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LOSS RESERVES: PERFORMANCE STANDARDS

C. K. KHURY

INTRODUCTION

Loss reserves have a significant impact on the reported operating results as well as on the financial condition of an insurer. Actuarial literature to date has focused on developing loss reserving methods [1]. The matter of assessing the condition [2] of loss reserves, on the other hand, has received relatively little attention.

The scarcity of material in this area seems to have given observers of our industry some sort of license to make periodic pronouncements [3] about the adequacy of loss reserves. Security analysts, for example, have made such statements and distributed them throughout the investment community and the insurance industry. The strength of these statements seems to derive mainly from the ability to give them a wide distribution; a subsequent section of this paper suggests that such statements can be speculative and highly misleading. Concurrently and separately, there appears to be within regulatory circles, particularly the National Association of Insurance Commissioners, a movement towards requiring an actuarial opinion on the condition of annual statement loss reserves.

Both conditions noted here suggest that published loss reserves are not viewed with a great deal of confidence, either by investors or by regulators. The problems caused by this lack of confidence are numerous. Two prominent classes of problems deserve mention: the many applications for rate increases which have been denied, or cut back, because of disagreements about the condition of loss reserves; and, the capacity which has been lost to the insurance industry because of failure to attract and retain investor capital due to investor anxiety about the condition of loss reserves.

What is the root cause of this lack of acceptance of the published loss reserve estimates? The major premise of this paper is that the rationale underlying the reserving process is not well understood generally by the ultimate users of the reserve estimate.

There appears to be a need to crystallize the rationale underlying the loss reserving process. This paper is aimed in that direction. Specifically, three concepts will be developed:

- The concept of the loss reserve as a combination of a point estimate *and* a confidence interval.
- The concept of developing *actuarial assumptions* as a necessary step in the loss reserving process.
- The concept of *actuarial gain/loss* in reporting financial results.

Methodologies illustrating these concepts will be introduced along with a description of opportunities for further research.

THE LOSS RESERVE AS A POINT ESTIMATE AND A CONFIDENCE INTERVAL

In the ordinary course of events, an insurer's estimate of its unpaid claim liability (case reserves plus supplemental [4] reserves) is reported in the Annual Statement as a single numerical value. Schedules O and P provide a means for reporting updates of this estimate. In this way the Annual Statement provides for a *retrospective test* of the accuracy of previously published loss reserve estimates. The Annual Statement, however, does not disclose the *quality* of the original estimate in relation to subsequent updates. In other words, if the original estimate ultimately turns out to have missed the mark by some amount, positive or negative, there is nothing in the Annual Statement to tell us whether the variation is within "acceptable" bounds. In the absence of some standard(s) of expected variation, of course, the issue is largely academic. In fact, if one is interested in evaluating the loss reserving performance, then defining a band of expected variation becomes a necessary adjunct to the statement of a point estimate of the unpaid claim liability.

In order to develop the loss reserve confidence interval in the context of this paper, it will be necessary to digress and discuss the pure premium confidence interval.

Expense loadings aside, the ratemaking process is concerned with estimating an ultimate pure premium per unit of exposure during a prospective experience period. In other words, the pure premium is the present value of *expected* claim payments. The pure premium estimate is based mainly on prior claim experience which has been projected forward to the applicable policy period. This pure premium is normally stated as a point estimate, for practical reasons that are obvious. None the less, since this point estimate is just that, *an estimate,* it has associated with it a process variance [5] which is a function of the underlying frequency and severity distributions. This process variance exists whether explicitly stated or not and it is generally equal to the compound variance of the underlying frequency and severity of loss.

First, let us consider the frequency element of the pure premium. Suppose the occurrence of claims is distributed according to a known risk process, F, and suppose we have a body of recent experience which has produced an observed frequency $\hat{f}$. For a given probability p, one can generate a confidence interval around $\hat{f}$, of radius $R(F, p)$. In other words, the true ultimate frequency, f, will lie somewhere within the interval $[\hat{f} \pm R(F, p)]$ with probability p.

The severity element of the pure premium is amenable to the same treatment with:

Severity distribution .... S
Observed severity .... $\hat{s}$
Selected probability .... p
Radius of the severity confidence interval
associated with S and p .... $R(S, p)$

Then for the given probability, the true ultimate severity, s, will be within the interval $[\hat{s} \pm R(S, p)]$ with probability p.

The pure premium confidence interval can now be constructed several different ways depending on the desired degree of precision. The endpoints of the simplest and most "liberal" interval are the products of the endpoints of the frequency and severity confidence intervals. On the other hand, the smallest and most "economical" confidence interval is based on the compound distribution of F and S. Often an explicit form for this distribution is not available. Mayerson, Jones, and Bowers [6], and Hewitt [7] have described procedures for determining the expected process variance under certain conditions, which can be extended to derive a confidence interval about the pure premium given a probability. In any event, between the two extremes lie many choices, with attendant varying degrees of economy, of confidence intervals which can be associated with the point estimate of the pure premium. The selected pure premium confidence interval will be a key input in the development of a confidence interval for the loss reserve point estimate.

Viewing the reserve situation from the threshold (January 1) of a given accident year, say 1978, suppose a pure premium, P, and a confidence interval of radius $R(P, p)$, corresponding to a selected probability p, have been developed. Also let the number of exposure units which are expected to be earned during 1978 be given by N. On January 1, 1978, the ultimate total incurred loss cost for accident year 1978 is expected to be in the interval $[NP \pm N^{1/2} \cdot R(P, p)]$ with probability p. The radius $[N^{1/2} \cdot R(P, p)]$ is associated with process variance. That is, if the *a priori* pure premium were known, the final result might still differ from the *a priori* level by as much as $[N^{1/2} \cdot R(P, p)]$ with probability p. In reality, however, there is still the uncertainty associated with parameter selection in the course of constructing P. In other words, the *a priori* frequency and severity do not exist, but have to be estimated. Thus, the aggregate expected variation of $(N \cdot P)$ is $[N^{1/2} \cdot R(P, p)]$ plus something to recognize *parameter* variance. The author has arbitrarily chosen $N^{1/2}$ as the factor by which the *a priori* radius has to be expanded. In other words, the ultimate total incurred loss will be in the interval $[NP \pm N \cdot R\ (P, p)]$ with probability p. The radius of this interval is $N^{1/2}[N^{1/2} \cdot R(P, p)] = [N \cdot R(P, p)]$.

Moving to January 1, 1979, the question of the rate (with its underlying pure premium) is now a matter of history. In other words, all policies written to become effective in 1978 at the pure premium P have been written, and all resultant earned exposures have been determined. Recalling that P is the sum of all present values of claims arising out of the N exposure units earned in 1978, the estimated value of P may be stated as follows:

$$P(1978,\ 1978) = \sum_{i=0}^{n} Pd(1978,\ 1978 + i,\ 1978), \text{ where:}$$

$P(x, y)$ = The pure premium for accident year x as calculated (estimated) on January 1, y. Thus, $P(1978, 1978)$ is the 1978 accident year pure premium as estimated on January 1, 1978. $P(1978, 1980)$ is the 1978 accident year pure premium estimated (recalculated) on January 1, 1980.

$Pd(x, y, z)$ = The present value (on January 1, x) of all claim payments made on behalf of accident year x, during year y as estimated on January 1, z. Thus, Pd(1978, 1980, 1978) is the present value (on January 1, 1978) of all claim payments to be paid on behalf of accident year 1978 during 1980 as estimated on January 1, 1978. Pd(1978, 1981, 1979) is the present value of all claim payments to be paid on behalf of accident year 1978 during 1981 as estimated on January 1, 1979.

n = The number of years needed to close out an accident year x counting from December 31, x.

Thus, on January 1, 1979, one is in fact able to compare the estimate Pd(1978, 1978, 1978) with actual experience, that is, with Pd(1978, 1978, 1979). One can construct Table 1 (letting $n = 3$ for this example).

TABLE 1

Projected	Actual
Pd(1978, 1978, 1978)	Pd(1978, 1978, 1979)
Pd(1978, 1979, 1978)	
Pd(1978, 1980, 1978)	
Pd(1978, 1981, 1978)	

On January 1, 1978, with little information about 1978, the range of the ultimate incurred loss was estimated to fall in the range $[N(P \pm R(P, p))]$ with probability p. On January 1, 1979, actual information about accident year 1978 becomes available; most of the claims have been reported and a portion of the severity has been incurred (the degree of f and s realized depends on the nature of the subject line of business). Recall that the issue at hand is "what kind of a confidence interval can be attached to the loss reserve estimate as of January 1, 1979?" [8]

The loss reserve for accident year 1978, valued as of January 1, 1979, can be viewed as the *newly* estimated

$$\sum_{i=1}^{3} Pd(1978, 1978 + i, 1979)$$

In other words, the reserving process on January 1, 1979 is equivalent to computing P(1978, 1979) based on all information available on January 1, 1978, *plus* all the new information acquired during 1978. It should be quite safe to assume that the quality of $P(1978, 1979)$ is no worse (and is probably better) than $P(1978, 1978)$. In other words, $P(1978, 1979)$ is closer than $P(1978, 1978)$ to the mark:

$$|P(1978, 1979) - P(1978, 1982)| \leq |P(1978, 1978) - P(1978, 1982)|$$

Now the perhaps obvious transition can be made from pricing to its sister process, reserving. The process of estimating $P(1978, 1979)$ is reduced to estimating $\Sigma_{i=1}^{3} Pd(1978, 1978 + i, 1979)$, since $Pd(1978, 1978, 1979)$ is already a known quantity. Thus, the comparison table (Table 1), shown earlier, can be extended into Table 2.

TABLE 2

Increments as of:

January 1, 1978	January 1, 1979
$Pd(1978, 1978, 1978)$	$Pd(1978, 1978, 1979)$ = History
$Pd(1978, 1979, 1978)$	$Pd(1978, 1979, 1979)$ = New Estimate
$Pd(1978, 1980, 1978)$	$Pd(1978, 1980, 1979)$ = New Estimate
$Pd(1978, 1981, 1978)$	$Pd(1978, 1981, 1979)$ = New Estimate
$P(1978, 1978)$	$P(1978, 1979)$ = New Estimate

The radius of the confidence interval associated with $P(1978, 1978)$ was given by $[N \cdot R(P, p)]$. The radius of the confidence interval associated with $P(1978, 1979)$ must be no greater than $[N \cdot R(P, p)]$. This is true because *more* information is available on January 1, 1979 for computing $P(1978, 1979)$ than was available on January 1, 1978 for computing $P(1978, 1978)$; both values represent attempts at hitting the same unknown, but fixed, bull's eye: $P(1978, 1982)$.

TABLE 3

Increments as of January 1,

1978	1979	1980	1981	1982
$Pd(1978, 1978, 1978)$	$Pd(1978, 1978, 1979)$	$Pd(1978, 1978, 1980)$	$Pd(1978, 1978, 1981)$	$Pd(1978, 1978, 1982)$
$Pd(1978, 1979, 1978)$	$Pd(1978, 1979, 1979)$	$Pd(1978, 1979, 1980)$	$Pd(1978, 1979, 1981)$	$Pd(1978, 1979, 1982)$
$Pd(1978, 1980, 1978)$	$Pd(1978, 1980, 1979)$	$Pd(1978, 1980, 1980)$	$Pd(1978, 1980, 1981)$	$Pd(1978, 1980, 1982)$
$Pd(1978, 1981, 1978)$	$Pd(1978, 1981, 1979)$	$Pd(1978, 1981, 1980)$	$Pd(1978, 1981, 1981)$	$Pd(1978, 1981, 1982)$
$P(1978, 1978)$	$P(1978, 1979)$	$P(1978, 1980)$	$P(1978, 1981)$	$P(1978, 1982)$

Extending Table 2 to an ultimate basis produces Table 3. The boxed amounts to the right of the dotted line are accident year 1978's loss reserves as they enter financial statements at successive year-ends. The values to the left of the dotted line are boxed for emphasis only, as they are values *implied* by the rates in use by the insurer and, as such, do not appear in any financial statements. Indexing $[N \cdot R(P, p)]$ in the same manner as P produces the following associations:

Valuation	Radius of Confidence Interval
$P(1978, 1978)$	$N \cdot R(P(1978, 1978), p) = N \cdot R(P, p)$
$P(1978, 1979)$	$N \cdot R(P(1978, 1979), p)$
$P(1978, 1980)$	$N \cdot R(P(1978, 1980), p)$
$P(1978, 1981)$	$N \cdot R(P(1978, 1981), p)$
$P(1978, 1982)$	$N \cdot R(P(1978, 1982), p) = 0$

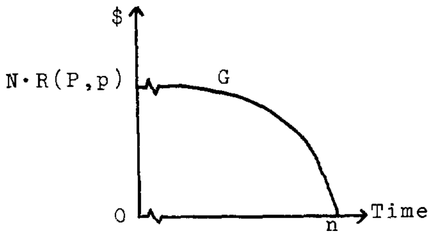
This illustrates the conclusion that the confidence interval associated with $P(1978, 1978 + i)$ must have a radius between 0 and $[N \cdot R(P, p)]$. The appropriate radius value (between 0 and $[N \cdot R(P, p)]$) can be determined by a function, $G(i)$, which satisfies the following conditions:

$G(i)$ exists on the interval $[0, n]$
$G(0) = [N \cdot R(P, p)]$
$G(n + 1) = 0$
$G(i) \geq G(j)$ whenever $i \leq j$

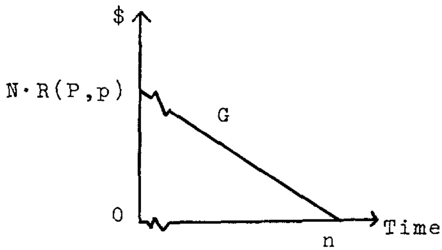
The choice of G should reflect the degree of conservatism the practitioner may wish to introduce into the reserving process in recognition of the volatility [9] of the subject line of business.

Figure 1 displays several possible forms of the function G. The form shown on Graph A in Figure 1 may be suitable for medical malpractice; Graph B may be suitable for workers' compensation; Graph C may be suitable for automobile property damage liability.

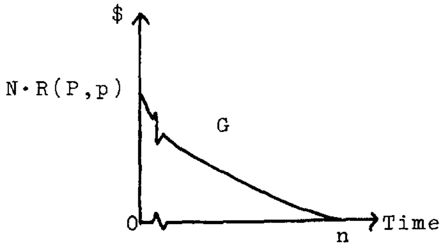
FIGURE 1



(A)



(B)



(C)

A numerical application of this process is provided below for illustrative purposes:

$$\begin{aligned}
\text{Accident Year} &= 1978 \\
N &= 10{,}000 \\
n &= 4 \text{ (accident year will be closed out on 12/31/82)} \\
P(1978, 1978) &= \$80 \\
p &= .85 \\
R(P, p) &= R(80, .85) = \$9 \\
N \cdot R(P, p) &= \$90{,}000 \\
Pd(1978, 1978, 1978) &= \$400{,}000 \\
Pd(1978, 1979, 1978) &= \$200{,}000 \\
Pd(1978, 1980, 1978) &= \$100{,}000 \\
Pd(1978, 1981, 1978) &= \$60{,}000 \\
Pd(1978, 1982, 1978) &= \$40{,}000 \\
G(i) &= [90{,}000/(i + 1)^2] - 500i \\
i &= 0, 1, 2, 3, 4, 5 = n + 1
\end{aligned}$$

Thus, the reserve amounts (projected as of January 1, 1978) and the radii of their projected confidence intervals are given by:

(1) Projected Valuation To Be Made As Of December 31	(2) i	(3) Expected Estimated Reserve	(4) Projected Confidence Interval $G(i)$	(5) % Swing (4) ÷ (3)
1978	1	$400,000	$22,000	5.5%
1979	2	200,000	9,000	4.5
1980	3	100,000	4,125	4.1
1981	4	40,000	1,600	4.0
1982	5	0	0	N/A

ACTUARIAL ASSUMPTIONS

At this point a brief digression from the property and casualty lines is in order. Consider the life insurance company statement. Policyholder reserves are by far the largest single liability. The derivation of these reserves is a mechanical process based on actuarial assumptions and well defined formulae. For each kind of policy that is in force, for example, actuarial assumptions are made (mortality, morbidity, interest, etc.), and the reserve is produced mechanically via various actuarial formulae.

The same is true in the valuation of pensions. To arrive at funding estimates, actuarial assumptions are made (mortality, morbidity, interest, employee turnover, etc.), and the funding estimate is produced mechanically. When one examines the process of estimating funding requirements, it quickly becomes apparent that this process is indeed very similar to loss reserving! In pension funding, both "frequency" (number of retired lives) and "severity" (the duration of an average retirement) are subject to frequent shifts. The pension actuary attempts to recognize these movements by reviewing and updating the plan's actuarial assumptions annually and making adjustments to the funding requirements based on those changes.

In many property and casualty lines, there normally exists a good deal of historical experience that can lend itself to a similar approach to the loss reserve estimation process. Consider, for example, two components of the pure premium, frequency and severity.

Frequency

Historical development (incidence) patterns of frequency over time can be arrayed so as to develop model frequency assumptions. For example, consider a given line of business, with a history of five completed accident years. Also, assume that all claims are reported within three years of occurrence.

Incidence of Frequency

Development Year	Accident Year				
	1	2	3	4	5
1	$f(1, 1)$	$f(2, 1)$	$f(3, 1)$	$f(4, 1)$	$f(5, 1)$
2	$f(1, 2)$	$f(2, 2)$	$f(3, 2)$	$f(4, 2)$	$f(5, 2)$
3	$f(1, 3)$	$f(2, 3)$	$f(3, 3)$	$f(4, 3)$	$f(5, 3)$

Several frequency models can be extracted from this history depending on environmental factors [10] as well as on the actuary's points of emphasis. One approach is to develop a frequency time index by dividing $f(k, i)$ by $f(k, 3)$ for every k and i:

Frequency Time Indices

Development Year	Accident Year 1	. . .	5	Model Frequency Time Index
1	$f(1, 1)/f(1, 3)$	. . .	$f(5, 1)/f(5, 3)$	$f(., 1)$
2	$f(1, 2)/f(1, 3)$	. . .	$f(5, 2)/f(5, 3)$	$f(., 2)$
3	$f(1, 3)/f(1, 3)$	. . .	$f(5, 3)/f(5, 3)$	$f(., 3)$

At this point, one should note that the extrapolation of model frequency time indices $f(., 1)$, $f(., 2)$, and $f(., 3)$ can be accomplished by taking the arithmetic mean for each of the development years; or by weighting the indices by an arithmetic series, by a geometric series, or by exposure units; or by using other approaches. The result, in any case, is the same: a frequency model has, in fact, been produced. Denote the general model by:

$$[F(L, m, T, n): f(., 1), f(., 2), \ldots, f(., n)]$$

This model is for line of business L, based on m years of experience ending with year T, and requires n years to develop f to a fully reported basis. For example, for line of business L, one might have:

$$[F(L, 5, 1974, 4): .80, .88, .95, 1.00]$$

Severity

The same construction applies to the severity element, producing the following general model:

$$[S(L, m', T', n'): s(., 1), s(., 2), \ldots, s(., n')]$$

Two other prominent factors need to be fixed as actuarial assumptions: interest and inflation. Also, in utilizing these assumptions, the need to develop a claim payout (cash flow) model will have to be met.

Interest

Since reserves represent funds held by the insurer, they will earn interest, regardless of to whom these funds (and, therefore, the interest) belong. An interest assumption, therefore, is needed to recognize future interest income on these loss reserves. There are those who believe that reserves should not be discounted for interest; for them an interest assumption of zero is suitable. An assumption must be made nevertheless. In this paper, interest will be treated as an assumption, i, which may take on any non-negative value. Because interest can vary from year to year, the assumption may be varied. Accordingly, denote the interest rate expected to prevail during year t by $i(t)$.

Inflation

It has been suggested often that loss reserves need not be discounted for interest because inflation acts as negative interest. This view may have validity as long as inflation and interest rates are identical. In all other cases, both factors need to be recognized separately. Denote the inflation assumption expected to prevail during year t by $j(t)$.

Payout Model

P can be constructed [11] in much the same way as S was, producing the following model increments:

$[A(L, m', T', n'): a(., 1), a(., 2), \ldots, a(., n')]$

where $a(., 2)$ represents the portion of an individual accident year that will be paid during the second year of development. Note the similarity of index construction to that underlying S.

Given that inflation acts on loss reserves in the same manner as "negative interest," the combined interest/inflation assumption may be constructed for year t, as $[i(t) - j(t)]$ and be denoted by $Z(t)$.

Given these assumptions: frequency, severity, payout model, interest, and inflation; the loss reserve estimate can be derived mechanically. For example, consider accident year 1978 as of December 31, 1978:

- Exposure units earned . . . . . . N
- Observed frequency . . . . $f(78, 1)$
- Observed severity . . . . $s(78, 1)$
- Observed payments . . . . $a(78, 1)$,

given the model assumptions developed earlier:

$F: f(., 1), f(., 2), \ldots, f(., n)$
$S: s(., 1), s(., 2), \ldots, s(., n')$
$A: a(., 1), a(., 2), \ldots, a(., n')$
$Z: z(79), z(80), \ldots, z(77 + n')$

Now the reserve can be generated directly as follows:

1. Determine the ultimate reserve assuming $z(t) = 0$ for all t:

   $$B = (N)[f(78, 1)f(., n)/f(., 1)][s(78, 1)s(., n')/s(., 1)] - a(78, 1)$$

2. Subdivide B into its payment increments using the payout model A:

   $$B_1 = (B)[a(., 2) - a(., 1)]/[a(., n') - a(., 1)]$$
   $$B_2 = (B)[a(., 3) - a(., 2)]/[a(., n') - a(., 1)]$$
   $$B_3 = (B)[a(., 4) - a(., 3)]/[a(., n') - a(., 1)]$$
   $$\vdots$$
   $$B_{n'-1} = (B)[a(., n') - a(., n' - 1)]/[a(., n') - a(., 1)]$$

3. Adjust the reserve for Z (assuming all the increments of B are paid on December 31 of the subject year [12]) and generate the present value of the final reserve for accident year 1978 as of December 31, 1978:

   Final Discounted Reserve =

   $$[1/(1 + z(79))]B_1 +$$
   $$[1/(1 + z(79))(1 + z(80))]B_2 +$$
   $$[1/(1 + z(79))(1 + z(80))(1 + z(81))]B_3 + \ldots =$$
   $$\sum_{q=1}^{n'-1} [1/(1 + z(79))(1 + z(80)) \ldots (1 + z(78 + q))]B_q$$

Under the arrangement described above, the pressure points underlying the reserving process are completely exposed; the focus is on the *assumptions* underlying the computations. Perhaps it is now clear why a security analyst should not assess the state of loss reserves based solely on the published reserve: he does not have access to a key part of the *prospective* reserve computation, namely, the actuarial assumptions. He is normally working with *retrospective* returns, which assess the adequacy of *past* reserves.

Knowledge of the adequacy level of past reserves, by itself, provides no information about the adequacy of current reserves. Knowledge of the assumptions underlying current reserves is needed before valid conclusions can be drawn about their condition. Viewed in this light, pronouncements about the adequacy of reserves by anyone not having access to the underlying assumptions are essentially numerology and have no foundation in fact. In this sense, statements by security analysts about the condition of loss reserves may generally be described as speculative and uninformed.

One last point: if the reserving process for the property and casualty lines becomes fully predicated on actuarial assumptions, it will have pulled alongside the life insurance reserving process. The significance of this observation lies in the fact that security analysts do not usually publish statements evaluating the adequacy of life insurance company reserves. They know neither the assumptions nor the formulae.

ACTUARIAL GAIN/LOSS

This section presumes that the estimate of the ultimate unpaid claim liability has been set as of December 31, t. During calendar year $(t + 1)$, the actual experience corresponding to this estimate can generate two effects on the financial results of an insurer:

- The effect of the difference between expected and actual claim payments. That is $|\hat{a} - a|$, actual development.
- The effect of any restatement of the remaining unpaid claim liability arising from changes in the underlying assumptions. That is, change in expected development.

The financial results for calendar year $(t + 1)$ are composed of the results for the most recent calendar/accident year, $(t + 1)$, and of the results generated by the two factors noted above in connection with the development of prior years' loss reserves. Because of this composition, the interpretation of current financial results is generally not favored with a great deal of clarity [13]. There appears to be a need to spell out [14] the composition of current financial results, distinguishing between those generated by current operations and those generated by loss development. In response to this need, this section contains one way in which this split can be effected and displayed in the annual statement.

Consider Exhibit I. While the construction is largely self-explanatory, the following comments may be helpful:

Line 1. $(t + 1)$ is the only year generating premium income during the subject year (hence the zero under "all other").

Line 2. From the moment a premium dollar is received, it generates investment income until it is fully earned. The total investment income generated by the premiums earned during $(t + 1)$ represents another source of premium-related income. As in the case of line 1, only calendar/accident year $(t + 1)$ generates this category of income.

Line 3. As the premium dollar is earned, the pure premium gradually becomes an incurred loss—partly paid, partly in case reserves, and partly in supplemental reserves. Until the pure premium is fully paid, it generates investment income. The investment income generated by the unpaid pure premium during the year $(t + 1)$ represents a source of income for both categories of experience periods: calendar/accident year $(t + 1)$ and all other accident years.

Line 10. The arithmetic is clear. The amount under the "all other" category represents the impact on current operations of loss reserve development, and it is proposed as the actuarial gain/loss realized during $(t + 1)$ as a result of loss reserve development. As mentioned earlier, this amount is composed of two segments due to:

$|\hat{a} - a|$, and
Changes in the December 31, t, reserve assumptions.

The exhibit might be even more striking if the actuarial gain/loss were split into its two components [15] and displayed in a footnote. In this way the impact of changes in assumptions would be plainly in view.

Although Exhibit I shows only one accident year split, there is no reason why it could not be extended to make use of several splits; the concepts are the same, and the actuarial gain/loss would be more precisely charged back to the appropriate accident period.

DISCUSSION, PROBLEMS, AND OPPORTUNITIES

Given the three concepts advanced here, the loss reserving process tends to take on a slightly different look. Exhibit II describes the input/output flowchart of the process. Of all the process steps, perhaps the fifth is the one requiring comment.

The chief executive might, with one stroke of the pen, unilaterally change the reserve estimate. While the right to do so is not at issue here, two consequences of such action should be spelled out:

- All rates which utilize the revised loss reserve estimate will be inadequate or redundant depending on which way the judgment is made.
- The accountability for the loss reserving performance will have shifted upward to the chief executive.

The first consequence has the greatest potential for immediate damage. Whether the rates are either inadequate or excessive, the "system" is out of synchronization. The ratemaking and reserving processes are joined together by many of the attributes joining the proverbial "chicken and egg" cycle. Because of this relationship, any change in the loss reserve estimate produced by the actuary should be made with the utmost care and with full awareness of its impact on the ratemaking operation.

The second consequence would emerge most prominently if and when the Annual Statement had to be certified. Can the actuary certify the judgment of the chief executive? There is a suggestion here that, if the Annual Statement has to be certified by an actuary, then the fifth step should be omitted from the reserving process. If the ratemaking consequence is not sufficient to remove this step, perhaps a certification requirement would be.

If the loss reserving process is fully predicated on actuarial assumptions as described here, then monitoring the performance of those making the assumption selections becomes a rather simple task. This can best be illustrated by the run-off chart illustrated in Exhibit III. The track record is plainly spelled out in terms of how the original assumptions fared. As a collorary to this application, one is able to test the ratemaking performance as well by inserting an additional column (in box) headed January 1, *t*. The assumptions in this column would be those underlying the original rate. In this manner the full interdependence of the ratemaking and reserving processes is further magnified.

Although the proposals advanced here stand alone, there still remain numerous opportunities for further research that would enhance the proposed procedures:

- The derivation of confidence intervals for the pure premium for different *classes* of business.
- The composition of confidence intervals for the loss reserve of *several* lines/classes of business.
- The development of *continuous* cash flow models for different lines of business.
- The manner of reporting loss reserve confidence intervals along with the attendant probabilities.
- Extension of the proposed concepts to lines of business insuring rare events—low frequency/high severity combinations.

These are but a few of the research possibilities connected with the concepts introduced in this paper.

SUMMARY

In this paper the loss reserving process is directly identified as a twin of the ratemaking process. Just as actuarial assumptions underlie the ratemaking process, it is suggested that actuarial assumptions underlie the loss reserving process. Just as the pure premium represents an estimate surrounded by a confidence interval, it is proposed that the loss reserve be defined as an estimate with its own confidence interval. Just as the actuary is normally accountable for the ratemaking performance, it is proposed that he also be held accountable for the loss reserving performance, along with full disclosure of how prior loss reserve estimates affected current financial results. For each of these concepts, an illustrative methodology is introduced.

It is this writer's belief that employing these ideas can enhance the clarity and prominence of the loss reserving process. Also, if and when a certification requirement should be introduced, these concepts should help in delineating the specific areas with which the actuary should deal. Finally, viewing the loss reserving process in the framework introduced here may sharpen the practitioner's awareness of the value of loss reserving performance standards, and in the process help motivate an even better work product.

NOTES AND REFERENCES

[1] D. Skurnick, "A Survey of Loss Reserving Methods," *PCAS* LX (1973).

[2] R. Salzmann, "How Adequate Are Loss and Loss Expense Liabilities?," *PCAS* LIX (1972).

[3] Lewinson, Brian, Kreisel, Nadler, and Balcarek, "How Do Investment Analysts View Company Loss Reserves?" Casualty Actuarial Society Loss Reserve Symposium, 1976.

[4] Supplemental reserves, as used here, denote the sum of the reserve for incurred but not reported claims and the reserve for future development on previously reported cases which are still open.

[5] C. Hewitt, "Credibility For Severity," *PCAS* LVII (1970).

[6] Mayerson, Jones and Bowers, "On the Credibility of the Pure Premium," *PCAS* LV (1968).

[7] C. Hewitt, "Loss Ratio Distributions: A Model," *PCAS* LIV (1967).

[8] January 1, 1979 is used instead of December 31, 1978 only as a matter of convenience.

[9] Volatility in reserve development may be deduced from the observed historical variance in age-to-age loss development factors.

[10] W. Fisher and E. Lester, "Loss Reserve Testing in a Changing Environment," *PCAS* LXII (1975).

[11] In order to simplify the process of developing present value reserves, it will be helpful to develop P assuming zero interest. This construction necessitates the discounting of all historical payout data to a present value basis.

[12] Using a continuous payment mode is not only possible, but preferable. The discrete case is used here solely to simplify the presentation.

[13] R. Balcarek, "Effect of Loss Reserve Margins in Calendar Year Results," *PCAS* LIII (1966).

[14] Above and beyond the reassembly of various Schedule O and Schedule P pieces.

[15] It is also possible to distribute each year's actuarial gain/loss among each of the five assumptions.

EXHIBIT I

SPLIT OF FINANCIAL RESULTS DURING CALENDAR YEAR $t + 1$
BETWEEN CURRENT OPERATIONS AND LOSS DEVELOPMENT

	Calendar/Accident Period		
	$(t + 1)$	All Other	All Years
1. Earned premiums	$10,000	$ 0	$10,000
2. Investment income on unearned premiums*	500	0	500
3. Investment income on loss reserves	300	1,500	1,800
4. Total income attributable to insurance operations	$10,800	$ 1,500	$12,300
5. Claim payments	$ 2,000	$ 5,000	$ 7,000
6. Loss reserves as of December 31, t	0	25,000	25,000
7. Loss reserves as of December 31, $(t + 1)$	4,000	22,000	26,000
8. Incurred losses [(5) + (7) − (6)]	$ 6,000	$ 2,000	$ 8,000
9. Incurred expenses**	$ 3,500	$ 600	$ 4,100
10. Net income due to insurance operations [(4) − (8) − (9)]	$ 1,300	$(1,100)	$ 200

* Only with respect to line 1. See narrative.

** Includes all loss adjustment expenses.

EXHIBIT II

Composition of a Typical Loss Reserving Cycle

Input	Processed by	Output
1. Day-to-day transactions of an insurance business.	Operating departments	Raw data
2. Environmental factors and nature of raw data.	Actuary	Assumptions
3. Raw data, assumptions, and method.	Actuary	Reserve point estimate
4. Loss ratio distributions and raw data.	Actuary	Confidence intervals
5. Reserve point estimate and confidence interval and ?	President	Final reserve estimate
6. Final reserve estimate.	Actuary	Annual Statement allocations and pricing inputs

EXHIBIT III

TESTING OF THE ACTUARIAL ASSUMPTIONS UNDERLYING THE RATES AND RESERVES OF ACCIDENT YEAR t

Assumptions		Valuation Date				
Category	Basis	$1.1.t$	$12.31.t$	$12.31.t+1$	$12.31.t+2$ . . .	$12.31.t+n-1$
1. Frequency	Ultimate	f	f_1	f_2	f_3	$\hat{f}$
2. Severity	Ultimate	s	s_1	s_2	s_3	$\hat{s}$
3. Interest	t	$i(t)$	$\hat{i}(t)$	$\hat{i}(t)$	$\hat{i}(t)$	$\hat{i}(t)$
	$t+1$	$i(t+1)$	$i_1(t+1)$	$\hat{i}(t+1)$	$\hat{i}(t+1)$	$\hat{i}(t+1)$
	$t+2$	$i(t+2)$	$i_1(t+2)$	$i_2(t+2)$	$\hat{i}(t+2)$	$\hat{i}(t+2)$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$t+n-1$	$i(t+n-1)$	$i_1(t+n-1)$	$i_2(t+n-1)$	$i_3(t+n-1)$	$\hat{i}(t+n-1)$
4. Inflation	t	$j(t)$	$\hat{j}(t)$	$\hat{j}(t)$	$\hat{j}(t)$	$\hat{j}(t)$
	$t+1$	$j(t+1)$	$j_1(t+1)$	$\hat{j}(t+1)$	$\hat{j}(t+1)$	$\hat{j}(t+1)$
	$t+2$	$j(t+2)$	$j_1(t+2)$	$j_2(t+2)$	$\hat{j}(t+2)$	$\hat{j}(t+2)$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$t+n-1$	$j(t+n-1)$	$j_1(t+n-1)$	$j_2(t+n-1)$	$j_3(t+n-1)$	$\hat{j}(t+n-1)$
5. Payout	t	$p(1)$	$\hat{p}(1)$	$\hat{p}(1)$	$\hat{p}(1)$	$\hat{p}(1)$
Increments	$t+1$	$p(2)$	$p_1(2)$	$\hat{p}(2)$	$\hat{p}(2)$	$\hat{p}(2)$
	$t+2$	$p(3)$	$p_1(3)$	$p_2(3)$	$\hat{p}(3)$	$\hat{p}(3)$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$t+n-1$	$p(n)$	$p_1(n)$	$p_2(n)$	$p_3(n)$	$\hat{p}(n)$
		Pricing Assumptions				

DISCUSSION BY J. R. BERQUIST

Mr. Khury has made a positive contribution to the expanding effort by casualty actuaries to replace over-emphasis on "seasoned judgment" by scientific method, and for this effort, he must be congratulated! His paper sets forth an approach for documentation of the actuaries' assumptions as to frequency, severity, inflation, payout patterns and the time value of money in an explicit manner. Furthermore, he has introduced a notational system for these expressions so that they can be developed mathematically. We commend him for that effort.

On the other hand, there is a possibility that the notational model is too restrictive to be of help in all but the more stable lines, i.e., those lines without the *critical* need for actuarial approaches. For example, extensions of the model to lines requiring an "n" of 20, not 3 or 4, will complicate the model manipulation considerably, since the extension for this time frame may introduce not only the additional terms but also the need for more complex assumptions as well.

I also find myself a bit critical of the implication that the mathematical problem is simpler than it may in fact be. An example would be the analogy to life or pension reserving procedures. This reviewer, who is 100% sympathetic to the steps toward "formula" approaches to loss reserving, is still of the opinion that our best efforts will be less than successful for some years to come unless we continue to combine practical judgments with the best of mathematical techniques.

Another area where Mr. Khury seems to be years ahead of the "state of the art" is his implication that there is a one-to-one correspondence of ratemaking subsets and reserving subsets. While this reviewer has consistently stated full agreement with that theoretical viewpoint, it must be realized that with the exception of one line, one state companies, there is usually not a one-to-one correspondence between rate setting subsets and reserving subsets. Again, that is a practical problem now and one that may vanish over time.

But although there are practical conditions, such as the development of a statistically rigorous estimator of the author's "G" function, which will make the transition to formularized reserves much more difficult than the paper implies, it does contain a framework which can enhance the actuarial computation of reserves. Except for the implication that the confidence interval can be determined more easily than most applications permit, the procedure outlined can be most helpful in understanding the theoretical implications of the process.

The actuarial gain/loss section sets forth a very useful process. For some time this reviewer has prepared exhibits for certain clients which define that component of the loss ratio which is due to effects of development on "old" claims and that component which is due to over or under reserving on "new" claims.

In summary, this reviewer would like to congratulate the author for clearly identifying the reserving process mathematically and for stating it so clearly. Even though it may be some time before we are equipped to handle all of the required mathematics so explicitly for all lines, the attempt to do so will be most helpful to understanding the underlying process. Finally, we find it encouraging to find an actuary who admits the close correspondence between ratemaking and reserve setting. It is a paper such as this one that can improve both.

METHODS FOR FITTING DISTRIBUTIONS TO INSURANCE LOSS DATA

CHARLES C. HEWITT, JR. AND BENJAMIN LEFKOWITZ

VOLUME LXVI

DISCUSSION BY LEE R. STEENECK

"While this paper, so suggestive of an austere scholarship, may seem directed to those of the avant-garde who delight in frolicking among the outer reaches of actuarial theory, Mr. Hewitt presents both a challenge and a promise to those members whose interests, like this reviewer's, may gravitate more towards the application of actuarial principles to current underwriting and rating problems." And so Robert Hurley began his review of an earlier Hewitt paper (*PCAS* LIII) titled "Distribution by Size of Risk—A Model."

I, too, have always been *intrigued* by actuarial theory put into practice to solve rating problems. Certainly the body of the Hewitt/Lefkowitz paper deals primarily with the practical manipulation of a fitted loss distribution's cumulative function for the purposes of determining deductible discounts, increased limits factors, and relative frequency and severity.

Lest the sharp reader point an accusing finger at me already, perhaps I should explain my use of the verb "intrigue." Although Webster's preferred definition is "to cheat or trick," certainly I imply no diabolic intent to actuarial theory. Rather, my "interest is aroused."

I have some expertise with models used to price deductibles and increased limits factors. It was in 1976 and the then recently past "first" modern era of double digit inflation that actuaries working with the ISO felt that an improvement on the uniform excess and so-called "layer-of-loss" approaches to increased limits rate-making were justly deserved. I became involved and was a past chairman of the standing ISO committee dealing with commercial lines increased limits rate-making. I chose to review this paper, comparing it with what has been done in the last several years at ISO. Please keep in mind that our individual pursuits were totally independent, but it will be shown that our thoughts followed similar courses. Only in the practical manipulation of the curve do we, in fact, see differences.

The Increased Limits Subcommittee has been working with general liability data. Our initial curve-fitting efforts centered around the log normal distribution to be used on "live" medical malpractice data. As work progressed on other lines, we noted the log-normal failed to adequately describe various loss processes. Frequently, there were too many smaller losses for a good fit. A second distribution, the Pareto, was then developed. But again, we have not been able to totally explain the loss process by this single distribution. Perhaps we should be investigating compound distributions as well. Instead, we have chosen to truncate the Pareto from below at a value of, say, $5,000 and assign a single probability mass for all losses in the range $1 to $5,000. Since this falls within basic limits, the distribution in the range lacks importance by comparison to the "tail" probabilities by loss amount.

As a reinsurance actuary, I am constantly asked to evaluate large loss potential given that an insured has suffered a variety of smaller losses. This is a problem most of us face at one point or another. Even ISO, with its substantial data base, is missing detail on large losses by either (a) their non-occurrence (the fact that we have not seen a whole distribution of losses larger than $300,000 each) or (b) the tendency of primary policy limits to cap those losses which have occurred and are reported. Also, losses from excess and umbrella policies cannot be used with other raw data for fitting purposes. They are just not available in sufficient detail to use. From my point of view, this is primarily where the Hewitt/Lefkowitz paper interests me—predicting the tail of the distribution.

As we follow the curve to the right beyond the fitted area, say, up to $300,000 limits, and move into the unknown larger loss area, the choice of curve is of primary importance. Whether it be gamma, log-gamma, log-normal, Pareto, etc., we are speculating on some increased limits losses. Substantial actuarial judgment is required.

The authors have analyzed the "tail" problem in a manner similar to ours at ISO. Those losses that are at policy limits are said to be censored. A particular curve is fit in such a way that the number of policy limits losses are retained and are said to come from somewhere within the smooth extrapolation of the curve beyond policy limits. The reasonability test is: do all the tail frequencies of the fitted curve, when added together, compare favorably to the number of losses at policy limits?

This sort of fitting process is performed many times on data split by policy limits. At ISO it is called a multi-censored model. Naturally, the lesser the policy limit below $300,000, the greater the number of losses being censored.

As I mentioned before, I am concentrating on increased limits rate-making. The derived deductible credit columns on Tables 2 and 3 in the paper lend themselves extremely well to this. The tables, of course, are unitized, as should be expected for cumulative distribution functions. Layers of loss as a percentage of the total are calculated by subtraction. At ISO, we use a variation on this theme. We choose to use average policy limits losses. They reflect losses uncapped by policy limits as well as those capped at the policy limits. Then increased limits factors (based on expected value pricing) are determined by policy limits average loss plus average allocated loss expense plus unallocated loss expense divided by basic limits average loss plus the same average allocated loss expense plus unallocated loss expense. Neither type of loss expense was included for discussion by the authors.

For all of us who enjoy working with calculators, we can derive some pleasure in manipulating the columns for deductible credit on Tables 2 and 3. But before I demonstrate one principle, let us briefly investigate whether 100% inflation is realistic (Table 2 to 3). At first glance it might appear high, but try raising 1.15 to the fifth power! Yes, for losses emanating from the 1975 policy year, if trend is level at 15% per annum losses will double when on-level calculations are performed for policy year 1980 rates. And policy year 1975 experience is a most integral part of rates and increased limits factors being made for 1980. The other point to consider is whether all losses trend by the same percentage regardless of amount. ISO has assumed so, based on a few limited tests. But certain lines do exhibit an apparent increasing trend by size of loss which requires further study.

Given the reasonability of Tables 2 and 3, let us price the time differential. From Table 2 it can be demonstrated that if basic limits are $10,000 then a $250,000 policy (exclusive of LAE) should be rated at 2.81 times basic limits. Roughly 36% of the total cost is in the basic limits area, that is, $100 for each $281. With inflation, under Table 3, the increased limit factor (exclusive of LAE) is 3.62. Only 28% of the total limits premium is in basic limits now, that is, $200 for each $724. Note that even though inflation is 100%, the excess of basic premium has risen nearly 200%, that is, from $181 to $524. If you are not acutely aware of the part excess limits experience plays on your underwriting results, you should be. The leveraged effect of inflation is a point well worth remembering.

The cumulative frequency of cases columns are also interesting. Again, if basic limits are $10,000, a full 82.15% of all cases had losses falling in basic limits (later to become 71.09% after inflation). The dollars of loss in basic limits during this period dropped from 25.88% to 17.54%. An analysis like this could be used to decide what limits should be called basic. Without raising the limit occasionally, the insurance principle of loss spread becomes much more dominant over equity for a class or territory.

The cumulative frequency of cases columns also have use for excess of loss reinsurers as they plan on claim staff size and other operational costs associated with servicing different retentions to different sized treaty reinsureds. One of the methods for selecting a reasonable retention has to do with reinsuring only a small proportion of the number of claims, i.e. the largest claims with the most effect on loss experience. Using relative frequencies on fitted curves, the excess of loss reinsurer could suggest reasonable increases in retention to simplify the dialog and paperwork associated with administering a less than reasonable reinsurance program.

Again, I thank Messrs. Hewitt and Lefkowitz for sharing with us the results of a no doubt time-consuming and expensive study. This paper should foster more "intelligent competition" in rates.

RISK IN A COMPLEX SOCIETY

ROBERT CLEMENTS

For you to understand what an intimidating assignment it is for me to stand here this morning in front of you — members of the highest professional organization within our fine industry — I must confess my woeful lack of formal professional insurance education. It consists of a single night school course at the College of Insurance in 1958. The subject was Large Commercial Risk Underwriting; what I remember best about the entire experience is having to write a paper which was to determine the course grade. In the daytime, I was a casualty underwriter, and in those years my company tried to make a five percent profit on liability insurance by keeping losses to forty-seven percent of premiums and expenses to forty-eight percent of premiums. These ratios were supplied to us by the National Bureau of Casualty Underwriters and the point of my paper, entitled "Underwriting the Expense Dollar," was to question the logic of training underwriters to spend one hundred percent of their effort forecasting losses, which are unknowable in any event, and zero time assessing expenses on an individual risk basis, even though expenses accounted for the larger dollar outlay and were not only knowable but controllable. Obviously, I earned a grade of "D" for the course, along with the comment that the paper constituted impertinent criticism while failing to demonstrate any assimilation of what had been taught.

As a result of that unhappy experience, I took up a career in Brokerage — a calling where academic ignorance and impertinence are not necessarily liabilities.

From the time I dropped out of insurance school twenty-two years ago, the expense ratio has been declining steadily, in part as the result of greater productivity within the industry, and in part because loss costs have inflated faster than expenses. Whatever the reason for this decline, it seems to me — since loss forecasting is clearly more difficult than expense forecasting — that there is a greater element of uncertainty associated with ratemaking, or pricing, now than ever before. In a sense, uncertainty is a lot of what I want to talk about this morning. Uncertainty and false perception, which seems to be the handmaiden of uncertainty. However, at this point, having admitted to a lack of academic preparation for an insurance career, I must also confess to rather narrow practical experience.

In my firm we are essentially involved in commercial insurance rather than personal insurance. In fact, seventy-five percent of our business is derived from clients who practice risk management in the sense that they are corporations or institutions with sufficient financial resources that they can and do employ not only insurance, but alternative devices (such as self-insurance and captives) for the administration of risk. Most of our other business comes from commercial accounts for whom insurance is the basic risk management solution. When I speak of insurance, therefore, I am basically talking about the complex business of commercial insurance. It is a significant distinction: personal insurance and commercial insurance are really two different businesses from the standpoint of consumer requirements, marketplace characteristics, and other factors. Unfortunately, it is a distinction drawn too infrequently by regulators and even by many in the business itself.

There are forces which now seem to be at work changing the basic structure of our business. We sense that these changes are taking place, but we are uncertain as to their precise nature and, therefore, as to the nature of the ultimate structure. This uncertainty is vexing and poses a number of questions to challenge us as we plan for the future. Thus, in addition to asking whether the rates are adequate to cover the losses, we ask:

— What is the future course of regulation in the United States?

— What facilities and resources are required for an insurer to compete in a commercial insurance market where many of the insureds now own insurance companies and where brokers, reinsurers, and independent adjustors all compete to sell services traditionally considered to be the inalienable responsibility of the underwriter?

— What resources must the broker possess to survive in the same complex environment?

— How do we respond to the twin challenges of advancing technology and unprecedented inflation?

FALSE PERCEPTIONS AND THE CONSEQUENCES THEREOF

It seems to me that the uncertainty and tension which today are pervasive in our little world of insurance are the result of a perception of the future as it may be; that is, they are the result of a perception based on analysis of the long-term consequences of the forces which seem to be working to change the basic structure of the business. In my personal opinion, some current generally held perceptions are false. I will list seven and then comment on each.

1 — Regulation can solve all problems.

2 — Investment income does not count in ratemaking.

3 — The market share of the agency-brokerage underwriters is shrinking, and inevitably so.

4 — Loss control and claim adjustment services are the natural province of insurers.

5 — The national brokers are embarked on a long-term policy of vertical integration.

6 — Size should be equated with efficiency in the delivery of services.

7 — The brokerage, or risk-service, element of the industry is inherently more profitable and more attractive than the underwriting, or risk-assumption, element of the industry.

