UNDERWRITING BETAS—THE SHADOWS OF GHOSTS

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Abstract

This paper critiques the methodologies used in prior studies to estimate underwriting betas for application in the "insurance CAPM." It argues that reliable estimates of underwriting betas do not exist. It also demonstrates the inapplicability of the CAPM to the yield to maturity of a bond or portfolio of bonds. Finally, it demonstrates that the assumption that the yield on a U.S. Treasury bill is risk-free for purposes of applying the CAPM implies that all U.S. Treasury securities, regardless of maturity, have betas of zero.

1. INTRODUCTION

The asset pricing models of modern finance theory are sometimes used in insurance rate hearings to determine the equilibrium rate of return to an insurer, and hence the premium level that is implied by that rate of return. One of those models, the Capital Asset Pricing Model (CAPM) (Sharpe, [21]), asserts that the equilibrium rate of return on any asset is given by

$$r_a = r_f + \beta_a(r_m - r_f) \tag{1.1}$$

where

r_a = expected rate of return on asset a,

r_f = risk-free rate of return,

r_m = expected rate of return on the market portfolio (the market portfolio is the portfolio that includes all risky securities, each in proportion to its market value;

i.e., U.S. stocks, foreign stocks, real estate, precious metals, etc.),

β_a = systematic risk or beta of asset a
$= \text{cov}(r_a,r_m)/\text{var}(r_m)$.

It is possible, under some assumptions, to derive an expression for the equilibrium underwriting rate of return for an insurer's total book of business as a percent of premium by applying the CAPM to an insurer and decomposing the total return into the sum of the underwriting return and the investment return. This results in the model that Cummins [7] has termed the "insurance CAPM." Further, with additional assumptions, estimates of equilibrium underwriting returns for individual lines of insurance can be obtained.

The "insurance CAPM" was first derived by Biger and Kahane [3]. Their version of the model not only assumed a world without taxes, but also that each dollar of premium is invested for exactly one year before being paid out in the form of loss or expense. This latter assumption was relaxed by Kahane in his paper, "The Theory of Insurance Risk Premiums—A Re-examination In The Light of Recent Developments in Capital Market Theory" [15]. Kahane's version of the model is

$$r_u = -kr_f + \beta_u(r_m - r_f) \tag{1.2}$$

where

r_u = expected underwriting return per dollar of premium,
β_u = systematic risk of underwriting
$= \text{cov}(r_u,r_m)/\text{var}(r_m)$,
k = a measure of the length of time that one dollar of premium is invested before being paid out in the form of loss or expense.

Other authors include taxes in their versions of the model. Both Hill [12] and Fairley [9], for example, include a single average tax rate in their versions of the model. Urrutia [24] recognizes two tax rates—one for underwriting income and another for investment income. Urrutia's model is

$$r_u = -kr_f(1-t_a) / (1-t_u) + t_a r_f / (s(1-t_u)) + \beta_u(r_m-r_f) \qquad (1.3)$$

where

$t_a =$ tax rate on investment income,

$t_u =$ tax rate on underwriting income,

$s =$ premium to equity ratio.

Although the "insurance CAPM" purports to give the equilibrium underwriting return, it is increasingly being used in insurance rate hearings to determine the "fair" underwriting return, and thereby, the implied premium level. The magnitude of return that is required in order to be a "fair" return is a constitutional question. An equilibrium return can be a "fair" return only if it meets the standards of "fair" returns that have been enunciated by the United States Supreme Court. It is not universally accepted that equilibrium returns meet the standards of "fair" returns. Whether or not they do, though a significant issue in its own right, is not the focus of this paper. Nor is it the intent of this paper to address the shortcomings of the models in their various forms. Rather, the intent of this paper is to focus solely on the problem of estimating underwriting betas.

It is common for expert witnesses who apply the "insurance CAPM" for the purpose of determining premium levels in insurance rate hearings to assume that underwriting betas are zero or slightly negative. Consider the following comments of two expert witnesses in a recent auto insurance rate hearing:

> Witness 1: "Empirical evidence shows that underwriting has no systematic risk. Remember, only systematic risk is compensable. Some studies have demonstrated very small and even negative underwriting risk, therefore we can assume it is not relevant."

> Witness 2: "The underwriting beta (β_u) is assumed to be zero precisely because no published study has shown that underwriting returns are related to market returns. This makes intuitive sense because one does not expect accidents to increase or decrease because stock market returns are increasing or decreasing."

