jennifer Graunas
Karen L. Greene
James C. Guszcza
Kandace A. Heiser

Scott E. Henck
Glenn R. Hiltpold
Michael F. Hobart
Keepyung B. Hong
Derek R. Hoyme
Theodore L. Husveth
Jennifer L. Ims
Gregory K. Jones
Jody L. Jordahl
Minas K. Kalachian
Michael A. Kaplan
Sean M. Kennedy
Joseph E. Kirsits
Vasilis Koutsaftis
Francis A. Laterza
Wei Li
Stephen L. Lienhard
Joseph B. Logsdon
Kathleen T. Logue
Richard P. Lonardo
John T. Maher
Steven Manilov
Jeffrey Margasak
Sarah P. Mathes
Jeffrey B. McDonald
Patrick A. McGoldrick
Jennifer A. McGrath
Rasa Varanka McKean
Vadim Y. Mezhebovsky
Rebecca E. Miller
Suzanne A. Mills
Ain H. Milner
Lori A. Moore
Rodney S. Morris
Saumya P. Nandi
Norman Niami
Rodrick R. Osborn
Wade H. Oshiro
Carolyn Pasquino
Michael A. Pauletti
Matthew J. Perkins
Christopher A. Pett
Jennifer A. Porter
Sherri L. Potter
Gregory T. Preble
Bill Premdas
Beth A. Pyle
Darryl L. Raines
Mary E. Reading
Mary Joseniae O. Reynolds
Danielle L. Richards
Arlene M. Richardson
Rhamonda J. Riggins
Keith A. Rogers
John D. Rosilier
Nancy Ross
Catherine Roy
Frederick D. Ryan
Margaret J. Sanchez
Jonathan A. Schriber
Richard H. Seward
Bintao Shi
Halina H. Smosna
Scott G. Sobel
Lisa V. Sockett
Michael D. Sowka
Sandra E. Starnes
Thomas L. Stramer
Lisa D. Strobel
Gil O. Student
Robert M. Thomas
Patrick Thorpe
Anil Varma
Gaetan R. Veilleux
Kevin K. Vesel
Natalie Vishnevsky
Ya-Feng Wang
Victoria K. Ward
William B. Westrate
William B. Wilder
Bruce Philip Williams
Giak Diang Tan Wong
Jonathan S. Woodruff
Stephen C. Young
Gene Q. Zhang

Part 4B

Madeeha Abdullah
Leah C. Adams
Mustafa Bin Ahmad
Thomas M. Ahmann
Genevieve L. Allen
Keith P. Allen
Brian C. Alvers
Madhu G. Amar
Frederick J. Andersen
Jonathan L. Ankney
Ka Ho Au Yeung
Patricia Azevedo
Monica S. Badlani
Paul C. Barone
Alison M. Bartlett

Martin J. Battle
Michael A. Bean
John R. Bedwell
Marie-Eve J. Belanger
Mariana Beytelman
Jennifer L. Blackmore
Brian E. Blatnik
Sebastien Blondeau
Alex Blundell
Luc Boissiere
David R. Border
Veronique Bouchard
Maureen A. Boyle
Christopher S. Bramstedt
Lisa K. Buege
Derek D. Burkhalter
Darryl D. Button
David R. Cabana
Daniel Cantin
Daniel A. Cantin
Scott W. Carpinteri
Alison S. Carter
Simon Castonguay
Hoi Leung Chan
Kin Sun Chan
Junie L. Chang
Yueh-Chi Chang
Sylvain Charbonneau
Julie Charron
Siu Kei Chau
Hung-Sheng Chen
Shang-Ting Chen
Lillian Cho
Jammy Chow
Oscar Chow

Scott A. Christensen
Michael Joseph Christian
Brenlee Claman
Isabelle Clement
Andrea D. Combs
John T. Condo
Kiera E. Cope
Gary C. Cosby
Christian Cote
Sandra Cote
Nicholas J. Craig
Judy Cui
M. Elizabeth Cunningham
Joseph W. Curran
William A. Da Silva
James C. Dahl
Matthew F. Daitch
Elise Dallain
Concetta A. DePaolo
Jeremy J. Derucki
Julien Descombes
Katherine Devlin
Lisa A. Dietrich
Stacey A. Dillabough
Jennifer A. Dolphin
Zelong Dong
Annie Doucet
Francois C. Doucet
Michael S. Downing
Scott H. Drab
Charles Dussault
Steven C. Ekblad
Glenn T. Elsey
Greg J. Engl

Stephanie Fadous
Kyle A. Falconbury
Weishu Fan
Kathleen M. Farrell
Michael W. Ferris
Kenneth D. Fikes
Eric Filion
Kristine M. Fitzgerald
Brian C. Flieder
Jean J. Forrest
Marc-Andre Fournier
Lyne Francoeur
Mark R. Frank
Christopher P. Freese
Claudia Gagne
David Gagnon
Julien Gagnon
Joseph B. Galbraith
Steve Galipeau
Marcia L. Gallos
Bradley G. Gipson
Pierre Girard
Todd B. Glassman
Jeffrey S. Goldin
Eric D. Golus
Melissa J. Goodson
Stacey C. Gotham
Lijia Guo
Rui Guo
Vivek Gupta
John T. Hanson
Valie R. Harley
Qing He
Kimberly A. Heiligenberg
Bradley R. Heinrichs

Hans Heldner
Kathryn E. Herzog
Milton G. Hickman
Kurt D. Hines
Kun Chi Bennet Ho
Michael F. Hobart
David E. Hodges
Gerald L. Hoeppner
Richard M. Holtz
Zhenjie Hou
Gerald K. Howard
Jui-Chun Hsu
Cheng-Jui Huang
Chris M. Hutzler
Ali Ishaq
Ian A. Jack
Christopher P. Jansen
Walter L. Jedziniak
Woan-Ling Jiang
William B. Johnson
Julie Joyal
Lawrence S. Katz
Carl L. Kennes
Rachel W. Killian
Phillip M. Kivarkis
Jason W. Kolysher
Laura L. Kozlevcar
Mark B. Kropf
Alexander Krutov
Claude Lachapelle
Eric Lacroix
Michael A. Lardis
Remi Laroche
Michelle A. Larson
Scot M. Larson
Bradley J. Lawson
Hangsuck Lee
Alfred Lerman
Antoine Letourneau
Han T. Liem
Kok Bin Liew
Joshua Y. Ligosky
Khang-Yee Lim
Han-Wei Lin
Hui-Ru Lin
Jennifer H. Lin
Erik F. Livingston
Diane M. Lloyd
Allen Chi Tat Lowe
Donald A. Luciak
Eric J. Lynn
Jesus A. Macaraeg
David D. Magee
Kerry A. Magnuson
Atul Malhotra
Joshua N. Mandell
Michel Rene Marcon
Christopher C. Mathewson
Timothy T. McKee
Heath W. Merlak
Stephanie Miller
Andrew A. Minten
Surenna Binte Mustafa
Jean-Philippe Nadeau
Christopher A. Najim
Saumya P. Nandi
Helen Patricia Neglia
Thomas E. Newgarden
Xiaoying Ni
Loren J. Nickel
Kenneth D. Nilsen
Joshua M. Nyros
Lawerence J. O'Brien
Matthew R. Ostiguy
Jeffrey A. Padavic
Rebekah L. Pagano
Dr. Jeffrey S. Pai
Michael T. Patterson
Manolis O. Paximadas
Nathalie Perreault
Kraig P. Peterson
Lynn A. Petros
Jorge E. Pizarro
Judy L. Pool
Amin Nizarali Punjani
Lovely G. Puthenveetil
Terry W. Quakenbush
Surendran Ramanathan
Srinivasa Ramanujam
Marco Ramsay
Frank S. Rau
Nigel K. Riley
Francis Carl Rivard
Choya A. Robinson
Cynthia V. Root
Brett A. Roush
Christian Rousseau
Catherine Roy
Gaetan P. Ruest
Pasquale Daniel Rulli
Salimah H. Samji
Jeremy N. Scharnick
Parr T. Schoolman
Barbara A. Scott
John R. Scudella
Amardip Sekhon
Nikolai D. Serykh

Steven R. Shallcross
Ishmael Sharara
Jee Shen
Jimmy Shkolyar
Deborah A. Shure
Paul Silberbush
Christian Simard
Lee O. Smith
Thomas M. Smith
Yun Song
Michael D. Sowka
Charly St. Martin
Michelle J. Steinborn
Peter H. Sun
Helaina I. Surabian
Michelle M. Syrotynski
Varsha A. Tantri
Sherman B. Tenorio
Sarah J. Thompson
Craig Tien
Stephen H. Tom
Tamara L. Trawick
Jonathan E. Trend
Sheng P. Tseng
Steve Turmel
Gaurav Upadhya
Martin Vachon
Martin Vezina
Timothy R. Wagenmaker
Jim Wagner
Muzammil Waheed
Lixin Wen
William B. Westrate
William B. Wilder
Bruce Philip Williams
Laura Markham Williams
Michael D. Williams
Paul D. Wirth
Jasper Wong
Vince Wong
Matthew L. Worthington
Peter R. Zeillmann
Yuanhan Zhang
Lianmin Zhou
Tong Zhou
Michael R. Ziegert

Part 5A

Jason R. Abrams
Michael D. Adams
Cheryl R. Agina
Julie A. Anderson
Kevin L. Anderson
Amy J. Antenen
Melissa J. Appenzeller
Wendy Lauren Artecona
Michael A. Bean
Patrick Beaudoin
Christopher D. Bohn
Mark E. Bohrer
Sherri Lynn Border
John R. Bower
Maureen A. Boyle
Jeremy James Brigham
Peter J. Brown
David C. Brueckman
Alan Burns
Anthony R. Bustillo
Joyce Chen
Wai Yip Chow
Marlene M. Collins
Jeffrey A. Courchene
William P. Cross
Michael J. Cummiskey
Mari A. Davidson
Michael Brad Delvaux
Jean-Francois Desrochers
Brian M. Donlan
Louis Christian Dupuis
Sophie Duval
Robert E. Farnam
Kathleen M. Farrell
Donovan M. Fraser
Kevin Jon Fried
Noelle Christine Fries
Donald M. Gambardella
Theresa Giunta
Christopher J. Graham
Robert A. Grocock
Curtis A. Grosse
Serhat Guven
Marcus R. Hamacher
Kimberly Baker Hand
David L. Handschke
Kendra S. Heidt
Deborah L. Herman

Amy L. Hicks
Kurt D. Hines
Mohammad A. Hussain
Christopher Donald Jacks
Michael S. Jarmusik
Susan K. Johnston
Bryon R. Jones
Mark C. Jones
William Rosco Jones
Brandon Daniel Keller
Jeffrey D. Kimble
Steven T. Knight
Robert A. Kranz
Scott C. Kurban
Elizabeth A. Kurina
Julie-Linda LaForce
Stephane Lalancette
Isabelle LaPalme
Aaron M. Larson
Dennis H. Lawton
Damon T. Lay
Eric F. Liland
Laura S. Marin
David E. Marra
Sarah P. Mathes
Laura A. Maxwell
Isaac Merchant
Todd A. Michalik
Ain H. Milner
Jonathan M. Moss
Charles A. Norton
Steven Brian Oakley
Nancy Eugenia O'Dell-Warren
Rick S. Pawelski
Mark Paykin
Christopher K. Perry
Michael R. Petrarca
Kristin S. Piltzecker
Beth A. Rasmussen
William J. Raymond
Ronald S. Rees
Sara Gay Reinmann
Nancy Ross
Jason R. Santos
Robert T. Schlotzhauer
Jeffery W. Scholl
Jonathan A. Schriber
Michael Robert Schummer
William Harold Scully
Tina Shaw
Lee O. Smith
John H. Soutar
Laura T. Sprouse
Laura B. Stein
Avivya Simon Stohl
Harlan H. Thacker
Charles A. Thayer
Michael S. Uchiyama
David M. Vogt
Karl C. Von Brockdorff
Josephine M. Waldman
Shannon A. Whalen
Arthur S. Whitson
Scott M. Woomer
Mihoko Yamazoe
Yuhong Yang
Mark K. Yasuda
Hau Leung Ying

