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FOREWORD

The Casualty Actuarial Society was organized in 1914 as the Casualty Actuarial and Statistical Society of America, with 97 charter members of the grade of Fellow; the Society adopted its present name on May 14, 1921.

Actuarial science originated in England in 1792, in the early days of life insurance. Due to the technical nature of the business, the first actuaries were mathematicians; eventually their numerical growth resulted in the formation of the Institute of Actuaries in England in 1848. The Faculty of Actuaries was founded in Scotland in 1856, followed in the United States by the Actuarial Society of America in 1889 and the American Institute of Actuaries in 1909. In 1949 the two American organizations were merged into the Society of Actuaries.

In the beginning of the twentieth century in the United States, problems requiring actuarial treatment were emerging in sickness, disability, and casualty insurance—particularly in workers' compensation—which was introduced in 1911. The differences between the new problems and those of traditional life insurance led to the organization of the Society. Dr. I. M. Rubinow, who was responsible for the Society's formation, became its first president. The object of the Society was, and is, the promotion of actuarial and statistical science as applied to insurance other than life insurance. Such promotion is accomplished by communication with those affected by insurance, presentation and discussion of papers, attendance at seminars and workshops, collection of a library, research, and other means.

Since the problems of workers' compensation were the most urgent, many of the Society's original members played a leading part in developing the scientific basis for that line of insurance. From the beginning, however, the Society has grown constantly, not only in membership, but also in range of interest and in scientific and related contributions to all lines of insurance other than life, including automobile, liability other than automobile, fire, homeowners and commercial multiple peril, and others. These contributions are found principally in original papers prepared by members of the Society and published in the annual *Proceedings*. The presidential addresses, also published in the *Proceedings*, have called attention to the most pressing actuarial problems, some of them still unsolved, that have faced the insurance industry over the years.

The membership of the Society includes actuaries employed by insurance companies, ratemaking organizations, national brokers, accounting firms, educational institutions, state insurance departments, and the federal government; it also includes independent consultants. The Society has two classes of members, Fellows and Associates. Both classes are achieved by successful completion of examinations, which are held in May and November in various cities of the United States and Canada.

The publications of the Society and their respective prices are listed in the *Yearbook* which is published annually. The *Syllabus of Examinations* outlines the course of study recommended for the examinations. Both the *Yearbook*, at a $10 charge, and the *Syllabus of Examinations*, without charge, may be obtained upon request to the Casualty Actuarial Society, One Penn Plaza, 250 West 34th Street, New York, New York 10119.

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NOTICE

Papers submitted to the *Proceedings of the Casualty Actuarial Society* are subject to review by the members of the Committee on Review of Papers and, where appropriate, additional individuals with expertise in the relevant topic. In order to qualify for publication, a paper must be relevant to casualty actuarial science, include original research ideas and/or techniques or have special educational value, and must not have been previously published or be concurrently considered for publication elsewhere. Specific instructions for preparation and submission of papers are included in the *Yearbook* of the Casualty Actuarial Society.

The Society is not responsible for statements of opinions expressed in the papers, criticisms, and discussions published in these *Proceedings*.

Volume LXXIII, Part 1 No. 139

PROCEEDINGS

May 11, 12, 13, 14, 1986

AN ACTUARIAL NOTE ON CREDIBILITY PARAMETERS

HOWARD C. MAHLER

Abstract

In this paper the relationship between the Bayesian credibility parameter, k, *and the classical credibility standard for full credibility,* F, *is examined from a practical standpoint. A very useful "rule of thumb" is developed.*

For most practical applications one can determine the F *that roughly corresponds to* k, *and vice versa. First convert* k *to a number of claims, if necessary, by multiplying by an expected frequency. Then take* F *equal to approximately eight times* k.

A few other interesting results are also derived. Among them is the effect of misestimating the Bayesian credibility parameter k. *The results of using credibility are relatively insensitive to misestimates of* k.

INTRODUCTION

Credibility concepts and formulas are used in many actuarial applications. In this paper some practical questions concerning the use of credibility will be explored. While a few results of theoretical interest are derived, the emphasis is strictly on the practical impacts. This paper assumes that the reader is already generally familiar with credibility. For those interested in the theoretical questions, there are many fine papers, some of which are listed in the references at the end of this paper.

The first question explored is the practical impact of choosing between classical and Bayesian credibility. The answer depends on the parameters used in the two credibility formulas. For a certain simple relationship between the parameters, the choice between classical and Bayesian credibility makes only a relatively small difference. For many practical applications this difference is acceptable.[1]

The second question explored is what is the practical impact of misestimating the Bayesian credibility parameter. The credibilities are relatively insensitive to misestimating this parameter.

CLASSICAL CREDIBILITY FORMULA

This paper assumes the following formula for the "classical" credibility Z_C.

$$Z_C = \begin{cases} (n/F)^{.5} & 0 \leq n \leq F \\ 1 & n \geq F \end{cases} \tag{1}$$

where n is the number of claims, and F is the so-called standard for full credibility. This formula is discussed further in [1] and [2].

BAYESIAN CREDIBILITY FORMULA

This paper assumes the following formula for the "Bayesian" credibility Z_B.

$$Z_B = \frac{P}{P + k}$$

where P is some measure of exposure such as payroll, premium, number of claims, etc. This formula and methods of deriving a value for k are discussed further in [3], [4], [5], [6], and [7].[2]

In many cases P is the number of claims, for example, when we are trying to estimate the average claim cost by class. In those cases where P is an

[1] The degree of accuracy required depends on the particular application. The differences in credibility are given in this paper. The question of whether the resulting differences in the quantity to be estimated are large or small will have to be decided on a case by case basis.

[2] An example of where a more complicated formula holds is given in Meyers [8].

exposure unit other than claims, the formula for credibility can be approximated by multiplying P and k by an estimate of the expected claim frequency.[3] Then

$$Z_B \cong \frac{n}{n + k'}$$

where n is the number of claims and k' is in units of claims; k' equals k times the expected frequency.

For simplicity, hereafter, we will assume a claim-based form of the formula for credibility, such as

$$Z_B = \frac{n}{n + k} \tag{2}$$

where n is the number of claims.

COMPARISON OF THE TWO FORMULAS

The formulas (1) and (2) were derived from different points of view or different methods. A discussion of these differences is beyond the scope of this paper. In spite of these differences, the two formulas yield curves with very similar shapes, as stated in Longley-Cook [1]. This is illustrated in Exhibit 1.

The credibility given by formula (1) is equal to the credibility given by formula (2) when

$$\frac{n}{n + k} = \left(\frac{n}{F}\right)^{.5}$$

$$k = F\ (n/F)^{.5}[1 - (n/F)^{.5}]$$

$$k = FZ_C(1 - Z_C).$$

Since we specifically have $Z_C = Z_B$, this can be written as

$$k = FZ(1 - Z). \tag{3}$$

If we define $R = F/k$, equation (3) can be rewritten as $1/R = Z(1 - Z)$. In other words, the curves given by formula (1) and formula (2) will cross at the

[3] This estimate need not be very accurate since the credibility is not very sensitive to the value of k as shown in a later section of this paper. Therefore, one can usually use a larger body of data to estimate the expected claim frequency sufficiently well for this purpose.

two points where the credibility has the values Z and $1 - Z$, provided we have

$$R = \frac{1}{Z(1 - Z)} . \tag{4}$$

That is, selecting the credibilities Z at which the classical and Bayesian credibilities are to be the same, yields the factor R that is used to relate the credibility parameters. Or, alternatively, given Bayesian parameter k and classical parameter F, formula (4) indicates the points at which the two will yield equivalent credibilities.

Choosing the value of R determines the two credibility values at which the two curves intersect. To cross near the middle,[4] take $1/R = (.5)\ (1 - .5)$ or $R = 4$. To cross near the ends, take $1/R = .1\ (1-.1)$ or $R \cong 11$. In the former case, the two curves are relatively far apart near the end points. In the latter case, the two curves are relatively far apart near the middle.

We are interested in having the two curves be "close" over the entire range of possible values for the credibility. One useful criterion, to define the concept of how close the two curves are, would be the maximum difference between the curves.

Thus, one might want to minimize the maximum difference between the two curves. Taking $R = 6.75$ does so, producing a maximum difference of 13%[5], as illustrated numerically in Exhibits 1 and 2. This is a relatively small difference in credibility. For many practical applications, it will make relatively little difference which credibility formula is utilized, provided that $R \cong 7$.

MINIMIZING VARIANCE

In Bayesian credibility theory, the credibility is chosen so as to minimize the variance of the estimate around the true result.[6] See, for example, the ISO Credibility White Paper [3].

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[4] Actually, in this particular case the two curves are tangent at a single point, $Z = 50\%$.

[5] This problem reduces to the solution of a fifth-degree equation. The solution via numerical analysis is $R \cong 6.757$. The maximum difference of 12.89% occurs at $r = R$ and $r = 1.5401$.

[6] The estimate given by Bayesian credibility is the least squares linear unbiased estimate.

Appendix I shows that if we use in place of the Bayesian credibility, Z_B, a different estimate, $Z_B + \Delta Z$, then the variance increases. The variance is given by a parabola.[7] For small changes from the optimal credibility, there is only a very small increase in the variance. Thus, for most applications, it will make no practical difference if the credibilities used differ slightly from optimal. The use of credibilities other than the optimal one still usually leads to a substantial decrease in variance compared to not using credibility at all. The relative increase in variance is given by

$$\frac{\Delta\ \text{Variance}}{\text{Variance}} = \frac{(\Delta Z)^2}{Z_B(1 - Z_B)} . \tag{5}$$

The full credibility standard that will produce the classical credibility curve with the smallest maximum relative increase in variance requires a choice of R that will minimize the maximum of

$$\frac{(Z_C - Z_B)^2}{Z_B(1 - Z_B)} .$$

The solution is $R = 8$. See Appendix II and Exhibit 3. The maximum increase in the variance in this case is only 12.5% = 1/8.

CHOOSING A RULE OF THUMB

A value of $R = 6.75$ minimizes the maximum difference between the classical and Bayesian credibility curves. However, taking $R = 8$ only increases this maximum difference from 13% to 17%. (See Exhibit 2.) On the other hand, taking $R = 6.75$ rather than $R = 8$, only increases the maximum variance to 1/6.75 = 14.8% from 1/8 = 12.5%. (See Appendix II.) Thus, either 7 or 8 would be equally good integral values of R for use as a general rule of thumb. They each have something to recommend themselves. The author is more concerned with the reduction in variance and thus prefers $R = 8$.

[7] This is the same result noted by Meyers [8]. Meyers' concept of efficiency is closely related to the variance of the estimate around the true result. One minus the efficiency is proportional to that variance.

EXAMPLES OF USES OF THE RULE OF THUMB

Example 1

You generally use Bayesian credibility methods to develop your territory relativities for private passenger automobile. However, you have to file for a rate change in one particular state where rates are tightly regulated. The insurance department refuses to accept anything but classical credibility methods.

Let's assume your Bayesian credibility parameter is 2500 car-years. Then, multiply this by the expected frequency and then by a factor of 8. If the expected frequency is 5%, then we get $2500 \times 5\% \times 8 = 1000$ claims. Thus you can use for your classical credibility standard roughly 1000 claims, for example, the traditional 1084. See Longley-Cook [1].

Example 2

You are computing estimated severities by classification for workers' compensation insurance, using an empirical Bayesian credibility method. When actually implementing the method, you find it is necessary to impose maximums and minimums on the computed values of k, the Bayesian credibility parameter. To aid you in choosing these values, you convert them to a classical credibility basis.

For example, $k = 350$ claims would correspond to a full credibility standard of $350 \times 8 = 2800$ claims. This could be thought of as a frequency standard of 1084, multiplied by a factor of 2.6 in order to convert it to a standard for severity. (2.6 can be thought of as the ratio of the variance of the severity to the square of the mean severity). See Longley-Cook [1].

THE EFFECT OF MISESTIMATING k

Quite often in the use of Bayesian credibility it is necessary to estimate k. For example, one might estimate k from the data as in either [3] or [7]. Fortunately, the results are not very sensitive to the value of k. Let $\tilde{k}$ be our estimate of the correct k.

Let $T = \tilde{k}/k$.

Then, as shown in Appendix III, the maximum difference in the credibility that results from $\tilde{k}$ as an estimate of k is

$$(\Delta Z)_{\mathrm{Max}} = \frac{T - 1}{(1 + \sqrt{T})^2}. \tag{6}$$

For values of T near 1, this is relatively small. (See Exhibit 4.) For example, if $T = 1.25$ or .8, then it is 6%. Even if $T = 2$ or $T = .5$, then the maximum difference is only 17%.[8] In other words, even if the estimated k is wrong by a factor of 2, the estimated credibilities are off by at most 17%.[9] For many practical purposes this is an acceptable difference.

In Appendix IV it is shown that the maximum change in variance is given by:

$$\left(\frac{\Delta V}{V}\right)_{Max} = \frac{(T-1)^2}{4T} \tag{7}$$

For values of T near 1, this is relatively small. (See Exhibit 5.) For example, if $T = 1.5$ or 2/3, then it is 4%. Even if $T = 2$ or .5, then the maximum relative increase in the variance is only $1/8 \cong 13\%$.[10] Once again, even if the estimated k is wrong by a factor of 2 in either direction, for many practical purposes the result is still acceptable.

CONCLUSION

For most practical applications, one can determine the standard for full credibility F that roughly corresponds to the Bayesian credibility parameter k, and vice versa. First convert k to a number of claims, if necessary, by multiplying by an expected frequency. Then take F equal to approximately eight times k.

When estimating the Bayesian credibility parameter k, the estimate need not be extremely precise. For many practical applications, the estimate of k can be wrong by as much as a factor of two in either direction and still produce a fairly good estimate of the quantity, e.g., frequency, severity, pure premium, etc., that credibility is being used to estimate.

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[8] For $T = 2$, this maximum difference occurs when the correct credibility is 58.6% and the estimated credibility is 41.4%. For $T = .5$, the correct and estimated credibilities are reversed.

[9] Of course, if the estimated k is wrong by more than a factor of 2, the estimated credibilities can be off by more than 17%.

[10] For $T = 2$, this maximum relative increase in variance occurs when the correct credibility is 2/3. For $T = .5$, this occurs when the correct credibility is 1/3.

ACKNOWLEDGEMENTS

The author wishes to thank Robert Conger and Richard Derrig for reading an earlier version of this paper and providing helpful comments. I also wish to thank Lesley Phipps for typing the paper and Margot Ghanouni for producing the accompanying graphs.

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[5] C. C. Hewitt, Jr., Discussion of Mayerson: "A Bayesian View of Credibility," *PCAS* LII, 1965, p. 121.

[6] C. C. Hewitt, Jr., "Credibility for Severity," *PCAS* LVII, 1970, p. 148.

[7] G. G. Meyers, "Empirical Bayesian Credibility for Workers' Compensation Classification Ratemaking," *PCAS* LXXI, 1984, p. 96.

[8] G. G. Meyers, "An Analysis of Experience Rating," *PCAS* LXXII, 1985, p. 278.

EXHIBIT 1
PART 1

ILLUSTRATIVE COMPARISON OF CREDIBILITIES
BAYESIAN CREDIBILITY WITH $k = 200$ VERSUS
CLASSICAL CREDIBILITY WITH VARIOUS VALUES OF F

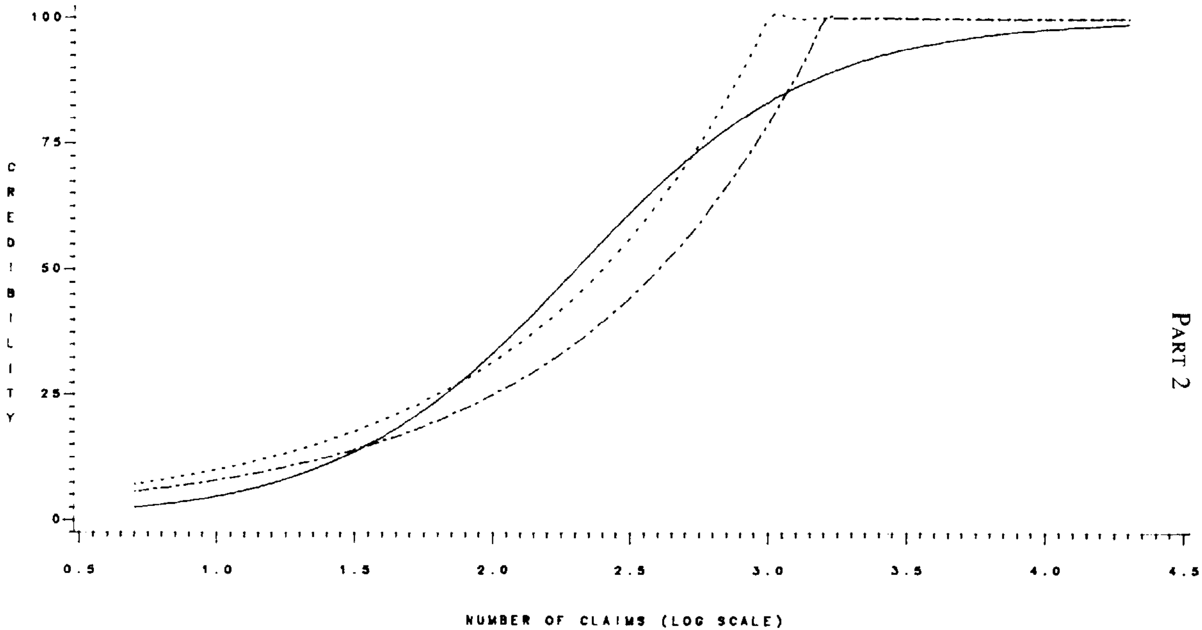
Claims	Bayesian Credibility $k = 200$	Classical Credibility $F = 1000$	$F = 1200$	$F = 1350$	$F = 1400$	$F = 1600$
5	2%	7%	6%	6%	6%	6%
10	5	10	9	9	8	8
20	9	14	13	12	12	11
30	13	17	16	15	15	14
40	17	20	18	17	17	16
50	20	22	20	19	19	18
60	23	24	22	21	21	19
70	26	26	24	23	22	21
80	29	28	26	24	24	22
90	31	30	27	26	25	24
100	33	32	29	27	27	25
125	38	35	32	30	30	28
150	43	39	35	33	33	31
175	47	42	38	36	35	33
200	50	45	41	38	38	35
250	56	50	46	43	42	40
300	60	55	50	47	46	43
350	64	59	54	51	50	47
400	67	63	58	54	53	50
450	69	67	61	58	57	53
500	71	71	65	61	60	56
600	75	77	71	67	65	61
700	78	84	76	72	71	66
800	80	89	82	77	76	71
900	82	95	87	82	80	75
1000	83	100	91	86	85	79
1200	86	100	100	94	93	87
1400	88	100	100	100	100	94
1600	89	100	100	100	100	100
1800	90	100	100	100	100	100
2000	91	100	100	100	100	100
3000	94	100	100	100	100	100
4000	95	100	100	100	100	100
5000	96	100	100	100	100	100
10000	98	100	100	100	100	100
20000	99	100	100	100	100	100

$$Z_B = \frac{n}{n + k}$$

$$Z_C = \begin{cases} (n/F)^{.5} & 0 \leqq n \leqq F \\ 1 & n \geqq F \end{cases}$$

EXHIBIT 1
PART 2

ILLUSTRATIVE COMPARISON OF CREDIBILITIES
BAYESIAN CREDIBILITY WITH K=200 VS. CLASSICAL CREDIBILITY
WITH VARIOUS VALUES OF F



(in Powers of 10)

DASHED LINE BASED ON CLASSICAL CREDIBILITY WITH CLAIMS = 1000.
VARIED LINE BASED ON CLASSICAL CREDIBILITY WITH CLAIMS = 1600.
CONTINUOUS LINE BASED ON A BAYESIAN CREDIBILITY WITH K = 200.

EXHIBIT 2
Part 1

Classical Credibility Minus Bayesian Credibility

r = Claims ÷ k	$R = 5$	$R = 6$	$R = 6.75$	$R = 7$	$R = 8$
.025	5%	4%	4%	4%	3%
.05	5	4	4	4	3
.10	5	4	3	3	2
.15	4	3	2	2	1
.20	3	2	1	0	−1
.25	2	0	−1	−1	−2
.30	1	1	−2	−2	−4
.35	1	−2	3	−4	−5
.40	0	3	−4	−5	−6
.45	−1	−4	−5	−6	−7
.50	−2	−4	6	−7	−8
.625	−3	−6	8	−9	11
.75	−4	−8	−10	−10	−12
.875	−5	−8	−11	−11	−14
1.00	−5	−9	−11	−12	−15
1.25	−6	−10	12	−13	16
1.50	−5	−10	−13	−14	−17
1.75	−4	−10	−13	−14	−17
2.00	−3	−9	−13	13	−17
2.25	−2	−8	−12	−13	−16
2.50	−1	−7	−11	−12	−16
3.00	2	−4	8	−10	−14
3.50	6	−1	−6	−7	−12
4.00	9	2	−3	−4	−9
4.50	13	5	0	−2	−7
5.00	17	8	3	1	−4
6.00	14	14	9	7	1
7.00	13	13	13	13	6
8.00	11	11	11	11	11
9.00	10	10	10	10	10
10.00	9	9	9	9	9
15.00	6	6	6	6	6
20.00	5	5	5	5	5
25.00	4	4	4	4	4
50.00	2	2	2	2	2
100.00	1	1	1	1	1

$$R = \frac{F}{k}$$

$$Z_C - Z_B = \begin{cases} \left(\dfrac{r}{R}\right)^{.5} - \dfrac{r}{1 + r} & 0 \leq r \leq R \\ \dfrac{1}{1 + r} & r \geq R \end{cases}$$

EXHIBIT 2
PART 2

CLASSICAL MINUS BAYESIAN CREDIBILITY

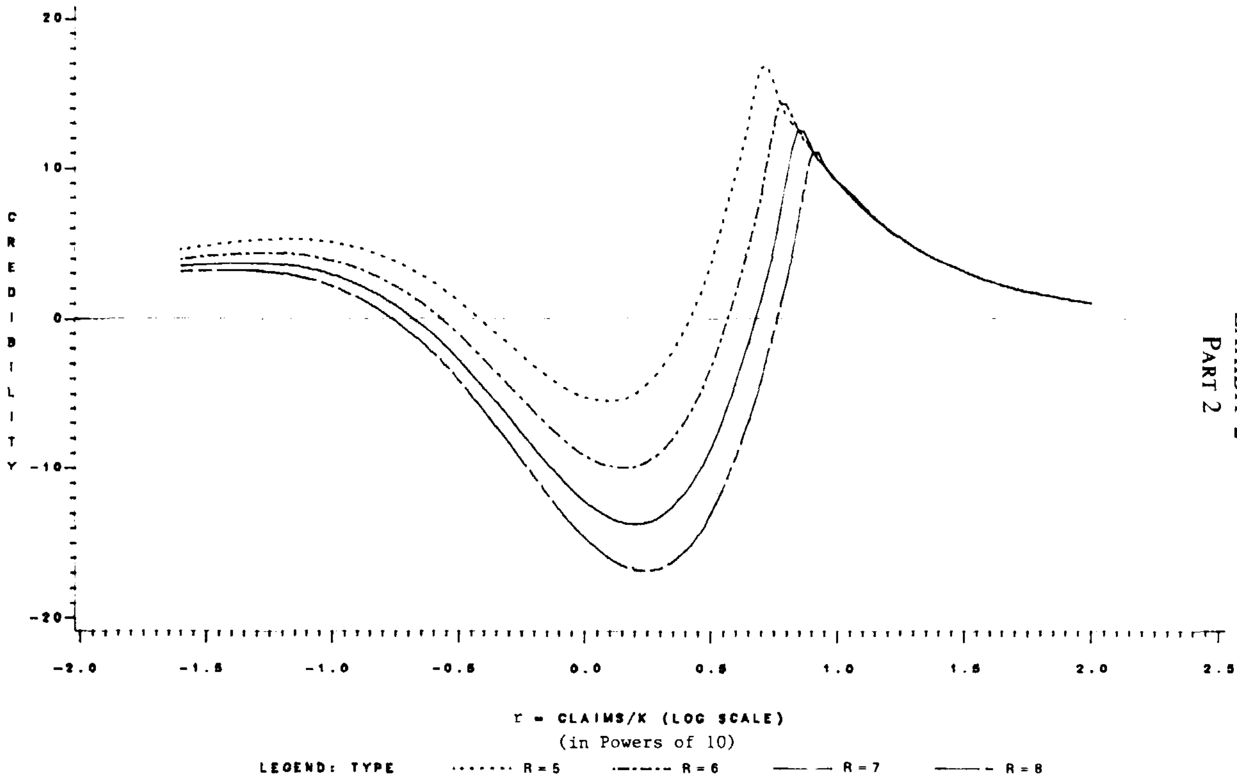


EXHIBIT 3
PART 1

INCREASE IN VARIANCE THROUGH USE OF CLASSICAL CREDIBILITY RATHER THAN BAYESIAN CREDIBILITY

r = Claims ÷ k	$R = 7$	$R = 8$	$R = 9$
0	14%	13%	11%
.0005	13	11	10
.001	12	10	9
.005	10	8	7
.010	8	7	6
.015	7	6	5
.02	6	5	4
.03	5	4	3
.04	4	3	2
.05	3	2	2
.10	1	1	0
.20	0	0	0
.30	0	1	1
.40	1	2	3
.50	2	3	4
.75	4	6	8
1.00	6	9	11
1.50	8	12	15
2.00	8	13	17
2.50	7	12	17
3.00	5	10	16
3.50	3	8	14
4.00	1	5	11
4.50	0	3	8
5.00	0	1	6
5.50	1	0	3
6.00	4	0	1
6.50	8	1	0
7.00	14	3	0
7.50	13	7	1
8.00	13	13	3
8.50	12	12	6
9.00	11	11	11
9.50	11	11	11
10.00	10	10	10
15.00	7	7	7
20.00	5	5	5
25.00	4	4	4
50.00	2	2	2
100.00	1	1	1

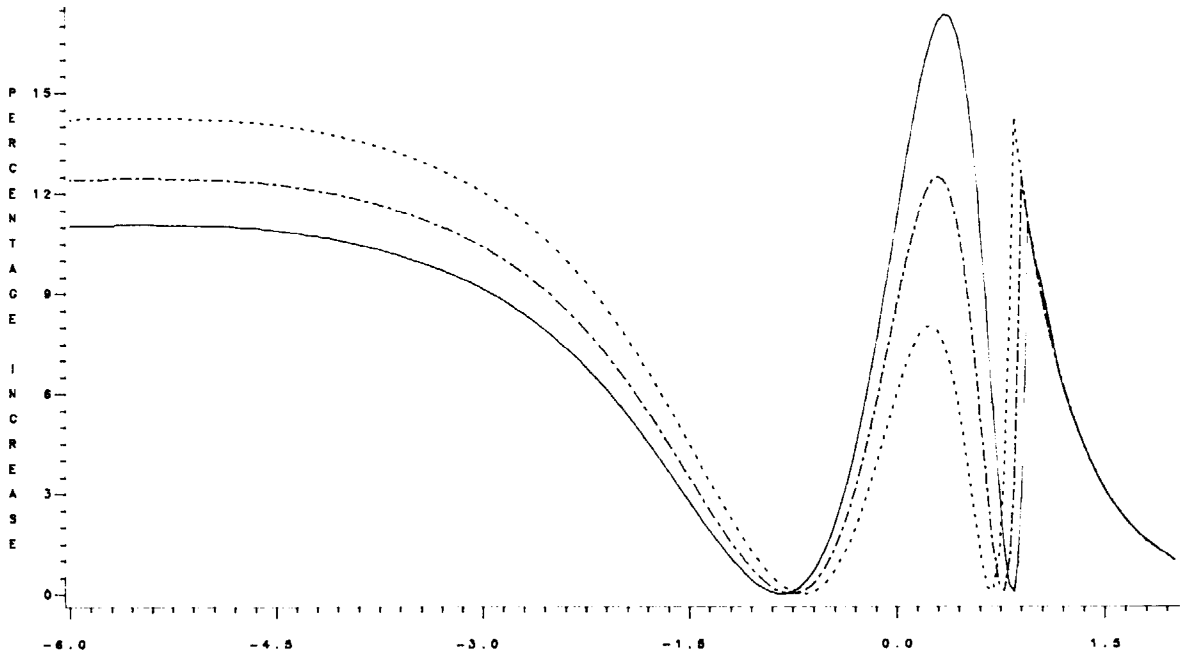
Note: The value given for $r = 0$ is actually the limit as $r \to 0$

$R = F/k$

$$\frac{\Delta V}{V} = \frac{(\Delta Z)^2}{Z_B(1 - Z_B)}$$ See Appendix II.

EXHIBIT 3
PART 2

INCREASE IN VARIANCE
THROUGH USE OF CLASSICAL CREDIBILITY
RATHER THAN BAYESIAN CREDIBILITY



r = CLAIMS/K (LOG SCALE)
(in Powers of 10)

LEGEND: TYPE .... R=7 —·—·— R=8 ——— R=9

EXHIBIT 4
PART 1

BAYESIAN CREDIBILITY

DIFFERENCE IN CREDIBILITY DUE TO MISESTIMATING k
ESTIMATED CREDIBILITY MINUS CORRECT CREDIBILITY

r	$T = 1/3$	$T = 1/2$	$T = 2/3$	$T = .8$	$T = 1.25$	$T = 1.5$	$T = 2$	$T = 3$
0	0%	0%	0%	0%	0%	0%	0%	0%
.01	2	1	0	0	0	0	0	−1
.02	4	2	1	0	0	−1	−1	−1
.05	8	4	2	1	−1	−2	−2	−3
.10	14	8	4	2	−2	−3	−4	−6
.25	23	13	7	4	−3	−6	−9	−12
.50	27	17	10	5	−5	−8	−13	−19
.75	26	17	10	6	−5	−10	−16	−23
1.00	25	17	10	6	−6	−10	−17	−25
1.50	22	15	9	5	−5	−10	−17	−27
2.00	19	13	8	5	−5	−10	−17	−27
5.00	10	8	5	3	−3	−6	−12	−21
10.00	6	4	3	2	−2	−4	−8	−14
20.00	3	2	2	1	−1	−2	−4	−8
50.00	1	1	1	0	0	−1	−2	−4
100.00	1	0	0	0	0	0	−1	−2

Note: r = Exposures ÷ k $\quad T = \dfrac{\tilde{k}}{k} = \dfrac{\text{Estimated Bayesian Credibility Parameter}}{\text{Correct Bayesian Credibility Paramer}}$

$$\Delta Z = \frac{r(1 - T)}{(1 + r)(T + r)}$$ See Appendix III.

EXHIBIT 4
PART 2

DIFFERENCE IN CREDIBILITY DUE TO MISESTIMATING K

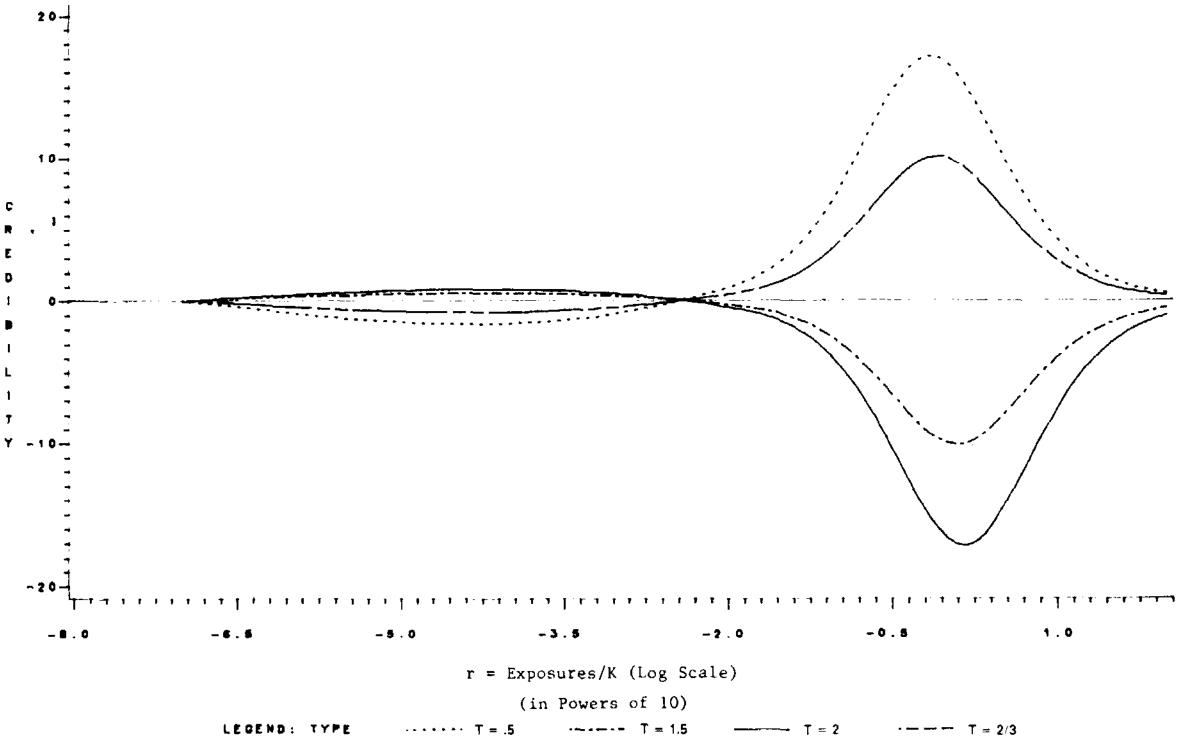


EXHIBIT 5
PART 1

BAYESIAN CREDIBILITY
INCREASE IN VARIANCE DUE TO MISESTIMATING k

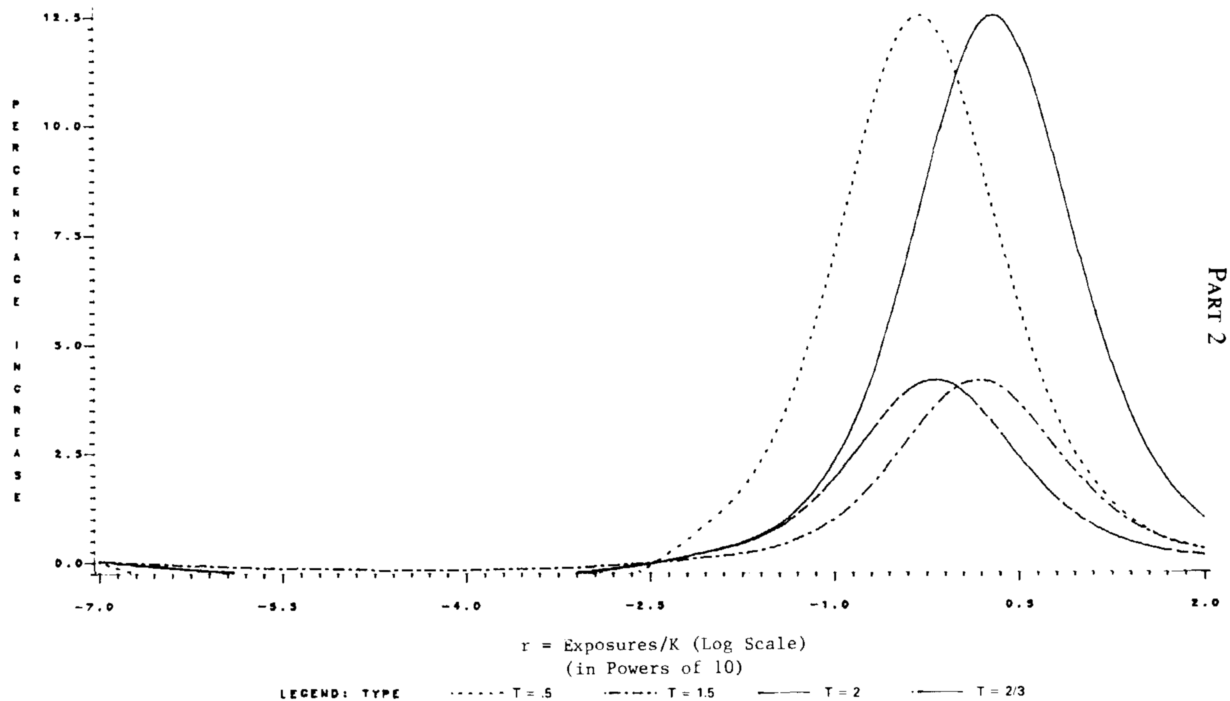
r	$T = 1/3$	$T = 1/2$	$T = .6$	$T = 2/3$	$T = .8$	$T = 1.25$	$T = 1.5$	$T = 1.75$	$T = 2$	$T = 3$
0	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
.01	4	1	0	0	0	0	0	0	0	0
.02	7	2	1	0	0	0	0	0	0	1
.05	15	4	2	1	0	0	1	1	1	2
.10	24	7	3	2	0	0	1	2	2	4
.25	33	11	6	3	1	1	2	4	5	9
.50	32	13	7	4	1	1	3	6	8	16
.75	28	12	7	4	1	1	4	7	10	21
1.00	25	11	6	4	1	1	4	7	11	25
1.50	20	9	5	4	1	1	4	8	12	30
2.00	16	8	5	3	1	1	4	8	13	32
5.00	8	4	3	2	1	1	3	6	10	31
10.00	4	2	1	1	0	0	2	4	7	24
20.00	2	1	1	1	0	0	1	2	4	15
50.00	1	0	0	0	0	0	0	1	2	7
100.00	0	0	0	0	0	0	0	1	1	4

Note: r = Exposures $\div$ k $\qquad T = \frac{\tilde{k}}{k} = \frac{\text{Estimated Bayesian Credibility Parameter}}{\text{Correct Bayesian Credibility Parameter}}$

$$\frac{\Delta V}{V} = \frac{r(T-1)^2}{(T+r)^2}$$ See Appendix IV.

EXHIBIT 5
PART 2

INCREASE IN VARIANCE
DUE TO MISESTIMATING K



APPENDIX I

This appendix derives an expression for the relative increase in variance that occurs when one uses a value for the credibility other than that indicated by Bayesian credibility. It is shown that the variance is given by a parabola.[11] The bottom of the parabola, i.e. minimum variance, occurs when the value for credibility indicated by Bayesian credibility is used. For different values near this, the increase in variance is relatively small.

Let X be a random variable whose distribution depends on a parameter θ. Let the mean of X for the value of the parameter θ be given by $\mu(\theta) = E[X/\theta]$.

Let F be an estimate of μ that gives weight a to the observed value X and weight $1 - a$ to the overall mean M.

$$F = aX + (1 - a)M$$

where $M = E(X) = E_\theta[E[X/\theta]]$.

F is a function of the parameter a.

We wish to determine the variance of the estimate F around the mean μ, averaged over all possible values of the parameter θ.

$$\begin{aligned}
\text{Let } V(a) &= E_\theta[E[(F - \mu)^2/\theta]]. \\
\text{Let } \tau^2 &= VAR_\theta[\mu(\theta)] = E_\theta[(\mu(\theta) - M)^2] = \text{“between variance”} \\
\delta^2 &= E_\theta[VAR[X/\theta]] = \text{“within variance.”} \\
F - \mu &= a(X - \mu) + (1 - a)(M - \mu) \\
(F - \mu)^2 &= a^2(X - \mu)^2 + (1 - a)^2(M - \mu)^2 \\
&\quad + 2a(1 - a)(X - \mu)(M - \mu) \\
E[(F - \mu)^2/\theta] &= a^2 VAR[X/\theta] + (1 - a)^2(M - \mu(\theta))^2 \\
V(a) &= E_\theta[E[(F - \mu)^2/\theta]] = a^2\delta^2 + (1 - a)^2\tau^2
\end{aligned}$$

Thus, this variance is given by a parabola in a. $V(a) = a^2\delta^2 + (1 - a)^2\tau^2$. It has a minimum when the derivative is zero.

$$0 = 2a\delta^2 - 2(1 - a)\tau^2$$

$$a = \frac{\tau^2}{\tau^2 + \delta^2}$$

[11] This well known result is given for example in Appendix B of Meyers [7].

$$V\left(\frac{\tau^2}{\tau^2 + \delta^2}\right) = \frac{\delta^2\tau^2}{\delta^2 + \tau^2}$$

Thus, combining the observed value with the overall mean reduces the variance. It is interesting to note in passing that

$$\frac{1}{\text{Minimum Variance}} = \frac{\tau^2 + \delta^2}{\delta^2\tau^2} = \frac{1}{\delta^2} + \frac{1}{\tau^2}$$

$$= \frac{1}{\text{the variance if you use observation}}$$

$$+ \frac{1}{\text{the variance if you use overall mean}} .$$

It is useful to think in terms of the reciprocals of the variance. We want to maximize the reciprocal variance by combining our two estimates. The maximum reciprocal variance is just the sum of the two individual reciprocal variances. Thus, the best that can be done is to double the reciprocal variance (when the two individual variances happen to be equal) and thus halve the variance.[12]

The usual expression for the Bayesian credibility is the value for the parameter a that gives the minimum variance, $Z_B = \tau^2/(\tau^2 + \delta^2)$.

The variance is larger than the minimum for $a = Z_B + \Delta Z$. In this case,

$$\Delta V = V(Z_B + \Delta Z) - V(Z_B) = V'(Z_B)\Delta Z + V''(Z_B)\,\frac{(\Delta Z)^2}{2}$$

where V' and V'' are the first and second derivatives, respectively. (Higher derivatives are zero since V is given by a parabola.) Then

$$\Delta V = (\Delta Z)^2(\delta^2 + \tau^2)$$

$$\frac{\Delta V}{V} = \frac{(\Delta Z)^2(\delta^2 + \tau^2)^2}{\delta^2\tau^2}$$

$$\frac{\Delta V}{V} = \frac{(\Delta Z)^2}{Z_B(1 - Z_B)} .$$

This is the desired expression for the relative increase in variance that occurs when the value used for the credibility is other than that indicated by Bayesian credibility.

[12] A related result is given in Appendix C of Chapter 2 of the ISO Credibility White Paper [3]. The optimal weights to assign to the individual estimates are inversely proportional to the variances.

APPENDIX II

This appendix explores the behavior of the expression derived in Appendix I for the relative change in variance. It is shown that $\Delta V/V$ has the smallest maximum for $R = 8$.

$$\text{Let } R = \frac{F}{k} = \frac{\text{Standard for Full Credibility}}{\text{Bayesian Credibility Parameter}}$$

$$r = \frac{n}{k} = \frac{\text{Number of Claims}}{\text{Bayesian Credibility Parameter}}.$$

$$\text{Let } g(r,R) = \frac{(Z_B - Z_C)^2}{(Z_B)(1 - Z_B)}.$$

This is the expression derived in Appendix I for $\Delta V/V$. However,

$$\frac{1}{Z_B} = \frac{n+k}{n} = 1 + \frac{1}{r} = \frac{1+r}{r}$$

$$\frac{1}{1 - Z_B} = \frac{n+k}{k} = 1 + r$$

$$Z_B - Z_C = \begin{cases} \dfrac{n}{n+k} - \left(\dfrac{n}{F}\right)^{.5} & 0 \leqq n \leqq F \\ \dfrac{n}{n+k} - 1 & F \geqq n \end{cases}$$

$$Z_B - Z_C = \begin{cases} \dfrac{r}{1+r} - \left(\dfrac{r}{R}\right)^{.5} & r \leqq R \\ -\dfrac{1}{1+r} & r \geqq R \end{cases}$$

Therefore, if $r \geqq R$,

$$g(r,R) = \left(-\frac{1}{1+r}\right)^2 \left(\frac{1+r}{r}\right)(1+r) = \frac{1}{r},$$

and, if $r \leqq R$,

$$g(r,R) = \left(\frac{r}{1+r} - \left(\frac{r}{R}\right)^{.5}\right)^2 \left(\frac{1+r}{r}\right)(1+r) = \left(\sqrt{r} - \frac{(1+r)}{\sqrt{R}}\right)^2.$$

For any given R, the local maximums on the interval $0 \leqq r \leqq R$ occur at $r = 0$, $r = R/4$, $r = R$.[13]

$$g(0,R) = g(R,R) = 1/R$$
$$g(\tfrac{1}{4}R,R) = 1/R + \tfrac{1}{2}(R/8 - 1)$$

Thus, $\text{MAXIMUM}_r\ g(r,R) = \begin{cases} 1/R & R \leqq 8 \\ 1/R + R/16 - \tfrac{1}{2} & R \geqq 8 \end{cases}$

Thus, $\text{MINIMUM}_R\ \text{MAXIMUM}_r\ g(r,R) = 1/8$, which occurs when $R = 8$.

[13] The first and last are endpoints. The second has $\partial g/\partial r = 0$. The other points where the partial derivative is zero are the minimums where $g = 0$. For $r \geqq R$, $g(r,R) = 1/r$ and is decreasing.

APPENDIX III

The appendix details the derivation of an expression for the maximum difference in Bayesian credibilities that occurs when an estimated value for the Bayesian credibility parameter k is used, rather than the correct value of the parameter.

$$\text{Let } T = \frac{\tilde{k}}{k} = \frac{\text{Estimate of Bayesian Credibility Parameter}}{\text{Correct Bayesian Credibility Parameter}}$$

$$r = \frac{n}{k} = \frac{\text{Exposures}}{\text{Correct Bayesian Credibility Parameter}} .$$

Then the difference in credibilities is

$$\Delta Z = \frac{n}{n + \tilde{k}} - \frac{n}{n + k} = \frac{r}{r + T} - \frac{r}{r + 1}$$

$$\Delta Z = \frac{r(1 - T)}{(1 + r)(T + r)} .$$

As expected, when k is overestimated, $(T > 1)$, the estimated credibility is too low, $(\Delta Z < 0)$.

Taking the partial derivative of ΔZ with respect to r indicates that ΔZ has a maximum when $r = T^{.5}$. The maximum value of $|\Delta Z|$ is

$$|\Delta Z|_{\text{Max}} = \frac{|T - 1|}{(T^{.5} + 1)^2} .$$

As expected, this quantity has a minimum value of zero at $T = 1$, i.e., when the Bayesian credibility parameter is correctly estimated. This expression has the same value for T and $1/T$. In other words, when k is misestimated by a given factor, the magnitude of the maximum difference in the credibility is the same whether k is overestimated or underestimated.

APPENDIX IV

This appendix derives an expression for the relative increase in variance that occurs when an estimated value for the Bayesian credibility parameter k is used, rather than the correct parameter value. An expression for the maximum relative increase in variance is also derived.

$$\text{Let } T = \frac{\bar{k}}{k} = \frac{\text{Estimate of Bayesian Credibility Parameter}}{\text{Correct Bayesian Credibility Parameter}}$$

$$r = \frac{n}{k} = \frac{\text{Exposures}}{\text{Correct Bayesian Credibility Parameter}} .$$

Then, from equation (5),

$$\frac{\Delta V}{V} = \frac{(\Delta Z)^2}{Z_B(1 - Z_B)} ,$$

but, as is shown in Appendix III,

$$\Delta Z = \frac{r(T - 1)}{(1 + r)(T + r)} .$$

Also note that

$$\frac{1}{Z_B} = \frac{n + k}{n} = \frac{1 + r}{r}$$

$$\frac{1}{1 - Z_B} = \frac{n + k}{k} = 1 + r$$

Substituting in equation (5) gives

$$\frac{\Delta V}{V} = \frac{r(T - 1)^2}{(T + r)^2}$$

Taking the partial derivative with respect to r indicates a maximum when $r = T$. Therefore, the maximum value of $\Delta V/V$ is

$$\left(\frac{\Delta V}{V}\right)_{\text{Max}} = \frac{(T - 1)^2}{4T} .$$

As expected, this quantity has a minimum value of zero at $T = 1$, i.e., when the Bayesian credibility parameter has been correctly estimated. This expression has the same value for T and $1/T$. In other words, the maximum relative increase in variance is the same whether k has been overestimated or underestimated by a given factor.

In Appendix III, the same behavior was noted for the maximum difference in credibility. The factor by which k is misestimated, rather than $/k - \bar{k}/$, the difference between the estimated and correct values, is the important quantity.[14]

[14] Therefore, we would expect that confidence intervals for k would not be symmetric around our best estimate. Rather, they should be larger on the high end and smaller on the low end. This behavior was noted in Section 7 of Meyers [8].

CLASSICAL PARTIAL CREDIBILITY WITH APPLICATION TO TREND

GARY G. VENTER

Abstract

Even with the recent advances in Bayesian credibility theory, there remain situations in which some may prefer the classical approach. Such situations may include data limitations, the failure of Bayesian model assumptions, the desire to incorporate a broader class of auxiliary information, ease of calculation and explanation, or just the force of tradition.

This paper discusses a probabilistic interpretation of the classical square root rule which provides some rationale for its use. The same rationale applied to trend projections leads to a similar rule, which utilizes the relative goodness of fit of the trend line.

While classical credibility for pure premiums is calculated from the volume of data used, the importance of volume is only in determining certain confidence intervals, which in turn determine credibility. In the trend model, the relative goodness of fit determines the confidence intervals. Using these confidence intervals in the same manner as in the pure premium case yields classical credibilities for the trend.

Volume is important here only to the extent that the stability it imparts contributes to the goodness of fit. As there may be other influences affecting the fit, volume alone does not guarantee high credibility in the trend case.

Credibility requirements under the Normal Power approximation also are reviewed. For these a partial credibility method different from the square root formula is indicated.

Partial credibility in the "classical" approach (Longley-Cook [5]) has often been presented in a somewhat ad hoc fashion, not particularly related to the statistical development of the full credibility standard.

An exception is provided by the "limited fluctuation" development (DeVylder [2] and Hossack, Pollard, and Zehnwirth [4]), which shows that the square root rule can be given a reasonable probabilistic interpretation when the full credibility standard is developed from a normal approximation to aggregate losses. The limited fluctuation concept is similar to an interpretation of credibility theory found in the 1932 *PCAS* (Perryman [8]).

The present paper outlines the limited fluctuation interpretation of credibility and uses it to develop a classical credibility approach to trend. As a further illustration, this method is extended to the computation of partial credibilities when the normal power approximation to aggregate losses is used to develop a full credibility standard (Mayerson, Jones, and Bowers [6]). To present this method clearly, a review of the standard credibility procedure is in order.

ELEMENTARY CREDIBILITY FROM AN ADVANCED STANDPOINT

The primary focus in classical credibility is the establishment of a full credibility standard. This is viewed as the expected number of claims needed to meet a predefined standard of stability of the aggregate losses. ("Aggregate losses" refers to the total dollar amount of the claims.) The standard is expressed in terms of confidence intervals. A typical standard would be that there be a 90% probability of observed aggregate losses for a year being within ±5% of the expected aggregate losses.

The limited fluctuation approach to partial credibility proceeds by establishing a confidence interval of the same precision and width as desired for full credibility, but centered at the credibility weighted estimate rather than at the observed mean. Some rationale for this method will be discussed below. This approach turns out to yield the square root rule for partial credibility in the case that aggregate losses are adequately approximated by a normal probability distribution. This distribution may not be a very good approximation in practice, but it is useful to illustrate the development of the theory. The development of the full credibility standard under this assumption proceeds as follows.

Since the normal distribution is symmetric about its mean, a 90% probability of the aggregate losses T being within $\pm k\mathrm{E}(T)$ of $\mathrm{E}(T)$ corresponds to a 95% probability of T being below $\mathrm{E}(T)(1 + k)$. In general, a probability p of T being within $\pm k\mathrm{E}(T)$ of $\mathrm{E}(T)$ translates to a probability of $.5(1 + p)$ of T being below $\mathrm{E}(T)(1 + k)$. For notational convenience, then, let $d = .5(1 + p)$ and y_d denote the dth quantile of the standard normal distribution, i.e., there is a probability d that a standard normal variate is less than y_d. For example, $y_{.95} = 1.645$.

Thus, to meet the standard of T being within $\pm k\mathrm{E}(T)$ of $\mathrm{E}(T)$ with probability p, $k\mathrm{E}(T)$ must equal y_d standard deviations of T, i.e., $k\mathrm{E}(T) = y_d\sqrt{\mathrm{Var}(T)}$. To express this standard in terms of the number of claims requires an expression for the variance of T in terms of the moments of N, the number of claims, and

X, the claim size. This expression, derived in Appendix 2, is

$$\mathrm{Var}(T) = \mathrm{Var}(X)\mathrm{E}(N) + \mathrm{Var}(N)\,\mathrm{E}(X)^2.$$

Thus, the full credibility requirement is

$$k^2\mathrm{E}(X)^2\mathrm{E}(N)^2 = y_d^2\,(\mathrm{Var}(X)\mathrm{E}(N) + \mathrm{Var}(N)\mathrm{E}(X)^2)$$

or

$$\mathrm{E}(N) = (y_d/k)^2((\mathrm{Var}(X)/\mathrm{E}(X)^2) + (\mathrm{Var}(N)/\mathrm{E}(N))).$$

Now, this is supposed to be an equation for E(N), but E(N) also occurs on the right side. However, the ratio Var(N)/E(N) can often be treated as a constant of the frequency distribution. In fact for a Poisson frequency, this constant is 1.0. The negative binomial distribution with parameters x and p has $\mathrm{E}(N) = x(1 - p)/p$ and $\mathrm{Var}\,(N) = x\,(1 - p)/p^2$, so the ratio of variance to mean is $1/p$. As long as p does not change, the expected number of claims can increase or decrease due to the x parameter without influencing the variance to mean ratio.