COMMENTARY

Regulation

The perception that regulation can solve all problems is undoubtedly the most narrowly held of all the ones on my list. It seems to exist primarily in the minds of regulators and a few legislators. Their premise seems to be that perfection of rate, form, and service is not only desirable but possible; the absence of such perfection in daily life is therefore not a sign of human fallibility but rather a sign of willful intent. The next logical step is to design perfection by law and regulation. This leads to well-meant but misdirected efforts such as the ones now going on in several states to restrict access to surplus lines markets and to bring surplus lines underwriters more directly under the purview of regulation.

The failure of regulators in most states to recognize that the needs of underwriters and consumers of commercial insurance are vastly different than the needs of underwriters and consumers of personal insurance contributes to uncertainty and presents a major stumbling block to the building of the kind of marketplace we're going to need in the 1980s. I might add that in a study commissioned last year by Marsh & McLennan, risk managers of American business, who the government says need more protection through regulation, voted overwhelmingly for less regulation.

Investment Income

Setting investment income aside so that it is not considered a factor in determining underwriting results causes three problems. First, it increases the likelihood of bad underwriting decisions. Failure to recognize investment income leads to the inefficient use of risk capital by influencing underwriters to decline business with significant profit potential while aggressively seeking lines with a relatively marginal profit opportunity. For example, the return on investment to an insurer writing automobile physical damage at a combined loss and expense ratio of 97.5% is clearly far less than the return on medical malpractice with a combined ratio of 102%.

Second, it forces insureds, who have no illusions about the time value of money, to seek alternatives to conventional risk transfer in circumstances where insurance would otherwise be their preference.

Finally, it creates a source of tension with consumers, regulators, and legislators. I am familiar with, and generally sympathetic to, all of the arguments against the inclusion of investment income in ratemaking. The fact is that the present state of the art in underwriting, particularly in so-called long-tail business, does not permit its inclusion on any rational basis.

Appropriate recognition of investment income seems to me to be perhaps the great challenge of the immediate future to your professional society. As a representative of the insurance consumer, I am quite prepared to endorse a ratemaking scheme which permits an exceptional return on investment to the risk-taker who binds himself to coverage years in advance of the potential loss payout, with no certainty about the effect of inflation on the ultimate loss cost or the value of the investment return during the premium holding period. If you can find a way to advance your technology to the point where this kind of return-on-investment ratemaking is feasible, you will have done a great deal to reduce the friction which complicates our relationship with regulators and consumers.

Direct Writers and the Imminent Demise of the Distribution System

One leading agency insurance company executive has referred frequently in public statements to what he calls "The Direct Writer Revolution." The idea seems to be that the ultimate distribution system has no intermediaries. Another leading executive talks about the future in terms of the one-stop shop where all of the consumer's insurance needs are solved in a single direct transaction. I think the two ideas go together, but *only* together. That is to say, the perfect distribution system is one which dispenses with intermediaries *only* if the vendor is in a position to answer all the needs of his customers. It's a wonderful concept, but I can think of a better one. If we could have a future without losses, then we could dispense with the entire system — even the one-stop shop would no longer be neccessary. Of course, we are going to have losses; losses in the future will be larger than those we've experienced historically. From where will the added capacity come? One solution might be to form a State Insurance Company, a true one-stop shop, which would take all the risks. That approach doesn't seem to work very well in practice, however. One reason is that the risks tend to be interstate and even international. More importantly, it forecloses the chance for the world to benefit from the influence of an entrepreneur with a better idea.

The alternative is to have many insurers competing freely with one another and, by innovation and specialization, seeking to out-perform all competitors. It is inconceivable to me that any of these insurers would determine the most efficient use of risk capital to be accepting all of the risks of the insurance consumer. Total account underwriting therefore tends to break down because it is vulnerable to the sharpshooting of specialty underwriters and is dependent on hired capacity, frequently in the form of facultative reinsurance, which lacks essential commitment.

My view of the future is that it holds very few one-stop shops, but an increasing number of complex business and personal risks. In such an environment, the intermediary is likely to continue to be of value to his client, the consumer.

Most of the discussion and prognostication relevant to the insurance distribution system lumps so-called direct writers on one side and so-called agency companies on the other. The agency companies are presumed to deal with both agents and brokers. In fact, however, there are not only agency companies which deal with agents and brokers, but also agency companies which deal exclusively with agents and other companies which deal exclusively with brokers. Of these three styles, the brokerage companies are the fastest growing, followed by the agency companies. The group which deals with brokers and agents through a single organizational setup is not only the slowest growing, but the least profitable. Some groups now seek the best of both worlds by operating both a brokerage division and an agency division.

Insurance Company Role

We now seem to have emerged from an environment which was pretty simple and clear-cut. It consisted of risks, risk-takers and middlemen. Now, due to a variety of factors, our world is much more complicated. Among the influences contributing to this complexity we might include inflation, social change, ineffective regulation and new technology, and, perhaps most of all, the development of risk management as a legitimate corporate management function. Many commercial insurance consumers now own insurance companies and thus are suppliers as well as consumers of the product. Many insurers, apparently disillusioned with risk-taking, seek to sell services while pursuing the policy of assuming as little underwriting risk as possible. All kinds of services, from reinsurance to salvage, and of course including loss control and claims adjustment, are now available from independent contractor specialists, who at one time made their facilities available exclusively to insurance companies. As they began in the mid-1980s to perceive the withdrawal of the big primary carriers, who had been their major clients, from the risk-taking function, these specialists came to the conclusion that if the real risk-taker is the insured — or, to put it more accurately, the self-insured — they had better make their facilities available directly to the insured. There is genuine disagreement and confusion in the matter of who is the logical purveyor of services to insureds, including especially self-insureds. I believe it would be safe to say that all brokers have traditionally felt that their part of the deal was to provide service to the insured, while the underwriter's part was the taking of risk. Inherent with assumption of risk is the provision of investment, actuarial, claim and loss control services. Traditionally, these services were considered to be primarily for the benefit of the underwriter as risk-taker. The underwriter's control of them was generally accepted, and quite properly so. In many instances today, however, the insured retains for his own account the principal primary risk. In such circumstances, loss control, claim adjustment, and even actuarial services are primarily for the benefit of the insured as principal risk-taker and, therefore, properly may be considered to be his to control. Thus, when an insured, or his broker, or an independent contractor hired by either of them, seeks to set up these facilities for his own account, what we have is not evidence of a competitive plan to enter the underwriting business, but a simple response to the fact that being at risk requires a certain amount of attendant administration on the part of the self-insured.

The Strategy of the National Brokers

No doubt some of you are familiar with the following definition of the term "Broker" from Samuel Johnson's 18th century dictionary. He wrote:

> *Broker — a lowly fellow, one who lives off the efforts of others: A procurer. Brokers, who having no stock of their own, set up a trade with that of another man and commonly abuse both sides to make a little paltry gain.*

My own personal definition of the word is much shorter. To me a Broker, most of all, is a person who makes commercial transactions happen. In specific insurance terms, that means primarily the assembling and organizing of risk-taking capacity in the most efficient, effective, and economic manner for the benefit of the Broker's client. A subsidiary, but related, brokerage function is the provision of risk management services to his client in specialized areas in which, for whatever reason, the client may choose not to perform these functions himself. Whether you use Samuel Johnson's definition or mine, risk assumption does not fit the model, and I seriously doubt that keeping the client's risks for his own account is part of the short- or long-run strategy of any thoughtful broker.

Size and Efficiency

I mentioned earlier what seems to be a prevailing tendency to correlate size with efficiency. The clearest manifestation of this is the presumption by certain carriers that economies will be introduced by the transfer of certain functions — notably clerical and administrative ones — from agents and brokers to themselves. According to figures from A. M. Best, the stock company expense ratio has declined steadily over twenty years from more than 36% in 1958 to 27.6% in 1978 — apparent evidence that the insurance product is being delivered to the consumer with ever-increasing efficiency. The fact of the improving expense ratio is widely understood. Of great importance, but less generally acknowledged, is the fact that the vast majority of this improvement has been accomplished by a reduction in the expenses of intermediaries. The insurer portion of the expense ratio was 15.9% in 1958 and declined only to 13.9% in 1978. During the same period, the portion attributed to brokers and agents dropped from 20.3% to 13.7%. It will no doubt come to many as a surprise that three-fourths of the increase in efficiency and productivity in the general insurance industry in the last two decades has been accomplished by intermediaries, while only one-quarter is reflected in insurer expenses. The real point, however, is that the price ought to reflect a process devoid of redundancies.

Brokers Have the Best End of the Business

In spite of the fact that average commission rates have fallen by more than one-third in barely twenty years, the general view of the insurance industry — both from within and without — is that brokers have the better part of the business; that, insulated from inflation and immune to the cycle of underwriting results, they are assured of predictable, steady growth. The other side of this coin is the proposition that the peril of inflation, and the fact of the cycle, are certain evidence that underwriting is probably inherently not a very good business and, in any event, is definitely not a good business to invest in due to overregulation, mismanagement on the part of some, excessive competition, and a variety of other factors. That this is the external view of the industry is obvious from the share valuation put on it by investors: the tendency today is for the share price of insurers to run at four to five times earnings, while the price for brokers more frequently tends to be in the neighborhood of eight to ten times earnings. The best pieces of evidence that the same view holds internally are the rapid expansion of insurers into the risk management service field, a new willingness on the part of many to unbundle the services of their claim and loss control departments, and a recently developed preference for large deductibles and cost-plus rating plans on the part of many underwriters who, at one time, argued that the introduction of even small deductibles posed a mortal threat to the integrity of the ratemaking process.

This theory that my part of the business is better than yours is the final perception that I want to challenge this morning, not because I have any lack of pride or confidence in the future of my business, but because I feel the idea that it is better than yours produces harmful tension and friction.

There is no reason why this should be, but the earnings multiple of a company's stock seems to have a big influence on whether the management of the enterprise views its business with pride or paranoia, depending on whether the multiple is high or low. Now there is a great danger of over-simplication in any analogy, so I want to be very careful about not going too far in comparing the business of securities analysis to the practice of astrology. Nevertheless, it does seem to be the case that in both callings the basic process is to look skyward in an effort to determine the significance of certain known earth-bound events on the future.

By this science, my business is determinued to be predictable and reliable; yours is determined to be cyclical and uncertain; both determinations are made in accordance with the number of days required for the earth to make a full trip around the sun. I think we might call this the planetary theory of security analysis. From my point of view as a broker, if the analyst is going to look towards heaven in order to assess my performance it is well for him to concentrate on the movement of the earth around the sun. A different planetary technique might focus on Mercury, which takes eighty-eight days to go around the sun. With that kind of a measurement period, my business looks very cyclical indeed; it regularly consists of one better-than-average ninety-day period, followed by two average periods, followed, finally, by one period which is much below average. Unfortunately for the insurance underwriter, it takes Mars less than two years to make one circle; it takes Jupiter twelve; there is nothing in between. What you need is to discover a planet which revolves in about three years, and promote a theory of analysis based on it.

In my company, we did a study which compared our results with those of the general insurance portion of fifteen of the largest underwriting groups over the last twenty years. We were just slightly ahead of the middle of the pack in terms of compound revenue growth, but next to last in compound earnings growth. When underwriting results were examined using a measurement period of thirty-six months, the cyclical impact of annual measurements was entirely smoothed, and earnings followed a steady and steep upward path. Looked at in such a manner, the business of insurance underwriting can be seen for the great growth industry that it is.

When I was starting out in the brokerage business, I used to be genuinely concerned about its future prospects. Our clients then, as now, were essentially businesses; also, then as now, they were mostly successful and growing. It seemed quite possible, at the time, that as these businesses continued to accumulate greater financial resources they might become less and less dependent on insurance for financial stability.

I remember toying with the hypothesis that a reasonable maximum premium expenditure for any risk might be about $1 million: that any foreseeable risk which could not be transferred for that amount or less could best be funded internally. In fact, last year we placed one risk with a $20 million deductible, yet a $30 million premium! I was not smart enough to divine the two factors which have been at work to make this a growth business and which are likely to continue to make it so in the future. First of all, contrary to the assumption of many, the technological advances which lead to corporate growth tend to increase the risk of loss and, accordingly, the need for insurance. For example, the growth of air transport does not breed larger fleets of small aircraft, but rather leads to the concentration of investment, and therefore risk, in larger and more expensive airplanes. The growth of demand for energy leads not to more wildcat wells and steam boilers, but to off-shore drilling platforms and nuclear power plants. In general, safety is a by-product of improved technology, but so is the increased concentration of capital investment. The result, in terms of property risks, is the happy (for underwriters) combination of increased demand for the insurance product accompanied by lower unit cost.

The other growth factor is sometimes referred to as social inflation or entitlement. Whatever we may choose to call it, there seems to be no doubt that, as the benefits of technological advances provide a society with increased material well-being, the citizenry of that society comes more and more to the expectation that reparations for its injured members can and ought to be payable. Risk of loss is consequently increased through adjudication and by legislation. This trend seems to have been more difficult than the first effect to foresee and quantify actuarially. In fact, the technology of casualty — especially general liability — underwriting lags dangerously behind the requirements of the marketplace. The technology of property underwriting is highly developed, to the point where the marketplace is well-served with broad forms, comparatively stable rates, and reasonably efficient loss adjustment procedures. In the liability field, the underwriter is handicapped by our inability to develop a generally acceptable definition of the occurrence which is supposed to trigger the coverage, and by almost total reliance on the extrapolation of past losses in ratemaking. Fortunately, there is some recent and welcome evidence that underwriters are beginning to have increased confidence in the tools of the trade manufactured for them by actuaries.

To sum up, though the future abounds with uncertainties, there is much about which to be optimistic. Insurance has been, is, and will very likely continue to be a growth industry and a profitable one. For many years, the amount of the gross national product expended for the protection and security of the underwriter's product has been steadily increasing. Given the likelihood of increased human expectation and continued technological advances, it is probable that this trend of growth above and beyond the average growth rate of industry will continue into the foreseeable future.

The principal uncertainty is our, or, more appropriately, your ability to measure the risks in a manner which will give the underwriter the confidence to take the risks and the consumer the willingness to pay the underwriter to do so — all in a manner seen by regulators to be fair to both sides. Thus, it seems particularly fitting, and a cause for much optimism, that you have chosen pricing as the theme of this conference. It has been my privilege to be here this morning. I thank you and wish you the most productive of sessions in the meetings to come.

MINUTES OF THE 1980 SPRING MEETING

May 11-14, 1980

CARIBE HILTON, SAN JUAN, PUERTO RICO

Sunday, May 11, 1980

The regular quarterly meeting of the Board of Directors was held from 1:00 p.m. to 5:00 p.m.

Registration took place from 4:00 p.m. to 7:30 p.m.

The President's reception for new Fellows and their spouses was held from 6:00 p.m. to 6:45 p.m.

A reception for members and guests was held from 6:30 p.m. to 7:30 p.m.

Monday, May 12, 1980

Registration was held from 7:30 a.m. to 8:30 a.m.

The Spring meeting was called to order at 8:30 a.m. President W. James MacGinnitie introduced the Honorable Rolando Cruz, Commissioner of Insurance of the Commonwealth of Puerto Rico, who welcomed the Society to Puerto Rico and spoke briefly about the actuary's contribution to the insurance industry in Puerto Rico.

Following Commissioner Cruz's remarks, President MacGinnitie asked the attendees to rise for a moment of reflection in remembrance of the Society's deceased members.

President MacGinnitie then read the names of the 38 new Associates, who rose to the acknowledgement of the assembly. As their names were called, each of the 13 new Fellows was asked to step forward to receive his or her diploma.

FELLOWS

Irene K. Bass
Albert J. Beer
Susan T. DiBattista
Michael C. Dolan
Doreen S. Faga
Robert A. Giambo
Francis J. Lattanzio
Mary L. O'Neil
Beatrice T. Rodgers
William J. Rowland
Oakley E. Van Slyke
Edward W. Weissner
Timothy L. Wisecarver

ASSOCIATES

Regina M. Berens
Nicholas M. Brown, Jr.
George Burger
Catherine J. Campbell
David G. Clark
Curtis G. Dean
George T. Dodd
John L. Doellman
Richard D. Easton
Grover M. Edie
David Engles
Bruce F. Friedberg
Irwin H. Goldfarb
Deborah A. Gorman
Roger M. Hayne
Bertram A. Horowitz
Russell T. John
Judy A. Johnson
Bruce R. Jones
Leon W. Koch
Michael R. Larsen
Yoong S. Lee
Carl J. Leo
Winsome Leong
Orin M. Linden
John S. Lombardo
Gail P. McDaniel
Dennis C. Mealy
Glenn G. Meyers
Emanuel Pinto
Louis G. Seguin
James Surrago
Kevin B. Thompson
Roger D. Walker
Thomas A. Weidman
William T. Weiland
Patrick B. Woods
Hank Youngerman

President MacGinnitie then introduced Mr. Walter L. Grace, President-Elect of the American Academy of Actuaries, who spoke about the involvement of the CAS in the Academy and the Academy's role of representing the actuarial profession to the public.

Secretary David P. Flynn then read a brief description of a new paper by Mr. C. K. Khury, Vice President, Prudential Property & Casualty Insurance Company, entitled "Loss Reserves: Performance Standards."

Following a short break, President MacGinnitie called the meeting to order.

Mr. Robert Clements, President of Marsh & McLennan, Inc., delivered the Keynote Address, entitled "Risk in a Complex Society."

After the Keynote Address, an informal discussion period was held.

The topic for the Call Paper program was "Pricing Property and Casualty Insurance Products." A summary of the sixteen call papers presented was given by Mr. Dale A. Nelson, Assistant Vice President and Actuary, State Farm Mutual Automobile Insurance Company.

At 11:30 a.m. the meeting recessed for an informal buffet luncheon.

The afternoon was dedicated to concurrent sessions for discussion of the Call Papers. The four session moderators were:

Session A:	James F. Brannigan Senior Vice President and Actuary H. F. Ahmanson & Company
Session B:	William Leslie, Jr. Consulting Actuary Tillinghast, Nelson & Warren, Inc.
Session C:	Lee R. Steeneck Assistant Vice President General Reinsurance Corporation
Session D:	Richard G. Woll Actuary Hartford Insurance Group

The call papers, their authors, and reviewers were as follows:

"Relativity Pricing Through Analysis of Variance," by Carl Chamberlain, INA (presented by Michael Dolan, INA), reviewed by Daniel Goddard, Industrial Indemnity Company.

"Expense Allocation in Insurance Ratemaking," by Diana Childs, INA and Ross A. Currie, INA, reviewed by David Klein, Hartford Insurance Group (presented by Richard G. Woll, Hartford Insurance Group).

"Impacts of State Regulation on the Marketing and and Pricing of Individual Health Insurance," by Charles Habeck, Milliman & Robertson, Inc., reviewed by Robert Schuler, Blue Cross of Western Pennsylvania.

"Credibility and Solvency," by Philip Heckman, CNA, reviewed by Janet Fagan, Home Insurance Company.

"Pricing for Corporate Objectives," by Frank J. Karlinski, Prudential Property & Casualty Insurance Company, reviewed by Robert A. Anker, American States Insurance Company.

"Ratemaking for the Personal Automobile Physical Damage Coverages," by John J. Kollar, Insurance Services Office, reviewed by Galen R. Barnes, Nationwide Insurance Companies.

"Uses of Closed Claim Data for Pricing," by R. Michael Lamb, Insurance Division of the State of Oregon, reviewed by Richard S. Biondi, Insurance Services Office.

"Rating Claims-Made Insurance Policies," by Joseph O. Marker, Westfield Companies, and F. James Mohl, St. Paul Fire & Marine Insurance Company, reviewed by Michael F. McManus, Chubb and Sons, Inc.

"An Analysis of Retrospective Rating," by Glenn G. Meyers, CNA, reviewed by James F. Golz, Wausau Insurance Companies.

"Estimating Aggregate Loss Probability and Increased Limit Factor," by Dr. Shaw Mong, Fred S. James & Company, reviewed by Robert S. Miccolis, Corroon & Black Corporation.

"Pricing Excess-of-Loss Casualty Working Cover Reinsurance Treaties," by Gary Patrik, Prudential Reinsurance Company, and Russell John, Prudential Reinsurance Company, reviewed by Jerry Miccolis, Tillinghast, Nelson & Warren, Inc.

"Experience Rates as Estimators: A Simulation of Their Bias and Variance," by James N. Stanard, Prudential Reinsurance Company, reviewed by John P. Robertson, Fireman's Fund Insurance Company.

"Actuarial Issues To Be Addressed in Pricing Insurance Coverages," by E. James Stergiou, Woodward & Fondiller, Inc., reviewed by Sheldon Rosenberg, Insurance Services Office and Aaron Halpert, Insurance Services Office.

"The Pricing of MediGap Contracts," by Emil J. Strug, Blue Cross/ Blue Shield of Massachusetts, reviewed by Robert F. Bartik, Kemper Insurance Group.

"Is Econometric Modeling Obsolete?" by Oakley E. Van Slyke, Warren, McVeigh & Griffin, reviewed by Michael Fusco, Insurance Services Office.

"Risk Classification Standards," by Michael A. Walters, Insurance Services Office, reviewed by Robert A. Bailey, National Association of Insurance Commissioners.

There was an informal discussion and coffee break in midafternoon, with the meeting adjourning for the day at 5:30 p.m.

Tuesday, May 13, 1980

The meeting reconvened at 8:30 a.m.

The morning meeting was a repeat of Monday afternoon's concurrent sessions for presentation of call papers.

An informal discussion and coffee break took place at midmorning.

The regular session reconvened at 2:30 p.m. with a workshop program. The workshop subjects and participants were:

Workshop 1 — "New Paper and Reviews"

Paper Presented:	"Loss Reserves: Performance Standards" by C. K. Khury Vice President Prudential Property & Casualty Insurance Company
Presented by:	David P. Flynn Vice President and Actuary U.S. Insurance Group
Reviewed by:	James R. Berquist Consulting Actuary Milliman & Robertson, Inc. presented by Douglas Kline
	Neil A. Bethel Consulting Actuary Tillinghast, Nelson & Warren, Inc.

James A. Hall
Director, Casualty Actuarial Consulting
Coopers & Lybrand

Workshop 2 — "Workers' Compensation Reserving"

Moderator: Richard W. Palczynski
Associate Actuary
The Travelers Insurance Companies

Members: Ronald C. Retterath
Vice President and Actuary
Wausau Insurance Companies

Joel S. Weiner
Associate Actuary
INA Corporation

Workshop 3 — "Statement of Actuarial Opinion for Fire & Casualty Insurance Company Annual Statement"

Moderators: Richard H. Snader
Vice President — Corporate Actuary
United States Fidelity & Guaranty Co.

Donald E. Trudeau
Manager
Peat, Marwick, Mitchell & Company

Committee meetings were held as scheduled.

A reception was held from 6:30 p.m. to 7:30 p.m.

Wednesday, May 14, 1980

President MacGinnitie reconvened the meeting at 8:30 a.m. and introduced William Haddon, Jr., M.D., President, Insurance Institute for Highway Safety, who presented a program entitled "Recent Loss Reduction Developments."

Following an informal discussion and coffee break, the meeting was reconvened at 10:15 a.m. for a Business Session. President MacGinnitie announced the awarding of the Michelbacher Prize for 1980 to Russell John and Gary Patrik for their paper, "Pricing Excess-of-Loss Casualty Working Cover Reinsurance Treaties."

The next item on the program was a panel entitled "Excess Profits."

Moderator:	Charles F. Cook Senior Vice President American International Underwriters
Panelists:	Galen R. Barnes Actuary: Property/Casualty Pricing Nationwide Insurance Companies
	Richard G. Woll Actuary Hartford Insurance Group
	C. Arthur Williams, Jr. Professor of Economics & Insurance University of Minnesota

The closing remarks were made by President W. James MacGinnitie, after which the meeting was adjourned at 12:00 noon.

The meeting was attended by 159 Fellows, 133 Associates, 12 subscribers, 16 guests, 1 student, and 149 spouses. The list of attendees follows.

FELLOWS

Alexander, L. M.
Angell, C. M.
Anker, R. A.
Arata, D. A.
Asch, N. E.
Bailey, R. A.
Barnes, G. R.
Bartik, R. F.
Bass, I. K.
Beer, A. J.
Bell, L. L.
Ben-Zvi, P. N.
Bergen, R. D.
Berquist, J. R.
Bethel, N. A.
Beverage, R. M.
Bill, R. A.
Biondi, R. S.
Blivess, M. P.
Bondy, M.
Bornhuetter, R. L.
Bovard, R. W.
Brannigan, J. F.
Carbaugh, A. B.
Childs, D. M.
Collins, D. J.
Conger, R. F.
Conners, J. B.
Cook, C. F.
Daino, R. A.
Davis, G. E.
DiBattista, S. T.
Dolan, M. C.
Donaldson, J. P.
Eddy, J. H.
Eland, D. D.
Eyers, R. G.
Faber, J. A.
Faga, D. S.
Fagan, J. L.
Fein, R. I.
Ferguson, R. E.
Flynn, D. P.
Fossa, E. F.
Foster, R. B.
Fowler, T. W.
Fresch, G. W.
Fusco, M.
Garand, C. P.
Giambo, R. A.
Gleeson, O. M.

FELLOWS

Goddard, D. C.
Golz, J. F.
Gottlieb, L. R.
Grannan, P. J.
Graves, J. S.
Grippa, A. J.
Hachemeister, C. A.
Hafling, D. N.
Hall, J. A., III
Hartman, D. G.
Harwayne, F.
Hazam, W. J.
Heer, E. L.
Hermes, T. M.
Honebein, C. W.
Hough, P. E.
Hoylman, D. J.
Hughey, M. S.
Jerabek, G. J.
Jones, A. G.
Karlinski, F. J.
Kates, P. B.
Kaufman, A.
Keene, V. S.
Kelly, A. E.
Kline, D. F.
Kollar, J. J.
Kormes, M.
Krause, G. A.
Kuehn, R. T.
Lamb, R. M.
Lehmann, S. G.
Leimkuhler, U. E.
Lerwick, S. N.
Leslie, W., Jr.
Levin, J. W.
Lino, R. A.
Lowe, R. F.
MacGinnitie, W. J.
Marker, J. O.
McClure, R. D.
McConnell, C. W., II
McManus, M. F.
Meeks, J. M.
Miccolis, J. A.
Mohl, F. J.
Moore, P. S.
Muetterties, J. H.
Munro, R. E.
Nelson, D. A.
Newlin, P. R.
Newman, S. H.
O'Brien, T. M.
O'Neil, M. L.
Palczynski, R. W.
Patrik, G. S.
Pearl, M. B.
Petersen, B. A.
Philbrick, S. W.
Phillips, H. J.
Quirin, A. J.
Reichle, K. A.
Retterath, R. C.
Richards, H. R.
Richardson, J. F.
Rodermund, M.
Rodgers, B. T.
Roland, W. P.
Rosenberg, S.
Roth, R. J.
Rowland, W. J.
Salzmann, R. E.
Scheibl, J. A.
Schuler, R. J.
Schultz, J. J., III
Schumi, J. R.
Sheppard, A. R.
Shoop, E. C.
Snader, R. H.
Squires, S. R.
Stanard, J. N.
Steeneck, L. R.
Steer, G. D.
Stergiou, E. J.
Stewart, C. W.
Streff, J. P.
Strug, E. J.
Switzer, V. J.
Taht, V.
Tatge, R. L.
Taylor, F. C.
Toothman, M. L.
Tverberg, G. E.
Van Slyke, O. E.
Venter, G. G.
Verhage, P. A.
Walsh, A. J.
Walters, M. A.
Weissner, E. W.
Williams, P. A.
Wilson, J. C.
Winkleman, J. J., Jr.
Wisecarver, T. L.
Woll, R. G.
Wood, C. P., Jr.
Wood, J. O.
Wright, W. C., III
Yoder, R. C.

ASSOCIATES

Andrus, W. R.
Applequist, V. H.
Austin, J. P.
Baer, D. L.
Berens, R. M.
Brewer, F. L.
Brooks, D. L.
Brown, N. M., Jr.
Burger, G.
Campbell, C. J.
Cheng, J. S.
Chorpita, F. M.
Christiansen, S. L.
Christie, J. K.
Cis, M. M.
Clark, D. G.
Cohen, H. L.
Cohen, H. S.
Connor, V. P.
Corr, F. X.
Covney, M. D.
Currie, R. A.
Davis, R. D.
Dean, C. G.
Degarmo, L. W.
Demers, D.
Dodd, G. T.
Doellman, J. L.
Duffy, T. J.
Duperreault, B.
Easton, R. D.
Edie, G. M.
Engles, D.
Fisher, R. S.
Flanagan, T. A.
Foote, J. M.
Friedberg, B. F.
Gerlach, S. B.
Godbold, M. J.
Godbold, N. T.
Goldfarb, I. H.
Gorman, D. A.
Granoff, G.
Gruber, C.
Harrison, E. E.
Head, T. F.
Heckman, P. E.
Henry, D. R.
Hobart, G. P.
Horowitz, B. A.
Javaruski, J. J.
Jensen, J. P.
John, R. T.
Johnson, J. A.
Johnston, D. J.
Johnston, T. S.
Jones, B. R.
Kaur, A. F.
Klingman, G. C.
Koch, L. W.
Kozik, T. J.
Lafontaine, G.
Lafrenaye, A. C.
Larose, J. G.
Larsen, M. R.
Lehman, M. R.
Leo, C. J.
Leong, W.
Linden, O. M.
Lo, R. W.
Lombardo, J. S.
Lommele, J. A.
Lowe, S. P.
Mathewson, S. B.
McConnell, D. M.
McDaniel, G. P.
McGovern, W. G.
Mealy, D. C.
Meyer, R. E.
Meyers, G. G.
Miccolis, R. S.
Moller, K. G.
Morell, R. K.
Mulder, E. T.
Murad, J. A.
Murphy, F. X., Jr.
Nash, R. K.
Neuhauser, F., Jr.
Nichols, R. S.
Niswander, R. E., Jr.
Parker, C. M.
Pastor, G. H.
Peacock, W. W.
Perry, L. A.
Philbrick, P. G.
Piazza, R. N.
Pinto, E.
Pilon, A.
Potok, C. M.
Potter, J. A.
Raid, G. A.
Reynolds, J. D.
Ritzenthaler, K. J.
Roach, R. F.
Robertson, J. P.
Rudduck, G. A.
Ryan, J. P.
Sandler, R. M.
Schneiker, H. C.
Schwartz, A. I.
Seguin, L. G.

ASSOCIATES

Shayer, N.
Silberstein, B.
Singer, P. E.
Skolnik, R. S.
Smith, F. A.
Sobel, M. J.
Stroud, R. A.
Surrago, J.
Swaziek, R. R.
Van Ark, W. R.
Wade, R. C.
Walker, R. D.
Wasserman, D. L.
Weidman, T. A.
Weiland, W. T.
Weiner, J. S.
Weller, A. O.
Wess, C.
Woods, P. B.
Young, R. G.
Youngerman, H.
Zatorski, R. T.

GUESTS — SUBSCRIBERS — STUDENTS

Allen, T. C.
Anderson, C. A.
Anderson, E. V.
Behan, D. F.
Belton, E. F.
Benktander, G.
Clarke, T. G.
Clements, R.
Countryman, G. L.
Cruz, R.
Grace, W. L.
Guarini, L. T.
Habeck, C.
Haddon, W., Jr.
Hager, G. A.
Hill, J.
Hinkle, T. C.
Kellison, S. G.
Kraysler, S. F.
Mong, S.
Odell, W. H.
Posnak, R. L.
Reall, G. F.
Rech, J. E.
Schiavo, M. F.
Smith, D. A.
Spangler, J. L.
Treloar, M.
Williams, C. A., Jr.

Respectfully submitted,
DAVID P. FLYNN,
Secretary

Volume LXVII, Part 2 No. 128

PROCEEDINGS

November 19, 20, 21, 1980

THE ACTUARY AND HIS PROFESSION: GROWTH, DEVELOPMENT, PROMISE

PRESIDENTIAL ADDRESS BY W. JAMES MacGINNITIE

It has become almost trite to observe that we live in an era of rapid change. Yet we need to recognize that change quite clearly, as it applies to our profession, if we are to understand where it is we are, and where we should be going. It was exactly twenty years ago this month that I wrote my first actuarial examination. In the ensuing score of years, our Society has more than doubled, and it has become increasingly difficult to get around at the receptions. The basic industry we serve has grown at a compound rate of 10% during that period. The business has spread from pure casualty to multiple line, and, in recent years, increasingly to a combined life and casualty operation. Liability business has grown more rapidly than property, and the relative shares have shifted dramatically. At the same time that the proportion has shifted to the longer tailed business, the tail itself has grown. Perhaps it would be more accurate to say that our perception of the length of the tail has grown, since it is now clear that the tail that lay ahead of us back in 1960 was a great deal longer than we ever imagined, even in our wildest nightmares. In two data bases with which I am familiar, we are still getting upward developments on the 1960 accident year. If I had answered one of those 1960 examination questions in a manner that indicated that I believed that a tail exceeding 20 years was appropriate, I suspect that I might have been given another opportunity to try again the following year.

In that 20 years, we have also experienced unprecedented rates of inflation, and, perhaps more importantly, the changes in the rate of inflation have become greater. A fairly persuasive argument can be made that we can handle a steady inflation rate, although that has never been tested at today's inflation levels. Accompanying these higher inflation levels have been higher rates of interest. Unfortunately, the interest rates seem to lag, with the result that negative real rates of return have been all too common in recent years.

Change in the computer area has also been extensive in the last twenty years. When I started, a great deal of information was handled on punch card equipment, and calculators produced ratios only slowly and noisily. Yet, today we can buy programmable calculators that, among other things, make our examination on compound interest obsolete.

In addition to the growth of our Society, there has also been a major shift in the employment of our members. They have increasingly gone to work for smaller and independent (as opposed to bureau) companies. More of you now work in the reinsurance field, and in consulting. Brokers and accounting firms increasingly employ you. Our Canadian membership has grown sharply, and many of you have substantial experience in the foreign and multinational side of the industry. And the increasing levels of responsibility held by members of our Society are a real credit to us all.

One area of significant change in recent years is education. Today's college graduates are better educated, particularly in mathematics and sciences, than were the students of a decade before. One can only pray that some day the same will be true of their writing and communications skills. But in mathematics, statistics, and computers, we face a great danger that our education and examination system will fail to keep up with the changes in the undergraduate curriculum, with the result that our profession will be less attractive to the most able students.

As an example, we continue to examine the subject of life contingencies, utilizing a special algebra and notation that were developed decades ago, when storage was cheap and computation was expensive, when addition and subtraction were significantly easier operations than multiplication, and division was to be avoided until there was nothing else that could be done. Yet, as any student will tell you, life contingencies problems are solved on the computer today in an entirely different manner; sometimes, however, the program has an additional routine that calculates the commutation function values so that the older actuaries will feel comfortable with the computer output. This is an area where change is needed in our education and examination system if we are to continue to attract good students. There is, perhaps belatedly, a new textbook under development that addresses the problem. In the compound interest topic, however, technology has really overwhelmed us; you can now buy a calculator for less than $100 that does it all more accurately, without tables, at any rate and term you can express in decimal form, and you don't have to move the bond to the coupon date first.

On the later examinations, our record is better. We hope no semiconductor manufacturer will decide to take the business risk of developing a single chip that is capable of producing an entire workers' compensation experience revision filing, but given the growing interest of corporate risk managers and financial officers, it might not be too farfetched to imagine an individual risk rating chip. Seriously, our later examination material is more challenging and current, due in part to the continued flow of new papers in the *Proceedings*. As in recent years, the discussion paper program has generated additional material that aids the education process. This past year we appointed a new Education Policy Committee, which will undertake the review of several of these questions. I am hopeful that they will be equal to the challenge, and that our education and examination system will attract the able students we need if we are to continue our growth and diversification.

Another result of the changes in recent years is that we are becoming a single actuarial profession, albeit with casualty, life, and pension specialties. The cross training between the life and casualty branches of many of the larger companies and consulting firms is one evidence of this. The growth of the common core to four examinations is another, and it is resulting in a greater overlap in membership between the Casualty Actuarial Society and the Society of Actuaries. The much closer working relationships between the various actuarial organizations on a variety of matters is also evidence of the growing oneness of the profession. It does, however, seem clear that a major reorganization of the profession in North America will not be seriously pursued in the near future. There will, though, be an increasing reliance and interdependence among the various bodies. I view this positively, and believe that all branches of the profession will benefit from the increased cooperation. Our long range goal should continue to be a reorganization that preserves the many advantages that we enjoy with a separate Casualty Actuarial Society. While there is no need to hurry, I suspect that before another decade has passed, it will come about.

The growing oneness of our profession is due in large part to the fact that we are in the process of becoming a *public* profession. This process is occurring in all branches of the profession. In life insurance, an actuarial signature on the statement has been required for several years. In the near future, the life actuary will also be required to take a more public posture on the dividend policy of his company. In pensions, ERISA has required an actuarial signature, and furthermore it specifies that the actuary shall act on behalf of the plan participants and beneficiaries. In the casualty area, we have a new annual statement instruction that requires the signature of a loss reserve specialist, and the only class of signators that is presumed to be qualified is actuaries.

(The history of how that instruction came to be in its present form, rather than some of the undesirable earlier versions, is an excellent example of the cooperation among the several branches of the profession. Both the American Academy and the Society of Actuaries helped to insure that the decision makers in the various insurance departments were well informed. The Academy's Committee on Relations with Accountants also assisted greatly.)

A characteristic of all these signature requirements is that the ultimate client of the signing actuary is no longer his employer, but rather is now the public that relies upon his signature. It is no longer the life insurance company whose statement is being signed, nor the employer whose pension plan is being evaluated, nor the casualty insurer. The actuary's clients, instead, are those policyholders and beneficiaries to whom the insurer or plan is obligated.

An important distinguishing characteristic of any profession is that its members put the interests of their clients ahead of their own; otherwise, it is no profession, but rather a group of skilled businessmen or tradesmen. The actuarial profession has historically been preponderantly a private profession. We worked as employees of insurance companies and their service organizations, such as rating bureaus. The client relationship was clearly to the employer. But the signature examples cited above are transforming the client relationship, because they are creating a public client beyond the employer, whose interests must be put ahead of both our own and our employers' interests.

Not all of our client relationships are becoming public, of course. Many of them can be expected to remain private. To draw an example from another public profession, the independent auditor has a public client relationship when he signs the opinion letter in the stockholders' report; but he still may offer other service and advice to his client on a private basis. So, too, will an actuary have a public client relationship when he signs the statement of opinion on loss reserves. But many other activities, such as advice on a competitive pricing strategy, will continue to be based on a private client relationship.

As we evolve from a private to a public profession, our loyalties and allegiance to the profession will grow relatively stronger. We will look to the profession to set the standards of conduct and to enforce them. Doing so is not an easy process, and we have had and are having our fair share of difficulties with it. I trust that in time we will succeed, if for no other reason than that the alternative of governmentally imposed standards is unacceptable to the majority of us.

Another result of our shifting loyalties is that we increasingly look to the profession to represent our interests to the various other groups with whom we must deal. In prior years, most casualty actuaries looked to their company and its trade association to deal with the NAIC or a legislative body. Today we increasingly look to the CAS and the Academy to fulfill this role where our professional actuarial interests are at stake. Those bodies were instrumental in developing a satisfactory reserve opinion instruction for the annual statement. We also have the Academy testifying in Washington on age and sex discrimination. Similarly, we look increasingly to the actuarial bodies to handle the relationships with accountants.

An important part of becoming a public profession is the establishment and enforcement of standards of conduct. The rest of my remarks are devoted to a discussion of some of the issues in the professional conduct area.

When I was teaching at the University of Michigan, we had a brown bag luncheon group of students that discussed questions of actuarial ethics. Our discussion material was the Guides and Opinions on Professional Conduct, and a series of case studies, usually less than a page in length, each containing an ethical question. In a typical situation, an actuary would find himself in a dilemma, where choosing the ethical solution might result in loss of his job. A lot of our luncheon discussion was spent clarifying the nature of the actuary's duty in the situation, and to whom he owed the duty. While I found the discussions interesting and helpful, it was my observation that ethical questions in the real world seldom arise in neat little one page summaries. Also, I felt that the students attending the luncheon were those who needed the discussion least.

I have also been privileged to participate in two panels on ethics at actuarial meetings. Both utilized short skits to create questions about the ethics of one of the characters. The audience's reaction was then sought on the degree of misconduct and fitting punishment: should the actuary involved be warned, admonished, reprimanded, suspended or expelled? Many of you were present at one of those sessions and may recall that the expressed standards of the audience were quite high. Furthermore, as the skits progressed the severity of the recommended punishment seemed to increase, to the place where it seemed that a sixth alternative might be needed, namely execution.

Before we take too much comfort in that audience reaction, however, I think we may need to look a little more closely at the kinds of issues that receive attention in the professional conduct area. In doing that, it may be helpful to categorize the issues into two main groups: procedural and substantive. In the procedural group, we refer to questions that seek to determine whether the actuary went about his assignment in the correct manner. A significant portion of our professional conduct guides and opinions are devoted to such procedural issues. Was the advertising professional? Did the report contain enough information so that another actuary could appraise the conclusions? Were the appropriate limitations and caveats expressed? These are important questions, and there are unfortunately too many times when the standards are not met. It is my impression that the majority of the questions and complaints reaching the discipline committees of the several actuarial bodies are procedural questions. I believe that our standards in the procedural area are in reasonably good shape, although, as usual, there is room for improvement.

We need to focus on the other category of professional conduct questions, the substantive questions. This is the area that asks not whether the actuary went about his assignment in the proper manner, but rather, did he complete it correctly? Was the actuarial content correct? In this area the questions become much more difficult. What we are really talking about is whether the work is actuarially sound. Here the Guides and Opinions of the American Academy give way to Recommendations and Interpretations. And, lo and behold, they even advise us, in the pension area, to eschew the phrase, "actuarial soundness."

The problem, of course, is in the elusive nature of the concept of actuarial soundness. How do you determine whether an actuary's analysis or recommendation is sound?

One potential course of action is to determine whether a correct method was used. This leads us to the development of standards of practice, which are expressed in Recommendations and Interpretations. In both the life and pension areas, considerable progress has been made; and with the advent of the casualty loss reserve opinion, it is vital that we develop the necessary standards in casualty loss reserving. Our initial efforts will not be perfect, of course.

Such standards would only provide safe harbor, and would not prohibit the use of alternative methods where they are warranted. Some prohibitions may be desirable, or at least a recommendation that an unsatisfactory method not be utilized except under carefully controlled circumstances. This is the negative approach, to be sure, but many of us have stronger beliefs about what is actuarially unsound than we do about what is sound.

This brings me to a point that merits particular attention. The underwriters who complain about our "actuarially unsound" judgments often say that "it may be actuarially unsound, but you have to introduce some business judgment." In my opinion, this assumes a false dichotomy between actuarial soundness and business judgment. The implication is often one of mutual exclusivity. Business judgment and actuarial soundness, in these discussions, become antithetical. More appropriate, I suggest, is the opposite. If it's actuarially sound, then it should be good business judgment; and it clearly is poor business judgment to implement something that is actuarially unsound. This may require a somewhat broader concept of actuarial soundness than some of us have used in the past. Marginal pricing, for example, rather than fully allocated costs, need not be seen as actuarially unsound. Or pricing to protect long-term market share. The training and expertise of the actuary is ideally suited to making such evaluations, and to their necessary quantification.

Part of what the underwriters are complaining about is our tendency to utilize the answer from our model (the black box) as the only actuarially sound estimate. As an illustration of the good job we've done selling the black box, I was recently involved in a hearing where a non-actuary was asked if he had fit a disputed trend line. No, he said, his assistant did. Was the assistant an actuary? No, again, but he used the actuary's machine. And what is an actuary's machine? You punch in the number, was the reply, and out comes the answer.

We need to recognize that the answer from the black box is at best the expected value of some distribution function. We often need to develop some estimate of that distribution function. We also need to recognize that there is a great deal of judgment involved in selecting or designing the black box, and in selecting the data we punch in. All of which needs to be factored into our decisions about actuarial soundness and business judgment.

We also need to work on the improvement of our models. All of us are able to suggest improvements in our own areas of expertise. The flow of new papers, and the outstanding success of our discussion paper program are very healthy aspects of our Society. And it is from this literature that our Recommendations and Interpretations will be distilled. There are some difficult areas ahead, however. We are grappling with the appropriate methods of handling investment income, in both pricing and reserving, but our progress has been slow, and the environment, with its increasing investment returns, is creating additional pressure. The area of risk classification is another difficult area where we are making progress, but perhaps too slowly.

The most disturbing development, however, may be found in a paper presented at the International Actuarial meetings this summer in Switzerland. Authored by Professor Jewell of the University of California at Berkeley, a past participant at our own meetings, the paper surveys the state of the art in actuarial models (or black boxes) and suggests that we are on the threshold of a major period of rapid change, wherein many of our models will be discarded because they no longer are valid in our changing world. The models that replace the old ones, suggests Professor Jewell, will be sounder and will draw on recent developments in statistics and management science, and will more effectively utilize the new computer capabilities. Professor Jewell's paper is most provocative, and I commend it to your attention. His challenge to the profession is very basic, and I hope that we will be equal to it.

All of these potential improvements in our models are vital. Many of them will help in our difficult task of developing standards of practice as they relate to assessments of actuarial soundness. It will not be easy, but it is necessary if we are to be successful as a public profession.

I would like to close with a story about a conversation I had several months ago with Haeworth Robertson, who was formerly the Chief Actuary of the Social Security Administration. I was lamenting the actuarial profession's lack of influence in the economic affairs of the nation. Considering the size of the asset pools of the casualty insurers, the life insurers and the pension plans for which actuaries serve as stewards, the influence seems small. Haeworth listened quietly, and then observed that my idea was reasonable, but that I had the wrong side of the balance sheet. Considering the liabilities which the actuaries are responsible for evaluating, the influence is clearly too small. As we become a more public profession, our influence will grow. Our challenge is to develop the standards of professional conduct that will enable us to soundly evaluate those liabilities.

ESTIMATING CASUALTY INSURANCE LOSS AMOUNT DISTRIBUTIONS

GARY PATRIK

I. INTRODUCTION

It is often necessary to estimate probability distributions to describe the loss processes covered by insurance contracts. For example, in order that the premium charged for a particular contract be correct according to any reasonable premium calculation principle, it must be based upon the underlying loss process for the contract. Practically, it is impossible to know the true underlying loss process, but a reasonably accurate estimate of this process can provide the basis for a reasonably accurate premium. One may discuss the loss process for an individual insured with a single coverage provided by a single contract, or for a group of insureds with multiple coverages provided by many contracts.

This paper considers the estimation of individual loss amount (severity) distributions. The term "loss amount" is used to signify the total settlement value of a single loss event. The term "contract" will be used to define any particular situation: the context should make clear whether individual or group, single or multiple coverages and contracts are being discussed.

I assume that for a particular contract at any point in time, there exists a probability distribution governing the loss amount for any loss event occurring at that time. There may be different distributions at different times, and they all may be interrelated and mutually dependent upon the number of events and their times of occurrence (Bühlmann (1970), p. 54ff). The distribution of loss amounts over a contract period is a function of the point-of-time distributions, the number of events and their timings.

This paper concentrates upon probability model-building and statistical techniques for estimating and testing the model parameters. I describe a general procedure for selecting a "best" parameterized model based upon loss amount data. This solves only part of a broader problem, which is to estimate loss amount distributions for future coverage periods or (future) final-valued loss amount distributions for past coverage periods where the losses are not all settled or even known. To solve this broader problem it is necessary to specify models of the overall insurance loss processes, defining how the future relates to the past and how the individual insured relates to the whole insurance portfolio. These models may be very simple or very complex, very loose or very mathematically precise, but we implicitly create them whenever we specify these future/past, individual/whole relationships. I will argue that a key component of the broader problem's solution is the use of probability models for loss amount distributions. Although this paper ignores some of the broader issues such as trend, loss development, population structure (classification), etc., I believe that precise model-building and testing would also resolve many of the problems connected with these.

This paper extends the work of Weissner (1978), who estimated report lag distributions from truncated data, to the estimation of loss amount distributions using censored and truncated data. The particular techniques were developed both for the estimation of commercial liability increased limits factors and for excess-of-loss reinsurance pricing. I would like to thank the following people for their contributions to this paper: Charles Hachemeister, Russell John, Mark Kleiman, Aaron Tenenbein, and Edward Weissner.

