A PROBABILISTIC MODEL FOR IBNR CLAIMS

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Abstract

IBNR reserves are presented as a stochastic variable. The model presented shows explicitly that the main factors contributing to IBNR reserves are number of claims, severity, and report lag distributions. The mean and variance of IBNR reserves are derived. Procedures to obtain an IBNR confidence interval are discussed. Two examples are provided on the use of the model. Suggestions are made as to how to obtain model parameters from actual insurance data.

1. INTRODUCTION

Accurate estimation of IBNR liabilities is a matter of concern for regulators, management, and investors in proper evaluation of financial statements of property-casualty insurance companies. Some commonly used methods to compute IBNR reserves were presented in Skurnick (1973), and in Bornhuetter and Ferguson (1972). In a survey of loss reserve methods, Skurnick (1973) mentioned the runoff method and the procedures that apply a factor to a *current* value of a base. Bornhuetter and Ferguson (1972) recommended procedures that initially require the computation of age-to-age factors derived from a loss development triangle. In a critique of reserve methodologies, Khury (1980) stated that reserve estimates are point estimates with no provision given for possible *variations* from their respective true values; he also stated that the *actuarial assumptions* used in determining reserve estimates are not mentioned explicitly.

Some commonly used procedures have two main shortcomings. First, a procedure that applies a constant factor to a current value of a base is ad hoc. For instance, statutory IBNR reserves for fidelity and surety coverages are computed as 10% and 5%, respectively, of premiums in force. Such an ad hoc procedure does not differentiate among companies with respect to underwriting practices, company operations, and management's attitude to risk bearing. Second, many of these procedures are a by-product of a *retrospective* reserve analysis (e.g., the runoff method or age-to-age factors derived from a loss development triangle). A retrospective reserve analysis provides information with regard to the adequacy or inadequacy of prior reserve estimates, but its implications about the accuracy of a current reserve are questionable.

Another more philosophical problem associated with retrospective procedures such as the runoff method or procedures based on age-to-age factors is that these procedures are not "statistical." A "statistical" procedure would consider an estimator that is usually unbiased and/or consistent; see Bickel and Doksum (1977). Statistical theory would guarantee that such estimators will be "about" the true parameter value or will "converge" to the true parameter value for large sample sizes (large volume of data). Even when adjusted for the volume of business or other pertinent facts, methods based on runoff procedures are not "on the average" guaranteed to estimate the true IBNR value. Similarly, procedures based on age-to-age factors, even when these factors are trended, cannot be relied on to estimate the true IBNR value correctly. Runoff procedures and procedures related to age-to-age factors may have an intuitive appeal for calculating IBNR. But there is no proof, at least to the extent of the author's knowledge, that these computational methods have desirable properties such as being unbiased and/or consistent.

In this presentation a probabilistic model, a statistical procedure, is developed that may be used as an alternative method for computing IBNR reserves.

2. IBNR MODEL

IBNR liability is presented as a stochastic variable. Parameters used in the model are distribution of number of claims, severity, and report lag by accident periods. These parameters (factors) are dependent on a company's mix of business written (current and past) and to some extent on a company's procedures for investigating and reporting claims. In this section, the probabilistic formulation of the model is considered. The specification of parameters has been delegated to another section. IBNR is presented as a finite sum of random variables. Each term in the finite sum is an "IBNR contribution by an accident period." These IBNR contributions are random sums (see Appendix B). Mean (expected value) and variance of IBNR have been derived.

Claims are grouped by accident periods. The unit of time for an accident period may be a month or a quarter. For the sake of simplicity it is assumed that each accident occurs at the middle of an accident period. It should be noted that when the accident period is one year, the assumption that all accidents occur at the midpoint of the accident year may be invalid for certain types of coverage because of seasonality and other pertinent facts. The "experience period" includes all the accident periods of interest. Diagram A is useful in presenting the "experience period."

DIAGRAM A

$\longleftarrow$ experience period $\longrightarrow$

$\longleftarrow$ c_i $\longrightarrow$

$s-1$ $\quad$ s	$i-1$ $\quad$ i	$t-1$ $\quad$ t
"initial" accident period	accident period i	"current" accident period

where $c_i = t - i + (1/2)$, a known constant,
$i = s, s+1, \ldots, t$.

The accident period i is the interval $(i-1, i]$. In this presentation, accident periods s and t represent "initial" and "current" periods, respectively.