Listening to these witnesses, one might get the impression that the issue is settled—that it is virtually certain that underwriting betas are indistinguishable from zero, and that this conclusion is supported by all of the studies that have addressed the issue. This, however, is not true. Not only do estimates of underwriting betas vary widely from study to study, but also numerous authors note the bias and inaccuracy inherent in the estimates themselves. This paper argues that reliable estimates of underwriting betas do not exist, and hence, the true values of those underwriting betas are unknown.

2. DISCERNING THE GHOST

Underwriting betas are not observable. If the underwriting operations of insurers were publicly traded, then historic underwriting betas might be estimable. All insurers, though, have investment operations as well as underwriting operations. This substantially complicates the estimation of underwriting betas.

A number of authors have tried to estimate underwriting betas using indirect methods (Biger and Kahane; Fairley; Hill; Cummins and Harrington, [6]; Cox and Rudd, [5]). The indirect methods employed are of two general types. One is to regress historic accounting underwriting returns on historic returns for some market index. In this paper, betas estimated using this technique are called "accounting betas." The second method of estimation is based on the notion that the equity beta of an insurer is a linear combination of an investment beta and an underwriting beta (or alternatively, an asset beta and a liability beta). Hence, the underwriting beta (liability beta) can be inferred from estimates of the investment portfolio beta (asset beta) and the equity beta of a publicly traded insurer. In this paper, betas estimated using this technique are called "inferred betas."

The results of these studies vary greatly across lines of insurance, firms, time, choice of the market portfolio, and estimation methodology. In spite of this variation, it is common, as stated earlier, for expert witnesses to assume that underwriting betas are zero or slightly negative. The support for this practice, if and when any is given, is to cite only those studies that affirm the witness's assertion.

Underlying the indirect methods of estimation are numerous unstated assumptions. Moreover, the magnitude of the estimation errors are unknown and potentially enormous. Given that these models and these estimates are increasingly being used to determine premium levels, a critical analysis of the reasonableness of these assumptions is necessary.

3. REVIEW OF THE LITERATURE

Biger and Kahane estimated two sets of accounting betas for each of eighteen lines of insurance. They regressed annual percentage underwriting profits, aggregated for all U.S. non-life stock insurance companies over the period from 1956-1973 (as reported in Best's *Property-Casualty Aggregates and Averages*), against two indices that served as proxies for the market portfolio. The first index is Moody's stock index including dividends. Hence, the market portfolio represented by this index is an all-equity portfolio. The betas corresponding to this portfolio range from -.109 to .199. Further, the betas for fifteen of the eighteen lines of insurance have an absolute value that is less than .100. The second index was constructed from Moody's stock index and the annual holding period returns on U.S. Treasury bonds. The Treasury bonds comprised 70% of the mixed portfolio and Moody's stock index comprised the remaining 30% of the portfolio. The betas that correspond to this mixed stock and bond market portfolio are more variable. They range from -.230 to .371. Biger and Kahane [p. 121] conclude, "systematic risk of underwriting profits approaches zero in most lines. Thus an intuitive solution for underwriting profit rates in these lines equal to minus the riskless interest rate is reasonable."

However, they go on to say [pp. 126-127] that:

> Evaluation of the systematic risk of underwriting, which is not based on market returns but on reported profits, may result in biased estimates of the coefficients . . . Several accounting procedures, unique to the insurance business, make the concept of profit or loss on any particular line of insurance less meaningful than the earnings per share figures for other business firms. In particular, the somewhat arbitrary allocation of overhead to individual lines makes the profit estimates even more questionable, as what is required are specific betas for specific lines. If one adds the empirical inconsistency between accounting betas and market betas for securities, reported in several studies, to these reservations, one must conclude that regulators should be cautious when accounting betas are used for the insurance lines in ratemaking.

Fairley estimates inferred underwriting betas for five lines of insurance. He first estimates beta for all lines combined. This estimate is inferred from the equity betas reported in the *Value Line Investment Survey* [1976] for nine predominantly property-liability stock insurance companies and an investment beta which is estimated using a subsample of the nine insurers. Betas for the various lines are estimated assuming that they are proportional to the ratio of liabilities to premiums. Fairley's estimates of the underwriting betas are as follows: .34 for auto bodily injury; .07 for auto property damage; .07 for homeowners; .34 for workers compensation; and .79 for medical malpractice.

Interestingly, Fairley has little confidence in the accuracy of accounting betas. He states [p. 205] that, "Accounting betas for liabilities or for underwriting determined by regressing annual accounting underwriting returns against annual market index returns are generally near zero in absolute value, though the possible downward bias in these estimates makes them suspect as estimates of true market betas."