Part 5B

Michael B. Adams
Mustafa Bin Ahmad
Ariff B. Alidina
Robert E. Allen
Silvia J. Alvarez
Mary P. Bayer
Rick D. Beam
Michael A. Bean
Patrick Beaudoin
Patrick Beaulieu
Chad M. Beehler
Ellen A. Berning
Mary Denise Boarman
Thomas L. Boyer
Bernardo Bracero
Jeremy James Brigham
Stephane Brisson
Claude B. Bunick
Angela D. Burgess
Kevin D. Burns
Matthew E. Butler
Heather M. Byrne
Mary L. Cahill
Mary Ellen Cardascia
Samuel C. Cargnel
Todd D. Cheema
Yvonne Wai Ying Cheng
Emily Y. Chien
Alan M. Chow

Andrew K. Chu
Julia F. Chu
Charles A. Cicci
Edward W. Clark
Steven A. Cohen
Peter J. Cooper
Leanne M. Cornell
Patrick J. Dubois
Nathalie Dufresne
Sophie Duval
Linda I. Eck
Matthew B. Feldman
Kenneth D. Fikes
Richard G. Fisher
Julia M. Ford
Mauricio Freyre
Cynthia Galvin
Donald M. Gambardella
Carol Ann Garney
Anne M. Garside
Matthew P. Gatsch
Kareen Gaudreault
Amy L. Gebauer
Micah R. Gentile
Laszlo Gere
Rainer Germann
Isabelle Gingras
Bradley G. Gipson
Alla Golonesky
Isabelle Groleau
Chantal Guillemette
James C. Guszcza
Marcus R. Hamacher
Gregory Hansen
Scott E. Haskell
Qing He
Glenn R. Hiltpold
Hsienwu Hsu
Charles B. Jin
Karen L. Jiron
Susan K. Johnston
Theodore A. Jones
Michael D. Kemp
Jeffrey D. Kimble
Anne Marie Klein
Sarah Krutov
Brandon E. Kubitz
Ignace Y. Kuchazik
Matthew R. Kuczwaj
Todd J. Kuhl
Margaret J. Kuperman
Richard A. Kutz
Chingyee Teresa Lam
Borwen Lee
Karen J. Lee
Monika Lietz
Dengxing Lin
David E. Marra
Julie Martineau
James J. Matusiak
David M. Maurer
Timothy J. McCarthy
Wayne H. McClary
Ian J. McCracken
Mea Theodore Mea
Suzanne A. Mills
Michael J. Miraglia
Jason E. Mitich
Alan E. Morris
Gwendolyn D. Moyer
Malongo Mukenge
Jennifer Y. Nei
James L. Norris
Miodrag Novakovic
James P. O'Donovan
Cosimo Pantaleo
Carolyn Pasquino
Rick S. Pawelski
Isabelle Perron
Michael R. Petrarca
Kristin S. Piltzecker
Jayne L. Plunkett
Peter V. Polanskyj
Sherman D. Power
Amy M. Quinn
Jamie Ramos
Neil W. Reiss
Brian E. Rhoads
Hany Rifai
Sophie Robichaud
Kathleen F. Robinson
Keith A. Rogers
Scott E. Root
Jaime J. Rosario
Beth K. Rossio
Brian Craig Ryder
Laura B. Sachs
Salimah H. Samji
James Charles Sandor
Jean Kim Scheible
Terri L. Schwomeyer
Peter A. Scourtis
William Harold Scully
Tina Shaw
Marina Sieh
Steven A. Smith
John H. Soutar

Harold L. Spangler
Daniel J. Spillane
Laura T. Sprouse
Benoit St-Aubin
Glenda M. Stalkfleet
Christine L. Steele-Koffke
David K. Steinhilber
Jonathan L. Summers
Karrie Lynn Swanson
Hung K. Tang
Eric D. Telhiard
Gary S. Traicoff
Andy K. Tran
James H. Tran
Michael C. Tranfaglia
Nathalie Tremblay
Richard A. Van Dyke
Tim A. Vargo
Gaetan R. Veilleux
Colleen Ohle Walker
Michael A. Wallace
Jamil Wardak
Chris J. Westermeyer
Patricia Cheryl White
Dana L. Winkler
Dean M. Winters
Yoke Wai Wong
Jonathan S. Woodruff
Yuhong Yang
Shawn M. Young
Yan Zhou

Part 6

Mustafa Bin Ahmad
Sajjad Ahmad
Anthony L. Alfieri
Jennifer A. Andrzejewski
John A. Annino
Anju Arora
Satya M. Arya
David Steen Atkinson
Nathalie J. Auger
Amy L. Baranek
Emmanuil Theodore Bardis
Anna Marie Beaton
Nicolas Beaupre
Andrew S. Becker
Jeremy T. Benson
Kristen M. Bessette
John T. Binder
Mario Binetti
Linda Jean Bjork
Kofi Boaitey
Mark E. Bohrer
Thomas S. Botsko
Lee M. Bowron
Thomas L. Boyer
Richard Albert Brassington
Rebecca Schafer Bredehoeft
Robert Lindsay Brown
James Douglas Buntine
Elise S. Burns
Hayden Heschel Burrus
Jennifer P. Capute
Allison F. Carp
Patrick J. Causgrove
Julia F. Chu
Louise Chung-Chum-Lam
Stephen Daniel Clapp
Jeffrey A. Clements
Jeffrey J. Clinch
Christopher Paul Coelho
Steven A. Cohen
Larry Kevin Conlee
Sean O. Cooper
David E. Corsi
Kathleen T. Cunningham
Jonathan Scott Curlee
Loren Rainard Danielson
Mary Katherine T. Dardis
Timothy Andrew Davis
Nancy K. DeGelleke
Patricia A. Deo-Campo Vuong
Lisa M. Diminich
Kevin G. Donovan
Kevin Francis Downs
Michael Edward Doyle
Tammi Beth Dulberger
Sophie Dulude
Ruchira Dutta
Mark Kelly Edmunds

Julie A. Ekdom
Brandon L. Emlen
Richard J. Engelhuber
Brian A. Evans
Stephen Charles Fiete
Kristine Marie Firminhac
Chauncey Fleetwood
Sy Foguel
Sean P. Forbes
Sarah Jane Fore
Hugo Fortin
Ronnie S. Fowler
Robert C. Fox
Timothy J. Friers
Neil P. Gibbons
Emily C. Gilde
Susan I. Gildea
Sanjay Godhwani
Olga Golod
Natasha C. Gonzalez
Christopher David Goodwin
Philippe Gosselin
Paul E. Green
David John Gronski
Jacqueline Lewis Gronski
Christopher Gerald Gross
Nasser Hadidi
Kenneth Jay Hammell
Michelle Lynne Harnick
Eric Christian Hassel
Christopher Ross Heim
Chad Alan Henemyer
Cynthia Jane Heyer
Allen J. Hope
Henry J. Itri
Jean-Claude Joseph Jacob
William R. Johnson
Burt D. Jones
Derek A. Jones
Robert C. Kane
Kimberly S. H. Kaune
Catherine L. Keenan
Ung Min Kim
Paul W. Kollner
Linda Kong
Bradley S. Kove
Richard Scott Krivo
Sarah Krutov
James Douglas Kunce
Bobb J. Lackey
Carl Lambert
Hugues Laquerre
Peter Latshaw
Bradley R. LeBlond
David Leblanc-Simard
Todd William Lehmann
Glen Alan Leibowitz
Charles Letourneau
Karen N. Levine
Craig Adam Levitz
Shiu-Shiung Lin
Diana M.S. Linehan
Rebecca M. Locks
Kenneth T. Lui
Allen S. Lynch
James P. Lynch
Kelly A. Lysaght
Kevin M. Madigan
Daniel Patrick Maguire
Jason A. Martin
Stephen Joseph McAnena
Timothy C. McAuliffe
William R. McClintock
Kirk F. Menanson
Stephanie J. Michalik
Eric Millaire-Morin
Michael J. Miller
Christopher James Monsour
Michael W. Morro
Lambert Morvan
Robert John Moss
Thomas M. Mount
Ethan Charles Mowry
Seth Wayne Myers
Charles P. Neeson
John-Giang L. Nguyen
Kari A. Nicholson
Michael D. Nielsen
Gregory P. Nini
John E. Noble
Christopher Maurice Norman
Corine Nutting
Laura E. Olis
David Anthony Ostrowski
Pierre Parenteau
M. Charles Parsons
James Alan Partridge

Lisa Michelle Pawlowski
Rosemary C. Peck
Jill E. Peppers
Julie Perron
David M. Pfahler
Jeffrey J. Pfluger
Anthony George Phillips
Jordan J. Pitz
David John Pochettino
Thomas L. Poklen
Kathy A. Poppe
Donald S. Priest
Warren T. Printz
William D. Rader
Ricardo A. Ramotar
Christopher David Randall
Sylvain Renaud
Mario Richard
Hany Rifai
Brad E. Rigotty
Karen Lynn Rivara
Kim R. Rosen
Richard A. Rosengarten
Robert R. Ross
Seth Andrew Ruff
Jennifer L. Rupprecht
Tracy A. Ryan
Shama S. Sabade
Joseph J. Sacala
Jason T. Sash
Raymond G. Scannapieco
Gary Frederick Scherer
Christy Beth Schreck
Annmarie Schuster
Nathan Alexander Schwartz
Steven George Searle
Ernest C. Segal
David G. Shafer
Seth Shenghit
Matthew Robert Sondag
Mark R. Strona
Brian Tohru Suzuki
Karrie Lynn Swanson
Nitin Talwalkar
Jonathan Garrett Taylor
Jennifer L. Throm
Andrea E. Trimble
Beth Susan Tropp
Brian K. Turner
Alice M. Underwood
Dennis R. Unver
Leslie Alan Vernon
Nathan Karl Voorhis
Kyle Jay Vrieze
Jon S. Walters
Douglas M. Warner
Matthew Joseph Wasta
Kevin E. Weathers
Kendall P. Williams
Joel F. Witt
Yoke Wai Wong
Linda Yang
Nora J. Young

Part 8

Rimma Abian
Timothy Paul Aman
Mark B. Anderson
Martin S. Arnold
Richard J. Babel
Robert Sidney Ballmer
Jonathan Everett Blake
Barry E. Blodgett
Kimberly Bowen
Kirsten Rose Brumley
Ron Brusky
Peter Vincent Burchett
Julie Burdick
Christopher J. Burkhalter
John Frederick Butcher
Michael E. Carpenter
Julie S. Chadowski
David A. Christhilf
Darrel W. Chvoy
Brian A. Clancy
William Brian Cody
Margaret Eleanor Conroy
Sheri L. Daubenmier
Jeffrey W. Davis
John D. Deacon
Robert G. Downs
Kimberly J. Drennan
Jeffrey Eddinger

