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FOREWORD

The Casualty Actuarial Society was organized in 1914 as the Casualty Actuarial and Statistical Society of America, with 97 charter members of the grade of Fellow; the Society adopted its present name on May 14, 1921.

Actuarial science originated in England in 1792, in the early days of life insurance. Due to the technical nature of the business, the first actuaries were mathematicians; eventually their numerical growth resulted in the formation of the Institute of Actuaries in England in 1848. The Faculty of Actuaries was founded in Scotland in 1856, followed in the United States by the Actuarial Society of America in 1889 and the American Institute of Actuaries in 1909. In 1949 the two American organizations were merged into the Society of Actuaries.

In the beginning of the twentieth century in the United States, problems requiring actuarial treatment were emerging in sickness, disability, and casualty insurance—particularly in workers' compensation—which was introduced in 1911. The differences between the new problems and those of traditional life insurance led to the organization of the Society. Dr. I. M. Rubinow, who was responsible for the Society's formation, became its first president. The object of the Society was, and is, the promotion of actuarial and statistical science as applied to insurance other than life insurance. Such promotion is accomplished by communication with those affected by insurance, presentation and discussion of papers, attendance at seminars and workshops, collection of a library, research, and other means.

Since the problems of workers' compensation were the most urgent, many of the Society's original members played a leading part in developing the scientific basis for that line of insurance. From the beginning, however, the Society has grown constantly, not only in membership, but also in range of interest and in scientific and related contributions to all lines of insurance other than life, including automobile, liability other than automobile, fire, homeowners and commercial multiple peril, and others. These contributions are found principally in original papers prepared by members of the Society and published in the annual *Proceedings*. The presidential addresses, also published in the *Proceedings*, have called attention to the most pressing actuarial problems, some of them still unsolved, that have faced the insurance industry over the years.

The membership of the Society includes actuaries employed by insurance companies, ratemaking organizations, national brokers, accounting firms, educational institutions, state insurance departments, and the federal government; it also includes independent consultants. The Society has two classes of members, Fellows and Associates. Both classes are achieved by successful completion of examinations, which are held in May and November in various cities of the United States and Canada.

The publications of the Society and their respective prices are listed in the *Yearbook* which is published annually. The *Syllabus of Examinations* outlines the course of study recommended for the examinations. Both the *Yearbook*, at a $10 charge, and the *Syllabus of Examinations*, without charge, may be obtained upon request to the Casualty Actuarial Society, One Penn Plaza, 250 West 34th Street, New York, New York 10119.

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NOTICE

Volume LXXI, Part 1 No. 135

PROCEEDINGS

May 13, 14, 15, 16, 1984

A NOTE ON LOSS DISTRIBUTIONS

J. GARY LAROSE

VOLUME LXIX

DISCUSSION BY STEPHEN W. PHILBRICK

This review will be divided into four sections. First, there are general comments about the paper; next, there are more specific comments and suggestions regarding standardized notation; third, there is a discussion of the Bickerstaff formula; and finally, the notation is extended to other actuarial concepts.

GENERAL COMMENTS

Over the years, many papers have been written on actuarial topics which relate to loss distributions of one form or another. Each author has been free to select the notation used to represent the various concepts, and this freedom has been exercised vigorously. Although this may have resulted in compact notation for a particular paper, the overall result is a plethora of "standards" which are often inconsistent.

Mr. LaRose has attempted to create some order out of this confusion and has succeeded admirably. He has developed a notation (based on the notation originally used by Finger [1]) and applied it to a wide variety of actuarial concepts.

The author actually accomplishes two important goals. First, and most obviously, the author succeeds in defining a reasonably concise notation which can be used to clearly represent many of the important actuarial concepts related to loss distributions. One measure of success is the compactness of the notation. In most cases, the resulting formula is quite compact. In the few exceptions, such as in the case of a disappearing deductible, the resulting formula is no more obscure than that using the original notation.

Second, the use of this standardized notation clearly points out the equivalence of certain actuarial concepts. Although the author makes this point in his conclusion, I think it deserves additional emphasis. The student who encounters Part 9 for the first time should find the going much easier when it is realized that excess ratios, table M charges, excess loss ratios, ELPF's, burning ratios, and stop loss factors are all related concepts.

STANDARDIZED NOTATION

The only concern I have is that this notation might become a de facto standard, without consideration of whether any improvement could be made. The review by Mr. Hewitt included some suggestions for alternative notation; I would like to add to this discussion.

The area defined by $X1(r)$ is referred to in statistics texts as the truncated distribution (with truncation point r) [2]. Similarly, the area defined by $X2(r)$ is referred to as the censored distribution (with censorship point r). Thus, the substitution of XT and XC for $X1$ and $X2$ would provide a useful mnemonic reference. The choice for $X3$ is not as obvious, but I suggest that XS would work.

As the use of risk theory becomes more widespread, we should extend our notation beyond concepts related to means and include variance concepts. One possibility would be to introduce the variables YT, YC and YS defined as follows:

$$YT(x) = \frac{1}{\beta} \int_0^x t^2 dF(t)$$

$$YC(x) = YT(x) + \frac{t^2}{\beta} \int_x^\infty dF(t)$$

$$YS(x) = 1 - YC(x) \quad \text{where } \beta = \textstyle\int_0^\infty t^2 dF(t)$$

Another possibility would be to define these variables using $(t - \alpha)^2$ instead of t^2, so that the variables represent percentages of the total variance, rather than percentages of the total sum of squares. More research needs to be done to determine which, if either, of these two possibilities would be preferable.

BICKERSTAFF

Mr. LaRose shows how the formula for net loss cost in Mr. Bickerstaff's paper [3] can be rewritten in his notation. Unfortunately, he has perpetuated the error in the original formula.

In the original paper, a formula is developed for the net loss cost of auto physical damage coverage. The original formula is reproduced here:

$$\begin{aligned} \textit{Net Loss Cost} = AC_n[&\alpha(1 + r)^{n-1} - DG(D) \\ &- \alpha(1 + r)^{n-1}H(D) - \alpha(1 + r)^{n-1}J(Ld^{n-1}) \\ &+ Ld^{n-1}\, G(Ld^{n-1})] \end{aligned}$$

The functions G, H, and J are related to the loss cost distribution and the first moment distribution. These distributions are based upon loss costs in year 0. To develop the correct loss costs in year n, two types of adjustments are needed.

1. The mean loss cost and list price must be adjusted for inflation and depreciation, respectively. These adjustments are well documented in the original paper.
2. The deductible and list price used as input to the functions must also be adjusted for inflation. This adjustment is not as well documented.

Because the distributions themselves are not changed when used to calculate results for year n, the input values must be stated in terms of year 0. (The impact of a \$100 deductible will be different in year n than in year 0.) The correct adjustment is to divide D and Ld^{n-1} by $(1 + r)^{n-1}$.

If the tables at the end of Bickerstaff's paper are examined, it will be clear that $D/(1 + r)^{n-1}$ is used, rather than D, even though the formula does not include the adjustment.

However, it does not appear that this adjustment was made to the list price. It may be that the factor d^{n-1} is intended to include this adjustment, although that does not appear likely from the text. The correct formula, reflecting these adjustments, is as follows:

$$\begin{aligned} \textit{Net Loss Cost} = AC_n[&\alpha(1 + r)^{n-1} - DG(D/(1 + r)^{n-1}) \\ &- \alpha(1 + r)^{n-1}\, H(D/(1 + r)^{n-1}) \\ &- \alpha(1 + r)^{n-1}J(Ld^{n-1}/(1 + r)^{n-1}) \\ &+ Ld^{n-1}G(Ld^{n-1}/(1 + r)^{n-1})] \end{aligned}$$

or, expressed in Mr. LaRose's notation:

$$\begin{aligned} Net\ Loss\ Cost = AC_n[&\alpha(1 + r)^{n-1} - D[1 - F(D/(1 + r)^{n-1})] \\ &- \alpha(1 + r)^{n-1}X1(D/(1 + r)^{n-1}) \\ &- \alpha(1 + r)^{n-1}[1 - X1(Ld^{n-1}/(1 + r)^{n-1})] \\ &+ Ld^{n-1}[1 - F(Ld^{n-1}/(1 + r)^{n-1})]] \end{aligned}$$

which can be simplified to:

$$Net\ Loss\ Cost = AC_n[\alpha(1 + r)^{n-1} - D[1 - F(D/(1 + r)^{n-1})$$
$$- \alpha(1 + r)^{n-1}X1(D/(1 + r)^{n-1})$$
$$- \alpha(1 + r)^{n-1}X3(Ld^{n-1}/(1 + r)^{n-1})]]$$

OTHER ACTUARIAL CONCEPTS

1. Workers' Compensation Experience Rating

Mr. LaRose indicates that the D-ratios in workers' compensation cannot be written in his notation. Although it is slightly awkward, the D-ratio can be written at least partly in his notation.

Recall that the formula for the primary portion of each loss is as follows [4]:

$$Ap = A \quad \text{when } A \leq I$$

$$Ap = \frac{A}{A + C}(I + C) \quad \text{when } A > I$$

The D-ratio, which is the ratio of the average primary losses to average total losses, can then be written as follows:

$$\text{D-ratio} = \frac{\int_0^I xdF(x) + I\int_I^\infty dF(x) + (I + C)\int_I^\infty (x/x + C)\, dF(x)}{\int_0^\infty xdF(x)}$$

The first two terms are $X2(I)$, so we can rewrite the formula as:

$$\text{D-ratio} = X2(I) + (I + C)\frac{\int_I^\infty (x/(x + C))\, dF(x)}{\int_0^\infty xdF(x)}$$

2. *Fratello Formula*

Subsequent to the completion of his paper, Mr. LaRose also used his notation to express the formula in Fratello's paper [5]. The results are shown below. It should be noted that, while the notation was originally used to study *loss* distributions, it can also be used to study other types of distributions as well (e.g., wage distributions as in Fratello).

Let α = average weekly wage
p = nominal % of compensation
A = minimum weekly benefit/p
B = maximum weekly benefit/p
$a = A/\alpha$
$b = B/\alpha$
t = weekly wage of a worker
$F(t)$ = c.d.f. of t

then, the limit factor is

$$X2(b) - X1(a) + a$$

3. *Table L*

The formulae used in Table L can be considered an extension of those used in Table M with the added consideration of individual loss limitations. However, a minor change to the LaRose notation is needed to express these formulae. If we write the expression for $X1(r)$ with the denominator written out, we have

$$X1(r) = \frac{\int_0^r xdF(x)}{\int_0^\infty xdF(x)}$$

Note in particular that the distribution, used in the numerator and denominator are identical.

If we examine the formulae used by Skurnick [6], we find that the denominator has been omitted (as it is equal to 1).

$$\psi^*(r) = \int_0^r (r - s)dF^*(S)$$

However, the omitted denominator is <u>not</u> $\int_0^\infty sdF^*(s)$ but $\int_0^\infty sdF(s)$. Here, the distributions in the numerator and denominator are different. We can overcome this by defining a new set of distributions as follows:

$$X1^*(x) = \frac{\int_0^x tdF^*(t)}{\int_0^\infty tdF(t)}$$

$$X2^*(x) = X1^*(x) + \frac{x(1 - F^*(x)}{\int_0^\infty tdF(t)}$$

$$X3^*(x) = \frac{\int_x^\infty (t - x)dF^*(t)}{\int_0^\infty tdF(t)}$$

In the specific case of Table L, the denominators are identically 1, so they may be omitted.

Now we can restate the Skurnick formulae in terms of this notation:

$$\begin{aligned}\phi^*(r) &= \int_r^\infty (s - r)dF^*(s) + k \\ &= X3^*(r) + k\end{aligned}$$

$$\begin{aligned}\psi^*(r) &= \int_0^\infty (r - s)dF^*(s) \\ &= rF^*(r) - X1^*(r) \\ &= r - X2^*(r)\end{aligned}$$

The relationship between the charge and the savings can also be derived. However, note that the relationship between $X3$ and $X2$ is slightly changed when we work with $X3^*$ and $X2^*$

$$\begin{aligned}X3^*(r) &= \int_r^\infty (t - r)dF^*(t) \\ &= \int_r^\infty tdF^*(t) - r\int_r^\infty dF^*(t) \\ &= 1 - k - X1^*(r) - r(1 - F^*(r)) \\ &= 1 - k - X2^*(r)\end{aligned}$$

Thus,

$$\begin{aligned}\phi^*(r) &= X3^*(r) + k \\ &= 1 - X2^*(r) \\ &= 1 - r + \psi^*(r)\end{aligned}$$

To be consistent with the notation I proposed earlier, I would suggest using *XTL*, *XCL*, and *XSL* instead of $X1^*$, $X2^*$, and $X3^*$ respectively, where *L* could be a mnemonic for either loss limitation or Table L.

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2. Kendall and Stuart, *The Advanced Theory of Statistics,* Volume 2, The MacMillan Company, New York, p. 551.
3. D. R. Bickerstaff, "Automobile Collision Deductibles and Repair Cost Groups: The Lognormal Model," *PCAS* LIX, 1972, p. 68.
4. R. H. Snader, "Fundamentals of Individual Risk Rating and Related Topics," CAS Study Note, Part III.
5. B. Fratello, "The Workmen's Compensation Injury Table and Standard Wage Distribution Table—Their Development and Use in Workmen's Compensation Ratemaking," *PCAS* XLII, 1955, p. 110.
6. D. Skurnick, "The California Table L," *PCAS* LXI, 1974, p. 117.

DURATION

BY RON FERGUSON

VOLUME LXX

DISCUSSION BY STEPHEN D'ARCY

Ron Ferguson has performed a valuable service to the CAS by encouraging actuaries to focus one eye on the investment side of insurance operations while keeping the other eye (hopefully the good one) on familiar underwriting terrain. Bond duration is an important component of investment performance and actuaries should be familiar with this concept. The explanations, examples, formulae, and references included in the paper provide the reader with a grasp of the fundamentals of duration and adequately achieve the objectives of this work. This discussion will expand on some of the weaknesses of the duration concept, propose an alternative investment strategy, and develop a procedure for calculating the duration of loss reserves.

Whereas an understanding of duration is essential to understand bond portfolio management, use of duration in practice does not assure investment success. Ferguson discusses some of the drawbacks of applying duration to immunize an investment portfolio, including the absence of long duration bonds; the need for continuously rebalancing the portfolio as time elapses and interest rates change; and the complications and costs introduced by call features, sinking funds, transaction costs, and taxes. A further, and more serious, disadvantage of duration results from the motivating factor behind duration. Duration is a useful concept when an investor's objective is to achieve a targeted nominal wealth position in the future regardless of interim interest rate changes. If interest rates fall so that cash flows generated by the investment are reinvested at lower-than-expected interest rates, then the value of the initial investment immediately rises to reflect the market value of an investment producing a stream of income above the new interest rate. This premium over the face value of the bond gradually reduces as the bond approaches maturity. However, since the bond matures after the time the wealth is needed under a duration-based investment strategy, the premium at that time is sufficient to offset the lower reinvestment returns. Conversely, an interim rise in interest rate produces greater reinvestment returns than expected, but those gains are offset by the discount from face value of the bond that remains at the time the wealth is needed. Under either condition, the terminal wealth position is at or near the target level.

For insurers, though, the amount of wealth required at a future date is not always independent of economic conditions. The value of losses payable in the future may be determined in part by the inflation rate prior to the time losses are paid. Inflation, which affects interest rates, may also affect the wealth needed. An investment strategy based on duration is intended to preserve nominal wealth, and not real wealth or purchasing power. Duration is a useful investment strategy only when the terminal wealth target is invariant with inflation. Although this is the case for some situations, such as total losses on stated value contracts, losses in excess of policy limits, and claim payments being processed for repairs already settled, not all loss settlements are independent of inflation that occurs subsequent to the date of loss and prior to the claim payment. The following situation describes the opposite extreme under which inflation has a direct effect on the loss settlement value.

Consider a simple example in which an insurer is reserving a claim for a class action suit against a drug manufacturer involving a product alleged to cause unintended side effects. The insurer estimates the cost of settlement (excluding interim loss adjustment expenses) at $10,000,000 and expects the claim to be settled in five years. Under current accounting procedures the insurer would establish a loss reserve of $10,000,000 for this claim. However, if management wanted to know how much cash had to be set aside now to cover the claim, a lower figure would be determined. Assuming the insurer wanted to minimize default risk by investing in U.S. Treasury issues and ignoring taxes (which may not be unreasonable in light of current tax loss carry forwards), the insurer could face a yield curve as illustrated in Table 1. The interest rate available on five year Treasury issues is 13.5 percent. If the insurer were to make the naive assumption that an investment in Treasury bonds that have a maturity of five years would alleviate all investment concerns, a problem arises in determining the proper discount rate. Discounting the claim at 13.5 percent for five years produces a present value of the claim of $5,309,097 ($10{,}000{,}000/(1.135)^5$). However, if the insurer followed what will be termed the maturity investment strategy of investing the present value of the claim in a five year issue, and reinvesting the interest payments when received for the time remaining in the five year period, the company will not achieve a $10,000,000 wealth position in five years if interest rates remain at current levels. The actual wealth position of the insurer in five years is shown in Table 2. For this calculation the convention used in Ferguson's paper, that interest is paid annually at the end of each year, is adopted. Interest received on the initial investment and subsequent reinvestments are invested at yields below 13.5 percent since the current yield curve is upward sloping (as it normally is), as shown in Table 1.

Table 2 illustrates that interest of $716,728 (.135 × 5,309,097) will be received at the end of the first year and reinvested at 13.3 percent for four years. At the end of the second year interest of $812,053 (.135 × 5,309,047 + .133 × 716,728) will be received and reinvested at 13.2 percent. The total amount available to the insurer at the end of five years is $9,956,402—and not $10,000,000—as a result of reinvestment of interest at rates lower than 13.5 percent. This $43,598 shortfall can be eliminated by investing $5,332,346 under the maturity investment strategy and, if current rates hold, $10,000,002 will be available in five years (Table 3). The proper discount rate should reflect the knowledge that the reinvestment rates are lower than the initial investment rate.

A naive duration strategy, without any rebalancing as time passes, can be adopted to eliminate the shortfall illustrated in Table 2 without any additional initial investment. If the insurer invests $5,309,097 in Treasury issues with a duration of five years rather than a maturity of five years, and reinvests each interest payment for the balance of the five year period,[1] the wealth position at the end of the five year period will be $10,021,098 (Table 4). The insurer initially purchases a 7.13 year issue, currently yielding 13.5 percent, which produces the same interest income stream as shown in Table 2. However, the initial investment would be worth $5,373,793 after five years as it represents a 2.13 year to maturity issue yielding 13.5 percent when the rate for this maturity issue is 12.85 percent (interpolated from the yield curve).[2]

Thus, duration can be used to assure the targeted wealth position if the yield curve does not shift. However, the motivating factor for duration is to assure that the targeted position is achieved despite changes in interest rates. For example, assume that interest rates increase across the entire yield curve by 7.5 percentage points immediately after the initial investment is made, and remain at the higher levels for the entire claim settlement period. Under the naive duration investment strategy, portfolio adjustments are not made despite the higher interest rates. Although this investment is not immunized against further changes in the interest rates, this example is only concerned with the effect of one sudden interest rate shift. The results are shown in Table 5.

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[1] The insurer could take advantage of the interest reinvestments to rebalance the duration closer to the remaining number of years in the claim period, but this method would complicate the example without much additional benefit.

[2] The formula for the price of a bond is

$$P = \sum_{t=1}^{n} \frac{CF_t}{(1 + y)^t}$$

The insurer would reinvest the interest at rates higher than expected, earning greater interest on interest. However, the value of the initial investment at the end of five years declines to $4,731,419 since it is paying below market rates for the remaining 2.13 years. The effects tend to cancel out, but leave the insurer slightly ($119,728) above the target. A maturity investment strategy would perform better than the duration strategy under increasing interest rates (and worse under declining rates) since the initial investment matures at the end of the five years avoiding the capital loss, whereas the reinvested interest would earn the higher than expected rates. As shown in Table 6, an investment of $5,332,346 for a five year term generates a terminal wealth position of $10,744,254 if interest rates were to increase 7.5 percentage points.

If the only goal of an insurer's investment policy were to generate a targeted wealth level at a given time, duration would be a useful strategy. However, for most situations insurers face the risk of claim settlement amount and time. For the example of the class action suit, the $10,000,000 loss reserve includes consideration of expected inflation over the settlement period. The final settlement will likely consist of specific damages, primarily medical costs, and general damages. Both values tend to increase with inflation, although obtaining an index to measure and project these changes has proven difficult.[3] Prior research has incorporated a proportional value between 0 and 1 that represents the inflation-sensitive component of loss reserves.[4] This value varies by line of business and over time. This review illustrates the extreme case under which inflation in claim costs is the same as the general rate of inflation. Based on finance theory, short term nominal interest rates are highly correlated with expected inflation rates. A good fit has been obtained for a 2 to 2.5 percentage point differential between short term U.S. Treasury issues and expected changes in the consumer price index.[5] However, expected inflation rates do not always correspond with experienced inflation rates, and substantial year to year variation from the normal differential occurs.

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[3] Norton E. Masterson, "Economic Factors in Property/Liability Insurance Claim Costs," *Best's Review Property/Casualty Insurance Edition*, Vol. 85, No. 2 (June, 1984), pp. 68–70.

[4] Robert P. Butsic, "The Effect of Inflation on Losses and Premiums for Property-Liability Insurers," *Casualty Actuarial Society Discussion Paper Program*, 1981, pp. 58–102; H. R. Folger, "Bond Portfolio Immunization, Inflation, and the Fisher Equation," *Journal of Risk and Insurance*, Vol. LI, No. 2 (June, 1984), pp. 244–264.

[5] W. E. Gibson, "Interest Rates and Inflationary Expectations: New Evidence," *American Economic Review*, Vol. 57 (December, 1972), pp. 854–865.

Accepting the accuracy of inflation expectations and the normal yield-inflation differential, the current short term interest rate of 9.7 percent for one month Treasury issues translates into an expected inflation rate of approximately 7.5 percent. The \$10,000,000 loss reserve should embody an inflation rate of 7.5 percent. If interest rates were to increase by 7.5 percentage points, the shift would most likely be caused by an equal increase in the expected inflation rate. The claim settlement would increase to \$14,010,282 ($10{,}000{,}000 \times (1.15)^5/(1.075)^5$). Under this circumstance, the naive duration strategy would generate a shortfall of \$3,890,554 since the "target" increased \$4,010,282. The maturity investment strategy performs only marginally better, with a shortfall of \$3,266,028.

Insurers can reduce the risk of inflation-driven claim settlements increasing beyond the level of funds dedicated to compensate them by adopting an alternative investment strategy. If the insurer were to invest all the initial capital to pay the claim short term, rather than for 5 or 7.13 years, all the proceeds could be reinvested at the current interest rates when rates change. This strategy outperforms the other investment strategies when investment rates rise and underperforms when the interest rates fall. However, rising or falling interest rates are likely to correspond with similar changes in the claim settlement value.

As short term rates yield 9.7 percent, the insurer would have to set aside \$6,294,582 ($10{,}000{,}000/(1.097)^5$) to generate \$10,000,000 in five years. This amount exceeds the maturity investment strategy by \$962,236 and the naive duration strategy by \$985,485, since one month Treasury rates are below longer term rates. The results of an instantaneous increase in interest rates by 7.5 percentage points immediately after the initial one month investment is made are illustrated in Table 7. The shortfall from the claim settlement inflated at a 15 percent rate is \$162,638, which is much less than the shortfall under the other investing strategies. This shortage occurs in part (\$71,256) since the insurer is locked into the initial 9.7 percent rate for one month with the remainder caused by the relationship between the increase in inflation and interest under a constant differential. Inflation increased 100 percent (7.5 to 15) whereas interest rates increased 77.3 percent (9.7 to 17.2).

Although the author believes a large increase in interest rates is more likely than a large decline, an interest rate drop is not inconceivable. For balance, the results of maturity, naive duration, and short term investing strategies under an instantaneous reduction in interest rates and inflation of 7.5 percentage points are shown in Tables 8, 9, and 10. The naive duration strategy produces both the highest terminal wealth position, \$10,164,134, and the one closest to

$10,000,000. Short term investing produces the lowest wealth, $7,059,331. However, if inflation were to decline 7.5 percentage points, the expected inflation rate would be 0, thus producing a claim settlement of $6,965,586 $(10{,}000{,}000/(1.075)^5)$. Thus, the short term investing strategy would produce a position closest to the final claim settlement.

The three investment strategies are compared on Table 11. Short term investing requires the greatest initial outlay of capital but always produces the terminal wealth position closest to the claim settlement. It is the most profitable investment strategy only if interest rates increase. The naive duration strategy requires the lowest initial outlay and produces the terminal wealth position closest to $10,000,000 if interest rates change, and produces the greatest wealth position if interest rates remain level or decline. However, this strategy produces the lowest terminal wealth if interest rates increase.

The other loss settlement risk faced by insurers is the timing of the settlement. Under the short term investing strategy, capital is always readily available. Under longer term investing if the claim is settled prior to the expected time, the bonds would have to be sold (or other capital diverted from investment) for which a capital gain or loss could occur depending on the direction in the change of interest rates. An early settlement coupled with higher interest and inflation rates would require the insurer to assume a capital loss on the initial investment simultaneously with a loss settlement in excess of the expected level.

Both the original paper and this review have concentrated on the use of duration for specific large claims. A far more common consideration for insurers is the development of an investment strategy to apply to the entire loss reserve. The formula for duration is:

$$\text{Duration} = \frac{\displaystyle\sum_{t=1}^{n} \frac{tCF_t}{(1+y)^t}}{\displaystyle\sum_{t=1}^{n} \frac{CF_t}{(1+y)^t}}$$

where CF_t = cash flow in year t
y = discount rate
t = year of cash flow
n = last year of cash flow

This formula can be applied to cash outflows (loss payments) just as readily as to cash inflows (investments).

The duration of a loss reserve will vary by insurer depending upon line of business mix and loss payment patterns. An example for automobile liability, the major component of loss reserves for the industry, is illustrated below. The payout ratios are derived from aggregate data published by Best's on Schedule P development for 200 representative insurers.[6] Based on the aggregate data, extrapolated until all losses are paid, the automobile liability payment development pattern is illustrated in Table 12. The current estimates of ultimate incurred losses by accident year are shown in Table 13.

The following notation is adopted for calculating the duration of the loss reserve:

P_i = percentage of ultimate incurred losses paid at the end of development year i
$\rho_i = P_i - P_{i-1}$ = percentage of ultimate incurred losses paid in development year i
L_x = ultimate incurred losses for accident year x
CF_t = cash flow (paid losses) in year t
a = latest accident year
y = discount rate

The future claim payments paid by year are projected as follows:

$$CF_{a+i} = \sum_{i=1}^{7} \sum_{j=1}^{7} L_{a+i-j}\rho_{i+j}$$

To determine the claims to be paid in 1983, sum the products of the 1982 accident year incurred losses multiplied by the percentage of incurred losses paid in development year 2, plus the 1981 accident year incurred losses multiplied by the percentage of incurred losses paid in development year 3, and so forth, through the 1976 accident year incurred losses multiplied by the percentage of incurred losses paid in development year 8. To determine claims to be paid in 1984, sum the product of the 1982 accident year losses multiplied by the percentage of incurred losses paid in development year 3, plus the 1981 accident year incurred losses multiplied by the percentage of incurred losses paid in development year 4, and so forth, through the 1977 accident year losses

[6] A. M. Best Company, "Casualty Loss Reserve Development," *Best's Insurance Management Reports Statistical Studies Property/Casualty*, Release Number 2 (January 23, 1984), p. 3.

paid in development year 8. Similarly, claims paid in 1985 through 1989 are determined. Performing these calculations produces the following cash flow:

Year	Cash Flow
1983	$12,249,322
1984	6,658,051
1985	4,022,837
1986	2,305,210
1987	1,274,849
1988	649,402
1989	257,541
Total	$27,417,212

The duration of this cash flow depends on the discount rate selected. Since the losses paid in a given year are not paid at the end of the year, as is assumed for bond investments, but paid throughout the year, the formula for determining the duration of this cash flow is:

$$\text{Duration} = \frac{\sum_{t=1}^{7} \frac{(t - \frac{1}{2})CF_t}{(1 + y)^{t-1/2}}}{\sum_{t=1}^{n} \frac{CF_t}{(1 + y)^{t-1/2}}}$$

The durations for automobile liability loss reserves for various discount rates are shown on Table 14. The longest duration, assuming a 0 percent discount rate, is only 1.65 years. Therefore, even a duration investing strategy for automobile liability reserves would suggest investing in relatively short maturity bonds.[7]

At the end of 1982, the property-liability insurance industry held 54.2 percent of its assets in bonds, and 58.6 percent of these bonds, or 31.8 percent

[7] A duration of 1.65 years can be achieved either by purchasing bonds with a maturity of approximately two years (the exact maturity depends on the interest rate) or by selecting a portfolio of bonds with different maturities such that the income generated by interest and maturing bonds matches the liabilities as these come due. Ferguson describes the latter case as cash flow matching. Both approaches depend on the liability not changing with inflation, as well as the other limitations of duration described by Ferguson and on the first page of this discussion.

of total assets, had maturities of over ten years.[8] This long term investment strategy has a high degree of risk. An increase in interest rate levels would reduce the market value of the bond portfolio. Loss reserves would either be unchanged, if inflation after the loss is reported does not affect the settlement, or increase in some proportion to the inflation rate. This discussion illustrates the situation where losses increase directly with inflation. If an insurer expects that its loss reserve estimates are adequate to pay all claims incurred to date regardless of future inflation rates, the company should adopt a duration investment strategy to avoid this potential risk. If claim settlements on these losses can be affected by future inflation, a short term investing strategy should be adopted. Under either condition, maturities should be reduced unless the insurer is willing to bet its solvency on the belief that interest rates and inflation will not increase.

[8] A. M. Best Company, "1982 Property/Casualty Bond Holdings," *Best's Insurance Management Reports Statistical Studies Property/Casualty,* Release Number 23 (December 19, 1983), p. 1.

TABLE 1

Representative Yield Curve
U.S. Treasury Issues in June, 1984

Investment Period	Yield
1 month	9.7%
3 months	10.0
6 months	11.3
9 months	11.9
1 year	12.1
1½ years	12.7
2 years	12.8
2½ years	13.0
3 years	13.2
3½ years	13.3
4 years	13.3
4½ years	13.5
5 years	13.5
6 years	13.5
7 years	13.5
8 years	13.5
9 years	13.5
10 years	13.5
20 years	13.5

Source: *Wall Street Journal,* "Treasury Issues/Bonds, Notes & Bills" (June 13, 1984), p. 37.

TABLE 2

MATURITY INVESTING—LEVEL INTEREST RATES
$5,309,097 INVESTED AT 13.5% FOR FIVE YEARS

Year	Interest Received		Reinvestment Period	Reinvestment Rate
1	$ 716,728		4 years	13.3%
2	812,053		3 years	13.2
3	919,244		2 years	12.8
4	1,036,907		1 year	12.1
5	1,162,373		—	—
	5,309,097	Initial investment		
	$9,956,402	Terminal wealth		

TABLE 3

MATURITY INVESTING—LEVEL INTEREST RATE
$5,332,346 INVESTED AT 13.5% FOR FIVE YEARS

Year	Interest Received		Reinvestment Period	Reinvestment Rate
1	$ 719,867		4 years	13.3%
2	815,609		3 years	13.2
3	923,269		2 years	12.8
4	1,041,448		1 year	12.1
5	1,167,463		—	—
	5,332,346	Initial investment		
	$10,000,002	Terminal wealth		

TABLE 4

Naive Duration Investing—Level Interest Rates
$5,309,097 invested at 13.5% for 7.13 years

Year	Interest Received		Reinvestment Period	Reinvestment Rate
1	$ 716,728		4 years	13.3%
2	812,053		3 years	13.2
3	919,244		2 years	12.8
4	1,036,907		1 year	12.1
5	1,162,373		—	—
	5,373,793*	Initial investment		
	$10,021,098	Terminal wealth		

$$* P = \frac{716{,}728}{1.1285} + \frac{716{,}728}{(1.1285)^2} + \frac{.13(716{,}728)}{(1.1285)^{2.13}} + \frac{5{,}309{,}097}{(1.1285)^{2.13}}$$

TABLE 5

Naive Duration Investing-Interest Rates Increase 7.5 Points
$5,309,097 invested at 13.5% for 7.13 years

Year	Interest Received		Reinvestment Period	Reinvestment Rate
1	$ 716,728		4 years	20.8%
2	865,807		3 years	20.7
3	1,045,029		2 years	20.3
4	1,257,170		1 year	19.6
5	1,503,575		—	—
	4,731,419*	Initial investment		
	$10,119,728	Terminal wealth		

$$* P = \frac{716{,}728}{1.2035} + \frac{716{,}728}{(1.2035)^2} + \frac{.13(716{,}728)}{(1.2035)^{2.13}} + \frac{5{,}309{,}097}{(1.2035)^{2.13}}$$

TABLE 6

Maturity Investing—Interest Rates Increase 7.5 Points
$5,332,346 invested at 13.5% for five years

Year	Interest Received		Reinvestment Period	Reinvestment Rate
1	$ 719,867		4 years	20.8%
2	869,599		3 years	20.7
3	1,049,606		2 years	20.3
4	1,262,676		1 year	19.6
5	1,510,160		—	—
	5,332,346	Initial investment		
	$10,744,254	Terminal wealth		

TABLE 7

Short Term Investing—Interest Rates Increase 7.5 Points
$6,294,582 invested at 9.7% for one month

Year	Amount Available for Reinvestment		Reinvestment Period	Reinvestment Rate
1	$ 7,339,483*		1 month	17.2%
2	8,601,874		1 month	17.2
3	10,081,396		1 month	17.2
4	11,815,396		1 month	17.2
5	13,847,644		1 month	17.2
	$13,847,644	Terminal wealth		

*Assumes one month at 9.7%, 11 months at 17.2% for 16.6% average during initial year.

TABLE 8

MATURITY INVESTING—INTEREST RATES DECLINE 7.5 POINTS
$5,332,346 INVESTED AT 13.5% FOR FIVE YEARS

Year	Interest Received		Reinvestment Period	Reinvestment Rate
1	$ 719,867		4 years	5.8%
2	761,619		3 years	5.7
3	805,031		2 years	5.3
4	847,698		1 year	4.6
5	886,692		—	—
	5,332,346	Initial investment		
	$9,353,253	Terminal wealth		

TABLE 9

NAIVE DURATION INVESTING—INTEREST RATES DECLINE 7.5 POINTS
$5,309,097 INVESTED AT 13.5% FOR 7.13 YEARS

Year	Interest Received		Reinvestment Period	Reinvestment Rate
1	$ 716,728		4 years	5.8%
2	758,298		3 years	5.7
3	801,521		2 years	5.3
4	844,002		1 year	4.6
5	882,826		—	—
	6,160,759*	Initial investment		
	$10,164,134	Total wealth		

$$* \; P = \frac{716{,}728}{1.0535} + \frac{716{,}728}{(1.0535)^2} + \frac{.13(716{,}728)}{(1.0535)^{2.13}} + \frac{5{,}309{,}097}{(1.0535)^{2.13}}$$

TABLE 10

Short Term Investing—Interest Rates Decline 7.5 Points
$6,294,582 invested at 9.7% for one month

Year	Amount Available for Reinvestment		Reinvestment Period	Reinvestment Rate
1	$6,470,830*		1 month	2.2%
2	6,613,188		1 month	2.2%
3	6,758,678		1 month	2.2%
4	6,907,369		1 month	2.2%
5	7,059,331		1 month	2.2%
	$7,059,331	Terminal wealth		

*Assumes one month at 9.7%, 11 months at 2.2% for 2.8% average during initial year.

TABLE 11

COMPARISON OF ADEQUACY OF TERMINAL WEALTH POSITIONS

		Level Rates Claim = $10,000,000		7.5 Point Increase Claim = $14,010,282		7.5 Point Decline Claim = $6,965,586	
Investment Strategy	Amount Invested	Terminal Wealth	Wealth-Claim	Terminal Wealth	Wealth-Claim	Terminal Wealth	Wealth-Claim
Maturity	$5,332,346	$10,000,002	$2	$10,744,254	−$3,266,028	$9,353,253	$2,387,667
Naive Duration	5,309,097	10,021,098	21,098	10,119,728	−3,890,554	10,164,134	3,198,548
Short Term	6,294,582	10,000,000	0	13,847,644	−162,638	7,059,331	93,745

TABLE 12

INDUSTRY PAYMENT DEVELOPMENT PATTERN—
AUTOMOBILE LIABILITY

Year of Payment	Symbol	Percentage of Ultimate Losses Paid
Accident year	ρ_1	36.80%
AY + 1	ρ_2	28.76
AY + 2	ρ_3	13.93
AY + 3	ρ_4	8.93
AY + 4	ρ_5	5.30
AY + 5*	ρ_6	3.18
AY + 6*	ρ_7	1.91
AY + 7*	ρ_8	1.19
Total		100.00%

*Projected at 60 percent of prior year's factor.
Source: A. M. Best Company, "Casualty Loss Reserve Development," *Best's Insurance Management Reports Statistical Studies Property/Casualty*, Release Number 2 (January 23, 1984), p.3.

TABLE 13

CURRENT ESTIMATE OF
ULTIMATE INCURRED LOSSES—
AUTOMOBILE LIABILITY

Accident Year	Ultimate Losses
1982	$21,642,097
1981	19,835,157
1980	17,460,403
1979	16,296,350
1978	14,490,255
1977	12,742,717
1976*	11,337,903

*Prior year estimated.
Source: A. M. Best Company, "Casualty Loss Reserve Development," *Best's Insurance Management Reports Statistical Studies Property/Casualty* Release Number 2 (January 23, 1984), p. 3

TABLE 14

DURATIONS OF AUTO LIABILITY LOSS RESERVES UNDER DIFFERENT DISCOUNT RATES

Year	Cash Flow
1	$12,249,322
2	6,658,051
3	4,022,837
4	2,305,210
5	1,274,849
6	649,402
7	257,541
Total	$27,417,212

Discount Rate	Duration
0%	1.65 years
5	1.56
10	1.48
15	1.41
20	1.35

TRANSFORMED BETA AND GAMMA DISTRIBUTIONS AND AGGREGATE LOSSES

GARY VENTER

VOLUME LXX

DISCUSSION BY ORIN M. LINDEN AND FRED KLINKER

One of the most important problems in collective risk theory has been the computation of the distribution of aggregate losses given individual frequency and severity distributions. Various approaches have been tried since the subject was first introduced by Filip Lundberg more than seventy-five years ago (Cramér [1]). These include approximation, simulation, and actual computation using numerical techniques. (A stochastic approach is also possible and the reviewers hope to discuss this in a later paper.) Approximations have been used with mixed success over the years. An appeal to the central limit theorem "justifies" a normal approximation if the number of claims is large (Beard, Pentikäinen, Pesonen [2]). This has not been satisfactory. Other approximations, such as normal power, Esscher, Gamma, Pareto, and just about any other distribution, have been used based on various theoretical (we can "prove" it) or empirical (it works) arguments. The use of these approximations has not been entirely satisfactory. The reviewers offer a reason for this later.

Another approach, the so called Monte Carlo simulation method, gives much better results. (For an elementary discussion of simulation see Gordon [3].) Simulation gives much better results but has three major drawbacks. First, it can be extraordinarily expensive in computer time, especially with large frequencies. Second, it's subject to the "whims" of the random number generators used. Third, it offers little insight into why a distribution behaves as it does. It has, however, been used very successfully and, up until very recently, it was the best alternative available in most cases.

In the last year or so two very good techniques have been introduced. The first, using a discrete density for the severities, uses a recursive formula and computes the aggregate loss density directly (Panjer [4]). The second, using a piece-wise linear severity, inverts the characteristic function of the distribution (Meyers and Heckman [5]). Both of these methods use numerical techniques. While the reviewers have not used these methods, we do feel that they are very good and that the problems associated with them are decidedly minor.

Despite Panjer's, and Meyers's and Heckman's results, there are very good reasons to have a good approximation formula. It's simple, quick, easy to use, and requires little mathematical knowledge to understand. In addition, for some applications, it's just as good as other techniques. Thus, a pricing formula may often be programmed into a hand calculator. In his paper Gary Venter proposes such an approximation using what he calls the Transformed Gamma Distribution (TGD). By adding a third parameter, α, to the ordinary gamma distribution the author can match up to three moments of the actual distribution. He writes down all the necessary formulas to compute the distribution and, as an example, applies it to the computation of excess ratios used to price aggregate stop loss insurance. The author then introduces the Transformed Beta Distribution (TBD) and explains that the combination of a TGD with a gamma, done in a certain way, produces a TBD. (This is similar to the combining of a Poisson frequency with a gamma to produce a negative binomial frequency.) This property is used to model one form of parameter uncertainty (that of λ). Going back to his prior example the author shows how incorporating such risk into his model almost doubles the expected excess loss over \$1,000,000 in this case. Finally, the author compares the TGD to the more exact computations provided by Meyers and Heckman. The TGD itself, while not fitting badly, doesn't fit extremely well either. However, the excess ratios computed from the fitted TGD are extremely close to the exact methods. We will comment on these two statements shortly and show how a much closer fit to the distribution may be obtained by using a sum of TGD's.

The paper provides a large amount of useful information. APL programs are presented to do most of the necessary computations including the solving of two simultaneous equations. The reviewers used these programs and had no trouble reproducing any of the work in the paper. The incomplete gamma program is especially nice to have. A discussion of Gaussian quadrature, for numerical integration, appears in Appendix F. These features make the paper a useful reference document.

Before getting to the heart of our review we will make a few remarks.

The author comments that to use the TGD the skewness must be greater than the coefficient of variation. We did not investigate this. If the author has a reason for this we'd like to see it. In any event this doesn't seem to be a large limitation. All the distributions we've used recently have had this property.

The part of the paper we find least convincing is the section dealing with parameter risk. The author seems very impressed with the transforming of a TGD into a TBD. So much, in fact, that he makes the assumption that λ is transformed gamma distributed. He is content to ignore uncertainty in α and r. This seems to be a somewhat artificial assumption. (It does, however, simplify the computations.) The expected value of the TGD is given by $E(X) = \Gamma(r + (1/\alpha))/\lambda\Gamma(r)$. Thus, a smaller λ implies a larger expected loss. Since most insurers don't go broke and most risks don't produce extraordinarily large losses, we would expect most λ's to be near or larger than the expected value of λ. That is we expect $P(\lambda > E(\lambda)) > .5$. Using the parameters in the example we compute $P(\lambda > 1.144\text{E-}6) = 1 - G(1.144\text{E-}6, 2.597, 1.47, 1{,}288{,}500) = .65$. This result is expected and calms the mind somewhat but we would have expected a larger percentage intuitively. We also believe uncertainty in α and r should be considered. Of course to do so would greatly complicate the calculations.

Earlier on we commented on the fit of a TGD to the actual distribution. Looking at the cumulative distribution offers no insight into the nature of the errors. We argue that, in general, the TGD, TBD, or any other mono-modal density can't fit the aggregate density function very well due to the presence of multiple modes on the density. (By this we do not mean the possibility of having zero loss with positive probability. This spike at the origin is properly accounted for by the author's model.) Exhibit I plots the actual density, from Exhibit 3 of the paper, against the transformed gamma approximation. The differences, due to the modes, are obvious. Exhibit II gives an even more severe case. Both of these distributions resemble those we've used.

We also show in the exhibits a modified TGD we've invented which retains much of the simplicity of the original model yet does a much better job in explaining the modes of the distribution. The actual model we used is

$$(1)\quad \tilde{F}(x) = \sum_{n=0}^{[x/m]} Q(n)\,[P(0|n)\pi(x - nm) + (1 - P(0|n))G(x - nm;\, r_n,\, \alpha_n,\, \lambda_n)]$$

Notation is as follows:

m	=	maximum possible loss per occurrence
$Q(n)$	=	probability of n occurrences of size m (total losses) in a time period
$P(0\|n)$	=	probability of no occurrences of size less than m (partial losses) given n total losses
$\pi(x)$	=	$\begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$
$G(x; r, \alpha, \lambda)$	=	the TGD

Appendix II describes the method of fitting the $(r_n, \alpha_n, \lambda_n)$'s. Appendix I gives formulas for $Q(n)$ and $P(0|n)$ for Poisson and negative binomial frequencies. Note that the above sum requires a maximum of $1 + [L/m]$ terms where L is the excess loss limit. In general no more than five terms are needed. All terms in the sum are readily calculable with just a little more programming than is necessary to compute $G(x; r, \alpha, \lambda)$ alone. In many cases $P(0|n)$ is small and thus the π-terms can be ignored. However, the required programming is so simple it's not necessary to do so.

The reviewers applied the above model to the cases shown in Exhibits I and II. A glance at these exhibits clearly indicates a substantial increase in accuracy. In particular, this approximation is able to pick up the multi-modal behavior of the aggregate density function. This is something that both the TGD and the TBD could not do.

We note that parameter risk can be considered in a way similar to that used in the paper. As will be seen later, $\tilde{F}$ has a very simple form in the case of a Poisson frequency. Thus, it is particularly simple to incorporate parameter risk. However, due to time constraints, we did not investigate this.

For completeness we consider the computation of excess ratios. Exhibits IV and V show comparisons of actual excess ratios to those computed from the TGD approximation and our modified TGD approximation. (Formulas to do the calculations for the modified TGD appear in Appendix III.) A look at these exhibits indicates that there is not much difference in using any of the three methods.

This result puzzled us at first, so we tried a fit to two other curves, a Pareto and a normal (see Exhibit IV). Excess ratios computed from the normal were also very close to the actual ratios. However, the more highly skewed Pareto provided ratios that were generally much higher. We speculate that the integral involved in the definition of the excess ratio smooths things out significantly, so that as long as the approximating curve isn't too highly skewed the formula for excess ratios is very robust. The performance of the Pareto supports this.

The form of our modified TGD is indicated by understanding the causes of multi-modality in the aggregate density. To do this we define additional notation as follows:

$S(X)$ = probability of an occurrence $< x$
P = probability of having an occurrence of at least size $M = 1 - S(M)$
$S_1(X) = \begin{cases} S(x)/(1-p) & x < m \\ 1 & x \geq m \end{cases}$
$n*$ as a superscript represents nth convolution
$P(n|n_0)$ = probability of n partial losses given n_0 total losses.

With the above notation the aggregate loss distribution is given by

$$F(X) = \sum_{n=0}^{\infty} P(n)S^{n^*}(x)$$

(Note that $S^{0^*}(x) = \pi(x)$)

In the following we consider separately the effects of partial losses and total losses. Clearly the conditional distribution of aggregate losses, given n partial losses and n_0 total losses, is given by $S_1^{n^*}(x - n_0m)$. Thus $F(X)$ can be written

$$F(X) = \sum_{n_0=0}^{\infty} Q(n_0)\left[\sum_{n=0}^{\infty} P(n|n_0)S_1^{n^*}(x - n_0m)\right]$$

$$= \sum_{n_0=0}^{\infty} Q(n_0)\left[P(0|n_0)\pi(x - n_0m) + \sum_{n=1}^{\infty} P(n|n_0)S_1^{n^*}(x - n_0m)\right]$$

Define $G(x|n_0) \equiv \dfrac{\sum_{n=1}^{\infty} P(n|n_0)S_1^{n^*}(x)}{1 - P(0|n_0)}$

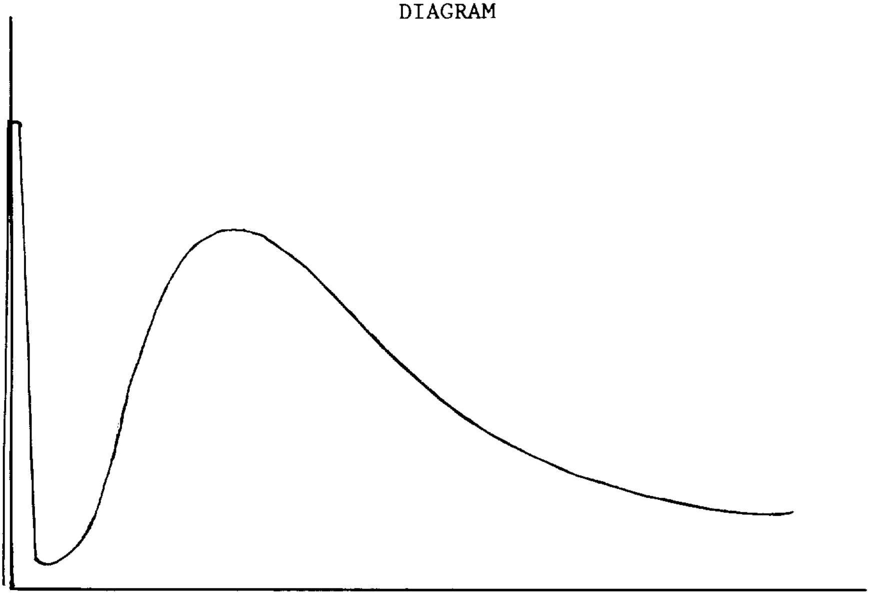
Then

$$(2)\ F(X) = \sum_{n_0=0}^{\infty} Q(n_0)[P(0|n_0)\pi(x - n_0m) + (1 - P(0|n_0))G(x - n_0m|n_0)]$$

The major problem arising in considering the modes of the density of $F(X)$ is in examining the fine structure of $G'(X)$. We believe that for any reasonable frequency and severity distributions (or combinations thereof) $G'(X)$ will have a primary mode that tends to dominate all of its secondary modes. (Consider, for example, a Poisson frequency and a gamma severity.) That is, we can think of $G'(X)$ as being essentially mono-modal. However, we should recognize that these secondary modes probably exist in most cases. They seem to give rise to much less important modes on the density of $F(X)$. Our simulation investigations tend to support this view.

With this in mind, we see that $F(X)$ is essentially a sum, weighted by the $Q(n_0)$'s, of distributions whose densities consist of a δ-function followed by a mono-modal distribution (see diagram).

DIAGRAM



Since the δ-functions have mass of only $P(0|n_0)$ they tend to have little effect on the shape of the density of $F(X)$. Thus, from (2), $F(X)$ will tend to have modes appearing at approximately the points where the $G'(x - n_0m|n_0)$ peak.

The above argument for the existence of modes hinges on the existence of a maximum loss. As a check Exhibit III shows the density of a distribution function with unlimited severity. The appearance of only one mode supports the argument.

The author recognizes a spike in the density of $F(X)$ at the origin and fits the rest of the distribution to a transformed gamma. What we do in (1) is recognize all spikes and fit each $G(x|n_0)$ to a TGD. Thus if $G(x; r_n, \alpha_n, \lambda_n)$ is fitted to replace $G(X|n_0)$ then (2) is transformed into (1) yielding our model.

In the case of a Poisson frequency,

$$P(n|n_0) = e^{-\omega(1-p)}(\omega(1 - p))^n/n!$$

independent of n_0. Hence $G(x|n_0) = G(x|0)$ is also independent of n_0. Thus, (2) becomes

$$(3)\ \tilde{F}(X) = \sum_{n=0}^{\infty} e^{-\omega p}(\omega p)^n/n![e^{-\omega(1-p)}\pi(x - nm) + (1 - e^{-\omega(1-p)})G(x - nm; r, \alpha, \lambda)]$$

where $G(x; r, \alpha, \lambda)$ fits to $G(x|0)$. This is the approximation used in Exhibits I and II.

REFERENCES

[1] H. Cramér, "Collective Risk Theory," *The Jubilee Volume of Försäkringsaktiebolaget Skandia,* 1955.

[2] R. E. Beard, T. Pentikäinen, and E. Pesonen, *Risk Theory,* 2nd Edition, Chapman and Hall, 1977.

[3] G. Gordon, *System Simulation,* 2nd Edition, Prentice Hall, 1978.

[4] H. H. Panjer, "Recursive Evaluation of a Family of Compound Distributions," *ASTIN Bulletin,* Vol. 12, No. 1, page 22.

[5] P. E. Heckman, and G. G. Meyers, "The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions," *PCAS* LXX (1983).

EXHIBIT IA

A Comparison of Average Densities

Aggregate Loss Interval (× 1000)	Average Densities[1] Characteristic Function Method[2] (× 10⁻⁶)	Transformed Gamma[3] (× 10⁻⁶)	Modified Transformed Gamma[4] (× 10⁻⁶)
0–25	2.032	2.484	2.264
25–50	3.132	2.556	2.724
50–75	2.872	2.540	2.784
75–100	2.668	2.500	2.696
100–125	2.452	2.436	2.536
125–150	2.216	2.352	2.328
150–175	1.992	2.252	2.100
175–200	1.788	2.148	1.860
200–225	1.604	2.028	1.628
225–250	1.436	1.908	1.400
250–275	1.944	1.776	1.944
275–300	2.088	1.652	1.912
300–325	1.808	1.524	1.760
325–350	1.588	1.396	1.584
350–375	1.376	1.276	1.408
375–400	1.192	1.152	1.228
400–425	1.024	1.040	1.064
425–450	.884	.932	.908
450–475	.760	.832	.768
475–500	.656	.740	.644
500–525	.668	.648	.660
525–550	.624	.572	.592
550–575	.524	.496	.512
575–600	.440	.432	.440
600–625	.368	.372	.376
625–650	.308	.324	.316
650–675	.256	.272	.260
675–700	.212	.232	.220
700–725	.180	.200	.180
725–750	.148	.164	.148

(1) Average Density = (difference of the values of the cumulative distribution at the endpoints of the interval)/25,000.

(2) From Venter, Exhibit 3, Page 1, Column 2.

(3) From Venter, Exhibit 3, Page 1, Column 6.

(4) See Exhibit IV, Note (2) for parameters.

Exhibit IB

Average Densities

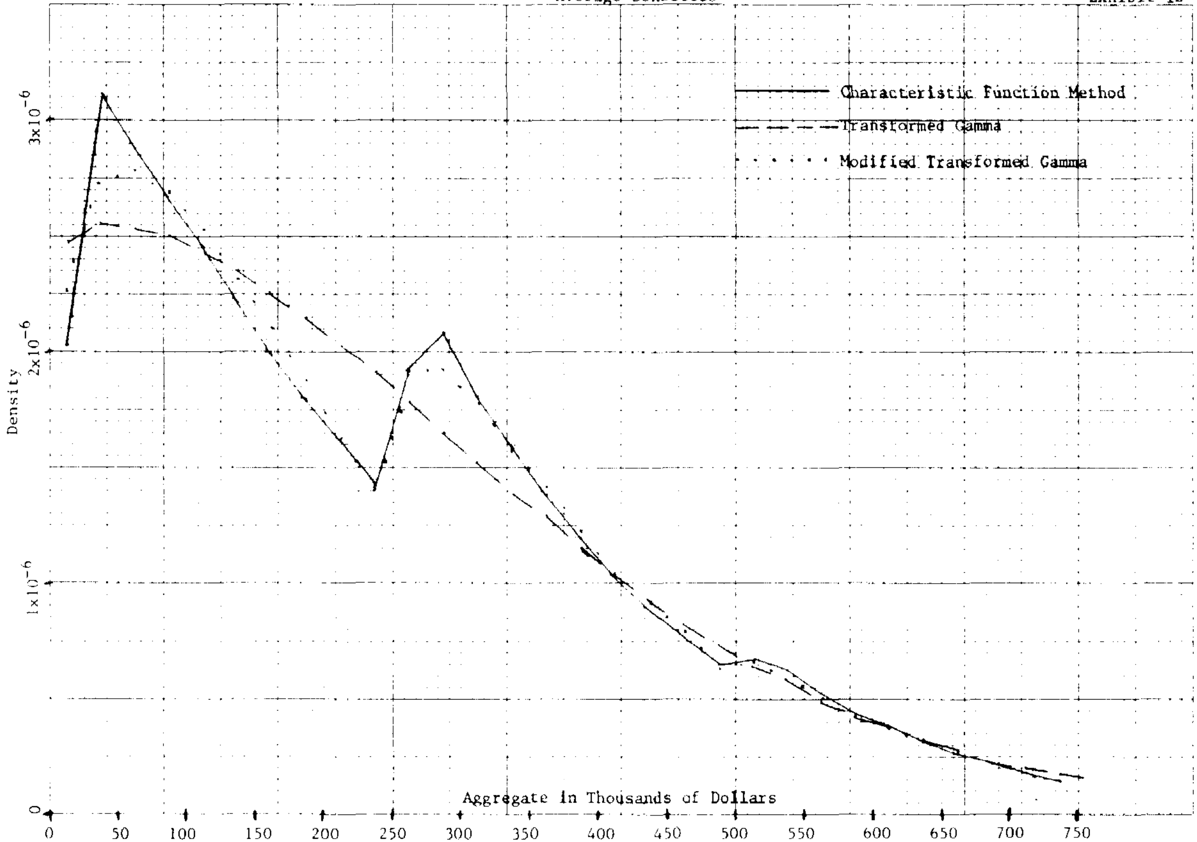


EXHIBIT IIA

A Comparison of Average Densities—Another Distribution

Aggregate Loss Interval (× 24,076)	Average Density: Simulation Method[1] (× 10^{-6})	Transformed Gamma[2] (× 10^{-6})	Modified Transformed Gamma[3] (× 10^{-6})
0–1	2.949	6.152	5.552
1–2	3.697	2.251	2.512
2–3	2.886	1.675	1.939
3–4	2.201	1.387	1.626
4–5	1.744	1.206	1.415
5–6	1.578	1.078	1.258
6–7	1.288	.9828	1.133
7–8	1.080	.9074	1.030
8–9	.9968	.8461	.9429
9–10	.7892	.7950	.8670
10–11	.7061	.7515	.8001
11–12	.6646	.7137	.7404
12–13	.6230	.6807	.6865
13–14	.5400	.6513	.6377
14–15	.5400	.6250	.5931
15–16	.4984	.6011	.5521
16–17	.4153	.5795	.5144
17–18	.3738	.5596	.4796
18–19	.3738	.5412	.4472
19–20	.3323	.5241	.4173
20–21	.2907	.5082	.3893
21–22	.3323	.4933	.3633
22–23	.2907	.4793	.3391
23–24	.2492	.4661	.3165
24–25	.2907	.4535	.2953
25–26	.2077	.4416	.2756
26–27	.2492	.4302	.2571
27–28	.2077	.4194	.2398

Notes appear on continuation of exhibit.

EXHIBIT IIA (continued)

A Comparison of Average Densities—Another Distribution

Aggregate Loss Interval (× 24,076)	Average Density: Simulation Method(1) (× 10^{-6})	Average Density: Transformed Gamma(2) (× 10^{-6})	Average Density: Modified Transformed Gamma(3) (× 10^{-6})
28–29	.2077	.4089	.2237
29–30	.1661	.3989	.2085
30–31	.2077	.3893	.1944
31–32	.1661	.3800	.1811
32–33	.1661	.3710	.1688
33–34	.1661	.3622	.1572
34–35	.1246	.3538	.1463
35–36	.1661	.3456	.1362
36–37	.1661	.3376	.1267
37–38	.1246	.3298	.1179
38–39	.1246	.3221	.1096
39–40	.1661	.3147	.1018
40–41	.1246	.3075	.0946
41–42	.4153	.3003	1.311
42–43	1.371	.2934	1.136
43–44	1.163	.2866	.8074
44–45	.9968	.2799	.6629
45–46	.7476	.2733	.5722
46–47	.6646	.2668	.5066
47–48	.4984	.2605	.4554
48–49	.4569	.2543	.4134
49–50	.4153	.2481	.3778
50–51	.3323	.2421	.3470
51–52	.2907	.2362	.3198
52–53	.2907	.2303	.2955
53–54	.2492	.2246	.2737
54–55	.2077	.2189	.2539
55–56	.1661	.2133	.2358

Notes appear on continuation of exhibit.

EXHIBIT IIA (continued)

A Comparison of Average Densities—Another Distribution

Aggregate Loss Interval (× 24,076)	Average Density: Simulation Method[1] (× 10^{-6})	Transformed Gamma[2] (× 10^{-6})	Modified Transformed Gamma[3] (× 10^{-6})
56–57	.2077	.2078	.2192
57–58	.1661	.2024	.2039
58–59	.1661	.1971	.1898
59–60	.1246	.1919	.1767
60–61	.1246	.1867	.1646
61–62	.0830	.1816	.1533
62–63	.1246	.1766	.1429
63–64	.1246	.1717	.1331
64–65	.0830	.1669	.1240
65–66	.1246	.1621	.1156
66–67	.0830	.1574	.1077
67–68	.0830	.1528	.1003
68–69	.0830	.1483	.09340
69–70	.08307	.1438	.08697
70–71	.08307	.1394	.08097
71–72	.08307	.1351	.07536
72–73	.04153	.1309	.07012
73–74	.08307	.1268	.06523
74–75	.04153	.1227	.06067
75–76	.08307	.1187	.05640
76–77	.08307	.1148	.05242
77–78	.04153	.1110	.04870
78–79	.04153	.1072	.04524
79–80	.04153	.1036	.04200
80–81	.04153	.09999	.03899
81–82	.04153	.09648	.03618
82–83	.04153	.09304	.03356
83–84	.2077	.08969	.3286

Notes appear on continuation of exhibit.