Hill presents accounting betas for 14 lines of insurance. They were calculated by regressing underwriting profit rates over the period from 1943 to 1973, as reported in Best's *Aggregates and Averages*, on the logarithm of the return on the market portfolio. Hill does not specify what comprises the market portfolio other than to note [p. 183] that, "The market return is the value weighted index computed by Ibbotsen and Sinquefield [1976]." The underwriting betas for the individual lines vary from -.212 to 1.013. Hill [p. 183] concludes, "Almost all the betas are insignificant. One might draw the weak conclusion that underwriting betas can be positive or negative and that they are generally fairly near zero." Nevertheless, he points out that, "There is a high probability that betas estimated from accounting data are biased towards zero."

Hill also presents inferred all-lines underwriting betas for six publicly traded insurers. They were calculated by regressing the underwriting return on the market return. The underwriting return is calculated by subtracting investment income and capital gains from the change in the market value of the firm plus dividends in each successive one-year period of time. The underwriting betas for the six firms range from -1.03 to .85 and average -.20.

Cummins and Harrington present two sets of all-lines accounting underwriting betas for 14 insurers for two periods of time. The two periods of time are from the first quarter of 1970 to the third quarter of 1975, and from the fourth quarter of 1975 to the second quarter of 1981. One set of betas is based on a regression of quarterly underwriting profits as a percent of earned premium, as reported by the A.M. Best Co., on the market return. The market portfolio is the value weighted index of the New York Stock Exchange and the American Stock Exchange common stocks. The second set of betas is based on a regression of quarterly underwriting profits on five lagged market returns, that is the return on the market portfolio for the current and prior four periods. The underwriting betas from this regression consist of the sum of the coefficients of the five market return variables.

The simple regression estimates are small in absolute value and most are negative. The 14 firm averages are -.05 and -.04 for the two periods of time. Twenty-one of the twenty-eight estimates are negative, and only seven have absolute values greater than .10. The estimates from the second regression are far more variable and average .49 and -1.18 for the two periods of time. Cummins and Harrington state [p. 16], "The results imply that underwriting betas may have been subject to significant instability during the 1970's. This finding suggests extreme caution if underwriting betas are to be used to establish fair profit margins in rate regulations."

They go on to state [p. 38], "Betas have not been stable during the 1970's and may shift again in the early to mid 1980's. Thus regulators should be extremely cautious in using ex post beta estimates to predict ex ante results. Betas also differ across insurers."

Cox and Rudd present two sets of all-lines accounting underwriting betas using the same regression models that Cummins and Harrington used, and one set of inferred underwriting betas for twenty-one insurers for two periods of time. The first period of time is from the first quarter of 1973 to the third quarter of 1977. The second period of time is from the fourth quarter of 1977 to the second quarter of 1982. The accounting betas were calculated using quarterly combined ratios with the Center for Research on Security Prices (CRSP) equally weighted stock index as the proxy for the market portfolio. The accounting betas based on the simple regression model average .068 and -.093 for the two periods of time. Most of the estimates have absolute values less than .100. The accounting betas based on the second regression average .024 and -.027 for the two periods. Most of these estimates also have absolute values less than .100.

The inferred betas were calculated based on data reported in Moody's *Bank and Finance Manual.* The inferred betas are far more variable. They average -.129 and -1.021 for the two periods of time, and range from a low of -2.076 to a high of .164. Cox and Rudd [p. 317] conclude: "Virtually no relationship is observed between the two types of estimates."

There is an extraordinary amount of variation in estimates of underwriting betas across lines of insurance, firms, time, choice of the market portfolio, and methodology. Estimates of underwriting betas are typically measured per dollar of premium. The investor, of course, pays market price when investing in the firm. Accordingly, the relevant risk to the investor is measured per dollar invested. Premium volume generally exceeds the market value of insurers. Hence, betas measured per dollar invested exceed those measured per dollar of premium. For example, if the ratio of premium to market value is two, then beta measured per dollar invested is exactly twice the value measured per dollar of premium. Measuring underwriting betas per dollar of premium thus reduces their apparant variability and contributes to the illusion, in at least some studies, that underwriting betas are near zero.

Is it reasonable to expect true market underwriting betas to vary so greatly? Or is it more reasonable to expect that such variation is caused by faulty estimation methods? If underwriting betas cannot be reliably and accurately estimated, little or no confidence can be placed in the appropriateness of the resulting premiums.

