VOLUME LXXV NUMBERS 143 AND 144

PROCEEDINGS

OF THE

Casualty Actuarial Society

ORGANIZED 1914

1988
VOLUME LXXV
Number 143 — May 1988
Number 144 — November 1988

COPYRIGHT — 1989
CASUALTY ACTUARIAL SOCIETY
ALL RIGHTS RESERVED

Library of Congress Catalog No. HG9956.C3
ISSN 0893-2980

This publication
is available in microform

University Microfilms International

300 North Zeeb Road	30-32 Mortimer Street
Dept. P.R.	Dept. P.R.
Ann Arbor, Mi. 48106	London WIN 7RA
U.S.A.	England

Printed for the Society by
Recording and Statistical Corporation
Boston, Massachusetts

FOREWORD

The Casualty Actuarial Society was organized in 1914 as the Casualty Actuarial and Statistical Society of America, with 97 charter members of the grade of Fellow; the Society adopted its present name on May 14, 1921.

Actuarial science originated in England in 1792, in the early days of life insurance. Due to the technical nature of the business, the first actuaries were mathematicians; eventually their numerical growth resulted in the formation of the Institute of Actuaries in England in 1848. The Faculty of Actuaries was founded in Scotland in 1856, followed in the United States by the Actuarial Society of America in 1889 and the American Institute of Actuaries in 1909. In 1949 the two American organizations were merged into the Society of Actuaries.

In the beginning of the twentieth century in the United States, problems requiring actuarial treatment were emerging in sickness, disability, and casualty insurance—particularly in workers' compensation—which was introduced in 1911. The differences between the new problems and those of traditional life insurance led to the organization of the Society. Dr. I. M. Rubinow, who was responsible for the Society's formation, became its first president. The object of the Society was, and is, the promotion of actuarial and statistical science as applied to insurance other than life insurance. Such promotion is accomplished by communication with those affected by insurance, presentation and discussion of papers, attendance at seminars and workshops, collection of a library, research, and other means.

Since the problems of workers' compensation were the most urgent, many of the Society's original members played a leading part in developing the scientific basis for that line of insurance. From the beginning, however, the Society has grown constantly, not only in membership, but also in range of interest and in scientific and related contributions to all lines of insurance other than life, including automobile, liability other than automobile, fire, homeowners and commercial multiple peril, and others. These contributions are found principally in original papers prepared by members of the Society and published in the annual *Proceedings*. The presidential addresses, also published in the *Proceedings*, have called attention to the most pressing actuarial problems, some of them still unsolved, that have faced the insurance industry over the years.

The membership of the Society includes actuaries employed by insurance companies, rate-making organizations, national brokers, accounting firms, educational institutions, state insurance departments, and the federal government; it also includes independent consultants. The Society has two classes of members, Fellows and Associates. Both classes are achieved by successful completion of examinations, which are held in May and November in various cities of the United States and Canada.

The publications of the Society and their respective prices are listed in the *Yearbook* which is published annually. The *Syllabus of Examinations* outlines the course of study recommended for the examinations. Both the *Yearbook*, at a $20 charge, and the *Syllabus of Examinations*, without charge, may be obtained upon request to the Casualty Actuarial Society, One Penn Plaza, 250 West 34th Street, New York, New York 10119.

JANUARY 1, 1989

*EXECUTIVE COUNCIL

KEVIN M. RYAN .... *President*
MICHAEL FUSCO .... *President-Elect*
ROBERT F. CONGER .... *Vice President-Administration*
CHARLES A. BRYAN .... *Vice President-Development*
MICHAEL L. TOOTHMAN .... *Vice President-Membership*
RICHARD I. FEIN .... *Vice President-Programs*

THE BOARD OF DIRECTORS

**Officers*

KEVIN M. RYAN .... *President*
MICHAEL FUSCO .... *President-Elect*

†Immediate Past President:

DAVID G. HARTMAN 1989

†Elected Directors:

IRENE K. BASS 1989
ALLAN M. KAUFMAN 1989
LEROY J. SIMON 1989
DAVID SKURNICK 1989
ALBERT J. BEER 1990
ALAN C. CURRY 1990
CHARLES L. MCCLENAHAN 1990
JEROME A. SCHEIBL 1990
WALTER J. FITZGIBBON, JR. 1991
CHARLES A. HACHEMEISTER 1991
STEVEN G. LEHMANN 1991
LEE R. STEENECK 1991

---

* Term expires at 1989 Annual Meeting. All members of the Executive Council are Officers. The Vice President-Administration also serves as Secretary and Treasurer.

† Term expires at Annual Meeting of year given.

1988 PROCEEDINGS
CONTENTS OF VOLUME LXXV

1988 PROCEEDINGS
CONTENTS OF VOLUME LXXV—CONTINUED

NOTICE

Papers submitted to the *Proceedings of the Casualty Actuarial Society* are subject to review by the members of the Committee on Review of Papers and, where appropriate, additional individuals with expertise in the relevant topic. In order to qualify for publication, a paper must be relevant to casualty actuarial science, include original research ideas and/or techniques or have special educational value, and must not have been previously published or be concurrently considered for publication elsewhere. Specific instructions for preparation and submission of papers are included in the *Yearbook* of the Casualty Actuarial Society.

The Society is not responsible for statements of opinions expressed in the articles, criticisms, and discussion published in these *Proceedings*.

Volume LXXV, Part 1 No. 143

PROCEEDINGS

May 8, 9, 10, 11, 1988

RECENT DEVELOPMENTS IN RESERVING FOR LOSSES IN THE LONDON REINSURANCE MARKET

HAROLD E. CLARKE

Abstract

The paper describes in detail a new method which can be applied by any insurance company to its own data to set reserves for outstanding losses (including IBNR) and to calculate a confidence interval for these reserves. The method has also opened up a whole range of interesting ways of looking at data. Although the method can be applied to any sort of business, it is particularly helpful in looking at long tail business, such as that written by reinsurers, for which other methods have proved less satisfactory. The methodology can also be applied by a supervisory authority to establish minimum reserving standards for companies where global general market data on run-offs for different classes of business is available. A new method of setting minimum reserves for individual syndicates based on the methodology in the paper is currently being tested by Lloyd's of London. This work is briefly described in the final section of the paper.

1. INTRODUCTION

This paper describes a system which our firm has developed and refined over the last five years to enable us to comment on reserves set up for outstanding and IBNR claims by companies writing marine, aviation, liability, and reinsurance accounts, or alternatively to advise on such reserves. The companies we have advised have been operating in the London market in the United Kingdom, of which Lloyd's is the center. The London market underwrites a significant part of the world's insurance and, in particular, its reinsurance, and is a dominating influence on insurance worldwide. Although the system described is particularly suitable for reserving for reinsurance accounts, it is also applicable to all other types of casualty business. The system is fully operational on our mainframe computer. It has been used many times and it is stable.

In the London market, details of numbers of claims are generally not available or not relevant. Data is usually available for each "account year," i.e., for all risks written in a particular accounting year which is usually a calendar year. The items normally available are:

(i) Premiums paid to date;
(ii) Claims paid to date; and
(iii) Claims outstanding, i.e., the case estimates for outstanding claims as notified by the brokers to the companies.

Further details of the constraints and problems posed by the data are given in Section 2.

The system had to be able to generate estimates of the reserves from this limited amount of data. The method works by estimating the ultimate loss ratio (ULR) for each account year, from which the necessary reserve is easily derived. An important innovation of the method is that a confidence interval is produced for the ULR and hence for the reserves. An outline of the method is given in Section 3, a detailed example in Section 4, and some further problems and considerations are discussed in Section 5. The method is very graphical and easy to see and present to actuaries and non-actuaries.

In the final section of the paper, Section 6, we describe an application of the method to setting minimum reserves at Lloyd's. This is currently being tested. The method can also be used in that way to set minimum reserves for companies operating in any insurance market where industry-wide statistics are available.

The method starts from an idea put forward in a paper by D.H. Craighead [1] to the Institute of Actuaries. Inside our firm we have considerably refined and extended this idea. A detailed description of the potential use of the method by Lloyd's together with an outline of the general method is given in the paper by my colleagues, S. Benjamin and L.M. Eagles [2], to the Institute of Actuaries. In this paper the emphasis is reversed with considerably greater detail being given about the general method. We also wish to thank A.B. English for the programming and application of the curve fitting algorithm and for much other programming.

2. DATA

As previously mentioned, the data available for setting reserves in the London market is sparser than that usually available from companies writing mainly domestic risks. The reasons are outlined below.

For the risks written in the London market, cover is usually given for one year. The premiums are typically received over a period of three years. This delay can be due, for instance, to excess of loss treaties being rated on a burning cost basis or to delays in monies being forwarded by brokers. The incidents which take place during the year of cover give rise to claims which may not be reported for many years and may then take several years to settle. The main reason for this delay is that the London market tends to deal in reinsurance where the information is "second-hand" in the sense that it comes from a primary insurer which may itself be subject to delays of information. For instance, suppose you are writing a catastrophe excess of loss treaty covering property damage exceeding $10 million in the aggregate for any one incident for a California company. The reinsurer may not hear anything from the California company until its own claims reach the agreed limit. The final outcome for the reinsurer in the London market may then take a long time to become fully known. Further, as this example illustrates, the concept of number of claims is not meaningful in this market.

Also, the risk will often be placed on a coinsurance basis, often with 20 or 30 different underwriters. Detailed data may be available to the leading underwriter, but that detailed information may not be available to others on the risk and will not be recorded centrally. Statistics have, in fact, tended to be subordinate to accounting data, which is therefore the only data commonly available. This has the problem that if an error is discovered in the statistics (e.g., an outstanding claim has been notified in Italian lire rather than U.S. dollars) it will be corrected from discovery, but the history will be left unchanged so that the statistics still reconcile with the published accounts.

The data is usually available for each account year. Thus, the method described in this paper will be presented for data collected on that basis. However, as will become clear, the method is equally applicable to data collected on an accident year basis. It is common for the data to be missing for early account years or early years of development, often due to computerization of the accounting function taking place at that point.

In the case of Lloyd's, further problems arise from the use of very broad risk categories which cannot be assumed to be homogeneous over time. The classic example of this is non-marine all other, which can include marine business written by non-marine syndicates. Further, the data collected centrally consist only of premiums received and claims paid, both net of reinsurance. After the end of the third year of development of an account year, future premiums received are set off against future claim payments in the statistics.

More information on the operation of the London market, in general, and Lloyd's, in particular, is given in the paper by D.H. Craighead [1].

The techniques described in this paper can be applied to gross data, net data, paid losses, or paid plus outstanding losses. That is why we have not defined closely the basis of the data.

3. SYSTEM REQUIREMENTS AND OUTLINE OF METHOD

For the data described in the previous section, most of the reserving methods commonly in use break down. We needed a method which:

(i) Was able to cope with long tail business.
(ii) Would use only information on premiums, paid claims, and claims outstanding as notified.
(iii) Could provide estimates where there were missing items of information from the run-off triangle.
(iv) Could handle multi-currency portfolios. Most of the companies whose reserving we examine write substantial U.S. dollar business even though they report in pounds sterling.

(v) Would enable us to set a range of values within which reserves would be acceptable. After all, no single estimate can be correct unless we have business which has completely run off. We would expect in the early years of development of an account year that the range would be relatively wide and would reduce as development increases.
(vi) Where necessary, would use market information or information from other similar businesses to establish reserves for a particular insurer.

It is vital that any system should be able to cope with all the preliminary data handling and be able to accept data in a variety of formats. In particular, the system needs to be flexible enough to deal with the following variations:

(a) The data can be either cumulative or incremental.
(b) Claims data can show paid claims and claims outstanding either separately or summed, and can be expressed either as loss ratios or cash.
(c) Development time intervals can be either quarters, half-years, or years.
(d) The data may be presented in a number of different currencies which the system must be able to combine at the user's discretion. (When currencies are combined, uniform exchange rates are assumed to apply for all periods of origin and development.)
(e) The data may be provided for a large number of separate categories in a variety of currencies. Again, at the user's option, the system must be able to combine any or all of the categories.

The system needs to be able to accommodate a variety of currencies because the London reinsurance market writes business internationally. It therefore accepts business in a wide variety of currencies. It is possible for a company to keep separate statistics for each of the currencies in which it does business. In practice, it is usual for a company to keep statistics in three currencies: U.S. dollars, Canadian dollars, and pounds sterling. In this case, currencies other than the first two are converted into pounds sterling at the exchange rate applying at the date of the relevant transaction.

A major consideration underlying our whole approach is that, for the classes of business we are considering, standard assumptions (e.g., homogeneous account from year to year, standard payout pattern, no change in speed of claims advice, etc.) would almost certainly all be violated. This suggests as a basic starting point that we examine the run-off of each account year separately. It also suggests that we look at the development of loss ratios rather than losses. Empirical considerations suggest that if we are seeking a smooth curve to fit the shape of the loss ratio at development time t, plotted against t, that curve will have a negative exponential shape.

In the remainder of this section we outline the reserving method we have developed to meet the above criteria. A worked example of the method is then given in Section 4 to expand on the outline.

(a) Run-off triangles are drawn up for as many account years as possible showing the development year by year (or quarter by quarter) of premiums and claims.
(b) An estimate of the ultimate premiums receivable is made for each account year. If we have to calculate the estimate, then we simply apply development factors calculated from the data without smoothing. Other methods could be used in appropriate circumstances. Often we use the underwriters' estimates, since they have a better feel for the way, in practice, policies are being signed down.
(c) The estimates of ultimate premiums are divided into the relevant claims to give a run-off triangle of loss ratios.
(d) Separately for each account year for which there is sufficient development (this depends on the length of the tail of the business), a curve of negative exponential form is fitted to the loss ratio development for that account year. From this curve a preliminary estimate of the ULR for that account year can be made. In certain cases we can fix some of the parameters in the negative exponential curve from our knowledge of the values of the parameters for the same class of business in other companies, or on an industry-wide basis. In the remainder of the paper this part of the process is referred to as "curve fitting."
(e) For each year of development, e.g., year r, we combine the results obtained in (d) to give a table of the loss ratios at the end of year r and the corresponding estimated ULR's. A line is

fitted to these points by standard linear regression techniques. Given the loss ratio at the end of development year r, a best estimate of the ULR for that account year can be obtained from the fitted line. A confidence limit for the ULR can also be obtained. In the remainder of the paper this part of the process is referred to as "line of best fit."

For an account year which is well developed, the estimate of the ULR is obtained from (d), so no range is quoted or usually needed. For a year with little development, the ULR and accompanying confidence interval from (e) is quoted. For intermediate years the method depends on one's judgment.

4. WORKED EXAMPLE TO ILLUSTRATE METHOD

The approach outlined in the previous section is illustrated below by means of an example based on typical medium tail data. The data is available for account years by quarters of development up to 1st July 1985. This is the date at which the reserves for outstanding claims are being calculated. For early years of development for the earlier account years the data is missing. It will be seen that this does not cause a problem to the system. The accompanying exhibits contain computer produced tables and graphs for the example. These are typical of the output produced by the computer system.

Estimating Ultimate Premiums

In this example we assume that no premiums are received after the end of development year 5. We thus need to estimate the ultimate premiums to be received for account years 1981 to 1984. (1985 is omitted from our consideration since halfway through the year is too early to establish reserves using this method). The estimates of ultimate premiums are given in Exhibit 1. The numbers above the dotted line are the cumulative premiums to date. The numbers below the dotted line are the estimates of cumulative premiums for future development years estimated by development factors. Thus, for each account year, the last number in the column of data for that year is the estimate of total premiums receivable that we intend to use for that year.

Triangle of Loss Ratios

The estimates of total premiums are divided into the cumulative development of incurred claims (i.e., claims paid plus notified claims outstanding) to generate the cumulative incurred loss ratios, based on ultimate premiums. Details of the loss ratios are given in Exhibit 2.

Estimation of ULR by Curve Fitting

We now make a first estimate of the ULR's for each account year by fitting a suitable curve to the loss ratio development for that account year. Over the years we have tried a number of different families of curves for this purpose. The family of curves should satisfy the two criteria:

(i) For an account year where the ULR is already known with a fair degree of certainty, the curve must level out at a value near that loss ratio.

(ii) For later account years the curve must fit the known data well and also allow for a reasonable amount of future development. In most cases this will mean a development period similar to the more fully developed years.

The curve we have found most suitable is:

$$L_t = A \times [1 - \exp(-[t/B]^C)].$$

where t is the development period and L_t is the loss ratio for that development period. There are three parameters: A, B, and C. A determines the ULR while B and C determine the length of the tail and the way in which it approaches the ULR. The curve was originally suggested in a paper by D.H. Craighead [1]. In Exhibits 3 and 4 we give examples of the effect on the shape of the curve of changing the parameters B and C. These illustrate the wide variety of run-off shapes which can be fitted by this curve.

This family of curves is used to give estimates of ULR's for account years 1971 to 1981. For later years, not enough development has yet taken place for a satisfactory curve to be fitted. In Exhibits 5 to 15, we give the graphs of the curves fitted (the solid curves) in this example together with the developed loss ratios. Each loss ratio is represented by a vertical line, with the dotted line joining up the developed loss ratios. The goodness of fit can be tested by eye by comparing the closeness of the dotted and solid curves. The comparison should obviously concentrate on the later years of development. At the bottom of each curve we give the values of A, B, and C fitted together with the mean squared error. In this particular example, C was set equal to 1.5 and only A and B were fitted. We discuss the selection of the parameters to be fitted and the choice of the developed loss ratios to be included in the fitting in Section 5. The graphs need not be studied in detail but should just be looked through quickly to see how well, in general, the curves fit the data.

On occasion, we have found that the graph produced by the computer does not suggest a smooth curve. Particularly when looking at incurred loss ratios we have found that the development can oscillate violently. An advantage of the system is that since it presents this in visual form it can be discussed with the underwriter. The most common explanations we have found for odd patterns are:

(a) Miscoding of data either by currency or category,
(b) Data corrections that have not been carried back to the beginning of the account year, and
(c) Delays in reinsurance recoveries.

Thus, the system is acting as a powerful check on the data.

In the particular example being used, it appears likely that initially some claims for 1978 development year 7 and 1980 development year 5 have been coded as 1979 development year 6 with the error not being fully corrected retrospectively.

Estimation of ULR's by "Line of Best Fit"

We have, so far, analyzed the run-off of one account year at a time. We now analyze the run-off by examining one development year at a time for all account years together. Thus we use all the information in the run-off triangle. For example, at development year 3 we have the following data:

Account year	Loss ratio at development year 3	Estimate ULR from previous curve fitting
1973	53.1%	91.0%
1974	65.8	92.1
1975	50.3	75.4
1976	43.6	70.2
1977	46.2	70.0
1979	73.5	103.8
1980	40.4	69.6
1981	39.1	72.2

Account years 1971 and 1972 are omitted because the loss ratios for early development years are missing, and 1978 is omitted because the run-off curve for that year seems to be a different shape from the other years.

The points are then plotted and the plot is examined to see if there is a statistically significant relationship between the loss ratio at development year 3 and the ULR. The method we use is to fit a regression line and test whether the gradient is significantly different from zero. In this case the regression line is:

$$\text{Estimated ULR} = 1.002 \times \text{Year 3 Loss Ratio} + 29.00\%.$$

The fitted line is shown in Exhibit 16, together with the eight points to which it was fitted.

To test if the gradient is significantly different from zero, we use a t-test, with the degrees of freedom equal to the number of points fitted minus two. In this case, we have $t_6 = 6.55$, which is significant at the 99% level. Thus, the line is a good fit and the gradient is non-zero, which supports the evidence available from inspection of the fitted line. As a general rule, as well as applying the t-test, one should also look at the graph of the relevant regression line to check that it appears reasonable to assume that the shape is significantly different from zero.

From the fitted line we can estimate the ULR for 1983 (where development year 3 is the latest known loss ratio) as:

$$\begin{aligned} \text{1983 ULR} &= 1.002 \times 39.57\% + 29.00\% \\ &= 68.65\%. \end{aligned}$$

Since we have fitted a regression line we can also construct a confidence interval for this estimate of the ULR. There are two alternative methods, one empirical and the other mathematical.

The empirical method is to take the historical point furthest from the regression line and state that the true result for the year is unlikely to fall outside the historical maximum. This gives a likely variation of the result of ± 8.8% in this particular case.

The mathematical method is to derive the statistical confidence interval from the regression line fit. We have found that a 90% confidence interval does the right job for our analyses of individual portfolios. This gives a confidence interval in the example of ± 10.9%. Obviously the width of the confidence interval depends on where the point lies on the regression line.

The choice of method is a matter of personal preference. The advantage of the maximum deviation is the ease of presentation to the underwriter of the rationale for the range. The advantage of the second method is that it is statistically based and does allow properly for the number of points to which the line is fitted. It should be noted that underlying the second method as well as the t-test is the assumption that the underwriting results for different account years are independent, identically distributed, random variables. Such limited investigations as we have carried out suggest that this is a reasonable assumption.

We have found in a number of cases that the gradient of the regression line is not significantly different from zero. This is particularly likely to be true for the most recent years of account. This implies that there is no correlation between the loss ratios at (say) year 3 and the ULR. In this case we would estimate the ULR as the average of the historic ULR's and obtain a confidence interval using the maximum deviation. In such a case it would obviously be desirable to adjust the ULR's to allow for changes in premium rates that may have taken place. However, in the London reinsurance market, the effects of changes in limits of cover, etc., make this a very difficult exercise to carry out. The fact that no correlation exists also tells us something very useful about the data for that account year. It says that effectively there is no information in the data showing the development of the account year so far to indicate how the year will ultimately turn out. Although this is a negative statement, we feel that it is a fact that is often not fully appreciated by management, particularly with regard to long tail business. However, in these cases it can usually be clearly demonstrated by the plots of loss ratios against ULR's that there is no relationship between the position at the end of the particular year of development and the ultimate outcome.

For our example, the regression lines fitted for development years 2 to 10 together with the account years for which they are fitted are shown in Exhibits 17 to 25. Looking at the regression lines you will see how the fit gets better as the development year increases. When we reach the year of development where the "tail" of claims has effectively run off, the loss ratio will equal the ULR. The regression line will pass through the origin of the graph and the slope of the line will be "1 in 1," i.e., 45°. You will see from Exhibit 25 that, for the class of business being used for the example, this position has almost been reached by the end of year 10. A summary of the lines fitted and the statistics is given in Exhibit 26. From this you will see that for 1984 it is not appropriate to fit a regression line, since the t-test statistic is not significant at the 95% level. Thus, for this year, an average ULR was used as described above. It will be seen from Exhibit 26 that the slopes of the regression lines range from about 0.7 to 1.5. The value of the slope can be interpreted as an indication of the "gearing" between the loss ratio at a particular development year and its ultimate value. Thus, if the slope is greater than one this means that, if you have a "bad" loss ratio at a particular point, the year will ultimately be proportionately much worse than if you had a "good" loss ratio at the same point. If the slope is less than one, the converse holds.

Final Estimates of ULR

In this example we consider that the estimates of ULR obtained from the curve fits are the appropriate ones to use for account years 1971 to 1977. Clearly, for the early account years, one cannot use the regression lines to estimate the ULR's since the lines would be based on too few data items to be credible. For account years 1979 to 1984, the results from the line of best fit seem most suitable. As previously stated, for 1978 the position is difficult because the shape is different from the other account years and we have therefore used the curve fit. Although no confidence interval can be calculated for this year, it is obvious from looking at the curve fit that, in order to convey the correct information to management, one should be quoted. This has arbitrarily been taken to be the same as 1979. We have on this occasion used 90% confidence intervals rather than maximum deviation intervals. The final results of the analysis are set out in Exhibit 27.

Further Considerations

We have already mentioned how this approach suggests how much information about the ULR is contained in the development to date of the relevant account year. The other useful thing that we find comes out of this approach is that it shows senior management that the estimate of the ULR is just that—an *estimate*. Thus, the actual result will be better or worse than that estimate. The confidence intervals give an indication to senior management of the range in which the result will lie. If the reserving model is correctly specified, then the confidence intervals will be accurate. In practice, the model is probably not specified exactly correctly, so the confidence intervals only give an indication of the likely range of possible outcomes. Despite this proviso, the confidence intervals do enable the management to assess the implications of establishing reserves based on particular estimates of the ULR. The closer to the upper limit of the ULR that the reserve is established, the more likely it is that the reserve will turn out to be more than adequate and the excess may be released as a profit in the future. The nearer to the lower limit of the range of the ULR that the reserve is established, the more likely that the reserve will turn out to be inadequate. That would mean that additional cash would have to be found in the future either by restricting dividend payments or raising new capital.

5. FURTHER DETAIL ON THE RESERVING METHOD

In this section we consider some of the practical problems that arise from using the approach to reserving discussed in the preceding two sections and describe some of the methods we have used to overcome these problems. Although a few of these problems and solutions were mentioned in the previous section, we have covered all of them in this section for completeness.

Problems Encountered with Curve Fitting

The exponential curve we fit has three parameters: A, B, and C. Initially, for each account year, we fit the curve allowing all three parameters to vary. This is because a free fit allows the curve to reflect the data as accurately as possible given the constraints of the curve. Further, a free fit permits the curves to reflect any lack of homogeneity in the data. Sometimes, where there is an error in the data or for some other reason, one can find that for particular account years the fit to the early years of development is satisfactory, but it is rather less good to the later years of development. In such cases, we fix either B or C in order to try to make the curve fit the later years of development better at the expense of a worse fit in the earlier years of development. We prefer to fix C as this allows more freedom in the shape of the curve than fixing B. If we have to fix a parameter for a particular account year and if most of the other account years are fitting well on a free fit, then we would take the values of the parameters of those other years into account when deciding on the values of the parameters to be fixed. If the parameters B or C all take similar values, then it is clear that all the account years are fairly homogenous so the choice of B or C is straightforward. In other cases it is less clear. If there is an obvious trend in the parameters, then that can be reflected in the choice of the values for the parameters for the account years for which the parameters have to be fixed. If there is no obvious trend, then it may be possible to obtain from the underwriter an indication of the relative lengths of the tails of the various account years. The judgment can then be incorporated in fixing a value of B or C for a particular account year. Alternatively, we would take into account the values of the parameters we have found suitable for similar classes of business either for other companies or on an industry-wide basis. The point to be emphasized is that, by fixing or not fixing some of the parameters as considered appropriate, one can allow for any homogeneity or lack of it in the data and also incorporate additional outside information.

As already mentioned, we do not fit curves to recent account years since, for such years, there is insufficient development to permit a curve to be fitted. For longer tail categories, we usually omit the first 8 to 12 quarters of data in fitting the curve to ensure that the fit is reasonable to the later development. This also solves the problem that for some of the earlier account years this early development can be missing from the data. Finally, we sometimes find that the curve is approaching the value of A slowly so that A is probably too high an estimate of the ULR. In such cases we assume that the development is completed after a reasonable period, say 15 to 20 years for the longer tail classes, and take the value of L_t for that development period as the estimate of the ULR.

Problems Encountered with "Line of Best Fit"

One important problem that is often encountered is where a particular account year has a significantly different speed of development from all the other account years for that class. This may be due, for example, to writing a peculiar treaty or treaties in that year. That such a thing is happening is usually clear from the graphs of the curves and the reason can often be found from discussion with the underwriter. In these cases, that account year is omitted from the calculation of the line of best fit. A good example of this was the omission of account year 1978 from the calculation of the lines of best fit in the previous section.

Another problem is where the data is very variable, particularly in the early years of development, so that there are significant random fluctuations on top of the basic run-off pattern. In this case, we have found that it is better to use the developed loss ratios obtained from the fitted curves rather than the actual values. This smoothes out the random fluctuations which one may consider are not being repeated in the account year for which one is using the line of best fit to calculate a ULR. Alternatively, the data for early years of development for some account years may be missing, and using the modelled data will permit the inclusion of those years in the calculation of the line of best fit. Because of the smoothing that takes place with modelled data, it will be found that the confidence intervals are narrower than those brought out by using the unadjusted data. They should, therefore, either be quoted with a cautionary note that they underestimate the true amount of fluctuation or not quoted at all.

It is interesting to compare the line of best fit approach with the approach using development factors. The development factor approach is equivalent to fitting a line for ULR against developed loss ratio that passes through the origin. Our experience is that for early years of development the lines of best fit often miss the origin by a wide margin.

However, as one progresses to the lines of best fit for the later development years, they become closer and closer to lines through the origin. If in looking at some lines of best fit we do not see this pattern, then this suggests that something is awry. The most probable reason is an error in the data.

As will be apparent from the example and the above discussion, the method is not an automatic method for setting loss reserves. It requires one to use one's judgment at all stages of the process. In particular, we have found that a careful study of the graphs of the curve fits and the linear regressions is very important in deciding upon an appropriate best estimate of the ULR and the accompanying confidence intervals. Although the method described uses a curve fitting approach to obtain the initial estimates of ULR's, there is no reason why alternative methods, as for example described in the paper by J.R. Berquist and R.E. Sherman [3], should not be used to obtain these initial estimates. However, we would emphasize that in practice we have found the curve fitting approach to be very flexible and more than adequate for calculating values of ULR's to use in the line of best fit. The alternative methods are found to be more necessary to assist in estimating the ULR's for the early account years where the line of best fit is not going to be used as part of the estimating process.

6. APPLICATION OF METHOD TO LLOYD'S

One important use of the method we have developed, and, in fact, one of the reasons for developing it, was to provide a new method for calculating the minimum reserves to be established by Lloyd's syndicates. This is described in considerable detail in the paper by S. Benjamin and L.M. Eagles [2] and we shall therefore give only a brief outline of the method for setting minimum reserves here.

The syndicates in Lloyd's are the bodies in Lloyd's equivalent to companies that underwrite the risks. Collectively, the syndicates constitute Lloyd's. The syndicates each maintain their own statistics and certain statistics are also collected centrally. Among other things, the central statistics are used to help set minimum levels of the reserves to be established by the syndicates for each account year.

The current method of setting minimum reserves is by the use of the "Lloyd's audit percentages" which are set by Lloyd's centrally. Under this present method, percentages are supplied for use as of the end of each calendar year, separately for each class of business and each account year in which business was written. The minimum reserve for claims outstanding and IBNR at the end of that calendar year for the class of business and account year is the premium advised to date multiplied by the relevant percentage. Thus, the minimum level for the total claims expected to be paid by the syndicate is the claims paid to date plus the minimum reserve. Suppose under the present method the paid loss ratio to date is 10% and the audit percentage for the minimum reserve is 78%. Then under the present method we have

$$\begin{aligned} &\text{Paid Loss Ratio} &&= 10\% \\ &\text{Reserve (Audit Percentage)} &&= \underline{78\%} \\ &\text{(Implied) ULR} &&= 88\% \end{aligned}$$

It will be clear that this method does not reflect the progress of the individual syndicates.

Under the proposed new method, two figures are used instead of one. In this particular case, instead of 78% the two figures are 3.4 and 33% and the calculation is as follows:

$$\begin{aligned} &\text{ULR} &&= 3.4 \times \text{Paid Loss Ratio} + 33\% && \\ & &&= 3.4 \times 10\% + 33\% &&= 67\% \\ &\text{Paid Loss Ratio} &&= && \underline{10\%} \\ &\text{(Implied) Reserve} &&= && 57\% \end{aligned}$$

Thus, two figures are provided for each class of business and account year for which currently one audit percentage is provided. The proposed new method has been tried on a limited experimental basis for three years. The evidence so far is favorable and the experiment is currently being widened to cover the whole market.

The two figures under the new method are calculated by applying the general method described in the preceding sections to the data collected centrally at Lloyd's for each class of business. For each class of business, if one carries out that process, one produces for each account year or year of development a line of best fit, together with an associated confidence interval, based on the point furthest from the line of best fit. The two numbers under the proposed method are the parameters that define the line of best fit. Thus, in the example, 3.4 gives the slope of the line and 33% its intercept on the vertical axis.

There was considerable discussion inside the working party which reported to the Audit Committee as to whether the line of best fit or one of the other lines should be used to set minimum reserves. In the end, the upper edge of the confidence interval seemed too high, the lower too low. The use of the line of best fit as a minimum allowed one to say that the total reserves set up in Lloyd's were at least as great as the average indicated by past experience, which seemed to be a useful statement to make. Underlying this approach to setting reserves is the assumption that for any class of business, the business written by a syndicate will be similar to that "written" by all of Lloyd's combined. Incorporating the paid loss ratio in the calculation of the ULR in the way proposed then allows the quality of the business written by a particular syndicate to be reflected in the ULR in what appears intuitively to be a reasonable way. Also, the new method would be easy to implement, requiring very little change by individual syndicates in the work they carry out.

In addition to being provided with the new figures for calculating the minimum reserves, the syndicates are also provided with graphs for each class of business and year of development showing:

(i) The lines of best fit together with the lines based on the point furthest from the line of best fit, and
(ii) The historical range of paid loss ratios.

Thus the syndicates are provided with graphs looking like Exhibit 28. The syndicates are being encouraged to plot their own data on the graphs to see how their experience compares with that of all of Lloyd's combined. It is hoped that as a result they will obtain useful information about their experience. For example if a syndicate's own path were narrow and different from the all-Lloyd's path, then that would demonstrate in a very vivid way that it was writing a different class of business.

Clearly, this approach can be adopted by any supervisory authority which wishes to set reserving standards for companies where global general market data of run-offs for the different classes of business is available.

REFERENCES

[1] D.H. Craighead, (1979) "Some Aspects of the London Reinsurance Market in Worldwide Short-Term Business," *J.I.A.* Vol. 106, Part III.

[2] S. Benjamin and L.M. Eagles, (1986) "Reserves in Lloyd's and the London Market," *J.I.A.* Vol. 113, Part II.

[3] J.R. Berquist and R.E. Sherman, (1978) "Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach," *P.C.A.S.* Vol. LXV.

EXHIBIT 1

ESTIMATION OF ULTIMATE PREMIUMS
DEVELOPMENT QUARTER 2

Account Year:

Development Year	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984
1				2,706	3,714	3,751	5,550	6,580	6,774	9,098	12,214	11,611	15,541	20,082
2			3,524	4,489	5,869	6,439	8,475	9,712	9,797	13,173	17,839	17,901	23,250	29,902
3		3,355	3,924	4,821	6,393	7,109	8,800	10,083	10,670	14,613	19,927	19,322	25,602	32,928
4	5,189	3,373	4,040	4,876	6,473	7,067	8,894	10,142	10,978	15,123	20,570	19,967	26,457	34,027
5	5,240	3,415	3,999	4,928	6,521	7,081	8,942	10,161	11,035	15,356	20,887	20,275	26,865	34,551
6	5,126	3,432	4,027	4,894	6,557	7,065	8,981	10,250	11,147					
7	5,279	3,449	4,024	4,911	6,570	7,091	9,006	10,329						
8	5,297	3,446	4,040	4,917	6,592	7,046	9,030							
9	5,300	3,452	4,035	4,896	6,580	7,070								
10	5,301	3,454	4,036	4,898	6,585									
11	5,288	3,455	4,037	4,894										
12	5,286	3,476	4,027											
13	5,284	3,474												
14	5,284													

EXHIBIT 2
LOSS RATIOS

Quarter of Development	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984
	Accident Years													
1					0.1	-0.6	1.2	0.1	[illegible]	0.0	0.1	0.0	0.1	0.9
2					0.8	-0.4	2.3	0.5	2.1	1.0	0.4	3.0	2.0	2.3
3					2.9	4.5	5.8	1.8	7.1	4.2	1.4	7.0	5.0	5.0
4				7.7	7.0	5.1	8.1	3.4	9.6	12.1	6.5	6.3	12.2	11.6
5				14.5	18.7	12.0	12.4	7.5	17.7	2.0	9.3	8.6	17.1	12.1
6				25.5	22.5	15.7	21.1	12.5	21.4	10.1	17.2	11.3	20.7	23.0
7				40.0	30.8	22.6	30.0	19.6	31.2	20.7	25.1	17.4	25.2	
8			41.6	48.2	39.1	37.3	35.6	26.0	63.[illegible]	29.9	30.7	25.4	29.6	
9			47.7	60.8	45.6	38.5	42.0	38.8	64.6	32.9	36.5	3.08	34.5	
10			53.1	65.8	50.3	43.6	46.2	40.7	73.5	40.4	39.1	33.5	39.6	
11			59.9	71.6	55.9	48.2	49.2	40.8	77.3	47.2	45.2	40.3		
12		55.9	64.9	76.0	61.5	52.3	51.0	46.8	75.5	44.2	47.8	43.8		
13		59.4	67.8	76.7	64.2	56.4	51.3	39.9	89.6	47.7	49.9	46.1		
14		61.9	73.5	79.1	67.4	58.3	52.9	41.9	87.2	50.5	53.2	47.5		
15		63.7	76.4	82.4	70.2	59.7	55.8	45.3	92.2	53.3	55.4			
16	63.2	65.2	78.3	83.4	71.4	60.9	60.7	43.6	92.6	55.7	56.6			
17	64.4	69.5	80.7	86.6	72.7	61.5	63.3	46.6	95.9	56.3	58.5			
18	65.5	69.1	81.4	85.0	72.6	64.3	65.4	48.8	96.2	58.2	60.6			
19	66.0	70.3	82.7	86.2	74.7	65.6	67.3	50.4	99.0	59.8				
20	66.5	71.1	84.1	86.6	75.5	67.3	68.7	51.7	107.3	60.5				
21	67.9	71.8	84.7	89.0	73.6	66.5	67.8	52.3	111.1	62.4				
22	68.4	72.4	87.2	89.2	72.9	67.2	69.5	56.1	107.7	63.8				
23	68.7	73.9	88.2	89.7	73.3	67.3	70.1	61.4	96.7					
24	68.0	74.1	88.2	90.5	73.0	67.9	71.3	57.9	96.0					
25	68.4	75.4	88.9	91.6	72.8	68.6	72.4	56.1	92.4					
26	68.6	75.6	89.8	92.5	73.8	69.1	72.1	55.0	96.[illegible]					
27	68.8	75.6	90.4	92.7	73.8	70.2	70.2	75.9						
28	69.9	75.6	91.0	93.1	74.0	71.4	69.7	82.0						
29	68.6	75.2	91.0	93.8	75.3	70.5	69.0	82.6						
30	69.3	75.2	91.6	93.8	75.7	72.0	68.5	86.3						
31	69.3	75.8	91.9	94.5	75.8	71.5	69.2							
32	69.2	75.9	91.7	94.1	76.2	70.6	68.6							
33	69.5	76.7	91.5	92.3	75.0	68.4	66.2							
34	69.8	78.2	91.7	92.8	75.3	68.6	65.0							
35	69.9	78.4	91.6	92.9	76.2	68.9								
36	69.6	78.6	91.0	92.5	75.5	68.8								
37	69.1	77.7	91.6	91.9	74.4	68.7								
38	68.9	77.8	91.6	91.9	74.4	69.0								
39	69.1	77.9	91.9	91.6	74.5									
40	69.1	78.6	91.6	91.6	75.1									
41	68.6	79.6	90.4	90.6	75.7									
42	68.7	79.9	90.4	90.6	75.7									
43	68.7	79.9	90.5	90.7										
44	68.6	79.9	90.6	90.7										
45	68.6	79.6	90.3	90.7										
46	69.3	79.6	90.3	90.7										
47	69.3	79.6	90.3											
48	69.5	79.6	90.0											
49	69.3	80.2	88.8											
50	69.3	81.1	88.8											
51	69.4	81.1												
52	69.4	81.3												
53	69.3	80.8												
54	69.5	80.6												
55	69.4													
56	69.4													
57	69.3													
58	69.4													

EXHIBIT 3

Effect on Shape of Curve $L_t = A \times [1 - \exp(-[t/B]^C)]$ of Changing Values of Parameter B

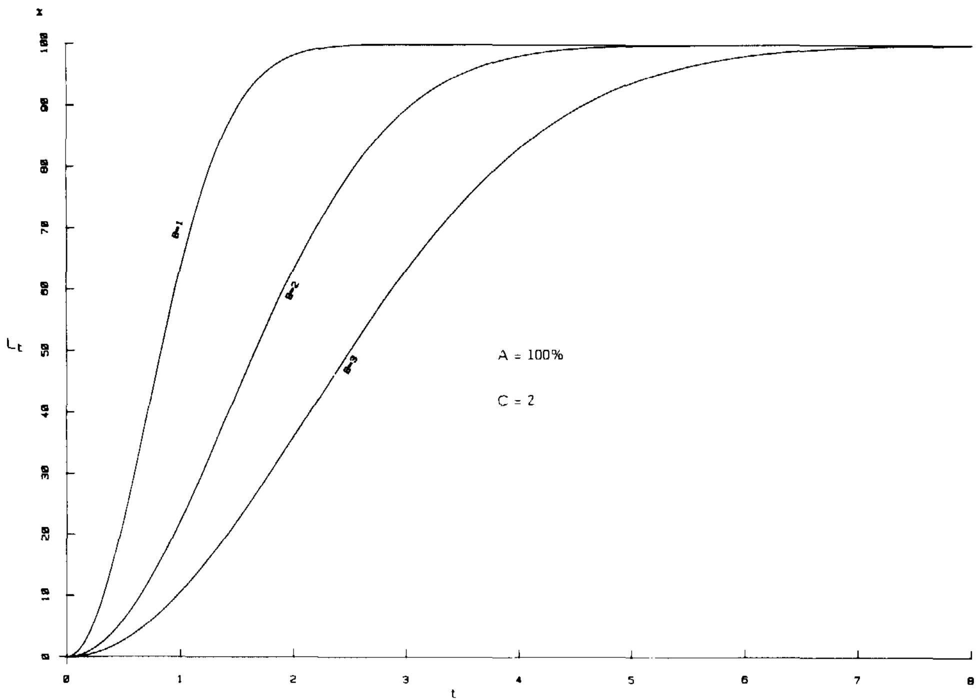


EXHIBIT 4

Effect on Shape of Curve $L_t = A \times [1 - \exp(-[t/B]^C)]$ of Changing Values of Parameter C

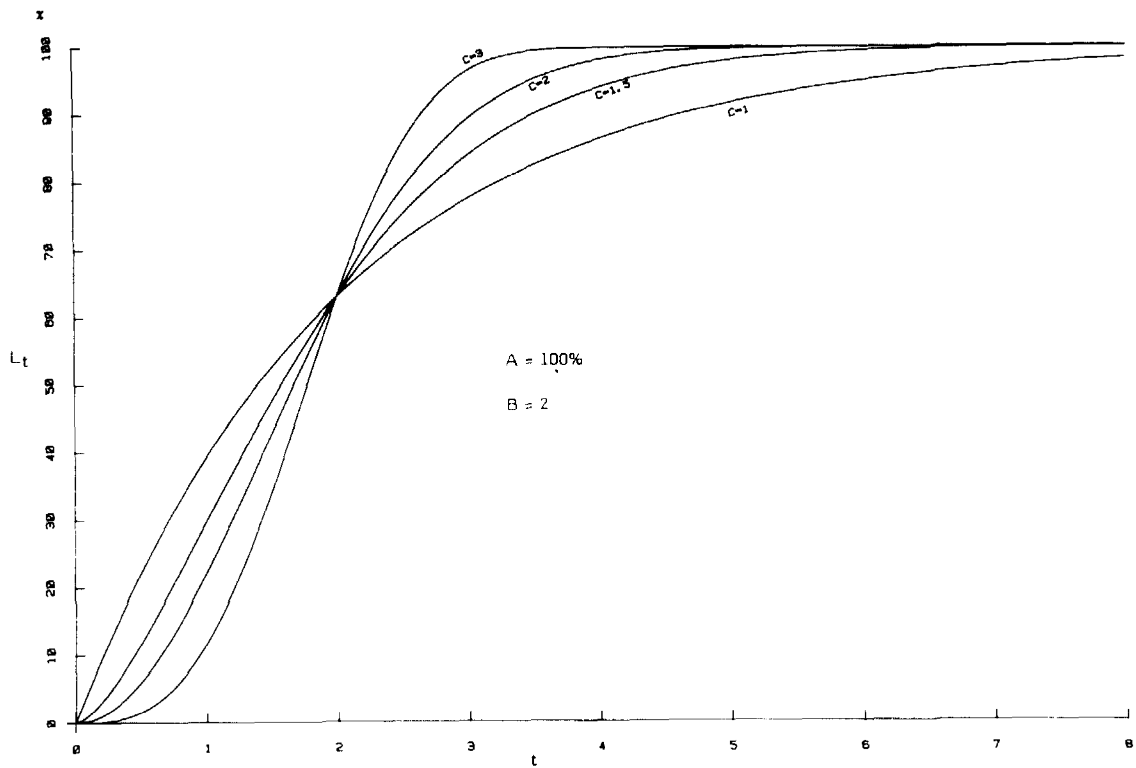
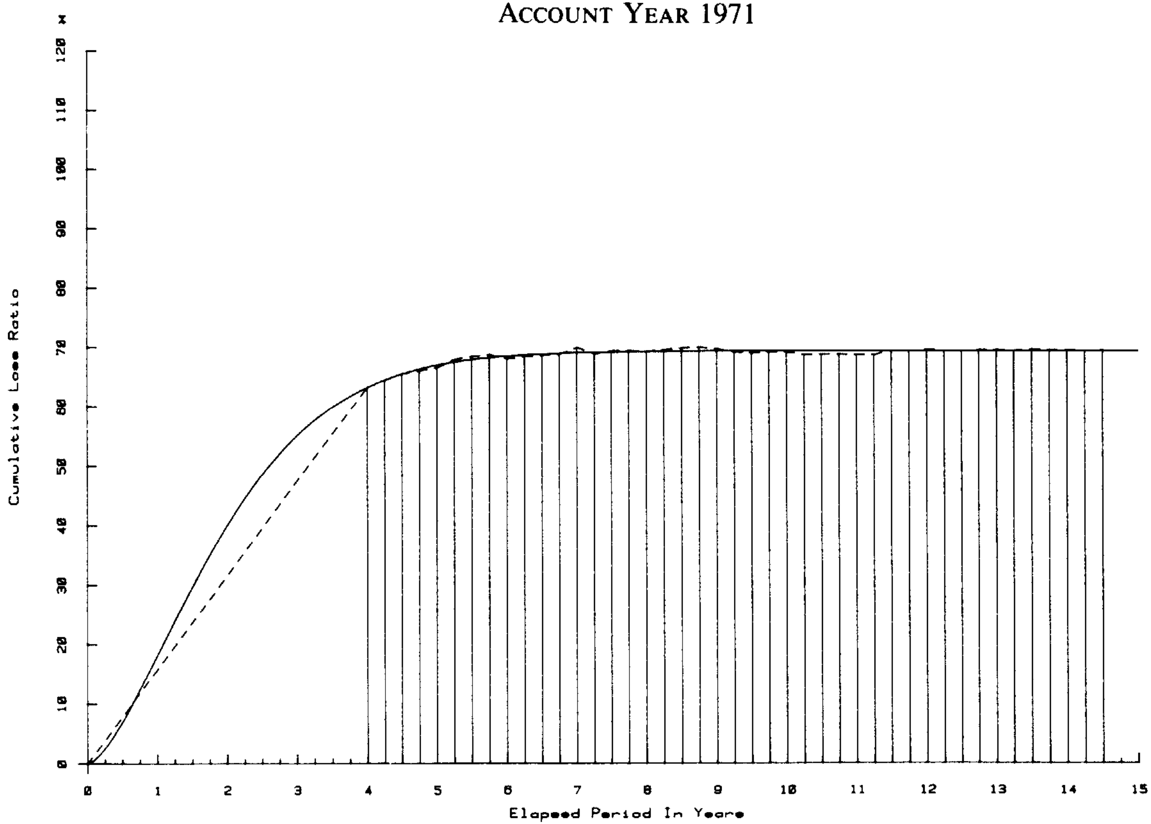
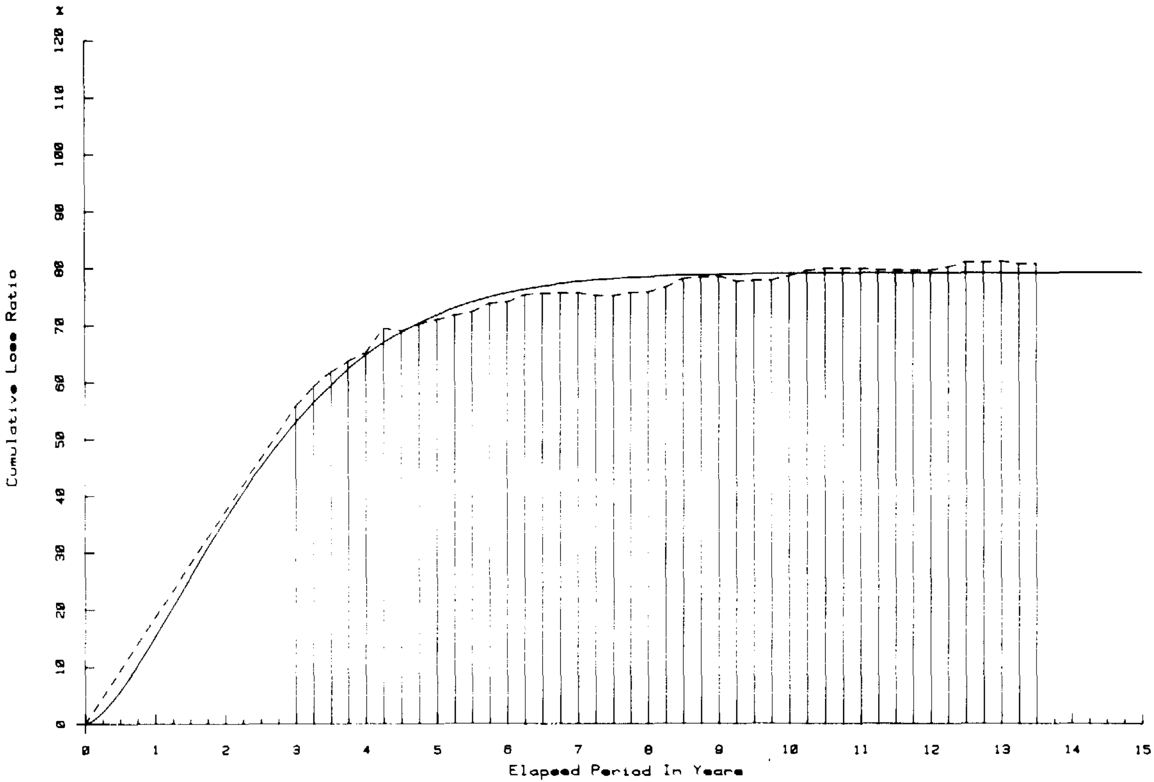


EXHIBIT 5
ACCOUNT YEAR 1971



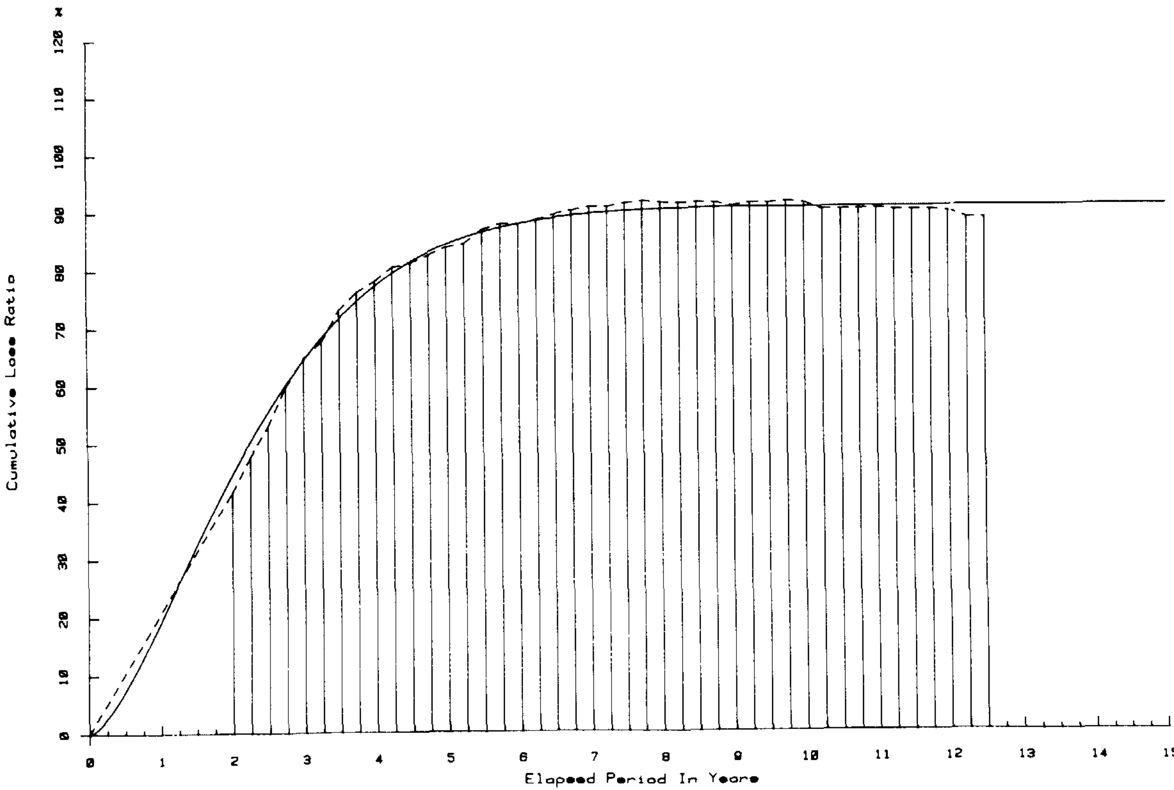
A = 69.3%, B = 2.21, C = 1.50, Mean squared error = 1.1

EXHIBIT 6
ACCOUNT YEAR 1972



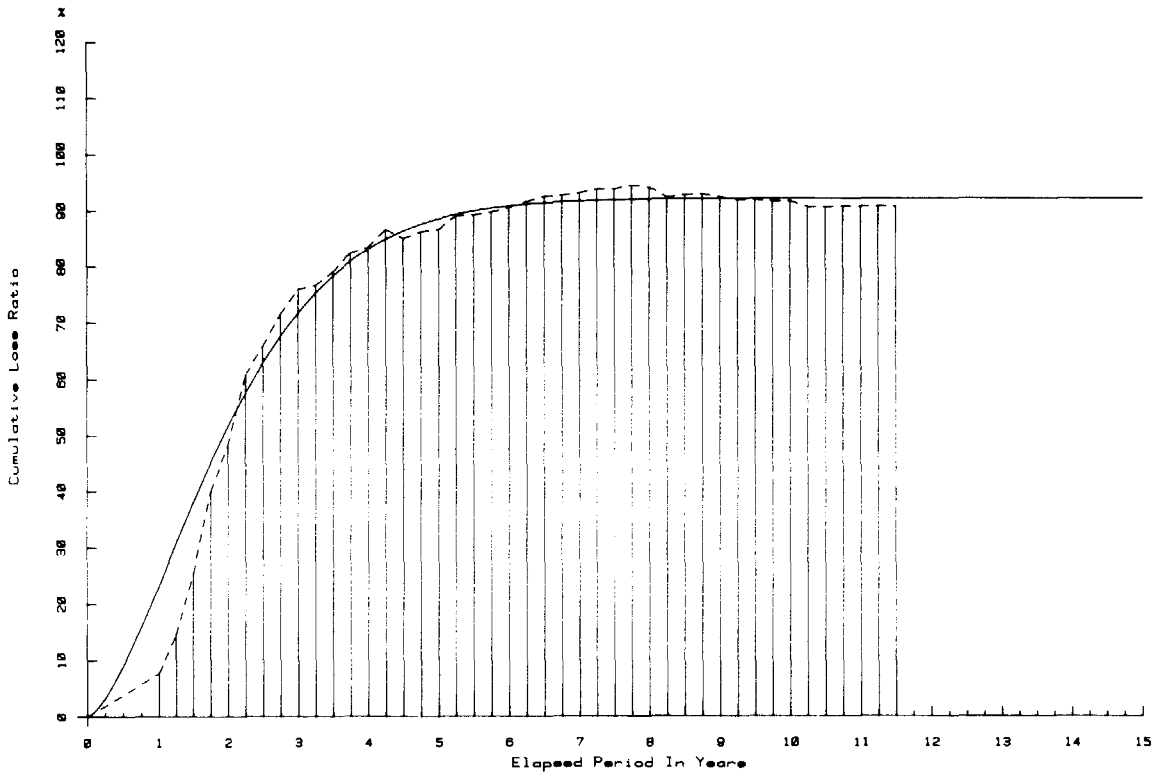
A = 79.2%, B = 2.80, C = 1.50, Mean squared error = 18.5

EXHIBIT 7
ACCOUNT YEAR 1973



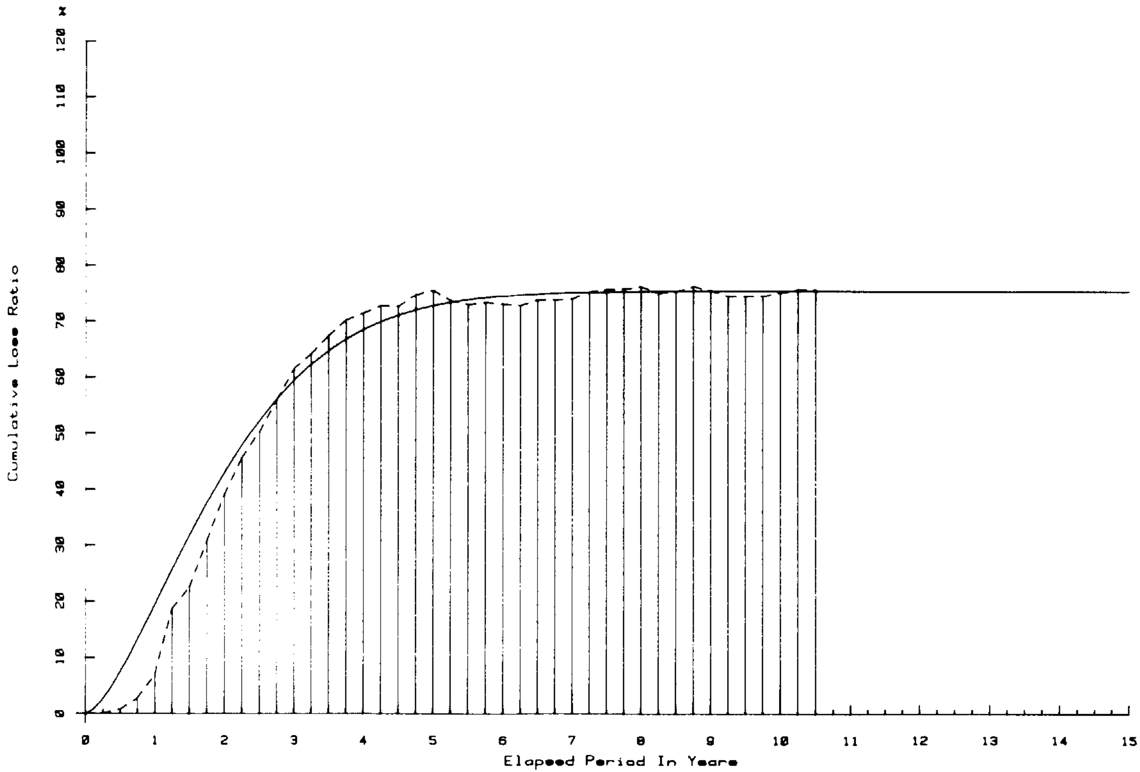
A = 91.0%, B = 2.60, C = 1.50, Mean squared error = 8.1

EXHIBIT 8
ACCOUNT YEAR 1974



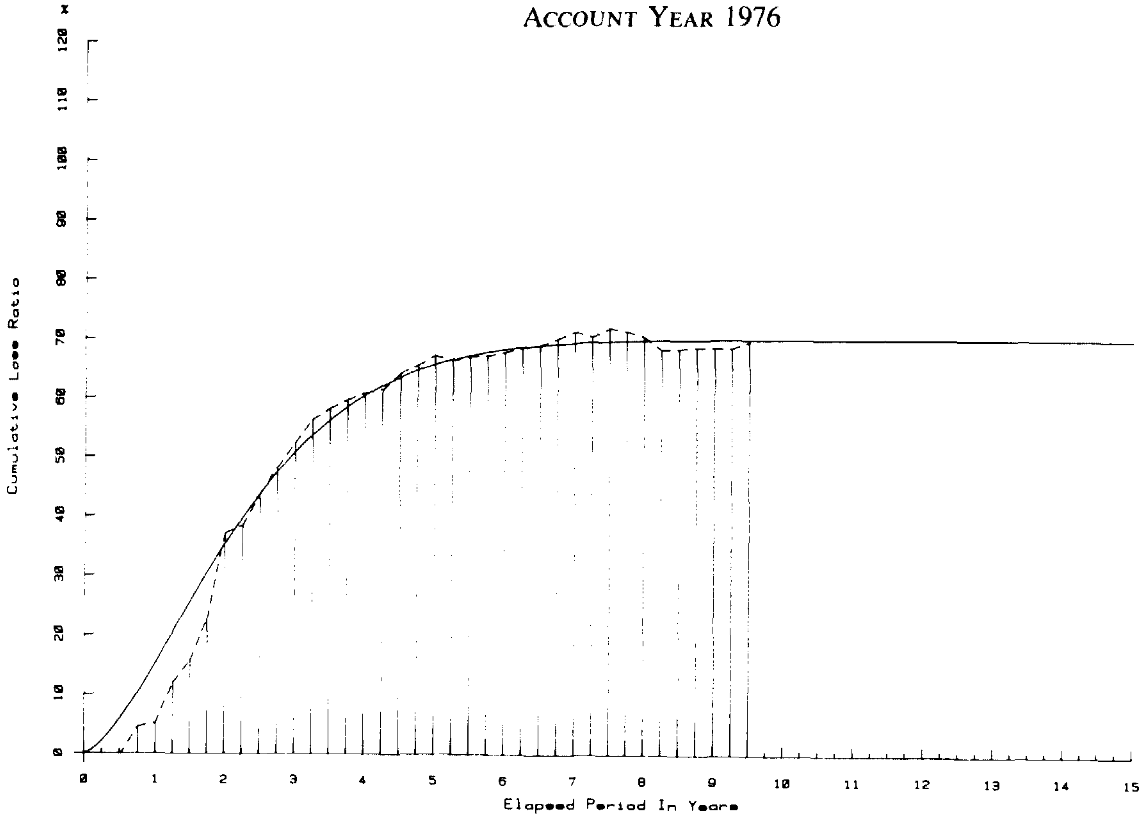
A = 92.1%, B = 2.28, C = 1.50, Mean squared error = 28.0

EXHIBIT 9
ACCOUNT YEAR 1975



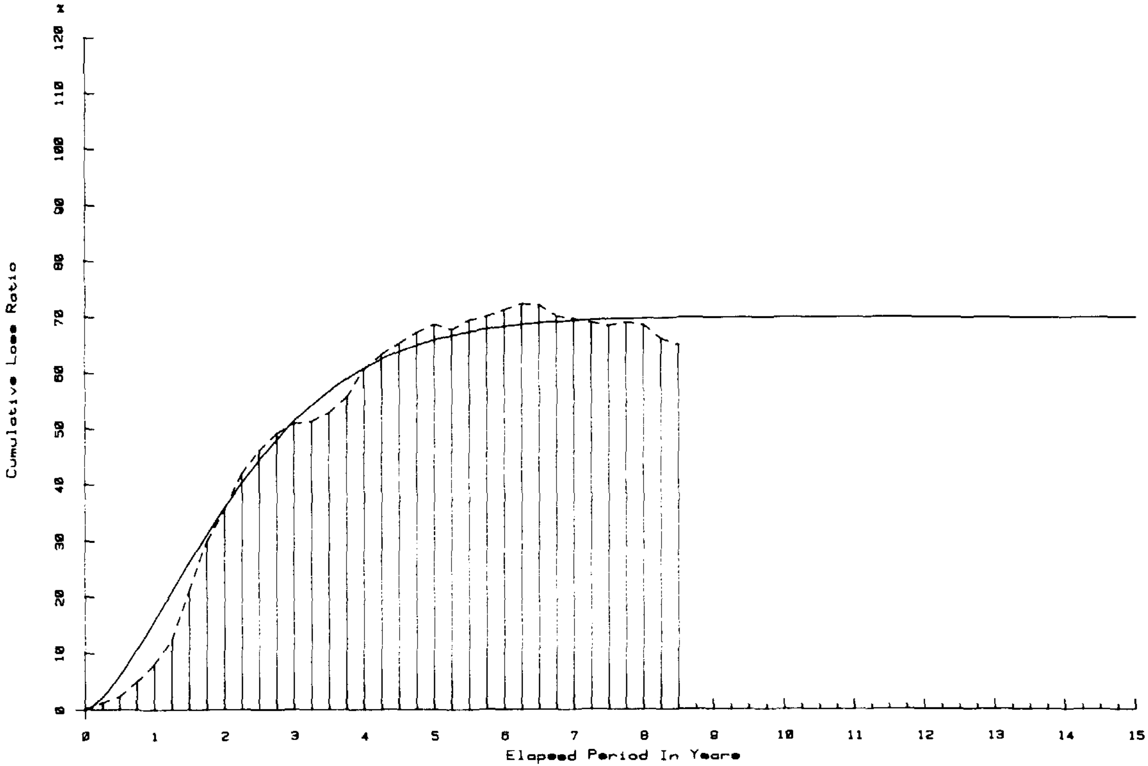
A = 75.4%, B = 2.24, C = 1.50, Mean squared error = 18.2