For any frequency distribution, increasing E(N) by adding independent identically distributed exposure units does not change this ratio, because Var(N) will increase proportionally. (For independent risks, $\mathrm{E}(N + M) = \mathrm{E}(M) + \mathrm{E}(N)$ and $\mathrm{Var}(N+M) = \mathrm{Var}(N)+\mathrm{Var}\,(M)$. From this it follows that if $\mathrm{Var}(N)/\mathrm{E}(N) = r = \mathrm{Var}(M)/\mathrm{E}(M)$, then also $\mathrm{Var}(N + M)/\mathrm{E}(N + M) = r$.) In more sophisticated models, large risks or portfolios are not assumed to behave as aggregations of independently distributed exposure units, and then this ratio is not a constant (Meyers and Schenker [7]). However, this constancy will be assumed here. Thus, the full credibility standard can be written as

$$\mathrm{E}(N) = c\,(y_d/k)^2,$$

where $c = (\mathrm{Var}(X)/\mathrm{E}(X)^2) + (\mathrm{Var}(N)/\mathrm{E}(N))$ is a constant of the distribution. The first term of c can be denoted as CV^2 with CV the severity coefficient of variation. For example, with a Poisson frequency, $c = 1 + CV^2$.

A standard example (Longley-Cook [5]) is given by a Poisson frequency and a severity distribution with $CV = 0$ (constant severity) and thus $c = 1$. Taking $y_d = 1.645$ and $k = .05$ then yields $\mathrm{E}(N) = 1082.4$. This might be a reasonable standard for claim frequency, or for aggregate losses with constant severity. To achieve the same confidence intervals, still with a Poisson frequency, this standard would have to be multiplied by $1 + CV^2$ to account for severity variation. In Longley-Cook [5], this factor is referred to as $1 + S_c^2/M_c^2$ and in Hossack [4] as $1 + (\sigma/m)^2$.

It is of interest to note that c is also invariant under scale changes in severity, since $\text{Var}(X)/\text{E}(X)^2$ has this invariance. A scale change is a transformation that affects every claim by a uniform factor, such as simple monetary inflation (Venter [11]). Real world inflation may affect different claim sizes differently, however (Rosenberg and Halpert [9]). $\text{Var}(X)/\text{E}(X)^2$ is invariant in this sense because numerator and denominator both change by r^2 under a scale change of r. Thus, for given constants p and k, the credibility standard will not change due to growth of the business (i.e., addition of independent identically distributed exposure units) or uniform inflation.

In practical applications, $\text{E}(N)$ is often estimated by the number of claims arising. Thus, for example, if 1,082 expected claims is the full credibility standard, a body of experience with 1,082 claims may be deemed fully credible. The model, however, specifies a standard in terms of the exact expected number of claims. Using an estimate of this expected number changes the confidence intervals. Expected claims of 1,000 or of 1,164, for example, could occasionally produce 1,082 claims. Using $k = y_a\sqrt{c}/\text{E}(N)$ yields k's of .052 and .048 for these two expected values. Thus, the confidence interval widths arising in practice may be slightly different than those contemplated by the theory. This problem seems to be minor, given the degree of judgment used to select k originally.

PARTIAL CREDIBILITY

When $\text{E}(N)$ is less than the full credibility standard, a weighting scheme is used to estimate $\text{E}(T)$. The estimate, u, is a weighted average of the observed aggregate claims T with v, a previous estimate of $\text{E}(T)$. The previous estimate v can be regarded as the best available estimate of $\text{E}(T)$ without the observation T. Thus

$$u = zT + (1 - z)v.$$

Under the limited fluctuation partial credibility approach, the weight z is calculated so that there will be a probability p of u being within $k\text{E}(T)$ of $z\text{E}(T) + (1 - z)v$, where p and k are the defining constants of the full credibility standard. Thus the credibility estimate u is, with probability p, within the originally desired distance $k\text{E}(T)$ of a weighted average of $\text{E}(T)$ and the previous estimate v.

For u to meet this criterion, zT must be within $k\text{E}(T)$ of $z\text{E}(T)$ with probability p, as can be seen from the definition of u. This is equivalent to requiring T to be within $(k/z)\mathrm{E}(T)$ of $\mathrm{E}(T)$ with probability p. But this is just the full credibility requirement with k replaced by k/z. Thus, under the above assumptions, the expected number of claims needed for credibility z is

$$\mathrm{E}(N) = c(y_d/(k/z))^2.$$

Comparing the resulting expected number of claims N_z needed for a credibility of z to the full credibility standard N_f yields that

$$N_z = z^2 N_f$$

or

$$z = \sqrt{N_z/N_f}.$$

That is, the credibility factor z for an expected number of claims N_z is just the square root of the ratio of N_z to the full credibility standard N_f, with a maximum of $z = 1$.

Also, since $(k/z)\mathrm{E}(T)$ is the width of the p confidence interval when $\mathrm{E}(N) = N_z$, then z is just the ratio of the target p confidence interval $k\mathrm{E}(T)$ to the wider p confidence interval around $\mathrm{E}(T)$, $(k/z)\ \mathrm{E}(T)$, that arises for $\mathrm{E}(N) = N_z$. As a result, the p confidence interval around $z\mathrm{E}(T)$ is of the targeted width $k\mathrm{E}(T)$, and thus there is a probability p of the credibility estimate u being within this target width of $z\mathrm{E}(T) + (1 - z)v$.

This gives a reasonable probabilistic interpretation to the square root rule for partial credibilities. It does not, however, rule out other possible partial credibility rules which also may be reasonable. The classical approach is essentially pragmatic, and does not claim optimality.

For an example, again assume Poisson frequency and constant severity, so $c = 1$. Suppose 683 claims are observed, and this is taken as the estimate of $\mathrm{E}(N)$. Using $k = y_d\sqrt{c/\mathrm{E}(N)} = .063$, a 90% confidence interval of 683 $(1 \pm .063) = 683 \pm 43$ is computed. However, suppose an interval half width of $(.05)\ (683) = 34$ is desired, which is smaller than the actual by the ratio of $.050/.063 = .79$. The 90% confidence interval around $.79N = (.79)\ (683)$ is of the desired half width $34 = (.79)\ (43)$. Adding the constant $(1 - .79)\ v$ does not change this half width. Thus, taking $z = .79$ meets the limited fluctuation criterion, and this z can be simply calculated as the square root of the ratio 683/1082.

It is sometimes claimed in casual conversation that the classical credibility criterion is biased against downward estimates. There are two lines of reasoning used for this. The first notes that a portfolio with a smaller expected number of claims has a smaller confidence interval radius than one with more expected claims, and thus asserts that it is unfair to give it lower credibility.

This argument in effect questions the use of a target confidence interval expressed as a percentage of the expected losses, and favors an absolute confidence interval. There are good reasons for using a relative confidence interval, however. For instance, the resources to absorb adverse fluctuations are usually available in approximate proportion to expected losses. These resources may include surplus, investment income, and a profit/contingency provision in the rates. It should also be noted that a criterion based on absolute confidence intervals would give the greatest weight to the smallest volumes of data, which is just the opposite of what is intended by credibility.

The other argument for bias applies when the actual rather than the expected number of claims is used for credibility. The model assumes random fluctuations occur equally on either side of the expected value. However, downward fluctuations get lower credibility than upward ones, giving the whole procedure a slight upward bias.

To illustrate this, consider a case where the full credibility standard is $1089 = 33^2$ claims, and $E(N) = 1000$. Assume also that the previous estimate $v = 1000$. The credibility should be .958 based on $E(N) = 1000$. However, if credibility is based on the actual number of claims it will usually differ somewhat from this value. The credibility z and credibility estimate u are shown below for several n's that could arise.

n	z	u	n	z	u
1023	.969	1022	977	.947	978
1063	.988	1062	937	.928	942
1088	1.000	1088	912	.915	919
		1047.6			954.8

As can be seen, the fluctuations above the expected value do produce slightly larger indicated changes than do those below the mean. In fact the average estimate produced is 1001, so there is a 0.1% expected upward bias in this case. The weights used for each row to compute this average are .4679, .3607, and .1713. These and the n's selected derive from the 6 point Gaussian quadrature integration procedure for the interval (905, 1095), which under the normal approximation contains about 99¾% of the values of n that could arise.

The 0.1% expected bias in this case comes about because the practice departs from the theory, i.e., the credibility is calculated based on the latest observed rather than the expected number of claims. This problem need not occur in other applications of the limited fluctuation theory which use some other estimate for expected claims.

To summarize classical normal approximation credibility, then, a full credibility standard is first established, based on a specified high probability of the data being within a specified narrow band around the expected value being estimated. Partial credibility standards are then derived by requiring that the credibility weighted estimate be within just as narrow a confidence band, but this confidence band is now centered at the credibility estimate. The partial credibility z then turns out to be the ratio of the width of the target full credibility confidence interval to the corresponding confidence interval produced by the actual data.

Does classical credibility theory make sense in this form, and if so, under what circumstances?

Assumptions for aggregate losses (e.g., approximately normally distributed) that lead to the confidence interval properties of the credibility estimator have been given, but the relationship between the observed aggregate losses, those being estimated, and the previous estimate need to be clarified in order to evaluate the methodology.

Without formulating a specific model, the credibility estimate seems useful when a situation like the following is involved.

Things (i.e, the underlying processes) tend to be fairly stable over time, but occasionally they change, and these changes are of varying degrees and directions. Observations fluctuate randomly around the underlying processes, and the degree of this latter fluctuation is fairly well known. Rates should respond to fundamental changes but not to fluctuations.

Under such a scenario, it seems reasonable to set up a target confidence criterion with respect to the random fluctuations so that the latest year's indication will be used at face value if the confidence interval this experience produces is tight enough, in reference to selected constants p and k. This would delimit the degree of random fluctuation that would be deemed acceptable.

At the other extreme, if no observation can be made, the previous estimate will continue to be used. Between these extremes, a weighted average of the observation and the previous estimate seems like a reasonable and appropriate choice. What should be the weights? One possibility is to attribute just enough weight to the last observation so that that observation gives the resulting weighted average only the degree of random fluctuation that has already been deemed acceptable. As the above analysis has demonstrated, this is the result the classical credibility procedure produces, given the assumptions involved.

Thus, although no claims about optimality are advanced, the classical procedure can at least be seen to have a reasonable probabilistic interpretation. It may be particularly useful when the premises of Bayesian credibility, such as homogenity over time, cannot be assumed to hold, when the data is not available to do a full Bayesian credibility analysis, and when the auxiliary data to be incorporated comes from a different source, such as broader economic indices.

In the next section, the above procedure will be used to develop a classical credibility standard for trend projections. In Appendix 1, it is used to produce partial credibility when the normal power approximation to aggregate losses is employed.

CREDIBILITY FOR A TIME TREND

To apply classical credibility to a trend projection, a full credibility standard relative to p and k must first be determined. In the classical spirit, this can be specified as follows: a projected point will be deemed fully credible relative to p and k if there is a probability of at least p that the actual value being projected will fall within $1 \pm k$ of the projected point.

Note that this standard is more restrictive than in the aggregate loss credibility framework in that it requires the realization of the random variable, not just its expected value, to be in the interval. Accordingly, a larger value of k may be deemed appropriate for a given p in this situation than for aggregate losses. There may be other ways to specify a reasonable full credibility standard, but the above definition will be used herein. As in the classical approach, the target confidence interval is expressed as a percentage of the estimate, which seems appropriate for most of the reasons advanced above.

There are standard statistical formulas, in texts covering regression, for calculating confidence intervals around a trended point. In general, these utilize the number of points in the experience period, the number of points forward the projection is carried, and the goodness of fit of the least squares line. Let us suppose, then, that the line is based on n equally spaced observed points and the projection of interest is m points beyond the midpoint of the observations. Goodness of fit will be measured by s, where $(n - 2)s^2 = SSR$, the sum of the squares of the residuals, i.e., the sum of the squared differences between the observed and fitted points. The $n - 2$ is an adjustment for degrees of freedom, because 2 parameters are required for fitting a line.

Under normal least squares assumptions, to be discussed further below, the usual formulas yield that the standard deviation of the projected point is

$$s\sqrt{1 + (1/n) + 12m^2/(n^3 - n)}$$

and the p confidence interval measures

$$t(d,n - 2)s\sqrt{1 + (1/n) + 12m^2/(n^3 - n)} \tag{1}$$

on each side of the projected point, where $t(d,n - 2)$ is the $100d$th percentile of the t distribution with $n - 2$ degrees of freedom, and, as before, $d = .5(1 + p)$. Formulas that reduce to these for a time trend can be found in many regression texts. The confidence interval incorporates both the variance of the subsequent point from its expected value on the line and the uncertainty as to where the line really is, since its parameters are estimated.

To use this confidence interval for credibility, it is first necessary to select p and k. For example, a 90% confidence interval of $\pm$ 10% of the projected value might be chosen as the full credibility standard. Then the actual p confidence interval is measured for the data at hand. Suppose, for example, the 90% confidence interval around the projected point is found in fact to be $\pm$ 12.5% of the projected value. Then, following the principles of classical credibility, the partial credibility for the particular case at hand would be the ratio of the full credibility interval to the actual interval. In this case the ratio is $.10/.125 = .80$, and thus the trend projection receives 80% credibility.

Applying a credibility factor in this manner limits the possible random deviation of the credibility weighted estimate to the targeted amount, i.e., to $\pm k$ of the projected point. However, the resulting confidence band, while of the desired width, is not centered on the value being estimated, but rather on the weighted average of this value with a previous estimate v. This is precisely what the classical procedure does in the aggregate loss case as well.

In other words, the credibility estimate is z times the projected point plus $1 - z$ times the prior expectation. The p confidence band around this estimate has been shrunk by a factor of z, which is chosen to give the resulting confidence band a width of k times the projected point. Then p expresses the probability that the credibility weighted average of the actual value being projected and the prior expectation will be within the given interval around the credibility estimate.

The theory does not specify what the prior estimate v should be, but it seems reasonable to stipulate that v is the best estimate available prior to the current projection. Possibilities may include a previous projection; a projection based on a wider population, e.g., countrywide data; or a projection based on a broader economic perspective, e.g., pure inflationary considerations.

An example of this method is given in Appendix 3, for a loss ratio trend. A loss ratio of .647 at current rate level is projected, with a 90% confidence interval of $\pm .159$. If the full credibility standard is taken to be a 90% confidence interval of $\pm .0647$, a credibility of $z = .0647/.159 = .41$ results. Thus $1 - z = .59$ will apply to the prior estimate. Suppose the prior estimate is $v = .620$. Then the credibility estimate is $u = (.59)(.620) + (.41)(.647) = .631$.

The probabilistic interpretation of this procedure is then as follows. There is a 90% probability that the expected loss ratio E being estimated is within .159 of .647. Thus, there is also a 90% probability that $.41E$ is within $(.41)(.159) = .065$ of $(.41)(.647) = .265$. Adding $.59v = .366$ to this shows that there is then a 90% probability that the credibility estimate $u = .265 + .59v$ is within .065 of $.41E + .59v$.

PROS AND CONS OF THE METHOD

The confidence interval approach to credibility for trend has several advantages and some disadvantages, as enumerated below. Some features of the method have positive and negative aspects, and thus are listed under both.

Advantages

1. The method is derived explicitly from a statistical model. Thus, it is possible to describe the estimate in probabilistic terms. It is not based on analogy or ad hoc reasoning.
2. Credibility bears a direct relationship to the goodness of fit of the trend line.
3. Since the model is simple, the concepts are relatively easy to explain and the estimation is not difficult to carry out.

4. The method leaves room for the informed judgment of trained experts, both in the selection of the full credibility standard and in the choice of the prior estimate. This makes the method responsive to the needs of different constituencies, which may have different evaluations of the applicability of the various sources of prior data, such as countrywide data or broader economic trends.

Disadvantages

1. The method does not optimize anything. This is in contrast to the modern least squares credibility approach, which does optimize a specific error function.
2. Subjective judgment is required. This is again in contrast to the least squares approach, in which all estimates are produced strictly from the data with no input from subjective probabilities called for. While informed judgment can truly be an advantage over purely data driven methods, judgment can be inconsistent over time and circumstances, and poor judgment can be a disadvantage.
3. The model requirements, while simple, are restrictive. The usual regression assumptions, for example, include normality of the residuals. This assumption can be tested, however, as is discussed further in Appendix 4. If normality is not found, it still may be possible to estimate confidence intervals by other means.

In summary, classical credibility, which can be thought of as a ratio of confidence intervals, can be extended directly to apply to trend. This has several advantages, including flexibility and ease of application and exposition. It is a pragmatic approach with a probabilistic interpretation, but is not derived as a statistical optimization. This leaves open the possibility that, under further assumptions about the statistical relationship between the data and a specific prior estimate, a different credibility procedure can be derived that optimizes a specified error measure.

LEAST SQUARES ASSUMPTIONS

The normal least squares assumptions provide that the various years' observations T_i are normally distributed random variables, each with the same variance, and with the expected value for each given as a linear function of time, i.e., $\mathrm{E}(T_i) = a + bi$.

In application, these assumptions may only hold as approximations. In some cases, for instance, the expected values may move as a non-linear function of time. Also, the data is often adjusted to remove systematic influences, e.g., rate changes and benefit changes, before the linear model is fit.

Further, nothing in the model assumptions requires the variance to be due to frequency and severity distributions alone. Price levels, the level of economic activity, and reserving changes could all contribute to the variance of the individual results from their expected values on the line. Thus the volume of experience underlying each point is not the sole determinant of the variance, and in fact may be overshadowed by other factors.

DEVELOPING A WORKING FORMULA

A projected point is fully credible p,k if the p confidence interval around the projected point has radius no more than k times that point. By (1) and the definition of s, this criterion will be fulfilled if

$$k\ PRO = t(d,n-2)\sqrt{(1+(1/n)+12m^2/(n^3/n))SSR/(n-2)}. \tag{2}$$

Here PRO denotes the projected point. Also, for credibility z, a confidence interval of (k/z) PRO is required, by the limited fluctuation principle. This can be expressed by substitution (k/z) for k in (2).

Rearranging terms then leads to

$$SSR_z/PRO^2 = k^2(n-2)/[z^2t(d,n-2)^2(1+(1/n)+12m^2/(n^3-n))] \tag{3}$$

where SSR_z is associated with credibility z. Thus

$$SSR_1/PRO^2 = k^2(n-2)/[t(d,n-2)^2(1+(1/n)+12m^2/(n^3-n))] \tag{4}$$

gives the full credibility standard relative sum of squared residuals in terms of p and k, the selected criteria; n, the number of points used to fit the line; and m, the number of points projected beyond the midpoint of the n original points.

Full credibility is expressed by a relative SSR, not an absolute SSR, because the target confidence interval is specified as a percentage of the projected value. As with credibility for aggregate losses, a smaller absolute confidence interval can lead to lower credibility if that interval is wider relative to the value being estimated, and again this appears to be entirely appropriate.

From (4), the full credibility relative SSR's are given for various increasingly specific assumptions below. First, take $p = .90$, so $d = .95$, and assume 5 points are used to fit the line so $n = 5$. Then $t(d, n - 2) = 2.353$ and:

$$SSR_1/PRO^2 = 5.418\ k^2/(12 + m^2).$$

If $k = .06$, this becomes

$$SSR_1/PRO^2 = .0195/(12 + m^2).$$

A typical projection may be to point 7.5, so $m = 4.5$ points beyond the midpoint of the data. This yields

$$SSR_1/PRO^2 = .0006.$$

This full credibility standard for the relative SSR only coincidentally is $.01k$. For $k = .05$, the standard is .0004, and for $k = .07$ it is .0008.

For a given PRO, (4) can be divided by (3) to yield

$$z = \sqrt{SSR_1/SSR_z},$$

which is the square root rule for partial credibility for trend. Here SSR_z is the actual SSR for the fitted line, and SSR_1 is the target relative SSR multiplied by PRO^2.

MAKING THE JUDGMENTS

Given the above working formulas, choosing p and k can be replaced by selecting a target full credibility relative SSR. This is perhaps a more reasonable judgment to make. Instead of picking p's and k's in advance, experienced actuaries, having a feel for the ratemaking process as a whole, and also for their corporate goals, may prefer to review a collection of fitted lines and select those which can be regarded as fully credible for ratemaking use. However, the resulting p's and k's may be a useful part of this review.

Such a process is also advantageous in that it is less tied to the normal distribution assumption of the model. The selection of the full credibility relative SSR can be made with recognition that the residuals may not be normally distributed, and that the confidence intervals involved might actually be wider than the model would predict for that relative SSR.

A judgment could also be made that a wider confidence interval may be acceptable when a longer projection is necessary, in recognition of the inherently greater uncertainty involved with a longer projection. One way to reflect this is to keep the target relative *SSR* constant under various projection periods.

Under these circumstances, the Actuarial Committee of the National Council on Compensation Insurance adopted a target relative *SSR* of .0006 for a five-year fitted trend line. As noted above, this results in $k = .060$, that is, a 90% confidence interval radius of 6.0% of the projected value when $m = 4.5$, which corresponds to a 2.5-year projection. For this relative *SSR* and $m = 5.5$, a 3.5-year projection, $k = .068$, that is, there is a 90% confidence interval radius of 6.8% of the projected value. Normally, workers' compensation ratemaking uses a projection period of 2.5 to 3.5 years.

The more general target relative *SSR* of $.0195/(12 + m^2)$ can be used for other projection periods. This maintains the target relative 90% confidence interval radius at 6.0% regardless of the length of the projection.

The Committee also noted that the above formula for the confidence interval around a projected point allows for random fluctuation of the projected loss ratio as well as for uncertainty about the parameters of the regression line. If only the latter were to be considered, the resulting confidence interval would actually be tighter than the formulas indicate.

This indication of a tighter interval may in part be counterbalanced by the possibility that residuals are not normally distributed. Although that distribution was not rejected by standard tests, the tests are not definitive in this context. To the extent that the residuals are from a skewed distribution, the target confidence interval may be wider than the formulas suggest.

Practical considerations such as these support the approach of selecting a target relative *SSR* based on informed judgment which considers, but is not strictly limited by, the implications of the statistical model.

The complement of credibility in this framework should apply to the best estimate of the trended point available prior to the projection that is being weighted. Logical candidates for this are projections based on the countrywide trend, the previous trend in the state, or broader economic indices. The assumption of no trend would not be appropriate unless there is an a priori reason to believe the trend is in fact flat. There may be, for example, good reason to believe this for the ratio of workers' compensation indemnity losses to payroll. However, as medical costs have been increasing faster than payroll in the economy at large, the ratio of medical losses to payroll could not be expected a priori to show no trend.

As medical benefits are quite similar across states, and are subject to similar inflationary influences, the latest available countrywide trend factor was selected as the prior estimate to be used with the complement of credibility for the medical pure premium trend. For indemnity trend, this was felt to be inappropriate, due to widely differing benefit laws. Zero trend was chosen as the prior estimate because of its a priori reasonableness.

APPENDIX 1

CREDIBILITY WITH THE NP APPROXIMATION

Mayerson, Jones, and Bowers [6] note that the normal approximation is inappropriate for casualty insurance aggregate claims distributions because these are almost always positively skewed. They suggest using the NP (normal power) approximation instead, although they never use that term. The NP adjusts the normal approximation for skewness. If t_d is the dth quantile of T, i.e., $\Pr(T \leqq t_d) = d$, and y_d is the dth quantile of the standard normal distribution, then the NP approximation is

$$t_d = \mathrm{E}(T)(1 + c_T(y_d + s_T(y_d^2 - 1)/6)), \tag{5}$$

where for any random variable X, c_X is the coefficient of variation (ratio of standard deviation to mean) and s_X is the coefficient of skewness (ratio of third central moment to the cube of the standard deviation). In this notation, the normal approximation is

$$t_d = \mathrm{E}(T)(1 + c_T y_d),$$

which is the NP with zero skewness.

The NP approximation arises from the first few terms of the Cornish-Fisher expansion, an infinite series expansion which expresses the percentiles of a distribution in terms of its moments. This is an alternating series expansion and is not necessarily convergent. Thus, adding more terms may or may not significantly improve the accuracy of this approximation. See Beard, Pentikainen, and Pesonen [1] for further discussion of this approximation.

Full Credibility with the NP

As with the normal approximation, the starting point for credibility is to find a full credibility standard such that T is within $\pm k\mathrm{E}(T)$ of $\mathrm{E}(T)$ with probability p. The NP does not in general provide a symmetric distribution of T around $\mathrm{E}(T)$; however, requiring T to be below $(1 + k)\mathrm{E}(T)$ with probability $d = (1 + p)/2$ is generally assumed to be sufficient for T to be within $\mathrm{E}(T) \pm k\mathrm{E}(T)$ with probability p for positively skewed aggregate claim distributions. This will be assumed for now, but it is discussed further below. With this assumption, the full credibility requirement gives the equation $\mathrm{E}(T)(1 + k) = t_d$, which must be solved for $\mathrm{E}(N)$ to get the full credibility standard. $\mathrm{E}(N)$ does not appear in this equation, but it is an element of both sides.

Employing the NP approximation (5) at this point and solving for k yields

$$k = c_T(y_d + s_T\,(y_d^2 - 1)/6).$$

To solve for E(N), c_T and s_T must be expressed in terms of N and X, i.e., frequency and severity. The methods of Appendix 2 provide the following formulas for the coefficients of variation and skewness of aggregate losses

$$c_T^2 = (c_x^2 + n_2)/\mathrm{E}(N), \text{ and}$$
$$s_T = (s_x c_x^3 + 3n_2 c_x^2 + n_3)/c_T^3 \mathrm{E}(N)^2,$$

where n_i is defined by $\mathrm{E}(N)n_i = \mathrm{E}(N - \mathrm{E}(N))^i$. For example, n_2 is the frequency ratio of variance to mean. For the Poisson, $n_2 = n_3 = 1$.

Introducing further notation for the numerators of the aggregate moments will simplify these expressions. Let

$$M_2 = c_x^2 + n_2, \text{ and}$$
$$M_3 = s_x c_x^3 + 3n_2 c_x^2 + n_3.$$

M_2 and M_3 are shape descriptors for the aggregate loss distribution. For example, M_2/E(N) is the square of the coefficient of variation of aggregate claims, and, in fact, M_2 is the adjustment factor c referred to above in the discussion of the normal approximation credibility formulas. M_3 is a third moment measure for aggregate losses, and comes up frequently in calculations. With this notation the formula for k becomes

$$k = y_d\sqrt{M_2/\mathrm{E}(N)} + (M_3/M_2)(y_d^2 - 1)/6\mathrm{E}(N).$$

This equation can be solved in general for the full credibility standard. Considering it to be a quadratic equation in $\sqrt{\mathrm{E}(N)}$ gives

$$2k\sqrt{\mathrm{E}(N)} = y_d\sqrt{M_2} + \sqrt{y_d^2 M_2 + 2k\,(y_d^2 - 1)M_3/3M_2}.$$

The resulting value of E(N) is the full credibility standard based on the NP approximation. Setting the last term under the square root to zero gives the formula for the normal approximation full credibility standard. Thus, that term is the end result of the NP adjustment.

Partial Credibilities

The limited fluctuation method can be used to calculate partial credibilities under the NP approximation. Following the development in the text, for an E(N) less than N_f, the partial credibility z represents a scaling factor that scales the p confidence interval that would arise for that number of claims down to the target p confidence interval of $k\mathrm{E}(N)$. The number of claims that generates a credibility of z, i.e. N_z, can be developed from the above full credibility formula by replacing k with k/z. This allows for the calculation of credibility tables, but no simple relationship, such as the square root rule of the normal approximation, is evident.

An example would probably be useful at this point. Consider a case with a Poisson frequency distribution and a lognormal severity with a coefficient of variation of 7.0. For the Poisson, $n_2 = n_3 = 1$. Thus $M_2 = 50$. For the lognormal generally, $s_x = c_x^3 + 3c_x$, so in this case $s_x = 364$. Thus $M_3 = 125{,}000$. Take a 90% confidence interval, so $y_d = 1.645$. Then $2k\sqrt{\mathrm{E}(N)} = 11.632 + \sqrt{135.3 + 2843k}$. If $k = .05$, $\mathrm{E}(N) = 80{,}026$ is then the full credibility standard. Replacing k by k/z gives the following standards N_z for partial credibility z

z:	.25	.50	.75
N_z:	9,103	25,786	49,468
z^2N_f:	5,001	20,007	45,015

The square root rule partial credibility criteria z^2N_f for this N_f are consistently lower.

The high credibility requirements in this example derive in part from the large severity CV assumed. For high limits of insurance or unlimited coverage, CV's of this magnitude have been reported by actuaries involved in various lines of commercial property and liability insurance (LeRoy Simon in his review [10] of the Mayerson, Jones, and Bowers paper).

Instead of the Poisson, a negative binomial frequency can be assumed. The negative binomial can be described by means of two parameters, x and p, so that $\Pr(N = n) = \binom{x + n - 1}{n} p^x(1 - p)^n$, with moments $\mathrm{E}(N) = x(1 - p)/p$, $n_2 = 1/p$, $n_3 = (2 - p)/p^2$. This illustrates that n_2 and n_3 can be considered fundamental measures of the shape of the frequency distribution, in that they are functions of p only, while the mean can be changed by moving x. Dropkin [3] found that $n_2 = 1.184$ in an automobile insurance study. This implies $p = .8446$ and so $n_3 = 1.620$. In the above example, this increases M_2 to 50.184 and M_3 to 125,027.668. Thus, $2k\sqrt{\mathrm{E}(N)} = 11.653 + \sqrt{135.8 + 2834k}$. For $k = .05$ this yields $\mathrm{E}(N) = 80{,}153$ expected claims for full credibility. Thus, in this case, the full credibility standard is not significantly changed by going to the negative binomial assumption.

Another example of a negative binomial frequency distribution is provided by Meyers and Schenker [7]. They discuss a workers' compensation setting in which the compound process of picking a risk at random from a class and observing its number of claims can be described by a negative binomial distribution. In their case each risk has a Poisson claim count distribution, and the risks' Poisson parameters are gamma distributed across the class. The resulting negative binomial distribution with mean $E(N)$ is estimated to have $n_2 = 1 + .037E(N)$. Since n_2 is increasing with risk size, very large values can arise for large risks. In this model, a portion of the uncertainty about a risk's claim count comes from the distribution of risks in the class, and this portion is not reduced by increasing the risk size. Essentially n_2 is no longer a fundamental frequency constant, but depends on $E(N)$.

Ignoring the context, however, suppose a negative binomial distribution is given with a constant but large n_2, say $n_2 = 51$. Then $p = 1/51$, and $n_3 = 5151$. In the above example these values give $M_2 = 100$ and $M_3 = 137{,}500$. Thus $2k\sqrt{E(N)} = 16.45 + \sqrt{270.6 + 1564k}$, and $k = .05$ gives $E(N) = 123{,}385$. Thus the negative binomial model does make a considerable difference when n_2 is large.

The lognormal assumption increases the skewness in these examples over what some other distributions would provide. A Weibull distribution with a CV of 7 has a shape parameter of .2678046 and thus skewness of 44.44. In the Poisson case above this reduces M_3 to 15,391. Thus $2k\sqrt{E(N)} = 11.632 + \sqrt{135.3 + 350.1k}$, or, for $k = .05$, $E(N) = 57{,}568$.

One-and Two-sided Intervals for Skewed Distributions

Previously it was stated that the NP approach to credibility usually assumes that if T has a probability of $d = (1 + p)/2$ of being below $(1 + k)E(T)$, then T will be within $\pm kE(T)$ of $E(T)$ with probability at least p. That this is not necessarily true for positively skewed distributions is shown in the following example.

Assume T is Pareto distributed with distribution function $F(t) = 1 - (1 + t/2.5)^{-3.5}$. Then $E(T) = 1.0$ and $F(1.0) = .6920$. Take $p = .5$, so a 50% symmetric confidence interval around 1.0 is sought. This interval is $1.0 \pm .6898$ as can be verified using $F(t)$. Since $p = .5$, $d = .75$, and since $F(1.215) = .75$, $t_1 = 1.215$. However, the probability of T being in the interval $1.0 \pm .215$ is less than 50%; in fact it is only 13.45%.

This arises because E(T) is well above the median of the distribution, so going only a small distance above E(T) reaches the 100dth percentile for this d. For this distribution, this situation holds up to a p = 72.14% confidence interval around the mean, i.e. up to 1.0 ± .8905. For this interval, the probability d of T being less than E(T)(1 + k) is given by (p + 1)/2 = 86.07% exactly. For higher p, the desired relationship does hold, i.e., being below the (1 + p)/2 quantile is enough to guarantee that the corresponding symmetric interval contains at least p in probability.

Since it is higher confidence intervals that are of interest in credibility, the assumed relationship would be fulfilled in this case. However, a more highly skewed distribution would place the mean at an even higher percentile, which would aggravate this problem. However, most loss distributions for which this credibility procedure is intended are not so highly skewed that this would be likely to occur. In fact the NP itself is of questionable accuracy for highly skewed distributions.

Applicability of the NP to Skewed Distributions

To investigate this, the percentiles of the several distributions are calculated directly and by the NP approximation. The Pareto distribution $F(t) = 1 - (1 + t/b)^{-s}$ has moments defined by

$$E(T^n) = \prod_{i=1}^{n} ib/(s - i).$$

Using these with the above values (b = 2.5, s = 3.5) yields, after some algebra, $c_T^2 = 7/3$ and $c_T s_T = 18$. Thus the NP approximation becomes

$$t_d = 3y_d^2 + 1.5275y_d - 2.$$

This is compared to the actual values of t_d for this distribution below.

d:	.10	.25	.50	.75	.90	.95
t_d NP:	.9698	−1.665	−2.000	.3952	4.885	8.630
t_d Actual:	.0764	.2142	.5475	1.215	2.327	3.384

The NP approximation is clearly not appropriate for this distribution. From the table, the NP might be reasonably accurate for a small range of values somewhere in between the 75th and 90th percentiles. For the right hand tail, it clearly overstates the percentiles. The problem here apparently is the high skewness.

Distributions of this great a skewness are not likely for large portfolios of risks, for which the NP was originally developed. The aggregate claim distribution for a small portfolio or a single risk could easily be this highly skewed, however, and use of the NP could lead to large errors in such a case.

For less skewed distributions (e.g., skewness below 1.0) the NP can be fairly accurate. Two distributions, the gamma and the Weibull, are compared below to their NP estimates. Both of these distributions are assumed to have mean 1 and standard deviation 1/3, which fixes their parameters. The gamma then has skewness of 2/3, while that for the Weibull is approximately .077. The percentiles are shown below.

d:	.01	.05	.25	.50	.75	.95	.99
t_d Gamma:	.390	.522	.760	.963	1.200	1.604	1.934
t_d NP:	.388	.515	.755	.963	1.205	1.611	1.939
t_d Weibull:	.277	.454	.765	.998	1.231	1.554	1.770
t_d NP:	.243	.459	.773	.996	1.223	1.556	1.794

The NP approximation is reasonably close for both distributions, although at the extremes it is better for the gamma than for the Weibull.

APPENDIX 2
FORMULA FOR VAR(T)

T is the sum of the individual claims X_i, where i runs from 1 to N, the number of claims. Since N is a random variable, both frequency and severity contribute to the variance of T. It is generally assumed that all claims have the same distribution, and that individual claim sizes are independent of each other and of N.

To compute the variance of T under these assumptions, begin by calculating $E(T^2/N = n)$, i.e., fix the number of claims at n and find $E((X_1 + \ldots + X_n)^2)$.

Expanding the square yields n^2 terms of the form X_iX_j. When $i = j$ the expected value of the term is $E(X^2)$. Otherwise, it is $E(X)^2$, since then X_i and X_j are independent. Thus

$$\begin{aligned} E((X_i + \ldots + X_n)^2) &= nE(X^2) + (n^2 - n)E(X)^2 \\ &= n\text{Var}(X) + n^2E(X)^2. \end{aligned}$$

Now, by general considerations of conditional expectations, $E(T^2) = E(E(T^2/N = n))$. Thus, taking the expected value of the above equation with respect to N gives

$$\begin{aligned} E(T^2) &= E(N)\text{Var}(X) + E(N^2)E(X)^2 \\ &= E(N)\text{Var}(X) + \text{Var}(N)E(X)^2 + E(N)^2E(X)^2. \end{aligned}$$

The last term is just $E(T)^2$. Subtracting it from both sides then yields

$$\text{Var}(T) = E(N)\text{Var}(X) + \text{Var}(N)E(X)^2.$$

APPENDIX 3

LOSS RATIO TREND EXAMPLE

The points labeled "Line" below were computed from the formula Line = .9684 − .0428 Year, and represent the least squares fit to "Data".

Year	Data	Line
1	.909	.926
2	.929	.883
3	.819	.840
4	.767	.797
5	.776	.754

The point for year 7.5 is projected to be .647, and the 90% confidence interval around this point is sought. The sum of the squared residuals is .00423, so $s = .03755$, since $3s^2 = .00423$. For year 7.5, $m = 7.5 - 3 = 4.5$, so $1 + 1/n + 12m^2/(n^3 - n) = 3.275 = 1.796^2$. Also, $t(.95,3) = 2.353$. Thus, the 90% confidence interval is $.647 \pm (2.353)(.03755)(1.796) = .647 \pm .159$.

APPENDIX 4

TESTING RESIDUALS FOR NORMALITY

As mentioned in the text, the confidence interval calculation relies on the assumption of normally distributed residuals. To some extent this assumption is testable, but for a trend based on a small number of data points, the tests are not particularly conclusive.

The SAS package provided a test of normality for small samples, namely the Shapiro-Wilk W statistic. W is the ratio of two estimates of the variance of the residuals, one (the numerator) based on order statistics, and the other the usual sample variance approach. This ratio is between 0 and 1, and small values lead to the rejection of normality.

For example, in Appendix 3, $W = .881$ was calculated by SAS. From the critical values provided by Shapiro and Wilk, the probability of a lower value of W arising from a sample of 5 from a truly normal population is 35%. This is not a low enough value to reject normality.

Since tests like this are not conclusive for small samples, one may want to appeal to general principles. In the case at hand, loss ratios are usually believed to have positively skewed distributions, so it may seem inappropriate to assume a normal distribution.

Three comments are in order, however:

1. In some cases the skewness may be small enough that the normal approximation is reasonable.

2. In some cases the deviations of the expected loss ratios for each year from the trend line may follow a normal distribution, and the deviation of the actual loss ratio from the expected for the year a positively skewed distribution. If the deviations of the expected from the line have a greater magnitude than the deviations of the actual from the expected, the normal approximation may not be too bad overall.

3. Confidence intervals using a skewness correction could possibly be developed in cases where a positive skewness is significant. In light of the role of informed judgment in selecting the full credibility standard, however, an explicit calculation of this type may not be required for moderately skewed distributions.

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ADDRESS TO NEW MEMBERS—MAY 12, 1986

CHARLES C. HEWITT, JR.

INTRODUCTION

It is a genuine pleasure for me to address the new members of the Casualty Actuarial Society today. While my remarks are primarily intended for the new Fellows and Associates, I hasten to recognize the role that your partners, spouses or otherwise, have played in the achievement which has just been recognized. So, my remarks are also addressed to those, here today, who have sacrificed in order that the person sitting next to you may now append the letters FCAS or ACAS to her name or his name.

Incidentally, for you spouses and partners of new Fellows, the old alibi, "But I have to study for exams" is no longer valid. Back to changing diapers and all the other joys of conjugal life.

When Phil Ben-Zvi called me to ask if I would accept this assignment I was particularly delighted for a very personal reason. One of the new Fellows is a young lady whom I recruited for the profession. Next to being a parent and enjoying the achievements of one's children, there is no greater satisfaction than participating in the success of someone for whom you have opened the door. The young lady knows who she is and I'm not going to embarrass her by identifying her publicly. But I do want to say to her, "Rhonda, you did it on your own and I'm very proud to be here today to participate in this important moment in your life!"

My remarks will be brief. I intend to cover three general areas which I will label COMMUNICATION, ACTUARY, and SPAN. The reason for this rather awkward choice of labels will become apparent later, although some of you may see through this selection of titles.

COMMUNICATION

Actuaries, as a class, are literal-minded people. They, too often, assume that words speak for themselves, thus ignoring the importance that tone of voice plays in oral communication, or that it is necessary to lay groundwork and provide emphasis in all forms of communication. They should be aware of "body language" as a means of understanding what others are trying to communicate. It took me thirty years of marriage to realize that my wife listened more closely to my tone of voice than to my words. It took me somewhat less time to understand that when she took out her emery boards and began working on her finger nails with a vigor that would have cut through solid oak, she was upset about something, but was not yet ready to discuss it.

In thinking about methods of communication, I am reminded of the story of the Irish priest whose sermons were constantly filled with vilification of the English. Word of this reached his bishop and the latter decided to attend mass on Palm Sunday and listen to the priest's sermon. As usual, the Irish priest managed to lambaste the English to a fare-thee-well. After mass the bishop took the priest aside. "My son," he said, "You do preach a good sermon, but do you really think it's proper to bring your political feelings into discussions of the Lord's work? Now, I would suggest that in the future you omit any reference to the English even though it's obvious your feelings on this subject are very strong." The following Sunday, Easter, the priest declared that his topic was to be the Last Supper. He described how Christ announced that one of his disciples had betrayed him, and how Christ proceeded to go around the supper table, one disciple at a time, asking who it was. "And each disciple answered firmly 'Not I, Lord' until Jesus came to Judas Iscariot. And, Jesus asked, 'Was it you, Judas, that betrayed me?' And, Judas replied, 'Blimey, guv'ner, it wasn't me!'"

Actuaries have a terrible time making themselves understood by people who are not actuaries! That bald statement of self-criticism is worth repeating. Yes, ACTUARIES HAVE A TERRIBLE TIME MAKING THEMSELVES UNDERSTOOD BY PEOPLE WHO ARE NOT ACTUARIES! Ask any senior member of our organization who is currently in an administrative position which factor most influences him or her when employing or promoting an otherwise qualified actuary, and he or she will readily identify the ability to explain ideas and results to others as paramount. Check our own *Proceedings* and find the number of times that presidential addresses take up this same issue.

To me, communication begins with putting oneself into the position of the person with whom you are communicating. What does that person expect to hear? Does the person have a lot of time or are they in a hurry? What analogies would be most readily appreciated? If you have an idea or set of facts that are worthy of being passed on, then, for heaven's sake, make the additional effort to pass them on properly. We all know the old philosophers' question, "If a tree falls in the forest and there is no one to hear it, is there any sound?" If you wish to pass on a thought or report something important, it may be lost if no one can understand you or if you can't make them want to listen.

ACTUARY

What does it mean to be an actuary? It means that you, and sometimes only you, will take the long-range point of view. Managers are here today and gone tomorrow, and most frequently look only for the short-term advantage that will further their own careers. Being an actuary means integrity; it means standing firm when you're in the right, not thinking any less of those who disagree with you, but trying to use facts and reason to overcome objections wrongfully come by.

Many of you will ultimately find yourselves in positions where very little of your day-to-day work is actuarial. Thumb through the CAS *Yearbook* and see how many of our members are in non-actuarial assignments. But as long as you bear the designation which you have studied so hard to achieve, and have received today, remember that other people think of you and respect you as an actuary. Others will continue to come to you for your actuarial advice or opinion.

Being an actuary is not unlike being a weather man. People will make snide remarks about you and your profession. Nevertheless, you will find that you have earned their respect for objectivity and honesty. Avoid the trap of telling people only what they want to hear. Learn to recognize the pros and cons in a decision; think them through and then be prepared to discuss both sides of an issue.

Speaking about being an actuary at all times, I came across the following statement which appeared recently in the *New York Times Sunday Magazine* in an article seriously questioning the need for liability insurance rate increases:

> *Actually liability awards are remarkably consistent. In constant dollars, the median award has hovered around $20,000 over the last 25 years, according to the Rand Corporation's Institute for Civil Justice, although the average award has risen appreciably, reflecting the impact of a few huge settlements.*

The context in which this statement is contained is clearly intended to make the layman feel that there is something called the 'median' which has a significant bearing on whether or not liability insurance rates should go up or down.

(For our guests, here today, who are not actuaries, let me say that the 'median' is simply the middle number in a string of numbers which have been arranged in order from smallest to largest or vice versa.)

To illustrate, come with me on a shopping trip to the supermarket. I'll try to be semi-realistic and yet keep my example simple. We buy a loaf of bread for 80 cents, a quart of milk for 70 cents, and a pound of coffee for $3.50. The median price of 70 cents, 80 cents and $3.50 is the middle amount—80 cents. Let's add up the bill. Ignoring sales tax, we'll expect to pay 70 cents plus 80 cents plus $3.50. That's $5.

Next week we'll go to the store and buy three loaves of bread at 80 cents per loaf, three quarts of milk at 70 cents, and three pounds of coffee at $3.50 per pound. The amounts, by item, are 70 cents, 70 cents, 70 cents, 80 cents, 80 cents, etc. Clearly the median price is still 80 cents. Would we expect the supermarket to charge $5, the same as last week?

Alternatively, suppose that bread and milk had stayed the same price but coffee had jumped to $4.50 a pound because of a freeze in Brazil. Then, if, next week, we bought the same items, but only one of each, would we expect the same total at the register, since the median price is still only 80 cents?

If the median liability award 'hovers' around $20,000, but the number of awards, or claims, doubles or triples, should we expect our bill for liability insurance to remain the same? If this median remains relatively constant but the larger awards get bigger and bigger, should we expect the cost of liability insurance to stay constant? Finally, if it's the large awards that are the major problem, should the cost of excess liability covers ignore this fact and not change?

The author of the *New York Times* article from which I quoted is a member of this Society. Unfortunately, this person would seem to have forgotten what it means to be an actuary. Now, I don't want my remarks to be misinterpreted as saying that you should never espouse a position that is unpopular with the majority of your actuarial brethren. Far from it; some of the older members will remember that Charlie Hewitt has been on the unpopular side of more than one issue. What I am saying is: get your facts straight and then interpret them objectively, i.e., actuarially.

SPAN

When congratulating new Fellows, my stock inquiry has always been, "Now what are you going to do with all your free time?" Recently, I received the reply, "Well, I'm certainly not going to read any more papers by Valerius!" Naively I responded, "You know I knew Valerius." Now, it should be explained that Nels Valerius is a fine old gentleman, and at last report was still living in Cheshire, Connecticut. He received his Fellowship in 1928.

The group of younger members with whom I was conversing looked aghast. One of them said, in disbelief, "You knew Valerius!?" When I nodded assent, the young member blurted out, "Boy, you're a real link with the past!" Now I must confess that I even knew Dorweiler—and he was the man who hired Valerius.

The point I'd like to make with you is that our careers as actuaries will span a considerable period of time; our lives will span an even longer period of time. Most of us focus, with the greatest intensity, on the present, and pay decreasing amounts of attention to either the past or the future. Picture a Normal curve with no beginning and no end, and with time as the x-axis. The present moment in time is the mode (median and mean, also). The height of this curve at any point in time can represent the effect that other times in our careers (or our lives) have upon our present actions and decisions. What we did or thought yesterday, or expect to do or think tomorrow will usually affect today's thought and actions far more than those things did one year ago or will do one year from now.

As you grow older you will appreciate that looking upon the full span of a career (or a life) will give a better perspective as to the importance of what's happening right now, or what happened yesterday, or what might happen tomorrow. Try to live your life and your careers without the perspective that today's deeds are all-important. Realize that what took place in the past has some importance, but with an ever-lessening intensity as we go backward in time. Similarly, although tomorrow seems awfully important and will be even more important when it becomes today, the other tomorrows further off must be acknowledged as having bearing on our actions and thoughts today.

CONCLUSION

COMMUNICATION, ACTUARY, SPAN—by now some of you may have perceived that the initial letters of these awkwardly chosen titles for my subjects spell *C A S*—for Casualty Actuarial Society. Once again, I'm reminded of a story; this time about the late Herman Hickman of whom I suspect most of you have never heard.

Herman was a 300-pound college and professional football player and sometime professional wrestler and during a brief period a notably unsuccessful football coach at Yale University. During this tenure as Yale's football coach, he told the story that during a halftime intermission he gave his team a pep talk in which he chose to use the letters of Yale—*Y A L E*—as the theme around which he would inspire his players to better deeds in the second half of the game.

"*Y*," said Herman, "is for You. You must get out there and fight, fight, fight. *A* is for All. All of us must give every ounce of our ability to win this football game. *L* is for Loyalty. It's our loyalty to dear old Yale that will enable us to go on to victory. *E* is for Each and Every one of us who must give his all to insure that we walk off the field today triumphant. *Y A L E*; those letters spell victory." Newly inspired, the Yale team charged out of the locker room. Trailing behind the rest of the team were two substitutes who had not played in the first half and had little prospect of playing at all. Unaware that the coach was immediately behind them, one sub turned to the other and said, "What did you think of the coach's pep talk?" The other replied, "All I can say is thank heavens we don't go to California Polytechnic University at San Luis Obispo."

It has been a pleasure and a privilege to address the new members of the Casualty Actuarial Society. You have my congratulations and my best wishes for both long and successful careers in whatever line of endeavor you may choose. Thank you.

MINUTES OF THE 1986 SPRING MEETING

May 11–14, 1986

HOTEL DEL CORONADO, CORONADO, CALIFORNIA

Sunday, May 11, 1986

The Board of Directors held their regular quarterly meeting from 12:00 noon to 4:00 p.m.

Registration was held from 3:00 p.m. to 5:30 p.m.

A presentation to the new Fellows and Associates on the workings of the Casualty Actuarial Society was conducted from 5:30 p.m. to 6:30 p.m. The vice presidents made short presentations concerning their areas of responsibility and the workings of the committees which report to each of them.

A general reception for all members and guests was held from 6:30 p.m. to 7:30 p.m.

Monday, May 12, 1986

Registration continued from 7:00 a.m. to 7:55 a.m.

President Phillip Ben-Zvi opened the meeting at 8:00 a.m. The first order of business was the admission of new members. Mr. Ben-Zvi recognized the 82 new Associates and presented diplomas to the 19 new Fellows. The names of these individuals follow.

FELLOWS

Mark S. Allaben	Allen A. Hall	William F. Murphy
Robert A. Bear	Gregory L. Hayward	Karen L. Nester
Janice L. Berry	Martin A. Lewis	Rhonda D. Port
Wallis A. Boyd	Barry C. Lipton	Michael B. Smith
Daniel B. Clark	Isaac Mashitz	Nancy R. Treitel
Kathleen F. Curran	Robert A. Miller, III	Charles S. White
James L. Dornfeld		

ASSOCIATES

Neil C. Aldin
Manuel Almagro, Jr.
Rebecca C. Amoroso
Mary V. Anderson
Kenneth Apfel
Richard V. Atkinson
James J. Callahan
Christopher S. Carlson
Louis-Philippe Caron
Michael J. Cascio
Sanders B. Cathcart
Ralph M. Cellars
David A. Christhilf
Susan J. Comstock
David B. Cox
Dan J. Davis
Raymond V. Debs
James M. Dekle
Michael J. Doyle
Jeffrey A. Englander
James E. Fletcher
Barbara L. Forbus
Richard Gauthier
James J. Gebhard
Peter M. Gidos
Steven A. Glicksman
Jeffrey H. Graham
Denis G. Guenthner
Randolph S. Hay
Joseph A. Herbers
Richard J. Hertling
Mark J. Homan
Wendy A. Johnson
Kenneth R. Kasner
Paul J. Kneuer
David Koegel
Rodney E. Kreps
John M. Kulik
Chung-Kuo Kuo
Mary Lou Lacek
Marthe A. Lacroix
Alain Lessard
Mark D. Lyons
Patrick Mailloux
Mary E. McCoy
Leonard L. Millar
Susan M. Miller
David F. Mohrman
Robert A. Mueller
Donald R. Musante
Richard T. Newell, Jr.
Henry E. Newman
Bruce E. Ollodart
Gregory V. Ostergren
Wade T. Overgaard
Anthony Peraine
Ronald D. Pridgeon
Boris Privman
Frank S. Rhodes
Denise E. Rice
James W. Rice
Robert S. Roesch
Donald D. Sandman
Mark W. Scully
Linda A. Shepard
George C. Sornberger
Bruce R. Spidell
Russell Steingiser
Russel L. Sutter
Suan-Boon Tan
Robert W. Thompson
Nanette Tingley
Ernest S. Tistan
Michel Trudeau
George W. Turner, Jr.
William J. Von Seggern
David G. Walker
Kelly A. Wargo
Dominic A. Weber
Arlene F. Woodruff
Chung-Ye Yen
James W. Yow

Mr. Ben-Zvi then introduced Charles Hewitt, who delivered a brief speech to the new members concerning the responsibilities of a casualty actuary.

Mr. Ben-Zvi then introduced Michael Fusco, Vice President of Programs, who gave a brief summary of the program content.

Mr. Ben-Zvi next introduced Stephen Philbrick, Chairman of the Committee on Review of Papers, who gave a brief summary of the new *Proceedings* papers.

Janet Fagan, Chairman of the Committee on Continuing Education, gave a brief summary of the Discussion Paper program and of the process of issuing the call for papers and reviewing the papers submitted.

Mr. Ben-Zvi concluded the business session at 9:00 a.m.

At 9:00 a.m., Mr. Leroy Simon moderated a panel entitled "Reinsurance – A Global Perspective." His panel consisted of:

James Meenaghan
President and CEO
John F. Sullivan Co.

Erkki Pesonen
Chairman of the Board
Kansa Group

Michael Fitt
President and CEO
Employers Reinsurance Corporation

The panelists commented on the current availability and affordability of reinsurance and implications for the primary domestic market.

Beginning at 11:00 a.m., there was a series of concurrent sessions, including eight Discussion Paper presentations, two *Proceedings* papers presentations, and four workshops.

The new *Proceedings* papers were:

1. "Classical Partial Credibility with Application to Trend"
   Author: Gary G. Venter
   Vice President & Actuary
   National Council on Compensation Insurance

2. "An Actuarial Note on Credibilty Parameters"
   Author: Howard C. Mahler
   Vice President & Actuary
   Massachusetts Rating Bureaus

The Discussion Papers presented were:

1. "The Operational Aspects of Outwards Reinsurance Treaties"
   Author: David S. Powell
   Tillinghast, Nelson & Warren, Inc.

2. "The Cost of Mixing Reinsurance"
   Author: Ronald F. Wiser
   St. Paul Fire and Marine Insurance Company

3. "Foreign Exchange Fluctuations in the Annual Statement"
   Author: Kirk G. Fleming
   Milliman & Robertson, Inc.