4. UNDERWRITING BETA—WHAT IS IT?

It is important to distinguish future time periods from historic time periods and expectations of future returns from realizations of those expectations. The CAPM is concerned with investors' expectations of future returns. The returns are expected since they are future returns—they have not yet been realized. Moreover, the returns that are realized may not equal those that are expected. Sharpe [21, pp. 85-86] states:

> Capital market theory concerns people's perceptions concerning opportunities. Actual results may (and usually will) diverge from predictions. The values of capital market theory are ex ante (before-the-fact) estimates. Observed values are ex post (after-the-fact) results. The portfolios that do, in fact, turn out to be efficient will lie along some line, but not necessarily the ex ante capital market line. In fact, the market portfolio invariably proves to be inefficient ex post. If the future could be predicted with certainty, investors would shun diversification—the optimal portfolio would contain only the security with the best (actual) performance.
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> But the future cannot be predicted with certainty. Ex ante estimates must be made. The lack of certainty provides the motivation for both portfolio theory and capital market theory.

According to the CAPM given in Equation 1.1, beta is the covariance of the expected future return on a security with the expected future return on the market portfolio divided by the variance of the expected future return on the market portfolio. The expectation is taken over all investors. The underwriting beta given in Equation 1.2 is the covariance of the investors's expected future underwriting return with the investors's expected future return on the market portfolio divided by the variance of the investors' expected future return on the market portfolio.

Since betas depend on expected future returns, in order to measure betas, those expected future returns must be known. The only way to learn about those expected future returns is to ask investors what they expect. No one has ever done such a study. Rather than measure betas, analysts typically estimate them using the ex post form of the CAPM. That is, betas are estimated using historic realized returns. The ex post form of the model requires additional assumptions that are not required by the ex ante form. According to Copeland and Weston [4, p. 205 and pp. 301-302], the ex post form of the model assumes that the return on any asset is a fair game.[1] Further, when betas that are estimated by using historic realized returns are used to establish premium levels for future periods, it is assumed that these historic estimates of beta apply to future periods.

It is not obvious that these two assumptions are reasonable. In fact, both of them are problematic. The fair game hypothesis, for example, requires that investors have unbiased estimates of expected future returns on each and every asset. In other words, investors must have perfect knowledge of the first moment of the probability distribution of future returns on every asset. This, of course, is highly unlikely. As some of the studies have shown, substantially different estimates of beta result from using different periods of time. Hence, even if investors possess perfect knowledge, the choice of the period of time that is used to estimate the historic beta is critical. Lengthening that period might lessen the chance that returns are not fair games, but it also increases the likelihood that beta has changed over the period.

Empirical applications of the ex post form of the CAPM are subject to unknown and potentially large amounts of estimation error. If any confidence is to be placed in the results, then the model must be validated in some way.

One way of validating the model might be to test how well the model can explain historic returns. The evidence is not reassuring. For example, Fama and French [10] found that historic betas were not able to explain historic returns. They found that size and the book-to-market equity ratio have greater explanatory power than historic betas. Perhaps it is the investors' imperfect knowledge, which prevents returns from being fair games, that limits the ability of historic estimates of beta to explain returns in capital markets. Alternatively,

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[1] A fair game model is one where, on average, across a large number of samples, the predicted future rate of return on an asset, conditioned on current information, is equal to the subsequent realized rate of return.

Bernstein [2, p .1] suggests that, "Despite all the mighty efforts of investment theory, we still do not have a firm handle on a quantitative gauge of risk." Beta may be the proper theoretical measure of risk, but reliable estimates of beta may not yet exist.

Historic returns may have some role in estimating betas, but if historic returns are used, then their use must be validated in some way. Without such validation the estimation error is unknown, and no confidence can be placed in the resulting estimates. For example, it would be enlightening to divide the data into two time periods and test how well estimates derived from the first period explain returns in the second. Rather than cross-validate their results, however, Cox and Rudd present two sets of inferred betas for two periods of time with very different results. Had they cross-validated their results, they may have concluded that estimates from the first period were unable to explain returns in the second period. Either the estimates are devoid of value, or underwriting betas vary enormously over relatively short time periods, and thus historic estimates bear no relation to future periods.

Cummins and Harrington also found that historic betas were not stable over time. If this is indeed true, then unless variation over time can be explained and predicted, historic betas have no relevance for determining premium levels.

There are potentially many ways to estimate betas other than naively using historic returns in a simple-minded way. Jorion [14], for example, uses empirical Bayes estimators. (The actuaries' knowledge of credibility theory uniquely qualifies them to contribute to this line of research.) Rosenberg [18] estimates prospective betas using fundamental factors. He also compares the ability of the predicted betas to explain returns versus that of historic betas. He concludes that the predicted betas are superior to historic betas in explaining subsequent returns. See also Rosenberg and McKibben [19], Rosenberg [17] and Rosenberg and Guy [20]. Whatever methodology is used to estimate underwriting betas, it must be validated in some way.

