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FOREWORD

The Casualty Actuarial Society was organized in 1914 as the Casualty Actuarial and Statistical Society of America, with 97 charter members of the grade of Fellow; the Society adopted its present name on May 14, 1921.

Actuarial science originated in England in 1792, in the early days of life insurance. Due to the technical nature of the business, the first actuaries were mathematicians; eventually their numerical growth resulted in the formation of the Institute of Actuaries in England in 1848. The Faculty of Actuaries was founded in Scotland in 1856, followed in the United States by the Actuarial Society of America in 1889 and the American Institute of Actuaries in 1909. In 1949 the two American organizations were merged into the Society of Actuaries.

In the beginning of the twentieth century in the United States, problems requiring actuarial treatment were emerging in sickness, disability, and casualty insurance—particularly in workers' compensation—which was introduced in 1911. The differences between the new problems and those of traditional life insurance led to the organization of the Society. Dr. I. M. Rubinow, who was responsible for the Society's formation, became its first president. The object of the Society was, and is, the promotion of actuarial and statistical science as applied to insurance other than life insurance. Such promotion is accomplished by communication with those affected by insurance, presentation and discussion of papers, attendance at seminars and workshops, collection of a library, research, and other means.

Since the problems of workers' compensation were the most urgent, many of the Society's original members played a leading part in developing the scientific basis for that line of insurance. From the beginning, however, the Society has grown constantly, not only in membership, but also in range of interest and in scientific and related contributions to all lines of insurance other than life, including automobile, liability other than automobile, fire, homeowners and commercial multiple peril, and others. These contributions are found principally in original papers prepared by members of the Society and published in the annual *Proceedings.* The presidential addresses, also published in the *Proceedings,* have called attention to the most pressing actuarial problems, some of them still unsolved, that have faced the insurance industry over the years.

The membership of the Society includes actuaries employed by insurance companies, ratemaking organizations, educational institutions, state insurance departments, and the federal government; it also includes independent consultants. The Society has two grades of members, Fellows and Associates. Both grades are achieved by successful completion of examinations, which are held in May and November in various cities of the United States and Canada.

The publications of the Society and their respective prices are listed in the *Yearbook* which is published annually. The *Syllabus of Examinations* outlines the course of study recommended for the examinations. Both the *Yearbook* and the *Syllabus of Examinations* may be obtained without charge upon request to the Secretary, Casualty Actuarial Society, One Penn Plaza, 250 West 34th Street, New York, New York 10119.

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Volume LXVIII, Part 1 No. 129

PROCEEDINGS

May 17, 18, 19, 20, 1981

RISK CLASSIFICATION STANDARDS

MICHAEL A. WALTERS

Abstract

The purpose of this paper is to examine insurance classifications in view of the statutory requirements that insurance rates not be unfairly discriminatory. More specifically, how does one decide what classification variables should be permissible and what definitions of classes are allowed?

The paper asserts that classifications should possess certain necessary and sufficient qualities called standards. Seven such standards are developed, which can be summarized into three broad categories, described as homogeneous, well-defined, and practical. Homogeneity is the most controversial, and the subject of much current debate about whether or how to measure it quantitatively, and about its relative importance.

Other qualities are discussed, none of which should be considered necessary conditions—for example, causality, controllability, incentive value, separation, and social acceptability.

The paper concludes with the perspective that sound classifications often conflict with the concept of affordability. Finally, classifications cannot and should not be used to try to solve all the problems of society.

INTRODUCTION

The escalating inflation of the past decade spawned complaints about more than just overall insurance rate increases. Unlike most other products, insurance costs depend upon buyer characteristics, so questions of fairness have naturally arisen as some insureds were confronted with four digit auto insurance prices along with double digit inflation. "Affordability," "availability," and "social acceptability" all became clichés of the late seventies. In particular, regulators, legislators, and other consumer advocates have focused increasing concern on the third requisite of virtually every state's mandate on insurance rates, that they "not be unfairly discriminatory."

Some critics have claimed that insurance rating methods, and classifications specifically, should be sensitive to consumer perceptions about what is fair. They suggest that classifications should possess qualities of reliability, causality, controllability, separation, and incentive value. Some of these proposals might be essential to the insurance process, while others may be merely sound business advice, and still others might be totally inappropriate.

A search through insurance and actuarial literature does not find an abundance of historical resource material relevant to, or in the language of, these current issues.[1] Some of the more persuasive reformers have, in fact, coined new phrases and fashioned new literature as the basis for change. From a social standpoint, some of the espoused changes may be genuine attempts to solve affordability problems in what is intended to be a "fair" manner. However, if the resulting mechanism violates the principles of insurance, it is not an insurance program. Therefore, it might not be under the jurisdiction of a state's insurance regulation.

A recent insurance monograph by Professor John Long elaborates on the problem.[2]

"It is fashionable to be critical of insurance theory and to blame the ills of the

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[1]From the author's viewpoint, research on this topic began prior to his testimony at a New York Insurance Department public hearing on classifications held in April, 1971. Much of the actual content of this paper was obtained from extensive testimony by the author at a May, 1979 hearing before the New Jersey Insurance Department. Some of the "standards" proposed are a composite obtained from the prior research of the literature (without specific reference). The list of references appended contains more recent readings which helped the author to articulate his opinions on this subject. This paper also represents some changes from an earlier version submitted for the 1980 CAS Discussion Paper Program and discussed at the May 1980 CAS meeting.

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[2]John D. Long, "Soft Spots in Insurance Theory," *Issues in Insurance*, Vol. II, 1978, p. 444.

insurance marketplace on the shortcomings of insurance theory. For example, one point of view is that the purpose of the insurance industry is to serve the needs of the public and that any inability of the industry to do so means that something is wrong with the underlying insurance theory . . .

"A case in point has to do with exposure to flood loss . . . The Congress has seen fit to provide a subsidy to eligible people who participate in what is called the national flood insurance program. This program raises the question of how much 'non-fortuitous' transfer of funds can occur in a transaction without causing the transaction to be something *other* than insurance . . . In the author's judgment, the federal flood program exceeds such limit and, therefore, is a type of welfare rather than a type of insurance. This classification is not to imply that because the flood program is not insurance it is 'bad.' The only point being made is that the subsidy for all participants by the taxpayers as a whole is so large that the arrangement is not insurance. Calling something insurance does not necessarily imbue it with the characteristics associated with insurance."

It is important therefore to distinguish those qualities which some would like to see an insurance classification system possess to achieve alternative goals, from those which are necessary and sufficient conditions, or standards, which flow from the nature of insurance. The purpose of this paper is to develop a set of these standards for insurance classifications, which have been implicitly used, or should be used, to evaluate compliance with insurance statutes.

NATURE OF INSURANCE

The purpose of insurance is to protect an insured from a large and fortuitous financial loss. It is achieved by contractually transferring the insured's uncertainty of loss to the insurer for the certainty of a smaller payment called the premium. This uncertainty of loss is called risk.

Since the insurer assumes the individual insured's risk of loss, the premium should be fundamentally based upon the expected value of an insured's losses. The expected loss for an insured is the average or probable number of losses (or claims) times the average cost of those claims. The premium should also include the expense of servicing the policy plus a margin for profit and contingency as a reward for taking the risk. The amount of this profit margin should depend upon two basic factors: the ability of the insurer to estimate the expected (or average) loss of the individuals insured,[3] and the amount of overall reduction of uncertainty accomplished by the pooling process.

[3]There is obviously more risk involved to the insurer than distinguishing one insured from another. The uncertainty of next year's inflation level, for example, affects the expected cost of individuals, but more or less to the same degree.

Insurers are not, of course, trying to predict the *actual* losses of each insured, only the *expected* loss. It is the variation of an individual's actual losses from his expected losses that motivates his purchase of insurance, while it is the variation of expected losses from individual to individual that motivates insurers to price insureds differently.

Although from an insured's standpoint the essence of insurance is the transfer of risk, a further value of insurance for society is the reduction of overall risk or uncertainty by pooling many insureds independently exposed to loss.

These risks in the pool do not have to be exactly the same types of risks for insurance to work, as witnessed by the success of Lloyd's of London, with a multiplicity of risks no two of whom may have been the same over the years. And certainly, insureds who are inherently different risks should not have to pay the same for the insurance process to work. But pooling works especially well within a given line of insurance, like private passenger auto insurance, when enough independent risks are pooled such that it is virtually impossible that they all will have accidents in the same year. In fact, the more risks that are written, the closer reality comes to the expected. This intuitively expresses the "law of large numbers."[4] Its first and perhaps best known application allows insurers to have more confidence that, once each risk has been reasonably priced, the *actual* losses on all those risks combined or pooled will come reasonably close to the combined *expected* losses at the end of the year.

This does not say that the pooling of risks is the same as pooling of losses. This latter term somehow may connote that everyone should share the costs equally. Insurance can work just as well even if every risk had a different expected loss, as long as you can reasonably estimate the expected losses.

Likewise, insurance does not require that each classification must be large enough to stand on its own. This fallacy says that individual classes cannot share the risk among other classes.[5] It would also deny the ability to summarize

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[4]D. B. Houston, "Risk, Insurance, and Sampling," *The Journal of Risk and Insurance*, Vol. XXXI No. 4, pp. 526–530.

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[5]See Stanford Research Institute, *The Role of Risk Classification in Property and Casualty Insurance*, Final Report, May 1976, p. 63: "Confusion surrounding the term 'classification' stems also from an association with the concept of pooling of risks to reduce the aggregate risk. Many people feel that the essence of classification lies in having large classes, the members of which share the total risk of the class (and supposedly do not share the risk of any other class). According to this incorrect view, classes must each have many members to pool risks; classes with too few members are therefore not 'credible' and are assumed to violate the basic principle of risk sharing."

across classes to gain additional information about other classes, such as pooling classification information within territory to determine territory rates, or territories within state to determine statewide rate levels.

Furthermore, some may believe that insurance is an instrument of social policy to compensate victims. This view treats the premiums as merely a means of accumulating funds to pay out losses in ways possibly fundamentally different from the relative risk that each insured presents to the pool. But trying to do something noble via the premium collection facilities of insurers does not make the resultant mechanism insurance, as cited earlier. Insurance is what it is—the transfer and reduction of risk; it is not a tax to redistribute wealth.

Thus, the expected loss of the individual is important to the pricing of insurance. But, being inherently unknowable, even by the insured himself, how do insurers infer this vital quantity? There are three basic methods.

First, they may use wisdom and experience as underwriters in exercising informed judgment about the nature of the insured and the exposure to loss and attendant hazards. From an insured's standpoint, with a primary desire to transfer the risk, as long as both parties agree that the price is reasonable, the insurance mechanism is working. A variation of this method when applied systematically with quantified parameters is called schedule rating.

The second basic method of inferring an individual risk's expected loss is to observe the insured's actual losses over a period of time. This gains certain additional information, picking up more of the subtleties of the risk that might not be obtained by logical, informed judgment. This is called experience rating as compared to schedule rating. However, this is not always reliable because information may be outdated, as the risk to be insured next year may have changed substantially. Furthermore, depending upon the frequency of accidents, it may take many years of observation to reasonably infer expected losses, given the dominance of randomness in the accident occurrence process.

The first two methods are usually more relevant to rating large commercial lines risks, where the published rates in the manuals can economically be adjusted to fit individual risks given the overall size of the premium involved. For smaller risks (usually personal lines risks) the rates in the manual are the final prices charged to the insured, with no individual risk modifications; thus, the importance of the next basic method, i.e. classification, in determining the right premium for the risk. Even for large risks, classification plays an important role in establishing a reasonable starting point for the individual risk rating process.

The third basic method of inferring expected losses is to observe the experience of a group of similar risks over a shorter and more recent period of time. This grouping of similar risks to estimate costs is called classification. Furthermore, this group observation process also involves the second use of the law of large numbers. The first use was that if you know the expected losses in advance, then the actual losses will tend to approximate the expected at the end of the year for the insurance enterprise as a whole. However, by observing a smaller number of similar risks over a short period of time you have more confidence that you have closely estimated the expected losses of the individuals in advance. This is especially important if the set of insureds can change from one year to the next. (This process of classifying is analogous to using stratified random sampling to gain more information when the size of the total sample is limited.[6])

There are some who feel that group inference for an individual member of a group is unfair per se, no matter how the groups are defined. This would seem to prohibit the use of any statistical-based knowledge throughout society, and is contradicted by all insurance statutes which allow, or even mandate, the use of classifications. The Stanford Research Institute (SRI) also clearly addressed this:

> ". . . the opinion that distinctions based on sex, or any other group variable, necessarily violate individual rights reflects ignorance of the basic rules of logical inference in that it would arbitrarily forbid the use of relevant information. It would be equally fallacious to reject a classification system based on socially acceptable variables because the results appear discriminatory. For example, a classification system may be built on use of car, mileage, merit rating, and other variables, excluding sex. However, when verifying the average rates according to sex one may discover significant differences between males and females. Refusing to allow such differences would be attempting to distort reality by choosing to be selectively blind."[7]

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[6]Houston, *op. cit.*, p. 534. Author's note: If the classes are fairly stable over time, they do not even need to have similar expected losses for the individuals within in order to gain a good estimate of the class average expected losses. Merely the variance of actual losses from the mean for each individual insured in the class should be similar. This results from the fact that insurance classification reviews use all the risks insured in each class.

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[7]Stanford Research Institute, *op. cit.*, p. 91.

CLASSIFICATION STANDARDS

So insurance classifications are seen as needed in the pricing of many kinds of insurance, helping to reduce overall risk, as well as enabling insureds to pay approximately in proportion to their relative hazard of loss. If there were no reflection of these relative costs by an insurer, it could risk insolvency if the distribution of exposures changed substantially. At a minimum, such an insurer will require a larger margin for profit and contingency to offset the much greater chance of adverse underwriting results.

At this point, it is important to distinguish risk classification from risk selection. Risk selection determines the set of insureds with whom the insurer decides to enter into a contractual relationship[8] and whom the classification system must price. Marketing gives a more general set of insureds and underwriting yields the specific insureds that need to be priced.

The fairness of marketing or underwriting rules is beyond the scope of this paper. However, once a risk is insured, it is reasonable that the standards for classifying that risk can and should be different from those of marketing or underwriting. Furthermore, once the classifications are established, there are also guidelines to follow in establishing the prices, or classification differentials, for the system. Guidelines regarding pricing structure (e.g. additive or multiplicative rating factors) or pricing estimation techniques are also beyond the scope of this paper. What this paper will focus on are the appropriate rules regarding selection of classification variables and the definition of classes at the very start of the classification rating process.

Given the preceding, the variables comprising a classification system should be chosen so that the following standards or conditions (in addition, of course, to any legal requirements regarding fair discrimination) are generally met:

1. Similar risks should be assigned to the same class with respect to each variable. Conversely, dissimilar risks should be assigned to different classes, so that there are no clearly identifiable subsets with a significantly different loss potential or expected loss in the same class.[9]

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[8]In some lines and states, an involuntary or "shared" market exists which requires participation by insurers in order to write voluntary business. This helps solve an availability problem for those not "selected" by insurers under usual markets.

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[9]It is important to stress the words "clearly identifiable" when dealing with the alleged overlap or heterogeneity of certain classes.

2. The common characteristics used to identify insureds as similar should reasonably relate to the potential for, or hazard of, loss.[10]
3. The classes should be exhaustive and mutually exclusive; that is, each insured should belong to at least one, but only one, class with respect to each rating variable.
4. There should be clear and objective phraseology in the definition of classes, with no ambiguity as to what class an individual insured belongs.
5. An insured should not be easily able to misrepresent or manipulate his classification.
6. The cost of administering a rating variable should be reasonable in relation to the benefits received.
7. The class rating factors should be susceptible to measurement by actual experience data.

The first standard is what is meant by *homogeneous* classes. Classes that are homogeneous will take fewer risks to obtain reasonable estimates of expected costs, and will minimize the ability of the competition to skim off better than average risks, thus changing the ultimate costs.

The second or "reasonable relationship" standard aids in maintaining homogeneous classes by avoiding spurious measures which likely have potentially identifiable subsets. Of course, if a strong statistical correlation persists over time, with no emergence of practical subdivisions, then the degree of perceived reasonableness may be enhanced over time, as well.

Homogeneity is also undergoing some current debate as to the possibility of statistical measurement.[11] While the scope of this paper precludes entering that debate, it is important to recall that one of the reasons for classifying is the impossibility of knowing a risk's true expected loss or accident likelihood. Given the randomness of accident and loss occurrence, and the fact that statistical tests must use actual loss distributions for individuals, it may be difficult to gain more than a glimpse or an insight into possible distributions of accident likelihoods within a class. This is especially true since assumptions must also be made about the functional form of the accident likelihood model (as well as of the loss severity model). Furthermore, the real test of homogeneity is in the most refined classification cell, not in the separate variables used in combination to classify the risk.

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[10]This is different from, and yet related to, what some others have used as the notion of causality, and will be covered in the section on non-standards.

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[11]Richard G. Woll, "A Study of Risk Assessment," *PCAS* LXVI, 1979.

It is also not necessary (nor even likely) for a classification to have identical expected losses for all risks within the class, even if true individual risk accident likelihood were "knowable." Even if statistical inferences can be made about the "true" distribution of risks with different expected losses within a classification, the lower expected loss insureds deduced to exist are not in any way identified (or identifiable) to the insurer or even known by the risks themselves. Therefore, it is bordering on a philosophical game to assert that such a class is too heterogeneous, and is therefore not permissible.

The SRI spoke to that fallacy as follows:

> "Indeed, the rationale that proscribing the use of certain rating variables is in the public interest because, under imperfect risk assessment systems, actuarial fairness is not achieved for some—albeit unidentifiable—individuals is fundamentally contradictory. It promotes a remedy for unfairness to some that increases the unfairness overall (by the same actuarial yardstick) and redistributes it."[12]

The third, fourth, and fifth standards deal with classes being *well-defined,* and help to ensure that each risk is actually placed in the right classification and to avoid unequal application of the classification system. The "exhaustive" quality allows more risks to be accepted and, once accepted, gives a complete method of rating them. "Exclusivity" precludes two different rates for the exact same risk. "No ambiguity" also prevents unequal treatment of the same risk, while protection from misrepresentation by insureds will keep the statistical data consistent as well as enhancing the equal treatment of insureds.

The dictionary definition of *practical* refers to "workable, useable, and sensible" and the final two standards deal with this goal. Being cost-effective is important because an inefficient system (or even attempts to be too precise) could increase total costs beyond the value of the information to be obtained. If, for example, it costs an insurer ten dollars on each policy to find only a small portion of risks who could save twenty dollars, it is not worth the effort.

In final perspective, one of the advantages of classifying is to use the law of large numbers on the actual observed experience of the past instead of relying too much on informed judgment. If there is no method of testing class-average prices by actual data, the system is closer to that of schedule rating. Of course, whether or not a classification rating factor is tested frequently depends upon the likelihood of change in a short period of time, and the relative size and importance of the rating factor.

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[12]SRI International, *Choice of a Regulatory Environment for Automobile Insurance,* May 1979, p. 58.

NON-STANDARDS

In this paper, the word "standards" has been used to denote a set of conditions for insurance classifications to generally meet, consistent with the nature of insurance as well as insurance statutes. However, the dictionary definition also includes "a basis of comparison in measuring or judging . . . quality." It is possible or indeed likely that other characteristics may be desirable for a classification system. Failure to include these in the basic standards means that it is felt that their presence is not as important in considering the classification system valid and appropriate.

Two different qualities that have been recently espoused are actually related—*controllability* and *incentive value*. By controllability is meant the ability of an insured to determine by his own efforts (presumably consciously) the class to which he is assigned. If that quality is present, it is argued, the insured will have the incentive to change to a lower rated class and thus reduce his own losses as well as the losses of the overall system.

One can sympathize with a risk that presents a much higher hazard, over which it has little or no control, but to deny use of that criterion, and make others with lower inherent risk subsidize the higher risk is, in effect, a denial of reality. In workers' compensation insurance, for example, the logging or lumbering industry has an inherently higher risk of injury to workers than clerical office work. Not to charge for that difference would be to contradict the essence of classification. Similarly, age in life insurance is an essential classification, yet is obviously uncontrollable. Controllability therefore is really extraneous, having benefits primarily in the area of public understanding.

Incentive value also has public appeal, and a variation of it is quite important to the overall insuring process. Whether it be classifications or exposure bases, or indeed the existence of insurance, the presence of an insurance contract should not encourage a laxity towards loss control or create a moral hazard of exaggerated or false claims.[13]

While incentive value could be a valuable addition to a rating system, it is not a necessary one, nor should classification plans be judged by it as a standard. Personal lines risks, for example, cannot be easily subjected to loss prevention measures like large commercial risks. Even so-called "merit rating" in automobile insurance may be nothing more than a theoretical incentive to prevent accidents. Few drivers wear seat belts despite the life saving evidence, so the

[13] C. A. Williams, et al., *Principles of Risk Management and Insurance*, Vol. I, 1978, p. 128.

prospect of saving a few dollars of insurance surcharge certainly will not induce a modification of driving behavior. In a Department of Transportation (DOT) study, a major conclusion in this area was also reached: "As long as deterrent measures concentrate on a punitive approach to the correction of 'driver error,' they are likely to remain relatively ineffective." [14] Of course, once an accident occurs, the fear of a surcharge may affect the reporting of accidents and submission of collision claims, but that may be in conflict with the liability insurance policy "condition" requiring notification of accidents.

Causality has also been recently cited as a desired quality for classifications to possess. It is defined as follows: "The actual or implied behavioral relationship between a particular rating factor and loss potential." [15] The use of the term "behavioral" makes this difficult to accept as a standard, because living in the river valley does not cause the river to flood, yet certainly increases the hazard involved in flood insurance.

Merit rating in auto insurance is almost totally non-causal. The fact that an insured has been involved in a past accident does not behaviorally cause him to get in the next one or even to have become a worse driver. And yet the same critics of current rating cite past accident record as an ideal rating variable.

Instead, a reasonable relationship to the hazard of loss, without such a rigid chain of causality or behavior, is more appropriate. As the earlier mentioned DOT study concluded: ". . . driver responsibility for crashes is rarely unilateral and is often impossible to isolate from the multiplicity of causes involved in almost every crash." [16]

By classifying risks, an insurer does not seek to determine the cause of the accidents. To the extent high risk insureds are identified, society may benefit by focusing attention on the need for possible remedies.

Separation has been defined as "a measure of whether classes are sufficiently different in their expected losses to warrant the setting of different premium

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[14] U.S. Department of Transportation, *Causality, Culpability and Deterrence in Highway Crashes*, 1970, p. 245.

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[15] "Final Report of the Rates and Rating Procedures Task Force" to the (NAIC) Automobile Insurance (D-3) Subcommittee, November 1978, p. 5.

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[16] Department of Transportation, *op. cit.*, p. 209.

rates."[17] This deals with the so-called "overlap" question where it is felt that if one class rate were close to another, some insureds in the first class would have accident likelihoods close to those in the second class, and therefore may be misclassified.

This is related to the homogeneity question. If the insureds who supposedly deserve to be in the second class *are not identifiable*, then it is questionable whether you can call them misclassified. Secondly, classifications with mean rates close together are not undesirable, if the hazard being reflected is a gradual one. Finally, even if some insureds in a $300 rated class truly deserve to be in a $305 class, the system is still working well from a cost/benefit standpoint. Therefore, the concept of separation does not appear very useful in the context of classification standards.

Reliability has also been a term which includes qualities that are objective, clearly defined, and easy to verify, all of which are consistent with the standards earlier mentioned, and about which there is little or no controversy.[18]

However, *social acceptability* and *admissibility* are terms which connote a variety of meanings and contexts regarding the use of insurance classifications. By way of perspective, it is one thing to give advice as to the public's view of certain rating variables among alternatives of equal value. It is quite something else to say that the unpopularity of some variables, as perceived subjectively by some, or even through public opinion polls, precludes their use. Rate adequacy and public acceptability are often in conflict.

The SRI report cited earlier suggested that insurers choose variables among the set of possible ones, without loss of precision, that are clearly explainable to the public, provide incentives for loss prevention, and are adjusted to social mores.[19] That this was meant as sound business advice, rather than a set of necessary conditions, is illustrated by their comments on the very next page:

> "On the other hand, the opinion that distinctions based on sex, or any other group variable, necessarily violate individual rights reflects ignorance of the basic rules of logical inference in that it would arbitrarily forbid the use of relevant information. It would be equally fallacious to reject a classification

[17] Division of Insurance, Commonwealth of Massachusetts, *Automobile Insurance Risk Classification: Equity and Accuracy*, 1978, p. 3.

[18] Division of Insurance, Commonwealth of Massachusetts, *op. cit.*, p. 3.

[19] Stanford Research Institute, *op. cit.*, pp. 89–90.

system based on socially acceptable variables because the results appear discriminatory. For example, a classification system may be built on use of car, mileage, merit rating, and other variables, excluding sex. However, when verifying the average rates according to sex one may discover significant differences between males and females. Refusing to allow such differences would be attempting to distort reality by choosing to be selectively blind.

"The use of rating territories is a case in point. Geographical divisions, however designed, are often correlated with sociodemographic factors such as income level and race because of natural aggregation or forced segregation according to these factors. Again we conclude that insurance companies should be free to delineate territories and assess territorial differences as well as they can. At the same time, insurance companies should recognize that it is in their best interest to be objective and use clearly relevant factors to define territories lest they be accused of invidious discrimination by the public."[20]

Moreover, in a later work, the SRI clearly stated: "The regulator's determination of what is unfairly discriminatory should relate only to the use of variables whose predictive validity cannot be substantiated and to unequal application of a classification system."[21] Furthermore, they put the context of extreme social intolerability in the legislative arena:

"One possible standard does exist for exception to the counsel that particular rating variables should not be proscribed. What we have called 'equal treatment' standard of fairness may precipitate a societal decision that the process of differentiating among individuals on the basis of certain variables is discriminatory and intolerable. This type of decision should be made on a specific, statutory basis. Once taken, it must be adhered to in private and public transactions alike and enforced by the insurance regulator. This is, in effect, a standard for conduct that by design transcends and preempts economic considerations. Because it is not applied without economic cost, however, insurance regulators and the industry should participate in and inform legislative deliberations that would ban the use of particular rating variables as discriminatory."[22]

Admissibility, as per the Massachusetts definition, begins with federal and state statutory requirements regarding discrimination and privacy, but continues in the social acceptability vein:

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[20]Stanford Research Institute, *op. cit.*, p. 91.

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[21]SRI International, *op. cit.*, p. 93.

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[22]SRI International, *op. cit.*, p. 94.

"There are also distinctions that, while not clearly illegal, are being increasingly questioned. These include sex, income, and marital status. Clearly, it is preferable to avoid such distinctions. Distinctions are best able to meet the test of admissibility if they are within an individual's ability to control and are causally related to the probability of loss. It would be undesirable, for example, to charge higher rates for redheads than brunettes even if it could be shown statistically that people with red hair have more accidents than those with brown hair."[23]

Use of the words "preference" and "desirability," from a perception of the public's view and using popular intuition about controllability and causality, again confirms that this characteristic is in the form of business marketplace advice. Insurers who can combine sound and relevant rating variables with the public's view of what is better will obviously be more successful. However, unless or until possible substitute variables are found which do not sacrifice accuracy and do not create subsidies, the failure to use appropriate, though unpopular, variables will only cause some individuals availability problems and still others to be overcharged relative to their risk.

REGULATION VERSUS COMPETITION

Given that insurance regulators must enforce the rate regulatory laws, a logical question to be asked is whether natural competitive forces will reinforce or conflict with the standards for insurance classifications.

Regarding homogeneity, it is obvious that the essence of competition will be to try to find rateable subsets of existing classifications to price more accurately and equitably (prices matching costs).

If classes are too broad, underwriters will tend to select risks out. However, it takes more discipline to define objective and practical new classifications to maximize the number of risks to be written voluntarily. If several different companies are licensed in a group under the same management control, the competitive drive for more homogeneity can be partially met by a different set of underwriting rules for each company in the group.

If there is only a strong statistical correlation for a particular variable, without an obvious relationship to hazard of loss, competitive forces will definitely strive to find a closer link. If no closer link is found over an extended period of time, the reasonableness of the relationship becomes much more established.

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[23]Division of Insurance, Commonwealth of Massachusetts, *op. cit.*, p. 4.

There is an analogy here with the statistical correlation between lung cancer and cigarette smoking, which, for many years, was not held to be a health hazard. In fact, there has yet to be found in human medicine a cause and effect link showing lung cancer resulting from tobacco smoking. Conceivably (but unlikely), cigarette smokers could have other characteristics related with carcinogens that are also less prevalent in non-smokers. The answer, of course, is not to avoid the use of statistical information until better data are found. Indeed, the U.S. Surgeon General and others have taken strong steps based mainly (and reasonably) on the statistical evidence. Even though the actual risk of death from lung cancer among the heaviest smokers is very small, it is many times that of non-smokers. Stated another way, most heavy smokers will not contract lung cancer; yet all of them have had certain privileges revoked and rights modified.

One can normally expect marketplace rewards for those who use well-defined class plans allowing equal treatment for all risks. However, there is a temptation to allow some ambiguity or subjectivity as a trade-off for additional costs needed to gain consistent information.

Regarding practicality, competitive forces will place a natural restraint on overspending to attain rating information. Part of the workability of classifications involves testing the rating factors with actual data to minimize the subjectivity of pricing. However, there is a potentially conflicting tendency to rely on judgment and assumptions to avoid the cost of truly testing for the appropriate price relationships. Of course, to the extent that other insurers find cost-effective ways of better measuring class relativities, then as long as there is the ability to exchange information, any pricing inequities will be short term.

Some examples of classifications which do not meet the standards might include the following:

1. The use of occupation as a rating variable for auto liability insurance may result in a problem with regard to meeting the ambiguity standard, both in splitting the population into exhaustive categories, as well as not having all cells likely being reasonably related to the hazard of loss.
2. Similarly, national origin (if not already proscribed by law) would have problems meeting the mutually exclusive and exhaustive standards.
3. Use of unverifiable criteria or too subjective wording, such as with psychological profiles, would also present major problems. The use of characteristics which are easily circumvented by some insureds and not others can favor the pricing of some to the detriment of others.

A class plan would not be homogeneous if it failed to reflect premium differences for identifiable and rateably different subsets within broader classifications. The degree of failure would depend upon the cost of determining the necessary information. From the insured's standpoint, the pricing impact of not subdividing depends upon the size of the subsets and the resulting differences in price for each of the subclasses. It may be that only a small amount of premium can be saved by refinement, if one of the subclasses is very large and also the lowest priced (such as rating by past accident record in auto insurance where accident-free or claim-free drivers usually save at most five percent over the cost of not having such a program).

However, if lower risk insureds were identified in a system and were rateable with a classification variable, the failure to reflect those differences would constitute a *subsidy*. If the set of insureds are not identifiable in advance, then there is no subsidy. For example, some have alleged that all of insurance is a subsidy since, as the reasoning goes, those who do not have accidents are subsidizing those who do. This is fallacious because you cannot identify *in advance* those who will have accidents. That is why people buy insurance. However, you can identify those with a higher likelihood of an accident, which is what classification is all about. Failure to classify would therefore be a subsidy by those with a lower loss likelihood of those with higher loss expectancy.

Some also allege that it is a cruel disservice to identify the high risk insureds in advance through refined classification plans. However, insurers should not be blamed for the existence of high risks in society. In a report from the Federal Trade Commission to the U.S. Department of Transportation in 1970, it was concluded that: "Regardless of law and underwriting systems, high risk drivers exist. The present system identifies them; it does not create them."[24] In fact what insurers do by keeping track of the sources of accidents is to help identify those segments of the population where loss prevention may be the answer rather than risk pooling. "In the interests of loss control and prevention, this high-risk group must be identified and treated before the accidents occur."[25] In other words, if high risk driving in high density areas produces an inordinate

[24]Report of the Division of Industry Analysis, Bureau of Economics, Federal Trade Commission to the Department of Transportation, *Price Variability in the Automobile Insurance Market*, August 1970, p. 144.

[25]Department of Transportation, *op. cit.*, p. 144.

amount of loss, perhaps more stringent licensing should be considered, or mass transportation improvements, or other alternatives; but do not hide the information. Until such time as the source of the problem is solved, to paraphrase the SRI report on risk classification, one should not legislate against the use of knowledge in a free society.[26]

SUMMARY

The purpose of this paper was to view the issue of reasonable classification from the perspective of the nature of insurance itself. In this way the qualities that many have felt classification ought to possess could be distinguished between the essential and the non-essential.

Much has been written in the past few years about what is fair or unfair, but this evaluation should not take place without an understanding of what classifications are designed to do in insurance. Affordability is one example of a quality which society might like insurance rates to have, but the essence of classifications serves to highlight high-risk, high-cost segments of the population. Unfortunately in that instance and in possibly others the solution to the problem may lie outside the scope of insurance classifications or even the insurance mechanism itself.

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[26]Stanford Research Institute, *The Role of Risk Classification in Property and Casualty Insurance*, 1976, Executive Summary Report, p. 25.

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DISCUSSION BY MICHAEL J. MILLER

The best time to reach agreement on the rules of the game is before the first pitch. A rules debate during the seventh inning of a close game may produce more heat than light.

The author acknowledges the current debate over risk classification and observes that some reformers have "fashioned new literature" to form the basis for their desired changes. He attempts to avoid this expediency by defining risk classification standards which flow from the nature of insurance and are consistent with insurance statutes. The seven standards suggested are summarized into three broad categories: homogeneous, well-defined, and practical.

The author also discusses seven additional characteristics: controllability, incentive value, causality, separation, reliability, social acceptability and admissibility. These are classifed as non-standards because, in the author's view, they are not as important in judging a risk classification plan.

The author concludes by discussing how competitive forces in the marketplace will tend to reinforce his risk classification standards.

We now have at least four treatises of relatively recent vintage that discuss risk classification standards: the Massachusetts report,[1] the Academy report,[2] Mr. Walters' paper, and the recent New Jersey decision.[3] As an aid in placing Mr. Walters' paper in perspective, it is instructive to compare the relative importance given to the various risk classification characteristics by each of the four authors (see Exhibit I).

All four agree that homogeneity of risks within a class is a desirable classification standard. Both the Massachusetts and New Jersey reports advocate the choice of a statistical model to directly assess the extent of homogeneity within a class. Mr. Walters advocates a method which essentially disproves the homogeneity of a broader class by attempting to identify homogeneous subsets of the

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[1] Division of Insurance, Commonwealth of Massachusetts, *Automobile Insurance Risk Classification: Equity and Accuracy*, 1978.

[2] American Academy of Actuaries, Committee on Risk Classification, *Risk Classification Statement of Principles*, June 1980.

[3] New Jersey Department of Insurance, "Final Determination—Analysis and Report," Hearing on Automobile Insurance Classifications and Related Methodologies, April 1981.

class. The Academy report is silent on the method of determining the extent of homogeneity, but does refer to the absence of clearly identifiable subsets.

Separation, or between-class differences in expected losses, is given high priority in the Massachusetts and New Jersey reports. Mr. Walters considers separation to be an insignificant non-standard. In his opinion, classifications with prices close together are acceptable if the price gradation is gradual. Similarly, the Academy report places emphasis on the smooth gradation of prices from class to class, but does state there should be few enough classes "so that differences in prices between classes are reasonably significant."[4]

According to the Massachusetts report a classification plan should provide a practical and reliable way of predicting losses. This reliability standard explicitly includes characteristics involving ease of administration and objectively defined distinctions which are easy to verify. The terminology "practical and reliable" seems to *imply* that the class plan should be economically feasible and provide credible experience data in order to accurately predict losses. Both the implied and explicit characteristics set forth by the Massachusetts standard of reliability are embodied in two New Jersey standards: reasonable relation to hazard of loss, and adequacy of definition. In these two standards the New Jersey report agrees that class definitions should be clear and objective, not subject to manipulation; should maximize inclusion of similar risks in the same class; should have a direct relation to vehicle operations; and should provide data sufficiently credible to derive accurate premiums. The New Jersey report states that the only cost trade-offs which can be measured are those affecting premium differences between classes and therefore that these are the only costs which should be considered in evaluating the economic feasibility of the classification plan. Mr. Walters agrees that class definitions should be clear and objective, not subject to manipulation; should be exhaustive; should have a reasonable relation to hazard of loss; should be cost-effective; and should provide data susceptible to measurement. The Academy endorses similar standards in its discussion of these characteristics: absence of manipulation, absence of ambiguity, measurability, credibility, predictive stability, expense, and constancy.

The Massachusetts report endorses incentive value as a standard. The New Jersey report does not set forth incentive value as a separate standard, but does endorse it as a desirable characteristic. Mr. Walters considers incentive value to be a non-standard, but nevertheless a desirable addition to a classification

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[4] American Academy, *Risk Classification*, p. 18.

plan. The Academy report indicates that hazard reduction incentives are desirable, but not necessary, in the design of a classification system.

The final standard set forth in the Massachusetts report is admissibility. This standard deals with issues of legality, social acceptability, and fairness in general. According to the Masschusetts report, if the class factors are subject to the control of the insured and are causally related to the hazard of loss, then the factors will be more admissible or acceptable to the public. The admissibility standard is embodied in the fairness standard of New Jersey. The fairness standard says that classifications must meet legal requirements and fairly address the responsibility issue. This responsibility issue concerns whether an individual should be accountable for the full extent of his inherent risk. Mr. Walters categorizes the characteristics of controllability, causality, social acceptability, and admissibility as non-standards. He observes that controllability and causality may be desirable in increasing public understanding. He agrees that using rating variables which are acceptable to the public makes good business sense, but he would not sacrifice accuracy to achieve popularity. The Academy report observes that public acceptability issues should be balanced with the economic effects, that causality should not be a requirement for a classification system, and that controllability may have both positive and negative aspects.

The Academy report discusses availability of coverage as a desirable characteristic of a classification system. Mr. Walters does not discuss this as a separate standard. He does acknowledge that the failure to use appropriate rating factors may cause availability problems for some individuals. Neither the Massachusetts nor the New Jersey report discusses the availability of coverage concept. In fact, both reports tend to downplay the role of economic forces in the marketplace.

It would appear that the authors of the four papers are in general agreement on standards pertaining to predictive accuracy and operational considerations (there is some disagreement with respect to separation and the importance of economic feasibility). The greatest disagreement arises with the concept of social or public acceptability. Both the Massachusetts and New Jersey reports rely heavily on the regulator's view of equity. The Academy recommends that regulatory restrictions on classification systems should balance public acceptability and economic considerations. Mr. Walters advocates a much heavier reliance on competitive forces.

In the concluding section of the paper, regulation versus competition, Mr. Walters concludes that a class plan would fail the homogeneity standard if it did not reflect premium differences for identifiable subsets within a broader classification. The degree of failure would be dependent upon the economic cost of maintaining the separate rating class. This situation raises an interesting actuarial and legal question. If a homogeneous subset of a broader classification is identified, is cost effective to maintain, and is predictively accurate, is it unfairly discriminatory to fail to reflect the difference in the price? Based upon a narrow reading of standards in the four treatises referred to in this review, an insurer, to avoid unfair discrimination, may be forced to separately rate an identifiable subset, even if that action placed the insurer at a competitive disadvantage. For that reason, this reviewer would suggest that competitive considerations should be given a more explicit position on any list of classification considerations.

EXHIBIT I

COMPARISON OF RISK CLASSIFICATION STANDARDS AND NON-STANDARDS

Massachusetts	New Jersey	Walters	American Academy
Homogeneity	Within class differences	Homogeneity	Homogeneity
Separation	Between class differences	(Separation)	Avoidance of extreme discontinuities
Reliability —Practical predictor —Clear and objective —Ease of administration —Reduce error or fraud	Reasonable relation to hazard Adequacy of definition	Clear and objective Manipulation Exhaustive Reasonable relationship Administrative cost Measurement (Reliability)	Manipulation Absence of ambiguity Measurability Credibility Predictive stability Expense Constancy
Incentive	(Incentive)	(Incentive)	(Incentive)
Admissibility —Legality —Social acceptability —Controllability —Causality	Fairness —Legality —Shared responsibility	(Admissibility) (Controllability) (Causality) (Social acceptability)	Public acceptability balanced with economic side effects (Causality) (Controllability)

(   ) denotes non-standard or not required.

COMPUTER SIMULATION AND THE ACTUARY: A STUDY IN REALIZABLE POTENTIAL

DAVID A. ARATA

Abstract

This paper argues that computer simulation is an underappreciated and, therefore, underutilized casualty actuarial resource. In so contending, "Computer Simulation and the Actuary" discusses five applications of Monte Carlo computer simulation to everyday actuarial problems: establishing full credibility standards; testing the solidity of new, limited purpose insurance companies; pricing difficult or catastrophic exposures; customizing casualty insurance charges and excess loss premium factors; and developing loss reserve confidence intervals.

Illustrations of appropriate simulation solutions to each of these problems are provided.

OVERVIEW

Computer simulation refers to the process of accurately describing a complex system in a computer language, inputting this program into a computer, and allowing the machine to mimic ("simulate") the performance of the system described. For example, computers can easily be programmed to simulate accident year loss experience, given specific claim frequency and severity assumptions.

Historically, simulation has been afforded relatively little attention in the actuarial literature. Moreover, although this technique has been employed by actuaries confronting problems not soluble by more traditional means, primary emphasis has been placed upon non-simulation pricing and reserving procedures.

Reasons for this reluctance to rely more heavily upon simulation in addressing actuarial problems have included the lack of an adequate computer, the expense of the computer's operation, and occasionally the actuary's unfamiliarity with programming languages. Also, the need for simulation approaches has been somewhat mitigated by the publication in these *Proceedings* of elegant and impressive analytical solutions to most really difficult pricing problems.

The above obstacles are fast disappearing. Cost and access problems, for instance, are being overcome by the widespread introduction of microcomputer systems. "Conversational" programming languages, such as the BASIC language in which all simulations presented in this paper are written, are easy to learn and available on most systems. Moreover, special simulation languages, such as GPSS and SIMSCRIPT, give some simulation capability to the actuary without extensive programming knowledge.

More importantly, today's property/casualty insurance business faces problems which can be solved better and sooner with the assistance of computer simulation. The following sections present five such problems along with examples of appropriate simulation solutions. In presenting these illustrations, this paper argues that it is both inevitable and desirable that computer simulation will become an increasingly important weapon in the casualty actuary's arsenal.

Outline of This Paper

- Section I describes how computer simulation may be used to rediscover and expand upon classic "limited fluctuation" credibility notions. In so doing, this section provides a foundation for the more complex simulation applications presented in Sections II–IV.
- Section II illustrates a method for extending Section I's loss simulation procedure to test the solidity of a newly-formed insurance company.
- Section III incorporates computer simulation into the pricing of pneumoconiosis (coal miner's "black lung") exposures. The techniques described in this section can be utilized in the pricing of virtually any new, unique, or catastrophic exposure.
- Section IV uses the results of Section I's loss simulations to illustrate a procedure for developing insurance charges for casualty individual risk rating programs. This section then concludes with an example of a possible use of computer simulation in computing loss reserve confidence intervals.

I. EVALUATING FULL CREDIBILITY STANDARDS[1]

Computer simulation provides an alternative method for establishing the fundamental notions of credibility theory. In addition, a simulation-based approach imparts greater flexibility, and thereby a means for expanding upon some of the basic actuarial developments in this area.

[1] Section I discusses how computer simulation can be used to develop and apply *limited fluctuation* credibility theory. For the interested reader, Appendix E illustrates how a computer can also assist the actuary in explaining and applying Bühlmann/Hewitt's *greatest accuracy* credibility model.

The Classic Credibility Problem

A casualty actuary draws reasonable conclusions based upon data. More precisely, he translates these data into estimates of some future variable, such as next year's Workers' Compensation loss ratio or the amount of self-insurance funding required by a large commercial risk.

Inherent in the above process is the actuary's determination of the credence to be placed in the underlying data. In making this decision, he uses his experience to select a realistic volume of data which he will consider to be fully representative of the variable being estimated. In establishing this "full credibility standard," the actuary balances the conflicting objectives of stability and responsiveness.

Once this full credibility requirement has been established, the actuary next determines the maximum probable error in his estimate, given a fully credible volume of data. If this maximum error is unacceptably high, the full credibility criterion is revised upward. The *classic credibility problem* refers to this problem of determining the probable maximum error in an estimate developed from "fully credible" data.

This section begins by examining traditional actuarial solutions to this problem. Results obtained are then compared with corresponding figures developed using computer simulation. Finally, the relative advantages and limitations of the two approaches are compared.

The Basic "Limited Fluctuation" Credibility Model

The simplest and most popular model for evaluating the potential error implied by a particular full credibility standard assumes that an individual risk's claim frequency is Poisson distributed, and that all losses are of some fixed amount.[2] Under these conditions, the volatility in an estimate developed from a specified volume of loss experience is calculated by means of a relatively simple formula.[3]

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[2] L. H. Longley-Cook, *An Introduction to Credibility Theory* (hereafter cited as "Longley-Cook").

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[3] Assuming that the expected number of claims can be estimated without error, the formula becomes Confidence Bounds = $\pm P/E^{1/2}$, where P is the appropriate z-statistic obtained from a standard normal distribution table, and E is the expected number of claims.

For example, selecting 1,082 claims as one's full credibility standard implies that the actual losses arising out of a fully credible sample will fall within ±5% of expected levels 90% of the time, given the previous frequency and severity assumptions.[4]

An Alternative Development

Given these simple frequency and severity assumptions, an alternative means of estimating the statistical reliability of a selected full credibility standard is possible. As indicated earlier, this second approach involves computer simulation.

To illustrate, assume that:

- claim frequency is Poisson distributed, and therefore approximately normally distributed, with a mean of 1,000 claims;
- all claims cost $5,000.

Given these conditions, one can easily program a computer to simulate 1,000 random trials ("years") of claim experience. A histogram of one such set of 1,000 simulations is presented as Chart 1, on the following page.

This chart reveals that simulated losses fall between $4,725,000 and $5,260,000 in 900 of the 1,000 trials. That is, given 1,000 trials, simulated losses fall between 94.5% and 105.2% of expected losses ($5 million) 90% of the time. Under Longley-Cook's formula, the corresponding theoretical limits are $4,740,000 and $5,260,000. Not surprisingly, the analytical and simulation approaches produce similar results.

[4] Longley-Cook, page 200. In particular, $5\% = 0.05 = 1.645/(1{,}082)^{1/2}$.

CHART 1: RESULTS OF 1,000 SIMULATIONS (CONSTANT SEVERITY)
EXPECTING 1,000 CLAIMS

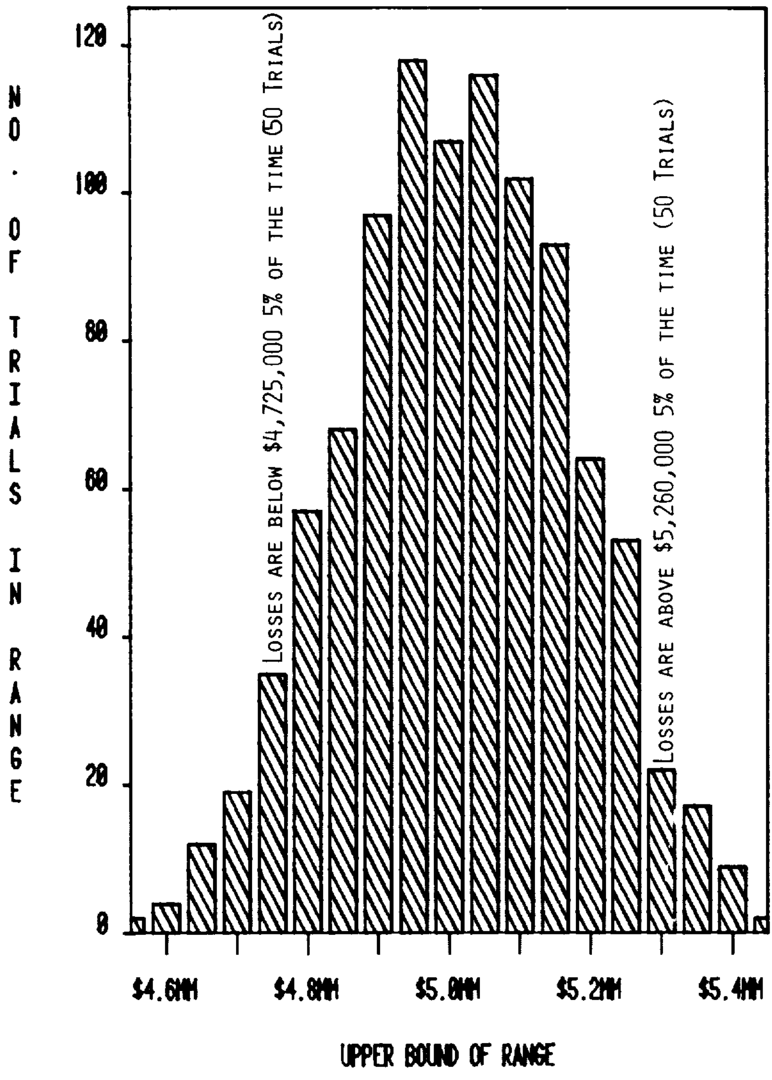


Table 1 extends this comparison to include other probability ranges obtained from this run.

TABLE 1

CONFIDENCE BOUNDS AS A PERCENTAGE OF EXPECTED LOSS*

Probability	Based on 1,000 Simulations: Lower Bound	Based on 1,000 Simulations: Upper Bound	Theoretical Values
99%	−8.1%	+8.2%	±8.1%
98%	−7.4%	+7.7%	±7.4%
95%	−6.5%	+6.3%	±6.2%
90%	−5.5%	+5.2%	±5.2%
80%	−4.3%	+4.1%	±4.1%

* Assuming constant severity and an expected frequency of 1,000 claims.

This correspondence between theoretical and simulation results usually improves as the number of simulations increases. For example, extending the prior run to 5,000 random trials generated a 90% probability range of $4,735,000–$5,255,000, slightly closer to the corresponding theoretical values.

Simulated confidence ranges for several other full credibility standards are provided in Appendix A.

More Sophisticated Credibility Models

Since a complete and simple analytical solution to the previous problem exists, one may question the usefulness and necessity of a simulation alternative. Indeed, were frequency and severity to behave as postulated in the first model, simulation would be a needless and expensive approach to a simple problem.

Unfortunately, frequency and severity usually do not behave as postulated in the basic credibility model. In particular, seldom are all claims the same size.[5]

When variability in both the frequency and severity distributions is considered, the simulation solution is generally preferable to an analytical approach

[5] Nor is the Poisson frequency assumption necessarily appropriate in all instances. See L. Simon, "Fitting Negative Binomial Distributions by the Method of Maximum Likelihood," *PCAS* XLVIII, 1961, page 45.

to the classic credibility problem. The next two subsections illustrate why this is so.

The Poisson/lognormal Model

A number of models which reflect variability in the size-of-claim distribution are presented in the actuarial literature. Of these, the Poisson/lognormal model suggested by Longley-Cook[6] and generalized by Mayerson, Jones, and Bowers[7] is among the most often cited.

Rather than assuming all claims to be of equal size, this model assumes that claim sizes are distributed according to a lognormal distribution. This assumption significantly complicates the derivation of appropriate formulas for determining the potential error associated with a particular full credibility standard. However, both papers conclude that a lognormal severity distribution increases the error calculated according to the simple credibility model by a factor of approximately $(1.0 + CV^2)^{1/2}$, where "CV" is the coefficient of variation[8] of the severity distribution.

For example, under the basic (constant claim size) model, choosing 1,000 claims as one's full credibility standard implies that the error in one's fully credible estimate will be less than 5.2% in nine of ten instances. By assuming a lognormal severity distribution with a coefficient of variation of 3.0, the error increases to approximately 16.4% (16.4% = 5.2% × $10^{1/2}$).

These results are easily confirmed by computer simulation. To illustrate, Chart 2 displays the distribution of 1,000 random trials developed assuming that:

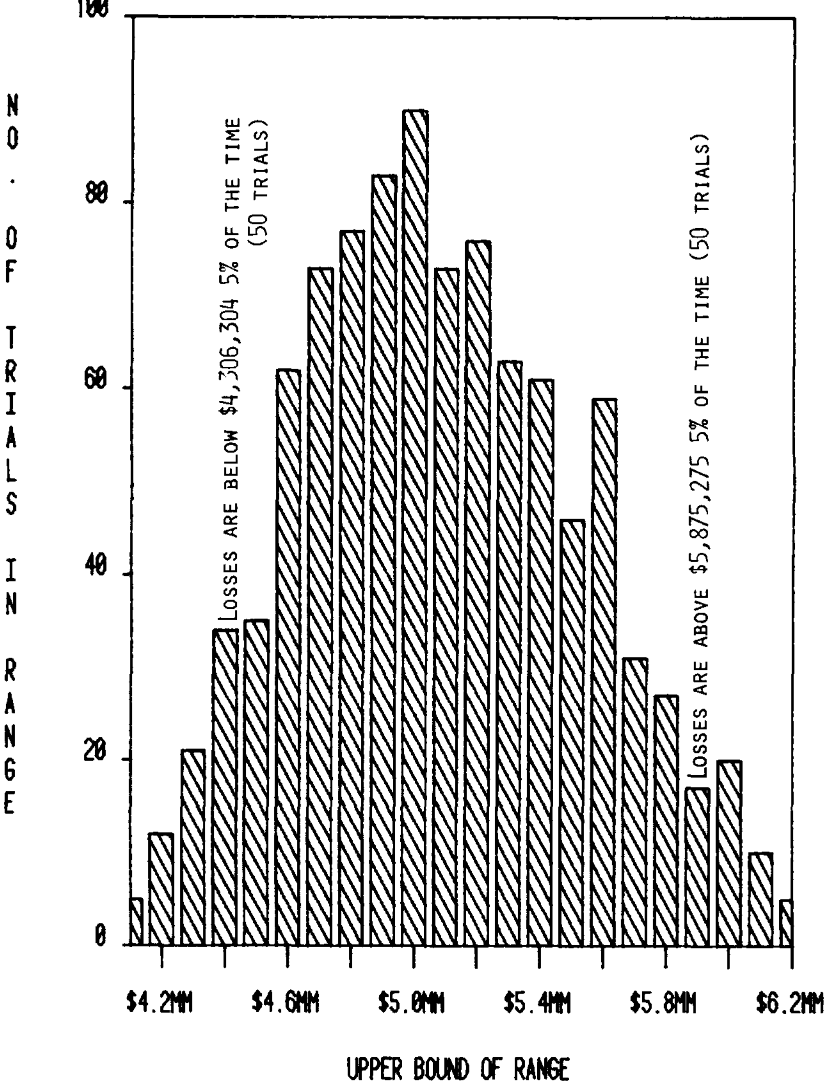
- claim frequency is once again Poisson distributed with a mean frequency of 1,000 claims,
- claim sizes are lognormally distributed with a mean of $5,000 and a coefficient of variation of 3.0,
- the number of claims (frequency) does not influence their average cost (severity), and
- the cost of a particular claim is independent of the cost of prior claims.

[6] Longley-Cook, page 220.