4. "Recent Developments in Reserving for Losses in the London Reinsurance Market"
   Author: Harold E. Clarke
   Bacon & Woodrow

5. "An Analysis of Excess Loss Development"
   Authors: Emanuel Pinto and Daniel F. Gogol
   Metropolitan Reinsurance Company

6. "Reserve Review of a Reinsurance Company"
   Author: Stephen W. Philbrick
   Tillinghast, Nelson & Warren, Inc.

7. "Reinsurance Pricing for the New Transitional Claims-Made G.L. Product"
   Author: Nolan E. Asch
   SCOR Reinsurance Co.

8. "Simulating Serious Workers' Compensation Claims"
   Authors: Gary G. Venter and William R. Gillam
   National Council on Compensation Insurance

The workshops covered the following topics:

1. "State-of-the-Art Homeowner's Ratemaking Techniques"
   Moderator: Charles A. Bryan
   Senior Vice President & Actuary
   USAA

   Panelists: Harry T. Byrne
   Actuary
   Aetna Life and Casualty

   John P. Drennan
   Assistant Vice President & Actuary
   Allstate Insurance Company

Harold N. Schneider
Vice President & Actuary
Farmers Insurance Group

2. "Application of Operations Research: A Syllabus Update"
   Moderator: Robert J. Finger
   Vice President & Actuary
   Future Cost Analysts

3. "How Reinsurance Really Works!"
   Moderator: Mary E. Hennessy
   Consulting Actuary
   Towers, Perrin, Forster & Crosby

   Panelists: Paul C. J. Markey
   Second Vice President
   Herbert Clough, Inc.

   Frank S. Wilkinson
   Partner
   E. W. Blanch

4. "The Federal Government as a Source of Reinsurance Capacity"
   (limited attendance workshop)
   Workshop Coordinator: Mavis A. Walters
   Senior Vice President
   Insurance Services Office

The President's Reception was held from 6:30 p.m. to 7:30 p.m.

Tuesday, May 13, 1986

Tuesday was devoted to a continuation of the concurrent sessions from Monday afternoon.

A general reception and western barbeque was held from 6:30 p.m. to 9:30 p.m.

Wednesday, May 14, 1986

Mr. Ben-Zvi reconvened the business session at 8:00 a.m. He awarded the Harold Schloss Scholarship to Mark Meyer.

Mr. Ben-Zvi introduced Patricia Furst, the chairman of the Michelbacher Committee. Ms. Furst briefly described the judging process and then awarded the Michelbacher Prize to Ronald Wiser, author of the Discussion Paper entitled "The Cost of Mixing Reinsurance."

Mr. Thomas Murrin then gave a brief summary of the activities of the Interim Actuarial Standards Board.

Mr. Ben-Zvi thanked those individuals who had planned the meeting and executed those plans. He then turned the podium over to Mr. Michael Walters. Mr. Walters introduced the first of the two panels, entitled "Should I Go Direct or Broker My Reinsurance?" The panel consisted of:

Patrick J. McFadden
Director – Reinsurance Brokerage Division
Towers, Perrin, Forster & Crosby

Tom N. Kellogg
Senior Vice President
General Reinsurance Corporation

At 9:30 a.m., a second panel was presented, moderated by Mr. Daniel McNamara, entitled "Does the United States Tort System Make Pricing Liability Insurance Impossible?" The panelists were:

Leslie Cheek
Vice President – Federal Affairs
Crum & Forster

Bruce Foudree
Insurance Commissioner
State of Iowa

Edward Hamilton
President
Hamilton, Rabinovitz, Szanton & Alschuler, Inc.

Mr. Ben-Zvi then closed the meeting, reminding all participants that the 1987 Discussion Paper subject is "The Financial Analysis of Insurance Companies." The meeting was adjourned at 11:15 a.m.

May 1986 Attendees

In attendance as indicated by the registration records were 303 Fellows; 195 Associates; and 42 guests, subscribers, and students. The list of their names follows.

FELLOWS

Addie, B. J.
Adler, M. J.
Alfuth, T. J.
Asch, N. E.
Atwood, C. R.
Barrow, B. H.
Bartlett, W. N.
Bass, I. K.
Bassman, B. C.
Basson, S. D.
Baum, E. J.
Bear, R. A.
Beer, A. J.
Belden, S. A.
Bell, L. L.
Bensimon, A. S.
Ben-Zvi, P. N.
Berquist, J. R.
Berry, J. L.
Bethel, N. A.
Beverage, R. M.
Bill, R. A.
Biondi, R. S.
Boccitto, B. L.
Boison, L. A., Jr.
Boone, J. P.
Bornhuetter, R. L.
Boulanger, F.
Bouska, A. S.
Boyd, W. A.
Bradshaw, J. G., Jr.
Braithwaite, P.
Brannigan, J. F.
Briere, R. S.
Brooks, D. L.
Bryan, C. A.
Bursley, K. H.
Byrne, H. T.
Cantin, C.
Captain, J. E.
Chansky, J. S.
Chanzit, L. G.
Cheng, J. S.
Chernick, D. R.
Childs, D. M.
Christiansen, S. L.
Christie, J. K.
Cis, M. M.
Clark, D. B.
Cohen, H. L.
Conger, R. F.
Cook, C. F.
Corr, F. X.
Covney, M. D.
Cundy, R. M.
Curley, J. O.
Curran, K. F.
Currie, R. A.
Curry, A. C.
Dahlquist, R. A.
Dean, C. G.
Dempster, H. V.
Deutsch, R. V.
Doellman, J. L.
Doepke, M. A.
Dolan, M. C.
Donaldson, J. P.
Dornfeld, J. L.
Dorval, B. T.
Downer, R. B.
Drennan, J. P.
Duffy, T. J.
Easton, R. D.
Egnasko, G. J.
Egnasko, V. M.
Ehlert, D. W.
Eldridge, D. J.
Engles, D.
Fagan, J. L.
Fallquist, R. J.
Fein, R. I.
Fiebrink, M. E.
Finger, R. J.
Fisher, R. S.
Fisher, W. H.
Flaherty, D. J.
Foote, J. M.
Forker, D. C.
Forney, J. R., Jr.
Foster, R. B.
Fresch, G. W.
Friedberg, B. F.
Furst, P. A.
Fusco, M.
Gallagher, C. A.
Gallagher, T. L.
Garand, C. P.
Gilles, J. A.
Ginsburgh, H. J.
Gleeson, O. M.
Gluck, S. M.
Goddard, D. C.

FELLOWS

Goldfarb, I. H.
Gorvett, K. P.
Gottlieb, L. R.
Grace, G. S.
Grady, D. J.
Graham, T. L.
Grannan, P. J.
Greco, R. E.
Hafling, D. N.
Hale, J. B.
Hall, A. A.
Hallstrom, R. C.
Haner, W. J.
Hanson, H. D.
Hanson, J. L.
Hartman, D. G.
Harwayne, F.
Haskell, G. E.
Hayne, R. M.
Hayward, G. L.
Heer, E. L.
Hennessy, M. E.
Henzler, P. J.
Herder, J. M.
Hewitt, C. C., Jr.
Hibberd, W. J.
Higgins, B. J.
Hillhouse, J. A.
Hough, P. E.
Hutter, H. E.
Ingco, A. M.
Irvan, R. P.
Jameson, S.
Jean, R. W.
Jerabek, G. J.
Johe, R. L.
Johnson, W. H., Jr.
Johnston, T. S.
Jones, B. R.
Kane, A. B.
Kaplan, R. S.
Karlinski, F. J., III
Kaufman, A. M.
Keatts, G. H.
Khury, C. K.
Kilbourne, F. W.
Knilans, K.
Kollar, J. J.
Kooken, M. W.
Kozik, T. J.
Lamb, R. M.
LaRose, J. G.
Larsen, M. R.
Lattanzio, S. P.
Lehmann, S. G.
Leong, W.
Levin, J. W.
Lewis, M. A.
Linden, O. M.
Lino, R.
Lino, R. A.
Lipton, B. C.
Lommele, J. A.
Longley-Cook, L. H.
Loucks, W. D., Jr.
Ludwig, S. J.
Lyle, A. C.
Mahler, H. C.
Makgill, S. S.
Marks, S. D.
Mashitz, I.
Mathewson, S. B.
Mayer, J. H.
McCarter, M. G.
McConnell, C. W.
McGovern, W. G.
McMurray, M. A.
McNamara, D. J.
McSally, M. J.
Mealy, D. C.
Meeks, J. M.
Mendelssohn, G. A.
Meyer, R. E.
Meyers, G. G.
Miccolis, J. A.
Miccolis, R. S.
Miller, D. L.
Miller, R. A., III
Miller, R. R.
Mills, R. J.
Moody, R. A.
Morgan, W. S.
Morison, G. D.
Muetterties, J. H.
Mulder, E. T.
Munro, R. E.
Murad, J. A.
Murphy, W. F.
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A FORMAL APPROACH TO CATASTROPHE RISK ASSESSMENT AND MANAGEMENT

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Abstract

Insurers paid $1.6 billion on property claims arising from catastrophes in 1984. Researchers have estimated that annual insured catastrophe losses could exceed $16 billion. Certainly, the financial implications for the insurance industry of losses of this magnitude would be severe; even industry losses much smaller in magnitude could cause financial difficulties for insurers who are heavily exposed to the risk of catastrophe losses.

The quantification of exposures to catastrophes and the estimation of expected and probable maximum losses on these exposures pose problems for actuaries. This paper presents a methodology based on Monte Carlo simulation for estimating the probability distributions of property losses from catastrophes, and discusses the uses of the probability distributions in management decision-making and planning.

INTRODUCTION

There were 28 catastrophes in 1984; they resulted in an estimated $1.6 billion of insured property damage. Most of these catastrophes were natural disasters such as hurricanes, tornadoes, winter storms, and floods. In 1985, Hurricane Elena caused over $543 million of insured losses, and a tornado outbreak affecting nine states caused insured damage of $231 million.

Hurricane Elena barely rated a three on a severity scale ranging from one to five, in which destruction from hurricanes increases exponentially with increasing severity. A hurricane that rated a four hit New York and New England in 1938; 600 people died and wind speeds of 183 mph caused hundreds of millions of dollars of damage.

If this storm were to strike again, dollar losses to the insurance industry could exceed ten billion given the current insured property values on Long Island and along the New England coast. Estimates of the dollar damages that will result if a major earthquake occurs in Northern or Southern California are even larger in magnitude.

A very severe hurricane or earthquake would produce a year of catastrophic loss experience lying in the upper tail of the probability distribution of annual losses from catastrophes. It is the opinion of the author that the 1984 catastrophe loss figure lies in the lower end of this distribution. However, the determination of the shape and the estimation of the parameters that describe this distribution are tasks that are not easily performed using standard actuarial methodologies. Yet since insurers require knowledge of their exposure to catastrophes and the probability distributions of annual catastrophe losses to make pricing, marketing, and reinsurance decisions, actuaries must be able to estimate the parameters of the distributions, including the expected and probable maximum losses.

Standard statistical approaches to loss estimation involve the use of historical data to estimate future losses. However, approaches that employ time series of past catastrophe losses can give poor estimates of potential catastrophe losses. Catastrophes are rare events so that the actual loss data are sparse and their accuracy is questionable; average recurrence intervals are long so that many exogenous variables can change in the time periods between occurrences. In particular, changing population distributions, changing building codes, and changing building repair costs alter the annual catastrophe loss distribution.

Since most catastrophes are caused by natural hazards, and since most natural hazards have geographical frequency and severity patterns associated with them, the population distribution impacts the damage-producing potentials of these hazards. A natural disaster results when a natural hazard occurs in a populated area. Changing population patterns necessarily alter the probability distribution of catastrophic losses. Since the average recurrence intervals of natural hazards in any particular area are long, patterns of insured property values may vary between occurrences to an extent that damage figures of historical occurrences have little predictive power. For example, the 1906 San Francisco earthquake caused losses of $364 million. In 1985 dollars, this equals $4.5 billion. Yet some have estimated that an earthquake of this size could cause damages exceeding $30 billion today.

It is primarily the influence of the geographic population distribution that renders time series models of natural catastrophe losses inadequate, although changing building codes also alter the loss-producing potentials of natural hazards. Over time, building materials and designs change, and new structures become more or less vulnerable to particular natural hazards than the old structures. Of course, changes in building repair costs also affect the dollar damages that could result from catastrophes.

The above issues do not render the estimation problem intractable, but they do indicate a need for an alternative methodology to approaches which employ historical catastrophe losses adjusted for inflation to estimate the probability distribution of losses. Even models which adjust historical losses for population shifts can give only very rough approximations of expected and probable maximum losses.

This paper presents a methodology based on Monte Carlo simulation, and it focuses on property damage arising from natural disasters. The next two sections discuss the simulation approach to catastrophe loss estimation. A windstorm example is then presented. Output analysis, model validation, and model uses are discussed in the following three sections.

THE SIMULATION APPROACH

The simulation approach is, very basically, the development of computer programs which describe or model the particular system under study. All of the system variables and their interrelationships are included. A high speed computer then "simulates" the activity of the system and outputs the measures of interest.

Simulation models may be deterministic or stochastic. Monte Carlo simulation models are stochastic models, and therefore, the variables which they include are random variables. Numbers are generated from the probability distributions of the random variables to assign values to the variables for each model simulation. These probability distributions are either standard statistical distributions (selected on the basis of good fits with empirical data) or actual empirical distributions.

Typically, many simulations or iterations are performed to derive estimates of the measures of interest from Monte Carlo simulation models. This is necessary to ensure that the output distribution has converged to the true distribution and that model derived estimates are "accurate." Obviously, the larger the variances of the model variables, the larger the number of model iterations necessary to reach convergence.

Computer simulation models can provide powerful tools for the analyses of a wide variety of problems, especially problems which involve solutions that are difficult to obtain analytically. Law and Kelton [8] state that "Most complex, real-world systems . . . cannot be accurately described by a mathematical model which can be evaluated analytically. Thus, a simulation is often the only type of investigation possible." The natural hazard loss-producing system is one such system.

THE NATURAL HAZARD SIMULATION MODEL

The natural hazard simulation model is a model of the natural disaster "system." The primary variables are meteorological or geophysical in nature. They may be classified as frequency or severity variables. The frequency variables determine the number of occurrences of the particular events within a given time period. Severity variables account for a hazard's force, size, and duration. These variables are, of course, random variables with stable (time independent[1]) probability distributions.

The model simulates the physical occurrences of the natural hazards by generating numbers from these probability distributions. Numbers are generated to assign values to each variable for each simulated occurrence. The probability distributions are estimated using historical data combined with the knowledge of authoritative meteorologists and geophysicists.

It is most efficient from a computational standpoint to generate numbers from the well-known statistical distributions. The empirical distributions formed by the raw data may be fit to these theoretical distributions using appropriate goodness-of-fit tests. If the data do fit any of these probability distributions, the moments of the distributions may be estimated and employed by the simulation model.

If the empirical data do not fit any theoretical distributions, the empirical distribution may be used for the generation of values for particular model variables. This procedure, however, has some drawbacks. First, since the sample is a collection of random data, a different sample could yield a very different

[1] There may be a time-space dependence with respect to earthquake severity.

empirical distribution. Second, the generation of random variables from an empirical distribution precludes the possibility of generating values of the variable outside of the observed range, and the observed range may not include all possible values of the variable. If the empirical data are sparse or do not fit theoretical distributions, knowledgeable physical scientists may provide information regarding the ranges of possible values of particular variables, as well as the shapes of the distributions and the most likely values of the variables.

Variables that change with time, e.g., the geographic distribution of exposure units, the insured property values, and the building construction types, are inputs into the model. The probability distribution of losses from natural hazards given these inputs is the model output. Per occurrence as well as annual aggregate distributions are estimated.

The model simulates the physical occurrences of the natural hazards and their effects on exposed properties thousands of times in order to estimate the distributions of losses. Thousands of iterations are performed to ensure that all possibilities have been simulated in accordance with the actual probabilities of occurrence and that the estimated distributions converge to the true distributions.

A WINDSTORM EXAMPLE

A model of the hurricane hazard has been developed and will be used to illustrate the Monte Carlo simulation approach. Exhibit I is a simplified flowchart of the computer model.

Most of the storm data used in the development of the model were obtained from the U.S. Department of Commerce. The data had been collected and analyzed by various agencies of the National Weather Service, and they included 86 years of history spanning the period 1900 to 1985. Complete and accurate meteorological data were available for most of the hurricanes that struck the U.S. in this time period.

A hurricane is a closed atmospheric circulation which develops over tropical waters and in which winds move counterclockwise around a center of pressure lower than the surrounding area. It is a severe tropical storm, with a center of pressure less than or equal to 29 (inches), which causes sustainable wind speeds of 74 mph or more. One hundred and thirteen hurricanes made landfall in the

U.S. during the sample period. One hundred and thirty-eight hurricanes either approached and bypassed (within 150 nautical miles), exited[2], or entered the U.S. during the period.

Annual Frequency

Referring to Exhibit I, the first step of the model (for each iteration) is the generation of the annual number of landfalling hurricanes. Table 1 shows the number of years in which the number of occurrences was 0, 1, 2, and so on. The historical data fit a negative binomial distribution with $s = 5$ and $p = .79$. The chi-square goodness-of-fit test statistic equals 2.923 which is not significant even at the $\alpha = .5$ level.

TABLE 1

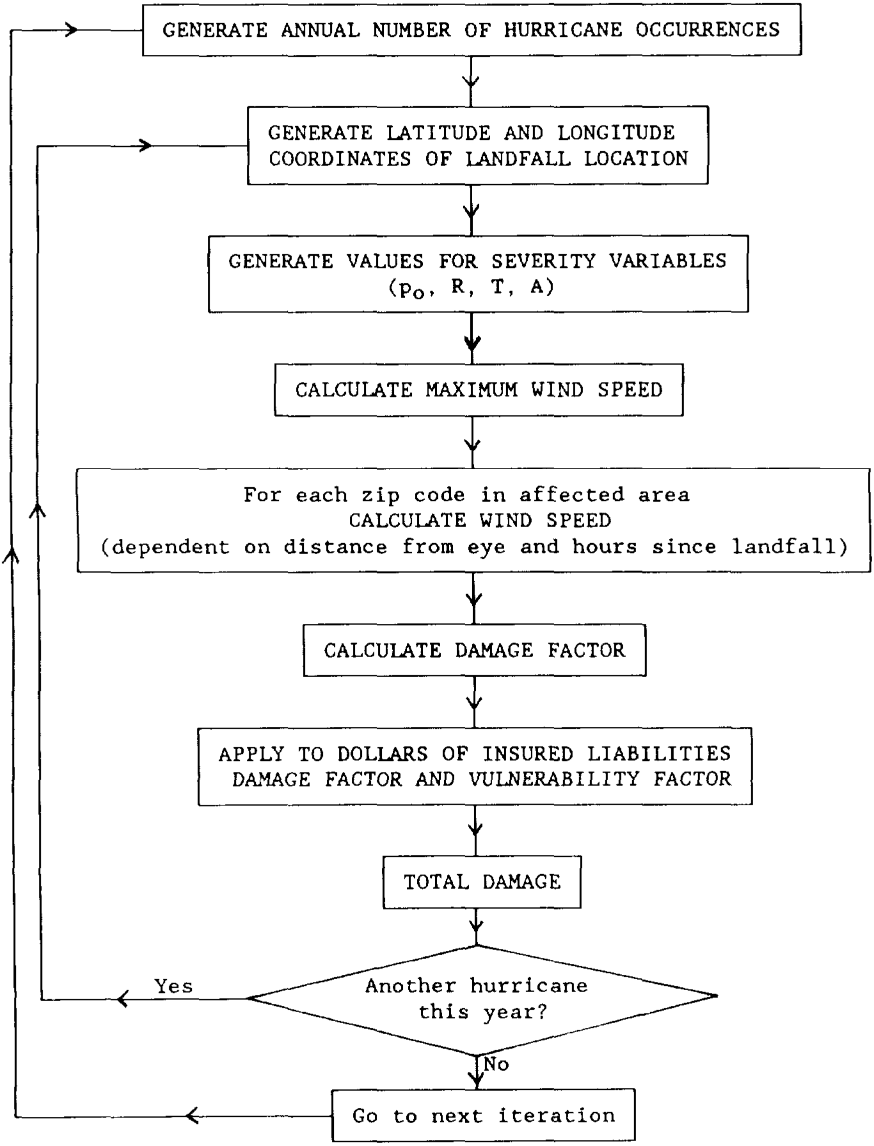
ANNUAL NUMBER OF HURRICANES LANDFALLING IN U.S.
(EXCLUDING EXITING STORMS)
1900–1985

No. Storms Per Year	Observed Occurrence	Relative Frequency	Neg. Bin. Rel. Freq.
0	26	.302	.308
1	29	.337	.323
2	18	.209	.204
3	6	.070	.100
4	6	.070	.042
>4	1	.012	.023

[2] An exiting storm is a hurricane that moves from land to sea and has a central pressure lower than 29 inches.

EXHIBIT I

MODEL FLOWCHART



Locational Frequency

The next step of the model is the determination of the landfall location of each storm. Hurricanes enter the U.S. from the Gulf and East Coasts. The map in Exhibit II shows the U.S. coastline from Texas to Maine divided into 31 smoothed 100 nautical mile segments.[3] The number of hurricanes that entered through each segment or bypassed within 150 nautical miles of the segment during the sample period is also shown.

The numbers indicate that there are variations in locational frequencies. In this case, it would not be correct to generate the landfall location from a distribution which assigns equal probabilities to all values, i.e., a uniform distribution. Neither would one want to use the actual numbers of storms to form the empirical distribution from which the landfall locations will be generated. This is because the selection of length of coastal segment is necessarily arbitrary. If a different length were used, the empirical distribution would be different. Additionally, although several segments are completely free of historical storm occurrences, it is not clear that the probability of hurricane landfall is zero in those areas.

To derive the model locational frequency distribution, the raw data on the numbers of occurrences were smoothed using a procedure selected on the basis of its ability to capture turning points in the data while smoothing slight variations. The coastline was redivided into 50 nautical mile segments, and the number of occurrences for each segment was set equal to the weighted average of 11 successive data points centered on that segment. The smoothed frequency values were obtained as follows:

$$F_i = \frac{\sum_{n=-5}^{5} W_n C_{i+n}}{\sum_{n=-5}^{5} W_n}$$

where C_i = the number of historical hurricane occurrences for the ith segment;

F_i = the smoothed frequency value for the ith segment; and,

W_n = .30, .252, .14, .028, −.04, −.03

for n = 0, ±1, ±2, ±3, ±4, ±5, respectively.

[3] The coastline is smoothed for irregularities such as inlets and bays.

EXHIBIT II

HURRICANES ENTERING OR BYPASSING THE U.S. 1900–1985

This is the preferred smoothing procedure in climatological analyses because the weighting scheme maintains the frequency and phase angle of the original series of numbers. The endpoints of the series are approximated so that each segment of the coast is assigned a relative frequency. The landfall location of each storm is generated from the thus derived locational frequency distribution.

Severity

Step three of the model is the generation of values for the severity variables. There are four primary variables which account for hurricane severity. These variables are: the minimum central pressure, the radius of maximum winds, the forward speed, and the angle at which the storm enters the coast, i.e., the track direction.

Central pressure (p_0) is defined as the sea-level pressure at the hurricane center or eye. This is the most important variable for computing hurricane wind speeds, and it is a universally accepted index of hurricane intensity. All else being equal, the square of the wind speed varies directly with Δp ($\Delta p = p_w - p_0$ where p_w is the peripheral pressure).

The radius of maximum winds (R) is the radial distance from the hurricane center to the band of strongest winds. Forward speed (T) refers to the rate of translation of the hurricane center from one geographical point to another. Track direction (A) is the path of forward movement along which the hurricane is traveling and is measured clockwise from north.

Hurricane severity varies by location as does frequency. In general, as latitude increases, average hurricane severity decreases. When a hurricane moves over cooler waters, its primary source of energy (latent heat from warm water vapor) is reduced so that the intensity of circulation decreases in the absence of outside forces. As such, the shapes and parameters of the severity variable probability distributions were estimated for each coastal location.

For each severity variable except track direction, samples of data points from 400 nautical mile segments of coastline were used to estimate the parameters of the distribution for each 100 nautical mile segment. Overlapping 400 nautical mile segments were centered on successive 100 nautical mile segments, the data were fit to theoretical statistical distributions, and the parameters were estimated.

The selection of 400 nautical mile lengths of coastline was somewhat arbitrary; 300, 400, and 500 nautical mile segments have all been used in climatological analyses of hurricane data. Obviously, shorter segments capture more of the variation in the historical data while larger segments increase the size and hence the credibility of the data sample used for estimation.

CENTRAL PRESSURE

The distribution of historical hurricane central pressures is a skewed distribution with an upper bound of 29 inches. Tropical storms with higher central pressures will in most cases not produce winds of hurricane force. Since the distribution is truncated at one end, the variable *Pdif* was modeled instead of p_0. *Pdif* was defined as 29 minus the central pressure of the storm. *Pdif* also has a skewed distribution so that the historical data were fit to both lognormal and Weibull distributions using the Kolmogorov-Smirnov goodness-of-fit test.

The Weibull distribution produced the best fit of the empirical data. Table 2 shows the estimated parameters, α and β, for each coastal segment along with the number of data points in each sample, N, and the goodness-of-fit test statistic, KS. No KS statistic was significant at the 99% confidence level.

RADIUS OF MAXIMUM WINDS

The distribution of R for each coastal segment is symmetrical around the average value. The normal distribution provided a good fit of the historical data, and the parameters of this distribution were estimated for each coastal segment. The mean value of R increases with increasing latitude. Exhibit III shows a plot of latitude versus the radius of maximum winds for the historical Gulf and East Coast hurricanes.

The radius of maximum winds seems to be positively correlated with central pressure as well as with latitude. Table 3 shows linear correlation coefficients (Pearson's) between the pairs of variables. Although tests of significance could not be performed on the correlation coefficients since it could not be assumed that pairs of variables form bivariate normal probability distributions, it is assumed that there is a positive correlation between p_0 and R. The meteorological literature on hurricanes supports this assumption.

TABLE 2

CENTRAL PRESSURE—WEIBULL DISTRIBUTION PARAMETER ESTIMATES FOR 100 NAUTICAL MILE SEGMENTS

100 n.mi. Segment	400 n.mi. Segment	α	β	N	KS
1	−150–250	2.020	1.080	9	.223
2	−50–350	1.773	0.974	16	.165
3	50–450	1.882	0.910	22	.147
4	150–550	1.819	0.906	22	.149
5	250–650	1.468	0.748	26	.111
6	350–750	1.350	0.801	23	.094
7	450–850	1.223	0.707	23	.066
8	550–950	1.270	0.690	25	.090
9	650–1050	1.128	0.572	23	.095
10	750–1150	1.161	0.573	20	.107
11	850–1250	1.251	0.426	18	.187
12	950–1350	1.296	0.624	16	.135
13	1050–1450	1.111	0.832	21	.139
14	1150–1550	1.545	0.875	28	.144
15	1250–1650	1.529	0.953	31	.116
16	1350–1750	1.423	0.838	24	.140
17	1450–1850	1.793	0.815	13	.141
18	1550–1950	1.534	0.485	8	.176
19	1650–2050	0.844	0.463	7	.166
20	1750–2150	1.007	0.563	12	.156
21	1850–2250	1.285	0.676	19	.207
22	1950–2350	1.204	0.655	18	.204
23	2050–2450	1.416	0.668	16	.202
24	2150–2550	1.455	0.628	12	.234
25	2250–2650	1.177	0.566	8	.296
26	2350–2750	1.556	0.663	9	.260
27	2450–2850	1.429	0.646	9	.277
28	2550–2950	1.325	0.596	10	.252
29	2550–2950	1.325	0.596	10	.252
30	2550–2950	1.325	0.596	10	.252
31	2550–2950	1.325	0.596	10	.252

EXHIBIT III

LATITUDE VS. RADIUS OF MAXIMUM WINDS

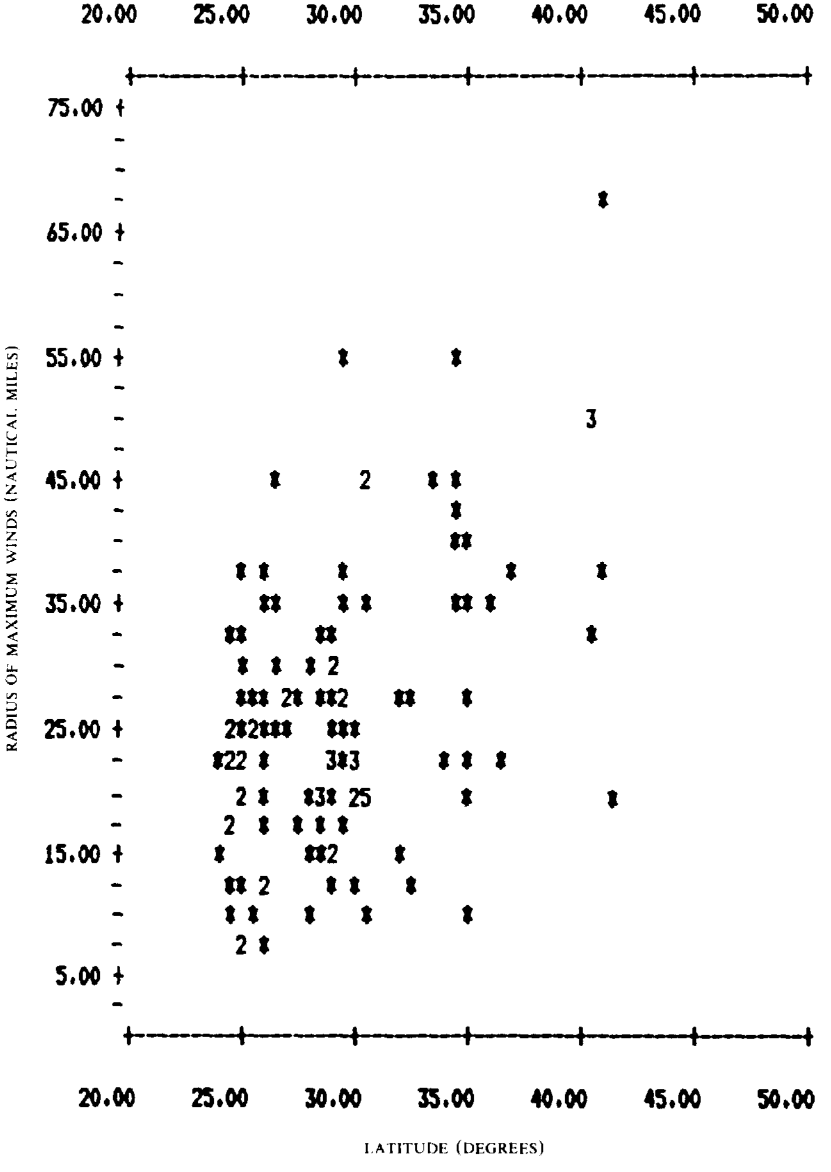


TABLE 3

LINEAR CORRELATION COEFFICIENTS

	East Coast Hurricanes						Gulf Coast Hurricanes				
	p_0	R	T	A	Lat		p_0	R	T	A	Lat
p_0		.27	−.04	−.08	.32	p_0		.31	.04	.17	.17
R			.35	.24	.49	R			.13	.04	.16
T				.42	.73	T				.06	.35
A					.50	A					.11

This correlation is accounted for by the model in two ways. First, since p_0 and R are both correlated with latitude and the distributions of p_0 and R have been estimated at various latitude points, the simulated values of the variables will necessarily be correlated. Also, the lower and upper bounds of simulated R values are determined by the value of p_0 for the simulated storm. As shown in Table 3, p_0 and R are positively correlated so that severe storms typically have smaller R's than weak storms.

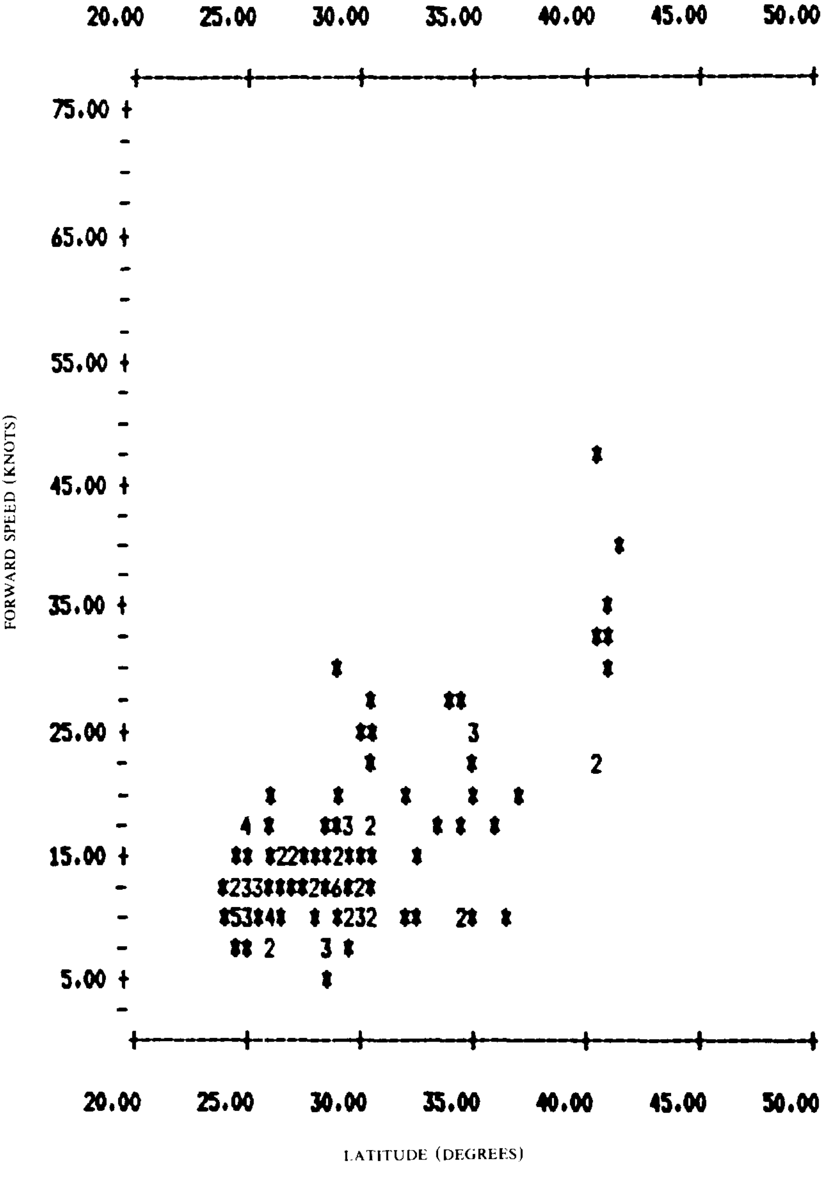
It should be noted at this point that the simulated values of all severity variables are bounded so that only storms with a nonzero probability of occurrence are simulated. The upper and lower bounds of the model variables have been determined somewhat subjectively by meteorologists who are experts on the subject of hurricanes. The model procedure is to regenerate values that are out of range rather than assign a value equal to the lower or upper bound of the range. This ensures that the simulated values will not be clustered at the endpoints of the ranges. Since the estimated distributions fit quite well, the simulated values fall within the acceptable range a high proportion of the time.

FORWARD SPEED

The historical data on forward speed fit lognormal distributions, and these distributions are employed by the model to generate values of T for each simulated storm. The average value of T increases with increasing latitude, and the lower and upper bounds of T are dependent on latitude. Exhibit IV is a plot of latitude versus forward speed for the historical storms.

EXHIBIT IV

LATITUDE VS. FORWARD SPEED



TRACK DIRECTION

Four hundred nautical mile segments were not used to estimate the parameters of the distributions of track direction for each coastal location. Since the orientation of the coastline influences the likely as well as the possible angles of entry at each coastal point, segments of varying length were employed. The length that was selected for each segment was the length of smoothed coastline with the same angle orientation as the segment of interest.

Track direction is distributed symmetrically around its average value, thus values for A are generated from the normal distribution. However, at some coastal locations, the standard deviation is quite wide relative to the range of possible values so that the distributional shape begins to tend to uniform. In these cases, a relatively high proportion of simulated values could need to be regenerated. For example, at three coastal segments, the range of possible values is only ± one standard deviation wide. Values for A could need to be regenerated 32% of the time for storms landfalling in these segments. Fortunately, the number of such segments is small.

Maximum Wind Speeds

Once values are obtained for all of the severity variables, the maximum sustained wind speed is calculated via straightforward meteorological formulas. The movement of the storm is next simulated by the computer model, and maximum wind speeds are calculated for each zip code area in the affected region.

The wind speed at each zip location is dependent on the distance of the location from R and on the hours since landfall. The wind speeds decrease as the distance from R increases and as the time since landfall increases.

Insured Damages

Dollar damages are estimated by applying damage and vulnerability factors to the insured property values in each zip code area. The damage factors are based on the results of engineering studies of the relationship between wind speed and structural damage. The vulnerability factors account for the variability in inflicted damage due to construction type and age. The dollar damages are accumulated for each storm.

Two thousand years of hurricane experience are simulated by the model. These two thousand iterations provide estimates of the complete probability distributions of annual hurricane losses and per occurrence losses from which expected and probable maximum loss estimates are derived.

OUTPUT ANALYSIS

Exhibit V shows the expected losses as well as the 80%, 90%, 95%, and 99% confidence level losses calculated as the 80th, 90th, 95th, and 99th percentile losses, respectively, for the geographical distribution of property exposures of a hypothetical company. The confidence level losses may be interpreted in two ways. A given confidence level loss shows the loss amount for which the probability of experiencing losses above that amount is 1.0 minus the particular confidence level. For example, for the loss distribution in Exhibit V, the probability of experiencing losses greater than $10 million is .20. The confidence level loss also shows the loss amount for which losses greater than that amount will be experienced, on average, once in every 1.0/(1.0 − confidence level) years. Again, from Exhibit V, losses greater than $10 million will be experienced once in every five years, on average. The loss distribution is highly skewed with a median value which is much below the mean and a high proportion of zero values.

EXHIBIT V

MODEL-GENERATED LOSS ESTIMATES (000'S)

Insured Liabilities	Expected Losses	Confidence Level Losses			
		80%	90%	95%	99%
7,170,753	9,011	10,003	24,179	44,827	117,946

Since the estimated loss distribution is so skewed, many model iterations are performed to ensure convergence to the true underlying loss distribution. Unfortunately, there is no straightforward formula for calculating the number of iterations necessary to obtain estimates with specific levels of precision. If computer resources are not a constraint, thousands of iterations should be performed to ensure convergence. If computing power is limited, iterations can be performed in groups of a hundred or so, and the distribution can be tested for significant changes after each group of iterations. When changes become arbitrarily small, the simulation run can be terminated.

MODEL VALIDATION

The validation of simulation models is often problematic. Since simulation models are representations of real world systems, they are usually simplifications of complex systems. As such, statistical tests of differences between actual data and simulated data will typically show statistically significant differences even if the simulation model is a good or at least "acceptable" representation of reality. As mentioned previously, simulation models are often built when no alternative means of analysis are available. The model builder must decide if model performance is acceptable or if more resources should be employed in improving the simulation model. The decision is more of a cost versus benefit decision than an accept versus reject decision.

In cases in which there is little actual data to compare to the simulated data, model validation is even more difficult. The natural hazard simulation model output, i.e., the catastrophe loss distribution, is an estimate of long run average costs given a particular geographical distribution of property exposures. It includes estimates of long run expected losses and probable maximum losses. There are no actual data to compare to the model output.

There are, however, two sets of assumptions to be tested. The first set includes all of the assumptions concerning the physical characteristics of the particular type of natural hazard. Do the physical characteristics of the simulated natural hazards match the characteristics of actual historical occurrences? If the probability distributions of the frequency and severity variables have been selected and estimated properly, simulated occurrences should be very similar to the historical occurrences.

In the hurricane model, the probability distributions of the model variables were fit to theoretical statistical distributions using the chi-square and Kolmogorov-Smirnov goodness-of-fit tests. Since the theoretical distributions were selected on the basis of a good fit with the empirical data, the simulated values of the variables match closely the historical values.

The second set of assumptions to be tested include all of the engineering assumptions which correlate the loss-producing phenomena with actual structural damage. These assumptions are more difficult to test empirically since actual loss data are needed. Testing requires the comparison of losses from particular natural catastrophes with the losses that the model would estimate for occurrences with the same physical characteristics, given the same geographical distributions of exposed properties. Frequently, these data are unavailable. If they are available, they are generally not available in the quantity necessary for statistical testing.

Results of these tests could be used to calibrate the model, however, it is not clear that the model builder would want to calibrate the model to a small number of actual data points. The objective of the model is to project long run average costs, not to predict losses from individual occurrences. There is so much randomness involved in a single occurrence that one cannot expect the model loss estimates to mirror exactly actual losses on each individual occurrence.

The question that arises then is whether or not the model is valid if it cannot be tested statistically. What is the value of the model if one cannot prove that its estimates are "correct"?

The nature of statistics is such that one can never prove that the sample is a true representation of the population. Statistical tests of significance merely provide confidence intervals for parameter estimates which are based on certain assumptions. These tests are used to choose between alternatives or competing hypotheses.

In the case of the catastrophe simulation model, there are no good alternative estimators. Yet there is a real need for the model output, i.e., an estimate of the catastrophe loss distribution. Insurers and reinsurers make decisions every day that affect the catastrophe loss distributions. They need to know how their decisions impact these distributions so that they can make the appropriate risk versus return trade-offs.

The degree of confidence that one has in model-generated estimates is a direct function of the level of confidence in the model assumptions. If each assumption has been tested for reasonability[4], then the model output should provide reasonable estimates. The area of validation of the natural hazard simulation model is an area worthy of further research.

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[4] There are several ways to test for reasonability. One way is to show experts in the field samples of simulated data and samples of actual data. If the experts cannot separate the actual data from the simulated data, the model builder can safely assume that the model is a good representation of reality.

MODEL USES

Knowledge of the probability distributions of property losses due to catastrophes enables management to plan for these events. The natural hazard simulation model helps insurers to manage their exposure to catastrophes; it serves as an aid to decision-making in the areas of pricing, marketing, and reinsurance buying and selling.

Pricing

The model-generated expected loss estimates can be used to calculate catastrophe premium loadings. Theoretically, if an insurer establishes a reserve for catastrophe losses and makes annual contributions equal to the annual expected losses, the insurer will break even with respect to catastrophe losses over the long run.

Of course, competitive factors influence the amount of freedom that an individual insurer has to set prices. If demand is very elastic, small increases in price will lead to large decreases in market share. Pricing can be used as part of marketing strategy to manage the geographical distribution of property exposures and hence the catastrophe loss distributions.

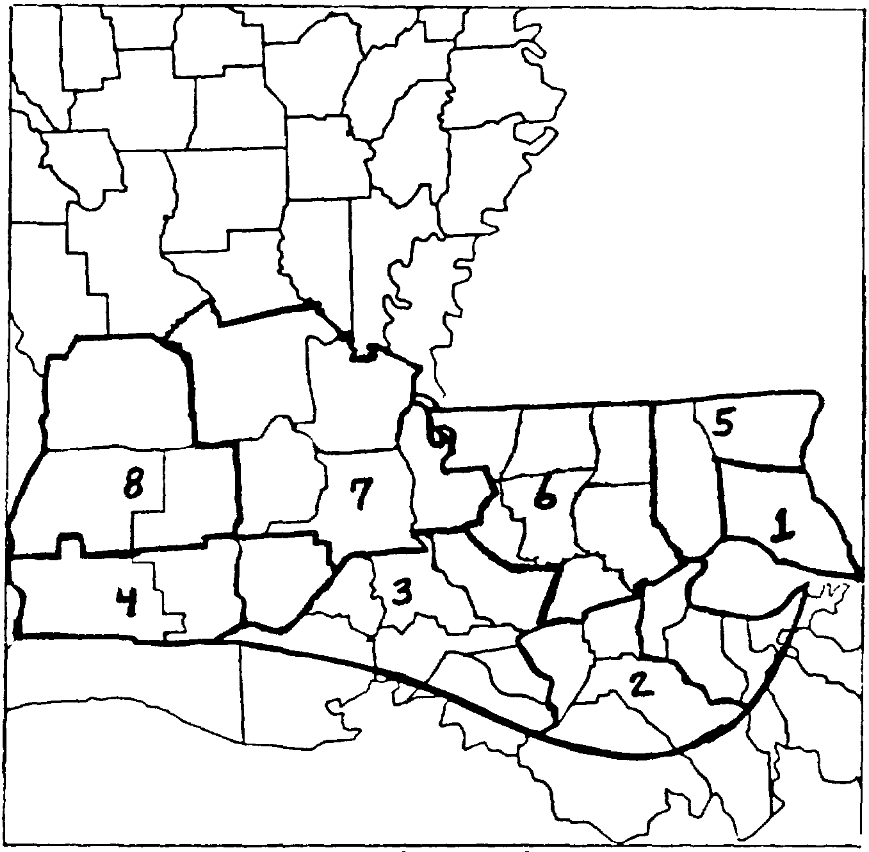
Marketing

The windstorm simulation model output as illustrated in Exhibit V shows the probability distribution of annual countrywide losses from the hurricane hazard. For marketing purposes, however, it may be more useful to divide the country into smaller zones so that the specific areas of high windstorm risk are clearly identifiable.

The computer model can be programmed to accumulate dollar damages by state, by country, or by any other geographical configuration. Exhibit VI shows the state of Louisiana divided into eight zones. The dollars of liability, i.e. exposure, the expected loss, and various confidence level losses[5] are shown for each zone. The figures clearly show that the higher risk areas are the coastal zones. The hurricane is at maximum force just as it crosses over land; as it travels inland, the storm dissipates because of the elimination of its primary energy source (kinetic energy from the sea) and because of surface frictional effects.

[5] It is interesting to note that for small geographic areas, the confidence level losses may be zero since the frequencies of hurricanes in specific locations are low.

EXHIBIT VI



LOUISIANA WINDSTORM ZONES

Zone	$ Exp	Expected Loss	Confidence Level Losses 80%	90%	95%	99%
LOUIS 1	90,417,112	256,512	0	276,770	1,947,396	4,938,375
LOUIS 2	9,210,113	25,540	0	12,932	213,693	537,371
LOUIS 3	56,674,660	94,866	0	31,306	653,101	2,098,500
LOUIS 4	50,672,900	71,042	0	0	234,088	1,722,377
LOUIS 5	79,796,656	80,965	0	0	547,837	2,021,005
LOUIS 6	176,149,552	231,604	0	0	598,946	6,823,092
LOUIS 7	40,664,716	47,598	0	0	193,227	1,309,985
LOUIS 8	33,114,748	16,552	0	0	5,991	772,278

Because all natural hazards have associated with them geographical frequency and severity patterns, they will produce gradations of damage or pockets of high risk and low risk. Management will want to avoid concentrations of property exposures in high risk areas, and the model output enables the development of marketing plans that are based on the long term profit potentials of various markets.

Property business in high risk areas may be very profitable in years of no natural hazard occurrences. As years pass and no catastrophes occur, insurers may begin to compete for the business in a high risk area. The competition may drive the profits as well as the catastrophe loadings to zero so that there are no resources available to cover the catastrophic losses when they occur. Knowledge of the probability distributions of losses from natural hazards in these areas enables insurers to resist the temptation to write business based on the very recent loss experience in these areas.

The natural hazard simulation model provides an excellent tool for evaluating the exposure to natural hazards resulting from alternative marketing plans. Alternative geographical distributions of property exposures may be input into the model to estimate the resulting catastrophe loss distributions.

Reinsurance

Pricing in accordance with expected losses does not eliminate the risk of large losses since catastrophes can occur when the loss fund is at a level that is not sufficient to cover all of the losses. Nor can marketing plans eliminate this risk since no area of the continental U.S. is free of natural hazards of all types. Insurers can use the probable maximum loss estimates to decide how much reinsurance to purchase for protection against large losses. An estimate of the probable maximum losses enables company management to make the appropriate risk versus return tradeoffs in evaluating reinsurance options.

SUMMARY AND CONCLUSIONS

Catastrophic events can impact significantly the results of property and casualty insurers. Since the losses resulting from the occurrences of catastrophes could affect adversely the financial condition of a company, management must plan for these events. In order to plan for these events, an estimate of the probability distribution of losses is needed.

The Monte Carlo simulation approach to the estimation of the probability distribution of catastrophe losses involves the development of computer models to simulate catastrophes. Each model is developed around the probability distributions of the random variables of the loss-producing "system."

There are several advantages of the simulation approach. First, it is able to capture the effects on the catastrophe loss distribution of changes over time in population patterns, building codes, and repair costs. Second, this estimation procedure provides management with a complete picture of the probability distribution of losses rather than just estimates of expected and probable maximum losses. And finally, the Monte Carlo simulation approach provides a framework for performing sensitivity analyses and "what-if" studies.

Disadvantages of the simulation approach include long model development time and potentially high development costs. Model validation is also problematic. However the benefits provided by the model and the value of the model output would seem to outweigh the costs. The simulation approach, while not perfect in an absolute sense, is far superior to competing approaches to catastrophe risk assessment and management.

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A PROBABILISTIC MODEL FOR IBNR CLAIMS

FARROKH GUIAHI

Abstract

IBNR reserves are presented as a stochastic variable. The model presented shows explicitly that the main factors contributing to IBNR reserves are number of claims, severity, and report lag distributions. The mean and variance of IBNR reserves are derived. Procedures to obtain an IBNR confidence interval are discussed. Two examples are provided on the use of the model. Suggestions are made as to how to obtain model parameters from actual insurance data.

1. INTRODUCTION

Accurate estimation of IBNR liabilities is a matter of concern for regulators, management, and investors in proper evaluation of financial statements of property-casualty insurance companies. Some commonly used methods to compute IBNR reserves were presented in Skurnick (1973), and in Bornhuetter and Ferguson (1972). In a survey of loss reserve methods, Skurnick (1973) mentioned the runoff method and the procedures that apply a factor to a *current* value of a base. Bornhuetter and Ferguson (1972) recommended procedures that initially require the computation of age-to-age factors derived from a loss development triangle. In a critique of reserve methodologies, Khury (1980) stated that reserve estimates are point estimates with no provision given for possible *variations* from their respective true values; he also stated that the *actuarial assumptions* used in determining reserve estimates are not mentioned explicitly.

Some commonly used procedures have two main shortcomings. First, a procedure that applies a constant factor to a current value of a base is ad hoc. For instance, statutory IBNR reserves for fidelity and surety coverages are computed as 10% and 5%, respectively, of premiums in force. Such an ad hoc procedure does not differentiate among companies with respect to underwriting practices, company operations, and management's attitude to risk bearing. Second, many of these procedures are a by-product of a *retrospective* reserve analysis (e.g., the runoff method or age-to-age factors derived from a loss development triangle). A retrospective reserve analysis provides information with regard to the adequacy or inadequacy of prior reserve estimates, but its implications about the accuracy of a current reserve are questionable.

Another more philosophical problem associated with retrospective procedures such as the runoff method or procedures based on age-to-age factors is that these procedures are not "statistical." A "statistical" procedure would consider an estimator that is usually unbiased and/or consistent; see Bickel and Doksum (1977). Statistical theory would guarantee that such estimators will be "about" the true parameter value or will "converge" to the true parameter value for large sample sizes (large volume of data). Even when adjusted for the volume of business or other pertinent facts, methods based on runoff procedures are not "on the average" guaranteed to estimate the true IBNR value. Similarly, procedures based on age-to-age factors, even when these factors are trended, cannot be relied on to estimate the true IBNR value correctly. Runoff procedures and procedures related to age-to-age factors may have an intuitive appeal for calculating IBNR. But there is no proof, at least to the extent of the author's knowledge, that these computational methods have desirable properties such as being unbiased and/or consistent.

In this presentation a probabilistic model, a statistical procedure, is developed that may be used as an alternative method for computing IBNR reserves.

2. IBNR MODEL

IBNR liability is presented as a stochastic variable. Parameters used in the model are distribution of number of claims, severity, and report lag by accident periods. These parameters (factors) are dependent on a company's mix of business written (current and past) and to some extent on a company's procedures for investigating and reporting claims. In this section, the probabilistic formulation of the model is considered. The specification of parameters has been delegated to another section. IBNR is presented as a finite sum of random variables. Each term in the finite sum is an "IBNR contribution by an accident period." These IBNR contributions are random sums (see Appendix B). Mean (expected value) and variance of IBNR have been derived.

Claims are grouped by accident periods. The unit of time for an accident period may be a month or a quarter. For the sake of simplicity it is assumed that each accident occurs at the middle of an accident period. It should be noted that when the accident period is one year, the assumption that all accidents occur at the midpoint of the accident year may be invalid for certain types of coverage because of seasonality and other pertinent facts. The "experience period" includes all the accident periods of interest. Diagram A is useful in presenting the "experience period."

DIAGRAM A

$\longleftarrow$ experience period $\longrightarrow$

$\longleftarrow$ c_i $\longrightarrow$

$s-1$	s	$i-1$	i	$t-1$	t
"initial" accident period		accident period i		"current" accident period	

where $c_i = t - i + (1/2)$, a known constant,
$i = s, s+1, \ldots, t$.

The accident period i is the interval $(i - 1, i]$. In this presentation, accident periods s and t represent "initial" and "current" periods, respectively.

The model assumptions and the main symbols used are as follows. For each accident period i,

(i) N_i, a random variable, denotes the number of accidents occurring;
(ii) corresponding to N_i, there are claim amounts X_{ij}, $j \leqslant N_i$, that are independent identically distributed (i.i.d.) random variables with the same probability distribution as X_i;
(iii) each claim X_{ij} corresponds to a report lag denoted by T_{ij}. For a given claim, the report lag is defined as the time difference between the accident date and the claim report date. The T_{ij}, $j \leqslant N_i$, are i.i.d. random variables with the same probability distribution as T_i;
(iv) it is assumed that N_i, X_{ij}, and T_{ij} are independent random variables for each $j \leqslant N_i$ and $i = s, s+1, \ldots, t$.