5. ACCOUNTING BETAS

The use of accounting underwriting betas to estimate true market underwriting betas suffers from a number of flaws. First, accounting underwriting betas are based on historic realized returns rather than on investors' expectations of future returns. As explained earlier, the estimation error is unknown and no confidence can be placed in the resulting estimates.

Second, the historic underwriting returns that are used are not discounted. The market, however, values future cash flows according to their discounted present value. It seems unlikely that undiscounted returns could accurately measure investors' expectations of discounted returns.

Some scholars and analysts have suggested that insurers deliberately smooth underwriting returns by manipulating loss reserves. A more significant source of smoothing of underwriting returns is that reported underwriting returns are undiscounted, and thus do not capture the volatility of interest rates. Another source of smoothing emanates from the way that insurers price their product. When determining premium levels, insurers typically consider investment income by using the portfolio (book) yield which is calculated using the book value of invested assets. Since long term bonds are a large part of most insurers' investment portfolios and are carried on the books at amortized cost rather than at market value, this treatment has the effect of smoothing away short term interest rate volatility and thereby introducing some stability to premium levels. Estimating risk by using a time series of returns where the variability has been smoothed away is obviously going to produce severely biased results.

If accounting betas are to have any value, they must accurately approximate market betas. Unfortunately, this is not the case, as historic returns to the Fortune 500 reveal. In the spring of each year since 1973, *Fortune* magazine reports the median return on equity and the median return to shareholders for the Fortune 500 [11]. Table 1 displays those returns as well as the returns to shareholders in the S&P 500 as reported in *Stocks Bonds Bills and Inflation 1992 Year-*

book [13]. Historic accounting and market betas are calculated for the Fortune 500 using the S&P 500 as a market proxy. The market beta for the Fortune 500 as measured by the median return is 1.00, while the accounting beta is -.02. Could it also be true that an accounting underwriting beta of -.02 corresponds to a market underwriting beta of 1.00? Accounting betas obviously do not provide reliable estimates of market betas.

TABLE 1

ACCOUNTING VS. MARKET BETAS—FORTUNE 500

	Fortune 500		
	Accounting Returns	Market Returns	
Year	Median Return on End of Year Equity	Median Total Return to Shareholders	Return on Market Portfolio S&P 500
1973	12.4%	-25.5%	-14.7%
1974	13.6	-22.4	-26.5
1975	11.6	51.2	37.2
1976	13.3	34.5	23.8
1977	13.5	-3.2	-7.2
1978	14.3	7.2	6.6
1979	15.9	21.3	18.4
1980	14.4	21.1	32.4
1981	13.8	-0.4	-4.9
1982	10.9	21.2	21.4
1983	10.6	30.2	22.5
1984	13.6	-0.8	6.3
1985	11.5	24.1	32.2
1986	11.6	15.5	18.5
1987	13.2	6.8	5.2
1988	16.2	14.1	16.8
1989	15.0	17.5	31.5
1990	13.0	-10.2	-3.2
1991	10.2	29.5	30.6
Standard Deviation	1.7	19.4	17.9
Correlation with market	-.20	.92	1.00
Beta	-.02	1.00	1.00

If the goal is to estimate systematic risk, then accounting returns are the wrong variables to study.

6. INFERRED UNDERWRITING BETAS

Since the total return to an investor consists of an underwriting return and an investment return, it follows that the equity beta of an insurer can be decomposed into a linear combination of an underwriting beta and an investment beta. In its simplest form the decomposition is as follows:

$$\beta_e = (A/E)\beta_a + (P/E)\beta_u \tag{6.1}$$

where

$\beta_e =$ equity beta,

$\beta_a =$ investment beta,

$\beta_u =$ underwriting beta,

$A =$ invested assets,

$E =$ equity,

$P =$ premium.

There are variations to this model. Some authors include a beta for the non-traded assets, and taxes need to be recognized. Nevertheless, for purposes of this discussion, this simple form will suffice.

Historic equity betas for publicly traded insurers can be computed from historic returns. Further, they are available from a number of investment advisory services and brokerage firms. To estimate the underwriting beta, then, it is necessary to estimate the investment beta and the two levers, *(A/E)* and *(P/E)*. At first blush, this method of estimating the underwriting beta seems simple and straightforward. However, it too is fraught with difficulties. This estimation method merely transfers the problems of estimation from underwriting betas to investment and equity betas. Moreover, any error in the estimation of the investment beta is leveraged by the ratio of invested assets to equity. This leveraging of the error can be quite substantial for some insurers, particularly those that write long-tail lines of insurance.