[7] A. L. Mayerson, D. A. Jones, and N. L. Bowers, Jr., "On the Credibility of the Pure Premium," *PCAS* LV, 1968, page 175 (hereafter cited as "Mayerson et al"). This paper's full credibility formulas can also apply for non-lognormal severity processes.

[8] A coefficient of variation is the standard deviation of a distribution divided by its mean.

CHART 2: RESULTS OF 1,000 SIMULATIONS (LOG-NORMAL SEVERITY)
EXPECTING 1,000 CLAIMS



The various confidence bounds read from this chart are compared with their corresponding theoretical approximations in Table 2.

TABLE 2

CONFIDENCE BOUNDS AS A PERCENTAGE OF EXPECTED LOSS*

	Based on 1,000 Simulations		Theoretical Values[9]
Probability	Lower Bound	Upper Bound	Approx. Bounds
99%	−21.5%	+27.5%	±25.7%
98%	−19.7%	+24.6%	±23.3%
95%	−16.2%	+20.2%	±19.6%
90%	−13.9%	+17.5%	±16.4%
80%	−10.9%	+13.4%	±12.8%

* Assuming an expected claim frequency of 1,000 claims and a lognormal severity distribution with a *CV* of 3.0.

Unlike the analytical derivation, the simulated results in Chart 2 reflect the slight skewness of the resulting pure premium distribution. This skewness is also evident in Table 2, wherein lower confidence bounds are closer to expected loss levels than their corresponding upper bounds.

Also in contrast to the traditional derivation, the procedure used to simulate confidence ranges under a lognormal severity assumption is essentially identical to the simulation technique employed in the first model. The ease with which simulation accommodates this added complication suggests that this technique might be employed to address problems which are not readily answerable by analytical methods.

[9] Determined according to the formula Bound = $\pm P \times 10^{1/2}/1{,}000^{1/2}$, where P is the appropriate z-statistic from a standard normal distribution table.

Another Model

This subsection discusses a credibility problem which does not lend itself to an easy or general analytical solution. Specifically, the following comments outline a solution to the classic credibility problem in a situation where the pure premium distribution is a product of a "compound" severity process.[10]

For example, situations sometimes arise wherein losses up to a certain level (say, $25,000) appear to be the product of a number of influences, whereas losses above this level seem to be influenced by totally different elements. In Workers' Compensation, for instance, smaller indemnity losses might be viewed as the product of the injured worker's wage, the state benefit level, and projected future movements in wage levels. Losses above a certain level, on the other hand, tend to be influenced mainly by such factors as quality of attorney and the liberalness of the Workers' Compensation administration in that particular state.

Under such situations, the size-of-loss distribution is really a "compound distribution," in the sense that it is a weighted average of two different severity distributions—a "primary" and an "excess" loss distribution. Intuitively, one suspects that a pure premium distribution resulting from a compound severity distribution is more volatile than the corresponding distribution developed under a simple size-of-loss assumption. The following paragraphs test this intuitive notion.

The form of most theoretical pure premium distributions resulting from known frequency processes and compound severity distributions tends to be formidable. Thus, explicit analytical solutions to the classic credibility problem are generally not available in such situations. Simulation, on the other hand, does not discriminate on the basis of complexity; hence, the simulation solutions obtained for earlier, simpler situations can be extended to take into account this added consideration.

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[10] For a discussion of several compound theoretical distributions, see C. C. Hewitt, Jr. and B. Lefkowitz, "Methods for Fitting Distributions to Insurance Loss Data," *PCAS* LXVI, 1979, page 139.

To illustrate this flexibility, Table 3 presents results of 1,000 random simulations which assume that:

- claim frequency is Poisson distributed with a mean of 1,000 claims,
- 95% of all claims are lognormally distributed with an average claim size of $5,000 and a coefficient of variation of 3.0,
- the remaining claims ("above $25,000") are Pareto distributed,[11]
- claim frequency is independent of claim severity, and
- the size of a loss is not influenced by the size of prior losses.

TABLE 3

CONFIDENCE BOUNDS* DEVELOPED WITH AND WITHOUT PARETO "TAIL"

	Without Tail (per Table 2)		With Tail	
Probability	Lower Bound	Upper Bound	Lower Bound	Upper Bound
99%	−21.5%	+27.5%	−30.5%	+37.7%
98%	−19.7%	+24.6%	−28.6%	+34.8%
95%	−16.2%	+20.2%	−24.4%	+29.1%
90%	−13.9%	+17.5%	−20.7%	+23.3%
80%	−10.9%	+13.4%	−16.2%	+18.4%

* Expressed as a percentage of expected loss.

This table clearly confirms our earlier supposition that a severity tail can add considerable volatility to a pure premium distribution.

Post-mortem: Section I

Section I describes how computer simulation may be used to develop approximate solutions to the classic credibility problem. In this process, it has become apparent that a simulation solution, unlike its analytical counterpart, is essentially pictorial. Specifically, each solution presented in Section I involved the computer's producing a histogram, from which this writer simply read his answer.

[11] That is, Y is distributed according to the formula $f(y) = 1.25y^{-2.25}$. In this formulation, "y" represents "normalized" losses in excess of $25,000; that is, Y = LOSS/25,000. Note that Y is never less than 1.

This intuitiveness carries with it obvious advantages for anyone charged with the difficult task of explaining the foundations of credibility theory to lay participants in the insurance process. Indeed, in such cases, one simulated picture may well be worth 1,082 words.

More significantly, however, the techniques used to generate Tables 1–3 can be modified to reflect almost any combination of theoretical or empirical frequency/severity assumptions. This inherent flexibility makes computer simulation an invaluable tool for applying and expanding traditional limited fluctuation credibility concepts, as well as for solving other difficult actuarial problems.

Illustrations of two such applications are provided in Sections II and III of this paper.

II. TESTING THE CAPITAL STRUCTURE OF A NEW INSURANCE COMPANY

Section I illustrates how a computer can be used to simulate a body of losses under assumed frequency and severity conditions. Simulating the ability of a new insurance company's capital structure[12] to meet its prospective loss obligations is a logical extension of this technique.

Accordingly, Section II uses simulation to test the capital structure of a hypothetical, limited-purpose "captive" insurance company. Again, results obtained under this approach are compared with those suggested by more traditional actuarial procedures.

The Company

The Consulting Actuaries' Reciprocal Exchange ("CARE") is being formed to provide a consistent and fairly priced market for Casualty Actuaries' Errors & Omissions coverage. Thus far, the steering committee examining the feasibility of this endeavor has agreed upon the following operational guidelines:

- the company will be domiciled offshore;
- the company will be a mutual insurance company, whose members will include consulting actuaries with three years of acceptable claim experience;
- the company will sell occurrence-basis Errors & Omissions policies;

[12] As used in this paper, "capital structure" includes the company's initial capitalization, as well as any other elements affecting its ability to pay losses. Such elements include retained earnings to date, applicable reinsurance arrangements, policyholder assessment provisions, and, of course, the company's underlying rate level.

- the company will be "capitalized" by means of a per-member capitalization fee, payable at a member's first policy inception;
- each member will be subject to a "solvency assessment," payable in the event that serious or sustained underwriting losses jeopardize the continued operation of the company on a sound basis;
- CARE will purchase only quota-share reinsurance because of reinsurer reluctance to participate on an excess-of-loss basis;
- to protect its solvency, the company will arrange to quota-share a substantial percentage of its exposure during its early years of operation.

In addition to the above seven constraints, the committee agrees upon the following preferences:

- a small initial capitalization fee, ideally \$500 or \$750;
- for marketing reasons, a maximum call provision of one year's premium;
- minimal use of quota-share reinsurance, since each dollar ceded costs CARE several cents.[13]

The committee retains an independent consultant to recommend the appropriate per-actuary rate, the maximum per-member assessment, a per-member capitalization fee, and the optimal amount of quota-share reinsurance which the company should purchase. The remainder of Section II illustrates how the consultant might use computer simulation to address these last three issues.

The Model: Underlying Assumptions

Of the four issues raised in the introduction, the first item—determining the proper per-actuary rate—is routinely accomplished by traditional actuarial means. For purposes of this example, assume that the consultant reviews the most recent three-year loss history of 1,000 prospective members, and thereby recommends a uniform annual rate of \$1,750 for \$3 million of occurrence-basis protection.

The consultant next turns his attention to the more complex and equally important questions concerning the proper assessment percentage, the initial capital contribution, and the percentage of business which the company should cede. Since these elements interact to jointly influence CARE's solidity, the consultant constructs a computer model to simultaneously address these three issues.

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[13] Expenses incurred less ceding commission allowance. CARE costs per dollar ceded are 7.5¢ (15¢ − 7.5¢) during first year, 4.5¢ during year 2, and 2.5¢ thereafter, per underlying assumptions 7 and 10.

The principal assumptions underlying this model are presented below.

1. *Distribution of the number of claims:* Since the base of insureds appears to be relatively homogeneous with a low expected frequency, the model assumes a Poisson claim frequency process.
2. *Distribution of claim amounts:* No credible Actuaries' Errors & Omissions size-of-loss information is available for this review. Fortunately, considerable size-of-loss data for other professional liability sublines are readily and generally available. Based on this information, the consultant hypothesizes a joint lognormal/Pareto severity distribution, as described below:

Claim-size Range	% of Claims	Distribution	Parameter
Below $500,000	98%	Lognormal	*CV* is 3.5
Above $500,000	2%	Pareto	Constant is 1.30

3. *Expected number of claims and average claim size:* Recall that a loss history was reviewed by the consultant. He estimated an average claim frequency of 2.5 claims/100 actuaries and a basic-limits[14] average claim size of approximately $45,000. Due to the underlying "parameter variance" in any distribution of sample means, these estimates are themselves subject to a certain amount of chance error. Accordingly, the consultant adjusts his simulation model to take into account the inherent error in his frequency and severity estimates.
   Specifically, the model assumes that these frequency and severity averages are normally distributed with standard errors of 0.2 claims/100 actuaries and $6,000, respectively. The means which underlie any particular trial's frequency and severity distributions reflect the consultant's initial estimates adjusted for this parameter error.
4. *Number of first-year participants:* Marketing intelligence estimates first-year participation of 1,000 actuaries, a level expected to continue through year five. Annual membership growth of 10% is projected for each of years six through ten.
5. *Frequency and severity trend:* No upward or downward trend in claim frequency is assumed. However, annual increases in E&O claim sizes are anticipated.

[14] "Basic-limits" losses are limited to $500,000 per occurrence.

Specifically, a 12% severity increase is assumed for year one. During subsequent years, the annual *change* in the claim inflation rate is assumed to be normally distributed with an average change of 0 and a standard deviation of 1 point.

6. *Collectibility of assessments:* Recognizing that the company would not be able to collect all assessments in the event a call is required, the model assumes an effective collection rate of 75%.
7. *Operating costs:* Administrative, underwriting, unallocated loss expense, and premium tax costs total 15% of premium during the first year, 12% in year 2, and 10% thereafter.
8. *Common inception date and policy term:* All CARE policies are to be written for one year, effective January 1.
9. *Rate level changes:* The $1,750 per-member rates will continue through the third year. Thereafter, annual 10% premium increases are assumed.[15]
10. *Ceding reinsurance commission:* CARE will receive a 7.5% commission on all quota-share reinsurance which it cedes.
11. *Payout of incurred losses:* Payout of a given policy-year's E&O losses is assumed to occur over five years, in 30/25/20/15/10 proportions.
12. *Interest earned on reserves, capital, and surplus:* The company's investable funds are assumed to earn interest at an annual rate of 10%.
13. *Federal income taxation:* Full (46%) corporate income taxation is assumed. To simplify computations, this taxation is assumed to occur during the year in which the corresponding income is earned.

[15] In practice, loss-sensitive pricing would probably be assumed.

The Model: Results

The consultant next uses his simulation model to carry out a first-level screening of the following twelve CARE operating scenarios:

Scenario	Quota-share Percentage	Per-member Capitalization Fee	Maximum Annual Policyholder Assessment (as a % of annual premium)
1	15%	$500	50%
2	15%	$500	100%
3	15%	$750	50%
4	15%	$750	100%
5	25%	$500	50%
6	25%	$500	100%
7	25%	$750	50%
8	25%	$750	100%
9	50%	$500	50%
10	50%	$500	100%
11	50%	$750	50%
12	50%	$750	100%

For each of these twelve scenarios, the model simulates 50 random "trials." Each trial consists of ten years' operating experience; for each year, net operating income is developed, and changes in CARE's policyholder surplus are recorded. To illustrate this technique, results of the first trial of Scenario 5 are presented in Table 4, on the following page.

TABLE 4

RESULTS OF FIRST TRIAL, FIFTH SCENARIO
(All dollar figures are in thousands)

	YEAR 1	2	3	4	5
Net Premium Earned	$1,313	$1,313	$1,313	$1,444	$1,588
Reins. Commission	$ 33	$ 33	$ 33	$ 36	$ 40
Investment Income	$ 81	$ 152	$ 210	$ 260	$ 321
Net Losses Incurred	$ 458	$1,266	$ 610	$1,572	$3,104
Expenses Incurred	$ 263	$ 210	$ 175	$ 193	$ 212
Fed. Income Taxes	$ 325	$ 10	$ 354	$ −11	$−629
Oper'tg Surplus @ Start	$ 500	$ 881	$ 893	$1,308	$1,295
Call Funds Required	$ 0	$ 0	$ 0	$ 0	$ 0
Oper'tg Surplus @ End	$ 881	$ 893	$1,308	$1,295	$ 557
Claim-cost inflation	12.0%	12.5%	12.0%	11.0%	11.3%
No. of members	1,000	1,000	1,000	1,000	1,000

	YEAR 6	7	8	9	10
Net Premium Earned	$1,922	$2,325	$2,813	$3,404	$4,118
Reins. Commission	$ 48	$ 58	$ 70	$ 85	$ 103
Investment Income	$ 358	$ 384	$ 438	$ 520	$ 627
Net Losses Incurred	$1,090	$1,961	$1,852	$2,799	$1,400
Expenses Incurred	$ 256	$ 310	$ 375	$ 454	$ 549
Fed. Income Taxes	$ 452	$ 228	$ 503	$ 348	$1,334
Oper'tg Surplus @ Start	$ 607*	$1,192*	$1,521*	$2,178*	$2,660*
Call Funds Required	$ 0	$ 0	$ 0	$ 0	$ 0
Oper'tg Surplus @ End	$1,137	$1,460	$2,112	$2,586	$4,225
Claim-cost inflation	11.7%	11.3%	11.4%	11.9%	10.6%
No. of members	1,100	1,210	1,331	1,464	1,611

* Includes $500/member assessment from new members.

Average annual surplus growth: 23.8%

Finally, each scenario is evaluated in terms of:

- the likelihood of CARE's avoiding a "capital call" (assessment),[16]
- the expected 10-year profitability of the operation as measured by surplus growth, and
- the consistency of CARE's year-to-year surplus growth.

This preliminary screening is carried out in Table 5.

TABLE 5

Summary Of Results Of Ten-Year Operating Simulation

	Adequacy of Call Provision		Profitability	
Scenario	No. of trials (out of 50) in which "call" is required	No. of trials in which "call" funds are *not* sufficient to offset surplus impairment(s)	Median 10-yr. surplus growth	Range in average surplus growth (Low/High)*
(1)	(2)	(3)	(4)	(5)
1 (15/500/50)	22	5	22.9%	Co. fails / 30.7%
2 (15/500/100)	23	None	23.1%	12.6% / 29.1%
3 (15/750/50)	12	6	19.7%	Co. fails / 24.9%
4 (15/750/100)	20	2	18.2%	None / 25.6%
5 (25/500/50)	20	8	21.0%	Co. fails / 27.0%
6 (25/500/100)	17	2	20.3%	None / 29.1%
7 (25/750/50)	18	6	16.2%	Co. fails / 22.9%
8 (25/750/100)	13	None	18.9%	10.1% / 23.8%
9 (50/500/50)	13	1	17.0%	3.9% / 24.9%
10 (50/500/100)	18	1	18.3%	11.9% / 24.2%
11 (50/750/50)	7	1	16.1%	6.4% / 19.2%
12 (50/750/100)	7	None	16.5%	2.0% / 19.9%

* Range represents the fifth lowest and fifth highest annual surplus growth rates recorded during the fifty trials.

[16] In this illustration, a call is required only in the event that CARE's policyholder surplus is exhausted. In practice, the company would empower its management to issue a call whenever surplus drops by some predetermined percentage (25–50%) during a specified period.

Recommendations

Given the previous criteria and the results presented in Table 5, the consultant narrows his field of possible recommendations to Scenarios 2, 6, 9, and 10. He cites the following reasons.

- Column (3) clearly establishes that a 50% call provision, in the absence of at least 50% quota-share reinsurance, does not provide sufficient contingent capitalization to assure the company's solidity. This observation eliminates Scenarios 1, 3, 5, and 7 from further consideration.
- Table 5 also demonstrates that a $750 per-member capitalization fee does not significantly improve CARE's operating integrity. On the other hand, higher initial capitalization reduces the company's premium/surplus leverage; Column (4) quantifies the negative impact of this added capitalization on CARE's annual surplus growth. Thus, Scenarios 4, 8, 11, and 12 are eliminated as possible candidates.

The consultant next reviews these findings with CARE's steering committee. During this review, the committee re-emphasizes its desire to avoid extensive reinsurance; accordingly, Scenarios 9 and 10 are dismissed. Moreover, the group asks the consultant to:

- extend his simulation analysis to 1,000 trials for each of the remaining two options, and
- analyze each of the remaining options under both the proposed $1,750/actuary base rate scenario, as well as under a $2,000 base rate assumption.

After reviewing this additional input, the committee adopts the sixth option (25% reinsurance, a $500 per-member capitalization charge, and a 100% call provision) along with a $2,000/actuary base rate.

Post-Mortem: Section II

This section extends the loss simulation techniques presented in Section I to include a consideration of inflation, reinsurance, corresponding premium movements, and cash flow. The simulation model which results from this extension provides an intuitively appealing method for testing the solidity of a new or existing casualty insurance company.

The reader will note that this approach to gauging an insurer's solidity bears little resemblance to traditional solvency testing procedures. The two, in fact, differ not only in form, but in what they are actually testing.

The first difference involves the form of the two procedures. Most traditional solvency tests, such as the NAIC Insurance Information Regulatory System ratios and the A. M. Best insurance company rating system, are designed for widespread application. In fact, many of these tests are conducted annually for all or most U.S. property/casualty carriers. It is doubtful that any organization could conduct meaningful solvency simulations on such a scale.

Beyond structural differences, however, the two approaches differ in what they attempt to measure. Specifically, traditional tests attempt to identify companies which are *already* experiencing surplus difficulty. The simulation approach suggested in this section, on the other hand, focuses primarily on the likelihood that a company may become insolvent. Also, simulation is designed to highlight steps which would reduce this probability.

Since traditional and simulation approaches measure different things, a direct comparison of the two is not meaningful. What is clear, however, is that a combination of the two methods produces a far better system than either approach alone provides. In particular, traditional ratio analysis is cost-effective for most large, established carriers, but is of little value to new or limited purpose insurance companies. For this latter group, simulation generally produces far more useful information.

A specific and needed application of a simulation approach to solvency testing concerns "captive" insurance companies. These carriers, which are increasing in number by approximately 100 to 150 per year, often find it difficult to convince the established reinsurance market of their legitimacy as insurance operations. In turn, this failure to gain market acceptance can seriously limit a captive's effectiveness, particularly during reinsurance negotiations and in its efforts to procure a book of "quality" non-related business.

For a captive in this position, a simulation analysis along the lines suggested in this section would either convincingly confirm the company's operating integrity, or provide information with which the carrier could judiciously strengthen its capital structure. In either case, the company would almost certainly improve its image and stature in the insurance market.

III. SIMULATION AS AN AID IN PRICING NEW, UNIQUE, OR CATASTROPHIC EXPOSURES

Computer simulation can also be used to improve pricing of exposures for which historical information is unavailable or not indicative of future experience. For example, the pricing of endemic disease exposures, such as coal miner's "black lung," textile worker's "brown lung," and asbestosis, can be improved with the aid of computer simulation.

To illustrate the use of simulation in pricing these exposures, Section III compares the current actuarial formula for pricing black lung (pneumoconiosis) coverage with an alternative procedure incorporating computer simulation. The advantages of the latter approach are highlighted.

Background

Title IV of The Federal Coal Mine Health and Safety Act of 1969 extended Workers' Compensation benefits to underground coal miners totally disabled by pneumoconiosis, a respiratory disease associated with dust levels in coal mines. The Act also provided benefits for the families of miners who died from the disease.

Understandably, pricing this coverage has proven to be a problem for the major Workers' Compensation ratemaking organizations. A partial listing of conditions complicating black lung pricing includes the facts that:

- there exists very little data on claim emergence patterns, even ten years after introduction of coverage;
- coverage is limited to death, permanent total disability, and medical benefits; therefore, the average undiscounted cost of black lung claims is currently estimated to be $200,000–400,000;
- the program provides for a dual benefit structure; affected miners qualify for the higher of Federal or state benefits;
- the Act's coverage continually changes, often retroactively;
- most importantly, the Act contains a (rebuttable) presumption that any miner with a respiratory impairment and a specified number of years of service in the mines is disabled from work-related black lung disability, and thereby entitled to black lung benefits.

Current Pricing Procedures

Given the previous considerations, a black lung pricing formula based entirely upon historical experience is neither possible nor appropriate. Thus, the National Council on Compensation Insurance has adopted a procedure which utilizes available actuarial and government statistics to estimate the appropriate *expected* pure premiums for coal mining classes.[17]

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[17] See Appendix B for a fuller discussion of the current NCCI formula.

This writer believes that the current expected loss black lung pricing formula is flawed, in that it ignores the interaction among variables affecting black lung costs. Instead, expected loss pricing focuses directly on the end products of the loss determination process—the number of claims filed and the average cost of these claims. In so doing, current procedures may exaggerate any underlying conservative biases on the part of the pricer, and thus result in his unintentionally overstating required pure premiums.

Simulation overcomes this problem by forcing the pricer to specify the assumed interaction among variables, and by displaying a range of possible outcomes consistent with these assumptions. As will be demonstrated, using a simulation based pricing formula narrows the range of reasonably foreseeable outcomes confronting the pricer, thus allowing him to select a saleable and reasonable pure premium which actual experience should not exceed in more than a specified percentage of instances.

The following illustration highlights these advantages.

The Model: Underlying Assumptions

Many factors interact to determine the discounted indemnity costs to be paid under the current black lung benefit system. The following eight items are among the more important of these influences. They provide the basis of the assumptions underlying the simulation model presented in this section. Appendix C contains a detailed discussion of each of these eight assumptions.

IMPORTANT FACTORS AFFECTING BLACK LUNG LOSSES

1. Frequency of retiring miners' filing of claims
2. Success rate among retiring miners who file claims
3. Miner mortality
4. Age of claimants
5. Number and nature of dependents
6. Wage inflation rate (for Federal benefit escalator)
7. Loss discounting percentage
8. Current and projected indemnity benefits

The Model: Results

Given the above assumptions, a computer program was written to simulate *one policy-year's black lung indemnity costs arising out of the retirement of 1,000 miners.* Table 6 presents results of 50 random trials carried out with losses discounted at 3.5%.

TABLE 6

Results of 50 Random Trials (Policy Years) of Black Lung Experience

	Wage Inflation				Miners Receiving Both State and Federal Benefits			
Trial	Yr. 1	High	Low	Losses Disctd. at	No. w/o Surv	No. with Surv	Total State Indemnity	Federal Supplement
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	8.0%	8.2%	1.4%	3.5%	29	39	$11,793,424	$ 422,171
2	8.0%	18.9%	6.2%	3.5%	108	106	$35,417,949	$28,329,620
3	8.0%	13.8%	1.5%	3.5%	61	59	$19,609,308	$ 802,263
4	8.0%	12.7%	0.0%	3.5%	197	181	$61,742,029	$ 5,382,631
5	8.0%	10.0%	0.0%	3.5%	29	38	$11,314,850	$ 315,453
6	8.0%	9.0%	0.0%	3.5%	79	64	$23,066,573	$ 1,260,823
7	8.0%	13.5%	0.1%	3.5%	58	41	$15,699,009	$ 3,997,199
8	8.0%	11.8%	3.8%	3.5%	83	72	$25,439,610	$ 3,073,195
9	8.0%	26.8%	6.6%	3.5%	88	80	$27,594,427	$ 7,417,803
10	8.0%	17.2%	8.0%	3.5%	121	101	$36,389,192	$24,345,390
11	8.0%	14.4%	0.9%	3.5%	48	39	$14,145,943	$ 335,548
12	8.0%	12.2%	5.8%	3.5%	90	100	$31,561,550	$ 7,606,673
13	8.0%	14.6%	6.0%	3.5%	74	86	$26,779,215	$ 3,657,434
14	8.0%	11.4%	0.0%	3.5%	110	111	$36,922,111	$ 5,088,262
15	8.0%	13.5%	0.0%	3.5%	78	96	$29,132,473	$ 4,098,705
16	8.0%	9.3%	1.6%	3.5%	73	90	$28,195,854	$ 249,448
17	8.0%	9.1%	0.0%	3.5%	25	25	$ 8,275,362	$ 273,957
18	8.0%	12.6%	4.8%	3.5%	168	173	$56,694,638	$11,261,580
19	8.0%	11.2%	2.3%	3.5%	36	51	$14,811,720	$ 706,279
20	8.0%	10.5%	2.8%	3.5%	51	74	$21,753,482	$ 2,595,748
21	8.0%	8.0%	0.0%	3.5%	100	110	$34,918,094	$ 6,552
22	8.0%	18.9%	8.0%	3.5%	66	35	$25,523,520	$ 9,524,768
23	8.0%	8.9%	0.0%	3.5%	69	74	$24,098,558	$ 0
24	8.0%	10.3%	1.7%	3.5%	84	67	$24,296,278	$ 2,303,286
25	8.0%	11.5%	6.0%	3.5%	142	141	$46,506,422	$ 5,975,460
26	8.0%	17.1%	6.7%	3.5%	146	137	$46,696,035	$ 5,743,238
27	8.0%	26.0%	8.0%	3.5%	104	104	$34,418,369	$14,616,021
28	8.0%	8.0%	0.0%	3.5%	58	41	$16,037,961	$ 22,121
29	8.0%	8.0%	0.0%	3.5%	82	83	$27,268,924	$ 143,290
30	8.0%	22.2%	5.8%	3.5%	56	72	$21,795,755	$ 5,302,253
31	8.0%	8.0%	2.2%	3.5%	79	82	$26,838,126	$ 272,082
32	8.0%	16.6%	3.1%	3.5%	48	38	$13,928,681	$ 732,148
33	8.0%	9.4%	1.5%	3.5%	61	60	$20,385,387	$ 1,403,695
34	8.0%	16.5%	8.0%	3.5%	74	84	$26,310,580	$ 4,360,145
35	8.0%	19.7%	4.2%	3.5%	62	57	$19,698,029	$ 3,586,797
36	8.0%	18.9%	6.3%	3.5%	117	113	$37,901,940	$ 5,632,761
37	8.0%	11.0%	3.2%	3.5%	73	69	$23,615,719	$ 2,907,338
38	8.0%	9.8%	0.0%	3.5%	28	32	$ 9,823,467	$ 4
39	8.0%	8.2%	0.0%	3.5%	34	59	$16,262,965	$ 106,774
40	8.0%	9.4%	2.6%	3.5%	81	81	$26,730,032	$ 1,070,919
41	8.0%	8.2%	0.0%	3.5%	91	81	$27,882,454	$ 0
42	8.0%	10.6%	0.0%	3.5%	38	48	$14,293,282	$ 772,804
43	8.0%	14.4%	2.0%	3.5%	86	97	$30,891,675	$12,989,713
44	8.0%	10.4%	3.9%	3.5%	86	84	$28,007,285	$ 2,108,072
45	8.0%	11.3%	0.0%	3.5%	45	36	$13,091,872	$ 2,222,252
46	8.0%	19.1%	8.0%	3.5%	93	79	$27,905,591	$ 8,345,419
47	8.0%	8.0%	0.3%	3.5%	85	79	$27,062,875	$ 31,161
48	8.0%	8.2%	0.0%	3.5%	103	96	$32,772,717	$ 613,959
49	8.0%	10.3%	0.0%	3.5%	71	55	$20,324,758	$ 772,486
50	8.0%	10.1%	0.0%	3.5%	24	29	$ 8,844,989	$ 152,660
Avg.					77	77	$25,809,421	$ 4,058,767

TABLE 6 cont.

RESULTS OF 50 RANDOM TRIALS (POLICY YEARS) OF BLACK LUNG EXPERIENCE

	Miners Receiving Only State Awards			Miners Receiving Only Federal Awards			All Miners	
Trial	No. w/o Surv.	No. with Surv.	Total State Indemnity	No. w/o Surv.	No. with Surv.	Total Federal Indemnity	No. Total Benef.	Total Cost
(1)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
1	156	146	$50,043,093	34	24	$ 6,675,629	428	$ 68,934,318
2	207	228	$73,168,306	11	10	$10,893,322	670	$147,809,200
3	165	132	$48,210,590	46	63	$13,818,380	526	$ 82,440,543
4	220	244	$77,521,904	14	8	$ 2,927,979	864	$147,574,546
5	151	155	$50,829,437	6	6	$ 1,437,476	385	$ 63,897,218
6	136	107	$39,214,016	19	25	$ 5,592,972	430	$ 69,134,384
7	156	133	$46,776,478	17	26	$ 8,440,706	431	$ 74,913,394
8	197	198	$65,582,734	22	25	$ 7,341,589	597	$101,437,130
9	197	185	$62,730,284	21	30	$ 9,926,297	601	$107,668,811
10	211	228	$73,061,427	2	4	$ 2,189,764	667	$135,985,774
11	189	190	$62,707,417	2	5	$ 991,287	473	$ 78,180,199
12	237	211	$72,991,772	8	13	$ 4,317,375	659	$116,477,371
13	162	141	$49,930,546	6	4	$ 1,572,640	473	$ 81,939,836
14	152	166	$53,086,514	106	88	$29,380,007	733	$124,476,896
15	183	183	$60,631,021	19	28	$ 7,733,378	587	$101,595,576
16	137	137	$44,960,660	69	73	$13,707,441	579	$ 87,113,404
17	111	101	$34,891,522	21	20	$ 4,788,235	303	$ 48,229,077
18	188	206	$65,710,368	48	34	$13,892,526	817	$147,559,112
19	227	197	$69,215,503	24	10	$ 3,609,897	545	$ 88,343,400
20	133	158	$49,170,926	56	55	$17,293,975	527	$ 90,814,132
21	234	212	$73,594,906	5	13	$ 1,681,190	674	$110,200,746
22	164	142	$49,827,723	7	7	$ 3,387,674	471	$ 88,263,686
23	250	274	$87,529,053	19	23	$ 3,524,010	709	$115,151,620
24	202	178	$62,469,631	77	55	$18,508,876	663	$107,578,072
25	269	286	$92,486,717	14	9	$ 3,454,796	861	$148,423,394
26	286	267	$90,594,835	8	9	$ 2,785,405	853	$145,819,512
27	238	208	$73,089,956	14	3	$ 3,890,234	671	$126,014,583
28	191	228	$70,586,766	18	12	$ 2,783,397	548	$ 89,430,247
29	127	131	$43,516,001	63	60	$12,591,221	546	$ 83,519,436
30	102	95	$32,258,242	43	47	$17,927,388	415	$ 77,283,640
31	163	145	$50,366,895	6	6	$ 1,236,084	481	$ 78,713,188
32	202	225	$71,403,735	18	21	$ 4,826,069	552	$ 90,890,635
33	157	152	$50,954,859	40	39	$10,417,018	509	$ 83,160,960
34	148	156	$50,412,515	7	3	$ 1,711,021	472	$ 82,794,263
35	303	254	$90,997,005	24	29	$ 8,850,339	729	$123,132,170
36	223	251	$78,745,314	32	31	$10,373,152	767	$132,653,168
37	161	165	$54,453,021	11	8	$ 2,692,884	487	$ 83,668,964
38	158	150	$50,694,697	15	11	$ 2,209,892	394	$ 62,728,060
39	201	201	$66,826,740	49	30	$ 8,045,087	574	$ 91,241,567
40	209	188	$65,022,151	26	26	$ 6,541,827	611	$ 99,364,928
41	167	201	$62,416,777	28	23	$ 4,738,808	591	$ 95,038,040
42	109	107	$35,797,900	38	31	$ 8,963,530	371	$ 59,827,517
43	182	194	$62,673,852	51	77	$33,699,587	687	$140,254,826
44	182	201	$63,325,629	25	18	$ 5,669,090	596	$ 99,110,077
45	185	201	$64,591,462	5	3	$ 1,311,648	475	$ 81,217,237
46	106	110	$35,917,009	23	24	$11,627,224	435	$ 83,795,244
47	151	217	$63,196,180	28	27	$ 5,020,107	587	$ 95,310,325
48	231	224	$75,000,411	8	9	$ 1,888,053	671	$110,275,142
49	139	115	$41,716,405	25	16	$ 4,522,414	421	$ 67,336,064
50	158	143	$48,791,959	19	10	$ 2,793,983	383	$ 60,583,591
Avg.	182	181	$60,193,857	25	24	$ 7,484,058	569	$ 97,546,104

Similar simulations were carried out under 0% and 7% loss discounting assumptions. Given the results of these runs and assuming both a $15,000 average miner's salary and a 1.5% retirement rate, the pure premiums in Table 7 were obtained.

TABLE 7

COMPARISON OF DISCOUNTED BLACK LUNG INDEMNITY PURE PREMIUMS

Loss Discounting Percentage	Based on Average Simulated Loss Level[18]	Based on 80th Percentile Loss Level	Based on Lowest Simulated Loss Level	Based on Highest Simulated Loss Level
0% (No disct.)	$15.72	$20.00	$7.23	$30.76
3.5% (NCCI)	9.75	12.31	4.82	14.84
7%	7.30	9.13	3.78	11.03

Interpretation and Significance of Results

Table 7 may be interpreted as follows. Consider the pure premiums displayed in the second (3.5% discount) row. When loaded 16% for expenses, these figures translate into black lung indemnity rates which range from $5.75 to nearly $18.00. From the viewpoint of the pricing actuary, however, the range of *selectable* rates runs from $11.60 (based upon the mean simulated loss level) to more conservative (80th percentile) estimates in the area of $14.50.

In this manner, incorporating computer simulation into black lung pricing enables the actuary to significantly narrow his range of potential loss (and thus rate) levels.

Post-Mortem: Section III

For practical and philosophical reasons, this paper does not suggest that computer simulation should diminish the current role of "traditional" insurance pricing formulas. However, using simulation to complement these formulas in the pricing of new or certain difficult exposures offers several obvious advantages.

In particular, by reflecting the often offsetting interactions among the many factors influencing such a coverage's ultimate cost, simulation usually enables the actuary to significantly narrow his range of potential prices. For instance,

[18] Pure premium (3.5% discount) = $97,546,104/10 million (units of $100 payroll). Similar computations apply for other discounting assumptions.

the $11.60–14.50 range of probable outcomes developed in the previous example is probably much smaller than the spread which one might intuitively expect, given earlier comments on the nature of this coverage and the problems encountered in its pricing.

More importantly, the simulation approach presented in this section requires an initial, detailed delineation of elements which affect the program's cost. Each of these go-in assumptions can readily be tested and appropriately modified as soon as meaningful experience becomes available. As a result, accurate pricing may occur more quickly by incorporating a simulation analysis into traditional expected loss pricing formulas.

IV. OTHER APPLICATIONS

Sections I–III present three applications of computer simulation to insurance pricing problems:

- in re-establishing and extending the fundamental notions of credibility theory,
- in assessing the solidity of an existing or contemplated property/casualty insurance company, and
- in pricing catastrophic exposures or hazards for which no relevant historical information is available.

Simulation can also assist the actuary in two other areas of historical and current concern: customizing individual risk insurance charges[19] and developing loss reserve confidence intervals. A brief discussion of these applications follows.

Customizing Insurance Charges and Excess Loss Premium Factors

The current "Table M" and "Table L" provide the insurance charges underlying most casualty retrospective rating plans, policyholder dividend schemes, and certain types of casualty premium allocation programs.

Historically, massive data requirements and other logistic problems have precluded regular periodic overhauls of the Tables or development of separate insurance charges for the non-Compensation casualty lines. As a result, the insurance charges currently used in most states (Table M) are grounded in the Workers' Compensation policy year experience of the early 1960's. Moreover, this same table often provides the charges used in automobile, burglary, and general liability retrospective rating programs.

[19] An *insurance charge at entry (or loss) ratio r* represents the proportion of a risk's losses which can be expected to fall above entry (loss) ratio *r*.

Simulation, on the other hand, is not constrained by these logistic complexities. In addition, the loss simulation techniques described in Section I of this report can easily be extended to include a calculation of insurance charges.[20]

To illustrate, consider a body of automobile exposures, each with an expected claim frequency of 500 claims and an average claim size of $500. Further, assume a Poisson/lognormal pure premium process with an underlying coefficient of variation of 3.25 for severity. Given these assumptions, loss experience of 1,000 trials (risks) is simulated as described in Section I. From these results, a table of insurance charges and excess loss premium factors[21] is generated by the standard formula.[22] Moreover, this same procedure is carried out with individual claims limited to $2,500.

The resulting insurance charges and ELPF's at selected entry ratios are compared with the corresponding 1977 Table M charges in Table 8.

TABLE 8

COMPARISON OF TABLE M CHARGES WITH SIMULATED VALUES

Entry Ratio	1977 Table M Charge (EL Group 19)	Simulated P.D. Insurance Charge (Unlim. Losses)	Simulated P.D. Charge with Losses Limited to $2,500	Indicated $2,500 ELPF ((4) − (3))
(1)	(2)	(3)	(4)	(5)
0	1.000	1.000	1.000	0
0.25	.750	.766	.766	0
0.50	.526	.532	.532	0
1.00	.190	.089	.217	.128
1.25	.105	.010	.217	.207
2.00	.029	0	.217	.217

[20] Viewing each trial as the experience of a single risk, the formula is

$$\text{Charge at entry ratio } r = \sum_{\substack{\text{trials where}\\ \text{losses exceed}\\ r^*(\text{Expected Losses})}} \{\text{Losses} - r^*(\text{Expected Losses})\} \Big/ \sum_{\substack{\text{all}\\ \text{trials}}} \text{Losses}.$$

[21] An *excess loss premium factor* ("ELPF") is the charge made for limiting a retrospectively rated risk's rateable losses to a per-occurrence amount, such as $25,000 or $50,000.

[22] See note 20.

The simulation approach to developing insurance charges offers three advantages over traditional means of developing Tables M or L:

1. The simulated factors are "customized" to reflect the specific frequency and severity characteristics of this particular insurance;
2. Updating (for inflation, etc.) is routine;
3. The table can be recast to reflect any desired level of loss limitation, thereby avoiding the classic problem of overlapping insurance charges and ELPF's.

Improving Loss Reserve Confidence Interval Calculations

A recent paper in these *Proceedings*[23] suggested a procedure for developing loss reserve confidence intervals from the corresponding pure premium confidence intervals underlying a given policy year's initial pricing. As illustrated in Section I of this report, simulation provides a means of improving the accuracy in one's estimate of these underlying pure premium ranges.

A description of how simulation might be used in the computation of loss reserve confidence intervals is provided as Appendix D.

V. CONCLUSION

The preceding pages discuss five specific areas—credibility theory, solvency testing, pricing new or difficult exposures, estimating individual risk rating charges, and developing confidence intervals around loss reserve estimates—in which computer simulation presents an opportunity for us to take a step toward overcoming traditional pricing and reserving obstacles. The recent introduction of inexpensive and highly efficient microcomputers provides the corresponding method and motive. Given method, opportunity, and motive, therefore, this writer believes that computer simulation will become a prominent (dominant?) actuarial tool during the 1980's.

This paper will be successful to the extent that it encourages other members of this Society to come forward with additional uses of computer simulation, or to offer improvements upon the applications suggested in this paper.

[23] C. K. Khury, "Loss Reserves: Performance Standards," *PCAS* LXVII, 1980, page 1.

APPENDIX A

Comparison of Simulated and Theoretical Confidence Intervals for Several Sample Sizes

		Constant Severity			Lognormal Severity*		
			Simulated Values (1,000 Trials)			Simulated Values (1,000 Trials)	
Expected Number of Claims	Conf. Range	Theor.	Lower	Upper	Theor.	Lower	Upper
500	99%	±11.5%	−13.2%	+11.8%	±36.4%	−25.2%	+44.8%
	98%	±10.4	−11.6	+11.0	±32.9	−25.0	+38.5
	95%	±8.8	−9.0	+8.2	±27.7	−21.3	+32.0
	90%	±7.4	−7.8	+7.2	±23.3	−18.5	+25.3
	80%	±5.7	−5.8	+5.8	±18.1	−15.5	+18.2
1,500	99%	±6.6%	−7.4%	+5.9%	±21.0%	−19.3%	+21.5%
	98%	±6.0	−6.7	+5.7	±19.0	−17.2	+19.3
	95%	±5.1	−5.5	+4.8	±16.0	−13.8	+16.3
	90%	±4.2	−4.5	+4.0	±13.4	−11.7	+13.5
	80%	±3.3	−3.5	+3.0	±10.5	−9.1	+10.6
2,500	99%	±5.2%	−5.2%	+5.1%	±16.3%	−13.1%	+20.0%
	98%	±4.7	−4.6	+4.5	±14.7	−12.0	+16.8
	95%	±3.9	−3.8	+3.7	±12.4	−10.2	+13.1
	90%	±3.3	−3.3	+3.1	±10.4	−8.3	+10.8
	80%	±2.6	−2.6	+2.6	±8.1	−6.8	+8.4

* Coefficient of variation is 3.0.

APPENDIX B

CURRENT NCCI BLACK LUNG PRICING PROCEDURES[24]

In developing black lung pure premiums, the National Council on Compensation Insurance:

1. Determines the average black lung indemnity cost using standard mortality assumptions (U.S. Life Total Population Tables, 1959–61), discounting assumptions (3.5%/year), and a black lung dependency distribution developed by the National Council;
2. Loads (1) for expenses (12.3% plus premium taxes, as of May, 1980);
3. Estimates the percentage of insured miners filing successful Compensation claims;
4. Multiplies (2) by (3) to obtain the expected cost per 100 miners;
5. Separately computes a medical pure premium;
6. Loads (5%?) for contingencies (e.g., mine closedowns which result in an unforeseeable outbreak of claims);
7. Converts (6) to a rate per $100 of payroll.[25]

[24] As described in Roy H. Kallop's *Black Lung Ratemaking*, a presentation to an industry symposium on black lung, St. Regis Hotel, New York City (May 19, 1980).

[25] The ratemaking formula described in Kallop's paper actually computes rates in two parts—one part paying for new claims, the second amortizing the cost of additional liabilities imposed by the black lung legislation effective March 1, 1978. This paper focuses exclusively on the National Council's calculation of premiums to pay for new claims.

APPENDIX C

EIGHT ASSUMPTIONS UNDERLYING BLACK LUNG LOSS SIMULATION

1. *Frequency of retiring miners' filing of claims.* Currently, a retiring miner has little reason not to file a black lung claim. Thus, a high but unknown percentage of retiring workers will probably file claims.
   The simulation presented in this paper assumes the following filing rates along with their respective likelihoods of occurrence.

Retiring Miners Filing Black Lung Claims	Likelihood of Occurring
5 of 10	25%
7 of 10	50%
9 of 10	25%

   Also, we assume that all claimants will file both state and Federal claims.

2. *Success rate among retiring miners who file claims.* The rate of successful claimants varies substantially by state. In the Federal area, currently high success ratios are expected to fall during the coming years. Thus, the following rates are assumed.

State Claims		Federal Claims	
Approval Rate	Assumed Likelihood	Approval Rate	Assumed Likelihood
5 of 10	25%	4 of 20	25%
7 of 10	50%	7 of 20	50%
9 of 10	25%	10 of 20	25%

   One consequence of this assumption should be noted. As mentioned earlier, a miner who successfully pursues a state and Federal black lung claim receives the higher of the state or Federal awards. Currently, most states' weekly benefits are higher than the Federal benefit; however, Federal amounts are annually escalated for inflation. Thus, miners qualifying for both types of benefits receive state benefits until Federal amounts exceed state levels, at which time the miner or his survivor receives a Federal supplement equal to the benefit difference.

It follows that the percentage of miners qualifying for both state and Federal awards is an important consideration in pricing black lung coverage. Assuming that a claimant's success or failure in pursuing a state claim does not affect the disposition of his Federal award, a sufficient number of simulations should produce a distribution of beneficiaries along the following lines.

Type of Benefits Received	% of Miners Filing Claims
Both State and Federal	25%
State only	45%
Federal only	10%
No benefits awarded	20%

3. *Mortality rates*. A 1977 study of miner mortality[26] revealed a significantly higher incidence of lung-related diseases in retiring underground coal miners. Accordingly, the following mortality assumptions are used in this illustration.

Mortality Assumption[27]	Probability
Retiring miner's life expectancy is five years less than that of a "typical" retiree	55%
Retiring miner's life expectancy is typical	35%
Retiring miner's expectancy is five years greater than that of a "typical" retiree	10%

4. *Age of claimants*. The simulation assumes that only retiring miners (pension age 57 or 62) file claims.
5. *Number of dependents*. As discussed earlier, benefits are paid to survivors of a deceased miner who was totally disabled from pneumoconiosis at the time of his death. Accordingly, the model presented in this paper

[26] National Institute for Occupational Safety and Health, *Mortality Among Coal Miners Covered by the UMWA Health and Retirement Funds*, March, 1977.

[27] "Typical" mortality as per the U.S. total population mortality table, 1969–71. Other mortality tables could, of course, be used.

assumes the following dependency distribution.

Number of Dependents	% of Retiring Miners
0	50%
1	50%

Also, instead of computing joint survivorship probabilities, the simulation program assigns an effective pension age of 57 years to miners with dependents, and 62 years to all other miners.

6. *Wage inflation (for Federal benefit escalator).* The simulation arbitrarily assumes the current year's wage inflation rate to be 8%. Also, to illustrate the flexibility of this approach to pricing, the *change* in annual wage inflation rates is assumed to be normally distributed with an average change of 0 and a standard deviation of 1 point. Negative inflation rates are not allowed.
7. *Loss discounting percentages.* In view of the dramatic impact of this assumption on the program's ultimate cost, separate simulations were carried out for discounting assumptions of 0% (losses not discounted), 3.5% (the current National Council assumption), and 7%.
8. *Black lung indemnity benefits.* This simulation attempts to price for black lung indemnity payments; a similar approach could, of course, be employed for pricing the medical component.
   All black lung beneficiaries are assumed to qualify for the following hypothetical state or Federal indemnity payments.

Maximum State Benefit		Maximum Federal Benefit	
With Dependent	No Dependent	With Dependent	No Dependent
$300/week	$225/week	$440/month	$340/month

APPENDIX D

ILLUSTRATION OF LOSS RESERVE CONFIDENCE INTERVAL CALCULATION USING COMPUTER SIMULATION

Note: All calculations presented in this appendix assume a Poisson frequency process (expecting 1,000 claims) and a lognormal claim size distribution with a *CV* of 3.0. For 1976, a $5,000 average claim size was anticipated; for 1977, the corresponding figure was \$5,500; for 1978, \$6,050; for 1979, \$6,655; and for 1980, \$7,320.

Step 1. Begin by modifying Table 2 to account for the *error in the initial estimate* of the expected frequency and claim cost. This "parameter error" increases the confidence ranges in Table 2 as indicated below.

SIMULATED CONFIDENCE BOUNDS AS A PERCENTAGE OF EXPECTED LOSSES

Conf.	Per Table 2—Reflects Process Variance Only		Adjusted to Reflect Parameter Error	
Range	Lower Bound	Upper Bound	Lower Bound	Upper Bound
99%	−21.5%	+27.5%	−30.2%	+38.0%
98%	−19.7	+24.6	−27.8	+32.3
95%	−16.2	+20.2	−24.3	+28.5
90%	−13.9	+17.5	−20.7	+23.2
80%	−10.9	+13.4	−17.3	+17.9

Step 2. Use the above results to compute the 90% confidence ranges about the go-in pure premium for each accident year in which losses remain outstanding.

Accident	Expected Losses at Policy	Approximate 90% Confidence Limit at Inception (above table)	
Year	Inception	Lower Bound	Upper Bound
1976	\$5,000,000	−\$1,035,000	+\$1,160,000
1977	\$5,500,000	−\$1,138,500	+\$1,276,000
1978	\$6,055,000	−\$1,253,500	+\$1,404,000
1979	\$6,655,000	−\$1,377,500	+\$1,544,000
1980	\$7,320,500	−\$1,515,500	+\$1,698,000

Step 3. Complete the calculation by assuming that

- loss reserve estimates improve in direct proportion to the time elapsed since policy inception, and
- all losses are settled within five years from date of occurrence.

Accident Year	90% Confidence Interval as of 1/1/81	
	Lower Bound	Upper Bound
1976	0	0
1977	−$228,000	+$255,000
1978	−$501,500	+$561,500
1979	−$826,500	+$926,500
1980	−$1,212,500	+$1,358,500
Total All Years	−$2,768,500	+$3,101,500

APPENDIX E

A COMPUTER APPROACH TO "GREATEST ACCURACY" CREDIBILITY THEORY

Section I of this paper illustrates how computer simulation can be used to develop and apply "limited fluctuation" credibility theory, as described by Longley-Cook, Mayerson, and Carlson.[28] This approach to establishing full credibility standards is basically a matter of developing confidence intervals, an application for which computer simulation is ideally suited.

Of course, a second credibility system—the "greatest accuracy" theory of A. L. Bailey,[29] Hewitt,[30] and others—has also gained wide acceptance within this Society. Under this second approach, credibility weights produce the best linear fit of observed pure premium data to conditional expectations of the pure premium over all possible data outcomes. Since confidence intervals are not involved in this formulation, the usefulness of a computer in developing greatest accuracy credibility factors is not readily apparent.

[28] Longley-Cook, page 196; Mayerson et al, page 175; T. O. Carlson, "Observations on Casualty Insurance Rate-Making Theory in the United States," *PCAS* LI, 1964, page 282.

[29] A. L. Bailey, "A Generalized Theory of Credibility," *PCAS* XXXII, 1945, page 13.

[30] C. C. Hewitt, Jr., "Credibility for Severity," *PCAS* LVII, 1970, page 148.

This Appendix illustrates how (non-simulation) computer techniques can also be used to explain and possibly expand upon the uses of the greatest accuracy credibility model.

"Greatest Accuracy" Credibility Theory Restated

As described in these *Proceedings*, the greatest accuracy credibility factor, z, is a number between 0 and 1.00 which minimizes the "mean square error," $M(z)$, given by

$$M(z) = \int_D \{E(H|d) - z \cdot d - (1.0 - z) \cdot E(H)\}^2 \cdot f(d)dd$$

$$\simeq \sum_d \{E(H|d) - z \cdot d - (1.0 - z) \cdot E(H)\}^2 \cdot \Pr(d).$$

"H" is here the prior estimate of an underlying parameter and "d" is actual observed data. $E(X|y)$, $E(X)$, $\Pr(x)$, and $f(x)$ have their usual interpretations.

Since this problem involves selecting a weight, z, which minimizes $M(z)$ over a large range of values, a computer approximation of z is feasible. An illustration follows.

The Problem

Assume that you are analyzing a body of 100 exposures with the following characteristics.

- You expect 0.25 claims per exposure (25 claims in your sample).
- A given exposure's expected frequency may be 0.15, 0.20, 0.25, 0.30, or 0.35 claims, with equal likelihood.
- All exposures have the same (but unknown) underlying frequency.
- The frequency process is Poisson.
- All claims cost $5,000.

Since severity is assumed constant, the following analysis deals exclusively with claim frequency. Clearly, the conclusions apply equally to a consideration of the pure premium.

Ignoring severity, $E(H) = 25$ claims. Moreover, the various values of $E(H|d)$ (d a value of D) can be determined as follows.

d	h	$\Pr(d\|h)$	$\Pr(h)$	$\Pr(d)$	$\Pr(h\|d)$*	$E(H\|d)$**
0 claims	15 claims	0	.20		0.99326	
	20 claims	0	.20		0.00669	
	25 claims	0	.20	0	0.00005	15.03392 claims
	30 claims	0	.20		0	
	35 claims	0	.20		0	
⋮						
20 claims	15 claims	.04181	.20		.21122	
	20 claims	.08884	.20		.44878	
	25 claims	.05192	.20	.03959	.26228	21.08226 claims
	30 claims	.01341	.20		.06775	
	35 claims	.00197	.20		.00996	
⋮						
25 claims	15 claims	.00498	.20		.02535	
	20 claims	.04459	.20		.22696	
	25 claims	.07952	.20	.03929	.40480	25.73962 claims
	30 claims	.05112	.20		.26019	
	35 claims	.01625	.20		.08270	
⋮						

* $\Pr(h|d) = \{\Pr(d|h) \cdot \Pr(h)\}/\Pr(d)$.

** $E(H|d) = \sum_h h \cdot \Pr(h|d)$.

Unfortunately, manually carrying out these calculations for all possible values of D is tedious and impractical. However, this routine is easily handled by a computer. With the assistance of an appropriately programmed machine, for example, the results in Table E1 were obtained for $d = 0, 1, 2, 3, \ldots,$ 59 claims.

TABLE E1

$E(H|d)$ AND $\Pr(d)$ GIVEN 0–59 CLAIMS

d (# of Claims)	$E(H\|d)$	$\Pr(d)$	d (# of Claims)	$E(H\|d)$	$\Pr(d)$
0 claims	15.03392	0	30 claims	29.63554	.03530
1	15.04528	0	31	30.25558	.03375
2	15.06044	.00001	32	30.81878	.03191
3	15.08069	.00003	33	31.32651	.02980
4	15.10772	.00013	34	31.78125	.02746
5	15.14379	.00040	35	32.18624	.02493
6	15.19187	.00101	36	32.54518	.02229
7	15.25587	.00218	37	32.86203	.01961
8	15.34087	.00416	38	33.14079	.01696
9	15.45337	.00709	39	33.38534	.01441
10	15.60155	.01096	40	33.59939	.01203
11	15.79532	.01555	41	33.78634	.00985
12	16.04625	.02046	42	33.94936	.00793
13	16.36697	.02526	43	34.09129	.00626
14	16.76991	.02953	44	34.21469	.00485
15	17.26539	.03301	45	34.32185	.00369
16	17.85911	.03563	46	34.41480	.00275
17	18.54995	.03743	47	34.49534	.00201
18	19.32882	.03857	48	34.56505	.00145
19	20.17966	.03924	49	34.62535	.00102
20	21.08226	.03959	50	34.67745	.00071
21	22.01589	.03974	51	34.72243	.00048
22	22.96208	.03977	52	34.76125	.00032
23	23.90576	.03971	53	34.79471	.00021
24	24.83492	.03955	54	34.82355	.00014
25	25.73962	.03929	55	34.84838	.00009
26	26.61130	.03890	56	34.86976	.00005
27	27.44238	.03834	57	34.88815	.00003
28	28.22649	.03757	58	34.90397	.00002
29	28.95864	.03657	59	34.91756	.00001

Having developed $E(H|d)$ and $\Pr(d)$ for all reasonably foreseeable outcomes of D, the computation of $M(z)$ becomes routine, if lamentably tedious. Again, however, a computer accomplishes the necessary calculations in a matter of microseconds. Since z is selected to minimize $M(z)$, it is easily seen from Table E2 that the appropriate greatest accuracy credibility is 0.67.

