INTEREST RATE RISK AND CAPITAL REQUIREMENTS FOR PROPERTY/CASUALTY INSURANCE COMPANIES

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Abstract

The advent of risk-based capital requirements and the potential expansion of the role of the Appointed Actuary demand expertise in evaluating the financial stability of insurance enterprises. Because of the growth of property/casualty loss reserves and the wide fluctuations in interest rates during the past two decades, asset-liability management is of increasing importance for casualty actuaries.

The American Academy of Actuaries task force on risk-based capital has provided the NAIC with a proposed "interest rate risk charge" for its risk-based capital formula. This paper reviews the theoretical development of an interest rate risk charge as well as its practical application for setting capital requirements.

Interest rate risk is the potentially adverse effect of a shift in market interest rates on the net worth of the insurance enterprise. For statutory risk-based capital requirements, interest rate risk depends on (i) the relative payment patterns of assets and liabilities, (ii) the statutory valuation rate for the assets, and (iii) the statutory valuation rate for the liabilities.

The paper also discusses the effects of numerous external factors—such as changes in market interest rates or changes in statutory valuation rates—on the magnitude of the interest rate risk, as well as several unresolved issues, such as the proper allocation of assets to cover loss liabilities. It concludes with an example illustrating the computation of an interest rate risk capital charge.

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1. INTRODUCTION

Asset-liability management considerations are gaining increasing prominence in evaluations of capital requirements and of the financial strength of property/casualty insurance enterprises. No longer may actuaries examine reserve adequacy in isolation, while investment officers examine investment strategy in isolation, and expect the combination to accurately portray the company's financial condition. Rather, the actuary and the investment analyst, working in tandem, must forecast the net effects of inflationary changes, interest rate changes, and macroeconomic conditions on policyholders' surplus and on economic net worth.

The goal is clear. For most casualty actuaries, however, the appropriate techniques for evaluating the effects of different financial scenarios on policyholders' surplus remain vague.

2. BACKGROUND

Life actuaries have long dealt with these issues (often termed "C-3 risk"), since interest rate changes have an immediate effect on life insurance company cash flows.[1] Many casualty actuaries, however, have only a rudimentary knowledge of the financial

[1] For a summary of the life actuarial risk categorization system of "C-1" through "C-4," see the report of the CAS Committee on Financial Analysis [8].

theory, and a vague understanding of its applicability to casualty company products. In fact, some casualty actuaries still conceive of interest rate risk solely as "duration mismatch." They reason that the greater the duration mismatch between assets and liabilities, the greater should be the capital requirements for interest rate risk.

This approach is misleading. First, optimal investment strategy does not imply duration matching. Insurance companies should indeed manage their assets in relationship to their liabilities, but analysis of durations is only one part of this process. In fact, an upward sloping yield curve, with higher yields for greater duration securities, along with the short durations of most property and casualty reserves, often implies that asset durations in excess of liability durations may simultaneously increase net investment income and lower the probability of insolvency.[2]

Second, standard asset-liability management theory uses market valuations. For monitoring the effects of interest rate changes on the adequacy of statutory surplus, one must incorporate the effects of the statutory valuation rates for assets and liabilities in the analysis.[3]

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[2]The term "reserves" in this paper refers to loss and allocated loss adjustment expense reserves, not to unearned premium reserves. In statutory accounting, unearned premium reserves are reported gross of prepaid acquisition expenses. This implicit solvency margin overwhelms any adverse effects from interest rate shifts, so no additional capital requirements would be appropriate. This is also the reason that unearned premium reserves are not considered in the NAIC's risk-based capital reserving risk charges or written premium risk charges; see Feldblum [16]. In an examination of the effects of interest rate changes and inflationary changes on the company economic value, of course, one must consider unearned premium reserves as well as future premium flows, particularly audit premiums and retrospective premiums; see Hodes, et al. [24].

[3]The "valuation rate" is the discount rate used to determine the present value of a future payment or disbursement. For instance, suppose a $10,000 claim will be paid one year hence. For statutory accounting purposes, a reserve of $10,000 must be booked. That is to say, the valuation rate is 0%. For the NAIC risk-based capital reserving risk, the held reserves are discounted at a 5% annual rate, so the valuation rate is 5% per annum. For internal management purposes, the company may wish to determine the economic effects of its insurance operations, so it may use a valuation rate equal to current market rates or current risk-free Treasury rates.

For an analysis similar to ours with regard to life insurance and annuity products, with consideration of both valuation rates and payment patterns, see Geyer [20].

Readers' Perspectives

Readers should consider this paper from two perspectives. First, it is an actuarial paper, explaining how interest rate risk *ought* to be treated in solvency monitoring. Second, it is a paper reviewing current regulatory developments, describing how interest rate risk is *now* being considered.[4]

In particular, *statutory* reserves have historically been reported at undiscounted values, and policies have not been subject to disintermediation by consumers. This makes the prop-

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[4]Two actuarial committees assisted the NAIC in developing an interest rate risk charge for the risk-based capital formula.

- The Investment Strategy Subcommittee of the CAS Committee on Valuation and Financial Analysis (VFAC) developed a theoretical foundation for interest rate risk.
- The American Academy of Actuaries (AAA) Task Force on Risk-Based Capital developed a practical interest rate risk charge for the NAIC risk-based capital formula.

The theory in this paper formed the core of the VFAC report. The practical procedures in this paper became the substance of the interest rate risk recommendation submitted by the AAA Task Force to the NAIC.

The major difference between the Task Force report from the method in this paper is that the Task Force report uses a calibration procedure based on an "expected policyholder deficit" analysis, as recommended by Robert Butsic. The calibration procedure defines the standard used to set the capital charges for each risk. For instance, a typical calibration procedure suggested by some European actuaries is to set capital charges such that the probability of ruin of the insurance company is less than a given percentage.

Butsic's calibration procedure uses a "deductible" offset to achieve a better fit between the expected policyholder deficit and the effects of the interest rate change. This calibration procedure is distinct from the concepts discussed in this paper. Moreover, it is specific to the NAIC's risk-based capital formula and to Butsic's expected policyholder deficit theory. Alternative calibration procedures can be used, such as the probability of ruin analyses favored by European actuaries or the judgmental "seven interest rate paths" in New York's Regulation 126. (On expected policyholder deficits, see Butsic [7] or Appendix B of Hodes, Feldblum, and Blumsohn [25]. For the use of probability of ruin analyses, see Pentikäinen, et al. [32]. For a description of New York's Regulation 126, see the Society of Actuaries examination study notes 443-84-88, "Description of New York Regulation 126" and 443-85-89, "Amendments to New York Regulation 126.")

This paper does not include a calibration procedure. The calibration procedure, or the capital standard, relates not to interest rate risk per se, but to the goals of financial regulation of insurance enterprises. Such regulation involves a trade-off between company solidity and consumer prices. The reduction in company failures resulting from more stringent capital requirements generally translates into higher premium rates and consolidation of the industry (as financially weak companies merge with stronger ones). This paper takes no position on where the socially optimal level of regulation lies.

erty/casualty situation much different from the life insurance case. For GAAP reporting, the SEC is now imposing requirements that certain fixed income assets must be held at market values, while liabilities must be held at book values, which generally means nominal values for loss reserves. As the discussion in this paper makes clear, it is necessary to consider the valuation standards to properly measure the effects of interest rate changes on the risk of statutory or GAAP insolvency.

Casualty actuaries will play a major role in debates on these issues. They must be well versed in both the theory and practice if their contributions are to be valuable.

Capital Requirements vs. Dynamic Financial Analysis

This paper deals with the capital requirements needed to guard against interest rate risk in a risk-based capital system. A recent paper by Hodes, et al. [24] from the 1996 Dynamic Financial Analysis (DFA) prize paper competition of the CAS dealt with a variety of financial modeling issues, one of which was the effect of a change in market interest rates and inflation rates on the cash flows and the economic net worth of an insurance company.

In certain respects, these two papers seem to address a similar topic. It is important, therefore, to clarify the differences between the objectives of each paper.

1. *Valuation Rates*: Capital requirements exist within an accounting framework. For interest rate risk, as discussed in this paper, the most important accounting consideration is the valuation of the insurer's assets and liabilities. A cash flow financial model is not tied to any specific accounting system, and there are no "book valuation rates" embedded in the model.

2. *Risks*: Capital requirements exist within a risk-based capital framework. Some of the causes that give rise to interest rate risk may also affect reserving risk or asset risks. When setting total capital requirements, one must

take care not to miss any risks and not to double count any risks.

The cash flow model in the DFA prize paper is a scenario testing model. It does not separately quantify different risks. Rather, it specifies various alternative scenarios, and it examines the influences on the company's combined cash flows.

3. *Generic Formulas*: Capital requirements must be relatively simple, formula-driven results that can be easily applied to any insurer. Dynamic financial analysis is insurer specific. Given the particular characteristics of the insurer under question, the financial model shows the expected future cash flows.

A comparison of underwriting risks and the related risk-based capital charges should clarify this distinction. The NAIC risk-based capital formula contains a "written premium charge," which quantifies the capital requirements needed to guard against adverse underwriting results in the coming year.[5] Hodes, et al. [24] discusses the use of financial models to examine the potential consequences of underwriting cycle downturns.

1. *Valuation*: The NAIC written premium charge is derived from Annual Statement data, is dependent upon book valuations of insurer liabilities, and uses a one-year time frame dependent upon NAIC examination cycles.[6]

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[5]For explanations of the charges in the current NAIC risk-based capital formula, see Feldblum [16].

[6]The book valuation rates are seen most clearly in two areas. (1) A dynamic financial model must examine the effects of an underwriting cycle downturn on both business in force and on new business, using market valuations (or cash flows) for all elements. But the unearned premium reserve is reported in statutory financial statements gross of all prepaid acquisition expenses. Since this implicit margin exceeds the capital requirements needed to hedge against adverse results, no additional charge is made in the NAIC risk-based capital formula. (2) Loss payments are discounted in the NAIC risk-based capital formula at a 5% per annum interest rate, regardless of prevailing market rates. A cash flow financial model shows results independent of any assumed interest rates. The company's current financial condition would be evaluated by discounting the cash flows at whatever rate is chosen by the valuation actuary.

The underwriting cycle downturn scenario in Hodes, et al. [24] is developed independently of Annual Statement data, shows cash flows that are not related to any book valuations, and uses a multi-year time frame, as is appropriate for the management of an insurance enterprise.

2. *Risks*: The NAIC risk-based capital formula quantifies "written premium risk" separately from the other risks faced by an insurance enterprise. For a risk-based capital formula, the appropriate questions are "Does written premium risk overlap with reserving risk?" or "Should there be a separate charge for catastrophe risk?" Underwriting cycle downturns, however, affect many parts of the insurance operation simultaneously, particularly if they are combined with business recessions. Hodes, et al. [24], following Feldblum [13], develops underwriting cycle scenarios based on multiple inter-connected elements of the insurer's operations.

3. *Generic*: The NAIC written premium risk is designed to be applicable to all insurers and to cover a variety of scenarios, whether soft markets or natural catastrophes. The scenario construction process of a financial model builds specific scenarios geared to the characteristics of the insurer under question, taking into account market conditions, concentrations of risk, and reinsurance arrangements.

In sum, a risk-based capital system and a dynamic financial model sit side by side in the same actuarial world: the valuation of insurance companies. However, they address different questions, they use different methods, and they sometimes produce dissimilar results. This paper examines the capital requirements for interest rate risk under the risk-based capital system currently in use by the NAIC. Hodes, et al. [24] examines the effects of numerous external influences, including changes in interest rates and inflation rates, on the cash flows and the economic net worth of an insurance enterprise.

3. FUNDAMENTALS

The previous sections of this paper have used terms like "asset-liability management" and "interest rate risk" without defining them. This section begins with a more careful treatment of these concepts.

Asset-Liability Management

The evolution of actuarial perspectives on underwriting and investment income may be divided into three stages.

1. *Dichotomy*: The earliest stage separated the insurance (or underwriting) functions from the banking (or investment) functions of the company. Underwriters, actuaries, and claims personnel strove for underwriting profits. Financial analysts and investment officers strove for banking profits.

   This dichotomy is oversimplified. From the earliest days of insurance, managers realized that underwriting losses may be offset by investment gains and that the profitability of the insurance enterprise depended on the interactions between the two. However, an integrated approach to underwriting and investment returns was lacking during this period.

2. *Global Integration*: The second stage conceived of the underwriting function as lending money to the investment function for the period between premium collection and loss payment. Many casualty actuaries conceived of the "loan" at a risk-free interest rate, following papers by Woll [37] and by Lowe [28]. The insurance operations were profitable if underwriting returns, plus the inter-

est at a risk-free rate on investable assets derived from insurance operations, were positive.[7] The investment operations were profitable if the realized returns minus the returns at a risk-free interest rate exceeded the expenses of the investment department.