II. MATHEMATICAL MODELS

There are compelling reasons to use mathematical models to describe insurance loss amount distributions. In general, a model is a simplified, idealized interpretation of reality. A mathematical model describes the behavior of a real system by use of mathematical symbols, functions and equations. All science is a continuing process of model-building and model-testing (Kuhn (1970)). Wagner (1969) describes the purpose of a model as follows:

> Constructing a model helps you put the complexities and possible uncertainties attending a decision-making problem into a logical framework amenable to comprehensive analysis. Such a model clarifies the decision alternatives and their anticipated effects . . . In short, the model is a vehicle for arriving at a well-structured view of reality. (p. 10)

But we must be careful: "The scientist who uses models in his reflections must always remain alert to the possibility that his questions are inspired only by properties of the model, having nothing directly to do with the subject matter itself" (Hanson (1971), p. 79). Because of this, some actuaries believe that we should not attempt to describe real loss amount distributions with mathematical models, but rather we should work with the raw data. This would be fine if all we wished to do was discuss sample realizations of historical loss amount distributions. However, as has been argued in the introduction, whenever we want to extend or compare our various information, we must use some kind of model. Since this is the case in insurance work when we want to predict future possibilities, it seems clear that we should use mathematical models to describe loss amount distributions.

However, we should not believe that we can build models which will completely describe reality.[1] What we can hope for instead is to discover models which describe the salient features of a real system with some degree of accuracy. Whenever we specify a model, we should test it to see if it adequately describes the real system it is meant to describe.

I suggest that the type of model we should construct for a given loss amount distribution should be a probability model with only as many parameters as warranted by the data against which we will be testing it. Too few parameters and it is unlikely to adequately describe any given loss amount distribution; too many parameters and it becomes difficult to understand, difficult to work with and difficult to specify and test. Some of the advantages of such a ''parsimonious'' probability model are as follows:

1. It can be easily understood. Its main characteristics can be clearly described and measured.
2. It can be easily manipulated. For instance, loss development and inflationary trends *might* be accounted for simply by adjusting a few parameters.
3. It can be easily extended to more general cases or to analogous cases in a consistent manner. For example, some knowledge of the distribution of loss amounts up to certain policy limits *might* indicate something about the tail of the unbounded distribution. Also, we might expect that the loss amount distributions for similar lines of insurance would have the same general form.

[1] Gödel's proof that any axiomatic system for the natural numbers must be incomplete (Gödel (1931), (1934) and Edwards (1967)) should lead us to suspect that if an abstract idealized mathematical system cannot be completely described, then any real system must be too complicated to be completely described by a model.

4. It can be easily restricted to particular cases in a consistent manner. For example, the distribution of Owners, Landlords and Tenants liability loss amounts for small grocery stores in Kansas *might* be a special case of the general countrywide distribution of Owners, Landlords and Tenants liability loss amounts. Also, the distribution of loss amounts for any particular policy limit *might* be a restriction of some general distribution of unlimited loss amounts.

5. It can be tested using explicit statistical methods. For example, the fit of any probability model to any set of loss amount data can be tested via the Kolmogorov-Smirnov statistic or by other statistical tests.

6. It can be used to compare or combine various contracts or sets of data. For example, for a given set of contracts, the probability models for various years can be explicitly compared in order to determine the effects of inflation. Or perhaps the relationship of the probability models for different contracts can be tested to see if it would be better to specify a single "credible" probability model for the group.

Many possible forms of probability models for loss amount distributions have appeared in the literature. The Bibliography is a fairly comprehensive listing of relevant English-language papers and books; Johnson and Kotz (1970) is especially useful.

The purpose of this paper is to describe a general procedure for selecting an adequate model for any particular loss amount distribution; I do not advocate any particular model. However, for illustration I will use the Pareto distribution of the second kind, also called the Lomax or Pearson Type VI distribution (Johnson and Kotz (1970), p. 233ff). Its cumulative distribution function (c.d.f.) for the random variable X is defined by:

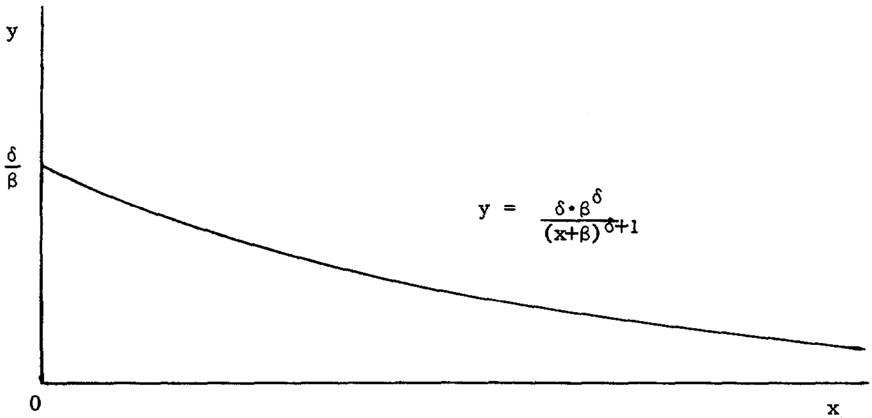
$$F(x \mid \beta, \delta) = \text{Prob}\,[X \leq x] = 1 - \left(\frac{\beta}{x + \beta}\right)^{\delta} \qquad \text{for all } x \geq 0 \qquad (2.1)$$

where $\beta > 0$, $\delta > 0$ are parameters.

This Pareto distribution is very easy to work with; a catalog of its main properties is given in Appendix A. The graph of its probability density function (p.d.f.) is shown in Figure 2.1.

FIGURE 2.1

PARETO DENSITY FUNCTION



In a slightly more complex form (to be discussed later), this model has been very useful both to myself and to the Insurance Services Office Increased Limits Subcommittee. It appears to more accurately account for large liability loss amounts than does the lognormal or other c.d.f.'s we have tested. Other investigators, such as Benckert and Sternberg (1957), Benktander and Segerdahl (1960), Mandelbrot (1964), Benckert and Jung (1974), Ramachandran (1974), and Shpilberg (1977) have found that this or the usual form of the Pareto describes fire loss data fairly well.

We will next consider how to estimate parameters for any probability model.

III. MAXIMUM LIKELIHOOD ESTIMATION

Suppose that we have postulated a probability model, such as the Pareto distribution (2.1), to describe a given loss amount distribution. The next step in our procedure should be to estimate values for the parameters of the model.

Suppose that we have a random sample of loss amounts and let us assume that they are properly adjusted to the level of the (future) distribution we are interested in. This is a strong assumption since, as already mentioned, we never see proper data because of the problems of individual loss reserve development, IBNR, and time and population differences. However, let us start here; later we will discuss a simple way of handling the time (trend) problem. We can estimate the parameters first via the method of maximum likelihood.

The situation is as follows:

1. We are given a sample $x_1, x_2, \ldots, x_n$ which we believe to be distributed according to a c.d.f. of a certain form, $F(x \mid \theta) = \text{Prob}\,[X \le x]$, where X designates the random variable and $\theta = \{\theta_1, \ldots, \theta_r\}$ designates the indeterminate general parameter (actually, a set of individual component parameters) for the c.d.f.
2. We want to find a paramter $\hat{\theta}$ such that the model with $\hat{\theta}$ as the value of θ "best" describes the data.

The method of maximum likelihood chooses that parameter $\hat{\theta}$ which maximizes the likelihood function:

$$L(\theta) = \prod_{i=1}^{n} f(x_i \mid \theta) \tag{3.1}$$

where $f(x_i \mid \theta)$ is either the probability of x_i given θ or the p.d.f. evaluated at x given θ, depending upon whether or not the distribution function has a jump or is absolutely continuous at x_i.

Thus, $\hat{\theta}$ is "best" in the sense of being "most likely" given $x_1, x_2, \ldots, x_n$.

For example, if $F(x \mid \theta)$ is our Pareto c.d.f. (2.1), then $\theta = \{\beta, \delta\}$ and

$$L(\beta, \delta) = \prod_{i=1}^{n} \frac{\delta \cdot \beta^{\delta}}{(x_i + \beta)^{\delta+1}} \tag{3.2}$$

Maximum likelihood estimation is a standard statistical method and much is known in general about the properties of maximum likelihood estimates (MLE's). Kendall and Stuart (1967), Dudewicz (1976) and many other standard statistical texts discuss the general MLE properties, and many papers discuss particular examples. Appendix B outlines the general properties and describes how to calculate the estimates in the case of r parameters. Essentially, we can expect that for large samples, MLE's will be more accurate than any other estimates. Because of this it is surprising that the maximum likelihood method has not been used more often by actuaries. I believe the reason for this is that MLE's are usually difficult to calculate—we must have detailed data and usually we must use some fancy iterative technique to approximate the MLE's. However, with modern computers the method is much easier; even mini-computers can be programmed to approximate MLE's very quickly for many of the standard probability models.

The standard procedure is to set the first partial derivatives of the natural logarithm of (3.1) equal to zero. These first partials are of the form:

$$\frac{\partial \log L(\theta)}{\partial \theta_j} = \sum_{i=1}^{n} f(x_i \mid \theta)^{-1} \cdot \left(\frac{\partial f(x_i \mid \theta)}{\partial \theta_j}\right) \tag{3.3}$$

for $j = 1, 2, \ldots, r$

Setting these equal to zero gives us a system of r equations in the r unknowns $\theta_1, \ldots, \theta_r$ which we can solve by the Newton-Raphson iterative technique outlined in Appendix B.

We will consider a few examples in this paper to illustrate that MLE's can be much better than the standard method-of-moments estimates most often used in the actuarial literature. These are not presented as proofs; the proofs are in the statistical texts. These are simply illustrations.

Let us begin with the simple case of our Pareto c.d.f. (2.1). Suppose the data are the set of loss amounts listed in Appendix C (column 1); these 200 values are computer-generated pseudo-random Paretian values with parameters $\beta = 25{,}000$ and $\delta = 1.5$. These are realistic parameters for commercial liability losses. We can easily compare the MLE's and the method-of-moments estimates to these values. To compute the MLE's we must maximize the likelihood function (3.2). It is equivalent to maximize the loglikelihood:

$$\begin{aligned} \log L &= \log L(\beta, \delta) \\ &= n \cdot \log \delta + n\delta \cdot \log \beta - (\delta + 1)\sum_{i=1}^{n} \log (x_i + \beta) \end{aligned} \tag{3.4}$$

If $\log L$ has second partial derivatives with respect to β and δ existing throughout its range, then a necessary condition for a point $(\hat{\beta}, \hat{\delta})$ to maximize $\log L$ is that the first partials evaluated at $(\hat{\beta}, \hat{\delta})$ be equal to zero. The first partials are:

$$\frac{\partial \log L}{\partial \beta} = n\delta\beta^{-1} - (\delta + 1) \cdot \sum_{i=1}^{n} (x_i + \beta)^{-1} \tag{3.5}$$

$$\frac{\partial \log L}{\partial \delta} = n\delta^{-1} + n \cdot \log \beta - \sum_{i=1}^{n} \log(x_i + \beta) \tag{3.6}$$

The second partials are:

$$\frac{\partial^2 \log L}{\partial \beta^2} = -n\delta \cdot \beta^{-2} + (\delta + 1) \cdot \sum_{i=1}^{n} (x_i + \beta)^{-2} \tag{3.7}$$

$$\frac{\partial^2 \log L}{\partial \delta \cdot \partial \beta} = \frac{\partial^2 \log L}{\partial \beta \cdot \partial \delta} = n\beta^{-1} - \sum_{i=1}^{n} (x_i + \beta)^{-1} \tag{3.8}$$

$$\frac{\partial^2 \log L}{\partial \delta^2} = -n\delta^{-2} \tag{3.9}$$

Since $\beta > 0$ and $\delta > 0$, the second partials exist throughout the range of $\log L$. Thus, setting (3.5) = 0 and (3.6) = 0 defines a point $(\hat{\beta}, \hat{\delta})$ which may maximize $\log L$. We should check to be sure that $(\hat{\beta}, \hat{\delta})$ indeed gives a maximum (see Appendix B). The equations can be solved by a simple iterative technique such as the Newton-Raphson technique (see Appendix B).

For our Pareto example, the calculated MLE's and the implied tail probabilities for amounts greater than 100,000 and 1,000,000 (calculated via (2.1)) are displayed in Table 3.1.

TABLE 3.1

PARETO

	β	δ	Prob [$X > 100{,}000$]	Prob [$X > 1{,}000{,}000$]
Model	25,000	1.500	.089	.004
MLE	26,297	1.586	.083	.003
Method-of-Moments	56,042	2.371	.088	.001

The corresponding method-of-moments estimates β', δ' are obtained by solving the two equations:

$$\frac{\beta'}{\delta' - 1} = \text{sample mean} = 40{,}880 \tag{3.10}$$

$$\frac{\delta' \cdot \beta'^2}{(\delta' - 2) \cdot (\delta' - 1)^2} = \text{sample variance} = 10.683 \times 10^9$$

The method-of-moments implied probability that $X > 100{,}000$ is close to the true value, but the implied tail probability beyond 1,000,000 is understated.

One property of the Pareto distribution is that any non-central moment $E[X^k]$ for the unbounded c.d.f. exists only if $k < \delta$ (see Appendix A). The method-of-moments estimates, by assuming the existence of the variance and thus of $E[X^2]$, automatically forces $\delta' > 2$. Consequently, the method-of-moments estimates based upon Pareto data with the true $\delta < 2$ will always produce an estimated c.d.f. with relatively fewer large losses than the true model. Let us note here that values of δ less than 2 are typical for liability loss amount data.

Next we will consider how to test a probability model with estimated parameters against the sample data, and we will discuss how we may select final models and parameter values.

IV. MODEL TESTING AND PARAMETER SELECTION

Now suppose that we have postulated a probability model to describe a particular loss amount distribution and from sample data we have calculated MLE's of the parameters. The next step in our procedure should be to test the model and perhaps to modify the parameters for other considerations, such as credibility.

We know that any model cannot be a perfect descriptor of reality, so we should only be looking for one that is good enough for the use to which it is to be put. I suggest that the following two tests are useful for determining whether or not a particular probability model with specified parameters adequately describes a random sample from a particular loss amount distribution:

Test 1: The Kolmogorov-Smirnov Test

This is a standard statistical test that attempts to decide whether or not a given sample was generated according to a specified c.d.f. See Massey (1951), Kendall and Stuart (1967) or Conover (1971) for good general discussions of the test. The test statistic is the maximum absolute difference between the specified c.d.f. and the sample c.d.f. That is, the test statistic D_n is defined by:

$$D_n = \max \{|F(x_i^- \mid \theta) - S_n(x_i^-)|, |F(x_i^+ \mid \theta) - S_n(x_i^+)|\} \tag{4.1}$$

where $\{x_i\}$ is the ordered sample $x_1 \leq x_2 \leq \cdots \leq x_n$

$S_n(x_i^-) = \dfrac{i-1}{n}$; the value of the sample c.d.f. before the jump at x_i

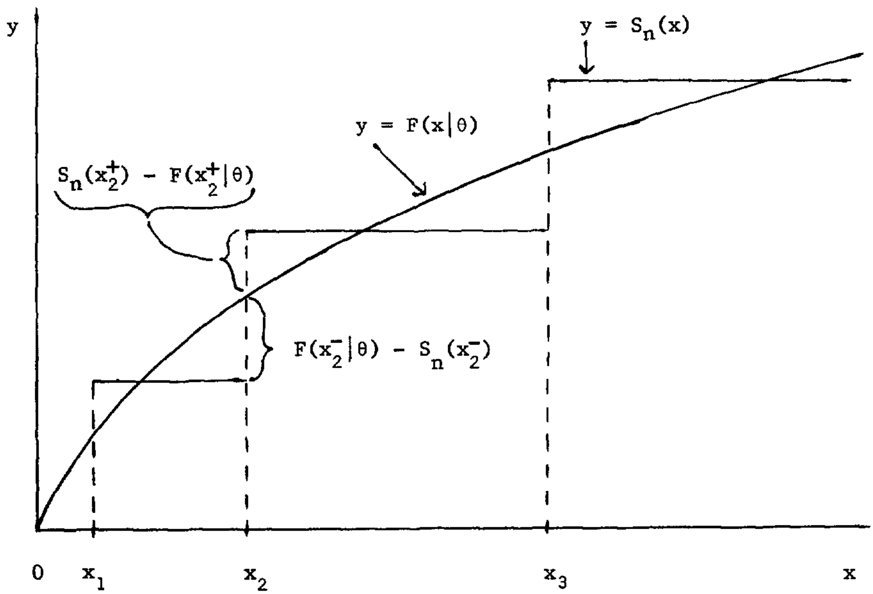
$S_n(x_i^+) = \dfrac{i}{n}$; the value after the jump

$F(x_i^- \mid \theta) = \lim_{x \to x_i^-} F(x \mid \theta)$; the limit from the left of the values of the specified c.d.f.

$F(x_i^+ \mid \theta) = \lim_{x \to x_i^+} F(x \mid \theta)$; the limit from the right.

FIGURE 4.1

K-S TEST



For any pre-specified confidence level, one rejects the hypothesis that the sample was generated according to a specified c.d.f. if the test statistic is greater than some critical value. Appendix C displays the K-S test of our Pareto c.d.f. with MLE parameters (Table 3.1) against the sample Pareto data. The K-S test statistic is .032 (Appendix C). Thus, using K-S test critical values from one of the aforementioned texts, we would not reject the hypothesis that the sample was generated by the specified Pareto c.d.f., if we were testing at a 5% significance level.[2]

[2] K-S test critical values should be smaller when the parameters are estimated from the sample. For example, see Lilliefors (1969) and Dropkin (1964).

The K-S test is more powerful than the chi-square test, since it takes into account the natural order of the data while the chi-square test ignores this order. See Massey (1951) and Conover (1971) for comparisons of the two tests. In fact, a problem with the K-S test is that in practice it seems to be too powerful for testing c.d.f.'s of loss amount distributions. At a 5% significance level it rejects any probability model yet tried for liability loss amount data. We will see an example of this later. Of course, the data we have may not be truly representative of the underlying loss amount distributions because of the previously mentioned problems (development, IBNR, etc.). I believe that we should continue to use the K-S test because its properties are well known, and the value of the K-S statistics can help us decide among different c.d.f.'s. However, we should have another test, to use in conjunction with the K-S test, which will not reject every probability model. The following test gives much useful information to an actuary.

Test 2: Expected Value Comparison (EVC) Test

This is a test of the expected value functions of the specified c.d.f. and the sample c.d.f. Define the following functions:

$$G(x \mid \theta) = \int_0^x X \cdot dF(X \mid \theta) + x \cdot (1 - F(x \mid \theta)) \tag{4.2}$$

$$G_n(x) = \frac{1}{n}\left\{\sum_{x_i \le x} x_i + x \cdot (\text{number of } x_i > x)\right\} \tag{4.3}$$

A suitable EVC test statistic might be the vector of values:

$$\left\{\frac{G(x_i \mid \theta) - G_n(x_i)}{G(x_i \mid \theta)}\right\} \tag{4.4}$$

where $x_1 \le x_2 \le \cdots \le x_n$ is again the sample.

This test statistic is simply the relative difference of the expected value functions $G(x \mid \theta)$ and $G_n(x)$ at each sample point. It is similar to the K-S test *vector* (not just a maximum). I don't know of any statistical work which investigates the properties of this statistic, but it is certainly a good statistic for actuaries who are interested in losses per layer. Appendix C (last column) displays the EVC statistic (as a percentage) for our Pareto c.d.f. with MLE parameters (Table 3.1) against the sample Pareto data. Note that for these 200 data points, the EVC statistic changes sign ten times and the largest absolute value is .0194 (−1.94%).

A reasonable decision rule might be: choose the probability model which has a low K-S test statistic and also has an EVC statistic which is close to the 0-vector and which has random-looking sign changes. We will see applications of this rule in the following sections.

After deciding which c.d.f. best describes a given sample of loss amounts, we should have further decision criteria which use additional information and knowledge to judge the reasonableness of the particular model. We want to know, for instance, that the particular model does not contradict our general knowledge of what loss amount distributions should look like based upon analogous data. Of the six advantages of mathematical models listed in section II, three have to do with extending to, restricting to, or comparing analogous cases. If we had a good broader model for how loss amount probability models should differ for different but similar contracts, we could test any particular c.d.f. against the general criteria. This is a deep problem in the realm of credibility theory, and it is certainly beyond the scope of this paper. However, in practice, since we actuaries do not yet have a comprehensive credibility model, we all use "actuarial judgment" to specify other pieces of the broader model.

Next we will consider some practical modeling and estimation problems and revise our basic probability model to handle them.

V. MODELING AND ESTIMATION PROBLEM 1: POLICY LIMITS BOUND THE LOSS AMOUNT DATA

For most lines of insurance, loss amounts are inherently bounded by policy limits. If the parameters of an unbounded c.d.f. are estimated from bounded data, then the c.d.f. with the estimated parameters may greatly understate the true tail of the loss amount distribution. This happens because the unbounded c.d.f. does not expect the tail of the loss amount distribution to be cut off by the policy limit.

For example, suppose our Pareto data in Appendix C (column 1) is limited to 200,000; we will then have 7 data points limited to the value 200,000. If the parameters are estimated by the method-of-moments formulas (3.10), we obtain the results displayed in Table 5.1. The "censored" MLE results will be derived later.

TABLE 5.1

PARETO (DATA LIMITED TO 200,000)

	β	δ	Prob $[X > 100{,}000]$	Prob $[X > 1{,}000{,}000]$
Model	25,000	1.500	.089	.004
Method-of-Moments	96,773	3.984	.059	.0001
"Censored" MLE	25,119	1.533	.085	.003

Since the lognormal model has been used so often for loss amount distributions, I thought that a lognormal example would also be instructive here. Appendix D lists 200 computer-generated pseudo-random lognormal values with parameters and tail probabilities given by $\mu = 9.0$ and $\sigma = 2.0$, where the parameterization used is the usual one with:

$$\text{Prob } [X \leq x] = \phi\left(\frac{\log x - \mu}{\sigma}\right) \tag{5.1}$$

where $\phi(y)$ is the normal (0, 1) c.d.f.

The standard method-of-moments estimates μ', σ' are obtained from the data limited to 200,000 by solving the two equations:

$$\exp\left\{\mu' + \frac{\sigma'^2}{2}\right\} = \text{sample mean} = 28{,}166 \tag{5.2}$$

$$(\exp\{\sigma'^2\} - 1) \cdot (\text{mean})^2 = \text{sample variance} = 2.204 \times 10^9$$

Solving these we obtain the results displayed in Table 5.2 as "Method-of-Moments I."

TABLE 5.2

LOGNORMAL (DATA LIMITED TO 200,000)

	μ	σ	Prob $[X > 100{,}000]$	Prob $[X > 1{,}000{,}000]$
Model	9.000	2.000	.104	.008
Method-of-Moments I	9.581	1.153	.047	.0001
Method-of-Moments II	8.950	1.897	.088	.005
"Censored" MLE	8.980	1.973	.100	.007

Alternative method-of-moments estimates μ'', σ'' can be obtained by considering the natural logarithms of the data limited to 200,000 to be normally distributed and taking the usual method-of-moments estimates for the normal distribution, e.g., μ'' = mean of the logs, etc. The results are displayed in Table 5.2 as "Method-of-Moments II." The "censored" MLE results will be derived later.

The c.d.f.'s with the method-of-moments estimated parameters underestimate the tail probabilities. These examples are important because this method has been used exactly as shown here so often in actuarial work.

Thus, our probability model must account for the effect of policy limits. When policy limits have been recognized in the actuarial literature, there seems to be a standardized model for liability losses. See Benktander and Segerdahl (1960), Lange (1969), Miccolis (1977) among others. They postulate that for liability loss amounts, for each particular type of business and type of coverage, there exists a unique underlying probability law dictating the distribution of loss amounts in the absence of policy limits; call this implied c.d.f. $F(x \mid \theta)$. The standard model hypothesizes that any policy limit c acts on the losses as a "censor" in the following sense: any loss which naturally would be greater than c is artificially limited to amount c. The bounded c.d.f. $F(x \mid \theta; c)$ for policy limit c can be written:

$$F(x \mid \theta; c) = \begin{cases} F(x \mid \theta) & \text{if } x < c \\ 1 & \text{if } x \geq c \end{cases} \tag{5.3}$$

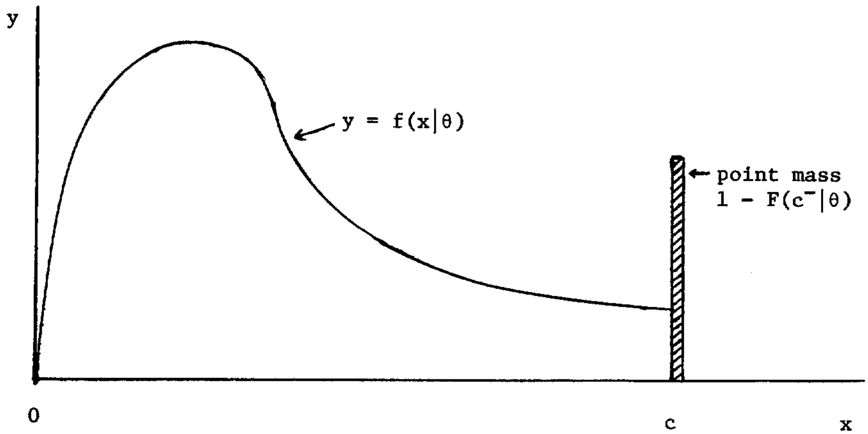
The probability function $f(x \mid \theta; c)$ is given by $f(x \mid \theta; c) = f(x \mid \theta)$ for $x < c$, and there is a discrete probability point mass $f(c \mid \theta; c) = 1 - F(c^- \mid \theta)$ at the point c, where $F(c^- \mid \theta) = \lim_{x \to c^-} F(x \mid \theta)$. The graph of $f(x \mid \theta; c)$ may be illustrated loosely by Figure 5.1.

There is no standard model for property loss amount distributions. See Benckert and Sternberg (1957), Bickerstaff (1972), Benckert (1962), Benckert and Jung (1974), Shpilberg (1977) among others. Many investigators have studied the distribution of the individual loss amount ratioed to the policy face amount for particular groups of fire insurance contracts. They have proposed either an inherently bounded c.d.f. such as the Beta distribution or some kind of censored model similar to the standard liability model.

We proceed with the censored model (5.3).

FIGURE 5.1

GRAPH OF $y = f(x|\theta; c)$



Suppose that $x_1, x_2, \ldots, x_n$ is a sample of loss amounts which we believe to be distributed according to a censored c.d.f. of the form (5.3), where c is the policy limit. We may reorder the x_i's so that the first $n - m$ are less than c and the remaining m are equal to c, i.e.,

$$x_1 \le x_2 \le \cdots \le x_{n-m} < c, \; x_{n-m+1} = \cdots = x_n = c.$$

In this case, the likelihood function for the censored c.d.f. (5.3) can be written:

$$L(\theta; c) = \left\{ \prod_{i=1}^{n-m} f(x_i \mid \theta) \right\} \cdot \{1 - F(c^- \mid \theta)\}^m \tag{5.4}$$

We can continue from here to find MLE's for the parameters by using the standard techniques discussed in Appendix B. Note that we will be solving a system of equations of the form:

$$\frac{\partial \log L}{\partial \theta_j} = \sum_{i=1}^{n-m} f(x_i \mid \theta)^{-1} \cdot \left(\frac{\partial f(x_i \mid \theta)}{\partial \theta_j} \right) - m \cdot \{1 - F(c^- \mid \theta)\}^{-1} \cdot \left(\frac{\partial F(c^- \mid \theta)}{\partial \theta_j} \right) \tag{5.5}$$

for $j = 1, 2, \ldots, r$

This is simply (3.3) with the addition of a censorship term. (Remember that n in (3.3) corresponds to $n - m$ in (5.5).) *Note that the resulting MLE's are the parameters for the unbounded c.d.f.* $F(x \mid \theta)$.

We can illustrate this with our Pareto example. The Pareto censored c.d.f. is given by:

$$F(x \mid \beta, \delta; c) = \begin{cases} 1 - [\beta/(x + \beta)]^{\delta} & \text{for } 0 \leq x < c \\ 1 & \text{for } x \geq c \end{cases} \tag{5.6}$$

where $\beta > 0$, $\delta > 0$

The loglikelihood function is then:

$$\log L = (n - m) \cdot \log \delta + n\delta \cdot \log \beta - m\delta \cdot \log (c + \beta) - (\delta + 1) \cdot \sum_{i=1}^{n-m} \log (x_i + \beta) \tag{5.7}$$

Note that this is simply (3.4) with the addition of the censorship term: $m\delta \cdot \log \beta - m\delta \cdot \log (c + \beta)$. (To see this, note that n in (3.4) becomes $n - m$ in (5.7).) We can compute the first and second partials as before and use a Newton-Raphson iteration to approximate the MLE's. For our Pareto data in Appendix C (column 1) censored to 200,000, we obtain MLE's and estimated tail probabilities displayed in Table 5.1 as "censored" MLE. These are quite different from the method-of-moments estimates in Table 5.1 and are quite close to the true values.

The MLE's and estimated tail probabilities for our lognormal data in Appendix D censored at 200,000 are displayed in Table 5.2 as "censored" MLE. Again, these are quite different from the method-of-moments estimates in Table 5.2 and are quite close to the true values. Of course, we could compute correct method-of-moments estimates accounting for the policy limit censorship. But the equations that must be solved are much more complicated than the general equation (5.5).

Insurance loss amount data are usually from a mixture of contracts with different policy limits. Since the standard liability model postulates a single underlying distribution $F(x \mid \theta)$ for unbounded loss amounts for a particular type of business at a particular time, the data from all policy limits should be used simultaneously to estimate the model for this distribution. The maximum likelihood method allows us to do this very easily.

Suppose that $\{x_{ki}\}$ is a sample of loss amounts which we believe to be distributed according to the same unbounded c.d.f. except that for each k, the x_{ki}'s are censored by policy limit c_k. Again, we may reorder the x_{ki}'s so that for each k, the first $n_k - m_k$ x_{ki}'s are strictly less than c_k and the remaining m_k are equal to c_k. In this case, the general likelihood function for the total sample is simply the product of the likelihood functions for each policy limit:

$$L(\theta; c_1, \ldots, c_s) = \prod_{k=1}^{s} L(\theta; c_k) \tag{5.8}$$

Since the general likelihood function is a product, the loglikelihood and all its partial derivatives will be sums of the individual censored components. Writing it all out results in equations terrifying to behold, but whose solution is really quite straightforward in practice. For example, (5.5) becomes:

$$\frac{\partial \log L}{\partial \theta_j} = \sum_{k=1}^{s} \left\{ \sum_{i=1}^{n_k - m_k} f(x_i \mid \theta)^{-1} \cdot \left(\frac{\partial f(x_i \mid \theta)}{\partial \theta_j} \right) - m_k (1 - F(c_k^- \mid \theta))^{-1} \cdot \left(\frac{\partial F(c_k^- \mid \theta)}{\partial \theta_j} \right) \right\} \tag{5.9}$$

for $j = 1, 2, \ldots, r$

There may be a problem with the MLE's in this general censored case. Since the general likelihood function (5.8) is a product of likelihood functions with respect to different c.d.f.'s (because of different censorship points), the properties discussed in Appendix B may not hold. The theoretical results on the properties of MLE's have been derived for a likelihood function with respect to a single c.d.f. I have not seen any derivation of the properties of MLE's for the general likelihood function (5.8). However, in practice thus far we have noticed no strange behavior of the resulting $\hat{\theta}$.

We will see an example of data from mixed policy limits in section VIII when we consider the problem of having data from many dates of occurrence.

VI. MODELING AND ESTIMATION PROBLEM 2: THERE ARE MORE SMALL LOSSES THAN CAN BE PREDICTED BY THE USUAL MODELS

A problem encountered when we attempt to describe liability loss amount data by one of the usual c.d.f.'s, such as the Pareto or lognormal, is that there are more small loss amounts than the model predicts. Appendix E, Part 1, displays an ISO Owners, Landlords and Tenants bodily injury liability loss amount data summary for policy limit $300,000 for policy year 1976 evaluated as of March 31, 1978. The number of losses below $8,000 is more than predicted by any model that I or the ISO Increased Limits Subcommittee have tried. Forcing one of these probability models to fit the total distribution will cause the model to greatly understate the potential tail.

To account for the many small "nuisance claims," Hewitt and Lefkowitz (1979) worked with mixed c.d.f.'s such as:

$$\text{Prob}\,[X \leq x] = p \cdot G(x \mid \theta_G) + (1 - p) \cdot H(x \mid \theta_H) \tag{6.1}$$

where $0 \leq p \leq 1$

$G(x \mid \theta_G)$ is gamma with parameter θ_G
$H(x \mid \theta_H)$ is loggamma or lognormal with parameter θ_H

The rationale for this model is that there may be two distinct loss amount generating processes, where some losses are "regular" large losses and may be described by a c.d.f. $H(x \mid \theta_H)$, while others are "nuisance" small losses which may be described by a c.d.f. $G(x \mid \theta_G)$.

For a sample $x_1, \ldots, x_n$ generated according to this loss amount distribution, the loglikelihood function is:

$$\log L = \sum_{i=1}^{n} \log \{p \cdot g(x_i \mid \theta_G) + (1 - p) \cdot h(x_i \mid \theta_H)\} \tag{6.2}$$

where g and h are the relevant p.d.f.'s.

We can certainly calculate MLE's for this model, although one can see that the equations will be complicated and that we will have many parameter components θ_G, θ_H and p to consider simultaneously.

A much simpler alternative model may be used if we are primarily interested in the large losses and thus want to concentrate upon estimating the tail of the loss amount distribution. This model assumes that the overall distribution splits into two distinct pieces above and below some truncation point t. The overall c.d.f. $F(x)$ can be written as follows:

$$F(x \mid \theta_G, \theta_H, t, p) = \tag{6.3}$$

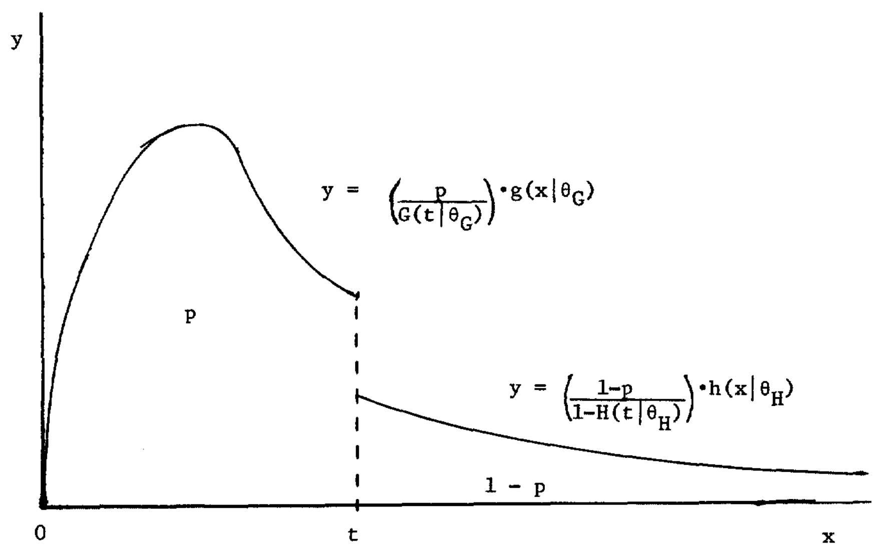
$$\begin{cases} \left(\dfrac{p}{G(t \mid \theta_G)}\right) \cdot G(x \mid \theta_G) & \text{for } x \le t \\ p + \left(\dfrac{1-p}{1 - H(t \mid \theta_H)}\right) \cdot (H(x \mid \theta_H) - H(t \mid \theta_H)) & \text{for } x > t \end{cases}$$

where $0 \le p \le 1$ and $t \ge 0$
$G(x \mid \theta_G)$ = small loss amount c.d.f.
$H(x \mid \theta_H)$ = large loss amount c.d.f.

If $g(x \mid \theta_G)$ and $h(x \mid \theta_H)$ are the respective p.d.f.'s, the graph of the overall p.d.f. $f(x \mid \theta_G, \theta_H, t, p)$ is shown in Figure 6.1.

FIGURE 6.1

GRAPH OF $y = f(x|\theta_G, \theta_H, t, p)$



The picture is intentionally drawn so that the graphs do not match up at t, i.e., $f(x \mid \theta_G, \theta_H, t, p)$ is not necessarily continuous at t. Unless we are pricing small deductibles, we are primarily interested in $H(x \mid \theta_H)$ and need only gross estimates of $G(x \mid \theta_G)$. In this case, there is no need to try to force continuity at t. For commercial liability data, good values for t seem to lie between \$2,000 and \$8,000.

In practice, it is convenient to specify a value for t so that it is no longer an indeterminant parameter. Maximum likelihood estimation for this model then becomes very simple, because the parameters θ_G, θ_H and p may all be estimated separately. That is, the following lemma holds:

Lemma: Assume that for the model (6.3), t is fixed and θ_G and θ_H are disjoint sets.[3] Suppose that $x_1, \ldots, x_n$ is a sample generated according to the model and $x_i \leq t$ for $i = 1, 2, \ldots, m$ and $x_i > t$ for $i = m + 1, m + 2, \ldots, n$. Then:

1. $\hat{p} = (m/n)$ is the MLE for p
2. The MLE's $\hat{\theta}_G$ and $\hat{\theta}_H$ are obtained independently from the subsamples $\{x_1, \ldots, x_m\}$ and $\{x_{m+1}, \ldots, x_n\}$ respectively.

The proof of this lemma is obvious once we write out the loglikelihood function:

$$\log L(\theta_G, \theta_H, p) = \sum_{i=1}^{n} \log f(x_i \mid \theta_G, \theta_H, p) \tag{6.4}$$

$$= \sum_{i=1}^{m} \log \left\{ \left(\frac{p}{G(t \mid \theta_G)} \right) \cdot g(x_i \mid \theta_G) \right\}$$

$$+ \sum_{i=m+1}^{n} \log \left\{ \left(\frac{1 - p}{1 - H(t \mid \theta_H)} \right) \cdot h(x_i \mid \theta_H) \right\}$$

$$= m \cdot \log p - m \cdot \log G(t \mid \theta_G) + \sum_{i=1}^{m} \log g(x_i \mid \theta_G)$$

$$+ (n - m) \cdot \log (1 - p) - (n - m) \cdot \log \{1 - H(t \mid \theta_H)\}$$

$$+ \sum_{i=m+1}^{n} \log h(x_i \mid \theta_H)$$

$$= m \cdot \log p + (n - m) \cdot \log (1 - p)$$

$$+ \sum_{i=1}^{m} \log g(x_i \mid \theta_G) - m \cdot \log G(t \mid \theta_G)$$

$$+ \sum_{i=m+1}^{n} \log h(x_i \mid \theta_H) - (n - m) \cdot \log \{1 - H(t \mid \theta_H)\}$$

[3] θ_G and θ_H have no elements in common.

Since the loglikelihood splits into three parts dependent upon p, θ_G, and θ_H respectively, then the first partial derivatives with respect to p, θ_G, and θ_H depend only upon those three parts respectively.

This model and the lemma allow us to split the Owners, Landlords and Tenants bodily injury liability loss data at $8,000, for example, and to estimate the distribution of the large loss amounts by:

1. estimating p from the relative number of large and small loss amounts;
2. estimating θ_H strictly from the large loss amounts.

If $H(x \mid \theta_H)$ is our Pareto c.d.f. censored at $c = \$300{,}000$ (5.6), then (6.4) and (5.4) together say that $\theta_H = \{\beta, \delta\}$ may be estimated from the loglikelihood:

$$\begin{aligned}
\log L &= \sum_{i=m+1}^{n-m'} \log h(x \mid \beta, \delta) + m' \cdot \log \{1 - H(c \mid \beta, \delta)\} \\
&\quad - (n - m) \cdot \log \{1 - H(t \mid \beta, \delta)\} \qquad (6.5) \\
&= \sum_{i=m+1}^{n-m'} \log \left(\frac{\delta\beta^\delta}{(x_i + \beta)^{\delta+1}}\right) + m' \cdot \log \left(\left(\frac{\beta}{c + \beta}\right)^\delta\right) \\
&\quad - (n - m) \cdot \log \left(\left(\frac{\beta}{c + \beta}\right)^\delta\right) \\
&= (n - m - m') \cdot \log \delta + (n - m - m') \cdot \log \beta \\
&\quad - (\delta + 1) \cdot \sum_{i=m+1}^{n-m'} \log (x_i + \beta) \\
&\quad + m'\delta \cdot \log \beta - m'\delta \cdot \log (c + \beta) - (n - m)\delta \cdot \log \beta \\
&\quad + (n - m)\delta \cdot \log (t + \beta) \\
&= (n - m - m') \cdot \log \delta - (\delta + 1) \cdot \sum_{i=m+1}^{n-m'} \log (x_i + \beta) \\
&\quad - m'\delta \cdot \log (c + \beta) + (n - m)\delta \cdot \log (t + \beta)
\end{aligned}$$

where $n - m$ = total number of $x_i > t$
m' = number of loss amounts equal to c
$t < x_i < c$ for $i = m + 1, m + 2, \ldots, n - m'$

The MLE's of β, δ and p for our Pareto c.d.f. fit to the O. L. & T. data excess of $8,000 in Appendix E, Part 1, are displayed in Table 6.1 as "Pareto MLE I" where for the fitted Pareto, the tail probabilities are:

$$P[X > x \mid \hat{\beta}, \hat{\delta}, \hat{p}] = (1 - \hat{p}) \cdot \left(\frac{t + \hat{\beta}}{x + \hat{\beta}}\right)^{\hat{\delta}} \qquad \text{from (6.3)}$$

TABLE 6.1

PARETO FIT TO O. L. & T. DATA

	β	δ	p	P[X > 100,000]	P[$X \geq$ 300,000]
Data	NA	NA	.950	.0016	.0004
Pareto MLE I	1,463	1.453	.950	.0016	.0003
Pareto MLE II	347	0.877	NA	.0069	.0027

Appendix E, Part 2, displays the results of the K-S test and the EVC test. The Pareto model, of course, fails the K-S test. But the EVC statistic does not look bad: there are eight sign changes, the last component is −.0187 (−1.87%) and the absolute maximum 0.056 (−5.60%) occurs in the same interval ($9,000 − $10,000) as the K-S maximum. The Pareto c.d.f. fits better at the upper end and the expected value functions coincide well. If we fit the Pareto model to the overall loss amount distribution, the results are worse. The results are displayed in Table 6.1 as "Pareto MLE II"; note how poorly this "untruncated" Pareto c.d.f. predicts the tail probabilities. Appendix E, Part 3, displays the K-S test and EVC test results for this "untruncated" Pareto. The results are opposite those which usually occur; in this particular case, both the tail probabilities and expected values are too high for the Pareto. The point here is that the "untruncated" results are misleading.

A word about the data. Remember that these are a full policy year of incurred loss amounts evaluated at 27 months and grouped by intervals. They are immature (individual loss reserve development), incomplete (IBNR), from loss events occurring over a two year period, and are not listed by individual loss amount for the MLE procedure. Thus any remarks regarding the approximation of the true underlying distribution are tentative. I decided to use undeveloped and incomplete data for this example so as not to get involved in the question of how to develop and complete it. Unsatisfactory though the data are, I hope that the example is illustrative.

A word about using data summarized by interval, as the O. L. & T. data. For c.d.f. $F(x \mid \theta)$, the likelihood function for interval data is based upon the discrete distribution of probability per interval. If the intervals are $(a_0, a_1]$, . . . , $(a_{s-1}, a_s]$ and a sample produces n_i losses in the ith interval, the true likelihood function is:

$$L(\theta) = \prod_{i=1}^{s} \left(\frac{F(a_i \mid \theta) - F(a_{i-1} \mid \theta)}{F(a_s \mid \theta) - F(a_0 \mid \theta)} \right)^{n_i} \tag{6.6}$$

Our MLE's based upon the O. L. & T. data, however, treated the data as if all the losses in each interval were concentrated at the average value for the interval. In Appendix E, Part 1, for example, the 58 losses in the interval 10,000 − 11,000 were assumed each to have value 10,430 and the "individual data point" loglikelihood function (6.5) was used. Our testing has shown that treating interval data this way gives good results as long as the intervals are fairly narrow.

This estimation technique for large loss data truncated below is also useful when dealing with excess-of-loss reinsurance coverage where the data are usually excess of some underlying retention. To illustrate its accuracy in estimating the "total distribution," we again turn to our Pareto data in Appendix C. Truncating below at 5,000 and censoring above again at 200,000, we calculate the MLE's and tail probabilities displayed in Table 6.2. The analogous estimates from the lognormal data in Appendix D truncated below at 5,000 and censored above at 200,000 are displayed in Table 6.3.

The lognormal estimates here are not as good as we have seen in previous cases. In both these examples, the corresponding method-of-moments estimates would be ridiculous if the lower truncation were not taken into account, and the formulas would be difficult if it were.

TABLE 6.2

PARETO (DATA TRUNCATED AT 5,000)

	β	δ	p	P[X > 100,000]	P[X > 1,000,000]
Model	25,000	1.500	.239	.089	.004
MLE	23,354	1.492	.235	.085	.004

TABLE 6.3

LOGNORMAL (DATA TRUNCATED AT 5,000)

	μ	σ	p	P[X > 100,000]	P[X > 1,000,000]
Model	9.00	2.000	.405	.104	.008
MLE	8.98	1.858	.370	.091	.005

VII. MODELING AND ESTIMATION PROBLEM 3: THE UNDERLYING LOSS AMOUNT DISTRIBUTIONS ARE NOT SMOOTH

Whenever we see detailed loss amount data, such as the Owners, Landlords and Tenants bodily injury liability loss interval data in Appendix E, Part 1, or individual loss amount data, we are immediately struck by the fact that the losses tend to cluster at certain round values such as $1,000, $10,000, $25,000, . . . , $100,000, etc. This clustering occurs even in mature loss amount data. Thus, it is apparent that any probability model which is to describe the data as exactly as possible cannot be a smooth c.d.f. such as the Pareto or lognormal.

An alternative to a smooth model might be a mixed c.d.f. similar to the Hewitt/Lefkowitz model (6.1) with $G(x \mid \theta_G)$ smooth and $H(x \mid \theta_H)$ discrete. If a mixed model is to fit loss amount data significantly better than a completely smooth model can, then it may need many parameters, perhaps one for each discrete cluster point. The likelihood equations (6.2) would be very difficult to solve. And even then would such a model provide any better prediction, through simple parameter changes, of future loss amount distributions? I believe that our data are inadequate to support such a model.

Even though our data seem to have cluster points, the last column of Appendix E, Part 2, shows that the Pareto c.d.f. describes the expected loss function fairly well. Remember that expected value is the most important component of most insurance premiums. The same exhibit shows how well the Pareto c.d.f. estimates the tail probabilities (see also Table 6.1); this is also an important aspect of insurance pricing. Since we apparently cannot specify a better model without great difficulty (remember the data problems), it looks as if we must be satisfied with smooth models for large loss amount distributions as long as the parameters are properly estimated.

VIII. MODELING AND ESTIMATION PROBLEM 4: THE DATA ARE FROM MANY OCCURRENCE DATES

To have enough data to be able to study loss amount distributions, we must of necessity use data from many accident occurrence dates. Suppose that we want to study the loss amount distribution at one point-of-time and suppose that we have specified a broader model which tells us how to trend the data from different occurrence dates to the level of this single date. Let us also suppose that besides being subject to various policy limits, our data are also larger than some common lower truncation value t. This situation is common in reinsurance, where our data are often excess of some specified retention level. Also this situation arises if we use a probability model such as (6.3) to separate large and small losses and we use a common split point t for all our data. In this case, for each occurrence date we must trend the value t along with the loss amounts and the policy limits to our single occurrence date. Thus our trended data have a mixture of trended lower truncation points and trended policy limits (censorship points). The method of maximum likelihood allows us to painlessly calculate parameter estimates for the single point-of-time model simultaneously from all these data.