TABLE E2

DISPLAY OF z AND $M(z)$

z	$M(z)$	z	$M(z)$
0.00	34.70944	0.51	3.21716
0.01	33.71701	0.52	2.98960
0.02	32.73957	0.53	2.77704
0.03	31.77713	0.54	2.57948
0.04	30.82969	0.55	2.39691
0.05	29.89724	0.56	2.22934
0.06	28.97980	0.57	2.07677
0.07	28.07735	0.58	1.93920
0.08	27.18989	0.59	1.81662
0.09	26.31744	0.60	1.70904
0.10	25.45998	0.61	1.61646
0.11	24.61752	0.62	1.53888
0.12	23.79006	0.63	1.47629
0.13	22.97760	0.64	1.42870
0.14	22.18013	0.65	1.39611
0.15	21.39766	0.66	1.37852
0.16	20.63019	<u>0.67</u>	<u>1.37593</u>
0.17	19.87772	0.68	1.38833
0.18	19.14024	0.69	1.41573
0.19	18.41776	0.70	1.45812
0.20	17.71028	0.71	1.51552
.		.	
.		.	
.		.	
0.48	3.98982	0.98	8.73426
0.49	3.71727	0.99	9.21158
0.50	3.45972	1.00	9.70391

Comparison with Theoretical Results

As expected, the previously derived greatest accuracy credibility factor ($z = 0.67$) can be verified analytically. Specifically, it is easily shown that this outcome would have resulted from Bühlmann's $z = N/(N + K)$ formulation, where K is the mean of process variance divided by the variance of the hypothetical means.[31]

- Mean of (Poisson) process variances = 0.25 claims^2/exposure.
- Variance of hypothetical means $= E(H^2) - (E(H))^2$
  $= 0.0675 - 0.0625 = 0.005.$
- Thus $K = 0.25/0.005 = 50.$
- Hence, for 100 exposures, $z = N/(N + K) = 100/150 = 0.67$, as per the previous development.

Advantages of a Computer-Based Approach

Section I suggested two advantages of a computer-based approach to determining limited fluctuation credibility standards:

- the computer approach is more intuitive, and therefore more easily presented and explained to non-actuarial users, and
- computer simulation provides a means of extending previous analytically derived results.

To a lesser extent, these same advantages can be realized by using a computer to develop greatest accuracy credibility factors.

Clearly, the computer approach is more intuitive than its analytical counterpart. In this writer's opinion, for example, tables along the lines of Table E2 aid considerably in explaining greatest accuracy factors.

Moreover, using a computer provides additional flexibility in the development of credibility formulas.

- By rerunning the necessary computer programs, the sensitivity of credibility factors to small changes in one's prior distribution assumption can readily be determined.
- Variations on greatest accuracy credibility formulas are easily accomplished. In particular, the previous procedure lends itself quite nicely to the development of credibility weights which minimize the mean square

[31] H. Bühlmann, *Mathematical Methods in Risk Theory* (1970), page 102.

error, $M(z)$, over a limited range of possible outcomes, instead of over all possible outcomes.

For instance, suppose that one wishes to determine the credibility factor which minimizes the mean square error of the preceding illustration over the range of outcomes 0, 1, 2, . . . , 25 claims. It can be verified by computer that the appropriate factor is $z = .71$.

The computer-based procedure outlined in this Appendix requires the derivation of $E(H|d)$. This additional information is usually helpful, if not directly applicable in all instances.

Conclusion

While *simulation* may not have direct application to greatest accuracy credibility theory, a computer can be used to explain and present these concepts. Moreover, while the theory behind the greatest accuracy credibility model is probably more advanced than its limited fluctuation counterpart, a computer may open the door to new and expanded applications of greatest accuracy theory.

GOOD AND BAD DRIVERS—A MARKOV MODEL OF ACCIDENT PRONENESS

EMILIO VENEZIAN

Abstract

Existing models of the distribution of accidents among a population of drivers do not account for both the differences among individuals and those among age groups. This paper proposes a simple model to simultaneously explain these variations.

The model assumes that all drivers begin at some early age as "bad" drivers. Subsequently, drivers switch at random from the "bad" state, with high accident probabilities per mile driven, to a "good" state with low accident probabilities. The opposite transition, from "good" to "bad" states, also occurs at random. As the proportion of good drivers increases with age, the average frequency declines with age. The author develops in his paper the explicit mathematical equations of the model and a method of parameter estimation.

The model leads to three conclusions:

1. Classification efficiency, as measured by the SRI formula, can never achieve 100%. An upper bound of classification efficiency exists because the actual state of the driver at the inception of coverage is not known.

2. Underwriting and other risk assessment methods that tend to separate drivers in the good state from those in the bad state will offset some of the weaknesses in classification, increasing the efficiency of the risk assessment process as a whole.

3. Even with "perfect" risk assessment, that is, with complete separation of drivers in the good and bad states, efficiency will not reach 100% because subsequent switching during the policy period will create heterogeneity.

INTRODUCTION

Many authors have developed models which attempt to describe the statistical distribution of accidents. The simplest, in a sense, is the Poisson model, which assumes the probability of any individual having an accident in any given time period to be the same for all individuals and all time periods. Data on accidents do not often fit the predictions from this model.[1]

One way to account for the difference between predictions and data is to appeal to differences between individuals in their probabilities of having accidents, also called their accident proneness. The convenient way to develop this type of model is to assume that accident proneness fits a gamma distribution,[2] under which assumption the observed numbers of accidents have a negative binomial distribution. This distribution accounts for data somewhat more successfully than does a Poisson model.[3] Additional assumptions are needed, however, if one wishes to use the model to yield information about the relationship of accidents to age, or about the autoregressive structure of accidents.

A second way to explain the difference between data and the Poisson model is to suppose that accident proneness increases with every accident. The Polya model assumes that the likelihood of an individual having an accident in a time interval increases linearly with the number of accidents that the individual had prior to the beginning of the interval. Under this assumption, also, the observed numbers of accidents have a negative binomial distribution.[4]

Statistically, therefore, this model would describe the distribution of the numbers of accidents in a group just as successfully as would an assumed gamma distribution of accident proneness. Moreover, both models imply that the likelihood of having an accident increases linearly with the prior number of accidents; in the Polya model this is a behavioral assumption, whereas in the

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[1] Hilary L. Seal, *Stochastic Theory of a Risk Business*, John Wiley & Sons, Inc., New York, 1969, pp. 12–29.

[2] Seal, *loc. cit.*; and Stanford Research Institute, "The Role of Risk Classifications in Property and Casualty Insurance: A Study of the Risk Assessment Process," Menlo Park, California, 1976.

[3] Seal, *loc. cit.*; Stanford Research Institute, *op. cit.*; and Donald C. Weber, "An Analysis of the California Driver Record Study in Context of a Classical Accident Model," *Accident Analysis and Prevention*, Vol. 44, 1972, pp. 109–116.

[4] William Feller, *An Introduction to Probability Theory and Its Applications*, John Wiley & Sons Inc., New York, 1968, Vol. 1, 3rd Edition, pp. 121, 142, 143; and Seal, *loc. cit.*

gamma model it is a consequence of the information contained in the prior history.[5] The Polya model inherently leads to the prediction that accident frequency will increase with age, a prediction which is wrong for automobile accidents.[6]

A third way to account for the data is to assume that accident proneness varies over time; an extreme of this model, in which accident proneness was viewed as an all-or-none variable, has been studied.[7] A somewhat different approach is to assume that accident proneness is either "high" or "low," so that there are "good" and "bad" drivers;[8] this can be viewed as a polarization of either the heterogeneous proneness model or the episodic proneness model. Neither of these models has the ability to predict the variation of accident proneness with age; they both provide information on the autoregressive structure of observed accidents.

My interest in developing a model that could describe the statistical characteristics of accident distribution and simultaneously provide information on the age structure of accident rates first arose in early 1976, while I was reviewing an early draft of the Stanford Research Institute (SRI) report.[9] The draft of that report stated that the datum ". . . contradicts the simplistic view of a driver population made up of 'good' drivers and some 'bad' drivers," but did not contain a test of this "simplistic view." I performed a crude test of this model on eight age groups; I noted in a memorandum to SRI that the fit appeared to be adequate for each group, and added:

> It is also interesting that the accident likelihood for "good" drivers is much the same at the various ages, as it is for "bad" drivers. This suggests, among other things, a Markov model in which there are "good" and "bad" drivers but switching occurs from "good" to "bad" and vice versa. Again, the implications for merit rating could be important.

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[5] Seal, *loc. cit.*; and Stanford Research Institute, *op. cit.*

[6] R. C. Peck, R. S. McBride, and R. S. Coppin, "The Distribution and Prediction of Driver Accident Frequencies," *Accident Analysis and Prevention*, Vol. 2, 1971, pp. 243–299; and Stanford Research Institute, *op. cit.*

[7] Seal, *loc. cit.*

[8] Seal, *loc. cit.*

[9] Stanford Research Institute, *op. cit.*

That suggestion lay dormant until a recent discussion of merit rating and the efficiency of risk classifications at the Risk Theory Seminar held under the auspices of the American Risk and Insurance Association.[10] Most discussants agreed that the view of classification efficiency taken in the SRI report was inadequate. The most heated arguments centered on whether Richard G. Woll's study[11] went far enough in correcting the errors inherent in the SRI measure of efficiency. The discussion was largely hampered by failures to distinguish between the "expected value of accident proneness" taken over a set of individuals at a given time and that taken over time for a given individual. The distinction is important because variance in the time-averaged proneness among individuals can be reduced, in principle at least, by both classification and underwriting selectivity, whereas variance in accident proneness resulting from future random events cannot be reduced by anything short of clairvoyance. My interest in the Markov model was revived when I realized that the model I had suggested could clarify some of these issues. I retained the assumption of two states, "good" and "bad," not for historical reasons but rather because the available data do not permit much discrimination. The revived model now has been developed to the extent that it is useful. Full development of the quantitative aspects of merit rating is still needed. At the current level, however, the model is useful as a framework for considering issues of classification and underwriting.

A MARKOV MODEL

Consider an individual who can be in one of two states, "good" or "bad." Assume that drivers have an accident probability of θ_1 per mile driven when they are in the "good" state and that the analogous quantity for the "bad" state is θ_2. Assume that the expected number of miles driven per unit time does not depend on the state (in fact, that complication could be accommodated readily). Also assume that in any time interval dt, an individual has a probability adt of changing from the "good" state to the "bad" state and a probability bdt of changing from the "bad" state to the "good" state.

The probability, $p(t)$, of being in the "good" state at time t is governed by the differential equation

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[10] The author is grateful to the institutions which support the Risk Theory Seminar and thereby create a forum for active exchange of views.

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[11] Richard G. Woll, "A Study of Risk Assessment," *PCAS* LXVI, 1979.

$$\frac{dp(t)}{dt} = -ap(t) + b(1 - p(t)) \tag{1}$$

For an individual who is known to be in the "bad" state at some initial time t_0, the solution of this equation is

$$p(t) = \frac{b}{a + b}(1 - e^{-(a+b)(t-t_0)}) \tag{2}$$

Averaging Equation 2 over the time period from t_1 to $t_1 + \Delta t$ yields the probability that the individual, known to be in the "bad" state at t_0, is in the "good" state during this interval:

$$p(t_1, \Delta t) = \frac{b}{a + b}\left[1 - \frac{1 - e^{-(a+b)\Delta t}}{(a + b)\Delta t} e^{-(a+b)(t_1-t_0)}\right] \tag{3}$$

If all we know is that the individual was in the "bad" state at t_0, then during the time interval from t_1 to $t_1 + \Delta t$ the expected value of the k^{th} moment of the accident proneness per mile driven is

$$E(\theta^k | t_1, \Delta t) = \theta_1^k p(t_1, \Delta t) + \theta_2^k[1 - p(t_1, \Delta t)] \tag{4}$$

This expression also is the expected value of θ^k, given t_1 and Δt, taken over individuals with the same θ_1, θ_2, a, b, and t_0.

The simplest assumption that can be made is that all parameters are the same for all individuals. This does not seem a realistic assumption *a priori*, but would be the most parsimonious one. A slightly more complex assumption is that t_0, the age at which people begin to switch from bad driving to good driving, relates to maturation so that t_0 for females may be somewhat lower than t_0 for males. Although this refinement still is very simplistic, it is of interest to develop the equations and test the ability of such a simple model to account for observations.

In most cases, data are available not by individual ages but only aggregated for all drivers within certain age spans, e.g., between ages t_1 and $t_1 + s$. Equation 3 can be modified to apply to the age span by averaging between the youngest and oldest ages included in the age span. If the age distribution within the span is uniform we obtain

$$p(t_1, \Delta t) = \frac{b}{a + b}\left[1 - \frac{1 - e^{-(a+b)\Delta t}}{(a + b)\Delta t} \frac{1 - e^{-(a+b)s}}{(a + b)s} e^{-(a+b)(t_1-t_0)}\right] \tag{5}$$

and Equation 4 needs no modification.

The expected accident proneness between ages t_1 and $t_1 + s$ per unit time $\dot{m}$ is

$$E(\phi(t_1)) = \dot{m}E(\theta)$$

$$= \dot{m}\left[\theta_1 \frac{b}{a+b} + \theta_2 \frac{a}{a+b} + f\frac{b}{a+b}(\theta_2 - \theta_1)e^{-(a+b)(t_1-t_0)}\right] \quad (6)$$

$$\text{where } f = \frac{1 - e^{-(a+b)\Delta t}}{(a+b)\Delta t} = \frac{1 - e^{-(a+b)s}}{(a+b)s} \quad (7)$$

For very large t_1 the expected accident proneness approaches an asymptotic value $E(\phi_a)$ given by

$$E(\phi_a) = \lim_{t_1\to\infty} E(\phi(t_1)) = \dot{m}\left(\theta_1 \frac{b}{a+b} + \theta_2 \frac{a}{a+b}\right) \quad (8)$$

Using this we can write Equation 6 in the form

$$\ln\,[E(\phi(t_1)) - E(\phi_a)] = \ln\left[\dot{m}f\frac{b}{a+b}(\theta_2 - \theta_1)\right]$$

$$+ (a+b)t_0 - (a+b)t_1 \quad (9)$$

The variance of the accident proneness per unit time is

$$V[\phi(t_1)] = \dot{m}[E(\theta^2) - E^2(\theta)]$$

$$= \dot{m}(\theta_2 - \theta_1)^2\left[\frac{ab}{(a+b)^2} + f\frac{b(b-a)}{(a+b)^2}e^{-(a+b)(t_1-t_0)} \quad (10)\right.$$

$$\left. -f^2\frac{b^2}{(a+b)^2}e^{-2(a+b)(t_1-t_0)}\right]$$

This variance also has an asymptotic value, $V(\phi_a)$:

$$V(\phi_a) = \lim_{t_1\to\infty} V(\phi(t_1)) = \dot{m}(\theta_2 - \theta_1)^2\frac{ab}{(a+b)^2} \quad (11)$$

For large values of t_1, the term $e^{-2(a+b)(t_1-t_0)}$ is much smaller than the other terms in Equation 10 so that we can rewrite this equation in the form

$$\ln[V(\phi(t_1) - V(\phi_a)] = \ln\left[\dot{m}(\theta_2 - \theta_1)^2 f \frac{b(b-a)}{(a+b)^2}\right]$$

$$+ (a+b)t_0 - (a+b)t_1 \qquad (12)$$

Equations 9 and 12 indicate that semilogarithmic plots of the differences between the quantities of interest and their asymptotic values would be useful in identifying whether the model is adequate.

DISTRIBUTION OF THE NUMBER OF ACCIDENTS

If the probability of an individual's having an accident during a time interval is governed by the Poisson distribution, then the number of accidents experienced by that individual will be Poisson distributed with a parameter equal to the realization of the total proneness, even if the accident proneness parameter varies over time. Thus, for an individual whose average proneness over an interval Δt turns out to be ϕ, the probability of x accidents is

$$p(x|\phi,\Delta t) = e^{-\phi\Delta t}\frac{(\phi\Delta t)^x}{x!} \qquad (13)$$

The k^{th} moment about the origin of the number of accidents for that individual is then

$$E(x^k|\phi,\Delta t) = e^{-\phi\Delta t}\sum_{x=0}^{\infty}\frac{x^k(\phi\Delta t)^x}{x!} \qquad (14)$$

$$= \phi\Delta t e^{-\phi\Delta t}\frac{\partial}{\partial(\phi\Delta t)}\sum_{x=0}^{\infty}\frac{x^{k-1}(\phi\Delta t)^x}{x!}$$

$$= (\phi\Delta t)\left[E(x^{k-1}|\phi\Delta t) + \frac{\partial}{\partial(\phi\Delta t)}E(x^{k-1}|\phi\Delta t)\right]$$

From this recursion equation we obtain

$$\begin{aligned}
E(x^0|\phi,\Delta t) &= 1\\
E(x^1|\phi,\Delta t) &= \phi\Delta t\\
E(x^2|\phi,\Delta t) &= (\phi\Delta t)^2 + (\phi\Delta t) \qquad (15)\\
E(x^3|\phi,\Delta t) &= (\phi\Delta t)^3 + 3(\phi\Delta t)^2 + (\phi\Delta t)\\
E(x^4|\phi,\Delta t) &= (\phi\Delta t)^4 + 6(\phi\Delta t)^3 + 7(\phi\Delta t)^2 + (\phi\Delta t)
\end{aligned}$$

From these expressions we find the moments of the distribution of the numbers of accidents by the equation

$$E(x^k) = \int_0^\infty E(x^k|\phi, \Delta t) g(\phi\Delta t) d(\phi\Delta t) \tag{16}$$

where $g(\phi\Delta t)$ is the probability density function of $\phi\Delta t$ taken over the same set of individuals as the number of accidents experienced in Δt.

Thus,

$$\left.\begin{aligned} E(x) &= \Delta t E(\phi) \\ E(x^2) &= (\Delta t)^2 E(\phi^2) + \Delta t E(\phi) \\ E(x^3) &= (\Delta t)^3 E(\phi^3) + 3(\Delta t)^2 E(\phi^2) + \Delta t E(\phi) \\ E(x^4) &= (\Delta t)^4 E(\phi^4) + 6(\Delta t)^3 E(\phi^3) + 7(\Delta t)^2 E(\phi^2) + \Delta t E(\phi) \end{aligned}\right\} \tag{17}$$

It follows from these equations that

$$E(x) = \Delta t E(\phi), \text{ and} \tag{18}$$

$$V(x) - E(x) = E(x^2) - E^2(x) - E(x) = (\Delta t)^2 V(\phi) \tag{19}$$

Equations 18 and 19 demonstrate that the model given in the preceding section specifies both the expected value of the number of accidents, $E(x)$, and the "excess variance," $V(x) - E(x)$, as a function of age.

Let $M(x)$ denote the mean number of accidents observed among N individuals, and $S(x)$ denote the calculated value of the excess variance; we hypothesize that these quantities follow the model described above. In order to test this hypothesis, we need estimates of the sampling variability of these quantities. For $M(x)$, the calculation is standard; the variance of $M(x)$ is simply $V(x) \div N$. For $S(x)$, the calculation is not as familiar since it must account not only for the variance of the sample estimates of the variance and the mean, but also for the covariance between these. The basic results can be obtained from most good books in statistics.[12] Neglecting terms of order N^{-2} we obtain

$$V(S(x)) = \frac{\mu_4(x) - \mu_2^2(x) - 2\mu_3(x) + \mu_2(x)}{N} \tag{20}$$

where $\mu_j(x)$ is the j^{th} central moment of x, for $j \geq 2$.

[12] Harald Cramer, *Mathematical Methods of Statistics*, Princeton University Press, Princeton, N.J., 1964, pp. 347–348.

After additional algebra to relate the moments of x to those of $\phi\Delta t$ we obtain

$$V(S(x)) = \frac{1}{N}[\mu_4(\phi\Delta t) - \mu_2^2(\phi\Delta t) + 4\mu_3(\phi\Delta t)$$

$$+ 2\mu_2(\phi\Delta t) + 4\mu_1(\phi\Delta t)\mu_2(\phi\Delta t) + 2\mu_1^2(\phi\Delta t)] \quad (21)$$

where $\mu_1(\phi\Delta t) = E(\phi\Delta t) = \Delta tE(\phi) = \mu_1(x)$.

In the special case of a completely homogeneous population, one in which the realization of ϕ is identical across all individuals, $S(x)$ reduces to

$$E_1(S(x)) = 0 \quad (22)$$

$$V_1(S(x)) = \frac{2}{N}\,\mu_1^2(\phi\Delta t) \simeq \frac{2}{N}\,\mu_1^2(x) \quad (23)$$

Since $S(x)$ is asymptotically normally distributed, the value

$$Z_1 = \frac{S(x)}{M(x)}\sqrt{\frac{N}{2}} \quad (24)$$

is, asymptotically, a unit variance normal deviate, and provides a test of significance for the excess variance against the null hypothesis of zero excess variance that corresponds to a Poisson process with no heterogeneity.

COMPARISON OF THE MODEL WITH DATA

In order to test the adequacy of the model we must compare the model predictions to data. A convenient set of data is that drawn from licensed drivers in California in 1961–1963.[13] The published data include the mean numbers of accidents by year of age for ages 17 through 30 and by five year age groups for ages 21 through 76. The data are available for males and females separately, and sufficient information is provided to allow the calculation of the excess variance for each sex in age groups spanning five years. The relevant data are shown in Tables I and II. It is of some interest that the excess variance greatly exceeds its standard deviation, as indicated by the large values of Z_1 found for most age groups in both sexes. This indicates that the excess variance does not arise from sampling variability.

[13] Peck, McBride, and Coppin, *loc. cit.*

TABLE I

AVERAGE NUMBER OF ACCIDENTS BY AGE AND SEX FOR AGES 17–30

	Males		Females	
Age Group	Number*	Average Accidents	Number*	Average Accidents
17	11**	0.727***	4**	0.250
18	1114	0.532	763	0.213
19	1399	0.476	955	0.219
20	1683	0.419	1146	0.198
21	1521	0.396	1182	0.149
22	1600	0.355	1182	0.163
23	1678	0.308	1182	0.129
24	1757	0.311	1182	0.126
25	1836	0.301	1182	0.123
26	1721	0.298	1315	0.113
27	1794	0.288	1315	0.124
28	1867	0.310	1315	0.094
29	1940	0.279	1315	0.132
30	2014	0.277	1315	0.129

* Estimated from totals for the three-year age spans (see text) by assuming numbers are linear with age and requiring that the mean average accidents for an age span be equal to the weighted mean of individual years; totals for age groups may differ from those in the original article because of rounding to the nearest integer.
** Smallest integer consistent with data given in original article.
*** Table 12 of Peck, McBride, and Coppin gives 0.737 for this value, which is inconsistent with the data for single and married males given separately in the same table. The value given here is consistent with the disaggregated data.

TABLE II

AVERAGE NUMBER OF ACCIDENTS AND EXCESS VARIANCE, BY SEX, FOR AGE GROUPS

Age Group	Males				Females			
	N	$M(x)$	$S(x)$*	$Z_1(x)$	N	$M(x)$	$S(x)$**	$Z_1(x)$
18–20	4196	0.468	0.062	6.1	2863	0.209	0.017	3.1
21–25	8392	0.332	0.054	10.5	5910	0.138	0.018	7.1
26–30	9336	0.290	0.047	11.1	6574	0.118	0.013	6.3
31–35	10200	0.256	0.058	16.2	7534	0.119	0.017	8.8
36–40	10573	0.250	0.039	11.3	8612	0.122	0.012	6.5
41–45	10127	0.231	0.041	12.6	8113	0.122	0.012	6.3
46–50	9041	0.234	0.031	8.9	6671	0.126	0.009	4.1
51–55	7466	0.226	0.034	9.2	5253	0.108	0.013	6.2
56–60	5949	0.224	0.023	5.6	3807	0.124	0.006	2.1
61–65	4608	0.226	0.038	8.1	2706	0.118	0.015	4.7
66–70	3419	0.193	0.030	6.4	1822	0.112	0.011	3.0
71–75	2027	0.179	0.010	1.8	952	0.136	0.007	1.1
≥ 76	1372	0.200	0.038	5.0	452	0.142	0.025	2.6

* Calculated from the distribution of male licenses by age and number of reported accidents given in Table 10 of Peck, McBride, and Coppin.
** Calculated from the distribution of female licenses by age and number of reported accidents given in Table 11 of Peck, McBride, and Coppin.

The values of the model parameters could be established by statistical fitting techniques. In view of the complexity of the task, however, we used a much simpler procedure, as described in the Appendix to this paper. The resulting parameter values are shown in Table III. For ease of comparison to the paper by Peck, McBride, and Coppin,[14] the parameters for males and females are shown in terms of the age recorded in that paper, which corresponds to two years more than the age t_1 used in our equations. Since the relevant variable is the difference $t_1 - t_0$, we can then use the age, as recorded in the article by Peck, McBride, and Coppin, with the value of t_0 from Table III to compute the relevant quantities.

TABLE III

VALUES OF COMPUTED PARAMETERS

Parameter	Value	Units
a	0.03	Per Year
b	0.17	Per Year
θ_1	4.20×10^{-6}	Per Mile
θ_2	18.76×10^{-6}	Per Mile
t_0, male	18.37	Years
t_0, female	16.02	Years

Figures 1a and 1b show the average numbers of accidents during a three-year period ($\Delta t = 3$) involving drivers in the age range 17 through 30 years, displayed by year of age ($s = 1$). The data for males are shown in Figure 1a; the data for females are shown in Figure 1b. In each case the asymptotic value has been subtracted from the observation and the range of plus and minus one standard deviation is shown. The lines shown in these figures represent Equations 6 and 16, with the relevant parameter values from Table III. The fit is generally adequate, though not outstanding.

Figures 2a and 2b display the corresponding data over the entire age range ($s = 3$ for 18 through 20 years, $s = 5$ for other ages). The line for males is in general agreement with the observations at all age groups up to the 61–65 year age group; beyond that, there may be some departure. In the case of females, however, the line follows the data only up to the 31–35 year age group, with what appear to be progressively larger departures after that age. More sophisticated fitting of the parameters would not improve the fit of the line to the data, since the data do not appear to be log-linear as implied by Equation 9.

[14] Peck, McBride, and Coppin, *loc. cit.*

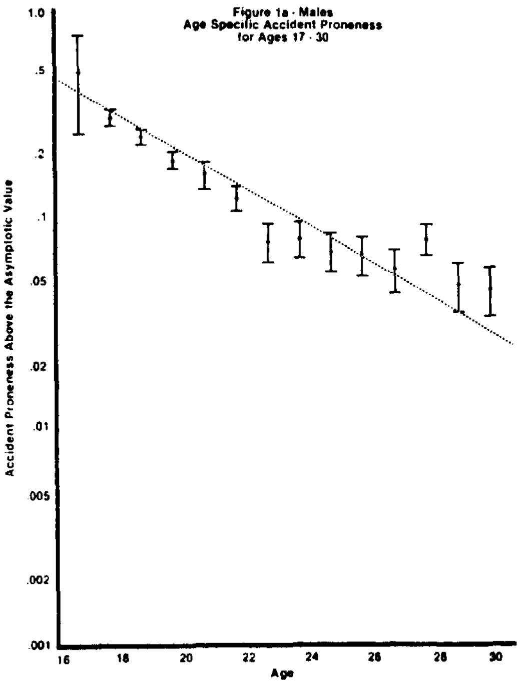

Figure 1a - Males
Age Specific Accident Proneness
for Ages 17 - 30

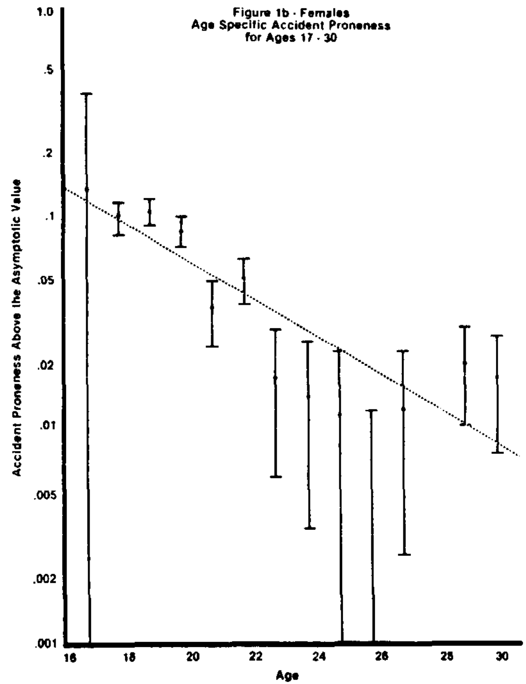

Figure 1b - Females
Age Specific Accident Proneness
for Ages 17 - 30

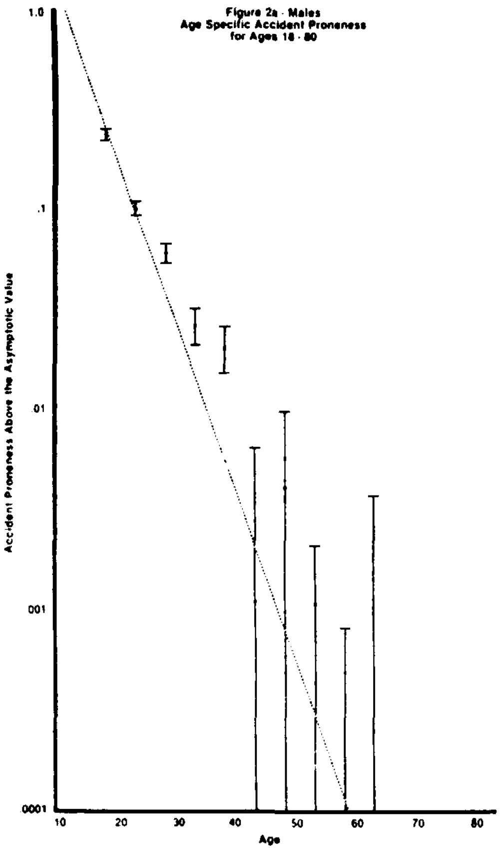
Figure 2a · Males
Age Specific Accident Proneness
for Ages 18 · 80
Accident Proneness Above the Asymptotic Value
1.0
.1
.01
.001
.0001
10
20
30
40
50
60
70
80
Age

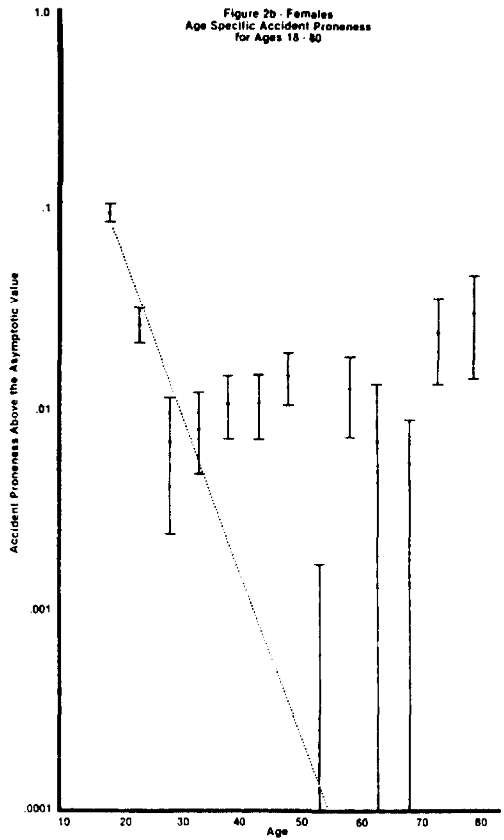
Figure 2b · Females
Age Specific Accident Proneness
for Ages 18 · 80
Accident Proneness Above the Asymptotic Value
1.0
.1
.01
.001
.0001
10
20
30
40
50
60
70
80
Age

Figures 3a and 3b show the data for the excess variance by age. The lines are calculated from Equations 8 and 17. The model agrees well with data for males but not particularly well with data for females. It is possible that the fit could be improved by relaxing the assumption that the mileage driven each year is *independent of age*. Using age-specific intensities of exposure would change the fitted values of all parameters, and might or might not lead to an improved fit. It would be feasible to examine this issue if questionnaire responses on mileage driven by a sample of the drivers studied were available. The exercise would be especially meaningful if the variance of mileage driven, as well as the mean, could be established for each age group and the equations were modified to allow for this variability.

DISCUSSION

The model presented here goes beyond the highly simplistic view of "good" and "bad" drivers by creating a model of transitions from one state to the other. The assumption that all drivers are "bad" at some suitably low age is then sufficient to account for differences in mean accident rates between age groups and heterogeneity within age groups. In this paper we have assumed that most parameters do not vary between individuals. More realistically, one might expect that mileage driven would be correlated not only with sex, as assumed here, but also with age, vehicle driven, and the characteristics of the territory in which most driving is done. Moreover, there is probably a quality weighting of the miles driven because of varying road and traffic characteristics. The values of the accident proneness parameters θ_1 and θ_2 may also vary across individuals and across driving environments. The characteristic age at maturation t_0 and the transition rates a and b also could be assumed to vary between individuals. A generalization of the model would include the specification of a joint distribution function for all these variables.

Keeping in mind the fact that the model is simplistic, it explains surprisingly well, within a simple theoretical structure, heterogeneity within and between ages and sexes. Because of this success, it is interesting to examine some implications of the model.

To begin with, the model is based on the assumption that an individual's accidents are generated by a Poisson process with time dependent parameters, yet the distribution of the numbers of accidents taken across individuals is not Poisson. Similarly, the numbers of accidents for a given individual taken over time subintervals will not exhibit a Poisson distribution around the mean for the interval as a whole, but will show clustering for subintervals during which the

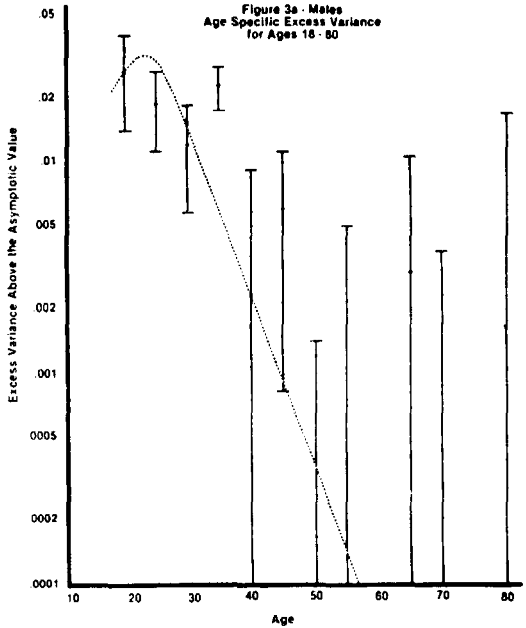
Figure 3a - Males
Age Specific Excess Variance
for Ages 18 - 80
Excess Variance Above the Asymptotic Value
.05
.02
.01
.005
.002
.001
.0005
.0002
.0001
10
20
30
40
50
60
70
80
Age

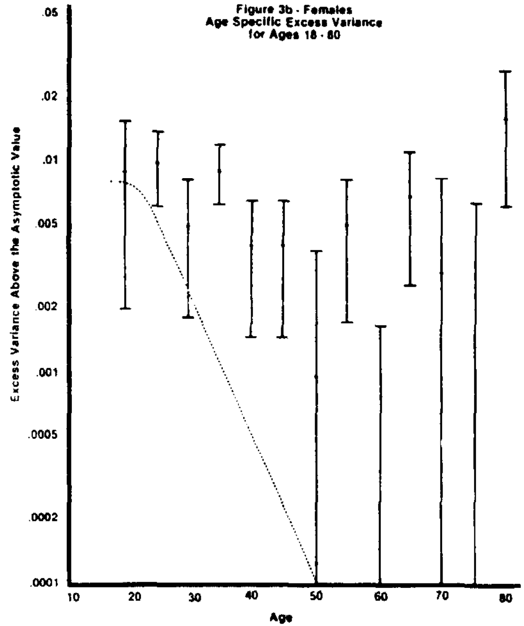
Figure 3b - Females
Age Specific Excess Variance
for Ages 18 - 80
Excess Variance Above the Asymptotic Value
.05
.02
.01
.005
.002
.001
.0005
.0002
.0001
10
20
30
40
50
60
70
80
Age

individual is in the "bad" driver state. An alternative statement of this last comment is that the distribution of times between accidents will not exhibit the exponential distribution that would be expected for a Poisson process of constant rate.

The model also illustrates the importance of maintaining clarity as to what is meant by "expected value." The expected value of the accident proneness per unit time for individuals of age t_1 is

$$E(\phi) = \dot{m}\left[\theta_1 \frac{b}{a+b} + \theta_2 \frac{a}{a+b} + \frac{b}{a+b}(\theta_2 - \theta_1)e^{-(a+b)(t_1-t_0)}\right] \tag{25}$$

when the averaging process is over individuals about whom no other information is available. For individuals known to be in the "good" state at age t_1, the expected proneness per unit time over the following T years is

$$E(\phi|\text{good}) = \dot{m}\left[\theta_1 \frac{b}{a+b} + \theta_2 \frac{a}{a+b} - \frac{a}{a+b}(\theta_2 - \theta_1)\frac{1-e^{-(a+b)T}}{(a+b)T}\right] \tag{26}$$

Similarly

$$E(\phi|\text{bad}) = \dot{m}\left[\theta_1 \frac{b}{a+b} + \theta_2 \frac{a}{a+b} + \frac{b}{a+b}(\theta_2 - \theta_1)\frac{1-e^{-(a+b)T}}{(a+b)T}\right] \tag{27}$$

Thus the expected value, taken over time, for a given individual (who must be in either one or the other state at age t_1) is not the same as the expected value taken over individuals, except in two special cases:

(1) $t_1 = t_0$ and $T = 0$, and

(2) $t_1 = T = \infty$

The variances will differ correspondingly. A group selected for identical ages and initial states will develop heterogeneity just because of the random changes of individuals within that group.

The model developed in this paper has interesting implications relative to the continuing controversy regarding classifications, homogeneity, and underwriting freedom.[15] It has become almost commonplace to say that the role of classification is not to predict the number of accidents that an individual will have during a time interval, but rather to predict the likelihood of that individual's having an accident.[16] By that criterion, 100% homogeneity, in the sense of no excess variance,[17] requires the identity of *realization* of proneness, not just identity in the *expected value* of proneness. This is unachievable.

The model has some important implications relative to merit rating. Individuals whose ages are close to t_0 will be in the "bad" driver state in almost every instance. Therefore, their prior accident records will contain additional information about their likely accident experience only to the extent that individuals differ with respect to mileage driven or other parameters assumed constant in this paper. For mature individuals, the situation is quite different. The age of mature individuals is not a good predictor of initial state, since at advanced ages very nearly 15% are in the "bad" state and 85% are in the "good" state. Among individuals who have just had an accident, nearly 45% will be in the "bad" state and only 55% will be in the "good" state. There is substantial persistence in a state; nearly 85% of the individuals in the "bad" state and 97% of those in the "good" state at any instant will remain in the state for at least one full year. Thus, a mature individual's prior accident record has substantial predictive value.

The model suggests that merit rating relativities will *increase* with age, and rapidly so at ages close to t_0. Though we have no data for drivers at ages close to t_0, the data from North Carolina,[18] shown in Figure 4, suggest that this prediction is correct. Further, the model suggests that the mileage driven by young people with accidents should be quite different from the mileage driven by young people without accidents; the difference should decrease with age.

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[15] Robert A. Bailey and LeRoy J. Simon, "Two Studies in Automobile Insurance Ratemaking," *The Astin Bulletin*, Vol. I, 1961, pp. 192–217.

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[16] Michael A. Walters, "Risk Classification Standards," *PCAS* LXVIII, 1981 and Richard G. Woll, *op. cit.*

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[17] Stanford Research Institute, *op. cit.*

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[18] J. Richard Stewart and B. J. Campbell, "The Statistical Association Between Past and Future Accidents and Violations," The University of North Carolina, Highway Safety Research Center, Chapel Hill, N.C., 1972.

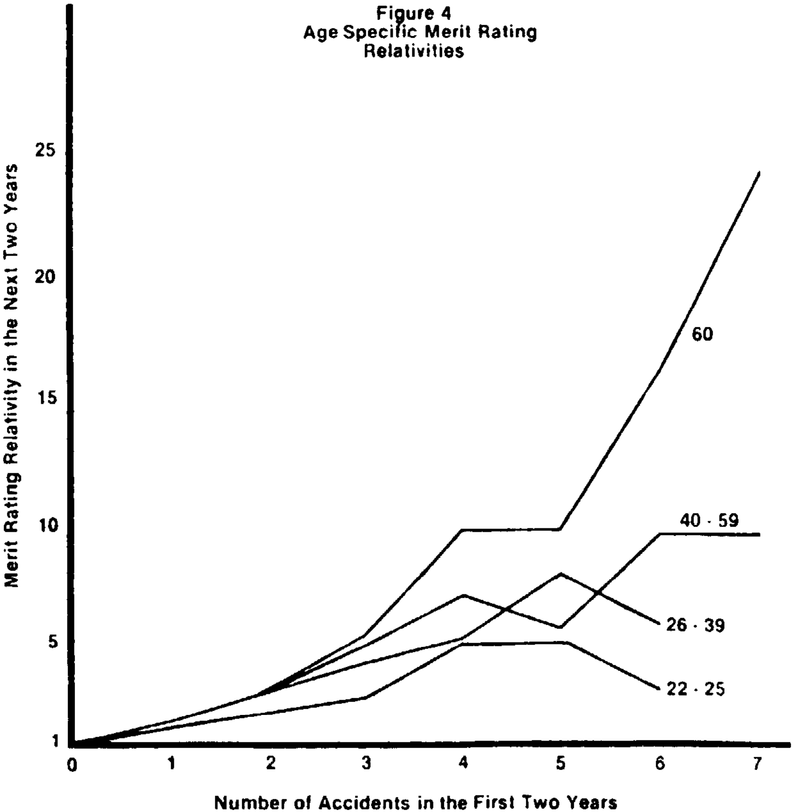

Figure 4
Age Specific Merit Rating
Relativities

In the model, the age relative to t_0 is the best objective *classification* predictor of an individual's state as "good" or "bad" driver. Prior accident record helps, but a complete development of the conditional probabilities would be needed to evaluate quantitatively the contribution of this variable to reduced heterogeneity. "Subjective" or "underwriting" judgments also could be used to determine whether an individual is in the "good" or "bad" state. Such judgments, if less than 100% efficient in separating drivers in the "good" state from those in the "bad" state, would have more impact if applied to mature drivers than if applied to young drivers. This contrasts with the usual perception of industry practices.

The applicability of models such as the one presented in this paper is often limited. In the case of automobile insurance, a major limitation is created by the fact that the coverage extends to an automobile and is not limited to a driver. Even if the model were an accurate representation of reality, its direct quantitative application to automobile insurance might not be warranted. The model does provide some interesting insights into the data that is needed to evaluate the model's validity. More sophisticated fitting of parameters seems much less important than assessing the interpersonal variation in all variables modelled and the impact of other variables, such as actual mileage driven.

APPENDIX

METHOD OF PARAMETER ESTIMATION

In order to check the fit of the model presented in this paper, the model's parameters must be determined. One parameter, mileage driven per year, is not readily accessible. This parameter is not of major importance, since it is merely a scaling factor; it is very convenient, however, since knowing the ratio of mileage driven by males to miles driven by females reduces by one the number of parameters to be estimated. I have used data based on a 1969–1970 survey by the Federal Highway Administration,[19] which indicates that, per person, per year, females drive approximately 48% of the mileage driven by males. The fitting was therefore performed on the basis that males drive 12,000 miles and females drive 5,800 miles per year.

The initial stage of the fitting relied on the fact that the mean numbers of accidents, minus the asymptotic value at high ages of the number of accidents,

[19] Motor Vehicle Manufacturers Association of the United States, Inc., "MVMA Motor Vehicle Facts & Figures 1978," Detroit, Michigan, p. 49. The information is based on unpublished data from the National Personal Transportation Survey conducted by the Bureau of the Census for the Federal Highway Administration, 1969–1970.

must be linear in a semilogarithmic plot. Since the asymptotic value is

$$\phi_a \Delta t = \dot{m}\Delta t \left(\theta_1 \frac{b}{a+b} + \theta_2 \frac{a}{a+b}\right), \tag{A1}$$

the value for females and males must be in the same ratio as the mileages. A few trials using the data for ages 17 through 30 gave $\phi_a \Delta t$ values of 0.23 for males and 0.11 for females. The slope of the semilogarithmic plots is $-(a + b)$, and the fact that both sets of data could be accommodated by $a + b = 0.20$ gave a preliminary indication that the model was promising and that the assumption of common parameters was tenable. This procedure also provided constraints on the parameters, giving two equations in four unknowns.

In order to determine all four of these unknowns, plus the values of t_0 for males and females, additional relationships were needed.

The asymptotic value of the excess variance provided one such relation:

$$S_a = S[x(\infty)] = (\dot{m}\Delta t)^2(\theta_2 - \theta_1)^2 \frac{ab}{(a+b)^2} \tag{A2}$$

Solving this with Equation A1 we obtain

$$\theta_2 = \frac{1}{\dot{m}\Delta t}\left[\phi_a \Delta t + \sqrt{\frac{b}{a}\, S_a}\right] \tag{A3}$$

Equation A3 allows solving for θ_2 if a value of b is assumed. Finally, we determined t_0 using the mean number of accidents at a recorded age[20] of 17 as estimated from the value at other ages:

$$\bar{x}_{17} - \phi_a = \dot{m}\Delta t \frac{b}{a+b}(\theta_2 - \theta_1) f e^{-(a+b)(t_1 - t_0)} \tag{A4}$$

This procedure was tried at various values of b until a reasonable fit, based on the excess variance at young ages, was obtained. A few trials sufficed. The selected parameters are shown below.

Parameter	Value	Units
a	0.03	Per Year
b	0.17	Per Year
θ_1	4.20×10^{-6}	Per Mile
θ_2	18.76×10^{-6}	Per Mile
t_0, male	18.37	Years
t_0, female	16.02	Years

[20] Recorded age is at the midpoint of the study for people in the middle of the age bracket. It therefore corresponds to $t_1 + 2$.

DISCUSSION BY DALE NELSON

"In any case, I am convinced that God does not play dice."
—A. Einstein, 1926.

In a 1970 paper,[1] Donald Weber presented a stochastic model of the automobile accident process. In that paper, Weber took age and gender differences into account in a deterministic fashion by means of some ad hoc rational functions of time. The present paper, on the other hand, deals with these differences explicitly though a stochastic model, by using a Markov process to describe how an individual's accident likelihood varies over time. The latter approach is much more satisfying since it recognizes that accident likelihoods do vary among individuals with otherwise identical risk characteristics.[2] In the light of current controversies over risk classification, this paper undoubtedly will be an important contribution to the literature, and it is certainly a timely and thought-provoking one.

Emilio Venezian's paper is not a particularly easy paper to read. One's inclination is either to be caught up in the intricacies of the model—and the attendant need to make the underlying assumptions more "realistic"—or to be turned off by the formidable mathematics involved, the latter being a characteristic of stochastic processes generally. My comments will steer clear of either extreme, and instead I will try to consider some of the implications of these kinds of models.

Individual Models

There are two ways that these models can be used to describe the accident process. The first, which is the way Venezian has developed his model, is to use the model to explain the expected behavior of an individual over time. The Poisson model is the most familiar of these models; in its simplest form it assumes that the accident likelihood is constant over time. This obviously is

[1] Donald C. Weber, "A Stochastic Approach to Automobile Compensation," *PCAS* LVII (1970), pp. 27–63.

[2] That is, identical in the sense that there are no clearly identifiable differences between individuals in a particular risk group. See Michael A. Walters, "Risk Classification Standards," *Pricing Property and Casualty Insurance Products*, 1980 Discussion Paper Program, Casualty Actuarial Society.

simplistic, but it does provide a useful model. The Polya model, which Venezian discusses briefly and which leads to one formulation of the negative binomial distribution, assumes that the individual's accident likelihood increases linearly with the number of prior accidents. In contrast, if the likelihood is assumed to decrease, rather than increase, the binomial model results.[3] A variety of other individual models are cited in the Ashton[4] and Seal[5] references.

All of these generalizations of the Poisson model are derived from a basic premise that an individual's future accident likelihood is somehow dependent on his/her prior record. Venezian's model, however, is premised on the assumption that the individual's future likelihood is dependent only on the person's present likelihood. Specifically, individuals are in one of two states—either they are "good" or they are "bad"—at any moment of time, with a fixed probability of moving to the other state at the next moment of time. Fitting this model to some California Driving Record data, the evaluated parameters indicate that the "bad" drivers have an accident likelihood which is about 4.5 times that of the "good" drivers; that there is, roughly, a .17 chance at any instant of time of someone in the "bad" state moving to the "good" state and a .03 chance of someone in the "good" state moving to the "bad" one; and that, under the assumption that everyone starts out in the "bad" state, girls start to move to the "good" state a couple years ahead of the boys. Because of the difference between the two transition probabilities mentioned above, there is a long-term drift toward the "good" state. After several years, you can expect about 85% (.85 = .17/(.03 + .17)) of the individuals to be in the "good" state and 15% to be in the "bad" state at any moment of time. Thus the model, at once, seems to account for the observed differences between males and females and between different age groups, and also allows for some "unexplainable" heterogeneity within these groups.

Or does it? Let's go back to the Markov assumption. It says that the probability of being in the "good" or "bad" state at the next moment of time is dependent only on the individual's current state. In other words, from the point of view of this model, there is no difference between an 18 year old male

[3] Winifred D. Ashton, *The Theory of Road Traffic Flow*, Methuen's Statistical Monograph Series, John Wiley and Sons, Inc., New York, 1966, pp. 148–168.

[4] *Ibid.*

[5] Hilary L. Seal, *Stochastic Theory of a Risk Business*, John Wiley and Sons, Inc., New York, 1969, pp. 12–29.

and a 55 year old female, for example, if it is known that presently both are in the same state (and, of course, are identical in all other respects, i.e., drive the same amount, drive in the same locale, etc.). How many underwriters—or actuaries, for that matter—will accept that conclusion? I know one group—the critics of risk classification—that would readily accept the conclusion, since it seemingly argues against the classification of individuals on the basis of personal characteristics.

Although not usually stated in so many words, underlying any classification plan is a belief that accident likelihoods are definite quantities and that changes in these likelihoods over time occur, if not deterministically, at least in some causal, generally non-random fashion. This is not the case under the author's model. But to the extent a person's accident likelihood is determined stochastically, I'm not sure classification is appropriate. Would you classify individuals as to whether they were more or less likely to have an accident if the mechanism involved were the toss of one of two biased coins and the choice of the coin, in turn, were the result of tossing yet another biased coin?

Aside from this consideration, I have some other difficulties with the model as outlined in the paper; these comments are more along the lines of making the assumptions more realistic. For instance, it is hard for me to visualize how persons can constantly be jumping back and forth between the "good" and "bad" states. This difficulty can be overcome by hypothesizing a larger number of states—perhaps even a continuum—and a more gradual drifting up and down the scale. Also, since a learning curve probably is involved—from both driving experience and driving record—the Markov assumptions perhaps could be relaxed to permit time-varying transition probabilities. However, this still leaves the process as being essentially a stochastic one—that is, one in which an individual's accident likelihood is as much a chance event as whether or not he/she actually has an accident. And, as discussed above, this seems to suggest the inappropriateness of risk classification or underwriting based on individual characteristics.

Cohort Models

There is a way of using the model, however, that seems much more satisfactory, and which overcomes this apparent conflict with risk classification. That would be to regard it as a description of the expected behavior over time of a group of individuals with similar risk characteristics. Thus the model no longer describes the behavior of individuals, but simply provides the expected distribution of these individuals among the "good" and "bad" states at various points in time. With this interpretation, the model says nothing about individual behavior, and in particular it does not imply that the movement between states is stochastic.

I think it is in the context of this interpretation of the model that much of the author's discussion has been written and, in any event, seems to hold.

The time-varying mix of business between "good" and "bad" states clearly implies the need for the fine-tuning of merit rating relativities. In fact, the model suggests that merit rating is of very little value for youthful risks, and that the use of such plans could best be limited to adults. Further, to the extent that underwriting is intended to identify the "good" risks, the model suggests that little would be accomplished by selective underwriting of youthful risks. Where both merit rating and underwriting are useful, the model implies an interesting but not surprising interaction between the two. Namely, the more successful the underwriting effort is at identifying "good" risks, the less important is the role of merit rating, and vice versa.

The validity of these implications is, of course, dependent on the validity of the model. However, as the author suggests, we probably will never be able to determine with any certainty which of several alternative models is the correct one—or even which of several possible explanations or interpretations of a particular model makes the most sense. The random element of the accident process is so large, and the available statistical estimation techniques are so robust, that empirical data "fit" a wide variety of models. For example, as has been frequently pointed out,[6,7] it generally is difficult to distinguish between a two state process such as the author is using and one which includes several (or possibly even a continuum of) states. These practical difficulties become virtually insurmountable as you complicate the models to make them more realistic (e.g., to measure the risk of insured cars, rather than individual operators; to reflect inflation, changes in driving patterns, changes in the types of cars on the road; etc.)

All this suggests that the value of these models is more in the qualitative insights that they can provide, than in any precise, verifiable predictions. As such they are more akin to economic models than to the types of models used in the physical sciences. This also means that to understand or test these models, you cannot rely on comparing predicted results with empirical evidence. You must know the underlying assumptions, and ask whether these make sense. In the present case, this also means that, as actuaries, we probably should know more about stochastic processes—and their mathematical development—than most of us do. The latter is tough going, but stimulating; I suspect the examination syllabus eventually will include a great deal more on these processes, simply as a matter of necessity.

The author is to be congratulated for a thought-provoking and timely contribution.

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[6] *Ibid.*

[7] Robert A. Giambo, "SRI and the Gamma Poisson: A Review of the Stanford Research Institute Report," presented at the Casualty Actuarial Society Risk Classification Conference, March 30–31, 1981. In his discussion, Mr. Giambo presented a simple example showing that the expected accident distribution for a class of "good" and "bad" drivers is, for all practical purposes, indistinguishable from one in which the accident likelihoods have a gamma density. This example and similar ones, which can be constructed easily, also shed light and cast doubt on some of the recent findings regarding class heterogeneity and overlap (e.g., Division of Insurance, Commonwealth of Massachusetts, *Automobile Insurance Risk Classification: Equity and Accuracy*, 1978; Department of Insurance, State of New Jersey, *Final Determination — Analysis and Report. In re: Hearing on Automobile Insurance Classifications and Related Methodologies*, 1981). In particular, these examples suggest that the relative variance of the accident likelihood distribution is not an adequate measurement of homogeneity, and that the degree of overlap between classes is greatly dependent on the assumptions regarding the underlying distribution of accident likelihoods. For instance, while there usually is a good deal of overlap between two different gamma densities, representing the distribution of accident likelihoods for two different classes, there may be little or no overlap if the two classes are each represented by "good-bad" distributions.