This perspective helped break down the wall between the underwriting and banking functions of insurance enterprises. However, this integration was only at a global level. It did not address the question: "How should investment strategy relate to underwriting strategy?"[8]

3. *Full Integration*: The third stage entails a more complete integration of the underwriting and investment functions. Consider two insurance enterprises. The first writes homeowners policies in Gulf Coast states. Over the long term, the insurance function is profitable. But the high risk of hurricanes makes liquidity an overriding concern for the investment department. Excessive use of private placements, real estate, and even publicly traded bonds with thin secondary markets may not be appropriate investments for this company.[9]

   The second company writes workers compensation policies for a stable customer base. Benefit payments, whose magnitudes are mandated by state law and not subject to jury discretion, are steady from year to year.

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[7] This is the perspective underlying the insurance pricing models of Kahane [26], Fairley [11], and Myers and Cohn [29]. These writers use a "risk adjustment" to the interest rate, generally based on the Capital Asset Pricing Model, though more recent analysis shows the effects to be insignificant (see Cummins and Harrington [9]; Feldblum [17]).

[8] Some actuaries examined the effects of external factors, such as federal income taxes, on underwriting and investment strategy, and asked: For given underwriting results, what investment strategy maximizes net after-tax income? See, for example, Gleeson and Lenrow [21] or Almagro and Ghezzi [1].

[9] Note that liquidity is distinct from duration. Many actuaries focus only on duration, arguing that the short duration of homeowners reserves makes private placements and real estate inappropriate investments. This intermingling of concepts simply confuses the issues. For writers of homeowners insurance, long-term Treasury bonds may be more appropriate investments than private placements, and robustly traded shares of common stock may be more appropriate than real estate.

Most insureds renew their policies each year, and loss costs increase predictably with inflation.

The insurance enterprise, seeking steady cash flows from assets, may invest in long-term and potentially high yielding securities to fund the expected benefit payments, along with investments in equities to capture uncertain but potentially lucrative capital gains. Once again, simple duration matching does not suffice for optimal investment strategy. If the policy renewal rate is high, and the insurer finds good investment opportunities among long duration securities, a considerable asset-liability "mismatch" may be appropriate.[10]

Interest Rate Shifts

The effect of interest rate changes on the value of the insurance enterprise is one aspect of asset-liability management. A rise in market interest rates will decrease the market value of fixed income securities. Conversely, a decline in market interest rates will increase the market value of fixed income securities. Similarly, a rise or decline in interest rates will decrease or increase the present value of fixed liability payments.

Many property/casualty insurers have more fixed income assets than they have fixed liability payments.[11] The effect of interest rate changes on the insurance company's underlying economic equity is similar to their effect on the insurer's investment portfolio: an unexpected rise in interest rates will decrease net equity, and a decline in interest rates will raise net equity. How-

[10]This mismatch refers to the durations of individual insurance contracts and investment vehicles. In practice, the mismatch may disappear when one compares the insurance and investment portfolios, *along with the renewal rates (or retention ratios) on the former*. For further discussion, see Panning [31].

[11]"Fixed liability payments" refers to obligations that are not affected by inflation between the valuation date and the payment date. For example, a traditional whole life contract promises fixed dollar amounts at the death of the policyholder. In contrast, workers compensation medical benefits are influenced by the inflation rate up to the date of the physician's services (or other medical bills).

ever, the dollar amount of this effect is dampened by the fixed liability payments.[12]

How should one determine the optimal attributes of the fixed income asset portfolio? The investment officer begins with three fundamental relationships and must address two questions.

Three Relationships

1. *Duration*: The market values of long duration securities are more affected by interest rate changes than are the market values of short duration securities. For instance, a 100 basis point increase in market interest rates may cause an 8% decline in the market value of a 20 year bond but only a 3.5% decline in the market value of a 5 year note.

2. *Yield Curve*: At most times, the yield curve is upward sloping. That is, long duration securities offer greater returns than do short duration securities. For instance, a 90 day Treasury bill may offer a yield of 5.5%, while a 30 year Treasury bond may offer a yield of 7.5%.[13]

3. *Risk*: Future changes in interest rates may increase or decrease the market value of the investment portfolio. The exposure to future (unknown) interest rate changes constitutes a risk, not a change in expectations. That is, this exposure affects the volatility of investment returns; it does *not* affect the expected value of investment returns.[14]

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[12]For further discussion of the effects of inflation on an insurance company's equity, see Butsic [6] and Noris [30].

[13]Shifts in the yield curve may also be accompanied by risks not explicitly measured by durations and not discussed here, such as spread risks, convexity, and the volatility of interest rates.

[14]This perspective is consistent with the "systematic risk" interpretation of the normally upward sloping yield curve, in that investors are compensated by higher yields for the increased risks of long maturity bonds. Other interpretations of the normally upward sloping yield curve, such as the "market segmentation" or "future expectations" views, do not necessarily see a significant difference in risk by maturity of the bond portfolio. For an overview of yield curve interpretations, see Gray [22].

In other words, lengthening the duration of the fixed income asset portfolio has two effects.[15]

a. The greater duration increases the expected investment yield, as long as the yield curve is upward sloping.

b. The greater duration increases the expected volatility of investment returns because of greater interest rate risk, though this does not affect the expected yield.

Two Questions

The investment officer must address two questions.

1. *Risk and Reward*: Greater duration securities offer higher yields but increase the risk caused by interest rate shifts. Given the various constraints imposed by the underwriting and investment strategies of the company, such as lines of business written, liquidity needs, safety of principal, and promised yields by type of security, what is the optimal trade-off between risk and reward when deciding on the appropriate characteristics of the fixed income securities portfolio?[16]

2. *Capital Requirements*: Interest rate shifts may affect the statutory net worth of the insurance enterprise and threaten its solvency. What asset characteristics minimize the risk of insolvency, or at least keep it within acceptable levels?

   One may rephrase this question in terms of capital requirements. An insurance enterprise holds surplus to guard against the risk of insolvency. The regulator may

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[15]Lengthening the duration of the corporate bond portfolio also increases the credit risk, since the probability of default increases as the time from issue increases. Default risks and other credit risks are separately evaluated by the NAIC's risk-based capital formula (though only with a one-year time horizon), and they are not discussed in this paper.

[16]For a comprehensive analytic framework for fixed income portfolio construction, see Fong [19].

ask: "How should the capital requirements relate to the effects of interest rate shifts on the statutory surplus of the company?"

Focus

These questions differ in two ways—in their focus on stockholders versus policyholders and in their focus on market values versus statutory values.

1. *Stockholders vs. Policyholders*: The first question ("risk and reward") deals with the value of the insurance enterprise. It is an internal question: How do we maximize the value of the firm? Most stockholders can diversify their own holdings. They are often less concerned with the unique risks of individual investments than with the expected returns from the investments.

   The second question ("capital requirements") deals with the security of the insurance promise. Policyholders and claimants want to be assured that the insurance enterprise will meet its obligations. The current earnings of the company are less important than its financial strength.

2. *Market Values vs. Statutory Values*: The first question ("risk and reward") deals either with market values or with internal accounting values. It is generally not concerned with statutory risk loads or statutory valuation rates. It asks how interest rate shifts affect either the market value of the insurance enterprise or management's perception of the worth of the insurance enterprise.

   The second question ("capital requirements") asks how interest rate shifts affect the potential for statutory insolvency. Statutory accounting may guard against the risk posed by interest rate shifts in several ways: by risk loads in the reserves, by differing valuation rates for assets and liabilities, and by additional capital requirements.

This paper deals with the second question (the solvency issues and the capital requirements), not the first question (the risk versus return relationship). Moreover, it assumes that there are no risk loads for reserves except for the differing valuation rates for assets and liabilities. Finally, it treats the valuation rates as given. Its focus is on capital requirements, not on recommended valuation rates or on risk loads.[17]

This last issue, capital requirements, is not necessarily more important than the others. However, it is the issue currently facing the NAIC, and it deserves a full and clear treatment on its own.

4. DURATION AND MARKET VALUES

Our analysis proceeds in three steps.

1. We examine the effects of a security's payment pattern on the sensitivity of its market value to shifts in interest rates. A "security" in this paper means either an asset or a liability. Most of the analysis deals with fixed income assets and fixed liability payments.[18]

   We refer to this as "duration analysis." The fundamentals of duration analysis are reviewed in Appendix A of this paper. More comprehensive treatments can be found in the actuarial and financial literature.[19]

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[17] On the appropriate valuation rate for reserves, see Butsic [5]. On the accounting treatment of risk loads for reserves, see Philbrick [33].

[18] On the effective duration of common stocks, see Leibowitz, Sorensen, Arnott, and Hanson [27]. On the implications for property/casualty insurance asset-liability management, see Feldblum [12].

[19] The seminal actuarial paper on duration analysis is Redington [34]. Good introductory treatments are Bierwag, Kaufman, and Toevs [4]; Ferguson [18] and the discussion by D'Arcy; Geyer [20]; and Tilley [36], along with the discussion by Hoiska.

2. We examine the effects of differing valuation rates used for assets and liabilities. For statutory accounting, the asset valuation rate is generally the yield rate at the time of purchase.[20] The liability valuation rate is set by regulatory prescription in the risk-based capital formula or in statutory accounting principles.

3. We combine the analysis of duration and valuation rates to determine the capital requirements needed to guard against the risk of interest rate shifts.

Characteristics of Duration

Three characteristics of duration are relevant to our discussion.

1. Since the weights used in the calculation of the duration of a security depend on the present values of the cash flows, not on their nominal values, the duration depends on both the cash payment pattern and the market interest rate. As the market interest rate changes, the duration of the security changes.

2. The statement that "the effect of interest rate shifts on the market price of the security is directly proportional to the duration of the security" is accurate for infinitesimal interest rate shifts. As market interest rates change, the duration of the security changes, so the effect on market value changes. If a decrease in market interest rates increases the duration, then the effects on market value of a decrease in market interest rates are magnified. Conversely, if an increase in market interest rates

[20] In statutory financial statements, investment grade fixed income securities are shown at amortized values. As a result, the valuation rate at the time of purchase, whether this be the initial issue or purchase in a secondary market, is retained for the life of the security.

In GAAP financial statements, bonds either "held for trading purposes" or "available for sale" are reported at market value; bonds intended to be held to maturity are reported at amortized values (SFAS 115).

decreases the duration, then the effects on market value of an increase in market interest rates are mitigated.

3. Durations may be determined explicitly for fixed income securities by the definition given above. *Implied* durations, determined from empirical relationships, may be ascribed to other types of securities, such as common stocks and real estate, and to property/casualty liabilities, such as personal auto loss reserves.

   For instance, suppose a bond with a duration of three years would have a three percent decline in market value for a one hundred basis point increase in the market interest rate. This relationship is determined mathematically, by computing present values of nominal cash payments at different interest rates.

   Personal auto loss reserves are at least partially inflation sensitive. Medical payments in tort liability states, for instance, depend in part upon jury awards at the date of settlement. The jury awards, in turn, are influenced by the rate of inflation, which is correlated (at least in the long run) with interest rates. In contrast, wage loss payments under no-fault compensation systems may be fixed at the time of accident, unless cost of living adjustments are built into the benefit schedule.

   A mathematical determination of the loss reserve duration is complex. However, if empirical studies show that the discounted value of personal auto loss reserves declines by three percent for each 100 basis point increase in interest rates, then we may say that the personal auto loss reserves have an effective duration of three years.

   This would be the theoretically correct approach to determining liability durations for interest rate risk capital requirements were the reserving risk charge in the risk-based capital formula to exclude the effects of inflation on adverse loss development. The actual structure of the reserving risk charge necessitates the exclusion of the inflation sensitivity of loss reserves from the calculation of the interest rate risk charge.

5. NOMINAL VS. EXPECTED PAYMENT PATTERNS

Heuristic illustrations, as well as most accounting exhibits, show *stated* payment patterns for fixed income securities. The stated payment patterns are the payment obligations stated in the debt instrument. Some actuaries, lacking practical investment experience, are tempted to use these accounting exhibits for interest rate risk analyses.

Investment analysts use *expected* payment patterns when performing asset-liability management studies. The expected payment patterns take into account refinancings, prepayments, call provisions, and default rates. For certain types of securities, they present a radically different picture of actual cash flows.

Mortgage-Backed Securities

As an illustration, suppose an insurer acquires a portfolio of mortgage-backed securities in 1984, with terms of 30 years and interest rates of 15% per annum. The stated maturity of this portfolio is indeed long.

But the duration of these assets is much shorter than the duration of bonds with the same maturities, for three reasons.

1. When homeowners move, the old mortgage is cancelled, and a new mortgage is issued on the new property. Americans move frequently, and the rate of mortgage cancellations and reissues is concomitantly high, leading to a much shorter duration for mortgage-backed securi-

ties than for bonds. This phenomenon exists even when market interest rates do not change.[21]

2. When market interest rates decline, homeowners are quick to refinance their mortgages. In fact, when interest rates decline sufficiently, mortgage borrowers are inundated with letters from mortgage brokers and banks offering refinancing advice and lower rates. This further reduces the average duration of mortgage-backed securities under certain interest rate paths.[22]

3. Bond principal is repaid in a lump sum at the maturity date. Mortgage principal has a fixed amortization schedule. It is repaid monthly, and it declines to zero over the duration of the mortgage, similar to a bond with a sinking fund. A bond and a mortgage may have the same "term to maturity," but the mortgage will have a much shorter duration.