The random variables N_i, X_i, and T_i, for $s \leqslant i \leqslant t$, correspond to the number of claims, the severity, and the report lag, respectively. N_i's are related to both frequency and volume (exposure). The values of X_{ij} correspond to their *ultimate* cost realizations. The probability distributions for N_i, X_i, and T_i can be different for each i. In Section 4, more information about the specification of N_i, X_i, and T_i distributions is provided. The assumption of independence, (iv) above, has two major implications: for each accident period, the number of claims is independent of claim amounts; and for each claim, the claim amount is independent of its report lag. If there is strong *empirical evidence* that for certain types of coverage a significant correlation (say, positive correlation) exists between claim amounts and their respective report lags, then the independence assumption, (iv) above, is violated and the derivations based on it are invalid. In such a case, one has to modify the model, or alternatively assess the sensitivity of the model to departures from independence.

Let I_A denote an indicator random variable for the event A. That is,

$$I_A = \begin{cases} 1, & \text{if } A \text{ occurs,} \\ 0, & \text{if } A \text{ does not occur.} \end{cases}$$

The following equation (1)

$$1 = I_{\{T_{ij} \leq c_i\}} + I_{\{T_{ij} > c_i\}} \tag{1}$$

implies that the claim X_{ij} is either a *reported claim* or an *IBNR claim* as of the end of accident period t. Let Y_i denote the contribution to IBNR liability from accident period i. Then,

$$Y_i = \sum_{j \leq N_i} X_{ij} I_{\{T_{ij} > c_i\}}. \tag{2}$$

Note that Y_i is a random sum (see Appendix B) (i.e., Y_i is the sum of random variables with the number of random variables contributing to the sum being random). IBNR as of the end of the "current" accident period t is defined as,

$$\text{IBNR} = \sum_{i=s}^{t} Y_i, \tag{3}$$

$$= \sum_{i=s}^{t} \sum_{j \leq N_i} X_{ij} I_{\{T_{ij} > c_i\}}. \tag{4}$$

Equation (3) presents IBNR as a sum of a finite number of random variables, where each random variable in the sum is a random sum denoting an accident period contribution to IBNR.

The mean and variance of Y_i, equation (2) above, are

$$\mathrm{E}(Y_i) = \mathrm{E}(N_i)\mathrm{E}(X_i)\mathrm{P}(T_i > c_i), \tag{5}$$

where $\mathrm{P}(T_i > c_i)$, in (5) above, denotes the probability that the random variable T_i exceeds the value c_i, and

$$\mathrm{Var}(Y_i) = \mathrm{E}(N_i)\mathrm{E}(X_i^2)\mathrm{P}(T_i > c_i) + [\mathrm{E}(X_i)\mathrm{P}(T_i > c_i)]^2[\mathrm{Var}(N_i) - \mathrm{E}(N_i)]. \tag{6}$$

Equations (5) and (6) are a consequence of (iv) above and Appendix B.

The Expected Value (Mean) of IBNR

Using (3) and (5), we have

$$\mathrm{E(IBNR)} = \sum_{i=s}^{t} \mathrm{E}(N_i)\mathrm{E}(X_i)\mathrm{P}(T_i > c_i), \tag{7}$$

$$= \sum_{i=s}^{t} w_i B_i, \tag{8}$$

where $B_i = \mathrm{E}(N_i)\mathrm{E}(X_i)$,
$w_i = \mathrm{P}(T_i > c_i)$.

B_i is "expected incurred losses" for the accident period i. If we use "expected incurred losses" as a *base*, it is clear from (8) that IBNR is a function of current and prior base values. Because it is common for IBNR estimates to be calculated from a base that is only a function of a single year, the above analysis, equation (8), implies that such procedures are inappropriate. The weights w_i can be computed using the report lag distribution(s), and their effect diminishes as we consider earlier accident periods. A deterministic procedure for calculating IBNR using lag probabilities, w_i above, has been presented by Patrik (1978).

The Variance of IBNR

Using the independence assumption about N_i, X_{ij}, and T_{ij}, and equations (3) and (6), we have

$$\mathrm{Var(IBNR)} = \sum_{i=s}^{t} \mathrm{E}(N_i)\mathrm{E}(X_i^2)\mathrm{P}(T_i > c_i) + \sum_{i=s}^{t} [\mathrm{E}(X_i)\mathrm{P}(T_i > c_i)]^2[\mathrm{Var}(N_i) - \mathrm{E}(N_i)].$$

If N_i's are Poisson random variables, equation (9) becomes

$$\mathrm{Var(IBNR)} = \sum_{i=s}^{t} \mathrm{E}(N_i)\mathrm{E}(X_i^2)\mathrm{P}(T_i > c_i). \tag{10}$$

One may be interested in Poisson number of claims for at least two reasons. First, if one expresses the parameter of a claim *process* in terms of the "operational" time rather than the "natural" time, then many claim count processes of interest are in fact *Poisson processes*. A claim count process is a stochastic process, $\{N(u),\ s - 1 \leq u \leq t\}$, where u is the parameter of the stochastic process. The parameter u denotes the time ("natural" time), and $N(u)$ is the number of claims (accumulated number of claims) at time u during the time interval $(s - 1, u]$. The number of claims in the accident period $(i - 1, i]$, N_i, can be expressed in terms of the claim count process by the following relationship: $N_i = N(i) - N(i - 1)$. For an elaborate discussion of "operational" time and claim count processes, the interested reader should refer to Bühlmann (1970). Second, the negative binomial is a suitable probability model for fitting claim count data; see Benjamin (1977). But negative binomial distribution arises from a Poisson random variable because of uncertainty in its parameter specification; see Longley-Cook (1962).

Equation (10) may be written as

$$\mathrm{Var(IBNR)} = \sum_{i=s}^{t} u_i B_i, \tag{11}$$

$$u_i = [\mathrm{E}(X_i^2)/\mathrm{E}(X_i)]\mathrm{P}(T_i > c_i).$$

Now the weights u_i depend on both severity and report lag distributions. When the number of claims has a Poisson distribution, the variance of IBNR can also be expressed in terms of current and prior values of a base. Moreover, in the case of the Poisson number of claims and the further assumption of a severity distribution, X, that does not change over the entire "experience period," we have

$$\begin{aligned}\mathrm{Var(IBNR)} &= \sum_{i=s}^{t} [\mathrm{E}(N_i)\mathrm{E}(X)\mathrm{P}(T_i > c_i)]\,[\mathrm{E}(X^2)/\mathrm{E}(X)], \\ &= \mathrm{E(IBNR)}\,[\mathrm{E}(X^2)/\mathrm{E}(X)].\end{aligned} \tag{12}$$

Equation (12) implies that the ratio of Var(IBNR) to E(IBNR) depends only on the severity in this case!

Derivation of an IBNR Confidence Interval

Some remarks on the derivation of a confidence interval for IBNR are appropriate at this time. In order to derive an *exact* confidence interval for IBNR reserves, it is necessary to know the distribution of IBNR. Note that IBNR is composed of a sum of a finite number of random variables, where each term in the sum is a random sum. Determining the exact distribution of a random sum is extremely difficult. It requires the evaluation of an infinite number of distributions where each one is a convolution of many distributions. This problem is well known in reinsurance, that is, the aggregate losses in stop-loss reinsurance arrangements are in fact a random sum; see Bühlmann (1970).

The distribution of IBNR can be analytically approximated by using the cumulative distribution of standard normal distribution, its derivatives, and the moments of IBNR. This approach is known as Edgeworth expansion and is discussed in Beard, Pentikäinen, and Pesonen (1969). This approximate distribution can be used to construct a confidence interval for IBNR.

Expressions for the mean and variance of IBNR have been given; now a crude IBNR confidence interval may be computed by using the Chebyshev inequality.

The author believes that a reasonably accurate IBNR confidence interval may be obtained by resorting to simulation. IBNR realizations can be generated and a "simulated" distribution computed by specifying an input scenario, that is, specification of the claim count, the severity, and the report lag distribution for each accident period, based on actual insurance data. Such a distribution may be used to derive a reasonable confidence interval for IBNR.

3. APPLICATION

In this section are two examples that use the preceding model. The specifications of input parameters in these examples are not based on any real insurance data, but are stated merely for *illustrative purposes* and *computational expediency*. Given specifications of input parameters should not be construed as model assumptions. The main assumptions of the model are in condition (iv) in Section 2: independence of claim count, severity, and report lag. A more appropriate use of the model would be to generate many IBNR values (realizations) by resorting to simulation based on input parameters derived from actual insurance data. Results of such a simulation may be used to provide an IBNR confidence interval and determine the sensitivity of IBNR to input assumptions.

Example A: Effect of Changes in Input Parameters on IBNR

IBNR, or more precisely, expected value of IBNR, can be calculated according to equation (7) in Section 2. Each IBNR computation requires an input scenario, that is, a specification of expected number of claims, mean severity, and report lag distribution for each accident period included in the experience period. In this example, we consider one input specification and refer to it as "Scenario A." We then investigate the effect of change(s) in input parameters relative to Scenario A on the value of IBNR. These investigations will show the sensitivity of IBNR value to changes in input parameters. In Table A, several deviations from Scenario A's input specifications are considered. In each case, a percentage change in IBNR value has been computed.

TABLE A

Scenario A:
(i) Growth in expected claim count is 6% annually.
(ii) Mean severity increases uniformly at the rate of 5% annually during the entire 10-year experience period.
(iii) Report lag distribution for each accident period is exponential with mean of 40 months.

Change in Input Assumptions Relative to Scenario A	*Percentage Change in IBNR
1. Change in growth rate for expected claim count from 6% to 9%.	24.1
2. Change in rate of increase in mean severity from 5% to 10% during the *second* 5-year experience period.	15.0
3. Change in mean lag from 40 to 50 months (assuming the distribution of lag remains exponential).	15.6
4. Changes in expected claim count and mean severity as in 1 and 2 above.	43.3
5. Changes in expected claim count and mean lag as in 1 and 3 above.	42.4
6. Changes in mean severity and mean lag as in 2 and 3 above.	31.8
7. Changes in expected claim count, mean severity, and mean lag as in 1, 2, and 3 above.	63.0

*To compute the percentage change, let $(\text{IBNR})_O$ and $(\text{IBNR})_A$ denote the value of mean IBNR according to Scenario O, that is any other scenario, and Scenario A, respectively. Then, the percentage change in IBNR is defined as

$$\{[(\text{IBNR})_O/(\text{IBNR})_A] - 1\} \times 100.$$

For more details on computation of the above percentages refer to Appendix A.

Example B: Projecting IBNR Values After Discontinuing Writing a Line of Business or a Coverage

Consider a situation in which at time t, end of the experience period, the insurer decides to discontinue writing a certain line of business or a coverage. The insurer may pay for IBNR claims as they are subsequently reported and settled, or the insurer may transfer the liability at a given price to an accommodating reinsurer. The rate of decline in IBNR, subsequent to discontinuation of coverage, is considered as follows.

Let E[IBNR(u,v)] denote the mean value of IBNR as of moment v evaluated at time u. Then, according to equation (7), we have

$$\mathrm{E}[\mathrm{IBNR}(t,t)] = \sum_{i=s}^{t} \mathrm{E}(N_i)\mathrm{E}(X_i)\mathrm{P}(T_i > c_i). \tag{13}$$

If coverage is discontinued at time t, $\mathrm{E}(N_i) = 0$, *for* $i > t$. The claim X_{ij} is an IBNR claim as of moment $t + 1$ if $T_{ij} > c_i + 1$. Thus,

$$\mathrm{E}[\mathrm{IBNR}(t,t + 1)] = \sum_{i=s}^{t} \mathrm{E}(N_i)\mathrm{E}(X_i)\mathrm{P}(T_{ij} > c_i + 1). \tag{14}$$

If T_i's are exponential with density $f(t)$,

$$f(t) = \vartheta e^{-\vartheta t},\ t > 0,$$

where the parameter ϑ is equal to 1/(mean lag). Then

$$\mathrm{P}(T_i > c_i + 1) = e^{-\vartheta(c_i + 1)} = e^{-\vartheta}\mathrm{P}(T_i > c_i). \tag{15}$$

Using (13), (14), and (15), we have

$$\mathrm{E}[\mathrm{IBNR}(t,t + 1)] = e^{-\vartheta}\mathrm{E}[\mathrm{IBNR}(t,t)];$$

similarly we have

$$\mathrm{E}[\mathrm{IBNR}(t,t + k)] = (e^{-\vartheta})^k\mathrm{E}[\mathrm{IBNR}(t,t)],\ \text{for } k = 1,2,\ldots \tag{16}$$

In particular, if the accident period is one month, then, according to equation (16), the projected value of IBNR a year after the evaluation date is equal to the current IBNR value multiplied by a factor (less than one) that is equal to

$$[e^{-1/(\text{mean lag})}]^{12} \tag{17}$$

The above factor is based on the premise that the lag distribution remains unchanged during the entire experience period and is exponential. Choosing an accident period of one month, IBNR is declining geometrically at an annual rate given by equation (17). Note that no restriction is put on the expected claim counts and the mean severity by accident periods. Table B shows one year decline factors for different mean lags assuming exponential lag distribution.

TABLE B

ONE YEAR DECLINE FACTOR

Mean Lag (Months)	Factor
10	.301
20	.549
30	.670
40	.741
50	.786

The emerged IBNR amounts in the respective future accident periods $t + 1$, $t + 2$, ..., are given by the following differences

$$E[\mathrm{IBNR}(t,t)] - E[\mathrm{IBNR}(t,t+1)],$$
$$E[\mathrm{IBNR}(t,t+1)] - E[\mathrm{IBNR}(t,t+2)],$$
$$\ldots,$$

based on our evaluation at time t. These emerged IBNR amounts may be used to give an estimate of a "discounted" IBNR.

4. SPECIFICATION OF MODEL PARAMETERS

For each accident period i, the specification of distributions for number of claims, severity, and report lag (i.e., N_i, X_i, and T_i) is required.

In determining N_i, the number of claims, distributions commonly fitted to insurance data are Poisson and negative binomial; see Benjamin (1977). In the case of Poisson, the only required input is the value of $E(N_i)$, the expected number of claims. $E(N_i)$ should not be based entirely on reported claims in accident period i, but adjusted for accident period i claims that will be subsequently reported. As Salzmann (1984) stated, "the extrapolation of the incurred count is straightforward and results are quite dependable."

The specification of claim distribution X_i is a more difficult task. Many parametric distributions have been fitted to claim data. Some popular distributions used are lognormal, Pareto, and gamma; see Beard, Pentikäinen, and Pesonen (1969). It is the author's belief that for earlier accident periods, the claim cost data are nearly "fully developed," and a parametric distribution fitted to individual claims (incurred losses) is the appropriate procedure. The term *earlier accident periods*, in the preceding sentence, depends on the circumstances of a given situation. It should be evaluated in terms of the volume of claim cost data and the claim settlement period relevant to that line of business. Finger (1976) wrote an interesting paper related to fitting a lognormal curve to claim data. For more recent accident periods, the claims are only "partially developed" and are not close to their "ultimate" cost values. A possible approach is to extrapolate (trend) the distribution of earlier periods to arrive at distributions for more recent periods. A procedure for *trending distributions* was presented by Rosenberg and Halpert (1981).

The distribution of report lag, T_i, can be obtained by a procedure outlined by Weissner (1978), where reported lags are fitted, by the method of maximum likelihood, to a parametric truncated distribution. The underlying report lag distribution is recovered by exploring the relationship between truncated and nontruncated distributions.

The last point to consider is the selection of an appropriate "experience period." Usually t is December 31 of the year of IBNR evaluation. The choice for s, the "initial" accident period, requires considerable judgment. For a new company or an existing company with a new line of business, the s should be the earliest possible period. In other cases, the choice of s depends on the report lag distribution. From equation (8), it is clear that for earlier accident periods, w_i is small because c_i is large, and consequently the contributions to IBNR from earlier accident periods tend to diminish. Thus, when IBNR is computed by lines of business or coverages, a judgmental choice with regard to the value of s should be made.

Finally, the distributions of N_i, X_i, and T_i are based on our knowledge at the end of the current period t. If the accident period is a month and IBNR is computed annually, at time $t + 12$, we have to update these distributions in the light of data gathered during period $(t, t + 12]$. Thus, the distributions for the claim count, the severity, and the report lag may be updated from one evaluation period to the next.

5. CONCLUSION

The model described in this paper has merits of its own in estimating IBNR reserves, particularly the following points. The model is not ad hoc because the parameters used are dependent on a company's book of business written, which is the most important factor in determining IBNR. The input parameters (distributions) may be continually updated from one evaluation to the next. If the company's operations change, or if other factors suggest an appreciable divergence from past development of input parameters, then, to the extent that these changes can be quantified, "historical" inputs should be replaced by these "subjective" inputs that incorporate the changes. The model is stochastically presented so that we can evaluate variability. The actuarial assumptions used are stated explicitly in terms of probability distributions for the number of claims, the severity, and the report lag. We have a tool, a stochastic model, to work with. More time can now be spent in examining the model assumptions and improving methods of estimating parameters from actual insurance data.

APPENDIX A

FORMULAS USED IN COMPUTING THE PERCENTAGES GIVEN IN TABLE A

Precise specification of the input parameters for the computation of percentages in Table A is given below. The accident period is assumed to be one month. Let $s = 1$ in equation (7); r_1 denotes the rate of growth for the expected number of claims; r_2 and r_3 denote the rate of growth of the mean severity *during the first and second five years* of the experience period, respectively. The input specifications are as follows:

$$E(N_i) = E(N)(1 + r_1)^{(i-1)/12}, \qquad \text{for } 1 \leq i \leq 120$$

$$E(X_i) = \begin{cases} E(X)(1 + r_2)^{(i-1)/12}, & \text{for } 1 \leq i \leq 60 \\ E(X)(1 + r_2)^{(60-1)/12}(1 + r_3)^{(i-60)/12}, & \text{for } 60 < i \leq 120 \end{cases}$$

where E(N) and E(X) denote the expected values of claim count and severity in the initial accident month. The lag distibution is selected to be exponential for each accident period with the density $f(t)$ as given in Example B. Using equation (7), the mean IBNR value is

$$E(\text{IBNR}) = E(N)E(X) \left\{ \sum_{i=1}^{60} (1 + r_1)^{(i-1)/12}(1 + r_2)^{(i-1)/12} e^{-\vartheta c_i} + \sum_{i=61}^{120} (1 + r_1)^{(i-1)/12}(1 + r_2)^{(60-1)/12}(1 + r_3)^{(i-60)/12} e^{-\vartheta c_i} \right\}$$

where $\vartheta = 1/(\text{mean lag})$.

For Scenario A, $r_1 = .06$, $r_2 = r_3 = .05$, with mean exponential lag of 40 months. For any other scenario, the input parameters that are *not* explicitly changed (see Table A) will be the same as those of Scenario A. In computing the percentage change in IBNR values, the E(N)E(X) term drops out.

APPENDIX B

MEAN AND VARIANCE OF A RANDOM SUM

In this appendix, we state (not derive) the appropriate expressions for the mean and variance of a random sum. The interested reader may refer to Feller (1971) or Mayerson, Jones, and Bowers (1968) for the derivation of the results stated below.

Let $Y, Y_1, Y_2, \ldots, Y_n, \ldots$ be independent and identically distributed random variables with finite first two moments. Let N denote a nonnegative integer-valued random variable with finite first two moments. A random sum, S_N, is defined as

$$S_N = \sum_{i \leq N} Y_i. \tag{B.1}$$

Let us assume that N and $Y_1, Y_2, \ldots$ are independent variables; then it can be shown—see Feller (1971)—that

$$\mathrm{E}(S_N) = \mathrm{E}(N)\mathrm{E}(Y), \tag{B.2}$$

$$\mathrm{Var}(S_N) = \mathrm{E}(N)\mathrm{Var}(Y) + [\mathrm{E}(Y)]^2\mathrm{Var}(N). \tag{B.3}$$

Equation (B.3) can be rewritten as

$$\mathrm{Var}(S_N) = \mathrm{E}(N)\mathrm{E}(Y^2) + [\mathrm{E}(Y)]^2[\mathrm{Var}(N) - \mathrm{E}(N)]. \tag{B.4}$$

If N is a Poisson random variable, the second term on the right-hand side of (B.4) is equal to zero.

It should be noted that for an indicator random variable I_A (see Section 2), we have

$$\mathrm{E}(I_A) = \mathrm{P}(A) \text{ , and}$$
$$\mathrm{E}(I_A^2) = \mathrm{P}(A) \text{ .}$$

These results concerning the mean and second moment (about zero) of the indicator random variable have been used in the derivation of equations (5) and (6) in Section 2.

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DISCUSSION BY RICHARD E. SHERMAN

In the paper, "A Probabilistic Model for IBNR Claims," a number of results of interest have been presented. The assumptions of the model have been clearly defined and several useful derivations worked out. It should be noted that this paper addresses "pure IBNR"—to the exclusion of reserves for adverse development on case reserves.

The author openly admits that application of the model is dependent on claim severities being independent of the report lag. Without exception, every set of casualty loss experience that this reviewer has studied (that contains sufficient detail to test the hypothesis of independence) indicates that claim severities increase markedly with report lag—up to some stage of development—and then tend to level off for later stages. Much of the more recent loss experience available to this reviewer is either confidential in nature or is based on too small a volume of claims. However, two generally available, though older, sources clearly demonstrate this phenomenon for a large body of data. Exhibit 1 presents the results of the NAIC Closed Claim Study (62,096 medical malpractice claims), and Exhibit 2 shows comparable data from the ISO's Products Liability Closed Claim Survey (12,213 claims).

This suggests that the derivations presented in this paper should be valid for that portion of the IBNR reserve associated with more mature accident years (where the claim severity of yet unsettled claims tends to be independent of the report lag). More specifically, the interesting and useful results for Example B should be valid for lines of business that were discontinued a number of years ago.

It also suggests that the derivations in this paper must undergo considerable modification before application to the IBNR reserve associated with the most recent accident years. Unfortunately, this latter portion tends to represent the bulk of the IBNR reserve for any long tail line.

The author expresses a number of appropriate misgivings about retrospective procedures such as runoff methods and age-to-age factor methods. What is unclear is whether the techniques presented in this paper would represent an approach that overcomes these misgivings. I do not sense that the methods presented in the paper will liberate the actuary from biases derived from past data and enable him/her to better foresee the future. On the other hand, the author's approaches and derivations do offer a refreshing perspective, and can serve as a basis for further advancements in IBNR analysis.

In the introduction, reference is made to IBNR reserves for fidelity and surety coverages. Ad hoc procedures, such as a fixed percentage of premiums in force are criticized for their failure to differentiate among companies on several counts. To the author's list, I would add the following: definition of accident date (especially for contract surety) and practices in setting case reserves.

EXHIBIT 1

RELATIONSHIP OF CLAIM SEVERITY AND REPORT LAG
NAIC CLOSED CLAIM STUDY
MEDICAL MALPRACTICE

Time from Incident to Report (Months)	Indemnity Paid (000's)	Number of Claims	Claim Severity
0– 6	$243,576	22,293	$10,926
7–12	138,435	10,370	13,350
13–24	234,814	15,089	15,562
25–36	134,054	8,631	15,532
37–48	60,456	2,732	22,129
Over 48	64,837	2,981	21,750
	$876,172	62,096	$14,110

EXHIBIT 2

Relationship of Claim Severity and Report Lag
ISO Products Liability Closed Claim Survey
Bodily Injury Liability Claims

Time from Incident to Report (Months)	Number of Claims	Claim Severity	Claim Severity ($25,000 Limit)
	(Trended for Severity)		
0	3,927	$ 2,834	$ 740
1–6	5,570	4,477	1,553
7–12	949	23,146	5,100
13–18	581	21,843	5,846
19–24	464	27,603	7,546
25–30	271	19,827	6,299
31–36	157	27,536	7,731
37–48	142	22,973	6,168
Over 48	152	102,136	7,874
	12,213	$ 9,171	$2,316
	(Untrended)		
0	3,927	$ 1,200	$ 622
1–6	5,570	2,292	1,211
7–12	949	9,659	3,956
13–18	581	10,314	4,265
19–24	464	10,572	5,178
25–30	271	8,452	4,292
31–36	157	6,802	4,490
37–48	142	7,408	3,852
Over 48	152	10,824	2,940
	12,213	$ 3,570	$1,694

DISCUSSION BY MARGARET WILKINSON TILLER

Mr. Guiahi's paper presents a model that is a good starting point for estimating the reserve associated with claims that have been incurred but are not reported. Since he refers only to "claim costs," it is not clear whether this reserve is for losses only, or losses and allocated loss adjustment expense. His technique can be applied to either or to allocated loss adjustment expenses only, provided that the model parameters are selected appropriately.

An important point to note is that the model does not produce an estimate of case reserve development, i.e., the difference between the ultimate value of claims and the total of the payments plus case reserves for claims that have been reported as of a given date. Again, the terms "value," "payments," and "case reserves" could refer to losses only, losses and allocated loss adjustment expenses, or allocated loss adjustment expenses only, as long as the definitions are consistent.

One of the main assumptions underlying Mr. Guiahi's model is that claim severity and report lag are independent. While this may be close to reality for a short-tailed line such as automobile property damage, it is probably not true for long-tailed lines such as medical malpractice and products liability. Mr. Guiahi points out that if there is empirical evidence that the assumptions are not valid, adjustments to the model must be made. He does not, however, explore what those adjustments are. For those lines of business in which claim severity and report lag are dependent or in which other model assumptions appear to be invalid the model can be used only as a starting point.

Mr. Guiahi states that his model overcomes many of the problems associated with retrospective reserve analysis (e.g., age-to-age factors derived from a loss development model). In particular: "A retrospective reserve analysis provides information with regard to the adequacy or inadequacy of prior reserve estimates, but its implications about the accuracy of a current reserve are questionable."

Any reserve analysis, including one based on Mr. Guiahi's model, assumes that the past is a good predictor of the future. Where known or suspected changes are taking place, a good actuary will modify the analysis techniques being used to reflect these changes as appropriate.

For example, if the number of claims and/or average claim size is increasing but the claim reporting pattern and the payment and case reserving practices have not changed, the loss development technique used on accident year reported losses to project ultimate losses for all incurred claims will not be affected by these changes. In Mr. Guiahi's model to project ultimate losses for incurred but not reported claims, these changes must be explicitly recognized and the model parameters adjusted accordingly.

If the claim reporting pattern is changing, the development technique can be used separately on accident year claims and average claim size to project ultimate losses for all incurred claims. This allows the development factors for claims to be modified so that the estimated ultimate claims reflect the change in the claim reporting pattern. In Mr. Guiahi's model to project ultimate losses for incurred but not reported claims, this change must also be explicitly recognized and the model parameters adjusted accordingly.

In summary, Mr. Guiahi's model is a good starting point for estimating reserves for losses, losses and allocated loss adjustment expenses, and allocated loss adjustment expenses only associated with claims that have been incurred but are not reported. To be of practical value, the model's assumptions should be evaluated carefully in light of empirical data and appropriate changes made to the model if the assumptions appear to be invalid. In addition, the reserve for case development must be estimated in order for the reserve picture to be complete.

THE CASH FLOW OF A RETROSPECTIVE RATING PLAN

GLENN MEYERS

Abstract

With current methodology, the parameters of a retrospective rating plan are calculated to place the plan in balance on an underwriting basis. This paper provides a way of calculating the present value of the retrospective premium. Using this methodology, one can compare the expected profitability of various retrospective rating plans on a discounted or operating basis. This includes paid loss retros. It is also possible to determine the parameters of a plan that will yield a predetermined operating profit.

This paper is an outgrowth of a project which I directed during my final year at CNA Insurance Companies. I worked very closely with John Meeks and Steve Maguire in developing the conceptual basis for what we called the "Account Pricing System." Many of these ideas originated with Brad Alpert before I was on this project. Steve and Ron Swanstrom wrote a program which made these ideas very workable in a production environment.

1. INTRODUCTION

In recent years, the state of the property and casualty insurance industry could be characterized by three highs: high combined ratios, high interest rates, and a high degree of competition. Insurance company managers know that a great deal of investment income can be made by writing insurance, and they are willing to lower prices in order to do this.

The question to be asked, then, is how much can rates be lowered and still maintain an acceptable overall profit? It should be noted that, in practice, actuaries do not have complete control of the pricing process. Underwriting and marketing personnel have considerable input. If actuaries do not calculate the contribution of investment income to the profitability of a line of insurance, someone else will. And the resulting "calculation" may amount to no more than a reaction to competitive pressures.

The question is not whether to reflect investment income in the calculation of rates. Instead the question is *how* to reflect investment income in the calculation of rates.

This paper considers the effect of investment income in the choice of the parameters of a retrospective rating plan. With current methodology, the parameters of a retrospective rating plan are chosen to place the plan in balance on a nominal, or underwriting basis. By this we mean that the expected retrospective premium is equal to the sum of the losses, expenses, and the anticipated profit. However, it is possible for different plans to have the same expected premium and have different cash flows.

For example, a plan with no maximum will have premium flowing in as long as losses develop, while a plan with a low maximum will stop producing premium as the insured breaks the maximum. Not all insureds will break the maximum, but there will, on average, be a faster premium flow for the low maximum plan because of the higher basic and the increased number of insureds who do break the maximum.

Other factors, such as the loss conversion factor and the minimum premium factor will also affect the cash flow of a retrospective rating plan.

This paper will provide a way of calculating the present value of the retrospective premium. Using this methodology, one can compare the profitability of various retrospective rating plans on a discounted or operating basis. This method also applies to paid loss retros. It is also possible to calculate parameters of a plan that will yield a predetermined operating profit.

The principal tool used will be the collective risk model. Excess pure premiums will be calculated for the insured at various stages of development. One can then calculate the expected retrospective premium at each stage, and obtain the present value of the retrospective premium.

This technique will enable the insurer to offer a standard incurred loss retro which is competitive with a paid loss retro. This alternative could help relieve some of the pressure that the Internal Revenue Service is putting on paid loss retros. In addition, it will become possible to price a retro with loss development factors. This will minimize the size of retrospective adjustments as time passes.

We begin by defining the parameters of a retrospective rating plan.

2. THE PARAMETERS DEFINED

The retrospective premium, R, for an insured is given by the following formula [1]:

$$R = (B + c \cdot E + c \cdot L) \cdot t.$$

R is subject to a maximum of G and a minimum of H.

B is the basic premium. Traditionally, B covers general expenses, profit, and the insurance charge (i.e., the net cost of the minimum and maximum premium provisions). There is no particular reason why B has to be set equal to these cost provisions. In its pure form, B is simply an amount used to determine the retrospective premium.

The factor c is called the loss conversion factor. Traditionally, c covers the loss adjustment expenses. Again, there is no reason why it has to be set equal to a loss adjustment factor. In its pure form, c is simply a factor used to determine the retrospective premium.

Many retrospective rating plans provide that no claim amount over a specified loss limit shall be used to calculate the retrospective premium. In this case, the expected value of the losses resulting from this provision must be added to the retrospective premium. This amount is denoted by E.

L represents the actual losses, subject to the per claim loss limit, incurred under the plan. Premium taxes are provided for by the factor t.

In order to keep this paper as simple as possible, we will not consider the effect of loss limits and premium taxes until the end of the paper. We shall also ignore the minimum premium. This results in a simplified formula for the retrospective premium:

$$R = B + c \cdot L,$$

subject to the maximum, G.

The timing of the retrospective premium payments is of particular importance. Recall that some claims are open a long time before final settlement. Thus, incurred losses are necessarily estimates of the final claims costs. Experience has shown these estimates are usually low, so one should expect the retrospective premium to increase over time. The first calculation is based on losses reported eighteen months after the effective date of the policy. Subsequent calculations are performed on a yearly basis. Payments typically lag three months behind the retrospective premium calculations.

It is usually required to make a premium payment before the first retrospective adjustment. Traditionally, this payment has the standard premium due on the effective date of the policy. More recently, the trend has been to pay an amount totaling less than the standard premium in installments.

We will be following a single hypothetical insured throughout this paper. The loss and expense information for this insured is given in the following table.

TABLE 1

	NOMINAL	PRESENT VALUE AT 8%
EXPECTED INCURRED LOSSES	$1,000,000	$820,000
EXPECTED LOSS ADJ. EXP.	100,000	87,000
OTHER EXPENSES	57,500	55,000
TOTAL	$1,157,500	$962,000

The expected incurred losses for each retrospective adjustment period are given in the following table.

TABLE 2

RETROSPECTIVE ADJUSTMENT	EXPECTED INCURRED LOSSES
#1 @ 18 MONTHS	$833,333
#2 @ 30 MONTHS	946,970
#3 @ 42 MONTHS	975,610
#4 @ 54 MONTHS	986,193
#5 @ 66 MONTHS	991,080
#6 @ 78 MONTHS	996,016
#7 @ 90 MONTHS	1,000,000

In order to calculate the average retrospective premium, one needs to have tables of excess pure premiums which correspond to each retrospective adjustment. These tables are provided in Exhibit 1. The Heckman-Meyers algorithm [2] was used to generate these tables. While the input for this algorithm could be provided, it seems just as easy to assume the tables are given. These tables provide excess pure premiums for loss amounts in increments of $10,000. Linear interpolation can be used to calculate excess pure premiums for loss amounts that are not a multiple of $10,000.

The average retrospective premium is calculated in the following manner [3]. Define the effective maximum to be equal to $(G - B)/c$, and let X be the excess pure premium for losses over the effective maximum. Then, the average retrospective premium is given by:

$$E[R] = B + c \cdot (E[L] - X).$$

The average retrospective premium must be calculated for each evaluation period.

As an example, assume B = \$232,450, G = \$1,500,000, $c = 1.1$, and $E[L]$ = \$1,000,000. Then the effective maximum equals \$1,152,320. By linear interpolation on Exhibit 1 (90 months), we find X = \$131,775 and $E[R]$ + \$1,187,500.

3. THE STANDARD INCURRED LOSS RETRO

We first calculate the expected underwriting profit for a standard incurred loss retro. We need only consider the seventh (final) retrospective adjustment for this calculation.

TABLE 3

BASIC	\$232,450
L.C.F.	1.1
MAXIMUM	\$1,500,000
E[R] @ 90 MTHS.	1,187,500
LOSS & EXPENSE	1,157,500
UNDERWRITING PROFIT	30,000

This plan was designed to yield approximately the 2.5% underwriting profit that is budgeted in standard Workers' Compensation rate filings.

Next, we calculate the expected operating profit for the same plan assuming an effective annual interest rate of 8%. That is to say, for example, that a payment due in three months is discounted at a rate of $1.08^{0.25}$. A deposit premium of \$960,000 is to be payable in six quarterly installments of \$160,000. The present value of the deposit premium is \$915,410. Additional amounts of premium due to retrospective adjustments are assumed to be paid three months after the calculation of the retrospective premium.

TABLE 4

BASIC	\$232,450
L.C.F.	1.1
MAXIMUM	\$1,500,000
DEPOSIT	960,000
E[R] @ 18 MTHS.	1,078,380
@ 30 MTHS.	1,155,720
@ 42 MTHS.	1,173,210
@ 54 MTHS.	1,179,480
@ 66 MTHS.	1,182,340
@ 78 MTHS.	1,185,200
@ 90 MTHS.	1,187,500
P.V. RETRO PREMIUM	1,103,720
P.V. LOSS & EXPENSE	962,000
OPERATING PROFIT	141,720

In this example we see that the standard rating method yields an operating profit of nearly 12% of the ultimate average retrospective premium. This is fine if the competition will allow it. If not, the insurance company management must decide what operating profit to seek.

Suppose management decides to seek an operating profit of \$100,000. Perhaps there is a vague notion that an underwriting profit of \$30,000 already anticipates a certain amount of investment income, and is not appropriate for an operating profit. Anyway, the question becomes one of selecting the basic premium that yields the desired operating profit. This can be done by repeating the calculations of Table 4 on a trial and error basis, although a numerical method may yield the desired solution more quickly [4]. The results of this process are in the following table.

TABLE 5

BASIC	$167,150
L.C.F.	1.1
MAXIMUM	$1,500,000
DEPOSIT	960,000
E[R] @ 18 MTHS.	1,024,100
@ 30 MTHS.	1,106,410
@ 42 MTHS.	1,125,210
@ 54 MTHS.	1,131,970
@ 66 MTHS.	1,135,050
@ 78 MTHS.	1,138,140
@ 90 MTHS.	1,140,620
P.V. RETRO PREMIUM	1,062,000
P.V. LOSS & EXPENSE	962,000
OPERATING PROFIT	100,000

Having described how to select the basic premium which yields a predetermined operating profit, it should be pointed out that it is possible to fix the basic premium and select the loss conversion factor which yields a predetermined operating profit.

Certain other cash flow provisions of a retrospective rating plan are often subject to negotiation between insurer and insured. Thus it seems appropriate that we show how to account for them.

4. RETRO DEVELOPMENT FACTORS

An optional provision of most retrospective rating plans is to adjust the incurred losses to their ultimate value by means of a loss (or retro) development factor. An advantage to the insured is that the retrospective premium is close to its ultimate value at the first retrospective adjustment. A disadvantage is that the insured must pay the premium sooner. To overcome this disadvantage, the insurer can offer to lower either the basic premium or the loss conversion factor.

In the following table we consider the latter option. The deposit premium is to be paid in installments as before. Although several retrospective adjustments are made, the contribution of the later adjustments is assumed to be negligible. The final table of excess pure premiums in Exhibit 1 (evaluated at 90 months) was used to calculate the average retrospective premium at the first adjustment.

TABLE 6

Basic	$167,150
L.C.F.	1.0775
Maximum	$1,500,000
Deposit	960,000
E[R] @ 18 mths.	1,127,730
P.V. Retro Premium	1,062,000
PV. Loss & Expense	962,000
Operating Profit	100,000

The results of this calculation should be directly comparable with the previous calculation (Table 5). The introduction of retro development factors caused about a 1.1% decrease in the average retrospective premium on a nominal basis.

The accuracy of this calculation depends upon our ability to calculate the proper loss development factors. Even if we get the correct overall loss development factors, changes in the shape of the aggregate loss distribution over time will affect the average retrospective premium. The author suspects that the result, over time, will be a thicker tail for the aggregate loss distribution, a higher excess pure premium, and a slight decrease in the average retrospective premium. Losses which are re-valued upward will be limited by the maximum premium, while losses which are valued downward will be unaffected. A full treatment of this effect is beyond the scope of this paper.

5. Paid Loss Retros

A very popular rating plan in recent years has been the so called "paid loss retro." While the details of the financial transactions may vary, a typical plan could work as follows. A basic premium is paid, possibly in installments. The retrospective premium based on paid losses is continuously paid from a special fund set up by the insured. At some point in time, usually 54 months after the effective date, the plan switches over to an ordinary incurred loss retro.

The continuous adjustment of the retrospective premium presents a technical problem. There is always the possibility that the insured will break the maximum on paid losses before the 54 month switchover. This could, in theory, require daily tables of excess pure premiums. In practice, the possibility of breaking the maximum before the switchover is considered remote, and is ignored in the following calculations. The average retrospective premium can then be estimated using ordinary loss payout patterns.

The effect of this simplifying assumption would be to overstate the average retrospective premium before the switchover. It will be corrected at the 54 month adjustment. The end result will be to overstate the present value of the average retrospective premium by the amount of interest earned on the excess pure premium before the switchover. This should be a negligible amount.

Let us assume that our hypothetical insured is expected to have paid $800,000 in losses by the switchover time, and that the present value of these payments is $720,000. Let us also assume that the basic premium is paid on the effective date of the plan. The following table describes the plan in detail.

TABLE 7

Basic	$ 215,170
L.C.F.	1.1
Maximum	$1,500,000
E[Paid R]	1,095,170
E[R] @ 54 mths.	1,167,130
@ 66 mths.	1,170,050
@ 78 mths.	1,172,980
@ 90 mths.	1,175,320
P.V. E[Paid R]	1,007,170
P.V. Retro Premium	1,062,000
P.V. Loss & Expense	962,000
Operating Profit	100,000

The results of this calculation should be directly comparable to the straight incurred loss retro (Table 5). The paid loss provision caused about a 3% increase in the average retrospective premium on a nominal basis.

6. EXCESS LOSS PREMIUM AND TAX MULTIPLIER

We did not consider the excess loss premium or the tax multiplier in the above calculations. The intent was to keep the discussion as simple as possible. We now show how to modify the calculation to take these into account.

On the premium side of the calculation, the only adjustment needed to handle the loss limit is to input a limited claim severity distribution into the Heckman-Meyers algorithm.

No adjustment is needed on the loss and expense side. Make note that the present value of the unlimited losses is still used.

A wrinkle in the above adjustment occurs when the excess layer is reinsured and one wants to incorporate the cost of reinsurance in the pricing. In this case one takes the sum of the present value of the limited losses and the cost of the reinsurance. This sum is used in place of the present value of the unlimited losses. A note of caution: the payout pattern for limited losses is faster than that of unlimited losses.

Premium taxes are paid on the basis of written premium. One should note that retrospective adjustments are also adjustments in written premium. The present value of the premium taxes can be calculated by using the average retrospective premium at each adjustment.

The following question should be asked at this point. Do we really need to have separate factors in the retrospective rating plan for excess losses and premium taxes?

Tax multipliers are not used in guaranteed cost plans, so why use them for retrospective rating? Rates for other guaranteed cost plans reflect premium taxes, and so could the basic premium and the loss conversion factor. Skurnick [5] put the excess premium into the basic premium for the California Table L, and there is no reason why this could not be done for all retrospective rating plans.

What really matters is that the present value of the retrospective premium is equal to the profit plus the present value of the losses and expenses. This can be accomplished by a proper selection of the basic premium and the loss conversion factor. The result will be a simpler formula for retrospective rating.

7. CONCLUSION

This paper is written under the premise that an explicit calculation of investment income is superior to the implicit recognition of investment income that some suggest is in many present rating formulas. We do not attempt to determine the proper operating profit. This task belongs to insurance company management and/or regulators. It does not belong to some ratemaking formula based on underwriting profit.

We have provided a methodology for finding the expected operating profit for a retrospective rating plan. This methodology is presently used by at least one major insurance company.

The author suspects that the more complicated versions of retrospective rating, such as paid loss retros, arose because the present plan does not allow for investment income. Now that the various versions of retrospective rating can be rated on a comparable basis, it is hoped that the more complicated versions will no longer be necessary. Retrospective rating can be made simple.

REFERENCES

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[4] R. L. Burden, J. D. Faires, and A. C. Reynolds, *Numerical Analysis*, 2nd Edition, Prindle, Weber & Schmidt, 1981, Ch. 2.

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EXHIBIT 1

Excess Pure Premiums

Losses Valued at 18 Months Expected Losses = $833,333			Losses Valued at 30 Months Expected Losses = $946,970		
Loss Amount	Cumulative Probability	Excess Pure Premium	Loss Amount	Cumulative Probability	Excess Pure Premium
$900,000	0.6508	$129,345	$900,000	0.5469	$196,000
910,000	0.6594	125,896	910,000	0.5561	191,516
920,000	0.6678	122,532	920,000	0.5653	187,123
930,000	0.6760	119,251	930,000	0.5742	182,820
940,000	0.6840	116,051	940,000	0.5831	178,607
950,000	0.6919	112,930	950,000	0.5918	174,481
960,000	0.6996	109,887	960,000	0.6003	170,442
970,000	0.7071	106,920	970,000	0.6088	166,487
980,000	0.7144	104,028	980,000	0.6170	162,616
990,000	0.7216	101,208	990,000	0.6252	158,827
1,000,000	0.7286	98,459	1,000,000	0.6332	155,119
1,010,000	0.7355	95,780	1,010,000	0.6410	151,490
1,020,000	0.7422	93,168	1,020,000	0.6487	147,939
1,030,000	0.7488	90,623	1,030,000	0.6563	144,464
1,040,000	0.7552	88,143	1,040,000	0.6638	141,064
1,050,000	0.7614	85,726	1,050,000	0.6711	137,739
1,060,000	0.7675	83,371	1,060,000	0.6782	134,485
1,070,000	0.7735	81,076	1,070,000	0.6853	131,303
1,080,000	0.7793	78,840	1,080,000	0.6922	128,190
1,090,000	0.7850	76,662	1,090,000	0.6989	125,145
1,100,000	0.7906	74,540	1,100,000	0.7056	122,168
1,110,000	0.7960	72,473	1,110,000	0.7121	119,256
1,120,000	0.8013	70,459	1,120,000	0.7185	116,409
1,130,000	0.8065	68,498	1,130,000	0.7247	113,625
1,140,000	0.8115	66,588	1,140,000	0.7309	110,903
1,150,000	0.8165	64,728	1,150,000	0.7369	108,241
1,160,000	0.8213	62,917	1,160,000	0.7427	105,639
1,170,000	0.8260	61,153	1,170,000	0.7485	103,095
1,180,000	0.8306	59,435	1,180,000	0.7542	100,609
1,190,000	0.8350	57,763	1,190,000	0.7597	98,178
1,200,000	0.8394	56,135	1,200,000	0.7651	95,802
1,210,000	0.8436	54,550	1,210,000	0.7704	93,479
1,220,000	0.8478	53,007	1,220,000	0.7756	91,209
1,230,000	0.8519	51,505	1,230,000	0.7807	88,991
1,240,000	0.8558	50,043	1,240,000	0.7857	86,823
1,250,000	0.8597	48,620	1,250,000	0.7906	84,704
1,260,000	0.8634	47,235	1,260,000	0.7954	82,634
1,270,000	0.8671	45,887	1,270,000	0.8001	80,611
1,280,000	0.8707	44,576	1,280,000	0.8046	78,635
1,290,000	0.8742	43,300	1,290,000	0.8091	76,703
1,300,000	0.8776	42,058	1,300,000	0.8135	74,816

EXHIBIT 1

EXCESS PURE PREMIUMS

Losses Valued at 42 Months Expected Losses = $975,610			Losses Valued at 54 Months Expected Losses = $986,193		
Loss Amount	Cumulative Probability	Excess Pure Premium	Loss Amount	Cumulative Probability	Excess Pure Premium
$900,000	0.5218	$214,600	$900,000	0.5127	$221,641
910,000	0.5311	209,865	910,000	0.5221	216,815
920,000	0.5403	205,223	920,000	0.5313	212,081
930,000	0.5494	200,672	930,000	0.5404	207,440
940,000	0.5584	196,210	940,000	0.5493	202,888
950,000	0.5672	191,838	950,000	0.5582	198,426
960,000	0.5759	187,553	960,000	0.5669	194,051
970,000	0.5844	183,355	970,000	0.5755	189,763
980,000	0.5928	179,241	980,000	0.5840	185,560
990,000	0.6011	175,211	990,000	0.5923	181,442
1,000,000	0.6093	171,263	1,000,000	0.6005	177,406
1,010,000	0.6173	167,396	1,010,000	0.6086	173,452
1,020,000	0.6252	163,608	1,020,000	0.6166	169,578
1,030,000	0.6330	159,899	1,030,000	0.6244	165,782
1,040,000	0.6406	156,267	1,040,000	0.6321	162,065
1,050,000	0.6481	152,711	1,050,000	0.6397	158,423
1,060,000	0.6555	149,229	1,060,000	0.6471	154,857
1,070,000	0.6627	145,820	1,070,000	0.6544	151,365
1,080,000	0.6698	142,483	1,080,000	0.6616	147,945
1,090,000	0.6768	139,216	1,090,000	0.6686	144,596
1,100,000	0.6837	136,019	1,100,000	0.6756	141,317
1,110,000	0.6904	132,889	1,110,000	0.6824	138,106
1,120,000	0.6970	129,826	1,120,000	0.6891	134,963
1,130,000	0.7035	126,829	1,130,000	0.6956	131,887
1,140,000	0.7099	123,895	1,140,000	0.7021	128,875
1,150,000	0.7161	121,025	1,150,000	0.7084	125,927
1,160,000	0.7222	118,216	1,160,000	0.7146	123,042
1,170,000	0.7282	115,468	1,170,000	0.7207	120,218
1,180,000	0.7341	112,779	1,180,000	0.7266	117,454
1,190,000	0.7399	110,149	1,190,000	0.7325	114,749
1,200,000	0.7455	107,576	1,200,000	0.7382	112,103
1,210,000	0.7511	105,058	1,210,000	0.7438	109,513
1,220,000	0.7565	102,596	1,220,000	0.7494	106,978
1,230,000	0.7618	100,188	1,230,000	0.7548	104,499
1,240,000	0.7670	97,832	1,240,000	0.7601	102,073
1,250,000	0.7722	95,528	1,250,000	0.7653	99,700
1,260,000	0.7772	93,274	1,260,000	0.7704	97,378
1,270,000	0.7821	91,070	1,270,000	0.7754	95,106
1,280,000	0.7869	88,915	1,280,000	0.7803	92,884
1,290,000	0.7916	86,808	1,290,000	0.7851	90,711
1,300,000	0.7962	84,747	1,300,000	0.7898	88,585

EXHIBIT 1

EXCESS PURE PREMIUMS

Losses Valued at 66 Months Expected Losses = $991,080			Losses Valued at 78 Months Expected Losses = $996,016		
Loss Amount	Cumulative Probability	Excess Pure Premium	Loss Amount	Cumulative Probability	Excess Pure Premium
$900,000	0.5086	$224,922	$900,000	0.5044	$228,254
910,000	0.5179	220,054	910,000	0.5137	223,345
920,000	0.5271	215,279	920,000	0.5229	218,528
930,000	0.5362	210,595	930,000	0.5320	213,803
940,000	0.5452	206,002	940,000	0.5410	209,168
950,000	0.5540	201,499	950,000	0.5499	204,622
960,000	0.5628	197,083	960,000	0.5586	200,165
970,000	0.5714	192,754	970,000	0.5673	195,795
980,000	0.5799	188,510	980,000	0.5758	191,510
990,000	0.5883	184,351	990,000	0.5842	187,310
1,000,000	0.5965	180,275	1,000,000	0.5924	183,193
1,010,000	0.6046	176,280	1,010,000	0.6006	179,158
1,020,000	0.6126	172,366	1,020,000	0.6086	175,203
1,030,000	0.6204	168,531	1,030,000	0.6164	171,328
1,040,000	0.6282	164,774	1,040,000	0.6242	167,532
1,050,000	0.6358	161,094	1,050,000	0.6318	163,812
1,060,000	0.6432	157,489	1,060,000	0.6393	160,167
1,070,000	0.6506	153,957	1,070,000	0.6467	156,597
1,080,000	0.6578	150,499	1,080,000	0.6539	153,100
1,090,000	0.6649	147,112	1,090,000	0.6611	149,675
1,100,000	0.6718	143,796	1,100,000	0.6681	146,321
1,110,000	0.6787	140,548	1,110,000	0.6749	143,036
1,120,000	0.6854	137,368	1,120,000	0.6817	139,818
1,130,000	0.6920	134,255	1,130,000	0.6883	136,668
1,140,000	0.6985	131,207	1,140,000	0.6948	133,584
1,150,000	0.7048	128,223	1,150,000	0.7012	130,564
1,160,000	0.7110	125,302	1,160,000	0.7075	127,607
1,170,000	0.7172	122,443	1,170,000	0.7136	124,712
1,180,000	0.7232	119,645	1,180,000	0.7197	121,879
1,190,000	0.7291	116,906	1,190,000	0.7256	119,105
1,200,000	0.7348	114,225	1,200,000	0.7314	116,390
1,210,000	0.7405	111,601	1,210,000	0.7371	113,732
1,220,000	0.7460	109,034	1,220,000	0.7427	111,131
1,230,000	0.7515	106,522	1,230,000	0.7482	108,585
1,240,000	0.7568	104,063	1,240,000	0.7536	106,094
1,250,000	0.7621	101,658	1,250,000	0.7588	103,656
1,260,000	0.7672	99,304	1,260,000	0.7640	101,270
1,270,000	0.7723	97,001	1,270,000	0.7691	98,936
1,280,000	0.7772	94,748	1,280,000	0.7741	96,651
1,290,000	0.7820	92,544	1,290,000	0.7789	94,416
1,300,000	0.7868	90,388	1,300,000	0.7837	92,229

EXHIBIT 1

Excess Pure Premiums

Losses Valued at 90 Months
Expected Losses = $1,000,000

Loss Amount	Cumulative Probability	Excess Pure Premium
$900,000	0.5010	$230,957
910,000	0.5103	226,014
920,000	0.5195	221,163
930,000	0.5287	216,405
940,000	0.5377	211,736
950,000	0.5465	207,157
960,000	0.5553	202,667
970,000	0.5640	198,263
980,000	0.5725	193,945
990,000	0.5809	189,712
1,000,000	0.5892	185,562
1,010,000	0.5973	181,494
1,020,000	0.6053	177,508
1,030,000	0.6132	173,600
1,040,000	0.6210	169,771
1,050,000	0.6286	166,020
1,060,000	0.6362	162,344
1,070,000	0.6436	158,742
1,080,000	0.6508	155,214
1,090,000	0.6580	151,758
1,100,000	0.6650	148,373
1,110,000	0.6719	145,057
1,120,000	0.6787	141,810
1,130,000	0.6853	138,630
1,140,000	0.6919	135,516
1,150,000	0.6983	132,467
1,160,000	0.7046	129,481
1,170,000	0.7108	126,558
1,180,000	0.7168	123,696
1,190,000	0.7228	120,894
1,200,000	0.7286	118,151
1,210,000	0.7344	115,466
1,220,000	0.7400	112,837
1,230,000	0.7455	110,265
1,240,000	0.7509	107,747
1,250,000	0.7562	105,283
1,260,000	0.7614	102,871
1,270,000	0.7665	100,511
1,280,000	0.7715	98,201
1,290,000	0.7765	95,941
1,300,000	0.7813	93,729

A BAYESIAN CREDIBILITY FORMULA FOR IBNR COUNTS

DR. IRA ROBBIN

Abstract

A formula for IBNR counts is derived as the credibility weighted average of three standard actuarial estimates:

Estimate	*IBNR Formula*
Pegged	*Initial Estimate of Ultimate* − *Reported to Date*
LDF	*(Reported to Date)* × (LDF − *1)*
Bornhuetter-Ferguson	*Initial Estimate of Ultimate* × *(1* − *1*/LDF)

Here LDF *denotes the age-to-ultimate development factor. The credibility weights vary by age of development in a methodical fashion reflecting prior belief in the reporting pattern and the estimate of ultimate.*

To derive the formula, IBNR is modelled as a parametrically dependent random variable. Bayes Theorem leads to a natural revision of the prior distribution of the parameters based on the data to date. Using the best least squares linear approximation to the true Bayesian estimate, and performing some algebraic manipulations, the credibility formula is obtained. While the formula could be applied in many ways, for demonstration purposes a fully automatic procedure is applied to three hypothetical triangles of data.