Similarly, the homogeneity depicted by a gamma density is vastly different than that represented by a "good-bad" distribution, even though their relative variances may be the same. Or, two different "good-bad" distributions may be equally homogeneous — in the sense of having a similar dispersion of accident likelihoods—yet have different relative variances.

INFLATION IMPLICATIONS FOR PROPERTY/CASUALTY INSURANCE

SUMMARY OF DISCUSSION PAPERS

RICHARD E. MUNRO

To say that inflation has become a way of life for us is a trite and tired expression which I could have left unsaid, but did not. Trite or not, it is a problem with which the property/casualty insurance industry has been contending and one which is not very likely to go away in the foreseeable future.

A well known and often stated fact is that the insurance industry must, in its current pricing activity, deal with *future* inflation to a greater degree than almost any other industry. How to do this without actually "fueling" inflation is a real challenge.

We must, of course, respond to different degrees of prospective inflationary impact on the separate elements of both pay-out and income. As many regulators and consumer advocates suggest, we might even consider the effect that increasing premiums have on policyholders. When, for example, might the insuring public arrive at the impression—hopefully a mistaken one—that private industry can do no better than the federal government in dealing with inflation?

Again, the response to the Committee on Continuing Education's call for papers and reviews has been fruitful and gratifying. You have had the opportunity to review thirteen papers and thirteen reviews which will form the basis for some constructive and stimulating discussions.

Current rating bureau procedures utilize inflation sensitive exposure bases—payroll and receipts. Richard S. Biondi and Kevin B. Thompson's paper reviews these bases from several perspectives.

First, the paper reviews the historical use of current inflation sensitive exposure bases and identifies three criteria that an exposure base must fulfill. The authors present several situations where current bases cause equity problems and highlight the need for underwriting judgment by the use of schedule rating plans, etc. It is interesting to note that several of the inequities discussed were also criticized by the recent United States Commerce Department study of product liability ratemaking. The paper studies the correlation between changes in losses and changes in exposures and discusses the impact on ratemaking of estimating future exposures. In conclusion, the authors discuss the possibility of extending inflation sensitive exposure bases to OL&T.

In review of this paper, Janet R. Nelson points out: "Messrs. Biondi and Thompson have provided us with a paper to initiate broader discussion on this subject. They do not propose answers to any of these questions." She goes on to critique some of the principal flaws in the paper and to point out some of the transition problems that will be encountered in modifying the exposure base.

The purpose of the paper written by Robert P. Butsic is to determine how inflation affects the important element of losses and consequently, how premiums are influenced.

The author begins with a model of a single policy and one pay-out pattern. He develops accident date and payment date inflation models and later combines them geometrically. The analysis shows that when claims costs are related to prices at the time of settlement, incurred losses may rise faster than the inflation rate at the time policies are sold.

Mr. Butsic studies the impact of changing inflation rates on payment patterns, loss reserves and needed income. By directly introducing investment income into the pricing calculation, the paper shows how inflation in claims costs is related to interest rates and how the combination of these two elements influences the competitive price to charge for the policy. The theoretical results of the various models developed are summarized into specific areas of practical application.

Mr. Butsic's paper was reviewed by Rafal J. Balcarek, whose concise and cogent analytical style is well known to all members of the Society. Mr. Balcarek speaks well of the Butsic paper; however, as might be expected, he raises some very real questions relating to the basic premises of Mr. Butsic's paper which are constructive and challenging to additional study in this area. The review adds important perspective to the Butsic paper.

In his paper, Stephen P. D'Arcy presents a model for the development of a method to "immunize" the property/liability insurance industry from inflation.

He presents a comparison of changes in inflation, measured as the December to December change in the CPI, with results from underwriting and investments. The paper analyzes the correlation of inflation with returns from underwriting, long-term bonds, common stocks and Treasury bills.

Mr. D'Arcy concludes that underwriting and investment returns are negatively correlated with inflation. Moreover, Treasury bills are positively correlated with inflation at a significant level. Based on these findings, the author postulates an investment strategy which will work to offset the negative correlation from underwriting with the positive correlation from Treasury bills.

This paper's reviewer, Roger C. Wade, directs his attention to some technical data problems in the paper related to insurance accounting. He goes on to suggest areas that are fruitful for future analysis.

The next paper, written by Robert P. Eramo, provides a description of actions taken by the Federal Reserve Board so less knowledgeable people can understand the impacts of Federal Reserve actions on our economy.

The paper is directed to intelligent lay people in our profession who may not be completely familiar with Federal Reserve actions and operations of the American banking system in general. Before discussing the sweeping October 1979 Federal Reserve Board policy change, Mr. Eramo presents the operations of the central bank, the definitions of money and definitions of price and monetary inflation. He concludes that money, the Federal Reserve system and high price inflation are not laws of nature to which we must resign ourselves. The rules of banking can be changed. This paper will provide some food for thought.

Paul M. Otteson states in his review that this paper is interesting and well worth thought and study. In addition, Paul presents some additional food for thought and raises equally challenging questions regarding the economic forces at work in our society.

Glenn A. Evans and Stanley K. Miyao began their study of the development of an inflation sensitive exposure base for Hospital Professional Liability Insurance in the mid-1970's during a time when loss cost inflation was greatly exceeding rate increase amounts for this line of insurance. Since loss cost inflation has not abated, the authors have studied a possible change in exposure base. The current exposure base of average daily census (average number of occupied beds per year) and outpatient visit counts is not sensitive to inflation. They suggest the use of gross patient revenue would ameliorate the need for large rate level changes and expedite regulatory approval.

To assess the effects of the conversion of the exposure base, Evans and Miyao have performed several calculations. These calculations indicate the degree of difficulty in the base conversion and the impact of the conversion on individual insureds.

In his review, Brian E. Scott points out some practical concerns he has relative to a new exposure base for this line. He also suggests several areas where supporting data or further development of points would have strengthened the paper.

Rates of return from the property and casualty insurance industry have historically been cyclical. Several studies have concluded that a six-year cycle exists. A paper written by Kaye D. James shows that around 1970 these cycles began to increase in amplitude while investment income began to grow. The purpose of this paper is to unravel the complex relationships that underlie the behavior of sources of income.

Ms. James analyzed the cycle to construct a causal model which could be used for predictive purposes. The hypotheses used are that the cycle is caused by changes in the price of insurance, both current and lagged; changes in the cost of insurance; and changes in the opportunities to earn higher investment income. The model developed offered support for the relationship between cycles and price and cost changes, but the direct link between the cycle and investment opportunity was not supported.

The reviewer, David J. Oakden, views this paper as a beginning. He offers a number of areas in which Ms. James's paper falls short, but in fairness he admits the difficulties inherent in the pursuit of causes of underwriting cycles. You will find both Ms. James's paper and Mr. Oakden's review to be thought provoking.

Alan E. Kaliski's paper discusses current automobile trend methodology and attempts to demonstrate that the current methodology is not adequately dealing with volatile and changing inflation rates. By way of example, the author develops a simple hypothetical model to illustrate the understatement of current cost factors when inflation rates vary by year from 5 to 13 percent. Mr. Kaliski suggests a refinement of current procedures to cope with varying inflation rates for current cost adjustments and argues that econometrics might well be the answer for the determination of future claims cost changes.

John B. Conners offers a kind review of the Kaliski paper, obviously with the knowledge that current methodologies for trend projection are woefully inadequate. The paper and the review point out that published literature and current ISO methodology are badly in need of updating.

John Kittel, in his paper, presents a review of current reserving methods for the unallocated loss expense reserve.

Loss adjustment expenses are divided into four basic expense groups—legal, independent adjuster costs, field adjuster costs, and claims department costs of operation. With this division Mr. Kittel studies current approaches and, based on a study of one company's loss department, derives reserving techniques suitable for measuring the expense reserve for each division.

Mr. Kittel concludes that many commonly used methods do not properly account for major factors which influence unallocated loss expense reserves. He suggests that an objective review of the loss department is necessary to determine which factors are significant. Until each significant factor is accounted for in the chosen reserving method, greater precision in estimating reserves cannot be achieved.

Richard Bill critiques Mr. Kittel's paper in an extensive review which in itself is well worth reading. Perhaps most importantly, Mr. Bill acknowledges that Mr. Kittel has brought to our attention a fallacy in the age-old method of establishing unallocated loss expense reserves.

Norton E. Masterson's paper measures over time the effects of inflation on individual lines of insurance as they compare with the Consumer Price Index.

First, the author reclassifies lines of insurance into various categories—personal injuries, autos, dwellings and other property. Mr. Masterson displays cost indexes as they compare to the CPI for factors which affect each of these lines. A separate section is devoted to workers' compensation. Loss adjustment expense indexes are studied as to factors which unduly increase costs. Expense ratio trends are analyzed. Hedges against inflation are studied as well as rate of return indexes for various investment types. An update of Mr. Masterson's earlier work is also given.

In his review, Dr. Richard I. Fein expresses my feelings accurately when he states, "one cannot help but be somewhat awed by the amount of effort and time that is consumed in the preparation and execution of this task." In his review, Dr. Fein presents some background on Index Number Theory and also relates several avenues of future work which offer potential.

William F. Richards wrote his paper to describe an explicit consideration of the amount of inflation loaded into loss reserves. He develops a technique which requires a conscious decision concerning the amount of inflation to be used in establishing loss reserves.

The technique involves removing the impact of inflation from the historical data triangle prior to forecasting the reserve and then replacing the effects of inflation by including an assumption of future inflation as a final step in the process. The author develops the appropriate "flying W" formulas and applies them to a theoretical automobile bodily injury payment triangle. He then tests the reasonability of his model and depicts resulting reserve amounts at various assumed inflation rates.

This 16-page paper was reviewed by Richard G. Woll in a 19-page review. In complete fairness to Mr. Woll, however, the bulk of the review does consist of tables of data. In his review, Mr. Woll acknowledges that the author has provided a useful introduction to an important subject. Mr. Woll's generous use of data and examples points out some of the difficulties involved in adjusting for the effects of inflation. He also suggests refinements which should broaden the utilization of Mr. Richards's methodology to apply under more general conditions.

As we are all quite aware, the carrying of bonds at amortized value versus market value during these inflationary times has resulted in an over-valuation of assets and net worth. In his paper, Martin Rosenberg describes the condition of the 100 largest property/casualty insurers whereby 64 percent of total admitted assets are bonds which represent 270 percent of statutory surplus. A 10 percent decline in market value below amortized value implies a substantial decline in surplus.

This reduction in the market value of insurers' assets led Mr. Rosenberg to inquire as to the optimum portfolio which seeks to maximize net worth in the short run, meets cash needs for claim and expense payments, and minimizes interest for loans to meet cash needs in case of shortfalls. To set the stage for the selection of the portfolio, Mr. Rosenberg sets up a model for investment decisions and cash pay-outs. Based on the timing of the model, he is then able to display how the portfolio would be chosen.

In his review, Donald E. Trudeau comments on the importance of knowing the efficiency level of the asset portfolio. He suggests a data development phase to aid in the evaluation of the efficiency level and provides guidelines for such an approach.

In a paper written by Sheldon Rosenberg and Aaron Halpert, we are provided a model for the adjustment of size of loss distributions for inflation.

They first lay out definitions of cumulative probability distributions to be used in their analysis. An analysis is made of the single uniform trend assumption model whereby each claim size is affected uniformly by inflation. A test of the validity of this model indicates to the authors that the assumption of uniform trend is not correct. A test for Products BI, OL&T BI and Hospital Professional shows the trend increases as the loss size increases. Based on their findings, Mr. Rosenberg and Mr. Halpert search for an alternate model, develop one, and test its accuracy against empirical results. This is a highly technical paper in which the authors provide a new method for several pricing algorithms.

The reviewer, Charles F. Cook, is extremely complimentary of this paper from both a theoretical and a practical standpoint.

Larry D. Shatoff, in his paper, develops a single model to illustrate the effects of changes in the inflation rate on reserves, prices and calendar year loss ratios.

The model assumes in early years that no inflation exists, followed by two years of inflation and then followed by years of no inflation again. Based on an assumed pay-out pattern, loss reserve developments are studied. When the inflation rate accelerates, under-reserving occurs. When the inflation rates decelerate, over-reserving occurs. By reviewing loss trends and estimated ultimate loss dollars, Mr. Shatoff also studies effects on pricing adequacy and measurement of calendar year income. This model clearly identifies the effects of changing inflation rates and hopefully will foster discussions of solutions to this problem.

E. LeRoy Heer, in his review, takes Mr. Shatoff to task for failing to recognize the important element that actuarial judgment should play in loss reserving and ratemaking. I'm sure you'll enjoy this lively exchange of views.

In summary, the papers presented, while all related to the subject of inflation, cover a broad spectrum of related issues. Perhaps most importantly, they are intended as *discussion* papers and form the nucleus of this week's program. I urge each of you to support this program with your attendance and active involvement in the discussion sessions.

A NOTE ON BASIC LIMITS TREND FACTORS

ROBERT J. FINGER

VOLUME LXIII

DISCUSSION BY JOHN P. ROBERTSON

Mr. Finger's paper makes the excellent point that the frequently noted fact that excess losses tend to rise faster than overall loss costs has a converse, namely, that basic limits costs tend to rise more slowly than overall loss costs. He then proceeds to give a method for modeling these changes and gives examples using the lognormal distribution.

There is one point, however, that needs to be clarified before the methods of the paper are used. This is that the *ART* computed by the method of the Appendix is not quite the same as the *ART* defined on page 109:

$$ART(R,i) = \frac{1}{i} \frac{(1 + i) \cdot X(R/(1 + i)) - X(R)}{X(R)}$$

The *ART* defined on page 109 is a "linear" *ART*. That is, if the factor for the total limits cost change (*TLCC*) is $1 + i$, then the factor for the basic limits cost change (*BLCC*) is given by $BLCC = 1 + ART \cdot i$. To see that this *ART* is not the one computed in the Appendix, consider the following example.

Start with a lognormal loss distribution with $CV = 0.4$ and ratio of basic limits to the total mean of 10. Then let total costs change by a factor of 100. This then makes the ratio of the basic limit to the total mean become 0.1. Since the $CV = 0.4$, there are very few claims that exceed ten times the mean or fall below one tenth of the mean. Thus, at first, one was paying practically the total amount of all claims since very few claims exceeded the basic limit. After the cost change, one pays one tenth of the total amount of losses, since one pays the basic limit on almost all claims and the basic limit is one tenth the mean claim cost. Basic limits costs have, therefore, increased by a factor of 10 $(=(100M \cdot 0.1) \div (M \cdot 1))$. The "linear" *ART* then satisfies $1 + ART \cdot 99 = 10$, or $ART = 1/11 = 0.09$. This can also be obtained by taking $(1/99) \cdot (100 \cdot (0.1) - 1) \div (1)$ per the second formula on page 109 ($X(10) = 1$, $X(0.1) = 0.1$, $i = 99$, $1 + i = 100$).

On the other hand, using the method of the Appendix, one computes the ART to be $(2.297 - 0) \div (2.303 - (-2.303)) = 0.50$. What is this ART?

I have learned through correspondence with Mr. Finger that the ART of the Appendix is an "exponential" one and satisfies $(TLCC)^{ART} = BLCC$. Note that $(100)^{0.50} = 10$. Call the linear one ART_L and the exponential one ART_E. Then $(1 + i)^{ART_E} = 1 + ART_L \cdot i = BLCC$, where $1 + i = TLCC$. Thus, given either ART, the other can be computed.

Regardless of ART used, we have

$$BLCC = \frac{TLCC \cdot X(R/TLCC)}{X(R)}$$

That is, the basic limits cost change is the total limits cost change times the change in the percentage of losses that are below the basic limit. R is the ratio of the basic limit to the unlimited mean before the cost change. This holds regardless of the form of the size of loss distribution.

I determined that Table II of the Appendix shows $\ln(R/X(R))$ and $\ln(R)$ for various R and CV for lognormal distributions. These can then be used to compute the ART_E by the method cited, since

$$ART_E = \frac{\ln(R/X(R)) - \ln((R/TLCC)/X(R/TLCC))}{\ln(R) - \ln(R/TLCC))} \quad \text{(By the rules given in the appendix)}$$

$$= \frac{\ln(TLCC \cdot X(R/TLCC)/X(R))}{\ln(TLCC)}$$

$$= \log_{TLCC}(TLCC \cdot X(R/TLCC)/X(R))$$

This implies $TLCC^{ART_E} = \dfrac{TLCC \cdot X(R/TLCC)}{X(R)} = BLCC$

which is the required relationship.

Table II should be labeled as showing

$$\int_{-\infty}^{\ln A} f(e^z)dz$$

This can be shown to be exactly equal to $\ln(A/X(A))$. The proof of this equivalence does not depend on the properties of the lognormal, but rather applies generally to all distributions. I will be happy to send this proof to anyone who requests it.

In order to follow example 1, it is useful to note that if $1 + i$ is the average *annual* total cost change, over n years, with *overall* total cost change $(1 + i)^n$, then the overall basic limit cost change is $(1 + i)^{nART_E}$. This gives an *annual* basic limits cost change of $(1 + i)^{ART_E}$ which is approximately $1 + ART_E \cdot i$. This is the reasoning that allows the ART_E to be applied to annual cost changes instead of overall cost changes.

Another item deserving of mention is the author's definition of B as a function of one variable, i.e. as $B(A/M)$ on page 107. This reviewer finds a definition of B as a two variable function, $B(A,M)$, more reasonable. The definition of B as a one variable function obscures the relationship $B(A,(1 + i)M) = (1 + i)B(A/(1 + i),M)$ which is needed for the derivation of the formula for ART_L on pages 108 and 109. By noting that $X(R) = B(RM,M)/T(M)$ and that this definition of $X(R)$ does not depend on the choice of M, the author's proof follows.

In summary, Mr. Finger has provided a mechanism for comparing basic limits cost changes to total limits cost changes. He points out that such changes can be modeled with the lognormal distribution, and in many cases it is possible to obtain useful results from such a model even when the shape of the lognormal cannot be determined exactly. This reviewer hopes that clarification of the above technical detail will help readers understand Mr. Finger's paper.

ESTIMATING CASUALTY INSURANCE LOSS AMOUNT DISTRIBUTIONS

GARY PATRIK

VOLUME LXVII

DISCUSSION BY STEPHEN PHILBRICK AND JEROME JURSCHAK

Gary Patrik has written a paper which is significant from several points of view. It provides:

- a well-conceived methodology for selecting a model for an empirical loss amount distribution;
- thoughtful remarks suggesting more than usual intuitive familiarity with the subject matter;
- a synthesis of a large body of existing literature interpreted to speak directly to the concerns of the actuary.

Mr. Patrik has discussed a number of reasons for seeking models for loss amount distributions. Successful model building requires a level of abstraction and understanding which goes beyond the mere analysis of data. Useful models have typically isolated those factors of marginal importance—the less cluttered the model, the more easily it can be communicated and the more likely cross-fertilization with other disciplines can be accomplished.

Since the K-S statistic is distribution-free and takes into account the natural ordering of the sample, it is a particularly useful goodness-of-fit test. However, the author states that it may be *too* powerful for certain actuarial considerations, since it has rejected (at the 5% level) all probability models yet tried. This observation is certainly not unexpected, partly for the reasons suggested by the author, but also due to clustering.

As practitioners, more than theoreticians, we know that real data rarely conform to the ideal. A number of arguments can be given as to why loss amounts cluster about particular levels. This observation is found frequently enough to be considered something of a norm for certain classes of business. This fact is a powerful argument in support of a statistical test which is less powerful than the K-S test. The χ^2 test, for example, is simple to use and easily communicated. While the choice of intervals is subject to manipulation, this liability can be an asset when dealing with the question of clustering. However, one must be careful, in any situation allowing manipulation, that adjustments which improve the fit can be realistically defended.

Another alternative is to modify the rejection percentile. Based on the expected discrepancies from the ideal, perhaps the K-S statistic should be used at other significance levels. In any case, the p-value (the smallest value of α for which the hypothesis would have been rejected) should be stated, thereby permitting different conclusions to investigators with differing qualitative assessments of the data itself.

It is also important to note a difference between the application of the K-S test to the simulated data in Appendix C and the application to the OL&T data. The simulated data consist of individual points, whereas the OL&T data is grouped (classified). Hoel (*Introduction to Mathematical Statistics*, p. 326) points out: " . . . the test then is no longer an exact one because the maximum difference for classified and unclassified data may not be the same; however, the discrepancy is usually slight if the classification is not too coarse." In the case of the OL&T data in Appendix E, Part 3, the first interval contains 41% of the data points. This is probably too coarse. However, if the point of the test is to compare the K-S statistic for competing distributions this may not be a problem.

One of the author's main conclusions is that the method of maximum likelihood should be used to estimate the parameters of the particular model. Although we agree with this conclusion, two points need to be stressed.

1. It must be recognized that comparison of method-of-moments estimates and the MLE estimates in Table 5.1 and Table 5.2 are not on the same basis. The MLE estimates are derived under the assumption that losses are censored. The method-of-moments calculations ignore this assumption. Hence, it is not surprising that the method-of-moments estimates are so poor. The author recognizes this fact, since he later states: ". . . we could compute correct method-of-moments estimates accounting for the policy limit censorship. But the equations that must be solved are much more complicated than the general equation (5.5)."
2. The maximum likelihood estimates for the parameters of the normal distribution are the sample mean and sample variance (Fraser, *Statistics—An Introduction*, p. 226). Hence, the MLE estimates are equivalent to the method-of-moments estimates. It then also follows that the method-of-moments applied to the logs of claim sizes (Method-of-Moments II

in Table 5.2 *if* it had been applied to the unlimited data) should be equivalent to the MLE estimates for the lognormal distribution. Note that this applies only to the unlimited distribution, not to a censored distribution.

The EVC test suggested by Mr. Patrik can be a very useful one. After all, as we are reminded, the expected value of loss is the most important component of most insurance premiums. One suggestion which would have the effect of making the computation of the vector statistic

$$\left\{\frac{G(x_i|\theta) - G_n(x_i)}{G(x_i|\theta)}\right\}$$

less cumbersome, and which would recognize the importance of policy limits, would be evaluation of the alternative statistic

$$\left\{\frac{G(P_i|\theta) - G_S(P_i)}{G(x_i|\theta)}\right\}$$

where the P_i, $i = 1, \ldots, L$ are some of the commonly used policy limits or retentions (such as \$100,000, \$250,000) and $G_S(P_i)$ is the sample average with censor P_i. It is the expected value of loss at policy limits which is a premier consideration.

Before ending with some comments on the use of the Pareto distribution, a few additional points will be discussed.

1. Our experience indicates that failure to modify data for trend and development before solving for the maximum likelihood estimates can produce future loss estimates differing significantly from those obtained with adjusted data. To the extent that IBNR losses tend to be larger than average, this would *partially* account for the observation that the data has too many small losses. However, note that even adjusting the individual claims for case development will not solve this problem. We suggest that unadjusted data be used for illustrative purposes only.
2. The author notes that the method-of-moments technique forces the value of δ to be greater than 2. This is a problem since typical values of δ are often less than 2. It should be noted that the single parameter Pareto with distribution function

   $$F(x) = 1 - x^{-\delta} \quad \delta > 1, x \geq 1$$

   has a less severe restriction, namely that $\delta > 1$. However, our experience generally indicates that the single parameter Pareto should be restricted to fitting excess losses, where the truncation point is approximately $10,000 or more.

3. In Table 6.1, the author fits the Pareto to the overall distribution and to the excess portion. The estimate of p for MLE I is .95 as shown in the table. It may be of interest to note that the value of p implied by MLE II, namely F(8000/347, .877) is equal to .9385. Although MLE II produces poor estimates of tail probabilities, it does a reasonably good job of estimating the proportion of losses less than the truncation point.
4. In Section VI, the author states that it is "convenient" to specify t (the truncation point at which the distribution splits into two distinct pieces) so that it is not an unknown. It should be pointed out that the choice of t is not an innocent one—different values of t can produce model estimates of tail probabilities which are quite different.

The final part of this review will deal directly with the Pareto distribution as a model for loss amount. While Mr. Patrik does not specifically advocate its use in any particular situation, he does state that both the ISO Increased Limits Subcommittee and he personally have found the two parameter Pareto very useful. The authors of this review have used the Pareto distribution to model large property and casualty losses in a wide range of circumstances including estimating property damage losses at large petrochemical complexes, forecasting corporate casualty losses excess of various self-insured retentions, pricing working cover excess of loss reinsurance, and establishing contributions to hospital trusts which serve to fund hospital professional liability losses. The particular model we have used is the single parameter distribution mentioned above.

In choosing to use almost exclusively the single parameter distribution, we have been guided by two considerations. First, its analytical form is simple enough to make the MLE parameter estimation routine ($\delta = n/\Sigma \ln x_i$) and to make accessible answers to such questions as sensitivity of forecast results to parameter value, the relationship between sample size and confidence in the parameter estimate, and the comparative impact on forecast losses of using unlimited, truncated, and censored distributions. Second, a single parameter gives a good fit to a variety of empirical data. For example, when fitting a one parameter Pareto distribution to the censored data in Appendix E, Part 1, the EVC statistic has components which range in magnitude from -6.04% to 2.72% (versus -5.60% to 1.71% for the two parameter model). This type of variation is small when compared to that inherent in the sampling distribution of δ itself. However, as mentioned earlier, the single parameter Pareto is generally appropriate only for excess losses.

Finally, we would like to discuss several areas in which additional research would be helpful.

1. Although the methodology for the calculation of MLE estimates of parameters should be well within the grasp of all actuaries, it might be the case that relatively few would spend the time necessary to pursue this concept. Is it possible that there are alternative methods which may sacrifice a little accuracy for a large savings in time and computation? For example, equating the 5th and 95th percentiles of the simulated data in Appendix C to the corresponding theoretical percentiles of the theoretical two parameter distribution and solving the resulting two equations yields parameter estimates $\beta = 28{,}339$ and $\delta = 1.623$ which in this case compare favorably to the actual values, as the probabilities that X is greater than 100,000 or 1,000,000 are .086 and .003 respectively. (See Quandt (1966) for additional discussion of this method.)
2. Suppose the estimates of parameters for a large set of data are calculated and also those for a small subset. For example, let the large set be all hospitals and the subset be a single hospital. Is it reasonable to derive parameter estimates for the single hospital by credibility weighting the two sets of parameters? If so, how does one determine the credibilities?
3. If parameters are estimated for various accident years, the values of the parameters will differ. To what extent can real changes in the shape of the distribution be measured by the changes in the parameter values? Equivalently, how sensitive are the parameter values to various sets of losses?
4. Can the concept of order statistics be used to draw inferences about the shape of the tail? For example, the expected largest loss from a finite sample generated by a Pareto distribution generally, in our experience, exceeds the greatest sample value. This may imply that the tail is too "thick," or possibly that a truncated (from above) Pareto is a better descriptor of reality.
5. In our experience, we have found that we can get reasonably good fits to loss data in excess of $25,000 with a one parameter Pareto (occasionally we split the distribution into two or more parts and estimate a sequence of parameters for a sequence of censored Pareto distributions). Although it is clear that two (or more) parameters are necessary to fit the distribution from ground zero, is it necessary for the distribution to have such a wide range? In many cases, an estimate of aggregate losses below some value will suffice; in other cases a different distribution may be a better choice for small losses. It may sound more complex to have two

distributions, one for losses up to a truncation point t, and another for losses in excess of t, but in fact the estimation of parameters may be easier.

Finally, we would like to make it clear that we do not advocate abandonment of a two parameter Pareto model. Anyone with the computer procedures for this distribution will certainly get good use out of them. We are merely suggesting to those without such techniques already developed, that there may be several suitable alternatives.

The following is a short extension to the bibliography in Mr. Patrik's paper.

Benktander, G. (1961). "On the Correlation in Results from Different Layers in Excess Reinsurance," *Astin Colloquium* 1961, pp. 203–209.

Haung, J. S. (1975). "A Note on Order Statistics from Pareto Distribution," *Scandinavian Actuarial Journal*, pp. 187–190.

Lwin, Thaung (1974). "Empirical Bayes Approach to Statistical Estimation in the Paretian Law," *Scandinavian Actuarial Journal*, pp. 221–236.

Malik, H. J. (1970). "Estimation of the Parameters of Pareto Distribution," *Metrika*, Vol. 15, pp. 126–132.

Quandt, R. E. (1966). "Old and New Methods of Estimation and the Pareto Distribution," *Metrika*, Vol. 10, pp. 55–82.

A METHOD FOR SETTING RETRO RESERVES

CHARLES H. BERRY

VOLUME LXVII

DISCUSSION BY ROY K. MORELL

All actuaries, I believe, would agree that a reserve formula of any kind which eliminates the need for actuarial judgment will never be devised. The author of this paper, I am quite sure, would be the first to agree. Nevertheless, this does not stop actuaries from developing formulae and procedures to help us with our work. This is because a formula helps us to organize our thoughts and exercise our judgment in an orderly manner. The procedures and formulae in this paper are no exception.

Before I attempt to make some constructive comments on the paper, I want to emphasize that I think the paper is a valuable addition to our literature. The method proposed for setting a reserve for retrospective rating adjustments is theoretically sound, easy to understand and apply, and very practical. It is a significant improvement over the method described in an earlier CAS paper. The remainder of this review will begin by raising some rather theoretical questions and then turn to some practical comments on the use and construction of the *DR1* and *DR2* formulae contained in the paper.

Theoretical Considerations

In discussing the relationship between deviation ratio and loss ratio, the paper states that this relationship is not perfect. This is in reference to the graph of these ratios using policy years 1967–72 (Exhibit I), in which the points, although highly correlated, do not all lie perfectly on the line. Is it possible that each of these points does lie perfectly on a line which describes the relationship between deviation ratio and loss ratio for the group of policies or set of circumstances which existed for that policy year? In other words, is it the subtle differences between the components and conditions of the various years which cause the points not to all lie on the same line? It is possible that the *DR1* formula is perfect but unknowable for a given year. What this suggests is that, prior to graphing, the points should be adjusted for any known differences of significance between the years. From a practical point of view, such adjustments would be very difficult, if not impossible, to make. The procedure suggested in the paper is reasonable and should generally provide an excellent estimate of the relationship between deviation ratio and loss ratio for future years.

Although possibly of theoretical value only, consideration should also be given to the perfect method of setting a reserve for retrospective rating adjustments. By this I mean a procedure which establishes a retrospective reserve for each individual account. Such a method could explicitly recognize all the individual characteristics of the policies which make up the group of policies for which a reserve is being set. Since any method which develops a reserve for a group is unavoidably imperfect, such an ideal system at least deserves mention. Obviously such a system would have many practical and some theoretical obstacles to it. However, given the computer technology available, these obstacles may be overcome.

The DRI Formula

When developing the *DRI* formula values, there are several pitfalls for which one must be alert when using the proposed method. The hazards are those changes in the conditions or characteristics of the policies which will affect the relationship between deviation ratio and loss ratio. Recognizing and adjusting for these changes requires the use of actuarial judgment. It will be sufficient to merely list some of the changes which will affect the *DRI* formula:

1. A major change in expense program, such as the one introduced by the National Council on Compensation Insurance in 1980.
2. A major change in the distribution of policies written by premium size.
3. A change in the distribution of loss limits purchased which would affect the percentage of total losses eliminated.
4. A major change in the distribution of minimum or maximum premium ratios purchased.
5. A sudden and significant change in Table M values used to determine insurance charge.
6. A change in the distribution or interaction of three year plans versus one year plans.
7. A change in the distribution or interaction of multi-line plans versus monoline plans.

One other comment on the *DRI* formula is appropriate. I think it is correct to set a minimum deviation ratio for the same reasons that a maximum is needed. Loss ratios on occasion can be extremely low and it is common practice to have a minimum premium for a retrospective policy.

The DR2 Formula

If earned standard premium (*ESP*) were being booked perfectly, one would expect to have less than 100% of the policy year *ESP* booked at any time prior to the end of the 24th month. Thus if the deviation projection factor (*DPF*) does in fact represent the reciprocal of the portion of first adjustments paid at a given time, it should be reduced at months 21, 22 and 23 to reflect the portion of total *ESP* earned at that point in time. For example, if at 21 months only 95% of the total *ESP* is earned and one sixth of all first adjustments are paid, then the *DPF* should be 5.70 (6.00 × 0.95) rather than 6.00.

The author is technically correct to include both a deviation projection factor (*DPF*) and a loss projection factor (*LPF*) in his *DR2* formula. However, a simpler and equally effective formula, for most situations, would be one which combines the *DPF* and *LPF* into a single *DPF* which would project deviations paid-to-date to ultimate deviations paid. This combined *DPF* would be greater than unity for early months and become less than unity around the 28th month for most companies. This simplification should be considered for use by those companies with very consistent patterns of paid deviation development.

For those who choose to retain the *LPF*, it should be pointed out that this factor relies heavily on consistent reserving by the claims department as well as a consistency in the emergence of late reported cases. For this reason, it deserves not only an annual retrospective review, but a review in prospective terms as well.

As in the case of the *DR1* formula, there are some changes to be aware of when developing the *DR2* formula values. One must be watchful for such changes and make appropriate adjustments when necessary. These are some of the changes which will affect the *DR2* formula values:

1. A change in the rate at which adjustments are processed.
2. A change in the use or magnitude of retrospective rating development factors.
3. A significant change in the distribution of policy anniversary dates.

A comment on the weight (*W2*) used to combine the *DR1* and *DR2* indications is also appropriate. The paper has chosen a weight which increases linearly between 21 and 60 months. However, the value and accuracy of the *DR2* indication increases very rapidly at first and then at a more gradual rate during the later months. Certainly by 33 months, when nearly all first adjustments have been processed, the *DR2* indication deserves an equal weight with *DR1*.

Conclusion

In conclusion, the author is to be commended for a well written paper of practical value. Despite some potential hazards in determining the formula values, the method outlined is technically sound and very reasonable. It does need to be emphasized that the values derived in the paper are only appropriate for the company whose data was used in the analysis. However, for those companies with their retrospective rating data separately available, the procedures outlined are easy to implement and will result in sound reserves.

IMPLICATIONS OF SALES AS AN EXPOSURE BASE FOR PRODUCTS LIABILITY

STEPHEN W. PHILBRICK

VOLUME LXVII

DISCUSSION BY ALBERT J. BEER

Mr. Philbrick's paper on sales as an exposure base for products liability represents a significant contribution to the *Proceedings* as a quantification of what heretofore had been held as a relatively subjective underwriting criterion. I found his presentation particularly interesting in the manner in which he demonstrated the problem with an illustrative example. While some readers may have felt the initial assumptions were oversimplified, I would disagree. Before a problem can be solved, it must be identified. All too often authors proceed immediately into a case study involving a number of complexities which tend to obscure the characteristics of the variable under investigation. The initial portion of this paper could be used by any number of underwriters, risk managers or interested insureds as a primer on the analysis of the amount of products that are currently in the stream of commerce. The remaining portion of the paper is well suited to the actuary or student who wishes to go beyond the initial assumptions and test the sensitivity of the various factors in the author's model. From a pedagogical point of view, I think the gradual introduction of complicating variables allows the reader to appreciate the role each concept plays in the total picture.

While I feel Mr. Philbrick did a fine job in analyzing the effect of "inventoried" sales on the "true" exposure, I must admit I was surprised that there was no mention of what may be an equally serious implication of sales as an exposure base. The author quotes Dorweiler where he states that a "good" exposure medium should satisfy at least two criteria:

1. The magnitude of the medium should vary with the hazard.
2. The medium should be practical and preferably already in use.

While the second criterion is certainly satisfied by sales, I question whether increased sales are, ipso facto, indicative of increased hazard. Many manufacturers of high-technology products spend a significant amount of funds on research and development. In addition, it is not uncommon for producers of manually operated equipment (e.g., snowblowers, drill presses, etc.) to design safety mechanisms which exceed governmental requirements or industry norms.

These additional costs are generally passed on to the purchaser in the form of higher prices. This may lead to an inequity in rating if one relies solely on sales as a measure of exposure. As an illustration, consider two manufacturers who produce items (*A* and *B*) in the same products rating classification. Item *A* is produced as cheaply as possible while item B has undergone rigorous testing and is equipped with a number of supplemental safety features. It is entirely possible that item *B* may have a sales price twice that of item *A* while it may represent only one-half the frequency and/or severity hazard. Using standard manual rating techniques, the premium arising from item *B* would be twice that of *A* while the pure premium ratio of *B* to *A* should actually be 1:2. In my admittedly extreme example not only does the medium (sales) fail to increase with the hazard but, in fact, they are inversely related. While experience rating should eventually reflect these differences, the inequities in the early years are never acknowledged.

The use of sales as a common exposure base within a classification is equivalent to assuming an average fixed price for each similar product. For example, $2,000 of lawnmower sales are assumed to represent the same exposure, regardless of manufacturer (e.g., ten mowers at an average price of $200). In reality, $2,000 in sales may represent anywhere from five very safe mowers to twenty hazardous pieces of equipment. The danger implicit in the assumption of an average price is discussed in another context when Mr. Philbrick discusses the growth patterns *g* in his computation of *v*: ". . . whenever growth patterns of a firm differ from those of the total industry, sales may *not* be a good measure of exposure." I believe the same conclusion is valid when the price per item for a firm differs from the industry average.

A common approach used today to price certain "a" rated risks is to measure the *number of units* manufactured and in the stream of commerce. While this concept helps reduce some of the inequity of a sales exposure base, it does not completely eliminate all bias. From a practical point of view, I would not advise a complete conversion to "number of units" as a new exposure base since the marginal improvement in accuracy may not compensate for the loss of sales as an inflation-sensitive exposure base.

The growing importance of the large commercial accounts and the concern for the financial stability of recently formed captives make it imperative that individual modifications from industry averages result in adequate yet competitive rates. Formal recognition of such pertinent characteristics as the concentration of products in the stream of commerce, which Mr. Philbrick discusses, or any number of other underwriting criteria will improve the art of rating and benefit both the insurer and the insured.

AN ANALYSIS OF RETROSPECTIVE RATING

GLENN G. MEYERS

VOLUME LXVII

DISCUSSION BY MARK E. FIEBRINK

In the opening of his paper "An Analysis of Retrospective Rating," Glenn Meyers asks the following question:

> "Should the present retrospective rating formula be modified to account for the claim severity distribution for the risk being insured, and for the loss limit chosen for the plan?"[1]

People experienced in pricing large casualty accounts are aware of these problems with the current rating formula. Reacting to competitive pressures, they are turning to the actuary and are no longer asking *should* the formula be changed, but *how* can it be changed to more equitably price the risk involved. These competitive pressures from within the industry and from outside of the industry in the form of self-insurance make this paper a timely and important contribution to our Society's literature.

Meyers begins by selecting three claim severity distributions reflecting low, medium, and high severity insureds. These hypothetical distributions are combined with a Poisson frequency distribution to demonstrate how our present retrospective rating formula fails to react to the differences in the severity distributions and, how it also can overcharge when loss limitations are included in the plan. Using several sets of retrospective rating plan parameters, he quantifies the retrospective rating premium adequacy for the three underlying severity distributions, with and without loss limitations. The author has found that with the proper excess loss premium factors, the remaining insurance charges are approximately equal regardless of the underlying severity distribution.

In the last part of his paper, the author discusses several possible changes to the retrospective rating formula. The alternative I believe holds the most promise is to generate a number of limited insurance charge tables to be used in conjunction with the full excess loss premium factors. For practical reasons,

[1] Glenn G. Meyers, "An Analysis of Retrospective Rating," *PCAS* LXVII, 1980, p. 110.

he suggests that the industry restrict the number of loss limitations to be offered. It might even be necessary to mandate one loss limitation if adverse selection causes the excess loss premium factors to be significantly inadequate.

Before addressing specific points in the paper, it is necessary to define some terms used throughout this discussion. The term "net insurance charge" refers to the provision built into the basic premium factor to collect the cost of limiting the retrospective premium to the maximum or minimum premium. The net insurance charge is equal to:

(Charge $-$ Savings) $\times$ (Permissible Loss Ratio) $\times$ (Loss Conversion Factor)

In a retrospective rating plan with a loss limitation, the net insurance charge includes a provision for limiting the losses per occurrence. The term "limited insurance charge" refers to the difference between the net insurance charge and the loss limitation charge [(Excess Loss Premium Factor) $\times$ (Loss Conversion Factor)].

Glenn Meyers' conclusion that the limited insurance charges are nearly equal for a given retrospective rating plan regardless of the underlying severity distribution is quite noteworthy. In an attempt to independently confirm this conclusion, I performed a similar hypothetical analysis for a \$250,000 policy. I based my work on a Poisson frequency distribution and a log-normal severity distribution. The mean of the medium severity distribution was varied by 50% to generate the low and high severity distributions.

Output from this exercise is displayed in Exhibits I and II. Exhibit I shows the resulting excess loss premium factors. Exhibit II displays the limited insurance charges by severity distribution for the retrospective rating plans selected by Meyers.

This simulated data only partially confirms the author's conclusion. As expected, the limited insurance charges for a given plan are of the same magnitude because fixed charges have been substituted for the most volatile parts of the severity distributions. However, the absolute value of the limited insurance charges generally increases with average severity in this data. This pattern is not as apparent in Exhibit XI in Meyers' paper.

The author also briefly discusses the impact of using a Poisson frequency distribution. While choosing the Poisson distribution because of its widespread use in actuarial literature, he does speculate that the conclusions he reached should hold even if some other distribution were used for the frequency. Exhibit

III displays net insurance charges for a \$250,000 policy using a Poisson frequency distribution and a negative binomial frequency distribution with a coefficient of variation (σ/μ) of .70. This selection for the coefficient of variation was not based on an empirical study, but was chosen to contrast the Poisson and negative binomial distributions. Higher insurance charges are generated with the negative binomial since the variance is significantly larger than the mean. In the Poisson distribution, the variance equals the mean.

Exhibit IV shows the limited insurance charges by severity distribution generated with the negative binomial frequency distribution. Note that the conclusion that these values are of the same magnitude regardless of the underlying severity distribution still holds. However, these limited insurance charges are greater than their counterparts generated with a Poisson frequency distribution. This stems from the fact that these limited insurance charges are the difference between the net insurance charges and the loss limitation charges. While the net insurance charges vary with the frequency distribution, the loss limitation charges do not, since the excess loss premium factor is a function of the underlying severity distribution only.

Note that the pattern of movement of the limited insurance charges in Exhibit IV is just the opposite of the pattern in Exhibit II. I do not attach any significance to this since the same coefficient of variation was used for the negative binomial frequency distribution in conjunction with each level of severity. One may argue, however, that the coefficient of variation should decrease with decreasing average severity since the expected number of claims increases to achieve \$150,000 (\$250,000 × .600) of expected losses. The question of the proper frequency distribution to employ should be investigated with actual data.

It appears that Glenn Meyers has gone to a great deal of work in calculating net insurance charges by setting the "retrospective premium adequacy" variable equal to 1.0 and using the Modified Regula Falsi method. However, given insurance charge information by entry ratio, one can solve for the net insurance charge in more traditional fashion.[2] Central to solving for the net insurance charge is the fact that retrospective rating is designed on the average to return the premium discount. Keeping this in mind, one can investigate a host of questions regarding the adequacy of premium generated under retrospective rating plans. The Appendix briefly discusses a few such questions related to the overlap of net insurance charges and loss limitation charges in the current formula.

[2] For two methods, see *Rating Supplement for Workers Compensation and Employers Liability Insurance Retrospective Rating Plan D*, issued by National Council on Compensation Insurance.

Following are two comments for the reader regarding the definition of the basic premium factor found in the paper. The basic premium factor, b, is defined as follows:

$$b = a + (c \times i)$$

The factor "a" provides for the "acquisition expenses, general underwriting expenses and profit." The loss conversion factor is represented by "c" and "i" stands for the "insurance charge." Thus, the insurance charge does not include the application of the loss conversion factor. The reader should be aware that definitions of insurance charge usually include the application of the loss conversion factor, contrary to the definition in the body of the paper. Keeping this in mind may help avoid some confusion.

The second comment concerns the definition of the expense portion of the basic premium factor as the provision for expenses other than loss adjustment expenses and taxes. This definition is true only if the selected loss conversion factor is equal to the ratio of losses plus loss adjustment expenses to losses contemplated in the expense table being used. While this definition may be useful as an educational tool for introducing the concept of retrospective rating, it doesn't lead to an appreciation of the flexibility available in the retrospective rating plan D through the interaction of the basic premium factor and the loss conversion factor.

Note that the author's suggested approach to adjusting insurance charge calculations fundamentally differs from the approach explored by Frank Harwayne and David Skurnick.[3] Whereas Harwayne and Skurnick propose the addition of an incremental charge to the Table M charge when a per accident limitation is imposed, Meyers proposes employing a modified insurance charge in addition to the loss limitation charge. In other words, Harwayne and Skurnick propose keeping Table M intact while Meyers proposes keeping the excess loss premium factor intact.

I favor Glenn Meyers' approach. In both approaches, the excess loss premium factor is assumed to be correct for the risk in question. Similarly, the Table M charge is acknowledged to be wrong when used in conjunction with a loss limitation. It therefore makes more sense to retain the excess loss premium factor and modify the insurance charge in attempting to avoid the "ruinous tide of paper."

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[3] See the discussions by Frank Harwayne and David Skurnick, *PCAS* LXII, 1975, p. 16.

I hope this paper will convince the reader that further investigations in this area with actual data are warranted. Although modification of the current retrospective rating plan formula will be expensive and time consuming, the resulting increase in pricing equity should be worth the investment. The industry has turned to our profession for some solutions to the problem. It is our responsibility to follow through on leads such as the one presented in this paper.

EXHIBIT I

Excess Loss Premium Factors by Severity Distribution

Loss Limitation	Severity Distribution Low	Severity Distribution Medium	Severity Distribution High
$ 10,000	.173	.304	.376
15,000	.128	.242	.310
20,000	.111	.215	.279
25,000	.097	.193	.253
30,000	.085	.173	.230
35,000	.075	.156	.211
40,000	.068	.143	.195
50,000	.056	.119	.169
75,000	.039	.089	.127
100,000	.029	.069	.101
150,000	.018	.047	.070
200,000	.013	.034	.052
250,000	.010	.026	.040
300,000	.007	.020	.032
500,000	.003	.009	.014
1,000,000	.000	.001	.002

Permissible Loss Ratio = .600

EXHIBIT II

LIMITED INSURANCE CHARGES BY SEVERITY DISTRIBUTION

Standard Premium = $250,000
Loss Limitation = $50,000
Poisson Frequency Distribution

Minimum Premium	Maximum Premium	Limited Insurance Charge: Low Severity	Medium Severity	High Severity
BxTM	1.00	.037	.053	.062
BxTM	1.20	.008	.012	.016
BxTM	1.40	.001	.003	.004
BxTM	1.60	.000	.000	.002
BxTM	1.80	.000	.000	.000
.60	1.00	.033	.046	.054
.60	1.20	.002	.000	.000
.60	1.40	−.004	−.009	−.013
.60	1.60	−.005	−.012	−.017
.60	1.80	−.005	−.012	−.017

Permissible Loss Ratio = .600
Loss Conversion Factor = 1.125
Tax Multiplier = 1.040

EXHIBIT III

NET INSURANCE CHARGES BY FREQUENCY DISTRIBUTION

Standard Premium = $250,000
Loss Limitation = $50,000
Medium Severity Risk

Minimum Premium	Maximum Premium	Net Insurance Charge: Poisson Frequency	Net Insurance Charge: Negative Binomial Frequency*
BxTM	1.00	.187	.298
BxTM	1.20	.146	.217
BxTM	1.40	.137	.180
BxTM	1.60	.134	.161
BxTM	1.80	.134	.151
.60	1.00	.180	.282
.60	1.20	.134	.167
.60	1.40	.125	.119
.60	1.60	.122	.092
.60	1.80	.122	.078

Permissible Loss Ratio = .600
Loss Conversion Factor = 1.125
Tax Multiplier = 1.040

* Coefficient of variation (σ/μ) = .70

EXHIBIT IV

COMPARISON OF LIMITED INSURANCE CHARGES BY SEVERITY DISTRIBUTION

Standard Premium = $250,000
Loss Limitation = $50,000
Negative Binomial Frequency Distribution*

Minimum Premium	Maximum Premium	Limited Insurance Charge		
		Low Severity	Medium Severity	High Severity
BxTM	1.00	.179	.164	.140
BxTM	1.20	.088	.083	.069
BxTM	1.40	.048	.046	.036
BxTM	1.60	.027	.027	.019
BxTM	1.80	.015	.017	.011
.60	1.00	.152	.148	.134
.60	1.20	.029	.033	.029
.60	1.40	−.030	−.015	−.014
.60	1.60	−.058	−.042	−.039
.60	1.80	−.073	−.056	−.054

Permissible Loss Ratio = .600
Loss Conversion Factor = 1.125
Tax Multiplier = 1.040

* Coefficient of variation (σ/μ) = .70

APPENDIX

This appendix outlines a method to quantify the impact of the overlap of insurance charges and loss limitation charges under the current retrospective rating plan formula. All calculations are performed at the $250,000 standard premium size and a $50,000 loss limitation, with the medium underlying severity distribution as presented in the body of this discussion. It is assumed that the average loss ratio is equal to the permissible loss ratio. The adequacy of premium generated under retrospective rating is measured against the targeted return of stock premium discount (15.5%). This 15.5% reflects the stock premium discounts under the workers compensation expense program effective April 15, 1975. Situations with inadequate insurance charges and inadequate excess loss premium factors are also explored.

These retrospective rating values are constant in all calculations:

Maximum Premium (*MAXPREM*) = 1.20
Minimum Premium (*MINPREM*) = .60
Loss Conversion Factor (*LCF*) = 1.125
Permissible Loss Ratio (*PLR*) = .60
Tax Multiplier (*TM*) = 1.040

The following items vary with the problem being solved:

Excess Loss Premium Factor (*ELPF*)

Basic Premium Factor (*b*)

Maximum Loss Ratio′ (*MAXLR′*): The loss ratio at the Maximum Premium if a loss limitation charge (*ELPF* × *LCF*) is used.

Minimum Loss Ratio′ (*MINLR′*): The loss ratio at the Minimum Premium if a loss limitation charge (*ELPF* × *LCF*) is used.

X′: The actual charge needed at the *MAXLR′*. Reflects the impact of the loss limitation charge.

S′: The actual savings realized at the *MINLR′*. Reflects the impact of the loss limitation charge.

Equations used to solve the problems:

(1): $MAXPREM = [b + ELPF \times LCF + MAXLR' \times LCF] \times (TM)$

(2): $MINPREM = [b + ELPF \times LCF + MINLR' \times LCF] \times (TM)$

(3): Average Retro Premium $= [b + ELPF \times LCF + (1.0 - X' + S') \times (PLR) \times (LCF)] \times (TM)$

Average Retro Premiums are calculated in the following situations with the current retrospective rating plan formula:

Situation	Net Insurance Charge	Loss Limitation Charge
A	Adequate	Adequate
B	Adequate	50% Inadequate
C	50% Inadequate	50% Inadequate

Item	Situation		
	A	B	C
b	.212	.212	.173
$ELPF \times LCF$	.134	.067	.067
$MAXLR'$	.718	.778	.812
$MINLR'$	.205	.265	.300
X'	.227	.217	.213
S'	.004	.013	.023
Average Retro Premium	.905	.850	.818

The average retro premiums should be compared to the targeted retro premium of .845 (1.0 − .155). In situation A, one sees that the impact of the overlap can be very significant if insurance charges and excess loss premium factors are both adequate. In situation B, the targeted return is almost achieved due to the inadequate loss limitation charge offsetting the overlap. Situation C indicates a 3 percent net premium deficiency (.818 ÷ .845 = .968) when both the net insurance charge and the loss limitation charge are 50 percent inadequate.

This technique is particularly useful in investigating the impact of retrospective rating under various assumptions regarding average loss ratios and insurance charge adequacy. Of course, it isn't necessary to include a loss limitation in the calculation or assume that the average loss ratio is equal to the permissible loss ratio.

MINUTES OF THE 1981 SPRING MEETING

May 17–20, 1981

THE HOMESTEAD, HOT SPRINGS, VIRGINIA

Sunday, May 17, 1981

The Board of Directors held their regular quarterly meeting from 1:00 p.m. to 5:00 p.m.

Registration took place from 4:00 p.m. to 7:30 p.m.

The Officers' Reception for new Fellows and their spouses was held from 6:00 p.m. to 6:45 p.m.

A reception for members and guests was held from 6:30 p.m. to 7:30 p.m.

Monday, May 18, 1981

Registration was held from 7:30 a.m. to 8:30 a.m.

The Spring Meeting was formally convened at 8:30 a.m. Following his opening remarks, President Jerome A. Scheibl introduced the Honorable James W. Newman, Jr., Commissioner of Insurance, Commonwealth of Virginia, who welcomed the Society to Hot Springs.

Upon thanking Commissioner Newman for his remarks, President Scheibl then read the names of the thirty-nine new Associates. Each new Associate in attendance rose as his or her name was called. Mr. Scheibl then asked each of the eleven new Fellows to step forward to receive his diploma.

The names of the eleven new Fellows and thirty-nine new Associates follow.

FELLOWS

Nicholas M. Brown, Jr.	John S. Lombardo	Martin Rosenberg
Russell T. John	Michael J. Miller	James D. Wickwire, Jr.
Alan R. Ledbetter	Ray E. Niswander, Jr.	Patrick B. Woods
Merlin R. Lehman	Jerry W. Rapp	

ASSOCIATES

Gary R. Abramson	Warren S. Ehrlich	Charles P. Orlowicz
Francois Betrand	Spencer M. Gluck	Karen A. Pachyn
Ralph S. Blanchard, III	Randall D. Holmberg	Leesa I. Pearce
LeRoy A. Boison, Jr.	Gary R. Josephson	Bernard A. Pelletier
James P. Boone	Glenn H. Keatts	John F. Ryan*
Jeanne H. Camp	Dennis L. Lange*	Robert L. Sanders
John D. Carponter	Joseph R. Luizzi	Mark J. Silverman
David R. Chernick	Virginia R. Lobosco	Stuart B. Suchoff
Allan Chuck	Kevin C. McAllister	Darlene P. Tom*
R. Kevin Clinton	Brian A. Montigney	Everett J. Truttmann
Barbara Colin*	Conrad P. Mueller*	Margaret E. Wilkinson
Shelley T. Davidson	Donna S. Munt	Michael L. Wiseman
Frank H. Douglas	James J. Muza*	John P. Yonkunas

*Not present.

Following the admission of new Fellows and Associates, President Scheibl called for reviews of papers previously submitted. No reviews were presented. Authors of the three new papers which were not associated with the Discussion Paper program were asked to present a short description of their papers. Mr. Scheibl then introduced Mr. David Arata, Mr. Robert Giambo (on behalf of Dr. Emilio Venezian) and Mr. Michael Walters. Each of these gentlemen presented a short review of the paper he was presenting on Tuesday afternoon.

At this time, President Scheibl introduced Mr. William A. Halvorson, President-Elect, American Academy of Actuaries, who spoke on current issues for the Academy. Excerpts of his remarks were later printed in *The Actuarial Review*.