EXHIBIT 10
ACCOUNT YEAR 1976



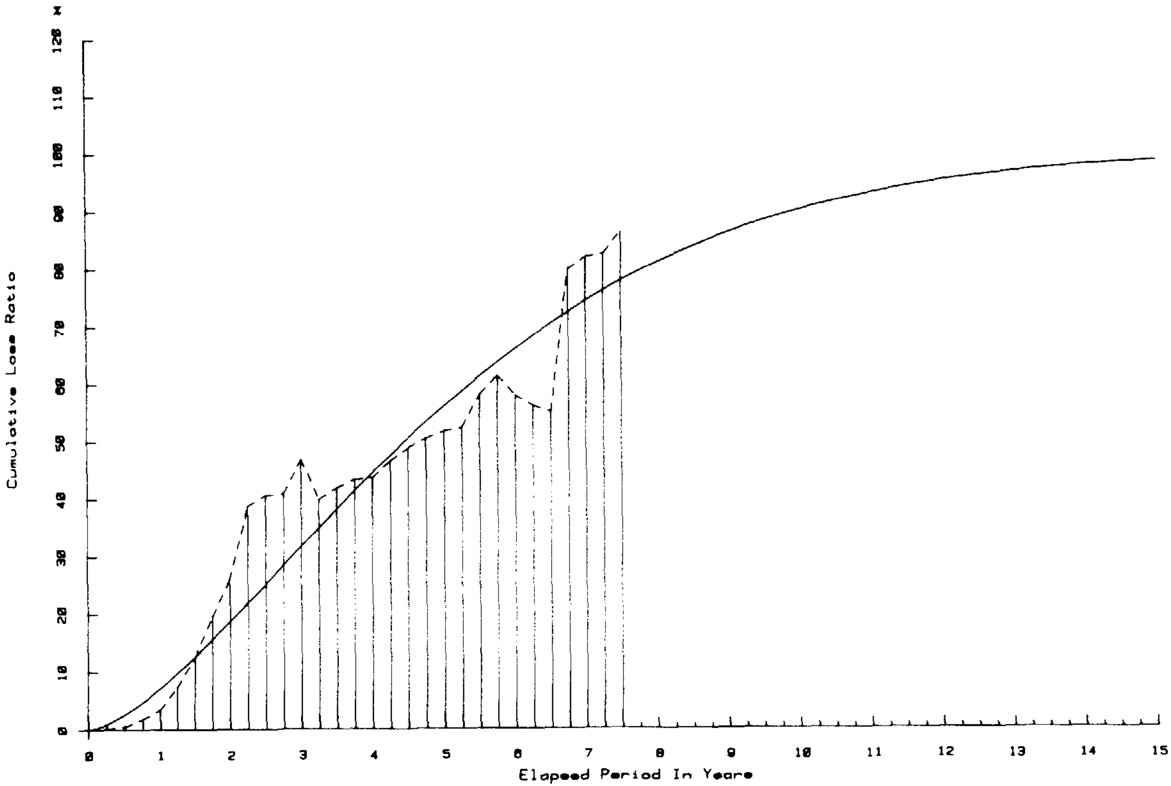
A = 70.2%, B = 2.54, C = 1.50, Mean squared error = 18.2

EXHIBIT 11
ACCOUNT YEAR 1977



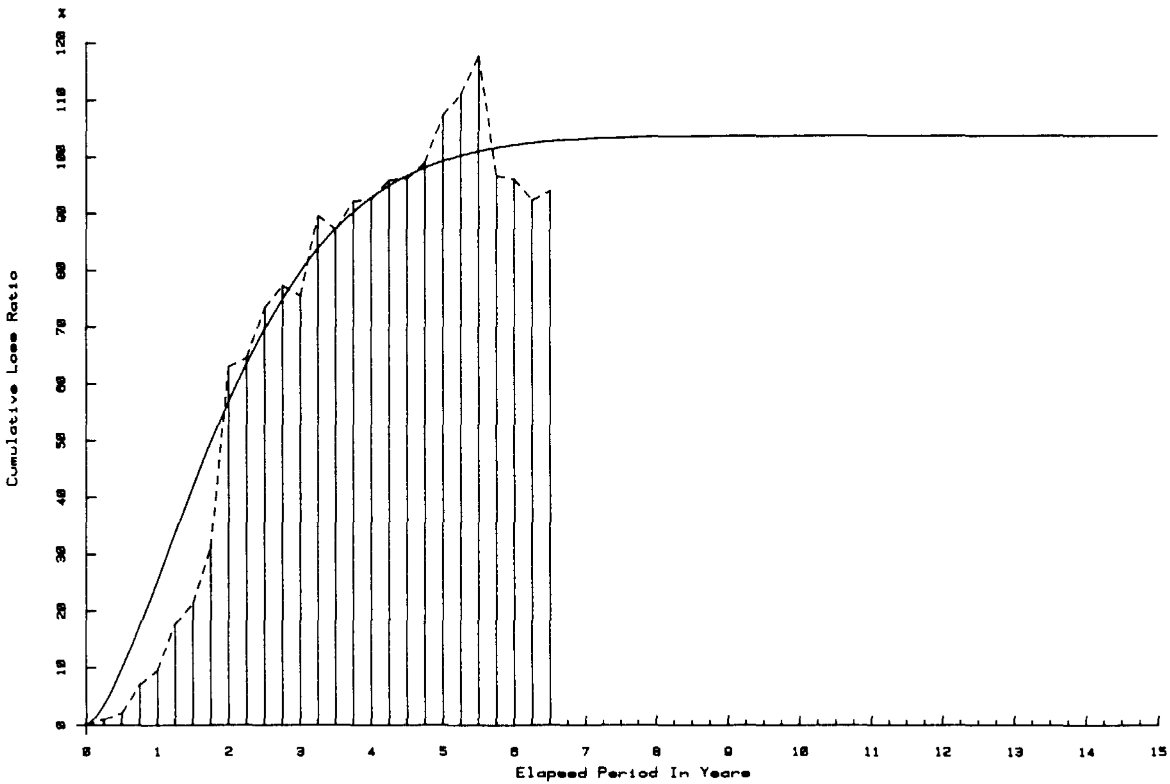
A = 70.0%, B = 2.49, C = 1.50, Mean squared error = 31.8

EXHIBIT 12
ACCOUNT YEAR 1978



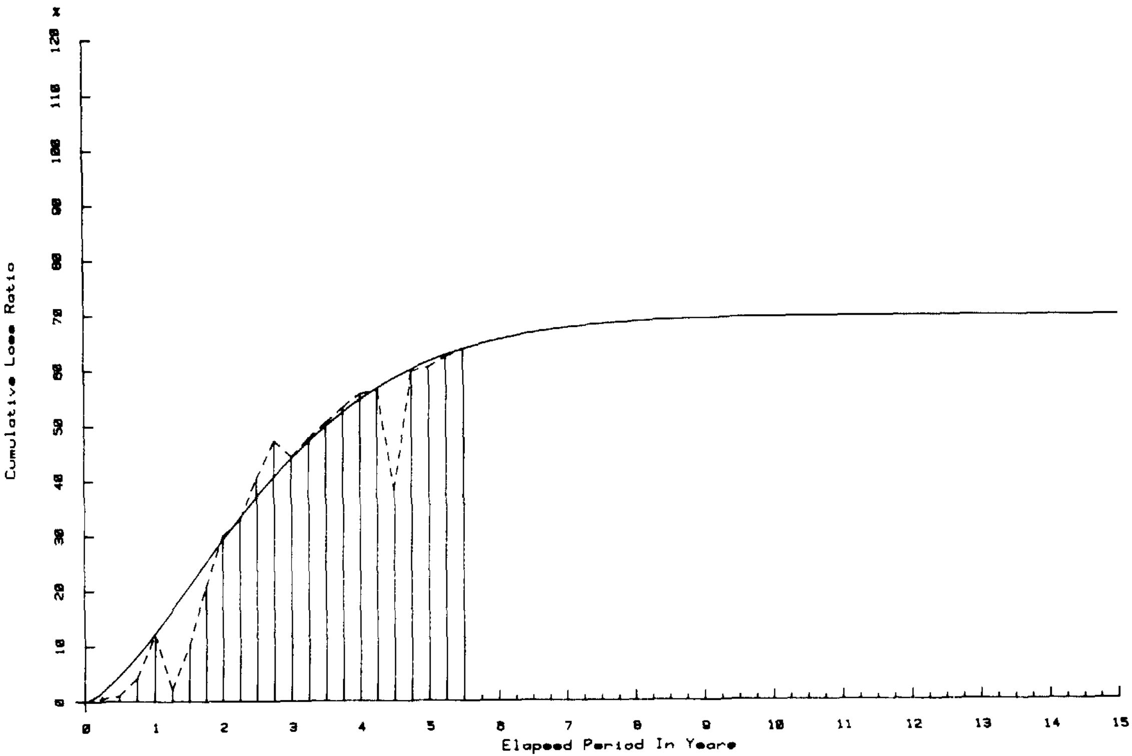
A = 99.9%, B = 5.69, C = 1.50, Mean squared error = 289.6

EXHIBIT 13
ACCOUNT YEAR 1979



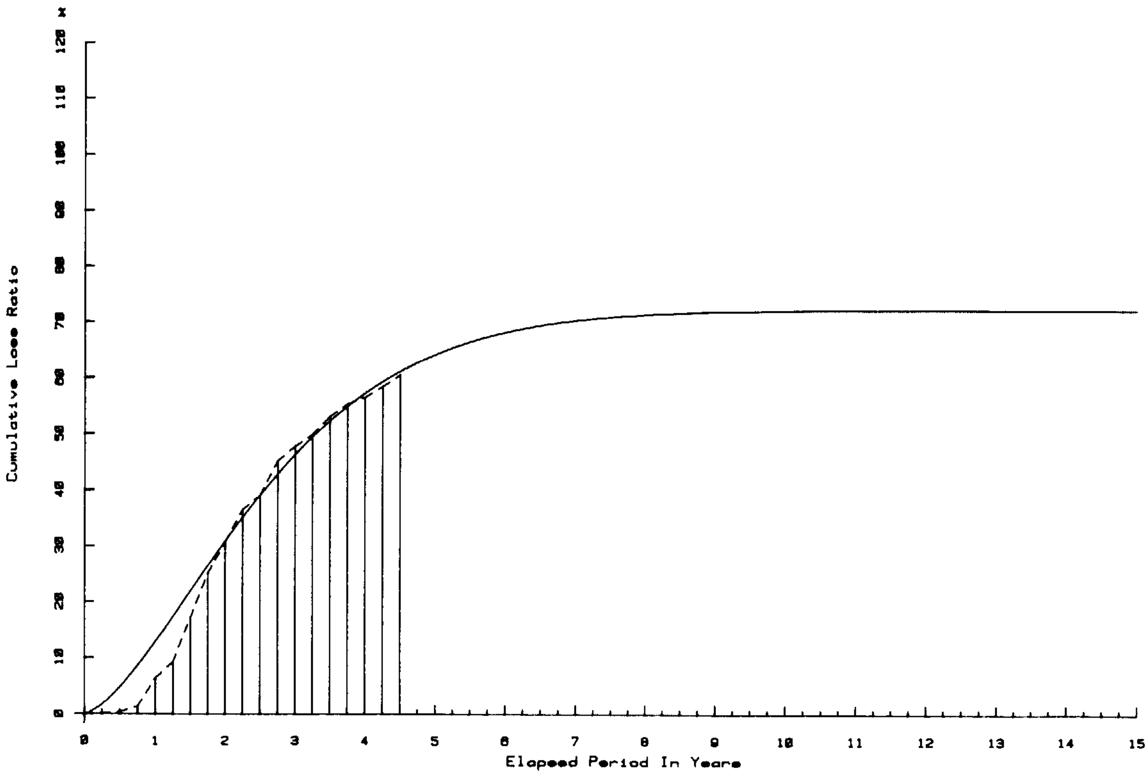
A = 103.8%, B = 2.33, C = 1.50, Mean squared error = 265.3

EXHIBIT 14
ACCOUNT YEAR 1980



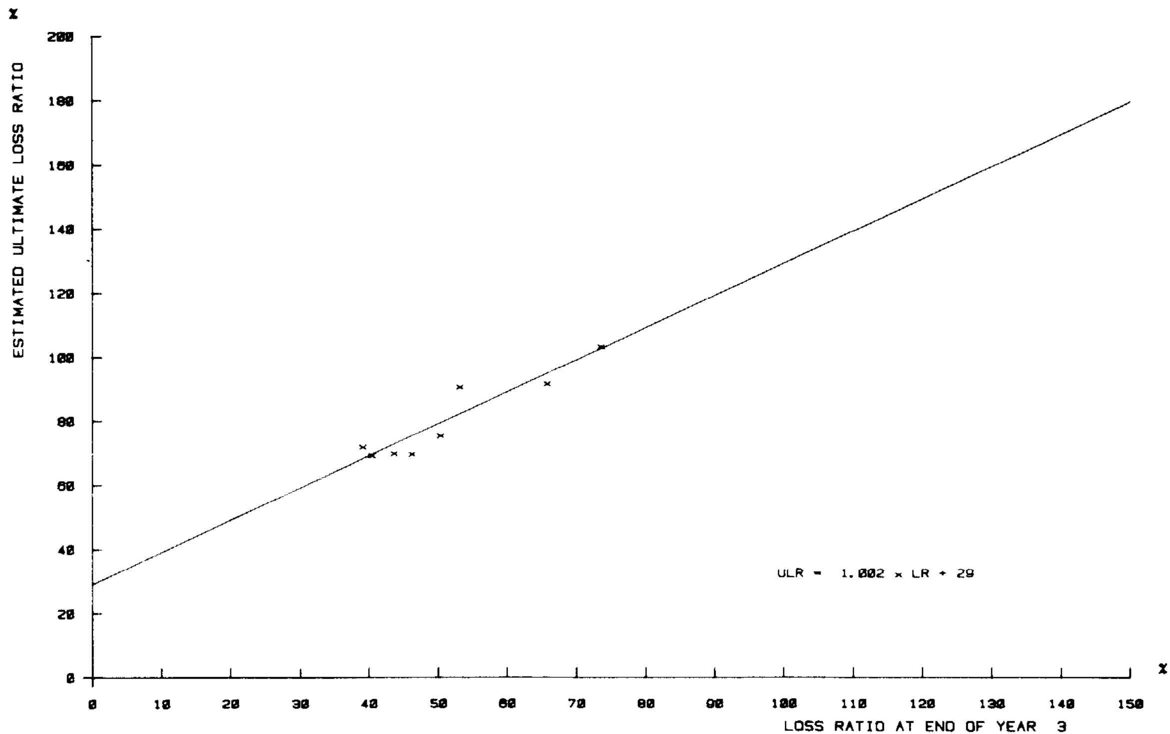
A = 69.6%, B = 3.00, C = 1.50, Mean squared error = 34.7

EXHIBIT 15
ACCOUNT YEAR 1981



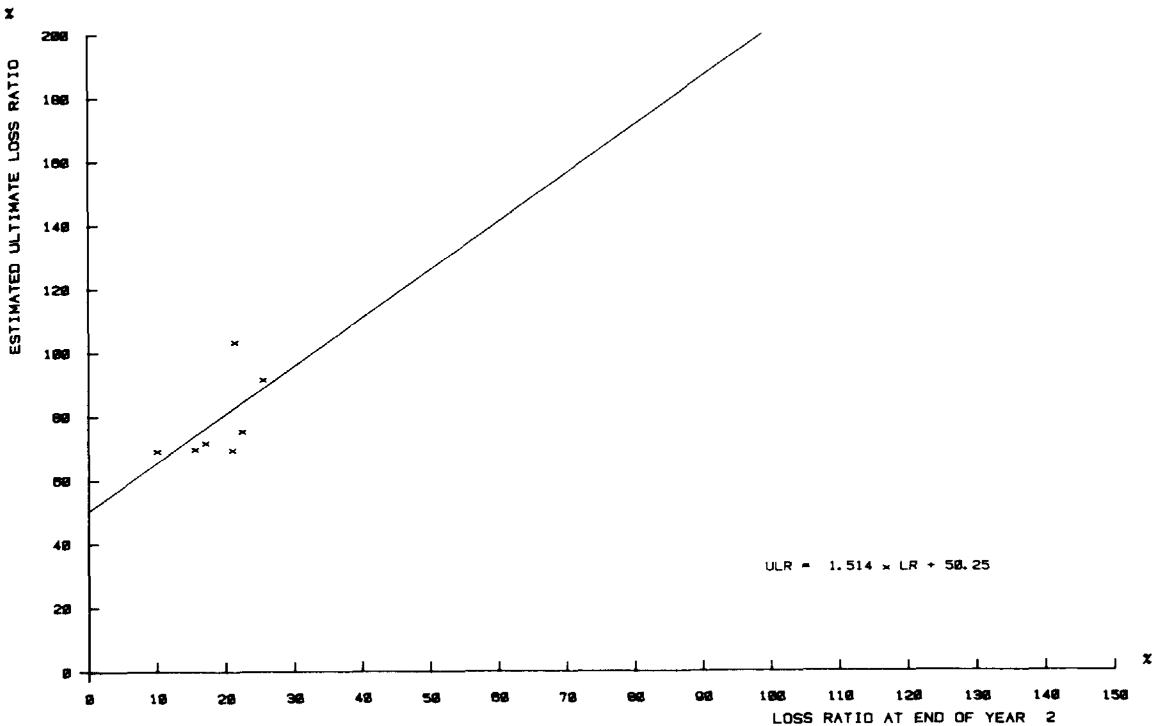
A = 72.2%, B = 2.95, C = 1.50, Mean squared error = 11.2

EXHIBIT 16
ESTIMATED ULTIMATE LOSS RATIO



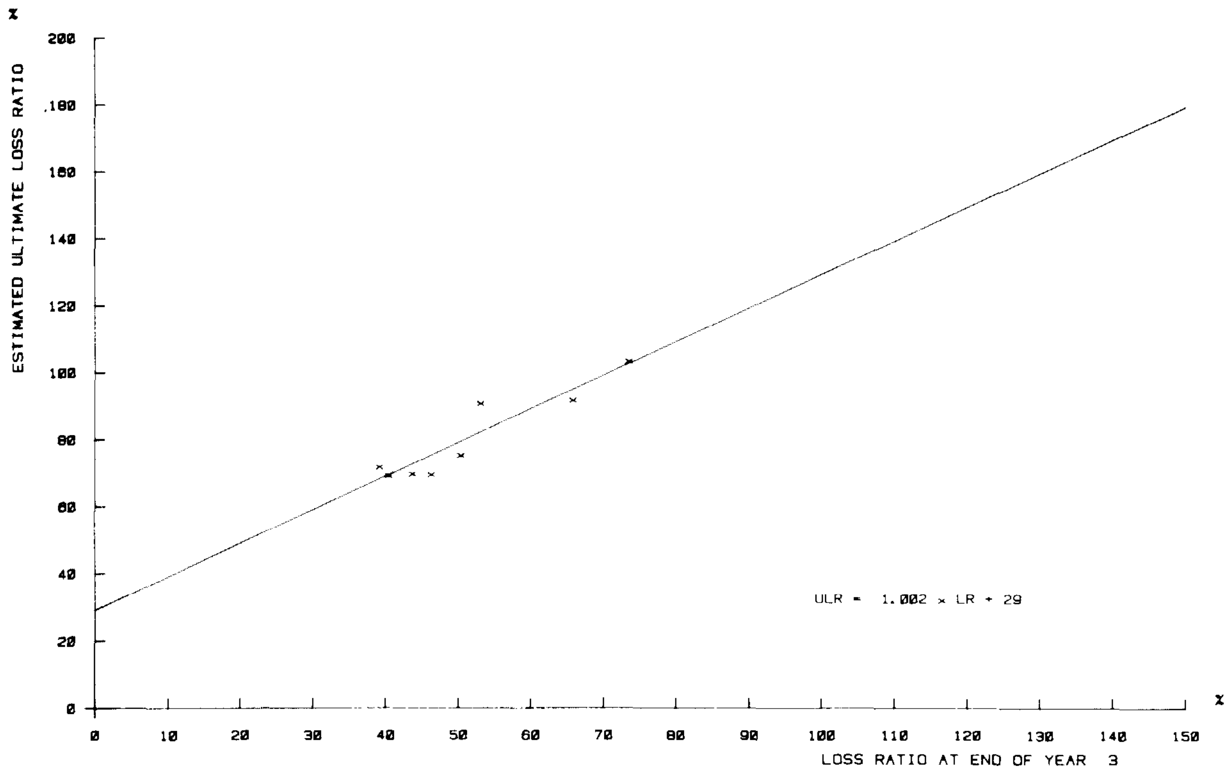
Account years fitted: 73, 74, 75, 76, 77, 79, 80, 81

EXHIBIT 17



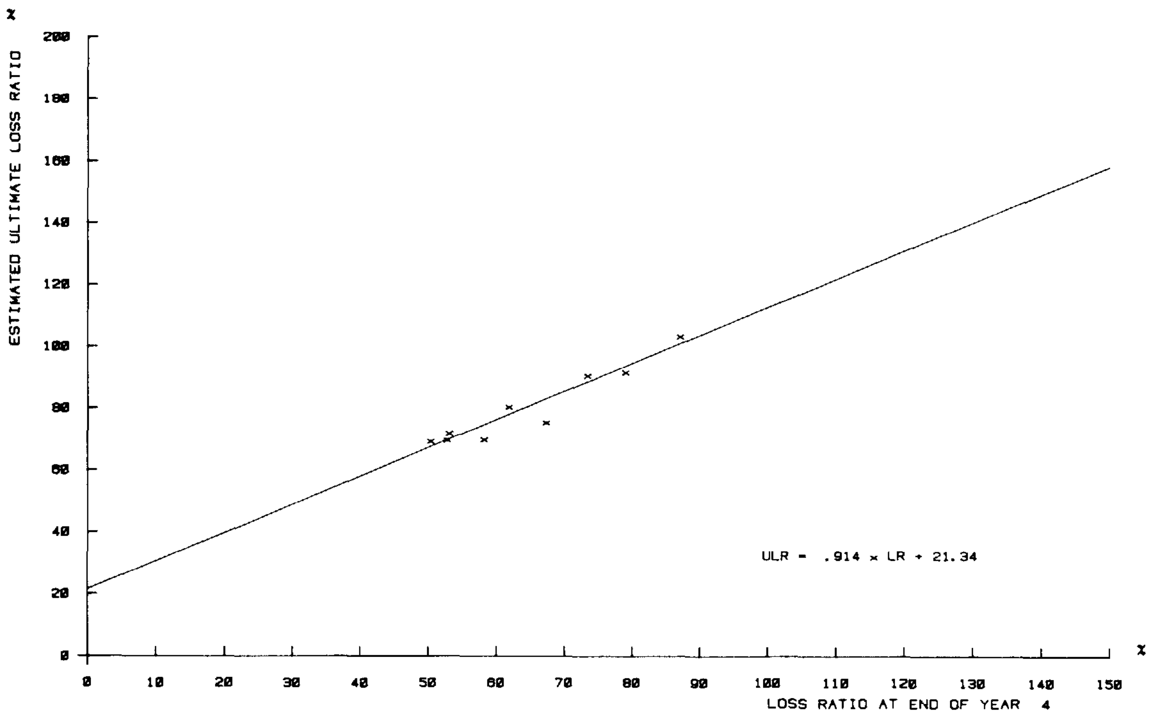
Account years fitted: 74, 75, 76, 77,79, 80, 81

EXHIBIT 18



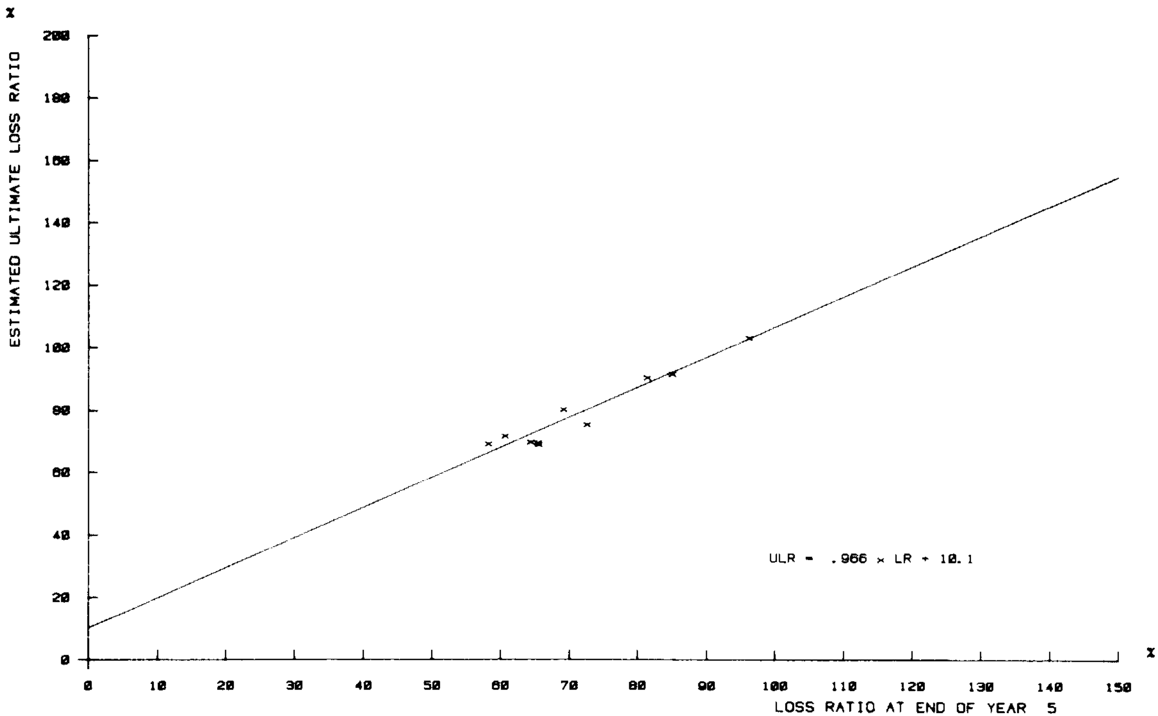
Account years fitted: 73, 74, 75, 76, 77, 79, 80, 81

EXHIBIT 19



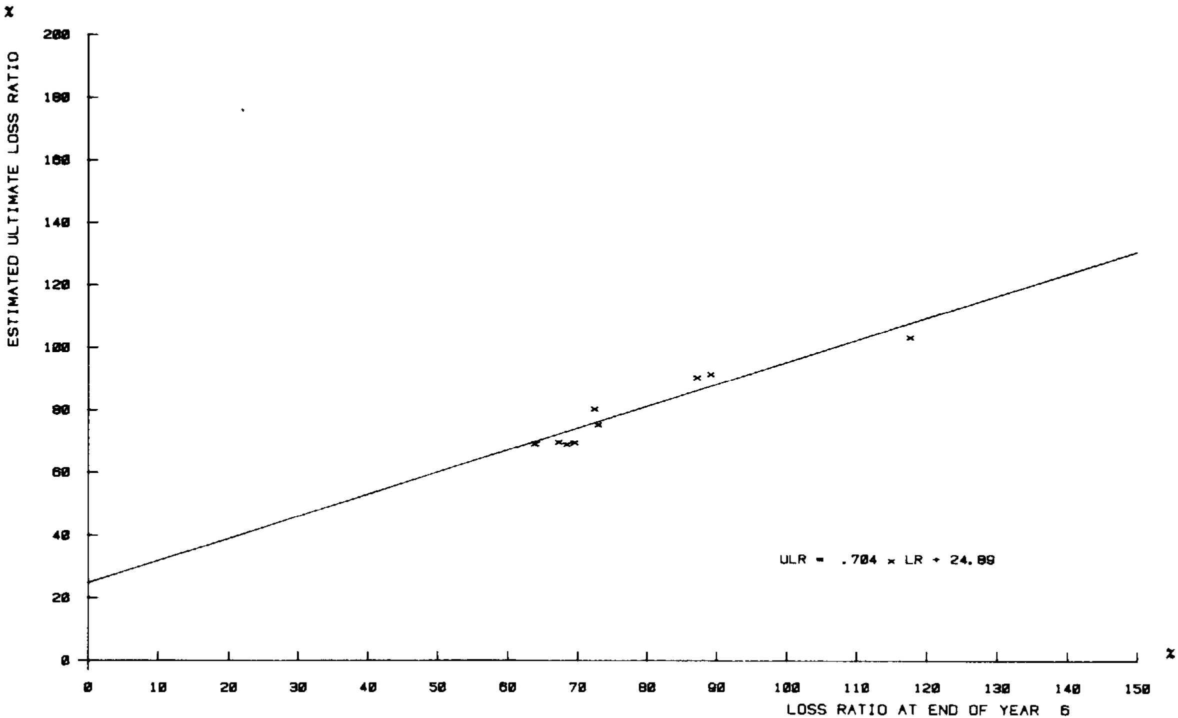
Account years fitted: 72, 73, 74, 75, 76, 77, 79, 80, 81

EXHIBIT 20



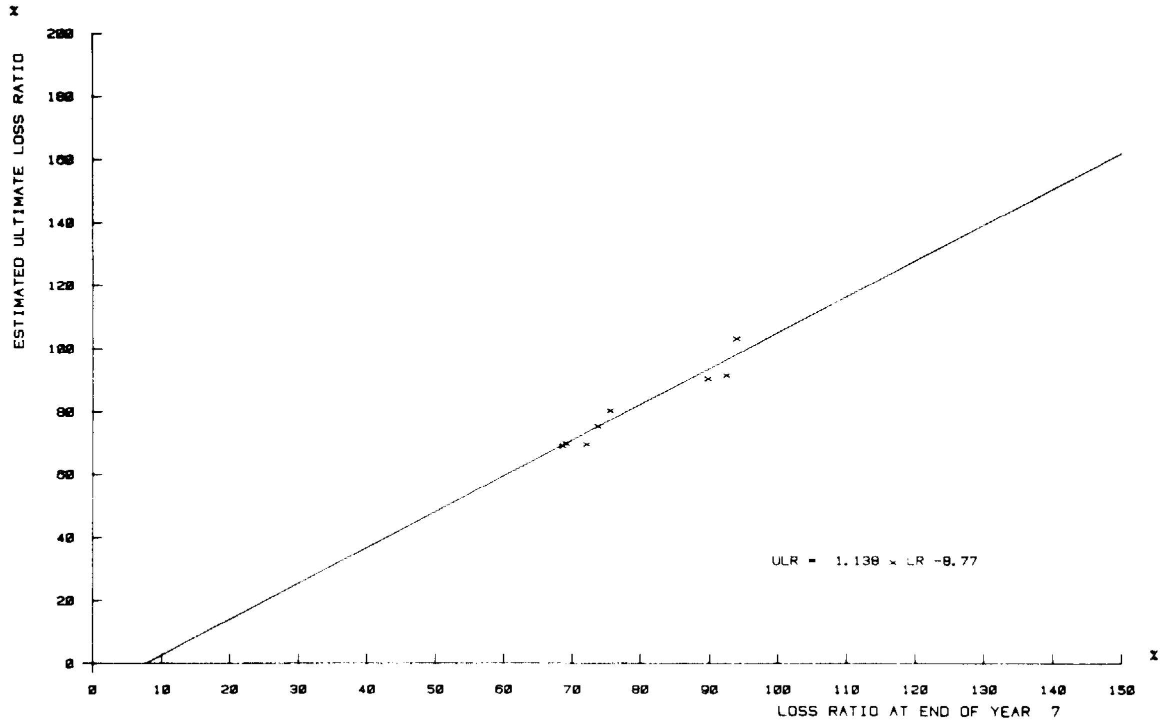
Account years fitted: 71, 72, 73, 74, 75, 76, 77, 79, 80, 81

EXHIBIT 21



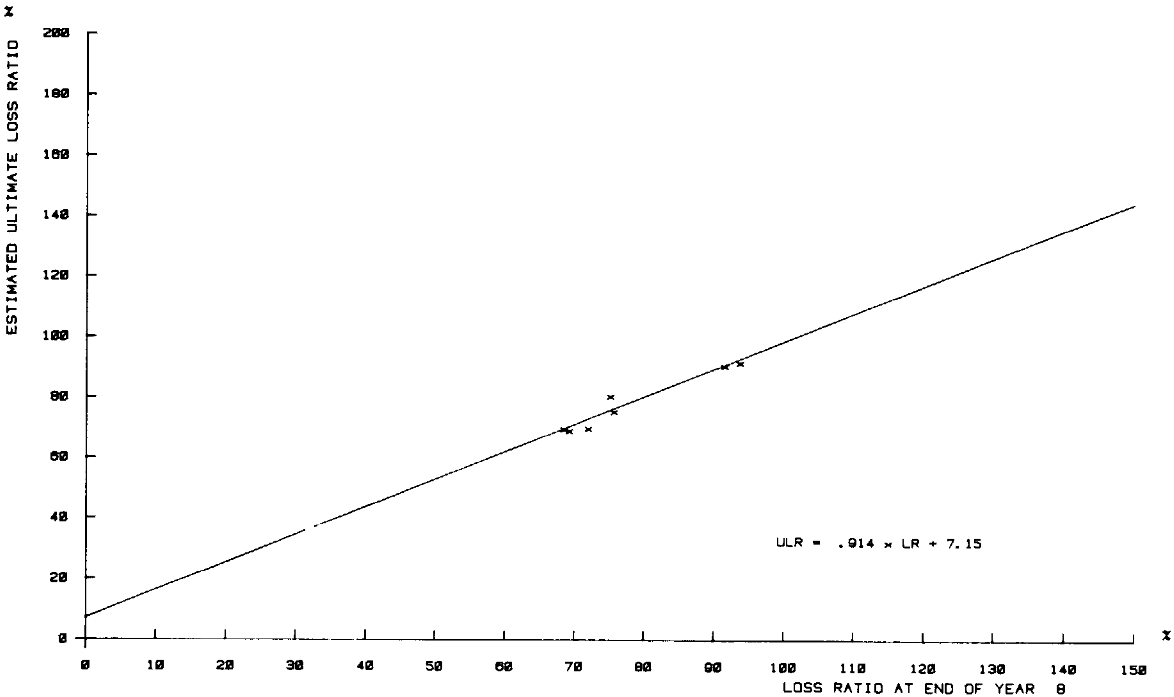
Account years fitted: 71, 72, 73, 74, 75, 76, 77, 79, 80

EXHIBIT 22



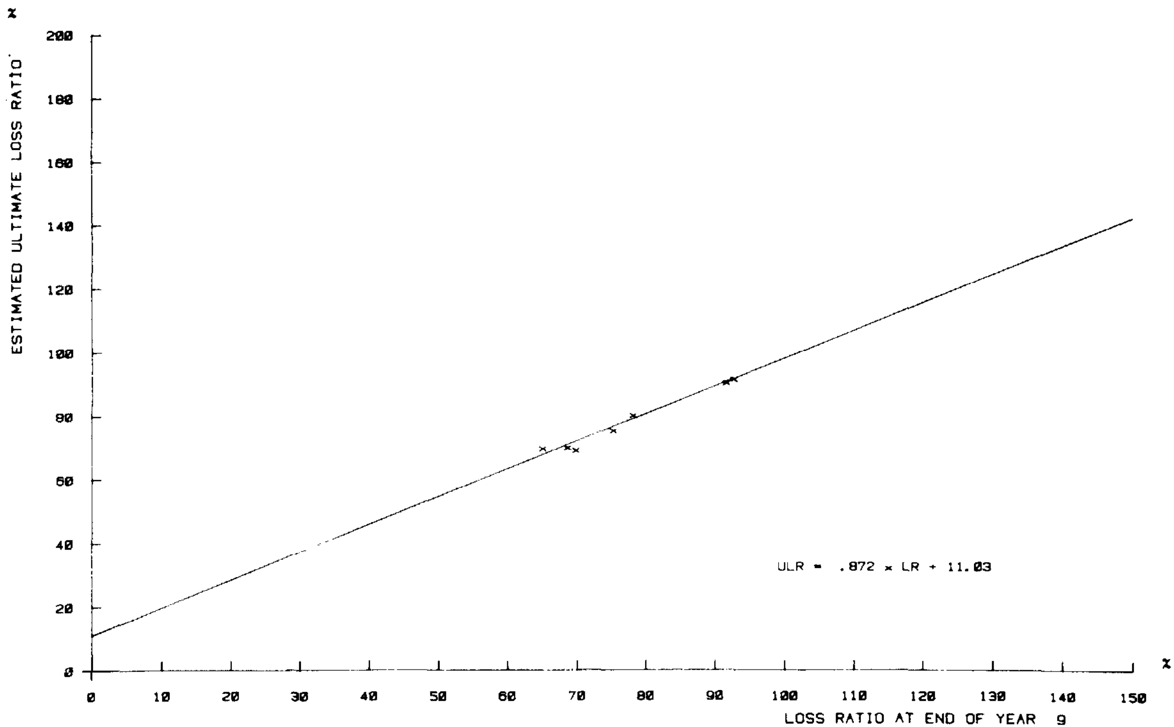
Account years fitted: 71, 72, 73, 74, 75, 76, 77, 79

EXHIBIT 23



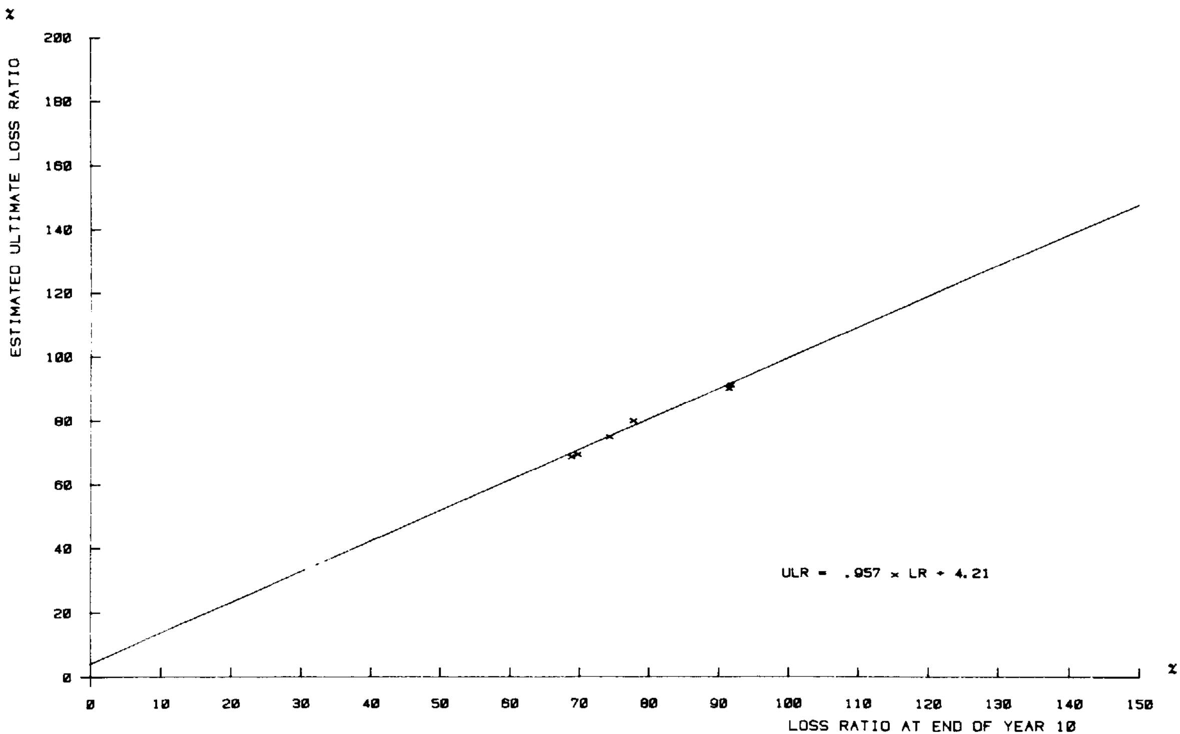
Account years fitted: 71, 72, 73, 74, 75, 76, 77

EXHIBIT 24



Account years fitted: 71, 72, 73, 74, 75, 76, 77

EXHIBIT 25



Account years fitted: 71, 72, 73, 74, 75, 76

EXHIBIT 26
SUMMARY OF REGRESSION LINES FITTED

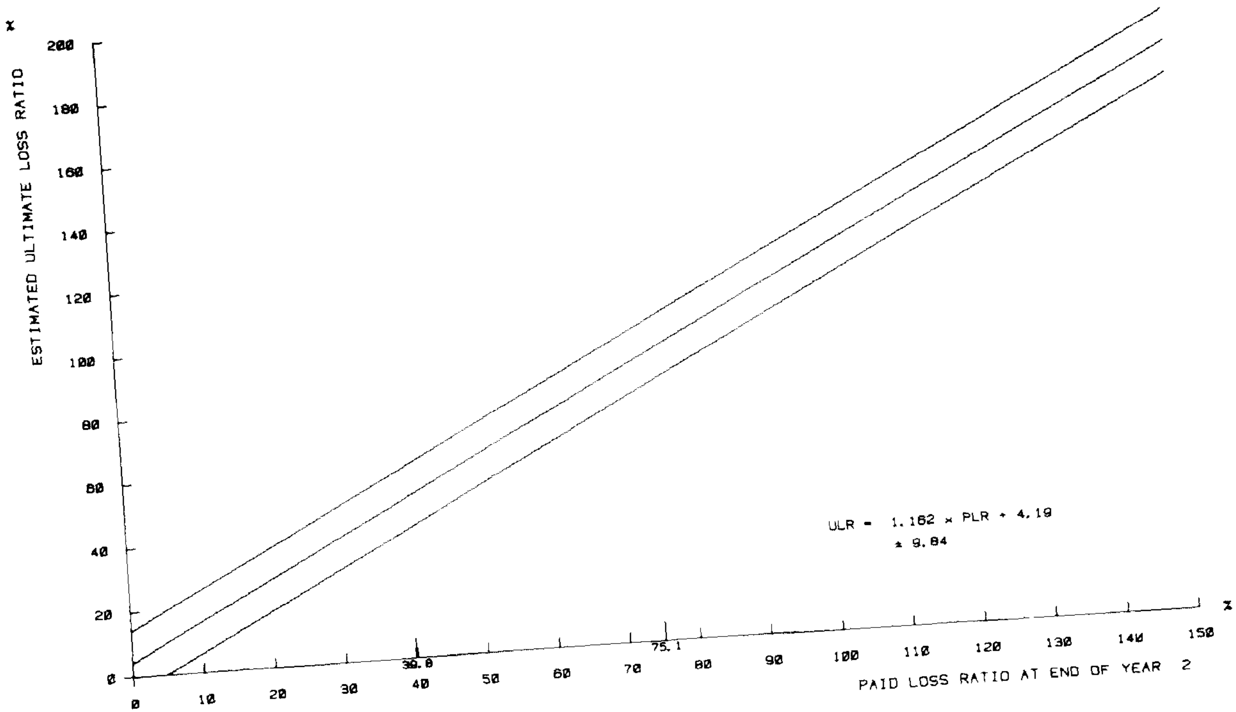
Account Year	Corresponding Development Year	Regression Line: Slope	Regression Line: Constant	t-test Statistic: Value	t-test Statistic: Degrees of Freedom
1984	2	1.514	50.25	1.58	5
1983	3	1.002	29.00	6.55	6
1982	4	.914	21.34	8.54	7
1981	5	.966	10.10	10.11	8
1980	6	.704	24.89	8.41	7
1979	7	1.138	−8.77	8.85	6
1978	8	.914	7.15	9.00	5
1977	9	.872	11.03	13.41	5
1976	10	.957	4.21	16.95	4

Account Year	Latest Loss Ratio	Estimated ULR	Maximum Deviation	90% Confidence Interval
1984	23.05%	85.15%	21.15%	27.07%
1983	39.57	68.65	8.75	10.86
1982	47.48	64.74	7.23	8.54
1981	60.63	68.67	4.55	7.05
1980	63.75	69.77	4.98	8.41
1979	93.97	98.17	5.63	8.40
1978	86.30	86.03	4.89	5.83
1977	64.96	67.68	2.48	4.14
1976	69.84	71.05	2.17	3.19

EXHIBIT 27
RECOMMENDED ESTIMATES OF ULR

Account Year	Loss Ratio to Date	Estimated ULR	Confidence Interval (+ or −)
1971	69.4%	69.4%	-
1972	80.8	80.8	-
1973	88.8	91.0	-
1974	90.7	92.1	-
1975	75.7	75.7	-
1976	69.8	70.2	-
1977	65.0	70.0	-
1978	86.3	99.9	8.4%
1979	94.0	98.2	8.4
1980	63.8	69.8	8.4
1981	60.6	68.7	7.0
1982	47.5	64.7	8.5
1983	39.6	68.6	10.9
1984	23.0	85.1	27.1

EXHIBIT 28
CATEGORY AVIATION SHORT TAIL (STERLING)
ALL LLOYDS BUSINESS



THE MATHEMATICS OF EXCESS OF LOSS COVERAGES AND RETROSPECTIVE RATING—A GRAPHICAL APPROACH

YOONG-SIN LEE

Abstract

The mathematics of excess of loss coverages and retrospective rating involves heavy algebra, mainly because the indemnity payment under such contracts assumes different functional forms in different parts of the loss size distribution. This paper presents a graphical approach to the theory, in which the indemnity payment under various conditions is represented by the regions in a graph described by the cumulative distribution function of the size of loss. Many intricate formulas and relations occurring in the two subjects, some expressible algebraically only in very complicated forms, can be understood simply and clearly through pictures. Treated visually in this paper are many mathematical relations and results included in the examination syllabus.

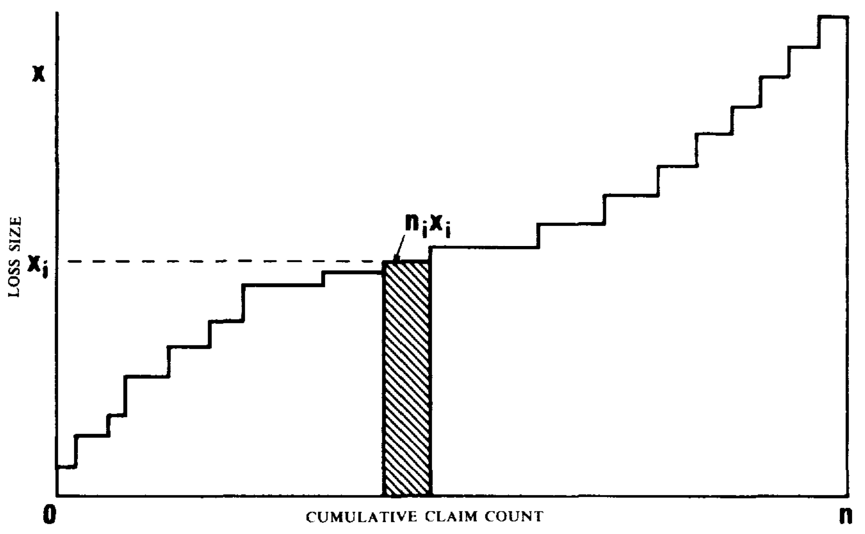
1. INTRODUCTION

The theory of excess of loss coverages and retrospective rating involves rather complicated mathematics. The underlying ideas in most cases are relatively simple, but the heavy algebra is often a great mental burden to the actuary and the student. This paper applies a graphical technique to excess of loss coverages and retrospective rating. Most of the algebraic results on these topics can be interpreted in graphic terms. The advantages of this approach are that the results so derived are easier to understand and the formulas can be easily remembered and written down.

Graphical methods are widely used in mathematics and statistics to visually present ideas which would otherwise be abstruse. Many mathematical ideas have geometric as well as symbolic interpretation. For example, the integral of a positive-valued function can be regarded as the area under the curve representing the function as well as the anti-derivative of the function. The use of diagrams and graphs to present numerical information in statistics is better known. Graphs in statistics are used to explain ideas such as density functions and cumulative distribution functions. In actuarial science, graphical methods have not been extensively utilized. A graphical device is presented herein for the explanation of the underlying mathematical ideas. It will not only provide powerful insight into the abstract relations, but also make the mathematical procedure much easier to follow compared with algebraic manipulations. For those who always prefer algebra, it will serve at least as a very useful supplement to the predominantly algebraic treatment that has been given to the subject in the literature.

To start with, consider a large number of losses, of sizes x_1, x_2, . . . , x_k, occurring n_1, n_2, . . . , n_k times, respectively, with $n = n_1 + \ldots + n_k$. In Figure 1 we represent these losses by means of a cumulative frequency curve, in which the ordinate represents the loss size, and the abscissa represents the cumulative number of losses, $c_i = n_1 + \ldots + n_i$, $i \leq k$. This representation is different from the usual form in statistical textbooks, where the abscissa and ordinate are reversed, but agrees with the representation in Snader [10]. (See also Philbrick [7].)

FIGURE 1
A CUMULATIVE FREQUENCY CURVE



The curve is a step function (with argument along the vertical axis) which has a jump of n_i at the point x_i. Consider the shaded vertical strip in the graph. It has an area equal to $n_i x_i$. Summing all such vertical strips we have

$$\text{Total amount of loss} = n_1 x_1 + \ldots + n_k x_k. \quad (1.1)$$

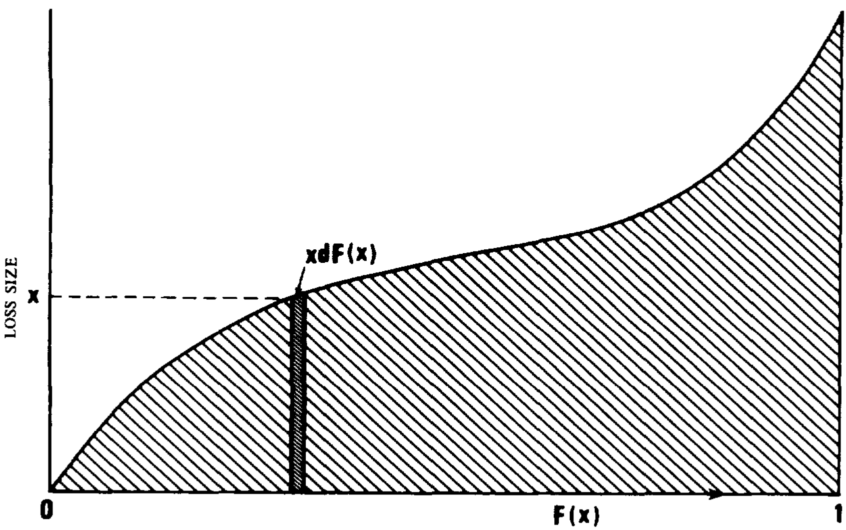
We may therefore interpret the area of the vertical strip corresponding to x_i as the amount of loss of size x_i, and the total enclosed area below the cumulative frequency curve as the total amount of loss. In fact, we have a new way of viewing the cumulative frequency function curve. This curve can be constructed by arranging the losses in ascending order of magnitude, and laying them from left to right with each loss occupying a unit horizontal length.

Now let X be a random variable representing the amount of loss incurred by a risk. Define the cumulative distribution function (cdf) $F(x)$ as

$$F(x) = \Pr(X \le x). \quad (1.2)$$

Figure 2 shows the graph of a continuous cdf. Consider the vertical strip

FIGURE 2
CDF CURVE AND EXPECTATION



in the graph, with area $xdF(x)$. If we sum up all these strips, we will obtain the expected value of X,

$$E(X) = \int_0^{\infty} xdF(x), \tag{1.3}$$

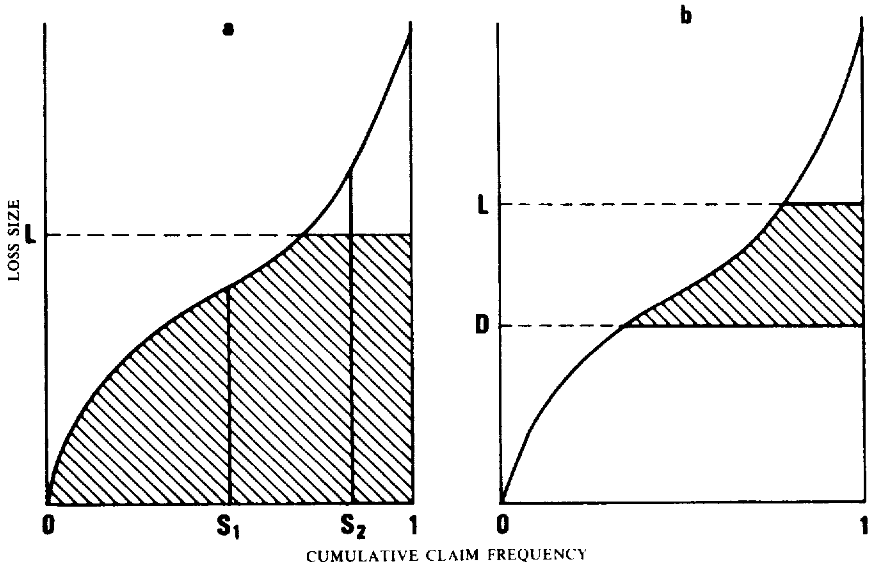
which is represented by the enclosed area below the cdf curve (the shaded area in the graph). We may interpret the expected loss as composed of losses of different sizes, and the strip $xdF(x)$ as the contribution from losses of size between x and $x+dx$. Throughout this paper, an expression such as $E\{X\}$ represents the expected value of a random variable X.

Limited Payments

As an immediate application, consider a coverage which pays for losses up to a limit L only. Figure 3(a) shows that a loss of size not more than L, such as S_1, is paid in full, while a loss of size S_2 which is greater than L, is paid only an amount L. By summing up vertical strips as before, except that strips with length greater than L are limited to length L, we obtain the expected payment per loss under such a coverage as the shaded area in Figure 3(a).

FIGURE 3

EXPECTED LOSS WITH (a) LIMIT AND (b) DEDUCTIBLE



Deductibles

Likewise, a coverage which pays for losses subject to a flat deductible D and up to limit L has expected payment per loss represented by the shaded area in Figure 3(b).

Size and Layer

As another application we first derive an integration identity. Consider Figure 4(a). The vertical strip has area $xdF(x)$ and the horizontal strip has area $G(x)\,dx$, where

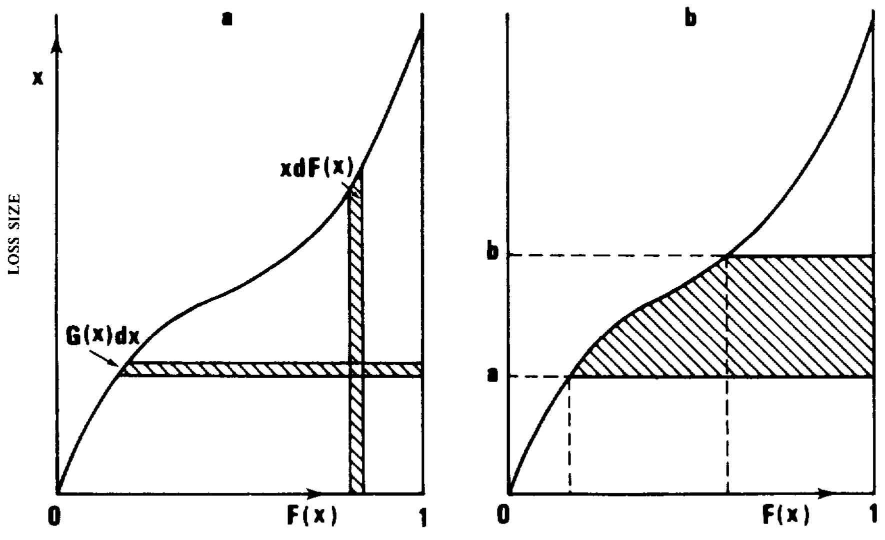
$$G(x) = 1 - F(x). \tag{1.4}$$

Summing up the vertical strips and the horizontal strips separately we have

$$\int_0^\infty xdF(x) = \int_0^\infty G(x)\,dx = E\{X\}, \tag{1.5}$$

because each of the integrals is equal to the enclosed area below the cdf curve, which, as we have seen, also represents the expected loss $E\{X\}$. The equality can also be algebraically derived via integration by parts.

FIGURE 4
SIZE AND LAYER VIEWS OF LOSSES



The two modes of summation correspond, in fact, to two views of the losses. The vertical strips group losses by size, whereas the horizontal strips group the loss amounts by layer. We may therefore call them the size method and the layer method. It is often more convenient to evaluate the expected loss in a layer by layer fashion, i.e. summing horizontal strips, than by the size method, i.e. summing vertical strips. For example, consider the layer of loss between a and b in Figure 4(b). The expected loss in this layer is represented by the shaded area. The layer method of summation gives simply

$$\int_a^b G(x)\,dx. \tag{1.6}$$

To express this integral by the size method is more difficult. A moment's reflection, with the help of Figure 4(b), yields the following expression for the integral:

$$\int_a^b x\,dF(x) + bG(b) - aG(a). \tag{1.7}$$

Again, the equality of the two expressions can be established via integration by parts.

The more complicated expression derived from the size method is the form commonly found in the literature. Although the integral associated with the layer method is simple in form, $G(x)$ is a function that is generally more difficult to integrate. This disadvantage disappears, however, when the distribution is given numerically, as, for example, when actual experience is used. The retrospective rating Table M and Table L have been constructed by the layer method; see Simon [8] and Skurnick [9]. We shall give the graphical interpretation later.

2. EXPECTED VALUE PREMIUM

Generally, given a loss X, a coverage would pay an amount depending on the value of X. We may represent this function by $g(X)$. The expected payment per loss is

$$E\{g(X)\} = \int_0^\infty g(x)\,dF(x). \tag{2.1}$$

The number of losses incurred by a risk in a policy period is a random variable, N, so that the total loss payment is

$$Y = \sum_{i=1}^{N} g\ (X_i), \tag{2.2}$$

which is the sum of a random number of random variables. It is customarily assumed that the loss severity X is distributed independently of the loss frequency N. With this assumption it can be shown that the expected payment in a policy period is

$$E\ \{Y\} = E\ \{N\} \cdot E\ \{g\ (X)\}, \tag{2.3}$$

which says that the expected value pure premium of a risk is the product of average frequency of loss and the average severity. (See Miccolis [5].)

Increased Limits Coverage

A liability insurance coverage is generally written to cover a loss in full up to a specified maximum dollar amount for any one loss. Let k be such a policy limit. We can express the payment function $g\ (X;\ k)$ of a loss X as

$$g\ (X;\ k) = \begin{cases} X, & 0 < X \le k \\ k & k < X \end{cases} \tag{2.4}$$

The expected payment per loss under this coverage can be expressed as

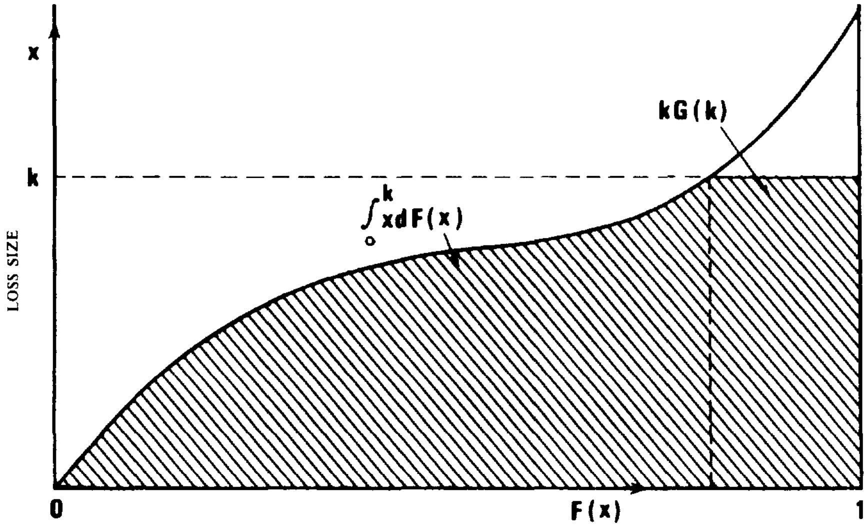
$$E\ \{g\ (X;\ k)\} = \int_0^k x dF\ (x) + kG\ (k). \tag{2.5}$$

The formula is demonstrated graphically in Figure 5, where the integral on the right is represented by the shaded area to the left of the broken vertical line, while the term $kG\ (k)$ is represented simply by the rectangle to the right of the line.

Rates are generally published for some standard limit called the basic limit; let this be b. Increased limits rates are expressed as a factor, $I\ (k)$, called the increased limits factor, to be applied to the basic limits pure premium rate. Thus

$$\begin{aligned} I\ (k) &= [E\ \{g\ (X;\ k)\} \cdot E\ \{N\}]\ /\ [E\ \{g\ (X;\ b)\} \cdot E\ \{N\}] \\ &= E\ \{g\ (X;\ k)\}\ /\ E\ \{g\ (X;\ b)\}, \end{aligned} \tag{2.6}$$

FIGURE 5
LOSSES WITH INDEMNITY LIMITED TO k



which depends on the distribution of size of loss only; see Miccolis [5]. The situation is demonstrated in Figure 6, where the increased limits factor is the ratio of the shaded area up to k versus the shaded area up to b. The picture also displays another property of the increased limits factor. Miccolis [5] shows that the derivative of I (k) can be expressed as

$$I'(k) = G\ (k)\ /\ E\ \{g\ (X;\ b)\}. \tag{2.7}$$

The picture shows that when k is increased by dk, the area representing the expected payment is increased by G (k) dk. Hence the result shown in Figure 6.

