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FOREWORD

Actuarial science originated in England in 1792 in the early days of life insurance. Because of the technical nature of the business, the first actuaries were mathematicians. Eventually, their numerical growth resulted in the formation of the Institute of Actuaries in England in 1848. Eight years later, in Scotland, the Faculty of Actuaries was formed. In the United States, the Actuarial Society of America was formed in 1889 and the American Institute of Actuaries in 1909. These two American organizations merged in 1949 to become the Society of Actuaries.

In the early years of the 20th Century in the United States, problems requiring actuarial treatment were emerging in sickness, disability, and casualty insurance—particularly in workers compensation, which was introduced in 1911. The differences between the new problems and those of traditional life insurance led to the organization of the Casualty Actuarial and Statistical Society of America in 1914. Dr. I. M. Rubinow, who was responsible for the Society's formation, became its first president. At the time of its formation, the Casualty Actuarial and Statistical Society of America had 97 charter members of the grade of Fellow. The Society adopted its present name, the Casualty Actuarial Society, on May 14, 1921.

The purpose of the Society is to advance the body of knowledge of actuarial science in applications other than life insurance, to establish and maintain standards of qualification for membership, to promote and maintain high standards of conduct and competence for the members, and to increase the awareness of actuarial science. The Society's activities in support of this purpose include communication with those affected by insurance, presentation and discussion of papers, attendance at seminars and workshops, collection of a library, research, and other means.

Since the problems of workers compensation were the most urgent at the time of the Society's formation, many of the Society's original members played a leading part in developing the scientific basis for that line of insurance. From the beginning, however, the Society has grown constantly, not only in membership, but also in range of interest and in scientific and related contributions to all lines of insurance other than life, including automobile, liability other than automobile, fire, homeowners, commercial multiple peril, and others. These contributions are found principally in original papers prepared by members of the Society and published annually in the *Proceedings of the Casualty Actuarial Society.* The presidential addresses, also published in the *Proceedings,* have called attention to the most pressing actuarial problems, some of them still unsolved, that have faced the industry over the years.

The membership of the Society includes actuaries employed by insurance companies, industry advisory organizations, national brokers, accounting firms, educational institutions, state insurance departments, and the federal government. It also includes independent consultants. The Society has two classes of members, Fellows and Associates. Both classes require successful completion of examinations, held in the spring and fall of each year in various cities of the United States, Canada, Bermuda, and selected overseas sites. In addition, Associateship requires completion of the CAS Course on Professionalism.

The publications of the Society and their respective prices are listed in the Society's *Yearbook*. The *Syllabus of Examinations* outlines the course of study recommended for the examinations. Both the *Yearbook,* at a charge of $40 (U.S.), and the *Syllabus of Examinations,* without charge, may be obtained from the Casualty Actuarial Society, 1100 North Glebe Road, Suite 600, Arlington, Virginia 22201.

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NOTICE

Papers submitted to the *Proceedings* of the Casualty Actuarial Society are subject to review by the members of the Committee on Review of Papers and, where appropriate, additional individuals with expertise in the relevant topics. In order to qualify for publication, a paper must be relevant to casualty actuarial science, include original research ideas and/or techniques, or have special educational value, and must not have been previously copyrighted or published or be concurrently considered for publication elsewhere. Specific instructions for preparation and submission of papers are included in the *Yearbook* of the Casualty Actuarial Society.

The Society is not responsible for statements of opinion expressed in the articles, criticisms, and discussions published in these *Proceedings*.

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THE COMPLEMENT OF CREDIBILITY

JOSEPH A. BOOR

Abstract

This paper explains the most commonly used complements of credibility and offers a comparison of the effectiveness of the various methods. It includes numerous examples. It covers credibility complements used in excess ratemaking as well as those used in first dollar ratemaking. It also offers six criteria for judging the effectiveness of various credibility complements. One criterion, statistical independence, has not previously been covered in the actuarial literature. This paper should explain all the common credibility complements to the actuarial student.

1. INTRODUCTION

Many actuarial papers discuss credibility. Actuaries use credibility when data is sparse and lacks statistical reliability. Specifically, actuaries use it when historical losses have a large error around the underlying expected losses (average of the distribution of potential loss costs) the actuary is estimating. They use a loss estimate such as

$$\text{estimate} = Z \times \text{historical losses} + (1 - Z) \times \text{ancillary statistic},$$

where Z is the credibility associated with the historical losses.

In those circumstances, the ancillary statistic that receives the remainder of the credibility can be more important than the data's credibility. For example, if the ratemaking statistic varies around the true expected losses with a standard deviation equal to its mean, it will probably receive a very low credibility. Therefore, the vast majority of the rate (in this context, expected loss estimate) will come from whatever statistic receives the complement of credibility. So, it is very important to use an effective statistic for the ancillary statistic (hereafter called the complement of credibility).

This paper will first discuss six desirable qualities for the complement: accuracy, unbiasedness, independence, availability of data, ease of computation, and an explainable relationship to the subject loss costs. It will do so in light of four basic areas: practical issues, competitive market issues, regulatory issues, and statistical issues. Then it will discuss several complements actuaries often use for first dollar (low or no deductible) losses. It will include a practical example of each complement. Also, it will discuss how well each complement possesses the six desirable qualities. Last, it will discuss several complements commonly used with losses excess of a high deductible or retention (with examples and discussions of how well each possesses the six desirable qualities).

2. FUNDAMENTAL PRINCIPLES—WHAT SHOULD THE ACTUARY CONSIDER?

There are four types of issues that any actuary must consider when choosing the complement: practical issues, competitive market issues, regulatory issues, and statistical issues.

Practical Issues

The easiest statistic to use is one that is readily available. Since some statistics require more complicated programming or expensive processing than others, some statistics are more readily available than others.

Ease of computation is another factor to consider. If a statistic is easy to compute, it is often easier to explain to management and customers. Since few actuaries have unlimited budgets, they usually weigh the time involved in computing a very accurate statistic against the increase in accuracy it generates. Also, when computations are easy to do, there is less chance of error.

Competitive Market Issues

Rates are rarely made in a vacuum. Generally, whatever rate the actuary produces will be subject to market competition. If the rate is too high, competitors can undercut the rate and still make a profit. That will cost the insurer customers and profit opportunities. If the rate is too low, the insurer will lose money. Therefore, in mathematical terms, the rate should be unbiased (neither too high nor too low over a large number of loss cost estimates) and accurate (the rate should have as low an error variance as possible around the future expected losses being estimated). Also, the difference between unbiasedness and accuracy is important. An unbiased statistic varies randomly about the following year's losses over many successive years, but it may not be close. An accurate statistic may average higher or lower than the following year's losses, but it is always close. Ultimately, the complement of the credibility should help make the rate as unbiased and accurate as possible.

Regulatory Issues

Usually, rates require some level of approval from insurance regulators. The classic rate regulatory law requires that rates be "neither inadequate, excessive, nor unfairly discriminatory." The principles of adequacy and non-excessiveness imply that rates should be as unbiased as possible.

Those principles could be stretched to imply that rates should be accurate. The argument goes as follows: Inaccurate rates create a greater risk of insolvency by causing random inadequacies. The law seeks to prevent insolvencies. Therefore, the law suggests rates should be as accurate as possible. Also, for most purposes, actuaries interpret "unfairly discriminatory" in the ratemaking context as "unbiased." Supposedly, if a rate truly reflects a class's probable loss experience, it is fair by definition.

A complement should have some logical relationship to the loss costs of the class or individual being rated. It is easier to explain to a regulator a rate for a class or individual that is consistent with the related loss costs.

Statistical Issues

Clearly, the actuary must attempt to produce the most accurate rate that is practical, but in doing so, the actuary must consider all the types of error that make up the prediction error. (The prediction error is the squared difference between the credibility weighted prediction and actual results.) There are, of course, the natural year-to-year variations in losses about the true mean due to process variance. There may also be errors because the predictor has a different mean than the losses (bias).

The error of the predictor may stem from the error of its components. The historical losses (the usual base statistic), when trended and developed, will contain prediction errors because the factors used to bring losses to a fully developed current cost level are different than what actually will happen (loss development and trend variance). When mathematical models of losses are used as complements, there may be errors in both the type of model used (model variance) and the specific parameters selected for the model (parameter variance). All of these (including any process error and bias of the complement) contribute to prediction error and reduce the accuracy of the prediction.

If the complement of the credibility is accurate in its own right and relatively independent of the base statistic (which receives the credibility), the resulting rate will be more accurate. The rationale involves statistical properties of credibility-weighted estimates. As Appendix A shows, if the optimum credibility for two unbiased statistics is used, then the prediction error (the variance of an estimate around next year's actual loss costs once they are known) of the credibility-weighted estimate is

$$\frac{\tau_1^2\tau_2^2(1-\rho^2)}{\tau_1^2+\tau_2^2-2\rho\tau_1\tau_2},$$

where

τ_1^2 is the average squared error (inaccuracy) of the base statistic as a stand-alone predictor of next year's mean loss costs (i.e., the expected squared difference between the base statistic and next year's actual eventual loss costs);

τ_2^2 is the average squared error (inaccuracy) of the complement of the credibility as a stand-alone predictor of next year's mean loss costs; and

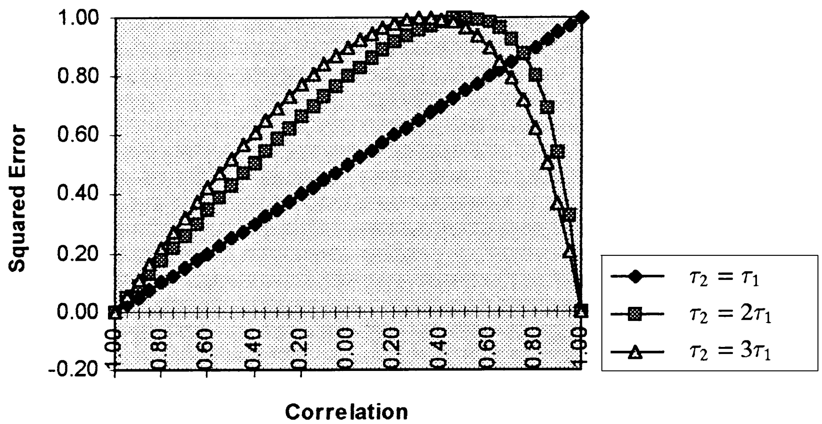
ρ is the correlation (interdependence) between the first statistic's prediction error (error in predicting next year's mean loss costs) and the second statistic's prediction error.

Reviewing that error expression shows that greater inaccuracy in either the base statistic or the complement of credibility will yield greater inaccuracy in the resulting prediction. The expression is symmetric in the two errors. Therefore, the accuracy of the complement of credibility is just as important as the accuracy of the base statistic.

The benefits of independence are more subtle. As it turns out, independence is most important when credibility is most important. That is, independence is most important for the intermediate credibilities (Z between 10% and 90%). As shown in Appendix B, that occurs when the largest standard predicting

FIGURE 1

PREDICTION ERROR AS A FUNCTION OF CORRELATION



error ($\sqrt{\text{inaccuracy}} = \tau$) is within two to three times[1] the smaller error. Consider the following graphs of the total prediction error by correlation for τ_2 = one, two, and three times τ_1.

As one can see in Figure 1, the predictions are generally best when there is actually a negative correlation between the two errors (that is, they offset), but that rarely occurs in practice. Generally, the complement of credibility will have some weak correlation with the base statistic. In that range, the prediction error is clearly lowest as the correlation is smaller. Further, the graph beyond the maximum error (correlations near unity) is misleading. Appendix B shows that the downward slope near unity brings negative credibilities. Those negative credibilities are clearly outside the general actuarial philosophy of credibility.

In fact, the example below illustrates the case of negative credibility.

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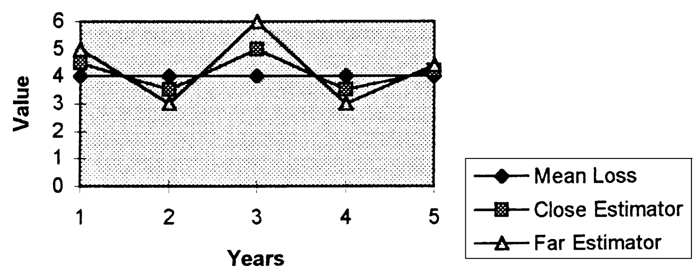
[1] Since Boor [1] shows that credibility is roughly proportional to the relative τ^2s, these examples cover credibilities between 10% and 90%. That range covers instances where credibility matters most.

FIGURE 2

NEGATIVELY CORRELATED ESTIMATORS



As one can see in Figure 2, the close estimator is always between the mean loss and the far estimator. In fact, the far estimator is always twice as far away as the close estimator. Thus, setting

$$\text{estimator} = x - (y - x) = 2x + (-1)y = Zx + (1 - Z)y$$

yields a perfect predictor with negative credibility. Of course, it is extremely rare for the errors of the two statistics to be as correlated as that of these two. In practice, one would rarely assume negative credibility.

Therefore, a complement of credibility is best when it is statistically independent of (that is, not related to) the base statistic.

Summary of Desirable Qualities

The previous sections show six desirable qualities for a complement of credibility:

1. accuracy as a predictor of next year's mean loss costs (i.e., low variance around next year's mean loss costs);
2. unbiasedness as a predictor of next year's mean subject expected losses (i.e., the differences between the predictor and the subsequent loss costs should average out near zero when the predictor is used a number of times);

3. independence from the base statistic;
4. availability of data;
5. ease of computation; and
6. explainable relationship to the subject loss costs.

3. FIRST DOLLAR RATEMAKING

First dollar ratemaking (as opposed to pricing above a very high deductible) generally uses historical loss data for the base statistic. Further, in first dollar ratemaking the historical losses are usually roughly the same magnitude as the true expected losses. Regulators are very concerned with whether or not a complement for first dollar ratemaking is related to the subject loss costs. They are usually less concerned about complements used in excess ratemaking.

There are a wide variety of techniques actuaries use to develop credibility complements. The following pages discuss some of the major methods in use. They are:

- using loss costs from a larger group including the class;
- using loss costs of a larger related class;
- Harwayne's method;
- trending present rates;
- applying the rate change from a larger group to present rates; and
- using competitors' rates.

Loss Costs of a Larger Group Including the Class–Bayesian Credibility

The most basic credibility complement comes from the most classic casualty actuarial technique: Bayesian credibility. In

Bayesian credibility, actuaries are typically either making rates for a large group of classes or making rates for a number of large insureds that belong to a single class. The classes (or individual insureds) do not contain enough exposure units for their historical loss data to reliably predict next year's mean loss costs. Therefore, actuaries supplement the classes' historical loss data by credibility weighting them with the loss costs of the entire group. Sometimes, as in Hurley [5], actuaries weight a class's losses with the losses of the same class in different states.

In mathematical terms, Hurley's loss cost estimate is

$$Z(L_c/E_c) + (1-Z)\left(\sum_i L_i \Big/ \sum_i E_i\right),$$

where

L_c is the historical loss costs for the subject class, c;

E_c is the historical exposure units for class c;

L_i is the historical loss costs for the ith class in the group;

E_i is the historical exposure units for the ith class in the group; and

Z is the credibility.

(For the rest of this paper, P_c will denote the historical loss rate for class c (L_c/E_c). P_g will do the same for the group's historical loss cost rate.)

A. Complement's Qualities

This complement has problems in two areas, accuracy and unbiasedness. The group mean loss costs may be the best available substitute. They may be unbiased with respect to all the information the actuary has when making the rate (e.g., historical loss data—the real means remain unknown). On the other hand, the actuary should believe that the true expected class losses will take a different value than the group expected losses. Therefore, this method contains an intrinsic bias and inaccuracy that is unknown.

This complement generally has some independence from the base statistic. As long as the base class does not predominate in the whole group, the process errors of all the other classes should be independent from that of the base class. Also, the error created by using the group mean instead of the class mean is independent of the base class process variance (error). To the extent that the actuary uses the same loss development, trend, and current level factors on the class and group, the error from those factors is interdependent between the class and group loss costs. On the other hand, one could view the ratemaking process as first estimating undeveloped, untrended historical expected losses at previous rates, and then applying adjustment factors. In the first part of that process, the predicting errors are nearly independent.

This complement performs well on availability and ease of computation. Generally, actuaries compute the group mean and group rate indication as the first stage of the pricing process for the entire line of business.

As long as all the classes in the group have something in common, a logical connection between the class's loss costs and those of the group is formed. However, that does not totally eliminate controversy from this credibility complement. Customers with good loss histories may complain that they are treated "just like everyone else." Overall, this complement has an average degree of relationship to the expected subject losses.

B. Choosing the Larger Group

When choosing a larger group, actuaries often use more years of data, a group of related classes (or all classes) within the same state or region, data from more insurers, or data from the same class for all of a state or region. An actuary should be careful when choosing which larger group to use. For example, given a choice between using same class data from other states (provinces) or other class data from the same state, the actuary should consider: Are the differences by state in loss levels more significant than the differences between class costs in the same state? (Usually, class differences are larger.) Can the other state's class data be adjusted to reflect the base state's loss levels (reducing bias)? Is there a group of classes in the state that the actuary would expect to have about the same loss costs (small bias)? All these factors merit consideration. The actuary should attempt to find the larger group statistic that has the least expected bias.

C. Example

Consider the data in Table 1.

TABLE 1

DATA FOR BAYESIAN CREDIBILITY COMPLEMENT

		Last Year's Data			Last Three Year's Data		
Rate Group	Class	Exposures	Losses	Pure Premium	Exposures	Losses	Pure Premium
A	1	100	5,000	$50	250	16,000	$64
	2	300	20,000	$67	850	55,000	$65
	3	400	19,000	$48	1,100	55,000	$50
	Subtotal	800	44,000	$55	2,200	126,000	$57
B	Subtotal	600	29,000	$48	1,700	55,000	$32
C	Subtotal	500	36,000	$72	1,400	120,000	$86
D	Subtotal	800	75,000	$94	2,300	200,000	$87
Total		2,700	184,000	$68	7,600	501,000	$66

If one is making rates for Class 1 in Rate Group A, one must first consider whether to use one-year or three-year historical losses. One must consider that the three-year pure premiums will be less affected by process variance (year-to-year fluctuations in experience due to small samples from the distribution of potential claims). On the other hand, sometimes the exposure base is large enough to minimize process variance, and societal events are causing pure premiums to change. In that situation, the one-year pure premiums are preferable.

Suppose one chooses the one-year pure premium ($50) for historical data. Using the three-year pure premium of the class ($64) for the complement would be inappropriate because the three-year pure premium is heavily interdependent with the one-year pure premium. Also, presumably the actuary has already decided that the three-year data is biased because of changes in loss cost levels. Thus, the actuary believes the three-year data does not add accuracy to the prediction. For the same reasons, the three-year rate group and grand total pure premiums would be inappropriate complements.

The next choice is between the rate group and grand total pure premiums. The decision involves a tradeoff between bias reduction and process variance reduction. The rate group data should reflect risks that are more similar to Class 1. Therefore, it should have less bias. On the other hand, the grand total data is spread over more risks, so it has less process variance.

In this example, the choice is more difficult. The one-year and three-year rate group pure premiums are very similar ($55 versus $57). For the other rate groups, there are more pronounced inconsistencies (i.e., $32 versus $48 for Rate Group B). The grand total shows it has little process variance, but it appears to contain roughly $15 of bias. The one-year Rate Group A pure premium of $55 is probably the best choice.

One could also consider using the three-year pure premium for historical losses. That does not preclude using the one-year rate group data as a complement. Using the one-year Rate Group A pure premium would simply assume that the total Rate Group A exposures were sufficient to minimize process variance. All things considered, it may be appropriate to use one-year data as a complement to three-year data.

Loss Costs of a Larger Related Class

Actuaries sometimes use the loss costs of a larger, but related class for the complement of credibility. For example, if a company writes very few picture framing stores but writes a large number of art stores, the actuary may choose to use the art store loss costs for the framing store complement of credibility. He may or may not make some adjustments to the art store loss costs to make them more applicable to framing stores. For example, he may wish to adjust for the minor woodworking exposure. Actuaries pricing general liability often use this "base class" (meaning the larger related class in this context) approach. See Lange [6] for an example of this.

A. Complement's Qualities

This approach has qualities similar to the large group complement. It is biased (though the bias and its direction are unknown), and thus it is inaccurate. The more the actuary adjusts the related class loss data to match the loss exposure in the subject base class, the more the bias is reduced. The independence may be less if the factor relating the classes generates high losses for the two classes simultaneously, but the actuary must be careful that this seeming independence is not just a simultaneous shift in the expected losses (which is not prediction error); it is usually an increase in expected losses.

This complement does not fare quite as well as the group mean in other categories. Data is not as readily available for this complement as the group mean, but if the company writes some related class, data should be available and already computed for that class's rates.

The computations involved in adjusting related class data may be more difficult. Any loss cost adjustments will require some extra work. Since there is some relationship between the base class and the related class (they must be related some way by definition), explaining this complement may be easier than explaining the larger group complement.

B. Example

Consider the case of the framing stores. Suppose the actuary wishes to estimate a fire rate for framing stores and already has a well-established rate for art stores. Perhaps the actuary sees that the only visible difference in exposure is the presence of wood and sawdust. He might choose to add a judgmental ten percent of the excess of the fire rate for lumberyards over the fire rate for art stores.

Harwayne's Method

Sometimes individual class data for a given state may be sparse, but the data for that class in other states may be affected by differences in the legal environment, traffic density, or other differences in the other state's overall level of loss costs. Harwayne's method [3] attempts to adjust for those state differences by adjusting the ancillary data from other states for differences in state loss costs levels. It is used extensively for making state class rates for workers compensation.

Harwayne's method uses a specific type of data from a related class. Usually, it is also a case of using loss costs from the larger group. In Harwayne's method, actuaries use countrywide (excepting the base state being reviewed) class data to supplement the loss cost data for each class, but they adjust countrywide data to remove overall loss cost differences between states (or provinces).

The process is as follows. First, the actuary determines what the total countrywide average pure premium would be if the countrywide data had the same percentage mixture of classes (class distribution) as the base state. The result reflects the base state class distribution but probably also reflects the differences in overall loss costs differences between states.

Next, actuaries use that difference in overall loss costs to adjust the countrywide class data to match the base state overall loss cost levels. They determine the ratio of overall state loss costs to overall (all classes) adjusted countrywide loss costs. Then they multiply that ratio times the countrywide base class loss costs to get the complement of credibility.

That is Harwayne's basic method. In a variant form, actuaries may adjust each state's loss costs individually to the base state level to eliminate biases due to different state distributions between classes. (Harwayne used this variant.) Then, actuaries compute the average class complement by weighting the individual state's adjusted loss costs. In another variant, actuaries adjust other states' historical loss ratios by class to match the base state's overall loss ratio. In either variant, the basic principles are the same.

A. Formulas

The simplified formula for Harwayne's method is as follows. Let:

$L_{c,s}$ denote the historical losses for class c in the base state s;

$E_{c,s}$ denote the associated exposure units;

$P_{c,s}$ denote the state pure premium for class c;

$L_{i,j}$ denote the historical losses for an arbitrary class i in some state j;

$E_{i,j}$ denote the associated exposure units; and

$P_{i,j}$ denote the state j pure premium for class i.

Actuaries compute the countrywide pure premium adjusted to the state class distribution. The first step is to compute the "state s" average pure premium (rate)

$$P_s = \sum_i L_{i,s} \Big/ \sum_i E_{i,s}.$$

The next step is to compute the countrywide rates by class

$$P_i = \sum_{j \neq s} L_{i,j} \Big/ \sum_{j \neq s} E_{i,j}.$$

Then, actuaries compute the countrywide rate using the state s distribution of exposures

$$\overline{P} = \sum_i E_{i,s} P_i \Big/ \sum_i E_{i,s}.$$

Thus, the overall pure premium adjustment factor is

$$F = P_s/\overline{P},$$

and the complement of the credibility for class c is assigned to $F \times P_c$.

Harwayne's more complicated (and more accurate) formula replaces the overall adjustments to countrywide data with separate adjustments for each state. That is, actuaries compute state overall means with the base state (s) class distribution

$$\overline{P}_j = \frac{\sum_m E_{m,s}P_{m,j}}{\sum_m E_{m,s}}.$$

Then, they compute individual state adjustment factors

$$F_j = P_s/\overline{P}_j.$$

Next, actuaries adjust each state's class c historical rates using the F_j's. That is, they compute the adjusted "state j" rates

$$P'_{c,j} = F_j P_{c,j}.$$

Then, actuaries weight them with the countrywide distribution among states

$$\text{Complement} = C = \frac{\sum_j E_{c,j}P'_{c,j}}{\sum_j E_{c,j}}.$$

The result is Harwayne's more complicated complement of credibility.

B. Complement's Qualities

This complement has very high statistical quality. Because Harwayne's method uses data from the same class in other states and attempts to adjust for state-to-state differences, it is very unbiased. It is also reasonably accurate as long as there is sufficient countrywide data to minimize process variance. Since the loss costs are from other states, their prediction errors (remaining bias) should be mostly independent of the base class process error in the base state. One exception might be where there is an across-the-board jump in all classes' loss costs in state *s* that alters the adjustment to the state experience level. On the other hand, across-the-board jumps usually flow through into the next year's expected losses, so they are rarely prediction errors.

This complement has a mixed performance on the less mathematical qualities. Data are usually available for this process, but the computations do take time and are complicated. Thankfully, they do bear a much more logical relationship to class loss costs in individual states than unadjusted countrywide statistics. On the other hand, this may be harder to explain because of complexity.

C. *Example*

Consider the data in Table 2. We will use it for Harwayne's method on Class 1 in State S.

For Harwayne's full method, one first computes

$$\overline{P}_T = \frac{100 \times 3.67 + 180 \times 4.00}{100 + 180} = 3.88,$$

and

$$\overline{P}_U = \frac{100 \times 2.22 + 180 \times 4.09}{100 + 180} = 3.42.$$

Then, one computes the state adjustment factors: $F_T = 2.86/3.88 = .737$ and $F_U = 2.86/3.42 = .836$. The next step is to compute the other states' adjusted Class 1 rates: $P'_{1,T} = .737 \times 3.67 = 2.70$ and $P'_{1,U} = .836 \times 2.22 = 1.86$. The last step is to weight the

TABLE 2

DATA FOR HARWAYNE'S METHOD

State s	Class c	Exposure E	Losses L	Pure Premium P
S	1	100	200	2.00
	2	180	600	3.33
	Subtotal	280	800	2.86
T	1	150	550	3.67
	2	300	1,200	4.00
	Subtotal	450	1,750	3.89
U	1	90	200	2.22
	2	220	900	4.09
	Subtotal	310	1,100	3.55
All	1	340	950	2.79
	2	700	2,700	3.86
	Total	1,040	3,650	3.51

two states' adjusted rates with their Class 1 exposures to produce

$$C = \frac{2.70 \times 150 + 1.86 \times 90}{150 + 90} = 2.39.$$

That is Harwayne's complement of the credibility.

Trended Present Rates

In some cases, especially countrywide rate indications, there is no larger group to use for the complement. Then, actuaries use present rates adjusted for inflation (trend) since the last rate change. If there is a difference between the last actuarial indication and the charged rate, actuaries build that in too. Essentially, this test allows some credibility procedure to dampen swings in the historical loss data, yet still forces the manual rates to keep up with inflation. This method was used to develop a second complement of credibility in Harwayne [3].

A. *Formula for the Complement*

The formula for this complement of credibility is

$$T^t \times R_L \times P_L \equiv P_C,$$

where:

T is the annual trend factor, expressed as one plus the rate of inflation. (This will usually be the same as the trend factor in the base indication.)

t is the number of years between the original target effective date of the current rates (not necessarily the date they actually went into effect) and the target effective date of the new rates. (This will often be different than the number of years in the base class trend. It is also usually different than the number of years between the experience period and the effective date of the new rates.)

R_L represents the loss costs presently in the rate manual.

P_L represents the last indicated pure premiums (loss costs).

P_C represents the pure premiums actually being charged in the current manual. This may differ from R_L because P_L and P_C may be taken over a broader group.

B. *Complement's Qualities*

This complement is not as desirable as the previous complements, but sometimes it may be the only alternative. It is less accurate for loss costs with high process variance. Process variance is presumably reflected in last year's rate. That is why it is primarily used for countrywide indications or state indications with voluminous data. It is unbiased in the sense that pure trended loss costs (i.e., with no updating for more current loss costs) are unbiased. Since it includes no process variance, it is mostly independent of the base statistic.

On the less mathematical side, this statistic performs fairly well. Everything an actuary needs to compute it is already in the base rate filing. Therefore, it is available and easy. There is one exception to this. Should one wish to analyze the effects of rate changes the company did not achieve at the level of individual classes, it may require more data than companies typically maintain. This statistic is also very logically related to the loss costs being analyzed. After all, the present rates are based on this complement.

C. Example

Consider the following data for 1996 policy rates:

Present pure premium rate—$120
Annual inflation (trend)—10%
Amount requested in last rate change—+20%
Effective date requested for last rate change—1/1/94
Amount approved by state regulators—+15%
Effective date actually implemented—3/1/94

The complement of the credibility would be

$$C = \$120 \times 1.1^2 \times \frac{1.20}{1.15} = \$152.$$

Rate Change from the Larger Group Applied to Present Rates

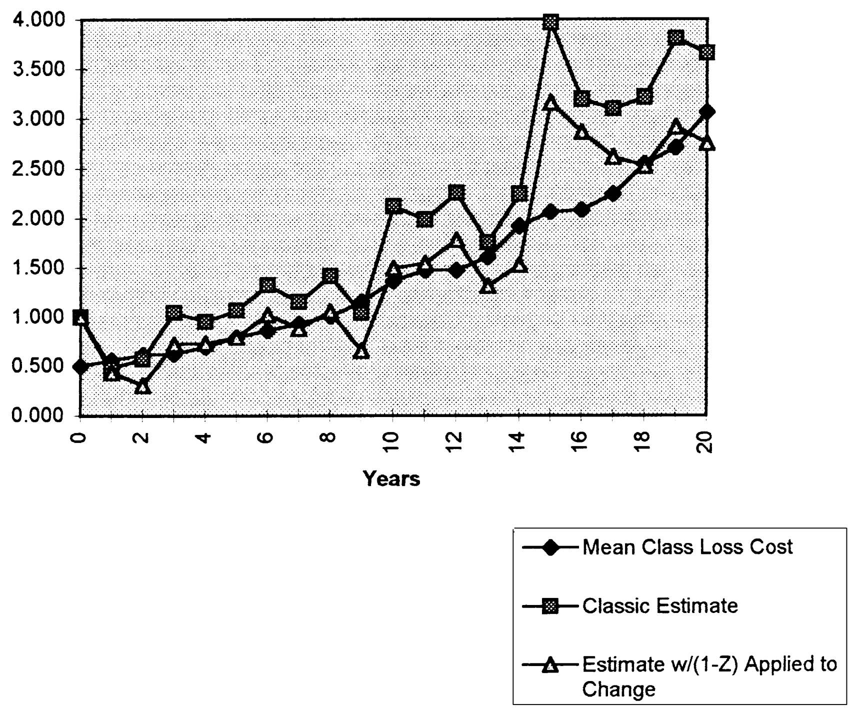
This complement is very similar to the Bayesian complement, but it does not have the substantial (though unknown) bias of the Bayesian complement. That is because the true class expected losses may be very different from the large group expected losses. This larger group test uses the large group rate change applied to present rates, instead of the large group historical loss data (Bayesian complement). Presumably, present rates are an unbiased predictor of the prior (i.e., before changes reflected in current ratemaking data) loss costs. As long as both rates need reasonably small changes, any bias in the overall larger group rate change as a predictor of the class rate change should be small. Also, using large group rate changes instead of straight trend allows the rate to mirror broad changes in loss cost levels that may not be reflected in trend.

A. *Example*

An example may help to illustrate how eliminating bias improves rate accuracy over time. Figure 3 shows group experience after simulation by successively applying $N(10\%, 0.25\%)$ (normal distribution with a mean of 10% and a standard deviation of

FIGURE 3

VALUE BEING ESTIMATED AND ESTIMATORS BY YEAR OF ITERATION



$\sqrt{0.25\%} = 5\%$) trends to a value starting at one. The true class expected losses were set at exactly half the group expected losses each and every year (a slightly unrealistic assumption). The historical class losses have a standard deviation of one-third the true expected losses for the class. A detailed chart of the values actually simulated is in Appendix C.

As Figure 3 shows, the Bayesian complement results in rates with consistent bias above the true expected losses. The complement based on applying group changes to present rates starts too high but very quickly becomes unbiased. It is almost always a better estimate.

B. Formula

This complement has a fairly straightforward formula. It is

$$R_c \times \left\{1 + \frac{(P_g - R_g)}{R_g}\right\},$$

where

R_c is the present manual loss cost rate for class c;

P_g is the present indicated loss cost rate for the entire group of classes; and

R_g is the present average loss cost rate for the entire group.

C. Complement's Qualities

This complement is a significant improvement over the Bayesian complement. It is largely unbiased. If the year-to-year changes are fairly small, it is very accurate over the long term (though often not as accurate as Harwayne's complement in practice). Also, since the complement is based on group variance, it is fairly independent. Since this complement requires a group rate change that must be calculated anyway, it is both available and easy to compute. Since it includes the present rate, it has a logical relationship to the class loss costs.

D. Numerical Example

Consider the data in Table 3. Using this data, the complement for Class 1 would be

$$\$750 \times (1 + (\$750 - \$782)/\$782) = \$719.$$

TABLE 3

DATA FOR APPLYING GROUP RATE CHANGE TO CLASS DATA

Class	Exposure	Losses	Indicated Pure Premium	Present Pure Premium	Underlying Losses
1	100	\$ 70,000	\$700	\$750	\$ 75,000
2	200	\$180,000	\$900	\$920	\$184,000
3	300	\$200,000	\$667	\$700	\$210,000
Total	600	\$450,000	\$750	\$782	\$469,000

Notes:—Both indicated and present pure premiums are at current cost levels.
—Underlying losses are extension of exposures by present premiums.
—Total present premium is ratio of total underlying to total exposures.

Using Competitors' Rates

New companies and companies with small volumes of data often find their own data too unreliable for ratemaking. Their actuaries use competitors' rates for the complement of credibility. They rationalize that if the competitor has a much larger number of exposures, the competitors' statistics have less process error. An actuary in this situation must consider that manual rates reflect marketing considerations, judgment, and the effects of the regulatory process, as well as loss cost statistics. Thus, competitors' rates have significant inaccuracies. They are also affected by differences in underwriting and claim practices between the subject company and its competitors. Competitors' rates probably have systematic bias as well. The actuary will often attempt to correct for those differences by using judgment, but those corrections and their size and direction may generate controversy. However, using competitors' rates may be the best viable alternative in some situations.

A. Complement's Qualities

Competitors' rates generally have prediction errors that are independent of the subject class loss costs. That is because their errors stem more from inter-company differences that are unrelated to subject company loss cost errors. They are often available from regulators, although the process may take some work. They are harder to use since they usually must be posted manually.

Regulators may complain that competitors' rates are unrelated to the subject company's own loss costs, but, if the company's own data is too unreliable, competitors' rates may be the only alternative.

B. Example

Consider a competitor's rate of \$100. Suppose a Schedule P analysis suggests the competitor will run a 75% loss ratio. Further, suppose one's own company has less underwriting expertise. Suppose one's own company expects ten percent more losses per exposure than the competitor. The complement would be $\$100 \times .75 \times 1.1 = \$83$.

Loss Ratio Methods

This paper discussed all the previous complements in terms of pure premium ratemaking. All the methods except the loss costs from a larger related class and competitors' rates also work with loss ratio methods. All the actuary needs to do is consider earned premium to be the exposure base. Replacing the exposure units with earned premium yields usable formulas.

4. SPECIFIC EXCESS RATEMAKING

Complements for excess ratemaking are structured around the special problems of excess ratemaking. Since specific excess policies only cover losses that exceed a very high per claim deductible (attachment point), there usually are very few actual claims in the historical loss data. Hence, actuaries will try to predict the volume of excess loss costs using the loss costs below the attachment point. For liability coverages, the loss development of excess claims may be very slow. That accentuates the sparsity of ratemaking data. Also, the inflation inherent in excess layers is different (usually higher) than that of total limits losses (see [2]). Since the "burning cost" (historical loss data) is an unreliable predictor, the statistic that receives the complement of credibility is especially important.

This paper will discuss four methods for determining excess credibility complements: using increased limits factors; derivation from a lower limits analysis; analysis reflecting the policy limits sold by the insurer; and the use of fitted curves.

Increased Limits Factors

When loss costs for the first dollar coverage up to the insurer's limit of liability are available, actuaries may use an increased limits factor approach. Actuaries multiply the "capped" loss costs by the increased limits factor for the attachment point plus the limit of liability. Then, they divide the result by the increased limits factor for the attachment point. That produces an estimate of loss costs from the first dollar up to the limit of liability. Then actuaries subtract the loss costs below the original attachment point. The remainder estimates the expected losses in the specific excess layer.

Actuaries use a variety of sources for increased limits factors. The Insurance Services Office publishes tables of estimated increased limits factors for products, completed operations, premises and operations liability, and manufacturers and contractors liability. The National Council on Compensation Insurance publishes excess loss pure premium factors that allow actuaries to compute increased limits factors for workers compensation. The *Proceedings of the Casualty Actuarial Society* may contain tables of property losses by ratio to probable maximum loss. Those can be converted to increased limits factors by using the factors for the ratio of the attachment point to the probable maximum loss (and the ratio of the attachment point plus the limit of liability to the probable maximum loss). Actuaries may compute increased limits factor tables using a company's own data (if the company sells enough specific excess). Actuaries may modify industry tables to reflect their company's loss cost history. Competitor prices may allow actuaries to estimate increased limits factors for obscure coverages. Actuaries would consider the ratios between competitor prices for various limits of liability.

A. Formula

The formula is as follows:

$$(P_A \times \mathrm{ILF}_{A+L} \equiv \mathrm{ILF}_A) - P_A \qquad \text{or} \qquad P_A \times \left(\frac{\mathrm{ILF}_{A+L}}{\mathrm{ILF}_A} - 1 \right).$$

In this case

P_A is the loss cost capped at the attachment point A (by convention, it is usually premium capped at the attachment point multiplied by the loss ratio the actuary projects);

ILF_{A+L} is the increased limits factor for the sum of the attachment point and the limit of liability L; and

ILF_A is the increased limits factor for the attachment point.

B. Complement's Qualities

As long as the insured being rated has a different loss severity distribution than the norm, this complement contains bias. In that likely event, it is also inaccurate. Further, it is not based on the individual insurer's own data. Actuaries must weigh those facts against the greater inaccuracy of burning cost statistics. When pricing specific excess insurance, actuaries must usually settle for less accurate and potentially biased estimators. That is because there are few highly accurate estimators available.

This complement's error is mostly independent of the burning cost error. This complement tends to contain a systematic (parameter-type) error rather than the process error inherent in burning cost. It is dependent on burning cost only to the extent that both are highly related to the losses below the attachment point.

Very few specific excess statistics are readily available or easy to compute. Considering the alternatives, the availability of industry increased limits tables (in the United States) makes this the easiest specific excess complement to compute. Also, the data for this test is available as long as premiums or loss costs capped at the attachment point are available.

The excess loss cost estimates this complement produces are more logically related to the losses below the attachment point than those above. That can be controversial with customers. It is a common problem with excess insurance pricing. However, burning cost is unreliable in isolation. Further, that problem is common to all excess complements.

C. Example

Consider Table 4.

TABLE 4

INCREASED LIMITS FACTORS

Limit of Liability	Increased Limits Factor
\$ 50,000	1.00
\$ 100,000	1.50
\$ 250,000	1.90
\$ 500,000	2.50
\$1,000,000	3.50

Suppose one wishes to estimate the layer between \$500,000 and \$1,000,000 given losses of \$2,000,000 capped at \$500,000 each.

The complement using increased limits would be

$$C = \$2{,}000{,}000 \times \left(\frac{3.5}{2.5} - 1\right) = \$800{,}000.$$

Lower Limits Analysis

Sometimes the historical losses near the attachment point may be too sparse to be reliable. So, an actuary may wish to base his complement on basic limits losses, where the basic limit is some low loss cap. In this case, the formula is almost exactly the same as that of the previous analysis. The actuary simply multiplies the historical basic limits losses by a difference of increased limits factors. Specifically, he multiplies basic limits losses by the difference between the increased limits factor for the attachment point plus the limit of liability and the increased limits factor for the attachment point. The result is the complement of credibility.

A. Formula

The formula is

$$P_b \times (\mathrm{ILF}_{A+L} - \mathrm{ILF}_A),$$

where

P_b represents the historical loss data with each loss capped at the basic limit b; and

ILF_{A+L} and ILF_A are as before.

Alternately, the actuary might choose to use a low capping limit d that is different from the basic limit underlying the increased limits table. Then, the formula would be

$$P_d \times \left(\frac{\mathrm{ILF}_{A+L}}{\mathrm{ILF}_d} - \frac{\mathrm{ILF}_A}{\mathrm{ILF}_d}\right).$$

B. Complement's Qualities

Actuaries must usually use judgment to decide whether loss costs capped at the attachment point or some lower limit are more accurate and unbiased predictors of the excess loss. Estimates made using the lower cap are more prone to bias. That is because using losses far below the attachment point accentuates the impact of variations in loss severity distributions. On the other hand, when there are few losses near the attachment point, historical losses limited to the attachment point may be unreliable and inaccurate predictors of future losses. Consequently, using higher loss caps may produce even more inaccurate predictors of excess losses.

By an argument similar to that of the previous test, this complement's errors are mostly independent of those of burning cost.

Generally, this complement features more available statistics and a slightly greater complexity. Basic limits losses may need to be coded for statistical reporting. They may be readily available for this complement. On the other hand, since insureds and reinsureds may place a higher priority on accounting for the total losses they retain, they are not as available as losses limited to the attachment point. The calculations are no more complicated for basic limits analysis than retained limits (attachment point) analysis. The only exception would be where actuaries must manually compute the loss costs between basic limits and the attachment point from a claims list.

As with the straight increased limits factor approach, this complement may generate controversy with customers because it is not based on actual burning cost.

C. Example

Suppose an actuary is estimating the losses between $500,000 and $1,000,000, and the actuary feels he can only rely on historical losses limited to $100,000. The estimated historical losses limited to $100,000 are $1,000,000. Then, using the increased limits factors from Table 4, he would calculate the complement at

$$C = \$1{,}000{,}000 \times \left(\frac{3.5}{1.5} - \frac{2.5}{1.5}\right) = \$666{,}667.$$

Limits Analysis

The previous approaches work well when losses limited to a single capping point are available, but sometimes they are not. Reinsurance customers generally sell policies with a wide variety of policy limits. Some of the policy limits will fall below (not expose) the attachment point. Some limits may extend beyond the sum of the attachment point and the reinsurer's limit of liability. In any event, each subject (first dollar) policy limit will require its own increased limits factor.

Therefore, actuaries analyze each limit of coverage separately. Generally, they assume that all the limits will experience the same loss ratio. Actuaries multiply the all limits combined (total limits) first dollar loss ratio times the premium in each first dollar limit to estimate the loss costs for that limit. Then, actuaries perform an increased limits factor analysis on each first dollar limit's loss costs separately.

A. Formula

The formula is as follows:

$$LR_T \times \sum_{d \geq A} W_d \frac{(\mathrm{ILF}_{\min(d,A+L)} - \mathrm{ILF}_A)}{\mathrm{ILF}_d},$$

where

LR_T is the estimated total limits loss ratio;

The "d" are all the policy limits the customer sells that exceed the attachment point ($\geq A$);

Each W_d is the premium volume the customer sells with policy limits of d; and

The ILF's have the same meaning as previously.

B. Complement's Qualities

Actuaries use this approach because it may be all that is available. Reinsureds may be unable to split their historical losses any more finely than losses that would have pierced the cover in the past versus all other losses. Since the total limits loss costs (which are almost always available, at least as an estimate) may include claims beyond the layer, it may be impossible to calculate losses limited to the attachment point. In any event, if some of the reinsured's policy limits are below the attachment point, they do not expose the layer and should be excluded from an increased limits factor calculation. Therefore, this may be the only available complement with low bias.

It is biased and inaccurate to the same extent that the previous increased-limits-factor-based complements were biased or inaccurate. It is more time-consuming to compute (unless the alternative is computing limited claims from claims lists). Further, it generates the same controversy as the other methods since it is not the same as the actual burning cost.

C. Example

Suppose an actuary is estimating the losses in a layer between \$250,000 and \$500,000. Breakdowns of losses by size are unavailable, but the actuary believes the loss ratio of the customer's entire business to be 70%. He does have a breakdown of premiums by limit of liability. Using that breakdown and the increased limits factors from Table 4, he computes the losses in the layer (see Table 5). He estimates the losses in the layer at \$86,400.

Fitted Curves

The problem with most of the previous complements is that they do not give special attention to the claims above or near the attachment point. As a result, they miss differences in loss severity distributions between insureds, but of course that must

TABLE 5

LIMITS ANALYSIS FOR LAYER BETWEEN $250,000 AND $500,000

Limit of Liability	Premium	Times 70% Loss Ratio	Increased Limits Factor	% in Layer	Loss in Layer
$ 250,000	$ 600,000	$420,000	1.9	0.00%	$ —
$ 500,000	$ 300,000	$210,000	2.5	24.00%	$50,400
$1,000,000	$ 300,000	$210,000	3.5	17.14%	$36,000
Total	$1,200,000	$840,000			$86,400

be counterbalanced against the fact that individual insureds' large claims histories usually lack credibility.

By fitting a family of loss severity curves to the distribution, actuaries make the most of the large claim data that is available. If the loss history shows no claims beyond the attachment point but many claims that are very near to the attachment point, a fitted curve will usually reflect that and project high loss costs in the subject layer. On the other hand, if there are few large claims close to the attachment point, the fitted curve will project low loss costs for the layer.

The details of how to fit curves are beyond the scope of this paper (see [4]), but it will provide an outline of how to use fitted curves in practice. After fitting and trending the curve, an actuary will use the curve to estimate what percentage of the curve's total loss costs lie in the subject layer. He may do this by evaluating the difference between the limited mean function $\int_{-\infty}^{L} xf(x)\,dx + (1+F(L))L$ at the attachment point and the attachment point plus the limit of liability. He would then divide the result by the total mean (or the mean when claims are capped at the typical policy limit) to get the percentage of the total loss costs that lie in the layer. Multiplying that percentage by the total claims cost yields the estimate of claim costs in the layer (for details, see [4]).

Of course, excess values from curve fits need extensive loss development as do burning costs. Actuaries may use excess loss development factors such as those published by the Reinsurance Association of America, or they may triangulate the fitted loss costs.

A. *Complement's Qualities*

This method is generally unbiased (except for concerns that the general shape of a family of curves may predispose the results for the family to estimated costs in particular layers that are either too high or too low). When there are few large claims, it is more accurate than burning cost. It is often more accurate than increased limits factors simply because it does a better job reflecting any general tendency towards large or small claims. On one hand, fitting curves forces data into a mold that may not fit the data. The actual loss severity distribution will almost certainly look very different from all the members of the family of curves. This "super-parameter" risk introduces error of its own. The "super-parameter" risk is totally distinct from process risk, and that makes the complement mostly independent. On the other hand, the presence or absence of burning cost claims in the layer can influence the curve fit heavily. Thus, this complement has somewhat more dependent (relative to burning cost) errors than the increased limits approaches.

Data availability and computational complexity are problems here. To fit a loss severity curve, an actuary must either use a detailed breakdown of all the claims into size ranges or use a listing of every single claim. Usually, that data is not readily available. Further, the processing required to fit curves requires fairly complex mathematical calculations. Besides the fact that complex calculations require special personnel, the complexity makes the results difficult to explain to lay people.

On one hand, this complement uses more of the insured's own data in and near the layer than any other excess complement. On the other hand, its complexity may make that fact difficult to communicate.

5. SUMMARY

The complement of the credibility deserves at least as much actuarial attention as the base statistic (historical loss data). Actuaries owe special attention to its unbiasedness and accuracy. In some cases, interdependence must be avoided. Further, any actuarial method must be implemented using reasonable labor on available statistics. Meeting those qualities may require statistics that make less explainable sense to lay people, but explainability must be considered, too.

This paper has detailed several statistics that are commonly used for the complement of credibility. Their use improves many actuarial projections considerably.

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APPENDIX A

THE ERROR IN CREDIBILITY ESTIMATES

This appendix will show that the error in an optimum credibility weighted estimate is

$$\Phi(x_1,x_2) = \frac{\tau_1^2\tau_2^2(1-\rho^2)}{\tau_1^2+\tau_2^2-2\rho\tau_1\tau_2}.$$

The proof involves three equations from Boor [1]:

$$\Phi(x_1,x_2) = Z\tau_1^2 + (1-Z)\tau_2^2 + (Z^2-Z)\delta_{1,2}^2 \qquad \text{(A.1)}$$

(p. 182, the simplified error of the credibility-weighted estimate);

$$Z = \frac{\tau_2^2-\tau_1^2+\delta_{1,2}^2}{2\delta_{1,2}^2} \qquad \text{(A.2)}$$

(p. 183, the formula for the optimum credibility); and

$$\delta_{1,2}^2 = \tau_1^2+\tau_2^2-2\mathrm{Cov}(x_1,x_2) \qquad \text{(A.3)}$$

(p. 179, the formula relating $\delta_{1,2}^2$ to the correlation).

In this case, τ_1, τ_2, and ρ are the same as they were in the body of the paper (the prediction errors of burning cost and the credibility complement and their correlation); $\Phi(x_1,x_2)$ is the minimum possible average squared prediction error from credibility weighting burning cost (x_1) and the credibility complement (x_2); and $\delta_{1,2}^2$ is the average squared difference between burning cost and the credibility complement.

Simple algebra on (A.1) allows one to pull out several terms that will create the numerator of (A.2).

$$\Phi(x_1,x_2) = -Z(\tau_2^2-\tau_1^2+\delta_{1,2}^2)+\tau_2^2+Z^2\delta_{1,2}^2$$

$$= -Z^2 2\delta_{1,2}^2+\tau_2^2+Z^2\delta_{1,2}^2 = \tau_2^2-Z^2\delta_{1,2}^2.$$

Using the definition of Z (Equation A.2) once again with some algebra gives

$$= \tau_2^2 - \frac{(\tau_2^2 - \tau_1^2 + \delta_{1,2}^2)^2}{4\delta_{1,2}^2}.$$

Using (A.3) and the relationship between the covariance and correlation gives

$$\begin{aligned}
&= \tau_2^2 - \frac{(\tau_2^2 - \tau_1^2 + \tau_1^2 + \tau_2^2 - 2\mathrm{Cov}(x_1, x_2))^2}{4\delta_{1,2}^2} \\
&= \tau_2^2 - \frac{(\tau_2^2 - \tau_1^2 + \tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2)^2}{4\delta_{1,2}^2} \\
&= \tau_2^2 - \frac{(2\tau_2^2 - 2\rho\tau_1\tau_2)^2}{4\delta_{1,2}^2} \\
&= \tau_2^2 - \frac{(\tau_2^2 - \rho\tau_1\tau_2)^2}{\delta_{1,2}^2}.
\end{aligned}$$

Then, more algebra gives

$$\begin{aligned}
&= \tau_2^2 \left(1 - \frac{(\tau_2 - \rho\tau_1)^2}{\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2}\right) \\
&= \frac{\tau_2^2}{\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2} \times (\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2 - \tau_2^2 + 2\rho\tau_1\tau_2 - \rho^2\tau_1^2) \\
&= \frac{\tau_2^2}{\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2} \times (\tau_1^2 - \rho^2\tau_1^2) \\
&= \frac{\tau_2^2\tau_1^2(1 - \rho^2)}{\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2},
\end{aligned}$$

and that is the error formula we sought to prove.

APPENDIX B

FOR CORRELATIONS NEAR UNITY, CREDIBILITY IS NEGATIVE

This appendix will show that whenever the correlation exceeds the point of maximum error, the credibility of one statistic is negative. To explain this principle, reviewing the graph of error by correlation will help.

As one can see in Figure 1, the prediction error is initially minimized when the correlation is negative. Then it increases until the error is maximized. Then the error decreases again beyond that maximum point. This section will show that the one credibility is actually negative beyond that maximum point.

As it happens, when $\tau_2 \geq \tau_1$, that maximum point is where $\rho = \tau_1/\tau_2$. And all correlations beyond that yield negative credibility for the complement. Alternately, when $\tau_1 \geq \tau_2$, $\rho = \tau_2/\tau_1 \leq 1$ is the point of maximum prediction error. Beyond that, the burning cost's credibility will be negative. But, this appendix must prove that.

It is easy to show that Φ has a maximum where $\rho = \tau_1/\tau_2$. One need only note that the function $\Phi(\rho)$ has a maximum where

$$0 = \frac{\partial\Phi}{\partial\rho} = \frac{2\rho(\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2) - 2\tau_1\tau_2(1-\rho^2)}{(\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2)^2}$$

(using the definition of $\Phi(\rho)$ from Appendix A). Using some algebra, that is equivalent to

$$0 = 2\rho\tau_1^2 + 2\rho\tau_2^2 - 4\rho^2\tau_1\tau_2 - 2\tau_1\tau_2 + 2\rho^2\tau_1\tau_2; \qquad \text{or}$$

$$0 = (\tau_1 - \rho\tau_2)(\tau_2 - \rho\tau_1).$$

So, the maximum is at τ_1/τ_2 or τ_2/τ_1, whichever is less than one.

To show that correlations beyond that maximum point result in negative credibilities, it suffices to show that they fulfill Boor's condition for negative credibility ([1], p.183):

$$\tau_2^2 \geq \tau_1^2 + \delta_{1,2}^2.$$

But that follows directly from Boor's equation relating the credibility and covariance ([1], p. 179). That is, since

$$\delta_{1,2}^2 = \tau_1^2 + \tau_2^2 - 2\mathrm{Cov}(x_1, x_2) = \tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2,$$

Boor's condition is equivalent to

$$\tau_2^2 \geq \tau_1^2 + \tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2.$$

Or,

$$\rho \geq \frac{\tau_1}{\tau_2};$$

that is, Boor's condition for negative credibility is fulfilled and fulfilled only for ρ beyond the point of maximum error. So, the correlations near unity yield negative credibilities.

APPENDIX C

DATA FOR EXAMPLE APPLYING COMPLEMENT TO GROUP RATE CHANGE

(a) Year	(b) Group N(.1,.0025) Trend	(c) Group Loss Cost	(d) Mean Class Loss Cost	(e) Class with Process Variance $N(0,((d)/3)^2)$	(f) Classic Z	(g) Classic Estimate	(h) Estimate with $(1-Z)$ Applied to Change
0	0.115	1.000	0.500	0.188	0.692	1.000	1.000
1	0.101	1.115	0.558	0.256	0.692	0.481	0.438
2	0.021	1.228	0.614	0.825	0.692	0.572	0.306
3	0.107	1.254	0.627	0.695	0.692	1.044	0.724
4	0.137	1.389	0.694	0.782	0.692	0.954	0.731
5	0.091	1.579	0.790	1.037	0.692	1.065	0.792
6	0.082	1.723	0.862	0.747	0.692	1.324	1.025
7	0.082	1.865	0.932	1.034	0.692	1.153	0.885
8	0.143	2.017	1.009	0.468	0.692	1.418	1.056
9	0.188	2.305	1.153	1.759	0.692	1.039	0.659
10	0.075	2.739	1.369	1.393	0.692	2.119	1.498
11	0.000	2.945	1.472	1.653	0.692	1.988	1.545
12	0.093	2.946	1.473	0.992	0.692	2.256	1.782
13	0.192	3.220	1.610	1.516	0.692	1.753	1.315
14	0.075	3.839	1.919	3.501	0.692	2.244	1.527
15	0.009	4.128	2.064	2.358	0.692	3.966	3.162
16	0.077	4.167	2.083	2.213	0.692	3.193	2.862
17	0.136	4.487	2.244	2.225	0.692	3.096	2.616
18	0.062	5.096	2.548	2.733	0.692	3.214	2.525
19	0.133	5.411	2.705	2.394	0.692	3.806	2.917
20	0.093	6.128	3.064	2.819	0.692	3.654	2.752

Notes:—Column (g) is [(f)*(previous (e)) + (1 – (f))*(previous (c))]* (1.10).
—Column (h) is {(f)*[previous (e) + (1 – (f))*[previous (h)*[1 + (previous (c)–1.1*next previous (c))/previous (c)]]}*(1.10).

PRICING TO OPTIMIZE AN INSURER'S RISK-RETURN RELATION

DANIEL F. GOGOL, PH.D.

Abstract

It is appealing to estimate loss discount rates and risk loads for categories of an insurer's premium by using the categories' contributions to surplus variation. However, as will be explained, there has been a theoretical obstacle to this approach.

This paper presents a method that overcomes the obstacle. It produces a surprisingly simple result. The risk load (in dollars) of a category is proportional to the covariance of the yearly return on surplus with the category's yearly profit.

The paper analyses the use of the above result to optimize an insurer's risk-return relation. Some examples of computations of risk loads and risk-based discount rates for losses are presented. The relationship between the method of this paper, the Capital Asset Pricing Model, and several other models is discussed.

1. INTRODUCTION

A few years ago, a Nobel Prize was awarded to Harry Markowitz [10] for developing a method of producing a diversified portfolio of stocks with the optimal relationship between expected rate of return and expected variability. In other words, Markowitz showed how to maximize the expected rate of return for a fixed amount of expected variability and, alternatively, how to minimize the variability at a fixed rate of return. Markowitz's method has been widely used by large investors because of their desire to lower the variability of their results.

Insurance company managers are also interested in reducing variability. Taking steps to reduce risk helps a company with its Best's rating and also increases the security of its employees and its policyholders. These actions help in attracting good business and retaining good employees, and produce increased profitability in the long run. Therefore, insurers generally require a greater profit margin on a risk with greater volatility.

Suppose that an insurer expects to write a certain volume and mix of business in the next year, and that the insurer has a certain target profit. The method presented in this paper produces a risk load for each risk such that the total expected profit equals the target and each risk is equally advantageous to the insurer in the following sense. If the insurer can charge more than the indicated risk load for any type of risk, then by increasing the proportion of that type of risk in the total book of business, the insurer can increase the expected return without increasing the surplus variability. Conversely, if the insurer charges less than the indicated price, increasing the proportion of that type of risk will decrease the expected return if variability is left constant.

The term "risk load" is sometimes given a different meaning than it is given above. Other meanings of the term include:

1. The risk load that a customer is willing to pay. This may be based on the market, or on the risk aversion of the customer.
2. The risk load that an underwriter desires, based on the possible effect that a contract may have on the total results of the contracts he or she has underwritten, or on the results of a profit center within the company.

The method presented here produces an indicated price for each risk by discounting losses and loss adjustment expenses at a risk-based rate and then adding a risk load as well as other expenses. As will be explained later, the risk loads and discount rates are produced by allocating surplus to categories of underwriting and loss reserves. This allocation is based on the contribution of these categories to surplus variability. The measure of surplus variability used in this paper is the "standard deviation of surplus," which is defined below.

Assume that, at each time t, there is an estimated value of surplus. Let the random variable X represent the estimated value one year in the future. The "standard deviation of surplus" is defined as the standard deviation of X.

A problem with allocating surplus based on each category's contribution to surplus variability is that the effect of a category on the standard deviation of surplus *cannot* be estimated by simply estimating the standard deviation of surplus with and without the category, and then taking the difference. The explanation of this is as follows. (See Gogol [7].)

The standard deviation of surplus equals the standard deviation of the sum of the effects on surplus of all the categories of underwriting, loss reserves, other liabilities, assets, and other sources of income and expense. Suppose those categories are arranged in a list. Suppose the effect of each category on the total standard deviation is defined as the difference between the standard deviation of the sum of the categories up to and including that category on the list, and the standard deviation of the sum of the categories prior to it on the list. The sum of all these "effects" equals the total standard deviation, but the effect of a particular category depends on the order of the list. (Suppose, for example, that there is a list of two independent categories each with standard deviation σ. The standard deviation of the sum is $2^{.5}\sigma$. The effect of the first category in the list is σ, and the effect of the second is $2^{.5}\sigma - \sigma$.)

This dependence on the order in which the categories are listed has been considered a barrier to using contribution to surplus variability to estimate required risk loads. This paper will propose a solution. The following quotations from Venter [12] give an interesting description of the problem.

In 1953, Harry Markowitz developed a way of selecting optimal holdings for each available security if you were clear about your preferred mean-variance trade-off. This has been applied to optimal line mix strategies for insurers as well.

It's tempting for actuaries to invent (or re-invent) the Mean-Variance Pricing Model (MVPM).

Presumably the change in variance of your whole portfolio of risks or securities is more important than that of the new entrant by itself.

MVPM could be applied to the portfolio with and without the new entrant, whose price then becomes the difference. But then the order of entry will influence the price, which it should not. Or you could estimate in advance the make-up of the portfolio and then pro-rate to each unit a credit based on the reduction in variance achieved by the combination. The mind boggles. Besides needing a fair way to allocate credits, which this theory does not provide, any difference from the predicted result will give the wrong price overall. Because of covariance, MVPM does not seem usable for pricing individual risks in a portfolio.

2. ESTIMATING RISK-BASED PREMIUM

A. *Return on Allocated Surplus*

The surplus considered in this paper is a type of adjusted surplus, using the market value of assets and a risk-based discounted value for loss reserves.[1] Statutory liabilities such as equity in the unearned premium reserve are included in the surplus. The value of the assets necessary to offset the discounted loss reserve lia-

[1] In this paper "loss reserves" will mean loss and loss adjustment expense reserves, net of ceded losses. "Earned premium" will refer to premium net of cessions.

bility is considered here to be greater than the discounted value of loss reserves at the "risk-free" interest rate (see Butsic [3]). This is because it would be necessary to pay an insurer more than this amount, as a reward for risk, in order for them to be willing to assume this liability. By using a lower discount rate to determine the loss reserve liability, the following is expected to occur. In the course of a year, the value of the offsetting assets is expected to grow at a greater rate of interest than was used to discount the liability, providing a profit for the risk of having the liability.

Suppose that each category of loss reserves is considered to be offset by an amount of assets that is equal to the risk-based discounted value of the reserves. The expected effect on surplus one year in the future of a category of discounted loss reserves and offsetting assets equals the accumulated value of the assets after one year of reserve payouts, minus the discounted value of the remaining reserves and the tax effects of the assets and liabilities.

The expected effect of a category of underwriting on the surplus one year in the future equals the effect of the premium minus the effect of the corresponding paid losses, discounted loss reserves, expenses, and taxes.

Suppose an amount of surplus is allocated to a category of underwriting, or to a category of loss reserves and offsetting assets. Then the expected return on the allocated amount during the year is the after-tax investment gain on it plus the expected effect of the category on surplus. The rate of return is the return divided by the amount of surplus.

B. Method of Allocation

Just as there is a probability distribution of the amount of surplus one year in the future, there are probability distributions of the effects on surplus of each category of underwriting, or of each category of discounted loss reserves and offsetting assets. A basic part of the method of this paper is the idea that the appropriate amount of surplus to allocate to a category of underwriting, or to a category of discounted loss reserves and matching assets, is equal to total surplus times

$$\frac{\text{Cov(total surplus, effect of category on total surplus)}}{\text{Var(total surplus)}}.$$

It will be shown below, by Theorem 1, that in a certain sense the above covariance of a category with surplus is proportional to the category's effect on surplus variability. It is shown by Theorem 2 that if surplus is allocated to each category of underwriting according to the above formula, and the appropriate risk-based loss discounting rate is used, the following is true. Each category will improve the risk-return relation of the insurer if, and only if, its rate of return on allocated surplus is greater than the rate of return on the total amount of surplus allocated to underwriting.

It is a property of covariance that the covariance with surplus of a sum of categories equals the sum of the covariances. Therefore, the surplus allocated to a sum of categories is the same whether the surplus is allocated based on the covariance of the sum, or allocated to each individual category based on its covariance. This would not be true if surplus were allocated in proportion to the standard deviation or variance of a category's effect on surplus.

Thus, the amount of surplus allocated to a category is independent of how finely the categories are subdivided. For example, the amount of surplus allocated to private passenger auto does not depend on whether it is considered to be one category or whether it is split into private passenger auto liability and private passenger physical damage.

Surplus variability is caused not only by underwriting and by loss reserves and offsetting assets, but also by other assets. If surplus is allocated to all sources of surplus variability, and these sources are referred to as "categories" 1 through n, then

$$\sum_{i=1}^{n} \text{Cov(surplus, effect of category } i \text{ on surplus)}$$
$$= \text{Cov}\left(\text{surplus, effect of}\left(\sum_{i=1}^{n} \text{category } i\right) \text{ on surplus}\right)$$
$$= \text{Cov(surplus, surplus)} = \text{Var(surplus)}.$$

Therefore, the proportions of surplus allocated to the categories sum to unity.

C. Risk-Based Underwriting Margin and Discount Rate

To understand how to apply the method of this paper, it is helpful to consider the following questions:

1. What risk-based discount rate should be used for loss reserves?
2. How much surplus should be allocated to loss reserves, and how much to underwriting?

Suppose the insurer's loss reserves are discounted, both at the beginning and end of the year, at a discount rate d. Suppose that, with this rate d, surplus is allocated by the above covariance formula to underwriting and to discounted loss reserves and offsetting assets. Lastly, suppose that the rate of return on allocated surplus from underwriting and from discounted loss reserves and offsetting assets is called rate R.

Call the amounts of surplus allocated to underwriting and to discounted loss reserves and offsetting assets S_u and S_r, respectively. It was mentioned above that the surplus allocated to a sum of categories by the covariance method is equal to the sum of the amounts allocated to the individual categories. Suppose for the moment that, for each category of loss reserves and offsetting assets, the discount rate d produces the same rate of return on allocated surplus. Since the sum of the amounts of surplus allocated to each category equals S_r, this rate of return equals R.

Suppose that, for some underwriting category c, the rate of return on the surplus allocated to the category, using the discount rate d, is R. Thus, the premium not only provides a rate of return on allocated surplus equal to the rate of return on S_r and S_u, but also provides for the offsetting assets for its loss reserves at the end of the year. Assuming that the required discount rate remains the same, these reserves and offsetting assets are expected to produce a rate of return R on allocated surplus in each following year. This is a key point, since it means that the expected effect on surplus of the loss reserve runoff from Category c neither helps nor hurts the insurer's risk-return relation.

It will be shown by Theorem 2 that in a certain sense the covariance method allocates surplus in proportion to a category's effect on surplus, and it follows that the Category c neither helps nor hurts the insurer's risk-return relation. This explains what conditions a category or contract must satisfy in order to help optimize that relation.

A discount rate d with the above properties may be found by iteration, as outlined below. (See Example A in Section 4 for additional explanation.) Suppose the insurer expects to earn a given amount of premium in the coming year, with a given expected loss ratio and expense ratio. Certain estimates are made relating to loss payout rates, loss reserve variability, asset variability, underwriting variability, and various correlations, and an initial value of the discount rate is selected.

The value of the discount rate affects the estimated amount of surplus as well as:

1. the covariance with surplus of the total effect on surplus of discounted loss reserves and offsetting assets;
2. the covariance with surplus of the total effect on surplus of all underwriting categories;
3. the total amounts of surplus allocated by the above two covariances; and
4. the rates of return on the above two amounts of surplus.

Iteration is used to find a discount rate d that makes the above two rates of return equal.

It isn't actually necessary to assume that a single discount rate d produces the same rate of return on the amounts of surplus allocated to each category of loss reserves and offsetting assets. The indicated discount rate may vary for different categories, and thus it may be appropriate to use different discount rates in estimating the required risk-based premiums for different underwriting categories. This would require a more complicated iteration than the one described above. This may not be preferable from a practical point of view. The need for a great deal of judgment in estimating covariances with surplus will be discussed further in Section 3E.

The theoretical significance of the allocation method is indicated by the following two theorems. The proofs[2] are in the Appendix.

Theorem 1

Using any discount rates for each category of loss reserves and for each category of underwriting, suppose a pro rata share of $1/n$ of each category of one year underwriting results, loss reserves and offsetting assets, and other assets, liabilities, expenses, and sources of income affecting surplus is added to a list, and

[2]It will be assumed in the proofs that the covariance of a category with surplus is not zero. The case in which the covariance equals zero will be left to the reader.

this is done n times. The limit as n approaches infinity of the total of the n effects of a category on the standard deviation of surplus,[3] divided by the total standard deviation of surplus, equals

$$\text{Cov(surplus, effect of category on surplus)/Var(surplus)}.$$

Theorem 2

Suppose that an insurer can charge more premium for a category of underwriting than the required risk-based premium described above. Then, by increasing the proportion of that category in the total book of business, the insurer can increase the expected return without increasing surplus variability. Specifically, there is some epsilon such that the expected return on surplus will increase if the following are assumed.

a. The premium for the category is increased by less than epsilon.

b. The expected underwriting return and the standard deviation of underwriting return for the category increase by the same proportion as the premium, and the correlation of its return with surplus is unchanged.

c. The rest of the insurer's premium is reduced by an amount such that total surplus variance remains the same.

d. The expected underwriting return and standard deviation of underwriting return for the rest of the premium decrease by the same proportion as the rest of the premium, and the correlation of its return with surplus is unchanged.

Conversely, a contract written at less than the required risk-based premium will decrease the expected return.

[3]The effect of each category on the standard deviation was defined in Section 1 as the difference between the standard deviation of the sum of the categories up to and including that category on the list, and the standard deviation of the sum of the categories prior to it.

3. DISCUSSION OF THE METHOD

A. Overall Premium Targets

The method presented above estimates the required risk-based premium for a contract or category, given certain overall expectations or targets of the insurer. These expected values or targets include the overall loss ratio, expense ratio, payout rate, and mix of business for the coming year. Covariances of categories with surplus are estimated based on these expected values. The method applies to individual underwriting decisions concerning contracts or categories of business, but it does not indicate what the overall mix or amount of premium should be. It is assumed that there are practical constraints against making drastic shifts in the current mix of business. An insurer is not free to simply choose any portfolio of business in the way that a stockholder can choose a portfolio of stocks.

If an insurer increases or decreases its premium, or changes the mix of business, these changes have an immediate effect, as well as an additional long term effect, on the insurer's combined ratio, total return on surplus, and variability of surplus. In the long run, increased variability can make an insurer less attractive to its employees and its clients, and can adversely affect its combined ratio and return on surplus.

If certain estimates are made, it is possible to use the Capital Asset Pricing Model (CAPM) to help in selecting the volume of premium which maximizes the market value of the insurer. This model (Lintner [9] and Sharpe [11]) will be discussed further in the last section of the paper. In actual practice, insurer managements are more likely to use informed judgment than CAPM.

B. One Year Variability

The one year time frame used for optimizing the risk-return relation is also intended to optimize this relation over the long term. Long term variability may be thought of as a sum of one year random variables.

Sometimes it may be more natural to estimate the long term variability for a category than to estimate the one year variability. Loss reserves for environmental and mass tort (E/MT) claims is an example of such a category. The estimate of the one year variability for E/MT reserves should be selected in a way that is consistent with estimated long term variability.

Let random variable X_i represent the effect of this category on surplus in the year i. Let Y_i equal $X_i - \mathrm{E}(X_i)$. $\mathrm{E}(X_{i+1})$ is based on the probability distribution of X_{i+1} at the end of year i, so the fact that X_i is greater or less than $\mathrm{E}(X_i)$ has no bearing on how X_{i+1} will differ from the mean of its distribution. Therefore, Y_{i+1} is independent of Y_i.

Similarly, for each integer $k > 1$, Y_{i+k} is independent of Y_i. Therefore, the sequence of observations $Y_1, Y_2, \ldots$ is a stochastic process for which each value is independent of previous values.

C. Loss Reserve Variability and Discounting

The estimates of loss reserves referred to in this paper are assumed to be unbiased, although annual statement estimates may be biased. Thus, the estimates do not necessarily equal the risk-based discounted values of annual statement estimates.

The reader may have noticed that the variability of loss reserves has been addressed in the paper, but not the variability of the unearned premium reserve. This is because the variability associated with this reserve is included in the underwriting variability for the coming year.

The definition of surplus in this paper uses a risk-based discounted value for the loss reserves. The corresponding value of surplus is not necessarily the market value of the insurer. For one thing, it excludes franchise value. However, it appears that optimizing the risk-return relation for this surplus, as discussed in this paper, should be a good approximation to optimizing the risk-return relation for market value.

D. Asset Variability

An attempt can be made to minimize the effects of interest rate variability on surplus. A relatively simple method is to choose a mix of assets with a "duration" (see Ferguson [5]) such that interest rate changes have the same effect on the value of assets as on the value of liabilities. To apply this duration method, using the definition of surplus in this paper, it is necessary to estimate the effect of interest rate changes on the risk-based loss discounting rate. The correlation between interest rates and inflation, and the effect of inflation on estimated loss reserves, must also be estimated.

An insurer may find that duration matching of assets and liabilities requires an asset portfolio with a shorter duration than is desired. Shorter duration bonds have a lower interest rate.

Changing the mix of assets, including stocks, can be used as a tool in attempting to optimize an insurer's risk-return relation. The correlation of the insurer's return with "market return" (i.e., the average return for the market of all capital assets) should be taken into account in such an attempt. This is discussed briefly in Section 5, which contains a comparison of the method of this paper with the Capital Asset Pricing Model. However, the subject of optimizing an insurer's mix of assets is beyond the scope of this paper.

E. Estimation Problems

The covariance between the effects on surplus of any two categories a and b will be denoted by $\mathrm{Cov}(a,b)$. The covariance of Category c with all other sources of surplus variability will be denoted by $\mathrm{Cov}(c,s-c)$.

Let the variance of the effect on surplus of a Category c be denoted by $(\sigma_c)^2$. Denote the correlation between the category and surplus by $\rho_{c,s}$. Note that

$$\mathrm{Cov}(c,s) = \mathrm{Cov}(c,c) + \mathrm{Cov}(c,s-c) = (\sigma_c)^2 + \sigma_c\sigma_{s-c}\rho_{c,s-c}.$$

Therefore, for a Category c that is small, the estimate of $\mathrm{Cov}(c,s)$ is very sensitive to the estimate of $\rho_{c,s-c}$. This is a problem, due to the low credibility of the related data. From a practical point of view, it is best to implement the method of this paper by starting with estimates relating to the largest categories.

For example, a practical first step would be to allocate surplus to the category of all loss reserves and offsetting assets and to the category of all underwriting. This determines the risk-based discount rate for the category of all loss reserves, and the risk-based profit margin on discounted underwriting results.

A reasonable second step would be to allocate surplus to the sum of all property underwriting categories and to the sum of all casualty underwriting categories. (Note that the sum of these two amounts of surplus equals the amount of surplus allocated in the first step to the category of all underwriting.) These allocations determine risk-based profit margins for property and casualty as a whole.

The problem of implementing the method is a vast one, and the examples in the next section are intended only as illustrations. In practice, it is necessary to use a considerable amount of judgment, in addition to making a study of relevant historical data.

4. EXAMPLES OF APPLICATIONS

A. *Overall Underwriting Risk Load and Overall Discount Rate*

Suppose the following for some insurer:

1. Risk-free interest rate on assets = 6%.
2. Loss reserves at start of year discounted at 3% = $500,000,000.
3. Discounted value of amount of loss reserves expected to be paid during year = $100,000,000.

4. Present discounted value of loss reserves not expected to be paid during year = $400,000,000.
5. Expected earned premium for coming year = $150,000,000.
6. Expected underwriting expenses to be incurred during year = $40,000,000.
7. Expected current accident year losses to be paid during year = $45,000,000.
8. Expected value of loss reserves at end of year for current accident year discounted at 3% = $50,000,000.
9. The pre-tax contributions to surplus of loss reserves and offsetting assets, and of underwriting, are in the same proportion as the corresponding after-tax effects.

Assume that the expected expense and loss ratios equal the targets that were discussed in Section 3A. "Risk load" will be taken to mean "risk-based underwriting margin," which was discussed in Section 2C. The after-tax effect on surplus of loss reserves and offsetting assets will be called the return from loss reserves. The after-tax effect on surplus of underwriting will be called underwriting return. These returns do not include investment income on allocated surplus.

Using the 3% discount rate, the expected one year pre-tax return from loss reserves and offsetting assets, assuming loss reserves paid during the year are paid on average in the middle of the year, is (as explained below):

$$(\$500{,}000{,}000)(1.06) - (\$100{,}000{,}000)(1.03)^{.5}(1.06)^{.5}$$
$$- (\$400{,}000{,}000)(1.03) = \$13{,}511{,}000.$$

By the end of the year, the $400 million in loss reserves that are not expected to be paid during the year grows to $400,000,000 × (1.03) due to one year's unwinding of discounting. The $400 million in offsetting assets grows, from investment income, to \$424 million, producing a pre-tax return of $\$400{,}000{,}000 \times (1.06 - 1.03)$. A loss reserve payment of $\$100{,}000{,}000 \times (1.03)^{.5}$ is made in the middle of the year (on average), reducing the assets that were offsetting those reserves to $\$100{,}000{,}000 \times ((1.06)^{.5} - (1.03)^{.5})$. By the end of the year, these assets grow by a factor of $(1.06)^{.5}$ to $\$100{,}000{,}000 \times ((1.06) - (1.03)^{.5}(1.06)^{.5})$. This expression plus the above $\$400{,}000{,}000 \times (1.06 - 1.03)$ is equal to the left side of the above equation.

If it is assumed, for the sake of simplicity, that the earned premium is received in the middle of the year, and that the underwriting expenses and accident year losses are paid in the middle of the year, then the expected pre-tax return on underwriting is

$$(1.06)^{.5}(\$150{,}000{,}000 - \$40{,}000{,}000 - \$45{,}000{,}000)$$

$$- \$50{,}000{,}000 = \$16{,}922{,}000.$$

Approaches to estimating the covariances of loss reserve return with surplus, and of underwriting return with surplus, will be discussed after the following brief description of the iterative process.

Suppose that, using the 3% discount rate, the above two covariances, respectively, are in the proportion $A:1$. The corresponding rates of return on allocated surplus are then in the proportion $13{,}511/A:16{,}922$. Call this proportion $B:1$. Suppose that using a 4% discount rate changes the proportion of rates of return from $B:1$ to $C:1$. Since the goal is to make the rates of return equal, a reasonable next step in the iteration would be

$$4\% + (3\% - 4\%)((1 - C)/(B - C)).$$

Suppose for the sake of illustration that the 3% rate is the solution to the iteration. It then follows from the formula for pre-tax return on underwriting that

$$\begin{aligned}\$150{,}000{,}000 &= \$40{,}000{,}000 + \$45{,}000{,}000\\ &\quad + (1.06)^{-.5}(\$50{,}000{,}000)\\ &\quad + (1.06)^{-.5}(\$16{,}922{,}000).\end{aligned}$$

In other words, the premium equals expected expenses (i.e., \$40,000,000) + expected discounted losses (i.e., $\$45{,}000{,}000 + (1.06)^{-.5}(\$50{,}000{,}000)$) + risk load (i.e., $(1.06)^{-.5}(\$16{,}922{,}000)$).

The covariance of the loss reserve return, and of the underwriting return, with surplus can be estimated based on the insurer's historical data. The insurer's loss reserve runoff variability, its loss ratio and expense ratio variability, the duration of its loss reserves, the duration of its assets, and the historical variability of interest rates are all relevant.

Variability in the loss reserve return is caused by differences between the estimated loss reserve and the one year runoff, changes in market values of offsetting assets, changes in estimated risk-based discount rates, and changes in estimated payout rates for loss reserves. To some extent, changes in asset values caused by interest rate changes are offset by corresponding changes in discount rates. Variability in the underwriting return results from variability in asset values, loss ratios, expense ratios, payout rates, and discount rates.

One way of estimating the covariances is as follows. For some period of years, estimates are made of what the expected increases in surplus, and the expected returns from loss reserves and underwriting, would have been at the beginning of each year. (Note that surplus is increased by the return on other assets as well as those offsetting reserves.) These estimates are then compared with what would have been estimated for each of those returns at the end of the same year.

For each year, all the above estimates can be brought to the level of the current year. The estimated loss reserves return for each year can be multiplied by a factor equal to the reserves at the beginning of the current year divided by the beginning reserves for the year. A similar on-level adjustment can be made for estimated underwriting return, based on the premium for the years. For the on-level factor for return on assets other than those offsetting reserves, the amount of those assets can be used. As mentioned above, the estimated increase in surplus is the sum of the above three estimated returns, so the on-level estimate is the sum of the three on-level estimates.

The covariances of the loss reserves and underwriting returns with surplus can then be estimated as shown in the example below. The example is intended to illustrate a method of computation, but in actual practice many more years of data would be used. For each year listed, each of three types of return for the year are estimated at 1/1 and then at 12/31. It is assumed that the 1/1 estimates equal the means of the probability distributions of possible 12/31 estimates.

TABLE 1

	Estimated Loss Reserve Return (000's)		Estimated Underwriting Return (000's)		Estimated Increase in Surplus (000's)	
Year	1/1	12/31	1/1	12/31	1/1	12/31
1990	$13,600	$12,800	$33,000	$28,600	$81,600	$75,600
1991	$13,200	$14,200	$31,400	$25,600	$80,800	$86,000
1992	$19,400	$18,600	$28,400	$39,600	$77,400	$81,900
1993	$17,000	$15,000	$21,400	$18,200	$62,200	$57,200
1994	$18,900	$14,400	$22,700	$24,200	$63,100	$59,500

Based on the data in Table 1, the estimated covariances with surplus are as follows (000,000's):

Loss Reserve Return:

$$(1/5)((12{,}800 - 13{,}600)(75{,}600 - 81{,}600)$$
$$+ (14{,}200 - 13{,}200)(86{,}000 - 80{,}800)$$
$$+ (18{,}600 - 19{,}400)(81{,}900 - 77{,}400)$$
$$+ (15{,}000 - 17{,}000)(57{,}200 - 62{,}200)$$
$$+ (14{,}400 - 18{,}900)(59{,}500 - 63{,}100)) = 6{,}520{,}000.$$

Underwriting Return:

$$(1/5)((28{,}600 - 33{,}000)(75{,}600 - 81{,}600)$$
$$+ (25{,}600 - 31{,}400)(86{,}000 - 80{,}800)$$
$$+ (39{,}600 - 28{,}400)(81{,}900 - 77{,}400)$$
$$+ (18{,}200 - 21{,}400)(57{,}200 - 62{,}200)$$
$$+ (24{,}200 - 22{,}700)(59{,}500 - 63{,}100)) = 11{,}448{,}000.$$

Another method of estimating the covariances of loss reserve return and underwriting return with surplus is to analyze the covariance structure and estimate the component parts.

Let σ_l, σ_u, and σ_a denote the standard deviations of the following random variables:

L: return from loss reserves;

U: return from underwriting; and

A: return on assets other than those offsetting loss reserves.

Let the correlations between the above returns be denoted by $\rho_{l,u}$, $\rho_{l,a}$, and $\rho_{u,a}$. Let $\mathrm{Cov}(L,S)$ and $\mathrm{Cov}(U,S)$ denote the covariances of the indicated returns with surplus. Then,

$$\begin{aligned}
\mathrm{Cov}(L,S) &= \mathrm{Cov}(L, L+U+A)\\
&= \mathrm{Cov}(L,L) + \mathrm{Cov}(L,U) + \mathrm{Cov}(L,A)\\
&= (\sigma_l)^2 + \sigma_l\sigma_u\rho_{l,u} + \sigma_l\sigma_a\rho_{l,a}, \qquad \text{and}\\
\mathrm{Cov}(U,S) &= \mathrm{Cov}(U, L+U+A)\\
&= \mathrm{Cov}(U,L) + \mathrm{Cov}(U,U) + \mathrm{Cov}(U,A)\\
&= \sigma_u\sigma_l\rho_{l,u} + (\sigma_u)^2 + \sigma_u\sigma_a\rho_{u,a}.
\end{aligned}$$

B. Risk Loads for Property and Casualty

Since 1980, the variation in industry casualty loss ratios has been much greater than the variation in property loss ratios. Also, casualty loss ratio variation has been significantly correlated with variation in loss reserve estimates. Both loss ratios and reserve estimates were affected by trends in loss severity.

Suppose that, for some insurer:

1. All premiums are either casualty or property.
2. The overall underwriting risk load (discussed in the previous example) is 8% of premium.
3. The covariances with casualty return and with property return of the return on assets other than those offsetting loss reserves are zero.
4. Expected property and casualty earned premiums are \$100,000,000 and \$150,000,000, respectively, and total risk-based discounted loss reserves are \$400,000,000.
5. The expected pre-tax returns from property and casualty premiums are in the same proportion as the corresponding after-tax returns.

6. The estimated covariances of property return, casualty return, and loss reserves return with each other are based on Table 2.

TABLE 2

Year	Change from 1/1 to 12/31 in Estimated Property Return (000's)	Change from 1/1 to 12/31 in Estimated Casualty Return (000's)	Change from 1/1 to 12/31 in Estimated Loss Reserves Return (000's)
1983	−\$2,500	−\$20,800	−\$14,600
1984	−\$6,100	−\$29,700	−\$16,400
1985	−\$400	\$6,100	\$1,300
1986	\$8,700	\$16,500	\$4,600
1987	\$4,100	\$28,800	\$8,900
1988	−\$600	\$6,200	\$1,400
1989	−\$500	\$1,500	\$4,800
1990	−\$6,000	−\$1,700	\$2,100
1991	−\$3,600	−\$1,400	\$5,700
1992	\$2,100	−\$2,500	\$5,900
1993	\$4,800	−\$3,800	\$1,200
1994	−\$1,500	\$900	−\$1,100

The covariance between any two of the returns in Table 2 is estimated by taking the average of the products of the numbers in each row of the two columns of returns. It is assumed that the 1/1 estimates equal the means of the probability distributions of possible 12/31 estimates. Let P, C, R, and A denote random variables which equal the returns from property, casualty, reserves, and other assets, and let S denote a random variable which equals the change in surplus. Then, since $\mathrm{Cov}(P,A)$ and $\mathrm{Cov}(C,A)$ are zero by Assumption 3 above,

$$\mathrm{Cov}(P,S) = \mathrm{Cov}(P,P) + \mathrm{Cov}(P,C) + \mathrm{Cov}(P,R) + \mathrm{Cov}(P,A)$$

$$= \mathrm{Var}(P) + \mathrm{Cov}(P,C) + \mathrm{Cov}(P,R) = 74.14 \text{ million},$$

and

$$\begin{aligned}\mathrm{Cov}(C,S) &= \mathrm{Cov}(C,P) + \mathrm{Cov}(C,C) + \mathrm{Cov}(C,R) + \mathrm{Cov}(C,A)\\ &= \mathrm{Cov}(C,P) + \mathrm{Var}(C) + \mathrm{Cov}(C,R) = 342.83 \text{ million.}\end{aligned}$$

The ratio of the risk load, in dollars, for property to that of casualty is $74.14:342.83$; i.e., $.216:1$. It was assumed above that overall underwriting risk load is 8% of premium, so if x represents the casualty risk load in dollars,

$$x + .216x = .08 \times (\$250{,}000{,}000)$$

$$x = \$16.447 \text{ million.}$$

Therefore, the risk loads for casualty and property, as percentages of premium, are, respectively, $16.447/150 = 11.0\%$, and $(.216(16.447))/100 = 3.6\%$.

Suppose that expenses are 30% of premium for both casualty and property, and that the respective risk-based present value factors for the losses are .800 and .970. It then follows, using the above risk loads of 11.0% and 3.6%, that the target combined ratio for casualty is given by

$$30 + (100 - 30 - 11)/.800 = 103.8,$$

and the target for property is given by

$$30 + (100 - 30 - 3.6)/.970 = 98.5.$$

C. Catastrophe Cover Risk Load

In this example, in order to estimate the value of a catastrophe cover to a ceding company, we will suppose that the ceding company re-assumes the cover, and we will estimate the required risk load.

Assume that:

1. The probability of zero losses to the catastrophe cover is .96, and the probability that the losses will be \$25 million

is .04. Therefore, the variance $(\sigma_c)^2$ of the losses is 24 trillion, and the expected losses are \$1 million.

2. Property premium earned for the year is \$100 million, and there is no casualty premium.
3. The standard deviation of pre-tax underwriting return is 15 million.
4. The expected pre-tax return from underwriting is \$8 million.
5. Taxes have the same proportional effect on the expected pre-tax returns on total premium and on the catastrophe cover, and on the standard deviations of the returns.
6. The covariance between the catastrophe cover's losses and losses net of the cover is equal to .50 times the variance of the cover's losses.
7. The discount rate for losses is zero.
8. Total underwriting return, and the return on the catastrophe cover, are statistically independent of non-underwriting sources of surplus variability.

It follows from 1 and 6 above that the covariance with surplus of the pre-tax return on the catastrophe cover is 24 trillion + .50(24 trillion); i.e., 36 trillion. It follows from 3 that the corresponding covariance for total underwriting is $(15 \text{ million})^2$; i.e., .225 trillion. Therefore, it follows from assumption 4 that the risk load for the catastrophe cover should be such that the pre-tax return from the catastrophe cover is given by $(36/225)(8 \text{ million}) =$ \$1.28 million. This is greater than the cover's expected losses.

The insurer may be able to cede the catastrophe cover for a price that is mutually beneficial to it and a reinsurer. For example, if a reinsurer is much larger and more diversified than the ceding company, and it pools its assumed catastrophe covers with other reinsurers, it may not require as great a risk load for the cover as would the ceding company.

D. Risk Load by Layer

Suppose that for some insurer:

1. All premium is property premium.
2. The accident year expected property losses for the \$500,000 excess of \$500,000 layer, and the 0 – \$500,000 layer, respectively, are \$10 million and \$90 million. Expected losses excess of \$1 million are zero.
3. The accident year property losses for each of the above layers are independent of all non-underwriting sources of surplus variation.
4. The discount rate is zero.
5. The coefficients of variation (ratios of standard deviations to means) of the higher and lower layers are .30 and .15, respectively.
6. The correlation between the two layers is .5.
7. Taxes have the same proportional effect on the returns of both layers.

Let σ_1 and σ_2 denote the standard deviations of the losses to the higher and lower layers, respectively. Let ρ denote the correlation. With the above assumptions, the pre-tax covariances with surplus for the higher and lower layers, respectively, are given by:

$$\begin{aligned}\sigma_1^2 + \rho\sigma_1\sigma_2 &= ((10 \text{ million})(.30))^2 \\ &\quad + (.5)(10 \text{ million})(.30)(90 \text{ million})(.15) \\ &= 29.25 \text{ trillion}, \qquad \text{and} \\ \sigma_2^2 + \rho\sigma_1\sigma_2 &= ((90 \text{ million})(.15))^2 \\ &\quad + (.5)(10 \text{ million})(.30)(90 \text{ million})(.15) \\ &= 202.5 \text{ trillion}.\end{aligned}$$

The allocated surplus for the 0 – $500,000 layer is 202.5/29.25 (i.e., 6.9) times as great as the allocated surplus for the $500,000 excess of $500,000 layer. The expected losses are nine times as great for the lower layer. Therefore, the required risk load, as a percentage of expected losses, is 1.3 (i.e., ((9)(29.25))/202.5) times as great for the higher layer as it is for the lower layer. This is expected due to the higher layer's larger coefficient of variation.

Note the contrast of the use of covariances to the use of variances or standard deviations. The covariances for the lower and higher layers are 202.5 trillion and 29.25 trillion, respectively. The corresponding variances are 182.25 trillion and 9 trillion, and the corresponding standard deviations are 13.5 million and 3 million. Thus the ratio of total risk loads, in dollars, for the lower and higher layers is about 7 for the covariance method, about 20 for the variance method, and exactly 4.5 for the standard deviation method.

5. SOME RELATED METHODS

It will be shown that the Capital Asset Pricing Model (CAPM) can be useful in selecting the overall premium and combined ratio targets that are used in this paper to set targets for individual categories. Also, the significance of the method of this paper from a CAPM perspective will be discussed.

According to CAPM, the price of a capital asset depends on its expected rate of return and the covariance of this rate with the overall rate of return on the market of all capital assets. (See Brealey and Myers [1], Lintner [9], and Sharpe [11].) There is some similarity between CAPM and the method presented here, since CAPM estimates prices based on the covariance of an asset with the market, and the method presented here estimates prices based on the covariance of a contract with surplus. The similarity is limited, however. The derivation of the CAPM formula for a capital asset uses the fact that holders of capital assets are able to use Markowitz diversification. The method presented here requires that the mix of business of an insurer be approximated in advance. The method applies to a risk-return optimization problem, but for an insurer with a stable, or almost stable, book of business.

According to CAPM, each asset j in the market of all capital assets will have a market price such that

$$E_j = R_f + (E_m - R_f)((\text{Cov}(R_j, R_m))/(\sigma_m)^2),$$

where

E_j = the expected rate of return on asset j,

E_m = the expected rate of return on the market portfolio,

σ_m = the standard deviation of the rate of return on the market portfolio,

R_f = the risk-free rate of return,

R_m = the market rate of return, and

R_j = the rate of return on asset j.

The market value of an insurer's assets, not including franchise value, minus its liabilities will be called the market value of its surplus. Suppose for the sake of illustration that for some insurer, the market value of surplus equals the market value of the insurer. In other words, the franchise value is zero. Suppose also that the expected market value of surplus one year in the future equals the expected market value of the insurer one year in the future. It then follows that the expected change in this value of surplus in the coming year, divided by the present surplus, is equal to E_j in the above formula if R_j represents the rate of return on the market value of the insurer.

This expected rate of return, which makes the market value of the insurer equal the runoff value (market value) of the assets and liabilities, could be considered to be the minimum acceptable expected return on surplus for the insurer.

Suppose that, due to a change in management, the expected change in surplus in the coming year increases, and there is no change in the expression R_f or

$$(E_m - R_f)(\mathrm{Cov}(R_j, R_m)/\sigma_m^2).$$

Since E_j does not change, the market value of the insurer theoretically increases and becomes greater than the market value of surplus. This creates what is known as franchise value.

The amount of premium that is required for a category in order to neither improve nor worsen the insurer's risk-return relation is not necessarily the same as the amount that neither increases nor decreases the market value of the insurer according to CAPM.

Suppose that surplus is allocated according to the method of this paper, and the estimated rate of return on the surplus allocated to a Category a is less than the rate of return of the insurer. Suppose also that, according to the application of CAPM to Category a and its allocated surplus, this rate of return is above the acceptable minimum for the insurer discussed above. Also, suppose that according to CAPM the rate of return of the insurer is equal to the acceptable minimum.

In the above example, Category a would be estimated by CAPM to increase the market value of the insurer if certain intangible effects of worsening the risk-return relation are ignored.

Advantages that the insurer gains by improving the risk-return relation were described in the second paragraph of the introduction to this paper. (The risk-return relation has an influence on policyholders, employees, and rating organizations.) In the long run, these advantages can translate into lower expected combined ratios. In the case of the above example, the long-term effects of worsening the risk-return relation should be weighed against a CAPM estimate that ignores them.

An insurer can also use CAPM to evaluate the effects on its market value of changes in its amount of written premium or the composition of its asset portfolio. Here again, the effects on the risk-return relation are important, as well as the effects on the CAPM estimate of market value. The intangible effects of variability on rating organizations, customers, and employees should be considered.

Kreps [8] presented a method of determining risk load by marginal surplus requirements. A problem with Kreps's method was discussed in the introduction. The sum of the effects of all categories on the standard deviation of surplus, as measured by Kreps, does not equal the total standard deviation. Kreps does not address the variability of loss reserves or the discounting of losses.

Feldblum [4] suggested a modified version of CAPM for determining risk loads for insurers:

> The market return R_m in the CAPM model should be replaced by the return on a fully diversified insurance portfolio.

Feldblum's method could be used to estimate required return on allocated surplus for an insurance contract. The subscript m for market is replaced in three places in the CAPM formula by i for insurance industry. Feldblum's method does not address the problem of discounting, but it could be expanded to do so.

Feldblum's method is somewhat similar to the method in this paper in that it addresses the problem, for an insurer, of optimizing the risk-return relation. The key difference between Feldblum's method and the method in this paper is the following: Feldblum's method evaluates insurance contracts for an insurer that is free to use an insurance analogue of Markowitz diversification to produce a portfolio of insurance contracts. (In actual practice, there are constraints on an insurer.) The method in this paper estimates the effect of a contract on surplus variance given an approximated mix of earned premium for the coming year.

Brubaker [2] and Ferrari [6] discuss methods of maximizing an insurer's profit, given a constraint on variance, by selecting an insurance portfolio. They don't address the problems of variability of loss reserves or discounting of losses. Underwriting profit margins by category are estimated prior to selecting the portfolio.

6. CONCLUSION

The method in this paper is an attempt to address the problem of risk-based pricing for an insurer in a way that is useful and also meaningful in the context of financial theory. Although there is considerable judgment and effort involved in applying the method, it provides a new theoretical framework for dealing with the challenge of improving an insurer's risk-return relation.

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APPENDIX

Proof of Theorem 1

Let the random variable X equal the effect of a Category x on surplus in a one year period. Let the random variable Y equal the combined effect of all other sources of surplus variation in the one year period.

Suppose a $1/n$ pro rata share of each category, including x, which contributes to surplus variation is added in any order. Suppose the process is repeated until Category x is about to be added for the $(k+1)^{\text{st}}$ time, where $k+1 \leq n$. Let V_1 denote the variance of the effect on surplus of the set of pro rata shares before x is added, and let V_2 denote the variance afterwards.

In the following argument, the expression $\approx$ will be used to indicate that the ratio of the expression on the left to the one on the right approaches 1 as k and n approach infinity. It can be seen that

$$V_1 \approx 2(k/n)^2 \rho_{x,y}\sigma_x\sigma_y + (k/n)^2\sigma_x^2 + (k/n)^2\sigma_y^2, \qquad \text{and}$$

$$V_2 \approx 2((k+1)/n)(k/n)\rho_{x,y}\sigma_x\sigma_y + ((k+1)/n)^2\sigma_x^2 + (k/n)^2\sigma_y^2.$$

The change in standard deviation, Δ Std. Dev., is $V_2^{.5} - V_1^{.5}$.

It follows from $(V_1^{.5} + \Delta \text{ Std. Dev.})^2 = V_2$ that

$$\begin{aligned}
\Delta \text{ Std. Dev.} &\approx .5((V_2 - V_1)/V_1^{.5}) \\
&\approx .5((2k/n^2)\rho_{x,y}\sigma_x\sigma_y + ((2k+1)/n^2)\sigma_x^2)/ \\
&\quad (2(k/n)^2\rho_{x,y}\sigma_x\sigma_y + (k/n)^2\sigma_x^2 + (k/n)^2\sigma_y^2)^{.5} \\
&\approx ((1/n)(\rho_{x,y}\sigma_x\sigma_y + \sigma_x^2))/(2\rho_{x,y}\sigma_x\sigma_y + \sigma_x^2 + \sigma_y^2)^{.5}.
\end{aligned}$$

Therefore, it can be seen that

$$\lim_{n\to\infty}\sum_{1}^{n}\Delta \text{ Std Dev.} = (\rho_{x,y}\sigma_x\sigma_y + \sigma_x^2)/(2\rho_{x,y}\sigma_x\sigma_y + \sigma_x^2 + \sigma_y^2)^{.5}.$$

$$= \text{Cov}(X+Y,X)/\text{Std. Dev. }(X+Y)$$

$$= \text{Cov}(\text{surplus},X)/\text{Std. Dev. (surplus)},$$

and

$$\left(\lim_{n\to\infty}\sum_{1}^{n}\Delta \text{ Std. Dev.}\right)/\text{Std. Dev. (surplus)}$$

$$= \text{Cov}(\text{surplus},X)/\text{Var(surplus)}.$$

Proof of Theorem 2

Let the random variable X equal the effect of the Category x on surplus in a one year period. Let the random variable S equal the change in surplus in the one year period.

It is assumed that the insurer gets more than the required risk-based premium for Category x. Therefore,

$$\text{E}(X) > \text{E}(S)(\text{Cov}(X,S)/\text{Var}(S)). \qquad \text{(A.1)}$$

It follows that,

$$\text{E}(S-X) = (\text{E}(S)-\text{E}(X)) < \text{E}(S)(1-(\text{Cov}(X,S)/\text{Var}(S)))$$

$$= \text{E}(S)(\text{Cov}(S-X,S)/\text{Var}(S)).$$

Therefore,

$$\text{E}(S-X) < \text{E}(S)(\text{Cov}(S-X,S)/\text{Var}(S)). \qquad \text{(A.2)}$$

Suppose the premium for Category x is multiplied by some number $1+a$, where $a>0$, and that the total premium for the rest of the book is multiplied by some number $1-b$, where $b>0$. Suppose also that the insurer's total surplus variance is unchanged. Therefore,

$$\begin{aligned}\mathrm{Var}(S) &= \sigma_x^2(1+a)^2 + (\sigma_{s-x})^2(1-b)^2 \\ &\quad + 2(1+a)(1-b)\rho_{x,s-x}\sigma_x\sigma_{s-x} \\ &= \sigma_x^2 + (\sigma_{s-x})^2 + 2\rho_{x,s-x}\sigma_x\sigma_{s-x}.\end{aligned}$$

Let $\Delta\mathrm{Var}(S)$ represent the first of the above two expressions minus the second. There is an expression $f(a,b)$ such that

$$\begin{aligned}0 &= \Delta\mathrm{Var}(S) \\ 0 &= \sigma_x^2(2a) + (\sigma_{s-x})^2(-2b) + (2a-2b)\rho_{x,s-x}\sigma_x\sigma_{s-x} + f(a,b) \\ 0 &= 2a\sigma_x(\sigma_x + \rho_{x,s-x}\sigma_{s-x}) - 2b\sigma_{s-x}(\sigma_{s-x} + \rho_{x,s-x}\sigma_x) + f(a,b) \\ 0 &= 2a(\mathrm{Cov}(X,S)) - 2b(\mathrm{Cov}(S-X,S)) + f(a,b)\end{aligned}$$

and the limit as a and b approach zero of $f(a,b)/a$, and of $f(a,b)/b$, is zero.

It follows from the above that

$$\begin{aligned}&a\mathrm{E}(S)(\mathrm{Cov}(X,S)/\mathrm{Var}(S)) \\ &\qquad = b\mathrm{E}(S)(\mathrm{Cov}(S-X,S)/\mathrm{Var}(S)) + g(a,b), \qquad \text{(A.3)}\end{aligned}$$

where $g(a,b)/a$ and $g(a,b)/b$ approach zero as a and b approach zero.

Now,

$$\begin{aligned}&\mathrm{E}((1+a)X + (1-b)(S-X)) \\ &\qquad = \mathrm{E}(X + (S-X) + aX - b(S-X)) \\ &\qquad = \mathrm{E}(S) + a\mathrm{E}(X) - b\mathrm{E}(S-X). \qquad \text{(A.4)}\end{aligned}$$

It follows from Inequalities A.1 and A.2 that the formula above equals

$$\begin{aligned} &\mathrm{E}(S) + a(\mathrm{E}(S)(\mathrm{Cov}(X,S)/\mathrm{Var}(S)) \\ &\qquad - b(\mathrm{E}(S)(\mathrm{Cov}(S-X,S)/\mathrm{Var}(S)) + ad + be, \end{aligned} \tag{A.5}$$

where $d > 0$ and $e > 0$.

It was mentioned above that $a > 0$ and $b > 0$. As a and b approach zero, d and e above remain constant and, by Equations A.3, A.4 and A.5,

$$\begin{aligned} &\mathrm{E}((1+a)X + (1-b)(S-X)) \\ &\qquad = \mathrm{E}(S) + g(a,b) + ad + be > \mathrm{E}(S). \end{aligned}$$

This completes the proof of Theorem 2 for the case in which Category x is written at more than the required risk-based premium. The proof of the converse is similar.

THE INTERACTION OF MAXIMUM PREMIUMS, MINIMUM PREMIUMS, AND ACCIDENT LIMITS IN RETROSPECTIVE RATING

HOWARD C. MAHLER

Abstract

This paper discusses the inaccuracies in workers compensation retrospective rating that resulted from the former method of separately calculating insurance charges from Table M and excess loss factors for loss limitations. These ideas have been previously presented by Glenn Meyers [1] and Ira Robbin [2]. However, this paper presents the ideas in a coherent fashion using Lee diagrams [3]. This should make these important ideas more accessible to CAS students while at the same time demonstrating the power of the techniques developed by Lee.

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1. RETROSPECTIVE RATING

As explained in Gillam and Snader [4], retrospective rating is an individual risk rating plan under which an insured's premium for a policy varies based on its experience during that policy period. For losses L, the retro premium, prior to the application of the minimum or maximum, is given by:

$$R = (b + ckE + cL)T,$$

where b is the basic premium, c is the loss conversion factor, k is the loss elimination ratio if an accident limit is selected,

E represents expected losses, and T is the tax multiplier. The retrospective premium varies between preselected minimum and maximum premiums. Usually, the plan is balanced to the guaranteed cost premium; i.e., the expected value of the retrospective premium should be equal to the standard premium less premium discounts. As explained in Gillam and Snader [4], this can be accomplished via the calculation of a net insurance charge using Table M.

Often the insurer and the insured agree to include in the retrospective rating plan an accident limit, which limits the dollars of loss that enter into the retrospective rating formula from any single accident. This stabilizes the insured's premium and protects the insured from the full impact of an extremely large accident. The imposition of an accident limit would also reduce the expected retrospective premium. Therefore, the insured must pay an additional amount for selecting an accident limit, so that the appropriate expected value of the retrospective premium is maintained. In the formula above, this impact was represented by the term $ckET$. Gillam [5] explains how excess loss factors (ELFs) can be used to quantify such an impact.

Skurnick [6] explains how Table L (which is based on the loss ratio distribution in the presence of an accident limitation) can be used to quantify the combined impact of the selection of minimum and maximum premiums together with the selection of an accident limit. Unfortunately, due to their interaction, the separate quantification of the effect of the former via Table M (which is based on the loss ratio distribution in the absence of an accident limitation) and of the latter via excess loss factors generally does *not* lead to the mathematically correct result (that is obtained in Skurnick via the use of Table L).

This paper will use Lee diagrams to explain this interaction and to illustrate how to quantify this error.

2. LEE DIAGRAMS

In Lee [3], a graphical technique is developed that is extremely useful for understanding retrospective rating.[1]

A key concept used in retrospective rating, as explained in Gillam and Snader [4], is the entry ratio. The entry ratio is defined as the observed loss ratio divided by the expected loss ratio. Equivalently, the entry ratio is the observed losses divided by the expected (unlimited) losses.

The Lee diagram has the entry ratio along the y-axis and probability along the x-axis. Figure 1 shows a relatively simple Lee diagram for retrospective rating without an accident limit. $F(x)$ is the cumulative distribution function for the (unlimited) entry ratios. Since the x-axis represents probability, as the entry ratio (y-value) increases, the distribution function approaches the vertical asymptote corresponding to a probability of unity. For an entry ratio of zero, the probability is zero in this example.[2] Generally, entry ratios are non-negative.[3]

Figure 1 is based on a simulation of 250 risks, each with an expected claim frequency of 100 accidents per year.[4] The

[1]Lee uses the same techniques to illustrate applications to size of loss distributions as well as to retrospective rating.

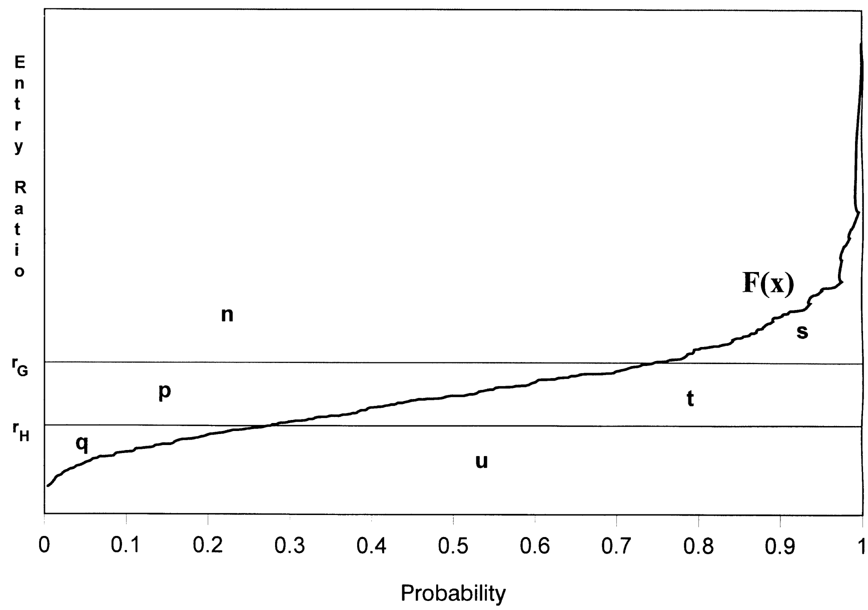
[2]For small risks, there is a significant probability of no losses in a year. For larger risks, such as those generally retrospectively rated, there is a small chance of no losses. For the example examined here, with 250 simulated risks with an average of 100 accidents each, the smallest observed entry ratio is .2177. Thus, in Figure 1, the cumulative distribution function $F(x)$ is zero for $x < .2177$. Since the entry ratios correspond to the vertical axis, the curve for $F(x)$ in Figure 1 hits the vertical axis at a height of about .2177.

[3]This follows from an assumption that actual losses are greater than or equal to zero and, therefore, entry ratios are greater than or equal to zero.

[4]In particular, the simulation employed a Poisson frequency assumption, based severity on random sampling from reported Massachusetts workers compensation claims, and assumed independence of frequency and severity. The Poisson frequency assumption was chosen for simplicity and may not reflect actual risks of this size. The results of this simulation are solely for illustrative purposes and many details of the behavior may not reflect actual insureds. However, we always expect $F(x)$ to be a non-decreasing function of x, even if, due to the limitations of the graphing software, it may not always appear to be so.

FIGURE 1

LEE DIAGRAM
RETROSPECTIVE RATING WITHOUT ACCIDENT LIMIT



Based on 250 simulated risks with an average of 100 accidents each.

diagram is intended solely for illustrative purposes; some details would differ depending on the particular risk process, but the general features would be retained. The same data was used as the basis for the diagram when an accident limit was imposed.

3. NOTATION

No Accident Limit

The notation used will, with minor exceptions, follow that in Skurnick [6]:

A	=	Actual (unlimited) losses for the risk for the policy period.
E	=	Expected (unlimited) losses.
r	=	A/E = (unlimited) entry ratio.
$f(r)$	=	The probability density function of entry ratios.
$F(r)$	=	The (cumulative) distribution function of entry ratios.
$\phi(r)$	=	The Table M charge for entry ratio r
	=	The charge for entry ratio r, computed from F
	=	$\int_r^{\infty}(s-r)f(s)\,ds = \int_r^{\infty}(1-F(s))\,ds.$
$\psi(r)$	=	The Table M savings for entry ratio r
	=	The savings for entry ratio r, computed from F
	=	$\int_0^{r}(r-s)f(s)\,ds = \int_0^{r}F(s)\,ds.$
$\{A\}$	=	The losses that effectively enter the retrospective rating calculation with maximum premium G and minimum premium H
	=	$\begin{cases} r_G E & \text{if} \quad A \geq r_G E \\ A & \text{if} \quad r_H E \leq A \leq r_G E \\ r_H E & \text{if} \quad A \leq r_H E. \end{cases}$
$\{r\}$	=	The entry ratio that effectively enters the retrospective rating calculation
	=	$\{A\}/E.$
G	=	Maximum premium.
r_G	=	Entry ratio corresponding to the maximum premium G. (The maximum premium G is attained when $A = r_G E$. Therefore, using the general formula for retrospective rating with $k = 0$, $r_G = G/cET - b/cE$.)
H	=	Minimum premium.
r_H	=	Entry ratio corresponding to the minimum premium H. (The minimum premium H is attained when $A = r_H E$. Therefore, using the general formula for retrospective rating with $k = 0$, $r_H = H/cET - b/cE$.)

As explained in Skurnick [6] and Gillam's review [7],

$$\mathrm{E}[\{r\}] = 1 + \psi_H - \phi_G.$$

With Accident Limit

A^* = The losses limited by the accident limit.
r^* = A^*/E = the limited entry ratio.
$f^*(r)$ = The density function for the limited entry ratios.
$F^*(r)$ = The distribution function for the limited entry ratios.
k = The loss elimination ratio[5] $= 1 - \mathrm{E}[A^*]/E$.
$\phi^*(r)$ = The Table L charge[6] for (limited) entry ratio r

$$= \int_r^\infty (s-r) f^*(s)\,ds + k$$

$$= \int_r^\infty (1 - F^*(s))\,ds + \int_0^\infty [F(s) - F^*(s)]\,ds.$$

$\psi^*(r)$ = The Table L savings for (limited) entry ratio r

$$= \int_0^r (r-s) f^*(s)\,ds$$

$$= \int_0^r F^*(s)\,ds.$$

4. LEE DIAGRAM, NO ACCIDENT LIMIT

In the case of no accident limit, the Lee diagram for retrospective rating (Figure 1) has horizontal lines corresponding to two entry ratios, r_G and r_H, related to a particular retrospective rating plan, and one distribution curve $F(x)$ for the (unlimited) entry ratios. This in general divides the diagram into six different non-overlapping areas, which have been labeled with small letters: n, p, q, s, t, and u.

---

[5] This is the portion of losses eliminated from the retrospective rating calculation. In other contexts, this would be referred to as the excess ratio, since it represents the portion of losses in excess of the accident limit.

[6] Note that the integral is similar to that for $\phi(r)$, except that it involves the density function for limited rather than unlimited entry ratios. Also note the extra term of the loss elimination ratio.

r_G is the entry ratio corresponding to the maximum premium for the particular retro plan. r_H is the entry ratio corresponding to the minimum premium. $r_G = L_G/E$ where E represents the expected (unlimited) losses and L_G represents those losses that correspond to the maximum premium G. As explained in Gillam and Snader [4], generally one selects the values of G and H and solves for the values of r_G and r_H. Herein, for simplicity it will be assumed that r_G and r_H are given.

The area under $F(x)$ is equal to the average (unlimited) entry ratio, which is 1.0 by definition. Therefore,

$$\text{Area } s + \text{Area } t + \text{Area } u = 1.$$

The insurance charge at r_G is the integral from r_G to infinity of $1 - F(x)$. Therefore, it is the area above r_G that is between $F(x)$ and the vertical line corresponding to Probability = 1. This area has been labeled s, and

$$\text{Area } s = \phi_G.$$

Similarly, the insurance charge at the minimum is the area above r_H and between $F(x)$ and 1. Thus,

$$\text{Area } s + \text{Area } t = \phi_H, \qquad \text{and}$$

$$\text{Area } t = \phi_H - \phi_G.$$

Also

$$\text{Area } u = 1 - \phi_H.$$

Similarly, one can get the savings in terms of areas on the diagram. The savings at the minimum are given by the integral from 0 to r_H of $F(x)$. This is the area between the vertical line at Probability = 0 and $F(x)$ that is below the horizontal line at r_H.

Thus

$$\text{Area } q = \psi_H.$$

Similarly,

$$\text{Area } p + \text{Area } q = \psi_G, \qquad \text{and}$$
$$\text{Area } p = \psi_G - \psi_H.$$

The net insurance charge is defined as the charge at the maximum minus the savings at the minimum:

$$\phi_G - \psi_H = \text{Area } s - \text{Area } q.$$

For small entry ratios, the insured pays the minimum premium, and therefore the insured pays the same premium as if it had an entry ratio of r_H. Similarly, for large entry ratios, the insured pays the same premium as if it had an entry ratio of r_G. Define the effective entry ratio as

$$\{r\} = \begin{cases} r_H & r \le r_H \\ r & r_H \le r \le r_G \\ r_G & r_G \le r. \end{cases}$$

$\{r\}$ measures how much the insured effectively pays for losses (other than indirectly through the net insurance charge).

Referring to the Lee diagram, $\mathrm{E}[\{r\}]$ is represented by the area below the line/curve going from left to right starting at r_H, going along the horizontal line until it meets $F(x)$, proceeding along $F(x)$ until it meets the horizontal line at r_G, and finally proceeding along the horizontal line at r_G.

Thus

$$\text{Area } q + \text{Area } t + \text{Area } u = \mathrm{E}[\{r\}].$$

In terms of entry ratios (and thus ignoring expenses and taxes) the insured pays $\mathrm{E}[\{r\}]$ + net insurance charge = Area q + Area t + Area u + Area s − Area q = Area s + Area t + Area u = 1, which balances to the guaranteed cost result. In other words, the expected premium ignoring expenses and taxes is expected losses.

TABLE 1

RETROSPECTIVE RATING PLAN WITH NO ACCIDENT LIMIT AS SHOWN IN FIGURE 1

Area	In Symbols	Size
n	N.A.	N.A.
p	$\psi_G - \psi_H$	.2549
q	ψ_H	.0469
s	ϕ_G	.1018
t	$\phi_H - \phi_G$	.2451
u	$1 - \phi_H$	.6531

Note: r_G = 1.20 and r_H = .70. Based on 250 simulated risks with an average of 100 accidents each. The average severity is about $10,500. The coefficient of variation of the severity is about 4.7. The skewness of the severity is about 20.6.

One should also note that the area under the horizontal line at r_G is equal to r_G, so:

$$\text{Area } p + \text{Area } q + \text{Area } t + \text{Area } u = r_G, \qquad \text{and}$$

$$\text{Area } q + \text{Area } u = r_H.$$

As pointed out in Lee [3], one can derive useful relationships easily using this diagram, for example, the fundamental relationship between charges and savings at a given entry ratio. Since Area $q = \psi_H$, and

$$\text{Area } u = 1 - (\text{Area } s + \text{Area } t) = 1 - \phi_H,$$

$$\text{therefore} \quad \psi_H + 1 - \phi_H = r_H.$$

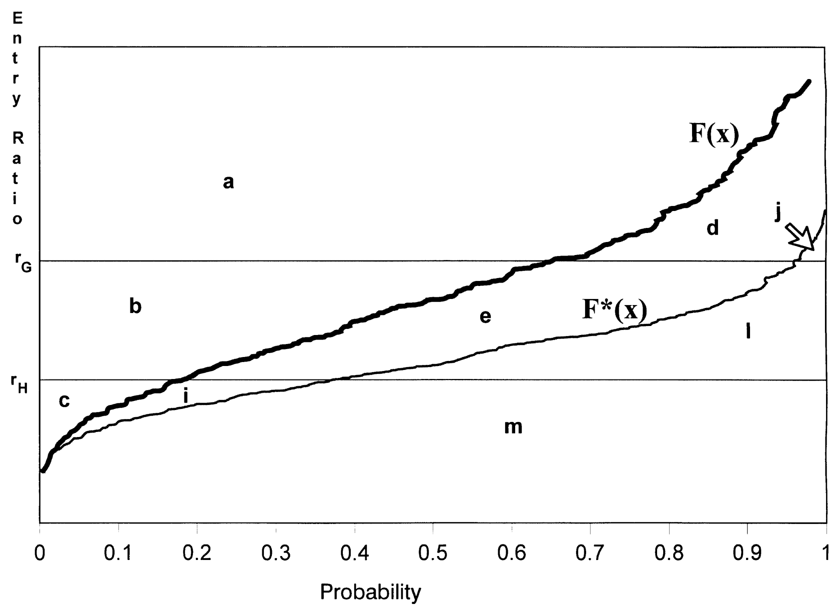
Table 1 summarizes this retrospective rating example.

5. LEE DIAGRAM, ACCIDENT LIMIT, TABLE L

The Lee diagram in Figure 2 relating to a specific accident limit has two distribution functions of entry ratios: $F(x)$ for unlimited losses and $F^*(x)$ for losses limited by the selected accident limit. For a given set of accidents, the unlimited losses are

FIGURE 2

LEE DIAGRAM, RETROSPECTIVE RATING WITH ACCIDENT LIMIT, TABLE L



Based on 250 simulated risks with an average of 100 accidents each, $100,000 Accident Limit, Loss Elimination Ratio of 0.314.

greater than or equal to the limited losses; the unlimited entry ratios are greater than or equal to the limited entry ratios. Thus $F(x)$ is above or equal to $F^*(x)$.

While $F(x)$ is usually above $F^*(x)$, for a sufficiently small entry ratio, $F(x)$ and $F^*(x)$ are identical. If the total unlimited losses for a risk in a year are less than the accident limit, we know none of the individual accidents can be affected by the accident limit. Thus, in this case, the limited and unlimited entry ratios are the same. In the particular example presented here, the average accident is about $10,500. For an expected frequency of 100, the expected losses are therefore about $1.05 million. A risk with total unlimited losses of \$100,000 or less is unaffected by a \$100,000 accident limit. Such a risk would have an entry ratio of 9.5% or less (\$100,000/\$1,050,000 = .095). Thus, for $x \le .095$, $F(x) \quad F^*(x)$. In fact, for the 250 simulated risks in this case $F(x) \quad F^*(x)$ for $x \le .30$.

In general, the curve for $F^*(x)$ branches off below the curve for $F(x)$ somewhere after the start. The higher the accident limit and/or the lower the expected losses the longer it takes for $F^*(x)$ to diverge. In this case, since the accident limit is small relative to the expected losses, $F^*(x)$ branches off relatively soon.

Since the average unlimited entry ratio is unity, the area under $F(x)$ is one, as it was for the previous Lee diagram.

The average limited entry ratio is $1 - k$, where k is the loss elimination ratio. (The average limited entry ratio = expected limited losses/expected unlimited losses = $1 - k$.) Thus the area under $F^*(x)$ is $1 - k$. (For an infinite limit, $k = 0$, which reduces to the unlimited case.) Therefore, the area between the $F(x)$ and $F^*(x)$ curves is always k. This result is very useful in working with the Lee diagram.

Using Figure 2 (with nine non-overlapping areas), as pointed out by Lee, one can derive many of the results in Skurnick [6], related to the use of Table L.[7]

$$\phi_G^* = k + \text{Area } j$$

$$= \text{Area } d + \text{Area } e + \text{Area } i + \text{Area } j.$$

$$\psi_H^* = \text{Area } c + \text{Area } i.$$

$$\text{Table L net insurance charge} = \phi_G^* - \psi_H^*$$

$$= \text{Area } d + \text{Area } e + \text{Area } j - \text{Area } c.$$

The expected value of the effective limited losses entering into the retrospective calculation is the area under the line

[7]Recall that the definition of the Table L charge ϕ^* is the sum of an integral (similar to the Table M charge) and the loss elimination ratio k.

starting at r_H, going horizontally until $F^*(x)$ is reached, along $F^*(x)$ until r_G is reached, and going horizontally until the vertical line Probability = 1 is reached. In other words, $\mathrm{E}[\{r^*\}] =$ Area c + Area i + Area l + Area m.

Using Table L, ignoring expenses and taxes, the insured pays on average the net Table L insurance charge plus $\mathrm{E}[\{r^*\}]$, which is

$$\text{Area } d + \text{Area } e + \text{Area } j - \text{Area } c + \text{Area } c$$

$$+ \text{Area } i + \text{Area } l + \text{Area } m$$

$$= \text{Area under } F(x) = 1.$$

Thus, the Table L plan balances to guaranteed cost; ignoring expenses and taxes, the insured pays for expected losses on average.

6. LEE DIAGRAM, ACCIDENT LIMIT, TABLE M

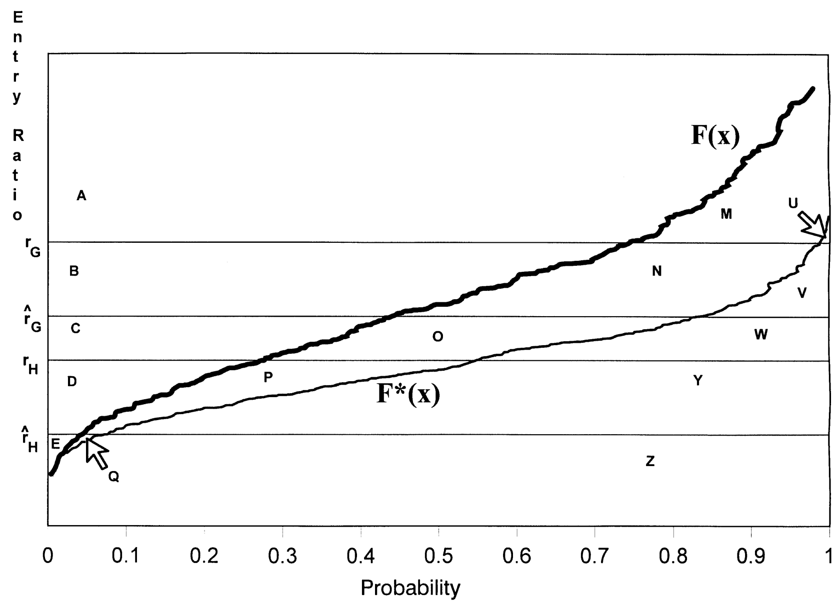
Figure 3 is a Lee diagram that is similar to Figure 2. There are horizontal lines corresponding to entry ratios r_G and r_H. However, there are also horizontal lines that correspond to additional entry ratios $\hat{r}_G$ and $\hat{r}_H$. We define $\hat{r}_G = \hat{L}_G/E$ where $\hat{L}_G$ is that level of (limited) losses such that including the charge for purchasing the accident limit but using the same basic premium as in the absence of the accident limit[8] we achieve the maximum premium.

It is assumed that the charge for the accident limit is the product of the loss conversion factor, tax multiplier, expected loss ratio, and loss elimination ratio. Thus we have ignored any risk loading that may be added to the excess loss factor. We assume the excess loss factor is kE.

[8]Using the same basic premium as in the absence of the accident limit results in an unbalanced plan, as will be discussed below.

FIGURE 3

LEE DIAGRAM, RETROSPECTIVE RATING WITH ACCIDENT LIMIT, SEPARATE USE OF TABLE M AND ELFS



Based on 250 simulated risks with an average of 100 accidents each, $100,000 Accident Limit, Loss Elimination Ratio of 0.314.

For (limited) losses L, the retro premium, prior to the application of the minimum or maximum, is given by

$$R = (b + ckE + cL)T,$$

where b is the basic premium, c is the loss conversion factor, and T is the tax multiplier. Thus we can solve for $\hat{L}_G$:

$$G = (b + ckE + c\hat{L}_G)T.$$

$$\hat{L}_G = \frac{G}{cT} - \frac{b}{c} - kE.$$

$$\hat{r}_G = \frac{\hat{L}_G}{E} = \frac{G}{cET} - \frac{b}{cE} - k.$$

Note that under the traditional (prior to insurance charge reflecting loss limitation[9]) Table M approach, the basic premium b is independent of the loss limit selected: the basic premium is calculated assuming no loss limit.[10] With no loss limit, $k = 0$ (there is no loss limitation charge), and therefore:

$$r_G = \frac{G}{cET} - \frac{b}{cE},$$

$$G = (b + cL_G)T,$$

$$L_G = \frac{G}{cT} - \frac{b}{c},$$

$$r_G = \frac{L_G}{E} = \frac{G}{cET} - \frac{b}{cE}.$$

Thus $\hat{r}_G = r_G - k$.

Similarly, with respect to the minimum rather than the maximum premium, $\hat{r}_H = r_H - k$. So, under the traditional Table M approach, the entry ratios $\hat{r}_G$ and $\hat{r}_H$ that actually achieve the maximum and minimum premiums are reduced by the loss elimination ratio k when a loss limit is selected, compared to the entry ratios r_G and r_H used in the calculation of the insurance charge that enters the basic. It is $\hat{r}_G$ and $\hat{r}_H$ that affect how often the maximum and minimum premiums are attained. Thus it is $\hat{r}_G$ and $\hat{r}_H$ rather than r_G and r_H that should be used to calculate the expected premiums when a loss limitation is selected.

The two distribution curves and four horizontal lines divide the Lee diagram (Figure 3) into a total of 15 different non-overlapping areas.[11] On Figure 3, they have been labeled using

[9] See Robbin [2] for an explanation of ICRLL (insurance charge reflecting loss limitation).
[10] See Meyers [1].
[11] In particular situations, some of these 15 areas will be of zero size.

capital letters. As in the previous Lee diagrams, there are various relationships that always hold.

Since the area under the $F(x)$ curve is unity,

$$\text{Area } M + \text{Area } N + \text{Area } O + \text{Area } P + \text{Area } Q + \text{Area } U + \text{Area } V + \text{Area } W + \text{Area } Y + \text{Area } Z = 1.$$

The area under the $F^*(x)$ curve is $1-k$:

$$\text{Area } U + \text{Area } V + \text{Area } W + \text{Area } Y + \text{Area } Z = 1-k.$$

The area between the $F(x)$ curve and the $F^*(x)$ curve is k:

$$\text{Area } M + \text{Area } N + \text{Area } O + \text{Area } P + \text{Area } Q = k.$$

The Table M insurance charge for r_G is the area above the line r_G between $F(x)$ and the line Probability = 1:

$$\text{Area } M + \text{Area } U = \phi_G.$$

Similarly,

$$\text{Area } M + \text{Area } N + \text{Area } O + \text{Area } U + \text{Area } V + \text{Area } W = \phi_H$$

The Table M savings for r_H is the area below the line r_H between $F(x)$ and the line Probability = 0:

$$\text{Area } D + \text{Area } E = \psi_H, \qquad \text{and}$$

$$\text{Area } B + \text{Area } C + \text{Area } D + \text{Area } E = \psi_G.$$

The Table L insurance charge $\phi^*(r)$ is defined as the sum of k and an integral which corresponds to the area between $F^*(x)$ and Probability = 1, *above* the line corresponding to the chosen entry ratio. Thus,

$$\begin{aligned}\phi^*_G &= k + \text{Area } U \\ &= \text{Area } M + \text{Area } N + \text{Area } O + \text{Area } P \\ &\quad + \text{Area } Q + \text{Area } U,\end{aligned}$$

where the area between $F(x)$ and $F^*(x)$ is k, the loss elimination ratio.

Similarly,

$$\phi^*_H = \text{Area } M + \text{Area } N + \text{Area } O + \text{Area } P + \text{Area } Q$$

$$+ \text{Area } U + \text{Area } V + \text{Area } W, \qquad \text{and}$$

$$\phi^*_H - \phi^*_G = \text{Area } V + \text{Area } W.$$

The Table L savings $\psi^*(r)$ are defined as an integral that corresponds to the area between $F^*(x)$ and Probability = 0, *below* the line corresponding to the chosen entry ratio.

$$\psi^*_H = \text{Area } D + \text{Area } E + \text{Area } P + \text{Area } Q,$$

$$\psi^*_G = \text{Area } B + \text{Area } C + \text{Area } D + \text{Area } E + \text{Area } N$$

$$+ \text{Area } O + \text{Area } P + \text{Area } Q, \qquad \text{and}$$

$$\psi^*_G - \psi^*_H = \text{Area } B + \text{Area } C + \text{Area } N + \text{Area } O.$$

The net Table L insurance charge is

$$\phi^*_G - \psi^*_H = \text{Area } M + \text{Area } N + \text{Area } O + \text{Area } U$$

$$- (\text{Area } D + \text{Area } E).$$

The expected value of the effective limited losses entering the plan (based on Table L) is the area under the line starting at r_H, going horizontally until $F^*(x)$ is reached, along $F^*(x)$ until r_G is reached, and going horizontally until the vertical line corresponding to Probability = 1 is reached:

$$\text{E}[\{r^*\}] = \text{Area } D + \text{Area } E + \text{Area } P + \text{Area } Q + \text{Area } V$$

$$+ \text{Area } W + \text{Area } Y + \text{Area } Z.$$

Using Table L, ignoring expenses and taxes, the insured pays on average the net Table L insurance charge plus $\text{E}[\{r^*\}]$, which is

$$\begin{aligned}\text{Area } D + \text{Area } E + \text{Area } P + \text{Area } Q + \text{Area } V \\ + \text{Area } W + \text{Area } Y + \text{Area } Z + \text{Area } M \\ + \text{Area } N + \text{Area } O + \text{Area } U - (\text{Area } D + \text{Area } E) \\ = \text{Area under } F(x) = 1.\end{aligned}$$

Thus, as was seen previously using Figure 2, the Table L plan balances to guaranteed cost; ignoring expenses and taxes, the insured pays for expected losses on average.

7. ERROR DUE TO INDEPENDENT USE OF TABLE M AND ELFs

As was seen above, the use of Table L produces a plan that balances to guaranteed cost. Ignoring expenses and taxes, the insured pays the expected losses on average.

In contrast, using Table M and ELFs independently, the retrospective rating plan will not, in general, balance to guaranteed cost. The net Table M insurance charge is

$$\phi_G - \psi_H = (\text{Area } M + \text{Area } U) - (\text{Area } D + \text{Area } E).$$

The expected value of the effective (limited) losses entering the plan is the area under the line starting at $\hat{r}_H$, going horizontally until $F^*(x)$ is reached, along $F^*(x)$ until $\hat{r}_G$ is reached, and going horizontally until the vertical line corresponding to Probability = 1 is reached. Note that we use $\hat{r}_H$ and $\hat{r}_G$, since these are the entry ratios at which the minimum and maximum premiums are attained when we add in the loss limitation charge. Thus, in this case,

$$\begin{aligned}\mathrm{E}[\{r^*\}] = \text{Area } E + \text{Area } Q + \text{Area } W \\ + \text{Area } Y + \text{Area } Z.\end{aligned}$$

Thus, the average amount paid by the insured, including the loss limitation charge k, is

$$
\begin{aligned}
&(\phi_G - \psi_H) + \mathrm{E}[\{r^*\}] + k \\
&\quad = [(\text{Area } M + \text{Area } U) - (\text{Area } D + \text{Area } E)] \\
&\qquad + [\text{Area } E + \text{Area } Q + \text{Area } W + \text{Area } Y + \text{Area } Z] \\
&\qquad + [\text{Area } M + \text{Area } N + \text{Area } O + \text{Area } P + \text{Area } Q] \\
&\quad = -\text{Area } D + 2(\text{Area } M) + \text{Area } N + \text{Area } O + \text{Area } P \\
&\qquad + 2(\text{Area } Q) + \text{Area } U + \text{Area } W + \text{Area } Y + \text{Area } Z.
\end{aligned}
$$

One desires that, ignoring expenses and taxes, the insured pays on average for expected losses. This corresponds to the areas on the Lee diagram adding to unity, the area under $F(x)$ (Area M through Area Z).

In general, for the Table M case, the insured pays on average an amount different than unity. Comparing to unity (the area under $F(x)$), we find the error to be:

$$
\begin{aligned}
\text{Error} &= (\text{Table M Case}) - (\text{Area under } F(x)) \\
&= \text{Area } M + \text{Area } Q - \text{Area } D - \text{Area } V \\
&= (\text{Area } M - \text{Area } V) - (\text{Area } D - \text{Area } Q).
\end{aligned}
$$

The error consists of four separate areas on the Lee diagram, Figure 3. Area M and Area V involve the interaction of the maximum premium and the accident limit. Similarly, Area D and Area Q involve the interaction of the minimum premium and the accident limit.

8. ERROR TERMS INVOLVING THE MAXIMUM

Area M enters into the error term due to some double counting in the separate calculation of the losses eliminated from the retrospective rating plan due to the maximum premium and the accident limit. When the insured has one or more large accidents and a large (unlimited) loss ratio, some of the same dollars will be eliminated by both the maximum premium and the accident limit.

For example, take an insured with \$1.3 million in small accidents and a single \$2 million accident. With expected losses of \$1 million, the unlimited entry ratio is 3.3. With a \$100,000 accident limit, the limited entry ratio is 1.4. With a maximum entry ratio of 1.2, at most \$1.2 million of losses enter the retro calculation. Thus the maximum premium has reduced the losses entering the retro by \$3.3 − \$1.2 = \$2.1 million. The accident limit has reduced the losses by \$3.3 − \$1.4 = \$1.9 million. The total reduction seen by the insured is only \$2.1 million, *not* the sum of the two separately calculated effects. It is such examples of double counting that explain why Area M appears in the error as an overcharge to the insured.

Area V enters into the error term with a minus sign. It is there because the maximum entry ratio r_G used in the calculation of the Table M insurance charge assuming no accident limit charge is not the entry ratio at which a retro with an accident limit charge achieves the maximum. With the accident limit charge, it is easier to hit the maximum. Therefore, the maximum has more effect than we had calculated. Thus we have undercharged the insured.

One can rewrite the terms in the error involving the maximum:

$$\begin{aligned}\text{Area } M - \text{Area } V &= (\text{Area } M + \text{Area } U) - (\text{Area } U + \text{Area } V)\\ &= \phi_G - (\phi^*_{\hat{G}} - k),\end{aligned}$$

where the notation $\phi^*_{\hat{G}} = \phi^*(\hat{r}_G)$ has been used. In the particular example here, as shown in Table 2, Area M − Area V = .1007 − .0227 = .0780. In general, we expect this difference $\phi_G - (\phi^*_{\hat{G}} - k)$ to be positive, representing an overcharge to the insured. In other words, we expect $\phi_G > \phi^*_{\hat{G}} - k$.

TABLE 2

RETROSPECTIVE RATING PLAN WITH $100,000 ACCIDENT LIMIT AS SHOWN IN FIGURE 3

Area	In Symbols	Size
A	N.A.	N.A.
B	$\psi_G - \psi_{\hat{G}}$	.1881
C	$\psi_{\hat{G}} - \psi_H$	.0668
D	$\psi_H - \psi_{\hat{H}}$	.0439
E	$\psi_{\hat{H}}$	.0030
M	$\phi_G - \phi_G^*$	.1007
N	$(\phi_{\hat{G}} - \phi_G) - (\phi_{\hat{G}}^* - \phi_G^*)$	.1032
O	$(\phi_H - \phi_{\hat{G}}) - (\phi_H^* - \phi_{\hat{G}}^*)$	.0620
P	$(\phi_{\hat{H}} - \phi_H) - (\phi_{\hat{H}}^* - \phi_H^*)$	.0467
Q	$k + \phi_{\hat{H}}^* - \phi_{\hat{H}}$	.0012
U	ϕ_G^*	.0011
V	$\phi_{\hat{G}}^* - \phi_G^*$	.0227
W	$\phi_H^* - \phi_{\hat{G}}^*$	.0572
Y	$\phi_{\hat{H}}^* - \phi_H^*$	.2234
Z	$1 - (k + \phi_{\hat{H}}^*)$	.3818

Note: $r_G = 1.20$, $r_H = .70$, $k = .314$, $r_{\hat{G}} = .886$, and $r_{\hat{H}} = .386$. Same simulated data as described in Table 1.

This can be seen by comparing the two terms. The former is the integral, from r_G to ∞, of the amount by which the (unlimited) entry ratio exceeds r_G. The latter is a similar integral, but starts at $\hat{r}_G = r_G - k$, and the integrand is the amount by which the limited entry ratio exceeds $\hat{r}_G$.

On average, the difference between unlimited and limited entry ratios is k, the loss elimination ratio. However, larger-than-average entry ratios are more likely to be associated with larger accidents and vice versa.[12] Thus the imposition of the accident

[12]This is the case in real world situations. One can construct mathematical situations where this is not true.

limit will reduce large entry ratios on average by more than k. Therefore we expect fewer risks to have limited entry ratios that exceed $\hat{r}_G = r_G - k$ than have unlimited entry ratios that exceed r_G. Also we expect the amount by which the limited entry ratios exceed $\hat{r}_G = r_G - k$ to be less on average than the amount by which the unlimited entry ratios exceed r_G.

In summary, we expect the range of the integral corresponding to $\phi^*_{\hat{G}} - k$ to have fewer risks than the range of the integral corresponding to ϕ_G, and we expect the integrand of $\phi^*_{\hat{G}} - k$ to be less than the integrand of ϕ_G. Thus we expect $\phi^*_{\hat{G}} - k$ to be less than ϕ_G, as stated above. Therefore, the portion of the error term involving the maximums is expected to be positive, representing an overcharge to the insured.

Since this portion of the error term is Area M − Area V, one can arrive at the same conclusion by observing that on the Lee diagram, Figure 3, Area M is greater than Area V. One can use a geometric argument to show that, in general, Area M + Area U is greater than Area V + Area U.

Area M + Area U and Area V + Area U each are approximately right triangles, except that rather than a straight line hypotenuse, one has a portion of the curve $F(x)$ or $F^*(x)$. Area M + Area U is larger because, as will be shown, it has both a larger height and larger width than Area V + Area U.

First, we note that on Figure 3, the curves $F(x)$ and $F^*(x)$ start off equal and get further apart vertically as we go to the right. The area between $F(x)$ and $F^*(x)$ is k, while the horizontal axis goes from zero to one. Therefore, the average vertical distance between $F(x)$ and $F^*(x)$ is k. Thus the vertical distance between $F(x)$ and $F^*(x)$ is greater than k near the right edge of Figure 3, while it is less than k near the left edge.

Now the left-hand vertex of Area M + Area U occurs where $F(x)$ crosses the horizontal line r_G, which in this case occurs at the point $(.745, 1.2)$. Since, in this portion of the diagram, the vertical distance between $F(x)$ and $F^*(x)$ is greater than k, the point where $F^*(x)$ attains the same level of probability .745 will be more than $k = .314$ lower. In this example, that point on $F^*(x)$ is at $(.745, .815)$.

The horizontal line corresponding to $\hat{r}_G$ at .886 is $k = .314$ below r_G at 1.200. Therefore, since $F^*(x)$ is increasing, it intersects the horizontal line $\hat{r}_G$ to the right of $(.745, .815)$. In this example, this intersection of $F^*(x)$ and $\hat{r}_G$ is at $(.829, .886)$ and represents the left-hand vertex of Area V + Area U.

In general, the left-hand vertex of Area V + Area U will be to the right of the left-hand vertex of Area M + Area U. Since both triangular shapes have Probability = 1 as their right-hand edge, Area V + Area U has a smaller width than Area M + Area U.

In addition, since $F^*(x)$ is more than k below $F(x)$ while $\hat{r}_G$ is k below r_G, Area V + Area U has a smaller height than Area M + Area U. Therefore, Area V + Area U with both a smaller height and width is smaller than Area M + Area U. Thus, it has been shown geometrically that the portion of the error term involving the maximums is expected to be positive, representing an overcharge to the insured.

9. ERROR TERMS INVOLVING THE MINIMUM

There are two areas in the error term that relate to the minimum, which are subtracted from the terms involving the maximum. The minimum terms are:

$$\text{Area } D - \text{Area } Q = (\text{Area } D + \text{Area } E) - (\text{Area } E + \text{Area } Q)$$
$$= \psi_H - \psi^*_{\hat{H}}.$$

We expect $\psi_H \geq \psi^*_{\hat{H}}$ or, equivalently, $\psi_H - \psi^*_{\hat{H}} \geq 0$. This follows from writing this difference of savings in terms of charges:[13]

$$\psi_H = \phi_H + r_H - 1, \qquad \text{and}$$

$$\psi^*_{\hat{H}} = \phi^*_{\hat{H}} + \hat{r}_H - 1 = (\phi^*_{\hat{H}} - k) + r_H - 1,$$

$$\text{therefore,} \quad \psi_H - \psi^*_{\hat{H}} = \phi_H - (\phi^*_{\hat{H}} - k).$$

But $\phi_H - (\phi^*_{\hat{H}} - k) \geq 0$ by the same reasoning that led to the conclusion that $\phi_G - (\phi^*_{\hat{G}} - k) > 0$.