After a short break, the Keynote Address was delivered by Dr. Pierre A. Rinfret, President, Rinfret Associates, Inc. Dr. Rinfret delivered a challenging, informative and entertaining address on the numerous causes and potential patterns of inflation.

At 10:15 a.m. an informal discussion was held.

At 10:45 a.m. a summary of the discussion papers was presented by Mr. Galen Barnes, Nationwide Insurance Company, on behalf of Mr. Richard Munro.

A luncheon break was held from 11:30 a.m. to 1:00 p.m.

The meeting resumed at 1:00 p.m. with concurrent sessions for the discussion papers. The discussion papers, their authors, and reviewers were as follows:

"Inflation Sensitive Exposure Bases for General Liability Insurance"
Authors: Richard S. Biondi, Insurance Services Office
Kevin B. Thompson, Insurance Services Office
Reviewer: Janet R. Nelson, St. Paul Fire & Marine Insurance Company

"The Effect of Inflation on Losses and Premiums for Property-Liability Insurers"
Author: Robert P. Butsic, Fireman's Fund Insurance Companies
Reviewer: Rafal J. Balcarek, Reliance Insurance Companies

"A Strategy for Property-Liability Insurers in Inflationary Times"
Author: Stephen P. D'Arcy, Consulting Actuary
Reviewer: Roger C. Wade, Frank B. Hall and Company, Inc.

"Money, Credit & Federal Reserve Policy Changes"
Author: Robert P. Eramo, Hanover Insurance Companies
Reviewer: Paul M. Otteson, Consultant

"Development of an Inflation-Sensitive Exposure Base for Hospital Professional Liability Insurance"
Authors: Glenn A. Evans, Argonaut Insurance Company
Stanley K. Miyao, Argonaut Insurance Company
Reviewer: Brian E. Scott, Aetna Life & Casualty

"Underwriting Cycles in the Property-Casualty Insurance Industry"
Author: Kaye D. James, Corroon and Black Corporation
Reviewer: David Oakden, Aetna Casualty Company of Canada

"The Responsiveness of Automobile Trend Factors"
Author: Alan E. Kaliski, Royal Insurance, presented by Wayne Fisher, Commercial Union Insurance Companies
Reviewer: John B. Conners, Liberty Mutual Insurance Company

"Unallocated Loss Adjustment Expense Reserves in an Inflationary Economic Environment"
Author: John Kittel, Middlesex Mutual Assurance Company
Reviewer: Richard Bill, Country Mutual Insurance Company

"Property-Casualty Insurance Inflation Indexes: Communicating with the Public"
Author: Norton E. Masterson, Consulting Actuary
Reviewer: Richard I. Fein, National Council on Compensation Insurance

"Evaluating the Impact of Inflation on Loss Reserves"
Author: William F. Richards, Aetna Life & Casualty
Reviewer: Richard G. Woll, Hartford Insurance Group

"Selection of the Optimum Asset Portfolio to Satisfy Cash Needs"
Author: Martin Rosenberg, Commercial Union Insurance Companies
Reviewer: Donald E. Trudeau, Peat, Marwick, Mitchell & Co.

"Adjusting Size of Loss Distributions for Trend"
Authors: Sheldon Rosenberg, Insurance Services Office
Aaron Halpert, Insurance Services Office
Reviewer: Charles F. Cook, American International Underwriters

"Loss Reserving and Ratemaking in an Inflationary Environment"
Author: Larry D. Shatoff, Towers, Perrin, Forster & Crosby
Reviewer: E. LeRoy Heer, W. R. Berkley Corporation

The moderators of the sessions were:

David R. Bickerstaff
Milliman & Robertson, Incorporated

Galen R. Barnes
Nationwide Insurance Companies

Robert L. Tatge
Farm Bureau Mutual Insurance Company

Michael L. Toothman
Great American Surplus Lines Insurance Company

An informal discussion was held at 3:00 p.m.

The day ended with the President's Reception from 6:30 p.m. to 7:30 p.m.

Tuesday, May 19, 1981

The meeting reconvened at 8:30 a.m. with a continuation of concurrent sessions for discussion papers.

A luncheon break was held from 12:00 noon to 2:30 p.m.

At this point, the meeting resumed with a workshop program and committee meetings. The workshops were held according to the following schedule:

Workshop 1 — "Refresher Course—Commercial Liability Ratemaking"

Members: Linda L. Bell
U.S. Insurance Group

Richard S. Biondi
Insurance Services Office

Workshop 2 — "Discussion on Proposed Revision to Article V of CAS Constitution Regarding Election of Officers and Directors"

Members: Robert B. Foster
Chairman, Ad Hoc Committee to Study the Nomination and Election Procedures

Jerome A. Scheibl
President, CAS

Workshop 3 — "Presentation of New Papers"

"Computer Simulation and the Actuary"

by David A. Arata
Marsh & McLennan, Incorporated

Reviewed by: Thomas V. Warthen
Tillinghast, Nelson & Warren, Inc.

"Good and Bad Drivers—A Markov Model of Accident Proneness"

by Dr. Emilio Venezian
Insurance Services Office

Reviewed by: Dale A. Nelson
State Farm Mutual Automobile Insurance Company. Mr. Nelson's review was presented by Mr. Steven Lehmann.

Sanford R. Squires
Commercial Union Insurance Companies

"Risk Classification Standards"

by Michael Walters
Insurance Services Office

Reviewed by: Michael J. Miller
State Farm Mutual Automobile Insurance Company

The day ended with a General Reception from 6:30 p.m. to 7:30 p.m.

Wednesday, May 20, 1981

The meeting reconvened at 8:30 a.m., opening with a panel discussion entitled "Risk Theory and Practice." Those participating were:

Moderator: David J. Grady
North American Reinsurance Corporation

Members: Jerry A. Miccolis
Tillinghast, Nelson & Warren, Inc.

Gary G. Venter
Prudential Reinsurance Company

Gary S. Patrik
Prudential Reinsurance Company

Lewis H. Roberts
Woodward and Fondiller, Inc.

At 9:45 a.m. the business session was reconvened.

The membership voted on the proposed amendment to Article V of the CAS Constitution. The proposal was passed with a 90.9% affirmative vote.

The Michelbacher Prize was awarded to Mr. Robert P. Butsic for his paper "The Effect of Inflation on Losses and Premiums for Property-Liability Insurers." It is worth noting that this makes Mr. Butsic a second-time winner of this prize, as he also won it in May 1979 for his paper entitled "Risk and Return for Property-Casualty Insurers."

At 10:45 a.m. a panel discussion entitled "Implications of 1980 Elections to the Property/Casualty Insurance Industry" was presented. The participants were:

Moderator: Stephen G. Kellison
Executive Director
American Academy of Actuaries

Members: Leslie Cheek, III, Vice President
Crum & Forster Corporation

Frank Nutter, General Counsel
Reinsurance Association of America

David Mathiasen
Office of Management and Budget

The closing remarks were made by President Jerome A. Scheibl who gave special thanks to Marty and Harriet Adler, Lenore Newman and Marlene Scheibl for their work with local arrangements. President Scheibl adjourned the Spring Meeting at 12:00 noon.

In attendance as indicated by registration records were 184 Fellows, 121 Associates, 22 guests, 14 subscribers, 7 students, and 132 spouses. A list of attendees follows.

FELLOWS

Adler, M.
Aldorisio, R. P.
Alexander, L. M.
Angell, C. M.
Anker, R. A.
Arata, D. A.
Asch, N. E.
Baer, D. L.
Bailey, R. A.
Balcarek, R. J.
Barnes, G. R.
Bartlett, W. N.
Bass, I. K.
Beer, A. J.
Bell, L. L.
Bennett, N. J.
Berquist, J. R.
Beverage, R. M.
Bickerstaff, D. R.
Bill, R. A.
Biondi, R. S.
Bondy, M.
Bornhuetter, R. L.
Bradley, D. R.
Brannigan, J. F.
Brown, N. M., Jr.
Brubaker, R. E.
Bryan, C. A.
Buck, J. E., Jr.
Carbaugh, A. B.
Cheng, L. W.
Childs, D. M.
Collins, D. J.
Conger, R. F.
Conners, J. B.
Cook, C. F.
Covney, M. D.
Curley, J. O.
Curry, A. C.
Curry, H. E.
Daino, R. A.
Dangelo, C. H.
D'Arcy, S. P.
Davis, G. E.
Demers, D.
Dempster, H. V., Jr.
Donaldson, J. P.
Eddy, J. H.

FELLOWS

Eland, D. D.
Eldridge, D. J.
Evans, G. A.
Eyers, R. G.
Faber, J. A.
Fagan, J.
Fallquist, R. J.
Fein, R. I.
Ferguson, R. E.
Fisher, W. H.
Fitzgibbon, W. J., Jr.
Flynn, D. P.
Foster, R. B.
Fowler, T. W.
Fresch, G. W.
Furst, P. A.
Garand, C. P.
Giambo, R. A.
Gleeson, O. M.
Grady, D. J.
Grannan, P. J.
Grippa, A. J.
Hachemeister, C. A.
Hafling, D. N.
Hall, J. A.
Hanson, H. D.
Hardy, H. R.
Hartman, D. G.
Harwayne, F.
Haseltine, D. S.
Hazam, W. J.
Heer, E. L.
Hermes, T. M.
Hewitt, C. C., Jr.
Honebein, C. W.
Hough, P. E.
Hoylman, D. J.
Irvan, R. P.
Jameson, S.
Jean, R. W.
Jerabek, G. J.
John, R. T.
Kaufman, A.
Kelly, A. E.
Khury, C. K.
Kilbourne, F. W.
Kist, F. O.
Kollar, J. J.
Krause, G. A.
Kuehn, R. T.
Ledbetter, A. R.
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Respectfully submitted,

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PROCEEDINGS

November 22, 23, 24, 1981

PRESERVING OUR HERITAGE

PRESIDENTIAL ADDRESS BY JEROME A. SCHEIBL

Enter upon your inheritance,
accept your responsibilities.
Sir Winston Churchill

We who have attained membership in this Society have inherited the legacies of those who have pioneered in our field. These legacies are the very tenets—the very foundations—of the casualty discipline of the actuarial profession. They have been developed through a careful blending of mathematics, the most demanding and precise of sciences, with the practicalities and eccentricities of the business world.

These legacies are ours to hold, to use and embellish, to nurture and defend, to share and to pass on to those who follow.

Our chosen stewardship comes at a time when all elements of society are being affected by the complexities of various changing influences.

These changes by themselves are not what make things so complex, but it is rather the uncertain pace of change, the uncertain directions of change and the whimsical interactions among the elements of change.

This erratic atmosphere threatens established institutions, including the professions, which rely heavily on continuity concepts to develop goals, plans and strategies to grow and, for many, merely to survive.

Peter Drucker refers to these as "turbulent times" and he warns that in times such as these, an enterprise must be managed both to withstand sudden blows and to avail itself of sudden unexpected opportunities. He cautions that for this to happen, fundamentals must be managed—and managed well.

Drucker's admonitions apply to both the Casualty Actuarial Society as an organization and the casualty discipline of the actuarial profession which it represents. In these turbulent times, we must be able to respond equally to sudden shocks and sudden opportunities as they affect our Society and our profession. We must manage our heritage effectively and efficiently.

I would like to expand on these thoughts a bit and then discuss some specifics as they relate to our organization and our profession.

Our Heritage

Sixty-seven years is not a very long time when speaking about heritage and tradition. Yet, in the 67 years of our Society's existence, it has witnessed a major evolutionary period for concepts in risk sharing, insurance regulation, financial management, social insurance and electronic data processing. The casualty insurance field has aged centuries in just a few short decades.

Surely the founders of our Society and some of their early successors never dreamed that adversarial proceedings in the ratemaking process would become more real than theoretical; that third-party liability cases would clog the court systems; that members of any profession would be prosecuted for malpractice at the insistence of laymen, except in the grossest of circumstances; or, for that matter, that first-party and third-party coverages would be written in the same policy.

The impracticalities of their day are the routines of today. Multiple regression analysis can now be done in seconds rather than days; mountains of data can be accessed at random with multicolored, numeric and graphic displays on hard copy or through a cathode-ray tube. Who in 1914 would have thought that calculators capable of highly complex manipulations and memory capacity could be purchased in drugstores and carried around in coat pockets?

The beginnings of our Society are well documented in Dudley Pruitt's paper "The First 50 Years" published in our 1964 *Proceedings*. For those of you who have not read it, I recommend it strongly. This paper provides an insight into the type of people who organized our Society and why they did it. It also describes the development of the casualty actuarial principles by some of our more prominent members.

I hesitate to cite examples of such contributions for fear of unintentionally omitting some names of the many who left their mark on the casualty actuarial discipline. Perhaps a tribute to them can best be summed up by quoting from Dudley Pruitt's article. He writes:

> "One of the benevolent dispositions of providence seems to be that when, in the course of human events it becomes necessary to have giants, giants are provided. So it was in the founding of our country, and so it was in the founding of our Society."

To that, I should add that giants have begotten giants. Our members over the years have contributed to the stature of our Society and profession by publishing textbooks, conducting research, presenting papers, participating in our meetings, counseling aspiring actuaries and demonstrating through their wisdom and demeanor that the actuary has a distinct and significant role in the casualty insurance business.

Their contributions have been most significant in developing risk classification systems, credibility measurements, experience and retrospective rating plans, loss distributions, loss reserve methods, ratemaking systems and, more recently, corporate models.

They have also instilled a sense of camaraderie into our profession that gives it a spirit that is hard to define. We have a real concern for others in our profession and we have certain traditions—both serious and nonsensical—that draw us together.

These Turbulent Times

Turbulent times imply turbulent environments. It is within these environments, as we experience them today, that we must recognize those occasions and circumstances that represent threats on the one hand and opportunities on the other. It is within these environments that we must decide whether we are to protect our heritage and embellish what we have chosen to inherit.

One need not be a historian or a futurist to realize that the present is anything but normal. Economically, we are witnessing the paradoxical coexistence of inflation and recession. Socially, we are seeing extreme poverty, famine and disease coexist side by side with prosperity and health, despite major advances in social consciousness, respect for human rights and medical technology. We are seeing developed countries with labor shortages, developing countries with raw material shortages, and underdeveloped countries in a state of chaos. An attitude of entitlement—where every wrong can be made right through monetary payment—permeates our times. Outspoken publics are raising their voices to be heard in both free and communist countries. There is a growing demand for personal freedom and individual independence at the same time that there is a growing movement to band together to support common causes or common hostilities.

These attitudinal and socioeconomic changes occur at the same time as the pace of scientific and technological advances have been spurred on by the necessities of the nuclear and space ages.

These many elements of change, and the uncertainty of their interactions, their pace and their direction, have made our world both confident and wary. We are awed at what we as a people can do when we exercise our God-given talents and freedoms. Yet, we are wary that the consequences of abusing these gifts may destroy our cherished institutions, our way of life and perhaps our very existence.

These turbulent times are the times in which we here have assumed the custodianship of our profession. Both our organization and our profession are affected. The impact is compounded, however, by factors unique to the business we serve—the business of assuming and sharing the fortuitous risks of our environment.

We must recognize and assess our ability to meet these challenges and opportunities head-on if our organization and profession are to prosper and grow.

I would like to explore with you what I believe to be some of the major challenges and opportunities facing the Casualty Actuarial Society and the casualty discipline of the actuarial profession to which the Society is dedicated. I'd like to look also at what is being done and what might be done to meet the challenges and to make the most of the opportunities.

First, let's look at our organization.

The Casualty Actuarial Society

The age of the electronic mathematician, the overcrowding of the more traditional professions and the growth of career opportunities in the actuarial profession have had an impact on the Casualty Actuarial Society. The accelerated growth of our organization has a potential shock effect. Our membership has practically doubled in the last ten years. Over half of our Associates have held the designation for fewer than six years; over half of our Fellows have held their Fellowships for fewer than five years. The average time since our active members received their Associate designations is only 11 years. This is paradoxical in a Society that will be celebrating its diamond jubilee just eight years from now.

We sought this growth—and now we welcome it. But with it have come new challenges to the preservation of the traditions and principles of our discipline which have been so carefully developed.

Our present organizational structure has been adequate for our Society over the past several years. We have gotten the job done without being overly pretentious. We have managed our affairs with a modest-sized Board of Directors, a fairly informal committee structure, and a heavy reliance on the time and energies of our officers and committee chairmen.

As our membership approaches the thousand mark, however, there is a point where this cozy arrangement must give way to a structure more in line with sound management practices, a structure which enables our Society to withstand the blows of our environment, take advantage of opportunities, and provide the services that our members have the right to expect.

This is something we should be able to do easily, because many of our members are engaged in various levels of corporate management. An *ad hoc* committee is currently reviewing our operations. Its recommendations will be considered by your Board of Directors over the next few months.

Because so many members are new to our organization, there is an understandable tendency for more and more of us to identify less and less with the contributions and personalities of those who have preceded us. The examination process preserves some of these ties as it force feeds our students. But this alone does not instill a loyalty to the profession; it merely tests competence.

There is much to be said for preserving our traditions and an appreciation for the work that has been done in developing the principles of our discipline. These traditions are a vital part of our heritage and the backbone of a unified organization. Some things are being done and some others should be considered to preserve them. Let me cite just a few.

- As we approach our 75th anniversary, we should consider publishing a monograph on the history of our Society as part of our observance of this milestone. It is not too early to engage an author, determine the funding arrangements and begin the research to produce a quality product. Such an endeavor is vital to perpetuating an appreciation for the legacies which we have inherited and will pass on to those who follow.

- We might also consider an honorarium to be awarded periodically to one of our retired members. This honorarium would cover expenses for attendance at one of our meetings. In return, the recipient would be expected to participate in the program. Many of our retired members have given so much to our Society and have become legends in their own time. It is a shame that we must deprive ourselves of their wisdom and counsel simply because of financial considerations or a feeling that they do not relate to our present Society. We need these people at our meetings for our own enlightenment and to demonstrate our appreciation for their contributions. I hope we can develop a program whereby we can all mutually benefit.

If you will permit a brief digression, I should remind you that the CAS Trust has been established to accept contributions and bequests to provide funding for projects such as these. So much for the commercial.

- Beginning with our next meeting, we will be providing an indoctrination workshop for our new Associates. These workshops will be a regular part of our program at each meeting and will more fully acquaint our new members with our Society's traditions and purpose and the responsibilities of a professional in our field. It will also provide a forum for an exchange of ideas on how our Society can best serve its members.
- A strong and vibrant organization can remain so in times of challenge and unrest only if it continues to serve the needs of its members as they perceive such needs. The CAS has traditionally provided these services by fostering scholarly and vocational dialog among its members and others who share in promoting the objectives of our Society.

  Turbulence has stimulated a movement towards specialization in the professions. Our own discipline has not escaped these influences. Many of our members, by design or necessity, have concentrated their interests in specialty areas. This, in turn, has changed their professional needs.

  Our organization has recognized these needs in different ways. First, we have provided for the formal organization of special interest groups to facilitate dialog among those whose interests are identified with specialty areas. I expect that the first of such groups will be organized and operating before our next meeting.

  Secondly, we have organized seminars on selected topics held at separate times and in separate locations from our semi-annual meetings. Our two seminars this year on Risk Classification and Loss Reserves, the latter cosponsored with the American Academy of Actuaries, were highly successful.

Thirdly, the discussion paper program instituted a few years ago continues to spew out papers in quantity, if not quality, on selected topics to form the basis for in-depth discussion at our spring meetings. Regretfully, not enough of these papers find their way into our *Proceedings*—but they do serve their purpose.

Finally, the tendency to specialize imposes a responsibility on our organization and its members to maintain a basic knowledge in phases of actuarial work in which we do not profess a special interest. This need has been filled by the recent practice of holding a basic refresher session on a selected topic at each of our meetings.

These activities demonstrate our Society's sensitivities to needs of its members and its ability to respond to such needs—a trait that is inherent in our heritage.

Earlier, I mentioned that our traditions fall into two categories, serious and nonsensical. We may be a studious organization—but we are certainly not stodgy. Throughout our *Proceedings*, there are references to dinners, humorous speeches, exuberant receptions and planned and unplanned recreational events of one sort or another. In more recent years, we have had all these things plus plays, musical reviews and a lot of gentle ribbing. These traditions are not ends unto themselves. They are instead our own way of communicating our warm feelings and respect for one another. Camaraderie is an important part of the Casualty Actuarial Society. It is a tradition that should be cherished and guarded.

I would like to move now from our organization itself to the discipline it serves—the casualty discipline of the actuarial profession.

The Casualty Actuary

The interactions of the various elements of our turbulent environment as predictors of future happenings almost defy analysis. Yet, in today's society, with its emphasis on planning and mapping strategies, such analyses are indispensable.

So it is too in the business of assuming, spreading and sharing the consequences of fortuitous risk—the business served by the actuarial profession. More than ever, insurance carriers, consumers, and governmental bodies are looking for guidance in minimizing the financial impact of fortuitous happenings. They are demanding and sometimes desperate in their search for professional assistance to sort through the complexities of risk analysis. They are faced with decisions regarding risk retention, affordable prices, discrimination, solvency, residual markets, adequate prices and the like—all requiring mathematical analyses of socioeconomic phenomena.

These demands have a mixed effect on the casualty discipline of the actuarial profession. On one hand, they have attracted many who are willing to subject themselves to specialized training and study, to rigorous examinations and to a code of professional standards and conduct. On the other hand, they have attracted many who have not chosen to make these commitments.

The threats to the casualty discipline of the actuarial profession come from both categories of practitioners; so do the contributions. We are willing to accept the fact that there are some qualified people working in the casualty actuarial field who are not members of our organization. However, we expect that such people not only see themselves as possessing a distinct knowledge, but also as accountable for their work product. It is essential that a professional be responsible for his or her work and be able to demonstrate that responsibility in some way. Responsibility and accountability are necessary qualifications to practice and must be demonstrated in some manner.

Poor service, poor advice to a client or employer on the part of one who calls himself or herself a qualified casualty actuary—or even hints at such qualifications—reflects on the integrity of the entire profession. This is bad when such service comes from those who know they are not qualified to provide it. It is worse when it comes from those who think they are qualified, but are not.

As members of the family of professions, we have certain obligations and rights with regard to professional courtesies. When we speak or write as actuaries, we must do so within the confines of our profession and our own individual capabilities. We have the right to expect that when it comes to actuarial matters, members of other professions will do likewise.

We are each entitled to express our opinions on any matter we choose. This is our right in free society. We do a disservice to our profession, however, when we use our status as actuaries as qualification to give expert advice on nonactuarial matters—whether we are otherwise qualified to give such advice or not. We should make it clear when we are speaking as actuaries on actuarial matters and when we are not.

By the same token, we should be alert to experts in other fields who use their professional status as qualifications to advise on matters that are actuarial in nature. If these people are accountable to their own profession and to the public they serve, they will qualify their advice as either within the limited scope of their expertise or as nonprofessional. If they are not accountable, their qualifications must be challenged publicly.

The demands for casualty actuarial advice have not only attracted members of other professions, but have appealed to some in other branches of the actuarial profession itself.

Giant strides have been made toward solidifying the actuarial profession in the last 20 years. Continuing dialog and jointly sponsored activities have developed a high level of understanding and mutual respect among the various North American bodies. The faith and trust we have in each other is exemplified by fairly common codes of professional conduct and discipline.

For the most part, we can be assured that most of those in other actuarial disciplines who advise in the casualty field do so fully cognizant of the limitations on their ability under their codes of conduct. There are others, however, who consider depth of knowledge in a certain area as sufficient qualification to advise on matters that require both depth and breadth of expertise. This is most noticeable in those lines where the occasion and extent of insured loss are governed by casualty insurance concepts while the payment of loss, once it occurs, can be more in the nature of a pension. In insurance parlance, we have become accustomed to referring to such losses as "benefits." This is unfortunate. Claim payments in such cases are not made under the principle of entitlement, but rather under the principle of indemnity. In order for payment to be made, a loss must occur; that is, a fortuitous event must take place. The uncertainty of loss occurrence and amount is the very essence of casualty and property insurance. There is some justification for utilizing other actuarial disciplines in these lines, but the scope of their application is limited.

Threats to our profession also come from within the casualty actuarial discipline itself.

Our clients, whether they be our employers or others, have a right to expect the full benefits of our individual capabilities and expertise as actuaries. That is what we have to offer; that is the basis for our compensation.

This competitive world of ours is full of temptations to impress our clients with gratuitous embellishments of our work product. I have already commented on the temptations to take advantage of our status by advising on matters of a nonactuarial nature or beyond our own level of competence. I am not suggesting that actuaries be gagged or deprived of the rights of free speech. I am suggesting, however, that we do a disservice to our clients and our profession when we imply that such advice is made within our capacities as professional actuaries and, therefore, is more credible than advice made by others.

There are temptations for members of our profession to sell their services to clients on the basis of their prejudices rather than on the basis of their competence and ability. Unfortunately, this is what clients may be looking for at times. Such practices should not be condoned by any of our members. They are potentially damaging to the actuarial image.

I hasten to add that I do not consider advocacy positions or viewpoints to be the same as prejudices. Prejudices are opinions formed beforehand without a knowledge or examination of the facts—a preconceived preference. This is hardly consistent with a profession that bases its work product on the analysis of factual data.

There may also be a temptation to impress clients by advancing new ideas and methods that have not been adequately tested or subjected to critical review of our peers. The advancement of these methods in a lay forum reflects on the integrity of the more traditional approaches. Further, it may raise doubts in some minds as to whether actuarial science is indeed as scientific as we claim it to be.

On the other hand, there may be traditionalists who insist on strict adherence to conventional methods. A public debate on whether such practices are realistic or not may also confuse the public. They too invite public criticism of our profession.

The CAS provides many opportunities for academic debate on actuarial topics. This is the proper arena to discuss innovations, sources of data, propriety of traditional methods and the like. This is where we should air the technical issues that affect our profession. While these issues may or may not be resolved in this arena, the debate may at least stimulate further research and study. Its effect on the profession may be positive rather than having a negative impact by raising doubts in the minds of those who cannot relate to the technical side of what may be emotional issues. The public is mystified enough at the complexities of our discipline. Instead of adding to that mystery, we should instill a feeling of confidence and appreciation for what actuarial science is, what it can do and what it cannot do.

These examples of threats from within are threats that we can and must manage for the good of our Society and the public we serve.

Conclusions

Drucker's turbulent times and their impact on the casualty discipline of the actuarial profession and the Casualty Actuarial Society itself are not theoretical concepts. They are very real as we see their effect on changing roles of rating bureaus, standards for carrier profits, erratic loss development patterns, trending of premiums and losses, carrier solvency, risk retention, reinsurance, adversarial relationships, and the like. We must heed Drucker's advice to manage and manage well to strengthen our stewardship under these conditions.

Our attention should focus on the following areas:

- The organizational structure of our Society. As I mentioned earlier, this is currently under review.
- Delineation and exposition of the principles of our discipline. Thanks to the work of an energetic *ad hoc* committee, the Board has approved a plan to develop a textbook of the survey type. This should not be confused with the earlier aborted efforts to develop an all-encompassing text on casualty contingencies.
- The codification of our standards of practice. These standards exist through custom and general acceptance. They are difficult to enforce in such a form. They must be set forth in writing and fully understood by all members of our profession.
- A greater awareness of our standards of conduct. A lot of work has gone into guides and opinions for professional conduct; however, these are just a lot of words unless they can be understood and somehow instilled into the way we go about our work.
- A more effective disciplinary procedure with emphasis on warning, counseling and speedy disposition of cases. Our present procedures are cumbersome, time-consuming and ineffective.
- Finally, an examination of the scope of our Society's mission. Our Board is currently reviewing the purpose and objectives of the CAS in our present environment. Careful thought is being given to what is needed to enhance our profession and how it can best be provided.

Our ability to identify and focus on issues as they arise is a sign of our ability to manage our affairs. So far, we have done quite well. The blows to our profession have been parried and we have been alert to our opportunities.

Our decisions to become members of the Casualty Actuarial Society were not altruistic; they were made to advance ourselves individually for personal gain. Each of us has sought professional status and each of us has attained it. The professional status was there to attain because of the dedication of those who have preceded us.

Francis Bacon wrote, "I hold every man to be a debtor to his profession." We owe a debt to our profession and those who have made it what it is. We can repay this debt by enriching the legacies which we have chosen to inherit. We must repay this debt by preserving our heritage.

ACTUARIAL VALUATION OF PROPERTY/CASUALTY INSURANCE COMPANIES

ROBERT W. STURGIS

Abstract

There has been a surge of insurance company acquisition and merger activity in the United States and Europe in recent years. Most of this activity has been in the life insurance area, but the pace of property/casualty activity has picked up recently, and there are predictions of heavy future activity.

The bibliography following this paper is not an exhaustive list of readings on the subject of actuarial valuations of insurance companies, but it represents an impressive library of actuarial readings on the subject of life company valuations. However, there is scant actuarial literature on the subject of casualty company valuations, and such discussions are absent from our *Proceedings*.

Evidence of the interest in this topic is the fact that the 21st International Congress of Actuaries held in June of 1980 had as its Topic 4, "Estimating the Value of Insurance Companies and Portfolios," with thirty papers presented. In his introductory remarks, J. B. R. Lieberman[1] suggested three general points for discussion. One of the three was: "How are non-life (property/casualty) insurance companies and portfolios valued in practice?" None of the thirty papers presented dealt specifically with property/casualty companies and, in spite of Mr. Lieberman's suggestion, the discussion was confined essentially to the life insurance business.

Accordingly, this paper is intended to set forth a basic method for the actuarial valuation of property/casualty companies.

[1] J. B. R. Lieberman, "Estimating the Value of Insurance Companies and Portfolios," (Topic 4), *Transactions of the 21st International Congress of Actuaries*, Introduction (June 19–26, 1980), p. 8.

ALTERNATIVE MEASURES OF VALUE

Mogens Andersen[2] in his paper points out the need to differentiate between the price a buyer is willing to pay and economic value. Bowles and Turner[3] go further in discussing this point. Purchase price is defined to be "the amount for which a company is, or is expected to be, purchased in an acquisition transaction." "In short, purchase price represents what an acceptable price is, or is expected to be, to *both* buyer and seller, and reflects the psychology of, and forces at work in, the marketplace." The authors define value, on the other hand, as the result of appraisals independently performed by the buyer and the seller. Value represents what an acceptable purchase price ought to be and "value determinations normally set the limits of purchase price acceptability." The authors proceed to describe in some detail five measures of value. These are summarized very briefly below.

Market Value is the value of outstanding shares of common stock. This measure is relevant since almost all acquisitions are consummated at a purchase price greater than market value.

Book Value is the amount of shareholders equity in the insurance company to be valued, on a GAAP or statutory basis. Since book value does not reflect any value for the company's ability to produce profitable business in the future, it may be a part of, but is not in itself a reasonable reflection of what an acceptable price would be.

Comparative Values are the ratios of purchase prices for recent company acquisitions to denominators such as market value, book value and earnings. For example, two comparative values that are representative of recent acquisitions are two times statutory net worth and ten times statutory earnings.

Dilution Value means the purchase price that would decrease the buyer's earnings per share or return on equity, whichever basis is used. Dilution value serves as an indicator of the maximum purchase price which would likely be tolerable to the buyer's shareholders and, thus, does represent a relevant consideration by the buyer in a purchase transaction.

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[2] M. Andersen, "Some Remarks on the Value of Insurance Companies and Portfolios," (Topic 4), *Transactions of the 21st International Congress of Actuaries*, (June 19–26, 1980), p. 1.

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[3] T. P. Bowles and S. H. Turner, "Acquisition of a Life Insurance Company: Determination of Value and Purchase Price," (Topic 4), *Transactions of the 21st International Congress of Actuaries*, (June 19–26, 1980), p. 84.

Economic Value is the book value plus the present worth (i.e., the capitalized value) of expected future earnings.

Of the measures of value enumerated above, only economic value fully satisfies our definition of value. The others place certain practical boundaries on the purchase price, but do not represent what an acceptable purchase price ought to be. Economic value is based upon a projection of future earnings, and as such, it is a determination which actuaries are most qualified to make.

ACTUARIAL DETERMINATION OF ECONOMIC VALUE

From a review of the actuarial readings on this subject, it appears that J. C. H. Anderson's[4] 1959 paper was the genesis for the current concept of actuarial valuations of life companies. In that paper Mr. Anderson pointed out that the value of a life insurance company must represent more than the total of its capital and surplus: "A more realistic value of an entire company must take account of its business in force and agency organization." Specifically, one must evaluate:

1. The present value of unrealized profits on business now in force, discounted at a rate representing adequate return to the investor on the total value; and,
2. The present value of profits on new business.

Future earnings can be capitalized at any desired rate of return. Selection of such a rate depends upon the buyer's desired return on investment and his assessment of risk. In particular, the less confidence one has in the projections of future earnings, the higher the risk rate of return should be in the discounting of those projections.

This general valuation concept has been adopted in all of the works reviewed by this author.

As Bowles and Turner[5] pointed out, the adopted concept requires that the determination should only include earnings *available* to the buyer. This suggests that earnings should be after federal income tax and should be statutory rather than GAAP, because such earnings are available for reinvestment in new busi-

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[4] James C. H. Anderson, "Gross Premium Calculations and Profit Measurement for Non-Participating Insurance," *Transactions, Society of Actuaries*, Vol. XI (1959), p. 378.

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[5] T. P. Bowles and S. H. Turner, *op. cit.*, p. 87.

ness and/or withdrawal from the company as shareholder dividends. It also suggests two alternative formulas:

1. The discounted value of maximum stockholder dividends; and,
2. Current net worth plus the discounted value of future earnings less cost of capital.

The first formula is based on the principle that only dividend income is available to the investors, and thus, only that should be considered. In other words, the economic value of net worth is best reflected by the earnings it produces by virtue of its investment in the insurance operation. Thus, the entire valuation is based upon projections of future earnings and is wholly dependent upon the particular selected risk rate of return.

The second alternative splits the economic value into component parts, and is the one most commonly adopted in the literature. The first component, net worth, is an accounting value, directly available from financial statements, and perhaps, subject to actuarial adjustment for reserve adequacy. This represents a significant portion of economic value and is not dependent on the selected risk rate of return. The third component, cost of capital, recognizes that the capital and surplus required to support the insurance operation will be required to be invested in a conservative manner. The cost of capital then is based upon the difference between the anticipated rate of return that will actually be realized on invested capital and surplus, and the rate of return it could be earning if invested elsewhere.

In the examples that follow, the second, or traditional, formula has been used. For a life insurance company, future earnings are usually based on separate valuations of the in-force business and new business. Here, the business in-force includes the renewals of current policyholders, since most individual life insurance business is issued with long term benefit and premium guarantees. As such, the value of the business in-force is often the largest part of the value of a life insurance company.

In property/casualty, coverage and premium guarantees seldom extend beyond one year, so that the business in-force is just the run out of the unearned premiums and the losses, expenses and investment income on premiums already written. In the example that follows, earnings on in-force and new business are calculated based on separate assumptions, but are combined in the determination of future earnings.

PROPERTY/CASUALTY MODEL

The exhibits that follow this paper present an example of a computer model for establishing a valuation of future earnings for a hypothetical company, W. C. Protective, writing only workers' compensation. In practice, the model will accommodate any number of lines.

The model is by underwriting, or policy, year. Accordingly, underwriting assumptions must be made for each policy year including past policy years for which loss reserves are still held. The example assumes a valuation at 12/31/81, and is based on the following underwriting assumptions:

1. *Coverage Term*—All policies are for one year terms and are issued evenly throughout the year.
2. *Reserve Runoff*—The ratios of loss and loss expense reserves to ultimate incurred at successive twelve month intervals from the beginning of the policy year are:

12 Mos.	.677	72 Mos.	.089
24 Mos.	.382	84 Mos.	.065
36 Mos.	.250	96 Mos.	.040
48 Mos.	.167	108 Mos.	.028
60 Mos.	.120	120 Mos.	.019

3. *Written Premium*—$40 million in 1982 followed by ten percent annual growth thereafter.
4. *Unearned Premiums*—Taken directly from the annual statement, assumed to be $11 million. (The unearned ratio is typically low for workers' compensation due to additional audit premiums which are fully earned.)
5. *Loss Reserves*—The actual loss and loss expense reserves ($53 million) held at 12/31/81 by accident year:

1981	$10 million	1977	$4 million
1980	17 million	1976	3 million
1979	11 million	1975	2 million
1978	6 million		

6. *Loss Ratios*—Assumed loss and loss expense ratios for all policy years:

1975	.75	1980	.75
1976	.77	1981	.77
1977	.75	1982	.75
1978	.70	Thereafter	.75
1979	.74		

7. *Acquisition Expense*—The ratio of those expenses to be related to written premiums is assumed to be 8% from 1975 through the end of the projection period.
8. *General Expense*—The ratio of all other expenses to be related to earned premiums is assumed to be 20% from 1975 through the end of the projection period. (The model is able to handle expenses related to incurred losses as well.)

For the purposes of this paper, underwriting selections are, of course, simple and illustrative only. In practice, they are the crux of the actuarial valuation. The further into the future the projections, the less reliable they are; but they are also less critical, because of the increasing impact of the present value discounts.

Projections of premium growth and underwriting ratios are typically based on comparisons of company versus industry performance. Often, long range financial plans of the company being valued will be available. These can be a valuable input to the process, but clearly cannot be relied on entirely.

In addition to the by-line projections enumerated above, companywide data and assumptions must be input. Since net worth will be accounted for separately, the model is initialized with zero capital and surplus. However, a theoretical surplus requirement is established at one third the annual written premium volume, and the "cost of capital" is set at 5% of that amount. In other words, the "required statutory surplus" could be earning an additional 5% interest, after tax, if it were available to invest elsewhere. Annual stockholder dividends are maintained at zero throughout the projection period.

Investment rates are expressed as return on total assets, rather than invested assets, and are net of investment expenses. In this example, one third of the company's assets are invested in non-taxables at six percent, and two thirds in taxables at ten percent. The federal tax rate is assumed to be 46% of taxable earnings.

The model was run for thirty future years plus reserve runoff thereafter, and the results, in balance sheet and income statement form, are shown in the attached exhibits and summarized in Table 1.

The statutory net worth of W. C. Protective, $15 million, is added to the above discounted adjusted earnings to produce a formula value of $34 to $76 million, depending upon the risk rate of return.

TABLE 1

Assumed Risk Rate of Return	Present Values (000's) Statutory Earnings	Cost of Capital	Adjusted Earnings
10%	$79,945	$18,788	$61,157
15%	40,870	9,972	30,898
20%	25,118	6,218	18,900

ADJUSTMENTS TO FORMULA VALUE

The valuation above is on a formula basis, with both current net worth and future earnings determined according to statutory accounting standards. There are several adjustments to this value that should either be made or called to the attention of the potential buyer as additional considerations.

From the example shown, it is obvious that the selected risk rate of return has a significant impact on the valuation of future earnings. The selected rate should be at a level above the risk-free rate of return (e.g. U.S. Treasury Notes) that can reasonably be expected throughout the projection period. This additional discount margin should reflect the uncertainty of actually achieving projected growth and profit levels. As pointed out, selection of the appropriate rate is often best left to the buyer based upon his own desired return on investment and assessment of risk.

In addition to producing values based on a range of discount rates, it is good practice to test the sensitivity of the model to future underwriting assumptions by running a series of alternative assumptions. If one assumes that strict underwriting and/or rating practices lead to lower loss ratios and depressed premium growth, there will be offsetting impacts on projections of future earnings. This fact, along with the impact of the discount rate, usually leads to the conclusion that the valuation is not unduly sensitive to a reasonable range of underwriting assumptions.

Any thorough valuation of a property/casualty company requires a thorough analysis of loss and loss expense reserves. In effect, the formula value assumes exact reserve adequacy. In this regard, the Schedule P penalty, if any, should be considered part of the company's reserves. Any reserve redundancy (inadequacy) should be added to (subtracted from) statutory net worth. Of course, the tax effect of any adjustment to reserve levels (as well as any other adjustments to net worth) should also be reflected.

There are often several accounting adjustments to statutory net worth that should be considered. These include non-admitted assets and special liabilities such as reinsurance from unauthorized reinsurers. Such adjustments should either be made by the actuary or simply highlighted as possible adjustments depending upon his knowledge of them.

Statutory accounting does not reflect any liability for incurred but undeclared policyholder dividends, since there is no binding obligation to pay them. Any such anticipated dividends should be reflected as an expense item in the underwriting assumptions.

Most property/casualty companies carry a substantial portfolio of bonds at book value. This should be pointed out to the client so that an adjustment to market value could be made if he deems that appropriate. However, it should also be pointed out that such an adjustment should carry with it a partially offsetting adjustment to the cost of surplus calculation. That is, our cost of surplus would be lower if we used a market, rather than a statutory, valuation of required capital and surplus.

All of the above assumes that we are dealing with an insurance company, but occasionally the company to be valued is a non-insurance holding company. Usually the actuary would confine himself to the valuation of the insurance subsidiaries, but if they make up the bulk of the holding company's operation, it may be desirable to value the entire operation. If there are any non-insurance subsidiaries they can be carried at book value and so noted to the buyer. As for the holding company itself, an adjustment should be made to reflect the difference between the actuary's valuation of the insurance subsidiaries and the value carried in the parent's financial statement.

There are, of course, adjustments and considerations other than the critical and directly measurable ones enumerated above. Many of these can only be gauged by the prospective buyer and involve operational and financial synergism with his existing operation. However, the actuary can provide input to these considerations with information on cash flows, tax loss carry forwards, etc.

SUMMARY

A major part of valuing a property/casualty company requires an evaluation of future earnings potential, which is a determination that actuaries are most qualified to make. This paper has presented a method for carrying out such a valuation by adapting classical life company valuation methods. While there is considerable fluctuation likely in actual future earnings, a range of reasonable present values can be established. Moreover, that range is typically narrower than the range of reasonable underwriting assumptions. Finally, several adjustments to the formula value were discussed. Depending upon their nature, these adjustments can best be made by the actuary, accountant or prospective buyer.

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EXHIBIT 1

PRESENT VALUES OF STATUTORY GAIN
WRITTEN PREMIUMS THROUGH 2012
W. C. PROTECTIVE

Year	Statutory Gain	Surplus Cost	Adjusted Gain	Dividend	Discount Factors @ 10.00%	Discount Factors @ 15.00%	Discount Factors @ 20.00%
1982	2678	667	2011	0	0.9090909	0.8695652	0.8333333
1983	3124	733	2391	0	0.8264463	0.7561437	0.6944444
1984	3350	807	2543	0	0.7513148	0.6575162	0.5787037
1985	3617	887	2730	0	0.6830135	0.5717532	0.4822531
1986	3913	976	2937	0	0.6209213	0.4971767	0.4018776
1987	4254	1074	3180	0	0.5644739	0.4323276	0.3348980
1988	4645	1181	3464	0	0.5131581	0.3759370	0.2790816
1989	5090	1299	3791	0	0.4665074	0.3269018	0.2325680
1990	5585	1429	4156	0	0.4240976	0.2842624	0.1938067
1991	6138	1572	4566	0	0.3855433	0.2471847	0.1615056
1992	6753	1729	5024	0	0.3504939	0.2149432	0.1345880
1993	7430	1902	5528	0	0.3186308	0.1869072	0.1121567
1994	8173	2092	6081	0	0.2896644	0.1625280	0.0934639
1995	8991	2302	6689	0	0.2633313	0.1413287	0.0778866
1996	9889	2532	7357	0	0.2393920	0.1228945	0.0649055
1997	10878	2785	8093	0	0.2176291	0.1068648	0.0540879
1998	11966	3063	8903	0	0.1978447	0.0929259	0.0450732
1999	13162	3370	9792	0	0.1798588	0.0808051	0.0375610
2000	14479	3707	10772	0	0.1635080	0.0702653	0.0313009
2001	15926	4077	11849	0	0.1486436	0.0611003	0.0260841
2002	17520	4485	13035	0	0.1351306	0.0531307	0.0217367
2003	19271	4934	14337	0	0.1228460	0.0462006	0.0181139
2004	21198	5427	15771	0	0.1116782	0.0401744	0.0150949
2005	23318	5970	17348	0	0.1015256	0.0349343	0.0125791
2006	25649	6566	19083	0	0.0922960	0.0303776	0.0104826
2007	28214	7223	20991	0	0.0839055	0.0264153	0.0087355
2008	31036	7945	23091	0	0.0762777	0.0229699	0.0072796
2009	34140	8740	25400	0	0.0693433	0.0199738	0.0060663
2010	37554	9614	27940	0	0.0630394	0.0173685	0.0050553
2011	41308	10575	30733	0	0.0573086	0.0151031	0.0042127
2012	45440	11633	33807	0	0.0520987	0.0131331	0.0035106
2013	56887	0	56887	0	0.0473624	0.0114201	0.0029255
2014	29656	0	29656	0	0.0430568	0.0099305	0.0024379
2015	19538	0	19538	0	0.0391425	0.0086352	0.0020316
2016	13344	0	13344	0	0.0355841	0.0075089	0.0016930
2017	9200	0	9200	0	0.0323492	0.0065295	0.0014103
2018	6251	0	6251	0	0.0294083	0.0056778	0.0011757
2019	4043	0	4043	0	0.0267349	0.0049372	0.0009797
2020	2432	0	2432	0	0.0243044	0.0042932	0.0008165
2021	1333	0	1333	0	0.0220949	0.0037332	0.0006804
2022	571	0	571	0	0.0200863	0.0032463	0.0005670
2023	125	0	125	0	0.0182603	0.0028229	0.0004725
PRESENT VALUES AT							
10.00%	79945	18788	61157	0			
15.00%	40870	9972	30898	0			
20.00%	25118	6218	18900	0			

EXHIBIT 2

Projection of Statutory Gain
Written Premiums Through 2012
W. C. Protective

Year	Written Premium	Earned Premium	Incurred Claims	Expenses	Investmt Income	Federal Tax	Statutory Gain	Surplus Cost	Dividend
1982	40000	32725	24719	9733	4821	416	2678	667	0
1983	44000	42167	31624	11953	5187	653	3124	733	0
1984	48400	46383	34788	13149	5594	690	3350	807	0
1985	53240	51022	38266	14463	6059	735	3617	887	0
1986	58564	56123	42094	15910	6581	787	3913	976	0
1987	64420	61736	46302	17501	7170	849	4254	1074	0
1988	70862	67909	50932	19251	7841	922	4645	1181	0
1989	77949	74701	56025	21176	8598	1008	5090	1299	0
1990	85744	82171	61628	23293	9440	1105	5585	1429	0
1991	94318	90388	67791	25623	10376	1212	6138	1572	0
1992	103750	99427	74569	28185	11415	1335	6753	1729	0
1993	114125	109369	82027	31004	12560	1468	7430	1902	0
1994	125537	120306	90230	34104	13816	1615	8173	2092	0
1995	138091	132337	99252	37514	15197	1777	8991	2302	0
1996	151900	145570	109178	41266	16717	1954	9889	2532	0
1997	167090	160128	120096	45392	18389	2151	10878	2785	0
1998	183799	176140	132105	49932	20228	2365	11966	3063	0
1999	202179	193754	145316	54925	22251	2602	13162	3370	0
2000	222397	213130	159847	60418	24476	2862	14479	3707	0
2001	244636	234442	175832	66459	26923	3148	15926	4077	0
2002	269100	257887	193415	73105	29616	3463	17520	4485	0
2003	296010	283675	212756	80416	32577	3809	19271	4934	0
2004	325611	312043	234033	88458	35835	4189	21198	5427	0
2005	358172	343247	257435	97303	39418	4609	23318	5970	0
2006	393989	377571	283179	107033	43360	5070	25649	6566	0
2007	433388	415329	311497	117737	47696	5577	28214	7223	0
2008	476727	456862	342647	129510	52466	6135	31036	7945	0
2009	524400	502549	376911	142462	57712	6748	34140	8740	0
2010	576840	552803	414602	156708	63484	7423	37554	9614	0
2011	634524	608083	456063	172379	69832	8165	41308	10575	0
2012	697976	668892	501669	189616	76815	8982	45440	11633	0
2013	0	319929	239947	63986	64419	23528	56887	0	0
2014	0	0	0	0	39596	9940	29656	0	0
2015	0	0	0	0	26087	6549	19538	0	0
2016	0	0	0	0	17816	4472	13344	0	0
2017	0	0	0	0	12283	3083	9200	0	0
2018	0	0	0	0	8346	2095	6251	0	0
2019	0	0	0	0	5398	1355	4043	0	0
2020	0	0	0	0	3248	815	2433	0	0
2021	0	0	0	0	1780	447	1333	0	0
2022	0	0	0	0	763	192	571	0	0
2023	0	0	0	0	167	42	125	0	0

EXHIBIT 3

INCOME AND EXPENSE BY YEAR*
W. C. PROTECTIVE

	1982	1983	1984	1985	1986	1987	1988
Written Premium	40,000	44,000	48,400	53,240	58,564	64,420	70,862
Unearned Premium							
Beginning of Year	11,060	18,335	20,168	22,185	24,403	26,844	29,528
End of Year	18,335	20,168	22,185	24,403	26,844	29,528	32,481
Earned Premium	32,725	42,167	46,383	51,022	56,123	61,736	67,909
Paid Claims	27,250	28,249	30,785	33,673	36,952	40,326	44,046
Claim Reserve							
Beginning of Year	53,000	50,469	53,844	57,847	62,440	67,582	73,558
End of Year	50,469	53,844	57,847	62,440	67,582	73,558	80,444
Incurred Claims	24,719	31,624	34,788	38,266	42,094	46,302	50,932
Percent of Earned	75.54%	75.00%	75.00%	75.00%	75.00%	75.00%	75.00%
Expenses Related to							
Written Premium	3,200	3,520	3,872	4,259	4,685	5,154	5,669
Earned Premium	6,533	8,433	9,277	10,204	11,225	12,347	13,582
Paid Claims	0	0	0	0	0	0	0
Total	9,733	11,953	13,149	14,463	15,910	17,501	19,251
Percent of Earned	29.74%	28.35%	28.35%	28.35%	28.35%	28.35%	28.35%
Total Claims & Exp.	34,452	43,577	47,937	52,729	58,004	63,803	70,183
Percent of Earned	105.28%	103.34%	103.35%	103.35%	103.35%	103.35%	103.35%
Underwriting Gain	(1,727)	(1,410)	(1,554)	(1,707)	(1,881)	(2,067)	(2,274)
Percent of Earned	(5.28)%	(3.34)%	(3.35)%	(3.35)%	(3.35)%	(3.35)%	(3.35)%
Investment Income	4,821	5,187	5,594	6,059	6,581	7,170	7,841
Percent of Earned	14.73%	12.30%	12.06%	11.88%	11.73%	11.61%	11.55%
Pre-Tax Gain	3,094	3,777	4,040	4,352	4,700	5,103	5,567
Percent of Earned	9.45%	8.96%	8.71%	8.53%	8.38%	8.27%	8.20%
Federal Income Tax	416	653	690	735	787	849	922
Percent of Earned	1.27%	1.55%	1.49%	1.44%	1.40%	1.38%	1.36%
After Tax Gain	2,678	3,124	3,350	3,617	3,913	4,254	4,645
Percent of Earned	8.18%	7.41%	7.22%	7.09%	6.97%	6.89%	6.84%
Surplus Cost	667	733	807	887	976	1,074	1,181
Percent of Earned	2.04%	1.74%	1.74%	1.74%	1.74%	1.74%	1.74%
Adjusted Gain	2,011	2,391	2,543	2,730	2,937	3,180	3,464
Percent of Earned	6.14%	5.67%	5.48%	5.35%	5.23%	5.15%	5.10%
Expense % Written	24.33%	27.17%	27.17%	27.17%	27.17%	27.17%	27.17%
Claim % Earned	75.54%	75.00%	75.00%	75.00%	75.00%	75.00%	75.00%
Total	99.87%	102.17%	102.17%	102.17%	102.17%	102.17%	102.17%

* Thirty years were actually run, of which seven are exhibited here.

EXHIBIT 4

Assets, Liabilities, and Value of In-Force*
W. C. Protective

	1982	1983	1984	1985	1986	1987	1988
Invested Assets**	68,804	74,012	80,032	86,843	94,426	103,086	112,925
Liabilities							
Unearned Premiums	18,335	20,168	22,185	24,403	26,844	29,528	32,481
Loss Reserves	50,469	53,844	57,847	62,440	67,582	73,558	80,444
Total Liabilities	68,804	74,012	80,032	86,843	94,426	103,086	112,925
Capital and Surplus							
Capital	0	0	0	0	0	0	0
Surplus	0	0	0	0	0	0	0
Total	0	0	0	0	0	0	0
Total Liabilities Capital & Surplus	68,804	74,012	80,032	86,843	94,426	103,086	112,925
Present Value of Future Statutory Gains from In-Force							
@ 10.00%	7,137	7,592	8,151	8,816	9,597	10,496	11,514
@ 15.00%	6,456	6,872	7,379	7,981	8,686	9,498	10,418
@ 20.00%	5,892	6,276	6,741	7,290	7,933	8,674	9,513
Capital & Surplus Plus Value of In-Force							
@ 10.00%	7,137	7,592	8,151	8,816	9,597	10,496	11,514
@ 15.00%	6,456	6,872	7,379	7,981	8,686	9,498	10,418
@ 20.00%	5,892	6,276	6,741	7,290	7,933	8,674	9,513
Surplus Reconciliation							
Beginning of Year	0	0	0	0	0	0	0
Underwriting Gain	⟨1,727⟩	⟨1,410⟩	⟨1,554⟩	⟨1,707⟩	⟨1,881⟩	⟨2,067⟩	⟨2,274⟩
Investment Income	4,821	5,187	5,594	6,059	6,581	7,170	7,841
Pre-Tax Gain	3,094	3,777	4,040	4,352	4,700	5,103	5,567
Fedl. Income Tax	416	653	690	735	787	849	922
Stockholder Divs.	0	0	0	0	0	0	0
End of Year	0	0	0	0	0	0	0

* Thirty years were actually run, of which seven are exhibited here.

** Since earnings are attributed to the investor, invested assets are deemed equal to liabilities.

CREDIBILITY-WEIGHTED TREND FACTORS

OAKLEY E. VAN SLYKE

Abstract

The credibility of trend lines is important because trend lines cannot be extrapolated reliably far into the future. Credibility-weighted trend factors can be calculated if two or more alternative assumptions are considered. The effects of changes in the goodness of fit of the trend lines being considered can also be explored.

This paper approaches the problem by ad hoc blending of alternative sets of hypotheses. The appropriateness of the method is argued by analogy with Empirical Bayesian credibility formulas. A specific example is used throughout.

In this example, a particular pair of alternative assumptions is considered—that there is no trend and that there is linear trend. The results suggest that an increase in the R^2 of the linear trend line may imply an increase in the credibility of the trend line, reliance on a greater amount of trend, or a more reliable resulting estimate. Which of these or which combination of these in the case depends on the data at hand. A greater R^2 does not necessarily imply greater credibility for trend.

The methods shown in this paper can be extended to other sets of assumptions, and other questions about the appropriateness of trend assumptions can also be studied.

