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FOREWORD

The Casualty Actuarial Society was organized in 1914 as the Casualty Actuarial and Statistical Society of America, with 97 charter members of the grade of Fellow; the Society adopted its present name on May 14, 1921.

Actuarial science originated in England in 1792, in the early days of life insurance. Due to the technical nature of the business, the first actuaries were mathematicians; eventually their numerical growth resulted in the formation of the Institute of Actuaries in England in 1848. The Faculty of Actuaries was founded in Scotland in 1856, followed in the United States by the Actuarial Society of America in 1889 and the American Institute of Actuaries in 1909. In 1949 the two American organizations were merged into the Society of Actuaries.

In the beginning of the twentieth century in the United States, problems requiring actuarial treatment were emerging in sickness, disability, and casualty insurance—particularly in workers' compensation—which was introduced in 1911. The differences between the new problems and those of traditional life insurance led to the organization of the Society. Dr. I. M. Rubinow, who was responsible for the Society's formation, became its first president. The object of the Society was, and is, the promotion of actuarial and statistical science as applied to insurance other than life insurance. Such promotion is accomplished by communication with those affected by insurance, presentation and discussion of papers, attendance at seminars and workshops, collection of a library, research, and other means.

Since the problems of workers' compensation were the most urgent, many of the Society's original members played a leading part in developing the scientific basis for that line of insurance. From the beginning, however, the Society has grown constantly, not only in membership, but also in range of interest and in scientific and related contributions to all lines of insurance other than life, including automobile, liability other than automobile, fire, homeowners and commercial multiple peril, and others. These contributions are found principally in original papers prepared by members of the Society and published in the annual *Proceedings*. The presidential addresses, also published in the *Proceedings*, have called attention to the most pressing actuarial problems, some of them still unsolved, that have faced the insurance industry over the years.

The membership of the Society includes actuaries employed by insurance companies, ratemaking organizations, national brokers, accounting firms, educational institutions, state insurance departments, and the federal government; it also includes independent consultants. The Society has two classes of members, Fellows and Associates. Both classes are achieved by successful completion of examinations, which are held in May and November in various cities of the United States and Canada.

The publications of the Society and their respective prices are listed in the *Yearbook* which is published annually. The *Syllabus of Examinations* outlines the course of study recommended for the examinations. Both the *Yearbook*, at a $10 charge, and the *Syllabus of Examinations*, without charge, may be obtained upon request to the Secretary, Casualty Actuarial Society, One Penn Plaza, 250 West 34th Street, New York, New York 10119.

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PROCEEDINGS

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SCALE ADJUSTMENTS TO EXCESS EXPECTED LOSSES

GARY G. VENTER

Abstract

Loss distributions underlying increased limits factors are usually based on countrywide data, as state/class information is generally regarded as too sparse for this purpose. Yet the state/class average severities may be reliable, and the countrywide distributions can be adjusted for differences in this average, for example, by assuming that all losses move in the same proportion.

Such a scale change model can yield state/class increased limits factors; however, in many cases a single factor can be derived to adjust the countrywide increased limit factor incremental differences for the state or class average severity differential. This serves to appreciably simplify the application of the scale change model.

A complication arises in the case of Commercial Automobile Combined Single Limits: besides the BI and PD average severity differentials, the relative frequencies of BI and PD losses vary by state/class, leading to differences in the overall BI–PD mix of losses, and thus in the relationship between basic limits and excess losses. With reasonable additional assumptions, the required change in CSL increased limit factor incremental differences can again be represented by a constant factor which, in this case, reflects mix as well as scale differences.

Introduction

Science may perhaps be distinguished from technology by the use of models versus the use of techniques. A model in this context is a conceptual framework used to help order and comprehend events; a technique is a procedure used to produce a result, and may or may not have a conceptual foundation. "Actuarial science" in many instances amounts to a collection of techniques with either unstated models or no models at all at their foundation and, thus, would probably be better labeled "actuarial technology." This technology has been relatively successful in many cases; nonetheless, in order to develop a true actuarial science, the conceptual framework behind our techniques needs to be formulated more explicitly.

A fairly satisfactory approach to the scientific method, advocated by Popper [5], is to advance the simplest models not contradicted by the existing evidence, where one criterion of simplicity is the ease by which the model could be falsified by future adverse observation. Thus, seeing a red cardinal leads to the hypothesis "all cardinals are red," rather than "all but three cardinals are red," because the former could be more easily falsified, i.e., only one non-red cardinal would be needed.

In the domain of loss severity, a simple model would be that all risks have the same claim size distribution. However, this model has often been falsified by the observance of different average loss values for different lines, and for different states and classes within lines.

In many cases, the simplest non-falsified hypothesis is that the observed differences in average severity apply uniformly to all losses, that is, that the shapes of the loss size distributions under investigation are the same and only scale differs. This hypothesis might apply to states or classes for a given line of insurance or for different time periods for a given state/line combination.

This scale change model has been widely used in casualty insurance, for example, by Finger [2] and Miccolis [3]. Its use is not restricted to severity distributions; for instance, interim updates of Table M by the National Council on Compensation Insurance have often implicitly applied this model to aggregate claim distributions. There is also evidence (e.g., see [6]) that the scale change hypothesis does *not* hold for variation over time for long tailed casualty business, i.e., there is more to trend than simple monetary inflation.

In this paper the mathematics of the scale change model is developed explicitly, and the implications of this model for excess expected losses are explored.

Increased Limits Factors

When comparing one state or class to another, a scale difference would affect the base rates charged. But what is less obvious is that it should also affect the increased limits factors applied to those rates.

For example, consider an automobile claim cost distribution expressed in dollars versus the same distribution expressed in Swiss francs. The increased limits factor for 100,000 over 10,000 would differ depending on what currency were being referred to. In fact, if a dollar were worth two Swiss francs, the factor for $100,000 over $10,000 would be the same as the factor for SF 200,000 over SF 20,000.

The same concept can apply to simple inflation. If the U.S. experiences a 20% inflation between 1982 and 1984, the factor for $120,000 over $12,000 in 1984 dollars would be the factor for $100,000 over $10,000 in 1982 dollars. In a similar vein, if every loss in California were to cost exactly 1.5 times as much as the same loss in Louisiana, then the ratio of expected losses limited to $150,000 to expected losses limited to $15,000 in California would be the same as the ratio at limits of $100,000 and $10,000 in Louisiana, assuming everything else remains the same.

The excess loss costs then will be doubly affected by a scale change, first due to the increase in primary expected losses and second due to a change in the ratio of excess to basic losses. Quantifying this effect will be discussed below.

Mathematical Development

The simplest form of scale change, as illustrated by the Swiss franc example, is when one random variable is a scalar multiple of another; e.g., $Y = kX$. Then $E(Y) = kE(X)$, $\sigma_Y = k\sigma_X$, etc. Interestingly enough $CV(Y) \equiv \sigma_Y \div E(Y) = \sigma_X \div E(X) = CV(X)$, i.e., the scale change does not affect the coefficient of variation. Also the skewness will not be affected. This partially expresses the idea that the shape of the distribution is not affected by a scale change.

Now consider the cumulative distribution functions F and G for X and Y respectively. By definition $G(a) = \Pr[Y \leq a] = \Pr[kX \leq a] = \Pr[X \leq a \div k] = F[a \div k]$. Note that the transformation $Y = kX$ corresponds to the inverse transformation $G(a) = F(a \div k)$ on the distribution functions.

In some cases of interest, the random variables under study will be defined on different spaces, e.g., accidents in Louisiana versus accidents in California, so they cannot always be thought of as multiples of each other. In these cases, the relationship between distribution functions can be used to specify what is meant by a scale change.

Thus, if for random variables X and Y with distribution functions F and G there is a constant k such that $G(a) = F(a \div k)$, then Y will be called a scale change of X with constant k.

It is easy to show from the chain rule that the probability density functions f and g satisfy $g(a) = f(a \div k) \div k$, and the relationships between the moments discussed above for $Y = kX$ are readily derived from this definition.

Excess Losses

To calculate the effect on excess loss costs, it is very useful to refer to the concept of limited mean for a loss severity distribution. Intuitively this is the average loss size for losses limited to some specific amount a. For a distribution function F with density f the average severity limited to a will be denoted by $S_F(a)$ and is defined as

$$S_F(a) = \int_o^a t\, f(t)\, dt + \int_a^\infty a\, f(t)\, dt.$$

By change of variable in integration, it is easy to show that $S_G(a) = kS_F(a \div k)$ when the conditions in the above definition of scale change hold.

Now, when F is the severity distribution function, the expected loss increased limits factor for limit a over limit b (i.e., b is basic limits) is just $I_F(a;b) = S_F(a) \div S_F(b)$. This is because multiplying the numerator and denominator by the expected number of losses yields losses limited to a divided by basic limits losses.

Thus, for a scale change, we have $S_G(a) = kS_F(a \div k)$ and $S_G(b) = kS_F(b \div k)$, so $I_G(a;b) = S_G(a) \div S_G(b) = S_F(a \div k) \div S_F(b \div k) = I_F(a \div k;b \div k)$. This implies that the expected loss increased limits factor after an upward scale change is the factor for the scaled down limits before the change, and vice versa for a downward change. Dividing both sides of this last relation by $I_F(a;b)$ and simplifying yields $I_G(a;b) \div I_F(a;b) = I_F(a \div k;b \div k) \div I_F(a;b) = I_F(b;b \div k) \div I_F(a;a \div k)$. This gives a factor for adjusting the increased limits factor at a over b for X to that for Y which depends just on the distribution function F. With basic limits b this factor will in general depend on the increased limit a.

Simplifying the Application

The formula above shows, for example, how, under the scale change model, the increased limits factors for a state relate to the state to countrywide average severity differential and the countrywide increased limits factors. However, having separate tables of increased limits factors for each state could prove unwieldy. It turns out that, for many severity distributions, a single factor can be derived for each state, independent of limits, that will closely approximate the state to countrywide ratio of the difference between increased limits factors. The closeness of the approximation will usually depend on how wide a range of limits is chosen, as will be seen below. Such factors could be used to calculate state excess charges directly from countrywide, without having state tables of increased limits factors.

To facilitate discussion, define the excess layer factor $L_F(c,a;b)$ to be the difference in increased limits factors $I_F(c;b) - I_F(a;b)$ where again b is the basic limit. Then L_F represents the ratio of layer expected losses to basic limits losses. Now what is the adjustment factor for a scale change with constant k? This is the ratio $L_G(c,a;b) \div L_F(c,a;b)$ which can be expressed as

$$\frac{S_F(c \div k) - S_F(a \div k)}{S_F(c) - S_F(a)} I_F(b,\ b \div k).$$

This can be proved by expressing everything in terms of the S_F and S_G functions.

Now the ratio $[S_F(c \div k) - S_F(a \div k)]/[S_F(c) - S_F(a)]$ does not depend too strongly on a and c when both limits are in a reasonable range. Thus, approximating this ratio by a single factor d_F will allow the adjustment factor $L_G(c,a;b) \div L_F(c,a;b)$ to be expressed as $d_F I_F(b;b \div k)$ independently of limits a and c.

To explore the range of variability for the ratio $[S_F(c \div k) - S_F(a \div k)]/[S_F(c) - S_F(a)]$ it will be calculated for some specific loss severity distributions.

First consider the Pareto distribution $F(x) = 1 - (x \div r)^{-q}$; $r,q > 0$. If losses follow this distribution, even if only for the range of interest (for example, \$100,000 to \$1,000,000), we can calculate the ratio as follows:

$$f(x) = (q \div r)(x \div r)^{-(q-1)}$$

$$S_F(c) - S_F(a) = \int_a^c r^q q t^{-q}\, dt + c\left(\frac{c}{r}\right)^{-q} - a\left(\frac{a}{r}\right)^{-q}$$

$$= \frac{r^q}{q-1}\,(a^{-(q-1)} - c^{-(q-1)});$$

and so

$$S_F(c \div k) - S_F(a \div k) = \frac{r^q}{q-1}\,((a \div k)^{-(q-1)} - (c \div k)^{-(q-1)})$$

$$= \frac{r^q}{(q-1)k^{q-1}}\,(a^{-(q-1)} - c^{-(q-1)}).$$

Thus,

$$\frac{L_G(c,a;b)}{L_F(c,a;b)} = \frac{S_F(c \div k) - S_F(a \div k)}{S_F(c) - S_F(a)}\, I_F(b;b \div k)$$

$$= k^{1-q} I_F(b;b \div k).$$

Thus, in this case, the adjustment does not depend at all on the limits a and c.

The above argument requires that losses follow the Pareto at least in between $a \div k$ and c. This has been reported informally to be a reasonably close but not exact form for several lines of casualty losses in working excess layers. Distributions giving closer fits will generally not have this property (of allowing an exact single factor adjustment for scale independently of layer), but if losses are close to the Pareto it is reasonable to believe that a single factor can be found which is close to proper for each layer.

Consider next the lognormal distribution. A *CV* of 4 with an expected loss size of $5,000 might represent a typical casualty line.

The ratios $[S_F(c \div k) - S_F(a \div k)]/[S_F(c) - S_F(a)]$ for a scale change (i.e., $k = 1.25$) calculated as above are shown for several excess layers in the top half of Appendix 1. (Recall that after the scale change a distribution with a mean of $6,250 and a *CV* of 4 results.) The ratios are computed as follows. First, the lognormal parameters $\mu = 7.10059$ and $\sigma = 1.68322$ were derived from the equations $\sigma^2 = \ln(1 + CV^2)$ and $\mu = \ln E(x) - \sigma^2/2$. To calculate $S_F(a)$ the formulas $F(a) = \Phi((\ln a - \mu)/\sigma)$ and $\int_0^a tf(t)dt = E(x)\,\Phi[((\ln a - \mu)/\sigma) - \sigma]$ from [1] were used, where Φ is the standard normal cumulative distribution function.

As the table shows, most factors lie in the 120% to 135% range. This may or may not be a small enough range to consider constant, depending on the uses to which this analysis is to be put. A factor of 1.275 applied uniformly to all excess layer expected losses would seem to be a reasonable figure.

The shifted Pareto distribution discussed in [4] is also treated in Appendix 1. That distribution function is of the form

$$F(x) = 1 - \left(\frac{\beta}{x + \beta}\right)^{\delta};\ \beta,\delta > 0.$$

For the shifted Pareto:

$$E(x) = \frac{\beta}{\delta - 1}\ (\delta > 1),\ CV^2 = \frac{\delta}{\delta - 2},\ \delta > 2,$$

and

$$\int_o^a xf(x)dx = \frac{\beta}{\delta - 1}\left[(1 - \left(\frac{\beta}{a + \beta}\right)^{\delta}\left(\frac{\beta + a\delta}{\beta}\right)\right].$$

From this it can be shown that

$$S_F(a) = \frac{\beta}{\delta - 1}\left(1 - \left(\frac{\beta}{a + \beta}\right)^{\delta - 1}\right).$$

The same mean and CV were assumed as in the lognormal case, and $\beta = 85{,}000/15$ and $\delta = 32/15$ were derived by matching moments. In this case, the ratios measured remained in the 126% to 128% area, as shown in Appendix 1. Thus, the constant ratio approximation is better for the shifted Pareto than for the lognormal. This distribution has generally been a more successful model for casualty loss severities than has the lognormal and is commonly used in increased limits ratemaking.

To obtain the final factor for adjusting excess layer factors for a scale change, the factor $I_F(b;b \div k)$ is needed for the basic limit b. Taking $b = 25{,}000$ gives 1.06 for the lognormal and 1.04 for the Pareto by applications of the above methods. The final adjustments to apply to excess layer factors are thus $1.06 \times 1.275 = 1.35$ for the lognormal and $1.04 \times 1.27 = 1.32$ for the shifted Pareto.

Calculating k

The factor k is the ratio of the average severity for Y to the average severity for X. One case of interest is where X is countrywide and Y is state loss size.

Then state excess factors can be calculated by applying a constant adjustment to the nationwide values by the method above. Estimating k is somewhat complicated by the fact that the average severities are available only for basic limits, i.e., $t = S_G(b) \div S_F(b)$ is known rather than $k = E(Y) \div E(X)$. Since $S_G(b) = kS_F(b \div k)$, it is possible to solve for k if the nationwide distribution F is known; e.g., $t = S_G(b) \div S_F(b) = kS_F(b \div k) \div S_F(b)$ or $tS_F(b) = kS_F(b \div k)$ gives an equation that can be solved for k if F and t are given.

For example in the Pareto case above ($\beta = 17{,}000/3$, $\delta = 32/15$),

$$S_F(b) = \frac{\beta}{\delta - 1}\left(1 - \frac{\beta}{b + \beta}\right)^{\delta - 1}$$

$$= 5000\ (1 - ((3b/17{,}000) + 1)^{-17/15}).$$

Suppose $t = 1.2$, i.e., the state in question has basic limit severity 20% above nationwide. Then with basic limits of \$25,000 the equation $tS_F(b) = kS_F(b \div k)$ becomes $1.2 \times 5{,}000\ (1 - ((75{,}000/17{,}000) + 1)^{-17/15}) = k \times 5{,}000\ (1 - ((75{,}000/k17{,}000) + 1)^{-17/15})$, i.e., $1.023 = k(1 - (1 + 4.412/k)^{-1.133})$. This can be solved iteratively to yield $k = 1.248$. Thus, with the given nationwide distribution and a state offset at basic limits of 1.2, a state scale factor of 1.248 results. Thus, from the above, an adjustment factor of approximately 1.32 should apply to excess layer factors.

A similar procedure could be used for other distributions. The calculation is somewhat easier for the Pareto because of the closed form for $S_F(a)$.

Scale Parameters

Often one of the parameters of a distribution can be used to effect a scale change. Such parameters are, therefore, called scale parameters. Beta for the Pareto and mu for the lognormal are examples. Thus, if F is the Pareto distribution function and $G(a) = F(a \div k)$, it is easy to see by direct substitution that G is a Pareto with just a different beta. A similar result can be derived for the lognormal and for many other distributions.

Combined Single Limits

For Commercial Automobile Combined Single Limits (CSL) an additional state offset to nationwide factors could be made to reflect the particular BI–PD mix in the state. For example, for the average state, \$5,000 limits PD losses might run about 80% of \$15,000 limits BI losses. Many companies have access to data of this type. One unpublished study available to the author indicated that for different states this percentage could fall anywhere in the range of 50% to 150%, or even outside this range, consistently by state. Possible explanations for this spread include differentials among the states in urban-rural mix and tort atmosphere. Property damage losses could reasonably be expected to predominate in urban areas where crowded conditions force lower speeds but lead to more encounters, while bodily injury may predominate on rural roads. So-called tort consciousness or propensity to sue could also lead to more bodily injury losses incurred in some areas.

Since BI and PD have quite different loss severity distributions, at least in average value, their mix could markedly affect the CSL severity distribution for the state.

Again the offset for state excess layer factors can be used as a single factor independent of layer for a suitable range of layers, but additional approximations are involved. Since there are numerous concepts to keep track of, some notation is necessary. Let B, P, and C refer to Bodily Injury, Property Damage, and Combined Single Limits, respectively, so $S_P(a)$ is Property Damage severity limited to a, $I_C(a;b)$ is the expected loss Combined Single Limits increased limits factor for a over b, and $L_B(c,a;b)$ is the Bodily Injury excess layer factor for the layer a to c with basic limits b. An asterisk will denote the concept for a state under consideration while non-asterisked variables will denote nationwide. A constant $t_C = L_{C^*}(c,a;b) \div L_C(c,a;b)$ is sought where t_B and t_P, the similar constants for BI and PD, have already been determined.

The first approximation needed for this is $N_CS_C(a) = uN_BS_B(a) + vN_PS_P(a)$, where N is the expected number of losses for each category. This expression says the CSL limited losses can be approximated as a linear combination of the BI and PD limited losses.

At a given limit the CSL expected losses should be less than the sum of BI and PD expected losses at the same limit, because the CSL limit applies to the BI plus PD total rather than to each separately. The constants u and v are discount factors to reflect this. An example is provided by the so-called single limit rule, which for many limits and states is equivalent to $u = 1$, $v = .91$. A more compact form of the above expression arises if we introduce the notation $D(a)$ for the total expected loss dollars limited to a, i.e., $D(a) = NS(a)$. Then $D_C(a) = uD_B(a) + vD_P(a)$ is the approximation noted.

Now,

$$L_C(c,a;b) = (D_C(c) - D_C(a)) \div D_C(b)$$

$$= \frac{u\,(D_B(c) - D_B(a)) + v\,(D_P(c) - D_P(a))}{u\,D_B(b) + v\,D_P(b)}$$

$$= \frac{uL_B(c,a;b)\,D_B(b) + v\,L_P(c,a;b)\,D_P(b)}{u\,D_B(b) + v\,D_P(b)}$$

$$= w\,L_B(c,a;b) + (1 - w)\,L_P(c,a;b)$$

where $w = u\,D_B(b) \div (u\,D_B(b) + v\,D_P(b)) = 1 \div (1 + rv \div u)$ where r is the ratio of PD to BI losses at limit b.

Now u and v are reasonably believable as constants among states; that is, even though BI and PD constitute different percentages of the CSL losses from one state to the next, the same percentages of BI and PD losses at a given limit are eliminated by the CSL approach. Nonetheless, w will vary by state due to the varying BI–PD mix r^*.

Thus $L_{C^*} = w^*L_{B^*} + (1 - w^*)\,L_{P^*}$, suppressing the $(c,a;\mathrm{b})$, and t_C, the factor being sought, may be expressed as

$$t_C = L_{C^*} \div L_C = \frac{w^*L_{B^*} + (1 - w^*)\,L_{P^*}}{w\,L_B + (1 - w)\,L_P}$$

$$= \frac{w^*}{w}\,\frac{(L_{B^*} \div L_B) + ((1 - w^*) \div w^*)(L_{P^*} \div L_P)(L_P \div L_B)}{1 + ((1 - w) \div w)(L_P \div L_B)}$$

$$= \frac{1 + rv \div u}{1 + r^*v \div u}\,\frac{t_B + t_P(r^*v \div u)\,L_P \div L_B}{1 + (rv \div u)\,L_P \div L_B}$$

In the last formula, only the nationwide ratio $L_P(c,a;b) \div L_B(c,a;b)$ depends on c and a. The second approximation is to use a constant to represent this ratio. In a test intended to be representative (see Appendix 2) this ratio was found to vary from .142 for the layer from \$750,000 = a to \$1,000,000 = c to .190 for the layer from \$100,000 = a to \$200,000 = c, where b = \$25,000. The actual ratio t_C varies less than this because the term containing $L_P \div L_B$ is added to a larger term in both numerator and denominator.

Thus to recapitulate,

$$t_C = \frac{1 + rv \div u}{1 + r^*v \div u}\,\frac{t_B + t_P(r^*v \div u)\,q}{1 + (rv \div u)\,q}$$

where r and r^* are the nationwide and state ratios of PD to BI expected losses at basic limit b, v and u are constants used to linearly approximate CSL expected losses at any limit by BI and PD losses at the same limit, and q is a point approximation of nationwide PD excess layer factors over BI excess layer factors at the same limits. An example is discussed in Appendix 2.

Final Notes

It should be noted that the single factor approximations discussed above do not apply to increased limits factors. Rather they apply to the excess layer factors which are differences between two increased limits factors. If the approximation is good in a range that includes basic limits, then the adjustment factor could be applied to the part above 1.0 of a given increased limits factor, because that would be the excess layer factor for the layer from basic limits to the given limit. Even if this approach is not reasonable, an adjustment to the increased limits factor is still in order, but a constant factor adjustment will not be appropriate.

It should also be emphasized that the above formulas relate only to the expected loss portion of the premium. Loss expense and risk load are also important elements of excess charges that ought to be considered when applying the scale change model to excess pricing. Loss expense can probably be handled in a way consistent with the above constant adjustment factor approach.

It is questionable whether the appropriate risk load for a layer is the difference between ground up risk loads at the layer limits, and, thus, the loading approach should tie in closely with the specific application being considered.

One area for further study is the determination of the single limit discounts u and v. Respective values of 1.0 and .91 reflect current conventions, but as single limit occurrence distributions become available, better measurements should be possible.

Finally, the scale model, while a good working hypothesis in many cases, is not universally applicable. It is probably better than the identical distribution model in instances where consistent average value differences have been observed; but where there is reason to suspect that shape differences may exist, they should be investigated. In many lines, variation between classes (e.g., heavy trucks versus vans) is an area where shape differences in severity distributions may be found.

APPENDIX 1

EFFECT OF 25% SCALE CHANGE ON LAYER SEVERITIES

$$\frac{S_F(c \div k) - S_F(a \div k)}{S_F(c) - S_F(a)}$$

Lognormal Distribution

$E(x) = 5{,}000 \qquad CV = 4$

$\mu = 7.10059 \qquad \sigma = 1.68322$

a = Lower Layer Limit (000)	c = Upper Layer Limit (000) 200	250	300	400	500	750	1000
100	1.198	1.205	1.211	1.219	1.224	1.231	1.235
200		1.241	1.248	1.259	1.267	1.278	1.284
250			1.260	1.271	1.280	1.292	1.299
300				1.281	1.291	1.304	1.312
400					1.307	1.323	1.332
500						1.335	1.346
750							1.371

Shifted Pareto Distribution

$E(x) = 5{,}000 \qquad CV = 4$

$$\beta = \frac{85{,}000}{15} \qquad \delta = \frac{32}{15}$$

a = Lower Layer Limit (000)	c = Upper Layer Limit (000) 200	250	300	400	500	750	1000
100	1.260	1.262	1.263	1.265	1.265	1.266	1.267
200		1.271	1.272	1.274	1.274	1.276	1.276
250			1.274	1.275	1.276	1.278	1.278
300				1.277	1.278	1.279	1.279
400					1.279	1.280	1.281
500						1.281	1.282
750							1.283

APPENDIX 2

COMBINED SINGLE LIMIT EXAMPLE

For nationwide PD severity the Pareto distribution $F_P(x) = 1 - (1 + x/\beta)^{-\delta}$ is used with $\beta = 335.023$ and $\delta = 1.35$. For BI a split Pareto severity distribution is used, i.e.,

$$F(x) = \begin{cases} 1.40935\ (1 - (1 + x/\beta)^{-\delta}) & x \leq 4{,}000 \\ 1 - .5913\ (1 + x/\beta)^{-\delta} & x \geq 4{,}000 \end{cases}$$

where $\beta = 5171.797$ and $\delta = 1.20848$. These parameters were chosen to be a realistic representation of the data once available. From the single limits rule $v \div u$ was taken at .91, and from a large unpublished sample a nationwide ratio of PD to BI \$25,000 losses of $r = .8$ was estimated. Suppose for a given state $t_B = 1.2$, $t_P = 1.0$, and $r^* = .6$ have been calculated. Then

$$t_C = \frac{1 + (.8)(.91)}{1 + (.6)(.91)} \times \frac{1.2 + (.6)(.91)q}{1 + (.8)(.91)q}\ .$$

By definition $q(c,a;b) = L_P(c,a;b) \div L_B(c,a;b)$ and $L(c,a;b) = (S(c) - S(a)) \div S(b)$. For PD,

$$S_P(a) = \frac{335.023}{.35}\left(1 - \left(\frac{a}{335.023} + 1\right)^{-.35}\right)$$

by the Pareto rule. A somewhat more complicated formula holds for S_B due to the split Pareto used. After some calculation, q is found to range from .142 to .190 for layers (a and c) in the \$100,000 to \$1,000,000 range. Selecting $q = .166$ yields $t_C = 1.287$. With q's of .142 and .190, t_C's of 1.294 and 1.280 arise respectively.

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A NOTE ON LOSS DISTRIBUTIONS

J. GARY LAROSE

Abstract

This paper presents a generalized notation in order to represent several actuarial rating values which are derived from loss distributions. Four functions are defined and then used to define various rating values such as Table M charges and savings, loss elimination ratios, increased limit factors, and excess loss premium factors. The notation has been adapted from a notation originally presented by R. J. Finger. Using this manner of presentation, a more unified approach to actuarial uses of loss distributions is possible. The paper should be of particular value to students of the Society.

I. INTRODUCTION

The topic of loss distributions has been and continues to be an important area of actuarial study. Many papers have been presented to this Society which discuss various actuarial applications utilizing loss distributions. Some of these papers appear on the CAS examination syllabus. In addition, the Actuarial Education and Research Foundation is currently preparing a textbook on loss distributions. It is the purpose of this paper to define some elementary functions which utilize an underlying loss distribution and then use these functions to generalize the derivation of several actuarial rating values. Using this manner of presentation, a more unified approach to actuarial uses of loss distributions is possible.

The term loss distribution is intended to be a general term. It could represent a per claimant loss distribution, a per occurrence loss distribution, a per risk annual loss ratio distribution, etc. The generality and wide application of the elementary functions result, in part, from the variety of types of specific loss distributions and probability models which could be considered in various areas of ratemaking. It should be noted that the functions presented are "distribution-free" in the sense that no particular probability law is assumed. In this paper we will use the terms "claim" and "loss" interchangeably.

II. ELEMENTARY FUNCTIONS

In this section, we give definitions for four elementary functions which utilize an underlying loss distribution and which will be used throughout the paper. We will use t to denote a loss variable and $f(t)$ to represent the probability density function (p.d.f.) of t. The domain of the functions is the non-negative real numbers and their range is the closed unit interval. In this paper we will use only continuous random variables; however, the discrete case is easily substituted. We now proceed with our definitions.

1. *Cumulative Distribution Function*
   This function represents the probability that a given loss size will be less than or equal to x.

$$F(x) = \int_o^x f(t)\,dt$$

2. *Basic Loss Function*
   This function represents the percentage of total losses generated by all claims which are smaller than some specified value x.

$$X1(x) = \frac{1}{\alpha} \int_o^x t \, dF(t),$$

$$\text{where} \quad \alpha = \int_o^\infty t \, dF(t) = \text{mean of the distribution.}$$

3. *Primary Loss Function*
This function represents the percentage of total losses generated by the aggregate amount of the first x dollars of each claim (the whole claim amount, if less than or equal to x).

$$X2(x) = \frac{1}{\alpha} \int_o^x t \, dF(t) + \frac{x}{\alpha} \int_x^\infty dF(t)$$

$$= X1(x) + \frac{x}{\alpha} [1 - F(x)]$$

4. *Excess Loss Function*
This function represents the percentage of total losses generated by the aggregate amount of the dollars of loss which exceed x per claim.

$$X3(x) = \frac{1}{\alpha} \int_x^\infty (t - x) \, dF(t)$$

$$= 1 - X1(x) - \frac{x}{\alpha} [1 - F(x)]$$

$$= 1 - X2(x)$$

III. FREQUENCY, SEVERITY, AND PURE PREMIUM

We would like to have an expression for the pure premium and its components in terms of the elementary functions. But first we need to make the following definitions.

R = retention (or deductible) amount
$E[n]$ = zero retention (or full coverage) frequency. Bickerstaff[1] calls this "absolute" frequency.
$p(R)$ = pure premium at retention level R

[1] D. R. Bickerstaff, "Automobile Collision Deductible and Repair Cost Groups: The Lognormal Model," *PCAS* LIX (1972), p. 68.

$$g(R) = \text{frequency at retention level } R = [1 - F(R)] \cdot E[n]$$
$$s(R) = \text{severity at retention level } R = \alpha \cdot X3(R) \,/\, [1 - F(R)]$$

Then,

$$\begin{aligned} p(R) &= g(R) \cdot s(R) \\ &= [(1 - F(R)) \cdot E[n]] \cdot [\alpha \cdot X3(R) \,/\, (1 - F(R))] \\ &= E[n] \cdot \alpha \cdot X3(R) \end{aligned}$$

We can define expected excess and expected primary losses as follows:

$$\begin{aligned} \text{expected primary losses} &= Ep = u \cdot p(0) - u \cdot p(R) = u \cdot E[n] \cdot \alpha \cdot X2(R) \\ \text{expected excess losses} &= Ee = u \cdot p(R) = u \cdot E[n] \cdot \alpha \cdot X3(R) \\ \text{expected losses} &= E = Ep + Ee = u \cdot p(0) = u \cdot E[n] \cdot \alpha \end{aligned}$$

where u = number of exposure units.

With these definitions and those of the preceding section, we are now ready to discuss some specific applications.

IV. LOSS ELIMINATION RATIOS

The Straight Deductible Loss Elimination Ratio

In his paper on automobile collision deductibles, Bickerstaff[2] defines a "first-dollar" loss elimination ratio (LER) as follows:

$$\begin{aligned} LER(D) &= \frac{D \cdot G(D) + \alpha \cdot H(D)}{\alpha} \\ &= H(D) + (D/\alpha)G(D) \\ &= (1/\alpha) \int_o^D t\, dF(t) + (D/\alpha)[1 - \int_o^D dF(t)] \\ &= X1(D) + (D/\alpha)[1 - F(D)] \\ &= X2(D) \end{aligned}$$

Bickerstaff takes $f(t)$ to be the lognormal p.d.f. and goes on to show that an adjustment to the loss cost, α, must be made in order to reflect an upper bound to the unlimited lognormal distribution. This is in recognition of the

[2] *ibid.*

practical fact that there exists a finite dollar bound, L, on the actual cash value of a vehicle. This adjustment can be calculated as:

$$\begin{aligned}\text{"adjustment"} &= \frac{\alpha \cdot J(L) - L \cdot G(L)}{\alpha} \\ &= J(L) - (L/\alpha)[1 - F(L)] \\ &= 1 - X1(L) - (L/\alpha)[1 - F(L)] \\ &= X3(L)\end{aligned}$$

We can now compute the net cost per claim ($NCPC$) for a given deductible D as follows:

$$\begin{aligned}NCPC(D,L) &= \alpha - \alpha \cdot X2(D) - \alpha \cdot X3(L) \\ &= \alpha[1 - X2(D) - X3(L)] \\ &= \alpha[X3(D) - X3(L)]\end{aligned}$$

(Note that $NCPC$ does not equal severity, as defined in the previous section.)

If we expand this formula and make the modifications Bickerstaff suggests, we can obtain a "complete" formula for net loss cost (i.e., pure premium).

$$\begin{aligned}NCPC(D,L) &= \alpha - \alpha \cdot X2(D) - \alpha \cdot X3(L) \\ &= \alpha - \alpha \cdot X1(D) - D[1 - F(D)] - \alpha[1 - X1(L)] \\ &\quad + L[1 - F(L)]\end{aligned}$$

We now substitute $\alpha(1 + r)^{n-1}$ for α, Ld^{n-1} for L, and multiply by AC_n to obtain the formula for net loss cost,

$$AC_n\{\alpha(1 + r)^{n-1} - \alpha(1 + r)^{n-1} X1(D) - D[1 - F(D)] - \alpha(1 + r)^{n-1} [1 - X1(Ld^{n-1})] + Ld^{n-1}[1 - F(Ld^{n-1})]\},$$

which the reader can verify is equivalent to the Bickerstaff formula.

Snader[3] gives a discrete formula for the straight deductible LER which can be generalized to $X2(D)$. This is straightforward and is left to the reader.

The Franchise Deductible Loss Elimination Ratio

The franchise deductible requires the insured to pay for losses less than or equal to the deductible amount, but when a loss exceeds the deductible, the

[3] R. H. Snader, "Fundamentals of Individual Risk Rating and Related Topics," CAS Study Note, Part III, p. 60.

loss is paid in full. The formula for the loss elimination ratio is:

$$\begin{aligned} LER(D) &= \frac{1}{\alpha} \int_{o}^{D} t\, dF(t) \\ &= X1(D). \end{aligned}$$

The Disappearing Deductible Loss Elimination Ratio

The discrete formula for this type of deductible is given by Snader.[4] Since the derivation of the equivalent form in terms of the elementary functions is rather cumbersome, only the formula will be given. This type of deductible is a straight deductible up to losses of amount D, there is a decreasing amount of deductible from D to an amount A (at which $D = 0$), and no deductible for losses in excess of A.

$$\begin{aligned} LER\ (D;A) = X1(A) &- \frac{A}{A - D} [X1(A) - X1(D)] \\ &+ \frac{A}{A - D} (D/\alpha)[F(A) - F(D)] \end{aligned}$$

V. EXPERIENCE RATING

D *Ratios*

Bailey[5] tells us that "any experience rating plan which uses a loss limitation must cope with D ratios." These ratios are necessary in order to split expected losses into expected primary and expected excess losses. However, there are several types of loss limitation which can be used to emphasize frequency of loss. One type is illustrated by the maximum single loss (*MSL*) limitation used in general liability.[6] In this case we have:

$$\begin{aligned} D \text{ ratio} &= Ep/E \\ &= [u \cdot E[n] \cdot \alpha \cdot X2(M)]/[u \cdot E[n] \cdot \alpha] \\ &= X2(M) \end{aligned}$$

where M = maximum single loss limitation.

[4] *ibid.*, Part III, p. 61.

[5] R. A. Bailey, "Experience Rating Reassessed," *PCAS* XLVIII (1961), p. 60.

[6] G. N. Alff, "Liability Experience Rating: Concepts and Structure," CPCU Journal, March, 1979, p. 44.

In workers' compensation, individual losses are split into primary and excess portions by the use of a formula in conjunction with additional dollar limits on multiple claimant cases, disease cases, etc. Currently, the formula primary portion of a loss is dependent on the size of the ground-up loss and, hence, is variable. This situation is similar to the deductible provisions of several of the crop-hail insurance policy forms.[7] In these cases, the formula given previously for Ep will not hold and, hence, we cannot obtain $X2(M)$ as a representation of the D ratio. Thus, this formula is not applicable when discussing workers' compensation experience rating, or any other plan not using a constant loss limitation, but it can be helpful in those plans which do use a constant loss limitation. This will also be the case with the excess ratio which we will discuss next.

Excess Ratios

In his paper on experience rating credibilities, Perryman[8] defines the excess ratio, r, to be the ratio of expected excess losses to expected losses. Hence,

$$\begin{aligned} r &= Ee/E \\ &= [u \cdot E[n] \cdot \alpha \cdot X3(M)]/[u \cdot E[n] \cdot \alpha] \\ &= X3(M) \end{aligned}$$

It should be emphasized that this formula is only valid when losses are limited by a constant amount such as an MSL limitation (see previous section). Since the excess ratio plays a part in two of Perryman's credibility formulae, we can see that, all other things being equal, the same forces which impact the loss distributions will also affect credibility values based on these formulae.

Values of g

These values are of more historical than practical interest; however, readings are currently on the examination syllabus which discuss the concept of a g value. The necessity for a g value arises from the possibility that primary credibility may exceed unity for sufficiently large values of the excess ratio, r, under Perryman's Formula II. By substituting $K_E = K \cdot (1 - W) + WgS$ for K when $Q \leq E$, we guarantee that primary credibility will not exceed unity.

[7] R. J. Roth, "The Rating of Crop-Hail Insurance," *PCAS* XLVII (1960), p. 108.

[8] F. S. Perryman, "Experience Rating Plan Credibilities," *PCAS* LVIII (1971), p. 143.

Perryman[9] defines g as:

$$g = \max\{r\}$$
$$= \max\{X3(M)\}$$

where r varies by classification.

Since g is a function of the excess ratio, this formula is valid only for constant amount loss limitations. It should be clear that, for a fixed M, the values of $X3(M)$ and hence g will increase under inflation and must be adjusted to reflect current conditions. Uhthoff[10] gives a good discussion of the impact of inflation upon these values and the implications of a failure to adjust certain experience rating values under changing conditions.

VI. RETROSPECTIVE RATING

Table M Charge

Snader[11] defines the "charge" (or excess pure premium ratio) at entry ratio r to be:

$$\phi(r) = \int_r^\infty (t - r)dF(t) / \int_o^\infty t\, dF(t)$$

$$= (1/\alpha) \int_r^\infty (t - r)dF(t)$$

$$= X3(r)$$

where r = entry ratio
= actual losses ÷ expected losses
= actual loss ratio ÷ expected loss ratio.

Since the Table M charge is based on a ratio to expected losses, we must multiply by the permissible loss ratio, E', to obtain a ratio to (standard) premium. Thus, the percentage charge (applicable to standard premium) for a maximum loss ratio is $E' \cdot X3(r)$ (exclusive of loss adjustment expenses).

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[9] *ibid.*

[10] D. R. Uhthoff, "The Compensation Experience Rating Plan—A Current Review," *PCAS* XLVI (1959), p. 285.

[11] Snader, *op. cit.*, Part II, p. 52.

Table M Saving

Snader[12] defines the "saving" at entry ratio r to be:

$$\begin{aligned}
\psi(r) &= \int_o^r (r - t)dF(t) / \int_o^\infty t\, dF(t) \\
&= (r/\alpha) \int_o^r dF(t) - (1/\alpha) \int_o^r t\, dF(t) \\
&= (r/\alpha)F(r) - X1(r) \\
&= 1 - X1(r) - (r/\alpha)[1 - F(r)] + (r/\alpha) - 1 \\
&= X3(r) + (r/\alpha) - 1 \\
&= \phi(r) + (r/\alpha) - 1
\end{aligned}$$

If $\alpha = 1$, then we obtain an important relationship between the charge and saving, namely,

$$\psi(r) = \phi(r) + r - 1$$

Excess Loss Ratio

Snader[13] defines the excess loss ratio for a given injury type and loss limitation, l, as follows:

$$\begin{aligned}
e^*(l) &= y - r^*x \\
&= (1/\alpha) \int_l^\infty t\, dF(t) - (l/\alpha) \int_l^\infty dF(t) \\
&= (1/\alpha) \int_l^\infty (t - l)\, dF(t) \\
&= X3(l)
\end{aligned}$$

where, $y = (1/\alpha) \int_l^\infty t\, dF(t)$

$$r^* = l/\alpha$$

$$x = \int_l^\infty dF(t).$$

[12] *ibid.*, Part II, p. 54.

[13] *ibid.*, Part II, p. 55.

Harwayne[14] describes a method of obtaining countrywide excess loss ratios using statewide tables of excess loss ratios based on ratios to the mean. If we let $r = l/\alpha$, then we can substitute r for l and obtain:

$$\begin{aligned} e^*(l) &= e^*(r) \\ &= X3(r) \end{aligned}$$

It should be noted that Skurnick[15] calls the excess loss ratio a loss elimination ratio (denoted k). This should not be confused with a deductible loss elimination ratio which is the complement of the excess loss ratio.

Excess Loss Premium Factor

We can now obtain the excess loss premium factor (*ELPF*) for a dollar loss limitation per claim under a retrospective rating plan (net of any expense items). Since $X3(l)$ is a ratio of excess losses to expected losses, we can transform this into a ratio of excess losses to premium by multiplying by the permissible loss ratio, E'. Hence,

$$\begin{aligned} ELPF &= E' \cdot X3(l) \\ &= E' \cdot X3(r) \end{aligned}$$

where $r = l/\alpha$.

VII. REINSURANCE

Excess of Loss Coverage

The term burning ratio (*BR*) could be used to describe the ratio of expected excess losses to expected losses. This can be written as:

$$\begin{aligned} BR &= Ee/E \\ &= [u \cdot E[n] \cdot \alpha \cdot X3(R)]/[u \cdot E[n] \cdot \alpha] \\ &= X3(R) \end{aligned}$$

In order to apply this ratio to subject premium we must multiply by the permissible loss ratio, E', underlying the primary rates. Compare this to our discussion of *ELPF*'s.

Ferguson[16] refers to burning cost (*BC*) as the ratio of unmodified excess

[14] F. Harwayne, "Accident Limitations for Retrospective Rating," *PCAS* LXIII (1976), p. 1.

[15] D. Skurnick, "The California Table L," *PCAS* LXI (1974), p. 117.

[16] R. E. Ferguson, "An Actuarial Note on Loss Rating," *PCAS* LXV (1978), p. 50.

losses to subject premium. Let us change this definition to include the modifications to excess losses which Ferguson discusses (e.g., trend and loss development factors). Then we see that,

$$
\begin{aligned}
(BC) \cdot P &= (BR) \cdot E' \cdot P \\
BC &= E' \cdot (BR) \\
&= E' \cdot X3(R)
\end{aligned}
$$

where P = subject premium.

In other words, burning cost is similar to an *ELPF* in retro rating and burning ratio is similar to the excess loss ratio.

In practice, the reinsurer will not accept unlimited exposure and thus the burning ratio would have to be modified for a reinsurer limit, L, as follows:

$$
\begin{aligned}
BR &= (1/\alpha) \int_R^{R+L} (t - R)\, dF(t) + (L/\alpha) \int_{R+L}^{\infty} dF(t) \\
&= (1/\alpha) \int_R^{\infty} (t - R)\, dF(t) - (1/\alpha) \int_{R+L}^{\infty} (t - R)\, dF(t) + (L/\alpha) \int_{R+L}^{\infty} dF(t) \\
&= \mathrm{X3}(R) - (1/\alpha) \int_{R+L}^{\infty} [t - (R + L)]\, dF(t) \\
&= X3(R) - X3(R + L) \\
&= X2(R + L) - X2(R)
\end{aligned}
$$

In his review of Ferguson, Patrik[17] gives a formula for expected aggregate losses excess of R with limit L as:

$$
\begin{aligned}
\text{“expected losses”} &= \int_R^{R+L} (t - R)\, dF(t) + L \int_{R+L}^{\infty} dF(t) \\
&= \alpha[X3(R) - X3(R + L)] \\
&= \alpha[X2(R + L) - X2(R)]
\end{aligned}
$$

Dividing this formula by α yields the above formula for the burning ratio with limit, L.

Stop Loss Coverage

In a previous section we discussed the Table M charge. This equals the percentage of expected losses which is expected to be incurred above a selected

[17] G. Patrik, discussion of “An Actuarial Note on Loss Rating” by R. E. Ferguson, *PCAS* LXV (1978), p. 56.

maximum loss ratio, r. We can show that this charge is equivalent to the charge necessary for a stop loss (aggregate excess) reinsurance contract. The only conceptual difference is in the definition of "claim" and "risk." Specifically, we can define "risk" to be a primary insurer with an underlying reinsurance program and "claim" to be an annual (aggregate) recoverable for net losses which exceed a specified loss ratio. Thus,

$$\begin{aligned}\text{stop loss ratio} &= \int_r^\infty (t - r)\, dF(t) / \int_o^\infty t\, dF(t) \\ &= (1/\alpha) \int_r^\infty (t - r)\, dF(t) \\ &= X3(r)\end{aligned}$$

where r = annual net loss ratio.

If there is a percentage participation (p) by the reinsured on excess losses and a reinsurer limit of $100L\%$, we would have

$$\text{stop loss ratio} = (1 - p)[X3(r) - X3(r + L)].$$

A stop loss premium factor can be obtained as the product of the permissible loss ratio, E', underlying the subject premium and the stop loss ratio, or $E' \cdot X3(r)$ (compare to the previous section).

VIII. EXCESS RATING

Increased Limit Factors

Miccolis[18] shows that increased limit factors can be obtained from a claim size distribution. If we let

$$\begin{aligned}E[g(x;k)] &= \int_o^k t\, dF(t) + k[1 - F(k)] \\ &= \alpha \cdot X2(k),\end{aligned}$$

then we can obtain a formula for an increased limit factor, $I(k)$.

[18] R. S. Miccolis, "On the Theory of Increased Limits and Excess of Loss Pricing," *PCAS* LXIV (1977), p. 27.

$$
\begin{aligned}
I(k) &= \frac{E[g(x;k)]}{E[g(x;b)]} \\
&= \frac{\alpha \cdot X2(k)}{\alpha \cdot X2(b)} \\
&= \frac{X2(k)}{X2(b)}
\end{aligned}
$$

Miccolis goes on to show that risk-adjusted increased limit factors can be obtained as:

$$
\begin{aligned}
I_r(k) &= \frac{E[g(x;k)] + \lambda \cdot E[g(x;k)^2]}{E[g(x;b)] + \lambda \cdot E[g(x;b)^2]} \\
&= \frac{\alpha \cdot X2(k) + \lambda \cdot E[g(x;k)^2]}{\alpha \cdot X2(b) + \lambda \cdot E[g(x;b)^2]} \\
&= \frac{X2(k) + \lambda \cdot E[g(x;k)^2]/\alpha}{X2(b) + \lambda \cdot E[g(x;b)^2]/\alpha}
\end{aligned}
$$

IX. MISCELLANEOUS

Relative Trend

In his paper on basic limits trend factors, Finger[19] defines the term relative trend (*RT*) to be the ratio of basic limits trend to total limits trend. In order to obtain a working formula, Finger defines the average relative trend (*ART*), for a particular period of time, which is the percentage increase in basic limits losses divided by the percentage increase in total limits losses. That is,

$$
ART(r) = \frac{B(vr) - B(r)}{B(r)} \div \frac{T(vr) - T(r)}{T(r)}
$$

where, r = basic limit ÷ mean
$v = (1 + i)^{-1}$
i = total limits trend over the period of time.

[19] R. L. Finger, "A Note on Basic Limits Trend Factors," *PCAS* LXIII (1976), p. 106.

We will define:

$$
\begin{aligned}
B(r) &= E[n] \cdot \alpha \cdot X2(r) \\
E(r) &= E[n] \cdot \alpha \cdot X3(r) \\
T(r) &= B(r) + E(r) = E[n] \cdot \alpha \cdot [X2(r) + X3(r)] = E[n] \cdot \alpha
\end{aligned}
$$

Under inflation, r will decrease to vr and α will increase to $\alpha(1 + i)$. Hence,

$$
\begin{aligned}
B(vr) &= E[n] \cdot \alpha \cdot (1 + i) \cdot X2(vr) \\
E(vr) &= E[n] \cdot \alpha \cdot (1 + i) \cdot X3(vr) \\
T(vr) &= E[n] \cdot \alpha \cdot (1 + i)
\end{aligned}
$$

We can now compute a working formula for ART.

$$
\begin{aligned}
ART(r) &= \frac{E[n] \cdot \alpha \cdot (1 + i) \cdot X2(vr) - E[n] \cdot \alpha \cdot X2(r)}{E[n] \cdot \alpha \cdot X2(r)} \\
&\div \frac{E[n] \cdot \alpha \cdot (1 + i) - E[n] \cdot \alpha}{E[n] \cdot \alpha} \\
&= \frac{(1 + i) \cdot X2(vr) - X2(r)}{i \cdot X2(r)}
\end{aligned}
$$

If we take the limit of ART as $i \to 0$, we obtain the relative trend prior to the inflation of the period of time assumed. Thus,

$$
\begin{aligned}
RT(r) &= \lim_{i \to 0} ART(r) \\
&= \lim_{i \to 0} (1/i) \cdot \frac{(1 + i) \cdot X2(vr) - X2(r)}{X2(r)} \\
&= \frac{X1(r)}{X2(r)} \qquad \text{(using L'Hôpital's Rule)}
\end{aligned}
$$

Depending on the particular application, either $RT(r)$ or $ART(r)$ may be needed.

X. CONCLUSION

We have discussed several original papers which have presented material to this Society relating to loss distributions. All of these papers are currently on the examination syllabus. However, this is not to imply that these are the only papers which utilize loss distributions. There are other papers currently in the

Proceedings, and there will most likely be future papers, dealing with this topic. Many of these papers could be analyzed using the generalized notation presented here. If this paper assists in the development of a clearer framework from which to understand the uses of loss distributions in casualty actuarial work, then the goal of the paper will have been reached. An appendix is included which gives a summary of the formulae presented. From this summary, it is clear that several rating concepts are mathematically (actuarially?) equivalent. The notation for the elementary functions is similar to that derived by R. J. Finger.

APPENDIX

SUMMARY OF FORMULAE

1.	Straight deductible *LER*	$X2(D)$
2.	Franchise deductible *LER*	$X1(D)$
3.	Disappearing deductible *LER*	see Section IV
4.	*D* ratio	$X2(M)$
5.	Excess ratio	$X3(M)$
6.	*g* value	$\max\ \{X3(M)\}$
7.	Table M charge	$X3(r)$
8.	Table M saving	$(r/\alpha)F(r) - X1(r)$
9.	Excess loss ratio	$X3(r)$
10.	Excess loss premium factor	$E' \cdot X3(r)$
11.	Burning ratio	$X3(R)$
12.	Burning cost	$E' \cdot X3(R)$
13.	Stop loss ratio	$X3(r)$
14.	Stop loss premium factor	$E' \cdot X3(r)$
15.	Increased limit factor	$X2(k)/X2(b)$
16.	Average relative trend	see Section IX
17.	Relative trend	$X1(r)/X2(r)$

A MODEL OF INDUSTRY GENERAL LIABILITY NET WRITTEN PREMIUMS

GREGORY N. ALFF AND JAMES R. NIKSTAD

Abstract

The paper presents an econometric model of industry general liability net written premiums. The model is fit using a multiple linear regression program. The reasons for using a log-differencing form are explored. Exposures, rate levels and pricing are the three most important influences on written premiums. Time series of values measuring these influences are compiled as input to the model. Several statistics are discussed that indicate an excellent fit to the data. Short term forecasts of the change in general liability written premiums are presented. The model's usefulness is in its ability to separate and quantify the impacts of exposure changes, rate level changes, and pricing cycle changes on general liability net written premiums.

I. PREFACE

Underwriting results for general liability during the last decade have been very volatile. Combined underwriting ratios in excess of 115% were common in the industry in 1974 and 1975. Three years later, in 1978, significant underwriting profits were typical. Many forces working together produce such swings in underwriting fortune, but the most important ingredient appears to be the pricing of the general liability insurance product. The question addressed by this paper is: What changes in underlying variables precipitate the irregular pattern of annual changes in general liability premiums?

II. THE MODEL

We chose what is basically a log-differencing form to model general liability premium changes. The equation for the model is:

$$\ln(CP) = b_1 \ln(CFS) + b_2 \ln(LR36/E \text{ lag } 2) + b_3 \ln(Price1 \text{ lag } 1) + b_4 \ln(Price2 \text{ lag } 3) + b_5 \text{ Dummy} + \text{error}.$$

In this equation, CP is the dependent variable, 1.0 plus the annual change in industry general liability net written premium. Alternatively, $\ln(CP)$ could be expressed as a difference, ln(current year written premium) minus ln(prior year written premium); hence, the term log-differencing.

The symbols b_1, b_2, b_3, b_4, and b_5 represent coefficients for the respective independent variables in the model. An error term is included here by convention; it serves as a reminder that the model does not describe the real world situation perfectly. The independent variables are discussed in the paper and defined in the appendix.

We chose to fit changes in the variables instead of the actual values of the variables. Fitting actual values of inflation sensitive variables can often lead to problems such as:

1. The colinearity of independent variables;
2. The model missing turning points; and,
3. The true magnitude of error being masked.

There are several important reasons for using the logarithmic form:

1. Coefficients are elasticities (discussed further in the appendix).
2. The fit is more robust.
3. We believe the independent variables should be applied multiplicatively.
4. An inflation-sensitive time series is deflated to a constant.

The log-differencing form of the equation used for modeling is shown above. Transformation from the logarithmic form shows the more direct equation with which we are working:

$$CP = CFS^{b_1} \cdot (LR36/E \text{ lag } 2)^{b_2} \cdot (Price1 \text{ lag } 1)^{b_3} \cdot (Price2 \text{ lag } 3)^{b_4} \cdot e^{b_5 \text{Dummy}} \cdot e^{\text{error}}.$$

The multiple regression modeling program, in a package produced by Data Resources, Incorporated (DRI) was used to compute the model coefficients. Exhibit I shows information defining the model and presenting important statistics concerning the model.

III. THOUGHTS UNDERLYING THE MODEL

This model explicitly considers three major influences on general liability premiums: changes in exposures, changes in rate level, and changes in underwriting pricing. Other influences are addressed in the last section of this paper.

Inflation has led to annual increases in payroll and sales exposure bases. ISO data indicates that these are the exposure bases for at least two-thirds of the general liability business. The entire effect of such exposure changes is generally converted into premium increases. Exposure also measures changes in more general economic conditions, such as periods of recession, which influence general liability premiums.

Rate revisions are the second major influence on premium changes. For a portion of the general liability business, rate revisions are relatively large because the exposure base used (area and frontage) is not inflation-sensitive. Rate revisions are necessary to adjust for the amount by which changes in the compounded levels of severity and frequency differ from the economic trend as measured by the exposure base.

Pricing is as important as the first two factors. There is a great deal of pricing flexibility available in the general liability line. Through the optional use of experience rating, schedule rating, loss rating, and "a" rating, the underwriter has a great deal of latitude in what he may charge for a particular liability exposure. Because of the relative inflexibility of pricing for workers' compensation and the often small volume of commercial auto insurance, the general liability line is used to compete in price for casualty accounts. This has been very apparent in the 1980 and 1981 commercial lines marketplace.

IV. REQUIREMENTS OF INDEPENDENT VARIABLES

In order to model the annual change in general liability net written premium, we wish to include at least one variable for each of the major influences described above. Values for each variable should be available for a significant number of years (say, 20) on a consistent basis, if possible. Data should be from recognized authoritative sources, such as the U.S. Department of Commerce or A. M. Best, if possible.

The correlation between independent variables should be low. That is to say, colinearity of independent variables should be minimized.

V. THE VARIABLES

The dependent variable we wish to model is 1.0 plus the annual change in industry general liability net written premium (excluding malpractice) and is called *CP*.

We expected to utilize at least three independent variables in order to include the influences of exposure, rate revisions, and underwriting pricing in the model. It was difficult to find a combination of independent variables to achieve a good fit in the model. We tried many variables, often specified in several different ways. Calculating the correlation coefficient between each proposed independent variable and the dependent variable helped to limit the search. The correlation coefficient between each pair of proposed independent variables pointed out potential problems with colinearity.

We finally arrived at four independent variables plus a "dummy" applied to two years. We chose annual change in final sales in the United States, *CFS*, for the exposure variable. The variable is input in the form, final sales in the year being modeled divided by the final sales in the previous year (*FS* divided by *FS* lag 1). The final sales variable is based on Department of Commerce statistics, is available for many years, is fairly stable and predictable, and is forecast by DRI.

The rate revision variable was the most difficult to specify. There is no long-term rate level index available as in workers' compensation. Rates are made separately for each of several sublines within general liability. Virtually all sources of data include malpractice through 1974. The variable we decided upon is the general liability Schedule P loss plus adjustment expense ratio as of 36 months divided by a permissible loss ratio. The variable is lagged two years since rates are made prospectively. There were significant obstacles in the way of compiling a reasonably long-term history of this variable. *LR36/E* is further described in the appendix.

To define a pricing variable or variables, we began with the premise that competition dictates pricing decisions. Corporate managements define underwriting or premium writing goals that are interpreted and pursued by field personnel. We see these decisions and goals as the main cause of the "underwriting pricing cycles" in general liability. Financial strength and recent underwriting results seem to be prime motivators in establishing pricing decisions and premium goals. We, therefore, arrived at two independent variables to include the effect of pricing in the model.

The first pricing variable, *Price*1, reflects financial strength. The basis of the variable is the premium-to-surplus ratio. The form of the variable entering the model is 1.0 plus the premium-to-surplus ratio minus a goal (or benchmark) premium-to-surplus ratio. The idea is to quantify how the industry views its financial strength. If this variable is less than 1.0, industry management will envisage financial strength and will be willing to compete vigorously for business. If the value is greater than 1.0, industry management will be concerned that their financial strength has eroded, price competition will subside, and there will be an increase in prices and premiums.

The establishment of the goal (or benchmark) premium-to-surplus ratio to be used in calculation of *Price*1 is somewhat problematic. The method by which the "goal" ratio is established is described in detail in the appendix. The variable enters the model lagged one year. This results from the time lag between the perception of a change in financial strength and the implementation of effective marketing programs.

The second pricing variable, *Price*2, deals with the effects of pricing on recent underwriting results. Many forms of variables were tried before arriving at what is essentially a modified time series variable. Premium changes in the second, third, and fourth prior years are significantly correlated with the premium change in the current year. The second prior year change is indirectly included in the ratemaking variable. Therefore, we concentrate on the third and fourth prior years. We adjust the third and fourth prior year premium changes by dividing by the change in the Consumer Price Index during the same two years. The variable is designed to measure the cycle in general liability underwriting pricing. Our logic is that when premium has been growing significantly faster than the CPI for three or four years, experience will improve and competition will intensify. On the other hand, when the ratio is low due to a soft general liability market, this cannot continue indefinitely. Deteriorating underwriting results will lead to a tightening of the general liability market and premium increases. This variable is more fully defined in the appendix.

The last independent variable entering the model is "Dummy," which equals 1.0 in 1971 and 1972 and zero for all other years. There were two disrupting influences which affected general liability premium changes in 1971 and 1972. First, federal price controls were a major influence which severely limited the magnitude of rate increases during these years. Second, the rate of premium growth for commercial multi-peril was greater in 1971 and 1972 than in any other years. This drained an abnormal amount of premium out of the general liability line in the same two years. Therefore, "Dummy" makes a special adjustment to the model in the 1971 and 1972 years.

It was determined that a constant did not improve the model. The major effect of inserting a constant was to replace other variables. Especially vulnerable to being excluded, based on its t-statistic, was the exposure variable, *CFS*. Since we believe it is more valuable to include the exposure variable without dilution by the constant, we eliminated the constant from the model.

Exhibit II shows historical values of the independent and dependent variables entering the model.

VI. THE MEANING OF THE STATISTICS

The statistics shown on Exhibit IA provide important information concerning the significance of the variables and the quality of the model.

The block of data at the top of Exhibit IA provides information regarding the five independent variables. The coefficients in Column 2 are calculated by multiple regression. They are the coefficients which result in fitted values which are closest to the actual values of the dependent variable. The sign of the coefficient for each variable agrees with our *a priori* expectations. An increase in final sales implies an increase in premium. A loss ratio larger than the permissible loss ratio in the rate revision variable implies an increase in premium. A premium-to-surplus ratio larger than the goal in the *Price*1 variable implies an increase in premium. Premium increases exceeding increases in the prior years' CPI by more than the average amount in the *Price*2 variable imply a premium decrease. Dummy equal to 1.0 in 1971 and 1972 has a negative coefficient, indicating a limiting of premium increases in those years.

