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FOREWORD

The Casualty Actuarial Society was organized in 1914 as the Casualty Actuarial and Statistical Society of America, with 97 charter members of the grade of Fellow; the Society adopted its present name on May 14, 1921.

Actuarial science originated in England in 1792, in the early days of life insurance. Due to the technical nature of the business, the first actuaries were mathematicians; eventually their numerical growth resulted in the formation of the Institute of Actuaries in England in 1848. The Faculty of Actuaries was founded in Scotland in 1856, followed in the United States by the Actuarial Society of America in 1889 and the American Institute of Actuaries in 1909. In 1949, the two American organizations were merged into the Society of Actuaries.

In the beginning of the 20th century in the United States, problems requiring actuarial treatment were emerging in sickness, disability, and casualty insurance—particularly in workers compensation—which was introduced in 1911. The differences between the new problems and those of traditional life insurance led to the organization of the Society. Dr. I. M. Rubinow, who was responsible for the Society's formation, became its first president. The object of the Society was, and is, the promotion of actuarial and statistical science as applied to insurance other than life insurance. Such promotion is accomplished by communication with those affected by insurance, presentation and discussion of papers, attendance at seminars and workshops, collection of a library, research, and other means.

Since the problems of workers compensation were the most urgent, many of the Society's original members played a leading part in developing the scientific basis for that line of insurance. From the beginning, however, the Society has grown constantly, not only in membership, but also in range of interest and in scientific and related contributions to all lines of insurance other than life, including automobile, liability other than automobile, fire, homeowners, commercial multiple peril, and others. These contributions are found principally in original papers prepared by members of the Society and published in the annual *Proceedings*. The presidential addresses, also published in the *Proceedings*, have called attention to the most pressing actuarial problems, some of them still unsolved, that have faced the insurance industry over the years.

The membership of the Society includes actuaries employed by insurance companies, ratemaking organizations, national brokers, accounting firms, educational institutions, state insurance departments, and the federal government; it also includes independent consultants. The Society has two classes of members, Fellows and Associates. Both classes are achieved by successful completion of examinations, which are held in May and November in various cities of the United States and Canada.

The publications of the Society and their respective prices are listed in the *Yearbook* which is published annually. The *Syllabus of Examinations* outlines the course of study recommended for the examinations. Both the *Yearbook*, at a $20 charge, and the *Syllabus of Examinations*, without charge, may be obtained upon request to the Casualty Actuarial Society, 1100 North Glebe Road, Suite 600, Arlington, Virginia 22201.

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NOTICE

Papers submitted to the *Proceedings of the Casualty Actuarial Society* are subject to review by the members of the Committee on Review of Papers and, where appropriate, additional individuals with expertise in the relevant topics. In order to qualify for publication, a paper must be relevant to casualty actuarial science, include original research ideas and/or techniques, or have special educational value, and must not have been previously published or be concurrently considered for publication elsewhere. Specific instructions for preparation and submission of papers are included in the *Yearbook* of the Casualty Actuarial Society.

The Society is not responsible for statements of opinions expressed in the articles, criticisms, and discussions published in these *Proceedings*.

Volume LXXVIII, Part 1 No. 148

PROCEEDINGS

May 19, 20, 21, 22, 1991

RETROSPECTIVE RATING: EXCESS LOSS FACTORS

WILLIAM R. GILLAM

Abstract

*This paper explains how Excess Loss Factors (*ELF*s) are computed. It is organized so the essential elements of the computation are described first, then the detailed origins of these elements are added.*

The detail may be found in the many spreadsheets used in the production of ELF*s. The writer has attempted to show how each fits into the structure and leads to the final values. The calculations are quite technical; to understand the whole, it may be necessary to trace the numbers through the spreadsheets using the text as a guide.*

The procedure for computing ELF*s has changed since it was last documented, the most significant revision occurring in 1986. In describing the parts that have changed, the author has supplied justifications, or at least rationales, for the particular changes.*

1. BACKGROUND

In Workers' Compensation, the premium paid by an employer for a one-year policy is a function of the exposure—the audited payroll during the year of coverage. This, of course, is only known some time after the policy is complete, at which time the final premium is calculated.

If, at the onset of the policy, the carrier and employer agree, the final premium can be a function not only of the payroll but also of the actual losses during the coverage period. An arrangement of this sort is formalized in the Retrospective Rating Plan as approved in most states. Ultimate premium is based on actual losses, expenses, and a net insurance charge to compensate for the application of maximum and minimum aggregate amounts. A detailed description of this plan may be found elsewhere. Exhibit 1 shows the basic symbolism.

For most insureds, the maximum premium can be a burdensome amount, but an amount they are reasonably confident they won't have to pay. There remains a fear that a single disastrous accident may cause enough loss by itself to result in the maximum. So the prudent insured may wish to select a "loss limit" or cap on individual losses that enter the retrospective premium formula. This can be done for a fee.

Charges for such excess coverage can be calculated using Excess Loss Factors (*ELF*s), listed for a variety of loss limits in the *Retrospective Rating Manual*. These vary by State and Hazard Group of the insured, as well as by loss limit. Hazard Group assignments are based on the classification of the insured with the most payroll (except certain administrative classifications). The grouping of classes by Hazard Group is done on the basis of relative expected severities.

This excess coverage attaches on a per-occurrence basis. All the loss excess of the loss limit due to an occurrence (possibly multiple-claim) is excluded from the calculation of the retrospective premium.

The *ELF* for a given loss limit can be applied to Standard Premium to generate the pure loss charge for the coverage. Several adjustments must be made for use in retrospective rating. Before multiplying the *ELF* by Standard Premium, a tabular factor called the Excess Loss Adjustment

Amount (*ELAA*) is subtracted. The Loss Conversion Factor is applied to provide expenses nominally varying with loss. The Tax Multiplier applied to the final retrospective premium compensates for taxes, loss assessments, and other miscellaneous items.

Consideration of how this fixed charge may overlap with the insurance charge in the Basic Premium will not be made in this paper, but can be found in Glenn Meyers [1]. Note that, at the time he wrote this paper, the charge for excess coverage was the Excess Loss Premium Factor (*ELPF*). Partly as a result of that paper, *ELPF*s were redefined to account for overlap, using tabular factors called Excess Loss Adjustment Amounts (*ELAA*s). Then

$$ELPF = ELF - ELAA. \tag{1}$$

The next section begins the dissection of the *ELF* computation for hypothetical State M.

2. LOADINGS IN *ELF*s

To present the procedure for calculation of *ELF*s, it will be easiest to start from the end and work backwards. This is because the manipulations necessary to put the data in the correct form are quite complex and, as such, could obscure what is a fairly simple computation.

Exhibit 2 is the final calculation of the *ELF*s in State M. This section covers the adjustment made to pure excess loss ratios for use in the Retrospective Rating Plan.

There is a page for each Hazard Group, but only Hazard Group II is shown. Incorporation of the variation by Hazard Group is the subject of Sections 6 and 7.

Average Excess Ratio [*Column* (14) = (5) + (9) + (13)]

The average excess ratio is the sum of partial excess ratios by claim type and is the portion of the total losses expected to exceed the retention in column (1) on a per-occurrence basis. The three claim types are: Fatal, Permanent Total (PT) or Major Permanent Partial (Major), and Minor Permanent Partial (Minor) or Temporary Total (TT). These are groupings of regular statistical plan injury types.

Permissible Loss Ratio [*Column* (15)]

The Permissible Loss Ratio (*PLR*), as appears here, is the factor applicable to Standard Premium to back into expected loss. In states where pure premium rates are produced, such as State M, this factor would be closer to unity, the complement of whatever loadings there may be in published loss costs. These are typically for loss adjustment expense and loss assessments.

The *PLR* is calculated by dividing the Target Cost Ratio (*TCR*), shown at the bottom of Exhibit 2, by the sum of the Loss Adjustment Expense Factor and the (Loss) Assessment Rate. The *TCR* is less than unity in states where rates are produced. The *PLR* will be an integral part of the rate filing to which the new *ELF*s are attached.

Indicated Excess Loss Factors [*Column* (16) = (14) × (15)]

When multiplied by Standard Premium, the indicated *ELF*s produce expected loss over the selected limit. Indicated Excess Loss Pure Premium Factors (*ELPPF*s) apply to pure premium rates. In State M, the National Council on Compensation Insurance (NCCI) disseminates pure premium rates, even though the standardized computer form shows "*ELF*" at the column heading.

Flat Loading [*Column* (17)]

The flat loading is 0.005, subject to a maximum of one-half of the indicated *ELF* in Column (16). The amount was established before the changes to the procedure made in 1986. It is based on judgment and is designed to compensate the insurer for parameter risk and antiselection.

Final Excess Loss Factors [*Column* (18) = (16) + (17)]

The Final *ELF*s are updated in the *Retrospective Rating Manual* at the time of an approved rate filing.

The following section explains how the partial excess ratios by claim type in Columns (5), (9), and (13) are computed.

3. CALCULATION OF PARTIAL EXCESS RATIOS

Columns (1) through (13) of Exhibit 2 provide the elements for column (14). They are grouped as follows:

Injury Type	Columns
Fatal	(2) thru (5)
PT/Major	(6) thru (9)
Minor/TT	(10) thru (13)

Loss Limit [*Column* (1)]

Any loss limit is possible, but limits from $25,000 to $1 million are the most common. These are shown in the *Retrospective Rating Manual*. Large carriers, excess insurers, and reinsurers frequently ask for information about higher retentions, and the NCCI has obliged by providing *ELF*s for retentions up to $10 million on request. These are output from the standard procedure.

Average Cost Per Case By Injury Type (Bottom of Exhibit 2)

The derivation of these values by injury type and Hazard Group may be found in Section 7.

Ratio to Average/1.1 [*Columns* (2), (6), (10)]

Central to the procedure is the translation of each dollar retention into an entry ratio calculated by dividing the retention by the average cost per claim by type. The claim size distributions underlying the excess ratios in columns (4), (8), and (12) are normalized so their means are unity, which facilitates the application. Using entry ratios automatically indexes the final *ELF* not only for the effect of inflation, but for the differences by State and Hazard Group. This technique was first documented by Frank Harwayne [2].

The distributions underlying columns (4), (8), and (12) are of individual claims by size. The factor of 1.1, applicable to the average claim cost by type, from the bottom of the exhibit, is used to adjust the Excess Ratios from a per-claim to a per-occurrence basis. This procedure, based on judgment, is thought to be an improvement over the former procedure. In the old procedure, a flat 1.1 factor was applied to the excess ratio for every retention. The new procedure results in a loading that varies so it is about 2% or 3% at the low retentions but increases to a level of more than 10% for retentions over $1 million.

Injury Weights [*Columns* (3), (7), (11)]

The final weights vary by Hazard Group. They result from a procedure described in Section 6 that adjusts countrywide relativities using state data. Each factor is a ratio:

$$\frac{\text{Expected Loss By Injury Type}}{\text{Expected Total Loss}}.$$

There is an implicit factor for Medical Only losses, but because these losses have a negligible excess ratio for the retentions normally used in retrospective rating, it is not applied. The final weights come from Exhibit 14.

Excess Ratios [*Columns* (4), (8), (12)]

The excess ratios in columns (4), (8), and (12) are based on size of claim distributions. Exhibit 3 (Parts 3, 4, and 5) shows the excess ratios applicable in State M. Section 4 describes the development of these tables.

Partial Excess Ratios [*Columns* (5), (9), (13)]

These are respective products:

$$(5) = (3) \times (4);$$
$$(9) = (7) \times (8);$$
$$(13) = (11) \times (12).$$

4. EXCESS RATIOS BY CLAIM TYPE

The Excess Ratios are based on parametrized loss distributions. In keeping with the "last is first" format of this paper, Exhibit 3 shows excerpts from tables of excess ratios used. Because these values are based on probability distributions, inconsistencies that can arise in the adjustment of empirical tables for trend and development are not present. Trend and development were each considered in the selected distributions, as explained below.

These distributions are based on distributions fitted to claims from the particular injury group in each of several states. No attempt was made to combine states. To select the final curves, the loss volume in each state, type of benefits (i.e., escalating or nonescalating), goodness of fit, and degrees of freedom were each considered. How these considerations were actually applied is outlined below.

For Fatal and PT/Major separately, consideration was given to whether the state had escalating, nonescalating, or limited benefits. (In states with limited benefits, escalation or nonescalation did not seem to be relevant.) Escalation can apply to Fatal (survivor) or PT (life pension) benefits, or both types of benefits, depending on state laws. Fits to data of several sample states showed states with escalating benefits had more skewed distributions than those with nonescalating benefits.

Somewhat surprisingly, states with aggregate limits on PT benefits gave rise to fitted distributions on PT/Major with higher skewness than states with nonescalating but unlimited benefits. The average size of the claim is surely smaller than it would be with no limit on benefits, but the skewness is still high. We believe this phenomenon is due to the combined effect of unlimited medical, which can be high on PT cases, and the accumulations of claims whose indemnity is capped by the limit value. In the final selections, two distributions were chosen for PT/Major claims: one for states with nonescalating but unlimited benefits, and one for states with either escalating or limited benefits.

In a similar way, two Fatal distributions were selected, but in this case limited benefit states were paired with nonescalating benefit states. Since fatal claims generally do not have a large medical component, this pairing need not be the same as for PT/Major. A single distribution for Minor/TT sufficed, making five in all.

To estimate the impact of loss development on size of claim distributions, the curves were fit to key states' data at successive maturities. Judgment was used to estimate the impact on the shape parameter, which usually progressed in such a way as to increase variance at more mature evaluations. Since most retro plans are closed out by the fifth year, and statistical plan data is not collected beyond that maturity, the selected parameters may not reflect ultimate development. If the *ELF*s are used for pricing excess of loss coverage, some consideration of development fifth to ultimate should be made.

Numerous loss distributions were each fitted to empirical data from the 1982 policy year. These distributions included:

1) Gamma
2) Transformed Gamma
3) Inverse Gamma
4) Inverse Transformed Gamma
5) Beta
6) Transformed Beta
7) Burr
8) Weibull
9) Pareto
10) Lognormal.

The forms of these distributions may be found in Exhibit 4. More detailed information about the distributions may be found in Robert Hogg and Stuart Klugman [3] and in Gary Venter [4].

Curves were fit using the method of Maximum Likelihood. Statistics for goodness of fit, including the negative log likelihood itself, were compared. The chi-square statistic was thought to be especially good for this application, as it measures relative rather than actual squared error. For the tail of the distribution, where probabilities are small, the difference between test data and the distribution is critical if we are to measure excess ratios accurately. How well the curve fits the data around the mean and median, where probabilities are large, is of less importance than the fit in the tail. An unweighted sum of squared residuals statistic would give most weight to the many claims near the middle range of sizes, which does not seem desirable. The chi-square statistic gives a more appropriate weighting.

Maximum Likelihood may be the best way to parametrize a curve, but not necessarily the best way to choose between alternative distributions, since the (log) likelihood statistic pertains in part to the characteristics of the curve being fit, not just the fit itself. Selection of the curve, then, was based primarily on the results of the chi-square test. Frequently, both statistics were best for the same curve, facilitating the choice. (For both statistics, the best is the lowest.)

Another criterion for selection was the number of parameters in the fitted distribution. A Transformed Gamma has three, while the Gamma has only two. If the latter fits nearly as well as the former, it is preferable to use the simpler one, as the additional degree of freedom provides little more information and a greater chance for spurious results.

This principle was applied in the selection of a Fatal curve, where the simple Gamma with two parameters fit nearly as well as the Transformed Gamma, which has three. Holding the first parameters of the Transformed Gamma to unity results in a Gamma.

Two sets of statistics for fits to Fatal claims can be seen in Exhibit 5. For this and the following two exhibits, the sample states A, B, C, etc., were arranged so that A, C, and G were judged to be bellwether examples of the jurisdiction type.

For PT/Major in nonescalating benefit states, the fit of the three-parameter Inverse Transformed Gamma was nearly as good as that of the four-parameter Transformed Beta, and sometimes better. This can be seen in Exhibit 6. Also in that exhibit, it may be observed that the chi-square statistic can blow up for distributions with too low a skewness to accommodate existing large claims.

Examples of the impact of loss development for PT and Major in escalating and limited benefit states may be found in Exhibit 7. This exhibit is one of many similar exhibits produced in the study. The choices of $\alpha = 3.20$ and $\rho = 0.64$ for the Inverse Transformed Gamma were made primarily by consideration of patterns in States A and G.

Of course, the value of β (the scale parameter) in the final curve for each claim type would be adjusted so that a mean of unity would result.

The final parameters for the five curves are shown in Exhibit 8. The next section shows the derivation of the state injury weights and average cost per claim by type. These are needed to produce the figures in columns (3), (7), and (11) and the entries at the bottom of Exhibit 2.

5. STATE INJURY WEIGHTS AND AVERAGE COST PER CASE BY CLAIM TYPE

There are very few serious claims by state, and especially few Fatal or Permanent Total. Although it is possible to separate them by Hazard Group, in most states the data is so thin that usual loss development techniques do not work well and actual average values are statistically unreliable. Hence, a single set of average values and claim type weights is estimated for the state, then spread to Hazard Group using countrywide relativities. Care must be taken in this spreading to see that recombinations of the Hazard Group numbers using weights taken from state data results in the known totals. This is described in the last two sections.

Exhibit 9 shows the calculations as applied in State M. The latest three available policy years are used. They are put on the latest law level, trended, and developed to ultimate separately, then combined for the average used in the *ELF* calculation.

Indemnity and medical losses are separately trended and put on current law level in columns (1) through (8). The losses are then combined and divided by the claim count to produce an "as of" severity in column (11). PT and Major are combined at this point. Factors for severity development to ultimate are applied to produce an estimate of ultimate severity by claim type for each policy year in column (13).

Columns (14) and (15) show aggregate loss development factors to be applied to the respective indemnity losses in column (4) and medical losses in column (8). These produce one-year total developed losses by type in column (16).

The final statewide numbers are a weighted three-year average set of severities by claim type, and three-year total injury weights by type found on Part 4 of Exhibit 9.

The loss severity development factors in column (12) are calculated on Fatal, Minor, and TT separately; but for PT and Major combined. The applicable age-to-age factors (*ATAF*) are an unweighted average of three *ATAF*s, calculated from four evaluations of statistical plan data. The average of three *ATAF*s provides some year-to-year stability in the calculation, since two of three factors in the average overlap from one year to the next.

Since the most mature evaluation of statistical plan data is the fifth report, development factors from fifth to ultimate are taken from financial data. For this application, it is assumed that all loss development beyond a fifth report is severity development on serious claim types.

Until recently, aggregate losses and claim counts were separately developed. The use of severity development reduces the problems of separate loss and claim count development associated with the (possibly frequent) reassignment of claims by type between reporting dates. Such shifting of categorizations would perhaps cancel out on average if the number of claims were large, but we found excessive year-to-year *ELF* volatility in the usual case of a small number of serious claims.

Trend is applied separately to indemnity and medical, as seen in columns (3) and (7) of Exhibit 9. Exhibit 10 shows the derivation of the trend factors.

6. DISTRIBUTION OF STATE INJURY WEIGHTS TO HAZARD GROUPS

Injury weights by type start with values derived in Exhibit 9, columns (14), (15), and (16). Losses by type are put on current law level, trended, and developed, for indemnity and medical separately, then combined in the last step. Losses from the three policy periods are added to provide three-year totals by type, all Hazard Groups combined, in Part 4 of Exhibit 9.

Losses are spread to the Hazard Groups using countrywide data. This data is in the form of partial loss ratios by injury type for each Hazard Group. These loss ratios, based on countrywide statistical plan data, may be seen in Exhibit 11. In this exhibit, the partial loss ratios of each injury type are rescaled so that they sum to 1.0 across the Hazard Groups using the following formula:

$$CLR_{I,H} = \frac{CL_{I,H}/CP_H}{\sum_{\text{Hazard Groups } H} CL_{I,H}/CP_H}\ ,$$

where CP_H is the countrywide premium for Hazard Group H from the experience period used and $CL_{I,H}$ is the countrywide losses for injury type I and Hazard Group H from the same time period. (The rescaling is gratuitous as it is repeated in the next step.)

Using a state distribution of premium by Hazard Group from the latest second report, found in Exhibit 12, a state distribution of losses by Hazard Group for each injury type is found:

$$L_{I,H} = \frac{CLR_{I,H} \cdot P_H}{\Sigma_H \, CLR_{I,H} \cdot P_H},$$

where P_H is now the state premium for Hazard Group H, and $CLR_{I,H}$ is the (relative) partial loss ratio from Exhibit 11. The resulting distributions are shown in Exhibit 13.

These $L_{I,H}$, or proportions of loss dollars by Hazard Group (within injury type), are applied to actual three-year state total losses by injury type from Exhibit 9 to produce the loss dollars by type of injury and Hazard Group in column(s) 2 of Exhibit 14. After each type of loss is distributed *across* the Hazard Groups, the *downward* distribution of losses by claim type is then calculated within each Hazard Group. Subtotals give the proportion of loss in the combined groups PT/Major and Minor/TT. With the associated Fatal weights, these become the injury weights in columns (3), (7), and (11) of Exhibit 2.

7. AVERAGE COST PER CASE BY CLAIM TYPE AND HAZARD GROUP

The state input data comes from Exhibit 9, which gives the statewide three-year average claim cost by injury type. The state premium distribution by Hazard Group comes from Exhibit 12. Exhibit 15 shows countrywide severity relativities for the serious claim types by Hazard Group, which are also needed.

The distribution of claims by Hazard Group differs by state. Hence it will not be correct to apply the relativities from Exhibit 15 to the average claim costs from Exhibit 9. An adjustment must be calculated for each claim type, so that the severity relativities will produce average severities by Hazard Group that are consistent with the overall state severities by type.

The correct weights are claim counts. However, because of the small sample size; i.e., a single claim type in one Hazard Group in one state, claim counts are too volatile. The weights used are the state premiums by Hazard Group from Exhibit 12. For the PT adjustment, we calculate:

$$0.977 = (0.017)(0.813) + \cdots + (0.032)(1.245) \tag{2}$$

in Section A of Exhibit 16. The relativities from Exhibit 15 are divided by these factors, respective of injury type, and the Hazard Group differentials become those in Section B of Exhibit 16. The differentials for the combined PT/Major group are then obtained by weighting the PT and Major differentials with injury weights, respective of Hazard Group, from Exhibit 14:

$$0.943 = [(0.057)(0.976) + (0.575)(0.940)]/[(0.057) + (0.575)]. \tag{3}$$

Using this factor from Exhibit 16 and the respective state severity from Exhibit 9, Part 4, the severity for PT/Major in Hazard Group II can be found at the bottom of Exhibit 2:

$$(0.943)(108{,}997) = 102{,}784. \tag{4}$$

For Minor/TT, no differentials are calculated, and the state average cost per case, as computed in Section 5, is used for all Hazard Groups.

This is the end of the technical presentation. A few comments about the final *ELF*s are in order:

1) For higher limits, the risk component (flat loading) becomes a significant portion of the charge. State M has higher excess ratios than many other states, so this becomes evident only above the $1,000,000 loss limit.
2) For all but the lowest limits, the excess ratio for PT/Major largely determines the final *ELF*.

REFERENCES

[1] Glenn G. Meyers, "An Analysis of Retrospective Rating," *PCAS* LXVII, 1980, p. 110.

[2] Frank Harwayne, "Accident Limitations for Retrospective Rating," *PCAS* LXIII, 1976, p. 1.

[3] Robert Hogg and Stuart Klugman, *Loss Distributions*, Wiley & Sons, 1984.

[4] Gary G. Venter, "Transformed Beta and Gamma Distributions and Aggregate Losses," *PCAS* LXX, 1983, p. 156.

EXHIBIT 1

BASIC RETROSPECTIVE RATING SYMBOLISM

The Retrospective Premium R is calculated after the end of the policy period by formula:

$$H \leq R = T\,(B + cL) \leq G,$$

where: H is the minimum premium;
T is the Tax Multiplier;
B is the Basic Premium;
c is the Loss Conversion Factor;
L is the actual losses during the period;
G is the maximum premium.

If a loss limit is selected:

$$H \leq R = T\,(\hat{B} + cELPF + c\hat{L}) \leq G,$$

where: $\hat{B}$ is the Basic Premium, respective of the selected loss limitation (in the current plan, $\hat{B}$ is not affected by the choice of loss limitations so $\hat{B} = B$);

$\hat{L}$ is the actual losses during the experience period, subject to a per occurrence limit;

$ELPF$ is the (net) charge for such loss limitations after correction for overlap with the insurance charge.

EXHIBIT 2

National Council on Compensation Insurance
State M Effective: 01/01/89
Limited Fatal Benefits—Nonescalating PT/Major Benefits
Excess Loss Factors Calculation
Hazard Group II

	Fatal				PT/Major				Minor/TT								
(1) Loss Limit	(2) Ratio To Ave. / 1.1	(3) Inj. Wgt.	(4) Excess Ratio	(5) Excess Ratio × Inj. Wt.	(6) Ratio To Ave. / 1.1	(7) Inj. Wgt.	(8) Excess Ratio	(9) Excess Ratio × Inj. Wt.	(10) Ratio To Ave. / 1.1	(11) Inj. Wgt.	(12) Excess Ratio	(13) Excess Ratio × Inj. Wt.	(14) Ave. XS Ratio	(15) PLR Excl. Asses.	(16) Ind. ELF (14) × (15)	(17) Flat Loading	(18) Final ELF (16) + (17)
$ 10,000	0.10	0.011	0.908	0.010	0.09	0.632	0.910	0.575	1.79	0.288	0.361	0.104	0.689	0.868	0.598	0.005	0.603
15,000	0,14		0.874	0.010	0.13		0.870	0.550	2.68		0.223	0.064	0.624		0.542	0.005	0.547
20.000	0.19		0.834	0.009	0.18		0.820	0.518	3.58		0.138	0.040	0.567		0.492	0.005	0.497
25,000	0.24		0.796	0.009	0.22		0.780	0.493	4.47		0.085	0.024	0.526		0.457	0.005	0.462
30,000	0.29		0.760	0.008	0.27		0.730	0.461	5.36		0.053	0.015	0.484		0.420	0.005	0.425
35,000	0.33		0.733	0.008	0.31		0.690	0.436	6.26		0.034	0.010	0.454		0.394	0.005	0.399
40,000	0.38		0.700	0.008	0.35		0.650	0.411	7.15		0.022	0.006	0.425		0.369	0.005	0.374
50,000	0.48		0.640	0.007	0.44		0.562	0.355	8.94		0.010	0.003	0.365		0.317	0.005	0.322
75,000	0.71		0.521	0.006	0.66		0.387	0.245	13.41		0.002	0.001	0.252		0.219	0.005	0.224
100,000	0.95		0.422	0.005	0.88		0.284	0.179	17.88		0.000	0.000	0.184		0.160	0.005	0.165
125,000	1.19		0.342	0.004	1.11		0.220	0.139	22.35		0.000	0.000	0.143		0.124	0.005	0.129
150,000	1.43		0.278	0.003	1.33		0.181	0.114	26.82		0.000	0.000	0.117		0.102	0.005	0.107
175,000	1.67		0.226	0.002	1.55		0.153	0.097	31.29		0.000	0.000	0.099		0.086	0.005	0.091
200,000	1.91		0.184	0.002	1.77		0.132	0.083	35.76		0.000	0.000	0.085		0.074	0.005	0.079
225,000	2.14		0.151	0.002	1.99		0.116	0.073	40.23		0.000	0.000	0.075		0.065	0.005	0.070
250,000	2.38		0.123	0.001	2.21		0.103	0.065	44.70		0.000	0.000	0.066		0.057	0.005	0.062
275,000	2.62		0.101	0.001	2.43		0.093	0.059	49.17		0.000	0.000	0.060		0.052	0.005	0.057
300,000	2.86		0.082	0.001	2.65		0.085	0.054	53.64		0.000	0.000	0.055		0.048	0.005	0.053
325,000	3.10		0.067	0.001	2.87		0.077	0.049	58.11		0.000	0.000	0.050		0.043	0.005	0.048
350,000	3.34		0.055	0.001	3.10		0.071	0.045	62.58		0.000	0.000	0.046		0.040	0.005	0.045

EXHIBIT 2

(CONTINUED)

	Fatal				PT/Major				Minor/TT								
(1) Loss Limit	(2) Ratio To Ave. / 1.1	(3) Inj. Wgt.	(4) Excess Ratio	(5) Excess Ratio × Inj. Wt.	(6) Ratio To Ave. / 1.1	(7) Inj. Wgt.	(8) Excess Ratio	(9) Excess Ratio × Inj. Wt.	(10) Ratio To Ave. / 1.1	(11) Inj. Wgt.	(12) Excess Ratio	(13) Excess Ratio × Inj. Wt.	(14) Ave. XS Ratio	(15) PLR Excl. Asses.	(16) Ind. ELF (14) × (15)	(17) Flat Loading	(18) Final ELF (16) + (17)
$ 375,000	3.57		0.045	0.000	3.32		0.066	0.042	67.06		0.000	0.000	0.042		0.036	0.005	0.041
400,000	3.81		0.037	0.000	3.54		0.062	0.039	71.53		0.000	0.000	0.039		0.034	0.005	0.039
425,000	4.05		0.031	0.000	3.76		0.058	0.037	76.00		0.000	0.000	0.037		0.032	0.005	0.037
450,000	4.29		0.025	0.000	3.98		0.054	0.034	80.47		0.000	0.000	0.034		0.030	0.005	0.035
475,000	4.53		0.021	0.000	4.20		0.051	0.032	84.94		0.000	0.000	0.032		0.028	0.005	0.033
500,000	4.77		0.017	0.000	4.42		0.048	0.030	89.41		0.000	0.000	0.030		0.026	0.005	0.031
600,000	5.72		0.008	0.000	5.31		0.039	0.025	107.29		0.000	0.000	0.025		0.022	0.005	0.027
700,000	6.67		0.004	0.000	6.19		0.033	0.021	125.17		0.000	0.000	0.021		0.018	0.005	0.023
800,000	7.63		0.002	0.000	7.08		0.029	0.018	143.05		0.000	0.000	0.018		0.016	0.005	0.021
900,000	8.58		0.001	0.000	7.96		0.025	0.016	160.93		0.000	0.000	0.016		0.014	0.005	0.019
1,000,000	9.53		0.000	0.000	8.84		0.023	0.015	178.81		0.000	0.000	0.015		0.013	0.005	0.018
2,000,000	19.06		0.000	0.000	17.69		0.011	0.007	357.63		0.000	0.000	0.007		0.006	0.003	0.009
3,000,000	28.60		0.000	0.000	26.53		0.007	0.004	536.44		0.000	0.000	0.004		0.003	0.002	0.005
4,000,000	38.13		0.000	0.000	35.38		0.005	0.003	715.26		0.000	0.000	0.003		0.003	0.002	0.005
5,000,000	47.66		0.000	0.000	44.22		0.004	0.003	894.07		0.000	0.000	0.003		0.003	0.002	0.005
6,000,000	57.19		0.000	0.000	53.07		0.003	0.002	1072.88		0.000	0.000	0.002		0.002	0.001	0.003
7,000,000	66.72		0.000	0.000	61.91		0.003	0.002	1251.70		0.000	0.000	0.002		0.002	0.001	0.003
8,000,000	76.26		0.000	0.000	70.76		0.002	0.001	1430.51		0.000	0.000	0.001		0.001	0.001	0.002
9,000,000	85.79		0.000	0.000	79.60		0.002	0.001	1609.33		0.000	0.000	0.001		0.001	0.001	0.002
10,000,000	95.32		0.000	0.000	88.45		0.002	0.001	1788.14		0.000	0.000	0.001		0.001	0.001	0.002

Fatal Average Cost Per Case:	$95,372	Target Cost Ratio:	1.0000
PT/Major Average Cost Per Case:	$102,784	Loss Adjustment Expense:	1.120
Minor/TT Average Cost Per Case:	$5,084	Assessment Factor:	0.032

EXHIBIT 3, PART 1

State Type: Escalating Benefits
Injury: Fatal
Distribution: Gamma (1.667, 0.6)

Mean = 1, Var. = 1.667, Coef. of Var. = 1.291,
Skewness = 2.582

Entry Ratio	Excess Ratio
0.25	.804
0.50	.659
0.75	.544
1.00	.452
1.25	.377
1.50	.315
1.75	.264
2.00	.222
2.25	.187
2.50	.157
2.75	.133
3.00	.112
3.25	.095
3.50	.080
3.75	.068
4.00	.058
4.25	.049
4.50	.041
4.75	.035
5.00	.030
5.25	.025
5.50	.022
5.75	.018
6.00	.016
6.25	.013
6.50	.011
6.75	.010
7.00	.008
7.25	.007
7.50	.006
7.75	.005
8.00	.004
9.00	.002
10.00	.001

EXHIBIT 3, PART 2

State Type: Escalating and Limited Benefits
Injury: Permanent Total and Major Permanent Partial
Distribution: Inverse Transformed Gamma
(3.2, 0.515, 0.64)

Mean = 1, Var. = 11.465, Coef. of Var. = 3.386,
Skewness: Undefined

Entry Ratio	Excess Ratio
1	.269
2	.132
3	.086
4	.064
5	.050
6	.042
7	.035
8	.031
9	.027
10	.024
11	.022
12	.020
13	.019
14	.017
15	.016
16	.015
17	.014
18	.013
19	.012
20	.012
25	.009
30	.008
35	.007
40	.006

EXHIBIT 3, PART 3

State Type: Nonescalating and Limited Benefits
Injury: Fatal
Distribution: Gamma (1.25, 0.8)

Mean = 1, Var. = 1.250, Coef. of Var. = 1.118,
Skewness = 2.236

Entry Ratio	Excess Ratio
0.25	.789
0.50	.628
0.75	.513
1.00	.404
1.25	.325
1.50	.262
1.75	.211
2.00	.170
2.25	.138
2.50	.112
2.75	.090
3.00	.073
3.25	.059
3.50	.048
3.75	.039
4.00	.032
4.25	.026
4.50	.021
4.75	.017
5.00	.014
5.25	.011
5.50	.009
5.75	.007
6.00	.006
6.25	.005
6.50	.004
6.75	.003
7.00	.003
7.50	.002
8.00	.001
9.00	.001
10.00	.000

EXHIBIT 3, PART 4

State Type: Nonescalating
Injury: Permanent Total and Major Permanent Partial
Distribution: Transformed Beta (7.0, 0.513, 1.28, 0.30)

Mean = 1, Var. = 5.045, Coef. of Var. = 2.246,
Skewness: Undefined

Entry Ratio	Excess Ratio
1	.247
2	.115
3	.074
4	.054
5	.042
6	.034
7	.029
8	.025
9	.022
10	.020
11	.018
12	.016
13	.015
14	.014
15	.013
16	.012
17	.011
18	.010
19	.010
20	.009
25	.007
30	.006
35	.005
40	.004

EXHIBIT 3, PART 5

State Type: All
Injury: Minor Permanent Partial and Temporary Total
Distribution: Transformed Beta (2.2, 7.24, 0.12, 2.9)

Mean = 1, Var. = 2.574, Coef. of Var. = 1.604,
Skewness = 2.914

Entry Ratio	Excess Ratio
1	.554
2	.322
3	.188
4	.110
5	.065
6	.039
7	.023
8	.015
9	.009
10	.006
11	.004
12	.003
13	.002
14	.001
15	.001
20	.000

EXHIBIT 4

Loss Distributions

For the following definitions, all of α, β, ρ, θ, and X are greater than zero.

1. Transformed Gamma

$$F(X;\alpha,\beta,\rho) = \int_0^{(X/\beta)^\alpha} \frac{\mu^{\rho-1}e^{-u}}{\Gamma(\rho)}\, du,\ X > 0$$

$$\mathrm{E}[F(X)] = \frac{\beta \cdot \Gamma(\rho + 1/\alpha)}{\Gamma(\rho)}$$

β is a scale parameter
If $\alpha = 1$, this is the Gamma Distribution, $\Gamma(X;\beta,\rho)$
If $\rho = 1$, this is the Weibull Distribution

2. Inverse Transformed Gamma

$$G(X;\alpha,\beta,\rho) = 1 - \int_0^{(\beta/X)^\alpha} \frac{\mu^{\rho-1}e^{-u}}{\Gamma(\rho)}\, du,\ X > 0$$

$$\mathrm{E}[G(X)] = \frac{\beta \cdot \Gamma(\rho - 1/\alpha)}{\Gamma(\rho)}$$

β is a scale parameter
If $\alpha = 1$, this is an Inverse Gamma Distribution
If $\rho = 1$, this is an Inverse Weibull Distribution

3. Transformed Beta

$$\hat{\beta}(X;\alpha,\beta,\rho,\theta) = \frac{\Gamma(\theta + \rho)}{\Gamma(\theta)\Gamma(\rho)} \int_0^{(X/\beta)^\alpha} t^{\rho-1}(1 + t)^{-(\rho+\theta)}\, dt,\ X > 0$$

$$\mathrm{E}[\hat{\beta}(X)] = \frac{\beta\Gamma(\rho + 1/\alpha)\Gamma(\theta - 1/\alpha)}{\Gamma(\rho)\Gamma(\theta)}$$

β is a scale parameter
If $\alpha = 1$, this is the Beta Distribution
If $\rho = 1$, this is the Burr Distribution
If $\rho = 1$, $\alpha = 1$, this is the Shifted Pareto

4. Lognormal

$$L(X;\alpha,\beta) = \frac{1}{\sqrt{2\pi}} \int_0^{(\ln X - \alpha)/\beta} e^{u^2/2}\, du,\ X > 0$$

$$\mathrm{E}[L(X)] = e^{[\alpha + \beta^2/2]}$$

EXHIBIT 5, PART 1

Fatal Loss Distribution
Curve Fit by Maximum Likelihood
Negative Log Likelihood

	DISTRIBUTION					
State	Gamma	T. Gamma	T. Beta	Pareto	Lognormal	Weibull
ESCALATING BENEFITS						
A	137	137	136	153	146	138
B	13	13	13	15	13	13
NONESCALATING BENEFITS						
C	144	142	143	154	156	145
D	111	111	120	131	124	120
E	88	85	86	91	97	87
LIMITED BENEFITS						
F	418	418	421	439	439	421
G	205	197	197	207	221	201
H	115	114	113	117	122	115

Lowest Statistics are Best

EXHIBIT 5, PART 2

Fatal Loss Distribution
Curve Fit by Maximum Likelihood
Chi-Square Statistics

	DISTRIBUTION					
State	Gamma	T. Gamma	T. Beta	Pareto	Lognormal	Weibull
ESCALATING BENEFITS						
A	8.9	9.9	11.6	68.1	23.5	12.3
B	5.9	5.6	5.5	33.8	5.6	5.9
NONESCALATING BENEFITS						
C	19.9	19.3	21.1	83.5	40.5	23.1
D	22.7	22.6	25.2	80.2	34.0	25.1
E	28.1	25.7	27.3	47.4	53.0	31.0
LIMITED BENEFITS						
F	23.9	24.2	27.1	66.9	48.8	26.9
G	69.8	58.6	57.3	83.6	111.9	70.2
H	20.1	19.5	18.6	26.8	35.6	21.6

Lowest Statistics are Best

EXHIBIT 6, PART 1

PT & Major Loss Distribution
Curve Fit by Maximum Likelihood
Negative Log Likelihood

	DISTRIBUTION							
State	Burr	Gamma	T. Gamma	I.T. Gamma	T. Beta	Pareto	Lognormal	Weibull
ESCALATING BENEFITS								
A	4,311	4,735	4,696	4,287	4,287	5,688	4,582	4,828
B	979	1,021	980	969	969	991	977	1,030
NONESCALATING BENEFITS								
C	2,272	2,702	2,521	2,260	2,260	2,398	2,329	2,580
D	2,620	3,123	3,110	2,625	2,619	2,821	2,765	3,110
E	724	856	851	728	724	782	770	863
F	9,027	9,906	9,393	9,020	9,025	9,857	9,338	10,290
LIMITED BENEFITS								
G	1,455	1,791	1,757	1,456	1,454	1,607	1,560	1,763
H	1,737	1,832	1,806	1,743	1,736	1,751	1,744	1,847

Lowest Statistics are Best

EXHIBIT 6, PART 2

PT & Major Loss Distribution
Curve Fit by Maximum Likelihood
Chi-Square Statistics

State	Burr	Gamma	T. Gamma	I.T. Gamma	T. Beta	Pareto	Lognormal	Weibull
				DISTRIBUTION				
ESCALATING BENEFITS								
A	95	3.35×10^{11}	2.35×10^{10}	65	65	21,191	9.67×10^{6}	1.66×10^{8}
B	39	494	54	24	24	96	44	591
NONESCALATING BENEFITS								
C	32	3,121	1.57×10^{9}	13	13	5,047	9,696	2.1×10^{9}
D	41	45,108	1.43×10^{12}	904	42	458	3,470	1.47×10^{12}
E	11	975	24,846	113	12	456	429	22,328
F	89	2.07×10^{9}	16,465	96	112	6,637	4,334	1.5×10^{13}
LIMITED BENEFITS								
G	5	906	1.74×10^{10}	8	4	362	1,892	1.2×10^{8}
H	31	1,029	2.41×10^{6}	90	31	175	84	7.8×10^{8}

Lowest Statistics are Best

EXHIBIT 7

Parameter Development
Inverse Transformed Gamma
Permanent Total and Major Permanent Partial

Escalating Benefits

	State A		
Parameters	α	β	ρ
1st Report	3.4725	23,638	.6948
2nd Report	3.0598	24,323	.7062
3rd Report	3.2537	23,627	.6392

	State B		
Parameters	α	β	ρ
1st Report	.7720	194,021	4.1473
2nd Report	.5156	1,348,999	6.3716
3rd Report	.7077	195,658	3.2564

Limited Benefits

	State G		
Parameters	α	β	ρ
1st Report	3.76	16,827	.5727
2nd Report	3.99	16,571	.5526
3rd Report	3.76	16,827	.5727

	State H		
Parameters	α	β	ρ
1st Report	.18	1.37×10^{15}	84.6606
2nd Report	.19	1.37×10^{15}	102.7719
3rd Report	.18	1.37×10^{15}	84.6606

EXHIBIT 8

Loss Distribution Models
Parameters Chosen

Type of State/Injury	Distribution	α	β	ρ	θ
1. Escalating Benefit/ Fatal	Gamma		1.667	.60	
2. Escalating and Limited Benefit/Permanent Total and Major PP	I.T.G.	3.20	.515	.64	
3. Nonescalating and Limited Benefit/Fatal	Gamma		1.250	0.80	
4. Nonescalating Benefit/ Permanent Total and Major PP	T. Beta	7.00	.513	1.28	0.3
5. All/Minor PP and Temporary Total	T. Beta	2.20	7.24	0.12	2.9

EXHIBIT 9, PART 1

National Council on Compensation Insurance

State M
Effective: 1/1/89
Policy Period: 4/1/85–3/31/86
Report: First

Excess Loss Factor Calculation
Average Cost Per Case

Type of Injury	(1) Indemnity Losses	(2) Amend. Factor	(3) Trend in Ind. Cost Per Case	(4) Indem. Trend on Level (1) × (2) × (3)	(5) Medical Losses	(6) Law Level Factor	(7) Medical Trend	(8) Medical Trended on Level (5) × (6) × (7)
Fatal	$8,904,969	1.073	1.292	$12,345,101	$472,879	1.000	1.326	$627,038
PT	3,372,190	1.022	1.292	4,452,721	3,714,911	1.000	1.326	4,925,972
Major	114,956,442	1.017	1.292	151,048,626	46,784,854	1.000	1.326	62,036,716
Minor	14,573,805	1.017	1.292	19,149,455	9,794,077	1.000	1.326	12,986,946
TT	61,806,919	1.016	1.292	81,132,212	57,376,307	1.000	1.326	76,080,983
Med. Only					27,520,731	1.000	1.326	36,492,489

Type of Injury	(9) Total Losses (4) + (8)	(10) No. of Claims	(11) Average Severity (9)/(10)	(12) Severity Dev. to Ult. Rpt.	(13) Developed Severity (11) × (12)	(14) Indemnity Dev. to Ult. Rpt.	(15) Medical Dev. to Ult. Report	(16) Total Developed (4) × (14) + (8) × (15)
Fatal	$12,972,139	103	125,943	0.869	109,444	1.027	1.532	$13,639,041
PT	9,378,693	41	83,508	1.333	111,316	4.711	3.638	38,897,455
Major	213,085,342	2,623				2.060	1.904	429,278,077
Minor	32,136,401	2,628	12,228	0.761	5,174	0.908	0.959	29,842,186
TT	157,213,195	30,998	5,072	0.951		0.959	0.959	150,767,454
Med. Only	36,492,489	xx	xx	xx	xx	1.000	0.959	34,996,297

EXHIBIT 9, PART 2

National Council on Compensation Insurance

State M
Effective: 1/1/89
Policy Period: 4/1/84–3/31/85
Report: Second

Excess Loss Factor Calculation
Average Cost Per Case

Type of Injury	(1) Indemnity Losses	(2) Amend. Factor	(3) Trend in Ind. Cost Per Case	(4) Indem. Trend on Level (1) × (2) × (3)	(5) Medical Losses	(6) Law Level Factor	(7) Medical Trend	(8) Medical Trended on Level (5) × (6) × (7)
Fatal	$6,989,165	1.090	1.370	$10,436,920	$1,287,604	1.000	1.419	$1,827,110
PT	6,951,686	1.026	1.370	9,771,429	11,951,469	1.000	1.419	16,959,135
Major	182,012,327	1.021	1.370	254,593,383	59,464,731	1.000	1.419	84,380,453
Minor	17,083,444	1.021	1.370	23,895,809	10,947,760	1.000	1.419	15,534,871
TT	54,847,614	1.020	1.370	76,644,056	48,598,297	1.000	1.419	68,960,983
Med. Only					29,022,162	1.000	1.419	41,182,448

	(9) Total Losses (4) + (8)	(10) No. of Claims	(11) Average Severity (9)/(10)	(12) Severity Dev. to Ult. Rpt.	(13) Developed Severity (11) × (12)	(14) Indemnity Dev. to Ult. Rpt.	(15) Medical Dev. to Ult. Report	(16) Total Developed (4) × (14) + (8) × (15)
Fatal	$12,264,030	105	116,800	0.896	104,653	0.963	1.340	$12,499,081
PT	26,730,564	61	94,254	1.190	112,162	2.651	2.917	75,373,855
Major	338,973,836	3,819				1.315	1.404	453,260,455
Minor	39,430,680	4,020	9,809	0.864	4,760	0.975	0.867	36,767,147
TT	145,605,039	33,794	4,309	1.002		0.990	0.867	135,666,788
Med. Only	41,182,448	xx	xx	xx	xx	1.000	0.867	35,705,182

EXHIBIT 9, PART 3

National Council on Compensation Insurance

State M
Effective: 1/1/89
Policy Period: 4/1/83–3/31/84
Report: Third

Excess Loss Factor Calculation
Average Cost Per Case

Type of Injury	(1) Indemnity Losses	(2) Amend. Factor	(3) Trend in Ind. Cost Per Case	(4) Indem. Trend on Level (1) × (2) × (3)	(5) Medical Losses	(6) Law Level Factor	(7) Medical Trend	(8) Medical Trended on Level (5) × (6) × (7)
Fatal	$6,257,156	1.109	1.452	$10,075,698	$624,136	1.000	1.518	$947,438
PT	6,086,216	1.027	1.452	9,075,790	2,863,209	1.000	1.518	4,346,351
Major	186,520,691	1.023	1.452	277,057,088	57,815,100	1.000	1.518	87,763,322
Minor	17,093,885	1.022	1.452	25,366,368	10,177,879	1.000	1.518	15,450,020
TT	49,286,232	1.021	1.452	73,066,445	42,863,196	1.000	1.518	65,066,332
Med. Only					29,338,432	1.000	1.518	44,535,740

Type of Injury	(9) Total Losses (4) + (8)	(10) No. of Claims	(11) Average Severity (9)/(10)	(12) Severity Dev. to Ult. Rpt.	(13) Developed Severity (11) × (12)	(14) Indemnity Dev. to Ult. Rpt.	(15) Medical Dev. to Ult. Report	(16) Total Developed (4) × (14) + (8) × (15)
Fatal	$11,023,136	107	103,020	0.982	101,166	1.006	1.274	$11,343,188
PT	13,422,141	44	97,510	1.069	104,238	1.490	1.834	21,494,135
Major	364,820,410	3,835				1.119	1.337	427,366,443
Minor	40,816,388	3,664	11,140	0.912	5,364	0.994	0.989	40,494,240
TT	138,132,777	29,309	4,713	1.011		0.989	0.989	136,613,316
Med. Only	44,535,740	xx	xx	xx	xx	1.000	0.989	44,045,847

EXHIBIT 9, PART 4

Three-Year Statewide Totals

State M

Losses by Injury Type	
Fatal	$37,481,310
PT	135,765,445
Major	1,309,904,975
Minor	107,103,573
TT	423,047,558

Average Cost Per Case	
Fatal	$105,035
PT/Major	108,997
Minor/TT	5,084

EXHIBIT 10

National Council on Compensation Insurance

State M
Effective: 1/1/89

Limited Fatal Benefits—Nonescalating PT/Major Benefits

Calculation of *ELF* Trend

Policy Period:	4/1/85–3/31/86 First Report	4/1/84–3/31/85 Second Report	4/1/83–3/31/84 Third Report
(1) Effective Date of Filing		1/1/89	
(2a) Midpoint of Filing		1/1/90	
(2b) Midpoint of Policy Period	4/1/86	4/1/85	4/1/84
(3) Benefit Level		1/1/89	
(4a) Yrs. from (2b) to (3)	2.75	3.75	4.75
(4b) Yrs. from (3) to (2a)	1	1	1
(5) Indemnity Trend (1.060**(4a)) × (1.101**(4b))	1.292	1.370	1.452
(6) Medical Average Charge—3/31/88		321.95	
(7) Medical Average Charge—3/31/83		230.93	
(8) Change over 5 Yrs. (6)/(7)		1.394	
(9a) Indicated Change Per Year (8) ** .2		1.069	
(9b) Limit on Change Per Year		1.070	
(10) Medical Trend ((9)**(4a)) × (1.101**(4b))	1.326	1.419	1.518

EXHIBIT 11

Type of Injury Loss Distribution Table

Countrywide

$CLR_{I,H}$

Hazard Group

Injury Type	I	II	III	IV
Fatal	0.086	0.128	0.282	0.504
PT	0.158	0.208	0.278	0.355
Major	0.224	0.228	0.288	0.260
Minor	0.310	0.283	0.226	0.181
TT	0.308	0.281	0.240	0.171
Med. Only	0.331	0.297	0.201	0.171

Based on countrywide Unit Statistical Plan summaries, policy year 1981 at second and third reports.

EXHIBIT 12

Premium Distribution by Hazard Group*

State M

Hazard Group	(1) Standard Premium	(2) Total Standard Premium	(3) P_H (1) ÷ (2)
I	35,912,865	2,095,858,472	0.017
II	988,939,212		0.472
III	1,003,721,317		0.479
IV	67,285,078		0.032

* Based on Unit Statistical Data excluding stevedoring for policy periods 4/1/82–3/31/83, 4/1/83–3/31/84, 4/1/84–3/31/85 (second reports).

EXHIBIT 13

Distribution of Losses by Hazard Group for Each Injury Type

State M

$L_{I,H}$

Hazard Group

Injury Type	I	II	III	IV
Fatal	0.007	0.284	0.633	0.076
PT	0.011	0.400	0.543	0.046
Major	0.015	0.418	0.535	0.032
Minor	0.021	0.528	0.428	0.023
TT	0.020	0.514	0.445	0.021
Med. Only	0.023	0.566	0.389	0.022

These numbers are derived from the state premium distribution and countrywide loss distribution. For each Hazard Group, the following procedure is utilized to obtain the distribution of losses within each injury type:

> The percentage of countrywide losses by Hazard Group (see Exhibit 11) is multiplied by the corresponding statewide ratio of standard earned premium to total (Exhibit 12). This is then divided by the sum of these calculations for all four Hazard Groups.