While the entry ratios greater than r_H are overall greater than average, they are much closer to average than those greater than r_G. The extent to which these entry ratios are greater than average was the central thread of the reasoning that led to the conclusion that $\phi_G - (\phi^*_{\hat{G}} - k) > 0$. Therefore, we expect the difference $\phi_H - (\phi^*_{\hat{H}} - k)$ to be smaller than the difference $\phi_G - (\phi^*_{\hat{G}} - k)$. In the particular example, $\phi_H - (\phi^*_{\hat{H}} - k) =$ Area D – Area Q = .0439 – .0012 = .0427, while $\phi_G - (\phi^*_{\hat{G}} - k)$ = .0780.

We note that Area D is analogous to Area V and enters into the error terms for the same reason. Area D relates to the fact that the minimum premium is achieved at $\hat{r}_H = r_H - k$ rather than r_H, when an accident limit charge is included in the retro premium. Thus, with an accident limit, there are fewer times where the insured pays more due to the imposition of a minimum. Therefore, we are crediting the insured with too much savings. Area $D = \psi_H - \psi_{\hat{H}}$ represents the resulting undercharge of the insured.

Area Q is analogous to Area M and enters the error term for the same reason. Area Q relates to the interaction of the minimum and the accident limit. Some of the benefit of the accident limit is lost to the insured, because reducing the losses that enter

[13]Both of these equations can be derived in the Lee diagram. The second follows from the fact that the area under the line $\hat{r}_H$ is Area E + (Area Q + Area Z), so that $\hat{r}_H = \psi^*_{\hat{H}} + (1 - \phi^*_{\hat{H}})$.

the retro has no effect if one is already below the point at which the minimum premium will be charged.

For example, assume the minimum entry ratio $\hat{r}_H$ corresponds to \$400,000 in losses, there is an accident limit of \$100,000, and an insured had a single \$250,000 accident. The insured will pay the minimum premium whether the full \$250,000 enters the retro calculation or the accident limited to \$100,000 enters the retro calculation. In this case the insured gained no benefit from the accident limit. Yet the ELF is based on the loss elimination ratio k, which includes as part of its average this \$150,000 reduction. Thus the insured is being charged for something which provides no benefit. Area $Q = \psi^*_{\hat{H}} - \psi_{\hat{H}}$ quantifies this overcharge to the insured.

10. ERROR TERM, SUMMARY

The error due to the separate use of Table M and ELFs has four terms:

$$\begin{aligned} \text{Error} &= (\text{Area } M - \text{Area } V) - (\text{Area } D - \text{Area } Q) \\ &= \{\phi_G - (\phi^*_{\hat{G}} - k)\} - \{\phi_H - (\phi^*_{\hat{H}} - k)\} \\ &= \{\phi_G - (\phi^*_{\hat{G}} - k)\} - (\psi_H - \psi^*_{\hat{H}}). \end{aligned}$$

The first two terms are related to the maximum premium and the second two terms are related to the minimum premium. In actual applications, we expect to find generally that the error is a difference of two positive terms with the first one being larger, resulting in a positive error. In general, we expect an overcharge to the insured.

In the particular example here, the error = .0780 – .0427 = 3.53% of expected losses.

It may also be useful to rewrite the error as

$$\text{Error} = (\phi_G - \psi_H) - (\phi^*_{\hat{G}} - k - \psi^*_{\hat{H}}).$$

In order to correct for this error one would remove the net Table M insurance charge $\phi_G - \psi_H$ and substitute the net Table L insurance charge $\phi^*_{\hat{G}} - \psi^*_{\hat{H}}$, excluding the charge for the loss limitation k, at a lower set of entry ratios $\hat{r}_G$ and $\hat{r}_H$.

11. CONCLUSION

The graphical methods in Lee have been used to demonstrate how to quantify the error that would result from a separate use of Table M and Excess Loss Factors. This error usually represents a net overcharge to the insured. There are two main concepts that are responsible for this error. First, the effect of the maximum or minimum premiums each interact with the effect of an accident limit; one must be careful to not count the same effect twice. Secondly, the addition of an accident limit charge into the retrospective rating formula lowers the entry ratios corresponding to the maximum and minimum premiums.

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ALLOCATED LOSS ADJUSTMENT EXPENSE LIABILITIES

RUTH E. SALZMANN

Abstract

This paper sets forth a simple, practical, and straightforward method of establishing liabilities for allocated loss adjustment expenses (ALAE). With a minimum of judgment, the process flows smoothly from main frame computer input data, to the actuary's spreadsheet, to the answer. For this reason, a monthly update is easy to produce, which makes it possible to reflect changes in level earlier and less abruptly than with less frequent reviews. This fluid process produces total ALAE liabilities by coverage that recognize the monthly aging progression of the component liabilities by accident year (including the stub periods for the latest accident year).

Most methodologies for quantifying ALAE liabilities are based upon measurable relationships between loss and ALAE; they are multiplicative processes.[1] These relationships are expressed as ratios of ALAE to losses by coverage by accident year, on either an incurred/incurred basis or an unpaid/unpaid basis. When incurred/incurred ratios are used, the ratios produce estimated ALAE incurred dollars, and the ALAE liabilities are derived by subtraction. When unpaid/unpaid ratios are used, the ratios produce the ALAE liabilities directly.

The underlying principle in these multiplicative processes is the following: "Because the smaller and easier claims (which are

[1]Less common methods are these: (1) when the loss and ALAE liabilities are estimated on a combined basis, the combined liability is allocated between the two on a basis that is consistent with historical relationships, and (2) when individual ALAE claim-file estimates are available, the ALAE liabilities may be established independently, using reserving methodologies to derive the bulk estimates needed for unreported ALAE.

settled faster) require proportionately less ALAE, the ratio of paid ALAE to paid losses generally increases with age of development" [1, p.6]. For this reason, the ratios are applied by accident year. When incurred/incurred ratios are used, this principle is not evident in the ratios, but is apparent in the resulting liability comparisons. When unpaid/unpaid ratios are used, this principle governs the estimating process.

This paper is not a critique of methodologies for estimating ALAE liabilities. Its purpose is to introduce a simplified application of sound methodology. Simplified procedures generally enjoy the advantages of faster compilations or unsophisticated computer adaptations, or both, which make it easier to frequently update the estimates. This application has these advantages.

When unpaid/unpaid ratios are used to estimate the ALAE liabilities, the estimated ratios are generally derived in one of two ways: (1) using restated unpaid/unpaid ratios from prior accident years at the same age of development, or (2) using age-adjusted calendar year paid/paid ratios [1, pp. 98–111]. The latter basis is used in this simplified procedure. It is particularly appropriate for a simple procedure because there are no estimates in paid/paid ratios.

Age-adjusted calendar year paid/paid ratios are derived by adjusting calendar year paid data to reflect only payments *subsequent to* specified accident year ages. (In relatively mature operations, the mix by age in the age-adjusted calendar year data should approximate the expected mix by age in the liabilities.) The procedure in this paper derives the age-adjusted paid data through successive subtractions of data younger than the specified accident year ages. Remainders are produced after each accident year subtraction, starting with the latest (least mature) accident year and ending with the eleventh latest accident year. These eleven sets of "subsequent-to" remainders for loss and ALAE produce the age-adjusted paid/paid ratios that correspond to the expected mix by age in the respective liabilities.

Because data for the latest calendar period are used, these ratios reflect current ALAE/loss payout relationships. Barring unusual circumstances, the estimated unpaid/unpaid ratios should at least equal these levels. Otherwise, the ALAE/loss relationship in the liabilities would be less than current payment ratios. The use of lower ratios would be justified only when singular settlements distort the data. In this event, a better choice would be to adjust the paid data. The use of higher ratios may be justified under special situations as well. For ongoing situations, however, it is reasonable to assume a continuation of the current paid/paid relationships. If so, the unpaid/unpaid ratios will equal the age-adjusted paid/paid ratios, and the resulting ALAE liabilities will approximate the same level of adequacy that exists in the loss liabilities.

The use of age-adjusted paid/paid ratios is not a common methodology, probably due to the fact that the published material on their derivation is rather complicated [1, pp. 197–199]. This paper intends to change that. Exhibits 1 through 4 illustrate the calculation of age-adjusted paid/paid ratios and their use in estimating ALAE labilities at both a year-end and interim evaluation date. A brief explanation of these exhibits follows:

1. Exhibit 1 shows the historical calendar year paid data in the accident year detail necessary to calculate age-adjusted paid/paid ratios as of July 31, 1994. (Because this exhibit includes the data needed as of December 31, 1993, a separate December 31, 1993 exhibit is unnecessary.) The exhibit includes data for the latest 36 months. Shorter calendar periods can be used if the data are sufficiently credible to do so. In the completion of each new exhibit, only the data for the latest calendar year are added; prior data are posted from the prior exhibits.

2. Exhibits 2 and 3 illustrate the calculation of the age-adjusted paid/paid ratios. Exhibit 2 shows the format used as of any year end. (December 31, 1993 is illus-

trated.) Exhibit 3 shows the format used as of any stub period. (July 31, 1994 is illustrated.) Line 1 includes the calendar year paid data for all accident years, producing the unadjusted paid/paid ratio for the latest 36 months. This ratio is informational, but it is interesting to compare this ratio with those that are age-adjusted. The subsequent lines illustrate the successive subtractions necessary to produce the age-adjusted paid/paid ratios. These ratios reflect the payment activity subsequent to the ages of the individual accident year components.

3. Exhibit 4 illustrates the calculation of ALAE liabilities as of December 31, 1993 and July 31, 1994. There is nothing new in this format. The ALAE labilities are derived by multiplying the loss liability for each accident year by the appropriate unpaid/unpaid ratio. As discussed earlier, the assumption in this calculation is that current age-adjusted paid/paid relationships will continue. Thus the unpaid/unpaid ratios will be those produced in Exhibits 2 and 3. These ratios can be transferred to Exhibit 4 generally without adjustment. Adjustments are necessary only when the ratios are believed to be inconsistent with the underlying principle that paid/paid ratios should not decrease as the age of development increases. Strictly interpreted, the principle applies to paid accumulations on closed claims only. When paid accumulations on both open and closed claims are used, explainable decreases can result. Most decreases, however, are likely to be the random behavior of data that are not fully credible. Thus, unless there is a continuing pattern of decreasing ratios, it is prudent to apply the principle and override any decreases that occur.[2] Two such overrides were made in

[2]For the purist who has data that include inventories of partial payments (ALAE and loss) on open claims, adjustments can be made to the paid data to produce aged paid-to-paid ratios on closed claims. These ratios are applied to gross loss reserves (which include partial payments), producing gross ALAE reserves. Net ALAE reserves are derived by subtracting partial ALAE payments on open claims.

Exhibit 4 and have been noted with an asterisk. After Exhibit 4 is completed, it is interesting to compare the liability/liability ratio for all accident years on the "Total" line with the unadjusted paid/paid ratio for all accident years on Line 1 in either Exhibit 2 or Exhibit 3. The difference, which is caused by the different mix by age in the two sets of data, emphasizes the importance of reflecting such differences when establishing the ALAE liabilities.

In conclusion, this paper provides a simple application of a sophisticated methodology for estimating ALAE liabilities. Because of its simplicity, the calculation can be made more frequently. The increased frequency creates a smooth change from evaluation date to evaluation date. By using updated data as frequently as monthly, one can see how easily this application could solve the problems of estimating the ALAE labilities for the latest accident year as it progresses from January to December. Because of its simplicity, this application can also serve as a means of testing the sufficiency of ALAE liabilities produced from other methodologies.

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EXHIBIT 1

PART 1

PAID LOSS HISTORY
GENERAL LIABILITY

Accident Year	1991 Calendar Period 7 Months	1991 Calendar Period 12 Months	1992 Calendar Period 7 Months	1992 Calendar Period 12 Months	1993 Calendar Period 7 Months	1993 Calendar Period 12 Months	1994 Calendar Period 7 Months
A/O	$ 947,102	$ 2,704,048	$ (163,886)	$ 823,924	$ 984,700	$ 2,906,899	$456,752
1981	(41,296)	(37,594)					
1982	1,016,202	1,097,762	(38,070)	2,091			
1983	968,230	1,099,984	(42,434)	(37,434)	163,685	177,941	
1984	954,099	1,096,096	569,818	640,983	178,477	1,047,694	145,628
1985	839,299	1,471,917	401,645	1,371,431	456,573	1,627,020	52,548
1986	2,668,201	3,549,585	1,304,505	1,569,400	380,483	408,226	77,379
1987	2,315,101	5,132,868	2,719,031	4,317,898	1,197,602	1,959,050	844,479
1988	2,000,265	3,466,625	1,185,204	2,074,387	2,510,750	3,158,500	660,135
1989	(30,294)	5,355,822	1,791,761	7,875,311	2,292,728	2,723,447	907,105
1990	1,119,225	1,907,138	1,152,013	2,503,478	2,136,841	3,181,337	4,198,602
1991	270,784	758,151	718,508	1,745,973	1,331,312	4,759,912	4,632,950
1992			366,651	882,847	556,904	1,162,908	1,522,449
1993					298,828	769,863	1,049,460
1994							271,518
TOTAL	$13,026,918	$27,602,402	$9,964,746	$23,770,289	$12,488,883	$23,882,797	$14,819,005

A/O = All Other Accident Years

EXHIBIT 1

PART 2

PAID ALAE HISTORY
GENERAL LIABILITY

Accident Year	1991 Calendar Period 7 Months	1991 Calendar Period 12 Months	1992 Calendar Period 7 Months	1992 Calendar Period 12 Months	1993 Calendar Period 7 Months	1993 Calendar Period 12 Months	1994 Calendar Period 7 Months
A/O	$ 851,690	$1,756,016	$ 521,645	$1,576,893	$1,125,830	$ 2,346,194	$1,101,555
1981	79,814	184,644					
1982	122,812	211,142	(39,777)	(35,926)			
1983	463,919	571,911	(36,363)	18,848	91,738	173,262	
1984	383,508	473,446	201,894	299,629	98,855	558,527	155,395
1985	487,820	837,936	337,515	665,911	404,721	744,480	5,233
1986	392,329	711,551	151,463	328,793	164,931	452,717	41,968
1987	672,324	1,324,508	529,592	917,211	408,359	696,751	170,804
1988	610,831	1,246,852	551,261	1,022,315	463,708	894,786	387,995
1989	764,654	1,458,817	1,033,379	1,761,475	742,136	1,221,764	453,216
1990	131,875	430,200	533,914	1,084,508	795,941	1,468,178	544,522
1991	19,256	71,604	150,818	597,953	1,154,530	2,021,536	1,592,673
1992			37,602	107,404	170,310	518,405	437,834
1993					32,034	72,476	113,119
1994							17,170
TOTAL	$4,980,832	$9,278,627	$3,972,943	$8,345,014	$5,653,093	$11,169,076	$5,021,484

A/O = All Other Accident Years

EXHIBIT 2

PART 1

AGED CALENDAR YEAR PAID RATIOS
ALAE DIVIDED BY LOSS
GENERAL LIABILITY
AS OF 12/31/93

		(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
		Calendar Year Paid ALAE ($000)				Calendar Year Paid Loss ($000)				Ratio
	Accident Year	1991	1992	1993	(a+b+c)	1991	1992	1993	(e+f+g)	(d)/(h)
1	Total	9,279	8,345	11,169	28,793	27,602	23,770	23,883	75,255	0.383
		detailed by accident year*				detailed by accident year*				
2	n	72	107	72	251	758	883	770	2,411	
3	Aged 1 Yr (1–2)	9,207	8,238	11,097	28,542	26,844	22,887	23,113	72,844	0.392
4	n–1	430	598	518	1,546	1,907	1,746	1,163	4,816	
5	Aged 2 Yrs (3–4)	8,777	7,640	10,579	26,996	24,937	21,141	21,950	68,028	0.397
6	n–2	1,459	1,085	2,022	4,566	5,356	2,504	4,760	12,620	
7	Aged 3 Yrs (5–6)	7,318	6,555	8,557	22,430	19,581	18,637	17,190	55,408	0.405
8	n–3	1,247	1,761	1,468	4,476	3,466	7,875	3,181	14,522	
9	Aged 4 Yrs (7–8)	6,071	4,794	7,089	17,954	16,115	10,762	14,009	40,886	0.439
10	n–4	1,324	1,022	1,222	3,568	5,133	2,074	2,724	9,931	
11	Aged 5 Yrs (9–10)	4,747	3,772	5,867	14,386	10,982	8,688	11,285	30,955	0.465

* where n = latest accident year; n–1 = second latest accident year; etc.

EXHIBIT 2

PART 2

Aged Calendar Year Paid Ratios
ALAE Divided by Loss
General Liability
as of 12/31/93

		(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
		Calendar Year Paid ALAE ($000)				Calendar Year Paid Loss ($000)				Ratio
	Accident Year	1991	1992	1993	(a+b+c)	1991	1992	1993	(e+f+g)	(d)/(h)
12	n–5	712	917	895	2,524	3,550	4,318	3,159	11,027	
13	Aged 6 Yrs (11–12)	4,035	2,855	4,972	11,862	7,432	4,370	8,126	19,928	0.595
14	n–6	838	329	697	1,864	1,472	1,569	1,959	5,000	
15	Aged 7 Yrs (13–14)	3,197	2,526	4,275	9,998	5,960	2,801	6,167	14,928	0.670
16	n–7	473	666	453	1,592	1,096	1,371	408	2,875	
17	Aged 8 Yrs (15–16)	2,724	1,860	3,822	8,406	4,864	1,430	5,759	12,053	0.697
18	n–8	572	300	745	1,617	1,100	641	1,627	3,368	
19	Aged 9 Yrs (17–18)	2,152	1,560	3,077	6,789	3,764	789	4,132	8,685	0.782
20	n–9	211	19	558	788	1,098	(37)	1,047	2,108	
21	Aged 10 Yrs (19–20)	1,941	1,541	2,519	6,001	2,666	826	3,085	6,577	0.912
22	n–10	185	(36)	173	322	(38)	2	178	142	
23	Aged 11 Yrs (21–22)	1,756	1,577	2,346	5,679	2,704	824	2,907	6,435	0.883

* where n = latest accident year; n–1 = second latest accident year; etc.

EXHIBIT 3

PART 1

Aged Calendar Year Paid Ratios
ALAE Divided by Loss
General Liability
as of 7/31/94

		(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)	(l)	(m)	(n)	(o)
		Calendar Year Paid ALAE ($000)							Calendar Year Paid Loss ($000)							
		1991	1992		1993		1994		1991	1992		1993		1994		
		Calendar Months						(a+b+c+	Calendar Months						(h+i+j+	Ratio
	Accident Year	last 5	first 7	last 5	first 7	last 5	first 7	d+e+f)	last 5	first 7	last 5	first 7	last 5	first 7	k+l+m)	(g)/(n)
1	Total	4,298	3,973	4,372	5,653	5,516	5,021	28,833	14,575	9,965	13,805	12,489	11,394	14,819	77,047	0.374
		detailed by accident year*							detailed by accident year*							
2	n		37		32		17	86		367		299		272	938	
3	Aged 7 Mos (1–2)	4,298	3,936	4,372	5,621	5,516	5,004	28,747	14,575	9,598	13,805	12,190	11,394	14,547	76,109	0.378
4	n	53		70		40		163	487		516		471		1,474	
5	n–1		151		170		113	434		719		557		1,049	2,325	
6	Aged 19 Mos (3– 4–5)	4,245	3,785	4,302	5,451	5,476	4,891	28,150	14,088	8,879	13,289	11,633	10,923	13,498	72,310	0.389
7	n–1	298		447		348		1,093	788		1,027		606		2,421	
8	n–2		534		1,154		438	2,126		1,152		1,331		1,522	4,005	
9	Aged 31 Mos (6–7–8)	3,947	3,251	3,855	4,297	5,128	4,453	24,931	13,300	7,727	12,262	10,302	10,317	11,976	65,884	0.378
10	n–2	694		551		867		2,112	5,386		1,352		3,429		10,167	
11	n–3		1,033		796		1,593	3,422		1,792		2,137		4,633	8,562	
12	Aged 43 Mos (9–10–11)	3,253	2,218	3,304	3,501	4,261	2,860	19,397	7,914	5,935	10,910	8,165	6,888	7,343	47,155	0.411
13	n–3	636		728		672		2,036	1,466		6,083		1,044		8,593	
14	n–4		551		742		544	1,837		1,185		2,293		4,199	7,677	
15	Aged 55 Mos (12–13–14)	2,617	1,667	2,576	2,759	3,589	2,316	15,524	6,448	4,750	4,827	5,872	5,844	3,144	30,885	0.503

* where n = latest accident year; n–1 = second latest accident year; etc.

EXHIBIT 3

PART 2

AGED CALENDAR YEAR PAID RATIOS
ALAE DIVIDED BY LOSS
GENERAL LIABILITY
AS OF 7/31/94

		(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)	(l)	(m)	(n)	(o)
		Calendar Year Paid ALAE ($000)							Calendar Year Paid Loss ($000)							
		1991	1992		1993		1994		1991	1992		1993		1994		
		Calendar Months						(a+b+c+	Calendar Months						(h+i+j+	Ratio
	Accident Year	last 5	first 7	last 5	first 7	last 5	first 7	d+e+f)	last 5	first 7	last 5	first 7	last 5	first 7	k+l+m)	(g)/(n)
16	n–4	652		471		480		1,603	2,818		889		431		4,138	
17	n–5		529		464		453	1,446		2,719		2,511		907	6,137	
18	Aged 67 Mos (15–16–17)	1,965	1,138	2,105	2,295	3,109	1,863	12,475	3,630	2,031	3,938	3,361	5,413	2,237	20,610	0.605
19	n–5	319		388		431		1,138	881		1,599		648		3,128	
20	n–6		152		408		388	948		1,304		1,198		660	3,162	
21	Aged 79 Mos (18–19–20)	1,646	986	1,717	1,887	2,678	1,475	10,389	2,749	727	2,339	2,163	4,765	1,577	14,320	0.725
22	n–6	350		177		289		816	633		265		761		1,659	
23	n–7		338		165		171	674		401		380		845	1,626	
24	Aged 91 Mos (21–22–23)	1,296	648	1,540	1,722	2,389	1,304	8,899	2,116	326	2,074	1,783	4,004	732	11,035	0.806
25	n–7	90		328		288		706	142		970		28		1,140	
26	n–8		202		405		42	649		570		457		77	1,104	
27	Aged 103 Mos (24–25–26)	1,206	446	1,212	1,317	2,101	1,262	7,544	1,974	(244)	1,104	1,326	3,976	655	8,791	0.858
28	n–8	108		98		340		546	132		71		1,170		1,373	
29	n–9		(36)		99		5	68		(42)		178		53	189	
30	Aged 115 Mos (27–28–29)	1,098	482	1,114	1,218	1,761	1,257	6,930	1,842	(202)	1,033	1,148	2,806	602	7,229	0.959
31	n–9	89		55		459		603	81		5		869		955	
32	n–10		(40)		92		155	207		(38)		163		145	270	
33	Aged 127 Mos (30–31–32)	1,009	522	1,059	1,126	1,302	1,102	6,120	1,761	(164)	1,028	985	1,937	457	6,004	1.019

* where n = latest accident year; n–1 = second latest accident year; etc.

EXHIBIT 4

ILLUSTRATIONS OF THE CALCULATION OF ALAE LIABILITIES GENERAL LIABILITY USING THE AGED PAID-TO-PAID RATIOS IN EXHIBITS 2 AND 3 ($000)

As of December 31, 1993				As of July 31, 1994			
	(1)	(2)	(3)		(4)	(5)	(6)
Acc. Year	Loss Liability	Aged Ratio (Exh. 2)	ALAE Liability (1)×(2)	Acc. Year	Loss Liability	Aged Ratio (Exh. 3)	ALAE Liability (4)×(5)
≤1983	21,359	.912*	19,479	≤1984	25,916	1.019	26,408
1984	4,446	.912	4,055	1985	5,585	.959	5,356
1985	5,490	.782	4,293	1986	6,581	.858	5,646
1986	6,099	.697	4,251	1987	7,446	.806	6,001
1987	8,068	.670	5,406	1988	10,095	.725	7,319
1988	9,302	.595	5,535	1989	14,348	.605	8,681
1989	15,308	.465	7,118	1990	15,511	.503	7,802
1990	19,656	.439	8,629	1991	18,186	.411	7,474
1991	21,730	.405	8,801	1992	18,666	.389*	7,261
1992	22,337	.397	8,868	1993	19,657	.389	7,647
1993	20,384	.392	7,991	1994	12,363	.378	4,673
Total	154,179	.548**	84,426	Total	154,354	.611**	94,268

* Manually adjusted so as not to be less than the next subsequent aged ratio.
** Calculated after Totals are established.

AN INTRODUCTION TO MARKOV CHAIN MONTE CARLO METHODS AND THEIR ACTUARIAL APPLICATIONS

DAVID P. M. SCOLLNIK

Department of Mathematics and Statistics
University of Calgary

Abstract

This paper introduces the readers of the Proceedings *to an important class of computer based simulation techniques known as Markov chain Monte Carlo (MCMC) methods. General properties characterizing these methods will be discussed, but the main emphasis will be placed on one MCMC method known as the Gibbs sampler. The Gibbs sampler permits one to simulate realizations from complicated stochastic models in high dimensions by making use of the model's associated full conditional distributions, which will generally have a much simpler and more manageable form. In its most extreme version, the Gibbs sampler reduces the analysis of a complicated multivariate stochastic model to the consideration of that model's associated univariate full conditional distributions.*

In this paper, the Gibbs sampler will be illustrated with four examples. The first three of these examples serve as rather elementary yet instructive applications of the Gibbs sampler. The fourth example describes a reasonably sophisticated application of the Gibbs sampler in the important arena of credibility for classification ratemaking via hierarchical models, and involves the Bayesian prediction of frequency counts in workers compensation insurance.

1. INTRODUCTION

The purpose of this paper is to acquaint the readership of the *Proceedings* with a class of simulation techniques known as Markov chain Monte Carlo (MCMC) methods. These methods permit a practitioner to simulate a dependent sequence of random draws from very complicated stochastic models. The main emphasis will be placed on one MCMC method known as the Gibbs sampler. It is not an understatement to say that several hundred papers relating to the Gibbs sampling methodology have appeared in the statistical literature since 1990. Yet, the Gibbs sampler has made only a handful of appearances within the actuarial literature to date. Carlin [3] used the Gibbs sampler in order to study the Bayesian state-space modeling of non-standard actuarial time series, and Carlin [4] used it to develop various Bayesian approaches to graduation. Klugman and Carlin [19] also used the Gibbs sampler in the arena of Bayesian graduation, this time concentrating on a hierarchical version of Whittaker-Henderson graduation. Scollnik [24] studied a simultaneous equations model for insurance ratemaking, and conducted a Bayesian analysis of this model with the Gibbs sampler.

This paper reviews the essential nature of the Gibbs sampling algorithm and illustrates its application with four examples of varying complexity. This paper is primarily expository, although references are provided to important theoretical results in the published literature. The reader is presumed to possess at least a passing familiarity with the material relating to statistical computing and stochastic simulation present in the syllabus for CAS Associateship Examination Part 4B. The theoretical content of the paper is mainly concentrated in Section 2, which provides a brief discussion of Markov chains and the properties of MCMC methods. Except for noting Equations 2.1 and 2.2 along with their interpretation, the reader may skip over Section 2 the first time through reading this paper. Section 3 formally introduces the Gibbs sampler and illustrates it with an example. Section 4 discusses some of the practical considerations related to the implementation of a Gibbs sampler. In Section 5, some aspects of Bayesian inference using Gibbs sampling are considered, and two final examples are presented. The first of these concerns the Bayesian estimation of the parameter for a size of loss distribution when grouped data are observed. The second addresses credibility for classification ratemaking via hierarchical models and involves the Bayesian prediction of frequency counts in workers compensation insurance. In Section 6 we conclude our presentation and point out some areas of application to be explored in the future.

Since the subject of MCMC methods is still foreign to most actuaries at this time, we will conclude this section with a simple introductory example, which we will return to in Section 3.

Example 1

This example starts by recalling that a generalized Pareto distribution can be constructed by mixing one gamma distribution with another gamma distribution in a certain manner. (See for example, Hogg and Klugman [16, pp. 53–54].) More precisely, if a loss random variable X has a conditional gamma (k,θ) distribution with density

$$f(x \mid \theta) = \frac{\theta^k}{\Gamma(k)} x^{k-1} \exp(-\theta x), \qquad 0 < x < \infty, \qquad (1.1)$$

and the mixing random variable θ has a marginal gamma (α, λ) distribution with density

$$f(\theta) = \frac{\lambda^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} \exp(-\lambda\theta), \qquad 0 < \theta < \infty,$$

then X has a marginal generalized Pareto (α, λ, k) distribution with density

$$f(x) = \frac{\Gamma(\alpha+k)\lambda^\alpha x^{k-1}}{\Gamma(\alpha)\Gamma(k)(\lambda+x)^{\alpha+k}}, \qquad 0 < x < \infty.$$

It also follows that the conditional distribution of θ given X is also given by a gamma distribution, namely,

$$f(\theta \mid x) \sim \text{gamma } (\alpha + k, \lambda + x), \qquad 0 < \theta < \infty. \qquad (1.2)$$

We will perform the following iterative sampling algorithm, which is based upon the conditional distributions appearing in Equations 1.1 and 1.2:

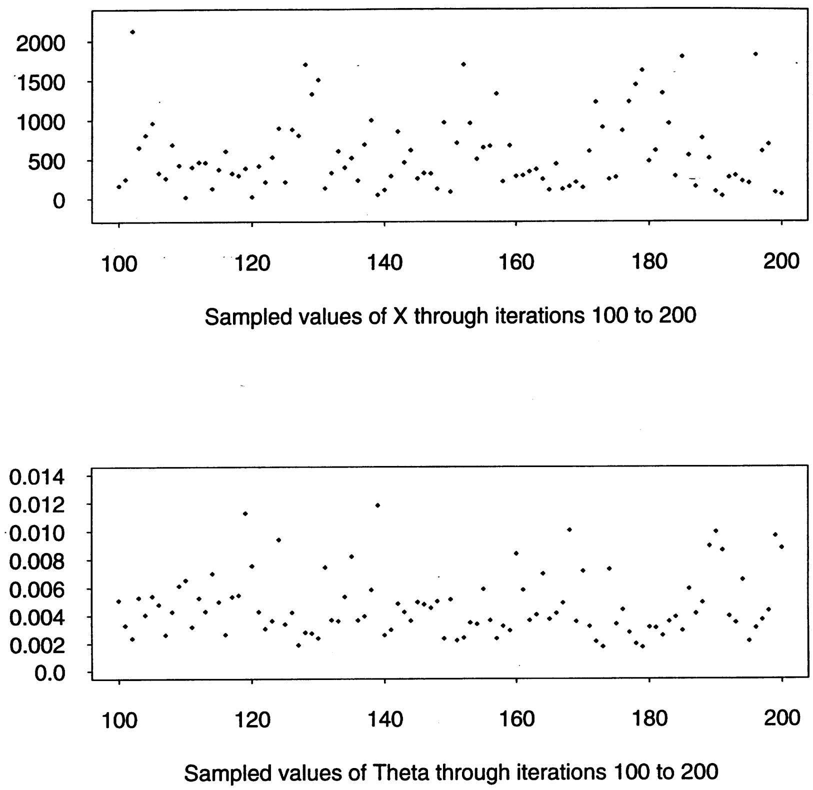
1. Select arbitrary starting values $X^{(0)}$ and $\theta^{(0)}$.
2. Set the counter index $i = 0$.
3. Sample $X^{(i+1)}$ from $f(x \mid \theta^{(i)}) \sim \text{gamma } (k, \theta^{(i)})$.
4. Sample $\theta^{(i+1)}$ from $f(\theta \mid X^{(i+1)}) \sim \text{gamma } (\alpha + k, \lambda + X^{(i+1)})$.
5. Set $i \leftarrow i + 1$ and return to Step 3.

For the sake of illustration, we assigned the model parameters $\alpha = 5$, $\lambda = 1000$ and $k = 2$ so that the marginal distribution of θ is gamma $(5, 1000)$ with mean 0.005 and the marginal distribution of X is generalized Pareto $(5, 1000, 2)$ with mean 500. We then ran the algorithm described above on a fast computer for a total of 500 iterations and stored the sequence of generated values $X^{(0)}, \theta^{(0)}, X^{(1)}, \theta^{(1)}, \ldots, X^{(499)}, \theta^{(499)}, X^{(500)}, \theta^{(500)}$. It must be emphasized that this sequence of random draws is clearly not independent, since $X^{(1)}$ depends upon $\theta^{(0)}$, $\theta^{(1)}$ depends upon $X^{(1)}$, and so forth. Our two starting values were arbitrarily selected to be $X^{(0)} = 20$ and $\theta^{(0)} = 10$. The sequence of sampled values for $X^{(i)}$ is plotted in Figure 1, along with the sequence of sampled values for $\theta^{(i)}$, for iterations 100 through 200. Both sequences do appear to be random, and some dependencies between successive values are discernible in places.

In Figure 2, we plot the histograms of the last 500 values appearing in each of the two sequences of sampled values (the starting values $X^{(0)}$ and $\theta^{(0)}$ were discarded at this point).

FIGURE 1

SAMPLE PATHS FOR $X^{(i)}$ AND $\theta^{(i)}$ IN EXAMPLE 1

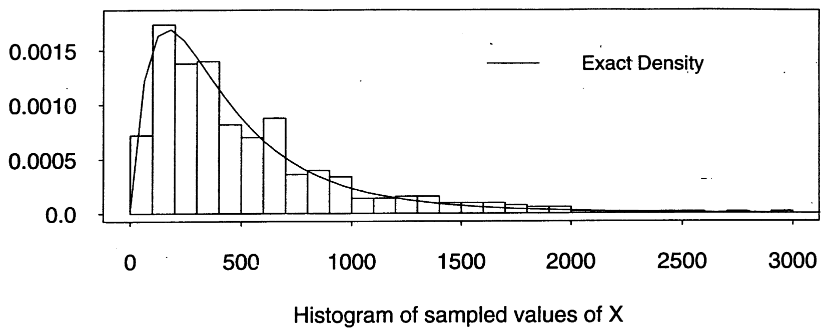


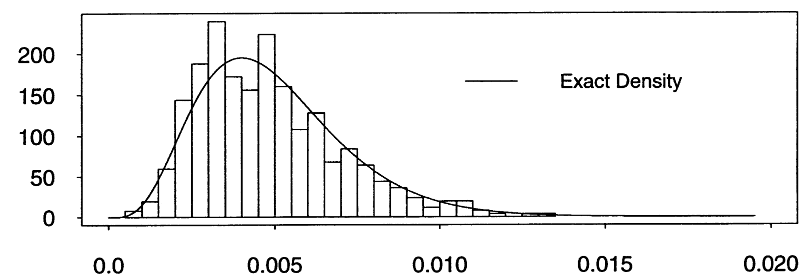
In each plot, we also overlay the actual density curve for the marginal distribution of either X or θ. Surprisingly, the dependent sampling scheme we implemented, which was based upon the full conditional distributions $f(\theta \mid x)$ and $f(x \mid \theta)$, appears to have generated random samples from the underlying marginal distributions.

Now, notice that the marginal distribution of X may be interpreted as the average of the conditional distribution of X given

FIGURE 2

HISTOGRAMS OF SAMPLE VALUES FOR X AND θ IN EXAMPLE 1





θ taken with respect to the marginal distribution of θ; that is,

$$f(x) = \int f(x \mid \theta) f(\theta) d\theta.$$

Since the sampled values of $\theta^{(i)}$ appear to constitute a random sample of sorts from the marginal distribution of θ, this suggests that a naive estimate of the value of the marginal density function for X at the point x might be constructed by taking the empirical average of $f(x \mid \theta^{(i)})$ over the sampled values for $\theta^{(i)}$. If $\theta^{(1)} =$

0.0055, for example, then

$$f(x \mid \theta^{(1)}) = 0.0055^2 x \exp(-0.0055x).$$

One does a similar computation for the other values of $\theta^{(i)}$ and averages to get

$$\hat{f}(x) = \frac{1}{500} \sum_{i=1}^{500} f(x \mid \theta^{(i)}). \tag{1.3}$$

Similarly, we might construct

$$\hat{f}(\theta) = \frac{1}{500} \sum_{i=1}^{500} f(\theta \mid X^{(i)}) \tag{1.4}$$

as a density estimate of $f(\theta)$. These estimated density functions are plotted in Figure 3 along with their exact counterparts, and it is evident that the estimated densities happen to be excellent.

2. MARKOV CHAIN MONTE CARLO

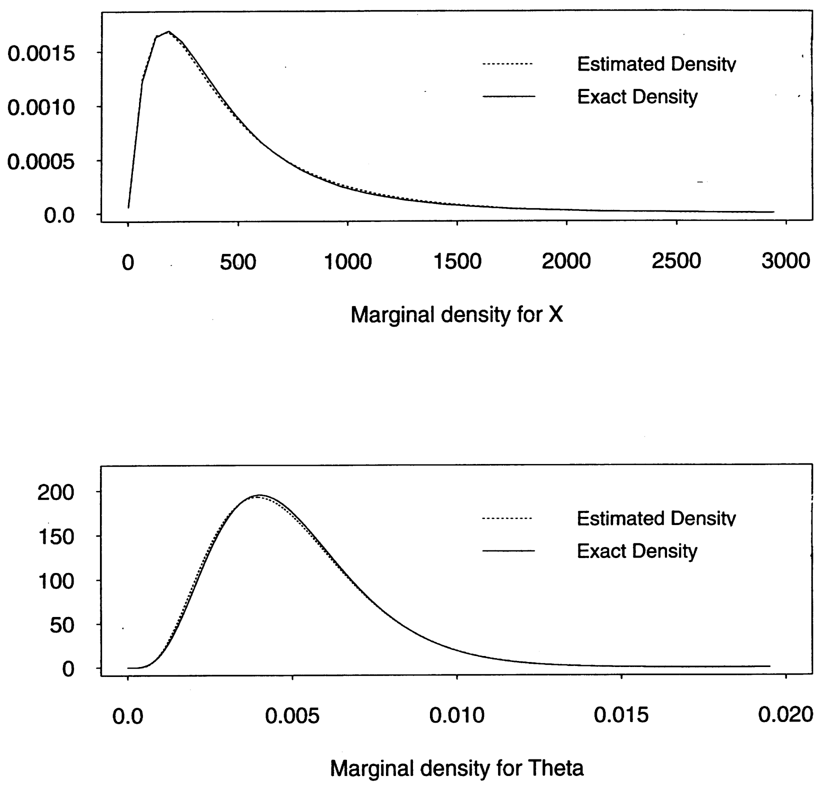
In the example of the previous section, we considered an iterative simulation scheme that generated two dependent sequences of random variates. Apparently, we were able to use these sequences in order to capture characteristics of the underlying joint distribution that defined the simulation scheme in the first place. In this section, we will discuss a few properties of certain simulation schemes that generate dependent sequences of random variates and note in what manner these dependent sequences may be used for making useful statistical inference. The main results are given by Equations 2.1 and 2.2.

Before we begin, it may prove useful to quickly, and very informally, review some elementary Markov chain theory. Tierney [31] provides a much more detailed and rigorous discussion of this material. A Markov chain is just a collection of random variables $\{X_n;\ n \geq 0\}$, with the distribution of the random variables on some space › $\subseteq R^k$ governed by the transition probabilities

$$\Pr(X_{n+1} \in A \mid X_0, \ldots, X_n) = K(X_n, A),$$

FIGURE 3

ESTIMATED AND EXACT MARGINAL DENSITIES FOR X AND θ IN EXAMPLE 1



where $A \subset$ › . Notice that the probability distribution of the next random variable in the sequence, given the current and past states, depends only upon the current state. This is known as the Markov property. The distribution of X_0 is known as the initial distribution of the Markov chain. The conditional distribution of X_n given X_0 is described by

$$\Pr(X_n \in A \mid X_0) = K^n(X_0, A),$$

where K^n denotes the nth application of K. An invariant distribution $\pi(x)$ for the Markov chain is a density satisfying

$$\pi(A) = \int K(x, A)\, \pi(x)\, dx,$$

and it is also an equilibrium distribution if

$$\lim_{n \to \infty} K^n(x, A) = \pi(A).$$

For simplicity, we are using the notation $\pi(x)$ to identify both the distribution and density for a random variable, trusting the precise meaning to be evident from the context. A Markov chain with invariant distribution $\pi(x)$ is irreducible if it has a positive probability of entering any state assigned positive probability by $\pi(x)$, regardless of the initial state or value of X_0. A chain is periodic if it can take on certain values only at regularly spaced intervals, and is aperiodic otherwise. If a Markov chain with a proper invariant distribution is both irreducible and aperiodic, then the invariant distribution is unique and it is also the equilibrium distribution of the chain.

A MCMC method is a sampling based simulation technique that may be used in order to generate a dependent sample from a certain distribution of interest. Formally, a MCMC method proceeds by first specifying an irreducible and aperiodic Markov chain with a unique invariant distribution $\pi(x)$ equal to the desired distribution of interest (or target distribution). Curiously, there are usually a number of easy ways in which to construct such a Markov chain. The next step is to simulate one or more realizations of this Markov chain on a fast computer. Each path of simulated values will form a dependent random sample from the distribution of interest, provided that certain regularity conditions are satisfied. Then these dependent sample paths may be utilized for inferential purposes in a variety of ways. In particular, if the Markov chain is aperiodic and irreducible, with unique invariant distribution $\pi(x)$, and $X^{(1)}, X^{(2)}, \ldots,$ is a realization of this chain, then known asymptotic results (e.g., Tierney [31] or Roberts and Smith [23]) tell us that:

$$X^{(t)} \xrightarrow{d} X \sim \pi(x) \qquad \text{as} \quad t \to \infty, \tag{2.1}$$

and

$$\frac{1}{t}\sum_{i=1}^{t} h(X^{(i)}) \to \mathrm{E}_{\pi}[h(X)] \qquad \text{as} \quad t \to \infty, \ \textit{almost surely}. \tag{2.2}$$

Equation 2.1 indicates that as t becomes moderately large, the value $X^{(t)}$ is very nearly a random draw from the distribution of interest. In practice, a value of $t \approx 10$ to 15 is often more than sufficient. This result also allows us to generate an approximately independent random sample from the distribution with density $f(x)$ by using only every kth value appearing in the sequence. The value of k should be taken to be large enough so that the sample autocorrelation function coefficients for the values appearing in the subsequence are reminiscent of those for a purely random process or a stochastically independent sequence, that is, until there are no significant autocorrelations at non-zero lags. This idea is illustrated in Example 2. Autocorrelation functions are covered in some depth in the course of reading for Associateship Examination Part 3A, *Applied Statistical Methods* (also see Miller and Wichern [21, pp. 333–337, 356–365]).

Equation 2.2 tells us that if h is an arbitrary π-integrable real-valued function of X, then the average of this function taken over the realized values of $X^{(t)}$ (the ergodic average of the function) converges (almost surely, as $t \to \infty$) to its expected value under the target density. In practice, usually the first 10 to 100 values of the simulation are discarded, in order to reduce the dependence of these estimates upon the selected starting values.

Notice that if $h(X)$ is taken to be the conditional density for some random variable Y given X, then Equation 2.2 suggests that the marginal density of Y may be estimated at the point y by averaging the conditional density $f(y \mid X)$ over the realized values $X^{(t)}$ (as in Gelfand and Smith [9, pp. 402–403]).

At this point, the reader is probably wondering how one would go about constructing a suitable Markov chain when a certain target density $\pi(x)$ is of interest. The so-called Gibbs sampler, a special kind of MCMC method, is one easy and very popular approach. The Gibbs sampler was introduced by Geman and Geman [11] in the context of image restoration, and its suitability for a wide range of problems in the field of Bayesian inference was recognized by Gelfand and Smith [9]. An elementary introduction to the Gibbs sampler is given in Casella and George [5], and those readers unfamiliar with the methodology are certainly encouraged to consult this reference. More sophisticated discussions of the Gibbs sampler and MCMC methods in general are given in Smith and Roberts [25], Tanner [29], and Tierney [31].

3. THE GIBBS SAMPLER

In order to formally introduce the Gibbs sampler, let us begin by letting the target distribution $\pi(x)$ now correspond to a joint distribution $\pi(x_1, x_2, \ldots, x_k)$. We assume that this joint distribution exists and is proper. Each of the x_i terms may represent either a single random variable or, more generally, a block of several random variables grouped together. Let $\pi(x_j)$ represent the marginal distribution of the jth block of variables, x_j, and let $\pi(x_j \mid x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_k)$ represent the full conditional distribution of the jth block of variables, given the remainder. Besag [2] observed that the collection of full conditional distributions uniquely determines the joint distribution, provided that the joint distribution is proper. The Gibbs sampler utilizes a set of full conditional distributions associated with the target distribution of interest in order to define a Markov chain with an invariant distribution equal to the target distribution. When we speak of a Gibbs sampler, we are actually referring to an implementation of the following iterative sampling scheme:

1. Select initial values $x^{(0)} = (x_1^{(0)}, x_2^{(0)}, \ldots, x_k^{(0)})$.
2. Set the counter index $i = 0$.
3. Simulate the sequence of random draws:

$$\begin{aligned}
x_1^{(i+1)} &\sim \pi(x_1 \mid x_2^{(i)}, \ldots, x_k^{(i)}),\\
x_2^{(i+1)} &\sim \pi(x_2 \mid x_1^{(i+1)}, x_3^{(i)}, \ldots, x_k^{(i)}),\\
x_3^{(i+1)} &\sim \pi(x_3 \mid x_1^{(i+1)}, x_2^{(i+1)}, x_4^{(i)}, \ldots, x_k^{(i)}),\\
&\vdots\\
x_k^{(i+1)} &\sim \pi(x_k \mid x_1^{(i+1)}, x_2^{(i+1)}, \ldots, x_{k-1}^{(i+1)}),
\end{aligned}$$

   and form

$$x^{(i+1)} = (x_1^{(i+1)}, x_2^{(i+1)}, \ldots, x_k^{(i+1)}).$$

4. Set $i \leftarrow i + 1$ and return to Step 3.

Notice that in Step 3 of the Gibbs sampling algorithm, we are required to sample random draws once from each of the full conditional distributions and that the values of the conditioning variables are sequentially updated, one by one. This sampling algorithm defines a valid MCMC method, and by its construction also ensures that the target distribution $\pi(x)$ is an invariant distribution of the Markov chain so defined (e.g., Tierney [31]). Mild regularity conditions (typically satisfied in practice) guarantee that Equations 2.1 and 2.2 will apply. Refer to Theorem 2 in Roberts and Smith [23] for one set of sufficient conditions. Notice that since Equation 2.1 implies that $x^{(i)}$ is very nearly a random draw from the joint distribution $\pi(x)$, it is also the case that each component $x_j^{(i)}$ is very nearly a random draw from the marginal distribution $\pi(x_j)$, for $j = 1,2,\ldots,k$ (provided throughout that i is sufficiently large). This is a useful result to note when simulation based inference is sought with respect to one or more of the marginal distributions.

Besag [2] observed the fact that the collection of full conditional distributions uniquely determines the joint distribution, provided that the joint distribution exists and is proper. However, it is not the case that a collection of proper full conditional distributions necessarily guarantees the *existence* of a proper joint distribution for the random variables involved. For example, note that

$$f(x_1,x_2) \propto \exp(-[x_1 + x_2]^2/2),$$

with $-\infty < x_1 < \infty$ and $-\infty < x_2 < \infty$, defines an improper joint distribution with two proper univariate normal full conditional distributions (Gelfand [8]). When a set of proper full conditional distributions fails to determine a proper joint distribution, any application of the Gibbs sampling algorithm to these full conditional distributions is to be avoided. If the Gibbs sampler was invoked under these circumstances, the algorithm may either fail to converge or else converge to a state that is not readily interpretable.

By now, perhaps the reader has noticed that the example presented in Section 1 really just amounted to an application of the Gibbs sampling algorithm to the two full conditional distributions $f(x \mid \theta)$ and $f(\theta \mid x)$ appearing in Equations 1.1 and 1.2. By construction, we ensured that the joint distribution $f(x,\theta)$ also existed as a proper distribution. From the discussion above, it follows that Equation 2.1 explains why the sequence of sampled values for $X^{(i)}$ and $\theta^{(i)}$ effectively constituted random samples from the marginal distributions of X and θ, respectively. Similarly, the two density estimates defined by Equations 1.3 and 1.4 performed as well as they did because of the result described by Equation 2.2.

We conclude this section with a second example, before discussing some of the practical issues relating to the implementation of a Gibbs sampler in Section 4.

Example 2

Consider the following distributional model:

$$f(y) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1}(1-y)^{\beta-1}, \qquad 0 \le y \le 1$$
$$\sim \text{beta } (\alpha,\beta); \tag{3.1}$$
$$f(n) = [\exp(\lambda)-1]^{-1}\frac{\lambda^n}{n!}, \qquad n = 1,2,\ldots$$
$$\sim \text{zero-truncated Poisson } (\lambda); \tag{3.2}$$
$$f(x \mid y,n) = \binom{n}{x} y^x(1-y)^{n-x}, \qquad x = 0,1,\ldots,n$$
$$\sim \text{binomial } (n,y). \tag{3.3}$$

We will assume that the random variables Y and N are independent, so that the proper joint distribution of X, Y, and Z obviously exists as the product of Equations 3.1, 3.2, and 3.3. In order to give the model above an actuarial interpretation, imagine that, conditional upon Y and N, the random variable X represents the number of policies generating a claim in a portfolio of N identical and independent policies, each with a claim probability equal to Y. A portfolio is characterized by the value of the parameters Y and N, which are random variables in their own right with independent beta and zero-truncated Poisson distributions, respectively. The marginal distribution of X describes the typical number of policies generating a claim in an arbitrary portfolio. Unfortunately, the marginal distribution of X cannot be obtained in a closed form. (The reader is invited to try.) In order to study the marginal distribution of X, we will consider an application of the Gibbs sampler.

For the model above, the following set of full conditional distributions may be derived in a straightforward fashion:

$$f(x \mid y, n) \sim \text{binomial } (n, y); \tag{3.4}$$

$$f(y \mid x, n) \sim \text{beta } (x + \alpha, n - x + \beta); \tag{3.5}$$

$$f(n \mid x, y) = \exp(-\lambda[1 - y]) \frac{(\lambda[1 - y])^{n-x}}{(n - x)!}, \quad n = x, x + 1, x + 2, \ldots \tag{3.6}$$

or

$$f(n - x \mid x, y) \sim \text{Poisson } (\lambda[1 - y]).$$

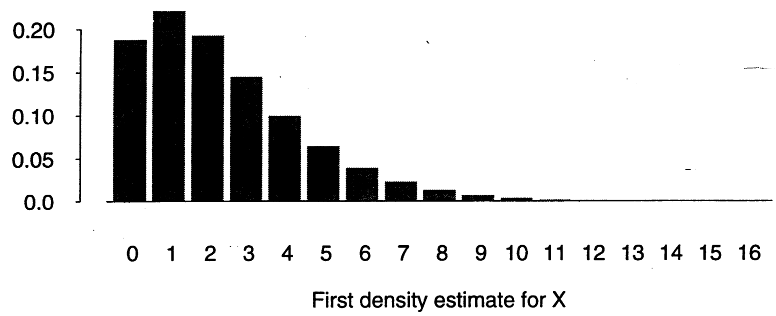
For the purpose of illustration, we set the model parameters equal to $\alpha = 2$, $\beta = 8$, and $\lambda = 12$, and initiated 5100 iterations of the Gibbs sampler using the full conditional distributions found in Equations 3.4, 3.5, and 3.6, with initial values $X^{(0)} = 4$, $Y^{(0)} = 0.5$, and $N^{(0)} = 50$. By averaging Equation 3.4 over the simulated values of $Y^{(i)}$ and $N^{(i)}$ in the spirit of Equation 2.2, after first discarding the initial 100 values in each sample path in order to 'burn-in' the Gibbs sampler and remove the effect of the starting values, a density estimate for the random variable X at the point x is given by the average of 5000 binomial distributions:

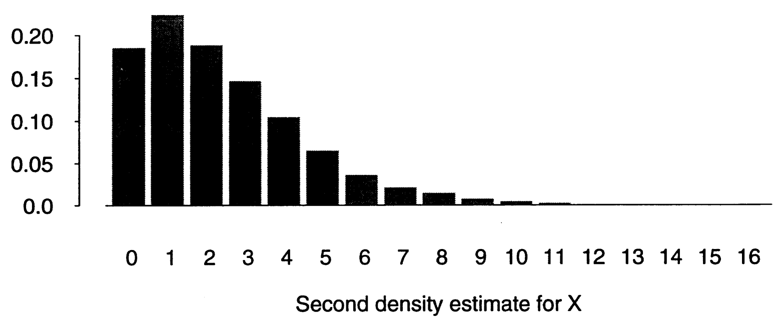
$$\hat{f}(x) = \frac{1}{5000} \sum_{i=101}^{5100} f(x \mid Y^{(i)}, N^{(i)}). \tag{3.7}$$

A plot of this density estimate appears in the upper half of Figure 4. For comparison, we also constructed a histogram estimate of the density for the random variable X on the basis of 1000 approximately independent realizations of this random variable. These 1000 approximately independent random draws were obtained by taking or accepting every fifth of the last 5000 values for X appearing in the simulation. (See the discussion in the next paragraph.) The resulting histogram density estimate appears as

FIGURE 4

TWO ESTIMATED DENSITIES FOR X IN EXAMPLE 2





the second plot in Figure 4, and we observe that it is consistent with the first estimate.

As previously mentioned in Section 2, thinning the sequence of simulated values output by a Gibbs sampler by accepting only every kth generated value reduces the serial correlation between the accepted values, and sample autocorrelation functions may be examined in order to assess the dependence in the thinned sequence (Miller and Wichern [21]). We applied this idea in the paragraph above to the last 5000 of the simulated values for X

output by the Gibbs sampler using $k = 5$, so that only every fifth of these generated values was accepted and so that 1000 values were accepted in total. The sample autocorrelation function for the original sequence of 5000 simulated values appears as the first plot in Figure 5. The heights of the twenty different spikes in this plot represent the values of the sample autocorrelation coefficients at lags 0 through 19 for this sequence of 5000 values, respectively. If this sequence of 5000 simulated values were independent, then all of the sample autocorrelations at non-zero lags should be close to zero. Spikes crossing either of the two horizontal dashed lines identify autocorrelation coefficients that are significantly different from zero (at the 95 percent level of significance). For this sequence of 5000 simulated values, we may observe that significant autocorrelations are identified at the non-zero lags 1 through 5, clearly demonstrating the dependent nature of this sequence. The sample autocorrelation function for the thinned sequence of 1000 simulated values appears as the second plot in Figure 5. This sample autocorrelation function is reminiscent of the function we would expect for a purely random process, since none of the autocorrelations at non-zero lags is significantly different from zero. This demonstrates that by thinning the original sequence of simulated values for X, we have indeed recovered an approximately independent random sample as claimed.

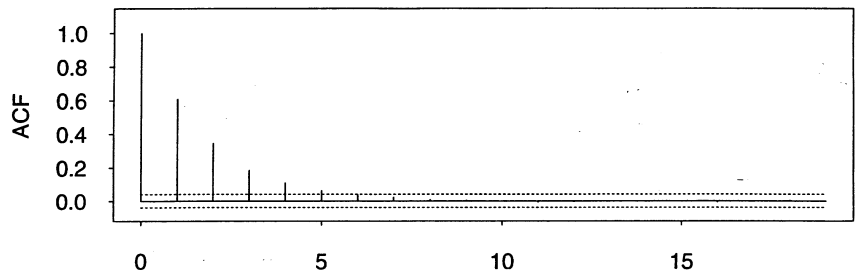
4. PRACTICAL CONSIDERATIONS RELATED TO GIBBS SAMPLING

There is one very simple and overriding reason for the recent popularity of the Gibbs sampler as a tool for statistical inference: it permits the analysis of any statistical model possessing a complicated multivariate distribution to be reduced to the analysis of its much simpler, and lower dimensional, full conditional distributions. In fact, all that is required is that we be able to iteratively sample a large number of random variates from these conditional distributions. Since

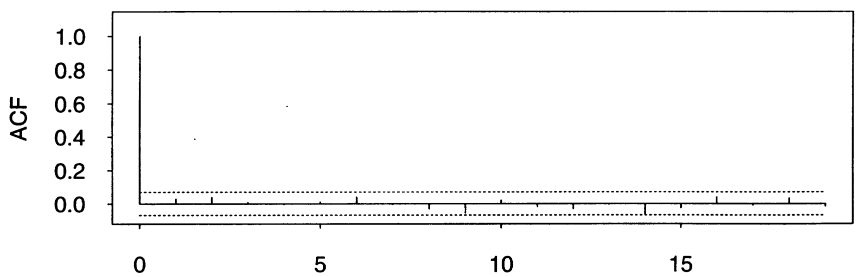
$$\pi(x_j \mid x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_k) \propto \pi(x), \tag{4.1}$$

FIGURE 5

TWO SAMPLE AUTOCORRELATION FUNCTIONS FOR X IN EXAMPLE 2



Sample ACF for the last 5000 values of X in the original sequence



Sample ACF for the last 1000 values of X in the subsequence

where $\pi(x)$ on the right-hand side is viewed as a function of x_j with all of the other arguments held fixed, we will always have the form of the full conditional distributions required to implement a Gibbs sampler immediately available (at least up to their normalizing constants) whenever the form of the target distribution is known. When a full conditional distribution is univariate, we will usually be able to generate random draws from it by making use of one of the algorithms for non-uniform random variate generation found in Devroye [7]. A number of these algorithms are also included in the syllabus for Associateship Examination Part 4B (e.g., Hogg and Klugman [16, pp. 69–75]). Gilks [12], Gilks and Wild [13], and Wild and Gilks [32] describe clever adaptive rejection sampling (ARS) methods that are very efficient when random draws are required from a univariate continuous distribution with a density that is concave with respect to its argument on the logarithmic scale, and these methods are becoming very popular as well. Many of these algorithms, including those for ARS in particular, do not even necessitate the calculation of the normalizing constants.

If one decides to implement a Gibbs sampler by coding it directly using a high-level programming language like APL, C, or FORTRAN, it will probably be necessary to code one or more of the algorithms for non-uniform random variate generation mentioned above. One way to avoid this bother is to make use of an existing software package for statistical computing, like S-Plus (Statistical Sciences Inc.) (Becker, Chambers, Wilks [1]) or Minitab (Minitab Inc.). Using a statistical computing package is often a convenient way in which to implement a Gibbs sampler, since random number generators for many standard distributions are often included in these packages. We implemented the Gibbs samplers for Examples 1 and 2 within the S-Plus programming environment using the random number generators rgamma, rbinom, and rpois, and each of the simulations took only seconds to run. On the other hand, intensive MCMC simulations for more complicated models often take minutes or hours to run when implemented using Minitab or S-Plus, but require only a few seconds or minutes to run when programmed in a high-level language like C or FORTRAN.

Specialized software for Gibbs sampling also exists. Foremost is the software package known as BUGS (Thomas, Spiegelhalter, and Gilks [30] and Gilks, Thomas, and Spiegelhalter [14]). Its name is an acronym for *Bayesian Inference Using Gibbs Sampling*, and BUGS is intended to be used for that purpose. BUGS will implement Bayesian inference using Gibbs sampling for a large class of full probability models in which all quantities are treated as random variables. This package is capable of analyzing very complicated models, and it appears to be competitive with C or FORTRAN in terms of raw speed. The BUGS software package is very convenient to use, insomuch as it provides a declarative language permitting the practitioner to make a straightforward specification of the statistical model at hand, following which the software automatically derives the associated full conditional distributions and selects appropriate sampling algorithms. Version 0.50 of this software is available free of charge for SUN Sparcstations and PC 386+387/486/586 platforms. Readers with access to the computer Internet may obtain BUGS, along with an instruction manual (Spiegelhalter, Thomas, Best, and Gilks [27]) and two volumes of worked examples (Spiegelhalter, Thomas, Best, and Gilks [26]) by anonymous ftp from *ftp.mrc-bsu.cam.ac.uk* in the directory *pub/methodology/bugs* or by accessing the uniform resource locator *http://www.mrc-bsu.cam.ac.uk* on the World Wide Web. These resources may also be obtained on disk from the developers for a small administrative fee. (For details, e-mail *bugs@mrc-bsu.cam.ac.uk*.) In Appendices A, B, and C, we provide illustrative BUGS programming code corresponding to Examples 3 and 4, which are themselves presented in Section 5. After reviewing Example 4, the reader will recognize that BUGS requires relatively few lines of code in order to implement a Gibbs sampler, even for a large and complicated model.

Recall that it will be necessary to run a Gibbs sampler for a little while in order to escape from the influence of the initial values and converge to the target distribution. Regardless of how one chooses to implement a Gibbs sampler, it will always be necessary to monitor this convergence. This is usually best diagnosed on the basis of the output from several independent replications of the Gibbs sampler, using widely dispersed starting values. If these Gibbs samplers have been left to run for a sufficiently long time so that convergence has been obtained, then the inferences drawn from each of the replications should be consistent, and virtually identical, with one another. In a similar vein, the behavior of the sampled values across replications at various iterations should be consistent when a large number of replications is considered. An ad hoc implementation of this idea is used in Example 3. More formal diagnostics are also available, and Cowles and Carlin [6] recently made a comparative review of a number of these. Two of the most popular are the methods proposed by Gelman and Rubin [10] and Raftery and Lewis [22]. An application of Gelman and Rubin's method may be found in Scollnik [24].

5. BAYESIAN ANALYSIS USING GIBBS SAMPLING

The Gibbs sampler has proven itself to be particularly suited for problems arising in the field of Bayesian statistical inference. Recall that a Bayesian analysis proceeds by assuming a model $f(Y \mid \theta)$ for the data Y conditional upon the unknown parameters θ. When $f(Y \mid \theta)$ is considered as a function of θ for fixed Y, it is referred to as the likelihood and is denoted by $L(\theta \mid Y)$ or $L(\theta)$. A prior probability distribution $f(\theta)$ describes our knowledge of the model parameters before the data is actually observed. Bayes' theorem allows us to combine the likelihood function with the prior in order to form the conditional distribution of θ given the observed data Y, that is,

$$f(\theta \mid Y) \propto f(\theta)L(\theta). \tag{5.1}$$

This conditional distribution is called the posterior distribution for the model parameters, and describes our updated knowledge of them after the data has been observed. Frequently, numerical methods are required in order to study posterior distributions with complicated forms. Following Equation 4.1 and its associated discussion, one may deduce that the Gibbs sampler is one method available for consideration. Other numerical methods that might be utilized in order to advance a Bayesian analysis include numerical quadrature and Monte Carlo integration, both of which are described in Klugman [18, Chapter 2]. One of the big advantages of the Gibbs sampler is that it is often far easier to implement than either of these other two methods. The Gibbs sampler is also flexible in the sense that its output may be used in order to make a variety of posterior and predictive inferences.

For example, imagine that we have implemented a Gibbs sampler generating values $\theta^{(i)}$ from $f(\theta \mid Y)$, provided that i is sufficiently large. Obviously, posterior inference with respect to θ may proceed on the basis of the sampled values $\theta^{(i)}$. However, if some transformation $\omega = \omega(\theta)$ of the model parameters is of interest as well, then posterior inference with respect to ω is immediately available on the basis of the transformed values $\omega^{(i)} = \omega(\theta^{(i)})$. Further, it will often be the case that the actuarial practitioner will be interested in making predictive inferences with respect to things like future claim frequencies, future size of losses, and so forth. Typically, the conditional model $f(Y_f \mid Y, \theta)$ for the future data Y_f given the past data Y and the model parameters θ will be available. The appropriate distribution upon which to base future inferences is the so-called predictive distribution with density

$$f(Y_f \mid Y) = \int f(Y_f \mid Y, \theta) f(\theta \mid Y)\, d\theta, \tag{5.2}$$

which describes our probabilistic knowledge of the future data given the observed data. An estimate of this predictive density is easily obtained by averaging $f(Y_f \mid Y, \theta)$ over the sampled values of $\theta^{(i)}$ in the sense of Equation 2.2. Recall that the density estimates appearing in Equations 1.3, 1.4, and 3.7 were all constructed in a like manner.

This section concludes with Examples 3 and 4. Example 3 involves the estimation of the parameter for a size of loss distribution when grouped data are observed. Example 4 addresses credibility for classification ratemaking via hierarchical models, and involves the prediction of frequency counts in workers compensation insurance. We will operate within the Bayesian paradigm for these examples and implement the Bayesian analyses using the Gibbs sampler.

Example 3

Assume that loss data has been generated according to the Pareto(θ, λ) distribution with density

$$f(x \mid \theta) = \frac{\theta \lambda^{\theta}}{(\lambda + x)^{\theta+1}}, \qquad 0 < x < \infty. \tag{5.3}$$

In order to simplify the presentation, we will assume that the parameter λ is known to be equal to 5000, so that the only uncertainty is with respect to the value of the parameter θ. Imagine that twenty-five independent observations are available in total, but that the data has been grouped in such a way so that we know only the class frequencies: 12, 8, 3, and 2 observations fall into the classes $(0, 1000]$, $(1000, 2000]$, $(2000, 3000]$, $(3000, \infty)$, respectively. Hogg and Klugman [16, pp. 81–84] consider maximum likelihood, minimum distance, and minimum chi-square estimation for grouped data problems like this when inference is sought with respect to the parameter θ. Below, we will consider how a Bayesian analysis might proceed.

Given the situation described in the paragraph above, the best likelihood function available is proportional to

$$L(\theta \mid \mathit{Obs.\ Data}) = \prod_{i=1}^{4} \left(\int_{c_{i-1}}^{c_i} f(x \mid \theta)\, dx \right)^{f_i}$$

with class limits $c_0 = 0$, $c_1 = 1000$, $c_2 = 2000$, $c_3 = 3000$, $c_4 = \infty$, and class frequencies $f_1 = 12$, $f_2 = 8$, $f_3 = 3$, $f_4 = 2$. Multiplying this likelihood function together with a prior density for θ will result in an expression proportional to the posterior density for θ given the observed data. Since the posterior distribution of θ is univariate, this posterior density might be evaluated in a straightforward fashion making use of numerical quadrature methods.

However, for the sake of illustration, we choose instead to implement the Bayesian analysis by utilizing the Gibbs sampler along with a process called data augmentation. Towards this end, let us first consider how the likelihood function would change if exact size of loss values supplementing or augmenting the twenty-five observed class frequencies were available as well. In this case, the likelihood function would be proportional to

$$L(\theta \mid \mathit{Obs.\ \&\ Aug.\ Data}) = \frac{\theta^{25}\lambda^{25\theta}}{\prod_{i=1}^{25}(\lambda + x_i)^{\theta+1}}.$$

Combining this likelihood function with the conjugate gamma(α,β) prior density for θ results in the posterior density

$$\begin{aligned} f(\theta \mid \mathit{Obs.\ \&\ Aug.\ Data}) & \\ &\propto \theta^{24+\alpha}\exp\left(-\theta\left(\beta - 25\ln\lambda + \sum_{i=1}^{25}\ln[\lambda + x_i]\right)\right) \\ &\sim \text{gamma}\left(25+\alpha, \beta - 25\ln\lambda + \sum_{i=1}^{25}\ln[\lambda + x_i]\right). \end{aligned} \quad (5.4)$$

Recall that a conjugate prior combines with the likelihood function in such a way so that the posterior distribution has the same form as the prior. For this example, we adopted the conjugate prior primarily for mathematical and expository convenience, and set $\alpha = \beta = 0.001$ so that our prior density for θ is very diffuse and noninformative with mean 1 and variance 1000. Although the the adoption of a diffuse conjugate prior is not uncommon when relatively little prior information is being assumed, in practice the practitioner should adopt whatever form of prior density that best describes the prior information actually available.

Now, the augmented data values are all independently distributed given the model parameters, and each is distributed according to Equation 5.3 but restricted to the appropriate class interval. In other words, the conditional distribution of the augmented data, given the model parameters and the observed class frequencies, is described by the following set of truncated Pareto distributions:

$$x_i \sim \text{truncated Pareto } (\theta, \lambda) \text{ on the interval } (0, 1000], \quad \text{for} \quad i = 1, 2, \ldots, 12; \tag{5.5}$$

$$x_i \sim \text{truncated Pareto } (\theta, \lambda) \text{ on the interval } (1000, 2000], \quad \text{for} \quad i = 13, 14, \ldots, 20; \tag{5.6}$$

$$x_i \sim \text{truncated Pareto } (\theta, \lambda) \text{ on the interval } (2000, 3000], \quad \text{for} \quad i = 21, 22, 23; \tag{5.7}$$

$$x_i \sim \text{truncated Pareto } (\theta, \lambda) \text{ on the interval } (3000, \infty), \quad \text{for} \quad i = 24, 25. \tag{5.8}$$

If a loss random variable has a Pareto distribution, with parameters θ and λ and a density function as in Equation 5.3, then the truncated density function of that random variable, on the restricted interval $(l, u]$, with $0 \leq l < u \leq \infty$, is given by

$$\frac{f(x \mid \theta)}{\Pr(l < X \leq u \mid \theta)}, \qquad l < x \leq u.$$

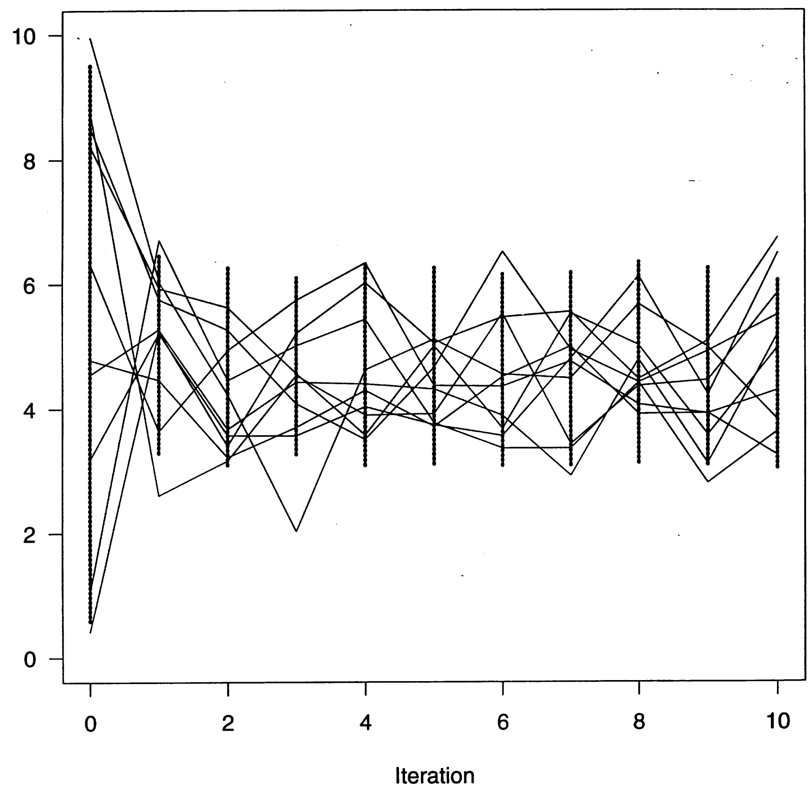
For example, the density function associated with the truncated Pareto distribution appearing in Equation 5.8 is given by

$$\frac{\theta(\lambda + 3000)^{\theta}}{(\lambda + x)^{\theta+1}}, \qquad 3000 < x < \infty.$$

By applying the Gibbs sampler to the 26 full conditional distributions defined by Equations 5.4 through 5.8, we are easily able to simulate a Markov chain with an invariant distribution equal to $p(\theta, \mathit{Aug.\ Data} \mid \mathit{Obs.\ Data})$. In order to make posterior inference with respect to θ, the parameter of interest, we initiated

FIGURE 6

TEN SAMPLE PATHS FOR θ IN EXAMPLE 3



1000 replications of this Markov chain, using randomly selected starting values for θ and the augmented data each time, and let each replication run for 10 iterations. Only the values generated in the final iteration of each replication will be used. The reader will recall that λ is equal to 5000 by assumption.

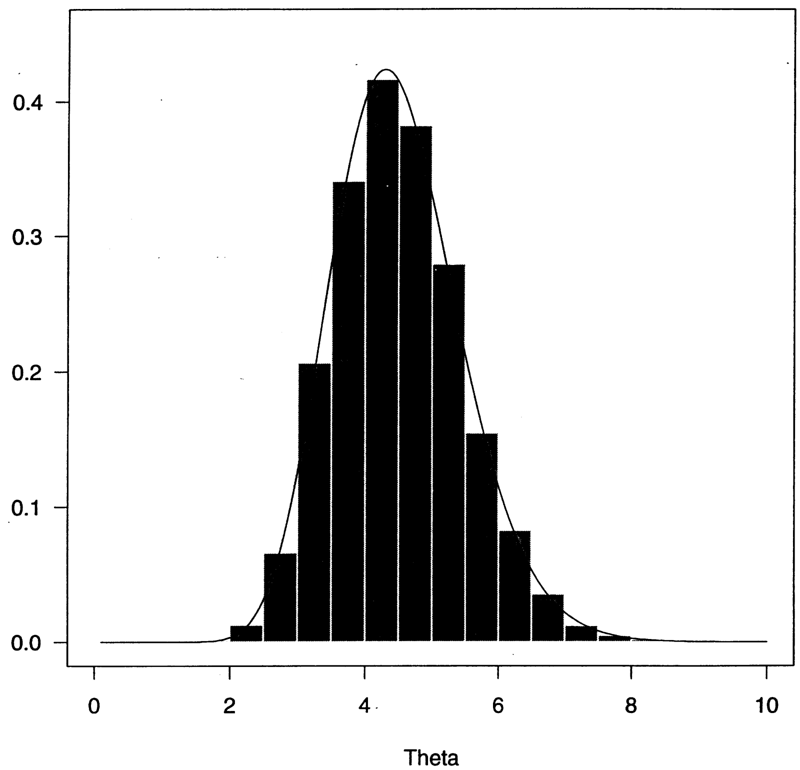
Ten arbitrarily selected sample paths for θ are plotted in Figure 6 for illustrative purposes. These 10 sample paths are typical of the entire collection of 1000 generated sample paths, and indicate that the simulated sequences stabilize almost immediately. In order to monitor convergence, we monitored the 5th and 95th empirical quantiles for θ over the 1000 replications for each iteration. The resulting 90% empirical confidence bands are plotted as vertical lines in Figure 6. These stabilize almost immediately as well, indicating that the Gibbs sampler was very quick to converge. Although we do not include the plots, the sample paths for the augmented data behaved similarly. By taking only the final value of θ appearing in each of the 1000 sample paths, we obtain an approximately independent random sample from the posterior distribution $p(\theta \mid Obs.\ Data)$. These values were used to construct the histogram of sampled values for θ appearing in Figure 7. Their sample mean and variance were 4.5097 and 0.9203, respectively. A smooth density estimate for θ was obtained by averaging Equation 5.4 over the corresponding 1000 sets of simulated values for the augmented data, and this estimate overlays the histogram in Figure 7. At this time we note that the data yielding the observed class frequencies used in this example were actually generated using a value of θ equal to 4.5, so that our posterior inference with respect to θ is certainly on the mark.

By monitoring the values taken on by the augmented data as the simulation proceeds, we can also make posterior inference with respect to the actual but unobserved sizes of loss. For example, by monitoring the values of the two losses appearing in the upper-most class, x_{24} and x_{25}, we can estimate the posterior probability that one or more of these two losses exceeded an upper limit of, say, 10,000 by simply observing the proportion of times this event occurred in the simulation. In fact, we observed that in only 124 of the 1000 final pairs of simulated values for x_{24} and x_{25} did at least one of these two values exceed 10,000. Thus, a simple estimate of the posterior probability of interest is given by the binomial proportion

$$\hat{p} = \frac{124}{1000} = 0.124,$$

FIGURE 7

HISTOGRAM OF SAMPLED VALUES AND A DENSITY ESTIMATE FOR θ IN EXAMPLE 3



which has an estimated standard error of

$$\left(\frac{\hat{p}(1-\hat{p})}{1000}\right)^{0.5} = \left(\frac{0.124 * 0.876}{1000}\right)^{0.5} = 0.0104.$$

The analysis described above was implemented using the statistical computing package S-Plus on a SUN Sparcstation LX (operating at 50 MHz). Five thousand iterations of the Gibbs sampler constructed for this problem took 380 seconds. By way of comparison, when implemented using BUGS on the same computer, 5000 iterations of the Gibbs sampler took 14 seconds. These times should be comparable to those one might encounter using a fast 486/586 PC. The BUGS code corresponding to this example appears in Appendix A.

Although we assumed that λ was known to be equal to 5000 in this example, treating it as a random parameter would have complicated the analysis only slightly. In this case, it would have been necessary to include random draws from the posterior full conditional distribution for λ, given θ and the augmented data, in the running of the Gibbs sampler. These univariate random draws might have been accomplished utilizing one of the strategies for random number generation described in Section 4. Of course, a prior distribution for the parameter λ would have to have been specified as well.

Example 4

For our last example, we will use the Gibbs sampler to analyze three models for claim frequency counts. These are the hierarchical normal, hierarchical first level Poisson, and the variance stabilized hierarchical normal models. The data corresponds to Data Set 2 in Klugman [18]. The observations are frequency counts in workers compensation insurance. The data were collected from 133 occupation classes over a seven-year period. The exposures are scaled payroll totals adjusted for inflation. The first two classes are given in Table 1, and the full data set may be found in Appendix F of Klugman [18]. Only the first six of the seven years will be used to analyze these models, and omitting those cases with zero exposure yields a total of 767 observations. The results of each model analysis will then be used to forecast the number of claims associated with the 128 classes with non-zero exposure in the seventh year. We will observe that the second of the three models (i.e., the hierarchical first level Poisson model) appears to have associated with it the best predictive performance in this context.

TABLE 1

WORKERS COMPENSATION INSURANCE FREQUENCIES

Class i	Year j	Exposure P_{ij}	Claims Y_{ij}
1	1	32.322	1
1	2	33.779	4
1	3	43.548	3
1	4	46.686	5
1	5	34.713	1
1	6	32.857	3
1	7	36.600	4
2	1	45.995	3
2	2	37.888	1
2	3	34.581	0
2	4	28.298	0
2	5	45.265	2
2	6	39.945	0
2	7	39.322	4
.	.	.	.
.	.	.	.
.	.	.	.

Klugman [17] argued that a hierarchical model is the most appropriate framework in which to implement credibility for classification ratemaking. In this spirit, Klugman [18] considered a Bayesian analysis of the workers compensation insurance frequency count data presently under consideration using a (one-way) hierarchical normal model (HNM), and demonstrated that this analysis might be implemented using any one of a number of numerical techniques, emphasizing numerical quadrature, Monte Carlo integration, or Tierney–Kadane's integral method. We will begin by considering the HNM as well, but we will implement its analysis using the Gibbs sampler. Letting x_{ij} denote Y_{ij}/P_{ij}, the relative frequency for class i and year j, the first two levels of the HNM we consider are described by

$$f(x_{ij} \mid \theta_i, \tau_1^2) \sim \text{normal } (\theta_i, P_{ij}\tau_1^2) \qquad (5.9)$$

$$\text{and} \qquad f(\theta_i \mid \mu, \tau_2^2) \sim \text{normal } (\mu, \tau_2^2). \tag{5.10}$$

Each of these normal densities is indexed by two parameters, a mean and a precision (i.e., inverse variance). The model parameter θ_i represents the true relative frequency for the ith class. The relative frequencies x_{ij}, for $i = 1,\ldots,133$ and $j = 1,\ldots,7$, are assumed to be independent across class and year, given the underlying model parameters θ_i, $i = 1,\ldots,133$, and τ_1^2. Similarly, the true frequencies θ_i, for $i = 1,\ldots,133$, are assumed to be independent, given the underlying parameters μ and τ_2^2. (Notice that under this model, negative claim frequencies are possible. For this reason, the HNM as presented is not entirely appropriate for modeling the non-negative workers compensation insurance frequency count data. We return to this point in the next paragraph.) In order to complete the model specification, Klugman [18] employed a constant improper prior density for the model parameters μ, τ_1^2 and τ_2^2. Instead, we adopt the diffuse but proper prior density described by

$$f(\mu) \sim \text{normal } (0, 0.001), \tag{5.11}$$

$$f(\tau_1^2) \sim \text{gamma } (0.001, 0.001), \tag{5.12}$$

$$f(\tau_2^2) \sim \text{gamma } (0.001, 0.001). \tag{5.13}$$

The assumption of a proper prior guarantees that the posterior distribution exists and is proper as well, and slightly simplifies the implementation of a Gibbs sampler. However, the precise form of the diffuse prior (and the selection of the prior density parameters) is not terribly important in this instance since the observed data comprises a rather large sample that will tend to dominate the prior information in any case. (Also, see the discussion in the next paragraph.) We assume prior independence between the model parameters μ, τ_1^2 and τ_2^2. If we assume that only the data corresponding to the first six of the seven years has been observed, then the posterior density for the model parameters is proportional to the product of terms appearing on the right-hand side of the expression

$$f(\theta_1,\theta_2,\ldots,\theta_{133},\mu,\tau_1^2,\tau_2^2 \mid Data)$$
$$\propto f(\mu)f(\tau_1^2)f(\tau_2^2)\prod_{i=1}^{133} f(\theta_i \mid \mu,\tau_2^2)\prod_{j=1}^{6} f(x_{ij} \mid \theta_i,\tau_1^2). \tag{5.14}$$

Recall that our objective is to forecast the number of claims associated with the 128 classes with non-zero exposure in the seventh year.

As previously remarked, the HNM as presented above is not entirely appropriate for modeling the non-negative workers compensation insurance frequency count data. Yet, we will continue with its analysis for the following reasons:

- the large amount of sample data available will tend to overwhelm the prior density and will also go a long way towards correcting the inadequacy of the model by assigning less posterior probability to parameter values that are likely to generate negative frequencies;
- it will be interesting to compare the results of the MCMC simulation based analysis of the HNM to the numerical analysis presented by Klugman [18];
- the MCMC simulation based analysis of the HNM provides a benchmark to which the MCMC simulation based analyses of the other two models may be compared; and
- the HNM is a very important model in its own right, and for this reason alone it is valuable and instructive to see how its Gibbs sampling based Bayesian analysis might proceed.

Having said this, there are at least two ways in which the basic HNM may be constructively modified if it is to be applied to frequency count data. The first solution is to adopt the recommendation made by Klugman [18, pp. 76–77] and transform the original data in some way so that the transformed data is more appropriately modeled using the HNM. This approach motivates the variance stabilized hierarchical normal model considered at the end of this section. The second solution involves adding restrictions to the HNM so that negative frequencies are prohibited. A simple way in which to do this is to replace the normal distributions appearing in Equations 5.9, 5.10, and 5.11 with normal distributions truncated below at zero. In fact, we analyzed this truncated HNM and observed that, although it did perform significantly better than the non-truncated HNM, it did not perform as well as the variance stabilized hierarchical normal model. For this reason, we will omit the details of the truncated HNM analysis.

In order to conduct a Bayesian analysis of the HNM described by Equations 5.9 to 5.13 using the Gibbs sampler, we are required to first derive the necessary full conditional distributions associated with this model. These may be derived by substituting Equations 5.9 through 5.13 into Equation 5.14, and then making use of the discussion following Equation 4.1. In this manner, for an arbitrary one of the 133 normal mean θ_i parameters we obtain

$$
\begin{aligned}
& f(\theta_i \mid Data;\ \theta_1,\ldots,\theta_{i-1},\theta_{i+1},\ldots,\theta_{133},\mu,\tau_1^2,\tau_2^2) \\
& \quad \propto f(\theta_i \mid \mu,\tau_2^2)\prod_{j=1}^{6} f(x_{ij} \mid \theta_i,\tau_1^2) \\
& \quad \propto \exp\left(-0.5\left[\tau_2^2(\theta_i-\mu)^2+\tau_1^2\sum_{j=1}^{6}P_{ij}(x_{ij}-\theta_i)^2\right]\right) \\
& \quad \sim \text{normal}\left(\left[\mu\tau_2^2+\tau_1^2\sum_{j=1}^{6}P_{ij}x_{ij}\right]\Bigg/\right. \\
& \qquad\qquad \left.\left[\tau_2^2+\tau_1^2\sum_{j=1}^{6}P_{ij}\right],\tau_2^2+\tau_1^2\sum_{j=1}^{6}P_{ij}\right).
\end{aligned}
$$

The recognition of the normal distribution in the last line of this derivation follows from completing the square in θ_i in the expression immediately above it. Observe that the full conditional distribution of θ_i is actually independent of any other parameter θ_j. For the parameter μ, we obtain

$$\begin{aligned} f(\mu \mid \mathit{Data};\ \theta_1,\ldots,\theta_{133},\tau_1^2,\tau_2^2) &\propto f(\mu)\prod_{i=1}^{133} f(\theta_i \mid \mu,\tau_2^2) \\ &\propto \exp\left(-0.5\left[0.001\mu^2 + \tau_2^2\sum_{i=1}^{133}(\theta_i-\mu)^2\right]\right) \\ &\sim \text{normal}\left(\left[\tau_2^2\sum_{i=1}^{133}\theta_i\right]\Big/[0.001+133\tau_2^2],\ 0.001+133\tau_2^2\right); \end{aligned}$$

and for the precision parameter τ_1^2, we have

$$\begin{aligned} f(\tau_1^2 \mid \mathit{Data};\ \theta_1,\ldots,\theta_{133},\mu,\tau_2^2) &\propto f(\tau_1^2)\prod_{i=1}^{133}\prod_{j=1}^{6} f(x_{ij} \mid \theta_i,\tau_1^2) \\ &\propto (\tau_1^2)^{0.001-1+767/2} \\ &\quad \times \exp\left(-\tau_1^2\left[0.001 + 0.5\sum_{i=1}^{133}\sum_{j=1}^{6}P_{ij}(x_{ij}-\theta_i)^2\right]\right) \\ &\sim \text{gamma}\left(383.501, 0.001 + 0.5\sum_{i=1}^{133}\sum_{j=1}^{6}P_{ij}(x_{ij}-\theta_i)^2\right). \end{aligned}$$

In the derivation of the full conditional distribution for τ_1^2 we made use of the fact that there were only 767 observations associated with non-zero exposures in the first six years. Finally, for the precision parameter τ_2^2, we have

$$f(\tau_2^2 \mid Data;\ \theta_1,\ldots,\theta_{133},\mu,\tau_1^2)$$

$$\propto f(\tau_2^2)\prod_{i=1}^{133} f(\theta_i \mid \mu,\tau_2^2)$$

$$\propto (\tau_2^2)^{0.001-1+133/2}\exp\left(-\tau_2^2\left[0.001+0.5\sum_{i=1}^{133}(\theta_i-\mu)^2\right]\right)$$

$$\sim \text{gamma}\left(66.501, 0.001+0.5\sum_{i=1}^{133}(\theta_i-\mu)^2\right).$$

Using these full conditional distributions, we can implement a Gibbs sampler for the model of interest by coding the appropriate random number generators in a high-level programming language like APL, C, or FORTRAN. However, for this moderately large model incorporating 136 random parameters, we prefer to use the BUGS software package mentioned in Section 4 and allow it to automatically select and program the necessary random number generators for us. In fact, since BUGS automatically determines and selects the appropriate random number generators for the full conditional distributions directly from Equations 5.9 through 5.13, we really did not have to derive these full conditional distributions ourselves, except perhaps to demonstrate how this task is accomplished.

Illustrative BUGS code corresponding to the HNM of interest is provided in Appendix B, and we used this programming code in conjunction with the BUGS software package in order to implement a Gibbs sampler for the problem at hand. We allowed the MCMC simulation to run for 20,000 iterations in order to "burn-in" the Gibbs sampler and remove the effect of the starting values, and then allowed it to run for an additional 5000 iterations in order to generate a dependent random sample of size

5000 from the joint posterior distribution of the model parameters $\theta_1, \theta_2, \ldots, \theta_{133}$, μ, τ_1^2, and τ_2^2, given the observed frequency counts. (This simulation took about 337 seconds on the same SUN Sparcstation LX we described previously.)

The values generated by this MCMC simulation may be used in order to make a wide variety of posterior and predictive inferences. For example, the expected number of claims for year seven in class i is given by $\mathrm{E}(Y_{i7} \mid P_{i7}, \theta_i) = P_{i7}\theta_i$. By multiplying each of the 5000 simulated values of θ_i with P_{i7}, we obtain 5000 realizations from the posterior distribution of $P_{i7}\theta_i$. Then posterior inference with respect to the expected number of claims for year seven in class i may proceed on the basis of this sample. We performed this procedure for five of the 128 classes with non-zero exposure in the seventh year, and have recorded in Table 2 the empirical mean, standard deviation, and several quantiles (i.e., the 2.5th, 50.0th, and 97.5th) for each of the resulting samples of size 5000. The five classes we selected are the same (non-degenerate) ones considered by Klugman [18, p. 128]. However, whereas Klugman provided only point estimates for the expected number of claims for year seven in each of these classes, we have been able to generate realizations from the posterior distribution of these expected claim numbers using MCMC. This allows us to observe, for instance, that both classes 70 and 112 have substantial posterior probability associated with negative expected number of claim values under the HNM, as is evidenced by Table 2. An overall measure of this model's prediction success is given by the statistic

$$OMPS = \sum_{P_{i7}>0} \frac{(P_{i7}\theta_i - Y_{i7})^2}{P_{i7}}. \tag{5.15}$$

(The name of this statistic is an abbreviation of Overall Measure of Prediction Success.) There are 128 terms in the summation, one for each of the 128 classes with non-zero exposure in the seventh year. Small values of $OMPS$ are indicative of a model with good overall prediction success. We obtained 5000

TABLE 2

EXPECTED WORKERS COMPENSATION INSURANCE FREQUENCIES (HIERARCHICAL NORMAL MODEL)

Class	Actual Values		Expected Number of Claims $P_{i7}\theta_i$				
	Exposure	Claims	Estimated Posterior Summary Statistics				
i	P_{i7}	Y_{i7}	Mean	S.D.	2.5%	50.0%	97.5%
11	229.83	8	10.19	2.98	4.34	10.21	16.04
20	1,315.37	22	41.38	5.57	30.62	41.38	52.30
70	54.81	0	0.61	1.20	−1.74	0.61	2.95
89	79.63	40	29.42	1.47	26.54	29.44	32.27
112	18,809.67	45	36.11	27.55	−19.02	36.47	89.43

realizations from the posterior distribution of *OMPS* by simply evaluating Equation 5.15 five thousand times, once using each of the 5000 joint realizations of $\theta_1, \theta_2, \ldots, \theta_{133}$ previously simulated from their posterior joint distribution. In Table 3, we present the empirical mean, standard deviation, and 2.5th, 50th and 97.5th quantiles for this sample of 5000 realizations from the posterior distribution of *OMPS*. We will return to this table after we introduce and analyze our second model. Incidentally, we remark that we checked our inferences throughout this example by independently replicating our entire MCMC simulation-based analysis several times, using different starting values for the Gibbs sampler each time.

Above, we concentrated on posterior inferences made with respect to the expected number of claims in year seven for various classes. As remarked at the start of Section 5, if we are interested in the future number of actual claims, then we should really be using the relevant predictive distribution in order to fashion our inferences. For the HNM presently under consideration, the distribution of the future number of claims for year seven in class i is independent of the data associated with the first six years provided that the underlying model parameters are

TABLE 3

AN OVERALL MEASURE OF PREDICTION SUCCESS (HIERARCHICAL NORMAL MODEL)

Estimated Posterior Summary Statistics				
Mean	S.D.	2.5%	50.0%	97.5%
16.49	1.23	14.16	16.48	18.96

known. This distribution is given by

$$f(Y_{i7} \mid \theta_i, \tau_1^2) \sim \text{normal}\left(P_{i7}\theta_i, \frac{\tau_1^2}{P_{i7}}\right), \tag{5.16}$$

in accord with Equation 5.9. Following Equation 5.2, it follows that the predictive distribution of the future number of claims for year seven in class i given the observed frequency counts over the first six years is

$$f(Y_{i7} \mid Data) = \int f(Y_{i7} \mid \theta_i, \tau_1^2) f(\theta_i, \tau_1^2 \mid Data)\, d\theta_i\, d\tau_1^2. \tag{5.17}$$

An estimate of this predictive distribution is easily obtained by simply averaging the density found in Equation 5.16 over the 5000 pairs of realized values for θ_i and τ_1^2 previously simulated from the posterior distribution of the model parameters.

Another way in which to proceed is by generating 5000 realizations of Y_{i7} according to the distribution in Equation 5.16, one realization per pair of values previously simulated for θ_i and τ_1^2. Then the 5000 simulated values of Y_{i7} represent a random sample from the predictive distribution in Equation 5.17, and the empirical distribution of this sample may be used to motivate predictive inference. Using this latter approach we simulated random samples of size 5000 from the predictive distribution in Equation 5.17 for each of the five classes we examined previously, and summary statistics for the samples from these predictive distributions appear in Table 4. From Table 4, we may

TABLE 4

PREDICTED WORKERS COMPENSATION INSURANCE FREQUENCIES (HIERARCHICAL NORMAL MODEL)

	Actual Values		Predicted Number of Claims Y_{i7}				
Class	Exposure	Claims	Estimated Predictive Summary Statistics				
i	P_{i7}	Y_{i7}	Mean	S.D.	2.5%	50.0%	97.5%
11	229.83	8	10.24	7.41	−4.12	10.22	24.92
20	1,315.37	22	41.46	17.04	8.03	41.39	75.22
70	54.81	0	0.63	3.46	−6.10	0.62	7.51
89	79.63	40	29.46	4.21	21.22	29.45	37.72
112	18,809.67	45	37.74	66.28	−92.32	38.25	167.52

observe that the predictive distributions associated with future claim frequencies exhibit greater variability than do the corresponding posterior distributions associated with the expected numbers of future claims. Also notice that 3 of the 5 classes (i.e., classes 11, 70, and 112) have substantial predictive probability associated with negative number of claim values in year seven under the HNM.

The second model we consider is a more realistic one for modeling frequency count data. This model is also hierarchical, and its first two levels are described by

$$f(Y_{ij} \mid \theta_i) \sim \text{Poisson } (P_{ij}\theta_i) \qquad (5.18)$$

$$\text{and} \qquad f(\ln\theta_i \mid \mu, \tau^2) \sim \text{normal } (\mu, \tau^2). \qquad (5.19)$$

The model parameter θ_i now represents the true Poisson claim frequency rate for the ith class with one unit of exposure. The frequency counts Y_{ij}, for $i = 1,\ldots,133$ and $j = 1,\ldots,7$, are assumed to be independent across class and year, given the underlying model parameters θ_i, $i = 1,\ldots,133$, and the Poisson claim frequency rate parameters θ_i, for $i = 1,\ldots,133$, are assumed to be independent, given the underlying parameters μ and τ^2. An obvious advantage of this model over the HNM considered previously is that the frequency counts are now being modeled at the first level with a discrete distribution on the non-negative integers. Assuming log-normal distributions as we have for the Poisson rate parameters implies that

$$\mathrm{E}(Y_{ij}) = \mathrm{E}(\mathrm{E}(Y_{ij} \mid \theta_i)) = P_{ij}\exp(\mu + 1/[2\tau^2]) = P_{ij}m \tag{5.20}$$

and

$$\begin{aligned}\mathrm{Var}(Y_{ij}) &= \mathrm{E}(\mathrm{Var}(Y_{ij} \mid \theta_i)) + \mathrm{Var}(\mathrm{E}(Y_{ij} \mid \theta_i)) \\ &= P_{ij}m + P_{ij}^2 m^2(\exp(1/\tau^2) - 1) > P_{ij}m, \end{aligned} \tag{5.21}$$

so that overdispersion is modeled in the count data. In order to complete the model specification, we will assume that the parameters μ and τ^2 are independent a priori, and adopt the diffuse but proper prior density described by

$$f(\mu) \sim \text{normal } (0, 0.001), \tag{5.22}$$

$$f(\tau^2) \sim \text{gamma } (0.001, 0.001). \tag{5.23}$$

If we assume that only the data corresponding to the first six of the seven years has been observed, then the posterior density for the model parameters is proportional to the product of terms appearing on the right-hand side of the expression

$$\begin{aligned}&f(\theta_1, \theta_2, \ldots, \theta_{133}, \mu, \tau^2 \mid Data) \\ &\quad \propto f(\mu) f(\tau^2) \prod_{i=1}^{133} \theta_i^{-1} f(\ln\theta_i \mid \mu, \tau^2) \prod_{j=1}^{6} f(Y_{ij} \mid \theta_i).\end{aligned} \tag{5.24}$$

The θ_i^{-1} terms appearing in this expression arise from the change in variable when passing from $\ln\theta_i$ to θ_i. As before, our objective is to forecast the number of claims associated with the 128 classes with non-zero exposure in the seventh year.

We will now conduct a Bayesian analysis of the (one-way) hierarchical model with a first level Poisson distribution, or hierarchical first level Poisson model (HFLPM), described above using the Gibbs sampler. Rather than derive the required full conditional distributions manually, and then program the necessary random number generators, we will let the BUGS software package do both tasks for us. Illustrative BUGS code corresponding to this model is provided in Appendix B, and we used this programming code in conjunction with the BUGS software package in order to implement a Gibbs sampler. As before, we allowed the MCMC simulation to run for 20,000 iterations in order to "burn-in" the Gibbs sampler and remove the effect of the starting values, and then allowed it to run for an additional 5000 iterations in order to generate a random sample of size 5000 from the joint posterior distribution of the model parameters $\theta_1, \theta_2, \ldots, \theta_{133}$, μ, and τ^2. This sample was used to implement the same sort of posterior and predictive inferences for the HFLPM as we did for the HNM considered previously.

We omit the specific details, but summaries of our estimated posterior and predictive inferences under the HFLPM are presented in Tables 5, 6, and 7. By comparing these summaries to those presented earlier in Tables 2, 3, and 4 for the HNM analysis, we are able to evaluate the relative performance of the two models. First of all, it is evident that the posterior and predictive distributions in which we are interested generally exhibit less variability under the HFLPM than under the HNM. Secondly, whereas the HNM permits negative relative frequencies and expected numbers of future claims, these are not a problem under the HFLPM. Finally, the posterior distribution of the *OMPS* statistic describing the overall measure of prediction success for a given model appears to be concentrated closer to zero under the HFLPM than under the HNM. In short, these observations suggest that the HFLPM may be a better model than the HNM for implementing credibility for classification ratemaking when the data is in terms of frequency counts.

Klugman [18, pp. 76–77, 152–153] also recognized that the HNM was inappropriate for modeling the workers compensation insurance frequency count data in its original form, and sug-

TABLE 5

EXPECTED WORKERS COMPENSATION INSURANCE FREQUENCIES (HIERARCHICAL FIRST LEVEL POISSON MODEL)

	Actual Values		Expected Number of Claims $P_{i7}\theta_i$				
Class	Exposure	Claims	Estimated Posterior Summary Statistics				
i	P_{i7}	Y_{i7}	Mean	S.D.	2.5%	50.0%	97.5%
11	229.83	8	10.10	1.47	7.46	10.02	13.20
20	1,315.37	22	41.31	2.20	37.16	41.28	45.81
70	54.81	0	0.37	0.21	0.09	0.32	0.90
89	79.63	40	33.85	2.08	29.96	33.79	37.96
112	18,809.67	45	36.09	2.69	31.02	35.98	41.55

TABLE 6

AN OVERALL MEASURE OF PREDICTION SUCCESS (HIERARCHICAL FIRST LEVEL POISSON MODEL)

Estimated Posterior Summary Statistics				
Mean	S.D.	2.5%	50.0%	97.5%
12.96	0.64	11.76	12.94	14.28

TABLE 7

PREDICTED WORKERS COMPENSATION INSURANCE FREQUENCIES (HIERARCHICAL FIRST LEVEL POISSON MODEL)

	Actual Values		Predicted Number of Claims Y_{i7}				
Class	Exposure	Claims	Estimated Predictive Summary Statistics				
i	P_{i7}	Y_{i7}	Mean	S.D.	2.5%	50.0%	97.5%
11	229.83	8	10.12	3.48	4	10	18
20	1,315.37	22	41.36	6.82	28	41	55
70	54.81	0	0.38	0.65	0	0	2
89	79.63	40	33.76	6.19	22	34	47
112	18,809.67	45	36.18	6.57	24	36	49

gested that the variance stabilizing transformation $z_{ij} = 2\sqrt{x_{ij}}$ should first be applied to the relative frequencies $x_{ij} = Y_{ij}/P_{ij}$, for $i = 1,\ldots,133$ and $j = 1,\ldots,7$, in order to produce values that are approximately normal. The interested reader is referred to Klugman [18, p. 77] for a discussion of the rationale justifying this transformation. Klugman shows that if we apply this transformation and let $\gamma_i = 2\sqrt{\theta_i}$, then the appropriate variance stabilized hierarchical normal model (VSHNM) for the workers compensation data has its first two levels described by

$$f(z_{ij} \mid \gamma_i) \sim \text{normal } (\gamma_i, P_{ij}) \tag{5.25}$$

and

$$f(\gamma_i \mid \mu, \tau^2) \sim \text{normal } (\mu, \tau^2). \tag{5.26}$$

As usual, these normal densities are indexed by two parameters, a mean and a precision (i.e., inverse variance). To complete the model specification, we adopt the diffuse proper priors

$$f(\mu) \sim \text{normal } (0, 0.001), \tag{5.27}$$

$$f(\tau^2) \sim \text{gamma } (0.001, 0.001), \tag{5.28}$$

and make our standard assumptions with respect to independence. We performed a Bayesian analysis of this model via the Gibbs sampler (using BUGS) and present summaries of the posterior and predictive analyses in Tables 8, 9, and 10. From Tables 3, 6, and 9, it appears that the VSHNM performed better than the original HNM, at least in terms of the posterior distribution of the statistic *OMPS* measuring overall prediction success, but not quite as well as the HFLPM. This observation is illustrated by Figure 8, in which we have plotted the estimated posterior distribution of *OMPS* resulting under each of the three hierarchical models.

6. CLOSING DISCUSSION

This paper focused on the MCMC method known as the Gibbs sampler. Other MCMC methods do exist. Perhaps the foremost of these is the so-called Metropolis-Hastings algorithm

FIGURE 8

ESTIMATED POSTERIOR DENSITIES FOR *OMPS* IN EXAMPLE 4

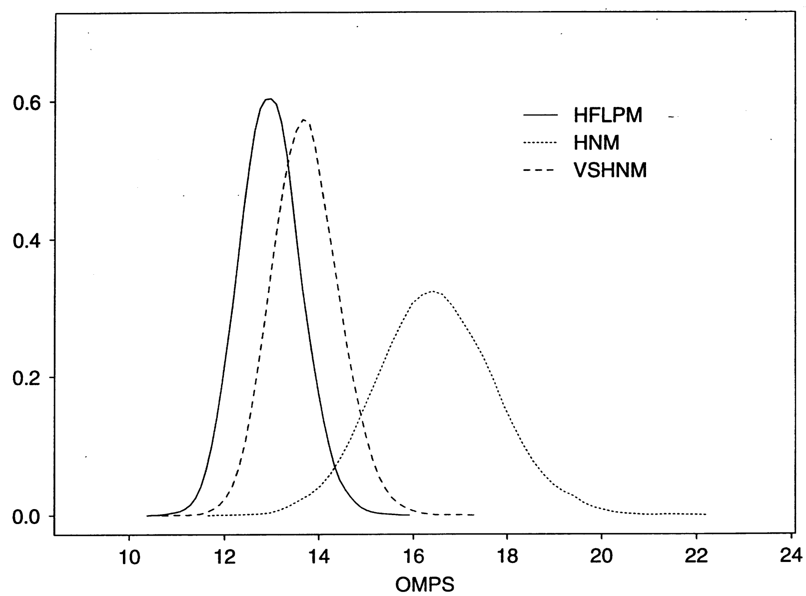


TABLE 8

EXPECTED WORKERS COMPENSATION INSURANCE FREQUENCIES
(VARIANCE STABILIZED HIERARCHICAL NORMAL MODEL)

	Actual Values		Expected Number of Claims $P_{i7}\theta_i$				
Class	Exposure	Claims	Estimated Posterior Summary Statistics				
i	P_{i7}	Y_{i7}	Mean	S.D.	2.5%	50.0%	97.5%
11	229.83	8	9.99	1.45	7.28	9.83	12.98
20	1,315.37	22	40.83	2.20	36.61	40.81	45.22
70	54.81	0	0.06	0.08	0	0.03	0.30
89	79.63	40	30.94	1.96	27.21	30.95	34.88
112	18,809.67	45	35.36	2.68	30.29	35.33	40.67

TABLE 9

AN OVERALL MEASURE OF PREDICTION SUCCESS
(VARIANCE STABILIZED HIERARCHICAL NORMAL MODEL)

Estimated Posterior Summary Statistics				
Mean	S.D.	2.5%	50.0%	97.5%
13.72	0.69	12.40	13.69	15.13

TABLE 10

PREDICTED WORKERS COMPENSATION INSURANCE FREQUENCIES
(VARIANCE STABILIZED HIERARCHICAL NORMAL MODEL)

	Actual Values		Predicted Number of Claims Y_{i7}				
Class	Exposure	Claims	Estimated Predictive Summary Statistics				
i	P_{i7}	Y_{i7}	Mean	S.D.	2.5%	50.0%	97.5%
11	229.83	8	10.16	3.50	4.33	9.82	18.11
20	1,315.37	22	41.21	6.61	29.26	40.84	54.62
70	54.81	0	0.31	0.43	0	0.14	1.54
89	79.63	40	31.25	5.82	20.50	31.00	43.13
112	18,809.67	45	35.69	6.49	24.00	35.37	49.29

(Metropolis, Rosenbluth, Rosenbluth, Teller, Teller [20]; Hastings [15]; Roberts and Smith [23]). There are also many other actuarial problems beyond those discussed in this paper for which MCMC methods have potential application. These include the simulation of the aggregate claims distribution, the analysis of stochastic claims reserving models, and the analysis of credibility models with state-space formulations. We hope to report upon some of these topics and applications in the future.

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APPENDIX A

The file in this appendix may be used in conjunction with the BUGS software in order to conduct Bayesian inference using Gibbs sampling for the model in Example 3.

```
# This is the BUGS file “lpareto.bug”.

model lpareto;
const
   cases=25, lambda=5000;
var
   x[cases], y[cases], theta;
data in “lpareto.dat”;
inits in “lpareto.in”;

{
   for (i in 1:12) {
      y[i]~dpar(theta, lambda) I(5000, 6000);
      x[i]<-y[i]-lambda;
      }
   for (i in 13:20) {
      y[i]~dpar(theta, lambda) I(6000, 7000);
      x[i]<-y[i]-lambda;
      }
   for (i in 21:23) {
      y[i]~dpar(theta, lambda) I(7000, 8000);
      x[i]<-y[i]-lambda;
      }
   for (i in 24:25) {
      y[i]~dpar(theta, lambda) I(8000,);
      x[i]<-y[i]-lambda;
      }
   theta~dgamma(0.001, 0.001);
}
```

APPENDIX B

The file in this appendix may be used in conjunction with the BUGS software in order to conduct Bayesian inference using Gibbs sampling for the first model in Example 4.

```
# This is the BUGS file "Normal.bug".

model Normal;
const
   cases=767, classes=133, years=6;
var
   loss[cases], payroll[cases], class[cases],
   x[cases], prec1[cases], theta[classes],
   mu, tau1, tau2,
   y11, y112, y70, y20, y89;
data in "Normal.dat";
inits in "Normal.in";

{
   for (i in 1:cases) {
      x[i]<-loss[i] / payroll[i];
      x[i]~dnorm(theta[class[i]], prec1[i]);
      prec1[i]<-tau1*payroll[i];
      }
   tau1~dgamma(0.001, 0.001);

   for (j in 1:classes) {
      theta[j]~dnorm(mu, tau2);
      }
   mu~dnorm(0, 0.001);
   tau2~dgamma(0.001, 0.001);
   y11<-229.83*theta[11];
   y112<-18809.67*theta[112];
   y70<-54.81*theta[70];
   y20<-1315.37*theta[20];
   y89<-79.63*theta[89];
}
```

APPENDIX C

The file in this appendix may be used in conjunction with the BUGS software in order to conduct Bayesian inference using Gibbs sampling for the second model in Example 4.

```
# This is the BUGS file "Poisson.bug".

model Poisson;
const
   cases=767, classes=133, years=6;
var
   loss[cases], payroll[cases], class[cases],
   lambda[cases], theta[cases], alpha[classes],
   mu, tau,
   y11, y112, y70, y20, y89;
data in "Poisson.dat";
inits in "Poisson.in";

{
   for (i in 1:cases) {
      loss[i]~dpois(lambda[i]);
      lambda[i]<-theta[class[i]]*payroll[i];
      }

   for (j in 1:classes) {
      log(theta[j])<-alpha[j];
      alpha[j]~dnorm(mu, tau);
      }
   mu~dnorm(0, 0.001);
   tau~dgamma(0.001, 0.001);
   y11<-229.83*theta[11];
   y112<-18809.67*theta[112];
   y70<-54.81*theta[70];
   y20<-1315.37*theta[20];
   y89<-79.63*theta[89];
}
```

ADDRESS TO NEW MEMBERS—MAY 13, 1996

MICHAEL L. TOOTHMAN

Congratulations to the 134 new Associates and the 18 new Fellows. This is a significant step for you. I'm glad that CAS President Al Beer has remembered the spouses or significant others and the people whom each of you has had for support. Those of us who have been in the actuarial profession for a while can cite many instances where that support has been the critical difference between achieving this milestone and not achieving it. And, sadly, I can also remember a few cases where lack of support has probably been the critical factor in an otherwise capable person's not reaching the milestone that you have reached. So we do want to remember those people today, as it is a significant achievement for them as well.

This is an important milestone for each of you. For the new Fellows, this is the culmination of many years of hard work. For the new Associates, the same statement would hold true, but we hope this is an interim step for you—that this will be just the first celebration and that, in another year or two, we will have the pleasure of welcoming you as new Fellows. For the new Associates, this is an important milestone because it represents your entrance into the Casualty Actuarial Society as a member and your entrance into the actuarial profession. With that comes many opportunities and responsibilities. You have chosen to make the actuarial profession your career, your life's work, or at least the basis and launching pad for your career; we welcome you to this profession. I hope you will find it as rewarding and as good a choice for you as I feel it has been for me, and as I imagine each of the people at the dais have found it to be as well.

For many of you, this is your first CAS meeting. Consider it a celebration of your achievement. We hope that you will enjoy it; that you'll learn something from the sessions; that you will gain some perspective; and that you will make some new friends.

For some of you, this is your introduction to the CAS. Each of you has been through the CAS Course on Professionalism, and some of you have been to exam seminars. However, this is probably the largest gathering of actuaries that you've ever experienced. So, I hope you'll make new friends here. You'll find as you go through your career that making new friends will be the part that you will remember the most, and that will mean the most, 20 years from now.

About a dozen years ago, the CAS began a tradition of asking a past president to offer some words of wisdom to the new Fellows and Associates. In fact, Tom Murrin was the first one who had this opportunity. Tom is here today, so I hope all of you will have a chance to visit with Tom during the meeting. He'll offer additional words of wisdom for you, and he's worth listening to and getting to know. The two presidents who spoke when I became an Associate and then a Fellow were LeRoy Simon and Charlie Hewitt. Now I suppose it wouldn't matter who the two presidents were, because a string of wonderful people have held that office, but I feel particularly privileged to have had Roy Simon and Charlie Hewitt as my two presidents. I still remember Roy's Presidential Address entitled "Know Thyself, Actuary." It should be recommended reading for all of you and can be found in the 1972 *Proceedings*. Roy is also here today, so please take the opportunity to meet Roy sometime during this meeting.

What do we expect of you as new members? First, I would say for the new Associates: Complete your exams. I have heard too many stories about a new Associate who is now going to take a break from the exams, or maybe is going to stop entirely. Don't do it. Please finish the exams. If you take a break, it will be that much more difficult to get back into them. There are only three, or in some cases, two or one exam left, so first complete those exams if you can. We encourage you all to do that. You are close to finishing the entire process, so do it now.

Second, we expect you to conduct yourselves as professionals. There are many ways to do that. Most importantly: Do your work well. We don't want to overlook that. Every day you are trading with my reputation as an actuary by what you do, just as everyone else in this room is trading with yours. We are all very inter-dependent. It is important to all of us, and to our reputations, that you do your work well. You will hear stories from time to time of an actuary who will say whatever his client wants him to say, or who will provide the answer that his boss wants to hear. That is not your responsibility as a professional. Sometimes you have to deliver tough news. So, do your work well. Do it technically correctly. Do the right thing as you make your judgments, because there is judgment in everything that you do. You have the benefit of many aids in doing that. The Standards of Practice are something you ought to read and study. Get to know them. Have them where you can reach them, and use them when necessary. They probably won't be a daily reference, but they ought not to gather dust on your bookshelf. The Code of Professional Conduct is critical. In terms of doing your work well, *how* you do it is perhaps as important as *what* you do. The code of conduct can be summarized by looking at just the first two precepts. Precept 1 reads: "An actuary shall act honestly and in a manner to uphold the reputation of the actuarial profession and to fulfill the profession's responsibility to the public." Precept 2 is like it: "An actuary shall perform professional services with integrity, skill and care." If you obey those two precepts, almost everything else will fall into place.

Third, give back to the profession. Al has already suggested that we've gained 18 new potential committee members today, and I hope we have 134 more new potential committee members on the way. For many of you that will mean starting with the CAS Examination Committee. Those of you who have finished the exams are perhaps in the best position to know how to make them better; how to write questions that will provide the right kind of discrimination; and how to help us determine who really knows the material well. So for many of you, the CAS Examination Committee will be the place to start your service to the profession. Furthermore, I hope that many of you will write papers and contribute to the literature of the profession. Perhaps at some point in the future, people taking the exams will see your material on the *CAS Syllabus of Examinations* and that will help them as they gain knowledge for their careers. Also, you can participate in the various sessions at CAS meetings and seminars.

Fourth, continue your education. You may think that your education is over as you finish the exams, but really it's just beginning. For some of you it may mean an additional degree program, such as an MBA; but your real education will continue every day in your work.

Earlier I mentioned Roy Simon's Presidential Address, "Know Thyself, Actuary." One thing I could add to that is "Know Your Business, Actuary." My pet peeve in this profession is the actuary who thinks that everything is in the numbers. An actuary who believes that a ten percent rate indication means that the only course of action, or even the correct course of action, is to increase rates by ten percent probably doesn't understand the underwriting and marketing aspects of this business. You need to understand the financial dynamics of the business and the people dynamics of the business. So continue your education in that fashion.

Twenty or twenty-five years from now, one of you may be here as President of the CAS. One of you may be giving this address. You may have written some papers that will be on the *Syllabus* that other actuaries will use to continue their education. Whatever you will have accomplished in your career at that point probably won't be nearly as important as the people relationships you will have established and the friendships that you will have made. It really is the most rewarding part of the profession. So, I encourage you to take advantage of this meeting to start that process.

On a personal note, I'll say that it was rather sad for me as I read the names of the new Fellows and new Associates and realized that there's only one among you whom I really know. I hope that by Wednesday I will have corrected that, and that I will have started the process of knowing several more of you. So, welcome to the CAS. Welcome to the actuarial profession, and enjoy the meeting.

MINUTES OF THE 1996 SPRING MEETING

May 12–15, 1996

J.W. MARRIOTT HOTEL, WASHINGTON, D.C.

Sunday, May 12, 1996

The Board of Directors held their regular quarterly meeting from noon to 5:00 p.m.

Registration was held from 4:00 p.m. to 6:00 p.m.

From 5:30 p.m. to 6:30 p.m., there was a special presentation to new Associates and their guests. All 1996 CAS Executive Council members briefly discussed their roles in the Society with the new members. In addition, David P. Flynn, who is a past president of the CAS, briefly discussed his role with the American Academy of Actuaries' Casualty Practice Council.

A welcome reception for all members and guests was held from 6:30 p.m. to 7:30 p.m.

Monday, May 13, 1996

Registration continued from 7:00 a.m. to 8:00 a.m.

CAS President Albert J. Beer opened the Business Session at 8:00 a.m. and recognized past presidents of the CAS who were in attendance at the meeting including: Irene K. Bass (1994), Phillip N. Ben-Zvi (1986), Ronald L. Bornhuetter (1976), Charles A. Bryan (1991), David P. Flynn (1993), Michael Fusco (1990), Allan M. Kaufman (1995), Frederick W. Kilbourne (1983), W. James MacGinnitie (1980), Thomas E. Murrin (1963-1964), Ruth E. Salzmann (1979), LeRoy J. Simon (1972), Jerome A. Scheibl (1981), and Michael L. Toothman (1992).

Mr. Beer also recognized special guests in the audience: Marc Fernet, President of the Canadian Institute of Actuaries; Wilson

Wyatt, Executive Director of the American Academy of Actuaries; and Sam Gutterman, President of the Society of Actuaries.

Paul Braithwaite, Susan T. Szkoda, John Kollar, and Michael Miller announced the 135 new Associates and the 19 new Fellows. The names of these individuals follow.

NEW FELLOWS

Daniel George Carr
Gary C.K. Cheung
Jo Ellen Cockley
Daniel Joseph Flick
Wayne Hommes
Charles N. Kasmer
Ann Louise Kiefer
Cheung S. Kwan
Mylène Labelle
Roland David Letourneau
Richard Stanley Light
Donald E. Manis
Kelly Jean Mathson
David William McLaughry
Scott M. Miller
Christina Lee Scannell
Jeanne Evelyn Swanson
Rae M. Taylor
Barry C. Zurbuchen

NEW ASSOCIATES

Jeffrey R. Adcock
Nathaniel James Babcock
Kimberly Moran Barnett
Elizabeth F. Bassett
David Bernard Bassi
Brian Keith Bell
Eric David Besman
Raju Bohra
Kimberly Ann Bowen
Charles Brindamour
Linda Marie Brockmeier
Lisa Ann Brown
Louis Michael Brown
Robert F. Brown
Kirsten Rose Brumley
Ron Brusky
Marian Margaret Burkart
Janet Pruitt Cappers
Joseph G. Cerreta
Hsiu-Mei Chang
Hong Chen
Michelle Codère
William Brian Cody
David Gary Cook
Matthew Dan Corwin
Jeffrey Wayne Davis
Raymond Victor De Jaco
Chris Dougherty
Peter Francis Drogan
David L. Drury
Louis Durocher
Dawn E. Elzinga
Vicki Agerton Fendley
John D. Ferraro
Mary Fleischli
Jeffrey M. Forden
Christian Fournier
Walter H. Fransen
Jean-Pierre Gagnon
Lynn Ann Gehant
Karl Goring
Jeffrey Shannon Goy
Mari Louise Gray
John A. Hagglund
Lynne Marie Halliwell
Alessandrea Corinne Handley
Gerald D. Hanlon
Ronald Joseph Herrig

Daniel Leo Hogan, Jr.
Wayne Hommes
Eric J. Hornick
Brett Horoff
Linda Marie Howell
Marie-Josèe Huard
Man-Gyu Hur
James Bernard Kahn
Anthony N. Katz
James Michael Kelly
Diane L. Kinner
Joseph P. Kirley
Brandelyn C. Klenner
Terri C. Kremenski
Steven M. Lacke
Jocelyn Laflamme
Steven W. Larson
Thomas Vuong Le
Guy Lecours
Jennifer M. Levine
Philip Lew
Lee C. Lloyd
Cara Mae Low
Robb W. Luck
William Richard Maag
Joseph A. Malsky
Betsy Fox Maniloff
Joseph Marracello
Bonnie Carole Maxie
David Molyneux
Matthew Stanley Mrozek
Karen Elaine Myers
Donna M. Nadeau
Kari S. Nelson
Catherine Anne Neufeld
Mindy Y. Nguyen
Kevin Jon Olsen
James David O'Malley
David J. Otto
Michael Guerin Owen
Erica Partosoedarso
Daniel Berenson Perry
Michael W. Phillips
Mitchell Stuart Pollack
Dale Steven Porfilio
David Scott Pugel
Patrice Raby
Kiran Rasaretnam
Raymond J. Reimer
Christopher Roy Ritter
Jeremy Roberts
Dave Harrison Rodriguez
Jean-Denis Roy
David L. Ruhm
Douglas A. Rupp
Romel Garry Salam
Cindy Rae Schauer
Christine Ellen Schindler
Jonathan N. Shampo
Kevin Huidong Shang
Kendra Barnes South
Caroline B. Spain
Theodore S. Spitalnick
William G. Stanfield
Christopher Mohr Steinbach
Curt A. Stewart
Lori Edith Stoeberl
Deborah L. Stone
Brian K. Sullivan
Mark Lynn Thompson
Diane Renee Thurston
Jennifer Marie Tornquist
Philippe Trahan
Joseph Daniel Tritz
Laura M. Turner
Mary Elizabeth Waak
Edward Harris Wagner
Benjamin A. Walden
Erica Lynn Weida
Denise R. Webb
Robert Gary Weinberg
Jennifer Naehr Williams
Bonnie Sue Wittman
Brandon L. Wolf
Kah-Leng Wong
Rick Allen Workman
Michele Nicole Yeagley
Richard Louis Zarnik

Mr. Beer then introduced Michael J. Toothman, a past president of the Society, who presented the Address to New Members.

Patrick J. Grannan, CAS Vice President-Programs and Communications, presented the highlights of the program.

David L. Miller, chairperson of the CAS Committee on Review of Papers, announced that four of the five accepted *Proceedings* papers would be presented at this meeting. In addition, one discussion of a paper that was published in the 1995 *Proceedings* would be presented at this meeting.

Gary Josephson gave a brief description of this year's Call Paper Program on Alternative Markets/Self Insurance. He announced that six of the ten call papers would be presented at this meeting, and that all ten call papers were bound in the 1996 *CAS Discussion Paper Program*.

Mr. Beer then began the presentation of awards. He explained that the CAS Harold W. Schloss Memorial Scholarship Fund benefits deserving and academically outstanding students in the actuarial program of the Department of Statistics and Actuarial Science at the University of Iowa. The student recipient is selected by the Trustees of the CAS Trust, based on the recommendation of the department chair at the University of Iowa. Mr. Beer announced that Tendra J. Cady is the recipient of the 1996 CAS Harold W. Schloss Memorial Scholarship. She will be presented with a $500 scholarship.

Mr. Beer also announced that Richard B. Amundson is the recipient of the 1996 Michelbacher Prize for his paper, "Residual Market Pricing." Mr. Beer explained that this award commemorates the work of Gustav F. Michelbacher and honors the author of the best paper submitted in response to a call for discussion papers. The papers are judged by a specially appointed committee on the basis of originality, research, readability, and completeness.

Mr. Beer then introduced Wilson Wyatt, Executive Director of the American Academy of Actuaries, who gave a presentation on casualty policy issues.

Mr. Beer then concluded the business session of the Spring Meeting by calling for a review of *Proceedings* papers. There were none presented at this time.

After a refreshment break, Mr. Beer introduced Patrick E. Kelly, the Interim Commissioner of Insurance for the District of Columbia Insurance Administration. Mr. Kelly gave a welcoming address to the meeting participants.

Afterward, Mr. Beer introduced the Featured Speaker, Peter B. Walker. Mr. Walker is the director of McKinsey and Company's New York office. McKinsey is a management consulting firm. During his 24 years with McKinsey, his clients have included property/casualty, life, and multi-line companies, as well as financial service conglomerates. He is the leader of McKinsey's North American insurance practice. He has authored a number of articles and is a frequent contributor to *Best's Review*.

The first General Session was held from 11:00 a.m. to 12:30 p.m.:

"Financial Restructuring"

Moderator: Joseph W. Brown
Chairman, President, and Chief Executive Officer
Talegen Holdings, Inc.

Panelists: David Walsh
DBG General Counsel
American International Group

George K. Bernstein
Attorney at Law

Douglas C. Moat
Chairman
The Manhattan Group

Stephan L. Christiansen
Senior Vice President
Conning & Company

After a luncheon, the afternoon was devoted to presentations of discussion papers and concurrent sessions. The call papers presented were:

1. “Residual Market Pricing”

   Author: Richard B. Amundson, FCAS
   Minnesota Department of Commerce

2. “A Stakeholder Approach to Risk Financing Programs”

   Authors: Stephen R. DiCenso, FCAS
   Deloitte & Touche LLP

   Michael R. Levin
   Deloitte & Touche LLP

3. “A Model for Estimating Loss Costs for Alternative Market Risks”

   Author: Joseph A. Herbers, ACAS
   Miller, Rapp, Herbers & Terry, Inc.

4. “A Casualty Actuary’s Guide to GASB Statement No. 10—Criteria for Determining Applicability of GASB 10 to Alternative Risk Programs and Suggested Guidelines for Actuarial Implementation’’

   Author: Roger C. Wade, ACAS
   KPMG Peat Marwick LLP

5. “Insurance Catastrophe Futures”

   Author: Robert P. Eramo, ACAS
   Johnson & Higgins

6. “A Buyer’s Guide for Options and Futures on a Catastrophe Index”

   Author: Glenn G. Meyers, FCAS
   Insurance Services Office, Inc.

The panel presentations covered the following topics:

1. Auto Safety Device Research

   Moderator: Steven F. Goldberg
   Senior Vice President and Chief Actuary
   United Services Automobile Association

   Panelists: Allan F. Williams, Ph.D.
   Senior Vice President, Research
   Insurance Institute for Highway Safety

   Wayne W. Sorenson
   Vice President, Research
   State Farm Mutual Automobile Insurance Company

   Timothy A. Hoyt
   Associate Vice President of Safety
   Nationwide Insurance Company

2. Questions and Answers with the CAS Board of Directors

   Moderator: Robert A. Anker
   (CAS President-Elect)
   President and Chief Operating Officer
   Lincoln National Corporation

   Panelists: David R. Chernick
   Assistant Vice President and Actuary
   Allstate Insurance Company

   John M. Purple
   Consulting Actuary
   Arthur Andersen LLP

   Kevin B. Thompson
   Assistant Vice President and Actuary
   Insurance Services Office, Inc.

3. Collecting From Your Reinsurer

Moderator: David S. Powell
Consulting Actuary
Tillinghast/Towers Perrin

Panelists: David M. Raim
Partner
Chadbourne & Parke LLP

Michael D. McNeely
Vice President, Controller and Treasurer
The Maryland Insurance Group

After a refreshment break from 3:00 p.m. to 3:30 p.m., presentations of call papers and concurrent sessions continued. Certain call papers and concurrent sessions presented earlier were repeated. Additional call papers presented from 3:30 p.m. to 5:00 p.m. were:

1. "Pricing Employment Practices Liability Exposures"

Authors: Brian Z. Brown, FCAS
Milliman & Robertson, Inc.

Chad C. Karls
Milliman & Robertson, Inc.

2. "TPA Service Pricing and Incentive Contracts"

Author: Hou-Wen Jeng, ACAS
Nationwide Insurance Company

3. "Cost Allocation Methods for Workers Compensation"

Author: George M. Levine, FCAS
KPMG Peat Marwick LLP

Additional concurrent sessions that were presented were:

1. The ASB and Financial Reporting Recommendation 8

Moderator: Martin Adler
Vice President and Actuary
GEICO

Panelists: Robert S. Miccolis
Senior Vice President and Actuary
Reliance Reinsurance Corporation

Patricia A. Teufel
Principal
KPMG Peat Marwick LLP

2. CAS Task Force on Education

Moderator: Steven G. Lehmann
Actuary
State Farm Mutual Automobile Insurance Company

Panelists: Claudette Cantin
Consulting Actuary
Tillinghast-Towers Perrin

John J. Kollar
Vice President
Insurance Services Office, Inc.

Michael A. LaMonica
Vice President and Actuary
Allstate Insurance Company

An officers' reception for new Fellows and guests was held from 5:30 p.m. to 6:30 p.m., and the general reception for all members and their guests was held from 6:30 p.m. to 7:30 p.m.

Tuesday, May 16, 1996

Two General Sessions were held simultaneously from 8:30 a.m. to 10:00 a.m. One was:

"Spreading Catastrophe Risk"

Moderator: John P. Drennan
Independent Consulting Actuary

Panelists: Jack F. Weber
Executive Director
National Disaster Coalition

Daniel Marshal
State of California Department of Insurance

Gary Grant
Vice President and Actuary
State Farm Fire & Casualty Company

The other session, presented simultaneously, was

"Environmental Liability and Superfund"

Moderator: Phillip N. Ben-Zvi
Partner
Coopers & Lybrand, L.L.P.

Panelists: The Honorable Michael G. Oxley (R-OH)
U.S. House of Representatives

Sanders B. Cathcart
Assistant Vice President and Actuary
Insurance Services Office, Inc.

William C. Aldrich
Vice President
ITT/Hartford Insurance Group

After a refreshment break, call paper presentations and concurrent sessions were held from 10:30 a.m. to noon. In addition to concurrent sessions and call papers presented the previous day, the following additional concurrent sessions and papers were presented:

1. The Internet and the Actuary

Moderator: Regina M. Berens
Consulting Actuary
MBA, Inc.

Panelists: James R. Garven, Ph.D.
Vice President
Strategic Concepts Corporation

Bradley Gorman
Computer Support Specialist
Casualty Actuarial Society

Christopher Diamantoukos
Senior Consulting Actuary
Ernst & Young LLP

Jerome E. Tuttle
Senior Vice President and Actuary
Mercantile & General Reinsurance Company

2. Managed Care and Auto Insurance

Moderator: Anthony J. Grippa
Principal
William M. Mercer, Inc.

Panelists: Nyshie Miller
Program Director, Auto
Sloans Lake Managed Care

Jane L. Renninger
Managing Director
Travelers/Aetna Property Casualty Corporation

David F. Snyder
Assistant General Counsel
American Insurance Association

3. Introduction to the CAS Examination Committee

Moderator: Beth E. Fitzgerald
Manager and Associate Actuary
Insurance Services Office, Inc.

Panelists: Charles D. Kline, Jr.
Assistant Vice President
GEICO

Virginia R. Prevosto
Assistant Vice President
Insurance Services Office, Inc.

The following additional call paper was presented:

1. "Statistical and Financial Aspects of Self-Insurance Funding"

   Author: Leigh J. Halliwell, ACAS
   Zurich Insurance Group

The following *Proceedings* papers were presented:

1. "Pricing to Optimize an Insurer's Risk Return Relationship"

   Author: Daniel F. Gogol
   Second Vice President
   General Reinsurance Corporation

2. Discussion of "A Simulation Test of Prediction Errors of Loss Reserve Estimation Techniques,"
   (by James Stanard, *PCAS* LXXII, 1985, p. 124)

   Discussion by: Edward F. Peck
   Manager
   John Deere Insurance Group

3. "Allocated Loss Adjustment Expense Liabilities"

   Author: Ruth E. Salzmann
   Retired
   Stevens Point, Wisconsin

Various CAS committees met from 1:00 p.m. to 5:00 p.m. In addition, a new concurrent session was held from 1:30 p.m. to 3:00 p.m.:

1. DFA Case Studies

   Moderator: Manuel Almagro, Jr.
   Consulting Actuary
   Tillinghast-Towers Perrin

   Panelists: Stephen M. Sonlin
   Vice President
   Falcon Asset Management

Stephen W. Philbrick
Consulting Actuary
Tillinghast-Towers Perrin

David B. Sommer
Consulting Actuary
Tillinghast-Towers Perrin

A mock trial was held from 1:30 p.m. to 4:30 p.m., during which CAS members portrayed U.S. Senators, Representatives, and witnesses in a mock situation that paralleled the formal proceedings of a Senate Committee hearing. This session was moderated by Jean Rosales of the American Academy of Actuaries.

All members and guests enjoyed a buffet dinner, music, and the movie "To Fly'' at the Smithsonian's National Air and Space Museum from 6:30 p.m. to 10:00 p.m.

Wednesday, May 17, 1996

Certain discussion papers and concurrent sessions that had been presented earlier during the meeting were repeated this morning from 8:30 a.m. to 10:00 a.m. Additional concurrent sessions presented were:

1. Equalization Reserves

Moderator: Jan A. Lommele
Principal
Deloitte & Touche LLP

Panelists: Vincent L. Laurenzano
Chief Examiner
New York Insurance Department

William Van Nostran
Government Relations
Cincinnati Companies

Mark A. Parkin
Partner
Deloitte & Touche LLP

2. CAS Committees on Continuing Education and Special Interest Seminars

   Panelists: Abbe S. Bensimon
   Vice President
   General Reinsurance Corporation

   Gary R. Josephson
   Consulting Actuary
   Milliman & Robertson, Inc.

The *Proceedings* Papers that were presented were:

1. "The Complement of Credibility"

   Author: Joseph A. Boor
   Consulting Actuary
   Actuarial & Technical Services, Inc.

2. "The Interaction of Maximum Premiums, Minimum Premiums, and Accident Limits in Retrospective Rating"

   Author: Howard C. Mahler
   Vice President and Actuary
   Workers' Compensation Rating & Inspection Bureau of Massachusetts

The final General Session was held from 10:30 a.m. to noon after a 30-minute refreshment break:

"Banks and Insurance: What's Ahead?"

Moderator: Charles A. Bryan
Partner
Ernst & Young LLP

Panelists: Frank J. Coyne
President
General Accident

Dennis R. Kosovac
President
Chemical Insurance Agency

Jonathan Plutzick
Managing Director
CS First Boston Corporation

John E. Washburn
Aon Risk Services

Mark Olson
Partner, National Director of Banking
Regulatory Affairs
Ernst & Young LLP

Susan T. Szkoda officially adjourned the 1996 CAS Spring Meeting at noon after closing remarks and an announcement of future CAS meetings.

May 1996 Attendees

The 1996 CAS Spring Meeting was attended by 214 Fellows, 169 Associates, and 85 Guests. The names of the Fellows and Associates in attendance follow:

FELLOWS

Martin Adler
Kristen M. Albright
William C. Aldrich
Gregory N. Alff
Manuel Almagro Jr.
Karen E. Amundsen
Richard B. Amundson
Robert A. Anker
Lawrence J. Artes
Albert J. Beer
Phillip N. Ben-Zvi
Abbe Sohne Bensimon
Regina M. Berens
Lisa M. Besman
David R. Bickerstaff
James E. Biller
Joseph A. Boor
Ronald L. Bornhuetter
Christopher K. Bozman
Nancy A. Braithwaite
Paul Braithwaite
Malcolm E. Brathwaite
Brian Z. Brown
Joseph W. Brown Jr.
Charles A. Bryan
Jeanne H. Camp
Claudette Cantin
William M. Carpenter
Daniel G. Carr
Lynn R. Carroll
Edward J. Carter
Sanders B. Cathcart
Michael J. Caulfield
Ralph M. Cellars
David R. Chernick
Stephan L. Christiansen
Allan Chuck
Mark M. Cis
Laura R. Claude
Michael A. Coca
Jo Ellen Cockley
Robert F. Conger
Mary L. Corbett
Susan L. Cross
Janice Z. Cutler
Ronald A. Dahlquist

Thomas J. DeFalco
Jerome A. Degerness
Stephen R. DiCenso
Christopher Diamantoukos
Kevin G. Dickson
Michael C. Dolan
James L. Dornfeld
John P. Drennan
Michael C. Dubin
Kenneth Easlon
Richard D. Easton
Paul E. Ericksen
Richard J. Fallquist
Richard I. Fein
Mark E. Fiebrink
Russell S. Fisher
Beth E. Fitzgerald
Daniel J. Flick
Claudia S. Forde
Kenneth R. Frohlich
Michael Fusco
Robert W. Gardner
Kim B. Garland
Bradley J. Gleason
Daniel C. Goddard
Steven F. Goldberg
James F. Golz
Timothy L. Graham
Patrick J. Grannan
Gary Grant
Gregory T. Graves
Eric L. Greenhill
Anthony J. Grippa
Sam Gutterman
Jeffrey L. Hanson
Thomas M. Hermes
Kathleen A. Hinds
Wayne Hommes
Mary T. Hosford
Douglas J. Hoylman
Brian A. Hughes
James G. Inkrott
Richard M. Jaeger
Eric J. Johnson
Laura A. Johnson
Warren H. Johnson Jr.
Thomas S. Johnston
Gary R. Josephson
Steven W. Judd
Allan M. Kaufman
Anne E. Kelly
Deborah E. Kenyon
Allan A. Kerin
C.K. Stan Khury
Frederick W. Kilbourne
Richard O. Kirste
Warren A. Klawitter
Joel M. Kleinman
Charles D. Kline Jr.
Leon W. Koch
John Joseph Kollar
Gustave A. Krause
Rodney E. Kreps
Andrew E. Kudera
Ronald T. Kuehn
Kay E. Kufera
John M. Kulik
Cheung S. Kwan
Michael A. LaMonica
Mylene J. Labelle
Christopher Lattin
Merlin R. Lehman
Steven G. Lehmann
Roland D. Letourneau
Joseph W. Levin
George M. Levine
Allen Lew
Richard S. Light
Orin M. Linden
Barry C. Lipton
Roy P. Livingston
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PERSONAL AUTOMOBILE PREMIUMS: AN ASSET SHARE PRICING APPROACH FOR PROPERTY/CASUALTY INSURANCE

SHOLOM FELDBLUM

Abstract

Asset share pricing models are used extensively in life and health insurance premium determination. In contrast, property/casualty ratemaking procedures consider only a single period of coverage. This is true for both traditional methods, such as loss ratio and pure premium ratemaking, and financial pricing models, such as discounted cash flow or internal rate of return models.

This paper provides a full discussion of property/casualty insurance asset share pricing procedures. Section 1 compares life insurance to casualty insurance pricing. It notes why asset share pricing is so important for the former, and how it applies to the latter as well. Section 2 describes the considerations essential for an asset share pricing model. Premiums, claim frequency, claim severity, expenses, and persistency rates must be examined by

time since inception of the policy. Appropriate discount rates must be selected for: (a) present values of the contract cash flows during each policy year, and (b) the present value of future earnings at the inception date of the policy.

Sections 3 through 7 present four illustrations of asset share pricing:

- *Section 3 is a general introduction.*
- *Section 4 illustrates pricing considerations for an expanding book of business. Since both loss costs and expense costs are higher for new business than for renewal business, traditional loss ratio or pure premium pricing methods show misleading rate indications.*
- *Section 5 discusses classification relativities. Since persistency rates and coverage combinations differ by classification, the traditional relativity analyses may be erroneous.*
- *Section 6 presents a competitive strategy illustration. Premium discounts and surcharges affect retention rates, particularly among policyholders who can obtain coverage elsewhere.*
- *Section 7 shows how underwriting cycle movements can be incorporated into pricing strategy. Expected future profits vary with the stage of the cycle; these future earnings and losses must be considered when setting premium rates.*

Section 8 discusses several types of profitability measures: returns on premium, returns on surplus or equity, internal rates of return, and the number of years until the policy becomes profitable. Traditional financial pricing models examine a single contract period and multiple loss payment periods. For asset share pricing, these models are expanded to consider multiple contract periods. For instance, the "return on premium" is the present value of future expected profits divided by the *present value of future expected premium, not the single period amounts used for operating ratios.*

Asset share models determine the long-run profitability of the insurance operations, the true task of the pricing actuary.

ACKNOWLEDGEMENTS

The author is indebted to Richard Woll and Stephen D'Arcy for inspiration and criticism of this paper. Ten years ago, Richard Woll was examining the effects of business volume growth on accounting profitability versus true profitability, and he demonstrated the powerful effects on the "costs of new business" (compare the first illustration in the paper). At about the same time, Professor D'Arcy was writing his papers on the "Aging Phenomenon" and on "Adverse Selection, Private Information, and Lowballing," which deal with some of the same issues as this paper covers, though it treats them differently. Professor D'Arcy sent early drafts of his papers to the author, he provided helpful critiques of the author's previous papers on this subject at a CAS conference, and he sent written comments on an earlier draft of this paper. The contributions of Richard Woll and Stephen D'Arcy greatly improved this paper.

1. INTRODUCTION

Asset share pricing models have long been used for life and health insurance premium determination. These models examine the profitability of the complete insurance contract from its inception to its final termination, including all renewals of the policy. That is to say, the life insurance pricing actuary does not evaluate the profitability of a block of policies in a given calendar year, policy year, or calendar/accident year. Indeed, such a valuation would not be meaningful, since a whole life insurance policy is expected to lose money in the initial year of issue but to make up for the loss in subsequent years. Rather, the life insurance actuary sets policy premiums to achieve an appropriate profit over the lifetime of the policy. Similarly, this paper applies asset share pricing methods to property/casualty lines of business.

Asset share pricing is especially important when cash flows and reported income vary by policy year. For instance, a whole life policy issued to a standard-rated thirty-year-old insured shows:

- high expense costs the first year (often greater than the gross premium),
- low mortality costs the first several years,
- higher mortality costs in later years, as the policyholder ages and the underwriting selection "wears off," and
- statutory benefit reserves that are somewhat redundant after the second or third year because of the conservative valuation of mortality tables and interest rates; during the first several years, preliminary term reserves reduce the statutory liability.[1]

In property/casualty insurance, loss ratio and pure premium ratemaking methods predominate. Financial pricing models are often used to set underwriting profit targets, although these methods, like the traditional property/casualty rate making techniques, presume an insurance contract in effect for a single policy period. Most financial pricing models examine the duration of loss payments, but they do not consider the duration of the insurance contract.[2]

Life Versus Casualty Ratemaking

The differing ratemaking philosophies for life and health insurance versus property/casualty insurance stem from several

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[1]On asset share pricing models for life insurance, see Anderson [8], Huffman [95], and Atkinson [10]; for health insurance, see Bluhm and Koppel [25]. Menge and Fischer [131, p. 131] explain the term "asset share" as "the equitable share of the policyholders in the assets of the company." Similarly, Atkinson [11] explains the term as "the share of assets allocable to each surviving unit."

[2]On the traditional ratemaking techniques, see McClenahan [129] and Feldblum [75]. On the development of financial pricing models, see Hanson [89], Webb [162], and Derrig [64]. For examples of the major models, see Fairley [67], Hill [92], NAIC [136], Urrutia [155], Myers and Cohn [135], Mahler [124], Woll [169], Butsic and Lerwick [39], Bingham ([20], [22]), Robbin [144], Feldblum [71], and Mahler [126]. For analyses of these models, see Hill and Modigliani [93], Derrig [65], Ang and Lai [9], D'Arcy and Doherty [61], Garven [85], D'Arcy and Garven [62], Mahler [125], and Cummins ([48], [50], [51]).

factors:

- *Cancellation:* Few individual life or health insurance policies may be canceled or non-renewed by the insurer, except for non-payment of premium. In property/casualty insurance, particularly in the commercial lines, the carrier has the right to terminate the policy at the renewal date and often to cancel the policy in mid-term.[3]
- *Claim costs:* Life and health insurance claim costs vary by duration since policy inception, for two reasons:
  - Policyholder age: mortality and morbidity costs rise as the insured ages.
  - Underwriting selection: medical questionnaires and examinations for life and health insurance lead to lower average initial benefit costs for insured lives. The effects of underwriting selection "wear off" after several years (Jacobs [106, p. 5]; Dahlman [55, p. 5]).