The equity beta applies to the market value of equity. Accordingly, the levers must also be valued at market. The market value of equity, however, is not available for many insurers since they are not publicly traded. Further, the market value of invested assets is not available for any insurer. The market values of some investments are reported. Stocks, for example, are carried on the books at market value. Insurers that are publicly traded report the market value of the bond portfolio in their annual report to shareholders. Some publicly traded insurers also report the market value of mortgage-backed securities in the shareholders' report. Similar information for insurers that are not publicly traded is not available.

For other assets, however, market values are simply unavailable. The market value of mortgage investments, for example, is generally not available regardless of whether the insurer is publicly traded. Real estate investments are carried on the books at cost less depreciation, rather than at market value. The market value of unconsolidated subsidiaries is generally unknown. Market values of other investments such as oil and gas partnerships, limited partnerships, etc. are unavailable. Thus, it is not possible to determine the proper values for the asset lever for any insurer nor the underwriting lever for most insurers.

The Equity Beta

Although historic equity betas can be computed and are available from a number of sources, they are of unknown quality. Many are based on simple regressions of historic returns. All of these estimates depend on the validity of the ex post form of the CAPM. As previously noted, the assumptions that underlie that model are problematic. Further, different analysts and firms calculate historic betas in different ways. For example, different proxies of the market portfolio and different holding periods are used. Theory provides no guidance as to which holding period should be used. Yet changing the holding period can cause significant changes in the estimates of beta. Longstaff [16], for example, states [p. 875]:

> The value of the market beta for firm *i* is a function of the length of the period over which returns are measured. Thus, betas estimated from daily returns need not equal betas estimated from monthly data, all other estimation problems aside. Perhaps even more important, the relative ranking of firms by betas estimated from daily data need not be the same as the ranking based on betas estimated from monthly returns.

It is well known that the composition of, and returns to, the proper market portfolio are unknown. Typically, a stock market index of some sort, usually a subsample of the entire stock market, such as the S&P 500 or the NYSE, is used as a proxy for the market portfolio. Underlying this practice is the assumption that residential and commercial real estate, farmland, foreign equities, foreign real estate, excluded U.S. equities (such as over the counter stocks and stocks traded on the American or other smaller exchanges), antiques, furs, paintings, precious metals, etc., have no discernable impact on estimates of beta. These excluded assets comprise a much larger part of the market portfolio than the stock indices used as its proxy. Is it reasonable to assume that the tail wags the dog?

Arguably, equity betas estimated by investment advisory services may be more accurate approximations of true equity betas, since investors pay for these services and presumably use them. However, there is great variation in the betas estimated by different firms. Table 2 displays the equity betas estimated by Value Line and Standard & Poors for those property-casualty insurers covered by Value Line which also have a beta published by Standard & Poors. The Value Line betas were published April 10, 1992 [1], and the Standard & Poors betas were current as of March 6, 1992 [23]. The average absolute value of the difference in the two estimates of beta is .26. Both firms use returns over a five year period for calculating betas. Value Line, however, uses a weekly holding period while Standard and Poors uses a monthly holding period.

TABLE 2

ESTIMATES OF BETA

Insurer	Betas Published By: Value Line	Betas Published By: Standard & Poors	Absolute Difference
Chubb	1.05	.67	.38
Cincinnati Financial	.80	.65	.15
Continental Corp.	1.05	1.02	.03
Frontier Insurance	.90	1.06	.16
Geico	.80	.70	.10
General Re	1.00	.68	.32
Orion Capital	1.10	1.27	.17
Progressive Corp.	.95	.52	.43
Safeco	1.15	.90	.25
St. Paul Cos.	1.05	.73	.32
20th Century	1.00	1.42	.42
USF&G	1.10	.70	.40
Average Absolute Difference			.26

Perhaps the consensus or average estimates of equity betas from all of the investment advisory services would provide truer estimates of investors' expected betas. This hypothesis, however, needs to be tested. In any case, estimating an equity beta is no simple task. The estimation error is unknown and potentially large. Use of the wrong equity beta obviously biases the estimate of the underwriting beta.

The Investment Beta

The investment portfolio beta is the weighted average of the betas of the securities in the portfolio. Bonds are a significant component of most insurers' portfolios. What is the beta of a bond portfolio? How is it estimated?