The expected cash flow pattern for a portfolio of securities therefore differs from the stated cash flow pattern. It is the expected cash flow pattern, under a variety of economic and interest rate scenarios, that is relevant for asset-liability management and the evaluation of interest rate risk.

Asset Cash Flows

The future appointed actuary performing asset adequacy analyses of a casualty insurance company's operations will rely on the expected cash flow patterns provided by investment personnel. But state regulators concerned with the effects of interest rate risk on risk-based capital requirements must rely on statutory financial data.

[21] However, when interest rates decline, homeowners find it easier to purchase new homes, so more old mortgages are cancelled, and new mortgages are issued.
[22] Fong [19] uses contingent claims analysis to estimate the effective durations for securities with issuer options such as calls and refinancings.

Schedule D, Part 1A, of the Fire and Casualty Annual Statement, which shows the maturity distribution of bonds, deals only with stated payment patterns, and it has a maturity schedule that is too coarse for quantification of interest rate risk. Instead, the asset cash flow exhibit reproduced here as Exhibit 1 is patterned after the supplementary asset schedule that was included in the 1995 risk-based capital submission.

The fixed income asset portfolio in Exhibit 1 consists primarily of long-term bonds and mortgages, with smaller amounts of short-term investments and collateral loans. The features of this supplementary asset schedule that are most important for interest rate risk analysis are listed below:

1. The supplementary asset schedule shows the aggregate cash flows themselves, not simply the parameters (such as coupon rates and maturities by security) that are needed to construct a cash flow schedule. This new format allows both the regulator and the actuary to directly address interest rate risk and asset-liability management issues.

2. The supplementary asset schedule shows the expected cash flows, based on the company's best estimate of expected prepayments and refinancings, not the stated cash flows in the bond indenture. For certain types of securities, such as mortgage-backed securities, the difference between stated maturities and expected maturities can be great. For instance, suppose the insurer purchases a portfolio of 30 year mortgages. Schedule D, Part 1A, shows the entire portfolio in the "over 20 years" category.

   But mortgages are often pre-paid or refinanced, even if interest rates do not change, because individuals move to different locations or purchase new homes. If the company expects 2% of the mortgages to be prepaid or refinanced in three years, an additional 3% to be prepaid or refinanced in four years, and so forth, these expectations are used in constructing the expected cash flows.[23]

3. Since the supplementary asset schedule shows cash flows, it clearly distinguishes between securities whose principal is repaid in a lump sum at the maturity date (e.g., government bonds) and securities whose principal is repaid by periodic installments (e.g., mortgages) or by contributions to a sinking fund (e.g., certain corporate bonds). Moreover, it shows interest and principal payments separately, and it shows the expected payment dates of each, not simply the "maturity date."

4. The supplementary asset schedule shows statement values and market values for the major classes of fixed income securities, thereby showing the magnitude of the valuation "cushion" or "deficiency" in the asset portfolio.[24]

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[23]The argument has been voiced that statutory exhibits should be based on auditable data, not on estimates, so exhibits using stated maturities are "better" than those using expected maturities. This argument is strange. Less meaningful data are not "better" simply because they can be audited. Consider Schedule P: the loss reserve entries are estimates, made either by claims examiners or by actuaries. These figures cannot be "audited," but they are essential for monitoring the financial condition of the company.

When entries cannot be audited, how might companies bolster regulators' confidence in their accuracy? Current statutory requirements for loss reserves suggest one means. Each year, the company's appointed actuary signs a statement of opinion certifying the reasonableness of the loss and loss adjustment expense reserve estimates. Similarly, an investment officer of the company might be required to sign a statement certifying the reasonableness of the fixed income cash flow estimates.

[24]Despite the complexity of Annual Statement Schedule D, statutory accounting does not provide this information. Schedule D, Part 1, has columns for book value (Column 4), par value (Column 5), market value (Column 7), and actual cost (Column 8). However, the "market value" column has the following Annual Statement instruction:

> Where a market value is published in the NAIC Valuation of Securities manual, it must be entered in Column 7. Where amortized value or any other value is used, insert a symbol alongside the amount reported.

Market values are published in the NAIC *Valuation of Securities Manual* for bonds that are *not* of investment grade. ("Investment grade" bonds are classes 1 and 2; "non-investment grade" bonds are classes 3 through 6.) For investment grade bonds, most companies show amortized values in Column 7, not market values. In Schedule D-M, market values are shown for the *aggregate* portfolio, not for individual securities or groups of securities.

5. Finally, the supplementary asset schedule shows a greater degree of refinement in the payment schedule than is available from the statutory exhibits.

In-house investment analysts performing an asset-liability management study would use more complete data than are contained in this schedule—just as actuaries performing loss reserve adequacy analyses use more complete data than are contained in Schedule P. This asset schedule serves as a one-page summary of the company's cash flow position, enabling regulators to better evaluate how changes in market interest rates may affect the company's financial solidity.[25]

6. RESERVE PAYMENT PATTERNS

The cash flow pattern from the investment portfolio differs by company based upon (a) the types of securities held and (b) the intent of the company to hold the assets to maturity or to trade them at earlier dates. Moreover, holding securities at amortized values means that the book values of the same security may differ by company based on the date at which the security was acquired. For quantifying interest rate risk, the calculation of asset cash flows and asset book values must be based on the individual company's data.

Loss reserves are different. Annual Statement Schedule P, Part 3, shows historical loss payout patterns by line of business. Although these patterns may differ by company, the differences are

[25]In practice, expected cash flows from fixed income securities vary as interest rates change. For instance, when interest rates decline, corporate bonds are more likely to be called, and mortgages are more likely to be refinanced. The supplementary asset schedule reproduced here as Exhibit 1 is but one piece of a more complete asset-liability management schedule developed by Alex Fontanes. This exhibit shows the expected cash flows if market interest rates remain at their current levels. For a more accurate interest rate risk analysis, one should also have corresponding exhibits showing the expected cash flows if interest rates increase or decrease by specified amounts, such as 200 basis points up or down.

not great, particularly for the high volume lines of business like personal automobile liability and workers compensation. Moreover, the data for small and even some medium size companies may not be sufficiently credible for independent analysis. Industry aggregate data from Best's *Aggregates and Averages* often provides more accurate projections of a company's future payment pattern.[26]

State regulators, seeking an interest rate risk component for the risk-based capital formula, prefer factors derived from industry data and applied uniformly to all insurers, whenever possible. This relieves regulators from the task of monitoring individual company calculations when there is little benefit of increased accuracy.

For the interest rate risk recommendations submitted to the NAIC by the AAA Task Force, reserve payout patterns by line of business were determined from industry data. Each company would weight these reserve payout patterns by its own mix of reserves by line of business. For instance, a company whose reserves were 85% personal auto liability, 5% personal auto physi-

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[26]In some instances, the insurance cash flow patterns do vary by company. Several examples should illustrate this.

1. In workers compensation, writers of large-dollar deductible policies, or writers of excess-of-loss reinsurance over high retentions, don't even begin to pay losses until years after the accident date, once the cumulative loss exceeds the deductible or the retention.
2. In workers compensation, the cash flow patterns differ greatly between one insurer writing small risks on prospectively rated policies with premiums paid upfront and a second insurer writing large risks on "cash-flow" retrospectively rated policies where the premiums are paid as the insurer pays the loss. For interest rate risk, our concern is with *net* insurance cash flows, or the difference between the payout pattern of loss reserves and the collection pattern of accrued retrospective premium reserves.
3. In general liability, the cash flows differ between (i) an insurer that has recently entered this line of business and that writes mostly retail risks and (ii) an insurer with significant toxic tort and environmental impairment liability exposures that may have loss reserve payout patterns extending 30 years into the future.

Thus, the generalizations in the text of this paper should be viewed with caution. They are meant to explain the genesis of the interest rate risk recommendations of the authors and of the American Academy of Actuaries. To the extent that better data become available, the procedures outlined here can be improved.

cal damage, and 10% homeowners, would use an 85%-5%-10% weighting of the industry aggregate reserve payout patterns for these lines of business.

The calculation of the reserve payout pattern by line of business is described in Appendix B. Since payout patterns do not vary much over time, this calculation need be performed only once, and then updated only if there is a substantial shift in the mix by class or the mix by state within a line, or if there is a regulatory or legal change that affects the payout pattern.[27]

7. VALUATION RATES AND STATUTORY SURPLUS

The previous sections of this paper have dealt with market values. They ask: “How do shifts in interest rates affect the market value of a security, or the economic value of the firm?” This is an important question in its own right, but it is only a stepping stone for our analysis. We wish to know: “How should interest rate risk affect the capital requirements of a property/casualty insurer?” To answer this we must first ask: “How do interest rate shifts affect the likelihood that the insurer’s assets will be insufficient to meet its liability obligations?”

Interest Rate Risk: An Illustration

How ought one to guard against interest rate risk? A better formulation of this question might be: How does statutory accounting presently deal with interest rate risk, and what further charges should be embedded in the risk-based capital formula?

Let us begin with an illustration. For heuristic purposes, the insurer in this example writes a single policy and purchases a single bond, though the extension to practical situations is straightforward.

[27] On the stability of payout patterns by line of business, see Woll [37].

The Scenario: Suppose an insurer expects to make a $1,000 personal auto liability payment two years hence. To fund the loss reserve, it purchases a ten year $1,000 par value 8% annual coupon bond. It intends to sell the bond after two years to pay the loss. It buys a ten year bond instead of a two year note to pick up the additional investment income on the longer term security.

Interest Rate Shifts: If interest rates do not deviate from the current market rate of 8% per annum, then the insurer sells the bond after two years for $1,000 to pay the loss. But if interest rates do shift, then the effects on the bond and the loss reserve are different. The bond has a Macauley duration of 7.25 years (see Appendix A). For simplicity, let us assume that the personal auto payment is fixed at $1,000, so the reserve duration is two years.

Since the asset has a greater duration than the liability has, interest rate shifts have a greater effect on the value of the asset than on the value of the liability. So we ask: "If interest rates rise to 10% immediately after purchase of the bond, will the asset still suffice to fund the loss payment?"

Reserve Valuation: The answer depends on the valuation rate for the loss reserves. The valuation rate differs between statutory accounting, the risk-based capital formula, and internal (management) accounting systems, so we treat them each in turn.

Statutory Accounting: Statutory accounting requires most loss reserves to be reported at undiscounted values. If a $1,000 payment is to be made two years hence, the full $1,000 must be set up as a loss reserve, and $1,000 in assets must be set aside to fund the loss. In our illustration, the cash flows are as follows:

- At issue of the $1,000 par value ten year 8% annual coupon bond, market interest rates are 8% per annum, so the purchase price is $1,000.

- Interest rates rise to 10% immediately after issue and remain at that level for the next two years.[28] The bond is then sold to fund the loss payment. The sale price in a 10% interest rate environment is

$$(\$80 \equiv 1.10) + (\$80 \equiv 1.10^2) + \cdots + (\$80 \equiv 1.10^7)$$

$$+ (\$1{,}080 \equiv 1.10^8) = \$893.30.$$

- The first year coupon of $80 is invested at 10% per annum to yield $88 at the end of the second year.

- The second year coupon of $80 is received right before the loss payment is made.

At the payment date, the insurer has $893.30 + $88 + $80 = $1,061.30 to fund the $1,000 loss. The excess of asset cash flows over the reserve obligation stems from the implicit interest margin in undiscounted reserves.

Let us not jump to the conclusion, however, that the use of undiscounted reserves in statutory reporting protects the insurer from interest rate risk. The "implicit interest margin" in the undiscounted reserves has several other functions, such as a cushion to protect against unexpected adverse loss development. The same margin cannot serve two purposes, and additional capital may be needed.

Internal Reporting: Suppose the insurer keeps a management accounting system for internal examination of profitability and financial solidity in which loss reserves are discounted at current market rates. The present value at the accident date of the $1,000 loss payment which will be made two years hence is

$$\$1{,}000 \equiv 1.08^2 = \$857.34,$$

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[28]In practice, of course, one would expect a gradual 200 basis point increase in interest rates over the course of the two years. The effect on the illustrations is not significant.

so only \$857.34 of ten year 8% annual coupon bonds are purchased to fund the loss.[29] The figures provided above must now be multiplied by 85.734%, and the available cash at the end of the second year is

$$0.85734 \times (\$893.30 + \$88 + \$80) = \$909.89.$$

This is insufficient to pay the \$1,000 loss by \$90.11.

The reader should not assume that this is a theoretical, academic scenario. On the contrary: if the entire implicit interest margin is needed to guard against unexpected adverse loss development, or "reserving risk," then there is nothing left to guard against interest rate risk. In other words, this is statutory accounting, not just hypothetical management accounting. The difference between the two scenarios depicted here may be viewed either as a change in the valuation rate or as a differing perception of the purpose of the implicit interest margin.

Risk-Based Capital: How much of the implicit interest margin is needed to guard against reserving risk? If nothing is needed, then the supporting assets exceed the required loss payments by \$61.30, even if interest rates climb to 10%. If all of it is needed for reserving risk, then the supporting assets are deficient by \$90.11 when interest rates climb to 10%.