  In property/casualty insurance, the relationship between expected losses and duration since policy inception is less apparent.
- *Expenses:* Expenses show a similar pattern. Whole life commission rates are high in the initial year but low for renewals.[4] For property/casualty companies using the independent agency distribution system, commission rates do not differ between the first year and renewal years.
- *Level premiums:* Much life insurance is provided by level premium contracts. The premium exceeds the anticipated benefits during the early policy years, when the insured is young and healthy. In later years, anticipated benefit costs exceed the

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[3]Renewability provisions in health insurance vary among contracts, though cancelable policies are proscribed in many jurisdictions (Barnhart [13]). Many states now proscribe mid-term cancellations of personal automobile policies; others, such as California or Massachusetts, prohibit even non-renewals.

[4]Lombardi and Wolfe [119]. Atkinson [11, p. 5] notes that traditional life insurance "acquisition costs usually exceed the first year premium by a wide margin. Acquisition costs may even exceed 200% of premium, especially for smaller policies."

premiums, and they are funded by the policy reserves built up in earlier years. In contrast, property/casualty insurance rates may be revised each year. No "policy reserves" are held to shift costs among accounting periods.

Developments in Casualty Insurance

These differences are valid, and asset share pricing is therefore more common for life and health insurance premium development. But property/casualty insurance is taking on several of the attributes that motivate asset share pricing.

- *Commissions:* Most personal lines insurance policies are now issued by direct writers, whose commission rates are higher in the first year than in renewal years.
- *Cancellations:* Although the insurer may have the right to cancel or non-renew the contract, it rarely does so. Profitability depends on the stability of the book of business, and carriers seek to strengthen policyholder loyalty.
- *Loss costs:* As will be discussed below, expected loss costs are greater for new business than for renewal business.[5]

The question faced by all insurers is the same: "*Is it profitable to write the insurance policy?*" A financially strong carrier does not focus on reported results or cash flows for the current year. Rather, it examines whether the stream of future profits, both from the original policy year and from renewal years, justifies underwriting the contract. Asset share pricing enables the actuary to provide quantitative estimates of long-term profitability.

2. ASSET SHARE COMPONENTS

Asset share pricing is not yet common in property/casualty insurance for several reasons:

[5]Most actuarial studies of this phenomenon have concentrated on personal automobile insurance. Unpublished studies by the author and his colleagues show the same phenomenon in other lines, particularly for workers compensation.

- The data needed are not always available.
- Casualty pricing techniques do not always take into account long-term profit considerations.
- The casualty insurance policy allows great flexibility in premiums and benefit levels.
- Liability claim costs are uncertain, both in magnitude and in timing.

This section examines the qualitative influences on the asset share pricing components, to lay the groundwork for the quantitative model that follows.

A. Premiums

Premiums for whole life policies are set at policy inception, and they continue unchanged until the termination or forfeiture of the contract. Premiums for renewable term life policies are generally guaranteed for the first several years and illustrated for an additional ten or fifteen years. Similarly, policyholder dividends on participating contracts are often illustrated for the first twenty years.[6]

Property/casualty insurance premiums may be revised each year or half-year, and insurers do not illustrate the expected future premiums. In fact, premiums fluctuate widely from year to year for a variety of reasons:

- Inflation raises loss costs, and premiums are adjusted accordingly. Life insurance benefits, in contrast, are often fixed in nominal terms.
- Underwriting cycles raise and lower the premiums charged, whether by manual rate revisions or individual risk rating ad-

[6]The NAIC Life Insurance Solicitation Model Regulation requires that insurers illustrate surrender cost and net payment cost indices for ten and twenty year durations (Black and Skipper [23]; see also Jensen [107, pp. 449–450]). Premiums for some newer contracts, such as indeterminate premium and universal life policies, are harder to project for future years.

justments. Underwriting cycles are not found in individual life insurance.

- The insured's classification or exposure may change from year to year. The personal auto insured may marry, the workers compensation insured may expand its operations, and the commercial property risk may install fire protection equipment.[7] The classification of the individual life policyholder generally does not change after inception of the policy.[8]

In sum, the level premiums for traditional whole life insurance policies, versus the variable premiums for casualty products, have contributed to the greater reliance of life actuaries on asset share pricing methods.

B. Claims

Mortality rates are stable from year to year, and the influences on mortality are well documented. We may not fully understand why sex has such a strong influence on mortality, but given an individual's age, sex, and physical condition, we can provide a life expectancy (Berin, Stolnitz, and Teitlebaum [18]). At the inception of the insurance policy, the actuary can estimate mortality rates for the insured's lifetime. Barring major wars or epidemics, the estimates should be accurate.

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[7] See, for instance, Feldblum [70]: "... average loss costs vary over the life of a policy. For example, many young unmarried men are carefree drivers, less concerned with safety than with presenting a courageous image. Once they have married, begun careers, and borne children, they feel more responsibility, both individual and financial, for their families—and their driving habits improve accordingly. When their children become adolescents and start driving the family cars, auto insurance loss costs climb rapidly. But when the children leave home and the insured retires, the automobiles may be unused except for shopping trips and weekend vacations; automobile accidents become rare. Finally, when the driver enters his or her 70s, physiological health deteriorates and reactions are slowed. If the insured continues to drive, accident frequency increases." Similarly, Whitehead [167, p. 312] writes: "Changes in inherent risk over time—the typical 'life-cycle' of an insured with respect of individual private passenger automobile insurance is for the level of inherent risk to decline as the age of the insured and his level of driving experience and competence increases (at least until a relatively advanced age)."

[8] Minor exceptions exist. For instance, a substandard rated policyholder may be rerated after several years upon submission of evidence of insurability (Woodman [171]). Re-entry term insurance allows reclassification at the end of each select period (Galt [84]; Jacobs [106]).

B.1. Casualty Claim Rates

Claim rates in casualty insurance are more variable and less well understood. Why do urban drivers have higher personal auto claim frequencies than suburban residents? Is traffic density higher in cities than in rural areas? Are road conditions worse in urban areas? Are suburban residents, who are friendly with the neighboring children, more careful drivers? Are there more attorneys in cities, and do they encourage accident victims to file claims? Does the type and extent of medical treatment differ between urban and rural areas? Are rural residents more familiar with insurance agents and brokers and less inclined to seek compensation from "impersonal" corporations?[9]

Claim rates in workers compensation vary with economic conditions and with the operations of the insured. During recessions, when layoffs or plant closings are anticipated, many employees file workers compensation claims for minor, non-disabling injuries that they would ignore in more prosperous times (Borba [27]; Boden and Fleischman [26]; Victor and Fleischman [158]; Victor [157]; NJCIRB [139]). When a firm expands quickly, with young, inexperienced workers, accidental injuries are more common (Worrall, Appel, and Butler ([172], [173]); NCCI [137, p. 34]; Walters [160, p. 22]; ISO [102]).

In the commercial liability lines (other liability, products liability, medical malpractice, and professional liability), statu-

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[9]Casualty actuaries are just beginning to examine these issues. On traffic density in urban and suburban areas, and on the contribution of suburban drivers to urban traffic, see Brissman [29]. The importance of attorneys can be seen by comparing claims represented by attorneys and those not represented in urban and rural areas (AIRAC [5], [6]; Feldblum [75]; IRC [99]). The effects of "claims consciousness," or the proclivity to file insurance claims, can be measured by the ratio of bodily injury claims to property damage claims. The frequency of PD claims is primarily determined by the incidence of physical accidents. The frequency of BI claims is affected by claims consciousness and attorney involvement as well. The ratio of BI to PD claims varies by jurisdiction, and it is higher in cities than in rural areas (IRC [98], [100], [101]; Woll [169]; Cummins and Tennyson [53]). The type of medical practitioner, such as physician, chiropractor, or physical therapist, affects both claim frequency and severity (Marter and Weisberg [127], [128]; Weisberg and Derrig [163], [164], [165]). For the corresponding influences on workers compensation, see Feldblum [75].

tory enactments and judicial precedents affect the frequency of claims. Congressional passage of the CERCLA (Comprehensive Environmental Response, Compensation, and Liability Act) in 1980, with strict, joint, several, and retroactive liability, encouraged the filing of environmental impairment claims (Hamilton and Routman [88], Miller [132]; Kunreuther and Gowda [112]; ISO [105]). State legislation modifying the statute of limitations and setting caps on awards has affected the filing of medical malpractice claims.

The stability of life insurance benefits versus the variability of casualty insurance losses is a second reason for the greater use by life actuaries of asset share pricing methods. However, the fundamental issue is not the predictability of losses but the relationship of losses and expenses to persistency. The asset share model examines a particular policy and asks: "*Is this risk's expected profitability above or below the average for other insureds in its class?*" To answer this question, we examine three items: relative loss costs by policy year, expenses by policy year, and persistency rates by policy year and by classification.

B.2. Policy Duration and Claim Frequency

Policy duration has a strong influence on claim frequency, particularly in personal automobile, where new insureds have higher average loss ratios than renewal policyholders. Conning and Company [47, pp. 10–11], note that "Companies have acknowledged results which show new business loss ratios varying from 10% higher to more than 30% higher, depending on the line of business and the underwriting year."[10] Older drivers, with lower average claim frequencies and loss ratios, are more common in an insurer's renewal book than in its new business (Feldblum [70]). Several personal auto writers provide "renewal

[10] So also Schraeder [149, p. 165]: "Experience has shown that new business, carefully underwritten, develops poorer overall results than that which has been reunderwritten, and the latter produces poorer results than that recorded by a seasoned or older book of underwriting risks."

discounts," which reflect the lower loss and expense costs after the first policy year.

B.3. Inexperience, Youth, Transience, and Vehicle Acquisition

The relationship between duration of the policy and expected claim frequency results from several factors. Drivers who apply for new auto insurance policies are likely to be inexperienced, young, or "transient" insureds. Also, they have often recently acquired the automobile itself, and they may be unaccustomed to the particular hazards of the vehicle.

- *Experience:* Good driving habits are acquired over time; safety precautions are "second nature" for the experienced driver. Many accidents result from carelessness, not recklessness, so inexperienced drivers have high claim frequencies (Bailey and Simon [12]).
- *Youth:* Young drivers, both male and female, have higher than average claim frequencies, even after adjusting for driving experience. Young drivers with their own residences or automobiles have relatively new auto insurance policies. (Adolescent drivers living at home may be insured on their parents' policies. Since these drivers have high average claim frequencies, they cause a temporary reversal in the generally inverse relationship of frequency with policy duration.[11])
- *Transience:* Many high risk drivers, such as young males, are "transient" insureds, in that they often drop their coverage with one carrier and purchase a policy from another. Termination rates for young male drivers are as high as 20–30% for several reasons:
    - Young male drivers are more likely to voluntarily cancel their policies, perhaps because they move to other locations,

[11] In general, claim frequency declines as the policy ages. But when adolescent children obtain licenses, claim frequency on the parents' policy increases. This is an example of a classification change, which overwhelms the normal decline in claim frequency. See below in the text.

they get married and switch to their wives' insurers, or they drop their coverage after an accident.

- Company underwriters are more likely to cancel the coverage of a young male driver than that of an adult driver, since the young male driver is more likely to have caused an accident and be considered too risky to insure.
- Young male drivers are likely to experience financial difficulties and fail to pay the required premiums.
- Young male drivers with high premium payments have more incentive to shop around for cheaper coverage.[12]

Many low-risk insureds, such as retired drivers in their 60s and 70s, have termination rates as low as 3 or 4%. Retired drivers have less information about marketplace prices, which younger persons may hear about at the workplace.[13] These low-risk "stable" insureds reduce the claim frequencies of renewal business compared to new business.

- *Acquisition of the Vehicle:* The duration since the inception of the policy is correlated with the time since acquisition of the automobile. Accident frequency often decreases with time since acquisition, as the insured becomes accustomed to the hazards of the particular vehicle. For instance, the insured may have purchased a second hand vehicle during the summer, only to discover that the car skids on icy December roads.

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[12]See Feldblum [68], particularly Figure 7 and the accompanying discussion. Similarly, D'Arcy and Doherty [60, p. 38] speak of "poor risks that move from insurer to insurer as their true risk exposure is discovered." D'Arcy [56, p. 28] lists four reasons for the higher loss ratios of new business: "The inability to surcharge new insureds properly since less information is available, the higher loss potential of insurance shoppers who regularly shift from insurer to insurer in search of bargain coverage, the fact that new insureds include a high proportion of risks not wanted by other insurers, and the possibility that new insureds may be individuals unfamiliar with local driving conditions."

[13]Many policy "terminations" for older drivers result from death, poor health, or other reasons that prevent them from driving, not because they find a cheaper rate with another carrier. Thus, these drivers are not "transient" insureds.

The age of the vehicle (not the time since acquisition) is a classification dimension for physical damage coverages, since the value of the car declines over time.[14] The time since acquisition of the vehicle, not its age, is important for liability coverages. The two classification dimensions are the same only when the insured purchases a new automobile. Contrast a recently acquired five-year-old car with a new model car bought two years ago. The two-year-old car would have the higher physical damage relativity, and the five-year-old car would have the higher liability relativity.[15]

B.4. Reunderwriting

The relationship between loss ratios and the duration since policy inception may also be affected by the carrier's reunderwriting actions. D'Arcy and Doherty [60] suggest that "the accumulation of private information by the contracting insurer" causes declining loss ratios as the policy ages. The importance of this private information depends on the insurer's underwriting philosophy and on the power of this information to predict future loss costs.[16]

In workers compensation, the loss engineering services provided by the insurer, as well as its encouragement of a safe work environment, reduce claim frequency among persisting insureds. Loss control studies can be expensive, and the insurance carrier lacks the incentive to undertake them for "transient" risks.

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[14]This is true for the "age rating system" that was the predominant pricing procedure for automobile physical damage coverages in the 1960s and 1970s. The "model year rating" system pioneered by the major direct writers in the 1980s assumes that the decline in the value of the vehicle over time is offset by inflationary increases in repair costs. See Chernick [44, pp. 10–11].

[15]These are loss cost relativities, not rate relativities. When setting rates, an insurer must decide whether to use these relativities or other risk classification systems. For the differences between loss cost relativities and rate relativities, see Section 5.

[16]"Underwriting terminations" are less important than voluntary terminations in explaining the differences between young male and adult persistency rates in personal automobile insurance (Feldblum [68], Figure 8). However, underwriting terminations weed out the particularly poor risks, and so they may have a larger effect on the relationship between loss ratios and the duration since policy inception.

Similarly, a successful loss control program initiated by the carrier will encourage the insured employer to retain the coverage.[17]

C. Expenses

Insurance expenses are greater in the year the policy is first issued than in renewal years because underwriting and acquisition expenses are incurred predominantly at policy inception.[18] This is true for both "per policy" expenses, such as the costs of underwriting and setting up files, and "percentage of premium" expenses, such as commissions and premium taxes.

C.1. Life Insurance Expenses

Premium determination for life insurance policies incorporates these expense differences by policy year. For instance, Jordan [108, p. 133] gives the following illustration of a gross premium calculation (see also Neill [138, pp. 53–56]):

$$G\ddot{a}_x \approx 1005\left(1+\frac{i}{2}\right) A_x + .75G + .2G(\ddot{a}_{x:\overline{2|}} - \ddot{a}_{x:\overline{1|}}) + .1G(\ddot{a}_{x:\overline{6|}} - \ddot{a}_{x:\overline{2|}}) + .05G(\ddot{a}_x - \ddot{a}_{x:\overline{6|}}) + 10 + 2a_x,$$

where G is the annual gross premium for \$1000 of insurance, a_x, $\ddot{a}_x$, and A_x are the standard annuity and cost of insurance

[17] The relationship between claim frequency and "transient" risks is also applicable to workers compensation. Commenting on the unprofitability of small workers compensation risks, Kormes [110, pp. 49–50] says: "... this group of risks, which unfortunately float from carrier to carrier, has a great influence on the unsatisfactory small risk situation ..."

Small enterprises that mushroom during prosperous years often fail when the economy sours. Since these firms lack the funds for needed workplace safety measures and their workforce often consists of inexperienced employees, their occupational injury rates are high. Those firms that fail face additional costs: Since the employee's alternative to insurance payments is unemployment, claim filings are high.

[18] Cf. Atkinson [11, p. 5]: "When a life insurance contract is sold, many expenses are incurred: marketing expenses, underwriting expenses, issue expenses, commissions and agent bonuses. These acquisition costs usually exceed the first year premium by a wide margin. Acquisition costs may even exceed 200% of premium, especially for smaller policies."

TABLE 1

ILLUSTRATIVE EXPENSE COSTS FOR A WHOLE LIFE POLICY

Policy Year	Percent of Premium Commissions	Percent of Premium Other	Percent of Face Value	Dollars per Policy
1	60%	5%	2.5%	$ 200
2	10	5	0.2	50
3	10	3	0.2	25
4	5	3	0.2	25

functions, and expenses are as follows:

per premium:	75% of the first premium, 20% of the second premium, 10% of the third through sixth premiums, and 5% of each premium thereafter;
per amount:	$10 at the beginning of the first year, and $2 at the beginning of each subsequent year per $1,000 of insurance;
per claim:	$5 per $1,000 of insurance as the cost of settlement.

An asset share pricing model uses a table of expense rates, which might begin as in Table 1 (Belth [15, pp. 22–24]).

C.2. Casualty Insurance Expenses

The loss ratio and pure premium methods that are used for casualty insurance ratemaking do not differentiate insurance expenses by policy year. An expected loss ratio is derived from company budgets (e.g., advertising), agency contracts (e.g., commissions), state statutes (e.g., premium taxes), and Insurance Expense Exhibit data (e.g., general expenses). The experience loss ratio, after trending, development, and similar adjustments, is compared to the expected loss ratio to determine the indicated rate change (Stern [151]; Lange [113]; Graves and Castillo [86]; McClenahan [129]; Brown [30]). This procedure treats all expenses identically, regardless of their actual incidence.

C.3. Policy Duration and Insurance Expenses

Property/casualty expense costs, like life insurance expense costs, are greater in the original year of issue than in renewal years.

- Underwriting expenses incurred predominantly in the first year include salaries, costs of policy issuance and underwriting reports (e.g., DMV reports for automobile insurance or credit reports for homeowners), and expenses allocated as overhead on salaries. Renewal underwriting may be only a perfunctory review of past loss experience.

- Loss control expenses incurred either at or before policy issuance include technical inspections (boiler and machinery), landfill inspections (environmental impairment), loss engineering services (workers compensation), financial analyses (mortgage guarantee), and building inspections (commercial fire). Few inspections are repeated at renewal dates. Those which are, such as some workplace safety inspections for workers compensation, are less comprehensive than the original underwriting inspection.

- Acquisition expenses for direct writers are greater in the first year than in renewal years. Three types of commission schedules are used in property/casualty insurance:

    - Independent agency companies pay level commissions, such as 15% or 20% of premium, in all years. The level commission structure is needed because the agent "owns the renewals" (*National Fire Insurance* case of 1904). That is, the insurer may not bypass the agent when renewing the policy. Rather, the agent may place the insurance with any carrier he or she represents, as long as the consumer agrees. A lower commission in renewal years would induce the agent to move the policy to a competing insurer and obtain a "first year" commission.

The level commission structure does not reflect the actual incidence of acquisition expenses, since agents spend more effort writing new policies than renewing existing policies. Because of this (and other reasons), many economists consider the independent agency system to be inefficient.[19] In the personal lines of business, direct writers are steadily gaining market share, and the level commission structure is becoming less important. As the asset share pricing model shows, a level commission structure works well for risks that terminate quickly. It works poorly for risks that endure with the carrier. In other words, a level commission structure is inappropriate for the persisting and profitable risks.

- Many direct writers pay commissions that vary by policy year: high first year commissions (20% to 25%) and low renewal commissions (2% to 5%). Since the insurer, who is the agent's sole employer, owns the renewals, the agent has no opportunity to move the policyholder to a competing carrier.

- Some direct writers have either a salaried sales force or a sales force that is compensated partly by commission and partly by salary. The acquisition costs incurred by the insurer may be determined by the actual incidence of these expenses. For instance, suppose the agent receives salary and benefits of $100,000 a year, and spends 80% of his or her time obtaining $500,000 of new business a year and 20% of his or her time servicing $2 million of renewal business. The insurer is paying the equivalent of a 16% commission

[19]The primary "other reasons" are the relative ease of automating a captive agency compared to an independent agency and the ability of direct writers to integrate distribution strategy with underwriting strategy. The efficiency of insurance distribution systems is a disputed issue; see Joskow [109], Cummins and VanDerhei [54], Cather, Gustavson, and Trieschmann [43], and Berger, Cummins, and Weiss [17].

on new business and a 1% commission on renewal business.[20]

- Most "other acquisition expenses," such as advertising, subsidies for new agents, and development costs for expanding or automating distributions systems, are expended at or before the inception date of the policy.

Casualty actuaries often differentiate between "fixed" and "variable" expenses. Variable expenses are those that are directly proportional to premium. Fixed expenses do not vary directly with premium: some are "per policy" expenses, such as some underwriting expenses, and some are "sunk costs" related to the block of business as a whole, such as certain advertising costs. The appropriate treatment of fixed and variable expenses is discussed in Section 4.

D. Persistency

Persistency rates, or retention rates, are the crux of asset share pricing models. Independent insurers pay careful attention to personal automobile retention rates, though rating bureaus have yet to incorporate them into their ratemaking procedures.

D.1. Policy Duration and Profitability

Persistency rates are most important when the net insurance income varies by duration since inception of the policy. Consider first a whole life insurance policy.

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[20]Formally, if x is the first year commission rate and y is the renewal commission rate, then we have the following:

- The total salary and benefits earned by the agent equals the implicit commission rates times the premium volume, or

$$\$500{,}000(x) + \$2{,}000{,}000(y) = \$100{,}000.$$

- The implicit commissions earned on new and renewal business should be proportional to the amount of time spent on these two components of the business, or

$$0.80 \equiv 0.20 = \$500{,}000(x) \equiv \$2{,}000{,}000(y).$$

Solving these two equations yields $x = 16\%$ and $y = 1\%$.

Net insurance income

= (premium collected + net investment income)

– (benefits paid + increase in policy reserves

+ incurred expenses + federal income taxes).

The standard non-forfeiture laws of each state cause the expected value of

(premium + net investment income)

– (benefits paid + increase in reserves)

to be rather level each year, whether the policyholder persists or terminates.[21]

D.2. Influences on Persistency Rates

Persistency rates vary widely by company. In personal auto, for instance, State Farm has high retention rates because: it targets a suburban and rural insured population; it offers low premium rates; and it provides renewal discounts.[22] Many independent agency companies have low retention rates because: the agents, who are not beholden to any particular carrier, can move the insured to whichever company offers the lowest rates; and these carriers use little consumer advertising.[23] The typical personal auto direct writer has retention rates of about 90%, ranging from under 85% in the first policy year to about 95% after ten years. In other words, termination rates (lapse rates) are over

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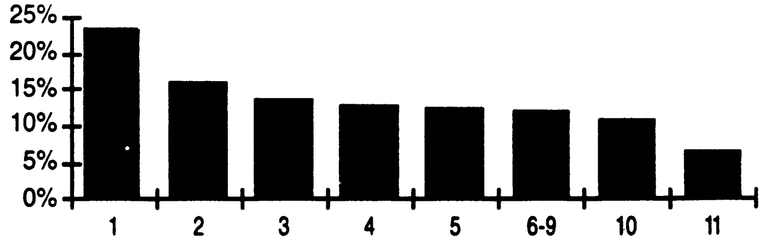
[21] The expected value will be level, but the actual value will vary, being lower in the year of death. Preliminary term policy reserves increase the value of net insurance income in the first policy year, though not enough to offset the higher underwriting and acquisition expenses.

[22] The terms "persistency rates" and "retention rates" are used interchangeably in this paper.

[23] Life insurance shows similar variability. With regard to whole life persistency, LIMRA [117, p. 286] notes: "Regardless of policy year, there is considerable variation in lapse experience across companies. For policy years one through ten, one quarter of the lapse rates are below ten percent. Another quarter of the lapse rates generally exceed twenty percent." See also Anderson [8, p. 373]; Winn et al. [168]; Moorehead [134, p. 295]; Belth [15, p. 19].

FIGURE 1

LONG-TERM ORDINARY LIFE LAPSE RATES



15% in the first policy year and decline to about 5% after ten years.

Persistency improves with duration since policy inception. Figure 1 shows industry-wide ordinary life insurance lapse rates (vertical axis) by policy year since inception (horizontal axis) (LIMRA [115, p. 338, Table 6]; Buck [33, p. 275]).

There is an intuitive relationship between duration and persistency for both life and casualty insurance. In the original year of issue, many policyholders are undecided about the relative value of the policy and the required premiums. Some insureds may decide that the insurance is not worthwhile; some may be dissatisfied with their carrier's service; some may believe the premium is too high and continue shopping for a lower rate; and some may be unable to afford any insurance. Thus, voluntary termination rates during the first year are high. In casualty lines of business, moreover, where underwriting terminations are permitted, carriers often reevaluate newly acquired risks that have had accidents in the first one or two policy years.

Once a policyholder has kept the policy for several years, it is likely that he or she will renew the contract for another year. The insured is probably satisfied with the carrier's service and finds the premiums reasonable and affordable. And unless the insured's classification changes, underwriting terminations are unlikely.[24]

D.3. Termination Rates and Probabilities of Termination

Persistency may be analyzed either by termination rates or by probabilities of termination. The *termination rate* is the number of terminations during a given renewal period divided by the sum of terminations during that period plus policies persisting through that period. The *probability of termination* is the number of terminations during a given renewal period divided by the number of originally issued policies in that cohort. (A cohort is a group of policies written in a given issue period.[25])

For instance, suppose an insurer writes 100 auto policies in 1990, 20 risks lapse the first year, 10 lapse the second year, and 5 lapse the third year. The termination rates are 20% [= 20 ≡ 100] the first year, 12.5% [= 10 ≡ 80] the second year, and 7.1% [= 5 ≡ 70] the third year. The probabilities of termination are 20% [= 20 ≡ 100] the first year, 10% [= 10 ≡ 100] the second year, and 5% [= 5 ≡ 100] the third year. Termination rates more clearly distinguish persistency patterns by classification.[26] Prob-

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[24]Classification changes are common in personal automobile. Most changes are from higher to lower rated classifications, such as a movement from youthful to adult driver, from unmarried to married driver, or from urban to suburban resident. These changes rarely provoke underwriting terminations. Some changes are to higher rated classifications: for example, an adolescent son may turn 16 and obtain a driver's license, the use of the car may switch from "pleasure" to "drive to work," or the insured may move from a low rated territory to a higher rated territory. These changes may lead to a re-evaluation of the risk. The most common impetus for re-underwriting, though, is not classification changes but poor claim experience, as noted in the text.

[25]Compare Huffman's distinction between asset shares and the asset fund [95, pp. 278, 279]. A_t is the "asset share per $1,000 unit of coverage *in force* at the end of policy year t." F_t is "the asset fund per I_0 *initially issued* units, accumulated at interest to duration t" (italics added). Huffman notes that "the asset share prorates funds among policyholders so that each gets its share; the asset fund does not, thereby measuring the accumulated funds held by the insurer."

[26]For instance, suppose 100 policies were issued to adult drivers and 100 policies were issued to young male drivers. By the fifth renewal, 20 of the adult drivers had lapsed, and 60 of the young male drivers had lapsed, leaving 80 adult drivers and 40 young male drivers. By the next renewal, an additional 5 adult drivers and 5 young male drivers terminate their coverage. The termination rates are 5 ≡ 80, or 6.25%, for adult drivers

abilities of termination, in certain analyses, provide a better portrayal of the insurer's profitability.[27]

D.4. Persistency by Classification

Persistency rates vary greatly by classification. In personal auto insurance, young male drivers have high termination rates, retired drivers have low termination rates, and middle-aged drivers are in between. Figure 2 shows average probabilities of termination for these three classifications.

The termination rate differences by classification, of course, are greater. The vertical axis in Figure 2 shows the probability of termination, and the horizontal axis shows the policy period since inception.[28]

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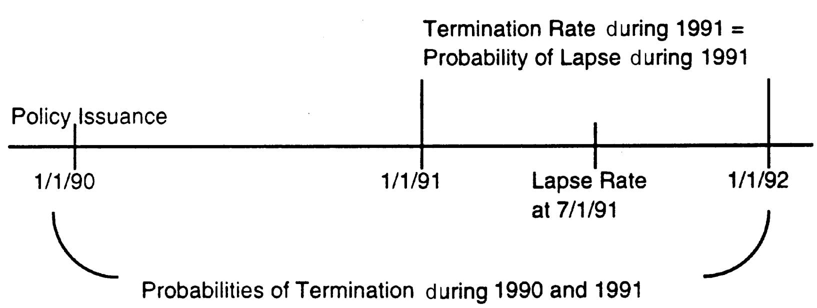
and $5 \equiv 40$, or 12.5%, for young male drivers. The probabilities of termination, however, are 5% for both groups of insureds.

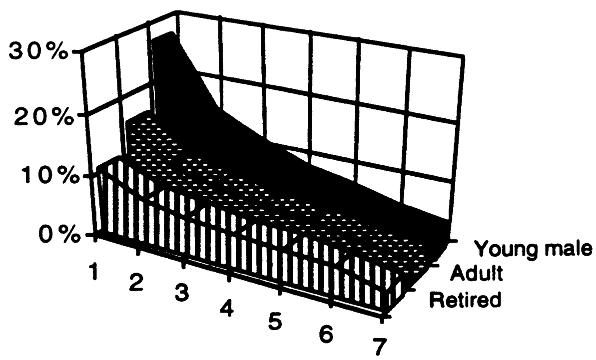
[27]The distinction between termination rates and probabilities of termination is taken from life insurance. The *mortality rate* is the annualized probability that an individual will die at a given time. The corresponding *probability* is the number of deaths at a given age divided by the number of insureds who have attained that age (Batten [14]; Atkinson [11, pp. 51–54]).

The use of these terms here is not identical to that in life insurance. The life insurance lapse rate pertains to a given moment of time. The life insurance probability of lapse is the percent of withdrawing policyholders during the year. The termination rate as used here is equivalent to the probability of lapse. The probability of termination as used here is the percent of original policyholders who terminate in a given year. The diagram below illustrates the use of these terms.



[28]See Feldblum [68] and [70]. LIMRA [116, Tables 8–10] shows similar relationships for long-term ordinary life insurance. Lapse rates for issue ages 20–29 are about double those for issue ages 50–59 at all policy durations.

FIGURE 2



Life insurance persistency patterns are analyzed by issue age, duration, interest rates, sex, rating (standard, preferred, and substandard), policy face amount, premium payment pattern (whole life versus limited payment life; annual, monthly, and payroll deduction), policy form (ordinary life, universal life, graded premium whole life, variable life, traditional term, select and ultimate term), distribution system (general agents, brokers, and branch offices), and numerous other variables.[29] Some of these dimensions are pertinent only to life insurance. For instance, if market interest rates rise faster than the credited rate on a universal life policy, lapse rates may increase. Other dimensions apply to casualty insurance as well. The relationship between the distri-

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LIMRA's most recent studies show lapse rates in the year of issue about 50 to 100% higher than those in the tenth and subsequent renewal years. Older persistency studies, such as Linton [118], Moorehead [133], and LIMRA's studies from the 1970s, show lapse rates in the year of issue about five times higher than those in the tenth and subsequent renewal years. (See LIMRA [117, p. 295, Table 2], for a comparison.) Persistency patterns are sensitive to external economic and social forces, so an unexamined extrapolation from historical experience may be misleading. Similar caution should be used when extrapolating from past personal auto experience.

[29]See Atkinson [10] and [11]. Belth [15, p. 18] notes additional dimensions, such as policyholder's income, occupation, previous ownership of life insurance, experience of the agent, and presence of policy loans. Bluhm and Koppel [25] discuss the variables affecting health insurance persistency patterns.

bution system and persistency patterns is particularly important for casualty insurance.

The dependence of persistency patterns on these dimensions warrants a careful analysis of the available experience. For an independent agency company to use persistency patterns derived from direct writers makes as much sense as for an insurer to use claim frequencies from adult drivers for young male insureds. Similarly, the persistency patterns between urban and rural territories may differ as much as loss costs differ between these territories. The termination rates used in Sections 4 through 7 are illustrative; only by coincidence would they be appropriate for a given company and a given block of policies.

E. Discount Rates

Asset share models examine cash flows and revenue streams over the lifetime of the policy. Future profits and losses of each policy year are discounted to the original issue date to determine present values.

E.1. Life Insurance Discount Rates

In non-participating whole life insurance contracts, both premiums and benefits are fixed at issue. Claims are paid soon after death, so there is no "settlement lag." The discount rate used to determine the present values of future premiums and benefits for statutory policy reserves is limited by the state's Standard Valuation Law. Life insurance policy reserves do not have the uncertainty of casualty insurance loss reserves, which are affected by inflation rates, court decisions, jury awards, and social expectations.

The life insurance actuary using an asset share model begins with known quantities: premium, death benefits, and policy reserves. With appropriate assumptions for mortality and withdrawal rates, he or she can determine statutory or GAAP book profits of each year. All that is needed is a discount rate to determine the present value of future earnings.

E.2. Casualty Insurance Issues

Casualty claims are not settled immediately after the accident. Under tort liability compensation systems, claim investigation, determination of liability, and legal negotiation and adjudication may delay settlements for months or years. In the no-fault lines of business, such as workers compensation and automobile personal injury protection, wage loss reimbursements are made only as the loss is accrued, so payments stretch out over years.

Property/liability insurance accounting, whether statutory or GAAP, records incurred losses on an undiscounted basis, resulting either in underwriting losses or in lower underwriting profits than if discounted loss reserves were held (Lowe and Philbrick [123]; Lowe [120], [122]). The investment income in the Annual Statement or in the Insurance Expense Exhibit—which may be viewed as offset to the underwriting loss—is the present investment income from the company's financial assets, not the investment income expected in the future (Feldblum [69], [74]; Bingham [19]). Property/liability insurance accounting, both statutory and GAAP, does not match the underwriting experience on a block of policies with the investment experience for the same block of policies. This matching, though, is essential for asset share pricing models. Several methods of matching underwriting and investment experience may be used:

- Record undiscounted incurred claims, but include an offsetting investment income account tied to the assets supporting the unpaid losses (Ferrari [77]; option three of Salzmann [147]; pricing models one and three of Robbin [144]).

- Record cash transactions, not the accounting statement incurred losses. The asset share model looks like an expanded (multi-period) internal rate of return model.[30]
- Record discounted loss reserves. The discount rates for unpaid losses may be market interest rates, risk-free rates, or "risk adjusted" rates.[31]

For simplicity, this paper uses the third method. The illustrations speak of "discounted incurred losses" without specifying the method of discounting. Note that the discount rate used to determine the present value of unpaid losses at the accident date need not be the same as the discount rate used to determine the present value of future earnings at the issue date.[32]

E.3. Rate Revisions and Rates

Casualty pricing methods often determine rate revisions and rate relativities, not actual rates. For instance, the actuary may determine that overall statewide rates should be increased 10%, or that the rate relativity for young male drivers should be changed from 1.750 to 1.850.

Asset share pricing determines rates, not rate revisions. Since there is no overall statewide rate, the actuary selects "pivotal" classifications for which an actual rate is determined. Interpola-

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[30]Internal rate of return and asset share pricing models, however, have different viewpoints. The internal rate of return model views the insurance transactions from the equityholder's perspective. It requires surplus commitment and equity flow assumptions (Feldblum [71]). The asset share model uses the insurance company's perspective and need not consider equity flows. For instance, Anderson [8] determines the ratio of the present value of profits to the present value of premium, not the return on investment or surplus. Thus, the asset share model is similar to a multi-period internal rate of return model in its construction, not in its perspective.

[31]Woll [170] and Bingham ([19], [20]) use risk-free rates. Fairley [67], Myers and Cohn [135], and Butsic [37] use risk adjusted rates, though they determine the adjustment differently. The need for risk margins is discussed in CAS Committee on Reserves ([40], [41]) and CAS Committee on the Theory of Risk [42]. See also D'Arcy ([57], [58]); Lowe [121]; FASB [80]; and Tiller, et al. [153].

[32]See Paquin [141] for a life insurance discussion of different discount rates for cash inflows and outflows. On the appropriate discount rate for determining present values of future uncertain profits, see also Shapiro [150].

tion and relativity analyses may be used for other (non-pivotal) classifications.

For instance, the life actuary may use an asset share model to determine whole life insurance rates for standard rated, non-smoking males at five year age intervals (e.g., ages 30, 35, 40). The mortality and persistency rates at these ages are derived from their own experience combined with the graduated experience for the entire insured population. Whole life insurance rates for a male aged 37 would be determined by interpolation of the rates for age 35 and age 40.

The same procedure is applicable to casualty ratemaking. Rates are determined for pivotal classifications, such as adult married drivers in a given group of territories, or young unmarried male drivers in an urban area.[33] To form the rates, one uses the experience of these classifications as well as the graduated experience of similar classifications. Rates are then determined for non-pivotal classifications by interpolation and relativity analyses.[34]

3. ASSET SHARE MODELING—FOUR ILLUSTRATIONS

Asset share modeling is particularly valuable when differences in termination rates influence expected profits. The first three illustrations in this paper show how an asset share model deals with such conditions. The fourth illustration shows how the movements of the underwriting cycle can be incorporated into policy pricing. The illustrations are as follows:

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[33]Thus, in appearance, asset share pricing is more akin to pure premium ratemaking than to loss ratio ratemaking. However, this similarity is deceptive. Both the pure premium approach and the loss ratio approach seek to estimate the expected loss costs during the future policy year. The asset share method assumes that the actuary has already estimated future loss costs, expense costs, and persistency rates, and now seeks to determine optimal premium rates.

[34]A similar procedure is used by Brubaker [31, pp. 107, 108]. Brubaker uses interpolation among "grid points" for geographic rating, similar to the interpolation among pivotal ages for asset share pricing.

1. *Business Expansion:* When an insurer begins writing in a new territory or policyholder classification, most risks are new business, with high loss and expense ratios. Traditional ratemaking procedures show high combined ratios, and the pricing actuary may conclude that the business is not profitable. But this is simply the cost of building an insurance portfolio. New business is generally "unprofitable," though the "loss" may be offset by the future profits in a stable renewal book. Asset share modeling helps the actuary determine the true profitability of the insurance writings.

2. *Classification Relativities:* Traditional ratemaking methods determine classification relativities from loss ratios, perhaps tempered with "expense flattening" procedures. Persistency differences among classifications can cause these methods to be misleading. If persistency is ignored, then rate relativities are too low for the poorly persisting classes and too high for the long-persisting classes. The illustration shows an asset share model determination of personal automobile classification relativities for young male drivers.

3. *Competitive Strategy:* Traditional ratemaking procedures match premiums to anticipated losses and expenses. They ignore the future profits and losses from expected renewals. Moreover, they ignore the effects of rate revisions on policyholder retention and new business production. A rate increase will reduce policyholder retention, particularly among the most profitable risks, who can obtain coverage from other carriers. Competitive pricing strategy is to raise or lower rates such that the expected changes in policyholder retention, new business production, and lifetime policy profits or losses will maximize long-term income. The illustration shows how asset share modeling determines the optimal retired driver discount in personal automobile insurance.

4. *Underwriting Cycles:* Market share and profit objectives are the linchpins of competitive strategy. Attempts to gain market share drive the soft phase of the cycle, and attempts to restore profits drive the hard phase. It is often unclear whether market share gains during the soft phase combined with profits on these policies during the hard phase will lead to satisfactory long-term income. Asset share modeling enables the actuary to quantify the effects of different pricing strategies on overall returns.

These illustrations demonstrate the power of the asset share pricing technique. Each illustration expands the scope of the issues being addressed:

- In the *business expansion* illustration, all the actuarial data are given. The rate levels, rate relativities, and classification scheme are predetermined. The pricing actuary uses the profitability measures provided by the asset share analysis to determine marketing strategy.

- In the *rate relativities* illustration, the classification scheme and business strategy are given, but not the rate levels or rate relativities. The pricing actuary uses the asset share analysis to determine class rates to achieve the desired profits from each group of insureds.

- In the *competitive strategy* illustration, neither the classification scheme nor the rate relativities are given. Rather, the pricing actuary uses the asset share analysis to determine the class groupings that will optimize the insurer's return.

- In the *underwriting cycles* illustrations, the issues are more general. The insurer must decide whether a particular line of insurance is expected to be profitable, and whether entry or exit from a given market is indicated.

4. ILLUSTRATION 1—BUSINESS EXPANSION

Company growth or contraction distorts reported financial results, particularly when the expected loss and expense ratios depend on the time since inception of the policy. Even without this dependence, business growth raises the statutory combined ratio, since loss reserves are held at undiscounted values and acquisition costs are written off when incurred. Deferring acquisition expenses and adding investment income, to give a "GAAP operating ratio," does not fully resolve the problem, since the investment income received in any calendar year derives from the business insured in the past. If the insurer is growing rapidly, the investment income received is smaller than the present value of the investment income expected from the current block of business.[35]

To circumvent this problem, the following illustrations assume that all figures are restated on a fully discounted basis. For instance, the $656 of the first policy year's losses in the "business expansion" illustration does not mean statutory incurred losses

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[35]Because premiums, losses, and insurance industry assets grew faster than after-tax investment returns during the 1970s and 1980s, statutory operating ratios were overstated by about 2.2 percentage points (Feldblum [74]; see also Butsic [38]).

The effects of business growth on statutory operating ratios can be grasped most easily by an illustration. In a steady state environment, with no growth, the statutory operating ratio equals the "true" operating ratio. Suppose the insurer writes $100 million of premium each January 1, has no expenses, pays $100 million of losses three years later, and earns a 5% investment yield. Each year, it holds about $300 million of loss reserves, on which it earns $15 million of investment income. (For simplicity, we have not assumed compounding of the investment balances.) The statutory operating ratio is

$$\$100\text{ M losses} - \$15\text{ M investment income} \equiv \$100\text{ M premium} = 85\%.$$

The present value of losses when the policy premium is collected is $85 million (again, assuming simple interest, not compound interest, for simplicity of illustration). The "true" operating ratio is also 85%.

What if the company's business volume expands? Consider the extreme case: what if the company begins writing the business this year? The "true" operating ratio is still 85%. But the company has only $100 million of reserves the first year, on which it earns $5 million of investment income, leading to a statutory operating ratio of 95%.

In practice, of course, the difference is not so great. But as long as a company is growing more quickly than the after-tax investment yield, the statutory operating ratios understate the company's true profitability.

of $656, but fully discounted losses of $656. Since the illustration uses a policy year model, not a calendar year model, there is no "property/casualty type" deferred acquisition cost. There is, of course, a "life insurance type" deferred acquisition cost, since underwriting and acquisition costs are higher in the original year of issue than in renewal years. The asset share pricing model incorporates this phenomenon, though without setting up an explicit asset.[36]

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[36]The difference between a "life insurance type" deferred policy acquisition cost (DPAC) and a "property/ casualty type" DPAC clarifies the workings of the asset share model. Suppose the insurer writes a personal auto policy on July 1, 1995, for a $1,000 premium, and it expects to renew the policy four times. Deferrable acquisition costs, such as agency commissions, are 24% the initial year and 6% in renewal years.

- Property/casualty statutory accounting says that all acquisition costs are written off when incurred. On July 1, 1995, the company collects $1,000 in premium, sets up an unearned premium reserve of $1,000, pays $240 in expenses, and shows an accounting loss of $240. Over the next twelve months, as the premium is earned, the unearned premium reserve declines to $0.
- Property/casualty GAAP statements show a deferred policy acquisition cost asset that is set up when the policy is issued and is taken down as the premium is earned. On July 1, 1995, the company collects $1,000 in premium, sets up an unearned premium reserve of $1,000, pays $240 in expenses, sets up a DPAC asset of $240, and shows no accounting loss or gain. Over the next twelve months, as the premium is earned, both the unearned premium reserve and the DPAC asset decline to $0. For instance, on December 31, 1995, the earned premium is $500, the unearned premium reserve is $500, and the DPAC asset is $120.

  On July 1, 1996, the company again collects $1,000 in premium, sets up an unearned premium reserve of $1,000, pays $60 in expenses, sets up a DPAC asset of $60, and shows no accounting loss or gain. The accounting follows the same procedures as in the initial policy year. There is no interaction between the initial year of issue and renewal years.
- Life insurance accounting, both statutory and GAAP, shows a DPAC asset that is set up when the policy is issued and is taken down over the lifetime of the policy. For simplicity, suppose that the company is certain that it will renew the policy exactly four times, and that the interest rate and inflation assumptions are both 0% per annum. The total acquisition expenses for this policy are $\$240 + 4 \times \$60 = \$480$. The policy persists five years, or 60 months, so these expenses must be amortized at $8 a month. On July 1, 1995, the company pays $240 in expenses and sets up a DPAC asset of $240. It reduces this asset by $8 a month, so on December 31, 1995, it has a DPAC asset of $192, not $120, and on June 30, 1996, the DPAC asset is $144, not $0. (In practice, of course, the amortization of the life insurance DPAC asset is more complex, depending on mortality and interest rate assumptions; see Tan [152].)

The asset share model is the pricing equivalent of the life insurance accounting system. It effectively "amortizes" the first year expenses over the lifetime of the policy when determining premium rates.

Growth in a New Territory

Suppose a profitable personal automobile direct writer expands into a new geographic area in 1992. To ensure an accurate financial appraisal of the expansion, all statistics on the new operation are separately recorded. "Fixed" costs peculiar to the expansion, such as subsidies for new agents, construction costs for a new branch office, and extra advertising expenses during the first year, are charged to a corporate account; they are not included in these statistics.

The insurer writes 10,000 policies in 1992, at an average annual premium of $800. The company is satisfied with the new business production, and 10,000 new policies are again written in 1993. In early 1994, the policy year 1992 results are tabulated and show a loss of $2.4 million after full discounting of loss reserves.

The insurer accepts the $2.4 million loss as "start-up" costs in addition to what it has budgeted to the corporate account, and it continues to add 10,000 new policies a year. But when policy year 1993 results, tabulated in early 1995, reveal an additional loss of $1.9 million, company management is concerned. In early 1996, policy year 1994 results show a further loss of $1.3 million. Company management concludes that it erred by expanding too rapidly, and the growth program is curtailed. The pricing actuary tries to explain about the cost of new business but is summarily dismissed.

Has the company indeed erred? The asset share model shows that the company is earning a 19% return on surplus, despite its inexperienced sales force and lack of name recognition in this area. The error lies in curtailing a successful program. Yet actuarial generalizations do not suffice. The true return and the cause of the reported losses must be clearly presented.[37]

[37] Brealey and Myers [28, pp. 272–275] present a similar illustration emphasizing the difference between economic (or true) earnings and book earnings.

Asset Share Assumptions

How can a 19% return on surplus be consistent with losses of $5.6 million in three years? Assume the following conditions for this block of business:

1. *Premiums:* The average policy premium is $800 in 1992. The loss cost trend is 10% per annum, and "fixed" expense costs are rising at 5% per annum. Regulators are not averse to insurers in this state, and the company expects average rate increases of 9% per annum.

2. *Losses:* The fully discounted loss ratio on new business is 82% in 1992, or an average of $656 a car. Loss costs are increasing at 10% per annum. The company expects the average loss costs on any policy to improve by 3% a year since policy inception, after adjusting for inflation. For example, the average loss cost for new business written in 1993 will be $\$656 \times 1.1 = \$722$. The average loss cost in 1993 for policies originally issued in 1992 will be $\$722 \equiv 1.03 = \$701$.[38]

3. *Expenses:* A direct writer has high expense costs the first year but low expense costs in renewal years. Simulated expense costs are shown in Table 2. Expenses which vary directly with premium (such as commissions and premium taxes) increase at the same rate as premium. We assume that "fixed" expenses, such as salaries and rent, increase at 5% per annum.

4. *Persistency:* Termination rates vary by company, geographic location, class of business, and various other dimensions. The pricing actuary has chosen termination rates based on prior experience, beginning at 15% in the

[38] A more realistic model would show a larger effect in the first few policy years and a smaller effect in later years. For instance, the improvement in average loss costs from policyholder persistency may be 7% in the first year, 5% in the next year, 4% in the next year, and gradually decline to 1% after ten years.

TABLE 2

ACQUISITION AND UNDERWRITING EXPENSES BY POLICY YEAR

	New Policies		Renewal Policies	
	Fixed Expense Provision	Variable Expense Provision	Fixed Expense Provision	Variable Expense Provision
Agency Commissions	0.0%	25.0%	0.0%	3.0%
Advertising and Other Acq.	5.0	0.0	0.0	0.0
General Expenses	12.0	3.0	3.0	1.0
Premium Tax	0.0	2.0	0.0	2.0
Taxes, Licenses, and Fees	0.8	0.2	0.8	0.2
Total Expenses	17.8%	30.2%	3.8%	6.2%

year the policy is originally issued and declining to 8% after 15 years.

5. *Present Values:* The company determines the present value of future earnings by discounting at its cost of capital, which is 12% in this illustration.

The Model

The asset share model is shown in Exhibit 1. The present values of current and future profits and premium are $480 and $5,012, respectively, for a return on sales of 9.6%. If the insurer has a premium to surplus ratio of two, then the return on surplus is 19.2%.[39]

[39]To estimate the total return on surplus, one must consider federal income taxes and the investment return on surplus funds. The investment return on surplus funds as a percentage of premiums depends on the premium to surplus ratio. Federal income taxes depend on a combination of tax strategy and investment strategy (see Almagro and Ghezzi [7] for details). To avoid additional complexities, the illustrations do not incorporate these items. In this example, the effects are largely offsetting. If the investment return on surplus funds is 9% per annum, and the marginal tax rate is 35%, then the before-tax return on surplus is 19.2% + 9.0% = 28.2%, and the after tax return is 65% × 28.2% = 18.3%. In general, however, the effects are not offsetting, and these items must be considered in pricing.

Let us consider each column in Exhibit 1.

1. Column 1 shows the year since the inception of the policy. The policy in this illustration was issued in 1992. The figures in the exhibit pertain to this cohort of policies only, not to policies issued previously or subsequently.

2. Column 2 shows the average premium: \$800 a car in 1992, increasing at 9% per annum.

3. Column 3 shows the average losses. The discounted loss ratio is 82% for new business, so 82% of \$800 is \$656. Losses increase at 10% per annum. At each renewal, loss experience is slightly better, because poor risks voluntarily terminate and reunderwriting efforts weed out unprofitable insureds. The illustration presumes that the average loss costs in any policy year are 3% lower than the average loss costs in the preceding policy year, after adjustment for loss cost trend.

   In this illustration, \$656 increased by 10% is \$722; \$722 decreased by 3% is \$701. Although the aggregate loss cost trend (10%) is greater than the premium trend (9%), the loss ratio for ten year old business (68% = 1,186 ≡ 1,738) is lower than the loss ratio for new business (82%).

4. Columns 4 through 7 show expenses. Expenses that vary directly with premium are 30.2% of premium in the year of issue and 6.2% in renewal years. Thus, 30.2% of \$800 is \$242, and 6.2% of \$872 is \$54.

   Fixed expenses average 17.8% of premium in the year of issue; 17.8% of \$800 is \$142. Fixed expenses for renewal years are now 3.8% of premium. Consider a policy first issued in a previous year having an \$800 premium this year. It would have fixed expenses of 3.8% × \$800 = \$30.40. This policy would have fixed expenses of $30.40 × 1.05 = $31.92 next year; $31.92 × 1.05 = $33.52 the succeeding year; and so forth.

Thus, in the asset share model, the renewal fixed expense column shows $0 in the initial year of issue, then $31.92 in the first renewal year, $33.52 in the second renewal year, and so forth (rounded).

5. Column 8 shows the expected persistency rate. The entries indicate that 85% of new policyholders persist into the second year; 86% of second year insureds persist into the third year; and so forth. The persistency rates in this illustration are low in the year of issue (85%) and increase gradually to 92% by the fifteenth year.
6. Column 9 shows the cumulative persistency rate, or the percentage of original insureds who persist into any policy year. For instance, 85% of original policyholders persist into the second year; 73.1% [= 0.85 × 0.86] of original policyholders persist into the third year; and so forth.
7. Column 10 shows the profit in each policy year. The profit is the product of the cumulative persistency rate and the policy year income, where the income equals premiums minus discounted losses minus expenses. For instance, in the third year, policy year income is $950 – $748 – $59 – $34 = $109. But only 73.1% of original policyholders persist into the third year, so 73.1% of $109 is $80.[40]
8. Column 11 shows the discount factors for future earnings. The company's cost of capital in this illustration is 12%, so Column 11 is 12% compounded annually (e.g., $1.12^2 = 1.25$).
9. Column 12 shows the present value of future earnings, or Column 10 divided by Column 11. Similarly, Column 13

[40]Premiums are assumed to be collected and expenses are assumed to be paid at the beginning of each policy year. Losses are discounted to the beginning of each policy year.

TABLE 3

RESULTS BY YEAR OF ISSUE AND POLICY YEAR SINCE INCEPTION ($000)

	Year Policies are Originally Issued			
Policy Year of Earnings	1992	1993	1994	Total
1992	−2,400			−2,400
1993	726	−2,625		−1,899
1994	803	743	−2,873	−1,327

shows the present value of future premiums, or Column 2 times Column 9 divided by Column 11. The totals of Columns 12 and 13 are $480 and $5,012, respectively. In other words, for a policy issued in 1992, the company expects to earn profits with a present value of $480 over the next 15 years. The present value of the premiums charged this insured, during the same period and with the same discount rate, is $5,012.

Accounting Results and Long-Term Profitability

The company reported earnings of −$5.6 million for the first three policy years, even after full discounting of losses. This is the result that traditional actuarial pricing techniques would show. Calendar year statutory financial statements, which use undiscounted loss reserves and write off all underwriting and acquisition expenses when incurred, show worse results.

The dependence of loss and expense ratios on the year since the policy was first issued explains the difference between the $5.6 million loss shown by traditional pricing analyses and the 19% return on surplus shown by the asset share model. The results by year of issue and by policy year since inception appear in Table 3.

The entries in the “1992” column are taken from Column 10 of Exhibit 1. The entries in the “1993” column are derived from an asset share model beginning one year later. Premiums begin 9% higher, losses begin 10% higher, and "fixed" expenses begin 5% higher. The entry in the "1994" column is derived from an asset share model beginning two years later.

Federal Income Taxes

To simplify the presentation, federal income taxes are not considered in these illustrations. The simplest way of incorporating income taxes is to multiply the "profit" column in the illustrations by the marginal tax rate. Thus, the pre-tax loss of $240 in the year of issue is an after tax loss of $156 (assuming a marginal tax rate of 35%). The pre-tax profit of $72.6 in the second policy year is an after-tax profit of $47.2.

With this procedure, the discount rate used to determine the present value of losses in Column 3 at the beginning of the corresponding policy year should be a before-tax discount rate appropriate for losses, and the discount rate used to determine the present value of profits at the original policy writing date in Column 11 should be an after-tax discount rate. If federal income taxes are first applied to the present value of profits in Column 12, then the discount rate in Column 11 should be a before-tax discount rate. In addition, the federal income taxes must also be applied to the present value of premiums in Column 13.

Alternatively, one could use after-tax values of premiums (revenues), losses, and expenses in Columns 2 through 7. In other words, the $800 of premium in the year of issue would be replaced by an after-tax revenue of $520. If this procedure is followed, then the discount rates used in Columns 3 and 11 should be after-tax discount rates.

Profitability Measures

Different measures of profitability can be incorporated in an asset share model. The illustration discounts future earnings at the company's cost of capital, implying that profits should be measured with a return on equity. To avoid the complexities of converting statutory surplus to GAAP equity, the illustration assumes that surplus equals equity and that the insurer writes at a two to one premium to surplus ratio.[41] Alternatively, one can use the premium to GAAP equity ratio for this insurer to directly obtain a return on equity.

One could also use asset share modeling to determine the "break-even" point. The company may ask: "Is writing insurance policies more profitable than simply investing the equity in financial securities of similar risk?" Assume that securities of similar risk are yielding 10% per annum. The insurer would use a 10% discount rate in Columns 3 and 11, discount losses to the same date as premiums are collected, and determine whether the present value of the total in Column 12 is greater or less than zero.

One can incorporate asset share pricing into an internal rate of return model. Instead of the "present value of losses" in Column 3, one would show several columns of cash transactions: losses paid, investments made, and investment income received. One would combine the cash transactions from the insurance operations with assumed equity flows and determine the internal rate of return to the equity providers (see Feldblum [71]).

In sum, asset share pricing is not restricted to any particular measure of profitability. Rather, whatever measure is used should be applied to the entire life of the policy, not to a single policy year or a single calendar year.

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[41] In practice, GAAP equity is generally greater than statutory surplus, because of deferred acquisition costs, non-admitted statutory assets, reinsurance penalties for unauthorized and slow-paying reinsurers and for overdue reinsurance recoverables, Schedule P penalties, and differences in the carrying value of subsidiaries. Offsetting these are the non-recognition of deferred federal tax liabilities on unrealized capital gains and the amortization of investment grade bonds in good standing under statutory accounting. See Holman and Stroup [94] and AICPA [4] for comparisons of statutory and GAAP accounting. Rosenthal [145] estimates that average GAAP equity is 25% greater than statutory surplus for property/casualty insurers. In addition, the economic net worth of the insurer is generally greater than GAAP equity because of the unrecognized interest discount in the loss reserves and because of the "goodwill" value of the distribution system (see ASB [1]).

5. ILLUSTRATION 2—CLASSIFICATION RELATIVITIES

Traditional ratemaking procedures determine classification relativities by comparing relative loss ratios or pure premiums among groups of insureds (Conger [45], Stern [151], Hurley [97], Harwayne [91], Finger [81]). For instance, if adult drivers (the "base" class) have average losses of $400 a year, and young male drivers have average losses of $900 a year, then young male drivers are assigned a classification relativity of 2.25. Similarly, if urban residents, with a territorial relativity of 1.50, have an average loss ratio of 70%, and the average loss ratio of all drivers in the state is 75%, then the territorial relativity for urban drivers should be reduced to 1.40 [= 1.50 × 70% ≡ 75%].

Persistency Effects on Ratemaking Assumptions

Classification ratemaking has been refined with expense flattening procedures that separate expenses into those that vary directly with premium, or "variable" expenses, and those that do not, or "fixed" expenses.[42] In the first example in the paragraph above, suppose that losses per driver average $500 a year, variable expenses average $150 a year, and fixed expenses average $100 a year. Variable expenses are $150 ≡ $750 (20.0%) of premium. Average losses are $400 for the base class and $900 for young male drivers, so the gross premiums are

Base class (adult drivers):

$$\text{premium} = \$400 + \$100 + 20\% \times \text{premium},$$

$$\text{or premium} = \$625.$$

Young male drivers:

$$\text{premium} = \$900 + \$100 + 20\% \times \text{premium},$$

$$\text{or premium} = \$1{,}250.$$

[42]On expense flattening procedures, see ISO [103]; Hunt [96]; Childs and Currie [45]; Wade [159]; Nodulman [140]; McClenahan [129]. The ratemaking terms "fixed" and "variable" expenses are not the same as the corresponding financial terms. The "fixed"

The classification relativity for young male drivers is 2.00 [= 1,250 ≡ 625].

These procedures fail to incorporate differences in persistency patterns among classes of insureds, resulting in inaccurate (and either unprofitable or uncompetitive) classification relativities. In any policy year, fixed expenses, as a percentage of total premium, are lower for young male drivers than for adult drivers, and variable expenses, as a percentage of total premium, are equal for the two classes. But young male drivers have higher termination rates than adult drivers have. Because of the higher termination rates, the ratio of total expenses to total premium *over the lifetime of the policy* is generally greater for young male drivers.[43]

Similar considerations apply to losses. Average losses, adjusted for loss cost trends, decline as the policy matures. The "business expansion" illustration assumed that average losses (after adjustment for trend) decline by 3% in each renewal year. Insureds who terminate quickly have "new business" loss ratios, which are generally higher than "renewal business" loss ratios.[44]

A Heuristic Example

The effects of persistency patterns on relative loss ratios by class depends on the type of classification system used. A simple (albeit unrealistic) example should clarify this.[45] Suppose

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expenses in actuarial ratemaking do vary with volume. However, they generally vary most closely with the number of policies, not with the dollar amount of premium.

[43]See Feldblum [68]. The generalization in the text is more applicable to direct writing insurers than to independent agency companies. Compare also Buck [32, p. 9]: "It is more expensive to handle a policy for a young, single male in a given territory than an adult policy in the same territory. This difference can be attributed to such factors as more frequent policy changes and flat cancellations in the youthful male policies."

[44]The cause and effect relationships are unclear. Perhaps young male drivers, who have higher loss ratios, have poorer persistency, so higher loss ratios also appear on new business. Or perhaps persisting drivers have lower loss ratios, so young male drivers, who terminate frequently, have higher loss ratios. As Stephen D'Arcy has pointed out to me, one must take care not to double count these effects. See also the following paragraphs in the text.

[45]The example is deliberately constructed to show a result opposite to the major conclusions in this paper, to demonstrate that careful analysis of each situation must be

average losses for adult drivers [the base class] are $500 a year, average losses for 17-year-old drivers are $1,000 a year, and all insureds persist for ten years. In other words, the 17-year-old drivers have twice the average loss costs of adult drivers. If all expenses vary with premium (i.e., there are no fixed expenses), their classification relativity should be 2.00.

But suppose that new business risks have average loss costs 25% higher than renewal business. All of the 17-year-old drivers are new business, but only 10% of the adult drivers are new business.[46] The 17-year-old drivers' average losses will drop to $800 during renewal years, so the 2.00 classification relativity is too high. An insurer can profit in the long-run by reducing the classification relativity for 17-year-old drivers and increasing its market share.[47]

Determinants of Rate Relativities

The correct relativity depends on the classification system, the average losses and persistency rates by classification, and

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undertaken. In general, however, reality has been in stark opposition to previous actuarial studies. Most analyses of "expense flattening" imply that high risk drivers are often overpriced, because their expense costs as a percentage of premium are less than those of lower risk drivers. In truth, when persistency rates are taken into account, many of these high risk drivers are *underpriced*, because their expense ratios over the policy lifetime are a greater percentage of premium than those of lower risk drivers.

[46] Adult drivers persist for ten years, so (in a steady state) 10% are in their first policy year, 10% in the second policy year, and so forth. This would be correct were there no switching of classifications. Since there is switching—that is, some adult drivers were first insured as young drivers—less than 10% of adult drivers are new business. If 25 is the minimum age for adult drivers, then drivers first insured below age 25 spend some renewal years in the adult classification but spend their first policy years as young drivers.

[47] This illustration is simplified for heuristic purposes. The actual analysis not only is more complex but may even lead to the opposite conclusion for two reasons. First, renewal loss experience may be better than new business loss experience because the renewal book has fewer 17-year-old drivers (among other reasons). This does *not* mean that when a group of 17-year-old drivers renew their policies, their loss experience will improve. Second, the illustration assumes that 17-year-old drivers and adult drivers have the same persistency rates. In fact, as this section shows, the different persistency rates among these classes affects the appropriate premium rate relativity.

The point of the simplified illustration in the text is two-fold: (1) persistency patterns cannot be ignored in determining rate relativities, and (2) the effect of persistency patterns, whether to increase or decrease the relativity, is not always obvious without careful actuarial analysis.

the strength of loss ratio improvement by policy year.[48] Asset share pricing models enable the actuary to determine accurate and profitable relativity factors.

This illustration compares young male drivers with adult drivers to determine the classification relativity factors. We need the following information, of which the second and third are essential for the asset share model:

1. the dimensions of the classification system,
2. the relative average loss costs of these two groups of insureds,
3. the relative average persistency rates of these two groups of insureds,
4. the strength of loss ratio improvement by policy year for these insureds.

The Classification System

The expected losses, expenses, and the current year's premium do not depend on the shape of the classification system. Future years' premium, however, are affected by such factors as renewal discounts and age boundaries between driver classes.[49]

For instance, suppose an asset share model is being used for an 18-year-old unmarried male driver. If the insurer differentiates between "males aged 25 and under" and "adult drivers," then this driver will spend 8 years in the "young male" classification. Since average losses decline rapidly between ages 17 and 25, his premium is probably too low for the first three or four years and

[48]The interrelationships among these dimensions are complex. For instance, a 22-year-old unmarried male driver who just completed college may have high expected losses. But if he is beginning a stable job, is engaged to be married, and is buying a house in a quiet suburb, his expected losses may drop quickly. In contrast, a 40-year-old married woman may have low expected losses, but she may show no loss ratio improvement for the next ten years.

[49]Persistency rates, which are influenced by relative future prices between the current insurer and its peer companies, also depend on the classification system.

too high for the subsequent four or five years. Termination rates are high for young male drivers but decrease with duration of the policy, so his expected termination rate will start high but decline markedly over the next eight years. A renewal discount will improve persistency but reduce renewal gross premiums.

Ideally, the classification system should be designed from the results of an asset share model. In practice, the classification system may be a "given" for the pricing actuary. In this section, the classification system is given. In the "competitive strategy" illustration (the following section), the classification system is designed from the asset share model.

Coverage Mix

Two types of differences affect classification relativities even for single policy year costs (that is, not considering persistency effects). First, average losses for any coverage vary by classification. For instance, young male drivers have higher expected bodily injury losses than adult drivers have. Second, the coverage mix varies by classification. For instance, young male drivers are less likely to purchase physical damage coverages or excess limits for liability coverages than adult drivers are.

If the ratio of expenses to premium did not vary with the coverage mix, or with the average loss per policy, then classification relativities would be similar to loss cost relativities. But fixed expenses do not vary directly with premium. They remain fixed regardless of the number of coverages, limits of liability, or deductibles chosen (Childs and Currie [45, pp. 53–54]).

Policy Basis Versus Coverage Basis Rate Relativities

We can use an asset share pricing model to develop rate relativities on either a policy basis or a coverage basis. The policy basis model compares losses and expenses for all coverages combined among classes of insureds. The resultant rate relativities must then be allocated to coverages. For instance, if the policy basis rate relativity for young male drivers is 2.0, and the premium volumes for liability and physical damage coverages are equal, the rate relativities by coverage might be 2.5 for liability and 1.5 for physical damage. When the coverage mix differs by classification, the allocation of the rate relativities may be complex.

The coverage basis model compares losses and expenses for an individual coverage among classes of insureds. The fixed expenses must be allocated to coverage before the asset share pricing model is used. Since some expenses do not vary with the number of coverages, the premiums rates are not additive: that is, there should be a "multiple coverages" discount. For instance, if the indicated rates are $500 for liability and $300 for physical damage, the correct rates might be $535 for liability alone, $325 for physical damage alone, and $780 for all coverages combined. Even when these differences are too small for practical application, the pricing actuary should know whether the rates are over- or under-stated for each classification and coverage combination.

Policy Basis Loss Cost Relativities

Policy basis loss cost differences between young male drivers and adult drivers depend on three factors:

1. *Young male driver rate relativities by coverage:* Average rate relativities for young male drivers are approximately 2.5 compared with the base classification rate (adult pleasure use). The rate relativities vary among insurers, depending on the definition of young male drivers (e.g., "25 and under," "29 and under," and so forth) and the other classification dimensions, such as years of driving experience and past accident history. Some states, such as New York, require separate relativities for comprehensive coverage, and some insurers use separate relativities in other states as well. The total average young

male driver rate relativity to that of all drivers is approximately 2.0.[50]

2. *Physical damage coverage by classification:* Young male drivers are more likely than other drivers to have liability coverage but no physical damage coverage because their premiums are high, they drive less valuable automobiles, and they may be less able to afford insurance.

3. *Average liability increased limits and physical damage deductibles:* Young male drivers have lower average liability limits and higher average physical damage deductibles for a given type of automobile. The higher average premiums for young male drivers, the fewer assets they have to protect, and the reluctance of company underwriters to provide high liability limits or full physical damage coverage to high risk drivers are the major reasons for this (Aetna [2, p. 26]).

For the "classification ratemaking" illustration, we use a coverage based asset share pricing model. Since the average coverage basis rate relativities are greater than the average policy basis rate relativities (about 2.0 : 1 versus 1.5 : 1), and much of the fixed expenses relate to per policy expenses, not per coverage expenses, we must adjust the per coverage fixed expenses by classification, assigning a higher dollar amount to young male drivers than to adult drivers.

An illustration should clarify this. Suppose class A purchases both liability and physical damage coverages, while class B, with a similar number of insureds, purchases only liability coverage.

[50]See ISO [104, pp. G-10–G-13]. ISO classifies young male drivers as (i) under 25 years of age if married or not the owner or principal operator of the vehicle and (ii) under 30 years of age if unmarried and the owner or principal operator. Rate relativities range from 1.15 for a 21 through 24-year-old "good student" married male using the automobile for pleasure use to 3.75 for a 17-year-old unmarried male driving his car to work and not eligible for a good student credit. Several jurisdictions, such as Massachusetts and California, prohibit classification by age, sex, or marital status. In these states, rate relativities are determined along other dimensions.

Expected losses and variable expenses are \$600 for each coverage and each classification, and per policy fixed expenses are \$100 per policy.

The ratio of fixed expenses to gross premiums for the entire line of business is 10% [= (\$100 + \$100) ≡ (\$600 + \$600 + \$600 + \$100 + \$100)].[51] Equivalently, fixed expenses are one ninth of losses plus variable expenses. If we used this ratio to assign fixed expenses by class, we would assign \$133 [= (\$600 + \$600) ≡ 9] to class A and \$67 [= \$600 ≡ 9] to class B.

Similarly, if we first allocated fixed expenses by coverage, we would assign \$133 to liability and \$67 to physical damage, since liability has twice the "losses plus variable expenses" that physical damage has. Splitting the \$133 equally between classes A and B gives the same result as before. The expense flattening procedure suggested by ISO [103] begins with fixed expenses by coverage, so it would not solve the problem outlined here.

But this allocation is not correct. Since class A has twice the premium per policy that coverage B has, the ratio of fixed expense to premium for class B should be twice that for class A. (This is an extended "expense flattening" procedure.) Thus, (\$600 + \$600)(x) + (\$600)($2x$) = \$200, or $x = 8.33\%$. For the liability coverage, the expense loadings should be (\$600)(8.33%) = \$50 for class A, and (\$600)(2)(8.33%) = \$100 for class B. For the physical damage coverages, the expense loading should be (\$600)(8.33%) = \$50 (for class A).

For the previous example in the text, adult drivers have about four thirds [2.0 ≡ 1.5] as much coverage per policy as young

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[51] This ratio is (Class A fixed expenses + Class B fixed expenses) ≡ total premium, where total premium equals

Class A liability loss costs plus variable expenses
+ Class A physical damage loss costs plus variable expenses
+ Class B liability loss costs plus variable expenses
+ Class A fixed expenses
+ Class B fixed expenses.

male drivers have. A precise quantification of the fixed expenses by class is difficult for several reasons.

- First, fixed expenses are not strictly "per policy" expenses. For example, underwriting efforts are greater for a policy with both liability and physical damage coverages than for a policy with only liability coverage.
- Second, many fixed expenses, such as underwriting expenses, vary with the quality and type of risk. Louis E. Buck, in summarizing the findings of the Aetna Automobile Insurance Affordability Task Force [32], said: "… there are differences by classification in the cost of handling policies. It is more expensive to handle a policy for a young, single male in a given territory than an adult policy in the same territory. This difference can be attributed to such factors as more frequent policy changes and flat cancellations in the youthful male policies." His accompanying statistics show policy processing costs to be 50% to 100% higher for youthful unmarried male drivers than for adult drivers. (See Aetna [2, p. 9].)

There is no rigorous quantification of fixed expenses by classification in this paper. However, the dollars of fixed expenses per coverage in each policy year in the asset share pricing model are higher for young male drivers than for adult drivers. Expense flattening procedures, which are incorporated automatically in the asset share pricing model, reduce the "proportional" fixed expense loading for young male drivers in each policy year. Persistency patterns raise the lifetime "proportional" fixed expense loading for these insureds compared to adult drivers. These effects can be seen in Exhibits 2 and 3.

Persistency by Classification

An insurer selling whole life coverage expects to show an accounting loss during the first policy year. For medically underwritten risks, the acquisition and underwriting costs generally exceed the first year premium. For guaranteed issue policies, adverse selection raises first year benefit costs. In either case, the loss turns into a profit as the policyholder persists.

Similarly, an insurer selling personal automobile coverage expects an accounting loss during the first policy year, since both expenses and loss costs are higher that year. As with life insurance, the loss turns into a profit as the policyholder persists.

Expected long-term profits depend upon the policyholder persistency rates, in addition to premium, loss, and expense levels. *Since persistency varies by classification, the rate relativities must consider persistency rates as well.*

Classification differences may be based on either current classification or original classification. In most lines of insurance, the classification does not change: a frame building does not develop into a masonry building (homeowners), a retailer does not become a manufacturer (workers compensation), an architect does not become a lawyer (professional liability). But personal automobile classifications do change, as young drivers become adults, as urban residents move to the suburbs, and as new cars age.

Young Male Drivers

Traditional ratemaking procedures consider current classifications. Premium rates decline when the young male driver marries or ages, not before. Asset share pricing models consider original classifications and expected future changes: if we write a policy now, what is the expected long-term income?[52]

Persistency rates by duration are most easily determined for current classifications, such as the percentage of young male

[52]Pricing decisions hinge on supply and demand considerations, though these factors are hard to include in traditional ratemaking methods. The insurer asks: "If we raise the premium, what happens to expected long-term income?" Raising premium helps the current year's income, but it lowers persistency. The next illustration, "competitive strategy," shows how asset share pricing models deal with this issue.

drivers in their fifth policy year who persist into their sixth year. But if the young male classification consists of male drivers under 25 years of age, the group considered in the previous sentence are drivers originally insured below 20 years of age. These drivers have different persistency rates from drivers originally insured from 22 to 24 years of age. The persistency of young male drivers in their fifth policy year does not tell us the expected fifth year persistency of young male drivers. We need persistency rates by original classification, not current classification.

Model Assumptions

For the asset share model, we begin with pivotal classifications: the adult pleasure use (the base class) and unmarried males aged 21 and 22 who drive to work. We need to know three differences by classification to form rate relativities: average loss costs, average fixed expense costs, and persistency rates. For this illustration, we assume the following differences; in actual pricing work, we would derive these from past experience:

1. Average liability loss costs are $400 per annum for adults and $1,000 per annum for young male drivers. Were all expenses proportional to premium, and were persistency rates the same for both classes, the rate relativity for young male drivers would be 2.5.
2. Average premium for all drivers is $550. Average first year fixed expenses are 17.8% of this, or $98. Adult drivers are less expensive to underwrite, especially per coverage. There are fewer underwriting rejections among adult drivers, and they purchase more coverages, so average fixed expenses per coverage is 10% less, or $88 per policy for the liability coverages. Conversely, young male drivers are more expensive to underwrite, especially per coverage. Underwriting rejections are more common, some applicants never remit the premiums, and many drivers purchase only basic limits liability cover-

TABLE 4

PERSISTENCY RATES BY DURATION AND CLASSIFICATION (AS PERCENTAGES)

Policy Year	1	2	3	4	5	6	7	8	9	10+
Young male	60	65	70	73	76	79	82	85	88	90
Adult	82	86	87	88	89	90	90	91	91	92

ages. Average fixed expenses for the liability coverages are 20% higher, or $117 per policy.[53]

3. Retention rates are higher for adult drivers than for young male drivers. We use the simulated rates in Table 4 to illustrate the asset share pricing model. Actual rates vary by insurer, distribution system, and classification plan, so these rates may not be appropriate for any given carrier.

The classification plan, average loss costs, average fixed expenses, and persistency rates are given. We assume that the insurer writes at a 2 : 1 premium to equity ratio and desires a pre-tax 14% return on equity from its insurance operations (i.e., excluding investment income on surplus funds). We use the asset share pricing model to determine a 7.0% return on premium for each class, and we then derive the rate relativities from the resulting premiums.

Exhibits 2 and 3 show the calculations. For each class, we select a starting gross premium and increase it 9% per annum, which determines the variable expenses in all future years. In the first year, fixed expenses are $88 for adults and $117 for

[53]See Aetna [2, p. 64]: "In considering how expenses should be allocated to policyholders, it must also be noted that the company must charge policyholders for the underwriting costs of *rejecting* applications. Thus, even if the actual costs of underwriting each accepted risk were known, the amount charged to a policyholder would have to exceed that actual cost to compensate for the costs associated with the applications of rejected applicants, from whom the company collects no premium."

young male drivers. We use the same ratio of renewal to first year fixed expenses as in the previous illustration, 3.8% to 17.8%, and increase the fixed expenses by 5% per annum. For adult drivers, $88 × 3.8% ≡ 17.8% = $19; this is then increased by 5% per annum to give all the fixed expense entries.

As before, the loss costs shown in the exhibit are discounted to the beginning of the corresponding policy year. The present values of future profits and premiums at the original policy issuance date are determined at a 12% interest rate, which is the assumed cost of capital. The original premium has been selected such that the ratio of the present value of all future profits to the present value of all future premiums is 7.0% for both classes.

Asset Share Results

The indicated premiums are $475 for adults and $1,272 for young male drivers. Note that:

- The loss cost relativity is 2.50, or $1,000 ≡ $400.
- The fixed expense cost relativity is 1.33, or $117 ≡ $88.
- The rate relativity is 2.68, or $1,272 ≡ $475.

Pricing procedures used in the 1960s would have set the rate relativity equal to the loss cost relativity, or 2.50. Since the fixed expense relativity is only 1.33, expense flattening procedures would have reduced the rate relativity. But the persistency differences between the two classes show that even the loss cost relativity is too low. A premium rate relativity of 2.68 is needed to equalize the returns between these two classes.

6. ILLUSTRATION 3—COMPETITIVE STRATEGY

The “business expansion” illustration presented in Section 4 took the environment as given and asked, “Is the growth strategy profitable?” The illustration in Section 5, “classification relativities," took the insured population as given and asked: "What prices are equitable?"

This is the traditional ratemaking perspective: the actuary aligns premiums with anticipated losses and expenses for a given insured population. *Competitive strategy reverses the question: "How can the pricing structure create a more profitable consumer base?"*

Some insurers have excelled at this task. New products, such as package policies in the commercial lines; modifications to existing products, such as replacement cost coverage for homeowners insurance; and classification revisions, such as retired driver discounts in personal automobile insurance, have spurred sustained growth for these carriers.

Two considerations should be kept in mind when seeking to change the insured population:

1. Any strategy may affect new business production or retention rates. For instance, the introduction of various professional liability coverages created a new clientele ("new business production"), whereas the expansion of experience rating plans increased renewals among desirable insureds ("retention rates"). Some new products, such as universal life insurance, serve both functions: they are savings vehicles for investors otherwise uninterested in life insurance, and they are replacement vehicles for insureds who might drop inefficient whole life policies.

2. Traditional ratemaking procedures are cost-based. The pricing actuary equates premiums with anticipated losses and expenses, so economic profits are eliminated. In practice, insurers seek to optimize certain goals, such as profits or market share. The price elasticity of demand becomes a crucial determinant of optimal strategy. That is, premium rates and relativities affect consumer

demand and the mix of insureds, thereby affecting insurer profitability.

Cars and Courage

> Although courage is a splendid attribute in its place, its place is not at the wheel of an automobile.
>
> — Ambrose Ryder [1935]

Early classification schemes had surcharges for older drivers: reactions slow as the body ages, and senior citizens lack the quick reflexes of their sons and daughters. Insurance experience, however, eventually showed the effects of youthful intrepidity, as Ambrose Ryder notes. The physical limitations of older drivers make them less capable of escaping from dangerous situations. But their awareness of these limitations make them less likely of entering into dangerous situations in the first place.[54]

The exposure to road hazards declines as drivers age. Older drivers, particularly after retirement, spend less time behind the wheel (Buck [32, p. 6]). They less frequently drive to work, take kids to amusement parks, or attend late parties.[55] As a re-

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[54]Ryder [146, p. 143] says: "The next question is whether a driver is a better risk because he reacts one-fifth of a second quicker than the average. Various devices have been on the market for testing the reaction times to danger signals. I think these are all very interesting and may possibly prove of value, but generally speaking the person who is quick on the trigger and who reacts very promptly is probably a less desirable risk than the more phlegmatic person who likes to think things over two or three times before he decides to do anything. The latter type will not react as quickly to the sudden danger that presents itself to his oncoming car but on the other hand neither will he be so likely to allow himself to get into a position where any sudden danger will arise that will require a one-tenth of a second reaction. Give me my choice and I will take the man who is not so quick on the trigger in everything he does in life.

"If the individual driver is going to be measured for his reactions to danger, it is even more important that he should be measured for his willingness to keep away from danger . . . . The timid soul is a much better risk than the daring young man who has the courage to drive his car at 90 miles per hour on a slippery road. The best type of risk, therefore, is the person who is really afraid to take unnecessary chances."

[55]Compare also IRC [99, p. 5], which examines auto injury rate by age of the victim: "The lowest percentage of injured persons fell into the oldest age groups, with eight percent age 55 to 64 and eight percent age 65 or older." Drivers make up a large percentage of auto accident victims, so the Insurance Research Council statistics are relevent for the analysis here, though the exact figures are not suitable.

sult, many insurers now provide discounts for older or retired drivers.

Older drivers not only have lower expected loss costs, they also have less impetus to price shop at renewal time. Younger drivers with high premiums have incentives to find lower cost coverage, and they hear about competing rates from friends at work. Older drivers, with lower premiums and often with less information about competing carriers, have less incentive and less opportunity to price shop.

This section examines the pricing of a retired driver discount. The relevant considerations for the asset share model include:

- expected loss costs by policyholder age,
- persistency rates by policyholder age and policy duration,
- price elasticity of demand: that is, the effects of price on retention rates.

An Illustration

The actual data used to price a retired driver discount are complex, though the principles are straightforward. To see their importance, let us consider a simple illustration, from both a traditional ratemaking perspective and from an asset share pricing perspective.

Suppose an automobile insurance policy is offered, with a life of five years. That is, each insured purchases coverage for six years, though not necessarily with the same carrier each year. Cost and persistency assumptions are as follows:

1. Expected loss plus expense costs, including a reasonable profit, are $100 the first year, $90 the second year, $80 the third year, $70 the fourth year, and $60 the fifth and sixth years.
2. The market is competitive, and consumers are most sensitive to price at early durations. Your major competitor

is offering the same product for $90 each year. If you price below the competitor's rate, your insureds will renew their policies. Moreover, you will attract 50% of your competitor's insureds in the first policy year, 25% in the second policy year, and none in subsequent policy years. If you price above your competitor's rate, you will attract none of your competitor's business, and you will lose 50% of your first year insureds and 25% of your second year insureds. If you price at the same level as your competitor, you will neither attract your competitor's insureds nor lose your own business.

3. You and your competitor each begin with 200 potential insureds. That is, if you charge equal rates, you will each have 200 insureds each year.

4. For simplicity, there is no time value of money. That is, interest and inflation rates are both 0%, and future events are certain. (The actual asset share pricing model, of course, determines present values of future profits and losses.)

These assumptions are summarized in Table 5.

The traditional ratemaking philosophy says that premiums should correspond to expected costs: $100 the first year declining to $60 the fifth and sixth years. With these rates, you will lose 100, or 50%, of your potential insureds the first year. In subsequent years, you will neither lose nor gain insureds, since in the second policy year you and your competitor have the same rates, and in the following policy years, insureds are not price sensitive. You will earn "normal" profits on this book of 100 insureds for six years, and you will have a 50% loss of market share.

But suppose you price the policy at $85 each year.

- The first year you attract 100 of your competitor's insureds and lose $15 on each policy.

TABLE 5

COMPETITIVE PRICING ILLUSTRATION

Policy Year	Expected Cost	Competitor's Rate	Effect of Rate Level on Retention and Production
1	$100	$90	50%
2	90	90	25
3	80	90	0
4	70	90	0
5	60	90	0
6	60	90	0

- The second year you attract 25 of your competitor's insureds and lose $5 on each policy.
- You retain these 325 policyholders for the next four years and earn $5, $15, $25, and $25 per insured each year.

Your net profit is:

$$(300)(-\$15) + (325)(-\$5) + (325)(+\$5) + (325)(+\$15) + (325)(+\$25) + (325)(+\$25) = \$16,625.$$

The factors used in this illustrations are oversimplified. For instance, the effects of rate level differences on business retention depend on the magnitude of the difference, not just on which competitor has the lower rate. But the principle is clear, and it is directly applicable to actual pricing problems: Since future profits are embedded in business renewals, long-term profits may be increased by incurring short-term losses to gain good risks.

Retired Drivers

The characteristics of this illustration are equally applicable to retired driver discounts:

1. Average loss costs decrease markedly as the policyholder ages. At age 55, the insured drives to work each day and

is exposed to road hazards. At age 65, the insured makes less use of the automobile and loss costs drop.

2. The price elasticity of demand, or the extent of comparison shopping, decreases as the policyholder ages. (Equivalently, "consumer loyalty" increases as the policyholder ages.) A driver is more likely to switch carriers at age 55 than at age 65 to obtain a lower rate.

Optimal pricing strategy calls for underpricing insureds in their 50s to gain market share among this desirable group, then reaping the profits when the policyholders advance into their 60s and 70s. Since expected loss costs decline when the driver retires, a level rate, or even a slightly decreasing rate, will cause the transition from losses to gains as the policyholder ages.

The pricing mechanics will be shown with an asset share model. The task of the actuary is not simply bringing premium to current level or developing losses to ultimate, so as to estimate future costs. Rather, optimizing long-term profits requires offering a discount before short-term data seem to justify it. The actuary must determine the initial age of the retired driver discount and its optimal magnitude, based on competitor actions and market share implications:[56]

- *Age:* The appropriate age for the retired driver discount is before actual retirement and even before any substantial decline in losses. The optimal age depends on the relationship between policyholder age and persistency and on the discounts offered by competitors, in addition to expected loss costs by age. (In the illustration above, termination rates drop from 50% in the first policy year to 0% in the third policy year. Actual termination rate differences are hardly so extreme.)

[56]Compare also Daykin, Pentikäinen, Pesonen [63, Chapter 14, Section 3], who use the theory of games in a multi-unit market model to simulate the effects of company rate changes, similar to the analysis in this paper. For the application of the theory of games to industrial economics, see Fudenberg and Tirole [83] or Tirole [154].

- *Magnitude:* The optimal size of the discount depends on the price elasticity of demand and the rate structures of peer companies, in addition to expected loss costs. In the illustration above, there is only one competitor, and demand is extremely elastic. In practice, one must examine the rate structures of one's competitors and estimate the effects of rate differences on retention rates and new business production.

Model Assumptions

To determine the optimal age and magnitude for the retired driver discount, the asset share pricing model requires two sets of assumptions. Some assumptions are grounded in empirical data; others must be projected by the actuary.

Loss Costs by Age of Policyholder

Many insurers examine loss costs by age of policyholder to support classification relativities. Table 6 shows loss ratio relativities by policyholder age, separately for new and renewal business.[57] The relativity shows the ratio of the loss ratio in that row to the average loss ratio for all rows combined.

The loss ratio relativities are similar to those in the heuristic illustration provided earlier: about unity for drivers below age 55, but dropping as low as 65% as the policyholder ages. The loss ratio differences are more pronounced for existing policyholders than for new insureds. For new business, the loss ratio relativities never dip below 82%. The loss ratio relativities for renewal policyholders are at or below this level from age 55 through age 74.

This difference makes sense, since the effects of aging differ among insureds. Some retired drivers drive less and drive more

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[57]The data are shown for all coverages combined. Actual experience differs somewhat by coverage and between frequency and severity. We use loss ratio *relativities* because (i) absolute dollar expected loss costs vary with inflation, with coverage, and with the policyholder mix, and (ii) absolute loss ratios vary with the stage of the underwriting cycle and with pricing strategy, but (iii) loss ratio relativities are relatively stable over time.

TABLE 6

LOSS RATIO RELATIVITIES BY POLICYHOLDER AGE

Policyholder Age	New Business LR Relativity	Renewal Business LR Relativity
20–49	1.02	1.03
50–54	1.00	0.98
55–59	0.94	0.83
60–64	0.84	0.72
65–69	0.82	0.65
70–74	0.98	0.76
75 & older	1.10	0.98
Total	1.00	1.00

carefully; these are the best risks. Others find their responses dulled, but they do not change their driving habits; these are dangerous insureds.

Why would a 65-year-old driver be looking for a new auto insurance policy? Many retired persons own their own homes and have close friends in their neighborhoods. They are not inclined to move elsewhere and begin new lives or careers—the most common motive for switching insurers. Those who do move often do so because of failing health. They join retirement communities, enter old age homes, or live with their children. They are not usually seeking new auto policies.

Insurers frequently review the policies of drivers who have had recent accidents. If the insurer believes the driver is too risky, it may terminate the policy or "discourage" renewal (e.g., by indifferent customer service). Some of the retired drivers seeking new automobile insurance policies have been considered poor risks by their former insurers.

Exposure distributions by age of the principal operator for new and renewal business reflect this. Among existing policyholders, older drivers form a large percentage of the population and are generally good risks. Among new insureds, older drivers

TABLE 7

PERSISTENCY RATES BY POLICYHOLDER AGE

Policyholder Age	50	54	58	62	66	70	74	78
Persistency Rate (%)	96	95	94	92	90	88	85	80

form a smaller percentage of the population. Some of these insureds are good risks; others are dangerous drivers.

For the asset share model, we use the loss ratio relativities for renewal business. The indicated retired driver discounts are not necessarily appropriate for new business. The criteria for the discount should be both the age of the policyholder and the number of years since inception of the policy.

Persistency Rates for Older Drivers

Retention rates improve as the policy ages and as the policyholder ages. Sections 4 and 5 show simulated persistency rates by policy duration for all drivers, adult drivers, and young male drivers. Simulated persistency rates for older drivers are shown in Table 7.

These persistency rates differ in two respects from those illustrated for adult drivers and for young male drivers in Section 5. First, most insureds aged 50 and over are mature renewal business, similar to the 10+ policy year duration category in Table 4. Thus, the persistency rates for insureds aged 50 through 66 are high. Second, as policyholders advance into their 70s, many stop driving because of death or ill health, so persistency rates drop.

In practice, the persistency rates depend upon the premium discount that is offered. If a 60-year-old driver pays $500 in premium, and a competing carrier offers the same policy for $450, the driver is unlikely to switch carriers. That is to say, price elasticity of demand is low, or policyholder loyalty is high.

TABLE 8

PERSISTENCY RATES BY POLICYHOLDER AGE

Policyholder Age	50	54	58	62	66	70	74	78
Persistency:								
with discount	98	97	96	94	92	90	85	80
without discount	90	85	80	75	80	80	85	80

However, if the competing carrier's premium is also $500, but it advertises a retired driver discount of 10%, the insured is more likely to switch carriers. The qualified insured views the retired driver discount as equitable; a carrier who does not offer it is seen as unfair.

We must therefore replace the persistency rates in Table 7 with a set of rows, showing persistency rates with no discount, with a 5% discount, with a 10% discount, and so forth. But these persistency rates depend on the discounts offered by other carriers. In other words, there are no absolute expected rates, since the expected rates depend on other carriers' discounts.

The difficulty in forecasting persistency rates highlights the importance of good assumptions. The persistency rate assumptions are subjective, at least until one develops the experience to justify them or to amend them. But they are essential for determining optimal prices.

For the asset share model, we assume two sets of persistency rates. One set, with lower rates, assumes that no premium discount is offered to older or retired drivers. The other set, with higher rates, assumes a 7.5% discount, which is the "market discount" in Table 8.

The persistency rates illustrated in Table 8 assume that most competing carriers offer a retired (or older) driver discount to policyholders aged 60, but only some of them offer discounts to policyholders in their early or mid-50s. Thus, persistency rates in the "without discount" scenario decline as the policyholder ages from the early 50s to the mid 60s. However, if a full discount is offered even to policyholders in their 50s, few of them switch carriers.

Determining the optimal premium discount requires several runs of the asset share pricing model, since the results depend on the actuary's assumptions. For instance, what effect does a 7.5% discount have on persistency rates? What effect do persistency rates have on average loss costs?[58] For simplicity, we use three iterations:

1. No carrier offers a retired driver discount.
2. Many peer companies offer the discount, but your company does not.
3. Your company offers a 7.5% discount, which is the prevailing "market" discount.

In each case, we use a 15 year asset share model for a cohort of insureds aged 52. We assume that persistency rates depend on the premium discount offered, but average loss costs do not.

Iteration 1. No Carriers Offer Discounts

Exhibit 4 shows the asset share model results for a cohort of 52-year-old drivers, assuming the persistency patterns in Table 7 and the loss ratio relativities in Table 6. Note several differences from the asset share model results in Section 4:

- The Section 4 illustration models new business production, so new business expense ratios are used for the first policy year. The cohort of 52-year-old drivers in this section consists of existing insureds, so only renewal business expense ratios are used.

[58]In life and health insurance, higher termination rates generally lead to higher mortality and morbidity costs, since insureds in poor health are more likely to retain their coverage. Health insurance actuaries refer to this phenomenon as "cumulative antiselection," following Bluhm [24].

- Average loss costs decrease sharply in the first few policy years but then level out. Section 4 used a 3% decline in average loss costs per policy year; this section uses a 1% decline, since most business is mature. In addition, the loss ratio improvements by policyholder age already reflect part of the loss cost improvements as the policy ages.

The model begins with average losses of $500 in the first year and average premium of $600. Because these are existing "high-quality" insureds, with high persistency rates and declining loss costs, profitability is good. The present value of profits over the next 15 years is $1,107, and the present value of premiums is $5,505, for a return on sales of 20%. This is not unusual. The insurer has already paid the high costs of new business production and is now earning the profits in the renewal book. Similarly, if one excludes the high first year costs in the "business expansion" illustration in Section 4, the return on sales is over 17%.

A return on premium measure of profitability is reasonable when market shares remain steady, not when market shares are affected by the rate structure. For instance, suppose an insurer writes 10,000 risks at a premium rate of $1,000 apiece, with an average loss plus expense cost of $900 per risk. The return on premium is 10%, or $1 million. Suppose also that if the insurer raises rates 50%, it loses most of its business. Only 2,500 of the poorer risks remain, with an average loss plus expense cost of $1,300 per risk. The return on sales has improved to 13.3% = [$200 ≡ $1,500], but the dollar amount of profits has declined to $500,000. The insurer's results have deteriorated, not improved.[59]

[59] If the decline in market share is not offset by increases elsewhere, the insurer's return on equity has decreased. For instance, if the insurer has $5 million in equity, then the return on equity is +20% before the rate revision and +10% after the rate revision. Some pricing actuaries are so used to "implied equity assumptions" that they presume that equity strictly follows the business volume. Alternatively, this assumes that equity is the major constraint on the volume of business written. In practice, other factors such as marketplace competition are more important constraints on business volume.

Iteration 2. Only Competitors Offer Discounts

The profitability of this business is good, so carriers seek to increase market share by offering retired driver discounts or older driver discounts. Your company wishes to retain its high profit margin, so it offers no discount.

Persistency rates drop sharply. Your insureds see the retired driver discounts offered by other carriers, and they perceive your stance as inequitable. Exhibit 5 shows the asset share pricing model results. The loss and expense ratios on any given policy have not changed, so the company retains the full profit margin. But retention rates are lower, as more insureds drop out each year. Although 42% of insureds persisted through the full 15 years in Iteration 1, now only 8% do so. The present value of future profits has declined from $1,107 per policy to $666 per policy.[60]

Iteration 3. You and Your Competitors Offer Discounts

To arrest the loss of market share, you offer a 7.5% discount to all drivers age 52 and over, which is the most common market discount (Exhibit 6). The premium discount pleases your insureds, so persistency rates are high. Expenses that are a function of premium, such as renewal commissions and premium taxes, also show a 7.5% decrease, but average loss costs and fixed expenses do not change.

The 7.5% discount cannot be justified on a short-term basis for drivers in their early to mid-50s. In fact, you show a loss of

[60]Since insureds in their 60s are more profitable than insureds in their 50s, the reduction in persistency has a greater effect on the present value of future profits than on the present value of future premiums. Thus, the return on premium declines from 20.1% to 16.7%.

The actual effects may be more adverse than the exhibits here imply. It may be that the better drivers are the ones most likely to find less expensive coverage elsewhere and therefore to terminate their policies. Bluhm [24] notes this for health insurance ("cumulative antiselection"). It is unclear how this affects personal automobile insurance.

$2 the first year and inadequate returns the next two years (4% on premium). But now 49% of insureds persist for 15 years, and the present value of future profits has increased to $797.

Other Advantages

Several other aspects of the retired driver discount have not been illustrated in Exhibits 4–6 but can be incorporated into the asset share pricing model.

1. The exhibits show only a 15 year illustration, as if all insureds terminated at age 67. But the insurer can expect another five or ten years of steady profits, so the difference between an 8% persistency rate in the no-discount case and a 49% persistency rate in the 7.5% discount case has a great effect on future earnings. Ideally, one should extend the pricing model until most business terminates.
2. The exhibits assume no change in the fixed expenses per policy regardless of market share. This is reasonable for premium collection costs, policy printing costs, and similar expenses. Corporate overhead expenses, however, increase as a percentage of premium (or on a per policy basis) when market share declines. Ideally, one should have three expense categories in the asset share pricing model: variable expenses, per policy expenses, and overhead expenses.
3. Several effects of policyholder satisfaction are difficult to quantify. If policyholders perceive the discount offered at age 52 and over as equitable, there may be fewer instances of fraudulent claims. In addition, persistency may improve slightly even for policyholders younger than 52, since they expect to eventually qualify for the discount.

These items should be considered when determining the optimal premium discount. Most important, though, is a structure that examines long-term profits and market share, such as an asset share model. Without it, the actuary is easily misled, unable to quantify the effects described in this section. With it, the actuary can project the true profitability of each risk.

7. ILLUSTRATION 4—UNDERWRITING CYCLES

Traditional ratemaking methods have no place for competitive pressures, marketplace prices, or consumer demand. Actuaries use volumes of data, established procedures for developing and trending losses, and careful analyses of required profit levels. Credibility formulas and actuarial judgment keep rates on a steady path, never deviating too far from either expected costs or past experience. And market prices seem to jump and skip in willful abandon.

The knowledgeable actuary does not expect market prices to adhere to rate recommendations. In a competitive industry, prices are set by the market. Actuaries tug at them, sometimes drawing them closer to costs, sometimes finding their efforts to be fruitless.

But the actuary also knows that rate recommendations must consider market prices. If competitors are charging $1,400 for a certain risk, few actuaries would recommend a rate of $1,100. If the insurer wishes to expand in this market, it might charge a rate of $1,300 and still earn profits on each risk. If the insurer believes that a rate cut will lead to matching cuts by competitors, it may continue with the $1,400 price.[61]

The actuary's rate recommendations are based on both expected costs and expected market prices. Market prices follow the course of the underwriting cycle. The future is not known with certainty, but its outline can be traced.

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[61] For the economic theory of pricing in anticipation of competitors' actions, see Tirole [154] and Scherer [148]. For the underlying mathematics, see Varian [156], Waterson [161], and Shapiro [150]. For a general business perspective, see Porter [143]. For applications to insurance, see Cummins, Harrington, and Klein [52] and Feldblum [76].

Indeed, its outline must be traced. Future losses are not known with certainty, so actuaries examine past claims, observed development patterns, and projected trends to estimate future costs. So too must actuaries consider competitive pressures and industry structure to project future marketplace prices.

Let us consider several illustrations. We begin with unrealistic assumptions, simply to clarify the themes. Suppose first that:

- Policyholder persistency is perfect: 100% retention rates each year.
- There is no time value of money. Alternatively, the expected annual increase in profits exactly matches the discount rate.[62]
- The course of the underwriting cycle is known with certainty.
- The industry alternates between soft (unprofitable) and hard (profitable) markets. The average profit exactly matches the insurer's target return.

Figure 3, which shows time along the horizontal axis and return on equity along the vertical axis, puts numbers on this illustration. The return on equity generated by this policy oscillates between 0% and 20%. The long-term return averages to 10%, regardless of when the policy is first issued.

The cycle has no effect on the insurer's underwriting decisions. The insurer may lose money in soft markets and make money in hard markets, but the long-term profits do not depend on when the policy is first written.

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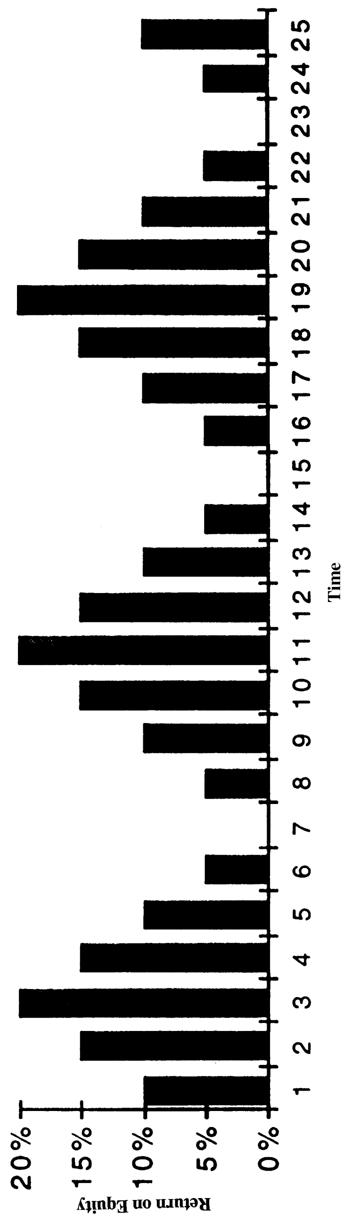
[62]In other words, suppose the financial analyst expects that all revenues and expenses will increase with inflation, but that all future profits should be discounted at the same rate. Modeling of the company's performance is simplified by assuming a 0% inflation rate. In practice, of course, the interest rate used for discounting the future profits is generally higher than the cost trends for revenues and expenses. The asset share exhibits therefore use distinct rates: the cost of capital for discounting future profits, loss cost trends, fixed expense cost trends, an expected rate of premium increases, and an implicit interest rate to determine the present value of losses.

FIGURE 3

UNDERWRITING CYCLE KNOWN AND NO DISCOUNTING



Traditional ratemaking procedures, which look at the future policy year in isolation, take no account of underwriting cycle movements. If underwriting results were poor during the experience period, a rate increase was "indicated." It made no difference whether the poor results during the experience period stemmed from inadvertent underestimates of loss costs or from conscious decisions to reduce rate levels.

The asset share approach expands the perspective. If underwriting results are poor right now because the underwriting cycle is at a nadir and the industry as a whole is suppressing rate levels, but the long-term outlook for the line of business is good, the proper pricing recommendation is generally *not* an immediate rate increase. As discussed in the previous illustration, setting rates at the actuarially adequate level without taking cognizance of market constraints may simply cause a loss of market share and thereby a loss of future profits.

Two characteristics of underwriting cycles support the asset share pricing approach:

1. Underwriting cycles are industry phenomena, not company phenomena.[63] Underwriting cycle fluctuations are not caused by individual company ratemaking "errors," which the pricing actuary should correct. On the contrary: "correction" of the "errors" simply prices the company out of the market. The prescient actuary "rides" the cycle; he or she does not swim against the current.

2. Following prices down in the underwriting cycle could be viewed as an effort to gain (or merely maintain) market share, and creating cyclical losses could be viewed as an effort to drive out new entrants, thereby protecting long-term profits. Underwriting cycles and asset share pricing techniques have similar underlying principles:

[63] See especially Daykin, Pentikäinen, and Pesonen [63, pp. 332–343]. Daykin, Pentikäinen, and Pesonen even provide a graph of six Finnish insurers, showing how the underwriting results of each insurer followed that of the five others.

> business decisions should be guided by long-term profits, not by short-term results.[64]

Let us now remove the unrealistic assumptions that we posited earlier:

- The retention rate is 90%. Expected profits decline each year because the insured may terminate the policy. The oscillatory pattern is dampened, as shown in Figure 4. The time value of money has two parts, which must also be incorporated.
- The insurer's cost of capital exceeds the expected (inflationary) increase in profits by five percentage points.[65]
- The course of the underwriting cycle is not certain. To offset the risk of uncertain future returns, the insurer discounts expected future returns by 5%.

The oscillatory pattern is further dampened, as shown in Figure 5. As one looks ten or twenty years into the future, most policyholders from the current cohort have terminated, and the profits actually achieved in those future years are deeply discounted.

In Figures 4 and 5, the point in the underwriting cycle at which the policy is issued affects the expected long-term return. The asset share model can be used to quantify the expected returns, using the same methods employed in the previous sections.

To model the effects of underwriting cycles, we begin with the standard asset share analysis shown in Exhibit 1. In Exhibit 1, premiums increase by 9% per annum. We now overlay an underwriting cycle pattern on the expected premiums. In Exhibit

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[64] For more complete discussions of underwriting cycles and business strategies, see Feldblum [76] or Harrington and Danzon [90].

[65] For companies of average risk, we would expect the cost of capital to exceed the inflation rate by the sum of the market risk premium and the *real* interest rate on short-term risk-free securities, such as Treasury bills. The former is generally estimated at about six to eight percentage points, and the latter is about two percentage points, giving an eight to ten point spread.

FIGURE 4

RETENTION LESS THAN 100%

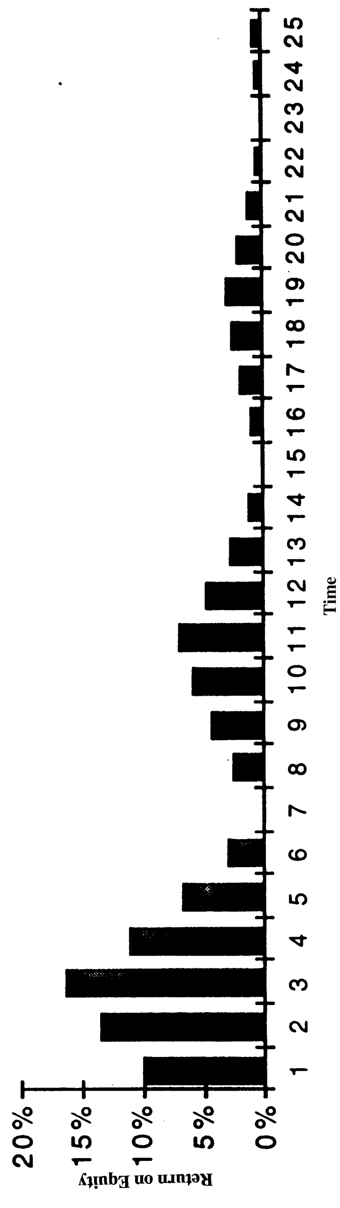
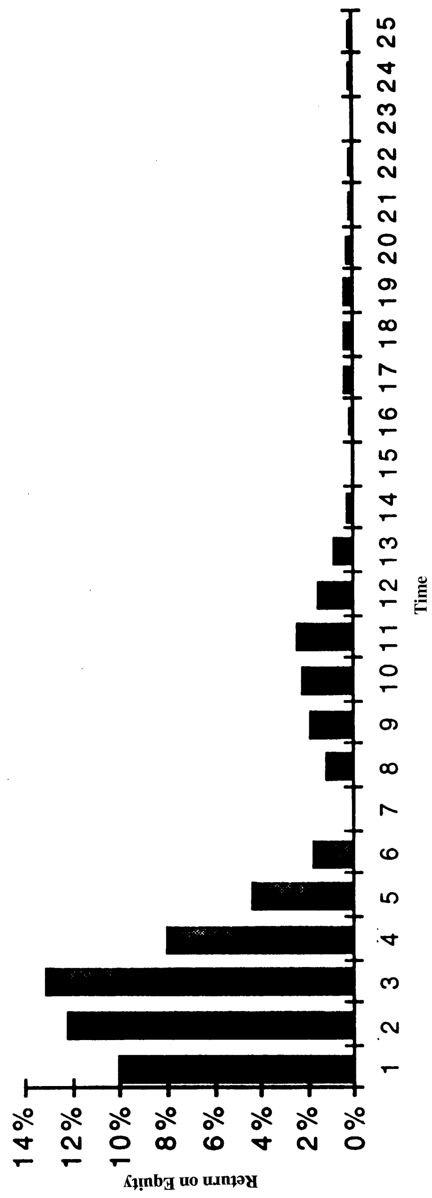


FIGURE 5

DISCOUNTING INTRODUCED



7, the pricing actuary presumes that the industry is now at the midpoint of the underwriting cycle, rates will increase to 30% above their long-term average (adjusted for inflation) over the next two years, then decrease to 30% below their long-term average over the next four years, and so forth. This is an eight year underwriting cycle, with the premiums in Exhibit 1 multiplied by the following factors:

Year	U/W Cycle Factor	Year	U/W Cycle Factor	Year	U/W Cycle Factor	Year	U/W Cycle Factor
1	1.00	5	1.00	9	1.00	13	1.00
2	1.15	6	0.85	10	1.15	14	0.85
3	1.30	7	0.70	11	1.30	15	0.70
4	1.15	8	0.85	12	1.15	16	0.85

Exhibit 8 shows an asset share exhibit with the same starting premium and assumptions for losses, expenses, persistency rates, and cost of capital, except now the company anticipates the underwriting cycle to be turning down. Exhibit 7 shows a "lifetime" return on premium of 14.0% [$730 ≡ $5,221], while Exhibit 8 shows a "lifetime" return on premium of 7.1% [$339 ≡ $4,803].[66]

The actuary does not try to change the course of the underwriting cycle; the solitary insurer cannot do this.[67] Rather, the pricing actuary sees underwriting cycles as constraints on the company's rate actions, and he or she sets premium levels, rate relativities, and various surcharges and discounts in that context.

8. PROFITABILITY MEASURES

Universally accepted standards for profit measurement in insurance do not exist. The traditional 5% or 2.5% underwriting

[66] An underwriting cycle with a premium swing of ∗ 30% is strong for personal auto insurance. It is mild compared to the general liability cycle of the early 1980s.

[67] However, "signaling" effects and market leadership movements can be potent; see Feldblum [76].

profit provision is no longer supported even by the NAIC, though a return on premium measure (in contrast to a return on equity measure) is advocated by several actuaries and economists (NAIC [136]; Woll [170]).

A common component of life insurance asset share profit measurement is the present value of future book profits (i.e., statutory profits). The rationale is that book profits determine the earnings available for stockholder dividends, so this measure is similar to financial measures of investor returns.[68]

Two differences between life and property/casualty insurers influence the optimal choice of profit measure:

1. Life insurers hold discounted policy reserves, with partial adjustment for deferred acquisition costs, so their book profits are similar to economic profits. Property/casualty insurers hold full value reserves with no offset for deferred acquisition costs, so book profits may differ greatly from economic profits.

2. The life insurance patterns of cash flows, adjusted for policyholder cash values, correspond to book profits. For

[68]See Anderson [8, p. 365]; Griffin, Jones, and Smith [87, p. 381]. See also Larner and Ryan [114, p. 448]: "The definition of economic or appraisal value as the present value of future net earnings streams taken at appropriate risk discount rates is generally accepted by actuaries and others as a natural one throughout the world in our experience .... Modern portfolio theory and other investment work provides a theoretical basis for the suggestion that the value of a company is the present value of its future net earnings." Actuarial Standard of Practice No. 19 concerning actuarial appraisals [1, p. 4, paragraph 5.2.1], notes the connection between book profits and investment returns: "*Distributable Earnings*—For insurance companies, statutory earnings form the basis for determining distributable earnings, since the availability of dividends to owners is constrained by the amount of accumulated earnings and minimum capital and surplus requirements, both of which must be determined on a statutory accounting basis .... Economic value generally is determined as the present value of future cash flows. Statutory accounting determines the earnings available to the owner. Hence, while future earnings calculated according to generally accepted accounting principles (GAAP) will often be of interest to the user of an actuarial appraisal, as may other patterns of earnings, the discounted present-value calculations contemplated within the definition of actuarial appraisal in this standard should be developed in consideration of statutory earnings, rather than some other basis."

instance, the first year "investment," corresponding to the first year book loss, is the first year cash outflow to agents and policyholders. Thus, investor returns correspond to book profits which correspond to actual patterns of cash flows and policyholder cash values.

Property/casualty insurance lacks this correspondence. First year cash flows are positive for the insurer. Capital to asset ratios, however, are high. The "investment" at the beginning of the insurance transaction is not simply the assets supporting the reserves, but also the investor capital "committed" to support the policy. In sum, the book profits for the insurer are not necessarily a good proxy for the implied equity transactions between the insurer and its stockholders.[69]

Measuring Rods

There are a variety of methods of adapting asset share profit measures for property/casualty operations. This paper uses economic profits instead of book profits by discounting the loss reserves. Profits may be measured in several ways:

- Profits may be measured as a return on surplus, using assumed premium to surplus (or reserves to surplus) leverage ratios (Butsic and Lerwick [39]; Bingham [19], [21]). This is the profit measure used in Section 4, the "business expansion" illustration. This is actually a return on sales measure, with an assumed turnover rate.
- Profits may be measured as the net present value of premiums minus the net present value of expenditures (losses, expenses,

[69] In contrast, life insurance capital to asset ratios are low, and surplus is needed more for asset risk and interest rate risk than for insurance risk. In other words, a "commitment of surplus" to support the insurance policy is less necessary. This difference can be seen most clearly in the risk-based capital formulas for life and property/casualty insurers. The property/casualty formula is dominated by underwriting risks (reserving risks and premium risks), whereas the life formula is dominated by asset risks (bond risks and equity risks); see Feldblum [73].

and taxes). Thus, Anderson [8], recommends that "the profit objective be defined by the criterion that the present value of the profits which will be received in the future be equal to the present value of the surplus depletion, with both present values based on a yield rate or yield rates which represent adequate return to the stockholders for the degree of risk incurred in expending surplus in the expectation of receiving future profits. That is, the present value of the entire series of profits and losses is zero." Surplus is relevant only for determining the taxes on investment income derived from capital (Myers and Cohn [135]).[70] This is similar to the dollar measure of profits in Section 6.

- Profits may be measured by a multi-period internal rate of return model, by showing:
  - the cash transaction between the insurer and its policyholders or claimants,
  - the investment transactions between the insurer and the financial markets, and
  - the implied equity transactions between the insurer and its stockholders (Cummins [50], [51]; Feldblum [71]).

  This procedure is the most accurate, since it determines the profit measure from all cash flows over the life of the policy. Other "multi-period" internal rate of return models show

[70] In other words, the surplus provided by equityholders is invested in financial markets and earns an appropriate return, which is returned to the equityholders. Were there no income taxes, there would be no need to consider the amount of surplus when pricing the policy. However, there are income taxes, and the investment income earned on equityholder supplied funds is taxed first at corporate rates before being returned to the equityholders. Equityholders would prefer to invest their funds themselves in the financial markets, rather than give them to an insurance company. Therefore, say Myers and Cohn, the policyholders must pay the tax on the investment income earned on policyholder supplied funds.

This argument by Myers and Cohn is true for all pricing models, not just for their risk-adjusted discounted cash flow procedure. The asset share exhibits shown in this paper are on a pre-tax basis. A major effect of putting the figures on a post-tax basis is the "double-taxation" of the investment income on equityholder supplied funds.

multiple periods from only one policy. This procedure shows multiple periods from each renewal. Nevertheless, its complexity may make this procedure less suitable for practical pricing work.

- Profits may be measured more simply, such as by the "discounted payback period," which is the number of years until the cumulative net present value of profits is positive (Atkinson [11, p. 18]). In the business expansion illustration, the cumulative net present value of profits is negative for the first four years and turns to a positive $11,000 in the fifth year. In other words, a policyholder must persist for at least five years before the transaction becomes profitable for the insurer.

Payback measures are sometimes criticized for their failure to consider the time value of money (Brealey and Myers [28]; Weston and Copeland [166]). This criticism is disingenuous: one need simply accumulate losses and profits at an appropriate interest rate to account for the time value of money. For instance, suppose a policy produces losses of $1,000 at the end of year 1, and then profits of $200 a year for the next ten years. Table 9 shows that the payback periods are six years at a 0% annual interest rate and nine years at a 10% interest rate.

9. CONCLUSION

Actuarial pricing must consider long-term profitability and market share objectives, not merely short-term accounting results. Considerations of persistency patterns, the variation of expected losses and expenses with the time since inception of the policy, and the use of a model that incorporates these effects are essential for accurate ratemaking.

This paper has presented the fundamentals of such an approach. It builds upon life insurance asset share techniques and adapts them for personal automobile business.

TABLE 9

PAYBACK PERIODS AT 0% AND 10% INTEREST RATES

Year	Cash Flow	Cumulative Cash Flow: 0% Interest		Cumulative Cash Flow: 10% Interest
1	−1,000	−1,000		−1,000
2	200	−800	−1,000 × 1.1 + 200 =	−900
3	200	−600	−900 × 1.1 + 200 =	−790
4	200	−400	−790 × 1.1 + 200 =	−669
5	200	−200	−669 × 1.1 + 200 =	−536
6	200	0	−536 × 1.1 + 200 =	−389
7	200	200	−389 × 1.1 + 200 =	−228
8	200	400	−228 × 1.1 + 200 =	−51
9	200	600	−51 × 1.1 + 200 =	144
10	200	800	144 × 1.1 + 200 =	358
11	200	1,000	358 × 1.1 + 200 =	594

Some of the specific techniques discussed above are new, but the underlying philosophy is not. Underwriters and salespersons of the major personal lines carriers base their marketing decisions upon intuitive estimates of long term results. Actuaries, seeking more accurate assessments, must strive to replace the intuition with facts.

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EXHIBIT 1

ASSET SHARE MODEL FOR COMPANY GROWTH

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Policy		PV of	Variable Expenses		Fixed Expenses		Persis-	Cumulative		Discount	PV of	PV of
Year	Premium	Losses	New	Renewal	New	Renewal	tency	Persistency	Profit	Factor	Profits	Premium
1	800	656	242	0	142	0	100%	100%	−240	1.00	−240	800
2	872	701	0	54	0	32	85%	85%	73	1.12	65	662
3	950	748	0	59	0	34	86%	73%	80	1.25	64	554
4	1,036	799	0	64	0	35	87%	64%	87	1.40	62	469
5	1,129	853	0	70	0	37	88%	56%	95	1.57	60	402
6	1,231	911	0	76	0	39	89%	50%	102	1.76	58	348
7	1,342	973	0	83	0	41	90%	45%	110	1.97	56	305
8	1,462	1,039	0	91	0	43	90%	40%	117	2.21	53	267
9	1,594	1,110	0	99	0	45	91%	37%	125	2.48	50	236
10	1,738	1,186	0	108	0	47	91%	33%	133	2.77	48	209
11	1,894	1,266	0	117	0	50	92%	31%	142	3.11	46	187
12	2,064	1,352	0	128	0	52	92%	38%	151	3.48	43	168
13	2,250	1,444	0	140	0	55	92%	26%	159	3.90	41	150
14	2,453	1,542	0	152	0	57	92%	24%	168	4.36	38	135
15	2,673	1,647	0	166	0	60	92%	22%	176	4.89	36	120
Total											$480	$5,012

Column 3, Present Value of Losses, is the present value at the beginning of that policy year.
Column 9, Cumulative Persistency, is the downward product of Column 8.
Column 10, Profit, equals Column 9 times {Column 2 minus the sum of Columns 3 through 7}.
Column 11, Discount Factor, is 12% a year compounded annually.
Column 12, Present Value of Profits, is Column 10 divided by Column 11.
Column 13, Present Value of Premium, is Column 2 times Column 9 divided by Column 11.

EXHIBIT 2

ADULT PLEASURE USE

(1) Policy Year	(2) Premium	(3) PV of Losses	(4) Variable Expenses New	(5) Variable Expenses Renewal	(6) Fixed Expenses New	(7) Fixed Expenses Renewal	(8) Persis- tency	(9) Cumulative Persistency	(10) Profit	(11) Discount Factor	(12) PV of Profits	(13) PV of Premium
1	475	400	143	0	88	0	100%	100%	−157	1.00	−157	475
2	518	427	0	32	0	20	82%	82%	32	1.12	28	379
3	564	456	0	35	0	21	86%	71%	37	1.25	29	317
4	615	487	0	38	0	22	87%	61%	42	1.40	30	269
5	670	520	0	42	0	23	88%	54%	46	1.57	29	230
6	731	556	0	45	0	24	89%	48%	51	1.76	29	199
7	796	593	0	49	0	25	90%	43%	56	1.97	28	174
8	868	634	0	54	0	26	90%	39%	60	2.21	27	153
9	946	677	0	59	0	28	91%	35%	65	2.48	26	135
10	1,031	723	0	64	0	29	91%	32%	69	2.77	25	120
11	1,124	772	0	70	0	31	92%	30%	75	3.11	24	107
12	1,225	824	0	76	0	32	92%	27%	80	3.48	23	96
13	1,336	880	0	83	0	34	92%	25%	85	3.90	22	86
14	1,456	940	0	90	0	35	92%	23%	90	4.36	21	77
15	1,587	1,004	0	98	0	37	92%	21%	95	4.89	19	69
Total											$204	$2,887

Column 2: First year premium is chosen such that the present value of profits (Column 12 total) = 7.0% of the present value of premium (Column 13 total). Subsequently, premiums increase 9% per annum.
Column 3: First year losses average $400; loss cost trend is +10% per annum; losses decrease 3% per annum as policy matures.
Columns 4 and 5: Variable expense ratio is 30.2% the first year and 6.2% in subsequent years.
Column 6: First year fixed expenses are $98 per policy, or 17.8% of the average premium for all drivers ($550). Fixed expenses for adult drivers are 10% lower, or $88 per policy.
Column 7: Fixed expenses in the first renewal year are $88x3.8% ÷ 17.8%. Subsequently, expenses increase 5% per annum.
Column 8: Assumed persistency rates for adult drivers. Column 9 = downward product of Column 8.
Column 10 = {Column 2 − sum (Columns 3 through 7)} x Column 9.
Column 11: Discount factor reflecting annual 12% cost of capital; e.g., 1.25 = 1.12 x 1.12.
Column 12 = Column 10 ÷ Column 11.
Column 13 = Column 2 x Column 9 ÷ Column 11.

EXHIBIT 3

YOUNG MALE DRIVERS

(1) Policy Year	(2) Premium	(3) PV of Losses	(4) Variable Expenses New	(5) Variable Expenses Renewal	(6) Fixed Expenses New	(7) Fixed Expenses Renewal	(8) Persis-tency	(9) Cumulative Persistency	(10) Profit	(11) Discount Factor	(12) PV of Profits	(13) PV of Premium
1	1,272	1,000	384	0	117	0	100%	100%	−230	1.00	−230	1272
2	1,386	1,068	0	86	0	26	60%	60%	124	1.12	110	743
3	1,511	1,141	0	94	0	28	65%	39%	97	1.25	78	470
4	1,647	1,218	0	102	0	29	70%	27%	81	1.40	58	320
5	1,796	1,301	0	111	0	30	73%	20%	70	1.57	45	227
6	1,957	1,389	0	121	0	32	76%	15%	63	1.76	36	168
7	2,133	1,484	0	132	0	34	79%	12%	58	1.97	29	129
8	2,325	1,584	0	144	0	35	82%	10%	55	2.21	25	103
9	2,535	1,692	0	157	0	37	85%	8%	54	2.48	22	85
10	2,763	1,807	0	171	0	39	88%	7%	55	2.77	20	73
11	3,011	1,930	0	187	0	41	90%	7%	56	3.11	18	64
12	3,282	2,061	0	204	0	43	90%	6%	58	3.48	17	56
13	3,578	2,201	0	222	0	45	90%	5%	59	3.90	15	49
14	3,900	2,351	0	242	0	47	90%	5%	61	4.36	14	43
15	4,251	2,511	0	264	0	50	90%	4%	62	4.89	13	38
Total											$269	$3,841

Column 2: First year premium is chosen such that the present value of profits (Column 12 total) = 7.0% of the present value of premium (Column 13 total). Subsequently, premiums increase 9% per annum.
Column 3: First year losses average $1,000; loss cost trend is +10% per annum; losses decrease 3% per annum as policy matures.
Columns 4 and 5: Variable expense ratio is 30.2% the first year and 6.2% in subsequent years.
Column 6: First year fixed expenses as $98 per policy, or 17.8% of the average premium for all drivers ($550). Fixed expenses for young male drivers are 20% higher, or $117 per policy.
Column 7: Fixed expenses in the first renewal year are $550x1.20x1.05x3.8%. Subsequently, expenses increase 5% per annum.
Column 8: Assumed persistency rates for young male drivers.
Column 9 = Downward product of Column 8.
Column 10 = {Column 2 − sum (Columns 3 through 7)}xColumn 9.
Column 11: Discount factor reflecting annual 12% cost of capital; e.g., 1.25 = 1.12x1.12.
Column 12 = Column 10 ÷ Column 11.
Column 13 = Column 2xColumn 9 ÷ Column 11.

EXHIBIT 4

NO CARRIERS OFFER DISCOUNTS

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
			Variable Expenses		Fixed Expenses								Loss
Policy Year	Premium	PV of Losses	New	Renewal	New	Renewal	Persis-tency	Cumulative Persistency	Profit	Discount Factor	PV of Profits	PV of Premium	Ratio Relativity
1	600	500	0	37	0	23	100%	100%	40	1.00	40	600	98%
2	654	528	0	41	0	24	96%	96%	59	1.12	53	561	95%
3	713	557	0	44	0	25	96%	92%	80	1.25	64	524	92%
4	777	586	0	48	0	26	95%	88%	102	1.40	72	484	89%
5	847	617	0	53	0	28	95%	83%	124	1.57	79	448	86%
6	923	649	0	57	0	29	95%	79%	149	1.76	84	414	83%
7	1,006	689	0	62	0	31	95%	75%	168	1.97	85	383	81%
8	1,097	732	0	68	0	32	95%	71%	189	2.21	85	354	79%
9	1,196	767	0	74	0	34	95%	68%	217	2.48	88	327	76%
10	1,303	813	0	81	0	35	94%	64%	238	2.77	86	299	74%
11	1,420	862	0	88	0	37	94%	60%	260	3.11	84	274	72%
12	1,548	912	0	96	0	39	93%	56%	279	3.48	80	248	70%
13	1,688	965	0	105	0	41	92%	51%	295	3.90	76	222	68%
14	1,839	1,036	0	114	0	43	91%	47%	301	4.36	69	196	67%
15	2,005	1,111	0	124	0	45	90%	42%	304	4.89	62	172	66%
Total											$1,107	$5,505	

Column 2: First year premium is set at $600; subsequent premiums increase 9% per annum.
Column 3: First year losses average $500; loss cost trend is +10% per annum; losses decrease 1% per annum as policy matures; and losses are adjusted by the change in Column 14 relativities. For instance, $528 = $500x1.1x0.99x0.95 ÷ 0.98.
Column 5: Variable expense ratio is 6.2% in renewal years.
Column 7: Fixed expenses in the first renewal year are $600x3.8% = $23. Subsequently, expenses increase 5% per annum.
Column 8: Assumed persistency rates for older drivers with mature policies.
Column 9 = Downward product of Column 8.
Column 10 = {Column 2 – sum (Columns 3 through 7)}xColumn 9.
Column 11: Discount factor reflecting annual 12% cost of capital; e.g., 1.25 = 1.12x1.12.
Column 12 = Column 10 ÷ Column 11.
Column 13 = Column 2xColumn 9 ÷ Column 11.
Column 14: Loss ratio relativities by age of insured: 52 years old in first policy year shown and 66 years old in last policy year shown.

EXHIBIT 5

ONLY COMPETITORS OFFER DISCOUNTS

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
			Variable Expenses		Fixed Expenses								Loss
Policy Year	Premium	PV of Losses	New	Renewal	New	Renewal	Persis-tency	Cumulative Persistency	Profit	Discount Factor	PV of Profits	PV of Premium	Ratio Relativity
1	600	500	0	37	0	23	100%	100%	40	1.00	40	600	98%
2	654	528	0	41	0	24	96%	96%	59	1.12	53	561	95%
3	713	557	0	44	0	25	94%	90%	78	1.25	62	513	92%
4	777	586	0	48	0	26	92%	83%	96	1.40	69	459	89%
5	847	617	0	53	0	28	90%	75%	112	1.57	71	402	86%
6	923	649	0	57	0	29	88%	66%	124	1.76	70	344	83%
7	1,006	689	0	62	0	31	85%	56%	125	1.97	63	285	81%
8	1,097	732	0	68	0	32	82%	46%	121	2.21	55	227	79%
9	1,196	767	0	74	0	34	80%	37%	118	2.48	47	177	76%
10	1,303	813	0	81	0	35	77%	28%	106	2.77	38	133	74%
11	1,420	862	0	88	0	37	75%	21%	92	3.11	30	97	72%
12	1,548	912	0	96	0	39	76%	16%	81	3.48	23	72	70%
13	1,688	965	0	105	0	41	77%	12%	71	3.90	18	54	68%
14	1,839	1,036	0	114	0	43	78%	10%	63	4.36	14	41	67%
15	2,005	1,111	0	124	0	45	80%	8%	56	4.89	11	32	66%
Total											$666	$3,996	

Column 2: First year premium is set at $600; subsequent premiums increase 9% per annum.
Column 3: First year losses average $500; loss cost trend is +10% per annum; losses decrease 1% per annum as policy matures; and losses are adjusted by the change in Column 14 relativities. For instance, $528 = $500x1.1x0.99x0.95 ÷ 0.98.
Column 5: Variable expense ratio is 6.2% in renewal years.
Column 7: Fixed expenses in the first renewal year are $600x3.8% = $23. Subsequently, expenses increase 5% per annum.
Column 8: Assumed persistency rates for older drivers with no premium discount.
Column 9 = Downward product of Column 8.
Column 10 = {Column 2 – sum (Columns 3 through 7)} x Column 9.
Column 11: Discount factor reflecting annual 12% cost of capital; e.g., 1.25 = 1.12x1.12.
Column 12 = Column 10 ÷ Column 11.
Column 13 = Column 2 x Column 9 ÷ Column 11.
Column 14: Loss ratio relativities by age of insured: 52 years old in first policy year shown and 66 years old in last policy year shown.

EXHIBIT 6

ALL CARRIERS OFFER DISCOUNTS

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
			Variable Expenses		Fixed Expenses								Loss
Policy Year	Premium	PV of Losses	New	Renewal	New	Renewal	Persis-tency	Cumulative Persistency	Profit	Discount Factor	PV of Profits	PV of Premium	Ratio Relativity
1	555	500	0	34	0	23	100%	100%	−2	1.00	−2	555	98%
2	605	528	0	38	0	24	98%	98%	15	1.12	14	529	95%
3	659	557	0	41	0	25	98%	96%	35	1.25	28	505	92%
4	719	586	0	45	0	26	97%	93%	57	1.40	41	477	89%
5	783	617	0	49	0	28	97%	90%	81	1.57	52	450	86%
6	854	649	0	53	0	29	96%	87%	107	1.76	61	420	83%
7	931	689	0	58	0	31	96%	83%	128	1.97	65	393	81%
8	1,015	732	0	63	0	32	95%	79%	148	2.21	67	363	79%
9	1,106	767	0	69	0	34	95%	75%	178	2.48	72	336	76%
10	1,205	813	0	75	0	35	94%	71%	199	2.77	72	307	74%
11	1,314	862	0	81	0	37	94%	66%	222	3.11	71	281	72%
12	1,432	912	0	89	0	39	93%	62%	242	3.48	70	254	70%
13	1,561	965	0	97	0	41	93%	57%	263	3.90	68	230	68%
14	1,702	1,036	0	105	0	43	92%	53%	273	4.36	63	206	67%
15	1,855	1,111	0	115	0	45	92%	49%	284	4.89	58	184	66%
Total											$797	$5,491	

Column 2: First year premium is set at $600; subsequent premiums increase 9% per annum; 7.5% discount applied to all premiums.
Column 3: First year losses average $500; loss cost trend is +10% per annum; losses decrease 1% per annum as policy matures; and losses are adjusted by the change in Column 14 relativities. For instance, $528 = $500x1.1x0.99x0.95 ÷ 0.98.
Column 5: Variable expense ratio is 6.2% in renewal years.
Column 7: Fixed expenses in the first renewal year are $600x3.8% = $23. Subsequently, expenses increase 5% per annum. The 7.5% premium discount does not affect fixed expenses.
Column 8: Assumed persistency rates for older drivers with 7.5% premium discount.
Column 9 = Downward product of Column 8.
Column 10 = {Column 2 − sum (Columns 3 through 7)} x Column 9.
Column 11: Discount factor reflecting annual 12% cost of capital; e.g., 1.25 = 1.12x1.12.
Column 12 = Column 10 ÷ Column 11.
Column 13 = Column 2 x Column 9 ÷ Column 11.
Column 14: Loss ratio relativities by age of insured: 52 years old in first policy year shown and 66 years old in last policy year shown.

EXHIBIT 7

UNDERWRITING CYCLE UPTURN

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
			Variable Expenses		Fixed Expenses								U/W
Policy Year	Premium	PV of Losses	New	Renewal	New	Renewal	Persistency	Cumulative Persistency	Profit	Discount Factor	PV of Profits	PV of Premium	Cycle Factor
1	800	656	242	0	88	0	100%	100%	−186	1.00	−186	800	1.00
2	1,003	701	0	62	0	32	85%	85%	177	1.12	158	761	1.15
3	1,236	748	0	77	0	34	86%	73%	276	1.25	220	720	1.30
4	1,191	799	0	74	0	35	87%	64%	180	1.40	128	539	1.15
5	1,129	853	0	70	0	37	88%	56%	95	1.57	60	402	1.00
6	1,046	911	0	65	0	39	89%	50%	16	1.76	9	296	0.85
7	939	973	0	58	0	41	90%	45%	−60	1.97	−30	213	0.70
8	1,243	1,039	0	77	0	43	90%	40%	34	2.21	15	227	0.85
9	1,594	1,110	0	99	0	45	91%	37%	125	2.48	50	236	1.00
10	1,998	1,186	0	124	0	47	91%	33%	214	2.77	77	241	1.15
11	2,462	1,266	0	153	0	50	92%	31%	305	3.11	98	244	1.30
12	2,374	1,352	0	147	0	52	92%	28%	233	3.48	67	193	1.15
13	2,250	1,444	0	140	0	55	92%	26%	159	3.90	41	150	1.00
14	2,085	1,542	0	129	0	57	92%	24%	85	4.36	20	114	0.85
15	1,871	1,647	0	116	0	60	92%	22%	11	4.89	2	84	0.70
Total											$730	$5,221	

Column 2: First year premium is \$800; subsequently, premiums increase 9% per annum. These premiums are then multiplied by the "underwriting cycle factor" in Column 14.
Column 3: First year discounted losses are \$656, for an 82% discounted loss ratio; loss cost trend is +10% per annum; losses decrease 3% per annum as the policy matures.
Columns 4 and 5: Variable expense ratio is 30.2% the first year and 6.2% in subsequent years.
Column 6: First year fixed expenses are 17.8% of the premium.
Column 7: Fixed expenses in the first renewal year are 800x1.05x3.8%. Subsequently, expenses increase 5% per annum.
Column 8: Assumed persistency rates by policy duration.
Column 9 = Downward product of Column 8.
Column 10 = {Column 2 − sum (Columns 3 through 7)}xColumn 9.
Column 11: Discount factor reflecting annual 12% cost of capital; e.g., 1.25 = 1.12x1.12.
Column 12 = Column 10 ÷ Column 11.
Column 13 = Column 2xColumn 9 ÷ Column 11.
Column 14: "Underwriting cycle factors," reflecting upward movement of an eight year cycle.

EXHIBIT 8

UNDERWRITING CYCLE DOWNTURN

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
			Variable Expenses		Fixed Expenses								U/W
Policy Year	Premium	PV of Losses	New	Renewal	New	Renewal	Persis-tency	Cumulative Persistency	Profit	Discount Factor	PV of Profits	PV of Premium	Cycle Factor
1	800	656	242	0	88	0	100%	100%	−186	1.00	−186	800	1.00
2	741	701	0	46	0	32	85%	85%	−32	1.12	−28	563	0.85
3	665	748	0	41	0	34	86%	73%	−115	1.25	−92	388	0.70
4	881	799	0	55	0	35	87%	64%	−5	1.40	−4	399	0.85
5	1,129	853	0	70	0	37	88%	56%	95	1.57	60	402	1.00
6	1,416	911	0	88	0	39	89%	50%	188	1.76	107	400	1.15
7	1,744	973	0	108	0	41	90%	45%	279	1.97	141	396	1.30
8	1,682	1,039	0	104	0	43	90%	40%	200	2.21	90	307	1.15
9	1,594	1,110	0	99	0	45	91%	37%	125	2.48	50	236	1.00
10	1,477	1,186	0	92	0	47	91%	33%	51	2.77	18	178	0.85
11	1,326	1,266	0	82	0	50	92%	31%	−22	3.11	−7	131	0.70
12	1,755	1,352	0	109	0	52	92%	28%	68	3.48	20	143	0.85
13	2,250	1,444	0	140	0	55	92%	26%	159	3.90	41	150	1.00
14	2,821	1,542	0	175	0	57	92%	24%	250	4.36	57	155	1.15
15	3,475	1,647	0	215	0	60	92%	22%	342	4.89	70	156	1.30
Total											$339	$4,803	

Column 2: First year premium is $800; subsequently, premiums increase 9% per annum. These premiums are then multiplied by the "underwriting cycle factor" in Column 14.
Column 3: First year discounted losses are $656, for an 82% discounted loss ratio; loss cost trend is +10% per annum; losses decrease 3% per annum as the policy matures.
Columns 4 and 5: Variable expense ratio is 30.2% the first year and 6.2% in subsequent years.
Column 6: First year fixed expenses are 17.8% of the premium.
Column 7: Fixed expenses in the first renewal year are 800x1.05x3.8%. Subsequently, expenses increase 5% per annum.
Column 8: Assumed persistency rates by policy duration.
Column 9 = Downward product of Column 8.
Column 10 = {Column 2 − sum (Columns 3 through 7)}xColumn 9.
Column 11: Discount factor reflecting annual 12% cost of capital; e.g., 1.25 = 1.12x1.12.
Column 12 = Column 10 ÷ Column 11.
Column 13 = Column 2xColumn 9 ÷ Column 11.
Column 14: "Underwriting cycle factors," reflecting downward movement of an eight year cycle.

NAIC PROPERTY/CASUALTY INSURANCE COMPANY RISK-BASED CAPITAL REQUIREMENTS

SHOLOM FELDBLUM

Abstract

The risk-based capital requirements adopted by the NAIC in 1994 are a major advance in the solvency regulation of property/casualty insurance companies. The components of the risk-based capital formula are grounded in actuarial and financial analyses of the risks faced by insurance companies and of the capital needed to guard against those risks.

The intricacy of the risk-based capital formula, the manifold considerations that shaped it, and the lack of explanation provided by the NAIC make the new capital requirements difficult to follow. This paper leads the reader through the formula, illuminating its workings and its rationale.

The paper first takes the reader through the components of the risk-based capital formula, as well as the "covariance adjustment" connecting them. The emphasis is on the development and justification of the charges, not simply on the accounting entries needed.

Casualty actuaries were instrumental in developing several components of the risk-based capital formula: the covariance adjustment, the offset for claims-made business, the offset for loss-sensitive contracts, the treatment of workers compensation tabular loss reserve discounts, and the additional charges for rapidly growing companies. In discussing the actuarial considerations in these five issues, the paper demonstrates how actuarial science has major practical implications for insurance regulation.

To be effective, the risk-based capital formula must be combined with statutory enactments empowering regula-

tory officials to take action against financially distressed companies. The paper explains the "action levels" in the NAIC Risk-Based Capital Model Act, as well as the various potential uses of the risk-based capital results.

The paper concludes with a fully documented illustration, showing how the Annual Statement figures are used to determine the risk-based capital ratio.

Expertise leads to authority. By fully understanding the NAIC capital requirements, casualty actuaries will be more qualified to suggest modifications in future years, as well as to develop their own models and standards for insurance company solvency monitoring.

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1. INTRODUCTION

Risk-based capital (RBC) standards for property/casualty insurance companies were adopted by the National Association of Insurance Commissioners (NAIC) in December 1993, effective for the 1994 and subsequent Annual Statements. Casualty actuaries were instrumental in developing the risk-based capital formula, and they are likely to be involved in determining capital strategies for their employers and clients.

Documentation of the risk-based capital formula has lagged behind its development. This paper explains the workings of the risk-based capital formula: the risks that are measured, the quantification techniques, and the actuarial or financial rationale for each component of the formula. Where appropriate, this paper discusses the arguments for and against various risk charges, and it explains the NAIC resolution of each controversy.

Instructions, Examples, and Analysis

Documentation of the NAIC risk-based capital formula comes in three varieties: instructions, examples, and analyses.

Instructions: Companies completing their risk-based capital report must know what numbers to enter on each line. This paper is not intended to serve this function. Rather, the official instructions for completing the report are found in the "NAIC Property/Casualty Risk-Based Capital Report, Including Overview and Instructions for Companies" (hereafter, *NAIC Instructions*), which is updated each year. If there are any discrepancies between this paper and the *NAIC Instructions*, the *NAIC Instructions* obviously govern.

Examples: Ambiguities in the risk-based capital formula are often resolved by clear examples. This paper includes exhibits for a simulated property/casualty insurance company to illustrate the workings of the risk-based capital formula.

Analyses: In-depth analysis of the risk-based capital charges may be found in the minutes of the NAIC Risk-Based Capital Working Group (NAIC Working Group), in the reports of the American Academy of Actuaries Task Force on Risk-Based Capital (AAA Task Force), and in the NAIC *Research Quarterly*. Many of these reports are difficult for outsiders to understand, since they presume a thorough familiarity with the topic at hand. This paper provides clear descriptions of the actuarial rationale for each charge in the NAIC formula and of the considerations involved in the development of the formula. The instructions and examples in this paper are secondary to the "analysis."

2. TYPES OF RISK

The property/casualty risk-based capital formula was developed from the corresponding life insurance formula. The life insurance formula groups risks into four categories, C-1 through C-4, which correspond roughly to asset risks, underwriting risks, interest rate risk, and other risks.

This structure was most evident in the first draft of the property/casualty formula, which was released in April 1991 [36], and it is retained in the NAIC "Risk-Based Capital Model Act."[1] The desire to have similar capital charges for life, health, and property/casualty insurers is referred to as a "seamless" capital requirement. In other words, the capital required to protect against any risk should not depend on whether the company is licensed as a life insurer or as a property/casualty insurer.

- For asset risks, which were considered similar for life and property/casualty companies, the capital charge was adopted without modification from the life formula, and the statistical analysis for the charges was done by the life actuarial advisory committee.[2]
- Underwriting risks are entirely different between life and property/casualty products. The property/casualty capital charges were developed by the NAIC Working Group and by the New York Insurance Department staff [33].
- Interest rate risk was not considered in the first draft of the property/casualty formula, though proposed capital charges have since been recommended by the AAA Task Force.

[1]See NAIC Risk-Based Capital Model Act, Section 2.C on pages 312-3 through 312-4.