The standard error of the coefficient of an independent variable is the estimated standard deviation of the coefficient. This statistic is used to test the significance of the coefficient of the independent variable. The "true" value of the coefficient is within two standard errors of the calculated coefficient 95% of the time.

This can be restated in terms of the t-statistic, which equals the coefficient divided by the standard error. If the t-statistic is greater than 2.0, the coefficient, and thus the variable, is said to be significant for the regression. The t-statistics show that all independent variables in this model are significant.

The F-statistic is the ratio of the explained variation to the unexplained variation of the dependent variable. Our F-statistic should be compared to a critical value for an F with 4 and 15 degrees of freedom. For alpha equal to .05, the critical value is 3.06. Since our F-statistic is greater than 3.06, the regression is significant.

The R^2 statistic is the common measure of the proportion of variance of the dependent variable accounted for by the relationship of the dependent variable to the independent variables. Regarding this model, it may be said that the model explains 90% of the annual change in written premium.

The R-Bar Squared statistic is the R^2 statistic adjusted for degrees of freedom. It may be thought of as R^2 refined for further accuracy. This statistic is defined in more detail in the appendix.

The Durbin-Watson statistic provides the standard test for autocorrelation. Autocorrelation occurs when the error between the fitted and actual value is not independent from one observation to the next. A Durbin-Watson statistic between 1.5 and 2.5 indicates that there is not serious autocorrelation. A Durbin-Watson outside this range indicates the probability of autocorrelation. The model described here has no significant autocorrelation indicated. This statistic is discussed in greater detail in the appendix.

The standard error of the regression measures how close the fitted values have been to the actual values for the history being modeled. This statistic is calculated so that, for 67% of the historical observations, the fitted value is within ± 1 standard error of the actual value. The fitted value is within ± 2 standard errors 95% of the time.

Exhibit IB shows the actual and fitted values of $\ln(CP)$. Exhibit IC shows a graph of these values.

VII. FORECASTS FROM THE MODEL

Given accurate forecasts of the independent variables, we believe this model will provide good indications of future annual changes in general liability written premiums. The model was originally fit with 20 data points (1960–1979). Although general liability written premiums had not shown an annual decrease in at least 25 years, the model correctly forecasted a negative change in premium in 1980. We believe that forecasts will improve when the model is refit with each new data point since "pure" general liability experience will be added.

Based on data through 1980, as shown on Exhibit II, the model forecasts that the change in industry general liability net written premium, excluding malpractice, for 1981 is −1.0%. A. M. Best Company, Inc., published estimates in January 1982 placing the change in premium at −2.8%. Thus, the model has again correctly indicated a decrease, apparently with accuracy within one standard error (3.5%). The authors find this result quite satisfactory.

Forecasts of 1982 general liability written premium have two potential sources of error. These are the error of the model and the error in forecasting the independent variables. We feel fairly comfortable with the DRI projections of final sales and the Consumer Price Index. However, it is necessary for us to select a premium-to-surplus ratio for 1981 and a general liability accident year loss ratio as of 36 months for 1980. We selected an increasing loss ratio and a slight decrease in the premium-to-surplus ratio as shown on Exhibit II. These inputs generate a forecast of a 9.0% increase in general liability written premiums for 1982.

The model leads us to believe that premiums will increase by more than 9.0% in 1983.

VIII. FINAL THOUGHTS

We recognize that this model has not explicitly included the effects of several other factors influencing general liability premiums. Among these factors are high deductible and captive modes of handling general liability exposures, movement to package policies, and the ebb and flow of retro adjustments. These factors are having some impact on premium changes. In general, we view changes based on these factors as being gradual. We believe that these gradual effects on premium are partially accounted for by the fitted values of the coefficients. In particular, we believe the coefficient of the exposure variable, *CFS*, would have been slightly larger if captives and high deductibles had not reduced premium changes in recent years.

If this model is reasonably accurate in describing the interrelationship of general liability pricing and the forces that drive it, the wide swings in underwriting results and market conditions are likely to continue for the near future. A cycle peak loss ratio is likely in 1982. This industry seems to have learned little from the lessons of the mid-1970's. Perhaps this model provides a first step for better understanding the "underwriting pricing cycle" for general liability. The challenge to the industry is to understand and control the factors causing the cycle so as to dampen its amplitude in the future.

APPENDIX

Definitions of Variables

CP — *CP* is the dependent variable. It is 1.0 plus the change in industry general liability calendar year net written premium. *CP* excludes medical malpractice for calendar years 1976 to 1980. It contains data for stock and mutual companies as compiled in *Best's Aggregates and Averages*.

CFS — *CFS* is the annual change in final sales. The values of the final sales variable were obtained from DRI and are in billions of dollars. Final sales data is compiled by the U.S. Department of Commerce, Bureau of Economic Analysis. It is a measure of the total final sales of the United States, where final means the last sale of a new product. For example, car sales to a consumer are included, but if General Motors buys a part for the starter of the car from the Bendix Corporation, this is excluded.

LR36/E — *LR36/E* is a proxy ratemaking variable. Ideally it would be the accident year loss and loss adjustment expense ratio as of 36 months, for GL (BI and PD), excluding malpractice, for the entire industry, divided by the permissible loss ratio. Permissible is assumed to be 62% (57% ISO permissible + 5% profit and contingency loading). This was our goal, but we ended up using approximations of these loss ratios in many instances.

We obtained our loss ratio data by compiling information from *Best's Reproductions of Annual Statements* for 26 major general liability writers. We compiled losses and premiums from annual statements for 12 years (1969–1980), obtaining somewhat more than 60% of the industry premium volume. In certain periods, the data we wanted was not available, so we were forced to use substitutes. The following is a list of situations where substitute data was used:

1. All data for 1974 and prior accident years includes medical malpractice.
2. Data from 1975 and prior annual statements excludes Commercial Union Insurance Company.
3. All data for accident year 1970 and prior excludes property damage.
4. Policy year data was used prior to 1969, as Schedule P was on a policy year basis.
5. For policy years 1962–1966, evaluations of the loss ratios later than 36 months were used, as the 36-month evaluations were not available to us.
6. For years prior to 1962, we had neither policy year nor accident year data available, so we assumed that the change in the accident year loss ratio was the same as the change in the calendar year loss ratio.

The reliability of this variable is reduced by the large number of adjustments that we found necessary. However, we believe it is better than using calendar year loss ratios throughout the period. This opinion is partially based on a recognition of the reserve strengthening which has occurred in the industry since 1973.

The variable is lagged two periods in the model for two reasons. First, the difference between the evaluation date of data entering ratemaking and the average effective date for policies utilizing revised rates based on the data is approximately two years. Second, the data entering ratemaking calculations is mainly from accident years lagged 2, 3, and 4 years from the effective year of the rates. However, premiums from calendar years lagged 3 to 4 years are included in the pricing cycle variable, *Price*2. Therefore, this variable concentrates on the accident year loss ratio lagged two years.

There is some overlap in function betweeen *LR*36/*E* and *Price*2.

*Price*1 — *Price*1 is a modified premium-to-surplus ratio which attempts to measure the premium-to-surplus ratio against a benchmark or goal. The premium-to-surplus ratio (*PSR*) was obtained from *Best's Aggregates and Averages*. There are two problems with using the *PSR* as given in Best's.

First, the series double counts surplus for members of an insurance group. However, the series is also available for eight years (1973–1980) on a consolidated basis (excluding the double counting). Over this time period, the ratio of the two *PSR's* (excluding to including double counting) is very stable at 1.265. Therefore, *PSR* as taken from *Best's Aggregates and Averages* was modified by a factor of 1.265 in our analysis.

A second problem concerning the premium-to-surplus ratio was encountered. We feel that the *PSR* that the industry used as a goal or benchmark changed during the period from 1961 to 1980. This was caused by:

1. A growing percentage of total business being casualty, which may be written at a higher *PSR* than property business.
2. Higher investment income caused by:
   a. A higher level of reserves in casualty lines.
   b. Higher interest rates.

To attempt to measure this change over time, we fit a least squares line between the premium-to-surplus ratio modified by a factor of 1.265 (*PSRM*) and time for the period 1945–1979. The fitted line is called "*PSRM* goal." The difference between PSRM and the fitted line (*PSRM* goal) is a measure of how strong the industry perceived itself to be.

*Price*1 is obtained by adding 1.0 to the residuals (actual minus fitted values) of the above regression. The 1.0 is added to make the variable appropriate to enter the model in log-differencing form.

The authors wish to acknowledge the help of James F. Golz who proposed the technique of fitting a least squares line to *PSRM* to remove the time trend.

Price2 — *Price2* is the ratio of the two-year change in general liability written premium to the two-year change in the consumer price index divided by the mean of this ratio over time.

$$Price2 = [(CP_t \cdot CP_{t-1})/(CC_t \cdot CC_{t-1})]/\text{Mean}$$

CP_t is the calendar year change in general liability written premium in year t.

CC_t is the one-year change in the CPI in year t.

Mean is the 23-year mean of $(CP_t \cdot CP_{t-1})/(CC_t \cdot CC_{t-1})$.

We calculated correlation coefficients between CP_t and CP_{t-x}/CC_{t-x} where x varied from 1 to 5. Significant correlations were found at $x = 2$, 3, and 4 years. Since the change in premium lagged two years was indirectly accounted for in our ratemaking variable, we used the lag 3 and lag 4 years relationship in this variable. The variable enters our regression lagged three years. The numerator of the variable entering the regression is the product of the ratios CP_t/CC_t lag 3 and CP_t/CC_t lag 4.

Dummy — Dummy is a variable equal to 1.0 in years 1971 and 1972 and zero in all other years. It is entered into the model to reflect circumstances unique to those two years. First, federal price controls severely limited rate increases filed and approved in 1971 and 1972. Second, the growth in CMP premiums was approximately 25% in each of those two years, but averaged approximately 16% in years prior and subsequent. Thus, more general liability premium than usual was lost to CMP during 1971 and 1972.

Elasticities

One advantage of using the log-differencing form is that the regression coefficients of the variables are the elasticities. An elasticity is the amount by which the dependent variable is changed by a 1% change in an independent variable. Thus, the coefficient of final sales (0.688) means that a 1% change in final sales causes a 0.688% change in written premium. This is a reasonable result since at least two-thirds of the general liability exposures are inflation sensitive.

R-Bar Squared

R-Bar Squared is a statistic used by DRI to measure the fit of a model. It is basically R^2 adjusted for degrees of freedom. The formula given by Johnston in *Econometric Methods* is:

$$R\text{-Bar Squared} = \frac{1 - K}{N - K} + \frac{R^2(N - 1)}{N - K}$$

where:

R^2 is the portion of the total sum of squares explained by the regression,
K is the number of parameters fit, and
N is the number of observations.

The following equivalent form shows that *R*-Bar Squared is always less than R^2:

$$R\text{-Bar Squared} = R^2 - \frac{(K - 1)(1 - R^2)}{N - K}$$

While R^2 always increases as more variables are added to the model, *R*-Bar Squared may decrease if the added variable has little value.

Durbin-Watson Statistic

The Durbin-Watson statistic is the standard test for autocorrelation. It is given by the following formula:

$$D = \frac{\sum_{t=1}^{N} (\hat{E}_t - \hat{E}_{t-1})^2}{\sum_{t=1}^{N} \hat{E}_t^2}$$

where:

$\hat{E}_t$ is the difference between the actual value and the fitted value for observation t.

D is approximately equal to $2(1 - R_1)$ where R_1 is the first order autocorrelation coefficient. Thus, D ranges from 0 to 4. The acceptable range for D is from 1.5 to 2.5. If D is in this range, no significant autocorrelation exists. It is interesting to note that this means that R_1 is between $-.25$ and $.25$. Thus, an acceptable Durbin-Watson implies a small first order autocorrelation coefficient.

EXHIBIT IA

MODEL OF CHANGE IN INDUSTRY GENERAL LIABILITY NET WRITTEN PREMIUM MULTIPLE LINEAR REGRESSION

ANNUAL (1961 TO 1980) 20 OBSERVATIONS
DEPENDENT VARIABLE: LN(*CP*)

Independent Variable	Coefficient	Standard Error	t-Statistic
ln(*CFS*)	$b_1 =$ .6884	.2368	2.907
ln(*LR*36/*E* lag 2)	$b_2 =$ .4361	.1063	4.103
ln(*Price*1 lag 1)	$b_3 =$ .0991	.0302	3.288
ln(*Price*2 lag 3)	$b_4 = -$.2586	.0783	−3.301
Dummy	$b_5 = -$.0866	.0270	−3.202

Regression Statistics

F-Statistic: 37.32
R Squared: .9087
R-Bar Squared: .8843

Durbin-Watson Statistic: 2.007
Standard Error of the Regression: .0348

EXHIBIT IB

LISTING OF ACTUAL AND FITTED VALUES OF THE REGRESSION

Dependent Variable: ln(*CP*)

Year	Actual	Fitted	Year	Actual	Fitted
1961	.061	.075	1971	.107	.115
1962	.035	.039	1972	.071	.063
1963	.030	−.003	1973	.056	.082
1964	.018	−.018	1974	.083	.139
1965	.023	.016	1975	.283	.272
1966	.059	.066	1976	.319	.297
1967	.096	.105	1977	.320	.273
1968	.097	.148	1978	.103	.161
1969	.158	.129	1979	.017	.000
1970	.222	.192	1980	−.025	−.049

EXHIBIT IC

GRAPH OF DEPENDENT VARIABLE

ln(CP)

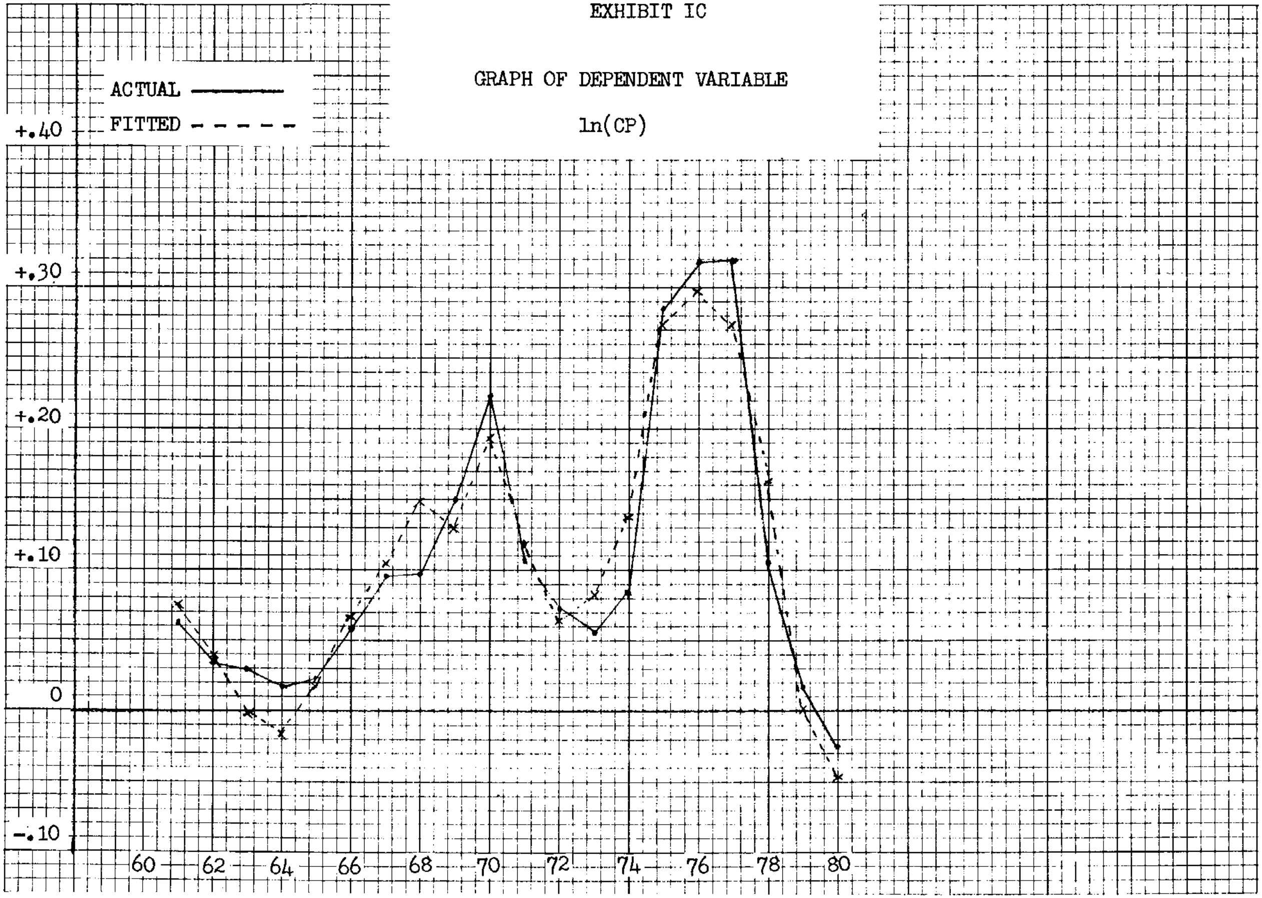

EXHIBIT II

DATA HISTORIES FOR VARIABLES USED IN MODELING ANNUAL CHANGE IN INDUSTRY GENERAL LIABILITY NET WRITTEN PREMIUM

Calendar Year	Change in GL Net Written Premium +1.00 *CP*	Change in Final Sales +1.00 *CFS*	Accident Year Loss & LAE Ratio as of 36 Months *LR*36	*LR*36/*E*	Best's Premium/ Surplus × 1.265 *PSRM*	*PSRM* Goal	*PSRM* − *PSRM* Goal +1.00 *Price*1	Change in CPI +1.00 *CC*	$[(CP_t \times CP_{t-1})/(CC_t \times CC_{t-1})]$ /Mean *Price*2
1956	1.094	—	—	—	1.466	1.636	.830	1.015	—
1957	1.093	—	—	—	1.727	1.662	1.065	1.035	.992
1958	1.067	—	.727	1.173	1.519	1.688	.832	1.028	.955
1959	1.112	—	.702	1.132	1.532	1.713	.819	1.008	.998
1960	1.117	1.044	.690	1.113	1.588	1.739	.849	1.016	1.056
1961	1.062	1.037	.644	1.039	1.341	1.764	.577	1.011	1.007
1962	1.035	1.070	.612	.987	1.435	1.790	.645	1.012	.937
1963	1.030	1.057	.637	1.027	1.380	1.815	.565	1.012	.907
1964	1.018	1.070	.664	1.071	1.364	1.841	.523	1.013	.891
1965	1.023	1.078	.683	1.102	1.483	1.866	.616	1.017	.881
1966	1.060	1.089	.735	1.185	1.796	1.892	.904	1.029	.903
1967	1.101	1.064	.740	1.194	1.723	1.917	.805	1.028	.906
1968	1.102	1.097	.821	1.324	1.723	1.943	.780	1.042	.986
1969	1.172	1.079	.821	1.324	2.211	1.969	1.243	1.054	1.025
1970	1.249	1.059	.769	1.240	2.245	1.994	1.251	1.059	1.142
1971	1.113	1.081	.741	1.195	1.986	2.020	.966	1.043	1.096
1972	1.073	1.099	.803	1.295	1.763	2.045	.718	1.033	.966
1973	1.057	1.112	.862	1.390	1.984	2.071	.913	1.062	.901
1974	1.087	1.086	.926	1.494	2.734	2.096	1.637	1.110	.849
1975	1.327	1.096	.838	1.352	2.498	2.122	1.376	1.091	1.038
1976	1.376	1.096	.718	1.158	2.457	2.147	1.309	1.058	1.378
1977	1.377	1.112	.609	.982	2.472	2.173	1.299	1.065	1.466
1978	1.108	1.125	.614	.990	2.315	2.199	1.116	1.077	1.160
1979	1.017	1.123	.639*	1.031*	2.123	2.224	.899	1.113	.819
1980	.975	1.098	.690*	1.113*	1.833	2.250	.583	1.135	.683
1981	.990f	1.104f			1.800f	2.276	.524f		
1982	1.090f	1.078f							

f = forecast
* = Estimated 36-month values

E = adjusted expected loss ratio
$E = .57 + .05 = .62$

Mean = mean of the series
$(CP_t \cdot CP_{t-1})/(CC_t \cdot CC_{t-1}) = 1.148$

THE OPTIMAL USE OF DEPOPULATION CREDITS IN THE PRIVATE PASSENGER AUTO RESIDUAL MARKET

THOMAS J. KOZIK

Abstract

This paper describes the depopulation credits that are available in the private passenger auto residual market plans of many states and develops two models that can be used by an insurer to optimize the use of those credits. Each model represents an extreme case, with the real world falling somewhere between the two extremes. An example of the use of the models also is included, as is some discussion of how to measure the benefit of optimally using depopulation credits.

INTRODUCTION

The private passenger automobile residual market plans of many states contain depopulation credit provisions. Under these provisions an insurer receives a fixed number of dollars credit against its residual market premium quota for each dollar of premium written voluntarily on specific categories of risks. In this paper, a risk that qualifies for such a credit is called an eligible risk. A company can use the depopulation credits on eligible risks to reduce its participation in the residual market. If the residual market in a particular state is consistently underpriced, this reduction in a company's quota can decrease its residual market underwriting losses. Thus, total underwriting income will be maximized by using the credits if the eligible risks written voluntarily by a company produce underwriting losses that are less than the reduction in underwriting losses attributable to the reduction in the residual market quota.

The problem, therefore, is determining which eligible risks to accept. The answer depends on the difference between total expected underwriting losses with the eligible risks written by the insurer in the voluntary market and total expected underwriting losses without them. This paper presents two models that give upper and lower bounds for the maximum loss and loss expense ratio that can be incurred on an eligible risk written voluntarily. The bounds are computed so as to maximize the total underwriting income of the company.

DEPOPULATION CREDITS AND RESIDUAL MARKET QUOTA

In most states an insurer can earn credits by voluntarily insuring the following types of risks:

- Class 2: Youthful male principal operator/youthful male household resident operator.
- Over 65: Operators aged 65 and over.
- Keep-out: Any risk who is a previously uninsured resident of a compulsory insurance state.
- Take-out: Any risk who is removed from the auto insurance plan (residual market) and written in the voluntary market.

Keep-out and Take-out credits are usually two-for-one credits. That is, two dollars of credit against the company's residual market premium quota is given for every dollar of premium written voluntarily on these risks. Class 2 and Over 65 credits are usually dollar-for-dollar credits. It is possible to "double-up" on credits. For example, a particular risk may be eligible for both Class 2 and Take-out credits. The company that voluntarily insures this risk gets both credits.

The formula used to determine each company's share of the residual market premium[1] is given below, based on the following notation:

$j = 1, 2, 3, 4$ denote the types of credits, i.e., $j = 1$ for Take-out credits, $j = 2$ for Keep-out credits, $j = 3$ for Class 2 credits and $j = 4$ for Over 65 credits;

$X_{j,i,y}$ denotes the premium eligible for type of credit j that is voluntarily written in year y by company i;

N_j denotes, for type of credit j, the number of credit dollars given per dollar of eligible premium written voluntarily;

$P_{i,y}$ denotes the voluntary market exposure penetration of company i in year y, i.e., the ratio of voluntary car-years insured by company i to the total number of car-years written in the voluntary market;

T_y denotes the total residual market premium to be assigned in the year y.

Take-out and Keep-out credits written in the current year reduce the current year's quota, while Class 2 and Over 65 credits take effect two years later. Thus the total credit in dollars for company i in year y is given by

$$K_{i,y} = \sum_{j=1}^{2} N_j X_{j,i,y} + \sum_{j=3}^{4} N_j X_{j,i,y-2} .$$

The credit in dollars for all companies combined is given by

$$K_y = \sum_i \sum_{j=1}^{2} N_j X_{j,i,y} + \sum_i \sum_{j=3}^{4} N_j X_{j,i,y-2}$$

The unadjusted quota for company i in year y is

$$q_{i,y} = \begin{cases} 0 \text{ if } P_{i,y-2}(T_y + K_y) - K_{i,y} \le 0 \\ \dfrac{P_{i,y-2}(T_y + K_y) - K_{i,y}}{T_y} & \text{otherwise.} \end{cases}$$

$P_{i,y-2}$ appears in the calculation of the quota for the year y because the most current data available for calculating P_i is two years old.

As long as the proper time relationships are kept in mind (i.e., the penetration ratio $P_{i,y}$ is calculated using two year old data, and some credits apply in the

[1] "Quota Determination and Quota Fulfillment," Automobile Insurance Plans Services Office, New York. The formula given applies in every state except Florida, Hawaii, Missouri, Texas, Maryland, Massachusetts, New Hampshire, North Carolina, and South Carolina.

year in which they are written while others apply two years later) we can, for simplicity, drop the year subscript. Thus we have

$$q_i = \begin{cases} 0 \text{ if } P_i(T + K) - K_i \leq 0 \\ \dfrac{P_i(T + K) - K_i}{T} \quad \text{otherwise.} \end{cases}$$

If $P_i(T + K) - K_i \geq 0$ for all i (that is, no company writes a number of credits greater than the number necessary to eliminate its residual market assignment), then

$$q_i = \frac{P_i(T + K) - K_i}{T} \quad \text{for all } i \text{, and}$$

$$\begin{aligned} \sum_i q_i &= \sum_i \left(\frac{P_i(T + K) - K_i}{T}\right) \\ &= \left((T + K) \sum_i P_i - \sum_i K_i\right) \Big/ T \\ &= \frac{T + K - K}{T} \\ &= 1 \end{aligned}$$

If $P_{i'}(T + K) - K_{i'} < 0$ for some i', then $q_{i'} = 0$ since negative quotas are not allowed and $\Sigma_i q_i > 1$. In this case the positive quotas are divided by $\Sigma_i q_i$ so that the adjusted quotas sum to one. The quotas are further adjusted for over-assignments and under-assignments made in previous years.

Note that, since the current year's quota q_i is calculated using a penetration ratio P_i that is two years old, the current year's quota is unaffected by the volume of business written in the current year. However, the volume of business written in the current year will affect the future residual market quota.

Adjustments for over- and under-assignments will be ignored in this paper because they do not affect a company's overall participation; these adjustments only alter the allocation of that participation by year. Also, it is assumed that $P_i(T + K) - K_i \geq 0$ for all companies.

Under these assumptions the quota for company i is simply $(P_i(T + K) - K_i)/T$. Note that the quota is a function of the total credits written by all insurers, and therefore its value for company i depends on the actions of other insurers. This fact complicates the analysis because it is not always possible to anticipate these actions. This problem is most pronounced when credits are offered for the first time, such as Keep-out credits offered in conjunction with the passage of a compulsory insurance law. In these instances, no history is available upon which to anticipate the actions of the other companies.

Company behavior is the feature which distinguishes the two models presented here. Each model assumes an extreme case, with the real world falling somewhere between the two extremes.

In both models it is assumed that all eligible risks must be written either voluntarily or through the residual market. In neither model does the acceptance criterion consider investment income.

FIRST MODEL

Let g_i = break-even loss and loss expense ratio for company i;
g_t = break-even loss and loss expense ratio for residual market business;
r_j = expected loss and loss expense ratio at voluntary rates of risks eligible for credit j;
r_t = expected loss and loss expense ratio of residual market business.

The variables g_i, g_t, r_j, and r_t will be called loss ratios, although it is understood that they include loss adjustment expense.

The first model assumes that every eligible risk is written voluntarily by some company. It does not matter, however, which company or companies chooses to write this business. Also, we assume that all companies charge the same rates. Thus, K is constant, but the K_i may vary. Hence, an increase in $K_{i'}$ for some i' results in a decrease in $K_{i''}$ for some i''.

Consider a risk that is eligible for type of credit j. Let the premium for this risk, if written voluntarily, be denoted by X_j.

If this risk is not written voluntarily by company i, then the residual market quota for company i is given by

$$(P_i(T + K) - K_i)$$

If this risk is voluntarily written by company i, then the residual market quota is given by

$$(P_i(T + K) - K_i - N_jX_j).$$

Thus, the reduction in the quota due to writing this risk is N_jX_j. Since residual market business produces an underwriting loss of $r_t - g_t$, the reduction in the underwriting loss of residual market business due to writing this risk is

$$N_jX_j\,(r_t - g_t).$$

The expected voluntary underwriting loss incurred on this risk is equal to

$$X_j(r_j - g_i).$$

This risk should be written voluntarily whenever

$$(r_j - g_i) \leq N_j(r_t - g_t),$$

which is equivalent to

$$r_j \leq g_i + N_j(r_t - g_t).$$

Thus, for each type of credit we have expressed the maximum expected loss ratio that minimizes the net underwriting loss as a function of one variable—the expected loss ratio, r_t, of the residual market business.

The assumption that all companies charge the same premiums is not crucial; approximate equality is sufficient. The second model will not require this assumption.

SECOND MODEL

In contrast to the first model, which assumed that any eligible risk rejected by company i would be voluntarily insured by some other company, the second model assumes that any eligible risk rejected by company i must obtain insurance through the residual market. Formally, it is assumed that $\bar{K} = \Sigma_{m \neq i} K_m$ is constant.

In the real world some of the rejected individuals will be voluntarily insured by other companies, and some will not. Clearly then, the real world may be approximated by a linear combination of the two models.

Consider a risk eligible for type of credit j that, if voluntarily written by company i, would produce premium D_i and a loss and loss expense ratio r_j. If this risk were written through an assigned risk plan, the premium would equal FD_i, where F is the assigned risk rate level factor for this risk relative to company i's rates. The loss ratio on this risk would then equal r_j/F.

If company i voluntarily writes this business, then its assigned risk quota is reduced because of the credits that it earns.

Company i's quota without this credit is

$$P_i\left(T + \bar{K} + \sum_j N_jX_{j,i}\right) - \sum_j N_jX_{j,i} .$$

Company i's quota with this credit is

$$P_i\left(T + \bar{K} + \sum_j N_jX_{j,i} + N_jD_i\right) - \sum_j N_jX_{j,i} - N_jD_i .$$

The reduction in company i's quota due to writing this business voluntarily is

$$-P_iN_jD_i + N_jD_i .$$

The expected underwriting loss incurred on this risk if written voluntarily is

$$D_i(r_j - g_i).$$

Thus, the net loss attributable to the decision to voluntarily write this risk is given by

$$L_1 = D_i(r_j - g_i) + (P_iN_jD_i - N_jD_i)(r_t - g_t).$$

If the eligible risk were not voluntarily written, then the assigned risk quota for the company would not change, but the size of the assigned risk pool would increase. Company i's share of the additional loss is

$$L_2 = P_iFD_i[(r_j/F) - g_t].$$

Whenever $L_1 < L_2$, overall losses can be reduced by voluntarily writing this risk. The inequality will be satisfied when

$(r_j - g_i) + (N_jP_i - N_j)(r_t - g_t) < P_iF[(r_j/F) - g_t]$, which is equivalent to

$$r_j < \frac{g_i - P_iFg_t - (N_jP_i - N_j)(r_t - g_t)}{(1 - P_i)} .$$

As in the first model, we have expressed the maximum loss ratio r_j that optimizes use of the credits as a function of one variable—the expected loss ratio for residual market business.

It is interesting to compare $\bar{r}_j$, the maximum loss ratio at which the risk should be written voluntarily, as calculated using the two models. For the first model we have

$$\bar{r}_{j,1} = g_i + N_j(r_t - g_t).$$

For the second model we have

$$\bar{r}_{j,2} = \frac{g_i - P_iFg_t - (N_jP_i - N_j)(r_t - g_t)}{1 - P_i} .$$

The difference is:

$$\bar{r}_{j,2} - \bar{r}_{j,1} = \frac{P_i(g_i - Fg_t)}{1 - P_i} .$$

Thus, the two models are equivalent when $g_i = Fg_t$, and the first model gives an upper bound when $g_i - Fg_t$ is negative.

EXAMPLE

The following fictitious example illustrates the two models. Suppose a company wants to determine whether or not it should voluntarily write a particular risk that is eligible for a two-for-one credit ($N_j = 2$). The company's breakeven loss and loss expense ratio, g_i, is .70; its breakeven loss and loss expense ratio for residual market business, g_t, is .75; the assigned risk rate level factor for this risk is 1.2; and it insures 5% of the voluntary market ($P_i = .05$). The total residual market loss experience is given in Table 1.

TABLE 1

ASSIGNED RISK EXPERIENCE

	Earned Premium	Incurred Loss And Loss Expense Ratio
1975	$20,000,000	95.0%
1976	30,000,000	102.0
1977	35,000,000	98.0
1978	45,000,000	106.0
Total	130,000,000	101.2

Take-out and Keep-out credits written in the current year reduce the current year's quota, while Class 2 and Over 65 credits reduce the quota two years later. Thus, it is necessary to estimate the assigned risk loss ratio that is expected to prevail either in the current year or two years later, depending on what type of credit is being considered. It is not possible to estimate this loss ratio with a great deal of precision. Nevertheless, by looking at the total plan experience in recent years and considering trends and the promptness with which assigned risk rate changes have been approved and implemented in the past, one can formulate expectations of the likely range of the assigned risk loss ratios in the near future. Continuing with the example, suppose the assigned risk loss ratio is expected to fall in the range 95.0% to 105.0% for the next several years.

Using the low end of this range, 95.0%, in the first model, we get

$$\begin{aligned}\bar{r}_{j,1} &= g_i + N(r_t - g_t)\\ &= .70 + 2(.95 - .75)\\ &= 1.100.\end{aligned}$$

The second model gives

$$\begin{aligned}\bar{r}_{j,2} &= \frac{g_i - P_iFg_t - (NP_i - N)(r_t - g_t)}{1 - P_i}\\ &= \frac{.70 -(.05)(1.2)(.75) - (2(.05) - 2)(.95 - .75)}{.95}\\ &= 1.089.\end{aligned}$$

The optimal value of $\bar{r}_j$ in the real world is probably between the above two values, say $\bar{r}_j = 1.095$. Thus, the company should voluntarily write this risk if the risk's expected loss ratio at voluntary rates is less than 109.5 percent.

How much money will a company save by following this rule? The savings can be estimated roughly as follows. If the expected loss ratio of a risk is less than g_i, then that risk will be written voluntarily whether or not it is eligible for a credit. Thus, those risks written because of the credit will have expected loss ratios lying in the interval from g_i to $\bar{r}_j$. If we assume that the loss ratios are uniformly distributed over this interval, then the expected loss ratio for the group is $(\bar{r}_j + g_i)/2$. Suppose 2000 risks written because of the credit have an average premium of $250 and an average expected loss ratio of 89.75 percent. Then the expected underwriting loss on this group is

$$2000 \times \$250\ (.8975 - .70) = \$98{,}750.$$

The reduction in the residual market expected underwriting loss is given by

$$2000 \times \$250 \times 2\ (.95 - .75) = \$200{,}000.$$

Thus, the use of the credits has reduced overall underwriting losses by an estimated $101,250.

The attractiveness of the credits increases as the expected assigned risk loss ratio increases and as the number of credits per dollar of premium increases.

SUMMARY

Both models have ignored the fact that a decision to write eligible risks voluntarily will increase the company's assigned risk quota because its voluntary business will increase.

The models developed here may not apply to the nine states that do not use the quota formula described above. Also, different types of credits may be offered in these states. In Massachusetts, for example, territory credits are available for voluntarily writing risks in Boston. However, it is possible to develop models for use in these nine states.

The optimal use of depopulation credits will not dramatically reduce underwriting losses in most states, but in those states with the largest and most underpriced residual markets, their use can be significant. If regulators expand the use of credits in an effort to depopulate the residual market, then the benefit of using those credits optimally will increase.

THE 1979 NCCI REMARRIAGE TABLE

PHILIP E. HECKMAN

Abstract

This is an account of the analytical work that eventuated in the table of the title. The problem and the available data are described, and several aspects of the analysis are dealt with in detail: historical studies of the data, the parametric model used to fit the data, the trend analysis that led to the final parameter values, and the population averaging carried out to fit the NCCI age distribution for claimants. A discussion of the actuarial valuation functions follows, including two never previously tabulated: the spouse's dowry and the automatic survivorship benefit.

(Editor's Note: Mr. Heckman's paper was originally presented at the November 1981 meeting of the Society.)

I. INTRODUCTION

An ever-increasing number of workers' compensation laws prescribe lifetime benefits for seriously injured claimants and, in fatal cases, benefits until death or remarriage for surviving spouses. For most purposes requiring prospective valuations of these cases, the annuity tables provided in the statistical plans of the National Council on Compensation Insurance (NCCI), or the independent bureaus, are used. While the numerical proportion of such cases is small (but growing), they nevertheless currently account for a substantial fraction (one-fourth to one-fifth) of the estimated incurred loss dollars.

Thus the financial effect of estimates drawn from these tables is significant and pervasive. The valuations in unit statistical reports affect the following:

- Experience modifications via the NCCI experience rating plan.
- Retrospective premiums under the standard retrospective rating plans.
- Classification relativities in the manual rates.

In addition, many companies use case reserves based on these tables as part of the case basis for their corporate loss reserves. (Approximately one-third of the pending dollars apply to such cases.) These in turn find their way into the financial aggregates used in determining overall manual rate levels.

Clearly, valuation tables with such broad influence on the business of compensation insurance deserve frequent review and careful attention. The NCCI has programs to carry out such review. The main purpose of this paper is to describe the outcome of recent efforts to bring up to date the remarriage assumptions used in computing the spouse's annuity table and to propose tables for valuing certain contingent benefits which are now either reserved by judgment or ignored.

The standard actuarial method for analyzing remarriage (or other) experience is to summarize it into absolute rates of decrement, independent of mortality and other effects, by the use of established exposure formulae. In contrast to mortality studies on the general population, where the experience is adequately described in terms of variation by attained age, it has been found in remarriage studies that variation by duration of widowhood is also significant, at least in the first five years (1). Thus the summaries are typically constructed as tables of annual rates by age at widowhood and duration of widowhood. The usual practice of appending an ultimate column, depending only on attained age, after a fixed term of duration, usually five years, was not followed in the present study, since the rates were modeled select at all durations and the ultimate column constructed from the model.

Such summaries of raw experience present a classical problem of statistical analysis: how to distinguish signal from noise. It is natural to assume that the observed terminations (remarriages) are generated by a binomial rate process with smooth variation in rates from one period to the next. The problem is to infer from the data just which process is at work.

In actuarial parlance, the body of technique applied to this end is termed "graduation." The alternatives available will be discussed in Section IV. Here we may simply remark that the present work differs from past studies in that we propose a *parametric model* for the force of remarriage as a function of age and duration, $\mu_{[x]+t}^{(r)}$. This is closely akin to the use of Gompertz's or Makeham's law in mortality studies. The model is fitted to the absolute annual rates of remarriage by a modified least squares procedure. This rather abstract and mechanical procedure is supplemented by graphical inspection of the results on the principle that no result is valid which fails to please the eye. The utility of this ponderous and difficult approach will become clear in later discussion of the trending procedure applied to the data.

While the present work is empirically independent of earlier studies, it is useful to review these works in order to judge the reasonableness of our results and to get a feeling for the variability of remarriage rates both historically and by group studied. References (1) through (4) give a partial bibliography of the classic studies. Much of the earlier work is ably summarized in reference (1).

The current NCCI valuation tables employ remarriage rates developed from experience under the United States Employee Compensation System (USECS) between 1916 and 1955 (1). Certain results of this study as well as the present work, summarized in Exhibit I, make clear that remarriage is a very volatile phenomenon, with substantial variations in aggregate rates observable on a fairly short time scale—say, five years. There is every reason to expect that remarriage experience is sensitive to trends in social attitudes and to shifts in compensation practice with the result that a fixed and rigid table of rates is likely to become obsolete rather rapidly.

It was in recognition of this likelihood that the NCCI Task Force on Mortality and Remarriage was formed in 1975 with the participation of Aetna Life and Casualty, Travelers, and INA. Due to prevailing record-keeping practices in the industry, remarriage data suitable for analysis are hard to come by. These were provided through the good offices of the administrators of the New York Aggregate Trust Fund (NYATF), which administers all fatal cases in New York for which awards have been made. NCCI staff and Travelers undertook to prepare the data. CNA joined the task force in the spring of 1978 and elected to provide research personnel and data processing support to the project. As noted above, the main result of these efforts is a parametric model for the force of remarriage as a function of age at accident and duration of claim. From this model, and from mortality assumptions to be described later, we have also generated actuarial functions to be used in valuing future benefits: annuities, dowries, and automatic survivorship benefits.

Though the NYATF provides a rather narrow sample, it is the only compensation data available in sufficient bulk to allow a detailed study. As will be discussed below, the use of a parametric model will facilitate later adjustments to the rates as bits and scraps of nationwide data become available. It should also allow treatment of regional variation, though such a study is far in the future.

This paper is structured so as to confine the copious technical detail to supporting appendices, one for each major section of text. Section II discusses the data used in the study. Section III deals with the analysis of the data. Section IV treats the formulation of the model, with the detailed form, parameter values, and supporting statistics given in Appendix A. Section V describes the fitting procedures and criteria employed, with details in Appendix B. Section VI then describes the trending procedure used to bring the model as near to present conditions as possible, while Appendix C outlines the supporting statistics vital to the interpretation of these results. Section VII presents the actuarial valuation functions which are our chief practical result, with details in Appendix D. The concluding Section VIII outlines future directions in data acquisition and surveillance of the remarriage phenomenon.

II. THE DATA

The data supplied by the administrators of the NYATF represent the detailed experience of the Fund on fatal cases arising from accidents between 1904 and early 1977. The NYATF deserves congratulations for conserving these data since they comprise the only such recent information available on compensation cases.

The information abstracted from these records by NCCI staff and Travelers employees consists of the following items:

a) Cause of termination
   1—Death
   2—Remarriage
   3—Other
   4—Open at end of study
b) Date of accident
c) Widow's birthday
d) Date of termination.

The total number of records on the tape transmitted to CNA was 10,673. Two records were excluded as implausible. The remainder represent 164,209 years of widowhood and 2,113 remarriages.

III. ANALYSIS

The preliminary analysis of the data consisted of preparing tables of observed annual rates and exposures. The absolute remarriage rates were extracted using conventional techniques, as described in reference (5), treating remarriages as valid terminations and all other terminations as withdrawals. The rates and exposures were tabulated by nearest year of age at accident and by year of duration of widowhood. The rules for reckoning exposures on these intervals are set forth in detail later in this section.

Tabulations were prepared for various ranges of date of accident. Aggregate average rates were computed on the actual exposures in each range of dates. The results are presented in Exhibit I along with similar results from USECS experience. This exhibit shows that the aggregate rates of remarriage were much lower before 1930 than after. A likely explanation for this break in the experience is the known fact that referral of cases to the Aggregate Trust Fund was optional and probably delayed during the earlier period, a possibility not considered in the tabulation of the data. In any event, it was decided to exclude accidents before 1930 from further analysis. In the trend study to be discussed in Section VI, the data were further restricted to cases arising in 1935 and after, the actual year when referral to the Fund became mandatory. Since referral dates are not included in the tabulations, we can only be certain of getting correct exposures if referral of all claims is required.

The next step of preliminary analysis consisted of preparing graphical displays of the rates observed for accidents from 1930 to 1977, summarized on five-year age intervals. An example of these displays, showing also the preferred fit of our proposed model, is given in Exhibit II. A key to interpretation of these graphs, with their supporting statistics is given later in this section. Similar displays showing the annual average force of remarriage were also prepared. These displays provided valuable insights into the structure of the data, leading eventually to a simple and successful parametric model. The salient features of the data may be outlined here. On careful inspection, the force of remarriage appears to consist of two distinct components:

- A short-duration component rising from zero at zero duration, peaking around two years, and falling off rapidly at longer durations. This component falls off rapidly at the higher ages;
- A long-duration component, peaking around five years of duration, falling off slowly at longer durations, and falling off slowly at higher ages.

The distinct age dependences make these components worth distinguishing. In the following, we shall refer to them as "prompt" and "delayed" components.

A. Basic Exposure Equation

In the ideal case that every life in the study is under continuous observation and that the expected force of decrement is known at all ages and durations, the expression for the expected number of terminations from a given cause takes on a particularly simple form. By continuous observation, we mean that all dates of entry and all terminations with dates and causes are known, for practical purposes, exactly. If these conditions are fulfilled, we have the basic relation,

$$\hat{N}^{(c)}_{[x]+t,\overline{h}|} = \int_0^h ds\ \mu^{(c)}_{[x]+t+s} \cdot \hat{l}_{[x]+t+s}$$

for the *expected* number of terminations from cause, (c), in the interval of duration t to $t+h$ among lives first exposed to the cause at age x. The force of decrement for the cause, (c), is $\mu^{(c)}_{[x]+t}$; $\hat{l}_{[x]+t}$ is the actual number of lives of age x at first exposure, under observation at exact duration t.

To the best of my knowledge, this relation has not appeared elsewhere. Formulas given in the actuarial literature deal with the situation where the empirical exposure function, $\hat{l}$, is sampled periodically and interest is focused on finding rates of termination based on finite periods.

While we could have used the basic equation in this study, we decided in favor of the more conventional tabulations in terms of annual rates, since the computations are more economical and the extra detail is not actually needed. These are treated in the next section.

B. Absolute Rates of Decrement

An absolute rate of decrement represents the probability of termination within a given period, supposing that all other causes of decrement are turned off. Assuming the two causes, mortality and remarriage, the probability of termination during the year for a life starting the year is

$$q_x^{(T)} = 1 - (1 - q_x'^{(r)})(1 - q_x'^{(m)}).$$

The segregation of this probability by cause invariably involves adopting a model to interpolate the absolute rates on partial years. That most frequently invoked is the Balducci hypothesis (5). To get accuracy better than first order in the rates being studied, one must model the several processes simultaneously. This is nearly never done and is probably not justified unless the data are plentiful, the rates of decrement large, and high accuracy imperative. If we content ourselves with first order accuracy and focus exclusively on one cause of decrement (remarriage in our case), then the same simple rules for reckoning exposure emerge no matter what interpolation model is assumed, Balducci or otherwise.

In a particular year of duration, assuming no entries during the year, these rules as they apply to our analysis are:

Case	Exposure
no termination	1
termination for remarriage	1
termination for any other cause	fraction of year actually exposed

If the number of remarriages is divided by the exposure thus compiled, the result is an estimate of the absolute rate of decrement that is accurate to first order in the rate. That is to say, if a rate of 0.1 is estimated, then the statistical bias in this estimate will be on the order of 0.01. Such inaccuracies are acceptable in most applications and were accepted in the present work.

C. Historical Study and Aggregate Rates

When the historical study, shown in Exhibit I, was carried out by segregating data records by ranges of accident date, the results were summarized as aggregate rates in two different ways. The first way, which produced what we call the crude remarriage rate (column (4) in Exhibit I), involved simply adding up all the remarriages in the period in question and dividing by the sum of all exposures tabulated by the rule for annual rates given above. The result is an average *annual* rate. If, say, a quinquennial rate is desired, a different tabulation of exposures must be performed.

The second way, which produces what we may call standard average rates, involves choosing a base period and using the exposures from the base period by age and duration with the observed rates from another period to deduce what results would be produced in the base period by the rates prevailing in the other period. This is achieved by extending the rates from the base period, summing, and dividing by the summed base period exposures. Cells in which no data appear in the measurement period are excluded from both numerator and denominator. This provides a rough but effective means of isolating real shifts in rates from mere shifts in the population of beneficiaries.

D. Statistics for Graphical Quinquennial Age Summaries

The graphical summaries of rates by quinquennial age groups (Exhibit II) show durations marked off in years on the abscissa. The scale factor printed out on each graph tells how many of the vertical divisions add up to unity. Some of the points, the reader will note, are bracketed by error flags. (In some cases these are too short to show up on the graph.) These extend one standard error in each direction from the observed value, spanning a range which includes the *true* value with something like two-to-one odds. If N remarriages occur in the year of duration on exposure, W, the standard error is calculated as

$$((N/W)(1 - N/W)/(W - 1))^{1/2} \qquad \text{[Ref. (6), p. 151]}$$

This is developed from a binomial model, ignoring the fact that W is non-integral due to our approximate method of filtering out the effects of the other causes of decrement. These flags are included to give a feeling for how seriously various features of the data are to be taken. In particular, bumps and wiggles in the data of a scale smaller than one or two flag spans (standard errors) can be ignored on purely statistical grounds as random fluctuations. (Other features can be excluded because it makes no financial sense to reproduce them in the model.) This is an essential and often ignored aspect of any empirical analysis. Note that our interpretation fails if no remarriages are observed. In this case, the needed information can be drawn from neighboring cells which do contain events. Naturally enough, if there are no exposures ($W=0$), no conclusion can be drawn.

IV. THE MODEL

In order to proceed with the analysis, it was then necessary to translate these observations into a specific model. The choice of a model was closely intertwined with another decision: what method of graduation to use.

The methods of graduation available fall into three general classes:

1. Optical: that is, graduation by inspection;
2. Algorithmic averaging: Whittaker-Henderson methods, moving average methods; and
3. Parametric modeling: fitting an analytical model to the observed rates.

The optical method has the virtue of directness since, regardless of the method used, results displeasing to the eye must be rejected. It is, however, very difficult to control such a procedure or even to characterize the quality of the fit. Algorithmic averaging methods have been known to produce useful results; but, when one searches for a statistical hypothesis—an underlying model—which could indicate the use of such methods, one is led to bizarre correlations among observations in neighboring cells. The parametric modeling method, by contrast, is in close harmony with the usual actuarial hypothesis as to what kind of process is taking place. On the other hand, it requires the use of a great deal of machinery: first, the model itself; second, a method of fitting to the observed data; last, a sensible criterion for assessing the quality of fit to determine whether the chosen method has anything to do with reality. In the present work, it was decided to adopt the third approach. The associated cost, while considerable, was justified by the ease with which the subsequent trending study could be carried out.

This decision made, one was then faced with the choice of which precise quantity to model. Our choice was to model the force of remarriage as a function of age at widowhood and duration. This is the actuary's term for the instantaneous fractional rate at which the population of widows is depleted by remarriage. Other effects, including mortality, may be included simply by adding in the associated force of decrement. Further, the force of remarriage is continuous in the time variables, easy to visualize, and can be manipulated freely by analytical and numerical techniques to yield any desired actuarial quantity. The greatest advantage of modeling the force, rather than rate or survival, is that there are no axiomatic constraints on the force except that it cannot be negative (i.e., the population cannot be increased by remarriage). These properties greatly simplify the task of fitting the data with reasonable parameter values. Reflecting our observations on the structure of the data, the model consist of two terms, one for the prompt component and one for the delayed component.

The mathematical form of the model, its preferred parameter values with associated statistics, and its relation to the annual rates of remarriage are shown in Appendix A.

This concrete definition of the model makes it easy to see its possibilities. The most interesting of these is the hypothesis that the age and duration dependence of these distinct terms is effectively universal while the strengths vary substantially in time and place. This hypothesis was investigated and, in my opinion, confirmed in our trending study, described in Section VI. The consequences of this are most intriguing, but first a discussion of fitting procedures is in order.

V. FITTING THE DATA

The model as described is elegantly tailored to the phenomenon being modeled, but it stands in very inconvenient mathematical form. Each of the nine parameters must be determined so that the overall fit is the best that can be achieved. First we must define what we mean by a good fit. This is usually done by specifying a loss function, which summarizes in a single value the deviation of the model from the data and is minimized by varying the parameters. Linear least squares is an example of such a procedure in which the best parameter values may be obtained by straightforward linear algebra. The present model is not linear in the parameters and must be optimized the hard way: we must carry out a full parameter search. Further, the choice of a loss function is not trivial. Some seemingly reasonable choices give absurd results; and, in practice, it was necessary to impose a criterion that the fit should please the eye before the loss function was deemed acceptable.

The stochastic process usually presupposed in situations like this is the binomial rate process with different rates in each of the age-duration cells containing exposure. This allows us to compute the mean and variance of the loss function, given the exposures in each annual cell, assuming that the model is correct. This gives us an additional criterion; for, if the excess of the best achievable value of the loss function over its expected mean value, measured in standard deviations, is too great, then the model has not yielded a convincing fit. For instance, if the best-fit loss function exceeds the mean by one standard deviation, then the probability is roughly one in six that, if the model is correct, random fluctuations in the data could produce a larger excess. One standard deviation seems reasonable as the maximum tolerable excess.

The mathematical details of the procedure are set forth in Appendix B, along with the statistics for the best fit.

VI. TRENDING

If the hypothesis can be upheld that secular variation in remarriage rates is describable by changes in our model coefficients, we have a method ready to make use of sparse but recent data. One may simply vary the coefficients of the model, while keeping the other seven parameters fixed at the values obtained in the fit to the NYATF data, to bring the rates into overall accord with the new data. It would be desirable to update the model in this fashion using data drawn from recent experience in the NCCI states, but such will not be available until the results of the NCCI Pension Study are analyzed. To provide a near-term solution, the Task Force decided to undertake a historical trend analysis of the NYATF data itself.

The data were compiled as described for the overall study, but in five-year segments by accident year, beginning in 1935. The partial period from 1975 to early 1977 contained no remarriages and thus could not be analyzed with confidence. The fitting procedure was then carried out on each segment independently, varying only the two coefficients. The results are tabulated in Exhibit III-A and displayed graphically in Exhibit III-B. The table shows the fitting statistics discussed previously. The last column shows the difference of the best fit and the expected value of the loss function, measured in standard deviations. One can see that the fits are better than expected on all the pieces except 1935–39. The fit for this early period is just marginal, but the fine results for the other periods yield a dramatic confirmation for our initial hypothesis. Indeed these fits are superior in quality to the overall fit for 1930–77, whose statistics are shown on the bottom line of Exhibit III-A.

The graphic display in Exhibit III-B is richly suggestive. The coefficient of the prompt component is plotted on the abscissa, that of the delayed component on the ordinate. The ellipse associated with each point is enclosed in a box two standard errors on a side and is presented as an approximate forty percent joint confidence region for the values of the coefficients. Doubling the scale of the ellipse gives roughly an eighty-six percent confidence region. The shape and direction of the ellipses give an idea of how the parameter estimates are correlated.

This graph makes it clear that there has been significant variation in these coefficients over the years. It also makes clear the gain in precision from looking at the more homogeneous five-year intervals. (The ellipse for the overall fit, 1930–77, covers nearly the entire display.)

As regards the pattern of variation of these numbers, it is difficult to argue for anything more systematic than a random walk, that is, a tendency for the leaps from one period to the next to be small ones. Consequently, the preferred near-term solution is to use the most recent set of values. Currently, the best estimate is the point for 1970–74, which sits in the midst of an elongated ellipse which betokens, naturally enough, a sparsity of information on what is going on at the longer duration. However, the coefficient for the prompt component is resolved well enough to be distinct from all other recent values; and that for the delayed component is well within the range of plausibility. Hence the Task Force has decided to proceed to construction of the rate table and the relevant actuarial functions using these values from 1970–74.

The details of the statistical underpinnings of this exhibit are given in Appendix C.

VII. TABLES

The choice of model and parameters outlined in the preceding determines the remarriage rates uniquely. A number of decisions remain, however, before we can specify the practical valuation tables that are needed in the current environment.

A. Mortality

Mortality rates have not come under close scrutiny in the present cycle of activity since it was felt that remarriage was a more urgent problem. What the Task Force is proposing at this turn is a simple update. The tables currently in use are based on the U.S. Life Table, 1959–61, White Females and Total Population. We propose to adopt in the update the U.S. Life Tables, 1969–71, All Females and Total Population (7). It is felt that the current racial composition of the population of workers' spouses is much closer than formerly to that of the total female population, so that there is no longer any justification for using White Female experience. The Task Force also proposes another update when the 1979–81 tables become available. Tentative proposals to trend the mortality rates will be discussed in the concluding section. The proposed rates are displayed in Exhibit IV-A.

B. Escalation

Certain jurisdictions prescribe that weekly benefits on certain categories of new and existing claims shall escalate annually in proportion to some index, usually the state average weekly wage. It is not feasible to predict the detailed fluctuations of such indices, nor would the result, if attainable, be useful in constructing valuation tables. The practical solution is to choose a reasonable average rate of escalation, assumed to apply indefinitely into the future. Nearly all of the affected NCCI jurisdictions have approved a six-percent rate for use in valuing future benefits. We have used this in the proposed tables, giving values per dollar of *present* annual benefit.

C. Benefits

Our basic objective is to provide subscribing carriers with a valuation basis for the long-term contingent benefits required by law. At this writing these include:

1. Life annuities to claimants and certain other beneficiaries,
2. Annuities for life unremarried to spouses of deceased workers,
3. Dowries payable on remarriage to spouses of deceased workers, and
4. The automatic survivorship benefit: a life annuity payable to the surviving spouse of a claimant who dies of causes unrelated to the accident.

One of our goals in the current revision is to propose tables for the latter two types which have not, until now, been provided for. The procedures for calculating these benefits are set forth in Appendix D.

D. Format

Remarriage rates typically show strong selection by duration during the first several years on claim. This is true of the USECS experience as well as the NYATF data. In the present study, the data were treated as select at all durations, whence the model can be extrapolated to all durations at fixed age without fear of mischief. One possible approach in building the table would be to keep as many select columns as one likes and then to average the benefit values for the advanced durations at fixed attained age using up-to-date population age distributions. This procedure, however, would make the definition of commutation functions impossible. The alternative approach, less attractive as a financial model, but more in accord with usual actuarial practice, is to average the model *rates* to create an ultimate column.

This approach allows much more to be done with the rates outside the computer while maintaining numerical reproducibility. On inspection of the observed and fitted NYATF rates, the Task Force concluded that variation at fixed attained age is insignificant at the fifth year of duration and beyond. Hence it was decided to publish the new rates in six-column format with select columns for valuation at durations zero through four years and an ultimate column for durations five and greater. This table was then written to a data file and read into a different program which produced the tables of actuarial functions. Hence the numerical results are all reproducible by hand calculation from the published rates. These rates are shown in Exhibit IV-B. The details of the population averaging are discussed in Appendix D.

E. Exhibits

The tables of actuarial functions, derived from the rates shown in Exhibits IV-A and B, are displayed in Exhibits IV-C through H in the following order.

IV-C. Spouse's select, $D^{(T)}_{[x]+t}$, combining mortality and remarriage.
IV-D. $N^{(T)}_{[x]+t}$, the upward sum of IV-C.
IV-E. Spouse's annuities payable continuously for life unremarried ($\bar{a}^{(T)}_{[x]+t}$) per dollar of annual benefit.
IV-F. Spouse's dowry insurance payable at remarriage ($\bar{A}^{(r)}_{[x]+t}$) per dollar at lump sum.
IV-G. Automatic survivorship benefit per dollar of prospective annual benefit, tabulated by attained age of injured worker and age differential of spouse.
IV-H. Claimant's annuities and commutation functions, $\bar{a}_x$, D_x, N_x, tabulated by attained age.

All these exhibits assume 3.5% interest, and all are recalculated to show values for six-percent escalation, as well as no escalation.

F. Comparison

A final display in Exhibit V shows a comparison of the proposed annuity values with those currently in use in the NCCI Statistical Plan. The proposed spouse's values are averaged over all durations at fixed attained age. Attained age distributions for spouses and for claimants (derived from the NCCI Injury Table age-at-accident distributions for widows and for permanent total injuries) are also shown and used to compute the overall averages at the bottom. Both averages are larger than the values for the current table, reflecting both the new mortality rates and the new remarriage rates, which are substantially lower on average than the USECS rates used previously. Note that these averages are based on the total current population of claimants and are intended to represent the effect of the proposed tables on the total reserve for future payments on tabular claims currently pending. The percentage effects are +4.6% on fatals and +2.5% on permanent disability claims. On an accident year incurred basis, the effects will be somewhat larger.

VIII. THE FUTURE

A historical view of remarriage experience shows that the problem cannot be left alone for very long; any particular tabulation can be expected to become obsolete on a time scale of about five years and should be reviewed at least that often. One of my hopes in submitting this work is that it may make this periodic review easier by allowing the use of sparse or fragmentary data to make simple adjustments to an already established form. Naturally the entire model should be reviewed on a longer time scale to check its general validity.

One adjustment that should be made as soon as practicable is to use data from the NCCI Pension Study to revalue the coefficients of the model. This adjustment would reduce the heavy dependence on New York experience, which may or may not be apt for application in the NCCI jurisdictions. Further in the future, after the improved NCCI data gathering and the statistical plan revisions have had time to take hold, one may envision valuation tables based entirely on NCCI experience. These actions become more important in view of the rapidly changing ratio of lifetime compensation claims to limited-payment claims.

Until an NCCI Mortality Study can be undertaken, it may be feasible to improve the valuation tables by trending the population rates to the period of application. The experience gained thereby would be useful also in future application to NCCI data.

It is a pleasure to thank Ed Seligman of CNA for helpful discussions; and Claus Metzner, Carl Meier, and Richard Palczynski, of the Task Force, and Frank Harwayne and Charles Gruber of NCCI, for their support and guidance in this work. My thanks also go to Barbara Dudman of CNA for typing the manuscript.