The risk-based capital formula provides an explicit answer. In the reserving risk calculations, the reported adverse loss development is offset by a 5% per annum discount.[30] In other words, this portion of the implicit interest discount is used to guard against

[29]Of course, bonds are not sold in denominations of \$857.34. In practical situations, however, the annual losses are in millions of dollars, and there is little difficulty in finding assets of the appropriate denominations. The simplified illustration in the text is for heuristic purposes only.

[30]The risk-based capital formula looks at the industry-wide adverse loss development by line of business over the past ten years from Schedule P data and selects the "worst case year." It then says: "This type of adverse loss development happened in the past; it might

adverse loss development. The remaining difference between the asset and liability valuation rates may be used to guard against interest rate risk.

Let us return to our illustration. At a 5% per annum valuation rate for liabilities (as is used in the risk-based capital formula), one need purchase a bond with a face value of $\$1{,}000 \equiv (1.05)^2 =$ \$907.03 to fund the expected loss. To determine whether the asset cash flows suffice to meet the liability obligations, we multiply the numbers given earlier by 90.703%:

$$0.90703 \times (\$893.30 + \$88 + \$80) = \$962.63.$$

At the end of the second year, the supporting assets are deficient by \$37.37 (= \$1,000 – \$962.63). In this scenario, the insurer needs \$37.37 of additional capital to guard against the risk of an unexpected 200 basis point increase in interest rates.

In sum, the reserve valuation rate is a critical factor determining the capital requirements to guard against interest rate risk. Table 1 summarizes the capital requirements needed in our simplified illustration for three valuation rates. (A positive capital requirement means that additional funds are needed to pay the loss when interest rates increase. A negative number means that the held reserves are more than sufficient to pay the loss even if interest rates increase.)

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happen again. Insurers need sufficient capital to protect them against such unexpected development."

Schedule P adverse loss development is on a nominal basis. Insurers report reserves on a full-value (undiscounted) basis. The difference between the economic (discounted) value and the full (undiscounted) value of the reserve is a cushion, or a risk margin, that also guards against unexpected adverse loss development.

The risk-based capital formula therefore offsets the observed "worst case year" adverse loss development with the "implicit investment income" in the undiscounted reserves. In the RBC formula, the IRS loss reserve payment patterns are combined with a fixed 5% interest rate to determine the amount of the reserve discount. Thus, the implied statutory reserve valuation rate for determining capital requirements is 5% per annum.

(The description in this footnote is over-simplified. The actual reserving risk calculations are more complex. For a more detailed description, see Feldblum [16]. For an actuarial evaluation of the capital needed to guard against reserving risk, using stochastic simulation to model loss development and an expected policyholder deficit analysis to calculate the resultant capital needs, see Hodes, Feldblum, and Blumsohn [25].)

TABLE 1

CAPITAL REQUIREMENTS

Valuation Rate	Capital Requirement
Statutory valuation (undiscounted)	($61.30)
Market valuation (fully discounted)	90.11
Risk-based capital (5% discount rate)	37.37

Inflation Sensitivity of Loss Reserves

An important issue is the proper method of dealing with the inflation sensitivity of casualty loss reserves. We analyze this issue in three steps.

1. If there were no reserving risk charge in the risk-based capital formula, then all effects of interest rate changes on either assets or liabilities would be incorporated in the interest rate risk charge. Since interest rates and inflation rates are correlated, and since most casualty loss reserves are affected by inflation through the payment date, a rise in interest rates causes both a decline in the market value of fixed income assets and a rise in the nominal value of casualty loss reserves.[31]

2. The reserving risk charge in the NAIC risk-based capital formula examines the historical adverse loss development by line of business. Ideally, the reserving risk charge should separate the historical adverse loss development into two components.

   - *True* adverse loss development stems from changes in the external environment, such as judicial decisions that were not anticipated by the insurance industry (as

[31] Indeed, Robert A. Bailey, the deputy insurance commissioner of the state of Michigan and a member of the NAIC working group on risk-based capital, argues that this is the proper manner of quantifying interest rate risk.

happened in certain pollution exposures), or misestimation of the reserve needs by company actuaries (as happened in medical malpractice).

- *Apparent* adverse loss development stems from changes in the inflation rate affecting nominal reserve needs. Since the discount rate generally follows the inflation rate, there may be no change in the present value of the loss reserve.

The first component—true adverse loss development—should be reflected in the reserving risk charge. The second component—apparent adverse loss development—should be excluded from the reserving risk charge and included in the interest rate risk charge. The second component does not change the true value of the loss reserve, so this is not a "reserving risk" that a well-managed company should guard against. The true risk here is that the rise in inflation, accompanied by a rise in interest rates, will cause a decline in the market value of fixed income assets even while it leaves the present value of loss reserves unchanged. This is interest rate risk.

3. The current reserving risk charge in the risk-based capital formula lumps all adverse loss development together. Risks may be recognized only once in the risk-based capital formula; to recognize them twice is double counting (see Hartman, et al. [23]). Since the effects of monetary inflation on loss reserves are reflected in the reserving risk charge, these effects should not be reflected in the interest rate risk charge.

Thus, the interest rate risk calculations seem to assume that inflation does not affect nominal loss reserves. In fact, of course, these calculations do not assume this. Rather, this effect of inflation is picked up elsewhere in the risk-based capital formula, not in the interest rate risk component.

The Covariance Adjustment

At first, one might suppose that it makes no difference whether the effects of inflation on loss reserves are reflected in the reserving risk charge or in the interest rate risk charge. Indeed, this would be true were the overall capital requirements an additive combination of the individual risk charges.

In fact, the individual risk charges are combined by a "square root rule" in the risk-based capital "covariance adjustment." This rule says that

$$\text{Total capital requirements} = R_0 + (R_1^2 + R_2^2 + R_3^2 + R_4^2 + R_5^2)^{0.5}$$

where R_4 is the reserving risk charge and R_1 is the asset risk charge for fixed income securities. The AAA Task Force has recommended that the interest rate risk charge be placed in the R_1 risk category.

Because of the square root rule, the marginal effect of each dollar of individual risk charge is proportional to the magnitude of its risk category. For most companies, the size of the reserving risk charge (R_4) is about ten times the size of the asset risk charge for fixed income securities (R_1). Thus, if the effect of inflation on reserves is placed in R_4, this risk has about ten times the effect on overall capital requirements than would be the case were it placed in R_1. For further explanation of the square root rule, see Butsic [7] and Feldblum [16].

Influences on Capital Requirements

Several factors affect the capital requirements for interest rate risk. We divide these factors into three groups:

1. attributes of the company's investment portfolio and liability portfolio, such as average durations and book rates of return;
2. changes in the investment environment affecting market values or payment patterns, such as market interest

rates and the availability and exercise of investor options; and

3. regulatory mandates regarding (a) the level of capital requirements, such as the degree of interest rate shifts, and (b) the valuation rate for liabilities.

The above categorization groups the factors affecting capital requirements into three types: (1) those under the control of the company, (2) those dependent upon the financial markets, and (3) those determined by the regulatory authorities.

1. Portfolio Attributes

1A. Interest rate risk increases as the difference between the average payment dates for assets and liabilities increases.

This is sometimes expressed as, “Interest rate risk varies with the duration mismatch.” This is true, if all other factors are held constant. But cash flow patterns are only one of the factors affecting interest rate risk. Since greater duration assets generally have higher yield rates, the actual effects of duration “mismatch” on interest rate risk are uncertain.

1B. Interest rate risk decreases as the spread between the book valuation rates for assets and liabilities increases.

Fixed income assets are held at amortized values on statutory balance sheets. The book valuation rate is the coupon rate for bonds bought at par or the yield rate at the purchase date for bonds bought at other values (usually in the secondary market).

Reserves are not traded in free markets; they have no coupon rates or yield rates. Rather, the book valuation rate is determined by statutory mandate. In the NAIC’s risk-based capital formula, reserves are valued at a 5% per annum discount rate. The NAIC may change this rate from time to time, though the changes will probably be infrequent, and the reserve valuation rate will presumably never exceed a risk-free interest rate.

The company has no control over the book valuation rate for liabilities in the NAIC's risk-based capital system. However, its investment decisions affect the book valuation rate for its asset portfolio, thereby affecting the spread between the asset valuation rate and the liability valuation rate.

The book valuation rate for assets will generally exceed the book valuation rate for liabilities. The larger the discrepancy, the greater the cushion already present in the statutory valuation rates, and the less need for an additional capital requirement.

2. Investment Environment

2A. Increases in market interest rates that are not reflected in the valuation rate of assets increase interest rate risk.

The interest rate risk quantification procedure described here measures the effects of a shift in interest rates from current market rates to "shocked" rates on the values of assets and liabilities. For most property/casualty insurers, the asset portfolio has a greater duration than the liability portfolio, so an increase in interest rates has a more adverse effect on the value of assets than on the value of liabilities.[32]

An increase in actual market interest rates between the time the assets were purchased and the date the solvency measurement is performed eats up some of the cushion generated by the difference between asset and liability valuation rates. The increase in actual market interest rates therefore increases the capital requirements needed to guard against interest rate risk.[33]

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[32]This is not always true. For instance, a workers compensation carrier with a heavy concentration in commercial paper, Treasury bills, and short-term mortgages may have an investment portfolio significantly shorter than its reserves portfolio. Thus, the generalization in the text should be treated with caution.

[33]The illustrations at the end of this paper demonstrate this effect. The capital required for an interest rate shift of 150 basis points is larger than the capital required for an interest rate shift of 100 basis points. But a shift of 150 basis points can also be viewed as an actual 50 basis point change in market interest rates accompanied by a 100 basis point shift test in the risk-based capital formula.

In general, interest rate changes are gradual, and the bond portfolio turns over steadily. An increase in market interest rates is usually accompanied by an increase in the book valuation rate for assets, which lowers interest rate risk (unless the valuation rate for reserves is raised as well).[34]

Over longer time horizons, the latter effect—valuation rate changes—is more powerful than the former effect—the effect of interest rates on the relative market values of assets and liabilities. At the extremes, the following two rules hold:

- Sudden and recent increases in market interest rates raise the interest rate risk charge.
- Gradual and extended increases in market interest rates lower the interest rate risk charge.

An example should clarify this. Suppose that reserves are valued at a 5% per annum discount rate, as in the current NAIC risk-based capital formula. A long maturity bond portfolio was bought in 1992, at an investment yield of 6% per annum. Capital requirements are being determined at December 31, 1995, when market interest rates for a similar bond portfolio are 8% per annum.[35]

The interest rate risk test measures the effect of a shift in interest rates from 8% to, say, 9% on the ability of the bond portfolio to support the reserve liabilities. The higher initial val-

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[34] A gradual climb in market interest rates, accompanied by a gradual turn-over of the bond portfolio, can still harm an insurance company whose assets have longer payout patterns than its liabilities. However, this gradual climb in market interest rates eats through the company's statutory surplus, since it may have sold some assets at a loss and its yield on recently maturing assets was lower than the yields on similar (recently issued) assets. The company now needs less capital to guard against interest rate risk, but it is financially weaker than before, since its statutory surplus has declined even more rapidly.

[35] The investment yield rates for the bond portfolio in this example are purely heuristic. They do not correspond to actual yields at the stated dates.

uation rate for assets (6%) than for liabilities (5%) provided an initial cushion. But the increase in market interest rates during the subsequent years ate through some of this cushion, since the long-maturity bonds were more adversely affected by the interest rate rise than the reserves were. Larger capital requirements are now indicated for interest rate risk than were indicated at December 31, 1992.

But suppose that the bond portfolio is being turned over from time to time. By December 31, 1995, when market interest rates are 8% per annum, the average valuation rate on the bonds may be 7% per annum. If so, the valuation rate cushion is larger, and the need for additional capital is reduced.

It is difficult to state general rules that will hold in all cases, since many of the relationships discussed above change over time and since so many factors are involved: the average payment date differences between assets and liabilities, the size of the initial cushion, the magnitude of the actual interest rate shift, and the rate of turnover of the investment portfolio. Each situation should be examined separately.

2B. As the difference between expected and nominal average payment dates for assets decreases, interest rate risk increases.

Interest rate risk varies with the difference between expected payment dates for assets and for liabilities. The expected average payment date for assets, which depends on the exercise of borrower options for prepayment, is shorter than the stated (nominal) average payment date. A decrease in this difference, caused by fewer borrower prepayments, increases the difference between asset and liability expected average payment dates, thereby increasing the interest rate risk.[36]

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[36]Again, this generalization assumes that the asset portfolio has a greater duration than the reserves portfolio, which is not necessarily true for all insurers.

An example should help clarify this. Suppose reserves with an average payment date of 4 years are backed by a portfolio of mortgage-backed securities. This portfolio has a nominal average payment date of 15 years, but an expected average payment date of 7 years, reflecting prepayments and refinancings.