Dawn E. Elzinga
Kristine Marie Esposito
Bruce D. Fell
Ginda Kaplan Fisher
John E. Gaines
David Evan Gansberg
Lynn A. Gehant
Thomas P. Gibbons
Julie Therese Gilbert
John T. Gleba
Daniel Cyrus Greer
Charles R. Grilliot
David Thomas Groff
Leigh Joseph Halliwell
Jodi J. Healy
William N. Herr
Jay T. Hieb
Christopher Todd Hochhausler
Jason N. Hoffman
Daniel L. Hogan
Eric J. Hornick
Man-Gyu Hur
Susan Elizabeth Innes
Brian J. Janitschke
Michael S. Johnson
Jeremy M. Jump
Ira Mitchell Kaplan
Hsien-Ming K. Keh
Lowell J. Keith
Brandon Daniel Keller
Claudia A. Krucher
Alexander Krutov
Jason Anthony Kundrot
Edward M. Kuss
Jean-Sebastien Lagarde
Stephen E. Lehecka
Todd William Lehmann
Steven J. Lesser
Janet G. Lindstrom
William R. Maag
Joseph A. Malsky
Patrice McCaulley
Richard Timmins McDonald
Allison Michelle McManus
Scott A. McPhee
Andrew Wakefield Moody
Jennifer Ann Moseley
Kevin T. Murphy
Jarow G. Myers
Mihaela Luminita S. O'Leary
Denise R. Olson
David J. Otto
Gerard J. Palisi
Joseph M. Palmer
Harry Todd Pearce
Glen-Roberts Pitruzzello
Richard A. Plano
Jennifer K. Price
Yves Provencher
Kara Lee Raiguel
Jennifer L. Reisig
Andrew Scott Ribaudo
Cynthia L. Rice
Paul J. Rogness
Nathan William Root
Christine R. Ross
Sandra L. Ross
Chet James Rublewski
Rajesh V. Sahasrabuddhe
Michael Shane
Huidong Kevin Shang
Meyer Shields
Kerry S. Shubat
Jay Matthew South
Caroline B. Spain
Alan M. Speert
Michael J. Sperduto
Carol A. Stevenson
Thomas Struppeck
Collin John Suttie
C. Steven Swalley
Christopher C. Swetonic
Chester J. Szczepanski
David M. Terne
Daniel A. Tess
Glenn Allen Tobleman
Joseph D. Tritz
Edward H. Wagner
Benjamin A. Walden
Robert J. Wallace
Jeffrey D. White
Trevar K. Withers
Tad E. Womack

Part 8C

Pierre Bourassa
Anthony E. Cappelletti
Heather L. Chalfant
Alana C. Farrell
Sylvain Fauchon
Walter H. Fransen
Jacqueline Frank Friedland
Lewis Y. Lee
Andrea Wynne Malyon
Benoit Morissette
Todd F. Orrett
Yves Raymond
Philippe Trahan
Christopher Brian Wei

Part 10

Jonathan David Adkisson
Larry D. Anderson
David B. Bassi
Andrea C. Bautista
Michael J. Bednarick
Michael J. Belfatti
Steven L. Berman
Suzanne E. Black
Daniel David Blau
Carol Ann Blomstrom
George Peter Bradley
Mary Hemerick Bready
Cary J. Breese
Charles Brindamour
Elliot R. Burn
J'ne Elizabeth Byckovski
Tania J. Cassell
Dennis K. Chan
Rita E. Ciccariello
Brian K. Ciferri
Jean Cloutier
Thomas P. Conway
Charles Anthony Dal Corobbo
Guy Rollin Danielson
Smitesh Dave
Elizabeth Bassett DePaolo
Brian Harris Deephouse
Jeffrey F. Deigl
Christopher S. Downey
Bernard Dupont
Carole M. Ferrero
Mary Elizabeth Fleischli
Christian Fournier
Kay L. Frerk
Kathy H. Garrigan
James B. Gilbert
Michael Ambrose Ginnelly
Moshe D. Goldberg
Annette J. Goodreau
Mari Louise Gray
Michael D. Green
Greg M. Haft
Paul James Hancock
Bradley Alan Hanson
David S. Harris
Michael B. Hawley
Daniel F. Henke
Thomas Gerald Hess
Amy Jean Himmelberger
Robert J. Hopper
Marie-Josee Huard
David Dennis Hudson
Paul Robert Hussian
Paul Ivanovskis
Christopher Donald Jacks
Randall A. Jacobson
Hou-Wen Jeng
Daniel Keith Johnson
Kurt Jeffrey Johnson
Mark Robert Johnson
Claudine Helene Kazanecki
Rebecca Anne Kennedy
Joan M. Klucarich
Eleni Kourou
Kenneth Allen Kurtzman
Salvatore T. LaDuca
Timothy J. Landick
Gregory D. Larcher

Steven Wayne Larson
Thomas C. Lee
P. Claude Lefebvre
Elizabeth Ann Lemaster
Jennifer McCullough Levine
Christina Link
Anthony L. Manzitto
Peter R. Martin
Michael Boyd Masters
Keith A. Mathre
Robert F. Maton
Douglas W. McKenzie
Jeffrey A. Mehalic
Daniel Julian Merk
Timothy Messier
Claus S. Metzner
David Molyneux
David Patrick Moore
Robert Joseph Moser
Matthew C. Mosher
Kimberly Joyce Mullins
Turhan E. Murguz
Timothy O. Muzzey
Vinay Nadkarni
Aaron West Newhoff
Mindy Y. Nguyen
Kevin Jon Olsen
Milary Nadean Olson
Dmitry E. Papush
Luba O. Pesis
William Peter
John S. Peters
Michael D. Price
Regina Marie Puglisi
Patrice Raby
Raymond J. Reimer
Andrew Scott Ribaudo
Meredith Gay Richardson
Gregory Riemer
David L. Ruhm
Joanne Emily Russell
Kevin L. Russell
Romel G. Salam
Elizabeth A. Sander
Stephen Paul Sauthoff
Christine E. Schindler
Terry Michael Seckel
Huidong Kevin Shang
Robert Daniel Share
Jeffrey Parviz Shirazi
Gary E. Shook
Jill C. Sidney
Lori Ann Snyder
Carl J. Sornson
Catherine E. Staats
Victoria Grossack Stachowski
Christopher M. Steinbach
Julia Causbie Stenberg
John A. Stenmark
Michael J. Steward
Deborah L. Stone
Kevin Douglas Strous
Thomas Struppeck
Steven John Symon
Laura Little Thorne
Linda Kay Torkelson
Theresa Ann Turnacioglu
Alice M. Underwood
Timothy J. Ungashick
Robert Ward Van Epps
Erica Lynn Weida
Geoffrey Todd Werner
Gayle Lynne Wiener
Elizabeth Ruth Wiesner
Roger Allan Yard
Gerald Thomas Yeung
Jeffery Michael Zacek
Doug A. Zearfoss
Alexander Guangjian Zhu

The following candidates were admitted as Fellows and Associates at the 1997 CAS Annual Meeting in November. By passing May 1997 examinations, these candidates successfully fulfilled the Society requirements for Fellowship or Associateship designations.

NEW FELLOWS

Jonathan David Adkisson
Timothy Paul Aman
Larry D. Anderson
Robert Sidney Ballmer II
Andrea C. Bautista
Steven L. Berman
Daniel David Blau
Carol Ann Blomstrom
Pierre Bourassa
George Peter Bradley
Mary Hemerick Bready
Kirsten Rose Brumley
Peter Vincent Burchett
John Frederick Butcher II
J'ne Elizabeth Byckovski
Dennis K. Chan
Rita E. Ciccariello
Jean Cloutier
William Brian Cody
Charles Anthony Dal Corobbo
Guy Rollin Danielson
Jeffrey F. Deigl
Robert G. Downs
Bernard Dupont
Jeffrey Eddinger
Carole M. Ferrero
Ginda Kaplan Fisher
Mary Elizabeth Fleischli
Christian Fournier
Julie Therese Gilbert
Michael Ambrose Ginnelly
Annette J. Goodreau
Mari Louise Gray
Leigh Joseph Halliwell
Paul James Hancock
Bradley Alan Hanson
David S. Harris
Amy Jean Himmelberger
Robert J. Hopper
David Dennis Hudson
Paul Robert Hussian
Hou-Wen Jeng
Daniel Keith Johnson
Kurt Jeffrey Johnson
Mark Robert Johnson
Ira Mitchell Kaplan
Lowell J. Keith
Rebecca Anne Kennedy
Joan M. Klucarich
Jason Anthony Kundrot
Edward M. Kuss
Salvatore T. LaDuca
Gregory D. Larcher
Elizabeth Ann Lemaster
Jennifer McCullough Levine
Keith A. Mathre
Robert F. Maton
Richard Timmins McDonald
Daniel Julian Merk
Timothy Messier
Claus S. Metzner
Andrew Wakefield Moody
Robert Joseph Moser
Kimberly Joyce Mullins
Turhan E. Murguz
Aaron West Newhoff
Kevin Jon Olsen
Milary Nadean Olson
Regina Marie Puglisi
Patrice Raby
Raymond J. Reimer
Andrew Scott Ribaudo

Meredith Gay Richardson
Gregory Riemer
Stephen Paul Sauthoff
Christine E. Schindler
Terry Michael Seckel
Huidong Kevin Shang
Robert Daniel Share
Jeffrey Parviz Shirazi
Gary E. Shook
Jill C. Sidney
Lori Ann Snyder
Carl J. Sornson
Victoria Grossack Stachowski
Julia Causbie Stenberg
John A. Stenmark
Deborah L. Stone
Kevin Douglas Strous
Thomas Struppeck
Collin John Suttie
Steven John Symon
Daniel A. Tess
Glenn Allen Tobleman
Linda Kay Torkelson
Philippe Trahan
Theresa Ann Turnacioglu
Robert Ward Van Epps
Erica Lynn Weida
Geoffrey Todd Werner
Jeffrey D. White
Gayle Lynne Wiener
Elizabeth Ruth Wiesner
Tad E. Womack
Roger Allan Yard
Gerald Thomas Yeung
Jeffery Michael Zacek
Doug A. Zearfoss
Alexander Guangjian Zhu

NEW ASSOCIATES

John Scott Alexander
Paul C. Barone
Anna Marie Beaton
Andrew S. Becker
Frank J. Bilotti
Linda Jean Bjork
Michael J. Bluzer
Sherri Lynn Border
Richard Albert Brassington
Rebecca Schafer Bredehoeft
Kevin D. Burns
Joyce Chen
Michael Joseph Christian
Stephen Daniel Clapp
Christopher Paul Coelho
Kathleen T. Cunningham
Michael J. Curcio
Kevin Francis Downs
Michael Edward Doyle
Sophie Dulude
Kristine Marie Firminhac
Chauncey Fleetwood
David Michael Flitman
Sy Foguel
Hugo Fortin
Kevin Jon Fried
Noelle Christine Fries
Micah R. Gentile
Susan I. Gildea
Christopher David Goodwin
Philippe Gosselin
Jacqueline Lewis Gronski
Christopher Gerald Gross
Nasser Hadidi
Kenneth Jay Hammell
Gregory Hansen
Michelle Lynne Harnick
Ia F. Hauck
Cynthia Jane Heyer
Ali Ishaq
Christopher Donald Jacks
Jean-Claude Joseph Jacob
Walter L. Jedziniak
William Rosco Jones
Robert B. Katzman

Brandon Daniel Keller
Linda Kong
Richard Scott Krivo
Alexander Krutov
Sarah Krutov
Kirk L. Kutch
Todd William Lehmann
Charles Letourneau
Marc E. Levine
Daniel Patrick Maguire
David E. Marra
William A. Mendralla
Richard Ernest Meuret
Stephanie J. Michalik
Michael J. Miller
Christopher James Monsour
Robert John Moss
Charles P. Neeson
Helen Patricia Neglia
Tieyan Tina Ni
Christopher Maurice Norman
Steven Brian Oakley
David Anthony Ostrowski
James Alan Partridge
Lisa Michelle Pawlowski
Mark Paykin
Julie Perron
Anthony George Phillips
David John Pochettino
Matthew H. Price
Denise Farnan Rosen
Tracy A. Ryan
Shama S. Sabade
Christy Beth Schreck
Michael Robert Schummer
William Harold Scully III
Halina H. Smosna
Avivya Simon Stohl
Brian Tohru Suzuki
Nitin Talwalkar
Jonathan Garrett Taylor
Alice M. Underwood
David M. Vogt
Nathan Karl Voorhis
Claude A. Wagner
Patricia Cheryl White
Bruce Philip Williams
Laura Markham Williams
Joel F. Witt
Yuhong Yang

The following candidates successfully completed the Parts of the Fall 1997 CAS Examinations that were held in October.