EXHIBIT 14

Combined Injury Weights

State M

Hazard Group I

(1) Type of Injury	(2) Total Incurred Losses	(3) Injury Weights
Fatal	262,369	0.008
PT	1,493,420	0.043
Major	19,648,575	0.565
PT/Major	21,141,995	0.608
Minor	2,249,175	0.065
TT	8,460,951	0.243
Minor/TT	10,710,126	0.308
Med. Only	2,639,188	xx
Total	34,753,678	xx

Hazard Group II

(1) Type of Injury	(2) Total Incurred Losses	(3) Injury Weights
Fatal	10,644,692	0.011
PT	54,306,178	0.057
Major	547,540,280	0.575
PT/Major	601,846,458	0.632
Minor	56,550,687	0.059
TT	217,446,445	0.229
Minor/TT	273,997,132	0.288
Med. Only	64,946,987	xx
Total	951,435,269	xx

Hazard Group III

(1) Type of Injury	(2) Total Incurred Losses	(3) Injury Weights
Fatal	23,725,669	0.022
PT	73,720,637	0.068
Major	700,799,162	0.651
PT/Major	774,519,799	0.719
Minor	45,840,329	0.043
TT	188,256,163	0.175
Minor/TT	234,096,492	0.218
Med. Only	44,636,710	xx
Total	1,076,978,670	xx

Hazard Group IV

(1) Type of Injury	(2) Total Incurred Losses	(3) Injury Weights
Fatal	2,848,580	0.044
PT	6,245,210	0.096
Major	41,916,959	0.646
PT/Major	48,162,169	0.742
Minor	2,463,382	0.038
TT	8,883,999	0.137
Minor/TT	11,347,381	0.175
Med. Only	2,524,441	xx
Total	64,882,571	xx

For each Hazard Group, the following procedure is utilized to obtain the distribution of losses:

The injury type total incurred losses from Exhibit 9, Part 4 are spread across the Hazard Groups using the distributions from Exhibit 13. Within each Hazard Group, percentages of loss by type of total are then calculated in column (3).

EXHIBIT 15

Severity Differential to Unweighted Average

Countrywide

	Hazard Group			
Injury Type	I	II	III	IV
Fatal	0.771	0.911	1.087	1.231
PT	0.813	0.954	0.988	1.245
Major	0.898	0.930	1.041	1.131

Based on 1981 statistical plan data, latest second and third reports.

EXHIBIT 16

Severity Differentials by Claim Type and Hazard Group

State M

(A) Adjustment Factors

Injury Type	
Fatal	1.003164
PT	0.977201
Major	0.989057

(B) Normalized Differentials

	Hazard Group			
Injury Type	I	II	III	IV
Fatal	0.769	0.908	1.084	1.227
PT	0.832	0.976	1.011	1.274
Major	0.908	0.940	1.053	1.144
PT/Major	0.903	0.943	1.048	1.161

(A) For each serious injury type, the countrywide Hazard Group unweighted average cost per case differential from Exhibit 15 is multiplied by percent of premium in the Hazard Group for that state from Exhibit 12. These products are summed to form the factors in (A).

(B) For each Hazard Group, the factors from Exhibit 15 are divided by the appropriate adjustment factor in Section A of this exhibit to produce differentials appropriate for State M.
For PT and Major injury types, combined differentials are derived by calculating weighted averages of two differentials by Hazard Group, using the factors from Exhibit 14 as weights.

EFFECTS OF VARIATIONS FROM GAMMA-POISSON ASSUMPTIONS

GARY G. VENTER

Abstract

Two types of variations from negative binomial frequency are considered: mixtures of Poisson other than Gamma, and Poisson parameters that change over time. Any severity distribution can be used instead of the Gamma as a mixing distribution, and Bayesian estimators are easy to calculate from the mixed probabilities. In the case of changing frequencies over time, the Gerber-Jones model is illustrated for calculating credibilities. The Bailey-Simon method is found to be useful for testing model assumptions.

1. INTRODUCTION

A model often used for experience rating assumes that each individual risk has its own Poisson distribution for number of claims, with a Gamma distribution across the population for the Poisson mean. This model has been known since at least 1920 (M. Greenwood and G. Yule [7]), and has been applied to insurance experience rating since at least 1929 (R. Keffer [10]). However, there is meager theoretical support for the Gamma distribution as a mixing function, and the main empirical support given in many studies is that it provides a better fit to the aggregate claim frequency distribution than that given by the assumption that all individuals have the same Poisson distribution; e.g., see Lester B. Dropkin [4], B. Nye and A. Hofflander [12], or R. Ellis, C. Gallup, and T. McGuire [5]. The Poisson assumption for each individual does have theoretical support, but not enough to be regarded as certain. For example, the Poisson parameter could vary over time in random ways, to be discussed further below.

Several alternative models, which, in many cases, fit better than the Poisson, have been presented in the literature; e.g., Gordon Willmot [19, 20], M. Ruohonen [14], W. Hürlimann [8]. Many of these are mixtures of the Poisson by other distributions, such as the inverse Gaussian, reciprocal inverse Gaussian, beta, uniform, noncentral chi-squared, and three-parameter origin shifted Gamma distributions.

The purpose of this paper is to explore the adequacy of the Poisson and Gamma assumptions, the information needed to verify them, and the experience rating consequences of using these assumptions when they do not apply. As will be seen below, there are substantial differences in the experience rating implications of models which have very similar predictions of the aggregate claim frequency distribution. Thus, a model which gives a good fit to this distribution does not necessarily give proper experience rating adjustments. In other words, a model that just fits better than the Poisson is not enough for experience rating use. More detailed records which track individuals over time are needed to determine how much credibility should be given to individual claim experience.

2. PRELIMINARY BACKGROUND

Suppose each risk has its own claim frequency distribution, constant over time, and that the mean of the individual risk annual claim frequency variances is s^2, and the variance of the risk means is t^2. Among linear estimators, the expected squared error in subsequent observations is minimized by the credibility estimator $zx + (1 - z)m$, where m is the overall mean, x is the individual risk annual frequency observed, and for n years of observations, $z = n/(n + K)$, with $K = s^2/t^2$. See, for example, A. Bailey [1], H. Bühlmann [3], W. Jewell [9]. If the restriction to linear estimators is removed, then the Bayesian predictive mean minimizes the expected squared error. Thus, when the Bayes estimator is linear in the observations, it must be the same as the credibility estimator.

This is the case with the Gamma-Poisson model. In fact, if the Gamma has parameters α and β, with mean α/β and variance α/β^2, the Bayesian predictive mean is

$$\frac{\alpha + nx}{\beta + n} = \frac{\alpha}{\beta} \cdot \frac{\beta}{\beta + n} + x \cdot \frac{n}{\beta + n},$$

which is the credibility estimator, as $m = s^2 = \alpha/\beta$ and $t^2 = \alpha/\beta$. The linearity of the Bayes estimate gives a degree of justification to this model. Besides being easy to calculate, it is also less likely than some nonlinear functions to take on exaggerated values in extreme cases. The fact that it is the best linear model implies that even if it is the wrong model, it is the best linear approximation to the Bayes estimator of the actual distribution. The mixed distribution is the negative binomial, with the probability of n claims given by

$$p_n = \frac{(\alpha + n - 1)!\beta^\alpha}{(1 + \beta)^{\alpha+n} n!(\alpha - 1)!},$$

or recursively by

$$p_o = \left[\frac{\beta}{1 + \beta}\right]^\alpha, \; p_{n+1} = p_n \frac{\alpha + n}{(n + 1)(1 + \beta)}.$$

3. VARIATIONS FROM GAMMA ASSUMPTION

First, the Poisson assumption will be retained, so that each individual is assumed to have a fixed Poisson probability for number of accidents, and variation from the Gamma assumption will be explored.

With the Poisson assumption, each risk has the same mean and variance, so that the average risk variance s^2 is the same as the average risk mean, m. Since the aggregate variance v is $s^2 + t^2$, t^2 is the difference between the aggregate variance and mean (assuming the variance exceeds the mean). Thus, the best linear estimator is the credibility estimator with $K = m/(v - m)$. This is also the Bayes estimator for the Gamma prior distribution having variance t^2.

Other prior distributions may also have variance t^2, but they have different Bayes estimators which are not linear functions of the observations. Two distributions will be shown below for which the aggregate probabilities are much the same as the Gamma provides, but the Bayes estimates are substantially different for some risks. Nonetheless, since the variances are the same, the predictive means from the Gamma prior will be the best linear approximation to the Bayes estimate for either distribution.

The first is the good-risk/bad-risk model. The population has two types of risks; 90% are good risks with a low probability of a claim, and the other 10% are bad risks with a high claim probability. As each risk is still assumed to be Poisson distributed, this model is the Poisson mixed by a two-point prior. If the two Poisson means are a and b, then, over all risks, the probability of n claims is $p_n = .9a^n e^{-a}/n! + .1b^n e^{-b}/n!$, the mean is $m = .9a + .1b$ and the variance is $t^2 = (.9)(.1)(b-a)^2$. Thus the method of moments estimators for a and b are: $a = m - t/3$ and $b = m + 3t$. Given a risk with k claims in n years, the probability (conditional on k) that it is a bad risk is

$$q_k = \frac{1}{1 + 9e^{n(b-a)}\left(\frac{a}{b}\right)^k},$$

by Bayes Theorem. The Bayesian predictive mean for that risk is thus $a(1 - q_k) + bq_k$.

The second model is the inverse Gaussian distribution, discussed in Willmot [19]. This can be parameterized with two parameters b and c with density

$$f(y) = (2\pi c)^{-.5}b^{-1}(b/y)^{1.5}e^{1/2c(2-y/b-b/y)},$$

mean $= b$, and variance $= t^2 = b^2c$. This is a somewhat more skewed distribution than the Gamma. In fact, the skewness is $3c^{1/2}$, and the Gamma with the same first two moments has skewness $2c^{1/2}$. The Poisson mixture has mean $= b$, variance $= b + b^2c$, and the probabilities, p_i, of i claims given by:

$$p_0 = e^{1/c[1-(1+2bc)^{.5}]};$$

$$p_1 = p_0 b(1 + 2bc)^{-.5};$$

$$p_n = \frac{2bc(n - 1)(n - 1.5)p_{n-1} + b^2 p_{n-2}}{(1 + 2bc)n(n - 1)}, \quad n > 1.$$

The inverse Gaussian is not obviously related to the Normal distribution. It gets its name from the fact that there is a different, but equivalent, way of parameterizing the distribution that looks like a Normal distribution if you switch the variable and one of the parameters.

The cumulative probabilities can be calculated using the standard Normal cdf $N(x)$ by

$$F(x) = N\left(\frac{x-b}{\sqrt{bcx}}\right) + e^{2/c}\left[1 - N\left(\frac{x+b}{\sqrt{bcx}}\right)\right].$$

As with any Poisson mixture, given a risk with n claims in a period, the Bayesian predictive mean for the number of claims to be observed in a future period is $(n+1)p_{n+1} \div p_n$. This can be readily verified as follows: let $f(\lambda)$ denote the density for the Poisson parameter λ. Then $p_n = 1/n! \int f(\lambda)e^{-\lambda}\lambda^n d\lambda$. The Bayes predictive mean given n claims is $E(N|n) = E(\lambda/n) = \int \lambda f(\lambda|n)d\lambda = 1/p_n n! \int \lambda f(\lambda)e^{-\lambda}\lambda^n d\lambda$, by Bayes theorem, and the result follows. This implies that any severity distribution can be used as the mixing distribution for a Poisson. The advantage of the inverse Gaussian is that p_n is given by the above recursive formula, while many other distributions would require numerical integration for this.

For the sake of comparison, the Gamma, two-point, and inverse Gaussian prior distributions will be fit to a sample of medical malpractice claims by the method of moments, so that the variances will be the same and thus the Gamma-Bayesian estimators will be the best linear approximation to the other two. The sample used is four years of closed claims data from 7,744 internists as reported in Ellis, Gallup, and McGuire [5]. This is for illustration only, as the use of closed claims for pricing insurance has been questioned on various grounds [11]. The number of doctors having various claim counts is shown below:

Number of claims	0	1	2	3	4	5	6
Number of doctors	7,299	386	52	5	1	1	0

This sample has mean .0664 and variance .0834. For the four-year period, by the Poisson assumption, $m = s^2 = .0664$, and thus $t^2 =$ variance $- s^2 = .0170$. Matching the moments m and t^2 will give priors for a four-year Poisson parameter. These prior distributions get the following parameters:

Gamma:	$\alpha = .260$	$\beta = 3.91$
Inverse Gaussian:	$b = .0664$	$c = 3.86$
Two-Point:	$a = .0229$	$b = .458$

Using the formulae above, these parameters result in the following Bayesian predictive means (i.e., the expected number of claims in four years) for risks having the number of claims shown. The percentage errors from using the Gamma when one of the other prior distributions is correct are also given.

BAYESIAN ESTIMATES

Number of Claims:	0	1	2	3	4	5	6
Gamma	.0530	.257	.460	.664	.868	1.070	1.270
Inverse Gaussian	.0540	.223	.521	.852	1.190	1.530	1.860
Two-Point	.0521	.280	.443	.457	.458	.458	.458
Error from using Gamma if true distribution is:							
Inverse Gaussian	−2%	15%	−12%	−22%	−27%	−30%	−32%
Two-Point	2%	−8%	4%	45%	90%	134%	177%

The overall distribution of number of claims predicted by each distribution is given below. These sample and fitted aggregate claim frequencies and the Bayesian means above are shown in Figures 1 and 2.

OVERALL CLAIM PROBABILITIES

Number of Claims:	0	1	2	3	4	5	6
Sample	.943	.0498	.00672	.000646	.000129	.0001290	.0000000
Gamma	.943	.0499	.00640	.000983	.000163	.0000283	.0000051
Inverse Gaussian	.942	.0509	.00568	.000987	.000210	.0000500	.0000127
Two-Point	.943	.0491	.00686	.001010	.000116	.0000106	.0000008
Single Poisson	.936	.0621	.00206	.000046	7.58E-7	1.01E-8	1.11E-10

If the Bayesian estimates are used as experience rating charges, the above two tables together show that small differences in the aggregate probabilities lead to fairly large differences in charges. On a percentage basis, the mixed distribution probabilities differ from each other mostly in the right tails, where the claim data is least reliable. This is also the

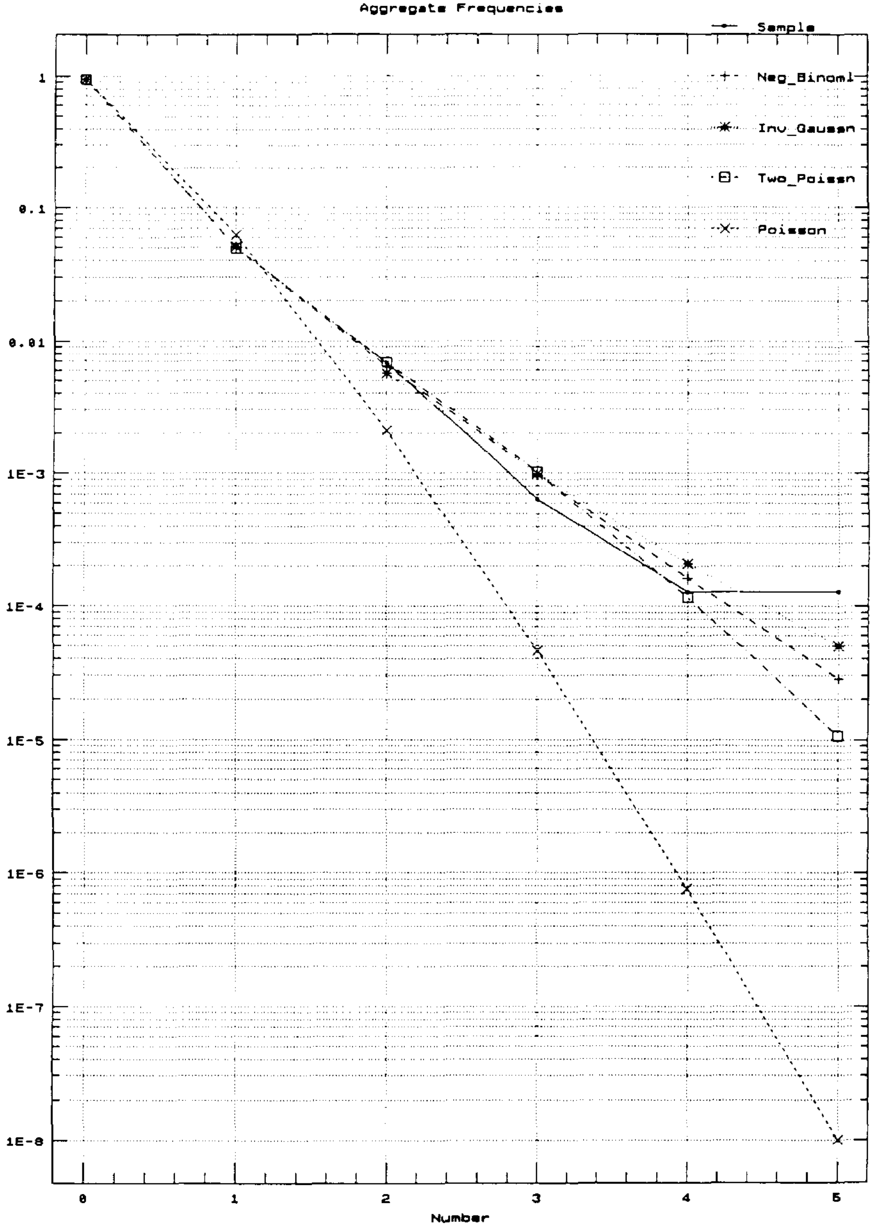
Aggregate Frequencies
Sample
Neg_Binoml
Inv_Gaussn
Two_Poissn
Poisson
1
0.1
0.01
1E-3
1E-4
1E-5
1E-6
1E-7
1E-8
0
1
2
3
4
5
Number

FIGURE 1

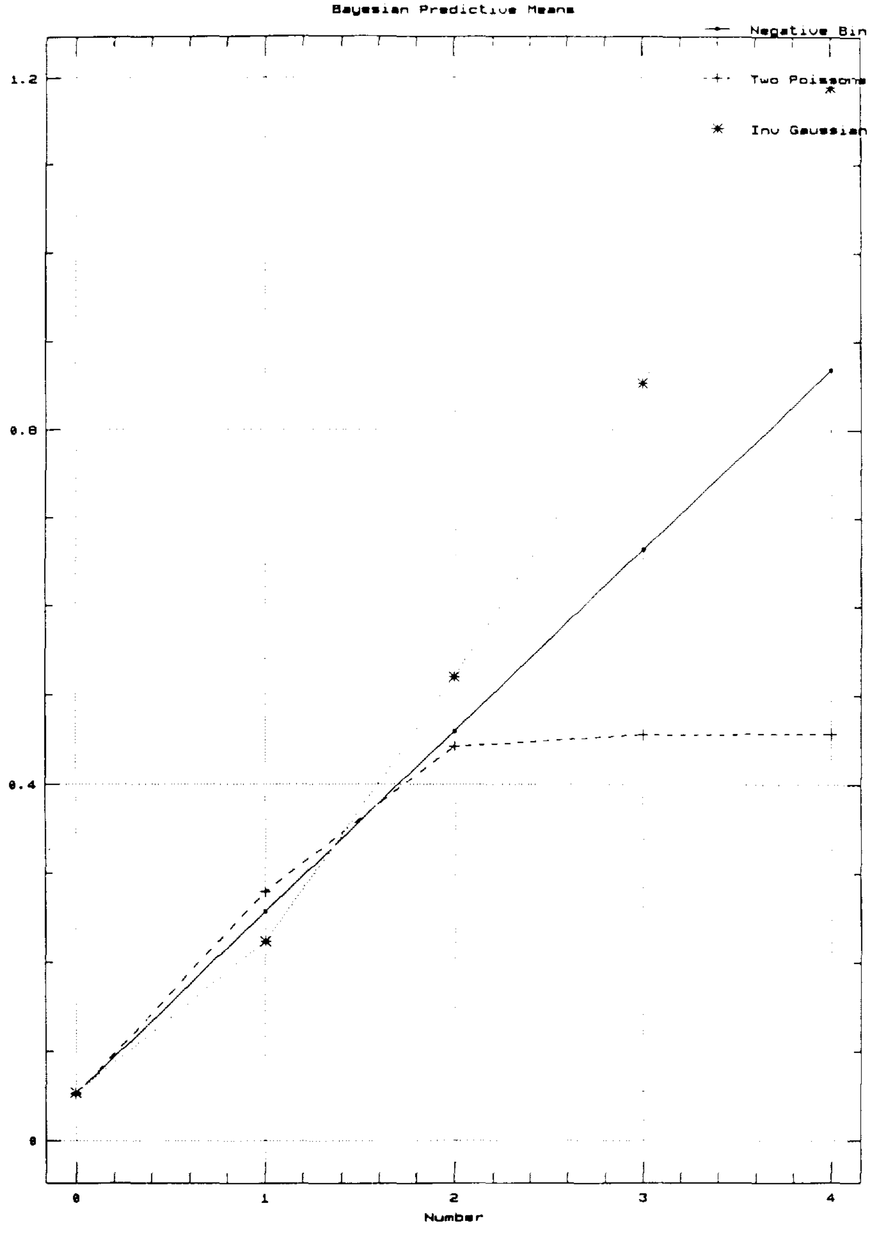
Bayesian Predictive Means
Negative Bin
Two Poissons
Inv Gaussian
1.2
0.8
0.4
0
0
1
2
3
4
Number

FIGURE 2

area in which the Bayesian estimates diverge the most. It is not clear that goodness-of-fit measures could be of much help in selecting among these distributions either, because without theoretical reason to support one prior distribution or another, a good fit in the left tails has questionable relevance to the right tails.

Nonetheless, as great as the differences are, they are small compared to those from using the overall mean of .0664 for each risk; i.e., not experience rating at all. If the population were known to consist of some mixture of risks, each with fixed Poisson distributed claims exposure, the two-point model might be the most justifiable in this case because the penalty for the right tail risks, whose exposures are not clearly understood, would be less. On the other hand, if it were known from other investigations that the high-frequency doctors were actually quite bad, a more heavily-tailed model might be justified.

4. VARIATIONS FROM POISSON ASSUMPTION

Two types of variation from the Poisson assumption are considered below. First, each risk may have some distribution of claim counts other than Poisson, with that distribution invariant over time. Second, each risk may have a fixed distribution for each time period, but the mean changes each period, with the degree of change coming from a distribution that is invariant over time.

The first case could arise if the risk has a Poisson distribution for each period, but the Poisson parameter is drawn at random each period from a prior distribution. The variance for a year would be the sum of the expected Poisson variance and the variance of the Poisson means from the prior. This is different from the second case because the Poisson parameters are drawn from a fixed distribution each year, while in the second case the incremental change in the parameter is so drawn. Both cases allow variation among risks in addition to the greater variation each risk can display due to the relaxation of the Poisson assumption. For instance, the good-risk/bad-risk model could end up being a mixture of two Negative Binomials instead of two Poissons. Since there are many possibilities for distributional assumptions, the analysis of these cases will be carried out only for the linear approximations to the Bayesian estimates, that is for the credibility estimators.

In the first case, the credibility estimator is the same as discussed above, $zx + (1 - z)m$, with $z = n/(n + s^2/t^2)$, again where s^2 is the expected individual risk variance and t^2 is the variance of the risk means.

This result did not depend on any Poisson assumption. However, without the Poisson assumption, it is not as easy to determine what s^2 and t^2 should be. They still add to the total variance v, but $v - m$ might not be a good estimator of t^2, as s^2 may be greater than m. In fact, in the extreme case, $s^2 = v$ and $t^2 = 0$, when all risks have the same (non-Poisson) distribution of claims. In this case, $z = 0$ and the best estimator for any risk is the overall mean m.

As Nye and Hofflander [13] point out, this is not an appealing model, because most people believe there is some inherent difference among risks. However, it is also plausible that risks would have some degree of instability over time as well, and the question becomes how much of the difference $v - m$ can be attributed to each effect. Data such as the distribution of claims in period 2 for the risks with no claims in period 1 would be useful for making such determinations. If the no-claim risks from period 1 had the same distribution in period 2 as did all the other risks, for example, it would lend support to the conclusion that all risks are fundamentally the same. On the other hand, if they had better-than-average experience, the credibility that should be attributed to those risks could then be estimated.

One model of the latter case is provided by H. Gerber and D. Jones [6]. The individual risk mean changes each year by a random amount taken from a distribution with mean 0 and variance d^2. For year 1, the risk means are distributed around an overall mean m with variance t^2. (Thus, for year 2, assuming independence, the mean is still m, but the variance is $t^2 + d^2$, etc.) The distribution of actual results around a risk's mean for a given year has variance s^2. Then, given a risk with losses X_i in year i, the linear least square estimator C_{i+1} for the next year's losses is given iteratively by:

$$C_1 = m; \qquad C_{i+1} = z_i X_i + (1 - z_i) C_i$$

where

$$z_1 = \frac{L}{L + 1}; \qquad z_{i+1} = \frac{z_i + J}{z_i + J + 1};$$

$$L = t^2/s^2; \qquad J = d^2/s^2.$$

This follows from [22: p. 428] by taking $v = s^2$ and $w = t^2$.

Gerber and Jones had a somewhat more general framework, in that m and t^2 were any prior mean and variance for the (conditional) mean of X_1, not necessarily arising from a distribution of risks around a grand mean. Working through the iterative definition gives

$$C_{i+1} = m \prod_{j=1}^{i} (1 - z_j) + \sum_{j=1}^{i} X_j z_j \prod_{h=j+1}^{i} (1 - z_h),$$

which shows how the credibility for an observation decreases in estimating ever later future observations. When all the past observations are 0, the estimate reduces to the first term above.

One study that provided data that could be used to evaluate the above cases was that of Robert Bailey and LeRoy Simon [2]. They estimated the credibility of one, two, and three years of driver experience as 1 minus the relative claim frequency of drivers with one or more, two or more, and three or more years without a claim prior to the experience period. The results for five driver classifications are shown below.

CREDIBILITY FOR CLAIM FREE EXPERIENCE

Class	1 year	2 years	3 years
1	.046	.068	.080
2	.045	.060	.068
3	.051	.068	.080
4	.071	.085	.099
5	.038	.050	.059

Bailey and Simon note that the additional credibility for years past the first is less than would be anticipated. Fitting $n/(n + K)$ to this data by row by least squares for K gives the percentage errors:

PERCENTAGE ERRORS IN CREDIBILITY WITH $n/(n + K)$

Class	1 year	2 years	3 years	K
1	−30	−8	+14	30.4
2	−38	−10	+16	35.1
3	−37	−8	+14	30.0
4	−41	−6	+17	23.2
5	−39	−9	+13	41.6

The large errors and systematic signs on the errors are indicative that the standard credibility model is inappropriate. Any other method of fitting the K's would have the same result. Also, the credibility from this model being too high in the third year and too low in the first indicates that the relevance of a year's data declines as it ages, which suggests that the changing mean model may apply. Fitting this by using least squares to find J and z_1 gives the following percentage errors:

PERCENTAGE CREDIBILITY ERRORS USING GERBER-JONES MODEL

Class	1 year	2 years	3 years	z_1	J
1	−4	+3	−1	.0098	.018
2	−9	+7	−1	.0035	.020
3	−8	+7	−3	.0065	.022
4	−13	+11	−3	.0024	.033
5	−11	+8	−2	.0044	.016

This fits much better, but it does use more parameters and still has systematic sign changes. The extra parameter helps because it does the right thing—it decreases the relative credibility for the older years, reflecting changing risk conditions over time. Nonetheless, there still seem to be aspects of the data that are not captured by the model.

The Bailey-Simon results support the use of changing parameter models over fixed parameter mixed models, Poisson or not. They also support the use of experience rating over not experience rating, and their work points to the kind of data that is needed to compute experience credits and debits.

Other studies using similarly detailed data have also rejected the stable Poisson assumption for automobile insurance. These include Venezian [16], who found that a two-point Poisson model with shifting driver probabilities between the two parameters fit well to California data from 1961–1963, and Richard Woll [21], who found problems with both the Gamma and the stable Poisson hypotheses using four years of North Carolina data published in 1970. These findings do not challenge the value of experience rating, but they do tend to reduce the credibilities that would apply.

The situation is not necessarily the same for medical malpractice, in that greater training is required prior to licensing, so learning by doing should have a smaller effect. However, there is anecdotal evidence to the contrary. In one study that followed individuals across time periods, Venezian, Nye, and Hofflander [18] were not able to reject the stable Poisson hypothesis using a chi-square test. However, the results of Venezian [17] suggest that there is not enough data in their sample to detect moderate deviations from Poisson by this test. Other tests, such as computing the Bailey-Simon credits and debits deserved by class, would be possible from their data. Another caveat is that since the sample contains only large claims, it may be better approximated by the Poisson than would data using all claims, due to the effect of the severity probability of a loss being large (see Joseph Schumi [15]).

In conclusion, aggregate frequencies are not adequate to verify either the Poisson or the Gamma hypothesis. Variations from the fixed Poisson assumption are likely, and would tend to lower the credibility which should be given to risk experience; variations from the Gamma assumption could lower or raise it. The Bailey-Simon method provides a good way to test proper credibilities, and the Gerber-Jones model gives a method to model changing frequencies over time.

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[17] Emilio C. Venezian, "The Distribution of Automobile Accidents—Are Relativities Stable over Time?" *PCAS* LXXVII, 1990.

[18] E. Venezian, B. Nye, and A. Hofflander, "The Distribution of Claims for Professional Malpractice: Some Statistical and Public Policy Aspects," *The Journal of Risk and Insurance* 61.4, 1989.

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DISCUSSION OF PAPER PUBLISHED IN VOLUME LXXVII

RISK LOADS FOR INSURERS

SHOLOM FELDBLUM

DISCUSSION BY STEPHEN W. PHILBRICK

1. INTRODUCTION

Mr. Feldblum has written a very interesting paper on the subject of risk loads. I am happy to see more written on this subject. His paper concentrates on risk load in the context of pricing. Because I believe that the risk load in pricing is inextricably linked to the risk margins in reserving, this paper will also add to the literature on that important subject.

However, I believe that Mr. Feldblum's enthusiasm to embrace Modern Portfolio Theory Methods has caused him to summarily dismiss other approaches a bit too quickly. It is only a slight overstatement to summarize Mr. Feldblum's paper as follows:

> There are five ways to calculate risk loads.
> Four are wrong; one is right.

I find that many of Mr. Feldblum's concerns are quite relevant and, to some degree, compelling. Many of the methodologies currently employed do suffer from incomplete theoretical justification. However, my opinion is that the conclusions are not nearly so black-and-white as Mr. Feldblum would have us believe.

I will offer my comments on each of the five methods as defined by Mr. Feldblum.

2. STANDARD DEVIATION AND VARIANCE METHODS

The most important comment (perhaps obvious to many) is that Mr. Feldblum's criticisms do not extend to *all* standard deviation and variance methods, but only to the specific methodology employed by the Insurance Services Office (ISO), which incorporates the process risk associated with the severity distribution. It is, of course, possible to incorporate frequency considerations into the calculations and, less easily, parameter risk considerations. One still might label these methods standard deviation and variance methods, and they might not suffer the same criticisms outlined by Mr. Feldblum.

I strongly share Mr. Feldblum's concern about the absence of parameter risk considerations in the risk load procedure. When the procedure was first implemented, I recall long conversations with a colleague where we attempted to determine whether the parameter risk might be even approximately coincident with the process risk. We concluded that parameter risk would be distributed across limits in a pattern differently than process risk; thus, the ISO procedure would not provide a surrogate for the total risk loading.

While I agree with Mr. Feldblum's concerns about parameter risk, I cannot agree with his statement, "In other words, the standard deviation of the individual's loss distribution is no guide even to the process risk faced by the insurer." He purports to show this by noting that the coefficient of variation (CV), or standard deviation divided by the mean, of 100 policies is vastly different than the CV of a single policy. This might be relevant if an insurer considered writing a single policy, but it does not.

The more relevant question is: If an insurer writes 100 policies and contemplates writing an additional policy, will the insurer's risk load requirements bear any relationship to the standard deviation or variance of the individual risk in question? This specific issue is explored in Rodney E. Kreps's recent paper [1].

The answer is yes, although the specific form of the answer surprised me. Suppose an insurer decides that its total risk load should be proportional to the variance of the aggregate distribution of its entire portfolio. Then it is reasonable to conclude that the risk load for an additional (marginal) insured should be proportional to the *marginal* increase in the aggregate variance. Assuming independence of risks, the marginal increase in aggregate variance is proportional to the variance of the individual risk. (This should hardly be surprising, as the marginal increase in aggregate variance is equivalent to the variance of the marginal risk.)

Alternatively, if the insurer decides that its total risk load should be proportional to the standard deviation of the aggregate distribution (a ruin theory approach), then the risk load for a marginal insured should be proportional to the marginal increase in the aggregate standard deviation. This increase is *also* proportional to the *variance* of the individual risk (not the standard deviation). (See Appendix for details.)

It should be noted that the calculations in the Appendix are done with the assumption of independence between risks, i.e., no covariance. The covariance term is incorporated in Mr. Kreps's paper [1]. The covariance terms should probably not be ignored in practice.

While Mr. Feldblum may be literally correct to say that the standard deviation of the individual risk is no guide to the insurer's process risk, the variance of the individual risk *is* such a guide.

3. UTILITY THEORY

I share all of the concerns laid out by Mr. Feldblum. While mathematically appealing, the practical problems are so difficult that I have never attempted to actually use utility theory in practice; nor have I read an exposition of such an attempt that satisfied *all* my concerns.

My only disagreement with Mr. Feldblum is his broad application of the concluding sentence of his introduction: "Only the last method, however, measures the true risk faced by insurers." Utility theory does, in fact, measure the true risk faced by insurers. Utility theory fails to be used commonly, not because it doesn't measure the true risk, but because of the practical problems associated with implementation.

4. PROBABILITY OF RUIN

Mr. Feldblum suggests that there are three ways to formulate the problem in the context of ruin theory. If one is limited to these three alternatives, one might indeed conclude that ruin theory is not up to the task of specifying risk loads. Let me suggest a fourth formulation of the problem that I believe falls within the sphere of ruin theory: For an insurer with a given portfolio of risks, what is the required amount of surplus plus risk loading necessary such that the probability of ruin is less than a given amount, and what is the proper relationship between the relative amounts of surplus and risk loading?

I believe that the above formulation may lead to practical solutions to the problem. (See Mr. Kreps's paper [1] for a specific exposition along this line.)

Mr. Feldblum's mathematical examples are not persuasive. In his first example, he is apparently attempting to prove that ruin theory applications would produce inappropriate or inconsistent risk loading requirements. While he concedes (in a footnote) that his examples are extreme, he suggests that similar conclusions will follow if one applies the analysis to "an insurer writing 1,000 policies." I disagree.

The calculations associated with an insurer writing only one or two risks are not a reliable guide to the calculations for an insurer that has already written 1,000 policies and is considering the addition of one of these two alternatives. Mr. Feldblum argued eloquently in his discussion of utility theory that wealth independence does not conform with reality. Mr. Feldblum should not then make the assumption he earlier refuted.

In his second mathematical example, he asks us to presume that a ruin theory calculation requires a risk load of "10% of premium on a $100,000 premium policy and 50% of premium on a $1,000,000 premium policy." Furthermore, the marketplace allows "only a 20% risk load on the latter policy." He then concludes that insurers would prefer the latter policy, thereby (apparently) proving that ruin theory is flawed. It is difficult to respond precisely without seeing the actual numbers that led to his required loads. I suspect that he may not be carefully distinguishing between required additional risk load and required additional surplus. His examples do not prove that ruin theory leads to inappropriate conclusions.

Ruin theory (in its present form) is not the solution to all problems. A personal concern is the overly simplistic binary division of the world into solvent and insolvent companies. Gradations of solvency are important, and not easily handled in ruin theory. Gradations of insolvency are also important, for a company that is "just barely" insolvent imposes a different burden on guaranty funds and society than a company that becomes insolvent by many millions of dollars.

5. REINSURANCE METHOD

The reinsurance method is far from perfect. For one reason, many reinsurance transactions are motivated in part by tax or regulatory considerations. This will distort the ability of the reinsurance transaction to provide a reliable guide to the appropriateness of risk loads. And, of course, Mr. Feldblum is technically correct in concluding that reinsurance does not "solve" the risk load problem; it merely transfers the problem to someone else. However, the reinsurance approach is valuable for two very different reasons:

1. It provides a powerful reality check for theoretically-based methods. I have seen a proposed theoretical method easily disproven by considering it in a reinsurance environment.

2. It provides real world answers in real situations. Consider a small insurer, wishing to issue policies with a $2,000,000 limit, but only able to retain $500,000 net. This insurer might set the price (including risk load) on the $1,500,000 excess of $500,000 equal to what its reinsurer is charging for that layer. It is not very meaningful to calculate a theoretical amount of risk load for that upper layer (unless the insurer can persuade the reinsurer to change its prices). The insurer is still left with the task of calculating the risk load on the net layer, but some of the original problem has been solved.

6. MODERN PORTFOLIO THEORY

Mr. Feldblum's discussion of this subject is a welcome addition to the actuarial literature. In the year that the CAS finally adds finance to its *Syllabus*, it is appropriate that we continue to explore the financial literature for useful tools. I am convinced that the Capital Asset Pricing Model (CAPM) is a useful tool for explaining concepts. I am less convinced that CAPM is the best tool for explicitly determining risk loads in practice.

For example, I am not yet ready to conclude that companies should write Aircraft and Surety (with betas of .07 and .04, respectively) at rates that generate returns equal to risk-free securities. Mr. Feldblum has provided us with much interesting and relevant background on CAPM, but he has left out the fact that major controversies arise over the actual application of the theory to specific problems, including insurance.

7. SUMMARY

Mr. Feldblum has given us much to think about regarding the subject of risk loads. He properly points out that some of the existing methodologies have various flaws, and a promising methodology (CAPM) should be explored further.

The subject of risk loads is critically important to the actuary. As a profession, actuaries need to refine, correct, or enhance all of our potential tools (including others not discussed here, such as option pricing theory). Eventually, we may settle on a single approach; but, at the present time, the choice is far from obvious.

APPENDIX

Assume an insurer considers writing one or the other of the two risks described by Mr. Feldblum in Table 1. The relevant information is repeated below:

Risk	Amount of Loss	Probability of Loss	Expected Value of Loss	Standard Deviation of Loss	Variance of Loss
A	$ 100,000	.01	$1,000	$ 9,950	$ 99,000,000
B	1,000,000	.001	1,000	31,607	999,000,000

Suppose that the insurer already writes 10,000 risks of type A. (The conclusions can also be made if the existing portfolio consists of type B risks, a mixture of each, or even a variety of different risks. I chose 10,000 type A risks to simplify the mathematics.) With 10,000 risks of type A, the aggregate parameters are as follows:

Expected Losses	$10,000,000
Variance	$990,000,000,000
Standard Deviation	$994,987
Coefficient of Variation	.099

If the insurer writes an additional risk of type A, obviously the total variance increases by $99,000,000. The standard deviation increases from $994,987 to $995,037, an increase of approximately $50.

If the insurer, instead, were to start with 10,000 risks of type A and write one additional risk of type B, the variance would increase by $999,000,000 and the standard deviation would increase from $994,987 to $995,489, an increase of approximately $502. This information is summarized as follows:

Risks	10,000 Type A	10,000 Type A Plus 1 Type A	10,000 Type A Plus 1 Type B
Expected Losses	$10,000,000	$10,001,000	$10,001,000
Variance	$990,000,000,000	$990,099,000,000	$990,999,000,000
Marginal Variance		$99,000,00	$999,000,000
Standard Deviation	$994,987.44	$995,037.19	$995,489.33
Marginal Standard Deviation		$49.75	$501.89

Note that the increase in standard deviation associated with risk B compared to risk A is in the same proportion as the relative variance of the individual risk. In both cases, this ratio is 10.09.

REFERENCE

[1] Rodney E. Kreps, "Reinsurer Risk Loads from Marginal Surplus Requirements," *PCAS* LXXVII, 1990, p. 197.

ADDRESS TO NEW MEMBERS—MAY 18, 1991

CARLTON W. HONEBEIN

As a native Californian, I'd like to welcome the new Fellows and Associates and everyone else in attendance to the "Sunshine State."

My accent may betray the fact that I wasn't born in California, but living out here for 25 years qualifies me as an honorary native.

The CAS started the procedure of having someone speak to the new members in 1984, and the following year, Stan Khury refined the process by deciding to have a past president—an old-timer—perform the duty. At that time I can remember thinking. . . . Yes, that's a good idea . . . to get one of the old-timers back and keep them involved in CAS meetings. So when Chuck Bryan called me to perform this function, I was both honored and depressed: honored to have the opportunity to speak to the CAS once again, but depressed to think that I was now an "old-timer."

You might think that, as you get older, school exams and actuarial exams all fade into one integrated activity. While I've managed to blank out individual exams, I do know one thing—in college I took exams to get out, but with the CAS, I took exams to get in.

When you graduated from college you received a diploma, you stopped paying tuition, and you participated in alumni functions sporadically. On the other hand, when you get your Associateship or Fellowship in the CAS, you get the privilege of paying annual dues; you acquire the responsibility of ethical and professional conduct; you shoulder the burden of ongoing education in order to maintain designation; and while participating in Society committees and governance is voluntary, it is strongly encouraged.

What else happens? Sooner or later at some cocktail party, someone will discover what you do for a living and exclaim—"Oh! You are an actuary. . ." and you'll have that ego rush we all feel when recognized for our mental capacities and what we have accomplished in passing the actuarial examinations. But that statement is also accompanied by negative connotations—pigeonholing you as introverted, shy, and non-communicative.

Decisions in business are made based upon lots of reasons. They are based on facts, proofs, competition, presentation, perception, forcefulness, and stubbornness. And, while actuaries usually have the facts, proofs, and perception, all too often these objective views are lost in the verbal barrage of the salespeople and underwriters.

Perhaps the actuary feels constrained in that his science is unable to quantify all the vagaries of the business world. But, this does not hamper the salesman who has learned to "go with his instincts" rather than just facts. Actuaries have to learn to do two things: first of all, to be forceful enough in their presentation so that the facts get proper weight in corporate decision making; and secondly, from time to time they have to go with their instincts, relying on judgment and perception just as the salesperson does.

Some time ago, I was chatting with Wes Kinder, a past California insurance commissioner, and he said to me, "You know, everybody thinks that actuaries are pessimists—but they really are optimists."

He reacted to my puzzled look by saying, "Actuaries always want to increase rates and raise reserves. Everybody thinks they're prophets of doom. But, hindsight shows that rates are almost always too low and the reserves inadequate. So actuaries are really optimists. They are just not as optimistic as everybody else."

That comment from Wes Kinder has always stuck with me. By becoming an actuary, you've accepted the challenge of foretelling the future, an impossible task. As I've always said of stockbrokers: if they were any good at picking stocks, they wouldn't have to work for commissions.

Perhaps you have heard the story which compares an insurance company to a car speeding down a highway. The president is at the wheel, the chief salesman has his foot on the gas pedal pushing it to the floor, the chief underwriter is fighting to get his foot on the brake, and they are all reacting to instructions being screamed by the actuary reading from a map he has drawn by looking out a back window.

Even if we can never see the future clearly, if actuaries are going to do their job right, they have got to look forward, not merely project the past. And that's the real challenge that all of you face in being successful actuaries.

In closing, I would like to welcome the new Associates to membership and to congratulate the new Fellows on their new elevated status. It's your Society and we need you to help keep it strong and viable.

But one last warning! This may happen to you. One day the telephone will ring and someone from the CAS Office will say, "Carl, in reviewing our records, we have found a terrible error. You did not pass Part 3 of the exams; so you are not a member of this Society until you take, and pass, that exam."

At that point, I wake up soaked with sweat and trembling with fear. Sometimes it seems so real. And I no longer can pass that exam. Not because the exams are harder, but I, as all of you will one day, grew out of that phase of life and into the next.

MINUTES OF THE 1991 SPRING MEETING

May 19–21, 1991

MARRIOTT'S DESERT SPRINGS, PALM DESERT, CALIFORNIA

Sunday, May 19, 1991

The Board of Directors held their regular quarterly meeting from Noon to 5:00 p.m.

Registration was held from 4:00 p.m. to 6:00 p.m.

From 5:30 p.m. to 6:30 p.m., there was a special presentation to new Associates and their guests. The session included an introduction to the standards of professional conduct and the CAS Committee structure.

A reception for all members and guests was held from 6:30 p.m. to 7:30 p.m.

A dinner for the Board of Directors and the members of the Executive Council was held from 7:30 p.m. to 10:00 p.m.

Monday, May 20, 1991

Registration continued from 7:00 a.m. to 8:00 a.m.

President Charles A. Bryan opened the meeting at 8:00 a.m. The first order of business was the admission of members. Mr. Bryan recognized the 76 new Associates and presented diplomas to the five new Fellows. The names of those individuals follow.

FELLOWS

James J. Gebhard	Valerie Schmid-Sadwin	Nancy P. Watkins
Melissa A. Salton	Warren B. Tucker	

ASSOCIATES

Marc J. Adee	Richard S. Goldfarb	Kenneth P. Quintilian
Karen L. Ayres	Charles T. Goldie	Eric K. Rabenold
Nathalie Begin	Matthew E. Golec	Donald K. Rainey
Thomas S. Boardman	Todd A. Gruenhagen	Elizabeth M. Riczko
Pierre Bourassa	Ellen M. Hardy	William E. Roche
Alicia E. Bowen	Jeffrey R. Ill	Bradley H. Rowe
Anthony J. Burke	Kathleen M. Ireland	Yves Saint-Loup
Janet L. Chaffee	Changseob Kim	Joanne Schlissel
Mario Champagne	Richard O. Kirste	Vincent M. Senia
Cindy C.M. Chu	David R. Kunze	Derrick D. Shannon
Dianne Costello	Frank O. Kwon	David A. Smith
Martin L. Couture	D. Scott Lamb	Keith R. Spalding
Kenneth M. Creighton	Mathieu Lamy	Stephen D. Stayton
Patrick K. Devlin	Nicholas J. Lannutti	Frederick M. Strauss
Kevin G. Dickson	Paul W. Lavrey	Rae M. Taylor
Pierre Dionne	Giuseppe F. Lepera	Peter S. Valentine
Yves Doyon	Donald F. Mango	Kenneth R. Van Laar
Patrick Dussault	Donald R. McKay	William Vasek
Brad C. Eastwood	Marlene D. Orr	Michael A. Visintainer
Charles C. Emma	Donald D. Palmer	Sebastian Vu
William E. Emmons	Chandrakant C. Patel	Kevin Wick
Yves Francoeur	Timothy B. Perr	Gnana K. Wignarajah
Louis Gariepy	Julia L. Perrine	John M. Woosley
Bruce R. Gifford	Sarah L. Peterson	Vincent F. Yezzi
Michael A. Ginnelly	Brian D. Poole	Sheng Hau Yu

Mr. Bryan introduced Carlton W. Honebein, a past President of the Society, who addressed the new members regarding their professional responsibilities.

Mr. Bryan introduced several guests: Terry Clarke from the British Institute of Actuaries and General Insurance Study Group (GISG); Daphne Bartlett, President of the Society of Actuaries; Ray Cole, President of the Conference of Actuaries in Public Practice; and Mavis A. Walters, President of the American Academy of Actuaries.

Robert F. Conger, Vice President-Administration, made a presentation on the new CAS Office. Mr. Conger also introduced James H.

"Tim" Tinsley, Executive Director of the CAS, who spoke about the opportunities for service to the membership that the new office presented.

Albert J. Beer, Vice President-Programs and Communications presented the highlights of the program.

Amy S. Bouska, a member of the Continuing Education Committee, presented a summary of the Discussion Paper Program.

Irene K. Bass, Vice President-Continuing Education, summarized the new *Proceedings* papers.

The Harold Schloss Award winner, La Tisha Booth, was announced.

Mr. Bryan concluded the business session at 9:30 a.m.

After a short break, Mr. Bryan introduced Henry Parker III, Senior Vice President and Managing Director of Chubb & Son Inc. Mr. Parker delivered an address on underwriting and brokering opportunities overseas, the barriers to doing business, and how to overcome them.

A panel presentation, "Solvency Regulation Around the World," followed. The panel was moderated by LeRoy J. Simon, Executive Consultant, Coopers & Lybrand. The panel members were: Christopher D. Daykin, Government Actuary, Government Actuary's Department, United Kingdom; John J. Gardner, Managing Director, Insurance Solvency International, Ltd.; and Robert M. Hammond, Deputy Superintendent–Insurance and Pensions Sector, Office of Superintendent of Financial Institutions, Canada.

A luncheon followed from 12:30 p.m. to 2:00 p.m.

Following lunch, the remainder of the afternoon was devoted to presentations of the discussion papers, the *Proceedings* papers, and six panel presentations.

The new *Proceedings* papers were:

1. "Retrospective Rating: Excess Loss Factors"

   Author: William R. Gillam
   National Council on Compensation Insurance

2. "Effects of Variations from Gamma-Poisson Assumptions"

   Author: Gary G. Venter
   Workers' Compensation Reinsurance Bureau

3. A Discussion of "Risk Load for Insurers"

   Author: Stephen W. Philbrick
   Tillinghast/Towers Perrin

The Discussion Papers presented were:

1. "Pricing Threshold No-Fault Automobile Insurance in Canada"

   Authors: Claudette Cantin
   Tillinghast/Towers Perrin

   David J. Oakden
   Tillinghast/Towers Perrin

2. "Auto Insurance in Italy"

   Authors: Terry G. Clarke
   Tillinghast/Towers Perrin

   Laura Salvatori
   Tillinghast/Towers Perrin

3. "Actuarial Aspects of Claims Reserving in the London Market"

   Authors: George Maher
   Tillinghast/Towers Perrin

   John P. Ryan
   Tillinghast/Towers Perrin

   Pierre A. Samson
   Tillinghast/Towers Perrin

4. "No-Claim Discount or Bonus/Malus Systems in Europe"

   Author: Guy H. Whitehead
   Bacon and Woodrow France

5. "Canadian Reserve Certification: Current Requirements and Practices"

   Author: Joanne S. Spalla
   Hartford Insurance Group

6. "Loss Reserve Opinion Requirements in the Principal Insurance Markets of the World"

   Author: Ralph L. Rathjen
   Tillinghast/Towers Perrin

7. "Some Aspects of Currencies and Exchange Rates in the London Market"

   Authors: Fred Duncan
   Sphere Drake Insurance, plc

   Roger M. Hayne
   Milliman & Robertson, Inc.

8. "A Closed System for Currency Fluctuation Control"

   Author: LeRoy J. Simon
   Coopers & Lybrand

9. "Solving the Problem of Foreign Exchange in Insurance"

   Author: Jay B. Morrow
   American International Group

10. "The Spiral in the Catastrophe Retrocessional Market"

    Authors: James N. Stanard
    F&G Re, Inc.

    Michael G. Wacek
    St. Paul Management, Ltd. (U.K.)

11. "Hurricane Sidney"

    Author: Colin J. W. Czapiewski
    Terra Nova Insurance Co., Ltd.

12. "Appraisal Values—A Comparison of European and North American Practice"

Authors: Ken Larner
Tillinghast/Towers Perrin

John P. Ryan
Tillinghast/Towers Perrin

13. "The Structure and Pricing of Savings-Type Policies in Japan"

Authors: Yasukazu Yoshizawa
Tokio Marine and Fire Insurance Co.

Daniel C. Goddard
Houston General Insurance Co.

The panel presentations covered the following topics:

1. "The Earthquake Peril—Progress Towards a Solution"

Moderator: John P. Drennan
Vice President and Actuary
Allstate Insurance Co.

Panelists: Eugene L. Lecomte
President
National Committee on Property Insurance

Richard J. Roth
Chief Property and Casualty Actuary
California Department of Insurance

Darrell W. Ehlert
Consultant
The Earthquake Project

2. "Qualification Standards and Continuing Education Requirements"

Panelists: Mavis A. Walters
President, American Academy of Actuaries
Executive Vice President
Insurance Services Office, Inc.

Irene K. Bass
CAS Vice President-Continuing Education
Principal
William M. Mercer, Inc.

Michael A. Walters
Chairman, Casualty Practice Council
Consulting Actuary
Tillinghast/Towers Perrin

3. "Risk Classification—The Science and the Fiction"

Moderator: John J. Kollar
Vice President and Actuary
Insurance Services Office, Inc.

Panelists: Frederick W. Kilbourne
President
The Kilbourne Co.