Introduction

Trend lines are used in ratemaking in virtually all lines of insurance. The purpose of introducing a calculation of trend into a rate derivation is to arrive at an estimate of future loss costs that reflects the changes in loss costs over time.

Trend was introduced into workers' compensation ratemaking in the late 1970's. An example of a trend calculation by the National Council on Compensation Insurance (NCCI) is shown in Exhibit I. This is a particularly good example of the calculation of a trend factor for two reasons. First, the various subtotals that go directly into the calculation of the trend line are shown explicitly. Second, the trend factor finally derived is a credibility-weighted trend factor, and such factors are the subject of this paper.

Problems with the Use of Trend Factors

The academic training of actuaries gives them a general awareness that trend lines cannot be extrapolated reliably very far into the future. Here "very far into the future" is a vague notion, but it clearly has something to do with the length of the time series that is used in the trend calculation.

In the case of workers' compensation data, there has traditionally been some doubt as to whether an underlying trend exists at all. The use of payroll as a measure of exposure and the special handling of law amendments were intended to encompass the economic changes that would affect losses. As economic indices are used more often in other lines in the coming years, these lines, too, will generate times series data in which there is some *a priori* doubt about the assumption that there is any remaining trend.

This situation has led to a study of the credibility of trend factors. To what extent should the trend forecast be relied on, and to what extent the historical average? The answer depends on the situation at hand and on the length of the time series and the goodness of fit of the trend line. There is a practical problem in tying these considerations together.

The NCCI has adopted a framework for computing the credibility-weighted trend factor. This is illustrated in Exhibit I. This paper is not intended to be a review or criticism of the NCCI method. It is intended rather to illustrate an alternate approach.[1]

Purpose

If the actuary does not use credibility-weighted trend factors, or something equivalent, he must rely on a single assumption about the population from which his sample data was drawn. He might assume, for example, that all of the sample values are from a population with a mean (expected value) that is unchanging. Or he might assume that the sample values are from a population with a mean that is changing steadily over time. He might assume that the steady change is linear, quadratic, exponential or some other form. Whatever assumption he makes, he must use the indicated results of that one assumption. One purpose of this paper is to show that the actuary's options are not so limited. The paper proposes a method for combining the projections from two or more sets of assumptions, rather than having to choose between them.

[1] Charles A. Hachmeister and G. C. Taylor have proposed other methods in papers in *Credibility: Theory and Applications*, P. M. Kahn, Ed., Academic Press, 1975.

Because of the reliance placed on the fraction of variance explained, R^2, in the application of trend factors derived by the regression analysis, this paper has a second purpose. It seeks to examine the implications of R^2 on (1) the credibility of the slope of the trend line, (2) the slope of the trend line and (3) the accuracy of the resulting forecast. By doing this for a particular application of the concepts of the first section, it intends to provide an example of how the effects of R^2 can be examined in other applications. This paper suggests some interesting conclusions. These are:

1. If only two alternative assumptions are considered—no trend and linear trend—and no *a priori* judgments are introduced, then the credibility-weighted trend factor declines asymptotically to zero as the length of the projection increases.
2. For these same two alternative assumptions, an increase in R^2 from one application to the next implies an increase in the credibility of the trend line, or reliance on a greater amount of trend, or a more reliable resulting estimate. A combination of these is also possible. Which of these three situations is really the case depends on the problem at hand. One cannot generally assume that a greater value of R^2 in one application than in another will imply greater credibility for trend.

Derivation of Credibility-Weighted Trend Factors

The purpose of this section is to show that it is not necessary to make a single assumption about the trend in order to estimate the value of a time series at some time in the future. This is shown by deriving a trend line by assuming that: (1) either there is no trend, or (2) there is a linear trend. The steps shown here could be extended to allow three or more assumptions to be reflected in the computation. Two assumptions are used to simplify the mathematics.

The projection for the value at time X depends on the assumption about trend that is being used. If the assumption that there is no trend is being used, the estimate of the value at any time in the future would be the average of the historical values, i.e.,

$$\hat{Y}(X) = \overline{Y} = (\Sigma Y_i)/n \qquad (1)$$

for all X.

(There is no discussion of maximum likelihood or minimum variance in this statement or those which follow. This would be a useful addition to this work. Also, it should be clear that all of the summations are for $i = 1, \ldots, n$.)

If the assumption is that there is linear trend, the estimate of the value at some time X would be

$$\hat{Y}(X) = \overline{Y} + \frac{\Sigma(X_i - \overline{X})(Y_i - \overline{Y})}{\Sigma(X_i - \overline{X})^2} \cdot (X - \overline{X}), \tag{2}$$

where $\overline{X} = (\Sigma X_i)/n$.

In the problem we are dealing with, we do not wish to choose between these estimates because that would be the same as choosing between the alternative assumptions. Instead, we wish to regard each estimate as a valid estimate based on the data at hand.

If each estimate is a valid estimate based on the data on hand, then we have no preconceived way of improving any of the estimates. We know of no correction terms which can be added *a priori* to improve either of the estimates. In other words, for each estimate

E [estimate of Y] $= Y$.

In statistical terms, each estimate is unbiased.

In most of our experience with estimators we are accustomed to the idea that only one of several alternative models can be unbiased. For example, if the model of linear trend is unbiased, the model of no trend must be biased. The formula omits the term for the trend component. How then, can each of the estimates be unbiased, as stated above? The answer is that we are not dealing with models in the formulation above. We are dealing only with empirical evidence and what can be learned from it. And given only the *data* at hand, each estimate is unbiased.[2]

---

[2] Consider a set of alternative states of the world, θ. Each value, θ_i, is associated with a particular model being valid. We do not know which value of θ exists for our problem, since we have only empirical evidence about the problem. The discussion above states that

$E[\overline{Y}|\theta_1] = Y$ and

$$E\left[\overline{Y} + \frac{\Sigma(X_i - \overline{X})(Y_i - \overline{Y})}{\Sigma(X_i - \overline{X})^2} \cdot (X - \overline{X})|\theta_2\right] = Y.$$

This does not imply that $E\left[\frac{\Sigma(X_i - \overline{X})(Y_i - \overline{Y})}{\Sigma(X_i - \overline{X})^2} \cdot (X - \overline{X})\right] = 0.$

The mathematics of the approach parallels that of empirical Bayes methods of Hans Bühlmann, *Mathematical Methods in Risk Theory*, Springer-Verlag, New York, New York, 1970, pp. 93–110.

A theorem of statistics states that if two estimators are unbiased and independent, then the minimum variance estimator is the weighted average of the two estimators with weights inversely proportional to the variances of the two (c.f., D. A. S. Fraser, *Probability and Statistics*, Duxbury Press, 1976, p. 382). This theorem can be applied to $\hat{Y}(X) - \bar{Y}$, which is zero in the first case and

$$\frac{\Sigma(X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma(X_i - \bar{X})^2} \,.\, (X - \bar{X}) \tag{3}$$

in the second case.

We have changed the definition of the problem now and ought to check that we are still solving the problem we want to solve. The new problem is to estimate the amount by which the time series will exceed its historical average (as it is known now) at some time in the future. This is not quite the same problem, but it certainly encompasses our reasons for using trend lines.

To apply the theorem we need to know only the variance associated with each estimate. The variance in the first estimate is the population sample variance,

$$V_A = \frac{\Sigma(Y_i - \bar{Y})^2}{n - 1} \tag{4}$$

The variance of the second, trended estimate is

$$V_T = \frac{\Sigma(Y_i - \bar{Y})^2}{n - 1}\left[\frac{1}{n} + \frac{(X - \bar{X})^2}{\Sigma(X_i - \bar{X})^2}\right] \tag{5}$$

The desired estimator of $\hat{Y}(X) - \bar{Y}$ is, therefore,

$$= \frac{\dfrac{1}{V_A} \cdot 0 + \dfrac{1}{V_T} \cdot \dfrac{\Sigma(X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma(X_i - \bar{X})^2} \cdot (X - \bar{X})}{\dfrac{1}{V_A} + \dfrac{1}{V_T}} \tag{6}$$

$$= \frac{V_A \cdot \dfrac{\Sigma(X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma(X_i - \bar{X})^2} \cdot (X + \bar{X})}{V_A + V_T}$$

$$= \frac{V_A \cdot \dfrac{\Sigma(X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma(X_i - \bar{X})^2} \cdot (X + \bar{X})}{V_A + V_A \cdot \left[\dfrac{1}{n} + \dfrac{(X - \bar{X})^2}{\Sigma(X_i - \bar{X})^2}\right]}$$

$$= \frac{\Sigma(X_i - \overline{X})(Y_i - \overline{Y})}{\frac{n+1}{n}\Sigma(X_i - \overline{X})^2 + (X - \overline{X})^2} \cdot (X - \overline{X} \qquad (7)$$

This is similar to the trend estimate. The difference is in the denominator of the slope, which now includes the term $1/n \cdot \Sigma(X_i - \overline{X})^2 + (X - \overline{X})^2$. This is a quadratically increasing function of $X - \overline{X}$, so the credibility-weighted trend line is a declining function of X as X moves away from $\overline{X}$. In fact, this estimate of $\hat{Y}(X) - \overline{Y}$ tends to zero as $X - \overline{X}$ gets very large, which means the credibility of the trend goes to zero as the extrapolation is taken far into the future.

Exhibit I provides the data for a numerical example. (We shall ignore the problems caused by autocorrelation in the observed values for loss ratios; they are beyond the scope of this paper.) The key values can be taken from Exhibit I as follows:

$$\begin{aligned} n &= 9 \\ \overline{X} &= 2 \\ \Sigma(X_i - \overline{X})(Y_i - \overline{Y}) &= \Sigma X_i Y_i - (\Sigma X_i)(\Sigma Y_i)/n \\ &= 11.354 - 18 - 5.334/9 \\ &= .686 \\ \Sigma(X_i - \overline{X})^2 &= \Sigma X_i^2 - (\Sigma X_i)^2/n \\ &= 51 - 18^2/9 \\ &= 15 \end{aligned}$$

The slope of the trend line, assuming a linear trend exists, is .686/15, or .0457. The height of the revised trend line, without assuming that a trend line exists (but assuming that if it does not there is no change in the expected value of the loss ratio over time), is

$$\hat{Y} - \overline{Y} = \frac{.686}{\frac{50}{3} + (X - \overline{X})^2} \cdot (X - \overline{X})$$

Extrapolated values of the time series of loss ratios are shown in Exhibit II.

For this set of data and this set of alternative assumptions, the credibility-weighted trend line is well below the linear regression trend line. This is because of the set of alternative assumptions used.

The trend, if any, could be exponential or quadratic, and considering these possibilities would raise the credibility-weighted trend line. *A priori* considerations could also lead one to give greater weight to the linear trend line. This paper does not advocate the use of the two-assumption formula in equation (7), but uses it to illustrate a general approach for determining credibility-weighted trend factors by averaging several separate projections using weights inversely proportional to each projection's variance.

There is another reason for the low trend line: the linear trend line is based on only nine data points. It is therefore not reliably estimated from the data alone.

The Effects of R^2 on the Credibility-Weighted Trend Factors

One would expect that the better the fit of the linear regression, the more credible the trend factors would be. This turns out to be the case, but only in a limited way. This section shows that for a given number of historical observations:

- If the slope of the trend line and the variance of the observations are held constant, an increase in R^2 implies an increase in the credibility of the trend line.
- If the variance of the independent variable and the variance of the observations (the dependent variable) are held constant, an increase in R^2 increases the slope of the trend line but not necessarily its credibility.
- If the variance of the independent variable and the slope are held constant, an increase in R^2 does not affect the credibility of the trend line. It does, however, increase the credibility of any forecasts based on the credibility-weighted trend line, the trend line or the simple average.

We must begin by deriving the credibility of the trend that is implicit in the credibility-weighted trend line. Equation (6) shows that the credibility of the trend estimate is

$$Z = \frac{\frac{1}{V_T}}{\frac{1}{V_A} + \frac{1}{V_T}}$$

This is what one would expect from the statistical theorem. This can be repressed in terms of the data as:

$$Z = \frac{V_A}{V_A + V_A\left[\frac{1}{n} + \frac{(X - \overline{X})^2}{\Sigma(X_i - \overline{X})^2}\right]}$$

$$= \frac{1}{1 + \frac{1}{n} + \frac{(X - \overline{X})^2}{\Sigma(X_i - \overline{X})^2}}$$

$$= \frac{n}{n + 1 + \frac{(X - \overline{X})^2}{\Sigma(X_i - \overline{X})^2/n}}$$

This is the familiar form for credibility. The number of points in the time series plays the role of exposure, n, and the "exposure constant" K is a function of the length of the extrapolation and the spread of the independent observations about their mean.

In terms of the data from which it is calculated, R^2 can be expressed as

$$R^2 = \frac{[\Sigma(X_i - \overline{X})(Y_i - \overline{Y})]^2}{\Sigma(X_i - \overline{X})^2\Sigma(Y_i - \overline{Y})^2}$$

An abbreviated notation will make the relationships clearer. Let

$$SS_{XY} = \Sigma(X_i - \overline{X})(Y_i - \overline{Y})$$

$$SS_X = \Sigma(X_i - \overline{X})^2$$

$$SS_Y = \Sigma(Y_i - \overline{Y})^2$$

Then

$$R^2 = \frac{SS^2{}_{XY}}{SS_X SS_Y}$$

$$Z = \frac{n}{n + 1 + \frac{(X - \overline{X})^2}{SS_X/n}}$$

The credibility-weighted trend factor is

$$\frac{SS_{XY}}{\frac{n+1}{n} SS_X + (X - \overline{X})^2} \cdot (X - \overline{X})$$

The trend factor itself is

$$\frac{SS_{XY}}{SS_X} \cdot (X - \overline{X})$$

and the slope of the trend line is SS_{XY}/SS_X.

If the slope of the trend line, SS_{XY}/SS_X, and the variance of the observations, $SS_Y/n-1$, are both held constant, then an increase in R^2 implies an increase in $SS_{XY}/n-1$. Since SS_{XY}/SS_X is constant, this implies an increase in $SS_X/n-1$. An increase in $SS_X/n-1$ implies an increase in Z, and the first point is established.

If the variance of the independent variable, $SS_X/n-1$, is held constant, z is a function of n and $(X - \bar{X})$ only. If $SS_X/n-1$ and the variance of the observations, $SS_Y/n-1$, are held constant, an increase in R^2 implies an increase in $SS_{XY}/n-1$, and hence of the trend factor itself. This establishes the second point.

If the variance of the independent variable, $SS_X/n-1$, and the slope, SS_{XY}/SS_X, are held constant, an increase in R^2 implies a decrease in $SS_Y/n-1$. This does not affect either the trend or the credibility of the trend. The variance of the credibility-weighted estimate is (see Fraser, op. cit):

$$\frac{1}{\dfrac{1}{V_A} + \dfrac{1}{V_T}} = \frac{V_A \cdot V_T}{V_A + V_T}$$

$$= \frac{V_A \cdot V_A \cdot \left[\dfrac{1}{n} + \dfrac{(X - \bar{X})^2}{SS_X}\right]}{V_A + V_A \cdot \left[\dfrac{1}{n} + \dfrac{(X - \bar{X})^2}{SS_X}\right]}$$

$$= [SS_Y/(n - 1)] \frac{\dfrac{1}{n} + \dfrac{(X - \bar{X})^2}{SS_X}}{1 + \dfrac{1}{n} + \dfrac{(X - \bar{X})^2}{SS_X}}$$

Therefore, a decrease in $SS_Y/n-1$ implies a decrease in the variance of the credibility-weighted estimate. The rest of the third point can be demonstrated using a similar analysis.

In summary, for a given number of observations, an increase in R^2 implies an increase in the credibility of the trend line, or reliance on a greater amount of trend, or a more reliable resulting estimate. A combination of these is also possible. Which of these three situations is really the case depends on the problem at hand.

These conclusions rest on the choice of alternative assumptions that was made. That choice was (1) that there is no trend, or (2) that there is linear trend. And the phrase "more reliable" is only valid in its least-squares sense. Still, these conclusions point up the fact that a greater R^2 does not necessarily imply greater credibility for trend.

Summary

The credibility of trend lines is important because trend lines cannot be extrapolated reliably far into the future. Credibility-weighted trend factors can be calculated if two or more alternative assumptions are considered. The effects of changes in the goodness of fit of the trend lines being considered can also be explored.

The methods shown in this paper can be extended to other sets of assumptions. Other questions about the factors that contribute to the appropriateness of trend assumptions can also be studied.

If a particular pair of alternative assumptions is considered—that there is no trend and that there is linear trend—an increase in the R^2 of the linear trend line may imply an increase in the credibility of the trend line, reliance on a greater amount of trend, or a more reliable resulting estimate. Which of these or which combination of these is the case depends on the data at hand. A greater R^2 does not necessarily imply greater credibility for trend.

EXHIBIT I

NATIONAL COUNCIL ON COMPENSATION INSURANCE

NATIONAL COUNCIL ON COMPENSATION INSURANCE

CALCULATION OF TREND FACTOR

(0) Temporal Rank	(1) Calendar Year	(2) Index (x) For (1)	(3) Standard Earned Premium	(4) Incurred Losses Incl. Loss Adj.	(5) Factor To Adjust Premium	(6) Factor To Adjust Losses	(7) Earned Premium On Level (3)x(5)	(8) Incurred Losses On Level (4)x(6)	(9) Loss Ratio (y) (8)÷(7)	(10) Loss Ratio Rank	(11) x^2 $(2)^2$	(12) xy (2)x(9)
1	1973	0.0	101,757,432	78,532,264	1.606	1.075	163,422,436	84,422,184	.517	1	0.00	0.000
	1973-74	0.5	109,194,609	82,846,099	1.536	1.070	167,722,919	88,645,326	.529		0.25	0.265
2	1974	1.0	110,495,110	82,016,281	1.477	1.069	163,201,277	87,675,404	.537	2	1.00	0.537
	1974-75	1.5	113,385,164	86,052,451	1.413	1.059	160,213,237	91,129,546	.569		2.25	0.854
3	1975	2.0	116,229,990	90,187,533	1.341	1.050	155,864,417	94,696,910	.608	4	4.00	1.216
	1975-76	2.5	125,236,306	89,761,215	1.270	1.048	159,050,109	94,069,753	.591		6.25	1.478
4	1976	3.0	141,708,133	97,926,553	1.205	1.042	170,758,300	102,039,468	.598	3	9.00	1.794
	1976-77	3.5	162,912,010	117,405,688	1.132	1.038	184,416,395	121,867,104	.661		12.25	2.314
5	1977	4.0	180,000,246	134,741,173	1.060	1.025	190,800,161	138,109,702	.724	5	16.00	2.896
TOTAL		18.0	xx	xx	xx	xx	xx	xx	5.334	xx	51.00	11.354

13. $D = \Sigma\ ((0) - (10)^2 = 0 + 0 + 1 + 1 + 0$ 2

14. Mid-point of Experience in Filing from 7-1-73 .... 4-1-77 or 3.7500

15. Mid-point of Period during which proposed rates effective .... 9-1-79 or 6.1667

16. Annual Increment in Loss Ratio = $B = \left[9\ \Sigma\ (12) - \Sigma\ (2)\ \Sigma\ (9)\right] \div \left[9\ \Sigma\ (11) - (\Sigma\ (2))^2\right]$ .... .0457

17. Loss Ratio at Base = $A = \left[\Sigma\ (9) - (16)\ \Sigma\ (2)\right] \div 9$ .... .5013

18. Probability of $D \leq (13)$ .... .0417

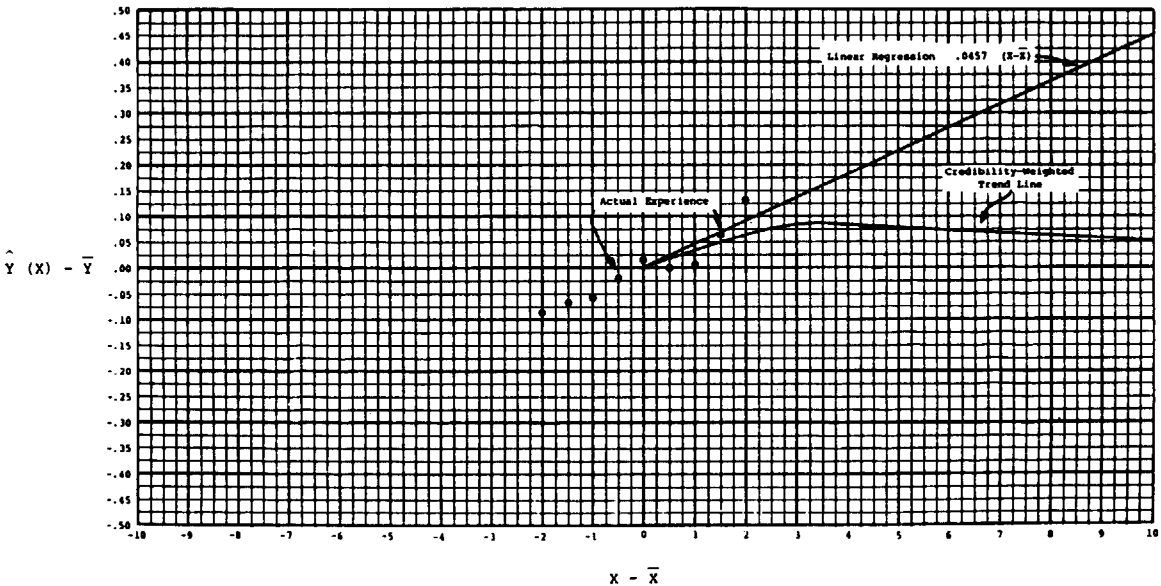
19. Credibility $\left[100\ (.5 - 2 \times (18))^{1/2}\right] \div .7$ 92%

20. Trend Factor prior to credibility $\left[(17) + (16)(15)\right] \div \left[(17) + (16)(14)\right]$ 1.163

21. Credibility weighted Trend Factor $\left[(19) \times (20)\right] + \left[(1.00 - (19)) \times 1.000\right]$ 1.150

EXHIBIT II

CREDIBILITY-WEIGHTED TREND LINE



RLS YARDSTICKS TO IDENTIFY FINANCIAL WEAKNESS

RUTH E. SALZMANN

Abstract

At the present time the regulators have two early warning systems to assist in identifying financially troubled insurers. These are the NAIC IRIS ratios[1] and the AIA Index of Financial Strength.[2] This paper recommends a third.

The goal of each of these systems is to identify the financially troubled company that can be helped to regain an acceptable financial footing. To identify financially strong companies serves little constructive purpose. The primary need is to identify those companies that can be salvaged. Quantitative yardsticks are never conclusive in themselves, nor will they uncover intentionally dishonest or fraudulent managements in sufficient time. The benefit, if there is to be any, will be in identifying potential insolvencies that can be prevented or in identifying insolvencies so as to minimize further loss.

There are perhaps seven areas of critical financial significance: reserve level, surplus level, liquidity, quality of assets, operating results, excessive growth, and reinsurance protection. The RLS yardsticks place primary emphasis on evaluations of reserve, liquidity, and surplus levels. These evaluations, all of which use data presented in the Annual Statement, are set forth in three exhibits producing two yardsticks. The exhibits at the end of this paper detail the arithmetic; the following comments explain the basis and rationale of those calculations.

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[1] National Association of Insurance Commissioners, "Using the NAIC Insurance Regulatory Information System, Property and Liability Edition," published annually.

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[2] Aetna Life and Casualty, "American Insurance Association, Property–Liability, Early Warning System Proposal," July 1978.

EXHIBIT R

Exhibit R evaluates reserve levels and provides input for Exhibits S and L. The calculation of reserve developments in Section I of Exhibit R is the same as the calculation of reserve developments in IRIS ratios 9 and 10 except that:

1. Reserve developments are compiled for the prior eight accounting dates[3] rather than for only the prior two accounting dates in IRIS ratios 9 and 10.
2. A reconciliation of Schedules O and P data is required before advancing in the calculation. This step is important to insure the integrity of subsequent calculations. From my experience, errors in accumulations of data in Schedules O and P are too frequent to omit such a check.

Once the reserve developments are calculated for prior accounting dates, an evaluation of current reserve levels can be made therefrom. Section II of Exhibit R is included for that purpose. This evaluation borrows from a prior paper of mine, "Schedule P on a Calendar/Accident Year Basis."[4] It was this paper that gave birth to the present Schedule P - Part 3 format. Schedule P - Part 3 sets forth data in a manner that assists in the evaluation of reported reserves as of the current accounting date. Such an evaluation is based on comparisons of current unpaid levels with restated unpaid levels of prior accident years at the same stage of development. These comparisons are detailed by coverage by accident year.

Exhibit R, like Schedule P - Part 3, provides data for comparisons of current unpaid levels with restated unpaid levels of prior reserve dates at the same stage of development. There are these two differences:

1. Schedule P - Part 3 sets forth data by coverage; Exhibit R, for all lines combined.
2. Both exhibits set forth paid and restated unpaid detail by age of development. Schedule P - Part 3 shows this detail for each accident year (n) with developments beginning $1/1/n$. Exhibit R shows this detail for each reserve date ($12/31/n$) with developments beginning $1/1/n+1$.

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[3] The maximum runoff period in Schedules O and P is eight years. Because Schedule O - Part 3 was not introduced until 1976, the maximum period of eight years will not become a reality for all lines until 12/31/83.

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[4] Ruth E. Salzmann, "Schedule P on a Calendar/Accident Year Basis," *PCAS* LIV (1967), p. 120.

Exhibit R - Section II and Schedule P - Part 3 both provide data to assist in a prospective evaluation of current reserve levels. IRIS ratio 11 is also a calculation of current reserve sufficiency. Section II of Exhibit R differs from this latter yardstick as follows:

1. Developed reserves for the prior eight reserve dates[5] are available in Exhibit R; only two prior reserve dates are available in IRIS ratio 11.
2. Paid and restated unpaid components of developed reserves are set forth in Exhibit R, thus enabling a more critical comparison with prior years at the same stage of development.
3. The acceptable current reserve level in IRIS ratio 11 is the average of the ratios of developed reserves to premiums earned for the two prior reserve dates. The determination of an acceptable reserve level in Exhibit R is not a precise calculation; it is derived after a progressive review process, starting with an evaluation of the current unpaid level in the oldest reserve date and proceeding to each subsequent reserve date in order (see Exhibit R-1).

Thus Exhibit R, as proposed, combines the best concepts in both Schedule P - Part 3 and IRIS ratio 11.

Exhibit R makes it possible to determine an acceptable reserve level by making comparisons in one or more of the following ways:

1. By comparing the variation or trend in ratios of developed reserves to calendar-year premiums earned for each of the eight prior reserve dates. This type of comparison is the common feature in Exhibit R and IRIS ratio 11.
2. By comparing current unpaid levels in developed reserves with restated unpaid levels at the same stage of development for prior reserve dates. This type of comparison is the common feature in Schedule P - Part 3 and Exhibit R. Though the format is common to both, there is an important distinction in the content. Exhibit R sets forth unpaid levels in developed reserve data, and Part 3 of Schedule P sets forth unpaid levels in developed accident year data.
3. By comparing unpaid increment levels (for the additional accident year) with restated unpaid increment levels at the same stage of development for prior reserve dates. A further explanation of this approach is in order. In the evaluation of reserve levels in Section II of Exhibit R, one readily realizes that the unpaid amount in current developments for reserve date

[5] As noted above, eight years will not become a reality until 12/31/83.

12/31/n is the sum of the unpaid amount in current developments for reserve date 12/31/$n-1$ plus the increment for accident year n. Paid dollars can also be sorted into accident year n and accident years $n-1$ and prior. Thus, the format of Section II makes it possible to compare the unpaid level for each accident year increment with the respective increments for prior accident years at the same stage of development. (Exhibit R-1 sets forth a strictly arithmetic procedure to illustrate this approach.)

When any of the above comparisons give cause to make an adjustment, such an adjustment can be entered on the additional line provided for that purpose in Exhibit R Section II. The analyst can use this space to override any current unpaid amount he deems necessary.

The review of reserve levels starts with the oldest reserve date and proceeds to each subsequent reserve date in order. Each review evaluates the current unpaid level in the developed reserves for that reserve date. Adjustments, or overriding of current data, can be made at any step in the review process. Such adjustments will then require recalculations of unpaid entries for earlier development dates before advancing to the next reserve date. This review process continues until reserve levels (line 24) for the current and immediately prior reserve dates can be accepted or adjusted for use in Exhibits S and L.

Although any of the three methods named above can be used to evaluate reserve levels in Section II of Exhibit R, the author prefers method 3. Method 1 is used in IRIS ratio 11, but calendar year premiums earned is a very crude yardstick for reserve levels; it is appropriate only when there is a consistent earned premium growth. Method 2 is an improvement on method 1 because it eliminates the calendar year premiums earned base and substitutes the "paid/unpaid status" as the basis for evaluation. Method 3 also uses paid/unpaid comparisons, but it adds a refinement to reflect changes in the age-of-claim mix due to variations in the impact of the latest accident year involved.

Method 3 is particularly helpful when material changes occur in the growth rates of calendar year premiums earned. This is because premiums earned affect new claim levels but not prior claim levels. In method 3, this impact can be quantified by an arithmetic approach which averages the respective unpaid levels of the prior two accident years (see Exhibit R-1); or one can use an arithmetic approach which trends such levels; or one can select values on the basis of judgment. Selecting values need not be based solely on a review of comparable unpaid levels; comparable paid activity levels for the added accident year also can be reviewed and used in the evaluation process. On whatever basis the analysis is made, Exhibit R provides an excellent format for evaluating and developing the reserve amounts needed for Exhibits S and L.

The above commentary sets forth the use of Exhibit R in the RLS System. A further use of Exhibit R becomes readily apparent. Section II, which sets forth the pay-out patterns of total reserves over subsequent calendar years, could serve as the basis for estimating future investment income attributable to such reserves. In my Presidential Address,[6] I suggested an accounting alternative to "discounted loss reserves" in fire/casualty financial reporting. This alternative would report the loss and loss expense reserves in ultimate dollars and then establish an asset or contra account for the investment income offset. The pay-out pattern in Section II of Exhibit R would provide the data necessary to quantify such an account.

EXHIBIT S

Exhibit S calculates the Index of the Surplus Position. The composition of this index is based on several considerations:

1. If loss and loss expense reserves can be combined with reported surplus in any analysis, one need not concern oneself with the level of current reserves.
2. If the level of current reserves is not a factor, then the Excess Statutory Reserves on page 3, line 16, can be added to surplus.
3. Traditionally, premium/surplus rules-of-thumb have been higher for casualty companies than those for fire companies. And Group A&H premium/surplus ratios, when addressed, generally have been higher than casualty. Thus, to the extent that the mix of business affects the volatility of results, such mix should be addressed in measuring the adequacy of a surplus position.
4. A surplus-aid reinsurance treaty is a useful and legitimate tool in the management of an insurance company; however, it is generally a recognition by management that the reported surplus would otherwise be at an undesirable level. Thus, any measurement of the adequacy of the surplus position should override this "managed" result.

[6] Ruth E. Salzmann, "Accountability: The Actuarial Imperative," *PCAS* LXVI (1979), p. 74.

Reflecting on these four matters, the author constructed the following formula:

$$\text{Index of Surplus Position} = \frac{\text{Pure Premium} - K + \text{Surplus}}{\text{Premiums Earned}}$$

$$= \frac{\text{O/S}_{12/31/n} + \text{Pd}_n - \text{Restated O/S}_{12/31/n-1} - K + \text{Restated Surplus}_{12/31/n}}{\text{P.E.}_{\cdot n}}$$

Where: $\text{P.E.}_{\cdot n}$ is subject to a maximum pure premium of 79%, and K is an additional risk provision for the more volatile exposures.

The formula does these things:

1. The formula establishes the inherent expense loading as a crude measurement of the surplus protection needed. The assumption underlying this premise is that the variation in the expense loading is a rough approximation of the variation in the volatility of underwriting results by major coverage grouping. The author makes this assumption, not because of any specific proof, but because the assumption is generally consistent with the traditional premium/surplus rules-of-thumb in current use. Criticisms of a strict adherence to the expense loading assumption can be accommodated by refinements as deemed necessary. The author recommends these two:

   a. The formula establishes a minimum level for premiums earned to protect against the extreme case where an excessive loss and loss expense ratio would otherwise allow a low or even negative surplus position. This minimum level was set at an estimated pure premium of 79%. (Step 10 in Exhibit S makes this calculation.) The 79% was derived by working backwards from a surplus-index floor of .957 and a 6-to-1 premium/surplus relationship. This calculation and the surplus-index floor are discussed in more detail later in the paper.

   b. The formula incorporates an adjustment for the more volatile exposures. This adjustment (K) increases the needed surplus level to the degree that such exposures are involved. The calculation of the current K factors is set forth in Exhibit S-1. Because the K factors compensate for the expected greater volatility in these lines, these factors are derived from respective standard deviations (σ's) of the loss and loss adjustment expense ratios. The K adjustment is the difference in percentage points that the number of σ's *needed* for each K exposure exceeds its respective expense loading percentage.

The number of σ's *needed* for each K line is set to be equivalent to the σ multiple in the expense loading for the total of "other" fire/casualty lines. (Footnote (c) in Exhibit S-1 details the lines included in "other.") Industry loss and loss expense ratios[7] for the last eight years were used in the calculations. (When more industry history becomes available, the number of years perhaps should be increased to ten or twelve.)

The industry expense-loading percentage for "other" lines is the complement of the average loss and loss expense ratio for the past eight years; it equates to 8.36 σ's of that loss and loss expense ratio history (see Exhibit S-1). To the extent that the expense loadings for Allied Lines, Farmowners, Homeowners, Reinsurance and International lines fall short of 8.36 σ's of their respective loss and loss expense ratio histories, the surplus level needed is increased by these K percentages of respective premiums earned.

The Reinsurance and International line was included as a K line even though the K factor in Exhibit S-1 is only 4.1 percentage points. When a longer base period becomes available, this line will undoubtedly show greater volatility and will require a higher K adjustment.

The K adjustments are made by line rather than as a group for two reasons. The first is that all four lines, albeit in varying degrees, are covers for catastrophe perils. For this reason, combining the coverages is not likely to reduce volatility or materially affect the total adjustment needed. The second reason is that the surplus needed by an individual insurer is more appropriately reflected by using separate K factors by line because the K adjustments vary by line and because the mix of these four lines varies by insurer.

2. The formula also modifies reported surplus to adjust for excess statutory reserves and surplus aid (as defined and quantified in Step D of IRIS ratio 3). The reasons for these adjustments were noted previously.

3. The formula, by using the modified expense loading assumption, makes it possible to combine current reserves and adjusted surplus in the numerator. (Only reserves as of the prior year-end, already one year developed, need further review and adjustment.) Thus, the Index of Surplus

[7] A. M. Best Company, "Aggregates & Averages, Property Casualty," 1978–1981.

Position neatly requires more reported surplus if current reserves are understated, and less reported surplus if current reserves are overstated.

This Index of Surplus Position combines the purposes of IRIS ratios 1, 3, 9, 10, and 11. The author suggests that a desirable index be greater than or equal to 1.04, with a suggested floor of .957. The calculation of the 1.04 equates to the 3-to-1 premium/surplus yardstick in IRIS ratio 1 except that earned rather than written premiums are used as a base (see Exhibit S-2). The .957 index floor equates to a 4-to-1 premium/surplus level, or 75% of the surplus level inherent in the 1.04 index. The .957 floor is then used to establish the maximum pure premium percentage included in the formula. This maximum should be at a level appropriate for traditionally high loss ratio lines such as Standard Group A&H insurance, where surplus requirements are generally lower. Assuming a 6-to-1 premium/surplus requirement, the maximum pure premium percentage becomes 79% (.957 − .167).

As of 12/31/80, the industry's premium written/surplus multiple, using Best's consolidated data,[8] was 1.83. The Index of Surplus Position calculated for the industry as of that date (assuming a modest 12/31/79 reserve inadequacy) was 1.28. This comparison does not mean that a 1.83 premium/surplus multiple is equivalent to an index of 1.28; it merely presents the relationship between the two yardsticks as of 12/31/80 given the formula components existing at that time.

EXHIBIT L

Exhibit L calculates the Index of Liquidity Position. Whereas the Index of Surplus Position measures the resources an insurer has to absorb above-average underwriting and investment losses, the Index of Liquidity Position measures the financial flexibility an insurer has to withstand unexpected changes in operational demands. Liquidity is the measurement of the nearness to cash of assets and liabilities. An insurer is exposed to insolvency hazards because of both insufficient surplus and insufficient financial flexibility levels.

The Index of Liquidity Position calculated in Exhibit L is a much-needed refinement of IRIS ratio 7. The proposed index matches the assets at the reporting date that will be available in the next year against the liabilities at the reporting date that will be due in the next year. Thus, assets are adjusted to include only those assets marketable or maturing in the subsequent year, and liabilities are adjusted to include only those liabilities which are due or are to be met in the subsequent year. This matching of maturities and obligations up

[8] *Ibid.*

to and including one year produces the Liquidity Index. As one can see, the new index falls between IRIS ratio 7 and the "acid test," or "quick-ratio test," in commercial accounting. As a result, the new index produces a much more sensitive measurement of liquidity than the measurement supplied by IRIS ratio 7.

To reduce liabilities to only those obligations in the forthcoming year, only the portion of the loss and loss expense reserves that will be paid within that next year need be included. Exhibit R, line 27, column 21[9] can be used to enter that estimated percentage. The amount of the adjusted reserves to be included in Exhibit L then becomes the product of that estimated percentage times the Analyst's Estimate of current reserves (Exhibit R, line 24, column 20).[10]

To determine the assets available in the forthcoming year, three adjustments are made:

1. Only bonds maturing in the next year are included. This amount can be obtained from Schedule D - Part 1A.
2. Only mortgage loans, collateral loans, and other invested assets stipulated as maturing in the next year are included. These amounts, if any, can be obtained from a review of Schedules B, BA - Part 1, and C - Part 1.
3. One year's investment income on "deferred" reserves is added. This treatment considers such income as an addition to accrued investment income.

Two further adjustments to assets are appropriate but have not been included in Exhibit L at this time due to inadequate financial reporting disclosures. These two items and the changes necessary for inclusion are described below:

1. An increase in assets for additional premiums on exposures already provided, but not yet booked. Some companies currently accrue such "receivables" even though there is no financial reporting standard for doing so. If a separate line (perhaps 8.3) were added on pages 2 and 12 for "premiums earned but not yet billed," this receivable could be entered and appropriately disclosed for all companies. (If line 8.3 is added, instructions for Exhibit L require no change.)
2. An adjustment in assets for the difference between the statement value and the market value of sinking fund preferred stocks. For purposes of

[9] Column 21 in the 12/31/81 exhibit; Column 23 in the 12/31/82 exhibit; Column 25 thereafter.

[10] Column 20 in the 12/31/81 exhibit; Column 22 in the 12/31/82 exhibit; Column 24 thereafter.

measuring liquidity, the market value is the more appropriate value. As market value is not currently reported for these stocks, a revision in the Schedule D Summary (page 29) is needed to provide this data. Exhibit L-1 illustrates such a format. (If Exhibit L-1 is adopted, the instructions for Exhibit L require no change.)

The above discussion describes how December 31 assets and liabilities can be adjusted so that maturities and obligations in the subsequent year can be matched. The ratio of the maturities to the obligations during this period produces the Index of Liquidity Position. The desirable level for this index is clearly greater than or equal to 1.00. An index of less than 1.00 indicates a lack of financial flexibility but does not necessarily indicate serious financial trouble. It means that an insurer must borrow cash flow from future business or create cash flow from liquidations of bond holdings with maturities beyond one year. Because of the availability of both of these options and because the index is an independent measurement at the present time, the author suggests an index floor of .8, with the expectation that this level be subject to change as experience dictates.

The Index of Surplus Position, described earlier, is a tool to measure the surplus level needed for domestic fire/casualty exposures. (As noted on Exhibit S, the data of a fire/casualty parent should include the data of its fire/casualty subsidiaries.) Surplus needs for exposures in life and international subsidiaries were not addressed. Although there may be substantive merit in recognizing such exposures, an adjustment was not included for two reasons: (1) the Consolidated Statement does not include such data at the present time and (2) audited data for the detail needed are not easily available. In Exhibit L bonds and stocks of parents, subsidiaries, and affiliates are excluded from "Assets Available." Thus life and international insurance subsidiaries, for the purposes of these measurements, are combined with non-insurance subsidiaries as restrictions on the insurer's liquidity position. For this and other reasons, the two yardsticks interact and both are relevant in determining the financial posture of an insurer.

COMBINED INDEX — A FUTURE POSSIBILITY

The foregoing section described the rationale for accepting an Index of Liquidity Position of less than 1.00 for regulatory action purposes. As indicated, some tolerance had to be allowed if the index were to stand alone.

It would be preferable, however, if the degree of tolerance in the Liquidity Index could be quantified. The tolerance level should not exceed the financial ability of the insurer to withstand the potential surplus impairment that would result from bond liquidations necessary to fund "unmatched" liabilities. In other words, the tolerance should not exceed the cushion in the insurer's Index of Surplus Position.

To provide for this interaction, a combined RLS index would be the ideal solution. The immediate problem, however, is that the measurement of the potential surplus penalty requires the availability of actual market value information on bond holdings. The market value data currently reported in the annual statement are neither complete nor suitable for this purpose.

Although the market value of the total bond portfolio could be approximated from a schedule setting forth yield/maturity combinations, the author is satisfied that the actual market value data currently reported, though incomplete, could be organized and used to approximate the surplus penalty. This could be done by constructing a new Schedule D - Part 1B. Using the same maturity year categories as in Part 1A, Part 1B would summarize and compare statement values with market values for those bonds with market values published in the NAIC Valuation of Securities Manual. Exhibit RLS-1 illustrates such a format.

From this comparative partial data, the amount of the surplus impairment could then be approximated. The amount of surplus impairment would equal the unrealized losses (excess of statement over market) beginning with maturities in the 1 year through 3 year category (lines 21/22 in Part 1B) and continuing through lines 31/32, 41/42, and 51/52[11] as necessary to reach the aggregate market value equivalent to the insufficiency of assets available in Exhibit L (line 4–line 13). Exhibit RLS-2 illustrates the format that could be used for such a calculation. The surplus penalty,[12] thus calculated, would then be subtracted from the numerator in the calculation of the Index of Surplus Position. With this modification, the Index of Surplus Position would become a combined RLS index, and the Liquidity Index calculation (line 14) would be omitted from Exhibit L, as Exhibit L would serve only as an input source for Exhibit RLS.

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[11] If and when the maturity categories in Part 1A are extended, both Exhibits RLS-1 and RLS-2 also should be extended at that time to be consistent with the revised maturity categories.

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[12] The surplus penalty is measured on a pre-tax basis. The underlying assumption is that the federal tax effect of any necessary liquidations will be reflected in the accrued tax liability of the liquidating year, not in that year's cash flow.

The yardstick levels for the combined RLS index could be the same as those previously described for the Index of Surplus Position. However, due to the fact that the RLS index reflects the impact of all three critical factors, a lower "Suggested Floor" certainly would be appropriate.

The single index, as noted, awaits future action and interest. Only when the necessary market value data are available in summarized form will a combined RLS index be feasible.

SUMMARY

This paper proposes an analytical technique composed of two indexes (at present) to aid in identifying financially weak property/casualty insurers. The new breed of insurance regulators wants more and more analyses up front with computer assistance, and less dependence on on-site triennial examinations. The goal, of course, is to make the regulatory examination process more cost effective. It is hoped that this paper will contribute to that evolution.

Exhibit R
(Section I)

____________________ Company

CALCULATION OF LOSS AND L. E. RESERVE DEVFLOPMENTS - ALL LINES*
As of 12/31/81
(000 omitted)

Acc. Yrs.	-1- ΣPd 12/31/80 (col. 2 last yr)	-2- ΣPd 12/31/81 (a)	-3- Reserve 12/31/81 (b)	-4- 1981 Cal. Yr. Pd (2 - 1)	-5- Paid Developments 12/31/80 (col. 6 last yr)	-6- Paid Developments 12/31/81 (4 + 5)	-7- Developed Reserve (3 + 6)	Post Cols. 6 & 7 to Sec. II, Col:
1. ≤ 1975								8
2. 1976					--	--	--	
3. ≤ 1976 (1+2)								10
4. 1977					--	--	--	
5. ≤ 1977 (3+4)								12
6. 1978					--	--	--	
7. ≤ 1978 (5+6)								14
8. 1979					--	--		
9. ≤ 1979 (7+8)								16
10. 1980					--	--	--	
11. ≤ 1980 (9+10)					--			18
12. 1981	--							
13. Total								

		Reconciliation	
18.	From Und. Exhs. (Parts 3, 3A, & 4)	(+)		
19.	F&S INR (Part 3A, Col. 4a & b, Lines 23 & 24)	(-)		
20.	Prop. D - prior to 1971 Acc. Yr.	(-)		
21.	Total			

Footnotes:

(a) Source: Schedule P - Part 1, Col. 6, plus Schedule O - Part [illegible], Cols. 6-3a.

(b) Source: Schedule P - Part 1, Cols. 9+10, plus Schedule O - Part 3, Cols. 9+10.

*Note: 1. This exhibit can be completed on an individual company, consolidated or pooled basis as deemed the most appropriate for evaluating reserve levels.

2. Sources, unless otherwise noted, are from the 1981 Annual Statement.

3. If this exhibit is completed for the first time as of 12/31/81, the historical paid data in Col. 5 must be derived from compilations of Col. 4 for each calendar year, 1976 through 1980. In the 1976 compilation, the entry in Line 1, Col. 4, is also entered in Col. 6. This same entry becomes the only entry in Col. 5 for the 1977 calculation. The 1977 calendar year compilation produces two entries (Lines 1 and 3) in Col. 6 to be carried forward to the 1978 calculation, etc.

Exhibit R
(Section II)

Company
LOSS AND L.E. RESERVE LEVEL EVALUATION - ALL LINES*
As of 12/31/81
(000 omitted)

	-8- 12/31/75 $	-9- %	-10- 12/31/76 $	-11- %	-12- 12/31/77 $	-13- %	-14- 12/31/78 $	-15- %	-16- 12/31/79 $	-17- %	-18- 12/31/80 $	-19- %	-20- 12/31/81 $	-21- %
RATIOS TO P.E.														
22. P.E. (c)		100.0		100.0		100.0		100.0		100.0		100.0	(d)	100.0
23. Developed Reserve (g)													(e)	
24. Analyst's Est. (i)													(h)	
PAID/UNPAID STATUS														
25. Developed Reserve (g)		100.0		100.0		100.0		100.0		100.0		100.0		
26. Analyst's Est. (i)		100.0		100.0		100.0		100.0		100.0		100.0		
After 1 Year														
27. Paid (c)											(f)		(j)	
28. Unpaid: (25)-(27)														
29. (26)-(27)											(h)			
After 2 Years														
30. Paid (c)									(f)					
31. Unpaid: (25)-(30)														
32. (26)-(30)									(h)					
After 3 Years														
33. Paid (c)							(f)							
34. Unpaid: (25)-(33)														
35. (26)-(33)							(h)							
After 4 Years														
36. Paid (c)					(f)									
37. Unpaid: (25)-(36)														
38. (26)-(36)					(h)									
After 5 Years														
39. Paid (c)			(f)											
40. Unpaid: (25)-(39)														
41. (26)-(39)			(h)											
After 6 Years														
42. Paid	(f)													
43. Unpaid: (25)-(42)														
44. Analyst's Est.	(h)													

Footnotes:
(c) Except for (d) and (f) entries, even-numbered columns are same as last year.
(d) Source: Page 4, Line 1, Col. 1.
(e) From Line 13, Col. 3, Section I.
(f) From Col. 6, Section I.
(g) From Col. 7, Section I, except for (e) entry.
(h) Analyst's Estimate after all other data company data has been posted and calculated.
(i) Complete by column after each (h) entry is made, equalling the sum of the (h) entry plus the respective paid $.
(j) Estimated.

*Note: 1. This exhibit can be completed on an individual company, consolidated or pooled basis as deemed the most appropriate for evaluating reserve levels.
2. Sources, unless otherwise noted, are from the 1981 Annual Statement.
3. If this exhibit is completed for the first time as of 12/31/81, the historical paid data referred to in Footnote (c) must be posted from the calculations of Col. 6 described in Footnote 3 of Section I.

Exhibit R-1

________________ Company

An Arithmetic Assist for "Analyst's Est." in Exhibit R, Section II

12/31/n (n = 1981)

1. Line 44, Col. 8: Enter Line 43 or $0, whichever greater ________

2. Line 41, Col. 10:
   a. If Line 40, Col. 11 equals or exceeds Line 41, Col. 9:
      Line 41, Col. 10 = Line 40, Col. 10 ________
   b. If Line 40, Col. 11 is less than Line 41, Col. 9:
      Line 41, Col. 10 = Line 39, Col. 10 ÷ (100.0% - Line 41, Col. 9) ________

3. Line 38, Col. 12:
   c. Calc. ratio: (Line 38, Col. 10 - Line 41, Col. 8) ÷ Line 22, Col. 10 ________
   d. Line 38, Col. 12 = Line 41, Col. 10 + (Step c X Line 22, Col. 12) ________

4. Line 35, Col. 14:
   a. Calc. ratio: (Line 35, Col. 12 - Line 38, Col. 10) ÷ Line 22, Col. 12 ________
   b. Calc. ratio: (Line 35, Col. 10 - Line 38, Col. 8) ÷ Line 22, Col. 10 ________
   c. 1/2 (a + b) ________
   d. Line 35, Col. 14 = Line 38, Col. 12 + (Step c X Line 22, Col. 14) ________

5. Line 32, Col. 16:
   a. Calc. ratio: (Line 32, Col. 14 - Line 35, Col. 12) ÷ Line 22, Col. 14 ________
   b. Calc. ratio: (Line 32, Col. 12 - Line 35, Col. 10) ÷ Line 22, Col. 12 ________
   c. 1/2 (a + b) ________
   d. Line 32, Col. 16 = Line 35, Col. 14 + (Step c X Line 22, Col. 16) ________

6. Line 29, Col. 18:
   a. Calc. ratio: (Line 29, Col. 16 - Line 32, Col. 14) ÷ Line 22, Col. 16 ________
   b. Calc. ratio: (Line 29, Col. 14 - Line 32, Col. 12) ÷ Line 22, Col. 14 ________
   c. 1/2 (a + b) ________
   d. Line 29, Col. 18 = Line 32, Col. 16 + (Step c X Line 22, Col. 18) ________

7. Line 24, Col. 20:
   a. Calc. ratio: (Line 24, Col. 18 - Line 29, Col. 16) ÷ Line 22, Col. 18 ________
   b. Calc. ratio: (Line 24, Col. 16 - Line 29, Col. 14) ÷ Line 22, Col. 16 ________
   c. 1/2 (a + b) ________
   d. Line 24, Col. 20 = Line 29, Col. 18 + (Step c X Line 22, Col. 20) ________

Exhibit S

____________________ Company

INDEX OF SURPLUS POSITION*

12/31/n (n = 1981)

Formula:

$$\frac{\text{Pure Premium} - K + \text{Surplus}}{\text{Premiums Earned}} = \frac{O/S_{12/31/n} + Pd_n - \text{Restated } O/S_{12/31/n-1} - K + \text{Restated Surplus}_{12/31/n}}{P.E._n}$$

Where:

K is an additional risk provision for the more volatile exposures

$P.E._n$ is subject to a maximum pure premium of 79%

Calculation

Numerator

1. O/S Loss and L. E. (Page 3, Lines 1 + 2) ______
2. Loss and Loss Expense Paid:
   a. Loss (Page 9, Col. 4, Line 31) ______
   b. Loss Expense (Page 11, Col. 1, Line 25) ______
   c. Total: a + b ______
3. Restated 12/31/n-1 O/S:**
   a. Line 24, Col. 18 : 12/31/n Exh. R ______
   b. Lines 19 + 20, Col. 3: 12/31/n-1 Exh. R ______
   c. Total: a + b ______
4. Premiums Earned (Page 7, Col. 4):
   a. All Lines (Line 31) ______
   b. Allied Lines (Line 2) ______
   c. Farmowners (Line 3) ______
   d. Homeowners (Line 4) ______
   e. Int'l & Reins (Lines 29 + 30) ______
5. Calculation of K:
   a. .324 X 4b, or $0 whichever greater ______
   b. .35 X 4c, or $0 whichever greater ______
   c. .141 X 4d, or $0 whichever greater ______
   d. .041 X 4e, or $0 whichever greater ______
   e. a + b + c + d ______
6. Excess statutory reserves (Page 3, Line 16) ______
7. Surplus (Page 3, Line 27) ______
8. Surplus Aid (Step D, IRIS Ratio 3) ______
9. Numerator: 1 + 2c - 3c - 5e + 6 + 7 - 8 ______

Denominator

10. Calculation of minimum P. E.:
    a. From 12/31/n Exh. R: Line 24, Col. 20 ______
    b. Lines 19 + 20, Col. 3 ______
    c. Pure Premium: 10a + 10b + 2c - 3c ______
    d. Minimum P. E.: 10c ÷ .79 ______
11. Denominator: 4a or 10d, whichever greater ______

Index

12. Index of Surplus Position: 9 ÷ 11 ______

    Desired ≧ 1.04

    Suggested Floor .957

*Note: 1. This exhibit should be completed on a consolidated basis for insurers with domestic fire/casualty subsidiaries.

2. Sources, unless otherwise noted, are from the 1981 Annual Statement.

**If Exhibit R is completed on a pooled basis and Exhibit S is not pooled, the appropriate pooled percentage should be applied to the a. and b. entries in Line 3.

Exhibit S-1

CALCULATION OF K FACTORS
(for use as of 12/31/81)

Industry Loss and L. E. Ratios
Source: "Best's Aggregates & Averages
Property-Casualty"
1978-1981

	Calendar Year	Allied Lines	Farm Owners	Home Owners	Reins. & Int'l.	All Lines Excluding[c]
	1973	45.5[a]	68.2	59.6	72.5[b]	70.3
	1974	64.0[a]	84.2	72.0	82.1[b]	75.3
	1975	59.4[a]	82.0	73.3	82.0[b]	80.1
	1976	52.6[a]	72.3	65.4	75.8[b]	76.7
	1977	48.1	67.5	60.3	76.9	71.6
	1978	57.5	66.5	60.2	74.7	70.7
	1979	69.0	64.2	67.6	74.4	73.0
	1980	71.6	80.6	73.9	76.1	74.1
1. Avg.		58.46	73.19	66.54	76.81	73.98
2. σ		8.845	7.394	5.694	3.265	3.113
3. Expense Loading:	100.00 - (1)	41.54	26.81	33.46	23.19	26.02
4. σ s in (3)		-	-	-	-	8.36
5. 8.36 x (2)		73.94	61.81	47.60	27.30	-
6. (5) - (3)		32.40	35.00	14.14	4.11	
7. rounded		32.4	35.0	14.1	4.1	

[a]Including Earthquake.
[b]Including Credit and Misc.
[c]All lines excluding those identified above and Group A&H and Factory Mutuals.