EXHIBIT IIA (continued)

A Comparison of Average Densities—Another Distribution

Aggregate Loss Interval (× 24,076)	Average Density: Simulation Method(1) (× 10^{-6})	Transformed Gamma(2) (× 10^{-6})	Modified Transformed Gamma(3) (× 10^{-6})
84–85	.2492	.08641	.1722
85–86	.2077	.08322	.1364
86–87	.1661	.08010	.1163
87–88	.1246	.07706	.1025
88–89	.08307	.07409	.09184
89–90	.08307	.07120	.08321
90–91	.08307	.06839	.07593
91–92	.08307	.06566	.06964
92–93	.08307	.06300	.06410
93–94	.08307	.06042	.05916
94–95	.04153	.05791	.05471
95–96	.04153	.05547	.05067
96–97	.04153	.05311	.04699
97–98	.04153	.05082	.04361
98–99	.04153	.04860	.04051
99–100	0	.04645	.03765

(1) This distribution is based on a Poisson frequency with mean 13.7376 and a Pareto severity
$F(X) = 1 - (B/(B + x))^{\delta}$ with $B = 264.7$ and $\delta = .45128063$
censored at 1,000,000.
The small scale fluctuations are due to our simulation routine which only calculates distributions to .001. Note that .001/24076 = .04153 E-6.

(2) See Exhibit V, Note (2) for parameters.

(3) See Exhibit V, Note (3) for parameters.

Exhibit IIB

Average Densities

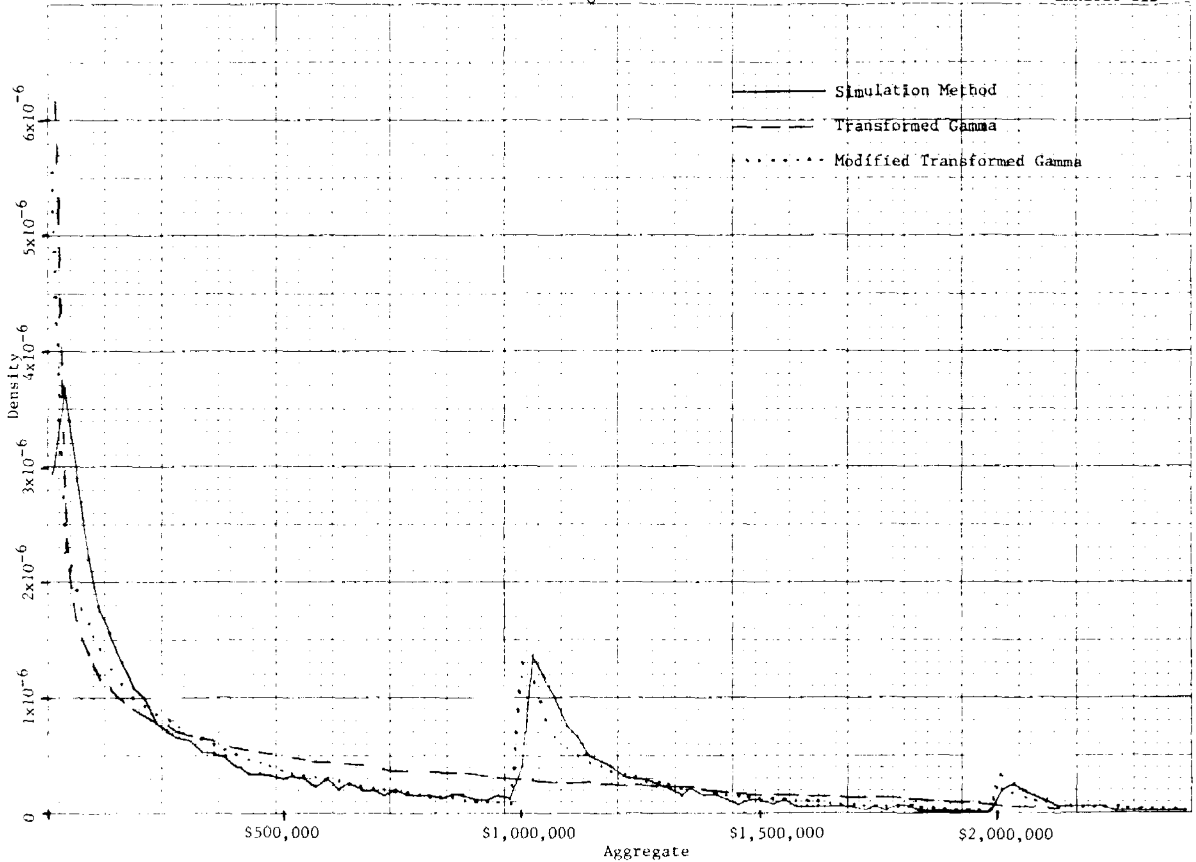


EXHIBIT IIIA

Average Density
Aggregate Distribution for a Severity Without a Censor[1]

Aggregate Loss Interval (× 120,380)	Simulation Method[2] (× 10^{-7})
0–1	.7476
1–2	5.732
2–3	12.63
3–4	16.20
4–5	14.12
5–6	11.05
6–7	7.809
7–8	5.150
8–9	3.323
9–10	1.994
10–11	1.412
11–12	.8307
12–13	.5815
13–14	.3323
14–15	.3323
15–16	.1661
16–17	.1661
17–18	.08307
18–19	.08307
19–20	0
20–21	.08307
21–22	0
22–23	.08307
23–24	0
24–25	0
25–26	0
26–27	.08307
27–28	0
28–29	0
29–30	0

(1) Poisson frequency with $\omega = 13.7376$ and a Pareto severity with $B = 65{,}721$ and $\delta = 2.5$ censored at 10^{12}.

(2) The small scale fluctuations are due to our simulation routine which only calculates distributions to .001. Note that .08307 E-7 = .001/120380.

Exhibit IIIB

Aggregate Densities

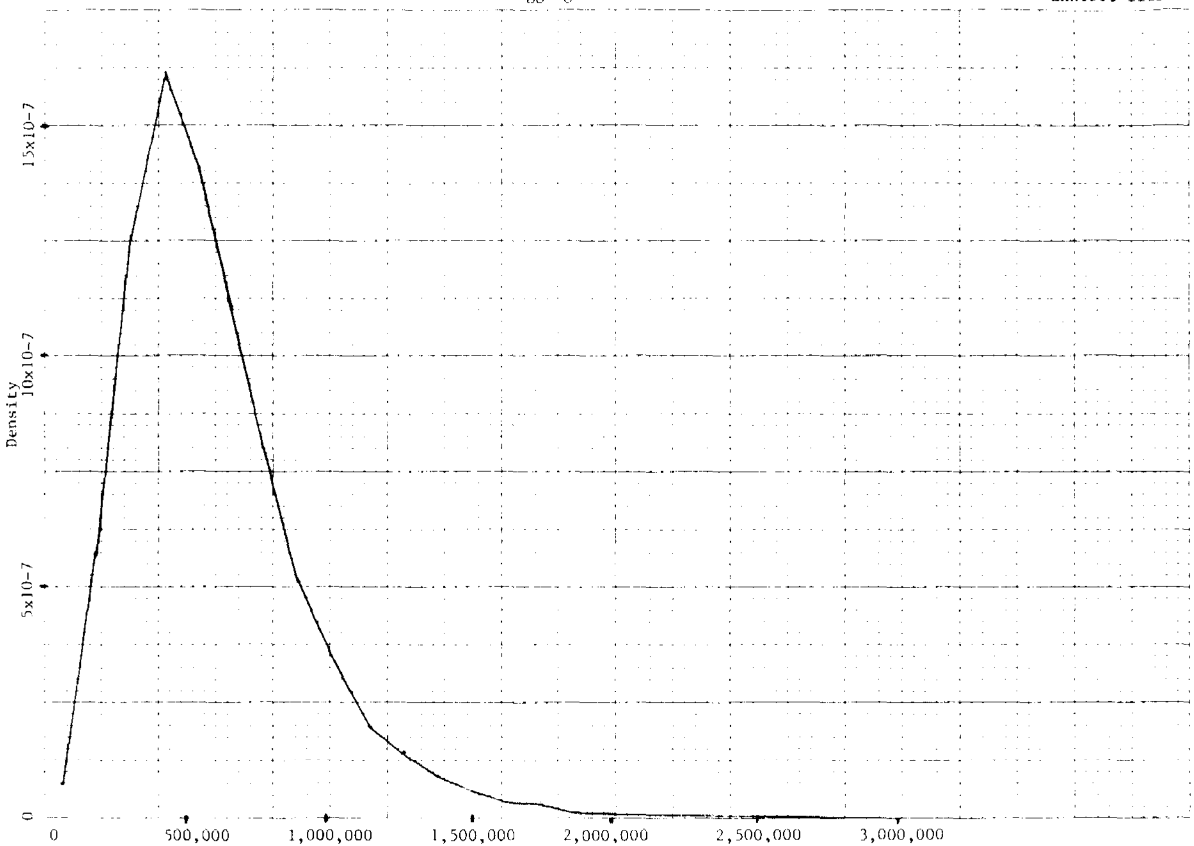


EXHIBIT IV

Comparison of Excess Ratios from Distributions in Venter's Exhibit 3

Aggregate Loss Amount (× 1000)	Characteristic Function Method[1]	TGD[1]	Modified TGD[2]	Normal[3]	Pareto[4]
25	.9016	.9031	.9026	.9033	.9062
50	.8107	.8125	.8116	.8131	.8236
75	.7273	.7283	.7276	.7292	.7506
100	.6507	.6503	.6504	.6517	.6859
125	.5806	.5786	.5798	.5801	.6282
150	.5163	.5129	.5152	.5145	.5768
175	.4573	.4529	.4562	.4546	.5307
200	.4030	.3984	.4022	.4001	.4893
225	.3529	.3491	.3525	.3507	.4521
250	.3066	.3047	.3066	.3062	.4185
275	.2642	.2650	.2648	.2662	.3881
300	.2273	.2295	.2279	.2305	.3605
325	.1951	.1981	.1955	.1987	.3354
350	.1672	.1702	.1674	.1706	.3126
375	.1431	.1457	.1430	.1458	.2917
400	.1221	.1243	.1219	.1241	.2727
425	.1039	.1055	.1036	.1051	.2552
450	.0880	.0893	.0878	.0887	.2392
475	.0742	.0752	.0741	.0745	.2244
500	.0622	.0631	.0622	.0622	.2109
525	.0518	.0528	.0519	.0518	.1984
550	.0430	.0439	.0432	.0429	.1868
575	.0357	.0364	.0358	.0353	.1761
600	.0296	.0301	.0296	.0290	.1662
625	.0245	.0247	.0245	.0237	.1570
650	.0202	.0203	.0202	.0192	.1485
675	.0167	.0165	.0166	.0155	.1406
700	.0137	.0134	.0136	.0125	.1332
725	.0112	.0109	.0112	.0100	.1263
750	.0091	.0088	.0091	.0080	.1199
775	.0074	.0070	.0074	.0063	.1139
800	.0060	.0056	.0059	.0050	.1082
825	.0048	.0045	.0048	.0039	.1030
850	.0039	.0035	.0039	.0030	.0980

Notes appear on following page.

EXHIBIT IV (continued)

Comparison of Excess Ratios from Distributions in Venter's Exhibit 3

(1) From Venter, Exhibit 3.

(2) Fit by method of Appendix II.
$\omega = 13.7376$
$m = 250{,}000$
$p = 0.0241$
$r = 0.7568$
$\alpha = 1.55601$
$\lambda = 4.3616\text{E-}6$

(3) Fit to match first two moments.

$$\text{Distribution Function} = \frac{1}{\sqrt{2\pi}\,\sigma(1 - \Phi(-\mu/\sigma))} \int_0^x \exp\left[\frac{-(t-\mu)^2}{2\sigma^2}\right] dt$$

$\Phi(x)$ = Standard Normal Distribution
$\mu = -31{,}828.4$
$\sigma = 327{,}408.6$

(4) Fit to match first two moments.
$F(X) = 1 - (B/(B + X))^{\delta}$
$B = 807{,}039$
$\delta = 4.22815586$

EXHIBIT V

COMPARISON OF EXCESS RATIOS FROM DISTRIBUTION IN EXHIBIT II

Aggregate Loss Amount ($\times 10^5$)	Simulation[1]	TGD[2]	Modified Transformed Gamma[3]
1	.8599	.8660	.8649
2	.7542	.7555	.7567
3	.6663	.6595	.6647
4	.5877	.5750	.5843
5	.5164	.5000	.5124
6	.4511	.4335	.4470
7	.3888	.3744	.3863
8	.3304	.3220	.3292
9	.2753	.2757	.2748
10	.2226	.2349	.2224
11	.1792	.1990	.1818
12	.1500	.1677	.1513
13	.1269	.1405	.1269
14	.1079	.1170	.1070
15	.0913	.0968	.0904
16	.0767	.0795	.0762
17	.0641	.0649	.0638
18	.0525	.0525	.0527
19	.0420	.0422	.0426
20	.0324	.0336	.0333
21	.0250	.0266	.0262
22	.0200	.0209	.0210
23	.0160	.0162	.0170
24	.0131	.0125	.0139
25	.0110	.0095	.0114

(1) ω = 13.7376 Poisson Frequency
m = 1,000,000 — B = 264.7
$S(X) = 1 - (B/(X + B))^{\delta}$ Pareto Severity — δ = .45128063

(2) ω = 13.7376 — α = 2.56852
r = 0.174667 — λ = 4.94882E-7

(3) ω = 13.7376 — r = 0.383347
m = 1,000,000 — α = 1.42077
p = 0.0243 — λ = 1.54E-6

APPENDIX I

$P(n)$, $Q(n)$, and $P(n|n_0)$

$P(n)$ is the probability of n losses in a time period; p is the probability of a total loss (of size m) given that a loss has occurred.

$Q(n)$ is the probability of n total losses. Then

$$Q(n) = \sum_{j=0}^{\infty} P(n + j) \binom{n + j}{n} p^n(1 - p)^j$$

$P(n|n_0)$ is the probability of n partial losses given that n_0 total losses have occurred. Then

$$P(n|n_0) = \binom{n + n_0}{n} p^{n_0}(1 - p)^n/Q(n_0)$$

If $P(n)$ is Poisson, then so are $Q(n)$ and $P(n|n_0)$. Likewise, $P(n)$ negative binomial implies that $Q(n)$ and $P(n|n_0)$ are also negative binomial. The form of the functions remains the same; only the parameters change.

	Poisson Parameter*	Negative Binomial Parameters**	
	α	α_1	α_2
$P(n)$	ω	x	q
$Q(n)$	ωp	x	$q/(p + q - pq)$
$P(n\|n_0)$	$\omega(1 - p)$	$x + n_0$	$p + q - pq$

Note the following interesting fact about the negative binomial case.

$$E(n|n_0) \equiv \sum_{n=0}^{\infty} nP(n|n_0) = (x + n_0) \left(\frac{1 - p - q + pq}{p + q - pq}\right)$$

As the number of total losses increases, so does the expected number of partial losses. This lends support to the usual interpretation of the negative binomial distribution as being associated with situations of positive contagion. (See for example Meyers and Heckman [5].)

* The form of the Poisson is Poisson $(n) = e^{-\alpha}\alpha^n/n!$

** Negative Binomial $(n) = \binom{n + \alpha_1 - 1}{n} \alpha_2^{\alpha_1}(1 - \alpha_2)^n$

APPENDIX II

Moments of $G(x|n_0)$

Recall $G(x|n_0) = \dfrac{1}{1 - P(0|n_0)} \sum_{n=1}^{\infty} P(n|n_0)\mathcal{S}_1^{n*}(x)$

where $\mathcal{S}_1^{n*}(x)$ is the n^{th} convolution of the cumulative distribution function of the partial losses. Setting $p = 1 - S(m) =$ the probability of a total loss,

$$\mathcal{S}_1(x) = \begin{cases} 0 & x < 0 \\ S(x)/(1 - p) & 0 \le x < m \\ 1 & m \le x \end{cases}$$

The program is as follows:

1) One already knows $E(n^j|n_0) = \sum_{n=0}^{\infty} n^j P(n|n_0)$

and $E(x^j) = (1 - p) \int_0^m x^j d\mathcal{S}_1(x) + pm^j$ for $j = 1$ to 3.

(If $P(n|n_0)$ is Poisson or negative binomial, then the $E(n^j|n_0)$ are tabulated, and presumably one has already calculated the $E(x^j)$.)

Calculate $E^*(n^j|n_0) = \dfrac{\sum_{n=1}^{\infty} n^j P(n|n_0)}{1 - P(0|n_0)} = \dfrac{E(n^j|n_0)}{1 - P(0|n_0)}$

and $E^*(x^j) = \int_0^m x^j d\mathcal{S}_1(x) = \dfrac{E(x^j) - pm^j}{1 - p}$ for $j = 1$ to 3.

2) $\mu_N = E(n|n_0)$
$\sigma_N^2 = E^*(n^2|n_0) - E^{*2}(n|n_0)$
$\gamma_N\sigma_N^3 = E^*(n^3|n_0) - 3E^*(n^2|n_0)E^*(n|n_0) + 2E^{*3}(n|n_0)$
$\mu_x = E^*(x)$
$\sigma_x^2 = E^*(x^2) - E^{*2}(x)$
$\gamma_x\sigma_x^3 = E^*(x^3) - 3E^*(x^2)E^*(x) + 2E^{*3}(x)$

3) Calculate for each n_0 needed, μ_L, σ_L, and γ_L of $G(x|n_0)$ function using the first three formulas of Venter's Appendix C.

4) Calculate the transformed Gamma parameters α_{n_0}, λ_{n_0}, and r_{n_0} by matching the three moments in (3).

Note that if $P(n)$, hence $P(n|n_0)$, is Poisson, then $P(N|n_0)$ and $G(x|n_0)$ are actually independent of n_0 and you need only calculate one triplet α, λ, r for all the G's.

APPENDIX III

Computation of Excess Ratios

Define $$\tilde{F}(X) = \sum_{n=0}^{[x/m]} Q(n)[P(0|n)\pi(x - nm) + (1 - P(0|n))G(x - nm; r_n, \alpha_n, \lambda_n)]$$

Then $E(x) = [mp + (1 - p)E_{\mathcal{G}_1}(x)]E(n)$

where $E_{\mathcal{G}_1}(x) = \int_0^\infty x d\mathcal{G}_1(x)$ and $E(n) = \sum_{n=0}^{\infty} nP(n)$.

(Note: The above must be proved and anyone wishing to see a proof can contact the reviewers.)

$$\text{Then } R(a) \equiv \int_a^\infty (x - a)d\tilde{F}(x)/E(x) = 1 - \frac{1}{E(x)} \sum_{n=0}^{[a/m]} Q(n) \Bigg[P(0|n)nm + (1 - P(0|n)) \left\{G\left(a - nm; r_n + \frac{1}{\alpha_n}, \alpha_n, \lambda_n\right) \frac{\Gamma(r_n + (1/\alpha_n))}{\lambda\Gamma(r_n)} + G(a - nm; r_n, \alpha_n, \lambda_n)nm\right\}\Bigg] - \frac{a}{E(x)}(1 - \tilde{F}(a))$$

Although this appears complicated it is really quite simple to compute since usually not many terms are needed.

In the case of a Poisson (with $E(n) = \omega$),

$P(n|n_0) = e^{-\omega(1-p)}(\omega(1 - p))^n/n!$

independent of n_0. Therefore λ_{n_0}, α_{n_0} and r_{n_0} are also independent of n_0.

$$\text{Then } \tilde{F}(X) = \sum_{n=0}^{[x/m]} e^{-\omega p} \frac{(\omega p)^n}{n!} [e^{-\omega(1-p)}\pi(x - nm) + (1 - e^{-\omega(1-p)})G(x - nm; r, \alpha, \lambda)]$$

$$E(X) = p\omega m + (1 - p)\omega E_{\mathcal{G}_1}(x)$$

$$= p\omega m + (1 - e^{-\omega(1-p)}) \frac{\Gamma(r + (1/\alpha))}{\lambda\Gamma(r)} \quad \dagger$$

$$R(a) = 1 - \frac{1}{E(X)} \sum_{n=0}^{[a/m]} e^{-\omega p} \frac{(\omega p)^n}{n!} \left[e^{-\omega(1-p)} nm \right.$$

$$+ (1 - e^{-\omega(1-p)}) \left(G \left(a - nm; r + \frac{1}{\alpha}, \alpha, \lambda \right) \frac{\Gamma(r + (1/\alpha))}{\lambda\Gamma(r)} \right.$$

$$\left. \left. + G(a - nm; r, \alpha, \lambda) nm \right) \right] - \frac{a}{E(X)} (1 - \bar{F}(a))$$

†Note: $(1 - e^{-\omega(1-p)}) \frac{\Gamma(r + (1/\alpha))}{\lambda\Gamma(r)}$

$$= (1 - e^{-\omega(1-p)}) \int_0^\infty x \, d \left(\frac{\sum_{n=1}^{\infty} P(n|n_0) \mathcal{G}_1^{n*}(x)}{1 - P(0|n_0)} \right)$$

$$= (1 - e^{-\omega(1-p)}) \sum_{n=1}^{\infty} e^{-\omega(1-p)} \frac{(\omega(1-p))^n}{n!} \frac{nE_{\mathcal{G}_1}(x)}{1 - e^{-\omega(1-p)}}$$

$$= (1 - p)\omega E_{\mathcal{G}_1}(x)$$

THE CALCULATION OF AGGREGATE LOSS DISTRIBUTIONS FROM CLAIM SEVERITY AND CLAIM COUNT DISTRIBUTIONS

PHILIP E. HECKMAN
GLENN G. MEYERS

VOLUME LXX

EDITOR'S NOTE

The following pages reproduce the exhibits associated with the paper "The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions" by Philip E. Heckman and Glenn G. Meyers (*PCAS* LXX, 1983). These exhibits were omitted from the original printing of the paper.

EXHIBIT I

CLAIM SEVERITY DISTRIBUTIONS FOR THE REMAINING EXHIBITS

COLLECTIVE RISK MODEL

CLAIM SEVERITY DISTRIBUTION

NAME: PRODUCTS BI

LOSS AMOUNT	CUMULATIVE PROBABILITY
0.0	0.0
1000.00	0.38935
5000.00	0.77870
6000.00	0.78438
7000.00	0.78981
8000.00	0.79498
9000.00	0.79993
10000.00	0.80466
12500.00	0.81564
15000.00	0.82553
17500.00	0.83449
20000.00	0.84264
25000.00	0.85690
35000.00	0.87927
50000.00	0.90280
75000.00	0.92739
100000.00	0.94256
125000.00	0.95278
150000.00	0.96009
175000.00	0.96556
200000.00	0.96979
225000.00	0.97316
250000.00	0.97590

SUMMARY STATISTICS:

SEVERITY MEAN = 18197.94
SEVERITY STD DEV = 48406.40

CLAIM SEVERITY DISTRIBUTION

NAME: WORKERS COMP

LOSS AMOUNT	CUMULATIVE PROBABILITY
0.0	0.0
25.00	0.20230
50.00	0.48880
100.00	0.71960
150.00	0.78150
200.00	0.81090
250.00	0.82890
300.00	0.84270
400.00	0.86090
500.00	0.87410
750.00	0.89600
1000.00	0.90980
1500.00	0.92720
2000.00	0.93921
2500.00	0.94758
3000.00	0.95381
4000.00	0.96257
5000.00	0.96851
6000.00	0.97283
7000.00	0.97613
8000.00	0.97875
9000.00	0.98087
10000.00	0.98262
12500.00	0.98594
15000.00	0.98825
17250.00	0.98984
20000.00	0.99132
25000.00	0.99322
30000.00	0.99451
40000.00	0.99613
50000.00	0.99710
75000.00	0.99835
100000.00	0.99896
150000.00	0.99944
250000.00	0.99978
350000.00	0.99988
500000.00	0.99995
750000.00	0.99998
1000000.00	0.99999
1500000.00	1.00000

SUMMARY STATISTICS:

SEVERITY MEAN = 985.15
SEVERITY STD DEV = 9812.41

EXHIBIT II

COLLECTIVE RISK MODEL

LINE	EXPECTED LOSS	CLAIM SEVERITY DISTRIBUTION	CONTAGION PARAMETER	CLAIM COUNT MEAN	CLAIM COUNT STD DEV
1	500000	PRODUCTS BI	0.0000	27.476	5.242

MIXING PARAMETER	0.0000
AGGREGATE MEAN	500000
AGGREGATE STD DEV	271071

AGGREGATE LOSS AMOUNT	ENTRY RATIO	CUMULATIVE PROBABILITY	EXCESS PURE PREMIUM	EXCESS PURE PREMIUM RATIO
50000.00	0.1000	0.0052	450056.03	0.9001
100000.00	0.2000	0.0320	400903.28	0.8018
150000.00	0.3000	0.0755	353535.23	0.7071
200000.00	0.4000	0.1295	308630.16	0.6173
250000.00	0.5000	0.1882	266560.19	0.5331
300000.00	0.6000	0.2510	227496.34	0.4550
350000.00	0.7000	0.3247	191859.59	0.3837
400000.00	0.8000	0.4029	160044.58	0.3201
450000.00	0.9000	0.4798	132125.88	0.2643
500000.00	1.0000	0.5518	107942.62	0.2159
550000.00	1.1000	0.6180	87204.77	0.1744
600000.00	1.2000	0.6802	69680.98	0.1394
650000.00	1.3000	0.7363	55121.80	0.1102
700000.00	1.4000	0.7854	43194.73	0.0864
750000.00	1.5000	0.8270	33533.74	0.0671
800000.00	1.6000	0.8619	25780.81	0.0516
850000.00	1.7000	0.8913	19632.21	0.0393
900000.00	1.8000	0.9154	14819.09	0.0296
950000.00	1.9000	0.9349	11093.50	0.0222
1000000.00	2.0000	0.9503	8237.03	0.0165
1050000.00	2.1000	0.9624	6064.95	0.0121
1100000.00	2.2000	0.9718	4429.20	0.0089
1150000.00	2.3000	0.9791	3209.97	0.0064
1200000.00	2.4000	0.9846	2309.49	0.0046
1250000.00	2.5000	0.9888	1649.77	0.0033
1300000.00	2.6000	0.9919	1169.94	0.0023
1350000.00	2.7000	0.9942	823.78	0.0016
1400000.00	2.8000	0.9958	576.14	0.0012
1450000.00	2.9000	0.9971	400.33	0.0008
1500000.00	3.0000	0.9979	276.38	0.0006
1550000.00	3.1000	0.9986	189.56	0.0004
1600000.00	3.2000	0.9990	129.17	0.0003
1650000.00	3.3000	0.9993	87.45	0.0002
1700000.00	3.4000	0.9995	58.83	0.0001
1750000.00	3.5000	0.9997	39.30	0.0001
1800000.00	3.6000	0.9998	26.06	0.0001
1850000.00	3.7000	0.9999	17.13	0.0000
1900000.00	3.8000	0.9999	11.16	0.0000
1950000.00	3.9000	0.9999	7.19	0.0000
2000000.00	4.0000	1.0000	4.58	0.0000

EXHIBIT III

COLLECTIVE RISK MODEL

LINE	EXPECTED LOSS	CLAIM SEVERITY DISTRIBUTION	CONTAGION PARAMETER	CLAIM COUNT MEAN	CLAIM COUNT STD DEV
1	500000	PRODUCTS BI	0.2500	27.476	14.704

MIXING PARAMETER 0.0000
AGGREGATE MEAN 500000
AGGREGATE STD DEV 368754

AGGREGATE LOSS AMOUNT	ENTRY RATIO	CUMULATIVE PROBABILITY	EXCESS PURE PREMIUM	EXCESS PURE PREMIUM RATIO
50000.00	0.1000	0.0563	451375.88	0.9028
100000.00	0.2000	0.1135	405615.08	0.8112
150000.00	0.3000	0.1708	362727.50	0.7255
200000.00	0.4000	0.2265	322670.09	0.6453
250000.00	0.5000	0.2793	285324.63	0.5706
300000.00	0.6000	0.3434	250861.35	0.5017
350000.00	0.7000	0.4075	219644.06	0.4393
400000.00	0.8000	0.4685	191560.03	0.3831
450000.00	0.9000	0.5252	166420.19	0.3328
500000.00	1.0000	0.5770	143995.20	0.2880
550000.00	1.1000	0.6264	124090.32	0.2482
600000.00	1.2000	0.6723	106575.73	0.2132
650000.00	1.3000	0.7139	91249.17	0.1825
700000.00	1.4000	0.7512	77895.65	0.1558
750000.00	1.5000	0.7844	66302.29	0.1326
800000.00	1.6000	0.8139	56271.91	0.1125
850000.00	1.7000	0.8400	47633.75	0.0953
900000.00	1.8000	0.8630	40223.35	0.0804
950000.00	1.9000	0.8831	33887.41	0.0678
1000000.00	2.0000	0.9005	28486.06	0.0570
1050000.00	2.1000	0.9155	23893.90	0.0478
1100000.00	2.2000	0.9285	20001.23	0.0400
1150000.00	2.3000	0.9396	16710.51	0.0334
1200000.00	2.4000	0.9491	13935.70	0.0279
1250000.00	2.5000	0.9573	11601.30	0.0232
1300000.00	2.6000	0.9642	9641.64	0.0193
1350000.00	2.7000	0.9700	8000.07	0.0160
1400000.00	2.8000	0.9750	6627.78	0.0133
1450000.00	2.9000	0.9791	5482.82	0.0110
1500000.00	3.0000	0.9826	4529.24	0.0091
1550000.00	3.1000	0.9856	3736.45	0.0075
1600000.00	3.2000	0.9880	3078.39	0.0062
1650000.00	3.3000	0.9901	2533.07	0.0051
1700000.00	3.4000	0.9918	2081.83	0.0042
1750000.00	3.5000	0.9932	1709.01	0.0034
1800000.00	3.6000	0.9944	1401.39	0.0028
1850000.00	3.7000	0.9954	1147.92	0.0023
1900000.00	3.8000	0.9962	939.31	0.0019
1950000.00	3.9000	0.9969	767.86	0.0015
2000000.00	4.0000	0.9975	627.10	0.0013
2050000.00	4.1000	0.9979	511.68	0.0010
2100000.00	4.2000	0.9983	417.14	0.0008
2150000.00	4.3000	0.9986	339.77	0.0007
2200000.00	4.4000	0.9989	276.54	0.0006
2250000.00	4.5000	0.9991	224.90	0.0004
2300000.00	4.6000	0.9992	182.78	0.0004
2350000.00	4.7000	0.9994	148.45	0.0003
2400000.00	4.8000	0.9995	120.51	0.0002
2450000.00	4.9000	0.9996	97.78	0.0002
2500000.00	5.0000	0.9997	79.33	0.0002

EXHIBIT IV

COLLECTIVE RISK MODEL

LINE	EXPECTED LOSS	CLAIM SEVERITY DISTRIBUTION	CONTAGION PARAMETER	CLAIM COUNT MEAN	CLAIM COUNT STD DEV
1	250000	PRODUCTS BI	0.0000	13.738	3.706

MIXING PARAMETER	0.0000
AGGREGATE MEAN	250000
AGGREGATE STD DEV	191676

AGGREGATE LOSS AMOUNT	ENTRY RATIO	CUMULATIVE PROBABILITY	EXCESS PURE PREMIUM	EXCESS PURE PREMIUM RATIO
25000.00	0.1000	0.0508	225403.80	0.9016
50000.00	0.2000	0.1291	202676.86	0.8107
75000.00	0.3000	0.2009	181812.52	0.7273
100000.00	0.4000	0.2676	162679.19	0.6507
125000.00	0.5000	0.3289	145147.95	0.5806
150000.00	0.6000	0.3843	129074.38	0.5163
175000.00	0.7000	0.4341	114315.48	0.4573
200000.00	0.8000	0.4788	100737.62	0.4030
225000.00	0.9000	0.5189	88218.38	0.3529
250000.00	1.0000	0.5548	76648.41	0.3066
275000.00	1.1000	0.6034	66060.59	0.2642
300000.00	1.2000	0.6556	56817.03	0.2273
325000.00	1.3000	0.7008	48785.07	0.1951
350000.00	1.4000	0.7405	41812.29	0.1672
375000.00	1.5000	0.7749	35764.65	0.1431
400000.00	1.6000	0.8047	30518.36	0.1221
425000.00	1.7000	0.8303	25963.29	0.1039
450000.00	1.8000	0.8524	22003.53	0.0880
475000.00	1.9000	0.8714	18556.24	0.0742
500000.00	2.0000	0.8878	15550.43	0.0622
525000.00	2.1000	0.9045	12946.55	0.0518
550000.00	2.2000	0.9201	10761.15	0.0430
575000.00	2.3000	0.9332	8932.75	0.0357
600000.00	2.4000	0.9442	7404.15	0.0296
625000.00	2.5000	0.9534	6127.36	0.0245
650000.00	2.6000	0.9611	5061.04	0.0202
675000.00	2.7000	0.9675	4170.32	0.0167
700000.00	2.8000	0.9728	3425.93	0.0137
725000.00	2.9000	0.9773	2803.56	0.0112
750000.00	3.0000	0.9810	2282.95	0.0091
775000.00	3.1000	0.9844	1849.60	0.0074
800000.00	3.2000	0.9873	1496.51	0.0060
825000.00	3.3000	0.9897	1209.42	0.0048
850000.00	3.4000	0.9916	976.16	0.0039
875000.00	3.5000	0.9932	786.79	0.0031
900000.00	3.6000	0.9945	633.11	0.0025
925000.00	3.7000	0.9955	508.41	0.0020
950000.00	3.8000	0.9964	407.26	0.0016
975000.00	3.9000	0.9970	325.21	0.0013
1000000.00	4.0000	0.9976	258.68	0.0010
1025000.00	4.1000	0.9981	204.95	0.0008
1050000.00	4.2000	0.9985	162.19	0.0006
1075000.00	4.3000	0.9988	128.22	0.0005
1100000.00	4.4000	0.9990	101.24	0.0004
1125000.00	4.5000	0.9992	79.83	0.0003
1150000.00	4.6000	0.9994	62.86	0.0003
1175000.00	4.7000	0.9995	49.41	0.0002
1200000.00	4.8000	0.9996	38.75	0.0002
1225000.00	4.9000	0.9997	30.33	0.0001
1250000.00	5.0000	0.9998	23.67	0.0001

EXHIBIT V

COLLECTIVE RISK MODEL

LINE	EXPECTED LOSS	CLAIM SEVERITY DISTRIBUTION	CONTAGION PARAMETER	CLAIM COUNT MEAN	CLAIM COUNT STD DEV
1	1000000	PRODUCTS BI	0.0000	54.951	7.413

MIXING PARAMETER 0.0000
AGGREGATE MEAN 1000000
AGGREGATE STD DEV 383352

AGGREGATE LOSS AMOUNT	ENTRY RATIO	CUMULATIVE PROBABILITY	EXCESS PURE PREMIUM	EXCESS PURE PREMIUM RATIO
100000.00	0.1000	0.0001	900000.86	0.9000
200000.00	0.2000	0.0026	800088.87	0.8001
300000.00	0.3000	0.0134	700794.65	0.7008
400000.00	0.4000	0.0379	603226.59	0.6032
500000.00	0.5000	0.0812	509004.78	0.5090
600000.00	0.6000	0.1457	420179.50	0.4202
700000.00	0.7000	0.2291	338775.36	0.3388
800000.00	0.8000	0.3268	266476.65	0.2665
900000.00	0.9000	0.4315	204363.65	0.2044
1000000.00	1.0000	0.5358	152759.95	0.1528
1100000.00	1.1000	0.6334	111301.59	0.1113
1200000.00	1.2000	0.7197	79067.18	0.0791
1300000.00	1.3000	0.7923	54787.70	0.0548
1400000.00	1.4000	0.8506	37052.22	0.0371
1500000.00	1.5000	0.8956	24471.64	0.0245
1600000.00	1.6000	0.9291	15795.43	0.0158
1700000.00	1.7000	0.9530	9970.78	0.0100
1800000.00	1.8000	0.9697	6159.75	0.0062
1900000.00	1.9000	0.9809	3726.91	0.0037
2000000.00	2.0000	0.9882	2209.92	0.0022
2100000.00	2.1000	0.9929	1285.15	0.0013
2200000.00	2.2000	0.9958	733.40	0.0007
2300000.00	2.3000	0.9976	410.99	0.0004
2400000.00	2.4000	0.9986	226.28	0.0002
2500000.00	2.5000	0.9992	122.48	0.0001
2600000.00	2.6000	0.9996	65.20	0.0001
2700000.00	2.7000	0.9998	34.15	0.0000
2800000.00	2.8000	0.9999	17.61	0.0000
2900000.00	2.9000	0.9999	8.93	0.0000
3000000.00	3.0000	1.0000	4.48	0.0000

EXHIBIT VI

COLLECTIVE RISK MODEL

```
LINE #  1  CLAIM SEVERITY DISTRIBUTION
NAME  AGE<30

  LOSS AMOUNT      CUMULATIVE PROBABILITY

        0.0            0.0
     7500.00           0.33000
    37500.00           0.86000
    67500.00           1.00000

SUMMARY STATISTICS

  SEVERITY MEAN =      20512.50
  SEVERITY STD DEV =   17025.34
```

```
LINE #  2  CLAIM SEVERITY DISTRIBUTION
NAME  AGE 30-34

  LOSS AMOUNT      CUMULATIVE PROBABILITY

        0.0            0.0
     9500.00           0.33000
    47500.00           0.86000
    85500.00           1.00000

SUMMARY STATISTICS

  SEVERITY MEAN =      25982.50
  SEVERITY STD DEV =   21565.44
```

```
LINE #  3  CLAIM SEVERITY DISTRIBUTION
NAME  AGE 35-39

  LOSS AMOUNT      CUMULATIVE PROBABILITY

        0.0            0.0
    10000.00           0.33000
    50000.00           0.86000
    90000.00           1.00000

SUMMARY STATISTICS

  SEVERITY MEAN =      27350.00
  SEVERITY STD DEV =   22700.46
```

```
LINE #  4  CLAIM SEVERITY DISTRIBUTION
NAME  AGE 40-44

  LOSS AMOUNT      CUMULATIVE PROBABILITY

        0.0            0.0
    11000.00           0.33000
    55000.00           0.86000
    99000.00           1.00000

SUMMARY STATISTICS

  SEVERITY MEAN =      30085.00
  SEVERITY STD DEV =   24970.50
```

```
LINE #  5  CLAIM SEVERITY DISTRIBUTION
NAME  AGE 45-49

  LOSS AMOUNT      CUMULATIVE PROBABILITY

        0.0            0.0
    12500.00           0.33000
    62500.00           0.86000
   112500.00           1.00000

SUMMARY STATISTICS

  SEVERITY MEAN =      34187.50
  SEVERITY STD DEV =   28375.57
```

```
LINE #  6  CLAIM SEVERITY DISTRIBUTION
NAME  AGE 50-54

  LOSS AMOUNT      CUMULATIVE PROBABILITY

        0.0            0.0
    12500.00           0.33000
    62500.00           0.86000
   112500.00           1.00000

SUMMARY STATISTICS

  SEVERITY MEAN =      34187.50
  SEVERITY STD DEV =   28375.57
```

```
LINE #  7  CLAIM SEVERITY DISTRIBUTION
NAME  AGE 55-59

  LOSS AMOUNT      CUMULATIVE PROBABILITY

        0.0            0.0
    13500.00           0.33000
    67500.00           0.86000
   121500.00           1.00000

SUMMARY STATISTICS

  SEVERITY MEAN =      36922.50
  SEVERITY STD DEV =   30645.62
```

```
LINE #  8  CLAIM SEVERITY DISTRIBUTION
NAME  AGE 60-64

  LOSS AMOUNT      CUMULATIVE PROBABILITY

        0.0            0.0
    13500.00           0.33000
    67500.00           0.86000
   121500.00           1.00000

SUMMARY STATISTICS

  SEVERITY MEAN =      36922.50
  SEVERITY STD DEV =   30645.62
```

```
LINE #  9  CLAIM SEVERITY DISTRIBUTION
NAME  AGE 65+

  LOSS AMOUNT      CUMULATIVE PROBABILITY

        0.0            0.0
    15000.00           0.33000
    39471.00           0.86000

SUMMARY STATISTICS

  SEVERITY MEAN =      22435.75
  SEVERITY STD DEV =   12613.05
```

EXHIBIT VI (cont.)

COLLECTIVE RISK MODEL

LINE	EXPECTED LOSS	CLAIM SEVERITY DISTRIBUTION	CONTAGION PARAMETER	CLAIM COUNT MEAN	CLAIM COUNT STD DEV
1	47086	AGE<30	-0.0005	2.295	1.514
2	36342	AGE 30-34	-0.0009	1.399	1.182
3	35830	AGE 35-39	-0.0010	1.310	1.144
4	54938	AGE 40-44	-0.0012	1.826	1.350
5	130126	AGE 45-49	-0.0010	3.982	1.991
6	270050	AGE 50-54	-0.0008	7.899	2.801
7	395471	AGE 55-59	-0.0010	10.711	3.255
8	258525	AGE 60-64	-0.0018	7.002	2.629
9	13247	AGE 65+	-0.0400	0.590	0.759

MIXING PARAMETER 0.0000
AGGREGATE MEAN 1247615
AGGREGATE STD DEV 268182

AGGREGATE LOSS AMOUNT	ENTRY RATIO	CUMULATIVE PROBABILITY	EXCESS PURE PREMIUM	EXCESS PURE PREMIUM RATIO
124761.50	0.1000	0.0000	1122853.49	0.9000
249523.00	0.2000	0.0000	998092.02	0.8000
374284.50	0.3000	0.0000	873331.76	0.7000
499046.00	0.4000	0.0006	748596.17	0.6000
623807.50	0.5000	0.0047	624091.03	0.5002
748569.00	0.6000	0.0226	500799.81	0.4014
873330.50	0.7000	0.0739	381593.41	0.3059
998092.00	0.8000	0.1776	271922.78	0.2180
1122853.50	0.9000	0.3342	178628.28	0.1432
1247615.00	1.0000	0.5180	106942.98	0.0857
1372376.50	1.1000	0.6913	57912.49	0.0464
1497138.00	1.2000	0.8256	28257.40	0.0226
1621899.50	1.3000	0.9129	12411.61	0.0099
1746661.00	1.4000	0.9615	4913.10	0.0039
1871422.50	1.5000	0.9848	1757.08	0.0014
1996184.00	1.6000	0.9946	569.57	0.0005
2120945.50	1.7000	0.9983	167.97	0.0001
2245707.00	1.8000	0.9995	45.23	0.0000
2370468.50	1.9000	0.9999	11.16	0.0000
2495230.00	2.0000	1.0000	2.52	0.0000

EXHIBIT VII

COLLECTIVE RISK MODEL

LINE # 2 CLAIM SEVERITY DISTRIBUTION
NAME: AGGPRODUCTS DI

LOSS AMOUNT	CUMULATIVE PROBABILITY
0.0	0.0
50000.00	0.05630
100000.00	0.11350
150000.00	0.17080
200000.00	0.22650
250000.00	0.27930
300000.00	0.34340
350000.00	0.40750
400000.00	0.46850
450000.00	0.52520
500000.00	0.57700
550000.00	0.62640
600000.00	0.67230
650000.00	0.71390
700000.00	0.75120
750000.00	0.78440
800000.00	0.81390
850000.00	0.84000
900000.00	0.86300
950000.00	0.88310
1000000.00	0.90050

SUMMARY STATISTICS:

SEVERITY MEAN = 471677.50
SEVERITY STD DEV = 302129.53

LINE	EXPECTED LOSS	CLAIM SEVERITY DISTRIBUTION	CONTAGION PARAMETER	CLAIM COUNT MEAN	CLAIM COUNT STD DEV
1	500000	WORKERS COMP	0.0500	507.536	115.703
2	471677	AGGPRODUCTS DI	-1.0000	1.000	0.000

MIXING PARAMETER 0.0000
AGGREGATE MEAN 971677
AGGREGATE STD DEV 391334

AGGREGATE LOSS AMOUNT	ENTRY RATIO	CUMULATIVE PROBABILITY	EXCESS PURE PREMIUM	EXCESS PURE PREMIUM RATIO
100000.00	0.1029	0.0000	871677.79	0.8971
200000.00	0.2058	0.0014	771716.06	0.7942
300000.00	0.3087	0.0129	672291.84	0.6919
400000.00	0.4117	0.0456	575012.81	0.5918
500000.00	0.5146	0.1026	482229.96	0.4963
600000.00	0.6175	0.1807	396242.53	0.4078
700000.00	0.7204	0.2734	318855.59	0.3281
800000.00	0.8233	0.3727	251134.30	0.2585
900000.00	0.9262	0.4715	193373.56	0.1990
1000000.00	1.0291	0.5644	145233.93	0.1495
1100000.00	1.1321	0.6478	105930.73	0.1090
1200000.00	1.2350	0.7234	74529.47	0.0767
1300000.00	1.3379	0.7958	50519.41	0.0520
1400000.00	1.4408	0.8595	33390.65	0.0344
1500000.00	1.5437	0.9078	21893.95	0.0225
1600000.00	1.6466	0.9407	14435.87	0.0149
1700000.00	1.7496	0.9618	9640.27	0.0099
1800000.00	1.8525	0.9750	6531.19	0.0067
1900000.00	1.9554	0.9833	4480.87	0.0046
2000000.00	2.0583	0.9887	3101.11	0.0032
2100000.00	2.1612	0.9922	2155.16	0.0022
2200000.00	2.2641	0.9945	1497.26	0.0015
2300000.00	2.3670	0.9961	1035.06	0.0011
2400000.00	2.4700	0.9973	709.00	0.0007
2500000.00	2.5729	0.9981	479.13	0.0005
2600000.00	2.6758	0.9987	318.19	0.0003
2700000.00	2.7787	0.9991	207.46	0.0002
2800000.00	2.8816	0.9994	133.15	0.0001
2900000.00	2.9845	0.9996	84.80	0.0001
3000000.00	3.0874	0.9998	54.02	0.0001
3100000.00	3.1904	0.9998	34.55	0.0000
3200000.00	3.2933	0.9999	22.33	0.0000
3300000.00	3.3962	0.9999	14.51	0.0000
3400000.00	3.4991	1.0000	9.52	0.0000
3500000.00	3.6020	1.0000	6.31	0.0000

EXHIBIT VIIIA

COLLECTIVE RISK MODEL

LINE	EXPECTED LOSS	CLAIM SEVERITY DISTRIBUTION	CONTAGION PARAMETER	CLAIM COUNT MEAN	CLAIM COUNT STD DEV
1	500000	WORKERS COMP	0.0500	507.536	115.703

MIXING PARAMETER	0.0500
AGGREGATE MEAN	500000
AGGREGATE STD DEV	278304

AGGREGATE LOSS AMOUNT	ENTRY RATIO	CUMULATIVE PROBABILITY	EXCESS PURE PREMIUM	EXCESS PURE PREMIUM RATIO
100000.00	0.2000	0.0020	400028.62	0.8001
150000.00	0.3000	0.0167	350409.99	0.7008
200000.00	0.4000	0.0582	302152.97	0.6043
250000.00	0.5000	0.1296	256731.38	0.5135
300000.00	0.6000	0.2229	215475.78	0.4310
350000.00	0.7000	0.3260	179181.62	0.3584
400000.00	0.8000	0.4284	148061.99	0.2961
450000.00	0.9000	0.5231	121890.37	0.2438
500000.00	1.0000	0.6066	100182.61	0.2004
550000.00	1.1000	0.6779	82345.54	0.1647
600000.00	1.2000	0.7374	67775.00	0.1355
650000.00	1.3000	0.7864	55910.65	0.1118
700000.00	1.4000	0.8262	46260.63	0.0925
750000.00	1.5000	0.8585	38407.99	0.0768
800000.00	1.6000	0.8845	32007.21	0.0640
850000.00	1.7000	0.9055	26776.33	0.0536
900000.00	1.8000	0.9224	22487.47	0.0450
950000.00	1.9000	0.9360	18957.66	0.0379
1000000.00	2.0000	0.9470	16040.54	0.0321
1050000.00	2.1000	0.9559	13619.25	0.0272
1100000.00	2.2000	0.9631	11600.54	0.0232
1150000.00	2.3000	0.9691	9909.96	0.0198
1200000.00	2.4000	0.9739	8487.99	0.0170
1250000.00	2.5000	0.9779	7286.92	0.0146
1300000.00	2.6000	0.9812	6268.44	0.0125
1350000.00	2.7000	0.9840	5401.65	0.0108
1400000.00	2.8000	0.9863	4661.53	0.0093
1450000.00	2.9000	0.9883	4027.72	0.0081
1500000.00	3.0000	0.9899	3483.56	0.0070
1600000.00	3.2000	0.9925	2611.80	0.0052
1700000.00	3.4000	0.9944	1962.32	0.0039
1800000.00	3.6000	0.9958	1476.33	0.0030
1900000.00	3.8000	0.9969	1111.75	0.0022
2000000.00	4.0000	0.9976	837.86	0.0017
2250000.00	4.5000	0.9988	414.75	0.0008
2500000.00	5.0000	0.9994	206.98	0.0004
2750000.00	5.5000	0.9997	104.49	0.0002
3000000.00	6.0000	0.9999	53.46	0.0001
3500000.00	7.0000	1.0000	14.80	0.0000

EXHIBIT VIIIB

COLLECTIVE RISK MODEL

LINE	EXPECTED LOSS	CLAIM SEVERITY DISTRIBUTION	CONTAGION PARAMETER	CLAIM COUNT MEAN	CLAIM COUNT STD DEV
1	500000	PRODUCTS BI	0.2500	27.476	14.704

MIXING PARAMETER	0.0500
AGGREGATE MEAN	500000
AGGREGATE STD DEV	394054

AGGREGATE LOSS AMOUNT	ENTRY RATIO	CUMULATIVE PROBABILITY	EXCESS PURE PREMIUM	EXCESS PURE PREMIUM RATIO
50000.00	0.1000	0.0590	451443.29	0.9029
100000.00	0.2000	0.1190	405889.22	0.8118
150000.00	0.3000	0.1785	363331.93	0.7267
200000.00	0.4000	0.2370	323717.22	0.6474
250000.00	0.5000	0.2979	287072.48	0.5741
300000.00	0.6000	0.3615	253551.37	0.5071
350000.00	0.7000	0.4244	223208.29	0.4464
400000.00	0.8000	0.4844	195943.85	0.3919
450000.00	0.9000	0.5403	171578.19	0.3432
500000.00	1.0000	0.5920	149904.46	0.2998
550000.00	1.1000	0.6394	130708.08	0.2614
600000.00	1.2000	0.6825	113772.55	0.2275
650000.00	1.3000	0.7213	98883.57	0.1978
700000.00	1.4000	0.7561	85833.47	0.1717
750000.00	1.5000	0.7870	74425.07	0.1489
800000.00	1.6000	0.8144	64474.34	0.1289
850000.00	1.7000	0.8386	55811.94	0.1116
900000.00	1.8000	0.8598	48283.68	0.0966
950000.00	1.9000	0.8784	41750.46	0.0835
1000000.00	2.0000	0.8947	36087.71	0.0722
1050000.00	2.1000	0.9089	31184.51	0.0624
1100000.00	2.2000	0.9212	26942.67	0.0539
1150000.00	2.3000	0.9319	23275.58	0.0466
1200000.00	2.4000	0.9412	20107.21	0.0402
1250000.00	2.5000	0.9492	17370.97	0.0347
1300000.00	2.6000	0.9561	15008.74	0.0300
1350000.00	2.7000	0.9622	12969.91	0.0259
1400000.00	2.8000	0.9673	11210.47	0.0224
1450000.00	2.9000	0.9718	9692.27	0.0194
1500000.00	3.0000	0.9757	8382.22	0.0168
1550000.00	3.1000	0.9790	7251.73	0.0145
1600000.00	3.2000	0.9819	6276.06	0.0126
1650000.00	3.3000	0.9844	5433.88	0.0109
1700000.00	3.4000	0.9865	4706.75	0.0094
1750000.00	3.5000	0.9883	4078.79	0.0082
1800000.00	3.6000	0.9899	3536.32	0.0071
1850000.00	3.7000	0.9913	3067.53	0.0061
1900000.00	3.8000	0.9925	2662.27	0.0053
1950000.00	3.9000	0.9935	2311.79	0.0046
2000000.00	4.0000	0.9944	2008.55	0.0040
2050000.00	4.1000	0.9951	1746.06	0.0035
2100000.00	4.2000	0.9958	1518.75	0.0030
2150000.00	4.3000	0.9963	1321.81	0.0026
2200000.00	4.4000	0.9968	1151.08	0.0023
2250000.00	4.5000	0.9972	1003.02	0.0020
2300000.00	4.6000	0.9976	874.53	0.0017
2350000.00	4.7000	0.9979	762.99	0.0015
2400000.00	4.8000	0.9982	666.09	0.0013
2450000.00	4.9000	0.9984	581.88	0.0012
2500000.00	5.0000	0.9986	508.66	0.0010

EXHIBIT VIIIC

COLLECTIVE RISK MODEL

LINE # 1 CLAIM SEVERITY DISTRIBUTION
NAME: AGGWORKERS COMP

LOSS AMOUNT	CUMULATIVE PROBABILITY
0.0	0.0
100000.00	0.00200
150000.00	0.01670
200000.00	0.05820
250000.00	0.12960
300000.00	0.22290
350000.00	0.32600
400000.00	0.42840
450000.00	0.52310
500000.00	0.60660
550000.00	0.67790
600000.00	0.73740
650000.00	0.78640
700000.00	0.82620
750000.00	0.85850
800000.00	0.88450
850000.00	0.90550
900000.00	0.92240
950000.00	0.93600
1000000.00	0.94700
1050000.00	0.95590
1100000.00	0.96310
1150000.00	0.96910
1200000.00	0.97390
1250000.00	0.97790
1300000.00	0.98120
1350000.00	0.98400
1400000.00	0.98630
1450000.00	0.98830
1500000.00	0.98990
1600000.00	0.99250
1700000.00	0.99440
1800000.00	0.99580
1900000.00	0.99690
2000000.00	0.99760
2250000.00	0.99880
2500000.00	0.99940
2750000.00	0.99970
3000000.00	0.99990
3500000.00	1.00000

SUMMARY STATISTICS

SEVERITY MEAN = 499980.00
SEVERITY STD DEV = 279204.82

LINE # 2 CLAIM SEVERITY DISTRIBUTION
NAME AGGPRODUCTS BI

LOSS AMOUNT	CUMULATIVE PROBABILITY
0.0	0.0
50000.00	0.05900
100000.00	0.11900
150000.00	0.17850
200000.00	0.23700
250000.00	0.29790
300000.00	0.36150
350000.00	0.42440
400000.00	0.48440
450000.00	0.54030
500000.00	0.59200
550000.00	0.63940
600000.00	0.68250
650000.00	0.72130
700000.00	0.75610
750000.00	0.78700
800000.00	0.81440
850000.00	0.83860
900000.00	0.85980
950000.00	0.87840
1000000.00	0.89470

SUMMARY STATISTICS

SEVERITY MEAN = 464057.50
SEVERITY STD DEV = 305133.12

EXHIBIT VIIIC (cont.)