Part 3B

Patrick B. Achey
Cheryl R. Agina
Greg A. Aikey
Richard T. Alden
Ashwin Arora
David M. Biewer
Neil M. Bodoff
Daniel J. Borzynkski
Steve E. Brasier
Maureen B. Brennan
John R. Broadrick
Robert E. Calkins
Jason A. Campbell
Gabriel F. Carrillo
John Celidonio
Patrick J. Charles
Peter S. Clarke
Gerald D. Cooper
Hugo Corbeil
Nicholas J. De Palma
David E. Dela Cruz
Jean-Francois Desrochers
William E. Doran
Derek D. Dunnagan
Louis Christian Dupuis
Sophie Duval
Ashifa Esmail
Nixon Etienne
Shelly A. Fowler

Rebecca E. Freitag
David Gagnon
Justin G. Gensler
Timothy S. Ghan
Emily C. Gilde
Isabelle Gingras
Isabelle Groleau
Travis Grulkowski
Chantal Guillemette
James C. Guszcza
Barry R. Haines
John G. Henares
Patricia A. Hladun
Melissa S. Holt
Douglas Bruce Homer
David W. Hurter
Patrice Jean
Luen Khaw
Christopher Kremer
Heather D. Lake
Carl Lambert
Maxime Lanctot
Michael A. Lardis
Geraldine Marie Z. Lejano
W. Scott Lennox
Joseph B. Logsdon
Kathleen T. Logue
Keyang Luo
Atul Malhotra
Steven Manilov
James J. Matusiak
Carolyn J. McElroy
Stephanie Miller
Christian Morency
Josee Morin
Surenna Binte Mustafa
Robert B. Newmarker
Charles A. Norton
Carolin G. Paidoussis
Stephen R. Prevatt
Leonid Rasin
Ronald S. Rees
Brian E. Rhoads
Hany Rifai
Wayne L. Rosen
Laura B. Sachs
Daniel David Schlemmer
Mike B. Schofield
Jonathan A. Schriber
Tammy L. Schwartz
Michelle Sheppard
Ju-Young Suh
Sara A. Trussoni
Peggy J. Urness
Seema M. Wadhwa
Tom C. Wang
Carolyn D. Wettstein
Joel D. Whitcraft
Gary A. Wick

Part 4A

Michael L. Alfred
Brian M. Ancharski
Nicki C. Austin
Vicki J. Bagley
Anna Bakman
John L. Baldan
Wendy A. Barone
Suzanne Barry
Michael A. Bean
Nathalie Belanger
Gregory A. Berman
Ellen A. Berning
Jay E. Blumenreich
Olivier Bouchard
Thomas G. Bowyer
Christopher S. Bramstedt
Erick A. Brandt
Peter J. Brown
Robert J. Brunson
Claude B. Bunick
Michael W. Buttke
Fatima E. Cadle
Mary Ellen Cardascia
Jennifer M. Carnahan
Scott W. Carpinteri
Simon Castonguay
Esther K.N. Chan
Jennifer A. Charlonne
Shu-Chuan Chen
Marcus K. Cheung
Alan M. Chow
Christian J. Coleianne
Kiera E. Cope
Leanne M. Cornell
Edgar Corredor
Michael J. Covert
Richard R. Crabb
Russell A. Creed

Arthur D. Cummings
Kelly K. Cusick
David B. Dalton
Stephen Darrow
Nicholas J. De Palma
Stephanie A. DeLuca
Peter R. DeMallie
D. Vance C. DeWitt
Paul B. Deemer
David E. Dela Cruz
Robert E. Dennison
Devin Derstine
Randi S. Deutsch
Christopher A. Donahue
William E. Doran
Elaine V. Eagle
John W. Elbl
Tricia G. English
William H. Erdman
Jeffrey S. Ernst
Kyle A. Falconbury
Solomon C. Feinberg
Elizabeth J. Fethkenher
Kristine M. Fitzgerald
Jeffrey R. Fleischer
Robin A. Fleming
Dennis Anthony Fong
Teresa M. Fox
Jeffrey A. Gabay
Patrick P. Gallagher
Cynthia Galvin
Michelle R. Garnock
Dustin W. Gary
Kareen Gaudreault
Amy L. Gebauer
Laszlo Gere
Shannon E. Gilbert
Matthew R. Gorrell
Elizabeth A. Grande
Donald B. Grimm
Stephanie A. Groharing
Marcus R. Hamacher
Valie R. Harley
Guo Harrison
Sonja M. Heiberg
Hans Heldner
Daniel D. Heyer
Amy L. Hicks
Joseph S. Highbarger
Carole K. L. Ho
Jeremy Hoch
Shaohe T. Huang
Bryan B. Jaicks
Charles B. Jin
Caroline F. Jo
Shantelle A. Johnson
Brian A. Junod
Kelly F. Kahling
Lawrence S. Katz
Lisa M. Kerns
Shenaz Keshwani
William F. Killian
Sang W. Kim
Jason A. Lauterbach
Daniel A. Levin
Joshua Y. Ligosky
Matthew A. Lillegard
Kenneth Liner
Jing Liu
Wing Lowe
Atul Malhotra
Joshua N. Mandell
Jeffrey L. Martin
Leroy H. Mattic
James J. Matusiak
Patricia McGahan
Shaun P. McGovern
Isaac Merchant
Ryan A. Michel
Scott P. Monard
Matthew E. Morin
John A. Nauss
Jennifer Y. Nei
Richard U. Newell
Linda C. Nichols
Loren J. Nickel
Sean R. Nimm
Kathy M. Nordness
Barbara B. O'Connor
Michael A. Onofrietti
Matthew R. Ostiguy
Patrick M. Padalik
Robin V. Padwa
Susan M. Pahl
Robert A. Painter
Joy-Ann C. Payne
Robert B. Penwick
Kraig P. Peterson
Jayne L. Plunkett
Sean E. Porreca
Lind R. Pratt
Leonid Rasin
Brian E. Rhoads
Benjamin L. Richards
Ezra J. Robison
Kelly J. Rosseland

Jason R. Santos
Jeremy N. Scharnick
Michelle Sheppard
Janel M. Sinacori
Vijayalakshimi Sridharan
Laura B. Stein
Kimberly A. Strauss
Thomas J. Stypla
Louis P. Sugarman
Piya R. Talwar
Neeza Thandi
Thomas E. Thun
Karen J. Triebe
Raymond D. Trogdon
Tammy Truong
Bruce C. H. Tse
Peggy J. Urness
John T. Volanski
Colleen Ohle Walker
Matthew J. Walter
Gregory A. Watson
Youcheng Wei
Brian D. White
Thomas J. White
Mark W. Whitford
Timothy P. Wiebe
Kaylie Wilson
Ann Min-Sze Wong
Mark K. Yasuda
Jacinthe Yelle
Joshua A. Youdovin
Raymond R. Y. Yung
Michael R. Zarember
Lianmin Zhou

Part 4B

Peter Abramovich
Molly Bush Acker
Syed W. Ahmed
George G. Alaishuski
Anthony R. Alvarez-Pedroso
Tomomi Arikawa
Colette F. Atkinson
Peter Attanasio
Alberto A. Autmezguine
Eynshteyn Averbukh
Brian J. Barth
Thierry Bedard
Byron N. Beebe
Minh P. Bennett
Jeremy T. Benson
Jason E. Berkey
David M. Biewer
Francois Blanchard
Wei Ming Bo
Christopher D. Bohn
Emilie Bouchard
Olivier Bouchard
Isabel Boyer
Lillian I. S. Brathwaite
Jason M. Bravo
Kevin P. Brennan
Jeremy James Brigham
John R. Broadrick
Robert J. Brunson
Yiwen Bu
Angela D. Burgess
Brian P. Bush
James E. Calton
Katherine S. Campbell
Samuel C. Cargnel
Lawrence S. Carson
Brian J. Cefola
Ching Chuen Chan
Juliza Chan
Wing Fai Chan
Shaoping T. Chang
Patrick J. Charles
Chu-Ka Chen
Rong Sen Chen
Wan Cheng
Chi Kau Cheung
Richard Chevalier
Ling-Yung Chiu
Tenny S. P. Chong
Hung-Yi Chou
MunLing Chung
Wesley G. Clifton
Bradley D. Crafton
Danielle R. Dallas
Rosy Danese
Scott H. Davis
Andrew J. Dierdorf
Patrick Dontigny
John L. Dowell
Yong Yao Du
Julie Duchesne
Derek D. Dunnagan
Jeffrey A. Dvinoff
Jean Robert Elie

Aleksandr Falikson
Brian A. Fannin
Jennifer L. Fitzpatrick
Daniel E. Flynn
John Fong
Michael Fong
Baruch M. Frankel
Mark J. Friedman
Freda W. Fu
Christophe Gaboriaud
Chris D. Garrett
Kareen Gaudreault
Martin Gelinas
Frederic Gendron
Saul Gercowsky
Shannon E. Gilbert
Patrick J. Gilhool
Meghan A. Gillin
Nathalie Giroux
Siti Jessimiah Goh
Stacey B. Goldstein
Lisa N. Guglietti
Steven M. Gutstein
Pavol Gvozdjak
Kimberly Baker Hand
Thomas N. Hanson
Delaine B. Hare
Michael N. Hartfield
Jeffery T. Hay
Arie Haziza
Scott E. Henck
Daniel D. Heyer
Martin W. Hill
Ka Lai Ho
Wen-Jung Ho
Lorin K. Hoeppner
Keepyung B. Hong
Kim W. Hoversten
Nai-Wen Hsu
Ching-Lu Huang
Chuang-Chi Huang
Christopher W. Hurst
Theodore L. Husveth
Ingab Hwang
Victoria K. Imperato
Craig D. Isaacs
Ray-Min Jao
Philippe Jodin
Naheed Kheraj
David R. Klauke
Christopher T. Knorr
Hon Keung Ko
Henry J. Konstanty
Tomasz J. Kozubowski
Emil B. Kraft
Seon-In Kwon
Stacy L. LaiFook
Chingyee Teresa Lam
Man Ching Lam
Ng Fei Lam
Chi Kin Lau
Suzanne R. Lavin
Wanna Law-Kam-Cio
Arthur H. Lee
Emil H. Lee
Johnny Lee
Patrick Lefebvre
Francois Lemieux
Francois G. Lemire
Shangjing Li
Stephen L. Lienhard
Kenneth Lin
Steven R. Lindley
Vadim Lipovetsky
Jing Liu
Ying Liu
June Lu
Yue Ma
Stacy R. Magliolo
Robert Mallette
Luis S. Marques
Sylvia S. Martin
Sarah P. Mathes
James J. Matusiak
Kevin P. McClanahan
Jeffrey B. McDonald
Brock T. McEwen
Jeffery E. McGill
Jennifer A. McGrath
Gary L. McWeeny
Vadim Y. Mezhebovsky
Rebecca E. Miller
Ain H. Milner
Jason E. Mitich
Camilo Mohipp
Michel Luc Montour
Jean-Gregoire G. Morand
Matthew E. Morin
David E. Moser
Timothy C. Mosler
David B. Mukerjee
William Na
Brian C. Neitzel
Chung Ping Ng
Richard Kwai-Fu Ng
Peter Nicolopoulos
Rodrick R. Osborn