I. INTRODUCTION

This paper will present a formula which estimates IBNR (Incurred But Not Reported) claim counts in terms of a credibility weighted average of more traditional actuarial estimates. The formula will be derived from a theoretical foundation using Bayesian analysis methods applied to claim count development models.

Before presenting the formula, it is instructive to review the traditional actuarial estimates under discussion. In the usual context, we are estimating IBNR counts for an exposure period at a certain stage of development. We are given, or can obtain some preliminary estimate of ultimate counts that does not depend on the count data reported to date. For instance, the preliminary estimate could be the product of expected frequency times exposures, where the expected frequency is calculated with data from prior exposure periods. We also have count data reported to date and a set of expected age-to-ultimate count loss development factors (*LDF*). With all this information, three different IBNR count estimates may be obtained for the exposure period in question at its current stage of development.

1. Pegged Method
   IBNR = Preliminary Estimate of Ultimate Counts
   − Counts Reported to Date
2. Loss Development Factor Method
   IBNR = Counts Reported to Date × (*LDF* − 1)
3. Bornhuetter-Ferguson Method
   IBNR = Preliminary Estimate of Ultimate Counts × (1 − 1/LDF)

To decide amongst these, the actuary has heretofore been forced to rely on qualitative reasoning. Such "actuarial judgement" is not necessarily the arbitrary Delphic process one might suppose. For instance, if the actuary knows from long experience that reporting patterns are generally stable, the *LDF* method would be preferred. If reporting patterns have characteristically been erratic and the preliminary estimate of ultimate counts is generally near the mark, the pegged estimate would be favored. Such qualitative reasoning involves implicit non-quantified assumptions regarding the stochastic variability of ultimate claim counts and reporting patterns. It also reflects the degree of confidence in the prelimary estimate of expected ultimate counts and in the expected *LDF*.

By constructing an explicitly stochastic claims development model, and making Bayesian prior assumptions on the parameters defining the model, one advances the art of reserving beyond the realm of qualitative guesswork. Theoretically, Bayes Theorem leads to revised IBNR estimates reflecting prior belief appropriately modified by the data to date. Unfortunately, the mathematics often becomes intractable. Thus, one is led to considering linear estimators with least squared error.

The simplest general estimator one obtains can be expressed as a credibility weighted average of the three traditional estimates. The credibility weights vary with the stage of development, so that, for instance, the pegged estimate might receive the most weight initially, the Bornhuetter-Ferguson estimate might predominate for a few subsequent periods, and the loss development estimate could have the most weight thereafter. This methodical evolution of credibility weights is perhaps the key practical advantage of the Bayesian approach. Based on our initial beliefs, we are able to decide when to give each method credence.

The object of this paper is to present the formula and demonstrate one method of applying it to a triangle of data. The method of application uses the data to approximate needed parameters, so that, in the end, one has an automated procedure for estimating IBNR counts. Other methods of application are possible.

Finally, it should be noted that the theory leads naturally to an estimate of the variance of the IBNR counts. This variance reflects both process and parameter uncertainty.

II. BAYESIAN ANALYSIS OF COUNT DEVELOPMENT MODELS

Let N denote the ultimate number of claims for a fixed set of exposures and write N_j for the counts reported in the j^{th} development period. Set $M_j = N_1 + \ldots + N_j$ so that M_j denotes the counts reported to date as of the end of the j^{th} period. Define the IBNR count as of the end of the j^{th} period as R_j. Thus, R_j can be written as the sum, $N_{j+1} + N_{j+2} + \ldots + N_u$, where u is the number of periods until ultimate, or one can write $R_j = N - M_j$.

Assume the N_j are (conditionally) independent Poisson random variables whose parameters we denote as n_j. It follows that N, M_j, and R_j are also Poisson distributed, since the sum of independent Poisson variables is Poisson. Let $n = n_1 + \ldots + n_u$ and define $p_j = n_j/n$. Thus, the sum of the p_j is unity. Also, set $q_j = p_{j+1} + \ldots + p_u$. We summarize the random variables thus far defined:

II.1. *Conditional Poisson Random Variables*

Variable	Description	Poisson Parameter
N_j	Counts Reported During Period j	$n_j = np_j$
M_j	Counts Reported as of Period j	$m_j = n(1 - q_j)$
R_j	IBNR Counts as of Period j	$r_j = nq_j$
N	Ultimate Counts	n

subject to constraints

(*i*) $0 \leq p_j \leq 1$

(*ii*) $p_1 + p_2 + \ldots + p_u = 1$

Next we define $LDF_j = N/M_j$ when M_j is strictly positive. Though not strictly true mathematically, we may from time to time estimate $\mathrm{E}(LDF_j)$ as $1/(1 - q_j)$.

It should be further noted that the parameter p_j is distinct from, but related to, the ratio random variable, N_j/N. Maintaining the assumption that the parameters n and p_j are fixed, one can show:

II.2. *Relation of p_j to N_j/N*

$$p_j = \mathrm{E}(N_j/N \mid N > 0)$$

Proof

See Appendix A.

Next, we allow the parameters n and p_j to vary according to some prior distribution whose density we write as $f(n,p)$. Unconditional expectation and variance formulas for N, N_j, M_j, and R_j can then be derived in terms of expectations and variances involving n, p_j, and q_j.

II.3. *Expectation and Variance Formulas*

(*i*) N

$$\mathrm{E}(N) = \mathrm{E}(n)$$
$$\mathrm{Var}(N) = \mathrm{E}(n) + \mathrm{Var}(n)$$

(*ii*) N_j

$$\mathrm{E}(N_j) = \mathrm{E}(p_jn)$$
$$\mathrm{Var}(N_j) = \mathrm{E}(p_jn) + \mathrm{Var}(p_jn)$$

(*iii*) M_j

$$\mathrm{E}(M_j) = \mathrm{E}[(1 - q_j)n]$$
$$\mathrm{Var}(M_j) = \mathrm{E}[1 - q_j)n] + \mathrm{Var}((1 - q_j)n)$$

(*iv*) R_j

$$\mathrm{E}(R_j) = \mathrm{E}(q_jn)$$
$$\mathrm{Var}(R_j) = \mathrm{E}(q_jn) + \mathrm{Var}(q_jn)$$

Proof

We prove only *(ii)* and leave the rest as an exercise for the reader. Consider

$$\begin{aligned}
E(N_j) &= E_{n,p}(E(N_j/n,p)) \\
&= E_{n,p}(np_j) = E(np_j) \\
E(N_j^2) &= E_{n,p}(E(N_j^2/n,p)) \\
&= E((np_j)^2) + E(np_j)
\end{aligned}$$

Thus,

$$\begin{aligned}
\mathrm{Var}(N_j) &= E(N_j^2) - (E(N_j))^2 \\
&= E((np_j)^2) + E(np_j) - (E(np_j))^2 \\
&= \mathrm{Var}(p_jn) + E(p_jn)
\end{aligned}$$

Before providing a simple example demonstrating these concepts, it should be noted that in writing $f(n,p)$ we have implicitly incorporated the constraints on the p parameters. In applications, these restrictions must be explicitly reflected. One way to do this is to define the p_j as functions of some other parameters in such a way that the constraints are automatically satisfied. Letting g denote these generating parameters, we may write $f(n,p(g))$ or $f(n,g)$.

Now, for a simple example to demonstrate these concepts suppose:

II.4. *Assumptions for Example*

(i) The prior distribution for n is a gamma with a mean of 1,000 and a variance of 10,000.

$$f(n) = \left(\frac{1}{10}\right)^{100} \frac{1}{99!}\, n^{99} \quad e^{-n/10}$$

$$E(n) = 1{,}000 \qquad E(n^2) = 1{,}010{,}000$$

(ii) (a) $\bar{p}$ and $\bar{q}$ are given via:

$$\begin{aligned}
p_1 &= 1 - g_1 & q_1 &= g_1 \\
p_2 &= g_1(1 - g_2) & q_2 &= g_1g_2 \\
p_3 &= g_1g_2 & q_3 &= 0
\end{aligned}$$

where $g_j \in (0,1)$. (Observe that the constraints on the p_j are automatically satisfied.)

(b) The prior joint distribution for g_1 and g_2 is

$$f(g_1,g_2) = 2(1 - g_2)$$

We compute the first and second moments of the p, $1 - q$, and q variables:

II.5. *First and Second Moments of p, $1 - q$, and q Variables in Example*

	First Moments			Second Moments		
j	$E(p_j)$	$E(1 - q_j)$	$E(q_j)$	$E(p_j^2)$	$E((1 - q)^2)$	$E(q_j^2)$
1	1/2	1/2	1/2	1/3	1/3	1/3
2	1/3	5/6	1/6	1/6	13/18	1/18
3	1/6	1	0	1/18	1	0

To show how these figures were obtained, we calculate $E(p_2^2)$ in detail.

$$\begin{aligned} E(p_2^2) &= \int_0^1 \int_0^1 g_1^2(1 - g_2)^2 2(1 - g_2)\, dg_1\, dg_2 \\ &= \left(g_1^3/3\Big|_0^1\right)\left(-2(1 - g_2)^4/4\Big|_0^1\right) \\ &= (1/3)(1/2) = 1/6 \end{aligned}$$

We are now in a position to compute the means, variances, and standard deviations of the various count random variables.

II.6. *Means, Variances, and Standard Deviations of N, N_j, M_j, and R_j in Example*

Means

$E(N) = 1{,}000$

j	$E(N_j)$	$E(M_j)$	$E(R_j)$
1	500	500	500
2	333	833	167
3	167	1,000	0

Variances

$\mathrm{Var}(N) = 11{,}000$

j	$\mathrm{Var}(N_j)$	$\mathrm{Var}(M_j)$	$\mathrm{Var}(R_j)$
1	87,167	87,167	87,167
2	57,558	35,889	28,502
3	28,502	11,000	0

Standard Deviations

$\mathrm{Var}^{1/2}(N) = 105$

j	$\mathrm{Var}^{1/2}(N_j)$	$\mathrm{Var}^{1/2}(M_j)$	$\mathrm{Var}^{1/2}(R_j)$
1	295	295	295
2	240	189	169
3	169	105	0

Again demonstrating one of the calculations in more detail, we compute:

$$\begin{aligned}\mathrm{Var}(N_2) &= \mathrm{Var}(np_2) + \mathrm{E}(np_2)\\ &= \mathrm{E}(n^2)\mathrm{E}(p_2^2) - \mathrm{E}(n)\mathrm{E}(p_2)^2 + \mathrm{E}(np_2)\\ &= (1{,}010{,}000)(1/6) - (333)^2 + 333 = 57{,}556\end{aligned}$$

We return now to the general presentation and follow the Bayesian approach by modifying our beliefs about the parameter distribution, $f(n,p)$, as more data becomes available. Let $f^{(0)}$ denote the prior density before any development has occurred, and let $f^{(j)}$ denote the revised density as of the end of the j^{th} period of development. Given development data ($N_1 = x_1, N_2 = x_2, \ldots, N_j = x_j$), Bayes Theorem allows one to derive the modified belief density, $f^{(j)}$, in sequential fashion.

II.7 *Bayes Revised Belief Density*

$$f^{(j)}(n,p) = c\ \mathrm{Prob}(N_j = x_j/n,p)\, f^{(j-1)}(n,p)$$

where c is a normalization constant, and

$$\mathrm{Prob}(N_j = x/n,p) = \exp(-np_j)(np_j)^x/x!$$

Equivalently, one can write

$$f^{(j)}(n,p) = c\ L(n,p/x_1, x_2, \ldots, x_j)f^{(0)}(n,p)$$

where c is some normalization constant and L is the likelihood function,

$$L = \prod_{i=1}^{j} \text{Prob}(N_i = x_i/n,p)$$

The revised belief density yields revised IBNR count estimates via II.3.

Thus, the IBNR count estimation problem is theoretically solved. Further, the variance equation in II.3 (iv) could be used to calculate the standard deviation of the IBNR estimate. This deviation would reflect both process and parameter uncertainty.

Returning to our example, our prior density is:

$$f^{(0)}(n,g_1,g_2) = 2(1 - g_2)\frac{1}{10^{100}}\frac{1}{99!}n^{99}e^{-n/10}$$

If we observe $N_1 = 400$, the revised parameter density would be

$$f^{(1)}(n, g_1, g_2) = c\ e^{-n(1-g_1)}(n(1 - g_1))^{400}(1 - g_2)\,n^{99}e^{-n/10}$$

where c is a normalizing constant. This density is rather inconvenient to work with.

Such difficulties are not peculiar to this example. Indeed, the computations become intractable in most interesting models. Thus, the formulas are difficult to apply and consequently of limited practical use. As is usually the case in Bayesian analysis, one is led to consider linear estimators.

III. LINEAR APPROXIMATION OF THE BAYESIAN ESTIMATOR

We first recall some general results of Bayesian credibility theory. Let X and Y be (possibly vector-valued) random variables, each parameterized by a common (vector) parameter. Assume the distribution of the parameter is governed by some underlying structure function. We consider linear estimators of Y given results for X. It is known that the linear estimator, Y^*, with least mean square error (against the Bayesian estimator) is given via:

III.1. *General Least Squares Linear Approximation*

$$Y^* = \mathrm{E}(Y) + C(X,Y')\mathrm{V}(X)^{-1}(X - \mathrm{E}(X))$$

where

X and Y are column vectors

X' = transpose of X

$C(X,Y) = \mathrm{Cov}(X,Y) = \mathrm{E}(XY') - \mathrm{E}(X)\mathrm{E}(Y')$

$\mathrm{V}(X) = C(X,X')$

Applying this result with $X = (N_1, \ldots, N_j)'$ and $Y = R_j$, we obtain:

III.2. *General Linear IBNR Count Estimator*

$$R_j^* = \mathrm{E}(R_j) +$$

$$(C(N_1, R_j), \ldots, C(N_j, R_j)) \begin{bmatrix} C(N_1,N_1) & \ldots & C(N_1,N_j) \\ & \ldots & \\ C(N_j, N_1) & \ldots & C(N_j, N_j) \end{bmatrix}^{-1} \begin{bmatrix} N_1 - \mathrm{E}(N_1) \\ \ldots \\ N_j - \mathrm{E}(N_j) \end{bmatrix}$$

The quantities in the above equation can be expressed in terms of expectations, variances, and covariances of the n, p, and q.

III.3. *Expectation Variance and Covariance Formulas*

(i) $\mathrm{E}(R_i) = \mathrm{E}(n)\, \mathrm{E}(q_i)$

(ii) For $i \geq j$

$$C(N_j, R_i) = \mathrm{E}(p_j)\, \mathrm{E}(q_i)\, \mathrm{V}(n) + \mathrm{E}(n^2)\, C(p_j,q_i)$$

(iii) $C(N_i, N_j) = \mathrm{E}(n^2)\, C(p_i, p_j) + \mathrm{E}(p_i)\, \mathrm{E}(p_j)\, \mathrm{V}(n) + \delta_{ij}\, \mathrm{E}(n)\, \mathrm{E}(p_i)$

where $\mathrm{V}(X) = \mathrm{Var}\,(X)$

$$\delta_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ otherwise} \end{cases}$$

Formula III.2 is thus reasonable to apply in practice and there is no necessity for further simplification due to computational considerations. However, with one additional simplification, we achieve a formula expressing the estimator as a credibility weighted average of the traditional actuarial estimators as discussed in the introduction.

Applying III.1 with $X = M_j$ and $Y = R_j$, and grouping terms appropriately (as shown in Appendix B), we obtain

III.4. *Credibility Weighting Formula for IBNR Counts*

$$R_j^* = Z_{nj}(\mathrm{E}(n) - M_j) + Z_{pj}\frac{M_j\mathrm{E}(q_j)}{1 - \mathrm{E}(q_j)} + (1 - Z_{nj} - Z_{pj})\,\mathrm{E}(n)\mathrm{E}(q_j)$$

where

$$Z_{nj} = \mathrm{E}(n^2)\mathrm{V}(1 - q_j)/D_j$$

$$Z_{pj} = \mathrm{E}(1 - q_j)^2\mathrm{V}(n)/D_j$$

and

$$D_j = \mathrm{E}(n^2)\mathrm{V}(1 - q_j) + \mathrm{E}(1 - q_j)^2\mathrm{V}(n) + \mathrm{E}(n)\mathrm{E}(1 - q_j)$$

Approximating $\mathrm{E}(q_j)$ via $(1 - 1/LDF_j)$, we have:

III.5. *Credibility Weighting Formula for IBNR Counts - LDF Notation*

$$\begin{aligned} R_j^* = {} & Z_{nj}\,(\mathrm{E}(n) - (M_j)) + Z_{pj}\,(M_j)\,(LDF_j - 1) \\ & + (1 - Z_{nj} - Z_{pj})\,\mathrm{E}(n)\,(1 - 1/LDF_j) \end{aligned}$$

This is the formula promised at the outset since in this notation the traditional estimates may be expressed as:

IBNR Estimate	Expression
Pegged	$\mathrm{E}(n) - M_j$
LDF	$M_j(LDF_j - 1)$
Bornhuetter-Ferguson	$\mathrm{E}(n)\,(1 - 1/LDF_j)$

There are several qualitative conclusions that can be drawn from the formula. First, if there is no parameter uncertainty with respect to both ultimate counts and reporting patterns, then the data to date is given no credibility. In that case, the formula reduces to a Bornhuetter-Ferguson type estimate.

If there is some parameter uncertainty regarding counts, but none regarding reporting patterns, then the formula become a weighted average of loss development factor and Bornhuetter-Ferguson estimates. As the count parameter uncertainty increases, the formula approaches a loss development factor estimate. Finally, if there is some parameter uncertainty about reporting patterns, but none regarding counts, then the formula becomes a weighted average of pegged and Bornhuetter-Ferguson estimates.

IV. APPLICATION

In this section, the formula will be applied to three triangles of hypothetical data. The first triangle was constructed so that the Bornhuetter-Ferguson method will work almost exactly. The second triangle was generated to have nearly constant age-to-age factors. The last triangle is obtained by averaging the counts from the original triangles.

The formula could be applied in many different ways. For instance, a pure Bayesian approach would entail making explicit assumptions for the forms and parameters of the prior distributions. The resulting system would then require actuarial judgement in setting the parameters appropriately each time it was run. While this would be the most theoretically pure method of application, it might be regarded as somewhat impractical.

In order to provide a reasonably convincing demonstration that the formula is of practical use, we proceed now to present a fully automatic method of application. Under this particular approach, we let the data dictate parameter values to the degree possible. We introduce explicit forms for prior distributions if needed, but let the data determine the parameters of the priors.

To begin the application in detail, assume that a triangle of data is given. Let N_{ij} denote the counts reported in the j^{th} development period for the i^{th} accident period, where $i = 1, 2, \ldots, u$ and $j = 1, 2, \ldots, u - i + 1$. Define M_{ij} and R_{ij} in a fashion analogous to the definitions of M_j and R_j in II.

Assume N_{ij} is (conditionally) Poisson distributed with parameter $n_{ij} = B_i w_{ij}$, where B_i denotes the exposures for the i^{th} accident year.

Define:

$$n_i = \sum_j n_{ij}$$

$$w_i = \sum_j w_{ij}$$

$$p_{ij} = w_{ij}/w_i$$

so that $n_{ij} = B_i w_i p_{ij}$

Now assume that each of the frequency parameters, w_i, is, in effect, drawn from a common distribution. Thus, a priori, we have $E(w_i) = E(w)$. Similar assumptions are made for the set of p_{ij} and the set of q_{ij} when i is fixed. Thus, we may write $E(p_{ij}) = E(p_j)$ and $E(q_{ij}) = E(q_j)$.

We next find maximum likelihood estimators, w_i^* and p_j^*, for w_i and p_j. The likelihood function is:

IV.1. *Likelihood Function*

$$L(\bar{p}, w/\bar{N}) = \prod_{i=1}^{u} \prod_{j=1}^{u-i+1} e^{-B_i w_i p_j}(B_i w_i p_j)^{N_{ij}}/N_{ij}!$$

subject to $p_1 + p_2 + \ldots + p_u = 1$

We maximize as usual by taking the natural log and then the necessary partial derivatives.

IV.2. *"Log Likelihood" and Partials*

$$\ln L = \sum_{i=1}^{u} \sum_{j=1}^{u-i+1} - B_i w_i p_j + N_{ij} \ln(w_i p_j)$$

$$+ \text{ independent terms of } w_i \text{ and } p_j$$

$$\frac{\partial \ln L}{\partial w_i} = \sum_{j=1}^{u-i+1} - B_i p_j + N_{ij}/w_i$$

$$\frac{\partial \ln L}{\partial p_j} = \sum_{i=1}^{u-j+1} - B_i w_i + N_{ij}/p_j$$

Utilizing the constraint, we solve the equations via numerical iteration to obtain w_i^* and p_j^* which satisfy:

IV.3. *Maximum Likelihood Estimates*

$$w_i^* = M_{i,u-i+1}/B_i(1 - q_{u-i+1}^*)$$

$$p_j^* = \left(\sum_{i=1}^{u-j+1} N_{ij}\right) \Big/ \left(\sum_{i=1}^{u-j+1} w_i^* B_i\right)$$

Using the maximum likelihood estimates just obtained, we approximate the frequency mean and frequency variance.

IV.4. *Frequency Mean and Variance Estimators*

$$E(w) \approx \bar{w} = \left[\frac{\Sigma B_i w_i^*(1 - q_{u-i+1}^*)}{\Sigma B_i(1 - q_{u-i+1}^*)}\right]$$

$$\text{Var}(w) \approx S_w^2 = \left[\frac{\Sigma B_i(1 - q_{u-i+1}^*)(w_i^* - \bar{w})^2}{\Sigma B_i(1 - q_{u-i+1}^*)}\right]$$

While this seems intuitively reasonable, the properties of this variance estimator need further investigation in the future. Perhaps it is biased.

To estimate the required second moments of the reporting pattern parameters, we assume that p_{ij} is Beta distributed with parameters $(Hp_j^*, H(1 - p_j^*))$. We further have that q_{ij} is Beta distributed with parameters $(Hq_j^*, H(1 - q_j^*))$. Note the use of the maximum likelihood estimates in defining the parameters of these Betas. Under these assumptions, we can obtain convenient expressions for the mean and variance of the reporting pattern parameters.

IV.5. *Mean and Variance of* p_{ij} *and* q_{ij}

$$E(p_{ij}) = p_j^* \qquad \text{Var}(p_{ij}) = p_j^*(1 - p_j^*)/(1 + H)$$

$$E(q_{ij}) = q_j^* \qquad \text{Var}(q_{ij}) = q_j^*(1 - q_j^*)/(1 + H)$$

Observe that the parameters of the reporting pattern have variances inversely proportional to H. To use the data to solve for H, we first estimate p_{ij} via: $\hat{p}_{ij} = N_{ij}/(M_{i,u-i+1} + B_i w_i^* q_j^*)$ and define

IV.6. *Estimator For Variance of Reporting Pattern Parameters*

$$S_p^2 = \left[\sum_{i=1}^{u} \sum_{j=1}^{u-i+1} B_i(\hat{p}_{ij} - p_j^*)^2\right] \Big/ \sum_i \sum_j B_i$$

Plugging the $\text{Var}(p_{ij})$ formula of IV.5 in place of $(\hat{p}_{ij} - p_j^*)^2$, we obtain the approximation

$$E(S_p^2) = \sum_{ij} B_i p_j^*(1 - p_j^*)/ \sum_{ij} B_i(1 + H).$$

Thus we derive an estimator for H:

IV.7. *Estimator for H*

$$H^* = \frac{\sum_{ij} B_i\, p_j^*\, (1 - p_j^*)}{S_p^2 \sum_{ij} B_i} - 1$$

As before, the author must caution that the theoretical vices or virtues of this estimator have not been investigated. It is probably biased toward overstating H and thus understating $\text{Var}(1 - q_j)$. This will tend to give too much credibility to the *LDF* method.

At this point, we have enough to estimate all the terms required in the credibility formulas.

IV.8. *Estimators for Terms in Credibility Formulas*

Notation Used in Chapter		
II	IV	Estimator
$E(n)$	$E(n_i)$	$B_i\bar{w}$
$Var(n)$	$Var(n_i)$	$B_i^2S_w^2$
$E(n^2)$	$E(n_i^2)$	$B_i^2S_w^2 + B_i^2\bar{w}^2$
$E(1 - q_j)$	$E(1 - q_{ij})$	$1 - q_j^*$
$Var(1 - q_j)$	$Var(1 - q_{ij})$	$(1 - q_j^*)q_j^*/(H^* + 1)$

These were used to obtain the Bayesian credibility IBNR estimates shown in the attached exhibits. While the credibilities are not 100% for the "right" method in the "pure" cases, they nonetheless show that the application methodology is at least somewhat responsive. The credibility estimated IBNR is in all cases reasonably close to the correct answer. Further, the correct answer is well within one standard deviation of the estimate. Finally, considered over all three examples, the credibility formula approach appears to perform better than any one of the methods alone. The reader will, of course, arrive at his or her own judgement.

V. CONCLUSION

To conclude, it is hoped that the proposed IBNR count formula will not only advance reserving theory, but will also prove of practical use. It settles old arguments about which of three traditional actuarial estimates should be employed by showing how they may be credibility weighted in a methodical fashion to obtain a final estimate. The credibility weights differ depending on the development period. Thus, the Bayesian credibility approach provides a far more subtle method than simply picking one set of credibility weights which would apply at every development period. The formula could be applied in many ways, but at least one practical application has been demonstrated with fairly good results.

APPENDIX A

Let N_1 and N_2 be two independent Poisson random variables with parameters n_1 and n_2, respectively. Set $n = n_1 + n_2$ and $p = n_1/n$. We consider the ratio random variable $N_1/(N_1 + N_2)$.

A.1. *Proposition on Expectation*

$$E(N_1/(N_1 + N_2)/N_1 + N_2 > 0) = p$$

Proof

$$E(N_1/(N_1 + N_2)/N_1 + N_2 > 0) \text{ Prob } (N_1 + N_2 > 0)$$

$$= e^{-n} \sum_{x=1}^{\infty} \sum_{y=0}^{\infty} (x/(x + y)) n_1^x n_2^y/(x!y!)$$

$$= e^{-n} \sum_{x=1}^{\infty} \sum_{z=x}^{\infty} (x/z) n_1^x n_2^{z-x}/(x!(z - x)!)$$

$$= e^{-n} \sum_{z=1}^{\infty} z^{-1}(z!)^{-1}(n)^z \sum_{x=1}^{z} \binom{z}{x} x\, p^x(1 - p)^{z-x}$$

$$= e^{-n} \sum_{z=1}^{\infty} z^{-1}(z!)(n)^z z_p = pe^{-n}(e^n - 1)$$

$$= p(1 - e^{-n})$$

The result follows since

$$\text{Prob } (N_1 + N_2 > 0) = 1 - e^{-n}$$

APPENDIX B
DERIVATION OF CREDIBILITY WEIGHTING FORMULA FROM GENERAL LINEAR LEAST SQUARE ERROR BAYESIAN APPROXIMATION

Applying the general formula yields

B.1.

$$R_j^* = \mathrm{E}(R_j) + C(M_j,R_j)C(M_j)^{-1}(M_j - \mathrm{E}(M_j))$$

Expressing the terms of B.1 using terms involving n and q_j,

B.2.

$$\begin{aligned}
\mathrm{E}(M_j) &= \mathrm{E}(n)\mathrm{E}(1 - q_j)\\
\mathrm{E}(R_j) &= \mathrm{E}(n)\mathrm{E}(q_j)\\
C(M_j,R_j) &= \mathrm{E}(n^2)\ \mathrm{E}((1 - q_j)q_j) - \mathrm{E}(n)^2\mathrm{E}(1 - q_j)\mathrm{E}(q_j)\\
C(M_j) &= \mathrm{E}(n^2)\ \mathrm{E}((1 - q_j)^2) + \mathrm{E}(n)\mathrm{E}(1 - q_j) - \mathrm{E}(n)^2\mathrm{E}(1 - q_j)^2
\end{aligned}$$

Simplify the second order terms as follows

B.3.

$$\begin{aligned}
\text{(i)}\ C(M_j,R_j) &= \mathrm{E}(n^2)\ \mathrm{E}((1 - q_j)q_j) - \mathrm{E}(n^2)\ \mathrm{E}(1 - q_j)\mathrm{E}(q_j)\\
&\quad + \mathrm{E}(n^2)\ \mathrm{E}(1 - q_j)\ \mathrm{E}(q_j) - \mathrm{E}(n)^2\mathrm{E}(1 - q_j)\mathrm{E}(q_j)\\
&= \mathrm{E}(n^2)\ \mathrm{E}(1 - q_j)^2 - \mathrm{E}(n^2)\ \mathrm{E}((1 - q_j)^2)\\
&\quad + \mathrm{V}(n)\mathrm{E}(1 - q_j)\ \mathrm{E}(q_j)\\
&= -\mathrm{E}(n^2)\ \mathrm{V}(1 - q_j) + \mathrm{V}(n)\mathrm{E}(1 - q_j)\mathrm{E}(q_j)\\
\text{(ii)}\qquad C(M_j) &= \mathrm{E}(n^2)\ \mathrm{E}((1 - q_j)^2) - \mathrm{E}(n^2)\mathrm{E}(1 - q_j)^2\\
&\quad + \mathrm{E}(n^2)\mathrm{E}(1 - q_j)^2 - \mathrm{E}(n)^2\mathrm{E}(1 - q_j)^2 + \mathrm{E}(n)\mathrm{E}(1 - q_j)\\
&= \mathrm{E}(n^2)\mathrm{V}(1 - qj) + \mathrm{E}(1 - q_j)^2\mathrm{V}(n) + \mathrm{E}(n)\mathrm{E}(1 - q_j)
\end{aligned}$$

Plugging into B.1 one finds

B.4.

$R_j^* = \mathrm{E}(n)\mathrm{E}(q_j)+$

$$\frac{\mathrm{V}(n)\mathrm{E}(q_j)\mathrm{E}(1-q_j)-\mathrm{E}(n^2)\mathrm{V}(1-q_j)}{\mathrm{E}(n^2)\mathrm{V}(1-q_j)+\mathrm{E}(1-q_j)^2\mathrm{V}(n)+\mathrm{E}(n)\mathrm{E}(1-q_j)}(M_j-\mathrm{E}(n)\mathrm{E}(1-q_j))$$

$$\begin{aligned} &= (\mathrm{E}(n)-M_j)\,(\mathrm{E}(n^2)\mathrm{V}(1-q_j))\,/\,D + (M_j\,\mathrm{V}(n)\,\mathrm{E}(q_j)\,\mathrm{E}(1-q_j))\,/\,D \\ &\quad - (\mathrm{E}(n)\,\mathrm{V}(n)\,\mathrm{E}(q_j)\,\mathrm{E}(1-q_j))\,/\,D \\ &\quad + \mathrm{E}(n)\mathrm{E}(q_j)\,(1+(\mathrm{V}(n)\mathrm{E}(q)\mathrm{E}(1-q_j) \\ &\quad - \mathrm{E}(n^2)\mathrm{V}(1-q_j))\,/\,D) \end{aligned}$$

$$\begin{aligned} &= (\mathrm{E}(n)-M_j)(\mathrm{E}(n^2)\mathrm{V}(1-q_j)\,/\,D) + M_j\frac{\mathrm{E}(q_j)}{\mathrm{E}(1-q_j)}\times\frac{\mathrm{V}(n)\mathrm{E}(1-q_j)^2}{D} \\ &\quad + \mathrm{E}(n)\mathrm{E}(q_j)\,(1-(\mathrm{V}(n)\mathrm{E}(1-q_j)\,(\mathrm{E}(q_j)-1)-\mathrm{E}(n^2)\mathrm{V}(1-q_j))/D) \end{aligned}$$

which simplifies immediately to III.4.

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EXHIBIT 1

SHEET 1

BORNHUETTER-FERGUSON DATA

BAYESIAN CREDIBILITY FORMULA
IBNR ESTIMATION
HYPOTHETICAL DATA

N(I,J)
COUNTS REPORTED DURING
DEVELOPMENT PERIOD *J*

ACCIDENT YEAR (*I*)	EXPOSURES	1	2	3	4	5	6	7	8
1	100	50	150	450	225	100	50	25	5
2	100	25	150	450	225	100	50	25	
3	100	75	150	450	225	100	50		
4	100	15	150	450	225	100			
5	100	50	150	450	225				
6	100	25	150	450					
7	100	75	150						
8	100	15							

M(I,J)
COUNTS REPORTED TO DATE
DEVELOPMENT PERIOD *J*

ACCIDENT YEAR (*I*)	EXPOSURES	1	2	3	4	5	6	7	8
1	100	50	200	650	875	975	1,025	1,050	1,055
2	100	25	175	625	850	950	1,000	1,025	
3	100	75	225	675	900	1,000	1,050		
4	100	15	165	615	840	940			
5	100	50	200	650	875				
6	100	25	175	625					
7	100	75	225						
8	100	15							

EXHIBIT 1

SHEET 2
BORNHUETTER-FERGUSON DATA

$N(I,J)/B(I)$
DEVELOPMENT PERIOD J

ACCIDENT YEAR (I)	1	2	3	4	5	6	7	8
1	0.500	1.500	4.500	2.250	1.000	0.500	0.250	0.050
2	0.250	1.500	4.500	2.250	1.000	0.500	0.250	
3	0.750	1.500	4.500	2.250	1.000	0.500		
4	0.150	1.500	4.500	2.250	1.000			
5	0.500	1.500	4.500	2.250				
6	0.250	1.500	4.500					
7	0.750	1.500						
8	0.150							

AGE-TO-AGE FACTORS
DEVELOPMENT PERIOD J

ACCIDENT YEAR (I)	1–2	2–3	3–4	4–5	5–6	6–7	7–8
1	4.000	3.250	1.346	1.114	1.051	1.024	1.005
2	7.000	3.571	1.360	1.118	1.053	1.025	
3	3.000	3.000	1.333	1.111	1.050		
4	11.000	3.727	1.366	1.119			
5	4.000	3.250	1.346				
6	7.000	3.571					
7	3.000						

EXHIBIT 1

SHEET 3
BORNHUETTER-FERGUSON DATA

IBNR ESTIMATES

ACCIDENT YEAR	REPORT PERIOD	REPORTED TO DATE	PEGGED METHOD	*LDF* METHOD	BORNHUETTER-FERGUSON METHOD	BAYESIAN CREDIBILITY METHOD	STANDARD DEV. OF BAYESIAN CRED. IBNR
1	8	1,055	−10	0	0	0	0
2	7	1,025	20	5	5	5	3
3	6	1,050	−5	31	30	31	8
4	5	940	105	77	80	78	13
5	4	875	170	181	179	181	22
6	3	625	420	393	404	398	38
7	2	225	820	1,009	855	897	67
8	1	15	1,030	341	1,001	948	76
TOTAL		5,810	2,551	2,038	2,553	2,537	

EXHIBIT 1

SHEET 4
BORNHUETTER-FERGUSON DATA

ESTIMATES OF ULTIMATE

ACCIDENT YEAR	PEGGED METHOD	*LDF* METHOD	BORNHUETTER-FERGUSON METHOD	BAYESIAN CREDIBILITY METHOD
1	1,045	1,055	1,055	1,055
2	1,045	1,030	1,030	1,030
3	1,045	1,081	1,080	1,081
4	1,045	1,017	1,020	1,018
5	1,045	1,056	1,054	1,056
6	1,045	1,018	1,029	1,023
7	1,045	1,234	1,080	1,122
8	1,045	356	1,016	963

CREDIBILITIES

REPORT PERIOD	PEGGED	*LDF*	B-F
1	0.43193	0.09885	0.46923
2	0.29120	0.33820	0.37060
3	0.08355	0.69136	0.22509
4	0.03106	0.78064	0.18830
5	0.01283	0.81165	0.17552
6	0.00468	0.82550	0.16981
7	0.00076	0.83218	0.16706
8	0.00000	0.83347	0.16653

EXHIBIT 1

SHEET 5
BORNHUETTER-FERGUSON DATA

MAXIMUM LIKELIHOOD ESTIMATES

ACCIDENT YEAR	MLE FREQUENCY $W(I)$	EXPOSURES $B(I)$	INITIAL ESTIMATED COUNT PARAMETER $(B(I) \times W(I)$*
1	10.550	100	1,055
2	10.299	100	1,030
3	10.810	100	1,081
4	10.173	100	1,017
5	10.560	100	1,056
6	10.175	100	1,017
7	12.274	100	1,227

ESTIMATED FREQUENCY MEAN	10.45106
ESTIMATED FREQUENCY VARIANCE	.52307

REPORT PERIOD	MLE $P(J)$	PERCENT REPORTED $E(P_j)$	PERCENT REPORTED TO DATE $E(1 - Q_j)$	PERCENT UNREPORTED $E(Q_j)$	AGE-TO-AGE FACTORS	FACTORS TO ULTIMATE
1	0.042	4.2	4.2	95.8	4.332	23.759
2	0.140	14.0	18.2	81.8	3.366	5.484
3	0.431	43.1	61.4	38.6	1.350	1.629
4	0.215	21.5	82.8	17.2	1.115	1.207
5	0.096	9.6	92.4	7.6	1.051	1.082
6	0.047	4.7	97.1	2.9	1.025	1.030
7	0.024	2.4	99.5	0.5	1.005	1.005
8	0.005	0.5	100.0	0.0	1.000	1.000
TOTAL	1.000					

EXHIBIT 1

SHEET 6

Bornhuetter-Ferguson Data

Report Pattern Parameters

Report Period	Beta Param (A)	Beta Param (B)	Beta Mean A ÷ (A + B)	Expected Pct Reported During Period	Expected Pct Reported to Date $E(1 - Q_j)$	Expected Pct Unrep. $E(Q_j)$	Expected Pct Unrep. Squared $E(Q_j^2)$	Pct Unrep. Var. $Var(Q_j)$	Pct Unrep. Stand Dev SD	Pct Reported to Date CV SD ÷ Mean	Pct Unrep. CV SD ÷ Mean
1	45.95	1,045.83	0.042	4.2	4.2	95.8	91.8	0.004	0.607	14.431	0.634
2	153.11	938.67	0.140	14.0	18.2	81.8	66.9	0.014	1.168	6.406	1.428
3	470.96	620.82	0.431	43.1	61.4	38.6	14.9	0.022	1.473	2.400	3.813
4	234.34	857.44	0.215	21.5	82.8	17.2	3.0	0.013	1.141	1.377	6.645
5	104.36	987.43	0.096	9.6	92.4	7.6	0.6	0.006	0.802	0.868	10.542
6	51.71	1,040.07	0.047	4.7	97.1	2.9	0.1	0.003	0.505	0.520	17.595
7	26.17	1,065.61	0.024	2.4	99.5	0.5	0.0	0.000	0.208	0.209	43.846
8	5.17	1,086.61	0.005	0.5	100.0	0.0	0.0	0.000	0.000	0.000	—

$H = 1{,}091.8$

EXHIBIT 2

SHEET 1
LDF DATA

BAYESIAN CREDIBILITY FORMULA
IBNR ESTIMATION
HYPOTHETICAL DATA

$N(I,J)$
COUNTS REPORTED DURING
DEVELOPMENT PERIOD J

ACCIDENT YEAR (I)	EXPOSURES	1	2	3	4	5	6	7	8
1	100	50	217	730	986	1,100	1,128	1,134	1,139
2	100	25	109	366	494	551	565	568	
3	100	75	325	1,094	1,477	1,647	1,689		
4	100	15	65	219	296	330			
5	100	50	217	730	986				
6	100	25	109	366					
7	100	75	325						
8	100	15							

$M(I,J)$
COUNTS REPORTED TO DATE
DEVELOPMENT PERIOD J

ACCIDENT YEAR (I)	EXPOSURES	1	2	3	4	5	6	7	8
1	100	50	167	513	256	114	28	6	5
2	100	25	84	257	128	57	14	3	
3	100	75	250	769	383	170	42		
4	100	15	50	154	77	34			
5	100	50	167	513	256				
6	100	25	84	257					
7	100	75	250						
8	100	15							

EXHIBIT 2

SHEET 2
LDF DATA

$N(I,J)/B(I)$
DEVELOPMENT PERIOD J

ACCIDENT YEAR (I)	1	2	3	4	5	6	7	8
1	0.500	1.670	5.130	2.560	1.140	0.280	0.060	0.050
2	0.250	0.840	2.570	1.280	0.570	0.140	0.030	
3	0.750	2.500	7.690	3.830	1.700	0.420		
4	0.150	0.500	1.540	0.770	0.340			
5	0.500	1.670	5.130	2.560				
6	0.250	0.840	2.570					
7	0.750	2.500						
8	0.150							

AGE-TO-AGE FACTORS
DEVELOPMENT PERIOD J

ACCIDENT YEAR (I)	1–2	2–3	3–4	4–5	5–6	6–7	7–8
1	4.340	3.364	1.351	1.116	1.025	1.005	1.004
2	4.360	3.358	1.350	1.115	1.025	1.005	
3	4.333	3.366	1.350	1.115	1.026		
4	4.333	3.369	1.352	1.115			
5	4.340	3.364	1.351				
6	4.360	3.358					
7	4.333						

EXHIBIT 2

SHEET 3
LDF DATA

IBNR ESTIMATES

ACCIDENT YEAR	REPORT PERIOD	REPORTED TO DATE	PEGGED METHOD	*LDF* METHOD	BORNHUETTER-FERGUSON METHOD	BAYESIAN CREDIBILITY METHOD	STANDARD DEV. OF BAYESIAN CRED. IBNR
1	8	1,139	−187	0	0	0	0
2	7	568	384	3	4	3	3
3	6	1,689	−737	16	9	16	6
4	5	330	622	12	33	12	18
5	4	986	−34	153	128	153	66
6	3	366	586	205	341	206	176
7	2	325	627	1,380	770	1,368	395
8	1	15	937	327	910	375	467
TOTAL		5,418	2,196	2,095	2,195	2,132	

EXHIBIT 2

SHEET 4
LDF DATA

ESTIMATES OF ULTIMATE

ACCIDENT YEAR	PEGGED METHOD	*LDF* METHOD	BORNHUETTER-FERGUSON METHOD	BAYESIAN CREDIBILITY METHOD
1	952	1,139	1,139	1,139
2	952	571	572	571
3	952	1,705	1,698	1,705
4	952	342	363	342
5	952	1,139	1,114	1,139
6	952	571	707	572
7	952	1,705	1,095	1,693
8	952	342	925	390

CREDIBILITIES

REPORT PERIOD	PEGGED	*LDF*	B-F
1	0.00004	0.91622	0.08374
2	0.00001	0.97936	0.02063
3	0.00000	0.99378	0.00622
4	0.00000	0.99539	0.00461
5	0.00000	0.99586	0.00414
6	0.00000	0.99596	0.00404
7	0.00000	0.99598	0.00402
8	0.00000	0.99600	0.00400

EXHIBIT 2

SHEET 5
LDF DATA

MAXIMUM LIKELIHOOD ESTIMATES

ACCIDENT YEAR	MLE FREQUENCY $W(I)$	EXPOSURES $B(I)$	INITIAL ESTIMATED COUNT PARAM $B(I) \times W(I)$*
1	11.390	100	1,139
2	5.705	100	571
3	17.055	100	1,705
4	3.417	100	342
5	11.390	100	1,139
6	5.711	100	571
7	17.059	100	1,706

ESTIMATED FREQUENCY MEAN	9.51743
ESTIMATED FREQUENCY VARIANCE	23.70887

REPORT PERIOD	MLE $P(J)$	PERCENT REPORTED $E(P_j)$	PERCENT REPORTED TO DATE $E(1 - Q_j)$	PERCENT UNREPORTED $E(Q_j)$	AGE-TO-AGE FACTORS	FACTORS TO ULTIMATE
1	0.044	4.4	4.4	95.6	4.340	22.768
2	0.147	14.7	19.1	80.9	3.364	5.246
3	0.451	45.1	64.1	35.9	1.350	1.560
4	0.225	22.5	86.6	13.4	1.115	1.155
5	0.100	10.0	96.6	3.4	1.025	1.035
6	0.025	2.5	99.0	1.0	1.005	1.010
7	0.005	0.5	99.6	0.4	1.004	1.004
8	0.004	0.4	100.0	0.0	1.000	1.000
TOTAL	1.000					

EXHIBIT 2

SHEET 6
LDF DATA

REPORT PATTERN PARAMETERS

REPORT PERIOD	BETA PARAM (A)	BETA PARAM (B)	BETA MEAN A ÷ (A + B)	EXPECTED PCT REPORTED DURING PERIOD	EXPECTED PCT REPORTED TO DATE $E(1 - Q_j)$	EXPECTED PCT UNREP. $E(Q_j)$	EXPECTED PCT UNREP. SQUARED $E(Q_j^2)$	PCT UNREP. VAR. $VAR(Q_j)$	PCT UNREP. STAND DEV SD	PCT REPORTED TO DATE CV SD ÷ MEAN	PCT UNREP. CV SD ÷ MEAN
1	97,715.43	2,127,084.44	0.044	4.4	4.4	95.6	91.4	0.000	0.014	0.313	1.4×10^{-7}
2	326,344.90	1,898,454.97	0.147	14.7	19.1	80.9	65.5	0.000	0.026	0.138	6.2×10^{-8}
3	1,002,477.70	1,222,322.16	0.451	45.1	64.1	35.9	12.9	0.000	0.032	0.050	2.3×10^{-8}
4	499,940.66	1,724,859.21	0.225	22.5	86.6	13.4	1.8	0.000	0.023	0.026	1.2×10^{-8}
5	222,108.92	2,002,690.95	0.100	10.0	96.6	3.4	0.1	0.000	0.012	0.013	5.7×10^{-9}
6	54,730.59	2,170,069.28	0.025	2.5	99.0	1.0	0.0	0.000	0.007	0.007	3.0×10^{-9}
7	11,714.15	2,213,085.72	0.005	0.5	99.6	0.4	0.0	0.000	0.004	0.004	1.9×10^{-9}
8	9,767.53	2,215,032.34	0.004	0.4	100.0	0.0	0.0	0.000	0.000	0.000	.

$H = 2,224,799.9$

EXHIBIT 3

SHEET 1
MIXED DATA

BAYESIAN CREDIBILITY FORMULA
IBNR ESTIMATION
HYPOTHETICAL DATA

N(I,J)
COUNTS REPORTED DURING
DEVELOPMENT PERIOD *J*

ACCIDENT YEAR (*I*)	EXPOSURES	1	2	3	4	5	6	7	8
1	100	50	159	482	241	107	39	16	5
2	100	25	117	354	177	79	32	14	
3	100	75	200	610	304	135	46		
4	100	15	100	302	151	67			
5	100	50	159	482	241				
6	100	25	117	354					
7	100	75	200						
8	100	15							

M(I,J)
COUNTS REPORTED TO DATE
DEVELOPMENT PERIOD *J*

ACCIDENT YEAR (*I*)	EXPOSURES	1	2	3	4	5	6	7	8
1	100	50	209	691	932	1,039	1,078	1,094	1,099
2	100	25	142	496	673	752	784	798	
3	100	75	275	885	1,189	1,324	1,370		
4	100	15	115	417	568	635			
5	100	50	209	691	932				
6	100	25	142	496					
7	100	75	275						
8	100	15							

EXHIBIT 3

SHEET 2
MIXED DATA

$N(I,J)/B(I)$
DEVELOPMENT PERIOD J

ACCIDENT YEAR (I)	1	2	3	4	5	6	7	8
1	0.500	1.590	4.820	2.410	1.070	0.390	0.160	0.050
2	0.250	1.170	3.540	1.770	0.790	0.320	0.140	
3	0.750	2.000	6.100	3.040	1.350	0.460		
4	0.150	1.000	3.020	1.510	0.670			
5	0.500	1.590	4.820	2.410				
6	0.250	1.170	3.540					
7	0.750	2.000						
8	0.150							

AGE-TO-AGE FACTORS
DEVELOPMENT PERIOD J

ACCIDENT YEAR (I)	1–2	2–3	3–4	4–5	5–6	6–7	7–8
1	4.180	3.306	1.349	1.115	1.038	1.015	1.005
2	5.680	3.493	1.357	1.117	1.043	1.018	
3	3.667	3.218	1.344	1.114	1.035		
4	7.667	3.626	1.362	1.118			
5	4.180	3.306	1.349				
6	5.680	3.493					
7	3.667						

EXHIBIT 3

SHEET 3

MIXED DATA

IBNR ESTIMATES

ACCIDENT YEAR	REPORT PERIOD	REPORTED TO DATE	PEGGED METHOD	*LDF* METHOD	BORNHUETTER-FERGUSON METHOD	BAYESIAN CREDIBILITY METHOD	STANDARD DEV. OF BAYESIAN CRED. IBNR
1	8	1,099	−100	0	0	0	0
2	7	798	201	4	5	4	3
3	6	1,370	−371	28	20	28	8
4	5	635	364	38	56	38	17
5	4	932	67	169	153	169	43
6	3	496	503	295	373	297	102
7	2	275	724	1,201	813	1,165	219
8	1	15	984	334	956	522	258
TOTAL		5,620	2,375	2,069	2,376	2,224	

EXHIBIT 3

SHEET 4

MIXED DATA

ESTIMATES OF ULTIMATE

ACCIDENT YEAR	PEGGED METHOD	*LDF* METHOD	BORNHUETTER-FERGUSON METHOD	BAYESIAN CREDIBILITY METHOD
1	999	1,099	1,099	1,099
2	999	802	803	802
3	999	1,398	1,390	1,398
4	999	673	691	673
5	999	1,101	1,085	1,101
6	999	791	869	793
7	999	1,476	1,088	1,440
8	999	349	971	537

CREDIBILITIES

REPORT PERIOD	PEGGED	*LDF*	B-F
1	0.07101	0.70066	0.22833
2	0.01814	0.91327	0.06859
3	0.00264	0.97558	0.02178
4	0.00081	0.98294	0.01625
5	0.00026	0.98513	0.01460
6	0.00009	0.98582	0.01408
7	0.00002	0.98611	0.01386
8	0.00000	0.98620	0.01380

EXHIBIT 3

SHEET 5
MIXED DATA

MAXIMUM LIKELIHOOD ESTIMATES

ACCIDENT YEAR	MLE FREQUENCY $W(I)$	EXPOSURES $B(I)$	INITIAL ESTIMATED COUNT PARAM $B(I) \times W(I)$*
1	10.990	100	1,099
2	8.016	100	802
3	13.984	100	1,398
4	6.725	100	672
5	11.007	100	1,101
6	7.907	100	791
7	14.711	100	1,471

ESTIMATED FREQUENCY MEAN	9.99352
ESTIMATED FREQUENCY VARIANCE	7.14026

REPORT PERIOD	MLE $P(J)$	PERCENT REPORTED $E(P_i)$	PERCENT REPORTED TO DATE $E(1 - Q_i)$	PERCENT UNREPORTED $E(Q_i)$	AGE-TO-AGE FACTORS	FACTORS TO ULTIMATE
1	0.043	4.3	4.3	95.7	4.339	23.284
2	0.143	14.3	18.6	81.4	3.364	5.366
3	0.441	44.1	62.7	37.3	1.350	1.595
4	0.220	22.0	84.7	15.3	1.115	1.181
5	0.098	9.8	94.4	5.6	1.038	1.059
6	0.035	3.5	98.0	2.0	1.016	1.021
7	0.016	1.6	99.5	0.5	1.005	1.005
8	0.005	0.5	100.0	0.0	1.000	1.000
TOTAL	1.000					

EXHIBIT 3

SHEET 6

MIXED DATA

REPORT PATTERN PARAMETERS

Report Period	Beta Param (A)	Beta Param (B)	Beta Mean A ÷ (A + B)	Expected Pct Reported During Period	Expected Pct Reported to Date $E(1 - Q_j)$	Expected Pct Unrep. $E(Q_j)$	Expected Pct Unrep. Squared $E(Q_j^2)$	Pct Unrep. Var. $Var(Q_j)$	Pct Unrep. Stand Dev SD	Pct Reported to Date CV SD ÷ Mean	Pct Unrep. CV SD ÷ Mean
1	141.48	3,152.57	0.043	4.3	4.3	95.7	91.6	0.001	0.353	8.224	0.369
2	472.38	2,821.66	0.143	14.3	18.6	81.4	66.2	0.005	0.678	3.640	0.834
3	1,451.43	1,842.62	0.441	44.1	62.7	37.3	13.9	0.007	0.842	1.344	2.259
4	723.27	2,570.77	0.220	22.0	84.7	15.3	2.4	0.004	0.628	0.742	4.092
5	321.73	2,972.31	0.098	9.8	94.4	5.6	0.3	0.002	0.400	0.423	7.167
6	116.79	3,177.25	0.035	3.5	98.0	2.0	0.0	0.001	0.246	0.251	12.094
7	51.98	3,242.06	0.016	1.6	99.5	0.5	0.0	0.000	0.117	0.118	25.772
8	14.98	3,279.06	0.005	0.5	100.0	0.0	0.0	0.000	0.000	0.000	—

$H = 3{,}294.0$

DISCUSSION BY GARY G. VENTER

If an indicator of a significant paper is that it opens the door for further research, Dr. Robbin's paper should stand the historical test. This review will emphasize generalizing the Poisson assumptions of the paper. Attention to optimal parameter estimation and other model assumptions may also prove fruitful, as may the quantification of uncertainty in the IBNR estimates.