COLLECTIVE RISK MODEL

LINE	EXPECTED LOSS	CLAIM SEVERITY DISTRIBUTION	CONTAGION PARAMETER	CLAIM COUNT MEAN	CLAIM COUNT STD DEV
1	499980	AGGWORKERS COMP	-1.0000	1.000	0.000
2	464057	AGGPRODUCTS BI	-1.0000	1.000	0.000

MIXING PARAMETER	0.0500
AGGREGATE MEAN	964037
AGGREGATE STD DEV	475482

AGGREGATE LOSS AMOUNT	ENTRY RATIO	CUMULATIVE PROBABILITY	EXCESS PURE PREMIUM	EXCESS PURE PREMIUM RATIO
100000.00	0.1037	0.0002	864042.82	0.8963
200000.00	0.2075	0.0052	764219.69	0.7927
300000.00	0.3112	0.0282	665690.29	0.6905
400000.00	0.4149	0.0767	570723.02	0.5920
500000.00	0.5187	0.1480	481793.22	0.4998
600000.00	0.6224	0.2348	400838.32	0.4158
700000.00	0.7261	0.3286	328977.39	0.3412
800000.00	0.8298	0.4226	266553.35	0.2765
900000.00	0.9336	0.5124	213347.26	0.2213
1000000.00	1.0373	0.5955	168805.70	0.1751
1100000.00	1.1410	0.6705	132180.58	0.1371
1200000.00	1.2448	0.7362	102593.67	0.1064
1300000.00	1.3485	0.7920	79085.13	0.0820
1400000.00	1.4522	0.8382	60674.39	0.0629
1500000.00	1.5560	0.8754	46424.32	0.0482
1600000.00	1.6597	0.9047	35491.54	0.0368
1700000.00	1.7634	0.9275	27153.98	0.0282
1800000.00	1.8671	0.9450	20817.57	0.0216
1900000.00	1.9709	0.9582	16008.18	0.0166
2000000.00	2.0746	0.9683	12355.99	0.0128
2100000.00	2.1783	0.9758	9577.22	0.0099
2200000.00	2.2821	0.9815	7456.65	0.0077
2300000.00	2.3858	0.9858	5832.22	0.0060
2400000.00	2.4895	0.9891	4582.47	0.0048
2500000.00	2.5933	0.9915	3616.50	0.0038
2600000.00	2.6970	0.9934	2866.29	0.0030
2700000.00	2.8007	0.9948	2280.82	0.0024
2800000.00	2.9045	0.9959	1821.77	0.0019
2900000.00	3.0082	0.9968	1460.19	0.0015
3000000.00	3.1119	0.9975	1174.14	0.0012
3100000.00	3.2156	0.9980	946.93	0.0010
3200000.00	3.3194	0.9984	765.77	0.0008
3300000.00	3.4231	0.9987	620.81	0.0006
3400000.00	3.5268	0.9990	504.43	0.0005
3500000.00	3.6306	0.9992	410.73	0.0004
3600000.00	3.7343	0.9993	335.08	0.0003
3700000.00	3.8380	0.9994	273.85	0.0003
3800000.00	3.9418	0.9996	224.17	0.0002
3900000.00	4.0455	0.9996	183.78	0.0002
4000000.00	4.1492	0.9997	150.88	0.0002
4100000.00	4.2529	0.9998	124.02	0.0001
4200000.00	4.3567	0.9998	102.07	0.0001
4300000.00	4.4604	0.9998	84.10	0.0001
4400000.00	4.5641	0.9999	69.36	0.0001
4500000.00	4.6679	0.9999	57.26	0.0001
4600000.00	4.7716	0.9999	47.33	0.0000
4700000.00	4.8753	0.9999	39.16	0.0000
4800000.00	4.9791	0.9999	32.45	0.0000
4900000.00	5.0828	1.0000	26.93	0.0000
5000000.00	5.1865	1.0000	22.41	0.0000

EXHIBIT IX

```
C***********************************************************************
C                            PROGRAM USED WITH
C           "THE CALCULATION OF AGGREGATE LOSS DISTRIBUTIONS
C           FROM CLAIM SEVERITY AND CLAIM COUNT DISTRIBUTIONS"
C                                   BY
C                  PHILIP HECKMAN AND GLENN MEYERS
C
C          THE PROGRAM IS WRITTEN IN IBM FORTRAN WITH G1 COMPILER.
C***********************************************************************
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 CUMPRB(128),AMT(128,32),PK(128,32)
      REAL*8 VARC(32),XLAM(32),SIGP(32),EXLOSS(32)
      REAL*8 A(257),T(256,5),F(256,5),G(256,5),X(512),ER(512)
      INTEGER NPTS(32)
      COMPLEX*16 NAME(32),EXHBT
C***********************************************************************
C     STEPS 1 AND 2
C***********************************************************************
      SIGSQA=0.0
      XMUA=0.0
      READ(3,1)EXHBT
C  EXHBT IS SUPPLIED TO IDENTIFY THE RUN
    1 FORMAT(2A8)
      READ(3,*)VARB
C  VARB=MIXING PARAMETER
      VARB=DMIN1(VARB,1.0-1D-7)
      VARB=DMAX1(VARB,1D-7)
      DO 10 N=1,32
      READ(3,*,END=20)EXLOSS(N),VARC(N)
C  EXLOSS=EXPECTED LOSSES FOR THIS LINE
C  VARC=CONTAGION PARAMETER FOR THIS LINE
      IF(DABS(VARC(N)).LT.1D-7)VARC(N)=1D-7
      READ(3,1)NAME(N)
C  NAME IS SUPPLIED BY THE USER TO IDENTIFY THE C.S.D.
      READ(3,*)NPTS(N)
C  NPTS IS THE NUMBER OF POINTS NEEDED TO SPECIFY THE C.S.D.
      AMT(1,N)=0.0
      CUMPRB(1)=0.0
      NPTS(N)=NPTS(N)+1
      X1=0.0
      X2=0.0
      NPT=NPTS(N)
      DO 3 I=2,NPT
      READ(3,*)AMT(I,N),CUMPRB(I)
C  AMT IS A CLAIM SEVERITY
C  CUMPRB IS THE CUMULATIVE PROBABILITY OF AMT
      PROB=CUMPRB(I)-CUMPRB(I-1)
      PK(I-1,N)=PROB/(AMT(I,N)-AMT(I-1,N))
      X1=X1+PROB*(AMT(I-1,N)+AMT(I,N))/2.
    3 X2=X2+PROB*(AMT(I,N)**2+AMT(I,N)*AMT(I-1,N)+AMT(I-1,N)**2)/3.
      PROB=1.0-CUMPRB(NPT)
```

```
      X1=X1+PROB*AMT(NPT,N)
      X2=X2+PROB*AMT(NPT,N)**2
      PK(NPT,N)=PROB
C  NOTE:UNUSUAL USE OF PK(NPT,N)
      SIGS=X2-X1**2
      XLAM(N)=EXLOSS(N)/X1
      SIGP(N)=DSQRT(XLAM(N)+VARC(N)*XLAM(N)**2)
      SIGSQA=XLAM(N)*(SIGS)+(X1*SIGP(N))**2+SIGSQA
      SIGS=DSQRT(SIGS)
      XMUA=X1*XLAM(N)+XMUA
C***********************************************************************
C     PRODUCE DISPLAY OF CLAIM SEVERITY DISTRIBUTION
C***********************************************************************
      WRITE(7,7)EXHBT,N,NAME(N)
    7 FORMAT('1',2A8,T31,'COLLECTIVE RISK MODEL'//
     &' LINE # ',I2,'  CLAIM SEVERITY DISTRIBUTION'/
     &' NAME: ',2A8//
     &'    LOSS AMOUNT      CUMULATIVE PROBABILITY'/)
      DO 8 I=1,NPT
    8 WRITE(7,9)AMT(I,N),CUMPRB(I)
    9 FORMAT(3X,F10.2,T27,F7.5)
   10 WRITE(7,11)X1,SIGS
   11 FORMAT(//' SUMMARY STATISTICS:'//'   SEVERITY MEAN =     ',F10.2/
     &'    SEVERITY STD DEV = ',F10.2/)
      WRITE(7,15)
   15 FORMAT(' ONLY 32 LINES ALLOWED')
C
   20 SIGSQA=VARB*XMUA**2+SIGSQA*(1.0+VARB)
      SIGA=DSQRT(SIGSQA)
      NL=N-1
C***********************************************************************
C     STEPS 3 AND 4
C***********************************************************************
      XMAX=0.0
      NUMX=1
      READ(2,*)ITYPE
C  ITYPE=1 IF AGGREGATE LOSS AMOUNT IS INPUT
C  ITYPE=2 IF ENTRY RATIO IS INPUT
      IF(ITYPE.EQ.2)GOTO 35
      IF(ITYPE.NE.1)STOP
   30 READ(2,*,END=50)X(NUMX)
C  X IS AN AGGREGATE LOSS AMOUNT
      ER(NUMX)=X(NUMX)/XMUA
      XMAX=DMAX1(XMAX,X(NUMX))
      NUMX=NUMX+1
      GOTO 30
   35 READ(2,*,END=50)ER(NUMX)
C  ER IS AN ENTRY RATIO
      X(NUMX)=ER(NUMX)*XMUA
      XMAX=DMAX1(XMAX,X(NUMX))
      NUMX=NUMX+1
      GOTO 35
   50 NUMX=NUMX-1
      H=2.*3.14159265*SIGA/XMAX
```

```
C***********************************************************************
C     STEP 5
C***********************************************************************
      A(1)=0.D
      DO 60 I=1,256
      DO 60 J=1,5
      F(I,J)=1.D
   60 G(I,J)=0.D
C
      DO 100 I=1,5
      A(I+1)=H/2.**(5-I)
  100 CALL GAUSS5(I,A,T,F,G,NPTS,AMT,PK,XLAM,VARC,SIGA,NL)
      DO 110 I=6,256
      A(I+1)=A(I)+H
      CALL GAUSS5(I,A,T,F,G,NPTS,AMT,PK,XLAM,VARC,SIGA,NL)
      E=D
      DO 105 J=1,5
  105 E=DMAX1(E,F(I,J)/T(I,J))
      IF(E.LT.D.00002)GOTO 120
  110 CONTINUE
  120 NINT=I
C***********************************************************************
C     PRODUCE DISPLAY OF OUTPUT
C***********************************************************************
  200 WRITE(7,201)EXHBT
  201 FORMAT('1',2A8,T31,'COLLECTIVE RISK MODEL'//
     &T9,'EXPECTED   CLAIM SEVERITY    CONTAGION   ',
     &'CLAIM COUNT   CLAIM COUNT'/
     &' LINE     LOSS       DISTRIBUTION      PARAMETER        ',
     &'MEAN        STD DEV'/)
      DO 210 I=1,NL
      IEX=IDINT(EXLOSS(I)+.5)
  210 WRITE(7,211)I,IEX,NAME(I),VARC(I),XLAM(I),SIGP(I)
  211 FORMAT(T3,I2,T9,I8,T20,2A8,T39,F7.4,T49,F10.3,T63,F10.3)
      IXMUA=IDINT(XMUA+.5)
      ISIGA=IDINT(SIGA+.5)
      WRITE(7,221)VARB,IXMUA,ISIGA
  221 FORMAT(//' MIXING PARAMETER',T22,F8.4/
     &' AGGREGATE MEAN   ',T22,I8/
     &' AGGREGATE STD DEV ',T22,I8//
     &'     AGGREGATE',6X,'ENTRY',5X,'CUMULATIVE',7X,'EXCESS PURE',5X,
     &'EXCESS PURE'/
     &'    LOSS AMOUNT',5X,'RATIO',5X,'PROBABILITY',8X,'PREMIUM',6X,
     &'PREMIUM RATIO'//)
C***********************************************************************
C     STEP 6
C***********************************************************************
      DO 310 I=1,NUMX
      CALL PCTEPP(X(I),VARB,XMUA,SIGA,A,T,F,G,NINT,PCT,EPP)
      TBM=EPP/XMUA
  310 WRITE(7,311)X(I),ER(I),PCT,EPP,TBM
  311 FORMAT(3X,F11.2,4X,F7.4,6X,F7.4,7X,F11.2,8X,F7.4)
C***********************************************************************
C     PRINT TECHNICAL INFORMATION
```

```
C*************************************************************************
      EPPERR=2*SIGA*E/(3.14159265*XMUA)
      WRITE(7,401)EXHBT,H,NINT,EPPERR
 401  FORMAT('1',2A8,T31,'COLLECTIVE RISK MODEL'//
     &' TECHNICAL INFORMATION'/
     &' H=',T45,F12.3/
     &' NUMBER OF INTERVALS=',T45,I12/
     &' ESTIMATED TRUNCATION ERROR IN EPP RATIO=',T45,F12.6)
      END
C
C     END OF MAIN PROGRAM - SUBROUTINES FOLLOW
C
C*************************************************************************
C     FIND POINTS WHERE THE AGGREGATE CHARACTERISTIC MUST BE EVALUATED
C     CALLED FROM THE MAIN PROGRAM
C*************************************************************************
      SUBROUTINE GAUSS5(I,A,T,F,G,NPTS,AMT,PK,XLAM,VARC,SIGA,NL)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 AMT(128,32),PK(128,32),VARC(32)
      REAL*8 A(1),T(256,5),F(256,5),G(256,5),S(5),XLAM(32)
      INTEGER NPTS(32)
      DATA S/-.90617985,-.53846931,0.0,.53846931,.90617985/
C
      DO 100 J=1,5
      T(I,J)=(A(I+1)-A(I))*S(J)/2.+(A(I+1)+A(I))/2.
      TS=T(I,J)/SIGA
      DO 100 K=1,NL
      CALL AGGCHR(NPTS,AMT,PK,K,XLAM,VARC,TS,FL,GL)
      F(I,J)=F(I,J)*FL
 100  G(I,J)=G(I,J)+GL
      RETURN
      END
C*************************************************************************
C     EVALUATE THE AGGREGATE CHARACTERISTIC
C     CALLED FROM GAUSS5
C*************************************************************************
      SUBROUTINE AGGCHR(NPTS,AMT,PK,K,XLAM,VARC,T,F,G)
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER NPTS(32)
      REAL*8 AMT(128,32),PK(128,32),XLAM(32),VARC(32),PZ(2)
      COMPLEX*16 Z
      EQUIVALENCE (PZ,Z)
C  PZ(1)=REAL PART OF Z.  PZ(2)=COMPLEX PART OF Z.
C
      CALL SEVCHR(NPTS,AMT,PK,K,T,XH,XK)
      PZ(1)=1.0-VARC(K)*XLAM(K)*XH
      PZ(2)=-VARC(K)*XLAM(K)*XK
      Z=-1./VARC(K)*CDLOG(Z)
C  LOG OF MODULUS=REAL PART OF COMPLEX LOG
C  ARGUMENT=COMPLEX PART OF COMPLEX LOG
      F=DEXP(PZ(1))
      G=PZ(2)
      RETURN
      END
```

```
C***********************************************************************
C     EVALUATE THE CHARACTERISTIC OF THE SEVERITY DISTRIBUTION
C     CALLED FROM AGGCHR
C***********************************************************************
      SUBROUTINE SEVCHR(NPTS,AMT,PK,K,T,XH,XK)
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER NPTS(32)
      REAL*8 AMT(128,32),PK(128,32)
C
      S2=0.0
      C2=1.0
      TH=0.0
      TK=0.0
      NPT=NPTS(K)
      DO 100 L=2,NPT
      A=AMT(L,K)*T
      S1=S2
      C1=C2
      S2=DSIN(A)
      C2=DCOS(A)
      TH=TH+PK(L-1,K)*(S2-S1)
  100 TK=TK+PK(L-1,K)*(C1-C2)
      XH=TH/T-1.0+PK(NPT,K)*C2
      XK=TK/T+PK(NPT,K)*S2
      RETURN
      END
C***********************************************************************
C     INTEGRATE TO GET CDF AND EXCESS PP BY GAUSSIAN QUADRATURE
C     CALLED FROM THE MAIN PROGRAM
C***********************************************************************
      SUBROUTINE PCTEPP(X,VARB,XMUA,SIGA,A,T,F,G,NINT,PCT,EPP)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 A(1),T(256,5),F(256,5),G(256,5),W(5)
      DATA W/.23692689,.47862867,.56888889,.47862867,.23692689/
C
      EPP=0.0
      PCT=0.0
      R=1.0+1.0/VARB
      DO 200 I=1,NINT
      P1=0.0
      P2=0.0
      DO 100 J=1,5
      XP1=1.0+(X*T(I,J)/(SIGA*R))**2
      ATX=DATAN(X*T(I,J)/(SIGA*R))
      P1=P1+W(J)*F(I,J)*XP1**((-1.-R)/2.)*DSIN((1.+R)*ATX-G(I,J))/T(I,J)
  100 P2=P2+W(J)*F(I,J)*(DCOS(G(I,J))-XP1**(-R/2.)*DCOS(R*ATX-G(I,J)))/
     &  T(I,J)**2
      PCT=PCT+(A(I+1)-A(I))*P1/2.
  200 EPP=EPP+(A(I+1)-A(I))*P2/2.
      PCT=.5+PCT/3.14159265
      EPP=XMUA-X/2.+EPP*SIGA/3.14159265
      RETURN
      END
```

MINUTES OF THE 1984 SPRING MEETING

May 13–16, 1984

CAMELBACK INN, SCOTTSDALE, ARIZONA

Sunday, May 13, 1984

The Board of Directors held their regular quarterly meeting from 1:00 p.m. to 4:00 p.m.

Registration was held from 4:00 p.m. to 6:30 p.m.

The Officers held a reception for new Fellows and their spouses from 5:30 p.m. to 6:30 p.m.

A general reception for all members and guests was held from 6:30 p.m. to 7:30 p.m.

Monday, May 14, 1984

Registration continued from 7:00 a.m. to 7:55 a.m.

President Carlton W. Honebein opened the meeting at 8:00 a.m. Mr. Chester Flaxmire, Special Assistant to the Director of the Arizona Insurance Department, welcomed our Society to Arizona.

Mr. Honebein then recognized the 60 new Associates and presented diplomas to the 18 new Fellows. The names of these individuals follow.

FELLOWS

Bonnie L. Boccitto	Adrienne B. Kane	Emanuel Pinto
Amy S. Bouska	Kyleen Knilans	Neal J. Schmidt
Lisa G. Chanzit	Michael W. Kooken	Joy A. Schwartzman
John D. Coffin	Thomas J. Kozik	Darlene P. Tom
George T. Dodd	Steven D. Marks	William F. Weimer
Thomas J. Duffy	Paul G. O'Connell	Michael L. Wiseman

ASSOCIATES

Bruce C. Anderson
Leo R. Bakel
Anthony J. Balchunas
Glenn R. Balling
Steven D. Basson
Robert A. Bear
Ina M. Becraft
Scott C. Belden
Janice L. Berry
William P. Biegaj
Bonnie L. Boccitto
Amy S. Bouska
Wallis A. Boyd
Susan E. Bryan
Kenrick A. Campbell
John E. Captain
Jeffrey R. Carlson
Joel S. Chansky
Jeanne D. Chiang
Martin W. Deede
Claude Désilets
Camille Dupuis
N. Paul Dyck
Paula L. Elliott
John R. Forney, Jr.
Gregory S. Grace
Ronald E. Greco
Gayle E. Haskell
Paul M. Hurley
Brenda J. Huyck
Andrew P. Johnson
Wayne S. Keller
Robert J. Kelley
Arthur B. Levenglick
Peter M. Licht
Dennis J. Loper
Daniel K. Lyons
Robert W. Matthews
Mary T. McQuilkin
Michael J. McSally
Gail A. Mendelssohn
John K. Mozeika
Karen L. Nester
Layne M. Onufer
Donald W. Palmer
Sylvie L. Paquette
Steven J. Peterson
Rhonda D. Port
Rajagopalan K. Raman
Ralph L. Rathjen
Randy J. Roth
Peter J. Schultheiss
Melvin S. Silver
Byron W. Smith
Judith P. Smith
Minh Trinh
Leigh M. Walker
Michael C. Walsh
Patricia J. Webster
Roy T. Woomer, III

Mr. Honebein concluded the business session at 9:00 a.m.

Dr. William C. Freund, Senior Vice President and Chief Economist of the New York Stock Exchange, delivered the Keynote Address. He offered his views on the future of financial service institutions in the 1980's.

At 10:30 a.m., Mr. W. James MacGinnitie moderated a panel on "Threats to Financial Solvency." His panel consisted of:

Charles F. Cook
President
American Universal Insurance Company

Richard Stewart
President
Stewart Economics, Inc.

James Wood
LeBoef, Lamb, Leiby & McRae

The panelists reviewed their thoughts on newly emerging threats to the financial solidity of the insurance industry.

The afternoon session consisted of a series of concurrent sessions, including six Discussion Paper Program presentations and five Workshops.

The Discussion Papers presented were:

1. "The Relationship Between Underwriting Profit and the Surplus Ratio: A Model"
   Author: Ray E. Niswander
   United States Automobile Association
   Reviewers: Ralph S. Blanchard, III and
   Claus S. Metzner
   Aetna Life & Casualty

2. "Loss Portfolios: Financial Reinsurance"
   Author: Lee R. Steeneck
   General Reinsurance Corporation
   Reviewer: Stephen J. Ludwig
   Hartford Insurance Group

3. "Property and Casualty Insurance: Solvency and Investments—Playing the Game"
   Author: Paul M. Otteson
   Consultant
   Reviewer: Robert P. Eramo
   Hanover Insurance Company

4. "Empirical Measure of Reserve Level Uncertainty Relative to Discounting and Financial Solvency for a Monoline Medical Profesional Liability Insurer"
   Authors: Allan Kaufman and David L. Wasserman
   Peat, Marwick, Mitchell & Co.
   Reviewer: William J. Hibberd
   Royal Insurance Company

5. "Evaluation of the Financial Condition of Insurance Companies—A Theoretical Approach"
   Author: Mary Lou O'Neil
   Prudential Property and Casualty Insurance Co.
   Reviewers: Alfred O. Weller
   BRI Coverage Corporation
   William F. Weimer
   F & G Reinsurance

6. "The Evaluation of an Insurance Company for an Acquisition Involving a Section 338 Tax Election"
   Authors: James A. Hall, Orin Linden, Stephen Gerard, and Michael Heitz
   Coopers & Lybrand
   Reviewers: Robert J. Finger
   Future Cost Analysts
   Randall E. Brubaker
   Fireman's Fund Insurance Companies

The Workshops covered the following topics:

1. "Mechanics of Solvency Measurement"
   Robert A. Brian
   Conning & Company

2. "Reinsurance for Financial Solvency"
   Gary S. Patrick and Heidi Hutter
   North American Reinsurance Co.

3. "Allocation of Surplus by Line of Business"
   Joel S. Wiener
   Towers, Perrin, Forster and Crosby

   Richard Derrig
   Masschusetts Rating Bureaus

4. "A 'Bring Your Calculator' Workshop on Asset/Liability Mismatch"
   Charles H. Berry
   Aetna Life and Casualty

5. Limited Attendance Workshop: "Loss Portfolio Transfers"
   Kirk G. Roeser—Workshop Coordinator
   Gill and Roeser, Inc.

A general reception was held from 6:30 p.m. to 7:30 p.m.

Tuesday, May 15, 1984

Tuesday was devoted to a continuation of the concurrent sessions from Monday afternoon.

There was a general reception and dinner held from 6:30 p.m. to 9:30 p.m.

Wednesday, May 16, 1984

From 9:00 a.m. to 9:55 a.m. there was a continuation of the concurrent sessions from Monday afternoon.

At 10:15 a.m. Mr. Honebein reconvened the business session. The Michelbacher Prize was awarded to Paul M. Otteson.

At 10:30 a.m., Mr. M. Stanley Hughey moderated a panel entitled "Applying Measures of Solvency." His panel consisted of:

Roger C. Day
Commissioner of Insurance
State of Utah

Stanford Miller
Consultant
Stanford Miller Consulting, Inc.

William Hartman
Alexander Brown & Co.

Mr. Honebein closed the meeting at 12:00 noon.

In attendance by registration records were 184 Fellows; 144 Associates; and 34 guests, subscribers and students. The list follows.

FELLOWS

Adler, M.
Alexander, L. M.
Alfuth, T. J.
Anker, R. A.
Arata, D. A.
Asch, N. E.
Atwood, C. R.
Barnes, G. R.
Bartlett, W. N.
Bass, I. K.
Bassman, B. C.
Bayley, T. R.
Beer, A. J.
Bell, L. L.
Bellinghausen, G. F.
Berquist, J. R.
Berry, C. H., III
Bethel, N. A.
Beverage, R. M.
Bill, R. A.
Blanchard, R. S., III
Boccitto, B. L.
Boison, L. A., Jr.
Bornhuetter, R. L.
Boulanger, F.
Bouska, A. S.
Bradley, D. R.
Bradshaw, J. G., Jr.
Brannigan, J. F.
Brian, R. A.
Brubaker, R. E.
Bryan, C. A.
Cantin, C.
Chanzit, L. G.
Cheng, J. S.
Childs, D. M.
Christie, J. K.
Clinton, R. K.
Coffin, J. D.
Collins, D. J.
Conger, R. F.
Conners, J. B.
Cook, C. F.
Corr, F. X.
Covney, M. D.
Cundy, R. M.
Curry, A. C.
Curry, H. E.
Dahlquist, R. A.
Davis, G. E.
Dean, C. G.
Degerness, J. A.
Dempster, H. V.
Dodd, G. T.
Doellman, J. L.
Donaldson, J. P.
Drennan, J. P.
Duffy, T. J.
Easton, R. D.
Egnasko, G. J.
Eldridge, D. J.
Engles, D.
Eyers, R. G.
Faber, J. A.
Fagan, J. L.
Fallquist, R. J.
Fein, R. I.
Ferguson, R. E.
Fiebrink, M. E.
Finger, R. J.
Fisher, R. S.
Fisher, W. H.
Flaherty, D. J.
Flynn, D. P.
Foote, J. M.
Fossa, E. F.
Fresch, G. W.
Friedberg, B. F.
Furst, P. A.
Fusco, M.
Gallagher, T. L.
Garand, C. P.
Gleeson, O. M.
Gluck, S. M.
Gottheim, E. F.
Grady, D. J.
Grannan, P. J.
Graves, J. S.
Grippa, A. J.
Groot, S. L.
Hachemeister, C. A.
Hafling, D. N.
Hall, J. A., III
Haner, W. J.
Hanson, H. D.
Hartman, D. G.
Harwayne, F.
Hayne, R. M.
Hazam, W. J.
Heer, E. L.
Henry, D. R.
Herder, J. M.
Herzfeld, J.
Hibberd, W. J.
Higgins, B. J.
Hillhouse, J. A.
Honebein, C. W.
Horowitz, B. A.
Hough, P. E.
Hughey, M. S.
Ingco, A. M.
Inkrott, J. G.
Irvan, R. P.
Jaeger, R. M.

FELLOWS

Jean, R. W.
Jerabek, G. J.
Johnson, W. H., Jr.
Jones, B. R.
Kallop, R. H.
Kane, A. B.
Khury, C. K.
Kilbourne, F. W.
Kist, F. O.
Kline, D. F.
Knilans, K.
Koch, L. W.
Kooken, M. W.
Kozik, T. J.
Krause, G. A.
LaRose, J. G.
Leong, W.
Levin, J. W.
Linden, O. M.
Lindquist, P. L.
Lommele, J. A.
Lowe, R. F.
Lowe, S. P.
Ludwig, S. J.
Lyle, A. C.
MacGinnitie, W. J.
Makgill, S. S.
Marks, S. D.
McCarter, M. G.
McClure, R. D.
McConnell, C. W.
McGovern, W. G.
McManus, M. F.
McMurray, M. A.
Mealy, D. C.
Miccolis, J. A.
Miccolis, R. S.
Mohl, F. J.
Muetterties, J. H.
Munro, R. E.
Murad, J. A.
Murrin, T. E.
Muza, J. J.
Myers, N. R.
Nelson, J. R.
Newlin, P. R.
Nickerson, G. V.
Niswander, R. E., Jr.
O'Connell, P. G.
O'Neil, M. L.
Otteson, P. M.
Pagnozzi, R. D.
Palm, R. G.
Pastor, G. H.
Patrik, G. S.
Pearl, M. B.
Petersen, B. A.
Petlick, S.
Petz, E. F.
Philbrick, S. W.
Phillips, H. J.
Pinto, E.
Pollack, R.
Pratt, J. J.
Quinlan, J. A.
Reichle, K. A.
Reynolds, J. J., III
Robertson, J. P.
Rogers, D. J.
Rosenberg, M.
Rosenberg, S.
Ross, J. P.
Roth, R. J., Jr.
Scheibl, J. A.
Schmidt, N. J.
Schneider, H. N.
Schumi, J. R.
Schwartzman, J. A.
Sheppard, A. R.
Sherman, R. E.
Shoop, E. C.
Simon, L. J.
Skurnick, D.
Smith, L. M.
Sobel, M. J.
Stanard, J. N.
Steeneck, L. R.
Steer, G. D.
Stephenson, E. A.
Stewart, C. W.
Streff, J. P.
Strug, E. J.
Tatge, R. L.
Tiller, M. W.
Tom, D. P.
Toothman, M. L.
Tverberg, G. E.
VanSlyke, O. E.
Venter, G. G.
Walker, R. D.
Walters, M. A.
Wasserman, D. L.
Webb, B. L.
Weimer, W. F.
Weller, A. O.
Wess, C.
Westerholm, D. C.
White, J.
Wiseman, M. L.
Woll, R. G.
Yonkunas, J. P.
Youngerman, H.
Young, B. G.
Zatorski, R. T.

ASSOCIATES

Amundson, R. B.
Anderson, B. C.
Andler, J. A.
Andrus, W. R.
Austin, J. P.
Bakel, L. R.
Balchunas, A. J.
Balling, G. R.
Basson, S. D.
Bear, R. A.
Becraft, I. M.
Belden, S. C.
Bell, A. A.
Bensimon, A. S.
Berry, J. L.
Biegaj, W. P.
Boyd, W. A.
Brooks, D. L.
Bryan, S. E.
Bursley, K. H.
Cadorine, A. R.
Campbell, K. A.
Captain, J. E.
Carbaugh, A. B.
Carlson, J. R.
Chansky, J. S.
Chiang, J. D.
Chorpita, F. M.
Chou, L. L.
Chou, P. S.
Cohen, A. I.
Colgren, K. D.
Connor, V. P.
Costner, J. E.
Currie, R. A.
Deconti, M. A.
Deede, M. W.
Degarmo, L. W.
Desilets, C.
Deutsch, R. V.
Dornfeld, J. L.
DuPuis, C.
Dyck, N. P.
Edie, G. M.
Egnasko, V. M.
Einck, N. R.
Elliott, P. L.
Eramo, R. P.
Fasking, D. D.
Fiebrink, D. C.
Forney, J. R., Jr.
Godbold, M. J.
Godbold, N. T.
Goldberg, T. L.
Grace, G. S.
Greco, R. E.
Gruber, C.
Hale, J. B.
Hall, A. A.
Harrison, E. E.
Haskell, G. E.
Henkes, J. P.
Henzler, P. J.
Hobart, G. P.
Hurley, J. D.
Hurley, P. M.
Hutter, H. E.
Huyck, B. J.
Javaruski, J. J.
Jersey, J. R.
Johnson, A. P.
Keller, W. S.
Kelley, R. J.
Kelly, M. K.
Klawitter, W. A.
Kolojay, T. M.
Koupf, G. I.
LaFrenaye, C.
Licht, P. M.
Lindquist, R. J.
Lis, R. S., Jr.
Liuzzi, J. R.
Livingston, R. P.
Loper, D. J.
Lyons, D. K.
Matthews, R. W.
McConnell, D. M.
McQuilkin, M. T.
McSally, M. J.
Mendelssohn, G. A.
Meyer, R. E.
Mittal, M. L.
Mokros, B. F.
Morgan, S. T.
Mozeika, J. K.
Mulder, E. T.
Napierski, J. D.
Neis, A. R.
Nelson, J. K.
Nester, K. L.
Neuhauser, F., Jr.
Nolan, J. D.
Ogden, D. F.
Onufer, L. M.
Paquette, S. L.
Pei, K.-J.
Penniman, K. T.
Peterson, S. J.
Petit, C. I.
Port, R. D.
Potok, C. M.
Potter, J. A.
Pulis, R. S.
Raman, R. K.

ASSOCIATES

Rathjen, R. L.
Reynolds, J. D.
Rosenberg, D. M.
Roth, R. J.
Rudduck, G. A.
Sandler, R. M.
Sansevero, M., Jr.
Schultheiss, P. J.
Sherman, O. L., Jr.
Silver, M. S.
Singer, P. E.
Skrodenis, D. P.
Smith, B. W.
Smith, J. P.
Smith, R. A.
Spalla, J. S.
Stadler, E.
Steinen, P. A.
Stroud, R. A.
Suchoff, S. B.
Torgrimson, D. A.
Trinh, M.
Urschel, F. A.
Varca, J. J.
Walsh, M. C.
Webster, P. J.
Weiner, J. S.
Wilson, O. T.
Woomer, R. T., III

GUESTS—STUDENTS—SUBSCRIBERS

Almer, M.
Anderson, E. V.
Arvanitis, R.
Bauer, B. P.
Boor, J. A.
Carlson, K. T.
Chang, C. E.
Curran, K. F.
Curtis, J. A.
Daniels, S.
Derrig, R. A.
Englander, J. A.
Eversmann, T. M.
Gamble, R. A.
Gutman, E.
Hager, G.
Johnson, C.
Kedrow, W. M.
Koester, S. M.
McMillen, R. H.
Metzner, C. S.
Novik, J. A.
O'Shea, H. J.
Radtke, W. L.
Roeser, K. G.
Schmidt, L. D.
Shapland, M. R.
Spangler, J. L.
Stenmark, J. A.
Thomas, A. M.
Vandernoth, J. P.
Vosburgh, J.
Weber, R. A.
Whitby, O.

Volume LXXI, Part 2 No. 136

PROCEEDINGS

November 11, 12, 13, 1984

PRESIDENTIAL ADDRESS—NOVEMBER 13, 1984
4891

CARLTON W. HONEBEIN

1984 marks the 70th year of the Casualty Actuarial Society, which was formed to meet the needs of a growing general insurance business in the United States. The CAS is unique in the world—it is the only actuarial society totally dedicated to property and casualty insurance.

As you may know, I recently attended the International Congress of Actuaries meeting in Sydney. I was frustrated there by the continued reference to "life" and "non-life" insurance, as if everything revolved around that side of the business.

I tried to put things in proper perspective by using the terms "casualty" and "non-casualty" insurance. Admittedly, that was a feeble attempt at equality. But it's particularly galling when you realize that our part of the insurance business dates back to the Phoenicians, while that upstart life insurance didn't come along until around the 17th century.

How is our 70-year-old Society doing? Pretty well, I think. Let me provide a thumbnail sketch of Society activities and see if you don't agree.

Our activities—split four ways:

- First is *Organization and Policy*. The new organizational structure was introduced just one year ago. Frankly, I was a "wait-and-see"-er, but now I have to tell you . . . I have become a strong supporter and proponent of this new structure. It's been my good fortune to observe firsthand and—

even more important—participate in this clearly more effective operation. More work is needed, particularly on determining clearly the policies of the CAS, but I'm confident this will be accomplished in the near future.

- Second is *Administration and Education*. We have initiated a complete review of the exam content and process, and we've developed a policy on meeting locations to avoid having them concentrated in one or two regions of the country. We've also begun to change the direction of the CAS Trust so it can better serve as a vital educational support activity for our membership.
- Third is the development of the *Casualty Actuarial Science*. We've created two new committees—Ratemaking and Management Information—and I'm optimistic that they will be as successful as the Reserve Committee in reaching out to everyone interested in casualty insurance. On the other hand, I must admit concern that our Society is not as effective as it once was in developing and sharing advances in actuarial science. Maybe we're influenced by open competition philosophies. I hope I'm wrong, but I would ask that particular attention be paid to this need in future years.
- Last is *Professionalism*. There is serious activity taking place on standards, loss reserving specialist qualifications, and valuation actuaries. I'm optimistic that the long-awaited day of recognition of the Actuary as a true professional is at hand.

As you look back, it's clear our society has made a lot of progress in the past 70 years, although sometimes progress isn't evident until after the fact. Kurt Vonnegut wrote—and I quote—"Every passing hour brings the Solar System forty-three thousand miles closer to Globular Cluster M-13 in Hercules—and still there are some misfits who insist that there is no such thing as progress."

Well, your Society is advancing in many areas, and to all who contributed their talent and energy, I extend my personal appreciation for a job well done. Sometimes it's difficult to see the progress, and harder even to measure it. But it's there and—the important thing—it's moving in the right direction.

The one-year term as president of the CAS doesn't allow one time to both initiate and complete major programs. Isaac Asimov dealt with this problem—in his "Foundation" quadrilogy—by what he called the Seldon Plan. This plan was based upon a new statistical mathematics: "Psychohistory"—you might call it the ultimate in actuarial mathematics for its accuracy in predicting future events. Psychohistory provided for regenerating a new universal society in the eyeblink time span of 1000 years.

You probably recall that outgoing presidents of the CAS have often called for some sort of sustained program or standing committee to address issues they've brought up in their departing remarks. Well, let me make it clear that I am *not* recommending a Committee on Psychohistory. I recognize that statistical mathematics may be attractive to actuaries, but I think we ought first to complete our work on risk theory and risk classifications.

George Orwell had a different vision of the future and he wrote a book about it back in 1949. Orwell dealt with a relatively short time-frame compared to Asimov's 1,000 years. While Asimov's epoch spanned centuries of ebb and flow, Orwell provided only a single snapshot—1984.

Orwell's 35-year look forward foresaw a world considerably different from the one we are experiencing today. One might wonder, then, why the book—with its rather sinister connotations for this year—has remained so popular.

I personally don't think it was due to any particularly effective marketing by his publisher. Rather I believe that, even though Orwell's environmental backdrop failed to materialize, the issues within the fabric of his story continue to be relevant.

These issues have relevance to the insurance business, and I'm going to spend a few minutes comparing Orwell's socio-economic views with some contemporary actuarial and insurance issues.

If it's been a few years since you've read *1984*—or even if you never got around to it—you probably can still relate to the concept of Big Brother that Orwell introduced in his book. I had developed my own image of Big Brother in my mind's eye, but while I was in Australia a new film version of *1984* was released, and my imagination has lost out to the filmmakers. The filmmakers chose a rather mild, soothing male image for Big Brother, not the authoritative, domineering one I had conceived.

By the way, since I had recently re-read *1984*, I found the movie terrific, but I'm not sure all the transitions and byplay would have been clear without the background of the book fresh in my mind.

Does the insurance industry operate under the watchful eye of a Big Brother and his Thought Police? Orwell must have anticipated the growth of regulatory control and intervention which exists today, and he might well have used the insurance business to illustrate his point.

Now *de*regulation is popular—and of course is the theme of our meeting—but this trend might better be termed *re*regulation. All those people in all those regulatory positions are not going to quietly walk away and give up their jobs. They'll just reapply their energies in other directions and employ different tactics to keep watch over the industry. So Big Brother is with us now—and will be for a long time to come.

What about our own professional regulation? Does Orwell apply here? Standards of practice, guides to professional conduct, discipline committees, enforcement . . . these are all of continuing interest, and even if we succeed in these needs we'll be far from Orwellian Big Brotherism.

But Orwell was really only highlighting the potential dangers. As long as standards and discipline are enlightened and controls do not constrict experimentation and progress, they can be for the better.

But, maybe some of the company people can't be so complacent. Orwell's Big Brother—or BB as he was called by the chanting citizens of Oceania in the movie version—might only have been a fictional character, but you have a real live BB. He's Bob Bailey (FCAS) of Best's, of course, who deals out those A's and B's and C's—striking almost as much terror in corporate management by a downgrading as BB's Thought Police did in Oceania.

Another of the popular recollections from *1984* is Newspeak—words like "doublethink" . . . "blackwhite" . . . "ungood" for bad . . . "doubleplusgood" for superior . . . and "Ingsoc" for English Socialism. Orwell was right that language would change and new words would come into use. But the thought control he envisioned by eliminating words and concepts was entirely wrong.

Vocabulary reconstruction in Orwell's plan was to eliminate thoughts and ideas. But that hasn't happened. In fact, we keep all the old words—and just add new ones as we need them. Words and terms like CMP . . . homeowners . . . moneyfund . . . stagflation . . . GAAP . . . and all the others that spring to mind.

Still, Orwell's concepts of doublethink and blackwhite do seem relevant to some current situations.

Doublethink, you'll recall, is the power to hold two contradictory beliefs in one's mind simultaneously—and accepting both of them. Blackwhite is demonstrated by calling black white in contradiction of the facts—believing black *is* white and forgetting that one believed the contrary.

So what has all this to do with insurance?

Think about some of the differences of opinion and interpretation that exist in our work. Two intelligent, knowledgeable actuaries can take the same data and the same set of facts with ostensibly the same objective and come up with rate changes for the same line of business varying from plus 100 percent to minus 25 percent.

A poor parallel to doublethink because there are two minds involved? Not when the information is provided to non-actuaries who believe they're getting the results of mathematical analysis. Small differences can be tolerated. But until we can eliminate analyses that directly contradict each other we'll always face a skeptical reception from the public.

What about all the underwriting and marketing decisions of which we're aware, that can only lead to red ink? What about all the capacity waiting to enter the market, guaranteeing that prices will remain too low?

Management must believe that losses are profits, forgetting that they once believed the contrary—a classic example of blackwhite.

Incongruous? Maybe, but all too often undertones of blackwhite must be the rationalization for acts that cause such major problems for our industry.

Orwell had a way with names. His Ministry of Peace directed the never-ending war. The Ministry of Truth refabricated the past. And of course Ingsoc was his shorthand for English Socialism. In a similar but more current vein, author Lawrence Saunders in his book *The Tomorrow File* renamed the Department of Health, Education and Welfare the Department of Bliss to acknowledge its operating philosophy toward recipients.

Well, with apologies to both Orwell and Saunders, in the spirit of *1984* I think it might be timely to consider developing names for our various actuarial societies that will be more effective and perhaps more appropriate in common usage. The names or initials we have now aren't particularly catchy or memorable—or even very pronounceable—and there's a lot of confusion in joint societal meetings as each of us refers to a particular organization as "The Society."

So how about:

Cassoc	—for the CAS
Soac	—for the Society of Actuaries
Canac	—for the Canadian Institute of Actuaries
Acac	—for the American Academy of Actuaries
I'd leave Capp	—for the Conference of Actuaries in Public Practice.

It might be a bit presumptuous to suggest names for the other societies. But how can the Academy pass up a name like Acac for the lobbyists of the profession?

A third major Orwellian theme in *1984* is: "Who controls the past controls the future; who controls the present controls the past."

You might recall that Orwell's chief character, Winston Smith, spent his days in the Ministry of Truth rewriting newspaper articles so that recorded history would always be consistent with and support the current posture of the government.

Farfetched? Perhaps a little. But how much time do corporate managements spend designing current operating results to be what they want them to be . . . rather than what they are?

Can we justify financial reinsurance for net income purposes rather than for surplus relief? Reserving—those of you who were at the 1984 Reserve Seminar will recall Charlie Hewitt's slide clearly depicting what can only be reserve management for earnings support.

One can read *1984* and be appalled by Orwell's concept of recorded history being destroyed at the direction of current leaders. But isn't that the same as taking actual results and changing them into something different—something more favorable, perhaps—and then having management and regulators make decisions from these altered data?

Orwell's approach was laborious and cumbersome. Some things being done today are sophisticated and imaginative. But the net effect might be just as sinister. We might want to paraphrase Orwell a bit to render his slogan appropriate for our business: "Who makes the past must predict the future; who makes the present makes the past."

There's one more Orwellian structure I'd like to discuss. It goes like this:

War is Peace
Freedom is Slavery
Ignorance is Strength.

Contradictions? Not so, according to Orwell. By some convoluted reasoning, Orwell argues that these aren't contradictions at all. He claims they are actual reasons why the governing party is and will continue to be successful.

Let me try some paraphrasing with slogans that seem particularly apt for the insurance business today:

Sales is Profitability
Surplus is Liability
Competition is Regulation.

Sales is Profitability doesn't sound controversial. In fact, it's the norm for most businesses. Simply, the more you sell the more you make. Your fixed expenses become a smaller proportion of the total, and variable expenses continue at their fixed percentage of sales.

But in the insurance business, variable expenses—which for the most part are losses—do not continue at a fixed rate. Very often they increase as sales increase. This phenomenon is frequently overlooked by new entrants into the insurance field—and sometimes even by experienced insurance management—and it is *never* admitted to by the sales force.

Our second slogan is *Surplus is Liability*. An accountant once said that insurance companies are unique because they grow by growing liabilities. The insurance business has been and continues to grow liabilities at a faster rate than premiums. This is in part due to increases in the longer-tail business written today, which in addition, not coincidentally, is also a greater challenge to reserve adequacy.

If these newly grown liabilities are understated—and that is the prevailing opinion—then the surplus becomes the liability. Not planned, not desirable, and not by the same name, but that seemingly solid surplus can be quickly swallowed by deficient loss reserves.

Finally, *Competition is Regulation*. You've all heard it, probably more times than you wish: "Competition is the best regulator." Well I'd like to ask: from whose point of view? The policyholder? The stockholder? Insurance company management? The regulator?

I don't think there's a clear answer on this bit of counterpoint. The *policyholder* is enjoying lower current prices, but he might suffer if his claims are not honored down the road.

The *stockholders* are seeing their investment value and dividends threatened—if not lowered—because company earnings have tumbled.

Management thinks it has won a victory. It's operating with greater freedom and more flexibility, but some have already faced a day of reckoning for their inability to cope with the new environment.

And *regulators* . . . well, interestingly, they may have experienced the least impact of all—as I said earlier, I believe we're seeing *re*regulation, not *de*regulation.

While I'm a believer in open competition—it goes hand in hand with apple pie and motherhood—I can't help wonder why so many companies and insureds find the California Workers' Compensation market so attractive.

So much for Orwell. 1984 will soon be over. My term as president of Cassoc will be over even sooner.

What about the next 35 years? 2019 has a pleasant, upbeat ring to it—a more spirited sound than the ominous 1984. Maybe there's a clue in that for us.

Futurist and science-fiction writers can be divided into two schools of thought—either things will get better or they will get worse. It's clear Orwell was on the wrong side—got it backwards you might say—and that's why I've titled this talk "4891."

Orwell's weighing of the issues led him to draw an ominous picture of the future. My own weighing—back in 1949, youthful as I was then, and now in 1984—leads me to an optimistic perspective about the future.

But, within the overall fabric of progress and prosperity, those haunting problems of business cycles, insurance cycles, competition for capital, excess capital, profitability, reserve adequacy, and even erratic entrepreneurial behavior will always be with us. And some of these problems will be quite acute, requiring drastic and immediate attention to drive the demons out.

And who better to call for speedy relief; who's got the training to analyze the problems; who's got the tools to provide the solutions? No, not "Ghostbusters"—Cassocers.

A NOTE REGARDING EVALUATION OF MULTIPLE REGRESSION MODELS

GREGORY N. ALFF

Abstract

Econometric multiple regression models are now commonplace aids to understanding variables affecting the insurance industry. For actuaries and other corporate management personnel to utilize these models to fullest advantage, it is necessary to be familiar with important regression statistics and to be able to critically evaluate model structure.

This paper discusses statistics for determining the strength or validity of a model. Special emphasis is given to the definition of the $\overline{R}^2$ statistic and its relationship to the R^2 and F statistics.

Exclusion of constants from causal models is recommended. Reasons for modeling change in dependent variable rather than level of the variable are considered.

"Who reads incessantly, and to his reading brings not
A spirit and judgment equal or superior, . . .
Uncertain and unsettled still remains,
Deep versed in books and shallow in himself."

John Milton
Paradise Regained

1. THE NEED FOR MODELING

It is not surprising to see rapid growth in the field of econometric research and modeling. Corporate management requires tools to enable it to evaluate economic projections and the probable consequences of alternative marketing and pricing decisions. Work has begun in this area. Econometric models of trends for rate making are now being formulated and utilized for exposures, claim severity, and claim frequency for many lines under the auspices of ISO. Actuaries on industry rate making committees have realized that neither linear nor exponential least squares procedures can be totally relied upon to yield realistic estimates of future trends in today's economic environment. What is needed is an understanding of the causal relationships between outside economic elements and those elements important to insurance rate making and pricing. One vehicle that can provide this understanding is the multiple regression model. In order to make more effective use of the models being developed, it is necessary to be familiar with important regression statistics and to be able to critically evaluate model structure.

II. TOOLS FOR EVALUATION

Actuaries and all levels of insurance management are continually being presented with new, purportedly improved, and ever-more complicated models. In their paper [1] Lommele and Sturgis discuss seven tests for determining the strength or validity of a model. They are as follows:

1. A t-test at the 95 percent level is used to test the importance of each independent variable. The usual standard for this test is $|t| \geq 2$ given at least 16 observations.
2. The sign of the t-test, indicating whether the independent variable's relation to the dependent variable is direct or inverse, should make good intuitive sense.
3. R^2, the coefficient of multiple determination, is a measure of the part of the variation in the dependent variable that is explained by the variation of the independent variables. There is no generally accepted standard of quality for R^2, rather it provides a measure for comparison of one model against another [2]. However, subjective standards do exist and are discussed in the next section in terms of $\bar{R}^2$.
4. The Durbin-Watson d statistic is used to test for autocorrelation in the residual or error terms. The d statistic is generally considered acceptable if $1.5 < d < 2.5$. A d outside this range would indicate probable serious autocorrelation of error terms.
5. Mean absolute error is an indicator of historical and recent accuracy. A more commonly calculated value is what is often referred to as the standard error of regression. It is calculated as: $SE_R = \sqrt{\Sigma(Y_i - \hat{Y}_i)^2/(N - K)}$, where $(N - K)$ is the degrees of freedom. This is a statistic useful for comparison of models, without a specific threshold for acceptance.
6. Correlation coefficients between each possible pair of variables from a model should show each independent variable to be more highly correlated with the dependent variable than with any other independent variable. If this is not the case colinearity may result, leading to low t-test

values for the two strongly-correlated independent variables as they compete for acceptance in the model.

7. The model as a whole should be intuitively sensible. This test is very important if the model is to gain acceptance with other potential users.

Information for the first five of these tests is often part of the model results presented by computer regression programs and in the published work of econometricians.

All seven tests are important considerations, but even with satisfactory indications from these tests, the model may still contain significant weaknesses.

III. R-BAR-SQUARED ($\overline{R}^2$)

If R^2 for a given model is .93, the person evaluating the model may be very impressed with the model. However, it is possible that he is being deceived. A better measure of fit is $\overline{R}^2$, which is R^2 adjusted for degrees of freedom [3]. Using $\overline{R}^2$ instead of R^2 guards against a model being "overspecified." Being "overspecified" basically means that the model has too many independent variables in conjunction with the given number of data observations, creating a problem with regard to degrees of freedom. A hint of this may come from the t-tests. If the t-test shows a marginal value or a value lower than acceptable at the 95 percent confidence level for a variable, overspecification may be the reason. Sometimes extra variables with questionable t-tests are left in the model because they improve the R^2. The $\overline{R}^2$ statistic will aid in evaluation of whether all variables should be allowed to remain in the model. Extra independent variables will often increase R^2, but $\overline{R}^2$ may decrease if the additional variable has little value.

The reason that $\overline{R}^2$ reacts differently than R^2 is that it is adjusted to account for degrees of freedom. A textbook [4] formula is:

$$\overline{R}^2 = \frac{1 - K}{N - K} + \frac{R^2(N - 1)}{N - K}$$

where: R^2 is the coefficient of multiple determination;
K is the number of independent variables, including any constant;
N is the number of observations.

But algebraically:

$$\bar{R}^2 = \frac{1 - K}{N - K} + \frac{R^2(N - 1)}{N - K} = \frac{1 - K + R^2N - R^2}{N - K}$$

$$= \frac{R^2N - R^2K + R^2K - K + 1 - R^2}{N - K}$$

$$= R^2 + \frac{K(R^2 - 1) - (R^2 - 1)}{N - K}$$

$$= R^2 - \frac{(K - 1)(1 - R^2)}{N - K}$$

Thus, $\bar{R}^2$ is equal to R^2, less a correction for degrees of freedom. Since each of the terms contained in the correction is positive, $\bar{R}^2$ will be less than R^2. The only exceptions are in the special cases when $R^2 = 1.0$ or $K = 1$, where the correction goes to zero and $\bar{R}^2 = R^2$.

The effect on $\bar{R}^2$ and acceptability of t-test values together should determine whether an additional variable is allowed in a model.

There are no generally accepted objective standards of quality for $\bar{R}^2$. However, subjective standards do exist among knowledgeable evaluaters. Such standards vary depending on the variable being modeled and the form and complexity of the model. Prior to examining the details of a simple model for the level of an inflation-sensitive dependent variable, my a priori expectation is that $\bar{R}^2$ should be greater than .90 for the model to be worth reviewing. This is because high values of $\bar{R}^2$ are relatively easy to achieve when modeling the level of such a dependent variable. For a model of change in the dependent variable incorporating a number of complex variable relationships, my expectations of $\bar{R}^2$ will not be as high. For some models of change in the dependent variable, any $\bar{R}^2$ greater than .80 may indicate a model well worth investigating in further detail.

The $\bar{R}^2$ statistic is most meaningful when used as a tool for comparison of competing models. Although $\bar{R}^2$ is an important statistic, it cannot stand alone. All the tests discussed in Section II are important in the evaluation of a given model or when comparing it to alternative models.

IV. $\bar{R}^2$ AND THE F-STATISTIC

Further algebraic substitution into the equation leads to an interesting relationship. The F statistic is defined as:

$$F = \frac{\Sigma(\hat{Y}_i - \bar{Y})^2/(K - 1)}{\Sigma(Y_i - \hat{Y}_i)^2/(N - K)} = \frac{\text{explained variance}}{\text{unexplained variance}}$$

and

$$R^2 = \frac{\Sigma(\hat{Y}_i - \bar{Y})^2}{\Sigma(Y_i - \bar{Y})^2} = \frac{\text{explained variation}}{\text{total variation}}$$

where:

Y_i is the dependent variable for point or year i;
$\hat{Y}_i$ is the fitted value;
$\bar{Y}$ is the mean of the Y_i values.

Then it can be shown that:

$$F = \frac{R^2/(K - 1)}{(1 - R^2)/(N - K)} \qquad \text{(see Appendix I)}$$

and by manipulating this formula,

$$\frac{(K - 1)(1 - R^2)}{(N - K)} = \frac{R^2}{F}$$

so finally,

$$\bar{R}^2 = R^2 - \frac{R^2}{F}$$

or $\bar{R}^2 = R^2(1 - 1/F)$

The F statistic is used to indicate the significance of the entire regression. With 11 or more observations, an $F \geq 5$ indicates a "significant" regression [5]. Note that given $F = 10$, then $\bar{R}^2 = .9R^2$. The example introduced in Section III where R^2 was .93 would be $.9 \times .93 = .84$ when adjusted for degrees of freedom. Such a model may not be quite so impressive when compared to another model that may be better specified by a different set of independent variables, and thus have a higher $\bar{R}^2$.

V. CONSTANT WEAKNESS

It is common in causal models to include a constant term. It is not unusual for the constant to have a strong t-test, indicating it is a strong contributor in the explanation of the level of, or change in, the dependent variable. Such a constant often may only be serving as a proxy for an economic variable that has historically shown stability or consistent period-to-period movement (depending on the form of the model equation). In an earlier paper [6] presenting a model of general liability written premium, it was noted that a constant did not improve that model. Rather, the major effect of inserting a constant was to replace one of the independent variables, as indicated by t-tests.

A constant does nothing to describe the underlying contributory causes of change in the dependent variable. Any independent variable which seems to have a logical causal effect on the dependent variable should be carefully tested. If the dependent variable and the constant are independently inserted in separate tests of the model, and the t-test for the independent variable is similar in strength to that for the constant, then the variable should be preferred. A stronger model may result from the inclusion of an explanatory variable, even if historically stable, because future movements in such a variable may prove important in the usefulness of the model as a predictor.

A constant may be statistically strong, but it does not help "explain" the movement in the dependent variable.

VI. MODELING CHANGE IN THE VARIABLE

Many models being presented use the level of the actual values over time as the dependent variable. In an earlier paper [7], it is suggested that fitting to actual values or levels of an inflation-sensitive variable can often lead to problems such as:

1. Causing colinearity of independent variables;
2. Misestimating turning points; or
3. Masking the true magnitude of error.

It is the third concern which is important in the context of this paper. The following is an example of a least squares linear regression fit to a set of actual values or levels:

Actual Value or Level	Fitted Value
200	205.6
220	220.4
245	235.2
260	250.0
250	264.8
275	279.6
300	294.4

$$R^2 = .922$$
$$\bar{R}^2 = .906$$

There is certainly an upward trend and the model appears to produce a good fit. But is management really concerned about the long-term trend, or is it perhaps more concerned with the change from one year to the next? If the concern is with annual changes—how does the above model perform?

Annual Change In Actual Values	Implied Annual Changes From Fitted Values
+ .100	+ .072
+ .114	+ .067
+ .061	+ .063
− .038	+ .059
+ .100	+ .056
+ .091	+ .053

$$R^2 = .051$$

If the concern is with annual change, there is a need to develop a causal model of annual change that can do a better job of projecting this uneven and possibly cyclical annual change series. This is illustrated by the graphs in Appendix II.

If the purpose of a model is to establish the direction and magnitude of a long-term trend, then modeling with actual value or level as the dependent variable may be sufficient. However, if points of fluctuation, turning points, or the magnitude of any individual points are important, then the model should be based on change in actual values as the dependent variable. In a long-term inflationary environment, modeling level of actual values is relatively easy and high (>.90) values of $\bar{R}^2$ should be expected. This is because the magnitude of variable values and underlying long-term trend mask the true annual movement in the dependent variable. As shown in the example above, modeling annual changes instead of level is one approach which will unmask the movement in the dependent variable. Detecting and defining causal relationships for a model of annual change in the dependent variable is more difficult. A model of annual change for a cyclical series in most cases should be preferred to a model of annual level because the value of $\bar{R}^2$ is more meaningful.

Another approach currently being utilized by actuaries working with loss severity trend is the removal of estimated underlying economic trend from the loss severity series by dividing severity values by index values from a deflator such as the GNP deflator. The underlying trend indicated by the indexed deflator is then set aside to be added back later in the analysis. This unmasks the true or residual trend in the insurance loss cost after stripping away the effects of general economic inflation. It is often difficult to develop a causal model with a high $\bar{R}^2$ to fit the residual annual change series. However, a clearer understanding of the causal effects of the independent variables is gained from the regression statistics of such a model.

The $\bar{R}^2$ statistic becomes more meaningful when it is not exaggerated by the effect of underlying long-term trend or general economic inflation.

VII. MODELS IN A DYNAMIC ENVIRONMENT

Even if a model of annual change does well in explaining a long-term historical cyclical pattern, its ability to predict future change should be carefully analyzed. The model of industry general liability premiums contained in the *Proceedings* [8] is a good example. That model fits 20 years of annual change data well. It predicted the first negative annual changes in written premium for 1980 and 1981, but the predicted return to strong positive premium increases in 1982 and 1983 did not happen. The economic environment changed dramatically, and strong surplus positions and industry competition for cash flow have not allowed premiums to rise. The model did include a variable to measure surplus position, but high investment yields and cash flow patterns were not directly accounted for. Did the model fail then? No, it provided an excellent explanation of premium changes for years 1962–1981, but this example clearly points out the need for continual adjustment and modification in a changing economic environment. The model must be modified if it is to be useful in the future. Any model should be reviewed regularly to be sure that the relationships on which the model is based continue to hold true.

Modeling can be used effectively to examine and better understand the relationships between elements in a complex and dynamic economy. This note emphasizes the $\overline{R}^2$ statistic as being one statistic and first difference in actual data as being one approach important to evaluating a multiple regression model. An understanding of important regression statistics and techniques for evaluation of model structure will enhance the usefulness of the modeling tool.

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APPENDIX I
DEFINITION OF THE F-STATISTIC IN TERMS OF R^2

$$F = \frac{\Sigma(\hat{Y}_i - \overline{Y})^2/(K - 1)}{\Sigma(Y_i - \hat{Y}_i)/(N - K)}$$

$$F = \frac{[\Sigma(\hat{Y}_i - \overline{Y})^2/\Sigma(Y_i - \overline{Y})^2]/(K - 1)}{[\Sigma(Y_i - \hat{Y}_i)^2/\Sigma(Y_i - \overline{Y})^2]/(N - K)}$$

We know that total variation = explained variation + unexplained variation,

$$\Sigma(Y_i - \overline{Y})^2 = \Sigma(\hat{Y}_i - \overline{Y})^2 + \Sigma(Y_i - \hat{Y}_i)^2$$

so

$$\Sigma(Y_i - \hat{Y}_i)^2 = \Sigma(Y_i - \overline{Y})^2 - \Sigma(\hat{Y}_i - \overline{Y})^2$$

and

$$R^2 = \frac{\Sigma(\hat{Y}_i - \overline{Y})^2}{\Sigma(Y_i - \overline{Y})^2}$$

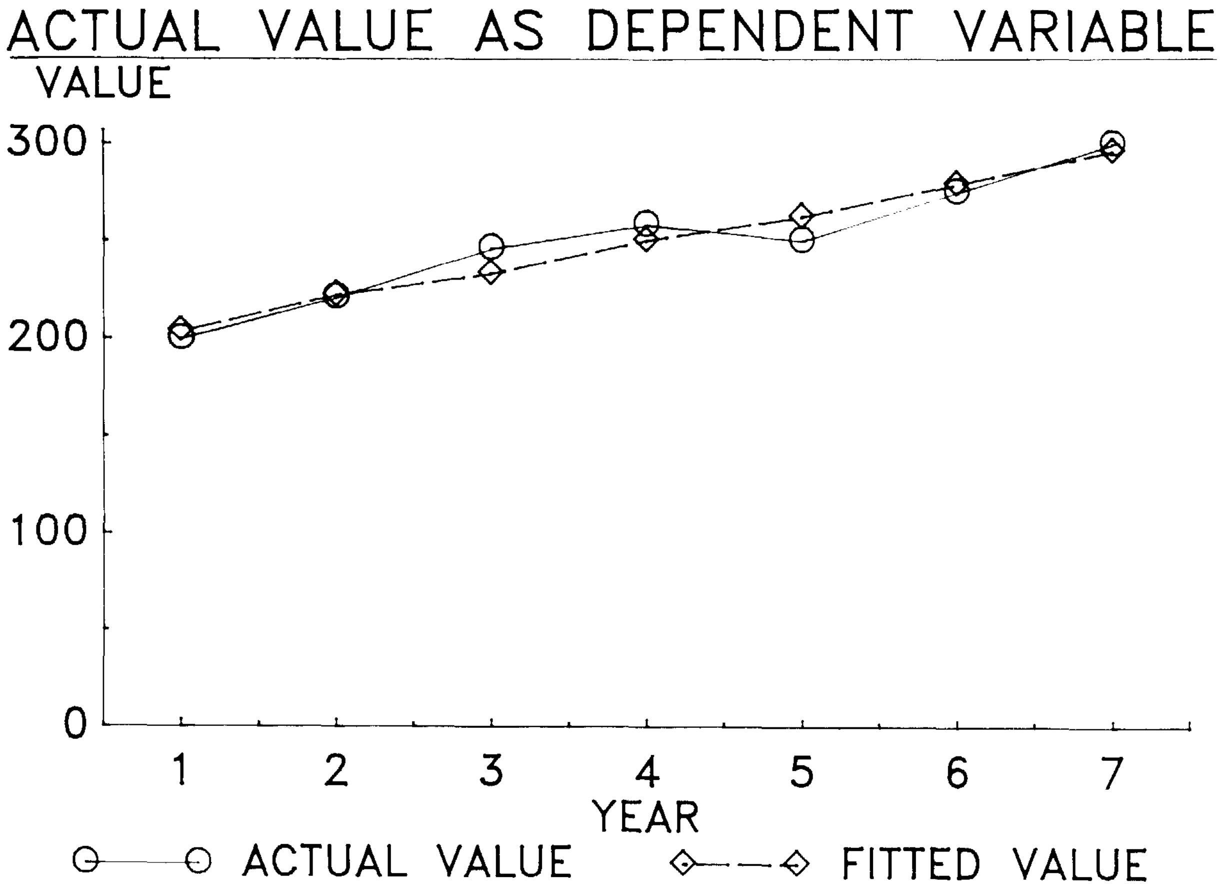
then by substitution,

$$F = \frac{R^2/(K - 1)}{\{[\Sigma(Y_i - \overline{Y})^2 - \Sigma(\hat{Y}_i - \overline{Y})^2]/\Sigma(Y_i - \overline{Y})^2\}/(N - K)}$$

Finally

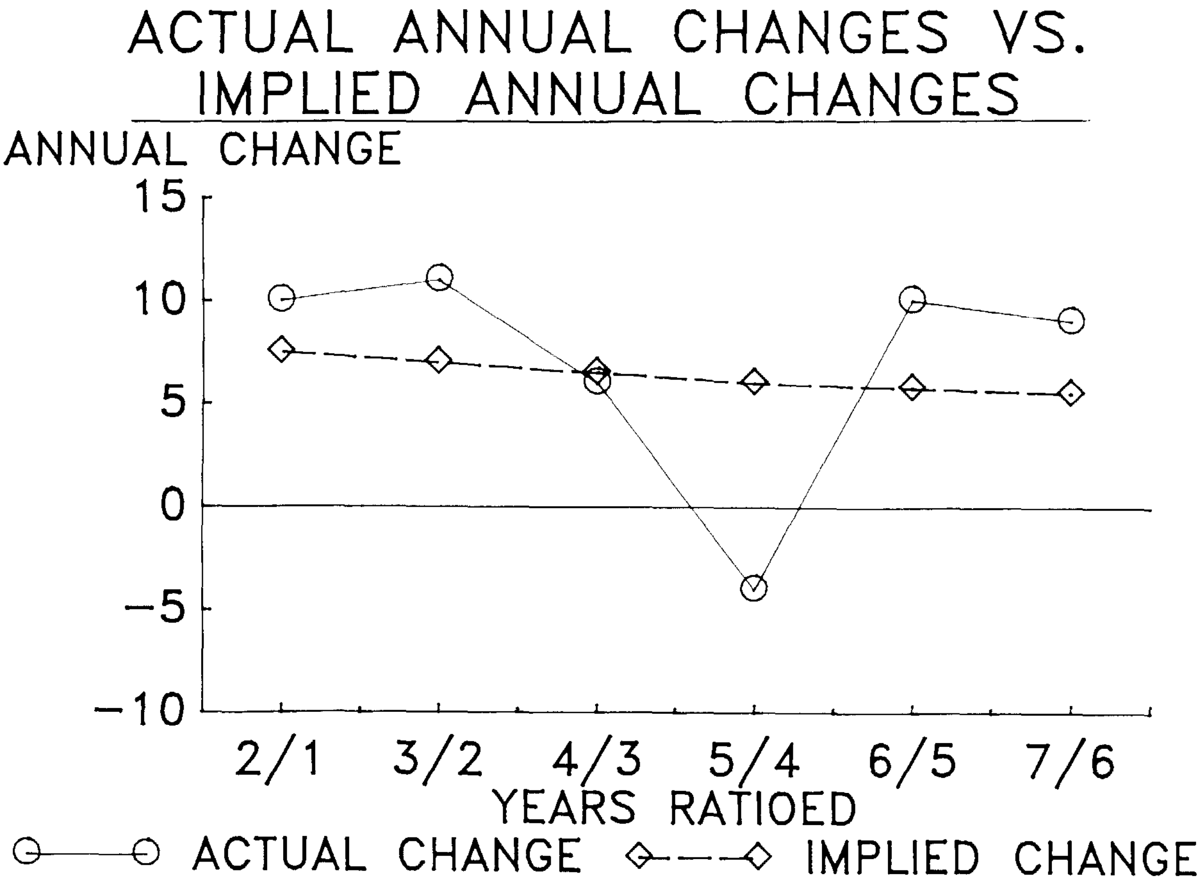
$$F = \frac{R^2/(K - 1)}{(1 - R^2)/(N - K)}$$

APPENDIX II
GRAPH I



APPENDIX II

GRAPH II



EMPIRICAL BAYESIAN CREDIBILITY FOR WORKERS' COMPENSATION CLASSIFICATION RATEMAKING

GLENN MEYERS

Abstract

This paper demonstrates how a company can derive accurate classification relativities. The method uses an empirical Bayesian credibility formula as taken from the paper "Credibility for Loss Ratios" by Buhlmann and Straub and modified by the ISO Credibility Subcommittee.