Stacey G. Oshanek
Chad M. Ott
Michael A.L. Palmer
Maruthy K. Pannala
Josee Patry
Bradley D. Peckinpaugh
Randall P. Petersen
Michael R. Petrarca
John M. Pickering
Richard B. Pitbladdo, Jr.
Mearl Platt
Ann E. Popovic
Jacques Potvin
Bill Premdas
Amanda L. Priesmeyer
John T. Raeihle
Chada S. Reddy
Qing Ren
Mary Joseniae O. Reynolds
Stephen M. Richard
Warren L. Rodericks
Kevin J. Ross
Stephan D. Sabourin
Prachi Sachdeva
Anthony N. Sammur
William R. Sarniak
Gily Savitch
Eric C. Sherman
Chung Fai Andrew Siu
Robert K. Smith
Man Chung So
Min-Wah Peter So
Laura T. Sprouse
Clay T. Stallard
Jeffrey M. Stelnik
Kelly L. Sterr
Adam D. Swope
Edward Sypher
Chun W. Sze
Marie-Claude Taillefer
Darrin M. Thomas
Boning Tong
Roger Tong
Gary S. Traicoff
Isabel Trepanier
Sara A. Trussoni
Hui-Ling Tsai
Man-Man Tsui
Kevin J. Vantil
Mark A. Verheyen
Peter R. Vita
Wade T. Warriner
Kevin Bruce Waterman
Chad D. Wilcox
Cindy Chen Wu
Li-Lin Wu
Shen-Chyun Wu
Lina Xu
Scott Ming Yan
Chih-Cheng Yang
Sheau-wen Yang
Li Chao Yao
Kelvin K. Yau
Tin Fu Yip
Yuen Ling Linda Yip
Jeng-Wei Yu
Wei Yu
Yuenian Yu
Suk Ping Yuen
Paolo Zadra
Laura M. Zalewski
Gene Q. Zhang
Qing Zhang
Yingjie Zhang
Fengkun Zhao
Basha H. Zharnest

Part 5A

Jodie Marie Agan
Mustafa Bin Ahmad
Faisal Ahmed
Ariff B. Alidina
Brian C. Alvers
Jonathan L. Ankney
Craig Victor Avitabile
Daniel M. Bankson
Alex G. Bedoway
Chad M. Beehler
Jody J. Bembenek
John T. Binder
Brad D. Birtz
Mary Denise Boarman
Thomas L. Boyer
Bernardo Bracero
Stephane Brisson
Derek D. Burkhalter
Matthew E. Butler
William Brent Carr
Scott A. Chaussee

Kin Lun Choi
Philip A. Clancey
Scott R. Clark
Spencer L. Coyle
Brian S. Donovan
Scott H. Drab
Kenneth D. Fikes
Jennifer L. Fitzpatrick
William J. Fogarty
Mauricio Freyre
Anne M. Garside
Ellen M. Gavin
Rainer Germann
Donald L. Glick
Joseph P. Greenwood
Daniel E. Greer
Caroline Gregoire
James C. Guszcza
Qing He
Jason C. Head
Jason B. Heissler
Kathryn E. Herzog
Milton G. Hickman
Melissa Kay Higgins
Glenn R. Hiltpold
Jesse T. Jacobs
Vibha N. Jayasinghe
Philip J. Jennings
Philippe Jodin
Steven M. Jokerst
Theodore A. Jones
Joseph E. Kirsits
James Douglas Kunce
Chingyee Teresa Lam
Travis J. Lappe
Borwen Lee
Craig Adam Levitz
James P. Lynch
Richard J. Manship
Kevin P. McClanahan
Paul B. Miles
Lambert Morvan
Norman Niami
Liam F. O'Connor
Wade H. Oshiro
Cosimo Pantaleo
Michael T. Patterson
Jeremy Parker Pecora
John M. Pergrossi
Lynellen M. Ramirez
Hany Rifai
Arnie W. Rippener
Salimah H. Samji
Ronald J. Schuler
Vladimir Shander
Meyer Shields
Steven A. Smith
Lora L. Smith-Sarfo
Matthew Robert Sondag
Michele L. Spale
Benoit St-Aubin
David K. Steinhilber
Jason D. Stubbs
Jonathan L. Summers
Duc M. Ta
Robert M. Thomas
Turgay F. Turnacioglu
Melodie A. Wakefield
Victoria K. Ward
Kelly M. Weber
Chris J. Westermeyer
Milton K. Wong
Jonathan S. Woodruff
Stephanie C. Young
Grace Zakaria

Part 5B

Jodie Marie Agan
Sajjad Ahmad
Genevieve L. Allen
Brian C. Alvers
Amy J. Antenen
Wendy Lauren Artecona
David Steen Atkinson
Daniel Bar-Yaacov
Rajesh K. Barnwal
Karen E. Bashe
Jonathan P. Berenbom
Jason E. Berkey
Daniel J. Berry
Kristen M. Bessette
Brad D. Birtz
David C. Brueckman
James Douglas Buntine
John C. Burkett
Derek D. Burkhalter
Alan Burns
Sarah Burns
Hayden Heschel Burrus

Wai Yip Chow
Benjamin W. Clark
Jeffrey R. Coker
Charles L. Costantini
Richard R. Crabb
Sandra Creaney
Marc-Andre Dallaire
Robert P. Daniel
Mari A. Davidson
Michael Brad Delvaux
Jeremy J. Derucki
Brian M. Donlan
Peter M. Doucette
Laura A. Esboldt
Dana M. Feldman
Gina C. Ferst
Karen L. Field
Sharon L. Fochi
William J. Fogarty
Graham S. Gersdorff
Patrick J. Gilhool
Christopher J. Graham
Christopher J. Grasso
Daniel Cyrus Greer
Edward Kofi Gyampo
Aaron G. Haning
Patricia W. Hardin
Jeffery T. Hay
Jason C. Head
James A. Heer
Kristina S. Heer
Kathryn E. Herzog
Michael F. Hobart
Joseph H. Hohman
Francis J. Houghton
Derek R. Hoyme
Todd D. Hubal
Carol I. Humphrey
Christopher W. Hurst
Scott R. Hurt
Michael S. Jarmusik
Scott R. Jean
John J. Karwath
Stacey M. Kidd
Chung H. Kim
Patricia Kinghorn
Joseph E. Kirsits
Henry J. Konstanty
James J. Konstanty
James Douglas Kunce
Francois Lacroix
Ravikumar Lakshminarayan
Travis J. Lappe
Sean R. Lawley
Wendy R. Leferson
Shangjing Li
Joshua Y. Ligosky
Joshua N. Mandell
Robert H. Marks
Emmanuel Matte
Randy D. Mattia
Stephen Joseph McAnena
Lawrence J. McTaggart
Alix M. Meyer
Kathleen C. Miller
Rebecca E. Miller
Ain H. Milner
David Patrick Moore
Amy J. Morehouse
Lambert Morvan
Seth Wayne Myers
Lester M. Y. Ng
Loren J. Nickel
Liam F. O'Connor
Nancy Eugenia O'Dell-Warren
Jason M. Olson
Gilbert Ouellet
Charles V. Petrizzi
Kathleen M. Rahilly-Van Buren
Lynellen M. Ramirez
Ricardo A. Ramotar
Christopher David Randall
Leonid Rasin
Mary E. Reading
Dean R. Reigner
Peggy-Anne K. Repella
Stephen D. Riihimaki
Michelle L. Sands
Jeremy N. Scharnick
Daniel David Schlemmer
Anand D. Shah
Junning Shi
Erica M. Sifen
Annemarie Sinclair
Jared M. Skowron
John J. Skowronski
Douglas E. Smith
Lee O. Smith
Michele L. Spale
Lisa C. Stanley
Gil O. Student

Helaina I. Surabian
Feifei Tan
Robert M. Van Brackle
Todd D. Vander Veen
Victoria K. Ward
Kelly M. Weber
Shannon A. Whalen
Paul D. Wilbert
William B. Wilder
Scott M. Woomer
Walter R. Wulliger
Christopher H. Yaure
Joshua A. Youdovin
Stephen C. Young
Christine Seung H. Yu
Gene Q. Zhang
Paul W. Zotti

Part 7

Michael B. Adams
Michael D. Adams
Mustafa Bin Ahmad
Stephen A. Alexander
Nancy Susan Allen
Anju Arora
Carl Xavier Ashenbrenner
Robert D. Bachler
Phillip Wesley Banet
Emmanuil Theodore Bardis
Michael William Barlow
Penelope A. Bierbaum
Gina Stroud Binder
Kevin Michael Bingham
Tony F. Bloemer
Maureen A. Boyle
Hayden Heschel Burrus
Matthew R. Carrier
Thomas Joseph Chisholm
Wanchin W. Chou
Jonathan Scott Curlee
Loren Rainard Danielson
Mary Katherine T. Dardis
Robert E. Davis
Timothy Andrew Davis
Nancy K. DeGelleke
Brian Harris Deephouse
Karen Denise Derstine
Donna K. DiBiaso
Sara Penina Drexler
Tammi Beth Dulberger
Francois Richard Dumontet
Mark Kelly Edmunds
James R. Elicker
Brandon L. Emlen
Juan Espadas
Brian A. Evans
Carolyn M. Falkenstern
Kathleen M. Farrell
Stephen Charles Fiete
Sarah Jane Fore
Timothy J. Friers
Donald M. Gambardella
Charles E. Gegax
James B. Gilbert
Bernard Harry Gilden
Bradley G. Gipson
Todd B. Glassman
Sanjay Godhwani
Natasha C. Gonzalez
Peter S. Gordon
Robert A. Grocock
David John Gronski
Brian T. Hanrahan
Michael S. Harrington
Gary M. Harvey
Eric Christian Hassel
William S. Hedges
Christopher Ross Heim
Kevin B. Held
Ronald L. Helmeci
Chad Alan Henemyer
Richard M. Holtz
Tina Tuyet Huynh
Susan Elizabeth Innes
Weidong Wayne Jiang
Anita J. Johnson
Susan K. Johnston
Daniel R. Kamen
Claudine Helene Kazanecki
Jeffrey D. Kimble
Kelly Martin Kingston

Andrew M. Koren
Scott C. Kurban
Bobb J. Lackey
Douglas H. Lacoss
Michael L. Laufer
Dennis H. Lawton
Manuel Alberto T. Leal
Daniel Leff
Glen Alan Leibowitz
John Norman Levy
Xiaoying Liang
Shiu-Shiung Lin
Diana M. S. Linehan
Victoria Suzanne Lusk
Allen S. Lynch
Kelly A. Lysaght
Kevin M. Madigan
James W. Mann
Albert Maroun
Stephen P. Marsden
Jennifer Ann McCurry
Mark Z. McGill
David Patrick Moore
Jennifer Ann Moseley
Ethan Charles Mowry
John V. Mulhall
Surenna Binte Mustafa
Jarow G. Myers
Seth Wayne Myers
Kari A. Nicholson
John E. Noble
Jason M. Nonis
Corine Nutting
Kathryn Ann Owsiany
M. Charles Parsons
David M. Pfahler
Richard Matthew Pilotte
Glen-Roberts Pitruzzello
Dylan P. Place
Sara Gay Reinmann
Brad E. Rigotty
Karen Lynn Rivara
Rebecca Lea Roever
Nathan William Root
Kim R. Rosen
Richard A. Rosengarten
Christina B. Rosenzweig
Brian P. Rucci
Seth Andrew Ruff
Brian Craig Ryder
James Charles Sandor
James C. Santo
Gary Frederick Scherer
Parr T. Schoolman
Nathan Alexander Schwartz
Stuart A. Schweidel
Peter A. Scourtis
Steven George Searle
Kelvin B. Sederburg
David G. Shafer
Alastair Charles Shore
John H. Soutar
Joy Magalit Suh
Karrie Lynn Swanson
Rachel Rene Tallarini
Varsha A. Tantri
Glenda Oliver Tennis
Laura Little Thorne
Andy K. Tran
Michael C. Tranfaglia
Beth Susan Tropp
Kris D. Troyer
Joel A. Vaag
Leslie Alan Vernon
Kyle Jay Vrieze
Tice R. Walker
Matthew Joseph Wasta
Lynne Karyl Wehmueller
Scott Werfel
Jo Dee Westbrook
Dean Allen Westpfahl
William B. Westrate
Matthew M. White
Vanessa Clare Whitlam-Jones
Kendall P. Williams
Dean M. Winters
Yoke Wai Wong
Jeffrey S. Wood
Linda Yang