The three way credibility weighting for IBNR is an interesting result of the paper. Credibility weights are specified for three estimators of IBNR:

(i) the original (e.g., pricing) expected claims less the observed claims to date;

(ii) the observed claims to date times a development factor; and,

(iii) the original expected claims less the expected claims to date.

To see the origin of these credibility weights, a slightly more general framework will be used here. A vector of parameters, u, is postulated to determine the distribution of N, the ultimate number of claims; M, the observed claims to date; and R, the IBNR claims.

It is assumed that M and R are conditionally independent given u. Further, n and q are functions of u, and s^2 is a positive constant with

$$\begin{aligned} E(N/u) &= n \\ E(M/u) &= n(1-q) \\ E(R/u) &= nq \\ EV(M/u) &= s^2 \end{aligned}$$

This last assumption generalizes the Poisson assumption of the paper, where the expected conditional variance of M was $EnE(1-q)$.

It is also assumed that u is a vector of random variables such that n and q are independent.

The fundamental credibility formula from Robbin, section III.1, is then invoked to estimate R:

$$R^* = ER + (M - EM)C(M,R)/VM.$$

From the assumptions, $ER = EnEq$ and $EM = EnE(1-q) = En(1-Eq) = En - EnEq$. Also $VM = EV(M/u) + VE(M/u) = s^2 + V(n(1-q)) = s^2 + E(n^2(1-q)^2) - E(n(1-q))^2$. Then by the reasoning of B.3.(ii) of the paper, $VM = s^2 + E(n^2)V(1-q) + E(1-q)^2Vn$.

These three components of the variance of the observed claims, when divided by that variance, will turn out to be the three credibility weights to be applied to the three IBNR estimators (i), (ii), and (iii), above. To see this, a general formula on covariances is used to compute $C(M,R)$:

$$C(M,\ R) = EC(M,R|u) + C(E(M|u),E(R|u)).$$

Because of the conditional independence of M and R, the first term is zero, and so

$$\begin{aligned} C(M,R) &= C(n(1-q),nq) \\ &= E(n(1-q)nq) - E(n(1-q))E(nq) \end{aligned}$$

Then, by the reasoning of B.3.(i) of the paper, $C(M,R) = VnEqE(1-q) - E(n^2)V(1-q)$. Plugging all of this back into the original credibility formula gives:

$$R^* = EnEq + (M + EnEq - En)[VnEqE(1-q) - E(n^2)V(1-q)]/VM.$$

This is regrouped into Robbin's three way credibility formula as follows: first combine the $EnEq$ terms; apply $M - En$ to the second term in brackets to yield $\underline{(En - M)E(n^2)V(1-q)/VM}$. When applied to the first term in brackets the M and En are separated, giving a) En combined with Eq and adding to the $EnEq$ component; and b) $\underline{(M[Eq/E(1-q)]VnE(1-q)^2/VM}$. The underlined terms are the IBNR estimators (i) and (ii) times credibility weights, where the weights are the second and third components of the variance VM above, divided by VM.

This interprets $Eq/E(1-q)$ as a development factor, and in fact by the hypotheses above, $ER/EM = Eq/E(1-q)$ and $EN/EM = 1/E(1-q)$. This corresponds to the method of estimating LDF's from several accident years' data by $\Sigma N_i/\Sigma M_i$, as recommended by Stanard (*PCAS* 1985). With this definition of the LDF, the mathematically imprecise estimate of the development factor used by Dr. Robbin becomes unnecessary.

Finally the remaining terms of R^* can be algebraically combined to yield the credibility weight of s^2/VM applied to $EnEq$. Writing $EnEq$ as $En - EnE(1-q)$ shows this term to be the original expected claims less the expected claims to date.

The assumption that M and R are conditionally independent may be somewhat limiting. The possibility that some claims come in earlier than usual, so fewer come in later (or vice versa), suggest that R and M are not unconditionally independent. Assuming they are conditionally independent then attributes their correlation to non-independent parameters. But this suggests that the parameters are different from year to year. If the claims reported before and after a given point are each modelled as conditionally independent draws from a fixed, possibly unknown, report lag distribution, a negative correlation between reported and unreported claims would not be anticipated.

Dr. Robbin is to be congratulated for this thought provoking and potentially useful paper. He has proven his main point: a Bayesian credibility formula for IBNR *does* count.

THE COST OF MIXING REINSURANCE

RONALD F. WISER

Abstract

Excess and surplus lines underwriters, and others, rely heavily on facultative reinsurance support as an important part of their underwriting function. Individual risks are often subject to multiple reinsurance transactions as a result of the underwriting process. The net retained by the underwriters for the company's account is then subject to the overall company reinsurance treaty. As a result, the final company net position has been layered in a complicated fashion. It is management's task to provide guidelines for the proper use of facultative proportional and excess reinsurance that achieves corporate risk and profitability objectives under such conditions.

This paper investigates the impact on profitability of a common reinsurance mixing situation. The impact on the stability function of excess reinsurance is quantified. General rules to guide practical use and evaluation of mixed situations are developed.

These results are equally applicable to property as well as casualty risks. The implications are valid for facultative reinsurance underwriters, and others that make heavy use of facultative proportional reinsurance arrangements.

INTRODUCTION

Many underwriters rely heavily on facultative reinsurance support as an important part of their underwriting function. This is especially the case in the excess and surplus lines and commercial property lines. Individual risks are often subject to multiple reinsurance transactions as a result of the initial underwriting process. The net exposure retained by the underwriters for the company's account is then subject to the overall company reinsurance treaty. As a result, the final company net retention has been layered in a complicated fashion. This complicated net position can lead to unexpected net loss ratio and combined ratio results.

The purpose of this paper is to investigate the consequences of one such reinsurance situation—the application of an excess of loss reinsurance treaty after the placement of proportional reinsurance on the same risk—and to investigate ways of managing this situation. We will take the viewpoint of the ceding company, although the subject is also of interest to the excess reinsurer. We will assume that, in general, the mixed reinsurance situation comes about through the application of proportional facultative reinsurance on individual risks, and the retained amounts are then subject to a corporate excess of loss treaty. In the case of a portfolio of risks, we assume the aggregate effect of individual facultative cessions can be adequately modeled by an average proportional retention applying to the entire portfolio.

The consequences of this mixed reinsurance situation are twofold:

Magnitude of net loss ratio: The application of proportional reinsurance below an excess of loss layer reduces the excess reinsurer's loss ratio and raises the ceding company's loss ratio. The expected loss ratio on the pro rata reinsurance is unchanged; it will always be the same as the gross loss ratio.

Stability of net loss ratio: While the purpose of excess of loss reinsurance is to provide stability to the net retained loss ratio, the application of proportional reinsurance under the excess of loss cover actually decreases the stability of the net loss ratio.

A heuristic argument can show that each of these effects is intuitively plausible. Actual examples will show the mechanics of both the magnitude and the stability effects. Beyond the examples, it is demonstrated that these are not isolated instances, but the effects can be mathematically shown to hold always. We will use the term "mixing reinsurance" or "mixing" to denote this scenario of applying an excess of loss reinsurance treaty *after* a proportional transaction.

Reasons for Mixing

As we investigate the implications of mixing proportional and excess reinsurance, we need to keep in mind the purpose for the particular mixing situations. Since all instances of mixing will penalize the net loss ratio to different extents, management must carefully evaluate whether the cost of mixing is justified by the advantage gained. Generally, senior management is heavily involved in negotiating and placing the major treaties of the company. Historically, lower levels of management have directed the use of facultative reinsurance. Often, the individual desk underwriter places quota share facultative reinsurance on a risk as he writes it.

The premise of this paper is that the *total* corporate reinsurance program (not just the major corporate treaties) must be actively managed to assure that corporate objectives are met. The interaction between proportional and excess reinsurance in the mixed case can be very significant. Management must institute guidelines and controls for use of proportional reinsurance which assure the objectives intended by placement of the corporate excess treaties are met. These objectives will generally be stated in the form of expected net loss ratio, or cost of reinsurance, and protection from large swings in net loss ratio (stability).

Some common reasons for the occurrence of mixed reinsurance situations are:

a) capacity;
b) net premium targets;
c) protecting the treaty;
d) sharing of layers; and,
e) commission overrides.

Capacity: An individual risk is too large to be retained net by the insurer. A proportion of the risk may be ceded on a quota share or surplus share basis to reduce its size. This is common on property risks. A mixed situation exists if the corporate property treaty is on an excess of loss basis.

Net Premium Targets: A corporate plan may call for a certain net premium increase that must be strictly adhered to (for instance, because of statutory income or surplus restrictions). If more gross premium is written than planned, the net target may be achieved by increased use of facultative proportional reinsurance. This strategy should be evaluated in light of the penalty imposed on the net loss ratio position.

Protecting the Treaty: If the rate on the excess treaty is clearly insufficient to absorb the exposure from a risk the insurer wishes to write, the excess loss potential can be scaled down by a facultative quota share placement to fit the treaty pricing. This comes about because proportional reinsurance changes the frequency and severity characteristics of the excess loss exposure. This is one case where mixing reinsurance may be the prescribed course of action to achieve the corporate objective of excess treaty perpetuation at a reasonable price.

Sharing of Layers: For any of the reasons above, the underwriter may substitute the direct writing of a proportional share of a risk in place of acceptance of the entire risk followed by a facultative quota share reinsurance transaction. This is, in fact, a disguised mixed reinsurance situation and is fully equivalent in its effect on net loss ratio and stability. The popularity of sharing layers increases as the facultative reinsurance market tightens. The normal operating procedure of the facultative reinsurance underwriter or the brokered treaty underwriter is to accept proportional shares of an excess layer. This is also a mixed reinsurance situation if an excess of loss treaty protects the reinsurer's net position.

Commission Overrides: In most cases, the proportional facultative reinsurer pays a ceding commission to the ceding company. This ceding commission is meant to cover direct commission costs, plus an additional "override" commission to cover the cedent's non-commission costs. The override has the effect of reducing the net expense ratio, and can even cause a negative net commission expense in some cases. A company, or an individual underwriter, may cede large amounts of facultative proportional reinsurance to obtain this override relief to the commission expense ratio.

A Simple Example: The magnitude effect can be demonstrated by inspecting a very simple situation. Suppose a ceding company has a size of loss distribution that allows only claim sizes of either $10,000 or $90,000, with equal probability. With an expected claim frequency of 48 claims per year, and an average claim size of $50,000, we have annual expected losses of $2,400,000 annually. If the company carries an excess of loss treaty with a $40,000 retention, the treaty reinsurer will have expected losses of $1,200,000 per year (24 claims at $50,000 each). Assuming an 80% expected loss ratio for both companies, the excess of loss reinsurer will expect a treaty rate of 50% of subject premium.

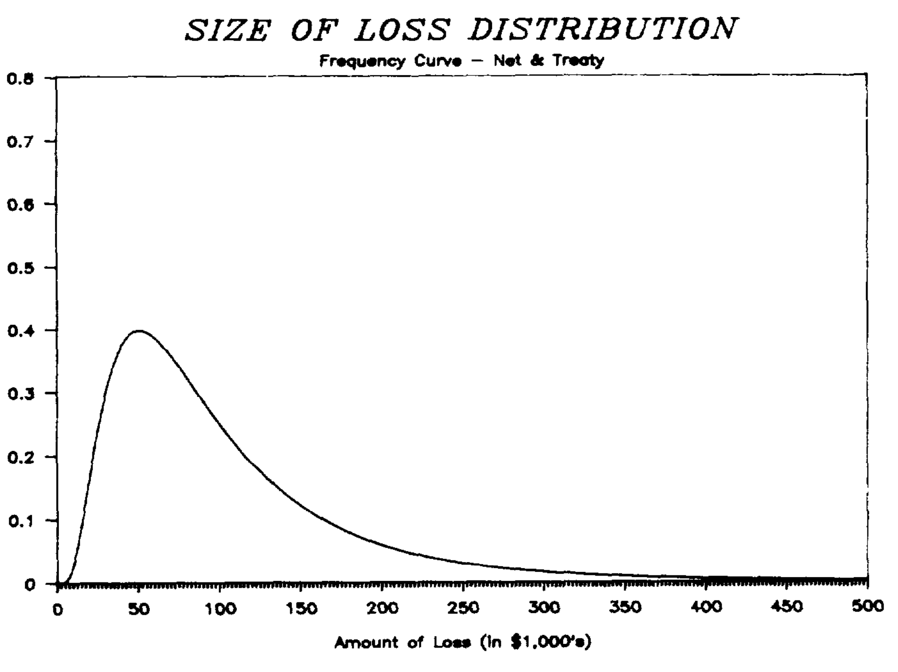
Now assume the underwriters writing this portfolio for the company place 50% quota share facultative reinsurance on every policy as they write it. The ceding company will retain 25% of gross premium, or $750,000, after paying for treaty and facultative reinsurance. The facultative reinsurer will pay half of every loss while the excess reinsurance only responds when the ceding company's 50% share of each loss penetrates the $40,000 retention. Since there are only 24 of these large losses expected, and after the proportional reinsurance they are $45,000 each, the excess reinsurer will have an expected incurred loss of $120,000. This will give it an expected loss ratio of 16% on the $750,000 of treaty premium. The ceding company will retain $1,080,000 of expected losses, for a loss ratio of 144% on its net retained premium of $750,000.

In this simplified example the two reinsurance negotiations have a combined unfavorable effect on the company. The treaty rate was correct for placement of 100% of each risk into the treaty. Because the underwriters did not tailor the facultative cessions to coordinate with the treaty rating, the company has suffered a penalty of 64 loss ratio points. Even though the direct business was correctly priced and evaluated, the net result is a totally unacceptable combined ratio. While the example is constructed to illustrate a point, actual variations on this situation can easily occur. In fact, every instance of an excess of loss reinsurance contract placed over proportional reinsurance works to the disadvantage of the net position, and thus the ceding company.

THE ROLE OF THE SIZE OF LOSS DISTRIBUTION

An inspection of a typical size of loss distribution indicates the underlying cause of mixing effects. Consider a size of loss frequency distribution of the amount of a single claim, as shown in Figure 1. The amount of loss can be read from the horizontal scale, and the relative frequency of such a loss amount

FIGURE 1

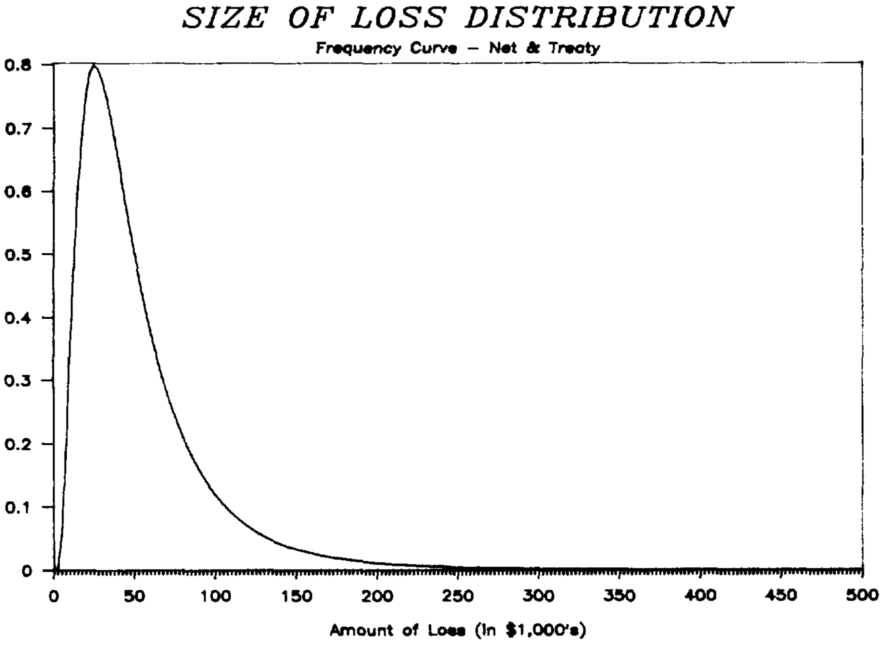


from the vertical scale. Figure 1 can also be used to determine the percent of total claim counts due to claims in a given range of amounts. For instance, we can see that loss over \$150,000 will represent 20% of the claims arising from this particular loss distribution. This is because the area under the size of loss curve above \$150,000 represents 20% of the total area under the curve.

The application of a 50% quota share reinsurance to this size of loss distribution essentially "shrinks" the curve horizontally, while maintaining its relative "shape," as shown in Figure 2.

Now consider the area of the "tail" of this new distribution over \$150,000. This area represented 20% of the total number of claims of the original loss distribution of Figure 1. The tail area of the "shrunken" distribution (Figure 2) over \$150,000, however, accounts for only 3.4% of total claims counts—much less than half of the original gross loss size distribution.

FIGURE 2



Thus, after the proportional "shrinking," the excess reinsurer will receive 50% of the premium that would have been received before proportional reinsurance was placed, but will experience much less penetration of its coverage layer than would have been expected in a situation without proportional reinsurance. In fact, the frequency of loss for the excess reinsurer after the 50% proportional reinsurance will be 17% (3.4% / 20%) of its original excess frequency. As a result, the excess reinsurer's expected net loss ratio after proportional reinsurance is now substantially improved over the experience before the proportional transactions.

Of course, this is simply a consequence of the nonlinear nature of the size of loss distribution. It is another way of stating that for large loss activity, a loss double a given size is experienced much less than half of the time.

Note also that the area under the curve of Figure 2 beyond \$150,000 is the same as the area under the curve of Figure 1 beyond \$300,000 (\$150,000 / 50%). Thus the excess rate over \$150,000, after a 50% quota share placement, should be the same as the excess rate for a \$300,000 retention with no quota share, ignoring risk charge and expense components, and the effect of the upper limit on the excess layer.

In understanding the impact of proportional reinsurance on the net position and the excess reinsurer, the fundamental relationship is the simple idea illustrated above. An excess retention of M after a proportional reinsurance retention of $100a\%$, is equivalent to an excess retention of M/a without proportional reinsurance. This result is shown as the Mixing Price Rule below.

This relationship is key in understanding how mixed reinsurance destabilizes net results. It seems intuitive, and can be shown mathematically (see the Appendix), that net aggregate loss results will show more stability (i.e., a lower coefficient of variation) under a \$150,000 retention than under a \$300,000 retention. In general, if an entire portfolio is proportionally reinsured to retain $100a\%$ of the total risk, with an excess of loss treaty with retention M, the stability of the portfolio's results will be identical to that of the same portfolio without proportional reinsurance and an excess loss limit of M/a. This result is shown as the Mixing Stability Rule below.

It is worth noting that the application of proportional reinsurance *after* the application of an excess of loss treaty does not change the magnitude of stability of the net loss ratio position. Hence, the order of application of reinsurance is extremely important.

Some simple examples will be instructive, and show situations where a disadvantageous net position can result in the ordinary course of business through mixing of reinsurance. This will be especially apparent if we consider the process of underwriting a single risk.

LOSS RATIO MAGNITUDE EFFECTS

A Casualty Example: Suppose an insurer is operating under an excess of loss treaty with \$2,000,000 limits, excess of a retention of \$250,000. The premium for this cover will be 30% of the subject premium that remains available for net and treaty, i.e., remaining after facultative placements.

The primary company underwriter writes an excess liability policy with limits of \$1,000,000, excess of a self-insured retention of \$100,000. He prices this at \$400,000, expecting a loss ratio of 60%. He pays a commission of 15%, and his internal expenses will account for another 10% of the gross premium. This leaves him with 15% (\$60,000) for profit and contingency load on this risk. This allows a 25% load on expected losses as a fluctuation margin. That is, the underwriter could suffer losses of up to \$300,000, or 125% of expected losses, before he has to dip into his surplus funds.

Next, he wishes to reduce his net and treaty exposure to this risk, so he arranges a facultative quota share placement of 50% of the risk. Thus, he is left with a \$500,000 exposure, net and treaty, and a subject premium for purposes of the excess treaty of \$200,000.

Generally, the cedent will receive a ceding commission that will cover his direct ceding commission costs (15% in this example), plus an "override" that is meant to cover the cedent's non-commission, or fixed, expenses. The override for this example will be 10%, which is identical to the ceding reinsurer's other expense ratio.

One can analyze the underwriter's net position before his facultative quota share placement. Assume that a lognormal distribution is an adequate model (Benckert [1]) for size of loss on this risk, with a mean claim size of \$30,000 and a coefficient of variation (CV) of 5.0. The following analysis of direct, reinsurance, and net results is summarized in Exhibit 1, the Mixing Cost Worksheet for this risk. Calculations on this exhibit are discussed below.

The size of loss assumption implies an average first-dollar claim severity of \$270,190 in the layer of interest, hence, an excess policy claim severity of \$170,190. Recall that this is the expected severity for all claims greater than

EXHIBIT 1

MIXING COST WORKSHEET

Policy: a casualty example without mixing

Input parameters:

Direct premium		$400,000
Policy limits		$1,000,000
Underlying retention		$100,000
Expected loss ratio		60.0%
Commission ratio		15.0%
Other expense ratio		10.0%
Reinsurance:		
Percent proportional		0.0%
Ceding commission		25.0%
Excess retention		$250,000
Excess limits		$2,000,000
Excess rate		30.0%
Ceding commission		0.0%
Loss distribution:	mean	$30,000
Lognormal	CV	5

Net results:

	Gross	Proportional	Excess	Net
Loss ratio	60.0%	NA	71.0%	55.3%
Expense ratio	25.0	NA	5.0	35.7
Combined ratio	85.0%	NA	76.0%	91.0%
Net underwriting profit				$25,144
Cost of Reinsurance:				
with mixing	$0	$0	$34,856	$34,856
Pure excess	0	0	34,856	34,856
Additional cost of reinsurance	$0	$0	$0	$0
Cost of Mixing Calculation:				
Actual cost of excess reinsurance			$34,856	
Cost based on subject premium			34,856	
Cost of mixing			$0	

$100,000, but with a maximum ceding carrier liability of $1,000,000 on those claims that are greater than $1,100,000 first-dollar. Expected losses of $240,000, (60% × $400,000) imply an expected claim frequency of 1.41 claims per annum on this risk for the excess carrier ($240,000/$170,190). This analysis is displayed on Exhibit 1.1.

Now the excess of loss reinsurer would assume all loss amounts over $350,000 first-dollar, up to a maximum policy limit loss of $1,100,000 first-dollar. Thus the excess of loss reinsurer will be providing the coverage for the layer from $350,000 first-dollar to $1,100,000 first-dollar for its $120,000 premium. Since 582 losses out of 10,000 exceed $100,000 first-dollar, and 118 losses out of 10,000 exceed $350,000 first-dollar, the excess of loss reinsurer's frequency will be 20% (118/582) of the direct reinsurer's frequency. Then, the reinsurer should expect 0.286 claims (1.41 × 20.3%) at an average severity of about $298,000 in the layer from $350,000 to $1,100,000 first-dollar. This implies a pure premium (expected losses) of about $85,000 (0.286 claims at $298,113 each), and an expected loss ratio of 71% for the excess of loss reinsurer. This analysis of the excess carrier's frequency and severity is displayed on Exhibit 1.3.

The primary company underwriter retains an expected loss cost of $155,000 and a net premium of $280,000, for an expected loss ratio of 55%. This would leave $25,000 for profit and contingency load on the net position, giving a 16% loading of expected losses for a fluctuation margin.

Thus, the primary company has paid 30% of its direct premium to the excess reinsurer. In return, its maximum exposure to loss from any one claim has been reduced from $1,000,000 to $250,000. The margin in the premium that is available to absorb fluctuations in results, however, has also decreased from 25% to 16%. In light of this reduction in the fluctuation loading, it is not immediately obvious whether the insurer is in a better position in terms of protection from random variation of results after this excess reinsurance transaction. As will be demonstrated below, however, excess of loss reinsurance decreases the probability of large aggregate losses to such a significant extent that this 16% risk margin actually reflects more safety than the gross position with its 25% margin.

On Exhibit 1 we have also calculated the cost of reinsurance. Of course, this is the *expected* cost of the reinsurance transaction. The actual cost in retrospect will vary considerably from year to year. The cost of reinsurance is

EXHIBIT 1.1

MIXING COST WORKSHEET
Casualty Example
Allocation of Layer Costs &
Determination of Net Position

Policy Parameters:	(a) Gross	(b) Proportional	(c) Excess	(d) Net
1. Premium	$400,000	$0	$120,000	$280,000
2. Commission	60,000	0	0	60,000
3. Other expenses	40,000	0	6,000	40,000
4. Expected losses	240,000	0	85,144	154,856
5. Profit/risk charge	60,000	0	28,856	
6. Retention	$100,000	NA	$250,000	$100,000
7. First-$ equivalent*	100,000	NA	350,000	100,000
8. Nominal layer width	1,000,000	0	2,000,000	250,000
9. First-$ equivalent*	1,100,000	NA	1,100,000	350,000
10. Effective layer width	1,000,000	0	750,000	250,000
11. First-$ equivalent*	1,100,000	NA	1,100,000	350,000
12. Claim severity	$170,192	$0	$298,113	$109,814
13. Claim frequency	1.410	1.410	0.286	1.410
14. Commission ratio	15.0%	25.0%	0.0%	21.4%
15. Other expense ratio	10.0%	3.0%	5.0%	14.3%
16. Premium rate	100.0%	0.0%	30.0%	70.0%
17. Fluctuation loading	25.0%	NA	33.9%	16.2%
18. Expected loss ratio	60.0%	NA	71.0%	55.3%
19. Combined ratio	85.0%	NA	76.0%	91.0%
20. Cost of reinsurance	$0	$0	$34,856	$34,856

* First-dollar equivalent is the amount of first dollar loss needed to hit this limit.

EXHIBIT 1.2

LOSS DISTRIBUTION TABLE

	Loss Amount x	Number Distribution $f\#(x)$	Amount Distribution $f\$(x)$
Primary retention	\$100,000	0.9417370	0.4069118
Reinsured's retention	350,000	0.9881997	0.6767204
Primary policy limit	1,100,000	0.9981221	0.8627949
Effective excess limit	1,100,000	0.9981221	0.8627949

Distribution type: lognormal

Distribution parameters:

mean = \$30,000 $\mu = 8.6799043$

CV = 5 $\sigma = 1.8050198$

EXHIBIT 1.3

DERIVATION OF LOSS CHARACTERISTICS FOR EXCESS TREATY

	(a) Amounts	(b) $f\#(x)$	(c) $f\$(x)$
1. Primary frequency	1.410		
First dollar equivalents:			
2. Primary retention	\$100,000	0.94173699	0.4069118
3. Primary policy limit	\$1,100,000	0.99812207	0.8627949
4. Reinsured's retention	\$350,000	0.98819966	0.6767204
5. Effective reinsurer limit	\$1,100,000	0.99812207	0.8627949
6. Ratio of excess carrier's frequency to primary frequency {1.0 − (4b)} / {1.0 − (2b)}	20.3%		
7. Excess layer frequency Expected claims per policy term (6) × (1)	0.286		
Severity calculations:			
8. Mean loss (SOL)	\$30,000		
9. Layer loss cost {(5c) − (4c)} × (8)	\$5,582		
10. Limit loss cost (5a) × {1 − (5b)}	\$2,066		
11. Number of layer losses (5b) − (4b)	0.992%		
12. Number of limit losses 1.0 − (5b)	0.188%		
13. Average severity of reinsured losses {(9) + (10)} / {(11) + (12)}	\$648,113		
14. Less: effective retention	\$350,000		
15. Excess layer severity (13) − (14)	\$298,113		
16. Percent pro rata reinsurance	0.0%		
17. Excess reinsurer's severity (15) × {1 − (16)}	\$298,113		

simply defined as the reinsurance premium paid, less the sum of ceding commissions received and expected reinsurance recoveries. Note that since reinsurance is a service that provides value to the cedent, we should expect a positive cost of reinsurance to be the hallmark of any long term reinsurance relationship. This definition of cost of reinsurance ignores investment income lost by the ceding carrier. This component may be required, however, to get realistic cost estimates.

The cost of excess reinsurance in this case is $34,856, which can be expressed as a cost of $87.14 per $1,000 of premium subject to the excess treaty.

The Effect of a Proportional Cession: Now consider the net position of the ceding underwriter after a 50% proportional reinsurance transaction on this policy. As shown in Exhibits 2-2.3, $200,000 net and treaty premium remains, of which $60,000 must go to the excess of loss reinsurer. Since all losses are 50% shared before application of this excess of loss treaty, a first-dollar loss of at least $600,000 is needed before the excess of loss reinsurance responds. Since such a loss occurs for only 52 claims out of every 10,000, the excess of loss reinsurer's frequency has been cut to 9% of the reinsured's frequency by use of the proportional reinsurance (Exhibit 2.3).

The average severity of losses greater than $600,000 limited at $1,100,000 is $900,586. These losses are 50% quota shared above $100,000, so the pro rata reinsurer and the reinsured evenly split the layer $500,000 excess of $100,000. The pro rata reinsurer and the excess reinsurer split the next $500,000 loss layer evenly. This leaves the excess of loss reinsurer with an average claim severity of $150,293 in its layer. With a claim frequency of 0.126 claims in the excess reinsurance layer, the excess reinsurer has an expected loss cost of only about $19,000. The reinsurer, however, has received $60,000 of premium for the excess reinsurance, so it has now improved its expected loss ratio position to 31.4%.

Who pays for this improvement of the excess reinsurer's loss ratio? Consider the proportional reinsurer's position. For 50% of the premium, the proportional reinsurer shares in all the gross losses equally. Thus, the expected losses of the proportional reinsurer are $120,000. This indicates an expected loss ratio of 60% for the pro rata reinsurer, the same as the gross loss ratio. In fact, the expected loss ratio of the quota share reinsurer will always be identical to that of the gross position.

EXHIBIT 2

MIXING COST WORKSHEET

Policy: a casualty example with mixing

Input parameters:

Direct premium		$400,000
Policy limits		$1,000,000
Underlying retention		$100,000
Expected loss ratio		60.0%
Commission ratio		15.0%
Other expense ratio		10.0%
Reinsurance:		
Percent proportional		50.0%
Ceding commission		25.0%
Excess retention		$250,000
Excess limits		$2,000,000
Excess rate		30.0%
Ceding commission		0.0%
Loss distribution:	mean	$30,000
Lognormal	CV	5

Net results:

	Gross	Proportional	Excess	Net
Loss ratio	60.0%	60.0%	31.5%	72.2%
Expense ratio	25.0	28.0	5.0	35.7
Combined ratio	85.0%	88.0%	36.5%	107.9%
Net underwriting profit				($11,081)
Cost of Reinsurance:				
with mixing	$0	$30,000	$41,081	$71,081
Pure excess	0	0	34,856	34,856
Additional cost of reinsurance	$0	$30,000	$6,225	$36,225
Cost of Mixing Calculation:				
Actual cost of excess reinsurance			$41,081	
Cost based on subject premium			17,428	
Cost of mixing			$23,653	

EXHIBIT 2.1

MIXING COST WORKSHEET

Casualty Example
Allocation of Layer Costs &
Determination of Net Position

Policy Parameters:	(a) Gross	(b) Proportional	(c) Excess	(d) Net
1. Premium	$400,000	$200,000	$60,000	$140,000
2. Commission	60,000	50,000	0	10,000
3. Other expenses	40,000	6,000	3,000	40,000
4. Expected losses	240,000	120,000	18,919	101,081
5. Profit/risk charge	60,000	24,000	38,081	(11,081)
6. Retention	$100,000	NA	$250,000	$100,000
7. First-$ equivalent*	100,000	NA	600,000	100,000
8. Nominal layer width	1,000,000	500,000	2,000,000	250,000
9. First-$ equivalent*	1,100,000	NA	1,100,000	350,000
10. Effective layer width	1,000,000	500,000	1,000,000	250,000
11. First-$ equivalent*	1,100,000	NA	1,100,000	350,000
12. Claim severity	$170,192	$85,096	$150,293	$71,680
13. Claim frequency	1.410	1.410	0.126	1.410
14. Commission ratio	15.0%	25.0%	0.0%	7.1%
15. Other expense ratio	10.0%	3.0%	5.0%	28.6%
16. Premium rate	100.0%	50.0%	30.0%	35.0%
17. Fluctuation loading	25.0%	20.0%	201.3%	−11.0%
18. Expected loss ratio	60.0%	60.0%	31.5%	72.2%
19. Combined ratio	85.0%	88.0%	36.5%	107.9%
20. Cost of reinsurance	$0	$30,000	$41,081	$71,081

* First-dollar equivalent is the amount of first dollar loss needed to hit this limit.

EXHIBIT 2.2

LOSS DISTRIBUTION TABLE

	Loss Amount x	Number Distribution $f\#(x)$	Amount Distribution $f\$(x)$
Primary retention	\$100,000	0.9417370	0.4069118
Reinsured's retention	600,000	0.9947991	0.7755223
Primary policy limit	1,100,000	0.9981221	0.8627949
Effective excess limit	1,100,000	0.9981221	0.8627949

Distribution type: lognormal

Distribution parameters:

mean =	\$30,000	$\mu = 8.6799043$
CV =	5	$\sigma = 1.8050198$

EXHIBIT 2.3

DERIVATION OF LOSS CHARACTERISTICS FOR EXCESS TREATY

	(a) Amounts	(b) $f\#(x)$	(c) $f\$(x)$
1. Primary frequency	1.410		
First dollar equivalents:			
2. Primary retention	\$100,000	0.94173699	0.4069118
3. Primary policy limit	\$1,100,000	0.99812207	0.8627949
4. Reinsured's retention	\$600,000	0.99479906	0.7755222
5. Effective reinsurer limit	\$1,100,000	0.99812207	0.8627949
6. Ratio of excess carrier's frequency to primary frequency {1.0 − (4b)} / {1.0 − (2b)}	8.9%		
7. Excess layer frequency Expected claims per policy term (6) × (1)	0.126		
Severity calculations:			
8. Mean loss (SOL)	\$30,000		
9. Layer loss cost {(5c) − (4c)} × (8)	\$2,618		
10. Limit loss cost (5a) × {1 − (5b)}	\$2,066		
11. Number of layer losses (5b) − (4b)	0.332%		
12. Number of limit losses 1.0 − (5b)	0.188%		
13. Average severity of reinsured losses {(9) + (10)} / {(11) + (12)}	\$900,586		
14. Less: effective retention	\$600,000		
15. Excess layer severity (13) − (14)	\$300,586		
16. Percent pro rata reinsurance	50.0%		
17. Excess reinsurer's severity (15) × {1 − (16)}	\$150,293		

Consider the net loss ratio, which was 60% gross and 55% net before any facultative placement. Of the total expected loss costs of \$240,000, the proportional reinsurer takes \$120,000 and the excess reinsurer assumes \$19,000. This leaves \$101,000 of expected losses for the reinsured's net position. Since \$140,000 of premium remains net, the expected net loss ratio is now 72%. This is substantially worse (17 loss ratio points) than the net loss ratio without any facultative proportional reinsurance. In addition, there is now no premium margin available for profit and contingency loading, since we are now at a combined ratio of 108%. Thus we see that use of proportional reinsurance below an excess of loss treaty simply moves loss dollars out of the excess reinsurer's account into the ceding insurer's account, without affecting the proportional reinsurer.

The Cost of Mixing: Notice that on Exhibit 2 we have calculated the Cost of Mixing. Recall that in the absence of any proportional reinsurance we calculated a cost of reinsurance of \$87.14 per \$1,000 of subject premium for the excess treaty. If we regard this cost as the reinsurer's price for providing an excess cover for this policy, we will hold this cost constant for any fraction of the policy that is retained after proportional reinsurance. This rate on the \$200,000 of subject premium implies a reinsurance cost of \$17,428 should be expected. In this mixed case, however, the actual cost for the excess reinsurance is \$41,081. We define the *Cost of Mixing* to be the difference of \$23,653. Note that this Cost of Mixing is greater than the underwriting loss on the policy of \$11,081. This implies that without the Cost of Mixing, this net position would have been profitable for the ceding company. The total cost of reinsurance in the mixed situation can also be decomposed as follows:

Cost of proportional reinsurance	\$30,000
Cost of excess reinsurance	17,428
Cost of mixing	23,653
Cost of total reinsurance	\$71,081

This example demonstrates a general principle that is independent of the choice of the size of loss distribution or policy parameters. A corollary of the Mixing Price Rule is that the net position after mixed reinsurance will always be worse than under a pure excess reinsurance. This rule states that the excess loss rate for an excess retention of M after a proportional retention of $100a\%$ must equal the loss rate for a pure excess retention of M/a.

The progressive deterioration of the loss ratio and combined ratio as the percent of proportional reinsurance increases can be seen in the table below. This table is for the casualty risk analyzed above, which has a gross expected loss ratio of 60%, with a gross combined ratio of 85%.

Percent Ceded	Net Loss Ratio	Expense Ratio	Combined Ratio
0%	55.3%	35.7%	91.0%
10	58.0	35.7	93.7
20	61.0	35.7	96.7
30	64.3	35.7	100.0
40	68.0	35.7	103.7
50	72.2	35.7	107.9
60	77.0	35.7	112.7
70	82.6	35.7	118.3
75	85.7	35.7	121.4
80	85.7	35.7	121.4
90	85.7	35.7	121.4

As the percent proportional ceded increases, losses are reduced for the excess reinsurer. These costs are shifted to the ceding company, and result in the increasing net loss ratio. Note that in the pure excess case, the loss ratio is reduced from 60% gross, to 55.3% net. The excess reinsurer, however, pays no ceding commission. This increases the expense ratio, and hence the net combined ratio.

When 75% of the risk is proportionally reinsured, no losses can penetrate the excess retention. This is simply because policy limits are $1,000,000, and the 25% of each loss retained net and treaty can never be greater then the $250,000 excess treaty retention. At this point, ceding larger shares of a risk no longer affects the net loss ratio.

THE MIXING PRICE RULE

The mean value of a random variable representing the size of claim after application of proportional reinsurance and excess of loss reinsurance can be expressed analytically. This allows the calculation of the loss cost portion of the excess reinsurance rate. The risk charge and expense load components of the reinsurance rate are ignored for the purposes of this demonstration.

Let $f(x)$ be the probability density function of X, the random variable representing the amount of one claim. We will assume $f(x)$ is appropriately truncated to reflect the policy limit issued by the ceding carrier. Let a be the fraction of each loss retained by the ceding insurer after proportional reinsurance, and M the retention under the excess reinsurance program. (This notation is identical to that used in Centeno [2].)

Then, if X is the gross claim size, the amount of claim after both reinsurances apply is given by

$$X(a,M) = \text{Min}\ (aX,M).$$

First, we establish the expected value of X under each single reinsurance type alone.

If only excess reinsurance applies,

$$E(\min(X,M)) = \int_0^M xf(x)dx + M \int_M^\infty f(x)dx.$$

If only proportional reinsurance applies,

$$E(aX) = a \int_0^\infty xf(x)dx.$$

It will also be useful to have an explicit formulation of the probability density of claim size subject to a proportional reinsurance. Let g_a be the density of x subject to proportional reinsurance that retains $100a\%$ of each claim.

Then $g_a(x) = 1/a\ f(x/a)$ will yield the expected value above. (Note: This is a probability density function since

$$\int g_a(x)dx = (1/a) \int f(x/a)dx.$$

Let $y = ax$; then $dy = adx$. Now we can substitute to obtain:

$$\begin{aligned} \int g_a(x)dx &= (1/a) \int f(y)ady \\ &= \int f(y)dy = 1.) \end{aligned}$$

Then applying excess of loss reinsurance to a claim after proportional reinsurance yields an expected value of

$$E(\min(aX,M)) = \int_0^M xg_a(x)dx + M \int_M^\infty g_a(x)dx.$$

Again set $ay = x$, so that $dx = ady$ and $x = M$ if and only if $y = M/a$. Rewrite these integrals in terms of the variable y.

$$
\begin{aligned}
E(\min(aX,M)) &= \int_0^{M/a} (ay)(1/a)\, f(y)ady + M \int_{M/a}^{\infty} (1/a)f(y)ady \\
&= a \int_0^{M/a} yf(y)dy + M \int_{M/a}^{\infty} f(y)dy \\
&= a[\int_0^{M/a} yf(y)dy + (M/a) \int_{M/a}^{\infty} f(y)dy] \\
&= aE(min(X,M/a))
\end{aligned}
$$

This means that the expected net value of the amount of a single loss subject to the combination of proportional reinsurance that retains $100a\%$ of each claim, and excess reinsurance that retains the first M amount of each claim, is equivalent to $100a\%$ of the expected value under an excess of loss reinsurance that retains that first M/a amount of each gross claim. This is a specific instance of the more general Mixing Moment Principle demonstrated below when we discuss stability.

Excess treaty premiums are usually calculated using a rate as a percent of subject premium.

Let *Rate* $XS(a,M)$ represent the excess rate for an excess retention M after a proportional retention of $100a\%$.

For purposes of simplifying the demonstration, recall that $f(x)$ reflects underlying primary policy limits and assume that the excess treaty limit extends above the primary policy limits. This allows us to ignore the truncation term due to the excess layer limit.

If we consider only the loss component of the excess premium rate, before any proportional reinsurance, the excess loss rate for limits of L over a retention of M will be

$$
Rate\ XS(1,M) = \frac{\int_M^{L+M} (x - M)\, f(x)dx + (L + M) \int_{L+M}^{\infty} f(x)dx}{Subject\ Premium},
$$

in the most general case.

This simplifies to $Rate\ XS(1,M) = \dfrac{\int_M^{\infty} (x - M)\, f(x)dx}{Subject\ Premium}$,

because of our assumptions.

After proportional reinsurance that retains $100a\%$ of each claim, let *Rate XS*(a,M) represent the rate. Then $100a\%$ of the prior subject premium is now subject premium for the excess treaty, and

$$Rate\ XS(a,M) = \frac{a[\int_{M/a}^{\infty} (x - M/a)f(x)dx]}{a(Subject\ Premium)}$$

$$= \frac{\int_{M/a}^{\infty} (x - M/a)f(x)dx}{Subject\ Premium} = Rate\ XS(1,M/a).$$

Thus, we can state the following:

Mixing Price Rule: The excess reinsurance loss rate for a retention M under a proportional reinsurance that retains $100a\%$ of each loss is identical to the excess loss rate over a retention of M/a, with no proportional reinsurance.

Note one simple implication of the Mixing Price Rule. The limited mean of a distribution F under limit M is given by

$$E_M(x) = \int_0^M x\ dF + M(1 - F(M))$$

and is the "complement" of the excess loss cost $\int_M^{\infty} (x - M)dF$.

Then the excess reinsurance loss rate under a mixed reinsurance case must be smaller than under pure excess if and only if the limited mean of the distribution limited at M/a is larger than the limited mean at M. Thus we have the following:

Mixing Loss Ratio Rule: If the limited mean of a loss distribution is a strictly increasing function of the limit, then the net loss ratio will always deteriorate under a mixed reinsurance case.

Only a most unusual loss distribution does not have the property of increasing limited means. Consider the following:

If $M_1 < M_2$ then

$$\begin{aligned}\int_{M_1}^{\infty} (x - M_1)dF &= \int_{M_1}^{M_2} (x - M_1)dF + \int_{M_2}^{\infty} (x - M_1)dF \\ &= \int_{M_1}^{M_2} (x - M_1)dF + \int_{M_2}^{\infty} (M_2 - M_1)dF + \\ &\qquad \int_{M_2}^{\infty} (x - M_2)dF \\ &> \int_{M_2}^{\infty} (x - M_2)dF,\end{aligned}$$

unless $\int_{M_1}^{M_2} (x - M)dF + \int_{M_2}^{\infty} (M_2 - M_1)dF = 0.$

The above sum of integrals is zero only if $dF = 0$ for $x \geq M_1$.

Thus if $M_1 < M_2$, then $\int_{M_1}^{\infty} (x - M_1)dF > \int_{M_2}^{\infty} (x - M_2)dF$; hence $E_{M1} \leq E_{M2}$ with equality only if $dF=0$ for $x \geq M_1$. In practice, equality will occur only when $f(x)$, the density associated with F, is truncated by policy limits.

We can write the full excess reinsurance rate as follows including the risk charge, $RC(a,M)$, and treaty expenses, Exp:

$$Rate\ XS(a,M) = \frac{a\int_{M/a}^{\infty}(x - M/a)f(x)dx + RC(a,M) + Exp}{a(Subject\ Premium)}$$

Without further information about the form of the risk charge, little more can be said about the excess rate. Note that Bühlmann [3] has identified four premium calculation principles based on the form of the risk charge. These principles calculate the risk charge on the expected value, standard deviation or variance of losses, or utility theory. If the premium calculation principle used in the excess rate is stated, then explicit calculations of equivalent excess rates in terms of the limit M/a are possible.

APPLICATIONS TO PROPERTY INSURANCE

The phenomenon described in the casualty example is due to the shape of the size of loss distribution. The same deterioration of net loss ratio due to mixed reinsurance situations will occur in property situations, if the underlying size of loss distributions follow any of the accepted probability models. A study of this subject done by Shpilberg [4] indicates that a loss distribution that falls between the lognormal and Pareto distributions in its tail behavior is an adequate model for fire insurance. The Mixing Price Rule discussion shows that if the limited mean is an increasing function of the limit M, any mixture of proportional and excess of loss reinsurance worsens the net loss ratio.

As we have seen, the limited mean condition is not very restrictive. Any reasonable choice of size of loss distribution, in particular the Pareto or lognormal, will satisfy this condition. Thus, the adverse consequences of mixing reinsurance will also hold for property risks.

There are, however, special characteristics of property risks that are notable. The policy limits of a property policy may be extremely large if there is a high Probable Maximum Loss level. The traditional approach to reducing this loss exposure to a level appropriate for an excess reinsurance treaty is the use of proportional reinsurance. Hence, a very high percentage of policy limits may be ceded before excess reinsurance.

Thus, property risks are a particularly fertile ground for finding examples of mixed reinsurance situations. The use of facultative reinsurance on large property risks is traditional and necessary to cut large policy limits down to net and treaty positions appropriate for the insurer's treaty capacity. This usage can have a substantial impact on the net loss ratio.

A property example will show net effects of proportional reinsurance similar to the casualty example already considered above.

Suppose the insurer has an excess of loss property treaty with $2,000,000 limits over a retention of $250,000, for this example. If a property risk requiring policy limits of $20 million is written, the underwriter must place $18 million of facultative reinsurance before he can place the remaining risk into his treaty. Most facultative property reinsurance has traditionally been on a proportional basis, resulting in a 90% cession to the facultative reinsurers.

If the gross premium for the risk is $500,000, we will cede $450,000 to the facultative reinsurers and retain $50,000 net as shown in Exhibit 3-3.4.

The results of the reinsurance can be quite different based on the type of property risk being underwritten. The differences we can attempt to model will be reflected in the Probable Maximum Loss (PML) potential, which should be closely related to the underlying size of loss distribution. The policy limits should also be based on the PML potential. For instance, if the risk consists of a single large warehouse, there is a potential probability of losing the entire insured value. For the purposes of this discussion we will model this by choosing a size of loss distribution with 1 chance in 10,000 of a $20,000,000 loss. A lognormal distribution with a mean of $67,500 and a coefficient of variation of 10 is used. The net expected loss ratio in this case is shown in Exhibit 3 as 74%, with a combined ratio of 110%.

As expected, this net position compares unfavorably to the gross position with an 85% combined ratio. Note that this example demonstrates a capacity problem, where facultative reinsurance *must* be used before the treaty can come into use. The use of excess of loss facultative reinsurance in place of proportional may improve these net positions, if such reinsurance is available at an appropriate price. If not, the only recourse to the underwriter is to price the gross risk appropriately to achieve his target 95% net combined ratio. A premium of $610,000 for this risk would be required to achieve a 95% combined ratio under this mixing situation with 90% proportional reinsurance. This would require pricing to a gross loss ratio of 49% and a gross combined ratio of 74% for the property. It is unlikely that the marketplace will allow such pricing.

EXHIBIT 3

MIXING COST WORKSHEET

Policy: a property example

Input parameters:

Direct premium		$500,000
Policy limits		$20,000,000
Underlying retention		$0
Expected loss ratio		60.0%
Commission ratio		15.0%
Other expense ratio		10.0%
Reinsurance:		
Percent proportional		90.0%
Ceding commission		25.0%
Excess retention		$250,000
Excess limits		$2,000,000
Excess rate		30.0%
Ceding commission		0.0%
Loss distribution:	mean	$67,500
Lognormal	CV	10

Net results:

	Gross	Proportional	Excess	Net
Loss ratio	60.0%	60.0%	27.8%	73.8%
Expense ratio	25.0	28.0	5.0	35.7
Combined ratio	85.0%	88.0%	32.8%	109.5%
Net underwriting profit				($3,336)
Cost of Reinsurance:				
with mixing	$0	$67,500	$10,836	$78,336
Pure excess	0	0	47,155	47,155
Additional cost of reinsurance	$0	$67,500	($36,319)	$31,181
Cost of Mixing Calculation:				
Actual cost of excess reinsurance			$10,836	
Cost based on subject premium			4,715	
Cost of mixing			$6,121	

EXHIBIT 3.1

MIXING COST WORKSHEET
Property Example
Allocation of Layer Costs &
Determination of Net Position

Policy Parameters:	(a) Gross	(b) Proportional	(c) Excess	(d) Net
1. Premium	$500,000	$450,000	$15,000	$35,000
2. Commission	75,000	112,500	0	(37,500)
3. Other expenses	50,000	13,500	750	50,000
4. Expected losses	300,000	270,000	4,164	25,836
5. Profit/risk charge	75,000	54,000	10,086	(3,336)
6. Retention	$0	NA	$250,000	$0
7. First-$ equivalent*	0	NA	2,500,000	0
8. Nominal layer width	20,000,000	18,000,000	2,000,000	250,000
9. First-$ equivalent*	20,000,000	NA	20,000,000	250,000
10. Effective layer width	20,000,000	18,000,000	20,000,000	250,000
11. First-$ equivalent*	20,000,000	NA	20,000,000	250,000
12. Claim severity	$65,577	$59,019	$310,572	$5,648
13. Claim frequency	4.575	4.575	0.013	4.575
14. Commission ratio	15.0%	25.0%	0.0%	−107.1%
15. Other expense ratio	10.0%	3.0%	5.0%	142.9%
16. Premium rate	100.0%	90.0%	30.0%	7.0%
17. Fluctuation loading	25.0%	20.0%	242.2%	−12.9%
18. Expected loss ratio	60.0%	60.0%	27.8%	73.8%
19. Combined ratio	85.0%	88.0%	32.8%	109.5%
20. Cost of reinsurance	$0	$67,500	$10,836	$78,336

* First-dollar equivalent is the amount of first dollar loss needed to hit this limit.

EXHIBIT 3.2

LOSS DISTRIBUTION TABLE

	Loss Amount x	Number Distribution $f\#(x)$	Amount Distribution $f\$(x)$
Primary retention	\$0	0.0000000	0.0000000
Reinsured's retention	2,500,000	0.9970693	0.7281287
Primary policy limit	20,000,000	0.9999017	0.9423854
Effective excess limit	20,000,000	0.9999017	0.9423854

Distribution type: lognormal

Distribution parameters:

mean=	\$67,500	$\mu = 8.8123226$
CV=	10	$\sigma = 2.1482831$

EXHIBIT 3.3

DERIVATION OF LOSS CHARACTERISTICS FOR EXCESS TREATY

	(a) Amounts	(b) $f\#(x)$	(c) $f\$(x)$
1. Primary frequency	4.575		
First dollar equivalents:			
2. Primary retention	\$0	0	0
3. Primary policy limit	\$20,000,000	0.99990169	0.9423854
4. Reinsured's retention	\$2,500,000	0.99706933	0.7281287
5. Effective reinsurer limit	\$20,000,000	0.99990169	0.9423854
6. Ratio of excess carrier's frequency to primary frequency {1.0 − (4b)} / {1.0 − (2b)}	0.3%		
7. Excess layer frequency Expected claims per policy term (6) × (1)	0.013		
Severity calculations:			
8. Mean loss (SOL)	\$67,500		
9. Layer loss cost {(5c) − (4c)} × (8)	\$14,462		
10. Limit loss cost (5a) × {1 − (5b)}	\$1,966		
11. Number of layer losses (5b) − (4b)	0.283%		
12. Number of limit losses 1.0 − (5b)	0.010%		
13. Average severity of reinsured losses {(9) + (10)} / {(11) + (12)}	\$5,605,719		
14. Less: effective retention	\$2,500,000		
15. Excess layer severity (13) − (14)	\$3,105,719		
16. Percent pro rata reinsurance	90.0%		
17. Excess reinsurer's severity (15) × {1 − (16)}	\$310,572		

EXHIBIT 3.4

DERIVATION OF LOSS CHARACTERISTICS FOR PRIMARY POLICY

	(a) Amounts	(b) $f\#(x)$	(c) $f\$(x)$
1. Expected losses	$300,000		
First dollar equivalents:			
2. Primary retention	$0	0	0
3. Primary policy limit	$20,000,000	0.99990169	0.9423854
Severity calculations			
4. Mean loss (SOL)	$67,500		
5. Layer loss cost {(3c) − (2c)} × (4)	$63,611		
6. Limit loss cost (3a) × {1 − (3b)}	$1,966		
7. Number of layer losses (3b) − (2b)	99.990%		
8. Number of limit losses 1.0 − (3b)	0.010%		
9. Average severity of primary losses {(5) + (6)} / {(7) + (8)}	$65,577		
10. Less: retention	$0		
11. Primary policy severity (9) − (10)	$65,577		
12. Primary policy frequency Expected claims per policy term (1) / (11)	4.575		

Note one very important implication of this example. We can no longer assume the underwriter can price this risk on the basis of gross frequency and severity characteristics alone. In order to achieve combined ratio results that allow long-run survival of the ceding insurer, the gross price must be set based on gross frequency and severity, the excess reinsurance rate, the amount of proportional reinsurance needed for capacity, and the ceding commission structures.