The data required for this method can be purchased from the National Council. A classification review is performed on three years of live data. Relativities predicted by both this method and the present ratemaking formula are compared with the actual relativities from a fourth year of data.

1. INTRODUCTION

Workers' Compensation has traditionally been a highly regulated line of insurance. Rates are usually recommended by the National Council on Compensation Insurance and, with regulatory approval, become the industrywide standard. While many states permit deviations, insurers have generally adhered to the standard rates. Insurers compete on price by offering various dividend plans.

With the creation of the model law for competitive rating in Workers' Compensation, this is rapidly changing. In order to promote a better business climate, many states have passed competitive rating laws.

Under a uniform pricing system, it is not necessary to have rates equal to the expected cost of writing the policy. But in a competitive environment, many economists, such as Paul Samuelson [1], assert that the price will be equal to the expected cost of writing the policy. While the present ratemaking formula, which is described by Kallop [2], makes no systematic deviation from expected cost pricing (on an underwriting basis), it is not obvious that these rates are the best estimates of the expected cost. The present ratemaking method has held up for a long time under a system of uniform ratemaking, but it remains to be seen how long it will hold up under the increased pressure of open competition.

In most states, all insurers report their experience to the National Council. This reporting takes two forms. First, insurers report their aggregate premium and loss experience. Since rates are uniform, it is not necessary to adjust premiums to a common rate level. Thus it is easier to estimate the overall needed rate change with this data. Second, insurers report loss and exposure experience for each insured on a policy year basis. While this data is not as timely as the financial aggregate data, it is more detailed. Because of its fine breakdown, it can be used for deriving class relativities.

The broad-based experience reported for Workers' Compensation should be compared to the experience reported for other lines. In private passenger automobile insurance, for example, many policies are written by independent insurers who do not report their experience. Many different classification systems and rating plans are used. Thus, combining experience is difficult, if not impossible. Because of this, it is difficult for many insurers to set accurate rates.

It can be argued that reporting experience on a standard basis can enhance competition by making it easier for insurers to enter the market. But the need to report experience on a standard basis can discourage insurers from trying innovative classification systems and rating plans. Clearly, some compromises must be made in order to obtain the greatest benefits from competitive rating.

To summarize, the economic incentive to calculate accurate rates for Workers' Compensation is stronger than ever before, and the volume and quality of data are better than in any other line of insurance. Also, methods of data processing are becoming cheaper and more flexible. Under these conditions, improvements in the accuracy of ratemaking can surely be made.

This paper addresses the problem of determining accurate classification relativities. The method used to derive classification relativities differs from the present method in its use of an empirical Bayesian credibility formula.

We begin with a description of the empirical Bayesian credibility formula. We then compare the accuracy of the classification relativities predicted using this formula with those predicted by the present ratemaking formula.

The theory described in this paper is applicable to both loss ratio and pure premium ratemaking. However, it makes no sense to credibility weight the pure premium of a class with a thirty cent rate with the pure premium of a class with a thirty dollar rate. This is frequently the case in Workers' Compensation. Thus, we describe the theory in terms of loss ratios.

The loss ratios are based on Unit Statistical Plan data. Since the overall rate change is determined externally (the National Council uses financial aggregates), these loss ratios are used to determine class relativities.

2. INFORMATION AND ESTIMATION

A general principle in statistical estimation theory is that more information about a certain quantity leads to a better estimate of that quantity. A goal of statistical estimation theory is to develop ways of using all sources of relevant information in arriving at an estimate. In this section we shall show how this principle applies to Bayesian estimation and credibility theory.

Our problem is to estimate the loss ratio for a class of insureds. We consider two sources of information that can be used to estimate the loss ratio.

First, we can use the historical loss ratios for the class. While this information has a direct relationship to the quantity being estimated, it can be subject to random fluctuation because of small volume.

Second, we can use the loss ratio for a group of similar classes. Because of the greater volume of experience, this information has less random fluctuation. However, it has a less direct relationship to the quantity being estimated. The classes in the group may simply have different loss ratios.

Each of these sources of information is relevant to the quantity being estimated. The problem we want to address becomes the following: how can one use both sources of information to derive an estimate of the loss ratio for a class?

We seek a mathematical solution to this problem. To solve this problem we must first specify a model that we feel resembles the situation. We must then specify the information that we have available. We then mathematically derive the best estimate of the loss ratio.

We begin by making the following assumptions.

1. The expected loss ratio, μ, is randomly selected from a distribution with mean M and variance τ^2.
2. Each loss ratio, X, is randomly selected from a distribution with mean μ, and variance σ^2.

This model bears a fair resemblance to our situation. We observe a class loss ratio, X, which fluctuates around the class's expected loss ratio, μ. Our second source of information is the loss ratio, M, for a group of classes. The possibility that classes in this group may have different loss ratios is represented by selecting μ at random from a specified distribution.

The problem is to estimate the true loss ratio for a given class. We now describe some solutions to this problem.

The Bayesian Solution

The Bayesian solution to this problem is to calculate the average μ for all classes with observed loss ratio X. We write this as $E[\mu|X]$. One must have a complete description of the distributions for X and μ to perform this calculation. For example, if we know that X and μ are normally distributed, it is demonstrated by Hoel [3] that

$$E[\mu|X] = \frac{\tau^2}{\tau^2 + \sigma^2} \cdot X + \frac{\sigma^2}{\tau^2 + \sigma^2} \cdot M.$$

Hewitt [4] and Mayerson [5] give the Bayesian solution for other distributional assumptions.

It should be noted that the Bayesian solution given above is a linear function of the observed loss ratio, X. While this is also true for many other Bayesian solutions, it is not true for all Bayesian solutions. Hewitt [6] gives an example where the Bayesian solution is not linear.

The Credibility Solution

The credibility solution, given by Buhlmann [7], is to use the linear approximation to the Bayesian solution which minimizes the expected squared error. As noted above, in many cases the credibility solution is identical to the Bayesian solution. While the credibility solution may not be as accurate as the Bayesian solution, it does not require as much information. One need not have a complete description of the distribution of X and μ. One need only have the values of M, τ^2 and σ^2. We will denote the credibility solution by $C[\mu|X]$.

The credibility solution can be stated as follows. Let

$$C[\mu|X] = A \cdot X + B.$$

We want to choose A and B so that

$$E[(C[\mu|X] - E[\mu|X])^2]$$

is minimized. The solution can be written in the following form.

$$C[\mu|X] = \frac{\tau^2}{\tau^2 + \sigma^2} \cdot X + \frac{\sigma^2}{\tau^2 + \sigma^2} \cdot M.$$

Define the credibility factor, Z, as follows:

$$Z = \frac{\tau^2}{\tau^2 + \sigma^2}$$

The credibility solution now takes the more familiar form:

$$C[\mu|X] = Z \cdot X + (1 - Z) \cdot M.$$

The credibility factor can be viewed as a measure which compares the variance of X with the variance of μ. A credibility factor close to zero indicates that the random fluctuations of individual class loss ratios are large compared to the true differences in loss ratios between classes in the group. A credibility factor close to one indicates just the opposite. Philbrick [8] discusses this aspect of credibility theory in detail.

A major problem with the credibility solution is that, in real life situations, one does not know M, τ^2 or σ^2. While it is possible to choose the unknown parameters by judgment, American actuaries have used a more direct approach; they choose the entire estimation formula by judgment. These formulas are generally referred to as the "classical" credibility formulas. The rationale for these formulas is given by Longley-Cook [9].

While the Bayesian and the credibility solutions provide considerable insight into the estimation process, one more step is needed. We must be able to form our estimates entirely from observations. This is the essence of the empirical Bayesian solution.

3. EMPIRICAL BAYESIAN CREDIBILITY

We begin our discussion of empirical Bayesian credibility with a description of the solution given by Buhlmann and Straub [10] in their landmark paper "Credibility for Loss Ratios." This solution has been amplified and modified by the Credibility Subcommittee of Insurance Services Office. Much of the following development is taken from a report written by the Credibility Subcommittee [11].

We begin by specifying the model underlying the empirical Bayesian credibility formula. Next, we give the credibility formula in terms of the parameters of the model. Finally, we show how to estimate the parameters of the model.

The Model

The formula requires the following data.

1. T years of experience for N classes.
2. The premium for class i in year t (denoted by P_{it}).
3. The loss ratio for class i in year t (denoted by X_{it}).

We make the following assumptions.

1. The expected loss ratio for class i, μ_i, is randomly selected from a distribution with mean M and variance τ^2.
2. Each loss ratio, X_{it}, is randomly selected from a distribution with mean μ_i and variance V_i^2/P_{it}.

Most actuaries would agree that the variability of a class loss ratio decreases as the size of the class increases. The assumption that the variance of the loss ratio is inversely proportional to the premium (i.e., $\text{Var}[X_{it}] = V_i^2/P_{it}$) is a simple way to approximate this relationship. Note that the constant of proportionality, V_i^2, can be different for each class.

It is unlikely that this relationship is precise. Meyers and Schenker [12] propose a model of the loss process in which the variance of the loss ratio is not inversely proportional to the premium. In this model the variance of the loss ratio can be written in the form $\text{Var}[X_{it}] = \alpha/P_{it} + \beta$. The constant term, β, is positive when there are additional, but unidentified, sources of variation. Examples of this could include changing economic conditions, or increased emphasis on loss control. Meyers [13] discusses how a positive constant term affects the credibility formula.

The Credibility Formula

For a given class, j, we want to find an estimate, $\hat{\mu}_j$, of the expected loss ratio, μ_j. Here, we present the formula given by Buhlmann and Straub [14].

The estimate is of the following form.

$$\hat{\mu}_j = \sum_i \sum_t A_{it} \cdot X_{it}$$

A_{it} is chosen to minimize $\text{E}[(\hat{\mu}_j - \mu_j)^2]$, subject to the constraint that $\text{E}[\hat{\mu}_j] = M$.

Note that all the observed loss ratios, X_{it}, contain some information about the expected loss ratio μ_j. The exact nature of this information is specified by the assumptions listed above and the accompanying mathematics. It should be noted that since the X_{jt}'s contain more information about μ_j than the other X_{it}'s, the A_{it}'s depend upon j.

Using the method of Lagrange multipliers, one can solve for the A_{it}'s. Buhlmann and Straub went one step further by algebraically manipulating the solution so as to express it in a form which resembles a standard credibility formula.

Let $P_{i.} = \sum_t P_{it}$ (total class premium),

$\bar{X}_{i.} = \sum_t P_{it} \cdot X_{it}/P_{i.}$ (premium weighted average of X_{it}),

$\Sigma^2 = \mathrm{E}[V_i^2]$

$K = \Sigma^2/\tau^2$ (credibility constant),

$Z_i = P_{i.}/(P_{i.} + K)$ (credibility factor), and

$\hat{M} = \sum_i Z_i \cdot \bar{X}_{i.} \Big/ \sum_i Z_i$ (credibility weighted average of $\bar{X}_{i.}$).

Then $\mu_j = Z_j \cdot \bar{X}_{j.} + (1 - Z_j) \cdot \hat{M}$.

There is one point that should not be overlooked. The complement of credibility is assigned to the *credibility-weighted* average loss ratio and not the premium-weighted average loss ratio as many would assume. The reason for this is simply that it is the solution to the minimization problem. It should be noted that $\hat{M}$ has some very nice properties.

First, it can be demonstrated [15] that

$$\sum_i \sum_t P_{it} \cdot \hat{\mu}_i = \sum_i \sum_t P_{it} \cdot X_{it}.$$

This means that the estimates of the class loss ratios are "in balance" with the overall loss ratio.

Second, it can be demonstrated [16] that $\hat{M}$ is the minimum variance unbiased estimate of M.

Estimating the Parameters

The following estimators of Σ^2 and τ^2 were derived by Buhlmann and Straub [17].

Let $P_{..} = \sum_i \sum_t P_{it}$ (total premium),

$$P2 = \sum_i P_{i.}^2 ,$$

$$\bar{X}_{..} = \sum_i \sum_t P_{it} \cdot X_{it}/P_{..} \quad \text{(premium-weighted average of } X_{it}\text{), and}$$

$$W = \sum_i P_{i.} \cdot (\bar{X}_{i.} - \bar{X}_{..})^2/(N - 1)$$

Then estimates for Σ^2 and τ^2 are given by

$$\hat{\Sigma}^2 = \frac{\sum_i \sum_t P_{it} \cdot (X_{it} - \bar{X}_{i.})^2}{\mathrm{N} \cdot \mathrm{T} - \mathrm{N}} \quad \text{and}$$

$$\hat{\tau}^2 = \frac{(W - \hat{\Sigma}^2) \cdot (N - 1) \cdot P_{..}}{P_{..}^2 - P2} .$$

Buhlmann and Straub then used $\hat{K} = \hat{\Sigma}^2/\hat{\tau}^2$ as their estimate of the credibility constant. The credibility of a class loss ratio becomes the following:

$$\hat{Z}_i^1 = \frac{P_{i.}}{P_{i.} + \hat{K}} .$$

The ISO Credibility Subcommittee modified this formula for the following reason. Even though $\hat{\Sigma}^2$ is an unbiased estimate of Σ^2, and $\hat{\tau}^2$ is an unbiased estimate of τ^2, it turns out that $\hat{Z}_i^1$ is a biased estimate of $Z_{i.}$ The modified formula, which attempts to correct for this bias, can be written as follows.

$$\hat{Z}_i = \frac{P_{i.}}{P_{i.} + \hat{K}} \cdot \frac{N - 3}{N} + \frac{3}{N}$$

This modification is identical to that given by Morris and Van Slyke [18]. A derivation of this modification is given by ISO [19]. This derivation makes a number of simplifying assumptions in addition to those already stated. They are as follows.

1. X_{it} is normally distributed.
2. μ_i is normally distributed.
3. Σ^2 is known.

Since these assumptions are somewhat restrictive, this correction for bias should be regarded as only approximate.

Under the above assumptions, it is not possible to correct for this bias when $N < 3$. Thus, one should not use this empirical Bayesian formula when there are three or fewer classes.

Note that the minimum credibility that is possible in this formula is $3/N$.

It is possible for the estimate, $\hat{\tau}^2$, to be negative. This can be disconcerting to those who think that estimates of a variance should be positive. However, this phenomenon does have a natural interpretation. If we assume that the X_{it}'s are normally distributed in addition to our stated assumptions, it is possible to test the hypothesis that all the μ_i's are equal. This test is referred to as analysis of variance (ANOVA), and is described by Freund and Littell [20]. This test calculates a statistic called the F statistic. Abnormally high values of the F statistic indicate that we should reject the hypothesis that all μ_i's are equal, while lower F values indicate failure to reject this hypothesis.

It turns out in our case that $F = W/\hat{\Sigma}^2$. Thus we have that $\hat{\tau}^2$ is negative if and only if F is less than one. Since under the null hypothesis, $E[F] = (N \cdot T - N)/(N \cdot T - N - 2) > 1$, a negative $\hat{\tau}^2$ indicates failure to reject the hypothesis that all μ_i's are equal.

Thus, we should assign a credibility of zero when $\hat{\tau}^2$ is negative.

One additional point should be made. The derivation of these estimators requires that the loss ratios for a given class are independent from one year to the next. Most ratemaking procedures in use at this time use loss ratios at "present rates." If rates are revised yearly, all but the most recent year of experience is used in calculating the present rate. The premium, and hence the loss ratio, for the most recent year will be influenced by the experience of the prior years. Thus, the independence assumption is violated!

The effect of using premium at present rates is to understate our estimate of τ^2. W is sharply reduced, while $\hat{\Sigma}^2$ will not be significantly affected. An extreme case results when all years of the current review were used in making the present rates, and a credibility of one was used. In this case, all the X_{it}'s are equal to the expected loss ratio, W is equal to zero and $\hat{\tau}^2$ is negative.

What to do about this problem is currently being debated by the Credibility Subcommittee. Some members feel that present rates should be used for estimating loss ratios, and the focus of the debate is on how to do this. In this paper we do not use present rates. Instead we use the most recent rates which were not based on the current experience.

It should be noted that if X_{it} is a pure premium rather than a loss ratio, the X_{it}'s will be independent, and it is not necessary to refer to older rates.

In summary, we have presented a credibility formula whose parameters are derived entirely from available data, and we have stated the assumptions that are used in deriving this formula. As is often the case in actuarial science, the model associated with these assumptions is necessarily simpler than the real world. However, this formula is easy to use and can produce accurate results, as we shall now demonstrate.

4. RATEMAKING WITH EMPIRICAL BAYESIAN CREDIBILITY

We now demonstrate how to use empirical Bayesian credibility in classification ratemaking.

The Data

Whenever the National Council files rates, it releases the raw data that underlie the rates. Recently, they began selling tapes containing loss and exposure data (Schedule Z), by class, derived from the Unit Statistical Plan. For this study, we obtained the tapes which correspond to the 1982 and 1983 rates for the state of Michigan.

The most recent rates which did not utilize any of the above data were those for the year 1979. Thus we calculate the premium by multiplying the payroll times the 1979 rate.

Below, we use the data on the first tape to calculate class relativities. Thus it is possible to make a direct comparison between the 1982 rates and the rates produced below. The tape which corresponds to the 1983 Michigan rates contained an additional year of data. We will use this additional year of data to compare the accuracy of the rates derived using the present ratemaking formula with those derived using empirical Bayesian credibility.

The losses were adjusted for law changes and loss development with factors taken from the 1982 Michigan rate filing. One technical point should be made here. The 1982 National Council rates do not reflect the modification due to (Michigan) Senate Bill 1044. This is appropriate since none of the experience reflects this bill and the adjustment was made outside the usual ratemaking formula.

Our purpose is to provide a direct comparison of ratemaking formulas, and so classes which presented special problems were deleted from this analysis. The special problems were of two kinds. First, many classes were absorbed into other classes between 1979 and 1982. It was felt that the 1979 rate for the new class could not be accurately estimated. Second, some classes contained disease elements which require special treatment. In practice, these problems must be dealt with. But that is beyond the scope of this paper.

Exhibit I shows the data used.

Determining the Class Loss Ratios

The empirical Bayesian credibility formula was applied to the data of Exhibit I with the following results.

$$N = 319$$
$$\hat{\Sigma}^2 = 92374$$
$$\hat{\tau}^2 = 0.019237$$
$$\hat{K} = 4801900$$
$$\hat{M} = 0.5822$$

For each class i, the credibilities, $\hat{Z}_i$, and the estimates, $\hat{\mu}_i$, are given in Exhibit I.

Distributing the Overall Rate Change

Even a moderately large insurer is unlikely to have exposure in all classes for which it must have a rate. Thus most insurers must obtain data similar to that described above in order to make independent rates for all classes. However, a company does not need data in such fine detail to determine the overall rate change.

As noted above, the National Council uses financial aggregate premium and loss experience to determine the overall rate change. Individual companies operating in a competitive environment invariably will have their own way of deriving the overall rate level. It is not our purpose to describe methods of determining the overall rate change. Instead we will describe how a company might distribute the overall rate change to the individual classes.

The procedure described below will produce estimates, $\hat{\mu}_i$, of the loss ratio at 1979 rates for each class i. Since it is quite likely that an insurer's payroll in the various classes will have changed since 1979, a logical procedure for determining the final rates might proceed as follows.

Let L = Total loss provision for the insurer's current book of business at the proposed rate level,
E_i = insurer's current payroll for class i and
R_i = 1979 rate for class i.

We define the rate adjustment factor, A, as follows.

$$A = L \Big/ \left(\sum_i E_i \cdot R_i \cdot \hat{\mu}_i \right)$$

The loss provision in the rate for class i is then given by the expression $R_i \cdot \hat{\mu}_i \cdot A$. If the loss provision in the rate for class i is defined in this manner, the total loss provision for the new class rates on the current book of business will be equal to L.

It should be noted that the estimates, $\hat{\mu}_i$, are really being used to determine class relativities.

5. TESTING CREDIBILITY FORMULAS

We shall now compare the accuracy of the rates produced by the empirical Bayesian credibility formula with those rates produced by the present ratemaking method.

The Underwriting Test

The accuracy of a ratemaking method can have a very important practical consequence. Suppose you are in an environment where some less accurate ratemaking method is being used. If you choose, or are required, to use the less accurate rates, you can use the more accurate rates to identify the better insureds. By writing these better insureds, you will have better than average underwriting results. Conversely, suppose you are able to use the rates indicated by the more accurate ratemaking method. You would then be charging a lower rate for the better insureds, and a higher rate for the worse insureds. You could then increase your writings for the better insureds and still make an adequate profit, while your competitors who use the other ratemaking method should write more of the worse insureds and make a less than adequate profit. A common phrase for this procedure is "skimming the cream."

Our first test will be based on this phenomenon, and will appropriately be called the "Underwriting Test." This test proceeds as follows. We first estimate the expected losses predicted by each formula for the test year. For each class, i, the expected losses are computed as follows.

Present Method:

$$\text{Expected Loss}_i = \text{Payroll}_i \cdot 1982\ \text{Rate}_i \cdot 0.769384$$

Empirical Bayesian Credibility:

$$\text{Expected Loss}_i = \text{Payroll}_i \cdot 1979\ \text{Rate}_i \cdot \hat{\mu}_i \cdot 1.053661$$

Since we are interested only in class relativities, we use the factors 0.769384 and 1.053661 to force the expected loss to sum to the total expected losses for the test year.

Next, we divide the classes into two groups. Group 1 consists of all classes for which the present ratemaking formula gives lower expected losses. Group 2 consists of all other classes.

For each group we then compare the ratio of actual losses for the test year to the expected losses predicted by both ratemaking formulas. The results are in the following table.

TABLE 1

UNDERWRITING TEST

	Group 1	Group 2	Total
1. # Classes	162	157	319
2. Actual Loss	216906003	199032667	415938670
3. Exp. Loss (Pres. Mthd.)	208238132	207700538	415938670
4. Exp. Loss (E. B. Cred.)	220310030	195628640	415938670
5. (2)/(3)	1.042	0.958	1.000
6. (2)/(4)	0.985	1.017	1.000

Line 5 of Table 1 shows that by using the present ratemaking formula and underwriting in favor of the Group 2 classes, one expects a better than average profit. Line 6 of Table 1 shows that by using the rates produced by the empirical Bayesian credibility formula, one could charge less than the rates produced by the present formula for the Group 2 classes and still make an average profit. Competitors with the same overall rate level who use the present ratemaking formula may end up writing a greater concentration of Group 1 classes and make less than their anticipated profit.

Thus we conclude that the empirical Bayesian credibility formula produced more accurate rates for this data.

We now address the statistical significance of this result. Our test is similar to the "bootstrap" technique described by Diaconis and Efron [21]. For our test, we constructed 2000 groups of insureds in which the members of the group were selected at random with a probability of 0.5. The loss ratios for each group were calculated and then listed by percentiles. These percentiles are given in Table 2.

TABLE 2

RANDOM LOSS RATIOS—
PRESENT RATEMAKING
METHOD

Percentile	Loss Ratio
.010	.939
.025	.949
.050	.957
.100	.965
.150	.971
.200	.976
.250	.980
.750	1.021
.800	1.027
.850	1.033
.900	1.041
.950	1.053
.975	1.064
.990	1.075

Looking at Table 2 we see that the Group 1 loss ratio for the present ratemaking method of 1.042 is near the 90^{th} percentile of the random loss ratio distribution. Similarly, we see that the Group 2 loss ratio of .958 for the present ratemaking method is close to the fifth percentile of the random loss ratio distribution.

Now there are two types of errors that can be made. A Type I error occurs when one keeps the present method when the empirical Bayesian method is better. A Type II error occurs when one changes from the present method to the empirical Bayesian method when the two methods are equally accurate. Table 2 shows that the probability of making a Type II error is less than one in ten. The probability of making a Type II error (i.e. the significance level) that should be required in order to change methods depends upon the relative costs of the two types of errors.

A single insurance company operating in a competitive environment may miss a good opportunity to expand in some profitable classes if it makes a Type I error, but should lose very little by committing a Type II error. A one in ten chance of making a Type II error should be sufficient to justify adopting the empirical Bayesian method.

A Type II error can be very costly for a rating bureau which is making an industrywide filing in a noncompetitive environment. Should the error be discovered after such a filing, the cost of returning to the present method can be enormous in time, money, and embarrassment. In such cases a one in ten chance of making a Type II error may not be sufficient to justify changing methods, and additional tests should be made. However, it should be noted that the cost of a Type I error is not insignificant. Companies can use the empirical Bayesian method for underwriting. There could be availability problems for some classes.

The table of loss ratio distributions for the empirical Bayesian credibility formula is similar to Table 2. The loss ratios of .985 for Group 1 and 1.017 for Group 2 are well within the normal range of fluctuation.

Mean Squared Error

A natural test for a ratemaking method is to measure how close the expected loss comes to the actual loss for the next year. With this in mind we calculate the following statistic.

$$MSE = \sum_i P_i \cdot (A_i/E_i - 1)^2/N$$

Where A_i = actual loss for class i
E_i = expected loss for class i
P_i = 1979 rate for class i times the payroll for class i
N = number of classes (319).

We shall refer to the number $P_i \cdot (A_i/E_i - 1)^2$ as the squared error for class i and we shall refer to MSE as the mean squared error.

The test statistics for the ratemaking methods considered above are given in the following table.

TABLE 3

	MSE
Empirical Bayesian Credibility	289651
Present Ratemaking Formula	298063

Here we see that the empirical Bayesian credibility formula produces the lower mean squared error.

To test if the differences between these mean squared errors are statistically significant we must consider the following.

1. The squared error for a class using one method is not independent of the squared error for the same class using another method.
2. The distribution of the squared errors is not normal.

A test that can work under these conditions is the Wilcoxon signed ranks test [22], which we now describe.

For a class i, let $SE1_i$ be the squared error for the present ratemaking method and let $SE2_i$ be the squared error for empirical Bayesian credibility. Let

$$DSE_i = SE1_i - SE2_i$$
$$R_i = \text{Rank}(|DSE_i|) \cdot \text{Sign}(DSE_i)$$
$$T = \sum_i R_i \Big/ \left(\text{Square root}\left(\sum_i R_i^2\right)\right)$$

We want to test the hypothesis

$$H_0\text{: } \mathrm{E}[SE1_i] = \mathrm{E}[SE2_i]$$

against the alternative hypothesis

$$H_1\text{: } \mathrm{E}[SE1_i] \neq \mathrm{E}[SE2_i].$$

For large N, we reject H_0 at the level of significance α if T lies below the $(\alpha/2)^{\text{th}}$ or above the $(1 - \alpha/2)^{\text{th}}$ percentile of the standard normal curve.

When comparing the MSE of the rates produced by the empirical Bayesian credibility formula with those produced by the present formula, we get

$T = .198$ which is at the 56^{th} percentile of the standard normal distribution. Thus we cannot reject H_0. Thus we conclude the expected mean squared errors are not significantly different.

Of the two tests conducted, the author considers the underwriting test to be the most relevant, since it corresponds directly to actions an insurance company can take. However the mean squared error test corresponds more closely to the criteria under which the empirical Bayesian credibility formula was derived, with the main difference being the substitution of actual loss ratios for "true" (but unmeasurable) loss ratios. This substitution adds a great deal of volatility to the test.

6. CONCLUSION

This paper describes how an empirical Bayesian credibility formula can be used to determine class relativities for Workers' Compensation insurance. Tests which compared the accuracy of this method with the present ratemaking method showed that the empirical Bayesian credibility formula produced more accurate rates.

The level of significance of these tests was sufficient for use by individual companies in a competitive environment, but the author would stop short of recommending industrywide use of this method in a highly-regulated noncompetitive environment until further tests are made.

However, it should be pointed out that if the empirical Bayesian approach is even marginally more accurate than the present approach, its accuracy should increase over time. One of the features of the approach described above is that it had to use the 1979 rates which were derived by the present ratemaking formula. If this method were adopted for the 1985 rates, the rates calculated above could be used in place of the 1979 rates. Gradually, the rates will become even more accurate.

Another advantage to the empirical Bayesian approach is that it calculates an optimal result based on an explicit set of assumptions. By knowing how well the assumptions are met, one can better decide when to adjust the calculated results on a judgemental basis, or when to derive a new formula based on alternative assumptions.

This author doubts that the above approach will be the last word in credibility theory, but it is hoped that this paper has set a standard that proposals for alternative formulas will follow. This standard is that the predictions should be tested on independent data. This standard is part of the scientific method and should be applied to actuarial science.

7. ACKNOWLEDGMENTS

The ratemaking method described in this paper is being used by my company. In developing this method I worked very closely with Burt Covitz. Burt's very detailed knowledge of Workers' Compensation ratemaking made this method much better than it might otherwise have been. Brad Alpert and Mike Kooken also contributed many valuable comments.

I have also profited tremendously by the very thorough work done by the staff of the ISO Credibility Subcommittee. ISO deserves to be commended for the resources committed to this subcommittee.

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9. NOTES ON EXHIBIT I

Exhibit I—Individual Classification Data and Results
List of Variables

CLASS	— NCCI class code
*PI*1	— Policy year starting 4/78 payroll times *RATE*79
*PI*2	— Policy year starting 4/77 payroll times *RATE*79
*PI*3	— Policy year starting 4/76 payroll times *RATE*79
*XI*1	— Policy year starting 4/78 loss developed from first report to ultimate divided by *PI*1
*XI*2	— Policy year starting 4/77 loss developed from second report to ultimate divided by *PI*2
*XI*3	— Policy year starting 4/76 loss developed from third report to ultimate divided by *PI*3
*RATE*79	— NCCI rate in effect for 1979
*RATE*82	— NCCI rate in effect for 1982 (Before S.B. 1044)
PAYROLL	— Payroll for policy year starting 4/79
ACTLOSS	— Policy year starting 4/79 loss
PI	— $P_{i.}$
XI	— $\bar{X}_{i.}$
ZI	— $\hat{Z}_i$ (credibility for class i)
UI	— $\hat{\mu}_i$ (credibility estimate for class i)
ELOSS	— Expected loss for policy year starting 4/79 predicted using *UI* (= *RATE*79**PAYROLL***UI**1.053661)
NCCIELOS	— Expected loss for policy year starting 4/79 predicted using NCCI rates (= *RATE*82**PAYROLL**0.769384)

EXHIBIT I

INDIVIDUAL CLASSIFICATION DATA AND RESULTS

CLASS	PI1	PI2	PI3	XI1	XI2	XI3	RATE79	RATE82	PAYROLL	ACTLOSS	PI	XI	ZI	UI	ELOSS	NCCIELOS
5	842937	864503	589346	0.277	0.506	0.144	6.73	3.81	165413	751702	2296787	0.329	0.330	0.499	585077	522010
11	1460127	1387183	1419262	0.674	0.248	0.374	6.73	3.81	297690	1238787	4266572	0.436	0.475	0.512	1081881	913874
34	268738	279120	308060	0.068	0.104	0.295	8.49	5.54	44732	51810	855917	0.161	0.159	0.515	206156	203078
35	1017404	959527	882704	0.253	0.501	0.408	5.57	2.95	230480	1495785	2859635	0.384	0.379	0.507	685882	554811
42	2779440	2437089	2092952	0.249	0.293	0.264	9.21	4.09	316174	1089411	7309481	0.268	0.607	0.391	1200498	1038688
106	1026492	1095728	1055057	0.562	0.376	0.412	15.79	10.96	63261	691857	3177277	0.448	0.404	0.528	555663	544961
128	1344541	1248626	1215192	0.384	0.242	0.469	12.56	6.82	105661	1068326	3808360	0.364	0.448	0.485	677807	571645
129	1653095	1947742	1787954	0.317	0.280	0.749	13.15	7.59	201655	2000139	5388791	0.447	0.533	0.510	1425600	1205471
130	1877080	1810503	1588122	0.941	0.685	1.157	13.38	14.67	169560	2286851	5275704	0.918	0.528	0.760	1815859	1989666
301	444153	375746	349813	0.570	0.072	0.687	8.70	7.26	47141	475465	1169711	0.445	0.203	0.554	239541	255929
908	1008855	995022	928189	0.037	0.070	0.009	53.00	40.00	20021	64676	2932066	0.039	0.385	0.373	417325	626255
909	420680	409890	385515	0.002	0.002	0.000	65.00	37.00	6597	3390	1216085	0.002	0.210	0.460	208075	191071
912	359024	399760	361456	0.198	0.037	0.232	152.00	159.00	2335	2250	1120240	0.152	0.197	0.497	186050	293009
913	1928964	2110020	1962774	0.029	0.061	0.045	138.00	101.00	14418	32915	6001758	0.045	0.560	0.282	590690	1155196
1320	559737	495926	463013	0.381	0.771	0.283	9.45	6.25	67905	881524	1518677	0.478	0.247	0.556	376303	332221
1322	778471	802038	595139	0.367	0.691	0.087	23.39	16.56	56306	671463	2175648	0.410	0.318	0.527	731756	627720
1463	677413	620715	677215	0.244	0.937	0.337	10.81	10.30	51946	188430	1975344	0.494	0.298	0.556	328861	390248
1470	858434	832018	614724	3.191	0.536	0.312	8.81	11.05	24078	26849	2305175	1.465	0.331	0.874	195381	191335
1654	203068	182355	169822	0.040	0.005	0.011	9.00	10.58	24376	133598	555245	0.020	0.112	0.519	120008	199446
1924	737381	792484	721109	0.258	0.426	0.159	9.22	6.75	13834	25908	2250975	0.286	0.326	0.486	65268	70172
1925	4825579	5451995	5318449	0.690	0.688	0.587	9.86	9.13	357602	1975059	15596023	0.654	0.767	0.637	2368092	2509787
2001	271555	222978	287299	1.636	2.840	0.233	5.40	6.44	59402	131157	781832	1.464	0.148	0.713	240927	289443
2002	228466	183177	154383	0.209	0.114	0.361	12.82	7.44	20080	77443	566026	0.219	0.114	0.541	146719	113146
2003	6162653	6825105	6530682	0.512	0.511	0.638	7.63	6.22	854319	5415420	19518440	0.554	0.804	0.559	3841239	3965999
2014	456234	426782	340787	0.246	1.248	0.458	7.59	6.04	70179	209135	1223803	0.654	0.211	0.597	335292	323904
2016	1164795	2808874	2677964	0.377	0.524	0.687	2.89	2.59	400986	505827	6651633	0.564	0.585	0.572	697956	765473
2039	307383	399821	398957	0.262	0.153	2.394	7.35	8.41	40028	26395	1106160	0.991	0.195	0.662	205217	248834
2041	684215	601603	467841	0.854	0.309	0.276	7.23	5.37	89960	231692	1753659	0.513	0.274	0.563	385916	375862
2065	200748	539073	532162	0.314	0.629	0.622	5.20	4.89	180348	445940	1271983	0.576	0.217	0.581	573990	668581
2070	1754005	2006385	2168261	1.701	0.666	0.708	5.99	7.29	349583	1988480	5928651	0.988	0.557	0.808	1782696	1887395
2081	1356370	1589739	1445421	0.594	0.510	0.656	16.11	13.10	82282	646318	4391529	0.584	0.483	0.583	814315	826122
2105	425216	309104	214443	0.235	0.324	0.517	7.27	4.84	58530	584773	948763	0.328	0.173	0.538	241327	219520
2110	880299	912997	772664	0.664	0.472	0.257	9.31	6.54	114640	730328	2565961	0.473	0.354	0.544	611373	579817
2111	2004770	1739229	1595198	0.807	0.830	0.714	8.66	9.35	222884	1848896	5339196	0.787	0.531	0.691	1404842	1654886
2112	225601	145303	242064	0.250	0.518	0.740	7.28	5.63	18799	219716	612968	0.507	0.122	0.573	82639	80874
2121	1616249	1426266	1301183	0.656	1.017	0.331	4.50	4.66	421140	601444	4343697	0.677	0.480	0.628	1253717	1355588
2143	921793	771171	622763	0.277	0.229	0.357	12.14	7.21	63674	294217	2315727	0.283	0.332	0.483	393319	352559
2150	59941	65115	110908	2.141	0.057	0.302	13.89	12.70	4992	4980	235964	0.702	0.056	0.589	43028	48210
2503	8920	8487	8695	0.136	0.025	0.039	0.68	0.73	11547	310	26103	0.068	0.015	0.575	4754	6291
2570	458832	416927	336260	1.811	1.288	1.188	10.25	11.86	29368	207151	1212019	1.458	0.209	0.765	242777	271817
2571	362058	356614	317728	1.484	1.400	2.317	7.68	8.87	38895	197500	1036400	1.710	0.185	0.791	249039	269241
2576	750939	669319	586290	0.641	1.249	0.433	9.79	7.82	48671	315221	2006548	0.783	0.301	0.643	322734	295533
2587	293177	234319	125571	1.483	1.690	0.540	4.18	4.11	66426	104180	653067	1.376	0.128	0.684	200073	212845
2683	83435	13788	44039	6.704	2.081	1.190	3.82	4.52	21216	98762	141262	4.534	0.038	0.731	62447	72916
2688	128785	248631	177659	0.155	0.441	0.451	4.35	2.86	42562	67237	555076	0.378	0.112	0.559	109117	95481
2702	1711439	2173770	2725812	0.372	0.471	0.240	47.85	13.62	26237	466217	6611021	0.350	0.583	0.447	591071	291900
2710	733400	808139	1039931	0.571	0.893	0.662	17.88	15.58	34301	106246	2581470	0.708	0.356	0.627	405254	431960
2731	259750	263581	263538	0.239	0.426	1.004	12.44	9.20	19562	6712	786869	0.558	0.149	0.579	148357	140878
2759	1478929	1572742	1549495	0.881	0.604	0.582	18.68	16.94	47023	383980	4601166	0.686	0.494	0.633	586209	647258
2797	2364423	3755144	2970229	0.505	0.420	0.499	11.12	7.47	119425	348353	9089797	0.468	0.658	0.507	709838	684408
2802	4805556	4841774	4342353	0.426	0.540	0.461	11.36	7.54	353227	1754127	13989682	0.477	0.747	0.503	2128660	2091224
2841	565878	731875	697613	1.723	0.416	0.509	10.12	9.12	56100	490748	1995366	0.819	0.300	0.653	390900	400907
2913	13284	110105	11821	0.000	0.095	0.000	5.16	4.09	23433	20017	135210	0.077	0.037	0.564	71827	73600
2923	678226	723997	721849	0.270	0.699	0.294	6.78	4.07	103876	334844	2124073	0.425	0.313	0.533	395420	319253
3004	1633796	1551150	1418600	0.566	0.422	0.090	5.45	3.90	309797	607108	4603546	0.371	0.494	0.478	849970	811255
3018	3288549	3031377	2628498	0.527	0.563	0.913	4.56	4.97	565089	1430447	8948423	0.653	0.654	0.628	1705781	1991793

EXHIBIT I (continued)

CLASS	PI1	PI2	PI3	XI1	XI2	XI3	RATE79	RATE82	PAYROLL	ACTLOSS	PI	XI	ZI	UI	ELOSS	NCCIELOS
3022	1540151	2449373	2055905	0.468	0.569	0.462	6.59	5.02	196702	248557	6045429	0.507	0.562	0.540	737551	725022
3027	794876	726565	1040278	0.751	1.720	0.619	6.39	4.97	114822	683576	2561718	0.972	0.354	0.720	556962	413873
3028	2117047	1836704	1695235	0.558	0.443	0.312	8.58	5.85	237855	571219	5648987	0.447	0.545	0.508	1093306	1030513
3030	3808876	3502056	3047891	0.477	0.523	0.799	16.08	13.10	197446	1932284	10358823	0.587	0.686	0.586	1959711	1858365
3064	155612	117603	130249	0.046	0.438	0.397	6.53	5.70	27554	38378	403464	0.274	0.086	0.556	105336	119302
3096	6984920	6725930	5413338	0.264	0.382	0.384	4.50	3.34	1795443	2849911	19124188	0.339	0.801	0.388	3299958	4501892
3110	3480299	3314510	3455698	0.545	0.762	0.616	15.80	16.15	201877	1421137	10250508	0.639	0.684	0.621	2088024	2252031
3111	452491	491198	660913	0.330	0.513	0.281	8.77	6.10	40034	461273	1604602	0.366	0.258	0.526	194757	182973
3114	442374	385490	438817	0.510	1.328	0.853	6.26	5.80	64350	42533	1266682	0.878	0.216	0.646	274248	281563
3116	6884538	6243554	5414449	0.519	0.714	0.585	4.50	3.34	1668460	2312161	18542541	0.604	0.796	0.599	4742654	4076124
3122	53374	168605	170861	0.590	0.459	0.097	5.54	4.45	22228	280460	392840	0.319	0.084	0.560	72671	75736
3131	1541606	1344256	1102200	0.820	0.641	0.946	7.23	7.39	192501	773025	3988062	0.794	0.459	0.680	996703	1099051
3132	5503567	5162026	4652120	0.295	0.473	0.279	9.45	4.90	495875	1499001	15317714	0.350	0.764	0.405	1999959	1799508
3146	10461448	10327501	10341663	0.642	0.575	0.598	6.34	5.44	1361354	5466606	31130611	0.605	0.868	0.602	5477101	5744591
3169	1263320	1103556	997050	0.784	0.818	0.167	8.63	6.95	162918	347298	3363925	0.612	0.418	0.595	881064	872144
3180	71111	52971	67226	1.084	0.766	0.570	5.78	5.83	9806	69270	191308	0.815	0.047	0.593	35430	44346
3188	1342651	2262766	2900956	0.250	0.467	0.630	12.23	8.68	94382	511048	6506374	0.495	0.579	0.532	646523	628503
3220	120876	116823	105879	0.245	0.003	0.041	4.55	3.91	24435	0	343579	0.100	0.076	0.546	63938	68709
3240	192267	519911	357551	0.577	0.543	1.075	5.71	6.49	29032	402242	1069729	0.727	0.190	0.610	106497	141726
3241	491883	557044	439154	0.520	0.239	0.153	9.85	6.13	51345	128496	1488081	0.306	0.244	0.515	274420	231984
3303	611573	500254	1387789	0.475	0.402	0.293	20.39	11.08	41157	775909	2499615	0.360	0.349	0.505	446256	342341
3306	100529	85929	59749	0.544	0.165	0.382	4.26	3.55	20005	23383	246208	0.372	0.058	0.570	51195	53947
3307	3526603	3389576	3011719	0.421	0.629	0.602	13.21	9.75	202341	1676944	9927897	0.547	0.677	0.558	1572891	1485384
3315	933144	905448	797898	0.434	0.796	1.513	3.00	3.70	325871	998596	2636490	0.885	0.361	0.691	712108	881089
3341	3072918	1782323	2073385	0.200	0.413	0.389	10.34	4.78	388072	1725585	6928626	0.311	0.595	0.421	1780030	1384467
3365	5239753	4020307	2634862	0.510	0.349	0.686	11.02	8.28	454320	2961773	11894922	0.495	0.715	0.520	2741949	2710325
3372	7957781	8035243	7368786	0.423	0.464	0.384	18.51	10.86	314904	2146146	23361810	0.425	0.831	0.452	2773853	2718522
3373	668306	713338	613002	0.682	0.424	0.647	29.18	21.72	14098	181411	1994647	0.579	0.300	0.581	251975	232597
3382	209007	140749	123465	1.144	0.332	1.127	3.39	3.06	28162	130	473222	0.898	0.098	0.613	61691	65526
3400	21847307	26284498	24347583	0.649	0.719	0.533	17.29	15.09	933011	8622676	72479387	0.635	0.938	0.632	10743705	11088736
3548	327022	288492	234682	0.513	0.285	0.250	5.84	3.25	63481	43622	850196	0.363	0.158	0.547	213860	157495
3559	370556	417932	1514657	0.780	0.369	0.680	4.24	3.57	88271	197004	2303145	0.640	0.331	0.601	237093	229626
3581	170491	150209	182260	1.426	0.347	0.810	3.47	3.26	47764	348487	502961	0.880	0.103	0.613	107058	117467
3612	4240098	4058378	3560756	0.644	0.349	0.454	4.66	3.43	602505	1992063	11859232	0.486	0.715	0.514	1519364	1507312
3620	2230698	2563176	2861172	0.507	0.288	0.563	17.03	12.03	113088	516514	7655045	0.455	0.618	0.503	1021646	964833
3628	8506844	8354286	7290971	0.450	0.638	0.568	4.72	3.34	2057991	3851358	24152100	0.551	0.836	0.556	5689534	4971061
3634	1022022	1092893	1016138	0.421	0.757	0.835	3.85	3.62	359172	684173	3131053	0.672	0.400	0.618	900969	973238
3635	3681690	2341564	3761357	0.561	0.657	0.535	5.90	4.86	532338	1623707	9784611	0.574	0.674	0.577	1908445	1905057
3638	1174961	1092317	864647	0.937	0.473	0.844	6.93	6.24	174342	982363	3131925	0.750	0.400	0.649	826537	820703
3643	4893011	4897730	3675854	0.877	0.571	0.708	5.80	5.81	660342	1677062	13466595	0.719	0.740	0.684	2758959	2897156
3647	176951	839659	672441	0.497	0.376	0.476	8.67	6.56	62129	506455	1689051	0.429	0.267	0.541	307185	300974
3648	2275335	2298681	2223042	0.974	1.094	0.617	7.15	8.13	158342	1517970	6797058	0.898	0.590	0.768	916691	1021213
3681	2248815	2214605	1804147	1.389	0.761	0.758	5.28	5.19	258808	574970	6267567	0.985	0.570	0.812	1169474	1072183
3685	1153182	1098880	862330	1.363	0.761	0.755	2.18	2.25	634293	995803	3114392	0.982	0.399	0.742	1081091	1073490
3719	214934	38736	127144	0.002	0.009	0.003	9.29	7.23	25039	80410	380814	0.003	0.082	0.535	131034	117753
3724	12702936	12741665	11011225	0.291	0.424	0.412	13.16	6.74	757668	3263651	36455826	0.374	0.885	0.398	4185168	3428900
3726	4343506	5820391	3645102	0.352	0.369	0.377	13.40	8.46	171646	900917	13808999	0.366	0.744	0.421	1020478	928836
3803	1607945	1327867	1034691	0.906	0.614	0.123	7.43	6.58	137	0	3970504	0.604	0.458	0.592	634	682
3807	572970	334702	499360	0.169	0.724	0.604	12.00	8.10	36475	157732	1407031	0.456	0.234	0.553	254858	219443
3808	673987	551242	381031	0.400	0.468	1.364	8.37	6.81	109735	662855	1606260	0.652	0.258	0.600	580905	568775
3821	1497541	1428867	1360542	0.162	0.481	0.312	17.97	8.10	78796	361175	4286950	0.316	0.477	0.455	679299	512118
3824	785816	1746149	1475725	0.449	0.528	0.507	10.88	8.25	49651	281979	4007690	0.505	0.460	0.547	311232	308717
3827	124113	100468	75771	0.045	0.704	0.025	3.44	2.93	42521	20023	300352	0.260	0.068	0.560	86375	88521
4034	4032715	4230401	3987395	0.357	0.482	0.330	16.76	9.39	224591	1291042	12250511	0.392	0.721	0.445	1764200	1573983
4112	1066	12027	9662	0.000	0.136	3.987	2.48	2.02	1992	405	22754	1.765	0.014	0.599	3118	3045
4130	1218479	1214366	1114642	0.464	0.350	0.251	7.41	4.69	179154	909047	3547487	0.358	0.430	0.486	679657	640147

EXHIBIT I (continued)

CLASS	PI1	PI2	PI3	XI1	XI2	XI3	RATE79	RATE82	PAYROLL	ACTLOSS	PI	XI	ZI	UI	ELOSS	NCCIELOS
4131	525750	474279	364110	0.862	0.332	0.473	7.57	6.08	58065	312722	1364139	0.574	0.229	0.580	268779	269233
4150	160788	152437	158834	0.111	0.063	0.146	1.66	1.13	96319	17592	472059	0.107	0.098	0.536	90237	84088
4206	495276	480714	403366	0.446	0.274	0.376	8.24	5.59	39003	0	1379356	0.365	0.230	0.532	180238	164481
4207	341426	398955	231045	0.236	0.700	0.152	7.15	4.73	25708	32668	971425	0.407	0.176	0.551	106782	90405
4240	118122	116680	97614	0.159	0.119	0.118	7.65	4.99	13887	131370	332416	0.133	0.074	0.549	61471	53482
4243	1791218	2280592	2987898	0.715	0.612	0.701	11.16	11.09	131087	1421198	7059708	0.676	0.599	0.638	983892	1089232
4244	3639612	3563932	3331370	0.569	0.756	0.442	13.75	11.94	230048	1307262	10534914	0.592	0.690	0.589	1963423	2074198
4250	355089	480466	700766	0.891	0.833	0.688	4.06	4.26	76089	809733	1536321	0.780	0.250	0.632	205600	240755
4251	586866	535595	452088	0.770	0.648	0.324	5.42	4.46	101807	180679	1574549	0.601	0.254	0.587	341254	349042
4253	161825	327854	509690	0.122	0.196	0.340	2.11	1.28	40715	33254	999369	0.258	0.180	0.524	47415	39162
4273	348897	317658	371601	0.202	0.599	0.317	8.46	5.83	41940	332286	1038156	0.364	0.186	0.542	202563	181608
4279	1814454	2074848	2422609	0.501	0.446	0.679	10.40	8.11	157608	712366	6311910	0.551	0.572	0.564	974977	990492
4304	1469845	1749031	1399756	0.742	0.959	1.407	4.57	5.49	234939	779353	4618632	1.026	0.495	0.802	907101	969235
4307	390753	369029	424123	1.201	1.638	0.726	5.45	5.53	66721	148888	1183905	1.167	0.205	0.702	269083	284486
4308	28684	34037	38308	0.344	2.060	0.359	1.05	0.89	21712	64911	101029	0.928	0.030	0.592	14234	14507
4351	68873	66931	55585	0.186	1.056	0.560	0.60	0.62	113193	273950	191390	0.599	0.047	0.583	41722	53204
4352	23348	21209	16847	0.097	0.190	0.101	1.44	1.68	11018	63676	61404	0.130	0.022	0.572	9568	14243
4360	39198	32830	27486	0.077	0.826	0.023	1.28	1.29	18432	905	99514	0.309	0.030	0.574	14273	17955
4361	687176	628267	600357	0.580	0.983	0.353	1.63	1.29	459255	255888	1915800	0.641	0.292	0.599	472811	469006
4420	4244831	4195915	4021782	0.966	1.373	1.530	12.59	16.38	301827	3854741	12462528	1.285	0.724	1.091	4370472	3427279
4452	2688568	2855786	2729080	0.418	0.211	0.593	10.77	8.16	250577	1235854	8273434	0.404	0.636	0.469	1333770	1623708
4459	1927416	2097828	1769049	0.466	0.408	0.638	9.37	6.50	196010	901221	5794293	0.498	0.551	0.536	1036478	1005514
4470	1057309	894172	1128722	0.887	0.569	0.344	5.93	5.03	93524	297598	3080203	0.596	0.397	0.587	343330	357695
4511	661689	623399	497505	1.133	1.815	0.910	1.49	1.72	377658	328422	1782594	1.309	0.278	0.784	464894	505396
4557	455048	357863	318264	2.015	0.383	1.014	5.39	5.77	72063	182149	1131175	1.217	0.198	0.708	289813	316472
4558	1965278	1872429	1729763	0.494	1.103	0.974	7.55	9.13	156903	908619	5567470	0.848	0.541	0.726	906406	1068127
4568	838165	593251	719917	0.369	0.436	0.505	9.11	5.81	115190	143728	2151333	0.433	0.316	0.535	591672	500466
4583	354034	389925	371991	0.452	0.392	0.373	9.97	6.99	31871	17459	1115950	0.405	0.196	0.547	183275	169726
4611	1019677	1343382	1204285	0.458	0.744	0.257	1.58	1.31	1040062	496022	3567344	0.498	0.432	0.546	945095	1042158
4635	541180	498965	578417	0.386	0.133	0.652	11.44	7.27	70313	213245	1618563	0.403	0.259	0.536	454179	386006
4665	605496	759192	672292	0.522	0.192	0.065	17.93	10.24	21043	36003	2036980	0.248	0.304	0.480	191014	161604
4692	290922	222811	211741	0.136	0.407	0.041	1.92	1.00	159006	44860	725474	0.191	0.139	0.528	169762	118904
4693	40360	37870	37103	0.062	0.056	0.089	3.42	2.94	23176	107704	115333	0.068	0.033	0.565	47224	52273
4720	1564747	1033927	673607	0.199	0.565	0.477	7.45	4.56	228338	210115	3272281	0.372	0.411	0.496	888728	787040
4740	564546	490967	397903	0.458	0.587	0.319	6.27	4.48	40595	148043	1453416	0.464	0.240	0.554	148533	132648
4741	195268	158205	139993	0.470	1.426	0.341	10.31	10.05	15138	56150	493466	0.740	0.102	0.598	98381	113768
4807	3866	3026	28814	0.062	0.211	3.983	12.91	9.28	113	130	35706	3.239	0.017	0.627	963	786
4808	23523	27786	24628	0.130	0.070	2.292	8.43	9.31	2542	790	75937	0.809	0.025	0.588	13274	17681
4809	109458	101500	92848	0.836	0.106	0.512	7.74	9.28	18796	9977	303806	0.493	0.068	0.576	88317	130821
4811	18440	31338	32477	0.062	2.297	1.089	6.63	9.15	6794	5431	82254	1.319	0.026	0.601	28548	47271
4812	1839	135091	174279	0.023	0.027	0.089	8.43	9.22	537	0	311209	0.062	0.070	0.546	2602	3675
4813	180091	337961	253529	0.195	0.681	2.656	7.74	9.22	25674	161811	771581	1.217	0.147	0.675	141385	179635
4815	1352555	2435726	2274498	0.827	0.550	1.062	8.57	9.06	254391	1054584	6062779	0.804	0.562	0.707	1623511	1769959
4817	8717	16598	12158	1.901	0.000	0.000	3.63	4.34	4408	862	37473	0.442	0.017	0.580	9777	14405
4819	32615	27210	43211	1.931	1.138	2.858	2.13	2.52	9558	50070	103036	2.110	0.030	0.628	13480	18332
4823	700	857	1242	0.077	43.849	0.083	1.75	2.08	913	0	2799	13.480	0.010	0.711	1197	1440
4902	309398	889486	731063	1.159	1.123	0.886	5.46	6.06	41034	241559	1929948	1.039	0.293	0.716	169090	192695
4923	126980	112054	89392	0.534	0.868	1.835	3.16	2.79	32605	130089	328427	1.002	0.073	0.613	66528	69443
5022	10317238	9343889	8578730	0.464	0.694	0.572	9.70	8.87	923710	7395982	28239857	0.573	0.856	0.574	5421145	5525538
5040	2186217	1669575	2953843	0.202	0.983	0.241	27.54	20.25	62695	275695	6809634	0.411	0.590	0.481	874863	789108
5057	3158515	3123971	2449114	0.394	0.610	0.802	19.50	19.43	173618	1452535	8731599	0.586	0.649	0.584	2085124	2172204
5102	1320230	1043734	1047667	0.486	0.593	0.132	9.39	7.21	120846	391810	3411630	0.410	0.421	0.510	609549	586572
5146	547945	681364	627500	0.802	0.782	0.396	9.46	10.21	85745	309271	1856810	0.658	0.286	0.604	516068	587318
5160	678035	568740	572779	0.264	0.261	0.782	5.72	5.93	110967	80293	1819554	0.426	0.282	0.538	360001	428019
5183	16842090	15442128	12637435	0.615	0.426	0.553	5.69	5.08	2610765	8861174	44921653	0.532	0.904	0.537	8409513	8561624
5188	725827	671872	599644	0.424	0.475	0.357	5.68	5.95	123638	284744	1997343	0.421	0.300	0.534	394960	476488

EXHIBIT I (continued)

CLASS	PI1	PI2	PI3	XI1	XI2	XI3	RATE79	RATE82	PAYROLL	ACTLOSS	PI	XI	ZI	UI	ELOSS	NCCIELOS
5190	10881175	10192630	7727757	0.553	0.423	0.397	5.22	3.85	1878430	4142254	28801562	0.465	0.858	0.482	4977017	4724015
5191	1577590	1574190	1449725	1.072	0.359	0.541	1.73	1.64	1130519	2099395	4601504	0.661	0.494	0.621	1279862	1391870
5192	1340288	1418106	1133537	0.639	0.851	0.996	4.38	4.64	268773	626916	3891931	0.820	0.453	0.690	855976	977222
5215	1868594	1424012	960914	0.626	0.417	0.728	10.79	9.91	117689	967602	4253520	0.579	0.475	0.581	777113	788621
5221	8552400	7873243	6233493	0.352	0.484	0.480	9.59	6.60	845125	5531666	22659136	0.433	0.827	0.459	3919012	3863239
5222	1266600	1448136	1511218	0.430	0.296	0.830	15.19	13.39	78829	726873	4225954	0.527	0.473	0.556	701626	697175
5223	297806	324050	289172	0.095	0.075	0.172	14.34	7.06	15346	8126	911028	0.112	0.167	0.503	116755	74636
5348	752918	707044	658042	0.621	0.376	2.547	6.16	7.99	120310	823951	2118004	1.138	0.313	0.756	590283	628989
5403	14427998	14164705	12141251	0.528	0.516	0.814	10.70	9.70	1229627	10847540	40733954	0.609	0.896	0.606	8402935	8238737
5437	1709170	1469425	1229307	0.718	0.461	0.504	6.12	5.81	264916	595404	4407902	0.572	0.484	0.577	986581	1038917
5443	169794	167766	179112	0.040	0.319	1.486	5.20	6.78	42220	45090	516672	0.632	0.106	0.587	135895	186191
5445	2300275	1793603	1445326	0.449	0.700	1.538	5.24	6.83	417280	2586405	5539204	0.814	0.540	0.707	1630189	1853808
5462	1107550	1004201	854136	0.417	0.546	0.486	8.21	7.29	146795	576960	2965887	0.481	0.388	0.543	689372	728075
5479	1500022	1437969	1085784	0.539	0.379	0.651	8.45	6.82	177678	1093476	4023776	0.512	0.461	0.550	869768	862570
5480	822192	985846	860072	0.438	0.381	0.235	10.68	7.13	85847	647937	2668109	0.351	0.363	0.498	481482	407436
5506	4950399	5203370	4849670	1.007	0.581	0.416	9.92	10.46	465543	4035222	15003439	0.668	0.760	0.648	3151809	3406432
5509	5326146	8081051	8021272	0.649	0.508	0.548	9.22	7.54	377439	3072049	21428469	0.558	0.819	0.563	2062943	2069825
5538	11450218	10996745	8919198	0.533	0.556	0.570	7.91	7.04	1517346	8697820	31366160	0.551	0.868	0.556	7026207	7166294
5551	9341150	8775226	6970244	0.383	0.488	0.359	27.72	17.46	357681	4308220	25086619	0.413	0.841	0.440	4594969	4482763
5606	5628626	4983361	3584432	0.416	0.371	0.377	5.28	3.36	1151756	1609170	14196420	0.390	0.750	0.438	2808325	2614429
5610	480126	513642	162137	0.455	0.054	0.898	8.15	6.25	31287	22261	1155905	0.339	0.202	0.533	143268	130729
5645	10470971	8724009	5879962	0.470	0.576	0.314	9.30	6.43	915514	5212561	25074942	0.470	0.841	0.488	4377823	4326434
5651	904120	660285	494851	0.542	0.741	0.246	9.30	6.43	102611	457269	2059255	0.535	0.307	0.568	570802	457797
6003	1132577	814322	675441	0.387	0.293	0.808	24.91	23.04	32833	40060	2622340	0.466	0.359	0.541	465841	487925
6204	1265337	1216055	968484	0.463	0.439	0.461	16.03	12.21	66579	643190	3449875	0.454	0.424	0.528	593760	562030
6217	11149911	10186845	8606849	0.469	0.321	0.288	12.72	7.54	915909	6072388	29943606	0.366	0.863	0.396	4860874	4706854
6229	872136	884324	731464	0.316	0.411	0.376	14.23	8.80	48864	329314	2487924	0.368	0.348	0.508	371950	297555
6233	773141	637802	621389	0.744	0.082	0.483	9.79	9.86	58756	163178	2032332	0.456	0.304	0.544	329680	371183
6306	7378522	4995746	3522883	0.408	0.356	0.566	19.16	13.18	306944	2509986	15897151	0.427	0.770	0.462	2865211	2667500
6319	2048806	1602707	1024810	0.286	0.469	0.519	12.09	7.86	169039	1020021	4676324	0.400	0.498	0.491	1057990	896035
6325	944919	968089	753434	0.549	0.585	1.202	8.81	10.02	138565	1410219	2666443	0.746	0.363	0.642	825656	957323
6400	372763	337771	272674	0.602	1.372	0.554	8.28	10.00	48531	164972	983208	0.853	0.178	0.630	266916	341040
6504	1485381	1984456	1774167	0.611	0.391	0.532	9.67	6.64	154186	725333	5244004	0.501	0.527	0.539	847424	801421
6834	1518155	1243933	1036946	0.915	0.839	1.459	5.09	5.87	214622	1103040	3799033	1.039	0.447	0.786	905080	992243
7219	41660087	42308736	33690828	0.585	0.592	0.608	10.82	10.82	3299246	24118052	117659651	0.594	0.961	0.594	22336456	22332516
7222	194184	402999	352555	0.273	0.770	0.251	20.10	17.18	14973	76756	949738	0.475	0.173	0.564	178777	200549
7230	2636169	2294613	1957225	0.549	0.440	0.577	3.69	2.92	794574	3404102	6888006	0.520	0.593	0.546	1685545	1623026
7360	418345	618610	571829	0.105	0.284	0.843	13.32	9.34	20695	36053	1608784	0.436	0.258	0.544	158152	154612
7380	11802698	12023099	11568520	0.836	0.737	0.711	4.83	5.22	2399602	10632558	35394317	0.762	0.882	0.740	9042432	9197067
7390	2591575	2050511	2008441	1.055	0.657	0.894	4.97	5.54	543336	3110476	6650528	0.884	0.585	0.758	2158368	2140682
7403	1008410	1063868	970189	0.423	0.638	0.409	3.85	2.96	428094	1128083	3042467	0.494	0.394	0.547	950765	894551
7405	11746	10824	5936	0.000	0.010	0.263	4.12	2.65	6602	19180	28506	0.058	0.015	0.574	16457	13159
7421	452951	420395	468875	0.102	2.278	1.658	6.20	7.51	68816	89361	1342221	1.327	0.226	0.750	337369	387599
7423	1274829	1074823	960187	1.166	0.327	0.572	5.60	4.59	181808	212770	3309840	0.721	0.414	0.640	686268	637670
7502	492558	374034	397113	0.540	0.291	0.948	4.38	3.94	105330	64283	1263705	0.594	0.216	0.585	284292	312769
7515	562484	537835	512213	0.097	0.080	0.308	4.66	2.32	74137	70549	1612531	0.158	0.258	0.473	172060	131299
7520	1044117	1168265	1248718	0.445	0.379	0.293	5.75	3.39	153349	733127	3461100	0.368	0.424	0.491	456416	402029
7538	1027057	1130487	1056093	0.420	0.622	0.451	16.86	15.82	44887	176855	3213637	0.501	0.407	0.549	438006	473089
7539	1459157	896546	1469559	0.773	0.355	0.181	4.87	3.56	318485	840458	3825262	0.448	0.449	0.522	852953	804101
7540	558864	568901	514413	0.394	0.388	0.668	7.61	5.84	85099	87548	1642178	0.478	0.262	0.555	378585	378255
7580	558444	601884	767212	0.701	0.247	0.747	5.10	4.19	86733	422196	1927540	0.577	0.293	0.581	270725	272555
7590	218462	190418	210226	1.694	0.414	0.926	8.91	7.37	27280	40845	619106	1.040	0.123	0.638	163481	158189
7600	1098243	1230926	1092591	0.868	0.493	0.465	2.70	2.28	408908	965146	3421760	0.604	0.422	0.591	688154	715254
7601	153075	173370	193791	4.912	0.706	6.877	21.55	26.11	9123	344277	520236	4.242	0.106	0.971	201178	166936
7610	929693	839075	711205	0.381	0.201	0.257	1.06	0.55	987596	914241	2479973	0.285	0.347	0.479	528472	427807
7704	4246260	4355054	4552100	0.632	0.535	0.423	7.98	5.97	407828	1978162	13153414	0.527	0.735	0.542	1858452	1791102