Part 7C

Nicolas Beaupre
Louise Chung-Chum-Lam
Alana C. Farrell
Hugues Laquerre
David Leblanc-Simard
Jean-Francois Ouellet
Pierre Parenteau
Asif M. Sardar
Benoit St-Aubin
Christopher Brian Wei

Part 9

John Scott Alexander
Ethan David Allen
Mark B. Anderson
Michele S. Arndt
William P. Ayres
Keith M. Barnes
David B. Bassi
Michael J. Bednarick
Wayne F. Berner
Frank J. Bilotti
Jonathan Everett Blake
Sherri L. Border
Michael D. Brannon
Richard Albert Brassington
Rebecca Schafer Bredehoeft
Cary J. Breese
Charles Brindamour
Lisa A. Brown
Robert F. Brown
Ron Brusky
Christopher J. Burkhalter
Michelle L. Busch
Tara E. Bush
Sharon C. Carroll
Bethany L. Cass
John S. Chittenden
Andrew K. Chu
J. Paul Cochran
Margaret Eleanor Conroy
David G. Cook
Christopher G. Cunniff
Michael Kevin Curry
Kenneth S. Dailey
Elizabeth Bassett DePaolo
John D. Deacon
Francis L. Decker
Christopher S. Downey
Michael Edward Doyle
Denis Dubois
Louis Durocher
Rachel Dutil
Kristine Marie Esposito
Ellen E. Evans
Tracy Marie Fleck
Chauncey Fleetwood
David Michael Flitman
Hugo Fortin
John E. Gaines
David Evan Gansberg
Susan I. Gildea
James W. Gillette
Mark A. Gorham
Karl Goring
Philippe Gosselin
Jay C. Gotelaere
David Thomas Groff
Christopher Gerald Gross
Julie K. Halper
Alexander Archibold Hammett
Gregory Hansen
Michelle Lynne Harnick
Steven Thomas Harr
Michael B. Hawley
Daniel F. Henke
William N. Herr
Ronald J. Herrig
Thomas Gerald Hess
Thomas E. Hettinger
Brett Horoff
Linda M. Howell
Marie-Josée Huard
Man-Gyu Hur
Brian L. Ingle
Paul Ivanovskis
Christopher Donald Jacks

Jeremy M. Jump
James B. Kahn
Anthony N. Katz
Brandon Daniel Keller
Mary D. Kroggel
Alexander Krutov
Robin M. LaPrete
Jean-Sebastien Lagarde
Steven Wayne Larson
Dawn M. Lawson
David Leblanc-Simard
P. Claude Lefebvre
Charles R. Lenz
Andre L'Esperance
Christina Link
Andrew M. Lloyd
Lee C. Lloyd
Jason K. Machtinger
Joseph A. Malsky
Andrea Wynne Malyon
Anthony L. Manzitto
David E. Marra
Anthony G. Martella
Bonnie C. Maxie
Ian J. McCracken
Mike K. McCutchan
Thomas S. McIntyre
Douglas W. McKenzie
Allison Michelle McManus
Scott A. McPhee
Paul D. Miotke
David Molyneux
Christopher James Monsour
Benoit Morissette
Roosevelt C. Mosley
Timothy O. Muzzey
Donna M. Nadeau
Vinay Nadkarni
Michael A. Nori
Mark A. O'Brien
Steven Brian Oakley
Kathleen C. Odomirok
Christopher Edward Olson
Denise R. Olson
David J. Otto
Michael G. Owen
Moshe C. Pascher
Lisa Michelle Pawlowski
Mark Paykin
Harry Todd Pearce
Tracie L. Pencak
Julie Perron
Daniel B. Perry
William Peter
John S. Peters
David John Pochettino
Jennifer K. Price
Michael D. Price
David S. Pugel
Kara Lee Raiguel
Tracy A. Ryan
Romel G. Salam
Bret Charles Shroyer
Caroline B. Spain
Alan M. Speert
Michael J. Sperduto
Christopher M. Steinbach
Carol A. Stevenson
Michael J. Steward
Curt A. Stewart
Lisa M. Sukow
Brian Tohru Suzuki
Roman Svirsky
Chester J. Szczepanski
Joy Y. Takahashi
Elizabeth Susan Tankersley
Michael J. Tempesta
David M. Terne
Jeffrey S. Trichon
Ching-Hom Rick Tzeng
Alice M. Underwood
Timothy J. Ungashick
Nathan Karl Voorhis
Robert J. Wallace
Patricia Cheryl White
Wendy Lynn Witmer
Joel F. Witt
Brandon L. Wolf
Simon Kai-Yip Wong
Yoke Wai Wong
Jeanne Lee Ying

NEW FELLOWS ADMITTED IN MAY 1997

First Row, from left: Eric J. Gesick, Andrew J. Doll, Marlene D. Orr, Raleigh Skaggs, Kathleen M. Pechan, Jean-Denis Roy, James M. MacPhee, Mark Joseph Moitoso. **Second row, from left:** Timothy Atwill, Steven Boyce White, Robert Emmett Quane III, Margaret Ann Brinkmann, Dale Steven Porfilio, **CAS President Robert A. Anker**, Mark L. Thompson, Alessandrea Corinne Handley, Floyd M. Yager, James F. Tygh.

NEW ASSOCIATES ADMITTED IN MAY 1997

First row, from left: Alexander Archibold Hammett, Miroslaw Wieczorek, Michael Victor Leybov, William J. Mazurek, Jason Israel, Kara Lee Raiguel, Joanne Emily Russell, Jeremy Michael Jump, Edmund L. Bouchie, Scott Andrew Kelly. **Second row, from left:** C. Steven Swalley, Wendy Lynn Witmer, Allison Michelle McManus, Sharon C. Carroll, Benedick Fidlow, Hugh Eric Burgess, Amy L. Hoffman, Rachel Dutil, Kristine Marie Esposito, Robin M. LaPrete. **Third row, from left:** Yin Lawn, Margaret Eleanor Conroy, Jay C. Gotelaere, Jeffrey S. Trichon, David Evan Gansberg, **CAS President Robert A. Anker**, Patricia Therrien, Michelle Luneau, Denis Dubois, Jane W. Hughes, Theresa Anne Christian. **Fourth row, from left:** Andrea Wynne Malyon, Vinay Nadkarni, Richard Joseph Castillo, John Edward Gaines, David E. Heppen, Paul David Miotke, Richard Bronislaus Puchalski, Simon Kai-Yip Wong, Michael A. Nori, Alan M. Speert, Cary J. Breese.

NEW ASSOCIATES ADMITTED IN MAY 1997

First row, from left: Alfred Denard Commodore, John D. Deacon, Rebecca J. Richard, Jean-Sebastien Lagarde, Kevin A. Lee. **Second row, from left:** Lisa M. Sukow, Kenneth S. Dailey, Janet G. Lindstrom, Martin Vezina, Jennifer K. Price, Deborah M. King, Janice C. Moskowitz, Dave R. Holmes. **Third row, from left:** Jennifer R. Ehrenfeld, Michael Shane, Benoit Morissette, **CAS President Robert A. Anker**, Mark Steven Wenger, Bradley H. Lemons, Marc Shamula. **Fourth row, from left:** David Thomas Groff, Adam Marshall Swartz, John W. Gradwell, Nathan R. Stein, Wayne F. Berner, Daniel J. Henderson, Mark B. Anderson.

NEW ASSOCIATES ADMITTED IN MAY 1997

First row, from left: Neal M. Leibowitz, Jerelyn S. Williams, G. Dennis Sparks, Ethan David Allen, Jonathan Everett Blake, Elizabeth Susan Tankersley, Timothy J. Ungashick. **Second row, from left:** Christina Link, Kimberly S. Troyer, Karen Lee Krainz, Sandra L. Ross, Bret Charles Shroyer, Rebecca Ruth Orsi, John R. Rohe, Sharon C. Dubin, Darci Z. Noonan, Joseph Gerard Evleth. **Third row, from left:** Katherine R.S. Smith, Karen E. Watson, Christopher Edward Olson, **CAS President Robert A. Anker**, Christopher J. Burkhalter, Wayne W. Edwards, Stephanie T. Carlson, Thomas Edward Hinds. **Fourth row, from left:** Michael James Moss, David Neal Kightlinger, Harry Todd Pearce, Jason Noah Masch, Phillip E. McKneely, William N. Herr Jr., Richard M. Chiarini, Christopher C. Swetonic. **New Associates admitted in May 1997 who are not pictured:** Timothy William Atwill, David John Braza, Margaret Ann Brinkmann, Tracy Marie Fleck, Allen Jay Gould, Luke Delaney Hodge, Paul Ivanovskis, George A. Kish, James M. MacPhee, Mark Joseph Moitoso, Mihaela Luminita S. O'Leary, John Sheldon Peters, Amy Ann Pitruzello, Patricia Ann Pyle, Lisa M. Scorzetti, Jeffrey F. Woodcock, Edward John Zonenberg.

NEW FELLOWS ADMITTED NOVEMBER 1997

First row, from left: Elizabeth Ann Lemaster, Julia C. Stenberg, Ira M. Kaplan, Meredith G. Richardson, Leigh J. Halliwell, **CAS President Robert A. Anker**, Linda Kay Torkelson, Gary E. Shook, Milary N. Olson, Carol Ann Blomstrom. **Second row, from left:** Jeffrey Eddinger, Glenn A. Tobleman, Robert W. Van Epps, Kurt J. Johnson, Carole M. Ferrero, Robert G. Downs, Andrew W. Moody, Raymond J. Reimer, Larry D. Anderson. **Third row, from left:** Robert D. Share, Richard T. McDonald, William B. Cody, Mark R. Johnson, George P. Bradley, Roger A. Yard, Robert S. Ballmer II, Keith A. Mathre, Kevin J. Olsen, Gregory D. Larcher.

NEW FELLOWS ADMITTED NOVEMBER 1997

First row, from left: Andrew S. Ribaudo, Gayle L. Wiener, Steven L. Berman, Mary H. Bready, Annette J. Goodreau, **CAS President Robert A. Anker**, Theresa A. Turnacioglu, Bradley A. Hanson, Deborah L. Stone, Ginda Kaplan Fisher. **Second row, from left:** Claus S. Metzner, Hou-Wen Jeng, Jeffery M. Zacek, Daniel K. Johnson, Rebecca A. Kennedy, Daniel J. Merk, Daniel D. Blau, Turhan E. Murguz. **Third row, from left:** David S. Harris, Robert E. Maton, Robert J. Moser, David D. Hudson, Timothy P. Aman, Bernard Dupont, Joan M. Klucarich, Doug A. Zearfoss. **Fourth row, from left:** Jeffrey F. Deigl, Carl J. Sornson, Aaron W. Newhoff, Robert J. Hopper, Peter V. Burchett, Jason A. Kundrot, Jeffrey D. White, Lori A. Snyder. **Fifth row, from left:** Steven J. Symon, Timothy Messier.

NEW FELLOWS ADMITTED NOVEMBER 1997

First row, from left: Victoria G. Stachowski, Jill C. Sidney, Erica L. Weida, Mary E. Fleischli, **CAS President Robert A. Anker**, Amy J. Himmelberger, Alexander G. Zhu, Paul J. Hancock, Regina M. Puglisi. **Second row, from left:** Kimberly J. Mullins, Kevin D. Strous, J'ne E. Byckovski, Jonathan D. Adkisson, Stephen P. Sauthoff, Charles A. Dal Corobbo, Elizabeth R. Wiesner. **Third row, from left:** Geoffrey T. Werner, Terry M. Seckel, Jeffrey P. Shirazi, Gregory Riemer, Tad E. Womack, Jean Cloutier, Collin J. Suttie, Edward M. Kuss.