[2]The major qualifications to this statement are that (i) the default risk charges for category 3, 4, and 5 bonds and (ii) the market risk charges for unaffiliated common stocks are half as large in the final property/casualty formula as those in the life formula. (The rationale for this difference is explained later in this paper.)

- The most important of the “other risks” is the credit risk charge for reinsurance recoverable.

A second draft of the formula, with significant changes from the earlier version, was released in June 1993, and it was adopted by the NAIC in December 1993 after several revisions. The most important change was the incorporation of a “covariance adjustment,” which necessitated a different structure for the capital charges. For instance, “asset risks” were divided into three categories: (i) unaffiliated fixed-income investments; (ii) unaffiliated equity investments, which were assumed to be independent risks; and (iii) affiliated investments, which did not enter the covariance adjustment at all. (See below for full treatment of the covariance adjustment.)

The risk-based capital requirements were first effective for the 1994 Annual Statement. Minor modifications continue to be made to the formula, though there are few significant differences to date between the 1994 and the subsequent formulas.

This paper presents the risk-based capital formula as adopted in December 1993, with emphasis on the evolution of several of these charges. When appropriate, the paper comments on a few formula modifications that have been made since the initial adoption.[3]

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[3]Most of the work on the risk-based capital formula was done in four committees:

- The *NAIC Property/Casualty Risk-Based Capital Working Group*, hereafter “NAIC Working Group”, chaired by Vincent Laurenzano of the New York Insurance Department. A corresponding working group for the life and health insurance risk-based capital requirements was chaired by Terence Lennon, also of the NY Insurance Department. In December 1993, with the adoption of the property/casualty risk-based capital requirements, the two groups were merged, under the chairmanship of Mr. Laurenzano. The first draft of the formula was developed by this group, working in conjunction with the staff of the NY Insurance Department. This group remains active, monitoring the effectiveness of the formula and overseeing its implementation in the various states.
- The *American Academy of Actuaries Task Force on Risk-Based Capital* (formerly the Actuarial Advisory Committee to the NAIC Risk-Based Capital Working Group), hereafter “AAA RBC Task Force.” From 1991 through 1993, during the development of the risk-based capital formula, this task force was chaired by David G. Hartman of the Chubb Group of Insurance Companies. Upon Mr. Hartman’s assumption of the

3. ASSET RISKS

The asset risk charges, which were largely adopted from the life insurance formula, stem from the charges in the life insurance Mandatory Securities Valuation Reserves (MSVR). The asset risk charges are the dominant piece of the life insurance risk-based capital formula, though they are of lesser importance for the property/casualty formula, for both practical and theoretical reasons.

- In practice, capital-to-asset ratios differ greatly between life and property/casualty companies. The average property/casualty company has assets about two to three times its capital, whereas life companies have about ten times as much assets as capital. A 5% asset risk charge for life companies translates into about 50% of surplus. The same charge for a property/casualty company is only about 10 to 15% of surplus.
- In theory, asset risks are more important for life insurance companies. Many life insurance products, particularly Universal Life, Variable Life, and Variable Annuities, are seen as a combination of insurance protection (against death or lack of income) and long-term investment (particularly when aided by the tax-deferred or tax-free inside build-up of policy cash values). When investment returns offered by the insurance prod-

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presidency of the American Academy of Actuaries in September 1993, chairmanship of the task force was passed to Frederick O. Kist of Coopers & Lybrand. Most of the "actuarial issues" discussed in this paper stemmed from work of this task force or of its members. This task force remains active, working particularly on several still unsettled issues, such as interest rate risk, liquidity risk, discount factors, and several aspects of the underwriting risk charges.

- The *Model Law Advisory Committee* to the NAIC Risk-Based Capital Working Group, chaired by William Murray of the Chubb Group of Insurance Companies. This committee was instrumental in developing the language for the Risk-Based Capital Model Act.
- The *Accounting Advisory Committee* to the NAIC Risk-Based Capital Working Group, chaired by Peter Storms of the Travelers Insurance Company. This committee was most active in developing capital charges for subsidiaries and affiliates and in revising the capital charge for unaffiliated common stocks.

The last two committees are no longer in existence, having been phased out when the NAIC disbanded all industry "advisory committees."

uct are not competitive, policyholders are more likely to exercise options such as policy loans and surrenders. Proper management of asset returns and asset risks is crucial for the life insurance company.

Property/casualty products, in contrast, are generally designed for insurance protection only, not for investment purposes. Moreover, there are few "policyholder options" in property/casualty products, so asset-liability management, with its balancing of yields and risks, is less essential for property/casualty insurance companies.

Unaffiliated Fixed Income Securities

The major risk for fixed income securities is default risk: the risk that the issuer will not make the required interest or principal payments. The risk factor varies by the NAIC bond class (or "asset class"). The factor ranges from 0% for Treasury securities, since the default risk is virtually non-existent, to 30% for bonds in NAIC Class 6, which are primarily bonds in or near default. The full set of risk-based capital default risk factors is shown in Table 1.[4]

[4]The *NAIC Instructions*, p. 2, explain that "these bond factors are based on cash flow modeling, using historically-adjusted default rates for each bond category. For each of 2,000 trials, annual economic conditions were generated for the ten-year modeling period. Each bond of a 400-bond portfolio was annually tested for default (based on a "roll of the dice") where the default probability varies by rating category and that year's economic environment. When a default takes place, the actual loss considers the expected principal loss by category, the time until the sale actually occurs, and the assumed tax consequences." (This analysis was performed by the actuarial advisory committee to the life insurance risk-based capital working group.)

For investment grade bonds (Classes 1 and 2), the factors in the property/casualty risk-based capital formula are the same as those in the life insurance formula, since these bonds are reported at amortized cost by both sets of insurers. Bonds below "investment grade" (Classes 3, 4, and 5) are reported at market value in the property/casualty statutory statement but may be reported at amortized value in the life insurance statutory statement. To use the same risk-based capital charges for the two sets of companies would amount to a double charge for property/casualty insurers. Consequently, the Class 3, 4, and 5 factors in the property/casualty formula are half as large as those in the life formula. This is the intent of the comment in the *NAIC Instructions* that "the factors for Classes 3 through 6 bonds recognize that the statement value of these bonds reflects a loss of value upon default by being marked to market."

TABLE 1

Bond Class	Risk-Based Capital Factor
Federal government bonds	0.0%
NAIC Class 1: Highest Quality	0.3%
NAIC Class 2: High Quality	1.0%
NAIC Class 3: Medium Quality	2.0%
NAIC Class 4: Low Quality	4.5%
NAIC Class 5: Lower Quality	10.0%
NAIC Class 6: In or Near Default	30.0%

Preferred Stocks

Preferred stocks are similar to bonds in that both provide a steady stream of interest or dividends. The risk-based capital factors for bonds were developed from a statistical analysis of the risk of default by rating class. Comparable data were not available for the default risk on preferred stocks. Instead, the NAIC Working Group assumed "that preferred stocks are somewhat more likely to default than bonds and that the loss on default would be somewhat higher than that experienced on bonds."

The capital charges for preferred stocks were therefore set equal to the capital charges on comparable bonds plus 2%, with two exceptions:

- There are no "federal government preferred stocks."
- The factors are capped at 30%, so the charge for "class 6 preferred stock" is 30%, not 32%.

"Insolvency risk," or "accounting risk," should be distinguished from "economic risk," or "pricing risk." Altman [2] argues that the higher default risk on lower quality securities is more than compensated for by the higher investment yield. However, the default rates on lower quality securities are not independent; rather, depressed economic conditions may lead to higher default rates on all bonds, and particularly on lower quality bonds. Thus, even if the "economic" risk is compensated for by the higher investment yield, the "insolvency" risk is not necessarily reduced. For further comments on this issue, see Feldblum [24].

TABLE 2

Preferred Stock Class	Risk-Based Capital Factor
NAIC Class 1: Highest Quality	2.3%
NAIC Class 2: High Quality	3.0%
NAIC Class 3: Medium Quality	4.0%
NAIC Class 4: Low Quality	6.5%
NAIC Class 5: Lower Quality	12.0%
NAIC Class 6: In or Near Default	30.0%

The complete list of charges by quality class is shown in Table 2.

In the life insurance risk-based capital formula, both the bond charges and the preferred stock charges are included in the "C-1" risk category (asset risks). In the property/casualty risk-based capital formula, the charges are the same except for the covariance adjustment. The bond charges are included in the "R_1" risk category (fixed-income securities) and the preferred stock charges are included in the "R_2" risk category (equities). (See Section 8 below for the classification of the capital charges into the R_0 through R_5 categories.)

Cash Risks

Cash deposited in a banking institution is subject to the risk that the cash may be uncollectible if the bank becomes insolvent. This is similar to the risk that bonds issued by a high quality corporation may default, so the NAIC Working Group chose a 0.3% charge for cash, similar to the charge on Class 1 bonds. Non-government money market funds, which are similar to cash deposits, have the same charge.

Bond Size Adjustment Factor

The bond size adjustment factor adjusts the risk-based capital charge to reflect the degree of diversification in the financial portfolio. The bond size adjustment factor decreases as the number of bond issuers increases.[5]

If the number of issuers is less than or equal to 50, the bond charge is multiplied by 250%. For the next 50 issuers, the adjustment is 130%. For instance, if the insurance company holds bonds from 80 issuers, the computation is

$$[(50 * 250\%) + (30 * 130\%)] \equiv (50 + 30) = 205\%.$$

This is the weighted average of the adjustment factor for the first 50 issuers and the corresponding factor for the next 30 issuers.

For issuers between 101 and 400, the adjustment factor is 100%. For all remaining issuers (i.e., issuers above 400), the adjustment factor is 90%.

The "bond size" factor is defined as the weighted average of the adjustment factors minus unity. For instance, suppose that the insurer's investment portfolio contains securities from 500 issuers subject to the bond size adjustment factor. The weighted average of the adjustment factors is

$$[(50 * 250\%) + (50 * 130\%) + (300 * 100\%) + (100 * 90\%)] \equiv (500) = 1.16.$$

The "size factor" is 1.16 – 1.00 = 0.16, or 16%. The "bond size factor RBC charge" is 16% of the "pre-size-factor bond RBC charge" for the bonds subject to the size factor. The "bond size factor RBC charge" is added to the "pre-size-factor bond RBC charge" to get the "total bond RBC charge."[6]

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[5]The number of bond issuers is based on the first six digits of the CUSIP number. In other words, one aggregates different bond series from the same issuer. Three types of bonds are not considered in determining the number of issuers and are not subject to the bond size factor:

- U.S. government bonds;
- Class 1 bonds that are issued by a U.S. government agency; and
- Bonds of parents, subsidiaries and affiliates.

[6]For property/casualty insurers, the bond size adjustment factor has little effect on the final risk-based capital ratios, though calculating the factor is time-consuming. The AAA Task Force is presently (mid-1996) preparing a recommendation that this factor be

Unaffiliated Common Stocks

The charge for unaffiliated common stocks elicited the most controversy among all the asset risk charges, leading eventually to different capital requirements in the life insurance and property/casualty risk-based capital formulas. The arguments summarized below are likely to re-emerge in the coming years, as the NAIC strives for a "seamless" formula: that is, a formula where the charge for a given risk does not depend on whether the company is licensed as a life insurer or as a property/casualty insurer. Moreover, these arguments show the different perspectives on asset-liability management, time horizons for solvency monitoring, and calibration methods for the charges that have influenced the risk-based capital formula.

The first (April 1991) draft of the property/casualty insurance risk-based capital formula had the same asset risk charges as the corresponding life insurance formula had. The life insurance formula has a 30% charge for investments in non-affiliated common stocks. Few life insurers have substantial common stock investments, so the magnitude of this charge elicited little industry opposition.[7]

In contrast, many property/casualty insurers have significant common stock holdings, and the original 30% common stock charge had a considerable effect on the risk-based capital requirements for property/casualty insurers. Some observers considered the charge to be excessive.

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dropped from the risk-based capital formula. Moreover, since the number of issuers subject to the bond size adjustment factor is not shown in the Annual Statement, errors in calculating the factor abound. Michael Barth, the research associate at the NAIC in charge of analyzing the risk-based capital results, has commented that "it is hard to argue that the bond size factor is meaningful when so many companies report it incorrectly" [4].

[7]The life insurance appointed actuary must prepare an asset adequacy analysis (in states that have adopted the NAIC's 1990 Standard Valuation Law) that compares the cash inflows from the investment portfolio with the cash outflows from benefit obligations. Such analyses are most easily prepared for fixed income securities, which have regular coupon or interest payments. They are harder to prepare for equity investments, many of which provide uncertain cash payments.

Three Perspectives

Members of three risk-based capital committees offered critiques of the 30% charge, leading to the reduction of the charge to 15% for property/casualty companies. Many regulators are uncomfortable with differing charges in the life insurance and property/casualty formulas for the same risk, and one can expect efforts in the coming years to equalize the charges in the two formulas.[8] The key issues involved are well represented by the following three perspectives on the common stock risk charge.

1. Robert Bailey, deputy insurance commissioner of the State of Michigan and a member of the NAIC Working Group, thought the 30% charge was too high, both for life insurers and for property/casualty insurers. However, since the life insurance risk-based capital actuarial advisory committee would not revise their 30% charge, Mr. Bailey recommended that this charge differ between life insurers and property/casualty insurers, for the following reason:

   > Many life insurers, especially those selling traditional whole-life insurance policies, have liabilities that are expressed in fixed dollar terms, such as $100,000 of life insurance. For such insurance contracts, common stocks can be a risky investment, since the market value of the stocks may fluctuate while the insurance liability remains fixed. Property/casualty insurers, however, have inflation-sensitive liabilities: when inflation accelerates, the dollar amount of required liability loss reserves also increases. Property/casualty insurers may use inflation-

[8] During late 1993, for instance, consideration was given to reducing the common stock charge in the life insurance risk-based capital formula as well. In early 1994, however, the life insurance actuarial advisory committee to the NAIC Working Group again concluded that 30% is an appropriate charge, and it should not be reduced to 15%.

sensitive assets, such as common stocks, to match their inflation-sensitive liabilities.[9]

2. William Panning (Hartford) and Peter Storms (Travelers), members of the Accounting Advisory Committee to the NAIC Working Group, reexamined the work of the life insurance risk-based capital actuarial advisory committee on common stock risks, using different investment years and different holding periods. Using 90% and 95% confidence intervals, they concluded that the 30% charge was excessive; a more appropriate number would be between 10% and 12%.

3. Robert Butsic of the Fireman's Fund Insurance Companies, a member of the AAA RBC Task Force, calibrated the common stock charge using a 1% "expected policyholder deficit." He also concluded that the 30% charge was excessive, and that a more appropriate number would be 15%.[10]

In early 1993, in light of these recommendations, the NAIC Working Group revised the non-affiliated common stock charge

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[9]On the inflation sensitivity of property/casualty loss reserves, see Butsic [10]. The inflation sensitivity of common stocks is a much debated issue; see Fama and Schwert [18] and Feldblum [19]. Bailey's position is best summed up in his July 6, 1992, letter to Sholom Feldblum: "I supported a lower RBC charge for common stocks for casualty insurers on the theoretical grounds that casualty insurers have a greater proportion of their liabilities that are inflation-sensitive and therefore need more assets that are inflation sensitive in the same direction."

[10]Butsic chose a 1% "expected policyholder deficit" (EPD) ratio because the reserving risk charges in the risk-based capital formula, when viewed from an expected policyholder deficit perspective, produce an expected policyholder deficit ratio of about 1%. See Butsic [11] for a discussion of the expected policyholder deficit concept and its application to risk-based capital requirements. Butsic argues that the various components of the risk-based capital formula should be internally consistent: each should be calibrated to approximately the same "solvency" level.

With regard to the Accounting Advisory Committee comments on the "holding period," see Butsic's Exhibit 4 and the related text regarding the "time horizon" for the risk-based capital system. For common stock investments and casualty loss reserves, the longer the time horizon, the greater the capital needed to satisfy a given EPD ratio.

for property/casualty companies to 15%. The NAIC interprets this charge by saying that "the factor for other unaffiliated common stock is based on studies which indicate that a 10% to 12% factor is needed to provide capital to cover approximately 95% of the greatest losses in common stock values over a one-year future period. The higher factor of 15% contained in the formula reflects the increased risk when testing a period in excess of one year."

Asset Concentration Factor

The "asset concentration factor" doubles the risk-based capital charges for the ten largest investments, with a maximum charge of 30% for any one security. Certain investments are not included in the asset concentration factor, such as Treasury securities, Class 1 bonds, affiliated investments, and home office real estate.[11]

The asset concentration factor may be viewed as an additional incentive for diversification, or as a penalty for investments in only a small number of securities.[12] To determine the asset concentration factor, investments are first aggregated by "name." For instance, suppose that the ABC Insurance Company owns $100,000 of common stock of the XYZ Corporation, $20,000 of preferred stock of the XYZ Corp., and $250,000 of XYZ bonds. The total "investment" in XYZ is therefore $370,000.

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[11] See the *NAIC Instructions*, p. 10, for the exact list of which investments are excluded from computation of the asset concentration factor.

[12] The asset concentration factor is a more flexible regulatory tool than the existing insurance company investment statutes in most states. Most current investment statutes *prohibit* investment of more than, say, 10%, of the insurer's surplus in stock of any one company, or ownership or control of more than say, 25%, of the stock of any one company. The risk-based capital formula does not prohibit any investments. It simply requires additional capital for an investment portfolio that seems insufficiently diversified, just as it requires more capital for an investment portfolio that seems more "risky." Note, however, that the risk-based capital formula does not replace existing state investment statutes, and NAIC efforts to strengthen investment regulation continue alongside the NAIC risk-based capital efforts.

The risk-based capital charges for the assets included in the ten largest investments are doubled, with the exceptions and limitations noted above. For the purposes of the covariance adjustment (see Section 8 below), the extra charges are included with the asset category in which each security is placed. Thus, in the XYZ Corp. example above, the asset concentration factor charges for the common stocks and preferred stocks are included in the R_2 category, and the asset concentration factor charge for the bonds is included in the R_1 category.

Interest Rate Risk

The risk-based capital formula has no charge for "interest rate risk," defined as the adverse effects on a company's statutory surplus that may be caused by a shift in market interest rates. In 1993 and 1994, the AAA RBC Task Force developed a charge for "interest rate risk" for possible inclusion in the risk-based capital formula. In June 1994, the NAIC Working Group voted to defer consideration of an "interest rate risk" charge until further data are compiled to evaluate its importance for property/casualty insurance companies.[13]

Insurance Affiliates

The risk-based capital charge for investments in subsidiaries was one of the most intensely contested issues in the NAIC formula. Many insurance "companies" are complex and layered organizations comprising dozens of legal entities. Initially some regulators desired high capital charges, as much as 100% of carrying value, for subsidiaries that they could not effectively regulate, such as off-shore insurance subsidiaries. Many U.S. in-

[13]For a complete discussion of this issue, see Hodes and Feldblum [30]. The corresponding A. M. Best capital adequacy measure, "BCAR," does contain an interest rate risk component; see Simpson and Kellogg [41], [42]. During the summer of 1996, the AAA RBC Task Force analyzed the asset exhibits included by property/casualty companies with their 1995 risk-based capital submissions and calculated the resulting interest rate risk charges. The results were consistent with expectations, and the task force has recommended that the interest rate risk charge be incorporated into the risk-based capital formula.

surers, however, retorted that such charges would hamper their ability to compete in international markets. The subsections below explain the general principles for treatment of subsidiaries in the final risk-based capital formula. A full analysis of the more complex insurance fleets must take into consideration the legal form and capital structure of each corporate entity.

Domestic Insurance Subsidiary

The charge for investments in insurance affiliates depends on whether the affiliate is U.S.-domiciled or an alien company. The risk-based capital requirement for a *domestic* insurance subsidiary is passed up to the parent. For instance, suppose that the Parent Insurance Company owns the Subsidiary Insurance Company. If the total risk-based capital requirement for Subsidiary is $10 million, then the risk-based capital charge to Parent for its investment in Subsidiary is $10 million.

Alien Insurance Subsidiary

The charge for alien insurance subsidiaries is 50% of the reported value of the enterprise or of the securities that it has issued, such as stocks or bonds. For instance, if Parent Insurance Company owns $12 million of stock issued by Off-Shore Subsidiary Insurance Company, or $12 million of bonds issued by Off-Shore Subsidiary Insurance Company, the risk-based capital charge to Parent Insurance Company is $6 million.

The NAIC Working Group would have liked to treat alien insurance affiliates in the same manner as it treats U.S.-domiciled insurance affiliates: that is, by passing up the risk-based capital requirement for the subsidiary to the parent. However, there is no risk-based capital requirement for an alien subsidiary, and because of the different accounting statements used in other countries, a risk-based capital equivalent would be difficult to determine. Since the average risk-based capital charge for U.S.-domiciled companies is about 50% of their carrying values, the

NAIC Working Group chose 50% as a proxy for the appropriate risk-based capital requirement for alien insurance companies.

Investment Subsidiary

The risk-based capital charge for an investment in an "investment subsidiary" is determined by "looking through" the subsidiary to its investment holdings. An investment subsidiary is "any subsidiary engaged … primarily in the ownership and management of investments for the insurer … " (*NAIC Instructions*, p. 4).[14]

For instance, suppose that the Parent Insurance Company has $10 billion of stocks and bonds. To more effectively manage its financial portfolio, it forms the Subsidiary Investment Fund, which invests in common stocks and bonds. Suppose also that if the Parent Insurance Company itself owned these stocks and bonds, the risk-based capital R_1 and R_2 charges for them would have been $300 million. Then the risk-based capital charge to Parent for its investment in the Subsidiary Investment Fund is $300 million. In other words, the risk-based capital charge to Parent for its investment in the Subsidiary Investment Fund is equal to the risk-based capital charge it would have had if it owned the specific securities held by the Subsidiary Investment Fund.

Non-Insurance Subsidiaries

The risk-based capital charge for an investment in a non-insurance affiliate, whether domestic or alien, is 22.5% of its carrying value.

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[14]Ron Sweet, Vice President of USAA's Capital Management Department, explains that many of these investment subsidiaries "have low capitalization and do not have operations of their own; they exist primarily to hold investment assets for the parent company." In particular, there are statutory limitations on the type of assets or the amount of liabilities that the investment subsidiary may have.

Three Principles

The actual risk-based capital charges for investments in affiliates are more complex than indicated by the brief descriptions above. See the *NAIC Instructions*, pp. 28–29, for the complete rules. Three additional principles cover most instances:

1. The risk-based capital charge for a parent company is generally capped at the carrying value of the subsidiary. For instance, suppose that Parent Insurance Co. owns Subsidiary Insurance Co., whose carrying value on Parent's books is $100 million. If Subsidiary has a total risk-based capital requirement of $125 million, then Subsidiary falls within the "company action level" of regulatory attention. However, the risk-based capital charge for Parent's investment in Subsidiary is only $100 million.

2. Suppose that Parent Insurance Co. owns Non-Insurance Holding Co., which in turn owns Subsidiary Insurance Co. In other words, Subsidiary Insurance Co. is *indirectly* owned by Parent Insurance Co. Moreover, suppose that Non-Insurance Holding Co. has a carrying value of $200 million, and that Subsidiary has a carrying value of $100 million and a risk-based capital requirement of $50 million. The risk-based capital charge to Parent for its investment in Non-Insurance Holding Co. is the risk-based capital requirement of Subsidiary (capped at its carrying value) plus 22.5% of the difference between the carrying values of Non-Insurance Holding Co. and of Subsidiary Insurance Co. In this illustration, the risk-based capital charge to Parent is $50 million + 22.5% ($200 million – $100 million) = $72.5 million.

3. If Parent Insurance Co. owns preferred stock or bonds of Affiliated Insurance Co., then the risk-based capital charge to Parent is limited to the smaller of (a) the car-

rying value of the preferred stock or bonds and (b) the amount of "excess risk-based capital" above the amounts allocated to common stock investments in affiliated insurance companies. The *NAIC Instructions* explain the computation of the "excess risk-based capital" (pp. 28, 29).

The proper classification of investments in affiliates is particularly important because of their treatment in the covariance adjustment, which is explained in Section 8. Investments in *insurance* affiliates and subsidiaries are included in the R_0 charge. Investments in *non-insurance* subsidiaries are included in either the R_1 or R_2 charge, depending on whether the investments are fixed income or equity securities. This difference is significant, since the R_1 or R_2 charges are included in the covariance adjustment, whereas the R_0 charge is not.

Off Balance Sheet Risks

Most of the risk-based capital charges relate to balance sheet entries. For instance, if the company has a balance sheet entry of $100 million of unaffiliated common stocks, the associated risk-based capital charge is $15 million.

Sometimes, a company may have assets or potential liabilities that are not shown on the balance sheet. For instance, suppose a class action sex discrimination lawsuit has been filed against the company by some of its employees, seeking $10 million in damages. The company believes the suit is groundless, and it makes no entry for this in its balance sheet. Nevertheless, it discloses the potential liability in the notes to its financial statements.

Three types of such "off balance sheet" items are shown in the Notes to the Financial Statements. A risk-based capital charge was judgmentally chosen as 1% of the amount that is shown in the notes (*NAIC Instructions*, p. 14):

- *Non-controlled assets*: These are assets which are not under the exclusive control of the company.[15]
- *Guarantees for affiliates*: These are guarantees made to affiliated companies that may have a material effect on the company's liabilities.
- *Contingent liabilities*: These are liabilities that are too uncertain for an entry in the company's balance sheet.

4. CREDIT RISK

The first (April 1991) draft of the risk-based capital formula, as well as the final version adopted in December 1993 (with the exceptions noted below), contained a 10% charge for reinsurance recoverables. No statistical rationale for this factor was put forth, and many reinsurers and trade organizations argue that the charge is excessive.

Rationale for the Reinsurance Charge

The apparently high charge on reinsurance recoverables was motivated by three considerations:

- Reinsurance collectibility problems contributed to several major insurance company insolvencies in the mid-1980s.[16]

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[15] This category actually encompasses two types of assets:

- Assets that do appear on the balance sheet but over which the company does not have exclusive control.
- Assets that the company has sold subject to a put option that is still in force. In other words, the purchaser has the right to sell the assets back to the company for the exercise price stated in the put option.

[16] The most commonly cited example of this was the Mission Insurance Company insolvency of the mid-1980s, which was the largest property/casualty insolvency until 1990, and which was a focus of Representative Dingell's Congressional scrutiny of state insurance department financial regulation. Interestingly, large recoveries in 1991, 1992, and 1994 have vastly reduced the cost of the Mission insolvency to only $111 million, removing it from the "top ten" highest cost insolvencies. (See Kenney [35].)

- Some financially troubled companies have allegedly used "sham" reinsurance transactions with affiliated companies to hide their financial problems.
- Many reinsurance contracts do not contain full risk transfer. For instance, a reinsurance treaty may specify that if losses are higher than expected, then the ceding company must remit additional premium to the reinsurer. Since there is no consideration of this additional reinsurance premium in the risk-based capital formula, the charge for reinsurance recoverables is set at a high ("conservative") level.[17]

Criticism of the Reinsurance Charge

In response, several criticisms were leveled against the charge for reinsurance recoverables in the risk-based capital formula.

1. *Incentives*: The high charge for reinsurance recoverables serves as a disincentive to reinsure primary insurance business. In practice, reinsurance is one of the primary tools for reducing risk, by transferring either layers of loss or proportional parts of the exposure to reinsurers. The high charge for reinsurance recoverables in the risk-based capital formula may exacerbate insolvency problems rather than reduce them.

   The NAIC Working Group has responded to this criticism by noting that the credit risk charge, even at 10%, is lower than most reserving risk charges. Insurers would still lower their capital requirements by reinsuring their business, even if not to the extent that they would like.[18]

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[17]For instance, suppose the reinsurance treaty provides no risk transfer at all. That is, the primary company reimburses the reinsurer for all losses if experience is poor and it receives all profits if experience is good. Then the appropriate solvency charge for reinsurance collectibles should be the same as the reserving risk charge, since the cession of the reserves to the reinsurer does not affect the primary company's obligations.

[18]See Laurenzano [37, p. 108]: "More importantly the current (risk-based capital) charge (for reinsurance recoverables) is less than the underwriting capital charge contained in the formula, thus leaving intact the incentive to reinsure and spread risk while discouraging wholesale reinsurance or excessive gross leverage."

2. *Quality of Reinsurer*: The risk-based capital charge does not differentiate by type of reinsurer. Reinsurance placed with well-capitalized domestic reinsurers is presumably less "risky" than reinsurance placed with small, unauthorized off-shore reinsurers.

   The AAA RBC Task Force recommended to the NAIC Working Group that the credit charge for reinsurance recoverables be graded by quality of the reinsurer. The NAIC Working Group felt that this would result in the NAIC becoming a rating agency for reinsurers, which would be inappropriate, so no change was made in the risk-based capital formula.

3. *Collateralization*: The risk-based capital formula does not differentiate between reinsurance recoverables that are secured (or "collateralized"), such as by letters of credit or by funds deposited with the ceding company, and reinsurance recoverables that are not secured. Collateralized reinsurance presents a lower credit risk than uncollateralized reinsurance, particularly when the ceding company controls both the collateral and the loss reserve evaluations.[19]

   Similar issues arise in the banking industry, where the risk-based capital requirements consider the collateralization of loans. Some actuaries have argued that since the property/casualty risk-based capital formula does not deal with this issue, an incentive for prudent collateralization is missing. Others have argued that collateral is generally sought only from unauthorized reinsurers (to avoid the "Schedule F penalty"). Reducing the risk-based capital charge for collateralized reinsurance recoverables might result in lower capital requirements for unauthorized reinsurance than for authorized reinsurance.

[19]This type of situation is particularly likely to arise when a captive reinsurer is wholly owned by a single parent.

A recurring question is how any capital requirement affects the costs of the insurance business. Consider the issue of risk-based capital requirements for collateralized reinsurance recoverables. On the one hand, requiring capital even for collateralized reinsurance recoverables may be unnecessary, thereby increasing the costs for an insurer seeking a specified return on its equity. On the other hand, differentiating between collateralized and uncollateralized reinsurance recoverables provides incentives to seek collateral for all recoverables, which also increases costs.

The RAA Study

In 1992, the Reinsurance Association of America (RAA) prepared a study on the insolvency risks facing reinsurers. This RAA report showed that failing reinsurers formed about 4% of the reinsurance industry by premium volume, implying that the appropriate risk-based capital charge for reinsurance recoverables should be about 4%.

The RAA noted two caveats on this implication:

- Although failing reinsurers had only 4% of the reinsurance premium volume, they had a far larger proportion of the reinsurance losses. This makes sense, since the financial reflection of insurance failure is a high ratio of losses to assets (or to equity, or to premium). Thus, the amount of uncollectible reinsurance would be greater than 4% of reinsurance recoverables, implying that a higher capital charge is needed.
- Not all reinsurance recoverables are uncollectible when a reinsurer fails. In many instances, a large proportion of the reinsurance obligations are indeed paid to ceding companies, particularly when the failed reinsurer is taken over by another company. Thus, the amount of uncollectible reinsurance would be less than 4% of reinsurance recoverables, implying that a lower capital charge is needed.

Some participants in the RAA study argued that these two effects are offsetting, though sufficient data to properly quantify them were lacking. In addition, the NAIC argued that the RAA study included data only for reinsurance companies, but it did not include coverage provided by the reinsurance departments of some large primary companies that failed, such as the Mission Insurance Company.

The RAA study, though, addresses expected uncollectibility amounts, not capital requirements. Consider the 15% risk-based capital charge on unaffiliated common stocks. The risk-based capital formula is not saying that the *expected* value of common stocks is 15% less than the reported value. In fact, the expected value equals the market value, which equals the reported value. Rather, the formula says: "Given the volatility of common stock values, companies should hold 15% of the reported value as capital to guard against surplus impairment resulting from stock market declines."

For common stocks and corporate bonds, there is sufficient financial data to allow rigorous "probability of ruin" and "expected policyholder deficit" analyses. For reinsurance recoverables, the historical data are poor, the marketplace changes continually, and there are so many other factors affecting expected collectibility (e.g., "quality of reinsurer") that statistical analyses of capital requirements are difficult. The NAIC Working Group therefore retained the 10% credit risk charge, despite its "judgmental" nature.

The Provision for Reinsurance

The statutory "provision for reinsurance" (that is, the "Schedule F penalty") is deducted from the reinsurance recoverables subject to a risk-based capital charge. To do otherwise would "double-count" the liability or the capital requirement.

Suppose the insurance company has a $100,000 recoverable from an unauthorized reinsurer, none of which is secured by funds withheld or letters of credit. The statement values on the balance sheet show the $100,000 as a contra-liability or as an asset, depending on whether the loss payment to the claimant has already been made. The "provision for reinsurance," however, sets a statutory liability of $100,000, thereby reducing policyholders' surplus by that amount.

Thus, in statutory accounting, there is no net receivable of $100,000, so there is no need for surplus to ensure that this receivable is collectible.

The same procedure is used for receivables from authorized insurers. In this case, the provision for reinsurance relates to overdue receivables and to receivables from slow-paying reinsurers. The provision for reinsurance is deducted from the reinsurance recoverable to determine the amount of the recoverable subject to the risk-based capital charge.[20]

Involuntary Market Pools

Several changes to the credit risk charge for reinsurance recoverables were made to the risk-based capital formula between the first (April 1991) and second (June 1993) drafts. The first draft of the risk-based capital formula imposed the 10% credit risk charge on reinsurance recoverable by servicing carriers from involuntary market pools.[21] Some insurers objected to this, noting that this credit risk charge would serve as a disincentive for companies to service the involuntary markets. The states already had enough consumer dissatisfaction with insurance availability, and exacerbating these problems by hampering the involuntary market mechanisms was not in the public interest.

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[20]For a complete explanation of the components of the provision for reinsurance and of their calculation, see Feldblum [21].

[21]When an employer is unable to obtain workers compensation insurance in the voluntary market, a policy is provided from the involuntary market pool. The servicing carrier collects the premium from the employer and remits it (minus an "expense allowance") to the pool. If the employer reports a claim, the servicing carrier pays the compensation benefits and bills them to the pool. For expected future payments on injuries that have already occurred, the servicing carrier sets up a "direct reserve" but cedes it to the pool. The original risk-based capital formula had a 10% credit risk charge on these recoverables from the pools.

Moreover, the credit risk charge guards against the risk of the primary company being unable to collect the reinsurance recoverables. But the involuntary pools impose joint and several liability on all member companies, and no state pool has defaulted on its obligations to servicing carriers.

In response to these criticisms, the NAIC Working Group eliminated the credit risk charge on reinsurance recoverable from involuntary pools, as well as from "public interest" voluntary pools. Public interest voluntary pools, such as the nuclear insurance pools, are pools designed to increase insurance availability for hard to service risks.[22]

Intercompany Pooling Agreements

The first (April 1991) draft of the risk-based capital formula imposed the 10% charge for reinsurance recoverables even on recoverables among affiliated insurers participating in intercompany pooling agreements. Some insurer groups objected, noting that:

- State rate regulations force them to use different legal entities to provide insurance to different classes of risks, such as "preferred" risks, "standard" risks, and "sub-standard" risks.

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[22]The term "public interest voluntary pool" is not used by the NAIC Working Group, since the term is too vague for objective measurement. Instead, these "voluntary market mechanism pools and associations" are defined by the NAIC Working Group (*NAIC Instructions*, p. 12) as "those which meet either of the two following sets of criteria:
Criteria #1

a. the members/reinsurers of the pool share pro rata in the experience of the pool; and
b. there are sufficient participants to provide a reasonably broad sharing of the risk, which shall be evidenced by a maximum 15% retention by any one participant.

Criteria #2

a. the purpose of the pool or association is to depopulate a residual market;
b. the pool or association must be specifically approved by the Commissioner of the domestic state;
c. liability of the reinsurers in the pool or association is joint and several;
d. at least five insurers participate in the pool; and
e. the premium volume of the pool or association exceeds $25 million."

- The intercompany pooling agreement reduces the risk to each individual legal entity. Furthermore, the risk to the *consolidated* enterprise is not increased by the intercompany pooling agreement, so why should its risk-based capital requirements increase?

The risk-based capital formula should encourage the use of such pooling agreements, not discourage their use. The NAIC Working Group agreed, and it eliminated the risk charge for reinsurance recoverable from affiliated U.S.-domiciled insurers.

Miscellaneous Receivables

There are two types of credit risks associated with various other receivables, such as "Receivables from Parents, Subsidiaries, and Affiliates" (Line 16 of Page 2 of the Annual Statement).

- The party owing the money may become insolvent and be unable to pay (or for other reasons may refuse to pay).
- The insurance company may have incorrectly estimated the receivable.

The risk-based capital charge for receivables was judgmentally chosen as 5% (see the *NAIC Instructions*, p. 12).

A lower charge is used for "Interest, Dividends, and Real Estate Income Due and Accrued." The risk here is primarily a default risk on the underlying securities. The charge chosen by the NAIC Working Group is the bond default charge for Class 2 bonds, or 1%.

5. UNDERWRITING RISKS

The charges for underwriting risks are the dominant portions of the risk-based capital formula. These charges have little similarity to the "C-2" charges in the life insurance formula. Most of the underwriting risk charges were developed by the staff of the New York Insurance Department or by the AAA RBC Task Force. Much controversy continues, both within the NAIC research department and among outside analysts, as to whether these charges accurately quantify the risks faced by insurance enterprises. Casualty actuaries who wish to influence solvency monitoring issues must understand the rationale for the current charges, their strengths and weaknesses, and the alternatives that have been proposed.[23]

Reserving Risk

The reserving risk charge in the risk-based capital formula measures the susceptibility of loss reserves to adverse developments. The charge is quantified separately by line of business, using Schedule P data for the past ten years.

The reserving risk charge does *not* attempt to measure the adequacy of reported reserves. Measurement of a company's loss reserve adequacy is handled by state financial examinations and by analysis of Schedule P, not by the risk-based capital formula.[24]

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[23]The internal NAIC assessment of the effectiveness of the underwriting risk components may be found in Barth [6]. An example of an outside assessment is Cummins, Harrington, and Klein [16].

[24]The June 1993 "Statement of the Property/Casualty Risk-Based Capital Working Group," p. 3, states: "The formula will assist regulators, but it is not, and was never intended to be, a panacea of solvency regulation. The risk-based capital requirements will be based upon data contained in the insurer's financial statement. A formula cannot assess the correctness of this data … "

This perspective is surprising to some observers, since the (unintended) effect may be to increase insolvency risks, not to decrease them. Cummins, Harrington, and Niehaus [14, pp. 435–436] state this succinctly:

> In addition, risk-based capital requirements by themselves will do little or nothing to help regulators determine whether an insurer's reported net worth is overstated. The great difficulty in determining whether an insurer's reported losses and loss reserves are significantly understated, especially for long-tailed lines with highly volatile costs, limits the ability of risk-based capital to encourage weak insurers to hold more capital and to assist regulators. In fact, poorly designed risk-based capital requirements could increase incentives for some insurers to under-report loss reserves in order to show lower required risk-based capital, higher capital relative to required risk-based capital, or both.

See the discussion below in this paper on the "incentives" produced by the reserving risk charges.

For most companies, the reserving risk charge is the dominant part of the risk-based capital requirements. Because of the importance of this charge, numerous criticisms have been leveled against the quantification method, and alternatives have been proposed. The following sections set forth its development and its rationale.

Industry Adverse Development

The reserving risk charge begins with the calculation of adverse loss development ratios by Schedule P line of business. This calculation was done by the NAIC staff in 1993, and the resulting charges were "frozen." Individual ratios may be updated by the NAIC as the need arises; this component of the reserving risk charge is *not* recalculated each year.

We begin with adverse loss development ratios by company and by Schedule P line of business.

- The numerator of this ratio is the increase in estimated ultimate incurred losses between two statement dates. This increase is determined from the historical data in Schedule P, Part 2.[25]
- The denominator of the ratio is the held loss reserves at the earlier statement date. The held loss reserves are determined by subtracting paid losses (Schedule P, Part 3) from incurred losses (Schedule P, Part 2).[26]

For example, suppose that at December 31, 1985, a company reports $10 million of "other liability" incurred losses and $4

[25] In actuarial parlance, "incurred loss development" is generally used to mean the change in reported losses between two valuation dates, where reported losses are paid losses plus case basis loss reserves. The adverse loss development used here refers to the change in *estimated ultimate losses* as shown in Schedule P, where the estimated ultimate losses include bulk reserves.

[26] In most actuarial analysis, the denominator of an incurred loss development ratio would be the incurred losses at the earlier reserve date, not the held reserves at the earlier reserve date. The risk-based capital reserving risk charge, however, is applied to held reserves, not to incurred losses, so held reserves are used as the denominator of the adverse loss development ratio.

million of "other liability" paid losses for accident years 1985 and prior. At December 31, 1991, the company reports $12 million of "other liability" incurred losses for accident years 1985 and prior. To ensure consistency, all these figures would be drawn from the 1991 Annual Statement.[27]

The risk-based capital formula would consider the following figures:

- The loss reserves at December 31, 1985, were $6 million ($10 million – $4 million).
- The adverse development is $2 million ($12 million – $10 million).
- The adverse development as a percentage of reserves is 33.3% ($2 million ≡ $6 million).

These calculations are performed separately by

a. Individual company (not company group),

b. Schedule P line of business, and

c. Statement date (e.g., changes in incurred losses for accident years 1982 and prior between statement dates 1982 and 1991, changes in incurred losses for accident years 1983 and prior between statement dates 1983 and 1991, and so forth).

Individual company ratios are averaged to determine the base industry reserve charges which are promulgated by the NAIC. In-

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[27]The 1991 Statement actually shows incurred losses only for accident years 1982 through 1991. The "prior years" line does not include all incurred losses. Rather, for accident years prior to 1982, the "prior years" row shows the loss reserve at each statement date plus the paid losses in calendar years 1982 and subsequent. However, since the risk-based capital calculation needs loss reserves, not incurred losses, this is not a problem. Although Schedule P data are not sufficient to determine incurred losses for the "prior years," they are sufficient to determine loss reserves.

The 1991 Schedule Ps were used for determining the ratios used in the current risk-based capital formula. If the NAIC decides to update any of these ratios, subsequent Schedule Ps would be used.

dividual companies need not perform these calculations. A high or low adverse development ratio for a specific company affects the industry charge in this part of the formula. This additional effect on the particular company's reserving risk charge is discussed below.

"Worst Case Year"

For each line of business, the individual company development ratios are averaged over all companies for each statement date. In other words, the derivation of the reserving risk charge begins with a three-dimensional matrix of adverse loss development ratios, with several thousand companies, nine statement dates, and fifteen Schedule P lines of business. The averaging across companies leaves a two-dimensional matrix, with nine statement dates and fifteen lines of business.

Simple averages are used, not weighted averages, so the adverse loss development for an insurer with $100,000 of reserves is given the same weight as that for an insurer with $10 billion of reserves.

Some actuaries have argued that

- because small insurers have greater random fluctuation in their adverse development ratios,[28] and
- because simple averages (not weighted averages) are used in the formula,

the "average industry-wide ratios" used in the formula are greater than the "industry aggregate" ratios (as might be determined from the industry data in Best's *Aggregates and Averages*) for

[28]See Lowe [39] and the studies by Allan Kaufman supporting additional charges on small and on rapidly growing companies (published in the *Proceedings of the NAIC*). Barth [5, p. 23], in reviewing the risk-based capital results submitted with the 1994 Annual Statements, similarly notes that: "The R_4 RBC (reserving risk) charge for companies with large reserves may be higher than necessary, relative to smaller companies."

most lines of business. Thus, the NAIC formula has an unjustified upward bias.

In rebuttal, the NAIC Working Group has argued that

- the simple averages are *not* uniformly higher than the weighted averages (the relationship varies by line of business), so there is not necessarily an "upward bias;" and
- using weighted averages would give undue influence to the results of the largest carriers.

The simple averages were therefore retained in the risk-based capital formula.[29]

The greatest average value is selected from among all the statement dates. For instance, suppose the average values of adverse loss development from the statement date below to December 31, 1991, over all companies in the industry, for a given Schedule P line of business are as follows:

Statement date: December 31,	'82	'83	'84	'85	'86	'87	'88	'89	'90
Avg. adverse development:	25%	22%	28%	32%	22%	16%	15%	8%	5%

The most severe adverse loss development as a percentage of original reserves (32%) occurred between December 31, 1985, and December 31, 1991. The value of 32% would be the "industry-wide adverse development" for this line of business.[30] The risk-based capital standards imply: "This adverse development happened in the past, so it might happen again. Insurers

[29]In addition, the NAIC Working Group notes that A. M. Best uses a slightly different population of companies than the NAIC Working Group uses, so the average figures that each derives may not match exactly.

[30]The *NAIC Instructions* use the term "Industry loss and loss adjustment expense risk-based capital percentage"; see Page 20, Line 4.

need sufficient capital to withstand adverse loss reserve development of this magnitude."

Interest Discount Factor

Statutory accounting requires that loss reserves be reported at undiscounted values. The "implicit interest margin," or the difference between the discounted value of the reserves and the undiscounted value of the reserves, serves as an implicit "cushion" for solvency.[31] Not taking this implicit "cushion" into account would double-count the required capital: an explicit capital requirement held as surplus and an implicit capital cushion held as reserves.

The implicit interest margin differs by line of business, depending on the loss payout pattern of the reserves. To quantify the loss payout pattern for most lines of business, the risk-based capital formula uses the same method as employed by the IRS for its loss reserve discounting procedure. The payout pattern is determined by comparing paid losses to incurred losses by accident year and line of business, using Best's *Aggregate and Averages* Schedule P Part 1 data.[32]

The IRS and the risk-based capital formula use different discount rates. For determining taxable income of property/casualty insurance companies, the IRS uses a sixty-month moving average of the Federal Midterm Rate, which is the rate on outstanding Treasury securities with remaining terms between 3 and 9 years. The risk-based capital formula uses a flat 5% discount rate.

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[31] The risk-based capital formula uses the term "adjustment for investment income"; see the *NAIC Instructions*, p. 17. Some actuaries consider this phrase inappropriate, since the risk-based capital formula is measuring the assumed present value of the reserves, not the actual investment income earned by the insurer. Nevertheless, actuaries should be aware of the terms used by the NAIC in the risk-based capital formula.

[32] For a description of the IRS procedure, see Gleeson and Lenrow [27] or Almagro and Ghezzi [1].

The "Net" Industry Charge

The reserving risk charges for private passenger automobile liability insurance in the risk-based capital formula should clarify these factors. The factor reflecting "worst case year" industry adverse development, before any adjustment for the implicit interest margin, is 25.4%.[33] The implicit interest adjustment, using the IRS discounting procedure with a flat 5% annual rate, implies that discounted auto liability reserves are only 92.1% of the undiscounted values. The reserving risk charge in the risk-based capital formula, assuming that there is no company adjustment (see below), is therefore

$$1.254 * 0.921 = 1.155$$

or 15.5% of reserves held.[34]

Company Differences

The 1.254 factor in the illustration above is the base "industry aggregate" figure used in the risk-based capital calculations. But companies differ both in their reserve estimation procedures and in the types of risks that they write. Some companies consistently report adequate full value reserves, and they show little adverse development in subsequent years. Other companies report less adequate reserves, as a result of either conscious management decisions or poor actuarial work, and they show significant adverse loss development in subsequent years.

The NAIC risk-based capital formula therefore compares the company's own average loss development by line of business over the past nine years to that of the industry. The average loss development is derived from Schedule P, Part 2. It is computed

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[33]This factor reflects *both* the worst case industry adverse development and the spreading of the reserving risk factors across lines of business (see below in the text).

[34]In the actual calculations, discussed below, the company adjustment is applied before the "adjustment for investment income." See the illustrations at the end of the paper for the exact sequence in which all factors are applied.

as

$$(\text{current incurred losses} - \text{initial incurred losses}) \equiv \text{initial incurred losses}$$

where

current incurred losses = the sum of incurred losses at the current statement date for the nine accident years prior to the current year, and

initial incurred losses = the sum of incurred losses at the initial statement dates for the nine accident years prior to the current year.

For instance, suppose the company shows the following figures in its 1997 Annual Statement for Schedule P, Part 2B ("private passenger auto liability/medical"):

1997 Schedule P, Part 2B ($000)

	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997
1988	500	500	490	510	515	525	530	530	530	530
1989		540	520	510	520	525	530	535	540	540
1990			580	585	600	605	605	610	605	610
1991				620	630	630	650	690	680	680
1992					660	670	700	705	705	710
1993						700	700	716	725	720
1994							750	745	745	740
1995								800	810	840
1996									850	870
1997										900

The "current incurred losses" are

$$\$(870 + 840 + 740 + 720 + 710 + 680 + 610 + 540 + 530) = \$6,240.$$

The "initial incurred losses" are

$$\$(850 + 800 + 750 + 700 + 660 + 620 + 580 + 540 + 500)$$
$$= \$6{,}000.$$

The company's average adverse development is therefore $(6{,}240 - 6{,}000) \equiv 6{,}000 = 4\%$, or a development factor of 1.040. Suppose that the corresponding industry average adverse development is 6.5%, or a development factor of 1.065. (See the *NAIC Instructions*, p. 20, line 1 for the actual industry average adverse development factors, which change from year to year.) The ratio of company to industry average adverse loss development is $1.040 \equiv 1.065$, or 0.977.

This factor is applied to the industry "worst case year" adverse loss development to give a company-specific worst case year adverse development factor of 0.248 $(0.254 * 0.977)$. A simple average is taken of the company-specific factor and the industry-wide factor to give the "company risk-based capital percentage" (*NAIC Instructions*, p. 17). This averaging may be conceived of as a credibility weighting of company adverse loss development and industry adverse loss development, with 50% credibility assigned to each component. In this illustration, the "company risk-based capital percentage" equals $(0.254 + 0.248) \equiv 2$, or 0.251.

This figure, plus unity, is multiplied by the "implicit interest margin" of 0.921 to give a final charge of $1.251 * 0.921 = 1.152$, or 15.2% of carried reserves.

Note carefully where industry data are used and where company-specific data are used. The "worst case" year is an industry concept; there is no company-specific worst case year in the risk-based capital formula. The company adjustment, which uses company-specific data, compares the particular company's average historical adverse development to the industry average historical adverse development. However, it does *not* substitute the individual company's worse case year for the industry's worst case year.

However, the industry worst case year is *not* developed from aggregate industry data. Rather, individual company data are used to determine the adverse reserve developments from each statement date to December 31, 1992 (the date used for the initial determination of the reserving risk charges). Simple (unweighted) averages of the individual company adverse developments are used to determine the industry adverse developments, after exclusion of "outlying" results.

Company Adverse Development

Note the two differences between the adverse loss development in the company adjustment and the loss development used for the industry charge of 1.254 in the example above:

- Adverse loss development in the calculation for the company adjustment is compared with initial *incurred losses*. The adverse development used to determine the industry "worst case year" charge is compared with initial *reserves*.
- For the company adjustment, the weighted average of the nine historical loss development factors is used, not the "worse case year." (Nine factors are used, since Part 2 of Schedule P shows ten statement dates. The tenth statement date is the current statement date, so there are nine periods for potential adverse development.) The weights for the average are the incurred losses at the initial statement dates.[35]

The *NAIC Instructions* show the weighted average historical loss development factors for the industry by line of business. Each insurer calculates its own weighted average historical loss development factors from the Part 2 exhibits of its Sched-

[35] This "weighted average" is equivalent to the formula in the text, which compares the total adverse development to the sum of the incurred losses at the initial statement dates, where the "initial statement date" differs for each accident year.

ule P (termed "company development" in the *NAIC Instructions*).[36]

Spreading Across Lines

The description above explains the quantification of the reserving risk charge for a single line of business. The resulting charges vary widely: for some lines of business they are high, and for others they are low. Part of the variation truly relates to the riskiness of the line of business, but part of the variation may be caused by random fluctuations in the historical data.

The risk-based capital formula therefore retains part of the reserving risk charge in the specific line of business and judgmentally spreads the rest over all lines. The basis of the allocation is the relationship of aggregate industry reserves by line of business to the sum of aggregate industry reserves for all lines of business.[37]

Written Premium Risk

The reserving risk charge guards against the risk that the company's past business will turn out to be less profitable than expected—i.e., that reserves will develop adversely. Equally important is the risk that the company's future business will be

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[36]The insurer must have experience for all ten accident years (for the ten year lines) in that line of business to use its own experience. Moreover, the insurer must not have initial negative incurred losses for any accident year, even if the current valuation of incurred losses is positive. See the *NAIC Instructions*, p. 16, for additional detail.

There is a slight mismatch in the dates of the adverse loss development. The company's average adverse loss development is based on the most recent Annual Statement. The industry average adverse loss development promulgated by the NAIC is based on data at least one year older. In general, this mismatch is not significant.

[37]Although there is a precise mathematical derivation of the individual line of business reserving risk factors, a strong dose of judgment was used in selecting the final risk factors for each line (that is, in "spreading" the total reserving risk capital requirement across lines of business). The NAIC Working Group has not disclosed the considerations affecting the spreading technique, other than to note the "basis of allocation." In general, it is not possible to exactly replicate the derivation of the final reserving risk factors, particularly for lines of business such as workers compensation, where significant adjustments were made to the standard procedure.

unprofitable, and that the company will have to cover underwriting losses with surplus funds.

One can develop capital charges to guard against potential underwriting losses over various time horizons, such as during the coming 12 months or during the coming five years. The risk-based capital formula uses a time horizon of one year: the potential underwriting losses to be considered are those that may occur during the next 12 months.[38]

Ideally, one would base the capital charge for future underwriting losses on the volume of business to be written during the coming year. As a proxy for the volume of business to be written during the coming year, the risk-based capital formula uses the volume of business written during the most recent calendar year. This future underwriting risk is termed "written premium risk."

The structure of the written premium risk charge is similar to that of the reserving risk charge. Average industry loss and loss adjustment expense ratios by accident year and by line of business are determined from Schedule P, Part 1, for the past ten years, by simple (unweighted) averages of individual company figures. The "worst case year," or the year with the highest average loss ratio, is selected.[39]

Interest Discount Factor

The Schedule P loss ratios are "ultimate" figures (also termed "nominal" figures, or "undiscounted" figures). Particularly for the long-tailed lines of business, the expected investment income resulting from the time lag between premium collection and loss payment is an important consideration in the profitability of a

[38]Contrast the 24-month time horizon used by the British Solvency Working Party, and the justification given for this by Daykin, Pentikäinen, and Pesonen [17], Chapter 14, Section 6.

[39]All references to "loss ratio" in this section refer to loss and loss adjustment expense ratios for net business, as shown in Schedule P, Part 1, Column 30.

book of business. The "worst case year" loss ratio is therefore multiplied by an investment income factor, which is derived from an IRS payment pattern and a 5% discount rate.

The "adjustment for investment income" used for the premium risk charge is not the same as the "adjustment for investment income" used for the reserving risk charge. The former reflects the expected investment income from policy inception to final loss payment for a newly issued block of business. The latter reflects the expected investment income on assets supporting the loss reserves currently held by the company for all accident years combined.

The relative magnitude of these two sets of figures depends on premium collection patterns and loss settlement patterns by line of business. The risk-based capital formula has greater premium risk "investment income adjustments" for workers compensation, medical malpractice, other liability, and products liability, but greater reserving risk "investment income adjustments" for homeowners, special liability, international, and reinsurance A and C.[40]

Company Experience

Just as is true for the reserving risk charge, the premium risk charge is adjusted for the company's own experience compared to that of the industry. Assume that for personal automobile insurance, the worst case year industry average loss ratio is 104.6%, and the average of all ten years' industry average loss ratios is 94.7%. Suppose also that a given company has a worst case year loss ratio of 110% and a ten year average loss ratio of 85%.

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[40] A "greater" adjustment means a smaller factor. For instance, the medical malpractice observed worst case adverse development used for the reserving risk charge is multiplied by an investment income adjustment factor of 80.8%, whereas the medical malpractice observed worst case loss ratio used for the premium risk charge is multiplied by an investment income adjustment factor of 77.8%. For a full discussion of this issue, see Woll [46].

The individual *company's* worst case year loss ratio is *not* used in the calculation; only the worst case year industry average loss ratio is used. However, the industry worst case year figure is adjusted for the individual company's average loss ratio compared with that of the industry, with equal weight given to industry and company experience. In this illustration, the ratio of company to industry average loss ratios is 0.898 ($0.850 \equiv 0.947$). To give equal weight to industry and company experience, the industry worst case year loss ratio is multiplied by a factor of

$$(1 + 0.898) \equiv 2, \quad \text{or} \quad 0.949,$$

giving an adjusted worst case year loss ratio of 99.3% (104.6% $*$0.949).[41]

For private passenger automobile liability, the "adjustment for investment income" factor is 0.924. The discounted worst case year loss ratio for this company's risk-based capital calculations is therefore

$$99.3\% * 0.924 = 91.8\%.$$

Combined Ratios

The company's (not the industry's) average expense ratio is added to this loss ratio to give a worst case year combined ratio. For instance, suppose that

[41] Unless the company has relatively complete experience, the adjustment outlined in the text is not made. Specifically, if for a given line of business the earned premium in any accident year is zero or negative, or the loss ratio in any accident year is zero or negative, no company adjustment is used. Furthermore, the risk-based capital formula uses what it terms a *de minimus* test, which is intended to avoid outlying values resulting from years with low premium volume. The *de minimus* test specifies that accident years with premium volume less than 20% of the average premium volume for all ten years should be excluded when calculating the company average loss ratio. Furthermore, if three or more years have premium volumes less than 20% of the ten year average, then no company adjustment is used in the premium risk charge. (For lines of business using only 5 accident years of data, if two or more years have a premium volume less than 20% of the five year average, no company adjustment is used.) Finally, all company loss ratios are capped at 300%, to avoid excessive charges resulting from random large losses in a small volume line of business. See the *NAIC Instructions*, p. 18, for the complete specifications.

- the industry's worst case year loss ratio, after adjustments for the individual company's experience and for the interest discount (expected investment income), for a particular line of business, is 94%, and
- the company's average expense ratio (for all lines combined) is 23%,

then the combined ratio used in the formula is 117% (94% + 23%). The written premium risk charge is calculated as the worst case year combined ratio minus unity. If the company wrote $50 million of business in this line in the most recent calendar year, then the capital requirement (before the adjustment for the premium concentration factor; see below) is $50 million $*$ 17% = $8.5 million.

The minimum written premium risk charge is 0%. For instance, if the industry worst case year discounted loss ratio is 84%, and the company's expense ratio is 13%, leading to a combined ratio of 97%, the written premium charge is 0%, not −3%.

In theory, one should add the expense ratio associated with the specific line of business to that line's loss ratio. However, expense ratios by line of business are not included in the Annual Statement, so an all lines combined expense ratio is used instead.[42]

The practical effect of using all-lines expense ratios is small: a lower expense ratio in one line of business is offset by a higher expense ratio in another line of business. The total risk-based capital requirement would differ only inasmuch as the premium risk charges for some lines are capped below at 0%. If the use of line-specific expense ratios would cause no line's written pre-

[42]Expense ratios by line of business are indeed found in the Insurance Expense Exhibit (IEE). However, the IEE is filed each year by April 1, one month after the Annual Statement and the risk-based capital calculations are due.

mium charge to fall below zero (and therefore be capped at zero), but the use of the all-lines expense ratio causes the written premium charge for one or more lines to be capped at zero, then the latter procedure will cause a higher all lines combined written premium charge than the former procedure does.

Note two differences between the loss portion and the expense portion of the combined ratio:

- The company average loss ratio and the industry average loss ratio are each given 50% weight in determining the loss portion of the combined ratio. For the expense portion, only company data are used, not industry data.
- The adjustment for investment income is applied only to the loss portion of the combined ratio, not to the expense portion. The adjustment for investment income is derived from a loss payout pattern, not a loss plus expense payout pattern. Although there is often a long lag between premium collection and loss settlement, most expenses are paid at about the same time as the premium is collected.

The offset for business written on loss-sensitive policy forms, the adjustment for business written on claims-made forms, the additional premium risk charge for rapidly growing companies, and the concentration adjustment to reflect diversification by line of business, are discussed below for both reserving risk and written premium risk.

Unearned Premium Reserves

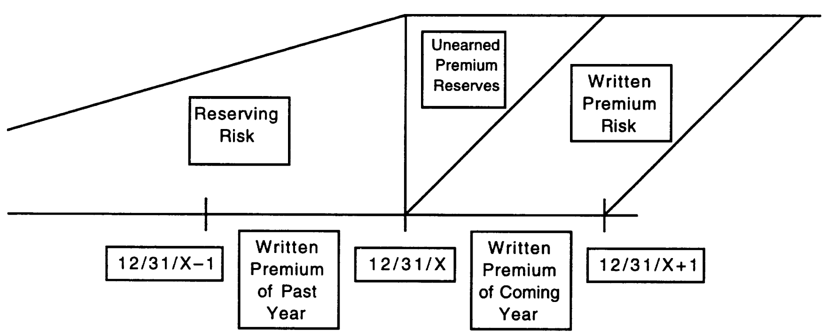
The previous sections have dealt with reserving risk and with written premium risk. Reserving risk is the risk that underwriting results might turn out to be worse than expected on insurance coverage that has already been earned but for which claims payments are not yet fully settled. Written premium risk is the risk that underwriting results may turn out to be worse than expected on the coming year's underwriting activities.

There is a risk intermediate between these two: the risk that underwriting results may turn out to be worse than expected on coverage that has already been written but has not yet been earned. Just as the insurer holds loss reserves for coverage that has already been earned but for which claims are not yet fully settled, the insurer holds unearned premium reserves for coverage that has been written but has not yet been earned. Just as the reserving risk charge protects against unanticipated adverse development on the loss reserves, should there not be a similar charge to protect against the possibility that the unearned premium reserves may be insufficient to fund the claims that will arise on this coverage?

Figure 1 shows this graphically. Note that the most recent year's written premium is a proxy for the coming year's written premium.

FIGURE 1

THE UNDERWRITING RISKS TIME LINE



Valuation Date: 12/31/X
Reserving Risk: Coverage written and earned, but not all claims settled yet
Unearned Premium Reserve: Coverage written but not yet earned
Written Premium Risk: Coverage to be written in coming year
(Written premium of the past year is a proxy for written premium of the coming year.)

Equity in the Unearned Premium Reserves

This is the underlying structure of the risk-based capital formula, and the first (April 1991) draft of the formula indeed had a charge applied to the unearned premium reserves. In fact, if insurance companies held "net" unearned premium reserves—that is, "net" of prepaid expenses—the factors used to compute the unearned premium reserves charge would be about the same size as the factors used to compute the written premium risk charge.

But statutory accounting requires insurance companies to hold unearned premium reserves *gross* of all prepaid expenses. Unlike GAAP, statutory accounting does not allow a deferred policy acquisition expense asset.

The objective of statutory accounting for unearned premium reserves is conservatism. For most companies, the gross unearned premium reserve is about 20% to 25% greater than the amount actually needed to fund future claims. This statutory margin is referred to as the "equity" in the unearned premium reserves.

For almost all lines of business, this margin is more than sufficient to guard against unexpectedly poor underwriting results on the unexpired portions of policies that have already been written. Just as the reserving risk charge and the written premium risk charge contain offsets for expected investment income, the unearned premium reserves risk charge in the first (April 1991) draft of the risk-based capital formula contained an offset for prepaid acquisition expenses. With this offset, the charge was either zero or insignificant for almost all lines of business.

To simplify the formula, the unearned premium reserves risk charge was deleted, since it did not contribute significantly to the final capital requirements. In the final draft of the formula, no relic of this charge remains, because statutory accounting already contains a more than sufficient margin for this risk.

Occurrence Policies versus Claims-Made Policies

The first (April 1991) draft of the risk-based capital formula had the same reserving risk charge and written premium risk charge for business written on claims-made forms as for business written on occurrence forms. Some insurers argued that the reserving risk and written premium risk are smaller for business written on claims-made forms, and that this should be reflected in the risk-based capital standards.[43]

Two issues are paramount here. Adverse loss development results

- from the emergence of incurred but not reported (IBNR) claims, and
- from development on reported claims ("inadequate" case reserves).

Business written on claims-made forms has the second type of development. In fact, since claims-made business often includes the more "risky" business, reserve estimates are uncertain, and development on reported claims may be great. However, claims-made business has little (if any) IBNR claim emergence, which is the primary cause of adverse loss development in general liability and medical malpractice. Thus, claims-made business should show less adverse loss development than occurrence business.

Since industry-wide Schedule P exhibits for occurrence business and claims-made business were not available separately until 1993, data from several large writers of claims-made general liability business were used to analyze adverse loss development. The data indicated that adverse loss development was indeed a lesser problem for claims-made business than for occurrence business, at least for medical malpractice. For other liability and

[43]Most of the work on this subject was done by Paul Braithwaite, a member of the AAA RBC Task Force. His reports on claims-made business and the underwriting risk charges can be found in the *Proceedings of the NAIC*.

for products liability, however, the data either did not show a significant difference or the available experience was too sparse to yield meaningful conclusions.

Quantification and Data

The NAIC Working Group responded favorably to the claims-made recommendation, but it asked two questions:

- First, how significant is the difference in adverse loss development between business written on occurrence forms and business written on claims-made forms?
- Second, if different reserving risk charges are incorporated in the risk-based capital standards for occurrence and claims-made business, how should the NAIC collect the data needed to quantify the offset?

The two questions are intertwined. Since loss triangles were not reported for occurrence and claims-made business separately in the Annual Statement prior to 1993, there were no industry-wide data for quantifying the appropriate factors. (Other reserving risk factors are determined from Schedule P data.)

Implementation and Adoption

To implement the recommendation of the AAA RBC Task Force, the NAIC Blanks Task Force split the Schedule P exhibits for three lines of business—other liability, products liability, and medical malpractice—into occurrence and claims-made sections. In conjunction with this, the 1992 Part 4 of Schedule P, which showed "claims-made experience," was eliminated, and the information regarding extended loss and expense reserves was moved to a Schedule P interrogatory. No claims-made experience is now shown for commercial multi-peril business, since claims-made business is not a large portion of this line.[44]

[44] Braithwaite estimates that claims-made business forms less than 0.5% of commercial multi-peril experience.

The final risk-based capital formula adopted by the NAIC in December 1993 reduces the reserving risk charge and the written premium risk charge for medical malpractice business written on claims-made forms compared to business written on occurrence forms by 20%.[45]

Offset for Loss-Sensitive Contracts

The reserving risk charge in the risk-based capital formula quantifies the amount of capital needed to guard against unexpected adverse loss development. This unexpected adverse loss development must be paid for with surplus funds, so insurers should hold sufficient capital to withstand this risk.

Similarly, the written premium risk charge quantifies the amount of capital needed to guard against unexpectedly poor underwriting results during the coming year. Once again, the underwriting losses must be paid for with surplus funds, so insurers should hold sufficient capital to withstand this risk.

Risks in Retrospective Plans

For business written on loss-sensitive contracts, such as retrospectively rated workers compensation policies, part of the adverse loss development on previously written business or the poor underwriting results on new business will be funded by additional ("retrospective") premiums. That is, if actual losses are worse than forecast, the insured is billed for additional premiums at the retrospective adjustment date. Depending on the plan parameters, the aggregate additional premium for a company

[45] As noted above in the text, the data provided by Paul Braithwaite showed the disparity between occurrence and claims-made forms to be greatest for medical malpractice business. The NAIC Working Group initially intended to consider whether a similar reduction in the charge is appropriate for other liability and products liability as well (based upon the new Schedule P data), and whether the 20% figure used for medical malpractice is indeed correct. Since the new Schedule P data do not justify a claims-made offset for other liability and products liability, it is not expected that the NAIC Working Group will extend the medical malpractice offset to these lines.

may be 70 percent to 80 percent of the adverse loss development on previously written business or of the worse than expected loss experience on new business. Thus, one needs less surplus to withstand the risks of adverse loss development or poor underwriting results on loss-sensitive contracts than on prospectively rated contracts.

Similarly, reinsurance treaties may use sliding scale commission rates. Suppose the primary insurer and the reinsurer anticipate a 70% loss ratio on the ceded business, and they would normally use a 30% reinsurance commission rate. The sliding scale formula may set the commission rate as

$$30\% + 0.5 * (70\% - \text{actual loss ratio}),$$

subject to a maximum of 50% and a minimum of 10%.

If the actual loss ratio on the ceded business is indeed 70%, the reinsurance commission is 30%. If the loss ratio is 90%, making the ceded business unprofitable, the reinsurance commission is only $30\% + 0.5\ (70\% - 90\%) = 20\%$. In this formula, the reinsurance commission is capped at 10% and 50%.

The initial work on this subject done by private insurance companies found a receptive audience on the NAIC Working Group. The working group asked the AAA RBC Task Force (at that time, its "actuarial advisory committee") to develop a full proposal.

Two issues were paramount in the deliberations of the NAIC Working Group: the magnitude of the offset and the definition of a loss-sensitive contract.

Magnitude of the Offset

The proper size of the offset depends on the sensitivity of retrospective (or other loss-sensitive) premiums to adverse loss development or to worse than expected underwriting results. In other words, if incurred losses increase by $1,000, how much additional premium will be collected?

The premium sensitivity depends on the parameters of the loss-sensitive contract, such as loss limits and maximum premium limits in retrospectively rated plans. For "wide swing" plans sold to "jumbo" accounts, with high loss limits and high maximum premium limits, premium sensitivity is high. Since losses and retrospective premiums are rarely capped, each dollar of additional loss, on average, leads to nearly a dollar of additional premium. For "narrow swing" plans sold to small accounts, with low loss limits and retrospective premiums severely constrained by the maximum premium limits, premium sensitivity is low.[46]

Several regulators were concerned with the "credit" risk on accrued retrospective premiums. In other words, if losses develop adversely, the insurer must pay the claims, but it may not be able to collect the additional premiums if the insured becomes bankrupt or is otherwise unwilling to pay.[47]

Definition of a Loss-Sensitive Contract

Several regulators were concerned that if risk-based capital charges are lower for business written on loss-sensitive contracts, many under-capitalized companies may attempt to portray their policies as loss-sensitive, even if the "loss-sensitive" part of the contract is not significant. In consultation with the AAA RBC Task Force, the NAIC Working Group drew up a definition of a

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[46]For a more complete treatment of the effects of loss-sensitive contracts on capital requirements, see Hodes, Feldblum, and Blumsohn [32]. For an analysis of the premium sensitivity for very small workers compensation retrospectively rated policies, see Bender [8].

In practice, numerous variables affect premium sensitivity. Hodes, Feldblum, and Blumsohn list "the plan parameters, the current loss ratio, and the maturity of the reserves." Bender [8, p. 36] lists "the retrospective rating formula, the aggregate loss ratio of the risks, and the distribution of the individual risks' loss ratios around the aggregate."

[47]On the credit risk for retrospectively rated policies, see Greene [28]. Industry-wide surveys of premium sensitivity were undertaken by the Tillinghast consulting firm, in a private study conducted by Stephen Lowe and Jon Michelson, and by the NAIC, in a study conducted by Robert Klein. Tillinghast's survey found that companies reported an average sensitivity of 65%. However, in consideration of the credit risk, and from a general desire to be "conservative," Tillinghast recommended an offset of only 45%, and this recommendation was adopted by the AAA RBC Task Force.

loss-sensitive contract to be used for risk-based capital purposes. The definition specifies six criteria that a contract must fulfill to be considered loss-sensitive:[48]

1. *An increase in losses can lead to an increase in net payment for that policy.* In other words, if the loss-sensitive item is not a monetary transaction, the contract is not loss-sensitive.
2. *The loss-sensitive payment must be at least 75% of the loss on primary business and at least 50% of the loss on reinsurance treaties, before the application of any limits.* In other words, if losses on a retrospectively rated workers compensation policy increase by $10,000, the retrospective premium must increase by at least $7,500, before the application of loss limits or maximum premium caps.
3. Maximum and minimum premiums, loss limits, and upper and lower bounds on the reinsurance commission may constrain an otherwise loss-sensitive contract. For a contract to be classified as loss-sensitive, *the "swing" of the plan must be at least 20% for primary business and 10% for reinsurance treaties.* In other words, the net amount payable when the loss experience is the worst possible must be at least 20% greater than the net amount payable when the loss experience is the best possible. For example, a retrospectively rated workers compensation policy with a minimum premium of $9,000 and a maximum premium of $10,000 would not qualify as loss-sensitive.
4. *The maximum net payment must be at least 15% greater than the expected net payment for primary business and at least 7.5% greater than the expected net payment for reinsurance treaties.* For example, a retrospectively rated workers compensation policy with a minimum premium

[48]These criteria are listed in the NAIC *Annual Statement Instructions* for Part 7 of Schedule P. For further discussion, see Feldblum [20].

of $5,000, an expected premium of $10,000, and a maximum premium of $11,000 would qualify as loss-sensitive under Criterion 3 but not under Criterion 4.

5. *The loss-sensitive payments must be either premiums or commissions.* In other words, a policy with loss-sensitive policyholder dividends does not qualify as loss-sensitive.

6. *The losses and the corresponding loss-sensitive payments must flow through the income statement and the balance sheet.* In other words, suppose the workers compensation policy has a large dollar deductible of $100,000. For losses below $100,000, the insurance company still settles the claim and pays the benefits, but the insured reimburses the insurer for these payments. One might characterize this policy as loss-sensitive, since the greater the losses paid by the insurer, the greater the payments made by the insured. However, these amounts do not flow through the income statement as incurred losses and as premiums, so the contract does not qualify as loss-sensitive.

The final version of the risk-based capital formula adopted by the NAIC in December 1993 contains an offset for business written on loss-sensitive contracts. The parameters, however, differ from those recommended by Tillinghast or by the AAA. For instance, the offset for primary insurance carriers is only 30% of the otherwise applicable risk-based capital reserving risk charge and written premium risk charge, not the 65% in the Tillinghast survey or the 45% recommended by the Tillinghast actuaries.[49] The offset for reinsurance companies is 15% of the otherwise applicable risk-based capital reserving risk charge and written

[49]The reasons for the choice of a low offset factor include the lack of credible industry-wide data, the inconsistencies in the definition of loss-sensitive contracts, and the desire to be conservative.

premium risk charge, corresponding to the lower sensitivity discussed in the Tillinghast report for reinsurance company loss-sensitive contracts.

The lower sensitivity for reinsurance contracts relates to the nature of these policies. Many loss-sensitive reinsurance treaties have sliding scale reinsurance commissions, where the commission amount depends on the loss ratio of the book of reinsured business. The sensitivity of the contract is therefore limited by the magnitude of the commission allowance. Moreover, the effects of a poor loss ratio on the reinsured business is sometimes spread over several years, so the immediate effect is further reduced.

Diversification by Line

Is adverse loss development in one line of business correlated with adverse loss development in other lines of business? Similarly, are poor underwriting results in one line of business correlated with poor underwriting results in other lines of business?

Consider adverse development. If the adverse loss development is caused by random loss fluctuations, one would expect little interdependence among lines of business. If adverse loss development is related to conscious company actions to smooth calendar year earnings, one would expect greater interdependence among lines of business.

The risk-based capital formula assumes a partial interdependence among lines of business. The total reserving risk charge determined by the procedure outlined above, after adjustment for loss-sensitive business and for business written on claims-made contracts, is multiplied by

$$70\% + 30\% * (\text{reserves in largest line of business} \equiv \text{total reserves for all lines}).$$

The ratio of the reserves in the largest line of business to the total reserves for all lines is termed the "loss concentration factor" (*NAIC Instructions*, p. 23, note 2). The actuarial justification for this formula may be found in Butsic [12]. Butsic finds that, "For both reserves and premiums, the average correlation between lines is about 40%, a number too low to lump all lines into a single independent category without adjustment, and too high to require independent line categories" [12, p. 181].[50]

For instance, suppose a personal lines carrier writes automobile, homeowners, commercial fire, and CMP business. It holds total reserves of \$800 million, of which \$600 million are for automobile liability. The total risk-based capital reserving risk component (after the appropriate adjustments) is multiplied by

$$70\% + 30\% * (600 \equiv 800) = 92.5\%.$$

Similarly, the written premium risk charge is adjusted by a "premium concentration factor," which is analogous to the loss concentration factor used in the reserving risk charge. The premium concentration factor is defined as the written premium in the largest line of business in the most recent calendar year divided by the total written premium in all lines of business. The written premium risk charge determined by the procedure outlined

[50]The NAIC research staff, upon reviewing 1994 risk-based capital results, found no significant correlations among lines of business. Thus, Wigger and Barth [44, p. 35], write:

> As the correlations show, there does not appear to be a strong relationship between the reserve development for any of the various lines of business. ... This means that, for individual companies, reserve development tends to be independent between lines, although individual companies may have different experience. The correlations are, on average, close to zero, even for those lines where one would expect to see some relationship (medical malpractice-occurrence and medical malpractice-claims made, for example). One can infer from these results that adverse development in one line of business does not mean that the other lines a company writes are any more likely to develop adversely.

Similarly, Barth [6] writes:

> Related research on the correlation of reserve risk between lines of business suggests that the loss concentration factor calculation understates the benefits of diversification.

above, after adjustment for loss-sensitive business and for business written on claims-made contracts, is multiplied by

$$70\% + 30\% * (\text{premium concentration factor}).$$

Growth Charges

The “growth charge controversy” was one of the most hotly debated in the development of the risk-based capital formula. Several private studies, such as that performed by the A. M. Best Corporation and those undertaken by some academicians, suggested that rapid growth was a prime cause of insurance company insolvency.[51] Since it takes many years before the true profitability of a book of business is known, particularly in the commercial liability lines of business, rapid growth may conceal financial weakness. Moreover, a rapidly growing company, particularly one that is relatively new to certain lines of business, may not be aware of the risks that it faces or of the potential severity of adverse loss development or underwriting cycle fluctuations.

Similar concerns were voiced about small companies. A small company, particularly if it is not well diversified, may be subject to great risks from random large losses or natural catastrophes. For instance, a small homeowners writer with business only in Florida and without adequate excess-of-loss reinsurance protection is more vulnerable to financial ruin from a hurricane than is a large and diversified multi-line writer with the same amount of Florida business.

The first (April 1991) draft of the risk-based capital formula had no additional charges for rapidly growing or small companies. The NAIC Working Group asked its actuarial advisory committee to develop a recommendation for such additional charges, if they were justified. In 1992, the actuarial recommendations

[51] See *Best's Insolvency Study* [9]. For an example of one academic study, see Willenborg [45].

were adopted by the NAIC Working Group. In 1993, however, after strong opposition by small insurance carriers, the NAIC dropped the small company charge (but retained the growth charge). Instead, it recommended that single state carriers with less than $2 million of annual premium be exempt from the risk-based capital requirements, subject to the decision of the state's insurance commissioner.

Determination of the Growth Charge

If rapidly growing companies have worse than average adverse loss development on their held reserves and worse than average underwriting results, the magnitude of these phenomena should be apparent in the historical experience. Allan Kaufman, a member of the AAA RBC Task Force, fit the following two regression equations to Annual Statement data:

$$\text{Reserve development bias} = A_0 * \text{growth} + B_{\text{line}}, \qquad \text{and}$$

$$\text{Loss ratio bias} = A_0 * \text{growth} + B_{\text{line}}.$$

"Growth" is defined as the "arithmetic average of the year-to-year changes in written premium for direct and assumed business for the group for the latest three years." All other risk-based capital charges are based on the characteristics of the individual company. The amount of growth is based on the growth of the corporate group. The reasoning is as follows:

- Solvency is an attribute of each legal entity. If there are two "sister companies," one domiciled in New York and one domiciled in Illinois, and the New York company becomes insolvent, the Illinois company is under no legal obligation to bail out the New York company. The capital held by the New York company should be sufficient to protect it from the risks it faces, without support from the Illinois company.
- A corporate group may sometimes shift business from one member company to another member company. In the example above, the corporate group may decide to enter the substandard

automobile insurance market and to use the New York company for standard business and the Illinois company for substandard business. The existing standard business in the Illinois company is shifted to the New York company. This gives the appearance of rapid growth in the New York company, but this is not the type of growth that causes "reserve development bias" or "loss ratio bias." The growth rate is therefore determined from consolidated group figures.

Monetary inflation and the expansion of the economy cause a certain amount of "normal" growth, which is not expected to cause "reserve development bias" or "loss ratio bias." The risk-based capital formula looks at "excess growth," which is the growth rate minus 10%. In addition, after averaging the most recent three years' growth rates, the formula caps the growth rate at 40%.[52]

The "bias" is the difference between (i) the observed reserve development or loss ratio for a particular company and (ii) the average reserve development or loss ratio for all companies. For instance, suppose that the average reserve development in a par-

[52] Kaufman ran the regression equations using both "total growth" and "excess growth" as the independent variable, with similar results. The figures shown in the text are his "total growth" results.

In theory, the amount of "normal growth" should depend on the inflation rate each year, as well as on real GNP growth and on the phase of the underwriting cycle. For instance, "normal growth" should be higher when monetary inflation is 15% per annum than when it is 5% per annum, all else being equal. However, a fixed normal growth rate of 10% per annum was chosen for simplicity, just as the flat 5% discount rate used to calculate the implicit interest margin was chosen for simplicity.

If a company has only three years of data, so only two annual growth rates can be calculated, the arithmetic average of these two growth rates is used, in lieu of the average of three growth rates that other companies use. If the company has only two years of data, a single growth rate is used.

The 40% cap on growth rates in the excess growth charge is applied to the average growth rate, not to the individual growth rates. For example, for a company with growth rates of 100%, 10%, and 10%, the average growth rate is 40%, which is not capped. Had we capped the individual growth rates, we would have 40%, 10%, and 10%, which gives an average of 20%.

The 40% capping is applied *before* the reduction by 10%. Thus, the maximum excess growth rate is 30%.

ticular line of business for all companies is 20%, and that the average reserve development in that line of business for companies with growth rates of +10% per annum is 25%. The reserve development "bias" is +5%, and the regression equation would indicate an A_0 value of +0.50 [since $0.50 * 10\% = 5\%$].

The reserve development bias regression equation produced an A_0 coefficient of 0.54, and Kaufman selected a value of 50% for the formula. The loss ratio bias regression equation produced an A_0 coefficient of 0.20, and Kaufman selected a value of 25% for the formula.[53]

Just as the reserving risk charges and the written premium risk charges are adjusted for the implicit interest margin in the reported reserves or in the underwriting results, the reserving risk "growth charge" and the written premium risk "growth charge" should be similarly adjusted. For simplicity, Kaufman [34, p. 3] chose a discount factor of 90% for all lines, which "approximates the average discount factor."

Adoption

The charge for rapidly growing companies was incorporated in the June 1993 version of the risk-based capital formula, which was adopted by the NAIC in December 1993. A company with an average three-year premium growth rate exceeding 10% per annum receives additional reserving risk and written premium risk charges.[54] The final formulas are

$$\text{Reserving risk growth charge} = 50\% * (\text{growth} - 10\%) * 90\%, \qquad \text{and}$$

[53]The full regression results, along with tests of significance and values for individual lines of business, may be found in Kaufman [34]. See particularly Appendix 1, Sheet 1, and Appendix 2, Sheet 2.

[54]The observant reader may note that since the underwriting risk charges are based on the experience of *all* companies, and the growth factors increase the charges for rapidly

$$\text{Written premium risk growth charge} = 25\% * (\text{growth} - 10\%) * 90\%.$$

6. THE COVARIANCE ADJUSTMENT

The first (April 1991) draft of the risk-based capital formula summed the individual charges to determine the company's capital requirements. Several actuaries, most notably Robert Butsic of the Fireman's Fund Insurance Companies, argued that the simple summation presumes that the various risks facing insurance enterprises might all occur simultaneously. In truth, these risks are at least partially independent. For instance, the risk of adverse loss development is independent of the risk of bond defaults or of stock market declines. Similarly, the risk of adverse development on personal automobile reserves is at least partially independent of the risk of adverse development on workers compensation reserves.

Even in 1991, of course, the NAIC Working Group recognized that a simple summation of charges was inappropriate for partially independent risks. However, good data quantifying the degree of independence were lacking, and there was no analysis of the proper method of combining the risk charges. Once Robert Butsic delivered his "Report on Covariance" to the NAIC Working Group, the risk-based capital formula was changed from a simple summation to Butsic's recommended formula.

In particular, Butsic [12, pp. 179, 180] notes the following relationships:

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growing companies, there should be an offsetting reduction in the factors for companies that are not rapidly growing. Indeed, Kaufman makes the same observation: "The otherwise applicable industry loss & lae reserve and premium charges by line should be reduced based on the amount of industry total RBC generated by the growth and size factors." No explicit reduction of this sort was made in the underwriting risk charges.

- Non-insurance asset risk (including credit risk) is independent of underwriting risk (reserves, premium, size, and growth risk).[55]
- Based on long-term historical data, the correlation between stock and bond returns is a rather weak 14%.
- Reserve and written premium risk are not very well correlated… From 1982 to 1991, the industry all-lines composite premium and reserve risk elements had only a 26% correlation.
- Based on judgment, reserve growth risk seems to be highly correlated with reserve risk.

Risk Categories

The various capital charges in the risk-based capital formula are first combined into six categories, termed R_0 through R_5, as follows:

R_0 :
- Investments in insurance affiliates
- Non-controlled assets
- Guarantees for affiliates
- Contingent liabilities

R_1 :
- Fixed income securities
    - Cash
    - Bonds
    - Bond size adjustment factor
    - Mortgage loans
- Short term investments
- Collateral loans
- Asset concentration adjustment for fixed income securities

[55]Butsic's report was written when there was a "size risk" component to the risk-based capital formula, so his formula has one term that is no longer present. Moreover, Butsic's paper does not have the adjustment for credit risk that is in the current formula.

R_2 :
- Equity investments
  - Common stocks
  - Preferred stocks
  - Real estate
- Other invested assets
- Aggregate write-ins for invested assets
- Asset concentration adjustment for equity investments

R_3 :
- Credit risk
  - Reinsurance recoverables
  - Other receivables

R_4 :
- Reserving risk
  - Basic reserving risk charge
  - Offset for loss-sensitive business
  - Adjustment for claims-made business
  - Loss concentration factor
  - Growth charge for reserving risk

R_5 :
- Written premium risk
  - Basic premium risk charge
  - Offset for loss-sensitive business
  - Adjustment for claims-made business
  - Premium concentration factor
  - Growth charge for premium risk

The proper categorization of the risk charges is essential for determining the overall capital requirements. Note particularly the following items.

1. After the credit risk charge has been calculated, one-half of this charge is removed from R_3 and added to R_4. This compensates for the inconsistency between (i) the interdependence of reserving risk and reinsurance collectibility risk and (ii) the lack of a covariance term in the square root rule (see the next two subsections).

2. The R_0 term appears *outside* the square root rule, whereas all the other terms appear *inside* the square root rule. This

makes it especially important to know which charges for affiliates appear in R_0. Charges for insurance subsidiaries, whether U.S.-based subsidiaries or alien subsidiaries, are included in R_0, so as to avoid a reduction of overall capital requirements by simple "layering" of the company's legal structure. Charges for non-insurance subsidiaries or affiliates appear in R_1 or R_2, depending on whether the insurer owns bonds or stock of the affiliate.

3. The determination of which investments are considered for the asset concentration factor is done for all assets combined, not separately for R_1 and R_2. The asset concentration factor charges are then separated into their R_1 and R_2 components for inclusion in the square root rule.

The Square Root Rule

Butsic recommended that the risk charges be combined by a square root rule. For instance, if there are two risk elements, with capital charges of \$3 million and \$4 million, then the total required capital would be $[(\$3 \text{ million})^2 + (\$4 \text{ million})^2]^{0.5} = \$5 \text{ million}$.

Of the six risk categories listed above, R_0 remains outside the square root rule and the remaining five risk categories are included inside the square root rule, or

$$\text{Total capital requirements} = R_0 + (R_1^2 + R_2^2 + R_3^2 + R_4^2 + R_5^2)^{0.5}.$$

In this statement of the formula, "R_3" means one-half of the credit risk charge, and "R_4" means the reserving risk charge plus the other half of the credit risk charge.

Three issues pertaining to this formula are particularly important:

- the lack of covariance terms in the square root rule;

- the exclusion of the R_0 charge from the square root rule; and
- the marginal capital effects of each risk element.

Covariance Terms

Let us return to the illustration above of two risk elements, "Risk A" and "Risk B," which have capital charges of $3 million and $4 million, respectively. A simple additive rule says that the total capital charge for the company should be $7 million. The square root rule says that the total capital charge for the company should be $5 million.

The true total capital requirement depends on:

- the meaning of the "capital requirement,"
- the probability distribution of each risk element, and
- the interdependence of the risk elements.

Suppose that the capital requirement for a given risk means that if the company had only that risk element and exactly that amount of capital, there is a 95% chance that the company would remain solvent over the coming year. For instance, if the company had only Risk A and exactly $3 million of capital, there is a 95% chance that it would remain solvent a year from now and a 5% chance that it would become insolvent.

If there are two risk elements that are perfectly correlated with each other, such that whenever the company lost money from risk element A it also lost money from risk element B, then the simple additive rule is correct. To have a 95% chance of remaining solvent over the coming year in the example above, the company needs $7 million of capital. If the two risk elements are at least partially independent of each other, then less than $7 million is needed. In this situation, the *complete* square root rule would say that the total capital requirement is

$$[(\$3\text{ million})^2 + 2 * \text{covariance (Risk A, Risk B)} * \$3\text{ million} * \$4\text{ million} + (\$4\text{ million})^2]^{0.5}$$

When risks A and B are perfectly correlated, this expression reduces to

$$[(\$3\text{ million})^2 + 2 * 1 * \$3\text{ million} * \$4\text{ million} + (\$4\text{ million})^2]^{0.5} = \$7\text{ million.}$$

When risks A and B are perfectly independent, this expression reduces to

$$[(\$3\text{ million})^2 + 2 * 0 * \$3\text{ million} * \$4\text{ million} + (\$4\text{ million})^2]^{0.5} = \$5\text{ million.}$$

Butsic [11] says that, "Knowing the degree of correlation between risk elements can be as important as knowing the risk of individual items." In his published paper, Butsic includes the covariance terms among all the risk elements. In his covariance adjustment for the NAIC risk-based capital formula, however, there are no correlation terms or covariance terms, as though all the risks were perfectly independent.

In practice, there is some dependence among the risk factors. For instance, during recessions, when bond default rates are higher than average, stock market declines are more likely. If the dependence among the risk factors is strong, the square root rule may underestimate the capital requirements.

In response, Butsic argues that

- The square root rule, by itself, overestimates the amount of capital needed to achieve a given "expected policyholder deficit" ratio if the risk elements have normal or lognormal probability distributions.[56]

[56]In other words, if a company with only risk element A needs $3 million of capital to achieve a 1% expected policyholder deficit (EPD) ratio, and a company with only risk

- The correlation among the risk factors is very weak, so the underestimate of the needed capital is small.
- The one important interdependence, between the risk of adverse reserve development and the risk of reinsurance collectibility, is accounted for by the movement of one-half of the credit risk charge into the reserving risk category.[57]

The first two effects are largely offsetting, so the unadjusted square root rule gives a reasonably accurate result.

The Charge for Subsidiaries

The risk charge for insurance subsidiaries—the R_0 charge—is outside the square root formula. The rationale for this is that the risk-based capital requirement for an insurance company should not depend upon the organizational structure of the company. If an insurance company forms a subsidiary with half of its business and half of its assets, its capital requirements should not change. If the R_0 charge were within the square root section of the formula, the total risk-based capital requirement for the group would decrease as more layers of ownership were introduced.

For example, suppose the only risk-based capital charge was a bond risk default charge (R_1) of \$10 million. If the company uses half the bonds to capitalize a subsidiary, the new charges would be R_0 of \$5 million and R_1 of \$5 million. If these two

element B needs \$4 million of capital to achieve a 1% EPD ratio, and if risk elements A and B are normally or lognormally distributed, then the amount of capital needed to achieve a 1% EPD ratio for a company with risk elements A and B is somewhat less than the amount of capital prescribed by the complete square root rule. Butsic [12, p. 185] shows the approximate amount of overstatement, separately for the normal and for the lognormal case. It is difficult to illustrate this effect, since in the simplest discrete cases (e.g., with binomial distributions for the risk elements), the square root rule generally *understates* the required capital.

[57]Reserving risk measures unanticipated adverse loss development. In some cases, when there is substantial unanticipated adverse loss development, as resulted from the passage of CERCLA (the "Superfund" legislation) in 1980, some reinsurers may become bankrupt or may be unwilling to pay claims, leading to high credit risk for reinsurance recoverables. In the absence of hard data showing the correlation between reserving risk and credit risk, the movement of one-half of the credit risk charge into the reserving risk category was a subjective method of accounting for this interdependence.

charges were now combined by the square root rule, the total risk-based capital requirement would be

$$[(\$5 \text{ million})^2 + (\$5 \text{ million})^2]^{0.5} = \$7.07 \text{ million}.$$

But the risks are not reduced simply by layering the insurance enterprise more finely. So the R_0 charge is kept outside the square root part of the formula. The risk-based capital requirement for the insurance enterprise remains \$5 million + \$5 million = \$10 million.[58]

Marginal Effects

An important implication of the square root rule for company strategy relates to the marginal effects of each risk charge on the total capital requirements. A preliminary illustration should clarify the meaning of marginal effects; the mathematical formula is derived afterwards.

> Suppose a company has two risk elements, A and B, with capital charges of \$10 million and \$2 million, respectively. The company can take on additional risk of either type A or type B, causing additional capital charges of \$1 million. That is, either risk A goes from \$10 million to \$11 million or risk B goes from \$2 million to \$3 million. What is the effect on the total capital requirements of the company?

If the risk charges are additive, such that the total capital requirements before the addition of the new risk elements is \$12 million, then it makes no difference whether Risk A is increased

[58]Butsic [12, p. 182] notes that consolidation of the risk-based capital categories for parents and affiliates and subsequent application of the square root rule is another means to deal with covariance among affiliates. Furthermore, Butsic points out that when the subsidiary is not a proportionate scaling of the parent company, the method described in the text (that is, keeping the R_0 charge outside the covariance formula) slightly overstates the theoretically correct risk-based capital requirements, whereas consolidation gives the correct figure.

from \$10 million to \$11 million or Risk B is increased from \$2 million to \$3 million. In either case, the new capital requirements are \$13 million.

If the square root rule is used, then the total capital requirements before the addition of the new risk element is

$$[(\$10 \text{ million})^2 + (\$2 \text{ million})^2]^{0.5} = \$10.198 \text{ million}.$$

If risk A is increased from \$10 million to \$11 million, the new capital requirements are

$$[(\$11 \text{ million})^2 + (\$2 \text{ million})^2]^{0.5} = \$11.180 \text{ million}.$$

The marginal effect of each extra dollar of Risk A is

$$(\$11.180 \text{ million} - \$10.198 \text{ million}) \equiv (\$11 \text{ million} - \$10 \text{ million}) = \$0.98.$$

If Risk B is increased from \$2 million to \$3 million, the new capital requirements are

$$[(\$10 \text{ million})^2 + (\$3 \text{ million})^2]^{0.5} = \$10.440 \text{ million}.$$

The marginal effect of each extra dollar of Risk B is

$$(\$10.440 \text{ million} - \$10.198 \text{ million}) \equiv (\$3 \text{ million} - \$2 \text{ million}) = \$0.24.$$

Adding a dollar to risk charge A has a far greater effect on the total capital requirements than adding a dollar to risk charge B has. In this discrete example, the marginal effect of increasing risk charge A is about four times as great as the marginal effect of increasing risk charge B.

In the continuous case, the ratio of the marginal effects equals the ratio of the original capital charges. In other words, if the illustration above were revised to ask: *What is the marginal effect of adding a single dollar of additional risk charge either to Risk A* *or to Risk B?* then the marginal effect of each extra dollar of risk charge for Risk A is five times as great as the marginal effect for Risk B, since the current risk charge for Risk A is five times as large as the risk charge for Risk B.

Mathematically, the covariance adjustment sets the total capital requirements as

$$\text{TCR} = \text{total capital requirements} = (\Sigma C_i^2)^{0.5},$$

where the C_i are the capital requirements for each individual risk. The *marginal* capital requirement for any risk j equals

$$\partial \text{TCR}/\partial C_j = 0.5(\Sigma C_i^2)^{-0.5} * 2C_j.$$

In other words, the marginal (post-covariance) charge for an additional dollar of any pre-covariance risk charge is proportional to the total dollars in that risk category. Risk categories with large pre-covariance charges, such as reserving risk, provide a high post-covariance contribution *for each dollar of risk charge*. Risk categories with low pre-covariance charges, such as default risk, provide a low post-covariance contribution *for each dollar of risk charge*.

There are several practical implications of this difference in marginal effects.

1. Before the covariance adjustment was included in the risk-based capital formula, financial analysts thought that changes in investment strategy could materially affect a company's capital requirements. For instance, some investment analysts envisioned high demand for bonds whose coupon payments were linked to a stock market index, thereby yielding returns similar to common stocks yet incurring the low capital charge afforded bonds. Once the covariance adjustment was incorporated into the formula, it became clear that the marginal effects of the asset risk charges were extremely small, thereby ren-

dering such investment ploys rather useless for reducing risk-based capital requirements.[59]

2. The risk-based capital requirements are dominated by the underwriting risk charges, and particularly by the reserving risk charge. Yet the reserving risk charges are perhaps the weakest link in the chain: it has been argued that the reserving risk charges are ad hoc extrapolations from historical happenstance, they do not adequately distinguish financially-troubled companies from sound companies, and they provide perverse incentives that may raise insolvency risks.

7. OTHER ISSUES

The previous sections of this paper describe the risk-based capital formula, along with the rationale for the various charges. Some charges, like the underwriting risk and the credit risk charges, were developed by the NAIC Working Group; some charges, like the asset risk charges, were developed by life insurance actuaries; and some charges, like the growth charges, the loss-sensitive contract offset, the claims-made business offset, and the covariance adjustment, were developed by members of the AAA RBC Task Force.

As noted previously, the reserving risk charge forms the dominant component of the risk-based capital requirements for most companies, and its pre-eminence is heightened by the square root rule. Many observers have criticized the magnitude and rationale of the reserving risk charges, as well as the unusual incentives provided by the formula.

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[59]In addition, the reduction in the common stock charge for property/casualty companies from 30% (which is the charge in the life insurance risk-based capital formula) to 15% makes the final effect of moving from stocks to bonds almost negligible. For further analysis, see Salomon Brothers, "Property/Casualty Risk-Based Capital: The Surprise on the Asset Side."

Casualty actuaries have been in the forefront in attempts to quantify reserving risk (or "reserve uncertainty"), and numerous reports have been published in recent years on this topic. The growing dissatisfaction with the underwriting risk charges, the recognition by the NAIC research staff of the weaknesses of these charges, and the development of sounder statistical techniques by practicing actuaries should soon stimulate the reexamination of these risk charges.

The following sections discuss several aspects of the underwriting risk charges that are essential for understanding the controversies on this issue. Some of the proposed revisions are discussed in the footnotes, though the text of this paper is restricted to the actual NAIC formula.

Quantifying the Capital Charge for Underwriting Risk

The risk-based capital formula balances three major considerations:

- *Accuracy*: The capital charges must accurately reflect the risks faced by insurance companies.
- *Simplicity*: The rationale for the capital charges should be understood by company executives and state regulators, not just by highly trained actuaries and financial analysts.
- *Incentives*: The risk-based capital formula should provide incentives for companies to strengthen their capital structures.[60]

These goals conflict at times. Improving the accuracy of the charges often requires more complex statistical formulas. The tension between the actuarial accuracy motivating some of the AAA reports and the desire for simplicity often underlying the NAIC Working Group decisions is perhaps most evident in the

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[60]See especially "Simplicity, Accuracy, and Incentives," a memorandum by Sholom Feldblum to the members of the actuarial advisory committee to the NAIC Working Group (January 29, 1992).

method of quantifying the capital charges for reserving risks and premium risks.

Worst Case Year

Some actuaries have criticized the NAIC's worst case year approach for measuring reserving risk and written premium risk for two reasons:

- *Theory*: The observed favorable or adverse development for a particular line of business over the past ten years may be due as much to historical happenstance as to true risk characteristics.[61]
- *Calibration*: Even if the observed adverse development is a good proxy for risk characteristics, the "optimal" or "required" capital may not be the same as the observed development.

Statistical Quantification

A subcommittee of the AAA RBC Task Force developed an alternative approach to quantifying the reserving and premium risk charges.[62] This approach considered the variances of reserve

[61] Lowe [39] summarizes this as follows:

"The current [risk-based capital] factors reflect the historical experience of the industry in the last underwriting down-cycle. In particular, they reflect the severe adverse reserve development that occurred in general liability, medical malpractice, and reinsurance, and the very severe loss ratios in malpractice and reinsurance....

"While the next down-cycle could easily be as severe, the specific forces that drive it will probably be different (as they are in each cycle), such that the incidence of adverse results by line will probably also be different....

"The methodology underlying the current factors, therefore, seems somewhat overly focused on the specifics of the recent past. While past experience is useful as a guide, it needs to be interpreted in terms of the current and future risks faced by the industry."

Compare, however, Vincent Laurenzano's implicit rejoinder to Lowe's argument [37, p. 102]: "The Working Group believes that it is more important for statutory capital standards to be based on past experience rather than on more subjective judgments as to the adequacy of current reserves, assumptions as to industry trends, or anticipated future business activities both as to the industry and individual companies."

[62] The subcommittee was chaired by Stephen Lowe, an actuary with Tillinghast. The final report of the subcommittee was discussed by the NAIC Working Group and subsequently published in the CAS *Forum* [39].

development and of loss ratios by line of business, as well as the effect of changing interest rates on statutory adverse development, and it combined these with an "expected policyholder deficit" concept to develop risk-based capital requirements.[63]

The NAIC reserving risk charges, despite their complexity, have shown surprisingly little predictive power. In a study of the NAIC risk-based capital formula, Cummins, Harrington, and Klein [16] assert that

> The loss reserve component of the NAIC risk-based capital formula, which accounts for half of industry risk-based capital, has virtually no predictive power in any of the tests we conducted.[64]

Michael Barth, the NAIC research associate currently responsible for monitoring the use of the risk-based capital formula, has expressed similar views [6, pp. 1, 2]:

> The current calculation does not track very well with the observed reserve risk for the individual lines of business, nor does the aggregate R_4 track very well with observed aggregate reserving risk.

Douglas M. Hodes, an actuary with the Liberty Mutual Insurance Company and a member of the life insurance risk-based capital actuarial advisory committee, issued a parallel report, using statistical methods similar to Lowe's but different assumptions. The resulting capital charges for reserving and written premium risk differed greatly from those arrived at by Lowe, particularly for the long-tailed lines of business.

Because of the complexity of these reports, and the lack of consensus among the actuarial community, the NAIC Working Group adhered to its original formula.

[63] For the use of expected policyholder deficit in devising capital requirements, see [11]. Robert Butsic, a member of the AAA RBC Task Force, developed the expected policyholder deficit component of Lowe's report as well.

Hodes, Feldblum, and Blumsohn [32] have combined the stochastic simulation techniques advocated by the British Solvency Working Party and the corresponding Finnish Working Party (and systematically laid out in Daykin, Pentikäinen, and Pesonen [17]) with Butsic's expected policyholder deficit procedures to develop capital requirements more in line with current actuarial thinking.

[64] See Cummins, Harrington, and Klein [16]. Robert Klein, a research economist with the NAIC, was in charge of assessing the effectiveness of the risk-based capital formula.

> Internal research supports these findings that the R_4 underwriting risk charge is not proportional to the observed reserving risk, on average.
>
> The graph compares total reserve error to total reserve RBC. The graph shows that there is no relationship between the observed reserve risk and the RBC requirement to support that risk.

Vincent Laurenzano points out that the underwriting risk charges were not designed to be predictive of reserve deficiencies. In fact, the RBC formula in general was not designed to be *predictive* of future insolvency, but rather to establish minimum capital requirements for insurers based on the risks contained in their balance sheets, including the risk that reserves may be understated.

Credibility

The development of the credibility component is instructive. The first (April 1991) draft of the risk-based capital formula used a "credibility weighting" of industry experience and the company's experience. The company's credibility varied from 0% to 50%, depending on the size of the company's reserves in the line of business. The largest companies received 50% credibility, while small companies received credibility factors close to 0%.

The full credibility standard (actually, the "50% credibility standard" here) was set by judgment, and the classical "square root" formula was used for partial credibility.[65] However, no analysis was done to justify the chosen full credibility standard or the partial credibility rule, so this element of the first draft formula was bereft of actuarial justification. Criticism of the formula, though, was strong, since small companies with good experience claimed that they were placed at a disadvantage

[65] See Longley-Cook [38] for further explanation of the classical full credibility standard and partial credibility rule.

compared with larger companies having the same experience. Since the risk-based capital formula was intended to set a company's capital requirements in accordance with the risks that it faced, this disparity was perceived as inequitable.[66]

Incentives

In theory, loss reserves are the result of a company's operations. Actuaries examine the premium and loss experience of the company and set reserves to cover the anticipated future obligations.

In practice, the adequacy of loss reserves may vary greatly from company to company, and even from year to year for a given company. These variations in reserve adequacy may affect the public's perception of the company's operations and its financial strength. A reduction in reserve adequacy—that is, a decrease in loss reserves (or in incurred losses)—shows up as an increase in statutory surplus. Conversely, an increase in reserve adequacy shows up as a decrease in statutory surplus.

Bulk reserve requirements cannot be estimated precisely, despite the reserve opinions written by actuaries or the financial audits performed by accountants. In fact, the bulk reserve is

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[66] Actuarial advances in credibility theory over the past two decades have made empirical recommendations increasingly difficult to justify. Twenty years ago, the development of a new classical credibility formula generally used a full credibility standard based on claim frequency only, assumed a normal distribution of claim counts, and required simply that the actuary select a confidence interval.

Current approaches to credibility correctly consider the classical theory as little more than ad hoc standards. Bayesian credibility theory compares the *relative* predictive power of alternative sets of data, not the *absolute* predictive power of either one. For further elaboration of classical versus Bayesian credibility theory, see Philbrick [40], Herzog [29], and Venter [43].

This difference between classical and Bayesian credibility can be seen in the reserving risk charge. The AAA RBC Task Force developed a report showing that the past adverse loss development of a company was not well correlated with future adverse loss development. The authors of this report therefore recommended that the company credibility for the reserving risk charge be reduced or eliminated. Modern credibility theory, however, says that the absolute predictive power of the company's past adverse development for future adverse development is irrelevant. Rather, the proper credibility value depends on the *relative* predictive power of company versus industry past adverse development for the company's future adverse development.

often judgmentally chosen from a range of possible values. It has been argued that some troubled companies have given themselves the "benefit of the doubt" and have chosen unrealistically low reserve estimates, which have the effect of hiding financial weakness. Since minimum surplus requirements were low in the past, there was little temptation for financially sound companies to underestimate bulk reserve needs simply to ensure sufficient surplus, as opposed to the incentive for financially unsound companies to underestimate reserves.[67]

The advent of risk-based capital requirements may dramatically change company behavior. Now even large statutory surplus amounts may be deemed insufficient, and this insufficiency may lead to regulatory intervention in the company's affairs.

Insurers seeking to avoid such regulatory intervention may attempt to modify their operations or their accounting practices to improve their risk-based capital ratios. Because of the structure of the risk-based capital charges, and particularly because of the covariance adjustment, changes in the asset portfolio have an extremely small effect on the final capital requirements. Similarly, the costs of modifying the company's reinsurance arrangements or its business strategies will often outweigh any short term risk-based capital benefits.

Reserve Strengthening and Weakening

The opposite is true for reserving practices. The risk-based capital formula adds additional incentives for companies to report inadequate reserves. Stephen Lowe has termed this the "triple whammy" of the reserving risk charge:

- Reducing reserves increases statutory surplus. This was true both before and after the implementation of risk-based capital requirements. The effect of the new requirements is that now even financially strong companies will be examining their risk-

[67]Moreover, understatement of loss reserves raises federal income tax liabilities. Financially sound companies therefore had little motivation to underestimate their reserves.

based capital ratios (i.e., the ratio of adjusted surplus to the risk-based capital requirements) and seeking the least costly and most effective ways to raise them.

- Reducing reported reserves lowers the reserving risk charge. Moreover, since the reserving risk charge is dominant for most companies, the covariance adjustment in the risk-based capital formula further increases the marginal effect of the reserving risk charge relative to other charges. In other words, each dollar reduction in the reserving risk charge has a much greater effect on the overall capital requirements than does a similar dollar reduction in the other risk-based capital charges.
- Reducing reported reserves also reduces the reported adverse development, particularly if one also allocates a larger percentage of the reserves to the most recent accident year in the Schedule P exhibits. In the past, reserve strengthening not only reduced capital needs (since reserves contained a healthier margin) but sometimes even demonstrated greater management honesty in reporting practices, thereby lessening the perceived need for a "capital cushion." Now companies may be loath to strengthen reserves, since this action will increase the company's reported adverse loss development, further increasing its reserving risk charge.

One solution to this problem is to base the reserving risk charge on indicated reserves, not reported reserves, where the indicated reserves are determined from a base independent of the company's reserving system (such as earned premiums). In 1992, Stephen Lowe, an actuary with Tillinghast, recommended a reserving risk charge based on a paid loss Bornhuetter-Ferguson estimate of indicated reserves, though the complexity of the calculation was inconsistent with the generic simplicity required for the risk-based capital formula. Dale Nelson, an actuary with State Farm, developed a "percent of premium" reserving risk charge for the AAA RBC Task Force. This kept the formula simple and eliminated the untoward incentives in the risk-based capital formula. Nevertheless, questions about the accuracy of this method persuaded the AAA RBC Task Force not to recommend it to the NAIC Working Group.

Moreover, the NAIC Working Group has argued that the alignment of reported reserves with indicated reserves is the task of the company's appointed actuary and the state insurance department's financial examiners, not that of the risk-based capital formula. The current requirements for a "Statement of Actuarial Opinion" regarding loss and loss adjustment expense reserve adequacy may help prevent deliberate reserve understatements, particularly if the actuarial community seeks to enforce the regulatory mandate. In addition, although reserve understatements may reduce the present company adjustment to the reserving risk charge, they increase future development and thereby the future reserving risk charge.

Workers Compensation and Tabular Discounts

The first draft of the risk-based capital formula led to an average (industry-wide) reserving risk charge for workers compensation of 0.4%, which seemed relatively low to some regulators and analysts.

Duration and Volatility

Some actuaries considered this factor appropriate, since the reserving risk charge reflects the *net* effect of unexpected adverse development and the implicit interest discount:

- Workers compensation indemnity payments are made over time as the injured workers' loss of income is realized, and medical payments are made when physicians' bills are submitted. Large awards are only occasionally paid as lump sums, in contrast to general liability or automobile liability claims. Rather, compensation claims are paid over long durations, as permanent disability payments or as lifetime pension awards. The resulting long reserve duration implies that the interest discount factor should be high for workers compensation.

- Payment patterns are statutorily mandated for workers compensation, so the volatility of adverse loss development is low. In lines of business subject to tort liability rules, an adverse court decision may dramatically change the insurer's liability, necessitating a revision of held reserves. In contrast, payments on workers compensation claims are set by statute and known to the insurer soon after the loss occurs. The expected future payments on reported cases may change if average durations of disability change. However, these changes are slow and incremental, unlike the sudden effects of court decisions.

The combined effect of these two characteristics—the long reserve duration and the statutorily mandated loss payments—produces a low reserving risk charge.

Aggregate industry experience is even more indicative of the stability of workers compensation reserves. The +0.4% reserving risk factor was derived from unweighted (simple) averages of individual company adverse developments. Using aggregate industry data, or weighted averages of individual company data, produced a large *negative* reserving risk charge for workers compensation.

In other lines of business, external factors may affect the entire industry's liabilities, such as the enactment of retroactive liability for environmental impairment exposures, which may raise the industry's products liability reserves, or a natural disaster, which may raise the industry's homeowners reserves. There are few such cataclysmic events that might raise the industry's workers compensation reserves.[68]

Criticisms

Other actuaries, as well as several regulators, considered the workers compensation reserving risk factor to be too low. Several

[68]For a more complete analysis of the factors affecting workers compensation reserve uncertainty, see Hodes, Feldblum, and Blumsohn [32].

arguments were given to support this view, such as

- The observed loss development in the 1980s does not reflect the riskiness of workers compensation in the 1990s.[69]
- Industry-wide workers compensation reserves are deficient, more so than in other lines of business. This argument was sometimes worded to say that compensation carriers hold discounted reserves, but do not disclose this fact in their financial statements.[70]
- Many insurance companies use tabular loss reserve discounts for their workers compensation lifetime pension claims. The reserving risk charge in the risk-based capital formula should reflect the use of tabular discounts.

Tabular Discounts

The last of the arguments listed above, the tabular discount argument, became the official reason for changing the workers compensation reserving risk charge, from 0.4% in 1992 to 11% in 1993. Some actuaries believe, however, that the first two reasons were the underlying impetus for the revision in the factor.

How does the valuation basis for loss reserves affect the appropriate reserving risk charge? Adverse loss development in statutory statements may result from two causes:

- reserve inadequacies, resulting either from unexpected developments or from poorly estimated (or consciously underestimated) initial reserves, or
- the "unwinding" of the interest discount on discounted reserves. Although the unwinding of an interest discount is expected, it appears as adverse development in Schedule P.

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[69] See, for example, Lowe [39, p. 112]. Other actuaries argue that this criticism is reversed, as the adverse economic results for workers compensation in the latter half of the 1980s have dissipated in the 1990s.

[70] See, for example, the August 1992 report of the NAIC Working Group regarding the workers compensation reserving risk charge.

For instance, suppose full value reserves are set up for a block of business in December 1993 for $10 million. In December 1994 the losses are paid, but because of an adverse court decision, the total settlement is $11 million. This is "true" adverse development.

Alternatively, discounted reserves of $10 million, at a 10% interest rate per annum, may be set up for a block of business in December 1993. In December 1994, the losses are paid for $11 million. If the discount is not "grossed up" in Schedule P, then the latter situation also shows the same adverse loss development: 10% of reserves.[71]

The rationale for the effect of tabular discounts on the appropriate reserving risk charge is as follows:

- For other lines of business, current reserves are assumed to be adequate, so no future development is expected. All reserve figures are at full (undiscounted) value, since the charges are determined from Part 2 of Schedule P, which has always been gross of non-tabular discounts.
- For workers compensation, to the extent that companies use tabular discounts, current reserves are reported on a discounted basis. Future development equal to the unwinding of the tabular discount is expected. To get the full "gross-of-discount" adverse loss development, one must add the expected future unwinding of the interest discount to the observed adverse loss development in the experience period.

[71] When the workers compensation reserving risk charge was developed in early 1992, all the Schedule P exhibits were net of tabular discount, though they were gross of non-tabular discount. Since 1994, Part 2 of Schedule P, from which the reserving risk charges were determined, has been changed to a gross-of-tabular-discount basis, though Part 1 of Schedule P remains net of tabular discount. For a complete analysis of the Schedule P reporting requirements and of the change in the treatment of tabular discounts, contrast Feldblum [20] which documents the 1996 Schedule P, with Feldblum [25], which documents the 1992 Schedule P.

To account for the expected future unwinding of the tabular interest discount, the workers compensation reserving risk charge was changed from 0.4% to 11.1%.[72]

Two aspects of the tabular discounts issue provoked heated debate within the actuarial community: the adjustments to surplus in the risk-based capital formula, and the definition of tabular discounts.

Surplus Adjustments

The risk-based capital formula removes non-tabular discounts from policyholders' surplus, but retains the tabular discounts. (See Section 9 below.) Non-tabular discounts are removed from surplus to place all companies on a "level playing field."[73] Tabular discounts are retained in surplus to place property/casualty insurers providing long-term disability benefits on a "level playing field" with life insurers providing similar benefits.

For instance, suppose a commercial lines insurer writing workers compensation and other liability business has $1 billion of surplus, $1.5 billion of other liability reserves, and $2 billion of workers compensation reserves. The compensation reserves are net of $100 million of tabular discounts on lifetime pen-

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[72]It is not clear how one should adjust the reserving risk charge for tabular discounts. Some actuaries have argued that the use of tabular discounts in workers compensation reserving should *lower* the reserving risk charge, not raise it, for the following reason.

Tabular loss reserve discounts are used for lifetime pension cases. The discounts are most frequently calculated at a 3.5% interest rate, which is the required rate for the unit statistical plan of the National Council on Compensation Insurance. Pension cases, which are permanent total disability and fatality claims, are extremely long-tailed, with average lifetimes of 30 to 40 years. The implicit interest margin in the risk-based capital formula uses the IRS loss reserve discounting procedures, which assumes that all claims are fully paid within 16 years. Extending the payment pattern from 16 years to the actual payment pattern of pension cases generates enough additional interest margin, because of the low tabular discount rate (3.5%) as compared to the risk-based capital discount rate (5.0%) or the actual portfolio yields received by insurance companies (about 7% in the 1990s), to compensate for the expected future unwinding of the interest discount.

For a complete analysis of the treatment of tabular discounts in the risk-based capital reserving risk charge, see Appendix A of Hodes, Feldblum, and Blumsohn [32].

[73]Compare the AAA "Conceptual Framework" white paper, from which several other studies of risk-based capital have borrowed the "level playing field" concept.

sion cases. In addition, this carrier has received permission from the insurance department in its domiciliary state to discount its other liability reserves and its remaining workers compensation reserves at a 4% discount rate. The amount of this "non-tabular" discount is $400 million.

The "adjusted policyholders' surplus" for the insurer's risk-based capital ratio calculation is $600 million, or $1 billion minus $400 million. In other words, the non-tabular discounts are subtracted from surplus, though the tabular discounts are not removed from surplus.

This adjustment to surplus for non-tabular discounts is phased-in over a five-year period. In the example above, $80 million is subtracted from surplus each year for five years, to get the risk-based capital "adjusted surplus." The reserving risk charge, however, is applied to reserves gross of any non-tabular discount, with no five-year phase-in.

Definition of Tabular Discounts

A company desirous of a higher risk-based capital ratio would prefer to label its interest discounts as "tabular" instead of as "non-tabular." Two questions arose:

- Do tabular discounts apply only to known cases (i.e., reported cases which are identified as lifetime pension), or to "unidentified pension cases" as well (i.e., claims that have not yet been coded as lifetime pension)?
- Do tabular discounts apply to indemnity (e.g., weekly disability) payments only, or to medical benefits as well (e.g., home nursing care for a quadriplegic)?

The NAIC Working Group stated that tabular discounts may be applied to all lifetime pension cases, whether already identified or not, but only to the indemnity benefits. The tabular discount on unidentified lifetime pension cases is determined by standard actuarial bulk reserving procedures.[74]

8. IMPLEMENTATION OF RISK-BASED CAPITAL REQUIREMENTS

The risk-based capital formula produces a number: a "risk-based capital requirement." Three questions arose repeatedly during the development of the risk-based capital standards:

- What exactly does the "risk-based capital requirement" or the "risk-based capital ratio" mean?
- What effect should these figures have on regulatory action?
- How will regulatory action be implemented?

Minimum, Target, and Triple-A Standards

Existing state statutes define the minimum amount of surplus that an insurance company must hold to obtain a license. The amount varies by state, and it depends on the lines of business which the insurer writes. It is generally quite low, ranging from about $1 million to about $5 million. It does not vary with the size of the company or with the particular risks which it faces.

These state statutes define minimum capital requirements. They make no attempt to define the "optimal" or "target" amount of capital which the company should hold.

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[74]The same definition has been adopted by the NAIC Blanks (EX4) Task Force for Schedule P reporting. A clear definition of tabular discounts is needed since Part 1 of Schedule P is net of tabular discounts but gross of non-tabular discounts; see Feldblum [20].

Note 19 to the Financial Statements requires disclosure of all loss reserve discounts, separately for tabular discounts and non-tabular discounts. The definition of tabular discounts follows that introduced by the risk-based capital formula; see Feldblum [26].

The Actuarial Committee's "White Paper"

One of the first projects undertaken by the actuarial advisory committee to the NAIC Working Group (now the AAA RBC Task Force) was to develop a "white paper" on risk-based capital requirements.

The actuarial advisory committee was being asked to study the parameters used in the risk-based capital formula that had been developed by the NAIC Working Group. The committee responded that the proper size of the parameters depended on the meaning of the risk-based capital standards:

- If the risk-based capital formula defined a minimum amount of capital that must be held by all companies to be allowed to operate, then low parameters were appropriate.
- If the risk-based capital formula defined a target amount of capital that represented the "optimal" capital position for an insurer, then higher parameters would be appropriate.
- If the risk-based capital formula represented a "Triple-A" standard, or an amount of capital that only the financially strongest companies would hold, then even higher parameters would be appropriate.

The NAIC Working Group did not initially address this issue. Rather, it implicitly responded by spelling out the regulatory and company actions necessitated by the ratio of actual surplus held to the risk-based capital standard. (See Section 9 below.) The June 1993 statement of the NAIC Working Group, however, says:[75]

> The Working Group believes the proposed formula provides a minimum threshold measure of capital adequacy and is not overly complex … Since the formula is intended to identify insurers that require regulatory

[75] *Proceedings of the NAIC*, 1993, Second Quarter, p. 565.

> attention and does not purport to compute a target level of capital ...

In other words, the risk-based capital formula is setting a minimum threshold for capital requirements, not a target level or a "Triple-A" level.[76]

9. REGULATORY ACTION

What effect do the risk-based capital standards have on regulatory and company actions?

Regulatory Hesitancy

Suppose you were the insurance commissioner in your state, and you have been informed that a medium-sized personal automobile writer domiciled there was in financial difficulty. You ponder how strenuously you should investigate this company:

- The company has over 1,000 employees in the state. If the company is liquidated, these individuals will be unemployed, increasing the discontent of the citizenry and reducing the state tax revenues.
- If the company does become bankrupt, outstanding claims will be paid by the state guaranty fund. The guaranty fund assesses all insurance companies doing business in the state, most of which are domiciled in other states.

The hesitancy of many state insurance departments to take action against financially troubled companies was a major impetus for the development of risk-based capital standards and requirements.

[76] Cummins, Harrington, and Niehaus [14], forcefully argue for a minimum threshold standard. See, for instance, [14, p. 443]. "The arguments in favor of a minimum threshold approach are compelling ... Fewer undesirable distortions in the decisions of sound insurers would result with a minimum threshold approach ..."

Levels of Action

Some regulators argue that insurance departments must be afforded great discretion in their dealings with domestic insurance companies. Other regulators have argued that certain actions must be required of regulators, particularly when the needed action is unpleasant.

The risk-based capital requirements are a compromise between these two viewpoints. There are four levels of regulatory action, depending on the relationship between the "adjusted surplus" held by the company and the "risk-based capital surplus." This ratio is termed the "risk-based capital ratio."

The ACL Level

The levels of regulatory action actually depend not on the risk-based capital ratio but on the relationship of the company's adjusted surplus to the risk-based capital "authorized control level" (ACL) benchmark. At first glance, this seems a superficial distinction, since the authorized control level is a percentage of the risk-based capital standard. In practice, it is easier to change the authorized control level than the risk-based capital formula itself, and thereby implicitly change all the regulatory action levels.

For example, during the first half of 1993, the ACL benchmark was expected to be 50% of the risk-based capital standards. This would have forced many companies into rehabilitation or liquidation, and may have led to substantial opposition to the new risk-based capital standards.

However, the June 1993 draft of the risk-based capital formula defined the ACL benchmark as 40% of the risk-based capital standards. At this level, only about half as many companies would have been forced into rehabilitation or liquidation; as a result, opposition to the new standards was muted.

In October 1993, the NAIC shifted back to a 50% ACL benchmark, with a two-year phase-in from 40% to 50%, thereby giving time to companies to strengthen their capital positions. By this time, the industry waters were placid, and in December 1993 the risk-based capital formula was adopted without significant opposition.

Four Action Levels

The NAIC envisions four levels of regulatory or company action, depending on the relationship of the company's actual (adjusted) surplus to its risk-based capital surplus. A property/casualty insurance company's actual surplus is adjusted for risk-based capital purposes by removing the amount of non-tabular loss reserve discounts from surplus (and adding them to reserves). Tabular loss reserve discounts do not affect the company's reported surplus for risk-based capital purposes.[77]

Company Action Level

The company action level is 75% to 100% of the risk-based capital standard, or 150% to 200% of the authorized control level benchmark. (The figures here assume an ACL benchmark equal to 50% of the risk-based capital surplus, as will be true at the end of the phase-in period.) If the company's adjusted surplus is within the company action range, no action is required of the state insurance department. Rather, the company must submit a plan of action to the insurance commissioner of the domiciliary state, explaining how the company intends to obtain the needed capital or to reduce its operations or risks to meet the risk-based capital standards.

Regulatory Action Level

The regulatory action level is 50% to 75% of the risk-based capital standard, or 100% to 150% of the ACL benchmark. The

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[77] In addition, a property/casualty insurance company which owns a life insurance subsidiary may make the same adjustments to its surplus that the life insurance subsidiary makes to its surplus. These adjustments are to add back the asset valuation reserve and one-half of the policyholder dividends liability.

company's action is the same as at the "company action level": it must submit a plan to the insurance commissioner explaining how it intends to raise its risk-based capital ratio. If the company's adjusted surplus is within the regulatory action level range, then the commissioner has the right to take corrective action against the company, such as by restricting new business. However, all action by the state insurance department is discretionary; nothing is mandated by the risk-based capital formula or associated statutes.

Authorized Control Level

The authorized control level is 35% to 50% of the risk-based capital standard, or 70% to 100% of the ACL benchmark. If the company's adjusted surplus is within the ACL range, regulatory action is still discretionary, but the insurance commissioner is "authorized" to take control of the company.

Mandatory Control Level

The extreme level of regulatory action, the mandatory control level, is below 35% of the risk-based capital requirements, or below 70% of the ACL benchmark. If the company's (adjusted) actual surplus is below 70% of the authorized control level, then the insurance commissioner of the domiciliary state *must* rehabilitate or liquidate the company.[78]

Implementation

Past NAIC practice has been to propose "model laws" that are enacted by each state's legislature. This procedure allows full state discretion in reformulating the statute, but it also leads to long delays and inconsistencies between states.

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[78]The actual wording of the NAIC Risk-Based Capital Model Act is quite complicated. See Sections 3 through 6, which contain detailed instructions for these four "event levels": company action level event, regulatory action level event, authorized control level event, and mandatory control level event (pp. 312-5 through 312-9 of the January 1995 version of the NAIC Model Regulation Service notebook).

Two changes have therefore been made for risk-based capital.

- The proposed model law will not specify the risk-based capital formula, since then each state legislature might make changes and pass a different formula. In addition, as changes are made to the risk-based capital formula, each state would have to modify its statutes. Rather, the model law will make the NAIC risk-based capital requirements the statutory capital requirements in that state.

  To ensure uniform adoption of the risk-based capital standards among the states, the statutory Annual Statement instructions were revised to require disclosure of the NAIC "authorized control level risk-based capital" and the "adjusted surplus" on lines 25 and 26 of the Five Year Historical Data exhibits on pages 22 and 23 of the Annual Statement. Each state already has legislation making the NAIC statutory blank the official insurance company accounting requirement. Thus, each state already has legislation requiring insurers to compute and disclose their risk-based capital figures.[79]

- In late 1990, the NAIC adopted a "Solvency Policing Agenda." One part of this agenda says that a state's insurance department will be accredited by the NAIC only if it passes the required model laws. In December 1993, the NAIC amended its life insurance risk-based capital model law to make it applicable to non-life insurers, and this new model law became part of the NAIC accreditation standard in June 1994. Since the states desire accreditation, passage of the risk-based capital model law should be swift.

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[79] A parallel procedure was used to force the adoption by all states of the Statement of Actuarial Opinion. In the 1970s and 1980s, many states independently passed legislation requiring an opinion regarding loss reserve adequacy from an actuary or a loss reserve specialist, leading to a motley set of requirements in these states. In 1991, the NAIC revised the Annual Statement Instructions to require a Statement of Actuarial Opinion, leading immediately to uniform requirements in all states.

Purposes of the Risk-Based Capital Standards

There are five potential uses of the risk-based capital standards. They are ranked below from the intended purposes to those uses that are expressly prohibited by the NAIC.

1. *Minimum Capital Requirements*: The risk-based capital requirements replace (or supplement) the existing ad hoc minimum capital and surplus requirements with standards that reflect the operations of each company and the risks that it faces.

2. *Solvency Monitoring*: The risk-based capital standards serve as an additional tool in the insurance commissioner's solvency monitoring repertoire, to be used in conjunction with more comprehensive financial examinations.

3. *Legal Authority*: The risk-based capital model act provides the insurance commissioner with legal authority to intervene in a company's operations if it appears to be financially troubled.

4. *Rate-Making*: The risk-based capital formula might be used to determine the needed capital for a "return on equity" rate filing.

5. *Marketing*: The risk-based capital ratio might be used as a marketing tool to identify "stronger" or "weaker" companies, either by the companies themselves or by independent agents and financial analysts.

The last two uses have been expressly prohibited by the NAIC, as illustrated by the June 1993 statement of the NAIC Working Group:

> Since the formula is intended to identify insurers that require regulatory attention and does not purport to compute a target level of capital, the Working Group

> does not believe the results of this formula should be used in setting or reviewing premium rates or in determining an appropriate rate of return for an insurer. Furthermore, this formula should not be used to rate insurers, as many other factors must be taken into consideration in such an evaluation.[80]

The first three purposes listed above are summarized by Vincent Laurenzano [37, p. 100] as follows:

> The primary objective of the NAIC's risk-based capital project is to raise the safety net that statutory surplus provides for policyholder obligations. This enhancement of statutory surplus is to be accomplished by replacing the current fixed minimum capital requirements with a flexible capital standard that is related to the size and the risk profile of an insurer's balance sheet and underwriting activities. For property and casualty insurers the intent is to set a threshold level of capital, based upon industry performance and individual insurer characteristics, which will raise the statutory capital level from its current generally low and arbitrary amounts to a realistic base. The proposed capital standard will enable regulators to more effectively use statutory remedies and, in conjunction with

[80]See also Sections 8B and 8C of the NAIC Risk-Based Capital Model Act:

- Section 8B: It is the judgment of the legislature that the comparison of an insurer's Total Adjusted Capital to any of its RBC levels is a regulatory tool which may indicate the need for possible corrective action with respect to the insurer, and is not intended as a means to rank insurers generally. Therefore … the making, publishing … of any advertisement, announcement, or statement … with regard to RBC levels of any insurer … is prohibited.
- It is the further judgment of the legislature that the RBC Instructions, RBC Reports, Adjusted RBC Reports, RBC Plans, and Revised RBC Plans are intended solely for use by the commissioner in monitoring the solvency of insurers and the need for possible corrective action with respect to insurers and shall not be used by the commissioner for ratemaking nor considered or introduced as evidence in any rate proceeding nor used by the commissioner to calculate or derive any elements of an appropriate premium level or rate of return for any line of insurance which an insurer or any affiliate is authorized to write.

the array of other solvency tools, hasten intervention into troubled situations.

Therefore the goals of the NAIC's risk-based capital project are to:

- relate capital and surplus requirements of an insurer to the risks inherent in its particular operations;
- establish a universally recognized capital standard; and
- provide regulators with the authority to enforce compliance with more appropriate capital requirements.[81]

10. CONCLUSION

The risk-based capital requirements are the product of the combined efforts of regulators and actuaries. Regulators had the authority to set capital requirements for insurance companies, and actuaries had the expertise to determine appropriate parameters for each charge.

Many parts of the risk-based capital formula reflect the contributions of casualty actuaries, from the six-category structure of the covariance adjustment to individual charges (such as the growth charge) or components of charges (such as the loss concentration factor, the claims-made business offset, or the loss-sensitive contract offset).

The present risk-based capital formula is but the first step in the actuarial analysis of financial strength. Numerous other for-

[81] A similar perspective is reflected in Barth [7, p. 3]: "The NAIC RBC system operates in two basic fashions. First, it acts as a tripwire system that gives regulators clear legal authority to intervene in the business affairs of an insurer that triggers one of the warning levels. As a tripwire system, RBC alerts regulators to undercapitalized companies while there is still time for the regulators to react quickly and effectively to minimize the overall costs associated with an insolvency. Secondly, the RBC results may be used to intervene in the management of a company that is found to be in hazardous condition during the course of an examination."

mulas and models are now being developed by actuaries under the rubric of "dynamic financial analysis." The development of the NAIC risk-based capital system shows the theoretical potential and practical limitations of one type of solvency monitoring system. Readers of this paper should now have a better grasp of what has been accomplished, as well as a determined but realistic view of what may yet be achieved.

11. AN ILLUSTRATION

The risk-based capital formula has many interlocking pieces. This section provides a fully documented illustration, showing the capital requirements for a hypothetical insurance company, to help the reader understand the components of the formula.

The NAIC provides a risk-based capital diskette to each domestic insurance company. The exhibits in this illustration are based directly on the NAIC diskette for the 1995 risk-based capital submission, which was due in early 1996.

Most of the NAIC diskette is automated: the company copies entries from the financial statements to the diskette, and the spreadsheet calculates the risk-based capital charges. For a few cells, such as the number of issuers for the bond size adjustment factor, there is no corresponding entry in the financial statements, and the company must provide the required figures.

The *NAIC Instructions* contain all the cross-references between the risk-based capital diskette and the Fire and Casualty Annual Statement. These cross-references are not repeated here.

Certain factors, such as the reserving risk industry-wide adverse development factors and the interest discount factors, are promulgated by the NAIC. The method of deriving these factors is covered in the text of this paper. Since many of these factors involved judgment, they cannot be replicated by others, and their derivation is not illustrated here. These factors are hard-coded into the NAIC diskette.

This illustration follows the rounding and presentation formats used in the NAIC diskette. In general, although intermediate values are shown in rounded format, actual values are kept with full precision and the final risk-based capital requirements are calculated to the dollar. Thus, there are numerous rounding discrepancies in the exhibits and the documentation. To replicate the final risk-based capital requirements, the reader should recalculate the intermediate values with greater precision.

Simplifications

There are several minor differences between the entries required of the company and the illustration shown here.

- For the reserving risk and written premium risk components, the company enters the historical information from Schedule P. The risk-based capital spreadsheet determines the "company average development" and the "company average loss ratio" by line of business. (The "industry average development" and the "industry average loss ratio" by line of business are promulgated by the NAIC, and they are hard-coded in the spreadsheet.) This illustration does not show the calculation of these factors, since the text of this paper provides an example. Instead, the illustration assumes that these figures are given.

  In addition, certain logical values are calculated by the spreadsheet. For instance, for the written premium charge, the spreadsheet seems to ask, "Does the company pass the *de minimus* test?" This is *not* an input cell. Rather, the user enters the Annual Statement premium figures for each accident year, and the spreadsheet determines if the company passes the *de minimus* test.[82]

- Certain exhibits are abbreviated in this illustration. For instance, the reserving risk and written premium risk charges

[82]The reader should consult the *NAIC Instructions* to see which cells must be entered directly and which are calculated by the spreadsheet. This paper is not intended as a "how-to manual" for completing the risk-based capital submission.

consider all the Schedule P lines of business. This illustration uses only the first six lines of business, and the documentation discusses only three of these. Showing more lines of business simply complicates the illustration and adds no more educational information.

- The risk-based capital charges for investments in affiliates can be exceedingly complex, particularly for large, multi-layered insurance groups. This illustration makes no attempt to cover the various potential situations. Rather, it assumes that the insurance company is the sole owner of several subsidiaries, whose book value and risk-based capital requirements are given. The intention is to illustrate how the risk-based capital formula deals with investments in affiliates, not to illustrate all the possible variations.

Order

This illustration follows the format of the NAIC exhibit. It covers

- asset risk charges for unaffiliated investments;
- investments in affiliates;
- credit risk charges: reinsurance recoverable and other receivables;
- reserving risk charges;
- written premium charges;
- off balance sheet risks and growth charges;
- the covariance adjustment; and
- summary.

The format of the NAIC exhibits is sometimes confusing. For instance, the "asset risk charges" exhibits have entries for both the R_1 and the R_2 risk components, and the exhibits do not always clearly separate them.

Investments

Calculation of the investment risk charges may be divided (conceptually) into three steps:

1. investments in unaffiliated enterprises;
2. adjustments to the RBC charges for these investments: the asset concentration factor and the bond size factor; and
3. investments in affiliated enterprises.

Unaffiliated Investments

The basic risk-based capital charge for investments in unaffiliated enterprises is the statement value of the investment times the RBC factor. The RBC factors differs (a) by type of investment and (b) by quality classification of the investment. Two additional charges are then included: a bond size factor charge and an asset concentration charge.

Bond Investments

Exhibit 1 shows the risk-based capital requirements for investments in bonds of unaffiliated enterprises.

- The company enters the statement values in the first numeric column.
- The RBC factors in the second numeric column are hard-coded into the spreadsheet.
- The risk-based capital charge in the third numeric column is the product of the entries in the first two columns.

The exhibits throughout this illustration are intended to highlight the major sources of risk, not necessarily to reflect prevalent industry practice. For instance, the bond risk charges are high only for bonds below investment grade quality. In this illustration, the company owns $35,000,000 of bonds that are “in or near default" (Class 06).[83] This set of bonds gives a risk-based capital charge of $10,500,000, which is almost half of the total bond charge (before the bond size adjustment factor) of $21,800,000 ($3,300,000 + $18,500,000).

Class 01 bonds are not subject to the bond size factor. For the remaining bonds, there are 227 issuers in this example. (The company must enter this number. It is not readily available from other Annual Statement exhibits, except by counting individual issuers.)

The bond size adjustment factor is calculated as

$$[(50 * 250\%) + (50 * 130\%) + (127 * 100\%)] \equiv 227 = 139.65\%.$$

In other words, the risk-based capital charge for bonds subject to the bond size factor, or $18,500,000, must be multiplied by 1.3965. The NAIC exhibit shows this as an additive factor, not a multiplicative factor. That is, the $18,500,000 is multiplied by 0.3965 to give $7,334,802, and this product is added to the other bond charges to give a total of $29,134,802. This figure, along with mortgages, other loans, and part of the asset concentration charge, becomes the R_1 component for the square root rule.

Unaffiliated Stock

The investments in unaffiliated stocks are divided between preferred stock and common stock, as shown in Exhibit 2. The risk-based capital charges for preferred stock are similar to those for corporate bonds with an additional 2% charge in each quality class (except for Class 06, which already has the maximum charge of 30%).

Investments in unaffiliated common stocks have a risk-based capital charge of 15%. Investments in non-government money market funds have a charge of 0.3%.

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[83]This illustration is heuristic only, with large amounts of Class 06 and Class 04 bonds (so that there are significant charges) and few other corporate bonds (so that there is a significant bond size factor).

The charges for preferred stock and common stock, along with the charges for other equities (such as real estate) and part of the asset concentration charge, becomes the R_2 component for the square root rule.

Other Investments

Exhibit 3 shows investments in several other types of securities, divided between long-term assets and short-term assets:

- real estate;
- mortgages;
- other long-term (Schedule BA) assets;
- collateral loans;
- cash; and
- other short-term investments.

As is true for investments in bonds and stocks, the RBC factors are hard-coded, the statement values are entered by the company, and the RBC charges are the products of these two figures. Some of these charges are included in the R_1 risk component and some are included in the R_2 risk component.

Asset Concentration Charges

The asset concentration worksheet doubles the risk-based capital charges for investments from the ten largest issuers. Investments that have less than a 1% risk-based capital charge, such as government bonds, are not included. Similarly, investments that already carry the maximum risk-based capital asset risk charge of 30%, such as Class 06 corporate bonds, are not included. In addition, affiliated common stock, preferred stock, affiliated bonds, and home office properties are excluded.

The remaining assets are grouped by issuer to determine the ten largest groups. The insurance company may hold both stocks and bonds from the same issuer, as in the first several examples in Exhibit 5. The stocks and bonds are combined to determine the ten largest issuers.

The asset concentration factors are shown in Exhibit 4. These are the same factors as for the original investments. Thus, the asset concentration procedure doubles the charge for these investments.

The "additional RBC" charges shown in Column 4 of Exhibit 5 are subtotaled into fixed income charges and equity charges, and they are included in the R_1 risk component and the R_2 risk component, respectively.

Investments in Subsidiaries

To illustrate the treatment of the risk-based capital charges for investments in affiliates, this illustration shows several subsidiaries: two directly-owned U.S. property/casualty insurance subsidiaries, one indirectly-owned U.S. property/casualty insurance subsidiary, the holding company that owns this property/casualty insurer and that has a value in excess of the indirectly-owned subsidiary, one alien insurer, and one investment subsidiary.

The risk-based capital charges are shown in Exhibit 6, and additional detail is shown in Exhibit 7. The charges for the insurance subsidiaries are included in the R_0 risk component, which is outside the square root procedure in the covariance adjustment. The charge for the holding company's value in excess of the indirectly-owned subsidiary is an equity charge, so it is included in the R_2 risk component. One "looks through" the investment subsidiary to the stocks (or bonds) that it owns. In other words, the equity risk charge for the stocks owned by this investment subsidiary is passed up to the parent company's R_2 risk category.

Because the R_2 risk component is relatively small in this illustration (as is true for most U.S. insurance companies) relative to the reserving risk charge and the written premium risk charge, the marginal effect of each dollar of R_2 risk charge after covariance is weak.

Thus, the relative effect of the risk charge for each affiliate is more extreme than it appears in Exhibit 6. The charges included in the R_0 risk component are powerful. The charges included in the R_2 risk component are diluted by the square root rule.

Credit Risk

The credit risk Exhibit 8 has two sections. The bottom section lists five miscellaneous receivables from page 2 of the Annual Statement:

- federal income tax recoverable (page 2, line 13);
- interest, dividends, and real estate income due and accrued (page 2, line 15);
- amounts recoverable from parents, subsidiaries, and affiliates (page 2, line 16);
- amounts receivable related to uninsured accident and health plans (page 2, line 18); and
- aggregate write-ins for other than invested assets (page 2, line 20).

The statement values in Column 4 are entered by the company from its Annual Statement balance sheet. The RBC factors in Column 5 are hard-coded in the spreadsheet. The risk-based capital requirements in Column 6 are the products of the entries in the preceding two columns.

Ceded Reinsurance

The top section of Exhibit 8 displays the charge for reinsurance recoverables, which is ten percent of the outstanding balance. As discussed in the text of this paper, there are several modifications to this charge.

- There is no charge for reinsurance recoverable from U.S. affiliates. As the first and fifth rows of this section indicate, recoverables from *non-U.S.* affiliates only are listed.
- There is no charge for reinsurance recoverables from involuntary (residual market) pools. As the third and seventh rows of this section of the exhibit indicate, recoverables from *voluntary* pools only are listed.
- There is no charge for reinsurance recoverables from certain voluntary pools and associations. The NAIC Working Group explains that

> Not all voluntary pools receive the reinsurance RBC charge. List those pools for which an exemption is claimed in the table below. The sum of the ceded balances in the table below and the sum of the ceded balances in the RBC table above should equal the total in lines 0799999 and 1699999 of Schedule F Part 3.

The Provision for Reinsurance

- The statutory provision for reinsurance (that is, the "Schedule F penalty") is deducted from the reinsurance recoverables before application of the risk-based capital charge.[84] To do otherwise would double-count the liability or the capital requirement.