If interest rates rise, there will be fewer prepayments and refinancings. The expected average payment date may lengthen to eight years instead of seven years, thereby increasing the interest rate risk.[37]

In summary, an increase in market interest rates has three effects:

- The valuation rate for assets gradually rises, lowering the interest rate risk charge and reducing the need for additional capital.
- The market value of existing assets falls more than the market value of reserves (as long as the assets have longer average maturities than the reserves), thereby reducing the cushion in the differing valuation rates and increasing the need for additional capital.
- Borrowers exercise their prepayment options less frequently, thereby lengthening the average payment date of assets, increasing interest rate risk, and increasing the need for additional capital.[38]

3. *Statutory Mandates*

Subjective changes in the parameters used for solvency monitoring affect the capital required to guard against interest rate risk in two ways.

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[37] See Appendix B, Exhibit 1, of Hodes, et al. [24], for the actual effects of a 200 basis point rise in market interest rates on a large portfolio of mortgage-backed securities.

[38] This last effect can be avoided to the extent that the insurer avoids corporate bonds with call provisions or personal mortgages with prepayment options.

3A. The larger the interest rate shift that is tested, the greater is the capital required for interest rate risk.

The magnitude of the interest rate shift may be decided by the regulator in a risk-based capital context or by the appointed actuary in a dynamic solvency testing context. The larger the interest rate increase, the larger the potential adverse effect on the company's surplus, and the more additional capital is needed. This issue of "calibration" is essential to a practical formula, though not to the procedure, so it is not further dealt with in this paper.[39]

3B. The higher the valuation rate used for reserves, the greater the interest rate risk.

The book valuation rate for reserves depends on the accounting system used. Statutory reporting uses undiscounted reserves. The NAIC's risk-based capital formula uses a 5% per annum discount rate. Many internal company valuation systems use market valuation rates, such as the risk-free rate on Treasury bills. The higher the valuation rate used for reserves, the smaller the valuation rate cushion between assets and liabilities, thereby increasing the additional capital needed to guard against interest rate risk.

The Management of Interest Rate Risk

Numerous factors affect interest rate risk. Some are controllable by the insurance company, such as the duration of the investment portfolio. Some are controllable by the regulator, such as the magnitude of the interest rate shift. And some reflect changing market conditions, such as the current market interest rate versus the valuation rate for assets.

[39]For the interest rate risk proposal of the AAA Task Force on Risk-Based Capital, Robert Butsic calibrated the capital requirements to a one percent expected policyholder deficit ratio. His resultant interest rate risk parameters were a 120 basis point interest rate increase along with a "deductible" equal to 3.5% of the loss reserve market value. For Butsic's derivation of these figures, see Appendix C of [3], particularly page C5 and Exhibits 4A and 4B.

The analyst should understand two aspects of each factor:

- The manner in which a movement in each factor affects interest rate risk. In particular, the analyst should understand whether any specific change will increase or decrease interest rate risk.
- The expected magnitude of the effects of a change in each factor. For instance, the analyst should know the expected effect of a one point change in the asset valuation rate on the capital requirements for interest rate risk. These magnitudes can not be stated as general rules, but must be examined for each company and for each book of business.

8. THE ALLOCATION OF ASSETS

One unresolved issue in the treatment of interest rate risk is the allocation of assets to specific liabilities.

> When determining interest rate risk, should supporting assets be assigned to each block of reserves, or should total assets be compared with total liabilities?

The resolution of this question depends on the goals of the analysis, such as generic monitoring of surplus adequacy by the risk-based capital formula vs. detailed analysis of the company's financial strength by the appointed actuary. We therefore present both sides of this issue.

The Rationale for Allocation

Three arguments favor the allocation of assets to specific liabilities:

1. Certain assets may notionally "support" each block of reserve liabilities, even if all assets ultimately stand behind all liabilities.

2. Asset-liability management suggests that different assets should be purchased to fund different blocks of business.
3. The corresponding life insurance company test, the "asset adequacy analysis," begins with an allocation of assets to blocks of reserves.

Supporting Assets: The illustrations in this paper portray the insurance company as purchasing a bond to fund a specific liability. The value of the bond is chosen to reflect the value of the liability, though the value of the bond depends on the valuation base for the liability.

This presentation serves a valuable analytical purpose. The fundamental issue underlying interest rate risk is whether the cash inflows from assets will support the required liability outflows even if market interest rates shift. Some actuaries argue that it is difficult to answer this question if one does not have a theoretical allocation of assets to blocks of business.

In practice, of course, this allocation does not occur, for two reasons:

- First, all the company's assets support each liability. Any allocation of assets is an accounting fiction, with no legal force. For instance, suppose a company sets up a case reserve of \$100,000 for a given accident, and it allocates assets to fund the loss payment. If two years later the company re-estimates the claim cost as \$500,000, an additional \$400,000 of assets should be allocated to this accident. The original loss estimate (and case reserve) of \$100,000 has no relevance. None of the company's assets has a higher or lower priority for supporting the reserve.
- Second, the company's investment department does not purchase assets to correspond with specific liabilities. Rather, the investment department has an overall sum of money, consisting of funds attributable to insurance transactions as well as funds attributable to capital and surplus. Moreover, it has in-

vestment guidelines, such as “All corporate bonds purchased should be in the two highest NAIC quality classes.”[40]

Asset-Liability Management: As noted above, the investment department will generally not purchase assets to correspond with specific loss reserves. But it will align the general characteristics of its investment portfolio with the attributes of its reserve portfolio.

For instance, suppose a hypothetical company writes workers compensation and homeowners insurance. Its investment philosophy may have three components:

1. For its workers compensation business, it desires long-term, high yielding, safe securities, with steady cash inflows. It chooses a mixture of private placements, mortgage-backed securities, and municipal bonds. The company has sacrificed liquidity and the opportunity for capital gains for steady, safe, and high returns, along with long-term tax advantages.
2. For its homeowners business, with its high catastrophe potential and consequent need for immediate cash payments, the investment department seeks liquid assets and chooses a mixture of Treasury securities and high-grade corporate bonds. The company has sacrificed the higher yields associated with private placements for the liquidity and the safety of principal associated with government and corporate bonds.[41]

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[40]The range of practice is actually broader than this paragraph implies. Life insurance companies often segment their asset portfolio by product type, and some property/casualty companies similarly segment their asset portfolio by major line of business. Segmentation, however, is far less common for property/casualty companies than for life companies.

[41]For the workers compensation business, the long-term, high-yielding securities are the assets backing the loss reserves. For the homeowners business, the liquid assets are protecting against the catastrophe risk, not against expected loss payments. Thus, these assets are effectively backing statutory surplus and the unearned premium reserve, not homeowners loss reserves.

3. For its remaining investments—that is, for assets associated with its surplus funds—the company chooses a mixture of common stock and real estate. Liquidity and steadiness of cash flows are not relevant for these monies. Rather, the company wishes to maximize its expected income, even if this strategy increases the variability of the value of its surplus.[42]

Asset Adequacy Analysis: One purpose of interest rate risk analyses is to examine whether the cash flows from assets will cover the required liability payments even under adverse future interest rate environments. This is the "asset adequacy analysis" required of the life insurance appointed actuary. We are simply carrying it to the property/casualty business, with adjustments as needed. It can be argued that just as the life actuary must first allocate assets to reserves to examine the sufficiency of future cash flows, so must the casualty actuary.

The Arguments Against Allocation

There are two principal arguments against allocation, one theoretical and one practical.

1. *Surplus Adequacy*: The purpose of a risk-based capital formula is to determine capital requirements to ensure surplus adequacy. Its purpose is not to determine risk margins to ensure reserve adequacy. Suppose the insurance company has the following attributes:

    - Assets consist of a $100 million bond portfolio. Liabilities consist of $80 million of reserves, so policyholders' surplus is $20 million.[43]

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[42]Compare Noris [30] for a similar investment strategy for a property/casualty insurer.

[43]These figures are the book values in the risk-based capital accounting system, not market values or statutory values. In the current NAIC risk-based capital system, the book value for assets equals the statutory value, and the book value for liabilities equals the reserves discounted at a 5% per annum interest rate.

- The interest rate risk analysis shows that the market value of the asset portfolio will decrease by 5% if interest rates rise by the "shock" amount, whereas the economic value of the liabilities will decrease by only 2%.
- The difference in book valuation rates between assets and liabilities provides a $2.4 million cushion.
- The company needs $20 million of capital to guard against other risks, such as reserving risk, default risk, and reinsurance risk.

If one asks, "Will the asset cash flows support the liability obligations even if interest rates rise?" the answer is yes. The decrease in the market value of liabilities is $1.6 million (= $80 million × 2%), and the decrease in the market value of the supporting assets is $4 million (= $80 million × 5%). The net decrease is $2.4 million, which is covered by the valuation rate margin.

If one asks, "Is the company holding sufficient capital to guard against the risks that it faces?" the answer is no. The entire $20 million of surplus is needed to guard against other risks. But if the bond portfolio declines in value by 5%, there is an additional decline in value that we have not considered above. The assets not supporting the reserves ($20 million) also decline by 5%, so surplus is only $19 million, not $20 million. The company needs an additional $1 million of capital to guard against the risks that it faces.[44]

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[44] Readers may ask: "Have you properly accounted for the differences between amortized value and market value in the valuation of the bond portfolio?" The answer is yes. The interest rate risk test compares the current book values of assets and liabilities with their market values at a higher interest rate. Thus, statutory accounting values are converted to market values for both assets and liabilities.

For the sake of simplicity, the example does not consider any "covariance" effects. In other words, since interest rate risk and reserving risk are not perfectly correlated, the total capital requirement is less than the sum of the capital requirements for each risk separately. The NAIC risk-based capital formula uses the "square root rule" developed

2. *Simplicity*: The allocation of assets to blocks of reserves is a complex process. In a solvency monitoring setting, companies will desire to allocate those assets to support their reserves that generate the lowest interest rate risk charges, regardless of which assets "ought" to correspond with specific reserves. To avoid additional complexities, regulators may wish to dispense with the allocation of assets.

9. SAMPLE CALCULATION

The previous sections of this paper have been explanatory, with simplified heuristic illustrations. This section provides a more complete example.

To quantify the capital requirements for interest rate risk, three sets of data are needed.

1. *Assets*
   a. One needs the expected cash flow patterns of the investment portfolio, including both interest and principal payments. Ideally, one wants expected cash flow patterns under various interest rate paths; see the comments above regarding mortgage-backed securities for the potential effects of interest rate changes on principal repayments. In practice, few property/casualty companies estimate projected asset cash flows under different interest rate scenarios.
   b. One needs either the statutory valuation rate for these assets or their statutory book value. Since the former may vary for each asset whereas the latter is published in the Annual Statement, it is simpler to use the latter.

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by Robert Butsic to combine the capital requirements for each risk component. Similarly, the AAA Task Force recommendations on interest rate risk placed this charge with the bond default risk charge, not with the reserving risk charge. See Feldblum [16] for further discussion of these issues.

2. *Liabilities*

a. One needs the expected cash flow patterns of the loss reserve portfolio. The appointed actuary of a large insurer may use internal data to determine these patterns, particularly for the long-tailed lines of business.[45] Financial regulators could use industry aggregate patterns from Schedule P data, either from Best's *Aggregates and Averages* or from the NAIC data tapes.[46]

b. One needs either the statutory valuation rate for these liabilities or their statutory book value. For the NAIC risk-based capital requirements, the statutory valuation rate is given as 5% per annum, though the resultant book value is nowhere published.[47] However, the NAIC publishes factors to convert the statutory

[45]Published industry data from Schedule P extends for only ten years. For workers compensation, the average payout of the reserves is about eight years from the valuation date. Short duration workers compensation claims (temporary total claims) are paid within a few months, and they form only a small portion of a company's year-end reserves. Lifetime pension claims (permanent total disability and fatality claims) have payment patterns extending for thirty or forty years, and they form a large portion of a company's year-end reserves. Schedule P data are inadequate for projecting the payment pattern of workers compensation reserves. Similar comments are true for medical malpractice, products liability, and excess-of-loss reinsurance.

[46]There is no mention here of potential variability of the amount and timing of liability payments under different interest rates. Asset cash flows are expected to change when interest rates shift even for fixed income securities because the issuers have options such as calls on corporate bonds and prepayments on mortgages and mortgage backed securities. Similarly, benefit payments and premium collections on life insurance and annuity products vary with the interest rate because policyholders have similar options: they may take policy loans, they may cash in the policy, or they may increase or decrease their premium payments on universal life and other indeterminate premium policies. On property and casualty insurance contracts, there are generally no policyowner options.

As noted elsewhere in the text, inflation does affect the magnitude of loss payments. Since inflation is correlated with interest rates, the magnitude of loss payments is also correlated with interest rates. However, the effects of inflation on the magnitude of loss payments are reflected in the reserving risk charge, so it cannot be "double counted" in the interest rate risk charge as well.

[47]The "book value" of liabilities is the implicit book value in the risk-based capital system. The risk-based capital formula discounts reserves at 5% per annum, to remove

(undiscounted) values of the loss reserves to the risk-based capital (discounted) values.

3. *Calibration*

   a. The capital requirements for interest rate risk depend on the severity of the interest rate shift to be guarded against.

   b. In theory, the severity of the interest rate shift that is selected depends on the type and magnitude of the solvency criterion, such as a 2% probability of ruin, or a 1% expected policyholder deficit. In practice, either the company or the regulator would select a "basis point shift" that is deemed to be sufficiently adverse yet realistic, such as a 150 basis point shift.