EXHIBIT I (continued)

CLASS	PI1	PI2	PI3	XI1	XI2	XI3	RATE79	RATE82	PAYROLL	ACTLOSS	PI	XI	ZI	UI	ELOSS	NCCIELOS
7720	8960542	10707376	10416162	0.759	0.640	0.562	5.27	4.85	1409822	6489913	30084080	0.649	0.864	0.640	5007390	5068850
7855	633968	626859	563345	0.391	0.347	0.185	26.27	15.85	21557	194835	1824173	0.312	0.282	0.506	301958	230221
8001	563211	503560	418058	0.301	0.250	1.260	2.59	1.69	242768	356233	1484829	0.553	0.243	0.575	381107	326477
8006	2306843	2035575	1575736	0.789	1.164	0.794	3.96	4.40	710474	1905943	5918154	0.920	0.556	0.770	2282538	2559021
8008	4827819	4825172	4500191	0.499	0.368	0.501	1.96	1.12	2448504	2681423	14153182	0.455	0.749	0.487	2461890	2370990
8010	4235800	3917308	3291833	0.516	0.565	0.560	2.62	1.91	1671705	3211085	11444941	0.546	0.707	0.556	2567879	2522521
8013	376639	331175	288103	1.151	0.621	0.739	0.81	0.80	493402	488083	995917	0.856	0.180	0.631	265858	314098
8017	8655017	7715904	6904165	0.813	0.554	0.603	2.26	1.81	4189113	6642611	23275086	0.665	0.831	0.651	6493368	6446930
8021	2066563	2290522	2378461	0.606	0.355	0.442	12.44	7.68	152737	765940	6735546	0.463	0.588	0.512	1024841	933423
8031	894430	863635	801416	0.423	1.393	0.475	5.84	5.19	166275	542852	2559480	0.767	0.354	0.647	662533	671427
8032	99148	110074	95410	1.306	2.108	0.787	1.98	2.23	62086	51907	304632	1.433	0.069	0.640	82967	111908
8033	10703932	9424974	7464023	0.970	0.591	0.877	4.12	4.32	2408123	4703833	27592929	0.815	0.853	0.781	8165487	8135199
8039	2521491	2980779	2896836	0.524	0.579	0.695	3.34	2.44	784983	1634935	8399106	0.603	0.640	0.595	1644567	1678634
8044	4103566	4231479	3428600	0.699	0.363	0.558	4.04	2.92	886878	3102440	11763644	0.537	0.713	0.550	2077145	2060731
8046	504131	446326	347115	0.357	0.201	0.458	3.19	1.90	170160	113045	1297572	0.330	0.220	0.527	301274	249027
8047	167798	188298	209453	0.212	0.174	0.095	4.38	2.55	33913	303747	565549	0.156	0.114	0.534	83533	68673
8050	499006	555130	506930	0.703	0.282	0.688	2.44	1.84	239716	138510	1561066	0.548	0.252	0.574	353538	362223
8058	784830	562159	420086	0.319	0.188	0.264	3.22	1.77	276992	299651	1767075	0.264	0.276	0.494	464728	392980
8102	96541	85570	73121	0.286	0.726	0.251	4.24	4.85	17474	8711	255232	0.423	0.059	0.573	44716	67299
8103	213741	332213	310834	0.354	0.603	0.662	11.54	7.97	6366	190061	856788	0.562	0.159	0.579	44824	40290
8106	11793278	11070371	10659259	0.457	0.542	0.412	19.10	12.62	490877	3737411	33522908	0.471	0.876	0.485	4787921	4628024
8107	3115221	2790487	2294525	0.401	0.647	0.709	6.56	5.44	465402	1222285	8200233	0.571	0.634	0.575	1850291	1843118
8111	936887	812610	767748	0.832	0.940	0.270	4.71	4.40	200507	677751	2517245	0.696	0.350	0.622	618861	667518
8116	1721662	1491526	1394697	0.586	0.598	0.351	6.11	4.35	298254	362050	4607885	0.519	0.495	0.551	1057743	1018710
8209	308896	266365	247365	1.036	0.249	1.362	8.96	7.79	47376	229746	822626	0.879	0.154	0.628	280924	295206
8215	204716	135213	109178	0.169	1.253	0.074	7.59	6.04	31597	538345	449107	0.472	0.094	0.572	144510	152020
8227	2564144	2147056	1632457	0.726	0.560	0.446	5.18	5.13	524388	900885	6343657	0.598	0.573	0.591	1691966	1776828
8233	25376	29450	21014	0.000	3.432	0.189	21.52	11.82	1234	211	75839	1.385	0.025	0.602	16855	11228
8235	418743	486868	430558	0.128	0.144	0.264	7.93	4.66	72075	257215	1336169	0.178	0.225	0.491	295825	264531
8263	95260	99711	160720	0.024	0.096	3.206	22.20	18.10	4106	31593	355692	1.482	0.078	0.652	62647	58600
8264	390122	331980	279266	0.933	1.072	0.590	23.01	19.19	17156	280027	1001368	0.883	0.180	0.636	264760	263607
8265	4770013	3911813	4270895	0.807	0.530	0.541	22.54	18.79	166582	1386262	12952721	0.635	0.732	0.621	2457687	2493318
8279	429795	346655	348981	0.278	0.471	0.347	10.28	7.32	33365	147449	1125432	0.359	0.198	0.538	194493	195357
8288	116271	85949	76630	0.080	0.181	4.532	5.49	6.24	16790	109564	278850	1.334	0.064	0.630	61209	83716
8291	161817	206003	197906	0.048	0.188	0.456	8.90	6.36	25995	170124	565727	0.242	0.114	0.543	132486	128375
8292	2898214	3040668	2571443	0.827	0.396	0.517	11.08	8.69	261585	1115014	8510325	0.579	0.643	0.580	1772327	1752679
8293	3203262	3199470	2750337	0.322	0.604	0.513	18.71	12.65	124897	1389675	9153069	0.478	0.659	0.514	1264627	1190742
8304	1538645	1543099	1514094	0.816	0.288	0.736	7.23	6.04	231210	786002	4595838	0.612	0.494	0.597	1051826	1081079
8350	3408926	3508183	3050023	0.509	0.581	0.639	7.69	6.37	414059	1599941	9967133	0.574	0.678	0.577	1934562	1945887
8385	1112465	1274049	1009490	0.695	1.016	0.306	10.35	9.20	114764	634085	3396004	0.700	0.420	0.632	790607	814072
8387	13382393	13958051	13017857	0.704	0.436	0.492	7.17	5.07	1708457	6933269	40358302	0.543	0.895	0.547	7063586	7030796
8392	375357	408405	484371	0.562	0.887	0.199	4.91	3.54	101123	306668	1268133	0.528	0.216	0.570	298438	284856
8393	4018520	3661562	2988391	0.580	0.464	0.469	6.23	4.48	562093	1914058	10668473	0.509	0.693	0.531	1960802	1899708
8500	1007959	877722	795934	0.382	0.741	0.849	19.28	17.22	44419	622893	2681615	0.638	0.364	0.603	543758	604906
8606	68108	777156	529766	0.034	0.634	0.246	14.43	9.07	8250	17363	1375030	0.455	0.230	0.553	69360	59058
8719	33601	41684	21714	0.003	5.144	0.007	3.43	2.80	9704	24884	96999	2.213	0.029	0.629	22079	21400
8720	405725	416034	388562	1.067	1.338	1.025	2.15	2.50	196499	84974	1210321	1.147	0.209	0.700	311665	384155
8742	15027070	14610494	12461295	0.664	0.463	0.551	0.85	0.69	17968009	9088589	42098860	0.561	0.899	0.563	9057893	9068473
8745	532432	331748	551806	0.163	0.419	0.349	7.11	3.91	39185	188785	1415986	0.295	0.235	0.515	151134	121918
8748	2195084	2176224	1800633	0.308	0.148	0.754	1.16	0.70	1525051	1170389	6171941	0.382	0.567	0.469	873588	783830
8800	180266	134787	123832	2.606	1.354	2.055	1.36	1.54	119315	172829	438885	2.066	0.092	0.719	122983	143688
8803	155186	142079	119842	0.503	1.113	0.916	0.17	0.20	1087727	314708	417106	0.830	0.089	0.604	117711	167902
8810	19674841	18494077	16281312	0.725	0.545	0.497	0.38	0.30	53363602	13813843	54450230	0.596	0.920	0.595	12709373	12699759
8829	16141273	15802318	14923641	0.753	0.651	0.716	7.40	5.86	2076701	12688059	46867232	0.707	0.908	0.696	11263976	11661565
8831	490844	445809	350497	0.277	0.806	0.850	2.36	1.83	223961	90361	1287150	0.616	0.219	0.590	328413	323128
8832	3216701	2658083	2092592	0.480	0.414	0.764	0.61	0.43	6248490	1092473	7967376	0.533	0.628	0.551	2213998	2088378

EXHIBIT I (continued)

CLASS	PI1	PI2	PI3	XI1	XI2	XI3	RATE79	RATE82	PAYROLL	ACTLOSS	PI	XI	ZI	UI	ELOSS	NCCIELOS
8833	4519987	7496166	7622172	0.580	0.528	0.748	1.62	1.33	2687172	2422170	19638325	0.625	0.805	0.617	2830469	2922525
8835	1542399	1656754	1496793	1.207	0.763	0.629	3.20	3.28	476618	1205124	4695946	0.866	0.499	0.724	1163587	1317573
8837	726135	929839	841591	0.494	0.930	0.510	5.43	4.31	124289	662406	2497566	0.662	0.348	0.610	433769	468411
8868	5881087	8037455	8405740	0.595	0.661	0.545	0.44	0.37	11920613	2492001	22324282	0.600	0.825	0.597	3298311	3417841
8901	182220	107554	104326	0.052	0.067	0.066	0.46	0.50	535544	456448	394100	0.060	0.085	0.538	139668	208128
9014	3713463	3724307	3138390	0.750	0.769	0.658	6.20	5.67	655949	3842141	10576160	0.730	0.691	0.684	2931264	3165815
9033	64844	41776	38445	0.225	0.255	2.647	3.75	3.62	16566	154485	145064	0.876	0.038	0.593	38849	46847
9040	2455879	4217572	4440302	0.581	0.872	1.045	5.32	5.55	396519	1987999	11113752	0.877	0.701	0.789	1753263	1917773
9052	4660359	4742673	4784389	0.587	0.634	0.774	6.19	4.77	698409	2557158	14187421	0.666	0.750	0.645	2936892	3066357
9058	1938505	1688192	1060873	0.768	0.797	0.959	4.02	3.76	462289	1704957	4687570	0.822	0.499	0.702	1373950	1471427
9060	2360563	2187406	1930050	0.607	0.395	0.675	3.99	2.84	664463	1387950	6478019	0.556	0.578	0.567	1583380	1563056
9061	1642685	1646871	1285497	0.658	0.536	0.593	3.48	2.60	538152	1228969	4575052	0.596	0.493	0.589	1162256	1150280
9063	482492	447299	451499	0.447	0.494	0.569	2.01	1.51	285828	293017	1381290	0.502	0.231	0.564	341237	354440
9079	30502044	30189414	25907410	0.509	0.479	0.514	4.02	2.21	7529275	22208301	86598869	0.500	0.948	0.504	16087046	15463790
9101	8527562	11719227	12239911	0.668	0.807	0.806	5.55	5.41	1363581	4334653	32486700	0.770	0.872	0.746	5950705	6190271
9102	1066629	1185761	1067543	0.770	0.340	0.294	4.53	2.99	163540	402062	3319933	0.463	0.414	0.533	416041	393584
9103	46305	74714	65678	0.292	0.813	0.467	5.55	4.26	5165	6552	186697	0.562	0.046	0.581	17559	17349
9154	710404	636055	568681	1.037	0.507	0.152	2.87	2.07	264885	267078	1915141	0.598	0.292	0.587	470137	435417
9156	78026	56303	48321	0.169	0.022	1.699	0.95	1.03	84759	23519	182650	0.528	0.046	0.580	49189	67583
9170	474549	538312	540464	1.255	0.420	0.262	40.22	34.27	9701	723697	1553324	0.620	0.252	0.592	243253	264815
9178	156237	122975	183758	0.153	0.220	0.132	8.16	5.48	12558	37990	462970	0.163	0.097	0.542	58493	53168
9179	686853	650957	665038	0.552	0.458	0.425	20.13	15.10	34808	227424	2002849	0.479	0.301	0.551	407014	394580
9180	477633	526911	406398	0.132	0.792	0.700	12.92	8.93	29940	534738	1410942	0.542	0.234	0.573	233475	213858
9182	60211	73786	66843	1.302	0.026	0.754	3.54	2.57	5912	955	200840	0.651	0.049	0.586	12913	11978
9186	234050	303365	267169	0.481	0.291	0.298	36.78	22.10	5385	47046	804583	0.349	0.152	0.547	114126	92044
9220	715318	736163	703966	0.733	0.545	0.489	6.53	5.46	110169	484504	2155447	0.589	0.316	0.584	443048	478658
9402	603852	553395	460424	1.041	0.409	0.469	6.23	5.37	80923	931210	1617670	0.662	0.259	0.603	320255	338446
9403	5951168	5701963	5470534	0.593	0.768	0.413	17.59	14.32	300128	3556465	17123666	0.594	0.783	0.591	3288505	3337157
9410	4074116	4801123	4587642	0.652	0.490	0.524	7.35	5.38	319975	2145720	13462881	0.551	0.740	0.559	1385258	1360005
9519	1780055	1807575	1592850	0.682	0.246	0.333	5.10	3.05	348285	969505	5180480	0.423	0.524	0.499	933486	806911
9521	1378453	1253369	1092400	0.528	0.713	1.222	4.78	5.76	269271	1002330	3724223	0.794	0.442	0.676	916473	1121684
9522	988738	967683	833211	0.481	0.643	0.396	4.22	3.13	183846	542245	2789633	0.512	0.373	0.556	454461	443236
9545	55891	51234	46135	0.173	0.034	1.308	5.20	6.50	18172	31938	153260	0.468	0.040	0.578	57516	80152
9549	399810	383087	411577	0.700	0.161	0.290	16.19	12.99	27024	166386	1194473	0.386	0.207	0.542	249704	240112
9558	337279	327073	329166	0.240	0.945	1.430	11.19	12.28	31828	207857	993519	0.866	0.179	0.633	237587	258826
9559	163032	134650	125235	0,203	0.102	0.361	5.80	4.59	26567	56806	422918	0.218	0.090	0.550	89228	91905
9586	678594	581205	516081	0.383	0.427	0.642	1.04	0.71	688485	304793	1775880	0.473	0.277	0.552	416461	393868
9600	1643	2750	1592	0.000	1.738	0.000	1.70	2.01	1108	143	5985	0.799	0.011	0.584	1161	1687
9620	573137	493516	451331	0.254	0.393	0.298	2.34	1.63	256258	606673	1517983	0.312	0.247	0.515	325704	330334

EXTRAPOLATING, SMOOTHING, AND INTERPOLATING DEVELOPMENT FACTORS

RICHARD E. SHERMAN

Abstract

The purpose of this paper is to provide a practical handbook describing simple yet accurate methods of extrapolating, smoothing, and interpolating development factors. It will focus on the inverse power curve, its properties, and examples of fits obtained to various types of loss experience. It will also illustrate usage of the inverse power curve in addressing a variety of actuarial problems, including the following:

- A lack of mature development experience.
- A lack of credible loss development data.
- Loss data at interim evaluation dates.
- Loss experience at odd, inconsistent evaluation dates.
- A need to break down annual development into quarterly or monthly segments.

The objective of this paper is to enhance the reader's capability in analyzing loss development.

INTRODUCTION

Development factor analysis is fundamental to most actuarial studies for ratemaking and reserving purposes. It is the purpose of this paper to materially enhance the reader's capability in analyzing loss development. A simple, general mathematical function, the inverse power curve, will be presented that usually fits loss experience as well as or better than other functions in common use today. Comparisons of goodness of fit using the inverse power curve and various other functions have been made based on incurred and paid losses, reported and paid claim counts, and primary and excess experience for workers' compensation, medical malpractice, automobile and general liability, automobile physical damage, fidelity, and surety. This is not a theoretical treatise so much as it is a practical guide aimed at presenting simple yet very accurate methods of extrapolating, smoothing, and interpolating development factors. We will focus on effective approaches to dealing with the following common actuarial problems:

- The most mature experience available still indicates the clear potential for further development (either upward or downward) to an ultimate basis.
- Only two or three development factors are available in the loss history, but there is still a need for a full profile of future loss development.
- Development factors for the later stages of development are sparse or fluctuate significantly and the reliability of selecting factors for the most mature stages of development on the basis of one or two historical factors is openly questionable.
- A given body of development data is based on relatively few claims and must be credibility weighted with external data sources while still preserving the *unique* characteristics of that experience.
- All prior development experience is on a year-end basis, but there is a need to incorporate the latest evaluation which is at some point in the middle of the year.
- Available loss experience is at odd, inconsistent evaluation dates.
- There is a need to estimate quarterly or monthly development, but only annual data is available.
- Accident or report quarter development factors are needed, but only annual factors for accident or report years are available.

An approach to dealing with each of these problems will be described in various sections of this paper. Although the examples in this paper are illustrated with the use of one type of mathematical function, many of the techniques can be used with a wide variety of other functions.

SECTION I

EXTRAPOLATION OF INCURRED LOSSES AND PAID LOSSES USING THE INVERSE POWER FUNCTION

The availability of a simple family of curves that closely fit loss development factors of all types for any line of business would be instrumental in advancing the quality of reserve and ratemaking analysis. Research indicates that the family of curves of the form, $1.0 + a(t + c)^{-b}$, which we shall call inverse power curves, comes closer to filling this need than other functions in use today. For example, a comparison of paid loss development factors for workers' compensation (accident year 1969 for the Wausau Insurance Companies) with approximations obtained by fitting the inverse power curve and five other mathematical functions is provided below.

	Development Factors						
Year of Develop-ment	Actual	Inverse Power	McClenahan[1]	Geo-metric[2]	Expo-nential Decay[3]	Log-Normal[4]	Loga-rithmic[5]
2:1	1.920	1.889	2.840	1.683	1.309	1.378	1.409
3:2	1.228	1.224	1.329	1.277	1.202	1.190	1.168
4:3	1.098	1.100	1.131	1.147	1.133	1.112	1.103
5:4	1.051	1.056	1.061	1.088	1.087	1.073	1.072
6:5	1.036	1.036	1.031	1.055	1.057	1.051	1.054
7:6	1.025	1.025	1.016	1.035	1.037	1.036	1.044
8:7	1.019	1.018	1.008	1.023	1.025	1.028	1.037
9:8	1.014	1.014	1.004	1.015	1.016	1.022	1.032
10:9	1.011	1.011	1.002	1.010	1.011	1.016	1.026
11:10	1.009	1.009	1.001	1.007	1.007	1.013	1.024
12:11	1.008	1.008	1.001	1.005	1.005	1.011	1.021
Chi-Square Statistic[6]		.001	.307	.039	.289	.216	.191

[1] Charles L. McClenahan, "A Mathematical Model for Loss Reserve Analysis," *PCAS* LXII, 1975, pp. 134–153.

[2] David Skurnick, Discussion of "A Mathematical Model for Loss Reserve Analysis," *PCAS* LXIII, 1976, pp. 125–127.

[3] Obtained by fitting an exponential curve of the form, $y = ae^{bt}$, to the development factors less one.

[4] Derived by fitting a log-normal distribution to the cumulative payments distribution, and then expressing the fitted distribution in terms of development of factors.

[5] Based on fitting a logarithmic curve of the form, $y = a + b \ln t$, to the cumulative payments distribution, and then expressing the fitted distribution in terms of development factors.

[6] Paul H. Hoel, *Introduction to Mathematical Statistics*, 1971, pp. 225–234.

The chi-square statistic for goodness of fit is substantially better for the inverse power curve than for the other functions. Similarly, the size of errors for the inverse power curve is also significantly less, as shown below.

Year of Develop-ment	Comparison of Curve Fit Errors					
	Inverse Power	McClenahan	Geo-metric	Expo-nential Decay	Log-Normal	Loga-rithmic
2:1	−.031	+.920	−.237	−.611	−.542	−.511
3:2	−.004	+.101	+.049	−.026	−.038	−.060
4:3	+.002	+.033	+.049	+.035	+.014	+.005
5:4	+.005	+.010	+.037	+.036	+.022	+.021
6:5	.000	−.005	+.019	+.021	+.015	+.018
7:6	.000	−.009	+.010	+.012	+.011	+.019
8:7	−.001	−.011	+.004	+.006	+.009	+.018
9.8	.000	−.010	+.001	+.002	+.008	+.018
10:9	.000	−.009	−.001	.000	+.005	+.015
11:10	.000	−.008	−.002	−.002	+.004	+.015
12:11	.000	−.007	−.003	−.003	+.003	+.013
Average Absolute Error	.004	.102	.037	.068	.061	.065

Another test of the appropriateness of various functions is the factor to ultimate they indicate. For this purpose we will truncate any development indicated past 80 years (since all permanent disability claimants will presumably have died within this period.) A comparison of development factors from 12 years to 80 years of development is as follows:

Indicated by Case Reserves	1.086
Inverse Power Curve	1.076
McClenahan	1.007
Geometric	1.011
Exponential	1.009
Log-Normal	1.047
Logarithmic	1.537

In the above example, historical patterns have shown that case reserves are adequate to cover IBNR losses as well as changes in reported reserves.

These results are representative of comparisons performed on both paid and incurred losses for most lines of business. This paper will focus on illustrating the usage of the inverse power curve to address a wide range of actuarial problems.

In the following example, incurred losses for an isolated accident year will be extrapolated to an ultimate basis using an inverse power function. The only information we are given is incurred losses for automobile bodily injury liability for accident year 1978 at the following evaluation dates:

Evaluation Date	Incurred Losses	Development Factor
12/31/78	$ 8,479,000	–
12/31/79	13,380,000	1.578
12/31/80	14,678,000	1.097
12/31/81	15,147,000	1.032

We will fit an inverse power curve to the development factors so that the factor at age t will be approximated by $(1 + at^{-b})$.

This fit can be performed in a least squares sense on a computer. For the sake of simplicity we will illustrate another method for fitting this curve which involves the use of only natural logarithms, exponentials, and linear regression. This method is displayed in Exhibit 1. First, we compute the reciprocals of each age of development (t) and we subtract 1.0 from each incurred loss development factor. The natural logarithms of $1/t$ and each development factor minus one are then calculated. A linear regression is then performed with $\ln(1/t)$ as the independent variable (x) and ln(factor − 1.0) as the dependent variable (y). In this case, the coefficient of determination (goodness of fit) was .99887. The values of a and b were obtained from the linear least squares trend line ($y = a + bx$) as 2.33259 and 4.19024, respectively. These parameters give us the following equation for the incurred loss development factor at age t:

$$1.0 + 10.30460t^{-4.19024}.$$

The extrapolated estimates in Exhibit 1 were easily obtained by first computing $1/t$ and $\ln(1/t)$ for each future age of development and then using the relationship

$$\ln(\text{development factor} - 1.0) = \ln a + b \ln(1/t)$$

from the linear regression to obtain the projections in column (4). These projections were then exponentiated to obtain the projected development factors (less one) in column (2). By adding one to each of these projected factors and taking their product, we obtain a factor to ultimate of 1.0257. This factor, when applied to the latest value of incurred losses for accident year 1978 of \$15,147,000, yields an estimated ultimate incurred loss of \$15,536,445.

Exhibit 2 provides a comparison of actual and fitted incurred loss development factors for automobile bodily injury liability, general liability, and workers' compensation over 10 to 15 years of development.

The goodness of fit of the inverse power curve can often be improved by adding a third parameter, making the function of the form:

$$1.0 + a\,(t + c)^{-b}.$$

In this case, we define a function, $f(c)$, to be the coefficient of determination (R^2) of the above inverse power curve. The value of $f(c)$ is estimated for a wide range of values of c and a local maximum can be found by numerical analysis techniques. For example, in Exhibit 2, $c = -1$ was used for general liability. This technique is often useful in obtaining a better fit for the earlier periods of development than for later periods. Variations in the c parameter usually have little impact on the projected factors for later periods of development, but have a major effect on varying the shape of the inverse power curve for the earliest periods of development. As an alternative to letting $c = -1$, we may simply redefine the values of t. For example, for the 2:1 development factor, we have defined t as being equal to 2 (its value at the end of the period of development). Alternatively, defining t as its value at the beginning of each development period would result in setting $c = 0$ for the examples in Exhibit 2 and would eliminate this third parameter.

To continue the previous example and to illustrate the versatility of the inverse power function, it will next be used to extrapolate paid losses to an ultimate basis using only the following information:

Evaluation Date	Incurred Paid	Development Factor
12/31/78	\$ 3,071,000	–
12/31/79	8,603,000	2.801
12/31/80	11,941,000	1.388
12/31/81	13,541,000	1.134

The method is identical to that used in projecting the incurred factors above and is illustrated in Exhibit 3. A coefficient of determination of .99998 was obtained, indicating an excellent fit. The product of all the extrapolated factors in column 2 is 1.1393, indicating an estimated ultimate loss of $15,427,261 ($13,541,000 × 1.1393). This closely compares with the incurred projection of $15,536,445 developed above.

SECTION II

SOME PROPERTIES OF THE INVERSE POWER FUNCTION

The inverse power curve possesses a characteristic which is essential to obtaining close approximations to actual loss development factors. To show this, let us define some terms. Let d_i represent the development factor for the i^{th} period of development. Let B_i be the "decay" ratio between (d_i − 1.0) and (d_{i-1} − 1.0). We have observed that a common characteristic of loss development data of any type is that B_i tends to increase asymptotically to 1.0 as i increases. This pattern can be verified from Exhibit 2 for general liability incurred losses as follows:

Years of Development	Decay Ratios (B_i) Actual	Smoothed[7]	Inverse Power
3	.333		.300
4	.663	.451	.496
5	.416	.519	.606
6	.506	.563	.675
7	.846	.722	.741
8	.879	.916	.765
9	1.034	.832	.794
10	.633	.785	.814
11	.737	.811	.834
12	1.143	.881	.848
13	.813	.950	.860
14	.923	.794	.870
15	.667		.879

[7] Each smoothed decay ratio is the third root of the product of the corresponding actual factor and the immediately preceding and immediately succeeding factor. For example, .451 = $(.333 \times .663 \times .416)^{1/3}$. This is also equivalent to taking the third root of the decay ratio between a given development factor minus one (d_i − 1.0) and the third subsequent development factor minus one (d_{i+3} − 1.0). For example, .451 = $(.077/.839)^{1/3}$. Both smoothing formulae are based on the assumption that there is a constant decay ratio applicable over a three-year period.

The inverse power curve satisfies this condition since

$$B_i = \frac{a(i)^{-b}}{a(i-1)^{-b}} = \left(\frac{i-1}{i}\right)^b = \left(1 - \frac{1}{i}\right)^b$$

and it is clear that $(1 - (1/i))^b$ increases to 1.0 as i increases.

One simple method of tail analysis assumes that B_i is constant (at least for the later periods of development). It is much more common for the decay ratios to increase than it is for them to remain constant. However, usage of a constant B_i (with a B_i based on more mature experience) can often serve to provide a lower bound for projections of future development.

In loss development experience we have reviewed, the earliest decay ratios are usually very low (.2 to .4) rising to the .7 to .9 range for later periods. It is this property of the inverse power curve which yields generally better fits than other functions. For example, consider the following comparison of decay ratios for the functions compared at the beginning of this paper.

	Decay Ratios						
Year of Develop-ment	Actual	Inverse Power	McClenahan	Geo-metric	Expo-nential Decay	Log-Normal	Loga-rithmic
3	.248	.252	.179	.406	.654	.503	.411
4	.430	.446	.398	.531	.654	.589	.613
5	.520	.560	.466	.599	.654	.652	.699
6	.706	.643	.508	.625	.654	.699	.750
7	.694	.694	.516	.636	.654	.706	.815
8	.760	.720	.500	.657	.654	.778	.841
9	.737	.778	.500	.652	.654	.786	.865
10	.786	.786	.500	.667	.654	.727	.813
11	.818	.818	.500	.700	.654	.813	.923
12	.889	.889	1.000	.714	.654	.846	.875

While many functions can fit loss development factors well over some segment of the history of development, few provide good fits over the entire history. It is the properties of the inverse power curve in terms of decay ratios, as noted above, as well as its flexibility in fitting the very large factors common at early stages of development, that make it a natural candidate for development factor analysis.

Because of the behavior of the decay ratios of the inverse power curve and their correspondence to this type of phenomenon in actual loss development experience, it is usually possible to obtain relatively good approximations of factors for later periods based solely on extrapolations of factors for earlier periods. For example, consider the general liability data in Exhibit 2 and extrapolations based only on the earliest factors:

	Extrapolation Based on			
Years of Development	First 2 Factors	First 3 Factors	First 4 Factors	Actual Factors
2	1.839	1.810	1.874	1.839
3	1.279	1.307	1.283	1.279
4	1.146	1.174	1.146	1.185
5	1.093	1.117	1.092	1.077
6	1.065	1.085	1.064	1.039
7	1.049	1.066	1.048	1.033
8	1.038	1.053	1.037	1.029
9	1.031	1.044	1.030	1.030
10	1.026	1.037	1.025	1.019
11	1.022	1.032	1.021	1.014
12	1.019	1.028	1.018	1.016
13	1.016	1.025	1.015	1.013
14	1.014	1.022	1.014	1.012
15	1.011	1.020	1.012	1.008

Naturally, the reliability of such projected factors is limited by the high degree of variability inherent in the first few factors and the sensitivity of any extrapolation technique to such variability.

While it would be highly desirable to derive a closed-form equation for the product of all extrapolated development factors as an estimate of the age-to-ultimate factor, the author has been unable to solve this problem. A simple program can be written to perform this otherwise cumbersome set of computations.

SECTION III

FITTING THE INVERSE POWER CURVE TO INCURRED LOSSES FROM THE REINSURANCE ASSOCIATION OF AMERICA EXPERIENCE

As an example of the goodness of fit of the inverse power function to excess experience, we have fitted curves to average incurred loss development factors from the 1983 edition of the *Loss Development Study* of the Reinsurance Association of America. In order to reduce fluctuations in this data before performing the curve fits, the mean factor for the latest 10 years was obtained for each year of development.

The curve fits shown in Exhibit 4 indicate that significant upward development is indicated beyond the most mature experience available for medical malpractice and workers' compensation. Upward development of 36.0% is projected for medical malpractice from 14 to 25 years of development. Upward development of 18.5% is estimated for workers' compensation from 25 to 50 years of development, which would no doubt be due to increasing medical costs and benefit changes on permanent disability cases.

SECTION IV

PROJECTING LOSSES IN A DYNAMIC ENVIRONMENT USING THE TWO-DIMENSIONAL INVERSE POWER FUNCTION

The accurate projection of losses in a dynamic environment can best be accomplished if a two-dimensional function can be found which closely approximates recent historical experience and which does not exhibit any detectable bias for any portion of that experience. In this section, the two dimensional inverse power function will be presented and tested and its derivation detailed. In keeping with the guidelines set forth earlier for keeping all analyses simple, we have limited our analytic tools to exponentials, natural logarithms, and linear least squares trend lines. The results are not perceptibly different from those which would be obtained from a computerized two-dimensional least squares fit and the added advantage of being able to perform all computations on a pocket calculator is achieved.

The data used in this test consisted of paid loss development factors for workers' compensation for accident years 1955 to 1980 from the Wausau Insurance Companies. The factors extended out to 12 years of development. The resultant two-dimensional inverse power curve took the following form:

$$PLDF_{AY,t} = 1.0 + (.819663 + .000983AY)t^{(-3.911356 + .027946AY)}$$

Exhibit 5 provides a comparison of the actual and fitted factors using the above function.

In this equation, t represents the year of development of the given paid factor minus 1.0. Thus, for the 2:1 factor, t equals 1.0. This is equivalent to setting $c = 1.0$ for the three-parameter function. AY represents the accident year, expressed in years since 1900. (Since each set of coefficients is defined in terms of a linear relationship, it does not matter how AY is defined in terms of the initial year.) For example, for accident year 1967, $AY = 67$. The above two-dimensional function may be viewed as a family of one-dimensional inverse power curves. Sample curves are as follows:

Accident Year	Inverse Power Curve
1957	$1.0 + .876t^{2.318}$
1962	$1.0 + .881t^{-2.179}$
1967	$1.0 + .886t^{-2.039}$
1972	$1.0 + .890t^{1.899}$
1977	$1.0 + .895t^{1.760}$

The above two-dimensional equation was derived by first estimating one-dimensional inverse power curves for the average factors for each of the following groups of accident years:

1955–1959
1960–1964
1965–1969
1970–1974

From these fits, the following inverse power curves were obtained:

Accident Years	Inverse Power Curve	Goodness of Fit
1955–59	$PLDF_t = 1.0 + .877134t^{2.321363}$	.997336
1960–64	$PLDF_t = 1.0 + .880757t^{2.175112}$	.998984
1965–69	$PLDF_t = 1.0 + .880758t^{2.037354}$	.999826
1970–74	$PLDF_t = 1.0 + .893510t^{-1.901515}$	.998100

Linear regression analysis was then applied to the set of coefficients of t, with AY as the independent variable, to obtain the equation:

$$\text{Coefficient of } t \text{ for accident year } AY = .819663 + .000983\ AY.$$

Likewise, a linear trend line was fitted to the exponents of t.

Exhibit 6 provides a test of potential bias which might result from fitting the two-dimensional function to the triangle of factors. There does not appear to be any detectable bias since there are not significant contiguous areas of the triangle for which the signs of the errors are consistently positive or negative.

SECTION V

SELECTING DEVELOPMENT FACTORS FOR THE MOST MATURE PERIODS OF DEVELOPMENT WHEN CREDIBILITY IS LOW

The top portion of Exhibit 7 presents the commonly accepted method for selecting development factors for the most mature periods of development. The arithmetic mean of these factors for each period of development is selected—unless that mean appears too far out of line. We might, for example, want to temper the 6:5 factor because of its unexpected magnitude.

Let us consider the reasonableness of this common practice. Of all of the mean $Y:X$ factors, the mean factors for the earlier periods of development are often more reliable indications of future development factors (*unless* some clear trend is present or the magnitude of development is large) than the later mean factors. The earlier mean factors are the average of a greater number of individual factors, each of which is the end result of more claims transactions than those for the later factors. For example, consider the following history of incurred loss development.

Accident Year	Incurred Losses (000's)					
	1	2	3	4	5	6
1976	1,234	2,340	2,789	2,873	2,841	3,517
1977	1,462	2,506	3,185	3,507	4,071	
1978	1,618	2,657	3,459	3,684		
1979	1,824	2,740	3,378			
1980	1,943	3,087				
1981	2,120					

Ratio of Total Incurred Losses	13,330 / 8,081	12,811 / 10,243	10,064 / 9,433	6,912 / 6,380	3,517 / 2,841
Dollar Weighted Average Development Factor	1.650	1.251	1.067	1.083	1.238

Relative Volume of Losses on which Average Factor is Based:

Numerator	1.000	.961	.755	.519	.264
Denominator	.789	1.000	.921	.623	.277

In the above example, the reliability which should be assigned to each successive factor (after the first) declines sharply. How do we recognize this in the commonly accepted procedures? Not only is it often not recognized, it is usually violated to a successively greater extent as factors are selected for the later periods of development. This process is culminated by placing full reliance on the sole factor available for the oldest period of development. Furthermore, this one factor is heavily impacted by only a few, generally large, claims.

An alternate method of selecting factors is displayed in Exhibit 7. As commonly done, the mean factors are first computed. An inverse power curve is then fitted to the mean factors for the first two periods of development to project the 4:3 factor. (Alternatively, the inverse power curve could be fitted to all the individual factors.) The selected factor (1.110) is then determined as the weighted average of the inverse power curve projection (1.125) and the arithmetic mean of the actual 4:3 factors (1.065). In this simple example, the weights used are the number of actual factors on which each estimate is based. In the case of the arithmetic mean, three factors were used in computing the mean and a weight of three is assigned to 1.065. Nine factors underlie the inverse power curve projection (five 2:1 factors and four 3:2 factors) and its estimate of 1.125 is assigned a weight of nine.

The above process is then repeated, with the next inverse power curve fitted to the first two mean factors and the selected 4:3 factor of 1.110. The projected factor of 1.063 from the curve fit is given a weight of 12, versus a weight of 2 for the mean factor of 1.075. The weighted average of 1.065 then becomes the selected factor. This process can be repeated ad infinitum to select development factors of greater stability and accuracy than can be typically obtained by selecting the mean factors for the most mature periods of development.

Let us further suppose that we have another body of experience for the same line of business. How can this information be properly combined with the more specific, but less credible data we have just analyzed? Of many approaches tried, the following appears to possess the greatest validity. We begin by comparing the residual factors (i.e., the development factor less 1.0) corresponding to the development factors:

Years of Development	Residual Factors		Ratio
	Company	"Industry"	
2:1	.669	.483	1.385
3:2	.250	.167	1.497
4:3	.110	.094	1.170
5:4	.065	.046	1.413
6:5	.054	.033	1.636

The arithmetic mean of the above ratios is 1.420; the median is 1.413; the arithmetic mean of the 3 middle ratios is 1.432. The stability of these ratios suggests that the company's residual factors tend to be about 42% higher than the "industry's." We may then use this assumption to further smooth the selected factors, and, perhaps more importantly, to project the development factors at later, yet to be experienced, stages of development:

Years of Development	"Industry" Factors	Smoothed Company Factors
2:1	1.483	1.686
3:2	1.167	1.237
4:3	1.094	1.133
5:4	1.046	1.065
6:5	1.033	1.047
7:6	1.028	1.040
8:7	1.019	1.027
9:8	1.012	1.017
10:9	1.009	1.013

SECTION VI

ESTIMATING QUARTERLY DEVELOPMENT FACTORS FROM ANNUAL FACTORS FOR A GIVEN ACCIDENT (REPORT) YEAR

In this section, a method will be presented for estimating quarterly development factors for a given accident (or report) year based only on annual development factors. The inverse power function is again used extensively. Applications for this technique appear in subsequent sections and include:

1) How to incorporate loss development information at odd evaluation dates. An example of this would be the inclusion of loss data as of June 30, 1983 in an analysis of annual development factors which are all at year end.
2) How to analyze loss development when all evaluation dates are odd. As an example, we will perform an analysis on accident years 1979–1982 incurred losses where the only data available is at the following evaluation dates: July 31, 1980, November 30, 1981 and April 30, 1982.
3) Performing more precise discount calculations by translating annual development factors into quarterly or monthly factors.

For simplicity in our current example, we will assume that the only information we have on accident year 1980 loss payments for workers' compensation is:

Evaluation Date	Cumulative Paid Losses	Paid Loss Development Factor
December 31, 1980	$11,300,000	—
December 31, 1981	25,817,000	2.285
December 31, 1982	35,040,000	1.357

In actuality, we have used data which includes quarterly evaluation dates and development factors, but we shall pretend that we do not have this and attempt to approximate it from the above information. The process is started by deriving two initial approximations of quarterly factors—one for each annual interval. Consider first calendar year 1981. There are four quarterly development factors we want to estimate, with t (in quarters as of the end of each period) equal to 5, 6, 7, and 8. The average t value for these factors is 6.5. We know that the product of these four quarterly factors is the annual factor of 2.285. A first approximation for the average of these four factors is the fourth root of 2.285, or 1.229. We assign this to the average t-value of these factors (6.5). Similarly, an average factor of 1.079 is estimated for 1982 and assigned to an average t-value of 10.5. With this, we have two points with which to determine a two-parameter inverse power curve ($1.0 + 14.583516\, t^{-2.219212}$), which forms the basis for our first approximation of the quarterly factors:

t	Factor	t	Factor
5	1.410	9	1.111
6	1.275	10	1.088
7	1.195	11	1.071
8	1.144	12	1.059

We note that in both cases, the product of these factors exceeds the annual factor, indicating the need for an improved approximation.

$$2.458 = 1.410 \times 1.275 \times 1.195 \times 1.144$$
$$1.371 = 1.111 \times 1.088 \times 1.071 \times 1.059$$

In the first case, the actual annual factor of 2.285 is .9296 of the above product of 2.458. The fourth root of .9296 (.9819) gives us an "average" correction factor to apply to our first set of approximations for calendar year 1981. Instead of applying this adjustment, it would be more accurate to distribute the total adjustment in proportion to the development factors less 1.0.

$$.410 + .275 + .195 + .144 = 1.024$$
$$1.0 - .9296 = .0704$$

$$(.410/1.024) \times .0704 = .0282$$
$$(.275/1.024) \times .0704 = .0189$$
$$(.195/1.024) \times .0704 = .0134$$
$$(.144/1.024) \times .0704 = .0099$$

$$(1.0 - .0282) \times 1.410 = 1.370$$
$$(1.0 - .0189) \times 1.275 = 1.251$$
$$(1.0 - .0134) \times 1.195 = 1.179$$
$$(1.0 - .0099) \times 1.144 = 1.133$$

After analogous adjustments to the quarterly factors for calendar year 1982, we have a full set of second approximations. We then fit an inverse power curve to this second set of approximations to smooth the factors and produce our third and final set of estimates.

	Approximations			Actual	
t	First	Second	Third	Factors	Error
2			3.500	3.531	−.031
3			2.067	1.971	+.096
4			1.585	1.657	−.072
5	1.410	1.370	1.366	1.382	−.016
6	1.275	1.251	1.251	1.245	+.006
7	1.195	1.179	1.181	1.160	+.021
8	1.144	1.133	1.137	1.145	−.008
9	1.111	1.110	1.107	1.112	−.005
10	1.088	1.087	1.086	1.079	+.007
11	1.071	1.070	1.070	1.063	+.007
12	1.059	1.058	1.058	1.064	−.006

The final set of approximations differs from the actual data to such a small degree that such differences may be attributable only to random fluctuations in the actual loss experience. If these approximations are used, we may, for example, refine present value calculations.

Payments During Calendar year	Present Value at 8% as of January 1, 1980: Based on Annual Payments	Present Value at 8% as of January 1, 1980: Based on Quarterly Payments
1980	96.23%	95.36%
1981	89.10	89.32
1982	82.50	82.71
	89.66%	89.51%

SECTION VII

INCORPORATING LOSS DEVELOPMENT DATA FROM ODD EVALUATION DATES

This section provides an application of the techniques of the last section to a very common problem. For illustration, let us assume that we have incurred losses for accident years 1980–82 as of each year end and have just received the latest evaluation (June 30, 1983). How do we incorporate this information which doesn't fit in our standard triangle? Without a systematic approach, this is typically a frustrating situation.

Accident Year	Incurred Losses (000's) as of *X* Months of Development			
	12	24	36	48
1980	$24,132	$40,746	$55,109	$62,328*
1981	27,782	45,929	55,712*	
1982	26,368	36,704*		
1983	15,961*			

*as of June 30, 1983

Accident Year	Incurred Loss Development Factors		
	24:12	36:24	48:36
1980	1.689	1.352	1.131*
1981	1.653	1.213*	
1982	1.392*		

*6-months factors

In the above situation, usage of the June 30, 1983 data seems particularly important since it provides half of the known development factors. The first step is to determine what time interval serves as the least common denominator for the time lags between any two successive evaluations. In this case, t is six months, so we define it in terms of six-month intervals. We use the same techniques as described in the last section to break down the annual data into semiannual factors. It may then be compared with the actual semiannual factors from the first half of 1983.

	Incurred Loss Development Factors (Y:X Months)					
Source of Factors	12:6	18:12	24:18	30:24	36:30	42:36
Breakdown of Annual Experience		1.352	1.243	1.182	1.144	
First Half of 1983		1.392		1.213		1.129
Inverse Power Curve Fitted to All of the Above Factors	1.618	1.368	1.255	1.192	1.152	1.125

The inverse power curve factors can then be used to project each year's losses as of June 30, 1983 to 42 months of development as well as to extrapolate losses to ultimate. In the above approach, we have effectively used all of the loss history available to make projections.

SECTION VIII

ANALYZING LOSS DEVELOPMENT WHEN ALL EVALUATION DATES ARE ODD

In the following example, we will deal with the analysis of loss development when the evaluation dates are completely inconsistent. For accident years 1979–82, the only evaluation dates available are July 31, 1980, November 30, 1981, and April 30, 1982. Since the dates are 16 months and 5 months apart, the least common denominator is one month and we must break down the data into monthly factors. We will denote each data point as a two-dimensional vector, with the first coordinate being the age of the accident year at the given evaluation date, and the second being incurred losses.

Accident Year	(Months of Development, Incurred Losses (000's))		
1979		(19,2413)	(35,3952)	(40,4245)
1980	(7,450)	(23,3120)	(28,3660)	
1981	(11,1201)	(16,2134)		
1982	(4,123)			

Accident Year	(Months of Development, Development Factor)		
1979		(35:19, 1.643)	(40:35, 1.073)
1980	(23:7, 6.933)	(28:23, 1.172)	
1981	(16:11, 1.777)		

For each development period, we derive a first approximation of a monthly incurred loss development factor for a month in the middle of the period by taking the n^{th} root of the development factor, where n is the length of the interval in months.

Accident Year	(Months of Development, Development Factor)		
1979		(27.5:26.5, 1.032)	(38:37, 1.014)
1980	(15.5:14.5, 1.129)	(26:25, 1.032)	
1981	(14:13, 1.122)		

An inverse power curve is then fitted to all of the above points to estimate monthly development factors up to 40 months. The factors from this curve are then accumulated to produce approximations of the actual factors.

Accident Year	(Months of Development, Development Factor)		
1979		(35:19, 1.690)	(40:35, 1.073)
1980	(23:7, 10.753)	(28:23, 1.184)	
1981	(16:11, 1.965)		

In this first iteration, our approximations are all significantly too high and we adjust our estimated monthly factors by correction factors equal to the n^{th} root of the quotient of the actual factor to the approximated factor. For example, the approximation of (23:7, 6.936) is (23:7, 10.753), so the correction factor is the 16^{th} root of (6.936/10.753), or .973. Thus, the new monthly development factor is revised from (15.5:14.5, 1.129) to (15.5:14.5, 1.129 × .973). With all of these new monthly factors, we fit another inverse power curve and estimate an entire new set of monthly factors, which are then used to approximate the known factors. This iteration process is repeated until there is no further improvement in minimizing the sum of the squares of the differences between the approximated factors and the known factors. In this case, the final curve is $(1.0 + 31.010659\, t^{-2.109624})$ and the sum of the squares of the differences is less than .001. With a full set of monthly factors, losses as of 4, 16, 29 and 40 months can be projected to ultimate.

SECTION IX

ESTIMATING QUARTERLY ACCIDENT QUARTER DEVELOPMENT FACTORS FROM ANNUAL ACCIDENT YEAR FACTORS

It is sometimes desirable to estimate quarterly development factors for individual accident quarters, but the only data available is that of annual development factors for separate accident years. In this section we will illustrate a procedure for deriving such a refinement in loss development history.

If quarterly factors are not available for each accident year, then they must first be estimated as in Section VI. We shall use the third approximation factors from that section as the starting point for our analysis. For simplicity, we will assume that the incurred (or paid) losses as of one quarter of development are the same for all four accident quarters. If d_i represents the i^{th} development factor and q represents losses as of one quarter of development, then incurred losses by accident quarter and quarter of development are as follows:

Accident Quarter	Quarters of Development				
	1	2	3	4	5
1	q	qd_1	qd_1d_2	$qd_1d_2d_3$	$qd_1d_2d_3d_4$
2	q	qd_1	qd_1d_2	$qd_1d_2d_3$	$qd_1d_2d_3d_4$
3	q	qd_1	qd_1d_2	$qd_1d_2d_3$	$qd_1d_2d_3d_4$
4	q	qd_1	qd_1d_2	$qd_1d_2d_3$	$qd_1d_2d_3d_4$

From the above, we can derive equations for each of the quarterly factors for the accident year:

$$(q + qd_1)/q = 3.500$$
$$(q + qd_1 + qd_1d_2)/(q + qd_1) = 2.067$$
$$(q + qd_1 + qd_1d_2 + qd_1d_2d_3)/(q + qd_1 + qd_1d_2) = 1.585$$
$$\frac{(qd_1 + qd_1d_2 + qd_1d_2d_3 + qd_1d_2d_3d_4)}{(q + qd_1 + qd_1d_2 + qd_1d_2d_3)} = 1.366$$

These equations can be solved successively to produce a first set of approximations of the quarterly accident quarter factors:

$$d_1 = 2.500 \qquad d_4 = 1.228$$
$$d_2 = 1.494 \qquad d_5 = 1.237$$
$$d_3 = 1.133 \qquad d_6 = 1.132$$

While these first approximations do not progress downward in a smooth fashion, an inverse power curve may be fitted to these approximations to add consistency. This second set of factors should be tested in relation to how closely they can reproduce the original accident year factors.

SECTION X

A SIMPLE, ALTERNATIVE METHOD FOR ESTIMATING DEVELOPMENT BEYOND THE MOST MATURE EXPERIENCE AVAILABLE

Because of the nature of the inverse power curve, it cannot be fitted to development factors less than 1.0, since this would involve taking the natural logarithm of a negative number. If development is generally upward, but there is an occasional factor less than 1.0, such factors can be removed by smoothing techniques (such as replacing d_i by $(d_{i-1}d_id_{i+1})^{1/3}$ or $(d_{i-2}d_{i-1}d_id_{i+1}d_{i+2})^{1/5}$). If incurred losses generally develop downward in some segment of the loss triangle, then an alternative method of extrapolation of losses is needed. Such a method is presented in this section. It is based on noting relationships between paid losses during a given development period (for a given accident or report period) and the change in outstanding losses during that same period.

It will be helpful to first present some mathematical notation. Loss payments during the i^{th} period of development will be denoted by P_i, and outstanding losses at the end of the i^{th} period of development by O_i. Incurred losses at the end of the t^{th} period of development are then equal to $O_t + \Sigma_{i=1}^{t} P_i$.

At the end of the t^{th} period of development, the ultimate value of unpaid losses is $\Sigma_{i=t+1}^{\infty} P_i$. We wish to find some equivalent expression for this in terms of O_t. Suppose that, after some stage of development, there is a constant relationship between P_i and $(O_{i-1}-O_i)$. That is, $P_i = \alpha(O_{i-1}-O_i)$. Then

$$\sum_{i=t+1}^{\infty} P_i = \sum_{i=t+1}^{\infty} \alpha(O_{i-1} - O_i) = \alpha \sum_{i=t+1}^{\infty} (O_{i-1} - O_i) = \alpha O_t$$

since O_t decreases to zero as t increases. If we can determine the value of α, the runoff ratio, then we have a quick estimate of the ultimate value of unpaid losses (αO_t), where O_t is the latest evaluation of outstanding losses. Estimating α is easy since we can obtain estimates of it for every development period and accident or report period:

$$\alpha = P_i/(O_{i-1}-O_i)$$

Suppose that we find that for the more mature periods of development that paid losses are generally 80% of the decline in outstanding losses. Then, assuming that the runoff ratio (α) is constant for all future periods of development, the ultimate value of unpaid losses is simply 80% of the latest value of outstanding losses.

Exhibits 8 through 10 present this application of the method to automobile liability data. With the consistent pattern of downward development of incurred losses shown in Exhibit 8, there is a need to anticipate further favorable development for accident year 1975. Exhibit 9 displays the calculation of runoff ratios for accident year 1975 while Exhibit 10 displays all available runoff ratios. A runoff ratio of 60% was selected on the basis of Exhibit 10, and application of this ratio to the latest outstanding losses for Accident Year 1975 produced an estimate ($3,919,000) of the ultimate value of outstanding losses. This estimate is equivalent to an incurred loss development factor to ultimate of .975, which has been applied in Exhibit 8.

Exhibit 11 displays runoff ratios for a company with severely deficient reserves. It should be noted that the runoff ratios never stabilize and continue to increase with age. In this case, application of some of the higher runoff ratios may only provide a lower bound for an estimate of ultimate losses.

Once the runoff ratios stabilize for all development periods beyond a certain point, the ultimate value of outstanding losses may be estimated by αO_t for each of the accident or report years which have reached that stage of maturity.

CONCLUDING REMARKS

It is hoped that the research and practical applications presented in this paper can serve as a foundation from which others can make further advancements in the field of loss development analysis.

EXHIBIT 1

EXTRAPOLATION OF INCURRED LOSS DEVELOPMENT FACTORS USING AN INVERSE POWER FUNCTION AUTOMOBILE BODILY INJURY LIABLITY—ACCIDENT YEAR 1978

Age (t)	(1) $1/t$	(2) Incurred Loss Development Factor − 1.0	(3) ln $(1/t)$	(4) ln (Development Factor − 1.0)
2	.500	0.578	−0.693	−0.548
3	.333	0.097	−1.100	−2.333
4	.250	0.032	−1.386	−3.442
		Extrapolated Estimates		
5	.200	0.0122	−1.609	−4.410
6	.167	0.0057	−1.792	−5.176
7	.143	0.0030	−1.946	−5.822
8	.125	0.0017	−2.079	−6.379
9	.111	0.0010	−2.197	−6.873
10	.100	0.0007	−2.303	−7.318
11	.091	0.0004	−2.398	−7.715
12	.083	0.0003	−2.485	−8.080
13	.077	0.0002	−2.565	−8.415
14	.071	0.0002	−2.639	−8.726
15	.067	0.0001	−2.708	−9.015

Notes

(1) The least squares regression was performed on the data for ages 2, 3, and 4, as shown above, which has been rounded to three places.

(2) The extrapolated estimates were derived from the least squares trend line ($y = a + bx$), with $a = 2.33259$ and $b = 4.19024$.

EXHIBIT 2

COMPARISON OF ACTUAL AND FITTED INCURRED LOSS DEVELOPMENT FACTORS USING AN INVERSE POWER FUNCTION

Years of Development	Auto Bodily Injury Liability		General Liability		Workers' Compensation	
	Actual	Fitted	Actual	Fitted	Actual	Fitted
2	1.634	1.680	1.839	1.886	1.493	1.490
3	1.094	1.077	1.279	1.266	1.167	1.159
4	1.025	1.022	1.185	1.132	1.094	1.082
5	1.008	1.009	1.077	1.080	1.046	1.052
6	1.003	1.004	1.039	1.054	1.033	1.036
7	1.003	1.002	1.033	1.040	1.028	1.027
8	1.001	1.002	1.029	1.030	1.019	1.021
9	1.000	1.001	1.030	1.024	1.012	1.017
10	1.001	1.001	1.019	1.020	1.010	1.014
11	—	—	1.014	1.016	1.011	1.012
12	—	—	1.016	1.014	1.010	1.010
13	—	—	1.013	1.012	1.009	1.009
14	—	—	1.012	1.010	1.008	1.008
15	—	—	1.008	1.009	1.007	1.007
Goodness of Fit (R^2)	.98462		.98278		.98551	
Parameters						
a =	.68047		.88614		.48984	
b =	3.14215		1.73380		1.62362	
c =	−1.00000		−1.00000		−1.00000	

Notes

(1) The actual factors above represent composite experience from five major carriers for each line of business.

(2) The goodness of fit is measured by the coefficient of determination (R^2).

EXHIBIT 3

EXTRAPOLATION OF PAID LOSS DEVELOPMENT FACTORS USING AN INVERSE POWER FUNCTION AUTOMOBILE BODILY INJURY LIABILITY—ACCIDENT YEAR 1978

Age (t)	(1) $1/t$	(2) Paid Loss Development Factor − 1.0	(3) ln ($1/t$)	(4) ln (Development Factor − 1.0)
2	.500	1.801	−0.693	+0.588
3	.333	0.388	−1.100	−0.947
4	.250	0.134	−1.386	−2.010
		Extrapolated Estimates		
5	.200	0.0578	−1.609	−2.850
6	.167	0.0291	−1.792	−3.536
7	.143	0.0163	−1.946	−4.114
8	.125	0.0099	−2.079	−4.613
9	.111	0.0064	−2.197	−5.055
10	.100	0.0043	−2.303	−5.453
11	.091	0.0030	−2.398	−5.809
12	.083	0.0022	−2.485	−6.135
13	.077	0.0016	−2.565	−6.435
14	.071	0.0012	−2.639	−6.713
15	.067	0.0009	−2.708	−6.972

Note

The extrapolated estimates were derived from the least squares trend line ($y = a + bx$), with $a = 3.18478$ and $b = 3.75038$.

EXHIBIT 4

COMPARISON OF ACTUAL AND FITTED INCURRED LOSS DEVELOPMENT FACTORS REINSURANCE ASSOCIATION OF AMERICA EXPERIENCE

Years of Development	Automobile Liability		General Liability		Medical Malpractice		Workers' Compensation	
	Actual*	Fitted	Actual*	Fitted	Actual*	Fitted	Actual*	Fitted
2:1	1.760	1.619	2.300	2.290	7.876	6.104	1.634	1.630
3:2	1.227	1.264	1.541	1.536	2.172	2.480	1.285	1.287
4:3	1.100	1.123	1.295	1.287	1.654	1.717	1.169	1.172
5:4	1.061	1.062	1.171	1.177	1.334	1.429	1.134	1.118
6:5	1.031	1.033	1.109	1.119	1.150	1.288	1.092	1.088
7:6	1.015	1.018	1.093	1.085	1.156	1.208	1.053	1.068
8:7	1.015	1.011	1.060	1.064	1.163	1.158	1.055	1.055
9:8	1.008	1.007	1.046	1.050	1.120	1.124	1.048	1.046
10:9	1.006	1.004	1.045	1.039	1.133	1.101	1.039	1.039
11:10	1.000	1.003	1.039	1.032	1.023	1.084	1.036	1.034
12:11	1.001	1.002	1.022	1.027	1.058	1.070	1.014	1.029
13:12	1.001	1.001	1.024	1.022	1.090	1.060	1.017	1.026
14:13	1.001	1.001	1.004	1.019	1.063	1.052	1.030	1.023
15:14	1.000	1.001	1.019	1.016	1.089	1.046	1.023	1.021
16:15	1.000	1.000	1.008	1.014		1.040	1.016	1.019
17:16	1.001	1.000	1.010	1.012		1.036	1.032	1.017
18:17	.999	1.000	1.008	1.011		1.032	1.005	1.016
19:18	1.000	1.000	1.018	1.010		1.029	1.021	1.015
20:19	1.000	1.000	1.004	1.009		1.027	1.015	1.014
21:20	.999	1.000	1.005	1.008		1.024	1.037	1.013
22:21	1.000	1.000	1.017	1.007		1.022	.996	1.012
23:22	1.000	1.000	1.000	1.006		1.020	1.038	1.011
24:23	1.000	1.000	.997	1.006		1.019	1.026	1.010
25:24	1.000	1.000	1.000	1.005		1.017	1.018	1.010

* These factors are the average of the latest 10 accident years for each given year of development from the 1983 edition of the RAA's *Loss Development Study*.