Let us illustrate this situation with our Pareto c.d.f. Let the x_{ki}'s represent the trended loss amounts. Assume that they are ordered so that for each k, the x_{ki}'s are larger than the trended lower truncation point t_k and are censored at the trended policy limit c_k. Also assume that for each k, the first $n_k - m_k$ x_{ki}'s lie strictly between t_k and c_k and the remaining m_k are equal to c_k. We assume that except for lower truncation and upper censorship, the x_{ki}'s are subject to the same underlying Pareto c.d.f. $F(x \mid \beta, \delta)$. Then using equations (5.8) and (6.5), with a suitable change in notation, the general likelihood function is:

$$L(\beta, \delta) = \prod_{k=1}^{s} L(\beta, \delta; t_k, c_k) \tag{8.1}$$

where the component loglikelihood function for each k is:

$$\begin{aligned} \log L(\beta, \delta; t_k, c_k) = {} & (n_k - m_k) \cdot \log \delta \\ & - (\delta + 1) \cdot \sum_{i=1}^{m_k} \log (x_{ki} + \beta) \\ & - m_k\delta \cdot \log (c_k + \beta) + n_k\delta \cdot \log (t_k + \beta) \end{aligned} \tag{8.2}$$

As in (5.9), the partial derivatives of the general loglikelihood will be the sum of the partials of the components in (8.2) for each truncation/censorship combination. The computer programming is straightforward.

For example, let us use the Owners, Landlords and Tenants bodily injury liability loss amount data in Appendices E and F for policy years 1975 and 1976 for policy limits of \$300,000 and \$500,000, and adjust by a trend factor of 18.9% per annum to an occurrence date of July 1, 1980. Since the individual loss occurrence dates are unknown, we will simply assume the average occurrence dates: for policy year 1975 this is January 1, 1976 and for policy year 1976 it is January 1, 1977. It would, of course, be better to know the occurrence month of each loss amount. We will use original lower truncation points for each year of \$8,000.

The simultaneous estimates for β, δ for July 1, 1980 are $\beta = 4{,}955$ and $\delta = 1.473$. The K-S test and EVC test results are displayed in Appendix G for the Pareto with parameters β, δ and each set of trended data. Note that in this case, the assumption that the loss amounts from different policy limits have the same underlying distribution looks like it may be false. The reason for this tentative conclusion is that the fitted Pareto greatly understates the expected values for higher limits of the \$500,000 policy limit trended data. This can be seen by studying the EVC test statistic in the last columns of Appendix G; the final value for trended policy year 1975 \$500,000 policy limit data is $-.298$ (Part 2) and the final value for trended policy year 1976 \$500,000 policy limit data is $-.178$ (Part 4).

The ISO Increased Limits Subcommittee has had mixed results when testing this assumption of a common loss amount distribution underlying different policy limits (except, of course, for the censorship at each limit). It is apparent that much more testing (and more careful model-building) needs to be done.

IX. CONCLUSION

This paper has presented a general procedure for selecting an adequate model for any particular loss amount distribution. The point of view is that we must use models whenever we want to extend or compare our various information, and moreover, we should use mathematical models. The procedure for finding an adequate model is to:

1. specify a particular probability model;
2. estimate parameters via the method of maximum likelihood;
3. test the model and select final parameters.

We have also discussed how to account for policy limits as censors, for too many small losses, for probability cluster points, and finally for loss amount inflation trends. The estimation technique discussed has been the method of maximum likelihood.

The Bibliography lists, beyond the direct references, many English-language papers and books which study casualty loss amount distributions or related problems. I trust that other American actuaries will find these references of interest.

APPENDIX A

Pareto (Lomax, Pearson Type VI) Distribution

$$F(x) = \text{Prob}\,[X \leq x] = 1 - \left(\frac{\beta}{x + \beta}\right)^{\delta} \qquad \text{for } x \geq 0 \tag{A1}$$

where $\beta > 0$, $\delta > 0$ are parameters.

$$f(x) = \frac{\delta \cdot \beta^{\delta}}{(x + \beta)^{\delta+1}} \text{ density} \tag{A2}$$

$$E[X] = \frac{\beta}{\delta - 1} \qquad \text{exists if } \delta > 1 \tag{A3}$$

$$\text{Var}\,[X] = \frac{\delta \cdot \beta^2}{(\delta - 2) \cdot (\delta - 1)^2} \qquad \text{exists if } \delta > 2 \tag{A4}$$

$$E[X^k] \text{ does not exist for } k \geq \delta \tag{A5}$$

$$\text{Prob}\,[X - t \leq x \mid X > t] = 1 - \left(\frac{\beta + t}{x + (\beta + t)}\right)^{\delta} \qquad \text{for } x \geq 0 \tag{A6}$$

Thus, a Pareto distribution excess of a lower truncation t is a Pareto distribution with new "beta parameter" $\beta + t$.

If $Y = tX$ for some $t > 0$, then (A7)

$$\text{Prob}\,[Y \leq y] = 1 - \left(\frac{\beta t}{y + \beta t}\right)^{\delta} \qquad \text{for } y \geq 0$$

Thus, if t is a trend factor and Y is the inflated value of X, then Y also has a Pareto distribution with new "beta parameter" βt. For any limit c, notate the integral of X^k from 0 to c by:

$$E[X^k; c] = \int_0^c X^k dF(X) \tag{A8}$$

Lemma: For any censor c, if k is a non-negative integer and $k - \delta$ is not a non-negative integer, then the integral of X^k from 0 to c is given by:

$$E[X^k; c] = \frac{k! \cdot \beta^k}{(\delta - 1)(\delta - 2) \cdots (\delta - k)}$$

$$- \delta \left(\frac{\beta}{c + \beta}\right)^{\delta} \left\{ \frac{(c + \beta)^k}{\delta - k} - \binom{k}{1} \frac{\beta(c + \beta)^{k-1}}{\delta - k + 1} + \cdots \right. \tag{A9}$$

$$\left. + (-1)^i \binom{k}{i} \frac{\beta^i (c + \beta)^{k-i}}{\delta - k + i} + \cdots + (-1)^k \left(\frac{\beta^k}{\delta}\right) \right\}$$

Proof: (This lemma and proof are due to Mark Kleiman)

$$E[X^k; c] = \int_0^c x^k \cdot \delta \cdot \beta^\delta \cdot (x + \beta)^{-\delta-1} dx$$

$$= \int_0^c \delta\beta^\delta (x + \beta)^{-\delta-1} \{(x + \beta) - \beta\}^k dx$$

$$= \int_0^c \delta\beta^\delta (x + \beta)^{-\delta-1} \left\{ (x + \beta)^k - \binom{k}{1} \beta(x + \beta)^{k-1} + \cdots \right.$$

$$\left. + (-1)^i \binom{k}{i} \beta^i (x + \beta)^{k-i} + \cdots + (-1)^k \beta^k \right\} dx$$

$$= \delta \int_0^c \left\{ \beta^\delta (x + \beta)^{k-\delta-1} - \binom{k}{1} \beta^{\delta+1} (x + \beta)^{k-\delta-2} + \cdots \right.$$

$$\left. + (-1)^i \binom{k}{i} \beta^{\delta+i} (x + \beta)^{k-\delta-i-1} + \cdots + (-1)^k \beta^{\delta+k} (x + \beta)^{-\delta-1} \right\} dx$$

$$= \delta \left\{ \left(\frac{\beta^\delta}{k - \delta}\right) \cdot (x + \beta)^{k-\delta} - \binom{k}{1} \left(\frac{\beta^{\delta+1}}{k - \delta - 1}\right) (x + \beta)^{k-\delta-1} + \cdots \right.$$

$$+ (-1)^i \binom{k}{i} \left(\frac{\beta^{\delta+i}}{k - \delta - i}\right) (x + \beta)^{k-\delta-i} + \cdots$$

$$\left. + (-1)^k \left(\frac{\beta^{\delta+k}}{\delta}\right) (x + \beta)^{-\delta} \right\} \Bigg|_0^c$$

$$= \delta \left\{ \left(\frac{\beta^\delta}{k - \delta}\right) [(c + \beta)^{k-\delta} - \beta^{k-\delta}] \right.$$
$$- \binom{k}{1}\left(\frac{\beta^{\delta+1}}{k - \delta - 1}\right) [(c + \beta)^{k-\delta-1} - \beta^{k-\delta-1}] + \cdots$$
$$+ (-1)^i \binom{k}{i}\left(\frac{\beta^{\delta+i}}{k - \delta - i}\right) [(c + \beta)^{k-\delta-i} - \beta^{k-\delta-i}] + \cdots$$
$$\left. + (-1)^k \left(\frac{\beta^{\delta+k}}{\delta}\right) [(c + \beta)^{-\delta} - \beta^{-\delta}] \right\}$$
$$= \delta\beta^k \left\{\frac{1}{\delta - k} - \binom{k}{1} \frac{1}{\delta - k + 1} + \cdots \right.$$
$$\left. + (-1)^i \binom{k}{i} \frac{1}{\delta - k + i} + \cdots + (-1)^k \frac{1}{\delta}\right\}$$
$$- \delta \left(\frac{\beta}{c + \beta}\right)^\delta \left\{ \frac{(c + \beta)^k}{\delta - k} + \cdots \right.$$
$$\left. + (-1)^i \binom{k}{i} \frac{\beta^i (c + \beta)^{k-i}}{\delta - k + i} + \cdots + (-1)^k \frac{\beta^k}{\delta}\right\}$$

We now want to prove that the first expression in braces in the last equality is equal to:

$$\frac{k!}{\delta(\delta - 1) \cdots (\delta - k)} \qquad \text{if } \delta - k \text{ is not a negative integer}$$

This is proved by judicious use of the binomial theorem and from the definitions of Gamma and Beta functions.

$$\frac{k!}{\delta(\delta - 1) \cdots (\delta - k)}$$
$$= \frac{\Gamma(k + 1) \cdot \Gamma(\delta - k)}{\Gamma(\delta + 1)} \qquad \text{if } \delta - k \text{ is not a negative integer}$$
$$= \int_0^1 (1 - x)^k x^{\delta-k-1} dx$$
$$= \int_0^1 \left\{1 - \binom{k}{1} x + \cdots + (-1)^i \binom{k}{i} x^i + \cdots + (-1)^k x^k\right\} x^{\delta-k-1} dx$$

$$= \int_0^1 \left\{ x^{\delta-k-1} - \binom{k}{1} x^{\delta-k} + \cdots + (-1)^i \binom{k}{i} x^{\delta-k+i-1} + \cdots \right.$$
$$\left. + (-1)^k x^{\delta-1} \right\} dx$$
$$= \left\{ \frac{x^{\delta-k}}{\delta - k} - \binom{k}{1} \frac{x^{\delta-k+1}}{\delta - k + 1} + \cdots \right.$$
$$\left. + (-1)^i \binom{k}{i} \frac{x^{\delta-k+i}}{\delta - k + i} + \cdots + (-1)^k \frac{x^{\delta}}{\delta} \right\} \Big|_0^1$$
$$= \frac{1}{\delta - k} - \binom{k}{1} \frac{1}{\delta - k + 1} + \cdots$$
$$+ (-1)^i \binom{k}{i} \frac{1}{\delta - k + i} + \cdots + (-1)^k \cdot \frac{1}{\delta}$$

Note: If $c < \infty$, then any integral $E[X^k; c]$ exists (is finite).
If $k - \delta$ is a non-negative integer, then $E[X^k; c]$ may be approximated for small $\epsilon > 0$ via:

$$E[X^k; c] \simeq \{E[X^{k-\epsilon}; c] + E[X^{k+\epsilon}; c]\}/2 \qquad \text{(A10)}$$

So, the lemma evaluation formula may be used.

Corollary: For any censor c:

$$E[X; c] = \frac{\beta}{\delta - 1} \left\{ 1 - \left(\frac{\beta}{c + \beta} \right)^{\delta-1} \left(\frac{\delta c + \beta}{c + \beta} \right) \right\} \qquad \text{if } \delta \neq 1 \qquad \text{(A11)}$$

APPENDIX B

Maximum Likelihood Estimation and Newton-Raphson Iteration

Given a sample $x_1, x_2, \ldots, x_n$ and general c.d.f. $F(x \mid \theta)$ with parameter $\theta = \{\theta_1, \ldots, \theta_r\}$ in some set Θ. The likelihood function is given by:

$$L(\theta) = \prod_{i=1}^{n} f(x_i \mid \theta) \qquad \text{(B1)}$$

where $f(x_i \mid \theta)$ is either the probability of x_i given θ or the p.d.f. evaluated at x_i given θ, depending upon whether or not the distribution function has a jump or is absolutely continuous at x_i.

The important properties of MLE's are (Kendall and Stuart (1967), p. 38ff and Dudewicz (1976), p. 193ff):

1. Under very general conditions, MLE's are consistent. That is, the MLE $\hat{\theta}$ converges in probability to θ_0, the true value of θ, as the sample size increases.

2. Under very general conditions, MLE's are consistent asymptotically normally distributed and efficient. That is, $\hat{\theta}$ is asymptotically (as sample size $n \to \infty$) normally distributed with mean θ_0 and covariance matrix equal to the inverse of the Fisher information matrix:

$$\text{Cov}\ \hat{\theta} \simeq I(\theta_0)^{-1} \qquad \text{(approximately)} \tag{B2}$$

where

$$I(\theta_0) = -E\left[\left(\left(\frac{\partial^2 \log L(\theta)}{\partial\theta_j \cdot \partial\theta_i}\bigg|_{\theta=\theta_0}\right)\right)\right] \tag{B3}$$

The determinant of Cov $\hat{\theta}$ becomes minimal as $n \to \infty$.

For our Pareto example (2.1), for sample size n we have:

$$I(\beta, \delta) = \begin{pmatrix} \dfrac{n\delta}{\beta(\delta + 2)} & \dfrac{-n}{\beta(\delta + 1)} \\ \dfrac{-n}{\beta(\delta + 1)} & \dfrac{n}{\delta^2} \end{pmatrix} \tag{B4}$$

So,

$$\text{Cov}\ (\hat{\beta}, \hat{\delta}) \simeq \frac{1}{n}\begin{pmatrix} \dfrac{\beta^2}{\delta}(\delta + 1)^2(\delta + 2) & \beta\delta(\delta + 1)(\delta + 2) \\ \beta\delta(\delta + 1)(\delta + 2) & \delta^2(\delta + 1)^2 \end{pmatrix} \tag{B5}$$

Finding a $\hat{\theta}$ which maximizes $L(\theta)$ is equivalent to finding a $\hat{\theta}$ which maximizes

$$\log L = \log L(\theta) = \sum_{i=1}^{n} \log f(x_i \mid \theta) \tag{B6}$$

If log L has second partial derivatives with respect to the θ_j's existing throughout its domain Θ, then a necessary condition for a point $\hat{\theta}$ to maximize log L is that the first partials evaluated at $\hat{\theta}$ be equal to zero:

$$\frac{\partial \log L}{\partial\theta_j}\bigg|_{\theta=\hat{\theta}} = 0 \qquad \text{for } j = 1, 2, \ldots, r \tag{B7}$$

If the matrix of second partials evaluated at $\hat{\theta}$ is negative definite, then $\hat{\theta}$ indeed maximizes log L.

Assuming that the matrix of second partials will be negative definite, we must find $\hat{\theta}$ which satisfies the system of equations (B7). Newton-Raphson iteration allows us to find a sequence of vectors $\theta^{(1)}$, $\theta^{(2)}$, . . . which may converge to a solution for any system of equations such as (B7). The only condition necessary is that the partials of the equations in (B7) with respect to each θ_j must exist for each $\theta^{(m)}$. See Conte and de Boor (1972).

Expressions for the second partial derivatives of log L are somewhat unwieldly. So we will simplify the notation of (B7) to a more general case: we assume that our problem is to find a point $\hat{\theta} = \{\hat{\theta}_1, \ldots, \hat{\theta}_r\}$ which is a solution for the following system of r equations:

$$\begin{aligned} \Psi_1(\theta) &= 0 \\ \Psi_2(\theta) &= 0 \\ \vdots \quad &\quad \vdots \\ \Psi_r(\theta) &= 0 \end{aligned} \tag{B8}$$

And we assume that the partials of the Ψ_i's with respect to the θ_j's exist throughout the domain Θ.

Start by selecting an initial value $\theta^{(1)}$. Then, in general, $\theta^{(m+1)}$ is obtained from $\theta^{(m)}$ by solving the following system of r equations in r unknowns $\theta_1^{(m+1)}, \ldots, \theta_r^{(m+1)}$:

$$\begin{aligned} \sum_{j=1}^{r} (\theta_j^{(m+1)} - \theta_j^{(m)}) \cdot \frac{\partial \Psi_1(\theta^{(m)})}{\partial \theta_j} &= -\Psi_1(\theta^{(m)}) \\ \vdots \qquad\qquad &\qquad \vdots \\ \sum_{j=1}^{r} (\theta_j^{(m+1)} - \theta_j^{(m)}) \cdot \frac{\partial \Psi_r(\theta^{(m)})}{\partial \theta_j} &= -\Psi_r(\theta^{(m)}) \end{aligned} \tag{B9}$$

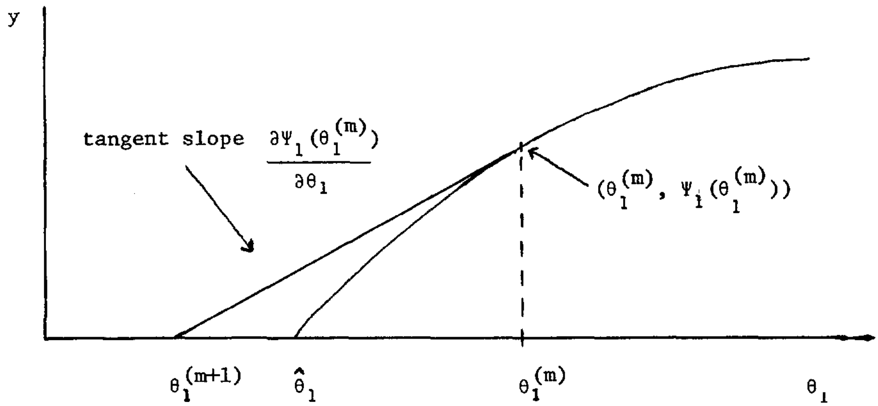
In the case that $r = 1$, we have the familiar solution:

$$\theta_1^{(m+1)} = \theta_1^{(m)} - \Psi_1(\theta_1^{(m)}) \cdot \left(\frac{\partial \Psi_1(\theta_1^{(m)})}{\partial \theta_1}\right)^{-1} \tag{B10}$$

The 1-dimensional case may be illustrated by Figure B1.

FIGURE B1

GRAPH OF $y = \Psi_1(\theta_1)$



The tangent through the point $(\theta_1^{(m)}, \Psi(\theta_1^{(m)}))$ intersects the θ_1-axis at the new θ_1 value, $\theta_1^{(m+1)}$. It should be clear that as long as $\Psi_1(\theta_1)$ is well-behaved, if $\theta_1^{(m)}$ is close to a zero $\hat{\theta}_1$ for $\Psi_1(\theta_1)$, then $\theta_1^{(m+1)}$ should be even closer, as in the figure.

In the general case, the solution to (B4) is obtained via Cramer's Rule as long as the matrix of second partials evaluated at $\theta^{(m)}$ is nonsingular (Herstein (1964), p. 288). For example, the case $r = 2$ is easily illustrated:

$$\theta_1^{(m+1)} = \theta_1^{(m)} - \frac{\Psi_1(\theta^{(m)}) \cdot \left(\dfrac{\partial\Psi_2(\theta^{(m)})}{\partial\theta_2}\right) - \left(\dfrac{\partial\Psi_1(\theta^{(m)})}{\partial\theta_2}\right) \cdot \Psi_2(\theta^{(m)})}{J(\theta^{(m)})} \quad \text{(B11)}$$

$$\theta_2^{(m+1)} = \theta_2^{(m)} - \frac{\Psi_2(\theta^{(m)}) \cdot \left(\dfrac{\partial\Psi_1(\theta^{(m)})}{\partial\theta_1}\right) - \left(\dfrac{\partial\Psi_2(\theta^{(m)})}{\partial\theta_1}\right) \cdot \Psi_1(\theta^{(m)})}{J(\theta^{(m)})}$$

where $J(\theta^{(m)})$ is the Jacobian evaluated at $\theta^{(m)}$:

$$J(\theta^{(m)}) = \left(\frac{\partial\Psi_1(\theta^{(m)})}{\partial\theta_1}\right) \cdot \left(\frac{\partial\Psi_2(\theta^{(m)})}{\partial\theta_2}\right) - \left(\frac{\partial\Psi_1(\theta^{(m)})}{\partial\theta_2}\right) \cdot \left(\frac{\partial\Psi_2(\theta^{(m)})}{\partial\theta_1}\right) \quad \text{(B12)}$$

This technique gives an iteration $\theta^{(1)}, \theta^{(2)}, \ldots$ which may be stopped when successive values $|\theta^{(m+1)} - \theta^{(m)}|$ are small enough according to some metric $|\theta|$.

APPENDIX C

Pareto

200 Psuedo-Random Values

$$\beta = 25{,}000 \qquad \delta = 1.5 \qquad P[X \le x] = 1 - \left(\frac{\beta}{x + \beta}\right)^{\delta}$$

	K-S Test			EVC Test		
				Means		EVC Statistic
X	Pareto CDF $F(X)$	Sample CDF $S(X)$	Maximum Difference	Pareto	Sample	(%)
9	.001	.005	−.004	8	9	−.03
151	.009	.010	.004	150	150	.02
225	.013	.015	.003	223	223	−.03
282	.017	.020	−.003	279	279	−.03
295	.018	.025	−.007	292	292	−.01
670	.039	.030	.014	656	658	−.20
772	.045	.035	.015	754	757	−.34
773	.045	.040	.010	755	757	−.34
845	.049	.045	.009	824	827	−.37
910	.053	.050	.008	885	889	−.38
915	.053	.055	.003	890	893	−.38
1,035	.059	.060	.004	1,003	1,007	−.35
1,092	.062	.065	−.003	1,057	1,060	−.34
1,151	.066	.070	−.004	1,112	1,116	−.32
1,208	.069	.075	−.006	1,165	1,169	−.29
1,231	.070	.080	−.010	1,187	1,190	−.28
1,548	.087	.085	.007	1,479	1,481	−.19
1,616	.090	.090	.005	1,541	1,544	−.19
1,638	.091	.095	−.004	1,561	1,564	−.19
1,643	.092	.100	−.008	1,565	1,568	−.19
1,727	.096	.105	−.009	1,641	1,644	−.15
1,786	.099	.110	−.011	1,695	1,697	−.12
1,866	.103	.115	−.012	1,767	1,768	−.07
2,005	.110	.120	−.010	1,891	1,891	−.01
2,049	.112	.125	−.013	1,930	1,930	.01
2,081	.114	.130	−.016	1,958	1,958	.03
2,108	.115	.135	−.020	1,982	1,981	.05
2,127	.116	.140	−.024	1,999	1,998	.07
2,469	.133	.145	−.012	2,298	2,292	.29
2,626	.140	.150	−.010	2,434	2,426	.33
2,945	.155	.155	.005	2,706	2,697	.33
2,949	.155	.160	−.005	2,709	2,700	.33
2,958	.156	.165	−.009	2,717	2,708	.33
3,036	.159	.170	−.011	2,783	2,773	.34
3,048	.160	.175	−.015	2,793	2,783	.35
3,285	.170	.180	−.010	2,991	2,979	.40
3,356	.173	.185	−.012	3,049	3,037	.41
3,428	.177	.190	−.013	3,109	3,095	.43
3,593	.184	.195	−.011	3,244	3,229	.46
3,796	.193	.200	−.007	3,409	3,393	.48
3,936	.198	.205	−.007	3,521	3,505	.48
3,952	.199	.210	−.011	3,534	3,517	.48
4,424	.219	.215	.009	3,908	3,890	.45
4,753	.232	.220	.017	4,163	4,148	.34
4,849	.235	.225	.015	4,236	4,223	.31
4,961	.240	.230	.015	4,322	4,310	.27
4,979	.240	.235	.010	4,335	4,324	.26
5,114	.246	.240	.011	4,437	4,427	.23
5,129	.246	.245	.006	4,449	4,439	.23
5,135	.246	.250	−.004	4,453	4,443	.23

APPENDIX C

Pareto

200 Psuedo-Random Values

	K-S Test			EVC Test		
				Means		EVC
X	Pareto CDF $F(X)$	Sample CDF $S(X)$	Maximum Difference	Pareto	Sample	Statistic (%)
5.195	.249	.255	−.006	4.498	4.488	.23
5.331	.254	.260	−.006	4.600	4.589	.24
5.662	.266	.265	.006	4.845	4.834	.22
5.666	.266	.270	−.004	4.848	4.837	.22
5.966	.277	.275	.007	5.067	5.056	.20
6.044	.280	.280	.005	5.123	5.113	.20
6.274	.288	.285	.008	5.288	5.278	.17
6.418	.293	.290	.008	5.390	5.381	.16
6.675	.301	.295	.011	5.571	5.564	.12
7.048	.314	.300	.019	5.829	5.827	.03
7.064	.314	.305	.014	5.840	5.838	.03
7.117	.316	.310	.011	5.876	5.875	.02
7.287	.322	.315	.012	5.992	5.992	−.01
7.343	.323	.320	.008	6.030	6.031	−.01
7.352	.324	.325	.004	6.036	6.037	−.01
7.366	.324	.330	−.006	6.045	6.046	−.01
7.613	.332	.335	−.003	6.211	6.212	.00
8.184	.349	.340	.014	6.588	6.591	−.05
8.247	.351	.345	.011	6.629	6.633	−.06
8.261	.352	.350	.007	6.638	6.642	−.06
8.265	.352	.355	−.003	6.640	6.645	−.06
8.274	.352	.360	−.008	6.646	6.651	−.06
8.920	.371	.365	.011	7.059	7.064	−.07
8.924	.371	.370	.006	7.061	7.066	−.07
8.960	.372	.375	−.003	7.084	7.089	−.07
9.035	.374	.380	−.006	7.131	7.136	−.07
9.045	.374	.385	−.011	7.137	7.142	−.07
9.205	.379	.390	−.011	7.237	7.241	−.05
9.323	.382	.395	−.013	7.310	7.313	−.03
10.128	.404	.400	.009	7.799	7.800	−.01
10.494	.413	.405	.013	8.015	8.019	−.05
10.535	.414	.410	.009	8.040	8.044	−.05
10.580	.415	.415	.005	8.066	8.070	−.05
10.652	.417	.420	−.003	8.108	8.112	−.05
11.295	.433	.425	.013	8.478	8.485	−.09
11.584	.439	.430	.014	8.641	8.651	−.12
11.967	.448	.435	.018	8.854	8.870	−.18
12.036	.450	.440	.015	8.892	8.909	−.19
12.826	.467	.445	.027	9.319	9.351	−.34
12.961	.470	.450	.025	9.391	9.426	−.37
13.041	.472	.455	.022	9.433	9.470	−.39
13.185	.475	.460	.020	9.509	9.549	−.41
13.187	.475	.465	.015	9.510	9.550	−.41
13.321	.478	.470	.013	9.580	9.621	−.43
13.690	.486	.475	.016	9.772	9.817	−.46
13.846	.489	.480	.014	9.852	9.899	−.48
13.998	.492	.485	.012	9.929	9.978	−.49
14.373	.499	.490	.014	10.118	10.171	−.52
14.447	.501	.495	.011	10.155	10.209	−.53
14.648	.505	.500	.010	10.255	10.310	−.54
14.699	.506	.505	.006	10.280	10.336	−.54
14.766	.507	.510	−.003	10.314	10.369	−.54
15.007	.511	.515	−.004	10.432	10.487	−.53
15.305	.517	.520	−.003	10.577	10.631	−.52
15.415	.519	.525	−.006	10.630	10.684	−.51

APPENDIX C

Pareto

200 Psuedo-Random Values

	K-S Test			EVC Test		
				Means		EVC
X	Pareto CDF $F(X)$	Sample CDF $S(X)$	Maximum Difference	Pareto	Sample	Statistic (%)
15,909	.528	.530	.003	10,865	10,919	-.50
15,983	.529	.535	-.006	10,900	10,954	-.49
16,282	.534	.540	-.006	11,040	11,093	-.48
16,479	.538	.545	-.007	11,131	11,183	-.47
16,685	.541	.550	-.009	11,226	11,277	-.45
16,710	.542	.555	-.013	11,238	11,288	-.45
16,818	.543	.560	-.017	11,287	11,336	-.44
17,036	.547	.565	-.018	11,386	11,432	-.40
17,241	.551	.570	-.019	11,479	11,521	-.37
17,428	.554	.575	-.021	11,563	11,602	-.34
18,259	.567	.580	-.013	11,928	11,955	-.23
18,452	.570	.585	-.015	12,011	12,036	-.21
19,123	.580	.590	-.010	12,297	12,315	-.15
19,755	.589	.595	-.006	12,559	12,574	-.11
19,862	.590	.600	-.010	12,603	12,617	-.11
19,905	.591	.605	-.014	12,621	12,634	-.10
19,985	.592	.610	-.018	12,654	12,666	-.10
20,146	.594	.615	-.021	12,719	12,729	-.07
20,275	.596	.620	-.024	12,771	12,778	-.05
20,390	.598	.625	-.027	12,818	12,822	-.03
21,889	.617	.630	-.013	13,406	13,384	.16
22,017	.619	.635	-.016	13,455	13,431	.17
22,172	.621	.640	-.019	13,514	13,488	.19
22,309	.623	.645	-.022	13,566	13,537	.21
22,362	.623	.650	-.027	13,586	13,556	.22
22,369	.623	.655	-.032	13,588	13,559	.22
23,919	.642	.660	-.018	14,158	14,093	.46
23,919	.642	.665	-.023	14,158	14,093	.46
24,469	.648	.670	-.022	14,353	14,278	.53
25,241	.656	.675	-.019	14,622	14,532	.61
25,384	.658	.680	-.022	14,671	14,579	.63
27,539	.679	.685	-.006	15,386	15,268	.76
28,520	.688	.690	.003	15,696	15,578	.75
28,545	.688	.695	-.007	15,704	15,585	.75
28,712	.690	.700	-.010	15,756	15,636	.76
30,016	.701	.705	-.004	16,153	16,027	.78
32,430	.720	.710	.015	16,851	16,740	.66
33,821	.731	.715	.021	17,232	17,143	.52
34,131	.733	.720	.018	17,316	17,231	.49
34,177	.733	.725	.013	17,328	17,244	.48
34,448	.735	.730	.010	17,400	17,319	.47
34,947	.738	.735	.008	17,531	17,453	.44
35,422	.742	.740	.007	17,655	17,579	.43
35,987	.745	.745	.005	17,800	17,726	.41
37,488	.755	.750	.010	18,175	18,109	.36
37,641	.756	.755	.006	18,213	18,147	.36
37,975	.758	.760	.003	18,294	18,229	.35
38,361	.760	.765	-.005	18,387	18,322	.36
38,498	.761	.770	-.009	18,420	18,354	.36
39,750	.768	.775	-.007	18,715	18,642	.39
40,137	.770	.780	-.010	18,804	18,729	.40
40,987	.775	.785	-.010	18,998	18,916	.43
43,817	.789	.790	.004	19,615	19,524	.46
44,606	.793	.795	.003	19,780	19,690	.46
45,150	.795	.800	-.005	19,892	19,802	.46

APPENDIX C

Pareto

200 Psuedo-Random Values

	K-S Test			EVC Test		
				Means		EVC Statistic (%)
X	Pareto CDF F(X)	Sample CDF S(X)	Maximum Difference	Pareto	Sample	
47,319	.805	.805	.005	20,326	20,235	.45
48,160	.808	.810	.003	20,489	20,399	.44
53,161	.827	.815	.017	21,401	21,350	.24
53,909	.829	.820	.014	21,529	21,488	.19
57,601	.841	.825	.021	22,137	22,152	−.07
59,057	.845	.830	.020	22,365	22,407	−.19
59,908	.848	.835	.018	22,495	22,552	−.25
62,120	.854	.840	.019	22,825	22,917	−.40
63,423	.857	.845	.017	23,013	23,125	−.49
64,290	.859	.850	.014	23,136	23,260	−.53
64,539	.860	.855	.010	23,171	23,297	−.54
66,841	.865	.860	.010	23,487	23,631	−.61
66,928	.866	.865	.006	23,499	23,643	−.61
69,947	.872	.870	.007	23,894	24,051	−.65
71,658	.876	.875	.006	24,110	24,273	−.68
71,830	.876	.880	−.004	24,131	24,295	−.68
72,206	.877	.885	−.008	24,178	24,340	−.67
74,097	.881	.890	−.009	24,407	24,557	−.62
74,454	.881	.895	−.014	24,450	24,596	−.60
74,806	.882	.900	−.018	24,491	24,633	−.58
97,519	.914	.905	.014	26,774	26,905	−.49
97,704	.915	.910	.010	26,790	26,922	−.50
98,579	.915	.915	.005	26,864	27,001	−.51
98,812	.916	.920	−.004	26,884	27,021	−.51
117,157	.932	.925	.012	28,270	28,488	−.77
118,540	.933	.930	.008	28,363	28,592	−.81
120,717	.935	.935	.005	28,507	28,745	−.83
126,789	.939	.940	.004	28,890	29,139	−.86
137,105	.945	.945	.005	29,490	29,758	−.91
151,162	.952	.950	.007	30,216	30,531	−1.04
158,996	.955	.955	.005	30,582	30,923	−1.11
165,704	.957	.960	−.003	30,877	31,225	−1.13
166,837	.958	.965	−.007	30,925	31,270	−1.12
210,571	.969	.970	.004	32,498	32,801	−.93
217,732	.971	.975	−.004	32,712	33,016	−.93
243,729	.975	.980	−.005	33.413	33,666	−.76
253,630	.977	.985	−.008	33,652	33,864	−.63
370,910	.987	.990	−.003	35,733	35,623	.31
616,233	.994	.995	.004	37,978	38,076	−.26
1,176,968	.998	1.000	.003	40,100	40,880	−1.94

Beta is 26,297
Delta is 1.586
The Truncation Point is 0
The Censorship Point is 10,000,000
The Sample Size is 200
Kolmogorov-Smirnov Test Statistic is 0.0317

APPENDIX D

Lognormal

200 Pseudo-Random Values

$$\mu = 9 \qquad \sigma = 2 \qquad \text{Prob}\,[X \leq x] = \phi\left(\frac{\log x - \mu}{\sigma}\right)$$

$$\text{where } \phi(y) = (2\pi)^{-1}\int_{-\infty}^{y} \exp\left\{-\frac{t^2}{2}\right\} dt$$

2	1,904	5,449	12,301	39,866
20	1,938	5,552	12,371	42,942
21	1,996	5,696	12,606	45,129
89	2,007	5,785	12,626	45,665
134	2,067	5,859	13,690	45,859
135	2,123	5,897	14,090	46,175
164	2,134	5,900	15,759	47,292
165	2,233	5,918	16,359	47,477
186	2,321	6,208	17,134	47,580
236	2,369	6,553	17,298	50,698
402	2,376	6,804	17,649	50,707
438	2,380	6,875	17,949	58,131
451	2,497	6,901	18,682	58,441
526	2,631	6,929	19,696	61,890
582	2,672	6,934	19,789	64,181
601	2,873	7,010	19,874	66,391
639	2,879	7,047	21,275	67,898
676	2,970	7,737	21,305	70,527
850	2,974	7,750	24,569	80,932
911	3,275	7,980	24,600	82,360
914	3,394	8,047	26,571	83,122
1,029	3,397	8,220	27,021	83,849
1,052	3,407	8,448	27,290	83,917
1,053	3,505	8,623	27,969	88,095
1,071	3,584	8,784	28,212	104,508
1,073	3,746	9,029	29,088	112,291
1,102	3,772	9,118	29,205	113,729
1,182	3,903	9,326	29,507	122,065
1,185	3,924	9,356	30,927	129,896
1,258	3,997	9,475	32,490	132,125
1,337	4,660	9,896	32,657	168,200
1,340	4,780	9,989	33,929	209,599
1,357	4,794	10,145	34,797	225,688
1,501	4,816	10,272	35,149	260,210
1,627	5,020	10,429	35,194	307,687
1,669	5,041	10,551	35,261	375,796
1,798	5,074	10,675	35,669	463,569
1,825	5,154	10,679	37,859	510,905
1,836	5,206	12,079	38,049	861,999
1,903	5,354	12,274	39,150	1,684,380

APPENDIX E

Part 1

ISO Owners, Landlords and Tenants Bodily Injury Liability

Loss Amount Data: Policy Year 1976 as of March 31, 1978
Policy Limit $300,000

Loss Amount	Number of Losses	Average Loss Amount
0- 250	10,075	88
250- 500	3,049	374
500- 1,000	3,263	783
1,000- 2,000	2,690	1,490
2,000- 3,000	1,498	2,543
3,000- 4,000	964	3,521
4,000- 5,000	794	4,777
5,000- 6,000	261	5,629
6,000- 7,000	191	6,600
7,000- 8,000	406	7,429
8,000- 9,000	114	8,500
9,000- 10,000	279	9,736
10,000- 11,000	58	10,430
11,000- 12,000	56	11,279
12,000- 13,000	47	12,572
13,000- 14,000	20	13,541
14,000- 15,000	151	14,965
15,000- 16,000	28	15,501
16,000- 17,000	16	16,471
17,000- 18,000	24	17,643
18,000- 19,000	9	18,713
19,000- 20,000	74	19,950
20,000- 21,000	16	20,445
21,000- 22,000	7	21,753
22,000- 23,000	12	22,658
23,000- 24,000	4	23,756
24,000- 25,000	70	24,960
25,000- 30,000	44	28,377
30,000- 35,000	30	33,888
35,000- 40,000	25	38,610
40,000- 45,000	15	43,106
45,000- 50,000	40	49,834
50,000- 55,000	3	51,146
55,000- 60,000	8	59,813
60,000- 65,000	4	64,247
65,000- 70,000	1	70,000
70,000- 75,000	8	75,000
75,000- 80,000	2	78,618
80,000- 85,000	5	82,425
95,000-100,000	12	99,404
100,000-110,000	3	104,556
110,000-120,000	1	120,000
120,000-130,000	5	124,463
140,000-150,000	4	150,000
150,000-160,000	4	155,128
160,000-170,000	1	161,000
170,000-180,000	2	173,398
190,000-200,000	2	197,495
230,000-240,000	2	233,449
240,000-250,000	1	250,000
250,000-260,000	1	252,800
270,000-280,000	1	273,747
280,000-290,000	1	287,540
300,000 and over	10	300,000
Total	24,411	2,279

APPENDIX E

Part 2

ISO Owners, Landlords and Tenants Bodily Injury Liability

Loss Amount Data: Policy Year 1976 as of March 31, 1978 Policy Limit $300,000

	K-S Test			EVC Test		
				Means		EVC Statistic
X	Pareto CDF $F(X)$	Sample CDF $S(X)$	Maximum Difference	Pareto	Sample	(%)
8,000	.000	.000	.000	0	0	.00
8,500	.072	.093	.072	481	500	−3.82
9,736	.217	.322	.124	1,534	1,620	−5.60
10,430	.283	.370	−.087	2,054	2,091	−1.77
11,279	.351	.416	−.065	2,633	2,626	.29
12,572	.436	.454	.021	3,415	3,381	.99
13,541	.488	.470	.034	3,936	3,910	.65
14,965	.551	.594	.081	4,618	4,664	−1.00
15,501	.572	.617	−.045	4,853	4,882	−.60
16,471	.605	.630	−.025	5,252	5,253	−.03
17,643	.640	.650	−.010	5,694	5,686	.13
18,713	.667	.657	.017	6,064	6,061	.05
19,950	.695	.718	.037	6,458	6,485	−.41
20,445	.705	.731	−.026	6,607	6,624	−.26
21,753	.729	.737	−.008	6,977	6,976	.02
22,658	.743	.747	.006	7,216	7,214	.03
23,756	.759	.750	.013	7,489	7,492	−.04
24,960	.775	.807	−.032	7,769	7,793	−.31
28,377	.812	.843	−.032	8,472	8,451	.24
33,888	.853	.868	−.015	9,389	9,314	.80
38,610	.877	.889	−.011	10,024	9,937	.86
43,106	.895	.901	.006	10,535	10,438	.91
49,834	.914	.934	−.019	11,173	11,106	.60
51,146	.917	.936	−.019	11,284	11,193	.81
59,813	.934	.943	−.009	11,924	11,747	1.49
64,247	.940	.946	−.006	12,204	12,001	1.66
70,000	.947	.947	.001	12,527	12,313	1.71
75,000	.952	.953	.005	12,779	12,579	1.57
78,618	.955	.955	.002	12,947	12,748	1.54
82,425	.958	.959	.003	13,113	12,920	1.47
99,404	.968	.969	.009	13,735	13,616	.87
104,556	.970	.971	.001	13,895	13,776	.85
120,000	.975	.972	.004	14,312	14,219	.65
124,463	.977	.976	.005	14,419	14,343	.52
150,000	.982	.980	.006	14,937	14,950	−.09
155,128	.983	.983	.004	15,026	15,056	−.20
161,000	.984	.984	.001	15,123	15,157	−.22
173,398	.986	.985	.002	15,312	15,360	−.31
197,495	.988	.987	.003	15,628	15,715	−.56
233,449	.991	.989	.004	16,009	16,187	−1.11
250,000	.991	.989	.003	16,157	16,377	−1.36
252,800	.992	.990	.002	16,181	16,407	−1.40
273,747	.993	.991	.002	16,346	16,613	−1.63
287,540	.993	.992	.002	16,446	16,737	−1.77
300,000	1.000	1.000	.000	16,530	16,839	−1.87

Beta is 1462.8
Delta is 1.4532
The Truncation Point is 8,000
The Censorship Point is 300,000
The Sample Size is 1,220
Kolmogorov-Smirnov Test Statistic is 0.1236

APPENDIX E

Part 3

ISO Owners, Landlords and Tenants Bodily Injury Liability

Loss Amount Data: Policy Year 1976 as of March 31, 1978
Policy Limit $300,000

	K-S Test			EVC Test		
				Means		EVC Statistic (%)
X	Pareto CDF F(X)	Sample CDF S(X)	Maximum Difference	Pareto	Sample	
88	.180	.413	−.233	79	88	−10.68
374	.473	.538	−.065	265	255	3.59
783	.645	.671	.107	440	444	−.91
1,490	.768	.781	.097	642	677	−5.50
2,543	.844	.843	.063	840	907	−7.94
3,521	.879	.882	.036	974	1,061	−8.89
4,777	.906	.915	.023	1,108	1,209	−9.09
5,629	.918	.926	−.008	1,183	1,281	−8.29
6,600	.928	.933	−.006	1,258	1,353	−7.59
7,429	.935	.950	−.016	1,315	1,409	−7.12
8,500	.942	.955	−.013	1,381	1,462	−5.86
9,736	.948	.966	−.018	1,449	1,518	−4.74
10,430	.951	.968	−.018	1,485	1,542	−3.85
11,279	.954	.971	−.017	1,525	1,568	−2.85
12,572	.958	.973	−.015	1,582	1,606	−1.55
13,541	.961	.974	−.013	1,621	1,633	−.71
14,965	.964	.980	−.016	1,675	1,670	.27
15,501	.965	.981	−.016	1,694	1,681	.75
16,471	.967	.982	−.015	1,727	1,700	1.58
17,643	.969	.983	−.014	1,765	1,721	2.47
18,713	.970	.983	−.013	1,798	1,740	3.20
19,950	.972	.986	−.014	1,834	1,761	3.94
20,445	.972	.987	−.014	1,847	1,768	4.28
21,753	.974	.987	−.013	1,883	1,786	5.14
22,658	.975	.987	−.013	1,906	1,798	5.68
23,756	.976	.988	−.012	1,933	1,812	6.29
24,960	.977	.990	−.014	1,962	1,827	6.89
28,377	.979	.992	−.013	2,037	1,860	8.71
33,888	.982	.993	−.011	2,143	1,903	11.22
38,610	.984	.994	−.010	2,223	1,934	12.99
43,106	.986	.995	−.010	2,291	1,959	14.49
49,834	.987	.997	−.009	2,382	1,992	16.38
51,146	.988	.997	−.009	2,399	1,997	16.77
59,813	.989	.997	−.008	2,500	2,024	19.03
64,247	.990	.997	−.008	2,547	2,037	20.01
70,000	.991	.997	−.007	2,604	2,053	21.16
75,000	.991	.998	−.007	2,650	2,066	22.03
78,618	.991	.998	−.006	2,681	2,074	22.63
82,425	.992	.998	−.006	2,713	2,083	23.23
99,404	.993	.998	−.005	2,842	2,118	25.48
104,556	.993	.999	−.005	2,877	2,126	26.11
120,000	.994	.999	−.005	2,974	2,148	27.78
124,463	.994	.999	−.005	3,000	2,154	28.20
150,000	.995	.999	−.004	3,135	2,184	30.33
155,128	.995	.999	−.004	3,160	2,190	30.70
161,000	.995	.999	−.004	3,187	2,195	31.14
173,398	.996	.999	−.004	3,242	2,205	32.00
197,495	.996	.999	−.003	3,340	2,223	33.45
233,449	.997	.999	−.003	3,468	2,246	35.23
250,000	.997	.999	−.003	3,521	2,256	35.94
252,800	.997	1.000	−.003	3,530	2,257	36.05
273,747	.997	1.000	−.002	3,593	2,267	36.88
287,540	.997	1.000	−.002	3,631	2,274	37.39
300,000	1.000	1.000	.000	3,665	2,279	37.82

Beta is 347.2
Delta is 0.8768

APPENDIX F

Part 1

ISO Owners, Landlords and Tenants Bodily Injury Liability

Loss Amount Data: Policy Year 1976 as of March 31, 1978
Policy Limit $500,000

Loss Amount	Number of Losses	Average Loss Amount
0- 250	3,977	83
250- 500	1,095	374
500- 1,000	1,152	774
1,000- 2,000	991	1,488
2,000- 3,000	594	2,520
3,000- 4,000	339	3,538
4,000- 5,000	307	4,770
5,000- 6,000	103	5,542
6,000- 7,000	79	6,477
7,000- 8,000	141	7,568
8,000- 9,000	52	8,674
9,000- 10,000	89	9,853
10,000- 11,000	23	10,420
11,000- 12,000	22	11,744
12,000- 13,000	23	12,551
13,000- 14,000	6	13,733
14,000- 15,000	51	14,960
15,000- 16,000	6	15,374
16,000- 17,000	5	16,700
17,000- 18,000	9	17,634
18,000- 19,000	3	18,801
19,000- 20,000	31	19,973
20,000- 21,000	2	20,502
21,000- 22,000	2	21,926
22,000- 23,000	5	22,530
23,000- 24,000	4	23,745
24,000- 25,000	31	24,968
25,000- 30,000	11	29,391
30,000- 35,000	18	34,249
35,000- 40,000	9	38,564
40,000- 45,000	4	43,718
45,000- 50,000	11	49,814
50,000- 55,000	3	52,333
55,000- 60,000	2	60,000
70,000- 75,000	9	74,750
75,000- 80,000	1	75,003
95,000-100,000	4	99,913
120,000-130,000	2	125,000
140,000-150,000	3	150,000
190,000-200,000	1	200,000
200,000-210,000	2	202,453
220,000-230,000	1	225,000
240,000-250,000	2	250,000
260,000-270,000	1	270,000
280,000-290,000	1	290,000
290,000-300,000	2	300,000
340,000-350,000	1	350,000
410,000-420,000	2	414,619
500,000 and over	0	500,000
Total	9,232	2,410