The excess reinsurance rate must also anticipate some use of facultative reinsurance for capacity purposes. Specifically, for property risks the excess rate must be calculated anticipating a certain amount of use of proportional reinsurance. This will be the case if a loss rating approach using past experience is used to calculate the excess rate, and this past period reflects a similar use of proportional reinsurance as anticipated for the next treaty year.

OTHER MAGNITUDE EFFECT CONSIDERATIONS

The net results of the casualty and property examples are not only a function of the percentage of proportional reinsurance used. Both the excess reinsurance rate and the ceding commission structure have an effect on the final net position. A detailed treatment of these subjects is not possible here, but some issues that relate to the magnitude effect will be mentioned.

The Excess Reinsurance Rate: In the casualty example, an excess treaty was specified with a $2,000,000 limit over a $250,000 retention. Depending on the underlying size of loss distribution one might assume that a "correct" excess loss rate could simply be calculated from the distribution statistics. However, the policy subject to the excess reinsurance could be any one of the following.

A primary policy with policy limits of $2,250,000 that uses the entire reinsurance layer of $2,000,000.

If the primary policy limits are only $1,000,000 the rate should be substantially different.

If the $1,000,000 policy limits are excess of a self insured retention of $100,000, the appropriate rate for the excess reinsurance would also be different.

If the ceding company writes an excess policy for $1,000,000 limits over a primary policy with $500,000 limits, the correct excess reinsurance rate is again different from any of the above.

One can immediately see that with no change in the underlying risk's loss potential (as characterized by its size of loss distribution), several different but "correct" excess reinsurance rates are possible. It becomes apparent that one cannot speak of a proper excess reinsurance rate on a portfolio without some measure of the anticipated underlying distributions of retentions and policy limits in the portfolio. Thus, the excess reinsurance rate must be formulated in anticipation of a certain portfolio structure.

This point has practical implications that generate mixing situations. Suppose an excess reinsurance program has been negotiated, with the parameters agreed to for two years forward. At the time of the negotiation, management of the ceding carrier fully intended to write a book of small surplus lines SMP risks. An excess and surplus lines carrier is usually very responsive to market opportunities; hence, six months into the program, management modifies its original marketing plan because conditions are excellent for obtaining strong rates on small casualty umbrellas. Management wants to take advantage of this opportunity. The original excess reinsurance rate, however, contemplated the SMP book and carried a provisional rate of 10%. The same calculations based on a book of small umbrella business would yield a proper rate of 35% for the excess reinsurance.

An excess reinsurance program can easily have 10 to 20 participants and have taken months of effort to place. Renegotiating the treaty at every shift in portfolio composition is not a realistic option. Furthermore, the excess and surplus lines market depends heavily on the reinsurance market for capacity. Many such companies may cede out 50% or more of their gross writings. Thus, including this umbrella book in the treaty at an inadequate excess rate is not a viable option for management concerned about maintaining a long term presence in the market with consistent reinsurer support.

As a practical matter, the ceding underwriter has little real choice but to attempt to "protect the treaty." As we have seen, the ceding underwriter has great control over his treaty loss ratio, through his use of proportional facultative reinsurance. By altering the percent of proportional reinsurance placed on a risk, the size of loss characteristics of the net position can be fit into the treaty rate structure.

Consider the casualty example given above to be representative of a typical umbrella policy. At a 10% rate, the excess reinsurer would receive $40,000 of premium and would have an expected loss ratio of 210% ($85,114 / $40,000), if no proportional reinsurance were placed. After the 50% proportional cession, however, the excess reinsurer would receive \$20,000 of premium at the 10% rate. With expected losses of \$18,853, this would yield an expected loss ratio of 94%, much better than the original 210%. Under the original scenario presented for the casualty example, the placement of 50% proportional reinsurance was not warranted. Under this new scenario, however, the 50% proportional reinsurance should clearly be placed before the identical policy is placed into the excess treaty. The cost of mixing in this case should be paid to the excess reinsurer to bolster an inadequate treaty rate for a risk not contemplated in the original treaty price.

Thus, the situation is manageable but becoming exceedingly complex. The underwriter must ascertain a correct price for the risk insured on a gross basis. This is no different from any underwriting situation. In addition, we again see that an essential part of the direct company's underwriting and pricing process must be the correct placement of reinsurance to achieve an acceptable net result. Even this, however, is not enough. The underwriter must also balance out his net position against the results he is passing on to the excess reinsurer. He must be able to maintain long-term acceptable results for his excess reinsurance support, in the face of continuing shifts in his portfolio composition due to market conditions.

The calculations we have made in our examples are complex and assume knowledge of the size of loss distribution underlying the policy. This is clearly an area where actuarial expertise can be applied to produce general guidelines and specific pricing procedures that aid in determining the net underwriting position. Without such pricing analysis available, management will have no effective way of controlling and evaluating the proper, coordinated use of proportional and excess reinsurance.

The Gearing Factor: The existence of the override in the ceding commission has been remarked on above. The purpose of the override is to reimburse the ceding company for the non-commission expenses it incurred in writing the direct business. Unfortunately, in times of excessive reinsurance capacity the override is used as a competitive tool by reinsurers. Thus, the casualty example considered above may be entitled to a 10% override based on the expense structure of the ceding carrier; however, a particularly aggressive reinsurer may offer an override of 15%. This, of course, makes the determination of the net position even less straightforward, and offers a powerful incentive to cede larger proportional reinsurance amounts.

The excessive override will tend to improve the combined ratio while the mixing effect will act to worsen the combined ratio. Hence, it becomes even more imperative to calculate the net position before a risk is bound and facultative arrangements settled. For instance, the 50% proportional reinsurance on the casualty risk with a 15% override would yield the same net loss ratio of 72.2%, but an improved net combined ratio of 100.8%. The effect on the property example with 90% ceded proportional reinsurance is even more leveraged, with a net loss ratio of 73.8%, but a net combined ratio of 45.2%, much improved from the original 110%.

The combined effect of an excessive override and a large percent of proportional ceded reinsurance may not only cancel out the mixing penalty, but also produce a favorable net combined ratio even when the direct risk is severely underpriced. For example, if the property risk example of Exhibit 3 were priced at a 100% gross loss ratio, the premium would be $300,000. Net retention after a 90% proportional reinsurance cession only would be $30,000 of written premium and expected losses. Expenses before ceding commission total 25% of gross premium, or $75,000. The ceding commission at a 15% override would total 30% of the $270,000 ceded premium, or $81,000. Thus, after the proportional cession the insurer would have net premium income of $30,000 and net costs as follows:

Net incurred losses:	$30,000
Direct expenses:	75,000
Ceding commission:	(81,000)
Net incurred costs	$24,000

This is equivalent to a combined ratio of 80%, a substantial improvement over the direct combined ratio of 125% at which the risk was written direct. This aspect of the override in proportional reinsurance has been termed the "Gearing Factor" by Buchanan [5]. The existence of the gearing factor effect can overwhelm the unfavorable mixing effects in the transaction.

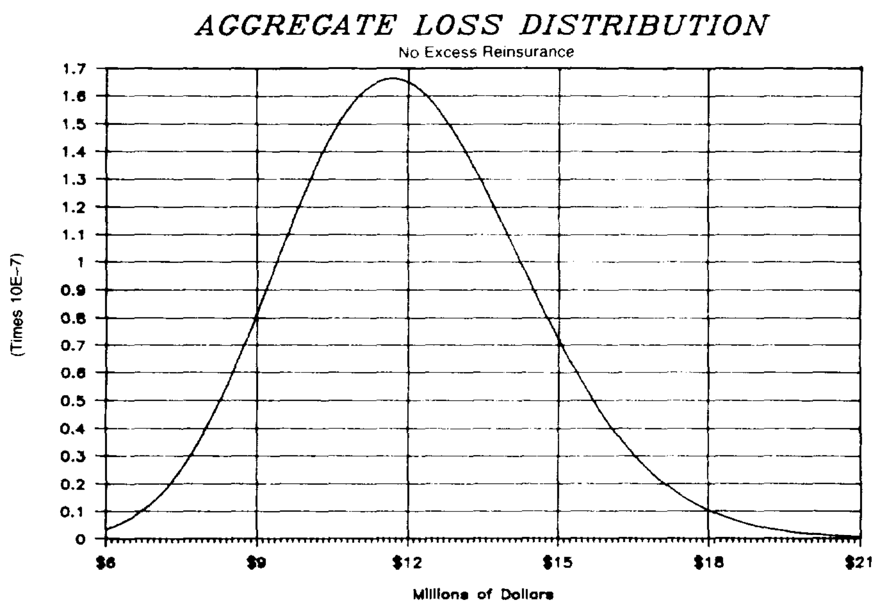
STABILITY EFFECTS

One of the less obvious effects of mixing proportional and excess of loss reinsurance types is the effect on the variation of the net loss ratio after reinsurance. The use of proportional reinsurance below an excess of loss treaty actually makes the resulting net aggregate loss costs more variable than would be the case under the excess treaty alone. This is significant because stability of net results is one of the most important benefits resulting from an excess reinsurance treaty. Any degradation of the stability "component" of the excess treaty "product" makes the treaty worth less.

We will use the casualty policy example to form a small portfolio that will allow us to investigate the impact on stability of mixing reinsurance. Assume we have a portfolio of 50 policies identical to the casualty example. Therefore, we have a book of excess casualty business that generates $20 million of gross premium and an average of 70.5 claims annually (50 × 1.410). These claims follow the lognormal size of loss distribution specified earlier, i.e. with a mean of $30,000 and a CV of 5.0. The expected loss ratios on this book of business are identical to those on the single policy—that is, 60% gross, 55% if only the excess treaty is applied but 72% in the mixed reinsurance case.

The aggregate loss distribution differs in the case of the portfolio and the single policy. As a simple demonstration, there is a substantial probability (24%) that the single policy will be loss-free. It is effectively impossible, however,

FIGURE 3



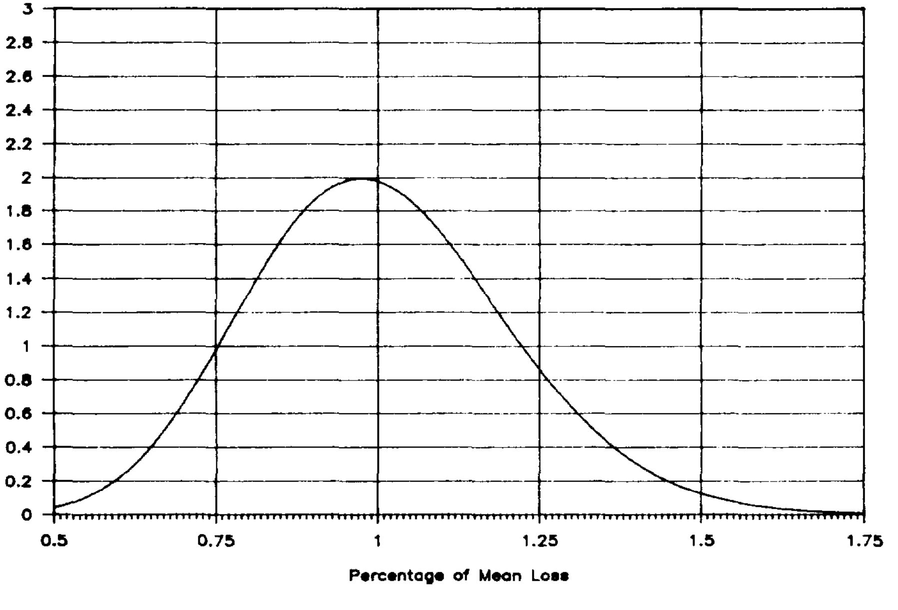
for the entire portfolio to be loss-free in any year (a probability of 2.4×10^{-31} of a loss-free year). The expected annual claim cost of the portfolio is \$12,000,000 (70.5 claims at \$170,200 each) and the aggregate losses of the portfolio are distributed as shown in Figure 3. All computations of aggregate loss distributions were made using the algorithm developed by Heckman and Meyers [6].

In order to make comparisons between aggregate loss distributions, we will normalize such distributions, by setting the mean aggregate loss to 100%, and present the probabilities of achieving various percentages of the mean. This maintains the relative shape of the distribution and facilitates the comparison of different distributions with various underlying aggregate loss means. The normalized aggregate distribution of the unreinsured portfolio above can be seen as Figure 4. This distribution has a coefficient of variation of 0.2.

FIGURE 4

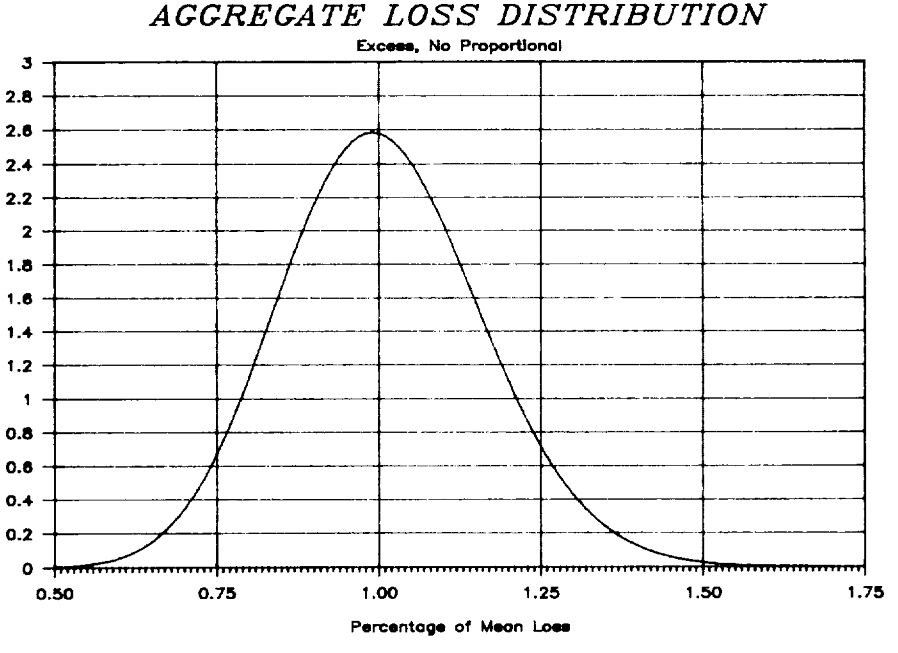
AGGREGATE LOSS DISTRIBUTION

No Excess Reinsurance



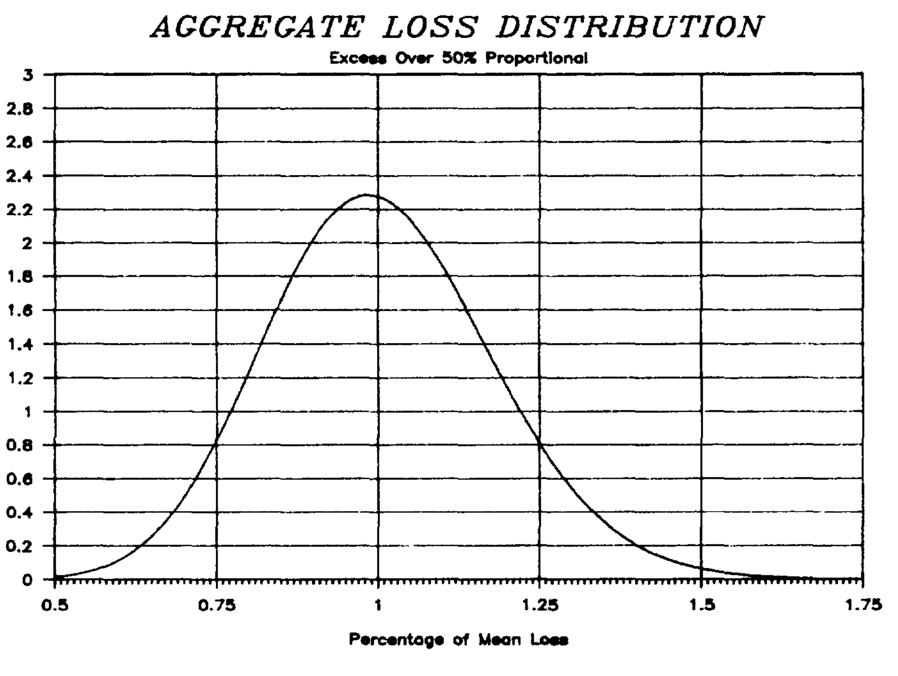
After placement of the excess treaty on this portfolio, the spread of the distribution is much reduced, as can be seen from Figure 5 below. Note that the probability of losses totalling over 150% of expected is substantially reduced by use of excess reinsurance, and the entire curve is distributed closer around its mean of 1.0. The coefficient of variation after excess reinsurance is reduced to 0.155.

FIGURE 5



Now, if the 50% proportional reinsurance is placed on each of the 50 policies in the portfolio, we obtain the aggregate loss distribution shown as Figure 6. This distribution clearly lies between the unlimited case and the pure excess case in its dispersion of possible loss amounts. Note the larger area under the curve over 150% of mean loss, for example, than under the pure excess treaty. The coefficient of variation has also increased to 0.175.

FIGURE 6



Since all aggregate distributions are normalized, they can be compared on the same scale as shown in Figure 7. This chart shows that the "spread" of possible results around the mean loss in the mixed case lies between the unlimited and pure net of excess distribution. In this sense, the stability paid for by purchase of excess reinsurance is "undone" by application of the proportional reinsurance.

Regarding the stability of the portfolio, we are most interested in the behavior of the aggregate loss distribution at the extreme right-hand tail. As shown in

FIGURE 7

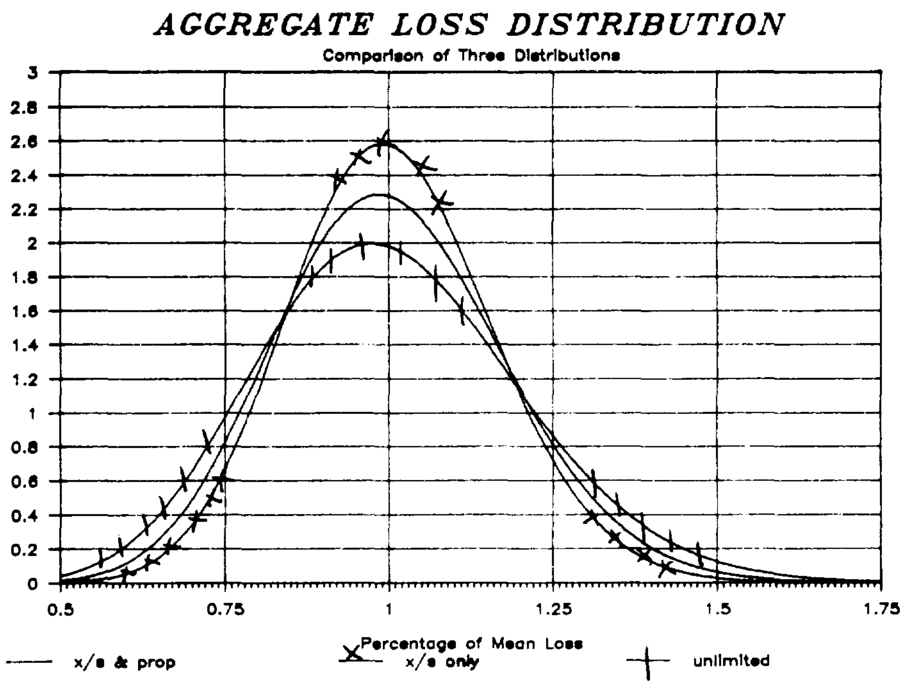
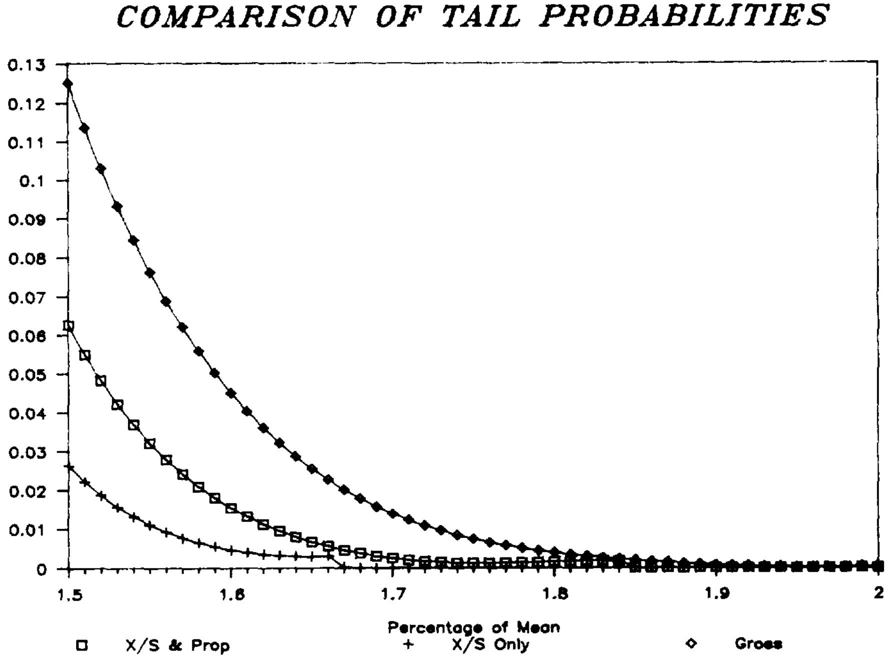


Figure 8, the tail behavior of the aggregate loss distribution in the mixed reinsurance case is substantially more severe than the pure excess treaty case.

The problem, of course, is that we are paying the same 30% rate of net and treaty premium for excess reinsurance protection in both the mixed reinsurance and pure excess cases. As Figure 8 shows, the protection from extreme fluctuations we receive for our 30% rate is substantially less in the mixed case.

While the normalized aggregate distributions are useful for comparing aggregate loss distributions with disparate means, it is also important to focus on

FIGURE 8



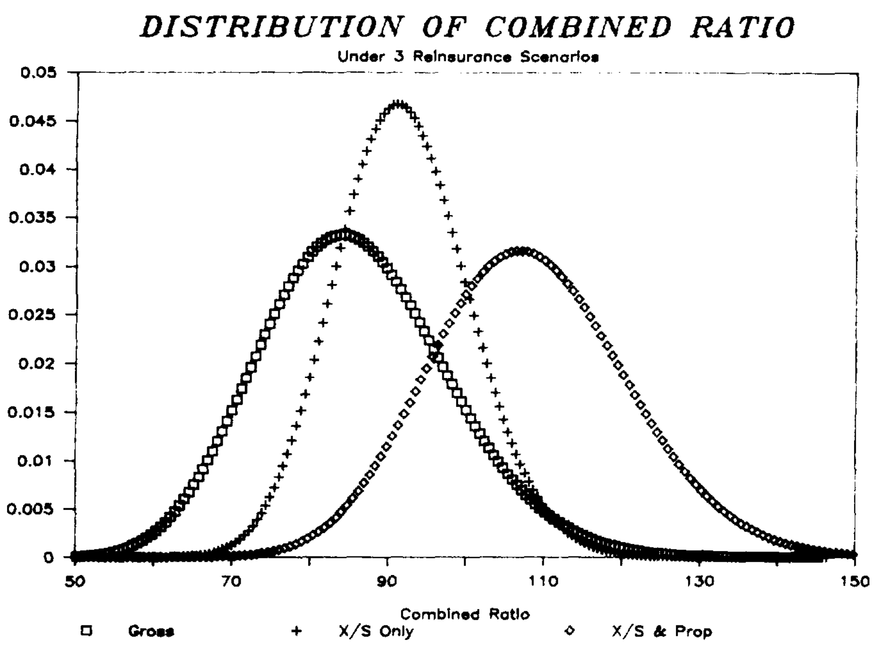
the bottom line—the distribution of combined ratios under the three different scenarios. The combined ratio becomes a random variable through the equation:

Combined Ratio = Expected Loss Ratio × Normalized Aggregate Loss Ratio + Expense Ratio.

Figure 9 shows the distribution of combined ratios for the three scenarios. Clearly, the range of alternatives under the mixed reinsurance scenario is the least desirable, not only in terms of its expected value, but also in terms of the probability of experiencing extremely adverse combined ratios. Note that there is little or no chance of a combined ratio over 120% in the case of the gross or pure excess case. The mixed case, however, leaves us exposed to a substantial probability that a combined ratio over 120% will be experienced.

Even the combined ratio comparison does not take the absolute scale into account. Dollar magnitudes are important, however, if we are to gauge the

FIGURE 9



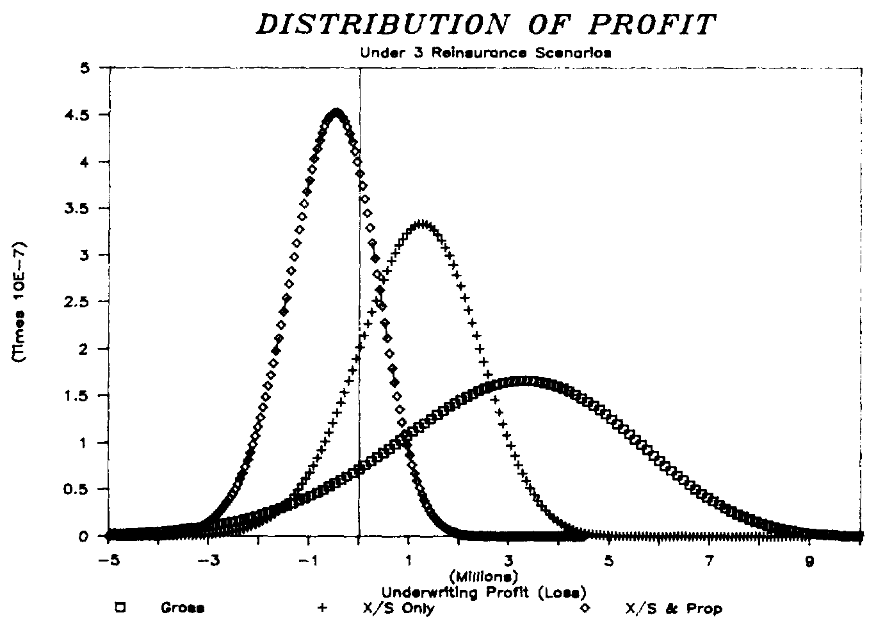
impact of the reinsurance programs on company surplus. An additional way of evaluating the bottom line is to simply review the distribution of statutory underwriting profit or loss. Profit can be represented as a random variable by:

Profit = Premium − Aggregate Losses − Expenses

where Aggregate Losses is the random variable we have been examining above, but not normalized. The resulting distribution is shown in Figure 10.

This chart is clearly of interest in evaluating ruin probabilities. Note that the gross loss distribution has a non-negligible probability of suffering an underwriting loss of over $4 million. The pure excess reinsurance makes a loss of over $3 million unlikely, and even the mixed case reduces the chance of suffering a $4 million underwriting loss significantly. The price that must be paid for this protection in the mixed case, however, is an expected underwriting loss. Thus the mixed case is clearly inferior to pure excess reinsurance in terms of both magnitude and stability of net underwriting results.

FIGURE 10



A usable table representing the tail probabilities for the three scenarios is presented below.

TAIL PROBABILITIES
Probabilities of Exceeding the Percent of Mean

		Type of Reinsurance	
Percent of Mean	Gross	Excess Over Proportional	Excess Only
125%	11.07%	8.15%	5.77%
130%	7.45	4.93	3.09
135%	4.85	2.84	1.55
140%	3.06	1.56	0.73
145%	1.87	0.82	0.32
150%	1.11	0.41	0.14
151%	1.00	0.36	0.11
152%	0.89	0.31	0.09
153%	0.80	0.27	0.08
154%	0.72	0.23	0.07
155%	0.64	0.20	0.05
Mean aggregate loss	$12,000,000	$5,054,050	$ 7,742,800
Net premium	20,000,000	7,000,000	14,000,000
Expenses	5,000,000	2,500,000	5,000,000
Expected U/W profit	$ 3,000,000	$ (554,050)	$ 1,257,200

Using this table it is possible to investigate alternate scenarios, using proportional only or excess of loss only, to achieve a desired risk level with net incurred loss. For instance, suppose that the 50% proportional reinsurance were placed in order to keep the probability of an extra $3,000,000 loss at about 1% or less. From the middle column, there is about a 1% probability of a loss over 142% of mean aggregate loss in the mixed reinsurance case. This corresponds to $2.1 million dollars of loss over the expected amount of $5,054,050. Taking expenses into account, about a 1% chance of suffering an underwriting loss of

$2.7 million is implied. Note that in order to achieve this protection, the company will have an expected underwriting loss of about $500,000.

Is there a more rewarding way to achieve the same risk position? There are at least two other reinsurance configurations that appear preferable. For instance, on a gross basis, there is a 1% probability of suffering a loss of $18,000,000 or higher. This is equivalent to a 1% chance of an underwriting loss of $3,000,000 or more. A 10% cession of this portfolio would reduce the 1% level of loss to $2.7 million, leaving an expected underwriting profit of $2.7 million. Even though the 90% proportional retention tail does not diminish as fast as the mixed case, the 1% level of risk is the same and expected profit is $3.2 million more.

Similarly, the 1% expected loss level for the excess of loss portfolio is 138% of the mean, or an underwriting loss of $1.7 million. Thus, the 1% loss level is much lower than the mixed reinsurance case, and the expected underwriting profit of $1.3 million is much higher than the mixed case.

To summarize, at the 1% probability of loss level we have inspected three alternatives, and the mixed case is the least desirable.

	90% Quota Share	$250,000 Excess Over 50% Proportional	$250,000 Excess Only
1% level of U/W loss	($2,700,000)	($2,700,000)	($1,700,000)
Expected profit	$2,700,000	($554,050)	$1,257,200

The simple calculations above hint at the complexity of the optimal reinsurance problem. Surprisingly, actuaries have studied this complex question extensively. See, for instance, Beard, Pentikainen, and Pesonen [7] for a bibliography. Three related results of interest are given:

1. For a fixed amount of reinsurance premium and ignoring risk loadings, aggregate stop loss is the optimum reinsurance to minimize the variance of net results [8].
2. With a risk load that increases with variance, proportional (quota-share) reinsurance is optimal to minimize the reinsurance cost for a given variance level [9].

Finally,

3. Allowing mixed reinsurance treaties and constraints on both mean and variance, in most cases pure excess of loss reinsurance is optimal to minimize the skewness of net aggregate losses [10].

THE MIXING STABILITY RULE

In a mixed reinsurance situation, a decrease in the amount retained after proportional reinsurance will decrease the stability of the net aggregate losses. In this sense proportional reinsurance will negate the major benefit of excess reinsurance.

As a measure of stability we will use the coefficient of variation of net aggregate loss results. Recall that if X is a random variable, we define

$$\mathrm{CV}\ (X) = \frac{\text{Standard Deviation } (X)}{\text{Mean } (X)}$$

Let X be the random variable representing the amount of one claim, and N be the random variable representing the number of claims in the experience period. Let M be amount retained under an excess of loss treaty, and $100a\%$ be the percent retained under proportional reinsurance.

Let $X(a,M) = \min(aX,M)$ represent the net amount of one claim under both reinsurances. This is the random variable of claim amount under the mixed reinsurance situation.

Let λ_k be the kth moment of N, the number of losses, and β_k the kth moment of X, the amount of loss. Then for any compound process Y defined by

$$Y = \sum_{i=1}^{N} X_1 i,$$

we know that

$$E(Y) = \lambda_1\beta_1 \text{ and,}$$

$$\mathrm{Var}\ (Y) = \lambda_1\ \mathrm{Var}\ (X) + \mathrm{Var}\ (N)\ \beta_1^2 \text{ (see Miccolis [11]).}$$

Thus,

$$\mathrm{Var}\ (Y) = \lambda_1\ (\beta_2 - \beta_1^2) + (\lambda_2 - \lambda_1^2)\ \beta_1^2$$

in terms of central moments.

And, in general,

$$\mathrm{CV}^2\,(Y) = \frac{\lambda_1\beta_2 + (\lambda_2-\lambda_1-\lambda_1^2)\,\beta_1^2}{(\lambda_1\beta_1)^2}\,,$$

which simplies to

$$\mathrm{CV}^2\,(Y) = \frac{\beta_2}{\lambda_1\beta_1^2} + \frac{\lambda_2-\lambda_1-\lambda_1^2}{\lambda_1^2}\,.$$

Both the mixing price and stability rules are essentially a result of the following relationship that holds for the kth central moment of $X(a,M)$, denoted by $\beta_k(a,M)$.

Mixing Moment Principle: $\beta_k(a,M) = a^k\beta_k(1,M/a)$
Proof: By definition,

$$\beta_k(a,M) = \textstyle\int_0^M x^k g_a(x)dx + M^k \int_M^\infty g_a(x)dx,$$

where $g_a(x) = (1/a)f(x/a)$ is the probability density of x under proportional reinsurance. If we set $ay = x$, then $ady = dx$, and $x = M$ if and only if $y = M/a$. Now rewrite β_k in terms of y,

$$\begin{aligned}\beta_k(a,M) &= \textstyle\int_0^{M/a}(ay)^k(1/a)f(y)ady + M^k \int_{M/a}^\infty(1/a)f(y)ady\\ &= a^k \textstyle\int_0^{M/a} y^k f(y)dy + M^k \int_{M/a}^\infty f(y)dy,\\ \beta_k(a,M) &= a^k[\textstyle\int_0^{M/a} y^k f(y)dy + (M/a)^k \int_{M/a}^\infty f(y)dy].\\ &= a^k\beta_k(1,M/a),\end{aligned}$$

which proves the result.

Following notation in Centeno [2], let $Y(a,M)$ represent net aggregate loss after application of both the proportional and excess reinsurance. Then

$$Y(a,M) = \textstyle\sum_{i=1}^N \min(aX_i,M).$$

We are interested in the stability of $Y(a,M)$ as a decreases. The following rule characterizes the stability of Y as a changes.

Mixing Stability Rule: The stability (coefficient of variation) of net aggregate losses after retention of $100a\%$ under proportional reinsurance and retention of M under an excess of loss treaty is equivalent to the stability of net aggregate losses under an excess treaty with a retention of M/a.

Proof: Write the coefficient of variation in terms of λ_i and $\beta_i(a,M)$,

$$\mathrm{CV}(Y(a,M)) = \frac{[\lambda_1\beta_2(a,M) + (\lambda_2 - \lambda_1 - \lambda_1^2)\ \beta_1(a,M)^2]^{1/2}}{\lambda_1\beta_1(a,M)}$$

$$= \frac{[\lambda_1 a^2\beta_2(1,M/a) + (\lambda_2 - \lambda_1 - \lambda_1^2)a^2\beta_1(1,M/a)^2]^{1/2}}{\lambda_1 a\beta_1(1,M/a)}$$

$$= \frac{[\lambda_1\beta_2(1,M/a) + (\lambda_2 - \lambda_1 - \lambda_1^2)\beta_1(1,M/a)^2]^{1/2}}{\lambda_1\beta_1(1,M/a)}$$

$$= \mathrm{CV}\ (Y(1,M/a)),$$

which proves the result.

We would suspect that the stability of net losses decreases as the retention of the excess of loss treaty increases. This is indeed the case, as shown in the Appendix. Thus, we can conclude that, in general, as the percent retained under proportional reinsurance decreases, and the excess of loss retention M remains fixed, the stabilty of net results of the portfolio decreases.

This shows that the situation of Figure 7 is not the result of any fortuitous choice of distributions or parameters. For any compound process, represented in general by $Y(a,M)$, the distribution of net results after mixed reinsurance will show more "spread" than the pure excess reinsurance case but less than the gross position.

CONCLUSION

The application of an excess of loss treaty after a proportional reinsurance transaction on a policy has been shown to have a significant adverse impact on the net expected loss ratio. In addition, the stability of net results sought from the excess of loss reinsurance is also adversely affected. The Mixing Price Rule and Mixing Stability Rule allow us to evaluate these effects of the mixing situation. The Cost of Mixing Worksheet allows us to calculate the net position in a mixed reinsurance situation. These three tools should allow the underwriter to make appropriate evaluations of pricing and facultative reinsurance decisions in individual risk situations.

From a broader management perspective, the mixing of reinsurance at the individual risk level presents a difficult management control issue. In a worst case scenario, if company underwriters were to make facultative reinsurance arrangements without proper coordination and direction from management, a substantial loss ratio penalty on the entire book of business could be expected. Extremely adverse fluctuations in net results would also be possible. The challenge for management is to establish guidelines and controls enabling underwriters to understand the structure and objectives of overall corporate reinsurance. The underwriters will then be able to make decisions on individual risk facultative reinsurance placements that work with, not against, the excess treaty. It is hoped that the ideas developed here will give actuaries a start in attempting to explore this aspect of the underwriting and pricing process.

Pricing a risk at a profitable direct premium is not sufficient to assure a net profit when significant amounts of different reinsurances apply. As our examples show, one can price the risk perfectly on a direct basis, yet still have an unfavorable net combined ratio, due to facultative placements with high mixing costs.

On a corporate level, the more subtle concept of probability of ruin comes into play. We have shown that unanticipated large amounts of proportional placements can destabilize net results significantly. While most insurance organizations are large enough to make the probability of ruin of academic interest only, the chance of suffering extremely large combined ratios increases as the share retained on a proportional basis decreases. The protection in the excess treaty is negated by proportional reinsurance.

Finally, most of the discussion has been from the viewpoint of the ceding company. The mixing cost, however, can work both ways. The excess treaty rate is calculated anticipating a certain percent of the book will be ceded proportionally before the treaty applies. If the ceding company finds that it can only cede a smaller than anticipated portion of its business facultatively, it will be putting larger shares of each risk into the treaty. This will result in a highly leveraged adverse loss ratio and destabilization effect on the excess treaty. This is a sensitive issue for both the excess reinsurer and the ceding company.

Pricing actuaries on both sides of the excess reinsurance treaty transaction have an interest in the mixing effects. The more use a ceding company makes of proportional reinsurance prior to the treaty, the more important the mixing effect becomes. An increased awareness of the effects of mixing should decrease the likelihood of unexpected adverse consequences to both treaty partners.

APPENDIX

Theorem: As the fraction a retained under proportional reinsurance decreases, the stability of the net aggregate losses decreases.

Proof: We wish to prove that as a decreases, the quantity $CV(Y(a,M))$ decreases. From the Mixing Stability Rule, it suffices to prove that if $M_1 < M_2$, then,

$$CV(Y(1,M_1)) < CV(Y(1,M_2)).$$

This is the case if

$$(\delta/\delta M)\ CV(Y(1,M)) > 0,$$

which is equivalent to

$$(\delta/\delta M)\ CV^2(Y(1,M)) > 0, \text{ because } CV \geq 0.$$

Let β_k represent $\beta_k(1,M)$; then

$$CV^2(Y(1,M)) = \frac{\lambda_1\beta_2 + (\lambda_2 - \lambda_1^2 - \lambda_1)\beta_1^2}{\lambda_1^2\beta_1^2}$$

$$= \frac{\beta_2}{\lambda_1\beta_1^2} + \frac{(\lambda_2 - \lambda_1^2 - \lambda_1)}{\lambda_1^2}.$$

Since only β_k is a function of M,

$$(\delta/\delta M)\ CV^2(Y(1,M)) = \frac{\lambda_1\beta_1^2\beta_2' - 2\beta_2\lambda_1\beta_1'\beta_1}{(\lambda_1\beta_1^2)^2}$$

$$= \frac{\beta_1\beta_2' - 2\beta_2\beta_1'}{\lambda_1\beta_1^3}.$$

Thus, $(\delta/\delta M)\ (CV^2(Y(1,M)) > 0$ if and only if

$$\beta_1\beta_2' - 2\beta_2\beta_1' > 0.$$

Now compute β_1' and β_2'.

$$(\delta/\delta M)\ \beta_1 = \delta/\delta M\ (\textstyle\int_0^M x\,dF + M(1 - F(M)))$$
$$= 1 - F(M), \text{ and}$$

$$(\delta/\delta M)\ \beta_2 = \delta/\delta M\ (\textstyle\int_0^M x^2\,dF + M^2(1 - F(M)))$$
$$= 2M(1 - F(M)).$$

$$\text{Let } I_1 = \int_0^M x\,dF \text{ and}$$
$$I_2 = \int_0^M x^2\,dF.$$

Then,
$$\beta_1\beta_2' = [I_1 + M(1 - F(M))]\,[2M(1 - F(M))], \text{ and}$$
$$2\beta_2\beta_1' = 2[I_2 + M^2(1 - F(M))]\,[1 - F(M)].$$

So,

$$\begin{aligned}\beta_1\beta_2' - 2\beta_2\beta_1' &= 2I_1M(1 - F(M)) - 2I_2(1 - F(M)) \\ &= 2(1 - F(M))\,(MI_1 - I_2) \\ &= 2(1 - F(M)) \int_0^M x(M - x)\,dF.\end{aligned}$$

Since $0 < x < M$, we know $M - x > 0$; hence, this integral is positive, and the result is proved.

(The author thanks professor Nasser Hadidi of the University of Wisconsin-Stout for his helpful discussions on this proof.)

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PRESIDENTIAL ADDRESS – NOVEMBER 11, 1986

YESTERDAY, TODAY AND TOMORROW

PHILLIP N. BEN-ZVI

One of the most common techniques which actuaries use is to examine past experience, evaluate current conditions, identify factors which are changing, and then use these to project the future. In my remarks today, I want to follow this same actuarial approach and first look back at history, then comment on the current situation, and finally make some predictions about the future.

All of us are familiar with the history of the Casualty Actuarial Society, which is summarized very well on the first page of our *Yearbook*. Our Society goes back to 1914; from that, one might assume that the first casualty actuaries probably began their practice a few years previously. The *Yearbook* then goes on to report that actuarial science originated in England in 1792, in the early days of life insurance. In preparing for this speech, I did some further research on actuarial history; and I am pleased to be able to tell you that not only is the actuarial profession far older than reported in the *Yearbook*, but the first actuary was a casualty actuary, not a life actuary. In fact, the actuarial profession goes back to Biblical times, and the name of the first actuary is a very well-known name – Joseph, that wild dresser with the technicolor coat.

I can see from the reactions of some of you that this is no surprise as there are clearly some Biblical scholars in this group. It is really all very obvious if you read the Bible carefully. If you recall, Joseph was a dreamer and an interpreter of dreams, in other words, clearly a practitioner of actuarial science. Furthermore, he was hated by his brothers – need I say more!

The Bible tells us quite a bit about Joseph's actuarial career. He was the first to identify the underwriting cycle, which at the time consisted of seven fat years followed by seven lean years. This led to his first assignment which was to price a new product, drought insurance. The premium was, of course, not paid in cash in those days, but was rather in the form of grain, and when his company put away reserves they did it literally – they put it into silos.

When pricing his product he built in an underwriting profit factor, and this allowed his company to have a consistently excellent total rate of return. He established his company's loss reserves on an ultimate basis. He did not believe in discounting to present value. A recently discovered papyrus contained the Annual Statement of his company and, believe it or not, it included an early version of Schedule P. And what a runoff it showed – the reserves were perfectly adequate at the end of the seven lean years of paid claims. Joseph maintained a solid balance sheet for his company and he had no hesitation in signing his statement of actuarial opinion. Perhaps the best proof of his abilities is that he was the first actuary to become president of his insurance company – in fact, of all of Egypt.

Of course, Joseph had some advantages. His company had no competitors and, therefore, as the actuary, he did not have to deal with marketing people. He also controlled the courts, and so, had no problems with attorneys. He had no difficulties with uncollectible reinsurance as "innocent capacity" had not yet been born. Finally, he had a tough bunch of claims adjusters working for him. In his time, pain and suffering was not something that inflated claims payments; rather, it was something that happened to the claimant if he tried to inflate the claim.

Several thousand years may have passed since Joseph's time and the world may have changed quite a bit, but the insurance industry hasn't really changed that much. We are still going through underwriting cycles and the latest one has had almost seven lean years. We have just come through a period of grossly inadequate prices and of clearly inadequate loss reserve levels. Various industry observers had estimated the reserve deficiency at anywhere from 10% to 20% or more, with much higher numbers for some lines of business. We have suffered through a period of horrendous underwriting results and extremely inadequate rates of return, culminating in a year in which the entire industry produced a net operating loss before adjusting for tax credits. We have had a record number of insolvencies of both primary and reinsurance companies, including some fairly sizeable ones.

It would be easy for us to sit here today feeling comfortable and enjoying improved industry results. We certainly see sharply improved commercial lines experience and there is no question that many companies have been strengthening their balance sheets. But many problems still remain, and many lines such as personal lines and workers' compensation have a long way to go before the results will be satisfactory. The uncertainty of the tort system still hangs over our heads as reforms thus far have been modest at best. Who is to say which substance will represent the next environmental pollution problem and what price the industry will have to pay to reimburse the injured parties? In personal lines, we need to deal with the impact of lower gas prices and greater automobile usage plus the proliferation of smaller, more damageable, and less protective automobiles. Reversing the recent trend towards a free market, regulatory pressures are becoming greater in reaction to the corrective actions taken by the insurance industry, and to the political pressures applied by those who oppose tort reform. As an industry, we also have to face an increased financial burden because of the new federal income tax law. At the same time, no one truly believes that loss reserves are yet adequate for the industry and we have certainly not seen the last of significant insurer insolvencies and their resulting impact on guarantee funds and uncollectible reinsurance. Indeed, the latter two are part of a vicious cycle which could in turn trigger even more insolvencies.

With this picture of the recent past and current conditions in mind, let's ask ourselves some questions. What has the track record of actuaries been during these lean years? How many of us, whether employed by companies, consulting firms, or regulators, can say that we made recommendations during these last few years that would have produced adequate prices, resulted in adequate reserves, avoided the poor returns that weakened many companies, and, in fact, avoided some of the insolvencies that occurred? Can all of the blame be laid at the doorstep of the top management of the insurance companies or the operating or marketing people in those firms? Are we really blameless as actuaries, or did our judgements and recommendations get colored by the events around us? Did our hearts often take over from our minds? Did some of us who are part of management confuse our management roles with our actuarial roles? In other words, are we comfortable that we have fulfilled our professional responsibilities during these lean years?

All of us in this room certainly recognize that actuarial work is a combination of art and science. Indeed, our entire educational process in the CAS reflects that reality. Our syllabus and our continuing education programs attempt to provide a knowledge of the needed mathematics and actuarial techiques, plus an overall understanding of all elements impacting the insurance business including policy coverages, underwriting, marketing, claims, regulation, and financial matters. That lengthy and continuing process of education is an attempt to provide us, as actuaries, with a broad view of every aspect of the business so that we can best make our actuarial judgements.

I feel relatively comfortable that we, as actuaries, know how to handle the science part of our responsibilities, difficult though it is. It has been the application of the actuarial art that has often been the source of our problems. Let's look at data as an example. Not only do we rarely have the right kind of data or enough of it to satisfy our needs, but it is in the interpretation of this information where the most professional judgement is required. Evaluating changes in the way in which business has been done or will be done is truly an actuarial art in that much of this is yet to be reflected in the available data. The values we place on the impact of underwriting actions that have been taken, marketing plans which may have changed the book of business, changes which may have been made by the claims department in adjusting or reserving cases, or even changes in processing policies, claims, or expenses are crucial. Finally, and most obviously, the judgements which we regularly make in selecting the most appropriate actuarial methodology and the most appropriate assumptions are a significant part of the actuarial art.

Our actuarial responsibility is, first and foremost, to provide the best estimate possible, ignoring any and all constraints. This may mean that actuarially indicated reserves may be much lower or much higher than those currently being carried by the company, and their use might present financial problems. It may mean that the indicated price is far higher or far lower than the market will bear. I do not in any way suggest that the best actuarial estimate should be made using a static approach. In fact, a very important part of the analysis is to help determine the optimal business strategy in a dynamic environment. In making the best estimate of the price which will produce the target rate of return, one has to consider the effects of the marketplace and the change in the mix of business that may result from a proposed set of rates.

A second but still very important actuarial responsibility is to determine the financial effects of strategies being considered by the employer or client. That may differ from the best estimate. The insurance business is a risk business and our role as actuaries is to evaluate those risks and provide that information to the client or employer. This will allow the management of the company to consider the risk versus reward tradeoffs in reaching a business decision.

We do no one any favors if we mingle real world constraints with our attempts to come up with a best estimate. If we do so, we quickly start to have our own judgements clouded and we begin to believe the resulting answers and think that they are indeed the best estimate. Similarly, our employers think that they are getting the best actuarial advice, when in fact they are not. Our role is to maintain our objectivity and our heads even when those around us may be losing theirs. Many actuaries have been gaining important managerial responsibilities. This could easily result in swelled heads, but even a swelled head cannot hold two hats at the same time. It is our actuarial responsibility to wear each of those hats, but only one at a time.

I think it is a safe prediction to say that we will soon have another opportunity to test ourselves on the proper exercise of actuarial responsibility. There will not be any gong that goes off to announce the beginning of the next cycle, but I think it is fair to assume that some elements will begin in the very near future. After all, cycles really begin when competitors perceive that some segments of business have reached, or will shortly reach, a point at which extraordinary profit opportunities are presented. In a business like ours, in which there are hundreds or even thousands of competitors, it requires only a small number of significant players to try to exploit those opportunities before we are off and running into a competitive period. This is actually a good and healthy part of our economic process. The danger begins, however, when the perceptions of potential profitability are wrong, and there is no appreciation of when the profits are disappearing as other competitors react in defense of their markets. Underwriting cycles develop segment by segment and heat up as more and more companies attack and defend their markets.

I would suggest that the actuary has a very vital role to play in this process and, in fact, can be the key to making this a healthy rather than a destructive process. First, the actuary must be involved in identifying the desirable segments and in quantifying the available profit margins. Next, he should establish a dynamic model that can provide insight into the most likely results as competitors react. His model must be able to quantify when the return becomes substandard and hence when opportunity turns into a problem. Finally, the actuary must regularly analyze the true results of the venture so that he can provide advice to management and allow them to take appropriate action based on sound financial input. The actuary's tools are becoming more sophisticated and actuaries are becoming more and more skilled. I strongly believe that if we carry out our actuarial responsibilities, we can help to minimize the amplitude of the next cycle.

It appears that one lesson the industry has learned from the last cycle is that there is a severe economic impact from operating in a business with so many competitors. When this is combined with the fact that insurance is a business which is perceived as having products which are commodities, and which has a substantial social content and a large degree of regulation, the results are almost inevitable. It produces an industry whose business is excessively competitive and one in which the competition is largely price-driven. It produces a business which is excessively cyclical; a business in which profits are always being squeezed and hence, tend to be inadequate even over a long period of time. And, finally, it produces a business in which there are more and more companies in weakened condition, leaving either a social problem or a burden to be borne by the remaining carriers.

What we have, therefore, been seeing and will continue to see is a movement to segment the marketplace and a tendency for companies to find niches over which they can have much greater control. There is also a movement to differentiate products – whether through changes in the insurance contract, through service, delivery mechanisms or any other approach which fertile minds can discover. Again, the identification of those profitable market segments is an actuarial challenge and responsibility; the development of the new products, and particularly their pricing, will require the greatest exercise of actuarial science and the most refined use of the actuarial art.

I am also convinced that we will be seeing more and more consolidation in the insurance industry. The industry simply cannot support so many carriers in the long run. Small and medium sized carriers will disappear at an accelerating rate and will be able to survive only if they turn themselves into insurance boutiques – that is, only if they offer something unique in the way of expertise, product, or service to their ultimate customers. Even the larger companies have discovered that they cannot be successful being all things to all people. Even those companies are trying to restructure themselves into conglomerations of specialty segments. Mergers and acquisitions will, therefore, increase greatly in the coming years and actuaries will be called upon to play an important role in those activities. It is clearly the actuary who is best able to assess the proper value of the companies involved. But actuaries will need to be much more knowledgeable in financial matters than we currently are. This is an area where our syllabus has traditionally been weak, though I am pleased to see that we have already begun to work on strengthening the financial content, and I believe we will see much more of this in the future. Presently, there are very few actuaries who serve as chief financial officers of their companies, but, in my opinion, actuaries have that unique combination of broad knowledge of the insurance business and quantitative skills which, if combined with an increase in financial expertise, make actuaries the ideal chief financial officers for insurance companies.

At the same time, our customers have become much more financially sophisticated. After years and even decades of modest inflation and low and stable interest rates, we saw a rapid acceleration in the late 1970s and the early 1980s, with inflation rates rising into the double digits and interest rates approaching 20%. No longer were people satisfied putting their savings into passbook accounts paying 4%. Consumers ran from bank to bank looking for higher yields on certificates of deposit, money market funds, tax deferred annuities, and all forms of high yielding securities or tax deferred investments. The so-called cash flow underwriting of the last cycle was not solely driven by individual companies' desires to increase their market shares and get the use of premium dollars in order to invest in high yield securities. It was also driven by the sophistication of the large commercial insured who became very conscious of the cost of money and wanted that to be reflected in his insurance arrangement. To some extent, even smaller commercial insureds and individuals became very conscious of the cost of money in the insurance process, as installment plans proliferated, often at little or no finance charge. During the last decade the genie escaped from the bottle, and make no mistake about it, we are never going to get the genie to go back into the bottle.

Even though interest rates have dropped dramatically and inflation has been reduced to a very modest and fairly stable level for the last couple of years, we can expect to see much more consciousness of the value of money. Not only will large commercial insureds be interested in cash flow type programs, but the same concepts will expand into the medium size account range. Insureds have also become more aware of alternatives available to them in managing their risk. Unless tax considerations make it undesirable to do so, more and more insureds will choose to retain or self insure the lower layers or relatively predictable levels of losses and will demand an unbundling of their insurance programs. Thus, companies will increasingly get into the sale of services with insurance provided only for the levels which the insured decides he cannot afford to retain. I believe the same will apply to the personal lines area, as individuals will increasingly be willing to self insure and raise average deductibles sharply. Finally, the products offered by our industry will cross traditional barriers with contracts beginning to encompass both the personal and commercial needs of the customer, property and casualty as well as life insurance needs, insurance plus other financial service needs, and all of this will become international in scope.