Michael J. Miller
Consulting Actuary
Tillinghast/Towers Perrin

4. "Guaranty Fund—Panel Discussion"

Moderator: James W. Yow
Assistant Vice President and Actuary
Aetna Life & Casualty

Panelists: Stephen W. Philbrick
Consulting Actuary
Tillinghast/Towers Perrin

Phillip W. Schwartz
Vice President Financial Reporting and Associate General Counsel
American Insurance Association

5. "Environmental Liability—Current Issues"

Moderator: Amy S. Bouska
Consulting Actuary
Tillinghast/Towers Perrin

Panelists: Don Schaefer
Field Manager/Complex Litigation
Aetna Life & Casualty

Orin M. Linden
Partner
Coopers & Lybrand

6. "Questions and Answers with the CAS Board of Directors"

Moderator: Michael L. Toothman
President-Elect

Panelists: Linda L. Bell
Member, Board of Directors

David J. Oakden
Member, Board of Directors

The officers held a reception for the new Fellows and their guests from 5:30 p.m. to 6:30 p.m.

A General Reception for all members and guests was held from 6:30 p.m. to 7:30 p.m.

Tuesday, May 21, 1991

Concurrent sessions were held from 8:30 a.m. to 10:00 a.m.

A panel presentation, "The Changing Face of Reinsurance," followed the morning sessions. The panel was moderated by Edmond F. Rondepierre, Senior Vice President, General Counsel and Secretary, General Reinsurance Corp. Members of the panel were Franklin Marstellar, Partner-Insurance Consulting Group, Price Waterhouse; Robert Hall, General

Counsel, American Re-Insurance; and David Wasserman, President, Centre Reinsurance U.S.

Tuesday afternoon was reserved for the various CAS committees to convene from Noon to 5:00 p.m.

Dinner and entertainment was a California barbecue held poolside from 6:30 p.m. to 10:00 p.m.

Wednesday, May 22, 1991

Concurrent sessions were held from 8:00 a.m. to 9:15 a.m.

A panel presentation, "Casualty Loss Reserve Opinions–Public Expectations," followed the sessions. The panel was moderated by David G. Hartman, Senior Vice President and Actuary, Chubb Group of Insurance Companies. The panel members were R. Michael Lamb, Casualty Actuary, Oregon Department of Insurance and Finance; W. James MacGinnitie, Consulting Actuary, Tillinghast/Towers Perrin; and Gary D. Simms, General Counsel and Director of Operations, American Academy of Actuaries.

The business session resumed at 11:00 a.m. with the presentation of the Michelbacher prize to Guy H. Whitehead.

The meeting was adjourned at 11:15 a.m. after closing remarks.

May 1991 Attendees

In attendance, as indicated by the registration records, were 280 Fellows; 193 Associates; and 43 guests, subscribers, and students. The list of their names follows.

FELLOWS

Abell, Casey
Alff, Greg
Alfuth, Terry
Allaire, Christiane
Anker, Bob
Apfel, Ken
Asch, Nolan
Atwood, Clarence
Bartlett, Bill
Bass, Irene
Bassman, Bruce
Baum, Edward
Beer, Al
Belden, Steve
Bell, Linda

FELLOWS

Bellafiore, Lenny
Bellusci, Dave
Ben-Zvi, Phil
Berquist, James R.
Berry, Janice
Bethel, Neil
Beverage, Richard
Bickerstaff, David
Bill, Richard
Blakinger, Jean
Blivess, Mike
Boison, LeRoy
Boor, Joe
Bornhuetter, Ron
Bothwell, Peter
Boulanger, François
Bouska, Amy
Boyd, Pete
Bradley, J. Scott
Braithwaite, Paul
Brannigan, Jim
Brehm, Paul
Brown, Brian
Brown, William
Bryan, Chuck
Buchanan, John
Burger, George
Cantin, Claudette
Captain, John
Carpenter, Bill
Chernick, David
Chiang, Jeanne
Childs, Diana
Conger, Bob
Covney, Michael
Cundy, Richard
Currie, Ross
Curry, Alan
Dahlquist, Ron
Daino, Bob
Davis, Larry
Dawson, John
Dean, Curtis Gary
DeLiberato, Bob
Dempster, Howard
DiGaetano, Mark
Dolan, Michael
Dornfeld, Jim
Downer, Robert
Drennan, John
Driedger, Karl
Dyck, Paul
Dye, Myron
Earwaker, Bruce
Ehlert, Darrell
Einck, Nancy
Englander, Jeffrey
Engles, David
Faga, Doreen
Fagan, Janet
Faltas, Bill
Fasking, Dennis
Fiebrink, Mark
Fisher, Wayne
Fitzgibbon, Walter
Foote, Jim
Forney, John
Fresch, Glenn
Furst, Pat
Fusco, Michael
Gallagher, Cecily
Gallagher, Tom
Gannon, Alice
Gapp, Steve
Garand, Chris
Gebhard, Jim
Gibson, Richard
Gillam, Judy
Gillam, William R.
Gillespie, Bryan
Glickman, Steven
Godbold, Terry
Goddard, Daniel
Goldberg, Len
Gorvett, Karen
Grace, Greg
Greco, Ron
Gutterman, Sam
Hachemeister, Charles
Haefner, Larry
Hafling, David
Hall, James
Haner, Walter
Harrison, David
Hartman, Dave
Hayne, Roger
Heer, LeRoy
Hennessy, Mary R.
Henry, Dennis
Henzler, Paul
Herder, John
Hines, Alan
Honebein, Carl
Hoppe, Ken
Howald, Ruth
Hutter, Heidi

FELLOWS

Jean, Ron
John, Russ
Johnson, Andrew
Johnson, Eric
Johnson, Larry
Johnson, Warren
Johnston, Thomas
Jones, Bruce
Jordan, Jeff
Josephson, Gary
Jovinelly, Ed
Joyce, John
Kane, Adrienne
Kaufman, Allan
Keen, Eric
Kilbourne, Fred
King, Mary Jean
Kist, Fred
Kollar, John
Koski, Mikhael
Koupf, Gary
Kozik, Thomas
Krause, Gus
Kreps, Rodney
Krissinger, Ken
Kucera, Jeff
Lalonde, David
Lamb, Dean
Lamb, Michael
Lamb, Scott
LaMonica, Mike
LaRose, Gary
Lebens, Joe
Lee, Robert
Lehmann, Steve
Leong, Winsome
Lerwick, Stuart N.
Levin, Joseph
Linden, Orin
Lino, Rich
Lommele, Jan
Ludwig, Steve
MacGinnitie, Jim
Marks, Steven
McAllister, Kevin
McConnell, Chuck
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McDermott, Sean
McDonald, Gary
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Morrow, Jay
Mucci, Bob
Murad, John
Murrin, Thomas
Muza, James
Nelson, Janet
Nemlick, Kenneth
Nester, Karen
Newlin, Patrick
Newman, Steven
Nickerson, Gary
Niswander, Ray
Normandin, Andre
Oakden, Dave
Onufer, Layne
Overgaard, Wade
Pagnozzi, Dick
Parker, Curtis
Peck, Steve
Pei, Kai
Philbrick, Steve
Pinto, Mel
Pratt, Joseph
Quirin, Al
Radach, Floyd
Radau, Chris
Raman, Ram
Rathjen, Ralph
Retterath, Ron
Robinson, Rich
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Roth, Richard, Jr.
Rowland, William
Ryan, Kevin
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Scheibl, Jerry
Scheuing, Jeffrey
Schmidt, Neal
Schultz, Roger
Schumi, Joseph R.
Schwartzman, Joy
Scott, Brian
Shapland, Mark
Sheppard, Alan
Sherman, Harvey
Sherman, Ollie

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Spalla, Joanne
Spidell, Bruce
Spiegler, David
Splitt, Dan
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Steeneck, Lee
Stewart, Walt
Strug, Emil
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Suchoff, Stuart
Surrago, Jim
Sutter, Russel
Switzer, Vern
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Teufel, Pat
Thorne, Joe
Tistan, Ernie
Tom, Darlene
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Trudeau, Michel
Tucker, Ben
Turner, George
Tuttle, Jerry
Van Slyke, Lee
Venter, Gary
Visintainer, Mike
Visintine, Jerry
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Walters, Mike
Walters, Mavis
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Winslow, Martha
Wiseman, Mike
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Yow, Heather
Ziccarelli, John

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Burns, Patrick
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Kolojay, Tim
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Kozlowski, Ronald
Kretsch, Dave
Kunze, Dave
Kwon, Frank
Lacefield, David
Lamy, Mathieu
Lannutti, Nick
Lavrey, Paul
Lemieux, Eric
Lepage, Pierre
Lepera, Giuseppe
Lew, Allen
Llewellyn, Barry
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McKay, Don
McPadden, Sean
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Roth, Scott

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Senia, Vincent
Shannon, Derrick
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Van De Water, John
Van Laar, Kenneth
Vanier, Anne-Marie
Varca, John
Vasek, Bill
Verges, Ricardo
Vu, Sebastian
Wacek, Mike
Wadman, Susan
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Wiegert, Paul
Wignarajah, Gnana
Wildman, Peter
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Wilson, Ollie
Yau, Michael
Yezzi, Vince
Yu, Sheng
Yunque, Mark

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PROCEEDINGS

November 10, 11, 12, 13, 1991

COMMUTATION PRICING IN THE POST TAX-REFORM ERA

VINCENT P. CONNOR
RICHARD A. OLSEN

Abstract

This paper discusses how a reinsurer prices the commutation of a group of claims. A commutation is an agreement between an insurer and a reinsurer in which one payment by the reinsurer settles a group of claims that have not been settled by (or perhaps reported to) the insurer. After discussing the reasons for commutations, an example is used to discuss the after-tax interest rate that is used to determine the present value of the claims. Also discussed is how to determine the value of the unwinding of the discount, as well as the tax on the underwriting gain/loss normally generated by a commutation. Also covered is a formula used to determine price and why the commutation price normally appears low to insurance companies. The second, more complicated example develops a commutation price for a typical property/casualty line. The overall discussion in this example touches upon a number of different points to keep in mind when pricing commutations. Some of these points include contract analysis, IBNR development, payment profile(s), and interest rate selection. An additional example comments on the effects on commutation pricing when the payment patterns and interest rates used to determine the present value of the losses are not *equal to those used to develop tax basis discounted reserves. The last part of the paper deals with sensitivity analysis where interest rates, tax rates, and payment profiles are varied to see their effect on the indicated price. While initially appearing complex, it is hoped that this step-by-step approach with examples will make this subject more understandable.*

1. INTRODUCTION

In today's marketplace, reinsurers receive premiums from ceding companies in exchange for a promise to make loss payments, under certain fortuitous conditions, at some future date. The conditions governing the timing and method of the loss payments are in the reinsurance contract. For the most part, reinsurance losses are paid shortly after the ceding company makes payments.

In response to its promise to reimburse the ceding company for future losses, the reinsurer sets up loss reserves. The level of the reserves is continually monitored and adjusted by the reinsurer as new information becomes available and actual loss payments are made. This process continues until the reinsurer's financial obligations to the ceding company are fulfilled.

Sometimes, though, the reinsurer and insurer form an agreement that lets the reinsurer pay for claims before they are actually paid by the ceding company. In essence, through this transaction, known as a commutation of claims, the reinsurer and insurer finalize the reinsurance agreement. This paper describes how to price commutations, with special attention being given to the effects of taxes on the pricing of commutations.

There are a number of reasons for commutations. Commutations can be promoted in order to improve the underwriting results of a contract, since the commutation price is normally less than the reserves carried. Commutations can evolve as a result of disagreements over the proper reserve to carry. Commutations can also arise out of different investment philosophies and forecasts of investment income. Different tax situations for insurer and reinsurer may also promote commutations. Commutations can also stem from insurer/reinsurer insolvencies and disputes over contract terms. For whatever reasons, reinsurers are occasionally asked to develop an overall commutation price for one or more claims.

As a start, consider this elementary case. Assume that the Random Reinsurance Corporation receives the following information regarding a requested commutation:

1. The commutation is for a single claim that occurred 1/10/89.
2. The current reserve is $100,000.
3. The claim will be paid in equal annual installments of $20,000 beginning 6/30/91.
4. Today's date is 6/30/90.

In order to develop an equitable figure, two questions have to be answered:

1. What are the costs of making payments according to the contract terms; i.e., no commutation?
2. What are the costs if there is a commutation?

The general approach is to develop a commutation price that balances these two costs.

2. COSTS OF NOT COMMUTING

Present Value of the Paid Loss

The first cost involved is the estimated five annual payments of $20,000. In order to express this figure in current dollars, thus taking into account the time value of money, the present value of the future loss payments should be calculated using an appropriate interest rate. The rate used should reflect current yields. This is because, to the extent possible, the commutation will be funded out of current cash flow. Even if current cash flow is not sufficient, and the reinsurer must sell securities, it will sell securities at a market price that will reflect current yields.

Before the Tax Reform Act of 1986, many insurance companies probably were not explicitly paying taxes on investment income.[1] As-

[1] This is because overall taxable income during the period was relatively low. This point should not be confused with the fact that a high implicit tax burden did exist. By investing in tax-exempt securities, the industry received a lower before-tax yield than it otherwise would have.

suming investments effectively yielded 8.5% for the five-year period, and that the investment income is re-invested at the same rate, then the present value of the loss is $78,813. However, as a result of the new tax law, taxes are now paid on investment income. Consequently, interest rates used for discounting must be after-tax interest rates.

Assume that, after consulting with tax and investment personnel, Random Reinsurance will be a regular taxpayer[2] at a 34% tax rate. Also assume an 8.5% nominal rate of return (before tax) for each of the six calendar years. Consequently, the after-tax interest rates becomes 5.61% (8.5% × (1 − .34)). As a result, the present value of the five loss payments becomes $85,149. If the company is a minimum taxpayer, it pays at a different tax rate than a regular taxpayer. Current law allows the company to recoup those additional taxes when it becomes a regular taxpayer. Recouping these adjustments can be reflected, but they complicate the calculations.

Thus, when performing the present value calculations, the two key considerations to remember regarding the after-tax interest rate are:

1. The future expected rate of return (before tax); and
2. The anticipated tax situation of the company. (Regarding this point, one item to keep in mind is whether or not the commutation will affect the anticipated tax situation.)

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[2] Property and casualty insurers are required to make two tax calculations a year. The tax is the higher of the regular calculation and the alternative minimum tax calculation. Regular taxable income is primarily statutory underwriting income, adjusted to discount the losses, plus investment income, excluding non-taxable municipal bond income, plus other adjustments. The regular income tax rate of 34% is used for income above $335,000. Alternative minimum taxable income is regular taxable income plus various tax preference items. The major tax preference item for a property and casualty insurer is expected to be 75% of the difference between Generally Accepted Accounting Principles (GAAP) book income and regular taxable income. The alternative minimum tax rate is 20%. A good description of the Tax Reform Act of 1986 as it applies to insurance companies is contained in "An Analysis of the Impact of the Tax Reform Act on the Property/Casualty Industry" by Owen M. Gleason and Gerald I. Lenrow in *Financial Analysis of Insurance Companies*, 1987 Discussion Paper Program, Casualty Actuarial Society, page 119. This paper also deals with the special requirements for municipal bond income and the development of tax-basis discounted reserves. Another good reference on this subject can be found in "Federal Income Taxes Provisions Affecting Property/Casualty Insurers" by Manuel Almagro and Thomas L. Ghezzi in *PCAS* LXXV, 1988, page 95.

Present Value of Tax Benefit on the Unwinding of the Discount

The next part of developing the cost of not commuting is to calculate the present value of the tax benefit on the unwinding of the discount.

Before the Tax Reform Act of 1986, the outstanding reserves for tax purposes were the same as those on the annual statement. The new law now requires the discounting of reserves.

Tax-basis discounted reserves can be based on individual company history or industry factors. Because losses are discounted, the tax-basis reserves will be less than current nominal reserves. Since the reinsurer expects to eventually pay out losses that equal current nominal reserves, the Random Reinsurance Corporation will, over time, realize a change in taxable income equal to the difference between the nominal and tax-basis reserves. This change in taxable income is expected to produce a tax benefit in total (although not necessarily in every calendar year) to the reinsurer. Consequently, this benefit should be reflected in the commutation price.

The amount of benefit that "unwinds," or is realized over each calendar year, will be equal to the change in tax-basis reserves plus the amount of calendar year payments; i.e., the tax-basis incurred. The change in taxes for the reinsurer will be equal to the change in taxable income multiplied by the anticipated tax rate for that particular calendar year. This assumes that there is sufficient taxable income to offset. The present value of these amounts is then calculated using the same after-tax interest rate as that assumed for the present value calculation of the paid losses. This calculation, using industry discount factors to calculate the tax-basis reserves, is presented in Table 1.

TABLE 1

Cal. Year	Paid in Cal. Yr.	Year End Reserve	Discount Factor	Disc. Tax Res.	Change in Tax Income	Tax Rate	Tax Ben.	Pres. Value[3]
1990	$ 0	$100,000	.79812	$79,812	$ —	—	$ —	$ —
1991	20,000	80,000	.77935	62,348	2,536	.34	862	816
1992	20,000	60,000	.75561	45,337	2,989	.34	1,016	911
1993	20,000	40,000	.73577	29,431	4,094	.34	1,392	1,182
1994	20,000	20,000	.70271	14,054	4,623	.34	1,572	1,264
1995	20,000	0	.68950	0	5,946	.34	2,022	1,539
Total	$100,000				$20,188		$6,864	$5,712

Thus, the present value of the tax benefit on the unwinding of the discount is calculated to be $5,712. The calendar year 1990 change in taxable income will be reflected elsewhere.

Therefore, the cost of not commuting is the present value of the losses, equal to $85,149, less the present value of the tax benefit on the unwinding of the discount, equal to $5,712. The resulting value of $79,437 is the amount of money the reinsurer needs to pay the claims. This amount, as it is increased by investment income earned as well as the tax benefit of the unwinding of the discount, will be sufficient for the payment of taxes on the investment income, as well as for payment of the loss, providing the assumptions are correct.

3. COSTS OF COMMUTING

The Commutation Price

This is the amount of money, to be calculated below, that the reinsurer will pay the ceding company to assume the nominal $100,000 liability.

[3] With estimated tax payments, assume that the benefit unwinds midway through the calendar year. The interest rates used to form the present value are the after-tax rates presented previously.

The Tax on the Underwriting Gain/Loss Generated by the Commutation

Before the Tax Reform Act of 1986, many insurance companies might not have been concerned about the taxable gain or loss generated by a commutation because they probably were not paying taxes. As a result of this legislation, most insurance companies are now paying taxes, or they soon will be. Consequently, the taxable underwriting gain or loss should be taken into account when pricing the commutation.

In order to quantify this amount, consider the following. If the commutation was done at year-end, the change in taxable income would be equal to the difference between the amount of tax-basis reserves taken down as a result of the commutation and the commutation price. Because taxes are calculated only once a year, a different approach is necessary when the commutation is not done at year-end.

The approach taken here is to contrast taxable income when there is *no* commutation against taxable income when there *is* a commutation. This comparison is shown in Exhibit 1. The exhibit shows the change in taxable income if the reinsurer does the commutation, which includes the unwinding of the discount, in the current calendar year. This change is equal to the estimated year-end tax-basis reserves plus the estimated 7/1/90–12/31/90 paid losses (assuming no commutation), less the commutation payment.

While appearing a little odd, the estimated 7/1/90–12/31/90 paid losses plus the year-end tax-basis outstanding can be viewed as an estimate of the tax-basis reserves at the time of the commutation. This calculation is consistent with the formula used at year-end. The appropriate tax rate can then be applied to this figure to determine the amount of taxes. If estimated taxes are paid over the calendar year, it is usually not necessary to discount the tax payment.

4. COST ANALYSIS

Once the above values have been calculated, except for the commutation price, formula equations can be set up to determine the commutation price by equating the cost of not commuting with the cost of commuting. Since the example does not include any profit or risk loading in the costs, this formula seeks to determine a point of indifference between commuting or not commuting.

This formula is given as follows:

Cost of not commuting = PV of paid losses −
PV of tax benefit on unwinding of discount
equals
Cost of commuting = Commutation price + tax on commutation
= Commutation price + tax rate × (expected payments, remainder of current calendar year + year-end tax-basis outstanding − commutation price).

Using the inputs:

Cost of not commuting = \$85,149 − \$5,712 = \$79,437
equals
Cost of commuting = Commutation price + .34 × (0 + \$79,812 − commutation price).

Then, using algebra, a commutation price of \$79,244 is determined.

Regarding this price, it is interesting to note that the commutation price may appear low, because the offer is less than the present value of the estimated paid losses. It can be noted that this will tend to happen, because of the tax effects created by the unwinding of the discount and the taxable gain generated by the transaction.

Now that a relatively elementary case has been analyzed and a good foundation has been laid, a more complicated example is considered. Assume the following information regarding a requested commutation is received:

1. The commutation is for a monoline, long-tailed, liability contract.
2. The current case reserves are as follows:

Accident Year	Reserves
1988	$ 5,000,000
1987	4,000,000
1986	6,000,000
1985	3,000,000
Total	$18,000,000

3. The timing of the individual claim payments is unknown.
4. Today's date is 6/30/90.

Given this data, a commutation price would be calculated as in the following paragraphs.

As a start, a thorough review of the contract would be performed. This investigation should include a detailed analysis of contract terms and limits, as well as discussions with various legal and underwriting personnel. In this way, potential areas of coverage dispute and confusion can be identified and appropriately resolved.

If there are adjustable features such as retrospectively-rated premium amounts payable by or to the reinsurer, these values should be included in the analysis. Sometimes these amounts are payable over time and therefore must be discounted. To keep this example simple, there will not be any adjustable features.

The next step is to estimate the IBNR reserves. In this calculation, any of the standard IBNR techniques could be used, and it is advisable to use more than one. If a loss development approach is taken and if the business is excess, it is important that excess loss development factors be used. Also, normally unallocated loss adjustment expense is not included in the contract. However, assuming expenses are not fixed, if the losses are commuted, Random Reinsurance will not have this expense. An estimate of this amount can also be included in the calculation.

For this example, assume that the estimates of IBNR, ALAE, and ULAE are as follows:

Accident Year	IBNR, ALAE, and ULAE	Case Reserves	Total Reserves
1988	$ 3,000,000	$ 5,000,000	$ 8,000,000
1987	2,000,000	4,000,000	6,000,000
1986	1,000,000	6,000,000	7,000,000
1985	500,000	3,000,000	3,500,000
Total	$ 6,500,000	$18,000,000	$24,500,000

Given this estimated total outstanding loss by accident year, the commutation cost analysis can now be started.

Present Value of the Paid Loss

In this case, because the timing of loss payments is not known, an estimate must be made of how the accident year reserves will pay out over future calendar years. In order to make this estimate, one would consider various economic, legal, and type of business factors; e.g., long-tailed versus short-tailed lines, proportional versus nonproportional reinsurance, and/or monoline versus multiline policy. Ideally, the estimated payment pattern would be based on the experience of the ceding company. However, reinsurance industry factors can also be used. As with the IBNR reserves, if the business is excess of a retention, an excess payment pattern must be used. If this contract covered multiple lines, several different payment patterns would be used for the projections.

Assume that the payment pattern displayed in Exhibit 2 is reflective of the type of business in this monoline contract. Based upon this pattern, an estimate of the future calendar year payment profile can be made. This calculation is displayed in Exhibit 3.

At this point, one must determine the present value of the estimated payments of the $24,500,000 in reserves. As in the elementary case, the interest rate(s) used must be reflective of the future expected rate of return (before tax) and the anticipated tax scenario of the reinsurer. For this calculation, assume a nominal 8% (before-tax) rate of return. Assume that the company anticipates that it will be a regular taxpayer for all the calendar years at a 34% tax rate. Based upon these inputs, the present value of the paid losses is calculated to be $19,641,000. This calculation is presented in Exhibit 4.

Present Value of Tax Benefit on the Unwinding of the Discount

This amount, whose calculation (using industry factors to calculate the tax-basis reserves) is displayed in Exhibit 5, emerges as a result of the difference between the tax-basis discounted reserves and the nominal reserves. This difference will be a reduction in taxable income in the future, and the present value of this amount can be determined.

As with the elementary case, the amount of benefit that "unwinds" or is realized over each ensuing calendar year will be equal to the estimated tax-basis incurred (change in tax-basis reserves plus estimated loss payments) multiplied by the anticipated calendar year tax rate. The present value of these amounts is then obtained using the same after-tax interest rates assumed in the calculation of the present value of the paid losses.

For this example, the taxable income effect of the unwinding of the discount is estimated to be $5,202,000 in Exhibit 5. The present value of the tax is calculated to be $1,363,000 in Exhibit 6.

Thus, for this case study, the cost of not commuting is equal to $18,278,000 (present value of paid loss less present value of tax benefit on unwinding of discount). Now that this value has been calculated, the rest of the analysis follows the same routine developed for the elementary case. Using the theoretical relationships that balance the two costs, the tax on the change in taxable income generated by the commutation, as well as the commutation price, can be easily calculated.

Cost of not commuting = PV of paid losses −
PV of tax benefit on unwinding of discount

equals

Cost of commuting = Commutation price + tax on commutation
= Commutation price + tax rate × (expected payments, remainder of current calendar year + year-end tax-basis outstanding − commutation price)

Using our inputs:

Cost of not commuting = \$19,641,000 − \$1,363,000 = \$18,278,000

equals

Cost of commuting = Commutation price + .34 × (2,070,000 + \$17,227,000 − commutation price)

Then, again using algebra, one arrives at a commutation price of \$17,753,000.

Exhibit 7 summarizes the information for this case in a useful format. The reinsurer is expected to make payments of \$24,500,000. Taking into account the time value of money, it is estimated that \$19,641,000 will be sufficient to fund these payments. Taking into account the benefit of the unwinding of the discount (\$1,363,000), only \$18,278,000 is necessary.

The reinsurer is willing to pay this amount, but must deduct the tax of \$525,000 due on the commutation to develop the indicated price of \$17,753,000. Please note that this is more than \$1,800,000 less than the present value of the losses.

One point worth emphasizing is that commutation pricing involves the use of two separate payment profiles and nominal interest rates. The first set is used to determine the present value of the paid losses. The second set is used to calculate discounted loss reserves for tax purposes. If the payment pattern and nominal interest rate used to determine the present value of the losses are identical to the factors used to develop the tax-basis discounted reserves, then the commutation price will equal the present value of the losses using the nominal interest rate. To demonstrate, consider the elementary case with the following adjustments to make the calculations a little easier:

1. The five annual payments of \$20,000 will begin 12/31/91.
2. Today's date is 12/31/90.

Using this information, Table 2 can be constructed.

TABLE 2

(1) Reserve/ Payment Date	(2) Loss Payment	(3) Nominal Reserve	(4) Discounted Tax Reserve	(5) Tax Basis Incurred	(6) Tax Credit	(7) After-Tax Payment
12/90	—	$100,000	$79,854	—	—	—
12/91	$20,000	80,000	66,243	$6,389	$2,172	$17,828
12/92	20,000	60,000	51,542	5,299	1,802	18,198
12/93	20,000	40,000	35,665	4,123	1,402	18,598
12/94	20,000	20,000	18,519	2,854	970	19,030
12/95	20,000	0	0	1,481	504	19,496

(8) Reserve/ Payment Date	(9) Interest Income	(10) Interest Income Tax	(11) After-Tax Interest Income	(12) Fund Liquidation
12/90	—	—	—	$79,854
12/91	$6,389	$2,172	$4,217	66,243
12/92	5,299	1,802	3,497	51,542
12/93	4,123	1,402	2,721	35,665
12/94	2,854	970	1,884	18,519
12/95	1,481	504	977	0

The five loss payments are paid out over the 12/91–12/95 period (columns 1 and 2). The corresponding reduction in the nominal reserves is given in column 3. For federal income tax purposes, the loss reserves are discounted at rates prescribed by the Internal Revenue Service (IRS). If the nominal interest rate is 8% and the IRS payment profile matches the payment schedule above, then the 12/90 tax-basis reserve (in column 4) can be calculated to be:

$$\$20{,}000 \times ((1.08)^{-1} + (1.08)^{-2} + (1.08)^{-3} + (1.08)^{-4} + (1.08)^{-5}),$$
or $79,854.

Column 5 shows the annual cost to the insurer: the loss payment plus the change in the tax-basis discounted reserve. For instance, in 1991, the insurer paid $20,000 and took down the discounted reserve from $79,854 to $66,243. The "cost" is $20,000 + ($66,243 − $79,854), or $6,389.

The annual cost to the insurer in column 5 provides a 34% tax credit shown in column 6. Column 7 shows the after-tax payment, or column 2 minus column 6. Column 12 shows the funds needed on 12/31/90 to fund the after-tax payments shown in column 7. For instance, the $79,854 invested on December 31, 1990, earns 8% interest, or $6,389 in 1991 (column 9). Federal income taxes of $2,172 must be paid on the interest income (column 10), so the after-tax return is $4,217 (column 11). The after-tax payment in 1991 is $17,828, so the value of the fund on December 31, 1992, is $79,854 + $4,217 − $17,828, or $66,243.

As a result, $79,854 is the present value of losses, with or without consideration of taxes, and is the amount of money needed to fund the loss payments. It is also the commutation price, using the equations presented earlier.

If the IRS payment profiles and interest rates equal the factors used to determine the present value of the losses, then the commutation price will equal the present value of the losses using the nominal interest rate. While this is an interesting result, this situation will rarely come about in practice for several reasons.

First, once an interest rate and payment pattern are published by the IRS for an accident year, they are fixed for all time. If the commutation is transacted several years after the accident year has expired, it would be unlikely that current yields match the IRS yield.

Second, the payment profiles used to discount loss reserves are subject to change once every five years. As a result of swings in underwriting, economic, and legal cycles, insurer policy retentions, limits, coverages, and appetite for reinsurance vary. Consequently, the tax-basis payment profiles, which would be based upon relatively old loss experience, may not be reflective of the current type of business being written.

Third, if the type of business subject to the commutation is reported under the reinsurance line in the annual statement, the reinsurer must use an industry aggregate payment profile to develop tax-basis reserves.

It does not have the option of using its own company experience. This IRS pattern may have little resemblance to the actual payment profile associated with the line of business.

As a result of all these factors, it is doubtful that the profiles used to determine the present value of the losses will match the profiles used to determine the tax-basis reserves. This mismatch in patterns and interest rates can lead to situations that can either promote or deter commutations.

5. CONCLUSION

Overall, while commutation pricing may appear quite complex, the study becomes much more manageable when the individual pieces are looked at one at a time. Throughout this paper, only a single set of assumptions has been made for each example. Due to the fact that the input values can vary substantially, risk loads may be considered for any or all of the following parameters:

1. IBNR.
2. After-tax interest rates.
3. Payment profile(s).

The amount of risk loading for each parameter can be set judgmentally by the actuary.[4] Due to the high variability associated with most of these parameters, though, it may be best to perform the analysis iteratively using different assumptions. If the study is programmed, perhaps using any one of the many spreadsheet software packages, this form of sensitivity analysis can be performed easily.

Exhibit 8 shows developed commutation prices for the first case using different interest rate and tax assumptions. Please note that the interest rate and tax assumptions given apply to all the calendar years. Exhibit 9 shows commutation prices for the second case using varying tax situations, interest rate, and payment profiles. Regarding these various outcomes, the following points can be noted:

[4] Robert Butsic suggests a method for doing this in "Determining the Proper Interest Rate for Loss Reserve Discounting: An Economic Approach" in *Evaluating Insurance Company Liabilities*, 1988 Discussion Paper Program, Casualty Actuarial Society, page 147.

1. The effects created by varying the payment schedules can be quite significant. Great care should be taken when the future payment stream is estimated.

2. In certain instances, the commutation price developed under this methodology can be negative. This can occur when there is a great mismatch between the payment profile/interest rate used to develop tax-basis discounted reserves and the payment profile/ interest rate used to calculate the present value of the losses. Specifically, the tax-basis discounted reserves are substantially higher than the present value of the losses. This leads to the tax on the underwriting gain/loss becoming greater than the cost of not commuting. In cases of reinsurance of long-tailed lines, such as workers' compensation, where the overall industry average reinsurance payment profile is quite short relative to the actual payment profile, negative commutation values can be expected frequently. In these situations, commutations are not favored.

There are a large number of assumptions made in pricing a commutation. The present value of the future expected losses is only the starting point in determining the price of the commutation. In addition to this, assumptions can include future yields and tax positions going out 30 years, or more. The use of a spreadsheet allows the actuary to vary assumptions and determine their effect on the indicated price. The bottom line is that the indicated commutation price is still an estimate based on many assumptions. Regarding this point, it must be stressed that the prices developed above are all theoretical. In the actual negotiation process between reinsured and reinsurer, both parties may have broad differences of opinion regarding any/all of the parameters. Also, if the motivation behind the commutation is insolvency, or threatened insolvency, the actual price may be much less than the theoretical price.

One last word of caution: It is usually a good idea to put a time limit on a commutation offer. Changes in economic outlook can affect any or all of the input parameters; e.g., interest rates, tax assumptions, etc. This can lead to significant changes in the commutation price.

EXHIBIT 1

Determination of Change in Taxable Income As a Result of a Commutation

No Commutation

A. Current Year Taxable Income = Change in Tax-Basis Reserves − Paid Losses in Current Year

= Beginning of Year Tax-Basis Reserves − Estimated Year-End Tax-Basis Reserves − Calendar Year Paid Losses prior to Date of Commutation − Expected Calendar Year Paid Losses after Date of Commutation

Commutation

B. Current Year Taxable Income = Change in Tax-Basis Reserves − Paid Losses in Current Year

= Beginning of Year Tax-Basis Reserves − Estimated Year-End Tax-Basis Reserves (=0) − Calendar Year Paid Losses prior to Date of Commutation − Expected Calendar Year Paid Losses after the Date of Commutation (=0) − Commutation Price

Change in Taxable Income as a result of a commutation equals B − A, which is:

Estimated Year-End Tax-Basis Reserves +
Calendar Year Paid Losses after the Date of Commutation − Commutation Price

EXHIBIT 2

Estimated Payment Profile
Random Reinsurance Corporation

Year	Payout Percentage
1	2.00%
2	3.00
3	16.00
4	11.00
5	10.00
6	10.00
7	9.00
8	8.00
9	6.00
10	5.00
11	4.00
12	3.00
13	3.00
14	3.00
15	2.00
16	2.00
17	1.00
18	1.00
19	1.00
Total	100.00%

EXHIBIT 3

Development of Future Paid Loss Stream
Random Reinsurance Corporation
(000 omitted)

Step 1: Develop Expected Nominal Paid Losses by Accident Year

Accident Year	Reserves at 6/30/90	Percentage of Total Acc. Yr. Losses Paid to Date	Expected Total Losses* for Accident Year
1985	$3,500	47.00%	$ 6,604
1986	7,000	37.00	11,111
1987	6,000	26.50	8,163
1988	8,000	13.00	9,195

Step 2: Develop Paid Loss Stream

Year	Expected Payout Pattern Acc. Yr. 1985–88	Calendar Year	Accident Yr. 1985 Payout Stream	Accident Yr. 1986 Payout Stream	Accident Yr. 1987 Payout Stream	Accident Yr. 1988 Payout Stream	Total
1	2%	1985	$ 132				$ 132
2	3	1986	198	$ 222			420
3	16	1987	1,057	333	$ 163		1,553
4	11	1988	726	1,778	245	$ 184	2,933
5	10	1989	660	1,222	1,306	276	3,465
6	10	1/1/90–6/30/90	330	556	449	736	2,070
7	9						
8	8	7/1/90–12/31/90	330	556	449	736	2,070
9	6	1991	594	1,111	816	1,011	3,533
10	5	1992	528	1,000	816	920	3,264
11	4	1993	396	889	735	920	2,939
12	3	1994	330	667	653	828	2,478
13	3	1995	264	556	490	736	2,045
14	3	1996	198	444	408	552	1,602
15	2	1997	198	333	327	460	1,318
16	2	1998	198	333	245	368	1,144
17	1	1999	132	333	245	276	986
18	1	2000	132	222	245	276	875
19	1	2001	66	222	163	276	727
		2002	66	111	163	184	524
Total	100%	2003	66	111	82	184	443
		2004	0	111	82	92	285
		2005	0	0	82	92	174
		2006	0	0	0	92	92
		2007	0	0	0	0	0
		2008	0	0	0	0	0
		Future Total**	$3,500	$7,000	$6,000	$8,000	$24,500

* Based on estimated payout pattern.

** Future Total does not include payments prior to 7/1/90.

Note: Paid in a Year = Total Expected Loss × Payout Percentage for Year.

EXHIBIT 4

PRESENT VALUE OF FUTURE PAID LOSSES
RANDOM REINSURANCE CORPORATION
(000 omitted)

Present Value Calendar Year	Paid Losses*	Nominal Interest Rate	Tax Factor	Net of Tax Rate	Discount Factor	Present Value of Paid
1990	$ 2,070	8.0%	0.660	5.3%	0.9872	$ 2,044
1991	3,533	8.0%	0.660	5.3%	0.9498	3,356
1992	3,264	8.0%	0.660	5.3%	0.9022	2,945
1993	2,939	8.0%	0.660	5.3%	0.8570	2,519
1994	2,478	8.0%	0.660	5.3%	0.8140	2,017
1995	2,045	8.0%	0.660	5.3%	0.7732	1,581
1996	1,602	8.0%	0.660	5.3%	0.7344	1,177
1997	1,318	8.0%	0.660	5.3%	0.6976	919
1998	1,144	8.0%	0.660	5.3%	0.6626	758
1999	986	8.0%	0.660	5.3%	0.6293	621
2000	875	8.0%	0.660	5.3%	0.5978	523
2001	727	8.0%	0.660	5.3%	0.5678	413
2002	524	8.0%	0.660	5.3%	0.5393	283
2003	443	8.0%	0.660	5.3%	0.5123	227
2004	285	8.0%	0.660	5.3%	0.4866	139
2005	174	8.0%	0.660	5.3%	0.4622	80
2006	92	8.0%	0.660	5.3%	0.4390	40
2007	0	8.0%	0.660	5.3%	0.4170	0
2008	0	8.0%	0.660	5.3%	0.3961	0
Total	$24,500					$19,641

* For 1990, assume payment is made 10/1/90. For all other years assume June 30.

EXHIBIT 5, Part 1

DETERMINATION OF UNWINDING OF DISCOUNT PAYOUT STREAM
RANDOM REINSURANCE CORPORATION
(000 omitted)

Calendar Year	(1) Nominal Paid Losses	(2) Accident Year 88 O/S at 12/31	(3) IRS Factor	(4) Tax-Basis Accident Year 88 O/S at 12/31	(5) Accident Year 87 O/S at 12/31	(6) IRS Factor	(7) Tax-Basis Accident Year 87 O/S at 12/31	(8) Accident Year 86 O/S at 12/31	(9) IRS Factor	(10) Tax-Basis Accident Year 86 O/S at 12/31	(11) Accident Year 85 O/S at 12/31	(12) IRS Factor	(13) Tax-Basis Accident Year 85 O/S at 12/31	(14) Nominal O/S at 12/31	(15) Total Tax-Basis O/S at 12/31	(16) Discount Unwind*
1990	$ 2,070	$7,264	0.787052	$5,717	$5,551	0.776806	$4,312	$6,444	0.758586	$4,889	$3,170	0.728501	$2,309	$22,430	$17,227	
1991	3,533	6,253	0.764042	4,777	4,735	0.758586	3,592	5,333	0.728501	3,885	2,575	0.716837	1,846	18,896	14,101	$ 407
1992	3,264	5,333	0.744839	3,972	3,918	0.728501	2,855	4,333	0.716837	3,106	2,047	0.713613	1,461	15,632	11,394	558
1993	2,939	4,414	0.712961	3,147	3,184	0.716837	2,282	3,444	0.713613	2,458	1,651	0.716331	1,183	12,693	9,070	615
1994	2,478	3,586	0.700375	2,512	2,531	0.713613	1,806	2,778	0.716331	1,990	1,321	0.746667	986	10,215	7,294	701
1995	2,045	2,851	0.696588	1,986	2,041	0.716331	1,462	2,222	0.746667	1,659	1,057	0.780160	824	8,170	5,931	683
1996	1,602	2,299	0.698986	1,607	1,633	0.746667	1,219	1,778	0.780160	1,387	858	0.817540	702	6,568	4,915	586
1997	1,318	1,839	0.730679	1,344	1,306	0.780160	1,019	1,444	0.817540	1,181	660	0.859831	568	5,250	4,111	514
1998	1,144	1,471	0.765829	1,127	1,061	0.817540	868	1,111	0.859831	955	462	0.908514	420	4,106	3,370	402
1999	986	1,195	0.805246	963	816	0.859831	702	778	0.908514	707	330	0.965834	319	3,120	2,690	307
2000	875	920	0.850059	782	571	0.908514	519	556	0.965834	537	198	0.965834	191	2,245	2,029	214
2001	727	644	0.901909	581	408	0.965834	394	333	0.965834	322	132	0.965834	128	1,517	1,424	123
2002	524	460	0.963277	443	245	0.965834	237	222	0.965834	215	66	0.965834	64	993	958	58
2003	443	276	0.963277	266	163	0.965834	158	111	0.965834	107	0	0.965834	0	550	531	16
2004	285	184	0.963277	177	82	0.965834	79	0	0.965834	0	0	0.965834	0	266	256	10
2005	174	92	0.963277	89	0	0.965834	0	0	0.965834	0	0	0.965834	0	92	89	6
2006	92	0	0.963277	0	0	0.965834	0	0	0.965834	0	0	0.965834	0	0	0	3
2007	0	0	0.963277	0	0	0.965834	0	0	0.965834	0	0	0.965834	0	0	0	0
2008	0	0	0.963277	0	0	0.965834	0	0	0.965834	0	0	0.965834	0	0	0	0
	$24,500															$5,202

* (16) = Paid in Current Year + Change in Tax-Basis Reserves = Tax-Basis Incurred.

EXHIBIT 5, Part 2

IRS DISCOUNT FACTORS*

Calendar Year	Accident Year 1988	Accident Years 1987 and Prior
AY + 0	0.835127	0.844514
AY + 1	0.805296	0.816121
AY + 2	0.787052	0.798700
AY + 3	0.764042	0.776806
AY + 4	0.744839	0.758586
AY + 5	0.712961	0.728501
AY + 6	0.700375	0.716837
AY + 7	0.696588	0.713613
AY + 8	0.698986	0.716331
AY + 9	0.730679	0.746667
AY + 10	0.765829	0.780160
AY + 11	0.805246	0.817540
AY + 12	0.850059	0.859831
AY + 13	0.901909	0.908514
AY + 14	0.963277	0.965834
AY + 15	0.963277	0.965834

* Composite Schedule P

EXHIBIT 6

PRESENT VALUE OF UNWINDING OF DISCOUNT
RANDOM REINSURANCE CORPORATION
(000 omitted)

Calendar Year	Nominal Unwinding Discount	Nominal Interest Rate*	Tax Factor	Net of Tax Rate	Discount Factor**	Present Value of UWD	Tax Rate	Present Value of Tax on UWD
1990		8.0%	0.660	5.3%			34%	
1991	$ 407	8.0%	0.660	5.3%	0.9498	$ 386	34%	$ 131
1992	558	8.0%	0.660	5.3%	0.9022	503	34%	171
1993	615	8.0%	0.660	5.3%	0.8570	527	34%	179
1994	701	8.0%	0.660	5.3%	0.8140	571	34%	194
1995	683	8.0%	0.660	5.3%	0.7732	528	34%	179
1996	586	8.0%	0.660	5.3%	0.7344	430	34%	146
1997	514	8.0%	0.660	5.3%	0.6976	359	34%	122
1998	402	8.0%	0.660	5.3%	0.6626	267	34%	91
1999	307	8.0%	0.660	5.3%	0.6293	193	34%	66
2000	214	8.0%	0.660	5.3%	0.5978	128	34%	43
2001	123	8.0%	0.660	5.3%	0.5678	70	34%	24
2002	58	8.0%	0.660	5.3%	0.5393	31	34%	11
2003	16	8.0%	0.660	5.3%	0.5123	8	34%	3
2004	10	8.0%	0.660	5.3%	0.4866	5	34%	2
2005	6	8.0%	0.660	5.3%	0.4622	3	34%	1
2006	3	8.0%	0.660	5.3%	0.4390	1	34%	1
2007	0	8.0%	0.660	5.3%	0.4170	0	34%	0
2008	0	8.0%	0.660	5.3%	0.3961	0	34%	0
Total	$5,202					$4,010		$1,363

* Tax factor and net of tax rate same as those used to present value the losses given in Exhibit 4.

** Assume discount unwinds on June 30 of each year.

EXHIBIT 7

General Summary
Random Reinsurance Corporation Commutation
(000 omitted)

Current Outstanding Losses	$24,500
PV* of Outstanding Losses	$19,641
PV of Tax Affected Unwinding of Discount	$1,363
Initial Cost without Commutation	$18,278
Tax on Underwriting Gain	$525
Balance Commutation Price	$17,753

* In the present value calculation, the discount factor is a function of our expected tax situation.

EXHIBIT 8

SENSITIVITY ANALYSIS
ELEMENTARY CASE

Tax Situation*	Nominal Interest Rate	Present Value	PV of Unwinding of Discount	Tax on Underwriting Gain	Commutation Price
Minimum	6%	$87,071	$3,447	$ (952)	$84,576
Minimum	7%	85,171	3,361	(501)	82,311
Minimum	8%	83,338	3,278	(63)	80,123
Minimum	9%	81,566	3,198	360	78,008
Minimum	10%	79,853	3,121	769	75,963
Regular	6%	89,137	6,019	(1,702)	84,820
Regular	7%	87,507	5,893	(928)	82,542
Regular	8%	85,923	5,772	(175)	80,326
Regular	9%	84,385	5,653	556	78,176
Regular	10%	82,890	5,538	1,268	76,084

* Minimum indicates 20% tax rate for all years; Regular indicates 34% tax rate for all years.

EXHIBIT 9
Part 1

SENSITIVITY ANALYSIS
PAYOUT PROFILE USING EXAMPLE PAYMENT PATTERN

Tax Situation*	Nominal Interest Rate	Present Value	PV of Unwinding of Discount	Tax on Underwriting Gain	Commutation Price
Minimum	6%	$20,005	$ 820	$ 28	$19,157
Minimum	7%	19,406	790	171	18,445
Minimum	8%	18,841	762	305	17,774
Minimum	9%	18,308	735	431	17,142
Minimum	10%	17,804	710	551	16,543
Regular	6%	20,674	1,451	38	19,185
Regular	7%	20,145	1,406	288	18,451
Regular	8%	19,641	1,363	525	17,753
Regular	9%	19,162	1,323	751	17,088
Regular	10%	18,705	1,284	967	16,454

* Minimum indicates 20% tax rate for all years; Regular indicates 34% tax rate for all years.

EXHIBIT 9
Part 2

SENSITIVITY ANALYSIS
PAYOUT PROFILE USING EXAMPLE PAYMENT PATTERN

Tax Situation*	Nominal Interest Rate	Present Value	PV of Unwinding of Discount	Tax on Underwriting Gain	Commutation Price
Minimum	6%	$19,402	$834	$ 126	$18,442
Minimum	7%	18,729	801	286	17,642
Minimum	8%	18,096	769	436	16,891
Minimum	9%	17,499	739	578	16,182
Minimum	10%	16,937	710	711	15,516
Regular	6%	20,155	1,482	206	18,467
Regular	7%	19,559	1,432	487	17,640
Regular	8%	18,993	1,383	754	16,856
Regular	9%	18,455	1,338	1,007	16,110
Regular	10%	17,943	1,294	1,248	15,401

* Minimum indicates 20% tax rate for all years; Regular indicates 34% tax rate for all years.

EXHIBIT 9
Part 3

SENSITIVITY ANALYSIS
PAYOUT PROFILE USING EXAMPLE PAYMENT PATTERN

Tax Situation*	Nominal Interest Rate	Present Value	PV of Unwinding of Discount	Tax on Underwriting Gain	Commutation Price
Minimum	6%	$21,969	$ 858	$ (383)	$21,494
Minimum	7%	21,596	839	(294)	21,051
Minimum	8%	21,235	822	(208)	20,621
Minimum	9%	20,886	804	(125)	20,207
Minimum	10%	20,548	788	(45)	19,805
Regular	6%	22,375	1,492	(671)	21,554
Regular	7%	22,055	1,465	(520)	21,110
Regular	8%	21,744	1,439	(373)	20,678
Regular	9%	21,441	1,414	(230)	20,257
Regular	10%	21,147	1,389	(91)	19,849

* Minimum indicates 20% tax rate for all years; Regular indicates 34% tax rate for all years.

EXHIBIT 9
Part 4

Additional Payment Patterns

Year	Slow Pattern	Fast Pattern
1	1.00%	5.00%
2	3.00	7.00
3	5.00	20.00
4	7.00	15.00
5	9.00	12.00
6	9.00	12.00
7	11.00	12.00
8	11.00	10.00
9	9.00	4.00
10	7.00	2.00
11	5.00	1.00
12	5.00	
13	4.00	
14	4.00	
15	3.00	
16	2.00	
17	2.00	
18	2.00	
19	1.00	
Total	100.00%	100.00%

AN EXPOSURE RATING APPROACH TO PRICING PROPERTY EXCESS-OF-LOSS REINSURANCE

STEPHEN J. LUDWIG

Abstract

Included in the 1963 Proceedings *is the paper, "Rating by Layer of Insurance," by Ruth E. Salzmann. In her paper, Salzmann examines the relationship between homeowners fire losses and the corresponding amount of insurance. Using 1960 accident year data from the Insurance Company of North America (INA), each homeowners fire claim was expressed as a percentage of the amount of insurance on the policy affording the coverage. An accumulated loss cost distribution by percentage of insured value was then developed. These distributions can be (and indeed still are) used to exposure rate property excess-of-loss reinsurance.*

In order to determine whether the relationship between size of loss and amount of insurance is a stable one over time, Salzmann's methodology has been applied to a more current set of data (Hartford Insurance Group homeowners losses for accident years 1984–1988). Any changes in this relationship over time would have obvious implications for any reinsurer currently using the Salzmann Tables to exposure rate property excess-of-loss reinsurance. Salzmann's methodology has also been applied to The Hartford's small commercial property book of business in order to determine whether the commercial property relationships of loss size to amount of insurance differ from those of homeowners.

1. INTRODUCTION

Included in the 1963 *Proceedings* is the paper, "Rating by Layer of Insurance" by Ruth E. Salzmann [1]. In this paper, Salzmann develops cumulative loss distributions by percentage of insured value, in order to demonstrate that there is a direct relationship between property size-of-loss distributions and the corresponding amounts at risk. As testimony to the thoroughness of her analysis, the "Salzmann Tables" contained in her paper are still used today by many reinsurers as one means of rating property excess-of-loss reinsurance.

However, in reviewing Salzmann's paper, it becomes evident that she never represented her study as the final word on property excess rating but, rather, intended it to be a modest first step into this arena. Furthermore, there are a number of important points not addressed by the study; therefore the continued use of these tables as a reinsurance rating tool is inappropriate. While the methodology employed by Salzmann is theoretically sound, the loss data used in her analysis differs significantly from that which is typically covered by a property excess-of-loss treaty. However, by applying Salzmann's methodology to a more appropriate set of loss data, it is possible to produce a revised set of tables that are directly applicable to the rating of property excess-of-loss reinsurance.

2. SALZMANN'S STUDY

In compiling the loss data for her study, Salzmann captured individual claim (and policy) information for each of the following variables:

Company:	INA
Line of Business:	Homeowners
Accident Year:	1960
Cause of Loss:	Fire
Coverage:	Building Losses Only (Coverage A)
Construction:	Frame, Brick
Protection:	Protected, Unprotected
Insured Values (Homeowners Coverage A Limit):	$10,000, $15,000, $20,000, $25,000

The stated reasons for selecting the homeowners line of business were that: (1) the insured value, or policy amount, was a fair approximation of the amount at risk; and (2) underinsurance, if any, would be relatively consistent by class, due to the built-in incentive to fully insure in order to satisfy the replacement cost clause, which comes into operation when the insured value equals 80% of the building's replacement cost. Also, only the building loss portion of each claim was considered, since it was felt that these losses would have the most direct relationship with the policy amount and thus provide the best basis for the study.