EXHIBIT 5

Comparison of Actual and Fitted Paid Loss Development Factors
Two-Dimensional Inverse Power Functions (Workers' Compensation)

Accident Year	Paid Loss Development Factors											
	2:1		3:2		4:3		5:4		6:5		7:6	
	Actual	Fitted	Actual	Fitted	Actual	Fitted	Actual	Fitted	Actual	Fitted	Actual	Fitted
1955	1.832	1.874	1.160	1.169	1.065	1.064	1.032	1.033	1.017	1.019	1.013	1.012
1956	1.807	1.875	1.167	1.172	1.064	1.066	1.042	1.034	1.024	1.020	1.017	1.013
1957	1.869	1.876	1.161	1.176	1.067	1.069	1.033	1.035	1.025	1.021	1.017	1.014
1958	1.863	1.877	1.182	1.179	1.079	1.071	1.039	1.037	1.023	1.022	1.016	1.014
1959	1.852	1.878	1.178	1.183	1.075	1.073	1.035	1.038	1.023	1.023	1.015	1.015
1960	1.897	1.879	1.181	1.187	1.073	1.075	1.037	1.040	1.024	1.024	1.018	1.016
1961	1.884	1.880	1.189	1.191	1.079	1.078	1.047	1.041	1.024	1.025	1.016	1.017
1962	1.871	1.881	1.201	1.195	1.073	1.080	1.045	1.043	1.029	1.026	1.022	1.018
1963	1.934	1.882	1.206	1.199	1.088	1.083	1.042	1.045	1.028	1.028	1.022	1.019
1964	1.827	1.883	1.198	1.203	1.074	1.086	1.045	1.047	1.028	1.029	1.019	1.020
1965	1.856	1.884	1.212	1.207	1.086	1.088	1.044	1.048	1.023	1.030	1.016	1.021
1966	1.893	1.885	1.213	1.211	1.090	1.091	1.050	1.050	1.032	1.032	1.023	1.022
1967	1.858	1.886	1.215	1.216	1.097	1.094	1.050	1.052	1.034	1.033	1.024	1.023
1968	1.879	1.887	1.229	1.220	1.100	1.097	1.060	1.055	1.035	1.035	1.027	1.024
1969	1.920	1.887	1.228	1.224	1.098	1.100	1.051	1.057	1.036	1.036	1.025	1.025
1970	1.870	1.888	1.219	1.229	1.091	1.104	1.055	1.059	1.036	1.038	1.029	1.027
1971	1.813	1.889	1.221	1.234	1.093	1.107	1.056	1.061	1.040	1.040	1.028	1.028
1972	1.906	1.890	1.240	1.239	1.110	1.110	1.062	1.064	1.042	1.042	1.035	1.030
1973	1.967	1.891	1.249	1.244	1.123	1.114	1.071	1.067	1.047	1.044	1.033	1.031
1974	1.926	1.892	1.253	1.249	1.117	1.118	1.073	1.069	1.048	1.046	1.034	1.033
1975	2.027	1.893	1.269	1.254	1.130	1.122	1.076	1.072	1.058	1.048		
1976	1.923	1.894	1.260	1.259	1.125	1.126	1.071	1.075				
1977	1.892	1.895	1.242	1.264	1.124	1.129						
1978	1.892	1.896	1.248	1.270								
1979	1.903	1.897										

EXHIBIT 5
(Continued)

Comparison of Actual and Fitted Paid Loss Development Factors
Two-Dimensional Inverse Power Function (Workers' Compensation)

Accident Year	Paid Loss Development Factors									
	8:7		9:8		10:9		11:10		12:11	
	Actual	Fitted	Actual	Fitted	Actual	Fitted	Actual	Fitted	Actual	Fitted
1955	1.010	1.009	1.005	1.006	1.004	1.005	1.003	1.004	1.002	1.003
1956	1.009	1.009	1.008	1.007	1.005	1.005	1.006	1.004	1.003	1.003
1957	1.011	1.010	1.006	1.007	1.004	1.005	1.005	1.004	1.003	1.003
1958	1.010	1.010	1.006	1.007	1.005	1.006	1.006	1.004	1.004	1.004
1959	1.010	1.011	1.007	1.008	1.006	1.006	1.004	1.005	1.004	1.004
1960	1.012	1.011	1.009	1.008	1.007	1.006	1.006	1.005	1.004	1.004
1961	1.015	1.012	1.008	1.009	1.006	1.007	1.006	1.005	1.005	1.004
1962	1.014	1.013	1.009	1.009	1.008	1.007	1.005	1.006	1.006	1.005
1963	1.013	1.013	1.010	1.010	1.009	1.008	1.008	1.006	1.007	1.005
1964	1.015	1.014	1.008	1.011	1.004	1.008	1.005	1.007	1.004	1.005
1965	1.013	1.015	1.010	1.011	1.008	1.009	1.005	1.007	1.006	1.006
1966	1.015	1.016	1.011	1.012	1.010	1.009	1.010	1.008	1.007	1.006
1967	1.016	1.017	1.015	1.013	1.010	1.010	1.009	1.008	1.008	1.007
1968	1.020	1.018	1.013	1.014	1.013	1.011	1.012	1.009	1.009	1.007
1969	1.019	1.019	1.014	1.014	1.011	1.011	1.009	1.009	1.008	1.008
1970	1.018	1.020	1.015	1.015	1.011	1.012	1.009	1.010		
1971	1.025	1.021	1.019	1.016	1.013	1.013				
1972	1.022	1.022	1.019	1.017						
1973	1.025	1.023								

EXHIBIT 6

Test of Bias: Signs of Errors
Fit of Two-Dimensional Inverse Power Function to Workers' Compensation Paid Loss Development Factors

	Years of Development										
	2:1	3:2	4:3	5:4	6:5	7:6	8:7	9:8	10:9	11:10	12:11
1955	+	+	−	+	+	−	−	+	+	+	+
1956	+	+	+	−	−	−	0	−	0	−	0
1957	+	+	+	+	−	−	−	+	+	−	0
1958	+	−	−	−	−	−	0	+	+	−	0
1959	+	+	−	+	0	0	+	+	0	+	0
1960	−	+	+	+	0	−	−	−	−	−	0
1961	−	+	−	−	+	+	−	+	+	−	−
1962	+	−	+	−	−	−	−	0	−	+	−
1963	−	−	−	+	0	−	0	0	−	−	−
1964	+	+	+	+	+	+	−	+	+	+	+
1965	+	−	+	+	+	+	+	+	+	+	0
1966	−	−	+	0	0	−	+	+	−	−	−
1967	+	+	−	+	−	−	+	−	0	−	−
1968	+	−	−	−	0	−	−	+	−	−	−
1969	−	−	+	+	0	0	0	0	0	0	0
1970	+	+	+	+	+	−	+	0	+	+	
1971	+	+	+	+	0	0	−	−	0		
1972	−	−	0	+	0	−	0	−			
1973	−	−	−	−	−	−	−				
1974	−	−	+	−	−	−					
1975	−	−	−	−	−						
1976	−	−	+	+							
1977	+	+	+								
1978	+	+									
1979	−										

EXHIBIT 7

ESTIMATION OF SELECTED DEVELOPMENT FACTORS USING THE INVERSE POWER CURVE

Workers' Compensation

Accident Year	Incurred Loss Development Factors				
	2:1	3:2	4:3	5:4	6:5
1976	1.896	1.192	1.030	.989	1.238
1977	1.714	1.271	1.101	1.161	
1978	1.642	1.302	1.065		
1979	1.502	1.233			
1980	1.589				
Average Factor	1.669	1.250	1.065	1.075	1.238
Fitted Curve—First 2 Factors ($ILDF = 1.0 + 3.584t^{-2.442}$)	1.669	1.250	1.125		
Weight for Average Factor			3/12		
Weighted Factor (3/12 × 1.065 + 9/12 × 1.125)			1.110		
Fitted Curve—First 2 Average Factors and Weighted 4:3 Factor ($ILDF = 1.0 + 4.117t^{-2.582}$)	1.683	1.238	1.113	1.063	
Weight for Average Factor				2/14	
Weighted Factor (2/14 × 1.075 + 12/14 × 1.063)				1.065	
Fitted Curve—First 2 Average Factors and Weighted 4:3 and 5:4 Factors ($ILDF = 1.0 + 4.040t^{-2.572}$)	1.680	1.239	1.114	1.064	1.041
Weight for Average Factor					1/15
Weighted Factor (1/15 × 1.238 + 14/15 × 1.041)					1.054
Selected Factors	1.669	1.250	1.110	1.065	1.054

EXHIBIT 8

Development of Accident Year Incurred Losses

Automobile Liability

Accident Year	Incurred Losses (000's) As of *X* Years of Development: 1	2	3	4	5	6	Factor To Ultimate	Projected Ultimate Incurred Losses
1975	121,943	116,946	113,249	110,057	106,055	103,343	.975	100,759
1976	129,645	125,138	121,514	115,652	111,277		.950	105,713
1977	146,500	139,283	131,289	124,856			.915	114,243
1978	157,940	148,253	140,551				.876	123,123
1979	158,590	153,068					.839	128,424
1980	168,432						.802	135,082

Accident Year	Incurred Loss Development Factors: 2:1	3:2	4:3	5:4	6:5	
1975	.959	.968	.972	.964	.974	
1976	.965	.971	.952	.962		
1977	.951	.943	.951			
1978	.939	.948				
1979	.965					
Average Factor	.956	.958	.958	.963	.974	.975

EXHIBIT 9

ESTIMATION OF RUNOFF RATIOS

Automobile Liability
Accident Year 1975

Evaluation As of December 31,	(1) Incurred Losses	(2) Cumulative Paid Losses	(3) Unpaid Losses (1)−(2)	(4) Change in Paid Losses	(5) Change in Unpaid Losses	(6) Runoff Ratio (4)/(5)
1975	$121,943	$36,710	$85,233			
1976	116,946	60,839	56,107	+$24,129	−$29,126	82.8%
1977	113,249	74,393	38,856	+ 13,554	− 17,251	78.6
1978	110,057	85,877	24,180	+ 11,484	− 14,676	78.3
1979	106,055	92,707	13,348	+ 6,830	− 10,832	63.1
1980	103,343	96,840	6,503	+ 4,133	− 6,845	60.4

Note
Amounts in columns 1 through 5 are in thousands of dollars.

EXHIBIT 10

RUNOFF RATIOS

Automobile Liability

Accident Year	Runoff Ratio During X Year of Development				
	2	3	4	5	6
1975	82.8%	78.6%	78.3%	63.1%	60.4%
1976	85.0	80.9	67.1	64.2	
1977	80.0	69.1	67.8		
1978	77.0	72.3			
1979	86.2				

EXHIBIT 11

ESTIMATION OF RUNOFF RATIOS

Example Company Nearing Receivership
Accident Year 1973

Evaluation As of December 31,	(1) Incurred Losses	(2) Cumulative Paid Losses	(3) Unpaid Losses (1)—(2)	(4) Change in Paid Losses	(5) Change in Unpaid Losses	(6) Runoff Ratio (4)/(5)
1973	$10,458	$ 2,987	$7,471			
1974	14,294	8,896	5,398	+$5,909	−$2,073	285.0%
1975	15,857	13,329	2,529	+ 4,433	− 2,870	154.5
1976	17,160	15,672	1,488	+ 2,343	− 1,040	225.3
1977	18,287	17,630	657	+ 1,958	− 831	235.6
1978	19,675	19,202	473	+ 1,572	− 184	854.3

Note

Amounts in columns 1 through 5 are in thousands of dollars.

KEYNOTE ADDRESS—NOVEMBER 12, 1984
REGULATION AND DEREGULATION

DR. ALFRED E. KAHN

Thank you very much for inviting me here. I presume you did so because of my accomplishments in deregulating the airlines—it can't have been because of my success in controlling inflation. But in the case of airline deregulation, one of my proudest moments was when I was introduced by the president of the American Economic Association. "This is Fred Kahn," he said, "Delta Airlines was not ready when he was."

Anyhow, I'd like to begin by pointing out to you that I am not a deregulatory nut. For example, I believe in government regulation to prevent false advertising. And I'm here under false advertising. The topic of my talk, I read, is "Regulation and Deregulation," with particular reference to the casualty insurance business. I know very little about the casualty insurance business. And while I will make some effort to draw parallels and welcome your questions, I don't promise to have any answers.

I'll try to show some of the relationships that may be relevant to the insurance industry, but my principal intention is to talk about the case for deregulation and try to show you some of the uniformities of the experience with regulation among such industries as airlines, trucking, railroads, communications, stock exchange brokers, and financial markets.

I just want to say one other thing at the outset: I am going to confine my attention, as these examples show, to strictly economic regulation. That is, where the government intervenes to decide who may enter an industry, what that person may sell, what the prices shall be, and so on. I am not going to talk about regulation to ensure nondiscrimination in employment, to ensure the rights of collective bargaining, safety in the workplace, or safety of consumer products or of drugs, or protection of the environment. (Incidentally, however, in many of these cases, as well, a perfectly respectable case can be made that we should be relying primarily on the competitive market and not entirely on regulation. That is to say, even in the case of airline safety, our principal protection surely is the fact that an airline can't have very many accidents before it goes out of business; therefore, you can rely very heavily on the self-interest of airlines to ensure safety. As I say, however, I've never been a total deregulatory nut. I don't believe in "Let the buyer beware," so far as airline safety is concerned.)

In any event, I am going to confine my attention to economic regulation. And even when we look at this kind of regulation, we have to recognize that it has been introduced in a variety of contexts, in a variety of times, and for a variety of reasons.

For example, my first major job was Chairman of the New York Public Service Commission, where I regulated very intensely the local generation, transmission, and distribution of electricity; telephone service; gas distribution; and provision of water. And essentially there, the logic of regulation was that these services were in the nature of a natural monopoly. It's not efficient to have more than one company stringing telephone wires down the street or laying gas mains in the streets. Therefore, direct government regulation is needed to protect the public against the unwanted effects of monopoly and to insure the benefits of competition.

In other circumstances, we've introduced regulation rather out of a general belief that uncontrolled competition is destructive of the quality of service, of its continuity, reliability, indeed, even of its safety.

For example, I believe in deregulation of taxis, for reasons that I would be glad to discuss, but at the same time, I have to recognize that there is a benefit of regulation to the unsuspecting public: when they get into a cab there is a name and a number of the driver there, and if the driver cheats them in any way, they can complain and there is the sanction that the license will be taken away. I remember reading a year or two ago about some visitors from Pakistan who were charged $350 for a cab ride from Kennedy Airport into downtown Manhattan. So, partly, the expectation is that with regulation you may get more continuous service, regularly scheduled service and safer service.

Finally, we've often had as a motive the desire to ensure universality of service. This motive is illustrated clearly in the telephone case, where regulation is introduced with the explicit intention of cross subsidization. That is to say, we overcharge interstate calling by as much as 60 percent (as of about two years ago) and the proceeds of that overcharge flow back to the states to hold down the basic monthly charge for telephone service. In this case, there is an explicit intention to have cross subsidization, in order to have service available to everybody.

You'll recognize the same argument, for example, in trucking, where the belief was widespread that we had to restrict entry into the lucrative markets, in order to enable the carriers there to obtain monopoly profits. And then we could tell those carriers to use those profits to finance continued operations in small towns, where they otherwise might not be able to make any money. As in communications, regulation kept competition out of the lucrative markets, purportedly in order to generate the requisite monopoly revenues.

Historically, AT&T's monopoly in long distance enabled it to charge the 60 percent markup and pass it on to hold down the basic monthly charge. One of the major problems in communications today is that we have introduced competition into the long distance business; the MCI's and the SPRINTs and the Satellite Business Systems are undercutting those long distance prices; and now we have to find some other way of responding to the drying up of that historic source of subsidy.

Sometimes the universality of service argument is genuine, but sometimes it is not. In the case of trucking, again, we discovered that the so-called "subsidy of service to small towns" was an absolute fraud. We asked the Interstate Commerce Commission if they knew who was serving small towns. They did not know: they had no record of who was serving individual towns. We then asked whether the ICC had ever denied an application to terminate service to a small town. The answer was that never in the ICC's history had it prevented anybody from dropping service to a small town. In those circumstances, the notion that truckers are being unwillingly forced to serve small towns as a price of having exclusive licenses to profitable routes falls down completely. Finally, the Department of Transportation selected a series of small towns to see who was serving them. It turned out they were not being served by the big truckers who were earning big returns on other routes. On the contrary, their service was principally by exempt carriers (that is to say, carriers of agricultural commodities, who were not regulated at all), by United Parcel Service, and about 10 percent on average by certified common carriers who specialized in those markets, presumably because they found it profitable. So in that case, the idea that the monopoly profits were supporting service to small towns was an absolute fraud.

In any event, though the motives have differed, the common and—so far as I know—universal characteristic of economic regulation is that it succeeds by suppressing competition, by licensing entry, in contrast with what is supposed to happen in a free enterprise system. Under that kind of regime, it wasn't enough if some investors thought they saw a market opportunity, raised the necessary funds, and undertook to enter the market at their own risk. No, they had to apply for a license. And typically those licensing procedures were subject to protest. By whom? By the people who already were in the market. And typically, the regulatory commission has a strong mandate to protect the financial soundness of the companies that are already there.

I wish I could describe to you the absurdity of those proceedings at the Civil Aeronautics Board. The applicants would hire expert economic witnesses to testify that it would be profitable for them to enter a particular market. It wasn't enough that *they* thought it was profitable. They had to convince *me*. The parade of expert witnesses on the one side would solemnly present estimates of expected revenues—very high; of costs—very low; the profits—enormous. Then the lawyers would come in from the airlines with whom the new entrants proposed to compete, accompanying *their* economic experts who would testify that the revenues would be negligible, the costs infinite, and the returns negative. (It was a great thing for economic experts!)

Moreover, the applicants had to prove that their proposed entry would not be injurious to the existing airlines. So the applicants would hire their experts who would testify that all the traffic they would get would be newly generated: 97 year old grandmothers who had never flown before would rush to their planes. And how much would they take from other carriers? Zero. But then, of course, the existing carriers would bring in their experts to testify that the applicant would generate no new traffic, that every passenger they got they would have stolen from the existing carriers.

The administrative law judge presumably would retire to his chambers with all this testimony and he would consult his navel. The outcome? In its 40-year history, the Civil Aeronautics Board never once certified a single new competitive trunk carrier.

So, the regulatory agencies operated by imposing restrictions on entry, in the interest of protecting the companies already there. They also systematically prohibited price competition. When the regulators gave licenses, they attached all sorts of strict limitations in order to keep companies from getting into each other's markets. For example:

> One trucker might haul logs cut crosswise but not lengthwise; another, logs cut lengthwise but not crosswise;
>
> One would be licensed to carry paint in 5-gallon cans but not 2-gallon cans; another in 2-gallon cans but not 5-gallon cans;
>
> One might carry pineapples mixed with bananas, and bananas alone, but not pineapples alone;
>
> Or a carrier might be authorized to truck steel I-beams from Albuquerque to Salt Lake City, but only by way of El Paso—and was prohibited from picking up anything in El Paso.

Those are real examples, and they are, I assure you, typical. I found myself often paraphrasing Bismarck, "Those who love sausages and laws should never see them made." Well, the same thing's true of regulation.

I'm going to give you just three more general, objective evidences of the validity of my contention, that the way in which these regulations proceeded was systematically to protect people from competition.

Number one, the certificates came to have value. The reason is of course that they were scarce: the demand exceeding the supply. A good example is the taxicab business in New York City, where the number of cabs is restricted to some 11,000, a total that has not changed for decades. A new entrant can pay $15,000 or $20,000 for a cab, but then must pay over $80,000 for the medallion that gives him the right to be in the cab business.

The practices of exempt (from regulation) carriers of agricultural commodities provide another good illustration. If such a trucker carried a truckload of oranges from Florida up to Boston, he could not load his empty truck with non-agricultural commodities for the return trip if he didn't have a license. Since business quite properly abhors unused capacity, these truckers shopped around for people who had the license and borrowed it for the return trip. A whole market developed in those licenses, and a typical price for that loan was 25 to 30 percent of gross revenues. That is a demonstration of the benefit to the people in the industry of having entry restricted.

Number two. A colleague of mine studied all the cases before the Interstate Commerce Commission for one whole year involving pricing of trucking services. You might think that the authentic purpose of regulation is to protect consumers. Protecting consumers means you set a ceiling on prices. He found that 95 percent of the cases involve not price ceilings, but price floors. That demonstrates that the purpose was not to protect consumers, at least not directly. It was to restrict price competition.

Number three. In each case of attempted deregulation, ask yourselves who was in favor of the regulation and who was opposed. The people who were opposed to my efforts to deregulate the airlines were the airlines and the pilots; the people who were opposed to our deregulation of trucking were the regulated truckers and the Teamsters, who enjoy pay on the order of 50 to 70 percent above comparable pay of people with comparable skills in unregulated industries. (The truckers on the outside, hoping to get into the business, were not opposed. Among our strong allies in favor of deregulating the trucking were the National Federation of Independent Businesses; and the National Federation of Minority Businesses, because deregulation offered a way in which minority people with relative little capital could hope to get into business, while regulation systematically prevented them from doing so.) In the airlines' case, United was the only airline that was interested in deregulation; that's because they never got a route. They always were being subjected to tight constraints, and they thought they could do better in a competitive market.

I suggest that those are three principal, and sufficient, demonstrations of the way in which regulation actually operates.

Now I don't list all these insanities simply in order to poke cheap fun at systems that gave us pretty good service; or to second-guess the case for imposing those regulations at the time when they were imposed. It's not surprising that four of the five major regulatory regimes we moved to dismantle in the Carter Administration (some of these efforts started under President Ford)—airlines, trucking, communications, financial markets, and railroads—were begun during the Great Depression of the '30s—in the Civil Aeronautics Act of 1938, the Motor Carrier Act of 1935, the Federal Communications Act of 1934, and the Banking Acts of 1933 and 1935.

Consider trucking as an extreme case. There is no question that there was real chaos in that industry during the Great Depression. With 25 percent of our workforce unemployed and with used trucks that could be picked up for virtually nothing, unemployed individuals were entering the trucking business and driving 12 to 14 hours a night. Schedules were unreliable, financial responsibility often nil. It could well be argued that competition was destructive at that time.

The fact remains that over the decades in the 1960s and 1970s, particularly, there developed a very widespread consensus among academic and other disinterested people that regulation, most obviously in transportation, had suppressed innovation, denied the public the benefits of competition, and fostered a wage-price spiral. Teamsters' wages, for example, increased something like 60 percent more rapidly than wages in industry generally during the 1970s.

The same thing happened in automobiles and steel where, again, the companies and unions were inadequately disciplined by price competition. No wonder these industries want protection against imports. The quotas on Japanese cars make it possible then to pay management multimillion dollar bonuses and to have average automobile workers' wages and fringes worth $23 an hour (that's $46,000 a year).

By suppressing price competition, regulation also encouraged wasteful competition in ways that inflated costs. The airlines, for example, prohibited from competing in price, competed by giving more sumptuous meals. So then, of course, in the international field, they regulated the size of the sandwiches. They competed by offering free in-flight entertainment. So in the international field, the government said, "You've got to get together and charge for those headsets." When the airlines say, "We're sorry, but international regulations require us to charge 3 bucks for the headset," do you really think they're sorry?

The airlines also competed, as I understand you do, by paying big commissions to travel agents. In the North Atlantic, for example, there were times when the airlines were giving commissions of 25 to 30 percent of the ticket price, in order to get the travel agents to shift people to their flights. I just read in October's *Fortune* magazine that the big airlines are paying 4 to 6 percent over and above the 10 percent commission right now to travel agents to steer people to their flights. That's a wasteful form of competition from the point of view of the public. It doesn't give the public the benefit. It gives it to a group of people who are in a position to direct traffic.

The airlines competed also by denser scheduling. In the absence of price competition, a carrier could break even on cross-country flights if the planes were only 35 percent full. So the airlines competed by scheduling more densely until, in fact, planes were flying across the country on average about 35 percent full. So regulation encouraged very wasteful and inefficient kinds of competition.

This kind of comprehensive regulation, which was naturally expanded wherever feasible to limit all these other forms of competition as well—travel agents' commissions and internationally the frequency of flights—promotes a very neat, tidy, stable environment. It should not be surprising, therefore, that once all these restraints were removed, there has been turmoil. For example, I just learned from the Air Traffic Transport Association that the average pay of pilots, with fringes, of the 20 leading airlines, is $111,000 a year. Meanwhile, the new airlines are paying pilots in the $25,000 to $35,000 range, and getting all they need coming out of the military.

So we have new airlines coming in, most of them nonunion, with far lower costs than the incumbents, cutting prices sharply. We have established companies not knowing how to exercise their pricing freedom, getting into and out of price wars. We have carriers exercising their newly-conferred freedom to enter new markets and pull out of others: within one year, for example, United Airlines pulled out of about 22 percent of its cities; within the next two years, it went back into half of them—it found that it made a mistake, because it wasn't able to feed itself traffic.

So no wonder it's been a mess in this period in all these deregulated industries; no wonder there has been a great deal of pain and turmoil. But behind the turmoil we see the social consequences of deregulation—preponderantly the beneficial consequences that we intended to achieve.

Let me summarize them briefly; in the process, I'll try to identify which are similar to your situation and which may be different.

Number one. There has, of course, been a striking deconcentration of markets. In airlines, the number of markets served by four or more competitors has increased very, very sharply. In the case of trucking, there are thousands of new carriers in the market. Moreover, existing truckers no longer have the former, insane restrictions on what they may carry or where they may go, and, therefore, the existing truckers, previously elaborately insulated from one another, have in effect all been thrown into competition with one another.

In consequence, there is an enormous increase in the intensity of competition of all kinds. Consider communications, for example. All you have to do is read the newspaper. Almost every day there is an announcement of some new venture. It used to be that we had a single monopoly, AT&T or the local telephone company, for voice communications. That market was separated from record (written) communications, telegrams, which were monopolized by Western Union. Western Union couldn't go into the telephone business; telephone companies couldn't go into Western Union's business. Similarly, we had broadcast and narrowcast communications defined so that they couldn't cross each other's boundaries. For a while, AT&T couldn't go into the satellite business, which was distinguished from terrestrial communications. Video—that's the cable TV companies—was separated from audio. Telephone companies were not permitted to go into the cable TV business, or computing; or data processing, storage, retrieval and communications. IBM had to stay here; AT&T had to stay there.

Technologically, those distinctions are totally meaningless today. And today, instead, everybody's pouring into all these fields. AT&T is into computers. Citicorp has put up its own communications system directly via satellite, totally bypassing the telephone company. Cable television companies are setting up combinations with MCI to provide end-to-end long-distance communications service. The coaxial cable can handle not only video, but also massive quantities of data. In New York City, Manhattan Cable has been handling all the local data communications business of Citicorp. Merrill Lynch and Western Union are laying fiber optic cable right down Wall Street to pick up all the data transmission business of the investment, banking and brokerage houses, totally circumventing New York Telephone. They will simply pipe all those messages out to a teleport on Staten Island, where, in conjunction with the New York Port Authority, they'll send it up to a satellite and then down to the receiving stations at the other end. Real estate companies have gone into the telecommunications business. If you watch the ads, you'll see announcements by Olympia & York, the biggest real estate developer in New York's metropolitan area, that it's setting up office projects in which it in effect provides telephone service, remote access data processing service, and teleconferencing facilities. They really are a diversified communications company.

So, to reiterate effect number one, there has been an enormous increase in competitive entry and intensity of competition.

Number two. Prices generally have come to be much more closely aligned with costs, although I'm going to point out that there are important exceptions to that.

Every statistical study I have seen demonstrates that, since deregulation, air fares have gone down the greater the *distance*, in reflection of the fact that costs per mile go down very sharply with distance, because of the very many costs that do not vary with length of trip—baggage handling, ticketing, reservations, check in, and so on. Air fares also have gone down with the *density*; that's because the denser the market, the bigger the planes you can use, and the bigger the plane, the less it costs per passenger seat per mile. If you can cram people into a 747, it costs only a fraction per mile of what it costs if you carry people in a little DC-9, let alone one of the tiny Swearingen Metros into which I now have to crawl when I fly out of Ithaca, New York.

Fares for business travel have gone down much less than fares for vacation travel. The reason is quite simple: business travel requires rather frequent service at convenient times to be able to get from one town to another, put in a day's work and get back. This means relatively small planes and relatively low load factors; vacationers, on the other hand, who generally spend their own money, for one thing, typically can be put in one 747, one time a day, maybe even an inconvenient time of day. The airlines rip out all the seats and put in space so that it's only suitable for people with no knees. And they fill those planes 90 to 95 percent full. That costs a fraction of what it costs to carry people between Ithaca, New York, and New York City in a DC-9, for example. So vacation travel fares have gone down.

Finally, and this is less obvious, the airlines have introduced vast differences between fares *on-peak* and *off-peak*. It is more costly to carry passengers on-peak: more travelers means the scheduling of more flights. Off-peak, there are empty seats, and the use of equipment that would otherwise be idle. These price differentials are partly visible: People Express charges $99 on-peak; $79 off-peak. Pricing on the shuttles, Eastern or New York Air, out of Boston is similar. But much more important and less obvious is the fact that the airlines control the number of discount seats that they make available, making very sophisticated use of their computers to decide how many, flight by flight. Naturally, they make very few discount seats available on-peak, and far more off-peak. So their actual realization, the average price that they get, varies widely from peak to off-peak. Eighty percent of travelers travel on discount seats these days. In short, prices track costs much more closely as a result of airline deregulation.

In the cases of trucking and rail I haven't seen systematic studies, but examples are evident. We have seen the introduction of lower fares for back-hauls. It used to be that the carriers were required to charge the same tariff per mile, no matter whether on a forward trip or a backward trip. Now they're competing the price down on the backhaul compared with the front haul, reflecting the availability of an empty truck or freight car; that's just the same as an on-peak, off-peak differential.

In communications, consider what is happening to the price for long distance service, because competition is coming in there; prices are being driven down to costs. And that is going to have the inevitable effect that the basic monthly charge will start rising to cost. And as an economist, I would have to say that it is a good thing.

Now don't get me wrong. This movement towards cost-based pricing has not been uniform. There is a great deal of price discrimination, for example, in all these areas. The most flagrant examples are in airlines where, depending on when you get your ticket (whether it's a week in advance or not), or whether you stay over a weekend, you may be paying only a fraction of what the person who sits next to you is paying. And that's really price discrimination. I gather that is something not available to you in casualty insurance because of the nature of your regulation.

On the other hand, a lot of the price differentials in airlines are not discriminatory. The fact, for example, that most discount seats are made available only off-peak reflects the fact that it really costs less to carry people off-peak. In the extreme case, if you take a standby fare, it costs the airline virtually nothing to carry you, because they don't have to carry you unless there's an empty seat that's going out anyhow. In large measure, the discount seats are anticipatory standbys. The airlines expect empty seats on the off-peak flights. Therefore, if you agree to go at that time, they'll give you a lower fare.

To some extent, these are cost-justified. But of course, there's also geographic discrimination. Each of you, I'm sure, can cite examples of cases in which you can go from New York to Florida or California for a fraction of what you pay on much shorter trips on thinner routes. Some of that differential is cost-justified for reasons I've given, but some of it obviously reflects differences in the effectiveness of competition. Understandably there's more competition on the thick routes and on the long routes, and that's why the benefits are more evident there. The 80 percent of travelers who are traveling on discount seats are surely seeing the benefit of competition. It tends to be the travelers on the thinner routes and the business travelers, the 15 or 20 percent, who are not seeing the benefits.

Number three. We have a lot of unbundling of services now, induced by competition, and travelers are being presented with a much greater variety of price and quality options. If you want very low fares, you can travel on People Express. You have trouble getting down the aisles because of the pack bags; there's no room for your knees; you pay to carry your own baggage; you bring your own sandwich or you pay for a meal. At the same time, you get low-cost, reliable air service with no gimmicks, no advance purchase, no required staying over a weekend. If you want more comforts, you go to the Eastern Shuttle. And if you want luxury service, you travel by New York Air, which decided that the only way it could hope to compete was to tear out all the seats and have much bigger spacing and leg room and have free drinks and have free *New York Times*. And that's a different kind of service from the Eastern Shuttle service. But the Eastern Shuttle has another different quality, a very important feature, that if you get there before flight time you go out. You don't need reservations.

Well, the same kinds of choices are available in the brokerage field. If you want just to have a consummation of a transaction you go to a discount broker. If you want the lush receiving rooms and the personal advice, you pay for it separately.

In the case of banking, you now get the advantages of getting interest on your deposits and, of course, you have all sorts of advantages in money market funds and NOW accounts, and a great variety of services. But the other side of the coin is that you now pay for banking services what it costs the bank to provide them. A lot of relatively modest-income people have been complaining about that. But competition is again coming to the rescue: many banks are introducing what they call "no frills" banking—the People Express of the banking business, in which you pay relatively low charges if you're willing to use the automatic teller machines. If you insist on personal service, you pay for it.

In the case of rails, piggybacking is much more common than it used to be. In communications, I scarcely need to mention the incredible burgeoning variety of tailor-made services that people are getting; it is no longer possible to call it *simply communications*.

These examples illustrate the third beneficial effect of competition. To review: number one is increased competition; number two is prices being better aligned with cost; number three, great varieties of price quality options.

Effect number four is improved efficiency as a result of the freedom of airlines to structure their routes; of truckers to use direct routes and to fill up with anything that they can pick up; the freedom of airlines to pull out of markets for which their equipment is unsuited, and to enter the markets for which their equipment is suited. That latter freedom has an adverse aspect in that the people who used to get jet service now are having to crawl into airplanes, but it is economic. There's no reason why I, living in Ithaca, enjoying the benefits of breathing fresh air and driving to work in 12 minutes, should be subsidized by the people who fly from Boston to Florida or Chicago to Hawaii, and given jets that I don't deserve to have.

In consequence of their newly obtained freedom and the pressures of price competition, the airlines are getting more working hours out of their planes. United Airlines, for example, is getting something like 25 percent more pay hours out of their planes because of their ability to schedule the way in which the planes are used rather than being restricted in where they might take them. On average, the carriers are also putting more seats in their planes, and they are filling more of their seats. Load factors have been higher in every single year since deregulation than in any pre-deregulation year since 1960, with the exception of one single year.

The peak, off-peak differentials that I have mentioned are contributing to these higher load factors by encouraging economy-minded travelers to help fill the planes off-peak.

The pressures of competition have resulted also in the abandonment of featherbedding practices. The Teamsters, under the pressure of competition, were forced to give up various provisions that the truckers had been trying to get rid of for 20 years. British Airways, in a period of three years, reduced the number of its employees from 59,000 to 37,500. American Airlines, in the same three-year period, cut its payroll from 42,000 to 35,000. Dick Ferris, of United Airlines, told me that they decreased their staff from 54,000 to 41,000.

Railroads have retired thousands of miles of track, which accounted for only about 2 or 3 percent of their total business. They had been forced previously to maintain trackage that wasn't even covering their variable costs.

In trucking, labor costs have declined by about 15 percent per ton mile carried.

Effect number five is that we've had downward pressure on inflated wages. I've already mentioned the pilots' wages, and of course, the public has seen the givebacks there. The Teamsters, similarly, whose contract in 1979 was one of the things that broke my wage-price standards, have in recent contracts been giving up cost-of-living adjustment clauses in order to retain their jobs. You had major givebacks in automobiles and steel, to the extent that you've had international competition there as well.

Effect number six is that prices in the markets experiencing intensified competition have all declined in real terms: they've risen less than the cost of living. In the case of airlines, the average per mile fare has gone down about 15 percent, in real terms, despite what has happened to the price of fuel. Now that does not mean the coach fare has gone down: it has gone up more than the Consumer Price Index; but the fact that roughly 15 percent of the people flew in discount seats before, whereas now it is 80, means that the average fare actually paid has declined relative to the CPI. The same pattern is evident in trucking. It seems not to be true of railroads—again not surprisingly, since the purpose of deregulation there was at least as much to free the financially weak carriers to increase rates where the traffic would bear it, as to reduce competitive rates.

The question of service quality is much more complicated and I cannot take the time to treat it adequately. There have been at least three or four surveys of shippers asking what has happened to the quality of trucking service. The majority of the respondents say there has been no observable change. But among those who say there is a change, three or four times as many say that there has been an improvement as report a deterioration.

In the case of airlines, some communities have lost services, no question about it, but many more communities, in all size ranges, have experienced an increase in the number of weekly departures. The loss of jets, I concede, is a genuine loss.

In communications, I've talked about the tailor-making of services, the various kinds of price-quality choices that are now available. Analyses of what has happened to service quality here is, however, complicated by the breakup of AT&T, which is not a necessary part of deregulation. We have been introducing competition into the communications business for the last 25 years, ever since we permitted companies to set up their own private microwave systems back in 1959. The breakup of AT&T has produced enormous confusion and delays; that's another story, which I don't have time to tell.

The story of the consequences of deregulation in communications really deserves separate, integrated considerations, because it is much more complicated than such cases as airplanes, trucking or stock exchange brokers. The main reason it's more complicated is that it has been impossible to deregulate completely: there are still important elements of monopoly. Because local telephone service is, in large degree, still a monopoly it's hard to imagine simply deregulating it. It's not even clear when AT&T can be deregulated to compete freely in the long distance business, though I think the time is getting very close.

So what we have there is a peculiar mess that is a consequence of our trying to run a system that is comprehensively regulated while also increasingly open to intense competition. And legislators, politicians and local regulators are still trying to get the advantage of squeezing subsidies out of that long distance stone, because the one thing they don't want, as political people, is to raise the basic monthly charge. So we find ourselves in this crazy situation in which people who will pay $5000 to install a bathroom when they build a house think that they're entitled to have the telephone installation for $50. The people who pay $15 or $25 a month for cable TV and Home Box Office think they have a God-given right to have their telephone service for $10 a month or less. Regulation has this crazy effect of making people think they are entitled to things below cost, even if they're very, very wealthy.

In conclusion, the greatest economic challenge confronting our country in the last 15 or 20 years is the challenge of stagflation. Only an ideologue or fool could be confident we've solved it: 4.5 percent inflation with 7.5 percent unemployment are far from Nirvana. What stagflation refers to is the dreary cycle in which we've experienced alternating periods of excessively unacceptable inflation followed by, and apparently curable only by, excessively high unemployment and stagnation, and all of this in the context of virtually no growth in productivity.

That problem, I suggest, has two major manifestations or components. One is productivity. Historically, output per worker has gone up on the average 2.5 to 3 percent per year, decade in and decade out, creating more goods and services for us to live better, and also financing government programs that modify poverty and relieve distress. Productivity growth declined to about 2 percent in the late 1960s and early 1970s, to 1 percent in the mid-1970s, and to 0 percent—no productivity growth—from 1978 to 1982. So productivity is a very serious part of the stagflation phenomenom.

The other component is the fact that we have become an entitlement society. That is to say, we are constantly, in varying ways, demanding a bigger and bigger share of the pie every year, because we've been accustomed every year to being better off. This attitude takes a lot of forms. Look at the problem we're having balancing the federal budget: everybody's in favor of reducing expenditures—except the expenditures that help his own particular group. Another manifestation of it is the wage-price spiral in which, regardless of what happens to productivity, wages are increased, then prices are increased, then wages are increased.

I know of no more effective cure for both loss of productivity growth and the wage-price spiral than the discipline of competition. We have seen what it has done to productivity in the cases of airlines, trucking, the railroads, and communications. We have seen what it has done by way of exerting powerful pressures to improve efficiency. We have seen what it has done by way of moderating the wage-price spiral.

Deregulation has inflicted a lot of pain and suffering in those industries, and I don't mean in any way to minimize them. But the suffering is the consequence of competition. That's why people are having trouble. That's why people are going bankrupt.

The social purpose of deregulation is to bring the public the benefit of competition. And that it has surely done.

Thank you very much.

I'd be very happy to try to respond to questions from the audience, but with the warning that I know very little about your industry.

QUESTION: Are there some additional industries which you see as particularly in need of deregulatory action?

DR. KAHN: Well, my first candidate is natural gas. I was the architect of the 2-price system for natural gas in the late 1950s. I never dreamed it would now be a 27-price system. The absurd situation that has developed in which we have something like 26 or 27 different kinds of natural gas, the same stuff, CH_4, coming out of the ground, priced in these various ways with a whole series of long-term contracts that have accumulated from the time when we had shortages. Some buyers are held to paying $1.50 a thousand cubic feet (the equivalent of $9 a barrel of oil) for the old gas. Meanwhile, people drilling much deeper, much more expensive new gas are charging $7, $8, $9 a thousand cubic feet (the equivalent of $40 to $60 a barrel of oil) or mixing it with the $1.50 and $2.00 gas and selling at $4 or $4.50.

There are incredible distortions in the market, which probably have been responsible for prices going up more than they otherwise would have. And competition will now work there, because gas prices now clearly are bumping up against and exceeding the ceiling of what the market will accept. Pipelines are having difficulty selling that gas, because they are competing with No. 2 fuel oil and No. 6 residual oil. So that I think that the public would be far better off in that case releasing that industry from regulation.

It is obvious that deregulation of insurance in some way is likewise in the cards. I'm very quickly out of my depth here, but the states' prohibitions on rebating by agents, for example, seem to me to have no economic justification. I must be much more cautious here, as I am about continued deregulation of financial markets. We do have a slightly different situation in each, which I recognize.

In the case of financial markets, we are dealing with the people's money. And our history is replete with illustrations of how cycles of speculative lending with other people's money have had the effect not merely of losing money for the investors, but of destroying the economy. That is to say, when bankers have their assets tied up in long-term obligations—mortgages, to take the extreme—and their liabilities are short term—people's demand deposits—and if you begin to get a collapse of confidence and people begin to come in and force the banks to liquidate, there's no way the banks can pay off. Banks fail. That results in liquidity crises. They have to call in their loans. Businesses have to dump their inventories. It has horrible effects. The question then is whether you can achieve the protective purposes for society while achieving the benefits of competition. Deposit Insurance is an obvious case.

As I turn to insurance, in the same way, I find intolerable the thought that people who think they have insurance might discover they don't have insurance because they bought it from someone who offered it to them at a lower price with an inadequate knowledge of what their costs really were going to be. Thus, I can't really envision deregulation of insurance unless there is some device for pooling or for back-up insurance of one kind or another.

I must confess, as well, from what little I know about the insurance case, that as to the positive social case for competition, I don't see the massive advantages by way of efficiency, because your major costs are not controllable. At the same time, I do see the desirability of customers having alternatives, and that's one of the advantages of price competition. One has to draw the line in ways that will preserve at least the social purposes of insurance, while providing the benefits of freedom of choice and pressures on insurers not to compete by raising agents' commissions.

QUESTION: My name is Gary Koupf, with the Motors Insurance Corporation. When you were talking about trucking deregulation and airline deregulation, I think there is one aspect that you didn't touch on at all: as the profit margins for these companies come down, as it must under deregulation, there must be pressures on these companies to cut back on activities such as maintenance of their equipment.

Now with banking, the most you're going to lose is money; and with insurance, the most you're going to lose is some money, as catastrophic as that may be; but with airline and trucking and railroad deregulation there is a big potential for catastrophic loss of life. Does the little bit of money that I save on an airline ticket justify that? I would rather have an airline pilot who is paid $111,000 a year than one who is paid $33,000 a year. I don't feel safe if an airline, in order to keep the profit margins up, is assigning 80 hours a week flying time to the pilot rather than 40 or 30, or whatever.

DR. KAHN: Would a salary of $200,000 be safer?

But seriously, I don't think anyone can deny with complete assurance that there may be some connection here, and I mentioned the trucking situation in the middle of the depression as an example.

I can't agree with your premise, however, that profits will almost certainly be lowered. The fact is that the airline industry all through its history has earned profits that were well below what's earned in industry, generally, because there seems to be something about the romance of aviation that induces people to want to invest in it irrationally.

In the period from 1970 to 1975, when the CAB practiced the most rigid tight regulation in its entire history—they refused, as a matter of policy, to entertain one single application for a change in route authority, so that when I arrived there, 632 applications were waiting approval—yet in that period, the trunk airlines of this country averaged a 3 percent return on equity. The reason is that they were competing in other ways. They were competing by offering more scheduling. They were competing by offering more munificient meals, by presenting in-flight entertainment, by bribing travel agents, and so on. So that regulation was not assuring financial soundness.

Moreover, we had the benefit of studies to see if there were any observable relationships over the preceding 40 years between safety or accidents and financial condition of either individual companies or the industry. They disclosed no relationship, neither over time, as the industry made more money and then made less money, nor as between profitable and unprofitable companies.

The reason for this pattern is fairly clear: you're not in business if you have one or two major accidents. Certainly, what's happened to Air Florida was very largely a consequence of that one accident.

Moreover, if you want safety, the way to achieve it is to regulate for safety. While we were deregulating economically, I went to the FAA and said, "Now, you must be even more vigilant than you were before in your safety regulation." The FAA is the one that sets limits on the number of hours that you may fly. If you're worried about trucking, have more road checks and enforce the laws.

Finally, we have now had something like six years of experience. I defy you to look at the statistics of airline accidents during those six years and see any evidence that the safety record of the industry has deteriorated. On the contrary, by every measure, it has improved.

The pilots, who are not exactly impartial parties, will point to Air Florida, and it may well be that the Air Florida case was one in which pilot inexperience or pressures were responsible, but the other biggest accident—the American Airlines crash in Chicago in 1979—involved very good members of the Air Line Pilots Association. And if you review other major accidents, they also involved members of the Air Line Pilots Association who were flying 45 hours a month and being paid the equivalent of $110,000.

So that, again, I'm not saying that you can't conceive of a situation in which, under extreme financial duress, there might not be a temptation to skimp on safety. But it has not shown up, and in any case you had extreme financial duress before deregulation. And although the airlines had financial troubles during the recent long, deep recession, in the last year they made record profits, and they're doing very well.

So that neither on the financial front, nor on the safety front, do I see any basis for concern, but I certainly favor the FAA regulating like mad.

QUESTION: I'm Dave Bradley from the Hartford Group.

We've heard you speak of the Steelworkers Union, the pilots union, and several other unions. Do you think labor unions have a valid social purpose in the United States today?

DR. KAHN: I certainly do!

All you have to do is look at Poland to get a picture of the importance of unions. I think it is terribly important that we preserve unions for a variety of reasons that I scarcely need describe. I'm an Old New Deal Democrat.

The fact is, however, that we have to recognize monopoly when we see it. When the automobile workers of this country, whose average pay was $22.80 an hour including fringes, ask us to protect their jobs by keeping out Japanese cars, that is costing the average purchaser of a car in the United States about $1500. We've built up a kind of elite of these highly organized workers in industries protected from competition, in which the monopoly profits have gone, not just to stockholders, not just to executives—look at those multimillion bonuses that the automobile company executives just paid themselves—but to the workers as well. And they're exploiting the school teachers and the hospital workers and the sanitation workers and the civil servants and the Ladies' Garment Workers, whose pay is half of theirs. If unions want protection from competition, then they should accept wage and price controls. We could treat them like public utilities, and not allow them to get wages more than the average of anything else.

That question has nothing to do with the question of the distress of certain areas of the country, which is real, and where I would do everything I could to help, but not by building permanently ensconced, protected monopoly positions, enabling them to exploit the rest of us. Do you know that, in general, if you want to buy a car in most parts of this country, you pay an availability charge of $1000 just to get a car and several thousand dollars for extras, even if you don't want them?

MEMBER OF THE AUDIENCE: I'd like to thank you very much, Professor Kahn, for a very stimulating talk. Those of us who have been involved with the insurance business for many years, I think saw a lot of parallels to the industries you described, and just about the time we thought we saw all the answers coming together, you turned the tides and exposed other problems that were just the opposite from what we expected. I think your talk has stimulated everybody's thinking on the subject. Thank you very much.

LUNCHEON ADDRESS—NOVEMBER 12, 1984
DEREGULATION IN THE INSURANCE INDUSTRY

MAURICE GREENBERG
PRESIDENT AND CHIEF EXECUTIVE OFFICER
AMERICAN INTERNATIONAL GROUP

Casualty Actuarial Society members; ladies and gentlemen. I'm pleased to be here to share with you some views about deregulation in the insurance industry. I came a little early and thus had an opportunity to listen to a portion of the panel discussion of "The Deregulation of U.S. Industry." I must say it didn't change my views but it did express rather clearly the dichotomy between complete deregulation and no deregulation. Of course, what will happen is something between those extremes.

The general thrust for deregulation in the last several years certainly has been by the federal government, specifically the executive branch. Under the Reagan administration there has been a general movement towards dismantling some of the shackles that have bound American businesses in a variety of industries.

The advocates of deregulation believe that market forces encourage far better decisions on how to run a business than does regulation, which is excessively costly, wasteful, and stifles change.

Insurance, as you know, has been regulated by the states. Over the last several years there have been some moves to modify and loosen some forms of state regulation. This certainly has been evident in the area of rate regulation. We no longer have a strict tariff; rather we have increasing flexibility in rates. Many in this room wish that weren't the case, given the financial results of the industry in the last several years.

This experience with deregulation points out something else. In the ideal world to have deregulation you need a mature industry that will act maturely, that simply will not use deregulation in any form either to be a predatory supplier of services—e.g., insurance—or to establish a monopolistic practice that ultimately will drive out the smaller operator.

It would seem that as some aspects of the financial services industry are deregulated, more oversight regulation needs to be introduced for other parts of the industry. Financial failures in the insurance industry, unfortunately, don't go away without someone paying for them. In our business, as you know, the insolvency funds are financed by assessing the industry to make up for insurers that have failed. It is very nice to say that an entrepreneur ought to have the right to succeed or fail in business—that's the way it should be in an ideal world. But that's not the way it is in the insurance industry. When an insurer fails, there is an assessment against the rest of the industry. Theoretically, that assessment is reflected in insurance rates and, theoretically again, the public pays for it in higher premiums. That theory really doesn't work out in practice very often. The industry winds up paying for the failures of individual companies. So there are indirect costs of deregulation that must be addressed in some forum.

Clearly, state regulation is going to remain. In the choice between federal and state regulation, I think there are not many who would advocate that we have one federal bureaucracy, no matter how small it might be, in a new deregulated environment. My view is that insurance is close to people and therefore it should be regulated at the state level, notwithstanding the problems associated with that type of regulation.

There are some facets of regulation that will have to be adjusted when deregulation gathers momentum, as it will later in this decade. Surely the oversight responsibilities—solvency monitoring and licensing, for example—are going to require a faster trigger point than currently exists. As some parts of business are deregulated to encourage more competition, we're going to need more attention to oversight responsibilities to avoid the failures that could accompany deregulation. I think that's one of the key discussion topics in several different legislative forums.

Now, with the major thrust in the last several years for deregulation of financial services or the convergence, if you will, of different institutions that want to sell each other's lines of business, the lines have become blurred. Banks certainly have been the driving force. Bank holding companies have been pressing for the authority to "be in the insurance business." "Be in the insurance business," is a term of art because I think some truly understand the nature of the risk and fear the idea of immediate expansion of their powers into the underwriting side of insurance. Most would like to tiptoe into the marketing of insurance first and then gradually expand into underwriting over a period of time as they begin to understand the nature of the beast. Not all, however, are approaching it that way. And so there is a difference of opinion as to the appropriate speed of entry.

Notwithstanding all the hullabaloo that has been raised that banks in insurance pose a new threat to the insurance industry, banks have been in the insurance business, in one form or another, for some years. Credit card solicitations have been going on for years. Credit life insurance sold by savings and loan associations has, in some instances, been related to fire insurance. It's not a new phenomenon. What is new is the manner in which it is being approached today as distinguished from approaches used in past years.

The driving forces are several. I suppose if you picked the single most important factor, it would be the dissatisfaction of everybody on both sides—the insurance industry and the bankers—with the distribution mechanism for personal lines insurance. They believe, rightly or wrongly, that the cost of delivering a personal lines policy is excessive: 30 percent or more of the premium dollar is absorbed by the acquisition expenses of that kind of product. Banks believe that their advanced state of communications technology—banks are more advanced than the insurance industry—would give the banks market segmentation capabilities. The banks are capable of delivering a policy in the personal lines field much more cheaply than the insurance industry currently does, and servicing it properly as well.

Now those are words in which the bankers believe; the proof will be in the eating. There are many issues that must be addressed before that dream can come true in any event. There's also a mistaken belief that insurance is a commodity and can be sold just like any other commodity. There are many who have held that belief and who wish they had not adopted that philosophy. Insurance is not a commodity. It's a risk product that requires underwriting and selection and proper pricing. Anything less than that simply won't wash. Treating insurance like a commodity might save money on the acquisition side but easily give it up on the underwriting side.

The other belief that the banks and others have held is that the insurance industry is a cash cow. I don't think too many of those cows are very rich in giving milk these days. With the recent financial results I believe there's been some moderation of that belief.

Another problem is that the banks are short of capital themselves. The banking system is extended. It has loan problems both in Latin America and in this country. The problems of the Continental Illinois were not the foreign loan debts; they were principally domestic debts. So there's that problem to be addressed as well.

Further, there is the whole problem of how, in a bank holding company, to insulate an insurer subsidiary from the problems of the bank or vice versa. That issue has not been resolved, even though it has been addressed at the Congressional level and at the New York State Commission on which I served. But there were no solutions at the moment to that problem, which will take legislative action to insure that a bank holding company that owns an insurer, and that gets in trouble on either one side or the other, won't be able to penetrate those lines between subsidiaries.

We also have the question of the insolvency funds. Will a bank holding company that owns an insurer have access to FDIC funds or federal reserve assistance—if it gets in trouble—where an insurer not so owned would not have access? That obviously would be unfair competition.

So, while the theory of deregulation and the theory of being able to market more effectively in a bank holding company system that has access to a major consumer list and that has built a confidence factor with that customer may seem plausible, there are problems to address.

The most likely products to be sold, in the first instance, will be interest-sensitive products. Those sales are currently underway. Sales of these products are happening now not only in bank systems but happening in stock brokerage firms. As the time value of money has become better understood in all financial institutions, the proliferation of products using the time value of money has gained momentum. What better vehicle through which to sell such products than vehicles that understand the time value of money far better than the insurance industry did, and who have sales organizations and distribution mechanisms that can do that?

So there's a natural allegiance between banks and those products. When you get beyond that, two key products will be homeowner or residential fire insurance policies and automobile insurance policies. There the entry of banks becomes a little slower. Certainly the homeowner and residential fire insurance may be the first out of the box in product innovation and in using the distribution mechanisms of these institutions.

Automobile insurance may be another matter. To begin with, the selection process—if banks are going to use any kind of underwriting judgment at all—would rule against writing everybody or a major swath cutting through the bank customer list. So selection is going to be a key element: will the banks say to a good customer for banking services that they won't write his automobile policy (or rather that of his 18-year-old son)? That will pose a problem. To believe, as some bankers do, that selection is really irrelevant because insurance is a commodity, and a profit can be made on the basis of sheer volume, is to invite a lesson that I think has been learned by a lot of the companies for which you work. It just doesn't work out that way. And so there's a learning process that has to be understood as well.

That learning process relates to the insolvency fund problem that I raised earlier. We can't afford to live for the next five or six years with the mistakes of yet another group of people who don't understand the insurance business. That problem has to be solved beforehand, and I think that it will.

The St. Germaine Committee in the Congress and the Garn Committee in the Senate both have had bills. Nothing happened in the last Congress. However, there is believed to be a growing consensus on a bill. I think the opening of insurance to banks may not be as broad as the banks have wanted in the first instance. On the other hand, I think the banks have other fish they want to fry first, and the banks are prepared to take underwriting of insurance as a second or third or fourth step, after they get into the distribution mechanisms and building infrastructure. But the interest of banks in insurance will not go away. If anybody thinks that this genie is going to be pushed back into the bottle, they're wrong. I think the dynamic forces have changed, and they're too strong to hold back.

Unquestionably there are problems. There always are problems when you're involved in change. But that doesn't mean change should be stifled. We must decide how the system is going to be regulated in the first instance. If a bank holding company that is regulated at the federal level has permission to buy an insurer, assuming the law is changed, the first question to be resolved is how that institution will be regulated and by whom. Will it be joint, state and federal? Will it be more, not less, regulation? I favor deregulation with proper control, with oversight by state regulators.

When the New York State Commission was meeting, while I voted in favor of banks coming into the insurance business, I had several reservations that went along with my approval. One was that this issue of regulation first be resolved. Two, that the insolvency issues be resolved. Three, that penetrating the corporate veil to protect the insurer from a bank subsidiary failure be addressed. And finally, that the underwriting powers of approval be postponed for five or six years until the banks had some knowledge of the instrumentality which they were trying to enter, but that banks be permitted into the agency or distribution system first.

Now, it seems to me also that you have to look at what the banks are doing internationally. What they've been prevented from doing domestically, they've not had as much resistance on internationally. The Federal Reserve Bank has limited bank expansion domestically but has been more generous to bank holding companies seeking to be in the insurance business outside the United States. Several have gained permission to buy life companies or limited fire and casualty powers outside this country. I think that an obvious strategy would be to get a lot of the insurance mechanism in place and then it becomes *fait accompli*—you're doing it and it's pretty hard to unscramble the eggs once they're scrambled. I suspect that process is underway.

I think a counterbalancing force is going to be in the next several years' financial results. Our industry is just facing up to its own problems; all of you know what they are. We've had the worst underwriting losses on record. It is estimated that in 1984 there will be about a $21 billion underwriting loss not covered by investment income. A loss of surplus at a time when rates are expanding and the lack of capital may prohibit many companies from getting whole or participating as they should in the recovery.

The banks also have their capital problems and are seeking ways to enhance capital. Another aspect is that several insurance companies or insurance groups have AAA ratings in Moody's and S&P. Very few banks have that kind of rating today. They haven't got the rating because they haven't got the capital. And so it becomes academic to think there's going to be a wholesale invasion of banks into insurance in the next several months or several years. There's a lot of repair work that has to be done to bank balance sheets and considerable understanding of the technology of regulation that I've been addressing for the last few moments.

In addition, there are a couple of other things that I should mention. The banking business is supposed to be a shorter term business than it has turned out to be on the lending side. The insurance industry has not solved its long-term liability pricing questions or the problems of defining the product itself. These problems are a long way from resolution. It seems to me that you can't ignore some of those issues and permit complete deregulation, forgetting some of the bitter lessons that have been learned.

In the final analysis, we all have to serve the public. They have to benefit one way or the other and I'm not sure they benefit from wholesale change without proper thinking in advance, if such wholesale change leads to insolvency. I favor deregulation, with all of the safeguards that I think are necessary.

We'll never get change, we'll never get progress, we'll never solve some of the issues that are related to acquisition expense, and we'll never force efficiency unless we permit change. Regulation, historically, has been stifling and costly. We've all lived through that, and I don't like it. I like to have a freer environment in which to do what I think is right; if I'm wrong I'm willing to pay the price. But I think we've got to protect the public and others in the insurance industry from sharing in the failure of an insurer who hasn't the maturity to do it right. And I hope that can be done.

I'd like to take a moment to discuss one other subject: possible tax law changes in Congress next year. There is a great deal of thought being given to taxing the fire and casualty industry on a discounted loss reserve basis. If you want to take this injured industry and bury it, that's the fastest way I know of doing it. I don't know how you can take liability reserves that we've historically been very poor at pricing, but that come out reasonably at the end of 15 or 20 years, and discount them in advance. They are discounted already. I think that this industry would be shattered by such a change. It would bring about more insolvencies than anything else that comes to mind. It would just add to the chaos and provide a false illusion that things are much better than they are.

MINUTES OF THE 1984 FALL MEETING

November 11–13, 1984

THE WESTIN HOTEL, BOSTON, MASSACHUSETTS

Sunday, November 11, 1984

The Board of Directors held their regular quarterly meeting from 1:00 p.m. to 4:00 p.m.

Registration was held from 4:00 p.m. to 6:30 p.m.

The Officers held a reception for new Fellows and their spouses from 5:30 p.m. to 6:30 p.m.

A general reception for all members and guests was held from 6:30 p.m. to 7:30 p.m.

Monday, November 12, 1984

Registration continued from 7:15 a.m. to 8:00 a.m.

President Carlton Honebein opened the meeting at 8:00 a.m. Dr. Stefan Peters of the Massachusetts Insurance Department welcomed our Society to Massachusetts.

Mr. Honebein announced the Harold W. Schloss Scholarship Fund. Mr. Schloss, a past president of our Society, died in 1979. His wife, Frances, presented a check from the Schloss estate to Phillip Ben-Zvi. The scholarship fund is intended for worthy actuarial students at the University of Iowa.

Mr. Honebein then recognized the 7 new Associates and presented diplomas to the 35 new Fellows. The names of these individuals follow.

FELLOWS

Edward J. Baum	Robert S. Briere	Paul J. Henzler
Abbe S. Bensimon	Dale L. Brooks	Larry D. Johnson
James P. Boone	David R. Chernick	Marvin A. Johnson
Peter T. Bothwell	Valere M. Egnasko	Jeffrey L. Kucera
David S. Bowen	Alice H. Gannon	William D. Louks, Jr.

Matthew P. Merlino	Bernard A. Pelletier	Kevin B. Thompson
Neil B. Miner	Frank D. Pierson	Frank J. Tresco
Peter J. Murdza, Jr.	Richard C. Plunkett	Richard L. Vaughan
Catharine L. Neale	Deborah M. Rosenberg	Michael G. Wacek
Raymond S. Nichols	Louis G. Séguin	Glenn M. Walker
Richard W. Nichols	Ollie L. Sherman, Jr.	David R. Whiting
Sylvie L. Paquette	Stuart B. Suchoff	

ASSOCIATES

Kenneth E. Carlton, III	John W. McClure, Jr.	Alan K. Putney
Vincent T. Donnelly	Clifford A. Pence, Jr.	Pamela J. Sealand
Israel Krakowski		

This was followed by a report by Frederick Kilbourne on the Board of Directors' meeting; a review by Stephan D'Arcy of Ronald Ferguson's "Duration" paper; and a summary by Charles Bryan of the new papers. Mr. Honebein then announced the results of the elections for Officers and Directors:

President-Elect
Phillip N. Ben-Zvi

Directors
Linda Bell
Michael Fusco
Kevin Ryan
Michael Toothman

From 9:15 a.m. to 10:15 a.m., Dr. Alfred Kahn, Professor of Political Economy at Cornell University, delivered the Keynote Address on the subject of the deregulation of American business.