Miccolis also discusses a consistency test for increased limits factors. A picture will provide much better insight into this question. In Figure 7, the enclosed region below the cdf curve is divided into horizontal panels which, for convenience of exposition, have equal width. The horizontal lines serve to subdivide a loss, such as L, into layers. With layers of equal width, the picture makes it quite plain that the expected payment in any layer is less than that in a preceding layer. If the layers

FIGURE 6
INCREASED LIMITS FACTOR

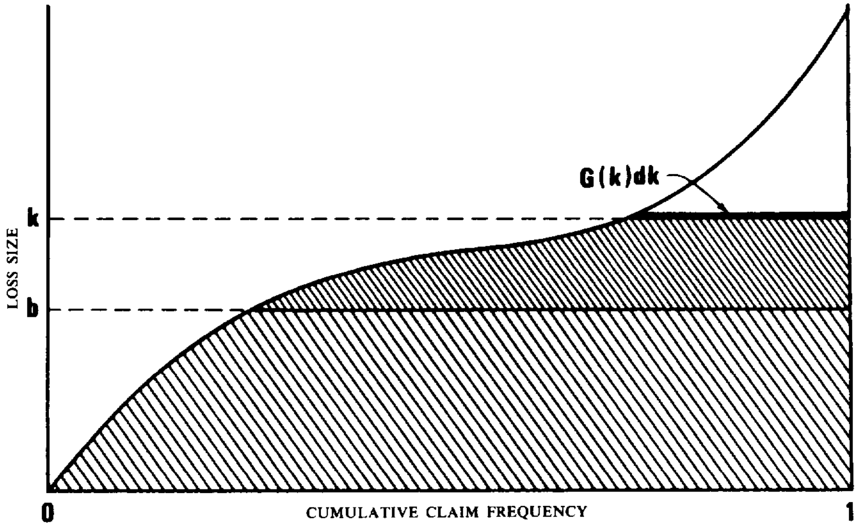
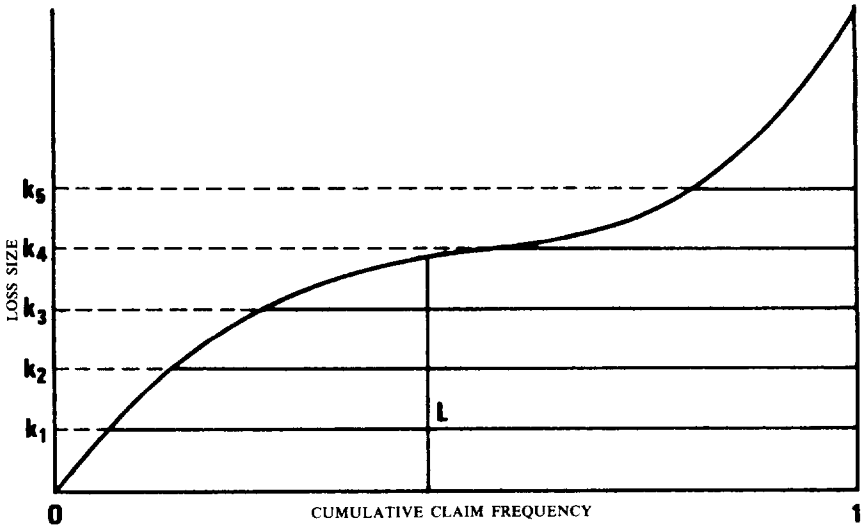


FIGURE 7
CONSISTENCY OF INCREASED LIMIT FACTOR



are of different widths, this property holds between the layers for the expected payment per unit coverage. Hence, the increased limits factor must increase at a decreasing rate as the limit increases. This is the consistency test. Actually, Figure 7 also shows that this is a common sense argument; a loss must have penetrated a lower layer before it reaches an upper layer.

Excess of Loss Coverage

An excess of loss contract generally covers losses in excess of a retention R, subject to a maximum limit L. The payment under such a contract may be expressed as a function of the loss X:

$$h\ (X;\ R,\ L) = \begin{cases} 0, & 0 < X \leq R \\ X - R, & R < X \leq S \\ L, & S < X, \end{cases} \tag{2.8}$$

where

$$S = R + L. \tag{2.9}$$

The situation may be described by means of the graph in Figure 8. For a loss such as represented by the line L_1 or L_2, the payment is represented by that portion of the line which falls inside the shaded region $BGEC$. The expected payment per ground-up claim under such contract has been derived in the literature by the size method, and can be expressed in many different forms; the following are given in Miccolis.

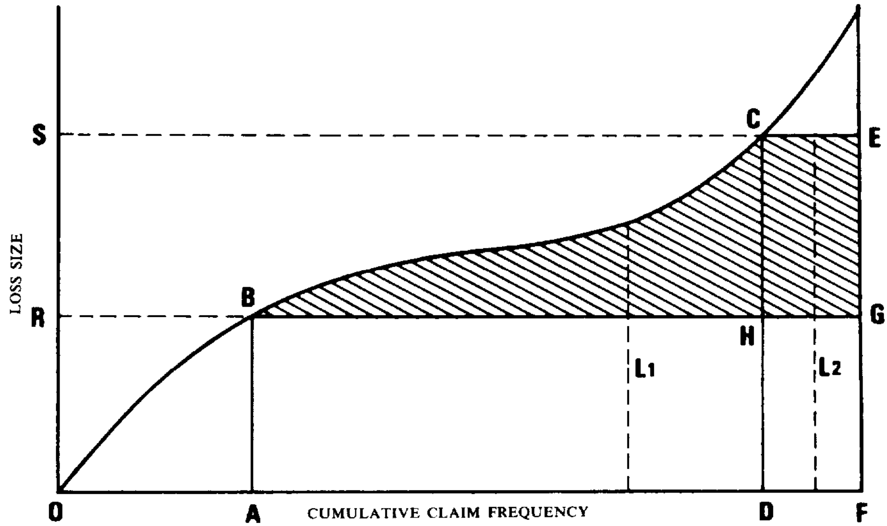
$$E\ \{h\ (X;\ R,\ L)\} = \int_R^S (x - R)\ dF\ (x) + LG\ (S) \tag{2.10}$$

$$= \int_R^S xdF\ (x) - R\ [F\ (S) - F\ (R)] + LG\ (S) \tag{2.11}$$

$$= \int_R^S xdF\ (x) + SG\ (S) - RG\ (R). \tag{2.12}$$

Figure 8 gives a simple graphical explanation of these integration results. They can be expressed in terms of the areas of the various regions shown in the graph, respectively as follows.

FIGURE 8
LOSSES WITH RETENTION AND LIMIT



$$E\{h(X; R, L)\} = BHC + HGEC \quad (2.13)$$

$$= ADCB - ADHB + HGEC \quad (2.14)$$

$$= ADCB + DFEC - AFGB. \quad (2.15)$$

Each of these is equal to the shaded area in the graph.

It is, of course, much easier to express the expected payment of such an excess of loss contract by the layer method:

$$E\{h(X; R, L)\} = \int_R^S G(x)\, dx. \quad (2.16)$$

The result is plain from Figure 8; it can also be derived from the integral expressions given above via integration by parts.

Relationships in the mathematics of excess of loss coverages could take on very complicated algebraic form, sometimes concealing the simplicity of the underlying idea. For example, Patrik [6] gives an expression for the expected loss excess of R subject to an upper limit of L in terms of $E\{X\} - R$ and other quantities. The average excess loss per ground-up claim is given by

$$E\{X\} - R + \Pr\{X \le R\} \cdot (R - E\{X | X \le R\}) - \Pr\{X \ge R+L\} \cdot [E\{X | X \ge R+L\} - (R+L)]. \quad (2.17)$$

This can be demonstrated by the graph in Figure 9 where A, B, C, and D represent areas of the respective regions. The above relation says simply that

$$B = (A + B + C) - (A + D) + D - C, \quad (2.18)$$

because

$$B = \text{expected excess loss} \quad (2.19)$$

$$A + B + C = E\{X\}, \text{ i.e. expected loss} \quad (2.20)$$

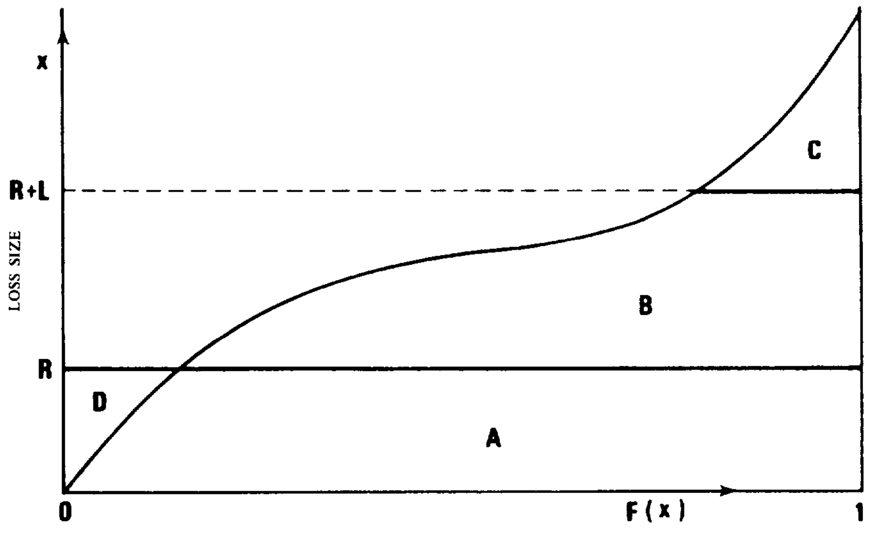
$$A + D = R \quad (2.21)$$

$$D = \Pr\{X \le R\} \cdot (R - E\{X | X \le R\}) \quad (2.22)$$

$$C = \Pr\{X \ge R+L\}\ [E\{X | X \ge R+L\} - (R+L)] \quad (2.23)$$

as is clear from the picture.

FIGURE 9
EXCESS OF LOSS COVERAGE



3. TREND

The effects of economic and social inflationary trends are to increase the size of losses. These effects act differently on the first dollar and the excess of loss coverages. Suppose the effect of inflation is, after a period of time, to change a loss of size x to a loss of size x', such that

$$x' = \alpha (x). \tag{3.1}$$

Assume that $\alpha (x)$ is a monotonic function, and let $F_1(x')$ be the cdf of x', i.e., the cdf after inflation. Then

$$F_1(x') = F (x), \tag{3.2}$$

and

$$F_1(\alpha (x)) = F (x). \tag{3.3}$$

The effect of inflation is demonstrated in Figure 10, where the lower curve represents the cdf before inflation, and the upper curve represents the cdf after inflation. The graph shows that a loss AB of size x becomes a loss AC of size x'. When, starting from the cdf curve $F (x)$, each size of loss, as represented by the vertical distance from the horizontal axis to the curve $F (x)$, is extended according to the function $x' = \alpha (x)$, we obtain the cdf curve after inflation. A simple case of inflation is one in which the loss is increased by a uniform multiplicative factor a, so that

$$x' = ax. \tag{3.4}$$

In this case, the cdf curve after inflation, $F_1(x')$, is obtained by extending each loss before inflation by a constant factor a.

It is well known that an excess of loss coverage is more seriously affected by inflation (assuming, for example, a uniform rate for all loss sizes); see, for example, Ferguson [2]. Figure 11 gives a dramatic demonstration of the leveraged effect of inflation on the excess of loss coverage. Let the rate of inflation be uniform for all sizes of loss, and the cdf curve after inflation be constructed from the curve before inflation as described above. The additional amount of loss resulting from inflation is shown in Figure 11 as the more heavily shaded region. If the retention R remains fixed, the expected excess loss payment is increased proportionally much more than indicated by the general rate of inflation.

FIGURE 10
EFFECT OF INFLATION

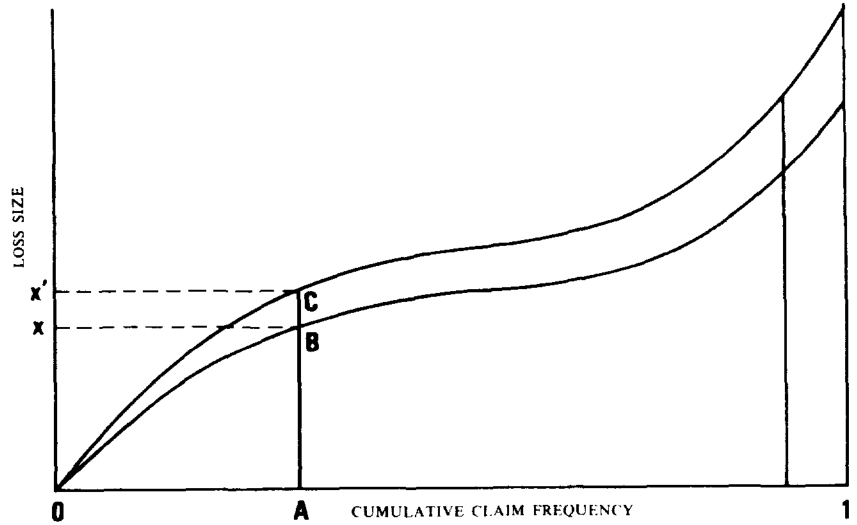
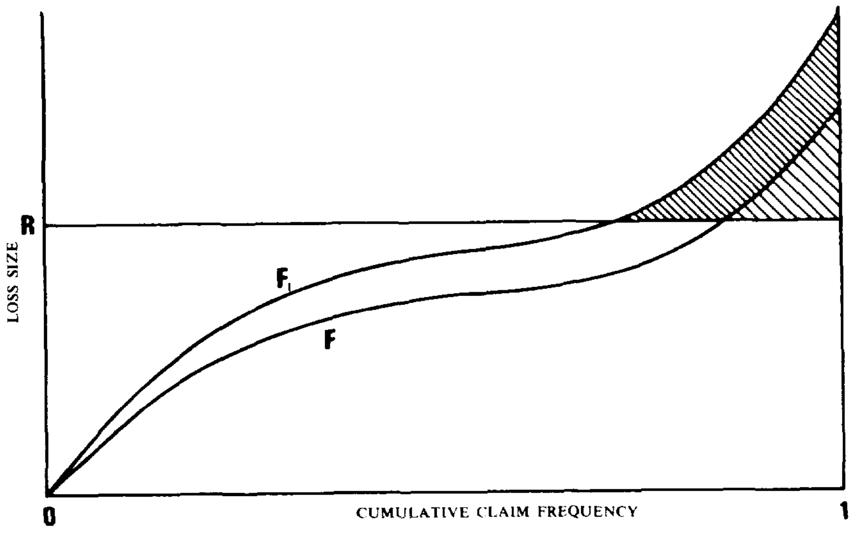


FIGURE 11
EFFECT OF INFLATION ON EXCESS LOSSES

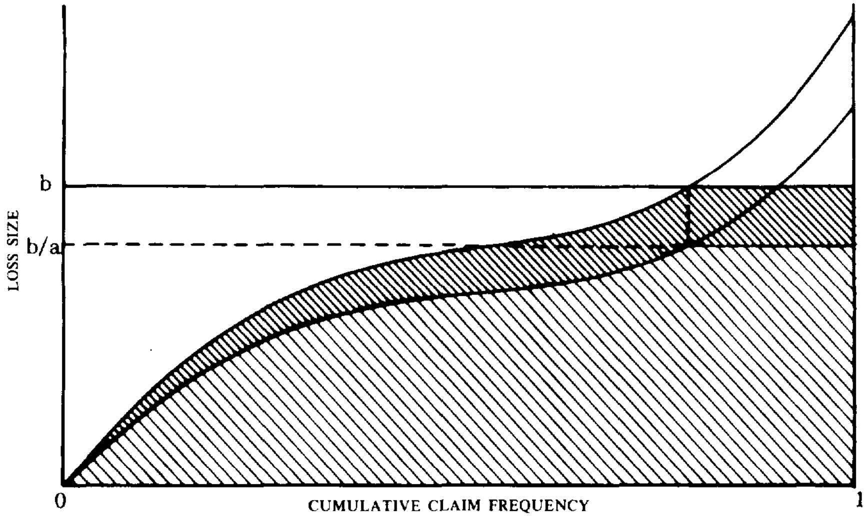


Since the total increase due to inflation is divided between the basic limits loss and the excess loss, the basic limits loss is expected to incur an inflationary increase at a lower rate than the total limits rate. This topic has been treated in Finger [3]. Figure 12 gives a graphical demonstration of this effect and also shows the following algebraic result (see, for example, Miccolis [5]):

$$E\{g(X'; b)\} = a\,E\{g(X; b/a)\}. \tag{3.5}$$

The picture says that the new expected basic limits loss, represented by the total shaded area, is equal to the old expected loss up to the limit b/a, represented by the more lightly shaded area, extended by a factor a. Any vertical line through the shaded region in Figure 12 would have its intercept in the more heavily shaded region equal to a times its intercept in the more lightly shaded region.

FIGURE 12
EFFECT OF INFLATION ON BASIC LIMIT LOSSES



The study of the effect of inflation on excess of loss coverages can lead to rather complicated algebraic expressions. For example, Ferguson [2] relates the pure premium of an excess of loss coverage with indexing to the pure premium of one without indexing, the difference being expressed as a discount on the coverage without indexing. In an excess of loss coverage with indexing, the retention increases with inflation. A moment's reflection shows that the discount can be determined by comparing the expected loss under one contract with that under another. Let $\bar{X}$ be the average excess loss trended and indexed, R be the retention, $a-1$ be the proportional increase due to inflationary trend, Δ' be excess cost (per claim) on claims that exceed the retention as a result of inflation, and k be the multiplying factor which is equal to $G(R)$. Then Figure 13 shows that

$$E\{L_0\} = k\bar{X} + k(a-1)R + c\Delta', \tag{3.6}$$

$$E\{L_I\} = k\bar{X}, \tag{3.7}$$

where $E\{L_0\}$ is the expected excess loss per ground-up claim without indexing and $c = G(R) - G(aR)$ and $E\{L_I\}$ the expected excess loss per ground-up claim with indexing. Thus,

$$\Delta = 1 - \frac{E\{L_I\}\,E\{N\}}{E\{L_0\}\,E\{N\}} \tag{3.8}$$

$$= 1 - \frac{k\bar{X}}{k\bar{X} + k(a-1)R + c\Delta'}$$

$$= 1 - \frac{1}{1 + R(a-1)/\bar{X} + c\Delta'/k\bar{X}}\,. \tag{3.9}$$

or,

$$D = 1 - \frac{1}{1 + R(a-1)/\bar{X}} \tag{3.10}$$

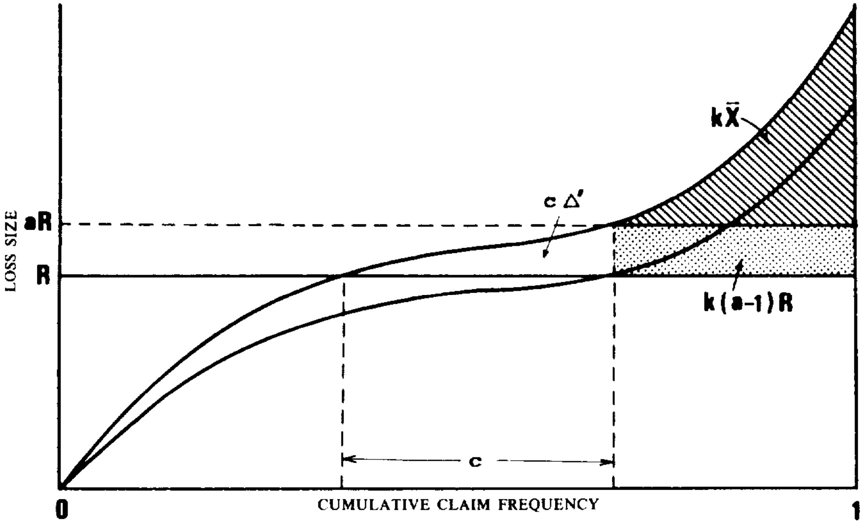
as proposed by Ferguson, neglecting the relatively small term involving Δ'.

4. RETROSPECTIVE RATING

The Excess Pure Premium Ratio

We first consider the mathematics of the excess pure premium ratio, commonly denoted by $\phi(r)$. This is defined to be a risk's average amount of loss in excess of r times its expected loss, divided by the expected loss. It is also known as the Table M charge, while the Table M savings,

FIGURE 13
INDEXING EXCESS OF LOSS COVERAGE



$\Psi(r)$, at the entry r (meaning r times the expected loss) is defined as the expected amount by which the risk's actual loss falls short of r times the expected loss, divided by the expected loss. More precisely, let

A = actual loss of the risk;
E = $E\{A\}$, the expected loss;
Y = A/E, actual loss in units of expected loss; and
$F(Y)$ = the cumulative distribution function of Y.

Then

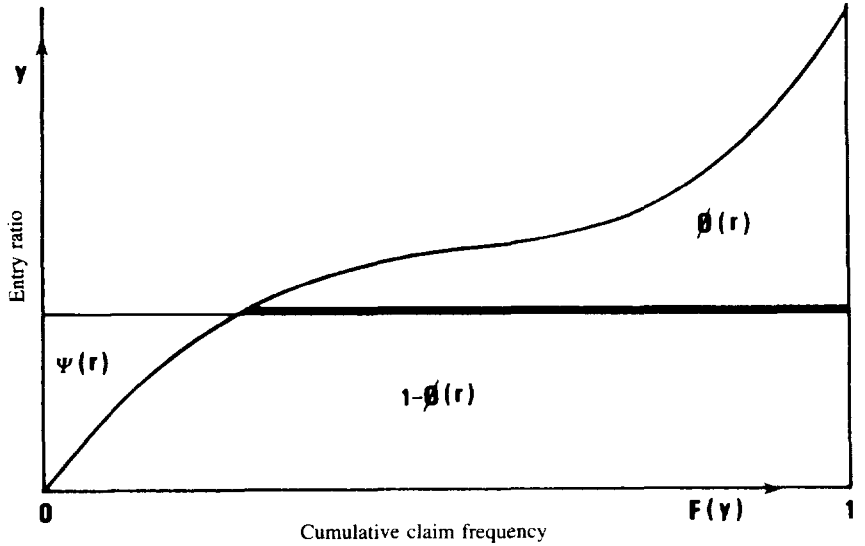
$$\phi(r) = \int_r^\infty (y - r)dF(y) \tag{4.1}$$

and

$$\psi(r) = \int_0^r (r - y)dF(y). \tag{4.2}$$

These functions are illustrated in Figure 14, where the cdf $F(y)$ is graphed against the entry ratio y. The functions $\phi(r)$ and $\psi(r)$ are represented by the areas indicated in the graph. A number of mathematical properties are now clearly demonstrated.

FIGURE 14
FUNCTIONS IN RETROSPECTIVE RATING



(1) By definition, the bounded area below the $F(y)$ curve is equal to 1. Hence $\phi(0) = 1$.
(2) $\phi(r)$ is a decreasing function of r, and $\phi(r) \to 0$ as $r \to \infty$.
(3) $\psi(r)$ is an increasing function of r; its value is unbounded as $r \to \infty$.
(4) Consider the small strip at $y = r$ in the graph. This shows that an increment dr from r will yield a decrease $G(r)dr$ in $\phi(r)$. Hence

$$\phi'(r) = (d/dr)\, \phi(r) = -G(r). \tag{4.3}$$

A second differentiation yields

$$\phi''(r) = f(r), \tag{4.4}$$

where $f(r)$ is the density function of the entry ratio, a result well known in the literature (Valerius [11]). Similarly, we may deduce from Figure 14 that

$$\psi'(r) = (d/dr)\, \psi(r) = F(r) \tag{4.5}$$

and

$$\psi''(r) = f(r). \tag{4.6}$$

(5) Consider the area of the rectangle on the interval from 0 to r in Figure 14. This gives the relation

$$r = [1 - \phi(r)] + \psi(r), \tag{4.7}$$

or

$$\psi(r) = \phi(r) + r - 1; \tag{4.8}$$

this is a fundamental relation connecting $\psi(r)$ and $\phi(r)$.

A result more general than (5) can also be obtained quite easily from Figure 15. Let

$$L = \begin{cases} r_1E & \text{if } A \leq r_1E \\ A & \text{if } r_1E < A \leq r_2E \\ r_2E & \text{if } r_2E < A. \end{cases} \tag{4.9}$$

Then the cdf of L/E can be represented by the solid line in Figure 15. The shaded area represents the quantity $E\{L\}/E$ and we have

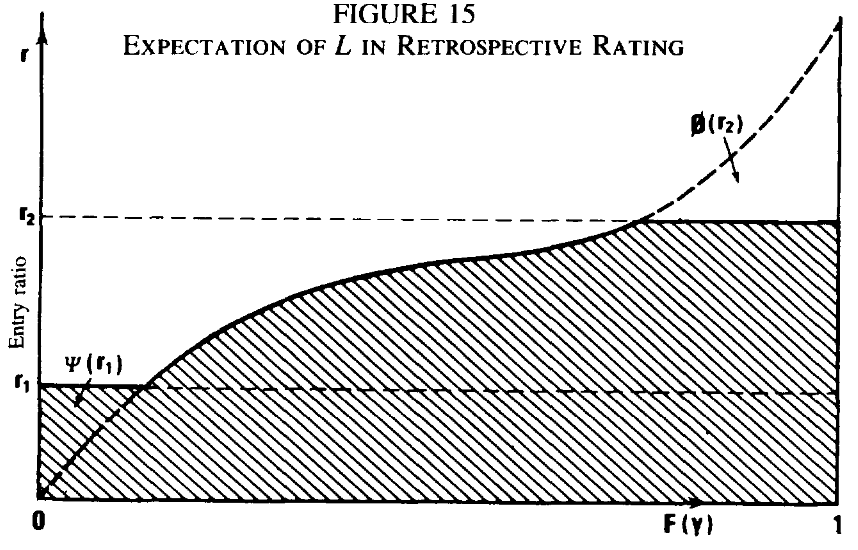
$$E\{L\}/E - \psi(r_1) + \phi(r_2) = 1, \tag{4.10}$$

or

$$E\{L\}/E = 1 + \psi(r_1) - \phi(r_2). \tag{4.11}$$

See Skurnick [9].

FIGURE 15
EXPECTATION OF L IN RETROSPECTIVE RATING



Retrospective Rating

In the Workers' Compensation Retrospective Rating Plan, the retrospective premium R is given by

$$R = b + CA, \tag{4.12}$$

subject to a maximum premium G and a minimum premium H, where b is the basic premium and C is the loss conversion factor (LCF), and where b is alternatively represented by

$$b = BP, \tag{4.13}$$

with P as the standard premium (before any applicable expense gradation) and B as the basic premium ratio. Let L_G be actual loss that will produce the maximum premium:

$$G = b + CL_G \tag{4.14}$$

and let

$$r_G = L_G/E. \tag{4.15}$$

Similarly, define L_H to be

$$H = b + CL_H, \tag{4.16}$$

$$r_H = L_H/E. \tag{4.17}$$

Further, let

$$L = \begin{cases} L_H & \text{if } A \le L_H \\ A & \text{if } L_H < A \le L_G \\ L_G & \text{if } L_G < A. \end{cases} \tag{4.18}$$

Then the retrospective premium can be represented by

$$R = b + CL. \tag{4.19}$$

For ease of exposition, we ignore the tax factor. If we identify r_H and r_G with r_1 and r_2, respectively, then Figure 16 shows the quantity $E\{L\}/E$ as the area of the shaded region $OFDCBA$. It then follows that

$$E\{L\} = E - \phi(r_G)E + \psi(r_H)E \tag{4.20}$$

$$= E - I, \tag{4.21}$$

where

$$I = E[\phi(r_G) - \psi(r_H)] \tag{4.22}$$

is called the net insurance charge of Table M. If the plan is to be balanced, the expected retrospective premium must be equal to the sum of the total expenses, e, and the expected loss, E:

$$E\{R\} = e + E. \tag{4.23}$$

On the other hand, it also follows from the above that

$$E\{R\} = b + C(E - I). \tag{4.24}$$

Equating these two quantities we obtain the basic premium in terms of the expense, expected loss, and the net insurance charge:

$$b + C\{E - I) = e + E \tag{4.25}$$

or

$$b = e - (C - 1)E + CI. \tag{4.26}$$

A formula relating the charge difference to the minimum premium, expected loss and expense provision has been used to facilitate the determination of retrospective rating values from specified maximum and minimum premiums. This formula can be derived with the help of Figure 16.

Consider the equation

$$R = b + CL \tag{4.27}$$

Taking the expectation and representing the expectation $E\{L\}/E$ by the shaded area of Figure 16 we have

$$e + E = b + CE[OFDCBA]. \tag{4.28}$$

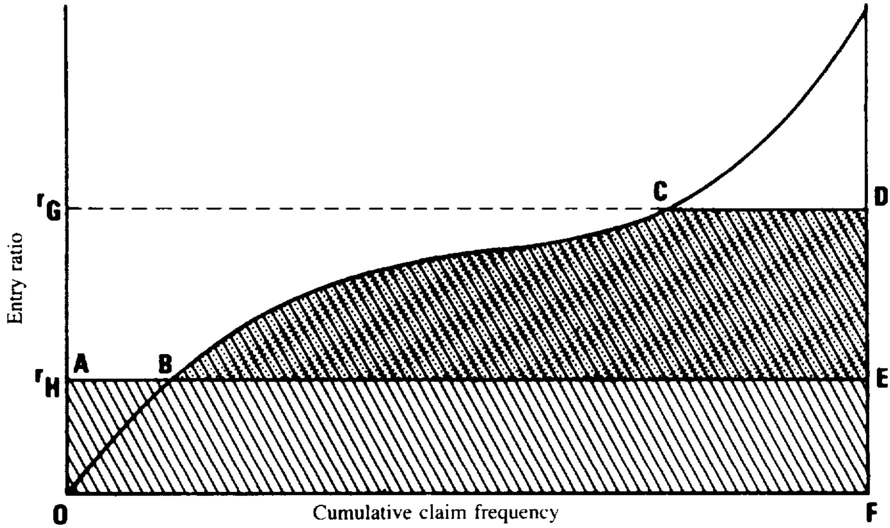
On the other hand, we have for the minimum premium H:

$$H = b + CEr_H \tag{4.29}$$
$$= b + CE\ [OFEA]. \tag{4.30}$$

Taking the difference on both sides of the two equations above we have

$$(e + E) - H = CE\ [BEDC] \tag{4.31}$$
$$= CE\ [\phi(r_H) - \phi(r_G)]. \tag{4.32}$$

FIGURE 16
RETROSPECTIVE RATING PREMIUM



This formula, together with the formula

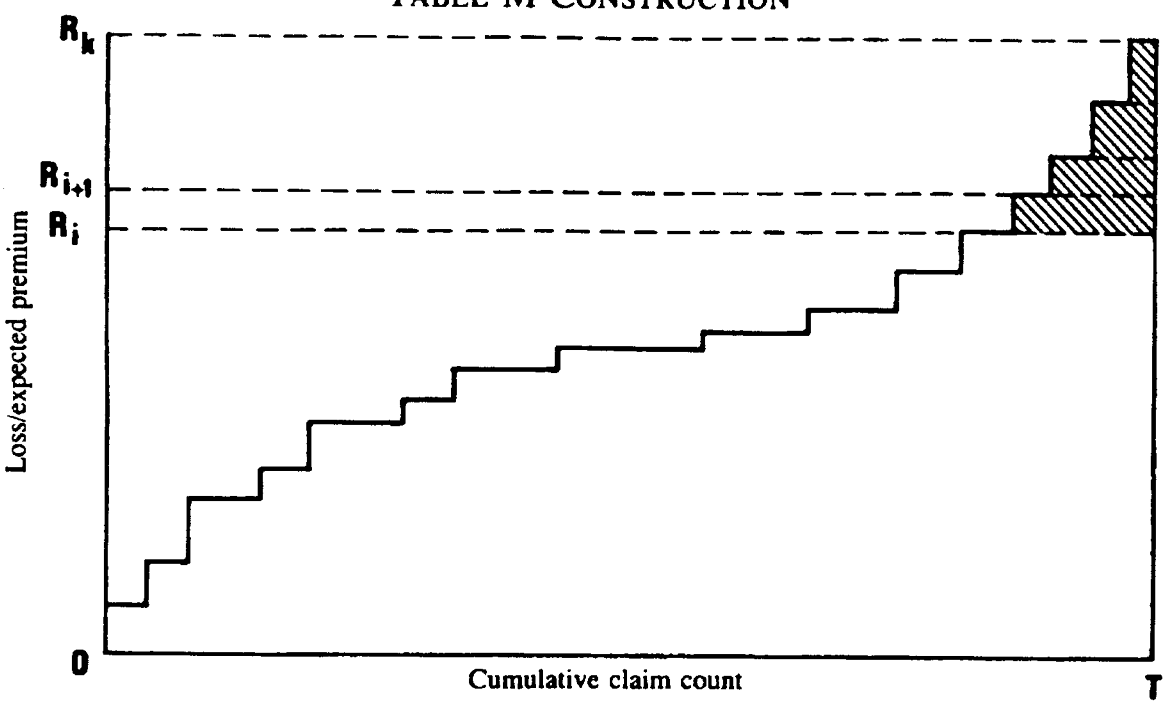
$$G - H = CE(r_G - r_H), \tag{4.33}$$

which is much easier to derive, can be used to determine the rating values given the maximum and minimum premiums. One may interpret the difference in charge, $\phi(r_H) - \phi(r_G)$, as indicated by the dotted area in Figure 16, to be the difference between the expected retrospective premium and the minimum premium, apart from a conversion factor CE.

Construction of Table M

A Table M has been constructed by Simon [8]; see also Skurnick [9]. The algebra involved in the construction procedure appears to be rather complicated. Actually, the idea is very simple when this is expressed in a graph. Figure 17 shows a cumulative frequency curve constructed from observed data on risks within a premium group. Let the loss ratios be arranged in ascending order: $R_1, R_2, \ldots, R_k$, with R_i occurring N_i times. Also let the total number of claims be $T = N_1 + \ldots + N_k$. The cumulative frequency up to R_i, i.e. $T_i = N_1 + \ldots + N_i$ is plotted against R_i for each i so as to form a step function whose abscissa in the interval (R_i, R_{i+1}) is the cumulative frequency T_i, as shown in Figure 17. We may think of this graph as a re-scaled version of the cdf curve plotted against the entry ratio. It now appears quite clear that the value of ϕ for the entry ratio corresponding to R_i is simply the shaded area in Figure 17 divided by the total enclosed area below the cumulative frequency curve. The entry ratio corresponding to R_i is simply R_i divided by the average loss ratio $\Sigma N_i R_i / T$.

FIGURE 17
TABLE M CONSTRUCTION



A convenient procedure to construct a Table M is to sum the horizontal strips downward, cumulatively, starting from the strip corresponding to (R_{k-1}, R_k), down to the strip corresponding to $(0, R_1)$. It is convenient also to sum the frequencies downward, cumulatively, because the cumulative sum of such frequencies down to and including N_{i+1} is the length of the strip corresponding to the interval (R_i, R_{i+1}). Thus let

$$S_{1,i} = \sum_{j=i+1}^{k} N_j \tag{4.34}$$

which is represented by the length of the strip on (R_i, R_{i+1}), and

$$S_{2,i} = S_{2,i+1} + S_{1,i} (R_{i+1} - R_i) \tag{4.35}$$

which describes the fact that the sum of the strips above R_i is obtained by adding the strip on (R_i, R_{i+1}) to the sum of the strips above R_{i+1}. The value of ϕ at the entry ratio corresponding to R_i is then $S_{2,i}/S_{2,0}$, with $S_{2,0}$ equal to the total area of all the strips. The entry ratio corresponding to R_i is obtained by normalization:

$$r_i = R_i \bigg/ \left(\frac{S_{2,0}}{S_{1,0}}\right) . \tag{4.36}$$

We may think of R_i as loss expressed in an arbitrary unit and the denominator as the expected loss in this unit. The procedure is described in algebraic form by Skurnick. It is easy to see that this is a layer approach.

Table L

A retrospective rating plan may provide for a per accident limit on losses. The table of charges which incorporates this per accident limitation is called Table L, which has been described by Skurnick [9]. Let A be the actual unlimited loss, as before, A^* be the actual limited loss, and $F^*(.)$ be the cdf of $Y^* = A^*/E$. Then the Table L charge is defined as (Skurnick)

$$\phi^*(r) = \int_r^\infty (y - r)dF^*(y) + k, \tag{4.37}$$

where k is the loss elimination ratio

$$k = [E - A^*]/E \tag{4.38}$$

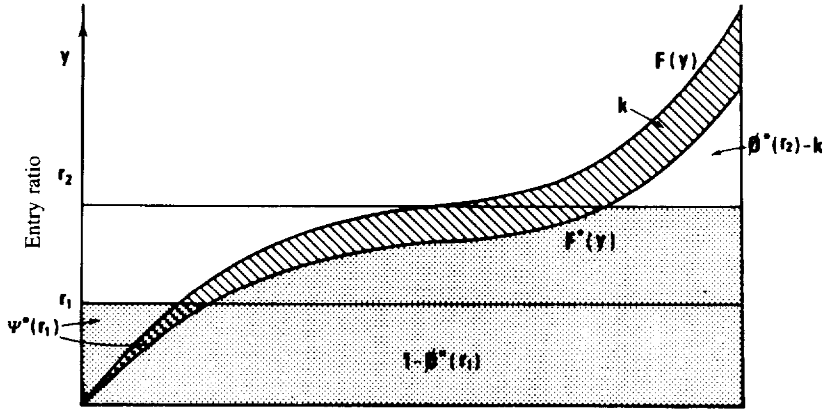
Further, the Table L savings are defined as

$$\psi^* = \int_0^r (r - y)dF^*(y). \tag{4.39}$$

In Figure 18 the curves for $F(y)$ and $F^*(y)$ are plotted against the entry ratio $r = A/E$. $F(y)$ is necessarily situated above $F^*(y)$, and by the definition of r, the enclosed area below the $F(y)$ curve is equal to 1, while the enclosed area below the $F^*(y)$ curve is $1 - k$. The area of the shaded belt is equal to the loss elimination ratio k. Many of the properties of the Table L charges, as presented by Skurnick [9], can be easily obtained from the graph. For example, consider the limited loss

$$L^* = \begin{cases} r_1E & \text{if } A^* \le r_1E \\ A^* & \text{if } r_1E < A^* \le r_2E \\ r_2E & \text{if } r_2E < A^*. \end{cases} \tag{4.40}$$

FIGURE 18
TABLE L FUNCTIONS



Then $E\{L^*\}/E$ is represented by the dotted area in Figure 18. We deduce that

$$E\{L^*\}/E - \psi^*(r_1) + [\phi^*(r_2) - k] = 1 - k \tag{4.41}$$

and hence

$$E\{L^*\}/E = 1 + \psi^*(r_1) - \phi^*(r_2), \tag{4.42}$$

as in Skurnick. As another example, identify r_1 and r_2, respectively, with r_H and r_G as defined before. Also let

$$R^* = b^* + CL^* \tag{4.43}$$

be the retrospective premium with per accident limitation. Then, combining the equation

$$E\{R^*\} = e + E = b^* + CEr_H + CE[\phi^*(r_H) - \phi^*(r_G)], \tag{4.44}$$

which follows from the fact that the expected retrospective premium is b^* plus the dotted area (converted), with the equation

$$H = b^* + CEr_H, \tag{4.45}$$

we have the Table L version of a familiar formula

$$e + E - H = CE\,[\phi^*(r_H) - \phi^*(r_G)], \tag{4.46}$$

the last factor on the right being represented by the dotted area between $r_1 = r_H$ and $r_2 = r_G$ in Figure 18. As a final example of the use of Figure 18, one may consider the construction of Table L. This can be done in a manner similar to the construction of Table M, except that the cumulative frequency function of the limited loss is used, and the final result has to be adjusted for the loss elimination factor k.

Asymptotic Behavior

As the premium size becomes large, the limiting form of the charge takes on a simple function. The graphs in Figure 19 help us to understand the asymptotic behavior. Consider the case with no per loss limitation.

FIGURE 19
LIMITING CASE IN RETROSPECTIVE RATING

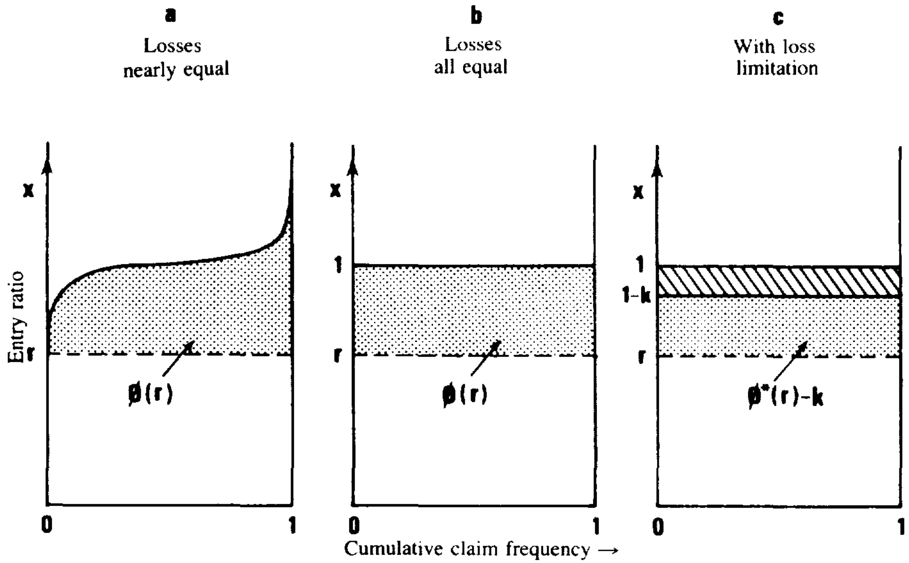


Figure 19(a) shows a cdf curve for losses which are nearly equal; here the $\phi(r)$ region almost forms a rectangle. When all losses are equal, the cdf $F(x)$ is a step function with a single jump at $x = 1$, as shown in Figure 19(b). The Table M charge, $\phi(r)$, at the entry point r is represented by the area of the rectangle between r and 1. Hence

$$\phi(r) = \begin{cases} 1 - r & r \leq 1 \\ 0 & 1 < r. \end{cases} \tag{4.47}$$

The limiting case with per loss limitation is shown in Figure 19(c). Here the cdf $F^*(x)$ is shown as the horizontal line $x = 1 - k$, where it has its single jump. The Table L charge, $\phi^*(r)$, is the area of the rectangle between r and $1 - k$, plus the loss elimination ratio k. Thus

$$\phi^*(r) = \begin{cases} 1 - r & r < 1 - k \\ k & 1 - k \leq r. \end{cases} \tag{4.48}$$

Other Applications

There are other interesting mathematical relationships in the mathematics of retrospective rating, and many such intricate relationships are presented in Carlson [1]. It is a great burden to follow the algebra of the many complicated relationships presented there. Most of these, however, become much clearer if we make use of the graphical approach adopted here. Rather than go through the numerous equations and formulas in Carlson, we present a particular example to illustrate the power of our graphical method. Let us pick, almost at random, equation (15a) in Carlson, which can be explained as follows. Let the minimum premium be greater than the basic premium, and the maximum premium be equal to the standard premium:

$$H > B,\ G = P. \tag{4.49}$$

Then, in Carlson's notation,

$$P - Rv = C(P's - H's) \tag{4.50}$$

$$= C(P' - H') - C(H'p - P'p). \qquad 4.51)$$

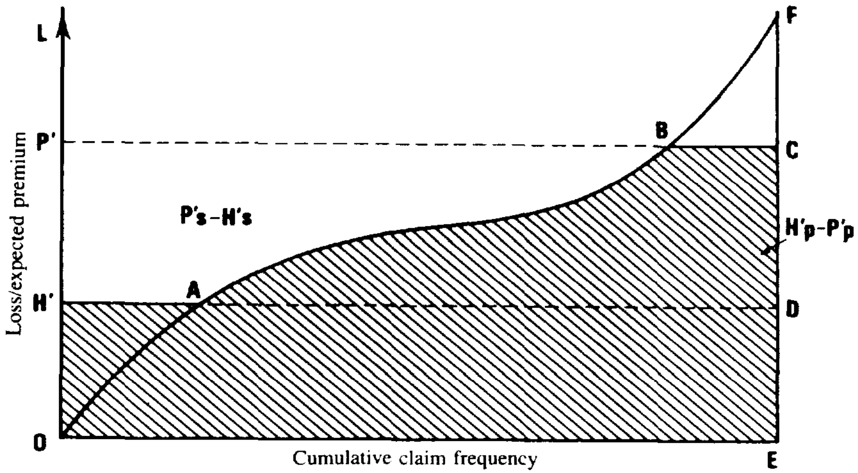
These equations follow immediately from Figure 20 with the following interpretation of Carlson's notations:

$$P \;= b + CP' \tag{4.52}$$

$$Rv = \text{expected retrospective premium} \tag{4.53}$$

$$= b + C[OECBAH'] \tag{4.54}$$

FIGURE 20
RELATIONSHIPS IN RETROSPECTIVE RATING



$$P's = OBP' \tag{4.55}$$

$$H's = OAH' \tag{4.56}$$

$$H'p = ADF \tag{4.57}$$

$$P'p = BCF. \tag{4.58}$$

5. CONCLUSION

This paper presents a graphical approach to the mathematics of excess of loss coverages and related topics. The graphs serve to simplify and clarify much of the complicated algebra which has hitherto been the sole vehicle to express the mathematical ideas involved. We hope this will become a useful addition to the actuarial tool box of the student and the practicing casualty actuary alike. This technique has been used in explaining the principles of coinsurance and its many properties (Lee [4]). Philbrick [7] uses the same idea to describe size of loss distributions.

REFERENCES

[1] Thomas O. Carlson, "An Actuarial Analysis of Retrospective Rating," *PCAS* XXVIII, 1941, p. 283.

[2] Ronald E. Ferguson, "Nonproportional Reinsurance and the Index Clause," *PCAS* LXI, 1974, p. 141.

[3] Robert J. Finger, "A Note on Basic Limits Trend Factors," *PCAS* LXIII, 1976, p. 34.

[4] Yoong-Sin Lee, "A Graphical Treatment of the Coinsurance Clause," *Journal of Risk and Insurance* LII, 1985, p. 644.

[5] Robert S. Miccolis, "On the Theory of Increased Limits and Excess of Loss Pricing," *PCAS* LXIV, 1977, p. 27.

[6] Gary S. Patrik, "Review of Ferguson: Actuarial Note on Loss Rating," *PCAS* LXV, 1978.

[7] Stephen W. Philbrick, "A Practical Guide to the Single Parameter Pareto Distribution," *PCAS* LXXII, 1985, p. 44.

[8] LeRoy J. Simon, "1965 Table M," *PCAS* LII, 1965, p. 1.

[9] David Skurnick, "The California Table L," *PCAS LXI*, 1974, p. 117.

[10] Richard H. Snader, "Fundamentals of Individual Risk Rating and Related Topics," Study Note for Parts 9b and 10, Casualty Actuarial Society.

[11] Nels M. Valerius, "Risk Distributions Underlying Insurance Charges in the Retrospective Rating Plan," *PCAS XXIX*, 1942, p. 96.

ADDRESS TO NEW MEMBERS—MAY 9, 1988

CHARGE TO THE NEW FELLOWS

W. JAMES MACGINNITIE

In preparation for this occasion, I looked back 25 years to 1963 when I was admitted as a new fellow. Perhaps in 2013, one of you will be at this podium. We had fewer new fellows then but, of course, the exams were tougher. The meeting drew about 175 attendees versus nearly 700 at this meeting. The cites in 1963 were the Catskills and Atlantic City, two resort areas in decline, whereas this year the Casualty Actuarial Society is meeting in Tampa and Montreal. In 1963, the premium volume for the insurance industry was less than $20 billion.

In the ensuing 25 years, the CAS has grown from less than 400 members to more than 1,400. Premium volume in the industry has grown at a compound rate in excess of 10% per year and now exceeds $200 billion in the data maintained by A. M. Best, plus a substantial amount of captive, self-insured and off-shore premium.

That growth is a function of many forces. One of those forces is the litigiousness of our society. I noted in my review of 1963 that the secretary of the CAS was authorized to take out the Society's first public liability policy. Another force is the growth of both population and the economy, with a great deal of the latter being inflationary in nature. The breakdown of the system of making rates in concert has also contributed to the growth of the CAS. The largest employer of CAS members is no longer a bureau or a company but is now a consulting firm, reflecting the rapid growth in that area of practice. The availability of data and computational machinery with which to manipulate it have also helped the growth. You can get more power in 15 pounds of laptop computer today than in a roomful of tube machines 25 years ago. But perhaps the largest contributor to the growth and membership of the CAS has been the perceived value of actuarial training, and the ability of those so trained to provide useful solutions to the many problems which beset the risk and insurance business.

Some things have not changed much in 25 years. In 1963, the President, Laurie Longley-Cook, reported ". . . greatly increased competition in the industry has forced rates for certain lines too low . . . other causes include inflation and greater claims consciousness." He went on to touch on "astronomical legal fees" and then observed ". . . ignorance and fear of loss of business sometimes lead to inadequate rate filings." The secretary mentioned problems of recruiting new members. A new booklet for recruiting was titled "Man With A Future;" at least gender is an area where some progress has been made. The Society also heard "a report on activities in cooperation with other actuarial organizations relating to the possible formation of some form of overall national actuarial organization or federation, and the related problem of accreditation of actuaries." Those activities led to the formation of the American Academy of Actuaries, in recognition of the increasingly public nature of the profession.

You are the beneficiaries of that growth and of the selfless work of generations that have gone before you. As you receive your Fellowship, you must now also assume the many responsibilities that go with the designation.

- Responsibility to continue your education, through study, discussion and participation in continuing education opportunities.
- Responsibility to extend the expertise of the profession through your research, both theoretical and applied, and through the sharing of your results at meetings such as this.
- Responsibility to recruit and train the next generation, which means for many of you work on the education and examination committees.
- Responsibility to contribute to the continued growth and development of the profession.
- Responsibility to conduct your affairs in a sound and ethical manner, and especially to recognize your fiduciary obligation for the financial soundness of the organizations you serve, with their promises to provide protection and payment far into the future.

The accomplishment for which we recognize you today is a substantial one. You and your families and associates can be justly proud. Twenty-five years from now, when you look back to today, much will have changed and much will remain unchanged. We trust that as you look back from that future perspective, your discharge of your responsibilities will give you as much pride then as your Fellowship does now.

MINUTES OF THE 1988 SPRING MEETING

May 8–11, 1988

SADDLEBROOK RESORT, WESLEY CHAPEL, FLORIDA

Sunday, May 8, 1988

The Board of Directors held their regular quarterly meeting from 1:00 P.M. to 4:00 P.M.

Registration was held from 4:00 P.M. to 6:30 P.M.

From 5:30 P.M. to 6:30 P.M. there was a special presentation to new Associates and their guests. This session included an introduction to standards of professional conduct and the CAS committee structure.

A reception/dinner for all members and guests was held poolside from 6:30 P.M. to 9:30 P.M.

Monday, May 9, 1988

Registration continued from 7:00 A.M. to 8:00 A.M.

President David G. Hartman opened the meeting at 8:00 A.M. The first order of business was the admission of members. Mr. Hartman recognized the 62 new Associates and presented diplomas to the 19 new Fellows. The names of those individuals follow.

FELLOWS

Mary V. Anderson	Marthe A. Lacroix	Linda A. Shepherd
Brian Y. Brown	Patrick Mailloux	Russel L. Sutter
William M. Carpenter	William J. Miller	Jean Vaillancourt
Sanders B. Cathcart	George N. Phillips	William J. VonSeggern
Bruce G. Earwaker	Donald D. Sandman	James C. Votta
Kenneth R. Kasner	Roger A. Schultz	Patricia J. Webster
Eric R. Keen		

ASSOCIATES

Jeffrey H. Adams	Susan M. Gozzo	Thomas G. Moylan
Lawrence J. Artes	Nancy A. Graves	Chris E. Nelson
Robert K. Bender	Bruce H. Green	Kenneth J. Nemlick
Kay E. Bennighof	James W. Haidu	Kwok C. Ng
Steven W. Book	James S. Higgins	Glen C. Nyce
Michael Caulfield	Alan M. Hines	George N. Phillips
Denis Cloutier	Jane E. Jasper	Denis Poirier
Joseph F. Cofield	Steven J. Johnston	Sasikala Raman
Steven L. Colin	John J. Joyce	Srinivasa Ramanujam
Kevin J. Conley	Chester T. Kido	Thomas E. Schadler
Alan M. Crowe	Constantine G. Koufacos	Valerie L. Schmid
Michael K. Curry	David A. Lalonde	Richard D. Schug
Robert N. Darby	Susan E. LaPointe	Steven A. Skov
Donna R. Dickinson	Richard Lebrun	John A. Stenmark
Mark DiGaetano	Cecilia M. LePere	Douglas N. Strommen
James Ely	Roland D. Letourneau	Ronald J. Swanstrom
Karen F. Evans	David J. Macesic	Guy Vezina
William G. Fanning	Michael W. Mahoney	Debra L. Werland
Beth E. Fitzgerald	James B. McCreesh	Peter W. Wildman
Richard J. Gergasko	William H. Mitchell	Heather E. Yow
Richard N. Gibson	Elena D. Mohler	

Mr. Hartman then introduced W. James MacGinnitie, a past President of the Society, who addressed the new members concerning their professional responsibilities.

Michael Fusco, Vice President-Programs, presented the highlights of the program.

Charles A. Bryan, Vice President-Development, summarized the new *Proceedings* paper.

John Purple, Chairman of the Committee on Continuing Education, presented a summary of the Discussion Paper program.

Mr. Hartman concluded the business session at 9:00 A.M. and introduced Arthur B. Laffer, professor at Pepperdine University and member of the Economic Policy Advisory Board to the President of the United States. Mr. Laffer delivered the keynote address, speaking on the Reagan

Administration's impact on the U.S. economy in terms of the budget deficit, the trade deficit, foreign investment, interest rates, and inflation.

A panel presentation, "The Trials and Tribulations of the Insurance Exchanges," followed. The panel was moderated by W. James MacGinnitie, Consulting Actuary, Tillinghast/Towers Perrin. The panel members were Edward F. Belton, Executive Vice President, PAFCO, and Former President, Ontario Insurance Exchange; Joseph Fays, President, New York Insurance Exchange; and James M. Skelton, President, Illinois Insurance Exchange.

A luncheon followed from Noon to 1:30 P.M. Concurrently, a separate luncheon was held for the new Fellows.

Following lunch, the members convened to hear the panel presentation, "Who Should Pay for Pollution Losses?" The panel was moderated by Richard A. Lino, Jr., Vice President and Actuary, Continental Insurance Group. Panelists were John J. Amore, Senior Vice President, American International Group; Joan B. Berkowitz, President and C.E.O., Risk Science International; William Hart, Esq., Partner, Anderson, Russell, Kill, Olick; and Donald V. Jernberg, Partner, Isham, Lincoln & Beale. The panelists discussed the assessment of environmental risk and the underwriting and marketing of environmental insurance.

The remainder of the afternoon was devoted to presentations of the discussion papers, one new *Proceedings* paper, four panel presentations, and four workshops.

The new *Proceedings* paper was:

"The Mathematics of Excess of Loss Coverages and Retrospective Rating—A Graphical Approach"

Author: Yoong S. Lee
National University of Singapore

The Discussion Papers presented were:

1. "Determination of Outstanding Liabilities for Unallocated Loss Adjustment Expenses"
   Author: Wendy A. Johnson
   The Wyatt Company

2. "Premium Deficiency Reserves"
   Authors: Terrence M. O'Brien
   Coopers & Lybrand

   John G. Aquino
   Coopers & Lybrand

3. "The Reserve for Unrecoverable Reinsurance"
   Author: William G. McGovern
   Peat, Marwick, Main & Co.

4. "Loss Reserve Certification—Standards and Issues"
   Author: Mark J. Sobel
   Touche Ross & Company

5. "Professional Guidance for Casualty Actuaries in Australia, Canada, United Kingdom & U.S.A."
   Authors: John P. Ryan
   Tillinghast/Towers Perrin

   D. Ian W. Reynolds
   City University of London

6. "Application of Collective Risk Theory to Estimate Variability in Loss Reserves"
   Author: Roger M. Hayne
   Milliman & Robertson, Inc.

7. "Risk Theoretic Issues in Loss Reserving: The Case of Workers' Compensation Pension Reserves"
   Author: Glenn G. Meyers
   University of Iowa

8. "Determining the Proper Interest Rate for Loss Reserve Discounting: An Economic Approach"
Author: Robert P. Butsic
Firemans Fund Insurance Company

9. "Discounting by Measuring the Asset Liability Mismatch"
Author: J. Scott Bradley
Tillinghast/Towers Perrin

10. "Adjusting Incurred Losses for Simultaneous Shifts in Payment Patterns and Case Reserve Adequacy Levels"
Authors: Kirk G. Fleming
Milliman & Robertson, Inc.

Jeffrey H. Mayer
Milliman & Robertson, Inc.

11. "Loss Estimation: The Exposure Approach"
Author: Mark W. Littmann
Aetna Life & Casualty

12. "Analysis of Loss Development Patterns Using Infinitely Decomposable Percent of Ultimate Curves"
Authors: I. Robbin
CIGNA Corporation

David Homer
CIGNA Corporation

13. "Evaluating Bodily Injury Liabilities Using a Claims Closure Model"
Authors: Martin Adler
Government Employees Insurance Company

Charles D. Kline, Jr.
Government Employees Insurance Company

14. "Evaluating Contingent Premium Liabilities For Excess-of-Loss Swing Plans"
    Author: David R. Bickerstaff
    Milliman & Robertson, Inc.

15. "Liabilities for Extended Reporting Endorsement Guarantees under Claims-made Policies"
    Author: Charles L. McClenahan
    Coopers & Lybrand

16. "An Integrated Approach to Reserve for Assumed Reinsurance"
    Author: Frank D. Pierson
    Centre Reinsurance Limited

17. "Reserves, Surplus, & Uncertainty"
    Authors: Aaron Halpert
    Peat, Marwick, Main & Co.

    Douglas W. Oliver
    Peat, Marwick, Main & Co.

The panel presentations covered the following topics:

1. "Concurrent Causation"
   Moderator: Gary Grant
   Assistant Vice President & Actuary
   State Farm Fire and Casualty Company

   Panelists: Cynthia B. Baldwin
   Second Vice President
   General Reinsurance Corporation

   Michael Bragg
   Fire and Casualty Claims Counsel
   State Farm Fire and Casualty Company

2. "Expert Systems in Insurance Underwriting"
   Moderator: Allan Chuck
   Assistant Vice President
   Firemans Fund Insurance Company

Panelists: Joseph J. DeSalvo
Director, Insurance Industry Decision Support Group
Coopers & Lybrand

Rob Elmore
Director of Marketing
Syntelligence

3. "Management of Insurance Company Assets"
Moderator: Owen M. Gleeson
President
Financial Analysis and Control Systems Corporation

Panelists: Stewart M. Coutts
Bacon & Woodrow

Allan R. Keith
Vice President
Alliance Capital Management Corporation

4. "Workers Compensation—An Update"
Moderator: Anthony J. Grippa
Principal
Mercer, Meidinger, Hansen

Panelists: William W. Davis
Vice President-Claims
American Reinsurance Corporation

James R. Rabenstine
Vice President and Manager
Liberty Mutual Insurance Group

The workshops covered the following topics:

1. "Open Session with the Education and Examination Committee"
Moderator: Michael L. Toothman
Vice President—Membership

Panelists: Richard S. Biondi
Chairman, Examination Committee

Steven G. Lehmann
Chairman, Syllabus Committee

David L. Miller
Chairman, Education Policy Committee

2. "Credibility Textbook Chapter"
Author: Gary G. Venter
President
Workers Compensation Reinsurance Bureau

3. "Manual Ratemaking Chapter"
Author: Charles L. McClenahan
Partner
Coopers & Lybrand

4. "Valuation Principles Statement"
Committee on Valuation Principles and Techniques
Representatives: Robert A. Miller, Chairman
Milliman & Robertson, Inc.

David P. Flynn
Industrial Indemnity Company

Robert S. Miccolis
Tillinghast/Towers Perrin

The officers held a reception for the new Fellows and their guests from 5:30 P.M. to 6:30 P.M.

The President's Reception for all members and guests was held from 6:30 P.M. to 7:30 P.M.

Dinner was held at Saddlebrook from 7:30 P.M. to 9:00 P.M.

Tuesday, May 10, 1988

Tuesday morning was devoted to a continuation of the Monday afternoon sessions.

Tuesday afternoon was reserved for the various CAS committees to convene from 12:00 P.M. to 5:00 P.M.

Dinner and entertainment was held at Busch Gardens from 6:00 P.M. to 11:00 P.M.

Wednesday, May 11, 1988

Concurrent sessions were continued from 8:30 A.M. to 9:45 A.M.

The business session resumed at 10:00 A.M. with a presentation of the Harold Schloss award to Trenton Werner. The Michelbacher prize was awarded to Robert P. Butsic. A panel presentation, "Trends in Financial Services," followed. The panel was moderated by David E. A. Carson, President, People's Savings Bank of Bridgeport, Connecticut. The two panelists were John G. Kneen, Manager, Eastern Region, Cresap, McCormick & Paget, and Thomas Chittenden, Vice President and General Counsel, Citibank.

The meeting was adjourned at 12:00 P.M. after closing remarks.

May, 1988 Attendees

In attendance, as indicated by the registration records, were 253 Fellows; 157 Associates; and 48 guests, subscribers, and students. The list of their names follows.

FELLOWS

Adler, M.
Alff, G. N.
Alfuth, T. J.
Allaben, M. S.
Asch, N. E.
Atkinson, R. V.
Bailey, V. M.
Barrette, R.
Bartlett, W. N.
Bassman, B. C.
Bear, R. A.
Beer, A. J.
Bell, L. L.
Bensimon, A. S.
Berquist, J. R.
Bethel, N. A.
Beverage, R. M.
Bickerstaff, D. R.
Biegaj, W. P.
Bill, R. A.
Biondi, R. S.
Biscoglia, T. J.
Bland, W. H.
Boccitto, B. L.
Boison, L. A., Jr.
Bornhuetter, R. L.
Boyd, W. A.
Brooks, D. L.
Brown, B. Z.
Brown, W. W.
Bryan, C. A.
Buck, J. E.
Bursley, K. H.
Carlson, J. R.
Carpenter, W. M.
Cathcart, S. B.
Chansky, J. S.
Chiang, J. D.
Chuck, A.
Clark, D. B.
Cohen, H. L.
Corr, F. X.
Cripe, F. F.
Curley, J. O.
Curran, K. F.
Curry, A. C.
Daino, R. A.
Degerness, J. A.

Demers, D.
Dempster, H. V.
Desilets, C.
Deutsch, R. V.
Donaldson, J. P.
Dornfeld, J. L.
Dorval, B. T.
Drennan, J. P.
Dye, M. L.
Easlon, K.
Ehrlich, W. S.
Eland, D. D.
Engles, D.
Eyers, R. G.
Faber, J. A.
Fagan, J. L.
Fein, R. I.
Fiebrink, M. E.
Finger, R. J.
Fisher, R. S.
Fitzgibbon, W. J.
Flaherty, D. J.
Flynn, D. P.
Frohlich, K. R.
Fusco, M.
Gallagher, T. L.
Giambo, R. A.
Gillam, W. R.
Gleeson, O. M.
Goddard, D. C.
Goldfarb, I. H.
Golz, J. F.
Gorvett, K. P.
Grady, D. J.
Graham, T. L.
Grannan, P. J.
Grant, G.
Graves, J. S.
Grippa, A. J.
Hafling, D. N.
Hall, J. A.
Hallstrom, R. C.
Haner, W. J.
Hartman, D. G.
Hein, T. T.
Hennessy, M. E.
Henry, D. R.
Henzler, P. J.
Herder, J. M.
Herman, S. C.
Hibberd, W. J.
Higgins, B. J.
Hillhouse, J. A.
Honebein, C. W.
Hosford, M. T.
Hughey, M. S.
Hutter, H. E.
Ingco, A. M.
Irvan, R. P.
Jameson, S.
Johnson, A. P.
Johnson, L. D.
Johnston, T. S.
Kallop, R. H.
Kasner, K. R.
Kaufman, A. M.
Keen, E. R.
Kelley, R. J.
Kelly, A. E.
Kist, F. O.
Kooken, M. W.
Koski, M. I.
Koupf, G. I.
Kozik, T. J.
Krause, G. A.
LaCroix, M. A.
LaRose, J. G.
Larsen, M. R.
Lee, R. H.
Lehman, M. R.
Lehmann, S. G.
Leong, W.
Levin, J. W.
Linden, O. M.
Lino, R. A.
Littmann, M. W.
Lombardo, J. S.
Lyle, A. C.
MacGinnitie, W. J.
Mahler, H. C.
Mailloux, P.
Makgill, S. S.
Marks, S. D.
Mayer, J. H.
McAllister, K. C.
McCarter, M. G.
McClenahan, C. L.
McClure, J. W., Jr.
McDonald, G. P.
McGovern, W. G.
McLean, G. E.
McManus, M. F.
McSally, M. J.
Meyer, R. E.
Meyers, G. G.
Miccolis, R. S.
Miller, D. L.
Miller, M. J.
Miller, P. D.
Miller, R. A., III
Miller, R. R.