NEW FELLOWS ADMITTED NOVEMBER 1997

First row, from left: Andrea C. Bautista, Christine E. Schindler, Rita E. Ciccariello, **CAS President Robert A. Anker**, Mari Louise Gray, Gerald T. Yeung. **Second row, from left:** Kevin H. Shang, Patrice Raby, Philippe Trahan, Christian Fournier, Salvator T. LaDuca, Julie T. Gilbert. **Third row, from left:** Lowell J. Keith, Guy R. Danielson, John A. Stenmark, Thomas Struppeck, Michael A. Ginnelly, Jennifer M. Levine. **New Fellows admitted in November 1997 who are not pictured:** Pierre Bourassa, Kirsten Rosa Brumley, John Frederick Butcher II, Dennis K. Chan, Paul Robert Hussian, and Daniel A. Tess.

NEW ASSOCIATES ADMITTED IN NOVEMBER 1997

First row, from left: Shama S. Sabade, Kathleen T. Cunningham, Bruce P. Williams, Lisa M. Pawlowski, Linda Kong, **CAS President Robert A. Anker**, Stephanie J. Michalik, Sherri L. Border, Richard E. Meuret, William A. Mendralla. **Second row, from left:** Joyce Chen, Kevin D. Burns, Philippe Gosselin, William H. Scully, Noelle C. Fries, Todd W. Lehmann, Brian T. Suzuki. **Third row, from left:** Jonathan G. Taylor, Frank J. Bilotti, Kevin J. Fried, Robert J. Moss, Nasser Hadidi, Denise F. Rosen, Christopher J. Monsour, Joel F. Witt. **Fourth row, from left:** Micah R. Gentile, Michael R. Schummer, Paul C. Barone, Steven B. Oakley, David M. Vogt, Anna M. Beaton, Yuhong Yang, Nitin Talwalkar.

NEW ASSOCIATES ADMITTED IN NOVEMBER 1997

First row, from left: Tracy A. Ryan, Laura M. Williams, Matthew H. Price, Halina H. Smosna, Michelle L. Harnick, **CAS President Robert A. Anker**, Julie Perron, Rebecca S. Bredehoeft, Christy B. Schreck, P. Cheryl White. **Second row, from left:** Susan I. Gildea, Richard A. Brassington, Christopher D. Jacks, Christopher P. Coelho, Richard S. Krivo, Alice M. Underwood, Andrew S. Becker, Jean-Claude J. Jacob, William R. Jones. **Third row, from left:** Michael J. Bluzer, Chauncey E. Fleetwood, Mark Paykin, Michael J. Christian, Brandon D. Keller, Gregory Hansen. **Fourth row, from left:** Stephen D. Clapp, Charles Letourneau, Christopher D. Goodwin, David J. Pochettino, Walter L. Jedziniak, Helen P. Neglia, Alexander Krutov, James A. Partridge, Charles P. Neeson.

NEW ASSOCIATES ADMITTED IN NOVEMBER 1997

First row, from left: Sophie Dulude, Avivya S. Stohl, Linda J. Bjork, **CAS President Robert A. Anker**, Cynthia J. Heyer, Jacqueline L. Gronski. **Second row, from left:** Christopher G. Gross, J. Scott Alexander, David E. Marra, Marc E. Levine, Sarah Krutov, Kevin F. Downs. **Third row, from left:** Michael J. Miller, David M. Flitman, Christopher M. Norman, Hugo Fortin, Nathan K. Voorhis.

New Associates admitted in November 1997 who are not pictured: Michael J. Curcio, Michael Edward Doyle, Kristine Marie Firminhac, Sy Foguel, Kenneth Jay Hammell, Ia F. Hauck, Ali Ishaq, Robert B. Katzman, Kirk L. Kutch, Daniel Patrick Maguire, Tieyan Tina Ni, David Anthony Ostrowski, Anthony George Phillips, and Claude A. Wagner.

OBITUARIES

ROBERT DRAYER BART, SR.
CLARENCE S. COATES
DOUGLAS CRITCHLEY
RICHARD C. ERNST
ALFRED VAN WORMER FAIRBANKS
GILBERT W. FITZHUGH
DAVE R. HOLMES
JOHN SEWARD MCGUINNESS
EARL H. NICHOLSON

ROBERT DRAYER BART, SR.
1913–1997

Robert Drayer Bart, Sr. died August 6, 1997 in West Bend, Indiana at the age of 83.

Born December 6, 1913 in Fort Wayne, Indiana, Bart entered Northwestern University in Evanston, Illinois in 1931. While a student majoring in economics and statistics, he met Ruth Sauhering. They were married on January 10, 1935.

His first position after college was with Kemper Insurance in Chicago where he stayed for nine years. During this time he successfully completed the Casualty Actuarial Society examinations and was admitted as a Fellow in 1942. During World War II, Bart did his part during the war doing actuarial work as a registered member for the National Roster of Scientific and Specialized Personnel.

Bart moved his family to West Bend, Indiana in 1944 to work for The West Bend Aluminum Company, initially as the office manager. He later served the company as comptroller, assistant treasurer, vice president, and finally as senior vice president. Bart retired from West Bend Aluminum in 1975 after 31 years of service.

Upon his retirement, he returned to actuarial work for a few years as an enrolled actuary under the Employee Retirement Income Security Act of 1974.

Bart was also very active in his community serving as secretary, treasurer, and president of the West Bend Country Club. His other community work included serving as president of the West Bend Noon Rotary Club, treasurer of the board of St. Joseph's Community Hospital, member of the board of West Bend Mutual Insurance Company, vice chairman of the Milwaukee School of Engineering Board of Regents, president of the Budget Executive Institute, and member of the board of directors of Milwaukee Blue Shield.

His wife, Ruth, died January 26, 1992. On December 18, 1993 he was married to Shelagh Brown Hazelrigg in West Bend.

Bart's grandfather, who was originally from Baden Baden, Germany, immigrated to the United States. He changed his family name from Von Barth to Bart. "It's a shame really but I suppose it made it easier in some way to be just Bart," said Mr. Bart's widow, Shelagh Hazelrigg Bart.

Robert D. Bart is survived by his wife, Shelagh, and three children: Robert D. Bart, Jr., MD, of Honolulu, Hawaii; Linda Karwath of Davenport, Iowa; and Patti Herman of West Bend, Indiana. Other family members include a stepson, Stephen Hazelrigg of McHenry, Illinois; seven grandchildren; three great-grandchildren; a brother Wayne L. Bart of San Angelo, Texas and other relatives and friends.

CLARENCE S. COATES
1901–1997

Clarence S. Coates died on November 4, 1997 in Arlington Heights, Illinois at the age of 96.

Born October 13, 1901 in Henley-on-Thames, England, Coates attended the University of California-Berkeley. On August 5, 1927, he married his wife Robina (Ina). Together they had two sons, William D. and Robert S.

In 1921, Coates became an Associate of the Society. The following year he passed Parts 1 and 2 of the Fellowship exam and was named a Fellow of the Casualty Actuarial Society—one of three new Fellows in 1922. When he became a FCAS, Coates was employed as an actuary by Western States Life Insurance Company, in San Francisco, California. His brother, Barrett N. Coates (FCAS 1918), was also employed by Western States at the same time as assistant secretary and actuary.

Coates' early career was centered in San Francisco where he worked for several companies including Federal Mutual Liability Insurance Company, Federal California Underwriters, and Lumbermen's Mutual Casualty Company (now Kemper Insurance Companies). In 1939 Coates moved to the Chicago, Illinois office of Lumbermen's Mutual and was promoted from statistician to assistant secretary. He was promoted in 1948 to third vice president then second vice president in 1958, and again to actuary in 1960. Coates retired in 1966.

A fellow Lumbermen's Mutual employee, Earl F. Petz (FCAS 1952), remembers an incident that illustrated Coates' attitude toward work. "When it was time for him to retire, I believe his last day was a Friday, and the normal routine was if you made it to lunch, then you went home. Not Clarence. He worked until it was quitting time and came in on Saturday to clean out his office. He was that conscientious," said Petz.

"His approach to being the boss was more the case of setting a great example and inspiring people to work instead of being a hard driver that cracked the whip," Petz continued. "You wanted to do the job for him and do it right because he was like that." Coates was also reported to have been a regular player in the daily bridge game at lunch time at Lumbermen's Mutual.

Another co-worker, M. Stanley Hughey (FCAS 1947) worked with Coates at Lumbermen's Mutual. He had been there a number of years at that point, Hughey reported. "Coates was always a very easy-to-deal-with type of guy but he was a stickler for details. When you took reports in to him and he cleared them, they were right, or else!"

Coates encouraged his son William to become an actuary, and in 1955, the Coates family produced a third actuary when William became an ACAS. Father and son worked together briefly. "We decided early on that we didn't want to work together," said William. "I felt I would be unable to reach my potential working under my dad and he felt he would have to treat me more fairly than otherwise because I was his son," Coates explained.

Coates' contributions to the Society included a discussion of Thomas Tarbell's paper, "Casualty Insurance Accounting and the Annual Statement Blank," in the 1941–1942 *Proceedings*. Coates enjoyed tennis and contributed to many community activities including serving on the board of a local hospice.

Coates is survived by his wife; two sons, Robert of Port Angeles, Washington, and William of Hinsdale, Illinois; and five grandchildren.

DOUGLAS CRITCHLEY
1920–1997

Douglas Critchley was born May 5, 1920 in Cheshire, England. He died January 10, 1997. He was 76.

Critchley attended Shrewsbury School in Shropshire, after which he served six years in the British Royal Navy during World War II. During the war, Critchley was fortunate to survive in a life raft after his ship was torpedoed. After the war, he turned down a chance to attend school at Oxford and instead opted to work for Royal Insurance Company in Liverpool, England. It was during this time that he began taking actuarial courses by correspondence.

In 1948, he transferred to his company's New York City office where he worked for seven years. During this time, Critchley completed his Casualty Actuarial Society exams, becoming a Fellow in 1952. On returning to England in the early 50s, Critchley took the final part of the Institute of Actuaries exams and became a member. Not long after returning to England, Critchley left Royal Insurance to become a partner in E.B. Savory & Co., a firm of stock brokers specializing in insurance investments. He later became senior partner in the company. He retired in 1977.

An active member of the Institute of Actuaries, Critchley participated in numerous discussions on economics and was a member of several dining clubs. He was also very fond of opera, which was a main source of relaxation for him. During his retirement, Critchley took an active interest in financing and encouraging small start-up businesses.

"He was very much loved by family and friends," said Sibyl Solomon, Critchley's sister. "He was a lovely person—very gentle." Throughout his life, Critchley was a generous giver to many charities. "He helped many people and is continuing to help many, many more through his will," said Solomon. Critchley is survived by his sister Sibyl and many other family members and friends.

RICHARD C. ERNST
1944–1997

Richard Ernst was born June 24, 1944, in Pittsburgh, Pennsylvania. He died September 17, 1997, from heart problems. He was 53.

A graduate of Duquesne University in Pittsburgh, Ernst earned a bachelor's degree in mathematics in 1967. After graduation, he taught math at Pittsburgh's North Catholic High School. At the school, he also chaired the math department and coached the football team. In 1972, he earned a master's degree in mathematics from the University of Notre Dame. That same year, he got his first actuarial job, working as an actuarial analyst for Aetna Insurance in Hartford, Connecticut. Ernst lived many years in Hartford also working for Connecticut General Life Insurance and finally as assistant vice president for CIGNA.

In 1987, he joined Reliance Insurance Company in Philadelphia to serve as vice president in charge of ratemaking and pricing. Ken Frohlich, FCAS, a former employer and colleague, made a special effort to recruit Ernst to join Reliance because of his exceptional communication skills and business acumen. "In addition to being a good actuary, he was a good business analyst. He was a class act all the way," said Frohlich. In 1991, Ernst joined Arthur Andersen in Philadelphia as an actuarial consultant.