The model assumptions and the main symbols used are as follows. For each accident period i,

(i) N_i, a random variable, denotes the number of accidents occurring;
(ii) corresponding to N_i, there are claim amounts X_{ij}, $j \leq N_i$, that are independent identically distributed (i.i.d.) random variables with the same probability distribution as X_i;
(iii) each claim X_{ij} corresponds to a report lag denoted by T_{ij}. For a given claim, the report lag is defined as the time difference between the accident date and the claim report date. The T_{ij}, $j \leq N_i$, are i.i.d. random variables with the same probability distribution as T_i;
(iv) it is assumed that N_i, X_{ij}, and T_{ij} are independent random variables for each $j \leq N_i$ and $i = s, s+1, \ldots, t$.

The random variables N_i, X_i, and T_i, for $s \leq i \leq t$, correspond to the number of claims, the severity, and the report lag, respectively. N_i's are related to both frequency and volume (exposure). The values of X_{ij} correspond to their *ultimate* cost realizations. The probability distributions for N_i, X_i, and T_i can be different for each i. In Section 4, more information about the specification of N_i, X_i, and T_i distributions is provided. The assumption of independence, (iv) above, has two major implications: for each accident period, the number of claims is independent of claim amounts; and for each claim, the claim amount is independent of its report lag. If there is strong *empirical evidence* that for certain types of coverage a significant correlation (say, positive correlation) exists between claim amounts and their respective report lags, then the independence assumption, (iv) above, is violated and the derivations based on it are invalid. In such a case, one has to modify the model, or alternatively assess the sensitivity of the model to departures from independence.

Let I_A denote an indicator random variable for the event A. That is,

$$I_A = \begin{cases} 1, & \text{if } A \text{ occurs,} \\ 0, & \text{if } A \text{ does not occur.} \end{cases}$$

The following equation (1)

$$1 = I_{\{T_{ij} \le c_i\}} + I_{\{T_{ij} > c_i\}} \tag{1}$$

implies that the claim X_{ij} is either a *reported claim* or an *IBNR claim* as of the end of accident period t. Let Y_i denote the contribution to IBNR liability from accident period i. Then,

$$Y_i = \sum_{j \le N_i} X_{ij} I_{\{T_{ij} > c_i\}}. \tag{2}$$

Note that Y_i is a random sum (see Appendix B) (i.e., Y_i is the sum of random variables with the number of random variables contributing to the sum being random). IBNR as of the end of the "current" accident period t is defined as,

$$\text{IBNR} = \sum_{i=s}^{t} Y_i, \tag{3}$$

$$= \sum_{i=s}^{t} \sum_{j \le N_i} X_{ij} I_{\{T_{ij} > c_i\}}. \tag{4}$$

Equation (3) presents IBNR as a sum of a finite number of random variables, where each random variable in the sum is a random sum denoting an accident period contribution to IBNR.

The mean and variance of Y_i, equation (2) above, are

$$\text{E}(Y_i) = \text{E}(N_i)\text{E}(X_i)\text{P}(T_i > c_i), \tag{5}$$

where $\text{P}(T_i > c_i)$, in (5) above, denotes the probability that the random variable T_i exceeds the value c_i, and

$$\text{Var}(Y_i) = \text{E}(N_i)\text{E}(X_i^2)\text{P}(T_i > c_i) + [\text{E}(X_i)\text{P}(T_i > c_i)]^2[\text{Var}(N_i) - \text{E}(N_i)]. \tag{6}$$

Equations (5) and (6) are a consequence of (iv) above and Appendix B.

The Expected Value (Mean) of IBNR

Using (3) and (5), we have

$$\mathrm{E(IBNR)} = \sum_{i=s}^{t} \mathrm{E}(N_i)\mathrm{E}(X_i)\mathrm{P}(T_i > c_i), \tag{7}$$

$$= \sum_{i=s}^{t} w_i B_i, \tag{8}$$

where $B_i = \mathrm{E}(N_i)\mathrm{E}(X_i)$,
$w_i = \mathrm{P}(T_i > c_i)$.

B_i is "expected incurred losses" for the accident period i. If we use "expected incurred losses" as a *base*, it is clear from (8) that IBNR is a function of current and prior base values. Because it is common for IBNR estimates to be calculated from a base that is only a function of a single year, the above analysis, equation (8), implies that such procedures are inappropriate. The weights w_i can be computed using the report lag distribution(s), and their effect diminishes as we consider earlier accident periods. A deterministic procedure for calculating IBNR using lag probabilities, w_i above, has been presented by Patrik (1978).