APPENDIX A
DETAILS OF THE MODEL

1. Forces of Decrement

Let us examine a closed population model in which remarriage acts specifically by age at widowhood and duration on claim and mortality acts specifically by attained age. In this situation, the *expected* number of widows going off claim between duration, t and $t + \delta t$, δt small, is

$$- \left(\frac{\partial}{\partial t} l^{(T)}_{[x]+t} \right) \delta t = \left(\mu^{(r)}_{[x]+t} + \mu^{(m)}_{x+t} \right) \cdot l^{(T)}_{[x]+t} \delta \mathrm{t},$$

where $l^{(T)}_{[x]+t}$ is the number of spouses, widowed at age x, remaining on claim at duration t; $\mu^{(r)}_{[x]+t}$ is the force of remarriage at duration t, on lives widowed at age x; and $\mu^{(m)}_{x+t}$ is the force of mortality acting at age $x + t$. The *expected* number of remarriages is $\mu^{(r)}_{[x]+t} \cdot l^{(T)}_{[x]+t} \cdot \delta t$ and so on. Always supposing that these two causes of decrement are the only ones acting, our differential equation can be intergrated very simply to give

$$l^{(T)}_{[x]+t} = l^{(T)}_{[x]} \exp \left\{ - \int_0^t ds \left[\mu^{(r)}_{[x]+s} + \mu^{(m)}_{x+s} \right] \right\} .$$

We may also separate these effects, and define absolute rates of decrement (annual):

$$q'^{(r)}_{[x]+t} = 1 - \exp \left\{ - \int_t^{t+1} ds \; \mu^{(r)}_{[x]+s} \right\} ,$$

$$q'^{(m)}_{x+t} = 1 - \exp \left\{ - \int_t^{t+1} ds \; \mu^{(m)}_{x+s} \right\} .$$

The first, given an appropriate expression for the force of remarriage, represents the expected values of the rates tabulated in our analysis of the remarriage data. The second corresponds to the rates to be found in our preferred mortality table. Given these two sets of rates, the values of $l^{(T)}_{[x]+t}$ at integral values of t can be reconstructed.

2. Proposed Model for the Force of Remarriage

We present here the detailed mathematical form of our proposed model for the force of remarriage with interpretations and preferred values of the parameters. The parameter subscripts reflect not the structure of the model but the order in which various features were added to it:

$$\mu^{(r)}_{[x]+t} = P_1 e^{-P_2 x} t^{P_8 - 1} e^{-P_3 t}$$
$$+ \Theta(t - P_7) \cdot P_4 e^{-P_5 x}(t - P_7)^{P_9 - 1} e^{-P_6(t - P_7)}, \ \Theta(z) = \begin{cases} 1, & z \geq 0 \\ 0, & z < 0. \end{cases}$$

The parameters are tabulated below:

		Accident Dates	
		1930–77	1970–74
Prompt Component:			$2.30\underline{9}57$
Coefficient:	P_1	$3.96\underline{7}77$	
Age dependence:	P_2	$.1032\underline{8}6$	
Duration dependence:	P_3	$.778\underline{5}77$	
Threshold behavior:	P_8	$2.31\underline{3}11$	
Delayed Component:			
Coefficient:	P_4	$6.64\underline{4}63$	$4.36\underline{4}33$
Age dependence:	P_5	$.078\underline{1}633$	
Duration dependence:	P_6	$.17\underline{1}092$	
Threshold behavior:	P_9	$1.80\underline{8}92$	
Threshold value:	P_7	$.58\underline{3}219$	

The last significant digit for each value is underscored. This corresponds to the finest step size used in the parameter search. The most recent coefficient values from the trending study are shown for comparison. For reference Table A-1 presents the correlation matrix of the parameter estimates, derived by the methods of Appendix C.

TABLE A-I

Correlation Matrix of Parameter Estimates

	1	2	3	4	5	6	7	8	9	Standard Error
1	1.0000	−.3750	−.5189	−.3517	−.2175	−.0141	−.6365	−.7130	−.02940	2.0105
2		1.0000	.3753	.5731	−.2260	−.5226	−.7495	.4696	−.4186	.023683
3			1.0000	.1431	.0341	.2162	−.4697	.7497	.1582	.22626
4				1.0000	.5184	−.8505	−.1542	−.1471	−.8806	3.6188
5					1.0000	−.1775	.2867	−.2839	−.2986	.005563
6						1.0000	.01349	.3761	.9612	.06033
7							1.0000	−.8470	−.1662	2.2944
8								1.0000	.4899	.70439
9									1.0000	.79877

A large positive (or negative) correlation between two parameters indicates that the corresponding linear combination of parameters is weakly determined. This means that, while the quality of fit may be acceptable, the model has too much freedom, and caution must be used when extrapolating beyond the data.

APPENDIX B
FITTING MACHINERY

1. Loss Function

As remarked in the main text, the variety of possible loss functions is bewildering and the choice depends in some degree on optical inspection of the results. We do have some axiomatic guidance, however. In the first place, we are dealing with a rate process in which the *observed rate* in each cell is statistically independent of what is going on in all other cells. This means that the preferred loss function should be a sum of terms each of which depends on events and exposures in one cell only. That is $\tilde{L} = \Sigma_{x,t}\tilde{L}_{x,t}$, where the tilde is used to emphasize that the object is a random variable. (Those familiar with Whittaker-Henderson and related methods will note that these methods violate this requirement by introducing terms involving local differences, thus assuming, very tacitly, an intricate pattern of correlations among nearby cells.) If we require, as is usual, that the loss function be quadratic, we are led to the form

$$\tilde{L} = \sum_{x,t} R_{x,t} \left(\frac{\tilde{N}_{x,t}}{W_{x,t}} - q'^{(r)}_{[x]+t}(\alpha)\right)^2$$

Here, $\tilde{N}_{x,t}$ is the number of remarriages in the given cell, and α denotes the set of parameters. This leaves open the choice of the weighting factor, $R_{x,t}$. Experience teaches that great trouble will ensue if $R_{x,t}$ depends on the parameters. This rules out the "minimum variance" weight, $W_{x,t}/q'^{(r)}_{[x]+t}(1 - q'^{(r)}_{[x]+t})$, [Ref. (6), p. 95]. This was tried, in fact, and gave catastrophically slow fall-off at long durations, where data are sparse. This leaves two possible choices, $R_{x,t} = W_{x,t}$, and $R_{x,t} = l_{[x]}$, the latter being the number of lives widowed at age x. The former alternative was chosen as being satisfactory, though one cannot say that it is the best possible. Thus the loss function that was finally used is:

$$\tilde{L} = \sum_{x,t} W_{x,t} \left(\frac{\tilde{N}_{x,t}}{W_{x,t}} - q'^{(r)}_{[x]+t}(\alpha)\right)^2$$

2. Fitting Statistics

The loss function can be minimized by parameter search to yield a best fit for any prescribed set of parameter step sizes, but we need also a basis for judging whether the fit achieved is a reasonable representation of the data. If it is implausible that the deviations from the model are mere "noise," then we must reject the model. As remarked before, the foremost criterion is optical: data and model must be presented to the eye and judgment made as to whether the model follows the shape of the data. However, we can also make use of mathematical-statistical criteria which reduce—but do not eliminate—the element of judgment.

Specifically, we can make use of our characterization of $\tilde{N}_{x,t}$ as a binomial random variable with rate parameter, $q'^{(r)}_{[x]+t}$. We again ignore the fact that $W_{x,t}$ is non-integral, and we also assume that the model, evaluated at the best fit values, is correct. Under these assumptions, we can calculate the risk function, that is, the expected value of the loss function: $L = E[\tilde{L}|\alpha_{bestfit}] = \Sigma_{x,t}V_{x,t}$, where $V_{x,t} \equiv q'^{(r)}_{[x]+t}(1 - q'^{(r)}_{[x]+t})$. Further, with considerably more algebra, we can find the variance of the loss function,

$$Var[\tilde{L}] = E[(\tilde{L} - L)^2|\alpha_{b.f.}] = \sum_{x,t}\left[\frac{V_{x,t}}{W_{x,t}} + 2V^2_{x,t}\left(1 - \frac{3}{W_{x,t}}\right)\right].$$

This gives the standard deviation directly: $SD[\tilde{L}] = Var[\tilde{L}]^{1/2}$.

One now has a scale on which to measure the excess of the best-fit loss function over the risk function. Its use in drawing inferences is discussed in the main text and illustrative values are shown in Exhibit III-A.

The reader should note that these estimates of the expected loss function and its variance give a *conservative* basis for evaluating the fit. This is because the estimate for the variance of observed rates in a single cell is based on the assumption of a homogenous population, which gives the smallest possible variance. When we test for quality of fit, we are simultaneously testing this assumption as well.

3. Fitting Procedures

The fits were carried out by a gradient parameter search at an interactive terminal. One begins by specifying starting parameter values and maximum step sizes. The program then computes the loss function and its finite gradient in the parameter space. It steps down this gradient direction, obeying the maximum step size constraints until it reaches a minimum. It then recomputes the gradient at the new position and repeats the procedure. This continues until no further improvement can be achieved. At this point, control returns to the programmer, who uses output of the loss function, risk function, and standard deviation as well as the parameter values to assess the situation and respecify the step sizes or decide to terminate. This approach would profit from more automatic adjustment of the step sizes, but it is adequate for our purposes.

APPENDIX C
FITTING STATISTICS FOR PARAMETERS

The stochastic model assumed in our treatment of the data is the following:

$$\frac{\tilde{N}_{x,t}}{W_{x,t}} = q_{x,t}(\underset{\sim}{\alpha}) + \tilde{\epsilon}_{x,t}(\underset{\sim}{\alpha}); \; - q_{x,t}(\alpha) \leq \tilde{\epsilon}_{x,t} \leq 1 - q_{xt}(\underset{\sim}{\alpha});$$

$$\langle \tilde{\epsilon}_{x,t} \rangle = 0, \; \langle \tilde{\epsilon}_{xt} \cdot \tilde{\epsilon}_{x't'} \rangle = \delta_{xx'}\delta_{tt'}\frac{q_{xt}(1 - q_{xt})}{W_{x,t}}$$

where x, t refer to annual intervals in age and duration;
$\tilde{N}$ is the observed number of remarriages;
W is the number of exposures, treated approximately as an integer;
$q(\underset{\sim}{\alpha})$ is the model value for the absolute annual rate of remarriage;
$\underset{\sim}{\alpha}$ is the vector of true parameter values.
$\tilde{\epsilon}$ is the true value of the fitting residual.
These random variables are skewed, but we have accepted the attendant imprecision in fitting for the sake of mathematical clarity.

Let $\hat{\underset{\sim}{\alpha}}$ denote an *estimate* of the parameter values; then $\hat{e} = e(\hat{\alpha}) = \tilde{N}/W - q(\hat{\underset{\sim}{\alpha}})$, is the *estimated* residual at a given (x, t). We adopt vector notation also on the data space (all values of x, t), using an underbar to denote vectors on this space. Letting W denote the diagonal exposure matrix, our chosen loss function is (transposition being denoted by a raised prime)

$$\bar{L} = \Sigma_{xt} W_{xt} \hat{e}_{xt}(\hat{\underset{\sim}{\alpha}})^2 = \underline{\hat{e}}' \cdot W \cdot \underline{\hat{e}}.$$

Also, in vector notation, we have the covariance matrix of residuals:

$$\langle \underline{\tilde{\epsilon}}\, \underline{\tilde{\epsilon}}' \rangle = \Omega, \; [\Omega]_{xt,x't'} = \delta_{xx'}\delta_{tt'} \frac{q_{xt}(1 - q_{xt})}{W_{xt}} .$$

The loss function is minimized by a choice of the parameters, $\hat{\underset{\sim}{\alpha}} = \hat{\underset{\sim}{\alpha}}_0$ such that

$$\underset{\sim}{e}'(\hat{\underset{\sim}{\alpha}}) \cdot W \cdot \frac{\partial \underset{\sim}{q}(\hat{\underset{\sim}{\alpha}})}{\partial(\hat{\underset{\sim}{\alpha}})}\bigg|_{\hat{\underset{\sim}{\alpha}}=\hat{\underset{\sim}{\alpha}}_0} = \underset{\sim}{O}.$$

Letting $\tilde{q} = \tilde{N}/W$, $\hat{\underset{\sim}{\nabla}} = \partial/\partial\hat{\underset{\sim}{\alpha}}$, we may restate the least squares condition:

$$(\hat{\underset{\sim}{\nabla}} \cdot \hat{\underset{\sim}{q}}') \cdot W \cdot (\tilde{\underset{\sim}{q}} - \hat{\underset{\sim}{q}}) = \underset{\sim}{O}.$$

Our goal is to find an appropriate expression for the $\hat{\underset{\sim}{\alpha}}$ in terms of the $\underset{\sim}{\alpha}$ and the true residuals, $\tilde{\underset{\sim}{\epsilon}}$. To achieve this, we expand in $(\hat{\underset{\sim}{\alpha}} - \underset{\sim}{\alpha})$, keeping only terms to first order in the residuals:

$$\hat{\underset{\sim}{q}} = \underset{\sim}{q} + (\hat{\underset{\sim}{\alpha}} - \underset{\sim}{\alpha}) \cdot \hat{\underset{\sim}{\nabla}}\underset{\sim}{q} + O(\epsilon^2),$$

whence the least squares condition becomes, approximately,

$$[\hat{\underset{\sim}{\nabla}}\underset{\sim}{q}' + O(\epsilon)] \cdot W \cdot [\tilde{\underset{\sim}{\epsilon}} - (\hat{\underset{\sim}{\alpha}} - \underset{\sim}{\alpha}) \cdot \hat{\underset{\sim}{\nabla}}\underset{\sim}{q} + O(\epsilon^2)] = \underset{\sim}{O},$$

or

$$\{(\hat{\underset{\sim}{\nabla}}\underset{\sim}{q}') \cdot W \cdot (\hat{\underset{\sim}{\nabla}}'\underset{\sim}{q})\} \cdot (\hat{\underset{\sim}{\alpha}}_0 - \underset{\sim}{\alpha}) = (\hat{\underset{\sim}{\nabla}}\underset{\sim}{q}') \cdot W \cdot \tilde{\underset{\sim}{\epsilon}}.$$

Denoting the parameter-space matrix in braces by $\mathcal{W}$, we obtain

$$\hat{\underset{\sim}{\alpha}}_0 - \underset{\sim}{\alpha} = \mathcal{W}^{-1} \cdot (\hat{\underset{\sim}{\nabla}}\underset{\sim}{q}') \cdot W \cdot \tilde{\underset{\sim}{\epsilon}} + O(\epsilon^2).$$

Whence the covariance matrix of the parameter estimates is

$$\begin{aligned}\langle(\hat{\underset{\sim}{\alpha}}_0 - \underset{\sim}{\alpha})(\hat{\underset{\sim}{\alpha}}_0 - \underset{\sim}{\alpha})'\rangle &\cong \mathcal{W}^{-1} \cdot (\hat{\underset{\sim}{\nabla}}\underset{\sim}{q}') \cdot W \cdot \langle\tilde{\underset{\sim}{\epsilon}}\tilde{\underset{\sim}{\epsilon}}'\rangle \cdot W \cdot \hat{\underset{\sim}{\nabla}}'q \cdot \mathcal{W}^{-1} \\ &= \mathcal{W}^{-1}\{(\hat{\underset{\sim}{\nabla}} \cdot \underset{\sim}{q}') \cdot W \cdot \Omega \cdot W \cdot (\hat{\underset{\sim}{\nabla}}'\underset{\sim}{q})\}\mathcal{W}^{-1} \\ &= \mathcal{W}^{-1}\mathcal{V}\mathcal{W}^{-1} = \hat{\Sigma}_0\end{aligned}$$

These parameter-space matrices can be expressed as

$$\mathcal{W} = \sum_{xt} W_{xt}(\hat{\underset{\sim}{\nabla}}\underset{\sim}{q}_{xt})(\hat{\underset{\sim}{\nabla}}'q_{xt});$$

$$\mathcal{V} = \sum_{xt} W_{xt}q_{xt}(1 - q_{xt})(\hat{\underset{\sim}{\nabla}}\underset{\sim}{q}_{xt})(\hat{\underset{\sim}{\nabla}}'q_{xt}).$$

These are compiled by a special-purpose program which evaluates the gradients by finite differences. The results appear in Exhibit III-B, where the ellipses around the data points represent the equation

$$(\hat{\underset{\sim}{\alpha}} - \hat{\underset{\sim}{\alpha}}_0)' \cdot \hat{\Sigma}_0^{-1} \cdot (\hat{\underset{\sim}{\alpha}} - \hat{\underset{\sim}{\alpha}}_0) = 1, \text{ and}$$

where $\hat{\Sigma}_0 = \mathcal{W}^{-1}\mathcal{V}\mathcal{W}^{-1}$, evaluated at the least squares parameter values.

The ellipses are enclosed by a box two standard deviations on a side. A suggested rule of inference is that points whose ellipses do not overlap are statistically distinct, while those whose ellipses do overlap are confounded (Rayleigh criterion). As remarked in the main text, the ellipses represent, approximately, a joint 40% confidence region. This rests on the assumption that the parameter errors have an approximate joint normal distribution. Such an assumption is reasonable since the parameter errors are the sum of many fitting residuals from individual data cells. Granting this, the quadratic form which defines the ellipses has a chi-square distribution with n degrees of freedom, where n is the number of parameters being examined. In Exhibit III-B, $n = 2$; and inspection of a chi-square table (8) gives a probability of .39347 that the true parameter values are contained in the ellipse. Doubling the size of the ellipse (chi-square = 4) gives 86% confidence.

The ellipsoidal region is used because it occupies minimal volume in the parameter space and thus is easy to characterize.

APPENDIX D
ACTUARIAL FUNCTIONS

1. Population Averaging

To carry out our analyses, we need some information about the current distribution of widowed spouses by attained age and by duration at fixed attained age. The first is needed to assess the financial consequences of the new remarriage and mortality assumptions, the second to evaluate the ultimate remarriage rates without relying solely on the population mix implicit in the data, which changes significantly over time and may not be appropriate in NCCI jurisdictions in the recent period.

Unfortunately this information is not directly available, though some will be as soon as the Pension Study results are usable. However, a reasonable approximation can be derived easily from the age-at-accident distribution in the recent NCCI Injury Table, granting a few assumptions.

Suppose that

1. The probability that the age at time of accident of a widowed spouse lies between x and $x + dx$ is $f_x(x)dx$;
2. a constant number of widowed spouses enters the population each year; and
3. the assumed mortality rates by attained age and the model remarriage rates, select at all durations, are appropriate for all such spouses.

Then the probability density of attained ages of widowed spouses now receiving benefits will be given by

$$f_z(z) = \frac{\int_0^z dt f_x(z - t) \cdot {}_tP^{(T)}_{[z-t]}}{\int_0^\infty dz' \int_0^{z'} dt f_x(z' - t) \cdot {}_tP^{(T)}_{[z'-t]}}$$

where ${}_tP^{(T)}_{[x]}$ is the probability that a spouse widowed at age x will survive unremarried to age $x + t$.

Under the same assumptions, the probability density of durations t, from time of accident for all spouses on claim at attained age z, will be

$$f_T(t|z) = \frac{f_x(z - t) \cdot {}_tP^{(T)}_{[z-t]}}{\int_0^z dt' f_x(z - t') \cdot {}_{t'}P^{(T)}_{[z-t']}}, \quad 0 \leq t \leq z.$$

This latter, summarized as probabilities on annual intervals, was used to average the model remarriage rates which appear as the ultimate column of Exhibit IV-B. The attained age distribution, used to evaluate overall average annuity values, is displayed in Exhibit V, along with a similar distribution for claimants derived from the Injury Table distribution for permanent totals.

2. *Annuity Values*

Annuity values are developed in the conventional way by first defining commutation functions.

$$D^{(T)}_{[x]+t} = D^{(T)}_{[x]+t+1}/v \cdot (1 - q'^{(m)}_{x+t})(1 - q'^{(r)}_{[x]+t}),$$

where superscript m indicates mortality and r remarriage. The primes are used to remined us that these are absolute rates. When the duration, t, is five years or greater, the remarriage rates from the ultimate column are used. The discount factor v, also contains the effect of escalation, if appropriate:

$$v = \frac{1 + r_e}{1 + i}.$$

The numerator function is

$$\overline{N}^{(T)}_{[x]+t} = \int_0^\infty ds\, D^{(T)}_{[x]+t+s} \doteqdot N^{(T)}_{[x]+t} - \frac{1}{2} D^{(T)}_{[x]+t}.$$

The values of $D^{(T)}_{[x]+t}$ are tabulated in Exhibit IV-C; $N^{(T)}_{[x]+t}$ are shown in IV-D. The continuous annuity commencing promptly,

$$\bar{a}^{(T)}_{[x]+t} = \int_0^\infty ds\, v^s \cdot {}_sP^{(T)}_{[x]+t} \doteqdot \frac{N^{(T)}_{[x]+t}}{D^{(T)}_{[x]+t}} - \frac{1}{2},$$

is shown in Exhibit IV-E.

3. Dowries

The dowry function tabulated in Exhibit IV-F represents the expected value per dollar of lump sum payable to a widowed spouse on remarriage. It has the structure of a partial insurance:

$$\bar{A}^{(r)}_{[x]+t} = \int_0^\infty ds\, v^s \cdot \mu^{(r)}_{[x]+t+s} \cdot {}_sP^{(T)}_{[x]+t}$$

$$\doteqdot \sum_{k=0}^{1} v^k \int_0^1 ds[v^s \cdot {}_{k+s}P^{(m)}_{x+t}] \cdot [\mu^{(r)}_{[x]+t+k+s} \cdot {}_{k+s}P^{(r)}_{[x]+t}]$$

$$\doteqdot \sum_{k=0}^{\infty} v^k \frac{1}{2} [1 + vP^{(m)}_{x+t+k}] \cdot [{}_kP^{(T)}_{[x]+t} \cdot q^{(r)}_{[x]+t+k}],$$

where we have used separate approximate annual averages of the factors in brackets. This may be evaluated from previously defined quantities if we take

$$\overline{M}^{(r)}_{[x]+t} \doteqdot \sum_{k=0}^{\infty} D^{(T)}_{[x]+t+k} \cdot q^{(r)}_{[x]+t+k} \quad [1 + v(1 - q^{(m)}_{x+t+k})];$$

$$\overline{A}^{(r)}_{[x]+t} = \overline{M}^{(r)}_{[x]+t}/D^{(T)}_{[x]+t}.$$

4. Automatic Survivorship Benefit

Recalling that this is a continuous annuity for life unremarried to the spouse on the death of the claimant for causes unrelated to the accident, we may write down

$$B_{x_1:x_2} = \int_0^\infty ds\, v^s_c[\mu^{(1)}_{x_1+s} \cdot {}_sP^{(1)}_{x_1}]\, [{}_sP^{(2)}{}_{x_2} \cdot \bar{a}^{(T),v}_{[x_2+s]}],$$

where

x_1 = attained age of claimant,
x_2 = attained age of spouse,
v_c = interest/escalation factor while claimant survives.
Superscripts (1) and (2) refer to the respective mortality assumptions (remarriage rates affect only the annuity function).

This expression has a structure similar to that of the dowry function and may be approximated by the same methods. The values are tabulated in Exhibit IV-G for three distinct escalation assumptions:

1. No escalation;
2. No escalation before claimant's death, 6% after; and
3. Uniform 6% escalation.

The latter two are both shown because of some fiscally significant uncertainties in interpretation of the benefit phrasing in the US Longshoremen's and Harbor Workers' Act.

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EXHIBIT I

HISTORICAL AGGREGATE REMARRIAGE RATES BY ACCIDENT PERIOD
NYATF DATA

Range of Accident Years	(1) Cases	(2) Remar-riages	(3) Exposure (years)	(4) Crude Remarriage Rates (2) ÷ (3)	(5) % Std. Err.	(6) Overlap† Exposure (years)	(7) Standard Population Remarriage Rates‡
1900–1919	151	22	4810	.0046	21	25970	.0071
1920–1929	219	18	7383	.0024	24	27802	.0010
1930–1941	1872	576	37368	.0154	4	31174	.0184
1942–1945	1162	356	21299	.0167	5	31133	.0267
1946–1960	3567	691	62061	.0111	4	31231	.0172
*1960–1976	3700	450	31289	.0144	5	31289	.0144
1900–1976	10671	2113	164209	.0129	2	31289	.0169

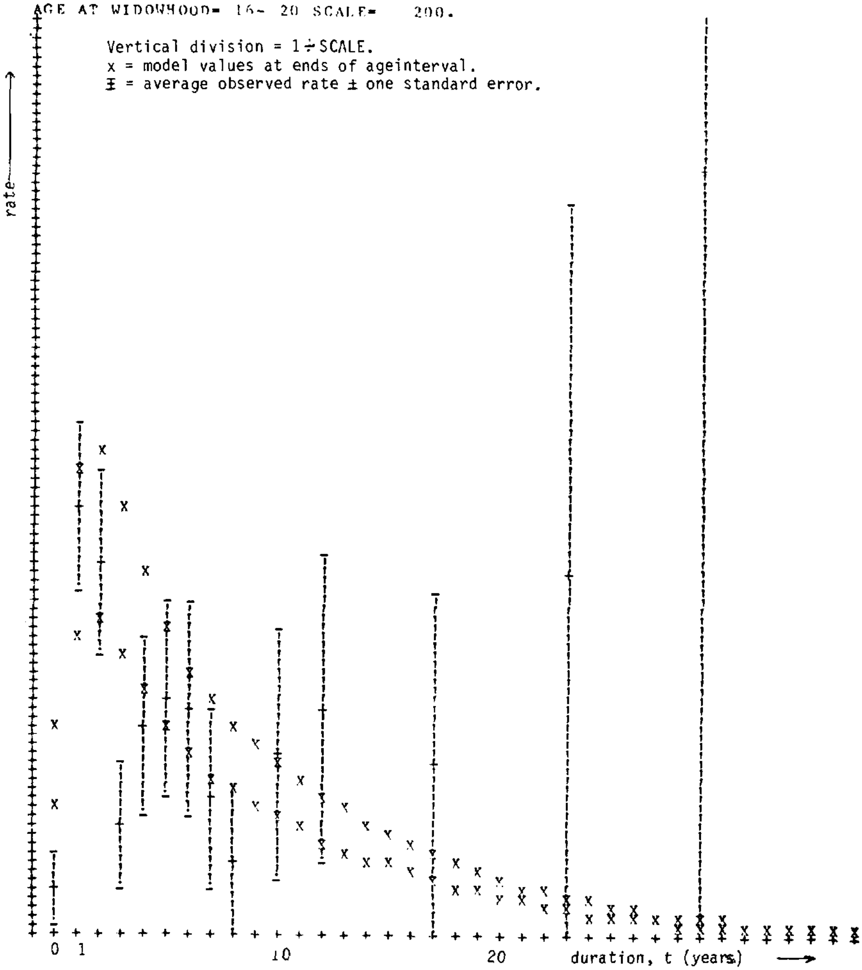
* Base period for standard exposures. † Exposures from base period, excluding annual cells with zero exposure in current period. ‡ Crude annual rates averaged over base period exposures.

USECS Data

Years	Exposures	Remarriages	Rate
1916–25	5794	209	.0361
1925–30	6741	114	.0169
1930–35	8907	105	.0117
1935–40	11273	129	.0114
1940–45	15486	382	.0247
1945–50	20505	530	.0258
1950–55	21794	269	.0123

EXHIBIT II

Remarriage Rates 1930-1977 by
Quinquennial Age Intervals
Preferred Model Fit at Beginning
and End of Interval



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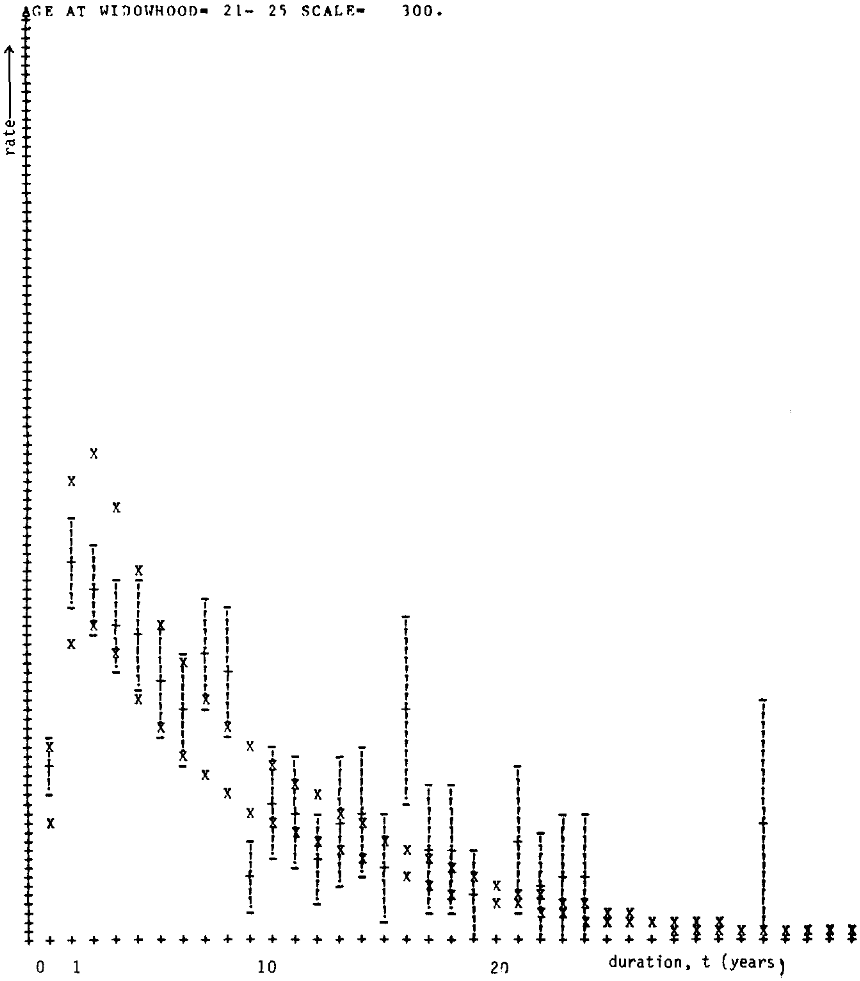

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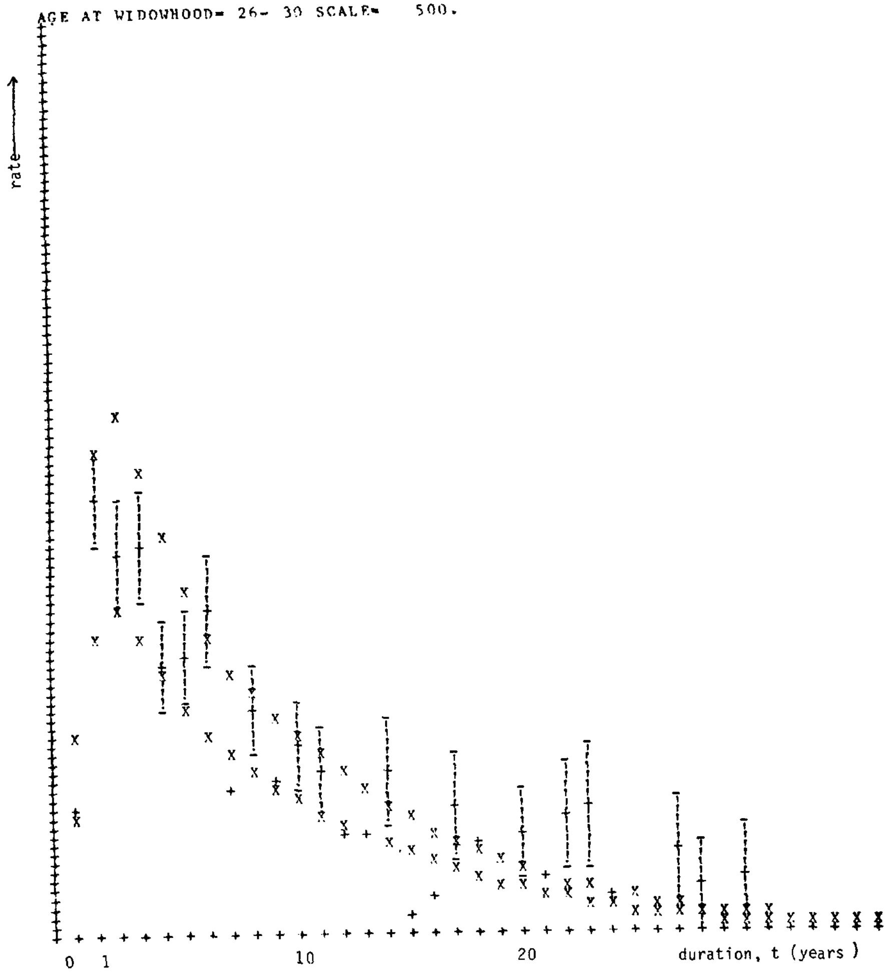

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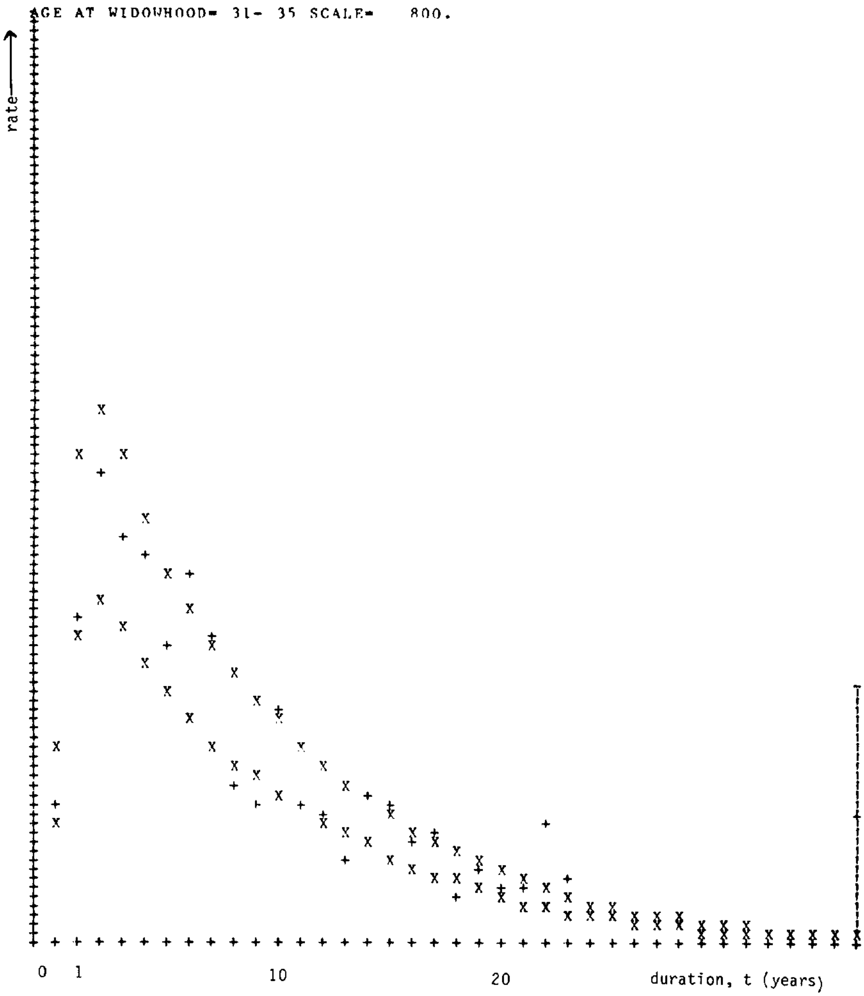

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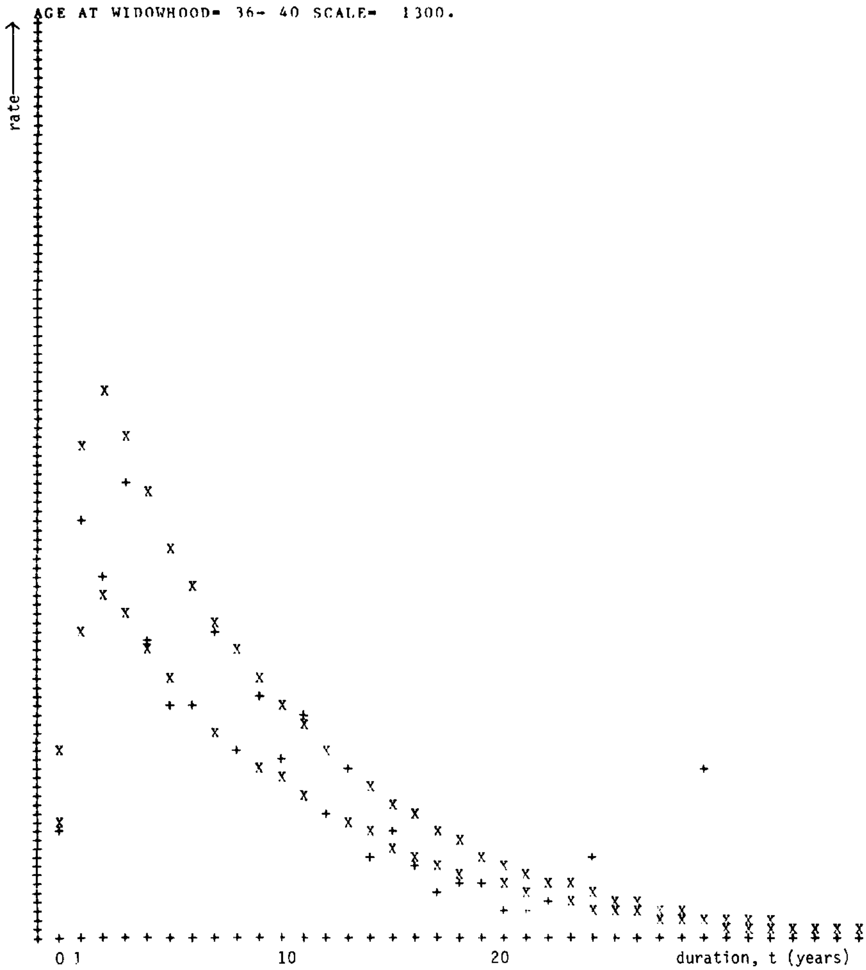

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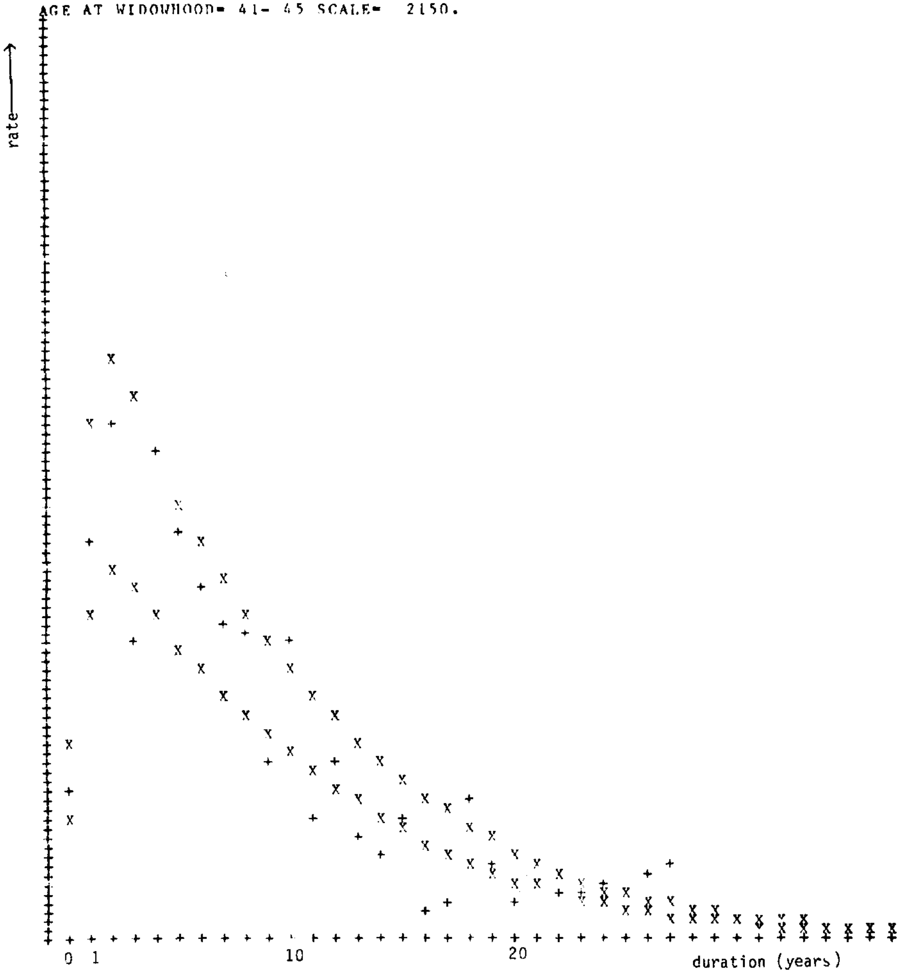

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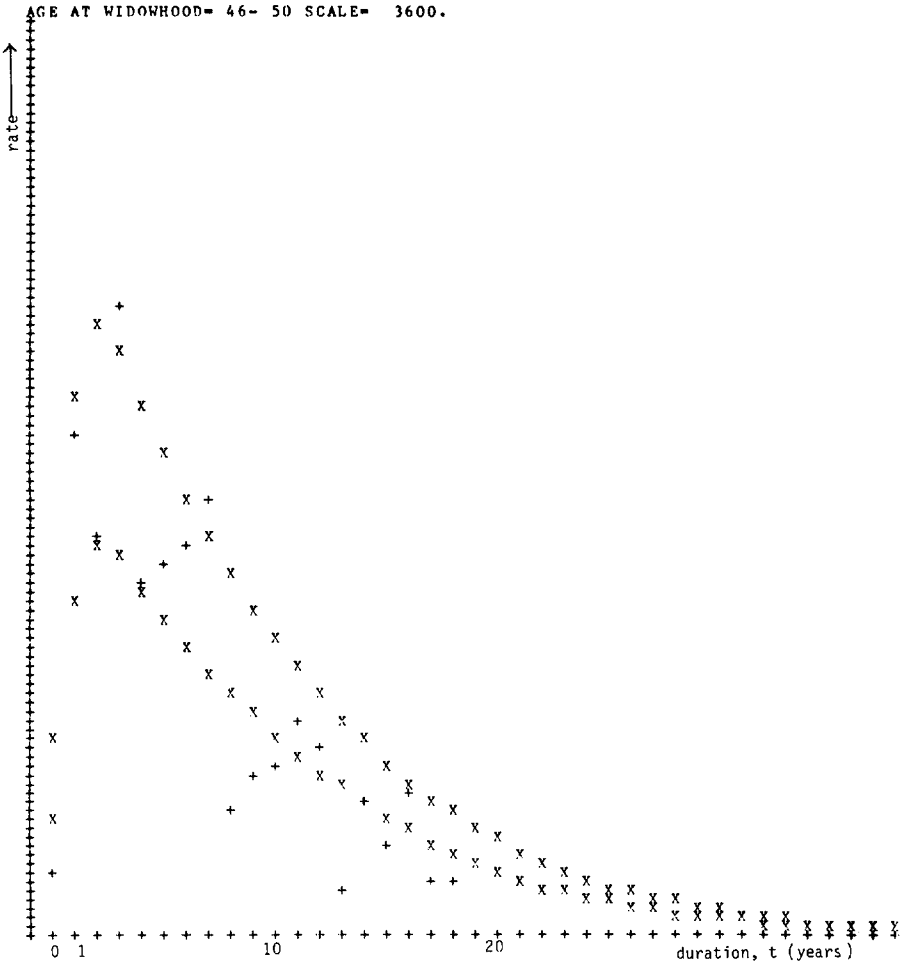

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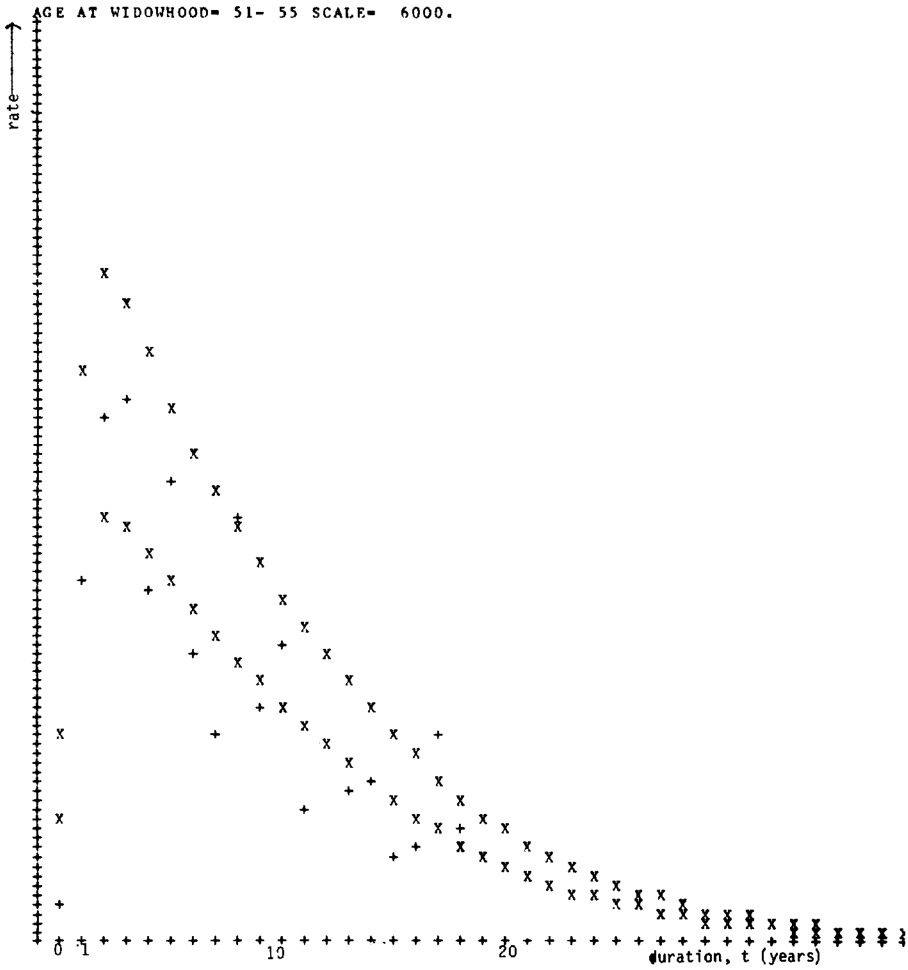

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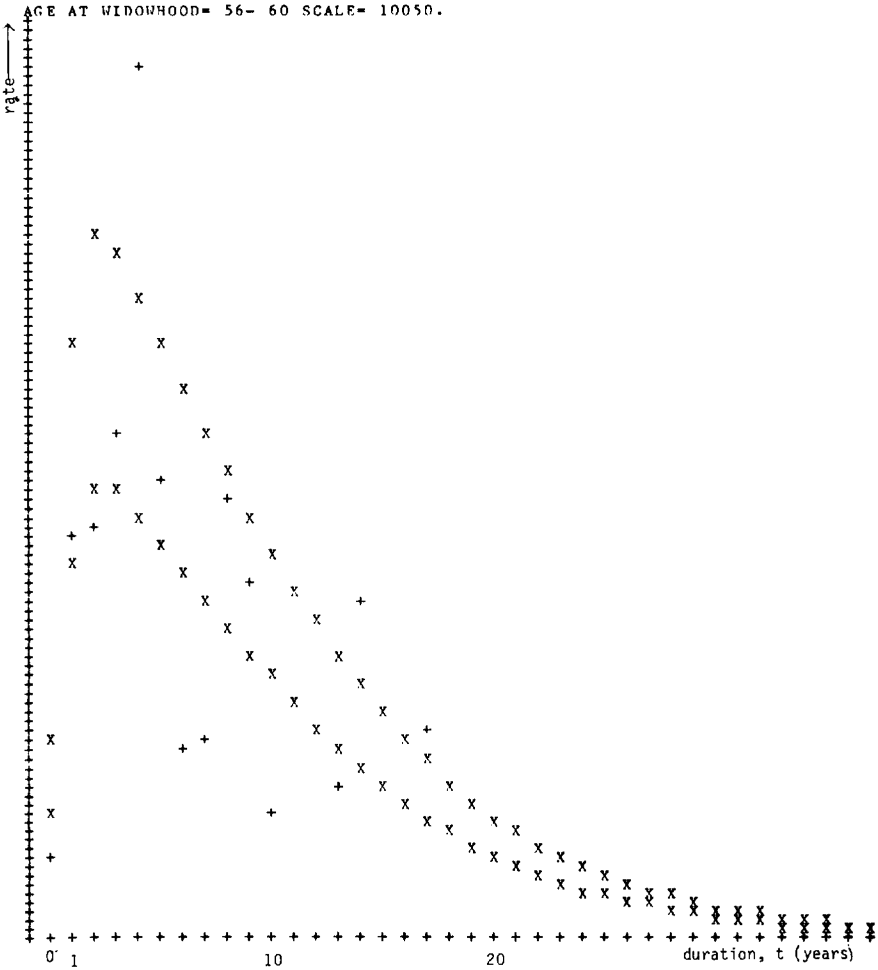


EXHIBIT III-A

REMARRIAGE TRENDING

Acc. Yr.	Exposure	P_1	P_4/P_1	P_4	Loss Function: Best Fit	Loss Function: Expected	Loss Function: Deviation
35–39	22603	2.91764	3.35049	9.77552	31.3013	28.6709	.99
40–44	25344	4.02828	2.74015	11.03809	31.2111	32.7288	−.54
45–49	25162	5.43403	1.06197	5.77078	22.3771	22.7655	−.16
50–54	23640	4.68924	1.29891	6.09090	20.2867	21.8005	−.59
55–59	18912	4.81341	.914997	4.40426	16.6808	18.4435	−.70
60–64	16480	4.11459	1.14955	4.72993	13.9554	16.3780	−1.04
65–69	11134	4.92020	.525821	2.58714	12.1081	13.2716	−.46
70–74	3643	2.30957	1.88967	4.36433	6.25557	6.82770	−.23
30–77	152016	3.97777	1.675	6.64463	29.0661	27.5244	.66

EXHIBIT III-B

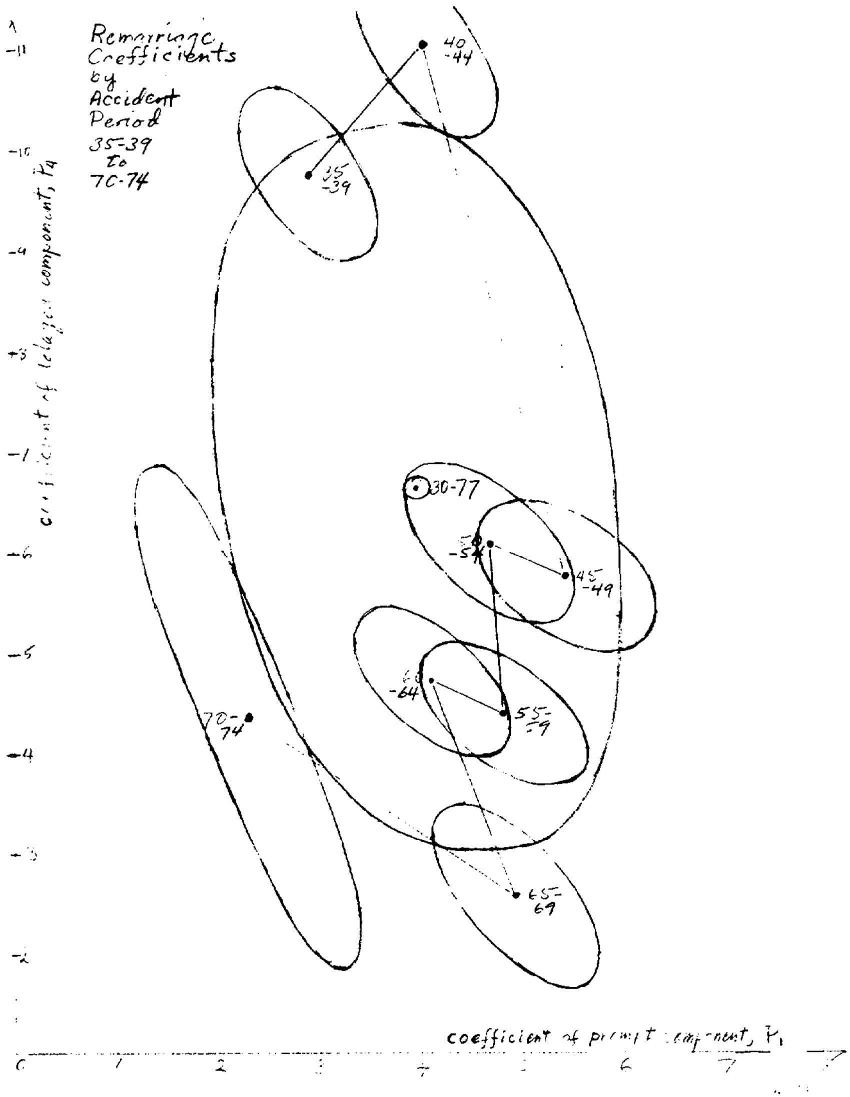


EXHIBIT IV-A

MORTALITY RATES: U.S. LIFE TABLES, 1969–71

Age	Tot Fem	Tot Pop	Age	Tot Fem	Tot Pop	Age	Tot Fem	Tot Pop
0	0.01746	0.02002	37	0.00180	0.00244	74	0.03893	0.05075
1	0.00116	0.00125	38	0.00197	0.00266	75	0.04325	0.05552
2	0.00077	0.00086	39	0.00215	0.00290	76	0.04790	0.06060
3	0.00060	0.00069	40	0.00233	0.00314	77	0.05295	0.06596
4	0.00051	0.00057	41	0.00251	0.00341	78	0.05840	0.07153
5	0.00043	0.00051	42	0.00273	0.00370	79	0.06432	0.07741
6	0.00038	0.00046	43	0.00297	0.00404	80	0.07097	0.08394
7	0.00034	0.00043	44	0.00325	0.00443	81	0.07834	0.09122
8	0.00031	0.00039	45	0.00354	0.00484	82	0.08612	0.09892
9	0.00028	0.00034	46	0.00384	0.00528	83	0.09419	0.10695
10	0.00026	0.00031	47	0.00416	0.00574	84	0.10275	0.11548
11	0.00025	0.00030	48	0.00449	0.00624	85	0.11282	0.12561
12	0.00027	0.00035	49	0.00484	0.00678	86	0.12462	0.13748
13	0.00033	0.00046	50	0.00523	0.00738	87	0.13685	0.14979
14	0.00040	0.00063	51	0.00565	0.00804	88	0.14859	0.16158
15	0.00049	0.00082	52	0.00611	0.00876	89	0.16006	0.17292
16	0.00058	0.00101	53	0.00660	0.00957	90	0.17264	0.18502
17	0.00066	0.00117	54	0.00712	0.01043	91	0.18718	0.19888
18	0.00069	0.00128	55	0.00768	0.01136	92	0.20243	0.21363
19	0.00071	0.00134	56	0.00829	0.01236	93	0.21750	0.22870
20	0.00072	0.00140	57	0.00894	0.01341	94	0.23186	0.24336
21	0.00073	0.00147	58	0.00962	0.01452	95	0.24584	0.25745
22	0.00075	0.00152	59	0.01035	0.01570	96	0.25854	0.26959
23	0.00077	0.00153	60	0.01113	0.01695	97	0.26980	0.28024
24	0.00079	0.00151	61	0.01200	0.01829	98	0.27996	0.28977
25	0.00081	0.00147	62	0.01298	0.01974	99	0.28949	0.29869
26	0.00083	0.00143	63	0.01411	0.02133	100	0.29836	0.30696
27	0.00086	0.00142	64	0.01538	0.02306	101	0.30659	0.31461
28	0.00090	0.00144	65	0.01678	0.02495	102	0.31420	0.32167
29	0.00096	0.00149	66	0.01832	0.02699	103	0.32122	0.32817
30	0.00102	0.00155	67	0.02004	0.02918	104	0.32768	0.33414
31	0.00110	0.00163	68	0.02195	0.03152	105	0.33361	0.33960
32	0.00119	0.00172	69	0.02407	0.03400	106	0.33904	0.34460
33	0.00129	0.00183	70	0.02632	0.03661	107	0.34401	0.34917
34	0.00140	0.00195	71	0.02879	0.03943	108	0.34855	0.35333
35	0.00152	0.00209	72	0.03165	0.04266	109	0.35269	0.35712
36	0.00165	0.00225	73	0.03503	0.04644			

EXHIBIT IV-B

SELECT ABSOLUTE RATES OF REMARRIAGE

AGE	0	1	2	3	4	ULT	AGE	AGE	0	1	2	3	4	ULT	AGE
16	0.06641	0.15455	0.16373	0.14581	0.12422	0.10604	21	61	0.00077	0.00240	0.00295	0.00298	0.00284	0.00186	66
17	0.06021	0.14129	0.15013	0.13396	0.11434	0.09613	22	62	0.00070	0.00220	0.00270	0.00273	0.00262	0.00168	67
18	0.05458	0.12909	0.13758	0.12301	0.10522	0.08732	23	63	0.00063	0.00200	0.00247	0.00251	0.00241	0.00152	68
19	0.04946	0.11789	0.12602	0.11292	0.09680	0.07966	24	64	0.00057	0.00183	0.00227	0.00231	0.00222	0.00136	69
20	0.04481	0.10761	0.11538	0.10362	0.08903	0.07284	25	65	0.00052	0.00167	0.00208	0.00212	0.00205	0.00123	70
21	0.04059	0.09818	0.10560	0.09506	0.08186	0.06687	26	66	0.00047	0.00153	0.00191	0.00195	0.00188	0.00110	71
22	0.03677	0.08955	0.09661	0.08718	0.07526	0.06139	27	67	0.00043	0.00140	0.00175	0.00179	0.00174	0.00098	72
23	0.03330	0.08165	0.08837	0.07994	0.06918	0.05624	28	68	0.00039	0.00128	0.00160	0.00165	0.00160	0.00087	73
24	0.03015	0.07443	0.08081	0.07329	0.06359	0.05150	29	69	0.00035	0.00117	0.00147	0.00152	0.00147	0.00078	74
25	0.02730	0.06784	0.07387	0.06718	0.05844	0.04716	30	70	0.00032	0.00107	0.00135	0.00139	0.00136	0.00069	75
26	0.02471	0.06181	0.06753	0.06157	0.05370	0.04316	31	71	0.00029	0.00098	0.00124	0.00128	0.00125	0.00061	76
27	0.02237	0.05631	0.06171	0.05643	0.04935	0.03949	32	72	0.00026	0.00089	0.00113	0.00118	0.00115	0.00054	77
28	0.02025	0.05129	0.05640	0.05171	0.04534	0.03614	33	73	0.00024	0.00082	0.00104	0.00108	0.00106	0.00048	78
29	0.01833	0.04672	0.05153	0.04738	0.04166	0.03308	34	74	0.00022	0.00075	0.00095	0.00100	0.00098	0.00042	79
30	0.01659	0.04255	0.04708	0.04341	0.03828	0.03026	35	75	0.00020	0.00068	0.00088	0.00092	0.00090	0.00037	80
31	0.01502	0.03875	0.04301	0.03978	0.03517	0.02767	36	76	0.00018	0.00063	0.00080	0.00084	0.00083	0.00033	81
32	0.01359	0.03529	0.03930	0.03645	0.03231	0.02528	37	77	0.00016	0.00057	0.00074	0.00078	0.00077	0.00028	82
33	0.01230	0.03213	0.03590	0.03340	0.02969	0.02309	38	78	0.00015	0.00052	0.00068	0.00071	0.00071	0.00025	83
34	0.01113	0.02926	0.03280	0.03060	0.02728	0.02109	39	79	0.00013	0.00048	0.00062	0.00066	0.00065	0.00022	84
35	0.01008	0.02664	0.02996	0.02804	0.02507	0.01926	40	80	0.00012	0.00044	0.00057	0.00061	0.00060	0.00019	85
36	0.00912	0.02426	0.02737	0.02569	0.02303	0.01759	41	81	0.00011	0.00040	0.00053	0.00056	0.00055	0.00017	86
37	0.00826	0.02209	0.02501	0.02355	0.02117	0.01607	42	82	0.00010	0.00037	0.00048	0.00051	0.00051	0.00014	87
38	0.00747	0.02012	0.02285	0.02158	0.01945	0.01468	43	83	0.00009	0.00034	0.00044	0.00047	0.00047	0.00013	88
39	0.00677	0.01832	0.02088	0.01978	0.01788	0.01342	44	84	0.00008	0.00031	0.00041	0.00044	0.00043	0.00011	89
40	0.00612	0.01669	0.01908	0.01813	0.01643	0.01227	45	85	0.00008	0.00028	0.00037	0.00040	0.00040	0.00009	90
41	0.00554	0.01520	0.01744	0.01662	0.01510	0.01122	46	86	0.00007	0.00026	0.00034	0.00037	0.00037	0.00008	91
42	0.00502	0.01385	0.01594	0.01523	0.01388	0.01027	47	87	0.00006	0.00024	0.00032	0.00034	0.00034	0.00007	92
43	0.00454	0.01261	0.01457	0.01396	0.01276	0.00940	48	88	0.00006	0.00022	0.00029	0.00031	0.00031	0.00006	93
44	0.00411	0.01149	0.01332	0.01280	0.01173	0.00860	49	89	0.00005	0.00020	0.00027	0.00029	0.00029	0.00005	94
45	0.00373	0.01047	0.01217	0.01174	0.01079	0.00788	50	90	0.00005	0.00018	0.00025	0.00027	0.00027	0.00005	95
46	0.00337	0.00954	0.01113	0.01077	0.00992	0.00721	51	91	0.00004	0.00017	0.00023	0.00024	0.00025	0.00004	96
47	0.00306	0.00870	0.01018	0.00987	0.00912	0.00661	52	92	0.00004	0.00015	0.00021	0.00023	0.00023	0.00003	97
48	0.00277	0.00793	0.00931	0.00906	0.00839	0.00605	53	93	0.00004	0.00014	0.00019	0.00021	0.00021	0.00003	98
49	0.00251	0.00723	0.00851	0.00831	0.00771	0.00554	54	94	0.00003	0.00013	0.00018	0.00019	0.00019	0.00003	99
50	0.00227	0.00659	0.00779	0.00762	0.00709	0.00507	55	95	0.00003	0.00012	0.00016	0.00018	0.00018	0.00002	100
51	0.00206	0.00601	0.00712	0.00699	0.00653	0.00465	56	96	0.00003	0.00011	0.00015	0.00016	0.00017	0.00002	101
52	0.00186	0.00548	0.00652	0.00642	0.00600	0.00425	57	97	0.00002	0.00010	0.00014	0.00015	0.00015	0.00002	102
53	0.00169	0.00500	0.00596	0.00589	0.00552	0.00389	58	98	0.00002	0.00009	0.00013	0.00014	0.00014	0.00001	103
54	0.00153	0.00456	0.00546	0.00541	0.00508	0.00356	59	99	0.00002	0.00008	0.00012	0.00013	0.00013	0.00001	104
55	0.00139	0.00416	0.00500	0.00496	0.00468	0.00325	60	100	0.00002	0.00008	0.00011	0.00012	0.00012	0.00001	105
56	0.00126	0.00379	0.00457	0.00456	0.00430	0.00297	61	101	0.00002	0.00007	0.00010	0.00011	0.00011	0.00001	106
57	0.00114	0.00346	0.00419	0.00418	0.00396	0.00271	62	102	0.00002	0.00007	0.00009	0.00010	0.00010	0.00001	107
58	0.00103	0.00316	0.00384	0.00384	0.00365	0.00247	63	103	0.00001	0.00006	0.00008	0.00009	0.00009	0.00001	108
59	0.00094	0.00288	0.00351	0.00353	0.00336	0.00225	64	104	0.00001	0.00006	0.00008	0.00008	0.00009	0.00001	109
60	0.00085	0.00263	0.00322	0.00324	0.00309	0.00205	65	105	0.00001	0.00005	0.00007	0.00008	0.00008	0.0	110