Historic estimates of bond betas can be computed. However, the estimate of beta varies according to the historic period that is used. For example, the beta of the annual return on long term Treasury bonds from 1926 to 1991, according to data reported in *Stocks Bonds Bills and Inflation 1992 Yearbook*, is .06. However, the estimate of the Treasury bond beta increases almost by a factor of five to .29 if it is based on data from 1970 to 1991. Both of these estimates use the S&P 500 as a proxy for the market. If historic estimates are to be used, then what is the appropriate time period? What assurance is there that the choice of the time period is consistent with the market's expectations?

If using historic estimates of bond betas is problematic, then perhaps the beta can be estimated from the yield to maturity of the bond and the current risk-free rate. Presumably the difference between these two yields is the product of the bond's beta and the market risk premium. One witness, in fact, in a recent auto insurance rate hearing estimated the bond portfolio beta of an insurer in this way. However, as is shown below, CAPM cannot explain the yield to maturity of a bond with a maturity that exceeds the holding period assumed by the CAPM.

If CAPM applies to the yield to maturity of a bond, then it must be able to explain the term structure of interest rates. The theories advanced to explain the term structure of interest rates (expectations theory, liquidity preference theory, and market segmentation theory), however, do not include the CAPM. Further, the implications of the CAPM are inconsistent with these theories. If CAPM applies to the yield to maturity of a bond, then that yield is the sum of a risk-free rate and a risk premium which is proportional to the bond's beta. It follows that risk is the only reason why yields on long term bonds differ from yields on short term instruments.

Consider that the yield on long term Treasury bonds in February, 1989 was approximately 8.8% as reported in the *Wall Street Journal*. The yield on ninety day Treasury bills was also approximately 8.8% and the yield on two year Treasury notes was approximately 9.2%

during that month. If CAPM applies to the yield to maturity of a bond, then it implies that although two year Treasury notes were risky at that point in time, long term Treasury bonds were not. Conversely, in April, 1993, the yield on ninety day Treasury bills was approximately 3.0%, and the yield on long term Treasury bonds was approximately 6.8%. CAPM thus implies that long term Treasury bonds were risky at that time. Hence, if CAPM applies to the yield to maturity of a bond, then bond betas are not stable over time and historic betas have no relevance for determining future premiums.

According to both the expectations theory and the liquidity preference theory, the yield to maturity of a long term bond depends on the market's expectations of future interest rates. Since CAPM, which is a single variable/single period model, does not capture the market's expectations of future interest rates, it cannot explain the yield to maturity of long term bonds with maturities that exceed the holding period assumed by the CAPM.

To demonstrate this, consider a default-free zero coupon bond that pays D dollars at the end of t years. The expected price of the bond at time j is

$$P_j = D/((1+_j r_{j+1})(1+_{j+1} r_{j+2})....(1+_{t-1} r_t)), \tag{6.2}$$

where

$P_j =$ expected price of bond at time j,

$D =$ payment from bond at time t,

${}_{i-1}r_i =$ forward interest rate for a default-free commitment made at time 0 to loan money at beginning of year i, and to be repaid with interest at end of year i.

Hence,

$$P_0 = D/((1+_0 r_1)(1+_1 r_2)....(1+_{t-1} r_t)) \tag{6.3}$$

and

$$P_1 = D/((1+{}_1r_2)(1+{}_2r_3)....(1+{}_{t-1}r_t)). \quad (6.4)$$

The CAPM is a single period model. In order to apply CAPM, it is necessary to specify the holding period. The holding period in turn determines the risk-free rate, since that rate must prevail over the holding period. The appropriate risk-free rate is thus the interest rate on a risk-free security with a maturity that matches the holding period. Suppose the CAPM with an annual holding period is applied to this bond. The expected return during the first year is

$$(P_1 - P_0)/P_0 = {}_0r_1 \, . \quad (6.5)$$

Thus the return that CAPM would try to explain is ${}_0r_1$. The yield to maturity, however, is given by

$$y_t = ((1+{}_0r_1)(1+{}_1r_2)....(1+{}_{t-1}+r_t))^{1/t} - 1. \quad (6.6)$$

In general, y_t does not equal ${}_0r_1$. Hence, the CAPM cannot explain the yield to maturity of long term bonds.

Note also that ${}_0r_1$ is simply the current interest rate on a default-free security with a maturity equal to the holding period assumed by the CAPM. If yields on default-free securities are assumed to be risk-free, as is commonly done, then ${}_0r_1$ is the appropriate risk-free rate for this application of the CAPM. Hence, the beta for this bond is zero. Further, it follows that by choosing a suitably short holding period, the beta of any default-free bond of any maturity is zero, since the bond is the sum of a portfolio of zero coupon bonds, all of whose betas are zero. This implies that an insurer that invests exclusively in U.S. Treasury securities, regardless of maturity, has an investment beta of zero. Accordingly, the investment portfolio betas of two insurers, one of which invests exclusively in U.S. Treasury bills while the other invests exclusively in thirty-year Treasury bonds, are both equal to zero, even though the latter may have a greater yield to maturity than the former.