APPENDIX F

Part 2

ISO Owners, Landlords and Tenants Bodily Injury Liability

Loss Amount Data: Policy Year 1975 as of March 31, 1978
Policy Limit $300,000

Loss Amount	Number of Losses	Average Loss Amount
0- 250	12,075	83
250- 500	3,420	381
500- 1,000	3,245	771
1,000- 2,000	2,623	1,509
2,000- 3,000	1,546	2,535
3,000- 4,000	877	3,557
4,000- 5,000	823	4,710
5,000- 6,000	308	5,471
6,000- 7,000	225	6,526
7,000- 8,000	384	7,451
8,000- 9,000	142	8,489
9,000- 10,000	279	9,792
10,000- 11,000	69	10,473
11,000- 12,000	76	11,711
12,000- 13,000	69	12,141
13,000- 14,000	30	13,750
14,000- 15,000	154	14,937
15,000- 16,000	32	15,574
16,000- 17,000	17	16,617
17,000- 18,000	33	17,601
18,000- 19,000	17	18,626
19,000- 20,000	91	19,907
20,000- 21,000	17	20,578
21,000- 22,000	9	21,900
22,000- 23,000	19	22,758
23,000- 24,000	12	23,667
24,000- 25,000	88	24,963
25,000- 30,000	65	28,364
30,000- 35,000	45	31,998
35,000- 40,000	41	39,018
40,000- 45,000	18	42,848
45,000- 50,000	61	49,721
50,000- 55,000	9	52,953
55,000- 60,000	6	59,568
60,000- 65,000	6	63,489
65,000- 70,000	6	67,598
70,000- 75,000	14	74,920
75,000- 80,000	2	78,260
80,000- 85,000	4	83,890
85,000- 90,000	7	88,705
90,000- 95,000	3	94,196
95,000-100,000	7	99,857
100,000-110,000	6	105,439
110,000-120,000	2	120,000
120,000-130,000	5	125,391
130,000-140,000	4	136,889
140,000-150,000	3	149,882
150,000-160,000	3	154,748
160,000-170,000	1	168,140
170,000-180,000	3	175,000
180,000-190,000	1	185,000
190,000-200,000	2	200,000
200,000-210,000	1	203,765
210,000-220,000	2	212,017
220,000-230,000	1	225,000
250,000-260,000	1	260,000
270,000-280,000	1	275,146
290,000-300,000	1	294,054
300,000 and over	6	300,000
Total	27,017	2,329

APPENDIX F

Part 3

ISO Owners, Landlords and Tenants Bodily Injury Liability

Loss Amount Data: Policy Year 1975 as of March 31, 1978
Policy Limit $500,000

Loss Amount	Number of Losses	Average Loss Amount
0- 250	3,286	78
250- 500	837	389
500- 1,000	928	774
1,000- 2,000	687	1,498
2,000- 3,000	412	2,577
3,000- 4,000	267	3,593
4,000- 5,000	263	4,757
5,000- 6,000	89	5,569
6,000- 7,000	64	6,631
7,000- 8,000	108	7,543
8,000- 9,000	35	8,563
9,000- 10,000	83	9,904
10,000- 11,000	15	10,432
11,000- 12,000	22	11,667
12,000- 13,000	22	12,624
13,000- 14,000	15	13,517
14,000- 15,000	52	14,945
15,000- 16,000	11	15,364
16,000- 17,000	5	16,749
17,000- 18,000	15	17,650
18,000- 19,000	1	19,000
19,000- 20,000	27	19,918
20,000- 21,000	7	20,351
21,000- 22,000	1	22,000
22,000- 23,000	4	22,487
23,000- 24,000	1	24,000
24,000- 25,000	33	24,980
25,000- 30,000	11	27,915
30,000- 35,000	9	33,655
35,000- 40,000	18	38,794
40,000- 45,000	7	43,611
45,000- 50,000	6	49,917
55,000- 60,000	2	60,000
60,000- 65,000	5	63,075
65,000- 70,000	2	67,090
70,000- 75,000	9	74,294
75,000- 80,000	3	75,900
80,000- 85,000	3	82,016
85,000- 90,000	2	87,505
95,000-100,000	5	98,586
110,000-120,000	3	116,177
120,000-130,000	1	123,528
140,000-150,000	1	150,000
150,000-160,000	2	150,100
240,000-250,000	1	250,000
290,000-300,000	1	300,000
300,000-310,000	1	309,000
330,000-340,000	1	335,675
340,000-350,000	1	349,910
480,000-490,000	1	483,840
500,000 and over	3	500,000
Total	7,388	2,849

APPENDIX G

Part 1

ISO Owners, Landlords and Tenants Bodily Injury Liability

Loss Amount Data: Policy Year 1975 as of March 31, 1978
Policy Limit $300,000
Trended to July 1, 1980 by 18.9% per annum

	K-S Test			EVC Test		
				Means		EVC
X	Pareto CDF F(X)	Sample CDF S(X)	Maximum Difference	Pareto	Sample	Statistic (%)
17,434	.000	.000	.000	0	0	.00
18,499	.066	.095	.066	1,028	1,064	−3.49
21,339	.211	.281	.116	3,466	3,635	−4.88
22,825	.272	.328	−.055	4,591	4,702	−2.42
25,523	.365	.378	.038	6,425	6,517	−1.43
26,460	.393	.424	−.032	7,006	7,099	−1.32
29,965	.480	.445	.056	8,975	9,116	−1.58
31,945	.521	.467	.076	9,962	10,216	−2.54
32,553	.532	.570	.065	10,250	10,540	−2.83
36,213	.592	.582	.022	11,848	12,113	−2.24
38,358	.622	.604	.040	12,691	13,011	−2.52
40,593	.649	.615	.045	13,506	13,897	−2.90
43,383	.678	.676	.063	14,444	14,971	−3.65
44,846	.692	.687	.016	14,905	15,445	−3.63
47,727	.716	.693	.029	15,756	16,347	−3.75
49,597	.731	.706	.037	16,273	16,920	−3.98
51,579	.744	.714	.039	16,793	17,503	−4.23
54,401	.762	.773	.048	17,489	18,311	−4.70
61,814	.800	.816	.027	19,105	19,995	−4.66
69,734	.830	.846	−.016	20,563	21,451	−4.32
85,031	.871	.874	.025	22,822	23,803	−4.30
93,379	.887	.886	.013	23,830	24,858	−4.31
108,357	.908	.926	.023	25,355	26,570	−4.79
115,401	.916	.932	−.016	25,973	27,088	−4.29
129,817	.929	.936	−.008	27,087	28,061	−3.60
138,363	.935	.941	−.005	27,668	28,604	−3.38
147,316	.941	.945	−.004	28,223	29,136	−3.23
163,273	.949	.954	−.005	29,104	30,022	−3.15
170,553	.952	.955	−.003	29,466	30,357	−3.03
182,822	.956	.958	−.002	30,028	30,907	−2.93
193,316	.960	.963	−.003	30,468	31,349	−2.89
205,281	.963	.965	−.002	30,929	31,797	−2.80
217,619	.966	.969	−.003	31,366	32,234	−2.77
229,784	.969	.973	−.005	31,763	32,608	−2.66
261,517	.974	.975	.001	32,671	33,456	−2.41
273,266	.976	.978	−.002	32,967	33,755	−2.39
298,323	.978	.981	−.002	33,542	34,308	−2.28
326,640	.981	.983	−.001	34,112	34,856	−2.18
337,243	.982	.985	−.003	34,308	35,041	−2.14
366,429	.984	.985	−.001	34,803	35,489	−1.97
381,379	.985	.987	−.002	35,035	35,709	−1.92
403,172	.986	.988	−.002	35,351	35,986	−1.80
435,861	.988	.989	−.002	35,780	36,379	−1.68
444,066	.988	.990	−.002	35,880	36,467	−1.64
462,049	.989	.991	−.003	36,092	36,647	−1.54
490,344	.990	.992	−.002	36,400	36,893	−1.35
566,620	.992	.993	−.001	37,117	37,505	−1.04
599,628	.992	.993	−.001	37,386	37,748	−.97
640,834	.993	.994	−.001	37,692	38,023	−.88
653,792	1.000	1.000	.000	37,782	38,101	−.85

Beta is 4955.2
Delta is 1.4728
The Truncation Point is 17,434
The Censorship Point is 653,792
The Sample Size is 1,496
Kolmogorov-Smirnov Test Statistic is 0.1159

APPENDIX G

Part 2

ISO Owners, Landlords and Tenants Bodily Injury Liability

Loss Amount Data: Policy Year 1975 as of March 31, 1978
Policy Limit $500,000
Trended to July 1, 1980 by 18.9% per annum

	K-S Test			EVC Test		
				Means		EVC Statistic
X	Pareto CDF $F(X)$	Sample CDF $S(X)$	Maximum Difference	Pareto	Sample	(%)
17,434	.000	.000	.000	0	0	.00
18,661	.076	.078	.076	1,179	1,227	−4.02
21,584	.222	.263	.143	3,657	3,921	−7.20
22,734	.269	.297	−.028	4,525	4,768	−5.37
25,425	.362	.346	.065	6,363	6,660	−4.67
27,510	.421	.395	.075	7,629	8,024	−5.17
29,457	.469	.429	.074	8,708	9,201	−5.67
32,569	.533	.545	.104	10,258	10,979	−7.03
33,482	.549	.569	−.020	10,677	11,396	−6.73
36,501	.596	.580	.027	11,965	12,696	−6.11
38,465	.623	.614	.043	12,731	13,520	−6.20
41,407	.658	.616	.044	13,788	14,656	−6.30
43,408	.678	.676	.062	14,452	15,424	−6.73
44,352	.687	.692	.011	14,751	15,730	−6.63
47,945	.718	.694	.026	15,818	16,837	−6.44
49,006	.726	.703	.032	16,112	17,161	−6.51
52,303	.749	.705	.046	16,976	18,140	−6.86
54,438	.762	.779	.057	17,498	18,769	−7.27
60,836	.796	.804	.017	18,907	20,183	−6.75
73,344	.842	.824	.038	21,154	22,640	−7.02
84,544	.870	.864	.046	22,759	24,615	−8.15
95,042	.890	.879	.026	24,016	26,044	−8.45
108,784	.909	.893	.029	25,394	27,701	−9.08
130,758	.930	.897	.037	27,153	30,055	−10.69
137,460	.934	.908	.037	27,609	30,743	−11.35
146,210	.940	.913	.031	28,157	31,544	−12.03
161,910	.948	.933	.035	29,034	32,911	−13.35
165,409	.950	.940	.017	29,213	33,145	−13.46
178,737	.955	.946	.015	29,847	33,948	−13.74
190,700	.959	.951	.013	30,362	34,589	−13.92
214,849	.965	.962	.015	31,271	35,775	−14.40
253,186	.973	.969	.011	32,448	37,230	−14.74
269,205	.975	.971	.006	32,867	37,730	−14.80
326,896	.981	.973	.010	34,117	39,404	−15.50
327,114	.981	.978	.008	34,121	39,410	−15.50
544,827	.991	.980	.013	36,928	44,270	−19.88
653,792	.993	.982	.013	37,782	46,459	−22.97
673,406	.993	.984	.011	37,914	46,809	−23.46
731,539	.994	.987	.010	38,274	47,718	−24.67
762,561	.995	.989	.008	38,449	48,133	−25.19
1,054,436	.997	.991	.008	39,707	51,391	−29.42
1,089,653	1.000	1.000	.000	39,825	51,705	−29.83

Beta is 4955.2
Delta is 1.4728
The Truncation Point is 17,434
The Censorship Point is 1,089,653
The Sample Size is 448
Kolmogorov-Smirnov Test Statistic is 0.1433

APPENDIX G

Part 3

ISO Owners, Landlords and Tenants Bodily Injury Liability

Loss Amount Data: Policy Year 1976 as of March 31, 1978
Policy Limit $300,000
Trended to July 1, 1980 by 18.9% per annum

	K-S Test			EVC Test		
				Means		EVC Statistic
X	Pareto CDF $F(X)$	Sample CDF $S(X)$	Maximum Difference	Pareto	Sample	(%)
14,663	.000	.000	.000	0	0	.00
15,580	.065	.094	.065	886	916	−3.43
17,845	.199	.324	−.125	2,846	2,969	−4.31
19,118	.260	.371	−.111	3,826	3,829	−.09
20,673	.325	.418	−.092	4,924	4,807	2.39
23,044	.408	.456	−.049	6,423	6,188	3.66
24,819	.459	.473	−.014	7,427	7,153	3.70
27,430	.522	.597	−.075	8,754	8,529	2.57
28,412	.543	.620	−.078	9,214	8,925	3.14
30,189	.576	.633	−.057	9,996	9,600	3.97
32,338	.612	.653	−.041	10,867	10,387	4.42
34,298	.640	.661	−.021	11,600	11,067	4.60
36,565	.669	.722	−.053	12,383	11,836	4.42
37,473	.679	.735	−.056	12,679	12,089	4.66
39,871	.704	.741	−.037	13,419	12,725	5.17
41,530	.719	.750	−.031	13,897	13,156	5.34
43,542	.736	.754	−.017	14,444	13,658	5.45
45,749	.753	.811	−.058	15,008	14,201	5.37
52,011	.792	.848	−.056	16,426	15,383	6.35
62,113	.836	.872	−.036	18,288	16,922	7.47
70,768	.863	.893	−.030	19,582	18,027	7.94
79,008	.883	.905	−.023	20,627	18,909	8.32
91,340	.904	.938	−.034	21,936	20,078	8.47
93,746	.907	.941	−.033	22,163	20,226	8.74
109,630	.926	.947	−.022	23,479	21,168	9.84
117,757	.933	.951	−.018	24,054	21,597	10.21
128,302	.940	.951	−.011	24,720	22,118	10.53
137,467	.946	.958	−.012	25,239	22,563	10.60
144,098	.950	.960	−.010	25,585	22,842	10.72
151,076	.953	.964	−.011	25,926	23,124	10.81
182,197	.964	.974	−.010	27,208	24,252	10.87
191,640	.966	.976	−.010	27,537	24,500	11.03
219,947	.972	.977	−.004	28,397	25,177	11.34
228,127	.974	.981	−.007	28,616	25,365	11.36
274,934	.980	.984	−.004	29,683	26,252	11.56
284,333	.981	.988	−.007	29,866	26,399	11.61
295,095	.982	.988	−.006	30,065	26,532	11.75
317,820	.984	.990	−.006	30,453	26,794	12.01
361,987	.987	.992	−.005	31,102	27,231	12.45
427,886	.990	.993	−.004	31,883	27,774	12.89
458,223	.991	.994	−.004	32,186	27,974	13.09
463,355	.991	.995	−.004	32,234	28,003	13.13
501,748	.992	.996	−.004	32,572	28,193	13.45
527,029	.992	.997	−.004	32,775	28,297	13.66
549,867	1.000	1.000	.000	32,947	28,372	13.88

Beta is 4955.2
Delta is 1.4728
The Truncation Point is 14,663
The Censorship Point is 549,867
The Sample Size is 1,214
Kolmogorov-Smirnov Test Statistic is 0.1251

APPENDIX G

Part 4

ISO Owners, Landlords and Tenants Bodily Injury Liability

Loss Amount Data: Policy Year 1976 as of March 31, 1978
Policy Limit $500,000
Trended to July 1, 1980 by 18.9% per annum

	K-S Teat			EVC Test		
				Means		
X	Pareto CDF *F*(*X*)	Sample CDF *S*(*X*)	Maximum Difference	Pareto	Sample	EVC Statistic (%)
14,663	.000	.000	.000	0	0	.00
15,899	.086	.114	.086	1,181	1,235	−4.61
18,060	.210	.309	−.100	3,017	3,150	−4.41
19,099	.259	.360	−.100	3,812	3,867	−1.46
21,526	.357	.408	−.051	5,486	5,422	1.17
23,005	.407	.458	−.052	6,400	6,298	1.59
25,172	.468	.471	.010	7,616	7,471	1.91
27,420	.522	.583	−.062	8,750	8,659	1.03
28,179	.538	.596	−.059	9,107	8,976	1.44
30,609	.584	.607	−.024	10,172	9,956	2.13
32,322	.611	.627	−.016	10,861	10,628	2.14
34,460	.642	.634	.015	11,658	11,426	2.00
36,608	.669	.702	.035	12,397	12,212	1.49
37,578	.680	.706	−.026	12,713	12,502	1.66
40,187	.707	.711	−.004	13,512	13,268	1.80
41,294	.717	.721	.007	13,831	13,589	1.75
43,522	.736	.730	.015	14,439	14,209	1.59
45,763	.753	.798	−.045	15,011	14,814	1.31
53,871	.802	.822	−.021	16,804	16,450	2.11
62,775	.839	.862	−.023	18,396	18,031	1.98
70,684	.863	.882	−.019	19,571	19,124	2.28
80,130	.885	.890	−.006	20,757	20,242	2.48
91,304	.904	.914	.014	21,932	21,468	2.12
95,921	.910	.921	−.011	22,361	21,863	2.23
109,973	.926	.925	.005	23,505	22,972	2.27
137,009	.946	.945	.020	25,215	24,988	.90
137,472	.946	.947	−.001	25,240	25,013	.90
183,129	.964	.956	.017	27,242	27,416	−.64
229,111	.974	.961	.018	28,642	29,433	−2.76
274,934	.980	.967	.020	29,683	31,242	−5.25
366,578	.987	.969	.020	31,163	34,256	−9.93
371,074	.987	.974	.018	31,222	34,394	−10.16
412,400	.989	.976	.015	31,716	35,482	−11.87
458,223	.991	.980	.015	32,186	36,587	−13.68
494,880	.992	.982	.011	32,515	37,311	−14.75
531,538	.992	.985	.010	32,810	37,954	−15.68
549,867	.993	.989	.008	32,947	38,235	−16.05
641,512	.994	.991	.005	33,543	39,240	−16.99
759,951	.995	.996	.004	34,150	40,279	−17.95
916,445	1.000	1.000	.000	34,769	40,965	−17.82

Beta is 4955.2
Delta is 1.4728
The Truncation Point is 14,663
The Censorship Point is 916,445
The Sample Size is 456
Kolmogorov-Smirnov Test Statistic is 0.1003

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AN ANALYSIS OF RETROSPECTIVE RATING

GLENN G. MEYERS

I. INTRODUCTION

The purpose of this paper is to address the following question: should the present retrospective rating formula be modified to account for the claim severity distribution for the risk being insured, and for the loss limit chosen for the plan? It will be shown that there are significant differences in premium adequacy that can be attributed to the above mentioned factors. Alternatives to the present formula will be proposed.

The Present Retrospective Rating Formula

The premium for an insured written under a retrospective rating plan is given by the following formula. This formula is generally used in Workers' Compensation insurance.

$$R = [(P \times b) + (P \times c \times e) + (c \times A)] \times t$$

subject to a minimum of $h \times P$ and a maximum of $g \times P$,

where:

R = Retrospective Premium,
P = Standard Premium,
b = Basic Premium Factor,
c = Loss Conversion Factor,
e = Excess Loss Premium Factor,
A = Actual Limited Losses,
t = Tax Multiplier,
h = Minimum Premium Factor, and
g = Maximum Premium Factor.

In some plans, losses arising out of a single accident are limited to a specified amount before entering the retrospective premium calculation. The excess loss premium factor provides for the cost of this loss limit.

The basic premium factor can be written as follows:

$b = a + (c \times i)$.

The factor a provides for acquisition expenses, general underwriting expenses and profit. The factor i is called the insurance charge. This factor provides for the net cost of limiting the retrospective premium between the minimum and maximum premiums.

The standard formula for calculating the insurance charge does not take into account the claim severity distribution of the individual insured, nor does it take into account the loss limit selected for the plan.[1] In other words, the insurance charge, as calculated by the standard formula, will be the same no matter what claim severity distribution applies to the insured, or what loss limit is used.

Given two insureds with the same expected loss, the loss experience will be more volatile for a high severity, low frequency insured than for a low severity, high frequency insured. Since a high severity, low frequency insured will "break the maximum" more often, he should have a higher insurance charge than an otherwise comparable low severity, high frequency insured.

The insurance charge includes a provision for that portion of the losses which exceed any potential loss limit. But, in a plan which has a loss limit, these losses are provided for by the excess loss premium factor. Thus, a plan with a loss limit should have a lower insurance charge than a plan with no loss limit.

It has long been recognized that these factors can significantly affect the adequacy of the retrospective premium. Perhaps the main reason the rating formula has not been modified is that it would involve making an already complex rating formula even more complex. According to one account, it could require 200,000 pages of tables to properly calculate the insurance charge.[2]

Another problem is inherent in the way data has been gathered under the present formula. The distribution of loss ratios is tabulated by direct observation. This allows one observation per insured each year. If one were to create categories of insureds and tabulate the experience for each of the categories, he might well find that the experience is not credible.

[1] National Council of Compensation Insurance, *Retrospective Rating Plan D*.

[2] An excellent discussion of these issues can be found in "The California Table L," *PCAS* LXI, by David Skurnick, and the ensuing discussions by Frank Harwayne and Richard H. Snader.

The general approach taken by this paper will be to build a mathematical model of the loss process. This model will be used to generate annual losses for different kinds of insureds. We will then quantify differences in premium adequacy that can be attributed to the factors mentioned above. Following that we will explore modifications to the current formula which can more adequately price a retrospective rating plan.

II. THE MODEL

The Generalized Poisson Distribution

The Generalized Poisson distribution will be used to model the loss process.[3] This model is based on the following assumptions:

1. The number of claims has a Poisson distribution, and
2. Claim severity is independent of claim frequency.

Three claim severity distributions have been selected. These distributions will represent a standard insured, a high severity insured and a low severity insured. The distributions are given in Exhibit I. These distributions are hypothetical ones selected by the author.

The following information is needed to generate a distribution of annual losses: (1) the expected losses, (2) the claim severity distribution, and (3) the loss limit. Sample values for the distribution are calculated by the following steps.

1. Calculate the average claim size from the claim severity distribution.
2. Calculate the parameter, λ, for the Poisson distribution, where

   λ = Expected Losses/Average Claim Size.

3. For each sample do the following.
   - 3.1 Randomly select the number of claims, n, from the Poisson distribution.
   - 3.2 Do the following n times.
     - 3.2.1 Randomly select a claim amount from the claim severity distribution.
     - 3.2.2 Adjust the claim amount for the loss limit.
   - 3.3 The sample loss amount is the sum of all claim amounts generated by step 3.2.

[3] R. E. Beard, T. Pentikainen and E. Pesonen, *Risk Theory*, Chapman and Hall Ltd. (1977), Ch. 3.

The annual loss distributions used in this paper are "empirical" ones consisting of 10,000 samples.

The use of the Poisson distribution for the number of claims deserves some comment. The author chose this distribution because of its widespread use in the actuarial literature. The author has no evidence that the Poisson distribution is the most appropriate. However, if some other distribution is chosen, one should expect only a slight increase in the variance of the annual loss distribution.[4] Thus the results of this paper should hold even if this assumption is changed.

The major results of this paper will be based on the difference between insureds represented by the claim severity distributions in Exhibit I. No attempt has been made to fit this model to live data.

However, it was the author's intention to select a realistic model. Using Exhibits II (Table A) and III, one can compare the results of this model with the present retrospective rating formula. Exhibit II (Table A) provides the excess loss premium factors derived from the claim severity distributions in Exhibit I. Exhibit III gives the insurance charges calculated using the standard formula and by a method (to be described below) using the claim severity distribution for the standard insured.

Adequacy of the Retrospective Premium

When given the parameters of the retrospective rating plan and the 10,000 loss samples generated by the model, it is possible to calculate the average retrospective premium generated by the plan. Similarly, one can calculate the average premium that would be generated by a "cost-plus" rating plan (i.e. a retrospective rating plan with no minimum or maximum premium). The premium for a "cost-plus" rating plan is given by the following formula:

$$CP = [(P \times a) + (P \times c \times e') + (c \times A)] \times t,$$

where e' is the "correct" excess loss premium factor as derived from the claim severity distribution.

The retrospective premium adequacy of a plan (*RPA*) can be defined as follows:

$$RPA = \frac{\text{Average ``Cost-Plus'' Premium}}{\text{Average Retrospective Premium}}$$

[4] R. S. Miccolis, "On the Theory of Increased Limits and Excess of Loss Pricing," *PCAS* LXIV, p. 43.

The retrospective premium adequacy of a plan is a measure of its profitability. If the retrospective premium adequacy is less than 1.00, the insurer should expect to make more than the budgeted profit. Conversely, if the retrospective premium adequacy is greater than 1.00, the insurer should expect to make less than the budgeted profit.

If all the parameters of a retrospective rating plan are given except the insurance charge, the retrospective premium adequacy can be thought of as a function of the insurance charge. To use the model to find the insurance charge one solves the equation $RPA(i) = 1$. This equation can be solved by standard numerical methods.[5] It should be pointed out that solving this equation by hand would be extremely difficult due to the large number of terms involved. However, solving this equation by computer has proved to be very speedy and reliable. It should also be pointed out that this method of finding the insurance charge can easily be adapted to other kinds of retrospective rating formulas.

III. AN ANALYSIS OF THE CURRENT FORMULA

Like it or not, we already have a formula for retrospective rating in use. With some minor exceptions, this formula is used on a countrywide basis for Workers' Compensation.

Since the price of a retrospective rating plan is fixed, the problem becomes one of risk selection. This section seeks to identify those insureds which can profitably be written under a retrospective rating plan.

Another particularly troublesome problem with the current formula is that many people feel that the excess loss premium factors currently in use are inadequate. This section will show how to quantify the effect of such an inadequacy.

A Model of the Current Procedure

Ideally, the current retrospective rating formula can be described as follows. A single loss distribution is chosen to represent all insureds with a given expected loss amount. The insurance charge is calculated from this loss distribution on the assumption that no loss limit will be used. This insurance charge is used whether or not a loss limit is actually used in the plan.

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[5] The author used the Modified Regula Falsi method, which is described in *Elementary Numerical Analysis: An Algorithmic Approach*, McGraw Hill Inc. (1972), by S. D. Conte and Carl de Boor.

The current formula will thus be modeled as follows. The standard claim severity distribution will be used to calculate insurance charges. They are given in the last column of Exhibit III. These insurance charges will be used to evaluate the retrospective premium adequacy of a plan no matter what the insured's claim severity distribution is and no matter what loss limit is selected.

Exhibit V shows the retrospective premium adequacy for the high and low severity insureds when there is no loss limit. As can be seen from this exhibit, there are substantial differences in the retrospective premium adequacy that can be attributed to differences in claim severity. Clearly it is not desirable for the insurer to write a high severity insured on such a retrospective rating plan.

Exhibit VI shows the retrospective premium adequacy for plans which have a loss limit. As can be seen from the exhibit, the overlap between the excess loss premium factor and the insurance charge results in a very favorable retrospective premium adequacy from the viewpoint of the insurer. This is true even for the high severity insureds which fared poorly when there were no loss limits.

The Effect of Inadequate Excess Loss Premium Factors

After examining Exhibit VI, one might conclude that an insurer should require loss limits on all retrospective rating plans. However, there are some problems with this strategy. In talking with various actuaries and underwriters who work in Workers' Compensation, the author has found many who believe that the excess loss premium factors currently in use are inadequate. To get some idea of the effect of inadequate excess loss premium factors, the author calculated the retrospective premium adequacy of plans with the excess loss premium factors cut in half. The results are shown in Exhibit VII.

The results of these calculations show that, in some cases, it still may be more profitable to write an insured with a loss limit. The profitability of a plan depends upon the balance between the amount of inadequacy in the excess loss premium factors and the redundancy in the insurance charge. This balance is more favorable to the insurer in plans with a low maximum premium. It should also be noted that this balance works against the insurer for the larger premium sizes.

If an underwriter is concerned about inadequate excess loss premium factors, he should encourage the insured to take a plan with a high maximum premium and no loss limit, or a plan with a low maximum premium and a loss limit. The author has discussed this underwriting strategy with both underwriting and marketing personnel. They thought that neither of these programs is marketable.

It should be clear why a plan with a high maximum would not sell. The marketability of the low maximum plan with a loss limit deserves some comment.

When deciding whether or not to purchase a plan with a loss limit, the insured will look at his past experience and see what he would have paid under each plan. Exhibits VIII and IX provide such a price comparison based on the 10,000 samples generated by the loss model. These exhibits show calculations of the retrospective premium at various percentiles. It should be noted that the insured in this example is paying \$25,062 in excess premium in the plan with a \$30,000 loss limit. In examining these exhibits one can see that at every percentile the insured would be paying a premium for the plan with a loss limit greater than or equal to the premium for the plan with no loss limit. The only time there is equality is when both plans pay the maximum premium.

Thus it appears that the normal insured would prefer the plan without a loss limit. However, a plan with a loss limit would be acceptable to an insured who has experienced a severe loss and is afraid of another one.

The possibility of adverse selection in plans with a loss limit is something that can be tested. What is required is a comparison between claim severity distributions for insureds who have and who have not purchased a plan with a loss limit. The author has not seen such a comparison.

Adverse selection could provide an explanation for inadequate excess loss premium factors.

IV. OTHER RETROSPECTIVE RATING FORMULAS

Insurance Charges Which Reflect Claim Severity and Loss Limits

Given the differences in the retrospective premium adequacy of the various plans mentioned above, it is natural to ask what the insurance charge should be in order to accurately reflect differences due to claim severity and loss limits. Exhibits X and XI provide the proper insurance charges.

The taking into account of differences due to claim severity presents the problem of rating different exposures which are under the same retrospective plan. To do this, one can simply sum the losses incurred by each separate exposure and then proceed as usual. Exhibit XV (Table A) provides calculations of insurance charges for an insured with standard premiums of \$150,000 in a class represented by the high severity distribution and $50,000 in each of the two classes represented by the low severity distribution and the standard distribution. This method can easily be generalized to cases where the expense factors and loss limits are different for each class.

While this method of calculating the insurance charge does not require an excessive number of tables, it does require a great deal of computer time. The overwhelming majority of the computer time is consumed by generating the distribution of annual losses. The author is aware of quicker ways to generate losses, which deserve serious consideration.[6]

Retrospective Rating Plans Which Require a Loss Limit

In his observations of Exhibit XI, the reader may have already noticed that the insurance charges for plans with the same standard premium and loss limit are nearly equal.[7] The difference in the price for insureds with different claim severity distributions can be attributed almost entirely to the excess loss premium factor. This is true because we are substituting a fixed excess premium for the most volatile part of the actual losses.

This observation suggests that, when using a fixed loss limit, one can devise a retrospective rating formula for which the differences in the insurance charges due to claim severity can be kept to an acceptable minimum. This plan would simply use the insurance charge calculated for the standard insured as the insurance charge for all insureds. Each insured would still use the appropriate excess loss premium factor. The retrospective premium adequacies for various insureds under such a plan are given in Exhibits XII and XV (Table B).

The author would also propose that the insured not be given a choice of loss limits. This would minimize the number of tables needed to calculate the insurance charge. The loss limit would be determined by the total expected losses of the insured. Furthermore, if it is determined that adverse selection is a cause of inadequate excess loss premium factors, it may be necessary to require that all insureds have the same loss limit.

If we are to require that a specific loss limit be used for a given insured, we should try to choose a loss limit that will be acceptable to a majority of the insureds. It may be desirable to calculate excess losses by the following formula.

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[6] R. E. Beard, T. Pentikainen and E. Pesonen, *op. cit.*, Ch. 7.

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[7] The reader should note the different definitions of the insurance charge that are in the literature. Skurnick's insurance charge provides for both the excess losses on individual claims and the effect of limiting the retrospective premium. Harwayne suggests reducing the excess loss premium factor to account for the overlap.

Let L be the total loss arising out of a single accident.

$$\text{If } L \leq A \begin{cases} \text{Primary Loss} = L \\ \text{Excess Loss} = 0 \end{cases}$$

$$\text{If } L > A \begin{cases} \text{Primary Loss} = (L \times B)/(L + B - A) \\ \text{Excess Loss} = L - \text{Primary Loss} \end{cases}$$

In this case we say the loss limit is $(A : B)$.

One can see that primary portion of the loss will be between A and B when the loss is greater than A. This formula is similar to the one used in multi-split experience rating for Workers' Compensation.

Exhibits XIII and XIV show calculations of the insurance charge and the retrospective premium adequacy for plans with a dual loss limit. It should be noted that a more restrictive loss limit allows less variance in the retrospective premium adequacy. The selection of a required loss limit will depend upon what will be acceptable to a majority of insureds and upon how much variance in the retrospective premium adequacy the insurer is willing to tolerate.

V. CONCLUSION

This paper discusses three options which can be taken with regard to the retrospective rating formula.

The first option is to leave the present formula unchanged. If this option is elected, a retrospective rating plan will produce premium deficiencies for high severity insureds, while it may produce premium redundancies for plans which have a loss limit. Such plans are not appropriate for high severity insureds.

The second option is to replace the present formula with one that properly accounts for claim severity and loss limits. This option would allow complete freedom in choosing the kind of plan to be used. The main drawback to this option is the large amount of computer time needed to calculate the insurance charge. It will be necessary to develop a more efficient loss generation program before this option can be implemented.

The third option is to restrict the number of plans available to the insured. This provides an immediate reduction in the number of tables needed. If we require that all retrospective rating plans have a loss limit, it turns out that the claim severity of an insured has only a slight effect on the insurance charge. Because of this it should not be necessary to have separate tables for each claim severity group in order to calculate the insurance charge. If a single loss limit is required, the resulting procedure should be no more complex than the present one. A single loss distribution and loss limit could be chosen to represent all insureds with a given expected loss amount.

This paper attempts to quantify the effect of each of these options. The author prefers a flexible formula like that mentioned in option two. Should this approach prove unworkable at the present time, the author would then choose option three. The present retrospective rating formula discards accuracy in order to maintain flexibility. The proposed formula discards flexibility in order to maintain accuracy.

This paper bases its conclusions on a computer simulation using hypothetical data. These techniques permitted a vast amount of experimentation with various retrospective rating plans. These conclusions are the results of this experimentation. Any concrete proposal for changing the current procedure must look at real data. The modification of the current procedure will be a very expensive and time consuming undertaking. It is hoped that this paper will convince the reader that such an undertaking is worth the effort.

The ideas expressed in this paper are the result of conversations the author has had with many people at his company. The author would like to thank these people for their contributions.

EXHIBIT I

CLAIM SEVERITY DISTRIBUTIONS

Claim Amount	Probability that a claim will be less than Column 1		
(1)	(2)	(3)	(4)
50	0.4310	0.3692	0.2464
100	0.5781	0.5147	0.4385
250	0.8561	0.8419	0.6195
500	0.8994	0.8835	0.8474
750	0.9175	0.9040	0.8684
1,000	0.9291	0.9155	0.8862
1,500	0.9455	0.9310	0.9050
2,500	0.9628	0.9495	0.9225
3,500	0.9718	0.9606	0.9348
5,000	0.9788	0.9704	0.9468
7,500	0.9846	0.9780	0.9592
10,000	0.9886	0.9824	0.9665
15,000	0.9935	0.9878	0.9748
25,000	0.9969	0.9936	0.9823
35,000	0.9982	0.9961	0.9862
50,000	0.9990	0.9977	0.9903
75,000	0.9995	0.9988	0.9941
100,000	0.9997	0.9992	0.9961
150,000	0.9998	0.9996	0.9977
250,000	1.0000	0.9998	0.9989
350,000	—	0.9999	0.9993
500,000	—	1.0000	1.0000
Mean	595	926	2269
Standard Deviation	4313	7608	16753

Column 2—Low Severity Insured
Column 3—Standard Insured
Column 4—High Severity Insured

It is assumed that the claim severity distribution is uniform between any two consecutive amounts in Column 1.

EXHIBIT II

TABLE A

Loss Limit	Excess Loss Premium Factor* Low Severity Insured	Standard Insured	High Severity Insured
10,000	0.191	0.270	0.391
15,000	0.146	0.222	0.353
20,000	0.118	0.187	0.322
25,000	0.098	0.162	0.296
30,000	0.084	0.143	0.274
40,000	0.064	0.116	0.237
50,000	0.052	0.098	0.208
75,000	0.033	0.070	0.156
100,000	0.023	0.053	0.124
150,000	0.010	0.034	0.083
200,000	0.003	0.023	0.056
250,000	—	0.015	0.038

TABLE B

Loss Limit**	Excess Loss Premium Factor* Low Severity Insured	Standard Insured	High Severity Insured
(2,000 : 20,000)	0.206	0.272	0.380
(5,000 : 60,000)	0.114	0.170	0.276
(10,000 : 100,000)	0.075	0.124	0.220
(10,000 : 20,000)	0.155	0.228	0.350
(30,000 : 60,000)	0.064	0.114	0.227
(50,000 : 100,000)	0.038	0.076	0.166

* Expected Loss Ratio = .600

** Excess losses for a dual loss limit $(A : B)$ are given by the following formula.

Let L be the total loss arising out of a single accident.

$$\text{If } L \leq A \begin{cases} \text{Primary Loss} = L \\ \text{Excess Loss} = 0 \end{cases}$$

$$\text{If } L > A \begin{cases} \text{Primary Loss} = (L \times B)/(L + B - A) \\ \text{Excess Loss} = L - \text{Primary Loss} \end{cases}$$

EXHIBIT III

COMPARISON OF INSURANCE-CHARGES INDICATED BY THE MODEL AND THE STANDARD FORMULA USING TABLE M.

Standard Premium = 50,000
No Loss Limit

Min.	Max.	Insurance Charge* Standard Formula	Model
BxTM	1.00	0.267	0.300
BxTM	1.20	0.173	0.219
BxTM	1.40	0.122	0.174
BxTM	1.60	0.090	0.144
BxTM	1.80	0.068	0.123
0.60	1.00	0.254	0.299
0.60	1.20	0.117	0.195
0.60	1.40	0.038	0.124
0.60	1.60	−0.016	0.071
0.60	1.80	−0.052	0.029

Standard Premium = 150,000
No Loss Limit

Min.	Max.	Insurance Charge* Standard Formula	Model
BxTM	1.00	0.173	0.179
BxTM	1.20	0.092	0.112
BxTM	1.40	0.059	0.079
BxTM	1.60	0.044	0.060
BxTM	1.80	0.029	0.047
0.60	1.00	0.150	0.171
0.60	1.20	0.047	0.087
0.60	1.40	0.000	0.043
0.60	1.60	−0.025	0.014
0.60	1.80	−0.042	−0.005

EXHIBIT III
(CONT.)

COMPARISON OF INSURANCE CHARGES INDICATED BY THE MODEL AND THE STANDARD FORMULA USING TABLE M.

Standard Premium = 250,000
No Loss Limit

		Insurance Charge*	
Min.	Max.	Standard Formula	Model
BxTM	1.00	0.130	0.128
BxTM	1.20	0.060	0.073
BxTM	1.40	0.033	0.048
BxTM	1.60	0.025	0.033
BxTM	1.80	0.015	0.023
0.60	1.00	0.099	0.119
0.60	1.20	0.012	0.054
0.60	1.40	−0.016	0.021
0.60	1.60	−0.032	0.001
0.60	1.80	−0.040	−0.014

* The parameters for the plans are given in Exhibit IV.

EXHIBIT IV

PARAMETERS FOR RETROSPECTIVE RATING PLANS

	Total Standard Premium		
	50,000	150,000	250,000
Expected Losses	30,000	90,000	150,000
Loss Conversion Factor (c)	1.125	1.125	1.125
Expense in Basic Premium Factor (a)	0.149	0.139	0.134
Tax Multiplier (t)	1.040	1.040	1.040

EXHIBIT V

Retrospective Premium Adequacy for Plans without a Loss Limit

Standard Premium = 50,000
No Loss Limit

		Retrospective Premium Adequacy*		
Min.	Max.	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.951	1.000	1.127
BxTM	1.20	0.936	1.000	1.161
BxTM	1.40	0.935	1.000	1.170
BxTM	1.60	0.937	1.000	1.170
BxTM	1.80	0.940	1.000	1.163
0.60	1.00	0.951	1.000	1.112
0.60	1.20	0.951	1.000	1.103
0.60	1.40	0.962	1.000	1.084
0.60	1.60	0.974	1.000	1.066
0.60	1.80	0.984	1.000	1.049

Standard Premium = 150,000
No Loss Limit

		Retrospective Premium Adequacy*		
Min.	Max.	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.951	1.000	1.119
BxTM	1.20	0.947	1.000	1.123
BxTM	1.40	0.953	1.000	1.113
BxTM	1.60	0.958	1.000	1.098
BxTM	1.80	0.962	1.000	1.085
0.60	1.00	0.956	1.000	1.078
0.60	1.20	0.964	1.000	1.052
0.60	1.40	0.976	1.000	1.028
0.60	1.60	0.987	1.000	1.008
0.60	1.80	0.994	1.000	0.992

EXHIBIT V
(CONT.)

RETROSPECTIVE PREMIUM ADEQUACY FOR PLANS WITH A LOSS LIMIT

Standard Premium = 250,000
No Loss Limit

		Retrospective Premium Adequacy*		
Min.	Max.	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.961	1.000	1.102
BxTM	1.20	0.961	1.000	1.095
BxTM	1.40	0.966	1.000	1.077
BxTM	1.60	0.972	1.000	1.061
BxTM	1.80	0.977	1.000	1.048
0.60	1.00	0.967	1.000	1.061
0.60	1.20	0.975	1.000	1.031
0.60	1.40	0.987	1.000	1.007
0.60	1.60	0.996	1.000	0.988
0.60	1.80	1.004	1.000	0.974

* The parameters for the plans are given in Exhibits III and IV.

EXHIBIT VI

RETROSPECTIVE PREMIUM ADEQUACY FOR PLANS WITH A LOSS LIMIT

Standard Premium = 50,000
Loss Limit = 10,000

		Retrospective Premium Adequacy*		
Min.	Max.	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.868	0.865	0.855
BxTM	1.20	0.814	0.811	0.800
BxTM	1.40	0.819	0.818	0.813
BxTM	1.60	0.838	0.838	0.836
BxTM	1.80	0.857	0.856	0.856
0.60	1.00	0.868	0.865	0.855
0.60	1.20	0.829	0.827	0.816
0.60	1.40	0.864	0.863	0.859
0.60	1.60	0.912	0.913	0.912
0.60	1.80	0.958	0.961	0.962

Standard Premium = 150,000
Loss Limit = 30,000

		Retrospective Premium Adequacy*		
Min.	Max.	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.904	0.908	0.901
BxTM	1.20	0.889	0.894	0.889
BxTM	1.40	0.906	0.909	0.907
BxTM	1.60	0.924	0.925	0.924
BxTM	1.80	0.939	0.939	0.939
0.60	1.00	0.908	0.912	0.905
0.60	1.20	0.912	0.916	0.914
0.60	1.40	0.944	0.945	0.947
0.60	1.60	0.974	0.973	0.977
0.60	1.80	0.995	0.994	0.999

EXHIBIT VI
(CONT.)

RETROSPECTIVE PREMIUM ADEQUACY FOR PLANS WITH A LOSS LIMIT

Standard Premium = 250,000
Loss Limit = 50,000

		Retrospective Premium Adequacy*		
Min.	Max.	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.925	0.931	0.937
BxTM	1.20	0.923	0.927	0.931
BxTM	1.40	0.940	0.941	0.944
BxTM	1.60	0.957	0.957	0.958
BxTM	1.80	0.969	0.969	0.969
0.60	1.00	0.931	0.936	0.943
0.60	1.20	0.942	0.944	0.948
0.60	1.40	0.970	0.967	0.969
0.60	1.60	0.992	0.988	0.987
0.60	1.80	1.010	1.005	1.003

* The parameters for the plans are given in Exhibits II (Table A), III and IV.

EXHIBIT VII

Retrospective Premium Adequacy for Plans with a Loss Limit and Inadequate Excess Loss Premium Factors

Standard Premium = 50,000
Loss Limit = 10,000

		Retrospective Premium Adequacy*		
Min.	Max.	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.899	0.914	0.936
BxTM	1.20	0.884	0.919	0.978
BxTM	1.40	0.910	0.955	1.031
BxTM	1.60	0.939	0.989	1.076
BxTM	1.80	0.964	1.017	1.110
0.60	1.00	0.899	0.914	0.937
0.60	1.20	0.906	0.944	1.009
0.60	1.40	0.963	1.013	1.102
0.60	1.60	1.021	1.073	1.166
0.60	1.80	1.069	1.121	1.213

Standard Premium = 150,000
Loss Limit = 30,000

		Retrospective Premium Adequacy*		
Min.	Max.	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.928	0.952	1.003
BxTM	1.20	0.930	0.967	1.048
BxTM	1.40	0.955	0.994	1.089
BxTM	1.60	0.976	1.016	1.120
BxTM	1.80	0.993	1.034	1.142
0.60	1.00	0.933	0.957	1.009
0.60	1.20	0.954	0.988	1.062
0.60	1.40	0.991	1.024	1.103
0.60	1.60	1.022	1.054	1.135
0.60	1.80	1.045	1.076	1.156

EXHIBIT VII
(CONT.)

RETROSPECTIVE PREMIUM ADEQUACY FOR PLANS WITH A LOSS LIMIT AND INADEQUATE EXCESS LOSS PREMIUM FACTORS

Standard Premium = 250,000
Loss Limit = 50,000

		Retrospective Premium Adequacy*		
Min.	Max.	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.943	0.968	1.028
BxTM	1.20	0.952	0.982	1.060
BxTM	1.40	0.972	1.004	1.088
BxTM	1.60	0.990	1.023	1.110
BxTM	1.80	1.004	1.038	1.127
0.60	1.00	0.950	0.974	1.027
0.60	1.20	0.970	0.996	1.056
0.60	1.40	1.000	1.024	1.083
0.60	1.60	1.023	1.045	1.102
0.60	1.80	1.042	1.063	1.118

* The parameters for the plans are given in Exhibits II (Table A), III and IV. The Excess Loss Premium Factors in Exhibit II (Table A) are multiplied by .5.

EXHIBIT VIII

DISTRIBUTION OF RETROSPECTIVE PREMIUM WITH 30,000 LOSS LIMIT—
STANDARD INSURED

1.	Standard Premium	150000
2.	Basic Premium (Excl. Ins. Chg. But Incl. Tax)	21684
3.	Basic Premium (Incl. 0.179 Ins. Chg. and Tax)	53098
4.	Excess Premium Generated by E.L.P.F. (Incl. Tax)	25062
5.	Needed Excess Premium (Incl. Tax)	25062
6.	Minimum Premium (= Line 3)	53098
7.	Maximum Premium (Line 1 × 1.000)	150000

(1) Probability that Subject Losses Are ≤ Col (2)*	(2) Losses Subject To Retro Rating*	(3) Retrospective Premium**	(4) Cost Plus Premium***	(5) Difference (3) − (4)
Min	10659	88819	57405	31414
.005	18287	96447	65033	31414
.010	20942	99102	67688	31414
.050	30342	108502	77088	31414
.100	37238	115398	83984	31414
.200	48255	126415	95001	31414
.300	57966	136126	104712	31414
.400	66673	144833	113419	31414
.500	75372	150000	122118	27882
.600	84315	150000	131061	18939
.700	95106	150000	141852	8148
.800	108743	150000	155489	−5489
.900	129005	150000	175751	−25751
.950	147786	150000	194532	−44532
.990	184776	150000	231522	−81522
.995	200951	150000	247697	−97697
Max	283075	150000	329821	−179821

* Subject Losses are adjusted to include L.A.E. and Taxes

** Retrospective Premium = Line 3 + Line 4 + Col (2)
Subject to Minimum and Maximum Premium

*** Cost Plus Premium = Line 2 + Line 5 + Col (2)

EXHIBIT IX

DISTRIBUTION OF RETROSPECTIVE PREMIUM WITH NO LOSS LIMIT—
STANDARD INSURED

1.	Standard Premium	150000
2.	Basic Premium (Excl. Ins. Chg. But Incl. Tax)	21684
3.	Basic Premium (Incl. 0.179 Ins. Chg. and Tax)	53098
4.	Excess Premium Generated by E.L.P.F. (Incl. Tax)	0
5.	Needed Excess Premium (Incl. Tax)	0
6.	Minimum Premium (= Line 3)	53098
7.	Maximum Premium (Line 1 × 1.000)	150000

(1) Probability that Subject Losses Are ≤ Col (2)*	(2) Losses Subject To Retro Rating*	(3) Retrospective Premium**	(4) Cost Plus Premium***	(5) Difference (3) − (4)
Min	10659	63757	32343	31414
.005	18287	71385	39971	31414
.010	20942	74040	42626	31414
.050	30342	83440	52026	31414
.100	37238	90336	58922	31414
.200	48273	101371	69957	31414
.300	58668	111766	80352	31414
.400	69178	122276	90862	31414
.500	81194	134292	102878	31414
.600	94581	147679	116265	31414
.700	112488	150000	134172	15828
.800	140164	150000	161848	−11848
.900	190628	150000	212312	−62312
.950	258305	150000	279989	−129989
.990	532459	150000	554143	−404143
.995	615667	150000	637351	−487351
Max	938677	150000	960361	−810361

* Subject Losses are adjusted to include L.A.E. and Taxes

** Retrospective Premium = Line 3 + Line 4 + Col (2)
Subject to Minimum and Maximum Premium

*** Cost Plus Premium = Line 2 + Line 5 + Col (2)

EXHIBIT X

INDICATED INSURANCE CHARGES

Standard Premium = 50,000
No Loss Limit

		Insurance Charge*		
Min.	Max.	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.230	0.300	0.424
BxTM	1.20	0.153	0.219	0.351
BxTM	1.40	0.113	0.174	0.305
BxTM	1.60	0.089	0.144	0.269
BxTM	1.80	0.072	0.123	0.241
0.60	1.00	0.226	0.299	0.424
0.60	1.20	0.129	0.195	0.351
0.60	1.40	0.071	0.124	0.289
0.60	1.60	0.034	0.071	0.224
0.60	1.80	0.006	0.029	0.159

Standard Premium = 150,000
No Loss Limit

		Insurance Charge*		
Min.	Max.	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.118	0.179	0.303
BxTM	1.20	0.063	0.112	0.217
BxTM	1.40	0.039	0.079	0.168
BxTM	1.60	0.026	0.060	0.135
BxTM	1.80	0.018	0.047	0.110
0.60	1.00	0.111	0.171	0.300
0.60	1.20	0.046	0.087	0.181
0.60	1.40	0.017	0.043	0.096
0.60	1.60	0.000	0.014	0.031
0.60	1.80	−0.012	−0.005	−0.021

EXHIBIT X
(CONT.)