EXHIBIT IV-C

SELECT D_X FOR LIFE UNREMARRIED @ 3.5%/0.0%

AGE	0	1	2	3	4	ULT	AGE	AGE	0	1	2	3	4	ULT	AGE
16	100000.0	90149.6	73591.0	59419.8	49004.6	41436.1	21	61	2645.0	2523.0	2400.2	2279.6	2162.2	2048.2	66
17	81027.2	73525.0	60959.5	50020.1	41824.4	35763.4	22	62	2517.7	2399.3	2280.5	2163.6	2049.7	1939.1	67
18	66760.1	60939.9	51242.0	42667.0	36126.8	31208.9	23	63	2394.7	2279.7	2164.4	2051.0	1940.5	1832.9	68
19	55831.8	51239.3	43638.8	36822.8	31536.5	27499.4	24	64	2275.8	2163.8	2051.8	1941.6	1834.1	1729.4	69
20	47307.0	43627.7	37588.9	32103.3	27782.2	24433.6	25	65	2160.4	2051.3	1942.3	1835.2	1730.6	1628.5	70
21	40554.6	37565.3	32707.0	28242.1	24673.6	21870.1	26	66	2048.4	1941.9	1835.8	1731.5	1629.5	1530.1	71
22	35127.1	32666.8	28713.6	25042.6	22068.5	19701.1	27	67	1939.5	1835.6	1732.1	1630.4	1531.1	1434.2	72
23	30713.4	28664.5	25413.8	22366.4	19866.0	17851.0	28	68	1833.5	1731.9	1631.0	1531.9	1435.1	1340.5	73
24	27088.5	25363.3	22663.3	20110.7	17991.1	16262.7	29	69	1730.1	1630.8	1532.4	1435.8	1341.3	1248.7	74
25	24081.4	22613.5	20349.7	18193.5	16382.6	14889.2	30	70	1629.3	1532.3	1436.3	1342.0	1249.5	1158.6	75
26	21562.8	20302.0	18387.2	16550.8	14992.1	13693.3	31	71	1531.0	1436.3	1342.5	1250.1	1159.3	1070.3	76
27	19435.6	18342.5	16709.3	15133.4	13782.5	12645.3	32	72	1435.2	1342.4	1250.5	1159.8	1070.9	984.0	77
28	17623.6	16667.8	15263.6	13901.4	12722.8	11721.3	33	73	1341.5	1250.5	1160.2	1071.4	984.5	899.9	78
29	16067.8	15225.2	14008.8	12823.5	11788.8	10901.5	34	74	1249.7	1160.2	1071.7	984.9	900.3	818.3	79
30	14721.6	13973.6	12912.3	11874.2	10960.5	10170.2	35	75	1159.6	1071.7	985.2	900.7	818.7	739.4	80
31	13549.3	12880.3	11948.3	11033.5	10221.9	9514.4	36	76	1071.2	985.2	901.0	819.0	739.8	663.5	81
32	12521.4	11919.3	11095.5	10284.6	9560.1	8923.6	37	77	984.8	901.0	819.2	740.1	663.8	590.6	82
33	11614.6	11069.6	10337.1	9614.3	8964.1	8388.7	38	78	900.7	819.3	740.3	664.0	590.9	521.4	83
34	10810.2	10313.9	9658.8	9011.2	8424.9	7902.3	39	79	819.0	740.3	664.2	591.1	521.6	456.2	84
35	10092.3	9638.0	9049.1	8465.9	7934.6	7458.0	40	80	740.1	664.2	591.2	521.8	456.4	395.4	85
36	9448.1	9030.5	8498.1	7970.3	7486.8	7050.5	41	81	664.1	591.3	521.9	456.5	395.5	338.8	86
37	8867.7	8481.7	7998.1	7518.2	7076.3	6675.5	42	82	591.1	521.9	456.6	395.6	339.0	286.5	87
38	8341.3	7983.2	7541.8	7103.7	6698.5	6328.8	43	83	521.8	456.6	395.7	339.1	286.6	238.9	88
39	7862.1	7528.6	7124.1	6722.6	6349.4	6007.1	44	84	456.6	395.8	339.1	286.7	239.0	196.5	89
40	7423.8	7112.2	6740.1	6370.5	6025.5	5707.5	45	85	395.7	339.2	286.8	239.1	196.6	159.5	90
41	7021.3	6729.4	6385.5	6044.0	5723.9	5427.5	46	86	339.1	286.8	239.1	196.6	159.5	127.5	91
42	6650.5	6375.9	6056.9	5740.1	5442.2	5165.2	47	87	286.8	239.1	196.7	159.5	127.5	100.1	92
43	6307.1	6048.1	5751.2	5456.3	5178.3	4918.8	48	88	239.1	196.7	159.6	127.5	100.1	77.1	93
44	5988.3	5743.3	5465.9	5190.7	4930.4	4686.6	49	89	196.7	159.6	127.5	100.1	77.1	58.3	94
45	5691.1	5458.8	5198.9	4941.3	4697.0	4467.5	50	90	159.6	127.6	100.2	77.2	58.3	43.3	95
46	5413.2	5192.5	4948.3	4706.6	4476.7	4260.0	51	91	127.5	100.2	77.2	58.3	43.3	31.5	96
47	5152.7	4942.5	4712.6	4485.1	4268.2	4063.2	52	92	100.2	77.2	58.3	43.3	31.5	22.6	97
48	4907.7	4707.4	4490.3	4275.6	4070.4	3876.0	53	93	77.2	58.3	43.3	31.5	22.6	15.9	98
49	4676.9	4485.5	4280.0	4076.9	3882.5	3697.7	54	94	58.3	43.3	31.5	22.6	15.9	11.1	99
50	4458.8	4275.8	4080.8	3888.2	3703.5	3527.6	55	95	43.3	31.5	22.6	15.9	11.1	7.6	100
51	4252.4	4077.0	3891.5	3708.5	3532.7	3364.9	56	96	31.5	22.6	15.9	11.1	7.6	5.2	101
52	4056.5	3888.1	3711.4	3537.1	3369.5	3209.2	57	97	22.6	15.9	11.1	7.6	5.2	3.5	102
53	3870.2	3708.3	3539.6	3373.4	3213.3	3059.9	58	98	15.9	11.1	7.6	5.2	3.5	2.3	103
54	3692.6	3537.0	3375.6	3216.8	3063.6	2916.6	59	99	11.1	7.6	5.2	3.5	2.3	1.5	104
55	3523.2	3373.2	3218.7	3066.6	2919.9	2778.9	60	100	7.6	5.2	3.5	2.3	1.5	1.0	105
56	3361.2	3216.5	3068.3	2922.6	2781.8	2646.4	61	101	5.2	3.5	2.3	1.5	1.0	0.6	106
57	3206.0	3066.4	2924.1	2784.2	2649.0	2518.7	62	102	3.5	2.3	1.5	1.0	0.6	0.4	107
58	3057.3	2922.5	2785.6	2651.2	2521.1	2395.4	63	103	2.3	1.5	1.0	0.6	0.4	0.3	108
59	2914.4	2784.1	2652.4	2523.0	2397.6	2276.1	64	104	1.5	1.0	0.6	0.4	0.3	0.2	109
60	2777.1	2651.1	2524.1	2399.3	2278.1	2160.5	65	105	1.0	0.6	0.4	0.3	0.2	0.1	110

EXHIBIT IV-C (cont'd)

SELECT D_X FOR LIFE UNREMARRIED @ 3.5%/6.0%

AGE	0	1	2	3	4	ULT	AGE
16	100000.0	95558.6	82686.9	70769.9	61867.2	55450.8	21
17	85888.9	82612.7	72603.7	63149.3	55970.5	50731.1	22
18	75011.7	72580.4	64691.9	57098.0	51246.5	46926.7	23
19	66496.6	64688.5	58398.6	52233.9	47419.3	43829.8	24
20	59724.0	58383.6	53320.5	48271.5	44280.6	41280.0	25
21	54271.1	53287.0	49179.2	45013.6	41685.6	39165.9	26
22	49828.5	49118.8	45765.1	42308.9	39521.3	37398.6	27
23	46181.6	45686.9	42936.0	40054.7	37711.6	35919.7	28
24	43175.0	42850.8	40586.6	38176.2	36201.6	34687.1	29
25	40685.0	40497.3	38629.7	36608.8	34942.8	33663.1	30
26	38615.8	38539.2	36998.6	35301.7	33895.8	32816.8	31
27	36894.6	36908.8	35639.6	34215.2	33030.5	32123.5	32
28	35462.2	35551.3	34509.4	33315.6	32320.4	31562.7	33
29	34271.4	34422.8	33572.9	32576.2	31744.5	31116.6	34
30	33284.2	33488.5	32801.9	31974.5	31284.9	30770.9	35
31	32471.7	32720.5	32174.0	31493.2	30927.5	30514.1	36
32	31808.7	32096.1	31670.4	31117.1	30660.4	30336.3	37
33	31275.6	31596.3	31275.9	30834.5	30474.2	30229.1	38
34	30855.8	31205.7	30977.1	30634.2	30359.3	30184.9	39
35	30535.2	30910.5	30763.0	30507.1	30308.1	30196.9	40
36	30301.5	30699.6	30623.2	30444.4	30313.4	30260.0	41
37	30146.2	30564.2	30550.7	30440.5	30370.7	30369.4	42
38	30058.0	30493.9	30536.4	30488.1	30474.1	30519.6	43
39	30031.4	30482.9	30575.9	30583.6	30619.0	30706.4	44
40	30058.4	30524.8	30663.1	30720.5	30800.4	30925.3	45
41	30134.6	30614.5	30793.1	30894.9	31014.1	31172.9	46
42	30255.5	30746.6	30960.9	31101.9	31257.0	31446.4	47
43	30414.9	30916.1	31162.0	31338.4	31525.7	31742.6	48
44	30610.1	31119.2	31393.1	31601.3	31817.5	32059.2	49
45	30836.7	31352.3	31651.4	31888.2	32130.2	32393.7	50
46	31090.5	31612.3	31933.6	32195.7	32460.4	32742.5	51
47	31369.9	31896.1	32236.9	32521.3	32805.7	33103.5	52
48	31671.5	32201.4	32559.3	32862.5	33163.0	33473.3	53
49	31992.4	32524.8	32896.6	33215.8	33529.3	33849.5	54
50	32331.1	32864.1	33247.2	33578.6	33902.3	34229.6	55
51	32684.5	33216.3	33607.6	33948.8	34279.9	34610.8	56
52	33049.2	33578.1	33975.0	34322.7	34657.8	34989.5	57
53	33423.0	33947.0	34346.8	34698.2	35034.2	35363.4	58
54	33803.4	34320.9	34720.9	35072.2	35405.7	35729.6	59
55	34187.5	34696.1	35093.0	35441.2	35769.8	36085.0	60
56	34572.0	35069.3	35460.4	35803.2	36123.0	36426.5	61
57	34954.9	35438.7	35821.2	36154.6	36462.7	36749.2	62
58	35332.9	35801.4	36172.0	36492.7	36783.9	37047.7	63
59	35703.2	36153.1	36508.8	36812.3	37080.8	37314.8	64
60	36062.6	36491.6	36827.4	37107.5	37346.2	37543.7	65

AGE	0	1	2	3	4	ULT	AGE
61	36407.9	36811.5	37122.0	37371.7	37573.4	37727.8	66
62	36734.9	37107.9	37385.6	37597.9	37756.6	37860.7	67
63	37037.0	37372.8	37611.5	37780.0	37888.4	37934.3	68
64	37308.8	37600.9	37793.7	37911.2	37960.9	37940.1	69
65	37542.6	37784.6	37924.9	37983.4	37966.3	37869.7	70
66	37731.3	37916.9	37996.3	37987.2	37894.3	37717.1	71
67	37869.1	37990.2	38000.6	37915.2	37741.3	37474.8	72
68	37946.9	37995.6	37928.1	37761.4	37498.1	37128.9	73
69	37956.8	37924.7	37774.2	37517.6	37151.2	36661.7	74
70	37889.8	37771.6	37530.0	37169.8	36683.0	36057.4	75
71	37740.6	37528.5	37182.0	36700.6	36077.6	35306.8	76
72	37500.3	37180.9	36712.4	36094.6	35325.9	34406.6	77
73	37156.7	36712.4	36105.8	35341.8	34424.5	33353.8	78
74	36691.1	36106.5	35352.8	34439.7	33370.5	32149.1	79
75	36087.7	35353.9	34450.0	33384.5	32164.6	30795.0	80
76	35337.2	34451.0	33393.9	32177.4	30809.1	29289.6	81
77	34436.9	33395.9	32186.8	30821.2	29302.5	27638.0	82
78	33383.5	32188.4	30829.5	29313.4	27649.9	25860.7	83
79	32177.8	30831.3	29321.0	27659.6	25871.0	23984.7	84
80	30822.4	29323.1	27666.5	25879.8	23993.8	22035.2	85
81	29315.4	27668.5	25886.1	24001.5	22043.2	20017.6	86
82	27661.8	25887.6	24006.8	22049.8	20024.4	17943.2	87
83	25882.4	24008.7	22054.6	20030.2	17949.2	15859.6	88
84	24004.4	22056.4	20034.4	17954.0	15864.3	13827.4	89
85	22052.6	20035.6	17957.4	15868.5	13831.4	11893.4	90
86	20033.0	17958.8	15871.5	13834.8	11896.7	10076.9	91
87	17956.7	15872.7	13837.3	11899.5	10079.5	8387.9	92
88	15870.8	13838.2	11901.4	10081.6	8389.9	6851.0	93
89	13836.9	11902.3	10083.3	8391.6	6852.6	5490.1	94
90	11901.4	10084.0	8393.0	6854.0	5491.3	4318.8	95
91	10083.1	8393.4	6854.9	5492.2	4319.7	3335.6	96
92	8392.8	6855.3	5493.0	4320.4	3336.2	2532.8	97
93	6854.9	5493.3	4320.9	3336.7	2533.3	1894.1	98
94	5492.9	4321.1	3337.1	2533.6	1894.4	1396.7	99
95	4320.9	3337.3	2533.9	1894.7	1396.9	1016.3	100
96	3337.1	2534.0	1894.8	1397.1	1016.5	730.3	101
97	2533.9	1894.9	1397.2	1016.6	730.4	518.6	102
98	1894.8	1397.3	1016.7	730.5	518.7	364.3	103
99	1397.3	1016.7	730.5	518.7	364.3	253.2	104
100	1016.7	730.6	518.8	364.3	253.2	174.4	105
101	730.6	518.8	364.4	253.3	174.4	119.0	106
102	518.8	364.4	253.3	174.4	119.0	80.5	107
103	364.4	253.3	174.4	119.0	80.6	54.1	108
104	253.3	174.4	119.0	80.6	54.1	36.1	109
105	174.4	119.0	80.6	54.1	36.1	23.9	110

EXHIBIT IV-D

SELECT N_X FOR LIFE UNREMARRIED @ 3.5%/0.0%

AGE	0	1	2	3	4	ULT	AGE	AGE	0	1	2	3	4	ULT	AGE
16	852344.	752344.	662195.	588604.	529184.	480179.	21	61	36769.	34124.	31601.	29201.	26921.	24759.	66
17	746100.	665072.	591547.	530588.	480568.	438743.	22	62	34122.	31604.	29205.	26924.	24761.	22711.	67
18	660716.	593955.	533016.	481774.	439107.	402980.	23	63	31602.	29208.	26928.	24764.	22713.	20772.	68
19	590840.	535008.	483769.	440130.	403307.	371771.	24	64	29206.	26930.	24767.	22715.	20773.	18939.	69
20	532681.	485374.	441746.	404157.	372054.	344272.	25	65	26930.	24769.	22718.	20776.	18940.	17210.	70
21	483580.	443026.	405461.	372754.	344512.	319838.	26	66	24769.	22720.	20778.	18942.	17211.	15581.	71
22	441587.	406459.	373793.	345079.	320036.	297968.	27	67	22720.	20780.	18945.	17213.	15582.	14051.	72
23	405291.	374577.	345913.	320499.	298133.	278267.	28	68	20780.	18947.	17215.	15584.	14052.	12617.	73
24	373633.	346544.	321181.	298518.	278407.	260416.	29	69	18947.	17217.	15586.	14054.	12618.	11277.	74
25	345774.	321692.	299079.	278729.	260536.	244153.	30	70	17217.	15588.	14056.	12619.	11277.	10028.	75
26	321059.	299496.	279194.	260807.	244256.	229264.	31	71	15588.	14057.	12621.	11279.	10028.	8869.	76
27	298974.	279538.	261196.	244486.	229353.	215571.	32	72	14058.	12622.	11280.	10030.	8870.	7799.	77
28	279105.	261481.	244813.	229549.	215648.	202925.	33	73	12623.	11281.	10031.	8871.	7799.	6815.	78
29	261118.	245050.	229825.	215816.	202993.	191204.	34	74	11282.	10032.	8872.	7800.	6815.	5915.	79
30	244745.	230023.	216049.	203137.	191263.	180302.	35	75	10033.	8873.	7801.	6816.	5915.	5097.	80
31	229766.	216216.	203336.	191388.	180354.	170132.	36	76	8873.	7802.	6817.	5916.	5097.	4357.	81
32	215999.	203477.	191558.	180462.	170178.	160618.	37	77	7803.	6818.	5917.	5098.	4358.	3694.	82
33	203294.	191679.	180610.	170273.	160658.	151694.	38	78	6818.	5918.	5098.	4358.	3694.	3103.	83
34	191524.	180714.	170400.	160742.	151730.	143306.	39	79	5918.	5099.	4359.	3694.	3103.	2582.	84
35	180583.	170491.	160853.	151804.	143338.	135403.	40	80	5099.	4359.	3695.	3104.	2582.	2126.	85
36	170379.	160931.	151900.	143402.	135432.	127945.	41	81	4359.	3695.	3104.	2582.	2126.	1730.	86
37	160837.	151969.	143487.	135489.	127971.	120895.	42	82	3696.	3104.	2583.	2126.	1730.	1391.	87
38	151888.	143546.	135563.	128021.	120918.	114219.	43	83	3105.	2583.	2126.	1731.	1391.	1105.	88
39	143477.	135615.	128087.	120962.	114240.	107890.	44	84	2583.	2127.	1731.	1392.	1105.	866.	89
40	135555.	128132.	121019.	114279.	107909.	101883.	45	85	2127.	1731.	1392.	1105.	866.	669.	90
41	128080.	121059.	114329.	107944.	101900.	96176.	46	86	1731.	1392.	1105.	866.	669.	510.	91
42	121014.	114363.	107987.	101931.	96190.	90748.	47	87	1392.	1105.	866.	670.	510.	382.	92
43	114324.	108017.	101969.	96218.	90761.	85583.	48	88	1105.	866.	670.	510.	382.	282.	93
44	107983.	101994.	96251.	90785.	85595.	80664.	49	89	866.	670.	510.	383.	282.	205.	94
45	101965.	96274.	90815.	85616.	80675.	75978.	50	90	670.	510.	383.	282.	205.	147.	95
46	96247.	90834.	85642.	80694.	75987.	71510.	51	91	510.	383.	282.	205.	147.	104.	96
47	90811.	85659.	80716.	76004.	71518.	67250.	52	92	383.	282.	205.	147.	104.	72.	97
48	85639.	80731.	76023.	71533.	67258.	63187.	53	93	282.	205.	147.	104.	72.	50.	98
49	80713.	76036.	71551.	67271.	63194.	59311.	54	94	205.	147.	104.	72.	50.	34.	99
50	76021.	71562.	67286.	63205.	59317.	55613.	55	95	147.	104.	72.	50.	34.	23.	100
51	71548.	67296.	63219.	59327.	55619.	52086.	56	96	104.	72.	50.	34.	23.	15.	101
52	67284.	63227.	59339.	55628.	52091.	48721.	57	97	72.	50.	34.	23.	15.	10.	102
53	63217.	59346.	55638.	52099.	48725.	45512.	58	98	50.	34.	23.	15.	10.	6.	103
54	59337.	55645.	52108.	48732.	45515.	42452.	59	99	34.	23.	15.	10.	6.	4.	104
55	55637.	52114.	48741.	45522.	42455.	39535.	60	100	23.	15.	10.	6.	4.	3.	105
56	52107.	48746.	45529.	42461.	39538.	36756.	61	101	15.	10.	6.	4.	3.	2.	106
57	48740.	45534.	42467.	39543.	36759.	34110.	62	102	10.	6.	4.	3.	2.	1.	107
58	45529.	42472.	39549.	36764.	34112.	31591.	63	103	6.	4.	3.	2.	1.	1.	108
59	42467.	39553.	36769.	34117.	31593.	29196.	64	104	4.	3.	2.	1.	1.	0.	109
60	39549.	36772.	34121.	31597.	29198.	26920.	65	105	3.	2.	1.	1.	0.	0.	110

EXHIBIT IV-D (cont'd)

SELECT N_X FOR LIFE UNREMARRIED @ 3.5%/6.0%

AGE	0	1	2	3	4	ULT	AGE	AGE	0	1	2	3	4	ULT	AGE
16	2777029.	2677029.	2581471.	2498784.	2428014.	2366147.	21	61	981675.	945267.	908455.	871333.	833961.	796388.	66
17	2670921.	2585032.	2502420.	2429816.	2366667.	2310696.	22	62	945243.	908508.	871400.	834015.	796417.	758660.	67
18	2580593.	2505582.	2433001.	2368310.	2311212.	2259965.	23	63	908489.	871452.	834079.	796468.	758688.	720799.	68
19	2502275.	2435778.	2371090.	2312691.	2260458.	2213038.	24	64	871441.	834132.	796531.	758737.	720826.	682865.	69
20	2433189.	2373465.	2315081.	2261761.	2213489.	2169208.	25	65	834127.	796584.	758800.	720875.	682891.	644925.	70
21	2371365.	2317094.	2263807.	2214628.	2169614.	2127928.	26	66	796581.	758850.	720933.	682937.	644950.	607055.	71
22	2315305.	2265477.	2216358.	2170593.	2128284.	2088763.	27	67	758855.	720986.	682995.	644995.	607080.	569338.	72
23	2263935.	2217753.	2172066.	2129130.	2089075.	2051364.	28	68	720993.	683047.	645051.	607123.	569362.	531863.	73
24	2216434.	2173259.	2130409.	2089822.	2051646.	2015444.	29	69	683059.	645102.	607178.	569403.	531886.	494735.	74
25	2172121.	2131436.	2090938.	2052309.	2015700.	1980757.	30	70	645117.	607227.	569456.	531926.	494756.	458073.	75
26	2130445.	2091829.	2053290.	2016291.	1980990.	1947094.	31	71	607245.	569504.	531976.	494794.	458093.	422015.	76
27	2090966.	2054071.	2017162.	1981523.	1947308.	1914277.	32	72	569523.	532022.	494842.	458129.	422035.	386709.	77
28	2053313.	2017850.	1982299.	1947790.	1914474.	1882154.	33	73	532043.	494887.	458174.	422068.	386727.	352302.	78
29	2017179.	1982907.	1948484.	1914912.	1882335.	1850591.	34	74	494909.	458218.	422111.	386758.	352319.	318948.	79
30	1982308.	1949024.	1915536.	1882734.	1850759.	1819474.	35	75	458240.	422152.	386798.	352348.	318964.	286799.	80
31	1948490.	1916019.	1883298.	1851124.	1819631.	1788703.	36	76	422173.	386836.	352385.	318991.	286813.	256004.	81
32	1915542.	1883733.	1851637.	1819967.	1788850.	1758189.	37	77	386858.	352421.	319025.	286838.	256017.	226714.	82
33	1883309.	1852034.	1820438.	1789162.	1758327.	1727853.	38	78	352441.	319058.	286869.	256040.	226726.	199076.	83
34	1851656.	1820800.	1789594.	1758617.	1727983.	1697624.	39	79	319076.	286899.	256067.	226746.	199087.	173216.	84
35	1820463.	1789928.	1759017.	1728254.	1697747.	1667439.	40	80	286917.	256094.	226771.	199105.	173225.	149231.	85
36	1789624.	1759323.	1728623.	1698000.	1667556.	1637242.	41	81	256110.	226795.	199127.	173240.	149239.	127196.	86
37	1759054.	1728908.	1698344.	1667793.	1637353.	1606982.	42	82	226809.	199147.	173259.	149252.	127203.	107178.	87
38	1728663.	1698605.	1668111.	1637575.	1607087.	1576613.	43	83	199160.	173278.	149269.	127214.	107184.	89235.	88
39	1698386.	1668355.	1637872.	1607296.	1576712.	1546093.	44	84	173289.	149285.	127228.	107194.	89240.	73375.	89
40	1668154.	1638096.	1607571.	1576908.	1546187.	1515387.	45	85	149294.	127241.	107205.	89248.	73379.	59548.	90
41	1637913.	1607778.	1577164.	1546370.	1515476.	1484461.	46	86	127250.	107217.	89258.	73386.	59551.	47655.	91
42	1607610.	1577355.	1546608.	1515647.	1484546.	1453289.	47	87	107223.	89267.	73394.	59557.	47657.	37578.	92
43	1577199.	1546784.	1515868.	1484706.	1453368.	1421842.	48	88	89272.	73401.	59563.	47661.	37580.	29190.	93
44	1546641.	1516031.	1484911.	1453518.	1421917.	1390100.	49	89	73406.	59569.	47666.	37583.	29191.	22339.	94
45	1515899.	1485062.	1453710.	1422059.	1390170.	1358040.	50	90	59572.	47671.	37587.	29194.	22340.	16849.	95
46	1484939.	1453849.	1422236.	1390303.	1358107.	1325647.	51	91	47673.	37590.	29197.	22342.	16850.	12530.	96
47	1453734.	1422364.	1390468.	1358231.	1325710.	1292904.	52	92	37592.	29199.	22344.	16851.	12531.	9194.	97
48	1422258.	1390587.	1358385.	1325826.	1292964.	1259801.	53	93	29201.	22346.	16853.	12532.	9195.	6662.	98
49	1390486.	1358494.	1325969.	1293072.	1259857.	1226327.	54	94	22347.	16854.	12533.	9196.	6662.	4767.	99
50	1358401.	1326070.	1293206.	1259959.	1226380.	1192478.	55	95	16854.	12534.	9196.	6662.	4768.	3371.	100
51	1325985.	1293301.	1260084.	1226477.	1192528.	1158248.	56	96	12534.	9197.	6663.	4768.	3371.	2354.	101
52	1293220.	1260171.	1226593.	1192618.	1158295.	1123637.	57	97	9197.	6663.	4768.	3371.	2355.	1624.	102
53	1260097.	1226674.	1192727.	1158380.	1123682.	1088648.	58	98	6664.	4769.	3371.	2355.	1624.	1106.	103
54	1226608.	1192804.	1158483.	1123762.	1088690.	1053285.	59	99	4769.	3372.	2355.	1624.	1106.	741.	104
55	1192742.	1158555.	1123859.	1088766.	1053325.	1017555.	60	100	3372.	2355.	1624.	1106.	741.	488.	105
56	1158498.	1123926.	1088857.	1053396.	1017593.	981470.	61	101	2355.	1624.	1106.	741.	488.	314.	106
57	1123875.	1088921.	1053482.	1017661.	981506.	945043.	62	102	1625.	1106.	741.	488.	314.	195.	107
58	1088877.	1053544.	1017743.	981571.	945078.	908294.	63	103	1106.	741.	488.	314.	195.	114.	108
59	1053505.	1017802.	981648.	945140.	908327.	871247.	64	104	741.	488.	314.	195.	114.	60.	109
60	1017767.	981705.	945213.	908386.	871278.	833932.	65	105	488.	314.	195.	114.	60.	24.	110

EXHIBIT IV-E

SELECT ANNUITY FOR LIFE UNREMARRIED @ 3.5%/0.0%

AGE	0	1	2	3	4	ULT	AGE	AGE	0	1	2	3	4	ULT	AGE
16	8.023	7.846	8.498	9.406	10.299	11.088	21	61	13.401	13.025	12.666	12.310	11.951	11.588	66
17	8.708	8.546	9.204	10.107	10.990	11.768	22	62	13.053	12.672	12.307	11.944	11.580	11.212	67
18	9.397	9.247	9.902	10.791	11.655	12.412	23	63	12.697	12.312	11.941	11.574	11.205	10.833	68
19	10.082	9.941	10.586	11.453	12.289	13.019	24	64	12.334	11.946	11.571	11.199	10.826	10.451	69
20	10.760	10.625	11.252	12.089	12.892	13.590	25	65	11.965	11.575	11.196	10.820	10.445	10.068	70
21	11.424	11.294	11.897	12.699	13.463	14.124	26	66	11.592	11.200	10.818	10.440	10.062	9.683	71
22	12.071	11.943	12.518	13.280	14.002	14.624	27	67	11.214	10.821	10.437	10.057	9.677	9.297	72
23	12.696	12.568	13.111	13.830	14.507	15.088	28	68	10.834	10.440	10.055	9.673	9.292	8.912	73
24	13.293	13.163	13.672	14.344	14.975	15.513	29	69	10.451	10.057	9.671	9.288	8.907	8.530	74
25	13.859	13.726	14.197	14.820	15.403	15.898	30	70	10.067	9.673	9.286	8.903	8.526	8.155	75
26	14.389	14.252	14.684	15.258	15.792	16.243	31	71	9.681	9.287	8.901	8.522	8.151	7.787	76
27	14.883	14.740	15.132	15.655	16.141	16.547	32	72	9.295	8.903	8.521	8.147	7.783	7.426	77
28	15.337	15.188	15.539	16.013	16.450	16.813	33	73	8.909	8.522	8.146	7.780	7.422	7.073	78
29	15.751	15.595	15.906	16.330	16.719	17.039	34	74	8.527	8.147	7.778	7.420	7.070	6.729	79
30	16.125	15.961	16.232	16.607	16.950	17.229	35	75	8.152	7.779	7.418	7.068	6.726	6.393	80
31	16.458	16.287	16.518	16.846	17.144	17.381	36	76	7.783	7.419	7.066	6.724	6.390	6.067	81
32	16.750	16.571	16.764	17.047	17.301	17.499	37	77	7.423	7.067	6.722	6.388	6.065	5.754	82
33	17.003	16.816	16.972	17.210	17.422	17.583	38	78	7.070	6.723	6.387	6.063	5.752	5.452	83
34	17.217	17.021	17.142	17.338	17.510	17.635	39	79	6.726	6.388	6.062	5.750	5.450	5.160	84
35	17.393	17.189	17.276	17.431	17.565	17.655	40	80	6.390	6.063	5.749	5.449	5.158	4.876	85
36	17.533	17.321	17.375	17.492	17.590	17.647	41	81	6.065	5.750	5.448	5.157	4.875	4.606	86
37	17.637	17.417	17.440	17.522	17.584	17.610	42	82	5.752	5.448	5.156	4.874	4.605	4.356	87
38	17.709	17.481	17.475	17.522	17.551	17.548	43	83	5.450	5.156	4.873	4.604	4.355	4.124	88
39	17.749	17.513	17.479	17.493	17.492	17.460	44	84	5.158	4.873	4.604	4.354	4.123	3.906	89
40	17.760	17.516	17.455	17.439	17.409	17.351	45	85	4.875	4.604	4.353	4.122	3.905	3.698	90
41	17.742	17.490	17.404	17.360	17.303	17.220	46	86	4.605	4.354	4.122	3.905	3.697	3.501	91
42	17.696	17.437	17.329	17.258	17.175	17.069	47	87	4.355	4.122	3.904	3.696	3.500	3.321	92
43	17.626	17.360	17.230	17.134	17.027	16.899	48	88	4.123	3.905	3.696	3.499	3.320	3.161	93
44	17.532	17.259	17.110	16.990	16.861	16.712	49	89	3.905	3.696	3.499	3.320	3.161	3.020	94
45	17.416	17.137	16.968	16.827	16.676	16.507	50	90	3.697	3.499	3.320	3.160	3.020	2.896	95
46	17.280	16.993	16.807	16.645	16.474	16.287	51	91	3.500	3.320	3.160	3.019	2.895	2.788	96
47	17.124	16.831	16.628	16.446	16.256	16.051	52	92	3.320	3.160	3.019	2.895	2.788	2.694	97
48	16.950	16.650	16.431	16.231	16.023	15.802	53	93	3.161	3.019	2.895	2.787	2.694	2.610	98
49	16.758	16.451	16.217	16.000	15.777	15.540	54	94	3.020	2.895	2.787	2.693	2.610	2.533	99
50	16.549	16.236	15.988	15.756	15.517	15.265	55	95	2.895	2.787	2.693	2.609	2.533	2.461	100
51	16.325	16.006	15.745	15.498	15.244	14.979	56	96	2.788	2.693	2.609	2.532	2.461	2.393	101
52	16.087	15.762	15.488	15.227	14.959	14.682	57	97	2.694	2.609	2.532	2.461	2.393	2.326	102
53	15.834	15.504	15.219	14.944	14.664	14.374	58	98	2.610	2.532	2.461	2.393	2.326	2.256	103
54	15.569	15.232	14.936	14.649	14.357	14.055	59	99	2.533	2.461	2.393	2.326	2.256	2.177	104
55	15.292	14.949	14.643	14.344	14.040	13.727	60	100	2.461	2.393	2.326	2.256	2.177	2.082	105
56	15.003	14.655	14.339	14.028	13.713	13.389	61	101	2.393	2.326	2.256	2.177	2.082	1.957	106
57	14.703	14.349	14.023	13.703	13.376	13.043	62	102	2.326	2.256	2.177	2.082	1.957	1.782	107
58	14.392	14.033	13.698	13.367	13.031	12.688	63	103	2.256	2.177	2.082	1.957	1.782	1.523	108
59	14.071	13.707	13.363	13.022	12.677	12.327	64	104	2.177	2.082	1.957	1.782	1.523	1.125	109
60	13.741	13.370	13.018	12.669	12.317	11.960	65	105	2.082	1.957	1.782	1.523	1.125	0.500	110

EXHIBIT IV-E (cont'd)

SELECT ANNUITY FOR LIFE UNREMARRIED @ 3.5%/6.0%

AGE	0	1	2	3	4	ULT	AGE	AGE	0	1	2	3	4	ULT	AGE
16	27.270	27.515	30.720	34.809	38.746	42.171	21	61	26.463	25.179	23.972	22.815	21.696	20.609	66
17	30.597	30.791	33.967	37.977	41.784	45.048	22	62	25.231	23.983	22.808	21.682	20.593	19.538	67
18	33.903	34.021	37.109	40.978	44.600	47.659	23	63	24.029	22.818	21.676	20.582	19.524	18.501	68
19	37.130	37.154	40.102	43.776	47.170	49.992	24	64	22.857	21.684	20.576	19.514	18.489	17.499	69
20	40.241	40.153	42.918	46.355	49.488	52.049	25	65	21.718	20.582	19.508	18.479	17.487	16.530	70
21	43.195	42.983	45.532	48.699	51.547	53.831	26	66	20.612	19.514	18.474	17.478	16.520	15.595	71
22	45.965	45.622	47.929	50.803	53.352	55.351	27	67	19.539	18.478	17.473	16.512	15.585	14.693	72
23	48.522	48.042	50.088	52.656	54.896	56.610	28	68	18.500	17.477	16.507	15.578	14.684	13.825	73
24	50.836	50.217	51.991	54.242	56.173	57.603	29	69	17.496	16.510	15.574	14.677	13.817	12.995	74
25	52.889	52.132	53.628	55.561	57.186	58.341	30	70	16.526	15.576	14.673	13.811	12.987	12.204	75
26	54.670	53.778	54.996	56.616	57.944	58.832	31	71	15.590	14.675	13.807	12.982	12.197	11.453	76
27	56.174	55.153	56.099	57.414	58.455	59.091	32	72	14.687	13.809	12.979	12.192	11.447	10.739	77
28	57.401	56.259	56.942	57.965	58.734	59.132	33	73	13.819	12.980	12.190	11.442	10.734	10.063	78
29	58.359	57.104	57.537	58.283	58.796	58.973	34	74	12.989	12.191	11.440	10.730	10.058	9.421	79
30	59.057	57.700	57.897	58.382	58.658	58.630	35	75	12.198	11.441	10.728	10.054	9.417	8.813	80
31	59.506	58.057	58.035	58.279	58.335	58.119	36	76	11.447	10.729	10.052	9.413	8.809	8.240	81
32	59.721	58.190	57.966	57.988	57.844	57.457	37	77	10.734	10.053	9.412	8.807	8.237	7.703	82
33	59.717	58.116	57.706	57.525	57.199	56.659	38	78	10.057	9.412	8.805	8.235	7.700	7.198	83
34	59.510	57.848	57.271	56.907	56.418	55.741	39	79	9.416	8.805	8.233	7.698	7.195	6.722	84
35	59.118	57.407	56.680	56.151	55.516	54.719	40	80	8.809	8.234	7.697	7.193	6.720	6.272	85
36	58.561	56.808	55.948	55.274	54.510	53.606	41	81	8.236	7.697	7.192	6.718	6.270	5.854	86
37	57.851	56.066	55.091	54.289	53.412	52.415	42	82	7.699	7.193	6.717	6.269	5.852	5.473	87
38	57.011	55.203	54.127	53.212	52.236	51.159	43	83	7.195	6.717	6.268	5.851	5.472	5.127	88
39	56.054	54.231	53.067	52.054	50.995	49.851	44	84	6.719	6.268	5.850	5.470	5.125	4.807	89
40	54.997	53.164	51.927	50.831	49.700	48.502	45	85	6.270	5.851	5.470	5.124	4.805	4.507	90
41	53.853	52.017	50.718	49.553	48.364	47.120	46	86	5.852	5.470	5.124	4.804	4.506	4.229	91
42	52.635	50.802	49.454	48.232	46.995	45.715	47	87	5.471	5.124	4.804	4.505	4.228	3.980	92
43	51.356	49.532	48.145	46.877	45.601	44.293	48	88	5.125	4.804	4.505	4.228	3.979	3.761	93
44	50.027	48.217	46.801	45.495	44.190	42.860	49	89	4.805	4.505	4.227	3.979	3.760	3.569	94
45	48.659	46.867	45.429	44.095	42.767	41.423	50	90	4.506	4.227	3.978	3.759	3.568	3.401	95
46	47.262	45.490	44.037	42.683	41.339	39.987	51	91	4.228	3.979	3.759	3.568	3.401	3.256	96
47	45.842	44.094	42.633	41.264	39.911	38.556	52	92	3.979	3.759	3.568	3.400	3.256	3.130	97
48	44.407	42.684	41.220	39.845	38.488	37.136	53	93	3.760	3.568	3.400	3.256	3.130	3.017	98
49	42.963	41.268	39.807	38.429	37.075	35.729	54	94	3.568	3.400	3.256	3.129	3.017	2.913	99
50	41.515	39.850	38.397	37.023	35.674	34.338	55	95	3.401	3.256	3.129	3.016	2.913	2.817	100
51	40.069	38.436	36.994	35.627	34.288	32.965	56	96	3.256	3.129	3.016	2.913	2.816	2.724	101
52	38.630	37.030	35.603	34.247	32.921	31.614	57	97	3.130	3.016	2.913	2.816	2.724	2.632	102
53	37.201	35.635	34.226	32.884	31.574	30.285	58	98	3.017	2.913	2.816	2.723	2.631	2.535	103
54	35.786	34.254	32.866	31.541	30.249	28.979	59	99	2.913	2.816	2.723	2.631	2.535	2.427	104
55	34.388	32.892	31.525	30.220	28.947	27.699	60	100	2.816	2.723	2.631	2.535	2.427	2.299	105
56	33.010	31.549	30.206	28.922	27.670	26.444	61	101	2.724	2.631	2.535	2.427	2.299	2.136	106
57	31.652	30.227	28.909	27.647	26.418	25.216	62	102	2.631	2.535	2.427	2.299	2.136	1.917	107
58	30.318	28.927	27.636	26.398	25.193	24.017	63	103	2.535	2.427	2.299	2.136	1.917	1.609	108
59	29.007	27.653	26.388	25.175	23.996	22.849	64	104	2.427	2.299	2.136	1.917	1.609	1.163	109
60	27.722	26.402	25.166	23.980	22.830	21.712	65	105	2.299	2.136	1.917	1.609	1.163	0.500	110

EXHIBIT IV-F

SELECT DOWRIES PAYABLE ON REMARRIAGE @ 3.5%/0.0%

AGE	0	1	2	3	4	ULT	AGE	AGE	0	1	2	3	4	ULT	AGE
16	0.7000	0.7041	0.6765	0.6386	0.6005	0.5658	21	61	0.0189	0.0190	0.0176	0.0155	0.0132	0.0110	66
17	0.6715	0.6748	0.6464	0.6080	0.5697	0.5349	22	62	0.0170	0.0171	0.0157	0.0138	0.0118	0.0097	67
18	0.6424	0.6449	0.6161	0.5776	0.5394	0.5047	23	63	0.0152	0.0153	0.0141	0.0123	0.0104	0.0086	68
19	0.6126	0.6147	0.5858	0.5474	0.5096	0.4753	24	64	0.0136	0.0137	0.0126	0.0110	0.0093	0.0075	69
20	0.5830	0.5844	0.5555	0.5177	0.4805	0.4469	25	65	0.0122	0.0123	0.0113	0.0098	0.0082	0.0066	70
21	0.5531	0.5540	0.5255	0.4884	0.4521	0.4193	26	66	0.0109	0.0110	0.0100	0.0087	0.0072	0.0057	71
22	0.5233	0.5238	0.4958	0.4596	0.4244	0.3925	27	67	0.0097	0.0098	0.0090	0.0077	0.0064	0.0050	72
23	0.4937	0.4940	0.4666	0.4315	0.3974	0.3666	28	68	0.0086	0.0087	0.0080	0.0068	0.0056	0.0043	73
24	0.4647	0.4647	0.4382	0.4043	0.3714	0.3418	29	69	0.0077	0.0078	0.0071	0.0060	0.0049	0.0037	74
25	0.4363	0.4361	0.4105	0.3780	0.3464	0.3180	30	70	0.0068	0.0069	0.0063	0.0053	0.0043	0.0032	75
26	0.4088	0.4084	0.3838	0.3527	0.3226	0.2954	31	71	0.0061	0.0062	0.0056	0.0047	0.0038	0.0028	76
27	0.3821	0.3816	0.3582	0.3285	0.2998	0.2740	32	72	0.0054	0.0055	0.0050	0.0042	0.0033	0.0024	77
28	0.3565	0.3559	0.3336	0.3055	0.2783	0.2537	33	73	0.0048	0.0049	0.0044	0.0037	0.0029	0.0020	78
29	0.3320	0.3314	0.3102	0.2836	0.2579	0.2346	34	74	0.0042	0.0043	0.0039	0.0032	0.0025	0.0017	79
30	0.3086	0.3080	0.2880	0.2629	0.2386	0.2166	35	75	0.0037	0.0038	0.0035	0.0029	0.0022	0.0015	80
31	0.2864	0.2857	0.2670	0.2434	0.2205	0.1998	36	76	0.0033	0.0034	0.0031	0.0025	0.0019	0.0012	81
32	0.2654	0.2647	0.2472	0.2250	0.2035	0.1840	37	77	0.0029	0.0030	0.0027	0.0022	0.0017	0.0010	82
33	0.2455	0.2449	0.2285	0.2078	0.1876	0.1694	38	78	0.0026	0.0027	0.0024	0.0020	0.0015	0.0009	83
34	0.2269	0.2264	0.2110	0.1916	0.1728	0.1557	39	79	0.0023	0.0024	0.0021	0.0017	0.0013	0.0007	84
35	0.2094	0.2089	0.1947	0.1766	0.1591	0.1430	40	80	0.0020	0.0021	0.0019	0.0015	0.0011	0.0006	85
36	0.1931	0.1926	0.1794	0.1626	0.1463	0.1313	41	81	0.0018	0.0019	0.0017	0.0014	0.0010	0.0005	86
37	0.1779	0.1775	0.1652	0.1496	0.1344	0.1204	42	82	0.0015	0.0016	0.0015	0.0012	0.0008	0.0004	87
38	0.1637	0.1633	0.1520	0.1375	0.1234	0.1104	43	83	0.0014	0.0015	0.0013	0.0011	0.0007	0.0004	88
39	0.1505	0.1502	0.1397	0.1263	0.1132	0.1011	44	84	0.0012	0.0013	0.0012	0.0009	0.0006	0.0003	89
40	0.1382	0.1380	0.1283	0.1160	0.1038	0.0926	45	85	0.0010	0.0011	0.0010	0.0008	0.0006	0.0002	90
41	0.1269	0.1267	0.1178	0.1064	0.0951	0.0847	46	86	0.0009	0.0010	0.0009	0.0007	0.0005	0.0002	91
42	0.1164	0.1162	0.1081	0.0975	0.0871	0.0774	47	87	0.0008	0.0009	0.0008	0.0007	0.0004	0.0002	92
43	0.1066	0.1065	0.0990	0.0893	0.0797	0.0707	48	88	0.0007	0.0008	0.0007	0.0006	0.0004	0.0001	93
44	0.0976	0.0976	0.0907	0.0817	0.0728	0.0645	49	89	0.0006	0.0007	0.0007	0.0005	0.0003	0.0001	94
45	0.0893	0.0893	0.0830	0.0748	0.0665	0.0588	50	90	0.0005	0.0006	0.0006	0.0005	0.0003	0.0001	95
46	0.0817	0.0817	0.0759	0.0683	0.0607	0.0536	51	91	0.0005	0.0005	0.0005	0.0004	0.0003	0.0001	96
47	0.0746	0.0747	0.0694	0.0624	0.0554	0.0488	52	92	0.0004	0.0005	0.0005	0.0004	0.0002	0.0001	97
48	0.0681	0.0682	0.0633	0.0569	0.0505	0.0444	53	93	0.0004	0.0004	0.0004	0.0003	0.0002	0.0001	98
49	0.0621	0.0622	0.0578	0.0519	0.0459	0.0403	54	94	0.0003	0.0004	0.0004	0.0003	0.0002	0.0001	99
50	0.0566	0.0567	0.0526	0.0472	0.0417	0.0365	55	95	0.0003	0.0003	0.0003	0.0003	0.0002	0.0000	100
51	0.0515	0.0516	0.0479	0.0430	0.0379	0.0331	56	96	0.0002	0.0003	0.0003	0.0003	0.0002	0.0000	101
52	0.0468	0.0470	0.0436	0.0390	0.0344	0.0299	57	97	0.0002	0.0003	0.0003	0.0002	0.0001	0.0000	102
53	0.0426	0.0427	0.0396	0.0354	0.0311	0.0270	58	98	0.0002	0.0002	0.0002	0.0002	0.0001	0.0000	103
54	0.0386	0.0388	0.0359	0.0321	0.0282	0.0244	59	99	0.0002	0.0002	0.0002	0.0002	0.0001	0.0000	104
55	0.0350	0.0352	0.0326	0.0291	0.0254	0.0219	60	100	0.0002	0.0002	0.0002	0.0002	0.0001	0.0000	105
56	0.0317	0.0318	0.0295	0.0263	0.0229	0.0197	61	101	0.0001	0.0002	0.0002	0.0002	0.0001	0.0000	106
57	0.0287	0.0288	0.0267	0.0237	0.0206	0.0176	62	102	0.0001	0.0002	0.0002	0.0001	0.0001	0.0000	107
58	0.0259	0.0260	0.0241	0.0214	0.0185	0.0157	63	103	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	108
59	0.0234	0.0235	0.0217	0.0192	0.0166	0.0140	64	104	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	109
60	0.0210	0.0212	0.0195	0.0172	0.0148	0.0125	65	105	0.0001	0.0001	0.0001	0.0001	0.0001	0.0	110

EXHIBIT IV-F (cont'd)

Select Dowries Payable on Remarriage @ 3.5%/6.0%

AGE	0	1	2	3	4	ULT	AGE
16	1.0159	0.9928	0.9666	0.9358	0.9018	0.8659	21
17	0.9916	0.9676	0.9383	0.9042	0.8672	0.8292	22
18	0.9641	0.9393	0.9073	0.8703	0.8310	0.7913	23
19	0.9337	0.9083	0.8740	0.8346	0.7935	0.7526	24
20	0.9006	0.8749	0.8388	0.7976	0.7552	0.7135	25
21	0.8654	0.8396	0.8021	0.7596	0.7164	0.6743	26
22	0.8284	0.8027	0.7642	0.7209	0.6774	0.6354	27
23	0.7901	0.7646	0.7256	0.6820	0.6385	0.5969	28
24	0.7508	0.7258	0.6868	0.6432	0.6001	0.5592	29
25	0.7111	0.6867	0.6479	0.6048	0.5625	0.5225	30
26	0.6714	0.6477	0.6095	0.5672	0.5258	0.4870	31
27	0.6319	0.6090	0.5717	0.5305	0.4904	0.4529	32
28	0.5931	0.5711	0.5349	0.4950	0.4563	0.4203	33
29	0.5551	0.5342	0.4993	0.4608	0.4237	0.3893	34
30	0.5182	0.4984	0.4649	0.4281	0.3926	0.3598	35
31	0.4827	0.4639	0.4319	0.3968	0.3631	0.3320	36
32	0.4485	0.4309	0.4005	0.3672	0.3352	0.3058	37
33	0.4159	0.3994	0.3707	0.3392	0.3090	0.2812	38
34	0.3850	0.3695	0.3425	0.3127	0.2844	0.2583	39
35	0.3557	0.3413	0.3159	0.2880	0.2613	0.2368	40
36	0.3281	0.3147	0.2909	0.2648	0.2398	0.2169	41
37	0.3021	0.2897	0.2675	0.2431	0.2198	0.1984	42
38	0.2778	0.2664	0.2457	0.2229	0.2012	0.1813	43
39	0.2551	0.2445	0.2253	0.2042	0.1840	0.1654	44
40	0.2339	0.2242	0.2064	0.1868	0.1680	0.1508	45
41	0.2142	0.2053	0.1888	0.1707	0.1533	0.1373	46
42	0.1959	0.1878	0.1726	0.1557	0.1397	0.1249	47
43	0.1789	0.1715	0.1575	0.1420	0.1271	0.1134	48
44	0.1632	0.1564	0.1436	0.1293	0.1155	0.1029	49
45	0.1487	0.1426	0.1307	0.1176	0.1049	0.0933	50
46	0.1353	0.1298	0.1189	0.1068	0.0951	0.0844	51
47	0.1230	0.1180	0.1080	0.0969	0.0862	0.0763	52
48	0.1117	0.1071	0.0980	0.0878	0.0780	0.0688	53
49	0.1013	0.0971	0.0888	0.0795	0.0704	0.0620	54
50	0.0917	0.0880	0.0804	0.0718	0.0635	0.0558	55
51	0.0830	0.0796	0.0727	0.0648	0.0572	0.0502	56
52	0.0749	0.0719	0.0656	0.0584	0.0515	0.0450	57
53	0.0676	0.0649	0.0591	0.0526	0.0462	0.0403	58
54	0.0609	0.0585	0.0533	0.0473	0.0414	0.0360	59
55	0.0548	0.0526	0.0479	0.0424	0.0371	0.0321	60
56	0.0492	0.0473	0.0430	0.0380	0.0331	0.0285	61
57	0.0441	0.0424	0.0385	0.0340	0.0295	0.0253	62
58	0.0395	0.0380	0.0345	0.0303	0.0262	0.0224	63
59	0.0353	0.0340	0.0308	0.0270	0.0233	0.0198	64
60	0.0315	0.0303	0.0274	0.0240	0.0206	0.0174	65

AGE	0	1	2	3	4	ULT	AGE
61	0.0281	0.0270	0.0244	0.0213	0.0182	0.0153	66
62	0.0250	0.0240	0.0217	0.0189	0.0160	0.0134	67
63	0.0222	0.0213	0.0192	0.0167	0.0141	0.0117	68
64	0.0197	0.0189	0.0170	0.0147	0.0124	0.0102	69
65	0.0174	0.0168	0.0151	0.0130	0.0108	0.0088	70
66	0.0154	0.0149	0.0133	0.0114	0.0095	0.0076	71
67	0.0136	0.0131	0.0117	0.0100	0.0082	0.0066	72
68	0.0120	0.0116	0.0103	0.0088	0.0072	0.0056	73
69	0.0106	0.0102	0.0091	0.0077	0.0062	0.0048	74
70	0.0093	0.0090	0.0080	0.0067	0.0054	0.0041	75
71	0.0082	0.0079	0.0070	0.0059	0.0047	0.0035	76
72	0.0072	0.0070	0.0062	0.0051	0.0040	0.0030	77
73	0.0063	0.0061	0.0054	0.0045	0.0035	0.0025	78
74	0.0055	0.0054	0.0048	0.0039	0.0030	0.0021	79
75	0.0048	0.0047	0.0042	0.0034	0.0026	0.0018	80
76	0.0042	0.0042	0.0037	0.0030	0.0023	0.0015	81
77	0.0037	0.0037	0.0032	0.0026	0.0019	0.0013	82
78	0.0033	0.0032	0.0028	0.0023	0.0017	0.0011	83
79	0.0028	0.0028	0.0025	0.0020	0.0015	0.0009	84
80	0.0025	0.0025	0.0022	0.0018	0.0013	0.0007	85
81	0.0022	0.0022	0.0019	0.0015	0.0011	0.0006	86
82	0.0019	0.0019	0.0017	0.0013	0.0009	0.0005	87
83	0.0017	0.0017	0.0015	0.0012	0.0008	0.0004	88
84	0.0014	0.0015	0.0013	0.0010	0.0007	0.0003	89
85	0.0013	0.0013	0.0012	0.0009	0.0006	0.0003	90
86	0.0011	0.0012	0.0010	0.0008	0.0005	0.0002	91
87	0.0010	0.0010	0.0009	0.0007	0.0005	0.0002	92
88	0.0008	0.0009	0.0008	0.0006	0.0004	0.0002	93
89	0.0007	0.0008	0.0007	0.0006	0.0004	0.0001	94
90	0.0006	0.0007	0.0006	0.0005	0.0003	0.0001	95
91	0.0005	0.0006	0.0006	0.0004	0.0003	0.0001	96
92	0.0005	0.0005	0.0005	0.0004	0.0003	0.0001	97
93	0.0004	0.0005	0.0004	0.0004	0.0002	0.0001	98
94	0.0004	0.0004	0.0004	0.0003	0.0002	0.0001	99
95	0.0003	0.0004	0.0004	0.0003	0.0002	0.0000	100
96	0.0003	0.0003	0.0003	0.0003	0.0002	0.0000	101
97	0.0002	0.0003	0.0003	0.0002	0.0002	0.0000	102
98	0.0002	0.0003	0.0003	0.0002	0.0001	0.0000	103
99	0.0002	0.0002	0.0002	0.0002	0.0001	0.0000	104
100	0.0002	0.0002	0.0002	0.0002	0.0001	0.0000	105
101	0.0002	0.0002	0.0002	0.0002	0.0001	0.0000	106
102	0.0001	0.0002	0.0002	0.0001	0.0001	0.0000	107
103	0.0001	0.0002	0.0002	0.0001	0.0001	0.0000	108
104	0.0001	0.0002	0.0001	0.0001	0.0001	0.0000	109
105	0.0001	0.0001	0.0001	0.0001	0.0001	0.0	110

EXHIBIT IV-G

AUTOMATIC SURVIVORSHIP BENEFIT @ 3.5%/0.0%, 0.0% BY AGE OF CLAIMANT & AGE DIFFERENCE (SPOUSE-CLAIMANT)

AGE	-5	-4	-3	-2	-1	0	AGE	AGE	-5	-4	-3	-2	-1	0	AGE
16	1.730	1.690	1.650	1.609	1.569	1.529	16	61	4.585	4.363	4.143	3.925	3.711	3.500	61
17	1.784	1.742	1.701	1.659	1.618	1.577	17	62	4.591	4.364	4.139	3.917	3.698	3.483	62
18	1.839	1.796	1.753	1.710	1.667	1.624	18	63	4.592	4.359	4.129	3.902	3.679	3.460	63
19	1.895	1.851	1.806	1.762	1.717	1.671	19	64	4.586	4.348	4.113	3.882	3.654	3.432	64
20	1.953	1.907	1.861	1.815	1.767	1.719	20	65	4.573	4.331	4.091	3.855	3.624	3.398	65
21	2.013	1.966	1.918	1.869	1.818	1.767	21	66	4.554	4.306	4.062	3.822	3.588	3.359	66
22	2.075	2.025	1.975	1.923	1.870	1.816	22	67	4.527	4.275	4.027	3.784	3.546	3.315	67
23	2.138	2.086	2.032	1.977	1.921	1.864	23	68	4.493	4.237	3.985	3.739	3.499	3.265	68
24	2.203	2.147	2.091	2.033	1.974	1.914	24	69	4.453	4.193	3.938	3.689	3.447	3.211	69
25	2.269	2.211	2.151	2.089	2.027	1.964	25	70	4.407	4.144	3.886	3.634	3.390	3.153	70
26	2.337	2.275	2.212	2.148	2.083	2.017	26	71	4.355	4.089	3.829	3.576	3.330	3.092	71
27	2.407	2.342	2.276	2.208	2.140	2.071	27	72	4.300	4.031	3.769	3.514	3.267	3.028	72
28	2.479	2.411	2.341	2.270	2.199	2.126	28	73	4.238	3.967	3.704	3.447	3.199	2.961	73
29	2.552	2.481	2.408	2.334	2.259	2.183	29	74	4.170	3.897	3.632	3.375	3.127	2.890	74
30	2.627	2.552	2.476	2.398	2.320	2.241	30	75	4.093	3.819	3.553	3.297	3.050	2.815	75
31	2.703	2.624	2.545	2.464	2.382	2.299	31	76	4.007	3.733	3.467	3.212	2.968	2.736	76
32	2.780	2.698	2.614	2.530	2.444	2.359	32	77	3.914	3.639	3.376	3.123	2.883	2.654	77
33	2.858	2.772	2.685	2.597	2.508	2.419	33	78	3.813	3.541	3.280	3.031	2.794	2.570	78
34	2.937	2.848	2.757	2.665	2.572	2.479	34	79	3.708	3.439	3.182	2.937	2.704	2.484	79
35	3.017	2.924	2.829	2.733	2.637	2.540	35	80	3.601	3.336	3.083	2.842	2.614	2.398	80
36	3.098	3.000	2.902	2.802	2.702	2.601	36	81	3.492	3.231	2.982	2.746	2.522	2.312	81
37	3.179	3.077	2.975	2.871	2.767	2.663	37	82	3.380	3.124	2.880	2.648	2.430	2.226	82
38	3.260	3.154	3.047	2.940	2.832	2.723	38	83	3.266	3.015	2.776	2.551	2.339	2.141	83
39	3.341	3.231	3.120	3.008	2.896	2.784	39	84	3.152	2.906	2.674	2.455	2.250	2.058	84
40	3.422	3.307	3.192	3.076	2.959	2.843	40	85	3.039	2.800	2.574	2.363	2.164	1.977	85
41	3.502	3.383	3.263	3.143	3.022	2.902	41	86	2.925	2.693	2.475	2.270	2.077	1.899	86
42	3.582	3.458	3.334	3.209	3.084	2.960	42	87	2.807	2.583	2.371	2.173	1.989	1.822	87
43	3.661	3.533	3.404	3.274	3.145	3.016	43	88	2.685	2.468	2.264	2.075	1.902	1.746	88
44	3.739	3.606	3.472	3.338	3.204	3.071	44	89	2.562	2.353	2.158	1.981	1.820	1.673	89
45	3.815	3.677	3.538	3.400	3.261	3.124	45	90	2.442	2.243	2.061	1.896	1.744	1.603	90
46	3.889	3.746	3.602	3.459	3.316	3.174	46	91	2.327	2.141	1.972	1.815	1.670	1.538	91
47	3.960	3.813	3.664	3.516	3.369	3.223	47	92	2.219	2.045	1.885	1.736	1.600	1.479	92
48	4.030	3.877	3.724	3.571	3.419	3.268	48	93	2.114	1.950	1.798	1.658	1.534	1.426	93
49	4.097	3.939	3.781	3.624	3.467	3.311	49	94	2.009	1.854	1.711	1.585	1.474	1.378	94
50	4.162	3.999	3.836	3.673	3.512	3.351	50	95	1.904	1.759	1.630	1.517	1.418	1.333	95
51	4.224	4.055	3.887	3.720	3.553	3.388	51	96	1.799	1.668	1.553	1.453	1.367	1.292	96
52	4.282	4.108	3.935	3.762	3.591	3.421	52	97	1.701	1.585	1.484	1.396	1.320	1.254	97
53	4.336	4.157	3.978	3.801	3.625	3.450	53	98	1.613	1.511	1.422	1.345	1.278	1.218	98
54	4.385	4.201	4.017	3.835	3.654	3.475	54	99	1.535	1.445	1.368	1.300	1.239	1.183	99
55	4.430	4.240	4.051	3.864	3.678	3.494	55	100	1.466	1.389	1.320	1.259	1.202	1.148	100
56	4.469	4.274	4.080	3.887	3.697	3.508	56	101	1.406	1.338	1.277	1.219	1.165	1.109	101
57	4.503	4.303	4.103	3.906	3.710	3.917	57	102	1.353	1.293	1.235	1.180	1.125	1.062	102
58	4.532	4.326	4.121	3.919	3.718	3.521	58	103	1.303	1.249	1.194	1.138	1.076	0.997	103
59	4.555	4.344	4.134	3.926	3.721	3.519	59	104	1.253	1.207	1.151	1.089	1.009	0.890	104
60	4.573	4.356	4.141	3.929	3.719	3.512	60	105	1.198	1.162	1.100	1.021	0.902	0.695	105