Inputs

The example here uses the following input data:

1. The fixed income securities investment portfolio consists primarily of long-term bonds and mortgages, along with smaller amounts of short-term mortgages, collateral loans, and other short-term investments.

2. The loss reserves are for personal automobile liability exposures. The payment patterns are derived from industry aggregate Schedule P data, as described earlier in this paper.

3. Our primary test is a 100 basis point rise in market interest rates. This is somewhat more conservative than the recommendation of the AAA Task Force, which

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the implicit interest margin in the undiscounted reserves on statutory statements. In other words, the risk-based capital formula determines capital requirements for a company whose reserves are discounted at 5% per annum, so this is the "book value" of the reserves used to determine interest rate risk.

used a 120 basis point rise combined with a 3.5% "deductible."[48]

"Market interest rates" is an amorphous concept. In practice, one must define the specific interest rate that is being used as a standard. One obvious choice is the federal midterm rate, which is the average rate on Treasury securities with remaining terms to maturity of between three and seven years. This is the rate on risk-free securities with terms to maturity about equal to the maturities of property/casualty liabilities. Moreover, this rate is used by the IRS in calculating discounted reserves. For the illustrations in this paper, we assume that the current market interest rate is 5.50% per annum.

We also show the effects of a more stringent interest rate shift, of 150 basis points and of 200 basis points.[49] As expected, the capital requirement increases as the interest rate shift grows larger.

Assets

The expected cash flows from the fixed income asset portfolio are derived from the asset schedule adopted by the NAIC for submission as a risk-based capital supplement to the Annual Statement for 1995 and subsequent years. This schedule is reproduced here as Exhibit 1, and it is discussed, along with the illustrative entries, earlier in this paper.[50]

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[48]The AAA recommendation was calibrated by Robert Butsic to produce a 1% expected policyholders deficit, since the reserving risk component of the NAIC risk-based capital formula implicitly came to this standard. Because of the complexity of the calibration issues, they are not treated in this paper.

[49]The 200 basis point shift is similar to the A. M. Best interest rate risk test.

[50]The asset schedule shows expected payments by year for the first four years, and then by groups of years (e.g., 4 to 7, 7 to 10) for the remaining durations. State regulators implementing a risk-based capital system would use these groupings when determining asset cash flow patterns. For instance, the assumed average payment date for the entire "4 to 7 years" cell is 5.5 years.

This information is superior to the information contained in previous insurance company financial statement submissions, but it is not perfect. For instance, the "4 to 7 years"

Allocation of Assets

The book value of the fixed income assets, \$163 million, exceeds the book value of the personal automobile loss reserves, \$139,970,000.[51] The illustration here shows the calculation of the capital requirements both with and without an allocation of assets to liabilities.

For the allocation of assets to liabilities, bonds with different maturities are divided pro-rata between the liability and surplus amounts. For instance, the \$21,672,000 in Column 4 of Exhibit 3, for the row "time of payment = 0.5 years," is calculated as:

$$\$25{,}238{,}000 \times (\$139{,}970{,}000 \equiv \$163{,}000{,}000) = \$21{,}672{,}000.$$

In a dynamic solvency testing environment, the appointed actuary would allocate assets to liabilities based on the company's asset-liability management strategy, not necessarily on a pro-rata basis. Such an allocation might assign more shorter-term securities to the personal auto reserves and more longer-term securities to surplus, which would reduce the interest rate risk charge.[52]

figure may be dominated by bonds maturing in just over 4 years or by bonds maturing in just under 7 years. The latter implies a longer asset duration, and it should (in theory) cause a higher interest rate risk capital charge.

The illustration here uses internal company data to more finely subdivide the expected payment dates of the fixed income security cash flows. For instance:

- The asset schedule in Exhibit 1 shows a total statement value (= book value) of \$163 million and a total nominal cash flow of \$238,159,000. Both of these figures are carried directly to Exhibit 3.
- The asset cash flow figures in Column 3 of Exhibit 3 for all "time of payment" rows except those for 4.5 to 9.5 years are taken directly from the asset schedule in Exhibit 1. For the six "time of payment" rows from 4.5 to 9.5 years, Exhibit 3 subdivides the aggregate figures in the asset schedule into yearly cells, using internal company information. For instance, the \$35,007,000 total expected cash flow in the "4 to 7 years" column in the asset schedule of Exhibit 1 is subdivided into three entries in Column 3 of Exhibit 3:
  - \$13,303,000 for a 4.5 years average time of payment,
  - \$11,552,000 for a 5.5 years average time of payment, and
  - \$10,152,000 for a 6.5 years average time of payment.

[51] The "book value" here is the statutory value for the assets (generally, amortized value), and the discounted value for the liabilities at a 5% per annum interest rate.

[52] Whether assets are allocated to liabilities depends on the purposes of the interest rate risk analysis. A valuation actuary dealing with interest rate risk in a dynamic financial

Liabilities

The entries in Column 2 of Exhibit 3, "Loss Reserve Payout," are derived in three steps.

1. The figures shown in Exhibit 3 are illustrative only. In practice, the statement value of the reserves, or $150,650,000 in these illustrations, should tie to the Schedule P totals from the company's Annual Statement, which shows undiscounted figures.[53] This statement value is the sum of the undiscounted cash flows, and is shown on the Total row of Exhibit 3.

2. The payment pattern for the loss reserve liabilities is determined from aggregate industry data, using Best's *Aggregates and Averages*, as shown in Exhibit 2. The entries in the first nine rows of Exhibit 3 are the payment pattern percentages by accident year from Exhibit 2 times the undiscounted reserve of $150,650,000 in Exhibit 3.[54]

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analysis setting is often helping management determine whether its investment strategy is appropriate, given the company's liability structure. In such a case, the actuary may notionally allocate assets to each block of reserves, to determine if there is a good fit between the two. The solvency regulator is concerned with the adequacy of the company's total capital, not with the appropriateness of its investment strategy. Asset allocation is less relevant to the regulator's concerns.

As noted earlier in the text, this paper examines interest rate risk from the regulator's viewpoint: capital requirements. The DFA perspective, which uses different techniques, may be seen in Hodes, et al. [24].

[53] For further discussion of the reporting of Annual Statement loss reserves gross or net of discount, see Feldblum [14].

[54] This paper views the company from a run-off perspective, as is appropriate for solvency monitoring; see Daykin, et al. [10]. From a "going-concern" perspective, the cash used to meet loss obligations comes (at least in part) from new premium inflows, not just from the assets currently held by the company.

Asset-liability management for a going concern is more complex than might be inferred from this paper, since it is affected by the renewal rates on the book of business and the sensitivity of premium rates to market interest rate changes. For the pricing side of this phenomenon, see Feldblum [15]; for the asset management implications, see Panning [31].

3. The "book value" of $139,970,000 in the last row of Column 1 in Exhibit 3 is the present value of the future cash flows discounted at 5% per annum. This illustration uses an actual discounting of the expected cash flows, as would be appropriate for appointed actuary work. For risk-based capital requirements, one would use the investment income offset factor in the RBC formula, which is a rough approximation based on the IRS loss reserve discounting procedure.[55]

Severity of the Test

The bottom half of Exhibit 3 has three columns showing the capital needed to guard against interest rate shifts of 100 basis points, 150 basis points, and 200 basis points, respectively. A comparison of the three columns shows the sensitivity of the capital requirement to the magnitude of the interest rate shift as summarized in Table 2 at the end of Section 9.

In each case, the current market interest rate is 5.5% per annum. Each column can be viewed in two fashions. The interest rate shift may be an assumed adverse scenario, and the company must hold capital to hedge against this adverse scenario. Alternatively, the "interest rate shift" may be—in part or in whole—an actual movement in market interest rates.

For instance, the right-most column may represent a current market interest rate of 5.5% per annum, with a 200 basis point shift in the risk-based capital test. Alternatively, the column may represent an actual change in market interest rates from 5.5% per annum to 6.5% shortly before the valuation date, and then

[55] For personal auto liability reserves, the investment income offset factor in the risk-based capital formula is 92.1%. The product of 92.1% and $150,650,000 is $138,748,650, which is approximately equal to the book value derived here. In practice, of course, the book value of the liabilities in the risk-based capital system is the undiscounted amount times the risk-based capital discount factor (the 92.1% shown directly above for personal auto liability). In the illustrations, we show the discounting at a 5% interest rate to highlight the factors affecting the interest rate risk charge.

an additional 100 basis point shift in the risk-based capital formula.

The alternative interpretation is realistic, representing a sudden change in market interest rates from 5.5% to 6.5%—say, in the half year preceding the valuation date. In these illustrations, the valuation rate of the assets is 5.22%, which makes more sense in a market interest rate environment of 5.5% than in a market interest rate environment of 6.5%.

A 5.22% average valuation rate for the relatively long-term securities in the asset portfolio implies a market interest rate of about 5% or less for medium term risk-free securities. This makes sense if the assets were bought over the preceding several years and the market interest rate has recently drifted upward to 5.5%. However, if the market interest rate has slowly drifted upward to 6.5% over the past several years, allowing for turnover of the asset portfolio to higher yielding securities, the asset valuation rate would probably be above 5.22%.

Capital Requirements

Rows A and B of Exhibit 3 show the capital required to guard against interest rate risk when assets are first allocated to liabilities. Rows C through F show the capital required if all fixed income assets are used, with no allocation to liabilities. Since the first computation does not consider the assets corresponding to the company's surplus, its resulting capital requirement, $7,017,000, is lower than that of the latter calculation, $8,640,000.

Rows A, B, D, and E show the present value of the asset and liability cash flows at the "shifted" or "shocked" interest rate (6.5%, 7.0%, and 7.5% per annum in the three columns). The following paragraphs document the calculations for the 6.5% per annum column.

1. The present value of the liability cash flows declines by 2.04%, from \$139,970,000 to \$137,120,000. The calculation is done by discounting each cash flow at 6.5% per annum. The reasonableness of the result can be checked by considering the adjusted Macauley durations. The liabilities have an adjusted Macauley duration of 1.39 years, which implies that a one hundred and fifty basis point increase in the discount rate causes a decline in present value of about 2.08%.

2. The present value of the asset cash flows declines by 7.05%, from \$139,970,000 to \$130,104,000. Again, the reasonableness of this figure can be checked by considering the adjusted Macauley durations.

    - The assets have an adjusted Macauley duration of 6.07 years.

    - The book value of the assets implies an average investment yield of 5.22% per annum.[56]

    - The shift to a 6.5% per annum discount rate is an increase of 1.28 percentage points, so the decline in market value of the assets should be about $1.28 \times 6.07 = 7.77\%$.

The combination of:

- the mismatch between asset and liability durations (6.07 years vs. 1.39 years), and

- the similarity of the statutory valuation rates (5.22% vs. 5.00%)

[56] In practice, the book value of the assets depends on the amortized value of each security, so the implied investment yield differs by security. The 5.22% yield is an aggregate figure. It says: "Given the future expected cash flows from this investment portfolio, what discount rate sets its present value equal to its statutory book value?"

leads to a significant interest rate risk charge. The charge, or the capital requirement, equals the change in the value of the assets minus the change in the value of the liabilities, or:

(book value of assets – present value
of assets at shifted interest rate)
– (book value of liabilities – present value
of liabilities at shifted interest rate).

When assets are first allocated to liabilities, the book values of the two are equal, so the capital requirement simplifies to:

present value of liabilities – present value of assets,

at the shifted interest rate. In this example, the capital requirement is

$$\$137{,}120{,}000 - \$130{,}104{,}000 = \$7{,}017{,}000.$$

When assets are not first allocated to liabilities, the total investment portfolio is considered. The book value of the investment portfolio is \$163,000,000, and its market value at a 6.5% per annum discount rate is \$151,510,000, so the capital requirement is

$$(\$163{,}000{,}000 - \$151{,}510{,}000) - (\$139{,}970{,}000 - \$137{,}120{,}000)$$
$$= \$8{,}640{,}000.$$

Other Interest Rate Shifts

The magnitude of the interest rate shift used in a risk-based capital setting is a calibration issue. This paper does not argue for any particular interest rate shift. However, the bottom half of Exhibit 3 shows the capital requirements if interest rate shifts of 100 basis points, 150 basis points, and 200 basis points are used, so that readers can see the effects of different interest rate shifts.

TABLE 2

CAPITAL REQUIREMENTS FOR INTEREST RATE SHIFTS

Basis Point Shift:	100 Basis Points	150 Basis Points	200 Basis Points
w/ allocation of assets	$7,017,000	$9,566,000	$11,934,000
w/o allocation of assets	8,640,000	11,760,000	14,666,000

Table 2 shows the capital requirements:

- for interest rate shifts of 100, 150, and 200 basis points, and
- with and without an allocation of assets to liabilities.

As expected, larger basis point shifts lead to larger capital requirements.

10. CONCLUSION

Asset-liability management is becoming an increasingly important aspect of insurance company investment strategy. Insurers hold enormous financial portfolios—both assets and liabilities—relative to their equity. Regulators are justifiably concerned about the effects of interest rate changes on the financial strength of the company and about the type of capital requirements needed to protect against interest rate risk.