Miller, S. M.
Miller, W. J.
Montgomery, W. D.
Moody, R. A.
Morell, R. K.
Muetterties, J. H.
Murad, J. A.
Murrin, T. E.
Muza, J. J.
Myers, N. R.
Myers, T. G.
Neis, A. R.
Newlin, P. R.
Nickerson, G. V.
Normandin, A.
Oakden, D. J.
O'Brien, T. M.
O'Connell, P. G.
Oien, R. G.
Onufer, L. M.
Palmer, D. W.
Pearl, M. B.
Petersen, B. A.
Petit, C. I.
Phillips, G. N.
Phillips, H. J.
Pierson, F. D.
Pratt, J. J.
Prevosto, V. R.
Pruiksma, G. J.
Purple, J. M.
Quirin, A. J.
Racine, A. R.
Riddlesworth, W. A.
Ross, G. A.
Roth, R. J., Jr.
Rowland, W. J.
Ruegg, M. A.
Ryan, K. M.
Sandman, D. D.
Scheibl, J. A.
Schmidt, N. J.
Schultz, R. A.
Schwartzman, J. A.
Shepherd, L. A.
Sherman, R. E.
Shoop, E. C.
Shrum, R. G.
Silver, M. S.
Simon, L. J.
Skurnick, D.
Smith, F. A.
Smith, L. M.
Snader, R. H.
Sobel, M. J.
Spalla, J. S.
Spitzer, C. R.
Stanard, J. N.
Steinen, P. A.
Stephenson, E. A.
Stewart, C. W.
Suchoff, S. B.
Surrago, J.
Sutter, R. L.
Switzer, V. J.
Szkoda, S. T.
Taylor, F. C.
Tiller, M. W.
Toothman, M. L.
Tresco, F. J.
Tuttle, J. E.
Tverberg, G. E.
Vaillancourt, J.
Venter, G. G.
Visintine, G. R.
Visner, S. M.
Von Seggern, W. J.
Votta, J. C.
Wacek, M. G.
Walters, M. A.
Walters, M. A.
Webster, P. J.
Weller, A. O.
White, J.
Whitman, M.
Willsey, R. L.
Wilson, J. C.
Wiseman, M. L.
Wiser, R. F.
Woll, R. G.
Woods, P. B.
Yow, J. W.
Zatorski, R. T.
Zory, P. B.

ASSOCIATES

Allard, J. E.
Amoroso, R. C.
Anderson, B. C.
Andler, J. A.
Andrus, W. R.
Applequist, V. H.
Aquino, J. G.
Artes, J. L.
Battaglin, B. H.
Bellafiore, L. A.
Bender, R. K.
Bennighof, K. E.
Billings, H. L.
Book, S. W.
Boor, J. A.
Buchanan, J. W.
Callahan, J. J.
Campbell, K. A.
Carlson, C. S.
Casale, K. N.
Caulfield, M.
Cellars, R. M.
Chorpita, F. M.
Cloutier, D.
Cofield, J. E.
Colin, S. L.
Conley, K. J.
Connor, V. P.
Crowe, A. M.
Curry, M. K.
Darby, R. N.
Davis, B. W.
Davis, R. D.
DeConti, M. A.
DiGaetano, M.
Donnelly, V. T.
Ely, J.
Englander, J. A.
Eramo, R. P.
Evans, K. F.
Fanning, W. G.
Fitzgerald, B. E.
Fleming, K. G.
Francis, L. A.
Friedman, H. H.
Gebhard, J. J.
Gerard, F. R.
Gergasko, R. J.
Gerlach, S. B.
Gibson, R. N.
Gidos, P. M.
Gillam, J. A.
Glicksman, S. A.
Goldberg, S. B.
Gorvett, R. W.
Gossrow, R. W.
Gozzo, S. M.
Graham, J. H.
Granoff, G.
Graves, N. A.
Griffith, R. E.
Haidu, J. W.
Halpert, A.
Harbage, R. A.
Harrison, D. C.
Hay, R. S.
Head, T. F.
Hines, A. M.
Jaso, R. J.
Jasper, J. E.
Jensen, J. P.
Johnson, R. W.
Johnson, W. A.
Johnston, S. J.
Joyce, J. J.
Kadison, J. P.
Kleinberg, J. J.
Kline, C. D.
Koegel, D.
Kolojay, T. M.
Konopa, M. E.
Koufacos, C. G.
Krissinger, K. R.
LaLonde, D. A.
LaPointe, S. E.
Lebens, J. R.
Lebrun, R.
Leiner, W. W., Jr.
LePere, C. M.
Letourneau, R. D.
Macesic, D. J.
Mahoney, M. W.
Matthews, R. W.
McConnell, D. M.
McCreesh, J. B.
Millman, N. L.
Mittal, M. L.
Mohler, E. D.
Montigney, B. A.
Moody, A. W.
Morgan, S. T.
Morrow, J. B.
Moylan, T. G.
Mucci, R. V.
Musulin, R. T.
Nelson, C. E.
Nelson, J. K.
Nemlick, J. K.
Newman, H. E.
Ng, K. C.
Nyce, G. C.

Orlowicz, C. P.
Ostergren, G. V.
Overgaard, W. T.
Poirier, D.
Potter, J. A.
Privman, B.
Pulis, R. S.
Quintano, R. A.
Reed, D. J.
Rice, D. E.
Rice, J. W.
Rice, W. V.
Robbins, K. B.
Robinson, R. D.
Roth, R. J.
Ryan, J. P.
Sandler, R. M.
Sansevero, M.
Schadler, T. E.
Schmid, V. L.
Schug, R. D.
Siczewicz, P. J.
Skolnik, R. D.
Skov, S. A.
Snow, D. C.
Sornberger, G. C.
Stenmark, J. A.
Strommen, D. N.
Svendsgaard, C.
Swanstrom, R. J.
Swisher, J. W.
Taylor, C. P.
Thompson, E. G.
Varca, J. J.
Vezina, G.
Wachter, C. J.
Walker, D. G.
Wallace, T. A.
Weber, D. A.
Werland, D. L.
Whatley, M. W.
Wildman, P. W.
Wilson, W. F.
Yard, R. A.
Yau, M. W.
Yow, H. E.

GUESTS—SUBSCRIBERS—STUDENTS

Amore, J. J.
Baldwin, C. B.
Belton, E. F.
Berkowitz, J. B.
Bradley, S. J.
Bragg, M.
Butsic, R. P.
Carroll, P.
Carson, D. E. A.
Chang, C. E.
Chittenden, T.
Coca, M.
Corbett, G.
Coutts, S. M.
Cushinsky, M.
Davis, W. W.
De Marlie, G.
De Salvo, J. J.
Elmore, R.
Eversmann, T. M.
Fays, J.
Fung, C. C. K.
Guarini, L. T.
Gutman, E.
Hart, W.
Homer, D. L.
Jernberg, D. V.
Keith, A. R.
Kneen, J. G.
Laffer, A. B.
Lee, Y. S.
Natte, B.
Norton, J.
Oliver, D. W.
Pagnozzi, D.
Parker, E.
Rabenstine, J. R.
Robbin, I.
Rorick, J. R.
Scruggs, M. L.
Skelton, J. M.
Smith, S. B.
Spangler, J. L.
Valliere, A.
Van Leer, P.
Wade, J.
Winn, J.
Wootton, G. A.

Volume LXXV, Part 2 No. 144

PROCEEDINGS

November 8, 9, 10, 1988

FEDERAL INCOME TAXES PROVISIONS AFFECTING PROPERTY/CASUALTY INSURERS

MANUEL ALMAGRO

THOMAS L. GHEZZI

Abstract

This paper describes the most significant provisions of the current tax code (referred to as the Internal Revenue Code of 1986) affecting property/casualty insurers, and provides an analysis of the impact which changes in certain investment, underwriting, and other operating results have on insurers' taxes and after-tax earnings. The paper includes simplified tax calculations that are not intended to list exhaustively or precisely all tax provisions nor are they intended to enable the reader to precisely calculate taxes for any given insurer. The examples are intended to illustrate the dynamics and interaction of the various provisions of the tax code.

The first section of the paper provides a brief description of the provisions of the Internal Revenue Code of 1986 that most affect property/casualty insurers. The initial section is followed by a hypothetical example of an insurance company's operating results for a given year and the simplified calculation of federal income tax and net income based on the hypothetical results. The example forms a "base scenario" which is later modified to illustrate the effect on federal income taxes and after-tax net income

of varying one or more of the base assumptions while the others are held constant. In particular, the effect on net income of changing the investment mix between taxable and tax-exempt bonds is illustrated. The underlying tax calculations are redone with new operating assumptions to illustrate the effect on net income of changing underwriting income, investment rates of return on taxable and tax-exempt bonds, overall growth, and average discount factor changes. Results are compared for short tail, medium tail, and long tail lines of business.

This paper focuses primarily on the permanent provisions of the Internal Revenue Code of 1986 and not specifically on the changes introduced by the Tax Reform Act of 1986. Appendix A provides an analysis of the impact of some of the more important components of the Act. Appendix B provides a detailed description of the calculation of loss and loss adjustment expense payment patterns and discount factors mandated by the tax code.

1. INTRODUCTION

Property/casualty insurance companies, like all other corporate entities operating within the United States, are subject to the provisions of the Internal Revenue Code (IRC). Unlike other taxpayers, the property/casualty industry is afforded special treatment whereby large tax deductions are allowed for unearned premium reserves and for loss and loss adjustment expense reserves. Since insurers generally receive a large portion of their revenue (i.e., premiums) near the effective date of a given insurance policy, but pay most costs related to the policy (i.e., loss and loss adjustment expense payments) at a later point in time, the reserve deductions result in a deferral of the recognition of income by insurers. This deferral was greatly reduced by the Tax Reform Act (TRA) of 1986 through several new provisions targeted at the property/casualty industry.

It is important for insurers to recognize the impact of the various tax code provisions on their federal income tax liabilities and on net income. Insurers derive a major portion of their total net income from investment income on bonds, which are either subject to income tax or are tax-exempt. Since the yield on tax-exempt instruments is usually less than the pre-tax return on taxable investments, the effect of the mix of taxable and tax-exempt investments on an insurer's overall tax bill becomes an important consideration in making investment decisions. Various other provisions of the Internal Revenue Code of 1986 tend to affect different lines of insurance differently. It is important for insurers to consider federal income taxes in pricing, marketing, and other operational decisions by line of business.

It must be noted that the various hypothetical examples used in this paper are based on simplified tax situations designed to illustrate the dynamics and interaction of the most important tax code provisions affecting property/casualty insurers. Of particular importance is the use of statutory income as the starting point for all tax calculations. The differences between statutory income and income calculated according to generally accepted accounting principles (GAAP) are ignored. It should be noted that the IRC requires the use of GAAP income in the alternative minimum taxable income calculations by companies that file GAAP financial statements. Therefore, insurers must not ignore the differences between statutory income and GAAP income. The differences between statutory accounting and GAAP accounting are fully described in Strain [1].

Other simplifying assumptions used in this paper are noted as appropriate.

2. PROVISIONS OF THE INTERNAL REVENUE CODE OF 1986 AFFECTING PROPERTY/CASUALTY INSURANCE COMPANIES

The following provisions of the Internal Revenue Code of 1986 are the most significant ones affecting the property/casualty insurance industry. The descriptions which follow highlight the major points, while other provisions applying to mutual insurers, special situations, or to relatively small segments of the industry are omitted. Fuller descriptions of these items are provided by the Commerce Clearing House [2] and by Gleeson and Lenrow [3].

2.1 Regular Taxable Income

The determination of regular taxable income of property/casualty insurers begins with statutory (pretax) income as calculated on page 4 of the NAIC Annual Statement. Several adjustments are made to the statutory result to derive regular taxable income. They are:

2.1.1 Revenue Offset

Statutory income includes the change in unearned premium reserve during the tax year as a deduction. Insurers' acquisition expenses, however, are generally incurred and deducted near the time premiums are collected. Therefore, the statutory calculation does not accurately match recognition of premium income with recognition of related expenses.

To approximately adjust for this mismatch, the IRC allows only 80% of the change in unearned premium reserve as a deduction. The limitation of the deduction is accomplished through an adjustment to statutory income, referred to as 'revenue offset,' whereby 20% of the unearned premium reserve change is added to statutory income for tax purposes.

The IRC of 1986 includes special transitional provisions enacted with the TRA of 1986 to add part of the unearned premium reserve held as of December 31, 1986 to taxable income. The transition rules call for one-sixth of 20% of the December 31, 1986 unearned premium reserve to be included in taxable income for each of the next six years (1987–1992). For simplicity, we ignore these provisions in the examples which follow. Additional details of the transition rules are discussed in Gleeson and Lenrow [3], [5], as well as in Appendix A.

2.1.2 Loss and Loss Adjustment Expense Reserve Discounting

Statutory accounting generally reflects all assets and liabilities at their full, ultimate (i.e., undiscounted) value. In particular, except in some relatively minor cases, loss and loss adjustment expense reserves are deducted from earnings without explicit consideration of the time value of money.

Regular taxable income also recognizes insurers' loss and loss adjustment expense reserves as a deduction, but the deduction is computed on a discounted basis. The discounting provision is intended to reflect the time value of money over the payout period of the reserves [4]. Separate discounting calculations are required for each accident year and for each line of business included in Schedules

O and P of the NAIC Annual Statement.

The components of the reserve discounting calculations are:

i) *Loss and Loss Adjustment Expense Payment Pattern*—Most insurers can use payment patterns based on either insurance industry aggregate Schedules O and P data or patterns calculated from their own Schedules O and P. The IRC includes detailed rules governing the precise calculation of payment patterns and insurers' options to choose the industry pattern or their own.

ii) *Interest Rate*—Explicit instructions for the calculation of the interest rate are included in the IRC. The rate is tied to the annual "Federal mid-term rate." The Federal mid-term rate is calculated each month by the United States Department of the Treasury and is based on the average market yield of outstanding Treasury securities with remaining maturity of between three and nine years.

A more complete description of the loss reserve discounting provisions contained in the TRA of 1986 is included in Appendix B.

2.1.3 Tax-Exempt Investment Income

Statutory income includes all investment income, regardless of the nature of the investments generating the income. Regular taxable income, however, recognizes the fact that income from bonds issued by state or local governments for "traditional governmental purposes" [4] is exempt from federal income taxes. Therefore, tax-exempt investment income is omitted from regular taxable income.

2.1.4 Dividends Received Deduction

Corporations that receive dividends related to their equity investments in other domestic corporations which are subject to federal income tax are generally allowed to exclude a portion of the dividends from regular taxable income. Amendments to the IRC of 1986 contained in the Omnibus Budget Reconciliation Act of 1987 provide that, in most cases, 70% of the dividends received are excluded from regular taxable income [6]. The purpose of the dividends received deduction is to partially offset the "triple taxation" that takes place when a corporation derives income from dividends paid by another corporation [7]. As in the case of tax-exempt investment income, the dividends received deduction reduces statutory income in the calculation of regular taxable income.

The aggregate amount of a corporation's dividends received deduction in any tax year is limited to 70% of the company's taxable income before the deduction. This limitation, however, does not apply if taxable income (prior to the deduction) is less than the unlimited dividends received deduction.

2.1.5 Proration of Tax-Exempt Income

The deduction for incurred losses is reduced by 15% of the sum of tax-exempt investment income and the dividends received deduction. Due to the potential limitation of the dividends received deduction, proration of tax-exempt investment income is calculated prior to calculation of the dividends received deduction. The proration provision applies only to tax-exempt income related to investments acquired after August 7, 1986.

While the IRC refers to proration amounts as reductions to the incurred loss deduction, we treat the amounts as additions to regular taxable income. The effect is identical since reserve discounting applies before the proration adjustment. Also, for simplicity, we assume all tax-exempt investment income and dividends received are related to investments acquired after August 7, 1986. Alternative assumptions are explored in Appendix A.

To summarize, regular taxable income =

statutory income + revenue offset − change in loss and loss adjustment expense reserve discount

-tax-exempt investment income + proration of tax-exempt investment income

-dividends received deduction + proration of dividends received deduction.

2.2 Alternative Minimum Taxable Income (AMTI)

All corporations must calculate regular taxable income as described above and alternative minimum taxable income (AMTI). Alternative minimum taxable income is equal to regular taxable income plus all or part of various "tax preference items."

For property/casualty insurers, the most significant tax preference item, referred to as the "book income preference," is calculated as the difference between "book income" and regular taxable income. Book income is pre-tax income as reported by the company in its usual financial reports to regulators or shareholders, or in reports prepared for other non-tax purposes. For property/casualty insurers, book income is usually either pre-tax statutory income or pre-tax income reported according to generally accepted accounting principles (GAAP income). Insurers that issue GAAP financial statements must use GAAP income as book income in calculating the book income preference item. (See Strain [1] for a discussion of differences between statutory and GAAP accounting principles.) For simplicity, we use statutory income in the calculation of the book income preference item. Fifty percent of the book income preference item is added to regular taxable income in calculating AMTI.

The most significant elements of the book income preference for property/casualty insurers are tax-exempt investment income and the dividends received deduction. The effect of these items is mitigated somewhat by revenue offset, loss reserve discounting, and proration, all of which generally increase regular taxable income but are excluded from book income. Other preference items exist but are ignored in the examples which follow. These additional preference items include certain types of accelerated depreciation and tax-exempt interest on private activity bonds. The entire amount of these preference items is included in determining AMTI.

Since all preference items other than the book income preference item are ignored in the tax examples which follow, we define AMTI to be equal to regular taxable income plus 50% of the book income preference item. Beginning with 1990, the book income preference item will be replaced by an "adjusted current earnings" (ACE) preference item, and the amount of ACE to include in AMTI will increase to 75%. At the time of this writing, the precise definition of ACE has not been determined by the Internal Revenue Service. Because of this, all examples used in this paper are based on the pre-1990 provisions. Further discussion of the 1990 provisions is included in Appendix A.

2.3 Carryovers

Carryovers are created when various provisions of the tax code result in less than full recognition of otherwise applicable tax deductions in a given tax year. Carryovers generated in a given year may be used to offset taxable income of several years prior to and subsequent to the year creating the carryover. Three major types of carryovers generally affect property/casualty insurers:

i) *Net Operating Loss Carryovers (NOL's)*-When regular taxable income is negative, a carryover equal to the amount of the loss is established. This carryover is referred to as a net operating loss carryover (NOL) and can be used to offset taxable income up to three years prior to the year creating the NOL and up to 15 years after.

ii) *Capital Loss Carryovers*-Losses incurred on the sale of assets may generally only be deducted by offsetting capital gains. If capital losses realized in a given tax year are greater than capital gains, the difference is established as a net capital loss carryover. Capital loss carryovers can only be used to offset realized capital gains income. They can be carried back three years and forward five years.

iii) *Minimum Tax Credit*-In a year in which the alternative minimum tax applies, the difference between the otherwise applicable regular tax and the alternative minimum tax generates a credit available to offset future years' regular tax. Unlike NOL's and capital loss carryovers, the minimum tax credit can only be carried forward, but it is available for an unlimited number of years. However, minimum tax credits generated as a result of the book income preference item may not be used after 1989.

The rules governing the establishment and use of carryovers are complex, and the TRA of 1986 established transition rules. A detailed description of all of the provisions is beyond the scope of this paper. Also, for simplicity, the examples which follow assume that no carryovers are available.

2.4 Regular Income Tax Rates

The general corporate tax rate of 34% applies to regular taxable income above $335,000. Lower marginal rates apply to income brackets below $75,000 and a slightly higher rate applies to taxable income between 75,000 and $335,000. The same rate structure applies to property/casualty insurers as applies to corporations generally. Exhibit 1 shows the marginal tax rates by income bracket. Exhibit 2 shows the effective tax rate produced by the marginal rates applied to regular taxable income at the upper end of each bracket.

We assume the corporate tax rate applicable to regular taxable income to be 34%. This ignores the differences at lower income brackets and transitional rates established by the TRA of 1986. Additional information on the transition rules is contained in Gleeson and Lenrow [3], [5].

2.5 *Alternative Minimum Tax Rate*

The alternative minimum tax rate is 20%. The rate applies to all "brackets" of AMTI, although a minor AMTI exemption of $40,000 applies. This exclusion is ignored in the calculations which follow.

2.6 *Federal Income Taxes*

The federal income tax due for a particular tax year is the higher of regular taxable income times the applicable regular tax rate and the alternative minimum taxable income times the alternative minimum tax rate.

3. SAMPLE INSURANCE COMPANY—BASE SCENARIO

The following assumptions are used to illustrate the calculation of federal income taxes for a hypothetical property/casualty insurer:

(1) Statutory Underwriting Profit/(Loss)	($15.0 million)
(2) Taxable Investment Income ($150 million invested at 10%)	15.0 million
(3) Tax-Exempt Investment Income ($50 million invested at 8%)	4.0 million
(4) Dividends Received ($100 million invested; 5% dividends)	5.0 million
(5) Realized Capital Gains	5.0 million
(6) Statutory Income (1) + (2) + (3) + (4) + (5)	$14.0 million
(7) Unearned Premium Reserve	
a. Beginning of Tax Year	$75.0 million
b. End of Tax Year	82.5 million

(8) Loss and Loss Adjustment Expense Reserve	
a. Beginning of Tax Year	$150.0 million
b. End of Tax Year	165.0 million
(9) Average Reserve Discount Factor	
a. Beginning of Tax Year	.8500
b. End of Tax Year	.8500

The loss and loss adjustment expense reserves and the average discount factor shown above represent all lines of business and all accident years combined to simplify the calculations which follow. In practice, separate discount factors are applied to reserves by line of business and accident year in the calculation of regular taxable income. Also, we assume that assets generating the tax-exempt income were all acquired after August 7, 1986.

These results yield the federal income tax calculations (note that all calculations are rounded to the nearest $0.1 million) shown on the adjoining page.

4. SAMPLE INSURANCE COMPANY—ALTERNATIVE ASSUMPTIONS EFFECT ON NET INCOME

Because of the interactive effects of the various provisions of the tax code, variations in the base assumptions do not always produce an intuitively obvious change in federal income taxes due, and hence, in net income. Changing the investment income assumptions is especially interesting since the tax code treats taxable investment income differently from tax-exempt income. As noted above, tax-exempt income is subject to proration and generates a tax preference item to be included in the calculation of alternative minimum taxable income. Also, a company's investment portfolio is, to a certain extent, controllable by the company, thereby becoming a variable that can be altered to maximize after-tax income.

In this section, the dynamics of investment mix are explored through analysis of a series of graphs showing net income as the y-axis and investment portfolio mix between taxable and tax-exempt bonds as the x-axis.

REGULAR TAX	
(1) Statutory Income	$14.0 million
(2) Revenue Offset ($82.5 million − $75.0 million) × 20%	1.5 million
(3) Reserve Discounting Effect ($165 million − $150 million) − [($165 million × .8500) − ($150 million × .8500)]	2.3 million
(4) Tax-Exempt Investment Income	4.0 million
(4a) Proration of Tax-Exempt Investment Income (4) × 15%	0.6 million
(5) Dividends Received Deduction $5.0 million × 70%	3.5 million
(5a) Proration of Dividends Received Deduction (5) × 15%	0.5 million
(6) Regular Taxable Income (1) + (2) + (3) − (4) − (5) + (4a) + (5a)	$11.4 million
(7) Regular Federal Income Tax (6) × 34%	$3.9 million
ALTERNATIVE MINIMUM TAX	
(8) Book Income Tax Preference Items (1) − (6), but not less than zero	$2.6 million
(9) Alternative Minimum Taxable Income (6) + ((8) × 50%)	$12.7 million
(10) Alternative Minimum Federal Income Tax (9) × 20%	$2.5 million
FEDERAL INCOME TAX DUE	
(11) Greater of Regular Tax and Alternative Minimum Tax: Max [(7), (10)]	$3.9 million
NET INCOME	
(12) Net Income (1) − (11)	$10.1 million

4.1 Variation in Investment Mix

The base scenario includes $150 million of the sample company's bond portfolio invested in taxables yielding 10% and the remainder ($50 million) of the portfolio in tax-exempt bonds yielding 8%. This bond portfolio results in taxable investment income of $15 million, tax-exempt investment income of $4 million, tax liability of $3.9 million, and net income of $10.1 million. In this scenario, the regular tax applies.

Since the regular tax calculaton applies to the base scenario, all taxable investment income and the prorated 15% of tax-exempt income is subject to the corporate tax rate of 34%. Therefore, the effective after-tax yields are 6.6% for taxables [10% − (10% × 34%)] and 7.6% for tax-exempts [8% − (15% × 8% × 34%)]. The after-tax relationship between taxable and tax-exempt bond yields implies that an additonal $10 of after-tax income is realized for every $1,000 shifted from taxable to tax-exempt securities [$1,000 × (7.6% − 6.6%)].

The relationship of after-tax yields might lead one to conclude that net income is maximized by full investment in tax-exempt bonds, and that the net income for the sample company of $10.1 million could be increased to $11.6 million by converting the $150 million of taxable bonds into tax-exempt investments. [$11.6 million = $10.1 million + ($150 million/$1,000) × $10]. Such a conclusion, however, ignores the alternative minimum tax provisions of the tax code.

Exhibit 3 graphically displays net income using all base assumptions but varying the bond portfolio mix. The base scenario is labeled on the net income curve with a "+". The various other components of the graph and all inflection points are labeled as follows:

4.1.1 Regular Tax Applies

For the sample insurance company, regular tax is greater than the alternative minimum tax as long as taxable investments make up between $200 million and approximately $80 million of the bond portfolio. The upward-sloping section of the net income curve represents these results.

On this section of the curve, the intuitive result referred to above holds. That is, the greater the investment in tax-exempts, the greater the after-tax income, due to the relationship between effective after-tax yields for taxable and tax-exempt bonds. The slope of this component is such that net income increases by $10 for every $1,000 of investment that is shifted from taxable to tax-exempt bonds. As the investment mix shifts, tax-exempt investment income rises, causing the book tax preference item to grow, while regular taxable income declines. Consequently, the alternative minimum tax gets closer to the regular tax.

4.1.2 Regular Tax = Alternative Minimum Tax

Exhibit 3 shows that the regular tax is equal to the alternative minimum tax when slightly less than $80 million of the bond portfolio is invested in taxables.

At this point, taxable investment income is about $8.0 million ($80 million × 10%) and tax-exempt income is about $9.6 million ($120 million × 8%). Combined with the statutory underwriting loss of $15 million, dividends received of $5 million, realized capital gain of $5 million, revenue offset of $1.5 million, reserve discounting effect of $2.3 million, dividends received deduction of $3.5 million, and total proration of $2.0 million [$1.5 million ($9.6 million × 15%) for tax-exempt income proration and $0.5 million ($3.5 million × 15%) for the dividends received deduction proration], these results yield regular taxable income of $5.3 million. The indicated regular tax is $1.8 million ($5.3 million × 34%).

Book income tax preference items total $7.3 million ($12.6 statutory income less $5.3 million regular taxable income), resulting in alternative minimum taxable income of $9.0 million ($5.3 million regular taxable income + 50% of $7.3 million in book tax preferences). The alternative minimum tax is $1.8 million ($9 million × 20%), which is the same as the regular tax.

Regular tax equals alternative minimum tax when alternative minimum taxable income is 70% greater than regular taxable income (e.g., $9 million is 70% greater than $5.3 million). This relationship is due to the fact that the regular tax rate of 34% is 70% greater than the AMT rate of 20%. At this point, net income for the sample company is approximately $10.8 million, or about $0.7 million (7%) greater than the base scenario result of $10.1 million.

4.1.3 Alternative Minimum Tax Applies

The alternative minimum tax applies to the sample insurance company when less than $80 million is invested in taxable bonds. When the alternative minimum tax applies, all taxable investment income and the prorated 15% of tax-exempt income are subject to the alternative minimum tax rate of 20%. In addition, half of the remainder of tax-exempt income is included in AMTI as a tax preference item and is therefore also taxed at 20%. Therefore, under the AMT situation, the effective after-tax yields are 8.0% for taxable income [10% − (10% × 20%)] and 7.1% for tax-exempts [8% − (15% × 8% × 20%) − (50% × 85% × 8% × 20%)]. This after-tax yield relationship implies that a reduction of $9 of after-tax income is realized for every $1,000 shifted from taxable to tax-exempt securities [$1,000 × (7.1% − 8.0%)].

Since further investment in tax-exempts beyond this point continues to increase tax preferences while regular taxable income is decreased, the alternative minimum tax continues to apply. These results imply that given the assumed relationship between pre-tax yields on taxables and tax-exempts, net income is maximized when regular tax equals the alternative minimum tax.

4.1.4 Limit on Dividends Received Deduction

The dividends received deduction is limited to 70% of the taxable income prior to the deduction. For the sample insurance company, the unlimited deduction is $3.5 million (70% of dividends received of $5 million). Based on these provisions, the limitation first applies when taxable income before the deduction is less than $5 million.

The limitation first takes place under our assumptions when the bond portfolio includes about $42 million invested in taxable securities. At this point, taxable investment income is about $4.2 million ($42 million × 10%) and tax-exempt income is about $12.6 million ($158 million × 8%). Combining these investment results with the other operating results noted above yields taxable income before the dividends received deduction of $4.8 million. This amount of income implies a limited dividends received deduction of $3.4 million ($4.8 million × 70%) instead of the unlimited $3.5 million. As Exhibit 3 shows, the declining net income caused by shifting away from higher-yielding taxable investments is accelerated by the partial loss of the dividends received deduction.

4.1.5 Dividends Received Deduction Restored

The limitation of the deduction is removed when taxable income before the deduction is less than the full deduction (i.e., for the sample insurance company, when income before the deduction is less than $3.5 million). This relationship occurs under our assumptions when the bond portfolio includes $26 million in taxables. The taxable investment income of $2.6 million plus proration of $2.1 million of the tax-exempt investment income ($174 million × 8% × 15%) combined with the underwriting and other results noted above gives taxable income of $3.4 million. This amount is less than the unlimited dividends received deduction, and the limitation is removed.

The results displayed on Exhibit 3 and described in this section show that the optimum net income for the sample insurance company occurs when regular tax equals the alternative minimum tax. Each additional $1,000 investment in taxables beyond the optimum level reduces net income by $10, and each $1,000 reduction in taxable investment holdings reduces net income by $9.

Further insight into these conclusions is provided by Exhibits 4 and 5. Exhibit 4 shows the movement in regular taxable income, book income, and AMTI as the investment portfolio is altered. Since regular taxable income excludes all but the prorated portion of tax-exempt investment income, it continually decreases as taxables are traded for tax-exempts. Book income includes the increased tax-exempt income and therefore declines only as the result of the lower pre-tax yield on tax-exempt investments. Consequently, the relationships of book to regular income and AMTI to regular income increase as taxable investments are swapped for tax-exempts.

Regular taxes and the alternative minimum tax are equal—and net income is maximized—when AMTI is 70% greater than regular taxable income. This point is noted on Exhibit 4.

Exhibit 5 shows the indicated regular tax (regular taxable income times 34%) and the alternative minimum tax (AMTI times 20%). The two tax lines intersect where the investment portfolio includes about $80 million in taxable bonds.

4.2 Variation in Underwriting Profit

Exhibits 6 and 7 display the net income curve with the underwriting income assumption changed from the base assumption of −$15 million to −$20 million and −$10 million, respectively. These examples assume that the change in underwriting loss levels occurs without affecting unearned premium or loss reserves.

The alternate assumptions cause the net income curves to shift vertically and horizontally, but the shapes of the curves are the same as for the base assumptions curve. The slopes of the various segments are unchanged since they are dependent upon taxable and tax-exempt yields which are not altered from the base level.

4.2.1 Larger Underwriting Loss

The effect of changing the base underwriting loss assumption of $15 million to an underwriting loss of $20 million is to shift the net income curve downward and to the left. At the base investment mix of $150 million in taxables and $50 million in tax-exempts, an underwriting loss of $20 million results in net income of about $6.8 million. This result compares to $10.1 million for the base scenario including an underwriting loss of $15 million. Since the regular tax calculation applies, the difference in net income is totally attributable to the additional loss less tax savings based on the regular rate of 34% [i.e., $6.8 million = $10.1 million − ($5 million × (100% − 34%))].

Since the larger underwriting loss serves to reduce regular taxable income by $5 million, fewer tax preferences are needed to obtain the 70% relationship between book income and regular taxable income required for regular tax to equal AMT. This implies that the optimum portfolio mix occurs at a greater proportion of taxable investments than is the case for the base scenario.

In our example, regular tax equals the alternative tax when the bond portfolio includes $116 million in taxables. At this point, regular taxable income is $3.4 million and alternative minimum taxable income is $5.9 million. These results yield taxes of $1.2 million and after-tax net income of just over $7.1 million. This result is displayed graphically on Exhibit 6. The details of the calculation are left for the reader.

4.2.2 Smaller Underwriting Loss

The effect of changing the base underwriting loss assumption of \$15 million to an underwriting loss of \$10 million is to shift the net income curve upward and to the right. At the base investment mix of \$150 million in taxables and \$50 million in tax-exempts, an underwriting loss of \$10 million results in net income of just under \$13.4 million. This net income result compares to \$10.1 million for the base scenario with an underwriting loss of \$15 million. Similar to the case just described, this net income result is totally attributable to the smaller loss less additional tax based on the regular rate of 34% [i.e., \$13.4 million = \$10.1 million + (\$5 million × (100% − 34%))].

The smaller underwriting loss causes the point where regular tax equals AMT to shift to the right relative to the base scenario since regular taxable income is greater by \$5 million. More tax preference items are needed to obtain the 70% relationship between AMTI and regular taxable income derived previously. The need for greater tax preference items implies that the optimum investment mix includes fewer taxables than was the case for the base scenario.

Under the −\$10 million underwriting result assumption, net income is maximized when the investment portfolio includes \$44 million in taxable bonds. The resultant regular taxable income is \$7.1 million, AMTI is \$12 million, and income tax is \$2.4 million. Net income is \$14.5 million. This result is shown graphically on Exhibit 7.

These findings show that increased underwriting losses make it necessary to invest more heavily in taxable bonds in order to maximize net income. Smaller losses call for more investment in tax-exempts. As will be shown later, these results are dependent upon the relationship between taxable and tax-exempt yields.

Variations in capital gains or losses have the same impact as variations in underwriting gains or losses on net income and taxes since capital gains are treated as regular income and are not subject to special deductions or proration. Except for the separate loss carryover provisions noted previously, there is no distinction between capital gains income and underwriting income in the calculation of federal income taxes.

4.3 Variation in Yields

Exhibits 8 through 12 show the net income curve using the base scenario for all variables except for taxable and tax-exempt yields. Each exhibit represents a different set of yields to display the effect on net income.

4.3.1 Taxable Yield: 12%; Tax-Exempt Yield: 10%

Under this variation displayed on Exhibit 8, both yields are increased by the same number of percentage points, thereby maintaining the same absolute pre-tax relationship as exists under the base scenario. The absolute difference in yields between taxable bonds and tax-exempt bonds is used here instead of the more commonly used percentage relationship in order to maintain the same pre-tax dollar effect of changing the investment mix between taxable and tax-exempt bonds. The after-tax effect of the change is to shift the net income curve upward and slightly to the right, and to change the shape.

The base bond portfolio includes $150 million in taxable investments and $50 million in tax-exempts. Applying the yields of 12% and 10%, respectively, results in taxable investment income of $18 million and tax-exempt investment income of $5 million. The resultant tax liability is $5.0 million and net income is about $13.0 million.

The increase from the base scenario net income of $10.1 million is attributable to the additional investment income of $4 million generated by the higher yields, offset somewhat by greater proration ($0.2 million), and subject to the regular tax rate of 34% [i.e., $13.0 million = $10.1 million + (($3 million additional taxable income − $0.2 million additional proration) × 66%) + $1 million additional tax-exempt income].

The point at which net income is maximized is shifted slightly under this alternative. The shift towards slightly greater investment in tax-exempts is primarily due to the increase in regular taxable income generated by the higher yields, thereby requiring proportionately greater tax preference income to give the 70% relationship between book income and regular taxable income needed to maximize net income. This is offset somewhat by the faster accumulation of tax preferences due to the higher yield on tax-exempt investments.

The shape of the net income curve is affected by the change in yield rates from the base scenario since, as shown previously, the slopes of the regular tax component and the alternative minimum tax component are functions of the taxable and tax-exempt yields. Taxable and tax-exempt yields of 12% and 10% result in after-tax yields of 7.9% for taxables and 9.5% for tax exempt bonds under the regular tax calculations. This implies that $16 of additional net income results from each $1,000 shift from taxable investments to tax-exempt investments. This is "steeper" than the base scenario result of $10 per $1,000 since more of the additional two points of tax-exempt interest is realized on an after-tax basis than is the case for the additional taxable yield.

The after-tax yields under the alternative minimum tax calculation are 9.6% for taxables and 8.9% for tax-exempt securities. This implies that net income decreases by $7 for each $1,000 of taxable bonds shifted to tax-exempt bonds.

4.3.2 Taxable Yield: 8%; Tax-Exempt Yield: 6%

Exhibit 9 shows the net income curve using the base assumptions but changing the yields to 8% for taxables and 6% for tax-exempts. This variation also retains the same absolute pre-tax differential as the base scenario, but at a lower absolute level. As observed with the last variation, the altered yield rates shift the curve and change its shape.

The observations made regarding the 12%/10% scenario generally apply to this variation as well, but the directions are reversed. For example, the base investment portfolio results in net income of about $7.1 million, or about $3.0 million less than the base assumptions; the optimum portfolio includes a slightly larger proportion of taxable investments due to the greater relative effect of tax preferences; the slope of the regular tax segment is not as steep as was the base scenario slope; and the slope of the AMT segment of the curve is steeper.

4.3.3 Taxable Yield: 10%; Tax-Exempt Yield: 9%

Exhibit 10 displays this scenario with its reduced difference in yields.

The most significant observations pertain to the slopes of the curve segments. The smaller difference between the pre-tax yields results in a steeper regular tax slope due to the greater relative after-tax return from tax-exempts. The nearly horizontal AMT segment indicates that the after-tax yield for taxables is nearly equal to the after-tax yield on tax-exempts.

The optimum portfolio is shifted slightly to the left due to the faster accumulation of tax preferences by the higher tax-exempt yield.

4.3.4 Taxable Yield: 10%; Tax-Exempt Yield: 7%

Exhibit 11 shows an alternative that increases the difference between taxable and tax-exempt yields.

The regular tax component is nearly horizontal, reflecting the fact that after-tax yields are nearly the same for taxable and tax-exempt bonds. The AMT component slope is steeper than the base scenario due to the significantly lower after-tax return on tax-exempts under this variation.

The maximum net income occurs with a slightly greater proportion of the bond investments in tax-exempt securities than in the base scenario. This is due to the slower accumulation of tax preferences generated by the lower tax-exempt yield. Very little difference exists among the net income results on the regular tax segment of the curve.

4.3.5 Taxable Yield: 10%; Tax-Exempt Yield: 6%

This alternative, displayed on Exhibit 12, widens the taxable/tax-exempt differential further. Although such a relationship between yields is not likely to exist for a significant period of time, the variation provides useful insights.

The effect is to make tax-exempt investments very undesirable relative to taxables. This case provides an exception to the general rule that net income is maximized when regular tax equals the alternative minimum tax. Due to the significantly greater after-tax return on taxable bonds compared to tax-exempts, the optimum strategy is to fully invest in taxables.

These examples illustrate the sensitivity of net income to the taxable and tax-exempt investment yields. In all cases, the net income curve is shifted and the shape is changed when yields are changed.

4.4 Variation in Growth Rate

The base scenario assumes that unearned premium reserves and loss reserves at the end of the tax year are 10% greater than the beginning reserves. This growth creates the change in unearned premium reserves used in the revenue offset calculation and the change in loss reserves used in the discounting calculations.

The base scenario generated a revenue offset addition to income of $1.5 million [($82.5 million − $75 million) × 20%]. The loss reserve discounting effect on income is $2.3 million [($165 million − $150 million) − (($165 million × .85) − ($150 million × .85))]. Variation in the assumed rate of reserve growth changes the components of the revenue offset and discounting calculations and therefore affects regular taxable income. Larger growth creates larger additions to taxable income and smaller (or negative) growth results in smaller additions.

Exhibit 13 displays the net income curve under five different growth scenarios. As the exhibit shows, the effect of larger growth is to shift the curve downward and to the right. Smaller growth shifts the curve upward and to the left. Since the effect of varying the growth assumption flows directly to regular taxable income, the observations made previously pertaining to the effect of varying underwriting income apply to the growth scenarios as well.

The implication of these results is that any action that has the effect of increasing premium and loss reserves during a given tax year also increases the effect of revenue offset and loss reserve discounting. While the overall deduction under an increasing growth scenario is also increased, the amount of the deduction is tempered.

Some examples of such actions which tend to increase reserves are:

a. growth in new business;
b. changing mix of business to longer tail lines of business;
c. assumed portfolio transfers;

d. loss reserve strengthening;
e. change from claims-made to occurrence coverage;
f. increasing policy term; and
g. higher limits of liability.

An insurer must realize that any of the above actions will affect net income by a greater degree than simply the absolute change in reserves. The additional net income effect is the result of the greater impact of revenue offset and reserve discounting on federal income taxes.

It should be noted that the results presented in Exhibit 13 assume the same underwriting loss of $15 million. Consequently, the varying growth assumptions likely imply varying loss ratios and/or combined ratios. Also, the simplifying assumptions are made that the composite discount factor and the overall investment income are the same as for the base scenario. In reality, growth would likely result in a different composite discount factor and in different investment income results.

4.5 Variation in Absolute Reserve Level

In addition to being sensitive to growth rates, the dollar difference between beginning and ending reserves is a function of the absolute size of these liabilities. For a given growth rate, a larger dollar reserve change results when the absolute size of beginning and ending reserves is larger. A smaller change results from lower reserve levels.

The net income curves displayed on Exhibit 14 assume varying relationships of loss reserves to premiums while all other base assumptions are held constant. The relationship to premiums is used to arbitrarily simulate short, medium, and long tail lines of business.

As the exhibit shows, short tail lines (e.g., those with reserves at the end of a particular year equal to half of the year's written premiums) derive greater net income from the same operating results due to the diminished effect of the loss reserve discounting provisions of the tax code. The effect, like that of varying underwriting results and growth, is to shift the curve vertically and horizontally. The basic shape, however, is unchanged from the base scenario.

4.6 Variation in Average Discount Factor

Variation in the average discount factor assumption has a direct effect on taxes and net income. The effect is due to the application of the discount factor to loss and loss adjustment expense reserves in the calculation of taxable income.

Exhibit 15 shows the net income curve under the base scenario using three average discount factor assumptions. The graph shows that the curve is shifted upwards and to the left as the discount factor increases. The opposite shift occurs when the average factor decreases. The effect of varying the average discount factor is to shift the curve vertically and horizontally while maintaining the same shape. The average discount factor can change due to shifts of loss reserves towards more or less mature accident years, shifts of business toward longer tail or shorter tail lines, changing payment pattern, changing discount rate, etc.

4.7 Variation in Average Discount Factor and Reserve Level Combined

Since it is most likely that lower average discount factors occur along with higher absolute reserve levels, Exhibit 16 shows net income curves which combine the results of Exhibits 14 and 15. In particular, the average discount factor of 0.9 is associated with the short tail assumption that reserves are 50% of premiums for the tax year; the base discount factor is combined with the base reserve-to-premium relationship of 1-to-1 to simulate the medium tail lines; and the 0.8 average discount factor is applied to the long tail scenario with reserves equal to twice the tax year's premium.

The results on Exhibit 16 are similar to those displayed on Exhibits 14 and 15, but the magnitude of the vertical and horizontal shifts for the short and long tail lines is greater due to the compounding effect of the discount factor and reserve level assumptions. The Exhibit 16 results indicate that significantly lower after-tax income is derived from long tail lines generating the same pre-tax results as short tail business. Each line assumes the same pre-tax statutory income of $14 million, but the after-tax results range from approximately $11.1 million for the short tail line to about $10.0 million for the long tail line. This relationship reflects the greater investment income potential present with the long tail line and shows the results of the TRA of 1986's attempt to match property/casualty insurers' liabilities and assets in deriving taxable income.

5. SUMMARY

The various provisions of the IRC of 1986 that affect property/casualty insurance companies provide opportunities for insurers to manage their federal income taxes and to maximize net income. While the ultimate degree of control an insurer can exercise on taxes is somewhat limited by external factors, significant differences in net income can result from different investment, underwriting, growth, and line of business strategies.

In this paper, a base scenario is developed for a hypothetical insurance company and simplified federal income tax and net income results are calculated. Various components of the base scenario are altered while all other base assumptions are held constant in order to isolate the impact of various factors on the sample company's after-tax income.

The base scenario includes the following:

Underwriting Income	($15.0 million)
Amount Invested in:	
Taxable Bonds	$150.0 million
Tax-Exempt Bonds	$ 50.0 million
Statutory Income	$ 14.0 million
Regular Taxable Income	$ 11.4 million
Federal Income Tax	$ 3.9 million
Net Income	$ 10.1 million

This scenario includes a taxable bond yield of 10% and a tax-exempt yield of 8%. In addition, the average composite discount factor is assumed to be 0.85. Variations in the base assumptions yield the results associated with maximized net income as shown on the adjoining page.

The results shown here indicate that changing the investment mix between taxable and tax-exempt bonds can mitigate the negative effects of worsening underwriting and investment yield results. For example, the net income associated with a $20 million underwriting loss ($7.1 million) is less than $5 million below the net income associated with a $15 million underwriting loss ($10.8 million). This is accomplished by shifting investment dollars from the lower-yielding tax-exempt bonds to the higher-yielding taxable bonds as underwriting results deteriorate. The opposite is true for improving underwriting results.

Optimum Bond Portfolio Mix

Assumption	Taxable	Tax-Exempt	Net Income
Base	$ 80 million	$120 million	$10.8 million
Underwriting Loss:			
($20 million)	$116 million	$ 84 million	$7.1 million
($10 million)	44 million	156 million	14.5 million
Investment Yields:			
12%; 10%	$ 78 million	$122 million	$14.3 million
8%; 6%	84 million	116 million	7.5 million
10%; 9%	84 million	116 million	11.9 million
10%; 7%	76 million	124 million	9.7 million
10%; 6%	200 million	0 million	9.7 million
Growth Rates:			
−5%	$152 million	$ 48 million	$12.0 million
NC	128 million	72 million	11.7 million
+5%	104 million	96 million	11.2 million
+15%	56 million	144 million	10.4 million
Reserves:			
50% of WrPr.	$ 94 million	$106 million	$11.1 million
200% of WrPr.	52 million	148 million	10.3 million
Avg. Discount Factor:			
.90	$ 90 million	$110 million	$11.0 million
.80	72 million	128 million	10.6 million
Reserves/Discount Factor Combined			
50%/.90	$100 million	$100 million	$11.2 million
200%/.80	34 million	166 million	10.0 million

The revenue offset and loss and loss adjustment expense discounting provisions of the IRC of 1986 produce different results for different growth rates and absolute reserve levels. Again, the bond portfolio can be used to mitigate some of the impact on net income of these provisions.

Lastly, the tax code provisions tend to affect net income from various lines of business differently. Those lines with large absolute reserve levels and with long payout patterns derive less net income from the same statutory income amount than low reserve/short payout lines derive. This result has significant pricing and profitability implications.

6. CONCLUSION

The Tax Reform Act of 1986 dramatically changed the impact of federal income taxes on property/casualty insurers. The changes generally serve to increase the amount of taxes paid by insurers and make it impossible for companies with profitable pre-tax earnings to avoid paying taxes through the use of tax-exempt securities.

In response to the new tax code, property/casualty insurers should carefully assess the tax implications of various marketing, investment, reinsurance, and pricing strategies. Careful tax planning, while no longer able to eliminate federal income tax payments in most instances, can materially increase after-tax earnings by carefully optimizing insurers' line of business and investment portfolios.

REFERENCES

[1] R. W. Strain, *Property-Liability Insurance Accounting*, Third Edition, Insurance Accounting and Systems Association, 1986.

[2] *Tax Reform Act of 1986*, Commerce Clearing House, Inc., October, 1986.

[3] Owen M. Gleeson and Gerald I. Lenrow, "An Analysis of the Impact of the Tax Reform Act on the Property/Casualty Industry," Casualty Actuarial Society 1987 Discussion Paper Program, p. 119.

[4] *Internal Revenue Code*, Volumes 1 and 2, Commerce Clearing House, January, 1988.

[5] Owen M. Gleeson and Gerald I. Lenrow, "Tax Reform: The Impact on Income Statements," *Best's Review*, July, 1987, p. 12.

[6] Edward A. Tepper, "Tax Reform Poses Obstacles for CEOs," *National Underwriter*, February 8, 1988, p. 50.

[7] R. W. Beckman, "Federal Income Taxes," *PCAS* LVIII, 1971, p. 1.

[8] Internal Revenue Service Advance Notice 88-100 on Discounted Unpaid Losses, issued August 17, 1988.

EXHIBIT 1

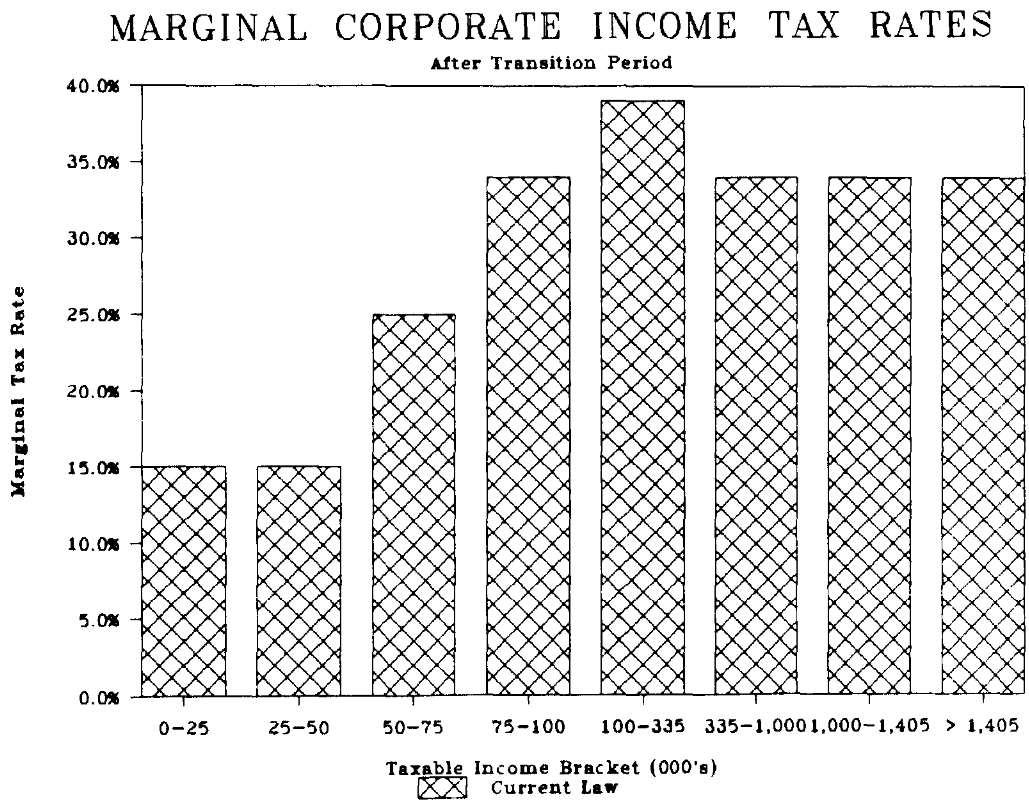

MARGINAL CORPORATE INCOME TAX RATES
After Transition Period
Marginal Tax Rate
40.0%
35.0%
30.0%
25.0%
20.0%
15.0%
10.0%
5.0%
0.0%
0-25
25-50
50-75
75-100
100-335
335-1,000
1,000-1,405
> 1,405
Taxable Income Bracket (000's)
Current Law

EXHIBIT 2

EFFECTIVE CORPORATE INCOME TAX RATES

After Transition Period

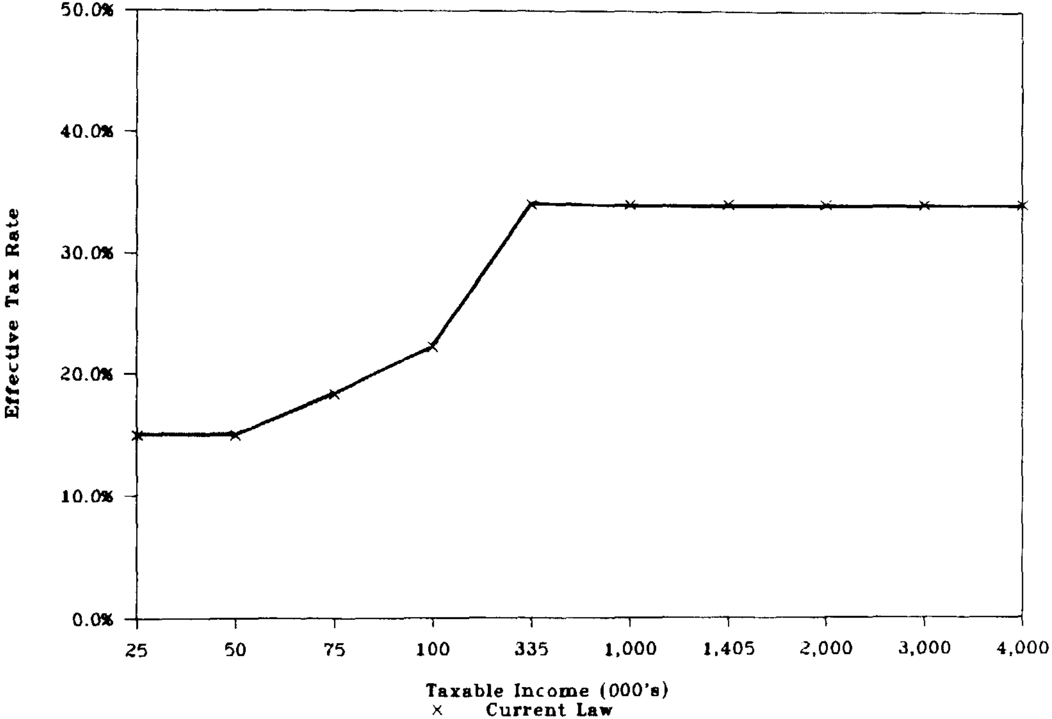


EXHIBIT 3

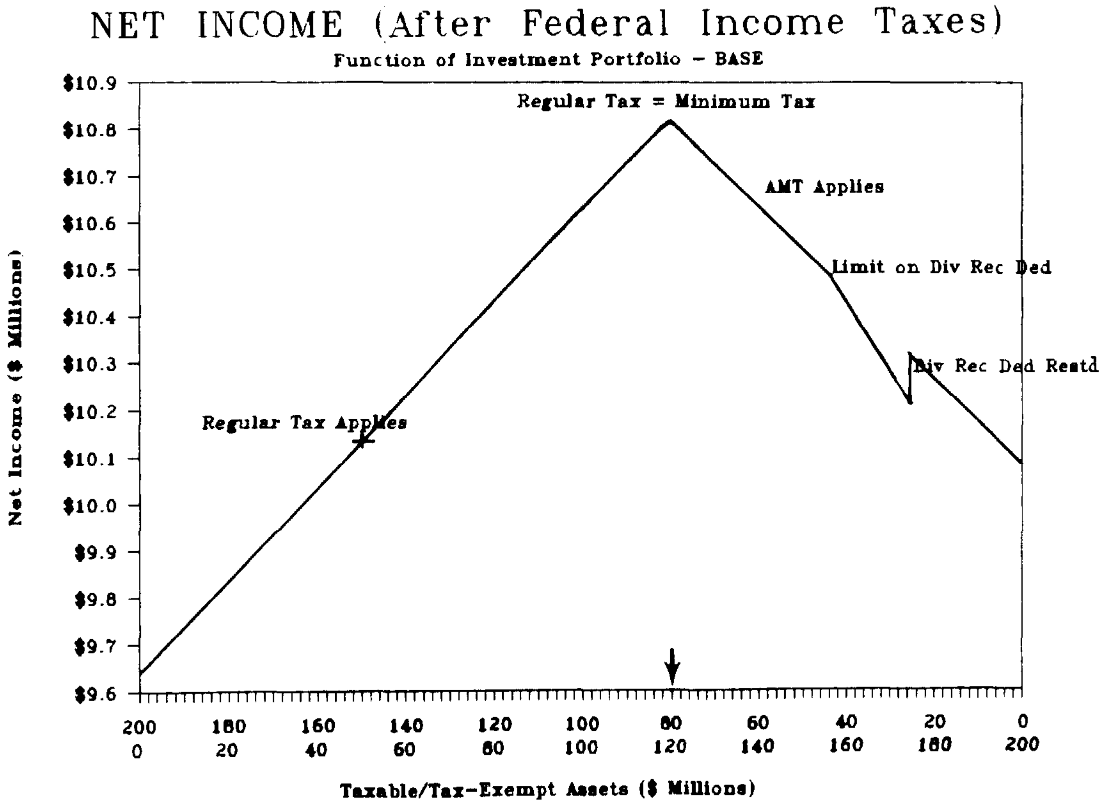
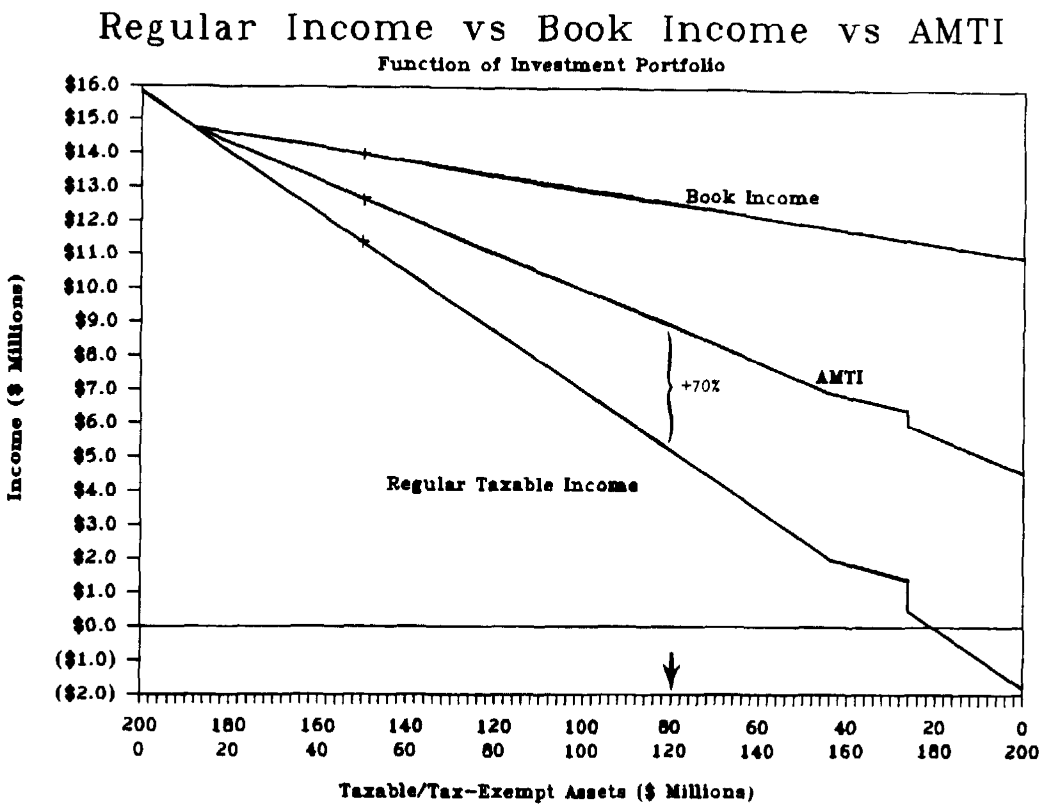

NET INCOME (After Federal Income Taxes)
Function of Investment Portfolio - BASE
Regular Tax = Minimum Tax
AMT Applies
Limit on Div Rec Ded
Div Rec Ded Restd
Regular Tax Applies
$10.9
$10.8
$10.7
$10.6
$10.5
$10.4
$10.3
$10.2
$10.1
$10.0
$9.9
$9.8
$9.7
$9.6
Net Income ($ Millions)
200 180 160 140 120 100 80 60 40 20 0
0 20 40 60 80 100 120 140 160 180 200
Taxable/Tax-Exempt Assets ($ Millions)

EXHIBIT 4



Regular Income vs Book Income vs AMTI
Function of Investment Portfolio
$16.0
$15.0
$14.0
$13.0
$12.0
$11.0
$10.0
$9.0
$8.0
$7.0
$6.0
$5.0
$4.0
$3.0
$2.0
$1.0
$0.0
($1.0)
($2.0)
Income ($ Millions)
Book Income
AMTI
+70%
Regular Taxable Income
200 180 160 140 120 100 80 60 40 20 0
0 20 40 60 80 100 120 140 160 180 200
Taxable/Tax-Exempt Assets ($ Millions)

EXHIBIT 5

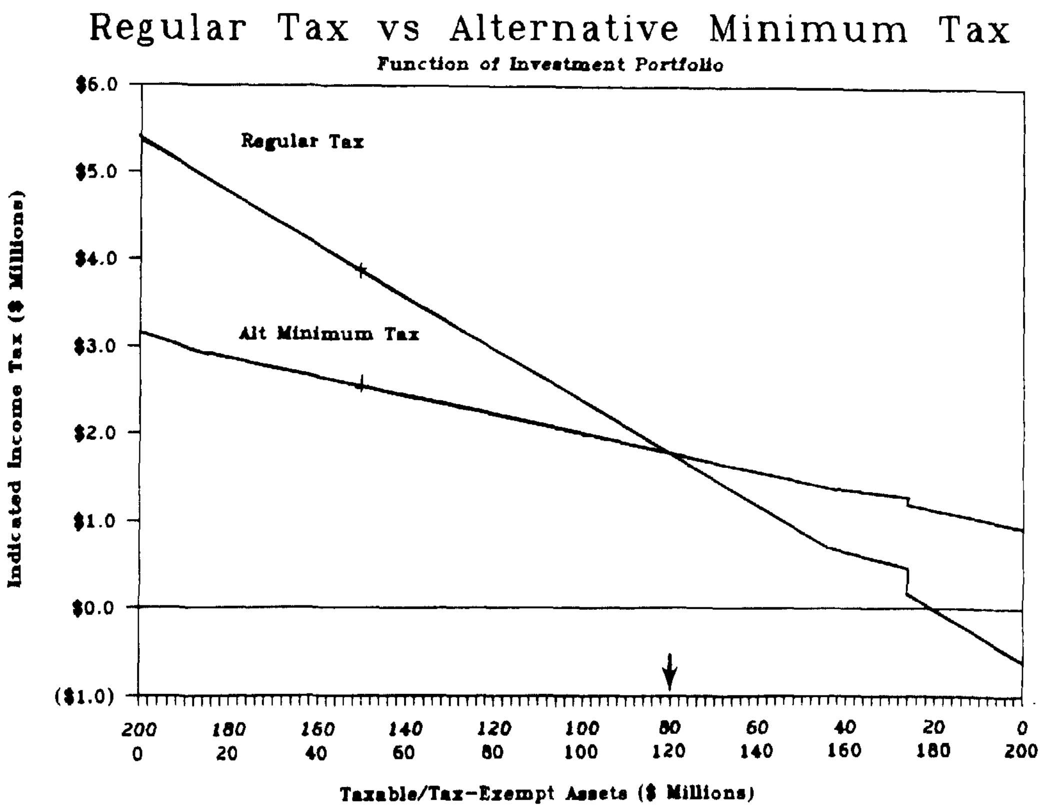

Regular Tax vs Alternative Minimum Tax
Function of Investment Portfolio
Regular Tax
Alt Minimum Tax
$6.0
$5.0
$4.0
$3.0
$2.0
$1.0
$0.0
($1.0)
Indicated Income Tax ($ Millions)
200 180 160 140 120 100 80 60 40 20 0
0 20 40 60 80 100 120 140 160 180 200
Taxable/Tax-Exempt Assets ($ Millions)