EXHIBIT IV-G (cont'd)

Automatic Survivorship Benefit @ 3.5%/0.0%, 6.0% by Age of Claimant & Age Difference (Spouse-Claimant)

AGE	-5	-4	-3	-2	-1	0	AGE	AGE	-5	-4	-3	-2	-1	0	AGE
16	4.330	4.208	4.088	3.972	3.858	3.746	16	61	8.254	7.726	7.218	6.730	6.262	5.815	61
17	4.458	4.332	4.208	4.087	3.969	3.854	17	62	8.159	7.629	7.118	6.629	6.160	5.712	62
18	4.587	4.456	4.328	4.202	4.080	3.958	18	63	8.055	7.522	7.010	6.519	6.050	5.602	63
19	4.718	4.582	4.449	4.320	4.189	4.058	19	64	7.940	7.405	6.892	6.401	5.932	5.485	64
20	4.852	4.712	4.574	4.436	4.296	4.156	20	65	7.814	7.279	6.766	6.275	5.807	5.362	65
21	4.990	4.845	4.698	4.550	4.401	4.251	21	66	7.678	7.143	6.630	6.141	5.675	5.233	66
22	5.132	4.977	4.820	4.662	4.503	4.343	22	67	7.532	6.997	6.486	5.999	5.537	5.097	67
23	5.274	5.108	4.940	4.771	4.602	4.432	23	68	7.377	6.844	6.336	5.852	5.392	4.957	68
24	5.415	5.238	5.059	4.880	4.700	4.520	24	69	7.214	6.684	6.179	5.699	5.244	4.814	69
25	5.558	5.369	5.180	4.989	4.799	4.609	25	70	7.045	6.519	6.018	5.542	5.092	4.667	70
26	5.702	5.503	5.302	5.101	4.901	4.701	26	71	6.872	6.350	5.854	5.383	4.939	4.520	71
27	5.850	5.639	5.427	5.216	5.005	4.796	27	72	6.696	6.178	5.688	5.224	4.785	4.372	72
28	5.999	5.777	5.555	5.333	5.112	4.893	28	73	6.515	6.004	5.519	5.061	4.629	4.223	73
29	6.150	5.917	5.683	5.451	5.220	4.991	29	74	6.327	5.823	5.345	4.894	4.470	4.073	74
30	6.301	6.056	5.812	5.568	5.327	5.089	30	75	6.131	5.634	5.164	4.722	4.307	3.921	75
31	6.452	6.195	5.940	5.686	5.435	5.187	31	76	5.926	5.437	4.976	4.544	4.142	3.767	76
32	6.602	6.334	6.067	5.803	5.541	5.284	32	77	5.713	5.234	4.785	4.365	3.974	3.611	77
33	6.751	6.471	6.193	5.918	5.647	5.380	33	78	5.495	5.028	4.592	4.186	3.807	3.456	78
34	6.899	6.607	6.318	6.032	5.751	5.475	34	79	5.276	4.824	4.401	4.008	3.642	3.303	79
35	7.044	6.741	6.440	6.144	5.853	5.567	35	80	5.060	4.622	4.214	3.833	3.480	3.154	80
36	7.187	6.872	6.560	6.254	5.953	5.658	36	81	4.847	4.423	4.029	3.662	3.322	3.008	81
37	7.327	7.000	6.677	6.360	6.049	5.745	37	82	4.635	4.226	3.845	3.492	3.166	2.867	82
38	7.462	7.123	6.790	6.462	6.141	5.828	38	83	4.425	4.031	3.665	3.327	3.016	2.730	83
39	7.592	7.242	6.897	6.559	6.229	5.907	39	84	4.221	3.842	3.492	3.169	2.872	2.598	84
40	7.716	7.354	6.999	6.651	6.311	5.980	40	85	4.023	3.661	3.326	3.018	2.734	2.473	85
41	7.835	7.461	7.095	6.737	6.388	6.048	41	86	3.830	3.484	3.165	2.871	2.600	2.354	86
42	7.947	7.563	7.186	6.819	6.460	6.112	42	87	3.636	3.307	3.003	2.722	2.467	2.239	87
43	8.054	7.658	7.271	6.894	6.527	6.170	43	88	3.441	3.128	2.839	2.576	2.340	2.129	88
44	8.152	7.746	7.349	6.962	6.586	6.221	44	89	3.250	2.952	2.681	2.438	2.220	2.023	89
45	8.242	7.825	7.418	7.022	6.637	6.265	45	90	3.067	2.788	2.538	2.313	2.110	1.924	90
46	8.322	7.895	7.478	7.074	6.681	6.301	46	91	2.895	2.638	2.407	2.198	2.006	1.834	91
47	8.393	7.956	7.530	7.117	6.717	6.329	47	92	2.735	2.498	2.283	2.086	1.908	1.752	92
48	8.455	8.008	7.574	7.153	6.745	6.350	48	93	2.584	2.363	2.161	1.979	1.818	1.679	93
49	8.507	8.052	7.609	7.180	6.765	6.364	49	94	2.436	2.230	2.043	1.879	1.737	1.614	94
50	8.551	8.086	7.636	7.199	6.777	6.369	50	95	2.291	2.100	1.933	1.787	1.662	1.554	95
51	8.584	8.111	7.653	7.209	6.780	6.367	51	96	2.149	1.979	1.831	1.703	1.593	1.498	96
52	8.606	8.125	7.660	7.209	6.774	6.355	52	97	2.019	1.869	1.739	1.627	1.531	1.447	97
53	8.617	8.129	7.656	7.199	6.759	6.334	53	98	1.902	1.771	1.658	1.560	1.475	1.399	98
54	8.614	8.119	7.640	7.178	6.732	6.303	54	99	1.800	1.685	1.587	1.501	1.423	1.352	99
55	8.599	8.098	7.613	7.145	6.695	6.262	55	100	1.710	1.611	1.524	1.446	1.374	1.305	100
56	8.572	8.064	7.574	7.102	6.647	6.211	56	101	1.632	1.545	1.466	1.394	1.324	1.254	101
57	8.531	8.018	7.524	7.047	6.589	6.149	57	102	1.562	1.485	1.412	1.342	1.271	1.193	102
58	8.478	7.961	7.462	6.982	6.521	6.079	58	103	1.497	1.428	1.357	1.287	1.209	1.112	103
59	8.414	7.893	7.390	6.907	6.443	5.999	59	104	1.433	1.372	1.301	1.223	1.126	0.987	104
60	8.340	7.814	7.309	6.823	6.357	5.911	60	105	1.363	1.314	1.235	1.139	1.000	0.768	105

EXHIBIT IV-G (cont'd)

AUTOMATIC SURVIVORSHIP BENEFIT @ 3.5%/6.0%, 6.0% BY AGE OF CLAIMANT & AGE DIFFERENCE (SPOUSE-CLAIMANT)

AGE	-5	-4	-3	-2	-1	0	AGE	AGE	-5	-4	-3	-2	-1	0	AGE
16	51.031	47.818	44.741	41.802	39.000	36.334	16	61	15.388	14.235	13.139	12.101	11.118	10.192	61
17	49.851	46.710	43.703	40.830	38.091	35.505	17	62	14.835	13.713	12.647	11.636	10.681	9.781	62
18	48.701	45.630	42.691	39.882	37.224	34.694	18	63	14.291	13.199	12.162	11.179	10.251	9.378	63
19	47.578	44.577	41.703	38.977	36.376	33.897	19	64	13.755	12.692	11.683	10.729	9.828	8.981	64
20	46.483	43.548	40.759	38.091	35.543	33.114	20	65	13.226	12.193	11.213	10.286	9.413	8.593	65
21	45.412	42.565	39.835	37.222	34.725	32.344	21	66	12.705	11.701	10.749	9.851	9.006	8.212	66
22	44.390	41.603	38.928	36.367	33.919	31.586	22	67	12.192	11.217	10.294	9.424	8.606	7.839	67
23	43.389	40.658	38.037	35.526	33.127	30.840	23	68	11.687	10.741	9.848	9.006	8.216	7.475	68
24	42.406	39.729	37.160	34.699	32.348	30.107	24	69	11.192	10.276	9.412	8.598	7.835	7.120	69
25	41.440	38.817	36.298	33.887	31.583	29.387	25	70	10.709	9.822	8.986	8.200	7.464	6.776	70
26	40.491	37.920	35.452	33.089	30.832	28.681	26	71	10.237	9.380	8.573	7.814	7.105	6.442	71
27	39.558	37.039	34.622	32.307	30.096	27.990	27	72	9.778	8.950	8.171	7.441	6.757	6.119	72
28	38.643	36.175	33.807	31.540	29.375	27.312	28	73	9.331	8.532	7.781	7.078	6.419	5.807	73
29	37.744	35.327	33.008	30.787	28.667	26.648	29	74	8.894	8.124	7.400	6.723	6.091	5.505	74
30	36.862	34.494	32.222	30.048	27.973	25.997	30	75	8.464	7.722	7.026	6.376	5.772	5.214	75
31	35.995	33.676	31.451	29.323	27.292	25.358	31	76	8.041	7.328	6.660	6.038	5.462	4.931	76
32	35.143	32.872	30.694	28.611	26.623	24.731	32	77	7.627	6.942	6.303	5.711	5.163	4.658	77
33	34.306	32.082	29.950	27.912	25.967	24.117	33	78	7.222	6.568	5.960	5.396	4.876	4.396	78
34	33.484	31.307	29.220	27.225	25.323	23.514	34	79	6.832	6.209	5.630	5.095	4.600	4.146	79
35	32.676	30.545	28.503	26.552	24.691	22.922	35	80	6.458	5.865	5.316	4.807	4.338	3.907	80
36	31.882	29.796	27.799	25.890	24.070	22.340	36	81	6.100	5.537	5.015	4.532	4.088	3.681	81
37	31.102	29.061	27.106	25.239	23.460	21.769	37	82	5.756	5.221	4.726	4.269	3.850	3.467	82
38	30.334	28.337	26.426	24.600	22.860	21.207	38	83	5.427	4.919	4.450	4.019	3.625	3.265	83
39	29.579	27.625	25.755	23.970	22.269	20.654	39	84	5.113	4.633	4.190	3.785	3.413	3.074	84
40	28.835	26.924	25.095	23.350	21.688	20.109	40	85	4.816	4.362	3.946	3.565	3.215	2.895	85
41	28.104	26.234	24.446	22.740	21.116	19.573	41	86	4.532	4.106	3.714	3.355	3.026	2.728	86
42	27.383	25.555	23.807	22.140	20.552	19.045	42	87	4.257	3.857	3.489	3.151	2.844	2.572	87
43	26.674	24.887	23.178	21.548	19.997	18.525	43	88	3.991	3.615	3.269	2.955	2.675	2.425	88
44	25.976	24.228	22.558	20.965	19.450	18.012	44	89	3.737	3.383	3.062	2.775	2.519	2.288	89
45	25.286	23.578	21.945	20.389	18.909	17.506	45	90	3.497	3.169	2.875	2.613	2.376	2.160	90
46	24.606	22.936	21.341	19.821	18.376	17.006	46	91	3.275	2.975	2.707	2.464	2.243	2.045	91
47	23.935	22.303	20.744	19.260	17.850	16.513	47	92	3.071	2.797	2.549	2.323	2.120	1.942	92
48	23.272	21.678	20.156	18.708	17.331	16.026	48	93	2.881	2.628	2.398	2.190	2.008	1.850	93
49	22.619	21.062	19.577	18.163	16.819	15.546	49	94	2.699	2.465	2.253	2.068	1.907	1.769	94
50	21.976	20.455	19.005	17.625	16.314	15.072	50	95	2.524	2.310	2.121	1.958	1.817	1.696	95
51	21.341	19.856	18.441	17.094	15.815	14.603	51	96	2.357	2.166	2.001	1.858	1.735	1.629	96
52	20.714	19.265	17.883	16.569	15.321	14.140	52	97	2.205	2.038	1.893	1.769	1.661	1.568	97
53	20.094	18.680	17.332	16.049	14.833	13.682	53	98	2.071	1.925	1.799	1.691	1.596	1.511	98
54	19.481	18.101	16.786	15.536	14.350	13.228	54	99	1.953	1.827	1.717	1.621	1.535	1.456	99
55	18.875	17.528	16.246	15.027	13.872	12.780	55	100	1.851	1.741	1.645	1.558	1.478	1.400	100
56	18.275	16.962	15.712	14.524	13.399	12.335	56	101	1.761	1.666	1.578	1.497	1.420	1.340	101
57	17.682	16.402	15.184	14.027	12.931	11.895	57	102	1.681	1.597	1.515	1.437	1.358	1.269	102
58	17.097	15.849	14.662	13.535	12.469	11.461	58	103	1.607	1.532	1.453	1.374	1.285	1.175	103
59	16.519	15.303	14.147	13.050	12.012	11.032	59	104	1.532	1.468	1.388	1.299	1.189	1.032	104
60	15.949	14.765	13.639	12.572	11.562	10.609	60	105	1.451	1.401	1.312	1.202	1.046	0.790	105

EXHIBIT IV-H

CLAIMANT ANNUITIES & COMMUTATION FUNCTIONS @ 3.5%/0.0%

AGE	ANNUITY	DX	NX	AGE	ANNUITY	DX	NX
16	24.193	100000.0	2469328.	61	12.424	17108.2	221110.
17	24.047	96520.8	2369328.	62	12.071	16227.4	204002.
18	23.900	93147.7	2272807.	63	11.718	15369.1	187775.
19	23.750	89882.6	2179660.	64	11.363	14532.7	172406.
20	23.596	86726.7	2089777.	65	11.009	13717.4	157873.
21	23.438	83676.6	2003050.	66	10.655	12922.9	144155.
22	23.276	80728.1	1919374.	67	10.302	12148.9	131233.
23	23.109	77879.6	1838646.	68	9.950	11395.5	119084.
24	22.936	75130.9	1760766.	69	9.599	10663.1	107688.
25	22.756	72480.6	1685635.	70	9.249	9952.3	97025.
26	22.569	69926.6	1613155.	71	8.899	9263.7	87073.
27	22.374	67465.3	1543228.	72	8.550	8597.5	77809.
28	22.172	65091.3	1475763.	73	8.203	7952.4	69212.
29	21.963	62799.6	1410671.	74	7.861	7326.7	61259.
30	21.747	60585.6	1347872.	75	7.526	6719.6	53933.
31	21.525	58446.0	1287286.	76	7.200	6131.9	47213.
32	21.297	56377.6	1228840.	77	6.881	5565.6	41081.
33	21.062	54377.4	1172463.	78	6.571	5022.7	35515.
34	20.820	52442.4	1118085.	79	6.268	4505.7	30493.
35	20.573	50570.2	1065643.	80	5.970	4016.3	25987.
36	20.319	48757.9	1015073.	81	5.681	3554.8	21971.
37	20.059	47003.1	966315.	82	5.400	3121.3	18416.
38	19.793	45302.8	919312.	83	5.128	2717.4	15295.
39	19.521	43654.4	874009.	84	4.864	2344.7	12577.
40	19.244	42055.9	830354.	85	4.607	2003.8	10233.
41	18.961	40506.1	788298.	86	4.361	1692.9	8229.
42	18.673	39002.9	747792.	87	4.133	1410.8	6536.
43	18.379	37544.5	708789.	88	3.923	1158.9	5125.
44	18.079	36128.3	671245.	89	3.725	938.8	3966.
45	17.776	34752.0	635117.	90	3.536	750.2	3028.
46	17.467	33414.3	600365.	91	3.355	590.7	2277.
47	17.154	32113.9	566950.	92	3.189	457.2	1687.
48	16.837	30849.8	534836.	93	3.039	347.4	1229.
49	16.515	29620.6	503987.	94	2.907	258.9	882.
50	16.188	28424.9	474366.	95	2.793	189.3	623.
51	15.858	27261.0	445941.	96	2.696	135.8	434.
52	15.525	26127.3	418680.	97	2.611	95.8	298.
53	15.188	25022.7	392553.	98	2.536	66.6	202.
54	14.849	23945.1	367530.	99	2.466	45.7	136.
55	14.508	22894.1	343585.	100	2.402	31.0	90.
56	14.164	21868.6	320691.	101	2.341	20.7	59.
57	13.820	20867.9	298823.	102	2.280	13.7	38.
58	13.473	19891.9	277955.	103	2.215	9.0	24.
59	13.125	18940.1	258063.	104	2.143	5.8	15.
60	12.775	18012.3	239123.	105	2.054	3.8	10.

EXHIBIT IV-H (cont'd)

Claimant Annuities & Commutation Functions @ 3.5%/6.0%

AGE	ANNUITY	DX	NX	AGE	ANNUITY	DX	NX
16	130.513	100000.0	13101253.	61	23.532	235488.3	5659216.
17	126.575	102312.0	13001253.	62	22.408	236765.3	5423728.
18	122.745	104660.7	12898941.	63	21.322	237697.6	5186963.
19	119.015	107051.6	12794280.	64	20.274	238246.5	4949265.
20	115.375	109490.4	12687229.	65	19.263	238374.6	4711019.
21	111.823	111978.1	12577738.	66	18.289	238041.4	4472644.
22	108.358	114514.3	12465760.	67	17.352	237211.2	4234603.
23	104.974	117102.1	12351246.	68	16.449	235852.0	3997392.
24	101.666	119747.2	12234144.	69	15.579	233935.2	3761540.
25	98.430	122454.4	12114397.	70	14.742	231439.9	3527604.
26	95.261	125227.9	11991942.	71	13.935	228352.6	3296164.
27	92.159	128069.4	11866714.	72	13.156	224646.9	3067812.
28	89.124	130976.6	11738645.	73	12.408	220258.3	2843165.
29	86.159	133947.1	11607668.	74	11.694	215102.6	2622907.
30	83.263	136978.1	11473721.	75	11.014	209118.2	2407804.
31	80.437	140069.3	11336743.	76	10.370	202278.7	2198686.
32	77.679	143218.8	11196674.	77	9.758	194610.5	1996407.
33	74.988	146425.9	11053455.	78	9.179	186164.6	1801797.
34	72.365	149688.3	10907029.	79	8.627	177023.4	1615632.
35	69.807	153005.1	10757341.	80	8.101	167264.9	1438609.
36	67.314	156373.3	10604336.	81	7.602	156925.8	1271344.
37	64.886	159790.1	10447962.	82	7.130	146055.7	1114418.
38	62.521	163250.5	10288172.	83	6.684	134786.8	968362.
39	60.220	166749.0	10124922.	84	6.262	123278.9	833575.
40	57.981	170281.5	9958173.	85	5.860	111676.5	710297.
41	55.802	173847.0	9787891.	86	5.486	100007.5	598620.
42	53.682	177439.1	9614044.	87	5.144	88342.0	498613.
43	51.621	181052.6	9436605.	88	4.833	76923.5	410271.
44	49.618	184676.8	9255553.	89	4.547	66052.0	333347.
45	47.673	188299.7	9070876.	90	4.277	55949.9	267295.
46	45.784	191914.6	8882576.	91	4.026	46699.4	211345.
47	43.951	195512.4	8690661.	92	3.797	38315.5	164646.
48	42.171	199085.6	8495149.	93	3.594	30858.0	126330.
49	40.444	202622.1	8296063.	94	3.417	24375.6	95472.
50	38.768	206109.4	8093441.	95	3.264	18889.1	71097.
51	37.143	209530.1	7887332.	96	3.134	14364.9	52207.
52	35.569	212865.9	7677802.	97	3.022	10745.7	37843.
53	34.044	216097.8	7464936.	98	2.921	7921.1	27097.
54	32.570	219199.6	7248838.	99	2.828	5761.7	19176.
55	31.143	222152.8	7029638.	100	2.741	4138.4	13414.
56	29.764	224934.2	6807486.	101	2.658	2937.3	9276.
57	28.432	227520.0	6582552.	102	2.574	2061.8	6338.
58	27.144	229890.9	6355032.	103	2.486	1432.4	4277.
59	25.899	232025.2	6125141.	104	2.386	985.6	2844.
60	24.695	233898.9	5893115.	105	2.265	672.1	1859.

EXHIBIT V

Comparison of Proposed and Current Annuity Values

Age	Spouse's Age Dist.	Spouse's Annuity		Claimant's Age Dist.	Claimant's Annuity		Spouse's Age Dist.	Spouse's Dowry
		Proposed	Current		Proposed	Current		
16	.000000	8.488	0.0	.000041	24.193	24.203	.000068	0.5778
17	.000000	8.898	0.0	.000134	24.047	24.053	.000199	0.5863
18	.000000	9.442	0.0	.000259	23.900	23.900	.000368	0.5922
19	.000000	10.085	0.0	.000416	23.750	23.744	.000580	0.5853
20	.000937	10.751	7.715	.000610	23.596	23.583	.000936	0.5694
21	.001189	11.427	8.360	.000848	23.438	23.419	.001188	0.5459
22	.001631	12.064	9.031	.001136	23.276	23.250	.001629	0.5212
23	.002099	12.664	9.717	.001464	23.109	23.076	.002087	0.4986
24	.002577	13.250	10.404	.001827	22.936	22.896	.002574	0.4755
25	.003055	13.814	11.071	.002220	22.756	22.710	.003051	0.4511
26	.003494	14.348	11.726	.002628	22.569	22.516	.003489	0.4263
27	.003899	14.851	12.365	.003051	22.374	22.315	.003894	0.4011
28	.004302	15.319	12.973	.003494	22.172	22.107	.004297	0.3754
29	.004694	15.744	13.564	.003959	21.963	21.892	.004688	0.3502
30	.005079	16.126	14.129	.004443	21.747	21.671	.005073	0.3260
31	.005442	16.465	14.655	.004943	21.525	21.444	.005436	0.3029
32	.005786	16.764	15.135	.005460	21.297	21.209	.005779	0.2800
33	.006136	17.024	15.573	.005994	21.062	20.968	.006129	0.2598
34	.006496	17.243	15.964	.006544	20.820	20.721	.006488	0.2399
35	.006972	17.423	16.300	.007112	20.573	20.467	.006864	0.2211
36	.007279	17.565	16.592	.007689	20.319	20.206	.007270	0.2036
37	.007718	17.669	16.836	.008277	20.059	19.938	.007709	0.1873
38	.008185	17.740	17.009	.008884	19.793	19.644	.008175	0.1723
39	.008684	17.778	17.141	.009510	19.521	19.384	.008674	0.1583
40	.009217	17.785	17.228	.010157	19.244	19.099	.009206	0.1454
41	.009796	17.763	17.270	.010836	18.961	18.808	.009784	0.1335
42	.010418	17.714	17.291	.011548	18.673	18.511	.010406	0.1225
43	.011075	17.639	17.258	.012278	18.379	18.209	.011062	0.1124
44	.011766	17.541	17.220	.013025	18.079	17.902	.011752	0.1031
45	.012486	17.421	17.145	.013784	17.776	17.590	.012471	0.0945
46	.013237	17.281	17.035	.014551	17.467	17.273	.013221	0.0866
47	.014020	17.122	16.907	.015326	17.154	16.951	.014003	0.0794
48	.014822	16.945	16.743	.016103	16.837	16.625	.014804	0.0727
49	.015640	16.750	16.559	.016880	16.515	16.295	.015621	0.0665
50	.016471	16.540	16.373	.017653	16.188	15.963	.016452	0.0608
51	.017319	16.314	16.148	.018411	15.858	15.629	.017298	0.0556
52	.018172	16.074	15.903	.019155	15.525	15.294	.018150	0.0508
53	.019017	15.821	15.654	.019886	15.188	14.955	.018994	0.0464
54	.019847	15.555	15.365	.020602	14.849	14.613	.019823	0.0423
55	.020653	15.277	15.084	.021299	14.508	14.266	.020628	0.0385

EXHIBIT V (cont'd)

Age	Spouse's Age Dist.	Spouse's Annuity Proposed	Spouse's Annuity Current	Claimant's Age Dist.	Claimant's Annuity Proposed	Claimant's Annuity Current	Spouse's Age Dist.	Spouse's Dowry
56	.021437	14.988	14.780	.021997	14.164	13.916	.021411	0.0351
57	.022190	14.688	14.470	.022688	13.820	13.561	.022163	0.0319
58	.022883	14.378	14.141	.023339	13.473	13.203	.022855	0.0289
59	.023500	14.058	13.788	.023937	13.125	12.844	.023471	0.0262
60	.024029	13.729	13.448	.024473	12.775	12.486	.024000	0.0237
61	.024415	13.390	13.078	.024933	12.424	12.128	.024385	0.0214
62	.024652	13.043	12.693	.025307	12.071	11.770	.024622	0.0193
63	.024788	12.688	12.340	.025582	11.718	11.412	.024758	0.0173
64	.024827	12.327	11.950	.025744	11.363	11.054	.024797	0.0155
65	.024773	11.960	11.555	.025781	11.009	10.696	.024743	0.0138
66	.024626	11.588	11.179	.025628	10.655	10.377	.024596	0.0123
67	.024393	11.212	10.780	.025289	10.302	9.978	.024364	0.0109
68	.024084	10.833	10.380	.024832	9.950	9.618	.024054	0.0096
69	.023702	10.451	9.968	.024275	9.599	9.260	.023674	0.0085
70	.023257	10.068	9.577	.023634	9.249	8.902	.023228	0.0074
71	.022766	9.683	9.170	.022955	8.899	8.546	.022738	0.0065
72	.022238	9.297	8.762	.022248	8.550	8.191	.022211	0.0056
73	.021655	8.912	8.359	.021483	8.203	7.838	.021629	0.0049
74	.021013	8.530	7.958	.020663	7.861	7.487	.020988	0.0042
75	.020311	8.155	7.565	.019785	7.526	7.140	.020286	0.0037
76	.019543	7.787	7.176	.018847	7.200	6.796	.019520	0.0032
77	.018710	7.426	6.791	.017856	6.881	6.454	.018688	0.0027
78	.017819	7.073	6.415	.016817	6.571	6.117	.017798	0.0023
79	.016875	6.729	6.047	.015741	6.268	5.787	.016854	0.0020
80	.015886	6.393	5.694	.014638	5.970	5.467	.015867	0.0017
81	.014865	6.067	5.357	.013518	5.681	5.162	.014847	0.0014
82	.013812	5.754	5.038	.012392	5.400	4.875	.013796	0.0012
83	.012734	5.452	4.733	.011266	5.128	4.602	.012719	0.0010
84	.011640	5.160	4.437	.010155	4.864	4.335	.011626	0.0009
85	.010545	4.876	4.139	.009072	4.607	4.066	.010532	0.0007
86	.009462	4.606	3.859	.008028	4.361	3.810	.009450	0.0006
87	.008394	4.356	3.601	.007023	4.133	3.570	.008384	0.0005
88	.007347	4.124	3.365	.006061	3.923	3.345	.007338	0.0004
89	.006342	3.906	3.149	.005156	3.725	3.136	.006334	0.0004
90	.005400	3.698	2.951	.004326	3.536	2.946	.005393	0.0003
91	.004535	3.501	2.780	.003580	3.355	2.776	.004530	0.0002
92	.003752	3.321	2.620	.002918	3.189	2.624	.003747	0.0002
93	.003050	3.161	2.493	.002337	3.039	2.490	.003046	0.0002
94	.002432	3.020	2.373	.001838	2.907	2.372	.002429	0.0001
95	.001903	2.896	2.268	.001418	2.793	2.266	.001901	0.0001
Average:		12.115	11.577		12.273	11.970		0.0527

DISCUSSION BY ALFRED O. WELLER

"The 1979 NCCI Remarriage Table" is a concise account of the construction of the remarriage table itself and corresponding revisions of the Unit Statistical Plan. The article is rich in actuarial concepts and sophisticated techniques. It is straightforward and well organized in its presentation of the extensive work underlying the 1979 NCCI Remarriage Table. This discussion deals with two aspects of this work: (1) the use of parametric models to understand insurance processes better and to overcome deficiencies in available data, and (2) standards for construction of Unit Statistical Plan tables.

The construction of the 1979 NCCI Remarriage Table employs actuarial techniques suited to the dearth of available data. Compared to the 1958 CSO Tables with 59 million life-years of exposure and 400,000 deaths, the 150,000 years of exposure and 2,000 remarriages for the 1979 NCCI Remarriage Table are few—so few that direct use of observed remarriage rates would have been inappropriate and inaccurate due to large sampling variation. For example, the observed remarriage rate for a forty-seven year old widow is 250% of the rate for a widow age forty-six (both women being widowed for four years). However, a larger sample is not possible.

In order to balance smoothness and fit properly (distinguish signal from noise), the 1979 NCCI Remarriage Table is derived from a continuous force of remarriage defined by nine parameters. These nine parameters are determined by minimizing a quadratic loss function. The expression for the force of remarriage is:

$$\begin{aligned}\mu^{(r)}_{[x]+t} &= \text{prompt component} && + \text{delayed component}\\ &= p_1\, e^{-p_2 x}\, t^{p_8-1}\, e^{-p_3 t} && + \theta\,(t - p_7)\, p_4\, e^{-p_5 x}\,(t - p_7)^{p_9-1} e^{-p_6(t-p_7)}\end{aligned}$$

where $\theta(t - p_7)$ is zero for t less than p_7 and unity otherwise. The superscript r denotes remarriage, the variable x the widow's age (nearest birthday) on being widowed, and the variable t the duration of widowhood.

Using the fitted values of parameters $p_1, p_2, \ldots, p_9$, the derived remarriage rates describe a general pattern of increasing rates during early widowhood followed by decreasing rates thereafter as the duration of widowhood increases. By analyzing derivatives (Exhibit I) it can be determined quickly that the force of remarriage increases during the first 1.6 years of widowhood, decreases after 5.3 years of widowhood, and attains a maximum (dependent upon age) between these two durations. Such a conclusion would not be as apparent from alternative methods of graduation—inspection of individual rates of remarriage would be necessary to determine if the statement were true. In this respect, the parametric approach facilitates sharper insight than does graduation using difference equations, moving averages, or graphical methods. This advantage is demonstrated also by the analysis of trends in remarriage rates in Section VI of the article.

In a sense the work of casualty actuaries always has concerned parameters and their estimation. After all, rates are derived from estimates of such well-known parameters as pure premiums and expense ratios. If pure premiums are all that is directly observed, then intervening variables (i.e., entities somewhere between cause and observation) such as claim frequency and loss severity escape attention. An increase in rates might be attributed to an increase in pure premiums, but the reasons for the increase in pure premiums cannot be identified without further research. With respect to any individual phenomenon such as loss ratio or remarriage rate, the use of intervening variables can serve several purposes including (a) definition of current limits of knowledge (ability to explain causes and predict effects), (b) identification of hypotheses to test the accuracy of knowledge, (c) summary of observations and effects, and (d) improved communication, as when two psychoanalysts discuss a libido. Most importantly, by identifying intervening variables in today's models, we build a link to causal influences and gradually extend the realm of actuarial science. For instance, for the 1979 NCCI Remarriage Table, the nine intervening variables define prompt and delayed components, streamline the analysis of trends, and identify testable hypotheses with respect to updating the Table.

The article proceeds logically from the analysis of remarriage data to the construction of revised Unit Statistical Plan tables. These tables have been incorporated in the Unit Statistical Plan to apply to loss valuations made on or after November 1, 1980. The revised Unit Statistical Plan tables differ from their predecessors in two key respects.

1. Sources.
   A. The 1979 NCCI Remarriage Table replaces graduated remarriage rates for the U.S. Employees' Compensation System (Table 7 of Actuarial Study No. 55 of the Social Security Administration).
   B. The 1969–71 U.S. Decennial Life Tables for Total Females and Total Population replace the 1959–61 U.S. Decennial Life Tables for White Females and Total Population, respectively.

2. Format.
   A. A select and ultimate format replaces tabular values keyed to the middle of three years used in experience rating.
   B. Tables have been introduced for lump sum remarriage awards (dowries) and procedures for survivorship benefits have been refined.

The central role of the Unit Statistical Plan (USP) in workers' compensation insurance sets the structural standards for the revised tables. For certain legal obligations, the Unit Statistical Plan assigns a standard tabular value, representative of a cohort of claimants, to corresponding losses for purposes of manual ratemaking, experience rating and retrospective rating. USP tables are used to value further benefits until the process of claim settlement establishes actual values for individual claims. These values generally will differ from USP tables. For example, a lump sum settlement or a structured settlement can close claims with present values for future benefits different from USP tables. When a claim is settled, its actual liquidated value is reported. Thus, USP tables are designed to value consistently a legal obligation in light of current conditions without specific assumptions regarding claims settlement.

The "1979 NCCI Remarriage Table" is an apt solution to a difficult problem. Through the Unit Statistical Plan it contributes to the accurate and equitable pricing of workers' compensation insurance. Further, it is a thorough account of valuable actuarial work. We are grateful to Phil Heckman for presenting it to the Casualty Actuarial Society.

EXHIBIT I

ANALYSIS OF DERIVATIVES

Given: $\mu^{(r)}_{\{x\}+t}$ = force of remarriage

$$= p_1\, e^{-p_2x}\, t^{p_8-1} e^{-p_3t} + \theta\{t - p_7\}\, p_4 e^{-p_5x} (t - p_7)^{p_9-1} e^{-p_6(t-p_7)}$$

Assuming $\theta\{t - p_7\} = 1$ and taking derivatives:

$$\frac{d}{dt}\mu^{(r)}_{x+t} = p_1\, e^{-p_2x}\, [(p_8 - 1)\, t^{p_8-2}\, e^{-p_3t} - p_3\, t^{p_8-1}\, e^{-p_3t}]$$

$$+ p_4\, e^{-p_5x}\, [(p_9 - 1)\,(t - p_7)^{p_9-2}\, e^{-p_6(t-p_7)} - p_6(t - p_7)^{p_9-1}\, e^{-p_6(t-p_7)}]$$

$$= p_1\, e^{-p_2x}\, t^{p_8-1}\, e^{-p_3t} \left(\frac{p_8 - 1}{t} - p_3\right)$$

$$+ p_4\, e^{-p_5x}\, (t - p_7)^{p_9-1}\, e^{-p_6(t-p_7)} \left(\frac{p_9 - 1}{t - p_7} - p_6\right)$$

Terms outside of large parentheses are positive for given parameter values. The derivative is set equal to zero and corresponding values of t determined.

Prompt component: $t = (p_8 - 1)/p_3$
$= (2.313 - 1)/.7786$
$= 1.686$

Delayed component: $t = p_7 + (p_9 - 1)/p_6$
$= .583 + (1.809 - 1)/.171$
$= 5.314$

It follows that for $t < 1.6$ the force is increasing and for $t > 5.3$ the force is decreasing. A maximum occurs between these values.

GENERAL LIABILITY RATEMAKING: AN UPDATE

MICHAEL MCMANUS

VOLUME LXVII

DISCUSSION BY WARREN JOHNSON

According to the author, the purpose of this paper is "to present a summary of the adjustments that have been made in the basic limits ratemaking methodology [in the fourteen years since Jeffrey T. Lange wrote "General Liability Insurance Ratemaking"] and the reasons for their introduction." The author has accomplished this stated purpose. All significant changes are discussed; some further changes that have occurred since the paper was written will be mentioned later in this review. Generally, each of these changes is well-documented with respect to both the new methodology that is used and the reasons for adopting the new methodology.

Undoubtedly this paper will be studied by students endeavoring to learn GL ratemaking. The author's decision to update rather than rewrite Lange's paper, while justifiable on the basis that much of what Lange wrote remains accurate and valid today, leaves the student in the unenviable position of having to learn this subject in a less than straightforward manner. First the student must master Lange's paper, which presupposes a knowledge of both Stern's "Ratemaking Procedures for Automobile Liability Insurance" (*PCAS* LII), and Benbrook's "The Advantages of Calendar-Accident Year Experience and the Need for Appropriate Trend and Projection Factors in the Determination of Automobile Liability Rates" (*PCAS* XLV). Then the student must read this update to sort out which of the 1966 procedures discussed by Lange remain valid, and which have been changed.

The author begins by stating that the industry has experienced a significant period of social and economic inflation. Presumably, social inflation refers to an increased propensity to sue by the public, and an increased willingness to award damages by the courts. We all have heard and read that these phenomena are occurring, and yet for GL other than professional liability, no significant upward trend in claim frequency is seen in the ratemaking data. On the other hand, Page 14 Annual Statement losses for GL have increased at a rate far greater than the general inflation rate.

A few comments with regard to McManus's section on loss development might be appropriate. The discussion of Medical Malpractice is up-to-date. For GL other than professional liability, Insurance Services Office (ISO) now has loss development data through 123 months of maturity for products liability and through 75 months of maturity for OL&T and M&C. (Bodily injury and property damage losses are available at each evaluation.) Ultimately, ISO expects to compile loss development data for these sublines through 135 months of maturity. The author refers to a procedure in which development beyond the last observed development interval is assumed to be equal to development in the last interval. It is not clear from the paper that this procedure has been used for BI as well as PD, and is still being used today for the sublines other than professional liability. In the loss development section, the author also refers to a theoretical problem in using data limited to a fixed dollar amount to calculate loss development factors. It would have been helpful if the author could have discussed how the problem is handled, or why it was dismissed.

In the section entitled "Definition of Basic Limits," an adjustment, due to the fact that a small number of insureds purchased policies with limits of less than $25,000, is discussed. This is a fairly minor technical point that could have been omitted. However, since the item is mentioned, I must point out that the adjustment is not properly described. The ratio of (a) the $25,000 increased limits factor (on a $5,000/$10,000 basis) to (b) the average increased limits factor for those insureds purchasing limits less than $25,000 was applied to the reported incurred losses above the $5,000 limit but below the $25,000 limit.

In the products liability section, it is mentioned that rates for the newly erected classifications were adjusted by overall trend factors during the period of time when there were no data available for these new classes. Although this is true, it was not the only procedure used. In some cases, a revised rate was selected by analogy to the indicated rates for marginally similar classifications.

The paragraph regarding classification ratemaking for products liability provides a discussion that is perhaps a bit too brief for a rather complicated and important issue. This omission of greater detail may be forgiven (although somewhat fortuitously) since this is one area in which the ratemaking methodology has been revised since Mr. McManus wrote his paper.

In the section entitled "Future Challenges," it is stated that "the resultant elimination of sublines will reduce the credibility problems that exist today." The reader might easily get the impression that combining things so as to increase the volume (number of claims) automatically increases credibility. This is not the case if dissimilar entities are combined; such combinations could reduce credibility by increasing the variance in the data (due to reduced homogeneity).

It is not my intent to document in complete technical detail those areas in which an update of the author's paper is in order, but rather to simply mention such areas and briefly describe what ISO has done.

The ISO General Liability Actuarial Subcommittee (GLAS) has adopted a Bayes credibility procedure for use in products liability classification ratemaking. Using this methodology, the credibility assigned to the experience of a class is a function not only of the experience for the individual class, but also of the experience for the class group within which the class falls. The greater the variance in loss ratios by year within a class (given a constant variance among classes), the lower the credibility assigned to that class's experience. The greater the variance in loss ratio between classifications within a class group (given a constant variance by year within class), the greater the credibility assigned to the individual class experience, since the class group loss ratio becomes a poorer predictor of the individual class's loss ratio. For a detailed discussion of this subject, refer to "Report of the Credibility Subcommittee: Development and Testing of Empirical Bayes Credibility Procedures for Classification Ratemaking," published by ISO in September, 1980.

An interesting and rather significant modification in the calculation of the overall rate change for products liability has been adopted by the GLAS. It has been observed that, to a certain extent, the pure premium using real exposures, i.e., exposures adjusted for inflation, varies inversely with the business cycle. This is because products liability coverage pays for occurrences during the policy period (regardless of date of sale); much of the products liability hazard results from goods sold in previous years. Thus, in an expansionary period exposures increase and losses increase, but not as much as exposures, leading to a reduction in pure premium. The opposite occurs during a period of recession. Although it is difficult to draw definitive conclusions from a limited number of data points, the attached exhibit and graphs tend to confirm the above hypothesis. It is often suggested that the number of claims tends to increase during poor economic times. Although this may be true to a limited extent for products liability, it is apparent that the largest part of the variation in claim frequency is due to movement in exposures, rather than claims. This problem has been addressed in the following manner. Exposure trend, which is based upon econometric forecasts, is calculated to reflect inflation only. In essence, a price deflator, properly weighted to be applicable to products liability, is used.

Claim frequency (number of claims related to dollars of premium at present rates adjusted to a common price level) is modeled against "Gross Investment in All Structures in 1972 Dollars" (a proxy for the business cycle), separately for BI and PD. Future claim frequencies are then calculated by using forecast values of "Gross Investment in All Structures in 1972 Dollars." Claim frequency trend factors are calculated not as annual percentage changes, but rather as specific factors to span the gap from each policy year in the experience period to the policy period for which the revised rates will apply. An analagous procedure will be used for M&C.

The final change in ratemaking methodology that I would like to mention pertains to claims made coverage. In 1975, a major shift began taking place from an occurrence form to a claims made form for professional liability. Approximately twenty percent of all Physicians, Surgeons and Dentists coverage is now written on a claims made form. ISO priced this coverage by establishing claims made multipliers that apply to rates for the occurrence form. These multipliers were based on a summarization of accident year losses by report year. These data were primarily from a single company that is a large writer of professional liability.

Once claims made data entered the ISO data base, the question arose as to how the data should be used, since they are too large a portion of the whole to be ignored. Two basic issues needed resolution. How should the claims made data be used for the basic rate level calculation? How should the claims made data be used in the trend calculation? Excluding the claims made data would distort the trend calculation because the entire trend curve would not be based on a consistent set of insureds. On the other hand, merely combining claims made and occurrence data in the frequency and severity trend calculations would produce a distortion because only the latest year(s) of the trend curves would contain claims made data. (The first year(s) of claims made data exhibits lower than average claim frequency and claim severity.)

Two solutions to the first problem were considered. The first was to restate the claims made data as if they had been written on an occurrence basis. Although difficult, this approach theoretically is possible. However, a necessary condition for the validity of this procedure is that the same insureds remain in the ISO data base year after year. The impossibility of such a consistent data base led to the rejection of this solution. The second alternative, which the Professional Liability Actuarial Subcommittee adopted, was to combine the claims made data directly with the occurrence data. Claims made losses, properly developed, are added to occurrence losses, properly developed. (Claims made losses develop only due to changes in reserves on known claims, since by definition there is no IBNR. A unity loss development factor for claims made losses was selected judgmentally, and will be used until an historical record is available for calculating actual claims made loss development factors.) Premium at present rates for claims made data (exposure times rate times claims made multiplier) is added to premium at present rates for occurrence data (exposure times rate).

The second problem was resolved by calculating trends from basic limit loss ratios at present rates (for occurrence and claims made data combined). This enabled the use of a consistent data base for all years in the trend calculation, and removed the distortion that claims made data would produce if severity and frequency trend were examined separately.

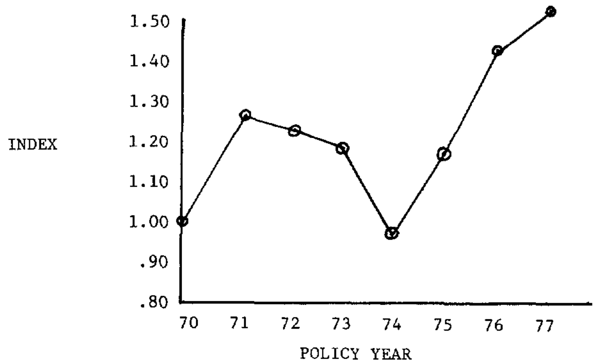
Both Mr. Lange and Mr. McManus have stated that GL ratemaking procedures will continue to change. Although these ongoing changes are in the nature of fine-tuning, rather than a complete overhaul, they are nonetheless both significant and frequent. Mr. McManus should be thanked for this effort to keep us up-to-date on the ratemaking procedures for this important line of insurance.

PRODUCTS LIABILITY CLAIM FREQUENCY CHANGES
BODILY INJURY AND PROPERTY DAMAGE—COMBINED
MANUALLY RATED CLASSES
COUNTRYWIDE

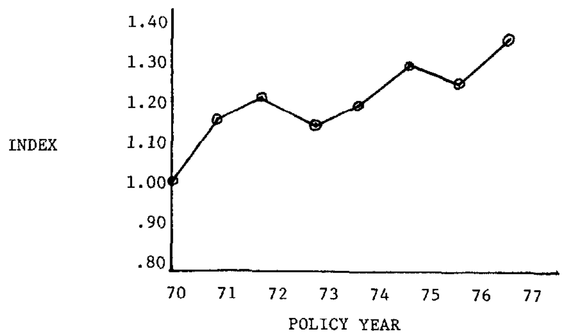
Policy Year	Premium at Present Rates* (millions)	Index to 1970	Number of Claims ϕ	Index to 1970	Claim Frequency#	Index to 1970
1970	$115.7	1.00	13,366	1.00	115.5	1.00
1971	145.5	1.26	15,491	1.16	106.5	.92
1972	141.8	1.23	16,212	1.21	114.3	.99
1973	136.0	1.18	15,281	1.14	112.4	.97
1974	112.0	.97	15,798	1.18	141.1	1.22
1975	135.8	1.17	17,276	1.29	127.2	1.10
1976	165.8	1.43	16,573	1.24	100.0	.87
1977	176.8	1.53	18,229	1.36	103.1	.89

* Trended to the price level anticipated 4/1/82.
ϕ Developed to an ultimate basis.
Number of Claims per $1,000,000 of premium at present rates.
Source: Insurance Services Office

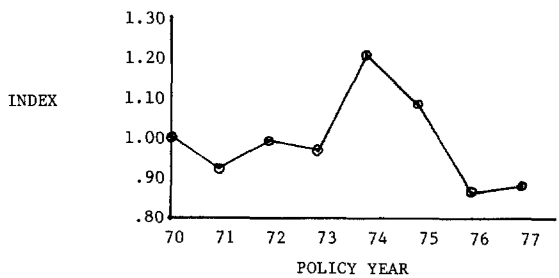
PRODUCTS LIABILITY



PREMIUM AT PRESENT RATES



NUMBER OF CLAIMS



CLAIM FREQUENCY

ACTUARIAL VALUATION OF PROPERTY/CASUALTY INSURANCE COMPANIES

ROBERT W. STURGIS

VOLUME LXVIII

DISCUSSION BY STEPHEN P. LOWE

"For what is Worth in anything,
But so much Money as 'twill bring."
—Samuel Butler

Mr. Sturgis's paper presents a comprehensive model for the actuarial valuation of a property/casualty acquisition candidate. As he points out, this topic is a new one to our *Proceedings*; it is therefore also likely to be a new topic to many members of the profession. Mr. Sturgis's paper presents another example of the expanding role of the property/casualty actuary and actuarial techniques in insurance and in the general economy.

Of particular interest to this reviewer are the relationships between the actuarial valuation process of a property/casualty company as presented in the model and the general economic principles that underlie any decision process relating to the sale or acquisition of an entity. This review will, therefore, focus initially on these relationships in an attempt to gain additional insight into the power and versatility of the actuarial valuation model.

THE ECONOMICS OF VALUE

Mr. Sturgis reviews several alternative measures of the value of an insurance company, including its market value as measured by outstanding common stock, its book value as measured by its balance sheet, and its comparative value as measured by analogy to recent purchase prices of similar companies. Two other measures of value are dilution value, representing the price above which the buyer's overall return on equity would be reduced; and economic value, defined as the present value of future earnings.

Mr. Sturgis points out that all these measures of value are related to, but not necessarily the same as, the purchase price the buyer is willing to pay for the company to be acquired.

This distinction, between the "value in use" and the "value in exchange" of an object, can be found throughout classical economics literature and is attributed to Aristotle. Value in use is the utility that something has in and of itself. Value in exchange is what that object will fetch in the marketplace. The former is intrinsic to the object; the latter is dependent on the relationship between supply and demand.

Value in use and value in exchange can be related by recognizing that in the marketplace a transaction will not be consummated unless *both the buyer and the seller receive a greater economic benefit than they give up*. This is possible because the two parties' valuations of the exchange are not the same. The seller of apples cannot possibly use all that he has; the cash that he obtains is, therefore, of greater value to him than the apple he sells. The hungry buyer is equally willing to part with a small amount of cash for the greater benefit gained from the apple that he obtains.

These concepts can be summarized by the following inequality relating price and economic value of an object being sold:

$$\begin{matrix}\textit{Value in Use} \\ \textit{to the Seller}\end{matrix} \leq \begin{matrix}\textit{Value in} \\ \textit{Exchange}\end{matrix} \leq \begin{matrix}\textit{Value in Use} \\ \textit{to the Buyer}\end{matrix}$$

If this inequality were not satisfied, no exchange would take place; each party would keep what he already has.

Of course it must be recognized that value in use is individual and subjective, being a function of relative needs, desires, preferences, and/or utilities at a given point in time.

CONSIDERATIONS IN VALUING A PROPERTY/CASUALTY COMPANY

In the context of property/casualty acquisitions, Mr. Sturgis has provided us with a model whereby value in use (the capitalized value of anticipated future earnings) can be quantified. However, he only touches on the considerations of the two frames of reference cited above, i.e., those of buyer and seller.

Mr. Sturgis's model can be run, at least in theory, in two different modes for the given acquisition being considered. First, a simulation depicting the seller's current use of the company can be run to determine the minimum price that would be prudently acceptable to the seller's management. Second, a simulation depicting the buyer's projected use of the company can be run to determine a corresponding maximum price.

The final price presumably would fall within the range imposed above, being dependent on such market factors as the availability of alternative acquisition candidates to the buyer (supply) and alternative purchasers to the seller (demand). Since this market lacks great numbers of buyers and sellers the relative urgency of the sale or purchase becomes an important related factor.

Of the two valuation modes above, the seller's current use of the company undoubtedly is the easier to simulate and project, since such a projection presumably involves a continuation of the status quo of the company. This is, of course, not necessarily the case; the alternative to sale might be liquidation.

On the other hand, the intended use of the acquisition by the buyer requires more careful consideration, centering on two principal areas:

1. Explicit changes in the operations of the acquired company.

   These may take the form either of planned changes imposed by the new management, or of changes precipitated directly by and resulting from the acquisition.

2. The revenue and earnings of the acquisition taken within the context of the new parent.

   Mr. Sturgis refers to this aspect at the conclusion of his paper when he alludes to the "operational and financial synergism with the existing operations." Key elements in this area would be presumed rates of premium growth and federal income taxes.

Outlined below are some specific operational aspects that might deserve consideration.

General Expenses. If the buyer is an existing insurance company the elimination of duplicative administrative activities may serve to reduce future costs in this area.

Reinsurance. The reinsurance program of the company being reviewed is based on the level of risk that it is willing to retain. The acquiring company may wish to alter the existing program to suit its own preferences. For example, a small stand-alone company being acquired by a larger one may have retentions substantially below those of its new parent.

Consolidation of Physical Plant. Mr. Sturgis points out that adjustments to the statutory net worth of a company should be made to reflect non-admitted assets. One specific and potentially significant item is the physical plant of the company under consideration: furniture, fixtures, and the excess of market over admitted value of the building itself, if owned. If the acquisition involves the eventual consolidation of operations it may be appropriate to include the additional value of these items with realizable earnings based on their expected time of sale. (Account also must be taken of the potential capital gains tax resulting from such a sale.)

Acquisition Costs. A rather fundamental operational question is whether or not the company being acquired will continue marketing its products in the manner currently employed; for example, the marketing approach might be modified to tie in with the approach used for the other products sold by the would-be parent (Sears/Allstate is perhaps the best illustration). If modification is planned, the future acquisition costs should reflect the modification.

When one company purchases another a reasonable assumption is that the motivation for the acquisition is the enhancement of existing operations through the potential synergism of their combination. This enhancement can involve several areas:

Marketing. The acquisition may add a complementary good or service that will enhance the marketing of existing product lines. Alternatively the acquisition may provide direct access to a new geographic market at less cost than building one from scratch.

Cash-Flow. Different industries require different amounts of cash. Such differences may be intrinsic, seasonal, or related to the business or marketing cycle of the products involved.

Smoothing of Earnings. Similarly, industries vary as to the sensitivity of their earnings to general business and economic conditions.

Federal Income Taxes. Both the tax treatment of the acquisition itself and the consolidation of returns subsequent to the acquisition may generate substantial benefits unattainable to the two entities separately.

Consideration of certain of the above areas may be outside the scope of an actuarial valuation, or indeed beyond the practical limits of any quantification process. However, it seems reasonable that a projection of future results could be performed so as to reflect the general intended use of the company by the buyer in these areas. Assumptions as to premium growth, lines of business written, underwriting and investment profitability can be constructed to conform to the buyer's as well as the seller's general business plan.

Such a valuation would not reflect fully the synergism of the acquisition, since the increased growth and profitability of the parent's original operation would not be included. The result would, therefore, fall below the true upper bound to the purchase price.

TAX CONSEQUENCES OF THE ACQUISITION

Certainly, the most complex aspect of any acquisition is an evaluation of its tax consequences. In illustrating his approach, Mr. Sturgis defined a single line insurance company subject to a 46% tax rate and with a fixed investment strategy of one-third of its assets in tax-exempts and the balance in taxables.

This approach treats the company being valued as a stand-alone entity for tax purposes. Such an approach is appropriate in presenting the seller's perspective, if in fact the company is a stand-alone entity.

In modeling the buyer's viewpoint, it is necessary to recognize that future earnings are subject to taxation within the context of the new parent's operations. The overall anticipated tax picture may in turn influence the assumed investment strategy. For example, if the new parent expects to show taxable losses on its operations, then the acquired company's investments presumably would be shifted to reflect a greater proportion of taxables. (This is a specific example of value in use to the buyer exceeding the seller's value in use.)

Finally, it is necessary to consider the tax consequences of the acquisition itself in obtaining a final value from the buyer's perspective. The acquisition can be handled in various ways, with differing tax consequences to the buyer (and seller). A discussion of acquisition tax issues and the implications of alternatives would be a suitable topic for an entire paper. Two key issues, for example, would be the treatment of existing loss carry-forwards and the tax-basis of the company's assets after the acquisition.

For an excellent introduction to the tax alternatives associated with acquisitions, the reader is referred to Lenrow, Milo, and Rua[1] which has an excellent chapter on this topic.

One specific tax option available to the purchasing corporation is the liquidation of the acquired company under Section 334(b)(2) of the Internal Revenue Code. Section 334(b)(2) provides that where property is received by a corporation upon the complete liquidation of another corporation, the basis of that property is the same as the basis of the stock acquired. This is important to the acquiring corporation because the subsequent depreciation of the property will be measured by the amount paid for the property rather than by the frequently much lower basis of the property in the hands of the acquired corporation.

[1] Gerald Lenrow, Ralph Milo, Anthony Rua, *Federal Income Taxation of Insurance Companies*, Third Edition, John Wiley & Sons, Inc., New York, 1979.

Under such a liquidation plan, the purchase price of the stock is allocated, based on fair market value, to all of the assets of the acquired company, including goodwill and the value of the company's existing business.

Since goodwill is not deductible or amortizable for tax purposes, it is important to properly value the other assets of the company. This includes the valuation of the existing business as a "wasting asset." The value of existing business can be considered a "wasting asset" only if it can be demonstrated that the business has a definite value distinct from goodwill and an ascertainable limited useful life.

A model of the form described by Mr. Sturgis can be used to determine the value of this item. Rather than being used to project the future earnings of all the company's business, the same model can be used to project the future earnings of only those portions that fall within the context of the "wasting asset."

OTHER ALTERNATIVE USES OF THE MODEL

The alternative use described above suggests that the model can be used to value various "blocks" of a company's book of business, rather than the company as a whole. This in turn suggests that the model can be used in a non-acquisition situation to evaluate and value alternative corporate strategies. By inputting alternative assumptions and comparing the resulting values, a company could evaluate the consequences of major marketing, underwriting, or financial decisions it is contemplating.

Several interesting uses might include:

A multiple lines national company considering the surrender of its license in a habitually unprofitable state could use the model to get a clearer picture of the potential impact on its overall operation.

An agency company considering conversion to direct writing could, similarly, evaluate the timing of the likely costs and benefits of such a conversion.

A company considering a change in claim settlement practices (such as a major program to lump-sum settle workers' compensation cases) could use the model to obtain a clearer picture of the overall consequences of such a change.

A company shifting from undiscounted to discounted loss reserves could evaluate the financial implications of such a move.

While the model might not be able to provide all the answers in situations such as those described above, it could be very useful by providing a baseline from which additional questions can be raised.

CONCLUSION

Mr. Sturgis's paper provides us with a new and powerful valuation technology. His paper illustrates the model's use in its "normative" state, but the model's uses extend to many different contexts, both within and outside the acquisition arena.

DISCUSSION BY ROBERT ROTHMAN AND ROBERT V. DEUTSCH

Introduction

The valuation of property/casualty insurance companies is a topic that has been neglected in the actuarial, financial, and economic communities. As Mr. Sturgis points out, there has been a notable increase in property/casualty insurance company acquisition and merger activity. Hence, his paper represents a needed and timely addition to the existing body of literature, and we hope that it provides the impetus for further research in this area.

Mr. Sturgis makes a number of points that we believe are important and that we will highlight in the following discussion. He concludes that a model based on a statutory earnings stream is appropriate for measuring the economic value of a firm. The use of statutory earnings to value an insurance company dates back to James Anderson's 1959 paper[1] and, to our knowledge, has not been contested as an accurate measure of value.

As an alternative, we believe that a model based on discounted cash flow has several advantages. Although such an approach has not been applied specifically to the property/casualty insurance industry, the use of discounted cash flow as a valuation technique has been well addressed and accepted by the business community, particularly in a capital budgeting framework. An application of this concept to a property/casualty company is discussed later in this review.

[1] James C. H. Anderson, "Gross Premium Calculations and Profit Measurement for Non-Participating Insurance," *Transactions, Society of Actuaries*, Vol. XI (1959), p. 357.

Valuation Measures

One of the objectives of Mr. Sturgis's paper is to focus on a valuation concept that determines what a purchase price "ought to be." His paper presents definitions of five alternative valuation measures and selects economic value as the most suitable in terms of the stated objective. We agree with this point, particularly from a potential buyer's perspective. Although the actual purchase price may differ from economic value, a determination of the economic value can provide the buyer with a useful benchmark from which to negotiate.

Mr. Sturgis defines economic value as "the book value plus the present worth of expected future earnings." He further points out that earnings should include only those available to the buyer and translates this to mean after-tax statutory earnings. We believe economic value is better defined as the present value of future cash flow. If the only cash flow available to the investor is the dividend stream, and dividends are limited to statutory earnings, then the two definitions of economic value result in essentially the same valuation. The treatment of book value is the only item that may cause the two definitions to produce different results.

Cash flow valuations frequently are used by the insurance industry for purposes other than the valuation of a company as a whole. For example, an actuary pricing a portfolio reinsurance transaction often will estimate the discounted value of the future loss payments.

In general, the property/casualty insurance industry is placing more emphasis on investment income in pricing its products than it has in the past. This shift in emphasis has been due to high interest rates, which reduce the present value of future loss payments and, therefore, reduce the premium required to produce a target rate of return. This concept of pricing for total return rather than underwriting profit—often called "cash flow underwriting"—is another example of the insurance industry's use of cash flow valuation techniques.

Why, then, has the insurance industry been reluctant to use cash flow techniques in valuing a company as a whole? Perhaps the reluctance stems from the belief that cash flow available to the investor is limited to statutory earnings. However, limiting cash flow to statutory earnings is unrealistic. It ignores the value of internally generated cash that can be invested by the owner at his discretion, within certain regulatory constraints. For example, excess cash can be used to finance further acquisitions or a wide variety of other investments.

Implications of Cash Flow Valuation

To develop further the concept of discretionary cash and some implications as to its use, assume that an owner invests a portion of his insurance company's portfolio to purchase another company that is not necessarily another insurance company. The remaining portion of the portfolio is invested in traditional securities such as government or corporate bonds. We submit that the value of the newly acquired company must be considered in the valuation of the property/casualty insurance company. The new company's assets, future earnings, and accumulation of wealth become available to the owner because the insurance company has provided the means of financing. Cash flow analysis permits the owner to measure properly the results of this concept.

Both uses of cash for investments, namely for discretionary investment and for traditional investment, can be valued by analyzing cash flow, but the discount rates used may be significantly different for different types of investments. The concept of valuing in-force business apart from new business also can be viewed in terms of cash flow. The value of in-force business is primarily the discounted investment portfolio runoff, net of the discounted loss payments. The discounted investment portfolio runoff is a function of the assets currently on the books and the rate at which those assets will be converted to cash. Net cash flow for new business is the discounted premium income less the discounted loss and expense outgo. Since greater uncertainty normally is associated with cash flows on new businesses, the discount rate for new business usually would be higher than the discount rate for business already on the books.