For each claim, the building loss was expressed as a percentage of the corresponding amount of insurance from the policy affording the coverage. By changing the claim size scale from a pure-dollar basis to a percentage-of-insured-value basis, the Table 1 claim count distribution was produced:

TABLE 1

CUMULATIVE CLAIM COUNT DISTRIBUTION BY PERCENT OF INSURED VALUE*

Loss as a Percent of Insured Value	Frame-Protected	Frame-Unprotected	Brick-Protected	Brick-Unprotected	Total
5%	92.0%	91.3%	93.9%	92.9%	92.3%
10	95.4	94.1	96.4	95.8	95.4
20	97.3	95.4	97.8	96.8	97.0
30	98.0	96.0	98.2	97.9	97.7
40	98.6	96.5	98.5	98.4	98.2
50	98.9	97.1	98.8	98.7	98.6
60	99.1	97.4	99.2	98.9	98.8
70	99.3	97.5	99.4	98.9	99.0
80	99.5	97.9	99.7	98.9	99.2
90	99.6	98.1	99.7	99.2	99.4
100	100.0	100.0	100.0	100.0	100.0

* Combined distribution for the $10,000; $15,000; $20,000; and $25,000 amounts of insurance.

In addition to examining the distribution of claim counts by percentage of insured value, Salzmann also produced a cumulative loss distribution by percentage of insured value. To derive the dollar amount of losses contained within the first *X*% of insured value, Salzmann combined two values: (1) *X*% of insured value, per claim, for those claims which exceeded *X*% of insured value, and (2) 100% of each claim's incurred loss, per claim, for those claims which did not exceed *X*% of insured value. The results of Salzmann's calculations are shown in Table 2.

TABLE 2

CUMULATIVE LOSS COST DISTRIBUTION BY PERCENT OF INSURED VALUE*

Loss as a Percent of Insured Value	Frame-Protected	Frame-Unprotected	Brick-Protected	Brick-Unprotected	Total
5%	42.8%	26.9%	39.3%	28.8%	38.1%
10	54.2	35.9	49.4	39.2	48.7
20	67.4	47.8	61.9	52.2	61.5
30	76.8	57.5	71.7	63.1	71.1
40	83.9	65.7	79.7	70.6	78.6
50	89.0	73.2	86.5	77.5	84.6
60	92.7	79.6	91.9	82.8	89.3
70	95.5	85.7	96.0	87.3	93.1
80	97.6	91.3	98.3	91.8	96.1
90	99.1	95.7	99.3	95.9	98.2
100	100.0	100.0	100.0	100.0	100.0

* Combined distribution for the $10,000; $15,000; $20,000; and $25,000 amounts of insurance.

By comparing the distributions derived for the various amount of insurance groups ($10,000; $15,000; $20,000; and $25,000), Salzmann concluded that the relationship between size-of-loss distributions and insured values was constant across all amounts of insurance. She also pointed out several potential uses for her tables, with one of them being their potential incorporation as a reinsurance rating tool. Some 30 years later, her tables are still considered to be a very useful source of reinsurance rating information.

3. USING SALZMANN TABLES TO PRICE REINSURANCE

Using Salzmann Tables to price property excess-of-loss reinsurance represents a so-called "exposure rating" technique. Exposure rating does not rely on the ceding company's actual loss history as a basis for developing a reinsurance rate but, rather, is based on its current (or projected treaty year) distribution of direct premium by policy limit. For each policy limit written by the ceding company, an estimate is made as to the proportion of losses that will fall within the reinsurance layer being priced. In casualty reinsurance, one standard method of estimating these proportions is through the use of increased limits factors, while in property reinsurance, Salzmann Tables serve an equivalent function.

An example of how Salzmann Tables are used to exposure rate a property reinsurance program is shown in Exhibit 1. The example is for a company which is considering purchasing a $100,000 excess of $100,000 reinsurance treaty to cover its homeowners property losses. The only input necessary to perform the exposure rating calculation is the ceding company's estimated distribution of premium by its Coverage A (Building) limits for the period to be covered by the treaty. Given this distribution of premium by Coverage A limits, the mechanics of calculating an exposure rate are straightforward. First, the ceding company's retention is expressed as a percentage of each of the Coverage A limits, yielding the percentages shown in Column 3. These percentages can be viewed simply as the portion of the total policy limit that is being retained by the ceding company. For example, for a $200,000 policy the ceding company retains the first 50% of the Coverage A limit, while for the lower limit policies, the ceding company retains anywhere from 100% to 400% of the Coverage A limit.

By using these relationships of percentage retention to Coverage A limit as entry values into the Salzmann Tables, the corresponding premium (loss) allocations can be determined. For example, if the ceding company retains the first 50% of a $200,000 policy, the Salzmann Tables indicate that they will be responsible for 89% of total loss. Thus, for any $200,000 policy, the ceding company should retain 89% of the total premium, while the reinsurer only needs 11% of total policy premium to cover losses in excess of 50% of the Coverage A limit. As detailed in Exhibit 1, since all of the other Coverage A limits are less than or

EXHIBIT 1

EXPOSURE RATING EXAMPLE—$100,000 EXCESS OF $100,000 LAYER

Coverage A Limit	Direct Premium	Ceding Co. Retention as a Percent of Coverage A Limit*	Percentage Allocation of Total Premium-Salzmann Table Frame-Protected	Ceding Co. Retention Plus Reinsurance Limit as a Percent of Coverage A Limit**	Percentage Allocation of Total Premium-Salzmann Table Frame-Protected	Exposure Factor (6) − (4)	Exposure Premium (2) × (7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$ 25,000	$ 200,000	400%	100%	800%	100%	0%	$ 0
50,000	200,000	200	100	400	100	0	0
75,000	200,000	133	100	267	100	0	0
100,000	200,000	100	100	200	100	0	0
200,000	200,000	50	89	100	100	11	22,000
	$1,000,000						$22,000

$$\text{Exposure Rate} = \frac{\$22{,}000 \times .60 \times 1.10}{\$1{,}000{,}000} \times 1.0 \times \frac{100}{80} = 1.82\%$$

* Column 3 = $100,000 ÷ Column 1

** Column 5 = $200,000 ÷ Column 1

equal to the ceding company's retention, the Salzmann Tables allocate 100% of the policy premium to the ceding company.

By using the Salzmann Tables, it is estimated that the primary company will collect $22,000 in direct premium to cover losses and expenses in the $100,000 excess of $100,000 layer. To convert this to a reinsurance premium, several additional adjustments are necessary:

1. Ceding company expenses (acquisition costs and other expenses) need to be removed. This can be accomplished by multiplying the gross exposure premium by the expected pure loss component (excluding loss adjustment expenses). For purposes of this example, assume an expected pure loss component of 60%.
2. If the reinsurer is to share the cost of allocated loss adjustment expenses, then an appropriate loading must be added to the reinsurance rate. For purposes of this example, the rate will be loaded by 10%.
3. The ceding company's rate adequacy needs to be assessed. If the ceding company's underlying rates are inadequate, the reinsurer's exposure premium resulting from use of the Salzmann Tables will also be inadequate by the same percentage. In this example, it is assumed that the underlying rates are adequate, so no adjustment is necessary; i.e., the adjustment factor = 1.0.
4. Finally, the reinsurer will include a loading for expenses and profit. For purposes of this example, it is assumed that this element represents 20% of the final reinsurance premium—this loading would be expressed as "100/80ths."

These adjustments result in a final indicated exposure rate of 1.82%:

$$\text{Exposure Rate} = \frac{\$22{,}000 \times .60 \times 1.10}{\$1{,}000{,}000} \times 1.0 \times \frac{100}{80} = 1.82\%$$

Thus, based on the ceding company's estimated distribution of direct premium by policy limit, an exposure rating estimate produced by using the Salzmann Tables indicates that the reinsurer needs only $18,200 to provide for both its expenses and for expected losses within the $100,000 excess of $100,000 layer.

As a second example of exposure rating using the Salzmann Tables, if the ceding company was considering a further reduction in its retention to $25,000, the cost of the additional necessary reinsurance ($75,000 excess of $25,000) would be estimated at 15.05% of its direct premium, or $150,500 (Exhibit 2). The ceding company may view this additional reinsurance purchase as both an effective, and relatively inexpensive, means of removing some unwanted volatility from its books.

The natural alternative to exposure rating is experience rating. In experience rating, the ceding company's actual claim history for the previous three to five accident years provides the basis for developing a reinsurance rate. In the simplest form of experience rating, actual historical losses are adjusted for inflation, on a claim-by-claim basis, from the date of loss up to the average loss date anticipated for the treaty. These trended claim values are then cast against the proposed reinsurance structure, to determine how they would impact both the $75,000 excess of $25,000 and $100,000 excess of $100,000 layers. On a trended basis, then, an estimate of the extent to which each accident year's actual reported claims would have impacted each of the reinsurance layers is produced. Excess loss development factors are then applied to these trended figures in order to produce an estimate of ultimate trended excess losses by layer for each accident year. By then comparing these accident year ultimate excess loss figures to their respective premium bases (with historical premiums adjusted to either present rate levels or proposed treaty year rate levels), a three- to five-year average burning cost can be developed. By loading this "trended and developed" burning cost for reinsurer expenses and profit an "experience rate" results.

A reinsurer will typically produce both an exposure rating estimate and an experience rating estimate for each layer of reinsurance. These two rating methodologies may not always produce similar answers, however. Determining which of the two estimates is more credible is not always a straightforward process. Generally, experience rating is useful only on working layers, while exposure rating theoretically works well on all layers. In our example, experience rating is apparently not well suited for the $100,000 excess of $100,000 layer, given that expected losses are only $13,200 ($22,000 × .60); experience rating may produce a useful pricing estimate for the $75,000 excess of $25,000 layer, where expected losses are $109,440 ($182,400 × .60). One method of com-

EXHIBIT 2

EXPOSURE RATING EXAMPLE—$75,000 EXCESS OF $25,000 LAYER

Coverage A Limit	Direct Premium	Ceding Co. Retention as a Percent of Coverage A Limit*	Percentage Allocation of Total Premium-Salzmann Table Frame-Protected	Ceding Co. Retention Plus Reinsurance Limit as a Percent of Coverage A Limit**	Percentage Allocation of Total Premium-Salzmann Table Frame-Protected	Exposure Factor (6) − (4)	Exposure Premium (2) × (7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$ 25,000	$ 200,000	100.0%	100.0%	400%	100%	0.0%	$ 0
50,000	200,000	50.0	89.0	200	100	11.0	22,000
75,000	200,000	33.3	79.2	133	100	20.8	41,600
100,000	200,000	25.0	72.1	100	100	27.9	55,800
200,000	200,000	12.5	57.5	50	89	31.5	63,000
	$1,000,000						$182,400

$$\text{Exposure Rate} = \frac{\$182,400 \times .60 \times 1.10}{\$1,000,000} \times 1.0 \times \frac{100}{80} = 15.05\%$$

* Column 3 = $ 25,000 ÷ Column 1

** Column 5 = $100,000 ÷ Column 1

bining experience and exposure rate estimates into a single estimate of reinsurance rate is described by Gary Patrik and Isaac Mashitz in their 1990 Discussion Paper "Credibility for Treaty Reinsurance Excess Pricing" [2].

4. COMMENTS ON SALZMANN'S ANALYSIS

Salzmann achieved her goal of demonstrating that there was a direct relationship between homeowners building size-of-loss distributions and their corresponding insured values. When viewed as a pricing tool for property excess-of-loss reinsurance, however, the Salzmann Tables are far from ideal, due to the following considerations:

1. Building Losses Only—By restricting her analysis to only the building loss portion of each homeowners claim, Salzmann was satisfied that losses would thereby have the most direct relationship with the policy amount. In a homeowners policy, however, all of the following property coverages are provided, and would typically be covered by a property excess-of-loss treaty:

   Coverage A: Building;

   Coverage B: Other Structures—Limit provided is 10% of the Coverage A limit;

   Coverage C: Contents—Limit provided is 50% of the Coverage A limit, unless Replacement Cost coverage is purchased, in which case the limit is increased to 70% of the Coverage A limit;

   Coverage D: Loss of Use—Limit provided is 20% of the Coverage A limit.

   Clearly, when considering a "total" homeowners property loss, we are not dealing with just a complete payment of the Coverage A limit, but rather we are looking at a loss which could go as high as two times the Coverage A limit. By considering building losses only, Salzmann did not cover this possibility.

2. Cause of Loss—In demonstrating that a direct relationship existed between building size-of-loss distributions and amounts at risk, Salzmann considered only one cause of loss—fire. Therefore, if Salzmann Tables are used to price a property excess-of-loss reinsurance treaty, an implicit assumption in that price is that all other causes of property losses will exhibit the same relationship between size of loss and amount at risk.

3. Line of Business—Salzmann makes the point in her article that a size-of-loss distribution developed from one population of risks may not be appropriate for another population of risks. Clearly, if Salzmann Tables are used to rate commercial property excess-of-loss treaties, an implicit assumption is that commercial risks possess the same size of loss to insured value relationships as do homeowners risks.

None of these three points should in any way be construed as a criticism of Salzmann, as she clearly stated the goal of her study. However, it seems clear that, due to the three points mentioned above, the way the Salzmann Tables are currently used to rate property excess-of-loss reinsurance is inappropriate.

5. AN UPDATED ANALYSIS OF PROPERTY LOSSES

In order to address the problems associated with using the Salzmann Tables as a reinsurance pricing tool, a number of steps were taken. First, an updated review of homeowners fire loss experience was performed, using Hartford Insurance Group data for the 1984–1988 accident years. Second, a similar review of homeowners loss experience was performed for (1) all wind losses, (2) all other property causes of loss, and (3) the 1989 Hurricane Hugo losses, in order to determine whether these distributions of loss as a percentage of insured value differed from those of the fire losses. Finally, a review of commercial property loss experience was also performed, again looking at fire, wind, all other property, and Hurricane Hugo losses.

6. HOMEOWNERS FIRE LOSS DISTRIBUTIONS

For all homeowners fire losses, individual claim information was obtained, with losses emanating from all of the property coverages (A, B, C, and D) being included. Losses were then restated as a percentage of the Coverage A limit, with the upper bound on an individual claim's ratio thereby being 200% of the Coverage A limit. As shown in Exhibit 5, by including all of the property coverages within the definition of loss, a much different cumulative claim count distribution emerges. For example, the percentage of claims that exceed the Coverage A limit (100%) varies from 1.3% for Brick-Protected to 8.0% for Frame-Unprotected, a possibility not considered by the Salzmann Tables. Also shown in Exhibits 5 and 6 (and all subsequent similar exhibits) are the claim counts/dollar values that make up the various distributions, so that an assessment of the credibility of each pattern can be made.

When the cumulative distribution of losses by percentage of insured value is examined, the difference becomes even more pronounced, with only 84.5% of total losses being contained within the Coverage A limit (Exhibit 6).

What are the implications of these revised homeowners fire loss tables? By returning to the example of the $100,000 excess of $100,000 layer, several significant changes become apparent. (See Exhibit 3.) As shown, the exposure rate of 7.41%, produced by using the revised homeowners property loss distributions, compares to a Salzmann Table exposure rate of 1.82%. This tremendous increase in the ceding company's exposure rate has two main sources. First, both the $75,000 and $100,000 policy limits represent an exposure to the layer, a fact which was not reflected in the Salzmann Tables. Second, the estimated exposure to the layer produced by the $200,000 policy limits more than doubled.

As an additional consideration, these revised tables also indicate that a homeowners policy carrying a $200,000 Coverage A limit represents a potential property loss which could reach as high as $400,000. The property reinsurance program, as currently structured, would leave the ceding company vulnerable to homeowners property losses within the

EXHIBIT 3

Exposure Rating Example—$100,000 Excess of $100,000 Layer

Coverage A Limit	Direct Premium	Ceding Co. Retention as a Percent of Coverage A Limit*	Percentage Allocation of Total Premium-Hartford Table Frame-Protected	Ceding Co. Retention Plus Reinsurance Limit as a Percent of Coverage A Limit**	Percentage Allocation of Total Premium-Hartford Table Frame-Protected	Exposure Factor (6) − (4)	Exposure Premium (2) × (7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$ 25,000	$ 200,000	400%	100.0%	800%	100.0%	0%	$ 0
50,000	200,000	200	100.0	400	100.0	0	0
75,000	200,000	133	93.4	267	100.0	6.6	13,200
100,000	200,000	100	84.2	200	100.0	15.8	31,600
200,000	200,000	50	61.7	100	84.2	22.5	45,000
	$1,000,000						$89,800

$$\text{Exposure Rate} = \frac{\$89{,}800 \times .60 \times 1.10}{\$1{,}000{,}000} \times 1.0 \times \frac{100}{80} = 7.41\%$$

* Column 3 = $100,000 ÷ Column 1

** Column 5 = $200,000 ÷ Column 1

EXHIBIT 4

EXPOSURE RATING EXAMPLE—$75,000 EXCESS OF $25,000 LAYER

Coverage A Limit	Direct Premium	Ceding Co. Retention as a Percent of Coverage A Limit*	Percentage Allocation of Total Premium-Hartford Table Frame-Protected	Ceding Co. Retention Plus Reinsurance Limit as a Percent of Coverage A Limit**	Percentage Allocation of Total Premium-Hartford Table Frame-Protected	Exposure Factor (6) − (4)	Exposure Premium (2) × (7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$ 25,000	$ 200,000	100.0%	84.2%	400%	100.0%	15.8%	$ 31,600
50,000	200,000	50.0	61.7	200	100.0	38.3	76,600
75,000	200,000	33.3	51.1	133	93.4	42.3	84,600
100,000	200,000	25.0	45.0	100	84.2	39.2	78,400
200,000	200,000	12.5	33.5	50	61.7	28.2	56,400
	$1,000,000						$327,600

$$\text{Exposure Rate} = \frac{\$327{,}600 \times .60 \times 1.10}{\$1{,}000{,}000} \times 1.0 \times \frac{100}{80} = 27.03\%$$

* Column 3 = $ 25,000 ÷ Column 1

** Column 5 = $100,000 ÷ Column 1

EXHIBIT 5

Cumulative Claim Count Distribution by Percent of Insured Value

Homeowners: Fire Losses Only

Loss as a Percent of Insured Value	Frame-Protected		Frame-Unprotected		Brick-Protected		Brick-Unprotected		Total	
	Hartford 1984–88	INA 1960	Hartford 1984–88	INA 1960	Hartford 1984–88	INA 1960	Hartford 1984–88	INA 1960	Hartford 1984–88	INA 1960
5%	85.5%	92.0%	82.1%	91.3%	91.4%	93.9%	89.6%	92.9%	88.0%	92.3%
10	90.3	95.4	85.7	94.1	94.9	96.4	92.6	95.8	92.2	95.4
20	93.4	97.3	87.9	95.4	96.8	97.8	94.0	96.8	94.7	97.0
30	94.4	98.0	89.0	96.0	97.2	98.2	94.2	97.9	95.4	97.7
40	95.0	98.6	89.4	96.5	97.5	98.5	94.6	98.4	95.8	98.2
50	95.5	98.9	89.8	97.1	97.8	98.8	95.0	98.7	96.3	98.6
60	95.9	99.1	90.4	97.4	98.0	99.2	95.2	98.9	96.6	98.8
70	96.3	99.3	90.9	97.5	98.2	99.4	95.6	98.9	96.9	99.0
80	96.7	99.5	91.1	97.9	98.4	99.7	95.6	98.9	97.1	99.2
90	97.0	99.6	91.7	98.1	98.5	99.7	95.8	99.2	97.4	99.4
100	97.2	100.0	92.0	100.0	98.7	100.0	96.1	100.0	97.6	100.0
110	97.6	100.0	92.2	100.0	98.9	100.0	96.6	100.0	97.9	100.0
120	97.9	100.0	92.5	100.0	99.1	100.0	96.8	100.0	98.2	100.0
130	98.2	100.0	93.4	100.0	99.2	100.0	96.9	100.0	98.4	100.0
140	98.5	100.0	94.7	100.0	99.3	100.0	97.3	100.0	98.7	100.0
150	98.9	100.0	95.8	100.0	99.5	100.0	97.7	100.0	99.0	100.0
160	99.3	100.0	98.0	100.0	99.7	100.0	98.1	100.0	99.4	100.0
170	99.5	100.0	98.8	100.0	99.8	100.0	99.0	100.0	99.6	100.0
180	99.7	100.0	99.7	100.0	100.0	100.0	99.4	100.0	99.8	100.0
190	99.9	100.0	100.0	100.0	100.0	100.0	100.0	100.0	99.9	100.0
200	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Total Claim Counts	16,289	4,862	1,367	1,333	14,381	1,432	968	378	33,005	8,005

EXHIBIT 6

CUMULATIVE LOSS COST DISTRIBUTION BY PERCENT OF INSURED VALUE

HOMEOWNERS: FIRE LOSSES ONLY

Loss as a Percent of Insured Value	Frame-Protected		Frame-Unprotected		Brick-Protected		Brick-Unprotected		Total	
	Hartford 1984–88	INA 1960	Hartford 1984–88	INA 1960	Hartford 1984–88	INA 1960	Hartford 1984–88	INA 1960	Hartford 1984–88	INA 1960
5%	23.2%	42.8%	13.6%	26.9%	32.3%	39.3%	18.4%	28.8%	25.1%	38.1%
10	30.9	54.2	19.0	35.9	39.9	49.4	23.6	39.2	32.5	48.7
20	41.1	67.4	27.6	47.8	49.2	61.9	31.6	52.2	42.2	61.5
30	48.8	76.8	35.2	57.5	56.4	71.7	38.3	63.1	49.7	71.1
40	55.6	83.9	42.3	65.7	62.9	79.7	44.7	70.6	56.4	78.6
50	61.7	89.0	49.1	73.2	68.3	86.5	50.6	77.5	62.3	84.6
60	67.1	92.7	55.3	79.6	73.1	91.9	56.1	82.8	67.6	89.3
70	72.1	95.5	61.2	85.7	77.3	96.0	61.3	87.3	72.4	93.1
80	76.5	97.6	66.7	91.3	81.3	98.3	66.3	91.8	76.8	96.1
90	80.6	99.1	71.9	95.7	84.9	99.3	71.2	95.9	80.9	98.2
100	84.2	100.0	76.7	100.0	88.0	100.0	75.9	100.0	84.5	100.0
110	87.5	100.0	81.3	100.0	90.8	100.0	80.1	100.0	87.7	100.0
120	90.3	100.0	85.8	100.0	93.1	100.0	84.0	100.0	90.6	100.0
130	92.7	100.0	89.9	100.0	94.9	100.0	87.7	100.0	93.0	100.0
140	94.8	100.0	93.4	100.0	96.5	100.0	91.1	100.0	95.1	100.0
150	96.5	100.0	96.2	100.0	97.9	100.0	94.2	100.0	96.8	100.0
160	97.7	100.0	98.2	100.0	98.8	100.0	96.8	100.0	98.1	100.0
170	98.6	100.0	99.3	100.0	99.4	100.0	98.5	100.0	98.9	100.0
180	99.2	100.0	99.8	100.0	99.7	100.0	99.7	100.0	99.4	100.0
190	99.6	100.0	100.0	100.0	99.9	100.0	100.0	100.0	99.7	100.0
200	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Total Loss Dollars	$94,022,331	$1,981,703	$12,798,859	$726,819	$49,739,143	$695,122	$5,873,890	$221,391	$162,434,223	$3,625,035

$200,000 excess of $200,000 layer. An obvious solution to this problem would be for the ceding company to purchase an additional layer of reinsurance protection.

If we look at the revised exposure rate for the $75,000 excess of $25,000 layer (Exhibit 4), the increase over the Salzmann Table estimate is less substantial, with a revised rate of 27.03%, compared to a Salzmann Table estimate of 15.05%.

7. HOMEOWNERS—ADDITIONAL PROPERTY LOSS DISTRIBUTIONS

In order to address the second problem associated with using the Salzmann Tables as a reinsurance pricing tool, an evaluation of homeowners wind losses was made. This was identical in every respect to the fire loss study, except for the removal of the protected/unprotected data split. Cumulative claim count and loss dollar distributions are shown in Exhibit 7. Clearly, the distribution of wind losses is dramatically different from that of the fire losses. However, it should be noted that the 1984–1988 period did not contain any significant catastrophes, so that the distributions shown in Exhibit 7 should be considered as essentially "non-catastrophe" wind distributions. By performing a review of the wind losses resulting from Hurricane Hugo (1989), one indication of the loss distribution resulting from a major windstorm catastrophe can be developed (Exhibit 8). Finally, all other property causes of loss were considered on a combined basis, with the resulting loss distribution being shown in Exhibit 9.

By comparing the loss cost distributions derived for the various causes of loss, it is clear that the Salzmann Tables, which consider fire losses only, are inappropriate for use as a reinsurance pricing tool.

TABLE 3

CUMULATIVE LOSS COST DISTRIBUTIONS BY PERCENT OF INSURED VALUE

Loss Size as a Percent of Insured Value	Fire	Wind	Hurricane Hugo	All Other
5%	25.1%	87.1%	54.0%	73.5%
10	32.5	93.4	70.0	81.0
20	42.2	95.9	81.5	86.0
30	49.7	96.9	86.8	88.6
40	56.4	97.6	90.1	90.4
50	62.3	98.0	92.5	92.0
60	67.6	98.4	94.1	93.2
70	72.4	98.7	95.5	94.3
80	76.8	98.9	96.5	95.3
90	80.9	99.1	97.4	96.1
100	84.5	99.2	98.2	96.9

8. HOMEOWNERS EXPOSURE RATING—AN EXAMPLE

Given the large differences that exist between the fire, wind, and all other loss distributions, the question becomes one of how this information can be combined into an effective rating plan for homeowners property excess-of-loss reinsurance. One possible method is outlined in the following example:

1. Obtain the ceding company's historical distribution of homeowners losses by cause of loss. For example, fire losses may represent 35% of total incurred losses historically, while wind losses (non-catastrophes) equal 15%, other property losses (theft, freeze, water, etc.) equal 35%, and liability losses equal 15%.

2. Calculate exposure rates for the reinsurance layer, using each of the fire, wind, and other property loss tables separately. It should be noted that, in this example, the exposure rates have been calculated using the "total" (all construction types/all protection

EXHIBIT 7

Cumulative Claim Count and Loss Cost Distributions by Percent of Insured Value

Homeowners: Wind Losses Only

Loss Size as a Percent of Insured Value	Frame Claim Counts	Frame Losses	Brick Claim Counts	Brick Losses	Total Claim Counts	Total Losses
5%	95.0%	86.7%	94.8%	87.8%	94.9%	87.1%
10	98.9	93.1	99.1	93.8	99.0	93.4
20	99.7	95.6	99.7	96.3	99.7	95.9
30	99.8	96.6	99.8	97.3	99.8	96.9
40	99.9	97.3	99.9	97.9	99.9	97.6
50	99.9	97.8	99.9	98.3	99.9	98.0
60	99.9	98.2	99.9	98.6	99.9	98.4
70	99.9	98.5	99.9	98.8	99.9	98.7
80	99.9	98.8	100.0	99.1	100.0	98.9
90	100.0	99.0	100.0	99.2	100.0	99.1
100	100.0*	99.2	100.0**	99.3	100.0	99.2
110	100.0	99.4	100.0	99.4	100.0	99.4
120	100.0	99.5	100.0	99.6	100.0	99.5
130	100.0	99.6	100.0	99.7	100.0	99.6
140	100.0	99.7	100.0	99.8	100.0	99.7
150	100.0	99.8	100.0	99.8	100.0	99.8
160	100.0	99.9	100.0	99.9	100.0	99.9
170	100.0	99.9	100.0	99.9	100.0	99.9
180	100.0	100.0	100.0	100.0	100.0	100.0
190	100.0	100.0	100.0	100.0	100.0	100.0
200	100.0	100.0	100.0	100.0	100.0	100.0
Total Claim Counts/ Loss Dollars	57,844	$70,170,726	27,698	$41,283,311	85,542	$111,454,037

* .04% of claims exceed 100% of insured value

** .03% of claims exceed 100% of insured value

EXHIBIT 8

CUMULATIVE CLAIM COUNT AND LOSS COST DISTRIBUTIONS BY PERCENT OF INSURED VALUE

HOMEOWNERS: HURRICANE HUGO LOSSES ONLY

Loss Size as a Percent of Insured Value	Frame		Brick		Total	
	Claim Counts	Losses	Claim Counts	Losses	Claim Counts	Losses
5%	69.2%	47.0%	74.4%	59.9%	72.3%	54.0%
10	86.5	62.3	90.2	76.6	88.7	70.0
20	94.7	75.1	97.3	87.0	96.2	81.5
30	96.9	81.5	98.5	91.2	97.8	86.8
40	97.8	85.8	99.0	93.8	98.5	90.1
50	98.6	89.0	99.4	95.5	99.1	92.5
60	98.7	91.3	99.6	96.4	99.2	94.1
70	98.9	93.5	99.7	97.2	99.3	95.5
80	99.2	95.0	99.7	97.8	99.5	96.5
90	99.3	96.2	99.7	98.4	99.5	97.4
100	99.4	97.3	99.7	99.0	99.6	98.2
110	99.6	98.2	99.9	99.5	99.7	98.9
120	99.7	98.7	100.0	99.8	99.8	99.3
130	99.7	99.1	100.0	99.8	99.9	99.5
140	99.8	99.5	100.0	99.9	99.9	99.7
150	99.8	99.7	100.0	99.9	99.9	99.8
160	99.9	99.9	100.0	100.0	100.0	99.9
170	99.9	99.9	100.0	100.0	100.0	100.0
180	100.0	100.0	100.0	100.0	100.0	100.0
190	100.0	100.0	100.0	100.0	100.0	100.0
200	100.0	100.0	100.0	100.0	100.0	100.0
Total Claim Counts/ Loss Dollars	1,869	$8,429,553	2,713	$9,943,900	4,582	$18,373,453

EXHIBIT 9

CUMULATIVE CLAIM COUNT AND LOSS COST DISTRIBUTIONS BY PERCENT OF INSURED VALUE

HOMEOWNERS: OTHER PROPERTY LOSSES ONLY

Loss Size as a Percent of Insured Value	Frame		Brick		Total	
	Claim Counts	Losses	Claim Counts	Losses	Claim Counts	Losses
5%	94.8%	72.5%	94.1%	75.6%	94.6%	73.5%
10	98.3	79.8	98.1	83.3	98.2	81.0
20	99.3	84.9	99.4	88.2	99.3	86.0
30	99.5	87.6	99.6	90.5	99.6	88.6
40	99.6	89.6	99.7	92.0	99.7	90.4
50	99.7	91.3	99.8	93.3	99.7	92.0
60	99.7	92.7	99.8	94.3	99.8	93.2
70	99.8	93.9	99.8	95.1	99.8	94.3
80	99.8	95.0	99.9	95.9	99.8	95.3
90	99.8	95.9	99.9	96.6	99.8	96.1
100	99.8	96.7	99.9	97.2	99.9	96.9
110	99.9	97.4	99.9	97.7	99.9	97.5
120	99.9	98.0	99.9	98.2	99.9	98.1
130	99.9	98.5	99.9	98.6	99.9	98.6
140	99.9	99.0	99.9	99.0	99.9	99.0
150	99.9	99.3	100.0	99.3	99.9	99.3
160	100.0	99.6	100.0	99.6	100.0	99.6
170	100.0	99.8	100.0	99.7	100.0	99.7
180	100.0	99.9	100.0	99.8	100.0	99.8
190	100.0	99.9	100.0	99.9	100.0	99.9
200	100.0	100.0	100.0	100.0	100.0	100.0
Total Claim Counts/ Loss Dollars	122,737	$191,655,726	66,250	$98,628,340	188,987	$290,284,066

classes) loss cost distributions for each cause of loss. This reflects the fact that reinsurers often have difficulty obtaining information regarding a ceding company's distribution of homeowners business by construction type or protection class. If this information is available for a particular ceding company, an additional step would be added to this exposure rating process, with the various construction/protection loss cost distributions being used to calculate an exposure rate for each cause of loss.

3. Produce a final exposure rate by weighting the exposure rates produced in Step 2 by the percentage weights obtained in Step 1. In the example:

Cause of Loss	Loss Weights	Exposure Rates*	
		$75,000 Excess of $25,000	$100,000 Excess of $100,000
Fire	35%	26.55%	7.26%
Wind	15	1.99	0.40
Other Property	35	6.59	1.53
Liability	15	N/A	N/A
Final Exposure Rate:		11.90%	3.14%

* Derived from total loss distributions in Exhibit 6 (Fire), Exhibit 7 (Wind), and Exhibit 9 (All Others).

This proposed rating methodology has several advantages over simply using the Salzmann Tables. First, it explicitly recognizes the fact that all causes of loss need to be considered, not just fire. If fire losses are only 35% of total losses historically, the exposure rate derived by application of the fire tables should only receive a 35% weight. Second, it recognizes that each cause of loss has its own unique loss distribution. Finally, by considering all of the homeowners property coverages (A–D), the revised tables are directly applicable to the rating of property excess-of-loss reinsurance, whereas the Salzmann Tables, based on building losses only, are not.

9. COMMERCIAL PROPERTY

In order to address the third problem associated with using the Salzmann Tables as a reinsurance pricing tool, an evaluation of commercial property loss experience was also made. In order to keep things on a manageable level, this analysis was performed on only the small commercial package segment, the so-called "Main Street" book that virtually every primary company professes to write, and virtually every reinsurer has targeted as its "niche." This analysis was further limited to only those policies covering a single location, so that losses and insured values (policy limits) would be directly comparable.

In addition to the multiple location problem, several other complicating factors exist in any analysis of commercial property loss experience. First, the coverages provided are not standard across all commercial property policies. Due to the fact that many commercial buildings are leased to tenants, some commercial policies (the owner's) may cover the structure itself, while other policies (the tenant's) may only include contents coverage. Second, even for those policies that provide both building and contents coverages, there isn't the same direct relationship between the building limit and the contents limit as there is with homeowners risks.

This lack of a direct percentage relationship with the building limit also extends to the time element (business interruption) coverages, which would typically be included in the definition of loss under a property excess-of-loss reinsurance agreement. A further complicating factor to consider is that while the population of homeowners risks represents a very homogeneous set of exposures (notwithstanding any protection class and/or construction type considerations), under a commercial property policy the class of business (e.g., retail, office, restaurant, etc.) being covered introduces an additional variable into the rating equation, resulting in a less homogeneous set of exposures. Finally, the range of insured values being covered by commercial property policies is much greater than that of homeowners, making it necessary to re-examine the question of whether the relationship between size of loss and insured value is constant across the entire range of insured values.

One possible approach to address the absence of a uniform relationship among the limits purchased by coverage within a single policy would be to segregate the historical loss experience into a number of building/contents/time element limits combinations, with cumulative loss cost distributions then being derived for each limits combination:

Building Limit	Contents Limit		Time Element Limit	
$ 0	$10,000; 20,000; . . .	500,000	$10,000; 20,000; . . .	500,000
25,000	0; 20,000; . . .	500,000	0; 10,000; . . .	500,000
50,000	0; 10,000; . . .	1,000,000	0; 10,000; . . .	1,000,000
.	. .	.	. .	.
.	. .	.	. .	.
.	. .	.	. .	.
1,000,000	0; 10,000; . . .	10,000,000	0; 10,000; . . .	10,000,000

The result of this exercise would be a separate "Salzmann Table" for each possible building/contents/time element limits combination. To then perform an exposure rating calculation, the only input required would be the ceding company's distribution of premium across the various limits combinations. Clearly, while this approach might produce the most accurate commercial property rating tool possible, a massive amount of loss data would be required to create such a system.

One possible means of condensing the analysis described above would be to produce a single combined building/contents/time element loss distribution for each class of commercial business; e.g., retail/wholesale; service/office; apartment/condominium; and restaurant. The assumption being made here is that since the underlying loss exposures are similar for each risk within a given class of business, there is likely to be a consistent relationship between the relative magnitudes of the building, contents, and time element limits required. By comparing the total loss generated from these three coverages to the total limits purchased, a cumulative loss cost distribution can be developed for each class of business.

Exhibits 10, 11, 12, and 13 detail the cumulative loss cost distributions derived for fire, wind, all other property, and Hurricane Hugo losses, with individual distributions having been developed for the four major classes of business. Several points should be noted regarding these distributions. First, the historical data indicates that class of business is a variable that should be considered in the reinsurance rating mechanism, as significant differences in the cumulative loss cost distributions have been developed. Second, as with the homeowners data, the cumulative loss cost distributions vary significantly by cause of loss. Finally, by comparing the commercial property loss cost distributions to both the Salzmann Table and the homeowners table, the need for separate, commercial property-only reinsurance rating tables becomes obvious.

TABLE 4

CUMULATIVE LOSS COST DISTRIBUTION BY PERCENT OF INSURED VALUE
FIRE LOSSES ONLY

Loss Cost as a % of Insured Value	Hartford Commercial Property Total	Hartford Homeowners Total	Salzmann Table Total
5%	51.2%	25.1%	38.1%
10	65.1	32.5	48.7
20	79.9	42.2	61.5
30	87.9	49.7	71.1
40	92.8	56.4	78.6
50	95.9	62.3	84.6
60	97.3	67.6	89.3
70	98.3	72.4	93.1
80	99.1	76.8	96.1
90	99.7	80.9	98.2
100	100.0	84.5	100.0

EXHIBIT 10

CUMULATIVE LOSS COST DISTRIBUTION BY PERCENT OF INSURED VALUE COMMERCIAL PROPERTY: FIRE LOSSES ONLY

Loss as a Percent of Insured Value	Retail/ Wholesale	Service/ Office	Apartment/ Condominium	Restaurant	Total
5%	44.2%	52.6%	60.0%	58.9%	51.2%
10	58.4	66.7	72.1	73.1	65.1
20	75.3	80.5	83.5	87.5	79.9
30	85.2	88.4	89.7	93.3	87.9
40	91.3	93.4	93.8	96.1	92.8
50	95.2	96.6	96.4	97.3	95.9
60	97.1	97.9	97.6	98.3	97.3
70	98.2	98.6	98.7	99.0	98.3
80	99.0	99.2	99.6	99.5	99.1
90	99.6	99.7	99.8	99.7	99.7
100	100.0	100.0	100.0	100.0	100.0
Total Loss Dollars	$43,970,963	$47,812,881	$11,548,944	$18,657,002	$121,989,790
Total Claim Counts	6,367	8,280	1,895	3,475	20,017

EXHIBIT 11

CUMULATIVE LOSS COST DISTRIBUTION BY PERCENT OF INSURED VALUE COMMERCIAL PROPERTY: WIND LOSSES ONLY

Loss as a Percent of Insured Value	Retail/ Wholesale	Service/ Office	Apartment/ Condominium	Restaurant	Total
5%	81.4%	79.4%	82.5%	90.6%	81.9%
10	87.3	87.2	85.8	93.6	87.9
20	91.7	94.5	90.1	96.2	93.2
30	94.3	97.9	93.2	97.3	96.0
40	96.0	98.8	96.2	98.1	97.5
50	97.6	99.3	99.2	98.4	98.6
60	98.5	99.6	100.0	98.8	99.2
70	98.9	99.9	100.0	99.1	99.5
80	99.3	99.9	100.0	99.5	99.7
90	99.6	100.0	100.0	99.8	99.9
100	100.0	100.0	100.0	100.0	100.0
Total Loss Dollars	$4,782,299	$5,583,213	$1,719,718	$1,848,131	$13,933,361
Total Claim Counts	1,547	1,832	625	764	4,768

EXHIBIT 12

Cumulative Loss Cost Distribution by Percent of Insured Value
Commercial Property: All Other Property Losses

Loss as a Percent of Insured Value	Retail/ Wholesale	Service/ Office	Apartment/ Condominium	Restaurant	Total
5%	74.7%	76.4%	96.7%	95.3%	79.0%
10	85.4	86.2	99.1	97.7	87.9
20	93.0	93.1	99.5	98.7	94.1
30	96.2	95.9	99.6	99.1	96.6
40	98.0	97.8	99.7	99.4	98.1
50	98.9	98.6	99.8	99.7	98.9
60	99.3	99.1	99.8	99.8	99.3
70	99.6	99.4	99.9	99.9	99.6
80	99.8	99.6	100.0	100.0	99.8
90	99.9	99.8	100.0	100.0	99.9
100	100.0	100.0	100.0	100.0	100.0
Total Loss Dollars	$23,299,486	$18,959,591	$1,226,169	$8,020,334	$51,505,580
Total Claim Counts	11,964	10,842	823	5,525	29,154

EXHIBIT 13

Cumulative Loss Cost Distribution by Percent of Insured Value
Commercial Property: Hurricane Hugo Losses Only

Loss as a Percent of Insured Value	Retail/ Wholesale	Service/ Office	Apartment/ Condominium	Restaurant	Total
5%					66.3%
10					80.6
20					90.8
30					96.3
40					97.9
50					98.8
60					99.2
70					99.6
80					99.8
90					99.9
100					100.0
Total Loss Dollars					$6,941,155
Total Claim Counts					946

Individual Class of Business loss cost distributions were not available.

10. COMMERCIAL PROPERTY EXPOSURE RATING—AN EXAMPLE

The commercial property exposure rating example is very similar to that set forth for homeowners. The steps involved in the exposure rating calculation are as follows:

1. For each commercial class of business written by the ceding company, obtain its distribution of premium by policy limit, with the policy limit being a combined building/contents/time element limit. For this example, assume the following premium distribution:

Policy Limit	Retail/ Wholesale	Service/ Office	Apartment/ Condomin-iums	Restaurant	Total
$ 25,000	$ 50,000	$ 50,000	$ 50,000	$ 50,000	$ 200,000
50,000	50,000	50,000	50,000	50,000	200,000
75,000	50,000	50,000	50,000	50,000	200,000
100,000	50,000	50,000	50,000	50,000	200,000
200,000	50,000	50,000	50,000	50,000	200,000
Total	$250,000	$250,000	$250,000	$250,000	$1,000,000

2. Obtain the ceding company's historical distribution of commercial property losses by cause of loss. While the distribution by cause of loss may vary by class of business, for simplicity it will be assumed that for each class of business, fire losses represent 40% of total incurred losses, while wind losses (non-catastrophes) equal 10%, other property losses equal 15%, and liability losses equal 35%.

3. For each class of business, calculate exposure rates for the reinsurance layer, using each of the fire, wind, and other property loss tables separately.

4. For each class of business, produce a weighted-average exposure rate by weighting the exposure rates produced in Step 3 by the percentage weights obtained in Step 2. For example, for the retail/wholesale class of business:

Cause of Loss	Loss Weights	Exposure Rates $75,000 Excess of $25,000	Exposure Rates $100,000 Excess of $100,000
Fire	40%	11.53%	0.79%
Wind	10	3.91	0.40
Other Property	15	3.51	0.18
Liability	35	N/A	N/A
Weighted-Average Exposure Rate:		5.53%	0.38%

5. At this point, each class of business has had a weighted-average (by cause of loss) exposure rate developed. These individual class of business exposure rates can now be combined into a total commercial property exposure rate:

	Class of Business Premium Weights	Exposure Rates $75,000 Excess of $25,000	Exposure Rates $100,000 Excess of $100,000
Retail/Wholesale	25%	5.53%	0.38%
Service/Office	25	4.44	0.27
Apartment/ Condominium	25	3.61	0.25
Restaurant	25	2.84	0.21
Total Commercial Property Exposure Rate:		4.11%	0.28%

As can be seen, the differences in exposure rates by class of business can be substantial.

This proposed rating methodology for commercial property explicitly accounts for differing size-of-loss distributions by cause of loss, while also recognizing the fact that these size-of-loss distributions have historically differed by class of business as well. While this represents a significant improvement over simply using the Salzmann Tables, there are still a number of unresolved issues that deserve further research.

The first issue is that of whether the size-of-loss distribution for contents-only policies differs significantly from that for policies containing building coverage. Based on the historical data, there does not appear to be a significant difference in these size-of-loss distributions. Exhibit 14 displays size-of-loss distributions for the retail/wholesale class of business for fire losses only. By comparing the size-of-loss distributions within a comparable amount of insurance range, it can be seen that the distributions are similar for the two types of coverages.

A second issue is that of whether the relationship between size of loss and insured value is constant across the entire range of insured values. Exhibit 14 indicates that the relationship is not constant for retail/wholesale fire losses, while Exhibit 15 indicates that on a total book of business basis, the relationship between size of loss and insured value is not constant for any cause of loss. These findings suggest that not only should class of business be considered in the rating methodology, but also that amount of insurance must be considered, through the implementation of separate exposure rating tables by amount of insurance for a given class of business. Exhibits 16, 17, and 18 provide information on these distributions by class of business and cause of loss.

A final issue is that not all commercial property classes of business have been considered in this study. Examples of classes that warrant additional study include manufacturing/contracting risks, and institutional risks (hospitals, schools, churches). By expanding the number of classes of commercial property risks, a more comprehensive and effective property exposure rating tool could be developed.

11. CONCLUSION

In the ongoing debate of art versus science, reinsurance rating remains as much of an art as ever. However, the continued use of Salzmann Tables, under the guise of introducing "science" into the rating equation, is ill-advised. Salzmann Tables are being used inappropriately in many property excess pricing applications today. While this may not pose a serious problem for the working layers of a treaty, due to the existence of a credible experience rate, their continued use on nonworking layers is inappropriate. Through the introduction of the revised homeowners loss tables, and the introduction of the commercial property tables, it is hoped that reinsurance actuaries and underwriters can move one step closer to the "science" end of the rating spectrum.

REFERENCES

[1] Ruth E. Salzmann, "Rating by Layer of Insurance," *PCAS*, Vol. L, 1963, p. 14.

[2] Gary S. Patrik and Isaac Mashitz, "Credibility for Treaty Reinsurance Excess Pricing," *Pricing*, CAS Discussion Paper Program, Vol. I, 1990, p. 317.