EXHIBIT 6

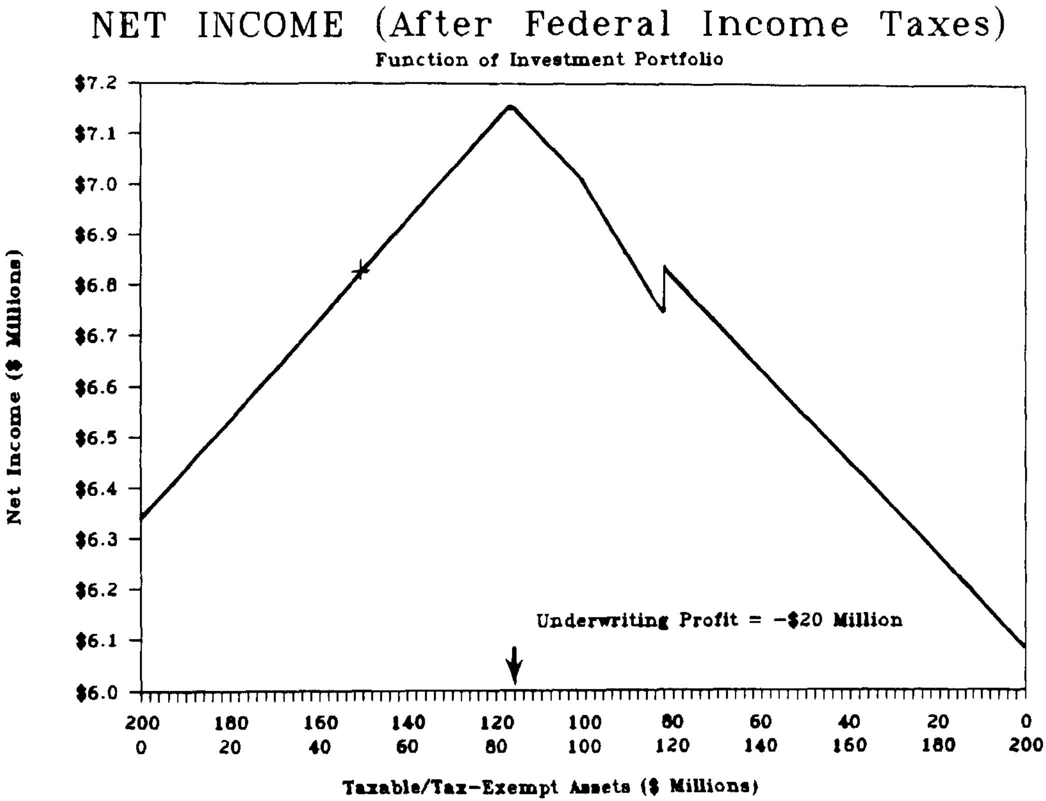

NET INCOME (After Federal Income Taxes)
Function of Investment Portfolio
$7.2
$7.1
$7.0
$6.9
$6.8
$6.7
$6.6
$6.5
$6.4
$6.3
$6.2
$6.1
$6.0
Net Income ($ Millions)
Underwriting Profit = -$20 Million
200 180 160 140 120 100 80 60 40 20 0
0 20 40 60 80 100 120 140 160 180 200
Taxable/Tax-Exempt Assets ($ Millions)

EXHIBIT 7

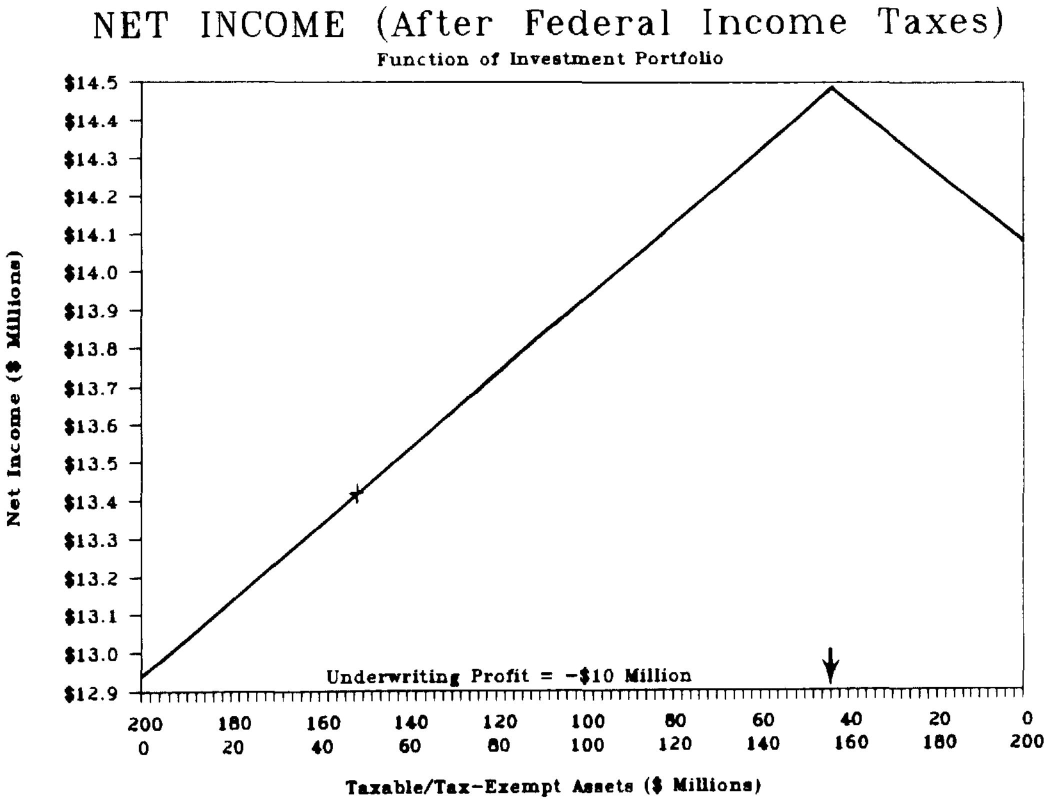

NET INCOME (After Federal Income Taxes)
Function of Investment Portfolio
Net Income ($ Millions)
$14.5
$14.4
$14.3
$14.2
$14.1
$14.0
$13.9
$13.8
$13.7
$13.6
$13.5
$13.4
$13.3
$13.2
$13.1
$13.0
$12.9
Underwriting Profit = −$10 Million
200 180 160 140 120 100 80 60 40 20 0
0 20 40 60 80 100 120 140 160 180 200
Taxable/Tax-Exempt Assets ($ Millions)

EXHIBIT 8

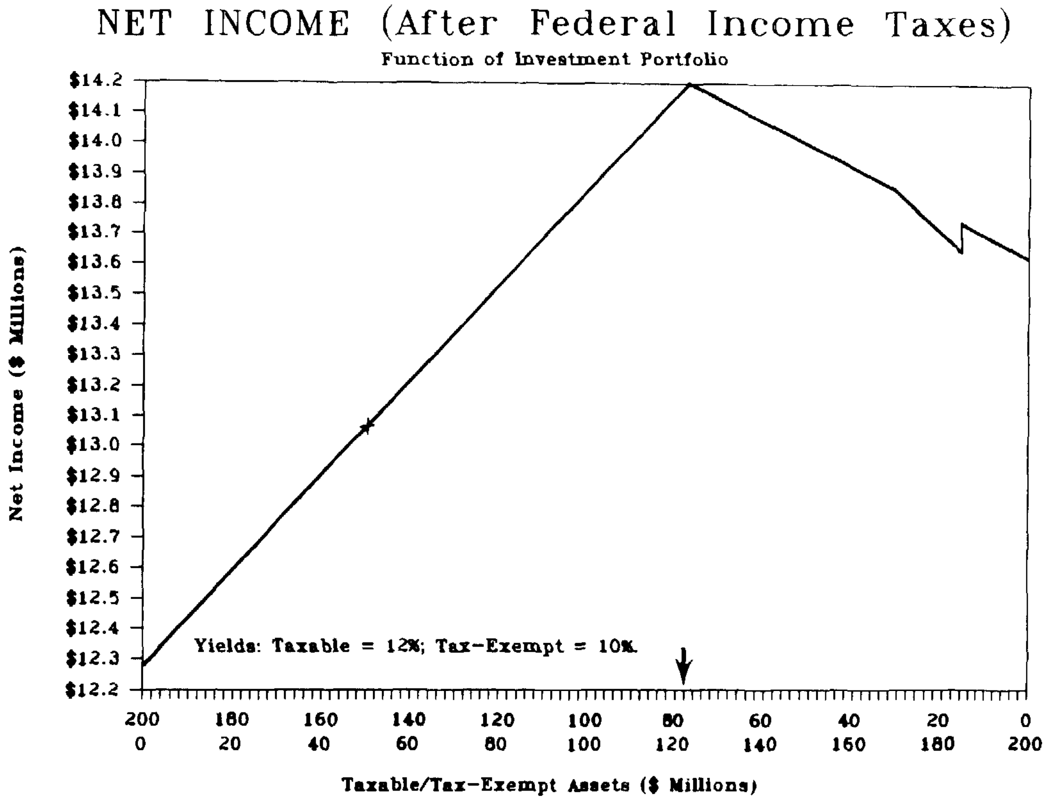

NET INCOME (After Federal Income Taxes)
Function of Investment Portfolio
$14.2
$14.1
$14.0
$13.9
$13.8
$13.7
$13.6
$13.5
$13.4
$13.3
$13.2
$13.1
$13.0
$12.9
$12.8
$12.7
$12.6
$12.5
$12.4
$12.3
$12.2
Net Income ($ Millions)
Yields: Taxable = 12%; Tax-Exempt = 10%.
200 180 160 140 120 100 80 60 40 20 0
0 20 40 60 80 100 120 140 160 180 200
Taxable/Tax-Exempt Assets ($ Millions)

EXHIBIT 9

NET INCOME (After Federal Income Taxes)

Function of Investment Portfolio

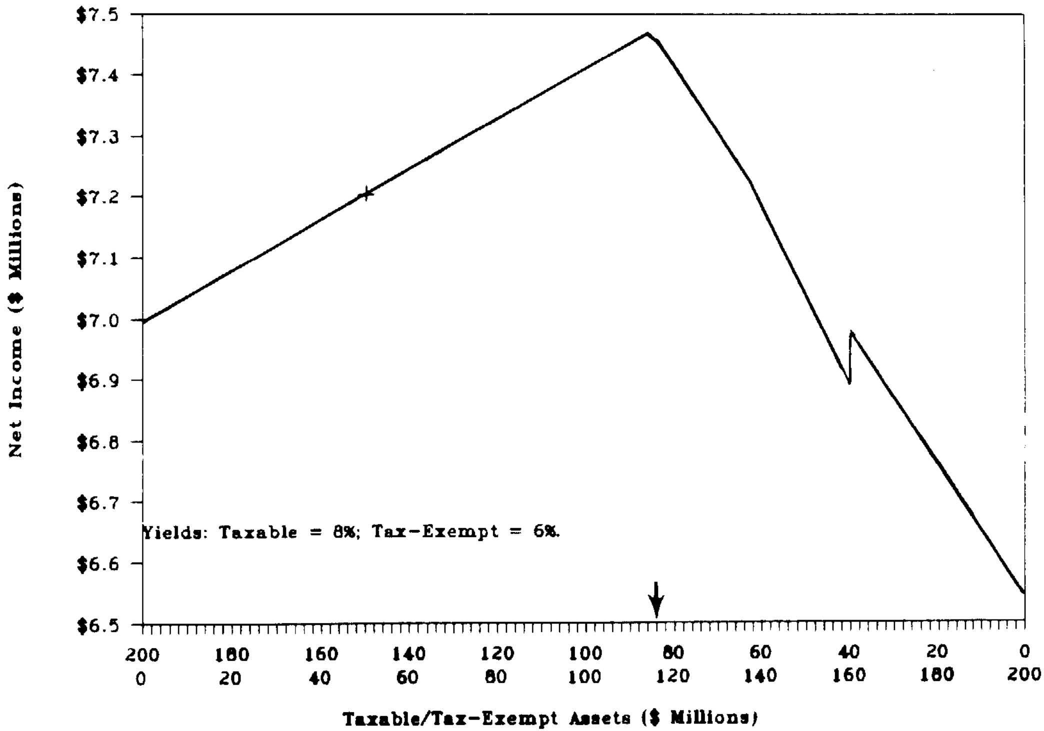
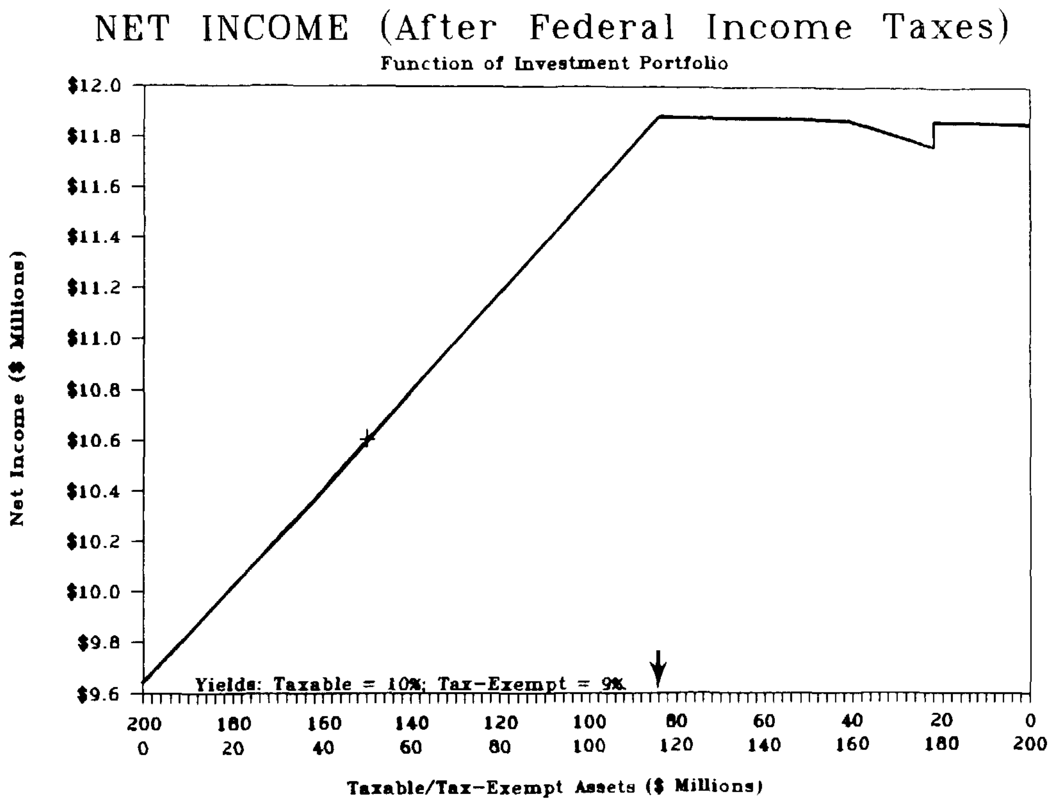


EXHIBIT 10



NET INCOME (After Federal Income Taxes)
Function of Investment Portfolio
Net Income ($ Millions)
$12.0
$11.8
$11.6
$11.4
$11.2
$11.0
$10.8
$10.6
$10.4
$10.2
$10.0
$9.8
$9.6
Yields: Taxable = 10%; Tax-Exempt = 9%
200 180 160 140 120 100 80 60 40 20 0
0 20 40 60 80 100 120 140 160 180 200
Taxable/Tax-Exempt Assets ($ Millions)

EXHIBIT 11

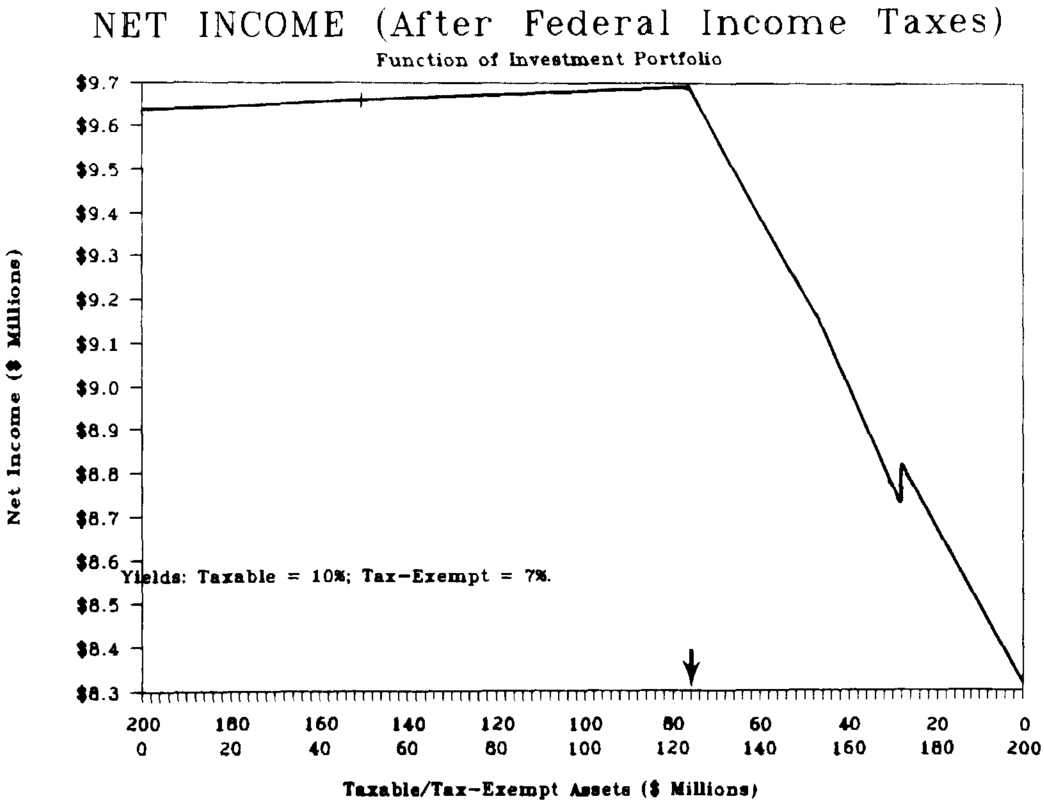

NET INCOME (After Federal Income Taxes)
Function of Investment Portfolio
$9.7
$9.6
$9.5
$9.4
$9.3
$9.2
$9.1
$9.0
$8.9
$8.8
$8.7
$8.6
$8.5
$8.4
$8.3
Net Income ($ Millions)
Yields: Taxable = 10%; Tax-Exempt = 7%.
200 180 160 140 120 100 80 60 40 20 0
0 20 40 60 80 100 120 140 160 180 200
Taxable/Tax-Exempt Assets ($ Millions)

EXHIBIT 12

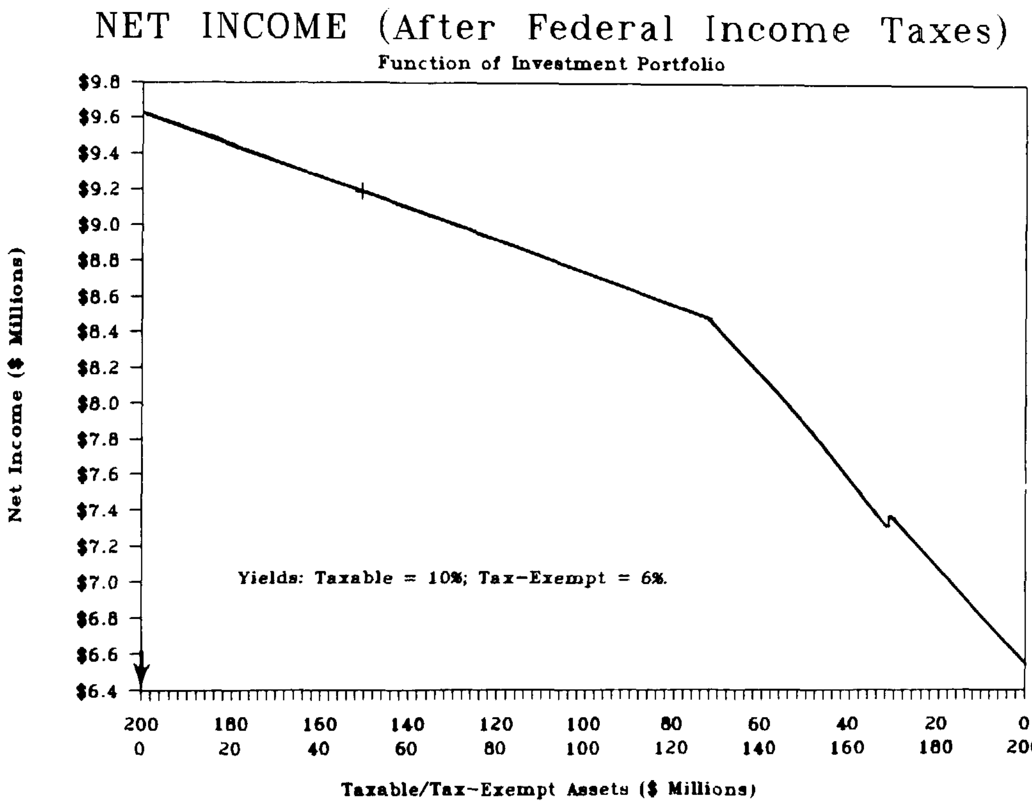

NET INCOME (After Federal Income Taxes)
Function of Investment Portfolio
Net Income ($ Millions)
$9.8
$9.6
$9.4
$9.2
$9.0
$8.8
$8.6
$8.4
$8.2
$8.0
$7.8
$7.6
$7.4
$7.2
$7.0
$6.8
$6.6
$6.4
Yields: Taxable = 10%; Tax-Exempt = 6%.
200 180 160 140 120 100 80 60 40 20 0
0 20 40 60 80 100 120 140 160 180 200
Taxable/Tax-Exempt Assets ($ Millions)

EXHIBIT 13

NET INCOME (After Federal Income Taxes)

Function of Investment Portfolio

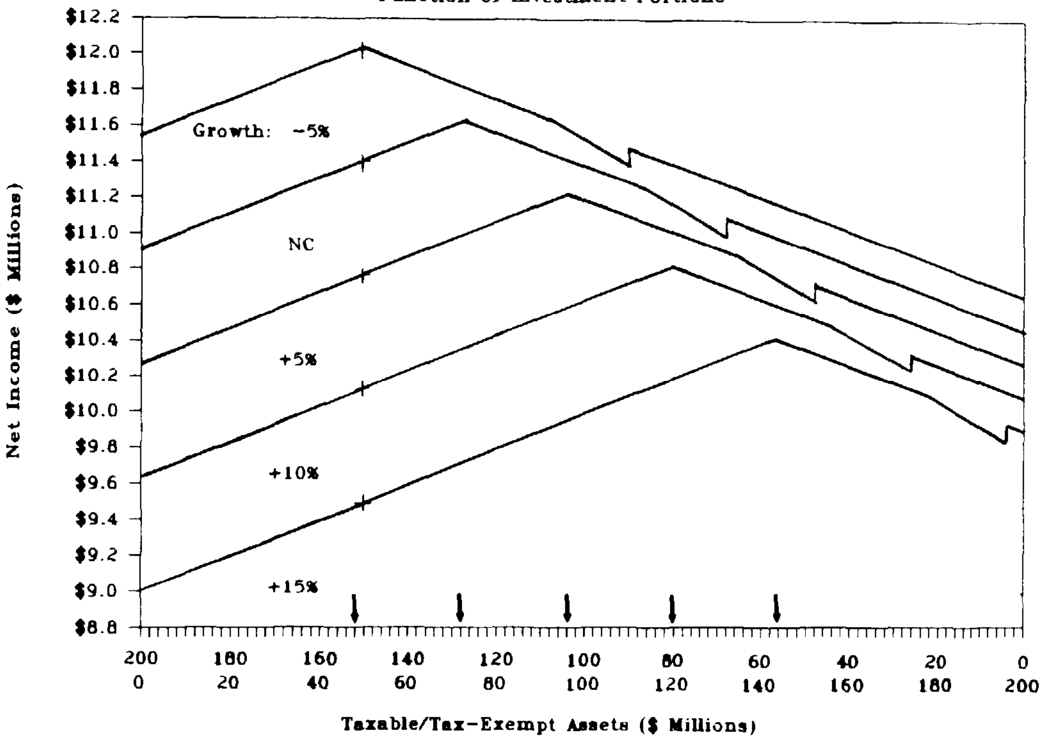


EXHIBIT 14

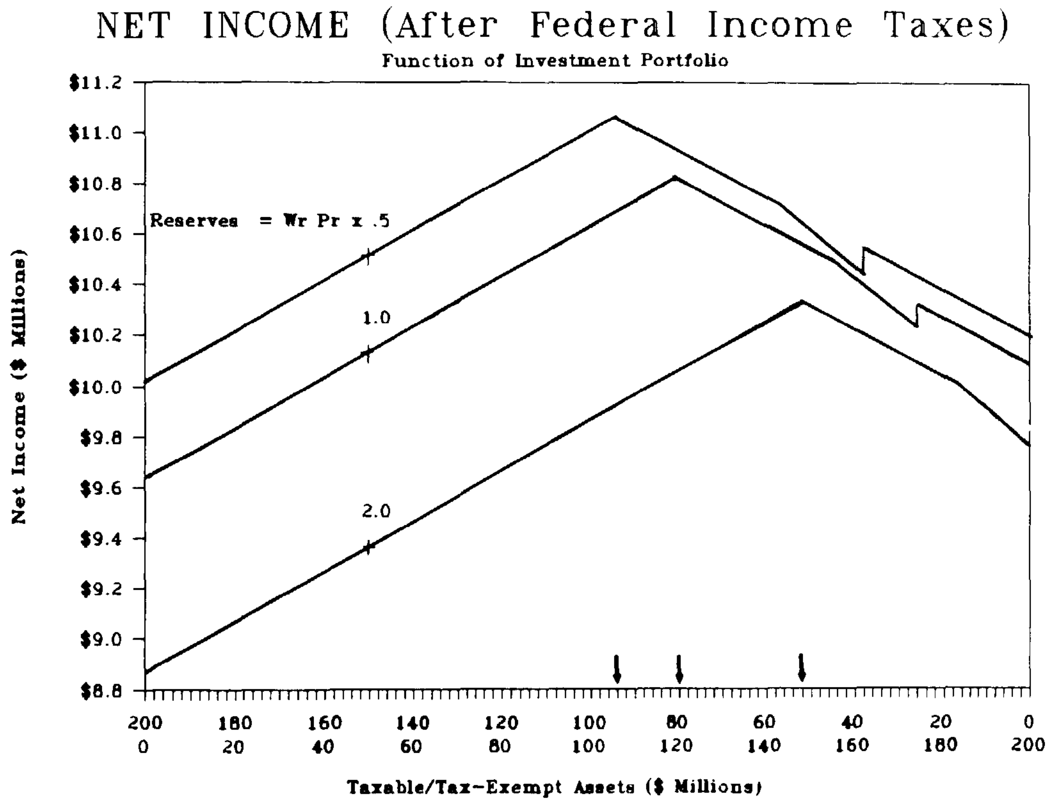

NET INCOME (After Federal Income Taxes)
Function of Investment Portfolio
Reserves = Wr Pr x .5
1.0
2.0
$11.2
$11.0
$10.8
$10.6
$10.4
$10.2
$10.0
$9.8
$9.6
$9.4
$9.2
$9.0
$8.8
Net Income ($ Millions)
200 180 160 140 120 100 80 60 40 20 0
0 20 40 60 80 100 120 140 160 180 200
Taxable/Tax-Exempt Assets ($ Millions)

EXHIBIT 15

NET INCOME (After Federal Income Taxes)

Function of Investment Portfolio

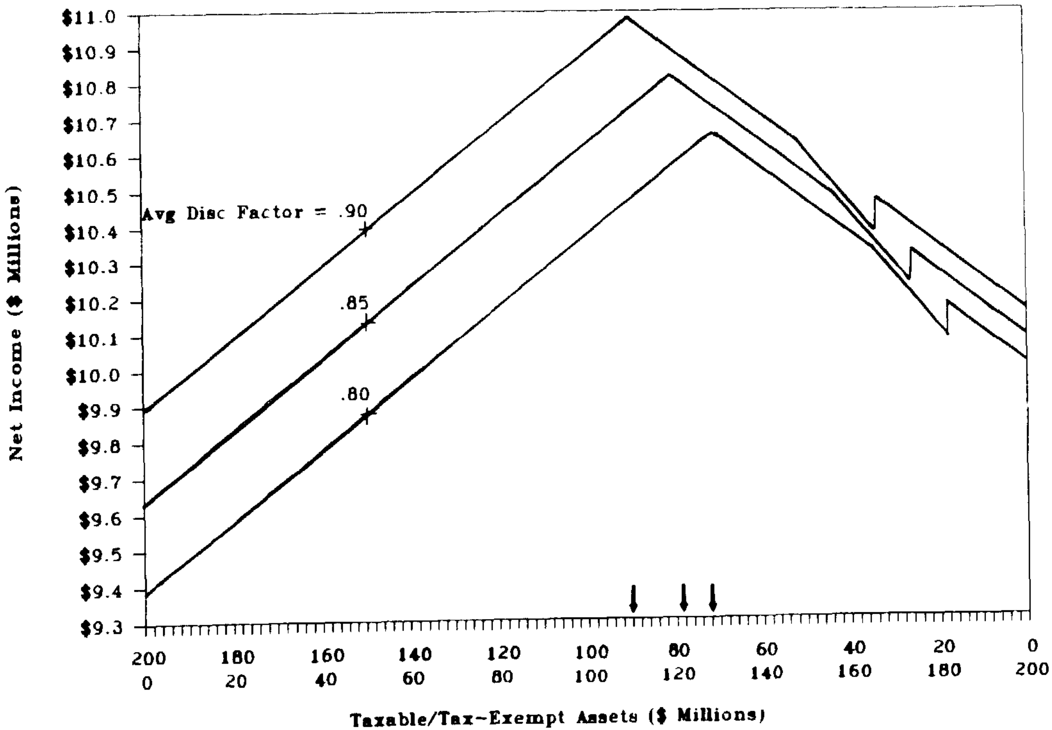
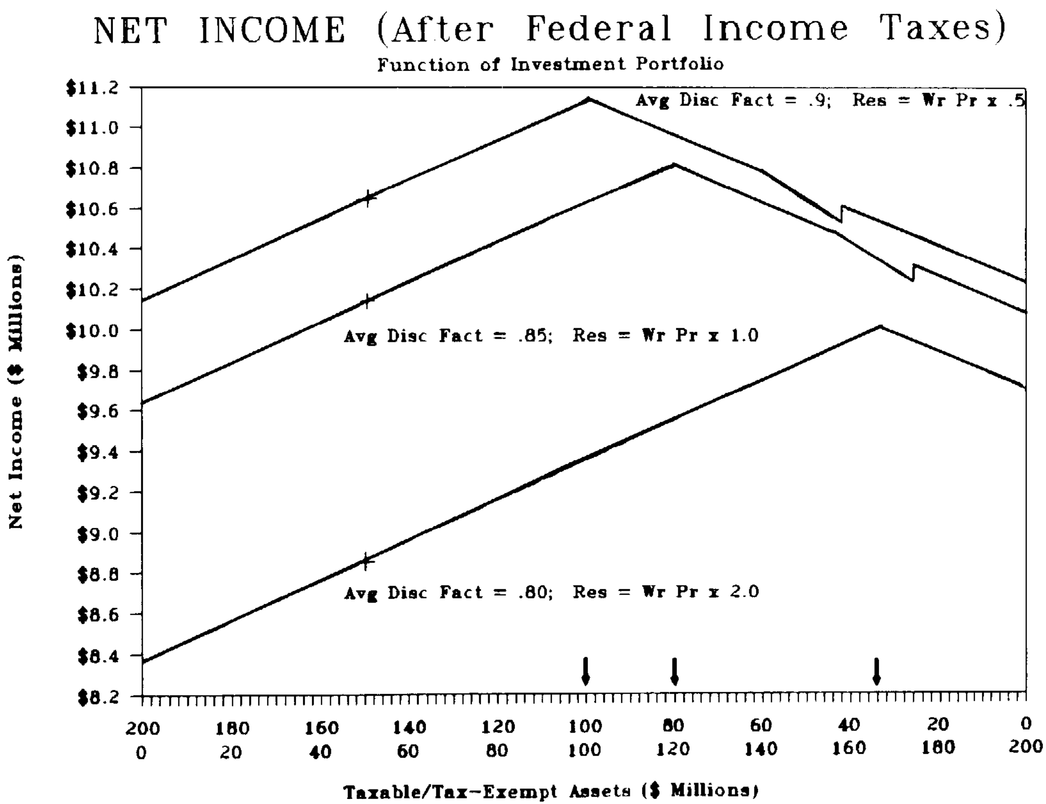


EXHIBIT 16


NET INCOME (After Federal Income Taxes)
Function of Investment Portfolio
Net Income ($ Millions)
$11.2
$11.0
$10.8
$10.6
$10.4
$10.2
$10.0
$9.8
$9.6
$9.4
$9.2
$9.0
$8.8
$8.6
$8.4
$8.2
Avg Disc Fact = .9; Res = Wr Pr x .5
Avg Disc Fact = .85; Res = Wr Pr x 1.0
Avg Disc Fact = .80; Res = Wr Pr x 2.0
200 180 160 140 120 100 80 60 40 20 0
0 20 40 60 80 100 120 140 160 180 200
Taxable/Tax-Exempt Assets ($ Millions)

APPENDIX A

TRANSITION PROVISIONS OF THE TAX REFORM ACT OF 1986

The Tax Reform Act of 1986 contains several transition rules which have a significant impact on property/casualty insurance companies. In this appendix, four of these rules are noted, but only two are discussed in detail.

First, the revenue offset provision of the TRA of 1986 redefines earned premium as written premium less 80.0% of the change in the unearned premium. This change is an attempt to better match the recognition of income with related expenses. In addition, the Internal Revenue Code requires that 20.0% of the unearned premium as of the 1986 calendar year-end be taken into income ratably over six years. In other words, approximately 3.3% of the 1986 year-end unearned premium is added to taxable income each year for the period 1987 through 1992.

The second transition item deals with tax-exempt investment income. As previously mentioned, the proration provision of the IRC calls for 15% of tax-exempt investment income to be deducted from incurred losses. This provision, however, applies only to investment income earned on tax-exempt investments purchased after August 7, 1986. Therefore, for a period of time, insurers will have a portion of tax-exempt investment income which is not subject to the proration provision. Exhibit A-1 shows a comparison of the net income curve assuming that all investment income is derived from investments purchased after August 7, 1986 (this is the base scenario) to the net income curve assuming all investments are purchased before August 7, 1986.

The third transition item is known as the "fresh start" provision. This provision "forgives" the discount in the 1986 year-end loss reserves by allowing a deduction of an amount equal to the difference between the undiscounted and discounted 1986 year-end loss reserves, during the 1987 tax year. The significance of the fresh start forgiveness can be seen by comparing the formula for calculating tax basis incurred losses with and without this provision:

Tax Basis Incurred Loss Without Fresh Start Forgiveness:
Paid Losses + (1987 Year-end Discounted Reserves − 1986 Year-end Undiscounted Reserves)

Tax Basis Incurred Loss With Fresh Start Forgiveness:
Paid Losses + (1987 Year-end Discounted Reserves − 1986 Year-end Undiscounted Reserves) + Fresh Start Amount

where the Fresh Start Amount = (1986 Year-end Undiscounted Reserves − 1986 Year-end Discounted Reserves)

Assuming a composite discount factor of .85, the fresh start amount adds 15% of the 1986 year-end loss reserves to incurred losses.

The discounting of loss reserves and the fresh start provision combine in a manner causing a double deduction of the fresh start amount. Exhibit A-2 shows the calculation of the fresh start amount and the contribution to the tax basis incurred loss from accident years 1986 and prior. Note that the total of the tax basis incurred losses equals the fresh start amount. This result should be no surprise since the tax basis incurred losses are a result of the emergence of the interest underlying the reserves. The 1986 reserves were deducted from taxable income in years prior to 1987. The tax incurred loss generated in 1987 and subsequent, as a result of emerging interest, thus constitutes a second deduction.

The final transition item to be discussed in this appendix is the change in the definition of alternative minimum taxable income (AMTI) that will occur with tax years beginning in 1990. For the purposes of this discussion, it is assumed that there are no preference items other than the book income preference item. This simplifying assumption is made so that we may set AMTI equal to regular taxable income prior to the book income preference item.

For tax years beginning in 1987, 1988, and 1989, the book income preference item is determined as follows. A factor of 50% is applied to the difference between book income and regular taxable income and the result is limited to positive values. AMTI is then the sum of regular taxable income and the book income preference item. The book income preference item ends with the 1989 tax year.

Starting with tax years beginning in 1990, the book income preference item is replaced with a preference item based on "adjusted current earnings" (ACE). The new preference item is determined by applying a factor of 75% to the difference between adjusted current earnings and regular taxable income. The result may be either positive or negative, but may only be negative to the extent that the cumulative value of the new preference item in prior years (1990 and subsequent) has been positive.

At the time of the writing of this paper, the definition of "adjusted current earnings" had not yet been determined. However, an early reading indicates that it will be book income restated using tax basis discounted reserves. Exhibit A-3 displays the net income curves under the pre-1990 AMTI provisions and under an approximate post-1990 AMTI approach. Exhibit A-3 uses statutory income adjusted to include discounted loss reserves as an approximation for ACE.

EXHIBIT A-1

NET INCOME (After Federal Income Taxes)

Function of Investment Portfolio

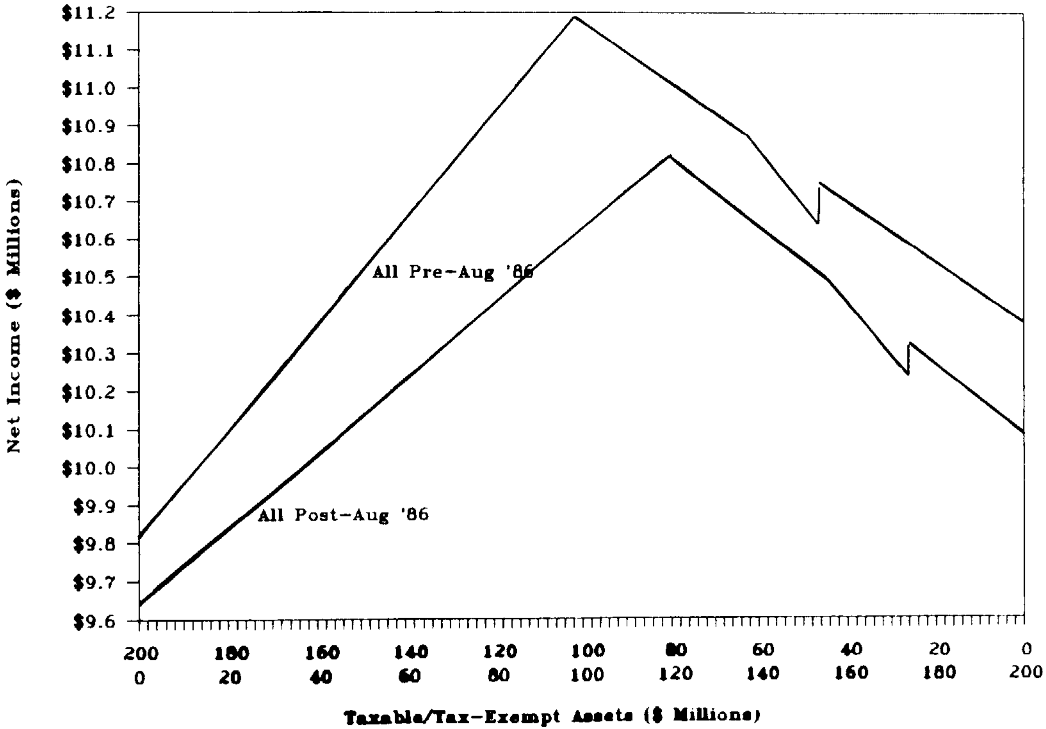


EXHIBIT A-2

Fresh Start Amount Determination and Tax Basis Incurred Loss

	Calendar Year Ending											
	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997
1. Undiscounted Loss & LAE Reserves	182,882	128,345	77,991	41,886	22,896	12,225	5,675	2,562	969	236	47	0
2. Discounted Loss & LAE Reserves	150,300	102,470	61,847	33,111	17,936	9,464	4,415	2,000	764	183	36	0
3. Fresh Start Amount	32,582											

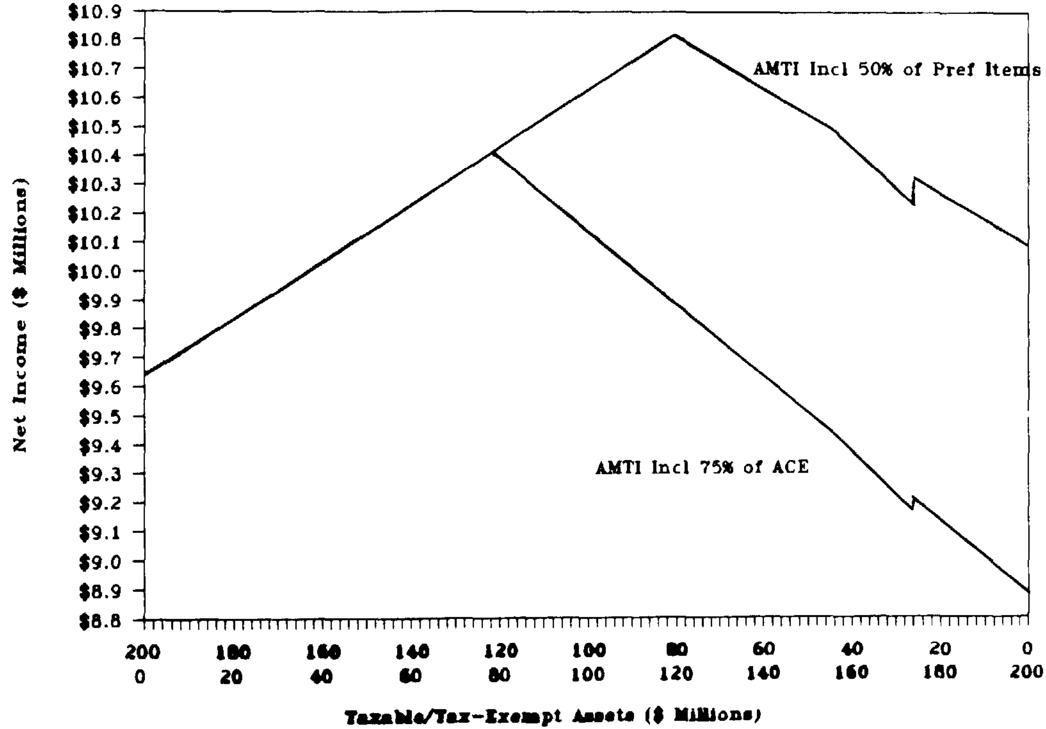
	Calendar Year											
	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	Total
4. Paid Loss & LAE	54,537	50,354	36,105	18,990	10,671	6,551	3,113	1,593	734	189	47	182,882
5. Change In Loss & LAE Reserve												
Undiscounted	(54,537)	(50,354)	(36,105)	(18,990)	(10,671)	(6,551)	(3,113)	(1,593)	(734)	(189)	(47)	
Discounted	(47,831)	(40,623)	(28,736)	(15,176)	(8,472)	(5,049)	(2,415)	(1,236)	(581)	(147)	(36)	
6. Incurred Loss & LAE												
Statutory Basis	0	0	0	0	0	0	0	0	0	0	0	
Tax Basis	6,707	9,731	7,370	3,815	2,199	1,502	698	357	153	42	11	32,582

NOTE: Amounts in thousands. Paid and Reserve numbers are for accident years 1986 and prior only

EXHIBIT A-3

NET INCOME (After Federal Income Taxes)

Function of Investment Portfolio



APPENDIX B
LOSS AND LOSS EXPENSE RESERVE DISCOUNTING

The discounting of loss and loss expense reserves, for tax purposes, is accomplished by applying discount factors to the full value reserves by accident year and Annual Statement line of business. It should be noted that if a company discounts reserves, the reserves are to be grossed up before applying tax discount factors. However, the company must have disclosed the amount of discount in order to be allowed to gross its reserves up.

The discount factors for each annual statement line depend on a payment pattern, which varies by line, and an interest rate. The IRS has prescribed very specific rules for the calculation of each of these two elements.

Payment Pattern

The IRC directs the Secretary of the Treasury to establish payment patterns for each line of business reported on Schedules O and P, for each determination year. A determination year is defined as one for which a payment pattern has been determined. Determination years start with 1987, and occur every fifth year thereafter.

Separate rules apply in the derivation of the payment pattern for Schedule P lines and for Schedule O lines. In both cases, the data for calculating the pattern comes from the most recently filed Annual Statement, creating a two-year lag between the year to which the pattern applies and the data on which the pattern is based.

If for a particular line of business (except for the International and Reinsurance lines reported on Schedule O) a company has sufficient prior loss experience to place it in the top 90% of all companies writing that line, it may use data from its own Annual Statement to determine the payment pattern for use in discounting reserves. In addition to the volume criterion, use of company data to derive payment patterns is allowed only if the company has written premium for the particular line of insurance for at least the number of years that unpaid losses are required to be reported for that line of business on the Annual Statement [8]. If a company does not have enough experience to determine a payment pattern based on its own data, it must use the patterns published by the

IRS. For the International and Reinsurance lines shown in Schedule O, the industry aggregate pattern based on all Schedule P lines combined must be used. Once the company has elected to use the company's own pattern or the industry pattern, that election is valid until the next determination year.

Companies electing to use the industry pattern must use it for the accident year coinciding with the determination year and the following four accident years. If the company pattern is elected, the pattern is revised for each subsequent accident year using the information in the most recently filed Annual Statement of the company. Once a payment pattern is established for an accident year, it is used for that accident year until all reserves are paid out.

Interest Rate

The interest rate promulgated by the IRS is based on a rolling sixty-month average of 100% of the midterm applicable federal rates (AFR) effective as of the beginning of each calendar month. The sixty-month period ends with the month before the beginning of the calendar year for which the determination is made. The midterm AFR is based on the yield of U.S. Government securities with maturities between three and nine years.

For accident years 1987 and prior, the rate is based on the AFR for calendar months of August through December of 1986, resulting in an interest rate of 7.20%. The rate for accident year 1988 is based on the AFR for calendar months of August 1986 through December 1987, resulting in an interest rate of 7.77%. The average continues to include more months each year until a maximum of sixty months is included. At that time the oldest month is dropped when a new month is added to the average.

Discount Factors for Schedule P Lines

Columns (1) through (8) of Exhibit B-1 display the derivation of the payment pattern for a Schedule P line of a hypothetical company. The incremental percentage paid in Column (5) is calculated by taking the first difference of Column (4). Subject to the two exceptions to be discussed later, any portion of losses unpaid after ten years of age is assumed paid in the following year. Thus, the adjusted incremental percentage paids in Column (8) reflect an additional 1.0% payment from the tenth prior year in order to make the sum of all the incremental percentages equal 100%.

The adjusted incremental percentage payments, along with the promulgated discount rate, determine the discount factors. Payments are assumed to be made in the middle of the year. For example, the discount factor for the seventh prior accident year is determined as follows:

$$\frac{(3.0 \times v^{1/2}) + (2.00 \times v^{3/2}) + (1.00 \times v^{5/2})}{(3.00 + 2.00 + 1.00)} = \frac{5.53988}{6.00000} = .923314,$$

where $v = (1.07200)^{-1}$.

There are two exceptions which may apply in determining the payment pattern for Schedule P lines. The first involves the extension of the pattern, up to an additional five years, in the case of long tail lines. A line is subject to this extension if the percentage outstanding after ten years exceeds the percentage paid during the tenth year. In this event, the percentage paid in the tenth year is used repeatedly for up to an additional five years or until the cumulative percentage reaches 100%. If, after repeated use of the tenth year payment, the cumulative percentage paid has not reached 100%, then the pattern is forced to reach 100% at the end of the sixteenth year. Actual reserves may not have been fully paid out, but for the purposes of determining discount factors the payment pattern is assumed to end. Any reserves outstanding beyond this age are assumed paid in the following year. Exhibit B-2 displays an example of the extension of payments using the data for the industry automobile liability line. Note that the percentage unpaid at the end of ten years is 1.02%, and exceeds the payment of 0.32% in the tenth year.

The second exception deals with reversals in the payment pattern. The Code makes specific provision for the case where the incremental percentage paid in the tenth year is negative. In the event this occurs, the last three years of incremental percentage payments are averaged and used repeatedly, until the cumulative pattern reaches 100%. If the three-year average is negative, then a four-year average is used and so forth. Exhibit B-3 displays an example of a line with negative percentage paid in the tenth year.

Other complications that require special handling as prescribed by IRS Notice 88-100 pertain to discount factors for accident years not separately reported in the Annual Statement and to zero or negative discount factors.

Discounted reserves for accident years not separately shown on the Annual Statement (i.e., "and prior" accident year reserves) are calculated by using a composite discount factor. The composite discount factor reflects the distribution among accident years of unpaid losses in the "and prior" category, and the discount factors applicable to each of the prior accident years. An example of the calculation of the composite discount factor for industry general liability data is shown in Exhibit B-4. Note that the composite factor changes over time.

When a company uses its own data to calculate the payment pattern for use in discounting, it is possible for zero or negative discount factors to arise (a zero discount factor = 1.0; a negative discount factor is one that is less than zero). This situation must be resolved by use of a "substitute discount factor," calculated by blending the positive discount factors immediately preceding and succeeding the zero or negative discount factor [8]. If two or more consecutive zero or negative discount factors occur, substitute factors are calculated starting with the most recent accident year generating the zero or negative factor, and moving to the older years until all zero or negative discount factors are eliminated. An example of this elimination is shown in Exhibit B-5.

Discount Factors for Schedule O Lines

Exhibit B-6 sets forth the calculation of discount factors for Schedule O lines, using the industry automobile physical damage data. As mentioned previously, special rules apply to the International and Reinsurance lines.

For Schedule O, a four-year payout is assumed, with reserves outstanding after two years paid equally over the last two years. For the physical damage line, 83.12% of the accident year losses are paid during the first twelve months. A disposal rate of 93.49% is applied to the outstanding portion of 16.88%, resulting in 15.78% paid in the second twelve months. The balance of the outstanding of 1.10% is assumed paid in equal amounts over the following two years.

EXHIBIT B-1

CALCULATION OF RESERVE DISCOUNT FACTORS
SCHEDULE P LINES
NO LONG TAIL EXTENSION
DISCOUNT RATE 7.20%

Accident Year (1)	Losses Paid (2)	Losses Incurred (3)	Cumulative Percentage Paid (2)/(3) (4)	Incremental Percentage Paid (5)	Percentage Unpaid 1.0 − (4) (6)
1975 & Prior	$100,000	$100,000	NA	NA	NA
1976	108,900	110,000	99.00%	2.00%-A	1.00%-B
1977	117,370	121,000	97.00	3.00	3.00
1978	125,114	133,100	94.00	3.00	6.00
1979	133,233	146,410	91.00	4.00	9.00
1980	140,114	161,051	87.00	4.00	13.00
1981	147,040	177,156	83.00	6.00	17.00
1982	150,051	194,872	77.00	10.00	23.00
1983	143,620	214,359	67.00	12.00	33.00
1984	129,687	235,795	55.00	25.00	45.00
1985	77,812	259,374	30.00	30.00	70.00

Accident Year (7)	Adjusted Incremental Percentage Paid (8)	Long Tail Extension of Payments (9)	Adjusted Percentage Unpaid (10)	Discounted Percentage Unpaid (11)	Loss Reserve Discount Factor (11)/(10) (12)
All Prior	NA	0.00%	0.00	0.00%	0.965834
14th Prior	NA	0.00	0.00	0.00	0.965834
13th Prior	NA	0.00	0.00	0.00	0.965834
12th Prior	NA	0.00	0.00	0.00	0.965834
11th Prior	NA	0.00	0.00	0.00	0.965834
10th Prior	1.00%		0.00	0.00	0.965834
9th Prior	2.00		1.00	0.97	0.965834
8th Prior	3.00		3.00	2.83	0.944211
7th Prior	3.00		6.00	5.54	0.923314
6th Prior	4.00		9.00	8.07	0.896145
5th Prior	4.00		13.00	11.39	0.875919
4th Prior	6.00		17.00	14.49	0.852087
3rd Prior	10.00		23.00	19.31	0.839460
2nd Prior	12.00		33.00	27.67	0.838459
1st Prior	25.00		45.00	37.40	0.831129
Current	30.00		70.00	59.03	0.843352
Total	100.00%	0.00%			

NOTES: (2) & (3) 1985 Annual Statement, Schedule P—Part 1, Columns 6 &11.
(5) First difference of Column (4).
(8) & (9) After application of extension and negative payment tests.
(10) Reverse sum of Columns (8) & (9).
(11) Column (8) & (9) discounted at the indicated discount rate.
A Percentage paid in penultimate year.
B Percentage unpaid at end of penultimate year.

EXHIBIT B-2

CALCULATION OF RESERVE DISCOUNT FACTORS
SCHEDULE P LINES
INDUSTRY—AUTO LIABILITY
DISCOUNT RATE 7.20%

Accident Year (1)	Losses Paid (2)	Losses Incurred (3)	Cumulative Percentage Paid (2)/(3) (4)	Incremental Percentage Paid (5)	Percentage Unpaid 1.0 − (4) (6)
1975 & Prior	$91,306,371	$91,545,592	NA	NA	NA
1976	11,389,407	11,506,437	98.98%	0.32%-A	1.02%-B
1977	12,853,464	13,027,563	98.66	0.23	1.34
1978	14,534,843	14,766,868	98.43	0.64	1.57
1979	16,266,022	16,633,374	97.79	1.24	2.21
1980	17,105,852	17,717,217	96.55	2.73	3.45
1981	18,974,882	20,225,872	93.81	4.76	6.19
1982	19,808,529	22,243,403	89.05	8.82	10.95
1983	20,047,428	24,986,353	80.23	15.03	19.77
1984	18,397,279	28,217,053	65.20	30.88	34.80
1985	10,734,519	31,281,287	34.32	34.32	65.68

Accident Year (7)	Adjusted Incremental Percentage Paid (8)	Long Tail Extension of Payments (9)	Adjusted Percentage Unpaid (10)	Discounted Percentage Unpaid (11)	Loss Reserve Discount Factor (11)/(10) (12)
All Prior	NA	0.00%	0.00	0.00%	0.965834
14th Prior	NA	0.00	0.00	0.00	0.965834
13th Prior	NA	0.06	0.00	0.00	0.965834
12th Prior	NA	0.32	0.06	0.06	0.965836
11th Prior	NA	0.32	0.38	0.36	0.955694
10th Prior	0.32%		0.70	0.65	0.925519
9th Prior	0.32		1.02	0.91	0.895529
8th Prior	0.23		1.34	1.16	0.866551
7th Prior	0.64		1.57	1.31	0.831890
6th Prior	1.24		2.21	1.83	0.830789
5th Prior	2.73		3.45	2.91	0.843689
4th Prior	4.76		6.19	5.36	0.866075
3rd Prior	8.82		10.95	9.60	0.876600
2nd Prior	15.03		19.77	17.47	0.883812
1st Prior	30.88		34.80	30.82	0.885530
Current	34.32		65.68	58.58	0.891776
Total	99.30%	0.70%			

NOTES: (2) & (3) 1985 Annual Statement, Schedule P—Part 1, Columns 6 & 11.
(5) First difference of Column (4).
(8) & (9) After application of extension and negative payment tests.
(10) Reverse sum of Columns (8) & (9).
(11) Column (8) & (9) discounted at the indicated discount rate.
A Percentage paid in penultimate year.
B Percentage unpaid at end of penultimate year.

EXHIBIT B-3

CALCULATION OF RESERVE DISCOUNT FACTORS
SCHEDULE P LINES
NEGATIVE PAID IN PENULTIMATE YEAR
DISCOUNT RATE 7.20%

Accident Year (1)	Losses Paid (2)	Losses Incurred (3)	Cumulative Percentage Paid (2)/(3) (4)	Incremental Percentage Paid (5)	Percentage Unpaid 1.0 − (4) (6)
1975 & Prior	$100,000	$100,000	NA	NA	NA
1976	105,600	110,000	96.00%	1.00%-A	4.00%-B
1977	117,370	121,000	97.00	3.00	3.00
1978	125,114	133,100	94.00	3.00	6.00
1979	133,233	146,410	91.00	4.00	9.00
1980	140,114	161,051	87.00	4.00	13.00
1981	147,040	177,156	83.00	6.00	17.00
1982	150,051	194,872	77.00	10.00	23.00
1983	143,620	214,359	67.00	12.00	33.00
1984	129,687	235,795	55.00	25.00	45.00
1985	77,812	259,374	30.00	30.00	70.00

Accident Year (7)	Adjusted Incremental Percentage Paid (8)	Long Tail Extension of Payments (9)	Adjusted Percentage Unpaid (10)	Discounted Percentage Unpaid (11)	Loss Reserve Discount Factor (11)/(10) (12)
All Prior	NA	0.00%	0.00	0.00%	0.965834
14th Prior	NA	0.00	0.00	0.00	0.965834
13th Prior	NA	0.00	0.00	0.00	0.965834
12th Prior	NA	0.67	0.00	0.00	0.965834
11th Prior	NA	1.67	0.67	0.64	0.965834
10th Prior	1.67%		2.33	2.21	0.947300
9th Prior	−1.00		4.00	3.67	0.917908
8th Prior	3.00		3.00	2.46	0.819732
7th Prior	3.00		6.00	5.19	0.865255
6th Prior	4.00		9.00	7.74	0.860039
5th Prior	4.00		13.00	11.08	0.852601
4th Prior	6.00		17.00	14.20	0.835454
3rd Prior	10.00		23.00	19.04	0.827992
2nd Prior	12.00		33.00	27.42	0.831003
1st Prior	25.00		45.00	37.17	0.826028
Current	30.00		70.00	58.82	0.840293
Total	97.67%	2.33%			

NOTES: (2) & (3) 1985 Annual Statement, Schedule P—Part 1, Columns 6 &11.
(5) First difference of Column (4).
(8) & (9) After application of extension and negative payment tests.
(10) Reverse sum of Columns (8) & (9).
(11) Column (8) & (9) discounted at the indicated discount rate.
A Percentage paid in penultimate year.
B Percentage unpaid at end of penultimate year.

EXHIBIT B-4

SHEET 1
CALCULATION OF RESERVE DISCOUNT FACTORS
SCHEDULE P LINES
INDUSTRY—GENERAL LIABILITY
DISCOUNT RATE 7.20%

Accident Year (1)	Losses Paid (2)	Losses Incurred (3)	Cumulative Percentage Paid (2)/(3) (4)	Incremental Percentage Paid (5)	Percentage Unpaid 1.0 − (4) (6)
1975 & Prior	$23,480,898	$25,101,360	NA	NA	NA
1976	2,702,169	3,081,827	87.68%	1.02%-A	12.32%-B
1977	2,812,829	3,245,716	86.66	2.17	13.34
1978	3,050,437	3,610,079	84.50	4.28	15.50
1979	3,339,115	4,162,493	80.22	5.11	19.78
1980	3,548,964	4,724,863	75.11	8.92	24.89
1981	3,429,366	5,180,556	66.20	10.99	33.80
1982	3,181,315	5,762,517	55.21	15.13	44.79
1983	2,493,908	6,222,045	40.08	14.69	59.92
1984	1,752,555	6,901,148	25.40	16.19	74.60
1985	824,218	8,957,695	9.20	9.20	90.80

Accident Year (7)	Adjusted Incremental Percentage Paid (8)	Long Tail Extension of Payments (9)	Adjusted Percentage Unpaid (10)	Discounted Percentage Unpaid (11)	Loss Reserve Discount Factor (11)/(10) (12)
All Prior	NA	7.23%	0.00%	0.00%	0.965834
14th Prior	NA	1.02	7.23	6.98	0.965834
13th Prior	NA	1.02	8.25	7.50	0.908971
12th Prior	NA	1.02	9.27	7.98	0.860875
11th Prior	NA	1.02	10.28	8.42	0.819168
10th Prior	1.02%		11.30	8.84	0.782316
9th Prior	1.02		12.32	9.23	0.749278
8th Prior	2.17		13.34	9.59	0.719322
7th Prior	4.28		15.50	11.04	0.712184
6th Prior	5.11		19.78	14.43	0.729563
5th Prior	8.92		24.89	18.39	0.739097
4th Prior	10.99		33.80	25.77	0.762351
3rd Prior	15.13		44.79	34.65	0.773635
2nd Prior	14.69		59.92	46.93	0.783308
1st Prior	16.19		74.60	57.97	0.776987
Current	9.20		90.80	69.71	0.767789
Total	88.70%	11.30%			

NOTES: (2) & (3) 1985 Annual Statement, Schedule P—Part 1, Columns 6 &11.
(5) First difference of Column (4).
(8) & (9) After application of extension and negative payment tests.
(10) Reverse sum of Columns (8) & (9).
(11) Column (8) & (9) discounted at the indicated discount rate.
A Percentage paid in penultimate year.
B Percentage unpaid at end of penultimate year.

EXHIBIT B-4

SHEET 2
EXAMPLE CALCULATION OF COMPOSITE DISCOUNT FACTOR FOR YEARS NOT REPORTED SEPARATELY
SCHEDULE P LINES
INDUSTRY—GENERAL LIABILITY
DISCOUNT RATE: 7.20%

Accident Year (1)	Nominal Percentage Unpaid (2)	Cumulative (3)	Discounted Percentage Unpaid (4)	Cumulative (5)	Composite Discount Factor (3)/(2) (6)
All Prior	0.00%		0.00%		0.965834
14th Prior	7.23	7.23%	6.98	6.98%	0.965834
13th Prior	8.25	15.48	7.50	14.48	0.935533
12th Prior	9.27	24.74	7.98	22.46	0.907575
11th Prior	10.28	35.03	8.42	30.88	0.881620

NOTES: i. In the 1987 Annual Statement, accident years 1976 & prior are not reported separately. The 1976 accident year would correspond with the 11th prior year.

ii. The nominal and discounted percentages unpaid are based on the payment pattern after adjustment for the negative payment and long tail extension tests.

EXHIBITS ARE CONTINUED ON
THE NEXT PAGE

EXHIBIT B-5

SHEET 1
EXAMPLE OF ELIMINATION OF NEGATIVE DISCOUNT FACTORS
SCHEDULE P LINES
NEGATIVE DISCOUNT FACTOR
DISCOUNT RATE: 7.20%

Accident Year (1)	Losses Paid (2)	Losses Incurred (3)	Cumulative Percentage Paid (2)/(3) (4)	Incremental Percentage Paid (5)	Percentage Unpaid 1.0 − (4) (6)
1975 & Prior	$25,000	$25,000	NA	NA	NA
1976	18,750	37,500	50.00%	−45.00%-A	50.00%-B
1977	53,438	56,250	95.00	10.00	5.00
1978	71,719	84,375	85.00	5.00	15.00
1979	101,250	126,563	80.00	5.00	20.00
1980	99,668	132,891	75.00	10.00	25.00
1981	90,698	139,535	65.00	10.00	35.00
1982	80,582	146,512	55.00	10.00	45.00
1983	69,227	153,838	45.00	15.00	55.00
1984	48,459	161,529	30.00	5.00	70.00
1985	42,401	169,606	25.00	25.00	75.00

Accident Year (7)	Adjusted Incremental Percentage Paid (8)	Long Tail Extension of Payments (9)	Adjusted Percentage Unpaid (10)	Discounted Percentage Unpaid (11)	Loss Reserve Discount Factor (11)/(10) (12)
All Prior	NA	46.43%	0.00%	0.00%	0.965834
14th Prior	NA	0.71	46.43	44.84	0.965834
13th Prior	NA	0.71	47.14	42.52	0.901948
12th Prior	NA	0.71	47.86	40.35	0.843227
11th Prior	NA	0.71	48.57	38.33	0.789228
10th Prior	0.71%		49.29	36.45	0.739548
9th Prior	−45.00		50.00	34.69	0.693819
8th Prior	10.00		5.00	−11.10	−2.220316
7th Prior	5.00		15.00	−0.70	−0.046507
6th Prior	5.00		20.00	4.18	0.208921
5th Prior	10.00		25.00	8.73	0.349078
4th Prior	10.00		35.00	17.80	0.508547
3rd Prior	10.00		45.00	26.26	0.583601
2nd Prior	15.00		55.00	34.16	0.621027
1st Prior	5.00		70.00	46.35	0.662142
Current	25.00		75.00	48.07	0.640881
Total	50.71	49.29%			

NOTES: (2) & (3) 1985 Annual Statement, Schedule P—Part 1, Columns 6 & 11.
(5) First difference of Column (4).
(8) & (9) After application of extension and negative payment tests.
(10) Reverse sum of Columns (8) & (9).
(11) Column (8) & (9) discounted at the indicated discount rate.
A Percentage paid in penultimate year.
B Percentage unpaid at end of penultimate year.