INDICATED INSURANCE CHARGES

Standard Premium = 250,000
No Loss Limit

		Insurance Charge*		
Min.	Max.	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.083	0.128	0.234
BxTM	1.20	0.039	0.073	0.154
BxTM	1.40	0.021	0.048	0.109
BxTM	1.60	0.011	0.033	0.080
BxTM	1.80	0.005	0.023	0.060
0.60	1.00	0.079	0.119	0.222
0.60	1.20	0.030	0.054	0.107
0.60	1.40	0.009	0.021	0.033
0.60	1.60	−0.003	0.001	−0.021
0.60	1.80	−0.010	−0.014	−0.061

* The parameters for the plan are given in Exhibit IV.

EXHIBIT XI

INDICATED INSURANCE CHARGES

Standard Premium = 50,000
Loss Limit = 10,000

		Insurance Charge*		
Min.	Max.	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.054	0.049	0.032
BxTM	1.20	0.013	0.012	0.006
BxTM	1.40	0.003	0.003	0.001
BxTM	1.60	0.001	0.001	0.000
BxTM	1.80	0.000	0.000	0.000
0.60	1.00	0.052	0.049	0.032
0.60	1.20	0.008	0.009	0.006
0.60	1.40	−0.004	0.000	0.001
0.60	1.60	−0.006	−0.003	0.000
0.60	1.80	−0.007	−0.004	0.000

Standard Premium = 150,000
Loss Limit = 30,000

		Insurance Charge*		
Min.	Max.	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.046	0.052	0.045
BxTM	1.20	0.010	0.013	0.011
BxTM	1.40	0.002	0.004	0.003
BxTM	1.60	0.000	0.001	0.001
BxTM	1.80	0.000	0.000	0.000
0.60	1.00	0.041	0.047	0.044
0.60	1.20	0.002	0.004	0.007
0.60	1.40	−0.006	−0.006	−0.003
0.60	1.60	−0.008	−0.009	−0.005
0.60	1.80	−0.009	−0.010	−0.006

EXHIBIT XI
(CONT.)

INDICATED INSURANCE CHARGES

Standard Premium = 250,000
Loss Limit = 50,000

		Insurance Charge*		
Min.	Max.	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.038	0.044	0.052
BxTM	1.20	0.007	0.010	0.013
BxTM	1.40	0.001	0.002	0.004
BxTM	1.60	0.000	0.000	0.001
BxTM	1.80	0.000	0.000	0.000
0.60	1.00	0.035	0.039	0.047
0.60	1.20	0.002	0.001	0.003
0.60	1.40	−0.004	−0.007	−0.007
0.60	1.60	−0.006	−0.009	−0.011
0.60	1.80	−0.006	−0.010	−0.011

* The parameters for the plan are given in Exhibits II (Table A) and IV.

EXHIBIT XII

RETROSPECTIVE PREMIUM ADEQUACY FOR ALTERNATE PLAN #1

Standard Premium = 50,000
Loss Limit = 10,000

		Retrospective Premium Adequacy*		
Min.	Max.	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	1.004	1.000	0.983
BxTM	1.20	1.002	1.000	0.993
BxTM	1.40	1.001	1.000	0.998
BxTM	1.60	1.000	1.000	0.999
BxTM	1.80	1.000	1.000	1.000
0.60	1.00	1.002	1.000	0.983
0.60	1.20	0.998	1.000	0.997
0.60	1.40	0.996	1.000	1.002
0.60	1.60	0.996	1.000	1.004
0.60	1.80	0.996	1.000	1.006

Standard Premium = 150,000
Loss Limit = 30,000

		Retrospective Premium Adequacy*		
Min.	Max.	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.994	1.000	0.994
BxTM	1.20	0.996	1.000	0.997
BxTM	1.40	0.998	1.000	0.998
BxTM	1.60	0.999	1.000	0.999
BxTM	1.80	1.000	1.000	1.000
0.60	1.00	0.995	1.000	0.998
0.60	1.20	0.998	1.000	1.003
0.60	1.40	0.999	1.000	1.003
0.60	1.60	1.001	1.000	1.004
0.60	1.80	1.001	1.000	1.005

EXHIBIT XII
(CONT.)

RETROSPECTIVE PREMIUM ADEQUACY FOR ALTERNATE PLAN #1

Standard Premium = 250,000
Loss Limit = 50,000

		Retrospective Premium Adequacy*		
Min.	Max.	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.994	1.000	1.008
BxTM	1.20	0.997	1.000	1.004
BxTM	1.40	0.999	1.000	1.002
BxTM	1.60	1.000	1.000	1.001
BxTM	1.80	1.000	1.000	1.000
0.60	1.00	0.996	1.000	1.007
0.60	1.20	1.001	1.000	1.003
0.60	1.40	1.003	1.000	1.000
0.60	1.60	1.004	1.000	0.998
0.60	1.80	1.005	1.000	0.998

* The insurance charges used are those of the Standard Insured in Exhibit XI. The parameters for the plan are given in Exhibits II (Table A) and IV.

EXHIBIT XIII

Retrospective Premium Adequacy for Alternate Plan #2

Standard Premium = 50,000
Loss Limit = (2,000 : 20,000)

			Retrospective Premium Adequacy*		
Min.	Max.	Insurance Charge*	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.055	0.999	1.000	0.992
BxTM	1.20	0.015	0.999	1.000	0.997
BxTM	1.40	0.005	0.999	1.000	0.998
BxTM	1.60	0.001	1.000	1.000	1.000
BxTM	1.80	0.000	1.001	1.000	1.000
0.60	1.00	0.055	0.998	1.000	0.992
0.60	1.20	0.014	0.996	1.000	0.998
0.60	1.40	0.002	0.997	1.000	1.002
0.60	1.60	−0.002	0.998	1.000	1.004
0.60	1.80	−0.003	0.998	1.000	1.004

Standard Premium = 150,000
Loss Limit = (5,000 : 60,000)

			Retrospective Premium Adequacy*		
Min.	Max.	Insurance Charge*	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.046	0.992	1.000	1.007
BxTM	1.20	0.012	0.994	1.000	1.003
BxTM	1.40	0.003	0.998	1.000	1.002
BxTM	1.60	0.001	0.999	1.000	1.000
BxTM	1.80	0.000	1.000	1.000	1.000
0.60	1.00	0.043	0.993	1.000	1.008
0.60	1.20	0.006	0.997	1.000	1.006
0.60	1.40	−0.003	1.000	1.000	1.003
0.60	1.60	−0.005	1.000	1.000	1.001
0.60	1.80	−0.006	1.001	1.000	1.001

EXHIBIT XIII
(CONT.)

RETROSPECTIVE PREMIUM ADEQUACY FOR ALTERNATE PLAN #2

Standard Premium = 250,000
Loss Limit = (10,000 : 100,000)

			Retrospective Premium Adequacy*		
Min.	Max.	Insurance Charge*	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.039	0.993	1.000	1.014
BxTM	1.20	0.008	0.997	1.000	1.009
BxTM	1.40	0.002	0.999	1.000	1.003
BxTM	1.60	0.000	1.000	1.000	1.002
BxTM	1.80	0.000	1.000	1.000	1.000
0.60	1.00	0.036	0.994	1.000	1.013
0.60	1.20	0.003	1.000	1.000	1.005
0.60	1.40	−0.004	1.002	1.000	1.000
0.60	1.60	−0.006	1.003	1.000	0.998
0.60	1.80	−0.006	1.003	1.000	0.997

* The parameters for the plan are given in Exhibits II (Table B) and IV.

EXHIBIT XIV

Retrospective Premium Adequacy for Alternate Plan #3

Standard Premium = 50,000
Loss Limit = (10,000 : 20,000)

			Retrospective Premium Adequacy*		
Min.	Max.	Insurance Charge*	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.078	1.000	1.000	0.987
BxTM	1.20	0.026	0.999	1.000	0.992
BxTM	1.40	0.010	0.998	1.000	0.995
BxTM	1.60	0.004	0.999	1.000	0.997
BxTM	1.80	0.001	1.000	1.000	1.000
0.60	1.00	0.077	0.998	1.000	0.988
0.60	1.20	0.019	0.995	1.000	1.000
0.60	1.40	0.001	0.995	1.000	1.009
0.60	1.60	−0.008	0.996	1.000	1.012
0.60	1.80	−0.011	0.996	1.000	1.014

Standard Premium = 150,000
Loss Limit = (30,000 : 60,000)

			Retrospective Premium Adequacy*		
Min.	Max.	Insurance Charge*	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.071	0.989	1.000	1.004
BxTM	1.20	0.022	0.992	1.000	1.004
BxTM	1.40	0.008	0.995	1.000	1.001
BxTM	1.60	0.003	0.998	1.000	1.000
BxTM	1.80	0.001	0.999	1.000	1.000
0.60	1.00	0.064	0.991	1.000	1.007
0.60	1.20	0.008	0.997	1.000	1.004
0.60	1.40	−0.009	1.001	1.000	1.002
0.60	1.60	−0.014	1.003	1.000	0.999
0.60	1.80	−0.016	1.004	1.000	0.999

EXHIBIT XIV
(CONT.)

RETROSPECTIVE PREMIUM ADEQUACY FOR ALTERNATE PLAN #3

Standard Premium = 250,000
Loss Limit = (50,000 : 100,000)

			Retrospective Premium Adequacy*		
Min.	Max.	Insurance Charge*	Low Severity Insured	Standard Insured	High Severity Insured
BxTM	1.00	0.058	0.980	1.000	1.019
BxTM	1.20	0.016	0.995	1.000	1.013
BxTM	1.40	0.005	0.997	1.000	1.006
BxTM	1.60	0.001	1.000	1.000	1.003
BxTM	1.80	0.000	1.000	1.000	1.002
0.60	1.00	0.051	0.994	1.000	1.014
0.60	1.20	0.004	1.001	1.000	1.003
0.60	1.40	−0.009	1.005	1.000	0.996
0.60	1.60	−0.013	1.007	1.000	0.992
0.60	1.80	−0.014	1.007	1.000	0.991

* The parameters for the plan are given in Exhibits II (Table B) and IV.

EXHIBIT XV

MULTI-EXPOSURE INSURED

Standard Premium for:	High Severity Exposure	=	150,000
	Standard Exposure	=	50,000
	Low Severity Exposure	=	50,000
	Total		250,000

TABLE A

Min.	Max.	Indicated Insurance Charge* No Loss Limit	50,000 Loss Limit
BxTM	1.00	0.183	0.047
BxTM	1.20	0.115	0.011
BxTM	1.40	0.080	0.002
BxTM	1.60	0.057	0.000
BxTM	1.80	0.042	0.000
0.60	1.00	0.175	0.044
0.60	1.20	0.086	0.003
0.60	1.40	0.033	−0.006
0.60	1.60	−0.002	−0.009
0.60	1.80	−0.028	−0.009

TABLE B

Loss Limit = 50,000

Min.	Max.	Insurance Charge**	Retrospective Premium Adequacy*
BxTM	1.00	0.044	1.001
BxTM	1.20	0.010	1.000
BxTM	1.40	0.002	1.000
BxTM	1.60	0.000	1.000
BxTM	1.80	0.000	0.999
0.60	1.00	0.039	1.001
0.60	1.20	0.001	1.001
0.60	1.40	−0.007	0.999
0.60	1.60	−0.009	0.999
0.60	1.80	−0.010	0.999

* The parameters for the plan are given in Exhibits II (Table A) and IV.

** From Exhibit XI.

GENERAL LIABILITY RATEMAKING: AN UPDATE

MICHAEL F. McMANUS

In the fourteen years since Jeffrey T. Lange wrote "General Liability Insurance Ratemaking," (1) the insurance industry has experienced a period of significant social and economic inflation. This has been evidenced by spiralling insurance claim costs, as well as by a rapidly growing number of claims, brought by an increasingly claims conscious public. The impact on the various General Liability lines of insurance has been a dramatic change in industry profitability, which in turn has presented severe challenges to the ratemaker.

Considering the turbulence of this fourteen year period, the adjustments that have been made to the actuarial methodology described by Lange have not been major, but they have served to improve the accuracy of the overall rate level calculation. The purpose of this paper is to present a summary of the adjustments that have been made in the basic limits ratemaking methodology and the reasons for their introduction. Recent revisions in increased limits ratemaking methodology are beyond the scope of this paper but are fully described in Robert S. Miccolis's paper "On the Theory of Increased Limits and Excess of Loss Pricing." (2)

Lange's excellent explanation of the general problems presented to the actuary by the various sublines and how the ratemaking methodology resolves them, especially with regard to classification ratemaking, should be read before reviewing the technical adjustments described in this paper. The methodology described is that of Insurance Services Office (ISO), which compiles ratemaking data and files rates for the great majority of General Liability insurers in the United States. The changes outlined in this paper were developed by ISO's Commercial Casualty Actuarial Subcommittee (CCAS) and its successor subcommittee, the General Liability Actuarial Subcommittee (GLAS), during the 1970's.

Before considering these changes, a review of the premium growth that has occurred in General Liability will help put the significance of ISO's General Liability ratemaking procedures into perspective.

Premium Growth

In 1966, according to Best's Executive Data Service, written premiums for General Liability (including Medical Malpractice) amounted to $1.2 billion, representing 5.7% of total Property/Casualty written premiums in the United States. By 1978, written premiums (including Medical Malpractice) had soared to $9.1 billion, comprising 11.2% of the industry's total writings.

On an individual risk basis, the rapid growth in the average premium that has resulted from the significant basic and increased limits rate increases implemented during this period has also made the application of experience rating plans much more frequent. As a result, their soundness has become more critical to overall industry profitability. While the technical adjustments that have been made to the General Liability Experience and Schedule Rating Plan are beyond the scope of this paper, the reader should be aware that significant revisions have been made to the plan, including higher premium eligibility requirements, introduction of trend and loss development factors, and revision of D-Ratios (3). In addition, the technical off-balance (the percentage difference between the actual charged premium—including Experience and Schedule Rating debits and credits—and the premium collectible at manual rate level) that existed in the plans was accommodated in the expected loss ratio. The impact of these changes on the actual premiums collected by the industry should not be underestimated, since as much as 75% of General Liability premiums are eligible for experience rating (4).

The adjustments made by ISO to the ratemaking procedures are described in the balance of this paper in two sections:

1. A description of general ratemaking adjustments that affect all General Liability sublines, and
2. An outline of specific adjustments that were made to the ratemaking process for each subline.

GENERAL RATEMAKING ADJUSTMENTS

Definition of Basic Limits

One of the first responses to the impact of inflation on General Liability ratemaking was the revision of the definition of basic limits for Bodily Injury (BI) coverages. Effective January 1, 1973, BI manual rates were revised to reflect a limit of $25,000 per occurrence, instead of the previous $5,000 per person and $10,000 per accident. Similarly, manual rates for Professional Liability sublines were adjusted to limits of $25,000 per claim and $75,000 in annual aggregate from the previous limits of $5,000 per person and $15,000 aggregate. In both cases, revised manual rates were determined by multiplying the prior basic limit rates, by state and class, by the appropriate increased limits factors.

The major motivation for the change in basic limits was the small number of insureds buying limits less than $25,000 as a result of the eroding effect of inflation on liability claim costs. A further consideration in the Professional Liability sublines was the fact that average paid claim costs were approaching $3,000, when all payments were limited to the $5,000 basic limit. As an increasing number of claims penetrated the basic limit, the impact of basic limits ratemaking by state was being surpassed by that of countrywide increased limits ratemaking. At the time, this was not yet a significant concern for the other General Liability sublines.

From a ratemaking viewpoint, the effect of the adjustment of $5,000 manual rates to a $25,000 basis was to allow basic limits rates by state and class to reflect to a greater extent the different claim severity levels that existed from state to state and from class to class. This occurred because the countrywide increased limits factor previously used to adjust $5,000 manual rates to a $25,000 basis was effectively reevaluated by state and by class. Since ratemaking data by state and class being reviewed at the time was actually reprocessed to determine losses up to $25,000 per occurrence, excess of $5,000, this reevaluation had a prompt impact.

The impact on this loss experience of the small number of insureds that had purchased limits less than $25,000 was also approximated after an examination of current policy limits distributions by subline. The average increased limits factor for those insureds purchasing limits less than $25,000 was applied to those reported incurred losses insured with limits above $5,000 but below $25,000. The CCAS felt this adjustment would reasonably approximate the increase in reported losses had all insureds been required to purchase limits of at least $25,000.

The change in basic limits, as well as other elements, necessitated adjustments to General Liability loss development and trend procedures; these adjustments are discussed in the next two sections.

Loss Development

Adjustments to the General Liability loss development procedure became necessary because of

1. The increased number, and dollar impact, of liability claims subject to lengthy litigation, and
2. The increasingly liberal interpretations of various aspects of tort law, e.g., statutes of limitation, by the courts (5).

Also, as noted in Robert J. Finger's article "Estimating Pure Premiums by Layer—An Approach" (6), there is a theoretical problem in using data limited to a fixed dollar amount to calculate loss development factors because the value of the fixed limit, expressed in constant dollars, is changing over time due to the impact of inflation on insurance claim costs.

In the past, policy year loss development for General Liability (excluding Medical Malpractice) was measured by state and class up to 39 months of maturity; further development was measured on a countrywide basis by subline (with no class detail) up to 63 months of maturity, which was considered to be an ultimate evaluation for all practical purposes.

As the observed countrywide developments beyond 39 months became more and more significant, the CCAS decided in 1974 to begin accumulating actual loss development by state and class beyond 39 months. Although countrywide loss development factors are still used for the non-Professional sublines, detailed loss development data is now available up to 111 months of maturity for Products Liability and is being compiled up to 123 months of maturity for all General Liability sublines, because of increased concern about the magnitude of the development "tail" in recent years. The importance of loss development in Medical Malpractice experience has long been recognized, and the period of measurement has been extended gradually from 75 months in the late 1960's to 135 months of maturity at the present time (see Exhibit 1).

In a number of cases, notably for the Property Damage Liability coverages, no development data beyond 39 months had been compiled. While extended development histories were being compiled, a procedure was used which assumed development beyond the last observed development interval to be equal to development in that last interval. Thus if a development factor of 1.02 has been observed between 27 and 39 months, that same factor is used to develop losses from 39 months to ultimate. The propriety of this approximation procedure was substantiated by actual extended developments (75 months and subsequent) available for Medical Malpractice (see Exhibit 1).

Trend Factors (7)

As might be expected, the effects of social and economic inflation on General Liability claim costs have necessitated significant changes to the trending procedures. Until 1974, the procedure outlined by Benbrook (8) was followed: calendar year average paid claim costs were fitted to a straight line of best fit, using the least squares methodology, and the trend was calculated as the average annual dollar change in the average fitted cost (the slope) divided by the midpoint of the fitted line.

Of course this procedure effectively implied that trend was decreasing on a percentage basis, since a fixed dollar amount, the slope, was related to a constantly increasing base. Therefore, in 1974, at the peak of an inflationary period, ISO's Actuarial Committee decided that the procedure should be revised to replace the least squares straight line with a least squares exponential curve of best fit (9), which produces a constant annual percent change between each pair of fitted values (10). This procedure was expected to provide a much more realistic measurement of the effects of inflation on insurance loss costs, and is the procedure still in use at this time.

A further problem resulted from the fact that, in times of changing claim frequency, using calendar year average paid claim cost to measure severity trend for liability lines is theoretically improper. This is because the significant time lag between occurrence and settlement of liability claims will produce a mix of small and large claims that will be paid in any given calendar year period. More severe claims are usually subject to litigation and will frequently take several years to be settled. As long as claim frequency is unchanging, the mix of claims remains relatively constant from year to year, and there is no problem. When claim frequency is increasing, however, an undue proportion of low valued, easily settled claims will be included in the most recent experience, distorting the average claim cost. This very phenomenon was observed by the CCAS in the calendar year average paid claim cost data for Medical Malpractice for calendar years 1974 and 1975, as shown in Exhibit 2. While all indications at that time pointed to rapidly rising claim costs, actual calendar year average paid claim costs were *decreasing*. Further study showed that this was caused by the problem described above. A theoretical model presented to the CCAS to more fully describe this problem is included here as an Appendix.

This situation was resolved in 1976 when the CCAS decided to measure severity trend for General Liability sublines using policy year incurred claim cost data rather than calendar year paid data. This procedure offers the following advantages over the prior methodology:

1. Policy year incurred claim costs present a more current indication of severity trends, since the most recent point includes only claims incurred in the most recent policy year, not claims incurred long ago and paid recently.
2. The distortion caused by changing claim frequencies is eliminated, since average costs are determined by the claims incurred in a given policy year, not by those that happen to be paid in a given calendar year.

The CCAS recognized that the introduction of outstanding losses into the trend procedure necessitated the application of loss development factors to obtain average severities for each policy year at comparable levels of maturity. While this does introduce some complexity into the procedure, the advantage of being able to use current outstanding losses was felt to overshadow this additional complexity. Any changes in individual company claim reserving practices were assumed to be negligible when experience was compiled on an industrywide basis. The CCAS also felt that the impact of this adjustment would be significant only for the most recent policy year or two.

While the change from calendar year to policy year data was being considered, detailed data by subline were reviewed, on both a paid and incurred basis. Significant differences between the magnitude of, and the rate of change in, average claim costs by subline were observed. Exhibit 3 details a comparison of these differences.

Until the time of this procedural change, separate trends had been calculated for all Professional Liability sublines combined, for Products Liability (when a trend procedure was initiated for that subline), and for all other sublines combined (based on calendar year average paid claim cost data combined for all sublines other than Professional and Products Liability). Because of the observed differences in trends between sublines, the CCAS decided that, coincident with the change to policy year incurred trend, the base for measuring trend would also be changed to reflect the experience of each individual subline.

One final revision was necessitated by the change to policy year incurred trend factors. Calendar year average paid claim cost data had been compiled on a semiannual basis and then, beginning in 1974, on a quarterly basis; these data were then combined so that the latest twelve overlapping quarterly year-ended points were used to calculate the trend. Since policy year data were available only for annual periods, the number of points to be used in the calculation needed to be reconsidered. Based on a judgmental consideration relative to the desired degree of responsiveness and stability, the CCAS decided to use six policy years for all sublines except Professional Liability, where a trend based on eight policy years was believed to be more appropriate because of greater volatility in the average claim costs.

All of the above discussion has addressed the measurement of severity trends. Historically the measurement of frequency trend for General Liability sublines has been difficult because of the multiplicity of exposure bases used. In 1975, however, a claim frequency trend procedure for the Professional Liability sublines was developed: claim frequency trend was measured by subline on a policy year incurred basis after developing incurred claim counts to an ultimate reported value.

Claim frequencies for other sublines are now reviewed using premium at present rates as the denominator (adjusted to current exposure levels where necessary) to avoid the problem of multiple exposure bases. At this time, frequency trend has not been reflected in any other ISO General Liability filing because the frequencies have appeared to be fairly constant.

Classifications

One other general area that has experienced significant revisions in the 1970's is that of General Liability classifications. The scope of these changes is discussed in this section. In general the approach described by Lange for determining General Liability classification rates is still in effect, although the procedure for Products Liability has recently been revised.

A significant change was made to the classifications themselves, effective January 1, 1974, when 5-digit Industrial Classification Codes (ICC's) replaced the prior 4-digit codes. This change was intended to allow the collection of more refined statistical experience by class. It had been observed in several instances that, as tort liability concepts expanded rapidly in the late 1960's and early 1970's, many exposures of a quite dissimilar nature were listed under the same class code. The 5-digit ICC System was selected to provide compatibility with liability data collected under the Commercial Risk Statistical Plan, which applied to most package business, and to allow the comparison of insurance statistics to statistics published by the Federal Government in ICC detail.

The expansion of the number of classes was most significant in the Manufacturers and Contractors subline, where the number of classes jumped from 192 to 498. The number of Owners, Landlords, and Tenants classes grew more modestly from 264 to 324. Classifications in the Products Liability Manual were extensively revised shortly after the introduction of the ICC System (effective May 29, 1974), creating classes for many newly developed products, so that a comparison of the number of classes before and after this change is not appropriate for this subline.

As several years of experience become available on the expanded class basis, improvements in the accuracy of General Liability class rates should become apparent.

OTHER DEVELOPMENTS

In addition to these general changes, there have been several other specific changes introduced by ISO for the individual General Liability sublines. This section describes these changes.

Owners, Landlords, and Tenants Liability

The procedure outlined by Lange for OL&T Bodily Injury ratemaking is still used today with the exception of the general changes described above and two other minor changes.

The number of class groups included in the statewide experience review increased to thirteen with the addition of the Hotel–Motel classifications. These classes were previously separately reviewed and filed. In 1971 the CCAS determined that they could be incorporated readily into the procedure used for the other twelve groups.

The second change affects the statewide rate level calculation in states with less than full credibility. The prior procedure, as described by Lange, had been to apply the complement of the state's credibility to the expected loss ratio (ELR) in such states. Of course this is equivalent to applying the complement of the credibility to no change in rate level. In times of rapidly rising costs, this procedure severely slowed movement towards an adequate rate level. In 1975, therefore, the CCAS adjusted the procedure; the ELR is now trended from the effective date of the last rate revision, or from the date of the last review if no revision was filed at that time. Thus, in the extreme example, if a state's experience had no credibility, rates would be adjusted by the overall trend since the last revision. Exhibit 4 provides an example of the new procedure.

Manufacturers and Contractors Liability

The unique three-way credibility weighting procedure suggested by Lange has proven to be a methodology capable of handling the great diversity of exposure among M&C classifications, many of which can be accorded only low credibility.

There have been modest changes in the way statewide rate levels have been determined. The few smaller states that were previously grouped together have been individually reviewed since 1973. The calculation of the trended ELR to which the complement of the state's credibility is assigned in low-credibility states was introduced in M&C as in OL&T. Also, as premium volume has grown steadily, the number of states with territorial rates has increased to 5; California, Florida, Illinois and Pennsylvania now are divided into rating territories in addition to New York.

A trend procedure was introduced for M&C in 1973, as increases in the severity of M&C claims were observed to exceed the increases in the exposure base, payroll. Previously severity trend and payroll trend were assumed to be equal. M&C severity trends are calculated in the manner described above (see also Exhibit 3), while payrolls are adjusted to the current level based on movements in the average wages of manufacturing and contract construction workers, as published by the Bureau of Labor Statistics. Exhibit 5 shows how this information is compiled.

An improvement in the exposure trending procedure was introduced in 1977. The observed difference between the average wage level for each policy year and the latest published wage level is used to adjust experience to current level. Trend *beyond* the latest point is based on the usual exponential extrapolation approach. Exhibit 5 displays the details of these calculations.

One other significant revision to M&C ratemaking has been necessitated by the gradual movement to an unlimited payroll basis of exposure. Non-executive payrolls were originally limited to $100 per week; this cap has gradually been increased in recognition of inflation's impact on average wage levels. In most states, the limitation was raised to $300 in the early 1970's and eliminated entirely in recent years.

Since these changes paralleled similar changes in the Workers' Compensation exposure definition, detailed state-by-state wage information collected by the National Council on Compensation Insurance has been used to adjust manual rates to reflect the new definition of payroll. Since the change from a $100 limit to a $300 limit was of the greatest significance, adjustments were usually determined by classification. The adjustment from $300 to unlimited payrolls had a much smaller impact (two to three percent) and was usually assumed to be the same for all classes in the state. Exhibit 6 provides an example of the procedure used by the National Council to calculate payroll offset factors.

Products Liability

The rapid increases in filed Products Liability rate levels in 1975 and 1976 were largely based on the procedure outlined in Lange's paper, with the addition of a trend procedure similar in approach to that outlined for M&C. Products exposures (sales or receipts for most classes) were adjusted using Consumer Price Index data for Commodities, which was found to represent a mix of products reasonably approximating that found in Products Liability Insurance data. This finding was made by the GLAS (11) following a review of the distribution of Products Liability premium for each major CPI Commodities component: food, apparel, other non-durables, durables, and all other. Elimination of the "all other" category produced the following comparison:

	Food	Apparel	Other Non-Durables	Durables
CPI Commodities Index	37.9%	12.5%	24.0%	25.6%
ISO Products Data	38.9	7.3	18.6	35.1

The GLAS felt these two sets of weights were sufficiently similar to permit the use of an unadjusted CPI Commodities index to measure Products Liability exposure trend.

As mentioned earlier, an extensive revision to the classifications in the Products Liability Manual was introduced in 1974. The main reasons for this revision were:

1. To create classifications for many newly developed products for which no current classification existed, and
2. To refine many existing classifications which were considered to be too broad in scope in the existing liability climate.

This classification revision presented severe challenges to the ratemaking process since experience was available only for the prior classifications, which in many cases were significantly different than the new classes. A careful mapping of new and old class codes was performed, and as much of the historical data as possible was used in subsequent rate reviews. For the newly erected classifications, manual rates, which were judgmentally established in most cases, were adjusted by the overall trend factors as experience was compiled under the new class definitions.

The ratemaking procedure discussed here only applies to manually rated classifications, which account for less than one-half the total monoline Products Liability premium volume. The remaining classifications are (a) rated, which means that the rate is judgmentally determined after the insurer evaluates the specific characteristics of the individual risk. While (a) rated classifications exist in every General Liability subline, they are of overall minor importance, except in the Products Liability subline. This is due to the extreme variation in Products Liability exposure that can be presented by two different manufacturers of the same product, and to variations in the relationship of current sales to sales in prior years. Given two manufacturers making the same durable product and having roughly the same volume of current sales, the insurer's exposure would be significantly different for a firm which had been making the product for 20 years than for a firm which had been manufacturing it only in recent years. This is because coverage is provided for all occurrences in the current policy year, regardless of when the product involved in the occurrence was manufactured.

With the heightened interest in Products Liability in recent years, the classification ratemaking procedure mentioned by Lange has been carefully studied. A revised procedure recently filed groups classifications by average pure premium and average claim size, within type of activity: manufacturing, contract construction, and wholesale and retail sales. The credibility procedure has also been revised to utilize a number of credibility tables, with the observed variation in loss ratio and claim severity determining the credibility table to be used; the former procedure used the 683 claim credibility standard for all classes.

Professional Liability

The rapid escalation in the cost and frequency of Professional Liability claims in the 1970's has made the Professional Liability sublines a more significant and much more visible piece of General Liability, so much so that they have been shown separately in the Annual Statement since 1975.

The overall rate level calculation procedure for the two major sublines, Hospitals, and Physicians, Surgeons and Dentists, was significantly revised by the GLAS in 1977 after careful study. It was first decided to change the basis of the rate level calculation from a 30%–70% weighting of the two latest policy years (which had been adopted from other General Liability sublines in the early 1970's) to an averaging of the three latest policy years. This was done to achieve greater stability in rate level indications.

The extreme lags between accident, report, and settlement dates for these sublines has long made the loss development procedure an extremely critical part of the ratemaking process. After analyzing loss development data by state, it became apparent to the GLAS that the use of countrywide loss development factors was inappropriate in many states. Since very few states had sufficient volume to allow using statewide loss development factors, each state was assigned to one of three groups of states for loss development purposes, based on the observed magnitude of the historical loss development factors. The general pattern of these groupings was such that the more urban, litigious states had the most significant loss development, while the more rural states showed very modest loss development. Examples of the relative magnitudes of development factors in these three groupings are shown in Exhibit 7.

The third major change was the shift to the policy year incurred severity trend procedure outlined earlier. The combined effect of these three changes has begun to result in more reasonable and stable rate level indications in many low credibility states.

There have been several other refinements made in each of the Professional Liability sublines; these are outlined below:

1. *Physicians, Surgeons, and Dentists:* Major expansions in classification codes were introduced in 1968 (mainly in the surgical area) and in 1976 (largely in the physicians area, where individual specialty codes were established) in order to provide a more precise measurement of insurance exposure. Dentists were incorporated into the overall Physicians and Surgeons review in 1976, in response to the extreme credibility problems that persisted for these classes. Dentists' rates are now related to the Physicians' rates in each state.
2. *Hospitals:* By 1973, the immunity status enjoyed by charitable hospitals, which was mentioned by Lange, had been overturned essentially in every state, either by legislative action or by judicial precedent. As a result, charitable hospitals' rates as well as for-profit hospitals' rates have been determined by state for some time.

   An additional revision to the Hospital Professional ratemaking procedure was introduced in 1975. Premiums generated by additional interests added to Hospital policies—employed doctors, nurses, technicians, maintenance employees, etc.—had become increasingly significant. As the loss potential for this coverage varied widely from hospital to hospital, the premium was (a) rated. Since all additional premiums were reported under one class code, with no exposure, the CCAS decided to incorporate this experience into the review by increasing premium at present rates for manually rated classes by the percentage of total Hospital Professional premiums represented by additional interest premiums. This procedure is illustrated in Exhibit 8. Losses reported under the additional interest class code are also included in the review.

3. *Druggists:* Rates for Druggists have been established on a countrywide basis since 1975, as experience for the previous two groups of states indicated no significant difference between the two groups. In addition, the basis for determining premiums was changed. In the past premiums were determined based on receipts, if receipts were greater than $100,000 per year, or else on a flat charge basis. The flat charge approach was discontinued in 1975, as very few risks were being written on this basis.

Special Multi-peril Policy Program

The Commercial Risks Statistical Plan was introduced in 1969 to collect ratemaking data on commercial package policies. While time revealed a number of problems with its design, particularly with regard to individual classification data, liability experience collected under CRSP has been used since 1975 to review the package discounts from monoline rate levels.

CRSP data for each of ISO's SMP Programs in each state is reviewed, after premiums are adjusted to current monoline rate levels. After reflection of SMP expense requirements and the effect of using rating plans, an indicated package discount by program is calculated. This indicated discount is compared to the current discount factor and a revised package discount factor selected. An example of these calculations for the SMP Motel/Hotel program is shown in Exhibit 9. This procedure has allowed the ratemaking process to reflect the differences between monoline and package experience.

FUTURE CHALLENGES

It should be clear from the foregoing that Lange's concluding comment that "General Liability ratemaking procedures are in a constant state of flux" (12) has proven to be very true. While the changes discussed in this paper have certainly improved the accuracy of General Liability ratemaking procedures, there are still significant areas needing further research and study.

Probably the largest single challenge ahead is that presented by the future availability of monoline and package General Liability data in compatible detail, as provided by the Commercial Statistical Plan, which became effective January

1, 1979. While discussion has already begun on whether (and how) to use package liability data in monoline ratemaking, it seems reasonably evident at this point that a final decision will not, and probably should not, be made until actual CSP data is available for analysis. Since this decision will likely affect the manner in which Commercial Package Policies are rated, underwritten, and marketed in the 1980's, very careful consideration of all implications of the decision is vital.

A second area of major importance is the ratemaking implications of the proposed consolidation of the various General Liability sublines (excluding Products Liability) into one policy and one rate, with a single, uniform, inflation-sensitive exposure base for each area of operation. Preliminary work on various aspects of this project is still underway; if, however, this approach is implemented, the importance of pricing the consolidation accurately initially and adjusting current ratemaking procedures to review the consolidation cannot be overemphasized. The resultant elimination of sublines will reduce the credibility problems that exist today, since losses will have to be assigned only to a particular operation, rather than to a particular coverage for that operation. This problem has been particularly chronic for the Contractual and Owners' or Contractors' Protective sublines. Shifting to an inflation-sensitive exposure base has the obvious advantage of keeping premiums up-to-date without the necessity of frequent rate filings.

Other areas requiring further work include developing reports to review experience written on both occurrence and claims-made policy forms; this is most critical for the Professional Liability sublines. The outlined changes in the Professional Liability area, namely grouping states for loss development and reviewing the appropriateness of the 30%–70% weighting of the latest two policy years for all states, should also be evaluated for possible use in other sublines.

In the area of trend, possible use of econometric procedures should continue to be explored, in order to develop a more responsive measurement of expected changes in loss levels.

One thing is clear: General Liability ratemaking procedures will continue to change in the years to come.

NOTES AND REFERENCES

(1) J. T. Lange, "General Liability Insurance Ratemaking," *PCAS* LIII (1966), p. 26.

(2) R. S. Miccolis, "On the Theory of Increased Limits and Excess of Loss Pricing," *PCAS* LXIV (1977), p. 27.

(3) For a definition of "D-ratios," see R. Snader, "Fundamentals of Individual, Risk Rating and Related Topics," Casualty Actuarial Society 1981 Study Kit on Advanced Ratemaking.

(4) Based on results of ISO's "Special Call for General Liability Expenses Distributed by Size of Risk," December 30, 1975.

(5) For example, see J. A. Dooley, *Modern Tort Law—Liability and Litigation,* Vol. I–III (1977), especially Chapters 32 and 34; and W. L. Prosser, *Handbook of the Law of Torts* (1971), especially Chapters 14 and 17.

(6) R. J. Finger, "Estimating Pure Premiums by Layer—An Approach," *PCAS* LXIII (1976), p. 34.

(7) This section is adopted from ISO GLAS "White Paper" "General Liability Insurance Average Claim Severity Trending Procedure," contained in the minutes of the September 20, 1977 meeting, distributed on October 11, 1977. The author contributed to the writing of the original "White Paper."

(8) P. Benbrook, "The Advantages of Calendar-Accident Year Experience and Need for Appropriate Trend and Projection Factors in the Determination of Automobile Liability Rates," *PCAS* XLV (1958), p. 20.

(9) For the mathematical derivation of an exponential curve of best fit, see P. G. Hoel, *Introduction to Mathematical Statistics* (Fourth Edition), p. 87 or R. V. Hogg & A. T. Craig, *Introduction to Mathematical Statistics* (Fourth Edition), p. 105.

(10) A similar change was instituted for all lines of insurance under ISO jurisdiction at the same time.

(11) See pages 44–58 of the ISO GLAS agenda for the May 10–11, 1977 meeting, distributed on April 29, 1977.

(12) Lange, op. cit., p. 53.

APPENDIX

*Model Illustrating the Impact of a Rapid Increase in Claim Frequency on Calendar Year Average Paid Claim Cost**

Claim frequency has increased rapidly for Medical Professional Liability insurance since 1972. This analysis considers the impact of claim frequency changes on average paid claim cost.

We make the following assumptions:

1. Policy year average incurred claim cost increases at a constant annual percentage rate *d*.
2. Claim frequency is constant for most of the period used to compute trend factors. During the latter portion of this period, it increases rapidly. Assume this increase is due to an increase in claims rather than to a decrease in exposures.
3. The increment in claims in recent years has a claim size distribution similar to the one which would have been observed had claim frequency remained constant.

The increase in claim frequency during recent years should not affect policy year average incurred claim cost; if other conditions remain unchanged, policy year average incurred claim cost will continue to increase at the annual rate *d*.

As long as claim frequency is constant and settlement procedures remain unchanged, calendar year average paid claim cost will increase at rate *d*. When claim frequency increases, a disproportionately large number of small claims from this increment in claims will be included in the immediate evaluations of calendar year average paid claim cost data. The large claims will take time to settle and will be included in future evaluations of average paid claim cost data because smaller claims are settled more quickly than larger claims.

The fact that an unusually large number of small claims will be included quickly in the calendar year paid claim cost data will lead to smaller values of average paid claim cost than would have been calculated if claim frequency had remained constant. The slope of the average paid claim cost curve will decrease, leading to an indicated average annual change in average paid claim cost which is smaller than *d*. Since policy year average incurred claim cost is still increasing at a rate of *d*, use of calendar year paid claim cost data to calculate trend factors would produce an inadequate rate level.

* This model was presented to ISO's Commercial Casualty Actuarial Subcommittee in a mailing dated September 22, 1976 and was originally prepared by Robert Bear of ISO.

As an illustration of the problem cited above, assume that the policy year 1972 average incurred claim cost was \$10,000. Assume that only the following two types of claims occur: small claims with incurred claim cost of \$1,000 in 1972, and large claims with incurred claim cost of \$20,000 in 1972. Incurred claim costs for both small and large claims are increasing at the constant rate of 10% per year. Assume the number of exposures remains constant throughout while the number of claims remains constant until 1974; in 1974, the number of claims increases 30%; the number of claims remains constant thereafter. Consequently, claim frequency is constant before 1974, increases 30% in 1974, and remains constant thereafter. Assume that small claims are settled immediately and that large claims take four years to settle; no loss development occurs. Finally, assume that the proportion of small and large claims remain constant from year to year.

Based on the 1972 average claim cost of \$10,000, the proportion of large claims can be obtained by solving the following equation:

$$\$20{,}000\,X + (1 - X)\,1000 = 10{,}000$$
$$X = .474$$

Consequently, 47.4% of all claims occurring in any policy year are large claims and 52.6% are small claims.

Let C denote the total number of claims occurring in any year prior to 1974. The total paid loss in calendar year 1972 is obtained by adding the costs of small claims occurring in 1972 to the incurred costs of large claims occurring in 1968:

$$\text{Calendar year 1972 losses} = \$1{,}000\,(.526C) + \$20{,}000\,(1.1)^{-4}\,(.474C)$$
$$= \$526C + \$6474.97C$$
$$= \$7000.97C$$

Thus, the average paid claim cost in 1972 is \$7,000.97.

The total paid claim cost for 1973 is obtained by adding the costs of small claims occurring in 1973 to the incurred costs of large claims occurring in 1969 (and settled in 1973):

$$\text{Calendar year 1973 losses} = \$1100\,(.526C) + \$20{,}000\,(1.1)^{-3}\,(.474C)$$
$$= \$7701.06C;$$

the average paid claim cost for 1973 is \$7701.06.

In policy year 1974, the total number of claims is $1.3C$. The total paid cost for calendar year 1974 is obtained by adding the costs of the .526 ($1.3C$) small claims occurring in 1974 to the incurred costs of the $.474C$ large claims which occurred in 1970 and were settled in 1974:

$$\text{Calendar year 1974 losses} = \$1210\,(.526)\,(1.3C) + \$20{,}000\,(1.1)^{-2}\,(.474C)$$
$$= \$8{,}662.11C$$

The average paid claim cost for 1974 is $\$8{,}662.11C/1.1578C = \$7481.53$. While average paid claim cost increases 10% from calendar year 1972 to 1973, it decreases 2.9% from calendar year 1973 to 1974. This drop is due solely to the jump in claim frequency in 1974 which results in a disproportionately large number of small claims being included in the paid claim cost data of 1974.

Values for average paid claim cost for succeeding calendar years are calculated similarly and are given in the table below, along with the annual changes in average paid claim cost.

Calendar Year	Average Paid Claim Cost	Percentage Increase
1975	8,229.68	10.0%
1976	9,052.64	10.0
1977	9,957.91	10.0
1978	12,402.64	24.6
1979	13,642.91	10.0
1980	15,007.20	10.0

Notice that average paid claim cost increases 24.6% from 1977 to 1978 due to the impact of the large number of big claims incurred in 1974 which are settled in 1978. Note also that average paid claim cost increases from 1972 to 1980 by a factor of $(1.1)^8$. Hence, average paid claim cost increases at an average annual rate of 10% from 1972 to 1980. However, the jump in claim frequency in 1974 produces a decrease in average paid claim cost in 1974 and a large increase in average paid claim cost in 1978. If only average paid claim cost data through 1974 or 1975 were used to compute an average annual change in average paid claim cost, the result would be significantly smaller than 10%.

EXHIBIT 1

COUNTRYWIDE LOSS DEVELOPMENT

PHYSICIANS, SURGEONS & DENTISTS—PROFESSIONAL LIABILITY INSURANCE

Policy Year Ending	Basic Limits Incurred Losses and Allocated Loss Adjustment Expense as of:									
	27 Months	39 Months	51 Months	63 Months	75 Months	87 Months	99 Months	111 Months	123 Months	135 Months
12/31/66									14,515,504	14,725,212
12/31/67								17,834,379	18,304,244	18,379,627
12/31/68							21,070,637	21,682,255	21,981,105	22,236,520
12/31/69						24,409,798	25,079,556	25,271,738	25,067,665	
12/31/70					29,504,055	29,968,671	30,839,537	30,626,180		
12/31/71				37,693,694	37,329,634	38,096,613	37,222,908			
12/31/72			49,654,899	55,410,098	55,758,608	56,378,770				
12/31/73		55,642,854	67,215,009	71,322,486	71,466,950					
12/31/74	43,796,302	76,611,689	95,590,512	92,353,039						
12/31/75	55,543,924	91,469,239	103,138,770							
12/31/76	41,626,398	63,260,930								

Policy Year Ending	Ratios								
	39:27	51:39	63:51	75:63	87:75	99:87	111:99	123:111	135:123
12/31/66									1.014
12/31/67								1.026	1.004
12/31/68							1.029	1.014	1.012
12/31/69						1.027	1.008	.992	
12/31/70					1.016	1.029	.993		
12/31/71				.990	1.021	.977			
12/31/72			1.116	1.006	1.011				
12/31/73		1.208	1.061	1.002					
12/31/74	1.749	1.248	.966						
12/31/75	1.647	1.128							
12/31/76	1.520								
3 Year Mean	1.639	1.195	1.048	.999	1.016	1.011	1.010	1.011	1.010

Policy Year Ending	27 to 39	39 to 51	51 to 63	63 to 75	75 to 87	87 to 99	99 to 111	111 to 123	123 to 135	Factor
12/31/73					1.016	1.011	1.010	1.011	1.010	1.059
12/31/74				.999	1.016	1.011	1.010	1.011	1.010	1.058
12/31/75			1.048	.999	1.016	1.011	1.010	1.011	1.010	1.109
12/31/76		1.195	1.048	.999	1.016	1.011	1.010	1.011	1.010	1.325
12/31/77	1.639	1.195	1.048	.999	1.016	1.011	1.010	1.011	1.010	2.172

Source: Insurance Services Office; includes all reporting companies.