EXHIBIT 14

Loss Cost Distribution as a Percent of Insured Value
Retail/Wholesale Risks: Fire Losses Only

	$1,000 to $25,000 Policy Limits Range		$25,000 to $100,000 Policy Limits Range		$100,000 to $300,000 Policy Limits Range		$300,000 to $1,000,000 Policy Limits Range		Greater than $1,000,000 Policy Limits Range	
Loss as a Percent of Insured Value	Contents Only Policies	All Other Policies	Contents Only Policies	All Other Policies	Contents Only Policies	All Other Policies	Contents Only Policies	All Other Policies	Contents Only Policies	All Other Policies
5%	22.7%	19.7%	38.3%	24.1%	41.9%	41.9%	49.4%	45.4%	93.9%	59.8%
10	36.2	35.9	52.2	34.4	56.1	55.9	63.3	60.1	100.0	74.9
20	52.7	60.4	68.3	50.5	74.1	72.9	78.5	77.6	100.0	91.7
30	64.6	76.8	78.8	62.5	84.8	83.1	86.8	88.3	100.0	97.9
40	73.9	83.8	86.1	71.2	91.3	88.9	94.3	94.5	100.0	100.0
50	81.5	86.5	92.0	78.9	95.7	93.1	98.3	98.1	100.0	100.0
60	86.5	89.2	94.6	84.5	97.6	95.8	99.4	99.5	100.0	100.0
70	90.5	91.9	96.4	89.1	98.5	97.3	100.0	99.9	100.0	100.0
80	94.2	94.6	98.0	93.3	99.2	98.6	100.0	100.0	100.0	100.0
90	97.4	97.3	99.1	96.9	99.7	99.6	100.0	100.0	100.0	100.0
100	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Total Loss Dollars	$1,238,692	$25,598	$6,002,156	$2,246,277	$9,698,540	$5,201,484	$3,892,854	$10,049,175	$157,517	$5,458,670
Total Claim Counts	364	6	1,451	270	1,396	884	470	1,060	31	435

EXHIBIT 15

Loss Cost Distribution as a Percent of Insured Value
All Commercial Property Classes of Business

Fire Losses Only
Amount of Insurance Ranges

Loss as a Percent of Insured Value	$1,000–$25,000	$25,000–$100,000	$100,000–$300,000	$300,000–$1,000,000	Greater Than $1,000,000
5%	24.2%	37.5%	45.0%	52.5%	75.3%
10	37.9	50.8	58.9	67.1	88.0
20	54.8	66.8	75.6	82.4	96.6
30	66.2	77.2	85.3	90.4	99.3
40	75.3	84.3	91.2	95.3	99.9
50	82.7	89.8	95.0	98.0	99.9
60	87.5	92.9	96.8	99.0	99.9
70	91.2	95.2	97.9	99.4	99.9
80	94.5	97.1	98.7	99.6	99.9
90	97.5	98.6	99.3	99.7	100.0
100	100.0	100.0	100.0	100.0	100.0

Wind Losses Only
Amount of Insurance Ranges

Loss as a Percent of Insured Value	$1,000–$25,000	$25,000–$100,000	$100,000–$300,000	$300,000–$1,000,000	Greater Than $1,000,000
5%	29.0%	63.3%	84.3%	82.5%	99.1%
10	45.2	74.5	91.9	87.0	100.0
20	65.0	82.8	97.3	92.5	100.0
30	76.8	87.4	99.5	95.8	100.0
40	84.7	90.4	100.0	97.6	100.0
50	89.7	92.8	100.0	99.3	100.0
60	93.1	94.8	100.0	100.0	100.0
70	95.6	96.6	100.0	100.0	100.0
80	97.3	97.9	100.0	100.0	100.0
90	98.7	99.2	100.0	100.0	100.0
100	100.0	100.0	100.0	100.0	100.0

All Other Losses
Amount of Insurance Ranges

Loss as a Percent of Insured Value	$1,000–$25,000	$25,000–$100,000	$100,000–$300,000	$300,000–$1,000,000	Greater Than $1,000,000
5%	38.9%	64.0%	82.8%	92.0%	98.9%
10	56.9	78.5	91.3	96.3	100.0
20	74.6	89.8	96.2	98.3	100.0
30	83.9	94.2	97.8	99.3	100.0
40	89.5	97.0	98.7	100.0	100.0
50	93.3	98.3	99.2	100.0	100.0
60	95.7	99.0	99.5	100.0	100.0
70	97.3	99.4	99.8	100.0	100.0
80	98.4	99.7	99.9	100.0	100.0
90	99.3	99.9	100.0	100.0	100.0
100	100.0	100.0	100.0	100.0	100.0

Underlying Loss Dollars/Claim Counts
Amount of Insurance Ranges

	$1,000–$25,000	$25,000–$100,000	$100,000–$300,000	$300,000–$1,000,000	Greater Than $1,000,000
Fire: Loss Dollars	$3,911,408	$17,945,684	$36,261,297	$44,669,984	$19,201,417
Fire: Claim Counts	992	4,255	6,404	6,014	2,352
Wind: Loss Dollars	$250,442	$1,894,150	$4,051,375	$5,736,626	$2,000,768
Wind: Claim Counts	110	834	1,694	1,604	526
Other: Loss Dollars	$3,150,320	$14,151,551	$16,043,644	$12,956,945	$5,203,120
Other: Claim Counts	1,971	8,060	9,150	7,176	2,797

EXHIBIT 16

Loss Cost Distribution as a Percent of Insured Value
Retail/Wholesale Risks Only

Fire Losses Only
Amount of Insurance Ranges

Loss as a Percent of Insured Value	$1,000–$25,000	$25,000–$100,000	$100,000–$300,000	$300,000–$1,000,000	Greater Than $1,000,000
5%	22.7%	35.1%	41.9%	46.5%	60.8%
10	36.2	48.3	56.1	61.0	75.6
20	52.9	64.4	73.7	77.9	91.9
30	64.9	75.2	84.2	87.9	98.0
40	74.0	82.8	90.4	94.4	100.0
50	81.6	89.1	94.8	98.2	100.0
60	86.5	92.4	97.0	99.5	100.0
70	90.5	94.8	98.1	99.9	100.0
80	94.2	97.0	99.0	100.0	100.0
90	97.4	98.6	99.7	100.0	100.0
100	100.0	100.0	100.0	100.0	100.0

Wind Losses Only
Amount of Insurance Ranges

Loss as a Percent of Insured Value	$1,000–$25,000	$25,000–$100,000	$100,000–$300,000	$300,000–$1,000,000	Greater Than $1,000,000
5%	35.3%	60.4%	86.5%	83.1%	97.6%
10	53.3	71.2	93.6	86.4	100.0
20	76.4	79.3	98.2	89.5	100.0
30	87.7	84.1	100.0	92.5	100.0
40	93.4	87.3	100.0	95.5	100.0
50	95.3	89.7	100.0	98.6	100.0
60	96.2	91.8	100.0	100.0	100.0
70	97.2	94.0	100.0	100.0	100.0
80	98.1	96.2	100.0	100.0	100.0
90	99.1	98.2	100.0	100.0	100.0
100	100.0	100.0	100.0	100.0	100.0

All Other Losses
Amount of Insurance Ranges

Loss as a Percent of Insured Value	$1,000–$25,000	$25,000–$100,000	$100,000–$300,000	$300,000–$1,000,000	Greater Than $1,000,000
5%	39.3%	62.4%	78.5%	86.3%	96.1%
10	57.5	76.8	88.6	93.6	100.0
20	75.5	88.6	95.2	97.3	100.0
30	85.0	93.4	97.4	99.1	100.0
40	90.4	96.5	98.4	100.0	100.0
50	94.3	98.0	99.1	100.0	100.0
60	96.5	98.6	99.6	100.0	100.0
70	97.8	99.1	99.9	100.0	100.0
80	98.9	99.5	100.0	100.0	100.0
90	99.5	99.8	100.0	100.0	100.0
100	100.0	100.0	100.0	100.0	100.0

Underlying Loss Dollars/Claim Counts
Amount of Insurance Ranges

	$1,000–$25,000	$25,000–$100,000	$100,000–$300,000	$300,000–$1,000,000	Greater Than $1,000,000
Fire: Loss Dollars	$1,264,290	$8,248,433	$14,900,024	$13,942,029	$5,616,187
Fire: Claim Counts	370	1,721	2,280	1,530	466
Wind: Loss Dollars	$98,439	$874,320	$1,491,976	$1,676,508	$641,056
Wind: Claim Counts	49	358	601	414	125
Other: Loss Dollars	$1,305,334	$6,958,824	$8,039,272	$5,542,385	$1,453,671
Other: Claim Counts	849	3,830	4,144	2,489	652

EXHIBIT 17

Loss Cost Distribution as a Percent of Insured Value
Service/Office Risks Only

Fire Losses Only
Amount of Insurance Ranges

Loss as a Percent of Insured Value	$1,000– $25,000	$25,000– $100,000	$100,000– $300,000	$300,000– $1,000,000	Greater Than $1,000,000
5%	25.6%	39.3%	44.9%	56.7%	78.0%
10	39.9	53.7	58.5	71.1	91.9
20	57.0	70.1	75.1	84.7	97.4
30	68.2	79.9	85.0	92.0	98.0
40	77.2	86.8	91.2	96.7	100.0
50	84.4	92.0	95.2	99.5	100.0
60	89.0	94.6	97.0	100.0	100.0
70	92.4	96.4	98.2	100.0	100.0
80	95.3	97.9	99.1	100.0	100.0
90	97.9	99.1	99.6	100.0	100.0
100	100.0	100.0	100.0	100.0	100.0

Wind Losses Only
Amount of Insurance Ranges

Loss as a Percent of Insured Value	$1,000– $25,000	$25,000– $100,000	$100,000– $300,000	$300,000– $1,000,000	Greater Than $1,000,000
5%	24.6%	64.9%	80.5%	79.8%	99.6%
10	39.2	77.9	89.8	86.9	100.0
20	57.4	87.7	95.5	96.3	100.0
30	69.9	91.9	98.9	100.0	100.0
40	79.3	94.4	100.0	100.0	100.0
50	86.5	96.9	100.0	100.0	100.0
60	91.5	98.7	100.0	100.0	100.0
70	95.0	99.9	100.0	100.0	100.0
80	97.1	100.0	100.0	100.0	100.0
90	98.5	100.0	100.0	100.0	100.0
100	100.0	100.0	100.0	100.0	100.0

All Other Losses
Amount of Insurance Ranges

Loss as a Percent of Insured Value	$1,000– $25,000	$25,000– $100,000	$100,000– $300,000	$300,000– $1,000,000	Greater Than $1,000,000
5%	38.0%	62.3%	83.8%	93.2%	100.0%
10	56.1	78.0	92.3	97.1	100.0
20	73.6	90.2	96.2	98.1	100.0
30	82.7	94.9	97.6	99.0	100.0
40	88.5	97.5	98.7	99.9	100.0
50	92.4	98.6	99.2	100.0	100.0
60	95.0	99.2	99.4	100.0	100.0
70	96.7	99.6	99.5	100.0	100.0
80	98.1	99.8	99.7	100.0	100.0
90	99.0	99.9	99.9	100.0	100.0
100	100.0	100.0	100.0	100.0	100.0

Underlying Loss Dollars/Claim Counts
Amount of Insurance Ranges

	$1,000– $25,000	$25,000– $100,000	$100,000– $300,000	$300,000– $1,000,000	Greater Than $1,000,000
Fire: Loss Dollars	$2,245,536	$7,464,529	$15,012,368	$15,691,044	$7,399,404
Fire: Claim Counts	562	1,938	2,665	2,340	775
Wind: Loss Dollars	$137,720	$709,647	$1,709,296	$2,279,065	$747,485
Wind: Claim Counts	52	343	648	629	160
Other: Loss Dollars	$1,731,669	$5,830,461	$6,022,589	$4,102,566	$1,272,306
Other: Claim Counts	1,024	3,219	3,443	2,471	685

EXHIBIT 18

Loss Cost Distribution as a Percent of Insured Value
Restaurant Risks Only

Fire Losses Only — Amount of Insurance Ranges

Loss as a Percent of Insured Value	$1,000–$25,000	$25,000–$100,000	$100,000–$300,000	$300,000–$1,000,000	Greater Than $1,000,000
5%	20.3%	38.9%	60.0%	51.9%	83.3%
10	30.7	49.8	73.9	68.9	93.3
20	46.2	64.1	87.9	86.9	100.0
30	57.7	73.8	93.5	94.6	100.0
40	67.6	80.1	96.9	97.6	100.0
50	75.3	84.6	98.3	98.7	100.0
60	80.8	88.5	99.1	99.4	100.0
70	85.8	92.3	99.5	100.0	100.0
80	90.5	95.5	100.0	100.0	100.0
90	95.3	97.8	100.0	100.0	100.0
100	100.0	100.0	100.0	100.0	100.0

Wind Losses Only — Amount of Insurance Ranges

Loss as a Percent of Insured Value	$1,000–$25,000	$25,000–$100,000	$100,000–$300,000	$300,000–$1,000,000	Greater Than $1,000,000
5%	23.6%	58.8%	87.1%	98.0%	100.0
10	41.2	67.1	91.7	100.0	100.0
20	56.0	74.2	99.2	100.0	100.0
30	68.4	80.6	100.0	100.0	100.0
40	76.5	86.1	100.0	100.0	100.0
50	80.7	88.7	100.0	100.0	100.0
60	84.8	91.2	100.0	100.0	100.0
70	88.9	93.8	100.0	100.0	100.0
80	93.1	96.3	100.0	100.0	100.0
90	97.2	98.9	100.0	100.0	100.0
100	100.0	100.0	100.0	100.0	100.0

All Other Losses — Amount of Insurance Ranges

Loss as a Percent of Insured Value	$1,000–$25,000	$25,000–$100,000	$100,000–$300,000	$300,000–$1,000,000	Greater Than $1,000,000
5%	49.5%	78.5%	97.9%	99.7%	100.0%
10	67.8	89.2	99.3	100.0	100.0
20	85.5	93.5	100.0	100.0	100.0
30	93.0	95.2	100.0	100.0	100.0
40	96.4	96.8	100.0	100.0	100.0
50	97.7	98.2	100.0	100.0	100.0
60	98.9	99.1	100.0	100.0	100.0
70	99.7	99.8	100.0	100.0	100.0
80	99.8	100.0	100.0	100.0	100.0
90	99.9	100.0	100.0	100.0	100.0
100	100.0	100.0	100.0	100.0	100.0

Underlying Loss Dollars/Claim Counts — Amount of Insurance Ranges

	$1,000–$25,000	$25,000–$100,000	$100,000–$300,000	$300,000–$1,000,000	Greater Than $1,000,000
Fire: Loss Dollars	$392,956	$1,856,015	$2,764,497	$9,306,687	$4,336,847
Fire: Claim Counts	53	495	829	1,242	856
Wind: Loss Dollars	$12,084	$236,522	$397,060	$798,827	$403,638
Wind: Claim Counts	6	93	195	285	185
Other: Loss Dollars	$103,325	$1,299,982	$1,630,079	$2,743,585	$2,243,363
Other: Claim Counts	94	966	1,298	1,832	1,335

THE CREDIBILITY OF A SINGLE PRIVATE PASSENGER DRIVER

HOWARD C. MAHLER

Abstract

The credibility of the experience of an individual driver is determined by analyzing the accident records of private passenger drivers. For the particular data set analyzed, the risk parameters were found to be relatively stable over time, resulting in significant credibility being assigned to older years of data.

1. INTRODUCTION

In this paper, the accident records of private passenger drivers are analyzed using the methods developed by this author (in Mahler [2]) in order to estimate the credibility of the experience of an individual driver. The analysis is done using only the following classification variables: gender, state of licensing, and being licensed over an entire 14-year span.[1]

The use of additional years of experience (more than 10) is found to add significant information and is projected to do so for longer periods of time. For this particular data set, the risk parameters were found to be relatively stable.[2]

2. THE DATA SET

The data analyzed are for California private passenger drivers [1]. The data show the number of accidents annually in 1961–1963 and 1969–1974, for a sample of drivers licensed from 1961 to 1974. Thus, there

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[1] Additional classification information was not available in the data set used.

[2] A larger data set, in terms of number of drivers, number of years of data, or classification information, may lead to a somewhat different conclusion.

are nine years of data for each driver, covering a 14-year period with a five year gap in the middle. The data are divided between male and female drivers. An extract from the data set is shown in Appendix A.

It should be noted that this data set allows an analysis only of accident frequency. No information is available on accident or claim severity.

3. CORRELATIONS

The correlations between years of data are shown in Exhibit 1. The key step in the analysis is to group together those pairs of years of data separated by the same number of years.[3] For example, there are five pairs separated by two years: 1961 and 1963, 1969 and 1971, 1970 and 1972, 1971 and 1973, and 1972 and 1974.[4]

The average correlations between pairs of years of data with different separations are shown in Exhibit 2. The correlations are all small, reflecting the low information-to-noise ratio. The correlations appear to decline gradually as the separation increases. This can be confirmed by fitting a linear regression to the average correlations or to the individual observed correlations.[5]

The results of fitting linear regressions to the individual observed correlations are:

Male Drivers: $\text{correlation}(\lambda) = .03515 - .00064\lambda$
Female Drivers: $\text{correlation}(\lambda) = .03102 - .00126\lambda$

Both regressions indicate a small, but significant, decline in the correlation as the separation between years λ increases.[6]

One can use these equations to approximate the covariance structure for separations of from one to 13 years. Also, it would not be unreasonable to use these equations to extrapolate the covariance structure for

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[3] Mahler [2] makes the assumption that the correlation depends solely on the number of years of separation.

[4] There is a gap in the data from 1964 to 1968.

[5] One could also fit a weighted regression to the average correlations with weights equal to the number of observations underlying each average. The results of any of these three regressions are very similar.

[6] Both are significant at a 0.5% level. The t-statistics are −2.81 and −4.02, respectively, for 34 degrees of freedom.

EXHIBIT 1

Correlations (Male Drivers)

	1961	1962	1963	1969	1970	1971	1972	1973	1974
1961	1.0000	.0426	.0387	.0261	.0330	.0391	.0285	.0314	.0258
1962	.0426	1.0000	.0384	.0228	.0267	.0405	.0257	.0226	.0332
1963	.0387	.0384	1.0000	.0299	.0374	.0246	.0269	.0185	.0285
1969	.0261	.0228	.0299	1.0000	.0304	.0320	.0302	.0279	.0240
1970	.0330	.0267	.0374	.0304	1.0000	.0269	.0350	.0388	.0407
1971	.0391	.0405	.0246	.0320	.0269	1.0000	.0350	.0291	.0340
1972	.0285	.0257	.0269	.0302	.0350	.0350	1.0000	.0363	.0358
1973	.0314	.0226	.0185	.0279	.0388	.0291	.0363	1.0000	.0342
1974	.0258	.0332	.0285	.0240	.0407	.0340	.0358	.0342	1.0000

Correlations (Female Drivers)

	1961	1962	1963	1969	1970	1971	1972	1973	1974
1961	1.0000	.0285	.0290	.0159	.0145	.0188	.0337	.0043	.0134
1962	.0285	1.0000	.0284	.0241	.0279	.0217	.0202	.0063	.0123
1963	.0290	.0284	1.0000	.0322	.0200	.0236	.0247	.0171	.0180
1969	.0159	.0241	.0322	1.0000	.0412	.0195	.0380	.0188	.0205
1970	.0145	.0279	.0200	.0412	1.0000	.0225	.0154	.0337	.0164
1971	.0188	.0217	.0236	.0195	.0225	1.0000	.0270	.0217	.0249
1972	.0337	.0202	.0247	.0380	.0154	.0270	1.0000	.0308	.0374
1973	.0043	.0063	.0171	.0188	.0337	.0217	.0308	1.0000	.0412
1974	.0134	.0123	.0180	.0205	.0164	.0249	.0374	.0412	1.0000

Note the gap in information from 1964 through 1968.

EXHIBIT 2

OBSERVED AVERAGE CORRELATIONS OF DRIVERS' EXPERIENCE OVER TIME

Difference Between Pairs of Years of Experience	Correlation		Number of Pairs of Years Observed
	Males	Females	
1	.0348	.0314	7
2	.0341	.0246	5
3	.0343	.0322	3
4	.0343	.0176	2
5	.0240	.0205	1
6	.0299	.0322	1
7	.0301	.0221	2
8	.0258	.0225	3
9	.0335	.0203	3
10	.0278	.0187	3
11	.0265	.0193	3
12	.0323	.0083	2
13	.0258	.0134	1

longer separations, provided one imposes the restriction that the correlations are not negative; i.e., that the correlations decline to zero and then remain there.[7] For the regression equation for male drivers, it takes 55 years for the correlation to decline to zero. For the female drivers, it takes 25 years for the correlation to decline to zero.

4. COVARIANCE STRUCTURE

In order to calculate the least squares credibilities, one has to estimate the covariance structure of the data. The required quantities are:

τ^2 = between variance;

$C(\lambda)$ = covariance for data for the same risk, λ years apart
= "within covariance;"

$C(0)$ = within variance.

The within covariances will be estimated in terms of the correlations discussed in the previous section:

$$C(\lambda) = \text{correlation}(\lambda) \times C(0);$$

$$\text{correlation}(\lambda) = \begin{cases} \text{MAX}[0,\ .03515 - .00064\lambda] & \text{Male Drivers;} \\ \text{MAX}[0,\ .03102 - .00126\lambda] & \text{Female Drivers.} \end{cases}$$

The variances are estimated in Appendix B. The results are:

	Within Risk Variance	Between Risk Variance
Male Drivers	.0724	.0116
Female Drivers	.0377	.0057

In both cases the within variance is larger than the between variance.[8]

[7] It would be equally valid to extrapolate using an exponential regression fit to the correlations, as well as other methods of extrapolation. The use of a linear extrapolation is judged to be sufficient to illustrate the general technique.

[8] The Bühlmann credibility parameter K is the ratio of the within variance to the between variance. In these cases K = 6.2 and 6.6. The Bühlmann credibility for N years of data is given by $Z = N/(N + K)$.

The resulting covariance structure is:

$$\tau^2 = \begin{cases} .0116 & \text{Male Drivers} \\ .0057 & \text{Female Drivers} \end{cases}$$

$$C(0) = \begin{cases} .0724 & \text{Male Drivers} \\ .0377 & \text{Female Drivers} \end{cases}$$

For $\lambda \geqq 1$:

$$C(\lambda) = \begin{cases} .0724 \times \text{MAX}[0,\ .03515 - .00064\lambda] & \text{Male} \\ .0377 \times \text{MAX}[0,\ .03102 - .00126\lambda] & \text{Female} \end{cases}$$

In addition, the following example, with much more quickly shifting risk parameters over time, will be provided for illustrative purposes of contrast. The assumed covariance structure is:

$$\tau^2 = .01;$$

$$C(0) = .07;$$

For $\lambda \geqq 1$:

$$C(\lambda) = .07 \times \text{MAX}[0,\ .5 - .05\lambda].$$

5. CREDIBILITIES

In the case of using the latest N years of data, with the complement of credibility given to the overall mean, Mahler[9] develops the following N linear equations in N unknowns which can be solved for the least squares (Bühlmann/Bayesian) credibilities:

$$\sum_{j=1}^{N} Z_j\,(\tau^2 + C(|i - j|)) = \tau^2 + C(N + \Delta - i) \qquad i = 1,2, \ldots N$$

where:

Z_j = the credibility assigned to year j, with $j = N$ the most recent year of data;

τ^2 = between variance;

[9] Equation 11.3 in Mahler [2].

$C(\lambda)$ = covariance for data for the same risk, λ years apart = "within covariance;"

$C(0)$ = within variance;

Δ = the length of time between the latest year of data used and the year being estimated.

Using the covariance structure from the previous section, these equations produce the credibilities shown in Exhibit 3. Given the relatively small amount of data used, the estimated credibilities are subject to a fair amount of uncertainty.[10]

For both the male and female drivers, the credibilities calculated for older years are relatively close to those for more recent years. The sum of the credibilities as shown in Exhibit 4 increases as the number of years of data increases in a manner that is not unexpected. For male drivers the total credibility is approximately $N/(N + 5)$. For female drivers the total credibility is approximately $N/(N + 6)$.

In the example for contrast, the most recent year gets much more weight than older years, since the correlations quickly decrease to zero. The sum of the credibilities is much higher for the use of between one and five years of data than is the case for the California data, since the correlations are higher in this example for contrast.

6. SQUARED ERRORS

Mahler[11] gives the following equation for the expected squared error between the observation and prediction:

[10] The values shown for the use of more than 15 years of data are subject to even more uncertainty, since they are based on an extrapolation of the covariance structure beyond that estimated from the data set.

[11] Equation 11.2 in Mahler [2].

$$V(Z) = \sum_{i=1}^{N} \sum_{j=1}^{N} Z_i Z_j(\tau^2 + C(|i - j|))$$

$$- 2 \sum_{i=1}^{N} Z_i(\tau^2 + C(N + \Delta - i))$$

$$+ \tau^2 + C(0),$$

where all the symbols are defined as before and Z_i is the credibility assigned to year i and the complement of credibility is given to the overall mean.

Exhibit 5 displays the squared errors corresponding to the use of the least squares credibilities calculated in the previous section. For both the male and female drivers, the squared errors decline slowly and at a gradually declining rate as more years of data are added. In the example for contrast, the squared error declines significantly with the use of a single year of data, then declines somewhat with the use of a few additional years, and then levels off more quickly than for the California driver data.

7. CONCLUSIONS

The data set analyzed in this paper was one of two analyzed in a paper by Emilio Venezian [3]. In this paper, the data is analyzed in a more detailed manner using the methods developed in Mahler [2]. This analysis leads to the conclusion that the risk parameters are shifting at a relatively slow rate, which explains why Dr. Venezian, for this data set, was not able to reject the hypothesis that relative accident rates are stable.

Given the relatively limited information available on each driver in this data set, additional years of each driver's past accident record provide useful information for predicting his or her future relative accident frequency. Therefore, accident records from 10 or 15 years ago would be given significant credibility. However, it is important to keep in mind that credibility is a relative concept. The 10-year-old accident information is being given significant weight, but only relative to the weight given

EXHIBIT 3, PART 1

MALE DRIVERS

Credibility (based on assumed covariance structure, $\Delta = 1$)

Years Between Data and Estimate	Number of Years of Data Used							
	1	2	3	4	5	10	15	20
1 (Most Recent)	16.8%	14.4%	12.6%	11.2%	10.1%	7.0%	5.5%	4.7%
2		14.3	12.5	11.1	10.0	6.9	5.4	4.6
3			12.5	11.1	10.0	6.8	5.3	4.5
4				11.0	9.9	6.7	5.2	4.4
5					9.9	6.6	5.1	4.3
6						6.6	5.0	4.2
7						6.5	5.0	4.1
8						6.4	4.9	4.0
9						6.4	4.8	4.0
10						6.4	4.8	3.9
11							4.7	3.8
12							4.7	3.8
13							4.6	3.7
14							4.6	3.7
15							4.6	3.6
16								3.6
17								3.6
18								3.5
19								3.5
20								3.5
Total Credibility	16.8%	28.7%	37.6%	44.4%	49.9%	66.3%	74.2%	79.0%

EXHIBIT 3, PART 2

Female Drivers

Credibility (based on assumed covariance structure, $\Delta = 1$)

Years Between Data and Estimate	Number of Years of Data Used: 1	2	3	4	5	10	15	20
1 (Most Recent)	15.7%	13.6%	12.0%	10.8%	9.8%	7.0%	5.8%	5.2%
2		13.5	11.9	10.6	9.7	6.9	5.6	5.0
3			11.8	10.5	9.5	6.7	5.4	4.8
4				10.4	9.4	6.5	5.2	4.6
5					9.3	6.4	5.1	4.4
6						6.2	4.9	4.3
7						6.1	4.8	4.1
8						6.0	4.7	4.0
9						6.0	4.5	3.8
10						5.9	4.4	3.7
11							4.3	3.6
12							4.3	3.5
13							4.2	3.4
14							4.1	3.3
15							4.1	3.2
16								3.1
17								3.1
18								3.0
19								3.0
20								2.9
Total Credibility	15.7%	27.1%	35.7%	42.3%	47.7%	63.7%	71.4%	76.0%

EXHIBIT 3, PART 3

EXAMPLE FOR CONTRAST

Credibility (based on assumed covariance structure, $\Delta = 1$)

Years Between Data and Estimate	Number of Years of Data Used: 1	2	3	4	5	10	15	20
1 (Most Recent)	51.9%	37.3%	32.7%	31.3%	31.0%	30.7%	30.2%	30.1%
2		28.2	22.2	20.2	19.7	19.6	19.6	19.5
3			16.1	13.2	12.4	12.5	12.8	12.7
4				8.8	7.5	7.8	8.2	8.1
5					4.1	4.7	5.0	4.9
6						2.6	2.5	2.6
7						.9	.5	.6
8						−.5	−1.4	−1.3
9						−2.1	−3.6	−3.6
10						−4.1	−6.6	−6.6
11							−.8	−.9
12							1.8	1.5
13							2.7	2.3
14							3.0	2.3
15							3.0	1.9
16								1.4
17								.9
18								.5
19								.4
20								.7
Total Credibility	51.9%	65.5%	71.1%	73.5%	74.7%	72.1%	76.9%	78.0%

EXHIBIT 4

SUM OF CREDIBILITIES OF THE INDIVIDUAL YEARS OF DATA

Number of Years of Data Used	Male Drivers	Female Drivers	Example For Contrast
1	16.8%	15.7%	51.9%
2	28.7	27.1	65.5
3	37.6	35.7	71.1
4	44.4	42.3	73.5
5	49.9	47.7	74.7
10	66.3	63.7	72.1
15	74.2	71.4	76.9
20	79.0	76.0	78.0
30	84.0	81.1	81.3
40	86.8	84.7	83.2
50	87.8	87.1	85.0
60	89.1	88.5	86.7

EXHIBIT 5

SQUARED ERRORS*

Number of Years of Data Used	Male Drivers	Female Drivers	Example For Contrast
0**	.0840	.0434	.0800
1	.0816	.0423	.0585
2	.0800	.0416	.0538
3	.0787	.0410	.0524
4	.0778	.0405	.0520
5	.0770	.0402	.0519
6	.0764	.0399	.0519
7	.0759	.0397	.0519
8	.0755	.0395	.0519
9	.0751	.0393	.0519
10	.0748	.0392	.0518
15	.0738	.0387	.0514
20	.0732	.0385	.0514
30	.0727	.0383	.0513
40	.0725	.0382	.0512
50	.0724	.0381	.0511
60	.0723	.0380	.0511

* Expected squared error between the observation and prediction, where the prediction employs the least squares credibilities.

** Relying solely on the overall mean, the expected squared error is the between variance plus the within variance.

to the other data that is available. The credibility depends on the value of the information contained in the overall mean, which is given the complement of credibility. This depends, in turn, on the classification information available. If, for example, data on the principal place of garaging of the car being driven or the age of the driver were available and incorporated in the analysis, then the credibility assigned to older accident data would differ.

This general method of analysis should be useful when applied to other sets of data.

REFERENCES

[1] K.W. Kwong, J. Kuan, and R.C. Peck, *Longitudinal Study of California Driver Accident Frequencies I: An Exploratory Multivariate Analysis,* Department of Motor Vehicles, State of California, Sacramento, California, 1976.

[2] H.C. Mahler, "An Example of Credibility and Shifting Risk Parameters," *PCAS* LXXVII, 1990, p. 225.

[3] E.C. Venezian, "The Distribution of Automobile Accidents . . . Are Relativities Stable over Time?" *PCAS* LXXVII, 1990, p. 309.

APPENDIX A

SAMPLE EXTRACT OF CALIFORNIA DRIVER ACCIDENT DATA*

Nine Year Total Number of Accidents	Single Year Accidents**	Male Drivers	Female Drivers
4	000010111	0	1
4	000011020	1	1
4	000012100	0	1
4	000020002	1	0
4	000020110	3	0

* Taken from Appendix I of Longitudinal Study of California Driver Accident Frequencies [1]. The various combinations of single year accidents that occurred for the 54,165 drivers in the sample are shown. The nine year total number of accidents observed ranged from 0 to 9.

** Columns 1 through 9 represent single year accident totals for years 1961, 1962, 1963, 1969, 1970, 1971, 1972, 1973, and 1974 respectively.

APPENDIX B

ANALYSIS OF VARIANCE

The total sum of squares of deviations from the grand mean for the data is given by:

$$\text{Total Sum of Squared Deviations} = \sum_i \sum_t X_{it}^2 - X^2/N,$$

where: $X = \sum_i \sum_t X_{it},$

$$N = \sum_i \sum_t 1.$$

$$\text{Within Risk Sum of Squared Deviations} = \sum_i \sum_t X_{it}^2 - \sum_i X_i^2/n_i$$

where: $X_i = \sum_t X_{it},$

$$n_i = \sum_t 1.$$

$$\text{Between Risk Sum of Squared Deviations} = \sum_i n_i\left(\frac{X_i}{n_i} - \frac{X}{N}\right)^2,$$

$$= \sum_i X_i^2/n_i - X^2/N.$$

Total Sum of Squares = Within Sum of Squares
+ Between Sum of Squares.

To get the variances, one divides each sum of squares by the product of (number of years of data − 1) × (number of drivers − 1). For the data sets examined here, the number years of data is nine. The number of male drivers is 30,293 and the number of female drivers is 23,872.

It should be noted that for the credibility analysis only the relative size of the variances is used. Therefore, as long as the sums of squared deviations are each divided by the same number, the result of the credibility analysis will be the same.

The sum of squared deviations are:

	Within	Between	Total
Male Drivers	17,555	2,815	20,370
Female Drivers	7,193	1,092	8,285

The resulting estimated variances are:

	Within Variance	Between Variance	Total Variance
Male Drivers	.0724	.0116	.0841
Female Drivers	.0377	.0057	.0434

THE COMPETITIVE MARKET EQUILIBRIUM RISK LOAD FORMULA FOR INCREASED LIMITS RATEMAKING

GLENN G. MEYERS

Abstract

Insurance Services Office, Inc. (ISO) has adopted a new risk load formula which is to become effective with 1991 advisory increased limits filings. This paper describes the underlying rationale of the new risk load formula. This formula differs from previous ISO formulas in that: (1) it is derived from competitive market assumptions; and (2) it recognizes the risks faced by the insurer in estimating the price of its product; i.e., parameter uncertainty. After the derivation of the formula, the paper will discuss considerations to be made by the insurer when using the formula. These considerations include excess-of-loss reinsurance.

1. INTRODUCTION

It is a common observation that, as the policy limit increases, the premium for a casualty insurance policy rises faster than its expected cost. This observation fits well with the economic principles of supply and demand. Policies with higher limits are perceived as being more risky. Insurers are more reluctant to sell them and insureds are more anxious to buy them. In the language of increased limits ratemaking, the additional premium to cover this increased risk is called the risk load. A risk load which rises faster than the expected cost as the policy limit increases is necessary if higher policy limits are to be made available.

In the late 1970s, Insurance Services Office, Inc. (ISO) introduced increased limits factors which were calculated with an explicit formula for the risk load.[1] In the years that followed, the formula has been refined

[1] See *Report of the Increased Limits Subcommittee: A Review of Increased Limits Ratemaking*, Insurance Services Office, Inc., 1980. The work done by ISO was based on Miccolis [1].

or revised a number of times, often with considerable debate. A major part of the debate has centered around whether or not the risk load formula met the demands of a competitive marketplace.

Effective with 1991 advisory increased limits filings, the ISO risk load formula has undergone still another change. As is the case with all advisory filings, each insurer must make its own decision to accept or modify the contents.

This paper describes the underlying rationale of the new risk load formula. This formula differs from previous ISO formulae in that: (1) it is derived from economic assumptions about the competitive market; and (2) it recognizes the risks faced by the insurer in estimating the price of its product; i.e., parameter uncertainty.

Table 1 illustrates the basic steps involved in the calculation of increased limits factors (ILFs).

TABLE 1

Policy Limit	Average Severity	ILF without Risk Load	Risk Load	ILF with Risk Load	Percent Risk Load
$ 25,000	$ 8,202	1.00	$ 281	1.00	3.42%
50,000	10,660	1.30	393	1.30	3.69
100,000	13,124	1.60	542	1.61	4.13
250,000	16,255	1.98	844	2.02	5.19
300,000	16,854	2.05	929	2.10	5.51
400,000	17,780	2.17	1,087	2.22	6.11
500,000	18,484	2.25	1,235	2.32	6.68
750,000	19,726	2.40	1,580	2.51	8.01
1,000,000	20,579	2.51	1,903	2.65	9.25
2,000,000	22,543	2.75	3,094	3.02	13.72

The policy limit refers to the maximum indemnity amount that will be paid for a single accident (or occurrence, in ISO terminology). The average severity is the average occurrence severity when subject to the given policy limit. The increased limits factor without risk load is the average severity at the increased limit divided by the average severity at the basic limit (usually $25,000). The risk load will be calculated on a per occurrence basis. The increased limits factor with risk load is the sum of the average severity and the risk load at the increased limit divided by the corresponding sum at the basic limit. Usually, loss adjustment expenses are included in the increased limits factor calculation but they will be ignored in this paper since they are not at issue here.

This paper will proceed by first developing the underlying economic rationale for the risk load formula. Next comes the description of the insurance risk. The risk load formula will then be derived, followed by considerations to be made by insurers when using the formula. These considerations include excess-of-loss reinsurance.

2. THE INCOMPLETENESS OF UTILITY THEORY

The original ISO risk load formula was based on the variance of the insured's losses. One possible economic basis for this formula comes from utility theory.[2] There are (at least) two questions addressed by utility theory that are relevant to insurance markets. The first question is: How much is a person willing to pay for insurance covering an uncertain loss? Utility theory provides an answer to this question by calculating a price so that the utility of insuring is equal to the expected utility of not insuring.[3] This mathematical exercise is usually not relevant in practice since the competitive nature of the insurance market often makes insurance available for less than the insured is willing to pay.

The second question is: How much premium must an insurer receive in order to be persuaded to take on the uncertain liability of an insurance policy? Utility theory provides an answer to this question by calculating a price so that the expected utility of not insuring the additional risk is equal to the expected utility of insuring.[4] This can be less than the price actually charged. The premium charged will be set by competitive market

[2] This is described by Bowers, Gerber, Hickman, Jones and Nesbitt [2]. For an exponential utility function and normal loss distribution, the variance-based risk load can be derived from utility theory (page 11). Exercise 1.10a shows that the variance based risk load can be used as an asymptotic approximation for any loss distribution or utility function.

[3] Bowers, *et al.*, *op. cit.*, Equation 1.3.1.

[4] Bowers, *et al.*, *op. cit.*, Equation 1.3.5.

forces or government regulation. If the chargeable premium is less than the insurer's utility calculation indicates, the insurer will not sell the policy.

Thus it can be seen that utility theory provides an upper and lower bound for the price of an insurance policy based on the risk preferences of the insurer and the insured. The actual price of the insurance policy depends upon market conditions; i.e., the supply and demand for insurance. The new risk load formula improves on the old by taking insurance market conditions into account. However, it should be noted that the supply and demand for insurance is influenced collectively by the attitudes toward risk of the insurers and the insureds.

3. THE INSURANCE MARKET

Insurance is a precondition for a great deal of economic activity. Financing for home and automobile ownership is usually contingent on obtaining insurance. Commercial enterprises can be liable for sums that could cripple the business operation. For example, employers are financially responsible for injuries to employees on the work premises and, in most instances, are required to purchase workers compensation insurance. Because insurance is a practical necessity, the demand for insurance might be assumed to be relatively inelastic. However, there is anecdotal evidence of insureds reducing, or even dropping, their coverage during periods of rapid price increases.

Property-casualty insurance companies in the United States number well over 1,000. These companies range from small specialty companies to large multiline companies. Entry to the insurance market is generally easy, and no single company has a dominant share of the market.

Some limitation to the supply of insurance comes from state regulators. They are interested in the solvency of the insurance companies under their jurisdiction and thus require the insurance company to have funds (i.e., surplus) available to pay for any excess of claim payments over collected premium. Surplus requirements usually are a function of the annual premium of the insurance company, although a more refined view holds that the required surplus should be a function of the variability of the total loss payments. James Stone [3], and R. Beard, T. Pentikainen, and E. Pesonen [4] provide some discussion of this view. Recently, the National Association of Insurance Commissioners formed a working group to develop risk-based capital and surplus requirements for insurers [5]. The total surplus provided by investors to the insurance companies (and consequently the total supply of insurance) depends upon the relative profitability of insurance and other investments.

The market structure of insurance, ease of entry into the business, and dependence of supply upon profitability indicate that the supply of insurance should be very elastic. Evidence of this proposition abounds in the several jurisdictions where regulatory price restraints have led to shortages in the voluntary insurance market.

The options available to an insurer in this environment are limited. The insurer has limited control over the prices of its products, because they are determined either by the competition or by government price regulation. The insurer *can* establish goals on how much insurance to write (within limits prescribed by state regulators). A multiline insurer can establish goals on how much insurance to write in each of several lines of insurance.

We summarize the above discussion by making the following assumptions. Admittedly, these assumptions may be somewhat stronger than the above discussion justifies, but it is believed that they are reasonable in light of the goal of deriving a workable risk load formula.[5]

1. The insurance market is competitive and efficient. The risk load cannot be influenced by the actions of a single insurer; i.e., insurers are price-takers, not price-makers.
2. The demand for each line/limit combination is known and fixed. That is, in deciding how much insurance to purchase, people and firms do not consider the cost of insurance.
3. Each insurer can decide how much insurance to write in each line of business and policy limit.
4. Each insurer is an efficient manager of its insurance portfolio. For the purpose of this paper, this means that each insurer will write the line/limit combinations in such a way as to maximize

[5] This paper has not addressed a large segment of financial theory which has been applied to the pricing of insurance policies, and which may have some bearing on the validity of these assumptions. A discussion of these issues is beyond the scope of this paper, but some issues are addressed separately by the author [6].

its total risk load subject to a constraint on the variance of its insurance portfolio.

5. The result of all insurers competing for business, as described above, will be an equilibrium characterized by the supply of insurance equaling the demand for insurance for each line/limit combination.

The fifth assumption requires additional discussion. This assumption should be viewed as an operational one. It was made to provide a useful tool to insurers. One can seriously question if insurance prices have ever been in equilibrium in recent history. If they do reach equilibrium, it is at best short-lived. The underwriting cycle is often presented as evidence of instability in insurance prices.

4. THE VARIABILITY OF INSURER LOSSES

We shall use the collective risk model with parameter uncertainty to describe the variability of insurer losses for a given line and policy limit. This model is described by the following algorithm.

1. Select χ at random from a distribution with mean 1 and variance c.
2. Select the occurrence count, K, at random from a distribution with mean $\chi \cdot n$ and variance $\chi \cdot n \cdot (1 + d)$.
3. Select α at random from a distribution with mean 1 and variance a.
4. Select occurrences, $Z_1, Z_2, \ldots, Z_K$, at random from a distribution with mean $\alpha \cdot \mu$ and variance $\alpha^2 \cdot \sigma^2$.
5. The total loss is given by: $X = \sum_{j=1}^{K} Z_j$

Actuaries have long recognized that a major part of the risk to insurers is that of estimating the cost of the insurance product. The technical term for this estimate of risk is parameter uncertainty. The random variables χ and α are introduced to model parameter uncertainty for the occurrence count and the occurrence severity distribution, respectively.

The expected occurrence count, n, will be used to quantify exposure. It will be very important to specify how the variance of the insurer-loss depends on exposure. Consider the case of a single unit of exposure. If there is no parameter uncertainty, we set

$$\mathrm{Var}[K] = 1 + d. \tag{4.1}$$

If we move to n independent units of exposure, we have

$$\mathrm{Var}[K] = n \cdot (1 + d), \tag{4.2}$$

and[6]

$$\mathrm{Var}[X] = n \cdot (\sigma^2 + \mu^2 \cdot (1 + d)). \tag{4.3}$$

It is important to note that the variance is a linear function of exposure when there is no parameter uncertainty.

When parameter uncertainty is introduced, the variance of the total loss is given by[7]

$$n \cdot u + n^2 \cdot v, \tag{4.4}$$

where

$$u = (\mu^2 \cdot (1 + d) + \sigma^2) \cdot (1 + a), \tag{4.5}$$

and

$$v = \mu^2 \cdot (a + c + a \cdot c). \tag{4.6}$$

When there is parameter uncertainty, the variance is a quadratic function of exposure. In practice, the values of a and c are relatively small and thus u is noticeably larger than v. For a small exposure; i.e., small n, parameter uncertainty is barely noticeable. However, as the exposure increases, parameter uncertainty becomes increasingly important.

[6] A special case of Equation 4.4 when $a = c = 0$.

[7] Demonstrated in Appendix A.

In order to incorporate Assumption 4 of Section 3, one must calculate the variance of the entire insurance portfolio. Use subscripts ranging from 1 to m to identify the parameters (e.g., n_i, c_i) of the various line/limit combinations. Different values of the subscript may denote completely different lines of insurance, such as commercial auto or products liability, or different policy limits within the same line. The parameters associated with the occurrence count distribution or the parameter uncertainty will be the same for each policy limit within a line of insurance. The occurrence severity distribution will be adjusted for each policy limit.

The variance for the entire portfolio of insurance is given by

$$\operatorname{Var}\left[\sum_{i=1}^{m} X_i\right] = \sum_{i=1}^{m} \sum_{j=1}^{m} \operatorname{Cov}[X_i, X_j]. \tag{4.7}$$

There are three cases to consider in the evaluation of $\operatorname{Cov}[X_i,X_j]$.

Case 1. $i = j$

In this case, $\operatorname{Cov}[X_i,X_j] = \operatorname{Var}[X_i]$.

Case 2. $i \neq j$, but the increased limits table of i is the same as that of j.

In this case, X_i and X_j will have the same underlying occurrence severity distribution, and the uncertainty random variables χ and α will be the same for X_i and X_j. However, X_i and X_j will be conditionally independent given χ and α.

Case 3. $i \neq j$ and the increased limits table for i is different from that of j.

In this case, assume that X_i and X_j are completely independent. Thus, $\operatorname{Cov}[X_i,X_j] = 0$.

The expressions for the covariance become:

$$\mathrm{Cov}[X_i,X_j] = n_i \cdot u_i + n_i^2 \cdot v_{ii}, \tag{4.8}$$

for Case 1; and

$$\mathrm{Cov}[X_i,X_j] = n_i \cdot n_j \cdot v_{ij}, \tag{4.9}$$

for Cases 2 and 3 ($v_{ij} = 0$ for Case 3).

The exact expressions for $\{u_i\}$ and $\{v_{ij}\}$ are given in Appendix A. Suffice it to say in the main text that they are similar to u and v in Equations 4.5 and 4.6.

At this point, it becomes more efficient to express our results in matrix notation. Set the column vector $\mathbf{U} = \{u_i\}$ and the matrix $\mathbf{V} = \{v_{ij}\}$. Also set the column vector $\mathbf{n} = \{n_i\}$. We then have:

$$\mathrm{Var}\left[\sum_{i=1}^{m} \sum_{j=1}^{m} X_{ij}\right] = \sum_{i=1}^{m} \sum_{j=1}^{m} \mathrm{Cov}[X_i,X_j] = \mathbf{n}^T \cdot \mathbf{U} + \mathbf{n}^T \cdot \mathbf{V} \cdot \mathbf{n}$$

Note that if there is no parameter uncertainty (i.e., $a = c = 0$), then $\mathbf{V} = \mathbf{0}$ but $\mathbf{U} \neq \mathbf{0}$. For this reason we say that $\mathbf{U}$ quantifies the process risk and $\mathbf{V}$ quantifies the parameter risk.

5. THE COMPETITIVE MARKET EQUILIBRIUM RISK LOAD FORMULA

Let the column vector $\mathbf{R} = \{r_i\}$ be the risk load per expected occurrence. As stated in the assumptions, the insurer attempts to maximize its total risk load, denoted by $\mathbf{n}^T \cdot \mathbf{R}$ in matrix notation, subject to the constraint that the variance of its total insurance portfolio cannot exceed a preset amount, A^2. The variance constraint is a function of the size (or surplus) of the insurer and of various other risks (such as investment risk) faced by the insurer. Since the market is competitive, the insurer cannot control $\mathbf{R}$, but it can control $\mathbf{n}$, the amount it insures in each line/limit combination. Mathematically, the problem the insurer faces can be expressed as follows.[8]

[8] The problem posed here is similar to that posed by R. E. Brubaker [7].

Choose **n** to maximize

$$\mathbf{n}^{\mathrm{T}} \cdot \mathbf{R}$$

subject to the constraint that[9]

$$\mathbf{n}^{\mathrm{T}} \cdot \mathbf{U} + \mathbf{n}^{\mathrm{T}} \cdot \mathbf{V} \cdot \mathbf{n} = A^2.$$

It is shown in Appendix B that **n** satisfies the equation[10]

$$\mathbf{n} = \frac{1}{2} \cdot \mathbf{V}^{-1}\left(\frac{\mathbf{R}}{\lambda} - \mathbf{U}\right) \tag{5.1}$$

where $$\lambda = \sqrt{\frac{\mathbf{R}^{\mathrm{T}} \cdot \mathbf{V}^{-1} \cdot \mathbf{R}}{4 \cdot A^2 + \mathbf{U}^{\mathrm{T}} \cdot \mathbf{V}^{-1} \cdot \mathbf{U}}}\,. \tag{5.2}$$

It would be useful to consider some simple examples at this point. Let's consider an insurer who writes four independent lines of insurance with parameters d and a set equal to zero. The remaining parameters are given in the first three columns of Table 2 below. The vector **u** is calculated using Equation 4.5. The matrix **V** is a diagonal matrix with the diagonal elements calculated by Equation 4.6. The variance constraint, A^2, was set equal to 10^{14} indicating that the insurer has sufficient surplus to cover a loss portfolio with a standard deviation of \$10,000,000. Using Equation 5.2[11] we obtain $\lambda = 1.952 \times 10^{-8}$.

Using the given risk loads for each line, in the column headed by r, one can then use Equation 5.1 to calculate the exposure, **n**, for each line to maximize the total risk load obtained by the insurer.

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[9] Philip E. Heckman in his paper "Some Unifying Remarks on Risk Load" (submitted for publication) has derived an alternative formulation which produces the same result. His formulation has the insurer minimizing the variance, $\mathbf{n}^{\mathrm{T}} \cdot \mathbf{U} + \mathbf{n}^{\mathrm{T}} \cdot \mathbf{V} \cdot \mathbf{n}$, is subject to the constraint that $\mathbf{n}^{\mathrm{T}} \cdot \mathbf{R} = \mathbf{P}$.

[10] Note that the matrix **V** may not have an inverse. If this is the case, interpret $\mathbf{x} = \mathbf{V}^{-1} \cdot \mathbf{y}$ as one of the many solutions of $\mathbf{y} = \mathbf{V} \cdot \mathbf{x}$. This case is treated rigorously in Appendix B.

[11] For a diagonal matrix, **V**,

$$\mathbf{x}^{\mathrm{T}} \cdot \mathbf{V}^{-1} \cdot \mathbf{x} = \sum_{i=1}^{m} x_i^2/v_{ii}.$$

TABLE 2

μ	σ	c	u	diag $\mathbf{V}$	r	n
10,000	30,000	0.010	1.000×10^9	1.000×10^6	250.00	5,904
20,000	100,000	0.010	1.040×10^{10}	4.000×10^6	500.00	1,902
10,000	30,000	0.030	1.000×10^9	3.000×10^6	250.00	1,968
20,000	100,000	0.030	1.040×10^{10}	1.200×10^7	500.00	634

Suppose, for the sake of discussion, that all insurers are identical to the one described by this example. If the total exposure demanded by all insureds was proportional to the exposure provided by the insurer described by Table 2, the market would be in equilibrium. However, if the total exposure demanded by all insureds was the same for each line of insurance, the market would not be in equilibrium. There would be a surplus of the first line, and a shortage of the last line.

Consider, instead, the case where our insurer is given the risk loads described by Table 3. All other conditions described above are the same. Using Equation 5.2, we obtain $\lambda = 2.017 \times 10^{-8}$. Equation 5.1 then gives the exposures needed to maximize the total risk load. If all insurers are identical and the total exposure demanded by all insureds was equal for each line of insurance, the market would be in equilibrium.

TABLE 3

μ	σ	c	u	diag $\mathbf{V}$	r	n
10,000	30,000	0.010	1.000×10^9	1.000×10^6	90.28	1,738
20,000	100,000	0.010	1.040×10^{10}	4.000×10^6	490.25	1,738
10,000	30,000	0.030	1.000×10^9	3.000×10^6	230.50	1,738
20,000	100,000	0.030	1.040×10^{10}	1.200×10^7	1051.13	1,738

The above examples illustrate the question to be addressed: *What risk load will result in market equilibrium?*

Assume that insurers 1, 2, . . . , g are seeking to maximize their total risk load by employing the strategy indicated by Equations 5.1 and 5.2. Assume further that **U** and **V** are the same for all insurers, but the j^{th} insurer has its own vector $\mathbf{n}(j)$ and its own λ_j. Also make the normative assumptions that: (1) all insurers are participating in all lines; and (2) the risk loads are the same for all insurers. (We will relax these two assumptions later.) We want to find the vector **R** that exists when the market is in equilibrium. Under equilibrium, the total insurance demanded must equal total insurance supplied which is given by:

$$\sum_{j=1}^{g} \mathbf{n}(j) = \frac{1}{2} \cdot \sum_{j=1}^{g} \mathbf{V}^{-1}\left(\frac{\mathbf{R}}{\lambda_j} - \mathbf{U}\right)$$

$$= \frac{1}{2} \cdot \mathbf{V}^{-1} \cdot \mathbf{R} \sum_{j=1}^{g} \frac{1}{\lambda_j} - \frac{g}{2} \cdot \mathbf{V}^{-1} \cdot \mathbf{U}.$$

Define

$$\bar{\lambda} \equiv \frac{g}{\sum_{j=1}^{g} \frac{1}{\lambda_j}} \tag{5.3}$$

and

$$\bar{\mathbf{n}} \equiv \frac{\sum_{j=1}^{g} \mathbf{n}(j)}{g} . \tag{5.4}$$

We then have that

$$\bar{\mathbf{n}} = \frac{1}{2} \cdot \mathbf{V}^{-1}\left(\frac{\mathbf{R}}{\bar{\lambda}} - \mathbf{U}\right) . \tag{5.5}$$

Solving for $\mathbf{R}$ yields

$$\mathbf{R} = \bar{\lambda} \cdot (\mathbf{U} + 2 \cdot \mathbf{V} \cdot \bar{\mathbf{n}}). \tag{5.6}$$

Some discussion about $\bar{\lambda}$ is in order. $\bar{\lambda}$ is the result, through Equations 5.2 and 5.3, of the variance constraints of all the insurance companies. While the variance constraints may provide a general description of how insurance companies operate, they are not sufficiently explicit to use Equations 5.2 and 5.3. However, it is possible to express $\bar{\lambda}$ in more concrete terms. Multiplying both sides of Equation 5.6 by $\bar{\mathbf{n}}^T$ yields:

$$\bar{\lambda} = \frac{\bar{\mathbf{n}}^T \cdot \mathbf{R}}{\bar{\mathbf{n}}^T \cdot (\mathbf{U} + 2 \cdot \mathbf{V} \cdot \bar{\mathbf{n}})} \equiv \frac{\text{Average Total Risk Load}}{\bar{\mathbf{n}}^T \cdot (\mathbf{U} + 2 \cdot \mathbf{V} \cdot \bar{\mathbf{n}})}. \tag{5.7}$$

The average total risk load can be derived from external considerations such as the overall profitability of the insurance industry.

6. INDIVIDUAL INSURER PRICING DECISIONS

Equation 5.6 was derived as a description of insurance market pricing. This section discusses its applicability as a tool for insurers to determine the price at which they will offer insurance.

Recall that Equation 5.6 was derived by making certain normative economic assumptions, namely that: (1) all insurers are participating in all lines/limits; and (2) the risk loads are the same for all insurers. One can argue that these assumptions are appropriate in the long run when the less efficient companies have been weeded out. Large multiline insurers are generally regarded as more efficient users of capital. Also, it is the total price of the product that is subject to competitive pressures. The marketplace might allow an insurer with an expense advantage to charge a greater risk load. But, in the long run, the insurers with an expense advantage should dominate the market.

As sensible as these normative assumptions may seem, they do not describe today's insurance market. Small specialty insurers are common and often successful. Direct writers consistently have held an expense advantage over the agency companies. While direct writers are growing, agency companies are concentrating on niches where they can provide superior service. To use this risk load formula as a pricing tool in today's marketplace, one should investigate what happens when the normative assumptions are relaxed.

First, relax the assumption that all insurers are participating in all lines/limits. Let $\mathbf{n}(j)$ be the exposure vector for the j^{th} company and let $\mathbf{I}_j$ be a diagonal matrix with the i^{th} diagonal element equal to 1 or 0 depending on whether or not the j^{th} company writes insurance in the i^{th} line/limit. As in the derivation of Equation 5.6, the total insurance demanded must equal the total insurance supplied which is given by

$$\sum_{j=1}^{g} \mathbf{n}(j) \cdot \mathbf{I}_j = \frac{1}{2} \cdot \sum_{j=1}^{g} \mathbf{V}^{-1} \left(\frac{\mathbf{R}}{\lambda_j} - \mathbf{U} \right) \cdot \mathbf{I}_j. \tag{6.1}$$

The effect of the $\mathbf{I}_j$ is to eliminate the j^{th} company's contribution to the line/limits it does not insure. Multiplying both sides of this equation by $\mathbf{V}$ and reordering some terms yields

$$\mathbf{R} \cdot \left(\sum_{j=1}^{g} \frac{\mathbf{I}_j}{\lambda_j} \right) = \mathbf{U} \cdot \left(\sum_{j=1}^{g} \mathbf{I}_j \right) + 2 \cdot \mathbf{V} \cdot \left(\sum_{j=1}^{g} \mathbf{n}(j) \cdot \mathbf{I}_j \right). \tag{6.2}$$

Let:

$\ell(i)$ be the set of insurers who write line/limit i,

g_i = number of insurers who write line/limit i,

$$\bar{\lambda}_i = \frac{g_i}{\sum_{j \in \ell(i)} \frac{1}{\lambda_j}},$$

$$\bar{\mathbf{n}}_i = \frac{\sum_{j=1}^{g} \mathbf{n}(j) \cdot \mathbf{I}_j}{g_i}.$$

Then the i^{th} component of Equation 6.2 can be written in the form

$$\mathbf{R}_i = \bar{\lambda}_i \cdot (\mathbf{U}_i + 2 \cdot (\mathbf{V} \cdot \bar{\mathbf{n}}_i)_i) \tag{6.3}$$

which resembles Equation 5.6, except that $\overline{\lambda}_i$ and $\overline{\mathbf{n}}$ can be different for each line i. $\overline{\mathbf{n}}_i$ can be interpreted as the average exposure vector over all companies that write line/limit i. The risk load multiplier, $\overline{\lambda}_i$, can be interpreted as the average λ_j over all companies who write line/limit i. In effect, this means that the risk load multiplier is strongly influenced by competitors.