EXHIBIT B-5

SHEET 2
EXAMPLE OF ELIMINATION OF NEGATIVE DISCOUNT FACTORS
SCHEDULE P LINES
NEGATIVE DISCOUNT FACTOR
DISCOUNT RATE: 7.20%

Accident Year (1)	Loss Reserve Discount Factor [Sheet 1, Col. (12)] (2)	Elimination of Most Recent Negative (3)	Elimination of Next Most Recent Negative (3)
All Prior	0.965834	0.965834	0.965834
14th Prior	0.965834	0.965834	0.965834
13th Prior	0.901948	0.901948	0.901948
12th Prior	0.843227	0.843227	0.843227
11th Prior	0.789228	0.789228	0.789228
10th Prior	0.739548	0.739548	0.739548
9th Prior	0.693819 *D*	0.693819 *G*	0.693819
8th Prior	−2.220316 *C*	−2.220316 *F*	0.532186 *H*
7th Prior	−0.046507 *B*	0.370554 *E*	0.370554
6th Prior	0.208921 *A*	0.208921	0.208921
5th Prior	0.349078	0.349078	0.349078
4th Prior	0.508547	0.508547	0.508547
3rd Prior	0.583601	0.583601	0.583601
2nd Prior	0.621027	0.621027	0.621027
1st Prior	0.662142	0.662142	0.662142
Current	0.640881	0.640881	0.640881

NOTES: i. B and C are the negative discount factors to be eliminated. ii. $F = A + [(D - A)/3]$. iii. $H = E + [(G - E)/2]$.

EXHIBIT B-6

CALCULATION OF RESERVE DISCOUNT FACTORS
SCHEDULE O LINES
INDUSTRY—AUTOMOBILE PHYSICAL DAMAGE

(1) Accident Year:	1985	1984	1983 & Prior
(2) Calendar Year 1985 Paid Losses:	13,876,758	1,743,502	(128,871)
(3) Unpaid Losses as of 12/31/85:	2,818,293	121,443	84,756
(4) Total:	16,695,051	1,864,945	(44,115)
(5) Percentage Paid/Disposal Rate:	83.12%	93.49%	NA

(6) Accident Year:	Current	1st Prior	2nd Prior	All Prior
(7) Unpaid at Beg. of Calendar Year:	100.00%	16.88%	1.10%	0.55%
(8) Disposal Rate:	83.12%	93.49%	50.00%	100.00%
(9) Incremental Percentage Paid:	83.12%	15.78%	0.55%	0.55%
(10) Cumulative Percentage Paid:	83.12%	98.90%	99.45%	100.00%
(11) Unpaid at End of Year:	16.88%	1.10%	0.55%	0.00%
(12) Discounted Unpaid at End of Year:	16.20%	1.03%	0.53%	0.00%
(13) Loss Reserve Discount Factor:	0.95964	0.93340	0.96583	0.96583

NOTES: (2) Payments net of salvage & subrogation.
(4) = (2) + (3).
(5) = (2) / (4).
(7) = (11) from previous column.
(8) = (5).
(9) = (7) × (8).
(10) = Cumulation of row (9).
(11) = 1 − (10).
(12) = Sum of discounted remaining incremental percentage payments.
(13) = (12) / (11).

THE CANADIAN CHARTER OF RIGHTS AND FREEDOMS ITS EFFECT ON THE CANADIAN AUTOMOBILE INSURANCE INDUSTRY

ROBERT L. BROWN

Equity is the correction of the law where it is defective by means of its universality.
—Aristotle

Abstract

With the existence of the new Canadian Charter of Rights and Freedoms, it is expected that many of the present risk classification parameters used by the Canadian automobile insurance industry will be challenged. Whether these challenges occur in court or in political forums, industry spokespersons should be prepared to present cogent and relevant comments on the pertinent issues. This paper is specifically designed to assist such persons.

1. INTRODUCTION

On April 17, 1982, Canada officially repatriated its constitution. Until that day, amendments to the constitution still technically required approval by the British parliament. It is now possible for Canada to amend its constitution unilaterally and internally.

Part of the repatriation process involved inclusion in the constitution of a new Charter of Rights and Freedoms (hereafter referred to as the Charter). The Charter has the potential to greatly affect the automobile insurance industry in Canada, particularly many of its presently accepted risk classification parameters (e.g., age, sex, and marital status).

Prior to the existence of the Charter, matters relevant to human rights usually found their way to provincial human rights commissions. The federal government, under John Diefenbaker, had passed a Bill of Rights in 1960, but it was given limited effect by the courts since it lacked the authority of a constitutional document.

Beginning in the early 1970's, the provincial governments amended their human rights codes to prohibit discrimination based on age, sex, and marital status. Prohibition against discrimination on the basis of race, religion, and various other factors had been in existence for some time.

Individual insurance contracts are generally subject to the sections of the provincial human rights codes prohibiting denial of services customarily available to the public or contracts offered to the public on the basis of the prohibited grounds for discrimination. Strictly speaking, individual insurance contracts have not come under the authority of federal legislation.

Some provinces specifically exempted insurance contracts from the discrimination provisions of their human rights codes within limits. For example, the Ontario Human Rights Code provides for limited exemptions "on reasonable and bona fide grounds because of age, sex, marital status, family status, or handicap." What is reasonable and bona fide obviously becomes the issue.

Unlike the provincial human rights codes, the Charter is not a statute; it is a part of Canada's constitution, and therefore, the supreme law of Canada. It is binding at both federal and provincial levels and overrides any statutes or laws to the extent that they are inconsistent with it.

At first, it might appear that as a result, the importance of the provincial human rights codes will fade significantly. In fact, just the opposite may be true. For reasons that will be explained in the next section, the Charter is not expected to apply to private contracts. For those transactions, rulings will still be made based on the provincial human rights codes. Because of the Charter, however, the power and extent of these provincial human rights codes may be broadened significantly.

For example, most provincial human rights codes specifically state that their rights apply only up to age 65. Because of the Charter, this restriction may be ruled invalid, extending the provincial codes' powers to all ages. It may also be possible to use the Charter to end the limited exemptions given to insurance contracts as previously mentioned.

In total, it is anticipated that for insurers, the provincial human rights codes will become more important and more contentious with the existence of the Charter. Present provincial human rights codes, as applicable to automobile insurance, have been summarized in Appendix A.

2. THE CHARTER

Three sections of the Charter are of prime importance to the general insurance industry.

Section 1—Rights and Freedoms in Canada

The Canadian Charter of Rights and Freedoms guarantees the rights and freedoms set out in it subject only to such reasonable limits prescribed by law as can be demonstrably justified in a free and democratic society.

Section 15 (1)—The Equality Section

Every individual is equal before and under the law and has the right to the equal protection and equal benefit of the law without discrimination and, in particular, without discrimination based on race, national or ethnic origin, color, religion, sex, age, or mental or physical disability. (Author's note: this is clearly not an exhaustive list.)

Section 32 (1)—Application of Charter

The Charter applies

(a) to the Parliament and government of Canada in respect of all matters within the authority of Parliament including all matters relating to the Yukon Territory and Northwest Territories; and
(b) to the legislature and government of each province in respect of all matters within the authority of the legislature of each province.

Other sections (not original wording) that may prove to be significant are: Section 15 (2), which specifically allows for affirmative action programs; Section 28, which says that notwithstanding anything in this Charter, the rights and freedoms referred to in it are guaranteed equally to male and female persons; and Section 33 (1), which allows Parliament (federal) and provincial legislatures to override the Charter or opt out.

Some important points need expansion here. First, the Charter applies only to the Parliament and government(s) of Canada and matters on which they have legislative authority. Thus, the Charter should not apply to private contracts or transactions. Legal opinion and some early cases agree with this interpretation. Hence, private contracts (transactions) will still look to the provincial human rights codes for guidance on matters of discrimination.

For automobile insurance, however, the matter of jurisdiction by the Charter may be a debatable point. If the insurance is regulated by the government(s), or if approval of contract forms, rates, etc., is required in any way, then it will be argued that the Charter will apply to such contracts/transactions. Professor Peter W. Hogg, Q.C., in an opinion written for the Insurance Bureau of Canada ([7], page 7) stated:

> "It is clear that an insurance company writing automobile insurance in a province in which there is no government participation in rate fixing is free of the Charter. But it is not clear what degree of governmental participation would be regarded as bringing the Charter to bear."

That will be a matter for the courts to ultimately adjudicate.

Putting Section 15 (1) and Section 1 together, we see that any case will involve two levels of proof. First, one must show that one has been discriminated against. Second, one must show, or refute the defense, that the discrimination was beyond the reasonable limits prescribed by law as can be demonstrably justified in a free and democratic society. Obviously, this leaves room for broad interpretation and will only be defined more clearly through judicial precedents.

It seems clear, however, that laws setting minimum ages for drinking, driving, and voting will have little chance of serious challenge. On the other hand, mandatory retirement and the use of age, sex, and marital status in setting automobile insurance rates are already being challenged as matters contrary to the Charter. It is expected that such cases will be allowed a final hearing in the Supreme Court. (The position with respect to risk classification for automobile insurance procedures will be discussed in detail later in the paper.)

Section 33 (1) would allow the provinces to declare an exemption for insurance contracts regardless of the Charter, but this would have to be done under the full glare of public scrutiny and should not be anticipated.

Finally, Section 28 seems to give special status to sexual equality. Section 28 was a late addition to the Charter and does not fit comfortably with the remainder of the document. It was inserted because of political pressure from the feminist lobby groups. Early opinion has it that the courts may not give this section as much weight as the feminists might hope. In particular, if one were to give a literal interpretation to Section 28, one would have to admit that Section 15 (2), which allows for affirmative action programs, would be inoperable and, in fact, meaningless. Thus, it is anticipated that, while Section 28 may give extra emphasis to sexual equality, its final interpretation will be as an integral part of the whole Charter.

In general, early cases indicate that the courts will give the Charter a broad interpretation attuned to changing circumstances as opposed to overly technical or literal interpretations. (In the words of Associate Chief Justice MacKinnon, "the letter killeth but the spirit giveth life.")

3. THE NEED FOR ADVOCATES

The Charter came into force on April 17, 1982, but the operation of Section 15 was postponed for three years to provide time for the various governments to make whatever changes were necessary to bring their laws into compliance.

While three years have come and gone, little of a substantive nature has occurred. Instead, the politicians seem to have decided to let many of the contentious issues be decided by the courts. There may be some wisdom in this ordering of events, as the courts could overturn legislation deemed to be inconsistent with the Charter anyway.

In this regard, both the politicians and the courts will be inviting the participation of industry experts. The remainder of this paper is devoted to identifying issues of which such experts should be aware in the hope of enhancing the industry's position; that is, that the disputed parameters should be allowed in automobile insurance risk classification.

4. A TWO-STAGE ISSUE

As referred to earlier, most issues will have to be approached in two steps. First, it has to be determined whether the guaranteed fundamental right or freedom has been infringed, breached, or denied. If the answer to that question is yes, then it must be determined whether the denial or limit is a reasonable one demonstrably justifiable in a free and democratic society.

At first glance, one might assume that on matters relevant to risk classification, the industry's involvement will begin at stage two. I would argue that this assumption is seriously in error.

To say that the industry will not become involved until stage two is to imply that we have admitted that we discriminate. That in itself might prove to be a serious tactical error. While we realize that the word discriminate has a positive definition ("the power of observing differences accurately, or of making exact distinctions"), society has decided that the word discrimination means "to discriminate against." Hence, by going directly to stage two, the industry immediately places itself on the defensive.

Instead, it seems prudent to start by saying that our present risk classification methods are totally in agreement with the intent of the Charter. In doing so, one should differentiate between treating people equally and treating them the same. The Charter should not be construed to mean that people should be treated the same or that being treated the same leads to equality.

Two examples might help. Having a weight/height criterion for entering into a career treats everyone the same, but discriminates against females. Having a written test criterion for entering into a university treats everyone the same, but discriminates against the blind.

Feminists in Canada are presently lobbying for pay equity legislation which would require equal pay for work of equal value. No one has suggested that everyone be paid the same, only that there be pay equity. Similarly, the insurance industry is in favor of premium equity, which requires equal premiums for risks of equal cost. Surely anyone in favor of pay equity must agree with the philosophy of premium equity.

> "Human rights should mean that everyone can enter a restaurant and get service. It should not mean that everyone has to order the same meal, nor pay the same price." ([8], page 10).

One must argue, then, that the present risk classification methods are in agreement with the Charter. These methods lead to economic equity which equates price and cost, and that equity is true equality. Treating people the same leads to superficial equality, but not true equality. Treating people equitably leads to true equality. With this argument, the industry places itself in total agreement with the Charter and starts on a truly positive basis.

Regardless of the acceptability of this argument, one must move along to address the stage two question. That is, whether or not our risk classification methods are deemed to be discriminatory, can they be "demonstrably justified in a free and democratic society?"

5. SECTION 1 ISSUES

Group Versus Individual Rights

One of the toughest aspects of arguing a case under the Charter is the fact that the Charter is very much a document about individual rights. On the other hand, the essence of insurance is the concept of the pooling of risk and the law of large numbers.

In entering a discussion on this issue, it is wise to point out that the pooling concept of insurance preceded the private insurance corporations. Centuries ago, people striving for economic security formed fraternals and cooperatives, so that a defined contribution from each member of the group could be used to reimburse the small number who met with economic loss through fire, death, disability, etc.

In fact, the present risk classification methods have moved this primitive pooling concept from one of pure single factor grouping (if you are in the group you share equally in the risk) to one where, to the extent possible before the outcome is known, individuals are truly treated as individuals.

And perhaps that is the crux of the matter. Underwriters and actuaries must price insurance coverages (to enable the transfer of economic risk) before the outcome is known. Hence, each individual is assessed according to some predictive characteristics. These include not only the disputed characteristics like age, sex, and marital status but also characteristics like driving record, use of car, geographic location, etc., which are not being disputed under the Charter. Underwriters get as much information on each individual as is economically feasible (risk classification will use finer and finer categories until the marginal cost of further refinement does not yield a corresponding increase of business to the seller) and then treat each individual truly as an individual. Underwriters also look for risk classification characteristics that are easy to ascertain and verify and are not subject to manipulation by the applicant. In this latter sense, age, sex, and marital status are preferred to variables such as miles driven.

At this time, one could also point out that the ultimate goal of those in favor of superficial equality is equality of outcome. It can be argued that the present risk classification methods, centering as they do on economic equity, also have as their goal equality of outcome, and, in that sense, we are in total agreement.

Age, Sex, and Marital Status

Many of the early discussions on the effect of the Charter on risk classification have centered on the issue of sex as a classification variable. This is partly because of the extra emphasis given to sexual equality in Section 28, but more likely because this is such an important political topic at this time. The discussion that follows will also center on sex, but the arguments presented apply equally as well to age and marital status.

The arguments with respect to the use of sex as a risk classification parameter usually center on whether one's sex *causes* the resultant risk profile or not. One may wish to argue that it is of little concern to the underwriter whether the risk profile is caused by the risk classification parameter or not; that, in fact, the underwriter is satisfied with correlations without cause and effect being necessary. For example, one can see a correlation between being in a hospital and mortality, but one would not conclude that being in a hospital *causes* one to die. However, the underwriter would claim that there is justification here to change one's risk class if one is presently in a hospital.

However, it would appear that the industry advocate may have to try to show that one's sex is, in fact, causal with respect to one's ultimate risk profile. It will be argued that the automobile insurance industry could use other classification parameters such as miles driven per year, driving record convictions, or number of claims as a substitute for sex. In fact, however, the industry already does vary its rates based on these parameters and can show that sex differentials are still appropriate even after all other differences have been accounted for, as illustrated in Table 1.

TABLE 1

COMPARISON OF MALE, FEMALE ACCIDENT FREQUENCIES WITHIN MILEAGE BANDS

Annual Mileage Band	Number of Accidents/Driver (3 Year Accident Record)		Number of Accidents/100,000 Miles (3 Year Accident Record)	
	Male	Female	Male	Female
0–2,499	.163	.079	8.15	3.95
2,500–4,999	.268	.103	7.15	2.75
5,000–7,499	.223	.152	3.57	2.43
7,500–9,999	.229	.179	2.62	2.05
10,000–14,999	.271	.242	2.17	1.94
15,000–19,999	.319	.249	1.82	1.42
20,000–24,999	.345	.299	1.53	1.33
25,000–29,999	.353	.277	1.28	1.01
30,000–39,999	.350	.271	1.00	0.77
40,000–49,999	.430	.273	0.96	0.61
50,000+	.563	.318	0.94	0.53

Source: U.S. Department of Transportation, Federal Highway Administration (with California Department of Public Works), 1973

The Canadian automobile insurance industry has been aware of the controversy surrounding the risk classification system that uses disputed parameters such as age, sex, and marital status for at least a decade. Although the industry has extensively investigated alternative risk classification models, no satisfactory alternatives have yet been found ([8], page 15).

This opinion is supported by many independent studies and reports. For example, in the Report of the Alberta Automobile Insurance Board respecting factors of age, sex, and marital status in automobile insurance rating:

> "We concluded that the research conducted by Ontario groups demonstrates that there are no variables capable of introduction to the system as true surrogates for the factors of age, sex, and marital status."

Further statistics can be cited. The 1983 Ontario Motor Vehicle Accident Facts Booklet shows a collision rate per 100 licensed drivers of 7.2 for male drivers of all ages and 3.3 for female drivers of all ages. Statistics from the TIRF [18] show that, in 1982, male drivers accounted for 60% of all traffic injuries in Canada and 75% of all traffic deaths. In 1982, 4 of every 5 drivers fatally injured in Canada were male and 2 of every 3 injured drivers were male. An Ontario Provincial Police survey of drivers charged with blood alcohol violations in the 1985 Christmas season showed that 94 per cent were male, and 30 per cent were single males under age 30 (the largest identified subgroup).

Similar statistics are shown for age in Table 2. Not only do young drivers have more collisions but, for the youngest drivers, the cost per accident is higher.

As to marital status, consider males aged 21 to 25. Those who are single have an expected accident frequency of 10.6 per every 100 cars insured while those who are married have a frequency of 8.8 accidents per 100 cars insured (source: Insurance Bureau of Canada).

Whether the parameter is age, sex, or marital status, the key to the argument is the same. That is, price should be a direct function of cost. An inevitable result is that equal risks are treated equally but unequal risks are treated unequally. The overriding criterion is economic equity which we argued earlier is a better attempt at true equality than the superficial equality that results from treating everyone the same.

In closing, one should mention that if the risk profile differences by sex, age, or marital status were to disappear over time (for whatever reason), the present risk classification methods would cause insurance rate differences to disappear also. That being the case, no specific legislation is required.

TABLE 2

ALBERTA
AGE AND SEX OF DRIVERS
INVOLVED IN SERIOUS COLLISIONS
1985

Age of Driver (Years)	Number of Collisions per 1,000 Licensed Drivers	
	Male	Female
Under 16	17.7	9.0
16–17	27.2	16.0
18–19	32.1	14.5
20–24	21.9	9.4
25–34	14.1	7.1
35–44	11.6	6.9
45–54	11.2	5.7
55–64	10.8	5.4
65+	10.6	5.6

Source: Alberta Traffic Collision Statistics, 1985

6. PUBLIC POLICY

While the question may not arise in a court proceeding, in many political forums the question of the ultimate effects of treating people the same (i.e., superficial equality) versus treating people equitably (i.e., true equality) may arise. This question is often asked by those concerned about social justice who feel that females have been "penalized" by our risk classification methods in the past and can now expect "the wrong to be righted."

Obviously, in a sex-neutral world, one would expect females to pay more for life insurance and automobile insurance but less for life annuities. While life insurance data are not available for Canada, it has been estimated that for the United States, females would pay $360 million more per annum for life insurance and $700 million more per annum for automobile insurance. The Insurance Bureau of Canada has found that if automobile insurance pricing stratification in Canada based on sex were abolished, females would pay 48 percent more for their insurance, while young males would pay 12 percent less. In a free market society, however, it is difficult to predict the exact outcome.

Under the present philosophy, risk classification methods arrive at an equitable value for benefits provided to any individual. No significant subsidies consciously exist. As already pointed out, if the risk differences between males and females were to disappear over time, the price differential would disappear also. However, if we are forced to treat individuals with superficial equality instead of true equality (e.g., provide the same rates to males and females regardless of the inherent risk), then subsidies will exist in that some individuals will pay less than their expected costs and some will pay more.

Such a system can exist within closed groups where free market consumer options are not available. For example, it is possible for unisex pensions to be available in the Canada/Quebec Pension Plan. It is also possible for an employer to pay equal benefits to females and males in a closed-group defined benefit pension plan, as is presently the case for 93.7% of private plan members in Canada.

However, such a system cannot exist for long in a free market place without serious consequences. Basic economic theory tells us that those who see the market price as being less than their expected cost will buy more units. Those who believe the price to be too high will buy fewer units, choose larger deductibles, or otherwise attempt to self-insure (e.g., drive without insurance). In particular, females will move to purchase lower limits of liability protection, higher deductibles, or even refrain from driving. Some young males, finding their insurance now affordable, will drive more. The public policy effect of these shifts actually could be an increased number of accidents, since high risk drivers would be encouraged to drive while the class of drivers representing a lower risk profile would be discouraged from driving. Consequently, average losses will rise, and in the next round of price determination, so too will prices.

The end result of this spiral is that, in the long run, the market price will reach an equilibrium equal to the fair price for the poorest risk in the group, while all other potential users of the insurance mechanism (to transfer risk) will remove themselves from the market for economic reasons. For example, if unisex automobile insurance rates are required by law, those rates could ultimately reach a new equilibrium at the previously existing single male rates or the industry may just allocate all male risks to the residual market (the Facility Association). In short, there will be few winners and many, many losers.

7. CONCLUSION

This paper has presented the philosophical arguments that could be used in presenting the actuarial argument in favor of the continued use of certain disputed parameters such as age, sex, and marital status, in the pricing of automobile insurance in Canada. What has not been presented are the economic and mathematical arguments that might center on the issue of the optimality of the present risk classification system. As stated in the American Academy of Actuaries document, "Risk Classification Statement of Principles" [2]:

> "There often is not a clear-cut optimal set of characteristics. Over time, in a perfectly competitive market, the optimal set of characteristics tends to emerge through the competitive mechanism. However, in practice, perfectly competitive markets are seldom achieved, and the risk characteristics commonly used reflect both observed fact and informed judgment."

Readers who have an interest in the mathematical optimality of competing risk classification systems, with imperfect information, are invited to read references [1], [5], [10], [12], and [19].

There is no doubt that the existence of the new Canadian Charter of Rights and Freedoms will lead to challenges to our present risk classification methods. Whether those challenges are in a court of law or in a political forum, the automobile insurance industry must be prepared with cogent and relevant arguments. We should also be seen to be proactive rather than reactive. It is the hope of the author that this paper will assist the Canadian industry in that regard.

REFERENCES

[1] G. Akerlof, "The Market for 'Lemons': Qualitative Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, August, 1970.

[2] American Academy of Actuaries, Committee on Risk Classification, "Risk Classification Statement of Principles," June, 1980.

[3] Canadian Institute of Actuaries, "A Generic Paper on Human Rights," June, 1986.

[4] Canadian Institute for the Administration of Justice, "The Canadian Charter of Rights and Freedoms," 1984.

[5] B. Dahlby, "Monopoly Versus Competition in an Insurance Market with Adverse Selection," *Journal of Risk and Insurance*, June, 1987.

[6] T. Flanagan, "Insurance, Human Rights, and Equality Rights in Canada: When is Discrimination 'Reasonable'?" *Canadian Journal of Political Science*, XVIII:4, December, 1985.

[7] P. W. Hogg, "Classification by Age, Sex and Marital Status for Automobile Insurance Rates," An opinion prepared for the Insurance Bureau of Canada, August, 1983.

[8] Insurance Bureau of Canada, "Classification and Rating Criteria for Automobile Insurance," February, 1982.

[9] Danielle Morin, "Consumerist Issues in the Canadian Life Insurance Industry," Society of Actuaries Part 6 Study Note 6-133-86.

[10] Mark V. Pauly, "Overinsurance and Public Provision of Insurance: The Roles of Moral Hazard and Adverse Selection," *Quarterly Journal of Economics*, 88, 1974, pp. 44–62.

[11] A. Samuel Rae, Jr. and Michael J. Trebilcock, "Rate Determination in the Automobile Insurance Industry in Ontario," Insurance Bureau of Canada, 1982.

[12] M. Rothchild and T. Stiglitz, "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *Quarterly Journal of Economics*, November, 1976.

[13] W. S. Tarnopolsky and G. A. Beaudoin, "The Canadian Charter of Rights and Freedoms," The Carswell Company Limited, 1982.

[14] W. S. Tarnopolsky, "Discrimination and the Law in Canada," Richard De Boo Limited, 1982.

[15] Traffic Injury Research Foundation, "The Young Driver Paradox," September, 1976.

[16] Traffic Injury Research Foundation, "Why Are Women Safer Drivers?" May, 1979.

[17] Traffic Injury Research Foundation, "Young Driver Accidents in Canada," November, 1981.

[18] Traffic Injury Research Foundation, "Trends in Road Accidents Involving Women," June, 1985.

[19] C. Wilson, "A Model of Insurance Markets with Incomplete Information," *Journal of Economic Theory*, December, 1977.

APPENDIX A

PROVINCIAL HUMAN RIGHTS LEGISLATION RELEVANT TO AUTOMOBILE INSURANCE

ALBERTA—THE INDIVIDUAL'S RIGHTS PROTECTION ACT

Section 3

"No person, directly or indirectly, alone or with another, by himself or by the interposition of another, shall

(a) deny to any person or class of persons any accommodation, services or facilities customarily available to the public, or

(b) discriminate against any person or class of persons with respect to any accommodation, services or facilities customarily available to the public,

because of the race, religious beliefs, color, sex, physical disability, ancestry or place of origin of that person or class of persons or of any other person or class of persons."

It should be noted that while there is no reference in Section 3 to discrimination by reason of age, the preamble to the Act does refer to age. It provides as follows:

"Whereas it is recognized in Alberta as a fundamental principle and as a matter of public policy that all persons are equal in dignity and rights without regard to race, religious beliefs, color, sex, physical disability, age, ancestry or place of origin . . ."

In 1985, the following provision was introduced allowing reasonable and justifiable contravention:

Section 11.1

"A contravention of this Act shall be deemed not to have occurred if the person who is alleged to have contravened the Act shows that the alleged contravention was reasonable and justifiable in the circumstances."

BRITISH COLUMBIA—HUMAN RIGHTS CODE

Section 3 was amended in 1984 to read as follows:

Section 3

"No person shall

(a) deny to a person or class of persons any accommodation, service or facility customarily available to the public, or

(b) discriminate against a person or class of persons with respect to any accommodation, service or facility customarily available to the public

because of race, color, ancestry, place of origin, religion, marital status, physical or mental disability or sex of that person or class of persons unless the discrimination relates, in the case of sex, to the maintenance of public decency or, in the case of sex or physical or mental disability, to the determination of premiums or benefits under contracts of life or health insurance."

It will be noted that age is not a factor to be taken into account nor are sex and physical or mental disability insofar as life or health insurance is concerned.

MANITOBA—HUMAN RIGHTS ACT

Section 3(1)

"No person shall

(a) deny to any person or class of persons any accommodation, service, or facility customarily available to the public; or

(b) discriminate against any person or class of persons with respect to any accommodation, service, or facility customarily available to the public,

unless reasonable cause exists for the denial or discrimination."

Section 3(2)

"For the purposes of subsection (1)

(a) the race, nationality, religion, color, sex, age, marital status, physical handicap, or ethnic or national origin of a person does not constitute reasonable cause; and

(b) the sex of any person does not constitute reasonable cause unless it relates to the maintenance of public decency."

Section 7(1)

"No person shall, in making available to any person, a contract that is offered to the public generally,

(a) discriminate against any person; or

(b) include terms or conditions in any such contract that discriminate against a person on the basis of race, nationality, religion, color, sex, age, marital status, physical handicap, ethnic or national origin of that person."

Section 7(2)

"No provision of Section 6 (which relates to discrimination prohibited in employment, advertising, etc.) or subsection (1) shall prohibit a distinction on the basis of age, sex, family status, physical handicap or marital status

(a) of any employee benefit plan or in any contract which provides an employee benefit plan, if the Commission is satisfied on the basis of the guidelines set out in the regulations that the distinction is not discriminatory or that the employee benefit can be provided only if the distinction is permitted; or

(b) in any contract which provides life insurance, accident and sickness insurance or a life annuity to a specified person where the contract is not part of an employee benefit plan, if the Commission is satisfied on the basis of guidelines set out in the regulations that the distinction is not discriminatory or that the insurance or annuity can be provided only if the distinction is permitted."

Section 7(3)

"Nothing in this Act prohibits a distinction on the basis of sex, age or marital status in any contract of automobile insurance offered or made available to the public under The Manitoba Public Insurance Corporation Act or The Insurance Act."

It will be noted that the Manitoba legislation does not prohibit the use of age, sex, or marital status in automobile insurance contracts.

Bill 47, The Human Rights Code, introduced on June 3, 1987, will bring substantial amendments to the present legislation. It will extend the elements of discrimination and retain "bona fide and reasonable" discrimination in several areas including life insurance and accident and sickness insurance.

PRINCE EDWARD ISLAND—HUMAN RIGHTS ACT

Section 2(1)

"No person shall discriminate

(a) against any individual or class of individuals with respect to enjoyment of accommodation, services and facilities to which members of the public have access; or

(b) with respect to the manner in which accommodations, services and facilities, to which members of the public have access, are provided to any individual or class of individuals."

Section 2(2)

"Subsection (1) does not prevent the denial or refusal of accommodation, services or facilities to a person on the basis of age if the accommodation, services or facilities are not available to that person by virtue of any enactment in force in the province."

Section 11

"The provisions of this Act relating to discrimination in relation to age or physical or mental handicap do not affect the operation of any bona fide retirement or pension plan or any bona fide group or employee insurance plan."

Section 14(1)(d)

Provides that Sections 2 to 13 do not apply to a refusal, limitation, specification, or preference based on a bona fide qualification.

NOVA SCOTIA—HUMAN RIGHTS ACT

Section 4

"No person shall

(a) deny to any individual or class of individuals enjoyment of accommodation, services and facilities, to which members of the public have access; or

(b) discriminate with respect to the manner in which accommodations, services and facilities, to which members of the public have access, are provided to any individual or class of individuals,

because of the race, religion, creed, color or ethnic or national origin of the individual or class of individuals."

Section 11(A)(1)

"No person shall deny to, or discriminate against, an individual or class of individuals, because of the sex of the individual or class of individuals, in providing or refusing to provide any of the following:

(a) accommodation, services and facilities customarily provided to members of the public;

(b) occupancy, or any term or condition of occupancy, of any commercial unit or self-contained dwelling unit;

(c) transfer any property or interest in property;

(d) employment, conditions of employment or continuing employment, or the use of application forms or advertising for employment, unless there is a bona fide occupational qualification based on sex."

Section 11(A)(2)

"No person or agency included in Subsection 2 of Section 8 or Sections 9, 10 or 11 shall discriminate against an individual or class of individuals because of the sex of the individual or class of individuals or on account of marital status."

NEW BRUNSWICK—HUMAN RIGHTS ACT

Section 5(1)

"No person, directly or indirectly, alone or with another, by himself or by the interposition of another, shall

(a) deny to any person or class of persons the accommodation, services, or facilities available in any place to which the public is customarily admitted, or

(b) discriminate against any person or class of persons with respect to the accommodations, services, or facilities available in any place to which the public is customarily admitted

because of race, color, religion, national origin, ancestry, place of origin, age, physical disability, mental disability, marital status or sex."

Section 5(2)

"Notwithstanding subsection (1), a limitation, specification, exclusion, denial or preference because of sex, physical disability or marital status shall be permitted if such limitation, specification, exclusion, denial or preference is based upon a bona fide qualification as determined by the Commission."

Note the bona fide qualification related to sex or marital status in Section 5(2).

NEWFOUNDLAND—HUMAN RIGHTS CODE

Section 7(1)

"No person shall deny to any person or class of persons admission to or enjoyment of accommodation, services or facilities available in any place to which the public is customarily admitted by reason only of the race, religion, religious creed, sex, marital status, physical disability, mental disability, political opinion, color or ethnic, national or social origin of such person or class of persons."

Section 7(2)

"Notwithstanding Subsection (1), a limitation, specification, exclusion, denial or preference because of physical disability, mental disability shall be permitted if such limitation, specification, exclusion, denial or preference is based upon a bona fide qualification as determined by the Commission."

SASKATCHEWAN—HUMAN RIGHTS CODE

Section 12(1)

"No person, directly or indirectly, alone or with another, by himself or by the interposition of another, shall:

(a) deny to any person or class of persons the accommodation, services, or facilities to which the public is customarily admitted or which are offered to the public; or

(b) discriminate against any person or class of persons with respect to the accommodation, services or facilities to which the public is customarily admitted or which are offered to the public;

because of the race, creed, religion, color, sex, marital status, physical disability, age, nationality, ancestry or place of origin of that person or class of persons or of any other person or class of persons."

Section 12(2)

"Subsection (1) does not apply to prevent the barring of any person because of his sex from any accommodation, services or facilities upon the ground of public decency."

Section 12(3)

"Subsection (1) does not apply to prevent the denial or refusal of any accommodation, services or facilities to a person on the basis of age, if the accommodation, services or facilities are not available to that person by virtue of any law or regulation in force in the province."

Section 15(1)

"No person shall, in making available to any person a contract that is offered to the public:

(a) discriminate against any person or class of persons; or

(b) include terms or conditions in any such contract that discriminate against a person or class of persons;

because of the race, creed, religion, color, sex, marital status, nationality, ancestry or place of origin of that persons or class of persons."

ONTARIO—THE HUMAN RIGHTS CODE

Section 1

"Every person has a right to equal treatment with respect to services, goods and facilities, without discrimination because of race, ancestry, place of origin, color, ethnic origin, citizenship, creed, sex, age, marital status, family status or handicap."

Section 3

"Every person having legal capacity has a right to contract on equal terms without discrimination because of race, ancestry, place of origin, color, ethnic origin, citizenship, creed, sex, age, marital status, family status or handicap."

Section 21

"The right under Sections 1 and 3 to equal treatment with respect to services and to contract on equal terms, without discrimination because of age, sex, marital status, family status or handicap, is not infringed where a contract of automobile, life, accident or sickness or disability insurance or a contract of group insurance between an insurer and an association or person other than an employer, or a life annuity, differentiates or makes a distinction, exclusion or preference on reasonable and bona fide grounds because of age, sex, marital status, family status or handicap."

It should be noted that Section 21 has been interpreted by the Ontario court in favor of insurers with respect to automobile insurance. This decision is presently under appeal.

QUEBEC—CHARTER OF HUMAN RIGHTS AND FREEDOMS

Section 10

"Every person has a right to full and equal recognition and exercise of his human rights and freedoms, without distinction, exclusion or preference based on race, color, sex, pregnancy, sexual orientation, civil status, age except as provided by law, religion, political convictions, language, ethnic or national origin, social condition, a handicap or the use of any means to palliate a handicap."

"Discrimination exists where such a distinction, exclusion or preference has the effect of nullifying or impairing such right."

Section 20

"A distinction, exclusion or preference based on the aptitudes or qualifications required for an employment, or justified by the charitable, philanthropic, religious, political or educational nature of a non-profit institution or of an institution devoted exclusively to the well being of an ethnic group, is deemed non-discriminatory."

(Not Proclaimed)

"[Similarly, under an insurance or pension contract, a social benefits plan or retirement, pension or insurance plan, or under a public pension or public insurance plan, a distinction, exclusion or preference based on risk determining factors or actuarial data fixed by regulation is deemed non-discriminatory]."

Section 90 (Prior Legislation And Still In Force)

"Sections 11, 13, 16, 17 and 19 of this Charter do not apply to pension plans, retirement plans, life insurance plans or any other plan or scheme of social benefits unless the discrimination is founded on race, color, religion, political convictions, language, ethnic or national origin or social condition."

In 1982, the Charter was amended substantially and Section 90 was to be repealed and replaced by Section 20, second paragraph. This provision has not been proclaimed and Section 90 is still in force with respect to discrimination in insurance contracts.

MINIMUM BIAS WITH GENERALIZED LINEAR MODELS

ROBERT L. BROWN

Abstract

The paper "Insurance Rates with Minimum Bias" by Robert A. Bailey [3] presents a methodology which is used by a large number of Canadian casualty actuaries to determine class and driving record differentials. In his paper, Bailey outlines four methods (two directly and two by reference to a previous paper by Bailey and Simon). No presentation has ever been made of an analysis of the applicability of these methods on Canadian data. Also, no attempt has been made within the Casualty Actuarial Society literature to augment Bailey's discussion using other statistical approaches now familiar to members of the Society.

This paper analyzes the four Bailey methodologies using Canadian data and then introduces five models using a modern statistical approach. (It should be noted that one of these statistical models turns out to be a reproduction of one of Bailey's models.)

The paper then gives a brief study of generalized linear models followed by an explanation of one possible way of mathematically modeling the specified Canadian data to the given models on the computer using a statistical software package called GLIM (Generalized Linear Interactive Modeling).

1. BACKGROUND

The concept of minimum bias was first introduced to insurance as a means of setting fair rates for groups of exposure units that could be classed in several different ways.

In his paper, "Insurance Rates with Minimum Bias" [3], Robert Bailey expresses the problem most eloquently:

"Although we may get a more reliable indicated adjustment for brick dwellings by combining all brick classes, and a more reliable indicated adjustment for small dwellings by combining all small dwelling classes, we cannot be so confident that the adjustment for brick dwellings and the adjustment for small dwellings will combine to produce the proper net adjustment for small brick dwellings. The data for small brick dwellings may be insufficient to be fully reliable but it will always provide some information. So we should look at it and take it into consideration. We should try to use a ratemaking system which, instead of producing each set of adjustments successively one after another, produces all sets of adjustments simultaneously. In this way the adjustments for brick dwellings and for small dwellings will both reflect the indication of small brick dwellings as well as the total for brick dwellings and the total for small dwellings. Such a system will produce a better result than a system which ignores the data in each subdivision."

In this 1963 paper, Bailey was actually expanding on work first presented in his 1960 paper, "Two Studies in Automobile Insurance Ratemaking," coauthored with LeRoy J. Simon [4].

In their 1960 paper, Bailey and Simon laid out four criteria for an acceptable set of relativities:

1. It should reproduce the experience for each class and merit rating (*driving record*) class and also the overall experience, i.e., be *balanced* for each class and in total.
2. It should reflect the relative *credibility* of the various groups involved.
3. It should provide a minimal amount of *departure* from the raw data for the maximum number of people.
4. It should produce a rate for each subgroup of risks which is close enough to the experience so that the differences could reasonably be caused by *chance*.

Using these criteria, the authors introduced four models (two multiplicative and two additive) that have proven very popular with actuaries.

Since the 1960 paper dealt with the same two variables for auto ratemaking (class and driving record) as we analyze in this paper, we will present Bailey's historical formulae as they would exist for two parameters.

Let r_{ij} be the factor from the actual experience that indicates losses for the n_{ij} risks that can be characterized by parameters x_i and y_j. Thus, for example, r_{ij} could be the average loss cost for cell (i,j) corresponding to class i and driving record j.

In the 1963 paper, Bailey introduces the multiplicative model whereby:

$$x_i = \frac{\sum_j n_{ij} r_{ij}}{\sum_j n_{ij} y_j} \tag{1.1}$$

and similarly for y_j $\left(\text{i.e., } y_j = \frac{\sum_i n_{ij} r_{ij}}{\sum_i n_{ij} x_i}\right)$.

We will refer to this later as Model 1.

He also introduces an additive model whereby:

$$x_i = \sum_j \frac{n_{ij}(r_{ij} - y_j)}{\sum_j n_{ij}} \tag{1.2}$$

and similarly for y_j. We will refer to this as Model 2.

From the 1960 paper, we also have two models, one multiplicative and one additive. For the multiplicative model, which we will refer to as Model 3, we have:

$$x_i = \left[\frac{\sum_j \frac{n_{ij} r_{ij}^2}{y_j}}{\sum_j n_{ij} y_j}\right]^{1/2} \tag{1.3}$$

and similarly for y_j.

For the additive model, which we will refer to as Model 4, we have:

$$\Delta x_i = \frac{\sum_j n_{ij}\left(\frac{r_{ij}}{x_i + y_j}\right)^2 - \sum_j n_{ij}}{2 \sum_j n_{ij}\left(\frac{r_{ij}}{x_i + y_j}\right)^2\left(\frac{1}{x_i + y_j}\right)}. \tag{1.4}$$

Finally, in the 1963 paper, Bailey introduces two tests that can be used to evaluate the appropriateness of the models. They are the chi-squared statistic and the absolute value statistic. For a multiplicative model, the respective formulae are:

$$\text{Absolute Value} = \frac{\sum_{ij} n_{ij} \left| r_{ij} - x_i y_j \right|}{\sum_{ij} n_{ij} r_{ij}} \tag{1.5}$$

$$\text{Chi-Squared} = \sum_{ij} \frac{n_{ij}(r_{ij} - x_i y_j)^2}{x_i y_j} \tag{1.6}$$

For an additive model, the respective formulae are:

$$\text{Absolute Value} = \frac{\sum_{ij} n_{ij} \left| r_{ij} - x_i - y_j \right|}{\sum_{ij} n_{ij} r_{ij}} \tag{1.7}$$

$$\text{Chi-Squared} = \sum_{ij} \frac{n_{ij}(r_{ij} - x_i - y_j)^2}{x_i + y_j} \tag{1.8}$$

2. INTRODUCTION

The data used in this paper were collected by the Insurers Advisory Organization (IAO) from third party auto liability totals for Canada for the years 1981, 1982, and 1983. The data have been grouped by class and driving record and their differentials have been determined according to class ($x_1, x_2, \ldots, x_{13}$) and driving record ($y_1, y_2, \ldots, y_5$). The differentials satisfy the objective of minimizing the bias in the rates.

Two main types of rate models are examined in the paper:

1) The multiplicative model; and
2) The additive model.

Under the multiplicative model, a driver in class i with driving record j will pay the rate

$$(BR_m) \times x_i y_j.$$

Under the additive model the same driver would pay the rate

$$(BR_a) + x_i + y_j,$$

where BR is a base rate and BR, x_i, and y_j vary by the model applied. Thus it can be seen that an entire rate manual can be constructed from 13+5 numbers. The only constraints placed on these 18 numbers are:

1) $\sum_{j=1}^{5} \sum_{i=1}^{13} n_{ij} f(x_i, y_j) =$ total loss dollars,

   where $f(x_i, y_j)$ is the premium that a class i driver with driving record j would pay; and
2) Each of the 65 premiums must be as "fair" as possible.

It is this second constraint that leads to the idea of minimum bias.

Robert Bailey introduced two different bias functions in his paper, "Insurance Rates with Minimum Bias." Each is a function of the new premium $f(x_i, y_j)$ and the expected unit loss costs which were written as r_{ij}. The two functions he introduced corresponded to the two different ratemaking models in use, the multiplicative and additive models. In each case the differential x_i or y_j was the one which minimized the total average difference in each class and driving record.

The average difference for class i for the multiplicative model is:

$$\frac{\sum_{j=1}^{5} n_{ij}(r_{ij} - f(x_i, y_j))}{\sum_{j=1}^{5} n_{ij} r_{ij}} . \tag{2.1}$$

Setting this equal to zero gives

$$\sum_{j=1}^{5} n_{ij}(r_{ij} - f(x_i, y_j)) = 0, \tag{2.2}$$

which implies

$$\sum_{j=1}^{5} n_{ij}r_{ij} = x_i \sum_{j=1}^{5} n_{ij}y_j. \tag{2.3}$$

This, in turn, gives the recursive method for calculating the x_i's and y_j's that are described in the two aforementioned papers.

Similarly, under the additive model, the average difference for class i is:

$$\frac{\sum_{j=1}^{5} n_{ij}(r_{ij} - (x_i + y_j))}{\sum_{j=1}^{5} n_{ij}r_{ij}}. \tag{2.4}$$

Setting this equal to zero gives

$$\sum_{j=1}^{5} n_{ij}(r_{ij} - y_j) = x_i \sum_{j=1}^{5} n_{ij}. \tag{2.5}$$

From this comes the second of Bailey's recursions.

More generally, one needs only to define a bias function,

$$f(r_{ij}, n_{ij}, x_i, y_j),$$

and then minimize

$$\sum_{i=1}^{13} \sum_{j=1}^{5} f(r_{ij}, n_{ij}, x_i, y_j) \tag{2.6}$$

with respect to $(x_1, \ldots, x_{13})$ and $(y_1, \ldots, y_5)$.

Thus it can be seen that Bailey's concept is simply an exercise in statistical modeling.

The second method of ratemaking is actually an exercise in statistically modeling the expected values (the r_{ij}'s) and then solving for the x_i's and y_j's so as to maximize the likelihood of the r_{ij}'s being generated by the model.

Thus if one assumes that the r_{ij}'s are independent observations from a random variable with distribution function

$$f^*(z, f(x_i, y_j)),$$

then the likelihood function becomes

$$\prod_{j=1}^{5} \prod_{i=1}^{13} \{f^*(r_{ij}, f(x_i, y_j))\}^{n_{ij}} = L. \tag{2.7}$$

Thus the model chosen is the one which maximizes L with respect to the x_i's and y_j's with the distribution f^* and the rate form $f(x_i,y_j)$ having already been chosen. More conveniently, one might choose to maximize the log likelihood function

$$\ln \{L\} = \sum_{j=1}^{5} \sum_{i=1}^{13} n_{ij} \ln \{f^*(r_{ij}, f(x_i,y_j))\} \tag{2.8}$$

so that

$$n_{ij} \ln \{f^*(r_{ij}, f(x_i, y_i))\} \tag{2.9}$$

takes the place of

$$f(r_{ij}, n_{ij}, x_i, y_j) \tag{2.10}$$

as the bias function, and we find the maximum value instead of the minimum.

From this analogy a system of 18 equations can be derived, all of which must be satisfied by the x's and y's.

$$\frac{\partial}{\partial x_i} \ln \{L\} = \sum_{j=1}^{5} \sum_{i=1}^{13} \frac{n_{ij} f_1(x_i,y_i)\, f_2^*\,(r_{ij}, f(x_i,y_i))}{f^*(r_{ij}, f(x_i,y_i))}\,, \tag{2.11}$$

and similarly for y_i.

Thus the maximum of $\ln \{L\}$ gives

$$\frac{\partial}{\partial x_i} \ln \{L\} = 0\,, \quad i=1, 2, \ldots, 13, \text{ and} \tag{2.12}$$

$$\frac{\partial}{\partial y_i} \ln \{L\} = 0\,, \quad j=1, 2, \ldots, 5. \tag{2.13}$$

From the nature of the expression of $\frac{\partial}{\partial x_i}\ln\{L\}$, it is easy to deduce that solving the 18 equations analytically is probably impossible and that some iterative method must be employed.

This paper explains how generalized linear models can be used to solve this problem.

3. THE LOSS COST APPROACH

The loss cost is defined as the incurred losses divided by the exposure units. There is a loss cost for each class and driving record combination which produces a matrix of loss costs. One's class is defined according to age, sex, marital status, and use of car (see Appendix A). Driving record is defined as the number of years of claim-free experience. For example, driving record 5 is defined as 5 years of claim-free experience. Statistics are available for driving record 0, 1, 2, 3, and 5.

By law, premium and claim statistics are collected by the Insurance Bureau of Canada in the seven provinces not operating under a government monopoly (the latter three being British Columbia, Saskatchewan, and Manitoba). The Province of Quebec has a government-administered no-fault system for bodily injury liability claims, so that the statistics published for class and driving record are usually analyzed for the six remaining provinces in total and for the last three policy years in total. This allows enough data for credibility. Many Canadian actuaries then derive pricing differentials for the two parameters, class and driving record, using a methodology consistent with that presented in Bailey's paper. The formulae from Bailey's 1963 *PCAS* paper have been reduced to two variables, x and y, representing class and driving record, the two parameters of interest. Note that the order of the variables is irrelevant.

The 13×5 matrix of loss costs becomes the parameter r_{ij} in all the formulae. The variable n_{ij} is the number of cars or exposure units. The computer model is then solved for the x_i and y_j differentials using the matrix of calculated loss costs as the r_{ij} and the exposure units as the n_{ij}.

Class 02 and driving record 3 define the base class and base driving records respectively. For multiplicative models, their differentials are each set to 1. In additive models, the base differentials are each set to

0. All differentials are normalized with the base differential. (See Appendix B for non-normalized and normalized differentials.)

The net premiums are calculated next, such that the total net premium income would equal the total dollars of loss. The premium in cell 023 (Class 02, Driving Record 3) is the base rate (BR). Using the formulae previously derived in the introduction,

$$BR_m \sum_{j=1}^{5} \sum_{i=1}^{13} n_{ij} x_i y_j = \text{incurred loss, for multiplicative model,} \quad (3.1)$$

and

$$\sum_{j=1}^{5} \sum_{i=1}^{13} (BR_a + x_i + y_j) n_{ij} = \text{incurred loss, for additive model,} \quad (3.2)$$

are the respective net premiums.

Bailey's 1963 paper introduces the following two models:

Model 1: Bailey's Minimum Bias Multiplicative Model

$$x_i = \frac{\sum_j n_{ij} r_{ij}}{\sum_j n_{ij} y_j} \quad (3.3)$$

and similarly for y_j. (Note that this model can be derived using maximum likelihood estimation for a Poisson distribution within a loglinear model, as shown later in the Statistical Approach section of the paper.)

Model 2: Bailey's Minimum Bias Additive Model

$$x_i = \frac{\sum_j n_{ij}(r_{ij} - y_j)}{\sum_j n_{ij}} \quad (3.4)$$

and similarly for y_j.

The 1960 paper by Bailey and Simon introduced two other methods, namely:

Model 3: Bailey and Simon—Multiplicative

$$x_i = \left[\frac{\sum_j \frac{n_{ij} r_{ij}^2}{y_j}}{\sum_j n_{ij} y_j} \right]^{1/2} \tag{3.5}$$

and similarly for y_j.

Model 4: Bailey and Simon—Additive

$$\Delta x_i = \frac{\sum_j n_{ij} \left(\frac{r_{ij}}{x_i + y_j} \right)^2 - \sum_j n_{ij}}{2 \sum_j n_{ij} \left(\frac{r_{ij}}{x_i + y_j} \right)^2 \left(\frac{1}{x_i + y_j} \right)} \cdot \tag{3.6}$$

and similarly for y_j.

For Model 4, different starting values of x converge to different non-normalized class and driving record relativities, but $x_i + y_j$ and the normalized class and driving record differentials are independent of the starting values of x and y.

These four classic methodologies were tested on the Canadian data split by rural and urban territories. As explained earlier, Bailey introduced two tests in his 1963 paper that can be used to evaluate the appropriateness of a model: the chi-squared statistic and the absolute value statistic. The chi-squared statistic and absolute value statistic for the first four models are as follows:

	Model			
	1	2	3	4
Urban Territories:				
Chi-Squared	6,684,350	56,886,610	6,552,692	10,854,933
Absolute Value	.05145	.05773	.05178	.06226
Rural Territories:				
Chi-Squared	7,101,723	115,079,807	6,459,712	8,309,002
Absolute Value	.06621	.07042	.07651	.08372

Note: In Canada, geographic territories are split depending on whether they are predominantly urban or predominantly rural in nature, and separate class and driving record relativities are accordingly derived.

4. THE LOSS RATIO APPROACH

In the loss cost approach described on the previous pages, it is assumed implicitly that the distribution of all other parameters is completely homogeneous across class and driving record. One approach to correct or adjust for heterogeneity in the distribution of any parameters not being directly analyzed would be to use the loss ratio method in defining the r_{ij} matrix for the minimum bias calculation.

The minimum bias analysis can be done using a loss ratio approach as follows. Given the 13 class differentials and the 5 driving record differentials provided by the IAO (see Appendix B), a 13 × 5 matrix of "existing differentials" is calculated for all 65 cells (note that cell 023 will equal 1.00). The loss ratios for these 65 cells are then calculated (incurred losses/earned premiums). Each of these entries is then divided by the loss ratio calculated for cell 023. This matrix multiplied by the "existing differential" matrix gives the "indicated differential" matrix, which is used as the r_{ij} in the minimum bias calculation. Before using the generalized linear modeling technique, the first iteration must be performed manually to convert the matrix of differentials into a matrix of rates.

This calculation can be done as follows:
Let BR = the base rate, where

$$BR \sum_{ij} n_{ij} r_{ij} = \text{incurred losses.} \tag{4.1}$$

Then,

$$BR \cdot r_{ij} = \text{the new } r_{ij} \text{ matrix.} \tag{4.2}$$

Generalized linear modeling can now be used in exactly the same manner as was used for the loss cost approach.

Using the loss ratio approach to calculate the r_{ij}'s, the results using the criteria outlined in Bailey's paper are as follows:

	Model			
	1	2	3	4
Urban Territories:				
Chi-Squared	6,689,226	108,368,525	6,553,952	10,828,905
Absolute Value	.05029	.05623	.05060	.06188
Rural Territories:				
Chi-Squared	5,741,837	129,531,557	5,224,229	6,754,593
Absolute Value	.06587	.07018	.07600	.08197

Using Bailey's criteria, the loss ratio method appears to provide slightly better results than the loss cost method in general.

5. A STATISTICAL APPROACH

The following is a summary of the results obtained from the minimum bias analysis for several possible models that can be derived using modern statistical formulae.

a) Maximum Likelihood Methods

If the losses for cell (i,j) are modeled by $L_{ij} = n_{ij}r_{ij}$, then $E(L_{ij}) = n_{ij}E(r_{ij})$.

Model 5: Exponential—Multiplicative

$L_{ij} \sim$ exponential and

$$E(L_{ij}) = n_{ij}E(r_{ij}) = n_{ij}x_iy_j \tag{5.1}$$

$$f(L_{ij}) = f(n_{ij}, r_{ij}) = \frac{1}{n_{ij}x_iy_j}\exp\left\{-\left(\frac{r_{ij}n_{ij}}{n_{ij}x_iy_j}\right)\right\}$$

$$= \frac{1}{n_{ij}x_iy_j}\exp\left\{-\left(\frac{r_{ij}}{x_iy_j}\right)\right\}. \tag{5.2}$$

The log likelihood function is

$$l = -\sum_i \sum_j \left(\ln n_{ij} + \ln x_i + \ln y_i + \frac{r_{ij}}{x_i y_j} \right) \tag{5.3}$$

$$\frac{\partial l}{\partial x_k} = 0 \Rightarrow \frac{1}{x_k} \sum_j 1 - \frac{1}{x_k^2} \sum_j \frac{r_{kj}}{y_j} = 0 \tag{5.4}$$

$$\Rightarrow x_i = \frac{\sum_j \frac{r_{ij}}{y_j}}{\sum_j 1} \tag{5.5}$$

and similarly for y_j.

Model 6: Normal—Multiplicative

$$L_{ij} \sim N(\mu_{ij}, \sigma^2) \tag{5.6}$$

$$\mu_{ij} = n_{ij} x_i y_j \tag{5.7}$$

$$f(L_{ij}) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{ -\frac{1}{2\sigma^2} (L_{ij} - \mu_{ij})^2 \right\}$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{ -\frac{1}{2\sigma^2} (r_{ij} n_{ij} - x_i y_j n_{ij})^2 \right\}. \tag{5.8}$$

The log likelihood function is

$$l = \sum_i \sum_j \{-\ln \sigma (\sqrt{2\pi})\} - \frac{1}{2\sigma^2} \sum_i \sum_j n_{ij}^2 (r_{ij} - x_i y_j)^2 \tag{5.9}$$

$$\frac{\partial l}{\partial x_k} = 0 \Rightarrow \sum_j n_{kj}^2 y_j (r_{kj} - x_k y_j) = 0 \tag{5.10}$$

$$\Rightarrow x_i = \frac{\sum_j n_{ij}^2 r_{ij} y_j}{\sum_j n_{ij}^2 y_j^2} \tag{5.11}$$

and similarly for y_j.

Model 7: Normal—Additive

$$L_{ij} \sim N(\mu_{ij}, \sigma^2) \tag{5.12}$$

$$\mu_{ij} = (x_i + y_j) n_{ij} \tag{5.13}$$

The log likelihood function is

$$l = \sum_i \sum_j \{-\ln(\sigma\sqrt{2\pi})\} - \frac{1}{2\sigma^2} \sum_i \sum_j n_{ij}^2 (r_{ij} - x_i - y_j)^2 \tag{5.14}$$

$$\frac{\partial l}{\partial x_k} = 0 \Rightarrow \sum_j n_{kj}^2 (r_{kj} - x_k - y_j) = 0 \tag{5.15}$$

$$\Rightarrow x_i = \frac{\sum_j n_{ij}^2 (r_{ij} - y_j)}{\sum_j n_{ij}^2} \tag{5.16}$$

and similarly for y_j.

Model 8: Poisson—Multiplicative (Bailey's Model 1)

We will now show that by using maximum likelihood estimation, a Poisson—Multiplicative model will reproduce Bailey's Model 1.

$$f(L_{ij}) = \frac{e^{-x_i y_j n_{ij}} (x_i y_j n_{ij})^{n_{ij} r_{ij}}}{(n_{ij} r_{ij})!} . \tag{5.17}$$

The log likelihood function is

$$l = \sum_i \sum_j \{(n_{ij} r_{ij} [\ln x_i + \ln y_j + \ln n_{ij}] - x_i y_j n_{ij} - \ln(n_{ij} r_{ij})!\} \tag{5.18}$$

$$\frac{\partial l}{\partial x_k} = 0 \Rightarrow \sum_j \left\{ \frac{n_{kj} r_{kj}}{x_k} - n_{kj} y_j \right\} = 0 \tag{5.19}$$

$$\Rightarrow x_i = \frac{\sum_j n_{ij} r_{ij}}{\sum_j n_{ij} y_j} \tag{5.20}$$

which is Bailey's Model 1.

b) Least Squares Estimate Methods
Model 9 LSE—Multiplicative

$$SS = \sum_i \sum_j n_{ij}(r_{ij} - x_i y_j)^2 \tag{5.21}$$

$$\frac{\partial SS}{\partial x_k} = 0 \Rightarrow \sum_j n_{kj} y_j (r_{kj} - x_k y_j) = 0 \tag{5.22}$$

$$\Rightarrow x_i = \frac{\sum_j n_{ij} r_{ij} y_j}{\sum_j n_{ij} y_j^2} \tag{5.23}$$

and similarly for y_j.

These models were applied to the Canadian data and the resulting chi-squared statistic and absolute value statistic are as follows:

	Model			
	5	6	7	9
Urban Territories:				
Chi-Squared	13,059,115	7,023,572	14,040,792	7,009,249
Absolute Value	.12810	.04175	.07145	.05621
Rural Territories:				
Chi-Squared	11,877,604	9,210,338	9,776,232	7,623,831
Absolute Value	.18830	.05155	.07633	.07757

6. GENERALIZED LINEAR MODELS

A generalized linear model is a probability distribution for an observed random variable vector Y given a set of explanatory vectors x_1, $x_2, \ldots, x_p$ which satisfy the following three conditions:

1) The distribution of each y_i of the vector Y ($i = 1, 2, \ldots, n$), given $x_{i1}, \ldots, x_{ip}$, belongs to an exponential family. The probability density function (pdf) for each y is of the form

$$\exp\left[\frac{y_i\theta_i - b(\theta_i)}{a(\phi)} + c(y_i, \phi)\right] \tag{6.1}$$

where θ, known as the *canonical* parameter, is a function of $x_{i1}, \ldots, x_{ip}$ that involves known parameters; and ϕ, known as the *dispersion* parameter, is constant for all i.

It can be shown that

$$\mu_i = E(y_i) = b'(\theta_i) \tag{6.2}$$

and

$$\mathrm{var}(Y_i) = b''(\theta_i)a(\phi). \tag{6.3}$$

2) The explanatory variables enter only as a linear sum of their effects, the linear predictor, η; hence, for each i,

$$\eta_i = \sum_{j=1}^{p} x_{ij}\beta_j = X\beta$$

where the β_j effects are the linear parameters to be estimated.

3) The expected value of each observation can be expressed as some function of its linear predictor, $\eta_i = g(\mu_i)$, where $g(.)$ is a monotonic and differentiable function known as the link function.

The link function is a transformation between the linear function and the mean. Those readers not familiar with generalized linear models are encouraged to read references [8] and [13].

7. GLIM

With the previous references, it would be possible for an actuary to program the technique known as generalized linear models with the information already provided. However, as one might expect, packages do exist to do this type of analysis. One of the best known and most complete and flexible is called the Generalized Linear Interactive Modeling (GLIM) system. It was designed by the Royal Statistical Society and is available at the address given at the end of the Bibliography. From this point on, GLIM will refer to the Royal Statistical Society program.

GLIM is a computerized statistical package which mathematically models a random quantity (dependent or response variables) and takes into account any related or covariate information (independent or explanatory variables). The model that is produced is the one that maximizes the log likelihood function over the given data set. This paper treats the pure premiums as the dependent variables and attempts to relate the corresponding class and driving differentials to these pure premiums.

8. BASIC COMPONENTS OF A GLIM PROGRAM

The general format of any GLIM program to be run in a batch environment using GLIM consists of a data definition section at the beginning, followed by the actual process or the body of the program, which performs the model fitting. The data definition directives describe the structure of the initial input data matrix, establish the variable labels for the input data, and read the data into the program work area. The GLIM commands which make up the body of the program provide the instructions for GLIM to do the statistical modeling and analysis. (Further details are available from the author.)

9. SUMMARY OF RESULTS

For this paper, a total of 12 models were fit using GLIM with a loss cost approach. Of these 12 models, 6 were run using urban data and 6 with rural data. Within each of the 2 sets of data, the output was further divided to include 3 probability distributions: Poisson, Gamma, and Normal. Finally, each distribution was run using a multiplicative approach and an additive approach.

The interesting statistics from the output are:

1) Deviance: This is the residual variance, or the variability not explained by the model. In particular it is twice the drop in log likelihood

between a model which fits the data perfectly (one parameter per observation) and the model actually fit. (This is similar to Bailey's chi-squared statistic.)
2) Degrees of Freedom
3) Fitted Values: This column gives the new premium rates as fitted by the model.
4) Estimate and Standard Error: These can be used to generate the differentials.

The deviances produced by GLIM for the 12 models are summarized below. Note that for two of the models (Poisson and Exponential—Additive using rural data) the deviances have been entered as "***" indicating that the fitted mean is out of range for the error distribution (i.e., an inappropriate model).

Deviance Under Loss Cost Approach

Distribution	Link	Model	Urban	Rural
Poisson	log	Multiplicative	6,596,200	5,295,126
Gamma*	log	Multiplicative	18,373	32,614
Normal	log	Multiplicative	3,413,386,183	1,518,522,878
Poisson	identity	Additive	10,422,477	***
Gamma*	identity	Additive	37,425	***
Normal	identity	Additive	4,084,117,310	1,902,075,827

*It should be noted that a Gamma distribution is of the form:

$$f(x;\alpha,\lambda) = \frac{\lambda^{\alpha} x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)}$$

and an Exponential distribution is of the form:

$$f(x;\lambda) = \lambda e^{-\lambda x}.$$

Thus an Exponential distribution is a Gamma distribution with the parameter $\alpha = 1$.