The Variance of IBNR

Using the independence assumption about N_i, X_{ij}, and T_{ij}, and equations (3) and (6), we have

$$\mathrm{Var(IBNR)} = \sum_{i=s}^{t} \mathrm{E}(N_i)\mathrm{E}(X_i^2)\mathrm{P}(T_i > c_i) + \sum_{i=s}^{t} [\mathrm{E}(X_i)\mathrm{P}(T_i > c_i)]^2[\mathrm{Var}(N_i) - \mathrm{E}(N_i)].$$

If N_i's are Poisson random variables, equation (9) becomes

$$\mathrm{Var(IBNR)} = \sum_{i=s}^{t} \mathrm{E}(N_i)\mathrm{E}(X_i^2)\mathrm{P}(T_i > c_i). \tag{10}$$

One may be interested in Poisson number of claims for at least two reasons. First, if one expresses the parameter of a claim *process* in terms of the "operational" time rather than the "natural" time, then many claim count processes of interest are in fact *Poisson processes*. A claim count process is a stochastic process, $\{N(u),\ s - 1 \leq u \leq t\}$, where u is the parameter of the stochastic process. The parameter u denotes the time ("natural" time), and $N(u)$ is the number of claims (accumulated number of claims) at time u during the time interval $(s - 1, u]$. The number of claims in the accident period $(i - 1, i]$, N_i, can be expressed in terms of the claim count process by the following relationship: $N_i = N(i) - N(i - 1)$. For an elaborate discussion of "operational" time and claim count processes, the interested reader should refer to Bühlmann (1970). Second, the negative binomial is a suitable probability model for fitting claim count data; see Benjamin (1977). But negative binomial distribution arises from a Poisson random variable because of uncertainty in its parameter specification; see Longley-Cook (1962).

Equation (10) may be written as

$$\text{Var(IBNR)} = \sum_{i=s}^{t} u_i B_i, \tag{11}$$

$$u_i = [\text{E}(X_i^2)/\text{E}(X_i)]\text{P}(T_i > c_i).$$

Now the weights u_i depend on both severity and report lag distributions. When the number of claims has a Poisson distribution, the variance of IBNR can also be expressed in terms of current and prior values of a base. Moreover, in the case of the Poisson number of claims and the further assumption of a severity distribution, X, that does not change over the entire "experience period," we have

$$\begin{aligned} \text{Var(IBNR)} &= \sum_{i=s}^{t} [\text{E}(N_i)\text{E}(X)\text{P}(T_i > c_i)]\ [\text{E}(X^2)/\text{E}(X)], \\ &= \text{E(IBNR)}\ [\text{E}(X^2)/\text{E}(X)]. \end{aligned} \tag{12}$$

Equation (12) implies that the ratio of Var(IBNR) to E(IBNR) depends only on the severity in this case!

Derivation of an IBNR Confidence Interval

Some remarks on the derivation of a confidence interval for IBNR are appropriate at this time. In order to derive an *exact* confidence interval for IBNR reserves, it is necessary to know the distribution of IBNR. Note that IBNR is composed of a sum of a finite number of random variables, where each term in the sum is a random sum. Determining the exact distribution of a random sum is extremely difficult. It requires the evaluation of an infinite number of distributions where each one is a convolution of many distributions. This problem is well known in reinsurance, that is, the aggregate losses in stop-loss reinsurance arrangements are in fact a random sum; see Bühlmann (1970).

The distribution of IBNR can be analytically approximated by using the cumulative distribution of standard normal distribution, its derivatives, and the moments of IBNR. This approach is known as Edgeworth expansion and is discussed in Beard, Pentikäinen, and Pesonen (1969). This approximate distribution can be used to construct a confidence interval for IBNR.

Expressions for the mean and variance of IBNR have been given; now a crude IBNR confidence interval may be computed by using the Chebyshev inequality.

The author believes that a reasonably accurate IBNR confidence interval may be obtained by resorting to simulation. IBNR realizations can be generated and a "simulated" distribution computed by specifying an input scenario, that is, specification of the claim count, the severity, and the report lag distribution for each accident period, based on actual insurance data. Such a distribution may be used to derive a reasonable confidence interval for IBNR.