We now relax the assumption that the risk load for each insurer will be the same for each line/limit. Let $\mathbf{R}(j)$ be the risk load vector for the j^{th} company. Setting the total insurance demanded equal to the total insurance supplied yields

$$\sum_{j=1}^{g} \mathbf{n}(j) \cdot \mathbf{I}_j = \frac{1}{2} \cdot \sum_{j=1}^{g} \mathbf{V}^{-1} \left(\frac{\mathbf{R}(j)}{\lambda_j} - \mathbf{U} \right) \cdot \mathbf{I}_j. \tag{6.4}$$

Multiplying both sides of this equation by $\mathbf{V}$ and reordering some terms yields

$$\sum_{j=1}^{g} \frac{\mathbf{R}(j) \cdot \mathbf{I}_j}{\lambda_j} = \mathbf{U} \cdot \left(\sum_{j=1}^{g} \mathbf{I}_j \right) + 2 \cdot \mathbf{V} \cdot \left(\sum_{j=1}^{g} \mathbf{n}(j) \cdot \mathbf{I}_j \right). \tag{6.5}$$

Since one cannot move the risk load vector outside the summation sign, a risk load equation with the form of Equation 5.6 or 6.3 is not possible. It is possible, however, for the risk load equation to be applicable to a segment of the line/limit's business. Consider, for example, the case when direct writers have an expense advantage and can command a higher profit. They will write as much insurance as is appropriate (perhaps governed by their variance constraints and Equations 5.1 and 5.2). Those insureds that remain will purchase their policies from agency companies. In effect, the line of insurance is segmented into two separate markets. One segment is serviced by the direct writers and the other segment is serviced by the agency companies. There may, or may not, be a qualitative difference between the two segments.

To summarize, the normative risk load formula given by Equation 5.6 may not be appropriate in all cases because of line specialization and/or segmentation. However, using Equation 5.6 with a risk load multiplier, $\overline{\lambda}$, that can vary by line of insurance may provide a usable risk load formula. The choice of $\bar{\lambda}$ will be influenced by competitive considerations. We will refer to Equation 5.6 as the Competitive Market Equilibrium (CME) risk load.

To date (mid-1991), ISO has filed the CME risk load for Commercial Auto, Premises/Operations General Liability, Products/Completed Operations, and Medical Malpractice. The same risk load multiplier is used for Commercial Auto Liability, Premises/Operations Liability, and Products/Completed Operations Liability. A different risk load multiplier is used for Medical Malpractice. The rationale for this is that, largely, the same companies compete for business in the first three lines but a different set of companies compete for business in the last line. It is likely that many insurers will be selecting their own risk load multipliers for each line of insurance.

7. AN ILLUSTRATIVE EXAMPLE

The risk loads in Table 1 were calculated by the CME formula. This section describes the calculations. Additional mathematical details are given in Appendix C. Since this paper was written to illustrate the concepts in the simplest way possible, the example shown below will not be identical to what ISO actually does in its advisory filings, but instead it will be a simpler analog.[12]

ISO publishes 19 separate increased limits tables for its standard commercial liability lines: three for Premises/Operations; three for Products/Completed Operations; and 13 for Commercial Auto. If ISO were to publish 10 increased limits factors for each table, there would be 190 separate line/limit combinations. At first glance it would appear that one has to work with a 190 × 190 matrix, **V**. But, as shown below, that is unnecessary.

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[12] There are two simplifications. The first is that this example uses a two-parameter Pareto rather than a five-parameter truncated Pareto. The second is that this example uses a simpler block structure in the matrix, **V**, than is used in ISO filings. The more complicated block structure is necessary because ISO estimates the occurrence severity distribution with countrywide data grouped by increased limits table within line, but does its basic limits ratemaking on statewide (countrywide for Products/Completed Operations) data grouped by line.

If the increased limits ratemaking is done independently by table, $v_{ij} = 0$ when i and j represent different tables. If the subscripts for each table are entered consecutively, the matrix **V** has a block diagonal structure. This block diagonal structure of **V** makes possible a useful simplification. This is best illustrated by way of example. Suppose there are two lines of insurance, each with two policy limits. Equation 5.6 would give:

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \bar{\lambda} \cdot \left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} + 2 \cdot \begin{bmatrix} v_{11} & v_{12} & 0 & 0 \\ v_{21} & v_{22} & 0 & 0 \\ 0 & 0 & v_{33} & v_{34} \\ 0 & 0 & v_{43} & v_{44} \end{bmatrix} \cdot \begin{bmatrix} \bar{n}_1 \\ \bar{n}_2 \\ \bar{n}_3 \\ \bar{n}_4 \end{bmatrix} \right) .$$

This equation produces the same results as:

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \bar{\lambda} \cdot \left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + 2 \cdot \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} \cdot \begin{bmatrix} \bar{n}_1 \\ \bar{n}_2 \end{bmatrix} \right) ,$$

and

$$\begin{bmatrix} r_3 \\ r_4 \end{bmatrix} = \bar{\lambda} \cdot \left(\begin{bmatrix} u_3 \\ u_4 \end{bmatrix} + 2 \cdot \begin{bmatrix} v_{33} & v_{34} \\ v_{43} & v_{44} \end{bmatrix} \cdot \begin{bmatrix} \bar{n}_3 \\ \bar{n}_4 \end{bmatrix} \right) .$$

This example demonstrates how, once $\bar{\lambda}$ is determined, the risk load equation can be applied to a single line of insurance without a detailed consideration of the other lines. The example given below illustrates how the formula works for a single line of insurance, but should be viewed in the above multiline context.

To construct an increased limits table with risk loads, one needs the following information:

1. The occurrence severity distribution, with uncertainty parameter a. (In our example, we use a Pareto distribution with cumulative distribution function:

$$S(z) = 1 - \left(\frac{b}{z + b}\right)^{q},$$

with $b = 5{,}000$ and $q = 1.1$. For the uncertainty parameter we use $a = .001$.)

2. The parameters d and c of the occurrence count distribution. (Recall that c is used to quantify parameter uncertainty in the count distribution. As illustrative values, we use $d = 0$ and $c = .02$.)

3. The exposure vector, $\bar{\mathbf{n}}$. (In practice, this can be estimated by first dividing the expected number of annual occurrences for a line by the number of insurers writing this line and using an all-industry policy limits distribution to distribute the expected claim count count to policy limit. The $\bar{\mathbf{n}}$ used is in Table 4 below.)

4. The risk load multiplier, $\bar{\lambda}$. (In this example, we used $\bar{\lambda} = 2 \times 10^{-7}$. In practice, this will be selected by individual insurers.)

The occurrence severity and count distributions are used to assemble the vector $\mathbf{U}$ and the matrix $\mathbf{V}$. The details of the calculations are provided in Appendix C. The risk load is then calculated using Equation 5.6 with the process risk vector defined as $\bar{\lambda} \cdot \mathbf{U}$, and the parameter risk vector defined as $\bar{\lambda} \cdot 2 \cdot \mathbf{V} \cdot \bar{\mathbf{n}}$. The results are in Table 4.

TABLE 4

Policy Limit	Average Severity	ILF without Risk Load	Process Risk	Parameter Risk	ILF with Risk Load	Percent Risk Load	$\bar{n}$
$ 25,000	$ 8,202	1.00	$ 28	$253	1.00	3.42%	2
50,000	10,660	1.30	64	330	1.30	3.69	2
100,000	13,124	1.60	135	407	1.61	4.13	10
250,000	16,255	1.98	339	505	2.02	5.19	2
300,000	16,854	2.05	404	524	2.10	5.51	24
400,000	17,780	2.17	533	553	2.22	6.11	2
500,000	18,484	2.25	659	575	2.32	6.68	70
750,000	19,726	2.40	965	615	2.51	8.01	8
1,000,000	20,579	2.51	1,262	641	2.65	9.25	70
2,000,000	22,543	2.75	2,391	703	3.02	13.72	10

8. THE RISK LOAD FOR EXCESS-OF-LOSS REINSURANCE

The conventional method of calculating increased limits factors for excess-of-loss reinsurance has been to subtract the ground-up increased limits factor for the retention point from the increased limits factor for the policy limit. For example, this method of calculating the increased limits factor for the layer between $500,000 and $1,000,000, using Table 4, yields the following:

TABLE 5

LAYERED INCREASED LIMIT FACTOR CALCULATION BY SUBTRACTION METHOD

Policy Limit	Average Severity	ILF without Risk Load	Process Risk	Parameter Risk	ILF with Risk Load	Percent Risk Load
$ 25,000	$ 8,202	1.00	$ 28	$253	1.00	3.42%
500,000	18,484	2.25	659	575	2.32	6.68
1,000,000	20,579	2.51	1,262	641	2.65	9.25
Layer $500,000 to $1,000,000	$ 2,096	0.26	$ 603	$ 66	0.33	31.92%

This method of calculating increased limits factors has the property that the price of a policy where the loss is shared between primary insurer and excess-of-loss reinsurer is the same as the price of a policy where the entire loss is retained by the primary insurer. From an economic point of view, it seems unlikely that the insurance market would supply both these options at the same price. There are two countervailing influences on the price which must be balanced. The first is the additional expense involved in reinsurance, and the second is the sharing of risk. Excess-of-loss reinsurance contracts are common because there is a sizable market segment for which the economic value of risk sharing is greater than the additional expense of reinsurance. The subtraction method of calculating increased limits factors for excess layers does not change layer prices to reflect the economic value of risk sharing when risks are so shared.[13]

The CME risk load applies for excess layers as well as for ground-up coverages. The formula presented in Appendix C has the lower and upper limits of the layer as input. Table 6 gives the result for the layer from $500,000 to $1,000,000.

TABLE 6

LAYERED INCREASED LIMIT FACTOR CALCULATION USING CME RISK LOAD

Policy Limit	Average Severity	ILF without Risk Load	Process Risk	Parameter Risk	ILF with Risk Load	Percent Risk Load	$\bar{n}'$	$\bar{n}$
$ 25,000	$ 8,202	1.00	$ 28	$253	1.00	3.42%	2	2
500,000	18,484	2.25	659	575	2.32	6.68	70	90
1,000,000	20,579	2.51	1,262	641	2.65	9.25	70	50
Layer $500,000 to $1,000,000	$ 2,096	0.26	$ 183	$ 66	0.28	11.90%	0	20

[13] The subtraction method is usually subject to judgmental revision. The author has found that most knowledgeable reinsurance actuaries will use the subtraction method on increased limits factors without the risk load (which is appropriate), and judgmentally add in their own risk load.

There are two observations that should be made about the CME risk load formula and layering. First, the total process risk load is reduced by layering, but the total parameter risk load remains the same. This is proved by Appendix D.[14] This reduction in the process risk load provides a quantification of the economic value of risk sharing. In the example above, the total process risk load is reduced from $1,262 to $842 (= 659 + 183). The final increased limits factor depends upon the total charge for reinsurance. Table 7 shows the increased limits factors after reinsurance for a variety of reinsurance expense charges for our example. If the reinsurance expense charge is less than $420 per expected occurrence (our unit of exposure), the increased limits factor with reinsurance is less than the increased limits factor without reinsurance, and thus it is more economical to reinsure.

TABLE 7

INCREASED LIMITS FACTORS WITH EXCESS REINSURANCE
PRIMARY LIMIT—$500,000

Policy Limit	Average Severity	Process Risk	Parameter Risk	Reinsurance Expense Charge	ILF with Reinsurance
$ 25,000	$ 8,202	$ 28	$253	$ 0	1.00
1,000,000	20,579	842	641	0	2.60
1,000,000	20,579	842	641	140	2.62
1,000,000	20,579	842	641	280	2.63
1,000,000	20,579	842	641	420	2.65
1,000,000	20,579	842	641	560	2.67

This example makes the very important point that the actuary should be aware of his company's reinsurance strategy when setting prices for increased limits.

The second observation has to do with the estimation of $\bar{\mathbf{n}}$. At first glance, it would seem necessary that the distribution of policy limits takes into account all excess-of-loss reinsurance arrangements. For ex-

[14] The result for process risk was originally demonstrated by Miccolis [1].

ample, in Table 6, the 70 units of exposure with a $1,000,000 policy limit could really consist of 50 units with no reinsurance, and 20 units with a primary insurer retention of $500,000 and an excess reinsurance policy covering the layer from $500,000 to $1,000,000. It is demonstrated in Appendix D that the CME risk load will be the same if we: (1) ignore excess reinsurance of the primary insurer; and (2) assume there is no reinsurance exposure in the excess limits. This is illustrated in the final two columns of Table 6.

There is one additional point to be discussed about layering: consistency. Consistency refers to the property that the price of a layer of constant width should not increase, as the initial attachment point increases. For example, the losses in the $250,000 excess of $750,000 layer will be no higher than the losses in the $250,000 excess of $500,000 layer. The consistency property states that the premium for the first layer should be no higher than the premium for the second layer. Since a loss in a higher layer is always less than or equal to a loss in a lower layer of equal width, it has been felt that increased limits factors should be consistent.

"Consistency tests" have historically been applied to increased limits factors using the subtraction method of calculating increased limits factors for layers. The justification for this practice only addresses losses. When consistency tests using the subtraction method have been applied to increased limits factors with risk loads, the consistency test would occasionally fail, and judgmental modifications to the increased limits factors were made.[15]

It is shown in Appendix E that the CME risk load will always produce consistent increased limits premiums.

[15] A discussion of the use of consistency tests is given by Rosenberg [8].

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APPENDIX A

DERIVATION OF VARIANCE FORMULAS

1. Unconditional variance of the occurrence count, K:

$$\begin{aligned} \text{Var}[K] &= \text{E}_\chi[\text{Var}[K|\chi]] + \text{Var}_\chi[\text{E}[K|\chi]] \\ &= \text{E}_\chi[\chi \cdot n \cdot (1 + d)] + \text{Var}_\chi[\chi \cdot n] \\ &= n \cdot (1 + d) + n^2 \cdot c \end{aligned} \tag{A.1}$$

2. Unconditional variance of the total loss, X, with parameter uncertainty for the occurrence count but without parameter uncertainty for severity:

$$\begin{aligned} \text{Var}[X] &= \text{E}_K[\text{Var}[X|K]] + \text{Var}_K[\text{E}[X|K]] \\ &= \text{E}_K[K \cdot \sigma^2] + \text{Var}_K[K \cdot \mu] \\ &= n \cdot \sigma^2 + n \cdot \mu^2 \cdot (1 + d) + n^2 \cdot \mu^2 \cdot c \\ &\quad \text{(from Eq. A.1)} \end{aligned} \tag{A.2}$$

3. Unconditional Variance for the total loss, X, with parameter uncertainty:

$$\begin{aligned} \text{Var}[X] &= \text{E}_\alpha[\text{Var}[X|\alpha]] + \text{Var}_\alpha[\text{E}[X|\alpha]] \\ &= \text{E}_\alpha[\alpha^2 \cdot (n \cdot \sigma^2 + n \cdot \mu^2 \cdot (1 + d) + n^2 \cdot \mu^2 \cdot c)] \\ &\quad + \text{Var}_\alpha[n \cdot \mu \cdot \alpha] \quad \text{(from Eq. A.2)} \\ &= n \cdot (\mu^2 \cdot (1 + d) + \sigma^2) \cdot (1 + a) + \\ &\quad n^2 \cdot \mu^2 \cdot (a + c + a \cdot c) \\ &= u \cdot n + v \cdot n^2 \end{aligned} \tag{A.3}$$

For the remainder of this appendix, replace Step 4 of the description of the collective risk model, in Section 4, with the following statement:

4′. Multiply the scale parameter of the occurrence severity distribution by α and select $Z_1, Z_2, \ldots, Z_K$ at random from the distribution.

This technical modification is necessary to remove the effect of parameter uncertainty on the policy limit. Otherwise it is equivalent to the original Step 4.

4. $\mathrm{Cov}[X_i,X_j]$ with parameter uncertainty:

$$\mathrm{Cov}[X_i,X_j] = \mathrm{E}_{\alpha,\chi}[\mathrm{Cov}[X_i,X_j|\alpha,\chi]] + \mathrm{Cov}_{\alpha,\chi}[\mathrm{E}[X_i|\alpha,\chi],\ \mathrm{E}[X_j|\alpha,\chi]] \qquad \text{(A.4)}$$

We now evaluate the first term of Equation A.4.

For Cases 2 and 3 ($i \neq j$), $\mathrm{Cov}[X_i,X_j|\alpha,\chi] = 0$, and so $\mathrm{E}_{\alpha,\chi}[\mathrm{Cov}[X_i,X_j|\alpha,\chi]] = 0$.

For Case 1 ($i = j$):

$$\begin{aligned}
\mathrm{E}_{\alpha,\chi}[\mathrm{Cov}[X_i,X_j|\alpha,\chi]] &= \mathrm{E}_{\alpha,\chi}[\mathrm{Var}[X_i|\alpha,\chi]] \\
&= \mathrm{E}_{\alpha,\chi}[\mathrm{E}_K[\mathrm{Var}[X_i|K,\alpha,\chi] + \mathrm{Var}_K[\mathrm{E}[X_i|K,\alpha,\chi]] \\
&= \mathrm{E}_{\alpha,\chi}[\mathrm{E}_K[K \cdot \mathrm{Var}[Z_i|\alpha,\chi]] + \mathrm{E}[Z_i|\alpha]^2 \cdot \mathrm{Var}_K[K|\chi]] \\
&= \mathrm{E}_{\alpha,\chi}[\chi \cdot n_i \cdot \mathrm{Var}[Z_i|\alpha] + \mathrm{E}[Z_i|\alpha]^2 \cdot \chi \cdot n_i \cdot (1 + d)] \\
&= n_i \cdot (\mathrm{E}_\alpha[\mathrm{Var}[Z_i|\alpha]] + \mathrm{E}_\alpha[\mathrm{E}[Z_i|\alpha]^2] \cdot (1 + d)) \\
&= n_i \cdot (\mathrm{E}_\alpha[\mathrm{E}[Z_i^2|\alpha]] + \mathrm{E}_\alpha[\mathrm{E}[Z_i|\alpha]^2] \cdot d) && \text{(A.5)} \\
&= n_i \cdot u_i && \text{(A.6)}
\end{aligned}$$

We now evaluate the second term of Equation A.4.

$$\begin{aligned}
&\mathrm{Cov}_{\alpha,\chi}[\mathrm{E}[X_i|\alpha,\chi],\mathrm{E}[X_j|\alpha,\chi]] \\
&= \mathrm{E}_{\alpha,\chi}[\mathrm{E}[X_i|\alpha,\chi] \cdot \mathrm{E}[X_j|\alpha,\chi] - \\
&\quad \mathrm{E}_{\alpha,\chi}[\mathrm{E}[X_i|\alpha,\chi]] \cdot \mathrm{E}_{\alpha,\chi}[\mathrm{E}[X_j|\alpha,\chi]] \\
&= \mathrm{E}_{\alpha,\chi}[\chi \cdot n_i \cdot \mathrm{E}[Z_i|\alpha] \cdot \chi \cdot n_j \cdot \mathrm{E}[Z_j|\alpha]] - \\
&\quad \mathrm{E}_{\alpha,\chi}[\chi \cdot n_i \cdot \mathrm{E}[Z_i|\alpha]] \cdot \mathrm{E}_{\alpha,\chi}[\chi \cdot n_j \cdot \mathrm{E}[Z_j|\alpha]] \\
&= n_i \cdot n_j \cdot ((1 + c) \cdot \\
&\quad \mathrm{E}_\alpha[\mathrm{E}[Z_i|\alpha] \cdot \mathrm{E}[Z_j|\alpha]] - \mathrm{E}_\alpha[\mathrm{E}[Z_i|\alpha]] \cdot \mathrm{E}_\alpha[\mathrm{E}[Z_j|\alpha]]) && \text{(A.7)} \\
&\equiv n_i \cdot n_j \cdot v_{ij} && \text{(A.8)}
\end{aligned}$$

This derivation applies for Cases 1 and 2 (i.e., the increased limits table for i and j is the same). For Case 3, $v_{ij} = 0$.

Combining Equations A.6 and A.8:

$\mathrm{Cov}[X_i,X_j] = n_i \cdot u_i + n_i^2 \cdot v_{ii}$ for Case 1; and

$\mathrm{Cov}[X_i,X_j] = n_i \cdot n_j \cdot v_{ij}$ for Cases 2 and 3 ($v_{ij} = 0$ for Case 3).

where u_i and v_{ij} are given in Equations A.5 and A.7.

APPENDIX B

DERIVATION OF THE RISK LOAD FORMULA

Our problem is to choose **n** which maximizes

$$\mathbf{n}^{\mathrm{T}} \cdot \mathbf{R}$$

subject to the constraint that

$$A^2 = \mathbf{n}^{\mathrm{T}} \cdot \mathbf{U} + \mathbf{n}^{\mathrm{T}} \cdot \mathbf{V} \cdot \mathbf{n}.$$

This can also be expressed as maximizing

$$\sum_{i=1}^{m} n_i \cdot r_i$$

subject to the constraint that

$$A^2 = \sum_{i=1}^{m} n_i \cdot u_i + \sum_{i=1}^{m} \sum_{j=1}^{m} n_i \cdot n_j \cdot v_{ij}\ .$$

To solve this, use the method of Lagrange multipliers. Set

$$L = \sum_{i=1}^{m} n_i \cdot r_i + \lambda \cdot \left(A^2 - \sum_{i=1}^{m} n_i \cdot u_i - \sum_{i=1}^{m} \sum_{j=1}^{m} n_i \cdot n_j \cdot v_{ij}\right).$$

By setting $\partial L / \partial \lambda = 0$, we see that

$$A^2 = \mathbf{n}^{\mathrm{T}} \cdot \mathbf{U} + \mathbf{n}^{\mathrm{T}} \cdot \mathbf{V} \cdot \mathbf{n}. \tag{B.1}$$

By setting $\partial L / \partial n_i = 0$ for each i, we see that the solution vector $\mathbf{n} = \{n_i\}$ satisfies the equations

$$r_i = \lambda \cdot \left(u_i + 2 \cdot \sum_{j=1}^{m} n_j \cdot v_{ij}\right) \quad \text{for each } i.$$

Expressing this in matrix notation we have that $\mathbf{n}$ is a solution to the equation

$$\mathbf{R} = \lambda \cdot (\mathbf{U} + 2 \cdot \mathbf{V} \cdot \mathbf{n}).$$

At this stage, the derivation will be easier to follow if one assumes that $\mathbf{V}$ is nonsingular. At the end of this appendix, it will be indicated how the equations must be adjusted for the case when $\mathbf{V}$ is singular.

Solving the above equation for $\mathbf{n}$ yields

$$\mathbf{n} = \frac{1}{2} \cdot \mathbf{V}^{-1} \left(\frac{\mathbf{R}}{\lambda} - \mathbf{U}\right). \tag{B.2}$$

Substituting the expression for $\mathbf{n}$ in Equation B.2 into Equation B.1 and solving for λ yields, after some algebra,

$$\lambda = \sqrt{\frac{\mathbf{R}^{\mathrm{T}} \cdot \mathbf{V}^{-1} \cdot \mathbf{R}}{4 \cdot A^2 + \mathbf{U}^{\mathrm{T}} \cdot \mathbf{V}^{-1} \cdot \mathbf{U}}} \tag{B.3}$$

$\mathbf{V}$ can be singular. Consider, for example, if the line and the limit for X_i are the same as the line and limit for X_j, then n_i and n_j could be any two numbers with the same sum. If $\mathbf{V}$ is singular, Equations B.2 and B.3 must be interpreted and derived differently. We now indicate how to do this.

First consider the case where the equation $\mathbf{V} \cdot \mathbf{r} = \mathbf{R}$ has infinitely many solutions. Let $\mathbf{r}$ be any one of the solutions. Let $\mathbf{K}$ be a matrix whose columns span the linear space of vectors, $\mathbf{x}$, such that $\mathbf{V} \cdot \mathbf{x} = 0$. Then every solution, $\mathbf{y}$, of the equation $\mathbf{V} \cdot \mathbf{y} = \mathbf{R}$ can be written in the form

$$\mathbf{y} = \mathbf{K} \cdot \mathbf{s} + \mathbf{r},$$

where $\mathbf{s}$ is a column vector with dimension equal to the number of rows of $\mathbf{K}$. Similarly, every solution, $\mathbf{z}$, of the equation $\mathbf{V} \cdot \mathbf{z} = \mathbf{U}$ can be written in the form

$$\mathbf{z} = \mathbf{K} \cdot \mathbf{t} + \mathbf{u}.$$

Let the vectors $\mathbf{r}$, $\mathbf{u}$, $\mathbf{s}$, and $\mathbf{t}$ be given. Define

$$\mathbf{V}^{-1} \cdot \mathbf{R} \equiv \mathbf{K} \cdot \mathbf{s} + \mathbf{r},$$

and

$$\mathbf{V}^{-1} \cdot \mathbf{U} \equiv \mathbf{K} \cdot \mathbf{t} + \mathbf{u}.$$

Using the alternative definitions and carefully working through the steps in deriving Equations B.2 and B.3 for the nonsingular case will yield the same identical equations.

Note that the $\mathbf{n}$ in Equation B.2 will depend on the choices of the vectors $\mathbf{r}$, $\mathbf{u}$, $\mathbf{s}$, and $\mathbf{t}$. However, the Lagrange multiplier, λ, will be the same in all cases since, for all vectors $\mathbf{s}$:

$$\begin{aligned} \mathbf{R}^T \cdot \mathbf{V}^{-1} \cdot \mathbf{R} &= \mathbf{R}^T \cdot (\mathbf{K} \cdot \mathbf{s} + \mathbf{r}) \\ &= (\mathbf{V} \cdot \mathbf{r})^T \cdot (\mathbf{K} \cdot \mathbf{s}) + \mathbf{R}^T \cdot \mathbf{r} \\ &= \mathbf{r}^T \cdot (\mathbf{V} \cdot \mathbf{K}) \cdot \mathbf{s} + \mathbf{R}^T \cdot \mathbf{r} \\ &= \mathbf{R}^T \cdot \mathbf{r}. \end{aligned}$$

Thus, $\mathbf{R}^T \cdot \mathbf{V}^{-1} \cdot \mathbf{R}$ is independent of the particular solution, $\mathbf{y}$, of $\mathbf{V} \cdot \mathbf{y} = \mathbf{R}$. A similar statement can be made about $\mathbf{U}^T \cdot \mathbf{V}^{-1} \cdot \mathbf{U}$. The uniqueness of λ follows from Equation B.3.

If $\mathbf{V}$ is singular, it is possible for there to be no solution to the equation $\mathbf{V} \cdot \mathbf{y} = \mathbf{R}$; i.e., the system of equations is inconsistent. Consider, for example, the case where the line and limit for X_i are the same as the line and limit for X_j, but $r_i \neq r_j$. In this case, it is clear what to do. If $r_i > r_j$, set $n_j = 0$, since one gets more premium in line i than in line j, with the same amount of risk. In general, it will be possible to eliminate various line/limits without reducing $\mathbf{n}^T \cdot \mathbf{R}$ and obtain a consistent set of equations.

Eliminating line/limits can also be appropriate even when $\mathbf{V}$ is nonsingular. It is possible for a (generally small) company to solve Equation B.2 and have negative exposures indicated for certain line/limits. Since an insurer sells insurance rather than buys insurance, this cannot happen. The solution is to eliminate line/limits when negative exposures are indicated.

An actual procedure for eliminating line/limits will not be specified here. However, it is clear that optimal solutions satisfying $n_i \geq 0$ for all i, and $A^2 = \mathbf{n}^T \cdot \mathbf{U} + \mathbf{n}^T \cdot \mathbf{V} \cdot \mathbf{n}$ will always exist (a continuous function will always have a maximum on a closed set). The method of Lagrange multipliers determines the optimal solution on the subset of line/limits, i, for which $n_i > 0$.

APPENDIX C

FORMULAE UNDERLYING THE ILLUSTRATIVE EXAMPLE

Let $f(\alpha)$ be the probability density function for α. From Equations A.6 and A.8, it follows that:

$$u_i = E_\alpha[E[Z_i^2|\alpha]] + d \cdot E_\alpha[E[Z_i|\alpha]^2]$$

$$= \int_0^\infty E[Z_i^2|\alpha]f(\alpha)d\alpha + d \cdot \int_0^\infty [E[Z_i|\alpha]^2]f(\alpha)d\alpha;$$

$$v_{ij} = (1 + c) \cdot E_\alpha[E[Z_i|\alpha] \cdot E[Z_j|\alpha]] - E_\alpha[E[Z_i|\alpha]] \cdot E_\alpha[E[Z_j|\alpha]]$$

$$= (1 + c) \cdot \int_0^\infty E[Z_i|\alpha] \cdot E[Z_j|\alpha]f(\alpha)d\alpha -$$

$$\int_0^\infty E[Z_i|\alpha]f(\alpha)d\alpha \cdot \int_0^\infty E[Z_j|\alpha]f(\alpha)d\alpha.$$

Let:

$$f(\alpha) = \frac{1}{\sqrt{2\pi a}} \cdot e^{-\frac{(\alpha - 1)^2}{2a}}.$$

The Hermite-Gauss three-point quadrature formula gives the approximations:[16]

$$u_i \approx \frac{1}{6} \cdot E[Z_i^2|\alpha_1] + \frac{2}{3} \cdot E[Z_i^2|\alpha_2] + \frac{1}{6} \cdot E[Z_i^2|\alpha_3]$$

$$+ d \cdot \left(\frac{1}{6} \cdot E[Z_i|\alpha_1]^2 + \frac{2}{3} \cdot E[Z_i|\alpha_2]^2 + \frac{1}{6} \cdot E[Z_i|\alpha_3]^2\right).$$

[16] The standard change of variables was used. See Ralston [9].

$$v_{ij} \approx (1 + c) \cdot \left(\frac{1}{6} \cdot \mathrm{E}[Z_i|\alpha_1] \cdot \mathrm{E}[Z_j|\alpha_1] + \frac{2}{3} \cdot \mathrm{E}[Z_i|\alpha_2] \cdot \mathrm{E}[Z_j|\alpha_2]\right.$$

$$\left. + \frac{1}{6} \cdot \mathrm{E}[Z_i|\alpha_3] \cdot \mathrm{E}[Z_j|\alpha_3]\right)$$

$$-\left(\frac{1}{6} \cdot \mathrm{E}[Z_i|\alpha_1] + \frac{2}{3} \cdot \mathrm{E}[Z_i|\alpha_2] + \frac{1}{6} \cdot \mathrm{E}[Z_j|\alpha_3]\right)$$

$$\cdot \left(\frac{1}{6} \cdot \mathrm{E}[Z_j|\alpha_1] + \frac{2}{3} \cdot \mathrm{E}[Z_j|\alpha_2] + \frac{1}{6} \cdot \mathrm{E}[Z_j|\alpha_3]\right),$$

where:

$$\alpha_1 = 1 - 1.224745 \cdot \sqrt{2a}; \quad \alpha_2 = 1; \quad \alpha_3 = 1 + 1.224745 \cdot \sqrt{2a}.$$

The occurrence severity distribution used in this paper is the Pareto distribution with c.d.f.:

$$S(z|\alpha) = 1 - \left(\frac{\alpha \cdot b}{z + \alpha \cdot b}\right)^q$$

Let LL_i and UL_i be the respective lower and upper policy limits corresponding to i. Then:[17]

$$\mathrm{E}[Z_i|\alpha] = \int_{LL_i}^{UL_i} (z - LL_i) \cdot \mathrm{d}S(z|\alpha) + (UL_i - LL_i) \cdot (1 - S(UL_i|\alpha))$$

$$= \int_{LL_i}^{UL_i} (1 - S(z|\alpha))\mathrm{d}\alpha$$

$$= \frac{(\alpha b)^q}{q - 1}\left(\frac{1}{(LL_i + \alpha b)^{q-1}} - \frac{1}{(UL_i + \alpha b)^{q-1}}\right), \quad q \neq 1,$$

and

[17] The proofs of lemmas E.1 and E.2 in Appendix E may provide some help in evaluating these integrals.

$$E[Z_i^2|\alpha] = \int_{LL_i}^{UL_i} (z - LL_i)^2 \cdot dS(z|\alpha) + (UL_i - LL_i)^2 \cdot (1 - S(UL_i|\alpha))$$

$$= \frac{2(\alpha b)^q}{q-1}\left[\frac{1}{q-2}\left(\frac{1}{(LL_i + \alpha b)^{q-2}} - \frac{1}{(UL_i + \alpha b)^{q-2}}\right) - \frac{UL_i - LL_i}{(UL_i + \alpha b)^{q-1}}\right], q \neq 1,2.$$

APPENDIX D

DEMONSTRATION OF RISK REDUCTION BY LAYERING

From Equation 5.6:

$$r_i = \overline{\lambda} \cdot (u_i + 2 \cdot (\mathbf{V} \cdot \overline{\mathbf{n}})_i).$$

Without loss of generality, it can be assumed that Z_1, Z_2, and Z_3 represent the occurrence severities in the layers from L to H, L to M, and M to H, respectively. To demonstrate risk reduction by layering, it must be shown that $r_1 > r_2 + r_3$. This will be done by showing that $u_1 > u_2 + u_3$ and that $(\mathbf{V} \cdot \overline{\mathbf{n}})_1 = (\mathbf{V} \cdot \overline{\mathbf{n}})_2 + (\mathbf{V} \cdot \overline{\mathbf{n}})_3$.

$$\begin{aligned}
u_1 &= \mathrm{E}_\alpha[\mathrm{E}[Z_1^2|\alpha] + \mathrm{E}[Z_1|\alpha]^2 \cdot d] \\
&= \mathrm{E}_\alpha[\mathrm{E}[(Z_2 + Z_3)^2|\alpha] + \mathrm{E}[Z_2 + Z_3|\alpha]^2 \cdot d] \\
&= \mathrm{E}_\alpha\Big(\mathrm{E}[Z_1^2|\alpha] + 2 \cdot \mathrm{E}[Z_2 \cdot Z_3|\alpha] + \mathrm{E}[Z_3^2|\alpha] \\
&\quad + (\mathrm{E}[Z_2|\alpha]^2 + 2 \cdot \mathrm{E}[Z_2|\alpha] \cdot \mathrm{E}[Z_3|\alpha] + \mathrm{E}[Z_3|\alpha]^2) \cdot d\Big) \\
&> \mathrm{E}_\alpha\Big(\mathrm{E}[Z_2^2|\alpha] + \mathrm{E}[Z_3^2|\alpha] + (\mathrm{E}[Z_2|\alpha]^2 + \mathrm{E}[Z_3|\alpha]^2) \cdot d\Big) \\
&= u_2 + u_3. \qquad \text{(D.1)}
\end{aligned}$$

$$\begin{aligned}
v_{1j} &= (1 + c) \cdot \mathrm{E}_\alpha[\mathrm{E}[Z_1|\alpha] \cdot \mathrm{E}[Z_j|\alpha]] \\
&\quad - \mathrm{E}_\alpha[\mathrm{E}[Z_1|\alpha]] \cdot \mathrm{E}_\alpha[\mathrm{E}[Z_j|\alpha]] \\
&= (1 + c) \cdot \mathrm{E}_\alpha[\mathrm{E}[Z_2 + Z_3|\alpha] \cdot \mathrm{E}[Z_j|\alpha]] \\
&\quad - \mathrm{E}_\alpha[\mathrm{E}[Z_2 + Z_3|\alpha]] \cdot \mathrm{E}_\alpha[\mathrm{E}[Z_j|\alpha]] \\
&= v_{2j} + v_{3j}. \qquad \text{(D.2)}
\end{aligned}$$

It then follows that:

$$(\mathbf{V} \cdot \overline{\mathbf{n}})_1 = (\mathbf{V} \cdot \overline{\mathbf{n}})_2 + (\mathbf{V} \cdot \overline{\mathbf{n}})_3. \qquad \text{(D.3)}$$

Equation D.1 shows that the total process risk is reduced by layering, while Equation D.3 shows that the total parameter risk remains constant with layering.

Equation D.2 makes possible a simplification in the tabulation of $\overline{\mathbf{n}}$. Let p represent the average exposure for those of whom the upper layer is covered by one company, and the lower layer is covered by another company. Define $\overline{\mathbf{n}}'$ so that $\overline{\mathbf{n}}_1' = \overline{\mathbf{n}}_1 + p$, $\overline{\mathbf{n}}_2' = \overline{\mathbf{n}}_2 - p$ and $\overline{\mathbf{n}}_3' = \overline{\mathbf{n}}_3 - p$. Since $v_{ij} = v_{ji}$, it follows from Equation D.2 that $\mathbf{V} \cdot \overline{\mathbf{n}} = \mathbf{V} \cdot \overline{\mathbf{n}}'$. In effect, this means that one can ignore the effect of excess reinsurance when estimating $\overline{\mathbf{n}}$, since the risk load will be the same as it would be if excess reinsurance were taken into account.

APPENDIX E

DEMONSTRATION OF CONSISTENCY

Let Z be a random variable with cumulative distribution function, $S(z)$. Let the layer moment functions be given by:

$$M_1(a,h) = \int_a^{a+h} (z - a) \cdot dS(z) + h \cdot (1 - S(a + h));$$

$$M_2(a,h) = \int_a^{a+h} (z - a)^2 \cdot dS(z) + h^2 \cdot (1 - S(a + h)).$$

Lemma E.1. $M_1(a,h)$ is a decreasing function of a.

Integration by parts yields:

$$M_1(a,h) = -(z - a) \cdot (1 - S(z))\Big|_a^{a+h} + \int_a^{a+h} (1 - S(z)) \cdot dz + h \cdot (1 - S(a + h))$$

$$= -h \cdot (1 - S(a + h)) + \int_a^{a+h} (1 - S(z)) \cdot dz + h \cdot (1 - S(a + h))$$

$$= \int_a^{a+h} (1 - S(z)) \cdot dz.$$

$$\frac{dM_1(a,h)}{da} = (1 - S(a + h)) - (1 - S(a))$$

$$= S(a) - S(a + h) < 0.$$

Thus, $M_1(a,h)$ is a decreasing function of a.

Lemma E.2. $M_2(a,h)$ is a decreasing function of a.

Integration by parts yields:

$$M_2(a,h) = -(z-a)^2 \cdot (1 - S(z))\big|_a^{a+h} + \int_a^{a+h} (1 - S(z)) \cdot \mathrm{d}z + h^2 \cdot (1 - S(a+h))$$

$$= -h^2 \cdot (1 - S(a+h)) + 2 \cdot \int_a^{a+h} (z-a) \cdot (1 - S(z)) \cdot \mathrm{d}z + h^2 \cdot (1 - S(a+h))$$

$$= 2 \cdot \int_a^{a+h} (z-a) \cdot (1 - S(z)) \cdot \mathrm{d}z$$

$$= 2 \cdot \int_a^{a+h} z \cdot (1 - S(z)) \cdot \mathrm{d}z - 2 \cdot a \cdot M_1(a,h)$$

$$\frac{\mathrm{d}M_2(a,h)}{\mathrm{d}a} = 2 \cdot (a+h) \cdot (1 - S(a+h)) - 2 \cdot a \cdot (1 - S(a)) - 2 \cdot a \cdot \frac{\mathrm{d}M_1(a,h)}{\mathrm{d}a} - 2 \cdot M_1(a,h)$$

$$= 2 \cdot h \cdot (1 - S(a+h)) - 2 \cdot \int_a^{a+h} (1 - S(z)) \cdot \mathrm{d}z$$

$$= 2 \cdot \int_a^{a+h} (S(z) - S(a+h)) \cdot \mathrm{d}z$$

< 0 since S is an increasing function.

Thus, $M_2(a,h)$ is a decreasing function of a.

We now turn to establishing the consistency of: (1) the expected loss; (2) the process risk; and (3) the parameter risk. Without loss of generality, one can assume that Z_1 is the occurrence severity for the layer from a_1 to $a_1 + h$ and Z_2 is the occurrence severity from the layer from a_2 to $a_2 + h$ with $a_1 < a_2$.

1. The consistency of the expected loss:

$$\mathrm{E}[Z_1] = \mathrm{E}_\alpha[M_1(a_1,h|\alpha)] > \mathrm{E}_\alpha[M_1(a_2,h|\alpha)] = \mathrm{E}[Z_2].$$

2. The consistency of process risk:

$$\begin{aligned} u_1 &= \mathrm{E}_\alpha[M_2(a_1,h|\alpha) + d \cdot M_1(a_1,h|\alpha)^2] \\ &> \mathrm{E}_\alpha[M_2(a_2,h|\alpha) + d \cdot M_1(a_2,h|\alpha)^2] \\ &= u_2 \end{aligned}$$

3. The consistency of parameter risk:

$$\begin{aligned} v_{1j} &= (1 + c) \cdot \mathrm{E}_\alpha[M_1(a_1,h|\alpha) \cdot \mathrm{E}[Z_j|\alpha]] \\ &\quad - \mathrm{E}_\alpha[M_1(a_1,h|\alpha)] \cdot \mathrm{E}_\alpha[\mathrm{E}[Z_j|\alpha]] \\ &> (1 + c) \cdot \mathrm{E}_\alpha[M_1(a_2,h|\alpha) \cdot \mathrm{E}[Z_j|\alpha]] \\ &\quad - \mathrm{E}_\alpha[M_1(a_2,h|\alpha)] \cdot \mathrm{E}_\alpha[\mathrm{E}[Z_j|\alpha]] \\ &= v_{2j}. \end{aligned}$$

It then follows that:

$$(\mathbf{V} \cdot \bar{\mathbf{n}})_1 > (\mathbf{V} \cdot \bar{\mathbf{n}})_2.$$

DISCUSSION OF PAPER PUBLISHED IN VOLUME LXXVI

EXPOSURE BASES REVISITED

AMY S. BOUSKA

DISCUSSION BY CHRISTOPHER DIAMANTOUKOS

If ye continue in my word, then are ye my disciples indeed; and ye shall know the truth, and the truth shall make you free.

John VIII, 31–32

I am grateful to David E. A. Sanders for reviewing my thoughts and helping me appreciate the scope and the difficulty of the problem in trying to measure exposures.

The focus of this paper is on some fairly difficult and sophisticated concepts that may not become obvious to the reader upon an initial reading. One key concept is that of "true exposure" as presented by Bouska, and its proxy provided by an exposure base. There are no complicated formulae presented in the paper, yet Bouska's observations that the subject is indeed complicated and intricate were realized during the discussions at the CAS Annual Meeting concurrent session where this paper was presented.

1. TRUE EXPOSURE

What is "exposure," or, better yet, what is "true exposure?" The paper offers little explanation beyond the "exposure to loss" definition first presented by Dorweiler [1].

The true exposure is a complex and changing characteristic of a risk. Intuitively, the true exposure of a risk can be viewed as the summation (integration) over the term of the policy of the random variable "insurable losses." The risk's "exposure to loss," its inherent insurable loss (risk pure premium), can change at any time during the policy period.

The "exposure" represents a measurable physical characteristic of the risk that is a dimensional translation of the expected value (mean) of the true exposure. The dimensions are dollars, which measure the mean true exposure, and whatever dimension the exposure is measured in: square feet, car-years, etc. The exposure pure premium, measured with respect to units of the exposure base, is the scalar reflecting the translation. For example, a class of risks might use miles driven as the exposure for automobile liability coverage. The exposure pure premium, $5 per 1,000 miles driven (a scalar of 5), translates the mean of the true exposure, measured in dollars, to units of the exposure base, units of 1,000 miles driven. To simplify analysis, true exposure might best be measured based on "full coverage" insurable losses, where coverage is defined by a specific combination of insured perils. Exposure pure premium estimates make use of risk characteristics, usually reflecting the entire term of the policy, such as classification and geographical location.

The term "risk" was mentioned several times in the last two paragraphs. The concept of an individual risk can be difficult to define and understand. Its definition is essential in order to measure true exposure or to count the number of units of an exposure base. The insuring of a large number of independent risks, or the "pooling of risks," is fundamental to making an insurance process work. The ideal is to pool (insure) a population of homogeneous risks. This ideal is rarely achieved, even at the classification level. Exposures measure differences in risk size. The exposures of an individual risk are clearly not independent, although rates are measured with respect to the pooled exposures of many risks.

Conceptually, for primary insurance coverages, risks can be thought of as representing the indivisible and independent entities to which coverages can be attached. Attachment results from the association of the coverage and the risk through the actual wording of the coverage form (insurance policy), particularly the insuring agreement. The association might be through the specification of an insured location, or the type of insured business activity conducted at certain premises of the insured, or the services provided by an insured at various points in time during the policy period, etc. For example, automobile liability coverage is attached to a vehicle and not to the insured.

Risks are normally independent of each other and form the basis from which true exposure is measured. Examples of familiar risk definitions include a building for fire coverage, a restaurant for premises liability coverage, and a vehicle for various forms of automobile liability and physical damage coverages. Bouska addresses the limitations on using exposures for "measuring" large risks.

An individual claim, or loss, is normally associated with one, and only one, risk. Exceptions to this include long-term exposure injuries, such as those resulting from exposure to asbestos. The injured claimant might make one "claim" resulting in payments from the several policies in force during the claimant's exposure to asbestos. In some cases there may be a single risk giving rise to the claimant's injuries over the course of several consecutive policies.

David E. A. Sanders of Eagle Star Insurance Company, London, England, has suggested a quadrilateral that integrates the concepts of risk, exposure, true exposure, and expected losses. For a policy period for a given risk, he relates the concept through a diagram:

$$\begin{array}{ccc} \text{Risk Period} & \longrightarrow & \text{Expected Losses} \\ \downarrow & & \uparrow \\ \text{Exposure Period} & \longrightarrow & \text{True Exposure} \end{array}$$

Each arrow represents a function unique to each risk and policy period. These functions transform, or map, one variable to another as indicated in the diagram. Risk period is identical to policy period. Exposure period reflects the time during the risk period that the risk is "exposed" to loss, as well as the variation in that exposure over time. The aggregation of independent risks to create an insurance process through application of the Law of Large Numbers (Central Limit Theorem) presents a major problem because these functions are different for each risk.

In order to help focus on the concept of true exposure and why it is so difficult to measure, mathematical notation will be introduced for the variables of a policy that are germane. This will also provide the groundwork for a theoretical presentation of different types of exposure bases. Let $\Pi(T)_i$ represent the insurable losses for risk i from the inception of a policy until time T. This will be termed the risk's pure premium when T equals the expiration date of the policy. It is a random variable representing the dollar value of the aggregate losses during the policy period. Of particular interest is the change in the risk's pure premium with respect to time; i.e., its derivative. It can be integrated over the term of the policy to obtain the mean of the risk's pure premium.

Note that the characteristics of the risk affecting its pure premium can change during the policy period. These characteristics can also be considered as dependent on time. Not included in this discussion are those characteristics affecting pure premium that are external to the risk, such as the judicial climate affecting liability awards. Bouska discusses risk characteristics in the section of her paper entitled "Problems: Complexity of Hazard." If it is assumed that there is a finite number n of such characteristics, and letting the n-dimensional vector $X_i(t,n) = (\chi_1(t), \chi_2(t), \ldots, \chi_n(t))$ represent their respective values at time t, an equation defining the mean of the true exposure is:

$$\Pi(T)_i = \int_0^T \int_0^\infty \nabla_t \Pi(X_i(t,n)) f(\nabla_t \Pi(X_i(t,n)), t) d(\nabla_t \Pi_i) dt. \tag{1.1}$$

$\nabla_t \Pi_i$ represents the change in the pure premium for risk i with respect to time during the policy period. It is the partial derivative, or gradient, of the risk pure premium with respect to time. It is a random variable whose density function may also change over time.

This equation sheds light on Bouska's observation that one can never know the true exposure or make measurements of its mean or moments. The digital limitations of the most extensive data gathering mechanisms can only approximate the measurements of all the variables (characteristics) that range over continua needed to completely measure the true exposure for any given risk, or even for an entire homogeneous class of risks. Examples of variables ranging over continua include time (obviously) and economic activity as reflected to varying degrees in commercial fire and general liability classifications. It is left to actuaries to use all available information and experience, including measurements of exposures and classifications, to find a "best" approximation to true exposure.

A simplifying assumption that can be made is that the risk pure premium does not change. Time intervals of coverage could be made as small as necessary to achieve any desired degree of conformity to this assumption. The assumption of a constant risk pure premium might be stronger for seasonal coverages if time intervals focus on the operations or values at greatest risk. Examples include summer months for seashore amusement parks or hurricane season where windstorm coverages are sold. Let $\tilde{f}$ represent this constant density function. The mean true exposure equation simplifies to:

$$\Pi(T)_i = \int_0^T \int_0^\infty \nabla_t \Pi(X_i(t,n)) \tilde{f}(\nabla_t \Pi(X_i(t,n))) d(\nabla_t \Pi_i) dt; \tag{1.2}$$

$$\Pi(T)_i = \int_0^T \mu[\nabla_t \Pi(X_i(t,n))] dt. \tag{1.3}$$

At any point in time, the mean pure premium gradient can be decomposed into the product of the means of its frequency (N) and severity (δ) components. If the dimensions of dollars of insurable losses, number of claims, and time are reflected properly, equation (1.3) becomes:

$$\Pi(T)_i = \int_0^T \mu[\nabla_t N(X_i(t,n))] \mu[\delta(X_i(t,n))] dt. \tag{1.4}$$

This equation represents the integration over time of the product of the change in the number of claims with the mean claim severity. Equation (1.4) should be compared with Bouska's equation (4):

> (number of exposure base units) × (expected number of losses per exposure base unit) × (expected dollars per loss) = expected losses

The integration over time has been substituted by the number of exposure base units in Bouska's equation (4). This is the result of the simplification offered by the use of exposures, discussed in the next section.

2. THE EXPOSURE SIMPLIFICATION

As mentioned earlier, the simplification to estimating the mean true exposure offered by the use of exposure bases is to determine some physically measurable risk characteristic that is directly proportional to the mean of the true exposure and use it as the exposure base. Note that the exposure base should ideally be susceptible to very accurate measurement and is not a transformation of the random variable true exposure. After classifying a risk, multiplying the exposure pure premium by the number of units of the exposure base for the policy period gives the expected pure premium for the risk. The chosen unit of the exposure base provides a numerical scaling to the values measured. It also determines the scalar value reflected in the dimensional translation between the exposure base and the true exposure.

To help explain the meaning of the simplification just presented, some assumptions will be made. First, the period of coverage will be taken to be one unit time period. Second, the characteristics of the risk are assumed to be fixed over the period of coverage. Third, the partial derivative of the risk pure premium with respect to time does not change over the period of coverage; i.e., it is a constant. Equation (1.4) then reduces to:

$$\Pi(1)_i = \mu[N(X_i)]\mu[\delta(X_i)] = \mu[\Pi(X_i)]. \tag{2.1}$$

Consider several examples of exposure bases. First, consider fire insurance, where the exposure is taken to be amount of insurance. In practice, the exposure is amount of insurance-years to take into account policy term and insured values that change significantly over time, for example, policies written on a reporting form basis. From a theoretical perspective, if full insurance to value is assumed (100% coinsurance) to be full coverage, then, at any point in time, current actual value is a better exposure base. The actual value should affect the mean severity of loss and not the frequency of loss. A normalized severity distribution of the type discussed by G. L. Head [2] could be used. The normalized severity ($\bar{\delta}$) ranges over the interval [0,1]. It represents severity as a percent of value. Multiplying by the number of units of the exposure base (ε) representing a constant actual value yields the expected severity.

By invoking the exposure simplification, the mean true exposure equation would then take the form:

$$\Pi(T)_i = \mu[N(X_i)]\mu[\bar{\delta}(X_i)]\varepsilon_i. \tag{2.2}$$

For liability coverages, increased limits factors are used to adjust base rates to reflect the expected increased severity of the higher limits purchased. Claim frequency is reflected by measuring units of the exposure base. One class of liability exposures involves a gradient of the exposure base with respect to time, such as sales per unit time. Integration will yield the total earned exposure over the policy period. Given an estimate of the mean frequency of claims per unit exposure and a constant gradient of the exposure with respect to time, the liability form of the mean true exposure equation for this first class of liability risks becomes:

$$\Pi(T)_i = \mu[\nu(X_i)]\mu[\delta(X_i)]\varepsilon_i. \tag{2.3}$$

An example of an exposure in this case might be sales in dollars over a period of one year. Note that the mean claim frequency per unit exposure is $\mu[\nu(X_i)]$.