Exhibit S-2

CALCULATION OF YARDSTICKS
FOR INDEX OF SURPLUS POSITION
(for use as of 12/31/81)

A. Industry Data (from "Best's Aggregates and Averages, Property-Casualty"):

1.	a.	1980 Premiums Earned:	All Lines*	$90,815,455
	b.		Allied Lines	1,516,847
	c.		Farmowners	530,107
	d.		Homeowners	9,276,151
	e.		Reins. & Other	3,379,827
2.	1973 - 1980 Avg. Loss & L.E. Ratio*			73.0%
3.	K adjustments (Exhibit S-1 factors)			$2,123,506
4.	(3) ÷ (1a)			2.34%

*excluding Factory Mutuals and Group A&H

B. Index of Surplus Position - Using a 3 to 1 Relationship of Premiums Earned to Surplus:

$$\frac{73.0 - 2.34 + 33.33}{100.0} = 1.04$$

C. Index of Surplus Position - Using a 4 to 1 Relationship of Premiums Earned to Surplus:

$$\frac{73.0 - 2.34 + 25.0}{100.0} = .957$$

D. Calculation of Maximum Pure Premium Percentage - Using a 6 to 1 Relationship:

$$.957 = \frac{X + 16.7}{100.0}; X = .79$$

No K factor was included because this calculation was based upon traditionally high loss ratio lines such as Standard Group A&H insurance which coverage was not included in the K adjustments.

Exhibit L

______________ Company
INDEX OF LIQUIDITY POSITION*
12/31/n (n = 1981)

Formula:

$$\frac{\text{12/31/n Assets Available Next Year}}{\text{12/31/n Liabilities Due Next Year}}$$

Calculation

Denominator

1. Loss and L. E. Reserve Payout next year: **
   a. From 12/31/n Exh. R: Line 24, Col. 20 ______
   b. Line 27, Col. 21 ______
   c. a X b ______
2. Unearned Premiums (Page 3, Line 10) ______
3. Misc. Liabilities (Page 3, Lines 3-9, 11, 17-22) [a] ______
4. Denominator: 1 through 3 ______

Numerator

5. Bonds Maturing next year (Page 30, Line 1, Cols. 2 + 3 + 4) ______
6. Stocks excl. Affiliates:
   a. Preferred:
      i. Page 29, Line 48, Col. 3 (+) ______
      ii. Page 29, Line 47, Col. 3 (-) ______
      iii. Total ______
   b. Common:
      i. Page 29, Line 66, Col. 3 (+) ______
      ii. Page 29, Line 65, Col. 3 (-) ______
      iii. Total ______
7. Cash (Page 2, Line 6) ______
8. Qualifying [b] items, if any ______
9. Uncollected Premiums Due (Page 2, Line 8) ______
10. Funds:
    a. Page 2, Line 9 (+) ______
    b. Page 3, Line 12 (-) ______
    c. Page 3, Line 13 (-) ______
    d. Total ______
11. Misc. Assets (Page 2, Lines 10-12 and 14-16) ______
12. Investment Income on L.&L. E. Reserve Funds held:
    a. Yield on one year paper (2/28/n+1) [c] ______
    b. 1a - 1c ______
    c. 12a X 12b ______
13. Numerator: 5 through 12 ______

Index

14. Index of Liquidity Position: 13 ÷ 4 ______
    Desired ≥ 1.00
    Suggested Floor .80

[a] Any liability with an offsetting write-in asset should be netted.
[b] Any invested assets stipulated as maturing next year in Schedules B, BA-Part 1, or C-Part 1. (Summarize individual company entries if on a consolidated basis.)
[c] Rate as of 2/27/81 for the 1980 calculation was .145.

*Note: 1. This exhibit should be completed on a consolidated basis for insurers with domestic fire/casualty subsidiaries.
2. Sources, unless otherwise noted, are from the 1981 Annual Statement.

**If Exhibit R is completed on a pooled basis and Exhibit L is not pooled, the appropriate pooled percentage should be applied to the a. and b. entries in Line 1.

Exhibit L-1

Form 2 ANNUAL STATEMENT FOR THE YEAR 1981 OF THE (Name) 29

SCHEDULE D—SUMMARY BY COUNTRY

Bonds and Stocks OWNED December 31 of Current Year

1 Description		2 Book Value	3 †Market Value (Excluding accrued interest)	4 Actual Cost (Excluding accrued interest)	5 Par Value of Bonds	6 *Amortized or Investment Value
BONDS Governments (Including all obligations guaranteed by governments)	1 United States					
	2 Canada					
	3 Other Countries					
	4 Totals					
States, Territories and Possessions (Direct and guaranteed)	5 United States					
	6 Canada					
	7 Other Countries					
	8 Totals					
Political Subdivisions of States, Territories and Possessions (Direct and guaranteed)	9 United States					
	10 Canada					
	11 Other Countries					
	12 Totals					
Special revenue and special assessment obligations and all non guaranteed obligations of agencies and authorities of governments and their political subdivisions	13 United States					
	14 Canada					
	15 Other Countries					
	16 Totals					
Railroads (unaffiliated)	17 United States					
	18 Canada					
	19 Other Countries					
	20 Totals					
Public Utilities (unaffiliated)	21 United States					
	22 Canada					
	23 Other Countries					
	24 Totals					
Industrial and Miscellaneous (unaffiliated)	25 United States					
	26 Canada					
	27 Other Countries					
	28 Totals					
Parents, Subsidiaries, and Affiliates	29 Totals					
	30. Total Bonds					
PREFERRED STOCKS Railroads (unaffiliated)	31 United States					
	32 Canada					
	33 Other Countries					
	34 Totals					
Public Utilities (unaffiliated)	35 United States					
	36 Canada					
	37 Other Countries					
	38 Totals					
Banks, Trust and Insurance Companies (unaffiliated)	39 United States					
	40 Canada					
	41 Other Countries					
	42 Totals					
Industrial and Miscellaneous (unaffiliated)	43 United States					
	44 Canada					
	45 Other Countries					
	46 Totals					
Parents, Subsidiaries, and Affiliates	47 Totals					
	48. Total Preferred Stocks					
COMMON STOCKS Railroads (unaffiliated)	49 United States					
	50 Canada					
	51 Other Countries					
	52 Totals					
Public Utilities (unaffiliated)	53 United States					
	54 Canada					
	55 Other Countries					
	56 Totals					
Banks, Trust and Insurance Companies (unaffiliated)	57 United States					
	58 Canada					
	59 Other Countries					
	60 Totals					
Industrial and Miscellaneous (unaffiliated)	61 United States					
	62 Canada					
	63 Other Countries					
	64 Totals					
Parents, Subsidiaries, and Affiliates	65 Totals					
	66. Total Common Stocks					
	67 Total Stocks					
	68 Total Bonds and Stocks					

†Statement value for preferred stocks. For certain bonds, values other than actual market may appear in this column (See Schedule D, Part 1, for details)

The aggregate value of bonds which are valued at other than actual market is $ ____________.

*Companies, societies, and associations which do not amortize their bonds should leave this column blank

SCHEDULE D—VERIFICATION BETWEEN YEARS

1. Book value of bonds and stocks, per Items 1 and 2, Col. 1, Exhibit 1, previous year ____________
2. Cost of bonds and stocks acquired, Col. 5, Part 3 ____________
3. Increase by adjustment in book value
   - (a) Col. 10, Part 1 ____________
   - (b) Col. 9, Part 2, Sec. 1 ____________
   - (c) Col. 8, Part 2, Sec. 2 ____________
   - (d) Col. 9, Part 4 ____________ ____________
4. Profit on disposal of bonds and stocks, Col. 11, Part 4 ____________
5. Total ____________
6. Deduct consideration for bonds and stocks disposed of, Col. 5, Part 4 ____________
7. Decrease by adjustment in book value
   - (a) Col. 11, Part 1 ____________
   - (b) Col. 10, Part 2, Sec. 1 ____________
   - (c) Col. 9, Part 2, Sec. 2 ____________
   - (d) Col. 10, Part 4 ____________ ____________
8. Loss on disposal of bonds and stocks, Col. 12, Part 4 ____________
9. Book value of bonds and stocks, per Items 1 and 2, Col. 1, Exhibit 1, current year ____________

Exhibit RLS

_____________________ Company
INDEX OF SURPLUS POSITION*
12/31/n (n = 1981)

Formula:

$$\frac{\text{Pure Premium} - K - SP + \text{Surplus}}{\text{Premiums Earned}} = \frac{O/S_{12/31/n} + Pd_n - \text{Restated } O/S_{12/31/n-1} - K - SP + \text{Restated Surplus}_{12/31/n}}{P.E._n}$$

Where:

K is an additional risk provision for the more volatile exposures,
$P.E._n$ is subject to a maximum pure premium of 79%, and
SP is a provision for the potential surplus penalty due to the *insufficiency of assets available.*

Calculation

Numerator

1. O/S Loss and L. E. (Page 3, Lines 1 + 2) ______
2. Loss and Loss Expense Paid:
   a. Loss (Page 9, Col. 4, Line 31) ______
   b. Loss Expense (Page 11, Col. 1, Line 25) ______
   c. Total: a + b ______
3. Restated 12/31/n-1 O/S:**
   a. Line 24, Col. 18 : 12/31/n Exh. R ______
   b. Lines 19 + 20, Col. 3: 12/31/n-1 Exh. R ______
   c. Total: a + b ______
4. Premiums Earned (Page 7, Col. 4):
   a. All Lines (Line 31) ______
   b. Allied Lines (Line 2) ______
   c. Farmowners (Line 3) ______
   d. Homeowners (Line 4) ______
   e. Int'l & Reins (Lines 29 + 30) ______
5. Calculation of K:
   a. .324 X 4b, or $0 whichever greater ______
   b. .35 X 4c, or $0 whichever greater ______
   c. .141 X 4d, or $0 whichever greater ______
   d. .041 X 4e, or $0 whichever greater ______
   e. a + b + c + d ______
6. Surplus Penalty, if any (Line 5f, Exh. RLS-2) ______
7. Excess statutory reserves (Page 3, Line 16) ______
8. Surplus (Page 3, Line 27) ______
9. Surplus Aid (Step D, IRIS Ratio 3) ______
10. Numerator: 1 + 2c - 3c - 5e - 6 + 7 + 8 - 9 ______

Denominator

11. Calculation of minimum P. E.:
    a. From 12/31/n Exh. R: Line 24, Col. 20 ______
    b. Lines 19 + 20, Col. 3 ______
    c. Pure Premium: 11a + 11b + 2c - 3c ______
    d. Minimum P. E.: 11c ÷ .79 ______
12. Denominator: 4a or 11d, whichever greater ______

Index

13. Index of Surplus Position: 10 ÷ 12 ______
    Desired ≧ 1.04
    Suggested Floor .957

*Note: 1. This exhibit should be completed on a consolidated basis for insurers with domestic fire/casualty subsidiaries.
2. Sources, unless otherwise noted, are from the 1981 Annual Statement.

**If Exhibit R is completed on a pooled basis and Exhibit RLS is not pooled, the appropriate pooled percentage should be applied to the a. and b. entries in Line 3.

Exhibit RLS-1

ANNUAL STATEMENT FOR THE YEAR 198_ OF THE ....
(Name)

30

1 Maturity *	2 Government Schedule D (Group 1)	3 Political Subdivisions, Governmental Agencies and Authorities (Groups 2, 3 & 4)	4 Other (Unaffiliated) (Groups 5, 6 & 7)	5 Parents, Subsidiaries and Affiliates (Group 8)	6 Total Bonds

SCHEDULE D - PART 1A
Maturity Distribution of Bonds Owned December 31, Current Year at Statement Values

(1981 schedule - no changes)

SCHEDULE D - PART 1B
Comparison of Market/Statement Values
by Maturity-Date Category in Part 1A
of Those Bonds with Market Values Published in the NAIC Valuation of Securities Manual

11. 1 year or less:	Market Value				X	
12.	Statement Value				X	
21. >1 yr; ≤3:	Market Value				X	
22.	Statement Value				X	
31. >3 yrs; ≤5:	Market Value				X	
32.	Statement Value				X	
41. >5 yrs; ≤10:	Market Value				X	
42.	Statement Value				X	
51. >10 yrs:	Market Value				X	
52.	Statement Value				X	
61. Total:	Market Value				X	
62.	Statement Value				X	

*If and when the maturity categories in Part 1A are extended, this exhibit should be extended at that time to be consistent with the revised maturity categories.

Exhibit RLS-2

________________Company
CALCULATION OF SURPLUS PENALTY (SP)
(to be completed only when Line 14, Exhibit L, is < 1.00)

From Schedule D - Part 1B, Col. 6:

1. Enter amounts:
   a. Line 21 . . . . __________
   b. Line 31 . . . . __________
   c. Line 41 . . . . __________
   d. Line 51 . . . . __________

2. Enter ratios:
   a. Line 22 ÷ Line 21, less 1.00 . . . . __________
   b. Line 32 ÷ Line 31, less 1.00 . . . . __________
   c. Line 42 ÷ Line 41, less 1.00 . . . . __________
   d. Line 52 ÷ Line 51, less 1.00 . . . . __________
   e. Enter the highest of a, b, c, or d . . . . . . __________

From Exhibit L:

3. Line 4 less Line 13 . . . . __________*

Calculations:

4. Allocation of Line 3 to:
   a. Line 1a penalty . . . . __________**
   b. Line 1b penalty . . . . __________**
   c. Line 1c penalty . . . . __________**
   d. Line 1d penalty . . . . __________**
   e. Remainder . . . . __________**

5. Calculation of Surplus Penalty:
   a. 2a X 4a . . . . __________
   b. 2b X 4b (if necessary) . . . . __________
   c. 2c X 4c (if necessary) . . . . __________
   d. 2d X 4d (if necessary) . . . . __________
   e. 2e X 4e (if necessary) . . . . __________
   f. Total Penalty (5a thru 5e) . . . . __________

*Must be a positive entry.

**If Line 3 is less than Line 1a, enter Line 3 in Line 4a and proceed to Line 5. If Line 3 is greater than Line 1a, enter the latter in Line 4a, and carry over the remainder to Line 4b. If the remainder is less than Line 1b, enter the remainder in Line 4b and proceed to Line 5. If the remainder is greater than Line 1b, enter the latter in Line 4b, and carry over the new remainder to Line 4c, etc.

Note: See footnote on Exhibit RLS-1. This exhibit should also be extended to be consistent with the revised maturity categories.

AN EXAMINATION OF CREDIBILITY CONCEPTS

STEPHEN W. PHILBRICK

Abstract

Credibility is one of the more important concepts in actuarial theory. However, it is one of the more complex concepts and is not as well understood as it should be. This paper takes a fresh look at some of the fundamentals of credibility theory in order to clarify and tie together various concepts.

Several loosely related approaches are taken. A new model is introduced to explain credibility concepts, an old model is discussed in more detail, and several potential ambiguities in the existing literature are directly addressed. This paper relies heavily on existing papers, particularly those on the *Syllabus*, and is intended to be read in conjunction with the various papers.

INTRODUCTION

The casual reader of articles on credibility is unlikely to come away with a lucid understanding of the true meaning of credibility. Consider the following observations.

Longley-Cook states: "While credibility and statistical variance are related, the former is meaningful only against a stated or implied background of the purpose for which the data are to be used and a consideration of the value of the prior knowledge available." He then goes on to establish a formula for full credibility based only on the properties of the observations, i.e., independent of the purpose of the data and the value of prior knowledge. When discussing partial credibility, he uses the formula $Z = n/(n + k)$ and notes that this never gives a value of 1.0, so he increases his partial credibilities by 50% to meet the full credibility standard. He then discusses an alternative (and inconsistent) approach, the so-called square root rule. Finally, he refers to Arthur Bailey's two types of credibility, "limited fluctuation credibility" and "greatest accuracy credibility," without fully explaining the differences.[1,2]

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[1] Laurence H. Longley-Cook, "An Introduction to Credibility Theory," *PCAS* XLIX, 1962, p. 194.

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[2] In his defense, it should be pointed out that he was not putting forth original theories; he was merely summarizing current practice.

Hewitt summarizes Mayerson (Lange quotes this summary) by stating that "credibility may under certain circumstances be a function of:

(1) sample size,
(2) underlying hazard (mean of prior distribution), and
(3) underlying dispersion (variance of prior distribution)."[3,4,5]

In Hewitt's review of Mayerson, Jones, and Bowers he states that: "There are, then, three variables which can affect credibility:

(i) number of observations,
(ii) variation in results (estimator for process variance), and
(iii) variation of hypotheses (variance of hypothetical means)."[6,7]

Mayerson *et al.* point out that existing standards are based only on numbers of claims and set out to establish a distribution-free standard for full credibility of the pure premium.[8] They define a standard of full credibility which is based upon the familiar P and K found in Longley-Cook. Hewitt's review claims that their standard is not distribution-free.[9] In his article with a similiar title, "Credibility for Severity," Hewitt never talks about P, discusses K (but this is not the same k as in Mayerson *et al.*) and never seems to talk about the number of claims or dollars needed for full credibility.[10]

It is not surprising that actuaries are not of a single mind when it comes to discussing credibility, since the various references are apparently inconsistent.

[3] Charles C. Hewitt, Jr., Discussion of "A Bayesian View of Credibility," *PCAS* LII, 1965.

[4] Allen L. Mayerson, "A Bayesian View of Credibility," *PCAS* LI, 1964.

[5] Jeffrey T. Lange, "Application of a Mathematical Concept of Risk," *The Journal of Risk and Insurance*, Volume XXXVI, No. 4, 1969, p. 385.

[6] Charles C. Hewitt, Jr., Discussion of "On the Credibility of the Pure Premium," *PCAS* LVI, 1969, p. 79.

[7] Allen L. Mayerson, Donald A. Jones, Newton L. Bowers, Jr., "On the Credibility of the Pure Premium," *PCAS* LV, 1968.

[8] Ibid., p. 175.

[9] Hewitt op. cit., p. 81.

[10] Charles C. Hewitt, Jr. "Credibility for Severity," *PCAS* LVII, 1970.

In this paper I would like to accomplish the following goal:

Explain credibility, via examples, so that the reader will have an understanding of the truė nature of credibility.

Being realistic, I will be satisfied if this article provides enough of a focal point so that by re-reading the various articles, you can bring together the various concepts. Each of the authors is essentially talking about the same thing, but they each make certain simplifying assumptions (some explicit, some implicit) which, in some cases, tend to oversimplify the concept; that is, some of the essence of credibility gets simplified into thin air.

The format of this paper will be as follows:

- Discuss the concept of credibility using a target-shooting example. This example is easy to follow and reasonably analogous to insurance situations.
- Expand the discussion using an example similar to Hewitt's die-spinner model. The model is slightly changed and the discussion, emphasizing a different look at essentially the same example, may be enlightening.
- Explain how ratemaking and experience rating credibility concepts differ and the impact this has on credibility formulas.
- Correct the misconception that large values of credibility are always desirable.
- Summarize some of the credibility articles which are required reading for the actuarial exams.
- Discuss some of the simplifying assumptions made by various authors that can lead to the apparent confusion pointed out in the beginning of this introduction.

CREDIBILITY AND MARKSMANSHIP

''And now for something completely different.''
-Monty Python

In this section an example will be presented, somewhat removed from the world of insurance, but one that I hope will give an insight into credibility. Consider the following situation. One of four people—*A*, *B*, *C* and *D*—will be chosen at random. The person chosen, whose identity will be unknown to you, will fire a gun at a target some distance away. Your task is to provide the best estimate of the location on the target which will be hit by his *next* shot after observing the location of the shot.

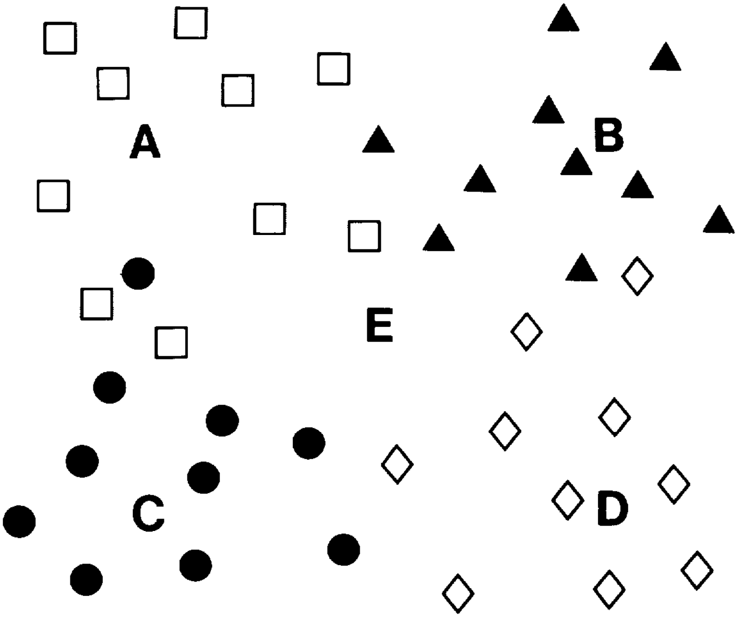
You also have some additional information. You have the results shown on Figure 1 of each of the four people firing a number of shots at an identical target. The squares represent the shots fired by person A, and the position marked A represents the center, or mean, of each of these points. Similarly, B is the center of the points marked by the triangles, C corresponds to the circles and D corresponds to the diamonds. The point E corresponds to the mean of all the points, or equivalently, the means of A, B, C, and D. Inspection will reveal that there is a clustering of the various symbols about their mean, although the clusters overlap. It can be presumed that each of the people is aiming for his respective mean, and the scattering is the result of random disturbances.

Prior to the observation of the shot, the best estimate must be based solely on the prior information; hence the best choice is E. Now we will consider the problem of making the best estimate after observation of a single shot, based on the current observation and the prior information.

If a strict Bayesian analysis procedure were followed, the next step would be to calculate the new probability that the shot was fired by A, B, C, or D, and then calculate an E based on these revised weights (see Hewitt for a discussion along this line). A Bayesian credibility approach would proceed as follows. Draw the straight line between the observed point and E. Determine the credibility Z of a single observation, and locate the point $100Z\%$ of the way from E to the observed point. The crucial point is the calculation of the credibility. In this example, the intent is not to do the explicit calculations, but to justify, on intuitive grounds, the calculations to be done in the next example.

Assume that the observed shot lies somewhere between A and E. Although a revised estimate would lie along the line connecting the observed point and E, it would probably not be far from E. Why? Although points in that region are more likely to have been produced by A, values corresponding to B and C are in the region, and D cannot be ruled out entirely. A is more likely; hence the revised estimate should be closer to A, but not much closer because the evidence for A is minimal.

FIGURE 1



Consider Figure 2. This figure was produced by four different people, A', B', C', and D'. Their "mean" shots were identical; hence E' coincides with E. However, these four are much better shots. Their shots cluster more closely around their mean. Mathematically, the process variance (the mean squared distances between the actual points and the mean points for each person) for each is reduced; therefore the expected value of the process variance is reduced.

If a shot is observed in the same place as before (somewhere between A' and E'), it is much more likely that the shot was fired by A', and the next predicted point will lie much closer to A' and much farther from E' than our previous prediction lay relative to A and E. (In a strict Bayesian analysis approach, the predicted point would probably lie even farther from E' than the observed point.)[11] Hence, the credibility attached to a single observation is increased when the process variance is decreased. Note that the variance of the hypothetical means, which is equal to the mean of the squares of the distances between E and A, B, C, and D, is unchanged between Figure 1 and Figure 2.

Now let us consider an example where the process variance is identical to that in Figure 1, but the variance of the hypothetical means is changed. Figure 3 shows such an example. In this case we can assume that our original persons A, B, C, and D are again shooting, but they are aiming for different points. We will call them (and their means) A'', B'', C'' and D'', to distinguish this example from the others.

In Figure 3, the clustering of shots around each of the means is similar to that in Figure 1, but the means are much farther apart, hence much farther removed from the population mean, E''. The variance of the hypothetical means will be much larger than in Figure 1. If a shot is observed somewhere between A'' and E'', it is more likely to have been fired by A'', so the predicted point will lie relatively closer to A'' than the predicted point in Figure 1 was to A. In other words, the credibility of the single observation is increased.

To this point, we have only examined the results of a single fired shot. If a number of shots were fired, the credibility attached to the mean of the observed shots would be greater than that for the single observation.

[11] This agrees with Hewitt's observation ("Credibility for Severity," p. 150) that the Bayesian resultant does not necessarily lie between the hypothetical mean and the observed result.

FIGURE 2

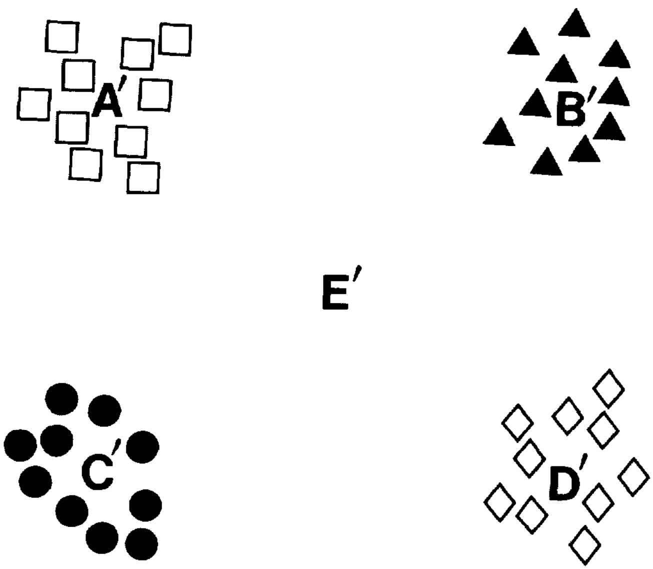
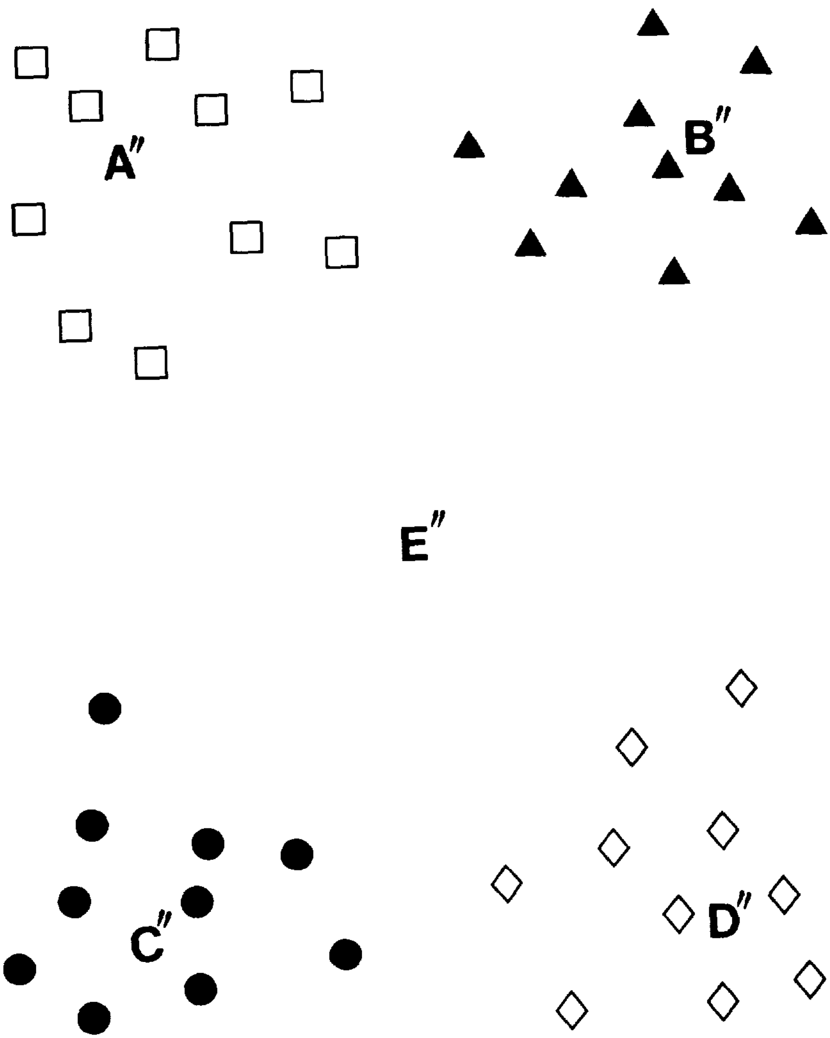


FIGURE 3



To summarize, it has been shown that, when projecting the location of a future shot based on current information and prior knowledge, the credibility attached to the current observations will increase with:

- Increasing number of observations,
- Decreasing process variance, and
- Increasing variance of the hypothetical means.

Finally, it will be helpful to analyze what happens at the extremes—as the three basic elements approach either zero or infinity. If no current observations are made, we have to rely totally on the prior information. This is equivalent to stating that $1 - Z = 1$, which implies that $Z = 0$. As the number of current observations goes to infinity, the pattern of shots will begin to resemble one of the four clusters, and the mean will tend toward the mean of the cluster. At the limit, the weight associated with the observed pattern will become one, hence, $Z = 1$ as n goes to infinity.

As the process variance goes to zero, the clusters will tend to shrink to single points. In terms of our example, we say that the marksmen are becoming better shots. At the limit, each of the four marksmen can hit the exact center of the target with every shot. The observation of a single shot will be sufficient to identify the marksman, and the next shot can be predicted with certainty. Hence, the credibility associated with the current observation goes to one as the process variance goes to zero. Conversely, as the process variance increases without bound, the clusters of shots tend to spread apart, and overlap one another. The observation of a single shot provides little information as to the identity of the marksman firing the shot, and the best estimate of the next shot will remain E. As a consequence, as the process variance goes to infinity, the credibility goes to zero.

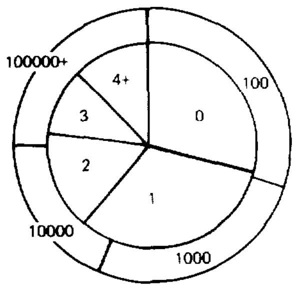
As the variance of the hypothetical means goes to zero, the clusters tend to move closer together. At the limit, each of the respective means coincides. When this happens, the observation of a shot will add nothing to our knowledge; the weight, or credibility, given to this observation will be zero. Increasing variance of the hypothetical means has the effect of moving the clusters apart. When they are sufficiently distant, a shot fired by one of the marksmen can be uniquely associated with one of the clusters. This situation also points out the difference between a pure Bayesian approach and Bayesian credibility. Using Bayesian credibility, the best estimate of the location will be the observed shot, because it has been given credibility equal to one. A pure Bayesian approach would select the mean of the cluster to which the observed shot is closest, rather than the position of the shot itself.

HEWITT REVISITED

In the following examples, the assumption is made that the process which creates losses can be modeled as a collection of spinners with inner and outer sections.[12] The universe is represented by the total collection of spinners and each individual risk corresponds to a single spinner. The inner portion of the spinner will be used to simulate the frequency of the risk and the outer section will be used to simulate the severity. An accident year consists of the selection of one (or more) of the risks (spinners), possibly at random, spinning once, observing the inner value (frequency), and spinning that many additional times, with each observation of the outer ring constituting a loss.

Figure 4 is a typical risk in the universe. In this risk, the probability of having exactly 0 claims is approximately 1/3, the probability of having exactly 1 claim is approximately 1/3, the probability of having exactly 2 claims is approximately 1/6, etc. For each claim, the probability of each of the possible severities equals the area corresponding to each value. (It would be trivial to extend to a continuous severity, but for simplicity, we will stick to the discrete case.)

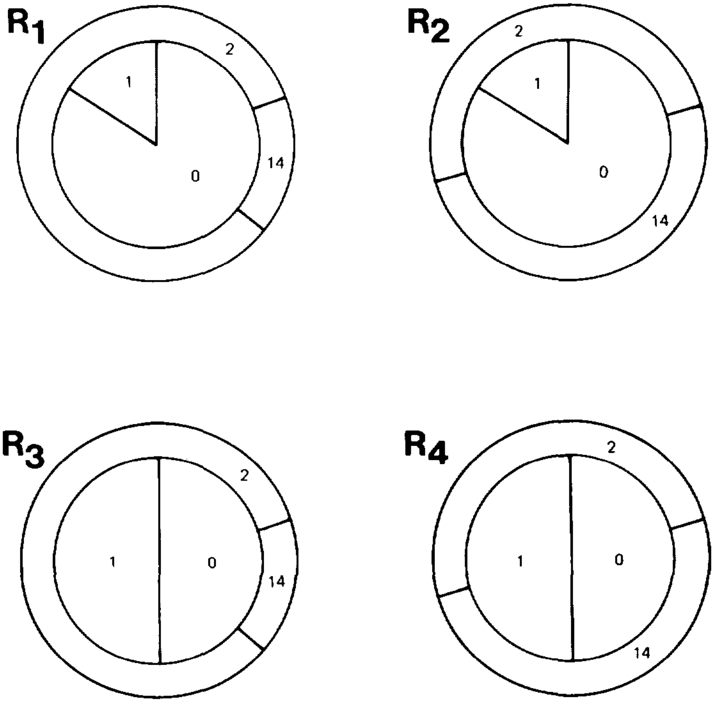
FIGURE 4



[12] The reader will notice the similarity between this and the examples used by Hewitt in "Credibility for Severity." This is intentional; his example is an excellent tool for explaining concepts. This paper is intended to expand on those ideas and provide additional insight into credibility concepts

In Figure 5 four spinners represent a universe of risks. The values associated with the areas in Figure 5 are either 1/6, 1/2 or 5/6. The universe has exactly one of each of the R_i and each has an equal probability of being chosen in a random sample.

FIGURE 5



This paper will take as assumptions the definitions and assumptions in Hewitt's "Credibility for Severity."[13] Briefly, a compromise estimate is chosen as a function of prior information (hypotheses) and current observations, according to the formula

$$C = ZR + (1 - Z)H$$

where R equals the mean of the observations,
H equals the mean of the hypotheses,
C equals the value of the compromise, and
Z is obtained from the formula $Z = n/(n + K)$.

The volume of observations is measured by n (number of trials or exposure units) and K is the K defined by Bühlmann:

$$K = \frac{\text{Expected value of process variance}}{\text{Variance of the hypothetical means}}.$$[14]

Although the terms "prior information" and "current observations" will generally be used, it should not be assumed that the two sets of data are necessarily different in time. For example, when calculating a class rate, the prior information might be the entire state (or countrywide) pure premium indication, and the current observations could be the specific class indications.

Further, it is important to note that the assumption is made that all parameters concerning the universe are known, although the identity of a particular risk chosen at random is not necessarily known. This implies that no parameter risk is involved in the example, only process risk.

The problem to be solved can be stated as follows. The universe has been described and all its parameters are known. From this information, hypotheses can be made regarding the correct premiums to be charged. A risk or risks are selected at random and observations of their experience are made. How can the prior knowledge (represented by the hypotheses) and the posterior knowledge (represented by the observations) be combined? From the point of view of an insurance company, assume that the universe, as defined, represents the universe of all possible insurable risks. Although there are four distinct types of risks (in terms of their loss process parameters), we will assume that these risks cannot be separately identified by any *a priori* characteristics other than historical experience. Hence, we can assume that there is only a single classification for

[13] Hewitt, op. cit., p. 149.

[14] Hans Bühlmann, "Experience Rating and Credibility," *The Astin Bulletin*, Volume IV, 1967.

the insurance company. The company must determine a rate and select a risk at random. The identity of the risk (in terms of its parameters) will remain unknown, although the actual experience will be observed. Based on the prior knowledge (of the universe) and the actual observations of this risk's experience, we wish to determine what is the best choice of a rate for *the same risk* in the subsequent year. (For convenience we ignore any timing problems related to calculating a new rate *after* observing the experience but *before* the new year commences.) It is vitally important to understand that the derived rate is applicable to the risk creating the experience, not to a new risk chosen at random.

Although it has been stated that the identity of the risk is not determinable, it will be instructive to first examine what action would be taken if the identity of the risk could be determined.

Assume that a risk is chosen and you know that you have selected R_1. You still could not predict with certainty the *actual* outcome of the loss process although you could calculate a mean value and a variance about the mean. If you had selected R_3, (and knew its identity) you could also calculate a mean and a variance but these would differ from the mean and variance of R_1. If you knew which risk you had, the choice of pure premium would be straightforward. You would set it equal to the mean of the risk.[15] However, suppose you chose a risk at random from the population. Before observing any loss experience of this risk, you would set the pure premium equal to the average of the means of the R_i, which is the same as equating it to the population mean.

There is an important difference between the two situations. In the first, where the identity is known, the actual experience will not exactly equal the expected in any one year, but over a long period of time, the average experience will tend toward the mean of the risk. In the second, the actual experience also will not reproduce the population mean, but, over the long run, the average experience will tend to the particular risk's mean, *not* to the universe's mean.

With enough observations of the risk experience, the cumulative mean of the observations will become arbitrarily close to the theoretical mean of that risk, and that value will be used for the pure premium. Before any observations are made, the mean of the population will be the best choice for the pure premium.

Credibility is concerned with the choice of the "best" pure premium based

[15] In order to make the example less complicated, risk loadings are being ignored. However, the extension of the example to a risk loading should be straightforward depending on one's choice of a risk measure.

upon a body of prior knowledge and a limited body of observations. It may be helpful to think of credibility in terms of the value of information. The prior knowledge has a certain amount of information about the "proper" pure premium and the actual observations also contain information. Credibility is concerned with the efficient blending of the information from the two sources.

Let us now examine our example in more detail. First calculate the pure premium that we would use for each R_i if we knew which R_i we had chosen. The mean pure premium is the product of the mean frequency and the mean severity. Thus, for R_1, the mean frequency is $1/6 \times 1 + 5/6 \times 0 = 1/6$, and the mean severity is $1/6 \times 14 + 5/6 \times 2 = 4$. Therefore, the mean pure premium is $1/6 \times 4 = 2/3$. Each of the others is calculated similarly. The details are shown in Table 1.

TABLE 1

	Frequency	Severity	Pure Premium
R_1	$1/6 \times 1 + 5/6 \times 0 = 1/6$	$1/6 \times 14 + 5/6 \times 2 = 4$	$1/6 \times 4 = 2/3$
R_2	$1/6 \times 1 + 5/6 \times 0 = 1/6$	$1/2 \times 14 + 1/2 \times 2 = 8$	$1/6 \times 8 = 4/3$
R_3	$1/2 \times 1 + 1/2 \times 0 = 1/2$	$1/6 \times 14 + 5/6 \times 2 = 4$	$1/2 \times 4 = 2$
R_4	$1/2 \times 1 + 1/2 \times 0 = 1/2$	$1/2 \times 14 + 1/2 \times 2 = 8$	$1/2 \times 8 = 4$

Assume that a risk is chosen at random and that risk is R_1 (although its identity is unknown to the observer). As observations are made, their cumulative average will tend to 2/3, although the average may vary significantly from 2/3 for the first few observations. As an example, the string of observations in Table 2 was randomly generated from R_1. Even after 10 trials, it is not clear that the choice of risk was R_1, because the cumulative average is greater than that expected for even R_2.

TABLE 2

Trial Number	1	2	3	4	5	6	7	8	9	10
Observation	0	0	0	14	0	0	0	2	0	0
Cumulative Average	0	0	0	3.50	2.80	2.33	2.00	2.00	1.78	1.60

Consider a different process as shown in Figure 6. The expected value of R'_1 is calculated in Table 3. Note that R'_1 has the same expected pure premium as R_1. Table 4 shows a string of observations from R'_1.

FIGURE 6

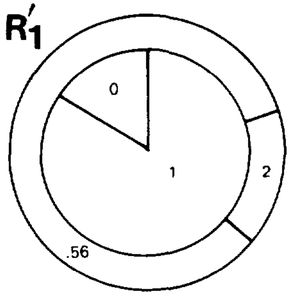


TABLE 3

	Frequency	Severity	Pure Premium
R'_1	$5/6 \times 1 + 1/6 \times 0 = 5/6$	$5/6 \times .56 + 1/6 \times 2 = .8$	$5/6 \times .8 = 2/3$

TABLE 4

Trial Number	1	2	3	4	5	6	7	8	9	10
Observation	.56	0	2	.56	.56	.56	.56	2	.56	.56
Cumulative Average	.56	.28	.85	.78	.74	.71	.69	.85	.82	.79

This time, it is much more obvious that we have chosen R'_1 rather than R_2, R_3 or R_4. Why should this be so, if R'_1 and R_1 have the same expected value? The answer lies in the variance of the process, appropriately called process variance. Let us now calculate the variance of the process for R'_1 and R_1. The formula for the variance of a compound process is

$$\sigma^2 = \text{E(Frequency)} \times \text{Var(Severity)} + \text{Var(Frequency)} \times \text{E}^2\text{(Severity)}.$$

Each of the components is a binomial process and can be calculated easily as shown in Table 5. Substituting these values into the formula, the process variance for the two risks is as follows.

R_1 $1/6 \times 20 + 5/36 \times (4)^2 = 5.56$
R'_1 $5/6 \times (.288) + 5/36 \times (.8)^2 = .329$

The process variance for R'_1 is much less than for R_1 which coincides with our observations.

TABLE 5

	R_1	R'_1
E (Frequency)	$(5/6) \times 0 + (1/6) \times 1 = 1/6$	$(5/6) \times 1 + (1/6) \times 0 = 5/6$
Var (Frequency)	$(1/6) \times (1 - 1/6) = 5/36$	$(5/6) \times (1 - 5/6) = 5/36$
E (Severity)	$(5/6) \times 2 + (1/6) \times 14 = 4$	$(5/6) \times .56 + (1/6) \times 2 = .8$
Var (Severity)	$((5/6) \times 2^2) + (1/6) \times 14^2 - 4^2 = 20$	$((5/6) \times (.56)^2) + (1/6) \times 2^2 - (.8)^2 = .288$

Roughly speaking, the process variance tells us "how far apart" the actual results can be for each trial. The smaller the variance, the closer the actual results will be to each other and to the expected. This is just another way of saying that the confidence interval around the expected (for a given probability) will be smaller for small variances.

We are now at the watershed between "classical" credibility and Bayesian credibility. Classical credibility continues down the road of confidence interval analysis, making various assumptions about the form of the distribution, deciding whether to include the claim severity or ignore it, and calculating the appropriate number of claims necessary to ensure that, with probability P, the actual claims (numbers or dollars) will be within $100K\%$ of the expected. The analysis then continues by arbitrarily assigning 100% credibility to this resulting value and exploring various ad hoc measures to calculate partial credibility. To the extent that the observations are not "fully credible," the complement of the credibility is assigned to the prior knowledge.

When a body of information receives less than 100% credibility, the implication is that the data is not "good" enough, that the variations from expected are too large to be acceptable for ratemaking. But when the complement of the credibility is assigned to prior knowledge, there is no discussion of whether the result of the combination is "good" enough in terms of the standard. And when data is assigned 100% credibility because the standard is met, there is no discussion of the fact that the result could be further improved with *some* weight assigned to prior knowledge.

We will now explore the path leading to Bayesian credibility. As we have seen, the process variance will tell us to what extent the actual results will tend to cluster around the mean. Another measure critical to the concept of credibility is the variance of the hypothetical means.

The hypothetical means are the expected values for each of the R_i. They have already been calculated as the pure premiums in Table 1. The variance of these values can be easily calculated; the result:

Variance of hypothetical means = 14/9.

The variance of the hypothetical means is a measure of the spread of the means—how far apart the means are from each other.

When we were examining the effects of process variance, we looked at two situations in which the process variances were different but the variances of the hypothetical means were the same. We found that we were more certain of the identity of the actual risk when the process variance was smaller. Now we will examine two situations with identical process variances but different variances of the hypothetical means.

The first situation will be the same as before; that is, the universe contains R_1, R_2, R_3, and R_4, and the same string of observations is randomly generated by R_1. In the second situation, we consider a universe containing R_1, and three new risks R'_2, R'_3, and R'_4. We assume that these new risks have the same process variance as their counterparts, but the pure premiums are 10, 20 and 30 respectively. The variance of the hypothetical means is now approximately 120. We can be more certain that the string of observations are generated by R_1 in the second situation, than in the first situation.

We have now demonstrated that it is easier to discern the true identity of an R_i chosen at random when

- the process variance becomes smaller, and
- the variance of the hypothetical means becomes larger.

Additionally, it is easier to discern the identity of the R_i as the number of observations increases.

Much of the emphasis has been placed on the ability to discern the true identity of the R_i. It is not necesssary to be absolutely certain of the true identity of the R_i; it is only necessary to change the *a priori* estimates of the probabilities of each of the R_i (this establishes the link with Bayesian analysis). With relatively small process variance and/or large variance of the hypothetical means, the *a posteriori* estimates of the probabilities of each R_i can be modified significantly from their *a priori* values. Because we presume that we have knowledge of the mean of the universe, and the mean of each of the risks, and are ignorant only of the actual identity of the R_i, it is that knowledge that will lead us to a better estimate of subsequent loss experience.

To recap, credibility should increase with

- the number of observations,
- decreasing process variance, and
- increasing variance of the hypothetical means.

The derivation of the proper function relating these variables can be found elsewhere and will be stated here without proof. Considering the complexity of the concepts, it is a remarkably simple formulation:

$$Z = n/(n + K)$$

where n is the number of observations,

K is $\dfrac{\text{expected value of the process variance}}{\text{variance of hypothetical means}}$, and

Z is the resulting credibility.

EXPERIENCE RATING CREDIBILITIES

It is interesting to compare the development of the credibility formulas used in experience rating with those used in ratemaking. The formula used in the worker's compensation experience rating plan has been essentially of the form $E/(E + K)$ since 1918.[16]

[16] Paul Dorweiler, "A Survey of Risk Credibility in Experience Rating," *PCAS* XXI, 1934 or *PCAS* LVIII, 1971.

Although various changes have been made to the formula to reflect different treatments of normal versus excess losses, the form has remained basically unchanged. It is also noteworthy that the formula is a function of *expected* losses, rather than *actual* losses, as is generally the rule for the ratemaking formulas. Although the form is the same as that discussed by Hewitt and Bühlmann, the value of K does not coincide with Bühlmann's derivation.[17,18]

According to Dorweiler, "the members of the committee, after consulting with underwriters, chose those curves which in their opinion produced the best results for the set of risks and thus established the constants K_1 and K_2 . . ." Later, the value of K was derived from an *ad hoc* selection of the "swing" of the plan.[19]

The concept of a Q-point and S-value, between which the value of Z no longer is calculated from $E/(E + K)$ but rises smoothly from $Q/(Q + K)$ to 1 as E varies between Q and S is not justified on theoretical grounds but can be justified on pragmatic grounds: for risks sufficiently large, the difference between the modifications resulting from the "correct" versus the *ad hoc* formula is insignificant and does not justify the additional computations necessary for the theoretically preferable formula. This pragmatic approach compares closely with the concept of full credibility described in Mayerson.[20]

Ratemaking formulas started with a formula for full credibility and then made adjustments to accommodate the need for partial credibility, whereas the experience rating formulas started with a formula for partial credibility and made adjustments to accommodate the practical need for full credibility. As trivial as this distinction may sound, it turns out to explain many of the historical problems with classical credibility. Assuming one accepts the formula $Z = E/(E + K)$, which approaches but never reaches unity, any attempt to define a unique full credibility standard is doomed to failure. Moreover, because the full credibility standard will not be based on $E/(E + K)$, the derivation of partial credibilities consistent with this formula will be more difficult.

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[17] Hewitt, op. cit.

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[18] Bühlmann, op. cit.

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[19] Francis S. Perryman, "Experience Rating Plan Credibilities," *PCAS* XXIV, 1937 or *PCAS* LVIII, 1971.

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[20] Allen L. Mayerson, op. cit.

A MISCONCEPTION

One of the misconceptions surrounding credibility is that a large value of credibility is a desirable situation. This sounds quite reasonable. After all, why would you prefer a situation where the experience has low credibility to one where the credibility is high? However, as will be shown, a situation where experience has low credibility can be preferable to the high credibility situation. I believe that this misconception rests on the confusion of the terms "credibility" and "confidence." The two terms sound similar but have different meanings. Credibility in the familiar sense (as opposed to its technical meaning) *is* almost a synonym for confidence. However, "credibility" is used in some places where the term "confidence" is meant.

Since the difference between these two terms is so important, it is appropriate to set down definitions of the terms.

> CREDIBILITY—The appropriate weight to be given to a statistic[21] of the experience in question *relative* to other experience.
>
> CONFIDENCE—The likelihood that a statistic is close to the theoretical value.

Several observations are pertinent.

- Credibility is a *relative* concept while confidence is an *absolute* concept.
- Credibility naturally produces values between 0 and 1. Confidence measures are not as well-behaved. Traditionally, confidence is measured as a probability P that the true mean is within $100K\%$ of the observed mean.
- Credibility can be thought of as relative confidence. Even though the mean of a particular set of observations has a low measure of confidence, if the prior information also has a low measure of confidence, the credibility of the current set may be high.

In the second example, the universe was assumed to consist of a single classification containing four elements. Let us redefine the universe to include a larger number of elements that have been partitioned into classifications. Assume that one of these classifications is comprised of the four elements in the

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[21] Recall that a statistic is simply a function of the observed values. Generally, we will be referring to the sample mean, but the concept of credibility should generalize to other statistics (with appropriate changes in the calculation of K).

original example. The credibility of a single observation can be calculated as follows.

$$Z = n/(n + K)$$

where $n = 1$,

expected value of process variance[22] = 154/9, and

variance of hypothetical means = 14/9;

$$\text{thus } Z = \frac{1}{1 + \frac{154/9}{14/9}} = \frac{1}{1 + 11} = \frac{1}{12}.$$

Suppose the original class of R_1, R_2, R_3, and R_4 is replaced by R_1, R'_2, R'_3, and R'_4. The expected value of the process variance is unchanged, but the variance of the hypothetical means is now 120 and

$$Z = \frac{1}{1 + \frac{154/9}{120}} \approx \frac{1}{1 + .143} \approx .88.$$

With our new classifications, we have 88% credibility for a single observation compared to 8% for the old classifications. Does this indicate a preferable situation? Absolutely not. The credibility is high because the new classification is much less homogeneous than the old one; the hypothetical means are much farther apart. The confidence surrounding the classification mean is extremely low. The absolute confidence of the observations has not changed, but the relative confidence has increased. Credibility is high, not because the sample information is so "good," but because the prior information is so "bad."

Does this mean low credibility is always desirable? Of course not. To understand when high credibility is desirable and when it is not, it will be helpful to examine our universe more closely.[23] Typically, our universe is composed of a number of classifications, each of which contains a number of

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[22] Although this value was not explicitly calculated in this paper, it is straightforward and can be calculated easily, or the reader may refer to Hewitt, "Credibility for Severity," p. 158.

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[23] Here, as before, the term "universe" is used in the mathematical sense. It includes not all possible things, but the entire set of items relevant to the question at hand. For example, if the question concerns automobile liability ratemaking, the universe would include experience relevant to automobile liability, but not homeowners experience.

individual risks so that our structure has three levels: risk, class, and universe.[24] There is an important distinction among these three levels. While there is generally no latitude as to the definitions of the universe or the individual risks, we are free to aggregate the individual risks into classes as we wish. Once a particular class plan has been chosen, there are two major uses of credibility:

- RATEMAKING—Ratemaking consists of two major steps. First, a new mean pure premium for the universe is calculated by credibility weighting the indications of the most recent data with the pure premium presently in use.[25] Second, the indications of each class are credibility weighted with the new mean premium for the universe to derive the new pure premiums for each class.
- EXPERIENCE RATING—Experience rating, or individual risk rating, consists of credibility weighting the actual experience of the risk with the pure premium of the particular risk's class.

The ultimate goal for each individual risk is a rate which is as close to the true mean of the risk as possible. Because the experience of each of the individuals is fixed, and, equivalently, the overall experience is fixed, the only variable is the class plan. Creation of a class plan is equivalent to a stratification of the universe of risks; hence the ideas in Lange's paper, "Implications of Sampling Theory. . .," are applicable. In this paper, he discusses a desirable property of a stratification, namely, that the resulting strata should be as homogeneous as possible.[26] Although Lange's immediate goal was to improve the estimate of the overall mean, while we are interested in the pure premium for the individual risk, the goals are consistent.

If we have homogeneous strata, or classes, then the experience of the individual risks within a class will be similar to each other, hence close to the class mean. But with homogeneous classes, the means of the various classes tend to be "farther apart" from each other than if we have non-homogeneous classes.[27] When considering the use of credibility in ratemaking, a credibility

[24] The situation where two levels of classifications exist (as distinct from a two-way class system), such as in Workers' Compensation, where individual risks make up classes that are aggregated into industry groups, is slightly more complicated and will not be addressed here.

[25] With appropriate adjustments for trend, development, etc.

[26] Jeffrey T. Lange, "Implications of Sampling Theory for Package Policy Ratemaking," *PCAS* LIII, 1966, p. 288.

[27] If this is not obvious, consider a class plan where individual risks are randomly assigned to classes. These classes will be quite non-homogeneous, and the experience of each class will tend to approximate that of the universe; hence the classes will tend to be "close to each other."

for each class must be calculated. Since in a "good" class plan, the experience of risks within a class is similar, the process risk (or "within" variance) will be smaller than if the classes were not homogeneous. Also, the variance of the hypothetical means (the "among" variance), which is the variance of the various classes, will be higher for a class plan with homogeneous classes. Hence, the calculation of K (to be used for credibility) for a class plan with homogeneous classes will result in a relatively small value of K, since

$$K = \frac{\text{expected value of process variance}}{\text{variance of hypothetical means}} .$$

A small value of K implies high credibility based on the formula

$$Z = n/(n + K).$$

So we see that, with a "good" class plan, the credibility of the class experience will be higher than for a poorer class plan.

The situation is different for individual risk rating. Here we are credibility weighting the individual experience with the class experience. The process variance refers to the variance of the individual risk's experience, while the variance of the hypothetical means is the variance of the means of the individual risks within the class. If a class plan is created that has a very non-homogeneous class, then the variance of the hypothetical means will be large, making K small, resulting in large credibilities for individual risk experience.

In summary, we desire a class plan with homogeneous classes, which results in classification experience that has *high* credibility, but individual risk experience with *low* credibility. The relatively low credibility assigned to the experience of a single car is not a cause for concern, but an indication that the class is doing a relatively good job.[28]

HISTORICAL PERSPECTIVE

The simple chart in Table 6 may be helpful to an understanding of the relationships among some of the articles on credibility.

[28] Robert A. Bailey and LeRoy J. Simon, "An Actuarial Note on the Credibility of Experience of A Single Private Passenger Car," *PCAS* XLVI, 1959, p. 159.

TABLE 6

	FREQUENCY ONLY	PURE PREMIUM
CLASSICAL	Longley-Cook	Mayerson, Jones, Bowers
BAYESIAN	Mayerson	Hewitt

This partitioning is reasonably accurate; some of the exceptions are:

1. Longley-Cook does suggest ways to handle the pure premium but does not go into detail.
2. Longley-Cook states the importance of the prior information but does not utilize it in any formulas.
3. Mayerson summarizes the classical view.

The introduction of this paper contains several apparently inconsistent statements regarding credibility. Much of the confusion surrounding credibility arises from two sources:

- primary focus on the properties of the current observations, and
- an attempt to tackle the full credibility standard before the partial credibility standard.