10. ADVANTAGES/DISADVANTAGES OF GLIM

Advantages

Once the user is familiar with the GLIM package, only a minimal amount of computer knowledge is necessary to utilize GLIM as an automobile ratemaking tool.

Once the GLIM system is in working order, it is easy to test different statistical models, as only minor changes must be made to the GLIM command file.

GLIM can reproduce results obtained by Bailey's multiplicative model because a GLIM Poisson log-linear model (under maximum likelihood estimation) is equivalent to Bailey's multiplicative model.

Disadvantages

To this point we have not been able to use GLIM to reproduce results obtained by Bailey's additive model, the Bailey and Simon additive model, or the Bailey and Simon multiplicative model. However, there should not be any need to produce premium rates using these models, since Bailey's multiplicative model (GLIM Poisson log-linear model) produces premium rates which provide a better fit to the data.

GLIM is a difficult package with which to become familiar. However, becoming familiar with GLIM should not be a problem, as this report answers most of the questions new GLIM users might ask.

Conclusion

In conclusion, it is safe to say that the advantages of GLIM outweigh the disadvantages, and, therefore, the GLIM statistical package could be used to determine net premium rates for automobile insurance. Further, once one is familiar with GLIM, many other property-casualty applications become apparent (e.g., loss reserving models).

REFERENCES

[1] Ajne, "A Note on the Multiplicative Rate-Making Model," *ASTIN Bulletin*, 8, 1974, p. 144.

[2] Andersen, "Multiplicative Poisson Models with Unequal Cell Rates," *Scandanavian Journal of Statistics*, 4, 1977, p. 153.

[3] R. A. Bailey, "Insurance Rates with Minimum Bias," *PCAS* L, 1963, p. 4.

[4] R. A. Bailey and L. Simon, "Two Studies in Automobile Insurance Ratemaking," *ASTIN Bulletin*, 1, 1960, p. 192.

[5] L. A. Baxter, S. M. Coutts, and G. A. F. Ross, "Applications of Linear Models in Motor Insurance," *Transactions of the 21st International Congress of Actuaries*, 2, 1979, p. 11.

[6] Peter Berg, "On the Loglinear Poisson and Gamma Model," *ASTIN Bulletin*, 11, 1980, p. 35.

[7] S. M. Coutts, "Motor Insurance Rating, An Actuarial Approach," *Journal of the Institute of Actuaries*, 111, 1984, p. 87.

[8] A. Dobson, *An Introduction to Statistical Modeling*, Chapman and Hall, 1983.

[9] J. van Eeghen, E. K. Greup, and J. A. Nijssen, "Rate Making," *Surveys of Actuarial Studies*, 2, Nationale-Nederlanden N.V., 1983.

[10] T. Grimes, "Claim Frequency Analysis in Motor Insurance," *Journal of the Institute of Actuaries' Students' Society*, 19, 1971, p. 147.

[11] P. D. Johnson and G. B. Hey, "Statistical Studies in Motor Insurance," *Journal of the Institute of Actuaries*, 97, 1971, p. 199.

[12] J. Jung, "On Automobile Insurance Ratemaking," *ASTIN Bulletin*, 5, 1968, p. 41.

[13] P. McCullagh and J. Nelder, *Generalized Linear Models*, Chapman and Hall, 1983.

[14] D. H. Reid, "The Limits of Resolution of a Statistical Rating System," *Transactions of the 22nd International Congress of Actuaries*, 4, 1984, p. 281.

Information regarding GLIM can be obtained from:

The Numerical Algorithms Group Ltd. (NAG)
Mayfield House
256 Bonbury Road
Oxford OX2 7DE
United Kingdom

APPENDIX A

PRIVATE PASSENGER AUTOMOBILE CLASSIFICATION

PLEASURE—NO MALES UNDER 25, NO FEMALE PRINCIPAL OPERATORS UNDER 25:

Class 01: No driving to work; annual mileage of 10,000 or less; 2 or fewer operators per automobile who have held valid operators' licenses for at least the past 3 years.

Class 02: Driving to work 10 miles or less one way permitted; unlimited annual mileage; 2 or fewer operators per automobile.

Class 03: Driving to work over 10 miles permitted; unmarried female occasional drivers under 25 may drive.

PLEASURE OR BUSINESS:

Class 06: Occasional male driver use—male under 25. (Note—the principal operator insures the automobile for use by all other drivers under Classes 01, 02, 03, or 07.)

Class 07: Business primarily; no male drivers under age 25.

PRINCIPAL OPERATORS UNDER 25 YEARS OF AGE:

MARRIED MALE:

Class 08: Ages 20 and under.

Class 09: Ages 21, 22, 23, or 24.

UNMARRIED MALE:

Class 10: Ages 18 and under.

Class 11: Ages 19 and 20.

Class 12: Ages 21 and 22.

Class 13: Ages 23 and 24.

FEMALES—MARRIED OR UNMARRIED:

Class 18: Ages 20 and under.

Class 19: Ages 21, 22, 23, or 24.

APPENDIX B
PAGE 1

EXPOSURES
URBAN

	Driving Record				
Class	5	3	2	1	0
1	1,032,596	69,952	7,176	6,531	7,531
2	908,551	92,324	12,630	11,138	8,376
3	171,145	22,770	2,333	2,275	2,115
6	22,509	67,929	7,527	8,865	4,315
7	101,962	13,586	1,177	1,214	3,025
8	238	1,471	118	119	57
9	22,395	7,768	890	682	397
10	439	6,876	1,448	1,096	516
11	2,406	17,515	1,421	1,112	874
12	25,362	16,827	1,756	1,420	950
13	37,145	11,345	1,201	981	648
18	2,374	17,957	2,447	1,738	900
19	50,032	18,679	2,212	1,669	905

RURAL

Class	Driving Record 5	3	2	1	0
1	588,554	34,156	3,137	2,674	2,853
2	390,669	32,182	4,398	3,768	2,520
3	72,173	9,898	764	732	651
6	6,489	31,307	5,587	6,441	1,902
7	30,164	3,073	231	220	434
8	125	1,239	133	95	45
9	15,172	5,554	578	412	290
10	104	3,473	1,028	700	240
11	552	9,296	853	647	428
12	10,957	6,982	771	589	380
13	14,504	3,922	482	370	233
18	722	9,028	1,447	1,077	400
19	20,085	7,739	979	753	355

APPENDIX B
PAGE 2

INCURRED LOSSES
URBAN

	Driving Record				
Class	5	3	2	1	0
1	$160,542,268	$18,776,760	$1,631,254	$1,497,881	$4,006,765
2	155,275,831	28,026,927	4,960,356	5,161,099	5,966,125
3	31,800,758	6,380,889	746,837	753,405	1,687,178
6	2,265,541	6,965,839	1,106,344	1,747,742	722,654
7	20,952,908	6,983,349	523,905	363,916	1,029,192
8	27,306	1,198,937	52,608	192,758	24,939
9	5,457,632	2,521,186	487,892	171,022	599,265
10	177,911	5,743,938	1,123,514	830,349	361,252
11	650,084	9,317,804	748,966	1,376,913	1,236,611
12	8,128,211	8,259,076	1,948,990	562,474	732,563
13	8,691,528	3,987,223	456,099	602,538	441,043
18	613,659	5,158,436	771,092	649,862	476,125
19	9,869,507	4,620,205	1,104,166	494,677	322,707

Rural

Class	5	3	2	1	0
1	$72,818,071	$6,569,811	$580,796	$671,249	$693,731
2	46,799,691	8,000,268	692,669	1,444,989	725,973
3	10,979,651	2,609,918	184,710	127,998	121,386
6	1,439,679	2,052,548	392,274	822,024	83,894
7	3,944,980	847,588	205,711	15,873	348,641
8	65,162	331,414	14,595	77,585	43,629
9	2,313,951	1,407,547	151,523	459,051	38,430
10	23,868	1,886,902	1,005,682	895,735	280,150
11	119,059	3,733,793	280,868	263,630	168,825
12	3,084,451	2,820,342	331,082	174,409	514,735
13	3,232,096	1,191,885	196,718	146,137	48,247
18	87,123	2,876,800	312,692	201,297	264,561
19	2,588,858	1,490,676	214,363	248,212	59,574

APPENDIX B
PAGE 3

INDICATED LOSS COSTS (r_{ij} FOR LOSS COST METHODS)*
URBAN

	Driving Record				
Class	5	3	2	1	0
1	$155.474	$268.423	$227.321	$229.349	$532.036
2	170.905	303.571	392.744	463.378	712.288
3	185.812	280.232	320.119	331.167	797.720
6	100.650	102.546	146.983	197.151	167.475
7	205.497	514.011	445.119	299.766	340.229
8	114.731	815.049	445.831	1,619.815	437.526
9	243.699	324.561	548.193	250.765	1,509.484
10	405.264	835.360	775.907	757.618	700.101
11	270.193	531.990	527.070	1,238.231	1,414.887
12	320.488	490.823	1,109.903	396.108	771.119
13	233.989	351.452	379.755	614.208	680.622
18	258.492	287.266	315.117	373.914	529.028
19	197.264	247.348	499.171	296.391	356.582

RURAL

Class	Driving Record 5	3	2	1	0
1	$123.724	$192.347	$185.144	$251.028	$243.158
2	119.794	248.594	157.496	383.490	288.085
3	152.130	263.681	241.767	174.861	186.461
6	221.865	65.562	70.212	127.624	44.108
7	130.784	275.818	890.524	72.150	803.320
8	521.296	267.485	109.737	816.684	969.533
9	152.515	253.429	262.151	1,114.201	132.517
10	229.500	543.306	978.290	1,279.621	1,167.292
11	215.687	401.656	329.271	407.465	394.451
12	281.505	403.945	429.419	296.110	1,354.566
13	222.842	303.897	408.129	394.965	207.069
18	120.669	318.653	216.097	186.905	661.402
19	128.895	192.619	218.961	329.631	167.814

*(Indicated Loss Costs)$_{ij}$ = (Incurred Losses)$_{ij}$ / (Exposures)$_{ij}$

APPENDIX B
PAGE 4

EARNED PREMIUMS
URBAN

	Driving Record				
Class	5	3	2	1	0
1	$235,547,350	$27,864,747	$3,400,585	$3,481,338	$5,389,670
2	243,651,153	43,214,401	7,044,434	7,003,389	7,036,261
3	46,094,341	10,671,357	1,309,985	1,435,101	1,780,991
6	3,067,070	15,693,178	2,082,685	2,752,201	1,780,886
7	31,782,258	7,443,208	765,162	882,174	3,025,876
8	87,819	905,370	87,173	95,278	60,002
9	7,085,253	4,253,265	579,097	501,015	391,990
10	275,491	7,173,186	1,803,896	1,523,661	971,610
11	1,313,681	16,004,319	1,541,004	1,354,574	1,417,867
12	9,971,261	11,404,388	1,403,404	1,279,771	1,153,945
13	14,047,048	7,442,591	929,577	853,985	758,385
18	647,226	8,212,282	1,337,674	1,070,350	740,322
19	13,409,777	8,580,997	1,215,449	1,027,302	753,915

RURAL

Class	Driving Record 5	3	2	1	0
1	$101,062,451	$9,321,598	$1,043,233	$1,033,029	$1,279,751
2	72,046,127	9,420,894	1,566,185	1,560,793	1,211,134
3	13,679,991	2,995,335	279,122	316,566	324,724
6	658,589	5,064,497	1,085,059	1,460,394	500,790
7	6,972,259	1,146,006	104,627	115,386	265,936
8	33,728	532,462	68,025	56,717	31,244
9	3,519,880	2,032,396	255,435	214,240	172,875
10	59,135	3,111,955	1,111,589	873,427	344,456
11	261,452	7,042,735	776,994	688,728	526,422
12	4,200,804	4,204,779	560,158	495,889	369,578
13	4,442,756	1,886,737	281,409	249,126	181,789
18	151,636	3,041,185	581,410	500,843	215,521
19	3,836,080	2,333,954	355,640	317,760	173,441

APPENDIX B
PAGE 5

CLASS AND DRIVING RECORD RELATIVITIES
UNDERLYING THE IAO PREMIUMS

Class	Urban	Rural
1	.86	.94
2	1.00	1.00
3	1.00	1.05
6	.50	.55
7	1.12	1.24
8	1.37	1.48
9	1.20	1.26
10	2.31	3.13
11	2.02	2.65
12	1.48	2.09
13	1.42	1.67
18	1.00	1.16
19	1.00	1.04

Driving Record	Urban	Rural
5	.58	.63
3	1.00	1.00
2	1.20	1.22
1	1.35	1.42
0	1.80	1.63

A NONPARAMETRIC APPROACH TO EVALUATING REINSURERS' RELATIVE FINANCIAL STRENGTH

STEPHEN J. LUDWIG
ROBERT F. McAULEY

Abstract

There have been a number of past attempts aimed at using financial data of individual companies to produce predictive models of insurance company solvency. These models have come in two forms: parametric and nonparametric. For example, the NAIC, with its Insurance Regulatory Information System, is taking a nonparametric approach to this problem, while the AIA has used a parametric approach in producing its formula for assessing an insurer's financial strength. However, when used to evaluate a reinsurer's *financial strength, these two systems have several shortcomings. For example, these models were developed by analyzing a primary company data base, and it is not clear whether a model created for primary companies will be effective when applied to reinsurance companies. Additionally, the criteria against which the models measure a company's financial strength are fixed, and thus do not reflect each year's changing economic conditions. Since economic conditions alter the value that a ratio can have, this could be a serious defect.*

The model that is presented in this article uses properties of a ranking distribution. The Wilcoxon rank sum test is initially used to determine which financial ratios have historically discriminated between "strong" and "weak" companies. For those ratios that are selected as good discriminators, the test ranks are summed for each company. This statistic is then used as the measure of relative financial strength. Since each year is considered separately, it is assumed that economic conditions of that year will affect all the companies' ratios similarly. It is hoped that this procedure will self-adjust in response to these variable conditions, and provide a more accurate and consistent indicator of a reinsurer's relative financial strength.

As with all projects like this one, there was data entry, data verification, programming and analysis. Much of this was completed by Lisa Pouliot, and the authors appreciate her contributions to this project.

1. INTRODUCTION

Evaluating the solvency of an individual insurance market is at best a difficult task. Due to the underwriting practices prevalent during the industry's recent past, many companies are still feeling the effects on their bottom line. The reinsurance market has been especially hard hit, with numerous companies withdrawing from the market either voluntarily or by state order, thus causing a capacity shortage in certain areas of reinsurance. At the same time, as both direct and reinsurance rates have rebounded, new untested capacity has started to enter into the reinsurance arena. While solvency-tracking models have been in place for many years, there have been no models developed specifically for the reinsurance industry. In this paper we present a nonparametric model for ranking reinsurance companies according to their relative financial strength, and compare its results to the NAIC model which has been used in the past, but which was not specifically developed for reinsurers. It should be noted here that in formulating this model, our goal was *not* to produce something which would replace all existing solvency-tracking systems. Rather, our intent, much like the NAIC's, was to produce a straightforward method for quickly developing a ranking based on relative financial strength, with the results being used to highlight those companies for which a more extensive reivew of the financials is urgently needed.

2. HISTORY

As mentioned above, a number of models have been produced in the past, none of which specifically addressed reinsurers. These models can generally be split into two broad categories: (1) nonparametric, and (2) parametric. A brief review of three of these models follows.

NAIC Insurance Regulatory Information System—Nonparametric

Established over a decade ago, the Insurance Regulatory Information System (IRIS) tests consist of the following eleven ratios which provide a quick overview of a company's operations:

	Usual Range
1. Net Written Premium to Surplus	≤ 300%
2. Change in Net Written Premium	−33% to +33%
3. Surplus Aid to Surplus	< 25%
4. Two-Year Overall Operating Ratio	< 100%
5. Investment Yield	≥ 6%
6. Change in Surplus	−10% to +50%
7. Liabilities to Liquid Assets	< 105%
8. Agents' Balances to Surplus	< 40%
9. One-Year Reserve Development to Surplus	< 25%
10. Two-Year Reserve Development to Surplus	< 25%
11. Estimated Current Reserve Deficiency to Surplus	< 25%

Usual ranges have been established for each ratio, and any company which falls outside of these ranges for four or more of these tests is classified as a "priority" company.

In applying these eleven tests to reinsurance companies, several shortcomings in this model become apparent. First, since only one set of usual ranges has been established for the entire insurance industry, they may not be stringent enough to identify "priority" reinsurance companies. For example, while a Net-Premium-to-Surplus ratio of 3.0 may be fine for a direct company, it may not be proper for a reinsurance company. Further, the criteria for passing a particular test could be very dependent on the year. For example, the investment yield ratio may have a very changeable range, depending, in part, on the prime interest rate and current tax laws as well as other undetermined factors. Short of a complete study each year, there may be no way to determine the usual range by year. However, no yearly adjustments are currently made to these ranges.

In order to assess the adequacy of the NAIC model, a data base was established for eighty-four domestic companies which predominantly wrote a reinsurance book and also had net written premiums of at least $1 million per year over the 1980–84 period. For these eighty-four companies which comprise our domestic reinsurance "industry," the distribution by number of test failures is shown below:

Number Outside of Usual Range	1980	1981	1982	1983	1984	1985
0	33	41	31	19	5	1
1	29	23	28	22	11	10
2	12	11	17	18	16	13
3	7	4	5	15	13	14
4	0	1	3	8	14	19
5+	3	4	0	2	25	27

As is shown, the first point at which a significant number of "priority" companies are identified is when year-end 1983 data is available, which would be early 1984. This can hardly be described as an "early warning". It is safe to say that a reinsurance buyer, relying solely on NAIC IRIS test results as an authoritative statement regarding the financial strength of its reinsurers, could have easily made a number of costly mistakes in its choice of reinsurers. Moreover, unlike individual policyholders who have the state guaranty funds to fall back on in case of a bad insurance-buying decision, an insurance company which makes a mistake in its choice of reinsurance markets has no such safety net.

A. M. Best Ratings

A. M. Best Inc., founded in 1899, annually assigns ratings to several thousand domestic insurers. These ratings range from a high of A^+ (Excellent) to a low C^- (Fair). There are also a number of conditions under which Best will not assign a letter rating, for example: NA-3 (Insufficient Experience), NA-6 (Reinsured by Unrated Reinsurer), NA-7 (Below Minimum Standards), or NA-10 (Under State Supervision).

As described in their literature, Best reviews a number of financial ratios as a part of its analysis, with these ratios coming from the following general areas:

1. Profitability Tests
2. Leverage Tests
3. Liquidity Tests
4. Loss Reserve Tests
5. Cash Flow Tests

A listing of the individual tests is given in Exhibit 1.

While details of their rating assignment methodology are confidential, there is no question that A. M. Best ratings have historically been heavily relied upon as an aid in making both insurance and reinsurance buying decisions. However, as with the NAIC IRIS test results, the question which must be addressed is whether or not the Best ratings are appropriate for the reinsurance industry in general, and whether or not they provided an early warning against some of the "weak" reinsurers. Shown below is the historical distribution of companies by Best rating for the eighty-four companies in our domestic reinsurance industry data base:

A. M. Best Rating	1980	1981	1982	1983	1984	1985
A^+	37	37	32	26	7	4
A	12	18	31	37	37	28
A^-	-	-	-	-	-	-
B^+	7	5	2	5	14	19
B	2	2	3	3	1	2
B^-	-	-	-	-	-	-
C^+	1	-	-	1	-	1
C	-	-	-	-	-	-
C^-	-	-	-	-	-	-
NA-3	25	22	16	9	7	4
NA-6	-	-	-	-	-	6
NA-7	-	-	-	1	10	6
NA-10	-	-	-	-	-	1
Other	-	-	-	2	8	13
Total	84	84	84	84	84	84

Similar to the IRIS tests, the A. M. Best Ratings do not show a significant downward movement until 1984.

AIA Formula—Parametric

The AIA model, developed by Aetna Life and Casualty and completed in 1978, uses a mathematical formula to differentiate between solvent and insolvent companies. This formula is:

$$
\begin{aligned}
\text{Company Score} = \; & 19.00916 \\
& -.11305 \times (\text{Two-year Operating Ratio}) \\
& -.04106 \times (\text{Liabilities to Liquid Assets}) \\
& -.06742 \times (\text{Change in Surplus}) \\
& -.00335 \times (\text{Net Written Premium to Loss and Loss} \\
& \qquad\qquad \text{Adjustment Expense Reserves}) \\
& -.07314 \times (\text{Change in Liability Mix})
\end{aligned}
$$

A company's score produced by this formula is then compared to the following index of financial strength:

Score (S)	Index of Financial Strength
$2.2 \leq S$	10 Very Strong
$1.4 \leq S < 2.2$	9
$1.0 \leq S < 1.4$	8
$.5 \leq S < 1.0$	7
$0 \leq S < .5$	6
$-.5 \leq S < 0$	5
$-1.0 \leq S < -.5$	4
$-1.4 \leq S < -1.0$	3
$-2.2 \leq S < -1.4$	2
$S < -2.2$	1 Very Weak

Due to the unavailability of some of the necessary data, we did not test the AIA model's predictive power. However, it is reasonable to assume that since this model was derived from a general insurance industry base and not from a specific reinsurance industry base, its results, much like the NAIC model, would not provide the necessary "early warning."

The choice of a regression model for this type of analysis may not be appropriate. The uses and assumptions under which a model would operate need to be examined. Also it should not be overlooked that companies can influence their published statistics. Given that a regression model must rely on only a few financial ratios so that collinearity will not cause overspecification, slight alterations of a particular ratio may have dramatic impacts on the final indication.

3. CHOICE OF MODEL

At the start, there was a good deal of support for selecting an AIA formula type approach. This parametric method was tested and found to be a poor predictor, since, in this case, the assumptions required by the model are not generally met. For example, the errors are not at all random or normally distributed about a mean of zero, in part, due to the difficulty in obtaining proper solvent and insolvent groups of companies from which the coefficient can be estimated. This step is critical, because an outlier can greatly impact the results. Even the notion that a company is either insolvent or solvent, with no possible middle ground, is extremely questionable. Solvency may only be in the eye of the beholder until certain obligations can no longer be met and nonpayments occur.

The estimates and subsequent predictions are further complicated because a regression requires a set of independent variables. This reduces the possible number of ratios that can be used without overspecifying the model. If one of the ratios used was influenced by a company for the sake of appearance, then the results could change dramatically. Clearly, the process used had to be as immune as possible to companies making small cosmetic modifications to their published results.

Exhibit 2 shows some of the results that were obtained using regression. The best fit equation for each year often consisted of different independent variables. Additionally, in many cases, even when the same variable was selected for different years, the magnitude or the sign of the coefficient was quite different. Consequently, this model was abandoned as unsatisfactory.

The NAIC's nonparametric method had a great deal of appeal. In fact, it was used with slight modifications. Instead of employing usual ranges, a ranking scheme was constructed that would dynamically update itself, rather than relying on out of date ranges. This method recognizes the difference between a greatly divergent value and a value which falls just outside of a usual range.

4. THE NONPARAMETRIC MODEL

Data was collected for eighty-four companies which predominantly wrote a reinsurance book, and which had net written premiums of at least $1 million for each of the years 1980–84. To obtain a fair comparison between companies, it was considered important that the data be uniformly collected. The *A. M. Best Trend Report* contains uniform data for five years of history, and provided the data for this analysis. The financial ratios that were tested are shown in Exhibit 3.

Selection of Ratios

To determine which financial ratios discriminated between financially strong and weak companies, a sample of both "strong" and "weak" companies had to be established. This was not a simple task, since if the financial well being of a company was easily determinable, there would be no reason to complete the analysis. Also, the selection of strong and weak companies for the sample should be random, so that no bias for size, age, or other attributes is inherent in the decisions. It was not considered necessary that a company actually be declared insolvent to be included in the sample of "weak" companies, but rather the selection was made following an in-depth review of the companies' annual statements, combined with informed judgments about the companies' management practices. Ten strong and ten weak companies comprised our sample for selecting financial ratios.

The selection of companies for the sample was, in part, a motivating influence on the procedures that were adopted. The regression model was heavily dependent on an absolute discrimination of strong from weak companies, in order to produce tight confidence intervals on the regression coefficients. While it was felt that the two groups were generally correct as chosen, they were neither the ten strongest nor the ten weakest companies in the population. For determining which test ratios were good discriminators, a test that was not quantitative, but only determined if the groups had different means was not only sufficient, but stood a better chance at improved resolution because it was less stringent. The Wilcoxon Rank Sum test was chosen for determining a financial ratio's ability to discriminate between groups.

Briefly, the Wilcoxon Rank Sum test works as follows. For a given ratio (e.g., gross leverage) and a given year (e.g., 1980) the twenty companies were ranked in ascending order. Then, the *ranks* of the ten "strong" companies and the *ranks* of the ten "weak" companies were summed. If the gross leverage ratio perfectly distinguished between the two groups back in 1980, the rank sums would be 55 and 155, respectively, for the "strong" and "weak" groups. For this particular example, the actual rank sums were 84 for the "strong" companies and 126 for the "weak" companies, which indicates that this ratio displayed substantial discriminative power as early as 1980. This procedure provides a statistical method for evaluating whether the two groups come from distributions with the same mean financial ratio. The resulting sums of 84/126 indicate that the probability of the rank sum being equal to or less than 84 would occur only 5.67% of the time, given that the means of the two groups were the same. This statement can be expressed equivalently as: the probability is 5.67% that the sum of the second group would be greater than or equal to 126. The percentages were calculated using the normal approximation. For the example of two groups of ten, the mean and variance are calculated as $M = N_1 \cdot (N_1 + N_2 + 1)/2$ and $V = N_1 \cdot N_2 \cdot (N_1 + N_2 + 1)/12$, respectively.

The results are $M = 105$, and $V = 175$, and for a sum of 126 the Z score

$$Z = \frac{(126 - 105)}{\sqrt{175}} = 1.59,$$

which corresponds to the 5.67% probability.

The Wilcoxon test was performed on each of the ratios for each of the five years of data, with the results being used to choose those ratios which consistently (1980–84) discriminated between the two groups of companies. The results of this Wilcoxon Rank Sum Test are shown in Exhibit 4. Based on this procedure, the following ten ratios were chosen as being "good" discriminators:

Gross Leverage
Surplus Aid to Surplus
Operating Ratio
Net Operating Income to Net Earned Premium
Yield on Investments
Premium Balances to Surplus
Ceded Leverage
1-Year Loss Development to Surplus
2-Year Loss Development to Surplus
Gross Leverage/Net Leverage

For those ratios which were not identified as being "good," an additional technique was employed. By redefining the test ratio to be the absolute value of the difference between the company's actual ratio and that ratio's median value, an attempt was made to highlight those additional ratios, such as Change in Net Written Premium, where the "strong" companies may be clustered around the median value, while the "weak" companies show up at both extremes. By redefining these remaining ratios and then performing the Wilcoxon test, the following ratios were also identified as "good" discriminators:

Change in Net Written Premium
Combined Ratio
Estimated Reserve Deficiency
% Change in Gross Leverage/% Change in Net Leverage

Exhibit 5 shows the results of the Wilcoxon test on the redefined ratios for each year. Thus, of the initial twenty-two ratios, fourteen of these have historically shown an ability to discriminate between "strong" and "weak" reinsurance companies.

Ranking Methodology

Given the fourteen ratios, our method for ranking the companies works as follows:

A. For each year of data (1980–84), the companies were ranked (1 through 84) for each of the fourteen ratios individually.

B. For each company and year, that company's average rank for the fourteen ratios was computed.

C. For each individual year, a final ranking of the companies was prepared by ordering the companies based on their 14-ratio average ranks.

D. Our "best guess" at ranking the companies was made by then taking a weighted average of the 1982, 1983 and 1984 individual year rankings, with relative weights of 1:2:4 used to arrive at a final ranking. These weights were selected using judgment.

It should be stated that each year's result is also viewed independently, and any company exhibiting a dramatic change in ranking from one year to the next is carefully examined. Attention is also given to a company whose rank increases or decreases steadily over time. Ideally, a "good" company is one that maintains an acceptable ranking consistently over time.

5. RESULTS

How well would this ranking technique have worked historically? We have tried to evaluate our results in several different ways as a means of answering this question. First of all, has there been any consistency to the rankings we have developed? To address this question we have (1) used the 1982–84 weighted average ranking as our "best guess" of the correct ranking, (2) eliminated the ten "strong" and ten "weak" companies from our eighty-four company data base, and (3) split the remaining sixty-four companies into thirds (top twenty-one, middle twenty-two, bottom twenty-one). We then looked back to see if these companies have historically fallen into the same categories based on the individual year rankings (1981–84). For example, Exhibit 6 shows that thirteen of the current top twenty-one companies were also ranked in the top twenty-one based on the 1981 data, while six were ranked in the middle twenty-two and two were ranked in the bottom twenty-one. More importantly, it shows that only one of the current bottom twenty-one was ranked in the top twenty-one based on 1981 data, while thirteen of the current bottom twenty-one would have already been placed in the bottom twenty-one based on 1981 data. If we look at 1982 data, seventeen of the current bottom twenty-one companies were already correctly identified.

A second, more important question is: while the rankings may have exhibited reasonable consistency over the years, are they correct? To help answer this question, we looked at the average Best rating historically assigned to companies in our top twenty-one, middle twenty-two and bottom twenty-one. We have assigned the following point scheme to the Best ratings:

A.M. Best Rating	Points
A^+	8
A, A^-	7
B^+	6
B, B^-	5
C^+	4
C, C^-	3
NA–7	2
NA–10	1
Liquidated	0

Using the 1982–84 weighted average ranks as a base, we have computed the average Best ratings historically assigned to the current top twenty-one, middle twenty-two, and bottom twenty-one companies. As a point of comparison, we have also displayed the average Best ratings of the ten "strong" and ten "weak" companies which were used to develop this model.

	Average Best Ratings					
	1980	1981	1982	1983	1984	1985
Top 21*	7.5	7.6	7.5	7.4	6.7	6.4
Middle 22*	7.5	7.6	7.5	7.4	6.8	6.3
Bottom 21*	7.1	7.2	7.1	6.4	3.9	3.7
"Strong" 10	7.8	7.6	7.6	7.7	7.6	7.3
"Weak" 10	7.1	7.2	6.9	6.0	3.6	2.5

*Based on 1982–84 weighted average ranking

The above table highlights the divergence of results between the nonparametric ranking model and the Best ratings. While the Best ratings become significantly lower in 1984 for the bottom twenty-one, there is never any differentiation in the Best ratings between the top twenty-one and the middle twenty-two. Therefore, from a reinsurance buyer's perspective, the average Best ratings conclude that given the same reinsurance cost, a buyer would be indifferent between using a middle twenty-two company or a top twenty-one company. The nonparametric model, however, explicitly differentiates between companies through the ranking procedure, thereby indicating that a reinsurance buyer should not be indifferent between choosing a middle twenty-two or a top twenty-one company.

If we rank the companies based solely on 1981 data, the same movement in the average Best ratings is again apparent:

	1980	1985
Top 21**	7.7	6.3
Middle 22**	7.4	5.6
Bottom 21**	7.1	4.3

**Based on 1981 ranking

As can be seen, based on 1981 data, the nonparametric model produces a ranking which is very successful in identifying those companies which are today carrying much lower Best ratings. While both the NAIC model and the A. M. Best ratings failed to recognize these potential problem companies until 1984, the nonparametric model would have already identified a majority of these companies in 1981. This would have truly constituted an "early warning."

6. ENHANCEMENTS

Although the current version of the nonparametric model appears to have worked well, several potential enhancements to the present model are readily apparent:

Additional Tests/Data Sources

One shortcoming of the *A.M. Best Trend Report* as a data source is that not all areas of a company's operations are equally covered. Several areas from which additional ratios should be tested are as follows:

1. Schedule F

   a. Ceded Leverage

      This test, even though it is currently being used by the model, might prove to be even more effective if it were split into two pieces—ceded to affiliates and ceded to nonaffiliates. Other possible splits might be ceded to authorized and ceded to unauthorized, or ceded to domestic and ceded to foreign.

b. Reinsurance Recoverables

Reinsurance recoverables could be split along the same lines as the ceded leverage tests, possibly resulting in a test which identifies those companies possessing the greatest uncollectible reinsurance exposure.

2. Property/Liability Premium Breakdown

Differences in the gross and net leverage positions of the various companies may be in some part attributable to their individual property/liability mixes of business. By identifying each company's property/liability split, more meaningful test results may occur. This information is available on a detailed basis in Part 2C—Premiums Written, and on a summarized basis in the Five-Year Historical Data in each company's Annual Statement.

Removal of Highly Correlated Tests

The model presented here may include tests that are so highly correlated they provide limited additional information. This is not felt to be a serious defect of the model, however, but merely a situation where one area of a company's operation (e.g., leverage) may be exerting a relatively greater influence on the final rankings than some other area (e.g., loss reserves).

Better Balance

As currently constructed, the fourteen tests contained in the nonparametric model are distributed as follows:

General Area Tested	Number of Tests
Leverage Tests	6
Profitability Tests	4
Loss Reserve Tests	3
Liquidity Tests	1
Cash Flow Tests	0

Future research should concentrate on identifying additional ratios from the liquidity and cash flow areas in order that a more balanced number of tests from each area can be achieved.

Miscellaneous Concerns

As it now stands, there are several areas which are not addressed by the model, and it is questionable whether several of these items are even quantifiable, and thus usable, by the model. The items are:

1. Geographical Distribution of Exposures

   While Schedule T lists direct premiums written by state, neither reinsurance assumed nor net premiums are shown. Therefore, it is impossible to assess a reinsurer's geographical distribution of exposures.

2. Parent Company Commitment

   The willingness of a parent company to back the obligations of its reinsurance subsidiary is often questionable. Regardless of the ranking assigned to a company by the nonparametric model, any information which reflects either positively or negatively on the parent's commitment to the insurance industry should be used to subjectively evaluate the rankings assigned by the model.

3. Management Philosophy

   The quality of management may not necessarily be reflected totally in a company's published financial statements. Although this does introduce another subjective element into the analysis, this is not an area which should be neglected when reviewing potential reinsurance markets.

7. CONCLUSION

It is unreasonable to assume that a solvency-tracking system established for the entire insurance industry would work equally as well, or at all, for the reinsurance industry. As a result of this study, five of the IRIS ratios were found to discriminate between strong and weak companies, while another two discriminated after being adjusted by that ratio's median value.

It was assumed that future insolvent companies could not be identified with certainty. Therefore, the objective was to find a screening process that could provide early warning as to which companies would most likely be subject to financial stress. These could then be more closely watched. If the market remains profitable, this may be unnecessary. But during a long-term low pricing cycle, this type of monitoring could save a company from poor reinsurance decisions.

REFERENCES

A. M. Best Company, *Best's Advance Rating Reports*, various years.

A. M. Best Company, *Best's Insurance Reports*, various years.

A. M. Best Company, *Best's Insurance Management Reports*, various years.

A. M. Best Company, *Best's Trend Reports*, various years.

M. Hollander and D. Wolfe, *Nonparametric Statistical Methods*, 1973, John Wiley & Sons, Inc.

Milliman & Robertson, Inc., "Property and Liability Solidity Testing Programs: An Analysis," Illinois Insurance Guaranty Fund, Illinois Department of Insurance, 1979.

National Association of Insurance Commissioners, "Using the NAIC Insurance Regulatory Information System: Property and Liability Edition," 1980 & 1986.

J. Trieschmann and G. Pinches, "A Multivariate Model for Predicting Financially Distressed P-L Insurers," *Journal of Risk and Insurance*, 1973, Vol. XL, pp. 327–338.

EXHIBIT 1

BEST'S ADVANCE RATING REPORTS TESTS

PROFITABILITY TESTS

1. Loss Ratio
2. Expense Ratio
3. Combined Ratio
4. Operating Ratio
5. Net Operating Income to Net Earned Premium
6. Yield on Investments
7. Change in Surplus
8. Return on Surplus

LEVERAGE TESTS

1. Change in Net Written Premium
2. Casualty % of Net Earned Premium
3. Direct Written Premium to Surplus
4. Net Written Premium to Surplus
5. Net Liabilities to Surplus
6. Net Leverage
7. Ceded Reinsurance Leverage
8. Gross Leverage
9. Surplus Aid to Surplus
10. Reinsurance Recoverable to Surplus

CASH FLOW TESTS

1. Net Cash Flow
2. Net Cash Flow to Quick Assets
3. Quick Liquidity

LIQUIDITY TESTS

1. Current Liquidity
2. Overall Liquidity
3. Agents' Balances to Surplus
4. Premium Balances to Surplus
5. Investment Leverage

LOSS RESERVE TESTS

1. Development to Surplus
2. Estimated Reserve Deficiency to Surplus
3. Loss Reserves to Surplus
4. Developed to Industry Average
5. Projected to Reported
6. Developed to Net Earned Premium
7. Change in Loss Reserves

EXHIBIT 2

MINIMUM SQUARED ERROR
FOUR PARAMETER REGRESSION EQUATIONS
BY YEAR

Year	Parameter	Coefficient
1980	Operating Ratio	−.160
	Investment Yield	1.907
	Gross Leverage	1.227
	Ceded Leverage	−4.570
1981	Operating Ratio	−.099
	Investment Yield	2.405
	Surplus Aid to Surplus	−.481
	Gross Leverage/Net Leverage	−5.464
1982	Net Income/Net EP	.403*
	Gross Leverage	.988
	2-yr Reserve Dev./Surplus	−.388
	Surplus Aid to Surplus	−.979
1983	Net WP/Surplus	5.602
	Agents' Balances/Surplus	.173
	Ceded Leverage	−5.320
	2-yr Reserve Dev./Surplus	−.142
1984	Net Income/Net EP	−.089*
	Return on Surplus	.397
	Premium Balances/Surplus	.080
	Investment Leverage	.101

*Note reversal of sign in coefficient between 1982 and 1984.

EXHIBIT 3

MEDIAN TEST SCORES FOR REINSURERS

Description	1980	1981	1982	1983	1984	1985
Change in Net WP	7	8	4	6	14	32
Premium to Surplus	1.4	1.3	1.2	1.2	1.4	1.6
Net Leverage	3.8	3.9	3.3	3.5	3.9	4.2
Gross Leverage	4.5	5.1	4.6	4.4	5.3	5.5
Surplus Aid to Surplus	1	1	1	1	1	1
Combined Ratio	107	107	114	117	132	120
Operating Ratio	89	89	93	99	113	102
Net Operating Inc. to Net EP	8	9	6	2	−9	−5
Yield on Investments	7.9	8.6	8.7	8.3	8.5	8.8
Change in Surplus	16	8	9	5	−13	7
Return on Surplus	15	9	10	7	−14	−1
Quick Liquidity	40	42	33	49	44	N/A
Overall Liquidity	123	119	117	121	105	N/A
Agents' Balances to Surplus	18	19	15	15	22	20
Prem. Balances to Surplus	22	22	21	17	19	16
Investment Leverage	26	26	24	25	24	25
Estimated Reserve Deficiency	−5	−6	−8	−2	8	15
Ceded Leverage	0.5	0.5	0.5	0.7	1.3	1.2
1-yr Reserve Dev. to Surplus	3	4	4	6	16	2
2-yr Reserve Dev. to Surplus	0	4	6	7	13	0
% Change in Gross Lev./ % Change in Net Lev.	—	1.01	1.00	1.01	0.77	1.03
Gross Leverage/Net Lev.	1.13	1.13	1.19	1.21	1.33	1.32

EXHIBIT 4

Wilcoxon Rank Sum Test Results
(Sum of Ranks for Strong Companies)/(Sum of Ranks for Weak Companies)

	1980	1981	1982	1983	1984
Change in Net Written Premium	103/107	97/113	120/90	118/92	120/90
Net Written Premium to Surplus	98/112	96/114	93/117	91/119	97/113
Net Leverage	106/104	104/106	102/108	98/112	93/117
Gross Leverage	84/126	83/127	77/133	78/132	90/120
Surplus Aid to Surplus	71/139	73/137	67/143	70/140	76/134
Combined Ratio	100/110	104/106	75/135	68/142	58/152
Operating Ratio	97/113	90/120	66/144	57/153	55/155
Net Operating Income to Net Earned Premium	116/94	122/88	142/68	144/66	155/55
Yield on Investments	116/94	120/90	124/86	117/93	112/98
Change in Surplus	112/88	104/106	120/90	127/83	138/72
Return on Surplus	120/90	102/108	144/66	131/79	153/57
Quick Liquidity	94/116	92/118	122/88	100/110	100/110
Overall Liquidity	90/120	89/121	111/99	123/87	130/80
Agents' Balances to Surplus	116/94	117/93	113/97	123/87	112/98
Premium Balances to Surplus	132/78	128/82	127/83	121/89	120/90
Investment Leverage	100/110	103/107	111/99	109/101	93/117
Estimated Reserve Deficiency	89/121	80/130	98/112	96/114	100/110
Ceded Leverage	63/147	62/148	62/148	65/145	80/130
1-Year Reserve Development to Surplus	96/114	95/115	69/141	55/155	79/131
2-Year Reserve Development to Surplus	109/101	93/117	68/142	57/153	69/141
% Change in Gross Leverage/% Change in Net Leverage	N/A	98/112	120/90	118/92	142/68
Gross Leverage/Net Leverage	60/150	59/151	63/147	64/146	69/141

EXHIBIT 5

WILCOXON RANK SUM TEST RESULTS
(Sum of Ranks for Strong Companies)/(Sum of Ranks for Weak Companies)
Based on Company Ratio Minus Industry Median

	1980	1981	1982	1983	1984
Change in Net Written Premium	83/127	113/97	78/132	80/130	91/119
Net Written Premium to Surplus	114/96	127/83	112/98	91/119	77/133
Net Leverage	102/108	124/86	98/112	108/102	70/140
Combined Ratio	90/120	78/132	75/135	90/120	88/122
Change in Surplus	94/116	85/125	101/109	92/118	59/151
Return on Surplus	109/101	93/117	88/122	84/126	101/109
Quick Liquidity	98/112	98/112	93/117	105/105	112/98
Overall Liquidity	104/106	121/89	124/86	118/92	103/107
Agents' Balances to Surplus	119/91	111/99	105/105	121/89	115/95
Investment Leverage	102/108	113/97	102/108	94/116	84/126
Estimated Reserve Deficiency	88/122	98/112	86/124	81/129	75/135
% Change in Gross Leverage/ % Change in Net Leverage	N/A	87/123	81/129	80/130	91/119

EXHIBIT 6

COMPARISON OF RESULTS
INDIVIDUAL YEAR RANKING VS. 1982/83/84 WEIGHTED AVERAGE RANKING

Individual Year Rankings	Group	1982/83/84 Weighted Average Ranking		
		Top 21	Middle 22	Bottom 21
1981	Top 21	13	7	1
	Middle 22	6	9	7
	Bottom 21	2	6	13
1982	Top 21	15	6	0
	Middle 22	4	14	4
	Bottom 21	2	2	17
1983	Top 21	15	6	0
	Middle 22	6	10	6
	Bottom 21	0	6	15
1984	Top 21	18	3	0
	Middle 22	3	16	3
	Bottom 21	0	3	18

ADDRESS TO NEW MEMBERS—NOVEMBER 9, 1988
MILESTONES, CROSSROADS AND CHALLENGES

JEROME A. SCHEIBL

This meeting marks the beginning of the 75th year for the Casualty Actuarial Society. As we celebrate our diamond anniversary next November, there will have been close to 2,000 of us who at one time or another had demonstrated the basic skills and knowledge that qualified us to practice as actuaries in the casualty field.

Today also marks a personal milestone for those of you who are receiving your diplomas as new fellows or associates at this meeting.

For you new fellows, this is a chance to get on with your careers without the annoyance of those pesky examinations, to gain experience in converting textbook applications into real life situations, and to reenter the world of reality and become reacquainted with family and friends after a long and sometimes tedious journey through the education and examination process. It also marks your assumption of new responsibilities to strengthen our profession through contributions to our literature, to share academic dialogue with colleagues, and to participate in the administrative functions and directions of the Society itself.

This is also an important milestone for you who are being recognized as new associates. You have not only demonstrated a level of knowledge that marks your emergence from the ranks of lay persons, but you have also assumed an allegiance to a code of professional conduct that will guide, and sometimes limit, your future activities.

Your achievement has been recognized by your peers. You are now counted among the over 1,400 members of our Society and the 13,000 plus members of the actuarial community in North America. You have qualified for what has been termed by the *Jobs Rated Almanac* as the best job in the United States—and I'm sure that that goes for Canada as well.

I hope you will pardon me while I digress for a moment to share a personal observation about that number one rating. When this study was publicized last spring, I was flattered, and I'm sure many of you were too. A few of my friends stopped questioning the legitimacy of the way I earn my daily bread. I even sensed a faint suggestion of respect at times.

My curiosity led me to look at the other end of the scale—at those jobs rated as the least desirable—to learn more about the significant contrasts that set actuaries apart from others. Here I found the migrant worker, the commercial fisherman, the dairy farmer, the cowboy, the lumberjack and the professional football player. (So much for free spirit!)

I should have left "well enough" alone. Many of the characteristics of those jobs seemed to fit what I had come to know as typical of the actuary's lot. The commercial fisherman, for instance, puts in long hours, does seasonal work, faces tough physical demands and must cope constantly with a high level of government regulation.

While we actuaries do not experience these characteristics to the same extent or in the same manner as commercial fishermen, there is a familiar ring to this list of "downside" attributes. Even the physical demands are a factor in their own way when one considers that it is common for actuaries to do some or all of their work in those cities ranked as the most stressful by *Psychology Today* magazine. I can list them, but that might do more harm than good.

Perhaps my nature as an actuary causes me to get carried away by things like this. I've been told that an actuary is one who, after satisfying him or herself that a solution to a problem will work in practice, then goes about trying to prove that it will work in theory. Some people may laugh at this, but it is our nature to be curious, to not accept things at face value, and to find just as much comfort in a world of abstraction as we do in a world of reality.

If there is one mark of an actuary that sets him or her apart from those in other professions and callings, I submit that it is this inherent quality to bridge the gap between abstract academic curiosity and the pragmatic demands of the business world and society. We must have a balanced concern for these disciplines and demands. Any other course may be the undoing of our profession.

The customary addresses of CAS presidents as they complete their terms of office often refer to milestones, crossroads and challenges, or words of similar import. They are usually cast in time frames of the past, present and/or future with threads of confidence, pride, concern and apprehension woven throughout. I don't have the slightest idea what Dave Hartman's focus will be as he shares his thoughts with us this noon, but I'm willing to bet that his talk will contain some of these elements.

We've been reminded by several past presidents on these occasions that, while our profession is strong in many ways, it is also vulnerable. Although we can point to innumerable technical achievements and contributions that we have made, we have lagged behind other professions in such areas as policing our professional conduct, establishing and maintaining standards of practice, attaining public recognition, fulfilling public demands for an adequate quantity and quality of practitioners, and similar attributes that mark an honored and responsible profession.

Our past presidents have reminded us repeatedly that we are continually at crossroads, where decisions must be made that can have profound effects on the future of our profession, our Society, and our careers. Each crossroad brings its own challenges, and each challenge brings its own opportunities to be converted into the milestones that mark the progress of our profession.

Many challenges are with us today and many new challenges lie on the horizon. There are growing demands for "certification" of loss reserves, policy questions concerning methods and practices related to discounting of liabilities, actuarial implications in the determination of social policies, and the developing interest of the federal government in the business of insurance, to name a few. Conceivably, these challenges could resolve themselves into assumptions of personal liability for reserve runoffs, federal or state examining and licensing of actuaries, or the imposition of illogical or otherwise unsound practice standards and principles, as expedient solutions to social and political problems.

It is just as conceivable that these challenges can become catalysts for increasing public awareness of the actuarial profession and the essential role it plays in today's environment. Whether they are opportunities or threats depends much on the professional manner in which they are handled.

When I gave my presidential address seven years ago, I enumerated six specific challenges as my legacy to the Society. These were not easy challenges. They involved such things as our organizational structure, development of a textbook, standards of practice, standards of conduct, disciplinary procedures, and a contemporary statement of our mission.

Significant progress has been made in each of these areas, thanks to the tireless efforts of many volunteers. I am happy to report that each of these challenges is being met in ways that far exceeded my expectations. This tells me something about the vitality of our organization and the enthusiastic efforts of our members to promote the professionalism and credibility needed to carve a prominent niche in today's business world and the society in which we live and work.

Congratulations again to the fall class of 1988! Welcome to your new status in our profession. And welcome to your new role in erecting the milestones, maneuvering through the crossroads, and meeting the challenges of the casualty discipline of the actuarial profession.

PRESIDENTIAL ADDRESS – NOVEMBER 9, 1988
COMPETITION, COMMUNICATION AND COOPERATION

DAVID G. HARTMAN

Three years ago, prior to Stan Khury's presidential address, he recognized all those present who had served the CAS. I would like to do the same today. Will everyone who has ever served on a committee of the CAS, or as an officer of a regional affiliate, please stand. Look around you. The key to the strength of the Casualty Actuarial Society is the high level of involvement by the members of our organization. As president, I have been especially able to see the significant contribution of our volunteer committee members and would like everyone to join me now in expressing our appreciation to those who have served or are now serving the CAS with a round of applause. Thank you.

Today I have the opportunity to mention some events that occurred during the past year, offer some personal observations, and, together with each of you, look forward to the future. Earlier this year the CAS Constitution was amended to reflect a revised statement of purpose. It reads:

> The purpose of the Casualty Actuarial Society is to advance the body of knowledge of actuarial science in applications other than life insurance, to establish and maintain standards of qualifications for membership, to promote and maintain high standards of conduct and competence for the members, and to increase the awareness of actuarial science.

So how did we, the officers and members, further the purpose of the CAS this past year? How have we positioned ourselves to advance the purpose in the future? To at least partially address these questions, I have selected three themes to address today.

Competition

The first theme is competition. I believe that competition is good. I believe in the free enterprise system. One of the personal characteristics that I look for in candidates for employment as actuarial students is competitiveness. As students preparing for actuarial examinations, they must have a competitive drive in order to be in the group of successful candidates who take CAS examinations.

During this past year there were two notable competitive events that occur only once every four years. They are the Games of the XXIV Olympiad in Calgary and Seoul and the presidential election in the United States. Unfortunately, the summer Olympic games will likely be remembered more for the scandals than for the triumphs and culminations of years of sacrifice, with personal best records for many participants. The presidential campaign has been one of the most negative ever, with personal attacks flying between George Bush and Michael Dukakis. The upcoming election on November 21 here in Canada has not been the most gentlemanly either.

As noted, one part of the purpose of the CAS is "to promote and maintain high standards of conduct and competence for the members." How will we casualty actuaries maintain these high standards while encouraging competition without scandal and negativism? Granted, we are not a group of amateurs, nor a group of politicians. However, to increase our professionalism, we have taken giant strides this year with the promulgation by the CAS Board of Directors of two statements of principles—one for ratemaking and one for reserving; a third on valuation principles is likely to be promulgated in 1989. The Canadian Institute of Actuaries is addressing standards of practice. Furthermore, the Actuarial Standards Board was formed as of July 1 by the American Academy of Actuaries. While we have had Guides to Professional Conduct for many years, and some standards of practice, the standards issued by the CIA and the Actuarial Standards Board should improve the professionalism of all actuaries. The statements of principles articulated by the CAS will serve as a foundation for the casualty standards of practice.

I am sure many of you have seen examples of work done in the casualty field that border on incompetent. It is difficult to maintain high standards of competence without generally accepted standards of practice. We want those standards to apply to competitive circumstances in the future so that we can compete competently and fairly and so that the public will hold our profession in high regard. The standards should not be so restrictive as to prohibit experimentation.

I salute the casualty actuaries working to improve our profession and to meet one part of the purpose of the CAS by serving on the Actuarial Standards Board, or on the various ASB operating committees, or developing CIA standards of practice. I also encourage each member of the CAS to review the exposure drafts of proposed standards of practice and to submit written comments as appropriate.

Communication

The second theme is communication. Again, one of the personal characteristics I look for in candidates for employment as actuarial students is the ability to clearly communicate. It has often been said that a person can be the world's greatest technician, but without being able to communicate the conclusions reached, that person will never achieve true maximum potential.

Actuarial students need to clearly communicate their accumulated knowledge in order to pass CAS exams. In many respects, practicing casualty actuaries must be effective sales people. We find ourselves selling the results of our complex analyses on reserve levels or pricing changes to our employers, regulators, and others. Past president LeRoy Simon has said our principal job is to educate the underwriter. Being able to clearly and convincingly communicate our conclusions is most important. However, as casualty actuaries, have we communicated in all the arenas that could benefit from the application of our actuarial expertise? The Council of Presidents Task Force on Strengthening the Actuarial Profession has evaluated the strengths and weaknesses of the profession. While it found many strengths, the single largest weakness is in the area of public interface. As a profession, we can do a better job of communicating with our various publics, such as insurance buyers and government agencies.

Probably the single largest news item relating to the U.S. property-casualty industry this past year was the filing of an anti-trust suit against several insurance carriers, reinsurers, and industry organizations by Attorneys General of eighteen states. The public must think that some very bad act was committed. When have so many Attorneys General brought suit against virtually an entire industry? Several observers conclude this was a politically motivated action during an election year. Possibly it was. However, the politicians did find a vulnerable target in the insurance industry.

Another example of the industry as a vulnerable target is the five voter initiatives dealing with property-casualty insurance on the California ballot yesterday. The citizens of California are frustrated with the rising cost of insurance. However, these initiatives indicate that the public does not have the knowledge or the understanding of the insurance industry, pricing, competition, and the realities of the free enterprise system.

Who has such knowledge and understanding? Who else is more trained in pricing, reserving, economics, insurance accounting, and insurance law than the casualty actuary? Who else can contribute more to increasing public understanding of property-casualty insurance than a well trained casualty actuary who can communicate clearly?

The opportunities for a casualty actuary to make a difference are too numerous to count. How many of you have tried to clarify insurance to your friends and neighbors? Who has become involved in local government? How many have written their state, provincial or federal representatives about insurance related topics such as excess profits legislation or McCarran-Ferguson? There are many situations where one seemingly insignificant person has made a real difference by speaking out. Part of the purpose of the CAS is to advance the body of actuarial science and to increase an awareness of actuarial science. We learn so much preparing for actuarial examinations and represent a valuable resource. Yet, prior to the number one ranking of the actuarial profession in the *Jobs Rated Almanac*, we actuaries remained a relatively unknown profession. We have so much to offer—let's communicate.

Cooperation

Moving from competition and communication, I would like to address my third theme—cooperation. Naturally, I would like new students to be cooperative individuals, but on this theme I would like to focus on increased cooperation between the CAS and the Society of Actuaries.

The purpose of the CAS is that of a learned body. It maintains a syllabus of readings for its qualifying examinations, stimulates research that is published in both refereed and unrefereed journals, and holds numerous meetings for continuing education—to name several characteristics of a learned body.

Meanwhile, the American Academy of Actuaries and the Canadian Institute of Actuaries are national public interface bodies. They are recognized by governments and other public groups as the organizations that speak for the actuarial profession. While each establishes its own qualification standards of practice, they both rely on the learned bodies to train their members. The profession needs the functions of both a learned body and a public interface body.

The Society of Actuaries is also a learned body supporting the American Academy and the Canadian Institute. Excellent dialogue has taken place between the CAS and SOA education and examination leaders this past year, but I wonder if there can be even more cooperation between the CAS and the Society of Actuaries on educational matters. As president this past year, I have seen clearly that the membership of the CAS has serious reservations about bringing our two organizations closer together. However, I would like each of you to step back and give fresh consideration to a proposal, which, to me, makes good sense. This proposal, made by fellow actuary Ardian Gill, is that the educational aspects of our profession be structured along the lines of a university.

Within this university there could be several colleges—one for the study of life actuarial topics, one for the study of pension actuarial topics, one for the study of health actuarial topics, one for the study of property-casualty actuarial topics, perhaps one concentrating on financial topics, and possibly others. Various majors could be offered within each college, with differences between U.S. and Canadian material. Each college would be solely responsible for the education in its field. Each would be governed by its own elected leaders.

The actuarial university would be primarily a degree granting organization with ancillary research functions and with responsibilities for symposia and other educational opportunities, much like the current meetings and seminars of the learned bodies.

The university would not have any qualification functions beyond certifying that an individual has satisfactorily completed several courses offered by the university. The function of deciding who are qualified to practice, and certifying them, would continue to lie with the Academy, or the Canadian Institute, or the Joint Board for the Enrollment of Actuaries. Those bodies would rely on the university to offer the courses needed and to examine proficiency with appropriate rigor in those courses. Similarly, standards of practice and discipline would remain with the national public interface bodies, which would have the right to expect that the underlying principles would emerge from the learned organizations.

Everyone matriculating in our university would be required to take certain core courses covered by examination Parts 1, 2, 3 and possibly Part 4, and would thereafter take courses in his or her major area. Students would be encouraged to take elective courses, including some in other colleges.

Just as in today's academic world where students earn doctorates, masters, bachelors and associates degrees, students in the actuarial university would earn various levels of degrees from the colleges of the actuarial university—fellowship, associateship, enrolled pension actuary and possibly others.

The key to this structure is recognizing the distinction between being certified as qualified to practice and acquiring the education that permits that qualification. Law, accounting and medical schools provide educations and grant degrees, but they do not certify anyone to practice.

Some casualty actuaries have raised the concern that the formation of such a university would cause the casualty actuary to lose recognition or identity. One has only to look at schools such as Wharton within the University of Pennsylvania to realize that an outstanding college within a university has much public recognition and identity. Furthermore, qualification standards established by the national public interface bodies would continue to be specific as to practice areas. Dividing the subjects currently covered by the Society of Actuaries into several colleges may be more traumatic to life actuaries than establishing a casualty college as a part of a university structure would be to us.

We casualty actuaries have much more in common with our brethren in the Society of Actuaries than we do with accountants, economists, MBA's and others who think they can do actuarial work better than actuaries. I would urge cooperation between us and hope that someday the educational aspects of our profession can achieve a structure similar to a university as outlined.

In summary, as casualty actuaries, we are faced with challenges in the areas of competition, communication and cooperation. These challenges apply, albeit in different ways, to the newest student all the way up to and including the most experienced Fellow. Much progress has been made in each of these areas during the past year, but more remains to be done. It is important for each person who completes his or her casualty actuarial examinations to realize that Fellowship is not the end. Rather, it is the foundation upon which to build in our competitive environment, which is so much in need of education about our complex insurance system, and where we are uniquely qualified to communicate. I believe that our profession would be even stronger if we could improve the cooperation between the Casualty Actuarial Society and the Society of Actuaries on educational matters within a university structure.

In closing, I would like to say that serving the Casualty Actuarial Society as its president this past year has been an honor, a privilege, a joy and even time-consuming. Please let me thank you, the members of the CAS, my employer and my employees, my wife, Kitty, and our sons, Tim and Andy, and God for giving me this opportunity to serve this organization I dearly love.

MINUTES OF THE 1988 ANNUAL MEETING

November 8–10, 1988

LE MERIDIEN HOTEL, MONTREAL, QUEBEC

Tuesday, November 8, 1988

The Board of Directors held their regular quarterly meeting from 1:00 P.M. to 4:00 P.M.

Registration was held from 4:00 P.M. to 6:30 P.M.

From 5:30 P.M. to 6:30 P.M. there was a special presentation to new Associates and their guests. This session included an introduction to standards of professional conduct and the CAS committee structure.

A general reception for all members and guests was held from 6:30 P.M. to 7:30 P.M.

Wednesday, November 9, 1988

Registration continued from 7:00 A.M. to 8:00 A.M.

President David G. Hartman opened the meeting at 8:00 A.M. The first order of business was the announcement of the election results. The new President-Elect is Michael Fusco. The new board members are Walter J. Fitzgibbon, Jr., Charles A. Hachemeister, Steven G. Lehmann, and Lee R. Steeneck.

The members of the Executive Council will be Vice President-Administration, Robert F. Conger; Vice President-Development, Charles A. Bryan; Vice President-Membership, Michael L. Toothman; and Vice President-Programs, Richard I. Fein.

Mr. Hartman recognized the twenty-six new Associates and presented diplomas to the forty-one new Fellows who were introduced by Kevin Ryan, President-Elect. The names of these individuals follow.

FELLOWS

Christiane Allaire	Joseph A. Boor	George R. Busche
Jean M. Blakinger	Joseph J. Boudreau	Ann M. Conway

Mark Crawshaw
Susan L. Cross
Robert V. DeLiberato
Christopher Diamantoukos
Paula L. Elliot
Janet M. Ericson
Gregory S. Girard
Ann V. Griffith
Linda M. Groh
Christy H. Gunn
Larry A. Haefner
David H. Hays
Clive L. Keatinge
Rodney E. Kreps
Dean K. Lamb
Alain Lessard
Mary F. Miller
David F. Mohrman
Robert V. Mucci
Nancy D. Mueller
Robert G. Muller
Wade T. Overgaard
Arthur C. Placek
Donald W. Procopio
Richard A. Quintano
Robert S. Roesch
Michel Trudeau
George W. Turner, Jr.
Andre Veilleux
Joseph L. Volponi
Gregory M. Wacker
Thomas A. Wallace
Robert G. Whitlock, Jr.
Arlene F. Woodruff
Roy T. Woomer, III

ASSOCIATES

Patrick J. Burns
David R. Clark
Gordon F. Diss
Dale R. Edlefson
Barry A. Franklin
David B. Gelinne
Bonnie S. Gill
Eric L. Greenhill
Mark A. Heise
John M. Hurley
Paul E. Kinson
Kenneth A. Klinger
John R. Kryczka
Steven E. Math
James P. McNichols
Robert J. Meyer
Robin Nesmith
Lynn Nielsen
Joanne M. Ottone
Katherine D. Radin
Edward C. Stone
Sharon K. Sublett
Christopher M. Suchar
Nancy P. Watkins
Leslie D. Weihrich
Martha A. Winslow

Mr. Hartman then introduced Jerome Scheibl, a past President of the Society, who delivered an address to the new Fellows and Associates.

Michael Fusco, Vice President-Programs, gave a brief summary of the program content.

Charles A. Bryan, Vice President-Development, discussed the new *Proceedings* papers to be presented at the meeting. He also presented the Dorweiler Prize to Richard H. Snader for his paper, "Reserving Long Term Medical Claims."

Mr. Hartman then called for reviews of previous papers from the floor. There were none.

He then introduced Irene Bass, Chairman of the Textbook Steering Committee, who spoke briefly about the textbook which is to be published in 1989.

Richard Snader, Vice President-Administration, then presented his report. Mr. Hartman concluded the business session at 9:30 A.M.

Mr. Hartman then introduced the featured speakers, Susan and Martin Tolchin, who spoke on the subject of foreign investment in the United States.

Following a brief refreshment time, there was a panel presentation entitled "Medical Costs." The panel was moderated by Allan Kaufman, and consisted of Robert T. C. Cone of Consolidated Healthcare, Inc., John H. Ferman, California Association of Hospitals, and Sam Gutterman of Price-Waterhouse. The panel discussed the factors affecting medical costs, the underlying medical cost trends, and potential coverage coordination.

Lunch was served from 12:30 P.M. to 2:00 P.M. The afternoon was devoted to presentations of four new *Proceedings* papers, four textbook chapter drafts, and five panel presentations.

The new *Proceedings* papers were:

1. "Minimum Bias with Generalized Linear Models"
   Author: Robert L. Brown
   University of Waterloo

2. "A Nonparametric Approach to Evaluating Reinsurers' Relative Financial Strength"
   Authors: Stephen J. Ludwig
   The Hartford Insurance Company

   Robert F. McAuley
   The Hartford Insurance Company

3. "The Canadian Charter of Rights & Freedoms—Its Effect on the Canadian Automobile Insurance Industry"
   Author: Robert L. Brown
   University of Waterloo

4. "Federal Income Taxes—Provisions Affecting Property/Casualty Insurers"
   Authors: Manuel Almagro, Jr.
   Tillinghast/Towers Perrin

   Thomas L. Ghezzi
   Tillinghast/Towers Perrin

The textbook chapters presented were:

1. "Classifications"
   Author: Robert J. Finger
   Mercer Meidinger Hansen
2. "Reinsurance"
   Author: Gary S. Patrik
   North American Reinsurance Corporation
3. "Reserving"
   Author: Ronald F. Wiser
   Progressive Casualty
4. "Individual Risk Ratemaking"
   Author: Margaret Wilkinson Tiller
   Tiller Consulting Group, Inc.