A second class of liability exposures involves bases such as square feet. These are constant level exposures represented as annualized values. In such cases, the gradient of the exposure with respect to time is zero. An example of a normalized exposure in this second class is number of square-feet-years. The actual normalized exposure reflects the annualized exposure basis used, such as per 1,000 square-feet-years. The analysis is similar to that of fire, substituting *frequency* proportional to the exposure base instead of *severity* being proportional to the exposure base as was the case in fire. The form is identical to equation (2.3) with the added simplification that the gradient of the frequency with respect to the constant level in force exposure is constant.

Finally, for workers compensation indemnity coverage, payroll can be considered as being directly proportional to the pure premium itself. The severity of indemnity losses increases with payroll per worker while the frequency of loss increases with workers per risk. Using payroll as the exposure base also employs a gradient of payroll with respect to time. The simplified mean true exposure equation becomes:

$$\Pi(1)_i = \mu[\pi(X_i)]\varepsilon_i. \tag{2.4}$$

Note that the mean pure premium per unit of exposure is represented by $\mu[\pi(X_i)]$. It is instructive to review Section 4 from the paper in light of the development above.

3. PROBLEMS IN CHOOSING AN EXPOSURE BASE

When Bouska presented the paper, she spoke of an emerging exposure base problem in aviation liability insurance. The paper's discussion of the workers compensation exposure base problem presents a theme similar to the aviation problem. Rate inequities in aviation are perceived due to the use of revenue passenger-miles as the exposure. Many causes of losses are related to takeoffs and landings. Hence, some commuter airlines, due to increased flight frequency, are thought to generate greater "exposure to loss" than might be indicated by using the existing exposure base. In addition, the growth of such commuter airlines results in a change to the underlying population upon which aviation liability rates are based. The previous underlying population that was used to set rates had a greater proportion of "long-distance" airlines.

This brief discussion of aviation liability insurance echoes observations made when the paper was presented: the measurement of true exposure must recognize frequency *and* severity components. Some exposure bases may be more responsive to frequency than they are to severity, or vice versa. The best solution to approximating the true exposure in some cases might be to utilize more than one exposure base. Two exposure bases might be used, one for frequency and the other for severity. For example, in the case of aviation liability insurance, the number of flights may be directly related to the frequency of losses while the number of passengers per flight relates to severity. Capturing the actual distribution of passengers per flight would focus even more sharply on true exposure.

The "dividing line" between exposure bases and rating variables that Bouska speaks of may need to be "crossed" sometimes. Workers compensation may have "solved" an "exposure base question" by addressing other parts of the rating system. Perhaps some changes in classification structure might solve some of the problems for some aviation liability risks. There is precedence in automobile liability where miles driven, a potential exposure base, perhaps a theoretically superior one in some respects, is reflected in classification values. There may be more than one acceptable solution for separating risk characteristics between potential exposure bases and rating variables.

Time was introduced as a continuum over which the change in pure premium could be measured. Some other continua were mentioned earlier. One discussed in the paper is size as measured in units of the exposure base. Size is a risk characteristic and there theoretically exists a function relating size to true exposure, or at least relating size to the mean true exposure. The paper discusses how less weight is put on the manual premium as the size of a risk increases. Size in the paper is measured by the mean of the risk's pure premium, which in turn depends on the true exposure. Bouska's observation of a decreased dependency of the charged premium (reflecting true exposure) with an increase in size of risk means that the number of units of the exposure base becomes less important. Greater credibility is given to a risk's experience as its actual loss (observed pure premium) experience increases.

The calculation of pure premiums, frequencies, and severities should still be of value in analyzing the true exposure of the large risk. The mean true exposure as a function of size does not necessarily have to approach a limiting value as size increases without bound. An alternative hypothesis is that the variation in the relationship between size and the mean true exposure becomes large enough to warrant giving increased credibility to the individual risk's loss experience, and less credibility to the exposure pure premium of the classification to which the risk belongs. Use of an exposure size function, one that measures the dependence of a risk's pure premium on its exposure size, could reinforce the use of exposures in the analysis of the very large risks. It would also be of interest if other risk characteristics have greater influence on the true exposure than size.

Another candidate for a continuum gradient is the rate of interest to discount loss payments if insurable losses are measured on a present value basis. This could be of particular interest in the long-tailed lines.

One of the problems discussed in Section 6 of the paper is that of temporal mismatch. The first example presented is that of claims-made policies. The problem involves the influence of the coverage provided on the measurement and choice of an exposure base. A careful description of coverage/policy forms is essential to clearly distinguish what is meant by coverage and what are considered coverage limitations, conditions, and exclusions. For the sake of this discussion, coverage refers to the underlying perils insured against without "limitations."

The temporal mismatch of claims-made coverage is caused by reporting limitations on covered claims. Another type of limitation that could cause temporal mismatch involves policy limits and aggregates. Recently introduced claims-made tail coverages allow the reinstatement of aggregate limits. Reinstatements have their greatest impact on rates, but they also affect understanding true exposure and how to select exposure bases. True "full coverage" would provide for unlimited reinstatements; but, in today's environment, aggregate limitations have become very important. The discussion by Bouska of products liability in this same section of the paper offers the "products in use during the year" alternative exposure base. This represents another case where true exposure could become a function of coverage limitations. The amount of "products used" for which liability coverage is provided could be limited.

Yet another example of temporal mismatch is provided in building fire coverage. If a building is significantly damaged by fire, there is clearly reduced value at risk until such time as the building is repaired. But what would happen if the building is fully restored before the end of the policy period? It would appear that coverage is reinstated for the duration of the policy and that a true exposure exists reflecting the restored value rather than the original insured value. The coverage rate might also be used to reflect this possibility.

The detailed mathematical presentation earlier dealt solely with approximations to the mean true exposure. There are density functions associated with the random variables used in the development that will affect the density functions of the true exposure. The estimation of the density function of aggregate losses is another approach that recognizes such randomness in the process. The approach is complicated if it must derive an estimate for a risk with an inherent risk pure premium that changes over time.

At the end of the subsection "General Liability: Area vs. Receipts," the author reflects on the passage of time as the answer to the question of how to derive a better exposure base. I agree; it would take considerable experience on a risk by risk basis, with the collection of several exposure base candidates, to answer the question. This was not done for general liability during the recent ISO conversion process.

Size is a key consideration when determining a good exposure base, or when choosing between exposure base candidates. Experience by individual risk over time is perhaps more important than experience by size groupings for making this determination. This is particularly true if the insured population included in an accident year evaluation of a size grouping changes from one year to the next. The focus would be on the relationship of size in units of the exposure base to the true exposure (or its mean). It would take extensive experience on a risk-by-risk basis to make such a determination because of the skewed nature and high variance of the distributions of risk pure premiums. However, any attempt at an analysis by risk over time would be hampered if the importance of exposures is played down because the individual "large" risk's influence on its expected pure premium increases.

The validity of basing experience rating for large risks on class pure premiums seems questionable if large risks are excluded from the underlying population of their "class." Furthermore, the appropriateness of class pure premiums can also be called into question when the underlying risk population is changing or eroding (depopulating) over time. Such erosion may have already happened in some classifications as a result of the migration of general liability risks and class-rated fire risks to the indivisible businessowners types of policies.

The use of an inflation-sensitive exposure can make it harder to understand the relationships over time between the exposure and the true exposure, pure premiums, or claim frequencies. For example, unadjusted claim frequencies per amounts of insurance over time for fire insurance should not be used at face value. The effects of trend (i.e., exposure trend reflecting increasing property values) must be factored out to allow for a consistent time series analysis. However, one can never tell exactly how much an exposure change for an individual risk is due to inflation, which affects all risks, and how much is due to a real increase in risk size. Some exposure bases directly reflect monetary inflation (at a minimum); e.g., "current" actual value.

However, a physical, measurable exposure would vary directly with the inherent pure premium of the risk as long as the insurance environment does not change drastically. Bouska speaks to this issue indirectly in her discussion of the workers compensation exposure base problem. Using physical exposure bases can separate exposure changes due to size from those due to economic or monetary inflation. Their use focuses on physical relationships between exposure and claim frequencies or severities. Applicable inflation can still be measured and reflected separately, at a minimum, through claim severities. Actual value in fire might be considered physical and measurable, but its current value is fairly subjective and susceptible to measurement error at policy inception.

The exposure base is not a transformation of the random variable true exposure as noted at the beginning of Section 2. This causes a large difference in the variances relative to the means (coefficients of variation, or CV) between the exposure (small CV) and the aggregate losses (large CV) of an individual risk. Hence, the appropriateness of an accurately measurable exposure base should be measured against the mean risk pure premium rather than trying to consider variations in one versus variations in the other.

The effect of exposures on credibility was not addressed in the paper. In an ideal model of an insurance process, pure premiums and claim frequencies are built up from homogenous risk populations. In practice, rates are generally determined per unit of exposure, not per risk. If increases in exposure truly translate to increases in insurable losses, then exposures will affect any credibility formulation of pure premiums. This can be addressed directly through the construction of an exposure-based credibility model or by separately considering the effect of changes in size of risk as measured by the exposure base. Exposures would be substituted for risks in the first case, along with a method of reflecting correlation among exposure units at the individual risk level. The second alternative focuses on risks as the population of the probability space, not exposures, for statistical inferences.

Each exposure unit is not independent of all other units whenever a risk can be greater than one exposure unit in size. This must be recognized as distinct from effects on statistical measurements caused by scaling alone through the selection of the unit employed for the exposure base. This is not an easy task. For example, to change an exposure basis from $1,000 to $100 of sales does not change the nature of risks ($1,000 of sales) that were formerly one but are now ten exposure units. If these risks were now treated as ten independent exposure units, estimates of classification pure premiums would be more credible and their coefficients of variation would decrease. It is also worth noting the difficulty in combining experience from classes that have different exposure bases within a coverage.

Briefly turning attention to reinsurance and excess of loss contracts, David E. A. Sanders notes that the underlying primary risks are not always independent. Catastrophes do indeed create a contagion effect, geographically causing many risks to suffer claims and losses. A new dimension must be addressed for these contracts making the measurement of true exposure and the choice of an exposure base a much more difficult task.

4. ARE EXPOSURES NECESSARY?

In the final analysis of the utility of exposures, it is their convenient physical reflection of increased true exposure rather than their use in calculating premiums from classification rates that is key. Were it not for exposures, rates would be stated as the average per risk without a convenient or reliable reflection of size differences except those afforded by experience rating. All variations would need to be measured on a risk-by-risk basis or through classification characteristics (variables). The use of an objective, physical measurement of size is also appealing in that it provides a convenient way to measure changes in aggregate pure premium without ignoring changes in coverage and business mix. There may be situations where it is worthwhile to separate policy experience into intervals smaller than one year in order to reflect any significant changes in true exposure.

There will continue to be other meanings employed for the term "exposure;" e.g., the total amount at risk in a hurricane-prone area or its use in specific fire rating schedules. However, it is the support of measuring "size of risk," which is synonymous with risk pure premium, that will continue to be the focal point of actuarial discussions.

The process of "rate times exposure" also has something more important about it than appearances might suggest. Measuring the *components* is important; i.e., the exposure and the rate, and not just the end product—premium. This helps assure that the process of setting rates is supported by the application of these same rates; that the way estimates of rates are determined is well founded. Bouska causes us to consider the critical role of exposures in property and casualty insurance. It is the *exposures* that ultimately affect the aggregate premiums and losses, to which so much attention is paid.

The paper takes casualty actuaries back to their roots to discuss a subject that may have been taken for granted. Bouska's discussions of the concept of "true exposure," the case of the large risks, and the types of problems exhibited by temporal mismatch, clearly focus on today's insurance environment. The research and experimentation needed to test theories and various exposure measures are very extensive and perhaps not easily supported by the statistical data gathering mechanisms currently in place. It is with interest that I await to see how the issues Bouska has raised play out over time.

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ADDRESS TO NEW MEMBERS—NOVEMBER 11, 1991

DANIEL J. McNAMARA

To each of our new Fellows and new Associates, I extend personal congratulations and a warm welcome from all the members of the Casualty Actuarial Society.

I am pleased to share this joyous day with you. I suspect, however, if you are like most new actuaries who have preceded you, you shall quickly forget these remarks. Indeed, CAS past presidents have sometimes been likened to the deceased at an Irish wake—where all the relatives and friends are convinced that their beloved has gone on to a better life and are in a mood to celebrate the event. They need a body to legitimize the party, but nobody expects the body to say very much.

While you will not recall the substance of my remarks a few months, or years, from now, I do hope to be remembered for something—and that is that I was brief. And so I make the same promise to you that Henry the Eighth uttered to each of his six wives on their wedding day—I shall not keep you long.

On similar previous occasions, other past presidents of our Society attempted, each in their own way, to put the reality of this significant achievement in perspective. You all know what perspective is, but let me elaborate on its meaning by reading to you a letter written by a college student at the end of her first semester:

Dear Mom and Dad:

You'll be relieved to know that I was able to get out before the flames reached my room in the dormitory. And the leap from the third floor window was not nearly so terrifying as you might suppose. The doctors have assured me that at my age no permanent damage from the break in my tibia and femur are to be anticipated, and I will be walking normally within six months. The boy who works in the garage across the street from the dormitory could not have been nicer. He is letting me share his room behind the grease rack, and he turned out to be a really caring person. He has, in fact, persuaded me to have the baby even though it may get him into trouble with his parole officer.

I'm just kidding, folks. But I did flunk English, and I wanted you to see it in perspective.

Love,
Jane

I do want to share with you some thoughts on two subjects I feel strongly about, and I hope you will bear with me.

Each of you has just received professional credentials and gained an analytical power and material security not available to those outside our profession. That power, especially for our 57 new Fellows, opens up career choices in abundance. Wealth and prominence lie ahead for many of you. Along with all these rewards, however, come certain dangers. The constant emphasis and pressure to instantly solve financial problems for your employer or your clients through artful modes of analysis may tempt some of you to ignore the ethics of the practice. And that is a serious problem at a time when every profession bristles with moral dilemmas and the public trust in all professionals has declined. The eternal quest for excellence, with personal integrity (i.e., character and professional competence), is what it's all about. Each of you must zealously guard and assume personal responsibility for your actuarial integrity so that the right you have now earned to be heard and to be influential will be preserved and enhanced. At the end of your careers, I suggest nothing will be more important to you.

Trying to do the right thing is still the best business, the shrewdest politics, and the strongest medicine for ethical headaches. I urge your personal active involvement in helping our profession prudently establish and vigorously enforce professional standards of practice. Members of the Casualty Actuarial Society are now in great demand, increasingly accountable, and very visible. I submit that each of you, as the future of our profession, has a serious obligation to continue our Society's proud history in this area and not tarnish the legacy that you have received today. Use this new power well and wisely.

As you begin your professional lives, your instinct will be to focus all of your considerable energies and talents on your careers and, after these examinations, revive your social lives. In my own actuarial and legal career, I have seen many new professionals unduly preoccupied with their own career aspirations and the needs and problems of the companies or clients they serve; showing little immediate concern over their obligation to that profession and the impact of that profession on society at large. There is absolutely nothing wrong with that, but, for each of you, I suggest it should not be enough. It is equally important that you also contribute some of your precious time and talent to making things better within your profession and in the world at large. Successful completion of all of the examinations has only given you the tools. Professional maturity begins with the realization that the ends and values to which you direct your talents are yours, and yours alone, to decide. For most of you, your contributions will most likely be on a voluntary basis. But let me assure you of one thing: this will be the most rewarding part of your lives, and you will receive back far more than you contribute. Whether you volunteer for the CAS Examination Committee, or tutor minority children, or serve on a school board—you will be satisfying both the need and the obligation in all of us to, in some small way, give something back to the world for the gifts we have received.

As each of you starts out on a wonderful journey over the next 25 to 30 years, I also urge that you never be satisfied with your success or discouraged by your failures. And none of you, I assure you, will be immune from career disappointments. Looking backward on my own insurance career, the inability of ISO, under my tenure, to successfully introduce into the marketplace a punitive damage coverage exclusion or a broadly applied claims-made policy is still costing dearly, not only insurance interests, but many other segments of our society.

None of you now know where your exciting journey will end. But one thing is certain—if you don't dream great dreams; if you don't take risks; if you don't push the boundaries of actuarial science back as far as you can; if you only listen to the little voice inside you that tells you to settle for a life of comfort and material well-being; if you accept the views of others who would attempt to limit you; then you will never know your true career destiny. So I encourage each of you to use all of your talents and to reach as high up and as far out as you can. If you do, no matter what the outcome, you will live a fuller and a more productive life.

I welcome you to the world of actuaries—it is an ever-changing, challenging, and very rewarding world. You should feel very proud today to be a part of the CAS heritage. May your careers blossom and prosper in the years ahead—the opportunities are almost infinite.

PRESIDENTIAL ADDRESS—NOVEMBER 11, 1991

THE MATURATION AND DESTINY OF THE CAS

CHARLES A. BRYAN

Good afternoon. This is my opportunity as President to share with you my thoughts on our CAS.

What a great *profession* this is. For the past 22 years, the CAS has been a constant in my life. Despite numerous career moves from New York to Illinois to California to Texas and back to New York, the CAS has always provided me with interesting colleagues, basic and ongoing education, friendships, and an identity not dependent on employment situation. Many of you have told me you feel the same way.

For five years, I have been privileged to serve as an officer and have seen what makes this Society work so well. And this year as President has given me special insight into how we have matured and what our destiny will be.

How has the CAS matured? Our infancy was from 1914 to 1929. We were small—14 charter members. We were very dependent on others—the members contributed their own money for many projects and companies sponsored items like nametags at meetings.

The next step was our childhood—1929 to 1960. We started in 1928 with 150 members and ended with 600 members in 1960. By then, we had developed some facility with our own language, such as credibility, indicated rates, required surplus, and so on.

Our adolescence was from 1960 to 1990. Those of us who have reared teenage children know that these years are characterized by an effort to learn more, act independently, and idealism. During these years we helped found the American Academy of Actuaries (AAA); put in place our current examination process; began our newsletter, the *Actuarial Review*; established regional affiliates; promulgated statements of principles; published a textbook; and became accepted as the experts in loss reserving and ratemaking. The culmination of our adolescent period came with the National Association of Insurance Commissioners (NAIC) actuarial opinion requirements in the United States and the appointed actuary requirements in Canada. Many of you participated in the battle to win approval of these requirements.

We have now reached full maturity—a strong, healthy, intelligent, elite group of 1,809 people who have reached adulthood at age 77.

As I began my term last November, the 1991 Executive Council pledged that we would take two important initiatives to take advantage of our new maturity: develop a more professional office, and establish a solid link with professional actuaries outside of North America.

I am pleased to report we have accomplished these objectives. In May we moved the office to Virginia, hired our first Executive Director, and upgraded the office staff. My thanks to Bob Conger for helping me accomplish this.

Throughout 1991, we have taken a number of steps to give the CAS a global outlook. For the first time, we are establishing very strong links with the General Insurance Study Group (GISG) in England. I participated in a panel discussion in Wales at the annual study group meeting just weeks ago. I am actively exploring ways we can increase the number of contacts between general insurance actuaries outside of North America and the CAS. Peter Johnson, who is here with us today, represents the general insurance actuaries of the United Kingdom.

As we look to the future as a mature organization, we will be most successful if we have a well articulated action plan to follow. As President, I have given this a great deal of thought and I am convinced we must do the following:

(1) Become fully committed to the global viewpoint. Adopting the global viewpoint requires us to: (a) invite overseas actuaries to our meetings and seminars; (b) publicize and attend ASTIN and AFIR meetings; (c) publish and read overseas papers on general insurance, such as those produced by GISG; (d) have official CAS representation at GISG meetings; (e) work to overcome differences in terminology; (f) consider restructuring the examinations into actuarial principles, actuarial science, and nation-specific topics; (g) include in our seminars and meetings a good

dose of overseas material. All of our vice presidents will play a role in leading the way to this global viewpoint—but the President must be the catalyst.

(2) Adopt the attitude that actuaries should actively guide the insurance mechanism. The CAS, through the AAA and the Canadian Institute of Actuaries (CIA), should lead in structuring, evaluating, and controlling the insurance mechanism. Actuarial people are best equipped to be the leaders of insurance—our 10 exams give us both a technical knowledge and a broad overview of how general insurance works. But, to be the leaders we must:

- Insist that certain activities, such as loss reserving and pricing, are primarily actuarial in nature. We have made great progress here. But we must exploit our temporary advantage.
- Research and publish on key aspects of insurance, so that we push the leading edge forward. The textbook was only a first step.
- Promote and encourage fully-credentialed actuaries to obtain additional relevant designations, such as CPCU or CPA or JD, so that we can make sure actuaries are viewed as experts in all aspects of insurance.
- Honor and utilize those of our members who have achieved great success in this profession and remain contributing members of the CAS. LeRoy Simon is the perfect example of this type of individual.
- Devote the extra time and effort to play a leading role in new areas, such as risk-based capital.

We should know enough to take a position and be committed enough to pursue that position.

(3) Demand more of ourselves; then demand even more. We live well, have good reputations, are well paid, and command considerable prestige. But that privileged position will end someday. To quote Hugh Scurfield's speech as president of the British Institute of Actuaries: "What you have inherited from your fathers, earn again for yourselves, or it will not be yours." We—

> you and I—are the only people who can save our profession from the atrophy that always threatens success. You and I must do it—no one else can, no one else will!

As I complete my term as President of the CAS, I wish to thank the Executive Council for a very successful 1991. I wish to thank my wife, Jean, who has made all the difference in my life and in helping me this year. And I wish to thank each of you for your many contributions. In closing, I will paraphrase a famous old saying attributed to General Douglas MacArthur but altered to fit this occasion: "Old actuaries never die, they just develop to ultimate."

MINUTES OF THE 1991 ANNUAL MEETING

November 10–13, 1991

WYNDHAM FRANKLIN PLAZA HOTEL
PHILADELPHIA, PENNSYLVANIA

Sunday, November 10, 1991

The Board of Directors held their regular quarterly meeting from 1:00 p.m. to 5:00 p.m.

Registration was held from 4:00 p.m. to 6:30 p.m.

From 5:30 p.m. to 6:30 p.m., there was a special presentation to new Associates and their guests. This session included an introduction to standards of professional conduct and the CAS committee structure. The Executive Council members were introduced and their roles explained as well.

A general reception for all members and guests was held from 6:30 p.m. to 7:30 p.m.

Monday, November 11, 1991

Registration continued from 7:00 a.m. to 8:00 a.m.

President Charles A. Bryan opened the meeting at 8:00 a.m. with the introduction of Constance Foster, Pennsylvania Insurance Commissioner. The welcoming address was provided by Foster.

After the welcoming address, Bryan continued the meeting with the first order of business being the announcement of the election results. The new President-Elect is David P. Flynn. The new board members are Ronald E. Ferguson, Heidi E. Hutter, Gary S. Patrik, and Sheldon C. Rosenberg.

The members of the Executive Council will be Vice President-Administration, John M. Purple; Vice President-Research and Development, Allan M. Kaufman; Vice President-Admissions, Steven G. Lehmann; Vice President-Continuing Education, Irene K. Bass; and Vice President-Programs and Communication, Albert J. Beer.

Bryan thanked the outgoing board members: Walter J. Fitzgibbon, Jr., Charles A. Hachemeister, David J. Oakden, and Lee R. Steeneck, as well as outgoing Vice President, Robert F. Conger. Peter Johnson, representing the Institute of Actuaries and the Faculty of Actuaries, was introduced as a special guest.

The next order of business was the admission of members. The 63 new Associates were recognized; and Bryan presented diplomas to the 59 new Fellows as they were introduced by Michael L. Toothman, President-Elect. The names of these individuals follow.

FELLOWS

Jeffrey H. Adams	Cynthia M. Grim	Pierre A. Samson
W. Brian Barnes	Marshall J. Grossack	Jeffrey W. Schmidt
Holly L. Billings	Pierre Lepage	Karen E. Schmitt
Gavin C. Blair	Allen Lew	Jeffory C. Schwandt
Jean-Francois Blais	Peter M. Licht	Susanne Sclafane
Paul Boisvert, Jr.	Andre Loisel	Alan R. Seeley
Charles H. Boucek	Brett A. MacKinnon	Ronald J. Swanstrom
Louis-Philippe Caron	Brian E. MacMahon	Suan-Boon Tan
Li-Chuan L. Chou	Brian A. Montigney	Michael T. S. Teng
David R. Clark	Daniel M. Murphy	Karen F. Terry
Janice Z. Cutler	G. Christopher Nyce	Mary L. Turner
Edward D. Dew	Joanne M. Ottone	Melanie A. Turvill
Dale R. Edlefson	Loren V. Petersen	Anne-Marie Vanier
Bob D. Effinger, Jr.	Jill Petker	Ricardo Verges
John S. Ewert	Deborah W. Price	Patrick M. Walton
Steven R. Fallon	Ronald D. Pridgeon	Peter W. Wildman
Luc Gagnon	Boris Privman	Chad C. Wischmeyer
David B. Gelinne	Timothy P. Quinn	Bill S. Yit
Alex R. Greene	Steven C. Rominske	Richard P. Yocius

ASSOCIATES

Martha E. Ashman
Mark S. Baker
Xavier Benarosch
John D. Booth
Mark L. Brannon
Paul A. Bukowski
William E. Burns
Carol A. Cavaliere
Jessalyn Chang
Victor G. dos Santos
William F. Dove
Maribeth Ebert
Jennifer L. Ermisch
Madelyn C. Faggella
Thomas R. Fauerbach
Carole M. Ferrero
Andrea Gardner
Bradley J. Gleason
Jonathan M. Harbus
Keith D. Holler
Beth M. Hostager
Anthony Iafrate
Anthony Iuliano
Edwin G. Jordan
Stephen H. Kantor
David L. Kaufman
Trina C. Kavacky
Richard F. Kohan
Adam J. Kreuser
James W. Larkin
Michael D. Larson
William G. Main
Donald E. Manis
Heidi J. McBride
Jeffrey F. McCarty
John W. McCutcheon, Jr.
John P. Mentz
Paul M. Merlino
Stephen J. Meyer
Linda K. Miller
Antoine A. Neghaiwi
Randy S. Nordquist
Margaret M. O'Connor
Ann E. Overturf
On Cheong Poon
Stuart Powers
Gregory Riemer
Laura A. Romine
Leigh A. Saunders Oates
David M. Shepherd
Lisa N. Steenken-Dennison
Paul J. Struzzieri
Joseph W. Tasker, III
Marianne Teetsel
Georgia A. Theocharides
Edward D. Thomas
Janet A. Trafecanty
Stacy L. Trowbridge
Dale G. Vincent, Jr.
Lawrence M. Walder
Patricia K. Walker
Teresa J. Williams
Charles J. Yesker

Bryan then introduced Daniel J. McNamara, a past President of the Society, who delivered an address to the new Fellows and Associates.

Conger, Vice President-Administration, gave the Secretary/Treasurer's Report.

Vice President-Research and Development, Kaufman, gave a brief report on the new research plan.

Matthew Rodermund was introduced by Bryan to present the first Matthew Rodermund Service Award. The award was presented to Robert Foster, who was unable to be present to accept the award.

Beer, Vice President-Programs and Communication, gave a brief summary of the program and Bass, Vice President-Continuing Education, introduced the *Proceedings* papers presented at the meeting. The Woodward-Fondiller award was presented to Robert A. Bear and Kenneth J. Nemlick.

Bryan then called for reviews of previous papers from the floor. One review was presented by Lee R. Steeneck. The business session was then closed.

After a brief refreshment, there was a panel presentation entitled "The Property and Casualty Industry into the 1990s—Ready or Not." The panel was moderated by LeRoy J. Simon and consisted of Edward B. Rust, Jr., of State Farm Insurance Companies; William R. Berkley of W. R. Berkley Corporation; Richard E. Stewart of Stewart Economics, Inc.; and Robert J. Vairo of Crum and Forster, Inc. The panel presented an executive view of current events and the future direction of the insurance and reinsurance industry in the United States and abroad.

Lunch was served from 12:15 to 1:30 p.m. Bryan presented the Presidential Address and passed the gavel to Toothman. The afteroon was devoted to presentations of the new *Proceedings* papers and panel sessions.

The new *Proceedings* papers were:

1. "Commutation Pricing in the Post Tax-Reform Era"

   Authors: Vincent P. Connor
   General Reinsurance Corporation

   Richard A. Olsen
   General Reinsurance Corporation

2. A Discussion of "Exposure Bases Revisited"

   Author: Christopher Diamantoukos
   CIGNA Property and Casualty Companies

3. "An Exposure Rating Approach to Pricing Property Excess-of-Loss Reinsurance"

Author: Stephen J. Ludwig
ITT/Hartford Insurance Group

4. "The Credibility of a Single Private Passenger Driver"

Author: Howard C. Mahler
The Workers' Compensation Rating and
Inspection Bureau of Massachusetts

5. "The Competitive Market Equilibrium Risk Load Formula for Increased Limits Ratemaking"

Author: Glenn G. Meyers
Insurance Services Office, Inc.

The panel presentations covered the following topics:

1. Costing of Auto No-Fault Law Changes

This session described techniques to estimate cost changes generated by No-Fault law revisions. Topics included descriptions of No-Fault compensation systems, data sources, data limitations, methods of analysis, and necessary assumptions. Costing examples were presented, as well as a review of actual versus predicted results for the 1989 Massachusetts law change.

Panelists: Robert T. Muleski
Liberty Mutual Insurance Company

Ruy A. Cardoso
Automobile Insurers Bureau of Massachusetts

Frederick F. Cripe
Allstate Insurance Company

Joseph A. Herbers
Tillinghast/Towers Perrin

2. Risk Classification Principles

The Risk Classification Research Committee is developing a set of principles for risk classification to supersede those published by the American Academy of Actuaries 10 years ago. The panelists introduced a Discussion Draft and solicited member input to be forwarded to the committee for discussion and consideration before issuing a formal Exposure Draft.

Panelists: Cecily A. Gallagher
Chairman-Committee on Risk Classification
Tillinghast/Towers Perrin

Peter T. Bothwell
Aetna Life & Casualty

Frank J. Karlinski, III
William M. Mercer, Inc.

3. The Impact of Medical Cost on Casualty Coverages—Recent Developments

This panel presented a review of recent developments in health insurance and the impact of these on casualty coverages, emphasizing the medical cost issues. The presentation focused on automobile and workers' compensation coverages.

Moderator: Allan M. Kaufman
Milliman & Robertson, Inc.

Panelists: William Wallace
State Farm Mutual Automobile Insurance Company

David Appel
Milliman & Robertson, Inc.

James M. Walter
Tufts Associated Health Plans, Inc.

4. Actuarial Board For Counseling and Discipline (ABCD)

In August the American Academy of Actuaries created the ABCD as an autonomous body, charged with investigating complaints lodged against actuaries and counseling them as to the applicable codes of conduct or standards of practice. The participants in this session discussed this new body and the proposed revisions in the code of conduct, which will be common for all professional organizations.

Panelists: David P. Flynn
Crum & Forster Corporation

Jerome A. Scheibl
Wausau Insurance Companies

5. NAIC Financial Database

This session provided information about how the NAIC office is organized and what services it performs for state regulators. Emphasis was placed on its financial data base and a discussion of current analytical tools used to identify potential problem companies.

Panelists: James Rose
National Association of Insurance Commissioners

Terry Boyer
National Association of Insurance Commissioners

6. Qualification Standards And Continuing Education Requirements

This panel discussed current minimum qualification standards for members who perform publicly required actuarial functions, current continuing education requirements, and methods of enforcement.

Panelists: Mavis A. Walters
Insurance Services Office

Irene K. Bass
William M. Mercer, Inc.

Michael A. Walters
Tillinghast/Towers Perrin

7. An Insurer's/Self-Insurer's Response to the Current Workers' Compensation Crisis

This session presented an overview of the financial results for workers' compensation in the commercial insurance market. A review of the growth of alternative risk financing was then discussed, quantifying the amount of premiums that are written through retrospectively rated insurance programs, high deductible insurance programs, self-insurance trusts, captives, and other forms of self-insurance. A review of a cost savings model applicable to high deductible or excess insurance products was also discussed.

Moderator: John P. Yonkunas
Tillinghast/Towers Perrin

Panelists: Ronald C. Retterrath
Wausau Insurance Companies

Dorothy A. Zelenko
General Reinsurance Corporation

8. Questions And Answers with the CAS Board of Directors

   Members of the Board of Directors discussed the status of issues of current interest. CAS members asked questions and expressed concerns and opinions.

   Moderator: Michael L. Toothman (President-Elect)
   Arthur Andersen & Co.

   Current Board Members:

   Charles A. Hachemeister (Elected 1988)
   F & G Re, Inc.

   Stephen W. Philbrick (Elected 1989)
   Tillinghast/Towers Perrin

   Robert A. Anker (Elected 1990)
   Lincoln National Corporation

The officers held a reception for the new Fellows and their guests from 5:30 p.m. to 6:30 p.m. The President's Reception for all members and guests was held from 6:30 p.m. to 7:30 p.m.

Tuesday, November 12, 1991

Mike Toothman introduced the general session entitled "Workers' Compensation Systems: Have They, Are They, and Will They Live Up To Expectations?" The moderator of the panel was Jerome A. Scheibl of Wausau Insurance Companies; panelists were Donald Elisburg, Esq., Center to Protect Workers' Rights; Therese A. Maloney, Liberty Mutual Insurance Company; Dr. John A. Gardner, Workers' Compensation Research Institute; and James W. Mackie, CPCU, ARM, ACME Markets, Inc. The panel discussed the various workers' compensation systems that have developed over the years, how well they have stood the test of time, how well they are meeting the demand of our current society, and what the outlook might be for the future.

A brief refreshment period was followed by continuation of the concurrent sessions and additional *Proceedings* papers.

The afternoon was reserved for committee meetings. A buffet dinner was held at the Franklin Institute Science Museum from 6:00 p.m. to 10:00 p.m.

Wednesday, November 13, 1991

Concurrent sessions continued from 8:00 a.m. to 9:30 a.m., including the presentation of one additional *Proceedings* paper.

Following refreshments, Toothman introduced Dr. John Stoessinger as the featured guest speaker. Dr. Stoessinger described the recent developments in the Soviet Union, Eastern Europe, and the Middle East. He related this information to the opportunities available to actuaries.

The meeting was adjourned at 11:15 a.m. after closing remarks.

November 1991 Attendees

In attendance, as indicated by registration records, were 275 Fellows; 133 Associates; and 80 guests, subscribers, and students. The list of their names follows.

FELLOWS

Adams, Jeff
Amundson, Rick
Angell, Charlie
Anker, Bob
Bailey, Vicky
Baily, Robert
Barnes, W. Brian
Barrow, Betty
Bashline, Donald
Bass, Irene
Baum, Edward
Bealer, Don
Bear, Bob
Beer, Al
Bell, Linda
Bill, Richard
Biller, Jim
Billings, Holly
Blair, Gavin
Blais, Jean-Francois
Blanchard, Ralph
Boisvert, Paul, Jr.
Boone, J. Parker
Bornhuetter, Ron
Braithwaite, Paul
Brathwaite, Malcolm
Bryan, Chuck
Cantin, Claudette
Cardoso, Ruy
Carroll, Lynn
Carter, Edward
Caulfield, Mike
Childs, Diana
Chou, Li-Chuan
Christiansen, Stephan
Christie, James K.

FELLOWS

Chuck, Allan
Cis, Mark
Clark, David
Cofield, Joseph
Conger, Bob
Connell, Eugene
Cook, Charles
Corr, Frank
Cripe, Fred
Crowe, Patrick
Curran, Kathleen
Currie, Ross
Curry, Alan
Cutler, Janice
Daino, Bob
Darby, Rob
DeFalco, Tom
Dempster, Howard
Dew, Ted
Diamantoukos, Chris
Dolan, Michael
Dornfeld, Jim
Duda, Diane
Duffy, Brian
Duffy, Tim
Dukatz, Judy
Easlon, Ken
Easton, Richard
Edlefson, Dale
Effinger, Bob
Egnasko, Gary
Egnasko, Valere
Eland, Doug
Ernst, Rich
Ewert, John
Faber, Jim
Fagan, Janet
Fallon, Steven
Fisher, Russell
Fisher, Wayne
Fitzgerald, Beth
Fitzgibbon, Walter
Flaherty, Dan
Flynn, David
Francis, Louise
Fusco, Michael
Gallagher, Cecily
Gannon, Alice
Gardner, Bob
Garland, Roberta
Gelinne, David
Girard, Greg
Goldberg, Steve
Golz, James
Graham, Tim
Grant, Gary
Graves, Greg
Graves, Nancy
Greene, Alex
Grim, Cindy
Grossack, Marshall
Hachemeister, Charlie
Hall, Allen
Hall, Jim
Hartman, Dave
Henry, Dennis
Hermes, Thomas
Hill, Anthony
Hosford, Mary
Hunt, Fred
Hutter, Heidi
Irvan, Bob
Johe, Dick
Johnson, Eric
Johnson, Marvin
Johnson, Warren
Karlinski, Frank
Kasner, Kenneth
Kaufman, Allan
Keatinge, Clive
Kelly, Anne
Kist, Fred
Kleinman, Joel
Kline, Chuck
Koupf, Gary
Krause, Gus
Kreps, Rodney
Kudera, Andy
Lehmann, Steve
Lepage, Pierre
Levin, Joseph
Lew, Allen
Licht, Peter
Livingston, Roy
Lo, Richard
Lockwood, Janet
Loisel, Andre
Lotkowski, Edward
Lowe, Robert
Ludwig, Steve
MacGinnitie, Jim
MacKinnon, Brett
MacMahon, Brian
Maher, Christopher
Mahler, Howard
Mathewson, Stuart
McCarter, Michael
McClure, John
McNamara, Dan
Meyer, Robert
Meyers, Glenn

FELLOWS

Miccolis, Robert
Miller, Dave
Miller, David
Miller, Mary Frances
Miller, Phil
Miller, Susan
Miller, William
Mohl, F. James
Montigney, Brian
Moore, Brian
Morison, George
Moylan, Tom
Mulder, Toni
Muleski, Bob
Munt, Donna S.
Murad, John
Murphy, Dan
Murphy, Francis X.
Murrin, Thomas E.
Myers, Thomas
Neis, Al
Nemlick, Kenneth
Newville, Ben
Nikstad, James
Niles, C., Jr.
Nyce, Christopher
Oakden, Dave
Ottone, Joanne
Patrik, Gary
Pearl, Marc
Pelletier, Bernie
Peraine, Tony
Petker, Jill
Philbrick, Stephen
Phillips, George
Piersol, Kim
Placek, Art
Plunkett, Richard
Pratt, Joseph
Prevosto, Virginia
Pridgeon, Ronald
Privman, Boris
Procopio, Don
Proska, Mark
Purple, John
Quinn, Timothy
Quintano, Richard
Radach, Floyd
Reale, Pam Sealand
Retterath, Ron
Robbins, Kevin
Robertson, John
Rodermund, Matthew
Roland, Paul
Rominske, Steve
Rosenberg, Deborah
Rosenberg, Sheldon
Ross, Gail
Roth, Richard
Roth, R. J., Jr.
Ryan, Kevin
Samson, Pierre
Scheibl, Jerry
Schmidt, Jeffrey
Schmitt, Karen
Schultheiss, Peter
Schultz, Ellen
Schwandt, Jeffory
Sclafane, Susanne
Scott, Bob
Seeley, Alan
Shepherd, Linda
Sherman, Harvey
Shoop, Ed
Shrum, Roy
Silver, Mel
Simon, LeRoy
Skurnick, David
Smith, Lee
Sobel, Mark
Steeneck, Lee
Steinen, Phillip
Steinert, Lawrence
Streff, James
Suchar, Chris
Svendsgaard, Chris
Switzer, Vernon
Tan, Suan-Boon
Taylor, Catherine
Taylor, Frank
Teng, Michael
Terrill, Kathleen
Terry, Karen
Thompson, Kevin
Toothman, Michael
Treitel, Nancy
Tresco, Frank
Turner, Marcie
Turvill, Melanie
Tverberg, Gail
Vanier, Anne-Marie
Verges, Ricardo
Votta, James
Wainscott, Bob
Walker, Glenn
Walker, Roger
Walters, Mavis
Walters, Mike
Walton, Patrick
Webb, Nina
Webster, Patsy

FELLOWS

Wess, Clifford
White, Chuck
White, William
Whitman, Mark
Wickwire, James
Wildman, Peter
Wilson, Ernie
Wischmeyer, Chad
Woll, Rich
Wrobel, Edward
Yatskowitz, Joel
Yingling, Mark
Yit, Bill
Yocius, Rich
Yonkunas, John
Zatorski, Rich
Zelenko, Dorothy

ASSOCIATES

Anderson, Bruce
Andler, Jim
Ashman, Martha
Beck, Doug
Benarosch, Xavier
Blank, Cara
Booth, John
Bowman, David
Brannon, Mark
Brauner, Jack
Bukowski, Paul
Burns, William
Cadorine, Art
Cavaliere, Carol
Chang, Jessalyn
Chorpita, Fred
Colgren, Karl
Creighton, Ken
Crifo, Dan
Curry, Michael
Curry, Robert
Dashoff, Todd
Debs, Raymond V.
Der, William
dos Santos, Victor
Dove, William
Ebert, Maribeth
Ermisch, Jenni
Eska, Catherine
Faggella, Madelyn
Fauerbach, Tom
Ferrero, Carole
Fitz, Loy
Fitzpatrick, Bill
Gardner, Andrea
Gidos, Peter
Gleason, Brad
Goldberg, Terry
Gould, Donald
Gutman, Ewa
Gwynn, Holmes
Halpert, Aaron
Harbus, Jonathan
Henry, Thomas
Herbers, Joe
Hofmann, Rich
Holler, Keith
Hostager, Beth
Iafrate, Anthony
Ireland, Kathy
Iuliano, Anthony
Jordan, Edwin
Kantor, Stephen
Kaufman, David
Kavacky, Trina
Kelly, Marty
Klawitter, Warren
Kleinberg, James
Koegel, David
Kohan, Rick
Kolojay, Timothy
Kreuser, Adam
Lacefield, David
Lamb, Scott
Lannutti, Nick
Larkin, Jim
Leccese, Nick
Liuzzi, Joe
Main, Bill
Malik, Sudershan
Manis, Don
Manley, Laura
McBride, Heidi J.
McCarty, Jeff
McCreesh, James
McCutcheon, John
McGee, Steve
Mentz, John
Meyer, Steve
Miller, Linda
Mittal, Madan
Moody, Andrew
Musulin, Rade
Neghaiwi, Tony
Nordquist, Randy
O'Brien, Margaret
O'Connor, Margaret
Orr, Marlene
Overturf, Ann
Pagliaccio, John
Poon, On Cheong
Powers, Stuart
Pulis, R. Stephen
Radin, Katy
Rahardjo, Kay
Rech, James
Reddig, Scott
Rhoads, Karin
Riff, Mayer
Rohn, Diane
Romito, Scott
Sadwin, Stu

ASSOCIATES

Saunders Oates, Leigh
Scanlon, Edmund
Schlenker, Sara
Schmidt, Lisa
Shapiro, Arlyn
Shepherd, David
Shook, Gary
Silverman, Janet
Snow, Dave
Stanco, Ed
Steenken-Dennison, Lisa
Struzzieri, Paul
Tasker, Trey
Teetsel, Marianne
Theocharides, Georgia
Thomas, Edward
Thompson, Gene
Thorne, Joseph
Tingley, Nanette
Trowbridge, Stacy
Valenti, Karen P.
Vincent, Dale
Walker, Christopher
Walker, Patricia
Washburn, Monty
Weinstein, Scott
White, Lawrence
Williams, Lawrence
Williams, Teresa
Wolter, Kathy
Yesker, Chas

REPORT OF THE VICE PRESIDENT-ADMINISTRATION

The objective of this report is to provide the membership with a brief summary of CAS activities since the last annual meeting.

The major activities of the CAS over these past 12 months fall into three general categories: education and professionalism, relationships with other actuarial organizations, and the establishment of a self-sufficient CAS Office.

EDUCATION AND PROFESSIONALISM

For the majority of our members and potential members, the most visible of these three categories is education and professionalism.

The interest in continuing education opportunities seems to increase each year, in part fueled by the continuing education requirements associated with our members maintaining American Academy of Actuaries (AAA) qualification to sign statements of actuarial opinion, but more fundamentally driven by the remarkable intellectual curiosity of actuaries. Unfortunately, the AAA's continuing education requirements also have produced some confusion; but a concurrent session offered at both the May and November meetings this year is intended to explain the requirements more completely.

The CAS continues to be a major provider of outstanding continuing education opportunities, and our members continue to be avid consumers of these opportunities. Meetings and seminars this year have included:

- the Spring Meeting in Palm Springs, attended by 486 members and 44 non-members;
- the Annual Meeting in Philadelphia, with advance registrations of 423 members and 80 non-members;
- the Casualty Loss Reserve Seminar in Arlington, Virginia, of which the CAS is a cosponsor, attended by 825;

- the Canadian Property and Casualty Insurance Liability Seminar in Montreal, sponsored by the CAS and the Canadian Institute of Actuaries, with 65 attendees;
- the Ratemaking Seminar, which attracted 585 registrants;
- the Rate of Return Seminar, attended by 180;
- the Risk Theory Seminar last month, which was attended by 62; and
- the Reinsurance Seminar, cosponsored by Casualty Actuaries in Reinsurance, which drew 190 attendees.

Publications distributed to the membership during the year have included the *Proceedings,* quarterly editions of the *Actuarial Review,* two editions of *Forum,* and the *Discussion Paper* book on International Issues, as well as the annual catalog of continuing education opportunities.

The role of regional affiliates in education and in other areas expanded further this year. Most of the regional affiliates held two meetings during the year. As noted above, one affiliate—Casualty Actuaries in Reinsurance—cosponsored a seminar with the CAS. In addition, the regional affiliates play a key role in conducting examination preparation seminars, in providing a network of local individuals who can respond to career inquiries from potential actuaries, and in reaching out to local regulators and universities through invitations to meetings. Although the regional affiliates are generally self-directed and self-sustaining, the CAS leadership lent its support by attending many of the regional affiliate meetings, and by creating a regional affiliate liaison position that serves as a clearinghouse for information between the CAS and the affiliates, as well as among the affiliates. The regional affiliates' leadership met together during the 1990 CAS Annual Meeting, and again several times during the year by teleconference. Finally, with respect to regional affiliates, we are pleased to welcome Casualty Actuaries of Bermuda (CABER), the newest affiliate.

Regarding professionalism of our members, the Board of Directors approved a Code of Conduct at its November 10, 1991 meeting. This Code, replacing the existing Guides and Opinions as to Professional Conduct, retains much of the content of its predecessors, but provides for a greater degree of uniformity in presentation with Codes of other actuarial organizations. At the same time, the Board prepared, for submission to the membership for approval, several revisions to the CAS Bylaws and Constitution. These revisions recognize the new Code of Conduct but, more importantly, introduce an important role for the profession-wide Actuarial Board for Counseling and Discipline (ABCD). Specifically, issues relating to the conduct of a CAS member in the United States would, under the proposed revisions, be referred to the ABCD. (In Canada, the Canadian Institute of Actuaries will play a similar role.) The ABCD's role is to investigate the conduct in question, to provide counseling if that course is indicated, or, in the case of a violation, to recommend action to the CAS Discipline Committee. The recommended action may involve a reprimand, suspension of membership privileges, or expulsion from the CAS. The CAS Board of Directors will have ultimate responsibility for any disciplinary actions taken. Perhaps more significantly for most of our members, the ABCD provides a resource to which each of us can turn when in need of advice or counseling to help us deal with a knotty problem. An informational session on the proposed changes will be held during the Annual Meeting in Philadelphia, and a mail ballot regarding the proposed changes will be distributed in November 1991.

Turning the focus to our prospective members, the CAS introduced the Course on Professionalism earlier this year. This course, a new requirement for Associateship, focuses on ethics and standards. Approximately 140 near-Associates attended the course, which was offered in New York, Chicago, and Los Angeles, during August 1991. I am pleased to report that the feedback has been excellent. Indeed, the level of interest in the course among *existing* members has been so high that a portion of the course will be offered as a limited-attendance workshop during the May 1992 CAS meeting in Chicago.

One important component of education and continuing education is the development of research to expand the existing body of knowledge. This year, the CAS embarked on a program of managed, funded research targeted to specific areas. To date, the projects initiated include a paper on "The Profit Provision in the Ratemaking Formula," expected to be completed shortly; a prize competition for papers addressing the variability of loss reserves; requests for proposals to author a paper on practical considerations in the use and disclosure of loss reserve risk margins, and proposals to prepare a synopsis and analysis of existing research on surplus requirements; and a discussion paper program on ratemaking topics.

Also underway, and to be presented for preliminary discussion during the 1991 Annual Meeting, is an initial working draft of risk classification principles.

Recognizing the importance of new research as a source for *Syllabus* material, the Executive Council created a formal liaison between the research committees and the Syllabus Committee earlier this year. In addition, in a continuing effort to enhance the quality of examinations, the CAS has begun using non-CAS academicians, on a selective basis, to assist in the development of examinations.

RELATIONSHIPS WITH OTHER ACTUARIAL ORGANIZATIONS

An area of particular attention this year has been the development and fostering of relationships with foreign actuarial organizations. We have established contact with actuarial organizations in 16 other countries, as well as several multinational bodies. We are now exchanging correspondence and literature on a regular basis with many of these organizations, and will be reprinting some of their materials in our own publications as well as adding them to our Bibliographies. The library in the CAS Office is accumulating the exchanged literature, and has catalogued it for easier access by our membership.

On a more personal level, President Chuck Bryan accepted invitations to attend a General Insurance Study Group (GISG) meeting in Wales, a working session of the Institute of Actuaries and Faculty of Actuaries, and a conference of the Institute of Actuaries. We are delighted to have Peter Johnson, chair of the British GISG, as our guest at the 1991 Annual Meeting of the CAS. We expect this to be the start of a tradition.

Here in North America, a healthy atmosphere of cooperation also exists among the various actuarial bodies. The "Working Agreement," defining a number of roles and responsibilities of each organization, as well as certain aspects of relationships among the actuarial organizations, was signed approximately one year ago. This document is serving as a useful blueprint for structuring various activities, and is now being complemented by a staff-level working agreement defining certain relationships and responsibilities of the administrative offices of the actuarial organizations.

The spirit of cooperation has been illustrated by the joint efforts of the CAS and the AAA to work with the National Association of Insurance Commissioners in crafting a definition of "qualified actuary" for purposes of actuarial opinions accompanying fire and casualty annual statements, by the establishment of a joint CAS-SOA committee on minority recruiting, by the cosponsorship of exams with the Canadian Institute of Actuaries, and by the identification of certain CAS and SOA exams that would be recognized for credit by the other organization.

The CAS also undertook this year to define the areas of CAS interest in health benefits. The identified areas are: health benefits as a significant component of casualty coverages, and health insurance as a coverage that can be provided by a casualty insurance company. Casualty actuaries should be familiar with the basics of the health care delivery and financing systems. Efforts will be made to assure the *Syllabus,* continuing education programs, and research support this need. It is expected that other dimensions of health benefits will be in the province of the SOA.

THE CAS OFFICE

In November 1990, the CAS Board authorized the Executive Council to commence the process of relocating the CAS administrative office from New York City to Arlington, Virginia. In the intervening 12 months, we selected a building, negotiated a lease, constructed our office space, and relocated all of our operations. Many CAS volunteers played key roles in making the relocation successful.

We are fortunate to have retained the services of three CAS employees: Michele Lombardo, Linda Burnett, and Kathy Spicer. Two of our long-time employees—Terry Cullinan and Gloria Sessa—decided not to relocate with us. We miss their contributions and their collective institutional memories, and thank them for their years with the CAS. Another relationship that ended with the relocation was our long-standing status as a guest/tenant of the National Council on Compensation Insurance (NCCI). We also thank the NCCI for many years of support.

The process of building a new staff has proceeded rapidly and successfully. In conjunction with the relocation, the CAS Board concluded that it was time for the CAS to hire an administrative Executive Director. Tim Tinsley was recruited for this position in May 1991.

Under Tim's leadership, the office has made rapid strides towards expanded and improved services to members, students, and committees; as well as increased responsiveness to requests and inquiries. The library facilities are available (in person or by mail) to all members and registered students, and conference room facilities are available to committees. Most significantly, with the expanded office capabilities, we are beginning to be able to relieve many committee members and volunteers of administrative aspects of their volunteer duties.

The increased level of office support will not reduce the importance of volunteer efforts by CAS members. Our Society is blessed with an extraordinarily high number of members willing to serve. During 1991, over 350 Fellows, or 33%, served on a committee, task force, or other volunteer position. The annual Participation Survey indicates continuing membership support for this level of effort and provides an efficient means for getting the right volunteers assigned to the right committees.