Longley-Cook stressed the importance of the value of prior information. But his statements were not motivated by the same reasons that caused us to examine the statistical properties of the prior distribution. In his example, he concluded that Oregon fire data is inappropriate for New York ratemaking, not because of the arguments discussed in this paper, but because of the lack of applicability to the existing problem.

The development of classical credibility is closely tied to the traditional concerns regarding the proper balance between responsiveness and stability. Large weights given to the more recent data, or to the specific class data, will tend to increase responsiveness and decrease stability. In addition, it was correctly perceived that it is easier to defend a rate when the data used to make the rate is "local," in terms of time, geography, or class. These considerations quite naturally led to the attempt to assign the maximum weight possible to the current observations, subject to a stability restriction. The calculations of classical credibilities outlined by Longley-Cook follow directly from these arguments. In addition, Arthur Bailey's limited fluctuation credibility and greatest accuracy credibility[29] provide the link between the responsiveness/stability argument and Longley-Cook's statements regarding prior information.

The issue, related to the use of a standard based on frequency but applied to the pure premium, was a bit of a dilemma. Based on the foundations of classical credibility, it was difficult to refute the arguments for consideration of severity on theoretical grounds. But to include severity would cause practical problems. If the stability constraints were unchanged, the standards would be considerably increased. This would reduce responsiveness and increase the weights needed for "external" data. In addition, most actuaries felt that the existing standards were fairly reasonable. In theory, the stability constraints could be altered so that the same standards would result. But then the actuaries would be in the uneviable position of trying to justify to management and regulators a major change in approach which creates an insignificant change in results. Although the subject continued to receive theoretical attention, the ratemakers took the only practical course—they ignored the issue.

Bayesian credibility provides solutions to some of the problems associated with classical credibility, but at a cost: it is not trivial to understand, nor easy to apply in practice. Hewitt's reviews and his paper have contributed significantly to this subject. Hewitt's first list of three critical variables is mentioned in this paper because it was quoted by Lange in "Application of a Mathematical Concept of Risk." Hewitt clears up any misconception arising from this list, but this clarification is contained in a footnote of a review and might be missed.[30] The reader is urged to read this footnote carefully. The second list, which is consistent with this paper, applies to the more general case.

SUMMARY

The use of classical credibility has served the actuary well for many years. But the increased refinement of actuarial science requires that we turn to the theoretically preferable Bayesian credibility. The increased scrutiny of our methods requires that we be able to defend and explain our methods. It is hoped that this paper has contributed to the understanding and explanation of credibility concepts.

[29] Arthur L. Bailey, "Sampling Theory in Casualty Insurance," *PCAS* XXIX, 1942, p. 50 and *PCAS* XXX, 1943, p. 31.

[30] Hewitt, Discussion of "On the Credibility of the Pure Premium," p. 78.

MINUTES OF THE 1981 FALL MEETING

November 22–24, 1981

NEW ORLEANS HILTON, NEW ORLEANS, LOUISIANA

Sunday, November 22, 1981

The Board of Directors held their regular quarterly meeting from 1:00 p.m. to 4:00 p.m.

Registration took place from 4:00 p.m. to 7:30 p.m.

The President's reception for new Fellows and their spouses was held from 5:30 p.m. to 6:30 p.m.

A general reception for all members and guests was held from 6:30 p.m. to 7:30 p.m.

Monday, November 23, 1981

The meeting opened with welcoming remarks from Sherman A. Bernard, Commissioner of Insurance, State of Louisiana.

President Scheibl then announced the results of the election:

President	Steven H. Newman
President-elect	Frederick W. Kilbourne
Vice President	Carlton W. Honebein
Secretary	Brian E. Scott
Treasurer	Michael A. Walters
Editor	C. K. Khury
General Chairman Education and Examination Committee	Phillip N. Ben-Zvi
Directors	Daniel J. Flaherty
	Robert W. Sturgis
	Robert A. Anker

President Scheibl also announced the appointment of Herbert J. Phillips as Assistant Treasurer and as Director to fill the unexpired term of Carlton W. Honebein.

President Scheibl then recognized the twenty new Associates and awarded diplomas to the thirty-two new Fellows. The names of these individuals follow.

FELLOWS

Barrow, Betty H.	Gottheim, Eric F.	Mathewson, Stuart B.
Campbell, Catherine J.	Hennessy, Mary E.	Myers, Nancy R.
Cheng, Joseph S.	Henry, Dennis R.	Piersol, Kim E.
Cloutier, Guy	Hibberd, William J.	Purple, John M.
Cohen, Howard L.	Ingco, Aguedo M.	Racine, Andre R.
Corr, Francis X.	Kleinman, Joel M.	Ransom, Gary K.
Crowe, Patrick J.	LaRose, J. Gary	Schwartz, Allan I.
Dean, Curtis G.	Lederman, Charles M.	Sobel, Mark J.
Doellman, John L.	Lee, Yoong S.	Wasserman, David L.
Drummond-Hay, Eric T.	Linden, Orin M.	Weller, Alfred O.
Dussault, Claude	Mahler, Howard C.	

ASSOCIATES

Alpert, Bradley K.	Henzler, Paul J.	Silverman, Janet K.
Baum, Edward J.	Jaso, Robert J.	Soul, Harry W.
Boley, Russell A.	LeClair, Peter T.	Wainscott, Robert H.
Bursley, Kevin H.	Mill, Ralph A.	Watford, James D.
Dornfeld, James L.	Miller, Allen H.	Wilson, Ronald L.
Edwalds, Thomas P.	Muleski, Robert T.	Young, Bryan G.
Esposito, David L.	Odell, W. H.	

President Scheibl then presented a gift to Ms. Edith Morabito in recognition of her 25 years of faithful service to our Society.

Mr. P. Adger Williams, President-elect of the American Academy of Actuaries, presented a report on the activities of the Academy.

Mr. David Flynn, Secretary, reported on the Society's activities during the year.

Mr. Michael Walters, Treasurer, presented the Treasurer's report.

Mr. Lewis Roberts presented the Woodward-Fondiller award to Mr. Stephen W. Philbrick for his paper "The Implication of Sales as an Exposure Base for Products Liability."

Mr. Scheibl presented the Paul Dorweiler award to Mr. Michael A. Walters for his paper "Risk Classification Standards."

From 10:00 a.m. to 11:30 a.m. a debate was conducted on "Pricing . . . for Underwriting Profit or Overall Return?" The participants were:

Moderator: Leroy J. Simon
Prudential Reinsurance Company

Members: Carlton W. Honebein
Fireman's Fund Insurance Companies

Allan J. Nadler
Goldman Sachs & Company

Mavis A. Walters
Insurance Services Office

W. James MacGinnittie
Tillinghast, Nelson & Warren, Inc.

David Haight
C. F. Industries

Michael J. Miller
State Farm Mutual Insurance Company

Following the lunch break, the meeting reconvened for concurrent workshop sessions from 1:00 p.m. to 5:00 p.m. The sessions were as follows.

Workshop A — "New Paper"
"The 1979 Remarriage Table,"
by Phillip E. Heckman
CNA Insurance Companies

Workshop B — "New Paper"
"Actuarial Valuation of Property/Casualty Insurance Companies,"
by Robert W. Sturgis
Tillinghast, Nelson & Warren, Inc.

Workshop C — "New Paper"
"Credibility-Weighted Trend Factors,"
by Oakley E. Van Slyke
Warren, McVeigh & Griffin

Workshop D — "New Paper"
"RLS Yardsticks to Identify Financial Weakness,"
by Ruth E. Salzmann
Sentry Insurance Group

Workshop E — "New Paper"
"An Examination of Credibility Concepts,"
by Stephen W. Philbrick
Peat, Marwick, Mitchell & Co.

Workshop F — "Pricing . . . for Underwriting Profit or Overall Return?"
This was a discussion of the morning panel with the panelists.

Workshop G — "Actuarial Issues in a Small Insurance Company"
Panelists: James G. Inkrott
Central Mutual Insurance Company

John J. Schultz III
California Casualty Group

Allan R. Sheppard
Scor Reinsurance Company

Joseph O. Marker
Westfield Companies

Workshop H — "Structuring a Basic Reinsurance Program"
Speaker: Christopher P. Garand
American Reinsurance Company

Workshop I — "Confidence Intervals"
Speaker: Margaret E. Wilkinson
Tillinghast, Nelson & Warren, Inc.

Workshop J — "Refresher Course in Workers' Compensation Ratemaking"
Speakers: Anthony J. Grippa
National Council on Compensation Insurance

Wayne H. Fisher
Continental Insurance Companies

The President's reception was held from 6:30 p.m. to 7:30 p.m.

Tuesday, November 24, 1981

The business session reconvened at 8:30 a.m.

From 9:00 a.m. to 10:00 a.m. a panel session was held on "Investments for Insurance Companies." Those participating were:

Moderator: Ronald L. Bornheutter
General Reinsurance Corporation

Panelists: Paul M. Otteson
Consultant

Anthony T. Cope
Wellington Management Company

Robert Stricker
Aetna Life and Casualty

At the conclusion of the panel, President Scheibl delivered his Presidential Address.

At 11:00 a.m. a panel discussion was held entitled "Medical Malpractice—Impending Crisis or Not?" Those participating were:

Panelists: David J. Hartman
Chubb & Son, Inc.

F. James Mohl
St. Paul Fire and Marine

James O. Wood
Tillinghast, Nelson & Warren, Inc.

After a lunch break, the meeting reconvened at 1:30 p.m. for a presentation on "Communications by Actuaries." The presentation was made by Linda Delgadillo, Director of Communications, Society of Actuaries.

The closing remarks were made by President-elect Steven H. Newman after which the meeting adjourned at 3:00 p.m.

In attendance, as indicated by registration records, were 189 Fellows, 100 Associates, 18 guests, 15 subscribers, 5 students, and 123 spouses. The list follows.

FELLOWS

Alfuth, T. J.
Anderson, D. R.
Anker, R. A.
Atwood, C. R.
Balcarek, R. J.
Balko, K. H.
Barrow, B. H.
Bass, I. K.
Bayley, T. R.
Beer, A. J.
Bell, L. L.
Belvin, W. H.
Ben-Zvi, P. N.
Berquist, J. R.
Bethel, N. A.
Biondi, R. S.
Bornhuetter, R. L.
Bradshaw, J. G., Jr.
Brown, J. W., Jr.
Campbell, C. J.
Carter, E. J.
Cheng, J. S.
Cis, M. M.
Cloutier, G.
Cohen, H. L.
Conger, R. F.
Corr, F. X.
Covney, M. D.
Crowe, P. J.
Davis, G. E.
Dean, C. G.
Degerness, J. A.
Dempster, H. V., Jr.
DiBattista, S. T.
Doellman, J. L.
Dolan, M. C.
Donaldson, J. P.
Drennan, J. P.
Dropkin, L. B.
Drummond-Hay, E. T.
Dussault, C.
Ehlert, D. W.
Faber, J. A.
Fallquist, R. J.
Farnam, W. E.
Ferguson, R. E.
Fiebrink, M. E.
Fisher, W. H.
Fitzgibbon, W. J., Jr.
Flaherty, D. J.
Flynn, D. P.
Fowler, T. W.
Fresch, G. W.
Frohlich, K. R.
Furst, P. A.
Fusco, M.
Garand, C. P.
Gibson, J. A., III
Gillespie, J. E.
Gleeson, O. M.
Goddard, D. C.
Goldberg, S. F.
Golz, J. F.
Gottheim, E. F.
Gottlieb, L. R.
Grannan, P. J.
Graves, J. S.
Groot, S. L.
Hachemeister, C. A.
Hafling, D. N.
Hartman, D. G.
Hazam, W. J.
Heer, E. L.
Henry, D. R.
Hennessy, M. R.
Herzfeld, J.
Hibberd, W. J.
Honebein, C. W.
Ingco, A.
Inkrott, J. G.
Jaeger, R. M.
Jerabek, G. J.
Kaliski, A. E.
Karlinski, F. J., III
Kaufman, A.
Kelly, A. E.
Khury, C. K.
Kilbourne, F. W.
Klaassen, E. J.
Kleinman, J. M.
Krause, G. A.
Larose, J. G.
Ledbetter, A. R.
Lederman, C. M.
Leimkuhler, U. E.
Lerwick, S. N.
Levin, J. W.
Linden, O. M.
Lino, R.
Lombardo, J. S.
Lowe, R. F.
Lowe, S. P.
MacGinnitie, W. J.
Mahler, H. C.
Marker, J. O.
Masterson, N. E.
Mathewson, S. B.
McCarter, M. G.
McClenahan, C. L.
McClure, R. D.
McLean, G. E.
McManus, M. F.
McMurray, M. A.
Miccolis, R. S.

FELLOWS

Miller, M. J.
Miller, P. D.
Mills, R. J.
Mohl, F. J.
Moore, B. C.
Moore, P. S.
Morison, G. D.
Munro, R. E.
Murray, E. R.
Myers, N. R.
Nash, R. K.
Nelson, D. A.
Newman, S. H.
Oakden, D. J.
O'Brien, T. M.
Otteson, P. M.
Pagnozzi, R. D.
Patrick, G. S.
Perkins, W. J.
Peters, S.
Petersen, B. A.
Philbrick, S. W.
Phillips, H. J.
Pierce, J.
Piersol, K. E.
Purple, J. M.
Racine, A.
Radach, F. R.
Ransom, G. K.
Reynolds, J. J., III
Richardson, J. F.
Rodermund, M.
Rogers, B. T.
Rosenberg, N.
Roth, R. J., Jr.
Rowland, W. J.
Salzmann, R. E.
Scheibl, J. A.
Schultz, J. J., III
Schumi, J. R.
Schwartz, A. I.
Scott, B. E.
Sheppard, A. R.
Simon, L. J.
Snader, R. H.
Sobel, M. J.
Stanard, J. N.
Stephenson, E. A.
Streff, J. P.
Sturgis, R. W.
Swift, J. A.
Tatge, R. L.
Taylor, J. C.
Teufel, P. A.
Thibault, A.
Toothman, M. L.
Trudeau, D. E.
Tuttle, J. E.
Van Slyke, O. E.
Venter, G. G.
Walsh, A. J.
Walters, Ma. A.
Walters, Mi. A.
Ward, M. R.
Wasserman, D.
Webb, B. L.
Weller, A. O.
Williams, P. A.
Wilson, J. C.
Wiser, R. F.
Wood, J. O.
Woods, P. B.
Zatorski, R. T.
Zelenko, D. A.
Zubulake, T. J.

ASSOCIATES

Alpert, B. K.
Andler, J. A.
Austin, J. P.
Baum, E. J.
Bealer, D. A.
Bell, A. A.
Brahmer, J. O.
Burger, G.
Bursley, K. H.
Cadorine, A. R.
Camp, J. H.
Chorpita, F. M.
Chou, P. S.
Ciezadlo, G. J.
Clark, D. G.
Cohen, A. I.
Connor, V. P.
Currie, R. A.
Dawson, J.
Degarmo, L. W.
Dodd, G. T.
Dornfeld, J. L.
Duffy, T. J.
Edwalds, T. P.
Einck, N. R.
Esposito, D. L.
Fasking, D. D.
Feldman, M. F.
Fisher, R. S.
Flack, P. R.
Flanagan, T. A.
Foley, C. D.
Gaillard, M. B.

ASSOCIATES

Granoff, G.
Gwynn, H. M.
Hallstrom, R. C.
Head, T. F.
Heckman, P. E.
Heersink, A. H.
Henzler, P. J.
Hine, C. A.
Hobart, G. P.
Hurley, J. D.
Jaso, R. J.
Jersey, J. R.
Johnson, L. D.
Johnson, M. A.
Johnston, T. S.
Klingman, G. C.
Knilans, K.
Kolojay, T. M.
Lamonica, M. A.
Lange, D. L.
Livingston, R. P.
Lommele, J. A.
Ludwig, S. J.
McDaniel, G. P.
McDonald, C.
Meyer, R. E.
Mill, R. A.
Miller, A. H.
Mokros, B. F.
Mulder, E. T.
Muleski, R. T.
Odell, W. H.
Pastor, G. H.
Peacock, W. W.
Pei, K-J.
Philbrick, P. G.
Piazza, R. N.
Pilon, A.
Pratt, J. J.
Pulis, R. S.
Rosa, D.
Sandler, R. M.
Sansevero, M., Jr.
Sherman, O. L., Jr.
Silverman, J. K.
Singer, P. E.
Skrodenis, D. P.
Smith, F. A.
Soul, H. W.
Surrago, J.
Sweeny, A. M.
Tom, D. P.
Torgrimson, D. A.
Urschel, F. A.
Van Ark, W. R.
Wade, R. C.
Waldman, R. H.
Walker, R. D.
Watford, J. D.
White, F. T.
White, J.
Whitman, M.
Wilkinson, M. E.
Wilson, R. L.
Yatskowitz, J. D.
Young, B. G.
Young, E. W.

GUESTS—STUDENTS—SUBSCRIBERS

Altschuler, M. C.
Belton, E. F.
Benson, D. W.
Bernard, S. A.
Brown, A. F., Jr.
Carpenter, J. G.
Chang, C. E.
Clarke, T. G.
Clowes, W. M.
Cope, A. T.
Curran, K. F.
Davies, R. W.
Delgadillo, L. M.
Deutsch, R. V.
Earls, R. R.
Eckilson, G. W.
Gutman, E.
Haight, D. R.
Hatfield, B. D.
Hendrickson, S. A.
Hoskins, R. H.
Hutter, H. E.
Jensen, P. A.
Keating, R. C.
Kimball, D. E.
MacKay, D. B.
Mitchell, J. A.
Nadler, A. J.
Pope, D. W.
Rech, J. E.
Reott, J. A.
Rothman, R.
Smith, D. A.
Spangler, J. L.
Steinhauser, J.
Stenmark, J. A.
Stricker, R.
Zanes, R. G.

Respectfully submitted

DAVID P. FLYNN
Secretary

REPORT OF THE SECRETARY

The purpose of this report is to provide the membership with information on the activity of the Society during the past year, particularly that of its Board of Directors and associated committees. Secretaries have learned to approach this task with some hesitation, as it is impossible to do justice to all of our members' individual contributions in a short narrative, and there were many such contributions.

Certainly the most pervasive change in the past year was the introduction of mail nominations and balloting to the election process. The new process increased the level of participation by our membership by 50% over that typically cast at our meetings. The total number of ballots cast for our November 1981 elections was 334.

We observed considerable activity this year in the areas of education and examination procedures and policy. Highlights of significant developments are summarized below.

(1) In March of this year, the Board adopted a formal Statement of Education Policy for the Casualty Actuarial Society. This statement is intended to provide a framework for the preparation of operating policies for the Education, Examination, and Continuing Education Committees. Most members will be pleased to learn that mandatory requirements for continuing education will not be established, with voluntary pursuit of knowledge being the preferred route. A copy of the Statement of Education Policy follows this report.

(2) To date, there is no basic textbook on the subject of actuarial science which either describes the historical development of the profession or addresses generic actuarial problems and applications such as ratemaking, reserves, and credibility. The need for such a source both for prospective and current members of the actuarial profession is readily apparent. An Ad Hoc Committee on Textbook Considerations was therefore established to produce a textbook, written primarily for the use of CAS students and members, which will serve as an introduction to casualty actuarial science. The committee to date has defined the contents and methods of production of this text, and will proceed along these lines to compile the necessary information.

(3) A great deal of time and effort is expended by our members who submit papers for the enrichment and education of our students and the entire Society. In recognition of these individuals, a plaque has been designed which will be presented to both past and future winners of the Michelbacher, Dorweiler and Woodward-Fondiller prizes.

(4) Formal recognition was also considered for Associates of the Society. In this light, the Board decided to reestablish the practice of awarding diplomas to new Associates. Current Associates will also be given the option to obtain certificates. Details about this program will be available in the near future.

(5) Over the years, the concept of introducing specialty tracks into the CAS examination program has been discussed for its potential value in work applications and qualifying requirements. Recently, the Board reaffirmed the recommendation of the Education Policy Committee that separate specialty tracks not be considered at this time. However, to address the need for a comprehensive examination program, the Education Committee will broaden the accident & health and Canadian content in the syllabus as they deem appropriate.

With respect to the internal organization and operating procedures of the Society, there were two major areas of activity.

(1) Due to the recent membership growth of our Society and our heavy reliance on volunteer committees to fulfill most of our organizational needs, the Board determined that a review of the efficiency of our organizational structure was required. In response to this concern, an Ad Hoc Committee on Management and Operations was established to conduct this review and propose a program consistent with sound management principles. An initial report was presented to the Board by the Committee at yesterday's Board meeting. This will be an important item in 1982.

(2) In July of this year, an Ad Hoc Committee was appointed to examine the possible need to recognize specialty and/or specialty interest groups within the CAS. While in their preliminary September report they encouraged greater use of workshops, seminars and theme meetings, the Committee also recommended the creation of a mechanism to organize formal groups that would be affiliated with and conform to the CAS and its Constitution and By-laws. The Board has therefore requested that the Committee formulate guidelines and a model constitution for the establishment of special interests sections.

The Long Range Planning Report of the American Academy of Actuaries was another topic of considerable discussion. The Board supported the Academy Statement of Purpose, but observed that the report, while consistent with the CAS Long Range Planning Committee Report, did not address the problem of qualified loss reserve specialists.

In general, the Academy Statement of Purpose dealt with the establishment of high standards of conduct and competence, and the interaction of actuaries with the public and other professions and organizations to fulfill public needs. The complete Academy report is available from the Academy or the Secretary's office for those who are interested in additional details.

With respect to the future directions of the Society, the Long Range Planning Committee recently offered their perspectives of the CAS for the next five to ten years. Some of the highlights of their forecasts are as follows.

(1) The CAS will continue as an independent society, primarily oriented toward property and casualty.
(2) The Society will be a recognized leader on property and casualty issues of an actuarial nature; it will become influential on legislative matters and be increasingly sought after for its comments and observations.
(3) While membership is expected to double by 1990, the financial needs of the society will increase substantially, necessitating a significant increase in annual membership dues.

Fortunately, the impact of these financial forecasts will not be felt immediately; there will be no increase in either 1982 CAS membership or examination fees. However, the Society of Actuaries will raise their 1982 examination fees for Parts 1–4. The new fees will be as follows:

Parts 1–3	$40.00
Part 4	$50.00

To mark the continued growth of our Society, we welcomed 59 new Associates and 43 new Fellows during 1981. Thus our total membership has grown to 940, comprised of 436 Associates and 504 Fellows. Exam enrollment for Parts 5–10 totaled 1371 in 1981, a 20% increase over 1980.

The Board has approved Toronto as the site of our November 1983 convention.

Finally, I wish to extend my thanks to all who have aided in the administrative work of the Society during the past year. Special thanks to Edith Morabito and Carol Olszewski of our New York office and to my own secretary, Pamela Sawas. Their work has again been invaluable and deeply appreciated.

Respectfully submitted,

DAVID P. FLYNN
Secretary

CASUALTY ACTUARIAL SOCIETY
STATEMENT OF EDUCATION POLICY

The Casualty Actuarial Society (CAS) is committed to the furtherance of actuarial knowledge through a comprehensive, integrated program of education and research and to the establishment of related professional standards.

The basic educational objectives of the CAS shall be:

1. To provide and foster a program of actuarial education leading to Fellowship in the CAS
   a. by defining the basic areas of knowledge and skills necessary to obtain the competence to practice in the various actuarial specialties,
   b. by defining standards of educational achievement required for membership in the CAS,
   c. by providing means of measuring educational achievement;
2. To provide and foster programs of actuarial education for members to update or expand upon their basic skills and knowledge;
3. To promote and foster educational activities and research which will expand and enhance the overall base of actuarial knowledge;
4. To provide mechanisms for disseminating to members and non-members resource material relating to actuarial topics of educational nature.

REPORT OF THE TREASURER

One of the responsibilities of the Treasurer is to present to the membership at the annual meeting an accounting of CAS fiscal matters. The detailed financial results and the audited accrual basis accounting statement as of September 30, 1981 are appended to this report. The highlights are as follows.

Fiscal 1981 ended with a surplus gain of $16,000, due mostly to an increase in interest earned plus a gain in the meetings account versus a budgeted loss. The 1982 budget, as approved by the Board of Directors, represents an increase in income and expenses, and targets for another surplus gain of smaller magnitude at the end of 1982, barring any surprises. No change in dues was needed, as some of the added costs were offset by increased revenues due to interest and a larger number of students taking exams. The surplus would cover about seven months of operating expenses, if there were no income.

In the spirit of conservative accounting, we have set up a new short term liability in the accrual-based income statement—namely, a reserve for prepaid examination fees. This means that fees received prior to September 30 (our fiscal year end) will be matched by an equal reserve until the exams are given. This has the effect of depressing accrued income in Fiscal 1981, since it defers the income for the November 1981 exam until Fiscal 1982. It should have little or no effect in future fiscal years' income statements, but will increase the stated liabilities at year end.

On the asset side, the CAS has reinvested its matured five-year 7% U.S. Treasury Note with another five-year note at twice the interest rate. The rest of the members' equity, plus virtually all the working capital, is now in money market funds or in six month certificates of deposit. Hence, total interest accrued in 1981 was almost half as much as dues income.

Looking at the changes over the past five fiscal years, surplus has grown since 1977 at an annual rate of 7%, while membership has grown at a 6% rate. The ten dollar dues increase in 1979, annualized over the entire period, is a 4% annual rate.

As a final perspective on how the CAS can operate with a relatively small budget, it is through the voluntary dedication of its principal resource—the members who serve on its committees. If this effort had to be recompensed, or staff had to be hired to replace it, the dues would probably increase tenfold.

Respectfully submitted,

MICHAEL A. WALTERS
Treasurer

FINANCIAL REPORT
Fiscal Year Ended 9/30/81

Income		Disbursements	
Dues	$ 59,418.40	Printing	$ 64,228.39
Exam fees	27,164.56	Office expenses	51,277.23
Meetings	77,011.25	Examination expenses	1,491.04
Sale of *Proceedings*	9,079.84	Meeting expenses	74,192.70
Sale of Readings	10,791.70	Library	258.00
Invitational program	5,489.26	Math. Assn. of America	1,500.00
Interest	24,006.31	Insurance	989.00
Actuarial Review	257.40	Publicity	2,500.00
Total	$ 213,218.72	Miscellaneous	1,224.71
		Total	$197,661.07

Income	$ 213,218.72
Disbursements	197,661.07
Change in CAS surplus	$ + 15,557.65

ACCOUNTING STATEMENT

Assets	9/30/80	9/30/81	Change
Bank accounts	$ 28,818.51	$ 8,238.38	$ − 20,580.13
Money market fund	65,064.06	119,685.71	+ 54,621.65
Bank Certificate of Deposit	—	20,000.00	+ 20,000.00
U.S. Treasury Notes	99,535.00	99,971.90	+ 436.90
Accrued income	17,040.00	9,144.00	− 7,896.00
Total	$210,457.57	$257,039.99	$ + 46,582.42
Liabilities			
Office services	$ 12,612.00	$ 35,149.00	$ + 22,537.00
Printing expenses	32,542.00	25,236.00	− 7,306.00
Examination expenses	300.00	1,148.00	+ 848.00
Meeting expenses	0	1,235.00	+ 1,235.00
Prepaid exam fees	0	7,764.00	+ 7,764.00
Other	500.00	2,932.00	+ 2,432.00
Total	$ 45,954.00	$ 73,464.00	$ + 27,510.00
Members' Equity			
Michelbacher fund	$ 36,266.88	$ 39,074.85	$ + 2,807.97
Dorweiler fund	7,757.60	8,439.40	+ 681.80
CAS trust	277.80	302.80	+ 25.00
CAS surplus	120,201.29	135,758.94	+ 15,557.65
Total	$164,503.57	$183,575.99	$ + 19,072.42

Michael A. Walters
Treasurer

This is to certify that the assets and accounts shown in the above financial statement have been audited and found to be correct.

Finance Committee
Walter J. Fitzgibbon, Jr., Chairman
Glenn W. Fresch
David M. Klein
James W. Thomas

1981 EXAMINATIONS—SUCCESSFUL CANDIDATES

Examinations for Parts 5, 7, and 9 of the Casualty Actuarial Society syllabus were held on November 16, and 17, 1981. Examinations for Parts 6, 8, and 10 were held on May 6, and 7, 1981.

Examinations for Parts 1, 2, 3, and 4 are jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries. These examinations were given in May and November 1981. Candidates who passed these examinations were listed in the joint releases of the two societies.

The Casualty Actuarial Society and the Society of Actuaries jointly awarded prizes to the undergraduates ranking the highest on the General Mathematics examination. For the May, 1981 examination, the $200 prize was awarded to Jeffrey E. Riley. The additional $100 prize winners were Anthony C. Carpentieri, Leon E. Gruenbaum, Christopher P. J. Paranicas, and Richard C. Payne. For the November, 1981 examinations, the $200 prize was awarded to Michael V. Finn. The additional $100 prize winners were David C. Cogburn, Randy A. Gomez, Mark A. Holman, and Amy Wu.

The following candidates were admitted as Fellows and Associates at the November, 1981 meeting as a result of their successful completion of the Society requirements in the May, 1981 examinations:

FELLOWS

Barrow, Betty H.
Campbell, Catherine J.
Cheng, Joseph S.
Cloutier, Guy
Cohen, Howard L.
Corr, Francis X.
Crowe, Patrick J.
Dean, Curtis G.
Doellman, John L.
Drummond-Hay, Eric T.
Dussault, Claude
Gottheim, Eric F.
Hennessy, Mary E.
Henry, Dennis R.
Hibberd, William J.
Ingco, Aguedo M.
Kleinman, Joel M.
LaRose, J. Gary
Lederman, Charles M.
Lee, Yoong S.
Linden, Orin M.
Mahler, Howard C.
Mathewson, Stuart B.
Myers, Nancy R.
Piersol, Kim E.
Purple, John M.
Racine, Andre R.
Ransom, Gary K.
Schwartz, Allan I.
Sobel, Mark J.
Wasserman, David L.
Weller, Alfred O.

ASSOCIATES

Alpert, Bradley K.
Baum, Edward J.
Boley, Russell A.
Bursley, Kevin H.
Dornfeld, James L.
Edwalds, Thomas P.
Esposito, David L.
Henzler, Paul J.
Jaso, Robert J.
LeClair, Peter T.
Mill, Ralph A.
Miller, Allen H.
Muleski, Robert T.
Odell, W. H.
Silverman, Janet K.
Soul, Harry W.
Wainscott, Robert H.
Watford, James D.
Wilson, Ronald L.
Young, Bryan G.

The following is the list of successful candidates in examinations held in May, 1981:

Part 6

Allaben, Mark S.
Alpert, Bradley K.
Atkinson, Roger A., III
Balling, Glenn R.
Baum, Edward J.
Becraft, Ina M.
Belden, Scott C.
Bensimon, Abbe S.
Bhagavatula, Raja R.
Biegaj, William P.
Boccitto, Bonnie L.
Boley, Russell A.
Boulanger, Francois
Bouska, Amy S.
Bowen, David S.
Brockmeier, Donald R.
Bujaucius, Gary S.
Bursley, Kevin H.
Busche, George R.
Campbell, Kenrick A.
Cantin, Claudette
Cathcart, Sanders B.
Coffin, John D.
Colin, Steven L.
Costner, James E.
Dashoff, Todd H.
DeLiberato, Robert V.
Domanico, Elaine M.
Dornfeld, James L.
Downer, Robert B.
Duffy, Brian
Edmondson, Alice H.
Edwalds, Thomas P.
Egnasko, Valere M.
Ellefson, Thomas J.
Elliott, Paula L.
Epstein, Michael
Esposito, David L.
Forde, Claudia S.
Fromentin, Pierre
Gattel, Lisa H.
Gerard, Felix R.
Gillam, William R.
Gilles, Joseph A.
Hall, Allen A.
Hanson, Jeffrey L.
Harwood, Catherine B.
Hayward, Gregory L.
Henzler, Paul J.
Hofmann, Richard A.
Holdredge, Wayne D.
Hoppe, Kenneth J.
Howald, Ruth A.
Hurley, Paul M.
Jackson, Vincent H.
Jaso, Robert J.
Kane, Adrienne B.
Keen, Eric R.
Keller, Wayne S.
Kelley, Kevin J.
Klawitter, Warren A.
Klinker, Fredrick L.
Kooken, Michael W.
Kostka, Thomas C.
Krestal, Stacy J.
LeClair, Peter T.
Lee, Robert H.
Lonergan, Kevin F.
Lonergan, Thomas X.
Loper, Dennis J.
Marks, Steven D.
Mashitz, Isaac
McGovern, Eugene
Mendelssohn, Gail A.
Mill, Ralph A.

Part 6

Miller, Allen H.
Miller, David L.
Milligan, Alfred W.
Mittal, Madan L.
Mozeika, John K.
Muleski, Robert T.
Narvell, John C.
Neale, Catharine L.
Nelson, Cheryl L.
Nikstad, James R.
O'Connell, Paul G.
Odell, W. H.
Ogden, Dale F.
Paglieri, Wayne C.
Parrish, Richard J.
Pierson, Frank D.
Potts, Cynthia M.
Raman, Rajagopalan K.
Rapoport, Andrew J.
Schmidt, Neal J.
Schultheiss, Peter J.
Siewert, Jerome J.
Silverman, Janet K.
Skaroff, Robert D.
Smith, Judith P.
Smith, Michael B.
Soul, Harry W.
Splitt, Daniel L.
Stadler-Hrbacek, E.
Theisen, Joseph P.
Thorrick, John P.
Townsend, Christopher J.
Visner, Steven M.
Wacek, Michael G.
Wainscott, Robert H.
Walker, Leigh M.
Washburn, Monty J.
Watford, James D.
Watson, Lois A.
Weimer, William F.
White, David L.
Whiting, David R.
Windwehr, Debra R.
Withers, David A.
Woomer, Roy T., III
Yen, Chung-Ye
Young, Bryan G.

Part 8

Addie, Barbara J.
Bashline, Donald T.
Briere, Robert S.
Carpenter, Thomas S.
Carponter, John D.
Chernick, David R.
Chuck, Allan
Ciezadlo, Gregory J.
Cimini, Edward D., Jr.
Clinton, R. Kevin
Connell, Eugene C.
Cundy, Richard M.
Currie, Ross A.
Dodd, George T.
Eagelfeld, Howard M.
Foote, James M.
Friedberg, Bruce F.
Gannon, Alice H.
Ghezzi, Thomas L.
Gillespie, Bryan C.
Gluck, Spencer M.
Hine, Cecily A.
Holmberg, Randall D.
Keatts, Glenn H.
Keller, Wayne S.
Knilans, Kyleen
Koski, Mikhael I.
LaMonica, Michael A.
Lobosco, Virginia R.
Ludwig, Stephen J.
Lyle, Aileen C.
Mahler, Howard C.
Matthews, Robert W.
McGovern, William G.
Mealy, Dennis C.
Miyao, Stanley K.
Moody, Rebecca A.
Murad, John A.
Murphy, William F.
Muza, James J.
Nichols, Richard W.
Onufer, Layne B.
Philbrick, Polly G.
Plunkett, Richard C.
Rudduck, George A.
Ryan, John P.
Scholl, David C.
Schwartz, Allan I.
Sherman, Ollie L., Jr.
Sobel, Mark J.
Tom, Darlene P.
Walker, Glenn M.
Warren, Jeffrey C.
Weidman, Thomas A.
Wess, Clifford
Whitman, Mark
Wiseman, Michael L.
Yingling, Mark E.
Yonkunas, John P.
Youngerman, Hank

Part 10

Barrow, Betty H.
Braithwaite, Paul
Burger, George
Campbell, Catherine J.
Cheng, Joseph S.
Christie, James K.
Cloutier, Guy
Cohen, Howard L.
Corr, Francis X.
Crowe, Patrick J.
Davis, Lawrence S.
Dean, Curtis G.
Doellman, John L.
Drummond-Hay, Eric T.
Duffy, Thomas J.
Dussault, Claude
Gottheim, Eric F.
Halpert, Aaron
Hennessy, Mary E.
Henry, Dennis R.
Hibberd, William J.
Hu, David D.
Ingco, Aguedo M.
Johnson, Warren H., Jr.
Josephson, Gary R.
Kleinman, Joel M.
Koch, Leon W.
Kozik, Thomas J.
LaRose, J. Gary
Larsen, Michael R.
Lederman, Charles M.
Lee, Yoong S.
Linden, Orin M.
Mahler, Howard C.
Mathewson, Stuart B.
Meyer, Robert E.
Meyers, Glenn G.
Myers, Nancy R.
Nickerson, Gary V.
Piersol, Kim E.
Pinto, Emanuel
Purple, John M.
Racine, Andre R.
Ransom, Gary K.
Roman, Spencer M.
Schwartz, Allan I.
Wasserman, David L.
Weller, Alfred O.
Whatley, Patrick L.

The following candidates will be admitted as Fellows and Associates at the May, 1982 meeting as a result of their successful completion of the Society requirements in the November, 1981 examinations:

FELLOWS

Bellinghausen, Gary F.
Christie, James K.
Haner, Walter J.
Herman, Steven C.
Johnston, Thomas S.
Judd, Steven W.
Koch, Leon W.
Larsen, Michael R.
Lommele, Jan A.
Meyers, Glenn G.
Nickerson, Gary V.
Whatley, Patrick L.

ASSOCIATES

Addie, Barbara J.
Belden, Stephen A.
Bensimon, Abbe S.
Biscoglia, Terry J.
Boulanger, Francois
Bowen, David S.
Braithwaite, Paul
Cantin, Claudette
Carpenter, Thomas S.
Chou, Li-Chuan L.
Cimini, Edward D., Jr.
Coffin, John D.
Costner, James E.
Dembiec, Linda A.
Downer, Robert B.
Eagelfeld, Howard M.
Edmondson, Alice H.
Egnasko, Valere M.
Faltas, Bill
Gillam, William R.
Gilles, Joseph A.
Gillespie, Bryan C.
Goldberg, Terry L.
Hofmann, Richard A.
Hoppe, Kenneth J.
Kolk, Stephen L.
Kollmar, Richard
Koupf, Gary I.
Leung, Kung L.
Lonergan, Kevin F.
Lonergan, Thomas X.
Lyle, Aileen C.
Martin, Paul C.
McIntosh, Karol A.
Mittal, Madan L.
Moody, Andrew W.
Murphy, William F.
Pelletier, Charles A.
Potts, Cynthia M.
Rosenberg, Deborah M.
Ross, Lois A.
Rowland, Vincent T., Jr.
Schwartzman, Joy A.
Siewert, Jerome J.
Splitt, Daniel L.
Stadler-Hrbacek E.
Tucker, Warren B.
Vitale, Lawrence A.
Whiting, David R.
Withers, David A.
Yingling, Mark E.
Youngner, Ruth E.

The following is the list of successful candidates in examinations held in November, 1981:

Part 5

Allaben, Adrienne C.
Bakel, Leo R.
Belden, Scott C.
Belth, Ann I.
Berry, Janice L.
Biegaj, William P.
Bothwell, Peter T.
Boyd, Wallis A.
Brown, Brian Y.
Chansky, Joel S.
Chanzit, Lisa G.
Chiang, Jeanne D.
Closter, Donald L.
Cox, David B.
Crane, Veronica K.
Dashoff, Todd H.
Deede, Martin W.
Della Penna, Paul F.
Domanico, Elaine M.
Donnelly, Vincent T.
Dye, Myron L.
Eckley, Douglas A.
Elliott, Paula L.
Forde, Claudia S.
Gapp, Steven A.
Glotzer, Leonard R.
Green, Bruce H.
Guernsey, Anne L.
Guiahi, Farrokh
Gunn, Christy H.
Hall, Allen A.
Hanson, Jeffrey L.
Hapke, Alan J.
Hauboldt, Richard H.
Hay, Gordon K.
Hayward, Gregory L.
Hein, Timothy T.
Henry, Thomas A.
Hurley, Paul M.
Hutter, Heidi E.
Jarvis, June V.
Johnson, Richard W.
Kadison, Jeffrey P.
Kane, Adrienne B.
Killick, David H.

Part 5

Klinker, Frederick L.
Koch, Joyce A.
Kooken, Michael W.
Kulik, John M.
Kuo, Chung-Kuo
Lacek, Mary Lou
Landuyt, Judith A.
Lebrun, Richard
Leccese, Nicholas M., Jr.
Lewis, Stephen H.
Li, Walter S.
Licitra, Sam F.
Lo, Eddy L.
Lyons, Daniel K.
MacDonald, Andrew M.
Manning, Clark P.
Marks, Steven D.
Matthews, Robert W.
McClinton, Mary L.
McDonald, Gary P.
Merlino, Matthew P.
Meyer, Jeanne R.
Mitchell, William H.
Morrow, Jay B.
Mugavero, Ann C.
Neale, Catharine L.
Nester, Karen L.
Newell, Richard T.
Noback, Jodee B.
Normandin, Andre
Onufer, Layne B.
Paglieri, Wayne C.
Paquette, Sylvie L.
Peterson, Steven J.
Placek, Arthur C.
Pridgeon, Ronald D.
Radin, Katherine D.
Raman, Rajagopalan K.
Ramanujam, Srinivasa
Rathjen, Ralph L.
Robbin, Ira L.
Robinson, Richard D.
Roth, Randy J.
Roupas, Theodore G.
Ruegg, Mark A.
Schmidt, Neal J.
Schnapp, Frederic F.
Schwab, Debbie
Sealand, Pamela J.
Sirkin, Jeffrey S.
Smith, Judith P.
Spalla, Joanne S.
Steinen, Phillip A.
Steingiser, Russell
Sterk, Joyce M.
Symnoski, Diane M.
Theisen, Joseph P.
Trinh, Minh
Vaillancourt, Jean
Vaughan, Richard L.
Vogel, Charles D.
Watkin, Mark
Weimer, William F.
Weishaus, Abraham S.
White, David L.
Williams, Lawrence
Willsey, Robert L.

Part 7

Addie, Barbara J.
Arvanitis, Robert J.
Barclay, David L.
Belden, Stephen A.
Bellinghausen, Gary F
Bennett, Robert S.
Bensimon, Abbe S.
Biscoglia, Terry J.
Boccitto, Bonnie L.
Boulanger, Francois
Bouska, Amy S.
Bowen, David S.
Braithwaite, Paul
Briere, Robert S.
Canetta, John A.
Cantin, Claudette
Carpenter, Thomas S.
Chou, Li-Chuan L.
Cimini, Edward D., Jr
Coffin, John D.
Colgren, Karl D.
Costner, James E.
Dembiec, Linda A.
Deutsch, Robert V.
Diss, Gordon F.
Downer, Robert B.
Eagelfeld, Howard M.
Edmondson, Alice H.
Egnasko, Valere M.
Faltas, Bill
Forbus, Barbara L.
Fueston, Loyd L., Jr.
Gillam, William R.
Gilles, Joseph A.
Gillespie, Bryan C.
Goldberg, Terry L.

Part 7

Halpern, Nina S.
Hofmann, Richard A.
Hoppe, Kenneth J.
Kolk, Stephen L.
Kollmar, Richard
Koupf, Gary I.
Leung, Kung L.
Levine, George M.
Lommele, Jan A.
Lonergan, Kevin F.
Lonergan, Thomas X.
Loucks, William D., Jr.
Lyle, Aileen C.
Martin, Paul C.
McIntosh, Karol A.
Miner, Neil B.
Mittal, Madan L.
Moody, Andrew W.
Mozeika, John K.
Murdza, Peter J., Jr.
Murphy, William F.
Narvell, John C.
Pelletier, Charles A.
Pierson, Frank D.
Potts, Cynthia M.
Rosenberg, Deborah M.
Ross, Lois A.
Rowland, Vincent T., Jr.
Schwartzman, Joy A.
Sellitti, Marie
Siewert, Jerome J.
Splitt, Daniel L.
Stadler-Hrbacek, E.
Stanco, Edward J.
Taylor, Thomas F.
Tresco, Frank J.
Tucker, Warren B.
Valenti, Anthony T.
Vitale, Lawrence A.
Wacek, Michael G.
Whiting, David R.
Wickman, Alan E.
Withers, David A.
Yingling, Mark E.
Youngner, Ruth E.

Part 9

Alff, Gregory N.
Bashline, Donald T.
Baum, Edward J.
Berens, Regina M.
Blanchard, Ralph S., III
Burger, George
Camp, Jeanne H.
Carponter, John D.
Christie, James K.
Ciezadlo, Gregory J.
Clinton, R. Kevin
Currie, Ross A.
Foote, James M.
Friedberg, Bruce F.
Haner, Walter J.
Herman, Steven C.
Hine, Cecily A.
Holmberg, Randall D.
Horowitz, Bertram A.
Jaso, Robert J.
Johnston, Thomas S.
Jones, Bruce R.
Josephson, Gary R.
Judd, Steven W.
Keatts, Glenn H.
Koch, Leon W.
Kucera, Jeffrey L.
Lange, Dennis L.
Larsen, Michael R.
Lobosco, Virginia R.
McAllister, Kevin C.
Meyer, Robert E.
Meyers, Glenn G.
Montigney, Brian A.
Morgan, William S.
Muleski, Robert T.
Muza, James J.
Nickerson, Gary V.
Nikstad, James R.
Pachyn, Karen A.
Pastor, Gerald H.
Pruiksma, Glenn J.
Sherman, Ollie L., Jr.
Silverman, Mark J.
Soul, Harry W.
Sweeny, Andrea M.
Van Ark, William R.
Weiland, William T.
Whatley, Patrick L.
Wilkinson, Margaret E.
Yonkunas, John P.
Young, Bryan G.

NEW FELLOWS ADMITTED MAY, 1981: Ten of the eleven new Fellows admitted at Hot Springs are shown with President Scheibl.

NEW ASSOCIATES ADMITTED MAY, 1981: Thirty-four of the thirty-nine new Associates admitted at Hot Springs are shown with President Scheibl.

NEW FELLOWS ADMITTED NOVEMBER, 1981: Thirty-one of the thirty-two new Fellows admitted at New Orleans are shown with President Scheibl.

NEW ASSOCIATES ADMITTED NOVEMBER, 1981: Seventeen of the twenty new Associates admitted at New Orleans are shown with President Scheibl.

OBITUARIES

EDWARD L. BOMSE
JAMES M. BUGBEE
SU TU CHEN
CHARLES M. GRAHAM
ALFRED N. GUERTIN
MILTON G. McDONALD
NORRIS E. SHEPPARD
HERBERT P. STELLWAGEN
ALEX C. WELLMAN

EDWARD L. BOMSE
1909–1981

Edward L. Bomse, an Associate of the Casualty Actuarial Society since 1934, died this past year at the age of 72.

A native of New York, Mr. Bomse graduated from New York University in 1929.

He joined the Mutual Casualty Insurance Rating Bureau in 1930, where he remained until 1936. From 1936 to 1945, he worked for the National Bureau of Casualty and Surety Underwriters. In 1945, he joined the Royal-Globe Insurance Companies, where he remained until his retirement in 1973.

Mr. Bomse also served as an instructor in casualty insurance from 1948 to 1960 at the School of Insurance, predecessor to the College of Insurance. He had a special ability to teach his subjects extremely well, and added to the reputation and growth of the College.

After his retirement, Mr. Bomse was able to indulge in his principal hobby, music. As well as being proficient in the piano, he was a member of a church choir and was also a member of the Choral Arts Society, whose membership consists primarily of teachers and other musical professionals.

He is survived by his wife, Iris; a son; a daughter; and three grandchildren.

JAMES M. BUGBEE
–1981

James M. Bugbee, an Associate of the Casualty Actuarial Society since 1924, died on August 21, 1981.

Mr. Bugbee graduated from the Massachusetts Institute of Technology in 1918 where he received a degree in Mining Engineering. In addition, he graduated with special training in French, history and literature from the Université de Torlouse and received a degree in Metallurgy from Harvard University in 1924.

Mr. Bugbee served with distinctions in front line combat in France during World War I.

Mr. Bugbee was employed by the Maryland Casualty Company from 1928 until his retirement in 1964. He became Vice President of the Automobile, Compensation and Liability Underwriting Division in 1957, a position he held until his retirement.

He is survived by a son.

SU TU CHEN
1897–1981

Su Tu Chen, an Associate of the Casualty Actuarial Society since 1927, died February 25, 1981, at the age of 84.

Prior to his retirement, he served as a consulting actuary, and earlier as an employee of V. Wingon Life Assurance Company Ltd. in Hong Kong.

He is survived by his wife, Florence, who resides in Natick, Massachusetts.

CHARLES M. GRAHAM
1900–1981

Charles M. Graham, a Fellow of the Casualty Actuarial Society since 1926, died on September 19, 1981 at the age of 81.

A native of Newark, New Jersey, Mr. Graham's career started at the National Council on Compensation Insurance. He then was employed by the New York State Insurance Fund. In 1945, he became Chief Self-Insurance Examiner of the Workmen's Compensation Board of New York, a position he held until he retired in 1957. At that time, he moved to Florida and entered consulting work. In 1959, he was named Fire and Casualty Actuary of the Florida Insurance Department, and in 1965 he took the same job with the South Carolina Insurance Department, becoming Chief Actuary of the Department in 1970. He returned to consulting work in Florida in 1971.

ALFRED N. GUERTIN
1900–1981

Alfred N. Guertin, an Associate of the Casualty Actuarial Society since 1935, died on March 27, 1981 at age 81.

Mr. Guertin graduated from Trinity College and was the recipient of an honorary degree and an Alumni Medal of Excellence from Trinity.

He was a member of the actuarial department of the Connecticut Mutual Life Insurance Company from 1922 to 1929. He then joined the New Jersey Department of Banking and Insurance in Trenton serving as chief assistant actuary from 1929 to 1932 and actuary until 1945. For the next twenty years he worked for the American Life Convention, Chicago, as an actuary and an expert on insurance matters.

Mr. Guertin also was a committee chairman of the National Association of Insurance Commissioners and his recommendations resulted in the enactment of standard nonforfeiture and valuation regulations, commonly known as the Guertin Laws, for the insurance industry.

In addition, he was President of Scholarships for Illinois Residents, Inc. as well as honorary chairman. He was a trustee of Sigma Nu Inc. Educational Foundation. For a period of time he acted as an advisor for the U.S. Treasury Department.

Mr. Guertin was the recipient of the Elizur Wright Insurance Literary Award, 1945 and the NAII Insurance Publications Award, 1965. He was elected to the Insurance Hall of Fame, The Ohio State University, in 1967.

Mr. Guertin was a Fellow of the Society of Actuaries and an Associate of the Institute of Actuaries of England. He served on the Board of Governors of the American Institute of Actuaries. He was a member of the American Academy of Actuaries, American Risk and Insurance Association and Sigma Nu.

He is survived by two sons, A. Thomas and Robert P.; and two grandchildren.

MILTON G. McDONALD
1912–1981

Milton G. (Jerry) McDonald, an Associate of the Casualty Actuarial Society since 1955, died September 28, 1981, in Boston, Massachusetts at the age of 69.

Mr. McDonald's retirement in 1980 as Deputy Commissioner and Actuary of the Massachusetts Division of Insurance marked the close of a 40-year career in the public service. During those years he served under 10 Commissioners and became primarily responsible for the development of automobile, workers' compensation and other casualty insurance rates in Massachusetts. He also conducted the Division's liaison with the Legislature.

Widely known in both regulatory and industry ranks, Mr. McDonald was a familiar figure at meetings of the National Association of Insurance Commissioners. His professional expertise, particularly in the field of compulsory automobile insurance, periodically found him on government consulting assignments in other states as well as in such outposts as the Virgin Islands and Bogota, Colombia.

Mr. McDonald was born in Cambridge, Massachusetts, and was graduated from the Massachusetts Institute of Technology in 1934. He served four years with the Army Corps of Engineers during World War II and was discharged with the rank of first lieutenant following assignment in the Aleutian Islands.

He is survived by his wife, Mary; five sons; a daughter; and 10 grandchildren.

NORRIS E. SHEPPARD
1897–1980

Norris E. Sheppard, an Associate of the Casualty Actuarial Society since 1924, died in Toronto, Canada on November 1, 1980.

Born in Clappisons Corner, Ontario, Canada in 1897, Mr. Sheppard earned B.A. and M.A. degrees from Victoria College.

He was a professor of mathematics at the University of Toronto for 50 years before retiring in 1969. He then worked as a consultant until 1977.

In 1945, he was one of three actuaries picked to establish a pension fund for the staff of the newly formed United Nations.

He is survived by his wife, Ruth; two sons; and four daughters.

HERBERT P. STELLWAGEN
1897–1981

Herbert P. Stellwagen, an Associate of the Casualty Actuarial Society since 1924, died in Bryn Mawr, Pennsylvania on May 14, 1981 at the age of 83.

Born in Brooklyn, New York on September 1, 1897, Mr. Stellwagen attended New York University. He graduated in 1918 as a member of the Phi Beta Kappa society.

Mr. Stellwagen joined the Indemnity Insurance Company of North America [an affiliate of the Insurance Company of North America (INA)] in 1929 following nine years with the National Bureau of Casualty and Surety Underwriters. In 1930, he was elected Vice President, and in 1941 he was elected Executive Vice President. In 1948, he was elected a director of the company. In 1956, he was elected Vice President and director of Life Insurance Company of America, a newly-formed, wholly-owned subsidiary of INA.

Mr. Stellwagen retired as Executive Vice President in 1963, but continued as a director until 1969.

In recognition of his work in the insurance field, he was made a life trustee of the American Insitute of Property and Liability Underwriters when the organization was founded in 1942. In addition, he was a trustee of the Williamson

Free School of Mechanical Trades, director of Provident Tradesman's Bank and Trust Company, Vice President and director of the Bryn Mawr Hospital, and a director of the Insurance Federation of Pennsylvania.

He is survived by his wife, Esther; two daughters, Anne S. Connor and Jane S. Polk; and three grandchildren.

ALEX C. WELLMAN
1903–1981

Alex C. Wellman, retired Vice Chairman of the Board of Protective Life Insurance Company, passed away in Birmingham, Alabama, on August 23, 1981 at the age of 78.

Mr. Wellman became an Associate of the Casualty Actuarial Society in 1925 and of the American Institute of Actuaries in 1926. He was also a member of the American Academy of Actuaries.

Mr. Wellman graduated from the University of Michigan in 1925, having done his undergraduate work at both Wayne University and the University of Michigan. After spending a year in the Actuarial Department of the Royal Union Life Insurance Company of Des Moines, Iowa, Mr. Wellman joined the Alabama National Life Insurance Company in Birmingham, Alabama in 1926, as Actuary. Following the merger of that company with Protective Life in 1927, he became Actuary for the consolidated company. He became a Vice President of Protective in 1930 and was elected to its Board of Directors in 1937. He became a Senior Vice President in 1955 and in 1967 was elected Vice Chairman of the Board. He served on the Board until his retirement in 1970.

Mr. Wellman was a major contributor to his company's successful emergence from the depression and to its development as a significant group insurer in the Southeast following the Second World War.

Alex Wellman was a member of the Advisory Board of the Birmingham Salvation Army and served as Chairman of its Home and Hospital Board. He also served as an Elder of the Sixth Avenue Presbyterian Church and was quite active in the Birmingham Kiwanis Club.

He is survived by his wife, a brother and a sister, three children and eight grandchildren.

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