EXHIBIT 1
continued

COMPARISON OF COUNTRYWIDE LOSS DEVELOPMENT FACTORS

PHYSICIANS, SURGEONS & DENTISTS PROFESSIONAL LIABILITY INSURANCE

Interval	Development Factor	Interval	Development Factor
75 to 87	1.016	87 to 135	1.042
87 to 99	1.011	99 to 135	1.031
99 to 111	1.010	111 to 135	1.021
111 to 123	1.011	123 to 135	1.010

EXHIBIT 2

Calculation of Annual Trend Factor Based on Average Paid Claim Cost Data

General Liability Insurance
Professional Liability
Basic Limits Bodily Injury

(1)	(2)	(3)	(4)	(5)
			Average Paid Claim Cost	
Twelve Months Ended	$25,000 Basic Limits Paid Losses*	Number of Paid Claims	Actual (2) ÷ (3)	Exponential Curve of Best Fit
6/30/70	$26,132,901	6,780	$3,854	$4,020.78
12/31/70	29,271,828	7,067	4,142	4,149.37
6/30/71	31,650,272	7,537	4,199	4,282.07
12/31/71	32,746,397	7,354	4,453	4,419.01
6/30/72	34,684,486	7,367	4,708	4,560.33
12/31/72	38,736,177	8,135	4,762	4,706.18
6/30/73	44,783,802	8,958	4,999	4,856.68
12/31/73	50,130,236	9,475	5,291	5,012.00
6/30/74	55,000,735	10,183	5,401	5,172.29
9/30/74	58,759,022	11,139	5,275	5,254.34
12/31/74	63,290,907	12,316	5,139	5,337.70
3/31/75	65,740,318	13,133	5,006	5,422.38

Average Annual Paid Claim Cost Trend Factor
[$4,282.07 ÷ $4,020.78] 1.065

* Excluding all loss adjustment expense.

Source: Insurance Services Office; includes all reporting companies.

EXHIBIT 3
Part 1

Comparison of Average Annual Trend Factors Paid vs. Incurred

General Liability Insurance

Subline	Paid*	Incurred*	See Exhibit 3 Part
OL&T—Bodily Injury, Basic Limits	1.131	1.102	2
OL&T—Bodily Injury, Total Limits	1.159	1.124	—
OL&T—Property Damage, Basic Limits	—	1.108	3
M&C—Bodily Injury, Basic Limits	1.076	1.118	4
M&C—Bodily Injury, Total Limits	1.102	1.175	—
M&C—Property Damage, Basic Limits	1.109	1.093	5
M&C—Property Damage, Total Limits	1.119	1.103	—
General Liability excluding Products and Professional—Bodily Injury			
Basic Limits	1.147	—	6
Total Limits	1.186	—	—
General Liability—Property Damage			
Basic Limits	1.121	—	7
Total Limits	1.167	—	—

* Paid trend factors are based on an exponential least squares fit of the 12 quarterly year ended average paid claim cost data points through March 31, 1976. Incurred trend factors reflect a similar fit applied to the average ultimate incurred claim costs for policy years 1970–1974, evaluated as of March 31, 1975.

Source: Insurance Services Office

EXHIBIT 3
Part 2

CALCULATION OF ANNUAL TREND FACTOR BASED ON INCURRED CLAIM COST DATA

GENERAL LIABILITY INSURANCE
OWNERS, LANDLORDS & TENANTS
BASIC LIMITS BODILY INJURY

(1)	(2)	(3)	(4)	(5)
			Average Incurred Claim Cost	
Policy Year Ending	Basic Limits Incurred Losses*	Number of Incurred Claims**	Actual (2) ÷ (3)	Exponential Curve of Best Fit
12/31/70	$ 86,168,004	62,234	$1,385	$1,370.09
12/31/71	91,968,722	59,696	1,541	1,509.81
12/31/72	110,828,095	69,533	1,594	1,663.77
12/31/73	109,756,890	61,003	1,799	1,833.42
12/31/74	111,756,464	53,656	2,083	2,020.38

Average Annual Incurred Claim Cost Trend Factor
(2,020.38 ÷ 1,833.42) 1.102

* Including all loss adjustment expense and developed to an ultimate settlement basis.

** As of 39 months.

Source: Insurance Services Office

EXHIBIT 3
Part 3

CALCULATION OF ANNUAL TREND FACTOR BASED ON INCURRED CLAIM COST DATA

GENERAL LIABILITY INSURANCE
OWNERS, LANDLORDS & TENANTS
BASIC LIMITS PROPERTY DAMAGE

(1)	(2)	(3)	(4)	(5)
			Average Incurred Claim Cost	
Policy Year Ending	Basic Limits Incurred Losses*	Number of Incurred Claims**	Actual (2) ÷ (3)	Exponential Curve of Best Fit
12/31/70	$11,154,844	29,972	$372	$376.08
12/31/71	12,692,026	29,144	435	416.79
12/31/72	15,620,690	35,450	441	461.90
12/31/73	18,902,315	36,597	516	511.90
12/31/74	20,691,387	36,236	571	567.31

Average Annual Incurred Claim Cost Trend Factor
(567.31 ÷ 511.90) 1.108

* Including all loss adjustment expense and developed to an ultimate settlement basis.

** As of 39 months.

Source: Insurance Services Office

EXHIBIT 3
Part 4

CALCULATION OF ANNUAL TREND FACTOR BASED ON INCURRED CLAIM COST DATA

GENERAL LIABILITY INSURANCE
MANUFACTURERS AND CONTRACTORS
BASIC LIMITS BODILY INJURY

(1)	(2)	(3)	(4)	(5)
			Average Incurred Claim Cost	
Policy Year Ending	Basic Limits Incurred Losses*	Number of Incurred Claims**	Actual (2) ÷ (3)	Exponential Curve of Best Fit
12/31/70	$ 70,429,232	19,553	$3,602	$3,650.94
12/31/71	73,480,091	18,026	4,076	4,082.16
12/31/72	96,626,729	20,507	4,712	4,564.30
12/31/73	101,668,362	20,026	5,077	5,103.38
12/31/74	111,899,044	19,841	5,640	5,706.14

Average Annual Incurred Claim Cost Trend Factor
(5,706.14 ÷ 5,103.38) 1.118

* Including all loss adjustment expense and developed to an ultimate settlement basis.

** As of 39 months.

Source: Insurance Services Office

EXHIBIT 3
Part 5

CALCULATION OF ANNUAL TREND FACTOR BASED ON INCURRED CLAIM COST DATA

GENERAL LIABILITY INSURANCE
MANUFACTURERS AND CONTRACTORS
BASIC LIMITS PROPERTY DAMAGE

(1)	(2)	(3)	(4)	(5)
			Average Incurred Claim Cost	
Policy Year Ending	Basic Limits Incurred Losses*	Number of Incurred Claims**	Actual (2) ÷ (3)	Exponential Curve of Best Fit
12/31/70	$39,108,952	99,876	$392	$397.94
12/31/71	39,377,197	88,825	443	434.86
12/31/72	49,375,642	102,400	482	475.20
12/31/73	55,979,114	110,408	507	519.29
12/31/74	62,467,019	109,376	571	567.46

Average Annual Incurred Claim Cost Trend Factor
(567.46 ÷ 519.29) 1.093

* Including all loss adjustment expense and developed to an ultimate settlement basis.

** As of 39 months.

Source: Insurance Services Office

EXHIBIT 3
Part 6

CALCULATION OF ANNUAL TREND FACTOR BASED ON AVERAGE PAID CLAIM COST DATA

GENERAL LIABILITY INSURANCE
ALL SUBLINES COMBINED EXCLUDING PROFESSIONAL & PRODUCTS LIABILITY
BASIC LIMITS BODILY INJURY

(1)	(2)	(3)	(4)	(5)
			Average Paid Claim Cost	
Calendar Year Ending	$25,000 Basic Limits Paid Losses*	Number of Paid Claims	Actual (2) ÷ (3)	Exponential Curve of Best Fit
6/30/72	$218,785,402	170,590	$1,283	1,245.80
12/31/72	228,390,998	172,958	1,320	1,334.44
6/30/73	246,497,863	171,315	1,439	1,429.38
12/31/73	262,432,437	172,434	1,522	1,531.08
6/30/74	258,890,150	163,442	1,584	1,640.01
9/30/74	266,634,243	161,270	1,653	1,697.35
12/31/74	264,654,012	153,195	1,728	1,756.70
3/31/75	280,023,470	152,924	1,831	1,818.12
6/30/75	279,295,068	144,922	1,927	1,881.68
9/30/75	278,496,210	139,359	1,998	1,947.47
12/31/75	284,878,602	139,439	2,043	2,015.56
3/31/76	273,230,393	132,522	2,062	2,086.03

Average Annual Paid Claim Cost Trend Factor
(1429.38 ÷ 1245.80) 1.147

* Excluding all loss adjustment expense.

Source: Insurnace Services Office; includes all reporting companies.

EXHIBIT 3
Part 7

CALCULATION OF ANNUAL TREND FACTOR BASED ON AVERAGE PAID CLAIM COST DATA

GENERAL LIABILITY INSURANCE
ALL GENERAL LIABILITY SUBLINES
BASIC LIMITS PROPERTY DAMAGE

(1)	(2)	(3)	(4)	(5)
			Average Paid Claim Cost	
Calendar Year Ending	$25,000 Basic Limits Paid Losses*	Number of Paid Claims	Actual (2) ÷ (3)	Exponential Curve of Best Fit
6/30/72	$58,363,769	172,951	$337	330.51
12/31/72	62,715,239	178,270	352	349.90
6/30/73	65,786,611	180,767	364	370.42
12/31/73	75,846,823	197,276	384	392.15
6/30/74	81,688,134	198,132	412	415.16
9/30/74	83,660,568	196,596	426	427.16
12/31/74	84,083,615	190,021	442	439.51
3/31/75	86,187,381	189,820	454	452.22
6/30/75	84,569,339	180,688	468	465.30
9/30/75	80,774,492	167,576	482	478.75
12/31/75	77,123,228	155,584	496	492.59
3/31/76	74,060,660	146,952	504	506.83

Average Annual Paid Claim Cost Trend Factor
(370.42 ÷ 330.51) 1.121

* Excluding all loss adjustment expense.

Source: Insurnace Services Office; includes all reporting companies.

EXHIBIT 4

Development of Statewide Rate Level Change

Owners, Landlords and Tenants Bodily Injury Liability Insurance

Premises and Operations (Subline Codes 314 & 326)
Class Groups 1–13
Maine (18)

(1) Policy Year Ending	(2) \$25,000 Basic Limits Earned Premium at Present Rates	(3) \$25,000 Basic Limits Incurred Losses*	(4) Number of Claims	(5) Loss and Loss Adjustment Ratio (3) ÷ (2)
12/31/76	\$1,807,819	\$1,586,273	302	.877
12/31/77	1,890,592	1,345,680	337	.712

(6)	Weighted Loss and Loss Adjustment Ratio at Present Rates (30% of Policy Year Ended 12/31/76 and 70% of Policy Year Ended 12/31/77)	.762
(7)	Expected Loss and Loss Adjustment Ratio	.570
(8)	Credibility Based on Latest Two Years' Number of Claims	.90
(9)	Indicated Rate Level Change {[(6) × (8)] + [(1.000 − (8)) × (7) × trend factor **]} ÷ (7)	1.314
(10)	Selected Statewide Rate Level Change	+31.4%

* Including all loss adjustment expense and developed to an ultimate basis. In addition, policy year losses have been trended from the average date of coverage to one year beyond an anticipated effective date of August 1, 1979. Actual loss severity trend as measured in Exhibit 3, was applied exponentially to bring losses to an October 1, 1978 level. In anticipation of positive effects that might be brought about by the voluntary Anti-Inflation Program, a reduced trend factor was selected to exponentially project losses beyond this date.

** Trends the expected loss ratio from one year after the last review or filing effective date to one year beyond an anticipated effective date of August 1, 1979, again reducing the trend from October 1, 1978 in anticipation of effects of the Anti-Inflation Program.

Source: Insurance Services Office; includes all reporting companies.

EXHIBIT 5
Part 1

CALCULATION OF AVERAGE ANNUAL CHANGE IN WAGE LEVELS

MANUFACTURERS & CONTRACTORS LIABILITY INSURANCE

(1) Quarter Ending	(2) Manufacturing Production Actual*	(3) (2) × .181**	(4) Contract Construction Actual*	(5) (4) × .819**	(6) (3) + (5)	(7) Exponential Curve of Best Fit
6/30/76	205.75	37.24	279.97	229.30	266.54	266.64
9/30/76	211.33	38.25	289.50	237.10	275.35	270.70
12/31/76	217.45	39.36	294.83	241.47	280.83	274.82
3/31/77	218.53	39.55	283.55	232.23	271.78	279.00
6/30/77	226.57	41.01	294.08	240.85	281.86	283.25
9/30/77	231.04	41.82	300.89	246.43	288.25	287.56
12/31/77	239.23	43.30	301.95	247.30	290.60	291.94
3/31/78	237.74	43.03	289.34	236.97	280.00	296.38
6/30/78	246.04	44.53	315.84	258.67	303.20	300.89
9/30/78	251.04	45.44	330.93	271.03	316.47	305.47
12/31/78	261.80	47.39	330.41	270.61	318.00	310.12
3/31/79	262.76	47.56	320.16	262.21	309.77	314.84

Average yearly change 1.062

* Source: *Monthly Labor Review* & Bureau of Labor Statistics

** Weights obtained from total collected premium.

Source: Insurance Services Office

EXHIBIT 5
Part 2

CALCULATION OF ANNUAL WAGE OFFSET FACTOR

MANUFACTURERS & CONTRACTORS LIABILITY

(1) Average wage level

Policy Year Ending	Average Wage Level*
12/31/73	216.24
12/31/74	229.73
12/31/75	244.29
12/31/76	261.36
12/31/77	276.92

(2) Average value as of 2/15/79** = 309.77 $\left\{ \begin{array}{l} 1/79 = 301.01 \\ 2/79 = 308.66 \\ 3/79 = 319.65 \end{array} \right\}$

(3) Indexing of policy year 1972-1976 to current (2/15/79) level

Policy Year Ending	Index (2) ÷ (1)
12/31/73	1.433
12/31/74	1.348
12/31/75	1.268
12/31/76	1.185
12/31/77	1.119

(4) Premium Trend: Trend from policy year to one year beyond anticipated effective date of 2/1/80.

Policy Year Ending	Premium Trend
12/31/73	$1.433 \times (1.062)^{1.958} = 1.612$
12/31/74	$1.348 \times (1.062)^{1.958} = 1.516$
12/31/75	$1.268 \times (1.062)^{1.958} = 1.426$
12/31/76	$1.185 \times (1.062)^{1.958} = 1.333$
12/31/77	$1.119 \times (1.062)^{1.958} = 1.259$

* Source: Bureau of Labor Statistics
** Source: *Monthly Labor Review*

EXHIBIT 6

DETERMINATION OF UNLIMITED PAYROLL OFFSET FACTORS SPECIAL STUDY FOR POLICY YEAR 1-1-70 TO 12-31-70 LOUISIANA*

Code No.	(1) Exec. Officer Payroll	(2) No. of Officers	(3) Other Payroll	(4) Total Payroll Above Limitation	(5) Payroll Subject to Limitation	(6) Offset (5) ÷ [(4) + (5)]
0005	11,707	1	40,077	51,784	2,158,006	.977
0006	71,729	117	380,997	452,726	12,039,809	.964
0008	0	0	0	0	76,730	1.000
0030	18,194	3	208,895	227,089	8,785,275	.975
0034	8,675	3	76,152	84,827	1,376,063	.942
0035	23,280	3	23,989	47,269	264,066	.848
0042	13,517	2	159,696	173,213	1,822,110	.913
0050	0	0	10,731	10,731	210,837	.952
0059	0	0	5,793	5,793	294,374	.981
0106	12,600	1	41,887	54,487	1,024,188	.949
0251	14,800	2	57,935	72,735	677,048	.903
0400	0	0	66,947	66,947	347,700	.839
0401	0	0	26,946	26,946	758,874	.966
1164	0	0	168,706	168,706	646,817	.793
1320	34,410	5	5,036,097	5,070,507	12,191,513	.706
1322	114,703	9	1,878,835	1,993,538	3,535,564	.639
1430	0	0	2,140	2,140	24,974	.921
1452	0	0	17,645	17,645	114,098	.866
1463	0	0	301,778	301,778	1,043,563	.776
1473	0	0	16,962	16,962	175,190	.912
1624	0	0	3,177	3,177	177,716	.982
1642	0	0	39,259	39,259	81,391	.675
1701	0	0	33,187	33,187	34,774	.512
1703	0	0	0	0	29,783	1.000
1803	4,880	1	34,204	39,084	208,725	.842
1852	0	0	0	0	2,099,743	1.000
2001	0	0	25,826	25,826	229,667	.899
2002	0	0	54,126	54,126	436,812	.890
2003	147,265	19	5,692,519	5,839,784	11,047,536	.654
2014	58,500	4	352,842	411,342	2,870,271	.875
2021	0	0	1,421,772	1,421,772	3,159,213	.690
2022	86,994	9	1,333,643	1,420,637	6,521,614	.821
2039	50,700	5	504,647	555,347	1,610,256	.744
2041	16,800	3	68,856	85,656	1,338,650	.940
2065	0	0	3,425	3,425	37,999	.917
2070	42,027	4	1,848,092	1,890,119	5,977,108	.760
2081	0	0	270,936	270,936	1,011,983	.789
2089	51,773	3	88,641	140,414	1,497,740	.914
2095	24,500	2	642,933	667,433	1,978,314	.748
2105	0	0	0	0	21,635	1.000

* Excerpted from the National Council on Compensation Insurance's 12/31/75 filing of Item B-1094, Amendment of Payroll Limitation Rules in Louisiana.

EXHIBIT 7
Part 1

PROFESSIONAL LIABILITY INSURANCE LOSS DEVELOPMENT

PHYSICIANS, SURGEONS & DENTISTS (STATES WITH LOW DEVELOPMENT)*

Basic Limits Incurred Losses and Allocated Loss Adjustment Expense as of:

Policy Year Ending	27 Months	39 Months	51 Months	63 Months	75 Months	87 Months	99 Months	111 Months	123 Months	135 Months
12/31/66						2,592,268	2,544,709	2,652,294	2,172,096	2,802,493
12/31/67					3,333,501	3,224,645	3,050,345	3,135,806	3,115,082	3,104,573
12/31/68				3,866,465	3,471,905	3,489,057	3,551,015	3,701,025	3,571,544	3,715,976
12/31/69			5,320,458	4,943,169	5,340,481	4,702,429	4,694,296	4,496,450	4,464,224	
12/31/70		6,106,561	6,774,965	6,784,410	6,179,470	5,931,344	5,696,357	5,929,706		
12/31/71	4,063,318	6,237,693	6,615,069	6,914,608	6,242,609	5,961,564	5,767,401			
12/31/72	6,082,197	10,075,310	10,752,535	10,520,629	9,854,569	10,206,054				
12/31/73	8,271,300	13,383,111	13,190,027	12,603,632	12,043,135					
12/31/74	11,841,959	18,209,291	19,013,225	17,106,396						
12/31/75	13,394,160	19,126,309	20,261,954							
12/31/76	7,456,558	11,306,954								

Ratios

Policy Year Ending	39:27	51:39	63:51	75:63	87:75	99:87	111:99	123:111	135:123
12/31/66						.982	1.042	1.023	1.033
12/31/67					.967	.946	1.028	.993	.997
12/31/68				.898	1.005	1.018	1.042	.965	1.040
12/31/69			.929	1.080	.881	.998	.958	.993	
12/31/70		1.109	1.001	.911	.960	.960	1.041		
12/31/71	1.535	1.060	1.045	.903	.955	.967			
12/31/72	1.657	1.067	.978	.937	1.036				
12/31/73	1.618	.986	.956	.956					
12/31/74	1.538	1.044	.900						
12/31/75	1.428	1.059							
12/31/76	1.516								
3 Year Mean	1.494	1.030	.945	.932	.984	.975	1.014	.984	1.023

Policy Year Ending	27 to 39	39 to 51	51 to 63	63 to 75	75 to 87	87 to 99	99 to 111	111 to 123	123 to 135	Factor
12/31/73					.984	.975	1.014	.984	1.023	.979
12/31/74				.932	.984	.975	1.014	.984	1.023	.913
12/31/75			.945	.932	.984	.975	1.014	.984	1.023	.862
12/31/76		1.030	.945	.932	.984	.975	1.014	.984	1.023	.888
12/31/77	1.494	1.030	.945	.932	.984	.975	1.014	.984	1.023	1.327

* Ala., Ark., Ga., Ky., La., Minn., N.C., Ohio, Tenn., Va.

Source: Insurance Services Office; includes all reporting companies.

EXHIBIT 7
Part 2

PROFESSIONAL LIABILITY INSURANCE LOSS DEVELOPMENT

PHYSICIANS, SURGEONS & DENTISTS
(STATES WITH MEDIUM DEVELOPMENT)*

Basic Limits Incurred Losses and Allocated Loss Adjustment Expense as of:

Policy Year Ending	27 Months	39 Months	51 Months	63 Months	75 Months	87 Months	99 Months	111 Months	123 Months	135 Months
12/31/66								4,965,395	4,914,066	4,970,428
12/31/67							6,042,997	6,269,797	6,436,210	6,333,321
12/31/68						7,546,617	7,773,144	7,845,349	7,916,681	7,970,198
12/31/69					8,533,546	9,021,410	9,234,803	9,087,059	9,125,212	
12/31/70				11,876,021	11,926,614	12,091,332	12,112,319	11,849,339		
12/31/71			15,023,441	15,960,740	15,742,204	15,563,035	15,525,421			
12/31/72		16,689,188	19,323,468	20,958,091	20,162,830	20,225,194				
12/31/73	12,675,832	20,811,842	26,018,108	25,577,158	26,216,658					
12/31/74	17,004,356	28,203,997	32,570,027	30,943,126						
12/31/75	21,946,572	32,632,978	35,844,172							
12/31/76	16,002,535	22,036,198								

Ratios

Policy Year Ending	39:27	51:39	63:51	75:63	87:75	99:87	111:99	123:111	135:123
12/31/66								.990	1.011
12/31/67							1.038	1.027	.984
12/31/68						1.030	1.009	1.009	1.007
12/31/69					1.057	1.024	.984	1.004	
12/31/70				1.004	1.014	1.002	.978		
12/31/71			1.062	.986	.989	.998			
12/31/72		1.158	1.085	.962	1.003				
12/31/73	1.642	1.250	.983	1.025					
12/31/74	1.659	1.155	.950						
12/31/75	1.487	1.098							
12/31/76	1.377								
3 Year Mean	1.508	1.168	1.006	.991	1.002	1.008	.990	1.013	1.001

Policy Year Ending	27 to 39	39 to 51	51 to 63	63 to 75	75 to 87	87 to 99	99 to 111	111 to 123	123 to 135	Factor
12/31/73					1.002	1.008	.990	1.013	1.001	1.014
12/31/74				.991	1.002	1.008	.990	1.013	1.001	1.005
12/31/75			1.006	.991	1.002	1.008	.990	1.013	1.001	1.011
12/31/76		1.168	1.006	.991	1.002	1.008	.990	1.013	1.001	1.181
12/31/77	1.508	1.168	1.006	.991	1.002	1.008	.990	1.013	1.001	1.780

* Colo., Dela., D.C., Fla., Ida., Ind., Iowa, Kans., Md., Miss., Mo., Mont., Neb., Nev., N.M., N.D., Okla., Ore., S.C., S.D., Tex., Utah, W.V., Wisc., Wyo., Haw., Alas., P.R.

Source: Insurance Services Office; includes all reporting companies.

EXHIBIT 7
Part 3

PROFESSIONAL LIABILITY INSURANCE LOSS DEVELOPMENT

PHYSICIANS, SURGEONS & DENTISTS
(STATES WITH HIGH DEVELOPMENT)*

Basic Limits Incurred Losses and Allocated Loss Adjustment Expense as of:

Policy Year Ending	27 Months	39 Months	51 Months	63 Months	75 Months	87 Months	99 Months	111 Months	123 Months	135 Months
12/31/66						6,267,160	6,648,473	6,703,874	6,889,342	6,952,291
12/31/67					7,596,832	7,984,313	8,271,555	8,428,776	8,752,952	8,941,733
12/31/68				8,416,766	8,808,923	9,440,258	9,746,478	10,135,881	10,492,880	10,550,346
12/31/69			8,746,086	10,169,826	10,899,551	10,685,959	11,150,457	11,688,229	11,478,229	
12/31/70		6,949,289	9,987,193	11,469,442	11,397,971	11,945,995	13,030,861	12,847,135		
12/31/71	4,608,189	8,963,200	13,304,111	14,818,346	15,344,821	16,572,014	15,930,086			
12/31/72	5,791,857	12,855,942	19,578,896	23,931,378	25,741,209	25,947,522				
12/31/73	10,021,758	21,447,901	28,006,874	33,141,696	33,207,157					
12/31/74	14,949,987	30,198,401	44,007,260	44,303,517						
12/31/75	20,203,192	39,709,952	47,032,644							
12/31/76	18,167,305	29,917,778								

Ratios

Policy Year Ending	39:27	51:39	63:51	75:63	87:75	99:87	111:99	123:111	135:123
12/31/66						1.061	1.008	1.028	1.009
12/31/67					1.051	1.036	1.019	1.038	1.022
12/31/68				1.047	1.072	1.032	1.040	1.035	1.005
12/31/69			1.163	1.072	.980	1.043	1.048	.982	
12/31/70		1.437	1.148	.994	1.048	1.091	.986		
12/31/71	1.945	1.484	1.114	1.036	1.080	.961			
12/31/72	2.220	1.523	1.222	1.076	1.008				
12/31/73	2.140	1.306	1.183	1.002					
12/31/74	2.020	1.457	1.007						
12/31/75	1.966	1.184							
12/31/76	1.647								
3 Year Mean	1.878	1.316	1.137	1.038	1.045	1.032	1.025	1.018	1.012

Policy Year Ending	27 to 39	39 to 51	51 to 63	63 to 75	75 to 87	87 to 99	99 to 111	111 to 123	123 to 135	Factor
12/31/73					1.045	1.032	1.025	1.018	1.012	1.139
12/31/74				1.038	1.045	1.032	1.025	1.018	1.012	1.182
12/31/75			1.137	1.038	1.045	1.032	1.025	1.018	1.012	1.344
12/31/76		1.316	1.137	1.038	1.045	1.032	1.025	1.018	1.012	1.769
12/31/77	1.878	1.316	1.137	1.038	1.045	1.032	1.025	1.018	1.012	3.322

* Ariz., Calif., Conn., Ill., Me., Mass., Mich., N.H., N.J., N.Y., Pa., R.I., Vt., Wash.

Source: Insurance Services Office; includes all reporting companies.

EXHIBIT 8

Development of Premium at Present Rates (Including Additional Interests)

Hospital Professional Liability Insurance New Jersey

(1) Policy Year	(2) Additional Interests Collected Premium	(3) Collected Premiums For Rated Classes	(4) Ratio (2) ÷ (3)
1974	$ 339,130	$ 1,619,212	.209
1975	466,527	2,161,940	.216
1976	950,267	6,662,552	.143
Total	$1,755,924	$10,443,704	.168

(1) Policy Year	(5) $25/75 Premium at Present Rates— Rated Classes	(6) Additional Interest Factor	(7) $25/75 Premium at Present Rates— All Classes (5) × (6)
1974	$5,783,630	1.143	$6,610,689
1975	7,797,548	1.143	8,912,597
1976	3,670,085*	1.143	4,194,907

* Mostly claims-made experience.

Source: Insurance Services Office

EXHIBIT 9

Calculation of Revised SMP Package Program Discounts

SMP Policy Program
Motel/Hotel Program
Indiana

Line of Business	(1) Adjusted SMP Loss and Loss Adjustment Ratio	(2) 1977 Adjusted Earned Premium Weight*
Fire	0.186	.461
Extended Coverage	0.593	.055
Casualty Other Than Automobile	1.091	.484

(3) Weighted Total SMP Program Loss and Loss Adjustment Ratio: Total ((1) × (2))	0.646
(4) Current Program Discount Complement	0.80
(5) Expected Loss and Loss Adjustment Ratio	0.570
(6) Revised SMP Package Program Discount 1.00 − ((3) × (4)/(5))	9.3%
(7) Factor to Adjust For Use of Rating Plans	0.92
(8) Revised Discount Including Rating Plan Effects 1.00 − [(7) × (1.00 − (6))]	16.6%
(9) Selected SMP Package Program Discount	15.0%

* Statewide

Source: Insurance Services Office

IMPLICATIONS OF SALES AS AN EXPOSURE BASE FOR PRODUCTS LIABILITY

STEPHEN W. PHILBRICK

In Dorweiler's classic article, "Notes On Exposure and Premium Bases," he defines the term exposure as follows:

> When *critical conditions* and *injurable objects* exist in such relationship that accidents may result there is said to be *exposure*. The term *critical conditions* is intended to cover, rather broadly, the presence of or the absence of anything, objective or subjective, generally external to the injurable object, which contributes to the accident frequency and/or the accident severity.[1]

This somewhat intangible concept will be referred to in the remainder of the paper as the "true exposure." It is obviously important to select an exposure medium which will accurately measure the true exposure. The selected medium is called "the premium basis"[2] by Dorweiler, and will be called the exposure base or exposure units in this paper. Dorweiler suggests two criteria for the determination of a good choice of an exposure medium:

1. Magnitude of Medium should vary with hazard.
2. The Medium should be practical and preferably already in use.[3]

Thus, payroll is a good measure of exposure for workers' compensation insurance since, for a given classification, higher payroll tends to indicate higher expected losses, and since payroll information is relatively easy to obtain.

Finally, Dorweiler states that "the hazard varies directly with the product of the three variables: *critical conditions*, *injurable objects*, and *period of time*."[4] The differing premium rates for different classifications within a line of insurance are recognition of the critical conditions variable while the other two variables are reflected in the exposure base. Thus, beds, doctors, cars and units are the oft-used but short-hand versions of the more technically correct bed-years, doctor-years, car-years, or unit-years. The partition of payroll into a quantity and a temporal variable is less obvious, but payroll can be viewed

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[1] P. Dorweiler, "Notes on Exposure and Premium Bases," *PCAS* LVIII (1971), p. 59.
[2] Ibid., p. 60.
[3] Ibid., p. 61.
[4] Ibid., p. 59.

either as a wage rate multiplied by length of time worked, or as a surrogate for person-hours or person-years. While the existence of a time component is necessary for most exposure bases, there are exceptions. Generally, these exceptions involve a single use or consumption, so that for fillings (propane tanks) or blood donations or food products, there is not really a time component.

The dominant exposure base for products liability insurance, dollars of sales, cannot easily be decomposed into the injurable objects and time components. Yet many products classifications which use sales as an exposure base do not fall into the above exception classes. This paper will explore the implications of using sales as an exposure base. Sales can be thought of as the product of the number of units sold and the average price per unit; in order to simplify the discussion, we will assume the price per unit is fixed, so that total sales and number of units sold can be used interchangeably.

The products liability policy as considered in this paper covers occurrences during the policy period. Occurrences are *not* limited to those resulting from products manufactured during the current policy period but rather could result from any products still in existence. Hence, the true exposure is more accurately a function of total sales to date, less "expired" products, where expired means consumed, destroyed, or otherwise disposed of. Although this measure would be a preferable exposure base from the standpoint of the first of Dorweiler's two criteria, since it more closely varies with the hazard, it has not been considered a practical medium since the information is not generally readily available. Sales data for the current year are more easily obtainable and are thus the preferred exposure base.

It will be helpful to examine the relationship between current year sales and products in use. For a product with a very short lifetime (e.g. batteries), the total number of units in use is small relative to the number of units sold during the year. At the other extreme, the number of drill presses in use is far in excess of the number sold in the current year. It should be clear that the ratio of true exposure to current year sales depends on the distribution of the lifetime of the product, and generally increases with the length of the expected lifetime. It also depends on the length of time that the company has been in business, as will be seen later.

The following crude example with overly restrictive assumptions shows the main implications of the use of sales as an exposure base; a slightly more refined model will be used to draw conclusions.

Consider the Widget Manufacturing Company, on which we impose the following assumptions:

1. The company's first year of operation is year 0.
2. The company's final year of operation is year 6.
3. All widgets are produced at the beginning of the year and have a lifetime of exactly 4 years.
4. The potential for a claim-producing occurrence is constant over the lifetime of the widget.
5. No inflation occurs (either in the cost of the widgets or in the size of the claims).
6. There are other companies producing widgets; the total number of widgets sold by the industry each year is constant and has been constant since at least four years prior to the first year of data used in the ratemaking calculations.
7. The products liability insurance rate (based on the experience of the entire widget industry and traditional insurance ratemaking procedures) is $1 per widget sold. Although this rate includes only the loss cost portion of the premium, it will be referred to as "the premium."
8. The company sells 100 widgets each year.

Based on these assumptions, Table 1 shows the life cycle of the widgets produced by this company.

TABLE 1

WIDGET MANUFACTURING COMPANY
PRODUCTS LIABILITY EXPOSURES AND PREMIUM

		Calendar Year									
		0	1	2	3	4	5	6	7	8	9
Year of Manufacture	0	100	100	100	100						
	1		100	100	100	100					
	2			100	100	100	100				
	3				100	100	100	100			
	4					100	100	100	100		
	5						100	100	100	100	
	6							100	100	100	100
True Exposure*		100	200	300	400	400	400	400	300	200	100
Calendar Year Widgets Sold		100	100	100	100	100	100	100	0	0	0
Premium Charged**		100	100	100	100	100	100	100	0	0	0
"Correct" Premium**		25	50	75	100	100	100	100	75	50	25

*Total number of widgets in existence.
**In dollars; see text for discussion.

The rows on Table 1 show how many of the widgets produced in a year were in use at various points in time; the columns display the total number of widgets in use during each calendar year. For each of the calendar years 3, 4, and 5, the number of true exposure units is 400, equal to the life of the product multiplied by annual number sold. This formula is valid (under the given assumptions) for all years in a steady-state situation (i.e., not including "start-up" years or "tail" years). The exposures used in the premium calculation (number sold) will then be one-fourth of the true exposures in all steady-state years. However, since the calculation of the insurance rate from the steady-state industry experience also uses the sales exposure base, the published rate applied to sales produces the appropriate premium in all steady-state years. (Note that this depends on the assumption regarding the level industry exposures.) In the years following cessation of production, there should be a premium charge equal to the steady state premium times .75, .50 and .25, for the first, second and third subsequent years, respectively. Equally important, it is clear that the start-up years should receive a premium *reduction*. The "correct" premiums are shown in Table 1 as the product of the true exposures and the true rate of $.25 per true exposure. This has several implications for current rating methodologies. It is apparent that it is appropriate to charge a premium following cessation of production, but under the given assumptions this premium is exactly equal to the premium credit that *should* have been given when the company began production. Hence, an insuring company covering the Widget Manufacturing Company's entire lifetime of production would receive the same total dollars under either rating system, but would receive them earlier under the present system. Under the given assumptions the only difference between the two methods, albeit a significant one, is investment income.

The situation is analogous to that of claims-made professional liability coverage. Some of the similarities are:

1. A claims-made policy covers a report year, claims reported during the current year based on the present and all prior occurrence years; a products policy covers an occurrence year, which consists of present year occurrences arising from the present and all prior "manufacturing" years.
2. A claims-made policy is incomplete in the sense that additional coverage is necessary beyond the expiration of the policy, even if the insured ceases practice. This is the so-called "occurrence tail," consisting of incurred claims that have not yet been reported ("made"). Occurrence coverage for products liability is incomplete in the sense that additional

coverage is necessary beyond the expiration of the policy, even if the insured ceases production. This "tail" consists of the occurrences arising in the future from products that have not yet expired.

3. The early claims-made years have a lower premium because there are fewer insured incidents from prior years than there are in a mature claims-made year. The early years of a products exposure *should* have a lower premium because there are fewer existing products in use than in a "mature" products year.

Now that the basic concepts have been covered, a more refined model will be constructed to examine the implications further. The discussion will make use of the following definitions:

Firm: A single insured, producing a single product. When a company makes products falling into differing classifications, the company will be considered as the sum of various firms.

Product: The output of a firm. Classifications for ratemaking will be assumed to consist of a single product, insofar as loss-producing potential and useful lifetimes are concerned.

Industry: All firms producing a given product. The industry can be viewed as the sum over all firms producing that product, or as the sum over all insurance companies for that particular classification.

Let $f(t)$ be the probability that a product expires exactly t years after being produced.

$$\text{Define } F(t) = \int_0^t f(s)ds \tag{1}$$

The function $F(t)$ represents the proportion of products that expire within t years after production.

$$\text{Define } G(t) = 1 - F(t) \tag{2}$$

The function $G(t)$ represents the proportion of products that are still in use t years after production. This function defines the distribution of the lifetime of the product.

Define $H(a, b)$ as the true exposure in the time interval (a, b) arising from production of a single unit at time $t = 0$. The true exposure is equal to the product of the length of time and the average number of products in use. The interval (a, b) has length $b - a$, and the average number of products in use can be easily calculated:

$$\text{Average number of products} = \frac{\int_a^b G(t)dt}{(b - a)}$$

Hence, the true exposure is:

$$H(a, b) = (b - a)\frac{\int_a^b G(t)dt}{(b - a)}$$

$$= \int_a^b G(t)dt \tag{3}$$

Appendix A provides further discussion of these functions and an alternate derivation of true exposure.

Since practical applications will generally be dealing with one-year units of time beginning at integral values of t, the following definition will simplify notation without sacrificing generality. Define

$$J(n) = H(n, n + 1) \tag{4}$$

$J(n)$ represents the true exposure in year n arising from a unit of production at time $t = 0$, where n will generally be assumed to be integer-valued. Assuming the distributions of the useful lifetimes of products manufactured at time $t = 0$ apply equally well for all manufacturing years, $J(n)$ can also be viewed as the exposure in year $m + n$ arising from unit production in year m.[5]

Define A_m as the number of products sold in year m. For this model, all production and sales are assumed to occur at the beginning of the period.

Consider policy year m. The true exposure is the sum of the exposure contributions of each of the manufacturing years. The current year's sales, A_m, multiplied by the exposure per unit, $J(0)$, yields $A_mJ(0)$; the previous year's output is A_{m-1} and the exposure per unit is $J(1)$. Hence, the total true exposure for policy year m is:

$$\text{True Exposure}_m = \sum_{k=0}^{\infty} A_{m-k}J(k) \tag{5}$$

[5] In actuality, the distributions may change over time due to improvements in the product. This could be included in the model by defining $J(m, n)$, but the less general model is presented for the sake of clarity.

(The upper limit of infinity is used for notational convenience only. The summation should be thought of as extending over the lifetime of the product, which is finite for virtually all products.)

Assume that production is increasing at the constant rate of $(1 + g)$ per year. Again, a more general model could be constructed by defining $g(m)$ as the growth rate in year m.

$$\text{Define } v = \frac{1}{1+g}; \tag{6}$$

then $\quad A_{m-1} = vA_m$

$$\text{and} \quad A_{m-k} = v^k A_m \tag{7}$$

Substituting (7) in (5) yields:

$$\text{True Exposure}_m = \sum_{k=0}^{\infty} v^k A_m J(k) \tag{8}$$

$$= A_m \sum_{k=0}^{\infty} v^k J(k) \tag{9}$$

Hence, the true exposure for year m is proportional to the traditional exposure, A_m, where the factor of proportionality is independent of m. Assuming that the total industry experience in the ratemaking base has the same distribution of lifetimes and the same growth factor as the particular firm examined above, it should be clear that rates determined by comparing past losses (adjusted for development and trend) to past sales, should be applicable to current sales, since the corrective factor of proportionality, $\Sigma_{k=0}^{\infty} v^k J(k)$, is the same for current sales as for past sales. Conversely, whenever growth patterns of a firm differ from those of the total industry, sales may *not* be a good measure of exposure.

The most extreme examples of the inappropriateness of a sales exposure base occur when a firm begins or ceases production. In the latter case, production in year m is zero, so the usual exposure measure will also be zero. The true exposure may, however, be significant. In policy year m, the first year following the end of production, the true exposure is the sum of the previous years' sales still in use:

$$\text{True Exposure}_m = \sum_{k=1}^{\infty} A_{m-k} J(k) \tag{10}$$

Again, assuming annual growth rate g,

$$A_{m-2} = vA_{m-1},$$

$$A_{m-k} = v^{k-1}A_{m-1},$$

and (10) can be written as:

$$\text{True Exposure}_m = A_{m-1} \sum_{k=1}^{\infty} v^{k-1}J(k) \tag{11}$$

Changing convention slightly, let A_m represent the normal production for year m under growth rate g, and the true exposure will be manipulated through the use of the index of summation. This allows (11) to be written as:

$$\text{True Exposure}_m = A_m \sum_{k=1}^{\infty} v^k J(k) \tag{12}$$

If the firm desires coverage in the first year following cessation of production, the appropriate factor to be applied to the current rates is the ratio of the true actual exposures to the true exposures contemplated in the rate:

$$\sum_{k=1}^{\infty} v^k J(k) \Big/ \sum_{k=0}^{\infty} v^k J(k) \tag{13}$$

Similarly, the factor to be applied in the nth year after the end of production is:

$$\sum_{k=n}^{\infty} v^k J(k) \Big/ \sum_{k=0}^{\infty} v^k J(k) \tag{14}$$

Of course, a problem occurs when one considers to what this factor should apply. In theory, it is applicable to the sales that *would* have occurred had production not been ceased. Since this is a subjective estimate, in practice, the most recent year's sales would probably be the best exposure to which this factor could be applied.

A similar type of analysis is required when a firm begins to produce a new product. (New product means new to the firm where an established products liability rate for the product already exists, as opposed to a completely new product requiring the calculation of an appropriate rate. The latter case is beyond the scope of this paper, although the ideas presented here should be valuable in the process of determining the new rate.) As above, A_m will refer to expected production in year m under growth assumption g, and actual production will be manipulated via the index. Suppose a firm begins production in year m. Production (sales) in year m will be A_m and the exposure will be $A_m \sum_{k=0}^{0} v^k J(k)$.

Hence, the appropriate rate in the first year of production is the normal rate multiplied by the factor:

$$\sum_{k=0}^{0} v^k J(k) \Big/ \sum_{k=0}^{\infty} v^k J(k) \tag{15}$$

Similarly, the factor applicable to the nth year of production will be:

$$\sum_{k=0}^{n} v^k J(k) \Big/ \sum_{k=0}^{\infty} v^k J(k) \tag{16}$$

Note that this factor becomes equal to one when n is equal to or greater than the lifetime of the longest lived product.

An example falling in between the two extremes of production startup or cessation is a firm with a growth rate g' differing from the industry growth g. The factor applicable to the industry rate will be the ratio of the actual true exposures to the true exposures implicit in the rates:

$$\sum_{k=0}^{\infty} (v')^k J(k) \Big/ \sum_{k=0}^{\infty} v^k J(k) \qquad \text{where } v' = \frac{1}{1+g'} \tag{17}$$

For a growth rate g' less than g, v' will be greater than v, hence the rate applicable to sales will be *greater* than the industry rate. For a firm growing faster than the industry, the correct premium rate will be less than the industry rate.

We now consider a less simplified but more realistic numerical example. The following assumptions are imposed on the Widget Manufacturing Company:

1. The company's first year of operation is year 0.
2. The company's final year of operation is year 7.
3. The company and industry growth rates are 10%.
4. The lifetimes of widgets are distributed as illustrated in Table 2 for widgets produced at the beginning of year 0. All failures are assumed to occur at the beginning of a year, so that column 1 contains the discrete counterparts of the $f(t)$, column 2 contains the $F(t)$, and column 3 contains the $G(t)$. Since failures all occur at the beginning of the year, $G(t)$ is constant throughout each year, and column 3 also represents the $J(t)$.
5. All other earlier assumptions hold.

TABLE 2

DISCRETE DISTRIBUTION OF WIDGET LIFETIMES

Year	(1) Proportion Failing in Year	(2) Cumulative Failures	(3) Proportion still in use 1.00 − (2)
0	.00	.00	1.00
1	.05	.05	.95
2	.10	.15	.85
3	.20	.35	.65
4	.30	.65	.35
5	.20	.85	.15
6	.10	.95	.05
7	.05	1.00	.00

Based upon these assumptions, Table 3, which displays Widget Manufacturing Company exposures by calendar year and production year, can be constructed.

TABLE 3

WIDGET MANUFACTURING COMPANY

PRODUCTS LIABILITY EXPOSURE

		Calendar Year													
		0	1	2	3	4	5	6	7	8	9	10	11	12	13
Year of	0	100.0	95.0	85.0	65.0	35.0	15.0	5.0	0	0	0	0	0	0	0
Manufacture	1		110.0	104.5	93.5	71.5	38.5	16.5	5.5	0	0	0	0	0	0
	2			121.0	115.0	102.9	78.7	42.4	18.2	6.1	0	0	0	0	0
	3				133.1	126.4	113.1	86.5	46.6	20.0	6.7	0	0	0	0
	4					146.4	139.1	124.4	95.2	51.2	22.0	7.3	0	0	0
	5						161.1	153.0	136.9	104.7	56.4	24.2	8.1	0	0
	6							177.2	168.3	150.6	115.2	62.0	26.6	8.9	0
	7								194.9	185.1	165.6	126.7	68.2	29.2	9.7
True Exposure		100.0	205.0	310.5	406.6	482.2	545.5	605.0	665.5	517.6	365.9	220.2	102.9	38.1	9.7

In this example, only calendar years 6 and 7 are in an equilibrium state, where the traditional rate is the correct rate. For all years in an equilibrium state, the ratio of true exposures to calendar year sales will be a constant; in this case:

$$605.0/177.2 = 665.5/194.9 = 3.41$$

(See Appendix B for more discussion of the ratio of true exposures to sales.)

In the prior Widget Manufacturing Company example, we assumed a premium rate of $1.00 per $100 of sales; the corresponding rate per true exposure was $.25. In the current example, this $.25 rate per true exposure implies a premium rate of $.853 per $100 of sales. Since losses are a function of true exposures rather than sales, the rate per unit of sales can be calculated by multiplying the rate per unit of true exposure ($.25 per $100) by the ratio of true exposures to sales. In the prior example, this factor was 4; the rate per $100 of sales was thus 4 × $.25 = $1.00. In this example, the factor is 3.41 so the rate per $100 of sales is 3.41 × $.25 = $.853. If the industry growth rate has been constant during the period between the first year used in ratemaking and the present, this calculation does not have to be made explicitly but will work out automatically. Note that a further implication of this discussion is that a material change in the industry growth rate will make the results of the present ratemaking methodology somewhat inappropriate.

Table 4 compares the premium that would normally be charged using sales as an exposure base and the "correct" premium using the true exposure.

Several observations can be made from Table 4. In the equilibrium years (6 and 7), the premium charged is the same under both measures of exposure. In the start-up years (0 through 5), the premium based on sales exceeds the correct premium, the difference being significant in the early years and decreasing over time. The correct premium exceeds the sales-based premium in the tail years, since there are no sales in those years. Note that the total premium over all years is *not* the same for the two exposure bases. The premiums are identical in all equilibrium years, but the excesses in the early years do not make up for the deficiencies of the later years. However, it cannot be concluded that traditional ratemaking does not provide enough premium over the life cycle of an insured. To answer this question, it is necessary to examine the nature of the

TABLE 4

WIDGET MANUFACTURING COMPANY

PRODUCTS LIABILITY PREMIUMS ON DIFFERENT EXPOSURE BASES

(1) Year	(2) Sales	(3) True Exposure	(4) Premium Based on Sales (2) × .854	(5) "Correct" Premium (3) × .25
0	100.0	100.0	85.4	25.0
1	110.0	205.0	93.9	51.3
2	121.0	310.5	103.3	77.6
3	133.1	406.6	113.7	101.6
4	146.4	482.2	125.0	120.6
5	161.1	545.5	137.6	136.4
6	177.2	605.0	151.3	151.3
7	194.9	665.5	166.4	166.4
Subtotal*	1143.6	3320.2	976.6	800.6
8	0	517.6	0	129.4
9	0	365.9	0	91.5
10	0	220.2	0	55.1
11	0	102.9	0	25.7
12	0	38.1	0	9.5
13	0	9.7	0	2.4
Total*	1143.6	4574.5	976.6	1143.6

* Totals may not add correctly due to rounding.

coverage for the "tail" years, in this case, years 8 through 13. Among the possibilities are:

1. If the firm has ceased coverage because it has gone out of business, it will probably not purchase insurance in years 8 through 13. In any event, the insurance company will not be liable for occurrences in the "tail"

years. In this case, the insurance company will have received \$976.60 but will only have to pay out \$800.60 (Table 4 subtotals through year 7).

2. If this product represents only a minor portion of the insured company's total sales, then the insurance company is likely to continue coverage on the discontinued product, even though there is no premium collected for this product during the tail years. In this case, the insurance company will receive \$976.60 but will have \$1143.60 in expected losses (but will also receive significant investment income).