The varied nature of the assets and liabilities comprising an insurer's financial portfolio, the differences between expected and stated cash flows, and the different statutory valuation rates used for assets and liabilities must be considered in the determination of interest rate risk and the associated capital requirements. This paper describes the procedure recommended for inclusion in the NAIC risk-based capital formula, and it provides an illustration for a sample company.

Neither the NAIC nor the American Academy of Actuaries has yet issued guidelines for quantifying interest rate risk for property/casualty insurance companies. Casualty actuaries must understand this subject thoroughly if they wish to participate in the industry discussions and to influence the coming professional and regulatory guidelines.

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EXHIBIT 1

Fixed Income Asset Cash Flow Schedule

Cash Flow Category	Statement Value	Market Value	Principal	1 year or less	1 to 2 years	2 to 3 years	3 to 4 years	4 to 7 years	7 to 10 years	10 to 15 years	15 to 20 years	20 to 30 years	Over 30 years	Total
Payment Range Midpoint (Years)				0.5	1.5	2.5	3.5	5.5	8.5	12.5	17.5	25	35	
PRINCIPAL PAYMENTS														
Bonds	90,000	95,000	100,000	2,000	5,000	6,000	12,000	12,000	14,000	14,000	15,000	20,000	0	100,000
Mortgages: Long Term	49,000	49,000	50,000	2,000	4,000	5,000	6,000	9,000	6,000	6,000	8,000	4,000	0	50,000
Mortgages: Other	9,000	9,000	10,000	1,000	2,000	3,000	2,000	2,000	0	0	0	0	0	10,000
Collateral Loans	5,000	5,000	5,000	2,000	1,000	1,000	1,000	0	0	0	0	0	0	5,000
Perpetual Preferred Stocks	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Mandatory Sinking Fund Preferred Stocks	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Short-term Investments	10,000	10,000	10,000	10,000	0	0	0	0	0	0	0	0	0	10,000
TOTAL	163,000	168,000	175,000	17,000	12,000	15,000	21,000	23,000	20,000	20,000	23,000	24,000	0	175,000
INTEREST PAYMENTS														
Bonds				5,066	4,850	4,578	4,018	8,668	6,429	5,638	3,318	1,659	0	44,224
Mortgages: Long Term				2,526	2,334	2,085	1,793	3,339	2,378	1,937	660	367	0	17,419
Mortgages: Other				482	378	217	110	0	0	0	0	0	0	1,187
Collateral Loans				164	110	55	0	0	0	0	0	0	0	329
Perpetual Preferred Stocks				0	0	0	0	0	0	0	0	0	0	0
Mandatory Sinking Fund Preferred Stocks				0	0	0	0	0	0	0	0	0	0	0
Short-term Investments				0	0	0	0	0	0	0	0	0	0	0
TOTAL				8,238	7,672	6,936	5,921	12,007	8,807	7,575	3,978	2,026	0	63,159
TOTAL CASH FLOW FOR PERIOD														
Bonds				7,066	9,850	10,578	16,018	20,668	20,429	19,638	18,318	21,659	0	144,224
Mortgages: Long Term				4,526	6,334	7,085	7,793	12,339	8,378	7,937	8,660	4,367	0	67,419
Mortgages: Other				1,482	2,378	3,217	2,110	2,000	0	0	0	0	0	11,187
Collateral Loans				2,164	1,110	1,055	1,000	0	0	0	0	0	0	5,329
Perpetual Preferred Stocks				0	0	0	0	0	0	0	0	0	0	0
Mandatory Sinking Fund Preferred Stocks				0	0	0	0	0	0	0	0	0	0	0
Short-term Investments				10,000	0	0	0	0	0	0	0	0	0	10,000
TOTAL				25,238	19,672	21,936	26,921	35,007	28,807	27,575	26,978	26,026	0	238,159

EXHIBIT 2

PRIVATE PASSENGER AUTO LIABILITY/MEDICAL
(FROM BEST'S *Aggregates and Averages*, 1994, IN $MILLIONS)

Accident Year	Age in Months 12	24	36	48	60	72	84	96	108	120
1984	7,107	13,731	16,905	18,677	19,655	20,147	20,396	20,542	20,619	20,661
1985	7,798	15,396	19,096	21,216	22,338	22,903	23,207	23,358	23,433	
1986	8,681	17,212	21,459	23,840	25,120	25,766	26,076	26,243		
1987	9,674	19,363	24,098	26,758	28,171	28,855	29,247			
1988	10,902	21,695	26,909	29,735	31,204	31,950				
1989	12,017	24,024	29,718	32,762	34,382					
1990	13,250	25,971	32,000	35,193						
1991	13,305	25,822	31,811							
1992	14,372	27,677								
1993	15,657									

Age-to-Age Factors

	12–24	24–36	36–48	48–60	60–72	72–84	84–96	96–108	108–120
1984	1.932	1.231	1.105	1.052	1.025	1.012	1.007	1.004	1.002
1985	1.974	1.240	1.111	1.053	1.025	1.013	1.007	1.003	
1986	1.983	1.247	1.111	1.054	1.026	1.012	1.006		
1987	2.002	1.245	1.110	1.053	1.024	1.014			
1988	1.990	1.240	1.105	1.049	1.024				
1989	1.999	1.237	1.102	1.049					
1990	1.960	1.232	1.100						
1991	1.941	1.232							
1992	1.926								

Age-to-Age Factors

	12–24	24–36	36–48	48–60	60–72	72–84	84–96	96–108	108–120	
										Age 120 assumed to be ultimate
Average ATA	1.967	1.238	1.106	1.052	1.025	1.013	1.007	1.003	1.002	↑
Age to Ult.	2.978	1.514	1.223	1.105	1.051	1.025	1.012	1.006	1.002	1.000
Payment Pattern	0.336	0.325	0.157	0.087	0.047	0.024	0.012	0.007	0.003	0.002

Payout of Company Reserves

	Reserve at 12/31/1993	Payout of Reserves, split by year 1994	1995	1996	1997	1998	1999	2000	2001	2002
1985	100	100								
1986	300	188	112							
1987	650	355	185	111						
1988	1,500	761	403	210	126					
1989	3,600	1,763	932	493	257	154				
1990	8,500	4,187	2,113	1,117	591	308	185			
1991	18,000	8,598	4,631	2,337	1,235	654	341	204		
1992	38,000	17,609	9,739	5,246	2,648	1,399	741	386	231	
1993	80,000	39,126	18,941	10,476	5,643	2,848	1,505	797	415	249
Total	150,650	72,686	37,056	19,990	10,500	5,363	2,771	1,387	647	249

EXHIBIT 3

CAPITAL REQUIREMENTS FOR INTEREST RATE RISK (IN THOUSANDS OF DOLLARS)

(1) Time of Payment	(2) Loss Reserve Payout	(3) Fixed Income Asset Inflows	(4) Pro-ration of Fixed Income Asset Inflows**
0.5	72,686	25,238	21,672
1.5	37,056	19,672	16,893
2.5	19,990	21,936	18,837
3.5	10,500	26,921	23,118
4.5	5,363	13,303	11,423
5.5	2,771	11,552	9,920
6.5	1,387	10,152	8,718
7.5	647	10,947	9,400
8.5	249	9,506	8,163
9.5		8,354	7,174
12.5		27,575	23,679
17.5		26,978	23,166
25.0		26,026	22,349
Total	150,650	238,159	204,510
Book Value*	139,970	163,000	139,970

*Book value of the assets is the Annual Statement value.
RBC book value of the liabilities is the present value at a 5% discount rate.

**Column 4 = Column 3 × book value of Column 2 ≡ book value of Column 3.

INTEREST RATE RISK CAPITAL REQUIREMENT USING ''MATCHED'' ASSETS AND LIABILITIES:

"Shocked" interest rate:	6.5%	7.0%	7.5%
A. PV of loss payments (Col 2):	137,120	136,202	135,300
B. PV of "matched" income flows (Col 4):	130,104	126,636	123,366
Capital Requirement [A – B]:	7,017	9,566	11,934

INTEREST RATE RISK CAPITAL REQUIREMENT USING ALL ASSETS AND LIABILITIES:

"Shocked" interest rate:	6.5%	7.0%	7.5%
C. PV of loss payments (Col 2):	137,120	136,202	135,300
D. Difference between C and BV of liabilities:	(2,850)	(3,768)	(4,670)
E. PV of "full" income flows (Col 3):	151,510	147,472	143,663
F. Difference between E and BV of assets:	(11,490)	(15,528)	(19,337)
Capital Requirement [D – F]:	8,640	11,760	14,666

APPENDIX A

DURATION AND MARKET VALUES

This paper deals with the effects of interest rate shifts on the net worth of an insurance company. "Duration" is a term denoting the sensitivity of the market value of a security to a shift in interest rates.

Duration analysis is widely used by life insurance actuaries and by investment personnel. The text of this paper assumes a general familiarity with this concept. This appendix provides the requisite background material for readers who have not previously worked with duration analysis.

Payment Patterns

Our fundamental question is often stated as follows: "How does the duration of a security affect the sensitivity of its market value to interest rate shifts?" In truth, this sentence is loosely worded. Duration does not affect market values when interest rates shift. Rather, duration is *defined* as the effect of interest rate shifts on market values. What we are really asking is: "How does the payment pattern of a security affect the change in market value resulting from an interest rate shift?" Or as financial analysts would phrase this, "How does the payment pattern of a security affect its effective duration?"[57]

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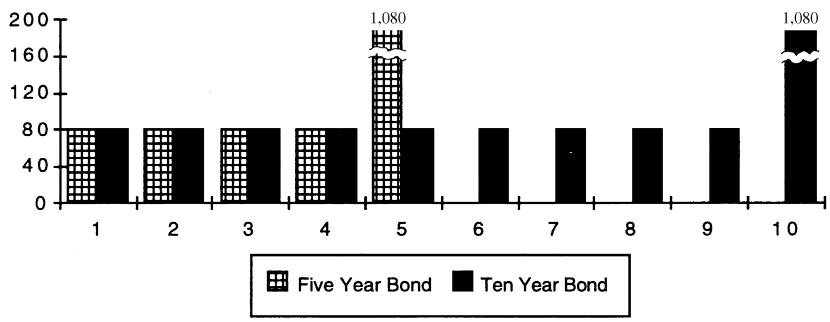
[57] Although the term "duration" has a temporal connotation, and it is commonly measured in units of years, it is a measure of sensitivity to changes in interest rates, not a measure of time. The term "duration" originally stemmed from the application of mathematical approaches to estimate the effects of changing investment conditions on the market value of a fixed income portfolio. The effects were dependent on the average time of the future payments, weighted by the present values of the cash flows. The term "duration" was both appropriate and intuitive to express this concept.

As our understanding of this concept evolved, and as theoreticians examined the effects of imbedded options on the financial relationships and the effects of the investment environment on more complex securities, such as equities, the temporal connotation of duration is sometimes more of a hamper than a benefit. Nevertheless, we have retained the use of this term here. In fact, because of the introductory nature of this paper, we have restricted the analysis to simple fixed income assets and liabilities, avoiding the complexities of inflation-sensitive equities and reserves.

FIGURE 1

CASH FLOWS: ANNUAL COUPONS AND PRINCIPAL REPAYMENTS



Consider two bonds, both with $1,000 par values and 8% annual coupons. Bond A has a five year term to maturity, and Bond B has a ten year term to maturity.

At issue, market interest rates are 8% per annum, and both bonds have a purchase price of $1,000. But if interest rates rise or decline after issue, the change in market value differs for the two bonds.

The market value of a bond is the present value of future cash payments, discounted at an appropriate capitalization rate. For simplicity, we assume that the yield curve is flat, so the appropriate capitalization rate is the market interest rate.[58]

Figure 1 shows the cash flows from the two bonds. Bond A pays $80 at the end of each year, plus a $1,000 repayment of

[58]As noted in the text of the paper, there is no single "market interest rate." Rather, interest rates vary with various attributes of each financial instrument, such as maturity, quality, liquidity, call provisions, and so forth. In this illustration, we assume that the yield curve is flat, so the interest rate will not vary by maturity. If the two bonds are similar in other respects, approximately the same interest rate is appropriate for both. In practice, of course, the bonds would not be similar in all other respects. For instance, longer maturity bonds have a higher risk of default, so they would have to offer a higher yield. See Altman [2].

principal at the end of five years. Bond B pays \$80 at the end of each year, plus a \$1,000 repayment of principal at the end of ten years.

Interest Rates and Market Values

At the issue date, the market interest rate (i.e., the capitalization rate for this bond) is 8% per annum. The market value of the five year bond is

$$(\$80 \equiv 1.08) + (\$80 \equiv 1.08^2) + \cdots + (\$80 \equiv 1.08^4)$$
$$+ (\$1{,}080 \equiv 1.08^5) = \$1{,}000.$$

Similarly, the market value of the ten year bond is

$$(\$80 \equiv 1.08) + (\$80 \equiv 1.08^2) + \cdots + (\$80 \equiv 1.08^9)$$
$$+ (\$1{,}080 \equiv 1.08^{10}) = \$1{,}000.$$

When interest rates rise, the market value of a bond declines, since future cash payments are worth less. The amount of the decline depends on the payment pattern. The further in the future the average payment is, the greater the decline in the present value.