3. APPLICATION

In this section are two examples that use the preceding model. The specifications of input parameters in these examples are not based on any real insurance data, but are stated merely for *illustrative purposes* and *computational expediency*. Given specifications of input parameters should not be construed as model assumptions. The main assumptions of the model are in condition (iv) in Section 2: independence of claim count, severity, and report lag. A more appropriate use of the model would be to generate many IBNR values (realizations) by resorting to simulation based on input parameters derived from actual insurance data. Results of such a simulation may be used to provide an IBNR confidence interval and determine the sensitivity of IBNR to input assumptions.

Example A: Effect of Changes in Input Parameters on IBNR

IBNR, or more precisely, expected value of IBNR, can be calculated according to equation (7) in Section 2. Each IBNR computation requires an input scenario, that is, a specification of expected number of claims, mean severity, and report lag distribution for each accident period included in the experience period. In this example, we consider one input specification and refer to it as "Scenario A." We then investigate the effect of change(s) in input parameters relative to Scenario A on the value of IBNR. These investigations will show the sensitivity of IBNR value to changes in input parameters. In Table A, several deviations from Scenario A's input specifications are considered. In each case, a percentage change in IBNR value has been computed.

TABLE A

Scenario A:
(i) Growth in expected claim count is 6% annually.
(ii) Mean severity increases uniformly at the rate of 5% annually during the entire 10-year experience period.
(iii) Report lag distribution for each accident period is exponential with mean of 40 months.

Change in Input Assumptions Relative to Scenario A	*Percentage Change in IBNR
1. Change in growth rate for expected claim count from 6% to 9%.	24.1
2. Change in rate of increase in mean severity from 5% to 10% during the *second* 5-year experience period.	15.0
3. Change in mean lag from 40 to 50 months (assuming the distribution of lag remains exponential).	15.6
4. Changes in expected claim count and mean severity as in 1 and 2 above.	43.3
5. Changes in expected claim count and mean lag as in 1 and 3 above.	42.4
6. Changes in mean severity and mean lag as in 2 and 3 above.	31.8
7. Changes in expected claim count, mean severity, and mean lag as in 1, 2, and 3 above.	63.0

*To compute the percentage change, let $(\mathrm{IBNR})_O$ and $(\mathrm{IBNR})_A$ denote the value of mean IBNR according to Scenario O, that is any other scenario, and Scenario A, respectively. Then, the percentage change in IBNR is defined as

$$\{[(\mathrm{IBNR})_O/(\mathrm{IBNR})_A] - 1\} \times 100.$$

For more details on computation of the above percentages refer to Appendix A.

Example B: Projecting IBNR Values After Discontinuing Writing a Line of Business or a Coverage

Consider a situation in which at time t, end of the experience period, the insurer decides to discontinue writing a certain line of business or a coverage. The insurer may pay for IBNR claims as they are subsequently reported and settled, or the insurer may transfer the liability at a given price to an accommodating reinsurer. The rate of decline in IBNR, subsequent to discontinuation of coverage, is considered as follows.

Let E[IBNR(u,v)] denote the mean value of IBNR as of moment v evaluated at time u. Then, according to equation (7), we have

$$\mathrm{E}[\mathrm{IBNR}(t,t)] = \sum_{i=s}^{t} \mathrm{E}(N_i)\mathrm{E}(X_i)\mathrm{P}(T_i > c_i). \tag{13}$$

If coverage is discontinued at time t, $\mathrm{E}(N_i) = 0$, *for* $i > t$. The claim X_{ij} is an IBNR claim as of moment $t + 1$ if $T_{ij} > c_i + 1$. Thus,

$$\mathrm{E}[\mathrm{IBNR}(t,t + 1)] = \sum_{i=s}^{t} \mathrm{E}(N_i)\mathrm{E}(X_i)\mathrm{P}(T_{ij} > c_i + 1). \tag{14}$$

If T_i's are exponential with density $f(t)$,

$$f(t) = \vartheta e^{-\vartheta t},\ t > 0,$$

where the parameter ϑ is equal to 1/(mean lag). Then

$$\mathrm{P}(T_i > c_i + 1) = e^{-\vartheta(c_i+1)} = e^{-\vartheta}\mathrm{P}(T_i > c_i). \tag{15}$$