In summary, to analyze properly cash flow one must distinguish between cash that is available to the investor for use at his discretion and cash that is locked into traditional investments in terms of both in-force business and new business. It is important to note that future investment income that could have been earned on discretionary cash is not a component of cash flow available to the buyer, since investment income is considered implicitly in the discount rate used to calculate the present value. This differs significantly from statutory income, which includes investment income explicitly. However, investment income is considered when determining the cash flow relating to locked-in assets since the cash invested in these assets is not a part of the cash flow; however, these assets do constitute cash flow upon reaching maturity.

Cash Flow Model

We developed a model to simulate both statutory earnings and cash flow with the ability to vary premium growth rates. For comparison purposes, the model utilized essentially the same assumptions employed by Mr. Sturgis. We analyzed the results of the model under three different premium growth patterns: constant premium volume, premium increases of 20% per year, and premium decreases of 20% per year. Valuations were made at two different valuation dates in order to consider a new company situation versus an established company with historical operating results. In addition, valuations were made using various discount rates.

We made the following observations:

—The results of the statutory and cash flow models differ dramatically depending on the valuation date and the discount rate.

—As one would expect, at a 0% discount rate the cash flow gain is negative because there is an underwriting loss.

—The impact of the underwriting loss is mitigated by the use of a positive discount rate and may become inconsequential depending on the premium growth rate.

—The cash flow is a function of premiums written and the loss payout pattern; the effect of a deferred payout of losses, generally considered beneficial, actually is contingent upon the premium growth rate.

In general, there is a tendency for statutory valuations to undervalue a company that is experiencing premium growth and to overvalue a company that is experiencing premium deterioration. The reason for this is that statutory accounting principles do not recognize revenues and expenses in the proper periods, i.e., premiums and losses are recognized over the policy period, while actually premiums are received at the beginning of the policy period and losses are paid over several years.

Other Considerations

As Mr. Sturgis notes, the selection of discount rates significantly affects the valuation. There are, of course, many reasonable discount rates from which to select, such as the opportunity cost, the cost of capital, etc. We suggest that a different discount rate be used for projected earnings many years after the valuation date because of a lack of credibility.

In addition to the discount rate, two other significant items are the underwriting assumptions and the variability of loss payouts. The latter, combined with historical investment decisions, becomes particularly important when valuing the in-force business using a cash flow analysis of assets and liabilities. Additional adjustments may be made based on many other factors, including underwriting risk, opportunity cost, the prospective buyer's utility function, and regulatory constraints.

Conclusions

The valuation of property/casualty insurance companies is a complex area that deserves more attention than can be afforded here. As Mr. Sturgis states, a significant component of valuing a property/casualty company should be a determination of the entity's economic value. To address this issue, he presents a model based on a discounted statutory earnings stream. We believe that the true economic value is more adequately measured by discounted cash flow than by discounted statutory earnings. We hope that the paper and this discussion stimulate further development of this important topic.

AN EXAMINATION OF CREDIBILITY CONCEPTS

STEPHEN W. PHILBRICK

VOLUME LXVIII

DISCUSSION BY THOMAS N. HERZOG

I would like to congratulate the author for his valuable contribution to our knowledge of credibility. The expanded discussion of the Hewitt examples and the figures in the first part of the paper are instructive and easy to understand.

The purpose of my discussion is to expand upon some of the ideas raised in Mr. Philbrick's paper. None of my ideas are new. Yet, because of their overriding importance in insurance ratemaking, I believe it is worthwhile to present them here.

My remarks are intended to reinforce and extend Mr. Philbrick's remarks as well as to clarify a few of his ideas that might benefit from being expressed in another fashion. I will begin with a few general remarks about Bayesian statistics and credibility.

Bayesian statistics enables us to combine our prior experience with our current observations in a unified and formal framework and forces us to make explicit our model as well as the underlying assumptions. This makes it easier to describe ratemaking procedures to other technicians, if not to those with less technical backgrounds.

The basic concept of Bayesian inference is that the prior knowledge (i.e., distribution) is modified by the current observations to produce the posterior distribution. For insurance ratemaking this means that the prior distribution of the rate (or pure premium) for each existing insurance policy is modified by the current (i.e., most recently available) experience to produce the posterior distribution of the insurance rate so that the new rate may be determined. The precision of the new rate can be estimated by examining its posterior distribution. For example, a posterior normal distribution with mean 10 and variance 4 leads to more precise estimates than does a posterior normal distribution with mean 10 and variance 25. (I do not think Mr. Philbrick makes this point quite as clearly as he might.) Currently, the risk loading in an insurance premium often is chosen to be proportional to the variance of the loss severity distribution (i.e., the probability distribution of the amount of loss on an individual claim). It may be preferable in the future to make the risk loading proportional to the variance of the posterior distribution of the pure premium since this relates to total losses during a policy period and incorporates the number as well as the amount of losses.

While Mayerson (1964) and Jewell (1976) show that, under certain conditions, the credibility formulas are exact Bayesian solutions (i.e., they are Bayesian conditional means), in other instances these formulas are just rough approximations. In addition, the concepts of full and zero credibilities are, of course, also just approximations, intended to make the life of the practicing actuary easier. When the actuary says the data are fully credible, he means that for practical purposes there is no reason to use a prior distribution because the weight to be given to the prior distribution would be almost zero. On the other hand, the actuary may have so little current data that he decides to give all the weight to the prior distribution and thereby avoids making computations having little or no impact on the result. We should add here that the Bayesian's focus is on the posterior distribution and this obviates the need for confidence intervals. In fact, Hogg and Craig (1970; Section 6.5) show how ill-conceived the notion of a confidence interval can be under certain circumstances. I don't know why Mr. Philbrick gets tangled up with confidence intervals rather than focusing on the posterior distribution.

Ideally, the actuary/Bayesian statistician should perform a full Bayesian analysis each time he calculates rates. There are, unfortunately, two potentially serious problems with this.

1. The construction of the posterior distribution may require calculations which are, for all practical purposes, impossible to carry out. For example, the composition of the appropriate (conjugate) prior distribution with the distribution of the observed data (i.e., the likelihood) may be a computational nightmare.
2. If the rate calculations (such as those in workers' compensation insurance) are to be understood and/or performed by a large number of nontechnical or semi-technical people, it may be completely unreasonable to expect them to follow the Bayesian procedure instead of the relatively simple credibility-based procedures now in existence.

While the potential problems listed above pertain currently, they may be of less consequence in the future. As the cost of computing continues to drop and as more people have immediate and easy access to high-powered computers, almost anyone may be able to input a few numbers, call a sophisticated computer program, and obtain rates in a few minutes or less. Thus, a complicated underlying procedure should be feasible.

Finally, I recommend that anyone with a serious interest in credibility read "A Survey of Credibility Theory" by Professor Jewell (1976). Of particular interest is his extension of the concept of credibility to "multidimensional credibility." I believe that this procedure, or something similar to it, has great potential for use in ratemaking. While the application of such procedures may be quite involved, the actuary must realize that he is dealing with difficult problems whose solutions may require a lot of careful thought. This is a challenge that actuaries must meet.

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2. A. L. Mayerson, "A Bayesian View of Credibility," *PCAS* LI, 1964.
3. W. S. Jewell, "A Survey of Credibility Theory," Operations Research Center #76-31, University of California, Berkeley, October 1976.

PREDICTING THE FUTURE IN THREE EASY STEPS

GERALD J. SULLIVAN

The title of my short talk this morning is, "Predicting the Future in Three Easy Steps." Predicting the future, after all, is one of the world's oldest and most honorable arts. This subject always puts me in mind of Sam Goldwyn's comment, "Never make forecasts—especially about the future."

The early actuaries tended to be in high positions related to the popular religion of the culture at the time. The art of early actuarial prediction was mathematically based and provided as strange, mysterious and terrifying a process as it does today.

What has all this to do with the theme of this meeting, "The Large Risk"? Well, with the large risk the actuary has an excellent opportunity to use his talents most effectively to predict the likely future course of that risk and therefore, to significantly add to the quality of the decisions made about that risk.

As you know, my perspective on this subject is that of an intermediary. While my discussing this subject with actuaries of your caliber is a bit like "carrying coal to Newcastle," this vantage point has allowed me an excellent opportunity to observe the work of many people working with large risks. From that vantage point I must say that I am constantly dismayed at how frequently one sees one or all of what I call the three cardinal sins of predicting the future.

The first one is: "Who was that masked man?" All sorts of things happened but nobody kept track of what it was, when it happened, or what its meaning really was.

Next is: "History will not repeat itself!" Some people seem to be firmly convinced that no matter how many times something has happened a certain way in the past, that everything has changed and there is no possible way it can likely happen that way again in the future. This is also known as the "Alice-In-Wonderland Syndrome." Don't get me wrong—change does occur and we have to realistically deal with that possibility. However, change occurs because something has caused it to occur—not because we, or our client, would prefer that history not repeat itself!

And finally: "If anyone can understand what I just said, I didn't do it right!"

Isn't it classic? We can do the best job in the world of gathering and analyzing just exactly the right data for a particular job and then describe it in such a complex and detailed manner that nobody, but *nobody,* can effectively use the information. Later this morning I'll briefly show you some tools we've developed to try to help with this problem.

But what about predicting the future in three easy steps?

Among the first actuaries were the Druids—the priests of the Celts in Europe, and then Ireland and Wales. They governed and predicted all kinds of events—the weather, the future. They fixed auspicious times for enterprises and wars and were the mediators for warring tribes (things haven't changed so much from 225 B.C.). They set currency rates, helped manage the treasury, and insured and funded some of the first trade routes in Europe. Unfortunately, they predicted the wrong outcome against Julius Ceasar, and suffered very heavy casualties. Thus began their demise. Some of the most consistent early actuaries are the ancestors of the people in Baru Koltok in Bali. According to this culture and its writings that stretch back more than 2,000 years, a special rite is performed during the festival of Eka Dasa Rudra by no less than 23 priests. The sacred writings of this culture call for the rite to be performed every 100 years, or when times are bad enough in order to reduce the risk of bad times. (I wonder if some insurance company presidents who just reported their first quarter results would like the phone number of these fellows?) In any event, the last time the rite was held was in 1963—just a few weeks later Mount Agung erupted killing more than 1,500 people. However, most of the time, when the event has been performed, it has resulted in good luck. (After all, even a good actuary with terrific tables and information can be wrong every 2,000 years or so.)

The Chinese have become most adept at predicting the kind of natural upheaval that set off the volcano in Bali. Perhaps one of the most fascinating actuarial documents in the world exists in China today. It is a collection of observations and records, meticulously kept for over 3,000 years. The reasons for keeping such a record were illustrated in China in 1556. It was the greatest natural disaster then known and it occurred in Shensi Province. During that year an earthquake hit and snuffed out the lives of 820,000 people. During a scientific visit to China in 1974, Dr. Barry Raleigh of the Geological Survey, and Dr. Lynn Sykes of Lamont-Doherty, witnessed three successful earthquake predictions by the Chinese. The Chinese now have a force of over 10,000 professionals in the field observing, recording and reporting earth tremors. But, what is most interesting, is that although there are 10 scientific methods which one can use to measure the earth's pulse, the Chinese still rely heavily on abnormal animal behavior—snakes crawling from their holes, rats leaving buildings, and birds refusing to roost.

Now for those of you who are beginning to wonder where this talk is leading, I appreciate your attention. You have not mistakenly stumbled into a historical society lecture. For those of you still interested in finding out how to predict the future in three easy steps, bear with me; after all, it took quite a while to develop this instant formula for success. So now on with some of the reasons for the three step program to develop actuarial predictions.

On hilltops in North America, archaelogists were puzzled for years by series of stone patterns which were found near sites where large Indian tribes stopped for certain seasons. The stone patterns ranged in age from 800 to 3,000 years old, and were used up until the time white men reached the northern hemisphere. All the patterns were laid out as a circle, with a mound in the top and dissected by stone lines.

One particularly interesting pattern was found on a remote peak high in the Bighorn Mountains of Wyoming. It rests on the shoulder of a flat near the top of a 10,000 foot mountain. It resembles a large 28 spoke wheel. It is 80 ft. across and has six rock piles unevenly spaced around the rim. Archaeologists in the 1800's were told by Indians that the unit was built by "people who had no iron." Others said that it was built to show the Indians how to build a tepee.

Although many archaelogists attempted to find answers, it wasn't until 1976 that John A. Eddy found an answer for one of these patterns, which were called "Medicine Wheels." The answer that Eddy found for one wheel, he then tested on others in Saskatchewan and the Great Plains area.

The answer was consistent in each case. The "Medicine Wheels," or stone circles, were used as astronomical devices. On the first day of autumn and spring the sun rises exactly east and sets exactly west. As spring wears on, sunrise moves farther north each day until late June, when it slows, stops, and begins to move southward again.

The day its northward motion stops, June 21, of course, is the summer solstice.

The Wheels were used as sighting tools. One of the piles of rocks was aligned with three others to mark the rising of three stars: Aldebaran in Taurus, Rigel in Orion, and Sirius in Canis Major. These three stars are the brightest in the region of the sky through which the sun passes in summer.

An Indian saying helped Eddy, "When the sun is highest, the growing earth is the strongest."

Descendants of the Indians recall little practical use of the sky or Medicine Wheels. The absence of this sky lore in historical records surely tells how fragile learning and communication are without the written word. Eddy and others have, however, since conjectured that the Wheels were used to help forecast the planting that was done by these nomadic Indians . . . also, possibly, to help them establish the time by which they should prepare to move to other grounds.

It is amazing, however, to note the mathematical and predictive usage similarities in the Medicine Wheel of the Northern Plains Indians, with the Great Aztec Calendar, and with the Circle of Stones of the Druids at Stonehenge, England.

The science of predictions, for all manner of things, has progressed through the ages. At the same time the Medicine Wheels were being used in North America, the Egyptians saw the skies as an enormous celestial vault, which they were sure was the spangled torso of the goddess "Nut." They observed, recorded and considered the information and used it to predict flood seasons, time for planting, auspicious times for trading, and by adding frogs, snakes and various other things, political upheavals. See—politics hasn't changed much either!

Great minds, such as Copernicus, Galileo and Kepler, assisted in altering the practice of using recorded knowledge, mathematics, and viewing the heavens from an occult predictive device into a more scientific tool; but it was Newton who brought the science of mathematics and prediction back to modern day reailty.

It was an unpredicted bubonic plague and 18 months in isolation in the country that finally brought Newton to his theory—not to mention the apple.

Not only did Newton determine the basic principles of light and color, he created integral and differential calculus and defined the workings of gravity. Obviously, calculus has become one of the important business tools of the century.

Much more recently another scientist, Roger Payne, completed 15 years of scientific study of the humpback whale. Payne has published works that definitively illustrate, in audio and mathematically graphic form, that humpback whales communicate through an eerie singing system.

His recordings have proven that not only do whales communicate, but also that they have a system which allows them to combine series of sounds, and then not only retain those sounds, but also to insert new "verses," so to speak, into the original series they created.

This brings me back to the present, and the opportunity to share with you my sure fire formula for successfully predicting the future in three easy steps.

The great actuary, I am told, is judged by the company he keeps solvent. And that is indeed a great challenge in the current market conditions.

Truly, the challenges and risks you face are sometimes more awesome, terrifying and challenging than those any ancient actuary was ever faced with.

Supertankers, plastics, pesticides, new wonder products, asbestosis, pharmaceuticals, robotics, and revolutionary cantilevered buildings were not realities the ancients had to deal with.

Floating platforms to drill for oil, satellites to transmit information, and 2 million dollar athletes were not part of the bargain back in 500 B.C. But, you have some advantages over the ancients. Computers capture more information and correlate it more quickly than did the scribe over a 50 year period working on papyrus. Hopefully, calculators, microfiche filing, and hordes of willing staff have assisted in making today's actuaries more perfect than ever before.

And thus, we arrive at the basis for the formula for accurate prediction of the future.

The three steps I am about to utilize are tools you know about. But, since I am a student of history and you are here, bear with me while I give you my point of view on the tools and how they can be put to use.

The three steps are:

1. Find the Data—Mathematical, Behavioral, Historical and Natural

More data for more things has been catalogued in the past 25 years than in the preceding 500 years. In addition, more knowledge about more things has been gained in the last 10 years than in the 1,000 years preceding this date. For example, in the area of medical malpractice there are detailed data banks of information available today which were nonexistent just 10 years ago.

In collecting data one should not look only to the obvious, but should collect all possible types of data and information that even remotely apply to the subject being studied.

Again looking to the area of medical malpractice, where I have some familiarity, we discovered several years ago that the then rapidly growing loss area of cardiac arrest could best be reduced by applying some basic anesthesiology techniques. This discovery would easily have been overlooked had we not had available to us extensive amounts and types of data about the cases we were studying.

The fact that the data you would like to study is not readily available should not be used as an excuse for presenting a poorly developed or weak thesis. This is especially true today with the large risk, for modern corporations and institutions are ferocious gatherers of information and data of all types. And if the particular risk you are dealing with in fact does not have the necessary data, that is still no excuse. Comparable data does exist somewhere and you are expected to find and apply it as best you can to your particular risk.

Which brings me to the next step:

2. Catalogue and Correlate the Data

You must, by the nature of your profession, collect the data on any given subject in such a way that you can digest it. (And, I am sure that some of you are driven to the point that oftentimes you feel like eating the paper in front of you.) Earlier I mentioned the Chinese and the catalogue of information that has been collected on earthquakes. The fact that the data was catalogued and correlated in a meaningful manner allowed the Chinese to predict the February 4, 1975 earthquake, which registered 7.3, five and one-half hours before it happened.

It took 3,000 years to collect the data, but it was correlated and put into modern form during the last 15 years.

Today, the large risk offers a unique opportunity to more accurately measure the liability in advance, but only if you have the data catalogued in a meaningful manner.

In the area of hospital professional liability exposure, we maintain a data bank which stretches back to 1956, including over 24,000 claims and covering some 52,000 beds. Numerous facts are kept on each case, and with that type of extensive information you would think that we had more than enough documentation to do any degree of analysis we wanted. But several years ago, thanks to a special study done to measure the likely outcome of applying a workers' compensation benefit schedule to malpractice losses, we discovered a whole new way to catalogue and analyze malpractice loss data.

The form you are now looking at on the screen is the form we are currently using to record information on potential malpractice situations. The key to this form is seen in the lower right hand corner. Essentially, this is a screening mechanism which has demonstrated its ability to cull out those medical incidents which will likely give rise to a malpractice action. The success of this screening device is demonstrated by the fact that it has been adopted by the American Hospital Association Joint Committee on Accreditation as the basis for its Quality Assurance Program. While we use the information to speed up and refine defense activities, as well as to develop ways to prevent these incidents from happening in the future, its significance to us here this morning is that there are in fact new and better ways to analyze data. We must be on the lookout for them all the time.

The last and most difficult step is:

3. Evaluate and Communicate the Information

Ancient actuaries, in predicing the outcome of wars, foretelling the most auspicious time for business dealings, and insuring trade caravans, used mathematics, past experience, the stars, gossip, snakes, eye of eel, and even blood to help assure the outcome as predicted.

The dramatic presentation of the information by the ancients tended to carry a lot of weight with both the buyer and seller. And, no doubt, both buyer and seller welcomed "divine" intervention in considering the risks.

Although we have evaluated the Medicine Wheels of the Northern Plains Indians, they failed to communicate the precise usage of the Wheels, and thus they are useless to us today. The 23 priests in Bali faithfully perform the special rites every hundred years, yet they have not correlated and catalogued presentation of the rite with the occurrence of disasters, and thus their efforts to alleviate risks are not worth much. The humpback whales spend their lifetime communicating long and loud, but we haven't the slightest notion as to what they are saying. Possibly, at some future date, the 15 years worth of data collection and correlation will be of some use, but at this time the message is a mystery.

Now, far be it from me to compare a humpback whale with a sophisticated 20th century actuary—but, communication is an important part of the actuarial process. Your job is to collect and correlate information and estimate the future as correctly as possible. Further, you evaluate the organization's potential liability arising from a possible risk. Then, you must communicate both the evaluation and prediction of risk. The best evaluation in the world is totally useless unless it is understood by both the buyer and the seller.

Henry Ford, II, who was a frustrated actuary, put it quite well, "Nobody can really guarantee the future. The best we can do is size up the chances, calculate the risks involved, estimate our ability to deal with them, and then make our plans with confidence." One of the great traits of the early Fords and, I have the feeling, Newton as well, was to put things simply. When it really came down to Newton's relating his theory of gravity, he utilized a very simple event to illustrate it to the public—the apple falling from the tree.

Insurance actuarial work requires like amounts of simplicity. At the same time, and this is the most important point, it must be recognized that the same data may very well have to be presented to different audiences in radically different ways in order to achieve the greatest communication.

For example an insurance underwriter, sophisticated in the ways of data interpretation and risk patterns, is readily able to interpolate and analyze the more traditional loss development data you see here on the screen.

Alternatively, the board of directors of a typical, large, long tail risk that must participate in risk handling decisions can have the same data presented to them in such a manner that they can easily grasp its essential realities. This next chart presents the same data we looked at earlier but in another form.

This can be used to easily and quickly communicate its essential underlying characteristics, thus, better risk handling decisions can be made. As you can see, the theory of trying to communicate with different audiences, in whatever manner is best to achieve maximum understanding from those audiences, frequently necessitates that you present the same data in very different ways to the various audiences. In fact, if you are really trying to maximize your level of communication, presenting the same data in the same way to different audiences should be the exception rather than the rule. Further, these simplified data presentation techniques can be used to demonstrate characteristics of risk that have, heretofore, only been discussed in lengthy prose. For example, the reality of the delay of claims reporting in long tail casualty business is much more apparent when presented in these loss development charts on a year by year basis.

With the excellent tools available today, there is no longer any reason why actuarial concepts are presented in anything but the simplest, most straightforward manner possible considering the capabilities of the audience.

Now I'd like to make a few comments about pricing. What we are concerned with here is the relationship between whatever liability or funding level appears realistic based on your analysis and what is really available in the marketplace. This is an especially critical issue in the market conditions we are facing today.

As you are well aware, it is not at all unusual to find that a good actuarial analysis of potential risk differs markedly from what the risk-bearing marketplace may be willing to charge to assume that risk. This situation is an equal problem, whether your client is a buyer or seller.

This is true because the problem occurs in both tight markets where the seller frequently overprices the assumption of risk, and in loose markets, such as we see today, where the seller is often significantly underpricing the assumption of risk.

What is your responsibility in these situations? Simple, "Tell it like it is!" That is the only way you can look out for your clients' best long-term interests. This is especially true where you are dealing with a long-tail, large risk where the available risk-bearing market is really quite limited. You may not think it is, but, believe me, in the real world it is!

Let me give you an example of what I mean.

If your analysis points to a liability or funding level of X, while at the same time some elements of the market are willing to assume that risk, say, at ½ or even ¼ X, is that good news?

Not if ½ X is so unrealistic that the risk-bearer will suffer enormous loss, for that will normally result in a significant overreaction and when the market turns, and it will, the buyer will usually find himself at the mercy of the market forces, and may very well find himself paying 2 to 4 X or more.

Wouldn't it be much better for the buyer to pay something much closer to X in all market conditions? This long-term strategy will *always* result in lower overall risk-bearing costs to your client than any short-term strategy. And don't fall for the line that it can't be done; it's been done for years by many who espouse a longer-term perspective and the stability that goes with it. But, because it's not a popular, trendy way to do things, you don't hear much of it.

This perspective does, however, require constant and extensive work with both the buyer and seller to achieve and maintain the fullest understanding of the real level of risk being faced.

Even sadder is the situation where someone opts for ½ X, only to find out several years later that the funding level was so unrealistic that the risk assumer is unable or unwilling to fulfill the promises of his contract. All the guarantee funds and reinsurance in the world cannot make a broken promise work; and this is not an idle comment about something that can't happen to you. There have been more fire and casualty company insolvencies in the U.S. in the last 12 months than in the preceding 5 years, and I can assure you this trend will only accelerate dramatically in the next year as the market begins to work its way out of its current catastrophic slump.

If you don't have good access to what is going on day-to-day in the marketplace, and of equal importance, to which security is good, and which is not so good, you have two choices. One, find a good source for this information and use it; or two, tell your client that your analysis does not take into consideration what is going on in the marketplace.

We may have the luxury of being able to analyze risk without the need to consider market conditions, but our clients do not have that luxury; they have to live with that market!

If your analysis for some reason cannot consider market conditions, you at least have the obligation to advise your client of that, and, further, to warn him that his risk handling decision must take market conditions into consideration.

So now, I would like to simply recap my formula:

Step 1: *Collect the data*—as much primary and peripheral data as necessary, as far back as possible as it relates to the subject. Collect mathematical data, behavioral data, exceptions to the rule, graphs, natural patterns and unnatural patterns.

Step 2: *Catalogue the data*—in a way that the lesser beings of the world can begin to deal with it. Catalogue and correlate the data to illustrate patterns, visualize new dimensions and contours.

Step 3: *Evaluate and communicate the data in light of your analysis*—being sure of your ground and your evaluation, simplify the information. Highlight the important information. If possible, define the reasons for the evaluation graphically and without resorting to snakes, charms or bloodletting, simply state the reasons for your prognostication. And finally, as simply as possible, sum up the evaluation in a few paragraphs or words relating the important facts which you know will affect the prediction, and stating simply the facts which may affect the risk.

MINUTES OF THE 1982 SPRING MEETING

May 23–26, 1982

THE BREAKERS, PALM BEACH, FLORIDA

Sunday, May 23, 1982

The Board of Directors held their regular quarterly meeting from 1:00 p.m. to 4:00 p.m.

Registration took place from 4:00 p.m. to 7:30 p.m.

The Officers' reception for new Fellows and their spouses was held from 6:00 p.m. to 7:00 p.m.

An informal reception for all members and guests was held from 7:00 p.m. to 8:30 p.m.

Monday, May 24, 1982

President Steven H. Newman opened the meeting at 8:30 a.m. and introduced Mr. Gary Granoff, Chief Actuary and Director, Rating Division, Florida Insurance Department. Mr. Granoff, a member of our Society, welcomed the Society to the State of Florida.

Mr. Newman then began the business session with the recognition of new Associates and an awarding of diplomas to all new Fellows. The names of the 12 new Fellows and 52 new Associates follow.

FELLOWS

Gary F. Bellinghausen	Thomas S. Johnston	Jan A. Lommele
James K. Christie	Steven W. Judd	Glenn G. Meyers
Walter J. Haner	Leon W. Koch	Gary V. Nickerson
Steven C. Herman	Michael R. Larsen	Patrick L. Whatley

ASSOCIATES

Barbara J. Addie	Bill Faltas	Andrew W. Moody
Stephen A. Belden	William R. Gillam	William F. Murphy
Abbe S. Bensimon	Joseph A. Gilles	Charles A. Pelletier
Terry J. Biscoglia	Bryan C. Gillespie	Cynthia M. Potts
Francois Boulanger	Terry L. Goldberg	Deborah M. Rosenberg
David S. Bowen	Richard A. Hofmann	Lois A. Ross
Paul Braithwaite	Kenneth J. Hoppe	Vincent T. Rowland, Jr.
Claudette Cantin	Stephen L. Kolk	Joy A. Schwartzman
Thomas S. Carpenter	Richard Kollmar	Jerome J. Siewert
Li-Chuan L. Chou	Gary I. Koupf	Daniel L. Splitt
Edward D. Cimini, Jr.	Kung L. Leung	Elisabeth Stadler
John D. Coffin	Kevin F. Lonergan	Warren B. Tucker
James E. Costner	Thomas X. Lonergan	Lawrence A. Vitale
Linda A. Dembiec	Aileen C. Lyle	David R. Whiting
Robert B. Downer	Paul C. Martin	David A. Withers
Howard M. Eagelfeld	Karol A. McIntosh	Mark E. Yingling
Alice H. Edmondson	Madan L. Mittal	Ruth E. Youngner
Valere M. Egnasko		

Reviews of papers previously submitted were then presented. Mr. Robert Deutsch and Mr. Robert Rothman reviewed Mr. Robert Sturgis's paper, "Actuarial Valuation of Property/Casualty Companies." Mr. Roy Morell reviewed Mr. Charles Berry's paper, "A Method for Setting Retro Reserves." Mr. Mark Fiebrink then reviewed Mr. Glenn Meyers's paper, "An Analysis of Retrospective Rating."

Following the review of old papers, there was an author's summary of new papers being presented as follows:

1. "A Note on Loss Distributions,"
   by J. Gary LaRose;
2. "A Model of Industry General Liability Net Written Premium,"
   by Gregory N. Alff and James R. Nikstad;
3. "Scale Adjustments to Excess Expected Losses,"
   by Gary G. Venter;
4. "The Optimal Use of Depopulation Credits in the Private Passenger Auto Residual Market,"
   by Thomas J. Kozik.

Mr. P. Adger Williams, President-elect of the American Academy of Actuaries, summarized the most significant current activities of the Academy.

Mr. George D. Morison presented the report of the Committee on Management. This committee recommended significant organizational changes to the Society which will be considered by the Board of Directors and the general membership later this year.

The keynote address for the meeting was then delivered by Mr. Gerald Sullivan of Gerald J. Sullivan & Associates.

Mr. Ronald Ferguson presented the summary of the eleven Discussion Papers to be presented during a later part of the meeting.

Following a lunch break, the meeting reconvened for concurrent workshop sessions for each of the Discussion Papers. The sessions were as follows:

1. "Federal Income Taxation of Self-Insurance Techniques"
   Author: Robert J. Finger, Consulting Casualty Actuaries
   Reviewer: Richard E. Sherman, Coopers & Lybrand

2. "A Note on Evaluating Aggregate Retentions for Special Risks"
   Author: Roy P. Livingston, CIGNA
   Reviewer: James J. Kleinberg, St. Paul Risk Services

3. "Parameter Uncertainty in the Collective Risk Model"
   Authors: Glenn G. Myers, CNA
   Nathaniel Schenker, CNA
   Reviewer: Roy K. Morell, Liberty Mutual

4. "Reinsuring the Capital/Specialty Company"
   Author: Lee R. Steeneck, General Reinsurance
   Reviewer: Alan R. Sheppard, Scor Reinsurance

5. "A Capacity Management Model Based on Utility Theory"
   Authors: Naomi Kleinman, Connecticut Mutual Life
   John Cozzolino, John Cozzolino & Associates
   Reviewer: Richard G. Woll, Hartford

6. "Transformed Beta and Gamma Distributions and Aggregate Losses"
   Author: Gary G. Venter, NCCI
   Reviewers: Orin M. Linden, The Home
   Fred Klinker, The Home

7. "Physician Professional Liability Insurers"
   Author: Allan Kaufman, Peat, Marwick, Mitchell
   Reviewer: Charles C. Hewitt, Jr., Metropolitan Reinsurance

8. "Optimization of Excess Portfolios"
   Authors: Philip E. Heckman, CNA
   Phillip Norton, CNA
   Reviewer: Lyle W. DeGarmo, American Agricultural

9. "Estimating Probable Maximum Loss with Order Statistics"
   Author: Margaret E. Wilkinson, Warren, McVeigh & Griffin
   Reviewer: Albert J. Beer, The College of Insurance

10. "Focused Marketing for Large Accounts"
    Author: Alfred O. Weller, Frank B. Hall
    Reviewer: James A. Hall, III, Coopers & Lybrand

11. "Optimum Use of Insurance, Loss Prevention, Loss Reduction and Self Insurance"
    Author: Martin Rosenberg, Reliance
    Reviewer: Frederick O. Kist, Tillinghast, Nelson & Warren

The Discussion Paper sessions were moderated by Messrs. C. K. Khury, Robert Anker, Robert Miccolis, Michael Toothman, Wayne Fisher, and LeRoy Heer.

A reception was then held for the attendees from 6:30 p.m. to 8:00 p.m.

Tuesday, May 25, 1982

The concurrent sessions on Discussion Papers were repeated from 8:30 a.m. to 12:00 noon.

Following a lunch break, concurrent workshops were conducted, as noted below.

1. New Paper
   "A Note on Loss Distributions"
   J. Gary LaRose
   Employers Reinsurance

2. New Paper
   "A Model of Industry General Liability Net Written Premium"
   Gregory N. Alff and James R. Nikstad
   Wausau Wausau

3. New Paper
   "Scale Adjustments to Excess Expected Losses"
   Gary G. Venter
   NCCI

4. New Paper
   "The Optimal Use of Depopulation Credits in the Private Passenger Auto Residual Market"
   Thomas J. Kozik
   Allstate

5. Refresher Course on Commercial Property Ratemaking
   Moderator: Milan E. Konopa
   Insurance Services Office

   William N. Bartlett
   Continental

   Michael C. Dolan
   CIGNA

6. Discussion of Committee on Management Report
   Moderator: C. K. Khury
   Prudential Property & Casualty

Wednesday, May 26, 1982

A panel discussion was conducted from 8:30 a.m. to 9:30 a.m. The topic and participants were:

"Federal Reserve Policy and the Economy"

Moderator: Sheldon Rosenberg
Insurance Services Office

Robert P. Eramo
Hanover

Stanley Wright
Data Resources

From 9:30 a.m. to 10:15 a.m. the business session continued with the presentation of the Michelbacher Award. The award was presented to Mr. Glenn G. Meyers.

Mr. Frederick W. Kilbourne then spoke to the membership on "Public Relations and Communications."

A panel discussion was conducted from 10:45 a.m. to 11:45 a.m. The topics and participants were:

"Treaty Reinsurance Negotiations"

Moderator: Russell S. Fisher
General Reinsurance

Michael D. Covney
North American Reinsurance

Frank Neuhauser, Jr.
AIG Risk Management

President Newman adjourned the meeting at 12:00 noon.

In attendance as indicated by registration records were 162 Fellows, 116 Associates, 10 students, 16 subscribers, and 20 guests. The list of attendees follows.

FELLOWS

Aldorisio, R.
Anker, R. A.
Asch, N. E.
Ashenberg, W. R.
Bartlett, W. N.
Bass, I.
Bassman, B. C.
Beer, A.
Bell, L. L.
Bellinghausen, G. F.
Bennett, N. J.
Bill, R.
Bishop, E. G.
Bornhuetter, R. L.
Bovard, R. W.
Brannigan, J. F.
Byrne, H. T.
Carbaugh, A. B.
Carter, E. J.
Cheng, J. S.
Christie, J. K.
Collins, D. J.
Conger, R. F.
Covney, M. D.
Crowe, P. J.
Demers, D.
Dempster, H. V.
Dolan, M. C.
Donaldson, J. P.
Evans, G.
Eyers, R. G.
Faber, J. A.
Faga, D.
Fagan, J.
Farnam, W. E.
Fein, R. I.
Ferguson, R. E.
Fiebrink, M. E.
Finger, R. J.
Fisher, W. H.
Flaherty, D. J.
Fresch, G. W.
Furst, P. A.
Gallagher, T.
Gillespie, J. E.
Graham, T.
Grannan, P.
Hachemeister, C. A.
Hafling, D. N.
Hall, III, J. A.
Haner, W.
Hartman, D. G.
Harwayne, F.
Hazam, W. J.
Heer, E. L.
Herman, S. C.
Hermes, T. M.
Hewitt, C.
Hibberd, W.
Honebein, C. W.
Ingco, A. M.
Irvan, R. P.
Jean, R. W.
Jerabek, G.
John, R. T.
Johnston, T. S.
Judd, S. W.
Kaliski, A.
Kallop, R. H.
Karlinski, F. J.
Kaufman, A.
Kelly, A.
Khury, C. K.
Kist, F. O.
Kleinman, J.
Kline, D. F.
Koch, L.
Krause, G. A.
Kuehn, R. T.
Lange, J.
LaRose, G.
Leimkuhler, U. E.
Lerwick, S. N.
Levin, J. W.
Linden, O. M.
Lo, R. W.
Lommele, J. A.
Lowe, S. P.
MacGinnitie, W. J.
Mahler, H. C.
Makgill, S.
McClenahan, C. L.
McClenahan, D.
McClure, R. D.
McConnell, C.
McManus, M. F.
Meyers, G.
Miccolis, R.
Miccolis, J. A.
Miller, D. L.
Miller, M.
Morell, R. K.
Morison, G. D.
Muetterties, J. H.
Munro, R. E.
Nash, R. K.
Neidermyer, J. R.
Nelson, J.
Newman, S.
Nickerson, G.
Niswander, R. E.
Oien, R. G.
Patrik, G.
Pearl, M. B.

Perkins, W. J.
Petersen, B.
Phillips, H. J.
Pollack, R.
Purple, J. M.
Reichle, K.
Reynolds, J.
Richards, H. R.
Richardson, J. F.
Rodermund, M.
Rogers, D. J.
Roland, W. P.
Rosenberg, M.
Rosenberg, N.
Rowland, W. J.
Ryan, K. M.
Salzmann, R. E.
Scheibl, J. A.
Schumi, J.
Scott, B. E.
Sheppard, A.
Sherman, R. E.
Shoop, E. C.
Simon, L. J.
Sobel, M. J.
Squires, S. R.
Stanard, J. N.
Stergiou, E. J.
Streff, J.
Strug, E. J.
Sturgis, R. W.
Swift, J. A.
Teufel, P. A.
Toothman, M. L.
Trudeau, D. E.
Tuttle, J. E.
Tverberg, G. E.
Van Slyke, O. E.
Venter, G.
Weissner, E. W.
Weller, A. O.
Whatley, P. L.
Williams, H. V.
Williams, P. A.
Wilson, J. C.
Wiser, R. F.
Woll, R. G.
Wulterkens, P.

ASSOCIATES

Addie, B.
Alff, G.
Andler, J.
Bashline, D.
Belden, S. A.
Bensimon, A. S.
Biscoglia, T. J.
Blanchard, R. S.
Boulanger, F.
Bowen, D. S.
Brooks, D. L.
Cantin, C.
Carpenter, T. S.
Chorpita, F.
Chou, L. L.
Cimini, Jr., E. D.
Coffin, J. D.
Cohen, H. S.
Connor, V. P.
Costner, J. E.
DeGarmo, L. W.
Dembiec, L. A.
Dodd, G. T.
Downer, R. B.
Driedger, K. H.
Duffy, T.
Duperreault, B.
Edie, G.
Edmondson, A.
Egnasko, G. J.
Egnasko, V.
Fisher, R. S.
Foote, J.
Friedberg, B.
Ghezzi, T. L.
Gillam, W. R.
Gillespie, B. C.
Godbold, M. J.
Godbold, N. T.
Goldberg, T.
Goldfarb, I. H.
Granoff, G.
Gruber, C.
Harrison, E.
Head, T. F.
Heckman, P. E.
Hobart, G. P.
Hofmann, R. A.
Hoppe, K.
Horowitz, B.
Hurley, J. D.
Jensen, J. P.
Kaur, A. F.
Keatts, G. H.
Kleinberg, J. J.
Kolk, S. L.
Kollmar, R.
Koupf, G. I.
Kozik, T.
Leo, C. J.
Leung, K. L.
Liuzzi, J. R.
Livingston, R. P.

Lonegran, K. F.
Lonegran, T. X.
Lyle, A. C.
Marino, J. F.
Marks, R. N.
Martin, P. C.
McConnell, D. M.
McGovern, W. G.
McHugh, R. J.
McIntosh, K.
Meyer, R. E.
Mill, R. A.
Millman, N. L.
Mittal, M. L.
Morgan, S. T.
Mulder, E. T.
Murad, J.
Nolan, J. D.
Parker, C. M.
Patterson, D. M.
Peacock, W. W.
Pei, K.
Pelletier, C. A.
Potts, C. M.
Pratt, J. J.
Ritzenthaler, K.
Rowland, V. T.
Sandler, R. M.
Sansevero, Jr., M.
Schneiker, H.
Schwartzman, J.
Siewert, J.
Silberstein, B.
Singer, P. E.
Skrodenis, D. P.
Splitt, D.
Stadler, E.
Suchoff, S. B.
Swisher, J. W.
Taranto, J.
Tucker, W. B.
Vitale, L.
Vogel, J. F.
Wade, R. C.
Walker, G.
Whatley, M. W.
Whiting, D.
Wilkinson, M.
Yingling, M. E.
Young, E. W.
Young, R. G.
Youngerman, H.
Youngner, R.

STUDENTS

Arvantis, R.
Boyd, W. A.
Colvin, S.
Deutsch, R. V.
Epstein, M.
Hutter, H. E.
Kane, A.
Neale, C.
Weimer, W. F.
Withers, D. A.

SUBSCRIBERS

Allen, T. C.
Bell, A. M.
Coutu, G. R.
Gutman, E.
Hager, G. A.
Hatfield, B. D.
Hopkovitz, M.
Koester, S. M.
Kraysler, S. F.
O'Shea, H. J.
Pope, D. W.
Posnak, R.
Reott, J. A.
Rothman, R.
Spangler, J. L.
Vandernoth, J. P.

GUESTS

Almer, M.
Belton, E. F.
Chang, C. E.
Cozzolino, J.
Duvall, R. M.
Eramo, R. P.
Grynkiewicz, C. M.
Grynkiewicz, M. C.
Jensen, P. A.
Jones, T. L.
Keating, R.
Kellison, S. G.
Knox, F.
Longcrier, R.
Lyle, T. A.
Rushton, I. L.
Swick, G. B.
Thomas, A. M.
Vosburgh, J.
Whitby, O.

BRIAN E. SCOTT
Secretary

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PROCEEDINGS

November 7, 8, 9, 1982

CYCLES AND TRENDS

PRESIDENTIAL ADDRESS BY STEVEN H. NEWMAN

This is quite a challenge. You assume a certain risk when you set out to speak at some length about our profession or our business, knowing that your words will be duly recorded in the *Proceedings* where they will most certainly be read for years . . . by future presidents preparing their own speeches.

Actuaries are trained to be critical, and I can easily imagine one of my successors approaching me some years from now and saying, "I found your address to be good and original, Steve. However, the part that was good was not original, and the part that was original was not good."

I have, in fact, borrowed my theme from an interesting conversation I had earlier this year with a gentleman trained as an economist, and now the Director of Research in the Home's investment department. We were talking about investments, but his basic premise would apply, I think, to many other areas. In a world where change is a given, there are, he had observed, two kinds of people—those who are predominantly "trend thinkers" and those who are predominantly "cycle thinkers."

Once something has started to change, trend thinkers expect the change to continue, and in the same direction. Jury awards have been getting bigger for years; trend thinkers expect that they'll be bigger still in the future. American heavy industry is declining and plants are closing; trend thinkers expect the decline to continue, and they look to the growing service sector to take its place in the American economy.

Cycle thinkers, on the other hand, think that most trends run just so long before they eventually slow, stop, and change direction. What goes up will in due course come back down, and vice versa. Americans have been spending successively higher percentages of their income on medical services each year. Cycle thinkers say it clearly can't go on forever, and sooner or later they expect the rise in the cost of health care relative to other items to go into reverse. Physical fitness has been enjoying a boom. Americans are losing weight, exercising in record numbers, and investing a fortune in sports equipment and health club memberships. Cycle thinking investors are putting their money elsewhere, trying to guess what everybody's favorite pastime will be next.

It's often difficult for the two types to coexist. My friend tells the story of the stockbroker, a cycle thinker, who calls his customer, a trend thinker, to recommend the purchase of United Widget. Following the broker's advice, the customer takes a position in the stock. A week later the broker calls back. "I have great news," he says. "United Widget is down 5 points and now you can buy some more at even less than you paid last week." You don't have to be a trend thinker to see where that conversation is headed!

In fact there are cycles, and there are trends, too. The problem is to tell which it is we have in a particular case. Is United Widget on a temporary downturn and about to make a comeback, or will it just keep right on declining into bankruptcy?

And, how about our own business? Right now, underwriting profits have just about disappeared from the books of the vast majority of our companies. Earlier this year, in their public pronouncements, the chief executives of some of the largest ones had declared this to be not just a down cycle but a permanent trend, a new reality. Total return, they said, had now been recognized as the important number; combined ratios under 100 were not necessary and indeed were a thing of the past. More recently, as underwriting losses are piling up and investment income isn't growing any more, attention is again being paid to those voices calling for a return to underwriting for profit. Will it happen?

I think a long term perspective is particularly useful in distinguishing trends from cycles. *Forbes* magazine runs a regular column called "Flashbacks" in which it reprints items from back issues of the magazine published 25 and 50 years before. It's interesting, sometimes surprising, and illustrates over and over again the truth of the subtitle: "The more things change, the more they stay the same." So once again I will borrow an idea from another and invite you to view some Flashbacks from our own history. Where were we ten years ago? Twenty-five years ago? Fifty years ago?

Ten years ago the CAS was here in San Francisco for three days at the Hotel St. Francis. The day before that November meeting opened, Richard Nixon and Spiro Agnew had been reelected in a landslide. The cover of *Time* magazine that week displayed a dove and the words "The Shape of Peace." The reference was to Vietnam and Henry Kissinger's promise that a settlement was imminent. The cover of *Time* the next week was a seagull, in honor of the unlikely bestseller *Jonathan Livingston Seagull*. Called by some "Horatio Alger in feathers," Jonathan's message to the world as he soared through the sky was "Find out what you love to do and do your darndest to make it happen." Remember those days?

Jonathan wasn't the only thing soaring. A week later the Dow would break 1000 for the first time. And stock insurance companies would finish the year with both the largest dollar underwriting profit and the largest investment profit in their history. The industry combined ratio for 1972 was 95.4, following a 95.8 for 1971.

The *National Underwriter* that week noted that the Colorado legislature voted "no" on no-fault auto insurance, and was the 30th state to reject the idea. "The year 1972 has belonged to the trial lawyers," T. Lawrence Jones of the A.I.A. said, lamenting the passing of what he thought might be the last year for states to control no-fault reform before the federal government took it out of their hands. Another article was titled "Products: Sick Line That Needs Nursing."

LeRoy Simon was President of the CAS that year. The membership was just over 500—about half of what it is today. There were seven new fellows including present Treasurer Mike Walters, a record 37 new associates, and nearly 300 in total at the meeting.

Six new papers were presented. All the topics are still current—IBNR, allocated loss expense reserves, nuclear property insurance, experience rating, catastrophe reinsurance and premium-to-surplus ratios. Two of them are still on the exam syllabus ten years later. And LeRoy Simon's presidential address entitled "Know Thyself, Actuary" could with very few changes be delivered today. Just two months ago I wrote in the Academy's newsletter of my concern about actuaries working outside their experience—benefits actuaries working in workers' compensation, for example. Roy, I now find, said the same thing in 1972. Earlier this year in our *Actuarial Review* I'd written about the many areas of insurance operations, besides pricing and reserves, where actuaries should be active. Roy made this point as well. At no time this year, though, did I take the opportunity to comment on the resounding success of our pricing activities as evidenced by the profit margins realized by our companies, as Roy did ten years ago, although he admitted to being more than a little surprised by it all. Can you imagine it? Five points, just like in the formulas!

There was no talk either of resounding success in "Doc" Masterson's presidential address 25 years ago in 1957. In those days, presidents addressed the Society in both May and November, and Doc's May address that year had been titled "Lessons from Adversity." Things were no better when the CAS met that November for two days at the Sheraton in Philadelphia.

1957 may have been a great year for Elvis Presley, who had eight gold records, but for a lot of others it was not the best of times. President Eisenhower, in the first year of his second term, had in September ordered 1000 federal troops into Little Rock, Arkansas to protect nine black school children. And later that fall Americans were stunned when the Soviet Union successfully launched Sputnik I and Sputnik II into earth orbit.

On the insurance front, after nearly 25 years of GAAP underwriting profits, the combined ratio the year before had reached 100.5. There had been an underwriting cycle all along, but the ups and downs represented more profit and less profit from underwriting, not losses. The stock companies' combined ratio for 1957 would "soar" to 102.9 by year-end, a level that would not be reached again until 1974.

Compulsory auto liability insurance had become the law in New York in February of 1957. The week of the CAS meeting the *National Underwriter* described the New York auto liability loss situation as "astonishingly bad." The New York Insurance Department turned down the request of the NBCU and the MIRB (these were ISO's stock and mutual predecessors) for a 9.5% rate increase even though the bureaus claimed it was half of what they needed. The department objected, according to the *National Underwriter*, both to the fact that the experience used—1955 and 1956—was too immature, with too many estimates, and not enough developed losses, and also that no account was taken of the new 1957 compulsory law.

The CAS had 330 members in 1957—roughly one-third of our present membership. About 100 were at the November meeting. Fourteen associates and eleven new fellows were admitted including two future CAS presidents—Ron Bornhuetter and Adger Williams. (Both would subsequently rise to head the entire profession as presidents of the American Academy of Actuaries, an entity that wasn't even conceived of back then.)

There were four new papers submitted, two on auto and one on fire ratemaking, and one on graduating the excess ratios in Table M. Today, the one that is perhaps most interesting concerned auto insurance classifications, and included a history of classifications up to then. The first—and at the time only—rating variable was introduced in 1921. It was "use," with a surcharge for "business use." "Driving record" was not used until 1929, and "age" debuted in 1939. "Mileage," "marital status," and "parenthood" (parents paid less) were first introduced as rating factors in 1953, with "sex" and "driver training" appearing in 1955. The author of the paper, Joseph Muir, was General Manager of MIRB and a new associate. "The philosophy of distributing loss experience among *all* insureds, irrespective of risk hazard," he wrote, "no longer prevails to any extent. It has been rejected in favor of a policy of fair discrimination with respect to rating criteria which are measurable in terms of loss costs." Ah, yes. We thought it was all settled.

But some things, perhaps, are settled. Doc Masterson, who was serving his second term as CAS President, entitled his presidential address that November "Professional Responsibilities of the Members of the Casualty Actuarial Society." Twenty-five years have not invalidated his main points. The professional actuarial society, he said, is a means through which existing knowledge is passed along and new ideas are tested. The CAS can be a particularly effective forum because its members represent companies, bureaus, consultants, regulators, and educators, and therefore many points of view. The practicing actuary uses the profession's collective knowledge and his judgment to analyze new situations for his company or client. Whatever his recommendations, he must be able to explain his reasoning with clarity to non-actuaries. That was a good summary of our purpose in 1957. It is still a good summary today.

And it is a good summary of what the CAS was about even fifty years ago. The year was 1932 and the Great Depression had grown progressively worse for three long years. Just ten days before the November meeting, Franklin Delano Roosevelt had defeated Herbert Hoover overwhelmingly. The platform he ran on included proposed federal old age and unemployment insurance, and more government responsibility for human welfare. It also called for a balanced budget, a sound currency, and more economy in government through a 25% *cut* in spending. But as Roosevelt's advisers said—platforms are to run on, not to stand on. Perhaps the most popular plank in the platform had been the one for the repeal of prohibition which, when it came, would make the world safe for official CAS cocktail parties, the first of which, I'm told, didn't occur until 1946.

The CAS in 1932 already had 306 members—just 24 fewer than 1957—but only 56 were present for the one-day meeting at the Hotel Pennsylvania in New York.

Thomas F. Tarbell, Casualty Actuary of the Travelers, was President of the CAS that year. His own career represented some of the diversity of experience Doc Masterson would cite 25 years later as a strength of the CAS. He had worked at the Mutual Life of New York and at the Aetna, and had been the actuary of the Connecticut Insurance Department, before going to the Travelers. A 1920 fellow by examination, he now set out in his presidential address to apply the profession's collective knowledge, and his judgment, to a new situation for his audience.

The new situation was deflation. Severe deflation. If you think of the CPI in 1929 as 100, it was 81 in 1932 and destined to go lower. When Tarbell spoke on "The Effect of Changes in Values on Casualty Insurance," he was talking about downward changes. Overall, he judged the effect to be adverse. Premium volume was way down—Tarbell predicted a 25% decline from 1931—and in spite of strenuous efforts to reduce expenses, you could not hope, he said, to bring them down immediately to the level of the reduced premium volume. The effects on losses were mixed. Tarbell thought declining values might be favorable for some first party coverages, uncertain for liability lines, and pretty poor for workers' compensation.

There were three new papers presented at the meeting, all worthy of note. A Canadian member discussed the new open rating law for automobile insurance in Ontario, wondering whether the victory over government regulation was worth the price of the rate-cutting that resulted. Does that question sound familiar to any workers compensation insurers today? Another paper discussed the new Wisconsin Unemployment Compensation Act, the first in the country. One in five Americans in the labor force was unemployed in 1932, and advocates of the new law pointed out that the burden of irregular employment fell directly on the worker. They demanded that business help to pay the social costs caused by their own irregular operations. Employers feared the increased cost of operating under the new law and the competitive disadvantage to Wisconsin businesses compared to other states which didn't require unemployment insurance. With our current high unemployment, the same arguments, pro and con, have been advanced about a proposed California law concerning workers' rights when plants are closed. And finally, even in those dreary days, the CAS was a forum for new ideas. It was at that meeting that Francis Perryman presented his classic paper "Some Notes on Credibility."

The CAS meeting was a news item in the *National Underwriter* that week. Its report mentioned two additional discussion topics—alternatives to payroll as an exposure base for workers compensation and (remember this is 1932) no-fault automobile insurance. And I must finally mention two other items that appeared among the usual complement of reports on the meetings of agents' associations. "The backers of the American agency system must continue their fight for its perpetuation," the president of the Tennessee agents warned. Particularly important in this regard, he thought, was the enforcement of the bank agency rule of the National Association of Insurance Agents which forbade the selling of insurance by banks. The president of the California Agents' Association was also wondering whether the American agency system would survive. The threat he saw came from the development of production branch offices and "the continuous trend of the company mind to reduce acquisition costs without regard to the ultimate effect on the producer."

At times it seems as if we have come full circle, doesn't it? Isn't it amazing how similar the problems in our industry have been during all these years? Comments made 50 years ago are repeated today as if presented for the first time. Have things in fact hardly changed? Is all of what we see part of a never-ending cycle?

To this we must answer: "No." There are real trends, permanent changes in the way things are or in the ways things are done. But just as we often overlook the fact that many of today's problems have been around before, we can also fail to recognize a real change—even after it has already happened. This last observation was the point that James Robinson, Chief Executive of American Express, was trying to get across when he read the following letter to insurance executives gathered for the annual meeting of the National Association of Casualty and Surety Agents & Executives at the Greenbrier last month.

"Dear President Andrew Jackson:

"The canal system of this country is being threatened by the spread of a new form of transportation known as railroads. The federal government must preserve the canals for the following reasons:

"*One*—If canal boats are supplanted by railroads, serious unemployment will result. Captains, cooks, drivers, repairmen and lock tenders will be left without means of livelihood, not to mention the numerous farmers now employed in growing hay for horses.

"*Two*—Boat builders would suffer and tow-line, whip and harness makers would be left destitute.

"*Three*—Canal boats are absolutely essential to the defense of the United States. In the event of the expected trouble with England, the Erie Canal would be the only means by which we would move the supplies so vital to waging modern war.

"For the above mentioned reasons, the government should create an Interstate Commerce Commission to protect the American people from the evils of railroads and to preserve the canals for posterity.

"As you may well know, Mr. President, railroad carriages are pulled at the enormous speed of 15 miles per hour by engines which in addition to endangering life and limb of passengers, roar and snort their way through the country-side setting fire to crops, scaring the livestock and frightening women and children. The Almighty certainly never intended that people should move at such break-neck speed."

Signed—Martin Van Buren, Governor of New York, January 1829.

Now Mr. Robinson's particular point, in the context of his talk, was that it was pointless to speculate on whether the insurance business would be affected if banks, securities firms, insurance brokers, and others, all started to view their business as (in the catch-all phrase) "financial services." His perspective was that such an outlook is now a fact of life, a trend, a present reality to be reckoned with. In fact, he implied that in a relatively short time the current provincial perspective of the insurance business would be viewed as a mockery, just as Governor Van Buren's canals are today.

So let us look at our whole industry in the same critical light. Which do we see among today's provocative issues that might be viewed as temporary (or cyclical) phenomena? Which are the here-today and here-to-stay realities, or the shape of things to come?

What about insurance as an integral part of the larger business called financial services? Or the entry of banks into the insurance business? Or the demise of the independent agency system? Or the vertical integration of insurance companies and producers?

Or, closer to home, what about GAAP versus statutory accounting? Or discounting long-tail liabilities? Or the inclusion of investment income projections in insurance pricing?

Time alone will provide the definitive answers to these questions. But I urge you to think about them, and to do more than think about them. Perhaps in some cases we can even influence the overall outcome by what we say and do.

But certainly, we can affect our own company's, or client's, or constituency's future by our ability to look forward, and to plan for the future as we see it unfolding.

For you see there's another noteworthy phenomenon around these days. *We're in the picture*. The CAS has grown remarkably in size and stature, particularly during the last 10 years. Its impact on the non-life insurance industry, both in the U.S. and abroad, and its importance within the business community, have grown as well. Our members are no longer merely technicians whose analyses are sometimes understood only by their peers. Today, many are presidents and chief executives of their companies, and others occupy very senior positions in state and federal government agencies.

Is this a cycle or a trend? Will our influence and importance continue to grow or, having risen, start to fall? More than anything else, this depends on us. As professional actuaries we are trained to be familiar with virtually every phase of the insurance business and the environment in which it operates. Our specialty, as I've heard President-Elect Fred Kilbourne say, is the science of risk assessment. These are excellent credentials today for a role of real leadership in our industry.

But credentials are not enough. If we would be leaders, we must think and act accordingly—take stands on controversial issues, make decisions, and follow through with responsible actions. And so our future course, I think, is in our own hands, both individually and as a Society. And this is quite a challenge.

A STRATEGY FOR PROPERTY-LIABILITY INSURERS IN INFLATIONARY TIMES

STEPHEN P. D'ARCY

Abstract

The primary business of the insurance industry is insurance underwriting. The insurance business is also engaged in the investment of funds generated by its underwriting activity as well as the capital and surplus. Thus, the operating results of insurers are affected by two components: underwriting results and investment returns. Historically, both of these components have been negatively correlated with the rate of inflation. Since insurers have considerable (but not complete) discretion in determining their investment mix, they are free to structure their investment portfolios to balance the adverse effects of inflation on underwriting profit margins. Thus, an investment strategy that correlates investment returns positively with the inflation rate is desirable during inflationary times. The purpose of this paper is to develop a method of inflation immunization for the property-liability insurance industry. The inflation immunized investment portfolio, based on experience during the period 1951 through 1981, involves a significant investment in Treasury bills. The strategy for reducing the effect of inflation on operating results presented in this analysis is one means by which insurers may cope with an inflationry environment.

Analysis of economic data indicates that inflation has both increased and become more variable over the past 15 years. Inflation has a considerable effect on insurance profitability by impacting both components of insurance operations, underwriting and investments. Since the elimination of inflation in the near future is unlikely despite progress in reducing the rate of inflation, the insurance industry must decide whether to continue to accept the risk of uncertain inflation or whether to protect itself against inflation. This paper presents a strategy for inflation immunization for the property-liability insurance industry and measures the cost of this strategy.

First the history of inflation in this country since 1926 is discussed. Then the correlation of each of the components of insurance operations with inflation is analyzed. Next the correlation of insurance investment returns with inflation is investigated by examining returns on long term bonds, common stocks, and Treasury bills. Following this discussion, an inflation immunization strategy for the insurance industry is developed. Portfolio theory is then introduced to develop an investment strategy that minimizes the effect of inflation on total insurance operations without diminishing the expected profitability. Next the inflation immmunization determination is updated using data through the end of 1981. Finally the results are summarized and some conclusions are offered. The method of determining the data and the sources are discussed in Appendix I. Summary statistics of the data are shown in Appendix II.

SECTION 1—INFLATION

Recent economic conditions have made the current rate of inflation a subject of common knowledge. A greater perception of the inflation issue can be obtained by viewing the inflation rate over an extended period of time. Figure 1 illustrates the yearly percentage change in Consumer Price Index measured from December to December for the period 1926 to 1981. This graph indicates that wide swings in the rate of inflation are not uncommon and that the relative price stability of the 1950s is more unusual than the extreme fluctuations of the 1970s.

The deficiencies of the Consumer Price Index (CPI) as an accurate measure of the true inflation rate are widely recognized, but no superior all-purpose inflation index is available.[1] The CPI is a monthly statistical measure of a market

[1] For a description of some of the problems with the CPI, see [8] and [9].

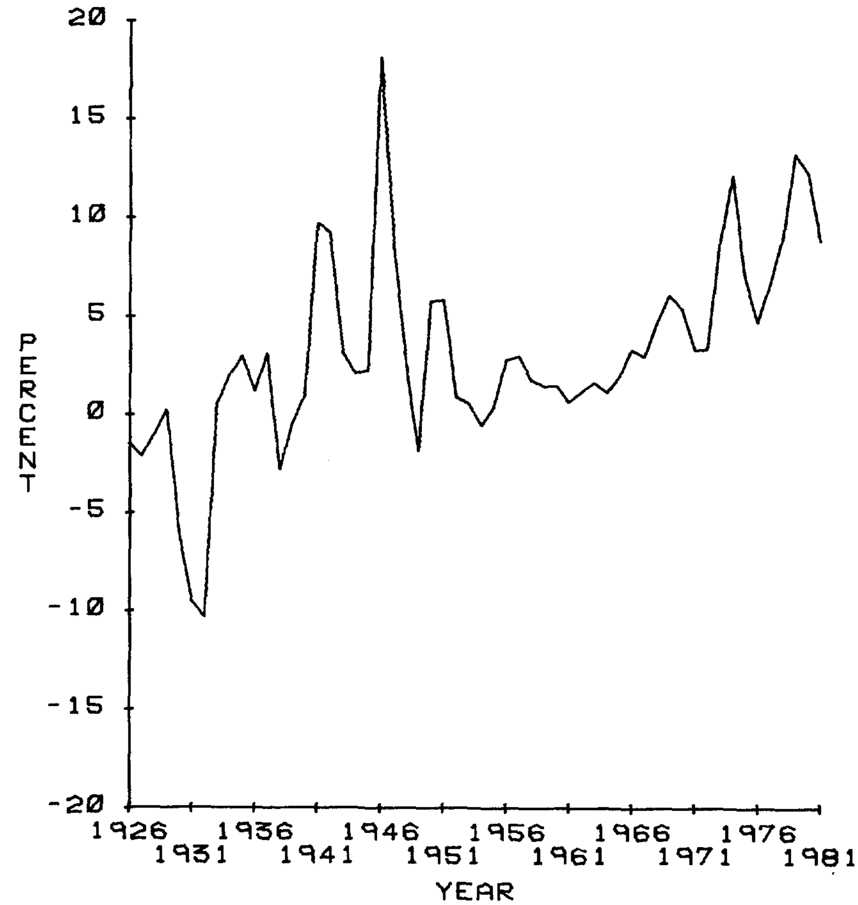

FIGURE 1

YEARLY PERCENTAGE CHANGE IN CPI (DEC.-DEC.)