For instance, if interest rates rise to 10% per annum immediately after the issue date, the market value of the five year bond declines to

$$(\$80 \equiv 1.10) + (\$80 \equiv 1.10^2) + \cdots + (\$80 \equiv 1.10^4)$$
$$+ (\$1{,}080 \equiv 1.10^5) = \$924.18,$$

and the market value of the ten year bond declines to

$$(\$80 \equiv 1.10) + (\$80 \equiv 1.10^2) + \cdots + (\$80 \equiv 1.10^9)$$
$$+ (\$1{,}080 \equiv 1.10^{10}) = \$877.11.$$

Conversely, if interest rates drop to 6% per annum immediately after the issue date, the market value of the five year bond rises to

$$(\$80 \equiv 1.06) + (\$80 \equiv 1.06^2) + \cdots + (\$80 \equiv 1.06^4)$$
$$+ (\$1{,}080 \equiv 1.06^5) = \$1{,}084.25,$$

and the market value of the ten year bond increases to

$$(\$80 \equiv 1.06) + (\$80 \equiv 1.06^2) + \cdots + (\$80 \equiv 1.06^9)$$
$$+ (\$1{,}080 \equiv 1.06^{10}) = \$1{,}147.20.$$

Zero-Coupon Bonds

Similar results hold for any characteristics that affect the payment pattern of the security. For instance, bonds with high coupon rates have a higher percentage of their cash flows during the term of the bond than do zero-coupon bonds, where the only cash inflow is the repayment of the principal, accumulated for interest, at the maturity date.[59]

For example, in an 8% per annum interest rate environment, a ten year $1,000 par value 8% annual coupon bond sells for $1,000. A ten year zero-coupon bond with a maturity value of $2,159 also sells for $1,000, since $\$2{,}159 \equiv 1.08^{10} = \$1{,}000$. But the zero-coupon bond is more strongly affected by interest rate shifts than is the annual-coupon bond. If the market interest rate rises to 10% immediately after issue, the market value of the zero-coupon bond drops to $832.36, as compared to $877.11 for the annual-coupon bond. If the market interest rate declines to 6% immediately after issue, the market value of the zero-coupon bond increases to $1,205.53, as compared to $1,147.20 for the annual-coupon bond.

Table A.1 summarizes the discussion above, showing the market value for these three bonds at three different market interest rates.

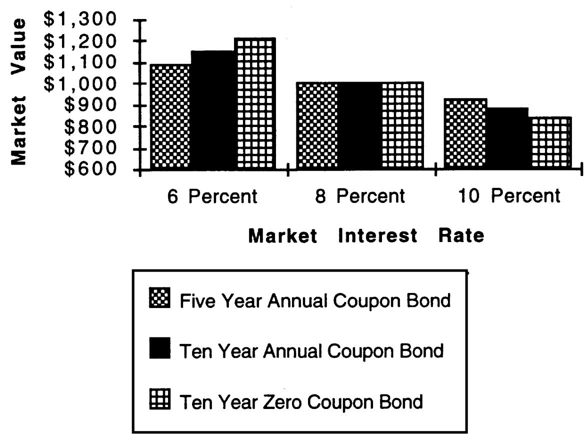
[59]For a full discussion of the factors affecting a bond's duration (coupon size, term to maturity, yield to maturity, sinking fund provisions, and call provisions), see Gray [22].

TABLE A.1

Market interest rate:	6%	8%	10%
Five year coupon bond	$1,084.25	$1,000.00	$924.18
Ten year coupon bond	$1,147.20	$1,000.00	$877.11
Ten year zero-coupon bond	$1,205.53	$1,000.00	$832.36

FIGURE 2

EFFECTS OF INTEREST RATE SHIFTS ON BOND MARKET VALUES



Interest Rate Shifts and Market Values

Figure 2 shows these effects graphically. Interest rate shifts have stronger effects on the market values of securities whose cash flows are further in the future, such as zero-coupon bonds versus coupon bonds, or ten year bonds versus five year notes.[60]

[60]The illustrations in this paper assume a flat yield curve. With a sloping yield curve, the results are slightly, but not significantly, different.

The effect of interest rate shifts on the market value of a security is expressed by the duration of the security.[61]

- The *Macauley duration* of a fixed income security is the weighted average of the cash flow dates, where the weights are the present values of the cash flows. The *adjusted* Macauley duration for an annual-coupon bond is the Macauley duration divided by one plus the interest rate. For instance, if the Macauley duration of an annual-coupon bond is 5 years in an 8% interest rate environment, the adjusted Macauley duration is $5 \equiv 1.08 = 4.63$ years.
- The effect of interest rate shifts on the market price of the security is directly proportional to the adjusted Macauley duration of the security. For instance, if the adjusted Macauley duration of a bond is 4.63 years and the market interest rate increases from 8% to 8.25%, then the market value of the bond decreases by approximately $4.63 \times 0.25\% = 1.16\%$.[62]

We illustrate these relationships with the ten year $1,000 par value 8% annual-coupon bond discussed above. Table A.2 shows:

- the year in which the cash flow occurs;
- the size of the cash flow;
- the present value factor at an interest rate of 8% per annum;

[61] The concept of duration is applicable to both assets and liabilities. As noted earlier, the duration of an asset reflects the sensitivity of the market value of the asset to marginal changes in interest rates. So too, the duration of a liability reflects the change in the present value of the liability in response to a marginal change in current interest rates.

[62] The adjusted Macauley duration for annual-coupon securities is the Macauley duration divided by one plus the interest rate. For small changes in market interest rates, the change in market value of the security is proportional to the adjusted Macauley duration times the change in the interest rate, or

$$\text{Change in market value} = (-1) \times (\text{Change in interest rate}) \times (\text{Adjusted duration}).$$

This approximation is exact for infinitesimally small changes in interest rates. As the interest rate changes, the duration of the security changes as well, so this formula becomes less exact.

TABLE A.2

Year (1)	Cash Flow (2)	Present Value Factor (3)	Weights (4) = [(2) ∗ (3)] ≡ Sum(2 ∗ 3)	Product (5) = (1) ∗ (4)
1	$80	92.59%	7.41%	0.0741
2	$80	85.73%	6.86	0.1372
3	$80	79.38%	6.35	0.1905
4	$80	73.50%	5.88	0.2352
5	$80	68.06%	5.45	0.2722
6	$80	63.02%	5.04	0.3025
7	$80	58.35%	4.67	0.3268
8	$80	54.03%	4.32	0.3458
9	$80	50.02%	4.00	0.3602
10	$1,080	46.32%	50.025	5.0025
Total			100.00%	7.2469

- the weights used in the calculation, or the present values of the cash flows; and
- the products of the weights and the years in which the cash flow occurs.

The Macauley duration for this bond is $724.69\% \equiv 100\% =$ 7.25 years. In an 8% interest rate environment, the adjusted Macauley duration is $7.25 \equiv 1.08 = 6.71$ years.

The approximate change in the market value of this bond is -6.71 times the change in interest rates. A 0.5% drop in interest rates should increase the market value by about 3.355% [= 0.5% ∗ 6.71].[63]

Characteristics of Duration

Three characteristics of duration are relevant to our discussion.

[63] At a 7.5% discount rate, the market value of the bond is $1,034.32, for an increase of 3.432% over its value at an 8% discount rate. This illustrates the earlier comment that the Macauley analysis is *exact* only for infinitesimally small changes, though it is a reasonable approximation for larger changes as well; see Ferguson [18].

1. Since the weights used in the calculation of the duration of a security depend on the present values of the cash flows, not on their nominal values, the duration depends on both the cash payment pattern and the market interest rate. As the market interest rate changes, the duration of the security changes.

2. The statement that "the effect of interest rate shifts on the market price of the security is directly proportional to the duration of the security" is accurate for infinitesimal interest rate shifts. As market interest rates change, the duration of the security changes, so the effect on market value changes. If a decrease in market interest rates increases the duration, then the effects on market value of a decrease in market interest rates are magnified. Conversely, if an increase in market interest rates decreases the duration, then the effects on market value of an increase in market interest rates are mitigated.

3. Durations may be determined explicitly for fixed income securities by the definition given above. *Effective durations*, determined from empirical relationships, may be ascribed to other types of securities, such as common stocks and real estate, and to property/casualty liabilities, such as personal auto loss reserves.

For instance, suppose a bond with a duration of three years would have a three percent decline in market value for a one hundred basis point increase in the market interest rate. This relationship is determined mathematically, by computing present values of nominal cash payments at different interest rates.

Personal auto loss reserves are at least partially inflation sensitive. Medical payments in tort liability states, for instance, depend in part upon jury awards at the date of settlement. The jury awards, in turn, are influenced by the rate of inflation, which is correlated (at least in the long run) with interest rates. In contrast, wage loss payments under no-fault compensation systems may be fixed at the time of accident, unless cost of living adjustments are built into the benefit schedule.

A mathematical determination of the loss reserve duration is complex. However, if empirical studies show that the discounted value of personal auto loss reserves declines by three percent for each 100 basis point increase in interest rates, then we may say that the personal auto loss reserves have an effective duration of three years.

APPENDIX B

RESERVE PAYOUT PATTERNS

The data needed to determine reserve payout patterns are taken from Annual Statement Schedule P, Part 3. Exhibit 2 shows the needed calculations, using industry data from the 1994 edition of Best's *Aggregates and Averages*.

The top-most triangle in Exhibit 2 shows the cumulative loss plus allocated loss adjustment expense payments by accident year and by development period. For instance, the top row in the triangle says that for accident year 1984 losses, $7.1 billion was paid in the first year (January 1, 1984, through December 31, 1984), $13.7 billion was paid in the first two years (1/1/84 through 12/31/85), and so forth.

The middle triangle in Exhibit 2 shows the age-to-age factors, or link ratios, for this block of reserves. Each age-to-age factor is the ratio, by accident year, of cumulative payments at one statement date to the cumulative payments at the previous statement state. For instance, the 1.932 factor for accident year 1984 in the "12 to 24" column means that the cumulative payments at 24 months for accident year 1984 ($13.7 billion) are 93.2% higher than the cumulative payments at 12 months for this accident year ($7.1 billion).

Three further rows appear at the bottom of the exhibit.

1. The average age-to-age (ATA) factor is the average of the individual accident year factors in the column above it. For instance, the 1.967 average age-to-age factor in the "12 to 24 months" column is the average of the column of factors beginning with 1.932 and ending with 1.926.
2. The age-to-ultimate factors are the backward product of the age-to-age factors, as illustrated in the following paragraph. The age-to-ultimate factors times the cumulative payments to date gives the expected ultimate losses.

In this illustration, no payments are expected past 10 years from the date of loss occurrence, so the final age-to-ultimate factor is unity. The age-to-ultimate factor in the penultimate column, 1.002, is the product of the average age-to-age factor in the same column (1.002) and the final age-to-ultimate factor (1.000). The preceding age-to-ultimate factor, 1.006, is the product of the two last age-to-age factors (1.003 and 1.002) and the final age-to-ultimate factor of 1.000. This procedure is used to derive all the age-to-ultimate factors.[64]

3. The final row in the middle section of Exhibit 2 shows the loss payment pattern. The 0.336 figure in the first column means that 33.6% of losses are paid in the first 12 months; the 0.325 figure in the next column means that 32.5% of losses are paid in the second 12 months; and so forth.

   These figures are derived from the age to ultimate factors directly above them. For instance, the 2.978 factor for "12 months to ultimate" means that for each dollar of loss paid in the first 12 months, 1.978 dollars will be paid in subsequent periods, for a total of 2.978 dollars. The percentage of losses paid in the first 12 months is therefore $1 \equiv 2.978 = 0.336$, or 33.6%.

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[64]For lines of business with payment patterns extending past ten years, such as workers compensation, general liability, or excess-of-loss reinsurance, a tail factor is needed. One procedure is to extend the loss triangles as far as possible from historical data and then to fit an inverse power curve to the observed age-to-age link ratio to project the tail development; see Sherman [35]. Hodes, Feldblum, and Blumsohn [25] apply this method to a large countrywide block of workers compensation business, using a 25-year historical triangle and then using an inverse power curve fit to extend the paid loss development up to as much as 70 years. (The exact length of the tail varies stochastically in that paper; see particularly Appendices C and D of that paper for the simulation technique.)

The book value of workers compensation reserves in the risk-based capital system uses the IRS loss reserve discounting procedure, which allows a pattern no longer than 16 years. Actual workers compensation payment patterns extend about 50 years. Thus, the difference between the risk-based capital book value and the actual market value of workers compensation reserves depends not only on the discount rate used but also on the assumed payment pattern.

The 1.514 age to ultimate factor in the second column means that for each dollar paid in the first 24 months, 0.514 dollars will be paid in subsequent periods, for a total of 1.514 dollars. The percentage of losses paid in the first 24 months is therefore $1 \equiv 1.514 = 0.661$, or 66.1%. Since 33.6% of losses are paid in the first 12 months, 32.5% of losses are paid in the next 12 months. This procedure is used to derive all the figures in the final row of the middle section of Exhibit 2.