Many insurers, of course, invest in municipal bonds, corporate bonds, and mortgages. Estimating the investment portfolio betas in these cases is no simple matter. Historic estimates are problematic since it is unknown which period of history is relevant. Further, CAPM is unable to explain the yield to maturity for these bonds since those yields depend on the market's expectations of future interest rates. Any error that is implicit in the estimate of the investment beta necessarily biases the estimate of the underwriting beta if it is inferred from the former. Moreover, the error in the investment beta is levered up by the ratio of invested assets to equity.

Some bonds that are subject to default risk can be expected to default. Accordingly, the yield to maturity overstates the expected yield on such a bond. The yield to maturity on risky bonds thus includes a default premium which is required to compensate the investor for the expected rate of default.

One way to estimate an upper bound for the beta of a risky bond (or portfolio of bonds) is to compare the yield to maturity (the taxable equivalent yield to maturity in the case of municipal bonds) of the bond with the yield to maturity of a U.S. Treasury bond with the same duration. The yields to maturity of both bonds capture the market's expectations of future interest rates. Hence, the difference in the yields is equal to the sum of the default premium and the risk premium. Since the risk premium is the product of the bond's beta and the market risk premium, it follows that the difference in the yields divided by the market risk premium is an upper bound for the bond's beta.

Contrary to this procedure, a witness in a recent auto insurance rate hearing estimated the beta of a long term high quality bond portfolio to be .24. At the time, Treasury bills and bonds were yielding 6.3% and 8.5% respectively, and long term corporate bonds were yielding 8.9%. Assuming the duration of the corporate bonds matched the duration of the government bonds and assuming a market risk premium of 8.6%, the implied upper bound for the corporate bond beta is .05. Overestimating the bond beta by inferring it from the yield differential over Treasury bills causes an underestimation of the underwriting beta since the levered investment beta is subtracted from the equity beta to get the levered underwriting beta. This further results in the determination of premium levels that are inadequate.

7. INTUITIVE CONSIDERATIONS

On the surface, the notion that underwriting betas for insurance are zero, since the occurrence or non-occurrence of accidents is unrelated to the performance of financial markets, has much intuitive appeal. A closer inspection, however, reveals that this notion is a bit too simple-minded. It confuses accidents with insurance claims, and completely overlooks the severity of those claims. Further, intuition suggests other reasons why underwriting betas might be positive.

There are a number of reasons why underwriting losses increase during times of economic malaise and high unemployment. To the extent that financial market performance is positively related to economic performance, this suggests that underwriting betas may be positive.

The conventional wisdom in the insurance industry is that theft, fire, and arson losses increase during times of high unemployment. Underwriting losses thus increase during such times for auto, homeowners, and commercial theft and fire insurance. When people are out of work, they are more likely to default on their debt. Thus, credit insurance and mortgage insurance losses increase during times of high unemployment. Drivers who are unemployed are more likely to drive without auto insurance, thus increasing the losses under uninsured motorist coverage. It is expected that fraud and misrepresentation increase during times of high unemployment. Misrepresentation such as not disclosing a young driver on an auto insurance policy or lying about the use, annual mileage, or territory of garaging of an insured vehicle deny the insurer the full premium that is required to insure the policy. This increases underwriting losses. It is also expected that when unemployment is high, claimants are more likely to pursue a claim and to exaggerate the value of that claim.

When interest rates are increasing, stock and bond markets tend to perform poorly. Underwriting losses, especially on long tail lines of insurance, also increase as interest rates rise. This suggests that underwriting returns may have positive betas.

Finally, catastrophes destroy business property and may depress economic activity from the resulting unemployment and business interruption.

Thus, the intuitive considerations are ambiguous. Intuition is insufficient to determine the value of underwriting betas.

8. CONCLUSIONS

The underwriting beta is a useful theoretical concept. However, it is not possible to measure it directly. The indirect methods that have been used to estimate underwriting betas are flawed and result in estimates that vary greatly across lines of insurance, firms, time, choice of the market portfolio, and estimation technique. Thus, reliable estimates of underwriting betas do not exist. Perhaps better methods of estimation may some day be developed. Until that time, however, underwriting betas will remain as visible as the shadows of ghosts.

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