  In the illustration, the unadjusted recoverable is shown in Column 1 in the upper half of the exhibit, the provision for reinsurance is shown in Column 3, and the difference, which is the "amount subject to RBC," is shown in Column 4.

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[84]For the computation of the "provision for reinsurance," see Feldblum [21].

For the unaffiliated reinsurers, there are various Schedule F penalties shown in the exhibit.

- The largest authorized unaffiliated reinsurer has been classified as "slow-paying" for this ceding company, and its balances are not secured. Thus, there is a large Schedule F penalty on line 2, and the ceded balances subject to RBC are $18,500,000.
- For the recoverables from domestic unaffiliated unauthorized reinsurers in line 6, or $10,000,000, 80% is secured by funds withheld or letters of credit. The Schedule F penalty is $2,000,000, and the ceded balances subject to risk-based capital charges equal $8,000,000.
- Very little of the recoverables from non-domestic unaffiliated unauthorized reinsurers in line 8, or $7,500,000, is secured by funds withheld or letters of credit. The Schedule F penalty is large ($6,500,000) and the ceded balances subject to risk-based capital charges are small ($1,000,000).

The figures in Column 4, the "amounts subject to RBC," are multiplied by the credit risk factor of 10% to give the figures in Column 5, the "RBC requirements." These amounts are summed to give the risk-based capital charge of $4,750,000 for reinsurance recoverables. To this is added the charge for miscellaneous recoverables to give the total credit risk RBC of $4,885,000.

Reserving Risk Charge

The risk-based capital underwriting risk charges use all the Schedule P lines of business. For simplicity, the first six of these lines are shown on Exhibit 9. In order to illustrate the various adjustments that must be considered, the computation of the charges for three of these lines is described below.

- *Private Passenger Automobile Liability* is the company's largest line in premium volume and second largest line in reserve volume. The underwriting risk charges for this line use the standard formula, with no adjustments (in this illustration) for loss-sensitive contracts or for claims-made business.

- *Workers Compensation*, one-fifth of whose business is written on retrospectively rated plans, receives the loss-sensitive contract offset on this portion.

- *Medical Malpractice* is partly written on occurrence forms, to which the full capital charges apply, and partly written on claims-made forms, to which the 20% claims-made reduction applies.

All dollar amount entries are in thousands, since the figures are from Schedule P, whose entries are in thousands of dollars. The final risk-based capital charges, however, are converted back to whole dollars.

The sixth row of figures on the exhibit shows "Loss + LAE Unpaid Sch P Part 1 (in 000s)." In the tenth and eleventh rows of figures, the company enters the percentage of reserves for accidents relating to loss-sensitive business, such as retrospectively rated workers compensation policies or reinsurance treaties with sliding scale commission rates. In 1994, this information was not found elsewhere in the Annual Statement. In 1995, a new Part 7 was added to Schedule P to provide this information (Part 7, Section 1, Column 4).[85] The illustration assumes that one-fifth of the workers compensation business is written on retrospectively rated plans, and one-fifth of the reserves are for accidents relating to such business.

The thirteenth row of figures shows the reserves relating to business written on claims-made forms. The claims-made risk charge reduction applies to medical malpractice business only.

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[85] See Feldblum [20] for further discussion of this statutory exhibit.

The illustration assumes that slightly more than half the company's medical malpractice business is written on claims-made forms, but claim reserves for these policies constitute only 25% of the line's reserves.

The figures appearing on the spreadsheet are rounded, though the actual entries and the computations use unrounded numbers. The use of the unrounded numbers in the spreadsheet cells enables us to obtain the "net loss + LAE RBC" of $320,157,630 in the Total column of Row 17.

Private Passenger Auto Liability

If there were no company adjustment, then the reserving risk charge would equal

$$\text{Reported Reserves} * [(1 + \text{RBC Charge})(\text{Discount Factor}) - 1].$$

As long as the company has the required historical experience, the RBC charge is modified by the company's own experience. The adjusted RBC charge is the average of the industry RBC charge and the company RBC charge. The company RBC charge is the industry RBC charge times the ratio of the company average development factor to the industry average development factor.

Exhibit 9 shows these computations. The "Industry RBC Percentage" of 0.254, which is based on 1982–1991 experience, is hardcoded into the spreadsheet; it does not change from year to year. The industry average development factor is based on Schedule P experience from the previous year's Annual Statements. Thus, for the 1995 risk-based capital spreadsheet, this figure is based on data from the 1994 Annual Statements. It is hard-coded into the spreadsheet, but it changes from year to year. For personal auto, the figure was 1.032 for the 1995 risk-based capital submission.

The "Company Average Development" is based on data from the current year's Annual Statement. It is not hard-coded into the spreadsheet, since it varies from company to company. In this illustration, the figure for personal auto is 1.150.

The ratio of the company average development to the industry average development is $1.150 \equiv 1.032 = 1.114$, as shown on Row 3. The adjusted RBC percentage (which is not shown on the spreadsheet) is $0.254 * 1.114 = 0.283$. The "Company RBC Percentage," which is the average of the adjusted RBC percentage and the industry RBC percentage, is $(0.254 + 0.283) \equiv 2 = 0.2685$. This figure is shown on Row 5.

The adjustment factor used to convert the "Industry RBC Percentage" to the "company RBC percentage" can also be computed in a single step as

$$[(\text{industry average development factor} + \text{company average development factor}) \equiv 2] \equiv \text{industry average development factor}.$$

In this illustration, the figures are

$$[(1.032 + 1.150) \equiv 2] \equiv 1.032 = 1.05717.$$

The "company RBC percentage" equals

$$0.254 * 1.05717 = 0.268.$$

The reserving risk charge equals

$$\text{Reported Reserves} * [(1 + \text{Company RBC Charge})(\text{Discount Factor}) - 1].$$

For this illustration, the figures are

$$\$600{,}000{,}000 * \{[1 + (0.268)](0.921) - 1\} = \$100{,}984{,}880.$$

Workers Compensation

Of the company's $1,250,000,000 in workers compensation reserves, 20%, or $250,000,000, is for accidents related to business written on retrospectively rated plans. This business gets the 30% reduction for loss-sensitive contracts,[86] so the final risk-based capital charge is multiplied by

$$1 - (30\%) * (20\%) = 94\%.$$

For workers compensation, the adjustment for the company's experience is

$$[(1.066 + 1.050) \equiv 2] \equiv 1.066 = 0.9925.$$

The reserving risk charge, before the offset for loss-sensitive contracts, equals

$$\text{Reported Reserves} * [(1 + \text{Company RBC Charge})(\text{Discount Factor}) - 1].$$

For this illustration, the figures are

$$\$1{,}250{,}000{,}000 * \{[1 + (0.9925)(0.273)](0.872) - 1\} = \$135{,}336{,}829.$$

Multiplying this figure by 94% yields the final risk-based capital charge of $127,216,620. The spreadsheet shows this computation in two steps. The loss-sensitive discount on Row 12 equals the percentage of loss-sensitive business times the loss-sensitive business offset factor times the "Base Loss + LAE Reserve RBC" (Row 9), or

$$20\% * 30\% * \$135{,}336{,}829 = \$8{,}120{,}210.$$

The final reserving risk charge is $135,336,829 – $8,120,210 = $127,216,620.

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[86] There are separate lines for the *direct* loss-sensitive business (Row 10) and the *assumed* loss-sensitive business (Row 11), since the loss-sensitive contract offset is 30% for direct business and 15% for assumed business. See the text of this paper for the justification of this differentiation.

Medical Malpractice

The medical malpractice charge has an offset of 20% for claims-made policies. Since 25% of the $400,000,000 of reserves (or $100,000,000) is for claims relating to business written on claims-made forms, as shown on Row 13, the final risk-based capital charge is multiplied by

$$1 - (20\%) * (25\%) = 95\%.$$

For medical malpractice, the adjustment for the company's experience is

$$[(1.028 + 1.200) \equiv 2] \equiv 1.028 = 1.084.$$

The reserving risk charge, before the offset for claims-made business, equals

$$\text{Reported Reserves} * [(1 + \text{Company RBC Charge})(\text{Discount Factor}) - 1].$$

For this illustration, the figures are

$$\$400{,}000{,}000 * \{[1 + (1.084)(0.565)](0.808) - 1\} = \$121{,}084{,}545.$$

Multiplying this figure by 95% yields the final risk-based capital charge of $115,030,318.

The spreadsheet shows the computation in two steps. The "Claims-Made Discount" on Row 14 equals the percentage of claims-made business times the claims-made offset factor times the "Base Loss + LAE Reserve RBC," or

$$25\% * 20\% * \$121{,}084{,}545 = \$6{,}054{,}227.$$

The final risk-based capital charge is $121,084,545 – $6,054,227 = $115,030,318.

The computations for the other lines shown on the exhibit, homeowners/farmowners, commercial auto liability, and commercial multi-peril, have no additional features beyond those already discussed.

Loss Concentration Factor

The sum of the reserving risk charges for the six lines of business is \$374,611,461, shown on Row 15. The loss concentration percentage is the ratio of unpaid losses and LAE for the largest line, or \$1,250,000,000 for workers compensation, to unpaid losses and LAE for all lines combined, or \$2,425,000,000. This ratio is 0.515464. This figure is *not* shown on the exhibit.

The adjustment for diversification by line, or the loss concentration factor, is

$$70\% + 30\% * (\text{Loss Concentration Percentage}).$$

In this illustration, the adjustment is

$$70\% + 30\% * 51.5464\% = 85.464\%.$$

Multiplying this factor by the unadjusted charge of \$374,611,461 gives the "Net Loss + LAE Risk-Based Capital Charge" of \$320,157,630 in the Total column of Row 17.

Written Premium Charge

Exhibit 10 shows the same lines of business as Exhibit 9. We discuss the same three lines as for the reserving risk charge.

The format of the written premium risk charge exhibit is similar to that of the reserving risk charge exhibit. The "Industry Loss & LAE Ratio" on Row 4 is based on 1982–1991 historical experience. It is hard-coded into the spreadsheet, and it is not updated each year.

The "Industry Average Loss and LAE Ratio" on Row 1 is based on Schedule P experience from the year prior to the current valuation date. It is hard-coded into the spreadsheet, but it is updated from year to year. For the 1995 risk-based capital computations, the entries are based on 1994 Schedule Ps.

The "Company Average Loss and LAE Ratio" on Row 2 is based on the company's current Schedule P experience. The "Company Underwriting Expense Ratio" on Row 6 is an all lines combined average (25% in this illustration), and it is based on information in the current Annual Statement.

Row 8 of the exhibit shows the net written premium by line of business in the most recent calendar year. (In the theory of the risk-based capital formula, this figure serves as a proxy for the net written premium in the coming twelve months.) Rows 10 and 11 show the percentage of premium written on loss-sensitive contracts. The figures are shown separately for direct business, which has an offset factor of 30%, and for assumed business, which has an offset factor of 15%.

Row 13 shows the percentage of claims-made written premium. This entry is relevant only for medical malpractice.

Private Passenger Auto Liability

If there were no company adjustment to the written premium risk charge factor, and no offsets for loss-sensitive contracts or for claims-made business, the written premium charge would be

$$\begin{aligned}&\text{Written Premium}\\&\quad *\{[(\text{Industry Loss \& LAE Ratio} * \text{Discount Factor})\\&\qquad + \text{Expense Ratio}] - 1\}\end{aligned}$$

The industry loss & LAE ratio is the "worst case year," not the average industry loss and LAE ratio. The discount factor is applied to the loss and LAE ratio, since loss and loss adjustment expenses are paid out over time. It is not applied to the expense ratio, since most expenses are paid out up front. Note the difference in the structure of the charge between the reserving risk charge and the written premium risk charge. The reserving risk RBC factor measures just the adverse development, so the discount factor is applied to "unity plus the RBC factor." In the written premium risk charge, there is no "unity plus" that needs to be added.

As long as the company has the required years of experience and it passes the *de minimus* test, the RBC charge factor is modified by the company's own experience. Specifically, the adjustment is

$$\begin{aligned}&[(\text{Industry Average Loss Ratio}\\&\quad + \text{Company Average Loss Ratio}) \equiv 2]\\&\qquad \equiv \text{Industry Average Loss Ratio.}\end{aligned}$$

In this illustration, the figures are

$$[(0.931 + 0.982) \equiv 2] \equiv 0.931 = 1.027.$$

The adjusted loss and LAE ratio is $1.046 * 1.027 = 1.075$. This is shown on Row 5 as the "Company RBC Loss & LAE Ratio."

The capital charge is

$$\begin{aligned}&\text{Written Premium}\\&\quad * \{[(\text{Adjusted RBC Charge Factor} * \text{Discount Factor})\\&\qquad + \text{Expense Ratio}] - 1\}\end{aligned}$$

In this illustration, the figures are

$$\begin{aligned}&\$800{,}000{,}000 * \{[(1.075 * 0.924) + 0.250] - 1\}\\&\quad = \$194{,}381{,}161.\end{aligned}$$

Workers Compensation

Of the company's $500,000,000 in workers compensation written premium, 20%, or $100,000,000, is written on retrospectively rated plans. This business gets the 30% reduction for loss-sensitive contracts, so the final risk-based capital charge is multiplied by

$$1 - (30\%) * (20\%) = 94\%.$$

The adjustment to the RBC Charge Factor is

$$[(0.901 + 0.850) \equiv 2] \equiv 0.901 = 0.9717.$$

The adjusted loss & LAE ratio is $0.9717 * 1.008 = 0.9795$.

The capital charge before the loss-sensitive contract offset is

$$\begin{aligned}&\text{Written Premium}\\ &\quad * \{[(\text{Adjusted RBC Charge Factor} * \text{Discount Factor})\\ &\qquad + \text{Expense Ratio}] - 1\}\end{aligned}$$

In this illustration, the figures are

$$\begin{aligned}&\$500{,}000{,}000 * \{[(0.9795 * 0.836) + 0.250] - 1\}\\ &\quad = \$34{,}419{,}170.\end{aligned}$$

Multiplying by 94% gives the final charge of $32,354,020. The spreadsheet shows this in two steps. The loss-sensitive discount is 6% of $34,419,170, or $2,065,150. The final written premium charge is $34,419,170 – $2,065,150 = $32,354,020.

Medical Malpractice

The medical malpractice charge has an offset of 20% for claims-made business. Since 53.3%, or $80,000,000, of the $150,000,000 premium is written on claims-made forms, the final risk-based capital charge is multiplied by

$$1 - (20\%) * (53.3\%) = 89.33\%.$$

The company adjustment to the RBC Charge Factor is

$$[(0.955 + 0.984) \equiv 2] \equiv 0.955 = 1.0152.$$

The adjusted loss & LAE ratio is $1.0152 * 1.472 = 1.494$.

The capital charge before the claims-made business offset is

$$\text{Written Premium} \\ * \{[(\text{Adjusted RBC Charge Factor} * \text{Discount Factor}) \\ + \text{Expense Ratio}] - 1\}.$$

In this illustration, the figures are

$$\$150{,}000{,}000 * \{[(1.494 * 0.778) + 0.250] - 1\} \\ = \$61{,}890{,}615.$$

Multiplying by 89.33% gives the final charge of $55,294,075. As for workers compensation, the spreadsheet shows the two-step format.

Premium Concentration Factor

The sum of the written premium risk-based capital charges for the six lines of business is $339,258,714. The premium concentration percentage is the ratio of written premium for the largest line, or $800,000,000 for private passenger auto liability, to written premium for all lines combined, or $1,800,000,000. This ratio is $8 \equiv 18 = 0.444$.

The adjustment for diversification by line, or the "premium concentration factor," is

$$70\% + 30\% * (\text{Premium Concentration Percentage}).$$

In this illustration, the adjustment is

$$70\% + 30\% * 44.4\% = 83.3\%.$$

Multiplying this factor by the unadjusted charge of $339,258,714 gives the net written premium risk-based capital charge of

$282,715,595 at the bottom of the exhibit (Row 17, Total column).

Off Balance Sheet Risks

The "miscellaneous off balance sheet items" in Exhibit 11 show three charges:

- noncontrolled assets, from General Interrogatory #20;
- guarantees for affiliates, from Note 4 to the financial statements; and
- contingent liabilities, from Note 8 to the financial statements.[87]

The risk-based capital factor is 1% for each of these, which is hard-coded into the second numeric column of the exhibit. The figures in the first numeric column, "Statement Value," are entered by the company. The RBC charges in the third numeric column are the products of the entries in the first two columns. These charges are included in the R_0 risk category.

The only miscellaneous off balance sheet item in this illustration stems from a suit against the company, unrelated to its insurance operations, seeking $15,000,000 in damages. The company believes that it has no liability; no entry is made to the balance sheet, though a disclosure is made in the notes to the financial statements. The risk-based capital charge is 1% of this amount, or $150,000.

Excessive Growth

The excessive growth RBC charge depends upon the rate of premium growth during the past three years for the group of which the company is a member. This is the only place where consolidated group figures are used in the risk-based capital calculation. Insurance company fleets sometimes shift an entire

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[87]The interrogatory numbers and financial statement note numbers are for the 1995 Annual Statement. The numbers may be different in subsequent years.

block of business from one member to another member. If individual company premium were used to determine excessive growth, this shift of business would show up as a surge in growth, when in fact there is no additional risk.

The excessive growth charge depends upon *gross* written premiums, not net written premiums. New insurers will often use much pro-rata reinsurance to lessen their risks and to gain underwriting assistance from the reinsurers. As these new insurers mature, they will eliminate much of the reinsurance coverage, in order to retain more of the profits from their book of business.

The excessive growth charge relates to the presumed unfamiliarity of the insurance company with the underwriting or reserving characteristics of a new book of business. This unfamiliarity is reflected in the growth of gross written premium, regardless of whether the insurer is reinsuring part of the risk. Of course, the growth charge is applied to net written premium and net loss reserves, so if the reinsurer has indeed transferred the underwriting and reserving risks to reinsurers, it will have no additional capital requirements.

Conversely, when the primary insurer takes down its reinsurance coverage, its net business has indeed increased. But this is not growth that reflects unfamiliarity with the characteristics of the book of business, so it does not affect the calculation of the growth charge factor. Of course, since the primary company is retaining more of the business, its risks have increased, so any growth charge factor that it does have (from increases in gross written premium), as well as the standard written premium and reserving risk factors, are applied to a larger volume of net written premium or net loss reserves.

The company enters the gross written premium figures for the consolidated group in the first four rows of Exhibit 12. The spreadsheet calculates:

- three individual year group growth rates (for the three most recent years);
- three average group growth rates (latest year, latest two years, and latest three years); and
- the selected group growth rate.

Each individual year's group growth rate is that year's gross written premium divided by the previous year's gross written premium. In the illustration, these rates are 18%, 17%, and 14%.

The average growth rates are arithmetic averages. For instance, the two year average is the average of the current year's group growth rate and the previous year's group growth rate.

The selected growth rate is the three-year average if it exists (i.e., if the group has been in business for more than three years). Otherwise, it is the two-year average, if it exists; otherwise it is the one-year rate, if it exists. In the illustration, the three-year average growth rate of +16.3% exists, so it is selected.

The excess growth rate is the selected growth rate minus 10%. The excess growth rate is capped below at 0% and above at 30%. (This is equivalent to capping the selected growth rate at 40%, as discussed in the text of this paper.) For a company that has been in business less than one year, the excess growth rate is 0%. In the illustration, +16.3% – 10.0% = +6.3%, which is the excess growth rate.

The company enters the current year's net loss and LAE unpaid and net written premium.

For the reserving risk portion, the Excessive Growth RBC Charge equals

$$\text{Excessive Growth Rate} * 45\% * \text{Unpaid Losses and LAE}$$
$$= 6.3\% * 45\% * \$2{,}425{,}000{,}000 = \$70{,}325{,}000.$$

In the illustration, the factor of "0.029" is derived as

$$\{[(18\% + 17\% + 14\%) \equiv 3] - 10\%\} * 0.45 = 2.85\%.$$

For the written premium risk portion, the Excessive Growth RBC Charge equals

$$\text{Excessive Growth Rate} * 22.5\% * \text{Net Written Premium}$$
$$= 6.3\% * 22.5\% * \$1{,}800{,}000{,}000 = \$25{,}200{,}000.$$

In the illustration, the factor of "0.014" is derived as

$$\{[(18\% + 17\% + 14\%) \equiv 3] - 10\%\} * 0.225 = 1.425\%.$$

For the covariance adjustment, the loss reserves excess growth RBC will be included in the R_4 risk category, and the written premium excess growth RBC will be included in the R_5 risk category.

Covariance Adjustment

In Exhibit 13, the individual risk-based capital charges are grouped into the six risk categories (total R_0, total R_1, etc.). Line 54, "Total RBC After Covariance," uses these subtotals in the square root rule, as

$$\text{Total RBC After Covariance}$$
$$= R_0 + (R_1^2 + R_2^2 + R_3^2 + R_4^2 + R_5^2)^{0.5}.$$

In the illustration, the total RBC after covariance is $948,037,136. In 1995, the "authorized control level RBC" is 45% of this amount, or $426,616,711, as shown on Line 55. In 1996 and subsequent years, the authorized control level RBC will be 50% of the total RBC after covariance.

Risk-Based Capital Ratio

The company's adjusted capital in this illustration is $1,335,000,000, as shown in Exhibit 14. This figure is derived from the company's policyholders' surplus as recorded in the Annual Statement, along with the adjustments noted in the text of this paper, such as the adjustments for loss reserve discounts.

Since the company's adjusted capital exceeds the company action level, no level of action is indicated. The company's risk-based capital ratio is $1,335,000,000 ≡ $426,616,711, or 3.13. In other words, the company is in reasonable financial condition, though it is not particularly strong.

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EXHIBIT 1

UNAFFILIATED BONDS

		(1) Statement Value		(2) Factor	(3) RBC Requirement
(1)	Class 01—U.S. Government—Direct and Guaranteed	1,200,000,000	×	0.000 =	0
(2)	Class 01—U.S. Government Agency (not backed by full faith and credit of the U.S. government)	1,100,000,000	×	0.003 =	3,300,000
(3)	Other Class 01 Unaffiliated Bonds	0	×	0.003 =	0
(4)	Class 02 Unaffiliated Bonds	350,000,000	×	0.010 =	3,500,000
(5)	Class 03 Unaffiliated Bonds	0	×	0.020 =	0
(6)	Class 04 Unaffiliated Bonds	100,000,000	×	0.045 =	4,500,000
(7)	Class 05 Unaffiliated Bonds	0	×	0.100 =	0
(8)	Class 06 Unaffiliated Bonds	35,000,000	×	0.300 =	10,500,000
(9)	Subtotal—Bonds Subject to Bond Size Factor	485,000,000			18,500,000
(10)	Number of Issuers	227			
(11)	Bond Size Factor				0.40
(12)	Bond Size Factor RBC				7,334,802
(13)	Total Unaffiliated Bonds RBC				29,134,802

EXHIBIT 2

Unaffiliated Preferred and Common Stock

	(1) Statement Value		(2) Factor	(3) RBC Requirement
Unaffiliated Preferred Stock				
(1) Class 01 Unaffiliated Preferred Stock	10,000,000	×	0.023 =	230,000
(2) Class 02 Unaffiliated Preferred Stock	5,000,000	×	0.030 =	150,000
(3) Class 03 Unaffiliated Preferred Stock	0	×	0.040 =	0
(4) Class 04 Unaffiliated Preferred Stock	0	×	0.065 =	0
(5) Class 05 Unaffiliated Preferred Stock	0	×	0.120 =	0
(6) Class 06 Unaffiliated Preferred Stock	0	×	0.300 =	0
(7) Total Unaffiliated Preferred Stock	15,000,000			380,000
Unaffiliated Common Stock				
(8) Non-government Money Market funds	20,000,000	×	0.003 =	60,000
(9) Other Unaffiliated Common Stock	350,000,000	×	0.150 =	52,500,000
(10) Total Unaffiliated Common Stock	370,000,000			52,560,000

EXHIBIT 3

OTHER LONG TERM ASSETS AND MISCELLANEOUS ASSETS

	LONG TERM ASSETS	(1) Statement Value		(2) Factor	(3) RBC Requirement
(1)	Company Occupied Real Estate	50,000,000	×	0.100 =	5,000,000
(2)	Encumbrances	0	×	0.100 =	0
(3)	Investment Real Estate	125,000,000	×	0.100 =	12,500,000
(4)	Encumbrances	0	×	0.100 =	0
(5)	Total Real Estate	175,000,000			17,500,000
(6)	Mortgage Loans	10,000,000	×	0.050 =	500,000
(7)	Schedule BA Assets	10,000,000	×	0.200 =	2,000,000
(8)	Total Long-Term Assets	195,000,000			20,000,000

	MISCELLANEOUS ASSETS	(1) Statement Value		(2) Factor	(3) RBC Requirement
(1)	Collateral Loans	2,500,000	×	0.050 =	125,000
(2)	Cash	5,000,000	×	0.003 =	15,000
(3)	Aggregate Write-ins for Invested Assets	7,500,000	×	0.050 =	375,000
(4)	Short-Term Investments	0	×	0.003 =	0
(5)	Total Miscellaneous Assets	15,000,000			515,000

EXHIBIT 4

Asset Concentration Factors

Type Asset	Factor
Class 02 Unaffiliated Bonds	0.010
Class 03 Unaffiliated Bonds	0.020
Class 04 Unaffiliated Bonds	0.045
Class 05 Unaffiliated Bonds	0.100
Unaffiliated Preferred Stock—Class 01	0.023
Unaffiliated Preferred Stock—Class 02	0.030
Unaffiliated Preferred Stock—Class 03	0.040
Unaffiliated Preferred Stock—Class 04	0.065
Unaffiliated Preferred Stock—Class 05	0.120
Real Estate Excluding Home Office	0.100
Real Estate Encumbrance Excluding Home Office	0.100
Schedule BA Assets	0.200
Aggregate Write-Ins for Invested Assets	0.050
Collateral Loans	0.050
Mortgages	0.050
Unaffiliated Common Stock	0.150

EXHIBIT 5

ASSET CONCENTRATION

(1)	(2) Statement Value	(3) Factor	(4) Additional RBC
ISSUER #1 Transient Industries			
Fixed Income-Type Assets			
1.01 Class 02 Unaffiliated Bonds	5,078,597	× 0.010 =	50,786
1.03 Class 04 Unaffiliated Bonds	4,278,072	× 0.045 =	192,513
1.07 SUBTOTAL—FIXED INCOME	9,356,669		243,299
Equity-Type Assets			
1.08 Unaffiliated Preferred Stock—Class 02	131,493	× 0.030 =	3,945
1.17 Unaffiliated Common Stock	2,806,391	× 0.150 =	420,959
1.18 SUBTOTAL—EQUITY	2,937,884		424,903
1.99 TOTAL—ISSUER #1	12,294,553		668,203
ISSUER #2 Insolvent Savings and Loan			
Fixed Income-Type Assets			
2.01 Class 02 Unaffiliated Bonds	1,344,445	× 0.010 =	13,444
2.03 Class 04 Unaffiliated Bonds	5,399,430	× 0.045 =	242,974
2.07 SUBTOTAL—FIXED INCOME	6,743,875		256,419
Equity-Type Assets			
2.08 Unaffiliated Preferred Stock—Class 01	1,866,501	× 0.023 =	42,930
2.09 Unaffiliated Preferred Stock—Class 02	499,999	× 0.030 =	15,000
2.18 SUBTOTAL—EQUITY	2,366,500		57,929
2.99 TOTAL—ISSUER #2	9,110,375		314,348

EXHIBIT 5

PART 2

(1)	(2) Statement Value	(3) Factor	(4) Additional RBC
ISSUER #3 Rapacious Development Corporation			
Fixed Income-Type Assets			
3.01 Class 02 Unaffiliated Bonds	2,968,829	× 0.010 =	29,688
3.07 SUBTOTAL—FIXED INCOME	2,968,829		29,688
Equity-Type Assets			
3.08 Unaffiliated Preferred Stock—Class 01	1,575,280	× 0.023 =	36,231
3.18 SUBTOTAL—EQUITY	1,575,280		36,231
3.99 TOTAL—ISSUER #3	4,544,109		65,920
ISSUER #4 Imperceptible Products			
Fixed Income-Type Assets			
4.01 Class 02 Unaffiliated Bonds	1,888,606	× 0.010 =	18,886
4.07 SUBTOTAL—FIXED INCOME	1,888,606		18,886
Equity-Type Assets			
4.08 Unaffiliated Preferred Stock—Class 01	745,152	× 0.023 =	17,138
4.18 SUBTOTAL—EQUITY	745,152		17,138
4.99 TOTAL—ISSUER #4	2,633,758		36,025
ISSUER #5 Brassbound Insurance Company			
Fixed Income-Type Assets			
5.01 Class 02 Unaffiliated Bonds	730,825	× 0.010 =	7,308
5.07 SUBTOTAL—FIXED INCOME	730,825		7,308
Equity-Type Assets			
5.08 Unaffiliated Preferred Stock—Class 01	407,194	× 0.023 =	9,365
5.18 SUBTOTAL—EQUITY	407,194		9,365
5.99 TOTAL—ISSUER #5	1,138,019		16,674

EXHIBIT 5

PART 3

ISSUER #6 Massachusetts Pork Authority			
Equity-Type Assets			
6.01 Class 02 Unaffiliated Bonds	923,456	× 0.010 =	9,235
6.07 SUBTOTAL—FIXED INCOME	923,456		9,235
6.99 TOTAL—ISSUER #6	923,456		9,235
ISSUER #7 Ingestme Food Corp.			
Equity-Type Assets			
7.17 Unaffiliated Common Stock	564,321	× 0.150 =	84,648
7.18 SUBTOTAL—EQUITY	564,321		84,648
7.99 TOTAL—ISSUER #7	564,321		84,648
ISSUER #8 DIS Information Processing			
Equity-Type Assets			
8.17 Unaffiliated Common Stock	249,666	× 0.150 =	37,450
8.18 SUBTOTAL—EQUITY	249,666		37,450
8.99 TOTAL—ISSUER #8	249,666		37,450
ISSUER #9 Gulf Bag			
9.17 Unaffiliated Common Stock	194,778	× 0.150 =	29,217
9.18 SUBTOTAL—EQUITY	194,778		29,217
9.99 TOTAL—ISSUER #9	194,778		29,217
ISSUER #10 Ennui Entertainment Industries			
10.17 Unaffiliated Common Stock	157,528	× 0.150 =	23,629
10.18 SUBTOTAL—EQUITY	157,528		23,629
10.99 TOTAL—ISSUER #10	157,528		23,629
11.07 SUBTOTAL—FIXED INCOME	22,612,260		564,835
11.18 SUBTOTAL—EQUITY	9,198,303		720,512
11.99 GRAND TOTAL—COMBINED ISSUERS	31,810,563		1,285,347

EXHIBIT 6

SUMMARY FOR SUBSIDIARY, CONTROLLED, AND AFFILIATED INVESTMENTS

	Affiliate Type	Affiliate Code	RBC Required for Affiliated Common Stock	RBC Required for Affiliated Preferred Stock	RBC Required for Affiliated Bonds	Total RBC Required
(01)	Direct P/C Subsidiaries—U.S.	1	203,919,032	5,100,000	10,024,303	219,043,335
(02)	Direct Life Subsidiaries—U.S.	2	0	0	0	0
(03)	Indirect P/C Subsidiaries—U.S.	3	189,973,343	0	0	189,973,343
(04)	Indirect Life Subsidiaries—U.S.	4	0	0	0	0
(05)	Investment Subsidiaries	5	17,500,000	0	0	17,500,000
(06)	Holding Company in Excess of Indirect Subsidiaries	6	9,485,912	0	0	9,485,912
(07)	Alien Insurers	7	28,875,134	0	0	28,875,134
(08)	N/A	8	—	—	—	—
(09)	Investment in Parent	9	0	0	0	0
(10)	Other Affiliate—P/C Insurance not subject to RBC	10	0	0	0	0
(11)	Other Affiliate—Life Insurance not subject to RBC	11	0	0	0	0
(12)	Other Affiliate—Non-insurer	12	0	0	0	0
(99)	Total	—	449,753,421	5,100,000	10,024,303	464,877,724

EXHIBIT 7

DETAILS FOR AFFILIATED BONDS AND STOCKS

(1) Name of Affiliate	(2) Affiliate Code	(3) NAIC Company Code or Alien ID Number	(4) Affiliate's RBC After Covariance	(5) Statement Value of Affiliate's Common Stock	(6) Total Value of Affiliate's Outstanding Common Stock	(7) Percent Owned	(8) Statement Value of Affiliate's Preferred Stock
Fenway Insurance Company	1	10000	131,450,121	157,869,234	157,869,234	100.0	0
Writeit Re	1	10001	87,593,214	72,468,911	72,468,911	100.0	5,100,000
Minuteman Insurance Company	3	10002	245,126,894	437,791,578	564,892,359	77.5	0
Goldfinger Inc.	5		17,500,000	125,000,000	125,000,000	100.0	0
ZZZ Holding Corp.	6		0	42,159,610	0	100.0	0
Norton Casualty of Calcutta	7	AA-10000	0	57,750,268	0	100.0	0
Total	—	—	481,670,229	893,039,601	—	—	5,100,000

(9) Total Value of Affiliate's Outstanding Preferred Stock	(10) Percent Owned	(11) Statement Value of Affiliate's Bonds	(12) Total Value of Affiliate's Outstanding Bonds	(13) Percent Owned	(14) RBC Required for Common Stock	(15) RBC Required for Preferred Stock	(16) RBC Required for Bonds	(17) RBC Required
0	100.0	0	0	100.0	131,450,121	0	0	131,450,121
5,100,000	100.0	15,275,625	15,275,625	100.0	72,468,911	5,100,000	10,024,303	87,593,214
0	100.0	0	0	100.0	189,973,343	0	0	189,973,343
0	100.0	0	0	100.0	17,500,000	0	0	17,500,000
0	100.0	0	0	100.0	9,485,912	0	0	9,485,912
0	100.0	0	0	100.0	28,875,134	0	0	28,875,134
—	—	15,275,625	—	—	449,753,421	5,100,000	10,024,303	464,877,724

EXHIBIT 8

CREDIT RISK FOR RECEIVABLES

	Reinsurance Recoverables	(1) Sched F Part 3 Statement Value	(2) Adjusted for Voluntary Pools	(3) Applicable Penalty	(4) Amount Subject to RBC		(5) Factor	(6) RBC Requirement
	Annual Statement Source							
(1)	L0399999—Alien Affiliated	5,000,000		0	5,000,000	×	0.100 =	500,000
(2)	L0599999—Unaffiliated U.S.	20,000,000		1,500,000	18,500,000	×	0.100 =	1,850,000
(3)	L0799999—Voluntary Pools	15,000,000	0	0	15,000,000	×	0.100 =	1,500,000
(4)	L0899999—Alien Unaffiliated	0		0	0	×	0.100 =	0
(5)	L1299999—Alien Affiliated	0		0	0	×	0.100 =	0
(6)	L1499999—Unaffiliated U.S.	10,000,000		2,000,000	8,000,000	×	0.100 =	800,000
(7)	L1699999—Voluntary Pools	0	0	0	0	×	0.100 =	0
(8)	L1799999—Alien Unaffiliated	7,500,000		6,500,000	1,000,000	×	0.100 =	100,000
(9)	Total Reinsurance Recoverables	57,500,000	0	10,000,000	47,500,000			4,750,000
(10)	Federal Income Tax Recoverable				0	×	0.050 =	0
(11)	Interest, Dividends, Real Estate Income Due & Accrued				1,000,000	×	0.010 =	10,000
(12)	Recoverables from Parent, Subsidiaries, Affiliations				2,000,000	×	0.050 =	100,000
(13)	Amounts Recoverable Relating to Uninsured A&H				0	×	0.050 =	0
(14)	Aggregate Write-Ins for Other Than Invested Assets				500,000	×	0.050 =	25,000
(15)	Total Credit Risk RBC Charge				51,000,000			4,885,000

EXHIBIT 9

UNDERWRITING RISK—RESERVES

	Schedule P Line of Business	(1) H/F	(2) PPA	(3) CA	(4) WC	(5) CMP	(6) Med Mal	(15) Total
(1)	Industry Average Development	0.994	1.032	1.075	1.066	1.038	1.028	—
(2)	Company Average Development	1.011	1.150	1.100	1.050	1.065	1.200	—
(3)	(2)/(1)	1.017	1.114	1.023	0.985	1.026	1.167	—
(4)	Industry Loss & LAE RBC Percentage	0.275	0.254	0.287	0.273	0.374	0.565	—
(5)	Company RBC Percentage	0.277	0.268	0.290	0.271	0.379	0.612	—
(6)	Loss + LAE Unpaid Schedule P Part 1 (000s)	50,000	600,000	100,000	1,250,000	25,000	400,000	2,425,000
(7)	Other Discount Amount Not Included in Loss + LAE Unpaid in Schedule P Part 1 (000s)	0	0	0	0	0	0	0
(8)	Adjustment for Investment Income	0.928	0.921	0.905	0.872	0.880	0.808	—
(9)	Base Loss + LAE Reserve RBC (000s)	9,269	100,985	16,776	135,337	5,335	121,085	388,786
(10)	Percentage Direct Loss-Sensitive	0.00	0.00	0.00	0.20	0.00	0.00	—
(11)	Percentage Assumed Loss-Sensitive	0.00	0.00	0.00	0.00	0.00	0.00	—
(12)	Loss-Sensitive Discount (000s)	0	0	0	8,120	0	0	8,120
(13)	Percentage Claims Made—Medical Malpractice	—	—	—	—	—	0.25	—
(14)	Claims Made Discount—Medical Malpractice (000s)	—	—	—	—	—	6,054	6,054
(15)	Loss + LAE RBC After Discount (000s)	9,269	100,985	16,776	127,217	5,335	115,030	374,611
(16)	Loss Concentration Factor							0.855
(17)	Net Loss + LAE RBC Charge							320,157,630

EXHIBIT 10

UNDERWRITING RISK—NET WRITTEN PREMIUMS

	Schedule P Line of Business	(1) H/F	(2) PPA	(3) CA	(4) WC	(5) CMP	(6) Med Mal	(15) Total
(1)	Industry Average Loss & LAE Ratio	0.808	0.931	0.818	0.901	0.689	0.955	—
(2)	Company Average Loss & LAE Ratio	0.805	0.982	0.980	0.850	0.912	0.984	—
(3)	(2)/(1)	0.996	1.055	1.198	0.943	1.324	1.030	—
(4)	Industry Loss & LAE Ratio	0.917	1.046	1.013	1.008	0.917	1.472	—
(5)	Company RBC Loss & LAE Ratio	0.915	1.075	1.113	0.979	1.066	1.494	—
(6)	Company Underwriting Expense Ratio	0.250	0.250	0.250	0.250	0.250	0.250	—
(7)	Adjustment for Investment Income	0.942	0.924	0.900	0.836	0.884	0.778	—
(8)	C/Y Net Written Premium (000s)	200,000	800,000	100,000	500,000	50,000	150,000	1,800,000
(9)	Base Written Premium RBC (000s)	22,442	194,381	25,198	34,419	9,591	61,891	347,921
(10)	Percentage Direct Loss-Sensitive WP	0.00	0.00	0.00	0.20	0.00	0.00	—
(11)	Percentage Assumed Loss-Sensitive WP	0.00	0.00	0.00	0.00	0.00	0.00	—
(12)	Loss-Sensitive Discount—WP (000s)	0	0	0	2,065	0	0	2,065
(13)	Percentage Claims Made WP—Medical Malpractice	—	—	—	—	—	0.53	—
(14)	Claims Made Discount—Medical Malpractice (000s)	—	—	—	—	—	6,598	6,598
(15)	NWP RBC after Discount (000s)	22,442	194,381	25,198	32,354	9,591	55,293	339,259
(16)	Premium Concentration Factor							0.833
(17)	Net Written Premium RBC Charge							282,715,595

EXHIBIT 11

MISCELLANEOUS OFF BALANCE SHEET ITEMS

		(1) Statement Value		(2) Factor	(3) RBC Requirement
(1)	Non-controlled Assets	0	×	0.010 =	0
(2)	Guarantees for Affiliates	0	×	0.010 =	0
(3)	Contingent Liabilities	15,000,000	×	0.010 =	150,000
(4)	Total Miscellaneous Off Balance Sheet Items	15,000,000			150,000

EXHIBIT 12

EXCESSIVE PREMIUM GROWTH

		(1) Company Gross Written Premiums	(2) Company Adjustments	(3) Group Gross Written Premiums	(4) Group Adjustments	(5) Selected Adjusted Gross Premiums	(6) Statement Value	(7) Factor	(8) RBC Requirement
(1)	1995	2,000,000,000	0	5,059,643,589	0	5,059,643,589			
(2)	1994	1,900,000,000	0	4,287,833,550	0	4,287,833,550			
(3)	1993	1,805,000,000	0	3,664,815,000	0	3,664,815,000			
(4)	1992	1,714,750,000	0	3,214,750,000	0	3,214,750,000			
(5)	1995 Growth Rate							0.180	
(6)	1994 Growth Rate							0.170	
(7)	1993 Growth Rate							0.140	
(8)	Three Year Average Growth Rate							0.163	
(9)	Two Year Average Growth Rate							0.175	
(10)	One Year Average Growth Rate							0.180	
(11)	Selected Average Growth Rate							0.163	
(12)	RBC Average Growth Rate = Line 11 − 10%, capped to fall between 0% and 30%							0.063	
(13)	Excessive Growth Charge Applied to Loss/LAE Reserve from Schedule P Part 1 Column 23 Line 12						2,425,000,000	0.029	70,325,000
(14)	Excessive Growth Charge Applied to Net Written Premiums from Underwriting & Investment Exhibit Part 2B Column 4 Line 32						1,800,000,000	0.014	25,200,000

EXHIBIT 13

PART 1

CALCULATION OF TOTAL RISK-BASED CAPITAL AFTER COVARIANCE

		RBC Amount
R_0—Asset Risk—Subsidiary Insurance Companies		
(1)	Affiliated U.S. P/C Insurers—Directly Owned	219,043,335
(2)	Affiliated U.S. P/C Insurers—Indirectly Owned	189,973,343
(3)	Affiliated U.S. Life Insurers—Directly Owned	0
(4)	Affiliated U.S. Life Insurers—Indirectly Owned	0
(5)	Affiliated Alien Insurers	28,875,134
(6)	Non-controlled Assets	0
(7)	Guarantees for Affiliates	0
(8)	Contingent Liabilities	150,000
(9)	Total R_0	438,041,812
R_1—Asset Risk—Fixed Income		
(10)	Class 01 U.S. Government Agency Bonds	3,300,000
(11)	Unaffiliated Bonds Subject to Size Factor	18,500,000
(12)	Bond Size Factor RBC Charge	7,334,802
(13)	Bonds—Affiliated Investment Subsidiary	0
(14)	Bonds—Affiliated Holding Company in excess of Insurance Subsidiaries	0
(15)	Bonds—Investment in Parent	0
(16)	Bonds—Affiliated U.S. P/C Not Subject To RBC	0
(17)	Bonds—Affiliated U.S. Life Not Subject To RBC	0
(18)	Bonds—Affiliated Non-insurer	0
(19)	Mortgage Loans	500,000
(20)	Collateral Loans	125,000
(21)	Cash	15,000
(22)	Short-Term Investments	0
(23)	Asset Concentration RBC—Fixed Income	564,835
(24)	Total R_1	30,339,637

EXHIBIT 13

PART 2

CALCULATION OF TOTAL RISK-BASED CAPITAL AFTER COVARIANCE

		RBC Amount
R_2—Asset Risk—Equity		
(25)	Common—Affiliated Investment Subsidiaries	17,500,000
(26)	Common—Affiliated Holding Company in excess of Insurance Subsidiaries	9,485,912
(27)	Common—Investment in Parent	0
(28)	Common—Affiliated U.S. P/C Not Subject To RBC	0
(29)	Common—Affiliated U.S. Life Not Subject To RBC	0
(30)	Common—Affiliated Non-insurer	0
(31)	Preferred—Affiliated Investment Subsidiaries	0
(32)	Preferred—Affiliated Holding Companies in excess of Insurance Subsidiaries	0
(33)	Preferred—Investment in Parent	0
(34)	Preferred—Affiliated U.S. P/C Not Subject To RBC	0
(35)	Preferred—Affiliated U.S. Life Not Subject To RBC	0
(36)	Preferred—Affiliated Non-insurer	0
(37)	Unaffiliated Common Stock	52,560,000
(38)	Unaffiliated Preferred Stock	380,000
(39)	Real Estate	17,500,000
(40)	Schedule BA Assets	2,000,000
(41)	Aggregate Write-ins for Invested Assets	375,000
(42)	Asset Concentration RBC—Equity	720,512
(43)	Total R_2	100,521,425
R_3—Asset Risk—Credit		
(44)	One half of Credit RBC Charge	2,442,500
R_4—Underwriting Risk—Reserves		
(45)	One half of Credit RBC Charge	2,442,500
(46)	Total Adjusted Unpaid LLAE Reserve RBC Charge	319,982,040
(47)	Excessive Growth Charge—Loss/LAE Reserve	70,325,000
(48)	A&H Claims Reserves Adjusted for LCF	0
(49)	Total R_4	392,749,540
R_5—Underwriting Risk—Net Written Premium		
(50)	Total Adjusted NWP RBC Charge	282,715,595
(51)	Excessive Growth Charge—Written Premiums	25,200,000
(52)	A&H Earned Premium Adjusted for PCF	0
(53)	Total R_5	307,915,595
(54)	Total RBC After Covariance	948,037,136
(55)	Authorized Control Level RBC	426,616,711

EXHIBIT 14

COMPARISON OF TOTAL ADJUSTED CAPITAL TO RISK-BASED CAPITAL

		Abbreviation	(1) Amount
(1)	Total Adjusted Capital		1,335,000,000
(2)	Company Action Level = 200% of Authorized Control Level	CAL	853,233,423
(3)	Regulatory Action Level = 150% of Authorized Control Level	RAL	639,925,067
(4)	Authorized Control Level = 100% of Authorized Control Level	ACL	426,616,711
(5)	Mandatory Control Level = 70% of Authorized Control Level	MCL	298,631,698
(6)	Level of Action, if Any		NONE
	The following numbers must be reported in the Five Year History Exhibit on the indicated line		
	Total Adjusted Surplus to Policyholders	1,335,000,000	
	Authorized Control Level Risk-Based Capital	426,616,711	

LOSS PREDICTION BY GENERALIZED LEAST SQUARES

LEIGH J. HALLIWELL

Abstract

The prediction of losses, whether for ratemaking or for reserving, is the quintessential activity of the actuary. The time-honored technique of loss development is the basis for the chain ladder and Bornhuetter–Ferguson methods. These methods, particularly the chain-ladder, have been subject to a great deal of statistical analysis since the mid-1980s. It is now thought by many that development factors obtained by least squares regression are unbiased. But this paper will argue that the linear modeling and the least squares estimation found in the literature to date have overlooked an important condition of the linear model. In particular, the models for development factors regress random variables against other random variables. Stochastic regressors violate the standard linear model. Moreover, the model assumes that the errors are uncorrelated, but stochastic regressors violate this assumption as well. This paper will show that what actuaries are really seeking is found in a general linear model; i.e., a model with nonstochastic regressors but with an error matrix that allows for correlation. An example will be presented.

1. A SIMPLE ILLUSTRATION OF LOSS ESTIMATION

Consider how actuaries might approach a simple loss reserving problem. Take an exposure period now at a certain age. Based upon our best knowledge heretofore, we have believed that \$100 of losses would ultimately be paid for this period. We know that \$60 has been paid to date. We have also looked into our records and have found that, on similar exposures at the same age, 50%

of the ultimate losses have been paid. How does this new information affect our prior estimate of $100 of losses?

First, we could rely on the statistic that 50% of the losses should have been paid by this time, which implies that we should revise our estimate of ultimate loss to $120. Actuaries would normally say that the development factor from this age to ultimate is 2.0. So, our paid losses should develop from $60 dollars to $\$60 \times 2.0 = \$120$. This is often called the chain ladder (CL) method of loss development.

Second, we could rely on the prior hypothesis that the ultimate loss will be $100, and assume that the accelerated payment of $60 to date will be countered by a decelerated payment of $40 henceforth.

As is so often the case, there is a third approach which mediates between the CL and the prior hypothesis methods. The CL method disregards the prior hypothesis, and sticking to the prior hypothesis disregards the payout statistic. Why not assume that the amount yet to be paid is half of the prior hypothesis estimate, or $50? This, plus the $60 already paid, makes for an ultimate loss of $110. This appealing solution is known as the Bornhuetter–Ferguson (BF) method, and has several variants.[1]

2. THE UPWARD BIAS OF THE CHAIN LADDER METHOD UNDER PLAUSIBLE CONDITIONS

James Stanard [10] simulated thousands of loss triangles, and developed these losses according to four methods, one of which was the CL. He concluded that the CL method was biased in the direction of overestimating ultimate losses. In his Appendix

[1] See Bornhuetter and Ferguson [2]. James Stanard [10, pp. 130f.] describes four loss development methods, the second of which is a "modified" BF method. His third method, called the "Cape Cod" method, is equivalent to what Gary Patrik [9, pp. 352–354] calls the Stanard-Bühlmann (SB) method, under the assumption that all accident years have the same prior expected losses. The SB method is a variant of the BF.

A, he shows why this method should be biased, but not whether the bias should be upward (overestimation) or downward (underestimation). In this section, we will show that under reasonable conditions, the bias is upward.

We have normal random variables $X_1 \sim [\mu_1, \sigma_1^2]$ and $X_2 \sim [\mu_2, \sigma_2^2]$. The correlation coefficient between the two is ρ. Let $Y_1 = e^{X_1}$ and $Y_2 = e^{X_2}$. Y_1 will represent the losses (whether paid or incurred) as of the earlier age; Y_2 as of the later. The assumption that losses are distributed lognormally is convenient for this demonstration, as well as frequently realistic. $X_2 - X_1$ is normally distributed as $[\mu_2 - \mu_1, \sigma_2^2 + \sigma_1^2 - 2\rho\sigma_1\sigma_2]$.

The development factor is an estimate of $\mathrm{E}[Y_2/Y_1]$. As Stanard shows in his appendix, the bias of the CL method depends on the relation between $\mathrm{E}[Y_1] \times \mathrm{E}[Y_2/Y_1]$ and $\mathrm{E}[Y_2]$. Given the lognormal assumption, $Y_2/Y_1 = e^{X_2 - X_1}$ is lognormal. Therefore,

$$\mathrm{E}[Y_2] = e^{\mu_2 + \sigma_2^2/2}, \qquad \text{and}$$

$$\begin{aligned}
\mathrm{E}[Y_1]\mathrm{E}\left[\frac{Y_2}{Y_1}\right] &= e^{\mu_1 + \sigma_1^2/2} e^{\mu_2 - \mu_1 + \sigma_2^2/2 + \sigma_1^2/2 - \rho\sigma_1\sigma_2} \\
&= e^{\mu_2 + \sigma_2^2/2 + \sigma_1^2 - \rho\sigma_1\sigma_2} \\
&= \mathrm{E}[Y_2] e^{\sigma_1^2 - \rho\sigma_1\sigma_2}.
\end{aligned}$$

So whether the CL method is biased downward, unbiased, or biased upward depends on whether $e^{\sigma_1^2 - \rho\sigma_1\sigma_2}$ is less than, equal to, or greater than one. And this depends on whether σ_1 is less than, equal to, or greater than $\rho\sigma_2$. Since ρ is less than one ($\rho = 1$ is unrealistic), σ_1 is greater than $\rho\sigma_2$ unless σ_2 is larger than σ_1. This means that the CL method is biased upward unless σ_2 is sufficiently larger than σ_1. The closer ρ is to zero, the less likely σ_2 will be sufficiently large; it is impossible when ρ is less than or equal to zero. Therefore, the CL method works best, or has the least upward bias, when the loss at the later time is highly positively correlated with the loss at the earlier time.

Furthermore, one very plausible assumption is that as a loss ages, its standard deviation remains proportional to its mean, or equivalently, that its coefficient of variation (CV) remains constant. Given the lognormal assumption, this means that:

$$\mathrm{CV}[Y_1] = \sqrt{e^{\sigma_1^2} - 1} = \sqrt{e^{\sigma_2^2} - 1} = \mathrm{CV}[Y_2],$$

which is true if and only if $\sigma_1 = \sigma_2$. But if $\sigma_1 = \sigma_2$, then σ_2 is not sufficiently large, and the CL method will be biased upward. Thus, we have some assumptions regarding lognormality and the coefficient of variation, having verisimilitude singly and together, under which the CL method must be biased upward.

3. AN ATTEMPT TO REHABILITATE THE CHAIN LADDER METHOD

Stanard's findings have disconcerted actuaries, who are very fond of using the CL method for estimating ultimate losses. The CL logic is simple and appealing. For example, "If half the losses should have been paid by now, and $60 have indeed been paid, then $120 should ultimately be paid." Moreover, the CL method makes no use of a prior hypothesis, so it seems to have the benefit of parsimony.[2] As for an upward bias, many actuaries would consider this to be a windfall since, if true, it would add an extra bit of conservatism to their estimates.[3]

[2]Recall Ockham's razor.

[3]It is ironic that although in theory and in simulation the CL method should be biased upward, in practice it frequently seems to be biased downward. Several years ago, while employed by NCCI, the author conducted a study of how accurately losses were developed in NCCI ratemaking. He found that the development was usually underestimated by five to ten percent. Of course, this is not really an indictment against our belief that the CL method is biased upward. Rather, it is reflective of the runaway conditions of workers compensation in the late 1980s; i.e., of the worsening conditions not reflected in projections of ultimate losses. It is assumed throughout this paper that all the rows of a loss triangle are commensurate (akin to one another), and that we are cognizant of, and can adjust for, the important exogenous effects on the losses. Doing justice to this assumption involves the hardest work of the actuary, and is more actuarial art than science.

However, most actuaries desire unbiased estimates—not just because of statistical purity, but also because of competitive pressures in business. If loss estimates need to be conservative, then the conservatism should be a deliberate and measured addition to an unbiased estimate. Therefore, Stanard's findings have been one impetus in the search for a better approach.

Daniel Murphy [8] has sought to extract unbiased loss development factors from loss triangles by the application of linear regression techniques. His model is $\mathbf{Y} = \mathbf{J}\alpha + \text{Prev}(\mathbf{Y})\beta + \mathbf{e}$, where α and β are the regression coefficients to be estimated and $\mathbf{Y}$, $\mathbf{J}$, Prev($\mathbf{Y}$), and $\mathbf{e}$ are $(t \times 1)$ vectors. Prev($\mathbf{Y}$) and $\mathbf{Y}$ are adjacent matching columns in the loss triangle, and $\mathbf{J}$ is a vector of ones, or an intercept vector. As for Var[$\mathbf{e}$], a $(t \times t)$ matrix, it is assumed to be diagonal; i.e., $\text{Cov}[\mathbf{e}_i, \mathbf{e}_j] = \sigma_i^2$ for $i = j$, but 0 otherwise.

Murphy [8, p. 187] appeals to the Gauss–Markov theorem in affirming that the least-squares estimates of the regression coefficients are best linear unbiased estimates (BLUE). From there, he fills in the loss triangle with supposedly unbiased estimates, and constructs a confidence interval for the aggregate incurred loss. However, it appears that Murphy has overlooked one of the conditions of the Gauss–Markov theorem, thus invalidating his claim of unbiasedness.

First, Murphy shows in his appendix that the familiar simple-average and weighted-average development factors fall out from a regression model with no intercept ($\alpha = 0$), given appropriate assumptions as to the σ_i^2 elements. This in itself should raise doubt: if a special case of the linear regression model reduces to the CL method which is biased, then how can the regression estimates be unbiased? One might be tempted to answer that the special case is biased, whereas the model with the intercept (nonzero α) is unbiased. However, the Gauss–Markov theorem, starting from the assumption that the linear model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}$ is well specified, where Var[$\mathbf{e}$] $= \boldsymbol{\Phi}$, proves $(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{Y})$

to be the best linear unbiased estimator of $\boldsymbol{\beta}$, irrespective of whether the regressor matrix $\mathbf{X}$ has a column of ones to serve as an intercept.[4]

The flaw in Murphy's claim of achieving unbiasedness is that his regressor matrix, the $(t \times k)$ matrix $\mathbf{X}$ in the model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}$, contains stochastic regressors; viz., one of its columns is Prev($\mathbf{Y}$), which is stochastic. George Judge [5, Ch. 13] discusses the ramifications of stochastic regressors at some length. In short, if the stochastic regressors are independent of the error vector, then the least-squares estimator is still unbiased. However, even in this case, the usual formulas for Var$[\hat{\boldsymbol{\beta}}]$ and for $\hat{\sigma}^2$ do not include the variation inherent in the fact that other values of the stochastic regressors could have been realized.

More to the point, Murphy's model is an example of what Judge [5, pp. 574–576] calls "partially independent stochastic regressors." Here Prev($\mathbf{Y}$) is not independent of all the error terms, and the most that can be said is that, under certain conditions, the least-squares estimator is consistent; i.e., asymptotically unbiased. This is the fundamental problem with the CL method. Rather than try to rehabilitate it, this paper introduces a different model that honors all the conditions of the Gauss–Markov theorem.

4. THE NECESSITY OF CONSIDERING EXPOSURE

Consider the conclusions of Stanard and Murphy as to their loss-development simulations:

> The common age-to-age factor approach (Method 1) is clearly inferior to the other three methods [Standard 10, p. 134].
>
> The performance of the incurred loss development technique based on the more general least squares

[4]For a proof of the Gauss–Markov theorem, see Appendix A.

> estimator may approach that of the Bornhuetter–Ferguson (BF) and Stanard–Bühlmann (SB) techniques in some situations [Murphy 8, p. 185].

What Murphy calls the BF and SB techniques correspond to Stanard's second and third methods. Stanard himself favors his fourth method, the additive model [10, pp. 131, 135]: "In fact, Method 4 may be completely unbiased." The SB and additive models can be considered variants of the BF.

The obvious question is this: If it is so hard to beat the BF method and its variants, then why continue to refine the CL method? Of course, loss development factors are used in some BF variants, as Murphy notes [8, p. 207]:

> The average bias of the BF and SB methods should be greater than zero as well because the LDFs on which they rely are themselves overstated more often than not.

But what is unique to the BF variants to give them a performance advantage over the CL method? The answer is that the BF variants incorporate prior knowledge, whether it be a prior estimate of incurred losses or a knowledge of exposure relativities.

I suspect that the desire to avoid relying on prior knowledge is one motive for actuaries to try to perfect the CL method, as if reliance on such knowledge would be tantamount to circular reasoning. However, what could be more axiomatic than a statement such as "Twice the exposure should produce twice the expected loss, all else being equal?" Nevertheless, this information is unknown to the CL method. But is it unknown, or just ignored? If there is enough information in the form of a loss triangle to produce development factors, then there must also be substantial knowledge of the underlying exposures. Otherwise, how would the actuary know that the rows of the triangle were commensurate, or that they represented the same process of development?

Substantial prior knowledge is implied in John Robertson's comment [11, p. 149]:

> Previous literature on reserving techniques generally has concentrated on overcoming the effects of changes in the underlying mix of business, changes in the individual claim reserving and settling policies, and changes in claims reporting systems. Most of this prior literature assumes that once the changes are accounted for and the data has been restated so as to have relatively constant underlying conditions, then any number of loss development methods can be applied to obtain valid forecasts.

Even Murphy, who has worked diligently to further the CL method, resorts to a knowledge of exposures in his argument for a non-zero intercept term [8, p. 204]:

> From Equation 2 one can see that the slope factor b_n does not depend on the exposure (N) but only on the reporting pattern, and that the constant a_n is proportional to the exposure. An increase in exposure from one accident year to the next will cause an upward, parallel shift in the development regression line.

The extent to which his simulated regression results outperform the BF method may be due not only to the extra parameter a_n, but also to a BF-like use of exposure.[5]

It is time to introduce a method that gives exposure its proper place.

---

[5] In his Section 5 [8, p. 204], Murphy considers the model $\mathbf{Y} = \mathbf{E}\alpha + \text{Prev}(\mathbf{Y})\beta + \mathbf{e}$, where **E** contains exposures for each row. It is unclear to the author whether he ever used this model in his simulations. Of course, if his simulated triangles had equal exposures in all rows, as did Stanard's, then the "**J**" and "**E**" models are equivalent. In the auto liability incurred loss and ALAE example (Figure 1A), exposures are obviously unequal, and the "**J**" model is used to produce estimates of $\mathbf{a}_{\text{LSL}}$ = \$374 and $\mathbf{b}_{\text{LSL}}$ = 2.027 for the 12 : 24 development [8, p. 190].

5. LOSS COVARIANCE VERSUS LOSS DEVELOPMENT

There is a distinction between loss covariance and loss development. To an actuary, loss development connotes the estimation of a loss as of time t_{i+1}, $X(t_{i+1})$, from the loss as of earlier times. In other words, there is some estimation function, f, such that $X(t_{i+1}) = f(X(t_1), X(t_2), \ldots, X(t_{i-1}), X(t_i)) + e_{i+1}$. Actuaries also simplify the functional form to $X(t_{i+1}) = f(X(t_i)) + e_{i+1}$. This simplification assumes that the most recent value of the loss is all-determinative of its future development; i.e., that the path the loss took in getting to $X(t_i)$ is irrelevant. Thomas Mack [7, p. 108] points out that this simplification may be inappropriate; however, without it, the functional forms could easily become overspecified. In any case, $X(t_{i+1}) = f(X(t_1), X(t_2), \ldots, X(t_{i-1}), X(t_i)) + e_{i+1}$ expresses the familiar and appealing concept of loss development. It is appealing because actuaries feel that earlier values of X should affect the later values. However, as was pointed out in Section 3, this entails estimation with stochastic regressors.

Loss covariance involves the following idea: Let $\mathbf{X}$ be an $(n \times 1)$ vector,

$$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix},$$

where x_i is the incremental loss, whether paid or incurred, during the ith time interval. Through research, we believe that we have a good idea of the mean and variance of $\mathbf{X}$, which depends on our knowledge of exposure, inflation, etc. Then, as x_is become known, the x_is still unknown can be considered elements of a conditional random vector. They are affected by the known elements in a Bayesian sense, through the variance matrix.

As an example, consider a two-part loss

$$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}\right),$$

which means that **X** is distributed as a bivariate normal random variable with the mu vector as its mean and the sigma matrix as its variance. The variance matrix must be symmetric, because $\sigma_{ij} = \mathrm{Cov}(x_i, x_j) = \mathrm{Cov}(x_j, x_i) = \sigma_{ji}$. Hence, $\sigma_{21} = \sigma_{12}$. The distribution of x_2 conditional on x_1 is:[6]

$$x_2 \mid x_1 \sim N\left(\mu_2 + \frac{\sigma_{12}}{\sigma_{11}}(x_1 - \mu_1), \sigma_{22} - \frac{\sigma_{12}^2}{\sigma_{11}}\right).$$

Knowledge of x_1 affects our expectation of x_2, and even lessens the variance of x_2. Since $\sigma_{11} = \sigma_1^2$ and $\sigma_{12} = \rho\sigma_1\sigma_2$, we can rewrite the conditional expectation as:

$$\mathrm{E}[x_2 \mid x_1] = \mu_2 + \rho\sigma_2\left(\frac{x_1 - \mu_1}{\sigma_1}\right).$$

Thus,

$$\frac{\mathrm{E}[x_2 \mid x_1] - \mu_2}{\sigma_2} = \rho\left(\frac{x_1 - \mu_1}{\sigma_1}\right).$$

Consider what this means: the conditional mean $\mathrm{E}[x_2 \mid x_1]$ will differ from the unconditional mean μ_2 in terms of standard deviation units (σ_2) by some proportion (ρ) of the standardized error of x_1. This is the essence of the loss covariance approach: that the known losses affect the unknown not through their absolute levels, but rather through a combination both of their relative departure from their expected values and of the covariance of the known with the unknown. The covariance defines the persistency of this departure.

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[6]For a general proof, see Appendix B. Johnson [4, p. 138] has another proof. The author's thinking was helped by Julien McKee's presentation at the 1994 Casualty Loss Reserve Seminar [6].

We do not yet have a linear model for loss prediction; nevertheless, we have uncovered a fundamental truth, a truth that has been missed in loss development methodology. Covariance is the link from one random variable to another. When actuaries seek to predict losses, they must consider how the known losses affect the unknown—and that involves covariance. Actuaries do not have to know how, or even whether, the known "causes" the unknown; the question of causality is academic. Prediction is a matter of covariance, which informs how one random variable is expected to differ from its mean, given the departure of another random variable from its mean. So covariance requires the estimation of means, which are functions of exogenous, known quantities such as exposures and price indices.

Let us solve for α and β in the equation $Y = \alpha + \beta X + e$, where $\mathrm{Cov}[X,e] = 0$, and $\mathrm{E}[e] = 0$. Applying the expectation operator, we have $\mathrm{E}[Y] = \alpha + \beta\mathrm{E}[X]$. Also, $\mathrm{Cov}[Y,X] = \beta\,\mathrm{Var}[X]$. Therefore, $\beta = \mathrm{Cov}[Y,X]/\mathrm{Var}[X]$, and $\alpha = \mathrm{E}[Y] - \beta\mathrm{E}[X]$. Moreover, $Y - \mathrm{E}[Y] = \beta(X - \mathrm{E}[X]) + e = \{\mathrm{Cov}[Y,X]/\mathrm{Var}[X]\}(X - \mathrm{E}[X]) + e$. What then is the function of the intercept α? Is it not to supply the proper combination of the mean values of the dependent and independent random variables? But the real relation is between the departures of the random variables from their means. In a well-constructed linear model, the intercept is replaced with expressions for mean values based on outside information. This amplifies the reason for the abolition of intercept terms advanced by Gregory Alff [1, p. 89], that "a constant does nothing to describe the underlying contributory causes of change in the dependent variable."

Though not a model, the example above encompasses the three approaches to loss reserving discussed in Section 1. Let us also treat σ_1 and σ_2 as equal. When ρ is positive, a greater than expected x_1 will raise $\mathrm{E}[x_2 \mid x_1]$. This is in keeping with the first approach, the CL method. When ρ is negative, a greater than expected x_1 will lower the conditional expectation. At the extreme, when $\rho = -1$, $\mathrm{E}[x_2 \mid x_1] = \mu_2 - (x_1 - \mu_1)$. In this case,

$x_1 + \mathrm{E}[x_2 \mid x_1] = \mu_1 + \mu_2$. All this keeps with the second approach, that of sticking to the prior hypothesis. When ρ is zero, $\mathrm{E}[x_2 \mid x_1]$ is unaffected, which keeps with the third approach, the BF method. A theory becomes very attractive when it unifies partial explanations. Such is the case with loss covariance. CL, prior hypothesis, or BF—which to choose? The answer will lie in a continuum dependent on the variance matrix of the incremental losses.[7]

6. A LINEAR MODEL OF LOSS COVARIANCE

The idea of loss covariance introduced in the previous section needs to be expanded before we consider a real-life example. Actuaries typically attempt to fill in a "loss rectangle" when all that is known is a triangular portion. The usual case is to have n observations of the earliest accident (or policy) time period, $n-1$ of the next, and so on until the latest time period, for which there is one observation. So there are $1 + 2 + \cdots + n = n(n+1)/2$ known cells, and $n(n-1)/2$ unknown cells in the $(n \times n)$ rectangle. In this discussion we are not concerned about extrapolating beyond the nth interval.[8] The (ij)th cell in the rectangle will

[7]Two more points in closing this section: A suitable variance matrix can make a conditional mean dependent on more than just the latest known loss, thus recognizing Thomas Mack's caveat mentioned earlier in this section. And second, Stanard's fourth model, the additive, which he claims to have performed best in his simulations [10, pp. 131, 135], is the method closest to the covariance method.

[8]So too Murphy: The model does not attempt to predict "beyond the triangle" [8, p. 205]. At this writing, the author is expecting the publication of a paper, "Statistical and Financial Aspects of Self-Insurance Funding," in the 1996 Discussion Paper Program. In Section 3 of that paper the author estimates losses from the 84th month (seventh report) to ultimate. Since the risk treated there had no loss history beyond 84 months, bureau data was invoked, according to which ninety percent of the losses were paid by the 84th month. One might interpret this to mean that there is a development factor from 84th to ultimate of $1.00/0.90 \approx 1.111$, and that the CL method with its bias resurfaces. Even if this were true, at least the use of the CL method would be restricted to a hopefully small role. However, in the paper just mentioned, cumulative predictions as of 84 months were not multiplied by 1.111. Rather, the pure premium for payments up to 84 months, for which an estimate had been derived, was divided by nine to arrive at an estimate of the pure premium for payments after 84 months. The payments after 84 months were then predicted as the product of exposures and the latter pure premium. Assumptions

contain Y_{ij}, the incremental loss of the ith accident period during the jth interval from the beginning of that accident period. The (ij) subscript is a link to much information about the distribution of Y_{ij}; e.g., information about the premium or exposure in the ith period, or inflation trends in absolute time (which is represented by $i + j$).

Now imagine the transpose of the ith row of the rectangle. This is an $(n \times 1)$ vector, the first $(n + 1 - i)$ elements of which are known. Take the known elements of each vector, stack them into an $(n[n + 1]/2 \times 1)$ vector, and call it $\mathbf{Y}_1$. Similarly, stack the unknown elements into an $(n[n - 1]/2 \times 1)$ vector, and call it $\mathbf{Y}_2$. Finally, stack $\mathbf{Y}_1$ on top of $\mathbf{Y}_2$, creating the partitioned $(n^2 \times 1)$ vector $\mathbf{Y}$. Each element of this $\mathbf{Y}$ was originally some Y_{ij} in the rectangle.

We can form the linear model $\mathbf{Y}_{(t\times 1)} = \mathbf{X}_{(t\times k)}\boldsymbol{\beta}_{(k\times 1)} + \mathbf{e}_{(t\times 1)}$, where $t = n^2$ and $\mathrm{Var}[\mathbf{e}] = \boldsymbol{\Sigma}_{(t\times t)}$. $\mathbf{X}$ is the design matrix, each row of which contains pertinent information affixed to the (ij)th location implicit in the same row of $\mathbf{Y}$. The variance, $\boldsymbol{\Sigma}$, determines how errors will influence one another. There is no reason why there cannot be correlations between errors of different accident periods (e.g., calendar-year effects), although it will not be considered in the following example. $\boldsymbol{\Sigma}$ has to be estimated with a minimum of parameters, so it is best to start with only correlation within accident periods.

The objectives are to estimate $\boldsymbol{\beta}$ and to predict the mean and the variance of $\mathbf{Y}_2$ conditional on $\mathbf{Y}_1$, which are done by the method of generalized least squares. The formulas for these objectives are derived in Appendix C. The model outlined here and treated in Appendix C is more general than the idea of the previous section in that (1) it provides for the estimation of unknown parameters, (2) it does not require that error terms be

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were specified as to the covariance of these payments with payments prior to 84 months, so that the payments to ultimate could be affected by the departures of the observations from their predicted values.

normally distributed, and (3) it allows for correlation between, as well as within, accident periods.

7. AN EXAMPLE

Exhibit 1 shows paid workers compensation indemnity losses for eight accident quarters at quarterly evaluations. The numbers above and to the left of the dotted line are actual observations; those below and to the right are projections based on the loss development factors in the bottom row. The development factors are weighted-averages between matched columns; e.g., 1.114 = (756,879 + 2,327,141)/(701,411 + 2,067,233). This is a typical example of the chain ladder method. Notice that the total penultimate (at 24 months) loss, 45,377,646, is obtained without any knowledge of exposures.

Exhibit 2 is an example of Stanard's additive model [10, p. 131]. Incremental losses are related to an exposure base, which in this case is on-level premium. For example, based on two observations, between eighteen and twenty-one months the indemnity payout of an accident quarter will be 0.69% of premium, (55,468 + 259,908)/(11,631,592 + 33,995,192). The incremental payments below and to the right of the dotted line can be projected, and a cumulative table can be constructed. The ultimate losses of the additive model are lower than those of the CL method for every accident quarter.

We will use the linear model $\mathbf{Y}_1 = \mathbf{X}_1\boldsymbol{\beta} + \mathbf{e}_1$, where $\mathbf{Y}_1$ and $\mathbf{X}_1$ are shown in Exhibit 3. We will assume, as is frequently done, that the variance of an observation is proportional to its exposure [Venter 12, p. 445]. The values in the column entitled "Scale $\mathbf{A}_1$" are the square roots of the respective premiums in $\mathbf{X}_1$. If we diagonalize Scale $\mathbf{A}_1$ and call it $\boldsymbol{\Lambda}_1$, then $\text{Var}[\mathbf{e}_1] = \sigma^2\boldsymbol{\Lambda}_1^2 = \sigma^2\boldsymbol{\Psi}_{11}$, where $\boldsymbol{\Psi}_{11}$ is defined as $\boldsymbol{\Lambda}_1^2$. The generalized least-squares estimator for $\boldsymbol{\beta}$, $(\mathbf{X}_1'\boldsymbol{\Psi}_{11}^{-1}\mathbf{X}_1)^{-1}(\mathbf{X}_1'\boldsymbol{\Psi}_{11}^{-1}\mathbf{Y}_1)$, turns out to be:

$$\hat{\boldsymbol{\beta}} = \begin{bmatrix} 0.0099 \\ 0.0196 \\ 0.0142 \\ 0.0123 \\ 0.0108 \\ 0.0096 \\ 0.0069 \\ 0.0061 \end{bmatrix}.$$

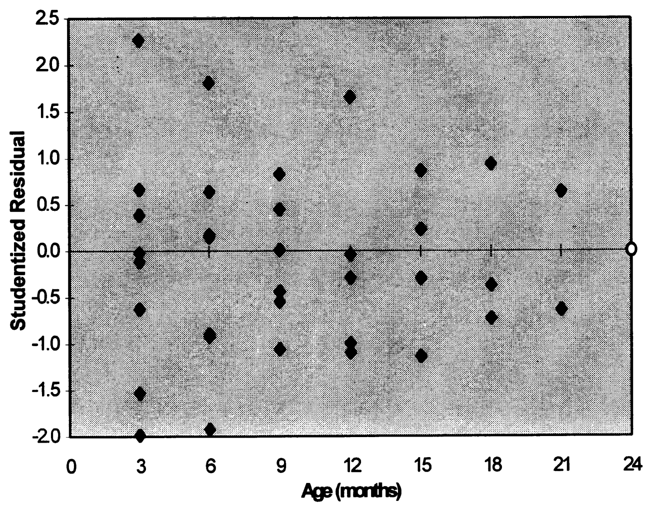
It should come as no surprise that these are the same coefficients as were obtained in Exhibit 2. The linear model with such a proportional variance produces weighted averages [7, pp. 111f]. The formula for the sample variance [5, p. 332] is: $\hat{\sigma}^2 = (\mathbf{Y} - \hat{\mathbf{Y}})'\Psi^{-1}(\mathbf{Y} - \hat{\mathbf{Y}})/(36 - 8)$, which is 176.3242.

Exhibit 4 contains results from the regression. $\mathbf{Y}$ is the same as $\mathbf{Y}_1$ in Exhibit 3; $\hat{\mathbf{Y}}$ is the fitted vector, or $\mathbf{X}_1\hat{\boldsymbol{\beta}}$; and $\hat{\mathbf{e}} = \mathbf{Y} - \hat{\mathbf{Y}}$. Appendix D derives the formula for Var[$\hat{\mathbf{e}}$]. The square roots of the diagonal elements of this matrix are contained in the column Std($\hat{\mathbf{e}}$). $\hat{\mathbf{e}}$ divided by these numbers forms the column Student($\hat{\mathbf{e}}$). If the model is homoskedastic, these studentized residuals should show no increase or decrease by accident quarter age. However, it appears from the graph in Figure 1 that the studentized residuals decrease by age. This is obvious from the following table of sample variances of the studentized residuals by age:

Age	Count	Variance	$\mathbf{Y}$ = Ln(Var)	$\hat{\mathbf{Y}}$	exp($\hat{\mathbf{Y}}$)
3	8	1.759	0.565	0.297	1.345
6	7	1.495	0.402	0.181	1.198
9	6	0.482	−0.729	0.065	1.067
12	5	1.226	0.204	−0.051	0.950
15	4	0.719	−0.329	−0.167	0.846
18	3	0.767	−0.265	−0.283	0.753
21	2	0.813	−0.206	−0.399	0.671

FIGURE 1

ACCIDENT QUARTER RESIDUALS

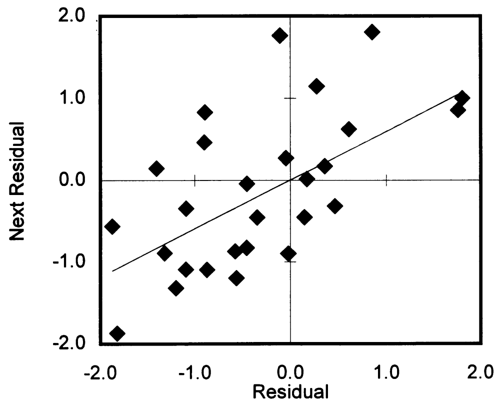


The table also shows that an exponential regression explains well the tapering off of the variance. Moreover, the variance can be predicted for age 24 months, which is 0.597. We now can remodel the variance of an observation as proportional not only to the premium, but also to the fitted or predicted sample variances. The square roots of these new variances are found in the column Scale B of Exhibit 3. For example, for AQ.Age 1.03, the standard deviation of the error is proportional to the square root of the product of 11,631,592 and 1.345, or 3,955.31.

The regression reweighted with Scale $\mathbf{B}_1$ diagonalized as $\mathbf{\Lambda}_1$, produces new standardized residuals that are homoskedastic. The estimate for β changes negligibly (no change within the first ten decimal places). The results of this regression are not shown; however, Figure 2 contains some of the studentized residuals. This exhibit shows that there is a relation between one studentized residual and the next; viz., that the next studentized resid-

FIGURE 2

CORRELATION OF RESIDUALS



ual tends to be 59.31% of the previous. Since we are dealing with studentized, homoskedastic residuals, whose variances should all be unity, the slope coefficient $\hat{\rho}$ should be a correlation coefficient [5, pp. 391f.].

Thus, we will use as our final model one whose error variance matrix is first-order autocorrelated within accident quarters. Exhibit 6 shows partitions of the correlation matrix $\mathbf{P}$, where ρ has been estimated to be 0.5931. It is not necessary to show $\mathbf{P}_{12}$, since it is the transpose of $\mathbf{P}_{21}$. For an explanation as to how first-order correlation produces correlation matrices such as these, see Judge [5, pp. 384–388]. Letting $\mathbf{\Lambda}_1$ and $\mathbf{\Lambda}_2$ be diagonalizations of Scale $\mathbf{B}_1$ and Scale $\mathbf{B}_2$ respectively, we can express the error variance matrix as:

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma}_{11} & \mathbf{\Sigma}_{12} \\ \mathbf{\Sigma}_{21} & \mathbf{\Sigma}_{22} \end{bmatrix} = \sigma^2 \begin{bmatrix} \mathbf{\Lambda}_1 & \\ & \mathbf{\Lambda}_2 \end{bmatrix} \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{\Lambda}_1 & \\ & \mathbf{\Lambda}_2 \end{bmatrix}$$

$$= \sigma^2 \begin{bmatrix} \mathbf{\Lambda}_1\mathbf{P}_{11}\mathbf{\Lambda}_1 & \mathbf{\Lambda}_1\mathbf{P}_{12}\mathbf{\Lambda}_2 \\ \mathbf{\Lambda}_2\mathbf{P}_{21}\mathbf{\Lambda}_1 & \mathbf{\Lambda}_2\mathbf{P}_{22}\mathbf{\Lambda}_2 \end{bmatrix} = \sigma^2 \begin{bmatrix} \mathbf{\Psi}_{11} & \mathbf{\Psi}_{12} \\ \mathbf{\Psi}_{21} & \mathbf{\Psi}_{22} \end{bmatrix}.$$

Therefore, we can estimate $\boldsymbol{\beta}$ as $(\mathbf{X}_1'\boldsymbol{\Psi}_{11}^{-1}\mathbf{X}_1)^{-1}(\mathbf{X}_1'\boldsymbol{\Psi}_{11}^{-1}\mathbf{Y}_1)$, or:

$$\hat{\boldsymbol{\beta}} = \begin{bmatrix} 0.0099 \\ 0.0199 \\ 0.0145 \\ 0.0125 \\ 0.0108 \\ 0.0100 \\ 0.0079 \\ 0.0078 \end{bmatrix}.$$

Also, the estimate for $\hat{\sigma}^2 = (\mathbf{Y}_1 - \mathbf{X}_1\hat{\boldsymbol{\beta}})'\boldsymbol{\Psi}_{11}^{-1}(\mathbf{Y}_1 - \mathbf{X}_1\hat{\boldsymbol{\beta}})/(36 - 8 - 2)$ is 149.9509. The denominator has two less degrees of freedom because two parameters were estimated in creating the correlation matrix; viz., the decay factor in the exponentially fitted variances and the correlation coefficient ρ. The predicted values of $\mathbf{Y}_2$ are calculated according to the formula derived in Appendix C: $E[\mathbf{Y}_2 \mid \mathbf{Y}_1] = \mathbf{X}_2\hat{\boldsymbol{\beta}} + \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}(\mathbf{Y}_1 - \mathbf{X}_1\hat{\boldsymbol{\beta}})$.

Exhibit 7 contains selected values from this final regression. The observed and predicted **Y**s are carried over to the incremental table of Exhibit 8. The cumulative table follows, and shows at age 24 the expected values of quarters 2 through 8. It can be seen that the estimates at age 24 for this method are higher than those of the additive method (Exhibit 2). They are lower than those of the CL method (Exhibit 1), except for quarters 2 and 3 (and even here the losses are only about 0.1 percent higher). Thus it seems that this "covariance method" mediates between a BF variant and the CL method.

Exhibit 9 is like Exhibit 8 except that it displays the $\mathbf{X}_1\hat{\boldsymbol{\beta}}$ and $\mathbf{X}_2\hat{\boldsymbol{\beta}}$ columns of Exhibit 7. These are predictions of $\mathbf{Y}_1$ and $\mathbf{Y}_2$ prior to any observation (of course, we needed observations in order to obtain $\hat{\boldsymbol{\beta}}$). Exhibit 9 helps us to see that the covariance is working in Exhibit 8. For example, the prediction of incremental AQ.Age 2.24 is 261,487. Ignoring observations of 2.03 through 2.21, the prediction would have been 266,326. Why is the a posteriori prediction less than the a priori? It is because the actual 2.21 observation of 259,908 is less than the a priori prediction of 268,555. So the covariance is carrying over to the prediction. AQ 8 is observed to commence with a payment higher than expected, and this excess is perpetuated in forecasts 8.06 through 8.24. However, the excess dampens over time, as expected.

A better but more complicated model would recognize a trend in the observed payments. By comparing Exhibits 2 and 9, one can see that the model tends to overestimate the payments of the first three accident quarters, and to underestimate those of the last five. So perhaps it is no surprise that for quarters 2 and 3 the model gives higher results than does the CL method. Building trend into the model, by applying some sort of inflation index to the exposures would probably lessen the estimate of ρ, and make better use of the error variance matrix. As it is now, it seems that the variance matrix is trying to chase the trend, as well as to capture covariance.

It should be noticed that the column totals of Exhibit 8 are identical to those of Exhibit 9. The author did not expect this, and checked the programming for errors (the work was done both on an Excel spreadsheet and in a SAS program,[9] and the results were the same). It is also of interest that the column rates are identical to the estimate of $\boldsymbol{\beta}$. The author thinks of these

[9] After the body of this paper was written, the author learned of a procedure in SAS, Proc Mixed, which has the ability to estimate simultaneously, by the method of maximum likelihood, both the regression coefficients and any parameters in the variance matrix. SAS users who will be using generalized least squares would do well to study the following SAS publications on Proc Mixed: SAS Institute Inc., SAS® Technical Report P-229, *SAS/STAT® Software: Changes and Enhancements, Release 6.07*, Cary, NC: SAS Institute Inc., 1992, ch. 16, and SAS Institute Inc., Introduction to the MIXED Procedure Course Notes, Cary, NC: SAS Institute Inc., 1995.

At the end of Chapter 16 of the Technical Report is an extensive bibliography of the literature devoted to the subject of mixed models, of which generalized least squares is a subset.

somewhat appealing qualities as "balance properties." Appendix E gives a demonstration of these properties, a demonstration that relies on the peculiarities of this example and so cannot be generalized.

Another interesting property, perhaps related to the column balance just described, is that within an accident quarter, the predictions depend only on the last observation of that quarter. Recall the prediction formula: $E[\mathbf{Y}_2 \mid \mathbf{Y}_1] = \mathbf{X}_2\hat{\beta} + \mathbf{\Sigma}_{21}\mathbf{\Sigma}_{11}^{-1}(\mathbf{Y}_1 - \mathbf{X}_1\hat{\beta})$. The matrix $\mathbf{\Sigma}_{21}\mathbf{\Sigma}_{11}^{-1}$ is zero only where $\mathbf{P}_{21}\mathbf{P}_{11}^{-1}$ is zero, since the $\mathbf{\Sigma}$s are $\mathbf{P}$s times diagonal scaling matrices. Exhibit 6 shows $\mathbf{P}_{21}\mathbf{P}_{11}^{-1}$. (For details about the inverse of a first-order autocorrelation matrix see Judge [5, p. 389].) From this it can be seen that the prediction adjustment of any AQ.Age is proportional to the proper power of ρ times the error of the last observation for that AQ. The errors of earlier ages, though correlated with the predictions, in the first-order autocorrelation model are impounded within, or built into, the error of the latest observation. This has a bearing on Mack's remark about path dependence [7, p. 108], discussed earlier in Section 5. If there is path independence in the first-order autocorrelation model, which is the most basic of generalized linear models, then perhaps actuaries have not been too remiss in developing losses from the last observation only.

The last column of the cumulative table of Exhibit 8 contains the standard deviations of the cumulative paid predictions at 24 months. Appendix C derives the formula for the variance of the predictions:

$$\mathrm{Var}[\mathbf{Y}_2 \mid \mathbf{Y}_1] = (\mathbf{X}_2 - \mathbf{\Sigma}_{21}\mathbf{\Sigma}_{11}^{-1}\mathbf{X}_1)\mathrm{Var}[\hat{\beta}](\mathbf{X}_2 - \mathbf{\Sigma}_{21}\mathbf{\Sigma}_{11}^{-1}\mathbf{X}_1)' + (\mathbf{\Sigma}_{22} - \mathbf{\Sigma}_{21}\mathbf{\Sigma}_{11}^{-1}\mathbf{\Sigma}_{12}).$$

This is a (28×28) matrix, too large to print, but whose row and column headings would be the same as those of $\mathbf{P}_{22}$ in Exhibit 6. The variance of the prediction of AQ 4 at 24 months, for example, is the variance of the sum of the predictions of 4.18, 4.21, and 4.24. This would be the sum of the nine variances and covariances which occupy the square whose diagonal is from (4.18,4.18) to (4.24,4.24). The square root of this number, 293,083, is found in the last column of Exhibit 8. The Total row contains the square root of the sum of all 784 elements of $\text{Var}[\mathbf{Y}_2 \mid \mathbf{Y}_1]$, which is the standard deviation of the sum of all the predictions.

If the errors were normally distributed, then the total predicted would be *t*-distributed with twenty-six degrees of freedom, with a mean of 41,778,516 and a standard deviation of 1,598,047. So, for example, the 95% upper bound for the total predicted would be $41{,}778{,}516 + 1{,}598{,}047 \times 1.706 = 44{,}504{,}784$. However, as stated in the appendices, the errors need not be normally distributed. We could just as easily assume that the total predicted is lognormal (41,778,516, 1,598,047). This is equivalent to $e^{N[\mu=17.54716,\sigma^2=0.001462]}$. The normal random variable has a 95th percentile at $17.54716 + (0.001462)^{1/2} \times 1.645 = 17.61006$. Therefore, the 95% upper bound with a lognormal distribution is 44,457,985.

8. CONCLUSION

Generalized least squares is a better method of loss prediction than the chain ladder method and the other loss development methods. Even when linear models are imposed on loss development methods, they incorporate stochastic regressors, and the estimates are not guaranteed to be either best or unbiased. The confidence intervals derived therefrom are not trustworthy. The fault lies in trying to make the level of one variable affect the level of the next, whereas the statistical idea is that the departure of one variable from its mean affects the departure of the next from its mean. This is the idea of covariance, and it is accommodated in the general linear model and generalized least squares estimation. It may not be easy to determine a good structure for the error variance matrix; but then again, the prediction of losses in itself is no easy feat.

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EXHIBIT 1

CHAIN LADDER METHOD

Cumulative Workers Compensation Indemnity Losses Paid

AQ	@3	@6	@9	@12	@15	@18	@21	@24
Qtr 1	87,248	275,126	393,511	492,620	605,497	701,411	756,879	827,621
Qtr 2	189,320	712,837	1,157,102	1,496,943	1,786,132	2,067,233	2,327,141	2,544,648
Qtr 3	392,599	1,457,421	2,182,953	2,840,352	3,440,079	4,017,704	4,475,360	4,893,652
Qtr 4	675,634	2,214,303	3,282,110	4,289,915	5,113,465	5,950,530	6,628,354	7,247,876
Qtr 5	720,152	2,052,734	3,134,652	4,004,034	4,805,445	5,592,088	6,229,082	6,811,286
Qtr 6	746,772	2,160,682	3,177,136	4,107,877	4,930,073	5,737,117	6,390,631	6,987,934
Qtr 7	769,063	2,218,298	3,331,899	4,307,979	5,170,225	6,016,581	6,701,929	7,328,328
Qtr 8	853,758	2,644,494	3,972,049	5,135,660	6,163,567	7,172,532	7,989,554	8,736,301
Total	4,434,546	13,735,895	20,631,412	26,675,379	32,014,483	37,255,195	41,498,932	45,377,646
Development Factor		3.097	1.502	1.293	1.200	1.164	1.114	1.093

EXHIBIT 2

ADDITIVE METHOD

Incremental Workers Compensation Indemnity Losses Paid

AQ	@3	@6	@9	@12	@15	@18	@21	@24	Premium
Qtr 1	87,248	187,878	118,385	99,109	112,877	95,914	55,468	70,742	11,631,592
Qtr 2	189,320	523,517	444,265	339,841	289,189	281,101	259,908	206,755	33,995,192
Qtr 3	392,599	1,064,822	725,532	657,399	599,727	577,625	371,000	326,440	53,674,098
Qtr 4	675,634	1,538,669	1,067,807	1,007,805	823,550	666,271	479,042	421,505	69,305,023
Qtr 5	720,152	1,332,582	1,081,918	869,382	788,639	700,314	503,518	443,042	72,846,114
Qtr 6	746,772	1,413,910	1,016,454	877,931	771,773	685,337	492,750	433,567	71,288,246
Qtr 7	769,063	1,449,235	1,006,824	870,551	765,286	679,577	488,608	429,922	70,689,000
Qtr 8	853,758	1,246,933	906,681	783,963	689,167	611,983	440,009	387,161	63,658,000
Total	4,434,546	8,757,546	6,367,866	5,505,980	4,840,208	4,298,123	3,090,303	2,719,133	447,087,265
Rate	0.0099	0.0196	0.0142	0.0123	0.0108	0.0096	0.0069	0.0061	

Cumulative Workers Compensation Indemnity Losses Paid

AQ	@3	@6	@9	@12	@15	@18	@21	@24
Qtr 1	87,248	275,126	393,511	492,620	605,497	701,411	756,879	827,621
Qtr 2	189,320	712,837	1,157,102	1,496,943	1,786,132	2,067,233	2,327,141	2,533,896
Qtr 3	392,599	1,457,421	2,182,953	2,840,352	3,440,079	4,017,704	4,388,704	4,715,143
Qtr 4	675,634	2,214,303	3,282,110	4,289,915	5,113,465	5,779,736	6,258,778	6,680,284
Qtr 5	720,152	2,052,734	3,134,652	4,004,034	4,792,673	5,492,987	5,996,505	6,439,547
Qtr 6	746,772	2,160,682	3,177,136	4,055,067	4,826,840	5,512,177	6,004,927	6,438,494
Qtr 7	769,063	2,218,298	3,225,122	4,095,673	4,860,958	5,540,535	6,029,143	6,459,065
Qtr 8	853,758	2,100,691	3,007,372	3,791,335	4,480,502	5,092,485	5,532,495	5,919,655
Total	4,434,546	13,192,092	19,559,958	25,065,938	29,906,146	34,204,269	37,294,572	40,013,705

EXHIBIT 3

PART 1

X_1, Y_1 MATRICES

AQ.Age	Y_1	X_1								Scale A_1	Scale B_1
1.03	87,248	11,631,592	0	0	0	0	0	0	0	3,410.51	3,955.81
1.06	187,878	0	11,631,592	0	0	0	0	0	0	3,410.51	3,732.86
1.09	118,385	0	0	11,631,592	0	0	0	0	0	3,410.51	3,522.47
1.12	99,109	0	0	0	11,631,592	0	0	0	0	3,410.51	3,323.94
1.15	112,877	0	0	0	0	11,631,592	0	0	0	3,410.51	3,136.59
1.18	95,914	0	0	0	0	0	11,631,592	0	0	3,410.51	2,959.81
1.21	55,468	0	0	0	0	0	0	11,631,592	0	3,410.51	2,792.99
1.24	70,742	0	0	0	0	0	0	0	11,631,592	3,410.51	2,635.58
2.03	189,320	33,995,192	0	0	0	0	0	0	0	5,830.54	6,762.77
2.06	523,517	0	33,995,192	0	0	0	0	0	0	5,830.54	6,381.61
2.09	444,265	0	0	33,995,192	0	0	0	0	0	5,830.54	6,021.94
2.12	339,841	0	0	0	33,995,192	0	0	0	0	5,830.54	5,682.53
2.15	289,189	0	0	0	0	33,995,192	0	0	0	5,830.54	5,362.26
2.18	281,101	0	0	0	0	0	33,995,192	0	0	5,830.54	5,060.03
2.21	259,908	0	0	0	0	0	0	33,995,192	0	5,830.54	4,774.84
3.03	392,599	53,674,098	0	0	0	0	0	0	0	7,326.26	8,497.64
3.06	1,064,822	0	53,674,098	0	0	0	0	0	0	7,326.26	8,018.70
3.09	725,532	0	0	53,674,098	0	0	0	0	0	7,326.26	7,566.76
3.12	657,399	0	0	0	53,674,098	0	0	0	0	7,326.26	7,140.28
3.15	599,727	0	0	0	0	53,674,098	0	0	0	7,326.26	6,737.85
3.18	577,625	0	0	0	0	0	53,674,098	0	0	7,326.26	6,358.09

EXHIBIT 3

PART 1—PAGE 2

AQ.Age	Y_1	X_1								Scale A_1	Scale B_1
4.03	675,634	69,305,023	0	0	0	0	0	0	0	8,324.96	9,656.02
4.06	1,538,669	0	69,305,023	0	0	0	0	0	0	8,324.96	9,111.79
4.09	1,067,807	0	0	69,305,023	0	0	0	0	0	8,324.96	8,598.24
4.12	1,007,805	0	0	0	69,305,023	0	0	0	0	8,324.96	8,113.63
4.15	823,550	0	0	0	0	69,305,023	0	0	0	8,324.96	7,656.34
5.03	720,152	72,846,114	0	0	0	0	0	0	0	8,534.99	9,899.63
5.06	1,332,582	0	72,846,114	0	0	0	0	0	0	8,534.99	9,341.68
5.09	1,081,918	0	0	72,846,114	0	0	0	0	0	8,534.99	8,815.17
5.12	869,382	0	0	0	72,846,114	0	0	0	0	8,534.99	8,318.33
6.03	746,772	71,288,246	0	0	0	0	0	0	0	8,443.24	9,793.20
6.06	1,413,910	0	71,288,246	0	0	0	0	0	0	8,443.24	9,241.25
6.09	1,016,454	0	0	71,288,246	0	0	0	0	0	8,443.24	8,720.40
7.03	769,063	70,689,000	0	0	0	0	0	0	0	8,407.68	9,751.96
7.06	1,449,235	0	70,689,000	0	0	0	0	0	0	8,407.68	9,202.32
8.03	853,758	63,658,000	0	0	0	0	0	0	0	7,978.60	9,254.27

EXHIBIT 3

PART 2
X_2, Y_2 MATRICES

AQ.Age	Y_2	X_2								Scale A_2	Scale B_2
2.24		0	0	0	0	0	0	0	33,995,192	5,830.54	4,505.73
3.21		0	0	0	0	0	0	53,674,098	0	7,326.26	5,999.74
3.24		0	0	0	0	0	0	0	53,674,098	7,326.26	5,661.59
4.18		0	0	0	0	0	69,305,023	0	0	8,324.96	7,224.82
4.21		0	0	0	0	0	0	69,305,023	0	8,324.96	6,817.62
4.24		0	0	0	0	0	0	0	69,305,023	8,324.96	6,433.37
5.15		0	0	0	0	72,846,114	0	0	0	8,534.99	7,849.50
5.18		0	0	0	0	0	72,846,114	0	0	8,534.99	7,407.09
5.21		0	0	0	0	0	0	72,846,114	0	8,534.99	6,989.62
5.24		0	0	0	0	0	0	0	72,846,114	8,534.99	6,595.68
6.12		0	0	0	71,288,246	0	0	0	0	8,443.24	8,228.90
6.15		0	0	0	0	71,288,246	0	0	0	8,443.24	7,765.11
6.18		0	0	0	0	0	71,288,246	0	0	8,443.24	7,327.46
6.21		0	0	0	0	0	0	71,288,246	0	8,443.24	6,914.48
6.24		0	0	0	0	0	0	0	71,288,246	8,443.24	6,524.77

EXHIBIT 3

PART 2—PAGE 2

AQ.Age	Y_2	X_2							Scale A_2	Scale B_2
7.09		0	070,689,000	0	0	0	0	0	8,407.68	8,683.67
7.12		0	0	070,689,000	0	0	0	0	8,407.68	8,194.25
7.15		0	0	0	070,689,000	0	0	0	8,407.68	7,732.41
7.18		0	0	0	0	070,689,000	0	0	8,407.68	7,296.60
7.21		0	0	0	0	0	070,689,000	0	8,407.68	6,885.35
7.24		0	0	0	0	0	0	070,689,000	8,407.68	6,497.29
8.06		063,658,000	0	0	0	0	0	0	7,978.60	8,732.69
8.09		0	063,658,000	0	0	0	0	0	7,978.60	8,240.50
8.12		0	0	063,658,000	0	0	0	0	7,978.60	7,776.06
8.15		0	0	0	063,658,000	0	0	0	7,978.60	7,337.79
8.18		0	0	0	0	063,658,000	0	0	7,978.60	6,924.22
8.21		0	0	0	0	0	063,658,000	0	7,978.60	6,533.97
8.24		0	0	0	0	0	0	063,658,000	7,978.60	6,165.70

EXHIBIT 4

FIRST REGRESSION

AQ	Age	**Y**	$\hat{\mathbf{Y}}$	$\hat{\mathbf{e}}$	Std($\hat{\mathbf{e}}$)	Student($\hat{\mathbf{e}}$)
Qtr 1	3	87,248	115,371	−28,123	44,694	−0.629
Qtr 1	6	187,878	227,840	−39,962	44,595	−0.896
Qtr 1	9	118,385	165,669	−47,284	44,437	−1.064
Qtr 1	12	99,109	143,246	−44,137	44,183	−0.999
Qtr 1	15	112,877	125,925	−13,048	43,697	−0.299
Qtr 1	18	95,914	111,822	−15,908	42,552	−0.374
Qtr 1	21	55,468	80,398	−24,930	39,091	−0.638
Qtr 1	24	70,742	70,742	0	0	0
Qtr 2	3	189,320	337,190	−147,870	74,420	−1.987
Qtr 2	6	523,517	665,898	−142,381	73,910	−1.926
Qtr 2	9	444,265	484,194	−39,929	73,093	−0.546
Qtr 2	12	339,841	418,658	−78,817	71,765	−1.098
Qtr 2	15	289,189	368,035	−78,846	69,178	−1.140
Qtr 2	18	281,101	326,817	−45,716	62,786	−0.728
Qtr 2	21	259,908	234,978	24,930	39,091	0.638
Qtr 3	3	392,599	532,380	−139,781	91,257	−1.532
Qtr 3	6	1,064,822	1,051,368	13,454	90,218	0.149
Qtr 3	9	725,532	764,480	−38,948	88,543	−0.440
Qtr 3	12	657,399	661,009	−3,610	85,792	−0.042
Qtr 3	15	599,727	581,081	18,646	80,320	0.232
Qtr 3	18	577,625	516,002	61,623	65,943	0.934
Qtr 4	3	675,634	687,419	−11,785	101,616	−0.116
Qtr 4	6	1,538,669	1,357,547	181,122	100,057	1.810
Qtr 4	9	1,067,807	987,112	80,695	97,530	0.827
Qtr 4	12	1,007,805	853,507	154,298	93,341	1.653
Qtr 4	15	823,550	750,303	73,247	84,836	0.863
Qtr 5	3	720,152	722,542	−2,390	103,690	−0.023
Qtr 5	6	1,332,582	1,426,910	−94,328	102,001	−0.925
Qtr 5	9	1,081,918	1,037,548	44,370	99,261	0.447
Qtr 5	12	869,382	897,116	−27,734	94,707	−0.293
Qtr 6	3	746,772	707,090	39,682	102,789	0.386
Qtr 6	6	1,413,910	1,396,394	17,516	101,157	0.173
Qtr 6	9	1,016,454	1,015,359	1,095	98,512	0.011
Qtr 7	3	769,063	701,146	67,917	102,438	0.663
Qtr 7	6	1,449,235	1,384,656	64,579	100,828	0.640
Qtr 8	3	853,758	631,408	222,350	98,114	2.266

EXHIBIT 5

AUTOCORRELATION (ρ)

AQ	Age	$\mathbf{X}$ = Student($\hat{\mathbf{e}}$)	$\mathbf{Y}$ = Next($\hat{\mathbf{e}}$)	$\hat{\mathbf{Y}} = \mathbf{X}\hat{\rho}$
Qtr 1	3	−0.577	−0.871	−0.342
Qtr 1	6	−0.871	−1.097	−0.517
Qtr 1	9	−1.097	−1.091	−0.650
Qtr 1	12	−1.091	−0.346	−0.647
Qtr 1	15	−0.346	−0.458	−0.205
Qtr 1	18	−0.458	−0.829	−0.272
Qtr 2	3	−1.823	−1.873	−1.081
Qtr 2	6	−1.873	−0.563	−1.111
Qtr 2	9	−0.563	−1.199	−0.334
Qtr 2	12	−1.199	−1.319	−0.711
Qtr 2	15	−1.319	−0.893	−0.782
Qtr 2	18	−0.893	0.829	−0.530
Qtr 3	3	−1.406	0.145	−0.834
Qtr 3	6	0.145	−0.453	0.086
Qtr 3	9	−0.453	−0.046	−0.269
Qtr 3	12	−0.046	0.269	−0.027
Qtr 3	15	0.269	1.146	0.159
Qtr 4	3	−0.106	1.760	−0.063
Qtr 4	6	1.760	0.853	1.044
Qtr 4	9	0.853	1.805	0.506
Qtr 4	12	1.805	0.999	1.071
Qtr 5	3	−0.021	−0.899	−0.013
Qtr 5	6	−0.899	0.461	−0.533
Qtr 5	9	0.461	−0.320	0.273
Qtr 6	3	0.354	0.168	0.210
Qtr 6	6	0.168	0.011	0.100
Qtr 7	3	0.608	0.623	0.361

EXHIBIT 6

PART 1

$\mathbf{P}_{11}$ CORRELATION MATRICES

P_{11}	1.03	1.06	1.09	1.12	1.15	1.18	1.21	1.24	2.03	2.06	2.09	2.12	2.15	2.18	2.21	3.03	3.06	3.09	3.12	3.15	3.18	4.03	4.06	4.09	4.12	4.15	5.03	5.06	5.09	5.12	6.03	6.06	6.09	7.03	7.06	8.03
1.03	ρ^0	ρ^1	ρ^2	ρ^3	ρ^4	ρ^5	ρ^6	ρ^7																												
1.06	ρ^1	ρ^0	ρ^1	ρ^2	ρ^3	ρ^4	ρ^5	ρ^6																												
1.09	ρ^2	ρ^1	ρ^0	ρ^1	ρ^2	ρ^3	ρ^4	ρ^5																												
1.12	ρ^3	ρ^2	ρ^1	ρ^0	ρ^1	ρ^2	ρ^3	ρ^4																												
1.15	ρ^4	ρ^3	ρ^2	ρ^1	ρ^0	ρ^1	ρ^2	ρ^3																												
1.18	ρ^5	ρ^4	ρ^3	ρ^2	ρ^1	ρ^0	ρ^1	ρ^2																												
1.21	ρ^6	ρ^5	ρ^4	ρ^3	ρ^2	ρ^1	ρ^0	ρ^1																												
1.24	ρ^7	ρ^6	ρ^5	ρ^4	ρ^3	ρ^2	ρ^1	ρ^0																												
2.03									ρ^0	ρ^1	ρ^2	ρ^3	ρ^4	ρ^5	ρ^6																					
2.06									ρ^1	ρ^0	ρ^1	ρ^2	ρ^3	ρ^4	ρ^5																					
2.09									ρ^2	ρ^1	ρ^0	ρ^1	ρ^2	ρ^3	ρ^4																					
2.12									ρ^3	ρ^2	ρ^1	ρ^0	ρ^1	ρ^2	ρ^3																					
2.15									ρ^4	ρ^3	ρ^2	ρ^1	ρ^0	ρ^1	ρ^2																					
2.18									ρ^5	ρ^4	ρ^3	ρ^2	ρ^1	ρ^0	ρ^1																					
2.21									ρ^6	ρ^5	ρ^4	ρ^3	ρ^2	ρ^1	ρ^0																					
3.03																ρ^0	ρ^1	ρ^2	ρ^3	ρ^4	ρ^5															
3.06																ρ^1	ρ^0	ρ^1	ρ^2	ρ^3	ρ^4															
3.09																ρ^2	ρ^1	ρ^0	ρ^1	ρ^2	ρ^3															
3.12																ρ^3	ρ^2	ρ^1	ρ^0	ρ^1	ρ^2															
3.15																ρ^4	ρ^3	ρ^2	ρ^1	ρ^0	ρ^1															
3.18																ρ^5	ρ^4	ρ^3	ρ^2	ρ^1	ρ^0															
4.03																						ρ^0	ρ^1	ρ^2	ρ^3	ρ^4										
4.06																						ρ^1	ρ^0	ρ^1	ρ^2	ρ^3										
4.09																						ρ^2	ρ^1	ρ^0	ρ^1	ρ^2										
4.12																						ρ^3	ρ^2	ρ^1	ρ^0	ρ^1										
4.15																						ρ^4	ρ^3	ρ^2	ρ^1	ρ^0										
5.03																											ρ^0	ρ^1	ρ^2	ρ^3						
5.06																											ρ^1	ρ^0	ρ^1	ρ^2						
5.09																											ρ^2	ρ^1	ρ^0	ρ^1						
5.12																											ρ^3	ρ^2	ρ^1	ρ^0						
6.03																															ρ^0	ρ^1	ρ^2			
6.06																															ρ^1	ρ^0	ρ^1			
6.09																															ρ^2	ρ^1	ρ^0			
7.03																																		ρ^0	ρ^1	
7.06																																		ρ^1	ρ^0	
8.03																																				ρ^0

EXHIBIT 6

PART 2

$\mathbf{P}_{21}$ Correlation Matrices

P_{21}	1.03	1.06	1.09	1.12	1.15	1.18	1.21	1.24	2.03	2.06	2.09	2.12	2.15	2.18	2.21	3.03	3.06	3.09	3.12	3.15	3.18	4.03	4.06	4.09	4.12	4.15	5.03	5.06	5.09	5.12	6.03	6.06	6.09	7.03	7.06	8.03
2.24									ρ^7	ρ^6	ρ^5	ρ^4	ρ^3	ρ^2	ρ^1																					
3.21																ρ^6	ρ^5	ρ^4	ρ^3	ρ^2	ρ^1															
3.24																ρ^7	ρ^6	ρ^5	ρ^4	ρ^3	ρ^2															
4.18																						ρ^5	ρ^4	ρ^3	ρ^2	ρ^1										
4.21																						ρ^6	ρ^5	ρ^4	ρ^3	ρ^2										
4.24																						ρ^7	ρ^6	ρ^5	ρ^4	ρ^3										
5.15																											ρ^4	ρ^3	ρ^2	ρ^1						
5.18																											ρ^5	ρ^4	ρ^3	ρ^2						
5.21																											ρ^6	ρ^5	ρ^4	ρ^3						
5.24																											ρ^7	ρ^6	ρ^5	ρ^4						
6.12																															ρ^3	ρ^2	ρ^1			
6.15																															ρ^4	ρ^3	ρ^2			
6.18																															ρ^5	ρ^4	ρ^3			
6.21																															ρ^6	ρ^5	ρ^4			
6.24																															ρ^7	ρ^6	ρ^5			
7.09																																		ρ^2	ρ^1	
7.12																																		ρ^3	ρ^2	
7.15																																		ρ^4	ρ^3	
7.18																																		ρ^5	ρ^4	
7.21																																		ρ^6	ρ^5	
7.24																																		ρ^7	ρ^6	
8.06																																				ρ^1
8.09																																				ρ^2
8.12																																				ρ^3
8.15																																				ρ^4
8.18																																				ρ^5
8.21																																				ρ^6
8.24																																				ρ^7

EXHIBIT 6

PART 3

$\mathbf{P}_{22}$ CORRELATION MATRICES

$\mathbf{P}_{22}$	2.24	3.21	3.24	4.18	4.21	4.24	5.15	5.18	5.21	5.24	6.12	6.15	6.18	6.21	6.24	7.09	7.12	7.15	7.18	7.21	7.24	8.06	8.09	8.12	8.15	8.18	8.21	8.24
2.24	ρ^0																											
3.21		ρ^0	ρ^1																									
3.24		ρ^1	ρ^0																									
4.18				ρ^0	ρ^1	ρ^2																						
4.21				ρ^1	ρ^0	ρ^1																						
4.24				ρ^2	ρ^1	ρ^0																						
5.15							ρ^0	ρ^1	ρ^2	ρ^3																		
5.18							ρ^1	ρ^0	ρ^1	ρ^2																		
5.21							ρ^2	ρ^1	ρ^0	ρ^1																		
5.24							ρ^3	ρ^2	ρ^1	ρ^0																		
6.12											ρ^0	ρ^1	ρ^2	ρ^3	ρ^4													
6.15											ρ^1	ρ^0	ρ^1	ρ^2	ρ^3													
6.18											ρ^2	ρ^1	ρ^0	ρ^1	ρ^2													
6.21											ρ^3	ρ^2	ρ^1	ρ^0	ρ^1													
6.24											ρ^4	ρ^3	ρ^2	ρ^1	ρ^0													
7.09																ρ^0	ρ^1	ρ^2	ρ^3	ρ^4	ρ^5							
7.12																ρ^1	ρ^0	ρ^1	ρ^2	ρ^3	ρ^4							
7.15																ρ^2	ρ^1	ρ^0	ρ^1	ρ^2	ρ^3							
7.18																ρ^3	ρ^2	ρ^1	ρ^0	ρ^1	ρ^2							
7.21																ρ^4	ρ^3	ρ^2	ρ^1	ρ^0	ρ^1							
7.24																ρ^5	ρ^4	ρ^3	ρ^2	ρ^1	ρ^0							
8.06																						ρ^0	ρ^1	ρ^2	ρ^3	ρ^4	ρ^5	ρ^6
8.09																						ρ^1	ρ^0	ρ^1	ρ^2	ρ^3	ρ^4	ρ^5
8.12																						ρ^2	ρ^1	ρ^0	ρ^1	ρ^2	ρ^3	ρ^4
8.15																						ρ^3	ρ^2	ρ^1	ρ^0	ρ^1	ρ^2	ρ^3
8.18																						ρ^4	ρ^3	ρ^2	ρ^1	ρ^0	ρ^1	ρ^2
8.21																						ρ^5	ρ^4	ρ^3	ρ^2	ρ^1	ρ^0	ρ^1
8.24																						ρ^6	ρ^5	ρ^4	ρ^3	ρ^2	ρ^1	ρ^0

EXHIBIT 6

PART 4

$\mathbf{P}_{21}\mathbf{P}_{11}^{-1}$ Correlation Matrices

$P_{21}P_{11}^{-1}$	1.03	1.06	1.09	1.12	1.15	1.18	1.21	1.24	2.03	2.06	2.09	2.12	2.15	2.18	2.21	3.03	3.06	3.09	3.12	3.15	3.18	4.03	4.06	4.09	4.12	4.15	5.03	5.06	5.09	5.12	6.03	6.06	6.09	7.03	7.06	8.03
2.24															ρ^1																					
3.21																					ρ^1															
3.24																					ρ^2															
4.18																										ρ^1										
4.21																										ρ^2										
4.24																										ρ^3										
5.15																														ρ^1						
5.18																														ρ^2						
5.21																														ρ^3						
5.24																														ρ^4						
6.12																																	ρ^1			
6.15																																	ρ^2			
6.18																																	ρ^3			
6.21																																	ρ^4			
6.24																																	ρ^5			
7.09																																			ρ^1	
7.12																																			ρ^2	
7.15																																			ρ^3	
7.18																																			ρ^4	
7.21																																			ρ^5	
7.24																																			ρ^6	
8.06																																				ρ^1
8.09																																				ρ^2
8.12																																				ρ^3
8.15																																				ρ^4
8.18																																				ρ^5
8.21																																				ρ^6
8.24																																				ρ^7

EXHIBIT 7

SELECTED VALUES

AQ.Age	$\mathbf{X}_1\hat{\boldsymbol{\beta}}$	$\mathbf{Y}_1$	AQ.Age	$\mathbf{X}_2\hat{\boldsymbol{\beta}}$	$E[\mathbf{Y}_2 \mid \mathbf{Y}_1]$
1.03	115,371	87,248	2.24	266,326	261,487
1.06	231,615	187,878	3.21	424,014	446,060
1.09	169,126	118,385	3.24	420,496	432,834
1.12	145,210	99,109	4.18	694,978	735,877
1.15	125,953	112,877	4.21	547,495	570,385
1.18	116,639	95,914	4.24	542,952	555,763
1.21	91,887	55,468	5.15	788,818	766,410
1.24	91,125	70,742	5.18	730,487	717,947
2.03	337,190	189,320	5.21	575,468	568,450
2.06	676,931	523,517	5.24	570,694	566,766
2.09	494,297	444,265	6.12	889,970	878,725
2.12	424,399	339,841	6.15	771,948	765,655
2.15	368,119	289,189	6.18	714,865	711,343
2.18	340,897	281,101	6.21	563,162	561,190
2.21	268,555	259,908	6.24	558,489	557,386
3.03	532,380	392,599	7.09	1,027,833	1,051,136
3.06	1,068,788	1,064,822	7.12	882,489	895,531
3.09	780,433	725,532	7.15	765,459	772,758
3.12	670,073	657,399	7.18	708,856	712,941
3.15	581,212	599,727	7.21	558,428	560,714
3.18	538,234	577,625	7.24	553,794	555,074
4.03	687,419	675,634	8.06	1,267,593	1,392,036
4.06	1,380,040	1,538,669	8.09	925,601	995,248
4.09	1,007,710	1,067,807	8.12	794,713	833,692
4.12	865,211	1,007,805	8.15	689,324	711,139
4.15	750,473	823,550	8.18	638,351	650,560
5.03	722,542	720,152	8.21	502,884	509,718
5.06	1,450,552	1,332,582	8.24	498,712	502,536
5.09	1,059,198	1,081,918			
5.12	909,418	869,382			
6.03	707,090	746,772			
6.06	1,419,531	1,413,910			
6.09	1,036,546	1,016,454			
7.03	701,146	769,063			
7.06	1,407,598	1,449,235			
8.03	631,408	853,758			

EXHIBIT 8

COVARIANCE METHOD

Incremental Workers Compensation Indemnity Losses Paid

AQ	@3	@6	@9	@12	@15	@18	@21	@24
Qtr 1	87,248	187,878	118,385	99,109	112,877	95,914	55,468	70,742
Qtr 2	189,320	523,517	444,265	339,841	289,189	281,101	259,908	261,487
Qtr 3	392,599	1,064,822	725,532	657,399	599,727	577,625	446,060	432,834
Qtr 4	675,634	1,538,669	1,067,807	1,007,805	823,550	735,877	570,385	555,763
Qtr 5	720,152	1,332,582	1,081,918	869,382	766,410	717,947	568,450	566,766
Qtr 6	746,772	1,413,910	1,016,454	878,725	765,655	711,343	561,190	557,386
Qtr 7	769,063	1,449,235	1,051,136	895,531	772,758	712,941	560,714	555,074
Qtr 8	853,758	1,392,036	995,248	833,692	711,139	650,560	509,718	502,536
Total	4,434,546	8,902,649	6,500,745	5,581,484	4,841,305	4,483,308	3,531,892	3,502,587
Rate	0.0099	0.0199	0.0145	0.0125	0.0108	0.0100	0.0079	0.0078

Cumulative Workers Compensation Indemnity Losses Paid

AQ	@3	@6	@9	@12	@15	@18	@21	@24	Standard Deviation of 24 Month Prediction
Qtr 1	87,248	275,126	393,511	492,620	605,497	701,411	756,879	827,621	
Qtr 2	189,320	712,837	1,157,102	1,496,943	1,786,132	2,067,233	2,327,141	2,588,628	87,982
Qtr 3	392,599	1,457,421	2,182,953	2,840,352	3,440,079	4,017,704	4,463,764	4,896,598	189,783
Qtr 4	675,634	2,214,303	3,282,110	4,289,915	5,113,465	5,849,342	6,419,727	6,975,489	293,083
Qtr 5	720,152	2,052,734	3,134,652	4,004,034	4,770,444	5,488,391	6,056,841	6,623,606	359,330
Qtr 6	746,772	2,160,682	3,177,136	4,055,861	4,821,515	5,532,858	6,094,049	6,651,434	405,187
Qtr 7	769,063	2,218,298	3,269,434	4,164,964	4,937,723	5,650,664	6,211,378	6,766,452	453,553
Qtr 8	853,758	2,245,794	3,241,042	4,074,734	4,785,873	5,436,433	5,946,151	6,448,687	470,040
Total	4,434,546	13,337,195	19,837,940	25,419,423	30,260,728	34,744,037	38,275,929	41,778,516	1,598,047

EXHIBIT 9

EXPECTED LOSSES PRIOR TO ANY OBSERVATIONS

Incremental Workers Compensation Indemnity Losses Paid

AQ	@3	@6	@9	@12	@15	@18	@21	@24
Qtr 1	115,371	231,615	169,126	145,210	125,953	116,639	91,887	91,125
Qtr 2	337,190	676,931	494,297	424,399	368,119	340,897	268,555	266,326
Qtr 3	532,380	1,068,788	780,433	670,073	581,212	538,234	424,014	420,496
Qtr 4	687,419	1,380,040	1,007,710	865,211	750,473	694,978	547,495	542,952
Qtr 5	722,542	1,450,552	1,059,198	909,418	788,818	730,487	575,468	570,694
Qtr 6	707,090	1,419,531	1,036,546	889,970	771,948	714,865	563,162	558,489
Qtr 7	701,146	1,407,598	1,027,833	882,489	765,459	708,856	558,428	553,794
Qtr 8	631,408	1,267,593	925,601	794,713	689,324	638,351	502,884	498,712
Total	4,434,546	8,902,649	6,500,745	5,581,484	4,841,305	4,483,308	3,531,892	3,502,587
Rate	0.0099	0.0199	0.0145	0.0125	0.0108	0.0100	0.0079	0.0078

Cumulative Workers Compensation Indemnity Losses Paid

AQ	@3	@6	@9	@12	@15	@18	@21	@24
Qtr 1	115,371	346,986	516,111	661,321	787,275	903,914	995,801	1,086,926
Qtr 2	337,190	1,014,121	1,508,418	1,932,818	2,300,936	2,641,834	2,910,388	3,176,715
Qtr 3	532,380	1,601,168	2,381,601	3,051,674	3,632,887	4,171,120	4,595,134	5,015,630
Qtr 4	687,419	2,067,459	3,075,169	3,940,380	4,690,853	5,385,831	5,933,325	6,476,277
Qtr 5	722,542	2,173,094	3,232,293	4,141,711	4,930,528	5,661,016	6,236,484	6,807,178
Qtr 6	707,090	2,126,621	3,163,168	4,053,137	4,825,085	5,539,951	6,103,112	6,661,601
Qtr 7	701,146	2,108,745	3,136,578	4,019,067	4,784,526	5,493,382	6,051,810	6,605,604
Qtr 8	631,408	1,899,001	2,824,602	3,619,315	4,308,639	4,946,989	5,449,874	5,948,585
Total	4,434,546	13,337,195	19,837,940	25,419,423	30,260,728	34,744,037	38,275,929	41,778,516

APPENDIX A

THE GAUSS–MARKOV THEOREM

This proof is an extension of the proof found in Judge [5, pp. 202–205]. Some matrix theory assumed here can be studied from that text, especially from its Appendix A. One principle merely stated here is that if $\mathbf{Z}$ is an $(n \times 1)$ random vector, distributed as $[\boldsymbol{\mu}, \boldsymbol{\Sigma}]$, and $\mathbf{A}$ is a nonstochastic $(m \times n)$ matrix, then $\mathbf{AZ} \sim [\mathbf{A}\boldsymbol{\mu}, \mathbf{A\Sigma A}']$. The distribution does not have to be normal.

We have a model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}$, where $\mathbf{e} \sim [\mathbf{0}, \boldsymbol{\Phi}]$, $\mathbf{Y}$ and $\mathbf{e}$ are $(t \times 1)$, $\mathbf{X}$ is $(t \times k)$, and $\boldsymbol{\beta}$ is $(k \times 1)$. $\mathbf{e}$ does not have to be normally distributed. The rank of $\mathbf{X}$ is k, and $\boldsymbol{\Phi}$ is positive definite. These are standard and nonrestrictive conditions. The last two conditions guarantee that $(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}$ exists, and that there is a nonsingular $(t \times t)$ matrix $\mathbf{W}$ such that $\mathbf{WW}' = \boldsymbol{\Phi}$. (See Appendix C regarding the Cholesky procedure.)

The generalized least-squares estimator is:

$$
\begin{aligned}
\hat{\boldsymbol{\beta}} &= (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{Y}) \\
&= (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X}\boldsymbol{\beta}) + (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{e}) \\
&= \boldsymbol{\beta} + (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{e}) \\
&\sim [\boldsymbol{\beta}, (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}(\mathbf{X}'\boldsymbol{\Phi}^{-1})\mathrm{Var}[\mathbf{e}](\boldsymbol{\Phi}^{-1}\mathbf{X})(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}] \\
&\sim [\boldsymbol{\beta}, (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}(\mathbf{X}'\boldsymbol{\Phi}^{-1})\boldsymbol{\Phi}(\boldsymbol{\Phi}^{-1}\mathbf{X})(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}] \\
&\sim [\boldsymbol{\beta}, (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}] \\
&\sim [\boldsymbol{\beta}, (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}].
\end{aligned}
$$

Now consider some alternative estimator:

$$\begin{aligned}\tilde{\boldsymbol{\beta}} &= \mathbf{AY}\\ &= \mathbf{AX}\boldsymbol{\beta} + \mathbf{Ae}\\ &\sim [\mathbf{AX}\boldsymbol{\beta}, \mathbf{A}\,\mathrm{Var}[\mathbf{e}]\mathbf{A}']\\ &\sim [\mathbf{AX}\boldsymbol{\beta}, \mathbf{A\Phi A}'].\end{aligned}$$

Both estimators are linear functions of $\mathbf{Y}$. The first is unbiased; the second is unbiased, whatever $\boldsymbol{\beta}$ may be, if and only if $\mathbf{AX}$ is the $(k \times k)$ identity matrix $\mathbf{I}_k$. Hence, $\mathbf{AX} = \mathbf{I}_k$.

So far we have two linear unbiased estimators of $\boldsymbol{\beta}$; we have the "LUE" of "BLUE." We show that the first estimator is better (or best of all) by showing that the difference of the variance matrices is nonnegative definite:

$$\begin{aligned}&\mathrm{Var}(\tilde{\boldsymbol{\beta}}) - \mathrm{Var}(\hat{\boldsymbol{\beta}})\\
&\quad= \mathbf{A\Phi A}' - (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\\
&\quad= \mathbf{A\Phi A}' - (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1} - (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1} + (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\\
&\quad= \mathbf{A\Phi A}' - \mathbf{AX}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1} - (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\mathbf{X}'\mathbf{A}'\\
&\qquad+ (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\\
&\quad= \mathbf{AWW}'\mathbf{A}' - \mathbf{AWW}^{-1}\mathbf{X}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\\
&\qquad- (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\mathbf{X}'(\mathbf{W}^{-1})'\mathbf{W}'\mathbf{A}'\\
&\qquad+ (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\mathbf{X}'(\mathbf{W}^{-1})'\mathbf{W}^{-1}\mathbf{X}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\\
&\quad= \{\mathbf{AW} - (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\mathbf{X}'(\mathbf{W}^{-1})'\}\\
&\qquad\times \{\mathbf{W}'\mathbf{A}' - \mathbf{W}^{-1}\mathbf{X}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\}\\
&\quad= \{\mathbf{AW} - (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\mathbf{X}'(\mathbf{W}^{-1})'\}\\
&\qquad\times \{\mathbf{AW} - (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\mathbf{X}'(\mathbf{W}^{-1})'\}'\\
&\quad\geq \mathbf{0}.\end{aligned}$$

The last line means that the matrix in the previous line is non-negative definite, which indeed it is since it is the product of a matrix and its transpose. Therefore, $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{Y})$ is BLUE.

APPENDIX B

THE CONDITIONAL MULTIVARIATE NORMAL DISTRIBUTION

We start with $\mathbf{X} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $\mathbf{X}$ and $\boldsymbol{\mu}$ are $(n \times 1)$, and $\boldsymbol{\Sigma}(n \times n)$ is symmetric and positive definite. Then we partition $\mathbf{X}$ into p known elements and q unknown $(p + q = n)$:

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{bmatrix} \sim N\left(\begin{bmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{bmatrix} \right),$$

where $\mathbf{X}_1$ and $\boldsymbol{\mu}_1$ are $(p \times 1)$, $\mathbf{X}_2$ and $\boldsymbol{\mu}_2$ are $(q \times 1)$, $\boldsymbol{\Sigma}_{11}$ is $(p \times p)$, $\boldsymbol{\Sigma}_{12}$ is $(p \times q)$, $\boldsymbol{\Sigma}_{21}$ is $(q \times p)$, and $\boldsymbol{\Sigma}_{22}$ is $(q \times q)$. Because $\boldsymbol{\Sigma}$ is symmetric, $\boldsymbol{\Sigma}_{11}$ and $\boldsymbol{\Sigma}_{22}$ are symmetric, and $\boldsymbol{\Sigma}_{21} = \boldsymbol{\Sigma}'_{12}$. Moreover, because $\boldsymbol{\Sigma}$ is positive definite, $\boldsymbol{\Sigma}_{11}$ and $\boldsymbol{\Sigma}_{22}$ are positive definite. Furthermore, $\boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21}$ and $\boldsymbol{\Sigma}_{22} - \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12}$ are symmetric and positive definite. Every positive definite matrix has an inverse. The probability density function for $\mathbf{X}$ is [Johnson 4, p. 128]:

$$f_{\mathbf{X}}(\mathbf{X}) = \frac{1}{(2\pi)^{n/2}|\boldsymbol{\Sigma}|^{1/2}} e^{-(1/2)(\mathbf{X}-\mu)'\boldsymbol{\Sigma}^{-1}(\mathbf{X}-\mu)}$$

$$\propto e^{-(1/2)(\mathbf{X}-\mu)'\boldsymbol{\Sigma}^{-1}(\mathbf{X}-\mu)}.$$

Let

$$\mathbf{A}_{(p\times p)} = (\boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21})^{-1}$$

and

$$\mathbf{D}_{(q\times q)} = (\boldsymbol{\Sigma}_{22} - \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12})^{-1}.$$

$\mathbf{A}$ and $\mathbf{D}$ must exist because they are inverses of positive definite matrices. They are also symmetric. An important equation is $\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12}\mathbf{D} = \mathbf{A}\boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}$, which is proven as follows:

$$
\begin{aligned}
\boldsymbol{\Sigma}_{11}^{-1}&\boldsymbol{\Sigma}_{12}\mathbf{D} - \mathbf{A}\boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1} \\
&= \mathbf{A}(\mathbf{A}^{-1}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\mathbf{D}^{-1})\mathbf{D} \\
&= \mathbf{A}([\boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21}]\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12} \\
&\qquad - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}[\boldsymbol{\Sigma}_{22} - \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12}])\mathbf{D} \\
&= \mathbf{A}([\boldsymbol{\Sigma}_{12} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12}] \\
&\qquad - [\boldsymbol{\Sigma}_{12} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12}])\mathbf{D} \\
&= \mathbf{A}(\mathbf{0})\mathbf{D} \\
&= \mathbf{0}.
\end{aligned}
$$

And let $\mathbf{B}_{(p\times q)} = -\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12}\mathbf{D} = -\mathbf{A}\boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}$, so $\mathbf{B}' = -\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21}\mathbf{A}$. It can be shown, from multiplying $\boldsymbol{\Sigma}$ by the following matrix and obtaining the identity matrix, that:

$$
\boldsymbol{\Sigma}^{-1} = \begin{bmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}' & \mathbf{D} \end{bmatrix}.
$$

Therefore,

$$
\begin{aligned}
(\mathbf{X}&-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{X}-\boldsymbol{\mu}) \\
&= (\mathbf{X}-\boldsymbol{\mu})' \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}' & \mathbf{D} \end{bmatrix} (\mathbf{X}-\boldsymbol{\mu}) \\
&= [(\mathbf{X}_1-\boldsymbol{\mu}_1)' \;\; (\mathbf{X}_2-\boldsymbol{\mu}_2)'] \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}' & \mathbf{D} \end{bmatrix} \begin{bmatrix} (\mathbf{X}_1-\boldsymbol{\mu}_1) \\ (\mathbf{X}_2-\boldsymbol{\mu}_2) \end{bmatrix} \\
&= (\mathbf{X}_1-\boldsymbol{\mu}_1)'\mathbf{A}(\mathbf{X}_1-\boldsymbol{\mu}_1) + (\mathbf{X}_1-\boldsymbol{\mu}_1)'\mathbf{B}(\mathbf{X}_2-\boldsymbol{\mu}_2) \\
&\quad + (\mathbf{X}_2-\boldsymbol{\mu}_2)'\mathbf{B}'(\mathbf{X}_1-\boldsymbol{\mu}_1) + (\mathbf{X}_2-\boldsymbol{\mu}_2)'\mathbf{D}(\mathbf{X}_2-\boldsymbol{\mu}_2).
\end{aligned}
$$

If $\mathbf{X}_1$ is given, then $(\mathbf{X}_1-\boldsymbol{\mu}_1)'\mathbf{A}(\mathbf{X}_1-\boldsymbol{\mu}_1)$ is constant, so:

$$
\begin{aligned}
f&_{\mathbf{X}_2|\mathbf{X}_1}(\mathbf{X}_2) \\
&\propto e^{-(1/2)[(\mathbf{X}_2-\boldsymbol{\mu}_2)'\mathbf{D}(\mathbf{X}_2-\boldsymbol{\mu}_2)+(\mathbf{X}_2-\boldsymbol{\mu}_2)'\mathbf{B}'(\mathbf{X}_1-\boldsymbol{\mu}_1)+(\mathbf{X}_1-\boldsymbol{\mu}_1)'\mathbf{B}(\mathbf{X}_2-\boldsymbol{\mu}_2)]}.
\end{aligned}
$$

Now, because $\mathbf{D}$ is symmetric and positive definite, there exists a nonsingular $\mathbf{W}_{(q\times q)}$ such that $\mathbf{W}'\mathbf{W} = \mathbf{D}$. Therefore,

$$
\begin{aligned}
&(\mathbf{X}_2-\mu_2)'\mathbf{D}(\mathbf{X}_2-\mu_2) + (\mathbf{X}_2-\mu_2)'\mathbf{B}'(\mathbf{X}_1-\mu_1)\\
&\qquad + (\mathbf{X}_1-\mu_1)'\mathbf{B}(\mathbf{X}_2-\mu_2)\\
&\quad = (\mathbf{X}_2-\mu_2)'\mathbf{W}'\mathbf{W}(\mathbf{X}_2-\mu_2)\\
&\qquad + (\mathbf{X}_2-\mu_2)'\mathbf{W}'(\mathbf{W}')^{-1}\mathbf{B}'(\mathbf{X}_1-\mu_1)\\
&\qquad + (\mathbf{X}_1-\mu_1)'\mathbf{B}\mathbf{W}^{-1}\mathbf{W}(\mathbf{X}_2-\mu_2)\\
&\quad = [(\mathbf{X}_2-\mu_2)'\mathbf{W}' + (\mathbf{X}_1-\mu_1)'\mathbf{B}\mathbf{W}^{-1}]\\
&\qquad \times [\mathbf{W}(\mathbf{X}_2-\mu_2) + (\mathbf{W}')^{-1}\mathbf{B}'(\text{"`}X_1-\mu_1)]\\
&\qquad - (\mathbf{X}_1-\mu_1)'\mathbf{B}\mathbf{W}^{-1}(\mathbf{W}')^{-1}\mathbf{B}'(\mathbf{X}_1-\mu_1).
\end{aligned}
$$

This expression goes into the exponent of the probability distribution. When $\mathbf{X}_1$ is given, the term after the minus sign in the last equation is constant. Therefore, dropping this term does not change the proportionality of the conditional distribution. So we continue:

$$
\begin{aligned}
&[(\mathbf{X}_2-\mu_2)'\mathbf{W}' + (\mathbf{X}_1-\mu_1)'\mathbf{B}\mathbf{W}^{-1}]\\
&\qquad \times [\mathbf{W}(\mathbf{X}_2-\mu_2) + (\mathbf{W}')^{-1}\mathbf{B}'(\mathbf{X}_1-\mu_1)]\\
&\quad = [(\mathbf{X}_2-\mu_2)' + (\mathbf{X}_1-\mu_1)'\mathbf{B}\mathbf{W}^{-1}(\mathbf{W}')^{-1}]\mathbf{W}'\mathbf{W}\\
&\qquad \times [(\mathbf{X}_2-\mu_2) + \mathbf{W}^{-1}(\mathbf{W}')^{-1}\mathbf{B}'(\mathbf{X}_1-\mu_1)]\\
&\quad = [(\mathbf{X}_2-\mu_2)' + (\mathbf{X}_1-\mu_1)'\mathbf{B}(\mathbf{W}'\mathbf{W})^{-1}]\mathbf{W}'\mathbf{W}\\
&\qquad \times [(\mathbf{X}_2-\mu_2) + (\mathbf{W}'\mathbf{W})^{-1}\mathbf{B}'(\mathbf{X}_1-\mu_1)]\\
&\quad = [(\mathbf{X}_2-\mu_2)' + (\mathbf{X}_1-\mu_1)'\mathbf{B}\mathbf{D}^{-1}]\mathbf{D}\\
&\qquad \times [(\mathbf{X}_2-\mu_2) + \mathbf{D}^{-1}\mathbf{B}'(\mathbf{X}_1-\mu_1)]\\
&\quad = [(\mathbf{X}_2-\mu_2) + \mathbf{D}^{-1}\mathbf{B}'(\mathbf{X}_1-\mu_1)]'(\mathbf{D}^{-1})^{-1}\\
&\qquad \times [(\mathbf{X}_2-\mu_2) + \mathbf{D}^{-1}\mathbf{B}'(\mathbf{X}_1-\mu_1)].
\end{aligned}
$$

Therefore,

$$f_{\mathbf{X}_2|\mathbf{X}_1}(\mathbf{X}_2) \propto e^{-(1/2)[(\mathbf{X}_2-\mu_2)+\mathbf{D}^{-1}\mathbf{B}'(\mathbf{X}_1-\mu_1)]'(\mathbf{D}^{-1})^{-1}[(\mathbf{X}_2-\mu_2)+\mathbf{D}^{-1}\mathbf{B}'(\mathbf{X}_1-\mu_1)]}.$$

This form is multivariate normal with the following characteristics:

$$\mathbf{X}_2 \mid \mathbf{X}_1 \sim N(\boldsymbol{\mu}_2 - \mathbf{D}^{-1}\mathbf{B}'(\mathbf{X}_1 - \boldsymbol{\mu}_1), \mathbf{D}^{-1}).$$

But $\mathbf{D}^{-1} = \boldsymbol{\Sigma}_{22} - \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12}$ and $\mathbf{D}^{-1}\mathbf{B}' = \mathbf{D}^{-1}(-\mathbf{D}\boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}) = -\boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}$. Therefore,

$$\mathbf{X}_2 \mid \mathbf{X}_1 \sim N(\boldsymbol{\mu}_2 + \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}(\mathbf{X}_1 - \boldsymbol{\mu}_1), \boldsymbol{\Sigma}_{22} - \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12}).$$

Finally, notice that:

$$\begin{aligned}\mathrm{Var}[\mathbf{X}_2] - \mathrm{Var}[\mathbf{X}_2 \mid \mathbf{X}_1] &= \boldsymbol{\Sigma}_{22} - (\boldsymbol{\Sigma}_{22} - \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12})\\ &= \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12}\\ &\geq \mathbf{0},\end{aligned}$$

because $\boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12}$ is nonnegative definite. Therefore, the conditional variance of $\mathbf{X}_2$ is less than or equal to the unconditional variance.

APPENDIX C

THE LEAST SQUARES PREDICTOR

This appendix relies much upon Appendix B, and is similar to the derivation by Judge [5, pp. 343–346]. We start with the standard linear model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}$, where $\mathbf{Y}$ and $\mathbf{e}$ are $(t \times 1)$, $\mathbf{X}$ is $(t \times k)$, and $\boldsymbol{\beta}$ is $(k \times 1)$. $\mathbf{e} \sim [\mathbf{0}, \boldsymbol{\Sigma}]$, not necessarily normal, where $\boldsymbol{\Sigma}$ is symmetric and positive definite. However, the first p rows of $\mathbf{Y}$ have been observed; the last $q = t - p$ rows are to be predicted. So the partitioned model is:

$$\begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{bmatrix} \boldsymbol{\beta} + \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{bmatrix},$$

where

$$\mathrm{Var}\begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{bmatrix}.$$

The four submatrices of $\boldsymbol{\Sigma}$ have all the properties described in Appendix B. Note that the known matrices are $\mathbf{Y}_1, \mathbf{X}_1, \mathbf{X}_2$, and $\boldsymbol{\Sigma}$. $\mathbf{Y}_1$ is known by observation, and $\boldsymbol{\Sigma}$ usually has to be estimated.

It is a theorem of matrix algebra that given a symmetric and positive definite $\boldsymbol{\Sigma}$, there exists a nonsingular, lower-triangular matrix $\mathbf{W}$, such that $\mathbf{W}\boldsymbol{\Sigma}\mathbf{W}' = \mathbf{I}$. Equivalently, $\boldsymbol{\Sigma}^{-1} = \mathbf{W}'\mathbf{W}$. A suitable matrix $\mathbf{W}$ can be found by the Cholesky procedure [Healy, 3, pp. 54f]. $\mathbf{W}$ can be partitioned as:

$$\begin{bmatrix} \mathbf{A}_{(p \times p)} & \mathbf{0}_{(p \times q)} \\ \mathbf{C}_{(q \times p)} & \mathbf{D}_{(q \times q)} \end{bmatrix},$$

where $\mathbf{A}$ and $\mathbf{D}$ are nonsingular and lower-triangular. Choose $\mathbf{A}$ such that $\mathbf{A}\boldsymbol{\Sigma}_{11}\mathbf{A}' = \mathbf{I}_p$, and $\mathbf{D}$ such that $\mathbf{D}(\boldsymbol{\Sigma}_{22} - \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12})\mathbf{D}' = \mathbf{I}_q$. The existence of suitable matrices $\mathbf{A}$ and $\mathbf{D}$ is guaranteed, since $\boldsymbol{\Sigma}_{11}$ and $\boldsymbol{\Sigma}_{22} - \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12}$ are both symmetric and positive definite. And let $\mathbf{C} = -\mathbf{D}\boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}$. Partitioned matrix multiplication will show:

$$\mathbf{W}\boldsymbol{\Sigma}\mathbf{W}' = \begin{bmatrix} \mathbf{A}_{(p\times p)} & \mathbf{0}_{(p\times q)} \\ \mathbf{C}_{(q\times p)} & \mathbf{D}_{(q\times q)} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{A}'_{(p\times p)} & \mathbf{C}'_{(p\times q)} \\ \mathbf{0}_{(q\times p)} & \mathbf{D}'_{(q\times q)} \end{bmatrix}$$
$$= \begin{bmatrix} \mathbf{I}_p & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_q \end{bmatrix} = \mathbf{I}_t.$$

The matrix $\mathbf{W}$ provides a convenient linear one-to-one transformation of the original model $\mathbf{WY} = \mathbf{WX}\boldsymbol{\beta} + \mathbf{We}$, where $\mathbf{We} \sim [\mathbf{0}, \mathbf{W}\boldsymbol{\Sigma}\mathbf{W}'] \sim [\mathbf{0}, \mathbf{I}_t]$. In partitioned matrices, we have:

$$\begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{bmatrix} \boldsymbol{\beta} + \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{bmatrix}, \qquad \text{and}$$
$$\begin{bmatrix} \mathbf{AY}_1 \\ \mathbf{CY}_1 + \mathbf{DY}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{AX}_1\boldsymbol{\beta} \\ (\mathbf{CX}_1 + \mathbf{DX}_2)\boldsymbol{\beta} \end{bmatrix} + \begin{bmatrix} \mathbf{Ae}_1 \\ \mathbf{Ce}_1 + \mathbf{De}_2 \end{bmatrix}.$$

Since $\mathbf{Y}_2$ is unknown, $\boldsymbol{\beta}$ must be estimated from the first p rows. And since $\mathbf{A}$ is nonsingular, the model for estimating $\boldsymbol{\beta}$ may be reduced to $\mathbf{Y}_1 = \mathbf{X}_1\boldsymbol{\beta} + \mathbf{e}_1$, where $\mathbf{e}_1 \sim [\mathbf{0}, \boldsymbol{\Sigma}_{11}]$. Therefore, the best linear unbiased estimator is (see Appendix A):

$$\begin{aligned} \hat{\boldsymbol{\beta}} &= (\mathbf{X}_1'\boldsymbol{\Sigma}_{11}^{-1}\mathbf{X}_1)^{-1}(\mathbf{X}_1'\boldsymbol{\Sigma}_{11}^{-1}\mathbf{Y}_1) \\ &= \boldsymbol{\beta} + (\mathbf{X}_1'\boldsymbol{\Sigma}_{11}^{-1}\mathbf{X}_1)^{-1}(\mathbf{X}_1'\boldsymbol{\Sigma}_{11}^{-1}\mathbf{e}_1) \\ &\sim [\boldsymbol{\beta}, (\mathbf{X}_1'\boldsymbol{\Sigma}_{11}^{-1}\mathbf{X}_1)^{-1}]. \end{aligned}$$

Now, instead of considering the predictor of $\mathbf{Y}_2$, let us consider the predictor of $\mathbf{CY}_1 + \mathbf{DY}_2$. This is easier because its error term, $\mathbf{Ce}_1 + \mathbf{De}_2$, is uncorrelated with the error term of the $\mathbf{AY}_1$, which is $\mathbf{Ae}_1$. This is the reason for finding $\mathbf{W}$ such that $\mathbf{W}\boldsymbol{\Sigma}\mathbf{W}' = \mathbf{I}$. Therefore, $\mathrm{Cov}[\mathbf{Ae}_1, \mathbf{Ce}_1 + \mathbf{De}_2] = \mathbf{0}_{(p\times q)}$. Moreover, premultiplying both sides of this equation by $\mathbf{A}^{-1}$ yields $\mathrm{Cov}[\mathbf{A}^{-1}\mathbf{Ae}_1, \mathbf{Ce}_1 + \mathbf{De}_2] = \mathrm{Cov}[\mathbf{e}_1, \mathbf{Ce}_1 + \mathbf{De}_2] = \mathbf{0}_{(p\times q)}$. And since $\hat{\boldsymbol{\beta}}$ is a linear combination of $\mathbf{e}_1$, this implies that

$\mathrm{Cov}[\hat{\boldsymbol{\beta}}, \mathbf{Ce}_1 + \mathbf{De}_2] = \mathbf{0}$. Hence:

$$\begin{aligned}\mathbf{CY}_1 + \mathbf{DY}_2 &= (\mathbf{CX}_1 + \mathbf{DX}_2)\boldsymbol{\beta} + (\mathbf{Ce}_1 + \mathbf{De}_2)\\ &= (\mathbf{CX}_1 + \mathbf{DX}_2)\hat{\boldsymbol{\beta}} - (\mathbf{CX}_1 + \mathbf{DX}_2)(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) + (\mathbf{Ce}_1 + \mathbf{De}_2)\\ &= (\mathbf{CX}_1 + \mathbf{DX}_2)\hat{\boldsymbol{\beta}} + \mathbf{h},\end{aligned}$$

where $\mathbf{h} = -(\mathbf{CX}_1 + \mathbf{DX}_2)(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) + (\mathbf{Ce}_1 + \mathbf{De}_2)$ is the total error term. $\mathrm{E}[\mathbf{h}] = \mathbf{0}_{(q\times 1)}$. And, because $\mathrm{Cov}[\hat{\boldsymbol{\beta}}, \mathbf{Ce}_1 + \mathbf{De}_2] = \mathbf{0}$,

$$\begin{aligned}\mathrm{Var}[\mathbf{h}] &= \mathrm{Var}[(\mathbf{CX}_1 + \mathbf{DX}_2)\hat{\boldsymbol{\beta}}] + \mathrm{Var}[\mathbf{Ce}_1 + \mathbf{De}_2]\\ &= (\mathbf{CX}_1 + \mathbf{DX}_2)\mathrm{Var}[\hat{\boldsymbol{\beta}}](\mathbf{CX}_1 + \mathbf{DX}_2)' + \mathbf{I}_q,\end{aligned}$$

where, of course, $\mathrm{Var}[\hat{\boldsymbol{\beta}}] = (\mathbf{X}_1'\boldsymbol{\Sigma}_{11}^{-1}\mathbf{X}_1)^{-1}$.