1926-1981

basket of items commonly purchased by urban workers. Measurement of certain items is lagged due to data collection procedures. The validity of the composite market basket for other segments of society, such as the retired or rural residents, is suspect. Norton Masterson [7] compiled a Claim Cost Index for property-liability insurers that more closely reflects inflation for insurers than the CPI. The CPI is used in this paper for lack of a better index to correlate not only with insurance underwriting, but also with investment returns.

Returning to Figure 1, it can be seen that prices declined significantly during the years 1930 to 1932, the onset of the Great Depression. Price changes then fluctuated in the range of plus to minus 3.0 percent until the beginning of World War II, and then increased significantly. Price controls instituted in 1942 restrained the rate of inflation until 1946 when their removal allowed the inflation rate to hit an as yet unsurpassed 18.2 percent. The ensuing period of relative price stability lasted until the late 1960s. Price controls during the period 1971 to 1973 again restrained the inflation rate until controls were lifted.

The change in prices in a price control period are not indicative of the true rate of inflation, according to Eugene Fama [3]. Fama contends that price controls substitute nonmonetary costs, such as waiting in line, shortages, and inconvenience, for monetary costs. Removal of controls then allows monetary costs, which are measured by the CPI and other price indexes, to catch up with the true cost of goods and services. This reasoning explains much of the variation in the rate of inflation just before, during, and immediately following price control periods. Nevertheless, the CPI still represents a measure of the cost of items to insurers for claims and, indirectly, wages, and for investors in determining interest rates and required rates of return. The prior inflation spikes of 1946 and 1974 can be explained by the lifting of price controls. The inflation rate of 1979 has the distinction of being the first time in 54 years that double digit inflation occurred other than as a result of ending price controls.

SECTION 2—COMPONENTS OF INSURANCE RISK

Insurance profitability is derived from the combination of two separate components, underwriting and investments. Underwriting profitability depends upon factors such as the adequacy of rate levels, competition, and catastrophe experience. Inflation affects underwriting profitability since, for those lines in which the price is not a function of the amount of coverage provided, rate level adjustments must continually be made to maintain adequate rates. Use of past data and delays, both internal and regulatory induced, produce inadequate rate levels under inflation. Automobile insurance provides a prime example. For coverages in which the insurance premium increases in line with inflation, the rate lag is less of a problem. Examples are inflation-adjusted Homeowners policies and business policies rated on the value of wages or sales.

Unanticipated inflation also affects loss reserve development. Loss reserves include a factor representing the expected rate of increase in claim costs. This factor can either be explicitly indicated and incorporated in the loss reserve determination or, more likely, it may simply be included implicitly in the loss reserving methodology. Loss reserves established based on paid or incurred loss development, for example, include as the expectation of future inflation a weighted average of past inflation rates. Unexpected changes in the inflation rate for claims will cause loss reserves to be deficient or redundant. This development will affect the calendar year combined ratio, commonly used to evaluate profitability and used in this paper. A higher than expected inflation rate will cause profitability to decrease, whereas a lower than expected inflation rate will increase profitability.

The statutory underwriting profit margin for stock property-liability insurers during the period 1926 to 1981 is shown in Figure 2, along with the change in the CPI each year. A pronounced negative relationship between the inflation rate and the underwriting profit margin is apparent by observing the extreme values. High underwriting profitability occurs in 1938, when price levels dropped. Underwriting profitability first reduced in 1942 after inflation increased, and then increased as inflation reduced in 1943. Underwriting profitability was high in 1948 and 1949 as inflation reduced. The pattern continued through the 1960s and 1970s with underwriting losses slightly lagging the inflation spurt in 1974 and reduction in 1976.

The pre-1933 period does not conform with the negative relationship outlined above. Underwriting profitability declined in 1930, 1931, and 1932 as price levels dropped substantially. One possible explanation for this atypical correspondence is the pervasive effect of the Depression. Despite price level reductions, economic conditions were so poor that insurance premium receipts declined, causing expense ratios to climb. Loss ratios jumped for Fire Insurance, Accident and Health, Workers' Compensation, and most substantially for Fidelity and Surety [2]. Depressed economic conditions led to increased losses in part from moral hazard, and likely would do so again under similar circumstances. However, because the concern here is for a strategy to deal with inflation, the deflationary period up through 1932 is not considered in developing the statistical relationships used in this model. Therefore, the usefulness of this

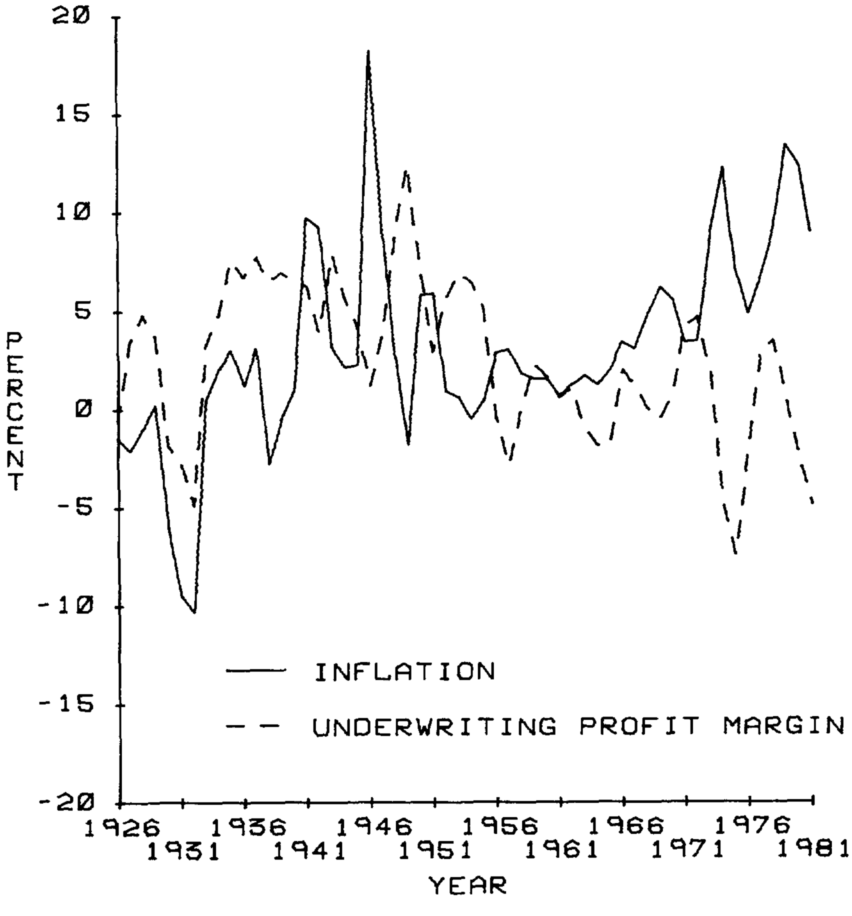

FIGURE 2

STATUTORY UNDERWRITING PROFIT MARGIN AND YEARLY PERCENTAGE CHANGE IN CPI (DEC.-DEC.) 1926-1981 model is restricted to inflationary conditions and does not necessarily apply to periods of deflation.

For the period 1933 to 1981, the relationship between underwriting profit margin and inflation, based on ordinary least squares regression,[2] can be expressed as:

$$\begin{aligned} UPM_t &= 4.36 - .389\, INF_t + e_t \\ T &= -3.079 \text{ (significant at the 1.0\% level)} \\ R^2 &= .168 \end{aligned}$$

where UPM = underwriting profit margin (statutory)
INF = inflation rate (percent change in the CPI)
e = error term

Later other variables will be introduced and incorporated in this analysis. Data for some of these variables are either not valid or not available prior to 1951 or after 1976. To simplify the presentation the same time period, 1951 through 1976, is used initially for all segments of the analysis to illustrate the methodology. The portion of this analysis for the variables where data are available through 1981 is updated later. For the common period 1951 through 1976, the relationship between underwriting profit margin and inflation was:

$$\begin{aligned} UPM_t &= 2.96 - .617\, INF_t + e_t \\ T &= -3.029 \text{ (significant at the 1.0\% level)} \\ R^2 &= .277 \end{aligned}$$

The significant negative relationship confirms the expected and observed negative correlation between underwriting profitability and inflation. The amount

[2] The use of regression methodology to analyze time series data depends on the consistency of the data base and the absence of nonrandom changes. Shifts in the line of business mix of property-liability insurers, the introduction of trend factors and loss development factors in ratemaking, and societal changes create the possibility of inappropriate results for the regression of underwriting profit margins against the inflation rate. However, analysis of the residuals of this relationship indicates no unusual patterns in recent years. The actual values do not consistently fall either above or below the fitted values. Thus, although this problem should be kept in mind while applying the techniques described in this paper, it does not appear to create serious problems for the data used here.

of variation in underwriting profitability that is explained by inflation (R^2 = .168 and .277) is not high, as many other factors impact insurance underwriting profitability. However, inflation does significantly affect underwriting profit margins.

Investment profit or loss, the other component of profitability for property-liability insurers, is the total of investment income (dividends or interest), realized capital gains or losses for bonds and real estate, and realized and unrealized capital gains and losses for stocks. Unrealized capital gains or losses on bonds that qualify for amortization valuation are not a factor in statutory investment profit or loss for insurers. Inflation tends to cause interest rates on bonds to increase, thus increasing investment returns. The loss in value on outstanding bonds that accompanies the increase in interest rates on new issues as inflation increases, although a consideration in overall financial planning for insurers, does not affect statutory accounting results if the loss is not realized. Variations in market values of stocks flow directly into overall insurance profitability.

When the realized losses on bonds and real estate plus the realized and unrealized losses on stocks exceed the investment income from dividends and interest, as occurred most recently in 1973 and 1974, the total investment return is negative. The investment income in this case is offset by the loss of principal producing negative total returns.

The insurance investment return may be calculated by dividing the investment profit or loss including investment income for each year by the mean investable assets of insurers for that year.[3] Some admitted assets for the insurance industry, such as premium balances, do not produce investment income. Investable assets for the industry have been approximated by multiplying total admitted assets by .90.[4] Insurance investment return for stock property-liability insurers during the period 1926 to 1981 is shown in Figure 3, again including the percent change in the CPI. Substantial variation in insurance investment return is evident, but the tendency of the rate of return to peak at inflation lows and hit a bottom at inflation peaks can be observed.

[3] All references to returns in this paper are to nominal rates of return.

[4] Data in *Best's Aggregates and Averages* [2] supports this approximation.

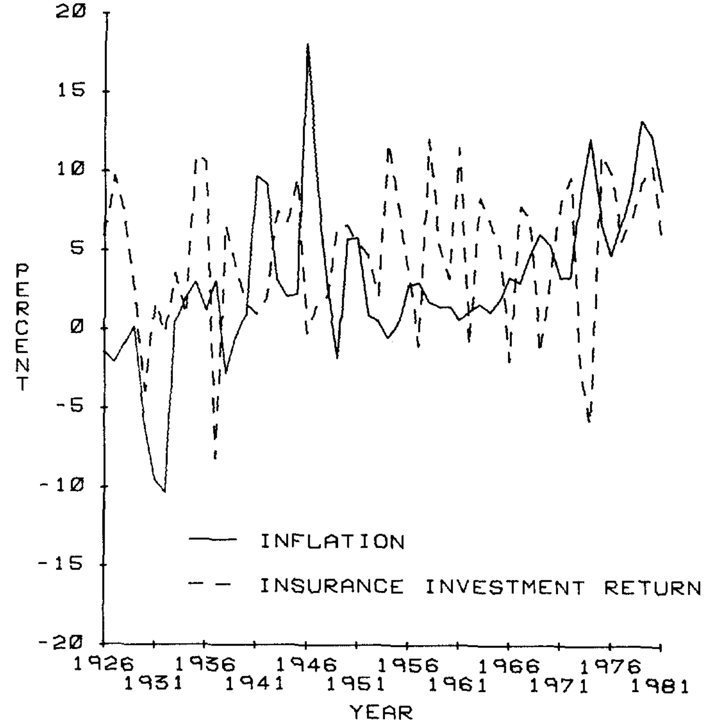

FIGURE 3

INSURANCE INVESTMENT RETURNS ON INVESTABLE ASSETS AND YEARLY PERCENTAGE CHANGE IN CPI (DEC.-DEC.)

1926-1981

Regressing insurance investment returns against inflation using the same procedure applied for underwriting profit margins shows the following results:

1933–1981

$IIR_t = 6.00 - .239\ INF_t + e_t$
$T = -1.513$ (not significant)
$R^2 = .046$

1951–1976

$IIR_t = 7.81 - .817\ INF_t + e_t$
$T = -2.646$ (significant at the 5.0% level)
$R^2 = .226$
where IIR = insurance investment return on mean investable assets.

Thus, inflation is negatively correlated with both insurance underwriting and insurance investment return. With both components of insurance operating results impacted adversely by inflation, inflation presents a severe threat to insurers. However, insurers are not forced to accept this fate. In the next section the investment returns of several investment alternatives are analyzed to give further consideration to the relationship between insurance investment returns and inflation.

SECTION 3—INVESTMENT RETURNS ON ALTERNATIVE INVESTMENTS

The insurance investment return determined previously is the average return of various investments. Insurers' assets consist of government and municipal bonds, corporate bonds, common and preferred stock, real estate, and other investments, as well as some non-income earning assets. The composition of stock insurers' investment portfolios has changed over time. The objective here is to isolate the effect of inflation on the investment returns of four different types of investments: long term government bonds, long term corporate bonds, common stocks, and Treasury bills. The returns include both interest income and changes in market value for the year. In insurance accounting, changes in market value for long term bonds are not included unless the bonds are sold. Thus, the returns on the long term bonds are not comparable to the statutory accounting conventions of the insurance industry, but do reflect the financial effects of long term bond investment. The method used in determining the rates of return and the sources of these data are specified in Appendix I.

Figure 4 illustrates the investment return on long term government bonds during the period 1926 through 1976. Figure 5 illustrates the return on long term corporate bonds. Figure 6 shows the return on common stocks during the period 1926 through 1981. Figure 7 indicates the return on U. S. Treasury bills during that same period. The inflation rate is included on each figure. The regression equations for each relationship are shown in Table 1.

TABLE 1

REGRESSION COEFFICIENTS
1951–1976

$LTG_t = 2.63 + .095\ INF_t + e_t$
$T = .205$ (not significant)
$R^2 = .002$
$LTC_t = 3.93 - .084\ INF_t + e_t$
$T = -.171$ (not significant)
$R^2 = .001$
$CS_t = 22.73 - 3.114\ INF_t + e_t$
$T = -2.675$ (significant at the 5.0% level)
$R^2 = .230$
$TB_t = 1.87 + .556\ INF_t + e_t$
$T = 7.594$ (significant at the 1.0% level)
$R^2 = .706$
where LTG = long term government bond returns
LTC = long term corporate bond returns
CS = common stock returns
TB = Treasury bill returns

Investment returns on long term government bonds and long term corporate bonds are not significantly correlated with inflation. However, common stock returns are significantly negatively correlated with inflation to the point that a 1 percent higher inflation rate reduces common stock returns by more than 3 percent.[5] The amount of variation explained by inflation is low ($R^2 = .230$) as many other factors affect stock prices.

[5] For an explanation of the basis of this relationship, as well as a review of the literature on this topic, see Feldstein [4].

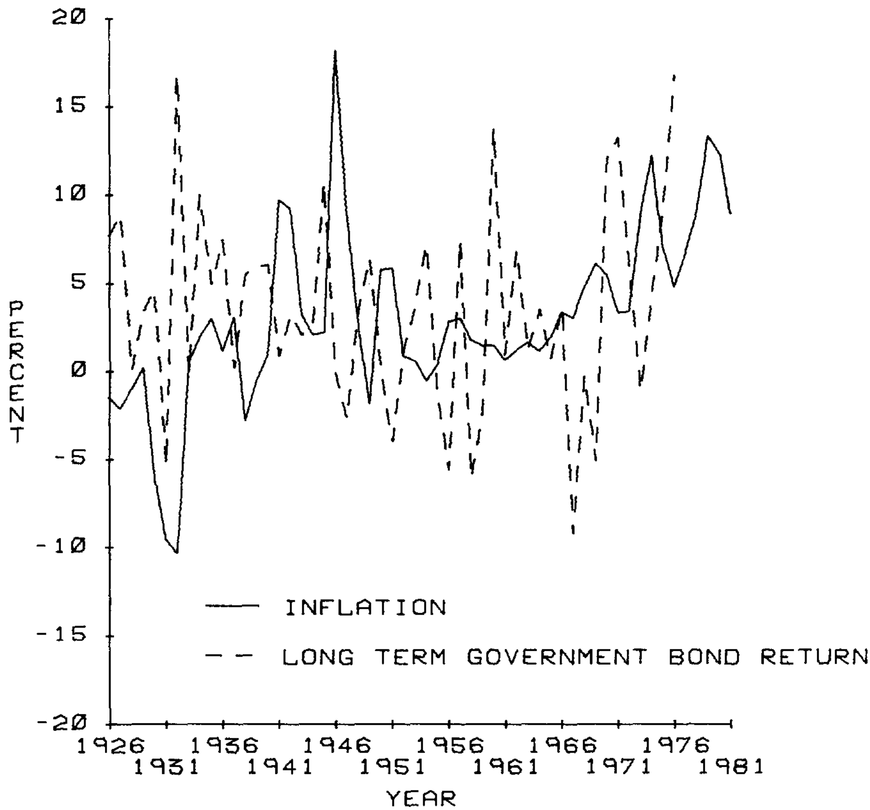

FIGURE 4

LONG TERM GOVERNMENT BOND RETURNS AND

YEARLY PERCENTAGE CHANGE IN CPI (DEC.-DEC.)

1926-1976

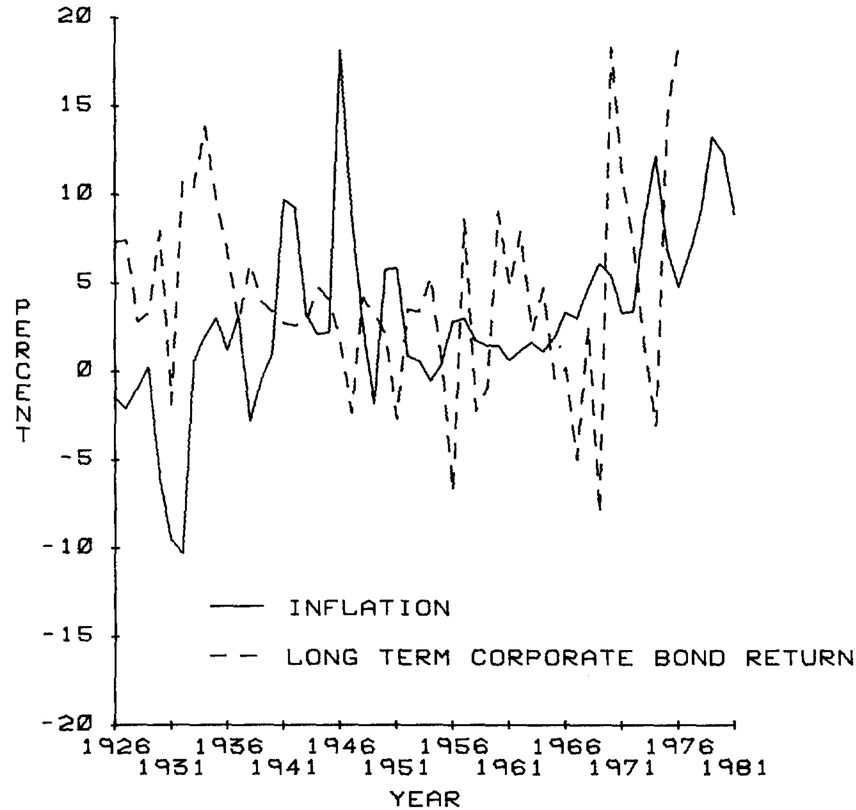

FIGURE 5

LONG TERM CORPORATE BOND RETURNS AND

YEARLY PERCENTAGE CHANGE IN CPI (DEC.-DEC.)

1926-1976

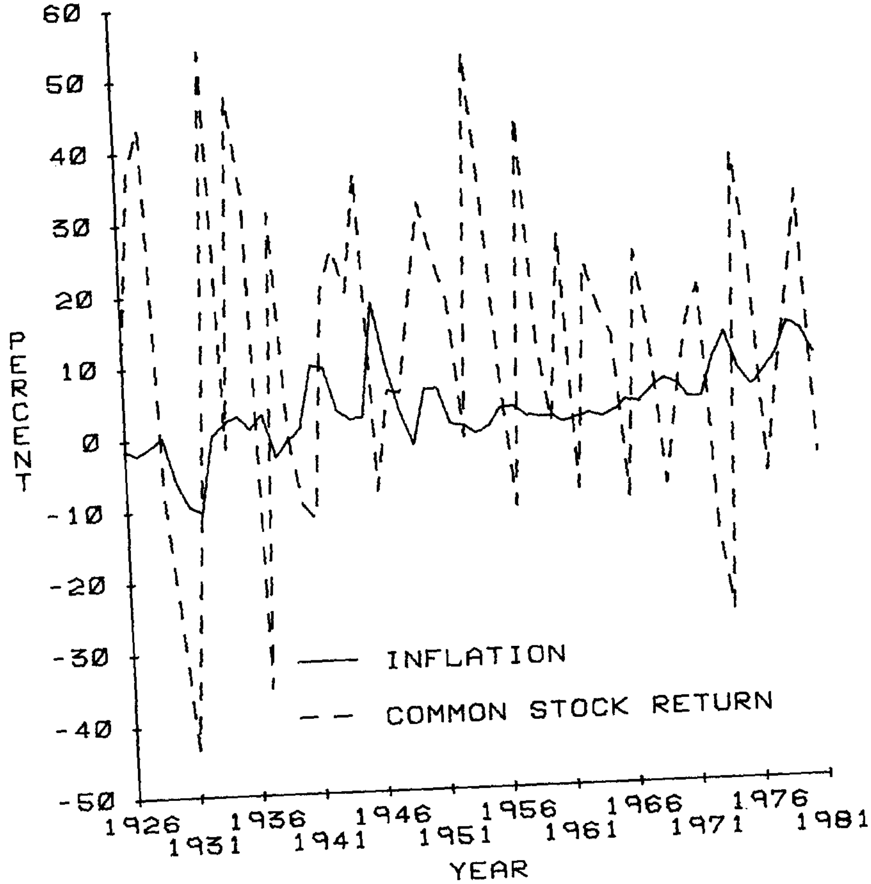

FIGURE 6

COMMON STOCK RETURNS AND

YEARLY PERCENTAGE CHANGE IN CPI (DEC.-DEC.)

1926-1981

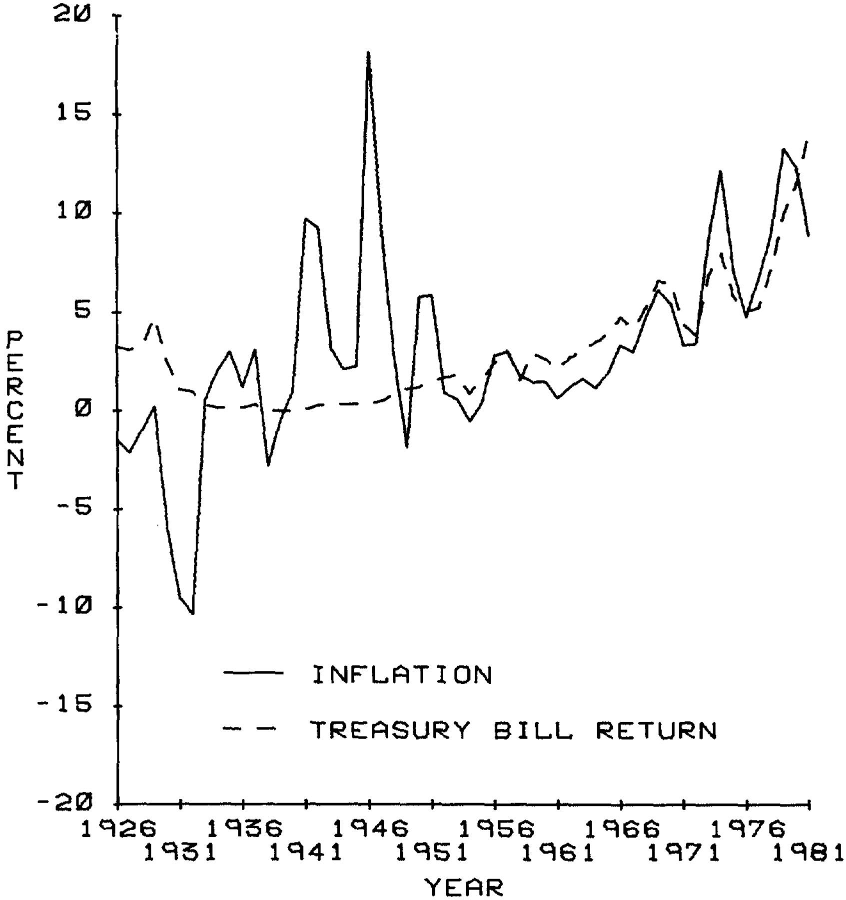

FIGURE 7

TREASURY BILL RETURNS AND

YEARLY PERCENTAGE CHANGE IN CPI (DEC.-DEC.)

1926-1981

Returns on Treasury bills, which are short term (1 to 3 month) investments, have been highly positively correlated with inflation since 1951. This relationship is expected and is explained by Fisher [5], Fama [3] and others. For high inflation rates investors demand a high interest rate to compensate for the loss of spending power. The nominal interest rate, according to the Fisher effect, is approximately equal to the anticipated inflation rate plus the desired real rate of return. This return would be available only on new bond investments, as previously purchased bonds would be locked into prior interest rates until maturity unless sold at the current market price. Short term investments avoid this lag. Prior to 1951, short term interest rates were intentionally held down by the Federal Reserve to accommodate government financing of social programs and the war debt. The Accord of 1951 supposedly ended the artificial suppression of short term interest rates. Experience prior to 1951, as can be seen from Figure 7, does not indicate a relationship between inflation and Treasury bill returns.

SECTION 4—INFLATION IMMUNIZATION

Insurance underwriting profit margins and current investment returns are both negatively correlated with the rate of inflation. Returns on Treasury bills are positively correlated with inflation. These opposite relationships can be utilized to immunize an insurer against the effect of inflation by properly structuring the investment portfolio. The adverse effects of inflation on underwriting and current investment returns can be offset by the beneficial effect of inflation on Treasury bill returns.

Since the assets of an insurer generally exceed the annual earned premium, the effect of a change in investment return has a greater impact on overall operating profitability than a similar change in underwriting profit margin. The leverage of total assets to earned premium varies over time. In 1980 the mean investable asset value for the year was 2.01 times the earned premium for that year for stock insurers [2]. This leverage factor is incorporated in the inflation immunization calculation.

In order to immunize an insurer from the effects of inflation, an investment portfolio must be chosen such that the impact of inflation on investment return offsets the effect of inflation on underwriting profit margin. The calculation involved in this determination is:

$$RUPM + RTB(L)(X) + RIIR(L)(1 - X) = 0.0 \qquad (1)$$

where $RUPM$ = regression coefficient for the effect of inflation on underwriting profit margins.
RTB = regression coefficient for the effect of inflation on Treasury bill returns
$RIIR$ = regression coefficient for the effect of inflation on insurance investment returns
L = leverage ratio (investable assets/earned premium)
X = portion of assets to be invested in Treasury bills

Substituting the regression coefficients calculated from the period 1951 through 1976 and the 1980 leverage ratio into equation 1 yields:

$$-.617 + .556(2.01)X - .817(2.01)(1 - X) = 0$$
$$X = .818$$

The inflation immunized investment portfolio for the stock insurance industry as of the end of 1980, based on relationships calculated on 1951 through 1976 data, would have involved investing 81.8 percent of investable assets in Treasury bills and leaving the remaining 18.2 percent of investable assets distributed as currently invested. Insurance operating results would continue to fluctuate, but variations would be independent of the rate of inflation. Insurers would be immunized against the effects of inflation to the extent that the historical relationships between inflation and the components of insurance profitability remain constant. Changes in line of business mix over time and other changes in insurance operations may affect the relationship of underwriting profitability to inflation and should be considered in determining the appropriate time period on which to base this analysis.

Immunization is not costless. To attract investors, risky investments are required to produce a higher expected return than less risky investments. Treasury bills, as a less risky investment than common stocks, produce a lower return in the long run. For the period 1951 through 1976, Treasury bills generated a mean annual return of 3.7 percent, compared with 12.3 percent for common stocks and 5.1 percent for aggregate insurance investment returns. If insurers had maintained 81.8 percent of their investable assets in Treasury bills during this period, the inflation immunized investment return would have been 4.0 percent. Based on the 1980 leverage ratio, this reduction of 1.1 percentage points in insurance investment returns would be equivalent to a 2.2 percentage points reduction in underwriting profit margin.

SECTION 5—PORTFOLIO THEORY

If the cost of inflation immunization is considered too high a price to pay to eliminate the effect of inflation on insurance company profitability, an alternative method is available to minimize the effect of inflation while still achieving the desired target rate of return. Mean-variance analysis is based on the premise that an investor given the option of different investment opportunities with equivalent expected returns will prefer the alternative with the lowest variance. Portfolio theory provides a method for determining the optimal investment mix to produce the lowest variance for a given expected rate of return.[6] The inputs required for this procedure are the expected return and variance for each investment option and the covariance between each pair of investments. Since the variance of total operating profitability is to be minimized, insurance underwriting is treated as an investment alternative, but the amount of premium is constrained.

The following terms will be used in this analysis:

$E(r_i)$ = expected return on investment i
X_i = proportion of the portfolio invested in i
S_i = standard deviation of return on investment i
$Cov(i,j)$ = covariance between returns on investments i and j

The objective of this determination is to minimize the variance of insurance profitability related to inflation. Therefore, the covariances between investments are determined by multiplying each of the regression coefficients for the investment option related to inflation by the variance of the rate of inflation; for example:

$$Cov(TB,CS) = (RTB)(RCS)(S^2_{INF})$$

The investment alternatives used in this example are insurance underwriting, long term government bonds, Treasury bills, long term corporate bonds, and common stocks. The expected returns, variances, and covariances are determined from the period 1951 through 1976. The 1980 leverage ratio is applied. The minimum variance investment mix is determined by solving the following equations:

Minimize:

$$\sum_{i=1}^{5} \sum_{j=1}^{5} X_i X_j Cov(i,j) \tag{2}$$

[6] For an introduction to the mathematics of portfolio theory, see Francis and Archer [6].

Subject to:

$$\sum_{i=1}^{5} X_i E(r_i) = (5.086)(2.01) = 10.22 \quad (3)$$

$$X_1 = 1.0 \quad (4)$$
$$X_2 + X_3 + X_4 + X_5 = 2.01 \quad (5)$$
$$X_2, X_3, X_4, X_5 \geq 0.0 \quad (6)$$
$$1 = UPM,\ 2 = LTG,\ 3 = TB,\ 4 = LTC,\ 5 = CS$$

Equation 2 indicates that the variance of the portfolio is to be minimized. Equation 3 requires the return on the portfolio from investments in long term government bonds, Treasury bills, long term corporate bonds, and common stocks to equal the target rate of return (the mean insurance investment return over the period) times the leverage factor.[7] Equation 4 constrains earned premium to its current proportion. Equation 5 requires the sum of the investments to equal the leverage factor. Equation 6 restricts investment to positive values.

The foregoing series of equations can be solved by quadratic programming. The solution to this system of equations is:

$$X_1 = 1.000$$
$$X_2 = 0.000$$
$$X_3 = 1.693$$
$$X_4 = 0.000$$
$$X_5 = 0.317$$

The minimum variance portfolio involves investing 84.2 percent of investable assets in Treasury bills and 15.8 percent of investable assets in common stock. No long term bonds are included in this inflation minimization portfolio.

SECTION 6—UPDATE

The regression coefficients of inflation related to profit margins, insurance investment returns, and common stock returns change considerably when the experience through 1981 is included, as shown in Appendix II. The regression

[7] The effect of taxes on investment income can be included in this determination by expanding Equation 3. The after tax expected returns of each investment alternative would be used rather than the total expected return. The target rate of return would be the historical after tax investment income return for the industry multiplied by the leverage factor. Although historical after tax investment income data are not published for the industry, individual insurers would have this information for their own use.

coefficient of inflation related to Treasury bill returns does not alter significantly for the updated period. Data are not available to extend the long term government and corporate bond returns through 1981.

Substituting into equation 1 the regression coefficients for the period 1951 through 1981 (shown in Appendix II) yields:

$$-.396 + .699(2.01)X - .178(2.01)(1 - X) = 0.0$$
$$X = .428$$

The inflation immunized portfolio based on this more recent experience involves investing 42.8 percent of investable assets in Treasury bills, leaving 57.2 percent as currently allocated. For the period 1951 through 1981, this investment portfolio would have yielded a 5.2 percent return, reduced from the actual 5.5 percent return on insurance investments. This decline of 0.3 percentage points would be equivalent to a 0.6 percentage points reduction in underwriting profit margin, based on the 1980 leverage ratio.

SECTION 7—SUMMARY AND CONCLUSIONS

Since historically both underwriting profit margins and investment returns have been negatively correlated with inflation, total insurance operating results have fluctuated significantly as the rate of inflation has changed. Returns on Treasury bills, however, are positively correlated with inflation. By structuring an insurer's investment portfolio to offset the effect of inflation on underwriting profitability, the effect of inflation on operating results can be eliminated. Depending on the period from which the data are based, the inflation immunized investment portfolio requires the insurer to allocate between 42.8 percent and 81.8 percent of investable assets to Treasury bills. This investment strategy would reduce investment returns by between 0.3 and 1.1 percentage points.

Alternatively, insurers can minimize the impact of inflation on operating results by restructuring the investment portfolio to achieve a target rate of return with minimum inflation induced variation. Based on the data from the period 1951 through 1976, this inflation minimization portfolio would involve investment in only Treasury bills (84.2 percent) and common stocks (15.8 percent).

A very serious problem would develop if insurers were to attempt to shift rapidly to the optimal portfolios presented in this paper. Old long term bonds have a market value well below the statutory amortized value used for convention valuation as a result of a general increase in interest rates. Surplus would be reduced or, for some insurers, eliminated if all currently held long term bonds were sold. Widespread sales would also greatly depress prices of long term securities, further eroding surplus. The only practical way for the insurance industry to achieve the desired investment mix would be to shift to the inflation immunized portfolio gradually by redirecting new funds and maturing issues. To a certain extent, insurers are locked into past investment policies, although such a problem can be avoided in the future.

Additional investment alternatives not considered in this paper could also offset the impact of inflation on underwriting profit margins and common stock returns. Commodity prices, since these reflect the cost of tangible products, and put options (which are the right to sell a stock at a given price), since put option prices increase as stock prices decline, are also likely to be positively correlated with the inflation rate. The financial futures market, operating since 1975, now allows investors the opportunity to hedge interest rate changes and changes in stock market index values.[8] An inflation immunized portfolio may include investment in these and other alternatives to the extent allowed by insurance investment regulation. Insurers have the ability to offset the adverse impact of inflation on underwriting profitability by structuring their investment portfolios so that investment returns are positively related to inflation. This strategy would reduce the variability of insurance operating profitability resulting from inflation. The property-liability insurance industry can cope with inflation.

[8] The author is indebted to Roger C. Wade for suggesting this alternative strategy. An introduction to this market is presented in Bacon and Williams [1].

APPENDIX I
DATA SOURCES

The three reference sources for obtaining or deriving the data used in this paper are:

1. *Best's Aggregates and Averages: Property-Casualty* (Oldwick, N.J.: A. M. Best Company, 1981, 1982)
2. Ibbotson, Roger G. and Rex A. Sinquefield, *Stocks, Bonds, Bills, and Inflation: The Past (1926–1976) and the Future (1977–2000)* (Charlottesville, Va.: Financial Analysts Research Foundation, 1977)
3. Standard and Poor's *Trade and Security Statistics* (Orange, Conn.: Standard and Poor's Corp., 1978, 1982)

The individual values were determined as follows:

1. Inflation: the percentage change in Consumer Price Index from December to December (Source 2 for 1926–1976; Source 3 for 1977–1981).
2. Underwriting profit margin: statutory underwriting profit margin for stock insurers (Source 1).
3. Insurance investment returns: statutory investment profit or loss including investment income for stock insurers as a percent of mean investable assets, with investable assets considered to be 90 percent of admitted assets (Source 1).
4. Long term government bond returns: total returns from interest and capital gains or losses on a 20 year term bond portfolio of U.S. Government bonds (Source 2).
5. Long term corporate bond returns: total returns from interest and capital gains or losses on the Salomon Brothers High Grade Long Term Corporate Bond Index and Standard and Poor's High Grade Corporate Composite yield data for 20 year maturities (Source 2).
6. Common stock returns: total returns from dividends and capital gains or losses based on the Standard and Poor's Composite Index (Source 2 for 1926–1976; Source 3 for 1977–1981).
7. Treasury bills: holding period returns on shortest term bills not less than one month to maturity held for one month (Source 2 for 1926–1976) and average yield on new issues of three month bills (Source 3 for 1977–1981).

APPENDIX II
SUMMARY STATISTICS

	1933–1981		1951–1976		1951–1981	
Variable	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
INF	4.19	4.26	3.34	2.92	4.43	3.80
UPM	2.72	4.05	0.90	3.42	0.74	3.40
IIR	5.00	4.72	5.09	5.01	5.52	4.75
LTG	3.27*	5.58*	2.95	6.64	NA	NA
LTC	4.04*	5.83*	3.65	6.98	NA	NA
CS	12.84	20.01	12.34	18.94	11.77	18.33
TB	3.09	3.23	3.73	1.93	4.68	3.09

* 1933–1976

REGRESSION COEFFICIENTS

$$Variable_t = a + b\,INF_t + e_t$$

Variable	*a*	*b*	*T*	R^2
1933–1981				
UPM	4.36	−.389	−3.079**	.168
IIR	6.00	−.239	−1.513	.046
CS	18.97	−1.463	−2.237*	.096
1933–1976				
LTG	4.08	−.230	−1.050	.026
LTC	4.89	−.241	−1.054	.026
1951–1976				
UPM	2.96	−.617	−3.029**	.277
IIR	7.81	−.817	−2.646*	.226
LTG	2.63	.095	.205	.002
LTC	3.93	−.084	−.171	.001
CS	22.73	−3.114	−2.675*	.230
TB	1.87	.556	7.594**	.706
1951–1981				
UPM	2.49	−.396	−2.657*	.196
IIR	6.31	−.178	−.777	.020
CS	17.98	−1.404	−1.638	.085
TB	1.58	.699	9.014**	.737

* = significant at the 5.0% level
** = significant at the 1.0% level

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A NOTE ON CALENDAR YEAR LOSS RATIOS

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Abstract

One important use of calendar year loss ratios is in the determination of rate changes. Two basic methods exist for calculating calendar year loss ratios. They are the standard calendar year loss ratio and the calendar year loss ratio by policy year contribution. This paper sets forth the mathematical definitions of these methods, examines the conditions under which the results equal those of a policy year or accident year approach, and examines the statistical variation of each method.

Introduction

Up until the early to mid 1970's, there was one basic method used to calculate calendar year loss ratios. This consisted of the paid losses plus change in loss reserves divided by the earned premium. At that point in time the National Council on Compensation Insurance (NCCI) introduced a new method of calculating calendar year loss ratios. This is referred to as calendar year loss ratios by policy year contributions. This calculation has been used by the NCCI in its rate filings since that time. The calendar year loss ratio is weighted 50%–50% with a policy year loss ratio in deriving the overall statewide rate change. However, no analysis has been presented as to why or if this procedure is superior. These are the questions examined herein.

Comparison of Average Results

The standard calendar year loss ratio on current benefit and rate level is

$$C_s = \sum_{i=0}^{-\infty} A_s \cdot (L_{i,1-i} - L_{i,-i})/P_s$$

where

C_s = standard calendar year loss ratio
A_s = factor to bring standard calendar year losses and premiums to current benefit and rate level
$L_{i,j}$ = reported incurred losses (includes a provision for IBNR) for policy year i evaluated at maturity j
P_s = calendar year earned premium.

It is well known that C_s will equal the ultimate accident year result if the *amount* of loss reserve adequacy has not changed over time.

The theoretical formula for the calendar year loss ratio by policy year contribution is:

$$C_p = \sum_{i=0}^{-\infty} A_i \cdot (L_{i,1-i} - L_{i,-i})/P_i$$

where

C_p = pure calendar year loss ratio by policy year contribution
A_i = factor to bring losses and premiums to a current benefit and rate level for policy year i
P_i = ultimate premium for policy year i

When put into this form it can be seen that C_p is really an estimate of the ultimate loss ratio for policy year (0) at the current benefit and rate level. The reserving method used in this formula relates developments in incurred losses between successive maturities to the earned premium for the particular policy year. By contrast, for policy years (-1) and (-2), in the NCCI rate filings, developments in incurred losses between successive maturities are related to the starting incurred loss value. Given this, one might question calling the result a calendar year loss ratio. However, the main purpose here is to examine under what conditions C_p gives an exact ultimate loss ratio. In Appendix I it is proven that C_p equals the ultimate policy year (0) on level loss ratio if the following two conditions hold:

(*i*) The ultimate on-level loss ratios for all policy years are equal.
(*ii*) The *percent* adequacy of the incurred losses for equal maturities is the same at successive policy years.

Hence, the standard calendar year approach is superior when the *amount* of incurred loss adequacy has not changed because it will then match the accident year loss ratio exactly. By contrast, the calendar year ratio by policy year contribution is more accurate when the *percent* of incurred loss adequacy has not changed since it will then match the policy year loss ratio exactly. In addition, for the policy year contribution method to be accurate, an additional condition must be imposed. We next examine the incurred loss adequacy conditions under which one method will be accurate and the other will not. These are set forth in Appendix III assuming an increasing premium volume. If premium volume is constant, then a constant amount adequacy will equal a constant percent adequacy. If premium volume is decreasing, then the low and high result would be interchanged.

The theoretical formula for the calendar year loss ratio by policy year contributions is not followed by the NCCI in its rate filings. The reason for this is that all loss developments past an 8th maturity are grouped together. The actual formula used by the NCCI is

$$C_n = \sum_{i=0}^{-7} A_i \cdot (L_{i,1-i} - L_{i,-i})/P_i$$

$$+ \sum_{i=-8}^{-\infty} A_{-8}/P_{-8} \cdot (L_{i,1-i} - L_{i,-i})$$

This formula is a hybrid of the standard calendar year loss ratio and the theoretical calendar year loss ratio by policy year contribution. In Appendix II it is shown that for this formula to provide the correct ultimate loss ratio, a constant percent incurred loss adequacy and on level loss ratio hold for maturities through 8. In addition, a constant amount incurred loss adequacy must hold after maturity 8. This would be expected in light of the conditions that underlie the components entering C_n.

Variance of Results

We next examine the statistical variance of the results under these two methods produced by random fluctuations in losses. It is shown in Appendix IV that the variance of C_p exceeds that of C_s when premiums are increasing, as has been the case for many years. This means that the pure calendar year loss ratio by policy year contributions will have larger swings from year to year than the standard calendar year loss ratio.

The reason for this is relatively simple. Theoretically, the same losses enter C_p and C_s. However, under C_p they are related to a smaller premium base and therefore have a larger variance. In practice, the actual losses entering may not be the same. This is because there are some companies that can report calendar year losses but are not able to split them into policy year components. Furthermore, it is relatively easy to show that Var (C_p) > Var (C_n) > Var (C_s).

Summary

The purpose of this paper is to compare the results of the calendar year loss ratio by policy year contribution and standard calendar year loss ratio calculations. In addition to the specific conclusions within, there is a universal one that can be drawn: No single ratemaking method can be best under all circumstances. The assumptions underlying each method have to be tested to see if they are met. If they are not, the extent of the deviation and the impact on the results need to be determined.

APPENDIX I

Derivation of Conditions Under Which Theoretical Calendar Year Loss Ratio by Policy Year Contributions Gives the Correct Result

We examine herein the conditions under which a pure calendar year loss ratio by policy year contribution will result in an unbiased result. An unbiased result is one in which all reserve adjustments for prior years cancel out. Hence, the loss ratio reflects only current underwriting conditions. The two necessary conditions are a constant on level ultimate loss ratio and a constant percent incurred loss adequacy for each year.

Let $L_{i,j}$ = incurred losses for policy year i evaluated at maturity j (These are the undeveloped incurred losses reported to the NCCI by individual companies. They include each company's own provision for case, reopened, and incurred but not reported loss reserves.)

P_i = ultimate premium for policy year i

$F_{i,j}$ = ratio of incurred losses evaluated at maturity j to ultimate incurred losses for policy year i

R_i = ultimate loss ratio for policy year i

A_i = factor to bring losses and premiums to a current benefit and rate level for policy year i

$L_{i,\infty}$ = ultimate incurred losses for policy year i

Maturity 1 is half a policy year, Maturity 2 is a just-completed policy year, etc. With the above definitions, we have:

$$L_{i,j} = F_{i,j} \cdot L_{i,\infty} \tag{1}$$

$$L_{i,\infty} = R_i \cdot P_i \tag{2}$$

$$L_{i,j} = F_{i,j} \cdot R_i \cdot P_i \tag{3}$$

The calendar year loss ratio by policy year contributions is:

$$\lim_{m \to -\infty} \sum_{i=0}^{m} A_i \cdot (L_{i,j+1} - L_{i,j})/P_i \tag{4}$$

where $i + j$ = constant, which because of the choice of indices above is 0.

$$\Rightarrow i + j = 0 \text{ or } j = -i \tag{5}$$

Substituting (3) and (5) into (4) we have:

$$\lim_{m \to -\infty} \sum_{i=0}^{m} A_i \cdot R_i(F_{i,1-i} - F_{i,-i})$$

if $A_i \cdot R_i = R'$ (constant on level loss ratio for all years)
(constant percent incurred loss adequacy) and $F_{i,j} = F'_j$

we have:

$$\lim_{m \to -\infty} R' \cdot \sum_{i=0}^{m} (F'_{1-i} - F'_i)$$

$$= R' \cdot \lim_{m \to -\infty} (F'_{1-m} - F'_0)$$

$$= R' \cdot (1 - 0) = R'.$$

APPENDIX II

Derivation of Conditions Under Which NCCI Calendar Year Loss Ratio by Policy Year Contributions Gives the Correct Result

In this Appendix we look at the conditions under which the NCCI calendar year loss ratio by policy year contribution will yield an unbiased result. We find that it is a combination of the conditions for the pure calendar year loss ratio by policy year contribution (Maturities 1 to 8) and the standard calendar year loss ratio (Maturities 8 and after).

$$C_n = \sum_{i=0}^{-7} A_i \cdot R_i \cdot (F_{i,1-i} - F_{i,-i})$$

$$+ \lim_{m \to -\infty} \sum_{i=-8}^{m} \frac{A_{-8}}{P_{-8}} \cdot R_i \cdot P_i \cdot (F_{i,1-i} - F_{i,-i})$$

Let $F_{i,j} = F'_j$ for $i = 0$ to -8 (constant percent incurred loss adequacy)

$A_i \cdot R_i = R'$ for $i = 0$ to -8 (constant on level loss ratio)

$R_i \cdot P_i \cdot F_{i,j} = L_{i,\infty} + E_j$ for $i = -8$ to $-\infty$ (constant amount incurred loss adequacy)

where E_j = amount by which the maturity j incurred losses differ from the ultimate incurred losses.

Then $C_n = R' \cdot (F'_1 - F'_0 + F'_2 - F'_1 + \ldots + F'_8 - F'_7)$

$$+ A_{-8}/P_{-8} \cdot \lim_{m \to \infty} (E_9 - E_8 + E_{10} - E_9 + \ldots + E_{m+1} - E_m)$$

$$C_n = R' \cdot F_8' - A_{-8}/P_{-8} \cdot E_8$$

$$E_8 = -L_{-8,\infty} + R_{-8} \cdot P_{-8} \cdot F_{-8,8}$$

$$\begin{aligned} C_n &= R' \cdot F_8' + A_{-8} \cdot L_{-8,\infty}/P_{-8} - A_{-8} \cdot R_{-8} \cdot F_{-8,8} \\ &= A_{-8} \cdot R_{-8} = R'. \end{aligned}$$

APPENDIX III

Comparison of Errors of Calendar Year Approaches Assuming Increasing Premium Volume

Theoretical Calendar Year Loss Ratio By Policy Year Contribution

Incurred Loss Adequacy	Constant Amount	Constant Percent
Excessive	Too Low	Exact
Inadequate	Too High	Exact

Standard Calendar Year Loss Ratio

Incurred Loss Adequacy	Constant Amount	Constant Percent
Excessive	Exact	Too High
Inadequate	Exact	Too Low

APPENDIX IV

Comparison of the Variances of Calendar Year Loss Ratio by Policy Year Contributions and the Standard Calendar Year Loss Ratios

Any type of loss ratio will include a certain amount of statistical variance due to random fluctuations in losses. The variances of the standard calendar year loss ratio and that of the pure calendar year loss ratio by policy year contributions are compared herein.

In addition to the definitions in Appendix I,

let C_p = calendar year loss ratio by policy year contributions
C_s = calendar year loss ratio calculated by standard methods
A_s = factor to bring standard calendar year losses and premiums to current benefit and rate level
P_s = standard calendar year premium
$D_{i,j}$ = difference in incurred losses for policy year i evaluated at maturities j and $j + 1$
$= L_{i,j+1} - L_{i,j}$

Then

$$C_p = \sum_{i=0}^{-\infty} A_i \cdot D_{i,-i}/P_i$$

$$\text{Var }(C_p) = \sum_{i=0}^{-\infty} \text{Var }(D_{i,-i})/(P_i/A_i)^2$$

assuming all the $D_{i,-i}$ are independent.

$$C_s = \sum_{i=0}^{-\infty} A_s \cdot D_{i,-i}/P_s$$

$$\text{Var }(C_s) = \sum_{i=0}^{-\infty} \text{Var }(D_{i,-i})/(P_s/A_s)^2$$

A number of items can be noted:

(*i*) Var $(D_{i,-i}) \geq 0$

(*ii*) For $i \leq -1$ it is almost certain that $P_i/A_i < P_s/A_s$ because of increasing premium volume.

(*iii*) Except in unusual cases, it is reasonable that:

$$P_s/A_s \approx (P_0/A_0 + P_{-1}/A_{-1})/2$$

(*iv*) It is reasonable to assume:
Var $(D_{0,0}) \leq$ Var $(D_{-1,1})$ since $D_{-1,1}$ includes reserve changes in addition to newly reported losses whereas $D_{0,0}$ includes only the latter.

(*v*) $1/(x + \epsilon) + 1/(x - \epsilon) > 1/(x/2)$ for $\epsilon \neq 0$
Given this it is easy to see that:
Var $(C_p) >$ Var (C_s)

ESTIMATING PROBABLE MAXIMUM LOSS WITH ORDER STATISTICS

MARGARET E. WILKINSON

Abstract

In the past there has been much discussion about the definition of probable maximum loss (PML), but little attention has been given to its quantification. This paper introduces the concept of order statistics as a tool to use in estimating the PML. Two different approaches, that of $X_{(n)}$, the largest sample value, and that of quantiles, lead to six specific methods to estimate the PML. Three of the methods require sample data, two of the methods require assumptions about the underlying distribution of the population and the frequency, and one of the methods requires only estimates of the mean and variance of the population and of the frequency. All six methods are illustrated using a particular size of loss distribution. The methods work equally as well if the distribution of size of loss as a percentage of value is available.

INTRODUCTION

The term PML is usually used in connection with property insurance, but it can also be applied to liability insurance. In fact, there is some controversy over whether the appropriate term, from a risk management viewpoint, is probable maximum loss, maximum possible loss, estimated maximum loss or one of many other similar phrases.

McGuinness [1] offers two definitions:

> "The probable maximum loss *for a property* is that proportion of total value of the property which will equal or exceed, in a stated proportion of all cases, the amount of loss from a specified peril or group of perils.
>
> "The probable maximum loss *under a given insurance contract* is that proportion of the limit of liability which will equal or exceed, in a stated proportion of all cases, the amount of any loss covered by a contract."

The first definition is pertinent to insureds and risk managers, while the second is pertinent to underwriters. These definitions were later combined by McGuinness [2] into one generalized definition:

> "The PML for a specified financial interest is that proportion of the total value of the interest which will equal or exceed, in a stated proportion of all cases, the amount of any financial loss to the interest from a specified event or group of events."

A guest reviewer [3] of McGuinness's paper, who is an underwriter, offered the following observations:

> "It is true that the definitions may vary between underwriters when put down in words, but I feel strongly that there is a universal meaning as to the end result which all underwriters expect PML to accomplish. . . . PML, no matter how you define it, is simply *Probable Maximum Loss*. It is neither *foreseeable* nor *possible* loss—rather, it is the maximum loss which *probably* will happen when, and if, the peril insured against actually occurs."

The concept of probable maximum loss used in this paper will not be defined separately from the definitions implied by the various measures to be discussed.

The PML depends upon (i) estimates of the likelihood that losses of various sizes will occur, (ii) the amount of losses and associated probabilities that the insured is willing to accept, and (iii) the amount of losses and associated probabilities that the underwriter is not willing to accept. Thus, the insured and the underwriter can have different estimates of the PML for the same loss exposure.

ORDER STATISTICS

Let $X_1, X_2, \ldots, X_n$ denote a random sample from a population with continuous cumulative distribution function F_X. Since F_X is continuous, the probability of any two sample values being equal is zero. Consequently, there exists a unique ordered arrangement of the sample. Let $X_{(1)}$ denote the smallest member of the set, $X_{(2)}$ the second smallest, etc. Then

$$X_{(1)} < X_{(2)} < \cdots < X_{(n)}$$

and these are called the *order statistics* from the random sample $X_1, X_2, \ldots, X_n$. For $1 \leq r \leq n$, $X_{(r)}$ is called the r^{th} *order statistic*.

Order statistics are particularly useful for studying certain phenomena because quite a few of the results concerning the properties of $X_{(r)}$ and the properties of functions of some subset of the order statistics are distribution-free. If an inference is distribution-free, assumptions regarding the underlying population are not necessary. The distribution-free inference is based on a random variable which has a distribution independent of the underlying population's distribution.

GENERAL RESULTS CONCERNING $X_{(n)}$

$X_{(n)}$ is the largest value of the sample. This is a good place to start since probable maximum loss is the worst loss likely to happen.

Distribution of $X_{(n)}$

The cumulative distribution function of $X_{(n)}$ is given by

$$\begin{aligned} F_{X_{(n)}}(x) &= \Pr\{X_{(n)} \leq x\} \\ &= \Pr\{\text{all } X_i \leq x\} \\ &= F_X^{\,n}(x) \end{aligned} \tag{1}$$

since the X_i's are independent. The corresponding density function is found by differentiating (1). It is easily verified that

$$f_{X_{(n)}}(x) = nf_X(x)F_X^{\,n-1}(x) \tag{2}$$

where f_X is the density function corresponding to F_X.

Moments of $X_{(n)}$

The exact moments of $X_{(n)}$ can be derived from the following equation:

$$E(X_{(n)}{}^k) = \int_{-\infty}^{\infty} x^k f_{X_{(n)}}(x)dx$$

$$= \int_{-\infty}^{\infty} nx^k f_X(x) F_X{}^{n-1}(x)dx. \tag{3}$$

This requires a specified distribution F_X and is of limited practical value due to the complexity of the integral involved.

There are large-sample approximations for the mean and variance of $X_{(n)}$ that are easily calculable. The approximations require two facts.

1. If $U_{(r)}$ denotes the r^{th} order statistic from a uniform distribution over the interval (0,1), then

   $X_{(r)} = F_X^{-1}(U_{(r)})$.

2. The Taylor's series expansion of a function $g(z)$ about a point μ is

   $$g(z) = g(\mu) + \sum_{i=1}^{\infty} \frac{(z-\mu)^i}{i!} g^{(i)}(\mu)$$

   where $g^{(i)}(\mu) = \frac{d^i g(z)}{dz^i} \Big|_{z=\mu}$.

   This series converges if

   $$\lim_{n\to\infty} \frac{(z-\mu)^n}{n!} g^{(n)}(z_1) = 0$$

   for $\mu < z_1 < z$.

The first requirement is due to the probability integral transformation and is proved in various statistical texts [4]. The second requirement is the standard Taylor's series expansion.

If the Taylor's series expansion is rewritten for a random variable Z with mean μ, and the expected value of both sides is taken, the result is

$$E[g(Z)] = g(\mu) + \frac{\text{var}(Z)}{2!} g^{(2)}(\mu)$$

$$+ \sum_{i=3}^{\infty} \frac{E[(Z-\mu)^i]}{i!} g^{(i)}(\mu).$$

So, a first approximation to $E[g(Z)]$ is $g(\mu)$, and a second approximation is

$$g(\mu) + \frac{\text{var}\,(Z)}{2!}\, g^{(2)}(\mu).$$

To find similar approximations for $\text{var}[g(Z)]$, form the difference $g(Z) - E[g(Z)]$, square it and take the expected value. The result is

$$\text{var}[g(Z)] = \text{var}(Z)\, [g^{(1)}(\mu)]^2 - \frac{1}{4}\, [g^{(2)}(\mu)]^2 \text{var}^2(Z) + E[h(Z)]$$

where $E[h(Z)]$ involves third or higher central moments of Z [5]. A first approximation to $\text{var}[g(Z)]$ is $\text{var}(Z)[g^{(1)}(\mu)]^2$, and a second approximation is $\text{var}(Z)[g^{(1)}(\mu)]^2 - (1/4)\, [g^{(2)}(\mu)]^2\, \text{var}^2(Z)$.

In order to apply these results to $X_{(n)}$, g is defined so that

$$g(u_{(n)}) = x_{(n)} = F_X^{-1}(u_{(n)})$$

where $u_{(n)} = F_X(x_{(n)})$. The appropriate moments [6] are

$$\mu = E[u_{(n)}] = n/(n + 1)$$

and

$$\text{var}[u_{(n)}] = \frac{n}{(n + 1)^2(n + 2)}\,.$$

The derivatives needed [7] are

$$g^{(1)}(\mu) = \{f_X[F_X^{-1}\,(n/(n + 1))]\}^{-1}$$

and

$$g^{(2)}(\mu) = -f_X'[F_X^{-1}\,(n/(n + 1))]\{f_X[F_X^{-1}\,(n/(n + 1))]\}^{-3}.$$

Substituting yields as first approximations:

$$E(X_{(n)}) \simeq F_X^{-1}\,(n/(n + 1)) \tag{4}$$

$$\text{var}(X_{(n)}) \simeq \frac{n}{(n + 1)^2(n + 2)}\,\{f_X[F_X^{-1}\,(n/(n + 1))]\}^{-2}. \tag{5}$$

Second approximations are similarly found by the appropriate substitutions.

Distribution-Free Bounds for $E(X_{(n)})$ [8]

If a variate X has a finite variance, the expected value of $X_{(n)}$ can not be arbitrarily large even if the range of X is unbounded.

From Equation (3), the expected value of $X_{(n)}$ is

$$E(X_{(n)}) = \int_{-\infty}^{\infty} nxF_X^{n-1}(x)f_X(x)\,dx.$$

Let $u = F_X(x)$ and standardize X to have mean 0 and variance 1. This means

$$E(X_{(n)}) = \int_0^1 nx(u)u^{n-1}du,$$

$$\int_0^1 x(u)du = 0,$$

$$\int_0^1 [x(u)]^2du = 1,$$

where $x(u)$ indicates that x is expressed as a function of u.

Schwartz's inequality states that

$$\int fg\,du \leq (\int f^2du \int g^2du)^{1/2}.$$

Let $f = x$ and $g = nu^{n-1} - 1$. Then

$$\int_0^1 x(nu^{n-1} - 1)du \leq \left(\int_0^1 x^2du \int_0^1 (nu^{n-1} - 1)^2du\right)^{1/2}.$$

Expanding yields

$$\int_0^1 xnu^{n-1}du - \int_0^1 x\,du$$

$$\leq \left(\int_0^1 x^2du\right)^{1/2}\left(\int_0^1 (n^2u^{2n-2} - 2nu^{n-1} + 1)du\right)^{1/2}.$$

Substituting for the various pieces gives

$$E(X_{(n)}) \leq \left(\int_0^1 (n^2u^{2n-2} - 2nu^{n-1} + 1)du\right)^{1/2}.$$

Hence

$$E(X_{(n)}) \leq (n - 1)/(2n - 1)^{1/2}.$$

If the mean and variance of the population are μ and σ^2, respectively, the result becomes

$$E(X_{(n)}) \leq \mu + (n - 1)\,\sigma/(2n - 1)^{1/2} \tag{6}$$

This result is distribution-free and requires only the knowledge of the mean and variance of the population, not its specific distribution.