Most of the membership is familiar with the name of Matt Rodermund, who served the CAS in many capacities over the years before retiring from his post as Editor of the *Actuarial Review* two years ago. In honor of Matt's years of volunteer service, and in recognition of the important role of volunteers in the CAS, the Board instituted the Matthew Rodermund Service Award, to recognize members who have made significant volunteer contributions to the actuarial profession. The first recipient is Bob Foster, who will be honored at the 1991 CAS Annual Meeting.

LEADERSHIP AND FINANCIAL CONDITION

The Board of Directors, with prime responsibility for setting policy, met four times in 1991. New members elected to the Board for next year include Ronald Ferguson, Heidi Hutter, Gary Patrik, and Sheldon Rosenberg. The membership elected David Flynn to the position of President-Elect, and Michael Toothman will be President for the 1991–1992 year.

The Executive Council, with primary responsibility for day-to-day operations, met several times during the year. The Board of Directors elected the following Vice Presidents for the coming year:

Vice President-Administration	John Purple
Vice President-Admissions	Steven Lehmann
Vice President-Continuing Education	Irene Bass
Vice President-Programs and Communications	Al Beer
Vice President-Research and Development	Allan Kaufman

In summary, the CAS continues to grow and thrive. During 1991, 139 new members joined our ranks, and 62 new Fellows were named. Our membership now stands at 1,047 Fellows and 761 Associates. The CAS remains financially healthy as well. A budget of approximately $1.3 million for the 1991–1992 year was approved by the Board of Directors. Dues for next year will be $215, an increase of $25; examination fees for Parts 4 through 10 will remain unchanged, as will the fee for the Academic Correspondent program. Also approved were a schedule of reduced examination fees for full-time college students and a reduced registration fee for actuarial students attending the 1991 CAS Annual Meeting in Philadelphia.

Finally, the Audit Committee examined the CAS books for fiscal year 1991 and found the accounts to be properly stated. The year ended with an increase in surplus of $318.72. In light of the expenses of moving the CAS Office, this result compares very favorably to a budgeted reduction in surplus for the year. Members' equity now stands at $595,944.43, subdivided as follows:

Michelbacher Fund	$ 80,171.51
Dorweiler Fund	6,983.26
CAS Trust	2,938.45
Scholarship Fund	7,996.49
Rodermund Fund	15,900.00
CLRS Fund	5,000.00
CAS Surplus	476,954.72
Total Members' Equity	$595,944.43

Respectfully submitted,

Robert F. Conger
Vice President-Administration

FINANCIAL REPORT
Fiscal Year Ended 9/30/91

OPERATING RESULTS BY FUNCTION

FUNCTION	INCOME	EXPENSE	DIFFERENCE
Exams	$540,078.61	$638,764.23 (a)	($98,685.62)
Member Services (b)	351,725.23	527,962.93	(176,237.70)
Programs	296,701.44	89,024.41 (c)	207,677.03
Other (d)	67,565.01	0.00	67,565.01
TOTAL:	$1,256,070.29	$1,255,751.57	$318.72 (e)

Notes: (a) Does not include exam-related expenses incurred by the Research & Development Function. (b) Areas under the supervision of VP-Administration and VP-Research & Development. (c) Does not include program-related expenses incurred by the Research & Development Function. (d) Investment income less foreign exchange and miscellaneous bank debits. (e) Change in CAS Surplus.

BALANCE SHEET

ASSETS	9/30/90	9/30/91	DIFFERENCE
Checking Account	$192,268.29	$278,635.93	$86,367.64
Bank Certificates of Deposit	100,000.00	0.00	(100,000.00)
U.S. Treasury Bills	806,526.28	767,724.58	(38,801.70)
Accrued Interest	22,895.21	15,157.44	(7,737.77)
Prepaid Meeting Exp.	0.00	10,212.23	10,212.23
Prepaid Seminar Exp.	0.00	490.00	490.00
CLRS Receivable	0.00	40,000.00	40,000.00
CLRS Fund	5,000.00	5,000.00	0.00
TOTAL ASSETS	$1,126,689.78	$1,117,220.18	($9,469.60)
LIABILITIES			
Office Expense	$114,138.60	$21,629.00	($92,509.60)
Printing Expense	239,306.11	185,700.00	(53,606.11)
Prepaid Exam Fees	166,698.00	244,353.20	77,655.20
Prepaid Inv. Program	12,685.00	0.00	(12,685.00)
Prepaid Nov. Mtg. Fees	12,460.00	5,591.00	(6,869.00)
Prepaid Seminar Fees	0.00	12,600.00	12,600.00
Mtg. and Sem. Expenses	0.00	1,402.55	1,402.55
Research	0.00	50,000.00	50,000.00
Other	5,264.61	0.00	(5,264.61)
TOTAL LIABILITIES	$550,552.32	$521,275.75	($29,276.57)
MEMBERS' EQUITY			
Michelbacher Fund	$76,654.10	$80,171.51	$3,517.41
Dorweiler Fund	7,531.38	6,983.26	(548.12)
CAS Trust	2,772.12	2,938.45	166.33
Scholarship Fund	7,543.86	7,996.49	452.63
Rodermund Fund	0.00	15,900.00	15,900.00
CLRS Fund	5,000.00	5,000.00	0.00
CAS Surplus	476,636.00	476,954.72	318.72
TOTAL EQUITY	$576,137.46	$595,944.43	$19,806.97

Robert F. Conger, Vice President-Administration

This is to certify that the assets shown in the above financial statement have been audited

1991 EXAMINATIONS—SUCCESSFUL CANDIDATES

Examinations for Parts 3B, 4, 6, 8, 8c (Canadian), and 10 of the Casualty Actuarial Society were held on May 6, 7, 8, 9, and 10. Examinations for Parts 3B, 5, 5A, 5B, 7, and 9 were held on November 4, 6, 7, and 8.

Examinations for Parts 1, 2, 3A, and 3C (SOA courses 100, 110, 120, and 135) are jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries. Parts 1 and 2 were given in February, May, and November 1991, and Parts 3A and 3C were given in May and November of 1991. Candidates who were successful on these examinations were listed in the joint releases of the two societies.

The Casualty Actuarial Society and the Society of Actuaries jointly awarded prizes to the undergraduates ranking the highest on the Part 1 examination.

For the February 1991 examination the $200 first prize was awarded to Zhen H. Liu. The $100 prize winners were Nancy H. Fu, Robert H. Graves, Danield J. Pasko, and Francois Theberge.

For the May 1991 examination the $200 first prize was awarded to Thomas P. Hayes. The $100 prize winners were Thian-Huat Ong, Robert T. Pope, Michail Sunitsky, and Barbara Zvan.

For the November 1991 examination the $200 first prize was awarded to Lawrence S. Carson. The $100 prize winners were James H. Carson, Luc Chen, Michael P. Jones, Jr., and Michael G. Steinberg.

As a result of successful completion of the Society requirements in the 1990 examinations, David P. Bechtel was admitted as a new Associate.

The following candidates were admitted as Fellows and Associates at the November 1991 meeting as a result of their successful completion of the Society requirements in the May 1991 examinations.

FELLOWS

Adams, Jeffrey H.
Barnes, W. Brian
Beaulieu, Karin H.
Billings, Holly L.
Blair, Gavin C.
Blais, Jean-Francois
Boisvert, Paul, Jr.
Boucek, Charles H.
Caron, Louis-Philippe
Chou, Li-Chuan L.
Clark, David R.
Cutler, Janice Z.
Dew, Edward D.
Edlefson, Dale R.
Effinger, Bob D., Jr.
Ewert, John S.
Fallon, Steven R.
Gagnon, Luc
Gelinne, David B.
Greene, Alex R.
Grim, Cynthia M.
Grossack, Marshall J.
Lepage, Pierre
Lew, Allen
Licht, Peter M.
Loisel, Andre
MacKinnon, Brett A.
MacMahon, Brian E.
Montigney, Brian A.
Murphy, Daniel M.
Nyce, G. Christopher
Ottone, Joanne M.
Pelly, Brian G.
Petersen, Loren V.
Petker, Jill
Price, Deborah W.
Pridgeon, Ronald D.
Privman, Boris
Quinn, Timothy P.
Rominske, Steven C.
Samson, Pierre A.
Schmidt, Jeffrey W.
Schmitt, Karen E.
Schwandt, Jeffory C.
Sclafane, Susanne
Seeley, Alan R.
Swanstrom, Ronald J.
Tan, Suan-Boon
Teng, Michael T.S.
Terry, Karen F.
Turner, Mary L.
Turvill, Melanie A.
Vanier, Anne-Marie
Verges, Ricardo
Walton, Patrick M.
Wildman, Peter W.
Wischmeyer, Chad C.
Yit, Bill S.
Yocius, Richard P.

ASSOCIATES

Ashman, Martha E.
Baker, Mark S.
Benarosch, Xavier
Booth, John D.
Brannon, Mark L.
Bukowski, Paul A.
Burns, William E.
Cavaliere, Carol A.
Chang, Jessalyn
dos Santos, Victor G.
Dove, William F.
Ebert, Maribeth
Ermisch, Jennifer L.
Faggella, Madelyn C.
Fauerbach, Thomas R.
Ferrero, Carole F.
Gardner, Andrea
Gleason, Bradley J.
Harbus, Jonathan M.
Holler, Keith D.
Hostager, Beth M.
Iafrate, Anthony
Iuliano, Anthony
Jordan, Edwin G.
Kantor, Stephen H.
Kaufman, David L.
Kavacky, Trina C.
Kohan, Richard F.
Kreuser, Adam J.
Larkin, James W.

ASSOCIATES

Larson, Michael D.	Nordquist, Randall S.	Tasker, Joseph W., III
Main, William G.	O'Connor, Margaret M.	Teetsel, Marianne
Manis, Donald E.	Overturf, Ann E.	Theocharides, Georgia A.
McBride, Heidi J.	Poon, On Cheong	Thomas, Edward D.
McCarty, Jeffrey F.	Powers, Stuart	Trafecanty, Janet A.
McCutcheon, John W., Jr.	Riemer, Gregory	Trowbridge, Stacy L.
Mentz, John P.	Romine, Laura A.	Vincent, Dale G., Jr.
Merlino, Paul M.	Saunders Oates, Leigh A.	Walder, Lawrence M.
Meyer, Stephen J.	Shepherd, David M.	Walker, Patricia K.
Miller, Linda K.	Steenken-Dennison, Lisa N.	Williams, Theresa J.
Neghaiwi, Antoine A.	Struzzieri, Paul J.	Yesker, Charles J.

The following is a list of successful candidates in examinations held in May 1991.

Abian, Rimma	4	Babcock, Nathan J.	3B	Bedard, Carole	4
Adams, Jeffrey	8	Baker, Mark S.	4	Begin, Nathalie	8c
Adee, Marc. J.	8	Ballmer, Robert S.	4	Behbahany, Saeeda	3B
Adkisson, Jonathan D.	3B	Banick, Timothy J.	3B,4	Beltz, Victoria A.	3B
Albright, Kristen M.	6	Barbanel, Emelia L.	4	Benarosch, Xavier	6
Allan, Christopher R.	6	Barnes, Joy A.	3B	Benedict, Douglas S.	6
Allison, Rhonda K.	6	Barnes, Keith M.	3B	Bensics, Frank G.	10
Ambrose, Rachelle R.	3B	Barnes, Walter B.	10	Bentley, Cynthia A.	4
Anand, Bijoy	4	Barrett, Rose D.	6	Berman, Steven L.	4
Anderson, Scott C.	6	Bartie, James M.	4	Berube, Julie	4
Angelina, Michael E.	3B	Baum, Philip A.	6	Bibbero, Herbert S.	6
Anson, Donald W.	3B	Beaulieu, Gregory S.	8	Billings, Holly L.	10
Arico, Nancy L.	4	Beaulieu, Karin H.	10	Binnig, Bruce E.	4
Arya, Satya M.	3B	Beaulieu, Martin	6	Biskner, Laverne J., III	6
Ashman, Martha E.	6	Beck, Douglas L.	8	Black, Suzanne E.	4
Atkinson, William M.	4	Becker, Allan R.	10	Blaesing, Steve D.	3B
Augustine, Lewis V.	4	Beckman, Brian P.	3B	Blair, Gavin C.	10
Ayres, Karen F.	8	Beckman, John A.	4	Blais, Annie	6

Blais, Jean-Francois	10
Blakeney, Gina L.	4
Blanco, Roberto G.	10
Blue, Betsy L.	6
Blumsohn, Gary	4
Boisvert, Paul	10
Bok, Ann M.	4
Bonsignore, John T.	3B
Booth, John D.	6
Borden, Sara	4
Botsko, Thomas S.	4
Boucek, Charles H.	10
Bouchard, Lloyd J.	8
Bouffard, Maurice P.	4
Bouvin, Erik R.	3B,6
Boyle, Thomas P.	3B
Bozman, Christopher K.	6
Brady, Kevin M.	6
Branagan, Betsy	6
Brancel, Robert E.	6
Brannon, Mark L.	6
Brassier, Dominique	10
Bray, David T.	3B,4
Bricker, Sherry B.	3B
Brindamour, Charles	4
Brissman, Mark D.	8
Brockmeier, Donald R.	8
Brooks, Ward M.	6
Brosius, J. Eric	4,6
Brown, Lisa A.	4
Brown, Stephanie J.	4
Brubaker, Lisa J.	4
Buckley, Russell J.	3B
Bukowski, Paul A.	6
Bunks, Laurie M.	3B
Burkart, Marian M.	3B
Burn, Elliot R.	6
Burnaford, Laura L.	4
Burns, Patrick J.	10
Burns, William E.	4
Bush, Tara E.	3B
Bustillo, Anthony R.	3B
Butler, John G.	3B
Cain, Mark J.	10
Calihan, Glen R.	3B
Callas, Linda E.	3B
Cappelletti, Anthony E.	4
Carey, Jeanne L.	6
Caron, Louis-Philippe	10
Carpine-Taber, Kristi I.	4
Carr, Daniel G.	3B
Carter, Victoria J.	6
Cash, David S.	3B,6
Cassuto, Irene A.	3B
Cavaliere, Carol A.	6
Celestin, Ramses T.	3B
Chaffee, Janet L.	8
Chan, Dennis	3B
Chang, Jessalyn	6
Chaussee, Scott A.	3B
Chen, Daoguang E.	6
Chen, Hong	4
Cheng, Peggy	3B
Chern, Jiunnjyh	4
Cherniawsky, David M.	3B
Chong-Kit, Roy A.	4
Chou, Li-Chuan L.	10
Chounard, Rene	3B
Christensen, Kathy	3B
Chuang, Wei	6
Chung, Kasing L.	6
Chvoy, Darrel W.	4
Ciardiello, Gary T.	6
Ciardiello, Susan D.	3B
Ciccariello, Rita E.	4
Cittone, Dawn L.	3B
Clancy, Brian A.	4
Clark, David R.	10
Clark, Denise R.	4
Cleary, Kay A.	4
Cler, Dennis J.	4
Codere, Michelle	6
Cody, William B.	4
Cole, Jeffrey R.	8
Coleman, Theresa A.	3B
Collins, Peter J.	6
Conlin, Pamela A.	4
Connor, Kathleen F.	4
Connors, Pamela A.	6
Considine, Rachelle	4
Conway, Thomas P.	6
Cooper, Craig A.	3B
Cooper, Kathleen M.	4
Cooper, Sharon L.	6
Cordner, Beverly E.	3B
Corwin, Matthew D.	3B
Cossette, Helene	4
Cote, Gregory L.	6
Couture, Martin L.	8
Cox, Brian K.	8
Cox, David B.	8
Cremin, Timothy J.	6
Cresswell, Catherine	3B
Cunningham, Kathleen T.	4
Curry, Robert J.	8

Cutler, Janice Z.	10	Dossett, A. Mark	3B	Fenstermacher, Kurt D.	3B
Czabaj, Daniel J.	10	Dove, William F.	6	Ferrara, John R.	6
Dagneau, Francois	8c	Dowell, William A.	4	Ferrero, Carole F.	6
Dallessio, Joyce A.	3B,4	Doyle, Peter I.	3B	Fescos, George	6
Datoo, Mujtaba H.	4	Drennan, Kimberly J.	3B	Finch, Stephen A.	6
Davenport, Mark A.	3B	Drolet, Daniel	3B,4	Finkelstein, Steven J.	4
Davies, Karen L.	6	Drolet, Pierre	3B	Finnerty, Deborah C.	8
Debigare, Manon	8c	Du, Thuan H.	4	Fisanick, William P.	3B
Delfino, Steven F.	3B	Duchna-Savrin, Mary Ann	6	Fischer, Brian C.	4
Delmastro, Anne M.	3B	Dulude, Sophie	4	Fitz, Loy W.	8
Demattei, Michael L.	6	Dumas, Francois	8c	Fitzgerald, Barbara	3B
Demers, Marie-Julie	6	Durand, Denis	4	Fitzgerald, Chantal N.	3B
Demski, Rachel R.	4	Dussault, Patrick	8c	Fitzpatrick, William G.	10
Denoncourt, Germain	8c	Ebert, Maribeth	4	Fletcher, James E.	10
Depolo, Catherine L.	3B	Eddinger, Jeffrey	3B	Foley, David A.	8
Desantis, Jean A.	4	Edlefson, Dale R.	10	Ford, Heather	3B
Desrochers, Mark R.	3B	Effinger, Bob D., Jr.	10	Forden, Jeffrey M.	3B
Devine, Mike	3B	Elinon, Melita M.	4	Fortin, France	8c
Devlin, Patrick K.	8	Embry, Krista J.	3B	Frank, Russell	4
Dew, Edward	10	Eng, Ching	3B	Franklin, Barry A.	8
Dibiase, Joseph A.	3B	Eriksen, Paul E.	3B	Frantom, Kirsten A.	3B
Dickmann, Kurt S.	4	Erlebacher, Alan J.	8	Fredrickson, Sandra L.	3B
Dickson, Kevin G.	8	Ermisch, Jennifer L.	6	Fried, Kevin J.	4
Dilapi, Anthony M.	3B	Eska, Catherine E.	10	Friedman, Keith E.	3B
Diminich, Lisa M.	3B	Eudy, Daniel A.	3B	Friess, Cynthia J.	4
Dionne, Michel	8c	Ewert, John S.	10	Fritz, Shina N.	3B
Dionne, Pierre	8c	Faggella, Madelyn C.	6	Fuller, Scott F.	4
Doedtman, Lisa A.	4	Fallon, Steven R.	8	Furrow, J'ne	3B
Doffing, Jeffrey E.	6	Fansler, Todd E.	4	Gagnon, Jean-Pierre	4
Doherty, Shawn F.	6	Farwell, Randall A.	8	Gagnon, Luc	10
Dollinger, Jeffrey L.	3B	Fauerbach, Thomas R.	4	Gamache, Nathalie	3B
Donnelly, Patricia J.	3B	Feathers, Renee L.	4	Gant, James E.	6
Donofrio, Pamela J.	3B	Feder, Denise A.	6	Garbus, Robert J.	3B
Dooley, Francis J.	3B	Fedor, David M.	4	Gardner, Andrea	6
dos Santos, Victor G.	6	Fenster, Craig M.	3B	Gauthier, Paul	4

Geering, Chris	4
Gegax, Charles E.	3B
Gehant, Lynn A.	4
Geist, Robert W.	4
Gelinne, David B.	10
George, Michael L.	3B
Germani, Margaret W.	6
Getz, Douglas S.	3B
Giargiana, Lauren B.	3B
Gibbons, Thomas P.	3B
Gilden, Bernard H.	3B
Gise, Mary K.	6
Glatz, Michael F.	4
Gleason, Bradley J.	6
Gleba, John T.	3B
Glenn, Donna L.	4
Glenn, Ronald E.	3B
Goldstein, Laurence B.	6
Goodrich, Carol A.	3B
Gorham, Mark A.	4
Goss, Linda M.	8c
Goulet, Vincent	4
Goyer, Odlie	6
Green, Michael D.	3B
Greene, Alex R.	10
Greenwood, Joseph P.	3B
Greig, Russell H.	3B
Grilliot, Charles R.	4
Grim, Cynthia M.	10
Gritz, Marcy G.	3B
Groeschen, Steven J.	6
Grossack, Marshall J.	10
Grossack, Victoria A.	4
Grove, M. Harlan	3B
Guimond, Alain	4
Gusler, Terry D.	8
Hackworth, David B.	3B
Hadidi, Nasser	4
Haidu, Jeffrey A.	3B
Hall, Marc S.	6
Halliwell, Leigh J.	6
Harbage, Robin A.	10
Harbus, Jonathan M.	6
Harnatkiewicz, Robert L.	4
Harris, Christopher L.	3B
Hausserman, Diane K.	8
Hay, Gordon K.	8c
Hay, Randolph S.	8
Hayden, Matthew T.	4
Hayes, Jonathan B.	4
Hedges, Barton W.	4
Hehr, Noel M.	4
Heirich, Fritz J.	4
Helou, Renee J.	3B,4
Hess, Sherry L.	3B
Highet, Thomas H.	6
Hightshue, James B.	3B
Hill, Betty-Jo	3B
Hill, Michael R.	6
Himmelberger, Amy J.	4
Hinds, Kathleen A.	8
Hinton, John V.	4
Hirsch, Michael B.	4
Hochler, Glenn S.	6
Holler, Keith D.	6
Holmberg, Mark P.	4
Homer, David L.	6
Homyak, Chet B.	6
Hornick, Eric J.	3B
Horovitz, Bernard R.	6
Hostager, Beth M.	6
Howard, Terrie L.	4
Howell, Linda M.	4
Hsieh, Po-Wo	4
Huey, Corine	3B
Hussian, Paul R.	6
Huynh, Tina T.	3B
Iafrate, Anthony	6
Ingle, Brian L.	6
Ireland, Kathleen M.	8
Israel, Jason	3B,4
Itri, Henry J.	3B
Iuliano, Anthony	6
Ivanovskis, Paul	3B
James, Peter H.	8
Jamroz, Christopher	3B
Janitschke, Brian	3B
Janssen, John F.	3B
Jean, Patrice	4
Jensen, Patrick C.	4
Jobidon, Christian	4
Johnson, Kurt	4
Jones, Terrell A.	8
Jordan, Edwin G.	6
Joynson, Hazel M.	4
Kahn, James B.	3B
Kaiser, Linda M.	4
Kanigowski, Barbara L.	3B
Kantor, Stephen H.	6
Kappeler, Gail	4
Karambelas, Panayotis N.	4
Karoski, Kathryn A.	4
Kasmer, Charles N.	3B
Kastan, Tony A.	4
Katz, Janet S.	3B,4

Kaufman, David L.	6	Kreuser, Adam J.	6	Lew, Allen	10
Kavacky, Trina C.	6	Kundrot, Jason A.	4	Licht, Peter M.	8
Kebodeaux, Gwenette T.	3B	Kunze, David R.	8	Light, Richard S.	3B
Kellner, Steven A.	4	Kuo, Dar-Jen D.	4	Liholt, Karen	3B
Kellner, Tony J.	8	Labelle, Mylene J.	6	Liles, Robert P.	3B
Kemp, Brian D.	3B	Lacke, Christine L.	4	Lin, Shu Ching	4
Kemp, Michael D.	3B	Laguarina, Richard V.	3B	Lin, Steven C.	4
Kenefick, Timothy P.	4	Lamb, D. Scott	8	Liu, Ling Ling	3B
Kennedy, Rebecca A.	6	Lambert, Josee	4	Livingstone, Paul R.	6
Kent, Susan E.	6	Lange, Alan E.	8	Lloyd, Andrew M.	4
Kerner, Michael G.	8	Lannutti, Nicholas J.	8	Lloyd, James A.	3B
Keyes, Tricia M.	3B	Larkin, James W.	6	Loisel, Andre	10
Kim, Ung M.	3B	Larson, David L.	4	Lowery, Robert G.	6
Kincaid, Bryan J.	8	Larson, Michael D.	6	Ly, Tai-Kuan	4
Kinson, Paul E.	8	Larson, Robert J.	4	Maala, Lennette U.	4
Kirschner, Gerald S.	6	Lattin, Christopher	6	MacFadden, Sally	6
Kirste, Richard O.	8	Laughlin, Laura L.	3B	MacKinnon, Brett A.	10
Kiscaden, Bradley J.	4	Lavigne, Nathalie M.	4	MacMahon, Brian E.	10
Klein, Michael	3B	Lavrey, Paul W.	8	Maguire, Richard	6
Kliethermes, Craig W.	6	Le, Eric	3B	Mah, Cornwell H.	3B
Klodnicki, Therese A.	4	Le, Thomas V.	4	Mahanna, Cathy A.	6
Knowling, Donna L.	3B	Leblanc-Simard, David	4	Maher, James M.	3B,4
Knull, Terry A.	4	Lee, Jennifer	3B	Main, William G.	6
Koch, John E.	3B	Lee, Lewis Y.	4	Maines, Gigi L.	3B
Koester, Timothy F.	6	Lee, Thomas	3B,6	Mainka, Daniel J.	3B,4
Kohan, Richard F.	4	Lefebvre, Marc-Andre	6	Malsky, Joseph A.	4
Korth, Louis K.	3B	Leikums, Robert	3B	Manis, Donald E.	6
Korthals, Gilbert M.	6	Lemay, Isabelle	4	Mann, Katherine A.	6
Koterman, Chris K.	4	Lents, Daniel E.	4	Maratea, Stephen N.	6
Kozik, James M.	3B	Lenz, Charles R.	3B	Marcks, Richard J.	4
Kozlowski, Ronald T.	8	Lepage, Pierre	10	Markowski, Sharon L.	4
Kratzer, Gary R.	3B	Lesage, Diane	4	Marlowe, Burton F.	8
Krause, Thomas F.	4	L'Esperance, Andre	3B,4	Marques, John C.	3B
Kraynyk, Myron W.	4	Letourneau, Roland D.	10	Martel, Dominique	4
Kretsch, David J.	10	Levine, Kenneth A.	3B	Martella, Anthony G.	4

Martin, Meredith J.	3B
Martin, Peter R.	3B
Martin, Scott A.	4
Martin, Suzanne	6
Masch, Jason N.	4
Massoni, Daniel J.	3B
Mathre, Keith A.	8
Mathson, Kelly J.	4
Matthew, Tracey L.	3B
Mattioli, Maria	6
Mays, Deann	3B
Mazurek, William J.	3B
McAuliffe, Timothy C.	3B
McBride, Heidi J.	6
McCarthy, Timothy L.	4
McCarty, Jeffrey F.	6
McClintock, William R.	3B
McCorkle, Teresa J.	6
McCutchan, Michael K.	4
McCutheon, John W.	6
McDonald, Richard T.	4
McElligott, Richard J.	4
McFarlane, Liam M.	8c
McGill, Cassandra M.	8
McGill, Mark Z., III	3B
McIntyre, Thomas S.	6
McMonigle, Kathleen A.	3B
McNeal, Lisa R.	4
McNeese, Dennis T.	8
McPherson, James C.	4
McWithey, Lynne S.	4
Mech, William T.	8
Meeks, Mary Jo	3B
Megens, Robert F.	4
Membrino, Conrad O.	6
Menard, Christian	3B
Mentz, John P.	6
Merberg, Mitchel L.	3B
Merk, Daniel J.	3B
Merlino, Paul M.	6
Messier, Timothy	4
Mestelle, Paul A.	4
Meyer, Stephen J.	6
Middough, Jennifer	4
Miller, Linda K.	6
Mitchell, Keith R.	3B
Mitzel, Charles B.	8
Mize, John H.	6
Monaghan, James E.	4
Montigney, Brian A.	8
Moore, Russell E.	6
Morin, Francois R.	6
Morissette, Benoit	4
Morissette, Francois L.	6
Morris, Dennis J.	3B
Morrow, Michelle M.	6
Moser, Robert J.	4
Mosher, Matthew C.	4
Moss, Jonathan M.	3B
Moss, Michael J.	3B
Moxon, Elayne	3B
Munson, Todd B.	8
Murphy, Daniel M.	10
Murray, Randy J.	4
Muzzey, Timothy O.	6
Nadeau, Donna W.	4
Naigles, Mark	6
Nance, Iris A.	3B
Narayan, Prakash	3B
Neary, James F.	3B
Neghaiwi, Antoine A.	6
Neidlinger, Melissa J.	3B
Neufeld, Catherine A.	4
Neveu, Martin	4
Newhoff, Aaron W.	4
Nguyen, John-Giang	4
Nicodemus, William F.	3B
Niemczyk, William A.	8
Njakou, Victor	6
Nonken, Peter M.	6
Noonan, Stephen	6
Nordquist, Randall S.	4
Nyce, Christopher G.	10
Nyren, Robert H.	4
O'Brien, Daniel E.	4
O'Brien, Margaret	10
O'Brien, Mark A.	3B
O'Connor, John F.	3B
O'Connor, Margaret M.	6
O'Keefe, Mary Beth	4
Oliveto, Helen S.	3B
Olson, Lowell D.	3B
O'Malley, James D.	4
Onnen, Douglas J.	4
Oostendorp, William L.	6
Osborn, Paul S.	4
O'Shea, Mary M.	3B
Ottone, Joanne M.	10
Overturf, Ann E.	6
Palo, Jennifer J.	4
Parenteau, Pierre	4
Pasley, Jacqueline E.	8
Patschak, Susan J.	10
Pattabiraman, Prabha	3B
Pauken, Patrick D.	3B

Paulauskis, Wayne V.	8
Pawlowski, Lisa M.	3B
Pearl, Charles C., Jr.	4
Peck, Edward F.	4
Pedrick, John R.	4
Pehrson, Karen L.	6
Pelly, Brian G.	10
Perreault, Kathleen H.	10
Perrine, Julia L.	10
Pestcoe, Marvin	10
Peter, William	4
Petersen, Loren V.	8
Petersen, Michael C.	3B
Petker, Jill	10
Petrocik, Michael J.	8
Phillips, Mark W.	6
Phillips, Michael W.	3B
Pickens, Dan C.	4
Piet, Marian R.	8
Pipia, Anthony J.	6
Pitruzzello, Glen-Roberts	4
Pittner, Frank P.	3B
Poe, Michael D.	8
Poirier, Gregory J.	4
Polfsky, Janice L.	4
Polson, Jennifer A.	8
Poon, On Cheong	6
Porcelli, Christine A.	6
Potter, Alan D.	3B
Poulin, Simon	4
Powers, C. Stuart	4,10
Powers, Michael R.	4
Powers, Tracey S.	3B
Prescott, Richard W.	8
Price, Deborah W.	10
Pridgeon, Ronald D.	10
Primack, Lazar M.	4
Principato, Armand	4
Privan, Mark	4
Privman, Boris	10
Puchalski, Richard B.	4
Pugel, David S.	4
Pulkstenis, Eduard J.	4
Pylman, Harry L.	4
Qiu, Cindy Q.	4
Quane, Robert E.	4
Quinn, Timothy P.	10
Quintilian, Kenneth P.	8
Raasch, Kelly L.	3B
Radigan, Kenneth D.	3B
Radin, Katherine D.	8
Raguse, Jeffrey C.	8
Rainbolt, Julianne	4
Rainey, Donald K.	8
Rath, Daniel D.	4
Rathgeber, John F.	10
Repella, Peggy-Ann	3B
Respler, Ellen J.	3B
Reynard, Alan T.	3B,4
Richardson, Meredith G.	4
Riemer, Gregory L.	6
Rioux, Jacques	4
Rivenburgh, Douglas S.	4
Roberts, Linda L.	6
Rohe, John R.	4
Rohn, Diane R.	8
Romanowski, James J.	4
Romine, Laura A.	6
Rominske, Steven C.	10
Romito, A. Scott	8
Rosenthal-Wiesner, Elizabeth	4
Ross, Christine R.	3B
Ross, Lisa M.	6
Roth, Daniel G.	6
Roy, Clement	3B
Rozema, Michael R.	3B
Ruggieri, Giuseppe A.	4
Rupert, Kenneth W., Jr.	4
Russell, Bryant E.	4
Russell, James V.	4
Russell, Kevin L.	4
Russell, Stephen P.	8
Ryan, Beverley K.	8
Ryan, Catherine L.	4
Ryan, Laura A.	3B
Sabiston, Cheryl	3B
Samorajski, Gregory S.	3B
Samson, Pierre A.	10
Sanders, Brandelyn C.	4
Santiago, Roxanne A.	4
Saunders, Leigh A.	6
Savage, David M.	8
Saylor, Letita M.	4
Scanlon, Edmund S.	8
Schadler, Thomas E.	8
Schattin, Risa R.	3B
Scheirer, Lori A.	3B
Schenk, Michael B.	3B,4
Schlenke, David O.	6
Schmidt, Jeffrey W.	10
Schmitt, Karen E.	10
Schofield, Dave B.	4
Scholdstrom, Ia F.	4
Schwandt, Jeffory C.	10
Schwanke, Peter R.	4

Sclafane, Susanne	10
Scorzetti, Lisa M.	3B
Scott, Jeffrey J.	4
Scruton, Gregory R.	3B,4
Scukas, Craig J.	3B
Seaman, Patrick J.	3B
Seeger, Jerylyn	3B
Seeley, Alan R.	10
Seiter, Margaret E.	10
Senak, Peter	6
Senia, Susan A.	3B
Senia, Vincent M.	8
Sexter, Alan J.	4
Sgaramella, Lisa A.	4
Shapiro, Scott A.	3B
Shaw, Dorothy M.	3B
Sheldon, Cynthia M.	6
Shen, Cheryl	4
Shepherd, David M.	6
Shubat, Kerry S.	4
Shupe, Paul O., Jr.	6
Siblik, Donna K.	3B
Siegel, Laura E.	3B
Simon, Christy L.	8
Sirkin, Jeffrey S.	6
Small, Deborah J.	4
Smith, Barbara J.	3B
Smith, Gerson	4
Smith, Larry K.	3B
Smith, Laura A.	3B
Smith, Michael R.	3B
Smith, Michelle	4
Smith, M. Kate	3B
Smolen, Tom A.	8
Snyder, Lori A.	3B
Sommer, David B.	6
Sopkowicz, John B.	4
Sornson, Carl J.	3B
Southwood, Klayton N.	3B
Spence, Calvin C., Jr.	3B
Sperduto, Michael J.	4
Stahley, Barbara A.	8
Staley, Ruth E.	4
Staudhammer, Christina L.	3B
Stauffer, Laurence H.	6
Stayton, Stephen D.	10
Steenken-Dennison, Lisa N.	6
Stein, Richard L.	3B
Steinberg, Barry P.	3B
Steward, Michael J.	6
Steig, Susan D.	3B
Stinde, Rita L.	3B
Stock, Richard A.	3B
Stoll, Brian M.	6
Stolle, Judy L.	4
Stone, Ilene G.	4
Storey, John E.	4
Storms, Theresa C.	3B
Strommen, Douglas N.	8
Strous, Kevin D.	3B
Struzzieri, Paul J.	4
Stubitz, Jayme P.	6
Stulman, Alan	4
Sullivan, Colleen M.	4
Swanson, Jeanne E.	4
Swanstrom, Ronald J.	10
Syrotynski, Michelle M.	3B
Tabor, Todd D.	4
Tait, Christopher	6
Takahashi, Joy Y.	4
Tan, Suan-Boon	10
Tang, Yuan-Yuan	8
Tardif, Francois	3B
Tasker, Joseph W., III	6
Teetsel, Marianne	6
Teng, Ting-Shih	8
Terne, David M.	3B
Terry, Karen F.	10
Tess, Daniel A.	3B
Thacker, Harlan H.	3B
Theocharides, Georgia A.	6
Thibodeau, Kellie A.	3B
Thill, Lawrence J.	3B
Thomas, Edward D.	6
Thomas, Richard D.	8
Thomson, Michelle A.	3B
Thorpe, Patrick	6
Thurston, Barbara H.	6
Timmins, Mont W.	4
Timney, Colleen A.	3B
Tio, Tony	4
Tobey, Dom M.	4
Tobleman, Glenn A.	4
Toce, Thomas C.	8
Toledano, Mike	6
Toney, Charles F.	4
Trafecanty, Janet A.	4
Tran, Ngoc H.	4
Traynor, Theresa A.	6
Tremblay, Nathalie	4
Trocchia, Thomas A.	3B
Trowbridge, Stacy L.	6
Trueman, Bonnie J.	3B
Tucker, Scotty M.	3B
Turcotte, Marie-Claire	4

Tures, Patrick N.	4
Turner, Mary L.	10
Turvill, Melanie A.	10
Tzeng, Ching-Hom	3B,6
Van de Water, John V.	10
Van Dreumel, Brian D.	3B
Van Epps, Robert W.	4
Van Kampen, Charles E.	4
Van Kley, Jeffrey A.	4
Vanier, Anne-Marie	8c,10
Vasek, William	8
Vaughn, Therese M.	4
Vaughn, Trent R.	3B
Verfurth, Jacqueline J.	3B,4
Verges, Ricardo	10
Vezina, Martin	4
Vidal, Cynthia L.	3B
Vincent, Dale G., Jr.	6
Violette, Jennifer A.	6
Visintainer, Michael A.	8
Vogel, Robert J.	3B
Vogt, David M.	6
Vonschaven, Jennifer A.	3B
Vu, Sebastian	8
Wacker, Wittie O.	3B
Wagner, Jennifer M.	6
Wahl, Jane A.	4
Walden, Benjamin A.	3B
Walder, Lawrence M.	4
Walker, Christopher P.	10
Walker, Patricia K.	6
Walton, Patrick M.	10
Wang, Alice M.	3B
Wang, Xiaochuan	4
Wanner, Gregory S.	6
Ward, Kimberley A.	6
Ware, Bryan C.	6
Webb, Jennifer M.	3B
Wegerich, Petra L.	3B
Weinberg, Robert G.	3B
Weinstein, Marjorie C.	8
Weitermann, Michael F.	3B
Welch, John P.	6
Wellington, Elizabeth A.	10
Weltmann, Nicholas L., Jr.	8
Wenitsky, Russell B.	10
Werland, Debra L.	8
Whelahan, Amy A.	4
White, Steven B.	4
White, Thomas J.	3B
Whitley, David L.	3B
Wieczorek, Miroslaw	4
Wildman, Peter W.	8
Williams, Marcia C.	4
Williamson, Jennifer S.	3B
Wilson, E. Jane	6
Wilson, Frances E.	3B
Wilt, William M.	4
Wischmeyer, Chad C.	8,10
Wittman, Bonnie S.	4
Wolf, Patricia L.	3B
Wolf, Robert F.	4
Wolfe, Beth M.	8
Wolter, Kathy A.	8
Womack, Tad E.	4
Wooley, Perry K.	4
Wright, Denise Y.	3B
Wright, John S.	4
Yager, Floyd M.	6
Yard, Roger A.	8
Yeagley, Michele N.	4
Yesker, Charles J.	6
Yeung, Gerald T.	3B
Yifru, Karen M.	3B
Ying, Jeanne Lee	4
Yit, Bill S.	10
Yocius, Richard P.	10
Yoder, Claude D.	4
Yongvanich, Thee	4
Yorty, Edward J.	3B
Young, Shawn M.	3B
Zaleski, Ronald J.	8
Zearfoss, Doug A.	4
Zhu, Lian	4
Zimmer, Ralph T.	4
Zonenberg, Edward J.	3B

The following candidates were admitted as Fellows and Associates as a result of their successful completion of the Society requirements in the November 1991 examinations.

FELLOWS

Becker, Allan R.
Blanco, Roberto G.
Burns, Patrick J.
Carlton, Kenneth E.
Czabaj, Daniel J.
Eska, Catherine E.
Fitzpatrick, William G.
Flannery, Nancy G.
Hughes, Brian A.
Ollodart, Bruce E.
Perrine, Julia L.
Polson, Jennifer A.
Stayton, Stephen D.
Vasek, William
Wellington, Elizabeth A.

ASSOCIATES

Bibbero, Herbert S.
Blackburn, Wayne E.
Brosius, J. Eric
Cash, David S.
Chan, Dennis K.
Chuang, Wei
Collins, Peter J.
Conway, Thomas P.
Cote, Gregory L.
Cox, Brian K.
Cremin, Timothy J.
Cuzzi, Gregory A.
DeMattei, Michael L.
Desson, Herbert G.
DiCenso, Stephen R.
Dionne, Michel
Doffing, Jeffrey E.
Feder, Denise A.
Garland, Kim B.
Groeschen, Steven J.
Guiahi, Farrokh
Gusler, Terry D.
Halliwell, Leigh J.
Hussian, Paul R.
Kent, Susan E.
Kenyon, Deborah E.
Koester, Timothy F.
Korthals, Gilbert M.
Laganiere, Benoit
Lange, Alan E.
Lattin, Christopher
Lefebvre, Marc-André
Martin, Suzanne
Moore, Russell E.
Morin, Francois R.
Murry, David A.
Njakou, Victor
Noonan, Stephen R.
Phillips, Mark W.
Ross, Lisa M.
Roth, Daniel G.
Rozema, Michael R.
Senak, Peter
Toce, Thomas C.
Ware, Bryan C.

The following 40 candidates, having been successful in completing the examinations will, upon completion of the Course on Professionalism and approval by the Executive Council, become members and be admitted as Associates of the Society.

Albright, Kristen M.	Gendron, Jeffrey C.	Nomicos, Kathleen F.
Bault, Todd R.	Goyer, Odile	Oostendorp, William L.
Blais, Annie	Homer, David L.	Phifer, Robert C.
Blau, Daniel D.	Jeng, Hou-Wen	Reinhardt, Karin R.
Blue, Betsy L.	Kesby, Kevin A.	Schlenke, David O.
Booher, John P.	Kirschner, Gerald S.	Share, Robert D.
Bozman, Christopher K.	Livingstone, Paul R.	Sommer, David B.
Christman, Bryan C.	Maguire, Richard	Thurston, Barbara H.
Chung, Kasing L.	Mann, Katherine A.	Toledano, Mike
Ciardiello, Gary T.	Marlo, Leslie R.	Vaughan, Therese M.
Daly, Michael K.	Mathre, Keith A.	Violette, Jennifer A.
Debigare, Manon	Mattiolo, Maria	Welch, John P.
Dubin, Michael C.	McIntyre, Thomas S.	White, Robert J.
Dumas, Francois	Mize, John H.	Wong, Windrie
Fung, Charles	Morissette, Francois L.	

The following is the list of successful candidates in examinations held in November 1991.

Abellera, Daniel N.	9	Allen, Danny M	9	Arnold, Richard T.	5A
Abian, Rimma	5A	Allen, Kay L.	3B	Asdhir, Bhim D.	7
Abramek, Kimberly M.	5A	Allen, Robert E.	3B	Atkinson, William M.	3B
Ackerman, Shawna S.	5	Alnes, Ann L.	7	Attoh-Okine, Ashaley N.	3B
Addiego, Mark A.	3B,5A	Alwis, K. Athula P.	5	Babcock, Helen A.	3B
Adee, Marc J.	9	Aman, Timothy P.	7	Banick, Timothy J.	5
Adkisson, Jonathan D.	5	Anderson, Kevin L.	3B	Barnes, Katharine	9
Agerton, Vicki L.	3B	Anderson, Larry D.	3B	Barone, Paul C.	3B
Ahearn, Elise M.	7	Andrzejewski, Jennifer A.	5A	Barre, Dana	5
Albright, Kristen M.	7	Angelina, Michael E.	5A	Bartie, James M.	3B
Allan, Christopher R.	3B	Annino, John A.	3B	Bault, Todd R.	7

Bazin, Dominic 5A
Beatman, Robert S. 5A
Beaulieu, Gregory S. 9
Becker, Allan R. 9
Begin, Nathalie 9
Behbahany, Saeeda 5
Benarosch, Xavier 9
Bentley, Cynthia A. 3B
Bergeron, Lyne 38
Berra, Paul S., Jr. 3B
Berry, Dan J. 3B
Besman, Eric D. 5A
Bibbero, Herbert S. 7
Bilot, Corey J. 3B,5
Bilotti, Frank J. 3B
Binkowitz, Abbe 5A
Binnig, Bruce E. 5A
Bisbee, Lisa J. 3B
Blackburn, Wayne E. 7
Blais, Annie 7
Blakeney, Gina L. 5B
Blanco, Roberto G. 9
Blau, Daniel D. 7
Blinn, Ming Y. 5A
Blondin, James F. 3B
Blue, Betsy L. 5A
Bohland, Jeannette M. 3B
Bond, Christina M. 3B
Bonds, Caleb M. 3B
Bono, Donna M. 3B
Bonsignore, John T. 5A
Booher, John P. 7
Borchert, Elizabeth S. 3B
Borden, Sara E. 5
Bosniack, Lesley R. 3B
Bouchard, Lloyd J. 5B,7
Bowen, Christopher L. 5A
Bowron, Lee M. 3B
Bozman, Christopher K. 7
Bradley, George P. 7
Bradley, Lori M. 5
Brannon, Michael D. 3B
Brasley, Donna D. 9
Brauner, Jack 9
Bray, David T. 5A
Brazee, Kevin J. 3B
Brieden, Lisa M. 5A
Brockmeier, Linda M. 5
Brooks, Ward M. 7
Brosius, J. Eric 7
Browan, David J. 5A
Brown, Stephanie J. 7
Bruce, Stephen J. 5A
Buckie, Patrick R. 3B
Burchett, Peter V. 3B
Burgess, Mark E. 3B
Burkart, Marian M. 5A
Burke, Anthony J. 9
Burnaford, Laura L. 5A
Burns, Patrick J. 9
Bush, Tara E. 5A
Butcher, John F., II 5A
Callas, Linda E. 5
Calton, James E. 5
Cappelletti, Anthony E. 7
Carlton, Kenneth E. 9
Carpenter, Michael E. 3B,5A
Carrier, Martin 5A
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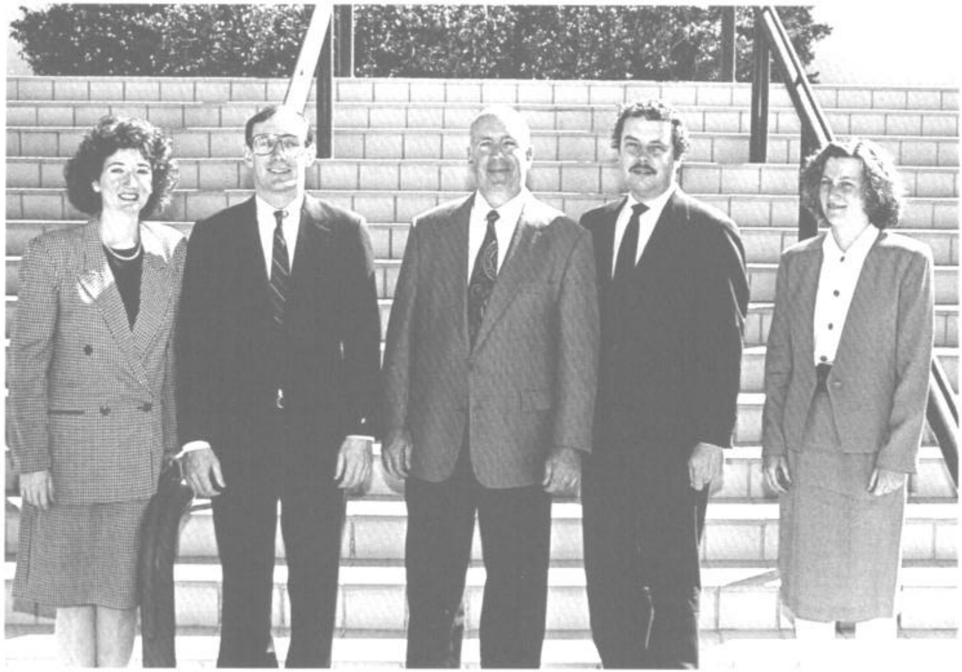
(Admitted in May 1991)
Group I

NEW ASSOCIATES

(Admitted in May 1991)
Group II

NEW FELLOWS

(Admitted in May 1991)



NEW ASSOCIATES

(Admitted in November 1991)

NEW FELLOWS

(Admitted in November 1991)

OBITUARIES

WILIIAM LESLIE, JR.

LEWIS A. VINCENT

HARRY V. WILLIAMS

WILLIAM LESLIE JR.
1918–1990

William Leslie Jr., a Fellow of the Casualty Actuarial Society since 1950, died on August 22, 1990, at the age of 71.

Mr. Leslie, an insurance executive, was general manager of the National Bureau of Casualty Underwriters and the National Council on Compensation Insurance. In his 40-year career in the insurance business, Mr. Leslie worked for Royal-Globe and the Continental Corporation in New York. After retiring from Continental as executive vice president, he joined Tillinghast, Nelson & Warren in Newport Beach, California, and was a director of Industrial Indemnity in San Francisco.

Mr. Leslie was President of the Casualty Actuarial Society and a charter member and Vice President of the American Academy of Actuaries.

A native of San Francisco, he was a graduate of Princeton University and attended Columbia Law School. In World War II, he was a naval aviator and flight instructor. In the Korean War, he was secretary of the National Defense Projects Plan, which negotiated with the Defense Department on behalf of the insurance industry.

LEWIS A. VINCENT
1905–1990

Lewis A. Vincent, a Fellow of the Casualty Actuarial Society since 1951, died on July 27, 1990, at the age of 84.

Lewis graduated from West Point in 1928 and accepted a Reserve commission in the Corps of Engineers, attaining the rank of Captain, C.E. Reserves. He then became a field engineer in the Chicago office of the National Board of Fire Underwriters. Four years later, he was transferred to New York as part of the general manager's staff. In 1937, Mr. Vincent was made assistant secretary. Five years later, he became assistant to the general manager. In 1945, he was in charge of the National Board's actuarial bureau and organized its present base of statistics. He became vice president of Continental Insurance Company at age 60.

During World War II (1940–45), Mr. Vincent was in Washington, D.C. on loan to the Office of the Chief of Engineers as a consultant to the War Department on fire protection. He also served as secretary of the committee that wrote the government's manual on fire protection for civil defense. In 1947, he was Assistant Executive Director of the President's Conference on Fire Prevention.

Lewis was a trustee of Underwriters Laboratories, Inc., a Director, Vice President and General Manager of the National Board of Fire Underwriters Building Corporation, a Director of Sanborn Map Company, a Director of First Pelham Corporation, a Director of Insurance Data Processing Center, a Director of American Standards Association, a Director of Insurance Society of New York, and a Director of the New Jersey State Safety Council.

Mr. Vincent was a member of the Society of American Military Engineers; a member of the Montclair, New Jersey Society of Engineers; member-at-large of the National Council of Boy Scouts of America; associate member of the Loss Executives Association; an honorary member of the Society of Insurance Accountants; member of the Army and

Navy Club (Washington, D.C.), California Lodge No. 1 F and A.M., Drug and Chemical Club of New York, Glen Ridge Country Club (Glen Ridge, New Jersey), Union League Club (Chicago), and the West Point Society of New York.

Lewis received the Gold Medal Award of the General Insurance Brokers Association of New York, Inc. in 1962, and was elected to membership in the Insurance Honor Society, Iota Nu Sigma, of New York University.

In 1970, Lewis retired from the Continental Insurance Company. He and his wife, Amy, moved to New London, New Hampshire, where he did volunteer work. In 1986, they moved to Dunwoody Village, a retirement home in Newtown Square, Pennsylvania.

HARRY V. WILLIAMS
1907–1991

Harry V. Williams, a Fellow of the Casualty Actuarial Society since 1935, died on May 10, 1991, at the age of 83.

Harry was a colleague of Matt McConnell at the National Council on Compensation Insurance when they both attained Associateship in 1934 and Fellowship in 1935. He was named statistician of the National Council in 1937. In 1939, he moved to the Hartford Accident and Indemnity, where he spent the remainder of his career. Given steadily increasing responsibilities, Harry became Vice President in 1955. He was chosen President of the Hartford Group in 1965, and President and Chairman of the Board in 1967. He held the position of President until 1973, and Chairman until 1976, and retired as a director in 1977.

Harry was elected to the CAS Council (Board) in 1943, and after two years of his three-year term he was elected Vice President. He served in that office until 1947. He was again elected to the Council for a three-year term in 1953.

Outside the CAS, Harry Williams served as the chairman of the American Insurance Association and the Health Association of America. He was also president of the National Automobile Underwriters Association.

Harry was very much in the limelight in 1969 during the negotiations between ITT and the Hartford Group to bring about one of the largest mergers on record. Subsequently, he was made a member of the ITT Board.

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