When $\mathbf{Y}_1$ is observed, $\hat{\boldsymbol{\beta}}$ is determined; and we have:

$$\begin{aligned}\mathbf{DY}_2 \mid \mathbf{Y}_1 &= (\mathbf{CX}_1 + \mathbf{DX}_2)\hat{\boldsymbol{\beta}} - \mathbf{CY}_1 + \mathbf{h}\\ \mathbf{Y}_2 \mid \mathbf{Y}_1 &= \mathbf{D}^{-1}(\mathbf{CX}_1 + \mathbf{DX}_2)\hat{\boldsymbol{\beta}} - \mathbf{D}^{-1}\mathbf{CY}_1 + \mathbf{D}^{-1}\mathbf{h}\\ &= \mathbf{D}^{-1}\mathbf{CX}_1\hat{\boldsymbol{\beta}} + \mathbf{X}_2\hat{\boldsymbol{\beta}} - \mathbf{D}^{-1}\mathbf{CY}_1 + \mathbf{D}^{-1}\mathbf{h}\\ &= \mathbf{X}_2\hat{\boldsymbol{\beta}} - \mathbf{D}^{-1}\mathbf{C}(\mathbf{Y}_1 - \mathbf{X}_1\hat{\boldsymbol{\beta}}) + \mathbf{D}^{-1}\mathbf{h}\\ &\sim [\mathbf{X}_2\hat{\boldsymbol{\beta}} - \mathbf{D}^{-1}\mathbf{C}(\mathbf{Y}_1 - \mathbf{X}_1\hat{\boldsymbol{\beta}}), \mathbf{D}^{-1}\mathrm{Var}[\mathbf{h}]\mathbf{D}'^{-1}].\end{aligned}$$

But $\mathbf{D}^{-1}\mathbf{C} = \mathbf{D}^{-1}(-\mathbf{D}\boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}) = -\boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}$. And

$$\begin{aligned}&\mathbf{D}^{-1}\mathrm{Var}[\mathbf{h}]\mathbf{D}'^{-1}\\ &\quad= \mathbf{D}^{-1}\{(\mathbf{CX}_1 + \mathbf{DX}_2)\mathrm{Var}[\hat{\boldsymbol{\beta}}](\mathbf{CX}_1 + \mathbf{DX}_2)' + \mathbf{I}_q\}\mathbf{D}'^{-1}\\ &\quad= \mathbf{D}^{-1}(\mathbf{CX}_1 + \mathbf{DX}_2)\mathrm{Var}[\hat{\boldsymbol{\beta}}](\mathbf{CX}_1 + \mathbf{DX}_2)'\mathbf{D}'^{-1} + \mathbf{D}^{-1}\mathbf{D}'^{-1}\\ &\quad= (\mathbf{D}^{-1}\mathbf{CX}_1 + \mathbf{X}_2)\mathrm{Var}[\hat{\boldsymbol{\beta}}](\mathbf{D}^{-1}\mathbf{CX}_1 + \mathbf{X}_2)' + (\mathbf{D}'\mathbf{D})^{-1}\\ &\quad= (-\boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\mathbf{X}_1 + \mathbf{X}_2)\mathrm{Var}[\hat{\boldsymbol{\beta}}](-\boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\mathbf{X}_1 + \mathbf{X}_2)' + (\mathbf{D}'\mathbf{D})^{-1}\\ &\quad= (\mathbf{X}_2 - \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\mathbf{X}_1)\mathrm{Var}[\hat{\boldsymbol{\beta}}](\mathbf{X}_2 - \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\mathbf{X}_1)'\\ &\qquad + (\boldsymbol{\Sigma}_{22} - \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12}).\end{aligned}$$

Therefore, per the Gauss–Markov theorem, $E[\mathbf{Y}_2 \mid \mathbf{Y}_1] = \mathbf{X}_2\hat{\boldsymbol{\beta}} + \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}(\mathbf{Y}_1 - \mathbf{X}_1\hat{\boldsymbol{\beta}})$ is the best linear unbiased predictor. Leaving aside the meaning of the square root of a variance matrix (i.e., a standard deviation matrix), we will write this as:

$$\boldsymbol{\Sigma}_{22}^{-0.5}(E[\mathbf{Y}_2 \mid \mathbf{Y}_1] - \mathbf{X}_2\hat{\boldsymbol{\beta}}) = \{\boldsymbol{\Sigma}_{22}^{-0.5}\boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-0.5}\}\{\boldsymbol{\Sigma}_{11}^{-0.5}(\mathbf{Y}_1 - \mathbf{X}_1\hat{\boldsymbol{\beta}})\}.$$

The terms on the ends of the equation look like standardized random vectors, and the middle term looks like a correlation matrix. This is a matrix generalization of the bivariate conditional expectation of Section 5 and Appendix B.

APPENDIX D

THE VARIANCE OF THE RESIDUALS

In Section 7, residuals were studentized; i.e., divided by the square root of the diagonal elements of a variance matrix. In this appendix, the expression for the variance of the residuals is derived.

We have the usual model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}$, where $\mathbf{e} \sim [\mathbf{0}, \boldsymbol{\Phi}]$. $\boldsymbol{\Phi}$ is symmetric and positive definite, and the rank of $\mathbf{X}_{(t\times k)}$ is k. These conditions guarantee that $(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}$ exists. We have shown in Appendix A:

$$\hat{\boldsymbol{\beta}} = \boldsymbol{\beta} + (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{e}) \sim [\boldsymbol{\beta}, (\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}].$$

By definition,

$$\begin{aligned}
\hat{\mathbf{e}} &= \mathbf{Y} - \hat{\mathbf{Y}} \\
&= \mathbf{X}\boldsymbol{\beta} + \mathbf{e} - \mathbf{X}\hat{\boldsymbol{\beta}} \\
&= \mathbf{e} - \mathbf{X}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) \\
&= \mathbf{e} - \mathbf{X}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{e}) \\
&= (\mathbf{I}_t - \mathbf{X}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\mathbf{X}'\boldsymbol{\Phi}^{-1})\mathbf{e}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\mathrm{Var}[\hat{\mathbf{e}}] \\
&= (\mathbf{I}_t - \mathbf{X}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\mathbf{X}'\boldsymbol{\Phi}^{-1})\mathrm{Var}[\mathbf{e}](\mathbf{I}_t - \mathbf{X}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\mathbf{X}'\boldsymbol{\Phi}^{-1})' \\
&= (\mathbf{I}_t - \mathbf{X}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\mathbf{X}'\boldsymbol{\Phi}^{-1})\boldsymbol{\Phi}(\mathbf{I}_t - \boldsymbol{\Phi}^{-1}\mathbf{X}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\mathbf{X}') \\
&= (\boldsymbol{\Phi} - \mathbf{X}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\mathbf{X}')(\mathbf{I}_t - \boldsymbol{\Phi}^{-1}\mathbf{X}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\mathbf{X}') \\
&= (\boldsymbol{\Phi} - \mathbf{X}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\mathbf{X}') - (\mathbf{X}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\mathbf{X}' - \mathbf{X}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\mathbf{X}') \\
&= \boldsymbol{\Phi} - \mathbf{X}(\mathbf{X}'\boldsymbol{\Phi}^{-1}\mathbf{X})^{-1}\mathbf{X}'.
\end{aligned}$$

If instead of $\boldsymbol{\Phi}$ we have $\sigma^2\boldsymbol{\Phi}$, with σ^2 unknown, we use the estimate for σ^2.

APPENDIX E

THE BALANCE PROPERTIES OF SECTION 7

The a priori predictions are expressed as a (64×1) partitioned vector

$$\begin{bmatrix} \mathbf{X}_1\hat{\boldsymbol{\beta}} \\ \mathbf{X}_2\hat{\boldsymbol{\beta}} \end{bmatrix}.$$

Let $\mathbf{E}_1$ be a (36×36) diagonal matrix whose diagonal elements are the exposures (or premiums) in $\mathbf{X}_1$. In other words, $\mathbf{E}_1$ is Scale $\mathbf{A}_1$ (Exhibit 3) diagonalized and squared. Since the exposure must be positive, $\mathbf{E}_1$ is nonsingular. Let $\mathbf{J}_1 = \mathbf{E}_1^{-1}\mathbf{X}_1$. $\mathbf{J}_1$ has ones where $\mathbf{X}_1$ has positive numbers, and like $\mathbf{X}_1$ is zero everywhere else. Let $\mathbf{E}_2$ (28×28) be similarly defined, but with respect to $\mathbf{X}_2$. And let $\mathbf{J}_2 = \mathbf{E}_2^{-1}\mathbf{X}_2$.

The column totals of the a priori predictions are represented by the (8×1) matrix

$$[\mathbf{J}_1' \quad \mathbf{J}_2'] \begin{bmatrix} \mathbf{X}_1\hat{\boldsymbol{\beta}} \\ \mathbf{X}_2\hat{\boldsymbol{\beta}} \end{bmatrix} = \mathbf{J}_1'\mathbf{X}_1\hat{\boldsymbol{\beta}} + \mathbf{J}_2'\mathbf{X}_2\hat{\boldsymbol{\beta}}.$$

Similarly, the column totals of the a posteriori predictions are

$$[\mathbf{J}_1' \quad \mathbf{J}_2'] \begin{bmatrix} \mathbf{Y}_1 \\ \mathrm{E}[\mathbf{Y}_2 \mid \mathbf{Y}_1] \end{bmatrix} = \mathbf{J}_1'\mathbf{Y}_1 + \mathbf{J}_2'\mathrm{E}[\mathbf{Y}_2 \mid \mathbf{Y}_1].$$

The first balance property to be demonstrated is that these two vectors are equal, or that their difference is $\mathbf{0}_{(8\times 1)}$:

$$\begin{aligned}
\mathbf{0} &= \mathbf{J}_1'\mathbf{Y}_1 + \mathbf{J}_2'\mathrm{E}[\mathbf{Y}_2 \mid \mathbf{Y}_1] - \mathbf{J}_1'\mathbf{X}_1\hat{\boldsymbol{\beta}} - \mathbf{J}_2'\mathbf{X}_2\hat{\boldsymbol{\beta}} \\
&= \mathbf{J}_1'(\mathbf{Y}_1 - \mathbf{X}_1\hat{\boldsymbol{\beta}}) + \mathbf{J}_2'(\mathrm{E}[\mathbf{Y}_2 \mid \mathbf{Y}_1] - \mathbf{X}_2\hat{\boldsymbol{\beta}}) \\
&= \mathbf{J}_1'(\mathbf{Y}_1 - \mathbf{X}_1\hat{\boldsymbol{\beta}}) + \mathbf{J}_2'(\boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}(\mathbf{Y}_1 - \mathbf{X}_1\hat{\boldsymbol{\beta}})) \\
&= (\mathbf{J}_1' + \mathbf{J}_2'\boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1})(\mathbf{Y}_1 - \mathbf{X}_1\hat{\boldsymbol{\beta}}) \\
&= (\mathbf{J}_1'\boldsymbol{\Sigma}_{11} + \mathbf{J}_2'\boldsymbol{\Sigma}_{21})\boldsymbol{\Sigma}_{11}^{-1}(\mathbf{Y}_1 - \mathbf{X}_1\hat{\boldsymbol{\beta}}).
\end{aligned}$$

But, since $\hat{\boldsymbol{\beta}} = (\mathbf{X}_1'\boldsymbol{\Sigma}_{11}^{-1}\mathbf{X}_1)^{-1}(\mathbf{X}_1'\boldsymbol{\Sigma}_{11}^{-1}\mathbf{Y}_1)$,

$$(\mathbf{X}_1'\boldsymbol{\Sigma}_{11}^{-1}\mathbf{X}_1)\hat{\boldsymbol{\beta}} = (\mathbf{X}_1'\boldsymbol{\Sigma}_{11}^{-1}\mathbf{Y}_1).$$

Hence,

$$\mathbf{0} = \mathbf{X}_1'\boldsymbol{\Sigma}_{11}^{-1}(\mathbf{Y}_1 - \mathbf{X}_1\hat{\beta}).$$

Therefore, if we can show that $(\mathbf{J}_1'\boldsymbol{\Sigma}_{11} + \mathbf{J}_2'\boldsymbol{\Sigma}_{21})$ can be factored as the product of some matrix and $\mathbf{X}_1'$, then we will have proved the first balance property.

We need to define some more notation. Let $\mathbf{L}_1$ be the (36×36) diagonal matrix of the variance relativities that were introduced into the weighted regression to remove heteroskedasticity. In other words, $\mathbf{E}_1\mathbf{L}_1$ is Scale $\mathbf{B}_1$ (Exhibit 3) diagonalized and squared. Of course, since both $\mathbf{E}_1$ and $\mathbf{L}_1$ are diagonal, $\mathbf{E}_1\mathbf{L}_1 = \mathbf{L}_1\mathbf{E}_1$.

Therefore, $\boldsymbol{\Sigma}_{11} = \mathbf{L}_1^{0.5}\mathbf{E}_1^{0.5}\mathbf{P}_{11}\mathbf{E}_1^{0.5}\mathbf{L}_1^{0.5}$. (To be more accurate, we should introduce the proportionality constant σ^2. However, the same constant would apply to $\boldsymbol{\Sigma}_{21}$, and the balance property is unaffected.) $\mathbf{P}_{11}$ is shown in Exhibit 6, and it is easy to see that $\mathbf{P}_{11}$ and $\mathbf{E}_1$ commute. Therefore, $\boldsymbol{\Sigma}_{11} = \mathbf{L}_1^{0.5}\mathbf{P}_{11}\mathbf{L}_1^{0.5}$ times $\mathbf{E}_1$, where $\mathbf{E}_1$ can be inserted anywhere after the equals sign. $\mathbf{P}_{11}$ and $\mathbf{L}_1$ do *not* commute, so the two factors of $\mathbf{L}_1^{0.5}$ cannot be combined. So, $\mathbf{J}_1'\boldsymbol{\Sigma}_{11} = \mathbf{J}_1'\mathbf{E}_1\mathbf{L}_1^{0.5}\mathbf{P}_{11}\mathbf{L}_1^{0.5} = \mathbf{X}_1'\mathbf{L}_1^{0.5}\mathbf{P}_{11}\mathbf{L}_1^{0.5}$.

Moreover, $\boldsymbol{\Sigma}_{21} = \mathbf{L}_2^{0.5}\mathbf{E}_2^{0.5}\mathbf{P}_{21}\mathbf{E}_1^{0.5}\mathbf{L}_1^{0.5}$, where $\mathbf{L}_2$ (28×28) is similar to $\mathbf{L}_1$ in that $\mathbf{E}_2\mathbf{L}_2$ is Scale $\mathbf{B}_2$ (Exhibit 3) diagonalized and squared. $\mathbf{P}_{21}$ is also shown in Exhibit 6, and it is not hard to see that $\mathbf{E}_2\mathbf{P}_{21} = \mathbf{E}_2^{0.5}\mathbf{P}_{21}\mathbf{E}_1^{0.5} = \mathbf{P}_{21}\mathbf{E}_1$. Of course, the $\mathbf{E}$s and the $\mathbf{L}$s commute. Therefore, $\mathbf{J}_2'\boldsymbol{\Sigma}_{21} = \mathbf{J}_2'\mathbf{E}_2\mathbf{L}_2^{0.5}\mathbf{P}_{21}\mathbf{L}_1^{0.5} = \mathbf{X}_2'\mathbf{L}_2^{0.5}\mathbf{P}_{21}\mathbf{L}_1^{0.5}$.

Let $\mathbf{L}$ without any subscript be the (8×8) diagonal matrix whose elements are the eight homoskedasticizing relativities. Again, it is not too hard to see that $\mathbf{L}\mathbf{X}_1' = \mathbf{X}_1'\mathbf{L}_1$, and that

$\mathbf{L}\mathbf{X}_2' = \mathbf{X}_2'\mathbf{L}_1$. Therefore,

$$\begin{aligned}
(\mathbf{J}_1'\boldsymbol{\Sigma}_{11} + \mathbf{J}_2'\boldsymbol{\Sigma}_{21}) &= \mathbf{X}_1'\mathbf{L}_1^{0.5}\mathbf{P}_{11}\mathbf{L}_1^{0.5} + \mathbf{X}_2'\mathbf{L}_2^{0.5}\mathbf{P}_{21}\mathbf{L}_1^{0.5} \\
&= \mathbf{L}^{0.5}\mathbf{X}_1'\mathbf{P}_{11}\mathbf{L}_1^{0.5} + \mathbf{L}^{0.5}\mathbf{X}_2'\mathbf{P}_{21}\mathbf{L}_1^{0.5} \\
&= \mathbf{L}^{0.5}(\mathbf{X}_1'\mathbf{P}_{11} + \mathbf{X}_2'\mathbf{P}_{21})\mathbf{L}_1^{0.5} \\
&= \mathbf{L}^{0.5}(\mathbf{J}_1'\mathbf{E}_1\mathbf{P}_{11} + \mathbf{J}_2'\mathbf{E}_2\mathbf{P}_{21})\mathbf{L}_1^{0.5} \\
&= \mathbf{L}^{0.5}(\mathbf{J}_1'\mathbf{P}_{11}\mathbf{E}_1 + \mathbf{J}_2'\mathbf{P}_{21}\mathbf{E}_1)\mathbf{L}_1^{0.5} \\
&= \mathbf{L}^{0.5}(\mathbf{J}_1'\mathbf{P}_{11} + \mathbf{J}_2'\mathbf{P}_{21})\mathbf{E}_1\mathbf{L}_1^{0.5}.
\end{aligned}$$

We will show that $(\mathbf{J}_1'\mathbf{P}_{11} + \mathbf{J}_2'\mathbf{P}_{21})$ can be factored as $\mathbf{Q}\mathbf{J}_1'$ for some $\mathbf{Q}$. Then

$$\begin{aligned}
(\mathbf{J}_1'\boldsymbol{\Sigma}_{11} + \mathbf{J}_2'\boldsymbol{\Sigma}_{21}) &= \mathbf{L}^{0.5}\mathbf{Q}\mathbf{J}_1'\mathbf{E}_1\mathbf{L}_1^{0.5} \\
&= \mathbf{L}^{0.5}\mathbf{Q}\mathbf{X}_1'\mathbf{L}_1^{0.5} \\
&= \mathbf{L}^{0.5}\mathbf{Q}\mathbf{L}^{0.5}\mathbf{X}_1'.
\end{aligned}$$

As we saw earlier, this will amount to a proof of the first balance property. The considerable maneuvering to this point is to show that the result is independent of exposure and variance relativity, as long as exposure is constant by accident period and variance relativity is constant by age.

The remainder of the proof relies on the fact that the error correlation matrix is first-order autoregressive by accident period. The (36×8) matrix $\mathbf{J}_1$ can be considered as a left-justified stack of identity matrices:

$$\mathbf{J}_1 = \begin{bmatrix} \mathbf{I}_8 \\ \mathbf{I}_7 \\ \vdots \\ \mathbf{I}_1 \end{bmatrix} = \text{Left}(\mathbf{I}_8, \mathbf{I}_7, \ldots, \mathbf{I}_1).$$

This notation can be made formal. The gaps to the right side caused by the decreasing dimensions of the identity matrices are filled with zeroes. Therefore,

$$\mathbf{J}_1' = [\mathbf{I}_8 \quad \mathbf{I}_7 \quad \cdots \quad \mathbf{I}_1] = \text{Top}(\mathbf{I}_8, \mathbf{I}_7, \ldots, \mathbf{I}_1).$$

We can write $\mathbf{P}_{11}$ as Diag($\mathbf{V}_8, \mathbf{V}_7, \ldots, \mathbf{V}_1$), where $\mathbf{V}_i$ is $(i \times i)$. As an example,

$$\mathbf{V}_4 = \begin{bmatrix} \rho^0 & \rho^1 & \rho^2 & \rho^3 \\ \rho^1 & \rho^0 & \rho^1 & \rho^2 \\ \rho^2 & \rho^1 & \rho^0 & \rho^1 \\ \rho^3 & \rho^2 & \rho^1 & \rho^0 \end{bmatrix}.$$

According to the rules of multiplying partitioned matrices, $\mathbf{J}_1'\mathbf{P}_{11} = \text{Top}(\mathbf{V}_8, \mathbf{V}_7, \ldots, \mathbf{V}_1)$.

Similarly, $\mathbf{J}_2 = \text{Right}(\mathbf{I}_1, \mathbf{I}_2, \ldots, \mathbf{I}_7)$. However, this is a (28×7) matrix. But we need to make it a (28×8) matrix by padding it with an extra leftmost column of zeroes. So we will say that $\mathbf{J}_2 = \mathbf{0}_{(28\times 1)} \,\|\, \text{Right}(\mathbf{I}_1, \mathbf{I}_2, \ldots, \mathbf{I}_7)$. And $\mathbf{J}_2' = \mathbf{0}_{(1\times 28)} // \text{Bottom}(\mathbf{I}_1, \mathbf{I}_2, \ldots, \mathbf{I}_7)$.

It is not so easy to see how $\mathbf{J}_2'\mathbf{P}_{21}$ works. However, just as premultiplying $\mathbf{P}_{11}$ by $\mathbf{J}_1'$ had the effect of elevating the submatrices of $\mathbf{P}_{11}$ to the top of an (8×36) matrix, so too a little thought will convince the reader that premultiplying $\mathbf{P}_{21}$ by $\mathbf{J}_2'$ has the effect of dropping the submatrices of $\mathbf{P}_{21}$ to the bottom of an (8×36) matrix.

The sparse, or zero, areas of $\mathbf{J}_1'\mathbf{P}_{11}$ and $\mathbf{J}_2'\mathbf{P}_{21}$ are complements of each other. The first eight columns of $\mathbf{J}_1'\mathbf{P}_{11} + \mathbf{J}_2'\mathbf{P}_{21}$ contain $\mathbf{V}_8$. The next seven columns contain $\mathbf{V}_8$ without its last column. The next six columns contain $\mathbf{V}_8$ without its last two columns. The pattern continues down to the last column, which contains the first column of $\mathbf{V}_8$. But this is also the result of multiplying $\mathbf{V}_8$ by Top($\mathbf{I}_8, \mathbf{I}_7, \ldots, \mathbf{I}_1$). Therefore,

$$\begin{aligned} \mathbf{J}_1'\mathbf{P}_{11} + \mathbf{J}_2'\mathbf{P}_{21} &= \mathbf{V}_8 \,\text{Top}(\mathbf{I}_8, \mathbf{I}_7, \ldots, \mathbf{I}_1) \\ &= \mathbf{Q}\mathbf{J}_1'. \end{aligned}$$

This is the factorization that we sought, so the proof is complete. Notice that even though we proved the property for eight accident periods, the proof can easily be generalized to any number of periods greater than or equal to two.

The second property can be treated succinctly.

$$[\mathbf{J}_1' \quad \mathbf{J}_2'] \begin{bmatrix} \mathbf{Y}_1 \\ \mathrm{E}[\mathbf{Y}_2 \mid \mathbf{Y}_1] \end{bmatrix}$$

represents the column totals of the a posteriori predictions. Each column total needs to be divided by the total exposure. As an (8×8) matrix, this is

$$[\mathbf{J}_1' \quad \mathbf{J}_2'] \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{bmatrix} = \mathbf{J}_1'\mathbf{X}_1 + \mathbf{J}_2'\mathbf{X}_2.$$

One can verify that this is an (8×8) diagonal matrix, each diagonal element of which is the sum of all accident period exposures. So the column weighted averages are

$$\begin{aligned}
&\left([\mathbf{J}_1' \quad \mathbf{J}_2'] \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{bmatrix}\right)^{-1} \left([\mathbf{J}_1' \quad \mathbf{J}_2'] \begin{bmatrix} \mathbf{Y}_1 \\ \mathrm{E}[\mathbf{Y}_2 \mid \mathbf{Y}_1] \end{bmatrix}\right) \\
&\quad = (\mathbf{J}_1'\mathbf{X}_1 + \mathbf{J}_2'\mathbf{X}_2)^{-1}(\mathbf{J}_1'\mathbf{Y}_1 + \mathbf{J}_2'\mathrm{E}[\mathbf{Y}_2 \mid \mathbf{Y}_1]) \\
&\quad = (\mathbf{J}_1'\mathbf{X}_1 + \mathbf{J}_2'\mathbf{X}_2)^{-1}(\mathbf{J}_1'\mathbf{X}_1\hat{\boldsymbol{\beta}} + \mathbf{J}_2'\mathbf{X}_2\hat{\boldsymbol{\beta}}) \\
&\quad = (\mathbf{J}_1'\mathbf{X}_1 + \mathbf{J}_2'\mathbf{X}_2)^{-1}(\mathbf{J}_1'\mathbf{X}_1 + \mathbf{J}_2'\mathbf{X}_2)\hat{\boldsymbol{\beta}} \\
&\quad = \hat{\boldsymbol{\beta}}.
\end{aligned}$$

INTEREST RATE RISK AND CAPITAL REQUIREMENTS FOR PROPERTY/CASUALTY INSURANCE COMPANIES

DOUGLAS M. HODES AND SHOLOM FELDBLUM

Abstract

The advent of risk-based capital requirements and the potential expansion of the role of the Appointed Actuary demand expertise in evaluating the financial stability of insurance enterprises. Because of the growth of property/casualty loss reserves and the wide fluctuations in interest rates during the past two decades, asset-liability management is of increasing importance for casualty actuaries.

The American Academy of Actuaries task force on risk-based capital has provided the NAIC with a proposed "interest rate risk charge" for its risk-based capital formula. This paper reviews the theoretical development of an interest rate risk charge as well as its practical application for setting capital requirements.

Interest rate risk is the potentially adverse effect of a shift in market interest rates on the net worth of the insurance enterprise. For statutory risk-based capital requirements, interest rate risk depends on (i) the relative payment patterns of assets and liabilities, (ii) the statutory valuation rate for the assets, and (iii) the statutory valuation rate for the liabilities.

The paper also discusses the effects of numerous external factors—such as changes in market interest rates or changes in statutory valuation rates—on the magnitude of the interest rate risk, as well as several unresolved issues, such as the proper allocation of assets to cover loss liabilities. It concludes with an example illustrating the computation of an interest rate risk capital charge.

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1. INTRODUCTION

Asset-liability management considerations are gaining increasing prominence in evaluations of capital requirements and of the financial strength of property/casualty insurance enterprises. No longer may actuaries examine reserve adequacy in isolation, while investment officers examine investment strategy in isolation, and expect the combination to accurately portray the company's financial condition. Rather, the actuary and the investment analyst, working in tandem, must forecast the net effects of inflationary changes, interest rate changes, and macroeconomic conditions on policyholders' surplus and on economic net worth.

The goal is clear. For most casualty actuaries, however, the appropriate techniques for evaluating the effects of different financial scenarios on policyholders' surplus remain vague.

2. BACKGROUND

Life actuaries have long dealt with these issues (often termed "C-3 risk"), since interest rate changes have an immediate effect on life insurance company cash flows.[1] Many casualty actuaries, however, have only a rudimentary knowledge of the financial

[1] For a summary of the life actuarial risk categorization system of "C-1" through "C-4," see the report of the CAS Committee on Financial Analysis [8].

theory, and a vague understanding of its applicability to casualty company products. In fact, some casualty actuaries still conceive of interest rate risk solely as "duration mismatch." They reason that the greater the duration mismatch between assets and liabilities, the greater should be the capital requirements for interest rate risk.

This approach is misleading. First, optimal investment strategy does not imply duration matching. Insurance companies should indeed manage their assets in relationship to their liabilities, but analysis of durations is only one part of this process. In fact, an upward sloping yield curve, with higher yields for greater duration securities, along with the short durations of most property and casualty reserves, often implies that asset durations in excess of liability durations may simultaneously increase net investment income and lower the probability of insolvency.[2]

Second, standard asset-liability management theory uses market valuations. For monitoring the effects of interest rate changes on the adequacy of statutory surplus, one must incorporate the effects of the statutory valuation rates for assets and liabilities in the analysis.[3]

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[2]The term "reserves" in this paper refers to loss and allocated loss adjustment expense reserves, not to unearned premium reserves. In statutory accounting, unearned premium reserves are reported gross of prepaid acquisition expenses. This implicit solvency margin overwhelms any adverse effects from interest rate shifts, so no additional capital requirements would be appropriate. This is also the reason that unearned premium reserves are not considered in the NAIC's risk-based capital reserving risk charges or written premium risk charges; see Feldblum [16]. In an examination of the effects of interest rate changes and inflationary changes on the company economic value, of course, one must consider unearned premium reserves as well as future premium flows, particularly audit premiums and retrospective premiums; see Hodes, et al. [24].

[3]The "valuation rate" is the discount rate used to determine the present value of a future payment or disbursement. For instance, suppose a $10,000 claim will be paid one year hence. For statutory accounting purposes, a reserve of $10,000 must be booked. That is to say, the valuation rate is 0%. For the NAIC risk-based capital reserving risk, the held reserves are discounted at a 5% annual rate, so the valuation rate is 5% per annum. For internal management purposes, the company may wish to determine the economic effects of its insurance operations, so it may use a valuation rate equal to current market rates or current risk-free Treasury rates.

For an analysis similar to ours with regard to life insurance and annuity products, with consideration of both valuation rates and payment patterns, see Geyer [20].

Readers' Perspectives

Readers should consider this paper from two perspectives. First, it is an actuarial paper, explaining how interest rate risk *ought* to be treated in solvency monitoring. Second, it is a paper reviewing current regulatory developments, describing how interest rate risk is *now* being considered.[4]

In particular, *statutory* reserves have historically been reported at undiscounted values, and policies have not been subject to disintermediation by consumers. This makes the prop-

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[4]Two actuarial committees assisted the NAIC in developing an interest rate risk charge for the risk-based capital formula.

- The Investment Strategy Subcommittee of the CAS Committee on Valuation and Financial Analysis (VFAC) developed a theoretical foundation for interest rate risk.
- The American Academy of Actuaries (AAA) Task Force on Risk-Based Capital developed a practical interest rate risk charge for the NAIC risk-based capital formula.

The theory in this paper formed the core of the VFAC report. The practical procedures in this paper became the substance of the interest rate risk recommendation submitted by the AAA Task Force to the NAIC.

The major difference between the Task Force report from the method in this paper is that the Task Force report uses a calibration procedure based on an "expected policyholder deficit" analysis, as recommended by Robert Butsic. The calibration procedure defines the standard used to set the capital charges for each risk. For instance, a typical calibration procedure suggested by some European actuaries is to set capital charges such that the probability of ruin of the insurance company is less than a given percentage.

Butsic's calibration procedure uses a "deductible" offset to achieve a better fit between the expected policyholder deficit and the effects of the interest rate change. This calibration procedure is distinct from the concepts discussed in this paper. Moreover, it is specific to the NAIC's risk-based capital formula and to Butsic's expected policyholder deficit theory. Alternative calibration procedures can be used, such as the probability of ruin analyses favored by European actuaries or the judgmental "seven interest rate paths" in New York's Regulation 126. (On expected policyholder deficits, see Butsic [7] or Appendix B of Hodes, Feldblum, and Blumsohn [25]. For the use of probability of ruin analyses, see Pentikäinen, et al. [32]. For a description of New York's Regulation 126, see the Society of Actuaries examination study notes 443-84-88, "Description of New York Regulation 126" and 443-85-89, "Amendments to New York Regulation 126.")

This paper does not include a calibration procedure. The calibration procedure, or the capital standard, relates not to interest rate risk per se, but to the goals of financial regulation of insurance enterprises. Such regulation involves a trade-off between company solidity and consumer prices. The reduction in company failures resulting from more stringent capital requirements generally translates into higher premium rates and consolidation of the industry (as financially weak companies merge with stronger ones). This paper takes no position on where the socially optimal level of regulation lies.

erty/casualty situation much different from the life insurance case. For GAAP reporting, the SEC is now imposing requirements that certain fixed income assets must be held at market values, while liabilities must be held at book values, which generally means nominal values for loss reserves. As the discussion in this paper makes clear, it is necessary to consider the valuation standards to properly measure the effects of interest rate changes on the risk of statutory or GAAP insolvency.

Casualty actuaries will play a major role in debates on these issues. They must be well versed in both the theory and practice if their contributions are to be valuable.

Capital Requirements vs. Dynamic Financial Analysis

This paper deals with the capital requirements needed to guard against interest rate risk in a risk-based capital system. A recent paper by Hodes, et al. [24] from the 1996 Dynamic Financial Analysis (DFA) prize paper competition of the CAS dealt with a variety of financial modeling issues, one of which was the effect of a change in market interest rates and inflation rates on the cash flows and the economic net worth of an insurance company.

In certain respects, these two papers seem to address a similar topic. It is important, therefore, to clarify the differences between the objectives of each paper.

1. *Valuation Rates*: Capital requirements exist within an accounting framework. For interest rate risk, as discussed in this paper, the most important accounting consideration is the valuation of the insurer's assets and liabilities. A cash flow financial model is not tied to any specific accounting system, and there are no "book valuation rates" embedded in the model.
2. *Risks*: Capital requirements exist within a risk-based capital framework. Some of the causes that give rise to interest rate risk may also affect reserving risk or asset risks. When setting total capital requirements, one must

take care not to miss any risks and not to double count any risks.

The cash flow model in the DFA prize paper is a scenario testing model. It does not separately quantify different risks. Rather, it specifies various alternative scenarios, and it examines the influences on the company's combined cash flows.

3. *Generic Formulas*: Capital requirements must be relatively simple, formula-driven results that can be easily applied to any insurer. Dynamic financial analysis is insurer specific. Given the particular characteristics of the insurer under question, the financial model shows the expected future cash flows.

A comparison of underwriting risks and the related risk-based capital charges should clarify this distinction. The NAIC risk-based capital formula contains a "written premium charge," which quantifies the capital requirements needed to guard against adverse underwriting results in the coming year.[5] Hodes, et al. [24] discusses the use of financial models to examine the potential consequences of underwriting cycle downturns.

1. *Valuation*: The NAIC written premium charge is derived from Annual Statement data, is dependent upon book valuations of insurer liabilities, and uses a one-year time frame dependent upon NAIC examination cycles.[6]

[5]For explanations of the charges in the current NAIC risk-based capital formula, see Feldblum [16].

[6]The book valuation rates are seen most clearly in two areas. (1) A dynamic financial model must examine the effects of an underwriting cycle downturn on both business in force and on new business, using market valuations (or cash flows) for all elements. But the unearned premium reserve is reported in statutory financial statements gross of all prepaid acquisition expenses. Since this implicit margin exceeds the capital requirements needed to hedge against adverse results, no additional charge is made in the NAIC risk-based capital formula. (2) Loss payments are discounted in the NAIC risk-based capital formula at a 5% per annum interest rate, regardless of prevailing market rates. A cash flow financial model shows results independent of any assumed interest rates. The company's current financial condition would be evaluated by discounting the cash flows at whatever rate is chosen by the valuation actuary.

The underwriting cycle downturn scenario in Hodes, et al. [24] is developed independently of Annual Statement data, shows cash flows that are not related to any book valuations, and uses a multi-year time frame, as is appropriate for the management of an insurance enterprise.

2. *Risks*: The NAIC risk-based capital formula quantifies "written premium risk" separately from the other risks faced by an insurance enterprise. For a risk-based capital formula, the appropriate questions are "Does written premium risk overlap with reserving risk?" or "Should there be a separate charge for catastrophe risk?" Underwriting cycle downturns, however, affect many parts of the insurance operation simultaneously, particularly if they are combined with business recessions. Hodes, et al. [24], following Feldblum [13], develops underwriting cycle scenarios based on multiple inter-connected elements of the insurer's operations.

3. *Generic*: The NAIC written premium risk is designed to be applicable to all insurers and to cover a variety of scenarios, whether soft markets or natural catastrophes. The scenario construction process of a financial model builds specific scenarios geared to the characteristics of the insurer under question, taking into account market conditions, concentrations of risk, and reinsurance arrangements.

In sum, a risk-based capital system and a dynamic financial model sit side by side in the same actuarial world: the valuation of insurance companies. However, they address different questions, they use different methods, and they sometimes produce dissimilar results. This paper examines the capital requirements for interest rate risk under the risk-based capital system currently in use by the NAIC. Hodes, et al. [24] examines the effects of numerous external influences, including changes in interest rates and inflation rates, on the cash flows and the economic net worth of an insurance enterprise.

3. FUNDAMENTALS

The previous sections of this paper have used terms like "asset-liability management" and "interest rate risk" without defining them. This section begins with a more careful treatment of these concepts.

Asset-Liability Management

The evolution of actuarial perspectives on underwriting and investment income may be divided into three stages.

1. *Dichotomy*: The earliest stage separated the insurance (or underwriting) functions from the banking (or investment) functions of the company. Underwriters, actuaries, and claims personnel strove for underwriting profits. Financial analysts and investment officers strove for banking profits.

   This dichotomy is oversimplified. From the earliest days of insurance, managers realized that underwriting losses may be offset by investment gains and that the profitability of the insurance enterprise depended on the interactions between the two. However, an integrated approach to underwriting and investment returns was lacking during this period.

2. *Global Integration*: The second stage conceived of the underwriting function as lending money to the investment function for the period between premium collection and loss payment. Many casualty actuaries conceived of the "loan" at a risk-free interest rate, following papers by Woll [37] and by Lowe [28]. The insurance operations were profitable if underwriting returns, plus the inter-

est at a risk-free rate on investable assets derived from insurance operations, were positive.[7] The investment operations were profitable if the realized returns minus the returns at a risk-free interest rate exceeded the expenses of the investment department.

This perspective helped break down the wall between the underwriting and banking functions of insurance enterprises. However, this integration was only at a global level. It did not address the question: "How should investment strategy relate to underwriting strategy?"[8]

3. *Full Integration*: The third stage entails a more complete integration of the underwriting and investment functions. Consider two insurance enterprises. The first writes homeowners policies in Gulf Coast states. Over the long term, the insurance function is profitable. But the high risk of hurricanes makes liquidity an overriding concern for the investment department. Excessive use of private placements, real estate, and even publicly traded bonds with thin secondary markets may not be appropriate investments for this company.[9]

   The second company writes workers compensation policies for a stable customer base. Benefit payments, whose magnitudes are mandated by state law and not subject to jury discretion, are steady from year to year.

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[7]This is the perspective underlying the insurance pricing models of Kahane [26], Fairley [11], and Myers and Cohn [29]. These writers use a "risk adjustment" to the interest rate, generally based on the Capital Asset Pricing Model, though more recent analysis shows the effects to be insignificant (see Cummins and Harrington [9]; Feldblum [17]).

[8]Some actuaries examined the effects of external factors, such as federal income taxes, on underwriting and investment strategy, and asked: For given underwriting results, what investment strategy maximizes net after-tax income? See, for example, Gleeson and Lenrow [21] or Almagro and Ghezzi [1].

[9]Note that liquidity is distinct from duration. Many actuaries focus only on duration, arguing that the short duration of homeowners reserves makes private placements and real estate inappropriate investments. This intermingling of concepts simply confuses the issues. For writers of homeowners insurance, long-term Treasury bonds may be more appropriate investments than private placements, and robustly traded shares of common stock may be more appropriate than real estate.

Most insureds renew their policies each year, and loss costs increase predictably with inflation.

The insurance enterprise, seeking steady cash flows from assets, may invest in long-term and potentially high yielding securities to fund the expected benefit payments, along with investments in equities to capture uncertain but potentially lucrative capital gains. Once again, simple duration matching does not suffice for optimal investment strategy. If the policy renewal rate is high, and the insurer finds good investment opportunities among long duration securities, a considerable asset-liability "mismatch" may be appropriate.[10]

Interest Rate Shifts

The effect of interest rate changes on the value of the insurance enterprise is one aspect of asset-liability management. A rise in market interest rates will decrease the market value of fixed income securities. Conversely, a decline in market interest rates will increase the market value of fixed income securities. Similarly, a rise or decline in interest rates will decrease or increase the present value of fixed liability payments.

Many property/casualty insurers have more fixed income assets than they have fixed liability payments.[11] The effect of interest rate changes on the insurance company's underlying economic equity is similar to their effect on the insurer's investment portfolio: an unexpected rise in interest rates will decrease net equity, and a decline in interest rates will raise net equity. How-

[10]This mismatch refers to the durations of individual insurance contracts and investment vehicles. In practice, the mismatch may disappear when one compares the insurance and investment portfolios, *along with the renewal rates (or retention ratios) on the former*. For further discussion, see Panning [31].

[11]"Fixed liability payments" refers to obligations that are not affected by inflation between the valuation date and the payment date. For example, a traditional whole life contract promises fixed dollar amounts at the death of the policyholder. In contrast, workers compensation medical benefits are influenced by the inflation rate up to the date of the physician's services (or other medical bills).

ever, the dollar amount of this effect is dampened by the fixed liability payments.[12]

How should one determine the optimal attributes of the fixed income asset portfolio? The investment officer begins with three fundamental relationships and must address two questions.

Three Relationships

1. *Duration*: The market values of long duration securities are more affected by interest rate changes than are the market values of short duration securities. For instance, a 100 basis point increase in market interest rates may cause an 8% decline in the market value of a 20 year bond but only a 3.5% decline in the market value of a 5 year note.
2. *Yield Curve*: At most times, the yield curve is upward sloping. That is, long duration securities offer greater returns than do short duration securities. For instance, a 90 day Treasury bill may offer a yield of 5.5%, while a 30 year Treasury bond may offer a yield of 7.5%.[13]
3. *Risk*: Future changes in interest rates may increase or decrease the market value of the investment portfolio. The exposure to future (unknown) interest rate changes constitutes a risk, not a change in expectations. That is, this exposure affects the volatility of investment returns; it does *not* affect the expected value of investment returns.[14]

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[12]For further discussion of the effects of inflation on an insurance company's equity, see Butsic [6] and Noris [30].

[13]Shifts in the yield curve may also be accompanied by risks not explicitly measured by durations and not discussed here, such as spread risks, convexity, and the volatility of interest rates.

[14]This perspective is consistent with the "systematic risk" interpretation of the normally upward sloping yield curve, in that investors are compensated by higher yields for the increased risks of long maturity bonds. Other interpretations of the normally upward sloping yield curve, such as the "market segmentation" or "future expectations" views, do not necessarily see a significant difference in risk by maturity of the bond portfolio. For an overview of yield curve interpretations, see Gray [22].

In other words, lengthening the duration of the fixed income asset portfolio has two effects.[15]

a. The greater duration increases the expected investment yield, as long as the yield curve is upward sloping.

b. The greater duration increases the expected volatility of investment returns because of greater interest rate risk, though this does not affect the expected yield.

Two Questions

The investment officer must address two questions.

1. *Risk and Reward*: Greater duration securities offer higher yields but increase the risk caused by interest rate shifts. Given the various constraints imposed by the underwriting and investment strategies of the company, such as lines of business written, liquidity needs, safety of principal, and promised yields by type of security, what is the optimal trade-off between risk and reward when deciding on the appropriate characteristics of the fixed income securities portfolio?[16]

2. *Capital Requirements*: Interest rate shifts may affect the statutory net worth of the insurance enterprise and threaten its solvency. What asset characteristics minimize the risk of insolvency, or at least keep it within acceptable levels?

   One may rephrase this question in terms of capital requirements. An insurance enterprise holds surplus to guard against the risk of insolvency. The regulator may

[15]Lengthening the duration of the corporate bond portfolio also increases the credit risk, since the probability of default increases as the time from issue increases. Default risks and other credit risks are separately evaluated by the NAIC's risk-based capital formula (though only with a one-year time horizon), and they are not discussed in this paper.

[16]For a comprehensive analytic framework for fixed income portfolio construction, see Fong [19].

ask: "How should the capital requirements relate to the effects of interest rate shifts on the statutory surplus of the company?"

Focus

These questions differ in two ways—in their focus on stockholders versus policyholders and in their focus on market values versus statutory values.

1. *Stockholders vs. Policyholders*: The first question ("risk and reward") deals with the value of the insurance enterprise. It is an internal question: How do we maximize the value of the firm? Most stockholders can diversify their own holdings. They are often less concerned with the unique risks of individual investments than with the expected returns from the investments.

   The second question ("capital requirements") deals with the security of the insurance promise. Policyholders and claimants want to be assured that the insurance enterprise will meet its obligations. The current earnings of the company are less important than its financial strength.

2. *Market Values vs. Statutory Values*: The first question ("risk and reward") deals either with market values or with internal accounting values. It is generally not concerned with statutory risk loads or statutory valuation rates. It asks how interest rate shifts affect either the market value of the insurance enterprise or management's perception of the worth of the insurance enterprise.

   The second question ("capital requirements") asks how interest rate shifts affect the potential for statutory insolvency. Statutory accounting may guard against the risk posed by interest rate shifts in several ways: by risk loads in the reserves, by differing valuation rates for assets and liabilities, and by additional capital requirements.

This paper deals with the second question (the solvency issues and the capital requirements), not the first question (the risk versus return relationship). Moreover, it assumes that there are no risk loads for reserves except for the differing valuation rates for assets and liabilities. Finally, it treats the valuation rates as given. Its focus is on capital requirements, not on recommended valuation rates or on risk loads.[17]

This last issue, capital requirements, is not necessarily more important than the others. However, it is the issue currently facing the NAIC, and it deserves a full and clear treatment on its own.

4. DURATION AND MARKET VALUES

Our analysis proceeds in three steps.

1. We examine the effects of a security's payment pattern on the sensitivity of its market value to shifts in interest rates. A "security" in this paper means either an asset or a liability. Most of the analysis deals with fixed income assets and fixed liability payments.[18]

   We refer to this as "duration analysis." The fundamentals of duration analysis are reviewed in Appendix A of this paper. More comprehensive treatments can be found in the actuarial and financial literature.[19]

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[17] On the appropriate valuation rate for reserves, see Butsic [5]. On the accounting treatment of risk loads for reserves, see Philbrick [33].

[18] On the effective duration of common stocks, see Leibowitz, Sorensen, Arnott, and Hanson [27]. On the implications for property/casualty insurance asset-liability management, see Feldblum [12].

[19] The seminal actuarial paper on duration analysis is Redington [34]. Good introductory treatments are Bierwag, Kaufman, and Toevs [4]; Ferguson [18] and the discussion by D'Arcy; Geyer [20]; and Tilley [36], along with the discussion by Hoiska.

2. We examine the effects of differing valuation rates used for assets and liabilities. For statutory accounting, the asset valuation rate is generally the yield rate at the time of purchase.[20] The liability valuation rate is set by regulatory prescription in the risk-based capital formula or in statutory accounting principles.

3. We combine the analysis of duration and valuation rates to determine the capital requirements needed to guard against the risk of interest rate shifts.

Characteristics of Duration

Three characteristics of duration are relevant to our discussion.

1. Since the weights used in the calculation of the duration of a security depend on the present values of the cash flows, not on their nominal values, the duration depends on both the cash payment pattern and the market interest rate. As the market interest rate changes, the duration of the security changes.

2. The statement that "the effect of interest rate shifts on the market price of the security is directly proportional to the duration of the security" is accurate for infinitesimal interest rate shifts. As market interest rates change, the duration of the security changes, so the effect on market value changes. If a decrease in market interest rates increases the duration, then the effects on market value of a decrease in market interest rates are magnified. Conversely, if an increase in market interest rates

[20]In statutory financial statements, investment grade fixed income securities are shown at amortized values. As a result, the valuation rate at the time of purchase, whether this be the initial issue or purchase in a secondary market, is retained for the life of the security.

In GAAP financial statements, bonds either "held for trading purposes" or "available for sale" are reported at market value; bonds intended to be held to maturity are reported at amortized values (SFAS 115).

decreases the duration, then the effects on market value of an increase in market interest rates are mitigated.

3. Durations may be determined explicitly for fixed income securities by the definition given above. *Implied* durations, determined from empirical relationships, may be ascribed to other types of securities, such as common stocks and real estate, and to property/casualty liabilities, such as personal auto loss reserves.

   For instance, suppose a bond with a duration of three years would have a three percent decline in market value for a one hundred basis point increase in the market interest rate. This relationship is determined mathematically, by computing present values of nominal cash payments at different interest rates.

   Personal auto loss reserves are at least partially inflation sensitive. Medical payments in tort liability states, for instance, depend in part upon jury awards at the date of settlement. The jury awards, in turn, are influenced by the rate of inflation, which is correlated (at least in the long run) with interest rates. In contrast, wage loss payments under no-fault compensation systems may be fixed at the time of accident, unless cost of living adjustments are built into the benefit schedule.

   A mathematical determination of the loss reserve duration is complex. However, if empirical studies show that the discounted value of personal auto loss reserves declines by three percent for each 100 basis point increase in interest rates, then we may say that the personal auto loss reserves have an effective duration of three years.

   This would be the theoretically correct approach to determining liability durations for interest rate risk capital requirements were the reserving risk charge in the risk-based capital formula to exclude the effects of inflation on adverse loss development. The actual structure of the reserving risk charge necessitates the exclusion of the inflation sensitivity of loss reserves from the calculation of the interest rate risk charge.

5. NOMINAL VS. EXPECTED PAYMENT PATTERNS

Heuristic illustrations, as well as most accounting exhibits, show *stated* payment patterns for fixed income securities. The stated payment patterns are the payment obligations stated in the debt instrument. Some actuaries, lacking practical investment experience, are tempted to use these accounting exhibits for interest rate risk analyses.

Investment analysts use *expected* payment patterns when performing asset-liability management studies. The expected payment patterns take into account refinancings, prepayments, call provisions, and default rates. For certain types of securities, they present a radically different picture of actual cash flows.

Mortgage-Backed Securities

As an illustration, suppose an insurer acquires a portfolio of mortgage-backed securities in 1984, with terms of 30 years and interest rates of 15% per annum. The stated maturity of this portfolio is indeed long.

But the duration of these assets is much shorter than the duration of bonds with the same maturities, for three reasons.

1. When homeowners move, the old mortgage is cancelled, and a new mortgage is issued on the new property. Americans move frequently, and the rate of mortgage cancellations and reissues is concomitantly high, leading to a much shorter duration for mortgage-backed securi-

ties than for bonds. This phenomenon exists even when market interest rates do not change.[21]

2. When market interest rates decline, homeowners are quick to refinance their mortgages. In fact, when interest rates decline sufficiently, mortgage borrowers are inundated with letters from mortgage brokers and banks offering refinancing advice and lower rates. This further reduces the average duration of mortgage-backed securities under certain interest rate paths.[22]

3. Bond principal is repaid in a lump sum at the maturity date. Mortgage principal has a fixed amortization schedule. It is repaid monthly, and it declines to zero over the duration of the mortgage, similar to a bond with a sinking fund. A bond and a mortgage may have the same "term to maturity," but the mortgage will have a much shorter duration.

The expected cash flow pattern for a portfolio of securities therefore differs from the stated cash flow pattern. It is the expected cash flow pattern, under a variety of economic and interest rate scenarios, that is relevant for asset-liability management and the evaluation of interest rate risk.

Asset Cash Flows

The future appointed actuary performing asset adequacy analyses of a casualty insurance company's operations will rely on the expected cash flow patterns provided by investment personnel. But state regulators concerned with the effects of interest rate risk on risk-based capital requirements must rely on statutory financial data.

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[21] However, when interest rates decline, homeowners find it easier to purchase new homes, so more old mortgages are cancelled, and new mortgages are issued.

[22] Fong [19] uses contingent claims analysis to estimate the effective durations for securities with issuer options such as calls and refinancings.

Schedule D, Part 1A, of the Fire and Casualty Annual Statement, which shows the maturity distribution of bonds, deals only with stated payment patterns, and it has a maturity schedule that is too coarse for quantification of interest rate risk. Instead, the asset cash flow exhibit reproduced here as Exhibit 1 is patterned after the supplementary asset schedule that was included in the 1995 risk-based capital submission.

The fixed income asset portfolio in Exhibit 1 consists primarily of long-term bonds and mortgages, with smaller amounts of short-term investments and collateral loans. The features of this supplementary asset schedule that are most important for interest rate risk analysis are listed below:

1. The supplementary asset schedule shows the aggregate cash flows themselves, not simply the parameters (such as coupon rates and maturities by security) that are needed to construct a cash flow schedule. This new format allows both the regulator and the actuary to directly address interest rate risk and asset-liability management issues.

2. The supplementary asset schedule shows the expected cash flows, based on the company's best estimate of expected prepayments and refinancings, not the stated cash flows in the bond indenture. For certain types of securities, such as mortgage-backed securities, the difference between stated maturities and expected maturities can be great. For instance, suppose the insurer purchases a portfolio of 30 year mortgages. Schedule D, Part 1A, shows the entire portfolio in the "over 20 years" category.

   But mortgages are often pre-paid or refinanced, even if interest rates do not change, because individuals move to different locations or purchase new homes. If the company expects 2% of the mortgages to be prepaid or refinanced in three years, an additional 3% to be prepaid or refinanced in four years, and so forth, these expectations are used in constructing the expected cash flows.[23]

3. Since the supplementary asset schedule shows cash flows, it clearly distinguishes between securities whose principal is repaid in a lump sum at the maturity date (e.g., government bonds) and securities whose principal is repaid by periodic installments (e.g., mortgages) or by contributions to a sinking fund (e.g., certain corporate bonds). Moreover, it shows interest and principal payments separately, and it shows the expected payment dates of each, not simply the "maturity date."

4. The supplementary asset schedule shows statement values and market values for the major classes of fixed income securities, thereby showing the magnitude of the valuation "cushion" or "deficiency" in the asset portfolio.[24]

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[23]The argument has been voiced that statutory exhibits should be based on auditable data, not on estimates, so exhibits using stated maturities are "better" than those using expected maturities. This argument is strange. Less meaningful data are not "better" simply because they can be audited. Consider Schedule P: the loss reserve entries are estimates, made either by claims examiners or by actuaries. These figures cannot be "audited," but they are essential for monitoring the financial condition of the company.

When entries cannot be audited, how might companies bolster regulators' confidence in their accuracy? Current statutory requirements for loss reserves suggest one means. Each year, the company's appointed actuary signs a statement of opinion certifying the reasonableness of the loss and loss adjustment expense reserve estimates. Similarly, an investment officer of the company might be required to sign a statement certifying the reasonableness of the fixed income cash flow estimates.

[24]Despite the complexity of Annual Statement Schedule D, statutory accounting does not provide this information. Schedule D, Part 1, has columns for book value (Column 4), par value (Column 5), market value (Column 7), and actual cost (Column 8). However, the "market value" column has the following Annual Statement instruction:

> Where a market value is published in the NAIC Valuation of Securities manual, it must be entered in Column 7. Where amortized value or any other value is used, insert a symbol alongside the amount reported.

Market values are published in the NAIC *Valuation of Securities Manual* for bonds that are *not* of investment grade. ("Investment grade" bonds are classes 1 and 2; "non-investment grade" bonds are classes 3 through 6.) For investment grade bonds, most companies show amortized values in Column 7, not market values. In Schedule D-M, market values are shown for the *aggregate* portfolio, not for individual securities or groups of securities.

5. Finally, the supplementary asset schedule shows a greater degree of refinement in the payment schedule than is available from the statutory exhibits.

In-house investment analysts performing an asset-liability management study would use more complete data than are contained in this schedule—just as actuaries performing loss reserve adequacy analyses use more complete data than are contained in Schedule P. This asset schedule serves as a one-page summary of the company's cash flow position, enabling regulators to better evaluate how changes in market interest rates may affect the company's financial solidity.[25]

6. RESERVE PAYMENT PATTERNS

The cash flow pattern from the investment portfolio differs by company based upon (a) the types of securities held and (b) the intent of the company to hold the assets to maturity or to trade them at earlier dates. Moreover, holding securities at amortized values means that the book values of the same security may differ by company based on the date at which the security was acquired. For quantifying interest rate risk, the calculation of asset cash flows and asset book values must be based on the individual company's data.

Loss reserves are different. Annual Statement Schedule P, Part 3, shows historical loss payout patterns by line of business. Although these patterns may differ by company, the differences are

[25] In practice, expected cash flows from fixed income securities vary as interest rates change. For instance, when interest rates decline, corporate bonds are more likely to be called, and mortgages are more likely to be refinanced. The supplementary asset schedule reproduced here as Exhibit 1 is but one piece of a more complete asset-liability management schedule developed by Alex Fontanes. This exhibit shows the expected cash flows if market interest rates remain at their current levels. For a more accurate interest rate risk analysis, one should also have corresponding exhibits showing the expected cash flows if interest rates increase or decrease by specified amounts, such as 200 basis points up or down.

not great, particularly for the high volume lines of business like personal automobile liability and workers compensation. Moreover, the data for small and even some medium size companies may not be sufficiently credible for independent analysis. Industry aggregate data from Best's *Aggregates and Averages* often provides more accurate projections of a company's future payment pattern.[26]

State regulators, seeking an interest rate risk component for the risk-based capital formula, prefer factors derived from industry data and applied uniformly to all insurers, whenever possible. This relieves regulators from the task of monitoring individual company calculations when there is little benefit of increased accuracy.

For the interest rate risk recommendations submitted to the NAIC by the AAA Task Force, reserve payout patterns by line of business were determined from industry data. Each company would weight these reserve payout patterns by its own mix of reserves by line of business. For instance, a company whose reserves were 85% personal auto liability, 5% personal auto physi-

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[26]In some instances, the insurance cash flow patterns do vary by company. Several examples should illustrate this.

1. In workers compensation, writers of large-dollar deductible policies, or writers of excess-of-loss reinsurance over high retentions, don't even begin to pay losses until years after the accident date, once the cumulative loss exceeds the deductible or the retention.
2. In workers compensation, the cash flow patterns differ greatly between one insurer writing small risks on prospectively rated policies with premiums paid upfront and a second insurer writing large risks on "cash-flow" retrospectively rated policies where the premiums are paid as the insurer pays the loss. For interest rate risk, our concern is with *net* insurance cash flows, or the difference between the payout pattern of loss reserves and the collection pattern of accrued retrospective premium reserves.
3. In general liability, the cash flows differ between (i) an insurer that has recently entered this line of business and that writes mostly retail risks and (ii) an insurer with significant toxic tort and environmental impairment liability exposures that may have loss reserve payout patterns extending 30 years into the future.

Thus, the generalizations in the text of this paper should be viewed with caution. They are meant to explain the genesis of the interest rate risk recommendations of the authors and of the American Academy of Actuaries. To the extent that better data become available, the procedures outlined here can be improved.

cal damage, and 10% homeowners, would use an 85%-5%-10% weighting of the industry aggregate reserve payout patterns for these lines of business.

The calculation of the reserve payout pattern by line of business is described in Appendix B. Since payout patterns do not vary much over time, this calculation need be performed only once, and then updated only if there is a substantial shift in the mix by class or the mix by state within a line, or if there is a regulatory or legal change that affects the payout pattern.[27]

7. VALUATION RATES AND STATUTORY SURPLUS

The previous sections of this paper have dealt with market values. They ask: "How do shifts in interest rates affect the market value of a security, or the economic value of the firm?" This is an important question in its own right, but it is only a stepping stone for our analysis. We wish to know: "How should interest rate risk affect the capital requirements of a property/casualty insurer?" To answer this we must first ask: "How do interest rate shifts affect the likelihood that the insurer's assets will be insufficient to meet its liability obligations?"

Interest Rate Risk: An Illustration

How ought one to guard against interest rate risk? A better formulation of this question might be: How does statutory accounting presently deal with interest rate risk, and what further charges should be embedded in the risk-based capital formula?

Let us begin with an illustration. For heuristic purposes, the insurer in this example writes a single policy and purchases a single bond, though the extension to practical situations is straightforward.

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[27] On the stability of payout patterns by line of business, see Woll [37].

The Scenario: Suppose an insurer expects to make a $1,000 personal auto liability payment two years hence. To fund the loss reserve, it purchases a ten year $1,000 par value 8% annual coupon bond. It intends to sell the bond after two years to pay the loss. It buys a ten year bond instead of a two year note to pick up the additional investment income on the longer term security.

Interest Rate Shifts: If interest rates do not deviate from the current market rate of 8% per annum, then the insurer sells the bond after two years for $1,000 to pay the loss. But if interest rates do shift, then the effects on the bond and the loss reserve are different. The bond has a Macauley duration of 7.25 years (see Appendix A). For simplicity, let us assume that the personal auto payment is fixed at $1,000, so the reserve duration is two years.

Since the asset has a greater duration than the liability has, interest rate shifts have a greater effect on the value of the asset than on the value of the liability. So we ask: "If interest rates rise to 10% immediately after purchase of the bond, will the asset still suffice to fund the loss payment?"

Reserve Valuation: The answer depends on the valuation rate for the loss reserves. The valuation rate differs between statutory accounting, the risk-based capital formula, and internal (management) accounting systems, so we treat them each in turn.

Statutory Accounting: Statutory accounting requires most loss reserves to be reported at undiscounted values. If a $1,000 payment is to be made two years hence, the full $1,000 must be set up as a loss reserve, and $1,000 in assets must be set aside to fund the loss. In our illustration, the cash flows are as follows:

- At issue of the $1,000 par value ten year 8% annual coupon bond, market interest rates are 8% per annum, so the purchase price is $1,000.

- Interest rates rise to 10% immediately after issue and remain at that level for the next two years.[28] The bond is then sold to fund the loss payment. The sale price in a 10% interest rate environment is

$$(\$80 \equiv 1.10) + (\$80 \equiv 1.10^2) + \cdots + (\$80 \equiv 1.10^7)$$
$$+ (\$1{,}080 \equiv 1.10^8) = \$893.30.$$

- The first year coupon of \$80 is invested at 10% per annum to yield \$88 at the end of the second year.

- The second year coupon of \$80 is received right before the loss payment is made.

At the payment date, the insurer has $\$893.30 + \$88 + \$80 =$ \$1,061.30 to fund the \$1,000 loss. The excess of asset cash flows over the reserve obligation stems from the implicit interest margin in undiscounted reserves.

Let us not jump to the conclusion, however, that the use of undiscounted reserves in statutory reporting protects the insurer from interest rate risk. The "implicit interest margin" in the undiscounted reserves has several other functions, such as a cushion to protect against unexpected adverse loss development. The same margin cannot serve two purposes, and additional capital may be needed.

Internal Reporting: Suppose the insurer keeps a management accounting system for internal examination of profitability and financial solidity in which loss reserves are discounted at current market rates. The present value at the accident date of the \$1,000 loss payment which will be made two years hence is

$$\$1{,}000 \equiv 1.08^2 = \$857.34,$$

[28] In practice, of course, one would expect a gradual 200 basis point increase in interest rates over the course of the two years. The effect on the illustrations is not significant.

so only \$857.34 of ten year 8% annual coupon bonds are purchased to fund the loss.[29] The figures provided above must now be multiplied by 85.734%, and the available cash at the end of the second year is

$$0.85734 \times (\$893.30 + \$88 + \$80) = \$909.89.$$

This is insufficient to pay the \$1,000 loss by \$90.11.

The reader should not assume that this is a theoretical, academic scenario. On the contrary: if the entire implicit interest margin is needed to guard against unexpected adverse loss development, or "reserving risk," then there is nothing left to guard against interest rate risk. In other words, this is statutory accounting, not just hypothetical management accounting. The difference between the two scenarios depicted here may be viewed either as a change in the valuation rate or as a differing perception of the purpose of the implicit interest margin.

Risk-Based Capital: How much of the implicit interest margin is needed to guard against reserving risk? If nothing is needed, then the supporting assets exceed the required loss payments by \$61.30, even if interest rates climb to 10%. If all of it is needed for reserving risk, then the supporting assets are deficient by \$90.11 when interest rates climb to 10%.

The risk-based capital formula provides an explicit answer. In the reserving risk calculations, the reported adverse loss development is offset by a 5% per annum discount.[30] In other words, this portion of the implicit interest discount is used to guard against

[29]Of course, bonds are not sold in denominations of \$857.34. In practical situations, however, the annual losses are in millions of dollars, and there is little difficulty in finding assets of the appropriate denominations. The simplified illustration in the text is for heuristic purposes only.

[30]The risk-based capital formula looks at the industry-wide adverse loss development by line of business over the past ten years from Schedule P data and selects the "worst case year." It then says: "This type of adverse loss development happened in the past; it might

adverse loss development. The remaining difference between the asset and liability valuation rates may be used to guard against interest rate risk.

Let us return to our illustration. At a 5% per annum valuation rate for liabilities (as is used in the risk-based capital formula), one need purchase a bond with a face value of $\$1{,}000 \equiv (1.05)^2 = \$907.03$ to fund the expected loss. To determine whether the asset cash flows suffice to meet the liability obligations, we multiply the numbers given earlier by 90.703%:

$$0.90703 \times (\$893.30 + \$88 + \$80) = \$962.63.$$

At the end of the second year, the supporting assets are deficient by \$37.37 (= \$1,000 – \$962.63). In this scenario, the insurer needs \$37.37 of additional capital to guard against the risk of an unexpected 200 basis point increase in interest rates.

In sum, the reserve valuation rate is a critical factor determining the capital requirements to guard against interest rate risk. Table 1 summarizes the capital requirements needed in our simplified illustration for three valuation rates. (A positive capital requirement means that additional funds are needed to pay the loss when interest rates increase. A negative number means that the held reserves are more than sufficient to pay the loss even if interest rates increase.)

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happen again. Insurers need sufficient capital to protect them against such unexpected development."

Schedule P adverse loss development is on a nominal basis. Insurers report reserves on a full-value (undiscounted) basis. The difference between the economic (discounted) value and the full (undiscounted) value of the reserve is a cushion, or a risk margin, that also guards against unexpected adverse loss development.

The risk-based capital formula therefore offsets the observed "worst case year" adverse loss development with the "implicit investment income" in the undiscounted reserves. In the RBC formula, the IRS loss reserve payment patterns are combined with a fixed 5% interest rate to determine the amount of the reserve discount. Thus, the implied statutory reserve valuation rate for determining capital requirements is 5% per annum.

(The description in this footnote is over-simplified. The actual reserving risk calculations are more complex. For a more detailed description, see Feldblum [16]. For an actuarial evaluation of the capital needed to guard against reserving risk, using stochastic simulation to model loss development and an expected policyholder deficit analysis to calculate the resultant capital needs, see Hodes, Feldblum, and Blumsohn [25].)

TABLE 1

CAPITAL REQUIREMENTS

Valuation Rate	Capital Requirement
Statutory valuation (undiscounted)	($61.30)
Market valuation (fully discounted)	90.11
Risk-based capital (5% discount rate)	37.37

Inflation Sensitivity of Loss Reserves

An important issue is the proper method of dealing with the inflation sensitivity of casualty loss reserves. We analyze this issue in three steps.

1. If there were no reserving risk charge in the risk-based capital formula, then all effects of interest rate changes on either assets or liabilities would be incorporated in the interest rate risk charge. Since interest rates and inflation rates are correlated, and since most casualty loss reserves are affected by inflation through the payment date, a rise in interest rates causes both a decline in the market value of fixed income assets and a rise in the nominal value of casualty loss reserves.[31]

2. The reserving risk charge in the NAIC risk-based capital formula examines the historical adverse loss development by line of business. Ideally, the reserving risk charge should separate the historical adverse loss development into two components.

   - *True* adverse loss development stems from changes in the external environment, such as judicial decisions that were not anticipated by the insurance industry (as

[31]Indeed, Robert A. Bailey, the deputy insurance commissioner of the state of Michigan and a member of the NAIC working group on risk-based capital, argues that this is the proper manner of quantifying interest rate risk.

happened in certain pollution exposures), or misestimation of the reserve needs by company actuaries (as happened in medical malpractice).

- *Apparent* adverse loss development stems from changes in the inflation rate affecting nominal reserve needs. Since the discount rate generally follows the inflation rate, there may be no change in the present value of the loss reserve.

The first component—true adverse loss development—should be reflected in the reserving risk charge. The second component—apparent adverse loss development—should be excluded from the reserving risk charge and included in the interest rate risk charge. The second component does not change the true value of the loss reserve, so this is not a "reserving risk" that a well-managed company should guard against. The true risk here is that the rise in inflation, accompanied by a rise in interest rates, will cause a decline in the market value of fixed income assets even while it leaves the present value of loss reserves unchanged. This is interest rate risk.

3. The current reserving risk charge in the risk-based capital formula lumps all adverse loss development together. Risks may be recognized only once in the risk-based capital formula; to recognize them twice is double counting (see Hartman, et al. [23]). Since the effects of monetary inflation on loss reserves are reflected in the reserving risk charge, these effects should not be reflected in the interest rate risk charge.

Thus, the interest rate risk calculations seem to assume that inflation does not affect nominal loss reserves. In fact, of course, these calculations do not assume this. Rather, this effect of inflation is picked up elsewhere in the risk-based capital formula, not in the interest rate risk component.

The Covariance Adjustment

At first, one might suppose that it makes no difference whether the effects of inflation on loss reserves are reflected in the reserving risk charge or in the interest rate risk charge. Indeed, this would be true were the overall capital requirements an additive combination of the individual risk charges.

In fact, the individual risk charges are combined by a "square root rule" in the risk-based capital "covariance adjustment." This rule says that

$$\text{Total capital requirements} = R_0 + (R_1^2 + R_2^2 + R_3^2 + R_4^2 + R_5^2)^{0.5}$$

where R_4 is the reserving risk charge and R_1 is the asset risk charge for fixed income securities. The AAA Task Force has recommended that the interest rate risk charge be placed in the R_1 risk category.

Because of the square root rule, the marginal effect of each dollar of individual risk charge is proportional to the magnitude of its risk category. For most companies, the size of the reserving risk charge (R_4) is about ten times the size of the asset risk charge for fixed income securities (R_1). Thus, if the effect of inflation on reserves is placed in R_4, this risk has about ten times the effect on overall capital requirements than would be the case were it placed in R_1. For further explanation of the square root rule, see Butsic [7] and Feldblum [16].

Influences on Capital Requirements

Several factors affect the capital requirements for interest rate risk. We divide these factors into three groups:

1. attributes of the company's investment portfolio and liability portfolio, such as average durations and book rates of return;
2. changes in the investment environment affecting market values or payment patterns, such as market interest

rates and the availability and exercise of investor options; and

3. regulatory mandates regarding (a) the level of capital requirements, such as the degree of interest rate shifts, and (b) the valuation rate for liabilities.

The above categorization groups the factors affecting capital requirements into three types: (1) those under the control of the company, (2) those dependent upon the financial markets, and (3) those determined by the regulatory authorities.

1. Portfolio Attributes

1A. Interest rate risk increases as the difference between the average payment dates for assets and liabilities increases.

This is sometimes expressed as, "Interest rate risk varies with the duration mismatch." This is true, if all other factors are held constant. But cash flow patterns are only one of the factors affecting interest rate risk. Since greater duration assets generally have higher yield rates, the actual effects of duration "mismatch" on interest rate risk are uncertain.

1B. Interest rate risk decreases as the spread between the book valuation rates for assets and liabilities increases.

Fixed income assets are held at amortized values on statutory balance sheets. The book valuation rate is the coupon rate for bonds bought at par or the yield rate at the purchase date for bonds bought at other values (usually in the secondary market).

Reserves are not traded in free markets; they have no coupon rates or yield rates. Rather, the book valuation rate is determined by statutory mandate. In the NAIC's risk-based capital formula, reserves are valued at a 5% per annum discount rate. The NAIC may change this rate from time to time, though the changes will probably be infrequent, and the reserve valuation rate will presumably never exceed a risk-free interest rate.

The company has no control over the book valuation rate for liabilities in the NAIC's risk-based capital system. However, its investment decisions affect the book valuation rate for its asset portfolio, thereby affecting the spread between the asset valuation rate and the liability valuation rate.

The book valuation rate for assets will generally exceed the book valuation rate for liabilities. The larger the discrepancy, the greater the cushion already present in the statutory valuation rates, and the less need for an additional capital requirement.

2. Investment Environment

2A. Increases in market interest rates that are not reflected in the valuation rate of assets increase interest rate risk.

The interest rate risk quantification procedure described here measures the effects of a shift in interest rates from current market rates to "shocked" rates on the values of assets and liabilities. For most property/casualty insurers, the asset portfolio has a greater duration than the liability portfolio, so an increase in interest rates has a more adverse effect on the value of assets than on the value of liabilities.[32]

An increase in actual market interest rates between the time the assets were purchased and the date the solvency measurement is performed eats up some of the cushion generated by the difference between asset and liability valuation rates. The increase in actual market interest rates therefore increases the capital requirements needed to guard against interest rate risk.[33]

[32]This is not always true. For instance, a workers compensation carrier with a heavy concentration in commercial paper, Treasury bills, and short-term mortgages may have an investment portfolio significantly shorter than its reserves portfolio. Thus, the generalization in the text should be treated with caution.

[33]The illustrations at the end of this paper demonstrate this effect. The capital required for an interest rate shift of 150 basis points is larger than the capital required for an interest rate shift of 100 basis points. But a shift of 150 basis points can also be viewed as an actual 50 basis point change in market interest rates accompanied by a 100 basis point shift test in the risk-based capital formula.

In general, interest rate changes are gradual, and the bond portfolio turns over steadily. An increase in market interest rates is usually accompanied by an increase in the book valuation rate for assets, which lowers interest rate risk (unless the valuation rate for reserves is raised as well).[34]

Over longer time horizons, the latter effect—valuation rate changes—is more powerful than the former effect—the effect of interest rates on the relative market values of assets and liabilities. At the extremes, the following two rules hold:

- Sudden and recent increases in market interest rates raise the interest rate risk charge.
- Gradual and extended increases in market interest rates lower the interest rate risk charge.

An example should clarify this. Suppose that reserves are valued at a 5% per annum discount rate, as in the current NAIC risk-based capital formula. A long maturity bond portfolio was bought in 1992, at an investment yield of 6% per annum. Capital requirements are being determined at December 31, 1995, when market interest rates for a similar bond portfolio are 8% per annum.[35]

The interest rate risk test measures the effect of a shift in interest rates from 8% to, say, 9% on the ability of the bond portfolio to support the reserve liabilities. The higher initial val-

[34]A gradual climb in market interest rates, accompanied by a gradual turn-over of the bond portfolio, can still harm an insurance company whose assets have longer payout patterns than its liabilities. However, this gradual climb in market interest rates eats through the company's statutory surplus, since it may have sold some assets at a loss and its yield on recently maturing assets was lower than the yields on similar (recently issued) assets. The company now needs less capital to guard against interest rate risk, but it is financially weaker than before, since its statutory surplus has declined even more rapidly.

[35]The investment yield rates for the bond portfolio in this example are purely heuristic. They do not correspond to actual yields at the stated dates.

uation rate for assets (6%) than for liabilities (5%) provided an initial cushion. But the increase in market interest rates during the subsequent years ate through some of this cushion, since the long-maturity bonds were more adversely affected by the interest rate rise than the reserves were. Larger capital requirements are now indicated for interest rate risk than were indicated at December 31, 1992.

But suppose that the bond portfolio is being turned over from time to time. By December 31, 1995, when market interest rates are 8% per annum, the average valuation rate on the bonds may be 7% per annum. If so, the valuation rate cushion is larger, and the need for additional capital is reduced.

It is difficult to state general rules that will hold in all cases, since many of the relationships discussed above change over time and since so many factors are involved: the average payment date differences between assets and liabilities, the size of the initial cushion, the magnitude of the actual interest rate shift, and the rate of turnover of the investment portfolio. Each situation should be examined separately.

2B. As the difference between expected and nominal average payment dates for assets decreases, interest rate risk increases.

Interest rate risk varies with the difference between expected payment dates for assets and for liabilities. The expected average payment date for assets, which depends on the exercise of borrower options for prepayment, is shorter than the stated (nominal) average payment date. A decrease in this difference, caused by fewer borrower prepayments, increases the difference between asset and liability expected average payment dates, thereby increasing the interest rate risk.[36]

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[36]Again, this generalization assumes that the asset portfolio has a greater duration than the reserves portfolio, which is not necessarily true for all insurers.

An example should help clarify this. Suppose reserves with an average payment date of 4 years are backed by a portfolio of mortgage-backed securities. This portfolio has a nominal average payment date of 15 years, but an expected average payment date of 7 years, reflecting prepayments and refinancings.

If interest rates rise, there will be fewer prepayments and refinancings. The expected average payment date may lengthen to eight years instead of seven years, thereby increasing the interest rate risk.[37]

In summary, an increase in market interest rates has three effects:

- The valuation rate for assets gradually rises, lowering the interest rate risk charge and reducing the need for additional capital.
- The market value of existing assets falls more than the market value of reserves (as long as the assets have longer average maturities than the reserves), thereby reducing the cushion in the differing valuation rates and increasing the need for additional capital.
- Borrowers exercise their prepayment options less frequently, thereby lengthening the average payment date of assets, increasing interest rate risk, and increasing the need for additional capital.[38]

3. Statutory Mandates

Subjective changes in the parameters used for solvency monitoring affect the capital required to guard against interest rate risk in two ways.

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[37] See Appendix B, Exhibit 1, of Hodes, et al. [24], for the actual effects of a 200 basis point rise in market interest rates on a large portfolio of mortgage-backed securities.

[38] This last effect can be avoided to the extent that the insurer avoids corporate bonds with call provisions or personal mortgages with prepayment options.

3A. The larger the interest rate shift that is tested, the greater is the capital required for interest rate risk.

The magnitude of the interest rate shift may be decided by the regulator in a risk-based capital context or by the appointed actuary in a dynamic solvency testing context. The larger the interest rate increase, the larger the potential adverse effect on the company's surplus, and the more additional capital is needed. This issue of "calibration" is essential to a practical formula, though not to the procedure, so it is not further dealt with in this paper.[39]

3B. The higher the valuation rate used for reserves, the greater the interest rate risk.

The book valuation rate for reserves depends on the accounting system used. Statutory reporting uses undiscounted reserves. The NAIC's risk-based capital formula uses a 5% per annum discount rate. Many internal company valuation systems use market valuation rates, such as the risk-free rate on Treasury bills. The higher the valuation rate used for reserves, the smaller the valuation rate cushion between assets and liabilities, thereby increasing the additional capital needed to guard against interest rate risk.

The Management of Interest Rate Risk

Numerous factors affect interest rate risk. Some are controllable by the insurance company, such as the duration of the investment portfolio. Some are controllable by the regulator, such as the magnitude of the interest rate shift. And some reflect changing market conditions, such as the current market interest rate versus the valuation rate for assets.

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[39]For the interest rate risk proposal of the AAA Task Force on Risk-Based Capital, Robert Butsic calibrated the capital requirements to a one percent expected policyholder deficit ratio. His resultant interest rate risk parameters were a 120 basis point interest rate increase along with a "deductible" equal to 3.5% of the loss reserve market value. For Butsic's derivation of these figures, see Appendix C of [3], particularly page C5 and Exhibits 4A and 4B.

The analyst should understand two aspects of each factor:

- The manner in which a movement in each factor affects interest rate risk. In particular, the analyst should understand whether any specific change will increase or decrease interest rate risk.
- The expected magnitude of the effects of a change in each factor. For instance, the analyst should know the expected effect of a one point change in the asset valuation rate on the capital requirements for interest rate risk. These magnitudes can not be stated as general rules, but must be examined for each company and for each book of business.

8. THE ALLOCATION OF ASSETS

One unresolved issue in the treatment of interest rate risk is the allocation of assets to specific liabilities.

> When determining interest rate risk, should supporting assets be assigned to each block of reserves, or should total assets be compared with total liabilities?

The resolution of this question depends on the goals of the analysis, such as generic monitoring of surplus adequacy by the risk-based capital formula vs. detailed analysis of the company's financial strength by the appointed actuary. We therefore present both sides of this issue.

The Rationale for Allocation

Three arguments favor the allocation of assets to specific liabilities:

1. Certain assets may notionally "support" each block of reserve liabilities, even if all assets ultimately stand behind all liabilities.

2. Asset-liability management suggests that different assets should be purchased to fund different blocks of business.
3. The corresponding life insurance company test, the "asset adequacy analysis," begins with an allocation of assets to blocks of reserves.

Supporting Assets: The illustrations in this paper portray the insurance company as purchasing a bond to fund a specific liability. The value of the bond is chosen to reflect the value of the liability, though the value of the bond depends on the valuation base for the liability.

This presentation serves a valuable analytical purpose. The fundamental issue underlying interest rate risk is whether the cash inflows from assets will support the required liability outflows even if market interest rates shift. Some actuaries argue that it is difficult to answer this question if one does not have a theoretical allocation of assets to blocks of business.

In practice, of course, this allocation does not occur, for two reasons:

- First, all the company's assets support each liability. Any allocation of assets is an accounting fiction, with no legal force. For instance, suppose a company sets up a case reserve of \$100,000 for a given accident, and it allocates assets to fund the loss payment. If two years later the company re-estimates the claim cost as \$500,000, an additional \$400,000 of assets should be allocated to this accident. The original loss estimate (and case reserve) of \$100,000 has no relevance. None of the company's assets has a higher or lower priority for supporting the reserve.
- Second, the company's investment department does not purchase assets to correspond with specific liabilities. Rather, the investment department has an overall sum of money, consisting of funds attributable to insurance transactions as well as funds attributable to capital and surplus. Moreover, it has in-

vestment guidelines, such as "All corporate bonds purchased should be in the two highest NAIC quality classes."[40]

Asset-Liability Management: As noted above, the investment department will generally not purchase assets to correspond with specific loss reserves. But it will align the general characteristics of its investment portfolio with the attributes of its reserve portfolio.

For instance, suppose a hypothetical company writes workers compensation and homeowners insurance. Its investment philosophy may have three components:

1. For its workers compensation business, it desires long-term, high yielding, safe securities, with steady cash inflows. It chooses a mixture of private placements, mortgage-backed securities, and municipal bonds. The company has sacrificed liquidity and the opportunity for capital gains for steady, safe, and high returns, along with long-term tax advantages.
2. For its homeowners business, with its high catastrophe potential and consequent need for immediate cash payments, the investment department seeks liquid assets and chooses a mixture of Treasury securities and high-grade corporate bonds. The company has sacrificed the higher yields associated with private placements for the liquidity and the safety of principal associated with government and corporate bonds.[41]

[40]The range of practice is actually broader than this paragraph implies. Life insurance companies often segment their asset portfolio by product type, and some property/casualty companies similarly segment their asset portfolio by major line of business. Segmentation, however, is far less common for property/casualty companies than for life companies.

[41]For the workers compensation business, the long-term, high-yielding securities are the assets backing the loss reserves. For the homeowners business, the liquid assets are protecting against the catastrophe risk, not against expected loss payments. Thus, these assets are effectively backing statutory surplus and the unearned premium reserve, not homeowners loss reserves.

3. For its remaining investments—that is, for assets associated with its surplus funds—the company chooses a mixture of common stock and real estate. Liquidity and steadiness of cash flows are not relevant for these monies. Rather, the company wishes to maximize its expected income, even if this strategy increases the variability of the value of its surplus.[42]

Asset Adequacy Analysis: One purpose of interest rate risk analyses is to examine whether the cash flows from assets will cover the required liability payments even under adverse future interest rate environments. This is the "asset adequacy analysis" required of the life insurance appointed actuary. We are simply carrying it to the property/casualty business, with adjustments as needed. It can be argued that just as the life actuary must first allocate assets to reserves to examine the sufficiency of future cash flows, so must the casualty actuary.

The Arguments Against Allocation

There are two principal arguments against allocation, one theoretical and one practical.

1. *Surplus Adequacy*: The purpose of a risk-based capital formula is to determine capital requirements to ensure surplus adequacy. Its purpose is not to determine risk margins to ensure reserve adequacy. Suppose the insurance company has the following attributes:

   - Assets consist of a $100 million bond portfolio. Liabilities consist of $80 million of reserves, so policyholders' surplus is $20 million.[43]

[42]Compare Noris [30] for a similar investment strategy for a property/casualty insurer.

[43]These figures are the book values in the risk-based capital accounting system, not market values or statutory values. In the current NAIC risk-based capital system, the book value for assets equals the statutory value, and the book value for liabilities equals the reserves discounted at a 5% per annum interest rate.

- The interest rate risk analysis shows that the market value of the asset portfolio will decrease by 5% if interest rates rise by the "shock" amount, whereas the economic value of the liabilities will decrease by only 2%.
- The difference in book valuation rates between assets and liabilities provides a $2.4 million cushion.
- The company needs $20 million of capital to guard against other risks, such as reserving risk, default risk, and reinsurance risk.

If one asks, "Will the asset cash flows support the liability obligations even if interest rates rise?" the answer is yes. The decrease in the market value of liabilities is $1.6 million (= $80 million × 2%), and the decrease in the market value of the supporting assets is $4 million (= $80 million × 5%). The net decrease is $2.4 million, which is covered by the valuation rate margin.

If one asks, "Is the company holding sufficient capital to guard against the risks that it faces?" the answer is no. The entire $20 million of surplus is needed to guard against other risks. But if the bond portfolio declines in value by 5%, there is an additional decline in value that we have not considered above. The assets not supporting the reserves ($20 million) also decline by 5%, so surplus is only $19 million, not $20 million. The company needs an additional $1 million of capital to guard against the risks that it faces.[44]

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[44] Readers may ask: "Have you properly accounted for the differences between amortized value and market value in the valuation of the bond portfolio?" The answer is yes. The interest rate risk test compares the current book values of assets and liabilities with their market values at a higher interest rate. Thus, statutory accounting values are converted to market values for both assets and liabilities.

For the sake of simplicity, the example does not consider any "covariance" effects. In other words, since interest rate risk and reserving risk are not perfectly correlated, the total capital requirement is less than the sum of the capital requirements for each risk separately. The NAIC risk-based capital formula uses the "square root rule" developed

2. *Simplicity*: The allocation of assets to blocks of reserves is a complex process. In a solvency monitoring setting, companies will desire to allocate those assets to support their reserves that generate the lowest interest rate risk charges, regardless of which assets "ought" to correspond with specific reserves. To avoid additional complexities, regulators may wish to dispense with the allocation of assets.

9. SAMPLE CALCULATION

The previous sections of this paper have been explanatory, with simplified heuristic illustrations. This section provides a more complete example.

To quantify the capital requirements for interest rate risk, three sets of data are needed.

1. *Assets*
   a. One needs the expected cash flow patterns of the investment portfolio, including both interest and principal payments. Ideally, one wants expected cash flow patterns under various interest rate paths; see the comments above regarding mortgage-backed securities for the potential effects of interest rate changes on principal repayments. In practice, few property/casualty companies estimate projected asset cash flows under different interest rate scenarios.
   b. One needs either the statutory valuation rate for these assets or their statutory book value. Since the former may vary for each asset whereas the latter is published in the Annual Statement, it is simpler to use the latter.

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by Robert Butsic to combine the capital requirements for each risk component. Similarly, the AAA Task Force recommendations on interest rate risk placed this charge with the bond default risk charge, not with the reserving risk charge. See Feldblum [16] for further discussion of these issues.

2. *Liabilities*

   a. One needs the expected cash flow patterns of the loss reserve portfolio. The appointed actuary of a large insurer may use internal data to determine these patterns, particularly for the long-tailed lines of business.[45] Financial regulators could use industry aggregate patterns from Schedule P data, either from Best's *Aggregates and Averages* or from the NAIC data tapes.[46]

   b. One needs either the statutory valuation rate for these liabilities or their statutory book value. For the NAIC risk-based capital requirements, the statutory valuation rate is given as 5% per annum, though the resultant book value is nowhere published.[47] However, the NAIC publishes factors to convert the statutory

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[45]Published industry data from Schedule P extends for only ten years. For workers compensation, the average payout of the reserves is about eight years from the valuation date. Short duration workers compensation claims (temporary total claims) are paid within a few months, and they form only a small portion of a company's year-end reserves. Lifetime pension claims (permanent total disability and fatality claims) have payment patterns extending for thirty or forty years, and they form a large portion of a company's year-end reserves. Schedule P data are inadequate for projecting the payment pattern of workers compensation reserves. Similar comments are true for medical malpractice, products liability, and excess-of-loss reinsurance.

[46]There is no mention here of potential variability of the amount and timing of liability payments under different interest rates. Asset cash flows are expected to change when interest rates shift even for fixed income securities because the issuers have options such as calls on corporate bonds and prepayments on mortgages and mortgage backed securities. Similarly, benefit payments and premium collections on life insurance and annuity products vary with the interest rate because policyholders have similar options: they may take policy loans, they may cash in the policy, or they may increase or decrease their premium payments on universal life and other indeterminate premium policies. On property and casualty insurance contracts, there are generally no policyowner options.

As noted elsewhere in the text, inflation does affect the magnitude of loss payments. Since inflation is correlated with interest rates, the magnitude of loss payments is also correlated with interest rates. However, the effects of inflation on the magnitude of loss payments are reflected in the reserving risk charge, so it cannot be "double counted" in the interest rate risk charge as well.

[47]The "book value" of liabilities is the implicit book value in the risk-based capital system. The risk-based capital formula discounts reserves at 5% per annum, to remove

(undiscounted) values of the loss reserves to the risk-based capital (discounted) values.

3. *Calibration*

   a. The capital requirements for interest rate risk depend on the severity of the interest rate shift to be guarded against.

   b. In theory, the severity of the interest rate shift that is selected depends on the type and magnitude of the solvency criterion, such as a 2% probability of ruin, or a 1% expected policyholder deficit. In practice, either the company or the regulator would select a "basis point shift" that is deemed to be sufficiently adverse yet realistic, such as a 150 basis point shift.

Inputs

The example here uses the following input data:

1. The fixed income securities investment portfolio consists primarily of long-term bonds and mortgages, along with smaller amounts of short-term mortgages, collateral loans, and other short-term investments.
2. The loss reserves are for personal automobile liability exposures. The payment patterns are derived from industry aggregate Schedule P data, as described earlier in this paper.
3. Our primary test is a 100 basis point rise in market interest rates. This is somewhat more conservative than the recommendation of the AAA Task Force, which

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the implicit interest margin in the undiscounted reserves on statutory statements. In other words, the risk-based capital formula determines capital requirements for a company whose reserves are discounted at 5% per annum, so this is the "book value" of the reserves used to determine interest rate risk.

used a 120 basis point rise combined with a 3.5% "deductible."[48]

"Market interest rates" is an amorphous concept. In practice, one must define the specific interest rate that is being used as a standard. One obvious choice is the federal midterm rate, which is the average rate on Treasury securities with remaining terms to maturity of between three and seven years. This is the rate on risk-free securities with terms to maturity about equal to the maturities of property/casualty liabilities. Moreover, this rate is used by the IRS in calculating discounted reserves. For the illustrations in this paper, we assume that the current market interest rate is 5.50% per annum.

We also show the effects of a more stringent interest rate shift, of 150 basis points and of 200 basis points.[49] As expected, the capital requirement increases as the interest rate shift grows larger.

Assets

The expected cash flows from the fixed income asset portfolio are derived from the asset schedule adopted by the NAIC for submission as a risk-based capital supplement to the Annual Statement for 1995 and subsequent years. This schedule is reproduced here as Exhibit 1, and it is discussed, along with the illustrative entries, earlier in this paper.[50]

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[48]The AAA recommendation was calibrated by Robert Butsic to produce a 1% expected policyholders deficit, since the reserving risk component of the NAIC risk-based capital formula implicitly came to this standard. Because of the complexity of the calibration issues, they are not treated in this paper.

[49]The 200 basis point shift is similar to the A. M. Best interest rate risk test.

[50]The asset schedule shows expected payments by year for the first four years, and then by groups of years (e.g., 4 to 7, 7 to 10) for the remaining durations. State regulators implementing a risk-based capital system would use these groupings when determining asset cash flow patterns. For instance, the assumed average payment date for the entire "4 to 7 years" cell is 5.5 years.

This information is superior to the information contained in previous insurance company financial statement submissions, but it is not perfect. For instance, the "4 to 7 years"

Allocation of Assets

The book value of the fixed income assets, \$163 million, exceeds the book value of the personal automobile loss reserves, \$139,970,000.[51] The illustration here shows the calculation of the capital requirements both with and without an allocation of assets to liabilities.

For the allocation of assets to liabilities, bonds with different maturities are divided pro-rata between the liability and surplus amounts. For instance, the \$21,672,000 in Column 4 of Exhibit 3, for the row "time of payment = 0.5 years," is calculated as:

$$\$25{,}238{,}000 \times (\$139{,}970{,}000 \equiv \$163{,}000{,}000) = \$21{,}672{,}000.$$

In a dynamic solvency testing environment, the appointed actuary would allocate assets to liabilities based on the company's asset-liability management strategy, not necessarily on a pro-rata basis. Such an allocation might assign more shorter-term securities to the personal auto reserves and more longer-term securities to surplus, which would reduce the interest rate risk charge.[52]

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figure may be dominated by bonds maturing in just over 4 years or by bonds maturing in just under 7 years. The latter implies a longer asset duration, and it should (in theory) cause a higher interest rate risk capital charge.

The illustration here uses internal company data to more finely subdivide the expected payment dates of the fixed income security cash flows. For instance:

- The asset schedule in Exhibit 1 shows a total statement value (= book value) of \$163 million and a total nominal cash flow of \$238,159,000. Both of these figures are carried directly to Exhibit 3.
- The asset cash flow figures in Column 3 of Exhibit 3 for all "time of payment" rows except those for 4.5 to 9.5 years are taken directly from the asset schedule in Exhibit 1. For the six "time of payment" rows from 4.5 to 9.5 years, Exhibit 3 subdivides the aggregate figures in the asset schedule into yearly cells, using internal company information. For instance, the \$35,007,000 total expected cash flow in the "4 to 7 years" column in the asset schedule of Exhibit 1 is subdivided into three entries in Column 3 of Exhibit 3:
  - \$13,303,000 for a 4.5 years average time of payment,
  - \$11,552,000 for a 5.5 years average time of payment, and
  - \$10,152,000 for a 6.5 years average time of payment.

[51] The "book value" here is the statutory value for the assets (generally, amortized value), and the discounted value for the liabilities at a 5% per annum interest rate.

[52] Whether assets are allocated to liabilities depends on the purposes of the interest rate risk analysis. A valuation actuary dealing with interest rate risk in a dynamic financial

Liabilities

The entries in Column 2 of Exhibit 3, "Loss Reserve Payout," are derived in three steps.

1. The figures shown in Exhibit 3 are illustrative only. In practice, the statement value of the reserves, or $150,650,000 in these illustrations, should tie to the Schedule P totals from the company's Annual Statement, which shows undiscounted figures.[53] This statement value is the sum of the undiscounted cash flows, and is shown on the Total row of Exhibit 3.

2. The payment pattern for the loss reserve liabilities is determined from aggregate industry data, using Best's *Aggregates and Averages*, as shown in Exhibit 2. The entries in the first nine rows of Exhibit 3 are the payment pattern percentages by accident year from Exhibit 2 times the undiscounted reserve of $150,650,000 in Exhibit 3.[54]

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analysis setting is often helping management determine whether its investment strategy is appropriate, given the company's liability structure. In such a case, the actuary may notionally allocate assets to each block of reserves, to determine if there is a good fit between the two. The solvency regulator is concerned with the adequacy of the company's total capital, not with the appropriateness of its investment strategy. Asset allocation is less relevant to the regulator's concerns.

As noted earlier in the text, this paper examines interest rate risk from the regulator's viewpoint: capital requirements. The DFA perspective, which uses different techniques, may be seen in Hodes, et al. [24].

[53]For further discussion of the reporting of Annual Statement loss reserves gross or net of discount, see Feldblum [14].

[54]This paper views the company from a run-off perspective, as is appropriate for solvency monitoring; see Daykin, et al. [10]. From a "going-concern" perspective, the cash used to meet loss obligations comes (at least in part) from new premium inflows, not just from the assets currently held by the company.

Asset-liability management for a going concern is more complex than might be inferred from this paper, since it is affected by the renewal rates on the book of business and the sensitivity of premium rates to market interest rate changes. For the pricing side of this phenomenon, see Feldblum [15]; for the asset management implications, see Panning [31].

3. The "book value" of $139,970,000 in the last row of Column 1 in Exhibit 3 is the present value of the future cash flows discounted at 5% per annum. This illustration uses an actual discounting of the expected cash flows, as would be appropriate for appointed actuary work. For risk-based capital requirements, one would use the investment income offset factor in the RBC formula, which is a rough approximation based on the IRS loss reserve discounting procedure.[55]

Severity of the Test

The bottom half of Exhibit 3 has three columns showing the capital needed to guard against interest rate shifts of 100 basis points, 150 basis points, and 200 basis points, respectively. A comparison of the three columns shows the sensitivity of the capital requirement to the magnitude of the interest rate shift as summarized in Table 2 at the end of Section 9.

In each case, the current market interest rate is 5.5% per annum. Each column can be viewed in two fashions. The interest rate shift may be an assumed adverse scenario, and the company must hold capital to hedge against this adverse scenario. Alternatively, the "interest rate shift" may be—in part or in whole—an actual movement in market interest rates.

For instance, the right-most column may represent a current market interest rate of 5.5% per annum, with a 200 basis point shift in the risk-based capital test. Alternatively, the column may represent an actual change in market interest rates from 5.5% per annum to 6.5% shortly before the valuation date, and then

[55] For personal auto liability reserves, the investment income offset factor in the risk-based capital formula is 92.1%. The product of 92.1% and $150,650,000 is $138,748,650, which is approximately equal to the book value derived here. In practice, of course, the book value of the liabilities in the risk-based capital system is the undiscounted amount times the risk-based capital discount factor (the 92.1% shown directly above for personal auto liability). In the illustrations, we show the discounting at a 5% interest rate to highlight the factors affecting the interest rate risk charge.

an additional 100 basis point shift in the risk-based capital formula.

The alternative interpretation is realistic, representing a sudden change in market interest rates from 5.5% to 6.5%—say, in the half year preceding the valuation date. In these illustrations, the valuation rate of the assets is 5.22%, which makes more sense in a market interest rate environment of 5.5% than in a market interest rate environment of 6.5%.

A 5.22% average valuation rate for the relatively long-term securities in the asset portfolio implies a market interest rate of about 5% or less for medium term risk-free securities. This makes sense if the assets were bought over the preceding several years and the market interest rate has recently drifted upward to 5.5%. However, if the market interest rate has slowly drifted upward to 6.5% over the past several years, allowing for turnover of the asset portfolio to higher yielding securities, the asset valuation rate would probably be above 5.22%.

Capital Requirements

Rows A and B of Exhibit 3 show the capital required to guard against interest rate risk when assets are first allocated to liabilities. Rows C through F show the capital required if all fixed income assets are used, with no allocation to liabilities. Since the first computation does not consider the assets corresponding to the company's surplus, its resulting capital requirement, $7,017,000, is lower than that of the latter calculation, $8,640,000.

Rows A, B, D, and E show the present value of the asset and liability cash flows at the "shifted" or "shocked" interest rate (6.5%, 7.0%, and 7.5% per annum in the three columns). The following paragraphs document the calculations for the 6.5% per annum column.

1. The present value of the liability cash flows declines by 2.04%, from $139,970,000 to $137,120,000. The calculation is done by discounting each cash flow at 6.5% per annum. The reasonableness of the result can be checked by considering the adjusted Macauley durations. The liabilities have an adjusted Macauley duration of 1.39 years, which implies that a one hundred and fifty basis point increase in the discount rate causes a decline in present value of about 2.08%.

2. The present value of the asset cash flows declines by 7.05%, from $139,970,000 to $130,104,000. Again, the reasonableness of this figure can be checked by considering the adjusted Macauley durations.

   - The assets have an adjusted Macauley duration of 6.07 years.
   - The book value of the assets implies an average investment yield of 5.22% per annum.[56]
   - The shift to a 6.5% per annum discount rate is an increase of 1.28 percentage points, so the decline in market value of the assets should be about $1.28 \times 6.07 = 7.77\%$.

The combination of:

- the mismatch between asset and liability durations (6.07 years vs. 1.39 years), and
- the similarity of the statutory valuation rates (5.22% vs. 5.00%)

[56]In practice, the book value of the assets depends on the amortized value of each security, so the implied investment yield differs by security. The 5.22% yield is an aggregate figure. It says: "Given the future expected cash flows from this investment portfolio, what discount rate sets its present value equal to its statutory book value?"

leads to a significant interest rate risk charge. The charge, or the capital requirement, equals the change in the value of the assets minus the change in the value of the liabilities, or:

(book value of assets – present value
of assets at shifted interest rate)
– (book value of liabilities – present value
of liabilities at shifted interest rate).

When assets are first allocated to liabilities, the book values of the two are equal, so the capital requirement simplifies to:

present value of liabilities – present value of assets,

at the shifted interest rate. In this example, the capital requirement is

$137,120,000 – $130,104,000 = $7,017,000.

When assets are not first allocated to liabilities, the total investment portfolio is considered. The book value of the investment portfolio is $163,000,000, and its market value at a 6.5% per annum discount rate is $151,510,000, so the capital requirement is

($163,000,000 – $151,510,000) – ($139,970,000 – $137,120,000)
= $8,640,000.

Other Interest Rate Shifts

The magnitude of the interest rate shift used in a risk-based capital setting is a calibration issue. This paper does not argue for any particular interest rate shift. However, the bottom half of Exhibit 3 shows the capital requirements if interest rate shifts of 100 basis points, 150 basis points, and 200 basis points are used, so that readers can see the effects of different interest rate shifts.

TABLE 2

CAPITAL REQUIREMENTS FOR INTEREST RATE SHIFTS

Basis Point Shift:	100 Basis Points	150 Basis Points	200 Basis Points
w/ allocation of assets	$7,017,000	$9,566,000	$11,934,000
w/o allocation of assets	8,640,000	11,760,000	14,666,000

Table 2 shows the capital requirements:

- for interest rate shifts of 100, 150, and 200 basis points, and
- with and without an allocation of assets to liabilities.

As expected, larger basis point shifts lead to larger capital requirements.

10. CONCLUSION

Asset-liability management is becoming an increasingly important aspect of insurance company investment strategy. Insurers hold enormous financial portfolios—both assets and liabilities—relative to their equity. Regulators are justifiably concerned about the effects of interest rate changes on the financial strength of the company and about the type of capital requirements needed to protect against interest rate risk.

The varied nature of the assets and liabilities comprising an insurer's financial portfolio, the differences between expected and stated cash flows, and the different statutory valuation rates used for assets and liabilities must be considered in the determination of interest rate risk and the associated capital requirements. This paper describes the procedure recommended for inclusion in the NAIC risk-based capital formula, and it provides an illustration for a sample company.

Neither the NAIC nor the American Academy of Actuaries has yet issued guidelines for quantifying interest rate risk for property/casualty insurance companies. Casualty actuaries must understand this subject thoroughly if they wish to participate in the industry discussions and to influence the coming professional and regulatory guidelines.

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EXHIBIT 1

Fixed Income Asset Cash Flow Schedule

Cash Flow Category	Statement Value	Market Value	Principal	1 year or less	1 to 2 years	2 to 3 years	3 to 4 years	4 to 7 years	7 to 10 years	10 to 15 years	15 to 20 years	20 to 30 years	Over 30 years	Total
Payment Range Midpoint (Years)				0.5	1.5	2.5	3.5	5.5	8.5	12.5	17.5	25	35	
PRINCIPAL PAYMENTS														
Bonds	90,000	95,000	100,000	2,000	5,000	6,000	12,000	12,000	14,000	14,000	15,000	20,000	0	100,000
Mortgages: Long Term	49,000	49,000	50,000	2,000	4,000	5,000	6,000	9,000	6,000	6,000	8,000	4,000	0	50,000
Mortgages: Other	9,000	9,000	10,000	1,000	2,000	3,000	2,000	2,000	0	0	0	0	0	10,000
Collateral Loans	5,000	5,000	5,000	2,000	1,000	1,000	1,000	0	0	0	0	0	0	5,000
Perpetual Preferred Stocks	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Mandatory Sinking Fund Preferred Stocks	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Short-term Investments	10,000	10,000	10,000	10,000	0	0	0	0	0	0	0	0	0	10,000
TOTAL	163,000	168,000	175,000	17,000	12,000	15,000	21,000	23,000	20,000	20,000	23,000	24,000	0	175,000
INTEREST PAYMENTS														
Bonds				5,066	4,850	4,578	4,018	8,668	6,429	5,638	3,318	1,659	0	44,224
Mortgages: Long Term				2,526	2,334	2,085	1,793	3,339	2,378	1,937	660	367	0	17,419
Mortgages: Other				482	378	217	110	0	0	0	0	0	0	1,187
Collateral Loans				164	110	55	0	0	0	0	0	0	0	329
Perpetual Preferred Stocks				0	0	0	0	0	0	0	0	0	0	0
Mandatory Sinking Fund Preferred Stocks				0	0	0	0	0	0	0	0	0	0	0
Short-term Investments				0	0	0	0	0	0	0	0	0	0	0
TOTAL				8,238	7,672	6,936	5,921	12,007	8,807	7,575	3,978	2,026	0	63,159
TOTAL CASH FLOW FOR PERIOD														
Bonds				7,066	9,850	10,578	16,018	20,668	20,429	19,638	18,318	21,659	0	144,224
Mortgages: Long Term				4,526	6,334	7,085	7,793	12,339	8,378	7,937	8,660	4,367	0	67,419
Mortgages: Other				1,482	2,378	3,217	2,110	2,000	0	0	0	0	0	11,187
Collateral Loans				2,164	1,110	1,055	1,000	0	0	0	0	0	0	5,329
Perpetual Preferred Stocks				0	0	0	0	0	0	0	0	0	0	0
Mandatory Sinking Fund Preferred Stocks				0	0	0	0	0	0	0	0	0	0	0
Short-term Investments				10,000	0	0	0	0	0	0	0	0	0	10,000
TOTAL				25,238	19,672	21,936	26,921	35,007	28,807	27,575	26,978	26,026	0	238,159

EXHIBIT 2

PRIVATE PASSENGER AUTO LIABILITY/MEDICAL
(FROM BEST'S *Aggregates and Averages*, 1994, IN $MILLIONS)

Accident Year	Age in Months 12	24	36	48	60	72	84	96	108	120
1984	7,107	13,731	16,905	18,677	19,655	20,147	20,396	20,542	20,619	20,661
1985	7,798	15,396	19,096	21,216	22,338	22,903	23,207	23,358	23,433	
1986	8,681	17,212	21,459	23,840	25,120	25,766	26,076	26,243		
1987	9,674	19,363	24,098	26,758	28,171	28,855	29,247			
1988	10,902	21,695	26,909	29,735	31,204	31,950				
1989	12,017	24,024	29,718	32,762	34,382					
1990	13,250	25,971	32,000	35,193						
1991	13,305	25,822	31,811							
1992	14,372	27,677								
1993	15,657									

Age-to-Age Factors

	12–24	24–36	36–48	48–60	60–72	72–84	84–96	96–108	108–120
1984	1.932	1.231	1.105	1.052	1.025	1.012	1.007	1.004	1.002
1985	1.974	1.240	1.111	1.053	1.025	1.013	1.007	1.003	
1986	1.983	1.247	1.111	1.054	1.026	1.012	1.006		
1987	2.002	1.245	1.110	1.053	1.024	1.014			
1988	1.990	1.240	1.105	1.049	1.024				
1989	1.999	1.237	1.102	1.049					
1990	1.960	1.232	1.100						
1991	1.941	1.232							
1992	1.926								

Age-to-Age Factors

	12–24	24–36	36–48	48–60	60–72	72–84	84–96	96–108	108–120	Age 120 assumed to be ultimate
Average ATA	1.967	1.238	1.106	1.052	1.025	1.013	1.007	1.003	1.002	↑
Age to Ult.	2.978	1.514	1.223	1.105	1.051	1.025	1.012	1.006	1.002	1.000
Payment Pattern	0.336	0.325	0.157	0.087	0.047	0.024	0.012	0.007	0.003	0.002

Payout of Company Reserves

	Reserve at 12/31/1993	Payout of Reserves, split by year 1994	1995	1996	1997	1998	1999	2000	2001	2002
1985	100	100								
1986	300	188	112							
1987	650	355	185	111						
1988	1,500	761	403	210	126					
1989	3,600	1,763	932	493	257	154				
1990	8,500	4,187	2,113	1,117	591	308	185			
1991	18,000	8,598	4,631	2,337	1,235	654	341	204		
1992	38,000	17,609	9,739	5,246	2,648	1,399	741	386	231	
1993	80,000	39,126	18,941	10,476	5,643	2,848	1,505	797	415	249
Total	150,650	72,686	37,056	19,990	10,500	5,363	2,771	1,387	647	249

EXHIBIT 3

CAPITAL REQUIREMENTS FOR INTEREST RATE RISK (IN THOUSANDS OF DOLLARS)

(1) Time of Payment	(2) Loss Reserve Payout	(3) Fixed Income Asset Inflows	(4) Pro-ration of Fixed Income Asset Inflows**
0.5	72,686	25,238	21,672
1.5	37,056	19,672	16,893
2.5	19,990	21,936	18,837
3.5	10,500	26,921	23,118
4.5	5,363	13,303	11,423
5.5	2,771	11,552	9,920
6.5	1,387	10,152	8,718
7.5	647	10,947	9,400
8.5	249	9,506	8,163
9.5		8,354	7,174
12.5		27,575	23,679
17.5		26,978	23,166
25.0		26,026	22,349
Total	150,650	238,159	204,510
Book Value*	139,970	163,000	139,970

*Book value of the assets is the Annual Statement value.
RBC book value of the liabilities is the present value at a 5% discount rate.

**Column 4 = Column 3 × book value of Column 2 ≡ book value of Column 3.

INTEREST RATE RISK CAPITAL REQUIREMENT USING ''MATCHED'' ASSETS AND LIABILITIES:

"Shocked" interest rate:	6.5%	7.0%	7.5%
A. PV of loss payments (Col 2):	137,120	136,202	135,300
B. PV of "matched" income flows (Col 4):	130,104	126,636	123,366
Capital Requirement [A – B]:	7,017	9,566	11,934

INTEREST RATE RISK CAPITAL REQUIREMENT USING ALL ASSETS AND LIABILITIES:

"Shocked" interest rate:	6.5%	7.0%	7.5%
C. PV of loss payments (Col 2):	137,120	136,202	135,300
D. Difference between C and BV of liabilities:	(2,850)	(3,768)	(4,670)
E. PV of "full" income flows (Col 3):	151,510	147,472	143,663
F. Difference between E and BV of assets:	(11,490)	(15,528)	(19,337)
Capital Requirement [D – F]:	8,640	11,760	14,666

APPENDIX A

DURATION AND MARKET VALUES

This paper deals with the effects of interest rate shifts on the net worth of an insurance company. "Duration" is a term denoting the sensitivity of the market value of a security to a shift in interest rates.

Duration analysis is widely used by life insurance actuaries and by investment personnel. The text of this paper assumes a general familiarity with this concept. This appendix provides the requisite background material for readers who have not previously worked with duration analysis.

Payment Patterns

Our fundamental question is often stated as follows: "How does the duration of a security affect the sensitivity of its market value to interest rate shifts?" In truth, this sentence is loosely worded. Duration does not affect market values when interest rates shift. Rather, duration is *defined* as the effect of interest rate shifts on market values. What we are really asking is: "How does the payment pattern of a security affect the change in market value resulting from an interest rate shift?" Or as financial analysts would phrase this, "How does the payment pattern of a security affect its effective duration?"[57]

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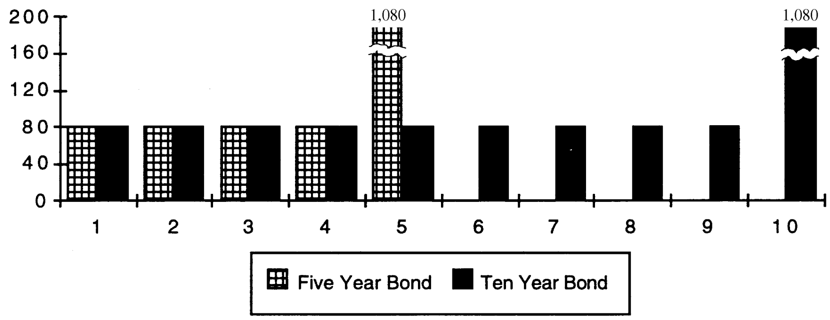
[57] Although the term "duration" has a temporal connotation, and it is commonly measured in units of years, it is a measure of sensitivity to changes in interest rates, not a measure of time. The term "duration" originally stemmed from the application of mathematical approaches to estimate the effects of changing investment conditions on the market value of a fixed income portfolio. The effects were dependent on the average time of the future payments, weighted by the present values of the cash flows. The term "duration" was both appropriate and intuitive to express this concept.

As our understanding of this concept evolved, and as theoreticians examined the effects of imbedded options on the financial relationships and the effects of the investment environment on more complex securities, such as equities, the temporal connotation of duration is sometimes more of a hamper than a benefit. Nevertheless, we have retained the use of this term here. In fact, because of the introductory nature of this paper, we have restricted the analysis to simple fixed income assets and liabilities, avoiding the complexities of inflation-sensitive equities and reserves.

FIGURE 1

CASH FLOWS: ANNUAL COUPONS AND PRINCIPAL REPAYMENTS



Consider two bonds, both with $1,000 par values and 8% annual coupons. Bond A has a five year term to maturity, and Bond B has a ten year term to maturity.

At issue, market interest rates are 8% per annum, and both bonds have a purchase price of $1,000. But if interest rates rise or decline after issue, the change in market value differs for the two bonds.

The market value of a bond is the present value of future cash payments, discounted at an appropriate capitalization rate. For simplicity, we assume that the yield curve is flat, so the appropriate capitalization rate is the market interest rate.[58]

Figure 1 shows the cash flows from the two bonds. Bond A pays $80 at the end of each year, plus a $1,000 repayment of

[58] As noted in the text of the paper, there is no single "market interest rate." Rather, interest rates vary with various attributes of each financial instrument, such as maturity, quality, liquidity, call provisions, and so forth. In this illustration, we assume that the yield curve is flat, so the interest rate will not vary by maturity. If the two bonds are similar in other respects, approximately the same interest rate is appropriate for both. In practice, of course, the bonds would not be similar in all other respects. For instance, longer maturity bonds have a higher risk of default, so they would have to offer a higher yield. See Altman [2].

principal at the end of five years. Bond B pays $80 at the end of each year, plus a $1,000 repayment of principal at the end of ten years.

Interest Rates and Market Values

At the issue date, the market interest rate (i.e., the capitalization rate for this bond) is 8% per annum. The market value of the five year bond is

$$(\$80 \equiv 1.08) + (\$80 \equiv 1.08^2) + \cdots + (\$80 \equiv 1.08^4) + (\$1{,}080 \equiv 1.08^5) = \$1{,}000.$$

Similarly, the market value of the ten year bond is

$$(\$80 \equiv 1.08) + (\$80 \equiv 1.08^2) + \cdots + (\$80 \equiv 1.08^9) + (\$1{,}080 \equiv 1.08^{10}) = \$1{,}000.$$

When interest rates rise, the market value of a bond declines, since future cash payments are worth less. The amount of the decline depends on the payment pattern. The further in the future the average payment is, the greater the decline in the present value.

For instance, if interest rates rise to 10% per annum immediately after the issue date, the market value of the five year bond declines to

$$(\$80 \equiv 1.10) + (\$80 \equiv 1.10^2) + \cdots + (\$80 \equiv 1.10^4) + (\$1{,}080 \equiv 1.10^5) = \$924.18,$$

and the market value of the ten year bond declines to

$$(\$80 \equiv 1.10) + (\$80 \equiv 1.10^2) + \cdots + (\$80 \equiv 1.10^9) + (\$1{,}080 \equiv 1.10^{10}) = \$877.11.$$

Conversely, if interest rates drop to 6% per annum immediately after the issue date, the market value of the five year bond rises to

$$(\$80 \equiv 1.06) + (\$80 \equiv 1.06^2) + \cdots + (\$80 \equiv 1.06^4)$$
$$+ (\$1{,}080 \equiv 1.06^5) = \$1{,}084.25,$$

and the market value of the ten year bond increases to

$$(\$80 \equiv 1.06) + (\$80 \equiv 1.06^2) + \cdots + (\$80 \equiv 1.06^9)$$
$$+ (\$1{,}080 \equiv 1.06^{10}) = \$1{,}147.20.$$

Zero-Coupon Bonds

Similar results hold for any characteristics that affect the payment pattern of the security. For instance, bonds with high coupon rates have a higher percentage of their cash flows during the term of the bond than do zero-coupon bonds, where the only cash inflow is the repayment of the principal, accumulated for interest, at the maturity date.[59]

For example, in an 8% per annum interest rate environment, a ten year $1,000 par value 8% annual coupon bond sells for $1,000. A ten year zero-coupon bond with a maturity value of $2,159 also sells for $1,000, since $\$2{,}159 \equiv 1.08^{10} = \$1{,}000$. But the zero-coupon bond is more strongly affected by interest rate shifts than is the annual-coupon bond. If the market interest rate rises to 10% immediately after issue, the market value of the zero-coupon bond drops to $832.36, as compared to $877.11 for the annual-coupon bond. If the market interest rate declines to 6% immediately after issue, the market value of the zero-coupon bond increases to $1,205.53, as compared to $1,147.20 for the annual-coupon bond.

Table A.1 summarizes the discussion above, showing the market value for these three bonds at three different market interest rates.

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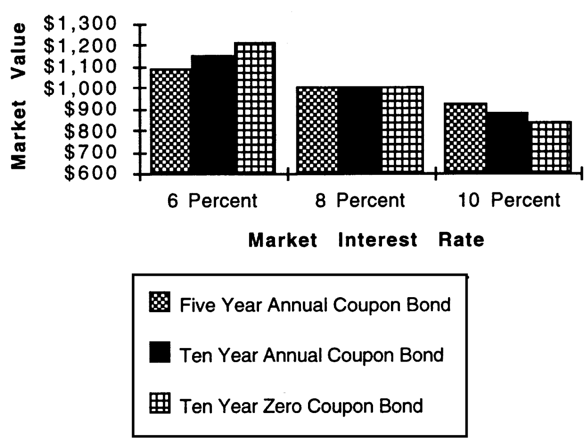
[59]For a full discussion of the factors affecting a bond's duration (coupon size, term to maturity, yield to maturity, sinking fund provisions, and call provisions), see Gray [22].

TABLE A.1

Market interest rate:	6%	8%	10%
Five year coupon bond	$1,084.25	$1,000.00	$924.18
Ten year coupon bond	$1,147.20	$1,000.00	$877.11
Ten year zero-coupon bond	$1,205.53	$1,000.00	$832.36

FIGURE 2

EFFECTS OF INTEREST RATE SHIFTS ON BOND MARKET VALUES



Interest Rate Shifts and Market Values

Figure 2 shows these effects graphically. Interest rate shifts have stronger effects on the market values of securities whose cash flows are further in the future, such as zero-coupon bonds versus coupon bonds, or ten year bonds versus five year notes.[60]

[60]The illustrations in this paper assume a flat yield curve. With a sloping yield curve, the results are slightly, but not significantly, different.

The effect of interest rate shifts on the market value of a security is expressed by the duration of the security.[61]

- The *Macauley duration* of a fixed income security is the weighted average of the cash flow dates, where the weights are the present values of the cash flows. The *adjusted* Macauley duration for an annual-coupon bond is the Macauley duration divided by one plus the interest rate. For instance, if the Macauley duration of an annual-coupon bond is 5 years in an 8% interest rate environment, the adjusted Macauley duration is $5 \equiv 1.08 = 4.63$ years.
- The effect of interest rate shifts on the market price of the security is directly proportional to the adjusted Macauley duration of the security. For instance, if the adjusted Macauley duration of a bond is 4.63 years and the market interest rate increases from 8% to 8.25%, then the market value of the bond decreases by approximately $4.63 \times 0.25\% = 1.16\%$.[62]

We illustrate these relationships with the ten year $1,000 par value 8% annual-coupon bond discussed above. Table A.2 shows:

- the year in which the cash flow occurs;
- the size of the cash flow;
- the present value factor at an interest rate of 8% per annum;

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[61] The concept of duration is applicable to both assets and liabilities. As noted earlier, the duration of an asset reflects the sensitivity of the market value of the asset to marginal changes in interest rates. So too, the duration of a liability reflects the change in the present value of the liability in response to a marginal change in current interest rates.

[62] The adjusted Macauley duration for annual-coupon securities is the Macauley duration divided by one plus the interest rate. For small changes in market interest rates, the change in market value of the security is proportional to the adjusted Macauley duration times the change in the interest rate, or

$$\text{Change in market value} = (-1) \times (\text{Change in interest rate}) \times (\text{Adjusted duration}).$$

This approximation is exact for infinitesimally small changes in interest rates. As the interest rate changes, the duration of the security changes as well, so this formula becomes less exact.

TABLE A.2

Year (1)	Cash Flow (2)	Present Value Factor (3)	Weights (4) = [(2)∗(3)] ≡Sum(2∗3)	Product (5) = (1)∗(4)
1	$80	92.59%	7.41%	0.0741
2	$80	85.73%	6.86	0.1372
3	$80	79.38%	6.35	0.1905
4	$80	73.50%	5.88	0.2352
5	$80	68.06%	5.45	0.2722
6	$80	63.02%	5.04	0.3025
7	$80	58.35%	4.67	0.3268
8	$80	54.03%	4.32	0.3458
9	$80	50.02%	4.00	0.3602
10	$1,080	46.32%	50.025	5.0025
Total			100.00%	7.2469

- the weights used in the calculation, or the present values of the cash flows; and
- the products of the weights and the years in which the cash flow occurs.

The Macauley duration for this bond is 724.69% ≡ 100% = 7.25 years. In an 8% interest rate environment, the adjusted Macauley duration is 7.25 ≡ 1.08 = 6.71 years.

The approximate change in the market value of this bond is −6.71 times the change in interest rates. A 0.5% drop in interest rates should increase the market value by about 3.355% [= 0.5%∗6.71].[63]

Characteristics of Duration

Three characteristics of duration are relevant to our discussion.

[63] At a 7.5% discount rate, the market value of the bond is $1,034.32, for an increase of 3.432% over its value at an 8% discount rate. This illustrates the earlier comment that the Macauley analysis is *exact* only for infinitesimally small changes, though it is a reasonable approximation for larger changes as well; see Ferguson [18].

1. Since the weights used in the calculation of the duration of a security depend on the present values of the cash flows, not on their nominal values, the duration depends on both the cash payment pattern and the market interest rate. As the market interest rate changes, the duration of the security changes.

2. The statement that "the effect of interest rate shifts on the market price of the security is directly proportional to the duration of the security" is accurate for infinitesimal interest rate shifts. As market interest rates change, the duration of the security changes, so the effect on market value changes. If a decrease in market interest rates increases the duration, then the effects on market value of a decrease in market interest rates are magnified. Conversely, if an increase in market interest rates decreases the duration, then the effects on market value of an increase in market interest rates are mitigated.

3. Durations may be determined explicitly for fixed income securities by the definition given above. *Effective durations*, determined from empirical relationships, may be ascribed to other types of securities, such as common stocks and real estate, and to property/casualty liabilities, such as personal auto loss reserves.

For instance, suppose a bond with a duration of three years would have a three percent decline in market value for a one hundred basis point increase in the market interest rate. This relationship is determined mathematically, by computing present values of nominal cash payments at different interest rates.

Personal auto loss reserves are at least partially inflation sensitive. Medical payments in tort liability states, for instance, depend in part upon jury awards at the date of settlement. The jury awards, in turn, are influenced by the rate of inflation, which is correlated (at least in the long run) with interest rates. In contrast, wage loss payments under no-fault compensation systems may be fixed at the time of accident, unless cost of living adjustments are built into the benefit schedule.

A mathematical determination of the loss reserve duration is complex. However, if empirical studies show that the discounted value of personal auto loss reserves declines by three percent for each 100 basis point increase in interest rates, then we may say that the personal auto loss reserves have an effective duration of three years.

APPENDIX B

RESERVE PAYOUT PATTERNS

The data needed to determine reserve payout patterns are taken from Annual Statement Schedule P, Part 3. Exhibit 2 shows the needed calculations, using industry data from the 1994 edition of Best's *Aggregates and Averages*.

The top-most triangle in Exhibit 2 shows the cumulative loss plus allocated loss adjustment expense payments by accident year and by development period. For instance, the top row in the triangle says that for accident year 1984 losses, $7.1 billion was paid in the first year (January 1, 1984, through December 31, 1984), $13.7 billion was paid in the first two years (1/1/84 through 12/31/85), and so forth.

The middle triangle in Exhibit 2 shows the age-to-age factors, or link ratios, for this block of reserves. Each age-to-age factor is the ratio, by accident year, of cumulative payments at one statement date to the cumulative payments at the previous statement state. For instance, the 1.932 factor for accident year 1984 in the "12 to 24" column means that the cumulative payments at 24 months for accident year 1984 ($13.7 billion) are 93.2% higher than the cumulative payments at 12 months for this accident year ($7.1 billion).

Three further rows appear at the bottom of the exhibit.

1. The average age-to-age (ATA) factor is the average of the individual accident year factors in the column above it. For instance, the 1.967 average age-to-age factor in the "12 to 24 months" column is the average of the column of factors beginning with 1.932 and ending with 1.926.
2. The age-to-ultimate factors are the backward product of the age-to-age factors, as illustrated in the following paragraph. The age-to-ultimate factors times the cumulative payments to date gives the expected ultimate losses.

In this illustration, no payments are expected past 10 years from the date of loss occurrence, so the final age-to-ultimate factor is unity. The age-to-ultimate factor in the penultimate column, 1.002, is the product of the average age-to-age factor in the same column (1.002) and the final age-to-ultimate factor (1.000). The preceding age-to-ultimate factor, 1.006, is the product of the two last age-to-age factors (1.003 and 1.002) and the final age-to-ultimate factor of 1.000. This procedure is used to derive all the age-to-ultimate factors.[64]

3. The final row in the middle section of Exhibit 2 shows the loss payment pattern. The 0.336 figure in the first column means that 33.6% of losses are paid in the first 12 months; the 0.325 figure in the next column means that 32.5% of losses are paid in the second 12 months; and so forth.

   These figures are derived from the age to ultimate factors directly above them. For instance, the 2.978 factor for "12 months to ultimate" means that for each dollar of loss paid in the first 12 months, 1.978 dollars will be paid in subsequent periods, for a total of 2.978 dollars. The percentage of losses paid in the first 12 months is therefore $1 \equiv 2.978 = 0.336$, or 33.6%.

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[64] For lines of business with payment patterns extending past ten years, such as workers compensation, general liability, or excess-of-loss reinsurance, a tail factor is needed. One procedure is to extend the loss triangles as far as possible from historical data and then to fit an inverse power curve to the observed age-to-age link ratio to project the tail development; see Sherman [35]. Hodes, Feldblum, and Blumsohn [25] apply this method to a large countrywide block of workers compensation business, using a 25-year historical triangle and then using an inverse power curve fit to extend the paid loss development up to as much as 70 years. (The exact length of the tail varies stochastically in that paper; see particularly Appendices C and D of that paper for the simulation technique.)

The book value of workers compensation reserves in the risk-based capital system uses the IRS loss reserve discounting procedure, which allows a pattern no longer than 16 years. Actual workers compensation payment patterns extend about 50 years. Thus, the difference between the risk-based capital book value and the actual market value of workers compensation reserves depends not only on the discount rate used but also on the assumed payment pattern.

The 1.514 age to ultimate factor in the second column means that for each dollar paid in the first 24 months, 0.514 dollars will be paid in subsequent periods, for a total of 1.514 dollars. The percentage of losses paid in the first 24 months is therefore $1 \equiv 1.514 = 0.661$, or 66.1%. Since 33.6% of losses are paid in the first 12 months, 32.5% of losses are paid in the next 12 months. This procedure is used to derive all the figures in the final row of the middle section of Exhibit 2.

THE COMPETITIVE MARKET EQUILIBRIUM RISK LOAD FORMULA FOR CATASTROPHE RATEMAKING

GLENN G. MEYERS

Abstract

The catastrophic losses caused by Hurricane Andrew and the Northridge Earthquake are leading many actuaries to reconsider their pricing formulas for insurance with a catastrophe exposure. Many of these formulas incorporate the results of computer simulation models for catastrophes. In a related development, many insurers are using a geographic information system to monitor their concentration of business in areas prone to catastrophic losses. While insurers would like to diversify their exposure, the insurance-buying public is not geographically diversified. As a result, insurers must take on greater risk if they are to meet the demand for insurance. This paper develops a risk load formula that uses a computer simulation model for catastrophes and considers geographic concentration as the main source of risk.

1. INTRODUCTION

Hurricane Andrew and the Northridge earthquake caused unprecedented catastrophic losses to the U.S. insurance industry and its reinsurers. These events revealed significant weaknesses in insurance practices in the United States. This paper will discuss a way to correct some of these weaknesses. It will focus on risk management practices from the point of view of the insurance company and suggest where these practices may lead.

Hurricane Andrew and the Northridge earthquake revealed that some insurers have been doing a poor job of diversifying their exposure to catastrophic losses. In response to this, a number of firms with sophisticated geographic mapping software have entered the market and are being kept very busy by insurers seeking to diversify their exposures.

However, the insurance-buying population itself is not geographically diversified. Therefore, insureds who live in densely populated areas will find it harder to obtain insurance, and hence the price of insurance will be higher for densely populated areas than for lightly populated areas. Since an insurer assumes a higher risk in writing geographically concentrated business, the portion of the price that varies by population density could well be called a "risk" load. This paper will propose a formula for calculating such a risk load. This formula will be called the Competitive Market Equilibrium (CME) risk load formula.

As we develop this risk load formula, it will become clear that an insurer who follows the strategy of geographically diversifying its exposure will have lower capital needs. However, the administrative expense involved in such diversification may discourage all but the very large insurers. Reinsurance can provide an economical alternative to direct diversification for smaller insurers. This paper will analyze the effect of various reinsurance strategies. Also, this paper will illustrate the use of some alternatives to reinsurance.

The insurance problems discussed here are certainly old ones, but this paper will cast new light on these problems through the use of geographic mapping technology and the resulting risk load formula.[1]

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[1] Schnieper [13] addresses many of the same problems as this paper. However, Schnieper assumes that the losses of individual insureds are uncorrelated. Many of the results of this paper reduce to Schnieper's results for uncorrelated losses.

2. GEOGRAPHIC INFORMATION SYSTEMS AND INSURANCE RATEMAKING

Catastrophic events happen so infrequently that the traditional actuarial methodology of extending past experience into the future is largely irrelevant. For example, no hurricane has made a direct hit on Miami in recorded insurance history. The same is true for Orlando. However, since Miami is on the coast and Orlando is well inland, no reasonable insurer would charge the same windstorm rates for the two cities. Moreover, data from past hurricanes is of questionable relevance since building practices have changed and the population density in coastal regions has increased in recent years. One can imagine making rates based on insured losses from the 1811–1812 New Madrid Earthquake, or the 1906 San Francisco Earthquake.

Recently, a number of firms have attempted to combine meteorological information, geological information, engineering expertise and insurance loss information to make insurance rates. The results usually take the form of computer simulated events. Exhibits 1 and 2 show the kind of information that typically goes into such an effort.

A geographic information system is a comprehensive database of geographical information. Typically, a geographic information system operates by taking an address and estimating its latitude and longitude. With the latitude and longitude, the system can link the address to other information such as distance to the ocean or distance from known seismic fault lines.

The computer simulated events can be combined with geographic exposure information provided by the insurer to produce a size of loss distribution for the insurer's book of business. This information can be used to evaluate its riskiness; price potential reinsurance contracts; and, as this paper will demonstrate, calculate a risk load.

3. ASSUMPTIONS ABOUT THE INSURANCE ENVIRONMENT

The CME risk load formula makes the following assumptions about the insurance environment.

1. An insurer's capital is a function of its insurance risk. The CME risk load formula is derived from the assumption that the amount of capital needed to support an insurer is a function of the variance of the insurer's total insurance portfolio. To write an additional insurance contract, the insurer must raise additional capital. However, the amount of capital that must be raised for a particular insurance contract may vary by insurer.
2. Each insurer will choose to write an insurance contract that will maximize the return on its required additional (or marginal) capital.
3. Insurers operate in a competitive market. The price for a particular insurance contract will be the same regardless of who insures it.

The CME risk load is then defined as the cost of the marginal capital needed to write the insurance contract.

The assumption that an insurer's capital is a function of the variance of its total insurance portfolio has precedent in both economic and actuarial theory. In their derivation of the Capital Asset Pricing Model (CAPM), Copeland and Weston [4, p. 187] assume that an investor's "utility is a function of the mean and variance of his end-of-period cash flows." In the CAPM, the role of the investor in selecting securities is very similar to the role of the insurer in selecting insurance contracts.

In a more direct treatment of insurance pricing, Ang and Lai [2] write: "The insurer's optimization problem can be written as one of maximizing its mean variance utility $U(E,V)$ subject to the budget constraint … ". They go on to derive a formula very similar to the CME.

Since the willingness of an insurer to take on risk increases with its capital, the role of the insurer's capital is similar to the role of the investor's, or the insurer's, utility function. For example, Kreps [8] assumes that the insurer's capital is proportional to the standard deviation (i.e., the square root of the variance) of the insurer's total loss distribution.

4. THE INSURER BEHAVIOR ASSUMPTIONS

In the course of doing business, an insurer gets the opportunity to expand its business by adding any one of a number of insurance contracts to its portfolio. For each contract it adds, it must add a given amount of capital. Let R be the risk load associated with a given contract. Since the insurer wants to maximize its marginal rate of return on capital, it will choose the contract for which

$$\frac{R}{\Delta\text{Capital}} \tag{4.1}$$

is a maximum.

Since the required capital is assumed to be a function of the variance of the total portfolio, we can rewrite Equation 4.1 to obtain:

$$\frac{R}{\Delta\text{Variance}} \cdot \frac{\Delta\text{Variance}}{\Delta\text{Capital}} \tag{4.2}$$

is a maximum.

Let the capital as a function of variance be given by $C(\text{Variance})$. If the marginal capital required for the insurance contract is small compared to the total variance, we can write:

$$\frac{\Delta\text{Capital}}{\Delta\text{Variance}} \approx C'(\text{Variance}).$$

Then we can approximate Equation 4.2 by:

$$\frac{R}{\Delta\text{Variance}} \cdot \frac{1}{C'(\text{Variance})} \tag{4.3}$$

is a maximum.

The increase in the variance of an insurer's portfolio brought on by the addition of an insurance contract could depend upon the other contracts in the portfolio. The amount of capital required for a given insurer should also depend on other factors, such as the quality of its assets and the variability of its loss reserves. Thus we allow this marginal variance to vary by insurer. The other uses of capital should not present any difficulties if we allow the function C(Variance) to differ by insurer.

At this point, we derive a general expression for the marginal variance due to an individual insurance contract.

Let: X_i = random losses for the ith group of existing contracts; and
Y = random losses for the additional contract under consideration.

Consider the following covariance matrix.

$$
\begin{matrix}
\mathrm{Cov}[X_1,X_1] & \cdots & \mathrm{Cov}[X_1,X_n] & \mathrm{Cov}[X_1,Y] \\
\vdots & \vdots & \vdots & \vdots \\
\mathrm{Cov}[X_n,X_1] & \cdots & \mathrm{Cov}[X_n,X_n] & \mathrm{Cov}[X_n,Y] \\
 & & & \\
\mathrm{Cov}[Y,X_1] & \cdots & \mathrm{Cov}[Y,X_n] & \mathrm{Cov}[Y,Y]
\end{matrix}
$$

The variance of the sum of random variables is the sum of the covariances in the covariance matrix of the variables. The sum of the covariances in the single framed box represents the total variance before introducing the new contract. The sum of the covariances in the double framed box represents the marginal variance of the new contract. Thus:

$$\Delta\mathrm{Variance} = \mathrm{Var}[Y] + 2 \cdot \sum_{i=1}^{n} \mathrm{Cov}[X_i, Y]. \qquad (4.4)$$

Since covariances are additive, the marginal variance does not depend upon the grouping of the X_is.

Combining Equations 4.3 and 4.4 yields the choice of insurance contracts for which

$$\frac{R}{\mathrm{Var}[Y] + 2\cdot\sum_{i=1}^{n}\mathrm{Cov}[X_i, Y]}\cdot\frac{1}{C'(\mathrm{Variance})} \tag{4.5}$$

is a maximum.

5. THE EFFECT OF GEOGRAPHIC CONCENTRATION

Suppose an insurer wants to start writing property insurance in areas with a catastrophe exposure. In accordance with Equation 4.1, a simple strategy would be to find the area where the marginal rate of return is the highest, and write as much as possible in that area. In this section, we argue that insurers will not do this. Instead, we argue that an insurer can maximize its marginal rate of return by spreading its writings geographically.

To illustrate, suppose one area has prospective insureds subject to a random loss, $U_1, U_2, \ldots$. Suppose further that another area has prospective insureds subject to a random loss, $V_1, V_2, \ldots$. We assume that all the Us are independent of the Vs, and that the Us and Vs are both independent of the losses arising from any other contracts the insurer is writing. Let the risk loads for writing a contract in the two areas be R_U and R_V respectively.

According to Equation 4.5, an insurer with no contracts in either area will decide to write its first contract by comparing[2]

$$\frac{R_U}{\mathrm{Var}[U_1]} \qquad \text{and} \qquad \frac{R_V}{\mathrm{Var}[V_1]}.$$

[2]We need not consider the term $1/C'(\mathrm{Variance})$ since it will be the same for each comparison.

Suppose that writing the Us gives the greatest return on marginal capital, and so the insurer writes the first U. Now let's suppose the insurer proceeds to write n Us. To decide what to write for its $n+1$st contract, the insurer compares

$$\frac{R_U}{\mathrm{Var}[U_{n+1}] + 2 \cdot \sum_{i=1}^{n} \mathrm{Cov}[U_i, U_{n+1}]} \qquad \text{and} \qquad \frac{R_V}{\mathrm{Var}[V_1]}.$$

Since all the Us are in the same area, we should expect them to have similar experience when a catastrophe hits. Thus $\mathrm{Cov}[U_i, U_j]$ will be positive for any i and j. As a result, the marginal rate of return will decrease as the insurer writes more Us. Thus, for some n, the marginal rate of return will be greater for writing a V.

We can extend this argument to many areas and lines of business, with the consequence that the insurer will seek to write the insurance contract that gives the greatest marginal rate of return. The process continues until:

$$\frac{R}{\Delta \mathrm{Capital}} = \frac{R}{\Delta \mathrm{Variance}} \cdot \frac{1}{C'(\mathrm{Variance})} = K \tag{5.1}$$

for all prospective insurance contracts.

K is the rate of return on the marginal capital to write the latest insurance contract. One should expect K to vary by insurer. If the insurer is new to the business, K could initially be very high. But a high K will attract more capital, enabling the insurer to expand its writings. As the insurer expands, it will eventually increase its concentration in all the areas in which it writes. As described above, the insurer's return on marginal capital will eventually decrease. When the insurer's volume has reached the point where it can no longer attract new capital, it will stop expanding.

Assume that K is the lowest rate at which the insurer can attract capital. It will then compete to write an insurance contract with risk load R and random loss Y if:

$$\frac{R}{\mathrm{Var}[Y] + 2\cdot\sum_{i=1}^{n}\mathrm{Cov}[X_i, Y]} \cdot \frac{1}{C'(\mathrm{Variance})} \geq K. \tag{5.2}$$

In a world of perfect competition, the needs of an individual insurer do not set the risk load, R. Instead, it is set by the insurance market. However, the insurer can control its concentration of business in a given area, and concentration is the relevant variable for the insurer seeking a competitive rate of return on marginal capital.

Back in the real world, insurance regulators have some influence on the insurance market. In addition to their traditional regulation of rates, some insurance regulators are putting restrictions on an insurer's withdrawal of coverage.

Equation 5.2 may provide an adequate description of insurer behavior for a given risk load, but it gives no hint about what an appropriate risk load might be. We now turn to that question.