GENERAL RESULTS FOR QUANTILES

Probable maximum loss has been defined as the worst loss likely to happen. If the sample under consideration has an unreasonably large loss, then using $X_{(n)}$ to estimate the PML would be unreasonable. In this case, quantiles could be used. The quantile approach would also be preferred if the insured was willing to accept more risk or the underwriter wanted to accept less risk. "More risk" and "less risk" used in this context are comparable to the expected retained losses implied by using $X_{(n)}$ to estimate the PML.

A quantile of a continuous distribution $f_X(x)$ of a random variable X is a real number which divides the area under the probability density function into two parts of specified amounts. Denote the p^{th} quantile by κ_p for $0 \leq p \leq 1$. Then κ_p is defined as any real number solution to the equation

$$F_X(\kappa_p) = p.$$

It is assumed that there is a unique solution to this equation, as there would be if F_X is strictly increasing.

Point Estimate for κ_p [9]

It can be shown that the r^{th} order statistic is a consistent estimator of the p^{th} quantile where $r/n = p$ remains fixed. A definition which provides a unique $X_{(r)}$ to estimate the p^{th} quantile is to choose r so that

$$r = \begin{cases} np & \text{if } np \text{ is an integer} \\ [np + 1] & \text{if } np \text{ is not an integer} \end{cases} \tag{7}$$

where $[x]$ denotes the greatest integer not exceeding x.

Distribution-Free Confidence Interval for κ_p [10]

Since consistency is only a large-sample property, it is desirable to have an interval estimate for κ_p with a known confidence coefficient for a given sample size. The objective is to find two numbers r and s, $r < s$, such that

$$P(X_{(r)} < \kappa_p < X_{(s)}) = 1 - \alpha$$

for some chosen number $0 < \alpha < 1$.

For all $r < s$,

$$P(X_{(r)} < \kappa_p < X_{(s)}) = P(X_{(r)} < \kappa_p) - P(X_{(s)} < \kappa_p).$$

Since F_X is a strictly increasing function,

$$X_{(r)} < \kappa_p \text{ if and only if } F_X(X_{(r)}) < F_X(\kappa_p) = p.$$

Thus,

$$\begin{aligned} P(X_{(r)} < \kappa_p < X_{(s)}) &= P[F_X(X_{(r)}) < p] - P[F_X(X_{(s)}) < p] \\ &= \int_0^p n \binom{n-1}{r-1} x^{r-1}(1-x)^{n-r} dx \\ &\qquad - \int_0^p n \binom{n-1}{s-1} x^{s-1}(1-x)^{n-s} dx. \end{aligned}$$

If this formula is integrated by parts the necessary number of times, the result is

$$P(X_{(r)} < \kappa_p < X_{(s)}) = \sum_{i=r}^{s-1} \binom{n}{i} p^i (1-p)^{n-i}. \tag{8}$$

This does not produce a unique solution for r and s. The narrowest interval is produced when $X_{(s)} - X_{(r)}$ is minimized. Alternatively, $s - r$ could be minimized. Also, a confidence interval produced by

$$\sum_{i=r}^{s-1} \binom{n}{i} p^i (1-p)^{n-i} = 1 - \alpha$$

is distribution-free.

The formula derived above can also be argued directly. For any p, $X_{(r)} < \kappa_p$ if and only if at least r of the sample values $X_1, X_2, \ldots, X_n$ are less than κ_p. The sample values are independent and can be classified according to whether they are less than κ_p. Thus, the n random variables can be considered the result of n independent trials of a Bernoulli variable with parameter p. The number of observations less than κ_p then has a binomial distribution with parameter p.

APPLICATION OF ORDER STATISTICS TO THE PML PROBLEM

The application of order statistics has various requirements depending on the approach taken. The PML can simply be estimated by $X_{(n)}$ if a reliable data set applicable to the particular problem is available. If the concern is to estimate the PML by using the expected value of $X_{(n)}$ or by constructing an interval around $X_{(n)}$ using the variance of $X_{(n)}$ and choosing the PML as the upper limit of this interval, the distribution of X, F_X, must be known (actually F_X^{-1}, f_X and f_X' are needed). If estimates of the mean and variance of F_X are available, derived either theoretically or from a data set, then the upper bound for $E(X_{(n)})$ could be used as the PML. If a data set is available but, for various reasons, the quantile approach is preferred, only the order statistics themselves are necessary to produce either a point estimate for the quantile or a confidence interval for the quantile. In the former case, the PML would be the quantile; in the latter case, the PML would be the upper bound of the confidence interval.

The data set or theoretical distribution used in estimating PML must be adjusted for trend. As there are several excellent papers [11] available on various methods of adjusting for trend, this paper will assume such adjustment has been made.

$X_{(n)}$ *as an Estimate for PML*

Exhibit I contains a list of 100 claims that are representative of a particular problem in which a PML estimate is needed. $X_{(n)}$ in this case is $X_{(100)}$ or \$576,525. Consequently the PML is \$576,525.

$E(X_{(n)})$ *as an Estimate for the PML*

The use of $E(X_{(n)})$ as an estimate for the PML requires F_X^{-1}. Suppose it is assumed that the data has a lognormal distribution. The mean is \$212,521 and the standard deviation is \$110,506. The corresponding normal distribution has a mean of 12.14714 and a standard deviation of .48920. From Equation (4), the approximation for the expected value of $X_{(n)}$ is

$$E(X_{(n)}) \simeq \Lambda_X^{-1}(n/(n+1)) = e^{[\sigma Z^{-1}(n/(n+1)) + \mu]}$$

where Λ_X is the lognormal distribution,
Z is the standard normal distribution,
μ is the mean of the normal distribution, and
σ is the standard deviation of the normal distribution.

If $n = 100$, the value of $Z^{-1}(.9901)$ is found from standard normal tables to be 2.33. The PML estimate is \$589,468.

The Upper Bound of an Interval Around $E(X_{(n)})$ *Using* $var(X_{(n)})$ *as an Estimate for the PML*

It is possible to choose k so that

$$E(X_{(n)}) + k(\mathrm{var}(X_{(n)}))^{1/2}$$

produces a reasonable estimate of the risk that is acceptable. If the prior example is continued, the $\mathrm{var}(X_{(n)})$ can be approximated using Equation (5):

$$\mathrm{var}(X_{(n)}) \simeq [100/(101)^2(102)]\ (\lambda_X(589{,}468))^{-2}$$

where λ_X is the density function corresponding to Λ_X. The formula for λ_X is

$$\lambda_X(x) = \frac{1}{x\sigma(2\pi)^{1/2}}\, e^{\{-(1/2\sigma^2)(\ln x - \mu)^2\}}.$$

The $(\mathrm{var}(X_{(n)}))^{1/2}$ is \$106,976 for this example. If k is chosen to be 2.0, the PML estimate is \$803,420.

The Distribution-Free Upper Bound of $E(X_{(n)})$ *as an Estimate for the PML*

The data shown in Exhibit I have a sample mean of \$212,521 and a sample standard deviation of \$110,506. Consequently,

$$E(X_{(100)}) \leq 212{,}521 + 99\ (110{,}506)/(199)^{1/2}.$$

The PML is thus \$988,044.

If sample data are not available, a mean, variance and number of claims could be chosen on some theoretical grounds and the upper bound calculated as shown above.

κ_p *as an Estimate for the PML*

Suppose it is decided that the .95 quantile will be used as the PML. If the sample data from Exhibit I are used, r is 95 (because $.95 \times 100 = 95$) and the PML $(X_{(95)})$ is \$434,449.

The Distribution-Free Upper Bound of κ_p *as an Estimate for the PML*

The estimate of κ_p for $p = .95$ based on the sample data is \$434,449. Now a confidence interval is desired around this estimate so that $\alpha = .10$. In other words, $r < s$ must be found so that

$$P(X_{(r)} < \kappa_p < X_{(s)}) = \sum_{i=r}^{s-1} \binom{n}{i} p^i (1-p)^{n-i} = .90.$$

We should also minimize $s - r$. Exhibit II shows $X_{(i)}$ and

$$\binom{n}{i} p^i(1 - p)^{n-i} \text{ for } i = 90, 91, \ldots, 100.$$

There are two possibilities for r and s:

$$P(X_{(91)} < \kappa_{.95} < X_{(99)}) = .934732$$

and

$$P(X_{(92)} < \kappa_{.95} < X_{(99)}) = .899831.$$

The second is closer to .90 and $s - r$ is 7. The first has an $s - r$ of 8. Even though the probabilities are so close, and the second probability is slightly less than .90, the second answer would be chosen because $s - r$ is minimized. The PML in this case is $X_{(99)}$ or $563,899.

In the above six examples a particular size of loss distribution was assumed. The PML estimates for the sample data are summarized in Exhibit III. While these estimates vary considerably, this is due to differing data and loss aversion considerations. The methods presented work equally well if the distribution of size of loss as a percentage of value is available. The former is more correct for liability insurance or for property insurance if the population has the same property value as the insured. The latter is more correct for property insurance where the property values differ among properties.

SUMMARY

This paper has presented two different approaches to the PML problem using order statistics: $X_{(n)}$ and quantiles. These approaches lead to six different methods for estimating the PML:

1. $X_{(n)}$,
2. $E(X_{(n)})$,
3. $E(X_{(n)}) + k(\text{var}(X_{(n)}))^{1/2}$,
4. distribution-free upper bound of $E(X_{(n)})$,
5. $X_{(r)}$ as an estimate of κ_p, and
6. distribution-free upper bound of κ_p.

Methods 1, 5 and 6 require sample data. Methods 2 and 3 require assumptions about n and the underlying distribution of the population. Method 4 requires only estimates of n and the mean and variance of the population. The choice of method would depend on availability of data, willingness to make assumptions about the underlying population, and the amount of losses and associated probabilities the insured is willing to accept or the underwriter is not willing to accept.

REFERENCES

[1] McGuinness, John S., "Is 'Probable Maximum Loss' (PML) A Useful Concept?" *Proceedings of the Casualty Actuarial Society,* Vol. LVI, 1969, p. 31.

[2] McGuinness, John S., "Author's Review of Discussions in Volume LVI, Pages 40–48," *Proceedings of the Casualty Actuarial Society,* Vol. LVII, 1970, p. 107.

[3] Black, Edward B., "Discussion by Edward B. Black," *Proceedings of the Casualty Actuarial Society,* Vol. LVI, 1969, p. 46.

[4] In particular, see Gibbons, Jean D., *Nonparametric Statistical Inference,* New York, 1971, p. 23.

[5] Ibid., p. 35.

[6] Ibid., pp. 32–33.

[7] Ibid., p. 37.

[8] David, Herbert A., *Order Statistics,* New York, 1981, pp. 56–59.

[9] Gibbons, op. cit., pp. 40–41.

[10] Ibid., pp. 41–43.

[11] For example, see Rosenberg, S. and Halpert, A. "Adjusting Size of Loss Distributions for Trend," *Inflation Implications for Property-Casualty Insurance,* Casualty Actuarial Society, 1981, p. 458.

EXHIBIT I

Ordered Sample Data

i	$X_{(i)}$	i	$X_{(i)}$
1	$ 19,874	51	$207,196
2	30,610	52	208,959
3	32,159	53	209,568
4	34,115	54	213,084
5	40,660	55	214,307
6	53,453	56	214,546
7	56,598	57	215,978
8	61,651	58	216,369
9	63,411	59	220,808
10	66,007	60	222,804
11	73,062	61	224,417
12	76,962	62	224,475
13	87,348	63	235,209
14	96,498	64	238,249
15	98,408	65	238,679
16	109,837	66	238,842
17	122,838	67	240,455
18	128,372	68	244,699
19	128,426	69	247,465
20	130,048	70	251,374
21	130,610	71	257,426
22	131,326	72	258,513
23	131,474	73	265,051
24	137,655	74	269,816
25	139,681	75	271,647
26	140,949	76	274,154
27	147,987	77	275,727
28	150,776	78	277,211
29	151,044	79	277,734
30	151,967	80	279,494

EXHIBIT I

ORDERED SAMPLE DATA

i	$X_{(i)}$	i	$X_{(i)}$
31	152,219	81	280,721
32	153,388	82	293,728
33	154,619	83	302,641
34	157,065	84	308,771
35	162,956	85	311,612
36	169,142	86	314,410
37	170,262	87	319,722
38	171,988	88	323,711
39	173,391	89	327,927
40	174,049	90	331,179
41	175,689	91	345,130
42	180,406	92	368,095
43	182,223	93	371,194
44	183,399	94	396,911
45	190,532	95	434,449
46	195,658	96	440,639
47	197,482	97	447,171
48	199,788	98	482,259
49	203,310	99	563,899
50	205,796	100	576,525

mean = \$212,521
standard deviation = \$110,506

EXHIBIT II

Binomial Probabilities for $n = 100$, $p = .95$

i	$X_{(i)}$	$\binom{100}{i}(.95)^i(.05)^{100-i}$
90	331,179	.016716
91	345,130	.034901
92	368,095	.064871
93	371,194	.106026
94	396,911	.150015
95	434,449	.180018
96	440,639	.178143
97	447,171	.139576
98	482,259	.081182
99	563,899	.031161
100	576,525	.005921

EXHIBIT III

Summary of Example PML Calculations

Method	PML Estimate
1. $X_{(n)}$	\$576,525
2. $E(X_{(n)})$	589,468
3. $E(X_{(n)}) + k(\text{var}(X_{(n)}))^{1/2}$	803,420
4.* upper bound of $E(X_{(n)})$	988,044
5. $X_{(r)}$ as an estimate of κ_p	434,449
6.* upper bound of κ_p	563,899

*These are distribution-free.

MINUTES OF THE 1982 FALL MEETING

November 7–9, 1982

HYATT ON UNION SQUARE, SAN FRANCISCO, CALIFORNIA

Sunday, November 7, 1982

The Board of Directors held their regular quarterly meeting from 1:00 p.m. to 4:00 p.m.

Registration took place from 4:00 p.m. to 6:30 p.m.

The President's reception for new Fellows and their spouses was held from 5:30 p.m. to 6:30 p.m.

A general reception for all members and guests was held from 6:30 p.m. to 7:30 p.m.

Monday, November 8, 1982

The meeting opened with welcoming remarks from Honorable Robert C. Quinn, Commissioner of Insurance, State of California.

President Newman then announced the results of the election:

President	Frederick W. Kilbourne
President-elect	Carlton W. Honebein
Vice President	C. K. Khury
Secretary	Brian E. Scott
Treasurer	Herbert J. Phillips
Editor	E. Frederick Fossa
General Chairman, Education and Examination Committee	Phillip N. Ben-Zvi
Directors	Martin Adler
	John B. Conners
	Stephen S. Makgill

President Newman then recognized the seventeen new Associates and awarded diplomas to the fifty-one new Fellows. The names of these individuals follow.

FELLOWS

Bashline, Donald T.
Bealer, Donald A.
Burger, George
Carponter, John D.
Clinton, R. Kevin
Connell, Eugene C.
Cundy, Richard M.
Dawson, John
Doepke, Mark A.
Easton, Richard D.
Egnasko, Gary J.
Engles, David
Friedberg, Bruce F.
Ghezzi, Thomas L.
Goldfarb, Irwin H.
Grant, Gary
Hallstrom, Robert C.
Heersink, Agnes H.
Hine, Cecily A.
Holmberg, Randall D.
Horowitz, Bertram A.
Jones, Bruce R.
Josephson, Gary R.
Keatts, Glenn H.
Koski, Mikhael I.
Lange, Dennis L.
Ludwig, Stephen J.
McGovern, William G.
Mealy, Dennis C.
Miller, Ronald R.
Moody, Rebecca A.
Murad, John A.
Muza, James J.
Pachyn, Karen A.
Parker, Curtis M.
Pastor, Gerald H.
Pratt, Joseph J.
Prevosto, Virginia R.
Robertson, John P.
Sweeney, Andrea M.
Van Ark, William R.
Walker, Roger D.
Warren, Jeffrey C.
Weidman, Thomas A.
Weiland, William T.
Wess, Clifford
White, Jonathan
Whitman, Mark
Wilkinson, Margaret E.
Yonkunas, John P.
Young, Bryan G.

ASSOCIATES

Amundson, Richard B.
Canetta, John A.
Colgren, Karl D.
Deutsch, Robert V.
Gapp, Steven A.
Halpern, Nina S.
Hapke, Alan J.
Harrison, David C.
Johnson, Richard W.
Levine, George M.
Loucks, Jr., William
Miner, Neil B.
Murdza, Jr., Peter J.
Robbins, Kevin B.
Tresco, Frank J.
Varca, John J.
Wickman, Alan E.

Mr. Steven H. Newman presented the Woodward-Fondiller award to Mr. Stephen W. Philbrick for his paper, "An Examination of Credibility Concepts."

Mr. Newman presented the Dorweiler award to Mr. Robert W. Sturgis for his paper, "Actuarial Valuation of Property/Casualty Insurance Companies."

From 9:30 a.m. to 10:30 a.m. a keynote address was given by Dr. Thomas Sowell, Senior Research Fellow, Hoover Institution.

From 11 a.m. to 11:55 a.m. there was a panel: Debate on Competitive Rating for Workers' Compensation.

Moderator: Kevin M. Ryan
President
National Council on Compensation Insurance

Members: J. Howard Bunn, Jr.
Vice President
National Association of Independent Insurers

Robert C. Gowdy
President
Industrial Underwriters, Inc.

Diane J. Plastino
Manager
Idaho State Insurance Fund

Roger Lawson
Assistant Vice President
Alliance of American Insurers

From 12:00 p.m. to 1:25 p.m. during the luncheon, the guest speaker was James J. Meenaghan, President, U.S. Property/Liability Operations, Fireman's Fund Insurance Companies.

From 1:30 p.m. to 2:30 p.m. a panel, "Generalized Models of the Insurance Business," was given by Professor William S. Jewell, Professor of Operations Research, University of California, Berkeley.

From 3:00 p.m. to 4:55 p.m. the meeting reconvened for concurrent workshop sessions as follows:

Workshop A — "Actuarial Considerations in a Competitive Environment for Workers' Compensation"

Moderator: Jerome A. Scheibl
Vice President
Wausau Insurance Companies

Members: A. Michael Lamb
Casualty Actuary—Insurance Division
Oregon Department of Commerce

Frank T. White
Vice President and Actuary
National Council of Compensation Insurance

Workshop B — "The Reinsurance Purchaser: Determination of Retention"

Speaker: Frank Neuhauser
Vice President and Actuary
AIG Risk Management

Workshop C — "The Reinsurance Seller: Pricing"

Speaker: Russell T. John
Assistant Actuary
Prudential Reinsurance Company

Workshop D — "Primer on Individual Risk Rating"

Speaker: Janet L. Fagan
Secretary and Actuary
Home Insurance Company

Workshop E — Discussion of Panel: "Generalized Models of the Insurance Business"

Speaker: Professor William S. Jewell
University of California, Berkeley

Workshop F — "New Paper"
"A Strategy for Property-Liability Insurers in Inflationary Times"
by Stephen D'Arcy
University of Illinois

Workshop G — "New Paper"
"Estimating Probable Maximum Loss Using Order Statistics"
by Margaret Wilkinson
Warren, McVeigh and Griffin

Workshop H — "New Paper"
"A Note on Calendar Year Loss Ratios"
by Allan Schwartz
Woodward and Fondiller

A general reception was held from 6:30 p.m. to 7:30 p.m.

Tuesday, November 9, 1982

The concurrent sessions reconvened at 8:30 a.m. to 9:25 a.m.

From 9:30 a.m. to 9:45 a.m. a report by the Ad Hoc Committee on Management and Reorganization was given.

From 9:45 a.m. to 10:00 a.m. there was a business session which included:

Committee Reports
Secretary's Report
Treasurer's Report

From 10:30 a.m. to 11:00 a.m. President Newman delivered his Presidential Address.

At 11:00 a.m. to 12:00 p.m. a panel discussion was given entitled "Tax Deductibility of Self Insured Reserves (HR 61114)." Those participating were:

Moderator: Alfred O. Weller
Vice President
Frank B. Hall and Company, Inc.

Panelists: W. James MacGinnitie
Consulting Actuary
Tillinghast, Nelson and Warren, Inc.

Alan Pearce
Assistant Treasurer
Foremost-McKesson, Inc.

Ronald W. Stasch
Corporate Risk Manager
Federal Mogul Corporation

Brenda Viehe-Naess
Senior Counsel
American Insurance Association

After a lunch break, the meeting reconvened at 1:30 p.m. to 2:45 p.m. for a panel discussion entitled "The Public Perception of the Actuary."

Moderator: Charles A. Hachemeister
Vice President and Actuary
Prudential Reinsurance Company

Panelists: Joseph Diamond
Editor
National Underwriter—Property/Casualty

William J. Shephard
Senior Vice President and Chief Financial Officer
Allianz Insurance Company

Daphne D. Bartlett
Vice President and Actuary
Transamerica Occidental Life
Chairperson: Society of Actuaries Committee on Public Relations.

The closing remarks were made by President Stephen H. Newman after which the meeting adjourned at 3:00 p.m.

In attendance, as indicated by registration records, were 223 Fellows, 101 Associates, 25 guests, 8 subscribers, 7 students, and 140 spouses. The list follows.

FELLOWS

Adler, M.
Alfuth, T. J.
Anderson, D. R.
Angell, C. M.
Atwood, C. R.
Balcarek, R. J.
Barker, L. M.
Barrette, R.
Bashline, D. T.
Bass, I. K.
Bayley, T. R.
Bealer, D. A.
Beer, A. J.
Bell, L. L.
Bennett, N. J.
Ben-Zvi, P. N.
Berquist, J. R.
Bethel, N. A.
Beverage, R. M.
Bill, R.
Bornhuetter, R. L.
Brannigan, J. F.
Brown, J. W.
Brubaker, R. E.
Buck, J. E., Jr.
Burger, G.
Carpoter, J. D.
Carter, E. J.
Cheng, J. S.
Cheng, L. W.
Childs, D.
Cis, M. M.
Cloutier, G.
Cohen, H. L.
Collins, D. J.
Conners, J. B.

Corr, F. X.
Covney, M. D.
Cundy, R. M.
Curry, A. C.
Dahlquist, R. A.
Dangelo, C. H.
D'Arcy, S. P.
Dawson, J.
Dean, C. G.
Doepke, M. A.
Dolan, M. C.
Donaldson, J. P.
Drennan, J. P.
Drobisch, M. R.
Dropkin, L. B.
Drummond-Hay, E. T.
Easton, R. D.
Egnasko, G. J.
Ehlert, D. W.
Eland, D. D.
Engles, D.
Eyers, R. G.
Faber, J. A.
Fagan, J. L.
Fallquist, R. J.
Fein, R. I.
Ferguson, R. E.
Finger, R. J.
Fisher, W. H.
Fitzgibbon, W. J., Jr.
Flaherty, D. J.
Ford, E. W.
Forker, D. C.
Fossa, E. F.
Foster, R. B.
Friedberg, B. F.
Furst, P. A.
Fusco, M.
Ghezzi, T. L.
Goldberg, S. F.
Gottlieb, L. R.
Gowdy, R. C.
Grannan, P. J.
Grant, G.
Graves, J. S.
Grippa, A. J.
Hachemeister, C. A.
Hafling, D. N.
Hallstrom, R. C.
Hartman, D. G.
Hazam, W. J.
Heer, E. L.
Heersink, A. H.
Hermes, T. M.
Herzfeld, J.
Hewitt, C. C., Jr.
Higgins, B. J.
Hine, C. A.
Holmberg, R. D.
Honebein, C. W.
Horowitz, B. A.
Hough, P. E.
Inkrott, J. G.
Jameson, S.
Jerabek, G. J.
Johe, R. L.
John, R. T.
Jones, B. R.
Josephson, G. R.
Kaufman, A.
Keatts, G. H.
Keene, V. S.
Kelly, A. E.
Khury, C. K.
Kilbourne, F. W.
Klaassen, E. J.
Koski, M. I.
Krause, G. A.
Kreuzer, J. H.
Kuehn, R. T.
Lamb, R. M.
Lange, D. L.
LaRose, J. G.
Larsen, M. R.
Lattanzio, S. P.
Lehmann, S. G.
Lerwick, S. N.
Leslie, W., Jr.
Levin, J. W.
Lino, R. A.
Liscord, P. S.
Lombardo, J. S.
Lowe, R. F.
Ludwig, S. J.
MacGinnitie, W. J.
Mahler, H. C.
Makgill, S. S.
Masterson, N. E.
Mathewson, S. B.
McCarter, M. G.
McClure, R. D.
McConnell, C. W., II
McGovern, W. G.
McMurray, M. A.
Mealy, D. C.
Meenaghan, J. J.
Miccolis, R. S.
Miller, M. J.
Miller, R. R.
Mohl, F. J.
Moody, R. A.
Moore, B. C.
Moore, P. S.
Morell, R. K.

Morison, G. D.
Muetterties, J. H.
Munro, R. E.
Murad, J. A.
Murray, E. R.
Muza, J. J.
Myers, N. R.
Nelson, J. R.
Newman, S. H.
Oakden, D. J.
Oien, R. G.
O'Neil, M. L.
Otteson, P. M.
Pachyn, K. A.
Parker, C. M.
Pastor, G. H.
Petersen, B. A.
Phillips, H. J.
Pollack, R.
Pratt, J. J.
Prevosto, V. R.
Price, E. E.
Radach, F. R.
Retterath, R. C.
Richards, H. R.
Robertson, J. P.
Rodermund, M.
Rosenberg, N.
Roth, R. J.
Ryan, K. M.
Salzmann, R. E.
Scheibl, J. A.
Schultz, J. J.
Schwartz, A. J.
Sherman, R. E.
Shoop, E. C.
Skurnick, D.
Sobel, M. J.
Spitzer, C. R.
Squires, S. R.
Stanard, J. N.
Steeneck, L. R.
Steer, G. D.
Streff, J. P.
Strug, E. J.
Sturgis, R. W.
Sweeny, A. M.
Switzer, V. J.
Tatge, R. L.
Van Ark, W. R.
Van Slyke, O. E.
Walker, R. D.
Walsh, A. J.
Walters, M. A.
Ward, M. R.
Warren, J. C.
Webb, B. L.
Weidman, T. A.
Weiland, W. T.
Weller, A. O.
White, J.
White, W. D.
Whitman, M.
Wilkinson, M. E.
Wilson, J. C.
Woll, R. G.
Wright, W. C.
Yoder, R. C.
Yonkunas, J. P.
Young, B. G.
Young, R. J., Jr.
Zelenko, D. A.
Zory, P. B.

ASSOCIATES

Amundson, R. B.
Applequist, V. H.
Austin, J. P.
Baum, E. J.
Bensimon, A. S.
Berens, R. M.
Bertrand, F.
Biller, J. E.
Boley, R. A.
Brahmer, J. O.
Bursley, K. H.
Camp, J. H.
Canetta, J. A.
Chernick, D. R.
Chorpita, F. M.
Christiansen, S. L.
Chuck, A.
Cimini, E. D., Jr.
Clark, D. G.
Cohen, A. I.
Colgren, K. D.
Colin, B.
Connor, V. P.
Costner, J. E.
Covitz, B.
Currie, R. A.
Davis, R. C.
DeGarmo, L. W.
Deutsch, R. V.
Dornfeld, J. L.
Driedger, K. H.
Egnasko, V. M.
Einck, N. R.
Evans, D. M.
Flack, P. R.

Gaillard, M. B.	Levine, G. M.	Robbins, K. B.
Gannon, A. H.	Loucks, W. D., Jr.	Sandler, R. M.
Gapp, S. A.	Meyer, R. E.	Sansevero, M., Jr.
Gilles, J. A.	Miller, R. A., III	Sawyer, J. S., III
Gluck, S. M.	Miner, N. B.	Schneider, H. N.
Gossrow, R. W.	Miyao, S. K.	Skolnik, R. S.
Halpern, N. S.	Moody, A. W.	Skrodenis, D. P.
Hapke, A. J.	Morgan, W. S.	Stadler, E.
Harrison, D. C.	Mueller, C. P.	Suchoff, S. B.
Hayne, R. M.	Mulder, E. T.	Thorne, J. O.
Henkes, J. P.	Muleski, R. T.	Tom, D. P.
Hobart, G. P.	Munt, D. S.	Tresco, F. J.
Hoppe, K. J.	Murphy, F. X., Jr.	Urschel, F. A.
Hurley, J. D.	Murphy, W. F.	Varca, J. J.
Jersey, J. R.	Nelson, J. K.	Wade, R. C.
Johnson, M. A.	Neuhauser, F., Jr.	Wainscott, R. H.
Johnson, R. W.	Nishio, J. A.	Weiner, J. S.
King, K. K.	Penniman, K. T.	White, F. T.
Klingman, G. C.	Pinto, E.	Whiting, D. R.
Knilans, K.	Raid, G. A.	Wilson, O. T.
Koupf, G. I.	Reinbolt, J. B.	Wilson, W. F.
Leong, W.	Reynolds, J. D.	Young, R. G.

GUESTS — STUDENTS — SUBSCRIBERS

Bartlett, D. D.	Johnson, E. J.	Renaud, P.
Bellusci, D.	Kellison, S. G.	Roy, T. S.
Benson, D. W.	Knox, F.	Sheppard, W. J.
Blumenkranz, S.	Kraysler, S. F.	Smith, D. A.
Bunn, J. H., Jr.	Lawson, R.	Souza, L.
Butsic, R. P.	Mabli, C. E.	Sowell, T.
Carpenter, J.	MacDonald, A.	Spangler, J. L.
Clarke, T. G.	Moran, J. W.	Stasch, R. W.
Diamond, J.	Newman, H.	Taylor, C. P.
Graves, G.	Pearce, A.	Viehe-Naess, B.
Gutman, E.	Plastino, D. J.	Volponi, J.
Herzog, T.	Posnak, R.	Weber, R. A.
Jensen, P. A.	Quinn, R. C.	Wilson, G. S.
Jewell, W. S.		

REPORT OF THE SECRETARY

This report is intended to provide the membership with a summary of the significant activities of the CAS during the past year.

Many of our members contribute to achievements of our Society through their participation on our Board and on our various committees. Time will not allow me to mention all of their contributions, but their participation is what makes our Society go. I will touch only on the highlights of the year.

A considerable amount of effort was directed at modifying the organizational structure of the CAS. This proposal was presented to the Board by the Ad Hoc Committee on Management and Operations. It was presented to the membership during our Spring meeting. The proposal, as modified, will significantly improve the day to day operation of our Society. The final touches will be put on the package for a membership vote in May.

The Board of Directors also approved changes to our Guides and Opinions as to Professional Conduct. These changes essentially bring our Guides and Opinions more into line with those of the American Academy of Actuaries, but preserve the wording unique to the CAS.

Our Society sent a gift to commemorate the 25th anniversary of ASTIN. The gift, an embroidered ASTIN flag, will decorate the lectern at all ASTIN events.

As usual, the area of education and examinations generated considerable activity during the year. It was decided to return to the practice of holding our own Part 4 examination. It was felt that the joint examination was not meeting the needs of casualty actuaries. Credit for Part 4 will be given only for the CAS exam, effective with the November 1982 examination.

The Board also adopted a policy on continuing education. The policy is as follows:

> "The Casualty Actuarial Society (CAS) believes that the broadening of one's professional knowledge is synonymous with the growth of the individual actuary.
>
> "Such expansion of knowledge may be gained in many different ways such as continuing practice, continuing formal education, guided self-study, reading of technical literature, participation in technical seminars including meetings sponsored by the CAS and its affiliates, or a combination of the above. Actuaries

who work primarily in a specialized area should strive to broaden the base of their technical skills through continuing education.

"The Society's 'Guides to Professional Conduct' restrain any member from practicing in areas in which he/she is not qualified to act as an expert, creating incentive for continuing education. It is the responsibility of the actuary to personally accept the task of continuing his/her professional development. The CAS does not require a written examination or other formal evidence of continuing education activities as a condition for continued membership.

"The CAS recognizes its obligation to provide a variety of opportunities for continuing education to its members and fulfills this through its Committee on Continuing Education."

During 1982 two CAS sponsored seminars were held. The second Loss Reserve Seminar and a new Reinsurance Seminar were offered to CAS members.

The Board authorized the creation of "Special Interest Sections" for the CAS. These affiliates will address the educational needs of special disciplines within our organization. The application of the first group, "Actuaries in Regulation," was adopted under this provision.

The Editorial Committee recommended and the Board approved the development of an Index to our *Proceedings*. The Index will categorize the content of the *Proceedings* in three sections: a Topical Section, a Biographical Section, and a Board and Committee Section. Work will proceed on this index during 1983.

Our membership ranks continued to grow with the admittance of 69 new Fellows and 63 new Associates, bringing our total membership to 1006.

Respectfully submitted,

BRIAN E. SCOTT
Secretary

REPORT OF THE TREASURER

This is my third and final Treasurer's report to the membership. In giving the highlights for this year, I would also like to put them in the context of the changes which took place over the past three years.

First of all, we ended another year with a surplus gain, implying the overall fiscal soundness of our activities. This makes the third straight year of a gain, totalling $38,000 over the past three years, a growth of 36% in CAS surplus. Including the other surplus funds the CAS has entrusted to it, our entire equity accounts have grown $50,000 in three years.

The principal reason for this beneficial result has been a conscious shift to higher yielding, but still secure, investments; i.e., money market funds, certificates of deposit, and a high yield U.S. Treasury Note. Four years ago the annual interest earned was about $10,000; whereas, this past year, we earned $32,000 interest from CAS surplus and another $6,000 for the Trust Funds. The high interest yields bring us more than half the amount that dues income is producing.

The dues schedule has been the same for the past three years, and in fiscal 1983 dues will again remain constant. We have locked in the high yields for the next few years, to protect against a decline in interest rates.

Next year's budget, while predicated on no increase in dues, is also expected to yield another increase in surplus, barring any major surprises. No increase in exam fees was needed as well, but there will be an increase in fees to non-members for some of our publications.

During the year a question was raised in the *Actuarial Review* about students possibly subsidizing Fellows, since exam revenues seemed to exceed exam expenses. However, as I responded in the August issue of the *Actuarial Review*, much of the CAS office expense should be charged to the administration of the exams and educational programs for the students. Further, it is worth reiterating that the work donated by the Fellows in designing and in administering the exams is one of our most valuable resources as a professional organization.

At this time, I would like to thank others who have, during these past few years, helped the Office of the Treasurer to run smoothly and successfully. First of all, our CAS Office staff in New York, Edith Morabito and Carol Olszewski, serve as an effective conduit for most of the bills paid and income received.

Walt Fitzgibbon and Glenn Fresch have served as chairmen of the Finance Committee during my three years as Treasurer, and provided expert financial counseling to this office. I would like to thank my own secretary, Mary Daraio, who has dedicated much of her time to the maintenance of the CAS financial records over the past three years.

And finally, in turning over the books and responsibilities to our new Treasurer, Herb Phillips, I would like to thank the members of the CAS for the privilege of serving as Treasurer these past three years.

Respectfully submitted,

MICHAEL A. WALTERS
Treasurer

FINANCIAL REPORT
Fiscal Year Ended 9/30/82 (Accrual Basis)

Income		Disbursements	
Dues	$ 63,830.60	Printing	$ 91,880.06
Exam fees	56,904.45	Office expenses	57,081.94
Meetings	76,278.05	Other exam expenses	3,833.42
Proceedings	8,628.10	Meeting expenses	80,546.00
Readings	11,257.49	Library	27.50
Invitational program	5,987.10	Math. Assn. of America	1,500.00
Interest	31,671.31	Insurance	2,804.55
Actuarial Review	246.00	Expenses—President	3,500.00
Other	596.04	Expenses—Pres.-Elect	2,500.00
Total	$ 255,399.14	Outside Services	750.00
		Miscellaneous	3,348.70
		Total	$247,772.17

Income	$ 255,399.14
Disbursements	247,772.17
Change in CAS Surplus	$ +7,626.97

ACCOUNTING STATEMENT (Accrual Basis)

Assets	9/30/81	9/30/82	Change
Checking account	$ 8,238.38	$ 586.76	$ −7,651.62
Money market funds	119,685.71	80,582.40	−39,103.31
Bank certificates of deposit	20,000.00	97,855.51	+77,855.51
U.S. Treasury note	99,971.90	99,971.90	0
Accrued income	9,144.00	5,324.00	−3,820.00
Total	$257,039.99	$284,320.57	$+27,280.58
Liabilities			
Office services	$ 35,149.00	$ 14,500.00	$−20,649.00
Printing expenses	25,236.00	41,379.13	+16,143.13
Examination expenses	1,148.00	0	−1,148.00
Meeting expenses & prepaid fees	1,235.00	8,038.63	+6,803.63
Prepaid exam fees	7,764.00	22,319.82	+14,555.82
Other	2,932.00	840.00	−2,092.00
Total	$ 73,464.00	$ 87,077.58	$+13,613.58
Members' Equity			
Michelbacher fund	$ 39,074.85	$ 43,678.40	$ +4,603.55
Dorweiler fund	8,439.40	8,836.52	+397.12
CAS trust	302.80	1,342.16	+1,039.36
CAS surplus	135,758.94	143,385.91	+7,626.97
Total	$183,575.99	$197,242.99	$+13,667.00

Michael A. Walters
Treasurer

This is to certify that the assets and accounts shown in the above financial statement have been audited and found to be correct.

Finance Committee
Glenn W. Fresch, Chairman
Douglas S. Haseltine
David M. Klein
William J. Rowland

1982 EXAMINATIONS—SUCCESSFUL CANDIDATES

Examinations for Parts 6, 8 and 10 of the Casualty Actuarial Society syllabus were held on May 5 and 6, 1982. Examinations for Parts 4, 5, 7 and 9 were held on November 1 and 3, 1982.

Examinations for Parts 1, 2, and 3 are jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries. These examinations were given in May and November of 1982. Candidates who passed these examinations were listed in the joint releases of the two societies.

The Casualty Actuarial Society and the Society of Actuaries jointly awarded prizes to the undergraduates ranking the highest on the General Mathematics examination. For the May, 1982 examination, the $200 prize was awarded to Zachary M. Franco. The additional $100 prize winners were Paul N. Feldman, Csaba P. Gabor, Fred C. Kuczmarski, and Evan W. Morton. For the November, 1982 examinations, the $200 prize was awarded to Kenneth R. Ballou. The additional $100 prize winners were Michael W. Bell, Bev I. Cope, Antone R. Costa, James G. Merickel, and Paul C. Wright.

The following candidates were admitted as Fellows and Associates at the November, 1982 meeting as a result of their successful completion of the Society requirements in the May, 1982 examinations.

FELLOWS

Bashline, Donald T.
Bealer, Donald A.
Burger, George
Carponter, John D.
Clinton, R. Kevin
Connell, Eugene C.
Cundy, Richard M.
Dawson, John
Doepke, Mark A.
Easton, Richard D.
Egnasko, Gary J.
Engles, David
Friedberg, Bruce F.
Ghezzi, Thomas L.
Goldfarb, Irwin H.
Grant, Gary
Hallstrom, Robert C.
Heersink, Agnes H.
Hine, Cecily A.
Holmberg, Randall D.
Horowitz, Bertram A.
Jones, Bruce R.
Josephson, Gary R.
Keatts, Glenn H.
Koski, Mikhael I.
Lange, Dennis L.
Ludwig, Stephen J.
McGovern, William G.
Mealy, Dennis C.
Miller, Ronald R.
Moody, Rebecca A.
Murad, John A.
Muza, James J.
Pachyn, Karen A.
Parker, Curtis M.
Pastor, Gerald H.
Pratt, Joseph J.
Prevosto, Virginia R.
Robertson, John P.
Sweeny, Andrea M.
Van Ark, William R.
Walker, Roger D.
Warren, Jeffrey C.
Weidman, Thomas A.
Weiland, William T.
Wess, Clifford
White, Jonathan
Whitman, Mark
Wilkinson, Margaret E.
Yonkunas, John P.
Young, Bryan G.

ASSOCIATES

Amundson, Richard B.
Canetta, John A.
Colgren, Karl D.
Deutsch, Robert V.
Gapp, Steven A.
Halpern, Nina S.
Hapke, Alan J.
Harrison, David C.
Johnson, Richard W.
Levine, George M.
Loucks, William
Miner, Neil B.
Murdza, Peter J., Jr.
Robbins, Kevin B.
Tresco, Frank J.
Varca, John J.
Wickman, Alan E.

The following is a list of successful candidates in examinations held in May, 1982.

Part 6

Abell, Ralph L.
Almagro, Manuel, Jr.
Amoroso, Rebecca C.
Amundson, Richard B.
Bakel, Leo R.
Basson, Steven D.
Bennett, Robert S.
Berry, Janice L.
Bothwell, Peter T.
Bray, Rosemary P.
Canetta, John A.
Carlson, Jeffrey R.
Chiang, Jeanne D.
Colgren, Karl D.
Costello, Diane
Cox, David B.
Cripe, Frederick F.
Debs, Raymond V.
Dekle, James M.
Deutsch, Robert V.
Donnelly, Vincent T.
Dye, Myron L.
Fitz, Loy W.
Forney, John R., Jr.
Gapp, Steven A.
Garelick, Mitchell I.
Gauthier, Richard
Grace, Gregory S.
Greaney, Kevin M.
Green, Bruce H.
Halpern, Nina S.
Hapke, Alan J.
Harrison, David C.
Haskell, Gayle E.
Hauboldt, Richard H.
Henry, Thomas A.
Hutter, Heidi E.
Johnson, Richard W.
Kaplan, Robert S.
Kelley, Robert J.
Koegel, David
Koufacos, Constantine G.
Krakowski, Israel
Kuo, Chung-Kuo
Laberge, Christian
Laurin, Pierre G.
Lebrun, Richard
Levenglick, Arthur B.
Levine, George M.
Licitra, Sam F.
Lyons, Daniel K.
Lyons, Mark D.
Lyons, Rebecca B.
McClure, John W., Jr.
McDermott, Thomas J., Jr.
McSally, Michael J.
Merlino, Matthew P.
Miller, William J.
Miner, Neil B.
Morrow, Jay B.
Murdza, Peter J., Jr.
Murry, Mary E.
Nester, Karen L.
Normandin, Andre
Palmer, Donald W.
Paquette, Sylvie L.
Pichler, Karen J.
Poirier, Denis
Pridgeon, Ronald D.
Rathjen, Ralph L.
Reott, Joel A.
Rice, James
Roach, William L.
Robbins, Kevin B.
Romito, A. Scott
Salton, Jeffrey C.
Schmidt, Lowell D.
Schultz, Roger A.
Scott, Diane D.
Sczech, James R.
Sellitti, Marie
Shapiro, Arlyn G.
Shapland, Mark R.
Smith, Richard A.
Spalla, Joanne S.
Spidell, Bruce R.
Steinen, Phillip A.
Stenmark, John A.
Symnoski, Diane
Tresco, Frank J.
Trinh, Minh
Trudeau, Michel
Varca, John J.
Vaughan, Richard L.
Vidal, Cynthia L.
Walsh, Michael C.
Weber, Dominic A.
White, Charles S.
Wick, Peter G.
Williams, Lawrence
Williams, Lincoln B.
Willsey, Robert L.
Woerner, Susan K.
Woodruff, Arlene F.
Yau, Michael W.

Part 8

Allaben, Mark S.
Atkinson, Roger A., III
Belden, Scott C.
Belden, Stephen A.
Bhagavatula, Raja R.
Biller, James E.
Blanchard, Ralph S., III
Boccitto, Bonnie L.
Boulanger, Francois
Bouska, Amy S.
Burger, George
Cantin, Claudette
Chanzit, Lisa G.
Coffin, John D.
Dawson, John
Domanico, Elaine M.
Downer, Robert B.
Ehrlich, Warren S.
Elia, Dominick A.
Forde, Claudia S.
Fueston, Loyd L., Jr.
Gilles, Joseph A.
Gorman, Linda A.
Grant, Gary
Hall, Allen A.
Hallstrom, Robert C.
Hayward, Gregory L.
Heard, Pamela B.
Holdredge, Wayne D.
Jones, Bruce R.
Josephson, Gary R.
Kane, Adrienne B.
Kooken, Michael W.
Lacefield, David W.
Leong, Winsome
Lonergan, Kevin F.
Marks, Steven D.
Mashitz, Isaac
McAllister, Kevin C.
McIntosh, Karol A.
Miller, Ronald R.
Muleski, Robert T.
Munt, Donna S.
Neale, Catharine L.
Nishio, Jo Anne
O'Connell, Paul G.
Pflum, Roberta J.
Port, Rhonda D.
Pruiksma, Glenn J.
Robertson, John P.
Ross, Lois A.
Sanders, Robert L.
Sarosi, Joseph F.
Schmidt, Neal J.
Schwartzman, Joy A.
Seguin, Louis G.
Silverman, Mark J.
Smith, Byron W.
Smith, Judith P.
Soul, Harry W.
Splitt, Daniel L.
Suchoff, Stuart B.
Surrago, James
Thompson, Kevin B.
Vaillancourt, Jean
Van Ark, William R.
Webster, Patricia J.
Weimer, William F.
White, David L.
Wilkinson, Margaret E.
Yatskowitz, Joel D.
Young, Bryan G.
Young, Edward W.
Zarnowski, James D.

Part 10

Addie, Barbara J.
Bashline, Donald T.
Bealer, Donald A.
Camp, Jeanne H.
Carpenter, Thomas S.
Carponter, John D.
Clinton, R. Kevin
Connell, Eugene C.
Cundy, Richard M.
Dawson, John
Doepke, Mark A.
Easton, Richard D.
Edie, Grover M.
Egnasko, Gary J.
Engles, David
Friedberg, Bruce F.
Ghezzi, Thomas L.
Gillespie, Bryan C.
Gluck, Spencer M.
Goldfarb, Irwin H.
Hallstrom, Robert C.
Hayne, Roger M.
Heersink, Agnes H.
Hine, Cecily A.
Holmberg, Randall D.
Horowitz, Bertram A.
Jones, Bruce R.
Keatts, Glenn H.
Koski, Mikhael I.
Lange, Dennis L.
Ludwig, Stephen J.
Lyle, Aileen C.
McGovern, William G.
Mealy, Dennis C.
Miller, Ronald R.
Moody, Rebecca A.
Mulder, Evelyn T.
Murad, John A.
Muza, James J.
Nikstad, James R.
Ogden, Dale F.
Pachyn, Karen A.
Parker, Curtis M.
Pastor, Gerald H.
Petit, Charles I.
Pratt, Joseph J.
Prevosto, Virginia R.
Raman, Rajagopalan K.
Robertson, John P.
Sweeny, Andrea M.
Walker, Roger D.
Warren, Jeffrey C.
Weidman, Thomas A.
Weiland, William T.
Wess, Clifford
White, Jonathan
Whitman, Mark
Wilkinson, Margaret E.
Wilson, Ronald L.
Yingling, Mark E.
Yonkunas, John P.
Young, Bryan G.
Youngerman, Hank

The following candidates will be admitted as Fellows and Associates at the May, 1983 meeting as a result of their successful completion of the Society requirements in the November, 1982 examinations.

FELLOWS

Addie, Barbara J.
Boison, LeRoy A., Jr.
Carpenter, Thomas S.
Davis, Lawrence S.
Fisher, Russell S.
Gillespie, Bryan C.
Gluck, Spencer M.
Gutterman, Sam
Johnson, Warren H., Jr.
Lyle, Aileen C.
Newville, Benjamin S.
Taranto, Joseph V.
Truttmann, Everett J.
Westerholm, David C.
Wilson, Ronald L.
Yingling, Mark E.
Young, Edward W.
Youngerman, Hank

ASSOCIATES

Bennett, Robert S.
Bhagavatula, Raja R.
Bothwell, Peter T.
Brockmeier, Donald R.
Bujaucius, Gary S.
Chanzit, Lisa G.
Duffy, Brian
Dye, Myron L.
Ellefson, Thomas J.
Epstein, Michael
Eramo, Robert P.
Fiebrink, Dianne C.
Forde, Claudia S.
Friedman, Howard H.
Fueston, Loyd L., Jr.
Gerard, Felix R.
Hall, Allen A.
Halpert, Aaron
Hanson, Jeffrey L.
Harwood, Catherine B.
Henry, Thomas A.
Hutter, Heidi E.
Kadison, Jeffrey P.
Kane, Adrienne B.
Klawitter, Warren A.
Kooken, Michael W.
Marks, Steven D.
Mashitz, Isaac
Merlino, Matthew P.
Miller, David L.
Morrow, Jay B.
Narvell, John C.
Neale, Catharine L.
Nichols, Richard W.
Nikstad, James R.
Normandin, Andre
O'Connell, Paul G.
Ogden, Dale F.
Pierson, Frank D.
Rapoport, Andrew J.
Schmidt, Neal J.
Sellitti, Marie
Sherman, Harvey A.
Smith, Richard A.
Spalla, Joanne S.
Stanco, Edward J.
Steinen, Phillip A.
Symnoski, Diane M.
Toczylowski, Deborah L.
Townsend, Christopher J.
Vaughan, Richard L.
Visner, Steven M.
Wacek, Michael G.
Weimer, William F.
White, Charles S.
White, David L.
Williams, Lawrence
Windwehr, Debra R.
Yau, Michael W.

The following is the list of successful candidates in examinations held in November, 1982.

Part 4

Aldin, Neil C.
Allaire, Christiane
Altabet, Meryl J.
Berger, Susan L.
Brissman, Mark D.
Brown, Brian Y.
Buchanan, John W.
Chabarek, Paul
Chansky, Joel S.
Clark, Daniel B.
Costello, Dianne
Creighton, Kenneth M.
Davis, James R.
DeFalco, Thomas J.
Der, William
Doe, David A.
Doyle, Michael J.
Dunlap, George T., IV
Eramo, Robert P.
Fanning, William G.
Fiebrink, Dianne C.
Fontaine, Andre F.
Fueston, Loyd L., Jr.
Glicksman, Steven A.
Graves, Nancy A.
Gunn, Christy H.
Hertling, Richard J.
Hill, Tony D.
Hollister, Jeanne M.
Holmes, Robert M., Jr.
Kumin, Cynthia L.
Lakins, William J.
Lemaire, Jacques
Liebers, Elise C.
Littmann, Mark W.
Mailloux, Patrick
McDermott, Sean P.
Menning, David L.
Michelson, Jon W.
Mischenko, Jean M.
Morrow, Catherine
Mugavero, Ann C.
Muller, Robert G.
Newell, Richard T., Jr.
Ng, Wai Hung
Nikstad, James R.
Patel, Bhikhabhai C.
Pechan, Kathleen M.
Peterson, Steven J.
Post, Jeffrey H.
Rosenbach, Allen
Santomenno, Sandra C.
Schilling, Gary M.
Schlenke, David O.
Siczewicz, Peter J.
Smith, David A.
Stauffer, Laurence H.
Svendsgaard, Christian
Tan, Suan-Boon
Treitel, Nancy R.
Visintine, Gerald R.
Wacek, Michael G.
Wilkins, Shaun W.
Wong, David T.
Wurmbrand, Laura

Part 5

Atkinson, Roger A., III
Basson, Steven D.
Becraft, Ina M.
Bellusci, David M.
Bennett, Robert S.
Bradford, David K.
Brodie, Pamela E.
Bryan, Susan E.
Captain, John E.
Carlson, Jeffrey R.
Carpenter, William M.
Cascio, Michael J.
Cathcart, Sanders B.
Collins, James J.
Cripe, Frederick F.
Desilets, Claude
Donelson, Norman E.
Dusold, Michael J.
Dyck, N. Paul
Earwaker, Bruce G.
Easlon, Kenneth
Eckstein, Daniel E.
Erickson, John A.
Feldblum, Sholom
Fromentin, Pierre
Gauthier, Richard
Goldberg, Steven B.
Greaney, Kevin M.
Griffin, Dale C.
Handte, Malcolm R.
Huyck, Brenda J.
Kartechner, John W.
Keen, Eric R.
Keepers, Lonnie L.
Keller, Wayne S.
Kelley, Robert J.
Krause, Ann M.
Kudera, Andrew E.
Lacroix, Marthe A.
Lee, Robert H.
Letourneau, Roland D.
Lewis, Martin A.
Lilly, Claude C.
Lindow, Nancy J.
Loper, Dennis J.
Lyons, Rebecca B.
Macasieb, Emma B.
Maguire, Brian P.
Maharajh, Bindranath
McClure, John W., Jr.
McKechnie, Ian M.
McQuilkin, Mary T.
McSally, Michael J.
Miller, David L.
Miller, John V.
Miller, William J.
Montgomery, Warren D.
Mucci, Robert V.
Mueller, Robert A.
Narvell, John C.
Newman, Henry E.
Ng, Kwok Ching
Ogden, Dale F.
Olson, Carol L.
Palmer, Donald W.
Pierson, Frank D.
Putney, Alan K.
Rech, James E.
Roesch, Robert S.
Schilling, Timothy L.
Schultheiss, Peter J.
Schultz, Roger A.
Schustak, Marlene D.
Sellitti, Marie
Shapiro, Arlyn G.
Shapland, Mark R.
Slusarski, John
Smith, Michael B.
Sornberger, George C.
Stanco, Edward J.
Vestal, Anne T.
Vogan, William E.
Volponi, Joseph L.
Walsh, Michael C.
Weber, Dominic A.
Whitlock, Robert G., Jr.
Woerner, Susan K.
Yard, Roger A.

Part 7

Anderson, Bruce C.
Bailey, Victoria M.
Bhagavatula, Raja R.
Bothwell, Peter T.
Brockmeier, Donald R.
Bujaucius, Gary S.
Chanzit, Lisa G.
Duffy, Brian
Dye, Myron L.
Eckley, Douglas A.
Ellefson, Thomas J.
Forde, Claudia S.
Friedman, Howard H.
Gerard, Felix R.
Griffith, Roger E.
Hall, Allen A.
Halpert, Aaron
Hanson, Jeffrey L.
Harwood, Catherine B.
Hein, Timothy T.
Henry, Thomas A.
Hutter, Heidi E.
Kadison, Jeffrey P.
Kane, Adrienne B.
Kaplan, Robert S.
Klawitter, Warren A.
Kooken, Michael W.
Krakowski, Israel
Levenglick, Arthur B.
Licht, Peter M.
Mair, Sharon A.
Marks, Steven D.
Mashitz, Isaac
Mayer, Jeffrey H.
McDonald, Gary P.
Merlino, Matthew P.
Morrow, Jay B.
Neale, Catharine L.
Nichols, Richard W.
Normandin, Andre
O'Connell, Paul G.
Pflum, Roberta J.
Rapoport, Andrew J.
Ruegg, Mark A.
Schmidt, Neal J.
Sherman, Harvey A.
Silver, Melvin S.
Sizer, Charles L.
Smith, Richard A.
Snow, David C.
Spalla, Joanne S.
Steinen, Phillip A.
Symnoski, Diane M.
Toczylowski, Deborah L.
Townsend, Christopher J.
Vaughan, Richard L.
Visner, Steven M.
Weimer, William F.
White, Charles S.
White, David L.
Williams, Lawrence
Windwehr, Debra R.
Yau, Michael W.

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Addie, Barbara J.
Alpert, Bradley K.
Belden, Stephen A.
Bensimon, Abbe S.
Boccitto, Bonnie L.
Boison, LeRoy A., Jr.
Boone, James P.
Boulanger, Francois
Bouska, Amy S.
Bowen, David S.
Braithwaite, Paul
Cantin, Claudette
Carpenter, Thomas S.
Colin, Barbara
Davis, Lawrence S.
DeConti, Michael A.
Downer, Robert B.
Egnasko, Valere M.
Fisher, Russell S.
Gannon, Alice H.
Gilles, Joseph A.
Gillespie, Bryan C.
Gluck, Spencer M.
Gutterman, Sam
Harrison, Eugene E.
Henzler, Paul J.
Javaruski, John J.
Johnson, Warren H., Jr.
Lonergan, Kevin F.
Loucks, William D., Jr.
Lyle, Aileen C.
Munt, Donna S.
Newville, Benjamin S.
Nishio, Jo Anne
Paquette, Sylvie L.
Rosenberg, Deborah M.
Splitt, Daniel L.
Suchoff, Stuart B.
Taranto, Joseph V.
Tresco, Frank J.
Truttmann, Everett J.
Walker, Glenn M.
Westerholm, David C.
Whiting, David R.
Wilson, Ronald L.
Yingling, Mark E.
Young, Edward W.
Youngerman, Hank

NEW FELLOWS ADMITTED MAY, 1982: Eleven of the twelve new Fellows admitted at Palm Beach are shown with President Newman.

NEW FELLOWS ADMITTED NOVEMBER, 1982: Forty-six of the fifty-one new Fellows admitted at San Francisco are shown.

NEW ASSOCIATES ADMITTED NOVEMBER, 1982: Fourteen of the seventeen new Associates admitted at San Francisco are shown.

OBITUARIES

JOHN W. CARLETON
1914–1982

John W. Carleton, a Fellow of the Casualty Actuarial Society since 1938, died on April 21, 1982 in Annandale, Virginia, at the age of 67.

Mr. Carleton, a native of California, was a 1935 graduate of the University of California at Berkeley.

His insurance career began in 1935 with the Fireman's Fund Indemnity Company. In 1939, he joined the California State Compensation Insurance Fund.

In 1941, he joined Liberty Mutual in the statistical department. After serving as an officer in the Navy in Washington, he returned to Liberty Mutual in 1946 and progressed in the actuarial and managerial departments. In 1955, he was elected a vice president. In 1968, he was named senior vice president, and, in 1970 he was elected a director of both Liberty Mutual Insurance Company and Liberty Mutual Fire Insurance Company.

After his retirement in 1979, he served as a consultant to the Alliance of American Insurers.

He is survived by his wife, Phoebe; two daughters; and three grandchildren.

CHARLES W. CROUSE
1907–1981

Charles W. Crouse, a Fellow of the Casualty Actuarial Society since 1946, passed away on December 9, 1981 at the age of 73.

Mr. Crouse was a native of Waynesboro, Pennsylvania. He graduated from Penn State University in 1929 with a B.S. degree in electrical engineering. He achieved a Masters degree in mathematics from the University of Pennsylvania in 1945.

Mr. Crouse started his insurance career in 1936 with the State Workmen's Insurance Fund of Pennsylvania. In 1938, he joined the American Casualty Company. In 1942, he joined the Pennsylvania Casualty Company in Philadelphia. In 1951, he joined the firm of C. E. Preslan & Co., Inc. as a consulting actuary. He remained there for 20 years. In 1971, he became an assistant professor of mathematics at Cleveland State University, a position he held until his retirement in 1977.

He is survived by his wife, Margaret; two daughters; one son; and five grandchildren.

JAMES H. DURKIN
1912–1982

James H. Durkin, an Associate of the Casualty Actuarial Society, died on September 8, 1982, in East Hampton, New York, at the age of 70.

A graduate of City College of New York, he entered the profession in 1957 by joining the staff of the consulting firm of Wolfe, Corcoran & Linder. In 1965, when that firm merged into Peat, Marwick, Mitchell & Company, he became a manager and consulting actuary with the latter organization. He remained there until his retirement in 1977.

His keen mind and wide interests led him into a number of actuarial paths. His two principal areas of interest were health insurance, primarily as a consultant to a large number of Blue Cross/Blue Shield plans, and medical malpractice insurance. He gained national recognition in both fields. His *Proceedings* paper, "A Glance at Prepaid Dental Insurance" won the Woodward-Fondiller prize in 1964. In the area of medical malpractice his advice and assistance was much sought after by medical organizations and insurers in many parts of the country. He continued his consulting activities beyond his retirement from his company and virtually up to the time of his death.

He is survived by his wife, Diana; and two daughters, Kathleen and Myra.

GARDNER V. FULLER
1896–1982

Gardner V. Fuller, a Fellow of the Casualty Actuarial Society since 1934, died on October 3, 1982, in Whitewater, Wisconsin. He was 86 years old.

A native of Pawtucket, Rhode Island, Mr. Fuller earned an engineering certificate from the Lowell Institute in Boston, Massachusetts.

Following military service with the Navy in World War I, Mr. Fuller was employed in the engineering field for a few years before joining the National Council on Compensation Insurance in New York. He was employed there for 20 years until 1944. He then joined the Kemper Insurance Company where he held various positions.

After his retirement, Mr. Fuller's hobby of monitoring and recording weather information led to his becoming the local weatherman for radio station WERL in Conover, Wisconsin.

He is survived by his wife, Edith; two sons; nine grandchildren; and four great-grandchildren.

JOHN A. MILLS
1904–1982

John A. Mills, a Fellow of the Casualty Actuarial Society since 1937, died on April 28, 1982, in Reeds Spring, Missouri. He was 77.

A native of Antwerp, Belgium, at the age of seven he emigrated to Chicago, Illinois, where he completed grammar school and two years of high school.

At the age of fifteen, he went to work for Lumbermen's Mutual Casualty Company as an office boy. During his forty-two years of service with that organization, he attended night school, became a citizen, achieved his Fellowship, and rose to the position of Vice President and Actuary.

His paper entitled, "The Effect of Daylight Savings Time on the Number of Motor Vehicle Accidents" was published in Volume XXVI of the *Proceedings*. He was instrumental in the preparation and initial release of "Regulation No. 30 and Expense Studies" published in 1947 by the Insurance Accounting and Statistical Association.

He is survived by two sons, Richard J. (FCAS 1957), John G. (Executive Director, Carlsbad Foundation); five grandchildren; and one great-grandson.

DONALD E. TRUDEAU
1935–1982

Donald E. Trudeau, a Fellow of the Casualty Actuarial Society since 1962, died suddenly on July 10, 1982. He was 47.

A native of New Bedford, Massachusetts, he attended Holy Cross and Fordham Universities.

His career started as a consultant in 1962 in Kansas City, Missouri. In 1973, he joined the American Mutual Insurance Companies, where he eventually became vice president and comptroller. In 1980, he joined Peat, Marwick, Mitchell in Chicago. From there, he joined the Home Insurance Company in mid-1981 as Vice President and Chief Actuary.

He is survived by his wife, Catherine; a son and a daughter.

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