The panel presentations covered the following topics:

1. Questions and Answers with the Board of Directors

   Members of the Board of Directors discussed the status of several issues currently before them. CAS members asked questions and expressed concerns and opinions.

   Moderator: Michael Fusco
   Vice President-Programs

   Panelists: Albert J. Beer
   Member, Board of Directors

   James R. Berquist
   Member, Board of Directors

Kevin M. Ryan
President-Elect

LeRoy J. Simon
Member, Board of Directors

2. Standards of Practice on Ratemaking and Reserving

The Casualty Committee of the Actuarial Standards Board invited comments on the exposure draft of the "Actuarial Standards of Practice for Documentation and Disclosure in Property and Casualty Insurance Ratemaking and Loss Reserving."

Moderator: Charles A. Bryan
United Services Automobile Association

Panelists: Michael J. Miller
Tillinghast/Towers Perrin

Alfred O. Weller
Ernst & Whinney

3. Actuarial Update on Information Systems

This session discussed how data collection systems and management information systems are designed and how they can be effectively managed.

Panelists: Glenn Pruiksma
Milliman & Robertson, Inc.

Mark Savory
Coopers & Lybrand

4. Procedures for Promulgating Principles

In 1988 the CAS voted to amend the Constitution confirming the authority of the Board of Directors to promulgate principles. The "Statement of Principles Regarding Property and Casualty Insurance Ratemaking" and the "Statement of Principles Regarding Property and Casualty Insurance Loss and Loss Adjustment Expense Reserves" were promulgated by the Board of Directors in May, 1988. This panel reviewed the procedures followed to promulgate these principles and discussed the historical development of these statements. Questions and comments from the audience were discussed.

Moderator: Mavis A. Walters
Insurance Services Office

Panelists: C. K. Khury
Mercer Meidinger Hansen

Charles L. McClenahan
Coopers & Lybrand

5. Alternative Distribution Mechanisms

This panel examined insurance distribution systems which have become alternatives to the traditional direct writing or agency sales approaches.

Moderator: Joel S. Weiner
CIGNA Property and Casualty Companies

Panelists: Leonard Samson
Tillinghast/Towers Perrin

Roger C. Wade
Frank B. Hall & Company, Inc.

The officers held a reception for new Fellows and their guests from 5:30 P.M. to 6:30 P.M.

The President's Reception for all members and guests was held from 6:30 P.M. to 7:30 P.M.

Thursday, November 10, 1988

A breakfast for new Fellows was held from 7:30 A.M. to 8:30 A.M.

Jacque Cloutier, the President of the Canadian Institute of Actuaries opened the joint meeting of the Canadian Institute of Actuaries and the

Casualty Actuarial Society with a welcoming address. David Hartman, President of the CAS, addressed the combined group and then introduced the panel entitled "Standards of Education and Practice." The moderator of the panel was Hugh White; panelists were Kenneth T. Clark and Michael D. Demner of the Canadian Institute of Actuaries, and Charles A. Bryan and Michael L. Toothman of the Casualty Actuarial Society. The panel discussed the current status and future direction of actuarial standards of education and practice in the United States and Canada.

Following a brief refreshment period, there were concurrent sessions.

1. Government Involvement in Automobile Insurance—Canada

   The panelists discussed the various automobile insurance systems in Canada resulting from the provinces' different regulations.

   Moderator: Herbert J. Phillips
   Coopers & Lybrand

   Panelists: Guy Cloutier
   Elite Insurance Management

   Daniel Demers
   The Laurentian Group

   James K. Christie
   Dominion of Canada Group

2. Government Involvement in Workers Compensation—Canada vs. U.S.

   The panelists contrasted the involvement of government in the Workers Compensation insurance systems of the two nations. The systems were compared with emphasis on areas such as coverage availability, benefit structure, costs to insureds, returns on investment, and solvency of insurers. Current problems of each system was emphasized.

   Moderator: Ronald C. Retterath
   Wausau Insurance Companies

Panelists: Chapin Clark
National Council on Compensation Insurance

Thomas Jenkins, Esq.
Lord, Bissel & Brook

John Neal
Workers Compensation Board
Province of Ontario

Lunch was served from 12:45 P.M. to 2:45 P.M. The guest speaker was Michael McKinsey, Director of the Office of the Superintendent of Financial Institutions.

Following lunch was a panel presentation entitled "Solvency of P&C Companies." This session presented lessons based on past failures of P&C companies, a discussion of current studies on the solvency of P&C companies in Europe, and experience and prospects for future work on this topic in North America. The panel consisted of the following individuals:

Moderator: Allan Brender
University of Waterloo

Panelists: Don Clon
Halifax Life

Chris Daykin
Government Actuary's Office, U.K.

Stephen Lowe
Tillinghast/Towers Perrin

The meeting adjourned at 4:00 P.M.

November, 1988 Attendees

In attendance, as indicated by registration records, were 226 Fellows; 82 Associates; and 105 guests, subscribers, and students. The list of their names follows.

FELLOWS

Aldin, N. C.
Allaire, C.
Amundson, R. B.
Asch, N. E.
Bailey, V. M.
Basson, S. D.
Bear, R. A.
Beer, A. J.
Ben-Zvi, P. N.
Bensimon, A. S.
Berens, R. M.
Berquist, J. R.
Berry, J. L.
Bertles, G. G.
Bill, R. A.
Blakinger, J. M.
Blodget, H. R.
Boison, L. A.
Boor, J. A.
Bornhuetter, R. L.
Boudreau, J. J.
Braithwaite, P.
Brian, R. A.
Briere, R. S.
Brooks, D. L.
Brubaker, R. E.
Bryan, C. A.
Bujaucius, G. S.
Carbaugh, A. B.
Chernick, D. R.
Childs, D. M.
Cis, M. M.
Conger, R. F.
Connell, E. C.
Conway, A. M.
Cook, C. F.
Crawshaw, M.
Cross, S. L.
Crowe, P. J.
Currie, R. A.
Curry, A. C.
Dahlquist, R. A.
Dean, C. G.
Deliberato, R. V.
Deutsch, R. V.
Diamantoukos, C.
Dodd, G. T.
Dolan, M. C.
Dornfeld, J. L.
Driedger, K. H.
Drummond-Hay, E.T.
Duda, D. S.
Duffy, T. J.
Earwaker, B. G.
Easton, R. D.
Elliott, P. L.
Ericson, J. M.
Faga, D. S.
Fasking, D. D.
Fein, R. I.
Finger, R. J.
Fisher, R. S.
Fitzgibbon, W. J.
Flynn, D. P.
Ford, E. W.
Forde, C. S.
Forker, D. C.
Forney, J. R.
Fresch, G. W.
Frohlich, K. R.
Furst, P. A.
Fusco, M.
Gallagher, C. A.
Gannon, A. H.
Gardner, R. W.
Ghezzi, T. L.
Gillespie, J. E.
Girard, G. S.
Goldberg, S. F.
Gottlieb, L. R.
Grace, G. S.
Grady, D. J.
Grannan, P. J.
Griffith, A. V.
Groh, L. M.
Groot, S. L.
Guenthner, D. G.
Gunn, C. H.
Gutterman, S.
Hachemeister, C. A.
Haefner, L. A.
Hafling, D. N.
Hale, J. B.
Hall, J. A.
Hanson, J. L.
Hartman, D. G.
Haseltine, D. S.
Haskell, G. E.
Hays, D. H.
Heer, E. L.
Herzfeld, J.
Hewitt, C. C.
Inkrott, J. G.
Irvan, R. P.
Jerabek, G. J.
Johnson, W. H.
Jones, B. R.
Josephson, G. R.
Kaufman, A. M.
Keatinge, C. L.
Keller, W. S.

Khury, C. K.
Kilbourne, F. W.
Kleinman, J. M.
Knilans, K.
Koch, L. W.
Kollar, J. J.
Koupf, G. I.
Krause, G. A.
Kreps, R. E.
Kudera, A. E.
Lamb, D. K.
Lamonica, M. A.
Larose, J. G.
Lehmann, S. G.
Lessard, A.
Levin, J. W.
Lindquist, P. L.
Lino, R. A.
Lipton, B. C.
Liscord, P. S.
Lockwood, J. G.
Lommele, J. A.
Lonergan, K. F.
Lotkowski, E. P.
Loucks, W. D.
Lowe, S. P.
Ludwig, S. J.
MacGinnitie, W. J.
Makgill, S. S.
Marshitz, I.
McClennahan, C. L.
Menning, D. L.
Meyer, R. E.
Miller, M. F.
Miller, R. R.
Miller, W. J.
Miner, N. B.
Mohrman, D. F.
Mucci, R. V.
Mueller, N. D.
Muller, R. G.
Munt, D. S.
Murdza, P. J.
Murrin, T. E.
Nester, K. L.
Niswander, R. E.
Noyce, J. W.
Overgaard, W. T.
Pagnozzi, R. D.
Patrik, G. S.
Peraine, A.
Pflum, R. J.
Phillips, H. J.
Placek, A. C.
Potts, C. M.
Procopio, D. W.
Pruiksma, G. J.
Quintano, R. A.
Racine, A. R.
Retterath, R. C.
Roberts, L. H.
Roesch, R. S.
Ross, G. M.
Roth, R. J., Jr.
Ryan, K. M.
Scheibl, J. A.
Schultheiss, P. J.
Schultz, R. A.
Sherman, H. A.
Sherman, O. L.
Shrum, R. G.
Simon, L. J.
Skurnick, D. S.
Smith, L. M.
Smith, R. A.
Snader, R. H.
Spidell, B. R.
Steeneck, L. R.
Steer, G. D.
Suchoff, S. B.
Taht, V.
Tatge, R. L.
Tiller, M. W.
Toothman, M. L.
Trudeau, M. W.
Turner, G. W.
Veilleux, A.
Volponi, J. L.
Wacker, G. M.
Walker, G. M.
Walker, R. D.
Wallace, T. A.
Walters, M. A.
Walters, M. A.
Weissner, E. W.
Weller, A. O.
White, C. S.
Whiting, D. R.
Whitlock, R. G.
Wickwire, J. D.
Wilson, R. L.
Wiser, R. F.
Woll, R. G.
Woodruff, A. F.
Woomer, R. T.
Wulterkens, P. E.
Yow, J. W.

ASSOCIATES

Balling, G. R.
Burns, P. J.
Cadorine, A. R.
Cardoso, R. A.
Carlton, K. E.
Chen, C.
Chorpita, F. M.
Clark, D. G.
Clark, D. R.
Cohen, A. I.
Costner, J. E.
Cutler, J. Z.
Dashoff, T. H.
Davis, D. J.
Diss, G. F.
Douglas, F. H.
Dupuis, C.
Edlefson, D. R.
Einck, N. R.
Eramo, R. P.
Francis, L. A.
Franklin, B. A.
Gauthier, R.
Gelinne, D. B.
Gendelman, N. J.
Gill, B. S.
Goldberg, L. R.
Goldberg, S. B.
Graves, G. T.
Greenhill, E. L.
Handte, M. R.
Harrison, E. E.
Head, T. F.
Hebert, N. P.
Heise, M. A.
Henry, T. A.
Jensen, J. P.
Kadison, J. P.
Kinson, P. E.
Kolojay, T. M.
Kryczka, J. R.
Kulik, J. M.
Kuo, C. K.
Lacek, M. L.
Leccese, N. M.
Lewandowski, J. J.
Licht, P. M.
Math, S. E.
McCoy, M. E.
McGovern, E.
McNichols, J. P.
Meyer, R. J.
Mozeika, J. K.
Mueller, R. A.
Nesmith, R.
Nielsen, L.
Nolan, J. D.
Ogden, D. F.
Ottone, J. M.
Peterson, S. J.
Pridgeon, R. D.
Putney, A. K.
Radin, K. D.
Rapoport, A. J.
Sansevero, M.
Schultze, M. E.
Shapland, M. R.
Skrodenis, D. P.
Snow, D. C.
Stadler, E.
Steingiser, R.
Stone, E. C.
Sublett, S. K.
Suchar, C. M.
Taylor, R. G.
Wade, R. C.
Watkins, N. P.
Weihrich, L. D.
Wick, P. G.
Wilson, E. I.
Winslow, M. A.
Yatskowitz, J. D.
Yow, H. E.

GUESTS – SUBSCRIBERS – STUDENTS

Binet, G.
Bouiassa, P.
Breton, M.
Brouillette, Y.
Butler, P.
Caron, L. P.
Cheng, J. S.
Clark, C.
Clause, R.
Clow, D. E.
Codere, M.
Cone, R. T. C.
Crawley, R. A.
Crouse, J. W.
Dannenberg, R.
Demner, M. D.
Deshaies, D. D.
Dodson, D.

Dorval, B. T.
Dufresne, J.
Dussault, C.
Everett, G.
Eversman, T.
Ferman, J.
Fung, C. C.
Gaudreault, A.
Graves, G.
Gutman, E.
Harrison, W.
Have, J. D.
Jenkins, T.
Joliuet, J. C.
Kaufman, D.
Knox, F.
Laing, C. E.
Lalonde, D. A.
Lepage, P.
Martin, C.
Metzner, C.
Oakden, D. J.
Ouimet, B. R.
Paterson, B.
Pawulski, K. T.
Perigny, I.
Plachy, R.
Potvin, R.
Radhakrishnan, R.
Samson, L.
Savory, M.
Smith, S. B.
Spangler, J.
Stenson, T.
Stockall, J. A.
Thibault, A. P.
Turvolgyi, S.
Vachon, D.
Van Leer, P.
Vilar, P.
Yves, R.

REPORT OF THE VICE PRESIDENT-ADMINISTRATION

This report is intended to provide the membership with a summary of the more important activities of the Society during the past year.

The CAS continued to grow during the 1988 fiscal year with 88 new Associates admitted and 60 Associates becoming new Fellows. Total membership now stands at 1,438. Although the number of new members admitted in 1988 is only slightly more than admitted in 1987, the number sitting for examinations has increased dramatically. More than 1,100 candidates registered for the November, 1988 examinations compared with 890 who sat for the November, 1987 examinations.

The Society also grew with respect to the breadth and variety of its members' activities. In 1988 a new Special Interest Section, Casualty Actuaries in Reinsurance (CARE), was formed.

The Board of Directors, with primary responsibility for setting overall CAS policy, met four times during the year. The significant actions taken by the Board were published in the *Actuarial Review*.

The Executive Council, with primary responsibility for day to day activities, also met four times during the year. The April meeting of the Executive Council was held in conjunction with the annual committee chairperson's meeting, and in March the CAS hosted a special triennial Joint Executive Committee meeting of all five of the North American actuarial organizations.

1988 was another active year for the CAS. The activities of the Board, the Executive Council and the many CAS committees included the following items.

Enhancement of the Body of CAS Knowledge

- Actuarial Principles

  Statements of principles regarding ratemaking and loss reserving were promulgated in May, 1988. An exposure draft on valuation principles was released in October with a comment deadline of December 31. Risk classification principles are being drafted. In addition, a "guidance paper" regarding management information was published in the Spring, 1988 issue of the *Actuarial Forum.*

- Actuarial Textbook

Drafts of all chapters have been completed, and seven of them have been exposed in the *Actuarial Forum.*

- Bibliographies

Two additional bibliographies, one on financial analysis and one on valuation, have been completed and are ready for distribution.

- *Actuarial Forum*

Three editions of the *Actuarial Forum*, which was authorized in 1987, were published during fiscal year 1988.

- Research

Funding of a paper on "Implications of Public vs. Private Insurance in Ratemaking" was authorized by the Board of Directors. An author is being sought. Papers on other topics are being written without a need for funding. Several papers are being solicited through AERF.

Seventeen papers were presented at the May 1988 meeting in connection with the discussion paper program on "Evaluating Insurance Company Liabilities." Fourteen papers have been submitted during 1988 for publication in the *Proceedings.* A Committee on Reserves work product concerning loss reserve discounting is expected to appear in the Spring, 1989 issue of the *Actuarial Forum.* The Financial Analysis Committee is working on an asset/liability matching model, also scheduled for publication in the Spring, 1989 *Actuarial Forum.* A call for discussion papers on valuation topics for presentation at the April, 1989 special interest seminar produced thirteen proposals.

Examinations, Education and Continuing Education

- Examinations and Syllabus

The Long Range Planning Subcommittee of the Syllabus Committee is preparing a report concerning examination content. Completion is expected in December. The Education Policy Committee prepared a report on examination structure, which was submitted to the Board at the September meeting. As a result of this report, the concept of smaller educational units was adopted and work begun on an implementation plan. In addition to the work being done on content and structure, a task force was appointed to study education and testing methods.

- Seminars

The CLRS was held in Atlanta in September, and set another attendance record. The Ratemaking Seminar held in March in Minneapolis was a resounding success and will become an annual event. The Canadian P&C Insurance Liabilities Seminar was held in October in Montreal. A Reinsurance Seminar was also held in October, and a Valuation Seminar is planned for April, 1989.

- Continuing Education Recognition

The American Academy of Actuaries authorized an exposure draft on continuing education recognition. The draft is linked to a second Academy exposure draft on qualification standards.

Organization and Staffing

- CAS Staff

Edith Morabito, after 33 years of faithful service to the CAS, is retiring as Manager of the Administrative Office effective December 1, 1988. Her position will be filled by Terry Cullinan, who currently serves as Financial Administrator.

- Task Force on the Future of the CAS Office

This task force was appointed early in 1988 fiscal year and charged with making both short term and long term recommendations regarding the CAS Office. Following the announcement of Ms. Morabito's retirement, the task force focused on short term considerations and reported its conclusions to the Executive Council in October. A plan for an orderly transition of responsibility was recommended and its essential elements were approved.

- COP Actuarial Services Group Task Force

This is a joint task force created by the Council of Presidents with members from several actuarial organizations. The task force is considering the formation of a centralized support and service office to efficiently provide common services (e.g. publications) to all North American actuarial organizations. The work of the task force is still in progress. Its conclusions will have an obvious impact on the long term scope of CAS Office activities.

- Organizational Review Task Force

This task force was formed in 1987 to review the CAS organizational structure. The task force completed its assignment and reported its conclusions to the Executive Council in October, 1988. In general, the task force concluded that the organizational structure that had been established in 1983 was working as effectively as originally intended. Its key recommendations were to add a fifth Vice President in order to distribute the workload and to make a provision allowing the Board to directly elect three of its twelve members. These recommendations will be considered by the membership in 1989.

- Committees

A new committee, the International Relations Committee, was authorized.

The Committee on Sites has been discharged. Its function will be performed by a new special appointment, the Sites Liaison, reporting to the Vice President-Administration.

Planning

- COP Task Force on Strengthening the Actuarial Profession

This task force, under the direction of the Council of Presidents, was formed in 1987 to explore ways to strengthen the actuarial profession as a whole and to consider whether restructuring the organization of the profession would help achieve this goal. The task force had three CAS representatives. In addition, a separate CAS ad hoc committee was formed to review the activities of the task force and advise the CAS Board of Directors. The work of the

COP Task Force is now complete and is being reviewed by the Boards of all North American actuarial organizations. The final report will be circulated to all actuaries in 1989. The focus of the report is on public interface.

Communication

- Public Relations

  The External Communications Committee has formulated a preliminary public relations plan and is working with the American Academy of Actuaries staff to implement it.

- Publicity

  A recruiting brochure focusing on the CAS is planned for development in 1989. Meanwhile, the CAS is still included in the joint CAS/SOA recruiting brochure. In addition, casualty companies have been added to a joint CAS/SOA booklet listing companies with actuarial training programs.

  A mailing to the National Association of College Admissions Counselors will be done by Regional Affiliates.

Elections

For 1989 the Boards of Directors elected the following Vice Presidents:

Vice President-Administration	Robert F. Conger
Vice President-Development	Charles A. Bryan
Vice President-Membership	Michael L. Toothman
Vice President-Programs	Richard I. Fein

The membership elected Michael Fusco to President-Elect and four new Board members: Walter Fitzgibbon, Charles Hachemeister, Steven Lehmann and Lee Steeneck.

Finances

The CAS financial condition remained strong in 1988. The surplus increase in 1988 was greater than anticipated, primarily due to the unusually large increase in examination registrants and higher than expected interest earnings on investments. Higher than expected income was partially offset by greater office and printing expenses. There were also two planned expenditures, one for research and one for a public relations study, that were not made in 1988 but will carry over into 1989. Despite favorable results achieved in 1988, dues and examination fee increases of $10 each will be needed to cover increased office expenses and printing costs. Dues will increase to $160, and examination fees will be $110. A surplus reduction of approximately $10,000 is expected in 1989.

The Audit Committee examined the CAS books for fiscal year 1988 and found the accounts to be properly stated. The year ended with an increase in surplus of $43,349.70. Members' equity now stands at $416,558.11, subdivided as follows:

Michelbacher Fund	$ 70,205.98
Dorweiler Fund	8,583.46
CAS Trust	2,467.18
Scholarship	7,159.01
CLRS Fund	5,000.00
CAS Surplus	323,142.48
Total Members' Equity	$416,558.11

This is my final report as Vice President-Administration, and I would like to take this oportunity to publicly express my sincere appreciation to those who have worked with me during the past three years. Thanks to Tony Grippa as Assistant Treasurer and to Bob Daino as Assistant Secretary. Thanks also to Glenn Keatts, who helped Bob with the Assistant Secretary's chores during this past year, and to all the committee members involved with the administration function.

The CAS Office staff members deserve our special thanks. They have done a marvelous job during a period of rapid growth and unprecedented changes in the organizational structure. And in particular, a debt of gratitude that can never be repaid is owed to Edee Morabito on the eve of her retirement as Manager of the CAS Office. As noted in Matt Rodermund's lead editorial in the November *Actuarial Review*, "No one in the entire Casualty Actuarial Society complex who might leave our midst—we repeat, no one—would be missed as sorely as Edee will be". A resolution expressing the Society's appreciation was passed by the Board of Directors at its November meeting.

Richard H. Snader
Vice President-Administration

FINANCIAL REPORT
FISCAL YEAR ENDED 9/30/88
OPERATING RESULTS BY FUNCTION

Function	Income	Disbursements	Net Result
Exams	$226,642.34	$167,598.10 (a)	$ 59,044.24
Member Services (b)	213,032.97	282,628.73	(69,595.76)
Programs	352,848.78	358,389.93	(5,541.15)
Other (c)	56,923.51	-0-	56,923.51
TOTAL	$849,447.60	$808,616.76	$ 40,830.84 (d)

Notes (a) Does not include exam related expenses incurred by the development function
(b) Areas under supervision of VP-Administration & VP-Development
(c) Investment income less Foreign Exchange and Miscellaneous Bank Debits
(d) Change in CAS Surplus

BALANCE SHEET

Assets	9/30/87	9/30/88	Change
Checking Account	$105,648.35	$146,753.74	$ 41,105.39
Money Market Fund	107,559.25	262,467.55	154,908.30
Bank Certificates of Deposit	0.00	0.00	0.00
U.S. Treasury Notes & Bills	443,631.19	561,270.60	117,639.41
Accrued Interest	15,934.63	9,309.90	(6,624.73)
CLRS Fund	5,000.00	5,000.00	0.00
TOTAL ASSETS	$677,773.42	$984,801.79	$307,028.37
LIABILITIES			
Office Expenses	$ 46,000.00	$ 65,000.00	$ 19,000.00
Printing Expenses	137,389.59	162,569.08	25,179.49
Prepaid Exam Fees	39,211.00	106,017.00	66,806.00
Prepaid Reading Fees	117.00	0.00	(117.00)
Meeting & Seminar Expenses & Prepaid Fees	(7,529.31)	95,595.66	103,124.97
Diamond Jubilee Expense	84,210.03	135,311.94	51,101.91
Other	5,166.70	3,750.00	(1,416.70)
TOTAL LIABILITIES	$304,565.01	$568,243.68	$263,678.67
MEMBERS EQUITY			
Michelbacher Fund	$ 67,175.45	$ 70,205.98	$ 3,030.53
Dorweiler Fund	9,168.31	8,583.46	(584.85)
CAS Trust	2,327.53	2,467.18	139.65
Scholarship Fund	7,225.48	7,159.01	(66.47)
CLRS Fund	5,000.00	5,000.00	0.00
CAS Surplus	282,311.64	323,142.48	40,830.84
TOTAL EQUITY	$373,208.41	$416,558.11	$ 43,349.70

Richard H. Snader
Vice President-Administration

This is to certify that the assets and accounts shown in the above financial statement have been audited and found to be correct.

AUDIT COMMITTEE
David M. Klein, Chairman
Albert J. Quinn
William J. Rowland
Charles Walter Stewart

1988 EXAMINATIONS—SUCCESSFUL CANDIDATES

Examinations for Parts 4, 6, 8 and 10 of the Casualty Actuarial Society were held on May 3, 4, 5 and 6. Examinations for Parts 5, 7 and 9 were held on November 2, 3 and 4.

Examinations for Parts 1, 2 and 3 (SOA courses 100, 110, 120, 130 and 135) are jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries. Parts 1 and 2 were given in February, May and November of 1988 and Part 3 was given in May and November of 1988. Candidates who were successful on these examinations were listed in the joint releases of the two societies.

The Casualty Actuarial Society and the Society of Actuaries jointly awarded prizes to the undergraduates ranking the highest on the Part 1 examination.

For the February, 1988 examination the $200 first prize was awarded to Eric Smith. The $100 prize winners were Eric Lemay, Denise Lobo, Ted Poon and Eric Reifschneider.

For the May, 1988 examination the $200 first prize was awarded to Daniel Nels Ropp. The $100 prize winners were Eric Crane, Bennett Eisenberg, Steven Eriebacker and Jamie Herzog.

For the November, 1988 examination the $200 first prize was awarded to Mark Motyka. The $100 prize winners were Keith Conrad, Paul Ericksen, Hap-Yan Lee and Michael Szydlo.

The following candidates were admitted as Fellows and Associates at the November, 1988 meeting as a result of their successful completion of the Society requirements in the May, 1988 examinations.

FELLOWS

Christiane Allaire	George R. Busche	Robert V. DeLiberato
Jean M. Blakinger	Ann M. Conway	Christopher Diamantoukos
Joseph A. Boor	Mark Crawshaw	Paula L. Elliott
Joseph J. Boudreau	Susan L. Cross	Janet M. Ericson

Gregory S. Girard
Ann V. Griffith
Linda M. Groh
Christy H. Gunn
Larry A. Haefner
David H. Hays
Clive L. Keatinge
Rodney E. Kreps
Dean K. Lamb
Alain Lessard
Mary F. Miller
David F. Mohrman
Robert V. Mucci
Nancy D. Mueller
Robert G. Muller
Wade T. Overgaard
Arthur C. Placek
Donald W. Procopio
Richard A. Quintano
Robert S. Roesch
Michel Trudeau
George W. Turner, Jr.
Andre Veilleux
Joseph L. Volponi
Gregory M. Wacker
Thomas A. Wallace
Robert G. Whitlock, Jr.
Arlene F. Woodruff
Roy T. Woomer, III

ASSOCIATES

Patrick J. Burns
David R. Clark
Gordon F. Diss
Dale R. Edlefson
Barry A. Franklin
David B. Gelinne
Bonnie S. Gill
Eric L. Greenhill
Mark A. Heise
John M. Hurley
Paul E. Kinson
Kenneth A. Klinger
John R. Kryczka
Steven E. Math
James P. McNichols
Robert J. Meyer
Robin Nesmith
Lynn Nielsen
Joanne M. Ottone
Katherine D. Radin
Edward C. Stone
Sharon K. Sublett
Christopher M. Suchar
Nancy P. Watkins
Leslie D. Weihrich
Martha A. Winslow

The following is the list of successful candidates in examinations held in May, 1988.

Part 4

Richard R. Anderson
Guy A. Avagliano
Karen F. Ayres
David W. Bahnemann
Jack Barnett
Martin Beaulieu
David P. Bechtel
Nathalie Begin
Anthony J. Benjamin
Jack J. Berger
Gavin C. Blair
Roberto G. Blanco
Thomas S. Boardman
John D. Booth
Yaakov B. Brauner
Patrick J. Burns
Terri L. Cartwright
Dennis K. Chan
Daoguang E. Chen
Dae-Ro Choi
Bryan C. Christman
Kasing L. Chung
Cindy C. Chu
Jerome M. Coleman
Jeffrey R. Cole
Pierre Couture
David A. Cullather
Robert J. Curry
David J. Darby
Stephen R. DiCenso

Francois Dumas
Brad C. Eastwood
Jeffrey Eddinger
Charles C. Emma
Beth L. Feller
David N. Fields
Doris Fortin
Yves Francoeur
Jacque B. Frank
Rebecca A. Fuxjaeger
Scott F. Galiardo
Bradford S. Gile
Beth M. Godt
Suzanne E. Graham
Cynthia M. Grim
Steven J. Groeschen
Carleton R. Grose
Jean-Francois Guimond
John M. Hagopian
George M. Hansen
Ellen M. Hardy
Thomas G. Hess
Anthony Iafrate
Thomas D. Isensee
Hou-Wen Jeng
Edwin G. Jordan
David L. Kaufman
Daniel R. Keddie
Tony J. Kellner
Bryan J. Kincaid
Jean-Raymond Kingsley
Paul E. Kinson
Leslie K. Kishi
Elizabeth Kolber
Ronald T. Kozlowski
David J. Kretsch
Howard A. Kunst
David R. Kunze
Benoit Laganiere
Alan E. Lange
Nicholas J. Lannutti
Christopher Lattin
France LeBlanc
Changsoo Lee
Louise Legros
Eric F. Lemieux
Marc E. Levine
Edward A. Lindsay
Brian E. MacMahon
Donald F. Mango
Guy Marineau
Cynthia E. Markey
Burton F. Marlowe
Matthew S. McPadden
William T. Mech
Robert J. Meyer
Charles B. Mitzel
Richard B. Moncher
Kelly L. Moore
Francois Morissette
Kevin J. Moynihan
David A. Murray
Prakash Narayan
William A. Niemczyk
Douglas W. Oliver
Ann Overturf
Mary M. O'Shea
Jacqueline E. Pasley
Timothy B. Perr
Jill Petker
Michael J. Petrocik
Jennifer A. Polson
Timothy P. Quinn
Robin A. Rabideau
Jeffrey C. Raguse
Kay K. Rahardjo
Victor U. Revilla
Karin M. Rhoads
Eric R. Riehl
Gregory L. Riemer
Mark R. Rodgers
Steven C. Rominske
Brian H. Rose
Paul D. Ross
Daniel G. Roth
Scott J. Roth
David A. Royce
Timothy J. Rundle
Stephen P. Russell
Stuart G. Sadwin
Karen E. Schmitt
Robert F. Scott
Margaret E. Seiter
Vincent M. Senia
Theodore R. Shalack
Marc Shamula
Christopher M. Smerald
Elizabeth A. Sogge
Thomas A. Stanford
Stephen D. Stayton
Elissa M. Sturm
Arumugam Suthanthiranathan
Christopher Tait
Joseph W. Tasker, III
Richard D. Thomas
Barbara H. Thurston
Elliot Trottier
Mary L. Turner
Melanie A. Turvill
Ricardo Verges
Patrick M. Walton
Peter C. Wei
Elizabeth A. Wellington
Nicholas L. Weltmann, Jr.
Robert J. White
Leigh F. Wickenden
Chad C. Wischmeyer

Part 6

Katherine Barnes
Walter B. Barnes
Allan R. Becker
Cara M. Blank
Lloyd J. Bouchard
Donna D. Brasley
Richard F. Burt, Jr.
Lynn R. Carroll
Martin Cauchon
Danielle Charest
David R. Clark
Michael A. Coca
Charles Cossette
Gregory A. Cuzzi
Daniel J. Czabaj
Edgar W. Davenport
Jeffrey F. Deigl
Edward D. Dew
Gordon F. Diss
Carol A. Dolan
Timothy B. Duffy
Denis Dumulon
Dale R. Edlefson
John W. Ellingrod
Catherine E. Eska
Steven R. Fallon
Kerry L. Fitzpatrick
William G. Fitzpatrick
Richard L. Fox
Barry A. Franklin
David B. Gelinne
Jeffrey C. Gendron
Bonnie S. Gill
Robert H. Goldberg
Eric L. Greenhill
Mark A. Heise
Todd J. Hess
John M. Hurley
Joanne K. Ikeda
Sadagopan S. Iyengar
James W. Jonske
Edward M. Jovinelly
Allan A. Kerin
Kevin A. Kesby
Daniel F. Kligman
Kenneth A. Klinger
John R. Kryczka
Pierre Lepage
Allen Lew
Siu K. Li
Emma B. Macasieb
James F. Mallon
Laura Manley
Steven E. Math
Malkie Mayer
Cassandra M. McGill
James P. McNichols
Elizabeth H. Mitchell
Daniel M. Murphy
Robin Nesmith
Lynn Nielsen
Jonathan Norton
Joanne M. Ottone
Donald D. Palmer
Wayne V. Paulauskis
Marvin Pestcoe
Loren V. Petersen
Robert C. Phifer
Michael D. Poe
Debbie W. Price
Katherine D. Radin
Allen D. Rosenbach
Kevin D. Rosenstein
Linda D. Ross
Pierre A. Samson
Edmund S. Scanlon
Barbara J. Schill
Jeffrey W. Schmidt
Susan C. Schoenberger
Robert F. Scott
Alan R. Seeley
Rial R. Simons
Christy L. Simon
Lawrence J. Steinert
Edward C. Stone
Frederick M. Strauss
Paul J. Struzzieri
Sharon K. Sublett
Christopher M. Suchar
Anuradha Sundram
Chester J. Szczepanski
William Vasek
Therese M. Vaughan
Ricardo Verges
Nancy P. Watkins
Leslie D. Weihrich
Russell B. Wenitsky
Martha A. Winslow
Windrie Wong
Richard P. Yocius

Part 8

John G. Aquino
Bruno P. Bauer
Kay E. Benninghof
Holly L. Billings
Joseph J. Boudreau
Teresa J. Caudill
Joseph F. Cofield
Susan L. Cross
Alan M. Crowe
Janice Z. Cutler
Robert N. Darby, Jr.
Carol Desbiens
Mark DiGaetano
John S. Ewert
Nancy G. Flannery
Nathan J. Gendelman
Richard N. Gibson
Steven A. Glicksman
Leonard R. Goldberg
Susan M. Gozzo
Gregory T. Graves
Alex R. Greene
Norman P. Hebert
Alan M. Hines
David D. Hu
Jane E. Jasper
Steven J. Johnston
Mary Jean King
Warren A. Klawitter
Charles D. Kline, Jr.
Rodney E. Kreps
Kenneth R. Krissinger
Paul E. Lacko
Joseph R. Lebens
Roland D. Letourneau
Christopher P. Maher
Michael W. Mahoney
Blaine C. Marles
Brett E. Miller
William H. Mitchell
Thomas G. Moylan
Chris E. Nelson
Kenneth J. Nemlick
Steve C. Peck
Brian G. Pelly
Isabelle Perigny
Mark R. Proska
Richard D. Robinson
Jeffrey C. Salton
Melissa A. Salton
Richard D. Schug
Mark E. Schultze
Marie Sellitti
Peter J. Siczewicz
Lisa A. Slotznick
Elisabeth Stadler
Judith E. Stoffel
Christian Svendsgaard
Angela E. Taylor
George W. Turner, Jr.
Guy Vezina
Bill S. Yit
Heather E. Yow

Part 10

Ralph L. Abell
Christiane Allaire
Kenneth Apfel
Jean M. Blakinger
Joseph A. Boor
George R. Busche
Ruy A. Cardoso
Michael J. Cascio
Kevin J. Conley
Ann M. Conway
Mark Crawshaw
Michael K. Curry
Dan J. Davis
Robert V. DeLiberato
Christopher Diamantoukos
Vincent T. Donnelly
Thomas J. Ellefson
Paula L. Elliott
Janet M. Ericson
William G. Fanning
Kirk G. Fleming
Barbara L. Forbus
Richard Gauthier
Judy A. Gillam
Gregory S. Girard
Ann V. Griffith
Linda M. Groh
Christy H. Gunn
Larry A. Haefner
James W. Haidu
David H. Hays
Richard J. Hertling
Brian A. Hughes
John J. Joyce
Clive L. Keatinge
Rodney E. Kreps
Dean K. Lamb
Alain Lessard
Mary E. McCoy

Mary F. Miller
David F. Mohrman
Robert V. Mucci
Nancy D. Mueller
Robert G. Muller
Richard T. Newell, Jr.
Wade T. Overgaard
Bruce Paterson
Arthur C. Placek
Donald W. Procopio
Richard A. Quintano
Robert S. Roesch
Eileen M. Sweeney
Robert W. Thompson
Ernest S. Tistan
Michel Trudeau
Andre Veilleux
Joseph L. Volponi
Christopher J. Wachter
Gregory M. Wacker
Thomas A. Wallace
Robert G. Whitlock, Jr.
Arlene F. Woodruff
Roy T. Woomer, III
Edward M. Wrobel, Jr.

The following candidates were admitted as Fellows and Associates as a result of their successful completion of the Society requirements in the November, 1988 examinations.

FELLOWS

Ralph L. Abell
Michael J. Cascio
James M. Dekle
Nancy R. Einck
Richard Gauthier
Kevin M. Greaney
Pierre G. Laurin
Robert W. Matthews
Jay B. Morrow
Richard T. Newell
Andrew J. Rapoport
Peter J. Siczewicz
Dominic A. Weber
Robin Marie Williams

ASSOCIATES

Walter B. Barnes
Bruno P. Bauer
Karin H. Beaulieu
Cara M. Blank
J. Scott Bradley
Mark D. Brissman
Jennifer S. Byington
Mark J. Cain
Lynn R. Carroll
Martin Cauchon
Teresa J. Caudill
Paul Chabarek
Danielle Charest
Guy R. Danielson
James R. Davis
Charles Desjardins
David K. Dineen
Timothy B. Duffy
Denis Dumulon
George T. Dunlap, IV
Dominick A. Elia
Alan J. Erlebacher
Steven R. Fallon
Loy W. Fitz
William G. Fitzpatrick
Nancy G. Flannery
Richard L. Fox
Luc Gagnon
Robert H. Goldberg
Ewa Gutman
Todd J. Hess
Brian A. Hughes
Nancy E. Kot
Christian Laberge
David W. Lacefield
Pierre Lepage
Barry I. Llewellyn
Mark J. Mahon
Sudershan K. Malik
Eduardo P. Marchena
Malkie Mayer
Jon W. Michelson
Brett E. Miller
Elizabeth H. Mitchell
Wai Hung Ng
Jonathan Norton
Kathleen M. Pechan
Brian G. Pelly
Isabelle Perigny
Loren V. Petersen
Michael D. Poe
Kathy Popejoy
Andre Premont
Christine E. Radau
Allen D. Rosenbach
Sandra Samson
Sandra C. Santomenno
Jeffrey W. Schmidt
Susan C. Schoenberger
Arthur J. Schwartz
Robert F. Scott
Michael L. Scruggs
Alan R. Seeley
David Spiegler
Barbara A. Stahley
Lawrence J. Steinert
Elaine E. Swords
Chester J. Szczepanski
John V. Van de Water
Ricardo Verges
Peter A. Weisenberger
Russell B. Wenitsky
Mary E. Wills
Gregory S. Wilson
Richard P. Yocius
Ronald J. Zaleski

The following is the list of successful candidates in examinations held in November, 1988.

Part 5

Mark S. Baker
David P. Bechtel
Gavin C. Blair
Jean-Francois Blais
Michael G. Blake
Roberto G. Blanco
Barry E. Blodgett
Thomas S. Boardman
John D. Booth
J. Scott Bradley
Donna D. Brasley
Kenneth A. Brehmer
James L. Bresnahan
Steven A. Briggs
Laura D. Brueckman
Christopher G. Brunnetti
Paul A. Bukowski
Anthony J. Burke
Todd S. Burrick
Mario Champagne
Gregory Christensen
Dennis J. Cler
Michael A. Coca
Michelle Codere
Brian K. Cox
Kenneth M. Creighton
M. Elizabeth Cunningham
Francois Dagneau
Thomas V. Daley
Jeffrey F. Deigl
Herb Desson
Edward D. Dew
D. Kevin Dineen
David A. Doe
Jeffrey E. Doffing
Carol A. Dolan
Jeffrey D. Donaldson
Alicia G. Doyle
Leonard G. Doyle
Ronald R. Earls
Brad C. Eastwood
Maribeth Ebert
Bob D. Effinger, Jr.
John W. Ellingrod
Charles C. Emma
Madelyn C. Faggella
Michele P. Fisher
Robert F. Flannery
Kelly F. Fogarty
David A. Foley
France Fortin
Vincent M. Franz
Louis Gariepy
Jeffrey C. Gendron
Bruce R. Gifford
Michael A. Ginnelly
Beth M. Godt
Dawson T. Grubbs
Deborah D. Haidu
Richard J. Haines
George M. Hansen
Jonathan M. Harbus
Thomas D. Herzfeld
Michael B. Hirsch
Anthony Iafrate
Joanne K. Ikeda
Kathleen M. Ireland
Thomas D. Isensee
Anthony Iuliano
Hou-Wen Jeng
Michael S. Johnson
Linda M. Kaiser
Trina C. Kavacky
Edith M. Keating
Daniel R. Keddie
Joseph P. Kilroy
Andrew M. Koren
Howard A. Kunst
Frank O. Kwon
Christian Laberge
Alan E. Lange
Nicholas J. Lannutti
James W. Larkin
France LeBlanc
Christine Lefebvre
Marc-Andre Lefebvre
Stephen E. Lehecka
Giuseppe F. LePera
Edward A. Lindsay
Frank K. Ling
Yuan-Long L. Liu
Andre Loisel
Brian E. MacMahon
James F. Mallon
Burton F. Marlowe
Eric Martel
Heidi J. McBride
Thomas J. McDermott, Jr.
Liam M. McFarlane
Van A. McNeal
Linda K. Miller
Robert L. Miller

Paul W. Mills
Richard B. Moncher
Kelly L. Moore
Francois Morissette
Todd B. Munson
Mark Naigles
John Nissenbaum
Keith R. Nystrom
Ann Overturf
Erica Partosoedarso
Jacqueline E. Pasley
John R. Pedrick
Robert L. Penick
Timothy B. Perr
Jill Petker
Michael J. Petrocik
Mark W. Phillips
Faith M. Pipitone
Jennifer A. Polson
Lisa M. Pouliot
C. Stuart Powers
Mark S. Quigley
Eric K. Rabenold
Christine E. Radau
Donald K. Rainey
John F. Rathgeber
Nancy L. Reker
Joel A. Reott
Elizabeth M. Riczko
Gregory L. Riemer
Michel Rivard
Linda Roberge
Diane R. Rohn
Allen D. Rosenbach
Bradley H. Rowe
John M. Ruane, Jr.
Stephen P. Russell
Maureen S. Ruth
Melissa A. Salton
Leigh A. Saunders
David M. Savage
Edmund S. Scanlon
Diann Schachtschneidner
Beth M. Schmiesing
Suzanne E. Schoo
Richard T. Schulz
Gordon L. Scott
Margaret E. Seiter
Vincent M. Senia
Charles L. Sizer
James J. Smaga
Christopher M. Smerald
Tom A. Smolen
Elizabeth L. Sogge
Angela K. Sparks
Thomas N. Stanford
Stephen D. Stayton
John P. Stefanek
Steven B. Strothman
Katie Suljak
Scott R. Sykes
Joseph W. Tasker, III
Georgia A. Theocharides
Barbara H. Thurston
Susan M. Treskolasky
Stacy L. Trowbridge
Mary L. Turner
Phillip C. Vigliaturo
Dale G. Vincent, Jr.
Sebastian Vu
Patrick M. Walton
Stephen D. Warfel
Marjorie C. Weinstein
Robert J. White
Gnana K. Wignarajah
William R. Wilkins
Calvin Wolcott
Sarah S. Wood
Betsy Woodard
Stephen K. Woodard
John M. Woosley
Robert S. Yenke
Charles J. Yesker
Vincent F. Yezzi, Jr.
Nancy E. Yost
Sheng H. Yu
Barry C. Zurbuchen

Part 7

Guy A. Avagliano
Walter B. Barnes
Bruno P. Bauer
Karin H. Beaulieu
Cara M. Blank
David R. Bowman
Dominique E. Brassier
Jack B. Brauner
Mark D. Brissman
Jennifer S. Byington
Mark J. Cain
Lynn R. Carroll
Martin Cauchon
Teresa J. Caudill
Paul Chabarek
Danielle Charest
Philip S. Chou
Kathleen F. Connor
Guy R. Danielson
James R. Davis
Germain Denoncourt

Charles Desjardins
Carol A. Dolan
Timothy B. Duffy
Denis Dumulon
George T. Dunlap, IV
Dominick A. Elia
Alan J. Erlebacher
Jennifer L. Ermisch
Steven R. Fallon
Thomas R. Fauerbach
Loy W. Fitz
William G. Fitzpatrick
Nancy G. Flannery
Richard L. Fox
Joyce M. Frank
Luc Gagnon
Robert H. Goldberg
Deborah A. Greenwood
Cynthia M. Grim
Carleton R. Gross
Ewa Gutman
Todd J. Hess
Brian A. Hughes
Vincent H. Jackson
Michael G. Kerner
Ho Kyong Kim
Richard F. Kohan
Nancy E. Kot
David J. Kretsch
David W. Lacefield
Pierre Lepage
Allen Lew
Barry I. Llewellyn
Mark J. Mahon
Sudershan K. Malik
Eduardo P. Marchena
Makie Mayer
Cassandra M. McGill
William T. Mech
Mark F. Mercier
Jon W. Michelson
Brett E. Miller
H. Elizabeth Mitchell
Kevin J. Moynihan
Wai Hung Ng
William A. Niemczyk
Jonathan Norton
Susan J. Patschak
Kathleen M. Pechan
Brian G. Pelly
Isabelle Perigny
Loren V. Petersen
Michael D. Poe
Kathy Popejoy
Andre Premont
Richard W. Prescott
Deborah W. Price
Kay K. Rahardjo
Scott E. Reddig
Margaret M. Reynolds
Steven C. Rominske
Sandra Samson
Sandra C. Santomenno
Jeffrey W. Schmidt
Karen E. Schmitt
Susan C. Schoenberger
Arthur J. Schwartz
Robert F. Scott
Michael L. Scruggs
Alan R. Seeley
Ahmad Shadman-Valavi
Michelle G. Sheng
Gary E. Shook
David Spiegler
Barbara A. Stahley
Andrew Stein
Lawrence J. Steinert
Paul J. Struzziere
Elaine E. Swords
Chester J. Szczepanski
Janet A. Trafecanty
John V. Van de Water
Anne-Marie Vanier
Ricardo Verges
Rebecca A. Wagner
Peter A. Weisenberger
Russell B. Wenitsky
Mary E. Wills
Gregory S. Wilson
Chad C. Wischmeyer
Kathy A. Wolter
Richard P. Yocius
Ronald J. Zaleski

Part 9

Ralph L. Abell
Manuel Almagro, Jr.
Rebecca C. Amoroso
Kenneth Apfel
Lawrence J. Artes
Ina M. Becraft
Robert K. Bender
Steven W. Book
John W. Buchanan
Michael J. Cascio
Li-Chuan L. Chou
Walter P. Cieslak
David R. Clark
Joseph F. Cofield
Alan M. Crowe

Janice Z. Cutler
Robert N. Darby, Jr.
Edgar W. Davenport
James M. Dekle
Carol Desbiens
Mark DiGaetano
Dale R. Edlefson
Nancy R. Einck
Karen F. Evans
Beth E. Fitzgerald
Barbara L. Forbus
Jacque B. Frank
Pierre Fromentin
Richard Gauthier
Bonnie S. Gill
Daniel F. Gogol
Nancy A. Graves
Kevin M. Greaney
Robin A. Harbage
Mark A. Heise
David R. Heyman
Brian A. Jones
Edward M. Jovinelly
Mary Jean King
Charles D. Kline, Jr.
Pierre G. Laurin
Peter M. Licht
Brett A. MacKinnon
Michael W. Mahoney
Robert W. Matthews
James B. McCreesh
Gail P. McDaniel
Sean P. McDermott
William H. Mitchell
Karl G. Moller, Jr.
Jay B. Morrow
Thomas G. Moylan
Mark W. Mulvaney
Chris E. Nelson
Kenneth J. Nemlick
Richard T. Newell, Jr.
Lynn Nielsen
Christopher G. Nyce
Susan L. Pino
Ronald D. Pridgeon
Boris Privman
Andrew J. Rapoport
Ralph L. Rathjen
Beverly K. Ryan
Jeffrey C. Salton
Sara E. Schlenker
Mark E. Schultze
Jeffory C. Schwandt
Susanne Sclafane
Kim A. Scott
Robert F. Scott
Peter J. Siczewicz
Edward C. Stone
Douglas N. Strommen
Ronald J. Swanstrom
Guy Vezina
Dominic A. Weber
Peter W. Wildman
Robin M. Williams
Martha A. Winslow
Roger A. Yard
Heather E. Yow

NEW FELLOWS ADMITTED MAY, 1988 (Left to Right): First Row: Roger A. Schultz, Linda A. Shepherd, Russel L. Sutter, Patricia J. Webster, Kenneth R. Kasner, Sanders B. Cathcart, Marthe A. Lacroix, Jean Vaillancourt, David G. Hartman (President); Second Row: George N. Phillips, Eric R. Keen, James C. Votta, Brian Z. Brown, Donald D. Sandman, William J. Miller, William M. Carpenter, Patrick Mailloux, William J. VonSeggern; Not Pictured: Mary V. Anderson and Bruce Earwaker

NEW ASSOCIATES ADMITTED MAY, 1988 (Left to Right): First Row: David G. Hartman (President), Kay E. Bennighof, Debra L. Werland, Denis Poirier, Guy Vezina, David J. Macesic, Richard Lebrun, Jane E. Jasper, Elena D. Mohler, Susan E. LaPointe, Susan M. Gozzo, Kwok Ching Ng, James Ely, George N. Phillips, Steven J. Johnston, Michael K. Curry, Peter W. Wildman; Second Row: Beth E. Fitzgerald, Steven L. Colin, Thomas G. Moylan, Chester T. Kido, Michael Caulfield, Alan M. Crowe, Michael W. Mahoney, Richard N. Gibson, Denis Cloutier, Heather E. Yow, William G. Fanning, Donna J. Reed, Alam M. Hines, Robert K. Bender, Douglas N. Strommen, Joseph F. Cofield, Constantine G. Koufacos, Robert N. Darby, Mark DiGaetano; Third Row: Nancy A. Graves, Kevin J. Conley, Christopher G. Nyce, Lawrence J. Artes, Ronald J. Swanstrom, Steven W. Book, John J. Joyce, Steven A. Skov, Valerie L. Schmid, Cecilia M. LePere, James W. Haidu, Karen F. Evans, Richard D. Schug, Chris E. Nelson, Kenneth J. Nemlick, James B. McCreesh, John A. Stenmark, Thomas E. Schadler, Roland D. Letourneau, David A. Lalonde, Richard J. Gergasko, Sr.; Not Pictured: Jeffrey H. Adams, Bruce H. Green, Sasikala Raman, Srinivasa Ramanujam, James S. Higgins, William H. Mitchell

NEW FELLOWS ADMITTED NOVEMBER, 1988 (Left to Right): First Row: Nancy D. Mueller, Paula L. Elliott, Ann M. Conway, Larry A. Haefner, David H. Hays, Ann V. Griffith, Christy H. Gunn, Jean M. Blakinger, Andre Veilleux, David G. Hartman (President); Second Row: David F. Mohrman, Gregory S. Girard, Christopher Diamantoukos, Mark Crawshaw, Clive L. Keatinge, Joseph A. Boor, Thomas A. Wallace, Robert V. Mucci, Robert S. Roesch, Alain Lessard, Robert V. DeLiberato, Michel Trudeau, Donald W. Procopio, Linda M. Groh, Susan L. Cross, Arlene F. Woodruff; Third Row: Jospeh L. Volponi, Dean K. Lamb, Arthur C. Placek, Mary F. Miller, Robert G. Muller, Richard A. Quintano, Wade T. Overgaard, Janet M. Ericson, Rodney E. Kreps, Joseph J. Boudreau, Gregory M. Wacker, Robert G. Whitlock, Jr., Ray T. Woomer, III, Christiane Allaire; Not Pictured: George R. Busche and George W. Turner, Jr.

NEW ASSOCIATES ADMITTED NOVEMBER, 1988 (Left to Right): First Row: David G. Hartman (President), Martha A. Winslow, Katherine D. Radin, Sharon K. Sublett, Nancy P. Watkins, Joanne M. Ottone; Second Row: Lynn Nielsen, John R. Kryczka, David R. Clark, Robin Nesmith, Bonnie S. Gill, Steven E. Math, Mark A. Heise, Eric L. Greenhill, Barry A. Franklin; Third Row: James P. McNichols, Leslie D. Weihrich, Edward C. Stone, Christopher M. Suchar, Gordon F. Diss, Paul E. Kinson, Dale R. Edlefson, David G. Gelinne, Patrick J. Burns; Not Pictured: John M. Hurley, Kenneth A. Klinger, Robert J. Meyer

OBITUARIES

Samuel N. Ain
Daniel Finkel
Joseph B. Glenn
Russell P. Goddard
Charles J. Haugh, Jr.
Richard D. McClure
Mary Jane Styczynski

SAMUEL N. AIN
1913–1988

Samuel N. Ain, an Associate of the Casualty Actuarial Society since 1939, died peacefully on February 1, 1988. Mr. Ain was also an Associate of the Society of Actuaries and a Fellow of the Conference of Actuaries in Public Practice.

Mr. Ain graduated from Brooklyn College with a B.S. degree in chemistry. He subsequently received a Masters Degree in actuarial mathematics from the University of Michigan in 1936. He began his actuarial career at the firm of George B. Buck, where he worked until being called to active duty in the United States Navy in 1941. He served in the Navy for four years as an officer on a destroyer and as a cryptoanalyst in Washington, D.C. He left the Navy with the rank of Lieutenant Commander.

After leaving the Navy, Mr. Ain worked as an actuary for four years with the Pension Trust Division of the Internal Revenue Service in Washington. In 1950, he established his own consulting firm in New York, which he operated until 1976. He merged his firm with The Wyatt Company, from which he retired on June 30, 1978.

He was a long time devoted member of Congregation Shearith Israel, the oldest congregation on the North American continent. He served ably as trustee, honorary trustee, and chairman of its Synagogue Insurance Committee.

Samuel Ain is survived by his wife Dorothy; his daughter Noa and son-in-law Olek, and granddaughter Julia; his son Jonathan; five sisters; and one brother. Mr. Ain has been described as a tower of strength, a man of intelligence, warmth, sensitivity, and compassion.

DANIEL FINKEL
1916–1987

Daniel Finkel, an Associate of the Casualty Actuarial Society since 1962, died on December 12, 1987.

Daniel was born in Brooklyn, New York. During the Depression, at one point, Dan was the only wage earner in his family. But he persevered in his studies, and graduated from the City College of New York with a degree in mathematics after eight years of night school attendance. Besides soon becoming an Associate of the Casualty Actuarial Society, Mr. Finkel was active in the American Mathematical Association, publishing articles and even a theorem that still carries his name.

During World War II, Mr. Finkel worked for the War Department. He protested the segregation of blacks, who were barred from the USO, and as a result, found he was being investigated as a subversive! After the war he worked for the State of New York. At age 62, he retired and eventually moved to California to be near his brother.

Daniel loved any sort of puzzle, including word play. He was interested in astronomy, history, and languages; his great hobby was stamp collecting. After retirement, he enrolled at San Jose State University for the Mathematics courses he'd always wanted to take. He enjoyed the San Jose Reperatory Company, and devoted his time to helping senior citizens as a tax advice volunteer.

Daniel is survived by a son, Alan H. Finkel, an Attorney in Los Angeles; and a brother, Herb, also of California.

JOSEPH B. GLENN
1905–1988

Joseph B. Glenn, a Fellow of the Casualty Actuarial Society since 1930, and a Fellow of the Society of Actuaries, died February 27, 1988 in Bethesda, Maryland after a lengthy illness.

Mr. Glenn was born in Vincennes, Indiana. He graduated from Indiana University in 1926 and began his actuarial career in the Actuarial Department of the Travelers Insurance Company in 1927. Mr. Glenn earned his Fellowship in both the Casualty Actuarial Society and the Society of Actuaries in 1930.

From 1934 to 1942, Mr. Glenn was Chief Actuary of the Railroad Retirement Board. He volunteered for military service during World War II and was assigned to the Navy's Construction Battalion, building airstrips in the Pacific.

From 1943 to his retirement in 1975, Mr. Glenn was the Chief Actuary for the Department of Defense. He was an actuarial pioneer in modeling and writing computer programs. All military legislation passed during this long formative period was analyzed by Mr. Glenn. Among other things, he played a major role in the development of the Career Compensation Act of 1949 which revamped the military compensation structure to provide the pay and allowance system still used today. In addition he acted as a consultant to the Korean Government in the development of a pension system. Mr. Glenn received several prestigious awards and citations from both the Secretary of Defense and the Congress.

He is survived by his widow, Carmella A. Glenn; his daughter, Mary Glenn Gordon; and his grandson, Bryan H. Gordon.

RUSSELL P. GODDARD
1907–1988

Russell P. Goddard, a Fellow of the Casualty Actuarial Society since 1931, died on July 15, 1987.

Russ started his actuarial career in the Casualty Actuarial Department of the Travelers Insurance Company after his graduation from Yale University. He had probably applied for a job in the Life Actuarial Department, where his brother David was employed. But, the Travelers had a rule against two relations working in the same department, and thus he became a casualty actuary, to our gain.

Russ gained much of his actuarial experience working for Harold Ginsburgh at the American Mutual Liability Insurance Company from 1931 until 1949. He represented the company on many actuarial and underwriting committees. He then went to the Pennsylvania Manufacturers' Association as Assistant to the President. He left there in 1960, to become Actuary of the New York Compensation Insurance Rating Board. In 1962, he moved to the consulting firm of Bowles, Andrews & Towne (a predecessor to Tillinghast) in Atlanta. He became an independent consultant in 1970, also in Atlanta. He accepted the post of Chief Actuary of the South Carolina Insurance Department in 1971, and remained there until he retired to Rhode Island in 1977.

Russ Goddard had been very active in the affairs of the Casualty Actuarial Society. He was Chairman of the Examination Committee before that position was an elected office of the Society. In the mid 40's, he served on the Council (now the Board of Directors), and in 1947 he was elected Vice President, serving a two year term. He was on the Committee on the Review of Papers in the 50's and later served as its Chairman. In 1959, he was elected Editor of the Society and he held that position for two years. He also contributed to the *Proceedings* through papers, book reviews, and discussions. His paper, "Policy Year Modification of Losses" won a Fondiller prize.

In the years when CAS meetings were smaller and more intimate, and when many of those in attendance would gather around a piano for an evening of singing, Russ Goddard would sometimes lift his resonant baritone voice in renditions of Scottish songs like "A Wee Deoch and Dorus," or "I Love A Lassie."

Russ is remembered as an intellect, a thinker, who was skilled with the English language. He played no small part in the growth and success of the casualty actuarial profession and he will be fondly remembered.

CHARLES J. HAUGH, JR.
1898–1988

Charles J. Haugh, Jr., a Fellow of the Casualty Actuarial Society since 1926, died on August 11, 1988.

Mr. Haugh was Secretary and Actuary of the North Dakota Workmen's Compensation Bureau when he received his Associateship in 1923. He was Assistant Actuary of the National Bureau of Casualty and Surety Underwriters when he attained his Fellowship in 1926. In 1931, he became Actuary of the National Bureau. He left the National Bureau in 1944 to become Secretary of the Compensation and Liability Department of the Travelers. He was made Second Vice President in 1954 and Vice President in 1956. He retired from the Travelers in 1963 and lived in Connecticut since then.

Charles Haugh was President of the Casualty Actuarial Society from 1945–1947. His conduct as President was distinguished both by the sharpness of his view of the insurance world and by his wit. He was presiding at the November, 1946 meeting when the Nominating Committee offered his name for a second term. Charlie then asked the members assembled if anybody had the temerity to make another nomination from the floor.

Mr. Haugh served on many committees of the Society. In 1931, he was elected to the Council (now the Board of Directors) for three years. From 1934 to 1936, he held the office of Vice President. He was again elected Vice President in 1942, served three years, and then became President in 1945. He contributed to the *Proceedings* through a number of papers and discussions on worker's compensation and automobile ratemaking.

Mr. Haugh lived a long and full life, and contributed much of it to the casualty actuarial profession.

RICHARD D. MCCLURE
1914–1988

Richard D. McClure, a Fellow of the Casualty Actuarial Society since 1963, died on October 17, 1988.

Dick was born on February 7, 1914 to Robert Louis Stevenson McClure and Edith Dodds McClure in New York state. His father was the godson to the famous author Robert Louis Stevenson.

Dick graduated from Alfred University in New York in 1937 with a B.S. in Mathematics. He was very proud of his math degree from this tiny university with an excellent reputation. One other benefit of his attendance here was that he met his wife-to-be, Skip.

He began his insurance career as a Commercial Lines Underwriter in the White Plains Office of American Mutual. During these years, Dick earned his Chartered Property and Casualty Underwriter designation and also began his intense study for the Casualty Actuarial Society exams.

Dick joined the Actuarial Department of the Kemper Group in mid-1965 in Long Grove, Illinois as manager of Research and Reinsurance. While at Kemper, Dick developed a comprehensive and detailed profit and loss report which today has become a widely distributed and quoted report at Kemper.

Upon his retirement from the Kemper Group, in early 1979, Dick returned to the northeast, this time to the mountains of Plymouth, New Hampshire. However, retirement did not last long—he soon became Executive Director of the Massachusetts Medical Malpractice Board.

In 1985, Dick and Skip moved to Lake Worth, Florida. However, retirement was still not a consideration—Dick joined Insurance Industry Consultants, Inc. in late 1985. He remained employed with this firm until his death on October 17, 1988.

Dick's contributions to the CAS were notable, both in service and in publications. He authored two papers and two reviews which were published in the Proceedings. He also served on panels and workshops, and from 1971–1974 was a member of the Finance Committee.

Dick put great emphasis on education—for himself, his sons and even co-workers. One of his hobbies, which continued throughout his career, was to solve challenging math problems. He would exchange ideas at all hours of the night with a son who was attending M.I.T. He also enthusiastically provided training to co-workers and subordinates.

Dick's personality was captivating and his drive was relentless. His high levels of excitement, intensity and dedication continued until the end.

Survivors include his wife Skip and four sons—Stephen, Fredrick, Christopher and Colin.

MARY JANE STYCZYNSKI
1953–1988

Mary Jane Styczynski, an Associate of the Casualty Actuarial Society since 1987, died on July 12, 1988 after a long bout with cancer. A graduate of Millersville State College at Millersville, Pennsylvania, with a degree in math, she joined State Farm in 1973 as a records clerk. In 1975, she moved to the Fire Actuarial Department as an actuarial trainee. She assumed the position of Associate Actuary at State Farm in 1987.

"Janie" was a member of the American Academy of Actuaries as well as the Casualty Actuarial Society. She is survived by her husband, Neil; a son, Matthew; and a daughter, Maura. Also surviving are her parents, Charles and Kathryn Smith of Westchester, Pennsylvania; two brothers Charles F. Smith, Jr. of Chicago, and Paul J. Smith of Blaine, Minnesota; and one sister, Joanne Roby of LeSueur, Minnesota.

INDEX TO VOLUME LXXV

INDEX-CONTINUED

EDITORIAL COMMITTEE, *PROCEEDINGS* Editors

RONALD E. GRECO, EDITOR

STEPHEN W. PHILBRICK *(ex officio)*
CATHERINE N. BISULCA
CLAUDIA S. FORDE
JAMES F. GOLZ
FRANK D. PIERSON
TIMOTHY L. SCHILLING
KATHLEEN W. TERRILL