From 10:30 a.m. to 12 noon, Mavis Walters moderated a panel on "The Deregulation of U.S. Industry." Her panel consisted of:

James Callison
Senior Vice President, General Counsel and Secretary
Delta Airlines

James Harkins
Managing Director—Traffic Services Division
American Trucking Associations

Hon. Stephen Kaufmann
Deputy Commissioner
Virginia Bureau of Insurance

Marc Rosen
Regional Director—Government Relations
American Telephone and Telegraph

The panelists presented their industries' experiences with deregulation and reacted to Dr. Kahn's Keynote Address.

A formal luncheon was served from 12:00 noon to 1:30 p.m. Mr. Maurice Greenberg, President and Chief Executive Officer of the American International Group delivered a speech summarizing his views of what deregulation might mean to the future of the insurance industry.

The afternoon was devoted to concurrent sessions, consisting of 7 Workshops, 3 American Academy of Actuaries presentations, and 3 new *Proceedings* papers.

The Workshops covered the following topics:

1. "The Evolving Law of Occupational and Latent Disease"
   William C. Aldrich—Moderator
   Vice President
   The Hartford Insurance Group

   Albert J. Millus
   Attorney at Law

   John Shea
   Vice President
   Aetna Life and Casualty

2. "Environmental Impairment Liability"
   Janet R. Nelson—Moderator
   Senior Vice President
   Atwater McMillian, Inc.

   Lynne Miller
   President
   Risk Science International

William Mahoney
Vice President
Marsh & McLennan, Inc.

John Tronzano
Vice President
Swett & Crawford Management Corp., Inc.

3. "The Alpha, Beta, Gammas of Loss Distributions"
Charles C. Hewitt, Jr.
President and CEO
Metropolitan Reinsurance Co.

4. "Actuaries and Their Computers"
Arthur I. Cohen—Moderator
Vice President
Pennsylvania Compensation Rating Bureau

Paul C. Martin
Senior Actuarial Assistant
USF & G

Richard G. Woll
Actuary
Hartford Insurance Group

Michael G. McCarter
Assistant Secretary
Reliance Insurance Co.

5. "Risk Theoretic Issues in Loss Reserving"
CAS Committee on the Theory of Risk

6. "The New CGL Policy"
Michael Fusco—Moderator
Senior Vice President
Insurance Services Office

Gregory N. Alff
Associate Actuary
Wausau Insurance Companies

Paul Lofgren
Assistant Secretary
Liberty Mutual

Dorothy A. Zelenko
Assistant Vice President
General Reinsurance Corporation

7. Limited Attendance Workshop: "Regulation"
Michael L. Toothman—Workshop Coordinator
Consulting Actuary
Tillinghast, Nelson & Warren

The American Academy of Actuaries presentations covered the following:

1. "Standards of Practice"
Bartley L. Munson—Moderator
Vice President and Actuary
Aid Association for Lutherans

Douglas C. Borton
Chief Actuary—Office of the President
G. B. Buck Consulting Actuaries

John H. Harding
Executive Vice President
National Life Insurance Co.

C. K. Khury
Vice President and Actuary
Prudential Property & Casualty

2. "Financial Reporting Developments"
Richard H. Snader—Moderator
Vice President and Corporate Actuary
USF & G

Robert H. Dobson
Consulting Actuary
Tillinghast, Nelson & Warren

Walter S. Rugland
Consulting Actuary
Milliman & Robertson

James F. A. Biggs
Principal
Peat, Marwick, Mitchell & Co.

3. "Taxes and the Actuary"
   James A. Faber—Moderator
   Principal
   Peat, Marwick, Mitchell & Co.

   Martin Adler
   Vice President and Actuary
   Government Employees Insurance Co.

   Jay A. Novik
   Vice President
   North American Reinsurance Corp.

   Richard S. Robertson
   Senior Vice President
   Lincoln National Corporation

The three new *Proceedings* papers were:

1. "Empirical Bayesian Credibility for Workers' Compensation Class Ratemaking"
   Glenn G. Myers
   Assistant Actuary
   CNA Insurance Companies

2. "A Note Regarding Evaluation of Multiple Regression Models"
   Gregory N. Alff
   Associate Actuary
   Wausau Insurance Companies

3. "Extrapolating, Smoothing and Interpolating Development Factors"
   Richard E. Sherman
   Senior Consultant
   Coopers & Lybrand

A general reception for all members and their guests was held from 6:30 p.m. to 7:30 p.m.

Tuesday, November 13, 1984

From 8:00 a.m. to 9:30 a.m., there was a continuation of Monday afternoon's concurrent sessions.

At 10:00 a.m., Mr. Honebein reconvened the business session. The Woodward-Fondiller prize was awarded to Albert J. Beer for his review of Margaret Wilkinson Tiller's paper, "Estimating Probable Maximum Loss with Order Statistics."

Norman Crowder then convened the business session of the American Academy of Actuaries.

Herbert Phillips presented the Casualty Actuarial Society report of the Vice President—Administration.

Mr. Honebein delivered the Presidential Address and closing remarks.

The meeting adjourned at 12:15 p.m.

In attendance as indicated by the registration records were 231 Fellows; 68 Associates; and 42 guests, subscribers and students. The list of their names follows.

FELLOWS

Adler, M.
Aldorisio, R. P.
Alff, G. N.
Anker, R. A.
Bailey, R. A.
Barrette, R.
Bashline, D. T.
Bass, I. K.
Baum, E. J.
Beer, A. J.
Belden, S. A.
Bell, L. L.
Ben-Zvi, P. N.
Bensimon, A. S.
Berquist, J. R.
Bertles, G. G.
Bevan, J. R.
Bill, R. A.
Biondi, R. S.
Boone, J. P.
Bornhuetter, R. L.
Bothwell, P. T.
Bradley, D. R.
Braithwaite, P.
Briere, R. S.
Brooks, D. L.
Brown, N. M., Jr.
Brown, W. W., Jr.
Bryan, C. A.
Camp, J. H.
Carpenter, T. S.
Chernick, D. R.
Ciezadlo, G. J.
Cohen, H. L.
Conger, R. F.
Conners, J. B.
Cook, C. F.
Crowe, P. J.
Daino, R. A.
D'Arcy, S. P.
Dawson, J.
Dieter, G. H., Jr.
Dolan, M. C.
Donaldson, J. P.
Downer, R. B.
Dropkin, L. B.
Drummond-Hay, E. T.
Easton, R. D.
Egnasko, G. J.
Egnasko, V. M.
Eland, D. D.
Evans, G. A.
Faber, J. A.
Fallquist, R. J.

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Fein, R. I.
Finger, R. J.
Fisher, R. S.
Fisher, W. H.
Fitzgibbon, W. J., Jr.
Flaherty, D. J.
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Ford, E. W.
Foster, R. B.
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Furst, P. A.
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Gannon, A. H.
Ghezzi, T. L.
Gleeson, O. M.
Goddard, D. C.
Golz, J. F.
Gorvett, K. P.
Gottlieb, L. R.
Grannan, P. J.
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Graves, C. H.
Grippa, A. J.
Hafling, D. N.
Hall, J. A., III
Hallstrom, R. C.
Hartman, D. G.
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Hazam, W. J.
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Horowitz, B. A.
Hughey, M. S.
Johe, R. L.
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Johnson, M. A.
Johnston, T. S.
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Kleinman, J. M.
Knilans, K.
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Kucera, J. L.
Lange, D. L.
LaRose, J. G.
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Lino, R. A.
Liscord, P. S.
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McClure, R. D.
McConnell, C. W.
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Miller, M. J.
Mills, R. J.
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Nelson, J. R.
Newman, S. H.
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Nichols, R. S.
Nichols, R. W.
Niles, C. L., Jr.
O'Brien, T. M.
Oien, R. G.
O'Neil, M. L.
Paquette, S. L.
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Ryan, K. M.
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Smith, L. M.
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Tresco, F. J.
Tuttle, J. E.
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Venter, G. G.
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Walker, G. M.
Walker, R. D.
Walters, M. A.
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Weller, A. O.
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Wilson, R. L.
Winkleman, J. J.
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Yingling, M. E.
Young, B. G.
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Zory, P. B.
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Chorpita, F. M.
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Silverman, J. K.
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GUESTS—STUDENTS—SUBSCRIBERS

Adams, M. H.
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Del Fave, L.
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Schloss, F. A.
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Thompson, R.
Treitel, N. R.
Votta, J.
Weinman, S.
Weiss, R. H.
Whitby, O.

REPORT OF THE VICE PRESIDENT—ADMINISTRATION

The purpose of this report is to provide the membership with a summary of the significant activities of the Society during the past year.

1984 was the year of reorganization for a society that has shown rapid growth in membership, and the reorganization was indeed quite necessary. The Board of Directors, which has the responsibilities of setting policy (which cannot be delegated), disciplining members, and electing the Vice Presidents, met four times during the year; and the newly created Executive Council, which has the prime responsibility for the day to day activities of the Society, met seven times. In this regard, it should be noted that the membership has now gone over the 1,100 mark, to approximately 1,115 members. From 1964 to 1974 the Society grew 54% and from 1974 to 1984 the growth was an amazing 83%. At the May meeting in Scottsdale, Arizona, 60 new Associates and 18 new Fellows were announced, and at this meeting in Boston there are 7 new Associates and 35 new Fellows. The growth continues: therefore the need for this reorganization.

The major policy statements approved by the Board of Directors this past year were:

1. Approval of the revised Guides to Professional Conduct and the interpretive opinions thereto. These will appear in the 1985 edition of the Yearbook.
2. Directing an ad hoc committee to outline various practical means for, and problems of, establishing a process to "qualify" loss reserve specialists. This report is due early in 1985.
3. Approval of policy guidelines for site selection for future CAS meetings. These guidelines include geographic distribution and frequency of meetings. This is a difficult task as CAS growth in membership and attendance continues.

At the first meeting of the Executive Council (the President, the President-Elect and the four Vice Presidents), the main item on the agenda was the setting of 1984 goals and their order of importance. The number one goal was that the CAS survive. Since we are now meeting in Boston a year later, the Council obviously has accomplished its prime goal.

During this period of reorganization and transition, many projects and tasks have been completed by many people. The Society now has an organization chart spelling out reporting authorities. Each block within the organization pyramid, from the Board of Directors, to the Executive Council, to the President, right down to every committee chairperson, has a position description stating its respective missions, duties and authorities. This manual is to be maintained by the Vice President—Administration. It is updated as needed to keep every position description current. Copies of position descriptions are available on request for newly elected or appointed officers, directors, or chairpersons.

One uncompleted task, which will be completed by the first of the year, is producing a Policy Manual. This document will contain, on a prospective basis, all the policy statements voted by the Board of Directors. It also will contain the organization chart and position descriptions already mentioned. It is proposed that this manual be given to each newly elected director on the Board so that he or she is well aware of the CAS structure, who is responsible for what, and the previous positions taken by the Board. In the past, newly elected directors were at a disadvantage for the first few meetings; we hope this manual will eliminate that problem.

At the September meeting of the Board of Directors, the four incumbent Vice Presidents were re-elected for another one year term. They are:

Vice President—Administration	Herbert J. Phillips
Vice President—Development	Robert A. Anker
Vice President—Membership	Wayne H. Fisher
Vice President—Programs	Michael A. Walters

In addition, the Board reviewed and approved the new budget presented by the Vice President—Administration. Unfortunately, this results in an increase in both dues and examination fees. Each will increase $20 for the new year. The major reasons for this increase are the cost and volume of printing and the increased cost of running the CAS office—telephone, rent, postage, and the like. It is an all too familiar problem to all of the membership in their respective company operations. However, even with these increases, the CAS fees are still below those of the other actuarial societies. This is a credit to the CAS members, who have demonstrated their willingness to volunteer their services to the CAS, rather than having to rely upon a larger paid staff to operate. As an example, the CAS is the only actuarial society without an Executive Director; the CAS performs through member volunteers and two very dedicated and capable women in the business office.

Finally, the newly formed Audit Committee examined the books of the Treasurer and they were found to be in order. The year ended with an increase in Members' Equity of only $1,815.33, much less than was budgeted. The major causes of this small growth were budget overruns on printing and less than expected income from the sale of CAS publications.

Members' Equity now stands at $208,362.25, subdivided into $54,791.76 for the Michelbacher Fund, $8,922.62 for the Dorweiler Fund, $1,810.64 for the CAS Trust, and $142,837.23 for CAS Surplus.

Respectfully submitted,

HERBERT J. PHILLIPS
Vice President—Administration

FINANCIAL REPORT
FISCAL YEAR ENDED 9/30/84 (ACCRUAL BASIS)

INCOME	
Dues	$ 91,976.18
Exam Fees	76,925.25
Meetings	154,880.27
Proceedings	6,950.00
Readings	10,075.95
Invitational Program	5,700.00
Interest	28,429.31
Actuarial Review	269.00
Yearbook	792.00
Foreign Exchange	(1,370.83)
Miscellaneous	2,528.95
Total	$377,156.08

DISBURSEMENTS	
Printing & Stationery	$119,645.27
Office Expenses	76,103.14
Exam Expenses	1,643.15
Meeting Expenses	155,929.28
Library	809.02
Insurance	7,089.45
Refund—Dues	315.00
Refund—Exam	1,465.00
Refund—Meeting	7,983.00
Refund—Reading	212.45
Math Assoc. of America	2,000.00
Expenses—President	5,000.00
Expenses—Pres.-Elect	2,500.00
Outside Services	0
Miscellaneous	639.07
Total	$381,333.83

Income	$377,156.08
Disbursements	381,333.83
Change in CAS Surplus	$ (4,177.75)

ACCOUNTING STATEMENT (ACCRUAL BASIS)

ASSETS	9/30/83	9/30/84	CHANGE
Checking Account	$ 8,553.45	$ 35,866.94	+27,313.49
Money Market Fund	31,883.98	61,930.52	+30,046.54
Bank Certificates of Deposit	100,115.58	102,573.00	+ 2,457.42
U.S. Treasury Notes	99,971.90	99,971.90	0
Accrued Income	14,658.01	24,216.75	+ 9,558.74
Total Assets	$255,182.92	$324,559.11	+69,376.19
LIABILITIES			
Office Expenses	$ 27,000.00	$ 28,000.00	$+ 1,000.00
Printing Expenses	0	62,000.00	+62,000.00
Prepaid Examination Expenses	0	(273.14)	(273.140)
Meeting Expenses & Prepaid Fees	4,500.00	(3,500.00)	(8,000.00)
Prepaid Exam Fees	17,136.00	29,970.00	+12,834.00
Other	0	0	0
Total Liabilities	$ 48,636.00	116,196.86	+67,560.86
MEMBERS' EQUITY			
Michelbacher Fund	$ 49,367.64	$ 54,791.76	$+ 5,424.12
Dorweiler Fund	8,547.66	8,922.62	+ 374.96
CAS Trust	1,616.64	1,810.64	+ 194.00
CAS Surplus	147,014.98	142,837.23	(4,177.75)
Totals	$206,546.92	208,362.25	+ 1,815.33

Herbert J. Phillips
Vice President—Administration

This is to certify that the assets and accounts shown in the above financial statement have been audited and found to be correct.

Audit Committee
Walter J. Fitzgibbon, Jr., Chairman
G. Gregory Bertles
David M. Klein

CASUALTY ACTUARIAL SOCIETY
COMMITTEE ON RESERVES

POSITION PAPER*
CLOSED CASE METHOD FOR REVIEWING THE ADEQUACY OF LOSS RESERVES

Comparison of the cost of closed claims to reserves has been used for many years, often simplistically, to evaluate loss reserve adequacy. Recently a particular "closed case" method, developed by the Internal Revenue Service, has received attention within the insurance industry. The Committee on Reserves has reviewed this method for its adherence to sound actuarial principles. The Committee finds that the closed case method is seriously inconsistent with the Casualty Actuarial Society's "Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Liabilities" and is inappropriate for testing the adequacy of loss reserves. The following statement expands upon this finding.

Description of Method

In its basic form the closed case method of testing loss reserves examines claims by line of business which were reported and case reserved, but unpaid, as of an earlier reserve evaluation date and which have been settled subsequently.

It develops an "experience rate" by dividing the amount reserved for these settled claims at the reserve evaluation date by the total amount paid on them subsequently. The experience rate is applied to (divided into) total reserves, reported and unreported, as of the current reserve date to adjust current reserves to an indicated zero redundancy/deficiency level. Typically, the earlier reserve date (test year) would precede the current date by five to seven years, and the experience rate would be the average of the rate developed for each of the test years.

Implicit Assumptions

Application of the closed case methodology carries certain implicit assumptions. For its indicated results to be valid, satisfactory testing of the acceptability of these assumptions would be necessary. Major implicit assumptions are:

* This is a position of the Committee on Reserves and of the Casualty Actuarial Society Board of Directors. It is not a position of the entire Society membership.

(a) The relative strength of case reserves at the earlier reserve evaluation date, for claims that are settled by the current reserve date, is comparable to that of total reserves at the current reserve date.

(b) The relative strength of the estimate for incurred but not reported (IBNR) claims at the current reserve date is comparable to that of the case reserves. The implication here is that the combined frequency and severity components of the IBNR reserve are comparable in strength to the severity component alone of case reserves. Alternatively, if the strength of the severity component of the IBNR reserve alone is comparable to that of the case reserves, then the frequency component is exact.

(c) The relative strength of the reserves for reinsurance assumed from all sources is comparable to that of the direct case reserves.

(d) Estimates of credits for ceded reinsurance are proportional to the direct case reserves and to assumed reinsurance in their impact on relative adequacy.

Adherence to Actuarial Principles

The "Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Liabilities" outlines a series of principles which must be considered for a reasonable and appropriate review of reserves. A comparison of these principles to the closed case method clearly illustrates that this method does not meet the criteria established by the CAS for proper review or establishment of reserves.

Key principles outlined in this statement and corresponding deficiencies in the closed case method are:

1. "Loss reserving procedures should operate on well-defined groups of losses" and give consideration to all elements of the total loss reserve.

   The closed case method:

   (a) gives no consideration to IBNR claims or reopened claims in the determination of the experience rate.

   (b) ignores the extent to which reinsurance arrangements applicable to claims outstanding at the current reserve date might differ from programs in place for claims in the test years and the effect such differences might have on claims emergence and development patterns.

(c) has drawbacks even as a means for testing only the case reserves. The implicit assumption that the relative strength of case reserves has remained constant is always questionable absent a review of average outstanding values over successive periods. Further, the method does not consider claims reserved at the test date but not yet settled nor any changes in the reserves thereon. These are the claims likely to be in litigation with their ultimate settled values less certain. For workers' compensation, permanent disability claims and even certain temporary disability claims would remain open and not considered even though periodic payments are being made on them. Additionally, if the case reserves are meant to contain a provision for reopened claims, the closed case method of testing would not consider this element since the reopened claims would not have been specifically case reserved at the reserve evaluation date.

2. "Understanding the trends and changes affecting the data base is a prerequisite to the application of actuarially sound reserving methods. A knowledge of changes in underwriting, claims handling, data processing and accounting, as well as changes in the legal and social environment affecting the experience is essential to the accurate interpretation and evaluation of observed data and the choice of reserving method."

   "It is not sufficient for the actuary merely to apply historical analytical procedures in the calculation of reserves. Whenever the impact of internal or external changes on claim data can be isolated or reasonably quantified, adjustment of the data is warranted before applying various reserving methods."

   "A competent actuary will ordinarily examine the indications of more than one method before arriving at an evaluation of an insurer's reserve liability for a specific group of claims."

   The closed case method:

   (a) does not recognize or adjust for changes in size of distribution, external influences, operational changes, reinsurance retention changes, aggregate limit changes, or other underlying changes affecting losses;

   (b) is a straight application of a formula with no consideration of trends or changes affecting the data;

   (c) is generally used as an only method rather than in conjunction with other reserving methods.

3. "The actuary should be conversant with the general characteristics of the insurance portfolio for which reserves are to be established." There should also be a thorough knowledge of claims practices. This principle implies that having this knowledge will affect one's reserve evaluation.

   The closed case method does not fulfill this requirement in that:

   (a) it ignores general characteristics of the nature of losses between various lines of business. The method is assumed to work equally well for low frequency/high severity lines as it does for high frequency/low severity lines of business;

   (b) out-of-the-ordinary claims practices, such as discounting loss reserves, are not given special recognition;

   (c) it provides no variation for differences in settlement patterns among different groups of claims, which is contrary to the Statement of Principles note that "the length of time that it normally takes for reported claims to be settled will affect the choice of the loss reserving procedure";

   (d) all data is treated to be fully credible, with no consideration given to the lack of credibility of indications based on small volumes of historical data.

Proponents' Viewpoint

Proponents of the closed case method argue that it is improper to use estimates to test reserves that are themselves estimates. They believe that the use of a test period of claims settlements produces a more accurate indicator by which to adjust current reserves. However, proper use of estimates in no way violates the Statement of Principles. Rather, the closed case method ignores significant information, which can be valuable when used with proper analytical techniques.

Committee Position

The Committee on Reserves believes that the closed case method of testing the adequacy of loss reserves, as described in the foregoing statement, does not conform to sound actuarial principles. While the method provides indications as to the historical adequacy of case reserves, such indications are incomplete and may be misleading. The committee has no objections to the underlying data used in the closed case method. However, they are appropriate only when used with proper actuarial techniques. In general, the committee finds that the closed case method is unsound and should not be used to evaluate total loss reserves.

SPECIAL MEETING ON PROPERTY-CASUALTY RESERVES

Editor's Note

The following is an edited transcript of a portion of the joint meeting of the Casualty Actuarial Society and the Canadian Institute of Actuaries held in Toronto, November 6–8, 1983.

Two sessions on the subject of loss reserving are included. The first session deals with the general principles involved in loss reserving. The second session contrasts Canadian requirements and practices with US traditions.

The transcripts have been edited to clarify references to visual aids used at the meeting and, in general, to translate the verbal presentations for the *Proceedings* reader.

FRED KILBOURNE:

I am Fred Kilbourne, President of the Casualty Actuarial Society for another twenty-four hours. This is the commencement of our joint meeting, being the last day of the CAS meeting and the first day of the Canadian Institute of Actuaries meeting. I'd like to welcome all who are joining us and turn the podium over to Chris Chapman, the President of the Canadian Institute of Actuaries.

CHRIS CHAPMAN

Thank you, Fred. I want to take this opportunity on behalf of the Canadian Institute of Actuaries to express our delight with this very unique commencement to our meeting. It's really very unusual and a very welcome way to begin a meeting in the Canadian Institute. We are very pleased that we are able to have this joint meeting. I have been working with the casualty people so much recently that I now am going to refer to you as *the* Society. In any event, welcome. We are very much looking forward to having participation in this joint meeting by the members of the Canadian Institute who are not members of the Casualty Actuarial Society.

SESSION 1—CONSIDERATIONS GOVERNING THE RESERVING PROCESS

MARTIN ADLER:

Good afternoon. I am Martin Adler and with me today is Dave Westerholm. The title of this panel is "Considerations in the Reserving Process" according to the CAS program. I believe that the CIA program calls it "Considerations Governing the Reserving Process." Either way I assume that my discussion will be relevant.

In this presentation we will focus on the Casualty Actuarial Society's *Statement of Principles on Loss Reserves*. Our purpose is to provide a foundation for tomorrow's panels on loss reserving techniques. We are going to be very basic.

Let's start out then by defining a loss reserve. I will define it as an amount set aside to settle a claim. The key characteristic of a loss reserve is that it is an estimated liability. That is very important to grasp. The precise amount needed to settle a claim cannot be known until after the claim is settled. Then why bother to set a reserve? Why not wait until the claim is settled and simply record the precise payment? I assume that everyone in the audience knows the answer, but let me repeat it anyhow. An insurance company needs to estimate its reserves in order to make a reasonably accurate evaluation of its financial position at any given time and, ultimately, to ensure its ability to discharge its fiduciary responsibility to pay the claim it has insured. Of course, what is most important to the insurer is not so much the reserve on any specific claim but the total loss reserve. The reserve on an individual claim is only a building block to determine that total loss reserve. The total loss reserve for a well-defined group of losses represents the amount that must be paid in the future to settle all losses which have occurred on, or prior to, a particular accounting date. It is estimated as of a given valuation date. Because reserves are estimates, the insurers estimate of the total loss reserve will likely change from one valuation date to another, as more facts become known.

Now let me make some distinctions among different types of dates which are pertinent in reserve evaluations: The *accounting* date identifies the group of losses for an accounting or statistical purpose. The *valuation* date is simply the date the evaluation was made on that group of losses. The *accident* date is the date on which the loss occurred; or, in the situation in which the loss results from an accumulation of exposure, it is the date on which the loss is deemed to have occurred. Finally, the *report* date is the date on which the loss was first reported to the company. In practice it is more likely to be the date on which it was first entered into the company's statistical records.

Exhibit 1 is meant to show graphically the distinctions between the various dates to which I have referred. The left most date represents the date on which the accident occurred. The next one, as we move right, represents the date on which the claim was considered by the company to have been reported, probably the date on which the company actually opened the file and entered the claim into its records. Further to the right are the dates on which various valuations were made. Of course, the company does not make evaluations only on the last date of the quarter, as shown here. The valuation at the end of the quarter would simply be the reserve on record at that date. The accounting date shown is the end of the year. The claim will fall into the category of claims accounted for as of that date. There will be subsequent evaluations of the claim until it is finally settled.

Let's go back now to the concept of the total loss reserve. There are five elements, although most companies will not use all five. Rather, as I will show, in practice companies use various combinations of the five. The elements are:

1. case reserves;
2. the provision for future development on known claims;
3. the reopened claims reserve;
4. the provision for claims incurred but not reported, commonly referred to as IBNR; and,
5. the provision for claims in transit.

Case reserves are set for known claims. They may be the values for individual claims assigned by claims adjusters; they may be set by formula; or they may be some combination of the two. Depending upon company practice, the individual estimates may or may not have a provision for development. If the case reserves are set by formula, they may be derived by averages applied to all claims in a specific category, or they may be derived by applying a single bulk amount to all claims in that category. To provide insight into reserving practices, I am going to draw a distinction between the adjuster's estimates, which I will call "pure case reserves" and reserves set by averages.

To get a better grasp of the distinctions let's look at the life cycle of a typical claim reserve on Exhibit 2. This is an automobile bodily injury claim for the example. The specific reserve arose from an accident which occurred on the evening of April 15th. It occurred in the United States as the driver was rushing to get his tax return postmarked by midnight. At the moment the accident took place, the claim to the company was incurred but not reported. The particular policyholder did not report the accident until two weeks later. From the moment the company received notification, the claim was deemed to be in transit. After going through the appropriate claims department procedures, the claim was opened and entered in the company's records, most likely into its computer system. Because of the company's practice, the reserve was set by average the moment it was entered into the computer. Approximately three months later the claims examiner established his first estimate of the ultimate cost of the individual claim. As soon as that estimate was entered into the computer the claim became what I call a "pure case reserve" in the more restricted sense—that is, it was based on an individual estimate. Four to five months later, when reviewing the file, the claims examiner revised the estimate, based on the emergence of more facts. About six months after that, the settlement was agreed upon. Up to this point the claim was still a case reserve. Not until payment was actually made was the claim closed, eliminating any reserve for it. At that point there was some small possibility that the claim would have to be reopened at a later date. The company, however, does not know in advance which claims will have to be reopened. If it did, those claims should not have been closed. The reserve for reopened claims thus is set by formula.

I have given some general idea of the manner in which a company reserves for known claims. What about IBNR? By its very nature IBNR must be set by formula. The formulas may be simple or complex. Company practices also vary considerably regarding the elements that are included in IBNR. The formula must take into account which elements the company includes. Here are various combinations which may be used:

1. "true" or "pure"—that is, claims not yet reported, nothing more;
2. true IBNR plus claims in transit, which is a more likely combination;
3. true IBNR plus claims in transit plus a provision for development of known claims;
4. true IBNR plus claims in transit plus a provision for reopened claims; or,
5. all of the above elements.

Now I would like to discuss the conceptual difference between reserve maintenance and reserve testing. I have identified the five elements of the total loss reserve. I also have pointed out that companies will use different approaches to develop those elements or combinations of those elements. Establishing and following procedures to build the elements is what I call "reserve maintenance."

How does the actuary or reserve specialist know that the procedures are establishing adequate reserves?

One way would be to wait for all the claims to settle and simply compare the amounts with the reserve set as of the valuation date. It should be obvious that the company cannot wait that long; at least not on a line where it takes years to settle the claims. The company cannot wait that long to know the answer, and the actuary would not have his job very long if he used that approach. What the actuary tries to do is test the reserves. By testing, I mean that he will apply various assumptions statistically to see how the claims will run off. This is called a *prospective* test. Before making the assumptions, he will likely look at how claims developed in the past in order to gain insight into the adequacy of the reserve methodology. This is referred to as a *retrospective* test. The actuary may not necessarily apply the test to each element of the claim reserves. He is more interested in testing the aggregate reserve—that is, the total reserve for that well-defined group of losses under consideration.

In testing, the actuary will focus on the development patterns. Exhibit 3 is a simplified example of development of claims for accidents which occurred during 1980 and are therefore referred to as the 1980 accident year. It is meant to represent a specific coverage and for this example I have used auto bodily injury liability.

The first line shows the amount paid for those 1980 accidents during each of the first four years from the beginning of the accident year. The second line shows the reserve for known claims at the end of each year. The third line shows the cumulative amount paid through the end of each year and the fourth line shows the cumulative incurred amount as of the end of the year. You will note that the cumulative incurred amount for the accident year is equal to the cumulative amount paid plus the reserve at the end of each year.

We have not previously defined incurred losses. The general definition is that incurred losses for a specified period of time equal the losses paid during the period plus the change in loss reserves over the period. Now since an accident year starts out with zero reserves, accident year incurred losses simply equal the cumulative amount paid plus the reserve at the end of the given period.

The bottom half of the exhibit shows the ratios between successive valuation dates. Thus the first entry on the payment line, 4.0, represents the growth in cumulative payment, from twelve months (the accident year's age at the end of the first year) to twenty-four months. That is, $4 million paid as of the end of 1981 for 1980 accidents is divided by $1 million paid as of the end of 1980.

Each of the other ratios represents the cumulative amount at the later age divided by the amount at the earlier age. Dave will discuss at greater length the use of such ratios.

There are two other concepts I would like to discuss. One relates to claim counts. The number of claims is a very useful parameter for the actuary in evaluating loss reserves. The amount of losses incurred during an accident period, and thus by implication the reserves for losses not yet paid, is a function of two things: the number of claims incurred, and the average size of those claims. Consistency in counting the claims is essential to measuring both of those elements. Here, too, company practices differ. Company guidelines vary on when to open a particular claim file. This is particularly true for those claims which the examiners estimate will never materialize, but have been reported just to put the company on notice. Some companies put such claims into a suspense category. But the distinctions do not stop there. Even in regard to claims which are opened, companies will differ on how they count the reported claims. Some companies assign one count per accident; whereas others assign a count for each claimant in the accident. Differences also exist in regard to the counting of closed claims. Depending upon the purpose, some reports may count claims as closed only if a payment has been made, whereas others count closures regardless of payment. If a claim is closed and then reopened, some companies' statistical systems count the claim again. These differences plus many others make comparisons between companies very difficult.

Reserves also must be maintained for the cost of settling the claims. These costs are referred to as loss adjustment expenses. Loss adjustment expenses are divided into two general categories: allocated and unallocated.

Allocated expenses are those which can be assigned to a specific claim. Examples of such expenses are attorneys' fees, legal expenses, court costs, witness fees, and (for some companies) independent adjusters' fees.

Unallocated expenses, on the other hand, cannot be assigned to a specific claim. One may think of them as overhead to the claims settlement process. The most specific costs are the salaries and related benefits of the claims department personnel. But there are also the general overhead for the claims department, the cars used by the adjusters, the rent charged for the space the department occupies, the supplies needed, and so forth. Some elements of company overhead also are charged to the claims function; and for some companies independent adjusters' fees are considered as unallocated, rather than allocated, expenses.

Dave Westerholm will discuss pertinent considerations in the actuary's evaluation. After Dave's presentation we will entertain questions from the floor.

DAVE WESTERHOLM:

Thank you, Marty. In this half of the presentation I will focus on some of the more important considerations that must be addressed in the loss reserving process. I will start out by discussing homogeneity and credibility: two key, but often conflicting, considerations in any reserve analysis. I will then move into data availability. The availability, or lack, of relevant data plays an important role in the kind of reserve analysis you can complete, and in the degree of credibility you can place in the resultant findings. Emergence, settlement, and development patterns will then be discussed. As Marty pointed out these are the key items on which the actuary will focus when doing his reserve testing and analysis. Next, internal and external considerations—the factors that impact the loss development pattern of a group of claims—will be discussed. Finally, we will get into the application of professional judgment. We will discuss the need to apply judgment throughout the reserving process which, as most of you know, is essential, since in very few cases can you rely strictly on the results of a mathematical formula or model.

You can't discuss the homogeneity and credibility considerations adequately without getting into the law of large numbers, which often is misinterpreted to mean "more is better." More specifically the law means that the larger the volume of a sample of homogeneous data, the closer the experience is likely to be to the expected value for the universe from which the sample is taken.

Arthur Bailey, in his paper "Sampling Theory in Casualty Insurance," stated that the losses incurred during a given time period never actually reflect the hazard covered but are always an isolated sample of all the possible amounts which could have been incurred. When you combine these two statements and apply them to the homogeneity and credibility considerations of loss reserving, they tell you to organize your reserving data into groups of claims that exhibit similar characteristics and that will yield statistically reliable, i.e. credible, loss development patterns.

Thus, when you are grouping claims for reserve analysis, you want to group them on the basis of relevant factors that will impact their loss development patterns: line of business (workers' compensation, general liability, homeowners, boiler and machinery); coverage (bodily injury, property damage); primary versus excess; personal versus commercial; size-of-loss distribution; or settlement pattern.

Homogeneity and credibility, as I mentioned earlier, are often conflicting considerations. Credibility is increased by the proper homogeneous grouping of claims and by increasing the number of claims analyzed within each group. Homogeneity is increased by refinement and fragmentation of the total data base. Thus, in your homogeneity consideration you can reach a point of refining your data to such an extent that the resultant groups are too small to provide any credible development patterns. Therefore, each reserve grouping requires a balancing of the statistical credibility and homogeneity considerations.

If you could measure these two factors, you would want to continue to refine your data until the marginal increase in homogeneity is offset by the marginal decrease in credibility. I leave it to the more statistically minded to come up with a procedure to do this effectively. I think a few examples might help emphasize this point.

Let's suppose your reserving data claims are represented as shown on Exhibit 4 and you have four different types of claims, A, B, C and D. You can try to set a reserve by looking at the loss development patterns in total or you can break them into the four pieces. Some of you might recognize this picture as being borrowed from Stephen Philbrick's article on credibility. Let's get into some more specific examples.

Let's consider general liability. You can look at your GL losses in total. I would not recommend this unless you absolutely must. A better idea is to break them into bodily injury and property damage components. Better still would be to break them into OL&T, M&C, products, professional, and all other components; and if you still go further, break these into their BI and PD components, as shown in Exhibit 5.

Consider one more example: automobile. You can look at auto in total, but again you would be better off at least splitting it into the private passenger and commercial pieces. If you go that far, why not break it into the liability and physical damage components? Once you have gone that far, what about BI, PD, comprehensive, and collision? Now if you are really getting carried away, you can continue until you get what is noted in the upper left hand corner of Exhibit 6: a single, 27 year old female farmer in Manhattan who drives a 1981 corvette and has one safe driver point. There are not a whole lot of us who can get down to that level of detail with any credibility.

It's the reservist's job to make sure that the data required for reserve analysis is available and reconcilable, or else take steps to see that such data and procedures are developed. I generally like to have the following data types available for the claim groupings used in any reserve analysis (Exhibit 7): paid losses, outstanding losses, incurred losses, paid allocated loss adjustment expense, reported counts, closed counts, reopened counts, outstanding counts, and earned and written premium and exposures. With regard to how the data set is organized, I would organize it by accident year—a record of losses for claims which have occurred during a given twelve month period regardless of when they are reported; by calendar year—a record of all loss transactions which have taken place during a given twelve month period regardless of when they occurred; or by policy year—a record of losses from claims arising from contracts which became effective during a given twelve month period. Report year or notice year I generally regard as a finer breakdown of policy and accident year data on the basis of date of loss and date of notice relativities. For some lines, especially some of the casualty lines, it would be very beneficial to have limited or layered losses. For example, I mean losses where you have segregated the first $100,000 of each loss.

Regarding the reconciliation of reserving data, the reserve groupings that you deal with generally represent aggregations of more detailed company financial records. The data used in your reserve analysis must be reconcilable to official company financial records. You must verify the internal consistency of all your reports, making sure that nothing has "fallen between the cracks." For example, if you are reserving general liability, you might decide to look at only products, umbrella excess, OL&T, and M&C. If that is all you do, you probably have forgotten about owners and contractors protective and contractual liability. You don't want to implicitly set a zero reserve, so it is always good to make sure you have accounted for all of the pieces of data. Make sure your inclusions and exclusions are reasonable and make sure you can balance them with other company records.

Generally, you never have all the data you want. I am sure some of the consultants in the audience could tell real horror stories regarding the data they had available, given the assignment with which they were charged. Generally, you don't have all the data types you want, or it's not organized the way you require. If you are lucky enough to get both of those, you probably don't have historical claim developments for as long as you would like. It's in situations like this where you have to adapt, improvise, or—to borrow a line from *Star Trek*—boldly go where no actuaries have ever gone before. Come up with some new procedures to fit the situation. I think one of the best things that you can do is to step back and recognize your limitations, recognize the biases and constraints that are introduced due to incomplete or limited data, and try, to the best extent possible to quantify them. If nothing else, try to get some feel for which way the available data is going to lead you.

As Marty mentioned earlier, when the actuary is testing and analyzing reserves, he is focusing on loss development patterns and must recognize and attempt to quantify relevant factors which could affect the reserve and expected future loss development patterns. When analyzing loss development patterns for a particular group of claims, it's often helpful to look separately at the factors affecting the emergence and settlement patterns that make up the group's total loss development pattern. Emergence is defined to be the time between the occurrence of a claim and when it is recorded on the company books. Settlement is the time between the reporting of a claim and when it is settled. I have shown a couple of examples on Exhibit 8. Auto physical damage generally displays a short time between the emergence of a claim and when it is settled. At the other extreme, where there is generally a long time between emergence and settlement, are products and medical malpractice. Later on I will discuss in detail some of the key factors that you should consider that will affect the loss development patterns you are analyzing.

Very basically, reserving boils down to predicting future loss development patterns from actual historical loss development patterns. The top half of Exhibit 9 is a triangle of incurred losses for accident years 1973 to 1982 at twelve month intervals. Below it are the incurred yearly loss development link ratios: 12–24 months, 24–36 months, 36–48 months, etc. As a reservist all you have to do, assuming ultimate at 72 months, is predict what each accident year loss will be at 72 months of development. Without knowing anything about loss reserving, anyone with some mathematical background could do a number of things with these loss development factors to predict future loss development trends. You can take simple averages of them, trend them, look at the most recent five factors, throw out the high and low and take an average of the middle three, or any number of things. However, it's a terribly uninformed way to go about doing things. What you want to do, is find out and quantify the effects of what is occurring today, and what will occur in the future, which will produce loss development patterns materially different from historical trends.

On Exhibit 10 are listed some of the internal considerations you need to address. Generally, the relative adequacy of case reserves is not terribly important to the reserving actuary as long as it doesn't change. A basic underlying premise when beginning most reserve analyses is that history will repeat itself. If the claim department *consistently* has overestimated or underestimated their case reserves, it will be reflected in your loss development patterns. What you don't want them to do is change it. If you found out that historically they have been 10% deficient on initial reserve estimates, the worst thing you can do is tell them because they will probably increase their reserves by 10% (no one wants to be "wrong") and you now will incorrectly build in another 10% development on top of that.

Other changes you have to consider are changes in claim handling procedures, such as when the claim department implements a fast track or average reserve valuation system, common for some auto physical damage types of claims. Changes in claim counting is another possibility. Has the claim department switched between a per accident or per claim type of counting or have they implemented a bulk reserving type of procedure? Do allocated loss adjustment expense payments reflect a change from pay-as-you-go throughout the life of the claim to a pay-at-time-of-closing procedure? Has there been an acceleration or slowdown in loss payments? Has there been an increase in the use of partial payments? What about the use of structured settlements? All of these factors can have a significant impact upon the loss development pattern you are analyzing. Has the claim department decided to adopt a get-tough claim litigation policy? What about the use of company adjusters versus independent adjusters? This will switch dollars between allocated and unallocated loss adjustment expense. Changes in pricing strategy: it is very important to find out what our counterparts in pricing are doing. Has there been a coverage that has been added on for free with the thought that it won't produce many claims? Have we tried to "buy" our way into the market? What about changes in underwriting programs or guidelines; changes in new versus renewal ratios; changes in the types of reinsurance and retention levels; changes in policy limits and deductibles? All these factors are internal to a company and definitely can affect the development patterns.

External factors include participation in voluntary pools and associations such as the National Workers' Compensation pool, assigned risk and fair plans—these are costs of doing business. Inflation, both economic, which can be measured, and social, which generally cannot be measured, are other external factors. What about claims consciousness of the public? How will that affect the counts and dollar amounts in a given line of insurance? Seasonality of loss experience is a factor you may or may not want to reflect. Legal or legislative changes can be a major external factor. If we ever get an asbestosis decision on which theory to use—manifestation, exposure, band theory, or a combination of all of them—it definitely will impact how much money a company will have to set up on reserve. The products liability model law, no fault, comparative versus contributory negligence; all of these laws will impact given lines of business to different degrees. The general state of the economy will impact workers' compensation, fidelity, and surety claim developments—both frequency and severity.

Ideally, you want to quantify the impact of all of these factors for each of your lines of business or at least recognize that a given factor can impact the line of business you are looking at.

To arrive at your final recommended loss reserve for a given line of business you may have used two, three, or half a dozen different techniques. The reserve you end up with is generally some combination or average of them and that's where your judgment comes in. You have to realize that when you finally recommend a reserve that it is a point estimate of a company's outstanding liability and that you have estimated it based on (hopefully) the best available data at the time. Given the nature of the line of business you are dealing with and the variability of the reserve indications, you want to move slowly towards the "correct" reserve.

Whenever possible, you want to measure the reasonableness of your loss reserve against relevant parameters such as premiums or exposures so you can come up with some sort of frequency, severity, or loss ratios that make sense. Ideally, you want to use one technique that relies on paid losses; one on incurred losses; one that utilizes counts multiplied by averages; and one that uses limited or layered losses, so that you expose yourself to the various biases that can impact your data and see what different results you achieve using each of these different techniques. Then try to reconcile the differences between them.

Finally, the underlying assumptions and methodologies that you use should be documented and subjected to a sensitivity analysis. You want to document, wherever possible, your underlying frequency and severity assumptions, so that you don't have to start your reserve analysis from scratch each time. You want to have some sort of report card to keep score of the accuracy of your assumptions.

At this time Marty and I will try to field any questions that you may have. Thank you.

SPEAKER UNIDENTIFIED:

What management is the final decision maker?

MARTIN ADLER:

At what level of management is the final decision made? I believe it depends upon which company one works for. At my particular company, it's finally made, or at least the final veto is, at the chief executive's desk. At other companies, it's at the chief actuary's desk. I am sure there are other variations as well.

SPEAKER UNIDENTIFIED:

What about the time value of money?

MARTIN ADLER:

The question relates to the time value of money. We did not define that in the presentation. You might say we did not touch that with a ten foot pole. There is a difference in the way the companies treat the time value of money or, to use the forbidden term, "discount reserves." Some set reserves without consideration of discounts and others do it either explicitly or implicitly. It's simply an additional consideration, with a lot of ramifications on its own. It would take quite a long session to go into what would be done with discounting. We have not even come to grips with a general question of whether or not it should be done.

It's my personal belief that the reserves should have a margin for adverse development because of what I consider the fiduciary nature of the insurer's obligation. I think that it should be in terms of the absolute amount of reserve estimated, and if it is discounted for any reason, that the discount rate be assumed conservatively. That is, it should be relatively low compared to what one might hope for in terms of the value of investing the money behind the reserves.

SPEAKER UNIDENTIFIED:

There didn't seem to be much in the presentation discussing the reserve for allocated expense. How does one approach the reserve for expense?

DAVID WESTERHOLM:

At my company we have by-line paid allocated expense development by accident year from 1965 to the present. I monitor, by accident year, paid allocated to paid loss ratios; project them out to ultimate; and at the same time monitor calendar year allocated paid-to-paid loss ratios. Thus, given an estimated ultimate incurred pure loss, I can expect X% of it to be an estimate of ultimate incurred ALAE.

ED SHOOP:

I guess I don't have so much of a question as I do an observation. May I get your or anybody else's reaction? In thinking about reserves and choosing methodologies and techniques and how you go about doing it, whether it's incurred, paid and so on, two things always seem to stand out and tend to be overriding considerations that you just couldn't ignore. One is that in the absence of anything changing the value of a claim between the time it is incurred and the time it is closed, by way of something like a benefit change, the ultimate value you predict for a group of claims shouldn't change, so that each time you do the reserve evaluation you should come up, not with the same reserve, but with the same ultimate values. The second characteristic is that those ultimates ought to be correct. Regardless of the methodology that you choose, if it's doing those two things for you reasonably well—always producing the same ultimates—and by retrospective testing those ultimates proving out to be pretty reasonable, I think that you have done a pretty good job at that and I would like the reaction of people in the audience or yourselves.

DAVID WESTERHOLM:

I agree, as long as you say that you use some retrospective tests on it so that the technique you use isn't so ignorant of what is happening out there that no matter what happens it will produce the same result until something really drastic happens in your development factors. If the reserving technique you are using continues to predict the same ultimate, you must ask if it is because it's a good technique or just blind to something that is happening out there in the real world. But if you are reasonably confident that it does react to movements out there in the real world, you should come up with the same ultimate, or reasonably close indications, each time.

MARTIN ADLER:

Ed, do you really think that the reserve patterns are so stable? Exhibit 9 is something that is probably more typical. In fact, it's my observation that it is a fairly stable pattern of development from year to year. But if you were selecting a number for the twelve to twenty-four month development, you would have five numbers, or a combination thereof, to choose from. It is highly unlikely that you are going to select a factor which would be a multiple of all the possible twelve to twenty-four, that is, the year-to-year development ratios, which will exactly reproduce your estimates. In fact, if I got exactly the same reserve estimate one year later I would begin to question whether I was being open-minded enough in my analysis of the reserves.

ED SHOOP:

Maybe I didn't make my observation clear, but what I am saying is that, given all claims incurred for accident year 1977, the way they have developed is from 27.3 to 38.9 and, I assume that the no change from 60 to 72 months of development occurs because all the claims finally closed by the end of the 72nd month. What I am saying is that every time you run your reserve evaluation for accident year 1972, you should have developed 38.9, and the test of the goodness, so to say, of the methodology is if in fact this occurs. If back in the year 1977 you are in fact estimating something around 40 million and you continue to do that throughout all the subsequent evaluations for that accident year, and you develop about the same ultimate and it doesn't change, that's one good test of methodology. The second one is, "Did you get the number right"? If you can do those two things right for any particular block of claims, I think you have a good method.

MARTIN ADLER:

That's true, but you continually have to make sure that nothing has changed in the operations of the company that would make that inapplicable as a predictor.

SPEAKER UNIDENTIFIED:

I didn't know if you had any comments regarding whether the actuary should make some judgment regarding the likelihood that a certain event would take place, for example, a class action against the industry that may be three years before final judgment is made.

MARTIN ADLER:

What we have is a particular problem that has emerged in the United States in recent years. I am not aware of the extent to which it may be a problem in Canada as well. I call it "changing the rules of the game after the game has been played." The claims department settles claims under an assumption that a law works a certain way and then finds out, as a result of a class action case, that the industry loses four or five years later that they settled the claims wrong and everything is reopened.

My general answer is that the company has to have some kind of reserve for that event. It is obviously very difficult to quantify. I even wonder whether it's an IBNR type of reserve or perhaps a reserve for an event that has not yet occurred but for which the company's already responsible—the event not having occurred is the court decision. The actuary has a responsibility to consider that, but it's a matter for all of management to try and make the best estimate of how much is going to be needed for that.

Do you have anything to add, Dave?

DAVE WESTERHOLM:

In terms of reserving for asbestosis claims, the actuary should establish estimates on the basis of both the manifestation and exposure theory. The recommended reserve necessarily involves considerable judgment and will in all likelihood be an appropriate compromise between each of the two theories and what the company can afford.

MARTIN ADLER:

I really don't think that the actuaries possess all the necessary wisdom within the organization. If they do, the organization is probably in trouble.

PAUL SINGER:

Should such a consideration be incorporated in loss reserves at all or should it be treated as the event to be disclosed by the auditors?

MARTIN ADLER:

The question is whether the consideration should be in the loss reserves at all or whether it's a contingency amount to be disclosed by the auditors. I don't think that the definitive ruling has come down on this. The events that give rise to this type of situation are still relatively new. I think somehow there must be a reserve. I am not sure whether anyone could have foreseen the emergence of the asbestos problem—certainly not the magnitude of it. But there are other things such as class action suits that have a material, but not devastating, impact on the company which one might consider in the overall IBNR reserve that the company sets.

SPEAKER UNIDENTIFIED:

By their nature, they may turn out to be zero or they may turn out to be catastrophic. Reserves are merely disclosed to the possibility.

I have a more general question along that line. If you have a ten percent chance that you are going to lose a $100 million case and the result will either be zero or a $100 million that you pay, what is the reserve you set? If you follow the usual actuarial formula you put up the expected loss of $10 million and if that is all that's involved and you don't have a spread of these things, your expected reserve is going to be wrong. It's either going to be too high by $10 million or too low by $90 million. This is a more philosophical question and I don't think that this panel on basics is really equipped to handle it.

Sooner or later we are going to be told, and I hope that the actuaries have input in deciding just how it is going to be handled.

SESSION 2—COMPARING AND CONTRASTING U.S. AND CANADIAN PRACTICES

HERBERT PHILLIPS:

Good afternoon, ladies and gentlemen. Welcome to the second of the four panels of this joint meeting between the CAS and the CIA. The subject for this second panel as it appears in the CAS brochure is "Analysis of U.S. and Canadian Reserving Practices." I think the one that is in the CIA program is possibly more descriptive of what will be covered here today and it is called "Compare and Contrast." The three panelists are gentlemen who have had insurance experience in both the United States and Canada, two having worked extensively in the United States as well as in Canada.

While we have a common border and it is undefended, economies that are interwoven closely, a common language and so on, there are many differences as respects insurance operations in general and loss reserving in particular. So I now would like to introduce each of the three panelists in the order in which they will make their presentations.

On my immediate left is Mr. David Oakden, actuary with the Aetna Casualty of Canada, who will speak first. On my right is Mr. David Atkins, a partner with Coopers Lybrand in Canada with accounts in both the United States and Canada. On the extreme left is Mr. Alain Thibault, a consulting actuary with Blondeau and Company. He was previously in the company ranks and also has worked extensively in both countries. So with that, I will turn the podium over to Mr. David Oakden.

DAVID OAKDEN:

Thanks, Herb. Before we get into the more technical presentations with Messrs. Atkins and Thibault, I am going to spend the next few minutes giving you an overview of the Canadian insurance scene. Before I get to the Canadian insurance market, let us start with the country itself.

Canada, with an area of 3.8 million square miles, is the world's second largest country, yet the population is a mere 24 million people. Canada stretches 4,000 miles from sea to sea and yet 90% of the population live within 100 miles of the U.S. border. This must rank as the world's narrowest and longest nation. However, while Canada has a small population, it has the world's ninth largest economy, and with annual premiums of $7.3 billion, is the fifth largest market for property-casualty insurance in the world. Politically, Canada is a federation of ten provinces and two northern jurisdictions. The system of government is based on English parliamentary democracy. There are basically three major political parties in Canada. The Liberals, who form the current government, are slightly left of centre; the Progressive Conservatives are slightly right of centre (at times they are slightly left of centre); and the New Democratic Party I would describe as a far left wing party. Fortunately, they are the smallest of the three major parties in Canada.

At the provincial level, there are two other parties which are fairly significant. The Social Credit Party, which is the current government in B.C., is a right wing party. The Party Quebecois, which is the current government in the province of Quebec, is left of centre, and some would say quite a bit left of centre. The PQ are a very independent Quebec party. At the present time, neither one of these two parties plays a factor at the federal level but that could change.

The Liberals, under Trudeau, form the current government and in fact, they have governed Canada for almost the entire century with just a few exceptions. However, at the provincial level the New Democratic Party (that's the left wing party), is very strong in Central and Western Canada. In fact, they form the current government in the province of Manitoba and they have also governed in Saskatchewan and British Columbia. The fact that these three provinces have provincial auto insurance plans is no coincidence. With the Party Quebecois in Quebec, politics in Canada are much further to the left than they are in the U.S.A.

The federal and provincial governments are known for their co-operation. This fact is clearly illustrated by the fact that it took a mere 115 years to agree on the Constitution.

Culturally, Canada is split between the French and English communities. I could go on for half an hour on this, but I will keep my comments brief.

Twenty-five percent of all Canadians, including at least twenty-five percent of the actuaries in the Canadian Institute, are French speaking. There are significant French Canadian minorities in all the provinces. French and English are both official languages of Canada. However, French is the official language in Quebec and in the remaining provinces English is the official language. This can, and does, create problems for companies operating in both Quebec and the remaining provinces. In fact, many companies get around this problem by operating only in Quebec, or only in the remaining provinces. Others have Quebec subsidiaries to handle the special problems of Quebec.

Another unique factor about Canada is its winter; and people do joke about the winter in Canada. All of Canada does experience a severe winter and in fact, Canada's capital city, Ottawa, has a colder winter temperature than Moscow (in spite of the fact that Moscow is colder than Canada on average). As a result of this, loss ratios in Canada are about 8–10% higher in the first and fourth quarters than they are in the second and third quarters. This is a factor which must be contemplated in setting year-end reserves. When I was working in the U.S., I did not notice any significant seasonal variation in the loss ratio, although I believe that some lines do experience some seasonal variations.

The Canadian legal system in all provinces but Quebec is, like the U.S. system, based on British Common Law. However, contingent fees are not permitted; Canadians are less litigious; pain and suffering awards have not exceeded $200,000; punitive damages have not yet arrived; and awards are generally much smaller than they are in the United States. We have not had a medical malpractice, products liability, or asbestos crisis. Our excess limit factors seem insanely low to U.S. actuaries. Someone last night was telling me they took about 25% of the U.S. excess limits factors for use in Canada. Also, our reserves have a much shorter tail on third party lines.

Canadians are great savers. The savings rate in Canada is 15% versus a rate of about 5% in the United States. This is partially due to the higher interest rates in Canada; the favourable tax treatment for investment income; and the fact that mortgages are not tax deductible. However, I believe this higher savings rate is due also to the fact that Canadians are more conservative with their money.

Canada has converted recently to the metric system, as some of you may have noticed when you listened to the weather in the morning. However, we have abandoned the decimal currency as our dollar is now worth 81¢.

Canada is a safer place to live than the United States. The murder rate is one-fifth of the U.S. level and that is an incredible difference for a country that has basically the same society. Serious crime is much lower and it is safe to walk the streets of our major cities. However, things are tending to trend towards the U.S. direction.

Now I will turn to the insurance market. The regulation of insurance is split between the federal and provincial governments, with the federal government being concerned with solvency and the provincial governments being concerned with rates and day-to-day matters. Regulation, especially at the federal level, has been strong, consistent, and fair. The Federal Department of Insurance, I believe, enjoys a very good reputation. At the present time, there are about 200 companies or groups operating in Canada competing for that market of about $7.3 billion. Most of them have federal licenses which permit them to operate in all ten provinces; however, some regional companies operate under provincial licenses which, in some cases, are less restrictive.

The Canadian market is dominated by foreign insurers. In fact, only six of the largest fifteen insurance companies in Canada are Canadian. Four others are British and four others are American. The British influence is especially strong in Canada and I feel this is responsible for many of the subtle differences that the American actuaries will notice between the U.S. and Canada. The lines of insurance written in Canada are similar to those written in the United States. The major exception is workers' compensation, which is run by provincial boards; and health insurance, which has been nationalized for hospitals' and physicians' fees. Automobile insurance, as I mentioned earlier, also has been nationalized in three provinces: British Columbia, Saskatchewan, and Manitoba. Even with the defeat of the socialist governments that enacted these laws, the auto plans in these provinces have not been dismantled and are still in effect. In addition, Quebec has taken over the bodily injury portion of automobile insurance.

On the brighter side, there is very little rate regulation in Canada. All lines except auto are open competition and auto rates are regulated in only three provinces: Alberta, New Brunswick, and Newfoundland. The residual automobile mechanism in Canada is the Facility, or in most provinces now the Facility Association, which is similar to a JUA. The Facility originated in Canada in 1967, however, it now has been replaced in all provinces except Quebec by the Facility Association. Both the Facility and the Facility Association, while they have provincial bodies, are national organizations and, while they are separate legal entities, they have the same general manager and the same managing staff. More than one consulting actuary with a Canadian client has had trouble interpreting the reports set out by the Facility and I would advise you all to study them very carefully if you find yourself in a similar situation.

The company interests in Canada are represented by two bureaus. First the Insurance Bureau of Canada, or the I.B.C. (as we refer to it), to which almost all companies in Canada belong. It is the industry's statistical arm and in addition handles legal, research, and public relations functions. The second organization is the Insurers' Advisory Organization of Canada, or the I.A.O. This represents about half the market and is responsible for ratemaking, engineering, and inspection.

The actuarial interests in Canada are represented by the Canadian Institute of Actuaries. A Fellow of the Casualty Actuarial Society working in Canada automatically qualifies for membership in the CIA. A foreign resident must demonstrate a need before he is permitted to join and, as I found out last night, he also must continue to demonstrate that need before we will let him stay in the organization. In addition, life actuaries must pass a foreign specialty exam before they are permitted to join the Canadian Institute of Actuaries. This applies to Canadian residents and foreign residents. I believe it is only a matter of time before casualty actuaries also are asked to pass a specialty exam. Associate actuaries are not permitted to join the CIA, however, associates who are resident in Canada are permitted to join as students.

The legal definition of an actuary in Canada is membership in the Canadian Institute of Actuaries. This places the Institute in a very strong position vis-a-vis the American Academy. The Institute has had a good relationship with the Department of Insurance and in the past has played an important role in developing insurance regulations. I believe that this role will continue. The Institute holds three meetings each year. With the increasing number of casualty actuaries in Canada there are usually several workshops of interest to the casualty actuaries.

I will conclude my talk today by mentioning some sources of statistics that are available to actuaries doing work in Canada.

First the Insurance Bureau of Canada, the industry statistical arm, publishes automobile, personal property and commercial statistics. These are referred to as the "Green," "Brown," and "Red Books," respectively. I should warn you, however, that you should consider these exhibits very carefully. They were designed for non-actuaries and as a result can be confusing. They contain actuarial adjustments, such as loss development factors, and the expense treatment is unusual. You should not waste any time looking through the Green Book for any age or symbol information.

The annual statement required by Canadian companies comes in a green cover and, for clarity, it also is referred to as the Green Book. The Federal Department of Insurance has a data base of almost all the information on the annual statement. This is available either on tape or on a time-sharing basis for a slight fee. In addition, the summary of this data plus corresponding data for some provincial insurers is contained in the "Track Report" which is published by Collander Publications Limited. The Department of Insurance also publishes a volume each year with a summary of the industry results.

Statistics Canada maintains a data base for property casualty companies which is continuous since 1966. Their exhibits contain a detailed balance sheet and a revenue statement for the industry, as well as loss ratios for automobile, property and liability. This information is available on a quarterly basis. Statistics Canada is also a good source of general economic data in Canada. In addition, the Canadian Institute of Actuaries publishes selected economic figures each year.

Each year, the Canadian Underwriter and the Canadian Insurance magazine publish summary data on each company and group. Charts ranking the companies and showing premiums by province also are included. Stone and Cox publish the "Brown Chart" which shows the premiums in Canada by company group and by line and also by province. The Facility and the Facility Association publish monthly and annual reports to the companies in Canada. Also, most provincial insurance departments publish annual summaries of the results in their province.

I have tried to cover a lot of ground in a very brief period of time. I trust that you are now all experts in the Canadian insurance scene but, on a serious side, I hope that I have been able to convey some of the unique characteristics of the Canadian insurance market. I will now turn the microphone over to David Atkins, who will describe the Canadian annual statement and perhaps, if we are lucky, convey some of that unique British influence that I mentioned previously.

DAVID ATKINS:

As Dave has indicated, there are two kinds of federal insurers. There is the Canadian company and there is the Canadian branch of a non-resident company.

Their reports are somewhat different. They were very different in the past and they have come together. They are reasonably similar now, except there are still some minor differences.

The next point is that the annual statements filed with the federal authorities are on the basis of generally accepted accounting principles. This is a major difference between Canada and the United States. The only exception to GAAP is that these companies do not consolidate the results of their subsidiaries. They show their results on what is called an "equity" basis. There is an option not to follow deferred tax accounting, although that is rare. Most Canadian casualty insurers follow deferred tax accounting, so it is a GAAP statement that you are looking at for federal companies.