Using (13), (14), and (15), we have

$$\mathrm{E}[\mathrm{IBNR}(t,t + 1)] = e^{-\vartheta}\mathrm{E}[\mathrm{IBNR}(t,t)];$$

similarly we have

$$\mathrm{E}[\mathrm{IBNR}(t,t + k)] = (e^{-\vartheta})^k\mathrm{E}[\mathrm{IBNR}(t,t)],\ \text{for } k = 1,2,\ldots \tag{16}$$

In particular, if the accident period is one month, then, according to equation (16), the projected value of IBNR a year after the evaluation date is equal to the current IBNR value multiplied by a factor (less than one) that is equal to

$$[e^{-1/(\text{mean lag})}]^{12} \tag{17}$$

The above factor is based on the premise that the lag distribution remains unchanged during the entire experience period and is exponential. Choosing an accident period of one month, IBNR is declining geometrically at an annual rate given by equation (17). Note that no restriction is put on the expected claim counts and the mean severity by accident periods. Table B shows one year decline factors for different mean lags assuming exponential lag distribution.

TABLE B

ONE YEAR DECLINE FACTOR

Mean Lag (Months)	Factor
10	.301
20	.549
30	.670
40	.741
50	.786

The emerged IBNR amounts in the respective future accident periods $t + 1$, $t + 2$, ..., are given by the following differences

$$\mathrm{E}[\mathrm{IBNR}(t,t)] - \mathrm{E}[\mathrm{IBNR}(t,t+1)],$$
$$\mathrm{E}[\mathrm{IBNR}(t,t+1)] - \mathrm{E}[\mathrm{IBNR}(t,t+2)],$$
$$\ldots,$$

based on our evaluation at time t. These emerged IBNR amounts may be used to give an estimate of a "discounted" IBNR.

4. SPECIFICATION OF MODEL PARAMETERS

For each accident period i, the specification of distributions for number of claims, severity, and report lag (i.e., N_i, X_i, and T_i) is required.

In determining N_i, the number of claims, distributions commonly fitted to insurance data are Poisson and negative binomial; see Benjamin (1977). In the case of Poisson, the only required input is the value of $\mathrm{E}(N_i)$, the expected number of claims. $\mathrm{E}(N_i)$ should not be based entirely on reported claims in accident period i, but adjusted for accident period i claims that will be subsequently reported. As Salzmann (1984) stated, "the extrapolation of the incurred count is straightforward and results are quite dependable."

The specification of claim distribution X_i is a more difficult task. Many parametric distributions have been fitted to claim data. Some popular distributions used are lognormal, Pareto, and gamma; see Beard, Pentikäinen, and Pesonen (1969). It is the author's belief that for earlier accident periods, the claim cost data are nearly "fully developed," and a parametric distribution fitted to individual claims (incurred losses) is the appropriate procedure. The term *earlier accident periods*, in the preceding sentence, depends on the circumstances of a given situation. It should be evaluated in terms of the volume of claim cost data and the claim settlement period relevant to that line of business. Finger (1976) wrote an interesting paper related to fitting a lognormal curve to claim data. For more recent accident periods, the claims are only "partially developed" and are not close to their "ultimate" cost values. A possible approach is to extrapolate (trend) the distribution of earlier periods to arrive at distributions for more recent periods. A procedure for *trending distributions* was presented by Rosenberg and Halpert (1981).

The distribution of report lag, T_i, can be obtained by a procedure outlined by Weissner (1978), where reported lags are fitted, by the method of maximum likelihood, to a parametric truncated distribution. The underlying report lag distribution is recovered by exploring the relationship between truncated and nontruncated distributions.

The last point to consider is the selection of an appropriate "experience period." Usually t is December 31 of the year of IBNR evaluation. The choice for s, the "initial" accident period, requires considerable judgment. For a new company or an existing company with a new line of business, the s should be the earliest possible period. In other cases, the choice of s depends on the report lag distribution. From equation (8), it is clear that for earlier accident periods, w_i is small because c_i is large, and consequently the contributions to IBNR from earlier accident periods tend to diminish. Thus, when IBNR is computed by lines of business or coverages, a judgmental choice with regard to the value of s should be made.

Finally, the distributions of N_i, X_i, and T_i are based on our knowledge at the end of the current period t. If the accident period is a month and IBNR is computed annually, at time $t + 12$, we have to update these distributions in the light of data gathered during period $(t, t + 12]$. Thus, the distributions for the claim count, the severity, and the report lag may be updated from one evaluation period to the next.