For actuaries, this process will have enormous implications. Our knowledge and our skills will have to expand rapidly to keep pace with these developments. In designing and pricing our products, we will no longer be able to assume relatively stable inflation or interest rates. We will have to develop the ability to assess the risks and determine the appropriate financial reward, explicitly dealing with this important variable. We have been through only one, relatively brief period of sharply changing inflation rates, and our track record of reserving and pricing in that environment was certainly very poor. Insurance companies in other countries deal with this problem constantly, and we will need to develop the sophisticated methodologies to separate this risk element and build it into our modeling approaches.

This changing environment also highlights the important task we face of learning to measure the true surplus needs of our business. For years we have lived by rules of thumb of premium to surplus relationships, as a rough measure of the capital requirements of our business. We have all known of the deficiencies in this approach and they have never been more obvious than in recent years. Whether explicitly or implicitly, all companies do, and will continue to do, business on a total return basis. That total return must be measured in relation to the capital requirements and then compared to an appropriate target return in order to retain and attract capital to our business. We can no longer live with simplistic measures of surplus or with relative surplus requirements for different lines of business. We need to develop and refine the theoretical means to determine absolute surplus needs and this must be done on a product by product basis. Clearly, the surplus requirements are quite different if an insurer is providing ground up coverage as compared to catastrophic layers only. If we fail to get our hands around this problem, we are not only doomed to go through further severe underwriting cycles, but also to see a large number of insurer insolvencies in the coming years.

In the long run, economic realities will break down most political and regulatory barriers. We are already seeing an increasing internationalization of our business and more is certain to come. Presently, North America generates the lion's share of the insurance business worldwide. While there will undoubtedly be future growth in our part of the world, the business here is relatively mature, but the growth opportunities in other parts of the world are simply enormous. This puts the CAS in a very interesting position. We remain the only actuarial organization in the world solely devoted to educating and accrediting actuaries specifically in the property and casualty insurance area. But almost all of our members, with only a literal handful of exceptions, reside in North America. Is actuarial science really nation-specific or is it merely some of the exam content and the language of our syllabus and exams that limit its scope? Shouldn't there be some way for the Casualty Actuarial Society to play a larger role in the education and development of casualty actuaries in other parts of the world? There are many problems to overcome in attempting this, not the least being political, but it is an area which I would recommend that our future leaders explore since we are in a unique position to aid in the development of the property and casualty insurance business throughout the world.

The insurance business has been through some tough times lately and more difficulties and challenges face us in the years ahead. If the business were simple, it would be no fun and certainly there would be little need for actuaries. However, just as the genie will never go back into the bottle, our business will never become simple again. In fact, quite the contrary. As the business becomes more complex and our customers more sophisticated, actuaries must do likewise. The ability to evaluate and quantify risk and the appropriate reward will differentiate the successful from the unsuccessful company. And actuaries will, therefore, play an increasingly key role in the business. I think that few, if any, of us have regretted our decisions to become actuaries. We are part of a fascinating, vital business and members of a vibrant and growing actuarial organization. We have come a long way since our first actuary, Joseph, and the future we face is bright and exciting.

MINUTES OF THE 1986 ANNUAL MEETING

November 9–11, 1986

OPRYLAND HOTEL, NASHVILLE, TENNESSEE

Sunday, November 9, 1986

The Board of Directors held their regular quarterly meeting from 12:00 p.m. to 4:00 p.m.

Registration was held from 4:00 p.m. to 6:30 p.m.

A presentation to the new Fellows and Associates on the workings of the Casualty Actuarial Society was held from 5:30 to 6:30 p.m. The Vice Presidents made short presentations concerning their areas of responsibility and the workings of the committees which report to each of them.

A general reception for all members and guests was held from 6:30 to 7:30 p.m.

Monday, November 10, 1986

Registration continued from 7:00 a.m. to 7:55 a.m.

President Phillip Ben-Zvi opened the meeting at 8:00 a.m. The first order of business was the admission of new members. Mr. Ben-Zvi recognized the 24 new Associates and presented diplomas to the 36 new Fellows. The names of these individuals follow.

FELLOWS

Amundson, Richard B.	Hollister, Jeanne M.	Littmann, Mark W.
Bailey, Victoria M.	Hosford, Mary T.	Livingston, Roy P.
Bellusci, David M.	Huyck, Brenda J.	Loper, Dennis J.
Chiang, Jeanne D.	Johnson, Andrew P.	Lyons, Daniel K.
Driedger, Karl H.	Kelley, Robert J.	Martin, Paul C.
Faltas, Bill	Klinker, Frederick L.	McClure, John W., Jr.
Forde, Claudia S.	Koupf, Gary I.	McDonald, Gary P.
Hankins, Susan E.	Krakowski, Israel	Menning, David L.

Myers, Thomas G.	Ruegg, Mark A.	Townsend, Christopher J.
Noyce, James W.	Silver, Melvin S.	Vitale, Lawrence A.
Potts, Cynthia M.	Terrill, Kathleen W.	Weinman, Stacy J.
Reppert, Daniel A.		

ASSOCIATES

Aquino, John G.	Griffith, Roger E.	Mulvaney, Mark W.
Atkinson, Roger A., III	Groh, Linda M.	Schwandt, Jeffory C.
Billings, Holly L.	Handte, Malcolm R.	Snow, David C.
Blakinger, Jean M.	Harbage, Robin A.	Svendsgaard, Christian
Davis, Brian W.	Hill, Tony D.	Sweeney, Eileen M.
Feldblum, Sholom	Johnson, Eric J.	Wachter, Christopher J.
Francis, Louise A.	Leiner, William W., Jr.	Wacker, Gregory M.
Gorvett, Richard W.	Mueller, Nancy D.	Whitehead, Guy H.

Mr. Ben-Zvi then introduced M. Stanley Hughey, who delivered a brief speech to the new members concerning the responsibilities of a casualty actuary.

Mr. Ben-Zvi then introduced Mike Fusco, Vice President of Programs, who gave a brief summary of the program content.

Mr. Ben-Zvi next introduced Stephen Philbrick, Chairman of the Committee on the Review of Papers, who gave a brief summary of the new *Proceedings* papers. Mr. Ben-Zvi then called for reviews of prior papers from those in the audience. There were none.

Mr. Ben-Zvi concluded the business session at 9:00 a.m.

At 9:00 a.m., Representative John J. LaFalce delivered the keynote speech.

At 10:30 a.m., Ms. Mavis Walters moderated a panel entitled "The Liability Crisis—Legislative, Regulatory and Company Perspectives." Her panel consisted of:

David Gates
Commissioner
Nevada Insurance Division

Judge Frederick B. Karl
Partner
Karl, McConnaughhay, Roland, Maida and Beal

Peter Lardner
President and CEO
Bituminous Casualty Corporation

The panelists reviewed their thoughts on the current liability crisis.

Lunch was served from 12:00 to 1:30 p.m. Mr. E. J. Fennell, a reinsurance consultant, delivered a luncheon speech.

Beginning at 1:30 p.m., there were a series of concurrent sessions, including five *Proceedings* paper presentations, and four workshops.

The new *Proceedings* papers presented were:

"A Probabilistic Model for IBNR Claims"
Farrokh Guiahi, Assistant Professor
Hofstra University

"The Cash Flow of a Retrospective Rating Plan"
Glenn G. Myers, Associate Professor
Dept. of Statistics and Actuarial Sciences
Division of Mathematical Science
The University of Iowa

"The Cost of Mixing Reinsurance"
Ronald F. Wiser, Senior Actuarial Officer
St. Paul Fire & Marine Insurance Co.

"A Bayesian Credibility Formula for IBNR Counts"
Dr. I. Robbin, Director & Actuarial Associate
CIGNA Corporation

"A Formal Approach to Catastrophe Risk Assessment and Management"
Karen M. Clark, Vice President
InSoft, Inc.

The workshops covered the following topics:

1. "An Actuary's Perspective on: Underwriting"

Moderator: Russell S. Fisher, Second Vice President
General Reinsurance Corporation

Panelists: Frank Neuhauser, Vice President & Actuary
AIG Risk Management

Dennis R. Henry, Vice President
Huggins Financial Services

2. "An Actuary's Perspective on: Data Management"
CAS Committee on Management Data and Information
Michael F. McManus, *Chairman*

Edward W. Ford	Donna S. Munt
Anthony J. Grippa	Raymond F. Nichols
Philip D. Miller	Glenn J. Pruiksma

3. "An Actuary's Perspective on: Marketing"

Moderator: David Skurnick, Vice President & Actuary
F & G Re

Panelists: James R. Young, Vice President – Sales
Allstate Insurance Company

David M. Klein, Second VP – Marketing
Hartford Insurance Company

4. "Personal Umbrella Ratemaking"

Moderator: Robert T. Muleski, Associate Actuary
Liberty Mutual

Panelists: Alice H. Gannon, Actuary
USAA

Lee R. Steeneck, Second Vice President
General Reinsurance Corporation

The President's Reception was held from 6:30 p.m. to 7:30 p.m.

Tuesday, November 11, 1986

Tuesday morning from 8:30 a.m. until 11:30 a.m. was devoted to a continuation of the concurrent sessions from Monday afternoon.

Mr. Ben-Zvi reconvened the business session at 11:45 a.m. and delivered his Presidential Address.

Lunch was served at 12:15 p.m.

At 1:30 p.m., Mr. Kevin Ryan moderated a panel entitled "How Will Tort Reform Affect Claim Costs and Insurance Availability?" His panel consisted of:

Anne E. Kelly
Assistant Chief Consulting Actuary
New York Insurance Department

Franklin W. Nutter
President
Alliance of American Insurers.

Sidney Gilreath
Parliamentarian
Association of Trial Lawyers of America

Mr. Ben-Zvi then closed the meeting and thanked those individuals who had planned the meeting and executed those plans. The meeting was adjourned at 3:15 p.m.

In attendance, as indicated by registration records, were 159 Fellows, 61 Associates, 9 Guests, 11 Subscribers, 7 Students, and 76 Spouses.

FELLOWS

Aldorisio, R. P.
Alff, G. N.
Amundson, R. B.
Bailey, R. A.
Bailey, V. M.
Bashline, D. T.
Bass, I. K.
Baum, E. J.
Bell, L. L.
Bennett, N. J.
Bensimon, A. S.
Ben-Zvi, P. N.
Bertles, G. G.
Bill, R. A.
Blanchard, R. S., III
Bornhuetter, R. L.
Bothwell, P. T.
Brooks, D. L.
Brubaker, R. E.
Bryan, C. A.
Ciezadlo, G. J.
Cis, M. M.
Crowe, P. J.
Curry, A. C.
Daino, R. A.
Davis, L. S.
Dean, C. G.
Degerness, J. A.
Donaldson, J. P.
Dornfeld, J. L.
Evans, G. A.
Eyers, R. G.
Fallquist, R. J.
Fein, R. I.
Ferguson, R. E.
Finger, R. J.
Fisher, R. S.
Fitzgibbon, W. J., Jr.
Ford, E. W.
Forde, C. S.
Fowler, T. W.
Furst, P. A.
Fusco, M.
Gannon, A. H.
Gleeson, O. M.
Golz, J. F.
Gottlieb, L. R.
Graves, J. S.
Grippa, A. J.
Hafling, D. N.
Hallstrom, R. C.
Hankins, S. E.
Hartman, D. G.
Harwayne, F.
Hein, T. T.
Henry, D. R.
Honebein, C. W.
Hoppe, K. J.
Hughey, M. S.
Huyck, B. J.
Inkrott, J. G.
Johe, R. L.
Johnson, A. P.
Johnson, L. D.
Johnson, M. A.
Kallop, R. H.
Kaufman, A. M.
Kelley, R. J.
Kelly, A. E.
Khury, C. K.
Kilbourne, F. W.
Klein, D. M.
Kleinman, J. M.
Klinker, F. L.
Koski, M. I.
Koupf, G. I.
Krakowski, I.
Krause, G. A.
Kucera, J. L.
Larose, J. G.
Lehman, M. R.
Levin, J. W.
Linden, O. M.
Lino, R. A.
Livingston, R. P.
Loper, D. J.
Lyons, D. K.
Macginnitie, W. J.
Mahler, H. C.
Marker, J. O.
Martin, P. C.
Mathewson, S. B.
McClure, J. W., Jr.
McDonald, G. P.
McManus, M. F.
Menning, D. L.
Meyer, R. E.
Meyers, G. G.
Miccolis, J. A.
Miccolis, R. S.
Miller, M. J.
Miller, P. D.
Mohl, F. J.
Mulder, E. T.
Muleski, R. T.

FELLOWS

Munro, R. E.
Munt, D. S.
Murdza, P. J., Jr.
Murrin, T. E.
Myers, T. G.
Newman, S. H.
Nichols, R. S.
Noyce, J. W.
Patrik, G. S.
Petersen, B. A.
Phillips, H. J.
Pinney, A. D.
Potts, C. M.
Pratt, J. J.
Pruiksma, G. J.
Purple, J. M.
Reppert, D. A.
Riddlesworth, W. A.
Rodermund, M.
Roth, R. J., Jr.
Ruegg, M. A.
Ryan, K. M.
Scheibl, J. A.
Schwartz, A. I.
Shrum, R. G.
Silver, M. S.
Simon, L. J.
Skurnick, D.
Smith, F. A.
Smith, L. M.
Snader, R. H.
Steeneck, L. R.
Strug, E. J.
Suchoff, S. B.
Surrago, J.
Terrill, K. W.
Tiller, M. W.
Toothman, M. L.
Townsend, C. J.
Tuttle, J. E.
Tverberg, G. E.
Van Slyke, O. E.
Venter, G. G.
Vitale, L. A.
Walters, M. A.
Walters, M. A.
Webb, B. L.
White, C. S.
White, D. L.
Wilson, J. C.
Wiseman, M. L.
Wiser, R. F.
Wright, W. C., III
Zatorski, R. T.

ASSOCIATES

Anderson, B. C.
Aquino, J. G.
Atkinson, R. A., III
Balling, G. R.
Billings, H. L.
Blakinger, J. M
Cadorine, A. R.
Chorpita, F. M.
Clark, D. G.
Comstock, S. J.
Connor, V. P.
Costner, J. E.
Crifo, D. A.
Douglas, F. H.
Driedger, K. H.
Easlon, K.
Feldblum, S.
Francis, L. A.
Gorvett, R. W.
Griffith, R. E.
Groh, L. M.
Harbage, R. A.
Head, T. F.
Hill, T. D.
Jaso, R. J.
Jensen, J. P.
Johnson, E. J.
Johnson, R. W.
Johnson, W. A.
Kollmar, R.
Lafrenaye, C.
Masella, N. M.
Montigney, B. A.
Mueller, N. D.
Mulvaney, M. W.
Napierski, J. D.
Neuhauser, F., Jr.
Orlowicz, C. P.
Pei, K. J.
Putney, A. K.
Riff, M.
Sansevero, M., Jr.
Schultz, R. A.
Schwandt, J. C.
Simons, M. M.
Snow, D. C.
Steinen, P. A.
Svendsgaard, C.
Sweeney, E. M.
Torgrimson, D. A.
Urschel, F. A.

ASSOCIATES

Van Cleave, M. E.	Wacker, G. M.	Whitehead, G. H.
Visner, S. M.	Waldman, R. H.	Wilson, O. T.
Von Seggern, W. J.	Webb, N. H.	Youngner, R. E.
Wachter, C. J.		

GUESTS – SUBSCRIBERS – STUDENTS

Booher, J. P.	Fujii, Y.	Robbin, I.
Brickman, S. J.	Gutman, E.	Roberts, J.
Clark, K.	Huang, M. I.	Santomenno, S.
Davis, B.	Ingraham, H. G., Jr.	Smith, D. A.
Demarlie, G.	Jensen, P. A.	Taylor, J.
Earls, R. R.	Lepere, C.	Thomas, A. M.
Fennell, E.	Maxon, R. G.	Van Leer, P.
Guiahi, F.	Michelson, J.	Wilson, G.
Franz, V.	Mohler, E.	Wright, J.

REPORT OF THE VICE PRESIDENT-ADMINISTRATION

The purpose of this report is to provide the membership with a brief summary of CAS activities since the last annual meeting.

1986 was a good year for the CAS with 106 new members admitted and total membership climbing to 1,275. Recognizing that continued rapid growth could impair the ability of the CAS office to continue its efficient operation, a study of the future of the CAS office was undertaken in 1985. During 1986, the following recommendations which emerged from the study were implemented:

- An additional staff member, Jennifer Dewar, joined the CAS office as Assistant Manager.
- New and expanded CAS office space has been occupied by the staff.
- Computer hardware (IBM PC-AT) and software have been acquired.
- All financial and accounting records have been automated.
- Examination registration and student examination history are being automated.

The CAS is also financially healthy. Despite the expansion of the CAS office, surplus was increased during fiscal year 1986. A budget for fiscal year 1987 approaching $600,000 was approved with no increase in dues or examination fees.

The Board of Directors, with prime responsibility for setting policy, met four times in 1986. A meeting of the CAS Regional Affiliates was held in conjunction with the February Board meeting. Several policy decisions were made. These policies were published in the *Actuarial Review* and also appear in the 1987 edition of the *Yearbook*.

The Executive Council, with primary responsibility for day to day activities, also met four times during the year. Continuing with the precedent established last year, the Committee Chairpersons meeting was held in conjunction with the April meeting of the Executive Council.

The activities of both the Board and the Council included the following items:

- Guidelines were established for maintaining the CAS surplus at an appropriate level.
- Guidelines were established for the administration of CAS trusts, memorials, and bequests.
- A policy for the investment of CAS funds was adopted.
- A policy was adopted allowing CAS meetings to be open for press coverage.
- Separate U.S. and Canadian Part 8 examinations were authorized beginning in 1987.
- The Syllabus was revised with respect to jointly administered examinations to reflect modifications resulting from the Society of Actuaries' flexible education program.
- The Board of Directors authorized exposure of an amendment to Article II of the CAS Constitution. Article II deals with the purpose of the CAS.
- A discussion draft of Ratemaking Principles was authorized for distribution to the membership.
- A Committee on Financial Analysis was appointed replacing the Committee on Financial Reporting Principles.
- A Committee on Valuation Principles and Techniques was created under the Vice President-Development. The appointment resulted from the recommendations of a Task Force appointed in 1985 to study and plan for CAS activities related to the valuation actuary issue. The purpose of the committee is to develop valuation principles and techniques applicable to property/casualty business and to monitor development in this area.
- The Casualty Actuaries of the Bay Area became a new regional affiliate.
- The CAS extended an invitation to ASTIN to hold a colloquium in the United States in November 1989.
- Joint sponsorship with the CIA of a Canadian loss reserve seminar was authorized.

For 1987, the Board of Directors elected the following Vice Presidents:

Vice President-Administration	Richard Snader
Vice President-Development	Charles Bryan
Vice President-Membership	Michael Toothman
Vice President-Programs	Michael Fusco

The membership elected David Hartman as President-Elect and four new Board members: Irene Bass, Allan Kaufman, LeRoy Simon and David Skurnick.

Finally, the Audit Committee examined the CAS books for fiscal year 1986 and found the accounts to be properly stated. The year ended with an increase in surplus of $61,905.48. Members' equity now stands as $330,563.38, subdivided as follows:

Michelbacher Fund	$64,351.65
Dorweiler Fund	9,703.83
CAS Trust	2,195.78
Scholarship Fund	7,288.19
CLRS Fund	5,000.00
CAS Surplus	242,023.93
	$330,563.38

Respectfully submitted,

RICHARD H. SNADER
Vice President-Administration

FINANCIAL REPORT
FISCAL YEAR ENDED 9/30/86 (ACCRUAL BASIS)

INCOME		DISBURSEMENTS	
Dues	$ 146,872.80	Printing & Stationery	$191,947.95
Exam Fees	137,324.72	Office Expenses	124,041.54
Meetings	217,893.34	Exam Expenses	1,934.49
Proceedings	9,456.50	Meeting Expenses	157,750.39
Readings	21,056.15	Library	705.38
Invitational Program	8,740.00	Insurance	8,778.65
Interest	35,809.76	Math. Assoc. of America	2,000.00
Actuarial Review	234.50	Pres. & Pres.-Elect Expenses	7,500.00
Yearbook	1,684.00	Diamond Jubilee Expense Reserve	18,729.37
Miscellaneous	(256.63)	Other	3,521.89
Total	$ 578,815.14	Total	$516,909.66

Income	$ 578,815.14
Expenses	516,909.66
Change in CAS Surplus	$ 61,905.48

ACCOUNTING STATEMENT (ACCRUAL BASIS)

ASSETS	9/30/85	9/30/86	CHANGE
Checking Account	$ 1,259.80	$ 63,313.38	$ 62,053.58
Money Market Fund	143,120.28	175,786.78	32,666.50
Bank Certificates of Deposit	0.00	100,000.00	100,000.00
U.S. Treasury Notes & Bills	222,926.78	243,247.83	20,321.05
Accrued Interest	11,684.06	9,443.40	(2,240.66)
CLRS Fund	0.00	5,000.00	5,000.00
Total Assets	$378,990.92	$596,791.39	$217,800.47

LIABILITIES			
Office Expenses	$ 30,000.00	$ 34,965.00	$ 4,965.00
Printing Expenses	30,611.00	146,133.06	115,522.06
Prepaid Examination Fees	45,767.00	55,712.20	9,945.20
Meeting Expenses & Prepaid Fees	13,813.02	10,435.38	(3,377.64)
Diamond Jubilee Expense Reserve	0	18,729.37	18,729.37
Other	0	253.00	253.00
Total Liabilities	$120,191.02	$266,228.01	$146,036.99

MEMBERS' EQUITY			
Michelbacher Fund	$ 59,681.87	$ 64,351.65	$ 4,669.78
Dorweiler Fund	9,881.80	9,703.83	(177.97)
CAS Trust	2,005.28	2,195.78	190.50
Scholarship Fund	7,112.50	7,288.19	175.69
CLRS Fund	0.00	5,000.00	5,000.00
CAS Surplus	180,118.45	242,023.93	61,905.48
Total	$258,799.90	$330,563.38	$ 71,763.48

Richard H. Snader
Vice President—Administration

This is to certify that the assets and accounts shown in the above financial statement have been audited and found to be correct

Audit Committee

David M. Klein, Chairman
Albert J. Quirin
William J. Rowland
Charles Walter Stewart

1986 EXAMINATIONS—SUCCESSFUL CANDIDATES

Examinations for Parts 4, 6, 8, and 10 of the Casualty Actuarial Society were held on May 6, 7, 8, and 9, 1986. Examinations for Parts 5, 7, and 9 were held on November 5, 6, and 7, 1986.

Examinations for Parts 1, 2, and 3 are jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries. These examinations were given in May and November of 1986. Candidates who passed these examinations were listed in the joint releases of the two societies.

The Casualty Actuarial Society and the Society of Actuaries jointly awarded prizes to the undergraduates ranking the highest on the General Mathematics examination. For the May, 1986 examination, the $200 prize was awarded to Robert B. Cumming. The additional $100 prize winners were Nathaniel G. Calvin, Ampon Dhamacharden, Christopher Lattin, Ralph L. Neill, and Michael Reid. For the November, 1986 examination, the $200 prize was awarded to Scott N. Wilson. The additional $100 prize winners were Andrew K. Fung, Siu C. Szeto, Thomas S. Watts, Thomas A. Zeller, and Josh A. Zirin.

The following candidates were admitted as Fellows and Associates at the November, 1986 meeting as a result of their successful completion of the Society requirements in the May, 1986 examinations.

FELLOWS

Amundson, Richard B.
Bailey, Victoria M.
Bellusci, David M.
Chiang, Jeanne D.
Driedger, Karl H.
Faltas, Bill
Forde, Claudia S.
Hankins, Susan E.
Hollister, Jeanne M.
Hosford, Mary T.
Huyck, Brenda J.
Johnson, Andrew P.
Kelley, Robert J.
Klinker, Frederick L.
Koupf, Gary I.
Krakowski, Israel
Littmann, Mark W.
Livingston, Roy P.
Loper, Dennis J.
Lyons, Daniel K.
Martin, Paul C.
McClure, John W., Jr.
McDonald, Gary P.
Menning, David L.
Myers, Thomas G.
Noyce, James W.
Potts, Cynthia M.
Reppert, Daniel A.
Ruegg, Mark A.
Silver, Melvin S.
Terrill, Kathleen W.
Townsend, Christopher J.
Vitale, Lawrence A.
Weinman, Stacy J.

ASSOCIATES

Aquino, John G.
Atkinson, Roger A., III
Billings, Holly L.
Blakinger, Jean M.
Davis, Brian W.
Feldblum, Sholom
Francis, Louise A.
Gorvett, Richard W.
Griffith, Roger E.
Groh, Linda M.
Handte, Malcolm R.
Harbage, Robin A.
Hill, Tony D.
Johnson, Eric J.
Leiner, William W., Jr.
Mueller, Nancy D.
Mulvaney, Mark W.
Schwandt, Jeffory C.
Snow, David C.
Svendsgaard, Christian
Sweeney, Eileen M.
Wachter, Christopher J.
Wacker, Gregory M.
Whitehead, Guy H.

The following is the list of successful candidates in examinations held in May, 1986

Part 4

Abellera, Daniel N.
Atkins, Heather E.
Atkinson, Roger A., III
Beaulieu, Gregory S.
Belleau, Richard
Blackburn, Wayne E.
Boisjoli, Marthe
Bonte, Sharon
Book, Steven W.
Bourassa, Pierre
Buckley, Joseph
Burt, Richard F., Jr.
Burrill, Linda J.
Cain, Mark J.
Carpentier, Marie
Casale, Kathleen N.
Chaffee, Janet L.
Champagne, Mario
Cloutier, Jean
Coca, Michael A.
Cofield, Joseph F.
Colton, Gary S.
Crowe, Alan M.
Daniels, Paul F.
Daoust, Alain
Darby, Robert N.
Dineen, David K.
Edlefson, Dale
Elliott, Angela F.
Ely, James
Emmons, William E.
Ewert, John S.
Fitzpatrick, Kerry L.
Gendelman, Nathan J.
Ghezzi, David J.
Giles, John S.
Gorvett, Richard W.
Gozzo, Susan M.
Griffith, Roger E.
Hebert, Norman P.
Hess, Todd J.
Higgins, James S.
Huang, Ming-I
Jovinelly, Edward M.
Klinger, Kenneth A.
Kopel, Noson
Kryczka, John R.
Larner, Kenneth P. W.
Laurin, Michel
Lepage, Pierre
Li, Siu Kuen
Maher, Christopher P.
Mahoney, Michael W.
Martin, Claude
Math, Steven
McKay, Donald R.
Moylan, Thomas G.
Nemlick, Kenneth J.
Nerone, Anthony J.
Nesmith, Robin
Nonken, Peter M.
Orrett, Todd F.
Palmer, Donald D.
Papadopoulos, Constantina
Peck, Steven C.
Pestcoe, Marvin
Plano, Richard A.
Pompeii, Peter A.
Royek, Peter A.
Santoro, Lawrence
Schmid, Valerie L.
Schoenberger, Susan C.

Schug, Richard D.
Schwab, Debbie
Sclafane, Susanne
Seeley, Alan R.
Seto, Hopland
Sheng, Michelle G.
Silverman, Jack
Simi, Laura J.
Sperger, Mary Jean
Stefanek, John P.
Steinert, Lawrence J.
Stoffel, Judith E.
Stone, Edward C.
Strommen, Douglas N.
Sublett, Sharon
Szczepanski, Chester J.
Vandermyde, Scott D.
Van de Water, John V.
Vasek, William
Weihrich, Leslie D.
White, William A.
Whitehead, Guy H.
Wildman, Peter W.
Wong, Windrie

Part 6

Aquino, John G.
Artes, Lawrence J.
Bennighof, Kay E.
Billings, Holly L.
Blakinger, Jean M.
Boisvert, Paul, Jr.
Boudreau, Joseph J.
Bourdon, Theresa A.
Brathwaite, Malcolm
Brehm, Paul J.
Caulfield, Michael
Conway, Ann M.
Cote, Jean
Crawshaw, Mark
Creighton, Kenneth M.
Cross, Susan L.
Davis, Brian W.
Davis, James R.
Desbiens, Carol
Doe, David A.
Dumontet, Francois R.
Erlebacher, Alan J.
Feldblum, Sholom
Francis, Louise A.
Frank, Jacqueline B.
Gergasko, Richard J.
Gibson, Richard N.
Girard, Gregory S.
Golberg, Leonard R.
Grab, Edward M.
Greene, Alex R.
Griffith, Ann V.
Groh, Linda M.
Haefner, Larry A.
Hampshire, Michael H.
Handte, Malcolm, R.
Harbage, Robin A.
Hawley, Karin S.
Hays, David H.
Heyman, David R.
Hill, Tony D.
Hines, Alan M.
Johnson, Eric J.
Johnston, Steven J.
Joyce, John J.
Keatinge, Clive L.
Kinson, Paul E.
Kohan, Richard F.
Krissinger, Kenneth R.
Lalonde, David A.
Lamb, Dean K.
LaPointe, Susan E.
Lebens, Joseph R.
Leiner, William W., Jr.
Mahon, Mark J.
Maud, Christine E.
Michelson, Jon W.
Miller, Mary F.
Mueller, Nancy D.
Mulvaney, Mark W.
Naylor, Walter R.
Nelson, Chris E.
Perigny, Isabelle
Pino, Susan L.
Proska, Mark R.
Raman, Sasikala
Rouillard, Marc L.
Salton, Melissa A.
Samson, Sandra
Schadler, Thomas E.
Schlenker, Sara E.
Schultze, Mark E.
Schwandt, Jeffory C.
Snow, David C.
Stahley, Barbara A.
Sterling, Mary E.
Svendsgaard, Christian
Sweeney, Eileen M.
Wachter, Christopher J.
Wacker, Gregory M.
Weisenberger, Peter A.
Werland, Debra L.
Yit, Bill S.

Part 8

Anderson, Mary V.
Atkinson, Richard V.
Boor, Joseph A.
Brown, Brian Y.
Busche, George R.
Carlton, Kenneth E.
Comstock, Susan J.
DeLiberato, Robert V.
Dezube, Janet B.
Dickinson, Donna R.
DiDonato, Anthony M.
Dodge, Scott H.
Ericson, Janet M.
Fanning, William G.
Fitzgerald, Beth E.
Gardner, Robert W.
Gevlin, James M.
Gunn, Christy H.
Haidu, James W.
Hankins, Susan E.
Hertling, Richard J.
Hughes, Brian A.
Jordan, Jeffrey R.
Klinker, Frederick L.
Krakowski, Israel
Lewandowski, John J.
Miller, Susan M.
Mohrman, David F.
Newell, Richard T., Jr.
Ollodart, Bruce E.
Pechan, Kathleen M.
Pence, Clifford A., Jr.
Peraine, Anthony A.
Peterson, Steven J.
Placek, Arthur C.
Post, Jeffrey H.
Procopio, Donald W.
Quintano, Richard A.
Robbins, Kevin B.
Roesch, Robert S.
Scheuing, Jeffrey R.
Schultz, Roger
Scott, Kim A.
Scully, Mark W.
Sealand, Pamela J.
Shapland, Mark R.
Slusarski, John
Spidell, Bruce R.
Sutter, Russel L.
Tan, Suan-Boon
Taylor, R. Glenn
Volponi, Joseph L.
VonSeggern, William J.
Wainscott, Robert H.
Walsh, Michael C.
Wargo, Kelly A.
Woerner, Susan K.
Yow, James W.

Part 10

Amundson, Richard B.
Bailey, Victoria M.
Bellusci, David M.
Buchanan, John W.
Carpenter, William M.
Chiang, Jeanne D.
Driedger, Karl H.
Englander, Jeffrey A.
Faltas, Bill
Forde, Claudia S.
Guenthner, Denis G.
Hankins, Susan E.
Hollister, Jeanne M.
Hosford, Mary T.
Huyck, Brenda J.
Johnson, Andrew P.
Kasner, Kenneth R.
Kelley, Robert J.
Kneuer, Paul J.
Koupf, Gary I.
Kudera, Andrew E.
Laurin, Pierre G.
Littmann, Mark W.
Livingston, Roy P.
Loper, Dennis J.
Lyons, Daniel K.
Mailloux, Patrick
Martin, Paul C.
McClure, John W., Jr.
McDonald, Gary P.
Menning, David L.
Miller, David L.
Miller, William J.
Morrow, Jay B.
Myers, Thomas G.
Noyce, James W.
Phillips, George N.
Potts, Cynthia M.
Reppert, Daniel A.
Ruegg, Mark A.
Siczewicz, Peter J.
Silver, Melvin S.
Terrill, Kathleen W.
Townsend, Christopher J.
Vitale, Lawrence A.
Votta, James C.
Weinman, Stacy J.
Williams, Robin M.

The following candidates will be admitted as Fellows and Associates at the May, 1987 meeting as a result of their successful completion of the Society requirements in the November, 1986 examinations.

FELLOWS

Aldin, Neil C.
Barclay, D. Lee
Chuck, Allan
Cripe, Frederick F.
Dye, Myron L.
Eagelfeld, Howard M.
Easlon, Kenneth
Edie, Grover M.
Gapp, Steven A.
Gruber, Charles
Guenthner, Denis G.
Homan, Mark J.
Howald, Ruth A.
Keller, Wayne S.
Kneuer, Paul J.
Lee, Robert H.
Montgomery, Warren D.
Onufer, Layne M.
Petit, Charles I.
Raman, Rajagopalan K.
Schilling, Timothy L.
Withers, David A.

ASSOCIATES

Abell, Ralph L.
Allaire, Christiane
Allard, Jean-Luc E.
Boisvert, Paul, Jr.
Boucek, Charles H.
Boudreau, Joseph J.
Bourdon, Theresa A.
Brathwaite, Malcolm E.
Brehm, Paul J.
Brutto, Richard S.
Buchanan, John W.
Cardoso, Ruy A.
Chen, Chyen
Cieslak, Walter P.
Conway, Ann M.
Crawshaw, Mark
Cross, Susan L.
Der, William
Desbiens, Carol
DiDonato, Anthony M.
Donelson, Norman E.
Ericson, Janet M.
Fromentin, Pierre
Girard, Gregory S.
Goldberg, Leonard R.
Graves, Gregory T.
Greene, Alex R.
Griffith, Ann V.
Haefner, Larry A.
Hays, David H.
Heyman, David R.
Keatinge, Clive L.
Keen, Eric R.
Klenow, Jerome F.
Krissinger, Kenneth R.
Lacko, Paul E.
Lamb, Dean K.
Laurin, Pierre G.
Lebens, Joseph R.
Leccese, Nicholas M., Jr.
Lewandowski, John J.
Licitra, Sam F.
Liebers, Elise C.
MacKinnon, Brett A.
McDermott, Sean P.
Miller, Mary F.
Pichler, Karen J.
Plano, Richard A.
Procopio, Donald W.
Proska, Mark R.
Schlenker, Sara E.
Schnapp, Frederic F.
Schwab, Debbie
Scott, Kim A.
Shapland, Mark R.
Siczewicz, Peter J.
Taylor, Craig P.
Taylor, R. Glenn
Veilleux, Andre
Votta, James C.
Weber, Robert A.
Wick, Peter G.
Williams, Lincoln B.
Williams, Robin M.
Wilson, Ernest I.
Yit, Bill S.

The following is the list of successful candidates in examinations held in November, 1986.

Part 5

Anderson, Richard R.
Ashman, Martha E.
Bahnemann, David W.
Barton, Frances H.
Beaulieu, Gregory S.
Beck, Douglas L.
Book, Steven W.
Bouchard, Lloyd J.
Bowman, David R.
Bradley, Tobe E.
Bryant, Debbie H.
Buchanan, John W.
Burns, Patrick J.
Burrill, Linda J.
Chaffee, Janet L.
Chan, Sammy S. Y.
Charbonneau, Scott K.
Charest, Danielle
Clark, David R.
Conley, Kevin J.
Cooper, Nancy L.
Cross, Susan L.
Crowe, Alan M.
Curry, Michael K.
Curry, Robert J.
Davenport, Edgar W.
Desnoyers, Lee A.
Ermisch, Jennifer L.
Eska, Catherine E.
Evensen, Philip A.
Fauerbach, Thomas R.
Feldmeier, Judith
Fields, David N.
Fontaine, Andre F.
Fox, Richard L.
Fung, Kai Y.
Gergasko, Richard J.
Giles, John S.
Goss, Linda M.
Graves, Nancy A.
Gray, Margaret O.
Greenwood, Deborah A.
Grossman, William G.
Hampshire, Michael H.
Heise, Mark A.
Hill, Robert C.
Hoerl, Frederick L.
Iyengar, Sadagopan S.
Johnson, Victor A.
Jones, Brian A.
Jones, William R.
Jonske, James W.
Jovinelly, Edward M.
Kangas, Patricia L.
Kantor, Stephen H.
Kellner, Tony J.
Kido, Chester T.
Kim, Ho K.
Kish, George A.
Klinger, Kenneth A.
Koester, Steven M.
Kot, Nancy E.
Lepage, Pierre
Leveille, Jean-Marc
Li, Siu Kuen
Liebers, Elise C.
Lin, Simon S.
Little, Laurie A.
Lombardi, Paul M.
Mahon, Mark J.
Manley, Laura
McDonnell, Janet A.
McGee, Stephen J.
McKay, Donald R.
Mech, William T.
Mercier, Mark F.
Mitchell, Sandra K.
Murray, David A.
Nesmith, Robin
Nevins, Richard N.
Orrett, Todd F.
Ottone, Joanne M.
Paffenbach, Teresa K.
Palmer, Joseph M.
Papadopolous, Constantina
Paterson, Bruce
Perigny, Isabelle
Poe, Michael D.
Punzak, John K.
Raman, Sasikala
Rau, Thomas O.
Raymond, Stephen E.
Reynolds, Margaret M.
Robertson, James
Roth, Scott J.
Samson, Pierre
Samson, Sandra
Schmidt, Jeffrey W.
Schmitt, Karen E.
Schoenberger, Susan C.
Schutte, Robert J.
Schwartz, Arthur J.
Seeley, Alan R.
Shook, Gary E.
Simons, Rial R.
Spore, Louis B.
Stahley, Barbara A.
Stauffer, Laurence H.

Strommen, Douglas N.
Sturm, Elissa M.
Subeck, Jeffrey L.
Suchar, Christopher M.
Thomas, Richard D.
Tscharke, Jennifer L.
Vandermyde, Scott D.
Van Laar, Kenneth R., Jr.
Verges, Ricardo
Vezina, Guy
Weber, Robert A.
Weihrich, Leslie D.
Weinstein, Scott P.
Wellington, Elizabeth A.
Weltmann, L. Nicholas, Jr.
Whalen, William T.
Wildman, Peter W.
Williams, Janice K.
Williams, Robin M.
Winslow, Martha A.
Wolter, Kathy A.
Wong, Windrie
Yuen, Benny S.
Zaleski, Ronald J.

Part 7

Abell, Ralph L.
Adams, Jeffrey
Allaire, Christiane
Allard, Jean-Luc E.
Boisvert, Paul, Jr.
Boucek, Charles H.
Boudreau, Joseph J.
Bourdon, Theresa A.
Brathwaite, Malcolm E.
Brehm, Paul J.
Brutto, Richard S.
Cadorine, Arthur R.
Cappers, Janet P.
Cardoso, Ruy A.
Chen, Chyen
Cieslak, Walter P.
Conway, Ann M.
Crawshaw, Mark
Der, William
Desbiens, Carol
DiDonato, Anthony M.
Donelson, Norman E.
Elliott, Angela F.
Ely, James
Ericson, Janet M.
Ewert, John S.
Frank, Jacque B.
Franklin, Barry A.
Fromentin, Pierre
Gaudreault, Andre
Gelinne, David B.
Gendelman, Nathan J.
Gibson, John F.
Girard, Gregory S.
Goldberg, Leonard R.
Graves, Gregory T.
Greene, Alex R.
Greenwalt, Anne G.
Griffith, Ann V.
Grossack, Marshall J.
Gruber, Charles
Haefner, Larry A.
Hays, David H.
Hebert, Norman P.
Heyman, David R.
Hroziencik, George A.
Jasper, Jane E.
Kartechner, John W.
Keatinge, Clive L.
Keen, Eric R.
Klenow, Jerome F.
Krissinger, Kenneth R.
Kryczka, John R.
Lacko, Paul E.
LaFrenaye, A. Claude
Lamb, Dean K.
Lamb, John A.
Laurin, Pierre G.
Lebens, Joseph R.
Leccese, Nicholas M., Jr
Lewandowski, John J.
Licitra, Sam F.
MacKinnon, Brett A.
Maher, Christopher P.
McDermott, Sean P.
McNichols, James P.
Miller, Mary F.
Naylor, Walter R.
Nielsen, Lynn
Palenik, Rudy A.
Paterson, Bruce
Peck, Steven C.
Pichler, Karen J.
Plano, Richard A.
Procopio, Donald W.
Proska, Mark R.
Radin, Katherine D.
Schlenker, Sara E.

Schnapp, Frederic F.
Schwab, Debbie
Sclafane, Susanne
Scott, Kim A.
Shapland, Mark R.
Siczewicz, Peter J.
Snook, Linda D.
Stoffel, Judith E.
Styczynski, Mary Jane
Taylor, Craig P.
Taylor, R. Glenn
Veilleux, Andre
Votta, James C.
Watkins, Nancy P.
Wick, Peter G.
Williams, Lincoln B.
Wilson, Ernest I.
Yates, Patricia E.
Yit, Bill S.

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Aldin, Neil C.
Aquino, John G.
Atkinson, Richard V.
Atkinson, Roger A., III
Balchunas, Anthony J.
Barclay, D. Lee
Billings, Holly L.
Cartmell, Andrew R.
Chuck, Allan
Comstock, Susan J.
Cripe, Frederick F.
Davis, Dan J.
DeFalco, Thomas J.
Diamantoukos, Christopher
Dye, Myron L.
Eagelfeld, Howard M.
Easlon, Kenneth
Edie, Grover M.
Englander, Jeffrey A.
Fasking, Dennis D.
Feldblum, Sholom
Francis, Louise A.
Gapp, Steven A.
Glicksman, Steven A.
Guenthner, Denis G.
Halpert, Aaron
Homan, Mark J.
Howald, Ruth A.
Johnson, Eric J.
Jordan, Jeffrey R.
Keller, Wayne S.
Kneuer, Paul J.
Kreps, Rodney E.
Lee, Robert H.
Lessard, Alain
Marles, Blaine C.
Miller, Susan M.
Mohrman, David F.
Montgomery, Warren D.
Mueller, Nancy D.
Muller, Robert G.
Onufer, Layne M.
Pence, Clifford A., Jr.
Peraine, Anthony A.
Petit, Charles I.
Phillips, George N.
Placek, Arthur C.
Quintano, Richard A.
Raman, Rajagopalan K.
Roesch, Robert S.
Schilling, Timothy L.
Scully, Mark W.
Spidell, Bruce R.
Steinen, Phillip A.
Svendsgaard, Christian
Tan, Suan-Boon
Trudeau, Michel
Turner, George W., Jr.
Visintine, Gerald R.
Wainscott, Robert H.
Wallace, Thomas A.
Wargo, Kelly A.
Whitehead, Guy H.
Withers, David A.
Wrobel, Edward M.
Yow, James W.

NEW FELLOWS ADMITTED MAY 1986 (Left to Right): First row: Phil Ben-Zvi (President), Martin Lewis, Robert Bear, Mark Allaben, James Dornfeld; Second row: Barry Lipton, Greg Hayward, Rhonda Port, Kathy Curran, Karen Nester; Third row: Allen Hall, Nancy Treitel, Robert Miller; Fourth row: Wallis Boyd, Isaac Mashitz, Michael Smith, William Murphy; Fifth row: Dan Clark, Janice Berry, Charles White.

NEW ASSOCIATES ADMITTED MAY 1986

NEW FELLOWS ADMITTED NOVEMBER, 1986 (Left to Right): First Row: Gary Koupf, Susan Hankins, Victoria Bailey, Brenda Huyck, Cynthia Potts, Phil Ben-Zvi (President), Daniel Reppert, Richard Amundson, Karl Driedger; Second Row: Claudia Forde, Mark Ruegg, Mary Hosford, Jim Noyce, Paul Martin, John McClure, Fred Klinker, Tom Myers; Third Row: Daniel Lyons, Kathleen Terrill, Andrew Johnson, Melvin Silver, Gary McDonald, Christopher Townsend, Lawrence Vitale; Fourth Row: Israel Krakowski, Roy Livingston, Robert Kelley, David Menning, Dennis Loper.

NEW ASSOCIATES ADMITTED NOVEMBER, 1986 (Left to Right): First Row: Nancy Mueller, Phil Ben-Zvi (President), Jean Blakinger, Louise Francis, Sholom Feldblum, Jeffory Schwandt; Second Row: Roger Griffith, Gregory Wacker, Mark Mulvaney, John Acquino, Christian Svendsgaard; Third Row: Holly Billings, Linda Groh, Eileen Sweeney, Christopher Wachter, David Snow; Fourth Row: Guy Whitehead, Eric Johnson, Richard Gorvett, Roger Atkinson, Brian Davis, Robin Harbage.

OBITUARIES

Bruce W. Batho
James B. Haley, Jr.
Mark Kormes
Murray W. Latimer
Richard Pennock
Rajaratnam Ratnaswamy
Harwood Rosser
David A. Tapley
Walter I. Wells

BRUCE W. BATHO
1908–1986

Bruce Max Willard Batho, an Associate of the Casualty Actuarial Society since 1940, died on February 9, 1986, at the age of 77.

Born in Winnipeg, Manitoba, Canada, Mr. Batho was educated at the University of Manitoba. He graduated magna cum laude with a business administration degree. He later moved to Springfield, Illinois, and became a United States citizen in 1938.

Mr. Batho was an assistant actuary for the Illinois State Insurance Department, and worked for Illinois insurance companies.

He joined Life of Georgia in 1944 as associate actuary and was elected vice president and actuary in 1954. A year later, he was elected to the board of directors. He was named comptroller in 1957, was elected to the executive committee in 1961, and was named executive vice president-Administration in 1963. During his career, he also served as chairman of the underwriting, claims, and profit sharing committees, and as a member of the finance committee, and common stock sub-committee. He retired in 1977, continuing as a director, and was named advisory director in May, 1979.

Mr. Batho had also served as board chairman of Insurance Systems of America, a national insurance consortium, from 1972 to 1975. He was also past president of the Chicago and Southeastern actuarial clubs.

Mr. Batho is survived by his wife, Mrs. Izora Powell Batho of Atlanta; a son, Norman of East Windsor, New Jersey; two daughters, Mrs. Mabel Green of Franklin, North Carolina, and Miss Barbara Batho of Atlanta; a brother, Elgin Batho of Cape Coral, Florida; 12 grandchildren, and four great-grandchildren.

JAMES B. HALEY, JR.
–1986

James B. Haley, Jr., an Associate of the Casualty Actuarial Society since 1950, and a Fellow since 1953, died recently. Mr. Haley's actuarial career started at Fireman's Fund. Upon achievement of Fellowship, he joined Argonaut, and served there as Actuary until 1958. He also worked 10 years for Coates, Herfurth and England in San Francisco. From 1969 to 1972, Mr. Haley was a consulting actuary. From 1973 to 1980, he was Vice President and Actuary with Employee Benefits Insurance Company. He returned to consulting in 1981.

MARK KORMES
1900–1985

Mark Kormes, a Fellow of the Casualty Actuarial Society since 1933, died in January, 1985. Mark worked as Associate Actuary for the Compensation Rating Board from 1933 to 1938. He was Director of Training and Organization for the NY State Insurance Fund from 1938 to 1940. Mark worked as a consulting actuary from 1940 until his retirement in 1980, serving as President of Actuarial Associates, Inc. in New York from 1960 to 1980.

Mark is remembered for his regular attendance at CAS meetings and his penchant for playing bridge there. Mark contributed a paper to the *Proceedings* on excess workers' compensation losses in 1948.

MURRAY W. LATIMER
1901–1985

Murray W. Latimer, a Fellow of the Casualty Actuarial Society since 1961, died in October, 1985. Prior to achieving membership in the Society, Mr. Latimer worked for several years for the US Railroad Retirement Board. In 1957, Murray joined the Industrial Relations Consultants. In 1968, Murray formed his own consulting organization in Washington, D.C. He retired in 1980.

RICHARD M. PENNOCK
1883–1976

Richard Pennock, became an Associate of the Casualty Actuarial Society in 1924. He served as Actuary for the Pennsylvania Manufacturers Association Insurance Company until he retired in 1950. Dick is remembered for his contributions to the committees of the Pennsylvania Workers Compensation Rating Bureau. He was soft spoken and reserved, but would expound his theses forcefully and defend them ably.

RAJARATNAM RATNASWAMY
1927–1986

Rajaratnam Ratnaswamy, an Associate of the Casualty Actuarial Society since 1965, died on February 28, 1986.

In 1956, Raj joined the Mutual Service Insurer Group. He worked there until 1964, when he joined the Detroit Auto Inner-Insurance Exchange. While there, he reviewed the book *The Regulation of Reciprocal Insurance Exchanges* for the 1968 *Proceedings*. From 1969 to 1984, he was at St. Paul Marine Insurance Group. In 1984, Raj joined the Michigan Millers Mutual Insurance Company as Actuary.

Raj is survived by his son, John, and a daughter, Mary.

HARWOOD ROSSER
1909–1986

Harwood Rosser, an Associate of the Casualty Actuarial Society since 1971, died on August 22, 1986, at the age of 86.

Mr. Rosser's history was quite varied. During college, Mr. Rosser received awards for swimming and poetry. He studied to be a concert pianist in the 1930's. Unfortunately, pianists were barely paid enough to keep eating. Mr. Rosser received a scholarship to Princeton to pursue a doctorate. Although he is credited with some original mathematics research, he did not complete his doctorate.

Mr. Rosser worked as an actuary for Gulf Life and Metropolitan Life. He also worked in the insurance departments in some Northeastern states. Most recently, he worked for the United States Department of Labor. He started there on June 2, 1975 in the Pension Welfare Benefits Administration as a consultant. He was converted to a career conditional appointment as an actuary in May, 1976. He retired from the Department of Labor on February 28, 1986.

Mr. Rosser represented the United States at a number of International Actuarial Association meetings. He also helped develop problems for some of the CAS examinations. He was noted for his keen sense of humor and his public speaking skills.

Mr. Rosser also served on the President's Council for the University of Florida.

Mr. Rosser is survived by a brother, Dr. J. Barkley Rosser; two sisters, Dr. Merryday Rosser, and Mrs. Julie Glenn McGuire; many cousins, and several nieces and nephews.

DAVID A. TAPLEY
–1981

David A. Tapley, a Fellow of the Casualty Actuarial Society since 1956, died on October 25, 1981.

David was a determined individualist, who was key to the development of the actuarial profession, as well as to the success of some of today's larger insurance companies. In his twenties, David was told he had a serious lung disease. He was not expected to live past 30, and became a forest ranger in Montana. Within a few years, he was running up and down the mountains without even breathing hard. Being cured, David entered the insurance business, and brought with him the same boundless energy that saved him in Montana.

In the early 40's, David worked for the Ohio Farm Bureau—now Nationwide—in the claims department. Harold Curry (FCAS, 1953) recognized Dave's unique mathematical talent and enticed him into the actuarial field. Dave eventually rose to the top actuarial position at the Ohio Farm Bureau. After World War II, the shift from a suppressed driving population to a widespread driving population forced many insurance companies out of business. Dave's astute actuarial and management skills probably helped prevent Ohio Farm Bureau from suffering the same fate. One example of his ingenuity was his use of raw cotton commodity price changes as an indicator for automobile rate making. His reason: they were the only prices not subject to government regulation and, therefore, the only measure of the true inflation rate.

Dave eventually followed Harold Curry to the State Farm actuarial department, and there worked on ways to build the successful insurance organization we know today. He became concerned about the fluctuation in the value of claim reserves, and "discovered" loss development patterns in automobile insurance at a time when IBNR was all but an unknown concept. In 1956, Dave was admitted to the Casualty Actuarial Society, after many years of practice in the field, based on his paper on loss reserving for automobile insurance. Many of the principles in David's paper are still in wide use today.

Dave left State Farm for a brief try at consulting in St. Louis. Disappointed with the amount of travel, Dave joined the Transamerica Group in Michigan. Dave's astute perception and management skills again came into play in turning

Transamerica into the successful corporation it is today. He became President of Transamerica Insurance Company, in Los Angeles, in 1968. David moved up to the Board of Directors at Transamerica and remained active there until his retirement in 1979.

Dave is survived by his son, Rice, and his daughter, Judith LaFollette.

WALTER I. WELLS
1901–1986

Walter I. Wells, an Associate of the Casualty Actuarial Society since 1930, and a Fellow of the Society of Actuaries, died on April 13, 1986.

Walter was born in Sackville, New Brunswick, Canada. At age 19, he taught grades 7 and 8 in Dorchester, New Brunswick, before earning a bachelor's degree in mathematics and physics in 1925 from the University of Toronto. He then entered the insurance arena at State Mutual Life Assurance Company of America. He worked there for two years before returning to Canada. Then, he joined Acadia University, in Nova Scotia, as a teacher of mathematics. He was also a Fellow in mathematics for a year at the University of Toronto.

In 1929, Walter joined Woodward, Fondiller, & Ryan, in New York City, as an associate actuary. From 1931 to 1945, he was head of sickness and accident underwriting at the Paul Revere Life Insurance Companies and the Massachusetts Protective Association.

Walter rejoined State Mutual in 1945 as an assistant actuary. In 1953, he was named director of the newly formed sickness and accident division—later, the health insurance division. In 1959, he became second vice president. He worked for State Mutual for 20 years, and served on the management council there. Walter retired in 1965, and returned to teaching mathematics and actuarial science at Worcester Polytechnic Institute.

Walter is survived by his wife, Lillian (Murdoch); a son, Richard; two daughters, Ann Bogle, and Ruth Zimmerman; 10 grandchildren; and three great-grandchildren.

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