There are two types of provincial company financial statements: those relating to Quebec, and those relating to the other provinces. These statements are not prepared on the basis of generally accepted accounting principles. In particular, provincial companies show unearned premiums on a discounted basis to allow for deferred policy acquisition costs on a national basis, which of course is not a generally accepted accounting principle. All Canadian and provincial companies require an audit from an independent firm of chartered accountants, and it is likely that all Canadian branches of foreign insurers also will require an audit. This is contained in a new bill, which no doubt Bob Hammond talked about yesterday.

Just before we proceed to the treatment of investments for federal companies, I would note that the provinces are getting together to advance the method whereby they require the companies within their jurisdiction to report in a special way in the area of investments. The provinces are beginning to recognize some form of unrealized gain or loss through the income statement of provincial companies. This is not yet law but, to a certain extent, the provincial Superintendents of Insurance are considering it seriously.

Back to the federal companies. I generally will restrict any discussion to federal companies. (When I don't mention the jurisdiction, it will be federal because most companies here are federal companies.)

As far as investments of federal companies are concerned, bonds are shown at amortized cost; that is, on a yield basis or a straight line basis. Stocks are shown at cost. The deferral or amortization basis, which I will explain, is permitted. When a bond is disposed of and there is a realized gain or loss, that realized gain or loss may be amortized to the date of maturity of the bond. This enables some recognition of the yield inherent in a realized gain or loss on a bond. There are some rules associated with the practice. There is normally a requirement for replacement by a similar security, and one certainly is not disposing bonds for trading purposes or to liquidate the portfolio.

There is an investment valuation reserve. This reserve recognizes market declines of investments. It is treated as an appropriation of surplus, not as a liability, and there is a gradual approach in recognizing market declines on stocks. I believe it is two or three years. (I think now it is three years.)

In Canada, expenses are allocated as to premium acquisition costs, claims, investments and general expenses. The premium acquisition costs are deferred in line with the unearned premiums and, of course, we go through the process of assessing the recoverability of deferred premium acquisition costs. Claim expenses include both external and internal adjustment expenses. In assessing the recoverability of deferred policy acquisition costs, accountants here do look at the yields on investments and use some form of a discount in trying to assess the recoverability of DPAC. If that is done, then that fact must be disclosed in the notes of the financial statements and the yield rate disclosed.

In regard to losses, there is a five year run off on exhibit 35 in the Annual Statement, which, incidentally, is not public information and is not obtainable from any of the sources mentioned by David Oakden. There is some discussion as to whether that exhibit will be available to the public in the future and, judging from the current attitude of officials of the Department of Insurance in Ottawa, I would say that it will become available. Incidentally, the exhibit will be breaking out reinsurance ceded and it also analyzes the IBNR inherent in the losses by year. So there will be far more disclosure of losses in Canada in the future, if the federal officials have their way.

Discounting of loss provisions, and I can use that expression as an accountant and not use loss "reserve," is permitted and it is a good principle. The only problem is in its application—in trying to assess the appropriate yield rate and in trying to assess the appropriate term. I have seen it done. It is extremely difficult and this is normally when I obtain the services of a casualty actuary.

We also have premium deficiency provisions in Canada. If there is a premium deficiency, first the deferred policy acquisition costs are written down and, when they have been written down, then a provision occurs up on the right-hand side of that balance sheet. Again, yield rates on investments are taken into account and, if that practice is followed, it should be disclosed along with the yield rate used.

As stated, only the main exhibits of these federal insurance companies are available to the public but the really interesting data still is hidden.

There has been much greater emphasis on reinsurance in Canada. We have had about nine company failures in the past fifteen years. These are relatively small companies and possibly three of those failures can be attributed to poor loss reserving. The vast majority of those failures have been the result of the inability to collect on unlicensed reinsurance, or a misunderstanding of terms and an unwillingness to pay on the part of the reinsurer. That has been the real problem in Canada—collectability of reinsurance—and, as in the United States, the notes to the financial statements of insurance companies should disclose the contingent liability of the netting of reinsurance against outstanding claims. That figure should be shown as a contingent liability.

In addition, it is likely that chief executive officers of insurance companies in Canada will be required to sign some kind of a memorandum or report setting out their existing reinsurance arrangements and their strategic plan for future reinsurance arrangements: net retention, and so on. That report will be submitted to the Superintendent of Insurance in Ottawa.

There will be some statement of existing reinsurance programs and impending and proposed reinsurance. We also, of course, are deeply influenced by the AICPA, such as the United States guideline on auditing for reinsurance. In other words, it is essential as an auditor that one finds in one's client the controls over reinsurance that one feels should be there. For example, where a company is ceding business into the reinsurance market, one assesses the reinsurer's ability to pay and meet commitments. In terms of assumed business, one should find controls assessing the timeliness and accuracy of reports received from ceding companies. Those controls should be in existence. We are very similar to the United States: our concerns are identical.

Turning to federal regulation: all federal insurers are subject to examination by the Federal Department of Insurance and, of course, to its supervision. These examinations are on the annual accounts, but they are often quite late. When you get an early examination, you can start worrying. If they delay that examination, you can relax a little bit. The examiners work closely with auditors. We do get calls from the Federal Department asking if they can look at certain files. Those files are never released without the client's permission. Frequently, however, the client is only too delighted that we can explain certain things to the Federal Examiners and, with our client's permission, we do that. So we work closely with them in that way. February 28 (like you in the States) is the deadline in Canada for submission of the annual statement; but, unlike you we get a 15 day grace period for reinsurance companies. They normally file on March 15.

In regard to Department of Insurance reserves: these are treated as an allocation of surplus, except for guarantee reserves which are treated as liabilities. These reserves include non-admitted assets such as over-ninety-day balances, furniture, fixtures, and prepaid expenses. There is a reserve for unlicensed reinsurance (I guess you would call it unauthorized reinsurance) which effectively is a reserve equivalent to the net amount that would be receivable from that market, if the company had to collect on every single reinsurance amount due to or from it on a wind-up. There is the investment valuation reserve that I mentioned earlier. The guarantee reserve for fidelity and surety is based normally on premium volume. There is a reserve for excessive deferred policy acquisition costs, and there are special solvency ratios used in Canada.

I would like to talk a little bit about these solvency ratios. You probably have heard about the 15% add-on for outstanding claims and you may have heard also of a potential 15% add-on for unearned premiums, dependent upon the loss ratios. Of course, these solvency tests are assessed after deduction of Department of Insurance reserves (i.e., on the free surplus and capital). Canada looks as if it is moving too towards the EEC solvency ratio, which is a volume-to-surplus type ratio, combining both premiums and claims. The European Economic Community ratio takes into account reinsurance, but only gives credit of up to 50% of it. It uses a three year average and, if losses exceed a given ratio, then there is a flip into claims so that claims become the basic method of computing surplus. So we are moving towards a EEC type of reserve in addition to our existing solvency ratios. One still sees the old three-for-one ratio being used as well (in the back pages of these annual statements). So those are some of the solvency ratios.

Concerning actuaries and auditors: the hallmark of a professional is to know when he's getting out of his depth. I think this applies to accountants as well as to actuaries. There is presently a joint task force of the CIA and the CICA, which is the Canadian Institute of Chartered Accountants. We are looking at the relationships of actuaries to auditors. Let me give you some ideas as to how we are pursuing this.

The auditor obviously needs the actuary in the life insurance environment, but we are not here to discuss that. The auditor definitely needs the actuary in some tricky areas of loss reserving and, when discounting is being used, I think a casualty actuary is vital. Certainly in assessing premium deficiencies, a casualty actuary is vital. Very frequently, the auditor needs the casualty actuary. I would think that the actuary would need the auditor when it comes to assessing the validity of data: assessing, for example, the solvency of reinsurers; or assessing the completeness, accuracy and validity of accounting transactions making up claims. We are working out ways in which we can use each other's services: not necessarily delineating lines of competence—that is always a dangerous thing to try to do—but rather addressing the manner in which we will be working with each other.

I think, viewed in that light, we have these professions working together. Both professions have a lot to give to the industry, providing that we can work together. I think that would be absolutely fabulous. We are working that way in Canada and its coming off very nicely. There will be a joint task force report, produced probably within the course of the next two months, to each professional body. That report will not be authoritative until the actuaries have decided to adopt it at their institute and the accountants have decided to adopt it as well. But we are moving ahead and it's a very good sign. Thank you very much.

ALAIN THIBAULT:

Thank you, Dave; ladies and gentlemen.

Well, you know being part of the minority can be at times a frustrating experience, and I would think that most people have experienced this at one time or another, or in one way or another during their lives. But frankly I have to admit that being a Canadian, French-speaking, property casualty consulting actuary is stretching the concept of minority status to its dangerous limit. The danger, of course, being falling into non-existence. Needless to say, I am reminded constantly of my humble position in our actuarial profession. I have come to take this philosophically. However, I have to say that I never have been as conscious of my position as the day when Carl Honebein, for whom I was working at the time at Fireman's Fund in San Francisco, got upset at me because he had just found out that I could not even qualify for his affirmative action goals. This is why I feel very privileged today to have a chance to be heard and I would like to thank Herb, the CAS, and all of you for the opportunity.

After these two excellent presentations I think we now have a pretty good overview of what the Canadian insurance and accounting environments are like. What I would like to do is give you my opinion of the state of loss reserving in Canada.

We have seen that there are many differences between our environments and in itself the existence of difference should not affect the theory and the objectives of loss reserving; but, in practice, it is having an impact on the development of this activity here in Canada and on its importance. It probably would be fair to say that generally in this country loss reserving as a rigorous science is in its infancy. Of course, some form of loss reserving does take place in every company. However, it is only in the most recent years that a handful of companies, mainly the larger ones, really have started to devote the time and efforts necessary to develop the information systems and also the reserving methodologies that are needed to control this area properly. Some of these companies have put in place practices that are sophisticated and could compare with what you would find in many of the larger U.S. companies. For the majority of companies in Canada, however, the loss reserving process is based strictly on the case-by-case approach and normally includes an IBNR provision which is determined in a more-or-less arbitrary manner. Overall analysis techniques are largely unknown. Even the use of fast track or average reserves is only starting to get wider acceptance. While the science of loss reserving in this country may not yet correspond exactly to the ideals that most of us in our areas are striving to attain, there are a number of practical reasons that can explain why reserving perhaps has not received so far the kind of attention that we think it deserves.

First of all, we should point out that there is in Canada an obvious shortage of qualified people, actuaries or others, who have not only the technical skills to establish a reserving process from scratch but also have obtained the experience and the status in their companies to get the support from their employers and the commitment of resources.

Although the property-casualty actuarial profession is growing at a substantial rate here, actuaries are still a relatively new and rare commodity. Since there is a lot of work to be done in all areas of our business and just a few of us to do it, the priorities have not always been placed on loss reserving. Probably another factor behind the lack of emphasis that has been placed on loss reserving is a relatively smaller exposure to long tail reserve development. To elaborate further on this, it might be helpful to briefly review some of the data that will give us a more concrete idea of the significance and makeup of loss reserves for the Canadian industry.

The figures I have compiled represent about 85% of the Canadian insurance industry and they include all Canadian federal companies and foreign insurers operating in Canada, but they exclude provincial companies. Total loss and loss expense reserves at the end of 1982 were approximately \$4.1 billion. This represents about 68% of the earned premium volume of \$6 billion for 1982. If we want to have a different measure of the significance of loss reserves, we can compare them with the industry's capital and surplus. With the latter accounting to about \$3.9 billion on a GAAP basis, as Dave has explained, we see that the reserve-to-equity ratio is almost one-for-one. If we were looking at equity on a traditional statutory accounting basis then the reserve-to-surplus ratio would be about 20 points higher.

If we look at the reserves by line of business, we see that auto liability and accident benefits represent by far the most important lines with about 40% of total reserves. General liability comes second with 23% of the reserves, and property follows closely at 22%, while all other lines combined represent about 15% of our reserves. As we can see, the lines that have a potential for a long term development represent about 63% of our reserves.

One last item I would like to review is the rate at which payments actually materialize. Since industry data are not available in this format I have obtained this information from a large company having a book of business that I believe is representative of the industry. These data show the cumulative percentage of accident year losses incurred which have been paid after 12, 24, 36 months, etc., for all lines of business combined. About 50% of our losses are paid the same year in which they have been incurred. This proportion increases to 82% twelve months later and two years after the close of the accident year almost 90% of the losses have been paid.

Although this conclusion does not necessarily apply in the case of each individual company, this quick analysis shows that our industry is not highly leveraged and the potential inadequacies in reserve levels probably could be absorbed without excessive pain. Further, we have mentioned that our exposure to long tail development is less than in the U.S. and accident year results materialize relatively quickly. In this context, perhaps it should not be surprising to find that the industry has not placed more emphasis on the development of improved reserving methodologies.

Why is the long term exposure relatively less significant in Canada than in the U.S.? Well, the fact that workers' compensation is not written in the private sector is certainly a part of the reason, but there are also a number of differences between our legal systems that can further explain the situation. For example, our courts generally have maintained a more conservative approach than in the U.S. and the concept of negligence has not been eroded to the same extent.

One important difference mentioned by Dave Oakden is that, unlike the U.S., in Canada juries usually are not involved in civil cases but only in criminal cases. The judge, who is less likely than the jury to be overly sympathetic to the plaintiff's case, fully controls the outcome of the trial and decides what damages are granted. Awards for pain and suffering generally are kept to a reasonable level. There is also a difference between our two countries in the way attorneys are compensated. In the U.S., it is common practice to have the attorney's compensation based on a percentage of whatever amount he is able to win for his client. With these contingent fees the claimant has little to lose by suing. In Canada this practice is prohibited and this will normally discourage most people unless they feel they have a strong case.

Perhaps because of these reasons, and also because of general public attitude, Canadians do not have the same propensity to claim for damages and take legal action. In general, our traditional emphasis has been on the interest of the collectivity as well as on individual rights. This has probably contributed further to keep the ultimate costs for the liability insurance system under greater control.

Another major reason for the slow development of loss reserving techniques in Canada probably depends on the structure of the market itself. A survey of all Canadian federal companies and foreign insurers, 280 companies altogether, indicates that the average loss reserve was about $14.5 million at the end of 1982. More than half of the companies had loss reserves smaller than $5 million and 75% of the companies had reserves of less than $25 million. There is obviously not much incentive for the vast majority of companies to develop any kind of complex reserving methodology.

The one last factor that may have contributed to the slow development of loss reserving techniques is the relatively confidential nature of insurance company results in Canada in comparison with the U.S. While a summary of each insurer's results is published each year by the Superintendent of Insurance, the annual statements themselves are not public and no data on the reserve developments are made available. Also, all but a few companies are either private companies or branch offices of foreign insurers, and do not have to make detailed financial statements available to the public at large. A company's reserve position is not, therefore, under constant scrutiny by stock analysts, competitors, and the public in general. Conversely, a CEO has no means of comparing the performance of his company from a reserving standpoint with that of his competitors. This reason, in addition to those mentioned earlier, illustrates why modern loss reserving techniques are only starting to be implemented. However, there are potential changes on the horizon that could significantly impact the attention that loss reserving has received and bring about a much more rapid development of this activity.

I think we have touched upon these developments but, at the risk of repeating what has already been said, I can give you a brief overview of what is coming. On September 20, 1982, probably as a direct consequence of the recent bankruptcies of two insurers—something that Canadians had almost come to forget could happen—the Federal Department of Insurance issued a memorandum outlining a series of proposed legislative changes that could have a significant impact on our industry. These proposals were designed to increase capitalization requirements, control the utilization of reinsurance, and tighten reporting requirements. The main changes proposed include an increase in the minimum capitalization requirement for a new company from $1 million to $5 million. Thereafter any company whose capital and surplus fell below $4 million would have its license automatically revoked. A new minimum capital formula also would be implemented for ongoing companies based on a combination of premiums and claims volume.

Reinsurance transactions also would be regulated. New and small companies could cede reinsurance only to authorized reinsurers and no company, with the exception of the new ones, would be allowed to cede more than 50% of its premiums. New companies for a period of five years would be allowed to reinsure up to 75%. A solvency guarantee fund would be created to which all federal and foreign insurers would have to contribute. Provincial companies would participate on an optional basis.

Another area of change that is of direct interest to the actuarial profession would require every insurer to have its loss and loss expense reserve, as well as its unearned premium reserve, certified by an actuary, which in Canada means a Fellow of the Canadian Institute of Actuaries. However, since the department recognizes that there is not a sufficient number of actuaries to fulfill the demand this would suddenly generate, the proposals also provide that a non-actuary meeting certain qualification requirements could certify a company's reserves if this company could demonstrate that it was unable to secure the services of a fully qualified actuary. The implication would be that, over time, the responsibility for the certification would be completely assumed by the actuarial profession as is the case in life insurance.

Already one year has gone by since these proposals have been made public and the necessary legislative amendments have not yet gone to Parliament. With the general improvement in the industry's results in 1982 and 1983, some of the pressure to get these changes enacted quickly may have disappeared. Whether or not, and at what time, actuarial reserve certification will become required remains unclear. However, the proposals already have opened an interesting debate on loss reserving and have created a greater awareness of the industry's needs and weaknesses in this area.

I spent the last few minutes talking about current reserving practices in our industry and observed that there is really a long way to go before loss reserving is performed on a scientific basis. The first major challenge that the valuation actuary will encounter in most companies will be the absence of the minimum information necessary to a reserve analysis. I would think that reserving standards would have to be phased in over a certain period of time as information systems are developed. It's not clear how difficult a job it will be to have those information systems implemented. While the industry does not oppose the principle of reserve certification, it seems obvious to me that few insurers, if any, initially perceive any benefit for themselves—especially if they previously have not deemed it desirable to put any more than a minimum effort into loss reserving. In this context it may be a very difficult task for the actuary to obtain the necessary support and financial commitment to make this exercise as worthwhile as it can be.

Another issue that we will face will be the size of the average company whose reserves we have to certify. As mentioned, 50% of the companies have reserves under $5 million and 75% are under $25 million. The question that arises concerns the role, from the loss reserving standpoint, and the cost benefit, I should say, of an actuary in a company with only $5 million in reserves spread over five different lines of business. At what reserve level do actuarial techniques start to have a minimum of statistical as well as practical meaning? At what point does our role really start to become different from that of the claims examiner or the accountant? Reserve certification will require our profession to do a serious introspection about the way we are to approach the small company situation. Some form of actuarial standards will have to be developed and it will be very important that we are able to recognize our strengths as well as our limitations.

Other issues that will arise relate to a variety of questions such as the role of the valuation actuary versus that of the auditor; and, the scope of certification with respect to reinsurance, especially in a heavily reinsured company. We have heard Bob Hammond tell us that he expects a valuation actuary to form an opinion on the soundness of the company's reinsurance program as well as on the recoverability of the reinsurance reserves. Needless to say, this will be a very challenging task for the actuary. Another issue will be the nature and difficulty of our involvement in determining unearned premium adequacy. How are we going to approach the case of a reinsurance company, for instance, or else a company with a large volume of commercial lines business in an environment where commercial lines pricing is not even controlled or monitored, which is the case in a lot of our companies? This actually means that the actuary will be asked to assess the company's underwriting practices and marketing strategy. Reserve certification, in the way it is being proposed, will pose a major challenge to our profession. To succeed we will need to be thoroughly familiar in all aspects of the company's operation. What I call the number-crunching approach is not going to do the job, and in addition, more than ever before, the ability to communicate effectively will be an indispensable asset.

I believe, as Bob Hammond mentioned, that certification could be a tremendous boost for our profession and ultimately a great benefit to our industry. The risk of failure will be equally significant and cannot be ignored. There is no doubt that to succeed we will require a great deal of dedication and leadership from every one of our members.

Thank you for the opportunity to share my thoughts on the state of loss reserving in Canada and the challenge faced by the property casualty actuary.

HERB PHILLIPS:

I believe the panel has done an excellent job by exposing, in about 70 minutes or so, the significant differences between the countries, and the credibility problem we definitely are going to have in Canada because of the overall size of the economy. I think that we have, with this presentation, presented to you the differences in the Canadian environment so you realize the potential problems, particularly those of you involved with United States or British subsidiaries.

SPEAKER UNIDENTIFIED:

In the proposals for certifying reserves and the valuation actuary, do they have to be independent or can they be employees of the company if they are a member of the CIA?

ALAIN THIBAULT:

I would have to think that an employee of a company could certify a reserve. I do not think that this has been conclusively determined yet.

FRED KILBOURNE:

This concludes the afternoon session. Thank you very much.

EXHIBIT 1

LIFE CYCLE OF A CLAIM RESERVE

RELEVANT DATES

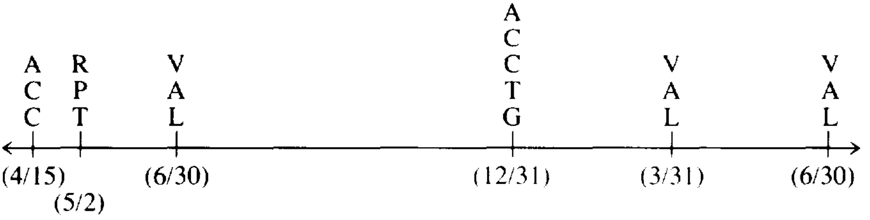


EXHIBIT 2

LIFE CYCLE OF A CLAIM RERSERVE

Date	Activity	Status
4/15/80	Accident Occurs	IBNR
4/30/80	Accident Reported	In Transit
5/02/80	Entered Into Records (System)	Avg. Reserve
7/28/80	Individual Reserve Estimate	Case Reserve
12/17/80	Estimate Revised	Case Reserve
6/04/81	Settlement Agreed	Case Reserve
6/11/81	Payment Made	Closed

EXHIBIT 3

1980 Accident Year Development

	Activity Year			
	1980	1981	1982	1983
Amount Paid ($000)	1,000	3,000	2,000	1,500
Reserve ($000)	5,000	3,500	2,500	1,500
Cumulative Paid	1,000	4,000	6,000	7,500
Cumulative Incurred	6,000	7,500	8,500	9,000

	Development Ratios		
	12-24	24-36	36-48
Payment	4.000	1.500	1.250
Incurred	1.250	1.133	1.059

EXHIBIT 4

Homogeneity & Credibility

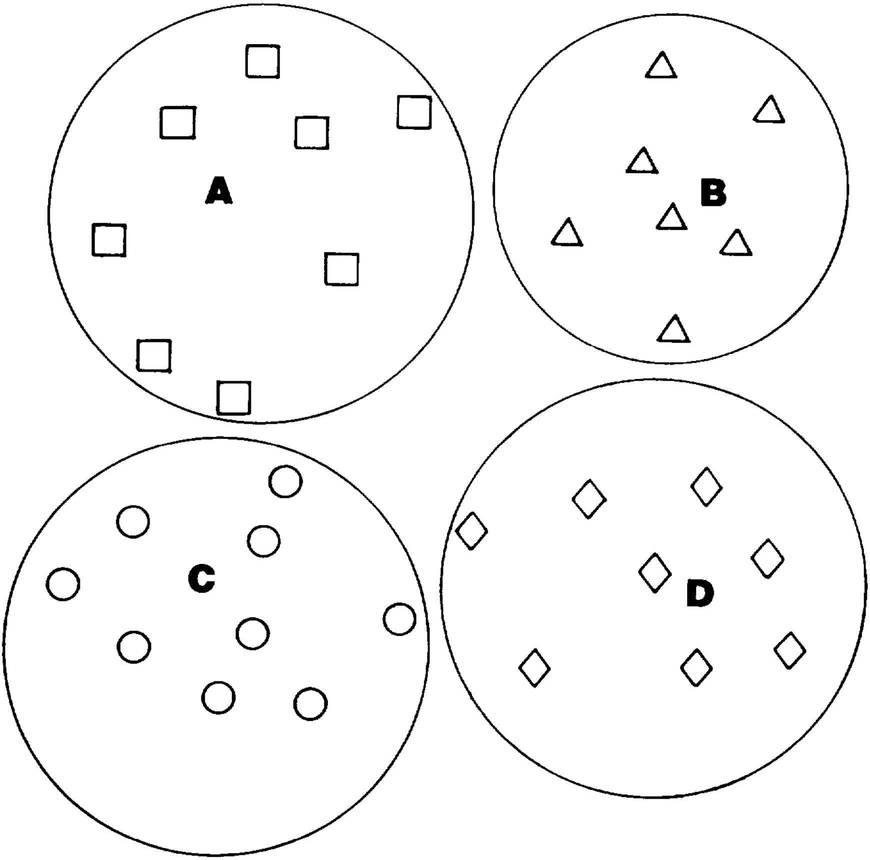


EXHIBIT 5

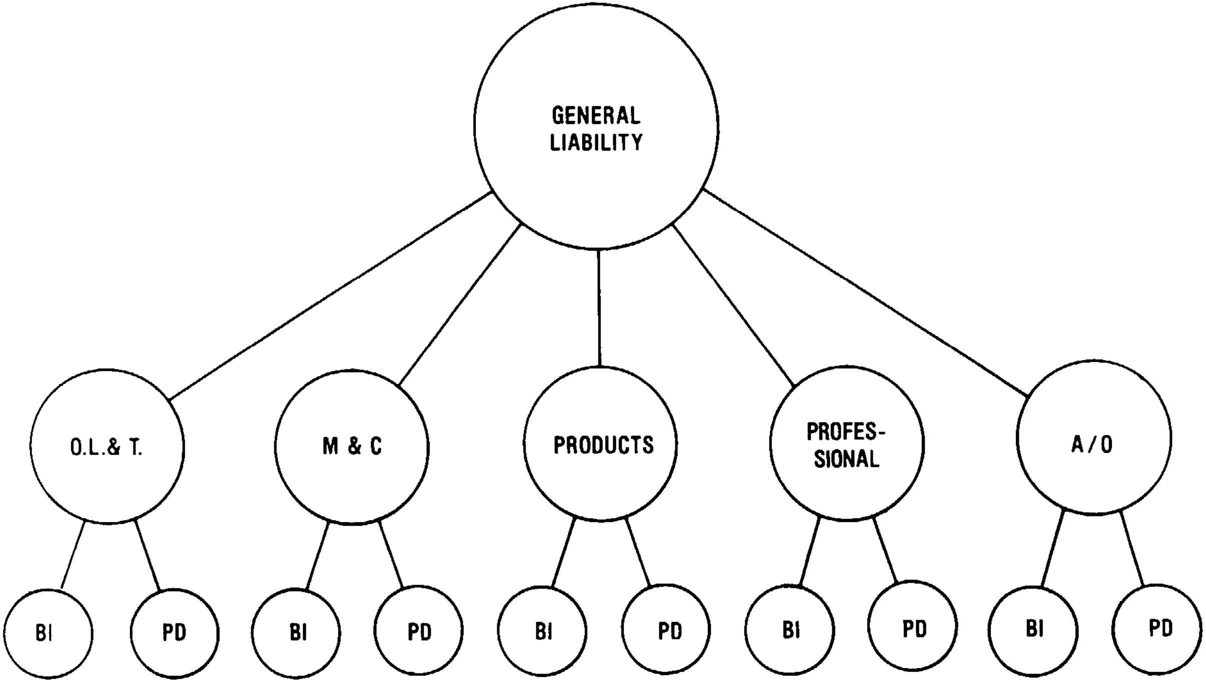

GENERAL LIABILITY
O.L.& T.
M & C
PRODUCTS
PROFES-SIONAL
A/O
BI
PD
BI
PD
BI
PD
BI
PD
BI
PD

EXHIBIT 6

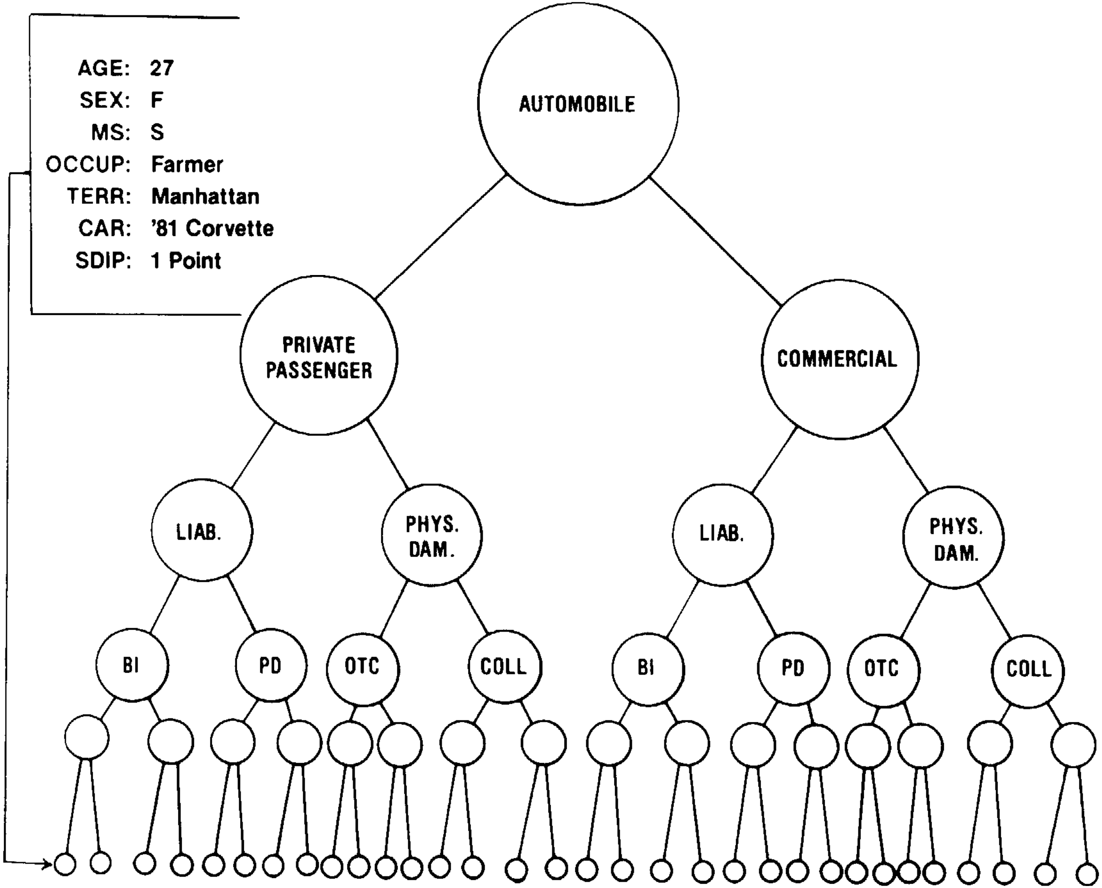

AGE: 27
SEX: F
MS: S
OCCUP: Farmer
TERR: Manhattan
CAR: '81 Corvette
SDIP: 1 Point
AUTOMOBILE
PRIVATE PASSENGER
COMMERCIAL
LIAB.
PHYS. DAM.
LIAB.
PHYS. DAM.
BI
PD
OTC
COLL
BI
PD
OTC
COLL

EXHIBIT 7

DATA AVAILABILITY

I. Data Types
   a. Paid Losses
   b. O/S Losses
   c. Incurred Losses
   d. Paid ALAE
   e. Reported Counts
   f. Closed Counts
   g. Reopened Counts
   h. O/S Counts
   i. Earned & Written Premium/Exposures

II. Data Organization
   a. Accident Year
   b. Calendar Year
   c. Policy Year
   d. Report Year
   e. Limited/Layered Losses

III. Reconciliation of Reserving Data

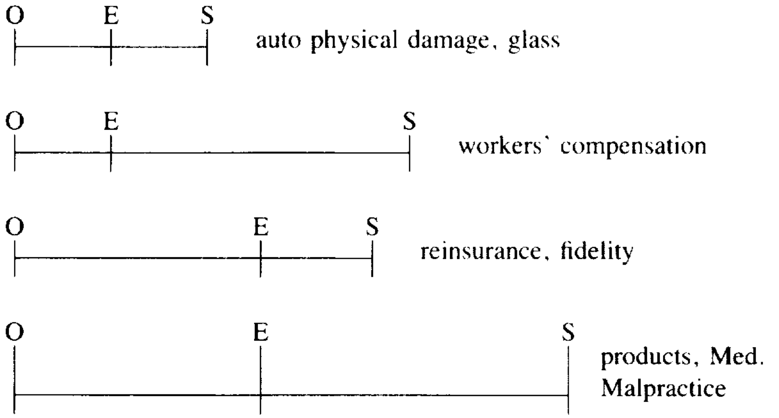
IV. Data Limitations/Incomplete Data

EXHIBIT 8

Emergence, Settlement, Development Patterns

Emergence: time between the occurrence of a claim and when it is recorded on the company books.

Settlement: time between the reporting of a claim and when it is settled (closed).



Development Pattern: historical record of the loss evaluations, from 1st reporting to closing, for a fixed group of claims.

EXHIBIT 9

CUMULATIVE ANNUAL INCURRED LOSS DEVELOPMENT
ACCIDENT YEARS 1973–1982

	Months of Development					
Acc. Yr.	12	24	36	48	60	72
1973	11900	14200	14240	14640	15100	15290
1974	16600	20500	22100	22740	23300	23640
1975	18690	24780	26740	28100	28600	28900
1976	22440	30540	32200	33200	33400	33800
1977	27290	35440	37600	38340	38900	38900
1978	32040	39100	39800	39940	40300	
1979	32640	38800	39510	40600		
1980	35280	43100	46210			
1981	36050	44400				
1982	48730					

	Loss Development Factors				
	12-24	24-36	36-48	48-60	60-72
1973	1.193	1.003	1.028	1.031	1.013
1974	1.235	1.078	1.029	1.025	1.015
1975	1.326	1.079	1.051	1.018	1.010
1976	1.361	1.054	1.031	1.006	1.012
1977	1.299	1.061	1.020	1.015	1.000
1978	1.220	1.018	1.004	1.009	
1979	1.189	1.018	1.028		
1980	1.222	1.072			
1981	1.232				
1982					

EXHIBIT 10

INTERNAL & EXTERNAL CONSIDERATIONS

Internal

I. Changes in Relative Adequacy of Case Reserves
II. Changes in Claim Handling Procedures
  a. Fast Track/Average Reserve Valuation System
  b. Claim Counting
  c. ALAE Payments
  d. Loss Payments
  e. Claim Litigation
  f. Company vs. Independent Adjusters
III. Changes in Pricing Strategy
IV. Changes in Underwriting Programs/Guidelines
V. Changes in New vs. Renewal Ratios
VI. Changes in Type of Reinsurance and Retention Levels
VII. Changes in Policy Limits and Deductibles

External

I. Participation in Involuntary Pools/Associations
II. Inflation
III. Claims Consciousness of Public
IV. Seasonality of Loss Experience
V. Legal/Legislative
VI. Economy

1984 EXAMINATIONS—SUCCESSFUL CANDIDATES

Examinations for Parts 4, 6, 8 and 10 of the Casualty Actuarial Society were held on May 8, 9, 10, and 11, 1984. Examinations for Parts 5, 7 and 9 were held on November 7, 8, and 9, 1984.

Examinations for Parts 1, 2 and 3 are jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries. These examinations were given in May and November of 1984. Candidates who passed these examinations were listed in the joint releases of the two societies.

The Casualty Actuarial Society and the Society of Actuaries jointly awarded prizes to the undergraduates ranking the highest on the General Mathematics examination. For the May, 1984 examination, the $200 prize was awarded to David I. Zuckerman. The additional $100 prize winners were Alan S. Edelman, Andrew E. Gelman, Patrick Godbout, and Marc Raymond. For the November, 1984 examination, the $200 prize was awarded to Nathaniel E. Glasser. The additional $100 prize winners were Brent A. Banister, Joel L. Coleman, Richard S. Margolin, and Daniel M. Wong.

The following candidates were admitted as Fellows and Associates at the November, 1984 meeting as a result of their successful completion of the Society requirements in the May, 1984 examinations.

FELLOWS

Edward J. Baum
Abbe S. Bensimon
James P. Boone
Peter T. Bothwell
David S. Bowen
Robert S. Briere
Dale L. Brooks
David R. Chernick
Valere M. Egnasko
Alice H. Gannon
Paul J. Henzler
Larry D. Johnson
Marvin A. Johnson
Jeffrey L. Kucera
William D. Louks, Jr.
Matthew P. Merlino
Neil B. Miner
Peter J. Murdza, Jr.
Catharine L. Neale
Raymond S. Nichols
Richard W. Nichols
Sylvie L. Paquette
Bernard A. Pelletier
Frank D. Pierson
Richard C. Plunkett
Deborah M. Rosenberg
Louis G. Séguin
Ollie L. Sherman, Jr.
Stuart B. Suchoff
Kevin B. Thompson
Frank J. Tresco
Richard L. Vaughan
Michael G. Wacek
Glenn M. Walker
David R. Whiting

ASSOCIATES

Kenneth E. Carlton, III
Vincent T. Donnelly
Israel Krakowski
John W. McClure, Jr.
Clifford A. Pence, Jr.
Alan K. Putney
Pamela J. Sealand

The following is the list of successful candidates in examinations held in May, 1984.

Part 4

Adams, Jeffrey
Atkinson, Richard V.
Barrows, Joanne K.
Bennighof, Kay E.
Boisvert, Paul, Jr.
Brathwaite, Malcolm E.
Brown, M. David R.
Brutto, Richard S.
Byington, Jennifer S.
Cardoso, Ruy
Carlson, Christopher S.
Caron, Philippe
Chan, Sammy S. Y.
Comstock, Susan J.
Cox, David B.
Dekle, James M.
Desbiens, Carol
Dodge, Scott H.
Donnelly, Vincent T.
Epstein, James C.
Ericson, Janet M.
Eschenbrenner, Denise A.
Fung, Kai Y.
Grab, Edward M.
Griesau, William J.
Grossman, William G.
Hampshire, Michael H.
Hartman, Don E.
Hayes, Thomas L.
Heyman, David R.
Hines, Alan M.
Jaeger, Mark
Jonske, James W.
Jordan, Jeffrey R.
Joyce, John J.
Keh, Hsien-Ming K.
Kish, George A.
Kligman, Daniel F.
Koufacos, Constantine G.
Krakowski, Israel
Krissinger, Kenneth R.
Labrie, Denis
Lewandowski, John J.
Luker, Christopher J.
Makuck, Brian D.
Mason, Fred M.
McClure, John W., Jr.
McCreesh, James B.
Miller, Susan M.
Mohrman, David F.
Mueller, Nancy D.
Mulvaney, Mark W.
Ollodart, Bruce E.
Overgaard, Wade T.
Penick, Robert L.
Perigny, Isabelle
Press, Edward R.
Privman, Boris
Procopio, Donald W.
Proska, Mark R.
Rhodes, Frank S.
Roberts, Jonathan S.
Roesch, Robert S.
Romito, A. Scott
Salton, Jeffrey C.
Samson, Sandra
Schadler, Thomas E.
Schmid, Christopher H.
Schwandt, Jeffory C.
Scully, Mark W.
Simons, Rial R.
Skov, Steven A.
Slusarski, John
Snook, Linda D.
Spidell, Bruce R.
Strauss, Frederick M.
Sutter, Russel L.
Swords, Elaine E.
Taylor, Richard G.
Von Seggern, William J.
Wargo, Kelly A.
Weisenberger, Peter A.
Wilson, Ernest I.
Woerner, Susan K.

Part 6

Allaire, Christiane
Bellusci, David M.
Brown, Brian Y.
Burns, William E.
Carlton, Kenneth E.
Cartmell, Andrew R.
Cassuto, Irene A.
Chabarek, Paul
Chen, Chyen
Der, William
Djordjevic, Nancy G.
Downing, Jeremiah M.
Dufresne, Jacques
Earls, Ronald R.
Fiore, David A.
Fitzgerald, Beth E.
Fonticella, Ross C.
Glicksman, Steven A.
Gogol, Daniel F.
Graham, Jeffrey H.
Griffith, Roger E.
Guenthner, Denis G.
Haidu, James W.
Hay, Gordon K.
Hay, Randolph S.
Hertling, Richard J.
Jarvis, June V.
Kartechner, John W.
Kneuer, Paul J.
Kot, Nancy E.
Kulik, John M.
Lacek, Mary Lou
Landuyt, Judith A.
Leccese, Nicholas M., Jr.
LePere, Cecilia M.
Lessard, Alain
Letourneau, Roland D.
Liebers, Elise C.
Macesic, David J.
MacKinnon, Brett A.
Mailloux, Patrick
McCoy, Mary E.
McDermott, Sean P.
Menning, David L.
Millar, Leonard L.
Muller, Robert G.
Musulin, Rade T.
Myers, Thomas G.
Newell, Richard T., Jr.
Ng, Wai Hung
Noyce, James W.
Pace, Michelle M.
Pechan, Kathleen M.
Pence, Clifford A., Jr.
Peraine, Anthony
Post, Jeffrey H.
Putney, Alan K.
Quintano, Richard A.
Ramanujam, Srinivasa
Scheuing, Jeffrey R.
Schiewer, Suzann P.
Schlissel, Joanne
Schnapp, Frederic F.
Schulz, Richard T.
Schwab, Debbie
Scott, Kim A.
Sealand, Pamela J.
Shepherd, Linda A.
Strenk, Frank W.
Terrill, Kathleen W.
Thompson, Robert W.
Tingley, Nanette
Turner, George W.
Veilleux, Andre
Visintine, Gerald R.
Volponi, Joseph L.
Votta, James
Walker, David G.
Weber, Robert A.
Whitlock, Robert G., Jr.
Williams, Robin M.

Part 8

Aldin, Neil C.
Bailey, Victoria M.
Barclay, D. Lee
Bear, Robert A.
Bensimon, Abbe S.
Bowen, David S.
Boyd, Wallis A.
Cadorine, Arthur R.
Captain, John E.
Carpenter, William M.
Cascio, Michael J.
Chansky, Joel S.
Chou, Li-Chuan L.
Clark, Daniel B.
Dashoff, Todd H.
Desilets, Claude
Deutsch, Robert V.
Driedger, Karl H.
Duffy, Brian
Dyck, N. Paul
Dye, Myron L.
Easlon, Kenneth
Fleming, Kirk G.
Friedman, Howard H.
Gauthier, Richard
Gerard, Felix R.
Grace, Gregory S.

Greaney, Kevin M.
Greco, Ronald E.
Halpern, Nina S.
Hapke, Alan J.
Harrison, David C.
Harwood, Catherine B.
Hein, Timothy T.
Homan, Mark J.
Huyck, Brenda J.
Johnson, Larry D.
Johnson, Richard W.
Kasner, Kenneth R.
Kelley, Robert J.
Laurin, Pierre G.
Lee, Robert H.
Lewis, Martin A.
Lipton, Barry
McDaniel, Gail P.
McQuilkin, Mary T.
Montgomery, Warren D.
Morrow, Jay B.
Mucci, Robert V.
Narvell, John C.
Normandin, Andre
Paquette, Sylvie L.
Pierson, Frank D.
Pulis, R. Stephen
Ruegg, Mark A.
Santomenno, Sandra C.
Schilling, Timothy L.
Sherman, Harvey A.
Smith, Richard A.
Steinen, Phillip A.
Steingiser, Russell
Treitel, Nancy R.
Visner, Steven M.
Wallace, Thomas A.
Weinman, Stacy J.
White, Charles S.
Woomer, Roy T., III

Part 10

Allaben, Mark S.
Baum, Edward J.
Belden, Scott C.
Bensimon, Abbe S.
Bertrand, Francois
Bhagavatula, Raja R.
Biscoglia, Terry J.
Boone, James P.
Bothwell, Peter T.
Briere, Robert S.
Brooks, Dale L.
Carlson, Jeffrey R.
Cathcart, Sanders B.
Chernick, David R.
Cripe, Frederick F.
Egnasko, Valere M.
Forney, John R., Jr.
Fueston, Loyd L., Jr.
Gannon, Alice H.
Gapp, Steven A.
Haskell, Gayle E.
Hayward, Gregory L.
Henzler, Paul J.
Howald, Ruth A.
Hutter, Heidi E.
Johnson, Marvin A.
Kaplan, Robert S.
Keen, Eric R.
Klinker, Frederick L.
Kucera, Jeffrey L.
Loucks, William D., Jr.
Mashitz, Isaac
Matthews, Robert W.
Mayer, Jeffrey H.
McSally, Michael J.
Merlino, Matthew P.
Miner, Neil B.
Morgan, William S.
Murdza, Peter J., Jr.
Murphy, William F.
Neale, Catharine L.
Nester, Karen L.
Nichols, Raymond S.
Nichols, Richard W.
Palmer, Donald W.
Pelletier, Bernard A.
Plunkett, Richard C.
Port, Rhonda D.
Rapoport, Andrew J.
Rosenberg, Deborah M.
Seguin, Louis G.
Sherman, Ollie L., Jr.
Sornberger, George C.
Suchoff, Stuart B.
Surrago, James
Symnoski, Diane M.
Thompson, Kevin B.
Tresco, Frank J.
Vaillancourt, Jean
Vaughan, Richard L.
Wacek, Michael G.
Walker, Glenn M.
Whiting, David R.
Withers, David A.

The following candidates will be admitted as Fellows and Associates at the May, 1985 meeting as a result of their successful completion of the Society requirements in the November, 1984 examinations.

FELLOWS

Bertrand, Francois
Bhagavatula, Raja R.
Biegaj, William P.
Biscoglia, Terry J.
Carlson, Jeffrey R.
Christiansen, Stephan L.
Ehrlich, Warren S.
Forney, John R., Jr.
Fueston, Loyd L., Jr.
Hapke, Alan J.
Hutter, Heidi E.
McSally, Michael J.
Meyer, Robert E.
Neis, Allan R.
Palmer, Donald W.
Ross, Lois A.
Surrago, James
Symnoski, Diane M.
White, David L.

ASSOCIATES

Allaben, Mark S.
Bellafiore, Leonard A.
Bellusci, David M.
Boor, Joseph A.
Brown, Brian Y.
Busche, George R.
Carpenter, William M.
Cartmell, Andrew R.
Clark, Daniel B.
Cripe, Frederick F.
Curran, Kathleen F.
Cutler, Janice Z.
Dashoff, Todd H.
DeFalco, Thomas J.
DeLiberato, Robert V.
Dufresne, Jacques
Earwaker, Bruce G.
Easlon, Kenneth
Fleming, Kirk G.
Gardner, Robert W.
Gogol, Daniel F.
Greaney, Kevin M.
Gunn, Christy H.
Hayward, Gregory L.
Holdredge, Wayne D.
Hollister, Jeanne M.
Howald, Ruth A.
Kline, Charles D., Jr.
Klinker, Fredrick L.
Lee, Robert H.
Lewis, Martin A.
Lipton, Barry
Littmann, Mark W.
Lyons, Rebecca B.
Maguire, Brian P.
McGovern, Eugene
Menning, David L.
Miller, William J.
Montgomery, Warren D.
Mucci, Robert V.
Muller, Robert G.
Myers, Thomas G.
Noyce, James W.
Placek, Arthur C.
Post, Jeffrey H.
Quintano, Richard A.
Reppert, Daniel A.
Robinson, Richard D.
Salton, Jeffrey C.
Sarosi, Joseph F.
Scheuing, Jeffrey R.
Schilling, Timothy L.
Scholl, David C.
Schultz, Roger A.
Shapiro, Arlyn G.
Slusarski, John
Smith, Michael B.
Somers, Edward C.
Theisen, Joseph P.
Treitel, Nancy R.
Vaillancourt, Jean
Visintine, Gerald R.
Volponi, Joseph L.
Weinman, Stacy J.
Whitlock, Robert G., Jr.
Willsey, Robert L.
Woerner, Susan K.

The following is a list of successful candidates in examinations held in November, 1984.

Part 5

Allard, Jean-Luc E.
Amoroso, Rebecca C.
Anderson, Mary V.
Aquino, John G.
Atkinson, Richard V.
Beaver, Arthur J.
Boucek, Charles H.
Carlson, Christopher S.
Caron, Philippe
Casale, Kathleen N.
Caxide, Alison G.
Colin, Steven L.
Comstock, Susan J.
Conway, Ann M.
Cox, Thomas
Danielson, Guy R.
Davis, Brian W.
Davis, Dan J.
Davis, James R.
Debs, Raymond V.
DeFalco, Thomas J.
Desbiens, Carol
Devine, Janice L.
Dickinson, Donna R.
Dumontet, Francois R.
Dunlap, George T., IV
Englander, Jeffrey A.
Epstein, James C.
Epstein, Reuben J.
Erlebacher, Alan J.
Ewert, John S.
Fanning, William G.
Fiore, David A.
Francis, Louise A.
Franklin, Barry A.
Fung, Charles C. K.
Gardner, Robert W.
Gelinne, David B.
Gevlin, James M.
Gibson, John F.
Gibson, Richard N.
Goldberg, Robert H.
Gozzo, Susan M.
Grab, Edward M.
Grose, Carleton R.
Grossack, Marshall J.
Haefner, Larry A.
Herbers, Joseph A.
Higdon, Barbara A.
Higgins, James S.
Hines, Alan M.
Hroziencik, George A.
Hughes, Brian A.
Jeffery, Philip W.
Johnson, Wendy A.
Jones, William R.
Joyce, John J.
Koegel, David
Kreps, Rodney E.
Krissinger, Kenneth R.
Laurin, Pierre G.
Lessard, Alain
Lipton, Barry
Macesic, David J.
MacKinnon, Brett A.
Maher, Christopher P.
Math, Steven
Mayer, Malkie
McCoy, Mary E.
McDermott, Sean P.
Miller, Susan M.
Mulvaney, Mark W.
Murry, Mary E.
Nielsen, Lynn
Ollodart, Bruce E.
Palenik, Rudy A.
Pechan, Kathleen M.
Peraine, Anthony
Pisarcik, Edward J., Jr.
Privman, Boris
Procopio, Donald W.
Rhodes, Frank S.
Romito, A. Scott
Rosenstein, Kevin D.
Sandman, Donald D.
Santomenno, Sandra C.
Schiewer, Suzann P.
Schill, Barbara J.
Schlenker, Sara E.
Schmid, Valerie L.
Schultze, Mark E.
Scott, Kim A.
Scruggs, Michael L.
Scully, Mark W.
Seaman, David A.
Siczewicz, Peter J.
Simi, Laura J.
Snow, David C.
Sperger, Mary Jean
Spidell, Bruce R.
Stoffel, Judy
Sutter, Russel L.
Sweeney, Eileen M.
Tan, Suan-Boon
Taylor, Craig P.

Taylor, Richard G.
Thompson, Robert W.
Trudeau, Michel
Turner, George W., Jr.
Votta, James
Wacker, Gregory M.
Wargo, Kelly A.
Weinman, Stacy J.
Weisenberger, Peter A.
White, Lawrence
Whitehead, Guy H.
Williams, Lincoln B.
Wilson, Ernest I.
Wilson, Theresa A.
Wrobel, Edward M.
Yates, Patricia E.
Yow, James W.

Part 7

Allaben, Mark S.
Bellafiore, Leonard A.
Bellusci, David M.
Blakinger, Jean M.
Boor, Joseph A.
Brown, Brian Y.
Busche, George R.
Carpenter, William M.
Cartmell, Andrew R.
Clark, Daniel B.
Cripe, Frederick F.
Curran, Kathleen F.
Cutler, Janice Z.
Dashoff, Todd H.
DeLiberato, Robert V.
Dezube, Janet B.
Dodge, Scott H.
Dufresne, Jacques
Earwaker, Bruce G.
Easlon, Kenneth
Farwell, Randall A.
Fleming, Kirk G.
Gebhard, James J.
Gogol, Daniel F.
Goldberg, Steven B.
Gorvett, Richard W.
Graham, Jeffrey H.
Greaney, Kevin M.
Groh, Linda M.
Gunn, Christy H.
Hay, Gordon K.
Hayward, Gregory L.
Holdredge, Wayne D.
Hollister, Jeanne M.
Howald, Ruth A.
Jordon, Jeffrey R.
Kline, Charles D., Jr.
Klinger, Kenneth A.
Klinker, Fredrick L.
Lee, Robert H.
LePere, Cecilia M.
Lewis, Martin A.
Littmann, Mark W.
Lyons, Rebecca B.
Maguire, Brian P.
Marles, Blaine C.
McGovern, Eugene
McKelvey, Therissa E.
Menning, David L.
Meyer, Robert E.
Millar, Leonard L.
Miller, William J.
Montgomery, Warren D.
Mucci, Robert V.
Muller, Robert G.
Myers, Thomas G.
Noyce, James W.
Placek, Arthur C.
Post, Jeffrey H.
Quintano, Richard A.
Reppert, Daniel A.
Robinson, Richard D.
Salton, Jeffrey C.
Sarosi, Joseph F.
Scheuing, Jeffrey R.
Schilling, Timothy L.
Scholl, David C.
Schultz, Roger A.
Shapiro, Arlyn G.
Slotznick, Lisa A.
Slusarski, John
Smith, Michael B.
Somers, Edward C.
Theisen, Joseph P.
Treitel, Nancy R.
Vaillancourt, Jean
Visintine, Gerald R.
Volponi, Joseph L.
Wallace, Thomas A.
Whitlock, Robert G., Jr.
Willsey, Robert L.
Woerner, Susan K.
Woodruff, Arlene F.

Part 9

Bailey, Victoria M.
Basson, Steven D.
Bennett, Robert S.
Bertrand, Francois
Bhagavatula, Raja R.
Biegaj, William P.
Biscoglia, Terry J.
Bryan, Susan E.
Bujaucius, Gary S.
Bursley, Kevin H.
Captain, John E.
Carlson, Jeffrey R.
Chansky, Joel S.
Christiansen, Stephan L.
Dembiec, Linda A.
Desilets, Claude
Duffy, Brian
Dyck, N. Paul
Ehrlich, Warren S.
Elliott, Paula L.
Faltas, Bill
Forde, Claudia S.
Forney, John R., Jr.
Fueston, Loyd L., Jr.
Grace, Gregory S.
Greco, Ronald E.
Handte, Malcolm R.
Hanson, Jeffrey L.
Hapke, Alan J.
Harrison, David C.
Haskell, Gayle E.
Hutter, Heidi E.
Huyck, Brenda J.
Kaplan, Robert S.
Krakowski, Israel
Kudera, Andrew E.
Levine, George M.
Martin, Paul C.
Mayer, Jeffrey H.
McClure, John W., Jr.
McQuilkin, Mary T.
McSally, Michael J.
Mendelssohn, Gail A.
Moody, Andrew W.
Mozeika, John K.
Mulder, Evelyn T.
Murphy, Francis X., Jr.
Narvell, John C.
Neis, Allan R.
Palmer, Donald W.
Pei, Kai-Jaung
Pflum, Roberta J.
Potts, Cynthia M.
Robbins, Kevin B.
Ross, Lois A.
Spalla, Joanne S.
Surrago, James
Symnoski, Diane M.
Trinh, Minh
White, David L.
Yatskowitz, Joel D.

NEW FELLOWS ADMITTED MAY 1984 (Left to right): Front row: Amy Bouska, Bonnie Boccitto, Kyleen Knilans, Lisa Gattel Chanzit, Adrienne Kane, Joy Schwartzman, Darlene Tom, William Weimer, George Dodd; Back row: Paul O'Connell, Tom Duffy, Thomas Kosik, Neil Schmidt, Emanuel Pinto, Michael Kooken, John Coffin, Steven Marks, Michael Wiseman.

NEW FELLOWS ADMITTED NOVEMBER 1984 (Left to Right): Front row: Valere Egnasko, Ollie L. Sherman, Jr., Parker Boone, Michael Wacek, Cathy Neale, Alice H. Gannon, Deborah M. Rosenberg, Sylvie Paquette, Abbe Bensimon, Glenn Walker; Middle row: Larry Johnson, Peter Bothwell, Richard Plunkett, Matthew Merlino, Paul J. Henzler, Frank J. Tresco, Raymond S. Nichols, David R. Chernick, Jeff Kucera, Stuart B. Suchoff, Neil Miner, David R. Whiting; Back row: Marvin A. Johnson, Bill Loucks, Edward Baum, Dale Brooks, Richard W. Nichols, Louis G. Séguin, Bernard A. Pelletier, Robert S. Briere, Kevin B. Thompson, Frank D. Pierson, Peter J. Murdza, Jr., Richard L. Vaughan.

NEW ASSOCIATES ADMITTED NOVEMBER 1984 (Left to Right): Pamela J. Sealand, John W. McClure, Jr., Clifford A. Pence, Jr., Alan K. Putner, Vincent T. Donnelly, Kenneth Carlton.

OBITUARIES

H. EARL CASSITY
MILTON HOROWITZ
JOSEPH J. MAGRATH
HENRY W. MENZEL
EDWARD R. MURRAY
WILLIAM F. POORMAN

H. EARL CASSITY
1939–1984

Earl Cassity, an Associate of the Casualty Actuarial Society since 1976, died suddenly on March 19, 1984 at the age of 44.

Earl held an M.S. in Mathematics from the University of Nevada in Reno and worked for the Department of Defense prior to beginning his actuarial career.

His first actuarial position was as an Actuarial Analyst for the Farmers Insurance Group, where he worked from 1972 to 1974. Following that, Earl worked in a variety of positions for several insurance organizations in California prior to joining Allianz Insurance Company as Casualty Actuary in 1981.

Earl was a member of the Insurance Accounting and Statistical Association as well as of the Casualty Actuarial Society.

MILTON HOROWITZ
1907–1984

Milton Horowitz, an Associate of the Casualty Actuarial Society since 1961, died on December 27, 1984 at the age of 77.

Following graduation from the College of the City of New York, Milton Horowitz received his initial appointment to the New York State Insurance Fund during the depression of the 1930's. He was selected for that position from the top group of successful candidates in a written examination with participants numbering in the thousands. In the decades that followed, as a result of progressive promotion examinations within the State Insurance Fund and Milton's membership in the Casualty Actuarial Society, he advanced to the position of Principal Actuary of the Fund. Milton retired from the Fund in November 1976 after 41 years of service.

He is survived by his wife, Rita; a daughter; and two grandchildren.

JOSEPH J. MAGRATH
1899–1983

Joseph Magrath, a Fellow of the Casualty Actuarial Society since 1958, died on July 7, 1983.

Joe began his insurance career at New York Life in 1916. In 1920 he moved into the public sector, working for the New York Public Service Commission from 1920 to 1921, and the New York Insurance Department from 1921 to 1937.

Joe pursued his college education in New York, attending Columbia and New York University in the evening.

In 1937, Joe joined the Marine Underwriting Department of Chubb & Son. He moved to the Investment Department in 1939 and later became Corporate Secretary.

Joe retired in 1964 and moved to Highland Beach, Florida, a location he loved dearly, where he contined his lifelong hobby of reading.

Joe is survived by his wife, Elizabeth; and son, Joseph.

HENRY W. MENZEL
1921–1984

Henry W. Menzel, a Fellow of the Casualty Actuarial Society since 1955, died suddenly on June 10, 1984.

Henry, more affectionately known to all as Hank, was born in Long Island City on July 11, 1921. He graduated Phi Beta Kappa as a mathematics major from the University of Pennsylvania. His actuarial career began with the Equitable in 1946, then with the National Bureau of Casualty Underwriters from 1947 to 1956 where he represented the Bureau on several of its committees. In 1956, he joined the Springfield-Monarch Insurance Companies to head its Actuarial and Research Division. In 1963, he returned to bureau activities as Actuary of the New York Compensation Insurance Rating Board, where he later held the position of President until July, 1971. He returned to the Insurance Services Office (successor to the National Bureau of Casualty Underwriters) as Vice President, from which position he retired in November, 1981.

Hank served as a member and chairman of the Examination Committee of the Casualty Actuarial Society and was a member of its Council (now Board of Directors); and was a charter member of the American Academy of Actuaries.

In all his dealings with people at every level Hank was a warm, considerate, outgoing person, always on the lookout to find positive satisfactory solutions to differences of view. He liked to do things for himself, from mastering actuarial intricacies to designing his home and caring for it. He will be missed by his many friends and associates.

He is survived by his wife Helen, his son Henry, Jr., daughter Christine, and two grandchildren.

EDWARD R. MURRAY
1938–1984

Edward R. Murray, a Fellow of the Casualty Actuarial Society since 1973, died on October 10, 1984 after a lengthy illness. He was 45.

Ed attended Loyola University in Chicago on a General Motors Scholarship, and received his B.S. degree in 1960.

Following two years of service in the Army, he joined Zurich Insurance Company as an Assistant Actuary.

In 1967, he moved to The Royal Insurance Company as Assistant Secretary-Special Accounts-Underwriting Division. In 1982 Ed joined Tokio Marine Management, Inc., where he advanced to the position of Vice President in the Actuarial Division.

Ed served on the Casualty Actuarial Society Committee on Sites, and Education and Examination Committee.

In the actuarial field, Ed's expertise was in dealing with loss reserving and large risks. In his personal life, Ed loved New York City and was a devotee of the theater.

Ed is survived by his parents.

WILLIAM F. POORMAN
–1984

William F. Poorman, an Associate of the Casualty Actuarial Society since 1922, died on February 22, 1984 at the age of 85.

William Poorman held a Master's degree in actuarial mathematics from the University of Michigan. His first actuarial position was with the Lincoln National Life Insurance Company.

William joined the Central Life Insurance Company as an actuary in 1925. He became a director in 1937, vice president and actuary in 1938, president in 1949, and chairman of the board in 1964. He retired in 1969.

William Poorman was considered an organizer of the Iowa Medical Service (Blue Shield) in 1945 and served on several local and national health advisory boards.

In his personal life he pursued such diverse hobbies as hunting, photography, and cattle ranching.

William Poorman's survivors include his wife, Zella.

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