6. THE COMPETITIVE MARKET ASSUMPTION

As almost everyone knows, any attempt to predict the behavior of the insurance market is dangerous. We make no claim of immunity from these dangers. However, thinking about the problem is better than ignoring it.

Suppose m insurers are competing for a given insurance contract. Let:

Y = random losses for the insurance contract under consideration;

X_{ij} = random losses for the existing contract of insurer j in group i;

R = risk load for the insurance contract, which we assume to be equal for all m insurers; and

$\lambda_j = K_j \cdot C'(\text{Variance}_j)$ for insurer j.

From Equation 5.2 we have

$$\frac{R}{\lambda_j} = \text{Var}[Y] + 2 \cdot \sum_{i=1}^{n} \text{Cov}[X_{ij}, Y].$$

Summing over the m insurers and dividing by m yields

$$R = \overline{\lambda} \cdot \left(\text{Var}[Y] + 2 \cdot \sum_{i=1}^{n} \text{Cov}[\overline{X}_i, Y] \right) \qquad (6.1)$$

where

$$\overline{\lambda} = \frac{1}{\dfrac{1}{m} \cdot \displaystyle\sum_{j=1}^{m} \frac{1}{\lambda_j}} \qquad \text{and} \qquad \overline{X}_i = \frac{1}{m} \cdot \sum_{j=1}^{m} X_{ij}.$$

Equation 6.1 is the competitive market equilibrium risk load formula.[3]

$\overline{\lambda}$ is called the risk load multiplier. As a consequence of Equation 5.2, the risk load multiplier is a function of the marginal rate of return, measured by K_j, and the marginal capital, measured by $C'(\text{Variance}_j)$, of each competitor.[4] The risk load also depends upon how the business written by competitors is related to, or covaries with, the contract under consideration.

7. THE RISK LOAD MULTIPLIER

Equation 6.1 shows that the risk load multiplier, $\overline{\lambda}$, depends upon the competition. Now it might be difficult for an insurer

[3]This formula gets its name from Meyers [10] although, on the surface, the derivation appears quite different. It was Heckman [6] who showed that the original Meyers formulation is equivalent to the return on marginal capital formulation used in this paper.

[4]Kreps [8] presents an alternative way to derive risk loads from marginal capital.

to obtain the λ_j of each of its competitors so, in practice, more informal competitive considerations might well be used. This section provides a formula to aid in the selection of a risk load multiplier.

Let K_j = expected total return of the jth insurer; and
C_j = capital of the jth insurer.

We now make two additional assumptions about the competing insurers:

1. The marginal return on capital is the same for all insurers. That is, $K_j \quad K$.
2. $C_j = C(\text{Variance}_j) = T \cdot \sqrt{\text{Variance}_j}$.

From the definition of λ_j, we obtain

$$\begin{aligned}
\lambda_j &= K \cdot C'(\text{Variance}_j) \\
&= K \cdot \frac{T}{2 \cdot \sqrt{\text{Variance}_j}} \\
&= \frac{K}{C_j} \cdot \frac{T^2}{2}. \qquad (7.1)
\end{aligned}$$

It follows from Equations 6.1 and 7.1 that

$$\begin{aligned}
\overline{\lambda} &= \frac{1}{\frac{1}{m} \cdot \sum_{j=1}^{m} \frac{1}{\lambda_j}} \\
&= \frac{m \cdot K \cdot T^2}{2 \cdot \sum_{j=1}^{m} C_j} \\
&\quad \frac{K \cdot T^2}{2 \cdot \overline{C}}, \qquad (7.2)
\end{aligned}$$

where $\overline{C} = 1/m \cdot \sum_{j=1}^{m} C_j$.

Thus under the additional assumptions of this section, it follows that the risk load multiplier is a function of:

K—the annual rate of return (before taxes);
$\overline{C}$—the average capital of the competitors; and
T—the coefficient of the capitalization function.

K and $\overline{C}$ can be estimated from publicly available data.

One possible way to choose T is so that S times the required capital is equal to Z standard deviations of the total loss distribution. That is:

$$S \cdot C_j = Z \cdot \sqrt{\text{Variance}_j},$$

which yields

$$T = \frac{Z}{S}. \tag{7.3}$$

In the examples below, $K = 20\%$, $\overline{C} = \$500{,}000{,}000$, $Z = 2$, and $S = 20\%$. This yields $\overline{\lambda} = 2 \cdot 10^{-8}$.

Here are some important caveats on the choice of the risk load multiplier.

1. While the capitalization function given in Assumption 2, above, is mathematically convenient, by no means is it universally recognized as the best. Other possible capitalization functions are based on the "probability of ruin" and the "expected policyholder deficit."[5]

2. An insurer must hold capital to write an insurance contract as long as potential liabilities remain. One year is usually sufficient for property insurance contracts, but for longer tailed lines of insurance, insurers must often hold some capital for several years. In this case, some modifications must be made to the formula for calculat-

[5]See, for example, Daykin, Pentikainen and Pesonen [5, p. 157], and the American Academy of Actuaries Property/Casualty Risk Based Capital Task Force [3, p. 123].

ing the risk load multiplier. This paper does not cover these modifications. Suffice it to say that the risk load multiplier should be higher for long tailed lines.

8. CALCULATING THE CATASTROPHE RISK LOAD

As described in Section 2, computer models can generate prospective catastrophe losses. To calculate the CME risk load, the information obtained from such a model should be organized in the following manner. Denote

- h as the natural event causing the catastrophe indexed from 1 to s, and
- i as the insured group indexed from 1 to n. Each group will have a class of business such as homeowners—wood frame houses, and a geographic unit such as ZIP code, associated with each i. (An alternative is to use two indices instead of one.) The class of business should be sufficiently homogeneous and the geographic unit should be small enough so that all properties in the insured group will have similar loss experience for a given event.

For each h and i, let

- p_h = the probability of the event h happening in a given year;
- d_{hi} = the loss per unit of exposure for insured group i, caused by event h; and
- $\overline{e}_i$ = the average number of exposure units in insured group i. This average is to be taken over all insurers competing for the insurance contract under consideration.

Assume that: (1) each event is independent of the other events; and (2) each event can happen at most once in a given year. These assumptions seem reasonable in light of the time needed to repair the property damage caused by a catastrophe, the shortness of the hurricane season, and the physical properties of earthquakes.

Let:

N_h = The random number of occurrences (either 0 or 1)[6] of event h; and

y_h = The damage caused by event h to the property being insured.

Define the random variables

$$Y = \sum_{h=1}^{s} y_h \cdot N_h \qquad \text{and} \qquad \overline{X}_i = \sum_{h=1}^{s} d_{hi} \cdot \overline{e}_i \cdot N_h.$$

Now derive the formula for the catastrophe risk load:

$$E[Y] = \sum_{h=1}^{s} y_h \cdot p_h \tag{8.1}$$

$$\begin{aligned} \mathrm{Var}[Y] &= \sum_{h=1}^{s} y_h^2 \cdot \mathrm{Var}[N_h] \\ &= \sum_{h=1}^{s} y_h^2 \cdot p_h \cdot (1 - p_h) \end{aligned} \tag{8.2}$$

$$\begin{aligned} \mathrm{Cov}[\overline{X}_i, Y] &= \sum_{h=1}^{s} \mathrm{Cov}[\overline{X}_i, Y_h] \\ &= \sum_{h=1}^{s} y_h \cdot d_{hi} \cdot \overline{e}_i \cdot \mathrm{Cov}[N_h, N_h] \\ &= \sum_{h=1}^{s} y_h \cdot d_{hi} \cdot \overline{e}_i \cdot p_h \cdot (1 - p_h). \end{aligned} \tag{8.3}$$

[6]Alternatively, N_h could have a Poisson distribution. But since catastrophic events are rare, the results would hard to distinguish from the chosen binomial model.

Combining Equations 6.1, 8.2 and 8.3 yields

$$R = \overline{\lambda} \cdot \left(\sum_{h=1}^{s} y_h^2 \cdot p_h \cdot (1 - p_h) + 2 \cdot \sum_{i=1}^{n} \sum_{h=1}^{s} y_h \cdot d_{hi} \cdot \overline{e}_i \cdot p_h \cdot (1 - p_h) \right) \tag{8.4}$$

as the formula for the catastrophe risk load.

9. AN ILLUSTRATIVE EXAMPLE

This section gives an example to illustrate some consequences of the risk load formula. Later, we will use this example to formulate hypotheses about the catastrophe exposure and propose ways to manage the catastrophe risk. It will require further work with a validated catastrophe model and real exposures to verify these hypotheses and justify the proposals.

Begin with a description of an imaginary state and the hurricanes that inflict damage on the property of its residents.

The State of Equilibrium is a rectangular state organized into 50 territories. It has an ocean on its east side and is isolated on its remaining three sides. Its property insurance is spread among various insurers that compete for business in every territory. Exhibit 3 provides a schematic map giving the average number of exposure units per insurer ($1,000's of insured value). Exhibit 3 shows that this state has a reasonable array of metropolitan areas, suburbs, and rural areas. The average number of exposure units per insurer is 2,500,000.

The State of Equilibrium is exposed to hurricanes that move in a westward path. Hurricanes occur at a rate of one out of every two years and come in various strengths. The damage caused by the hurricane can span a width of either one or two territories. Each landfall has the same probability of being hit. The losses due to each hurricane decrease as the storm goes inland, with the loss cost decreasing to 70% of the loss cost of the territory bordering on the east. The overall statewide average loss cost is $4 per $1,000 of insurance.

The appendix gives the parameters, p_h and d_{hi}, of the hurricanes.

Using Equation 8.4, risk loads are calculated for a \$100,000 property for each territory. The risk load multiplier, $\overline{\lambda}$, is set equal to 2×10^{-8}. Exhibit 3 shows these risk loads expressed as percentages of the expected losses.

Here are some general comments about these risk loads.

1. Higher risk loads are associated with the more densely populated territories. For example, Territory 25 has a higher risk load than Territory 15, even though the expected loss for a single exposure in each of these two territories is the same.
2. Proximity to a densely populated territory increases the risk load. For example, Territory 20 has the same population density as Territory 15, yet Territory 20 has a higher risk load than Territory 15. This is because some hurricanes hit both Territories 25 and 20, but no hurricanes hit both Territories 25 and 15.
3. Distance from a densely populated territory does not guarantee a lower risk load. For example, Territory 21 has a higher risk load than Territory 11, even though each territory is geographically isolated from a major population center. This is because Territory 21 is behind Territory 25, and these two territories are exposed to the same storm paths.
4. The risk load decreases slightly as a percentage of expected loss as we move inland. Equation 6.1 shows that we can divide the risk load into two parts:

   $$\overline{\lambda} \cdot \mathrm{Var}[Y] \qquad \text{and} \qquad \overline{\lambda} \cdot 2 \cdot \sum_{i=1}^{n} \mathrm{Cov}[\overline{X}_i, Y].$$

   The risk load percentage due to the first part decreases from 0.03% to 0.01% as we move inland. The risk load percentage due to the second part remains the same as we move inland.

The magnitude of the risk loads in this hypothetical example are much larger than the customary "cost of capital" provisions in property (primary) insurance rates. The overall average risk load for this example is 172% of the expected loss. The remainder of this section discusses what one should expect as the overall average magnitude of the risk load.

Probably the most debatable part of the formula comes with the selection of the risk load multiplier. The risk load multiplier used depends on admittedly arbitrary risk based capital requirements presented in Equation 7.3. But, as Section 7 shows, the risk load multiplier also depends upon the properties of the competitors, the return on marginal capital, and the amount of time the insurer must hold capital to fulfill the obligations of the insurance contract. Unless the set of competitors differs noticeably by line of insurance, the risk load multiplier should not depend upon the line.

We argue that a catastrophe exposure can have a much larger overall risk load than a normal exposure. To see this, compare the variance added by a well-diversified insurer in the above example with the variance added by a fire insurer.

For the insurer with exposures equal to those given in Exhibit 3, expected losses are \$10,000,000 and variance of the loss is 4.28×10^{14}. Consider a claim severity distribution with an expected loss of \$8,000 and a standard deviation of \$24,000. This claim severity distribution is typical of that for fire insurance.[7] If the insurer expects 1,250 claims, the expected loss will be \$10,000,000. For simplicity, assume both the hurricane losses and the fire losses are independent of the other losses the insurer

[7]This distribution is from the "Total" Column of Exhibit 5 in Ludwig [9] and scaled to a homeowners policy with \$100,000 of insurance. The mean and standard deviation of the distribution are rounded to the nearest \$1,000.

anticipates. Then the relative risk load between the hurricane and fire exposures equals the quotient of their respective variances (see Table 1).

TABLE 1

Parameter Risk	Fire Insurance Variance[8]	Relative Cat/Fire Risk Load
None	8.00×10^{11}	535
Low	2.80×10^{12}	153
Moderate	4.80×10^{12}	89

If these examples are anywhere near realistic, one must conclude that either fire risk loads should be near zero, or that catastrophe risk loads are very large. In practice, the catastrophe risk loads could be significantly smaller—or larger—than the risk loads in this example.

10. ALLOCATING SURPLUS

An alternative to using a risk load is to allocate surplus to an individual contract, and include the cost of this allocated capital in place of the risk load. This practice is controversial because it implies a monoline auto insurer with a surplus of $X is equivalent to a multiline insurer with a surplus of $X allocated to auto insurance. Many, including this author, believe this is not a valid comparison. However, a proper use of allocated surplus does provide a way to pass down the insurer's goal for its overall rate of return to individual product managers. Given its increasing popularity, it should be addressed.

The purpose of this section is (1) to demonstrate that for any risk load formula, there is an equivalent surplus allocation for-

[8]These variances are calculated with Equation 4.4 in Meyers [10], using $b = 0$ and $c =$ 0.00, 0.02, and 0.04 respectively.

mula; and (2) to derive the surplus allocation formula that is equivalent to the CME risk load formula.

Let R_i be the risk load for the ith insurance contract. Let R be the total risk load charged by the insurer, that is:

$$R = \sum_{i=1}^{n} R_i.$$

An insurer with capital C can then "allocate" C_i of its capital to the ith contract in proportion to the risk load R_i, that is:

$$C_i \quad C \cdot \frac{R_i}{R}. \tag{10.1}$$

Conversely, an insurer that derives its allocated capital C_i and its overall risk load R from a different source can then calculate the "risk load" for the ith contract by setting:

$$R_i \quad R \cdot \frac{C_i}{C}. \tag{10.2}$$

By design, these formulas make the return on allocated capital equal to the return on total capital.

This equivalence of risk load formulas with surplus allocation formulas is not a deep thought. It is merely a tautology designed to bring together two schools of thought on pricing for risk.

Now according to Equation 5.1:

$$R_i = K \cdot C'(V) \cdot \Delta V_i \tag{10.3}$$

where V is the variance of the insurer's loss portfolio and ΔV_i is the marginal variance due to the ith insurance contract. Since $K \cdot C'(V)$ is constant across all insurance contracts, allocating surplus in proportion to the risk loads is equivalent to allocating surplus in proportion to the marginal variances.

The actual allocation formulas can now be derived.

According to Equation 4.4:

$$\Delta V_i = \mathrm{Var}[X_i] + 2 \cdot \sum_{\substack{j=1 \\ j \neq i}}^{n} \mathrm{Cov}[X_j, X_i].$$

It can be demonstrated that:

$$\sum_{i=1}^{n} \Delta V_i = V + 2 \cdot \sum_{i=1}^{n} \sum_{j=1}^{i-1} \mathrm{Cov}[X_i, X_j], \tag{10.4}$$

and therefore:

$$C_i = C \cdot \left(\frac{\mathrm{Var}[X_i] + 2 \cdot \sum_{\substack{j=1 \\ j \neq i}}^{n} \mathrm{Cov}[X_j, X_i]}{V + 2 \cdot \sum_{i=1}^{n} \sum_{j=1}^{i-1} \mathrm{Cov}[X_i, X_j]} \right). \tag{10.5}$$

11. MANAGING THE CATASTROPHE RISK

To compete effectively in the insurance market, an insurer must provide its product for the lowest cost. This cost includes the cost of capital, which is provided by the risk load. Reinsurance can reduce the need for capital, and an insurer who effectively uses reinsurance can provide catastrophe insurance for a lower cost. However reinsurance has its own costs. This section examines how insurers and reinsurers may work together to provide coverage for the least cost.

Case 1—"Local" Reinsurance

By "local" reinsurance, we mean that the primary insurers and the reinsurers are operating in the same market. Since all reinsurers are competing for the same insurance contract, we assume that each of them uses the same risk load multiplier.

Let

$$Y = Y_1 + \cdots + Y_g$$

where Y_k is the amount paid by the kth reinsurance contract.

As a matter of convenience, we will only consider contracts for which $\mathrm{Cov}[Y_k, Y_j] \geq 0$. This is true for quota share and excess of loss reinsurance contracts where an increase in Y_k is never associated with a decrease in Y_j.

We have

$$\begin{aligned} \mathrm{Var}[Y] &= \sum_{k=1}^{g} \mathrm{Var}[Y_k] + 2 \cdot \sum_{k=2}^{g} \sum_{j=1}^{k-1} \mathrm{Cov}[Y_k, Y_j] \\ &\geq \sum_{k=1}^{g} \mathrm{Var}[Y_k]. \end{aligned} \tag{11.1}$$

Thus the variance part of the risk load,

$$\overline{\lambda} \cdot \mathrm{Var}[Y], \tag{11.2}$$

is reduced when the loss Y is distributed among the g insurers.

We also have

$$\mathrm{Cov}[\overline{X}_i, Y] = \sum_{k=1}^{g} \mathrm{Cov}[\overline{X}_i, Y_k] \tag{11.3}$$

for all i.

Thus, the covariance part of the risk load,

$$2 \cdot \overline{\lambda} \cdot \sum_{i=1}^{n} \mathrm{Cov}[\overline{X}_i, Y] \tag{11.4}$$

is *not* reduced when the loss Y is distributed among the g insurers.

Now examine how much the risk load can be reduced by sharing the loss among g insurers. Suppose an insured faces a random loss Y. If the loss Y is split equally among g insurers instead of kept exclusively with a single insurer, the total risk load is reduced by

$$\overline{\lambda}\cdot\left(\mathrm{Var}[Y]-g\cdot\mathrm{Var}\left[\frac{Y}{g}\right]\right)=\overline{\lambda}\cdot\mathrm{Var}[Y]\cdot\left(1-\frac{1}{g}\right). \tag{11.5}$$

Equation 11.5 represents the theoretical maximum that the variance part of the risk load can be reduced by sharing the loss among g insurers. Consider the case of $g = 2$. We have

$$\begin{aligned}\mathrm{Var}[Y] &= \mathrm{Var}[Y_1]+2\cdot\mathrm{Cov}[Y_1,Y_2]+\mathrm{Var}[Y_2]\\ &= \mathrm{Var}[Y_1]+2\cdot\rho\cdot\sqrt{\mathrm{Var}[Y_1]\cdot\mathrm{Var}[Y_2]}+\mathrm{Var}[Y_2],\end{aligned} \tag{11.6}$$

where ρ is the coefficient of correlation between Y_1 and Y_2. Let $g = \sqrt{\mathrm{Var}[Y_1]/\mathrm{Var}[Y]}$, $Y_1' = g\cdot Y$, and $Y_2' = (1-g)\cdot Y$. We have $\mathrm{Var}[Y_1'] = \mathrm{Var}[Y_1]$, and the coefficient of correlation between Y_1' and Y_2' is 1. Since Equation 11.6 must hold for Y_1' and Y_2', we must have that $\mathrm{Var}[Y_2'] \leq \mathrm{Var}[Y_2]$. Thus we can replace any shared contract by a proportional contract with a total risk load at least as small.

Thus, the maximum reduction of risk load will occur with a proportional sharing contract of the form $Y_1 = g\cdot Y$ and $Y_2 = (1-g)\cdot Y$. In this case the reduction is

$$2\cdot p\cdot(1-p)\cdot\mathrm{Var}[Y]. \tag{11.7}$$

This expression is maximized when $g = 1/2$. Thus the maximum reduction in the risk load is:

$$\frac{\mathrm{Var}[Y]}{2}. \tag{11.8}$$

If $g > 2$, any two insurers with different liabilities can get together and reduce their joint share by each taking 1/2 of their joint liability. If each insurer takes $1/g$ of the total liability, no

TABLE 2

REINSURANCE PRICES FOR SAMPLE BOOKS OF BUSINESS IN THE STATE OF EQUILIBRIUM

Book	Exposure Distribution	Expected Loss (000)	Total Risk Load (000)	Percentage Risk Load	Variance Risk Load	Covariance Risk Load
1	Industry	2,500	4,696	187.8%	16.5%	171.3%
2	Territory 25	2,500	8,741	349.6	93.4	256.3
3	Uniform	2,500	3,717	148.7	11.9	136.8
4	Industry	5,000	10,219	204.4	33.1	171.3
5	Industry	1,250	2,245	179.6	8.3	171.3

reduction in the total risk load can occur. Thus Equation 11.5 gives the theoretical maximum reduction in the risk load by g insurers.[9]

In theory, the variance part of the risk load can be eliminated entirely by increasing g indefinitely. In practice, g will not be increased indefinitely because of the transaction costs involved in reinsuring. If the transaction costs of adding a reinsurer exceed the corresponding reduction in the risk load, it will not be economical to add that reinsurer to the contract. The expense of reducing the risk load will exceed the cost of capital needed to bear the risk.

We now continue the illustrative example started in Section 9. Suppose an insurer wants to reinsure all its property insurance in the State of Equilibrium. Table 2 gives the expected losses and the risk loads for various books of business when a single reinsurer takes all the business.

The first book of business consists of 6,250,000 units of exposure, distributed among the territories in proportion to the entire

[9]The variance part of the risk load is the same as the variance principle for calculating premiums. The analogous result for the variance principle is well known. See Daykin, Pentikainen and Pesonen [5, Chapter 6] for a standard reference on this subject.

industry. The total risk load for reinsuring the entire book of business equals 187.8% of the expected loss. The variance part of the risk load equals 16.5% of the expected loss. The second book consists of 3,549,523 units of exposure concentrated in Territory 25. The third book consists of 6,398,443 units of exposure, uniformly spread over the 50 territories. We chose these exposure levels so that the expected loss is the same for the first three cases.

Books 4 and 5 illustrate the effect of changing the overall exposure level while maintaining the same relative concentration as Book 1. The covariance risk load is a constant percentage of the expected loss. However, the variance risk load, expressed as a percentage of the expected loss, increases directly with the overall exposure level.[10] Thus, an insurer may expand more efficiently by moving into other geographic regions or to other lines of business. Such a decision will depend upon the other costs of doing business.

The single (or direct) reinsurer arrangement described in Table 2 may not be the most efficient one available. In fact, most catastrophe reinsurance is done through the brokerage market. To continue our example, assume that the reinsurance broker charges an additional commission (above that of the direct reinsurer) equal to 10% of the expected loss. Assume also that each reinsurer involved in the contract incurs an additional expense equal to 0.5% of the expected loss. Then the minimum risk load plus transaction cost occurs when

$$\text{Broker's Commission } \% + \frac{\text{Variance Risk Load } \%}{g} + 0.5\% \cdot g \tag{11.9}$$

is a minimum.

[10]Part of this effect may be an artifact of this example. Here we assume that each hurricane inflicts damages on all properties in a territory in a constant, non-random manner. A more detailed model might include some random effects of hurricanes on the property in a given territory.

TABLE 3

SAMPLE REINSURANCE ARRANGEMENT FOR BOOK 1—
INDUSTRY EXPOSURE DISTRIBUTION PRIMARY INSURER
RETAINS 10% OF ALL LOSSES

Layer	Expected Loss	Total Risk Load	Percentage Risk Load	Variance Risk Load	Covariance Risk Load
0 } 2,000,000	755,870	706,169	93.4%	1.8%	91.7%
2,000,000 } 6,000,000	723,195	1,154,388	159.6	4.8	154.8
6,000,000 } 12,000,000	489,581	1,181,366	241.3	8.4	232.9
12,000,000 } 20,000,000	247,524	797,542	322.2	11.1	311.1
20,000,000 } 30,000,000	33,830	133,824	395.6	7.7	387.9
Total	2,250,000	3,973,288	176.6%	5.3%	171.3%

For the least concentrated example, Book 2 of Table 2, the minimum variance risk load plus brokerage expense is $10 + 11.9/5 + 0.5 \cdot 5 = 14.9\%$. This does not compare favorably with the 11.9% original reducible risk load and so the contract will stay with the direct reinsurer.

In Book 1, the insurer follows the industry concentration. The minimum reducible risk load plus brokerage expense is $10 + 16.5/6 + 0.5 \cdot 6 = 15.8\%$. This is slightly lower than the 16.5% original reducible risk load, and so further investigation is called for. In practice, reinsurers rarely use this optimal contract. (Could it be that reinsurance underwriters don't believe actuarial theory?) Reinsurers usually require the primary insurer to retain a certain proportion of the loss, to assure diligence in adjusting claims. The remaining losses are parceled out in various layers.

Suppose that the broker comes up with the agreement described in Table 3. With this agreement, the total reducible

TABLE 4

SAMPLE REINSURANCE ARRANGEMENT FOR BOOK 2—ALL EXPOSURE IN TERRITORY 25 PRIMARY INSURER RETAINS 10% OF ALL LOSSES

Layer	Expected Loss	Total Risk Load	Percentage Risk Load	Variance Risk Load	Covariance Risk Load
0 – 4,000,000	227,184	474,078	208.7%	6.7%	201.9%
4,000,000 – 12,000,000	454,369	978,807	215.4	13.5	201.9
12,000,000 – 24,000,000	546,325	1,391,386	254.7	19.3	235.4
24,000,000 – 40,000,000	552,566	1,655,379	299.6	25.5	274.1
40,000,000 – 60,000,000	390,499	1,393,837	356.9	30.1	326.8
60,000,000 – 84,000,000	79,057	331,920	419.9	24.3	395.6
Total	2,250,000	6,225,408	276.7%	20.4%	256.3%

risk load plus brokerage expense is $10 + 5.3 + 0.5 \cdot 5 = 17.8\%$. This does not compare favorably with the original 16.5% reducible risk load, so the contract will stay with the direct reinsurer.

In Book 2 of Table 2, all the primary insurer's business was in Territory 25. The minimum variance risk load plus brokerage expense is $10 + 93.4/14 + 0.5 \cdot 14 = 23.7\%$. This compares favorably with the 93.4% original reducible risk load, so further investigation is necessary.

Suppose that the broker comes up with the agreement described in Table 4. With this arrangement, the total variance risk load plus brokerage expense is $10 + 20.4 + 0.5 \cdot 6 = 33.4\%$. This compares very favorably with the original 93.4% variance risk load, so the brokered contract is sold. Note that the cost of the

TABLE 5

SAMPLE REINSURANCE ARRANGEMENT FOR BOOK 1—INDUSTRY EXPOSURE DISTRIBUTION PRIMARY INSURER RETAINS 10% OF ALL LOSSES

Layer	Expected Loss	Total Risk Load	Percentage Risk Load	Variance Risk Load	Covariance Risk Load
0 – 2,000,000	755,870	151,866	20.1%	1.8%	18.3%
2,000,000 – 6,000,000	723,195	258,660	35.8	4.8	31.0
6,000,000 – 12,000,000	489,581	269,020	54.9	8.4	46.6
12,000,000 – 20,000,000	247,524	181,511	73.3	11.1	62.2
20,000,000 – 30,000,000	33,830	28,851	85.3	7.7	77.6
Total	2,250,000	889,909	39.6%	5.3%	34.3%

brokered contract differs from that of the optimal contract. The broker may be able to come up with a better contract.

As these examples show, "local" reinsurance helps very little when the insureds are geographically diversified, but it can help when the insureds are geographically concentrated. But does it help enough? We move on to the next case.

Case 2—"Global" Reinsurance

By "global" reinsurance, we mean that the reinsurer's market covers a much larger area than the primary insurer's market. This case is certainly closer to the norm for catastrophe reinsurance.

As Section 6 shows, the risk load depends upon how the business of competitors is related to, or covaries with, the contract

TABLE 6

SAMPLE REINSURANCE ARRANGEMENT FOR BOOK 2—
ALL EXPOSURE IN TERRITORY 25 PRIMARY INSURER
RETAINS 10% OF ALL LOSSES

Layer	Expected Loss	Total Risk Load	Percentage Risk Load	Variance Risk Load	Covariance Risk Load
0 – 4,000,000	227,184	107,076	47.1%	6.7%	40.4%
4,000,000 – 12,000,000	454,369	244,801	53.9	13.5	40.4
12,000,000 – 24,000,000	546,325	362,621	66.4	19.3	47.1
24,000,000 – 40,000,000	552,566	443,874	80.3	25.5	54.8
40,000,000 – 60,000,000	390,499	372,777	95.5	30.1	65.4
60,000,000 – 84,000,000	79,057	81,734	103.4	24.3	79.1
Total	2,250,000	1,612,883	71.7%	20.4%	51.3%

under consideration. Global reinsurers should have a very diversified book of business. A fairly large portion of the business should be independent of the primary insurer's business. We now illustrate this effect with the examples described in Tables 3 and 4, with one change. The average exposure in the State of Equilibrium of the competing reinsurers is lower by a factor of five. The remaining exposures of the competing reinsurers have losses independent of the losses in the State of Equilibrium. Assume no change in the capital requirements or the average size of the competing reinsurers. Thus the risk load multiplier remains the same (see Table 5). Here we see that "global" reinsurance can have a dramatic effect on the overall risk load. By comparing Tables 2 through 4 with Tables 5 through 7, it would appear that an insurer could compete far more effectively with the aid of a "global" reinsurer.

TABLE 7

SAMPLE REINSURANCE ARRANGEMENT FOR BOOK 3—UNIFORM EXPOSURE DISTRIBUTION PRIMARY INSURER RETAINS 10% OF ALL LOSSES

Layer	Expected Loss	Total Risk Load	Percentage Risk Load	Variance Risk Load	Covariance Risk Load
0 – 2,000,000	823,024	153,419	18.6%	1.7%	16.9%
2,000,000 – 6,000,000	776,838	248,637	32.0	4.4	27.6
6,000,000 – 12,000,000	578,988	274,577	47.4	8.2	39.2
12,000,000 – 20,000,000	71,151	37,899	53.3	4.7	48.5
Total	2,250,000	714,532	31.8%	4.4%	27.4%

12. THE COMPOUNDING EFFECT OF BUILDING CODES

So far, we have only discussed the insurance side of risk management. This section discusses the effects of loss mitigation efforts.

Assume the existence of a loss mitigation technology that can reduce the expected loss to each insured by a factor of v. If Y is the loss random variable for the insured, the expected loss after loss mitigation is $v \cdot E[Y]$. Since loss mitigation is intended to reduce losses, $v < 1$.

Under normal conditions,[11] an insurer will reduce its rate by a factor of v when there is convincing evidence that the insured's expected losses are reduced by a factor of v. However, as we shall argue, the positive effects of loss mitigation are compounded when a catastrophe exposure is present.

[11] Here we ignore considerations such as fixed expenses which figure into pricing deductibles.

In the discussion that follows, R will be the risk load that applies before any loss mitigation measures take place.

If only one insured takes the loss mitigation measure, the risk load, R_M, for that insured becomes

$$\begin{aligned} R_M &= \overline{\lambda} \cdot \left(\mathrm{Var}[v \cdot Y] + 2 \cdot \sum_{i=1}^{n} \mathrm{Cov}[\overline{X}_i, v \cdot Y] \right) \\ &= v \cdot \overline{\lambda} \cdot \left(v \cdot \mathrm{Var}[Y] + 2 \cdot \sum_{i=1}^{n} \mathrm{Cov}[\overline{X}_i, Y] \right) \\ &\approx v \cdot R. \end{aligned} \tag{12.1}$$

This last approximation is good for individual properties which are part of a catastrophe exposure. In this case, as discussed in Section 9, the covariance risk load is much larger than the variance risk load.

If all insureds take the loss mitigation measure, the risk load, R_M, for an insured becomes

$$\begin{aligned} R_M &= \overline{\lambda} \cdot \left(\mathrm{Var}[v \cdot Y] + 2 \cdot \sum_{i=1}^{n} \mathrm{Cov}[v \cdot \overline{X}_i, v \cdot Y] \right) \\ &= v^2 \cdot \overline{\lambda} \cdot \left(\mathrm{Var}[Y] + 2 \cdot \sum_{i=1}^{n} \mathrm{Cov}[\overline{X}_i, Y] \right) \\ &= v^2 \cdot R. \end{aligned} \tag{12.2}$$

As argued above, the risk load can be a significant part of the overall property rate. Thus the message contained in Equations 12.1 and 12.2 is that the premium for an individual insured can be significantly reduced if its neighbors also take steps to mitigate losses. All insureds have an interest in community-wide loss mitigation. Effective building codes are one way to express this interest.

13. CONCLUDING REMARKS

This paper has derived the Competitive Market Equilibrium risk load formula from standard competitive market economic assumptions, as they apply to the business of insurance. The paper applies the risk load formula to lines of business with a significant catastrophe exposure. The formula uses output from newly developed catastrophe models. The key idea is as follows:

> The marginal capital needed to support an insurance contract increases with the concentration of exposure.

We define the risk load as the cost of marginal capital needed to support the insurance contract. The Competitive Market Equilibrium (CME) risk load is the risk load that matches the supply and demand for insurance.

Through examples, the possibility that the risk load can be very high relative to the expected loss is raised. Rather than pass this risk load on to the insured, cooperative risk management arrangements can result in significantly lower risk loads.

This paper provides a way to balance price, concentration, and the transaction costs of reinsurance.

Market equilibrium is a rare phenomenon in real economic behavior. Shocks to the system happen too often for an equilibrium to develop. However, the examples in this paper show that the CME risk load formula can provide guidance for pricing and managing the catastrophe risk in an evolving insurance market.

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EXHIBIT 1

MODELING THE EFFECTS OF EARTHQUAKES (SHAKE DAMAGE) ON INSURED LOSSES

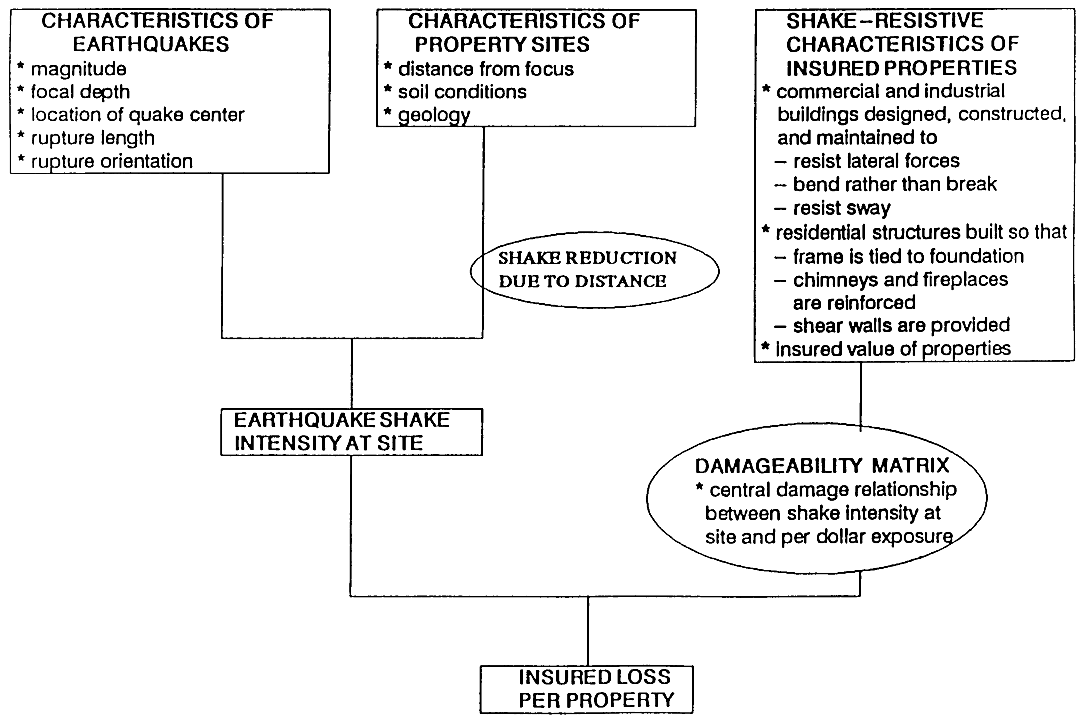

EXHIBIT 2

MODELING THE EFFECTS OF HURRICANE WINDS ON INSURED LOSSES

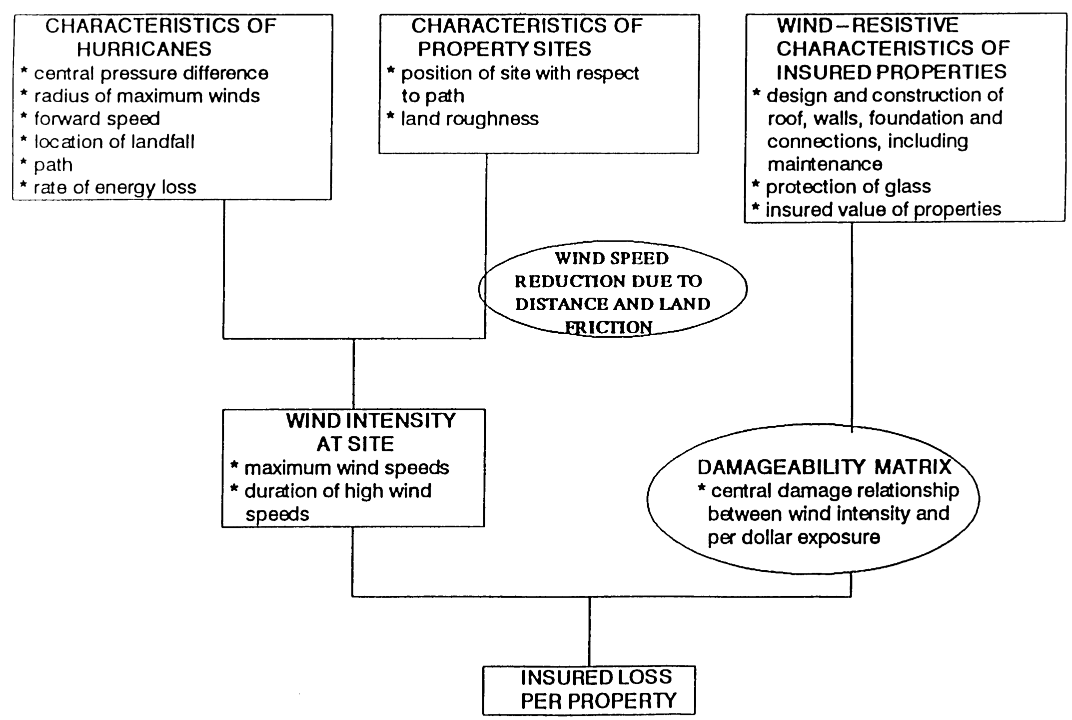

EXHIBIT 3

MAP OF THE STATE OF EQUILIBRIUM

Key

WW XXXXXX YYY ZZZ.ZZ%	WW = Territory	XXXXXX = $\frac{\text{Total Number of Exposure Units}}{\text{Number of Insurers}}$
	YYY = Expected Loss for 100 Exposure Units	ZZZ.ZZ% = $\frac{\text{Risk load}}{\text{Expected Loss}}$ (%)

N
W ← • → E
S

Inland #4	Inland #3	Inland #2	Inland #1	Landfall	Ocean
1 25000 169 85.74%	2 75000 242 85.74%	3 75000 345 85.75%	4 25000 493 85.75%	5 25000 704 85.76%	\/\/\/ \/\/\/
6 25000 169 101.10%	7 75000 242 101.10%	8 75000 345 101.11%	9 25000 493 101.11%	10 25000 704 101.12%	\/\/\/ \/\/\/
11 25000 169 78.15%	12 25000 242 78.16%	13 25000 345 78.16%	14 25000 493 78.17%	15 25000 704 78.17%	\/\/\/ \/\/\/
16 25000 169 144.26%	17 25000 242 144.26%	18 25000 345 144.26%	19 25000 493 144.27%	20 25000 704 144.28%	\/\/\/ \/\/\/
21 25000 169 256.26%	22 25000 242 256.26%	23 25000 345 256.26%	24 225000 493 256.27%	25 225000 704 256.28%	\/\/\/ \/\/\/
26 25000 169 144.26%	27 25000 242 144.26%	28 25000 345 144.26%	29 25000 493 144.27%	30 25000 704 144.28%	\/\/\/ \/\/\/
31 25000 169 100.61%	32 25000 242 100.61%	33 25000 345 100.62%	34 25000 493 100.62%	35 25000 704 100.63%	\/\/\/ \/\/\/
36 125000 169 179.41%	37 25000 242 179.41%	38 125000 345 179.41%	39 125000 493 179.42%	40 25000 704 179.43%	\/\/\/ \/\/\/
41 125000 169 183.21%	42 25000 242 183.21%	43 125000 345 183.21%	44 125000 493 183.22%	45 25000 704 183.23%	\/\/\/ \/\/\/
46 25000 169 94.70%	47 75000 242 94.70%	48 25000 345 94.71%	49 25000 493 94.71%	50 25000 704 94.72%	\/\/\/ \/\/\/

APPENDIX

PARAMETERS FOR SAMPLE HURRICANES

The sample hurricanes used in this paper travel from east to west. As a hurricane moves inland, the damage per exposure unit, d_{hi}, is multiplied by 0.7 as it crosses each territory.

Hurricane Number h	Landfall Territory i	Average Damage Per Exposure Unit d_{hi}	Annual Probability p_h
1	5	41.46	0.01618123
2	5	82.91	0.01294498
3	5	124.37	0.00485437
4	10	41.46	0.01618123
5	10	82.91	0.01294498
6	10	124.37	0.00485437
7	15	41.46	0.01618123
8	15	82.91	0.01294498
9	15	124.37	0.00485437
10	20	41.46	0.01618123
11	20	82.91	0.01294498
12	20	124.37	0.00485437
13	25	41.46	0.01618123
14	25	82.91	0.01294498
15	25	124.37	0.00485437
16	30	41.46	0.01618123
17	30	82.91	0.01294498
18	30	124.37	0.00485437
19	35	41.46	0.01618123
20	35	82.91	0.01294498
21	35	124.37	0.00485437
22	40	41.46	0.01618123
23	40	82.91	0.01294498
24	40	124.37	0.00485437
25	45	41.46	0.01618123
26	45	82.91	0.01294498
27	45	124.37	0.00485437
28	50	41.46	0.01618123
29	50	82.91	0.01294498
30	50	124.37	0.00485437
31	5 10	124.37	0.00485437

PARAMETERS FOR SAMPLE HURRICANES

Continued

Hurricane Number h	Landfall Territory i		Average Damage Per Exposure Unit d_{hi}	Annual Probability p_h
32	5	10	165.82	0.00647249
33	5	10	207.28	0.00323625
34	10	15	124.37	0.00485437
35	10	15	165.82	0.00647249
36	10	15	207.28	0.00323625
37	15	20	124.37	0.00485437
38	15	20	165.82	0.00647249
39	15	20	207.28	0.00323625
40	20	25	124.37	0.00485437
41	20	25	165.82	0.00647249
42	20	25	207.28	0.00323625
43	25	30	124.37	0.00485437
44	25	30	165.82	0.00647249
45	25	30	207.28	0.00323625
46	30	35	124.37	0.00485437
47	30	35	165.82	0.00647249
48	30	35	207.28	0.00323625
49	35	40	124.37	0.00485437
50	35	40	165.82	0.00647249
51	35	40	207.28	0.00323625
52	40	45	124.37	0.00485437
53	40	45	165.82	0.00647249
54	40	45	207.28	0.00323625
55	45	50	124.37	0.00485437
56	45	50	165.82	0.00647249
57	45	50	207.28	0.00323625
58		5	124.37	0.00485437
59		5	165.82	0.00647249
60		5	207.28	0.00323625
61		50	124.37	0.00485437
62		50	165.82	0.00647249
63		50	207.28	0.00323625

DISCUSSION BY JAMES E. GANT

1. INTRODUCTION

Mr. Meyers has laid out a framework within the context of his previous contribution, the Competitive Market Equilibrium risk load formula, that reasonably accounts for the increase in the variance of expected losses due to the effects of geographic concentration of an insurer's portfolio. The key idea that the author expresses may seem apparent to most actuaries, but the author's use of statistical notation to establish his point is impressive. In Mr. Meyers' words,

> "The marginal capital needed to support an insurance contract increases with the concentration of exposure."

The author has also addressed the timely need of how to calculate a risk load for catastrophic lines of insurance covering both earthquake and hurricane losses, including a brief summary of computer simulation models that have been offered as a solution to critical issues in pricing for these catastrophic lines. The use of geographic information systems within the overall framework of the risk load calculation is also proposed.

2. THE MAIN SOURCE OF RISK

Mr. Meyers uses the output of a computer simulation model, and then develops a risk load formula that has as its main source of risk for catastrophe lines the geographic concentration of an insurer's portfolio. The author has made a strong argument for the fact that catastrophic risk loads are large in comparison to normal risk loads, given concentrations of writings lying on a well-frequented storm track for hurricanes. One could make a similar argument regarding a risky portfolio in close proximity to an active earthquake fault historically causing massive damage to lives and property. This does not prove that geographic concentrations are the main source of risk, however, and actuaries should consider other possible sources of risk connected with catastrophe lines.

Specifically, the types of risk that should be considered when pricing catastrophe coverage include, but may not be limited to, the following:

1. The uncertainty of earthquake and hurricane prediction: In this regard, the Northridge earthquake occurred along a previously unknown fault. Were the geographic concentrations in the Northridge area a main source of risk before the event, or only after the event occurred? Or should all geographic concentrations, regardless of whether the prevailing wisdom of where any "safe harbors" are located, be considered at risk? On the subject of the hurricane peril, Clark [2] has addressed the impact global warming may have on both the frequency and severity of future hurricanes. Should insurers include a provision in the rates for an increase in severity in these storms due to global warming?

2. The time-dependence question: Are earthquake or hurricane events independent in time? This question may be raised in different ways. One way is to ask whether an area that has been struck by an earthquake or a hurricane is either less likely or more likely to be struck again in the relatively near future. Another way is to question whether cycles of earthquake or hurricane activity can be expected to occur, and the question may be raised regarding either the frequency or the severity of a series of events that are in some way causally connected.

3. The demand for insurance: Mr. Meyers' Competitive Market Equilibrium risk load formula assumes that the market can work effectively to reach an equilibrium be-

tween the demand and the supply of insurance. The occurrence of events such as Hurricane Andrew or the Northridge earthquake create disequilibrium. The demand for insurance increases as homeowners recognize the need to reduce the risk of a major loss. At the same time, the supply of insurance decreases as insurers recognize the need to reduce the risk presented by geographic concentrations of their portfolios.

4. Social pressures to offer coverage for catastrophes at an affordable price: The ability of insurers to control geographic concentrations through price increases or withdrawing from the market may be limited by political forces. Alternative funding mechanisms may be created that may or may not reduce the ultimate amount of risk borne by insurers.

5. The computer simulation models and sources of error: In calculating risk loads for most lines of insurance, some actuarial methods generally measure process risk and ignore parameter risk, or use approximations to normal or log-normal distributions that reduce parameter risk to an acceptable level.[1] These assumptions are not valid for catastrophe risks such as earthquakes or hurricanes when computer simulation models are used.

Some of the above sources of risk may be interconnected. The interest in computer simulation models and the perceived need to abandon historical methods of pricing for catastrophe risks may be symptoms of disequilibrium in the insurance market. All dimensions of risk need to be considered carefully before using computer simulation models in pricing.

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[1] However, as Heckman [4] points out, "parameter uncertainty is the prime determinant of the risk load in a consistent and market-viable scheme." See also Feldblum [3] and Philbrick [5].

The author's choice is to treat the output of the computer simulation model as a series of events that are time-independent, identically distributed random variables so that the overall frequency is binomial-distributed. These are reasonable assumptions given a scarcity of raw, unprocessed data. However, the computer simulation models do not rely on the scarcity of data, but upon an abundance of scientific, engineering, geophysical, and meteorological knowledge applied to a historical record of sparse data.[2] The models also rely on an insurer's database for coverage-related data. The accuracy of the model's output is directly related to the accuracy and level of detail that an insurer can provide about its portfolio.

3. COMPUTER SIMULATION MODELS AND PARAMETER ERRORS

Actuarial familiarity with computer simulation modeling of catastrophes is needed to assess both the ability of the models to accurately forecast potential losses and the sensitivity of the models to parameter error. Actuaries should insist on treating the output from computer simulation models the same way we treat all data:

- How credible or reliable is the data?
- How variable are the results?
- How sensitive is the output to parameter selection?
- How much confidence is there that the expected annual loss produced as an output by the model equals the true value of the future annual losses?

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[2] See Risk Management Solutions, Inc. [6]. RMS treats the output of their model, IRAS, as coming from a binomial distribution to calculate mean values and 90th percentile values. The study also details the complex inputs used in the model.

For example, the current state of earthquake prediction relies in part on a simple formula to compute return periods, that in turn represent the frequency with which a given event of a given magnitude will occur at a location along an active fault.[3] The equation assumes a log-linear relationship between two parameters, the earthquake magnitude, and the return period. Tables 1 and 2 show sample values for the initial parameters, α and β, and the corresponding relationship between predicted magnitudes and return periods. The exact values of α and β are unknown, and their estimates do not have any high degree of precision. Adjusting the parameters by as little as 5% may

TABLE 1

EARTHQUAKE FORMULA

Earthquake formula using loglinear relationship of magnitude and the reciprocal of the return period:

$$N(m) = \alpha + \beta \times M$$

$$\text{100-year return period: } (\log(.01) = \alpha + \beta \times M)$$

The parameters α and β are determined by examining the historical data and geologic characteristics of the fault. Estimates are typically given as a range of values corresponding to the 100-year event, M_0. Table 1 shows the predicted magnitudes of the 100-year event for a mythical fault having the following estimated range for α and β:

M_0		α	
β	5.550	5.843	6.126
−1.5428	6.6	6.8	7.0
−1.6200	6.3	6.4	6.6
−1.6966	6.0	6.2	6.3

Computer simulation models will choose a single point estimate for α and β for each known historical fault. For example, α might be selected to equal 6.126 and beta selected to equal −1.6966. The model would predict that the fault would generate a 6.3 magnitude event with probability 1%.

[3] See Bolt [1] for a generic version of this formula.

TABLE 2

VARIATION IN RETURN

Table 2 shows the variation in the return periods and probabilities given:

1. Selections of α and β from Table 1.
2. Possible error of plus or minus 5% in either direction.
3. A magnitude 7.1 event.
4. $\log p = \alpha + \beta \times 7.1$.
5. Return period = $1/p$.

Return periods and associated probabilities of a 7.1 magnitude quake:

$R(7.1)$		α			Average % across rows	Resulting Relative error
β		5.820	6.126	6.432		
−1.6118	return	277	204	150		
	frequency	0.36%	0.49%	0.67%	0.51%	82.57%
−1.6966	return	506	372	274		
	frequency	0.20%	0.27%	0.36%	0.28%	
−1.7814	return	924	680	501		
	frequency	0.11%	0.15%	0.20%	0.15%	45.29%
Avg. % down columns		0.22%	0.30%	0.41%		
Relative error		26.40%		35.89%		

The model would assume a probability of 0.27% that a magnitude 7.1 earthquake would occur along the mythical fault. However, if both α and β were 5% above or below their "true" values, the probability might be as high as 0.67% or as low as 0.11%. In the former case, the assumed probability would be less than half the true frequency. In the latter case, the assumed probability would be more than double the true frequency.

magnify the error in the resulting return periods by over 100%. This is a case of extreme sensitivity to initial parameter selection that the actuary should consider in the selection of a risk load.

Hurricane prediction is also fraught with uncertainty. One component of the disaster potential that is difficult for the computer simulation models to manage is the storm surge and how high it could rise, since the role of the tidal forces must also be fit into the equations. Return periods are likewise not known with precision, and the chief problem confronting the computer simulation models is whether the historical data upon which the model's parameter selections have been based have been drawn from a long enough period of time to represent a random sequence of all potential hurricane events.

The author's suggested approach is to develop a risk load using the output from computer simulation models. This implies that the risk load is dependent only on the variance of the expected annual loss. But the catastrophic risk threatens not only the stability of individual insurers and the insurance industry, but also their survival. Some portion of the risk load must account for the contingency of underestimating the "Great Earthquake" or "Great Hurricane." One possible solution is to use the models to compute a worst case scenario event that uses less conservative frequency and severity assumptions than are used to generate the expected annual loss.

4. APPROPRIATE USES OF COMPUTER SIMULATION MODELS

The focus on using computer simulation models to calculate appropriate risk loads for catastrophic lines may shift attention from other possible uses of such models in pricing. Among these uses are the following:

1. Territory relativities for earthquake premiums. Computer simulation models can be used to determine the relative potential for loss between territories. Care should be taken to ensure against bias in measurement. Should the individual insurer's portfolio be used or should a simulated book of business representative of the demographics of the state be used instead? Are the mean damage ratio assumptions and construction type assumptions used in the simulations unbiased with respect to territory

(e.g., has the model accounted for possible correlation of size of dwelling and territory)?

2. Variance of expected annual losses. Should a simplifying assumption as to the underlying distribution of the model's output be used? Or should the variance of the actual simulation runs or stochastic model be used?

3. Scenario analysis. The model can test the effects of changing deductibles, limits, or other coverage features. Conversely, actuaries can test the reasonableness of the model's output by changing these and other inputs pertaining to the inventory database.

4. Ex ante tests. How well does the model do in predicting actual losses given the event that occurred and its location?

5. Evaluating reinsurance program costs. Models could be used to demonstrate how exposed an individual insurer is in a catastrophe-prone area. Claims of careful underwriting can be analyzed. The adequacy of limits of coverage can also be analyzed. Risk loads might be based on the individual insurer's portfolio spread.

6. Portfolio management. The costs of writing business in catastrophe-prone areas may be better understood.

5. CONCLUSION

Mr. Meyers has bravely proposed a way of calculating risk loads for catastrophe lines that treats the task as a tractable problem; he deserves special commendation for a distinguished effort. The Competitive Market Equilibrium risk load approach is a promising one that recognizes the importance of including parameter risk in the risk load. However, the author omitted consideration of all the risk factors involved in writing catastrophe lines of insurance.

It is important for actuaries to consider all the sources of risk in the calculation of a risk load for catastrophe lines. This is true whether or not computer simulation models are used. When computer simulation models are used for calculating risk loads, the impact of geographic concentrations of exposure certainly needs to be included. Some measure of the parameter variance within the model needs to be included as well. Other sources of parameter risk exist that may be part of the underlying assumptions of the model or that may lie outside the focus of current computer simulation models. Without accounting for these other sources of risk, the associated risk loads will be less than adequate.

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ESTIMATING THE PREMIUM ASSET ON RETROSPECTIVELY RATED POLICIES

MICHAEL T. S. TENG AND MIRIAM E. PERKINS

Abstract

This paper presents a method for estimating the premium asset on retrospectively rated policies, using the functional relationship between the losses and the retrospective premium. This relationship is examined using the historical premium and loss development data and the retro rating parameters sold in the underlying policy. The cumulative ratio of premium development to loss development, when applied to the expected future loss emergence, gives the expected future premium development on the retro rated policies. The sum of all future premium development is the premium asset.

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1. INTRODUCTION

On retrospectively rated policies, premium that the insurer expects to collect based on the expected ultimate loss experience, less the premium that the insurer has already booked, is called the premium asset. Many insurers call this the Earned But Not Reported premium (EBNR). The admitted portion of the premium asset appears on the balance sheet as the "Asset for Accrued Retrospective Premiums."

In recent years, retro rated policies have become popular for several reasons.

1. A retro rated policy returns premium to the insured for good loss experience. This feature is attractive for a customer who anticipates favorable loss experience through loss control and loss management. By offering retro rated policies, the insurer may be able to attract these good customers.

2. A growing number of commercial insurance buyers are taking advantage of the cash flow feature in a retro rated policy. A retro rated policy allows the insured to pay premium as losses are reported or paid, depending on the contract, rather than paying all premiums up front. This allows the insured to hold on to cash longer.

3. Inflation, rate regulations, uncertainty in claims compensability, increasing utilization of the insurance benefits, and growing attorney involvement have made the cost of insurance much harder to predict today than in the past. Since the premium for a retro rated policy varies directly with the insured's actual loss experience, writing retro policies allows an insurer to shift a large portion of the actual risk to the insured. This makes the insurer more willing to write insurance.

As a result of the growth of retro rated policies, estimating the premium asset for them is a growing need for many commercial lines insurers. This asset frequently exceeds 10% of surplus. Despite the growing importance of the premium asset, there have been few articles written on this subject. Berry [1] and Fitzgibbon [2] have presented methods of calculating the "retro reserve," defined as the difference between the *premium deviation to date* and the *ultimate premium deviation*.[1] The retro reserve is the negative equivalent of the premium asset referred

[1]The ultimate premium deviation is the amount by which the ultimate premium for a retro rated policy is expected to differ from the standard premium (manual premium adjusted for experience rating). The premium deviation to date is the amount by which the currently booked premium differs from the standard premium.

to in this paper. Their approach is to analyze the historical relationship between the loss ratio and the premium deviation using statistical techniques, and then apply such a relationship to the projected loss ratio to calculate a projected ultimate premium deviation. This ultimate premium deviation is then reduced by the premium deviation to date to produce the retro reserve. Berry uses a second approach, which is to estimate ultimate premium using the historical premium emergence pattern, and then subtract current premium to get the retro reserve.

While the statistical methods presented in [1] and [2] may be theoretically sound, they lack intuitive appeal, particularly as they relate to how a retro rating formula actually works. On a retro rated policy, premium is calculated as a function of loss. This function is composed of retro rating parameters such as the loss conversion factor, tax multiplier, retro minimum, and retro maximum; they define how much premium an insurer can collect given a certain amount of loss. Therefore, the premium asset on a retro rated policy should be established as a function of reported losses and the reserve for loss development, where this function is defined by the retro rating parameters.

This paper will present, through an example, a method of calculating the premium asset as a function of current losses, expected future loss emergence, and the retro rating parameters. Specifically, the method looks at how premiums develop as losses develop. The relationship can be expressed as the ratio of premium development to loss development, referred to here as the PDLD ratio. There are two methods of calculating the PDLD ratio—from historical premium and loss development data, and from the retro rating parameters. The latter approach will be developed first, and will be followed by the calculation of the PDLD ratios from historical data. Once the relationship between premium and loss is determined, it can be applied to the expected future loss development to get the expected future premium development. The sum of all future premium development is the premium asset.

This method applies only to retro rated polices (or similar loss sensitive rating plans), and not to prospectively rated policies. There may be a premium asset on prospectively rated polices due to changes in exposure, but this topic will not be discussed here. This method is intended to be applied to an aggregate book of business, or large segment of a book of business, rather than at the individual policy level.

2. THE FORMULA APPROACH TO CALCULATING PDLD RATIOS

The first step is to derive the formula for a PDLD ratio. This starts with the first retro adjustment. On a retro rated policy, the premium calculation is based on a retro formula. A commonly used formula is

$$P_n = [BP + (CL_n \times LCF)] \times TM, \qquad (2.1)$$

where

P_n = Premium at the n^{th} retro adjustment,

BP = Basic premium,

CL_n = Capped loss at the n^{th} adjustment[2],

LCF = Loss conversion factor, and

TM = Tax multiplier.

For example, P_1 denotes the premium computed for the first retro adjustment; P_2 denotes the premium computed for the second retro adjustment. Note that BP, LCF, and TM typically stay the same throughout all retro adjustments. For a more thorough discussion of the retro rating formula, see Gillam and Snader [3].

Using formula (2.1) and denoting L_1 as the amount of loss developed for the first retro adjustment, the first PDLD ratio

[2]Losses that contribute to additional premium: these are total losses subject to a minimum and a maximum amount corresponding to the plan minimum and maximum premiums. Individual claims may also be capped by a per accident limitation, which limits the adverse impact of any single large claim on the premium calculation.

can be stated as follows:

$$P_1/L_1 = [BP + (CL_1 \times LCF)] \times TM/L_1$$
$$= [(BP/L_1) \times TM] + [(CL_1/L_1) \times LCF \times TM]. \quad (2.2)$$

The first term of this formula is $(BP/L_1) \times TM$. This is basic premium divided by the loss emerged for the first retro adjustment times the retro tax multiplier. One can approximate this as

$$BP \times TM/(SP \times ELR \times \%Loss_1), \quad (2.3)$$

where

SP = Standard premium,[3]

ELR = Expected loss ratio

= Expected ultimate loss divided by standard premium, and

$\%Loss_1$ = Expected percentage of loss emerged for the first adjustment.

Formula 2.3 is equivalent to $(BP/SP) \times TM/(ELR \times \%Loss_1)$, which is the basic premium factor in a retro rating formula times the tax multiplier, divided by the expected loss ratio emerged for the first retro adjustment. The expected loss ratio for the first retro adjustment would depend on the ultimate expected loss ratio and the percentage of losses emerged at the first adjustment. Typically, losses emerged as of 18 months are used to compute the first retro adjustment.

In Formula 2.2, the term CL_1/L_1 is the ratio of capped losses to uncapped losses. This ratio is referred to as the *loss capping ratio*. Capped losses are losses that contribute to an additional

[3] Manual premium adjusted for experience rating.

premium. Any change in loss, where total loss exceeds the minimum and is below the maximum, will result in additional premium. Conceptually one can view the difference between the capped loss (CL) and the uncapped loss (L) as the portion of loss outside the boundaries of the retro maximum and minimum. On plans that cap the losses with a per accident loss limit, the capped loss would also exclude the losses exceeding this limit, since they do not contribute to additional premium. The loss capping ratio usually decreases as the data becomes more mature. This is because an increasing portion of the loss development occurs outside of loss limitations. The loss capping ratio can be derived by comparing the capped and the uncapped loss development, if such data are available; often they are not. In this paper, the loss capping ratio is derived using a loss ratio distribution. Because the explanation of this method is somewhat detailed, it is presented after the example of the PDLD ratio calculation, in Section 5.

If the loss data used is already capped (i.e., L_n equals CL_n for all n), then the loss capping ratio will be one. Otherwise, this ratio will have to be estimated. The example assumes that the loss capping ratio is 0.85 for losses developed through the first retro adjustment. This means that 15 percent of the losses developed through the first retro adjustment are eliminated by the net effect of the retro maximums, minimums, and per accident limitations.

To show how Formula 2.2 can be used to estimate the PDLD ratio, the example assumes the following retro rating parameters:

$$\text{Basic premium factor} = 0.20$$

$$\text{Expected loss ratio} = 0.70$$

$$\text{Loss conversion factor} = 1.20$$

$$\text{Tax multiplier} = 1.03$$

$$\%\text{Loss}_1 = 78.4\%.$$

These retro rating parameters may be computed as the average of the sold retro parameters. Substituting these values into Formula 2.2, one gets a PDLD ratio for the first retro adjustment of

$$[0.20 \times 1.03/(0.70 \times 78.4\%)] + (0.85 \times 1.20 \times 1.03) = 1.426.$$

The PDLD ratio for the second retro adjustment period refers to the *incremental premiums* developed between the first and the second retro adjustments, divided by the *incremental losses* developed between these two adjustments. Typically, successive retro adjustments occur at one year intervals. One can view the PDLD ratio for the second retro adjustment period as the ratio of the *change in premium* divided by the *change in loss*. Algebraically, this equals

$$\begin{aligned}(P_2 - P_1)/(L_2 - L_1)& \\ &= (CL_2 - CL_1) \times LCF \times TM/(L_2 - L_1) \\ &= [(CL_2 - CL_1)/(L_2 - L_1)] \times LCF \times TM. \qquad (2.4)\end{aligned}$$

This example assumes an incremental loss capping ratio of 0.58 for the second retro adjustment period. Substituting this loss capping ratio and the retro rating parameters into Formula 2.4, one gets a PDLD ratio of $0.58 \times 1.20 \times 1.03 = 0.717$. The PDLD ratios for the third and subsequent retro adjustments are calculated in a similar manner.

The advantage of using the retro formula to estimate the PDLD ratio is that it responds to changes in the retro rating parameters that are sold, whereas the PDLD ratios derived from the historical data may not be indicative of the future PDLD ratios. If the retro rating parameters change significantly over time, one should give more weight to the PDLD ratios derived by formula than those derived from the historical data. A summary of the formula PDLD ratios is shown in Exhibit 4, Part 2.

When possible one should retrospectively test the PDLD ratios derived by formula against actual emergence in the subsequent retro adjustment periods to determine if any bias exists. A possible source of bias is the use of average parameters for the LCF, tax multiplier, maximum, minimum, and per accident limitation. One should study the appropriateness of the selections and adjust them as necessary. Such a study could lead to better parameter selections and more accurate premium estimates.

3. THE EMPIRICAL APPROACH TO CALCULATING PDLD RATIOS

The use of empirical data is another way to calculate the PDLD ratios. Two types of data are needed for the empirical approach: booked premium development and reported loss development.[4] For the example presented in this paper, premium booked by policy effective quarter by valuation quarter is displayed in Exhibit 6 and reported loss data is shown in Exhibit 7. The calculation of the PDLD ratios is shown in Exhibit 4. The PDLD ratio after the sixth retro adjustment is selected at zero, which assumes that there are no further retro adjustments.[5]

Data should be segregated into homogeneous groups by size of account and by the type of rating plan sold. When appropriate, other criteria should be used in grouping the data. Policies are grouped based on the calendar quarter in which they became effective. These groups will be referred to as policy effective quarters. The first policy effective quarter of 1994 will be

[4]Booked premium on a retro rated policy is the premium computed using the retro rating formula and the most recent loss valuation. Reported loss is the amount of loss that has been reported to the insurer. It does not include future loss development for unreported claims, for such losses are often not entered into the premium calculation.

[5]The NCCI and ISO retrospective rating manuals prescribe a maximum premium adjustment period of 3 to 4 years. The actual maximum adjustment period varies from one retro policy to another. A maximum premium adjustment period of six years is common among major commerical line retro policies. However, due to increasing uncertainty of loss costs and growing usage of cash flow financing of premiums, retro policies will probably be written with longer premium adjustment periods in the future.

denoted as 1994.1, the second quarter will be denoted as 1994.2, and so on.

The first retro premium computation is usually based on losses developed through 18 months. However, it takes time to do the retro calculation and to record adjusted premiums. This paper assumes that due to time lags in processing and recording, premiums are recorded 3 to 9 months following the recording of losses. Therefore, it is assumed that premiums booked through 27 months are the result of the first retro adjustment. Since retro adjustments are usually done in annual intervals, premiums recorded through 39 months would be the result of the second retro adjustment, using losses evaluated at 30 months. Premiums recorded through 51 months would be the result of the third retro adjustment, using losses evaluated at 42 months, and so on. In practice, the actual length of the retro adjustment period and the premium booking lag may vary from one insurer to another.

The PDLD ratio for the first retro adjustment equals premiums booked through 27 months divided by losses reported through 18 months. At the first retro adjustment period, the PDLD ratio indicated by an overall average of the historical data is 1.460 (see Exhibit 4, Part 1). However, there is an upward trend in the responsiveness of premium to loss over the latest several policy quarters and these PDLD ratios are higher than the historical average. Such a trend could be the result of more liberal retro rating parameters (higher maximum, minimum, or per accident limitation), but this is probably not the case here since the PDLD ratio calculated by formula is 1.426 and it reflects the plan parameters currently being sold. A more likely explanation for the trend is an improvement in loss experience, either due to chance or to known changes in the system such as workers compensation reform. A larger portion of the loss is within the boundaries of the retro maximum and the per accident limitation, resulting in more additional premium per dollar of loss. The formula approach will not reflect a change in loss experience unless the formula is revised. (This revision is discussed in Section 5.) In recognition of these changing conditions, a PDLD ratio of 1.750 was selected for the first adjustment.

The PDLD ratio for the second retro adjustment period is the *incremental premiums* developed between the first and the second retro adjustments divided by the *incremental losses* developed between these two adjustments. It is assumed that losses developed through 30 months are used to calculate the premiums for the second retro adjustment and that the resulting premiums are booked at the 39 month valuation. The selected PDLD ratio from historical data is 0.700, which is close to the formula ratio of 0.717. The PDLD ratios from the two methods also compare closely at the third adjustment.

The historical PDLD ratios may fluctuate significantly after the first retro adjustment period. This is because the premium and loss development on a few policies can be a large component of the total incremental development on policy quarter data. Historical PDLD ratios for an individual policy quarter could even be negative in spite of upward aggregate loss development—this could happen when there is upward development in high loss layers (resulting in no additional premium) and downward development (and return premium) on layers that are still within loss limitations. Where the historical PDLD ratios fluctuate significantly, one should use an average of as many historical data points as possible. In situations like this, the PDLD ratios derived by formula may provide a better indication of the relationship between premium and loss.

In the example, the historical and formula PDLD ratios begin to diverge after the third retro adjustment period. Several factors could be contributing to this. First, since the historical ratios are lower than the formula ratios, worse than expected loss experience during the mid-1980s may have caused a larger portion of the loss to be outside the boundaries of the retro maximum and the per accident limitation than the formula approach would predict. This is the opposite situation from the one described at the first retro adjustment period above. Second, average retrospective rating parameters may be changing over time. In the case of shifting parameters over time, a single selected PDLD ratio may not be the best estimate of development for all exposure periods. As with loss development analysis, the actuary must decide how best to develop each period to "square the triangle." For the fourth through sixth adjustment periods, the PDLD ratios were selected between those indicated by the two methods.

4. CUMULATIVE PDLD RATIOS

The ultimate goal of this method is to estimate the premium asset, which is the sum of all future premium adjustments based on the expected future loss emergence. As shown before, the relationship between premium and loss can be expressed by the PDLD ratios. However, the PDLD ratios are incremental factors. To estimate how much premium can be expected based on all future loss development, one needs to calculate the cumulative PDLD ratios, or the CPDLD ratios.

A CPDLD ratio is the average of the PDLD ratios in all subsequent retro adjustment periods, weighted by the percentage of losses to emerge in each period. For instance, the CPDLD ratio at the second retro adjustment is the average of the PDLD ratios for the second and subsequent retro adjustment periods, weighted by the percentage of losses emerged in each period. The CPDLD ratio at the third adjustment is the average of the PDLD ratios for the third and subsequent retro adjustment periods, weighted by the percentage of losses emerged in each period. The loss emergence pattern is shown at the bottom of Exhibit 7.

Using the loss emergence pattern derived from the loss development data in Exhibit 7 and the selected PDLD ratios from Exhibit 4, one can calculate the CPDLD ratios. For example, the first CPDLD ratio equals 1.492, which is computed as follows:

$$\frac{(1.750 \times 78.4\% + 0.700 \times 9.3\% + 0.550 \times 4.4\% + 0.450 \times 2.9\% + 0.400 \times 3.0\% + 0.350 \times 1.6\%)}{(78.4\% + 9.3\% + 4.4\% + 2.9\% + 3.0\% + 1.6\% + 0.4\%)}.$$

The second CPDLD ratio is 0.556, which is computed as follows:

$$\frac{(0.700 \times 9.3\% + 0.550 \times 4.4\% + 0.450 \times 2.9\% + 0.400 \times 3.0\% + 0.350 \times 1.6\%)}{(9.3\% + 4.4\% + 2.9\% + 3.0\% + 1.6\% + 0.4\%)}.$$

The calculation of the remaining CPDLD ratios is shown in Exhibit 3.

The CPDLD ratio tells how much premium an insurer can expect to collect for a dollar of loss that has yet to emerge. For instance, the first CPDLD ratio is 1.492, which means that each dollar of loss emerged provides the insurer one dollar and 49 cents of premium. The second CPDLD ratio is 0.556, which means that after the first retro adjustment, each additional dollar of loss provides the insurer 56 cents of premium.

The relationship of premium development to loss development is usually greater than unity at the first retro adjustment. This is because the basic premium is included in the first retro premium computation, and because only a small portion of loss is limited by the retro maximum or per accident limitation at this early maturity. The application of the loss conversion factor and the tax multiplier results in more than a dollar of premium per dollar of loss. As time goes on, however, a decreasing portion of incremental loss development results in additional premium. Incremental premium, equal to the loss capping ratio times LCF and TM, will generally be less than loss and hence the CPDLD ratios should be less than 1.0 at the later adjustments.

Having calculated the CPDLD ratios, the next step is to multiply these ratios by the expected future loss emergence to get the expected future premiums. Adding future premiums to the booked premiums gives ultimate premiums. For example, at 12/31/94, policy effective quarters 1993.1 through 1994.4 have not yet had the first retro adjustment (they are all less than 27 months old). The expected loss amount for these policy effective quarters, as computed in Exhibit 2, is $280,844,000 ($196,767,000 from 1993, plus $84,077,000 from 1994). Since the marginal premium per dollar of loss is $1.492, this means $280,844,000×1.492 or $419,019,000 of future premium is expected. Since there was no prior retro adjustment, the expected ultimate premium for these policy effective quarters is $419,019,000.

At 12/31/94, policy quarters 1992.1 through 1992.4 have had one retro adjustment (they are older than 27 months but not yet 39 months old). For these policy periods, the expected amount of loss yet to emerge is $50,747,000 (see Exhibit 2). Exhibit 3 shows that for each dollar of loss emerged after the first retro adjustment, the insurer can expect $0.556 of premium. This means the insurer can expect to collect $50,747,000×0.556 or $28,216,000 in additional premium. Adding this to the $328,778,000 of premium booked from the first retro adjustment (the premium for 1992.1 through 1992.4 evaluated as of 27 months), gives an expected ultimate premium of $356,993,000. Exhibit 1 shows the calculation of the ultimate premium for each policy period.

The final step is to subtract premium booked as of 12/31/94 from the estimated ultimate premium to get the premium asset as of 12/31/94. The sum of the premium assets for all policy periods as calculated in Exhibit 1 is $43 million.

Note that the premiums booked as of 12/31/94 (Column (7) of Exhibit 1) are close to but not equal to the premiums booked from the prior retro adjustments (Column (5) of Exhibit 1). This may be due to differences in the timing of retro adjustments, minor premium adjustments, or interim premium booking that occurs between the regularly scheduled retro adjustments.

5. LOSS CAPPING RATIO

We now return to the subject of the loss capping ratio. The loss capping ratio, CL/L, is the ratio of capped loss development to uncapped loss development. This term is essential to the calculation of the PDLD ratio, which expresses the relationship between premium development and loss development on a retro rated policy. Capped loss development includes the effect of the retro maximum and minimum, and the per accident loss limit. It is often difficult to obtain capped loss development data, especially as it pertains to losses eliminated by the retro maximum and minimum. Hence, it may be necessary to use a Table M[6] approach to estimate the impact of the retro plan maximum and minimum on loss development. If a per accident limit is purchased, the treatment of the losses eliminated by the limit is similar to that for losses eliminated by retro maximum and minimum.

The loss capping ratio can be solved for using the relationship

$$\text{CLR} = \text{LR}(1 - \chi - \text{LER}),$$

where

$$\begin{aligned}
\chi &= \text{Table M net insurance charge} \\
&= \text{Table M charge at max} - \text{Table M savings at min}, \\
\text{LER} &= \text{Percent of losses eliminated due to the per accident limitation}, \\
\text{CLR} &= \text{capped loss ratio} \\
&= \text{capped loss divided by standard premium, and} \\
\text{LR} &= \text{uncapped loss ratio} \\
&= \text{uncapped loss divided by standard premium.}
\end{aligned}$$

[6] Also called the Table of Insurance Charges. Table M is used to calculate the insurance charge associated with a retro plan's maximum and minimum. Gillam and Snader [3] give a detailed description of this table.

The loss capping ratio is then:

$$\text{CLR/LR} = (1 - \chi - \text{LER}). \tag{5.1}$$

To calculate the loss capping ratio, one needs the net insurance charge at each retro adjustment period. The insurance charge is typically determined from the values of the retro rating parameters sold under the plan and the presumed loss ratio distribution underlying Table M. However, the percentage of losses actually affected by the retro maximum or minimum will differ from expected due to the random nature of insurance losses and the fact that losses are not at their ultimate valuation. Therefore, the charge and savings computed at each retro adjustment period should be a function of the actual loss ratio as opposed to the expected ultimate loss ratio under the plan.

If it is assumed that the loss ratio probability distribution function has the same shape throughout all development stages, then at each retro adjustment one may enter Table M by defining two entry ratios:

Entry ratio at the max = (loss ratio at max/actual loss ratio), and

Entry ratio at the min = (loss ratio at min/actual loss ratio).

Loss ratios at the retro maximum and minimum should be estimated from the sold retro rating parameters. The loss ratio at maximum is the standard premium loss ratio at which the net retro premium reaches the maximum premium; for this example, we assume it is 1.200. Similarly, the loss ratio at minimum is the standard premium loss ratio at which the net retro premium reaches the minimum premium; for this example, we assume it is 0.100.

The actual loss ratio may be computed by dividing the actual loss at each retro adjustment period by the standard premium. Alternatively, it can be estimated as the expected loss ratio (expected ultimate loss divided by standard premium) times the expected percentage of losses emerged at each retro adjustment. For instance, if the expected loss ratio is 0.700 and 78.4% of losses emerge by the first retro adjustment, one can estimate the actual loss ratio at the first retro adjustment to be $0.700 \times 78.4\%$, or 0.549.

If actual loss experience differs from the expected experience underlying Table M, one should multiply the estimate of the actual loss ratio by a factor representing the relationship between actual and expected losses. For example, if the original expected loss ratio was 0.700 but actual loss experience produces an average loss ratio of 0.800, multiply 0.549 by a factor of 0.800/0.700. Such an adjustment factor is needed to calculate the correct entry ratios for Table M.

The two entry ratios for the first retro adjustment can be computed as:

$$\text{Entry ratio at the max} = (1.200/0.549) = 2.19, \text{ and}$$

$$\text{Entry ratio at the min} = (0.100/0.549) = 0.18.$$

Table M also requires one to estimate the average size of the accounts insured by the retro rated policies. For this example, the average size is assumed to be $750,000 in standard premium. This may be estimated from the sold policy information. The use of the average policy size is another potential source of bias between the PDLD ratios calculated using the formula method and the PDLD ratios that actually emerge. One way to reduce this bias is by grouping the data according to policy size. The net insurance charge for a $750,000 account at 2.19 and 0.18 entry ratios is calculated to be 0.109. This is shown in Exhibit 5.

In the event that a per accident loss limit is sold, losses eliminated by such limit divided by total losses should also be considered in the calculation of the loss capping ratio. Furthermore, the Table M insurance charge should be adjusted to reflect the per accident loss limit. One method of making such an adjustment is presented by Robbin [4]. In this example we assume that 4.2% of losses are eliminated by the per accident limitation as of the first retro adjustment. Thus, the loss capping ratio at the first retro adjustment is one minus 0.109 (the net insurance charge) minus 0.042 (the per-accident loss elimination ratio), or 85%. Loss capping ratios for the second and subsequent retro adjustment periods are calculated in Exhibit 5.

By using Table M to calculate the loss capping ratios, one major assumption is that the loss ratio probability distribution function underlying Table M is appropriate for all retro adjustment periods. This may not be true. The procedure can be refined by using a loss ratio distribution that is more appropriate for each retro adjustment period. Such distributions may be calculated from empirical data at the proper evaluation dates, and be used to replace or modify the Table M distribution, depending on the credibility of the empirical data.

Thus far the loss capping ratios calculated are those developed as of each retro adjustment. Since the PDLD ratios are incremental, one needs to calculate the incremental loss capping ratios, using the loss capping ratios developed through each retro adjustment. This is done by algebraic manipulation. For example, the incremental loss capping ratio for the second retro adjustment period is $[(CL_2 - CL_1)/(L_2 - L_1)]$ which may be stated as

$$\frac{[(CL_2/L_2) \times (ELR \times \%Loss_2) - (CL_1/L_1) \times (ELR \times \%Loss_1)]}{[(ELR \times \%Loss_2) - (ELR \times \%Loss_1)]}. \tag{5.2}$$

Note L_n is the amount of losses emerged as of the nth retro adjustment, and CL_n/L_n is the loss capping ratio developed as of the nth retro adjustment. The ELR is the expected loss ratio, and $\%Loss_n$ is the expected percentage of losses emerged as of the nth retro adjustment. The incremental loss capping ratios are calculated in Exhibit 5.

6. FURTHER ISSUES

The method described in this paper can be used to calculate the premium asset for all types of loss-sensitive rating plans, as long as the rating formula reflects what is being sold to the insured. Further issues to think about are:

1. The definition of loss may include allocated loss adjustment expense (ALAE). Frequently, retro rated policies are written with ALAE included in the definition of loss. This allows the insurer to pass on to the insured not only losses, but attorney expenses as well. The loss data used in computing the PDLD ratios should be consistent with that used in the rating plan.
2. Changes in the mix of business may change the PDLD ratio. Changes in the mix of business by state, industry group, or even geographical region can alter the average rating parameters sold and the underlying claim frequency and claim severity. This will in turn affect how sensitive the premium is to loss.
3. Collectibility of premium should be considered. When the premium asset is secured, there is little question as to its collectibility. If a portion of the premium asset is not secured, then a provision should be made to anticipate bad debt.

REFERENCES

[1] Berry, Charles H., "A Method of Setting Retro Reserves," *PCAS* LXVII, 1980, pp. 226–238.

[2] Fitzgibbon, W. J. Jr., "Reserving for Retrospective Returns," *PCAS* LII, 1965, pp. 203–214.

[3] Gillam, W. R. and R. H. Snader, "Fundamentals of Individual Risk Rating," National Council on Compensation Insurance, 1992.

[4] Robbin, Dr. I., "Overlap Revisited—The 'Insurance Charge Reflecting Loss Limitation' Procedure," 1990 CAS Discussion Paper Program, Vol. II, pp. 809–855.

EXHIBIT 1

CALCULATION OF FUTURE PREMIUM EMERGENCE AND PREMIUM ASSET

(dollars in thousands)

Policy Periods (1)	Expected Future Loss Emergence (2)	CPDLD Ratios (3)	Expected Future Premium (2)x(3) (4)	Premiums Booked from Prior Adjustment (5)	Estimated Total Premium (4)+(5) (6)	Premium Booked as of 12/94 (7)	Premium Asset (6)-(7) (8)
1987.1 to 1987.4	-262	0.000	0	494,927	494,927	494,927	0
1988.1 to 1988.4	3,282	0.285	935	467,388	468,324	467,796	528
1989.1 to 1989.4	9,146	0.354	3,238	460,660	463,897	460,716	3,181
1990.1 to 1990.4	21,347	0.390	8,325	453,525	461,850	452,520	9,331
1991.1 to 1991.4	25,397	0.447	11,352	336,654	348,007	337,966	10,041
1992.1 to 1992.4	50,747	0.556	28,216	328,778	356,993	330,216	26,777
1993.1 to 1994.4	280,844	1.492	419,019	0	419,019	425,590	-6,570
					3,013,018	2,969,730	43,288

Notes:

(2) From Exhibit 2, Column (7a).
(3) From Exhibit 3, Column (7).
(5) From Exhibit 4.
(7) From the latest diagonal of Exhibit 6.

EXHIBIT 2

LOSS PROJECTIONS

(dollars in thousands)

Policy Eff. Quarter (1)	Losses Reported as of 12/94 (2)	Loss Develop. Factors (3)	Percent Earned as of 12/94 (4)	Ultimate Losses (2)x(3)x(4) (5)	Annual Total (5a)	Losses Reported at Prior Retro Adjust. (6)	Annual Total (6a)	Expected Loss Emer- gence (5)-(6) (7)	Annual Total (7a)
1987.1	102,064	1.000	100%	102,064		102,059		5	
1987.2	65,339	1.000	100%	65,339		65,264		75	
1987.3	155,738	1.000	100%	155,738		155,950		-212	
1987.4	94,067	1.000	100%	94,067	417,208	94,197	417,470	-130	-262
1988.1	88,908	1.001	100%	89,040		87,781		1,259	
1988.2	57,763	1.002	100%	57,880		58,054		-174	
1988.3	152,121	1.004	100%	152,683		151,031		1,652	
1988.4	96,809	1.010	100%	97,734	397,337	97,189	394,055	545	3,282
1989.1	81,384	1.010	100%	82,203		80,475		1,728	
1989.2	55,898	1.012	100%	56,582		55,541		1,041	
1989.3	131,539	1.020	100%	134,190		130,944		3,246	
1989.4	91,423	1.030	100%	94,189	367,164	91,058	358,018	3,131	9,146
1990.1	89,715	1.032	100%	92,599		87,639		4,960	
1990.2	56,032	1.039	100%	58,239		54,473		3,766	
1990.3	118,268	1.052	100%	124,399		117,202		7,197	
1990.4	73,037	1.063	100%	77,660	352,897	72,236	331,550	5,424	21,347
1991.1	60,399	1.067	100%	64,424		57,620		6,804	
1991.2	34,136	1.068	100%	36,459		33,064		3,395	
1991.3	60,696	1.085	100%	65,883		58,493		7,390	

EXHIBIT 2

PART 2

Policy Eff. Quarter (1)	Losses Reported as of 12/94 (2)	Loss Develop. Factors (3)	Percent Earned as of 12/94 (4)	Ultimate Losses (2)x(3)x(4) (5)	Annual Total (5a)	Losses Reported at Prior Retro Adjust. (6)	Annual Total (6a)	Expected Loss Emer-gence (5)-(6) (7)	Annual Total (7a)
1991.4	61,068	1.100	100%	67,187	233,953	59,380	208,556	7,807	25,397
1992.1	53,455	1.107	100%	59,152		49,161		9,991	
1992.2	37,393	1.114	100%	41,652		33,060		8,592	
1992.3	62,118	1.140	100%	70,830		53,069		17,761	
1992.4	50,766	1.166	100%	59,188	230,822	44,785	180,075	14,403	50,747
1993.1	39,519	1.184	100%	46,799		0		46,799	
1993.2	30,286	1.215	100%	36,793		0		36,793	
1993.3	51,005	1.276	100%	65,093		0		65,093	
1993.4	34,516	1.393	100%	48,082	196,767	0	0	48,082	196,767
1994.1	21,189	1.661	90%	31,672		0		31,672	
1994.2	11,381	2.416	70%	19,245		0		19,245	
1994.3	14,339	4.397	45%	28,372		0		28,372	
1994.4	1,862	12.856	20%	4,788	84,077	0	0	4,788	84,077
TOTAL	2,118,031			2,247,065	2,196,148	1,889,725	1,889,725	357,340	306,423

Notes:

(2) Figures on the latest diagonal of the loss data in Exhibit 7.

(3) Derived from loss development data in Exhibit 7.

(4) These earning ratios reflect the fact that policies written in the latest four quarters are not fully earned.

(6) These represent losses recorded as of prior retro adjustments (Exhibit 7). Policy effective quarters 1993.1 through 1994.4 would not have had any retro adjustments as of 12/31/94; therefore, the losses recorded are 0. Policy effective quarters 1992.1 through 1992.4 would have had one retro adjustment; therefore, losses evaluated at 18 months were entered into this column.

EXHIBIT 3

CPDLD RATIO CALCULATION

Retro Adjustment Periods (1)	Selected PDLD Ratios (2)	% Loss Emerged (3)	PDLD Ratio x Loss Emg (2)x(3) (4)	Upward Cumulative of Col. (4) (5)	Upward Cumulative of Col. (3) (6)	CPDLD Ratios (5)/(6) (7)
First	1.750	78.4%	1.371	1.492	100.0%	1.492
Second	0.700	9.3%	0.065	0.120	21.6%	0.556
Third	0.550	4.4%	0.024	0.055	12.3%	0.447
Fourth	0.450	2.9%	0.013	0.031	7.9%	0.390
Fifth	0.400	3.0%	0.012	0.017	4.9%	0.354
Sixth	0.350	1.6%	0.006	0.006	2.0%	0.285
Subsequent	0.000	0.4%	0.000	0.000	0.4%	0.000

Notes:

(2) From Exhibit 4.

(3) From Exhibit 7.

EXHIBIT 4

PART 1

PDLD RATIO CALCULATION

(dollars in thousands)

Policy Eff. Quarter	First Retro Adjustment Loss 0-18	First Retro Adjustment Prem 0-27	First Retro Adjustment PDLD Ratio	Second Retro Adjustment Loss 19-30	Second Retro Adjustment Prem 28-39	Second Retro Adjustment PDLD Ratio	Third Retro Adjustment Loss 31-42	Third Retro Adjustment Prem 40-51	Third Retro Adjustment PDLD Ratio
1983.3	42,461	52,436	1.235	5,515	4,012	0.727	4,533	2,351	0.519
1983.4	20,151	26,222	1.301	2,738	2,722	0.994	1,480	576	0.389
1984.1	23,076	29,189	1.265	2,142	1,927	0.900	2,076	1,086	0.523
1984.2	19,243	23,422	1.217	1,032	1,904	1.844	507	740	1.461
1984.3	54,927	69,310	1.262	8,900	6,371	0.716	3,804	3,432	0.902
1984.4	33,393	43,305	1.297	4,308	3,189	0.740	2,819	1,274	0.452
1985.1	46,100	59,203	1.284	3,384	3,349	0.990	2,312	1,347	0.583
1985.2	27,696	38,717	1.398	2,679	2,120	0.791	2,675	1,687	0.631
1985.3	96,041	133,094	1.386	9,717	7,926	0.816	6,465	3,054	0.472
1985.4	49,481	66,351	1.341	7,193	4,063	0.565	4,268	2,560	0.600
1986.1	63,095	87,173	1.382	5,865	4,249	0.724	4,045	2,298	0.568
1986.2	42,163	57,654	1.367	3,904	2,283	0.585	3,882	1,981	0.510
1986.3	115,105	160,838	1.397	12,006	10,917	0.909	12,037	7,932	0.659
1986.4	58,712	84,641	1.442	6,627	3,536	0.534	3,737	3,579	0.958
1987.1	77,373	103,693	1.340	7,879	8,776	1.114	4,795	2,987	0.623
1987.2	49,770	68,397	1.374	4,867	3,467	0.712	4,029	993	0.246
1987.3	120,053	171,434	1.428	15,117	9,858	0.652	8,909	4,189	0.470
1987.4	73,502	101,483	1.381	7,479	5,701	0.762	5,101	2,290	0.449
1988.1	71,999	98,806	1.372	6,083	4,745	0.780	4,138	1,006	0.243
1988.2	45,861	63,885	1.393	5,253	2,688	0.512	3,392	853	0.252
1988.3	115,461	161,154	1.428	13,462	6,642	0.652	7,128	2,854	0.470
1988.4	79,063	109,253	1.382	7,723	3,974	0.515	5,082	2,604	0.512

EXHIBIT 4

PART 1—PAGE 2

1989.1	71,471	99,777	1.396	3,744	913	0.244	4,462	2,131	0.478
1989.2	49,486	67,553	1.365	2,976	2,923	0.982	2,834	1,906	0.672
1989.3	108,330	153,443	1.416	12,886	6,525	0.506	6,733	5,732	0.851
1989.4	72,082	104,838	1.454	10,903	8,693	0.797	7,288	4,224	0.580
1990.1	76,452	107,468	1.406	8,989	7,055	0.785	2,199	319	0.145
1990.2	46,393	70,127	1.512	6,513	5,695	0.874	1,566	1,159	0.740
1990.3	102,035	158,027	1.549	13,486	6,788	0.503	1,682	901	0.536
1990.4	57,548	91,918	1.597	8,522	2,840	0.333	6,166	1,227	0.199
1991.1	54,037	81,901	1.516	3,583	2,595	0.724			
1991.2	30,240	54,045	1.787	2,824	666	0.236			
1991.3	55,325	94,797	1.713	3,168	2,552	0.805			
1991.4	54,302	97,650	1.798	5,078	2,448	0.482			
1992.1	49,161	82,057	1.669						
1992.2	33,060	59,279	1.793						
1992.3	53,069	99,074	1.867						
1992.4	44,785	88,367	1.973						
Selection Based on Historical Averages									
		Average All	1.460			0.730			0.556
		Weighted Average All	1.455			0.680			0.532
		Selected	1.750			0.700			0.550
Selection Based on Retro Formula									
		LCF	1.20			1.20			1.20
		TM	1.03			1.03			1.03
		Loss Capping Ratio	85%			58%			45%
		Implied PDLD Ratio	1.426 *			0.717			0.556
Final Selection			1.750			0.700			0.550

* Also assumes a basic premium factor of 0.2, an expected loss ratio of 0.7, and an expected loss emergence of 78.4% at first adjustment.

EXHIBIT 4

PART 2

PDLD RATIO CALCULATION

(dollars in thousands)

Policy	Fourth Retro Adjustment			Fifth Retro Adjustment			Sixth Retro Adjustment		
Eff.	Loss	Prem	PDLD	Loss	Prem	PDLD	Loss	Prem	PDLD
Quarter	43-54	52-63	Ratio	55-66	64-75	Ratio	67-78	76-87	Ratio
1983.3	1,925	763	0.397	2,057	712	0.346	1,170	75	0.064
1983.4	1,078	662	0.615	64	56	0.867	525	186	0.355
1984.1	1,139	883	0.776	827	526	0.636	1,123	-103	-0.092
1984.2	1,137	573	0.504	906	593	0.655	165	15	0.088
1984.3	2,949	1,159	0.393	2,619	635	0.243	2,475	137	0.055
1984.4	1,424	206	0.145	1,378	46	0.033	1,329	86	0.065
1985.1	1,538	267	0.173	2,265	120	0.053	528	615	1.165
1985.2	2,026	773	0.381	1,730	189	0.109	1,072	210	0.196
1985.3	6,525	2,670	0.409	6,604	2,611	0.395	3,566	155	0.043
1985.4	3,049	1,196	0.392	2,194	1,091	0.497	2,533	958	0.378
1986.1	1,700	1,243	0.731	3,519	874	0.248	1,477	621	0.421
1986.2	2,480	63	0.025	1,476	888	0.601	1,969	194	0.099
1986.3	5,380	2,703	0.502	8,623	1,693	0.196	4,364	1,601	0.367
1986.4	3,316	561	0.169	3,032	728	0.240	1,907	84	0.044
1987.1	5,508	1,796	0.326	4,720	1,522	0.322	1,784	69	0.039

EXHIBIT 4

PART 2—PAGE 2

1987.2	2,521	207	0.082	2,970	869	0.293	1,107	416	0.375
1987.3	7,089	2,571	0.363	3,589	2,532	0.705	1,191	-320	-0.268
1987.4	4,456	1,199	0.269	3,277	572	0.175	381	226	0.593
1988.1	3,267	1,498	0.458	2,294	82	0.036			
1988.2	2,461	894	0.363	1,086	102	0.094			
1988.3	6,284	3,014	0.363	8,696	108	0.012			
1988.4	4,351	2,528	0.581	970	698	0.720			
1989.1	798	339	0.425						
1989.2	245	147	0.601						
1989.3	2,996	1,043	0.348						
1989.4	785	472	0.601						
Selection Based on Historical Averages									
Average All			0.400			0.340			0.222
Weighted Average All			0.385			0.266			0.182
Selected			0.400			0.300			0.200
Selection Based on Retro Formula									
LCF			1.20			1.20			1.20
TM			1.03			1.03			1.03
Loss Capping Ratio			40%			40%			40%
Implied PDLD Ratio			0.494			0.494			0.494
Final Selection			0.450			0.400			0.350

EXHIBIT 5

PART 1

LOSS CAPPING RATIO CALCULATION

(with per accident limitation)

Retro Adjustment (1)	Ultimate Standard Premium Loss Ratio (2)	Percent of Total Losses Emerged (3)	Emerged Loss Ratio (2)x(3) (4)	Loss Ratio at Retro Maximum (5)	Loss Ratio at Retro Minimum (6)	Entry Ratio at Retro Maximum (5)/(4) (7)	Entry Ratio at Retro Minimum (6)/(4) (8)
First	0.700	78.4%	0.549	1.200	0.100	2.19	0.18
Second	0.700	87.7%	0.614	1.200	0.100	1.95	0.16
Third	0.700	92.1%	0.645	1.200	0.100	1.86	0.16
Fourth	0.700	95.1%	0.665	1.200	0.100	1.80	0.15
Fifth	0.700	98.0%	0.686	1.200	0.100	1.75	0.15
Sixth	0.700	99.6%	0.697	1.200	0.100	1.72	0.14
Subsequent	0.700	100.0%	0.700	1.200	0.100	1.71	0.14

EXHIBIT 5

PART 2

LOSS CAPPING RATIO CALCULATION

(with per accident limitation)

Retro Adjustment (1)	Insurance Charge at Retro Maximum (9)	Insurance Saving at Retro Minimum (10)	% of Losses Eliminated by Retro Max/Min (9)-(10) (11)	Loss Elimination Ratio from Per Accident Limitation (12)	Cumulative Loss Capping Ratios 1.0-(11)-(12) (13)	Incremental Loss Capping Ratios (14)	Selected Incremental Loss Capping Ratios (15)
First	0.113	0.004	10.9%	4.2%	84.9%	84.9%	85.0%
Second	0.133	0.003	13.0%	5.0%	82.0%	58.0%	58.0%
Third	0.142	0.003	13.9%	5.9%	80.2%	45.0%	45.0%
Fourth	0.148	0.003	14.5%	6.5%	79.0%	39.6%	40.0%
Fifth	0.154	0.003	15.1%	7.1%	77.8%	38.7%	40.0%
Sixth	0.156	0.002	15.4%	7.4%	77.2%	41.7%	40.0%
Subsequent	0.158	0.002	15.6%	7.5%	76.9%	3.3%	0.0%

Notes:

(2) By judgment.

(3) Based on loss development pattern. See Exhibit 7.

(5),(6) Based on the retro rating values on the policies sold.

(9),(10) From NCCI Table of Insurance Charges, assuming $750,000 standard premium at the entry ratios listed in Columns (7) and (8), with losses used for loss group estimation adjusted for the per accident limitation.

(12) From a study of the percentage of losses eliminated due to per accident limitation.

(14) = [(13)x(4) - (Prior 13)x(Prior 4)] / [(4) - (Prior 4)].

(15) By judgment.

EXHIBIT 6

PART 1

BOOKED PREMIUM

(dollars in thousands)

POL EFF	EVALUATED AT (MONTHS)													
QUARTER	3	6	9	12	15	18	21	24	27	30	33	36	39	42
1983.3	18,087	23,481	33,550	40,867	45,075	49,911	50,174	50,629	52,436	52,388	52,428	53,735	56,448	56,442
1983.4	7,545	10,684	15,697	19,696	22,407	24,608	25,438	25,367	26,222	26,135	26,109	26,653	28,944	28,886
1984.1	7,930	13,516	19,135	24,101	25,408	28,018	28,777	28,947	29,189	28,967	28,960	29,563	31,116	32,031
1984.2	6,277	10,386	14,770	18,422	20,366	22,441	22,258	23,261	23,422	23,608	23,714	24,239	25,326	25,319
1984.3	20,221	31,438	45,913	56,490	61,851	66,697	67,910	68,094	69,310	69,648	69,804	73,363	75,681	75,727
1984.4	9,581	21,089	29,224	38,056	41,299	43,344	43,489	43,897	43,305	43,851	43,794	44,982	46,495	46,645
1985.1	15,110	28,734	40,623	49,872	52,336	57,881	58,742	56,075	59,203	59,164	59,814	60,692	62,552	62,780
1985.2	9,345	19,304	26,189	33,115	37,010	37,650	39,114	40,131	38,717	38,855	39,247	39,753	40,836	40,780
1985.3	43,187	69,712	99,219	120,931	128,832	138,488	138,401	132,641	133,094	134,019	134,241	138,147	141,020	141,198
1985.4	18,627	36,106	51,122	61,567	63,169	67,119	68,046	67,651	66,351	67,117	67,131	68,357	70,414	70,703
1986.1	27,390	46,053	62,065	76,616	79,958	83,746	88,382	83,949	87,173	87,630	87,438	88,964	91,422	91,780
1986.2	15,906	28,997	39,082	48,402	53,764	56,713	59,748	58,482	57,654	57,171	57,156	57,801	59,937	59,994
1986.3	75,944	99,936	128,593	152,088	157,264	167,176	168,696	160,320	160,838	161,506	161,702	167,381	171,755	173,178
1986.4	34,837	47,808	61,795	75,233	79,268	83,931	85,404	82,732	84,641	84,662	83,954	85,334	88,177	89,027
1987.1	43,330	57,756	74,709	90,700	96,064	103,569	105,732	100,641	103,693	105,325	105,992	107,900	112,468	113,333
1987.2	21,776	37,152	46,687	57,190	62,681	67,875	69,209	68,077	68,397	67,799	68,083	70,573	71,863	71,866
1987.3	81,929	97,806	130,510	154,795	165,210	174,935	172,841	172,374	171,434	170,357	170,034	176,167	181,292	181,375
1987.4	40,213	56,338	71,591	88,365	91,969	98,370	99,258	99,738	101,483	101,762	101,940	104,615	107,184	107.014

EXHIBIT 6

PART 1—PAGE 2

1988.1	42,723	54,489	70,630	85,547	90,156	96,939	97,210	97,349	98,806	99,398	99,426	101,516	103,551	103,824
1988.2	22,257	34,740	43,526	53,014	58,682	62,781	62,671	62,232	63,885	63,771	63,912	66,090	66,573	66,593
1988.3	72,106	88,814	119,264	144,686	150,957	155,317	157,645	157,669	161,154	159,364	159,056	163,070	167,796	167,503
1988.4	39,617	54,419	70,154	90,176	96,199	100,143	101,400	103,318	109,253	109,553	109,928	111,601	113,227	113,259
1989.1	39,587	52,001	70,440	86,851	90,443	92,953	95,370	95,048	99,777	99,118	98,874	99,321	100,691	100,996
1989.2	22,202	34,226	45,230	58,091	61,548	63,860	65,452	65,312	67,553	67,514	67,325	68,442	70,476	70,691
1989.3	65,381	87,768	116,546	143,674	150,281	154,254	155,500	153,310	153,443	152,289	151,898	153,635	159,968	162,505
1989.4	39,328	59,317	79,875	101,224	107,600	109,565	108,541	107,785	104,838	105,096	106,006	106,285	113,531	113,513
1990.1	41,480	58,906	82,540	101,782	105,099	109,581	107,738	108,319	107,468	108,595	108,481	112,426	114,523	114,013
1990.2	24,980	44,112	57,612	69,544	72,525	73,996	74,014	72,759	70,127	71,153	71,796	73,669	75,822	75,729
1990.3	63,128	90,937	124,828	150,974	157,555	164,785	161,333	157,804	158,027	154,559	155,939	161,526	164,815	164,057
1990.4	39,431	56,685	75,859	96,782	102,985	103,811	105,995	102,283	91,918	90,293	89,057	92,789	94,759	94,621
1991.1	33,478	44,023	65,178	77,912	80,929	82,418	85,231	83,748	81,901	81,593	81,741	83,321	84,497	84,444
1991.2	14,280	28,297	40,717	52,368	56,505	57,790	58,229	54,008	54,045	53,647	53,094	54,363	54,711	54,940
1991.3	45,699	57,962	76,470	90,684	94,509	99,243	96,431	97,296	94,797	95,483	95,037	95,884	97,348	97,351
1991.4	40,472	55,839	71,234	96,004	103,650	110,764	108,317	107,345	97,650	97,275	97,453	98,816	100,098	
1992.1	33,004	48,424	63,007	77,569	80,923	83,160	82,743	79,426	82,057	81,207	82,305	83,654		
1992.2	15,320	28,687	42,953	55,414	59,215	59,989	61,432	62,709	59,279	59,531	59,450			
1992.3	43,587	62,778	79,724	96,319	102,225	102,001	100,433	101,661	99,074	98,745				
1992.4	39,809	55,591	64,921	84,643	91,176	95,711	95,421	96,272	88,367					
1993.1	23,915	33,188	43,949	53,188	56,028	57,200	57,259	57,304						
1993.2	18,976	30,311	39,234	46,519	49,357	50,776	50,887							
1993.3	45,269	55,794	70,411	84,974	87,886	89,838								
1993.4	28,913	40,173	51,901	64,487	69,025									
1994.1	24,902	33,572	41,305	49,737										
1994.2	20,642	28,772	34,494											
1994.3	42,916	55,331												
1994.4	18,975													

EXHIBIT 6

PART 2

BOOKED PREMIUM

(dollars in thousands)

POL EFF	EVALUATED AT (MONTHS)														
QUARTER	45	48	51	54	57	60	63	66	69	72	75	78	81	84	87
1983.3	56,698	57,448	58,799	58,904	58,875	59,306	59,562	59,301	59,335	59,492	60,275	60,309	60,321	60,221	60,350
1983.4	28,923	29,122	29,520	29,490	29,463	29,492	30,182	30,048	30,048	30,204	30,238	30,172	30,164	30,041	30,424
1984.1	31,993	31,896	32,202	32,473	32,489	32,513	33,085	32,997	33,046	33,223	33,612	33,612	33,612	33,735	33,509
1984.2	25,433	25,717	26,066	26,095	26,330	26,365	26,639	26,622	26,944	26,961	27,232	27,241	27,070	27,086	27,247
1984.3	76,163	77,812	79,113	79,140	79,358	79,730	80,272	79,940	80,017	80,328	80,907	80,813	80,850	80,934	81,044
1984.4	46,535	47,080	47,768	47,815	47,798	47,833	47,974	47,844	47,424	47,527	48,020	47,977	47,971	47,873	48,106
1985.1	62,880	63,239	63,899	64,075	64,072	63,911	64,166	63,818	63,785	63,794	64,286	64,360	64,348	64,502	64,901
1985.2	41,156	41,412	42,523	42,446	42,635	42,814	43,296	42,989	42,975	43,080	43,485	43,517	43,371	43,682	43,695
1985.3	141,185	143,140	144,073	144,137	144,236	145,032	146,743	146,395	146,591	147,980	149,354	149,163	149,109	149,224	149,509
1985.4	70,606	71,420	72,974	72,741	73,185	73,342	74,170	73,882	73,688	74,169	75,261	75,226	74,893	75,060	76,219
1986.1	91,767	92,532	93,720	94,000	93,988	94,319	94,963	94,514	94,611	94,869	95,837	95,762	95,762	96,108	96,458
1986.2	60,463	61,202	61,919	61,909	61,941	61,664	61,982	61,909	62,439	62,630	62,869	63,097	63,109	63,093	63,064
1986.3	173,315	174,277	179,686	179,875	180,311	181,404	182,390	181,594	181,531	181,942	184,082	184,903	184,896	185,286	185,683
1986.4	89,240	90,156	91,756	91,817	91,734	91,698	92,317	92,091	91,988	92,023	93,045	93,071	93,055	92,899	93,129
1987.1	113,294	114,273	115,455	115,468	115,768	116,457	117,251	116,837	116,912	117,887	118,773	118,924	118,864	118,536	118,843
1987.2	71,824	72,598	72,856	73,264	73,165	72,941	73,063	72,576	72,622	73,138	73,933	74,079	74,081	74,175	74,348

EXHIBIT 6

PART 2—PAGE 2

1987.3	181,456	183,852	185,481	185,695	185,960	185,921	188,052	188,062	187,991	188,412	190,584	190,639	190,735	190,555	190,265
1987.4	106,769	108,691	109,474	109,590	109,674	109,553	110,673	110,406	110,472	110,766	111,245	111,243	111,236	111,279	111,472
1988.1	103,719	103,937	104,557	104,769	104,822	104,662	106,055	105,656	105,466	105,683	106,137	106,218	106,230	106,292	
1988.2	66,453	66,542	67,426	67,585	67,579	68,302	68,320	68,055	67,964	67,923	68,422	68,448	68,462		
1988.3	167,536	168,059	170,650	172,206	172,539	172,583	173,664	172,956	172,995	172,960	173,772	173,984			
1988.4	113,847	114,158	115,831	115,967	115,909	116,218	118,360	117,566	117,541	117,189	119,058				
1989.1	101,165	102,126	102,822	102,919	102,750	102,955	103,161	103,367	103,574	103,781					
1989.2	70,687	72,016	72,382	72,362	72,385	72,457	72,529	72,602	72,675						
1989.3	162,799	163,947	165,700	166,040	165,811	165,390	166,743	166,034							
1989.4	113,494	115,806	117,754	117,872	117,990	118,108	118,226								
1990.1	114,032	114,738	114,842	113,920	113,959	113,443									
1990.2	75,670	76,523	76,981	77,086	76,983										
1990.3	164,447	164,886	165,716	166,107											
1990.4	94,655	94,805	95,986												
1991.1	84,616	85,643													
1991.2	54,874														

EXHIBIT 7

PART 1

REPORTED LOSSES

(dollars in thousands)

POL EFF QUARTER	EVALUATED AT (MONTHS) 3	6	9	12	15	18	21	24	27	30	33	36	39	42
1983.3	5,121	15,662	24,950	36,667	41,044	42,461	44,191	45,528	46,321	47,976	48,898	49,439	50,413	52,508
1983.4	1,336	5,853	10,153	15,218	18,928	20,151	21,293	21,821	22,244	22,889	23,270	23,466	23,802	24,369
1984.1	2,746	6,798	11,408	17,227	20,523	23,076	23,954	24,351	24,730	25,218	25,765	26,065	26,897	27,294
1984.2	1,393	5,284	8,929	14,264	18,651	19,243	20,080	20,759	20,954	20,276	20,143	20,504	20,549	20,782
1984.3	6,618	17,632	31,538	46,077	51,318	54,927	58,153	59,814	61,636	63,827	65,053	65,301	66,273	67,631
1984.4	2,417	9,115	17,939	24,030	30,204	33,393	35,277	36,237	36,887	37,701	38,636	39,640	39,583	40,520
1985.1	3,847	13,981	22,898	34,132	41,523	46,100	46,809	47,718	48,853	49,485	50,490	50,804	50,748	51,796
1985.2	2,164	6,559	12,772	20,433	26,052	27,696	28,720	29,155	29,654	30,375	31,281	31,597	32,095	33,051
1985.3	11,514	34,201	57,070	84,782	92,911	96,041	98,225	101,501	103,660	105,758	106,336	108,721	110,196	112,223
1985.4	4,252	14,692	28,032	38,282	46,101	49,481	52,737	54,006	55,152	56,674	58,126	59,382	59,953	60,942
1986.1	6,670	20,522	32,059	46,939	56,233	63,095	63,724	65,044	65,887	68,960	69,613	70,511	71,368	73,005
1986.2	3,531	10,917	19,770	31,772	40,050	42,163	43,055	43,374	44,117	46,067	46,648	47,551	47,994	49,949
1986.3	14,331	38,985	63,615	99,612	109,338	115,105	118,102	121,840	124,300	127,112	129,821	131,477	133,952	139,149
1986.4	4,768	17,534	32,808	45,213	54,351	58,712	61,941	62,548	63,559	65,340	66,158	66,925	67,544	69,077
1987.1	8,142	23,354	38,500	56,764	67,709	77,373	79,521	81,580	83,222	85,252	86,910	87,978	88,660	90,047
1987.2	4,329	13,671	25,907	37,740	49,263	49,770	50,940	52,428	53,018	54,637	56,396	56,994	57,460	58,666
1987.3	13,373	36,138	63,600	101,166	113,655	120,053	124,721	128,107	130,850	135,171	137,613	139,866	141,475	144,080
1987.4	6,190	20,923	41,460	56,846	67,724	73,502	76,331	76,489	78,388	80,981	82,914	84,044	84,277	86,082
1988.1	6,916	20,545	34,772	50,554	63,347	71,999	73,728	74,396	75,834	78,082	79,412	80,223	80,531	82,220
1988.2	4,087	11,179	21,183	34,005	44,314	45,861	46,854	47,968	49,482	51,114	52,248	53,089	53,407	54,507
1988.3	12,952	35,571	63,806	98,684	110,159	115,461	120,036	123,446	124,724	128,922	131,384	133,661	134,948	136,050
1988.4	5,451	22,014	40,250	57,358	71,190	79,063	85,346	85,263	86,493	86,786	88,053	88,746	89,616	91,868

EXHIBIT 7

PART 1—PAGE 2

1989.1	7,869	21,725	34,204	50,512	62,907	71,471	71,946	72,275	72,442	75,215	76,405	77,279	77,522	79,677
1989.2	3,615	12,348	23,914	36,257	46,573	49,486	50,566	50,507	51,567	52,462	54,081	54,467	54,940	55,296
1989.3	11,397	33,593	61,655	92,142	102,949	108,330	112,488	116,584	119,203	121,216	124,649	125,375	126,285	127,949
1989.4	4,842	19,715	39,063	52,236	64,224	72,082	76,852	78,864	80,320	82,985	87,691	88,199	88,955	90,273
1990.1	9,511	25,559	39,469	56,952	68,215	76,452	78,668	79,502	81,249	85,440	86,905	86,250	86,565	87,639
1990.2	3,836	12,606	22,351	33,833	42,925	46,393	48,984	50,954	52,012	52,906	53,893	53,431	53,847	54,473
1990.3	11,677	32,321	57,374	83,863	94,633	102,035	106,144	109,426	112,652	115,521	116,449	116,762	116,920	117,202
1990.4	5,196	14,997	30,880	42,939	52,459	57,548	62,316	63,183	64,537	66,070	70,479	70,789	71,707	72,236
1991.1	7,133	19,024	29,513	39,914	47,837	54,037	56,184	56,093	56,697	57,620	59,265	59,537	59,577	60,025
1991.2	2,129	6,883	13,419	20,833	28,953	30,240	31,317	32,484	32,779	33,064	33,768	33,584	33,779	33,771
1991.3	5,053	14,983	30,591	47,533	53,959	55,325	55,647	57,417	57,666	58,493	59,106	59,916	59,814	60,696
1991.4	3,797	14,259	28,024	40,545	48,147	54,302	58,775	58,620	58,270	59,380	59,800	60,963	61,068	
1992.1	6,135	16,850	27,327	36,669	43,512	49,161	51,595	52,060	51,888	52,340	53,333	53,455		
1992.2	3,052	8,282	16,036	24,417	31,064	33,060	35,210	35,543	36,320	36,680	37,393			
1992.3	4,619	15,383	31,122	45,310	50,764	53,069	55,737	59,254	60,047	62,118				
1992.4	3,596	12,562	23,922	33,822	41,006	44,785	47,814	49,581	50,766					
1993.1	3,786	12,318	18,973	26,870	31,880	34,297	36,340	39,519						
1993.2	2,091	7,172	12,834	20,247	24,915	27,847	30,286							
1993.3	5,349	14,393	25,756	40,138	46,561	51,005								
1993.4	2,881	10,585	20,030	27,906	34,516									
1994.1	6,241	9,031	14,755	21,189										
1994.2	1,357	6,287	11,381											
1994.3	5,083	14,339												
1994.4	1,862													
Wt'd Avg 16	2.924	1.820	1.454	1.192	1.092	1.050	1.026	1.016	1.022	1.024	1.007	1.006	1.014	1.016
Selected	2.924	1.820	1.454	1.192	1.092	1.050	1.026	1.016	1.022	1.024	1.007	1.006	1.014	1.016
Cumulative	12.856	4.397	2.416	1.661	1.393	1.276	1.215	1.184	1.166	1.140	1.114	1.107	1.100	1.085
% Emerged	7.8%	22.7%	41.4%	60.2%	71.8%	78.4%	82.3%	84.4%	85.8%	87.7%	89.8%	90.4%	90.9%	92.1%

EXHIBIT 7

PART 2

REPORTED LOSSES

(dollars in thousands)

POL EFF QUARTER	EVALUATED AT (MONTHS) 45	48	51	54	57	60	63	66	69	72	75	78	81	84	87
1983.3	53,010	52,952	52,987	54,433	54,844	54,908	55,464	56,489	56,740	56,907	56,968	57,660	58,013	58,319	58,893
1983.4	24,713	24,840	25,085	25,447	25,598	25,913	25,430	25,511	25,527	25,784	25,905	26,036	26,257	26,439	26,396
1984.1	27,873	28,168	28,054	28,433	28,783	28,888	28,887	29,261	29,942	29,959	29,857	30,384	30,332	30,414	30,496
1984.2	21,110	21,056	21,408	21,919	22,129	22,130	22,260	22,825	22,942	22,721	22,697	22,990	23,351	23,331	23,639
1984.3	68,177	68,656	69,571	70,581	70,598	70,630	70,699	73,199	73,429	73,673	74,127	75,675	76,027	76,207	76,291
1984.4	41,687	41,786	41,658	41,944	42,229	42,602	42,760	43,323	43,870	44,107	44,278	44,652	45,046	45,296	45,746
1985.1	52,028	52,557	52,925	53,334	54,047	54,796	55,246	55,600	55,619	55,744	55,635	56,128	56,031	56,762	56,822
1985.2	33,713	33,797	34,161	35,077	35,629	35,888	35,816	36,806	37,422	37,734	37,721	37,878	37,977	38,073	38,029
1985.3	113,596	114,633	115,792	118,749	120,458	121,027	121,826	125,352	126,865	127,252	127,387	128,918	129,448	129,815	130,355
1985.4	61,543	62,769	63,048	63,992	64,759	65,053	65,140	66,186	67,419	68,036	68,140	68,719	69,068	69,417	69,467
1986.1	73,711	73,961	73,350	74,704	76,207	76,344	77,005	78,223	79,734	80,044	80,055	79,700	79,828	80,048	80,598
1986.2	50,459	50,798	51,492	52,429	52,185	52,937	53,475	53,905	54,533	54,659	54,858	55,874	55,290	55,033	55,170
1986.3	141,803	141,276	142,446	144,528	148,673	149,820	150,424	153,151	154,266	155,533	156,039	157,514	158,046	157,870	158,282
1986.4	70,913	71,454	71,585	72,392	73,624	73,956	74,188	75,425	76,073	76,103	76,481	77,332	77,791	77,708	77,451
1987.1	91,273	91,974	92,426	95,555	96,936	97,662	98,212	100,275	100,547	100,919	101,009	102,059	102,416	102,215	102,064
1987.2	59,490	59,750	60,308	61,187	62,269	62,746	62,840	64,157	64,589	64,652	64,690	65,264	65,408	65,176	65,339
1987.3	146,880	148,007	148,826	151,169	152,187	152,686	153,406	154,758	155,466	155,732	156,009	155,950	156,064	155,915	155,738
1987.4	88,294	89,146	89,288	90,538	92,262	92,860	92,825	93,815	93,787	93,977	93,987	94,197	94,251	94,124	94,067

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1988.1	83,299	83,605	84,078	85,487	85,962	86,393	86,795	87,781	89,697	89,780	89,457	89,473	89,042	88,908	
1988.2	55,421	55,937	56,157	56,968	57,732	57,913	57,606	58,054	58,287	58,285	57,871	57,691	57,763		
1988.3	139,179	139,464	140,598	142,334	145,198	150,268	150,526	151,031	151,733	151,487	151,590	152,121			
1988.4	92,734	93,316	94,645	96,219	97,757	97,788	97,436	97,189	97,467	97,076	96,809				
1989.1	80,263	79,879	79,483	80,475	81,137	81,113	80,897	81,253	81,546	81,384					
1989.2	55,591	55,467	55,322	55,541	55,780	55,751	55,663	55,652	55,898						
1989.3	129,957	129,598	129,944	130,944	131,516	131,650	131,233	131,539							
1989.4	91,621	91,261	91,296	91,058	91,020	91,296	91,423								
1990.1	91,080	90,996	90,437	89,844	89,689	89,715									
1990.2	55,529	55,502	55,535	55,556	56,032										
1990.3	117,673	117,528	117,746	118,268											
1990.4	73,643	73,101	73,037												
1991.1	60,440	60,399													
1991.2	34,136														
Wt'd Avg 16	1.001	1.003	1.011	1.012	1.007	1.002	1.010	1.008	1.002	1.000	1.006	1.002	1.001	1.001	
Selected	1.001	1.003	1.011	1.012	1.007	1.002	1.010	1.008	1.002	1.000	1.006	1.002	1.001	1.001	Tail
Cumulative	1.068	1.067	1.063	1.052	1.039	1.032	1.030	1.020	1.012	1.010	1.010	1.004	1.002	1.001	1.000
% Emerged	93.6%	93.8%	94.0%	95.1%	96.2%	96.9%	97.1%	98.0%	98.8%	99.0%	99.1%	99.6%	99.8%	99.9%	100.0%

DISCUSSION OF PAPER PUBLISHED IN VOLUME LXXXI

UNDERWRITING BETAS—THE SHADOWS OF GHOSTS

THOMAS J. KOZIK

DISCUSSION BY SHOLOM FELDBLUM

Abstract

For fifteen years, academicians peering into insurance ratemaking have led us on a chase of underwriting betas, in pursuit of economic and normative notions of "equilibrium rates of return" and "fair rates of return." Underwriting betas, we were told, would elevate actuarial ratemaking to financial pricing—if only we could grasp hold of these will-o'-the-wisp emanations from modern portfolio theory. Now Tom Kozik tells us: "Leave the chase, for these betas are ghosts."

1. ACADEMICIANS AND PRACTITIONERS

Are we to be downcast—for we will never catch our prey? Or are we to be bemused—for underwriting betas were never more than academic diversions from marketplace pricing?

What exactly are underwriting betas? The Capital Asset Pricing Model (CAPM) says that the expected return on an asset is a linear function of that asset's risk. The risk to be considered, however, is only systematic risk, or non-diversifiable risk. This is the price volatility that all assets have in common, and it is measured by the covariance of the asset's return with the market return. Unique risks, or risks specific to an individual asset, are not relevant for forecasting expected returns. Investors, says the CAPM, are not rewarded for risks that can be eliminated by portfolio diversification.

Those are market betas; what about underwriting betas? Financial analysts have extended modern portfolio theory from investment securities to corporate operations. The expected return on any project, we are told, depends on the project's beta, or the systematic risk of the project. Thus, the return on underwriting operations depends on the systematic risk of underwriting, or the covariance of underwriting returns with the general market returns.

Herewith began the search for underwriting betas. Academicians led the effort, with studies of accounting betas and inferred betas, annual betas and quarterly betas, positive betas, negative betas, and null betas.

Practitioners, however, use a simple rule of thumb: New concepts are not accepted solely on the basis of obscure mathematical formulas. Rather, they must also make intuitive sense. Consider underwriting betas. How is the profitability of insurance operations related to market returns? Well, in prosperous years people drive more and take more vacations, so perhaps auto liability claims increase. Also, in prosperous years, firms hire many inexperienced employees, so perhaps workers compensation claims increase. But during recessions, thefts become more frequent, so perhaps auto comprehensive claims increase. And during recessions, injured employees stay longer on disability (since there are fewer jobs to return to), so perhaps workers compensation claim severity increases.

The intuition is muddled. There are no convincing arguments to support either a strongly positive or a strongly negative correlation between underwriting returns and market returns. And the empirical studies? The empirical studies are equally lame, showing only insignificant relationships between underwriting returns and economic conditions.

Insurance regulation, of course, is rarely hindered by mundane facts. In 1976, James Stone, the Insurance Commissioner of Massachusetts, mandated that premium rates be set on the basis of modern portfolio theory. Fairley's work [6], whose formula for underwriting profit margins rested on his estimates of underwriting betas, became the basis of Massachusetts automobile and workers compensation ratemaking. In 1982, Fairley's formula was replaced by that of Stewart Myers and Richard Cohn [10], whose discounted cash flow model remains the lynchpin of Massachusetts bureau pricing to this day.

Myers and Cohn, both of whom are "efficient market theorists," used Fairley's estimates of underwriting betas to develop the appropriate discount rate for insurance losses. Their paper makes no attempt to advance the theory of underwriting betas. In fact, they note explicitly that their pricing model is entirely distinct from the choice of the discount rate. They used Fairley's estimate of the appropriate discount rate simply because it was already accepted by the Massachusetts Insurance Department.

Nevertheless, many actuaries associate underwriting betas with the Myers–Cohn discounted cash flow pricing model. And in fact, the annual Massachusetts hearings on automobile and workers compensation rates are replete with testimony on underwriting betas, market risk premiums, and risk-free rates.

Tom Kozik writes: "…these estimates [of underwriting betas] are increasingly being used to determine premium levels… ." Not quite. They are used only in Massachusetts, one of the last bastions of rigid rate regulation, by the auto and workers compensation rating bureaus, to whose rates all companies must adhere. Actuaries in private firms have little regard anymore for underwriting betas, and even many academicians now find the use of underwriting betas to be unproductive (see especially [4]).

In fact, the workers compensation ratemaking bureau in Massachusetts, after a thorough investigation of underwriting betas, has reached a conclusion similar to Mr. Kozik's. One can get almost any estimate one wants of underwriting betas, so pronouncements on their true values are unconvincing. Indeed, the Massachusetts bureau is now looking into replacing the Myers–Cohn pricing model with other actuarial techniques, such as the internal rate of return pricing model—assuming the Commonwealth allows it to do so.

2. ACTUARIAL RISK LOADS

The underwriting beta theorists tell us that expected returns depend on systematic risk; no reward is provided for diversifiable risk. These actuaries, then, who dismiss the theory of underwriting betas—do they believe that there is no risk in insurance operations, and that companies need no reward to compensate them for underwriting risk?

Quite the contrary. Actuarial risk theory has been aligned with the practitioners—has followed the observed practices of insurance firms. Insurers are loath to accept large risks with great uncertainty, regardless of whether this uncertainty is correlated with market returns.

What is the actuary's task here? Insurers are risk averse, no less so than other economic entities. Insurers will enter into insurance contracts with highly uncertain payoffs if they are appropriately compensated for doing so. Only the most naive of intellectuals would say to them: "You insurers are all misbehaving. Underwriting betas are insignificant, so you should accept these contracts with nothing but a risk-free return."

The practicing actuary muses: "The economic reality is that insurers demand a return even for uncertainty that is not correlated with market returns. But insurance company managements take crude guesses at the size of the needed returns. Sometimes they are too high, and they can't sell the policy; sometimes they are too low, and they lose money on the policies that they do sell. So let us quantify the needed risk loads. We will provide formulas and estimates. The ultimate test, however, remains the marketplace."[1]

Actuarial risk load theory has stayed away from underwriting betas, from Robert Miccolis's 1977 *Proceedings* paper on increased limits, through Robert Butsic's 1988 discussion paper on loss reserve discounting, Rodney Kreps's 1990 *Proceedings* paper on reinsurer risk loads, Sholom Feldblum's 1990 *Proceedings* paper on risk loads for insurers, and Stephen Philbrick's 1994 *Forum* paper on accounting for risk margins, as well as his discussion of the Feldblum paper [2, 7, 8, 9, 11, 12, 13].

3. UNDERLYING PRINCIPLES

Critics of actuarial risk theory argue that the papers listed above use divergent measures of risk—standard deviations, variances, and analogues of the CAPM beta—with no systematic principles underlying them. Instead of an actuarial theory of risk loads, there are bits and pieces of disjointed actuarial insights.

How timely it is, then, that the CAS is publishing Todd Bault's discussion of "Risk Loads for Insurers" [1] alongside Mr. Kozik's paper on "Underwriting Betas." Mr. Bault's masterful synthesis of the risk load papers shows that the disparate measures used by these actuaries are all variations on a theme, with the choice of measure dependent upon the correlation of a new risk's variability with that of the insurer's existing portfolio.

Devotees of underwriting betas are pursuing a theory long since refuted by reality. The developers of actuarial risk loads are laying a solid foundation for insurance pricing.

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[1] Particularly telling is D'Arcy and Garven's [5] discovery that, of the pricing models which they examined, the Myers–Cohn model—whether used with negative underwriting betas or null underwriting betas—was the *least* successful in predicting actual underwriting results.

4. THE REGULATOR'S PERSPECTIVE

"Wait," say the critics. "The actuarial risk load measures take the viewpoint of insurers. Insurers seek higher profit margins for all their contracts, regardless of the risks for which a reward is truly deserved. But insurance regulators are faced with a normative question. They must determine 'fair premiums,' which—as Mr. Kozik says—'meet the standards of fair returns that have been enunciated by the United States Supreme Court.'"

What, then, is the regulator's task? And what measure of risk is most relevant for the insurance regulator? The micromanagement of premium rates should not be the purview of the regulator. In the U.S. economy, markets are the arbiters of prices. When competition is robust, this price arbitration is efficient. And if one values the efficient and voluntary transfer of goods among economic entities, then the market's price arbitration is more "equitable" than the machinations of insurance regulators.

Should the regulator take no interest in risk loads? On the contrary: the regulator's primary responsibility is to mitigate the adverse consequences of insurance failures. Robert Butsic, in "Solvency Measurement for Property-Liability Risk Based Capital Applications," argues that capital requirements should be related to "expected policyholder deficit ratios" [3]. The capital requirements, it turns out, depend upon the variability of the insurer's operations, not upon the covariance of underwriting returns with market returns.[2]

5. CONCLUSION

Tom Kozik tells us to abandon the race, for we are chasing after ghosts. His advice is sound, but perhaps unneeded, for he is the solitary runner.

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[2]Myers and Cohn [10] note explicitly that their model is incomplete in that it does not consider the risk of insolvency.

But risk is becoming an increasingly important element in the casualty actuary's world: for premium determination, for loss reserve setting, and for capital requirements. Actuaries have pursued this subject along different paths, some convergent and some divergent, but never intersecting with underwriting betas. Five years ago, this subject was seen as the province of the pure actuary. Now, the quantification of risk is the practitioner's task: "What risk load is needed for discounted reserves? How should capital requirements relate to underwriting risk? How should risk loads differ between ground-up and large deductible policies?" These questions demand living answers, not ghosts.

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ADDRESS TO NEW MEMBERS—NOVEMBER 11, 1996

MICHAEL FUSCO

Let me begin by thanking CAS President Albert J. Beer for inviting me to address this distinguished and large new group. There is a bit of nostalgia here, for this tradition of asking a past president to welcome the new Associates and Fellows to the CAS began when Al was Chairman (and it was Chairman, not Chairperson back then) of the Program Committee and I was the Vice President of Programs for the CAS. Tom Murrin was the guinea pig and it has worked out so well that we are almost out of past presidents. So, Tom, get ready for round two.

To the 85 new Associates and 104 new Fellows (with one overlap), it is my privilege and pleasure to congratulate you on this significant milestone and to welcome you to the CAS. And I congratulate the accompanying persons as well for their part in the process.

How can I best describe the functions the Casualty Actuarial Society performs? It conducts examinations, holds lots of meetings, and stimulates research. You are all now a part of this and there is a give and take to it. Simply put—the Fellows (including the new Fellows) *give* the exams; the candidates (and that includes the new Associates for a short while) *take* them. The three R's that we learned in elementary school are applicable here too—the 'rithmetic is on the exams, all of us should be reading relevant papers to stay current, and a few of us will be 'riting those papers. And, a select fewer will have the good fortune to be selected as award winners. I can only hope that several of you will experience the pride of being recognized in that way by your professional peers.

The CAS is run by volunteers and I urge you to become one of them. There was a time when the CAS was much smaller in size; when it had only one vice president, not five; when it conducted fewer exams (and ones that were not partitioned); and when there was no Course on Professionalism. These and other changes occurred to enhance the organization, and the changes were made by the volunteers. My advice to you is don't watch the next series of changes happen—make them happen.

Your volunteer effort could come in the form of debt because we all owe the CAS, or it could come in the form of desire—the desire to keep on improving things for the next generation of actuaries. If the contribution comes from the latter source, you will feel a lot better for it.

I congratulate you on passing these difficult exams. It took hard work and brains to pass them, and as a result, you earned your initials—ACAS or FCAS. However, it is not your initials that employers will be buying—it is your mental and analytical prowess. It is analogous to professional sports teams who pay athletes for their physical skills, not for their trophies. The message is simple—physical skills and mental skills are items that are for sale in this world. But one thing that is not for sale is your character.

Your integrity cannot be bought. You should be paid to sign a loss reserve opinion, but not coerced to sign someone else's. I urge you to read the Code of Professional Conduct. It defines the boundaries of appropriate professional behavior. Study those boundaries, but not for the purpose of determining just how close you can get to them. I do look forward to getting to know all of you better in the coming years, but not in my role as Chairperson of the CAS Discipline Committee.

The seven to ten exams that you passed were rigorous, but not all-inclusive. You studied ratemaking and reserving and financial analysis as well as accounting, law, and insurance. You learned how to value an acquisition in financial terms; as the years pass, you will learn how to value mergers and acquisitions in human terms. Even if you never work for a company that is an acquirer or an acquiree, you will learn about process reengineering and all of its jargon like downsizing (rightsizing), strategic focus, and my personal favorite, *paradigm*. You may think the actuarial exams were real life experiences, but more are yet to come.

The CAS was formed in 1914 and each generation helped prepare the next one. Soon we will be admitting our first 21st century actuaries into the CAS. They will ask you new Fellows and Associates whether you would still become an actuary if you had it to do over again. I asked that very question of five past presidents of the CAS at a ratemaking seminar panel last March, and four of five of them answered in the affirmative. I urge you to take your own poll by asking the blue badges here at this meeting, and I expect you will be pleased with the responses.

I am a fortunate actuary in that I have an authentic crystal ball in my office. If I may suggest to the accompanying persons, this may be a very appropriate gift to a give to a Fellow for a special occasion. Actuaries are not fortune tellers, but once in a while, we make predictions. I turned to my crystal ball for two. The first has to do with time. All those hours studying for exams are now available to you. Whether they are taken up by work, by family and friends, or by the CAS is up to you. My prediction—in fact, my guarantee—is that with all that newly-found time, you still won't mow the lawn or whatever else the exams allowed you to escape.

My second prediction has to do with your careers. They will take diverse paths—many of you will stay within the property/casualty insurance industry that we actuaries have served so well for so long. Just as likely, there will be some of you who will travel the other road—who will apply your actuarial skills and your work ethic to industries different from the property/casualty industry. Both groups will be successful and I predict several CEOs from both groups.

I welcome you to the CAS, an organization of tradition and camaraderie. Your first opportunity to share in this might be the reception for new Fellows this evening when you can tell each other all of the actuary jokes you heard through the years. By the way, the one about an actuary carrying a bomb on a plane is no longer in vogue. The team spirit will continue as you join committees, or serve on panels, or even hold reunions, as past presidents do once every five years at a dinner when they repeat bad jokes about actuaries.

The CAS Executive Council has its own camaraderie and its own coach—this year our Joe Torre is Al Beer. Even though he's had a great year, he's retiring. Bob Anker will be the coach for the 1997 season and Mavis Walters the year after that. I wish them luck. And, speaking of Joe Torre, as a boy who grew up in the Bronx I must comment that for most of the Fellows in this room (including Al Beer), this is the first time they can say as Fellows that the Yankees are world champs. I sincerely hope that next year's new Fellows do not have to wait very long to say it also.

Congratulations one last time! Enjoy yourselves at this meeting, your work throughout your careers, and your profession for the rest of your lives. And, to some of my teammates in this room, I look forward to having drinks with you in the Paradigm Room later today.

PRESIDENTIAL ADDRESS—NOVEMBER 11, 1996

THE CAS LEGACY

ALBERT J. BEER

This aspect of the program is usually set aside to provide the outgoing CAS president with an opportunity to wax philosophic on those issues that he or she feels have long-term importance to the CAS. Realistically, however, I am sure it's a safe bet that very few in the audience today have emotionally vivid recollections of any prior presidential addresses. And, going way out on a limb, I will boldly predict that very few, if any, of you will someday have this presentation committed to memory.

Tempered by this expectation, I realize that the most I could ever hope to achieve today is to share with you my passion for our profession, thereby hopefully stimulating some thoughts and maybe, just maybe, even provoking some productive activity. An enormous feeling of accomplishment would come years from now, when these words were long forgotten, if one of you were to stop me in the hall or on an elevator at one of these meetings and tell me that I "made a difference."

I know that I am forever grateful to those actuaries who have made a difference in my career: Mike Fusco, Kevin Ryan, Tom Murrin, and Fred Kilbourne, who each showed me that inspirational leadership can easily accommodate a wonderful self-deprecating sense of humor. (Who can ever forget Fred's sage observation—"There are three kinds of actuaries: those that can count and those that can't.") Jim MacGinnitie, Chuck Bryan, Dave Hartman, Stan Khury, and Allan Kaufman—all of whom represent a tireless dedication to serving the profession, often at great personal sacrifice. Mary Hennessy, Mike Toothman, Mike Walters, and Dave Flynn—who showed me that the qualities of personal integrity and caring for people are not inversely related to financial success.

I have also learned that an important component in the formula for success is the ability to be humbled—which is why I am convinced that God gifted me with my dear son, Tommy. I distinctly remember one snowy morning when Tom was about twelve. When he asked for a ride to school, for some unknown reason I decided to playfully remind him that, "When Abraham Lincoln was your age, he walked seven miles through the snow to school." Tommy quickly answered, "So what's your point? When he was your age, he was President of the United States!" You've got to love kids.

Whichever qualities we personally identify as being worthy of emulating, these actuaries and hundreds of others have been inspirations to our profession and constitute what I refer to as…

The CAS Legacy

One of the easiest mistakes to make as a member of the actuarial profession is to take the prestige afforded to us for granted. I've had the good fortune of serving in a wide variety of roles, including pricing, reserving, underwriting, and management. There is little doubt in my mind that the FCAS is regarded as the most prestigious designation within the property/casualty insurance industry. Whether it be at our desks, a conference room, or a board room, our colleagues bestow upon us a special respect that is clearly unique to the actuarial profession.

Think back to some of your own experiences. How often have you perceived a noticeable reaction when either you or someone else present is introduced as an actuary? This intangible "presumption of intelligence" is an invaluable asset that is rarely bestowed elsewhere in the business environment and often grants one an initial level of credibility which would otherwise have to be earned over a prolonged period of time. Of course, I realize that this imputed credibility ultimately needs to be affirmed by a quality work product in order to be sustained. But the initial acceptance of one's credentials can often make a significant difference in the efficiency and, yes, even the success of any project, and the actuarial stereotype, as frustrating as it can be at times, often provides a perceptible advantage in the business environment.

When I share these thoughts about "imputed credibility," I think back to one particular experience in my childhood that has had a strong influence on my choices in life. When I was growing up in Queens in the late '50s and early '60s, I was obsessed with playing professional baseball. One of the factors contributing to this passion was the fact that a major league baseball player actually lived on the next block. Unfortunately, he played on teams that were brutally inept and, every year of his career, the games he played in August and September were meaningless. I'll never forget his demeanor during every World Series, year after year longing for the opportunity to play in important games and to be respected as a champion. Here was a person living every child's dream, yet feeling unfulfilled because of the lack of quality of the organization with which he was associated. Although living in New York was an important factor, it was actually this appreciation of his continual frustration that caused me to coin a phrase at the age of seven that everyone who has ever worked with me in the business world will recognize as one of my constant refrains—"If I'm going to play, I'm going to play for the Yankees!"

I realize many of you would challenge the metaphor (Lord knows, there were many years in the '70s and '80s when even the Yankees were far from "Yankee-like"), but I think we all can relate to the concept of striving to be the best. Although working for second class organizations may help build character, give me a championship environment every time. I want to emphasize that my definition of "championship caliber" is not necessarily defined by financial success. I simply am suggesting that the desire to be respected for hard work, integrity, and quality of work product should be absolutely uncompromised in any endeavor, whether it be volunteer work or high finance. I've been fortunate to have had the opportunity to work for "Yankee-class" employers and, even more importantly, to have been blessed with the ability to achieve membership in the CAS, a world-championship organization. By the way, I find it more than just a little ironic that I stand before you today concluding my year of presidency, a year in which the Yankees won the World Series . . . for the first time since the year I received my Associateship and was admitted as a member of the CAS.

Getting back to the key issue, it is obvious that the actuarial profession achieved such esteemed status through the effort and dedication of those who have gone before us. In this context, it is extremely important that we all recognize the huge debt of gratitude we owe our predecessors for crafting the foundation upon which our renowned profession rests. We are all obligated to continue this great tradition and should each be personally committed to enhancing the status of the actuary well into the future. We each share the significant responsibility of protecting and nurturing this gift of our CAS legacy.

Education

Our mission statement affirms that the CAS is the learned body of the casualty actuarial profession. As such, it is our responsibility to continue to enhance actuarial knowledge through research and education. And never before in our history has knowledge been so valuable.

I recently came across some interesting work by the management consultant, Price Pritchett. In it he makes some fascinating observations regarding the way in which we, the global community, have viewed education. In an agrarian economy (for example, the U.S. pre-1850), education was generally provided by a school system that was largely supported by the family and/or the church. In this environment, most children received a somewhat informal education during the ages of seven to approximately fourteen, at which point they were deemed to be prepared for an entire lifetime of productive work. In the industrial economy, the environment within which most of us grew up, education became the responsibility of society. (Remember Hillary Rodham Clinton's concept of the "village"?) The generally accepted norm to which we aspired was to expect schooling to run roughly from the ages of six to 22, with some "retooling" given infrequently during one's career(s).

Today, we have evolved out of the industrial age and have moved into the "information age." To support this point, consider the fact that 1991 marked the first year in history that more money was spent globally on computers and communication than on all construction, industrial, and farm equipment combined. And who can deny that today we have more computer-literate first graders than computer-literate first grade teachers.

This information era will force us all to be *perpetual* students. The idea of someday "finishing your education" has become obsolete as we begin to recognize that, in this world of accelerating change, a significant amount of the knowledge we work so hard to attain actually has a limited "shelf life." The usefulness of certain information in our service economy can diminish very quickly. Just think about the advances in technology that have occurred in our own life time. There is more computing power in most of today's hand calculators than existed in the entire world prior to 1945. Another interesting aspect of this revolution is the fact that a significant part of the responsibility for effective education has been transferred to the individual and the employer. In fact, employee learning is the fastest growing segment of education. Knowledge is power, not only for the individual, but also for the employer. The concept of a finite amount of schooling is being replaced by a continuous educational process that occurs on the job, at seminars, and in professional organizations, using interactive technology and a whole host of innovative learning tools.

This environment is both the greatest challenge and the greatest opportunity for the CAS. Our educational process has always been one that develops independent and continual value-added learning, and our members are well prepared for the demands of new projects, cross-training, and career shifts. Unfortunately, adaptive skills are no longer enough to guarantee success in the economic environment in which we live. In just the past year, I have seen two articles written by highly trained actuaries who have been "re-engineered" out of their jobs. On the brighter side, the fact that they are both now gainfully employed highlights the real point of this issue—professional knowledge will continue to be one of the most valuable resources that a person can possess. And the CAS has established itself as the preeminent learned body of worldwide casualty actuarial education. However, it must be realized that this educational process should be continually refreshed and made current at an ever-increasing rate. In many ways, the CAS has a distinct advantage over many other professional organizations in that the educational processes that we have established do not require revolutionary changes to accommodate the information age. The need to provide knowledge that is useful and broad-based in its applicability will force us to constantly challenge the scope of our work, force us to develop ever-widening skill sets, and motivate us to apply our educational foundation to many more non-traditional areas, both inside and outside the insurance industry. The narrow perspective that we and many others have of our industry was driven home a few years ago when I solemnly gathered my family together to announce that I had left consulting to join the reinsurance industry. Kevin, my precious son who has been cursed with his father's sense of humor, asked, "What's reinsurance?" In my most pedagogical tone I explained that it was the insurance of insurance companies—to which he replied "Oh, I get it. Boring squared!"