Ernst, who became a Fellow of the CAS in May 1978, was very active with CAS committees. Most notably, Ernst served as chair of the CAS External Communications Committee from 1992 to 1994. He was also a member of the Editorial Committee of the *Actuarial Review* from 1984 to 1994 and the Examinations Committee from 1983 to 1984. He was also a member of the American Academy of Actuaries.

Having a talent and passion for photography, particularly, sports photography, Ernst became active in the Chester County

Camera Club. He later served as club president from 1996 to 1997. During his year as president, he was instrumental in developing the "Focus on Longwood," an international photo competition sponsored by the club. Ernst developed a way to level scores among the many judging panels for the competition. Fellow club member Marc Horton said that Ernst drew on his actuarial prowess to create a "laureate award-worthy mathematical scheme to achieve fairness for all entries." In addition to his work on the competition, Ernst wrote the curriculum and taught the Camera Club's first basic photography course.

The Chester County Camera Club held a memorial service for Ernst on September 24, 1997. Horton recalled that Ernst was one of the Club's first members. "It was clear that he was a man of integrity; someone you could rely on to do more than promised," said Horton. "During the Club's growth years that followed, Rich became the man of the hour."

To honor Ernst's memory, the Camera Club inaugurated the "Richard Ernst Memorial Photographer of the Year" award. Each year, the club will present the award to the photographer who has received the greatest score from competitions during the year. Award recipients will have their names added to the award so that it will be a constantly growing memorial.

Ernst is survived by his spouse of 27 years, Charlotte Joan Krym Ernst; two sons, Andrew and Kenneth; and a brother.

ALFRED VAN WORMER FAIRBANKS
1918–1997

Alfred V. Fairbanks died June 30, 1997 in Springfield, Massachusetts. He was 79.

Fairbanks was a lifelong resident of Springfield, Massachusetts, where he was born on May 8, 1918. In 1936, he joined Monarch Life Insurance Company in Springfield. Fairbanks worked for Monarch for 47 years before retiring in 1983.

He attended Foster Memorial Church and was a graduate of Classical High School. During World War II, he served in the Army Air Forces. Among his community service activities, he was involved in the Hampden Masonic Lodge, the Scottish Rite Bodies, and the Melha Temple.

Fairbanks earned his Fellowship in the Casualty Actuarial Society in 1951 and was a member of the American Academy of Actuaries. In addition, he was a contributor to the *Proceedings*, Volume XLII, presenting his paper "Notes on Noncancellable Health and Accident Ratemaking" at the CAS Annual Meeting on November 17, 1955.

His survivors include his wife Rita Ethier McKenna Fairbanks; a son, Gregg of Norway, Maine; and two stepdaughters, Suzanne Robitaille of East Longmeadow, Massachusetts, and Jeanne Broderick of Wilbraham, Massachusetts.

GILBERT W. FITZHUGH
1909–1997

Gilbert Fitzhugh died December 29, 1997, after a long battle with Alzheimer's disease. He was 88.

Fitzhugh was born July 8, 1909 in Brooklyn, New York. He graduated from Princeton in 1930 with a bachelor of science in mathematics. He met and married his wife Léa when she was transferred from the London office of Metropolitan Life to Manhattan. They had two children.

At age 23, Fitzhugh passed his examinations for Fellowship in the Society of Actuaries (SOA), becoming the youngest person to do so. Because of an SOA rule barring admission to those under the age of 25, Fitzhugh had to wait before becoming an official member. He later was elected SOA president in 1965. Fitzhugh became a Fellow of the CAS in November 1935. He was also a Fellow of the Canadian Institute of Actuaries and a member of the American Academy of Actuaries.

Fitzhugh began a long career with Metropolitan Life Insurance Company, joining the company in 1930 as a clerk in its social insurance division. He served Met Life for 33 years before being appointed the company's chairman of the board and chief executive in 1963. While he headed Met Life, the company added automobile and home insurance. Fitzhugh retired in 1973 after 43 years.

Fitzhugh was involved in creating political change during the 1960s and early 1970s. In 1967, Fitzhugh announced the formation of a $1 billion urban renewal fund in which insurance companies would invest to restore run-down housing. President Lyndon B. Johnson, whose own urban renewal programs met with fierce opposition from Congress, called the fund "a historic contribution to our country." Today, the fund has grown to $2 billion.

From 1971 to 1972, Fitzhugh led a government-appointed panel assessing the procurement practices of the Pentagon. Some believed that the panel exposed some of the worst faults in the Pentagon's procurement system.

An adventure lover, Fitzhugh scaled both Mount Marcy in New York's Adirondack Mountains and Mount Whitney in California. Fitzhugh's son, Gilbert, recounted a story illustrating his father's adventurous streak: "In 1931, Washington's birthday was a Monday. To see if they could do it, Dad and a friend left work Friday afternoon, took the ferry to New Jersey, drove to Miami, had dates (short ones!) with a couple of friends, and drove back in time for work Tuesday morning. With no Interstate. In a Model A Ford. On plank road through the Georgia swamps in the middle of the night. Never over 50 miles an hour, and often much slower. Boiling out the alcohol anti-freeze going down, and replacing it coming back. And stopping every 500 miles to change the oil. Now...how many of today's FCASs would want to insure these two idiots?"

Fitzhugh made a point to encourage others to take up actuarial work. Charles C. Hewitt, Jr., FCAS, whose first job was with Met Life working for Fitzhugh, credits Fitzhugh with helping him pursue an actuarial career. "I think it was Bacon who said, 'I hold every man to be a debtor to his profession.'...Gilbert Fitzhugh paid his dues many times over when he took the time...to visit a college campus and then to give a seventeen-year-old boy a summer job in actuarial work," said Hewitt.

In addition to his son, Gilbert of Morristown, New Jersey, Fitzhugh is survived by his wife, the former Léa Van Ingh of Hightstown; a daughter, Léa Fitzhugh Welch of Yakima, Washington; a sister, Sarah Fitzhugh Dawson; two grandchildren; three step-grandchildren; two great-grandchildren; and one step-grandchild.

DAVE R. HOLMES
1950–1997

Dave R. Holmes died accidentally on October 10, 1997 at his home in Bristol, Connecticut. He was 47. Holmes had recently become a member of the Casualty Actuarial Society, obtaining his Associateship in May 1997.

He was born April 27, 1950, in Burley, Idaho to Ralph Norman and Maxine Maughan Holmes. He attended schools in Burley and graduated from Burley High School in 1968.

After high school, he attended the California Institute of Technology and then transferred to the University of Idaho where he received his degree in mathematics in 1973. He received his Ph.D. in mathematics from the University of California at Riverside in 1981. The title of his doctoral thesis is "An Extension to *N* dimensions of the Von Karman Equations." In 1986 he received a masters degree in physics from the University of California-Riverside.

Holmes came into the actuarial profession as a career change from engineering and statistical work. He worked for TRW for seven years doing systems engineering and statistical analysis of ICBM flight and ground test data. After leaving TRW, he began studying to pass the actuarial exams. Holmes started in 1990 and became an Associate in 1997. During this time he worked for Zenith Insurance Company for five years as an actuarial assistant writing data retrieval computer programs and doing other projects for the company's special situations division. In August 1997, he moved to Connecticut to work for the research division of The Hartford as a computer and math specialist.

Holmes was enthusiastic about his change in career. "I know that Dave enjoyed his dealings with [the CAS] and was looking forward to being a part of the group and making contributions," wrote Holmes' sister Yvonne Hallock.

A black belt in karate, Holmes enjoyed the exercise and camaraderie of the sport and participated in many tournaments and championships over the years. He had advanced to teaching karate classes occasionally.

He enjoyed the outdoors and took many trips to mountains and deserts in California, Idaho, Alaska, and Utah. Holmes especially enjoyed backpacking trips to Idaho's Sawtooth Mountains and Alaska's wilderness area. Through his many photographs, Holmes tried to capture the scale and grandeur of the spectacular scenery he visited.

His survivors include his father; two sisters, Yvonne of Coeur d'Alene, Idaho, and Valerie Callow of Pocatello, Idaho; three brothers, Richard Pullman of Boise, Idaho, Scott Sherrod of Layton, Utah, and Paul Holmes of Burley; seven nieces and six nephews. He was preceded in death by his mother, Maxine, and stepmother, Jean Wolf Holmes.

JOHN SEWARD McGUINNESS
1922–1997

Dr. John S. McGuinness died April 29, 1997 at the age of 79 in Scotch Plains, New Jersey.

A World War II veteran, McGuinness was born March 12, 1922 in Kingston, Pennsylvania. He married and became a father, pursued advanced degrees, conducted management research, formed his own business, and served as a leader and member of several organizations.

On November 23, 1957, he married Shirley Paige Campbell. They had three children, Brian B. McGuinness, Ann M. Werner, and Lauren K. McGuinness.

After graduating from Broadway High School in Seattle, Washington in 1941, McGuinness attended the University of Washington in Seattle (1941–43), the University of Pittsburgh (1943–44), and the University of California at Berkeley where he earned his bachelor of science degree (1948) and MBA (1949). He did some post-graduate work at the University of Zurich in Switzerland (1949–50) before attending Stanford University (1950–1951) where he received his doctorate degree (1955). In addition, Dr. McGuinness graduated from the U.S. Army War College in 1971.

During World War II, Dr. McGuinness served in the U.S. Army from 1943–1946 in Europe. He continued his military service in the U.S. Army Reserves and was awarded the Meritorious Service Medal. He retired as a colonel.

A 30-year member of Fanwood Presbyterian Church, he served as an elder, usher, and foundation trustee. Other church activities included the men's Bible study group, men's chorus, food bank, and the annual Thanksgiving dinner.

McGuinness was a Fellow of the Casualty Actuarial Society (FCAS 1960), an emeritus member of the Canadian Institute of Actuaries, and a member of the American Academy of Actuaries. He was very active in the Society for the Advance of Management, serving as the organization's international vice president (1972–74), international president (1979–80), and as international chairman of the board (1980–81). He also served as president of the Society of Insurance Research (1972) and as treasurer for the Research Officers Association (1984–86). McGuiness' other affiliations included the International Actuarial Association, Actuarial Studies in Non-Life Insurance, Association of Actuarial Approach to Financial Risks, American Statistical Association, Confederation Internationale des Officiers de Research, Swiss Association of Actuaries, Order of Kentucky Colonels, International Insurance Society, Beta Gamma Sigma, Alpha Kappa Psi, and Delta Tau Delta.

An author of many articles and professional papers on managerial and actuarial subjects, McGuinness wrote *Top Management Organization and Control of Insurance Companies*, which was translated into Japanese.

McGuinness held numerous positions in the U.S. and also worked in Europe before forming his own actuarial and management company, John S. McGuinness Associates-Consultants, in 1964. He served as company president until the time of his death.

He is survived by his wife and children as well as a brother, James B. McGuinness; a sister Grace M. Torgan; and a grandson.

EARL H. NICHOLSON
1900–1997

Earl H. Nicholson died October 27, 1997 in Reno, Nevada. He was 97.

A native of Lowell, Michigan, Nicholson was born March 9, 1900 to Judd B. and Ella Hemingway Nicholson.

Nicholson worked as an actuary and assistant insurance commissioner for the state of Nevada from 1960 until his retirement in 1971. Before moving to Nevada, Nicholson worked for Joseph Froggett Company in New York City for 35 years.

Nicholson served in World War I and was a member of the Unitarian Universalist Fellowship. He received bachelor's and master's degrees from the University of Michigan in Ann Arbor.

His wife, Esther A., a son James B., and a sister Lucretia N. Straatsma, preceded him in death. His survivors include two daughters, Nancy N. Carlton of Pikeville, Kentucky, and Linda N. Roe of Eugene, Oregon; seven grandchildren; and seven great-grandchildren.

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