5. CONCLUSION

The model described in this paper has merits of its own in estimating IBNR reserves, particularly the following points. The model is not ad hoc because the parameters used are dependent on a company's book of business written, which is the most important factor in determining IBNR. The input parameters (distributions) may be continually updated from one evaluation to the next. If the company's operations change, or if other factors suggest an appreciable divergence from past development of input parameters, then, to the extent that these changes can be quantified, "historical" inputs should be replaced by these "subjective" inputs that incorporate the changes. The model is stochastically presented so that we can evaluate variability. The actuarial assumptions used are stated explicitly in terms of probability distributions for the number of claims, the severity, and the report lag. We have a tool, a stochastic model, to work with. More time can now be spent in examining the model assumptions and improving methods of estimating parameters from actual insurance data.

APPENDIX A

FORMULAS USED IN COMPUTING THE PERCENTAGES GIVEN IN TABLE A

Precise specification of the input parameters for the computation of percentages in Table A is given below. The accident period is assumed to be one month. Let $s = 1$ in equation (7); r_1 denotes the rate of growth for the expected number of claims; r_2 and r_3 denote the rate of growth of the mean severity during the first and second five years of the experience period, respectively. The input specifications are as follows:

$$E(N_i) = E(N)(1 + r_1)^{(i-1)/12}, \qquad \text{for } 1 \leq i \leq 120$$

$$E(X_i) = \begin{cases} E(X)(1 + r_2)^{(i-1)/12}, & \text{for } 1 \leq i \leq 60 \\ E(X)(1 + r_2)^{(60-1)/12}(1 + r_3)^{(i-60)/12}, & \text{for } 60 < i \leq 120 \end{cases}$$

where $E(N)$ and $E(X)$ denote the expected values of claim count and severity in the initial accident month. The lag distibution is selected to be exponential for each accident period with the density $f(t)$ as given in Example B. Using equation (7), the mean IBNR value is

$$E(\text{IBNR}) = E(N)E(X) \left\{ \sum_{i=1}^{60} (1 + r_1)^{(i-1)/12}(1 + r_2)^{(i-1)/12} e^{-\vartheta c_i} \right.$$
$$\left. + \sum_{i=61}^{120} (1 + r_1)^{(i-1)/12}(1 + r_2)^{(60-1)/12}(1 + r_3)^{(i-60)/12} e^{-\vartheta c_i} \right\}$$

where $\vartheta = 1/(\text{mean lag})$.

For Scenario A, $r_1 = .06$, $r_2 = r_3 = .05$, with mean exponential lag of 40 months. For any other scenario, the input parameters that are *not* explicitly changed (see Table A) will be the same as those of Scenario A. In computing the percentage change in IBNR values, the $E(N)E(X)$ term drops out.

APPENDIX B

MEAN AND VARIANCE OF A RANDOM SUM

In this appendix, we state (not derive) the appropriate expressions for the mean and variance of a random sum. The interested reader may refer to Feller (1971) or Mayerson, Jones, and Bowers (1968) for the derivation of the results stated below.

Let Y, Y_1, Y_2, ..., Y_n, ... be independent and identically distributed random variables with finite first two moments. Let N denote a nonnegative integer-valued random variable with finite first two moments. A random sum, S_N, is defined as

$$S_N = \sum_{i \le N} Y_i. \tag{B.1}$$

Let us assume that N and Y_1, Y_2, ...are independent variables; then it can be shown—see Feller (1971)—that

$$\mathrm{E}(S_N) = \mathrm{E}(N)\mathrm{E}(Y), \tag{B.2}$$

$$\mathrm{Var}(S_N) = \mathrm{E}(N)\mathrm{Var}(Y) + [\mathrm{E}(Y)]^2\mathrm{Var}(N). \tag{B.3}$$

Equation (B.3) can be rewritten as

$$\mathrm{Var}(S_N) = \mathrm{E}(N)\mathrm{E}(Y^2) + [\mathrm{E}(Y)]^2[\mathrm{Var}(N) - \mathrm{E}(N)]. \tag{B.4}$$

If N is a Poisson random variable, the second term on the right-hand side of (B.4) is equal to zero.

It should be noted that for an indicator random variable I_A (see Section 2), we have

$$\mathrm{E}(I_A) = \mathrm{P}(A)\ , \text{ and}$$
$$\mathrm{E}(I_A^2) = \mathrm{P}(A)\ .$$

These results concerning the mean and second moment (about zero) of the indicator random variable have been used in the derivation of equations (5) and (6) in Section 2.

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