In the latest issue of the *Actuarial Review*, I shared with you the story of the young child who was excitedly drawing trees with purple crayons. When the "educated" parent happened by and pointed out that trees were, in fact, green, the child meekly pointed outside to a magnificent maple tree, glowing purple in the late fall sunset. We are at one of the most exciting times in the history of the actuarial profession and yet, perhaps, all we allow ourselves to see are "green trees." As the next century quickly approaches, if we are going to carry on the CAS legacy, we must begin to allow ourselves to have "purple tree" thoughts. We must constantly strive to seek new ways to make our skills and knowledge valuable. As your career grows, think of ways to challenge yourself! Volunteer for projects that frighten you intellectually! Don't always evaluate personal opportunities in light of short-term economics. And never, *ever*, stop learning!

This year as your president has been one of the most stimulating of my professional career, but I dread to think how much less effective it might have been if not for the support of the CAS office. I want to personally thank Alison, Jane, Jennifer, Mike, Paula, Tom, Todd—and Kathy Spicer who has had to put up with my perfectionist tendencies longer than the rest. In particular I want to single out Tim Tinsley who, in his own quietly professional way, has left an indelible mark of efficiency on the CAS. I know this isn't his style, but I'd like to ask Tim to stand as we give him a warm round of applause to acknowledge his ongoing contribution to our organization.

I once read a *Harvard Business Review* article that pointed out that when a chief executive is gifted with a bright, energetic management team, the most effective management style that can be adopted is one of "Just keep the herd moving west!" Well, let me tell you, "This here cowboy ain't no dummy!" My Executive Council—Sue, Paul, Mike, Pat, John, and Bob—were a pleasure to work with and are overwhelmingly responsible for the successes that were accomplished this year. Thank you all for your tireless support.

I've already regaled you with tales of my "sweet little boys," but I want to especially thank my daughter Katie for giving up some serious quality time with Dad this year. Katie, I'm sorry if I missed helping you with some homework and came late to a game or two, but I couldn't be any prouder of your success in school and the fact that I can (very objectively) declare you to be one of the best sixth grade point guards and shortstops in the Northeast!

And finally, a word about my lovely wife, Mary. This year has been a difficult time to be married to me. (Come to think of it, given my personality, she could rightfully say that none of our 22 years together has exactly been a day at the beach!) But CAS activities, in addition to some intense travel and business commitments, have really taxed her patience and I want to say how much I appreciate her unending support and encouragement. It takes a very unusual person to be completely at ease in all circumstances, whether it's in an evening gown at a Wall Street celebration or in a Yankee tee shirt lying in bed with me at 11 o'clock watching Seinfeld reruns. There are not very many people who can carry on informed conversations, with equal aplomb, about both leveraged buyouts and the "Soup Nazi." I'm proud to be married to the greatest wife and mother on earth. And like most significant others, her contribution to the CAS is enormous—yet goes largely unrecognized. Please help me rectify that in some small way by acknowledging my wife, Mary. I love you, babe.

And so to my son, Tom, I say "No, I never will become President of the United States—but I did get the chance to serve as president of one of the most prestigious professional organizations in the world" and, for that privilege, I want to sincerely thank all of you. The challenge I leave you with today is simple—never stop trying to make a difference and always be ready to see the "purple trees."

MINUTES OF THE 1996 CAS ANNUAL MEETING

November 10–13, 1996

BOCA RATON RESORT & CLUB, BOCA RATON, FLORIDA

Sunday, November 10, 1996

The Board of Directors held their regular quarterly meeting from noon to 5:00 p.m.

Registration was held from 4:00 p.m. to 6:00 p.m.

From 5:30 p.m. to 6:30 p.m., there was a special presentation to new Associates and their guests. All 1996 CAS Executive Council members briefly discussed their roles in the Society with the new members. In addition, David P. Flynn, who is a past president of the CAS, briefly discussed his role on the American Academy of Actuaries' (AAA) Casualty Practice Council.

A welcome reception for all members and guests was held from 6:30 p.m. to 7:30 p.m.

Monday, November 11, 1996

Registration continued from 7:00 a.m. to 8:00 a.m.

CAS President Albert J. Beer opened the business session at 8:00 a.m. and recognized past presidents of the CAS who were in attendance at the meeting, including Ronald L. Bornhuetter (1975), David P. Flynn (1992), Michael Fusco (1989), David G. Hartman (1987), Charles C. Hewitt, Jr. (1972), Allan M. Kaufman (1995), Thomas E. Murrin (1963–1964), and Michael L. Toothman (1991).

Mr. Beer also recognized special guests in the audience: Christopher D. Daykin, Immediate Past President of the Institute of Actuaries in the United Kingdom and Deputy Chairman of the International Forum for Actuarial Associations (IFAA); Willy Lenearts, Secretary-Treasurer of the International Actuarial Association; Masaaki Fujikura of the Institute of Actuaries of Japan; Wilson W. Wyatt, Jr., Executive Director of the American Academy of Actuaries; Harry H. Panjer, President-Elect of the Canadian Institute of Actuaries; and David M. Holland, President of the Society of Actuaries.

Mr. Beer then announced the results of the CAS elections. The next President will be Robert A. Anker, and the President-Elect will be Mavis A. Walters. Members of the Executive Council for 1996–1997 will be Paul Braithwaite, Vice President–Administration; Kevin B. Thompson, Vice President–Admissions; Susan T. Szkoda, Vice President–Continuing Education; Patrick J. Grannan, Vice President–Programs and Communication; and Robert S. Miccolis, Vice President–Research and Development. New members of the CAS Board of Directors will be Sholom Feldblum, Alice H. Gannon, David N. Hafling, and Richard J. Roth, Jr.

Paul Braithwaite, John J. Kollar, and Michael Miller announced the new Associates and the new Fellows. The names of these individuals follow.

NEW FELLOWS

Shawna Ackerman
Mark A. Addiego
Elise M. Ahearn
Craig A. Allen
Scott C. Anderson
Steven D. Armstrong
Douglas S. Benedict
Wayne E. Blackburn
Annie Blais
Ward M. Brooks
Tracy L. Brooks-Szegda
Lisa J. Brubaker
Richard F. Burt, Jr.
Douglas A. Carlone
Carol A. Cavaliere
Maureen A. Cavanaugh
Heather L. Chalfant
Kasing Leonard Chung
Frank S. Conde
Brian C. Cornelison
Catherine Cresswell
Joyce A. Dallessio
Behram M. Dinshaw
Jeffrey D. Donaldson
Norman E. Donelson
Yves Doyon
David M. Elkins
Martin A. Epstein
James G. Evans
Judith M. Feldmeier
John R. Ferrara
Kirsten A. Frantom
James E. Gant
Nicholas P. Giuntini
Richard W. Gorvett
Russell H. Greig, Jr.
Terry D. Gusler
Michele P. Gust

Elizabeth E. L. Hansen
Robin A. Harbage
Barton W. Hedges
Kirsten Costello Hernan
Betty-Jo Hill
David L. Homer
Sandra L. Hunt
F. Judy Jao
Christian Jobidon
Stephen H. Kantor
Timothy P. Kenefick
Michael B. Kessler
Timothy F. Koester
Louis K. Korth
Gary R. Kratzer
Howard A. Kunst
Bertrand J. LaChance
Benoit Laganiere
Matthew G. Lange
John P. Lebens
David R. Lesieur
Kenneth A. Levine
James M. Maher
Leslie R. Marlo
Kelly S. McKeethan
Robert F. Megens
Stephen V. Merkey
Camille Diane Minogue
Madan L. Mittal
Kenneth B. Morgan, Jr.
Giovanni A. Muzzarelli
David Y. Na
Peter M. Nonken
Marc F. Oberholtzer
Douglas J. Onnen
Melinda H. Oosten
Nicholas H. Pastor
Clifford A. Pence, Jr.
Daniel A. Powell
Andrew T. Rippert
Brad M. Ritter
Tracey S. Ritter
Douglas S. Rivenburgh
Sallie S. Robinson
Jay Andrew Rosen
James B. Rowland
Kenneth W. Rupert, Jr.
Jason L. Russ
David A. Russell
Sean W. Russell
Melodee J. Saunders
Letitia M. Saylor
Sara E. Schlenker
Peter R. Schwanke
Michelle G. Sheng
Elissa M. Sirovatka
Raleigh R. Skaggs, Jr.
Patricia E. Smolen
Brian M. Stoll
Edward D. Thomas
Janet A. Trafecanty
Joseph W. Wallen
John M. Woosley
Cheng-Sheng P. Wu
Edward J. Yorty
Joshua A. Zirin

NEW ASSOCIATES

Mohammed Q. Ashab
Richard J. Babel
Keith M. Barnes
Michael J. Bednarick
Michael J. Belfatti
Bruce E. Binnig
Lesley R. Bosniack
Bethany L. Cass
Henry H. Chen
Sally M. Cohen
Richard J. Currie
Sheri L. Daubenmier
John T. Devereux
Patricia J. Donnelly
Christopher S. Downey
Kevin M. Dyke
Anthony D. Edwards
Ellen E. Evans
Sylvain Fauchon
David I. Frank
Kathy H. Garrigan
Abbe B. Gasparro
James W. Gillette, Jr.
Moshe D. Goldberg
Michael D. Green
Greg M. Haft
Scott T. Hallworth
Michael B. Hawley
Jodi J. Healy
Thomas E. Hettinger

John F. Janssen	Michael B. McKnight	Cynthia L. Rice
Joseph W. Janzen	Jeffrey A. Mehalic	Chet James Rublewski
Brian E. Johnson	Jennifer Middough	Elizabeth A. Sander
Chad C. Karls	Alison M. Milford	Timothy D. Schutz
Mary C. Kellstrom	Stephen A. Moffett	Scott A. Shapiro
John Hun Kim	Lisa J. Moorey	Jill C. Sidney
Martin T. King	Roosevelt C. Mosley	Jeffery J. Smith
Elina L. Koganski	Prakash Narayan	Jay Matthew South
Andre L'Esperance	Richard D. Olsen	Catherine E. Staats
Timothy J. Landick	Abha B. Patel	Carol A. Stevenson
Betty F. Lee	Tracie L. Pencak	Roman Svirsky
Ramona C. Lee	Miriam E. Perkins	John L. Tedeschi
Brian P. LePage	Luba Pesis	Michael J. Tempesta
Steven J. Lesser	Ellen K. Pierce	Kai Lee Tse
Leslie A. Martin	Igor Pogrebinsky	Marie-Claire Turcotte
Claudia A. McCarthy	Anthony E. Ptasznik	David S. Wolfe
Patrice McCaulley	Ni Qin-Feng	Alexander G. Zhu
Douglas W. McKenzie	Andrew S. Ribaudo	

Mr. Beer then introduced Michael Fusco, a past president of the Society, who presented the Address to New Members.

Patrick J. Grannan, CAS Vice President–Programs and Communications, spoke to the meeting participants about the highlights of this meeting and what was planned in the program.

Mr. Beer then announced the recipient of the 1996 CAS Matthew S. Rodermund Service Award, Walter J. Fitzgibbon. David G. Hartman presented the 1996 CAS Charles A. Hachemeister Prize to Gregory C. Taylor for his paper, "Modelling Mortgage Insurance Claims Experience: A Case Study." The paper is published in the CAS Forum, Winter 1997 Edition, Including the Ratemaking Call Papers.

Mr. Beer then requested a moment of silence in honor of those CAS members who had passed away since November 1996. They are: Dewey G. Williams, E. Frederick Fossa, and Albert J. Walsh.

Paul Braithwaite then presented the Report of the Vice President–Administration.

Patrick J. Grannan then told attendees about the highlights of the upcoming program.

David L. Miller, chairperson of the CAS Committee on Review of Papers, announced that six *Proceedings* papers and two discussions of *Proceedings* papers would be presented at this meeting. Mr. Miller then presented the 1996 CAS Dorweiler Prize to Clive L. Keatinge for his paper, "Balancing Transaction Costs and Risk Load in Risk Sharing Arrangements."

Larry D. Zimpleman, President-Elect of the AAA, provided an update on the Casualty Policy Issues.

Christopher D. Daykin, Chairman of the IFAA, gave a presentation on the internationalization of the actuarial profession.

Mr. Beer then concluded the business session of the Annual Meeting by calling for a review of *Proceedings* papers.

After a refreshment break, Mr. Beer introduced the featured speaker, Mark Shields, a nationally syndicated political columnist.

The first general session was held from 11:00 a.m. to 12:30 p.m.:

Lloyd's Reconstruction and Renewal: A Progress Report

Moderator: Robert V. Deutsch
Executive Vice President, Chief Actuary, and Chief Financial Officer
Executive Risk, Inc.

Panelists: Donald J. Greene
Senior Partner
LeBoeuf, Lamb, Greene & Macrae

Alan Punter, Ph.D.
Managing Director
Alexander Howden Developments

David Shipley
Deputy Underwriter
Harvey Bowring & Others Syndicate

Following the general session, CAS President Albert J. Beer gave his address at the luncheon. After his address, Mr. Beer officially passed the CAS presidential gavel on to new CAS President Robert A. Anker.

After the luncheon, the afternoon was devoted to concurrent sessions. The panel presentations covered the following topics:

1. The Future of State Subsequent Injury Funds

   Moderator: Michael C. Dubin
   Manager and Consulting Actuary
   Watson Wyatt Worldwide

   Panelists: Abbe S. Bensimon
   Vice President
   General Re Strategic Solutions, Inc.

   William J. Miller
   Team Leader and Actuary
   National Council on Compensation Insurance, Inc.

   G. Kevin Saba
   Assistant State Treasurer
   Connecticut Second Injury Fund

2. Reinsurance and Alternative Funding Mechanisms

   Moderator: Robert V. Arvanitis
   Investment Banker
   Guy Carpenter & Company, Inc.

   Panelists: Paul J. Kneuer
   Vice President and Actuary
   Holborn Corporation

Scott C. Stevenson
Vice President
E. W. Blanch Capital Risk Solutions

Timothy Van Housen
Vice President, Client Strategies Group
Merrill Lynch

3. Territorial Ratemaking

Moderator: Michael A. LaMonica
Vice President and Actuary
Allstate Insurance Company

Panelists: Randall E. Brubaker
Consulting Actuary

Judith M. Feldmeier
Assistant Vice President and Chief Actuary
AAA Michigan

Debra L. Werland
Executive Director
United Services Automobile Association

4. Property Insurance in Florida

Moderator: Chad C. Wischmeyer
Principal
William M. Mercer, Inc.

Panelists: Dale S. Hammond
President and Chief Executive Officer
First Floridian Auto and Home Insurance Company

Cecil L. Pierce
Director of De-Population Programs
Florida Residential Property and Casualty JUA

Daniel Sumner
General Counsel
Florida Department of Insurance

5. Technological Innovation in Data Management

Moderator: Arthur R. Cadorine
Assistant Vice President and Associate Actuary
Insurance Services Office, Inc.

Panelists: J. Michael Boa
Communications and Research Coordinator
Casualty Actuarial Society

Peter Corbett
Principal Consultant
Price Waterhouse LLP

Israel Krakowski
Pricing Director
Northbrook Property and Casualty

Robert H. Waldman
Second Vice President
General Reinsurance Corporation

6. Q & A with the CAS Board of Directors

Moderator: Robert A. Anker
President
Lincoln National Corporation

Panelists: Regina M. Berens
Consulting Actuary
MBA, Inc.

Claudette Cantin
Consulting Actuary
Tillinghast-Towers Perrin

David L. Miller
Second Vice President and Chief Actuary
Commercial Union Insurance Company

The 1996 CAS Hachemeister Prize was also presented at this time:

"Modelling Mortgage Insurance Claims Experience: A Case Study"

by Gregory C. Taylor
Tillinghast-Towers Perrin
Sydney, Australia

After a refreshment break from 3:15 p.m. to 3:45 p.m., concurrent sessions continued, and a *Proceedings* author gave a presentation of two of his papers. Certain concurrent sessions presented earlier were repeated. Additional concurrent sessions presented from 3:45 p.m. to 5:15 p.m. were:

1. 24-Hour Coverage

Moderator: Karen P. Gorvett
Vice President and Actuary
SCOR Reinsurance Company

Panelists: Philip S. Borba, Ph.D.
Senior Consultant
Milliman & Robertson, Inc.

Seth B. Madnick
Partner
Musick, Peeler & Garrett

James W. Stevens
Management Review Specialist
Florida Department of Insurance

2. Use of Credit Reports in Personal Lines Underwriting

Moderator: John J. Kollar
Vice President
Insurance Services Office, Inc.

Panelists: Birny Birnbaum
Consulting Economist

Lamont D. Boyd
Senior Marketing Representative
FAIR, Isaac

Richard A. Smith
Senior Actuary
Allstate Insurance Company

3. Introduction to the CAS Examination Committee

Moderator: David L. Menning
Senior Associate Actuary
State Farm Mutual Automobile Insurance Company

Panelists: J. Thomas Downey
Manager, Admissions
Casualty Actuarial Society

Thomas G. Myers
Vice President
Prudential Property & Casualty Insurance Company

Richard P. Yocius
Actuary
Allstate Insurance Company

4. American Academy of Actuaries' Casualty Practice Council

Moderators: David P. Flynn
Director
American Re Asset Management, Inc.

Michael L. Toothman
Partner
Arthur Andersen LLP

The following Proceedings papers were presented:

1. "Interest Rate Risk and Capital Requirements for Property/Casualty Insurance Companies"

by Sholom Feldblum
Assistant Vice President and Associate Actuary
Liberty Mutual Group

2. "NAIC Property/Casualty Insurance Company Risk-Based Capital Requirements"

   by Sholom Feldblum
   Assistant Vice President and Associate Actuary
   Liberty Mutual Group

A reception for new Fellows and guests was held from 5:45 p.m. to 6:30 p.m., and the general reception for all members and their guests was held from 6:30 p.m. to 7:30 p.m.

Tuesday, November 12, 1996

Registration and a continental breakfast took place from 7:30 a.m. to 8:30 a.m.:

Two general sessions were held simultaneously from 8:30 a.m. to 10:00 a.m.:

Recent Developments in Targeting Underserved Urban Markets

Moderator: Steven L. Groot
President
Allstate Indemnity Company

Panelists: Daryll Fletcher
Assistant Vice President for Urban Markets
Allstate Indemnity Company

Alexander B. "Pete" Grannis
Chairperson of the Insurance Committee
New York State Assembly

Robin A. Harbage
Actuary
Progressive Insurance Company

The Actuary and Technology

Moderator: Myron L. Dye
Deputy CIO and Vice President
USAA Information Services

Panelists: Lauren M. Bloom
General Counsel
American Academy of Actuaries

Joseph A. Kazenas
I/T Manager
USAA Information Services

Joel S. Weiner
Manager
Coopers & Lybrand, L.L.P.

After a refreshment break, concurrent sessions were held from 10:30 a.m. to noon. In addition to concurrent sessions that were presented the previous day, the following four concurrent sessions, and two additional *Proceedings* papers, were presented.

1. Developments in Workers Compensation Reserving

Moderator: Anthony J. Grippa
Principal
William M. Mercer, Inc.

Panelists: Gary Blumsohn, Ph.D.
Associate Actuary
Liberty Mutual Group

Sean Downes
Vice President
Risk Data Corporation

2. Actuaries in Other Countries

Moderator: Karl P. Murphy
English Wright & Brockman

Panelists: Michael Brockman
English Wright & Brockman

3. Codification of Statutory Accounting

Moderator: Lisa Slotznick
Director
Coopers & Lybrand, L.L.P.

Panelists: Joseph Pomilia
Director of Financial Reporting and Taxation
National Association of Independent Insurers

Phillip L. Schwartz
Vice President of Financial Reporting and Associate General Counsel
American Insurance Association

John Tinsley
Special Deputy-Examination
Delaware Insurance Department

4. American Academy of Actuaries' Council on Professionalism

Moderator: Ken W. Hartwell
Sun Life of Canada

The following *Proceedings* papers were presented:

1. "Loss Prediction by Generalized Least Squares"

by Leigh J. Halliwell
Regional Actuary of Latin America
Zurich Compania de Seguros, Mexico

2. Discussion of "Underwriting Betas—The Shadows of Ghosts," by Thomas J. Kozik, PCAS LXXXI, 1994

Discussion by Sholom Feldblum
Assistant Vice President and Associate Actuary
Liberty Mutual Group

Various CAS committees met from 1:00 p.m. to 5:00 p.m. In addition, the tennis and golf tournaments were held during that time.

All meeting participants and their guests enjoyed a clam bake buffet dinner at the Boca Beach Club from 7:00 to 10:00 p.m.

Wednesday, November 13, 1996

A continental breakfast was held from 7:30 to 8:30 a.m.

In addition to concurrent sessions that had been given previously and which were repeated, one additional concurrent session and two additional *Proceedings* papers were presented. The concurrent session was:

1. Long-Term Changes in Climate and their Effect on Insurance

   Moderator: Ronald T. Kozlowski
   Consulting Actuary
   Tillinghast-Towers Perrin

   Panelists: Anthony H. Knap, Ph.D.
   Director/Senior Scientist
   Bermuda Biological Station for Research, Inc.

   Christopher W. Landsea, Ph.D.
   Research Meteorologist
   National Oceanic and Atmospheric Administration

   Eric F. Lemieux
   Vice President–Actuary
   Hamilton Services, Ltd.

The following *Proceedings* papers were presented:

1. "Personal Automobile Premiums: An Asset Share Pricing Approach for Property/Casualty Insurance"

   by Sholom Feldblum
   Assistant Vice President and Associate Actuary
   Liberty Mutual Group

2. "The Competitive Market Equilibrium Risk Load Formula for Catastrophe Ratemaking"

   by Glenn G. Meyers
   Assistant Vice President
   Insurance Services Office, Inc.

3. "Estimating the Premium Asset on Retrospectively Rated Policies"

   by Miriam Perkins
   Assistant Actuary
   Liberty Mutual Group

   Michael T. S. Teng
   Associate Actuary
   Liberty Mutual Group

The final general session was held from 10:30 a.m. to noon after a 30-minute refreshment break:

DFA, the Actuary, and Strategic Planning

Moderator: Allan M. Kaufman
Principal
Milliman & Robertson, Inc.

Panelists: Robert B. Downer
Vice President
Farmers Insurance Group

Stephen P. Lowe
Consulting Actuary
Tillinghast-Towers Perrin

Eric M. Simpson
Vice President
A.M. Best Company

Susan T. Szkoda
President
Szkoda Actuarial Services, Inc.

After the general session, Albert J. Beer announced future CAS meetings and seminars, and officially adjourned the 1996 CAS Annual Meeting at 12:05 p.m.

Attendees of the 1996 CAS Annual Meeting

The 1996 CAS Annual Meeting was attended by 359 Fellows, 199 Associates, and 241 Guests. The names of the Fellows and Associates in attendance follow.

FELLOWS

Shawna Ackerman
Mark A. Addiego
Elise M. Ahearn
Craig A. Allen
Kerry F. Allison
Scott C. Anderson
Robert A. Anker
Timothy J. Banick
Allan R. Becker
Albert J. Beer
Linda L. Bell
Douglas S. Benedict
Robert S. Bennett
Abbe Sohne Bensimon
Regina M. Berens
G. Gregory Bertles
Richard M. Beverage
Richard A. Bill
James E. Biller
Wayne E. Blackburn
Annie Blais
Robert G. Blanco
William H. Bland
Cara M. Blank
Michael P. Blivess
Gary Blumsohn
LeRoy A. Boison, Jr.
Ronald L. Bornhuetter
Peter T. Bothwell
Charles H. Boucek
David S. Bowen
Paul Braithwaite
Yaakov B. Brauner
Paul J. Brehm
Ward M. Brooks
Tracy L. Brooks-Szegda
William W. Brown, Jr.
Lisa J. Brubaker
Randall E. Brubaker
James E. Buck
George Burger
Patrick J. Burns
Claudette Cantin
Douglas A. Carlone
Christopher S. Carlson
Michael J. Cascio
Carol A. Cavaliere
Maureen A. Cavanaugh
Francis D. Cerasoli
Heather L. Chalfant
David R. Chernick
Gary C. K. Cheung
Allan Chuck
Denis Cloutier
Frank S. Conde
Robert F. Conger
Brian C. Cornelison
Francis X. Corr
Martin L. Couture
Catherine Cresswell
Frederick F. Cripe
Alan C. Curry
Joyce A. Dallessio
Curtis Gary Dean
Martin W. Deede
Jerome A. Degerness
Linda A. Dembiec
Howard V. Dempster
Lisa Nan Dennison
Robert V. Deutsch
Edward D. Dew
Anthony M. DiDonato
Behram M. Dinshaw

George T. Dodd
Michael C. Dolan
Jeffrey D. Donaldson
Norman E. Donelson
Victor G. dos Santos
Robert B. Downer
Michael C. Dubin
Myron L. Dye
Dale R. Edlefson
Gary J. Egnasko
Valere M. Egnasko
Douglas D. Eland
David M. Elkins
Jeffrey A. Englander
David Engles
Martin A. Epstein
Dianne L. Estrada
James G. Evans
Doreen S. Faga
Dennis D. Fasking
Sholom Feldblum
Judith M. Feldmeier
John R. Ferrara
Russell S. Fisher
David P. Flynn
Kirsten A. Frantom
Kenneth R. Frohlich
Michael Fusco
Scott F. Galiardo
James J. Gebhard
John A. Gibson III
Bruce R. Gifford
Judy A. Gillam
William R. Gillam
Mary K. Gise
Nicholas P. Giuntini
Olivia Wacker Giuntini
Spencer M. Gluck
Karen Pachyn Gorvett
Susan M. Gozzo-Andrews
Patrick J. Grannan
Gregory T. Graves
Ronald E. Greco
Russell H. Greig, Jr.
Cynthia M. Grim
Anthony J. Grippa
Linda M. Groh
Steven L. Groot
Terry D. Gusler
Michele P. Gust
David N. Hafling
Elizabeth E. L. Hansen
George M. Hansen
Robin A. Harbage
Christopher L. Harris
David C. Harrison
David G. Hartman
Gregory L. Hayward
Barton W. Hedges
E. LeRoy Heer
Kirsten Costello Hernan
James S. Higgins
Betty-Jo Hill
Mark J. Homan
David L. Homer
Sandra L. Hunt
James G. Inkrott
F. Judy Jao
Christian Jobidon
Eric J. Johnson
Jeffrey R. Jordan
Stephen H. Kantor
Frank J. Karlinski III
Kenneth R. Kasner
Allan M. Kaufman
Clive L. Keatinge
Anne E. Kelly
Brian Danforth Kemp
Timothy P. Kenefick
Michael B. Kessler
Joe C. Kim
Frederick O. Kist
Joel M. Kleinman
Paul J. Kneuer
Terry A. Knull
Timothy F. Koester
John J. Kollar
Louis K. Korth
Thomas J. Kozik
Ronald T. Kozlowski
Israel Krakowski
Gustave A. Krause
David J. Kretsch
Jeffrey L. Kucera
Andrew E. Kudera
Howard A. Kunst
David R. Kunze
Bertrand J. LaChance
Michael A. LaMonica
Blair W. Laddusaw
Dean K. Lamb
Matthew G. Lange
Nicholas J. Lannutti
James W. Larkin
Francis J. Lattanzio
Pierre Guy Laurin

Marc-Andre Lefebvre
Steven G. Lehmann
Urban E. Leimkuhler, Jr.
Eric F. Lemieux
Kenneth A. Levine
John J. Lewandowski
Elise C. Liebers
Stephanie J. Lippl
Stephen P. Lowe
Aileen C. Lyle
James M. Maher
Barbara S. Mahoney
Lawrence F. Marcus
Leslie R. Marlo
Isaac Mashitz
Steven E. Math
Kelly J. Mathson
Jeffrey H. Mayer
Michael G. McCarter
Charles L. McClenahan
Liam Michael McFarlane
Kelly S. McKeethan
Dennis T. McNeese
William T. Mech
David L. Menning
Stephen V. Merkey
Glenn G. Meyers
Robert S. Miccolis
David L. Miller
Mary Frances Miller
Philip D. Miller
William J. Miller
Camille Diane Minogue
Madan L. Mittal
Richard B. Moncher
Phillip S. Moore
Russell E. Moore
Robert V. Mucci
Evelyn Toni Mulder
Thomas E. Murrin
James J. Muza
Giovanni A. Muzzarelli
Nancy R. Myers
Robert J. Myers
Thomas G. Myers
David Y. Na
Kenneth J. Nemlick
Karen L. Nester
John Nissenbaum
Victor A. Njakou
Peter M. Nonken
David J. Oakden
Marc Freeman Oberholtzer
Kathy A. Olcese
Melinda H. Oosten
William L. Oostendorp
Timothy A. Paddock
Robert G. Palm
Donald D. Palmer
Jennifer J. Palo
Nicholas H. Pastor
Charles C. Pearl, Jr.
Marc B. Pearl
Wende A. Pemrick
Clifford A. Pence, Jr.
Melanie Turvill Pennington
Andre Perez
Jill Petker
Steven Petlick
Arthur C. Placek
Daniel A. Powell
Arlie J. Proctor
Mark R. Proska
John M. Purple
Alan K. Putney
Mark S. Quigley
Richard A. Quintano
Donald K. Rainey
Andrew J. Rapoport
Jerry W. Rapp
Donna J. Reed
David E. Renze
Ronald C. Retterath
Donald A. Riggins
Brad M. Ritter
Tracey S. Ritter
Douglas S. Rivenburgh
Sallie S. Robinson
Sharon K. Robinson
William P. Roland
Jay Andrew Rosen
Allen D. Rosenbach
Deborah M. Rosenberg
Sheldon Rosenberg
Gail M. Ross
Lois A. Ross
Bradley H. Rowe
James B. Rowland
Kenneth W. Rupert, Jr.
Jason L. Russ
David A. Russell
James V. Russell
Sean W. Russell

Melodee J. Saunders
Letitia M. Saylor
Christina L. Scannell
Sara E. Schlenker
David C. Scholl
Debbie Schwab
Peter R. Schwanke
Susanne Sclafane
Kim A. Scott
Margaret E. Seiter
Michelle G. Sheng
Edward C. Shoop
Melvin S. Silver
Christy L. Simon
Rial R. Simons
Elissa M. Sirovatka
Raleigh R. Skaggs, Jr.
David Skurnick
Lisa A. Slotznick
Christopher M. Smerald
Michael Bayard Smith
Richard A. Smith
Richard H. Snader
Bruce R. Spidell
Elisabeth Stadler
Barbara A. Stahley
Douglas W. Stang
Grant D. Steer
Phillip A. Steinen
Elton A. Stephenson
Brian M. Stoll
Edward C. Stone
Stuart B. Suchoff
Mary T. Sullivan
James Surrago
John A. Swift
Susan T. Szkoda
Frank C. Taylor
Michael T. S. Teng
Alain Thibault
Edward D. Thomas
Kevin B. Thompson
Thomas C. Toce
Michael L. Toothman
Cynthia J. Traczyk
Janet A. Trafecanty
William R. Van Ark
John V. Van de Water
Gary G. Venter
Jennifer A. Violette
Gerald R. Visintine
Lawrence A. Vitale
James C. Votta
Michael G. Wacek
Gregory M. Wacker
Robert H. Wainscott
Christopher P. Walker
Thomas A. Wallace
Joseph W. Wallen
Lisa Marie Walsh
Mavis A. Walters
Patrick M. Walton
William F. Weimer
L. Nicholas Weltmann, Jr.
Debra L. Werland
Robert G. Whitlock, Jr.
Peter W. Wildman
Gregory S. Wilson
Chad C. Wischmeyer
Beth M. Wolfe
Richard G. Woll
Cheng-Sheng P. Wu
Paul E. Wulterkens
Richard P. Yocius
Edward J. Yorty
Ronald J. Zaleski
Joshua A. Zirin

ASSOCIATES

Christopher R. Allan
Nancy L. Arico
Mohammed Q. Ashab
Richard J. Babel
Glenn R. Balling
Joanne Balling
Michael J. Bednarick
Michael J. Belfatti
Brian K. Bell
Bruce E. Binnig
Raju Bohra
Ann M. Bok
Lesley R. Bosniack
Erik R. Bouvin
David R. Bowman
Robert E. Brancel
Steven A. Briggs
Donald R. Brockmeier
Elliot R. Burn
J’ne E. Byckovski
Arthur R. Cadorine

Kenrick A. Campbell
Bethany L. Cass
Paul A. Chabarek
Henry H. Chen
Philip S. Chou
J. Paul Cochran
Sally M. Cohen
Vincent P. Connor
Warren P. Cooper
J. Edward Costner
Richard J. Currie
Sheri L. Daubenmier
James R. Davis
Raymond V. DeJaco
Jeffrey F. Deigl
John T. Devereux
Gordon F. Diss
David A. Doe
Patricia J. Donnelly
William A. Dowell
Christopher S. Downey
Kevin M. Dyke
Jeffrey Eddinger
Thomas P. Edwalds
Dawn E. Elzinga
William E. Emmons
Robert P. Eramo
David L. Esposito
Ellen E. Evans
Charles V. Faerber
Sylvain Fauchon
Karen M. Fenrich
David I. Frank
Mary B. Gaillard
Bernard J. Galiley
Kathy H. Garrigan
Abbe B. Gasparro
Eric J. Gesick
Nathan Terry Godbold
Moshe D. Goldberg
Terry L. Goldberg
Gary Granoff
Michael D. Green
Steven A. Green
Ewa Gutman
Greg M. Haft
Leigh Joseph Halliwell
Scott T. Hallworth
Scott J. Hartzler
Michael B. Hawley
Jodi J. Healy
Paul D. Henning
Joseph A. Herbers
Thomas E. Hettinger
Jason N. Hoffman
Bernard R. Horovitz
David B. Hostetter
Jeffrey R. Hughes
Jeffrey R. Ill
John F. Janssen
Joseph W. Janzen
Brian E. Johnson
Kurt J. Johnson
Mark R. Johnson
Daniel J. Johnston
Philip A. Kane IV
Chad C. Karls
David L. Kaufman
Mark J. Kaufman
Hsien-Ming K. Keh
Mary C. Kellstrom
Steven A. Kelner
John H. Kim
Martin T. King
Elina L. Koganski
Richard Kollmar
Frank O. Kwon
Timothy J. Landick
Thomas V. Le
Brian P. LePage
Betty F. Lee
Ramona C. Lee
P. Claude Lefebvre
Stephen E. Lehecka
Elizabeth Ann Lemaster
Carl J. Leo
Steven J. Lesser
Lee C. Lloyd
Ronald P. Lowe, Jr.
Robb W. Luck
Cornwell H. Mah
Janice L. Marks
Leslie A. Martin
Malkie Mayer
Dee Dee Mays
Claudia A. McCarthy
Patrice McCaulley
Heather L. McIntosh
Douglas W. McKenzie
Jeffrey A. Mehalic
Jennifer Middough
Alison M. Milford
Neil L. Millman
Stephen A. Moffett
Andrew W. Moody
Lisa J. Moorey
Roosevelt C. Mosley

Raymond D. Muller
Kevin T. Murphy
Donna M. Nadeau
Prakash Narayan
John K. Nelson
Henry E. Newman
Douglas W. Oliver
Richard A. Olsen
Richard D. Olsen
Charles P. Orlowicz
Abha B. Patel
Kathleen M. Pechan
Tracie L. Pencak
Miriam E. Perkins
Luba Pesis
Ellen K. Pierce
Richard A. Plano
Igor Pogrebinsky
Michael D. Price
Anthony E. Ptasznik
Ni Qin-Feng
Thomas O. Rau
Andrew S. Ribaudo
Cynthia L. Rice
Christine R. Ross
Scott J. Roth
Chet James Rublewski
Elizabeth A. Sander
Michael Sansevero, Jr.
Sandra C. Santomenno
Timothy D. Schutz
Michael L. Scruggs
Barbara A. Seiffertt
Jeffrey P. Shirazi
Kerry S. Shubat
Jill C. Sidney
Janet K. Silverman
Charles Leo Sizer
Gina L. B. Smith
Jeffery J. Smith
David C. Snow
Jay Matthew South
Calvin C. Spence, Jr.
Catherine E. Staats
Carol A. Stevenson
Michael J. Steward II
Roman Svirsky
John L. Tedeschi
Michael J. Tempesta
Joseph P. Theisen
Robert W. Thompson
Kai Lee Tse
Marie-Claire Turcotte
James F. Tygh
Cynthia L. Vidal
Phillip C. Vigliaturo
Benjamin A. Walden
Robert H. Waldman
Patricia K. Walker
Gregory S. Wanner
Linda F. Ward
Joel S. Weiner
David L. Whitley
William Robert Wilkins
Mary E. Wills
Bonnie S. Wittman
Calvin Wolcott
Robert F. Wolf
David S. Wolfe
Robert S. Yenke
Alex Zhu

REPORT OF THE VICE PRESIDENT–ADMINISTRATION

In order to provide a framework for addressing several important issues facing the Casualty Actuarial Society (CAS) and its members, the Board of Directors and the Long Range Planning Committee worked throughout the year to develop a strategic plan for the CAS. This plan was approved by the Board in September 1996 and is being mailed to the membership. Our activities this year have centered around the four purposes of the CAS as stated in our constitution.

Our first purpose is to advance the body of knowledge of actuarial science in applications other than life insurance. In support of this purpose, the CAS has made significant progress in the area of research this year. As one measure of the pace of activity, the CAS completed five different prize programs and funded research projects in 1996, and another five are underway. Topics included dynamic financial models, real-world valuation of property/casualty companies, ratemaking issues such as catastrophes and territorial relativities, and reserving for workers compensation and mega-risks.

In addition, the CAS Board laid the groundwork for future research by deciding to join with the SOA in supporting a new actuarial foundation. This new organization will support research to extend the frontiers of actuarial knowledge.

Our second purpose is to establish and maintain minimum standards of qualification for membership. The admissions committees have addressed this objective by making continuous improvements to the *Syllabus* and examination process, while overseeing the testing of approximately 6,300 candidates this year. During 1996, members of the CAS Educational Task Force continued to identify needed skills for actuaries of the future and to shape our education process to develop those skills. Preliminary draft course descriptions for several examination parts are now under review by our admissions committees. The Task Force members are working in cooperation with the SOA Design Team to explore potential joint-sponsorship of several of the new exams.

Our third purpose is to promote and maintain high standards of conduct and competence for the members. To help fulfill this purpose, continuing education opportunities have increased. A wide variety of topics was offered this year, including sessions on catastrophe issues, capital and reinsurance, and emerging technologies.

Our fourth purpose is to increase the awareness of actuarial science. The CAS has addressed this by continuing to provide financial support to Forecast 2000—a project which promotes the image of the actuary and reaches out to public policy makers.

We also took a major step forward this year in our ability to reach out to various audiences by establishing the new CAS Web Site—www.casact.org—in early October. Current features include a "Students' Corner," a "Members Only" section with searchable *Yearbook* listings, a research section, and a section for people considering a career as an actuary. Already, 550 members have registered a password, and we expect explosive growth in the content and usage of the web site over the next few years.

The CAS office continues to provide excellent support and to expand its services and capabilities. New services this year included supporting the Web site, producing the *Student Newsletter*, instituting credit card payment options, and implementing conference call services for committee meetings.

In closing, I am pleased to report that our membership has continued to grow and that our financials are well in order. We have added a total of 218 new Associates and 123 new Fellows this year, which increases our membership to 2,705.

Preliminary financial statements show that fiscal year 1996 ended with net income of $459,000. Members' equity now stands at about $2.1 million. Due largely to the tremendous efforts and sacrifices of our numerous volunteers, the CAS has had an active and productive year, and has emerged stronger and more focused than before.

Respectfully submitted,
Paul Braithwaite
Vice President–Administration

FINANCIAL REPORT
FISCAL YEAR ENDED 9/30/96

OPERATING RESULTS BY FUNCTION

FUNCTION	*INCOME*	*EXPENSE*	*DIFFERENCE*
Membership Services	$ 799,765	$ 847,521 (a)	$ (47,756)
Seminars	989,810	739,432	250,378
Meetings	588,508	561,957	26,551
Exams	2,135,055	1,944,322 (b)	190,733
Publications	61,054	22,199	38,855
TOTAL	$ 4,574,192	$ 4,115,431	$ 458,760 (c)

NOTES: (a) Includes expense of $9,274 to adjust marketable securities to market value (SFAS 124).
(b) Includes $1,279,000 of Volunteer Services for income and expense.
(c) Change in CAS Surplus net of $52,000 of interfund transfers ($50,000 to Research Fund and $2,000 to ASTIN fund).

BALANCE SHEET

ASSETS	*9/30/95*	*9/30/96*	*DIFFERENCE*
Checking Account	$ 50,260	$ 149,550	$ 99,290
T-Bills/Notes	1,992,419	2,595,152	602,733
Accrued Interest	54,661	45,728	(8,933)
CLRS Deposit	5,000	0	(5,000)
Prepaid Expenses	23,810	28,405	4,595
Prepaid Insurance	7,949	8,256	307
Accounts Receivable	45,000	8,555	(36,445)
Computers, Furniture	259,800	253,266	(6,534)
Less: Accumulated Depreciation	(192,299)	(199,649)	(7,350)
TOTAL ASSETS	$ 2,246,600	$ 2,889,263	$ 642,663

LIABILITIES	*9/30/95*	*9/30/96*	*DIFFERENCE*
Exam Fees Deferred	$ 315,087	$ 328,948	$ 13,861
Annual Meeting Fees Deferred	38,359	62,675	24,316
Seminar Fees Deferred	48,028	67,376	19,348
Accounts Payable and Accrued Expenses	134,589	269,427	134,838
Deferred Rent	39,002	33,407	(5,595)
Accrued Pension	36,101	45,692	9,591
TOTAL LIABILITIES	$ 611,166	$ 807,525	$ 196,359

MEMBERS' EQUITY			
Unrestricted	*9/30/95*	*9/30/96*	*DIFFERENCE*
CAS Surplus	$ 1,324,641	$ 1,766,753	$ 442,111
CLRS Fund	5,000	0	(5,000)
Michelbacher Fund	91,292	94,856	3,564
Dorweiler Fund	5,115	4,371	(744)
CAS Trust	3,469	3,641	172
Research Fund	180,665	185,404	4,739
ASTIN Fund	4,000	6,000	2,000
Subtotal Unrestricted	1,614,182	2,061,025	446,843
Temporarily Restricted			
Scholarship Fund	7,319	7,182	(137)
Rodermund Fund	13,934	13,531	(403)
Subtotal Restricted	21,253	20,713	(540)
TOTAL EQUITY	$ 1,635,434	$ 2,081,738	$ 446,304

Paul Braithwaite, Vice President–Administration

This is to certify that the assets and accounts shown in the above financial statement have been audited and found to be correct.

CAS Audit Committee: Robert F. Conger, Chairperson;
Regina M. Berens, Anthony J. Grippa, and William M. Rowland.

Examinations for Parts 3B, 4A, 4B, 5A, 5B, 6, 8, 8C (Canadian), and 10 of the Casualty Actuarial Society were held on April 29, 30, and May 1, 2, and 3, 1996. Examinations for Parts 3B, 4A, 4B, 5A, 5B, 7, and 9 of the Casualty Actuarial Society were held on October 28, 29, 30, and 31, 1996.

Examinations for Parts 1, 2, 3A, and 3C (SOA courses 100, 110, 120, and 135) are jointly-sponsored by the Casualty Actuarial Society and the Society of Actuaries. Parts 1 and 2 were given in February, May, and November of 1996, and Parts 3A and 3C were given in May and November of 1996. Candidates who were successful on these examinations were listed in joint releases of the two societies.

The Casualty Actuarial Society and the Society of Actuaries jointly awarded prizes to the undergraduates ranking the highest on the Part 1 CAS Examination.

For the February 1996 Part 1 CAS Examination, the $200 first prize winner was Lily H. Fang of Simon Fraser University. The $100 second prize winners were: Francis J. Conlan, California State University; Michael T. Doberenz, Texas A & M University; Desmond Mak, University of Calgary; Alex Mints, Stuyvesant High School, Brooklyn; John M. Pickering, Central Washington University; and Daniel A. Stronger and Vasisht M. Vadi, both of Stuyvesant High School, New York City.

For the May 1996 Part 1 CAS Examination, the $200 first prize winners were: Na Jia, People's University of China; and Wei Sun and Yu Xiang, both of Shanghai University of Finance and Economics. The $100 second prize winners were: Xu Gu, Hui He, and Yan Zhang, all from Shanghai University of Finance and Economics.

For the November 1996 Part 1 CAS Examination, the $200 first prize winners were: Ka Ming Chow, University of Hong Kong; Ying Guo, Renmin University; and Junfeng Xie, Zhongshan University. The $100 second prize winners were: Hai Dong Li, Zhongshan University; Michael Liberov, City University of New York; Zhanzhong Liu, Hengchang Pan, and Zhu Tang Ye, all of Zhongshan University; and Dongfeng Zheng, Shanghai University.

The following candidates were admitted as Fellows and Associates at the 1996 CAS Spring Meeting in May. By passing November 1995 CAS examinations, these candidates successfully fulfilled the Society requirements for Fellowship or Associateship designations.

NEW FELLOWS

Daniel G. Carr	Cheung S. Kwan	Scott M. Miller
Gary C. K. Cheung	Mylene J. Labelle	Christina L. Scannell
Jo Ellen Cockley	Roland D. Letourneau	Jeanne E. Swanson
Daniel J. Flick	Richard S. Light	Rae M. Taylor
Wayne Hommes	Donald E. Manis	Barry C. Zurbuchen
Charles N. Kasmer	Kelly J. Mathson	
Ann L. Kiefer	David W. McLaughry	

NEW ASSOCIATES

Jeff R. Adcock	Marian M. Burkart	Louis Durocher
Nathan James Babcock	Janet P. Cappers	Dawn E. Elzinga
Kimberly M. Barnett	Joseph G. Cerreta	Vicki A. Fendley
David B. Bassi	Hsiu-Mei Chang	John D. Ferraro
Brian K. Bell	Hong Chen	Mary E. Fleischli
Eric D. Besman	Michelle Codere	Jeffrey M. Forden
Raju Bohra	William B. Cody	Christian Fournier
Kimberly Bowen	David G. Cook	Walter H. Fransen
Charles Brindamour	Matthew D. Corwin	Jean-Pierre Gagnon
Linda M. Brockmeier	Jeffrey W. Davis	Lynn A. Gehant
Lisa A. Brown	Raymond V. DeJaco	Karl Goring
Louis M. Brown	Elizabeth B. DePaolo	Jeffrey S. Goy
Robert F. Brown	John C. Dougherty	Mari L. Gray
Kirsten R. Brumley	Peter F. Drogan	John A. Hagglund
Ron Brusky	David L. Drury	Lynne M. Halliwell

Alessandrea C. Handley
Gerald D. Hanlon
Ronald J. Herrig
Daniel L. Hogan, Jr.
Eric J. Hornick
Brett Horoff
Linda M. Howell
Marie-Josee Huard
Man-Gyu Hur
James B. Kahn
Anthony N. Katz
James M. Kelly
Diane L. Kinner
Joseph P. Kirley
Brandelyn C. Klenner
Terri C. Kremenski
Steven M. Lacke
Jocelyn Laflamme
Steven W. Larson
Thomas V. Le
Guy Lecours
Jennifer M. Levine
Philip Lew
Lee C. Lloyd
Cara M. Low
Robb W. Luck
William R. Maag
Joseph A. Malsky
Betsy F. Maniloff
Joseph Marracello
Bonnie C. Maxie
David Molyneux
Matthew S. Mrozek
Karen E. Myers
Donna M. Nadeau
Kari S. Nelson
Catherine A. Neufeld
Mindy Y. Nguyen
James D. O'Malley
Kevin J. Olsen
David J. Otto
Michael G. Owen
Erica Partosoedarso
Daniel B. Perry
Michael W. Phillips
Mitchell S. Pollack
Dale S. Porfilio
David S. Pugel
Patrice Raby
Kiran Rasaretnam
Raymond J. Reimer
Christopher R. Ritter
Jeremy Roberts
Dave H. Rodriguez
Jean-Denis Roy
David L. Ruhm
Douglas A. Rupp
Romel G. Salam
Cindy R. Schauer
Christine E. Schindler
Jonathan N. Shampo
Kevin H. Shang
Kendra Barnes South
Caroline B. Spain
Theodore S. Spitalnick
William G. Stanfield
Christopher M. Steinbach
Curt A. Stewart
Lori E. Stoeberl
Deborah L. Stone
Brian K. Sullivan
Mark L. Thompson
Diane R. Thurston
Jennifer M. Tornquist
Philippe Trahan
Joseph D. Tritz
Laura M. Turner
Mary Elizabeth Waak
Edward H. Wagner
Benjamin A. Walden
Denise R. Webb
Erica L. Weida
Robert G. Weinberg
Jennifer N. Williams
Bonnie S. Wittman
Brandon L. Wolf
Kah-Leng Wong
Rick A. Workman
Michele N. Yeagley
Richard L. Zarnik

The following candidates successfully completed the Parts of the Spring 1996 CAS Examinations that were held in April and May of 1996.

Part 3B

Leah C. Adams
Michael B. Adams
Michael D. Adams
Alison M. Adrian
Michael L. Alfred
Ariff B. Alidina
Nicki C. Austin
Matt K. Bailey
John M. Barish
Nancy Barry
Nicolas Beaupre
Benjamin Beckman
John R. Bedwell
Shawn P. Beenken
Christine L. Berg
William C. Blackmon
Michael D. Blohm
Maureen A. Boyle
David C. Brueckman
John C. Burkett
Donia N. Burris
Alexander A. Bushel
Heather M. Byrne
Caryn C. Carmean
Dennis W. Carter
Daniel P. Checkman
Henry H. Chen
Karen M. Chleborad
John Clara
Laurel A. Cleary
Kiera E. Cope
Jeffrey A. Courchene
Kathleen T. Cunningham
David W. Dahlen
Amy L. DeHart
Krikor Derderian
Jeremy J. Derucki
Denis Dubois
Nathalie Dufresne
Yvonne M. Duncan
Gregory L. Dunn
Jeffrey A. Dvinoff
Kim M. Dymond
Mark Kelly Edmunds
Brian Elliott
Brandon Emlen
Kelly F. Farrell
Dana M. Feldman
Amy Elizabeth Fey
Kenneth D. Fikes
Richard G. Fisher
Martin Fortin
Jennifer J. Foshee
Jeffrey A. Gabay
Matthew P. Gatsch
Ellen M. Gavin
Patrick J. Gilhool
James W. Gillette, Jr.
Stephanie A. Groharing
Brian O. Haaseth
Philip S. Haynes
Kathryn E. Herzog
Lori Heslin
Glenn R. Hiltpold
Richard M. Holtz
Francis J. Houghton, Jr.
Derek R. Hoyme
Carol I. Humphrey
James W. Hunt
Scott R. Hurt
Jodie M. Hyland-Agan
Jennifer L. Ims
Ronald J. Jankoski
Michael S. Jarmusik
Joseph B. Johnson
William B. Johnson
Gregory K. Jones
David M. Judge
Vasilios Kakavetsis
Inessa Kantarovich
Douglas H. Kemppainen
Sean Kennedy
Sayeh Khavary
John Hun Kim
Young Y. Kim
Jeffrey D. Kimble
Omar A. Kitchlew
Henry J. Konstanty
Julie-Linda LaForce
Isabelle LaPalme

Stephane Lalancette
Jean-Francois Larochelle
Aaron M. Larson
Francis A. Laterza
Borwen Lee
Robin R. Lee
John N. Levy
Wei Li
Erik F. Livingston
Wing Lowe
Jason K. Machtinger
Kevin M. Madigan
Jodi McFarland
Patrick A. McGoldrick
Katherine F. Messerschmidt
Jennifer Middough
Silvan George Murray
John A. Nauss
Lester M. Y. Ng
Jill A. Nielsen
Kathy M. Nordness
Miodrag Novakovic
Nancy E. O'Dell-Warren
James M. Owen
Abha B. Patel
Brian S. Piccolo
David J. Pochettino
Natasha Pogrebinsky
Peter V. Polanskyj
Sherman D. Power
Gregory T. Preble
Bill Premdas
Warren T. Printz
Marie-Josee Racine
Kathleen M. Rahilly-Van Buren
R. Rebecca Raynor
Jeanne M. Rea
Mario Richard
Sophie Robichaud
Keith A. Rogers
Charles A. Romberger
Scott E. Root
Jaime J. Rosario
Nancy Ross
Nichole M. Runnels
Ray M. Saathoff
Elizabeth A. Sander
James C. Santo
Robert T. Schlotzhauer
Richard T. Schneider
Nathan A. Schwartz
Andrea T. Shafer
Scott M. Shannon
Junning Shi
Janel Sinacori
Yury G. Sivin
Sarah A. Skees
Lee O. Smith
Duff C. Sorli
Laura T. Sprouse
Anya K. Sri-Skanda-Rajah
Michael W. Starke
William J. Stone
Jason D. Stubbs
Carol G. Swaniker
Christian A. Thielman
Christopher S. Throckmorton
Gary S. Traicoff
John D. Trauffer
David Traugott
Tamara L. Trawick
Alison Tremblay
Tammy Truong
Gaetan R. Veilleux
Nathan K. Voorhis
Victoria K. Ward
Jennifer L. Weiler
Chris J. Westermeyer
William B. Westrate
Michael D. Williams
Jason L. Wrather
Jimmy L. Wright
Tricia G. Yonemoto
Joshua A. Youdovin
Stephen C. Young
Jane E. Zawistowski
Xu Zhang

Part 4A

Anurag Ahluwalia
Robert E. Allen
Melissa J. Appenzeller
Stephane Arvanitis
Julia E. Baker
Maura Curran Baker
Brent W. Barney
Chad M. Beehler
John T. Binder
Allan L. Bittner
Brenda A. Brazil
Angela D. Burgess
Alison S. Carter
Dushyant Chadha
Ruth J. Chang
Ying-Yuan Chen
Scott R. Clark
Jeffrey A. Clements
Eric J. Clymer
Marlene M. Collins
Kristine A. Compton
Christopher W. Cooney
Deanna L. Crist
Janet M. Curlee
Robert P. Daniel
Timothy A. Davis
Vickie L. Davis
Krikor Derderian
Jeremy J. Derucki
Christina M. Dethloff
Timothy M. DiLellio
Nilesh O. Dihora
Kenneth R. Dipierro
Donald R. Duley
Dana L. Eisenberg
Steven C. Ekblad
Julie A. Ekdom
Joseph G. Evleth
Weishu Fan
Mark T. Ford
Valerie A. Frate
Francois Fugere
Rosemary D. Gabriel
Anne M. Garside
Graham S. Gersdorff
Emily C. Gilde
Genady Grabarnik
Christopher J. Graham
Caroline Gregoire
Robert A. Grocock
Marie-France Groulx
Lisa N. Guglietti
David D. Hall
Kimberly Baker Hand
Jeffery T. Hay
Dianne Henry
Kathryn E. Herzog
Keith Hitchon
David E. Hodges
Jennifer L. Huber
Vibha N. Jayasinghe
Scott R. Jean
Susan K. Johnston
Elena Y. Karzhitskaya
Stephen Kcenich
Jeanne M. Keller
Chung H. Kim
Anne Marie Klein
Kirk L. Kutch
Maxime Lanctot
Travis J. Lappe
Scot M. Larson
Rocky S. Latronica
Steven R. Lindley
Diana M. S. Linehan
Bradley W. Lippowiths
William F. Loyd
Gary A. Luhovey
Kelly A. Lysaght
Elisabetta Manduchi
Timothy J. McCarthy
Jason E. Mitich
Christopher J. Monsour
Christian Morency
Robert B. Newmarker
Kari A. Nicholson
James L. Norris
Liam F. O'Connor
Sheri L. Oleshko
Jason M. Olson
Jean-Pierre Paquet
Michael T. Patterson
Isabelle Perron
Terry C. Pfeifer
Brian S. Piccolo
Kevin M. Pilarski
Jorge E. Pizarro
Paul M. Pleva
Peter V. Polanskyj
Terry W. Quakenbush
Amy M. Quinn

Christopher D. Randall
Ronald S. Rees
Choya A. Robinson
Denise A. Rosengrant
Seth A. Ruff
Julie C. Russell
Michal Ryduchowski
Deborah M. Schienvar
Nathan A. Schwartz
David G. Shafer
Marina Sieh
Laura Smith
Jason T. Sokol
Matthew R. Sondag
Harold L. Spangler, Jr.
David K. Steinhilber
Helaina I. Surabian
Brian T. Suzuki
Karrie L. Swanson
Edward T. Sweeney
Julie Ann Swisher
Stephen J. Talley
Nelson C. E. Townsend
Alice M. Underwood
Steven J. Vercellini
Leslie A. Vernon
Jeffrey J. Voss
Kyle J. Vrieze
Janet L. Wang
Jamil Wardak
David W. Warren
Dana S. Weisbrot
Paul D. Wilbert
Chad D. Wilcox
Shawn A. Wilkin
Joel F. Witt
Michael J. Yates
Hau L. Ying
Xu Zhang
Yan Zhou

Part 4B

Jason R. Abrams
Silvia J. Alvarez
Denise M. Ambrogio
Kevin L. Anderson
Melissa J. Appenzeller
Scott A. Armstrong
David S. Atkinson
Jane L. Attenweiler
Michael Augustine
Mary G. Avila
Richard J. Babel
Maura Curran Baker
Gregory K. Bangs
Rick Beam
Robert S. Beatman
Patrick Beaudoin
Stephane Beaulieu
Andrew S. Becker
Alison J. Begley
Johanne Belleau
Rosalie A. Beltramo
Herve H. Benitah
Mark D. Bequette
Scott L. Berlin
Ellen A. Berning
Allan L. Bittner
Mary Denise Boarman
Kristi M. Bohn
Alex Bondarev
Anthony C. Borelli
Dominque Boucher
Anthony J. Brantzeg
Joanne M. Briody
Robert J. Bryant
Claude B. Bunick
Donna L. Burchfield
Paul S. Burnell
Mary Ellen Cardascia
Christopher S. Carlson
Trevor Cartlidge
Bethany L. Cass
Celine Castonguay
John Celidonio
Bosco Lai-Shun Chan
Shiu Hang Chan
Yiu Fai Chan
Wayland Chau
Lai-Chin Cheah
Hongyan Chen
Wei-Hsin Chen
Yen-Cheng Chen
George Chow
Wai Yip Chow
Chris A. Christaki
James C. Christou
Andrew K. Chu
Julia F. Chu

Matthew R. Coleman
Sean O. Cooper
David J. Covey
Karen M. Cranston
William P. Cross
Loren R. Danielson
Todd J. Dembroski
Alain Desgagne
Jean-Francois Desrochers
Jean Dessureault
Francis J. Dooley
Neil P. Duffy
Sonia L. Duguay
Tammi B. Dulberger
Mark D. Edwards
Charlotte H. Ege
James R. Elicker
Warren C. Eng
Mark D. Epstein
Danielle T. Fairburn
William S. Fairburn
Robert E. Farnam
Susan M. Farquhar
Olga Faytlina
Richard D. Fearrington
Solomon C. Feinberg
Mitchell D. Fellen
Brian D. Ferguson
Ariel V. Fernando
Philip P. Ferrari
Elizabeth J. Fethkenher
Ethan S. Fish
Philip K. Fosu
Scott N. Fullerton
Cynthia Galvin
Patricia Geroulis
Esther Gewirtz
Neil P. Gibbons
Susan I. Gildea
Gianni Gioseffini
Nabiha Glennon
Andrew S. Golfin, Jr.
Paul J. Goodman
Christopher J. Grasso
Paul E. Green
John V. Grosso
Tieliu Guo
Rebecca N. Hai
David L. Handschke
Aaron G. Haning
Chuck A. Harvey
Eric C. Hassel
Michael Hebert
Jonathan S. Hede
James A. Heer
Sonja M. Heiberg
Christopher R. Heim
Michael Joseph Helewa
Timothy S. Herron
Amy L. Hicks
Glenn R. Hiltpold
Keith Hitchon
Patricia A. Hladun
Kan Tak Ho
May S. Ho
Candace Yolande Howell
Benoit Hudon
Carol I. Humphrey
Chi Yuen Hung
Paul Ivanovskis
Laura E. Jackson
James D. Jacobs
Diane M. Jakubiak
Scott R. Jean
Xiaohu Jiang
Steven M. Jokerst
Robert M. Jurgensmeier
Elena Y. Karzhitskaya
Hideaki Kazeno
Samuel J. Keller
Frederic Kibrite
Jeong Ryool Kim
Jeffrey D. Kimble
Patricia Kinghorn
Jennifer E. Kish
Jennifer S. Kjellsen
Scott M. Klabacha
David Kodama
James J. Konstanty
Matthew E. Kropp
Alex G. Kuhel
Yi-Fen Kung
Scott C. Kurban
Julie-Linda LaForce
Barbara LaVoie
Jean-Francois Lafleur
You Kim Lai
Stephane Lalancette
Michelle J. Lansink
Serge Lapierre
Peter Latshaw
Fong Ying Lisa Lau
Nadine Lavoie
Anh Tu Le

Brian P. LePage
Jean-Claude Lebel
Chanseo Lee
Hung Tak Lee
Ken Lee
Ramona C. Lee
Why-Chong Leem
Christian Lemay
Samantha Levin
Douglas A. Levy
Yishiaw Lin
Yung-Chang Lin
Zinoviy Lipkin
Fang-Lan J. Liu
Rebecca M. Locks
Gilles Lortie
Anna S. Loy
Hsi-Yen Lu
Nelson T. Lu
Yu Luo
Nhon T. Ly
James P. Lynch
Gang Ma
Geoffery W. G. Macdonell
Adrian Mackaay
Jeffrey D. Mackey
Kevin M. Madigan
Julie C. Mark
Richard A. Marko
Heather D. Marsh
Francois Martel
Meredith J. Martin
Rosemary C. Martin
Monique Masterson
Michael J. Mastricolo
Erik R. Mattson
David M. Maurer
Stephen J. McAnena
David S. McComb
Sheldon W. McDonald
Debra L. McGill
Smith W. McKee
Martin Menard
Ross H. Michehl
Matthew G. Mignault
Rose L. Miller
Richard G. Millilo
Suzanne A. Mills
Seng Yew Moi
Christopher J. Monsour
Matthew K. Moran
Vincent Morin
Lambert Morvan
Michael J. Moss
Gwendolyn D. Moyer
Malongo Mukenge
Seth W. Myers
Carole Nader
Achilles M. Natsis
Matthew R. Naughton
Sue A. Nielson
Gregory P. Nini
Chris M. Norman
Liam F. O'Connor
Shana R. O'Dell
Nancy E. O'Dell-Warren
Michael S. O'Reilly
Scott Orr
Wade H. Oshiro
Keith D. Osinski
Sophie Ouellet
Meiqing Pan
Yuhuan Niu Pandit
Ranjana S. Pannett
M. Charles Parsons
Carolyn Pasquino
Nilesh T. Patel
Jill E. Peppers
Christopher K. Perry
Claude Pichet
Isabelle Plourde
Feliks Podgaits
Darlene Pogrebinsky
Igor Pogrebinsky
Jonathan M. Pollio
Daniel W. L. Poon
Kristopher K. Presler
Diane R. Quigley
Daniel J. Rachfalski
Karen L. Raham
Evan P. Reese
Jean-Francois Reid
Kenneth F. Reiskytl
Cynthia L. Rice
Hany Rifai
Jay S. Rine
Marn Rivelle
Nancy Ross
Adam L. Rudin
Seth A. Ruff
Kelli R. Rumrill
Josef W. Rutkowski
Tracy A. Ryan
Derek T. Rylicki
Laura B. Sachs

Maher Saleh
Edith Samuels
Robert Sanche
Michelle L. Sands
James C. Santo
Raymond G. Scannapieco
Daniel D. Schlemmer
Mike B. Schofield
Tina Shaw
Seth Shenghit
Gazi Sher
Paul O. Shupe
Donna K. Siblik
Laura Smith
Neal M. Smith
William L. Smith
Lora L. Smith-Sarfo
Pierre St-Onge
Steven Strasberg
Christopher S. Strohl
Gary A. Sudbeck
Lisa M. Sukow
Pierre F. Suter
Ronnie L. Tan
Man-Kit Simon Tang
Eric D. Telhiard
Alexandra Tempelmann
Charles A. Thayer
Richard J. Theisen
Robert M. Thomas II
James A. Vallee
Alain Valois
Justin M. Van Opdorp
Robert M. VanBrackle
Jennifer A. Vandeleest
Karl C. Von Brockdorff
Kyle J. Vrieze
Melodie A. Wakefield
Hung-Ju Wang
Douglas M. Warner
Todd M. Watts
Charles M. Wells
Roy A. Wells
Amy Whinston
Marc I. Whinston
Carolyn White
Thomas J. White
Arthur S. Whitson
Gloria C. Wijangco
Timothy J. Wilder
Michelle Willcutt
Stephen P. Windsor
Dean M. Winters
Gretchen L. Wolfer
Sarah E. Woolley
Scott M. Woomer
Jennifer A. Wooster
Mark K. Yasuda
Joshua A. Youdovin
Jianhua Yu
Grace Zakaria
Kenneth X. Zheng

Part 5A

Ethan D. Allen
Deborah J. Almeida
Gwendolyn Lilly Anderson
Mary K. Anderson
David S. Atkinson
Robert D. Bachler
Frank J. Barnes
Keith M. Barnes
Andrew S. Becker
Esther Becker
David J. Belany
Kristen M. Bessette
Kevin M. Bingham
Linda J. Bjork
Lesley R. Bosniack
Rebecca S. Bredehoeft
Allison F. Carp
Nathalie Charbonneau
Ja-Lin Chen
Andrew K. Chu
Kevin M. Cleary
Susan M. Cleaver
Eric J. Clymer
Richard Jason Cook
Christopher W. Cooney
Sean O. Cooper
Paul T. Cucchiara
Kathleen T. Cunningham
Jonathan S. Curlee
Willie L. Davis
Anthony M. Di Lapi

Ryan M. Diehl
Tammi B. Dulberger
Jeffrey A. Dvinoff
Kevin M. Dyke
Anthony D. Edwards
James R. Elicker
Donna L. Emmerling
Ellen E. Evans
Danielle T. Fairburn
Carolyn M. Falkenstern
Julia M. Ford
Isabelle Gaumond
Sanjay Godhwani
Philippe Gosselin
Michael D. Green
Greg M. Haft
Susan M. Harris
Michael B. Hawley
Jodi J. Healy
William N. Herr, Jr.
Thomas E. Hettinger
Jamison J. Ihrke
Karen L. Jiron
William R. Johnson
Lawrence S. Katz
Dennis J. Keegan
Kathryn E. Keehn
Mary C. Kellstrom
Douglas H. Kemppainen
David N. Kightlinger
Young Y. Kim
Kristie L. Klekotka
Wendy A. Knopf
Tanya M. Kovacevich
Margaret J. Kuperman
Douglas H. Lacoss
Hugues Laquerre
Nathalie M. Lavigne
Eric T. Le
Sue Jean Lee
Brendan M. Leonard
Karen N. Levine
Steven E. Levitt
Cherry W. Lo
Rosemary C. Martin
James C. McPherson
Alison M. Milford
Scott P. Monard
Seth W. Myers
Jennifer L. Nelson
Gregory P. Nini
John E. Noble
Kelly A. Paluzzi
M. Charles Parsons
Javanika Patel
Bruce G. Pendergast
Anthony E. Ptasznik
Edward L. Pyle
Christopher D. Randall
Teresa M. Reis
Joanne E. Reitz
Brian E. Rhoads
Karen L. Rivara
Kathleen F. Robinson
Chet James Rublewski
Brian P. Rucci
Joseph J. Sacala
Jason T. Sash
Jeremy N. Scharnick
Gary F. Scherer
Daniel D. Schlemmer
Michael F. Schrah
Bradley J. Schroer
Anastasios Serafim
Bintao Shi
Aviva Shneider
Sven Sinclair
John J. Skowronski
Michael W. Starke
Gary A. Sudbeck
Roxann P. Swenson
Stephen J. Talley
Jonathan G. Taylor
W. Mont Timmins
Timothy J. Ungashick
Joel A. Vaag
Leslie A. Vernon
Nathan K. Voorhis
Douglas M. Warner
Vanessa C. Whitlam-Jones
Craig R. Whittinghill
Bruce P. Williams
Jerelyn S. Williams
Donald S. Wroe
Ruth Zea
Yin Zhang

Part 5B

Julie A. Anderson
Kevin L. Anderson
Mark B. Anderson
Carl X. Ashenbrenner
Afrouz Assadian
Scott P. Augutis
Patrick Barbeau
Keith M. Barnes
Nicolas Beaupre
David J. Belany
Jody J. Bembenek
Christopher D. Bohn
Mark E. Bohrer
Caleb M. Bonds
Cary J. Breese
Karen A. Brostrom
Randall T. Buda
Donia N. Burris
Anthony R. Bustillo
Sandra J. Callanan
Stephanie T. Carlson
Milissa D. Carter
Bethany L. Cass
Patrick J. Causgrove
John Celidonio
Lori Anne Cieri
Michele Cohen
Larry Kevin Conlee
Costas A. Constantinou
Richard Jason Cook
Christopher W. Cooney
Sharon R. Corrigan
David E. Corsi
Tina M. Costantino
Spencer L. Coyle
William P. Cross
Michael J. Cummiskey
Sheri L. Daubenmier
Timothy A. Davis
Nicholas J. De Palma
Nancy K. DeGelleke
D. Vance C. DeWitt
Romulo N. Deo-Campo Vuong
Jean-Francois Desrochers
Ryan M. Diehl
Melodee S. Dixon
Denis Dubois
Julie A. Ekdom
Greg J. Engl
Todd E. Fansler
Kathleen M. Farrell
Chauncey E. Fleetwood
Sean P. Forbes
Hugo Fortin
Susan I. Gildea
Matthew J. Gillette
Sanjay Godhwani
Gary J. Goldsmith
Peter S. Gordon
Philippe Gosselin
Stephanie A. Gould
Curtis A. Grosse
Greg M. Haft
Michael B. Hawley
James D. Heidt
Jason B. Heissler
William N. Herr, Jr.
Rusty A. Husted
Joseph W. Janzen
Philip J. Jennings
Kathleen M. Johnson
Steven M. Jokerst
Bryon R. Jones
Lawrence S. Katz
Lisa M. Kerns
Gary G. Kilb
Jill E. Kirby
Kristie L. Klekotka
Steven T. Knight
Elina L. Koganski
Scott C. Kurban
Nathan P. LaCombe
Bobb J. Lackey
Jean-Sebastien Lagace
Hugues Laquerre
Damon T. Lay
Brendan M. Leonard
Craig A. Levitz
Eric F. Liland
Jason K. Machtinger
Thomas J. Macintyre
Jeffrey S. Magrane
Daniel Patrick Maguire
Alexander P. Maizys
Jason A. Martin
Michael J. Mastricolo
Sarah P. Mathes
Daniel E. Mayost
George J. McCloskey

Marci A. Meyer
Alison M. Milford
Lisa J. Moorey
Erica F. Morrone
Jennifer L. Nelson
Kari A. Nicholson
Michael D. Nielsen
Jason M. Nonis
Avital Ohayon
Randall W. Oja
Sheri L. Oleshko
Michael T. Patterson
Kimberly A. Paulson
Jeremy P. Pecora
Luba Pesis
Andrea L. Phillips
Jordan J. Pitz
Dylan P. Place
Anthony E. Ptasznik
Harry L. Pylman
John T. Raeihle
Beth A. Rasmussen
Karen L. Rivara
Delia E. Roberts
Adam J. Rosowicz
Tracy A. Ryan
Rachel Samoil
Jason R. Santos
Frances G. Sarrel
Jason T. Sash
Gary F. Scherer
Christy B. Schreck
Annmarie Schuster
Ernest C. Segal
Seth Shenghit
Glenn D. Shippey
Jill C. Sidney
Robert K. Smith
Thomas M. Smith
Matthew R. Sondag
Michael W. Starke
Barry P. Steinberg
T. Matthew Steve
Karen M. Strand
Jonathan G. Taylor
Craig Tien
Colleen A. Timney
John D. Trauffer
Turgay F. Turnacioglu
David Uhland
Susan B. Van Horn
Janet L. Wang
Helen R. Wargel
David W. Warren
Kevin E. Weathers
William B. Westrate
Wendy L. Witmer
Stephanie C. Young
Kathermina Lily Yuen
Grace Zakaria
Michael R. Zarember

Part 6

Ethan D. Allen
Mark B. Anderson
Paul D. Anderson
Amy P. Angell
Wendy L. Artecona
Mohammed Q. Ashab
Carl X. Ashenbrenner
Craig V. Avitabile
Daniel M. Bankson
Michael W. Barlow
Paul C. Barone
Mary P. Bayer
Michael J. Bednarick
Michael J. Belfatti
Sheila J. Bertelsen
Frank J. Bilotti
Gina S. Binder
Bruce E. Binnig
Jonathan E. Blake
Mariano R. Blanco
Michael J. Bluzer
Daniel R. Boerboom
David R. Border
Edmund L. Bouchie
Stephane Brisson
Karen A. Brostrom
Julie Burdick
Hugh E. Burgess
Christopher J. Burkhalter
Kevin D. Burns
Stephanie T. Carlson
Sharon C. Carroll
Joyce Chen
Richard M. Chiarini
Wanchin W. Chou

Michael J. Christian
Theresa A. Christian
Brian K. Ciferri
Sally M. Cohen
Margaret E. Conroy
Jose R. Couret
Michael J. Curcio
Richard J. Currie
Harin A. De Silva
John D. Deacon
Michael B. Delvaux
Sharon D. Devanna
John T. Devereux
Mike Devine
Donna K. DiBiaso
Patricia J. Donnelly
Christopher S. Downey
Sharon C. Dubin
Rachel Dutil
Wayne W. Edwards
Kristine M. Esposito
Jonathan Palmer Evans
Alana C. Farrell
Sylvain Fauchon
Brian M. Fernandes
Benedick Fidlow
Tracy M. Fleck
David M. Flitman
David I. Frank
Donovan M. Fraser
Kevin J. Fried
Noelle C. Fries
Shina N. Fritz
John E. Gaines
James M. Gallagher
Sherri L. Galles
David E. Gansberg
Michael A. Garcia
Kathy H. Garrigan
Abbe B. Gasparro
Micah R. Gentile
Cary W. Ginter
Theresa Giunta
Moshe D. Goldberg
Jie Gong
Jay C. Gotelaere
Allen J. Gould
Daniel C. Greer
David T. Groff
Scott T. Hallworth
Alex A. Hammett
Gregory Hansen
Scott W. Hanson
Bryan Hartigan
Shlomo O. Haviv
Daniel J. Henderson
David E. Heppen
Jay T. Hieb
Christopher T. Hochhausler
Luke D. Hodge
Amy L. Hoffman
Todd H. Hoivik
Jane W. Hughes
Tina T. Huynh
Susan E. Innes
C. M. Ali Ishaq
Christopher D. Jacks
John F. Janssen
Brian E. Johnson
William Rosco Jones
Jeremy M. Jump
Chad C. Karls
Claudine H. Kazanecki
Brandon D. Keller
James F. King
Martin T. King
Kelly Martin Kingston
Andrew M. Koren
Myron W. Kraynyk
Alexander Krutov
Robin M. LaPrete
Jean-Sebastien Lagace
Timothy J. Landick
Yin Lawn
Dennis H. Lawton
Manuel Alberto T. Leal
Betty F. Lee
Daniel Leff
Neal M. Leibowitz
Bradley H. Lemons
Sally M. Levy
Michael Leybov
Janet G. Lindstrom
Michelle Luneau
Vahan A. Mahdasian
James W. Mann
David E. Marra
Leslie A. Martin
Julie Martineau
William J. Mazurek
Claudia A. McCarthy
Patrice McCaulley
Jennifer A. McCurry
Rasa Varanka McKean
Ian M. McKechnie
Douglas W. McKenzie
Michael B. McKnight

Allison M. McManus
Sarah K. McNair-Grove
Scott A. McPhee
Jeffrey A. Mehalic
William A. Mendralla
Richard E. Meuret
Paul D. Miotke
Stephen A. Moffett
David P. Moore
Jennifer A. Moseley
Roosevelt C. Mosley
Jarow G. Myers
Prakash Narayan
Helen P. Neglia
Michael D. Neubauer
Tieyan Tina Ni
Jason M. Nonis
Darci Z. Noonan
Michael A. Nori
Richard D. Olsen
Christopher E. Olson
Rebecca R. Orsi
Leo M. Orth, Jr.
Alan M. Pakula
Gerard J. Palisi
Michael A. Pauletti
Mark Paykin
Harry T. Pearce
Tracie L. Pencak
John M. Pergrossi
Miriam E. Perkins
Judy D. Perr
Ellen K. Pierce
Richard M. Pilotte
Amy A. Pitruzzello
Jennifer K. Price
Troy J. Pritchett
John K. Punzak
Harry L. Pylman
Ni Qin-Feng
Kara L. Raiguel
Andrew S. Ribaudo
Rebecca J. Richard
Nathan W. Root
Janelle P. Rotondi
William P. Rudolph
Joanne E. Russell
Brian C. Ryder
James C. Sandor
Asif M. Sardar
Michael A. Sce
Lawrence M. Schober
Michael R. Schummer
Timothy D. Schutz
Stuart A. Schweidel
Peter A. Scourtis
William H. Scully III
Meyer Shields
Aviva Shneider
Alastair Shore
Rebecca L. Simons
Donna L. Sleeth
Jeffery J. Smith
Scott G. Sobel
Jay Matthew South
George Dennis Sparks
Alan M. Speert
Benoit St-Aubin
Catherine E. Staats
Christine L. Steele-Koffke
Carol A. Stevenson
Joy M. Suh
Elizabeth A. Sullivan
Jonathan L. Summers
Roman Svirsky
Christopher C. Swetonic
Elizabeth S. Tankersley
Varsha A. Tantri
John L. Tedeschi
Michael J. Tempesta
Glenda O. Tennis
Jo D. Thiel
Sadhana Tiwari
Andy K. Tran
Jeffrey S. Trichon
Kimberly S. Troyer
Kai Lee Tse
Marie-Claire Turcotte
Jacqueline J. Verfurth
Claude A. Wagner
Colleen Ohle Walker
Tice R. Walker
Robert J. Wallace
Karen E. Watson
Lynne K. Wehmueller
Dean A. Westpfahl
Patricia C. White
Miroslaw Wieczorek
Bruce P. Williams
Wendy L. Witmer
David S. Wolfe
Simon Wong
Yuhong Yang
Alexander G. Zhu
Edward J. Zonenberg

Part 8

Jonathan D. Adkisson
Elise M. Ahearn
Christopher R. Allan
Larry D. Anderson
Michael J. Andring
Martha E. Ashman
Lewis V. Augustine
Barry Luke Bablin
Andrea C. Bautista
Cynthia A. Bentley
Wayne F. Berner
Lisa A. Bjorkman
Carol A. Blomstrom
Ann M. Bok
Douglas J. Bradac
Michael D. Brannon
James L. Bresnahan
Tracy L. Brooks-Szegda
Conni J. Brown
Russell J. Buckley
Tara E. Bush
J'ne E. Byckovski
Douglas A. Carlone
Peggy Cheng
Rita E. Ciccariello
J. Paul Cochran
Frank S. Conde
Brian C. Cornelison
Kenneth M. Creighton
Catherine Cresswell
Claudia Barry Cunniff
Charles A. Dal Corobbo
Elizabeth B. DePaolo
Brian H. Deephouse
Yves Doyon
Jennifer R. Ehrenfeld
Martin A. Epstein
Kendra M. Felisky-Watson
Mary E. Fleischli
Christian Fournier
Margaret Wendy Germani
Nicholas P. Giuntini
Annette J. Goodreau
Richard W. Gorvett
Mari L. Gray
Daniel E. Greer
Russell H. Greig, Jr.
Robin A. Harbage
Ellen M. Hardy
David S. Harris
Daniel F. Henke
Robert J. Hopper
Brett Horoff
Linda M. Howell
David D. Hudson
Walter L. Jedziniak
James B. Kahn
Thomas P. Kenia
Deborah M. King
Bradley J. Kiscaden
Gary R. Kratzer
Brian S. Krick
Salvatore T. LaDuca
Gregory D. Larcher
John P. Lebens
Guy Lecours
Kevin A. Lee
Elizabeth Ann Lemaster
Jennifer M. Levine
Christina Link
Richard B. Lord
Gary P. Maile
Leslie R. Marlo
Anthony G. Martella, Jr.
Robert F. Maton
Thomas S. McIntyre
Van A. McNeal
Robert F. Megens
James R. Merz
Madan L. Mittal
David Molyneux
Anne Hoban Moore
Matthew S. Mrozek
Turhan E. Murguz
Giovanni A. Muzzarelli
Vinay Nadkarni
Catherine A. Neufeld
Hiep T. Nguyen
Mark A. O'Brien
Marc F. Oberholtzer
Kevin J. Olsen
Milary N. Olson
Marlene D. Orr
John S. Peters
David S. Pugel
Patrice Raby

Srinivasa Ramanujam
Raymond J. Reimer
Natalie J. Rekittke
Meredith G. Richardson
Dennis L. Rivenburgh, Jr.
Douglas S. Rivenburgh
Sallie S. Robinson
Jay Andrew Rosen
Daniel G. Roth
Peter A. Royek
Jason L. Russ
Thomas A. Ryan
Manalur S. Sandilya
Linda M. K. Saunders
Melodee J. Saunders
Christine E. Schindler
Robert J. Schutte
Peter R. Schwanke
Craig J. Scukas
Terry M. Seckel
Jeffrey P. Shirazi
Jill C. Sidney
Lori A. Snyder
Klayton N. Southwood
Victoria G. Stachowski
Julia Causbie Stenberg
Deborah L. Stone
Kevin D. Strous
Christopher Tait
Janet A. Trafecanty
Robert W. Van Epps
Jeffrey A. Van Kley
Jennifer S. Vincent
Mary Elizabeth Waak
Joseph W. Wallen
Linda F. Ward
Stephen D. Warfel
Steven B. White
Elizabeth R. Wiesner
William Robert Wilkins
Michael J. Williams
Kirby W. Wisian
Jeffrey F. Woodcock
Cheng-Sheng P. Wu
Gerald T. Yeung
Edward J. Yorty
Benny S. Yuen
Doug A. Zearfoss
Alexander G. Zhu

Part 8C

Yves Doyon
Christian Fournier
David S. Harris
Guy Lecours
Kevin A. Lee
Robert F. Megens
Patrice Raby
Srinivasa Ramanujam

Part 10

Shawna Ackerman
Mark A. Addiego
Craig A. Allen
John P. Alltop
Timothy P. Aman
Scott C. Anderson
Steven D. Armstrong
Timothy W. Atwill
Douglas S. Benedict
Wayne E. Blackburn
Annie Blais
Pierre Bourassa
Margaret A. Brinkmann
Ward M. Brooks
Lisa J. Brubaker
Kirsten R. Brumley
Peter V. Burchett
Richard F. Burt, Jr.
John F. Butcher II
Carol A. Cavaliere
Maureen A. Cavanaugh
Heather L. Chalfant
Jean-Francois Chalifoux
Kasing Leonard Chung
Gary T. Ciardiello
William B. Cody
Brian C. Cornelison
Kenneth S. Dailey
Joyce A. Dallessio

Behram M. Dinshaw
Andrew J. Doll
Jeffrey D. Donaldson
Norman E. Donelson
David M. Elkins
James G. Evans
Judith M. Feldmeier
Bruce D. Fell
John R. Ferrara
John D. Ferraro
Stephen A. Finch
Ginda Kaplan Fisher
Kirsten A. Frantom
James E. Gant
Eric J. Gesick
Julie Terese Gilbert
Marvin Harlan Grove
Terry D. Gusler
Michele P. Gust
Alessandrea C. Handley
Elizabeth E. L. Hansen
Steven T. Harr
Barton W. Hedges
Kirsten Costello Hernan
Betty-Jo Hill
David L. Homer
Sandra L. Hunt
Man-Gyu Hur
Jason Israel
F. Judy Jao
Christian Jobidon
Stephen H. Kantor
Ira M. Kaplan
Lowell J. Keith
Steven A. Kelner
Timothy P. Kenefick
Michael B. Kessler
Joseph P. Kilroy
Timothy F. Koester
Louis K. Korth
Jason A. Kundrot
Howard A. Kunst
Edward M. Kuss
Andre L'Esperance
Bertrand J. LaChance
Benoit Laganiere
Matthew G. Lange
Scott J. Lefkowitz
David R. Lesieur
Kenneth A. Levine
James M. MacPhee
James M. Maher
Richard J. Marcks
Kelly S. McKeethan
Stephen V. Merkey
Camille Diane Minogue
Mark J. Moitoso
Kenneth B. Morgan, Jr.
Francois L. Morissette
David Y. Na
Peter M. Nonken
Douglas J. Onnen
Melinda H. Oosten
Nathalie Ouellet
Nicholas H. Pastor
Clifford A. Pence, Jr.
Gregory J. Poirier
Dale S. Porfilio
Daniel A. Powell
Robert E. Quane III
Kiran Rasaretnam
Andrew T. Rippert
Brad M. Ritter
Tracey S. Ritter
James B. Rowland
Jean-Denis Roy
Kenneth W. Rupert, Jr.
Jason L. Russ
David A. Russell
Sean W. Russell
Letitia M. Saylor
Sara E. Schlenker
Matt J. Schmitt
Michelle G. Sheng
Elissa M. Sirovatka
Raleigh R. Skaggs, Jr.
L. Kevin Smith
M. Kate Smith
Patricia E. Smolen
Brian M. Stoll
Sebastian Yuan Yew Tan
Daniel A. Tess
Edward D. Thomas
Mark L. Thompson
Glenn A. Tobleman
Philippe Trahan
James F. Tygh
Jeffrey D. White
Robert F. Wolf
John M. Woosley
Joshua A. Zirin

The following candidates were admitted as Fellows and Associates at the 1996 CAS Annual Meeting in November. By passing May 1996 examinations, these candidates successfully fulfilled the Society requirements for Fellowship or Associateship designations.

NEW FELLOWS

Shawna Ackerman
Mark A. Addiego
Elise M. Ahearn
Craig A. Allen
Scott C. Anderson
Steven D. Armstrong
Douglas S. Benedict
Wayne E. Blackburn
Annie Blais
Ward M. Brooks
Tracy L. Brooks-Szegda
Lisa J. Brubaker
Richard F. Burt, Jr.
Douglas A. Carlone
Carol A. Cavaliere
Maureen A. Cavanaugh
Heather L. Chalfant
Kasing Leonard Chung
Frank S. Conde
Brian C. Cornelison
Catherine Cresswell
Joyce A. Dallessio
Behram M. Dinshaw
Jeffrey D. Donaldson
Norman E. Donelson
Yves Doyon
David M. Elkins
Martin A. Epstein
James G. Evans
Judith M. Feldmeier
John R. Ferrara
Kirsten A. Frantom
James E. Gant
Nicholas P. Giuntini
Richard W. Gorvett
Russell H. Greig, Jr.
Terry D. Gusler
Michele P. Gust
Elizabeth E. L. Hansen
Robin A. Harbage
Barton W. Hedges
Kirsten Costello Hernan
Betty-Jo Hill
David L. Homer
Sandra L. Hunt
F. Judy Jao
Christian Jobidon
Stephen H. Kantor
Timothy P. Kenefick
Michael B. Kessler
Timothy F. Koester
Louis K. Korth
Gary R. Kratzer
Howard A. Kunst
Bertrand J. LaChance
Benoit Laganiere
Matthew G. Lange
John P. Lebens
David R. Lesieur
Kenneth A. Levine
James M. Maher
Leslie R. Marlo
Kelly S. McKeethan
Robert F. Megens
Stephen V. Merkey
Camille Diane Minogue
Madan L. Mittal
Kenneth B. Morgan, Jr.
Giovanni A. Muzzarelli
David Y. Na
Peter M. Nonken
Marc F. Oberholtzer
Douglas J. Onnen
Melinda H. Oosten
Nicholas H. Pastor
Clifford A. Pence, Jr.
Daniel A. Powell
Andrew T. Rippert
Brad M. Ritter
Tracey S. Ritter

Douglas S. Rivenburgh
Sallie S. Robinson
Jay Andrew Rosen
James B. Rowland
Kenneth W. Rupert, Jr.
Jason L. Russ
David A. Russell
Sean W. Russell
Melodee J. Saunders
Letitia M. Saylor
Sara E. Schlenker
Peter R. Schwanke
Michelle G. Sheng
Elissa M. Sirovatka
Raleigh R. Skaggs, Jr.
Patricia E. Smolen
Brian M. Stoll
Edward D. Thomas
Janet A. Trafecanty
Joseph W. Wallen
John M. Woosley
Cheng-Sheng P. Wu
Edward J. Yorty
Joshua A. Zirin

NEW ASSOCIATES

Mohammed Q. Ashab
Richard J. Babel
Keith M. Barnes
Michael J. Bednarick
Michael J. Belfatti
Bruce E. Binnig
Lesley R. Bosniack
Bethany L. Cass
Henry H. Chen
Sally M. Cohen
Richard J. Currie
Sheri L. Daubenmier
John T. Devereux
Patricia J. Donnelly
Christopher S. Downey
Kevin M. Dyke
Anthony D. Edwards
Ellen E. Evans
Sylvain Fauchon
David I. Frank
Kathy H. Garrigan
Abbe B. Gasparro
James W. Gillette, Jr.
Moshe D. Goldberg
Michael D. Green
Greg M. Haft
Scott T. Hallworth
Michael B. Hawley
Jodi J. Healy
Thomas E. Hettinger
John F. Janssen
Joseph W. Janzen
Brian E. Johnson
Chad C. Karls
Mary C. Kellstrom
John Hun Kim
Martin T. King
Elina L. Koganski
Andre L'Esperance
Timothy J. Landick
Betty F. Lee
Ramona C. Lee
Brian P. LePage
Steven J. Lesser
Leslie A. Martin
Claudia A. McCarthy
Patrice McCaulley
Douglas W. McKenzie
Michael B. McKnight
Jeffrey A. Mehalic
Jennifer Middough
Alison M. Milford
Stephen A. Moffett
Lisa J. Moorey
Roosevelt C. Mosley
Prakash Narayan
Richard D. Olsen
Abha B. Patel
Tracie L. Pencak
Miriam E. Perkins
Luba Pesis
Ellen K. Pierce
Igor Pogrebinsky
Anthony E. Ptasznik
Ni Qin-Feng
Andrew S. Ribaudo
Cynthia L. Rice
Chet James Rublewski
Elizabeth A. Sander
Timothy D. Schutz
Scott A. Shapiro
Jill C. Sidney
Jeffery J. Smith
Jay Matthew South
Catherine E. Staats

Carol A. Stevenson
Roman Svirsky
John L. Tedeschi
Michael J. Tempesta
Kai Lee Tse
Marie-Claire Turcotte
David S. Wolfe
Alexander G. Zhu

The following candidates successfully completed the Parts of the Fall 1996 CAS Examinations that were held in October 1996.

Part 3B

Andrea Ondine Ahern
John Scott Alexander
Rohana S. Ambagasitiya
Brian M. Ancharski
Amy J. Antenen
Gary A. Arens
Peter Attanasio
Julia E. Baker
Gregory K. Bangs
Daniel M. Bankson
Patrick Beaudoin
Minh P. Bennett
Jeremy T. Benson
Jason E. Berkey
Amy L. Borkowski
John R. Bower
Mary J. Boyd
Margaret A. Brinkmann
Kevin C. Burke
Derek D. Burkhalter
Samuel C. Cargnel
Matthew J. Cavanaugh
John Chan
John Y. J. Chan
Jamie Chow
Julia F. Chu
John R. Cloutier
Sheldon Cohen
Richard Jason Cook
Kathleen M. Cooper
Charles L. Costantini
Russell A. Creed
Stephen Darrow
Conrad K. Davids
Stephanie A. DeLuca
Concetta A. DePaolo
Romulo N. Deo-Campo Vuong
Devin Derstine
Timothy M. Devine
Tanuja S. Dharmadhikari
Scott H. Drab
Dagmar Dugan
Robert N. Edwards
Richard Engelhuber
Greg J. Engl
Laura A. Esboldt
Weishu Fan
Doris A. Filzen
Jennifer L. Fitzpatrick
Richard G. Fleissner
Jon P. Fortney
Donald M. Gambardella
Amy L. Gebauer
Laszlo Gere
Bradley G. Gipson
Donald L. Glick
William G. Golush
Rebecca N. Hai
Joseph P. Hannigan
Valie R. Harley
Richard A. Haugen
Qing He
Kimberly A. Heiligenberg
Kevin A. Hilferty
Kurt D. Hines
Luke D. Hodge
Jane W. Hughes
Theodore L. Husveth
Jamison J. Ihrke
Xiaohu Jiang
Charles B. Jin
Susan K. Johnston
Mark C. Jones
Dana F. Joseph
Diana L. Jud

Robert C. Kane
Paul W. Kollner
Brandon E. Kubitz
Kirk L. Kutch
Richard A. Kutz
Nathan P. LaCombe
Francois Lacroix
Stephen J. Langlois
Travis J. Lappe
Hugues Laquerre
Henry T. Lee
Jeffrey Leeds
Jing Liu
John R. McCollough
Jeffrey B. McDonald
Jennifer A. McGrath
Ellen E. Mercer
Eric Millaire-Morin
Rebecca E. Miller
Ain H. Milner
Jason E. Mitich
Mark J. Moitoso
Lambert Morvan
John V. Mulhall
Brian J. Mullen
Michael D. Nielsen
Mark J. Noble
Derek M. Osborne
Robin V. Padwa
Jeff D. Paggi
Cosimo Pantaleo
Philip J. Panther
Pierre Parenteau
Lorie A. Pate
Matthew J. Perkins
Charles V. Petrizzi
Kevin M. Pilarski
Jamie Ramos
Eric W. L. Ratti
Mary E. Reading
Sara Gay Reinmann
Mary Joseniae O. Reynolds
Richard G. Rhode
Marie R. Ricciuti
Lynn M. Richardson
Michele S. Rosenberg
Giuseppe Russo
Salimah H. Samji
Jason R. Santos
Jeremy N. Scharnick
Michael F. Schrah
George M. Schrode
Patricia Marie Schultz
Stuart A. Schweidel
Frank W. Shermoen
Jennifer L. Smith
Stacy L. Stalker
Jeff B. Sturtridge
Gary A. Sudbeck
John L. Terlisner
Wesley K. Thompson
Jennifer L. Throm
Paul A. Vendetti
Natalie Vishnevsky
David M. Vogt
Karl C. Von Brockdorff
Melodie A. Wakefield
Matthew J. Walter
Janet L. Wang
Wade T. Warriner
Chang-Hsien Wei
Julie A. Wheeler
Miroslaw Wieczorek
Paul D. Wilbert
Jonathan S. Woodruff
Jacinthe Yelle
Michael G. Young
Stephanie C. Young

Part 4A

John Scott Alexander
Jonathan L. Ankney
Afrouz Assadian
Eynshteyn Averbukh
Darci L. Axness
Rick Beam
Esther Becker
Richard D. Behnke
Marie-Eve J. Belanger
Johanne Belleau
Lisa M. Bellotti
Heather L. Bennett
Jesse A. Beohm
Sarah J. Billings
Maureen A. Boyle
Richard A. Brassington
Jeremy James Brigham
Bruce D. Browning
Matthew R. Carrier
Daniel P. Checkman
Tracy L. Child

Benjamin W. Clark
Edward W. Clark
Mary Jo Curcio
Marc-Andre Dallaire
David A. DeNicola
Stefvan S. Drezek
Jeffrey A. Dvinoff
Richard Engelhuber
Kathleen M. Farrell
Dana M. Feldman
Kenneth D. Fikes
William J. Fogarty
Joseph B. Galbraith
Donald M. Gambardella
Genevieve Garon
Leslie A. George
Rainer Germann
Daniel J. Gieske
Patrick J. Gilhool
Bradley G. Gipson
David Patrick Glenn
Joseph E. Goldman
Alla Golonesky
Heather J. Gordon
Christopher J. Grasso
Amanda J. Gress
Laurie L. Griffin
Rebecca N. Hai
Stacey M. Haller
David S. Hart
Qing He
Kimberly Head
Mary-Reem Helewa
Martin W. Hill
Laurent Holleville
Hsienwu Hsu
Carol I. Humphrey
Paul Ivanovskis
Jesse T. Jacobs
Philippe Jodin
Brian B. Johnson
William B. Johnson
Steven M. Jokerst
Barbara L. Kanigowski
Jennifer E. Kish
Henry J. Konstanty
Julia M. Lavolpe
Marc R. Levinsky
David G. Lim
Dengxing Lin
Erik F. Livingston
Tony Lu
James P. Lynch
Kevin M. Madigan
Deep Mandal
Luis S. Marques
Victor Mata
Matthew M. McKenzie
Ellen E. Mercer
Joshua C. Mermelstein
Pantelis N. Messolonghitis
Todd A. Michalik
Stephanie Miller
Amy J. Morehouse
Brian C. Neitzel
Lowell D. Nelson
Van Y. Nguyen
Kristina S. Nolan
Charles A. Norton
Brian D. Peckingpaugh
Richard B. Pitbladdo, Jr.
Lovely G. Puthenveetil
John T. Raeihle
Stephen D. Riihimaki
Scott I. Rosenthal
Janelle P. Rotondi
Laura B. Sachs
James C. Santo
Jason T. Sash
Daniel D. Schlemmer
Richard T. Schneider
Amy V. Shakow
Steven R. Shallcross
Junning Shi
Maria Shlyankevich
Paul Silberbush
Lee O. Smith
Anthony A. Solak
Laura T. Sprouse
Michelle J. Steinborn
Jonathan L. Summers
Sarah J. Thompson
Gary S. Traicoff
Tamara L. Trawick
Isabel Trepanier
Andrea E. Trimble
Sara A. Trussoni
David Uhland
Benoit Vaillancourt
Chris J. Westermeyer
Michael D. Williams
Walter R. Wulliger
Mihoko Yamazoe
Michael G. Young

Part 4B

Michael B. Adams
Dariush Akhtari
Joe Alaimo
Rohana S. Ambagasitiya
Brian G. Anast
John A. Annino
Todd M. Aron
Anju Arora
James P. Ash
Francois Audet
Cameron Avestan
Jelena B. Babic
Brice A. Ballard
Pascal Barriere
Chad M. Beehler
Rowen B. Bell
Rex V. Belleza
David J. Bergeron
Luc Bergeron
David A. Berry
George N. Berry
Heather A. Bertellotti
Eric B. Best
Yongyi Bi
John T. Binder
Brad D. Birtz
Tony F. Bloemer
Mark A. Bonsall
Thomas L. Boyer II
Christopher C. Brand
Marie-Helene Brassard
Benoit Briere
David C. Brueckman
Matthew E. Butler
Stephen J. Calfo
Mario R. Candia
Tamela Canora
Darryl A. Cardozo
Allison F. Carp
Dennis W. Carter
Richard J. Castillo
Daniel Castilloux
Brendan P. Cavanaugh
Chun Rui Chang
Mark J. Chartier
Chi-Chung Chen
Henry Hai-Tao Chen
Eric Cheng
Hsiang-Kun Kurt Cheng
Yao-Wen Cheng
Winnie Wing Dee Cheung
Chow Shun Chi
Nicholas J. Chocas
Derek H. Chow
Eric A. Christensen
Mein Sze Chwei
Charles A. Cicci
Christopher E. Clark
Edward W. Clark
Scott R. Clark
Eric J. Clymer
Timothy J. Connell
Andrea L. Connolly
Melanie Coulombe
Jeffrey A. Courchene
Patrick Couture
Brenda K. Cox
Richard S. Crandall
Tighe C. Crovetti
Michael C. Dakin
Francois Dalceggio
John E. Daniel
Robert P. Daniel
Timothy D. Daniels
Pamela J. Dannelly
Joshua P. Dau
Timothy A. Davis
Gregory J. DeJong
Isabelle Desrosiers
Timothy M. DiLellio
Ryan M. Diehl
Gioivanni Dimeo
Marc J. Dinerman
Wendy L. Dong
Jeremiah M. Downing
Patricia L. Drajin
Bertin Drolet
Patrick J. Dubois
Louis Christian Dupuis
Marc Dussault
Andrew D. Eastman
Thomas E. Ellenwood
Brandon Emlen
Richard Engelhuber
Xavier J. Erhart
Joseph G. Evleth
Joseph R. Fairchild
Anthony V. Ferraro
Karen L. Field

Paul Filkiewicz
Louis Fiset
Laura Jarrait Flora
Janice E. Floyd
Ronnie S. Fowler
Aaron D. Fried
Francois Fugere
Donald M. Gambardella
Anne M. Garside
Cynthia Gaudreault
Rainer Germann
Graham S. Gersdorff
Emily C. Gilde
Robert Gordon
Christopher J. Graham
Sarwar Grami
Kevin D. Gray
Naomi S. Greenberg
Stephanie A. Greer
Anna M. Griffel
Jeanne M. Guernsey
James C. Guszcza
Richard Dale Hall
Faisal O. Hamid
Dawn M. Happ
Rena Hartstein
Ronald E. Hauser
Jason C. Head
John G. Henares
Stephen J. Higgins, Jr.
Kerry P. Hindsley
Rahim Hirji
Chi-Hing Ho
Bradford K. Hoagland
Iyad Hourani
Hung-Tai Hsu
Hong-Chih Huang
Mohammad A. Hussain
Jodie M. Hyland-Agan
Emidio Iacobucci
Jennifer L. Ims
Mohammad J. Iqbal
Fan Jiang
Tricia L. Johnson
Rick Mark Johnston
Susan K. Johnston
Bryon R. Jones
James L. Jones
Mark C. Jones
Mark D. Justus
Kyewook Kang
Mary Jo Kannon
Thomas W. Keenleyside
Matthew H. Kerbel
Steven D. Kim
Jill E. Kirby
Joseph E. Kirsits
Susan L. Klein
Brian J. Klimek
Tanya M. Kovacevich
Margarita Kritikos
Thomas P. Kurt
Richard V. LaGuarina
Isabelle LaPalme
Yuh-Jin Lai
Timothy W. Lant
Travis J. Lappe
Aaron M. Larson
Justin M. Laughlin
Don Le
Mo Chi Myra Lee
Wai Yuk Lee
Yuet Hwa Lee
Wendy R. Leferson
Raymond S. Len
Koon Sing Li
Raymond A. Li
Wei Li
Dengxing Lin
Shan Lin
Waijye Lin
Diana M. S. Linehan
Christina Link
Bradley W. Lippowiths
William J. Lisk
Jin Liu
Yen-Ping Lo
Richard P. Lonardo
Diana K. Long
Shih-Nin Low
Wing Lowe
William F. Loyd
Wen-He Lu
Wai Ting Lui
Paul K. Luk
Jinxia Ma
Thomas J. Macintyre
Stephen M. Madden
Yoshimori Maeda
Sanjay M. Mahboobani
Sylvain Mailhot
Maryse Malo
James W. Mann
Warren A. Manners
Craig David McKall

Joel D. McMann
Lawrence J. McTaggart III
Genevieve Meloche
Cecilia S. Mendoza
David Miller
Daniel M. Millman
John Lincoln Mitchell
James J. Moloney
Robert A. Mone
Mariyam Moonis
Christian Morency
Rodney S. Morris
Catherine A. Morse
Antoanetta Moushmof
John V. Mulhall
Vinay Nadkarni
Gretchen A. Neal
Mark R. Nebelung
Jennifer L. Nelson
Robert B. Newmarker
Michael S. N. Ngai
David P. Noga
Kristina S. Nolan
Sylvain Nolet
Ann Marie O'Grady
Frank G. O'Neill
John S. Oh
Yohann Ouaknine
Rebecca A. Owen
Clare E. Pace
Kim Page
Jean-Pierre Paquet
Dorina Paritsky
Wendy W. Peng
Pascal Pepin
Dianna K. Perrie
Kevin T. Peterson
Andrea L. Phillips
Kingsada Phraxayavong
Paul M. Pierantozzi
Marylene Plante
Paul M. Pleva
Derek C. Popkes
Andreas Prinsen
Nathalie Proteau
Francois Racicot-Daignault
Marie-Josee Racine
Michael L. Rasmussen
Anita M. Recchio
Timothy O. Reed
Benjamin A. Reich
Sara Gay Reinmann
Neil W. Reiss
Carole M. Richards
Rick L. Richmond
Stephen D. Riihimaki
Arnie W. Rippener
Marie-Luc Robert
Delia E. Roberts
Wayne L. Rosen
Benjamin G. Rosenblum
Sandra L. Ross
Douglas I. Roth
Ray M. Saathoff
Michael Sakoulas
Farid Sandoghdar
Jason T. Sash
Larry E. Scheinson
Mark D. Scherr
Steven M. Schienvar
Robert T. Schlotzhauer
Kenn F. Schroder
Bradley J. Schroer
Nathan A. Schwartz
Stuart A. Schweidel
Lisa M. Scorzetti
Ernest C. Segal
Jill A. Serin
Larry J. Seymour
David G. Shafer
Parag S. Shah
Sunil B. Shah
Maanaaz S. Shamji
Hoque A. Sharif
Brad J. Sherfey
John Shey
Idan Shlesinger
Minhome Shou
Marina Sieh
Irwin Silber
Bradley H. Simanek
Rebecca L. Simons
Joseph A. Smalley
Madhumathi Soundararajan
John H. Soutar
Anya K. Sri-Skanda-Rajah
Anneliese R. St. Rose
Shawn M. Stackhouse
Nathan R. Stein
David K. Steinhilber
Desmond H. Sutherland

Brian T. Suzuki
Joseph C. M. Tang
Ranee Thiagarajah
Colleen A. Timney
Carol W. Tom Pay Shun
John D. Trauffer
Louis Tremblay
Melissa K. Trost
Chen-Hsiu Tsai
King Piu Tse
Lazar R. Turetsky
Alexander Ungerer
Lawrence A. Vann
Mathieu Vezina
Michael J. Villa
Charles K. Vogl
Tice R. Walker
Keith A. Walsh
Joseph K. Wang
Victoria K. Ward
Chris J. Westermeyer
Sandor Weyers
Paul D. Wilbert
Suzanne E. Wille
Melvin A. Williams
John J. Wood
Jonathan S. Woodruff
Jimmy L. Wright
Marcus J. Wright
Philip S. Wunderlich
James A. Wynstra
Renqiu Xiao
Yu-Chen Yang
Jason B. Yao
Lo Shan Yau
Anthony C. Yoder
Stephen C. Young
Chi Keung Yuen
Frank Jun Zhang
Shihao Zhuo
Jian Zou

Part 5A

Silvia J. Alvarez
Rohana S. Ambagasitiya
Anju Arora
Satya M. Arya
Maura Curran Baker
Kim G. Balls
Patrick Barbeau
Daniel J. Berry
Sheila J. Bertelsen
Mario Binetti
Josee Bolduc
Caleb M. Bonds
David J. Braza
Elise S. Burns
Lisa A. Cabral
Yvonne W. Y. Cheng
Emily Y. Chien
Karen M. Chleborad
Heng Seong Cho
Julia F. Chu
Steven A. Cohen
Nancy J. Collings
Larry Kevin Conlee
John E. Daniel
Loren R. Danielson
Mark A. Davenport
D. Vance C. DeWitt
John D. Deacon
Junko K. Ferguson
Lawrence K. Fink
Ronnie S. Fowler
Rosemary D. Gabriel
Matthew P. Gatsch
Micah R. Gentile
Siddhartha Ghosh
Isabelle Gingras
John P. Gots
Elizabeth A. Grande
David B. Hackworth
Jeffery T. Hay
James A. Heer
Chad A. Henemyer
David E. Heppen
Joseph H. Hohman
Elizabeth J. Hudson
Scott R. Hurt
Rusty A. Husted
Philip M. Imm
Joseph M. Izzo
Gregory O. Jaynes
Weidong Wayne Jiang
Derek A. Jones
Patricia Kinghorn
Anne Marie Klein
Ravikumar Lakshminarayan
Mai B. Lam
James P. Leise
Christian Lemay

Rebecca M. Locks
James M. MacPhee
Atul Malhotra
James W. Mann
David M. Maurer
Ian J. McCracken
Kirk F. Menanson
Jill M. Merchant
Suzanne A. Mills
Melissa R. Montante
Malongo Mukenge
Kari A. Nicholson
Jason M. Nonis
Miodrag Novakovic
Brett M. Nunes
Sheri L. Oleshko
Helen S. Oliveto
Moshe C. Pascher
Wendy W. Peng
Sylvain Perrier
John S. Peters
Kevin T. Peterson
Kraig P. Peterson
David R. Picking
Jordan J. Pitz
Peter V. Polanskyj
Charlene M. Pratt
Donald S. Priest
John T. Raeihle
Sylvain Renaud
Peggy-Anne K. Repella
Mario Richard
David C. Riek
Seth A. Ruff
Frederick D. Ryan
Frances G. Sarrel
Jennifer A. Scher
Nathan A. Schwartz
Michael Shane
Seth Shenghit
Joseph A. Smalley
Anya K. Sri-Skanda-Rajah
Karrie L. Swanson
Edward T. Sweeney
Edward Sypher
Nitin Talwalkar
Gary S. Traicoff
Michael C. Tranfaglia
David Uhland
Alice M. Underwood
Richard A. Van Dyke
Kyle J. Vrieze
Robert J. Wallace
Mark S. Wenger
William B. Westrate
Karin H. Wohlgemuth
Christopher H. Yaure

Part 5B

Jason R. Abrams
Michael D. Adams
Cheryl R. Agina
Sarah Albro
Deborah J. Almeida
Jonathan L. Ankney
Melissa J. Appenzeller
Suzanne Barry
Heather L. Bennett
James H. Bennett
John T. Binder
Brian A. Bingham
Tony F. Bloemer
Edmund L. Bouchie
Lee M. Bowron
David J. Braza
Brenda A. Brazil
Paul E. Budde
Jennifer P. Capute
Jennifer A. Charlonne
Marcus K. Cheung
John Clara
Jason T. Clarke
Kevin M. Cleary
Craig A. Cooper
Hugo Corbeil
Jeffrey A. Courchene
Hall D. Crowder
Kristin J. Dale
Loren R. Danielson
Crystal Dawn Danner
Dawne L. Davenport
Alain P. DesChatelets
Tammi B. Dulberger
Gregory L. Dunn
Louis Christian Dupuis
Ruchira Dutta
Brandon Emlen
Melissa M. Emmendorfer
Donna L. Emmerling
Ashifa Esmail

Danielle T. Fairburn
Carolyn M. Falkenstern
Robert E. Farnam
Patrick V. Fasciano
David E. Gansberg
Ellen M. Gavin
Theresa Giunta
Andrew S. Golfin, Jr.
Teresa L. Golin
Melanie T. Green
Joseph P. Greenwood
Caroline Gregoire
Kimberly Baker Hand
David L. Handschke
Chad A. Henemyer
Deborah L. Herman
Melissa Higgins
Margaret M. Hook
Caleb E. Huntington
Weidong Wayne Jiang
Philippe Jodin
Burt D. Jones
Claudine H. Kazanecki
Dennis J. Keegan
Kathryn E. Keehn
Brandon D. Keller
Alexander Krutov
Isabelle LaPalme
Dejya Debra Lai
Michael A. Lardis
Aaron M. Larson
Francis A. Laterza
Chang H. Lee
Christian Lemay
Karen N. Levine
Diana M. S. Linehan
Cherry W. Lo
Serge M. Lobanov
Aviva Lubin
James P. Lynch
Susan A. Lynch
James R. Lyter
Kevin M. Madigan
David D. Magee
Richard J. Manship
Michelle M. Marabella
Rosemary C. Martin
Martin Menard
Padmanabhan Menon
Eric Millaire-Morin
Matthew K. Moran
Christian Morency
John A. Nauss
Norman Niami
Charles A. Norton
Mihaela L. O'Leary
Wade H. Oshiro
James M. Owen
Robert A. Painter
Robert B. Penwick
Jill E. Peppers
Christopher K. Perry
John S. Peters
Brian S. Piccolo
Christopher J. Poteet
Donald S. Priest
Ronald S. Rees
Sylvain Renaud
Robert R. Ross
Piya Roy
Seth A. Ruff
Joseph J. Sacala
James C. Santo
Jeffery W. Scholl
Parr T. Schoolman
Jonathan A. Schriber
Ronald J. Schuler
Nathan A. Schwartz
John R. Scudella
Michele Segreti
Ba M. Sein
Darrel W. Senior
Linda R. Shahmoon
Joseph A. Smalley
Mark A. Smith
Laura B. Stein
Roxann P. Swenson
Stephen J. Talley
Robert M. Thomas II
Sadhana Tiwari
Andrea E. Trimble
Elaine Ching Tse
Brian K. Turner
Kieh T. Ty
David S. Udall
Alice M. Underwood
Leslie A. Vernon
Lidia E. Villasenor
John T. Volanski
Kyle J. Vrieze
Melodie A. Wakefield
Douglas M. Warner
Arthur S. Whitson
Michael D. Williams

Karin H. Wohlgemuth
Elissa C. Wolf
Linda Yang
Mark K. Yasuda
Xu Zhang
Yin Zhang

Part 7

Ethan D. Allen
Mark B. Anderson
Jennifer A. Andrzejewski
Amy P. Angell
Wendy L. Artecona
David S. Atkinson
Timothy W. Atwill
Craig V. Avitabile
Anna Marie Beaton
Andrew S. Becker
Wayne F. Berner
Frank J. Bilotti
Linda J. Bjork
Jonathan E. Blake
Michael J. Bluzer
Rebecca S. Bredehoeft
Cary J. Breese
James D. Buntine
Hugh E. Burgess
John C. Burkett
Christopher J. Burkhalter
Kevin D. Burns
Stephanie T. Carlson
Sharon C. Carroll
Joyce Chen
Richard M. Chiarini
Michael J. Christian
Theresa A. Christian
Andrew K. Chu
Christopher P. Coelho
Alfred D. Commodore
Margaret E. Conroy
Christopher W. Cooney
Kathleen T. Cunningham
Michael J. Curcio
Kenneth S. Dailey
Douglas L. Dee
Michael B. Delvaux
Kevin F. Downs
Michael E. Doyle
Sharon C. Dubin
Denis Dubois
Nathalie Dufresne
Sophie Dulude
Rachel Dutil
Wayne W. Edwards
Jennifer R. Ehrenfeld
Jane Eichmann
Kristine M. Esposito
Jonathan Palmer Evans
Benedick Fidlow
William M. Finn
Tracy M. Fleck
Chauncey E. Fleetwood
David M. Flitman
Hugo Fortin
Mauricio Freyre
Noelle C. Fries
John E. Gaines
Sherri L. Galles
Susan I. Gildea
Philippe Gosselin
Jay C. Gotelaere
Allen J. Gould
John W. Gradwell
Francis X. Gribbon
David T. Groff
Jacqueline L. Gronski
Kenneth J. Hammell
Alex A. Hammett
Gregory Hansen
Michelle L. Harnick
Ia F. Hauck
Daniel J. Henderson
William N. Herr, Jr.
Thomas E. Hinds
Christopher T. Hochhausler
Amy L. Hoffman
Dave R. Holmes
Jason Israel
Christopher D. Jacks
Jean-Claude J. Jacob
Walter L. Jedziniak
William Rosco Jones
Jeremy M. Jump
Robert B. Katzman
Brandon D. Keller
Scott A. Kelly
David N. Kightlinger
Deborah M. King
George A. Kish

Bradley S. Kove
Karen L. Krainz
Richard S. Krivo
Alexander Krutov
Sarah Krutov
James D. Kunce
Robin M. LaPrete
Jean-Sebastien Lagarde
Yin Lawn
Kevin A. Lee
Todd W. Lehmann
Neal M. Leibowitz
Bradley H. Lemons
Charles Letourneau
Craig A. Levitz
Michael Leybov
Janet G. Lindstrom
Michelle Luneau
Jason K. Machtinger
Daniel Patrick Maguire
Andrea W. Malyon
David E. Marra
Jason N. Masch
William J. Mazurek
Stephen J. McAnena
Ian J. McCracken
Phillip E. McKneely
Allison M. McManus
William A. Mendralla
Richard E. Meuret
Paul D. Miotke
Christopher J. Monsour
Benoit Morissette
Janice C. Moskowitz
Michael J. Moss
Robert J. Moss
Tieyan Tina Ni
Darci Z. Noonan
Michael A. Nori
Chris M. Norman
Steven B. Oakley
Randall W. Oja
Christopher E. Olson
Rebecca R. Orsi
David A. Ostrowski
Moshe C. Pascher
Javanika Patel
Mark Paykin
Harry T. Pearce
Julie Perron
Anthony G. Phillips
Amy A. Pitruzzello
Jennifer K. Price
Matthew H. Price
Richard B. Puchalski
Patricia A. Pyle
Kara L. Raiguel
Ricardo A. Ramotar
Christopher D. Randall
Rebecca J. Richard
Hany Rifai
John R. Rohe
Denise F. Rosen
Tracy A. Ryan
Christy B. Schreck
Michael R. Schummer
William H. Scully III
Marc Shamula
Vladimir Shander
Kelli D. Shepard-El
Meyer Shields
Aviva Shneider
Bret C. Shroyer
Katherine R. S. Smith
Matthew R. Sondag
George Dennis Sparks
Alan M. Speert
Stephen J. Streff
Lisa M. Sukow
Brian T. Suzuki
C. Steven Swalley
Adam M. Swartz
Christopher C. Swetonic
Nitin Talwalkar
Elizabeth S. Tankersley
Jonathan G. Taylor
Patricia Therrien
Jeffrey S. Trichon
Kimberly S. Troyer
Alice M. Underwood
Timothy J. Ungashick
Steven J. Vercellini
Martin Vezina
Nathan K. Voorhis
Claude A. Wagner
Karen E. Watson
Patricia C. White
Bruce P. Williams
Jerelyn S. Williams
L. Alicia Williams
Laura M. Williams
Wendy L. Witmer
Joel F. Witt
Simon Wong
Jeffrey F. Woodcock
Yuhong Yang
Edward J. Zonenberg

Part 9

Rimma Abian
Jeff R. Adcock
Michael J. Andring
Martin S. Arnold
Timothy W. Atwill
Barry Luke Bablin
Robert S. Ballmer II
Karen L. Barrett
Rose D. Barrett
Michael J. Belfatti
Bruce J. Bergeron
Suzanne E. Black
Pierre Bourassa
Kimberly Bowen
Douglas J. Bradac
Mary Hemerick Bready
Kirsten R. Brumley
John F. Butcher II
J'ne E. Byckovski
Kristi Irene Carpine-Taber
Debra S. Charlop
Brian A. Clancy
William B. Cody
Maryellen J. Coggins
Pamela A. Conlin
Claudia Barry Cunniff
Jeffrey W. Davis
Raymond V. DeJaco
Laura B. Deterding
Sean R. Devlin
Andrew J. Doll
Barry P. Drobes
David L. Drury
Mary Ann Duchna-Savrin
Bernard Dupont
Dawn E. Elzinga
Alana C. Farrell
Sylvain Fauchon
Vicki A. Fendley
Karen M. Fenrich
Stephen A. Finch
Steven J. Finkelstein
Kristine M. Firminhac
Mary E. Fleischli
Christian Fournier
Walter H. Fransen
Gary J. Ganci
Kathy H. Garrigan
Christine A. Gennett
Eric J. Gesick
Thomas P. Gibbons
Michael A. Ginnelly
John T. Gleba
Moshe D. Goldberg
Matthew E. Golec
Mari L. Gray
John E. Green
Steven A. Green
Daniel C. Greer
Daniel E. Greer
Greg M. Haft
Lynne M. Halliwell
Scott T. Hallworth
Alessandrea C. Handley
Brian D. Haney
Robert L. Harnatkiewicz
Jodi J. Healy
Cynthia J. Heyer
Jason N. Hoffman
Daniel L. Hogan, Jr.
David D. Hudson
Suzanne G. James
Joseph W. Janzen
Patrice Jean
Daniel K. Johnson
Chad C. Karls
Hsien-Ming K. Keh
Mary C. Kellstrom
Thomas P. Kenia
Susan E. Kent
Bradley J. Kiscaden
Elina L. Koganski
Richard F. Kohan
Timothy J. Landick
Gregory D. Larcher
Thomas V. Le
Guy Lecours
Isabelle Lemay
Daniel E. Lents
Steven J. Lesser
Jennifer M. Levine
Siu K. Li
Robert G. Lowery
Robb W. Luck
William R. Maag
Gary P. Maile
Leslie A. Martin

Scott A. Martin
Keith A. Mathre
Charles L. McGuire III
James R. Merz
Timothy Messier
Stephen A. Moffett
Anne Hoban Moore
Matthew C. Mosher
Matthew S. Mrozek
Kevin T. Murphy
Catherine A. Neufeld
Mindy Y. Nguyen
Randy S. Nordquist
James L. Nutting
Mihaela L. O'Leary
James D. O'Malley
Kevin J. Olsen
Richard D. Olsen
Marlene D. Orr
Dmitry E. Papush
Abha B. Patel
Kathleen M. Pechan
Ellen K. Pierce
Igor Pogrebinsky
Mitchell S. Pollack
Dale S. Porfilio
Anthony E. Ptasznik
Robert E. Quane III
Patrice Raby
Kiran Rasaretnam
Peter S. Rauner
Raymond J. Reimer
Andrew S. Ribaudo
Dennis L. Rivenburgh, Jr.
David A. Rosenzweig
Jean-Denis Roy
Michael R. Rozema
Chet James Rublewski
David L. Ruhm
Elizabeth A. Sander
Stephen Paul Sauthoff
Michael B. Schenk
Frederic F. Schnapp
Timothy D. Schutz
Terry M. Seckel
Kevin H. Shang
Jill C. Sidney
Gerson Smith
Carl J. Sornson
Angela Kaye Sparks
Louis B. Spore
Catherine E. Staats
Julia Causbie Stenberg
Deborah L. Stone
Scott J. Swanay
Steven J. Symon
Sebastian Yuan Yew Tan
Mark L. Thompson
Jennifer M. Tornquist
Philippe Trahan
Laura M. Turner
James F. Tygh
Edward H. Wagner
Benjamin A. Walden
Robert J. Walling III
Erica L. Weida
Jeffrey D. White
Steven B. White
Elizabeth R. Wiesner
William Robert Wilkins
Robin Davis Williams
Robert F. Wolf
Floyd M. Yager
Charles J. Yesker

NEW FELLOWS ADMITTED IN MAY 1996

Front row, from left: Jo Ellen Cockley, David W. McLaughry, Rae M. Taylor, Roland D. Letourneau. **Second row, from left:** Jeanne E. Swanson, Cheung S. Kwan, **CAS President Albert J. Beer**, Donald E. Manis, Mylene Labelle, Scott M. Miller. **Third row, from left:** Barry C. Zurbuchen, Daniel J. Flick, Richard S. Light, Daniel G. Carr, Wayne Hommes.

New Fellows admitted in May 1996 who are not pictured: Charles N. Kasmer, Ann Louise Kiefer, Kelly Jean Mathson, and Christina Lee Scannell.

NEW ASSOCIATES ADMITTED IN MAY 1996

Front row, from left: Diane R. Thurston, Eric D. Besman, William R. Maag, Brian K. Sullivan, David Molyneux, Christian Fournier, Jean-Denis Roy, Nathaniel J. Babcock. **Second row, from left:** Joseph G. Cerreta, Kimberly Ann Bowen, Kimberly Moran Barnett, Jeffrey W. Davis, Marie-Josee Huard, **CAS President Albert J. Beer**, Janet Pruitt Cappers, Chris Dougherty, Guy Lecours, Erica Partosoedarso. **Third row, from left:** Jeffrey Shannon Goy, Linda Marie Howell, Louis Durocher, Denise R. Webb, Douglas A. Rupp, David L. Drury, Romel Garry Salam, Patrice Raby. **Fourth row, from left:** Matthew D. Corwin, Gerald D. Hanlon, Ronald J. Herrig, David G. Cook, Christopher M. Steinbach, Rick A. Workman, James M. Kelly.

New Associates Admitted in May 1996

Front row, from left: Lisa A. Brown, Brandelyn C. Klenner, Caroline B. Spain, Lynne M. Halliwell, Terri C. Kremenski, Daniel Berenson Perry, Kah-Leng Wong, Vicki Agerton Fendley. **Second row, from left:** John A. Haggland, Hsiu-Mei Chang, Linda M. Brockmeier, David B. Bassi, Eric J. Hornick, **CAS President Albert J. Beer**, Steven M. Lacke, Kirsten R. Brumley, Louis M. Brown, Daniel Leo Hogan Jr. **Third row, from left:** Kiran Rasaretnam, Raymond V. De Jaco, Karl Goring, Philip Lew, Benjamin A. Walden, Mindy Y. Nguyen, Diane L. Kinner, Deborah L. Stone. **Fourth row, from left:** Alessandrea C. Handley, Phillippe Trahan, Matthew S. Mrozek, Michael G. Owen, Ron Brusky, William G. Stanfield, Mitchell S. Pollack.

NEW ASSOCIATES ADMITTED IN MAY 1996

Front row, from left: Jennifer N. Williams, Karen E. Myers, Laura M. Turner, Catherine A. Neufeld, Lori E. Stoeberl, Betsy Fox Maniloff, Kendra Barnes South, Bonnie S. Wittman. **Second row, from left:** Hong Chen, Jennifer M. Levine, Jean-Pierre Gagnon, William B. Cody, Marian M. Burkart, **CAS President Albert J. Beer**, Kevin H. Shang, James B. Kahn, Curt A. Stewart, Richard L. Zarnik. **Third row, from left:** Robert G. Weinberg, Kevin J. Olsen, Charles Brindamour, Lynn Ann Gehant, Elizabeth F. Bassett, Mary Fleischli, Mary Elizabeth Waak, Joseph A. Malsky **Fourth row, from left:** James D. O'Malley, Man-Gyu Hur, Joseph Marracello, David S. Pugel, Mark L. Thompson, Dale S. Porfilio, Raymond J. Reimer.

NEW ASSOCIATES ADMITTED IN MAY 1996

Front row, from left: Cindy R. Schauer, Christopher R. Ritter, Jeffrey R. Adcock, Christine E. Schindler, Theodore S. Spitalnick, Cara Mae Low, Jeffrey M. Forden. **Second row, from left:** Joseph D. Tritz, Kari S. Nelson, Dave Harrison Rodriguez, **CAS President Albert J. Beer**, Anthony N. Katz, Jennifer M. Tornquist, Brandon L. Wolf. **Third row, from left:** Steven W. Larson, Joseph P. Kirley, John D. Ferraro, Edward H. Wagner, Peter F. Drogan, Robert F. Brown.
New Associates admitted in May 1996 who are not pictured: Brian Keith Bell, Raju Bohra, Michelle Codere, Walter Fransen, Mari Louise Gray, Brett Horoff, Jocelyn LaFlamme, Thomas Vuong Le, Lee C. Lloyd, Robb W. Luck, Bonnie Carole Maxie, Donna Nadeau, David J. Otto, Michael W. Phillips, Jeremy Roberts, David L. Ruhm, Jonathan N. Shampo, Erica Lynn Weida, Michele Nicole Yeagley.

New Fellows Admitted November 1996

First row, from left: Douglas A. Carlone, Elissa M. Sirovatka, **CAS President Albert J. Beer**, Heather Lee Chalfant, Kirsten A. Frantom, Howard Allen Kunst, Louis Konrad Korth. **Second row, from left:** Nicholas H. Pastor, Barton Walter Hedges, Sean William Russell, Tracy Lynn Brooks-Szegda, Joyce Ann Dallessio. **Third row, from left:** Russell H. Grieg Jr., Terry D. Gusler, Maureen Anne Cavanaugh, Edward Daniel Thomas, Jason Louis Russ, Judith Michalovko Feldmeier. **Fourth row, from left:** Stephen Vernon Merkey, Sallie Smith Robinson, Melodee Jane Saunders, Stephen Howard Kantor, Timothy P. Kenefick. **Fifth row, from left:** Giovanni A. Muzzarelli, Craig A. Allen, Elizabeth E. Leyda Hansen, Behram Mehelli Dinshaw, Camille Diane Minogue, Brian M. Stoll. **Sixth row, from left:** Wayne Edward Blackburn, Norman E. Donelson, Brad Michael Ritter, Tracey Suzanne Ritter, Jay Andrew Rosen. **Seventh row, from left:** Melinda Helen Oosten, Daniel A. Powell, David Young Na, Cheng-Sheng Peter Wu, Michael B. Kessler, Sara Elizabeth Schlenker. **Eight row, from left:** Kelly S. McKeethan, Madan Lal Mittal, Mark A. Addiego, Bertrand Jean LaChance, Brian Cornelison. **Ninth row, from left:** Kenneth Ari Levine, James B. Rowland, James Gordon Evans, Joseph W. Wallen, Scott C. Anderson, James Michael Maher.

NEW FELLOWS ADMITTED NOVEMBER 1996

First row, from left: Michele P. Gust, Lisa Jenny Brubaker, John R. Ferrara, **CAS President Albert J. Beer**, Robin Austin Harbage. **Second row, from left:** Janet Ann Trafecanty, Peter Robert Schwanke, Fong-Yee Judy Jao, Kenneth W. Rupert Jr., Peter Max Nonken. **Third row, from left:** Betty-Jo Hill, Sandra L. Hunt, Gary C. K. Cheung, Douglas Stephen Benedict, Leslie Roberta Marlo, Letitia May Saylor. **Fourth row, from left:** Carol A. Cavaliere, Annie Blais, Clifford Arthur Pence Jr., Shawna Sue Ackerman, Michelle G. Sheng. **Fifth row, from left:** Elise Marie Ahearn, Douglas S. Rivenburgh, Christian Jobidon, Ward Brooks, Jeffrey David Donaldson, Martin Arthur Epstein. **Sixth row, from left:** Nicholas P. Giuntini, Matthew G. Lange, David L. Homer, David A. Russell, Kirsten Costello Hernan. **Seventh row, from left:** Marc Freeman Oberholtzer, David Michael Elkins, Frank Samuel Conde, Timothy F. Koester, Edward Johnson Yorty, Catherine Cresswell. **New Fellows admitted in November 1996 who are not pictured:** Steven D. Armstrong, Richard F. Burt Jr., Kasing Leonard Chung, Yves Doyon, James E. Gant, Richard W. Gorvett, Gary R. Kratzer, Benoit Laganiere, John P. Lebens, David R. Lesieur, Robert F. Megens, Kenneth B. Morgan Jr., Douglas J. Onnen, Andrew T. Rippert, Raleigh R. Skaggs, Jr., Patricia E. Smolen, John M. Woosley, and Joshua A. Zirin.

NEW ASSOCIATES ADMITTED IN NOVEMBER 1996

First row, from left: Sherrill Lynne Daubenmier, Sally Marie Cohen, **CAS President Albert J. Beer**, Richard Dean Olsen, Christopher Sean Downey. **Second row, from left:** Chet James Rublewski, Ellen Erway Evans, Henry Houng-Shing Chen, Jeffery Joseph Smith, Roman Svirsky, Patrice McCaulley. **Third row, from left:** John Hun Kim, David S. Wolfe, Andrew S. Ribaudo, Timothy J. Landick, Cynthia Lou Rice. **Fourth row, from left:** Lisa Joy Moorey, Chad Christian Karls, Ramona C. Lee, Igor Pogrebinsky, Douglas W. McKenzie, Prakash Narayan. **Fifth row, from left:** Jodi J. Healy, Tracie Lyn Pencak, Joseph William Janzen, Luba O. Pesis, Abha B. Patel. **Sixth row, from left:** Greg M. Haft, Marie-Claire Turcotte, Elina L. Koganski, Sylvain Fauchon, Dawn Elizabeth Elzinga, Jay Matthew South. **Seventh row, from left:** Roosevelt Charles Mosley Jr., David I. Frank, Elizabeth Ann Sander, Steven J. Lesser, Anthony Edward Ptasznik. **Eighth row, from left:** Claudia Anita McCarthy, Ellen Katharine Pierce, Thomas Edwin Hettinger, Alexander Guangjian Zhu, Michael Joseph Tempesta, Martin Thomas King.

New Associates Admitted in November 1996

First row, from left: Bruce Emerson Binnig, **CAS President Albert J. Beer**. **Second row, from left:** Stephen Andrew Moffett, Ni Qin-Feng, Patricia Jane Donnelly, Bethany L. Cass, John Leo Tedeschi. **Third row, from left:** Jeffrey A. Mehalic, Moshe David Goldberg, Michael D. Green, Kai Lee Tse, Jill C. Sidney, Kathy H. Garrigan. **Fourth row, from left:** Richard John Babel, Abbe B. Gasparro, Carol Ann Stevenson, Michael J. Bednarick, Lesley R. Bosniack. **Fifth row, from left:** Jennifer Middough, Miriam E. Perkins, Brian Eric Johnson, Michael James Belfatti, Leslie Ann Martin, Betty F. Lee. **Sixth row, from left:** Kevin Michael Dyke, Timothy Daniel Schutz, Richard J. Currie, Mary Christine Kellstrom, Alison Marie Milford. **Seventh row, from left:** Michael B. Hawley, Mohammed Quamrul Ashab, Catherine Elaine Staats, John T. Devereux, Scott Tige Hallworth, Brian Patrick LePage.
New Associates admitted in November 1996 who are not pictured: Keith M. Barnes, Anthony D. Edwards, James W. Gillette Jr., John F. Janssen, Andre L'Esperance, Michael B. McKnight, and Scott A. Shapiro.

OBITUARIES

E. Frederick Fossa
Albert J. Walsh
Dewey G. Williams
Hubert W. Yount

E. FREDERICK FOSSA
1942–1996

E. Frederick Fossa, a Principal of Milliman & Robertson, Inc., Wakefield, Mass., and an active volunteer with the CAS, died suddenly of a heart attack on July 31, 1996, while on a business trip in San Francisco, California. He was 54.

A Massachusetts native, Fred was born in Beverly, Mass., and attended St. John's Preparatory School in Danvers, Mass., and Merrimack College in North Andover, Mass.

At the time of receiving his ACAS in 1968, Fred had already begun his actuarial career with The Employers'-Commercial Union Insurance Group, in Boston, Mass. He held many positions at the firm including Vice President and Senior Actuary and Senior Vice President and Actuary. He became a Member of the American Academy of Actuaries in 1971.

In 1973, The Employers'-Commercial Union changed its name to Commercial Union Assurance Companies. Also that year, Fred became a Fellow of the Casualty Actuarial Society.

In 1980 Fred became a consulting actuary with Tillinghast, Nelson & Warren in Newton, Mass. He became an Associate of the Conference of Consulting Actuaries in 1982. In 1983 he moved to Hanseco Insurance Co. (now known as John Hancock Property & Casualty Insurance Company), in Boston, as Vice President and Actuary. He was promoted to Senior Vice President and Actuary.

In 1987, Fred returned to consulting when he opened the Boston office of Milliman & Robertson, Inc. (M&R). Within 10 years, through his leadership, energy, and enthusiasm, Fred built one of the largest casualty consulting practices within M&R. Fred also helped build a number of other consulting practices in the U.S. and abroad. His entrepreneurial spirit knew no bounds—no project was too big or too difficult to accept. Client development and service gave him great joy. He had a very wide circle of friends and touched many lives in positive ways—as a mentor, as a business partner, and as a friend.

In 1989, he became a Principal of M&R and in 1996 he was elected to the company's Board of Directors.

Fred's contributions to the CAS were extensive. From 1974 until 1976 he served on the Education and Examination Committee—Examinations. From 1976 through 1978, he served on the Education and Examination Committee—Education. He became the *Proceedings* Editor for the Editorial Committee in 1979 and held that position until 1983. Also, from 1979 until 1984, Fred served on the Committee on Loss Reserves (later renamed the Committee on Reserves).

In 1982, he served as an *ex officio* member of the Committee on Review of Papers and began his two-year term as Chairperson of the Editorial Committee.

Fred was the CAS Representative to the Joint Task Force on Loss Reserve Seminars and the Representative to the Actuarial Education and Research Foundation from 1982 until 1983. He was a Director of the CAS from 1983 until 1985.

From 1983 until 1984, Fred was a member of the CAS Long Range Planning Committee. He served on the Task Force on Canadian Casualty Loss Reserve Seminar from 1987 until 1989. Also, in 1988 he served on the Joint Organizing Committee for Property and Casualty Foundation. He became a member of the Joint Program Committee on the Appointed Actuary Seminar in 1989.

Fred is survived by his children, Justin E. Fossa and Sasha L. Fossa of Rockport, Mass., two brothers, Joseph P. Fossa of Arizona and Angelo E. Fossa of Florida, and four sisters, Priscilla M. Aucone and Constance Knudson of Beverly, Madeline J. Ryan of Peabody, Mass., and Amelia J. Moroni of Florida.

ALBERT J. WALSH, JR.
1930–1996

Albert J. Walsh, Jr., a Fellow of the Society since 1962 and a charter member of the American Academy of Actuaries, died on July 23, 1996 at the age of 66.

Walsh was born June 24, 1930 in Providence, R.I. He graduated from Harvard College in 1951 with a degree in economics. Walsh became a Chartered Property and Casualty Underwriter in 1954.

He received his ACAS designation in 1961. At that time he was employed by Liberty Mutual Insurance Company in Boston, Mass., as an associate actuary. The following year, Walsh became a Fellow of the CAS. In 1964 he was promoted to Assistant Vice President at Liberty Mutual. The next year he became Vice President of Reliance Insurance Company in Philadelphia, Pa. He remained there for four years until becoming Vice President and General Manager of the Interinsurance Exchange of the Automobile Club of Southern California in Los Angeles, Calif., in 1969. Walsh held that position until his retirement in 1986.

Walsh served on two CAS committees during his actuarial career. From 1967 until 1968, he was a member of the Publicity Committee and from 1969 until 1970 he served on the Finance Committee.

He is survived by his wife, Lucretia G. Walsh, of Pasadena, Calif., three children: Holiday Walsh, of Oakland, Calif.; Bruce Walsh, of Wakefield, R.I.; and Heather Walsh, of Orlando, Fla.; as well as two brothers and a sister.

DEWEY G. WILLIAMS
1925–1996

Dewey G. Williams, the first person from the State of Texas to be admitted to the CAS by examinations, died April 4, 1996, of chronic lymphatic leukemia and congestive heart failure. He was 70.

Born July 22, 1925, in Sulfur Springs, Texas, Williams served from 1943 through 1946 in the U.S. Air Force. He graduated with a bachelor of science degree in mathematics from Southern Methodist University, Dallas, Texas, in 1949.

Williams began his long career with the Dallas-based Texas Employers Insurance Association (later known as Employers Insurance of Texas) soon after graduation. His boss was John F. Stephens. Stephens remembers his first meeting with Williams, "I hired Williams in 1949 as a trainee in the actuarial department of our company. In the first and only interview I had with Dewey, he said his math professor at SMU had told him to go interview with H.L. Hunt (one of the richest men in the country) because he wanted a math major to calculate the odds on horse races. Dewey said he did not go for the interview because he wanted something a little more stable (no pun intended)."

In 1954 Williams became an Associate of the Society and at that time held the position of Staff Actuary with his company. He was promoted to Assistant Actuary in 1956, and to Manager/Actuarial Department in 1959. Williams was named Assistant Secretary in 1962 and the following year he became a Fellow of the CAS.

In 1967, Williams was promoted to Vice President, Actuary and in 1975 to Senior Vice President. In 1979, he was named Executive Vice President and was appointed to the company's Board of Directors and Executive Committees. Williams was finally promoted to President in 1981, a position he held until his retirement in 1993.

Williams was active in the insurance industry. He served on two CAS committees: the Publicity Committee from 1966 until 1968, and the Special Task Force to Study Recruitment of New Candidates to the Profession in 1971. Williams was also a charter member of the American Academy of Actuaries and served on the Board of Directors for the National Council on Compensation Insurance.

Williams also contributed his time and energy to the Dallas community. He was the Chairman of the Board of the American Cancer Society, Dallas Central Unit and a member of the Board of Directors of the Baylor University Medical Center Foundation. In addition, he was a member of the Dallas Citizens Council and the Dallas Chamber of Commerce (serving on its finance committee). He also served on the Advisory Council of Communities Foundation of Texas and on the Board of Directors for the Baylor Dental School.

Stephens reported that Williams loved to play golf, especially with his sons. "Later in life he got interested in ballroom dancing with his wife. He had a large dance floor installed in a new home for entertaining the members of their dance club."

He is survived by his wife of 53 years, Betty S. Williams, of Dallas; three children: Sharon Williams Bothe, of Bedford, Texas; Paul David Williams, of Dallas; and Don Allan Williams, of Plano, Texas; and six grandchildren: Darren, John, Susan, and Janet Bothe, Thomas Finney and Brad Williams.

HUBERT W. YOUNT
1900–1996

Hubert W. Yount, a Fellow of the Society since 1953 and a charter member of the American Academy of Actuaries, died on December 1, 1996, after a long illness.

Yount was born in Biltmore, North Carolina, on March 8, 1900. He graduated from Ohio State University in 1921 with a Bachelor of Science degree. During college, Yount served in ROTC, but World War I ended the day he reported for active duty. He earned his Master of Science degree from Massachusetts State College in 1923. He married Ruth Millicent Carpenter on October 25, 1924.

At the onset of his career in 1923, Yount was an instructor in the Agricultural Economics Department at the Massachusetts State College. He was promoted to Assistant Research Professor in 1925 and to Assistant Professor in 1927.

He joined Liberty Mutual Insurance Company, Boston, Mass., and began his steady climb to executive management as Supervisor of Research in 1929. He was promoted to Associate Actuary in 1932 and to Actuary the following year. Yount was named Vice President and Actuary in 1934. Nine years later, in 1943, Yount was promoted to Vice President and Underwriting Manager. He was named Vice President of Liberty Mutual Fire Insurance Company in 1956 and Executive Vice President of Liberty Mutual Insurance Company the next year. He was named to the Executive Council in 1963 and retired in 1963.

Yount also lent his experience to many civic and professional organizations. Yount served on the CAS Committee on Social Insurance in 1964. He was also a member of the Corporation of the Massachusetts Memorial Hospital, a member of the American Statistical Association, and a member of the Board of Governors of the Insurance Institute of America. He served as a trustee, a member of the Executive Committee, chairman, and president for the American Institute for Property and Liability Underwriters, Inc.; director and president of the American Mutual Insurance Alliance; director and president of the National Association of Automotive Mutual Insurance Companies; and director and president of the National Association of Mutual Casualty Companies. He was also a director and president of the Atomic Industrial Forum, Inc., a member of the Insurance Executives Advisory Group to the Atomic Energy Commission, and a member of the Board of Trustees of the Insurance Society of New York. He also a member of Alpha Zeta and Phi Kappa Phi, The Algonquin, and the Brae Burn Country Club.

According to his daughter, Elizabeth Yount Black, Hubert Yount liked challenges, to travel, and to meet people. “I have in my possession a folder of wonderful letters sent to him following his retirement, from people he had been in contact with not just at Liberty Mutual, but from other firms and in other capacities. It shows a side of my father I was not privileged to know.”

Yount is survived by his daughter, Elizabeth Yount Black, of Jacksonville, Florida; four grandchildren, and eight greatgrandchildren.

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