3. If the product represents a major portion of the company's sales but the company remains in business after discontinuing this product, the insurer may refuse to provide coverage for claims arising from this product occurring after year 7, unless additional premium payments are made. Assuming that the insurer can estimate the appropriate premium for the tail years as \$343.00 (\$1143.60 − \$800.60), the insurer will receive \$1319.60 (976.60 + \$343.00) for \$1143.60 in expected losses. Although an informed insured might realize that there were overcharges in the early years, the arguments will be useless if there has been a change in insurers. However, the insured may be in a position to demand premium credits for its newer products, since the reasoning used by the insurer to charge premium for the "tail" years is identical to the reasons for expecting a credit in the early years.

One other relationship on this table should be pointed out. The "excess" premium in year 0 is 85.4 − 25.0 = 60.4. The "deficiency" in year 8 is 129.4 − 0 = 129.4. The difference between the two is attributable to growth in production: $60.4 \times (1.1)^8 = 129.4$. A similar relationship holds true for years 1 and 9, 2 and 10, etc. The excess premium arises from an overstating of exposures in the early years, while the deficiency arises from an understating of exposures. But the understating of exposures occurs eight years later than the overstating, by which time the exposures have grown by the factor $(1.1)^8$. It should be clear that, with an assumption of no growth (as in the first example given), the excesses and deficiencies cancel out. In either case, investment income and changing carriers complicate the analysis.

In actual practice, the distortions caused by the use of sales will be less than indicated in this example. Normally, a company produces a number of products and the elimination of a single product would have a relatively small effect. It is also likely that an eliminated product will be replaced by another, new product. To the extent that the two products have the same distribution of lifetimes and the same potential for loss, the errors will cancel each other. In the case where a company completely ceases production (if, for example, it goes out of business), it may not even purchase insurance.

Despite the significant potential distortions which may result from the use of sales as an exposure base, it is not the intention of this paper to suggest that a change in exposure bases is necessary. The problems involved in attempting to objectively measure the true exposure would outweigh the benefits in most cases. Rather, it is the intention of this article to outline the implications of sales exposure bases so that the effects will be understood and the appropriate actions can be undertaken in the extreme cases. A few examples may help explain typical problems that may be encountered.

1. If a manufacturer has *recently* discontinued a hazardous product, losses will still continue to occur even though there are no sales. If the firm is being experience-rated, the indicated modification will be too low and a schedule debit may be appropriate. A few years later, the situation is reversed. The loss experience, but not the exposures, of the discontinued product will be included in the experience rating calculations. Since the future losses arising from that discontinued product should decrease, the experience modification is now too high and a schedule credit may be appropriate. Similarly, a firm adding a major product should get a schedule credit from the manual rate. At the very least, this paper will be helpful for understanding and explaining to the insured why changes in the experience modification are occurring.
2. A manufacturer recently requested advice as to whether it should join a captive insurer, since its loss experience was significantly better than that contemplated in the existing premium rates. (The premium rates were developed for that particular industry by a specialty company.) However, the firm had been in existence for only three years, and was producing a product with an expected lifetime of fifteen to twenty years. This paper makes it clear that the experience during these early years of production *should* be significantly better than that of established manufacturers of the same product. For the purpose of illustration, assume that the lifetime of each product is exactly fifteen years, and there has been no growth in sales for the industry. The established manufacturers will have fifteen units of true exposure for every one sold in the current year, while the firm in question only has three units of true exposure for each one sold in this year. The true exposure of the new firm (in this year) is only 20% of the exposure of the established firms: the firm

deserves an 80% credit from the manual rate. The amount of credit should decrease over time as the true exposures increase. It was recommended that the firm should not join the association captive, where it would share disproportionately in the other members' losses, but rather the firm should remain with its insurer and try to negotiate a premium credit.

3. Projections of losses for a manufacturer typically involve a regression line fit to the history of annual loss pure premiums (losses divided by exposures). In the start-up years of a product, such an analysis may indicate a significantly increasing pure premium. However, if the firm is nearing the equilibrium stage, the pure premiums should begin to plateau (ignoring inflation), rather than continue their steep rise. In the absence of an understanding of the implications of this paper, artificially high losses may be projected. Similarly, an understanding of these concepts will aid in projecting future losses arising from a discontinued product.
4. Another manufacturer is producing a product with a lifetime in excess of one hundred years, but has been producing the product for only thirty years. It should be clear that the total true exposure is increasing each year, even if sales are constant. If the nature of true exposure were not considered, it would be difficult to understand why losses are growing each year, even after adjusting for inflation and sales growth.
5. If products produced now have longer lifetimes than products produced in the past, the true exposure will increase even though sales (inflation adjusted) are not increasing.
6. For a product such as an elevator, it is likely that the exposure to loss is not constant over the elevator's lifetime, but more concentrated in the later years. A firm may not have increasing sales now, but may find it has an increasing ''inventory'' of older elevators which are more likely to produce losses.

Summary

The use of sales as an exposure base for products liability insurance can have a distorting effect under certain circumstances. However, it is neither necessary nor feasible to change the exposure base. As long as the effects of this distortion are understood, the impact can be estimated and corrected. Certainly the effects are not trivial to calculate, since this calculation requires an estimate of the distribution of the lifetime of a product, but even crude estimates will result in more accurate premiums in some of the extreme cases.

APPENDIX A

Derivation of True Exposure Arising from a Product

We have defined $f(t)$ to be the probability density function of the lifetime of a product manufactured at time 0. It can be viewed either as the probability of expiration at time t or as the portion of products that have a lifetime of length t. Since the true exposure is the product of the number of objects and the length of their lifetimes, the exposure arising from a single unit of production can be calculated by multiplying the lifetime t by the portion of products with lifetime t, and summing over all values of t. Hence,

$$\text{True exposure from unit of production} = \int_0^\infty tf(t)dt$$

For the purposes of this paper, it is necessary to calculate the exposure during the time interval $(0, t)$. The expression

$$\int_0^t sf(s)ds$$

does *not* represent the total exposure in the interval $(0, t)$; it represents only the exposure arising from products which expire at or before time t. The exposure from the products still unexpired must be added. Since the portion of products still unexpired is $G(t)$, the total exposure during the time interval $(0, t)$, is

$$tG(t) + \int_0^t sf(s)ds$$

The following derivation will show that $\int_0^t G(s)ds$ is equal to the above expression, and hence, is equivalent to the true exposure during the time period $(0, t)$ from a product manufactured at time 0.

From equation (2),

$$G(s) = 1 - F(s).$$

From equation (1),

$$G(s) = 1 - \int_0^s f(r)dr.$$

Differentiating both sides,

$$G'(s) = -f(s).$$

Multiply by s,

$sG'(s) = -sf(s)$.

Integrate over $(0, t)$,

$$\int_0^t sG'(s) = \int_0^t -sf(s)$$

Integrating by parts yields

$$sG(s)\Big|_0^t - \int_0^t G(s)ds = -\int_0^t sf(s)ds$$

or

$$\int_0^t G(s)ds = tG(t) + \int_0^t sf(s)ds.$$

The right-hand side of this equation has a useful verbal interpretation. The true exposure between zero and t is the sum of two pieces:

1. The portion of products still in use at time t, multiplied by the length of exposure: $tG(t)$.
2. The exposure arising from products which expire during the period: $\int_0^t sf(s)ds$.

APPENDIX B

True Exposure and Sales

The relationship between the true exposure in a year, $\Sigma_{k=0}^{\infty} A_{m-k}J(k)$, and the sales in the year, A_m, is affected by two factors:

1. The distribution of the lifetime of the product. This factor is reflected in the function $J(k)$.
2. The growth pattern of the calendar year sales, reflected in the A_{m-k}.

If the number of products sold does not change from year to year (as in the first example given in the text), then $A_{m-k} = A_m$ for all k. Then the ratio of true exposure to sales is given by

$$\frac{\sum_{k=0}^{\infty} A_{m-k}J(k)}{A_m} = \frac{\sum_{k=0}^{\infty} A_m J(k)}{A_m} = A_m \frac{\sum_{k=0}^{\infty} J(k)}{A_m} = \sum_{k=0}^{\infty} J(k)$$

In this circumstance, the ratio is solely dependent on the function $J(k)$. Rewriting,

$$\begin{aligned}\sum_{k=0}^{\infty} J(k) &= \sum_{k=0}^{\infty} H(k, k + 1)\\ &= H(0, 1) + H(1, 2) + \ldots\\ &= \int_0^1 G(s)ds + \int_1^2 G(s)ds + \ldots\\ &= \int_0^{\infty} G(s)ds\end{aligned}$$

As shown in Appendix A, this expression represents the expected or average lifetime of the product. Hence, under the condition of constant sales, the ratio of true exposure to sales will be equal to the expected lifetime of the product.

If sales are not constant, the relationship becomes more complicated. However, it can be said that if sales are growing at a constant rate, then the ratio of true exposure to sales will be less than the expected lifetime of the product. This can easily be seen using (9). The ratio of true exposure to sales will be

$$A_m \frac{\sum_{k=0}^{\infty} v^k J(k)}{A_m} = \sum_{k=0}^{\infty} v^k J(k);$$

when the rate of increase $(1 + g)$ is greater than one, v is less than one and $\Sigma_{k=0}^{\infty} v^k J(k)$ will be strictly less than $\Sigma_{k=0}^{\infty} J(k)$.

As a specific example, consider the equilibrium state of Widget Manufacturing Company in the main text. The ratio of true exposures to sales was 665.5/194.9 = 3.41.

The numerator can be broken into its components (refer to Table 3 in the text): 665.5 = 5.5 + 18.2 + 46.6 + 95.2 + 136.9 + 168.3 + 194.9.

This can be further decomposed as:

$$\frac{(.05)(194.9)}{(1.1)^6} + \frac{(.15)(194.9)}{(1.1)^5} + \frac{(.35)(194.9)}{(1.1)^4} + \frac{(.65)(194.9)}{(1.1)^3}$$

$$+ \frac{(.85)(194.9)}{(1.1)^2} + \frac{(.95)(194.9)}{(1.1)^1} + \frac{(1.00)(194.9)}{(1.1)^0}$$

which should be recognizable as $\Sigma_{k=0}^{\infty} v^k J(k) A_m$.

If there were no growth in sales, the denominators in the above expression would be unity, and the sum would reduce to (4.0) × (194.9) = 779.6; the effect of growth in sales is to reduce the contribution of prior years sales to true exposures (i.e. reduce the numerator), without affecting the denominator.

DETERMINING ULTIMATE CLAIM LIABILITIES FOR HEALTH INSURANCE COVERAGES

EMIL J. STRUG

I. INTRODUCTION

The purpose of this paper is to add another chapter to the fund of knowledge being accumulated on loss reserving techniques. Except for Paul Otteson's paper on Group Accident and Health Hospital Therapeutic Benefits (*PCAS* XLI, 1954), nothing has been published in the *Proceedings* in recent years on the methods employed to develop ultimate health insurance loss costs.

The *Proceedings* of the Society have analyzed and presented various methods of establishing ultimate loss costs for various lines of insurance. Generally, the same techniques are used in developing ultimate loss costs for health insurance benefits. The items of most interest are probably the settlement patterns of the various health coverages and any seasonal or cyclical patterns which they display.

II. RESERVES FOR VARIOUS LINES OF BUSINESS

The lines of business for which loss reserves are developed within the author's scope of operation are:

1. Hospital Benefits
2. Physician Benefits
3. Extended Benefits (Superimposed Major Medical)
4. Dental Benefits

Hospital benefits refer to those benefits provided by a general hospital on an inpatient and outpatient basis.

Physician benefits are those medical and surgical benefits rendered by a physician in a general hospital (inpatient or outpatient), physician's office, or patient's home, excluding such items as routine physicals, immunization, etc.

Extended benefits deal with such items as admissions to a mental institution, physician home and office visits, private duty nursing, drugs, prosthetic devices, etc. In addition, complementary programs to Medicare are considered as an extended benefit.

Dental benefits refer to those procedures performed by a dentist, primarily in an office setting, for the dental needs of the insured such as cleanings, fillings, extractions, prosthodontics, etc.

Beside the fact that these are considered as separate lines of business, they are reserved separately as each displays a different development pattern. If these lines were not segregated, a significant change in one of the elements could affect the overall results. This same problem can arise within a line of insurance if there is a significant variation in the development of ultimate values due to different reporting and settlement patterns for certain types of claims. This is the case for hospital claims, where the reporting and processing of inpatient claims differs dramatically from outpatient claims.

One might question the separation of physician and dental benefits for reserving purposes. Dental is a new and expanding line of insurance. Dental claims have shown a faster development pattern than medical/surgical claims, probably reflecting the more efficient billing systems employed by dentists which make it possible to report and collect low dollar but high volume claims from their patients on a timely basis.

Within the Extended Benefit category, Medicare complementary programs for age 65 and over and complementary programs for under age 65 are reserved separately as each develops a different reporting and settlement pattern.

To illustrate the different time span required for full development, we have taken an incurred calendar quarter (first quarter of 1975) and aged or tracked it for eight quarters (24 months), calculating each stage of paid development based upon the ultimate incurred liability. Claim payments are assigned to the month and year in which the claim occurred. The results are shown in tabular as well as graphical form.

No. of Quarters Developed	Hospital				Extended Benefits	
	Inpatient	Outpatient	Physician	Dental	Under 65	65 and Over
0	49.36%	39.60%	19.79%	43.25%	1.86%	32.02%
1	94.39%	82.54%	84.44%	88.73%	26.38%	80.25%
2	98.35%	91.64%	94.67%	95.93%	43.41%	90.16%
3	99.11%	94.64%	97.22%	98.17%	51.26%	93.90%
4	99.85%	96.55%	98.68%	99.48%	68.81%	96.43%
5	100.03%	98.15%	99.41%	99.61%	77.05%	98.07%
6	99.98%	98.89%	99.68%	99.87%	80.96%	98.77%
7	99.95%	99.33%	99.78%	100.00%	83.64%	99.32%
8	99.96%	99.67%	99.91%	100.00%	86.50%	99.61%

The table and especially the graphs (see Exhibits 1 and 2) depict quite vividly the variance in development patterns for each of the lines of insurance.

It should be noted that the data used is taken from the actual records of the corporation for which the author provides actuarial services.

Under "Hospital Inpatient" you will note that the factor at the fifth quarter of development is in excess of 100% and then drops for subsequent quarters to below 100%. The aberration is a result of coordination of benefits, subrogation, and Workers' Compensation recoveries which were recorded after the fifth quarter.

The different patterns displayed most likely reflect the benefit structure and the attitudinal differences of the providers (who bill the corporations directly) and the insureds. The reporting patterns generally reflect the cash flow needs of the providers and the insureds which are at times influenced by the general economy.

It should be noted that individual case reserves are not used in developing total estimated incurred claims liabilities. Ultimate loss costs are determined by formula. At one time case reserves were established but, due to the volume of claims and the attendant maintenance of the values and files, the company chose to discontinue the method.

III. METHODS USED TO ESTABLISH ULTIMATE INCURRED VALUES

Before describing briefly the methods used in establishing ultimate loss cost values, some comments as to the overall approach in setting reserves are appropriate.

In establishing loss reserves the results produced can be segmented into two categories. The categories are defined by the age of the claim and are determined by the historic pattern of development displayed for each line being evaluated. The two breaks are:

1. claims for which subsequent development is predictable, and
2. claims in the early stages of development which are subject to distortions of reporting and processing.

The primary method used to develop ultimate values is the use of completion or projection factors developed from triangulation or completion tables.

If the amount for incurred period A_{y-1} is assumed to be fully developed at time period m, then the projection or completion factor for time period $y - 1$ would be

$$\sum_0^{m-1} A_{y-1} \left(\sum_0^{m} A_y \Big/ \sum_0^{m-1} A_y \right) .$$

The factor for time period $y - 2$ would be

$$\sum_0^{m-2} A_{y-2} \left[\sum_0^{m-1} A_{y-1} \left(\sum_0^{m} A_y \Big/ \sum_0^{m-1} A_y \right) \Big/ \sum_0^{m-2} A_{y-1} \right] .$$

This calculation is carried on until the value for y_0 is determined. The completion factor for y_0 can apply to a single month, a quarter, a year, or a series of months where each month in the grouping is at the equivalent stage of reporting and development.

The broader the base used in developing these completion factors, the more stable are the results for months with reasonable degrees of development. The use of broader time bases obviously requires the use of older data. As a result, these factors are less responsive to current changes in reporting and payment patterns when applied to more recent reporting periods. In situations where it is known that reporting or processing has shown a new and stable trend, more reliance can be placed upon the factor developed using more recent time frames. For incurred periods with low and slow development, such as the initial incurred period, results based upon completion factors are erratic and generally unreliable. For these periods an alternate technique is used which we call the ratio method.

The ratio method, as the name implies, develops historic ratios or indices of various incurred periods to a base period for which the ultimate values are considered to be most accurate. These indices are then applied to more recent time periods to develop estimates of current ultimates. For example, if the ultimate incurred amount to be estimated is the 3rd quarter of 1975, which we will denote as 3Q75, and we have data going back to the fourth quarter of 1971 (4Q71), the following ratios of incurred amounts would be calculated:

3Q72/4Q71	3Q73/4Q72	3Q74/4Q73
3Q72/1Q72	3Q73/1Q73	3Q74/1Q74
3Q72/2Q72	3Q73/2Q73	3Q74/2Q74

By applying the factors in the first row to 4Q74, the second row to 1Q75, and the third row to 2Q75, a set of values representing the estimated amounts for the 3rd quarter of 1975 are produced.

If exposure, utilization, and cost trends have some consistency during the historic and current periods, there will be a clustering of values. It is obvious that if any of these elements departs dramatically from or shows no consistency with prior patterns, then this method is rendered misleading. In many cases it is possible to adjust the value for changes in exposure, utilization, and cost trends and to produce a meaningful result.

Variations of the completion method and, where appropriate, the ratio method are used to produce first estimates of the total incurred liability. In some situations, especially for new benefit offerings, a loss ratio method is used. Here the earned premium is multiplied by the expected loss ratio to produce the estimated incurred amount. Inherent in the process is the reevaluation of previously estimated incurred values. The analysis is extended back into time to a period where no additional development is expected.

IV. CALCULATION OF RESERVES

Estimated ultimate claim amounts are calculated using for the most part some or all of the previously described methods. In the first pass at the estimations there is no adjustment to the values. The initial results are then tempered or adjusted to reflect conditions which render the results useless or suspect for certain periods of time. Such conditions include the imposition of cost controls, the removal of cost controls, dramatic changes in exposure, or extraordinary inflationary factors as they apply to medical care. At this point there is the blending of the art and science of reserving.

In the development of total ultimate loss cost for each line of insurance, those methods which historically have produced the most consistent results are used.

At this stage of the review we attempt to refine the calculations, or our selection of a value, based upon internal and external forces which may have had an influence upon the calculation.

Internal forces affecting the values would be claim receipts, claim inventories, and processing cycles. External forces from providers would be such items as reporting cycles and increases in hospital and physician costs. Other external conditions such as government controls, weather, and postal strikes have an impact upon the insured, insurer, and provider. An evaluation of internal processing cycles is made to determine if the values calculated via the projection route will tend to overstate or understate. Reporting cycles are analyzed in a similar fashion. In most cases it is impossible to quantify the results, but the movement will generally indicate which of the results is likely to be the most accurate. In recent years coordination of benefits and subrogation activities have been intensified. Until the rate of recovery becomes somewhat constant, manual adjustment to estimated ultimate values are made.

External provider activities relative to cost are monitored by evaluating reports of past and current items such as hospital charges, physician charges, dental charges, the various elements of the medical component of the CPI (local and national), and hospital costs issued by the American Hospital Association (AHA).

In conjunction with the development of loss reserves, analyses are performed to determine the adequacy of current rates and to evaluate utilization and cost trends. These trend factors are applied to project ultimate loss costs for past periods to current periods. A comparison of these results to those produced via the routine techniques is made to determine the reasonableness of the results in conjunction with patterns of prior periods.

Where appropriate, pure premiums are calculated by dividing ultimate amounts by exposure for periods for which the values are considered to be stable, and these are projected to current periods of time. By multiplying the projected pure premiums by the exposure, an ultimate claim cost is produced.

V. APPLICATIONS

For Hospital and Physician Benefits ultimate values are calculated by multiplying estimated claim counts times estimated claim costs and by projecting claims paid by incurred period to their estimated ultimate values. As previously mentioned, claim counts and amounts are estimated using two projection techniques.

The first technique uses factors developed by use of twelve month moving data with each accident month within the twelve months being at the same stage of payment. The time span used encompasses 44 months from the oldest to the current accident month. This technique has the advantage of averaging out aberrations which might occur on a monthly or quarterly basis and provides a high degree of stabilization in the factors developed. It is, however, less responsive to current changes in reporting and processing cycles.

The second approach uses unweighted calendar-accident periods to develop projection factors. This technique has the advantage of reflecting recent changes in reporting, processing, or seasonal patterns. This approach is, however, subject to aberrations due to any non-repetitive occurrence which happened in any of the prior accident periods used to calculate the applicable projection factors. Under this method the estimated number of claims, average claim cost, and total ultimate claim cost are developed for each accident quarter. The average claim cost is used in the moving average calculations described in the previous paragraph.

In addition to calculating claim counts, amounts, etc., pure premiums by incurred period are developed by using the same technique. The estimated ultimate claim amount is calculated by extending the estimated ultimate pure premium by the earned exposure for the accident period.

The ratio method, as described in Section III, is used to develop estimated ultimate amounts as well as estimated ultimate pure premium by incurred period. The pure premiums are extended by the earned exposure for the corresponding incurred period to produce ultimate amounts.

In developing the ultimate loss costs, values are generally calculated using claim counts and claim costs as well as total dollars.

If we examine the graphs portraying the development pattern of each of the lines of insurance and their subdivisions, some insight will be gained as to the timing of the use of the two general techniques.

Except for Extended Benefits for under age 65, all the elements show that after six months of development 90% or more of the ultimate loss cost incurred has been paid. The use of projection or completion factors at this stage of development and beyond proves to produce very accurate and stable results.

At the zero stage of development the percent incurred, reported, and paid for all breaks is less than 50%. The results at this point tend to be quite erratic. For this reason the ratio method is almost universally employed along with judgment.

Between the third and sixth months of development, the ratio and both projection methods are used. Based upon claim notices received, claim dollars paid and claim inventories, a judgment is made as to the validity of the values produced by each of the projection methods. For Extended Benefits under age 65, the ratio and projection methods are employed for 18 months, at which time the projection factor takes over. However, substantial judgment enters into the choice of values depending upon the three elements enumerated earlier in the paragraph.

VI. GENERAL COMMENTS

To aid in the determination of current loss reserve values a series of exhibits containing pertinent data are prepared. The material falls into five general categories:

1. service and claim counts,
2. average claim and service costs,
3. claim receipts and inventories,
4. cycle time for claims submission and processing, and
5. changes in exposure.

These data are depicted in graphical form in Exhibits 3 through 15B.

In analyzing ultimate amounts from current periods, these items of information provide insight as to possible aberrations in the formula results as well as logical relationships from one period of time to the next. For example, a decrease in exposure and claim receipts would indicate that one should expect a decrease in incurred claims. If the cycle time from incurred date to paid date has been shortened, then generally the formula approach will overstate the reserves. The analysis of the runoff of claims paid using claims incurred and paid-to-date for a current period and the runoff for the comparable period a year before provides a benchmark as to the minimum value one might normally expect. By adjusting the prior year's runoff for inflation and exposure and any payment aberrations, a ballpark estimate of the ultimate loss cost can be obtained.

These graphs may be of significant interest to those unfamiliar with health coverages. The patterns indicate that for the most part the elements involved in determining ultimate loss costs develop relatively fast for health coverages with the exception of major medical for under age 65. The development patterns are probably more analogous to property damage lines than to liability coverages with a "long tail."

A definite seasonal pattern is shown for all lines. Costs are very sensitive to external economic forces as well as frequency or utilization. The latter is not as discernible as cost but it does display some cyclical tendencies based upon unemployment cycles.

VII. SUMMARY

The presentations portray the various methods currently being employed to determine loss reserves. With the availability of time-sharing computers, modeling techniques can be applied to develop estimated ultimate values from which loss reserves can be produced. The advantages to a computer based model are obvious as it allows one to measure the impact of the change in variables upon the final results within a short span of time.

As was stated in the introduction, this presentation was not intended to be all-inclusive. Alternate techniques are constantly being applied and evaluated as to consistency and accuracy. Data bases are being constructed to allow for automation of the reserve calculation and, in time, to allow for modeling. The approaches presented may appear to be rudimentary and unsophisticated, but over time the results produced have been satisfactory.

EXHIBIT 1

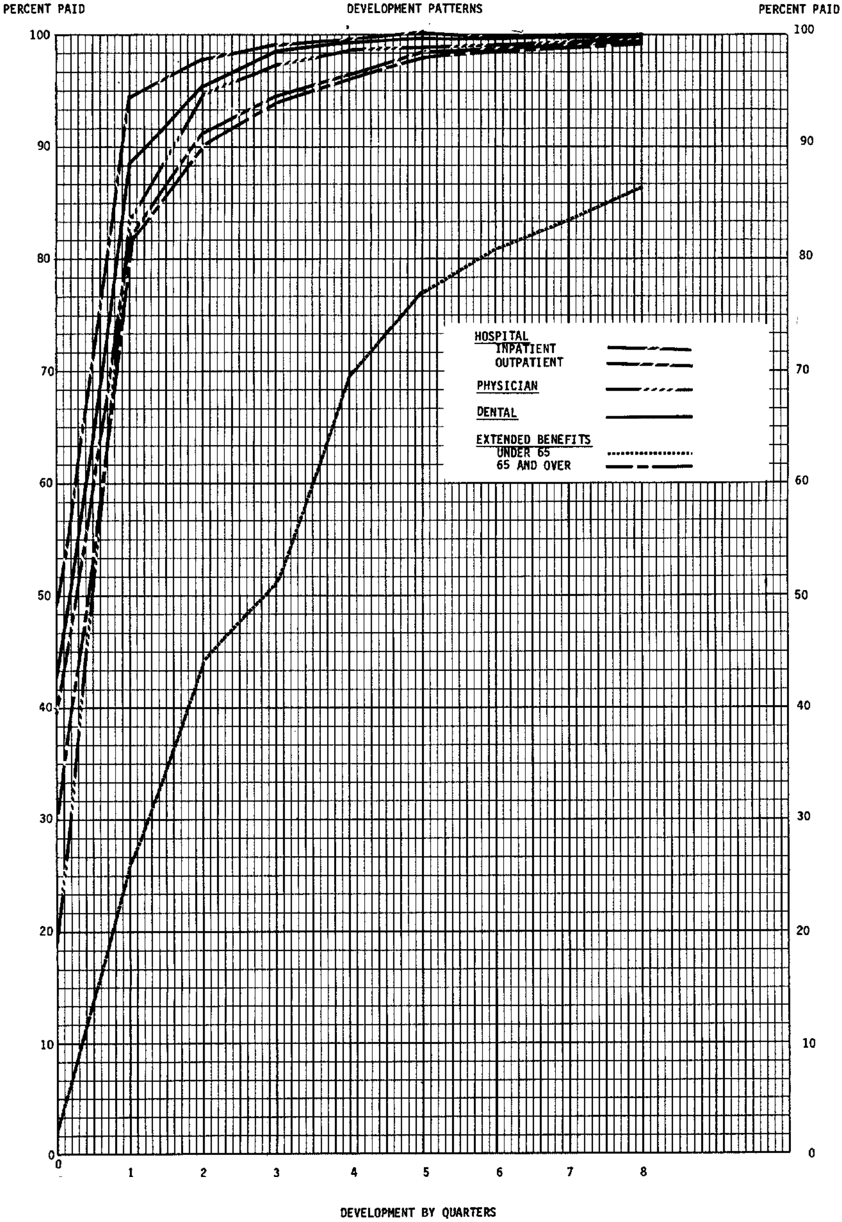
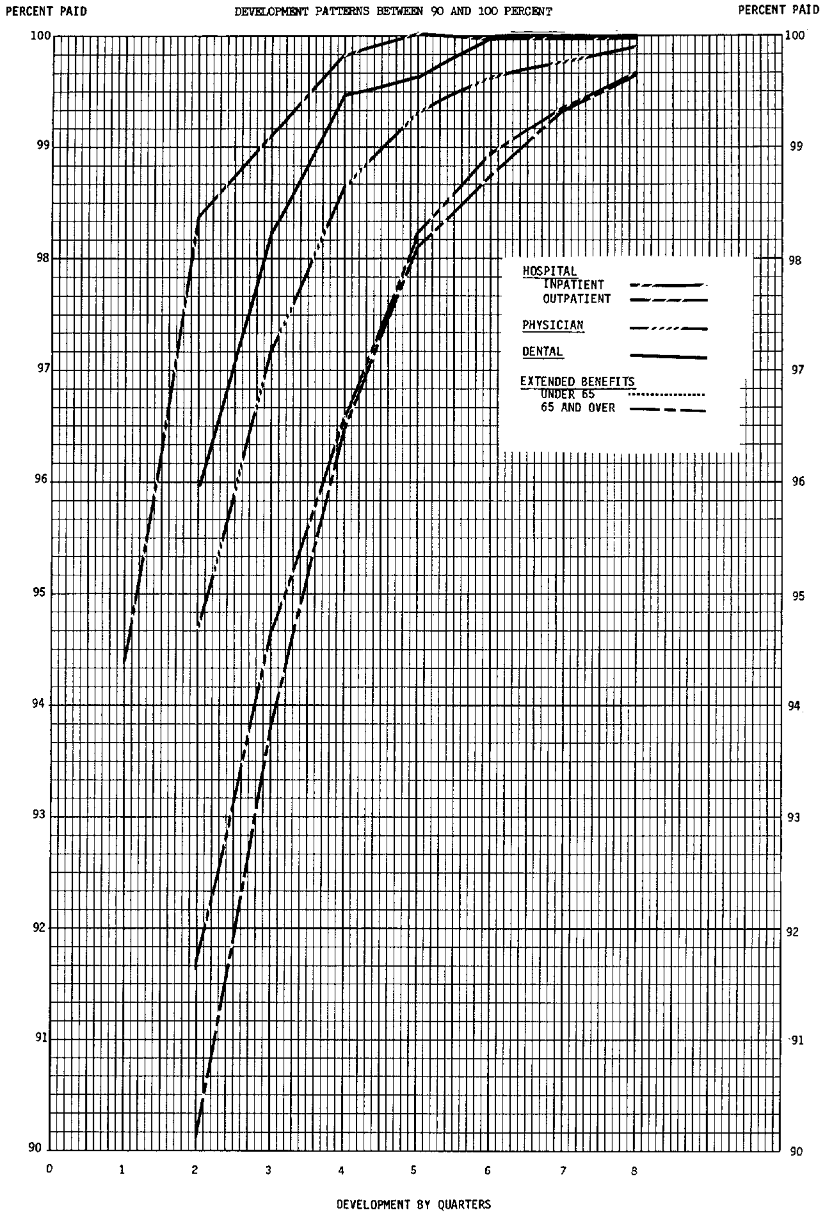

PERCENT PAID
DEVELOPMENT PATTERNS
PERCENT PAID
100
90
80
70
60
50
40
30
20
10
0
0
1
2
3
4
5
6
7
8
HOSPITAL
INPATIENT
OUTPATIENT
PHYSICIAN
DENTAL
EXTENDED BENEFITS
UNDER 65
65 AND OVER
DEVELOPMENT BY QUARTERS

EXHIBIT 2



PERCENT PAID
DEVELOPMENT PATTERNS BETWEEN 90 AND 100 PERCENT
PERCENT PAID
100
99
98
97
96
95
94
93
92
91
90
HOSPITAL
INPATIENT
OUTPATIENT
PHYSICIAN
DENTAL
EXTENDED BENEFITS
UNDER 65
65 AND OVER
0
1
2
3
4
5
6
7
8
DEVELOPMENT BY QUARTERS

EXHIBIT 3

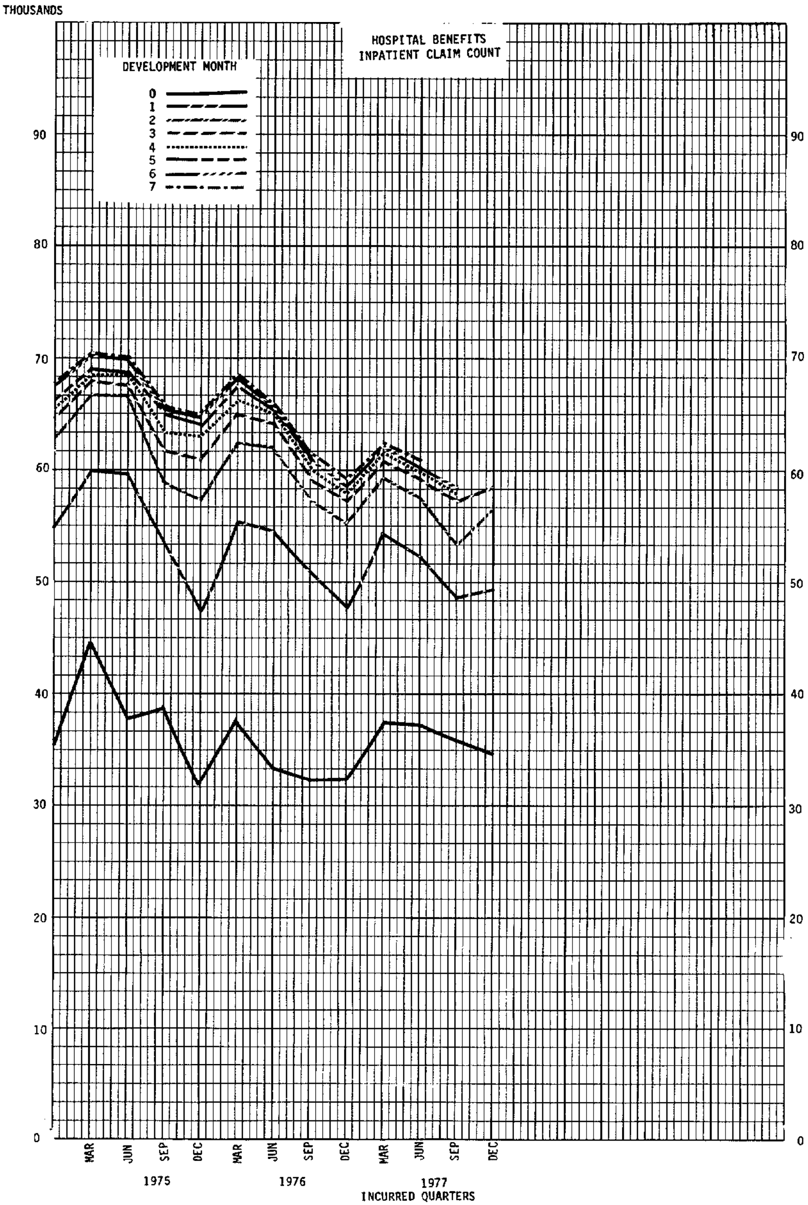

THOUSANDS
HOSPITAL BENEFITS
INPATIENT CLAIM COUNT
DEVELOPMENT MONTH
0
1
2
3
4
5
6
7
90
80
70
60
50
40
30
20
10
0
MAR
JUN
SEP
DEC
1975
1976
1977
INCURRED QUARTERS

EXHIBIT 4

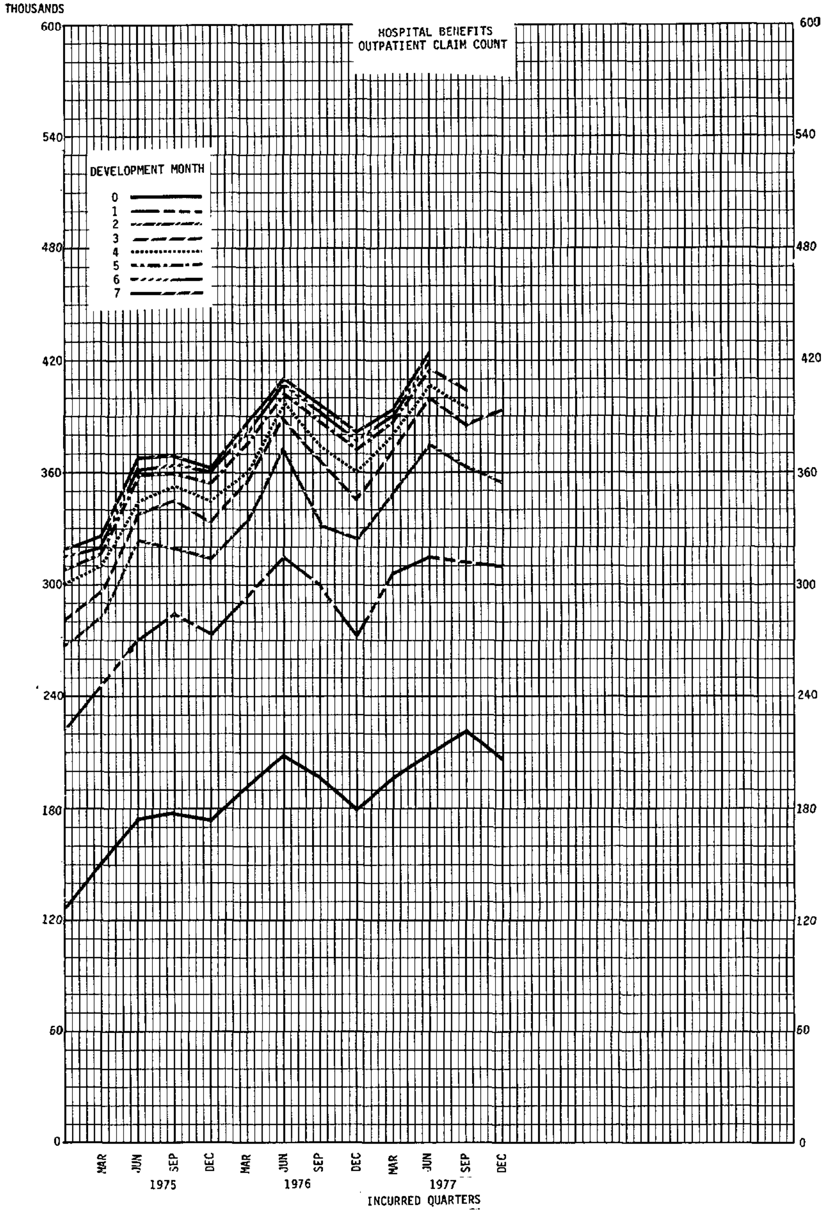

THOUSANDS
HOSPITAL BENEFITS
OUTPATIENT CLAIM COUNT
DEVELOPMENT MONTH
0
1
2
3
4
5
6
7
600
540
480
420
360
300
240
180
120
60
0
MAR
JUN
SEP
DEC
1975
1976
1977
INCURRED QUARTERS

EXHIBIT 5

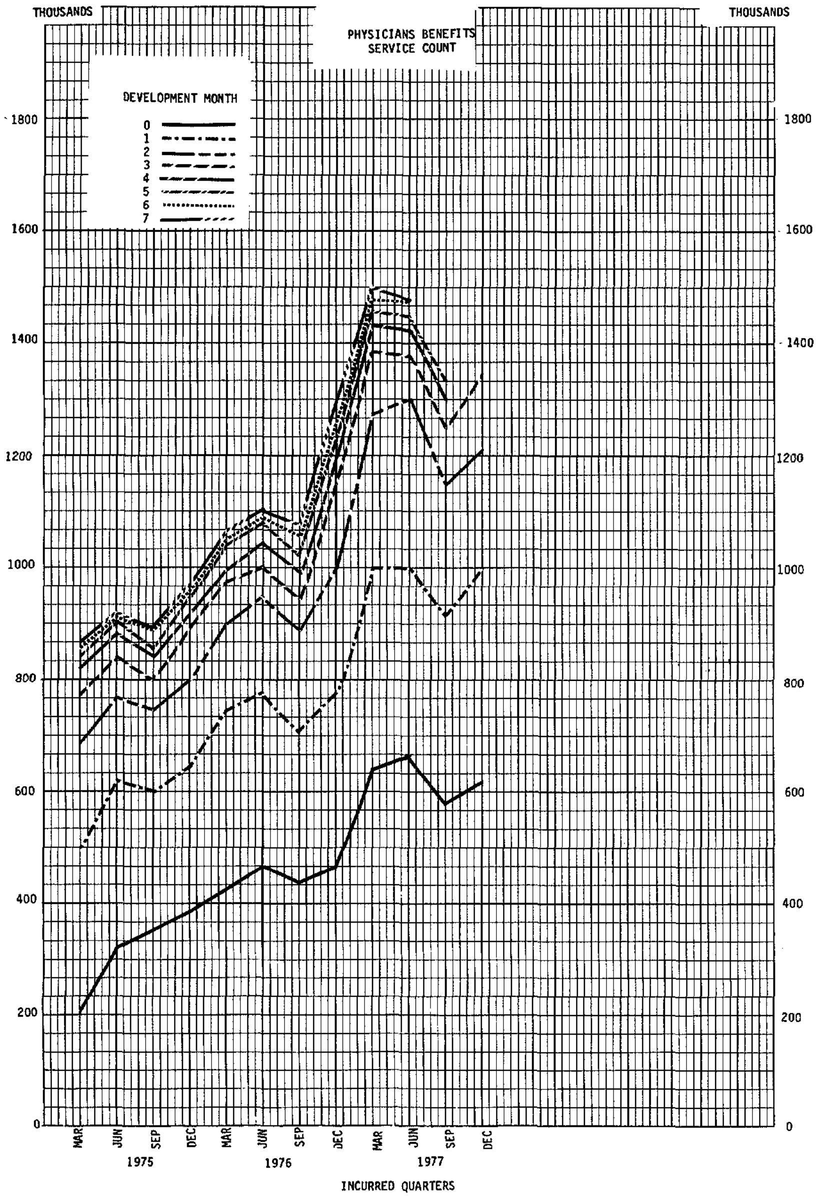

THOUSANDS
THOUSANDS
PHYSICIANS BENEFITS
SERVICE COUNT
DEVELOPMENT MONTH
0
1
2
3
4
5
6
7
1800
1600
1400
1200
1000
800
600
400
200
0
MAR
JUN
SEP
DEC
MAR
JUN
SEP
DEC
MAR
JUN
SEP
DEC
1975
1976
1977
INCURRED QUARTERS

EXHIBIT 6

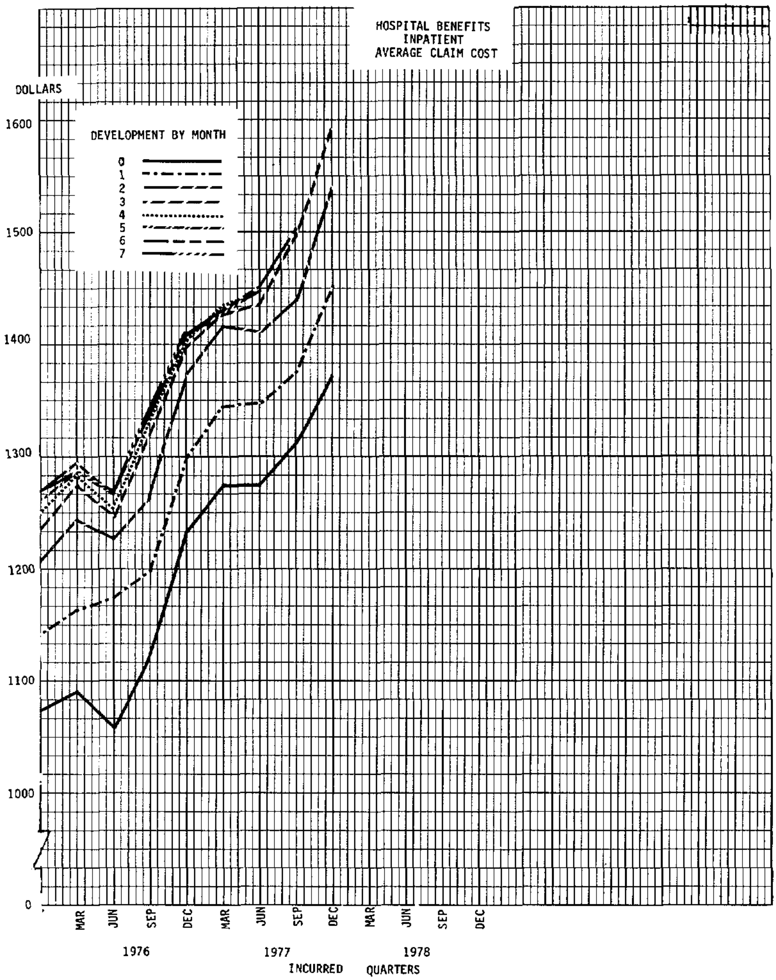

HOSPITAL BENEFITS
INPATIENT
AVERAGE CLAIM COST
DOLLARS
DEVELOPMENT BY MONTH
0
1
2
3
4
5
6
7
1600
1500
1400
1300
1200
1100
1000
0
MAR
JUN
SEP
DEC
MAR
JUN
SEP
DEC
MAR
JUN
SEP
DEC
1976
1977
1978
INCURRED QUARTERS

EXHIBIT 7

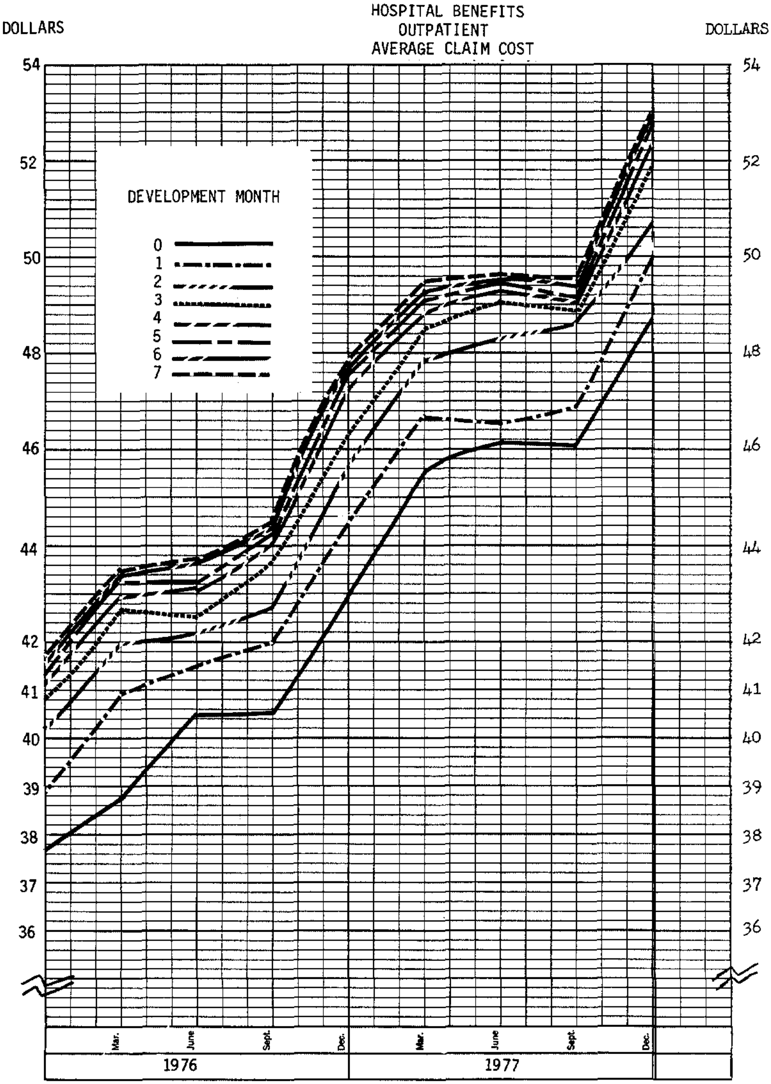

HOSPITAL BENEFITS
OUTPATIENT
AVERAGE CLAIM COST
DOLLARS
DOLLARS
54
52
50
48
46
44
42
41
40
39
38
37
36
DEVELOPMENT MONTH
0
1
2
3
4
5
6
7
Mar.
June
Sept.
Dec.
Mar.
June
Sept.
Dec.
1976
1977

EXHIBIT 8

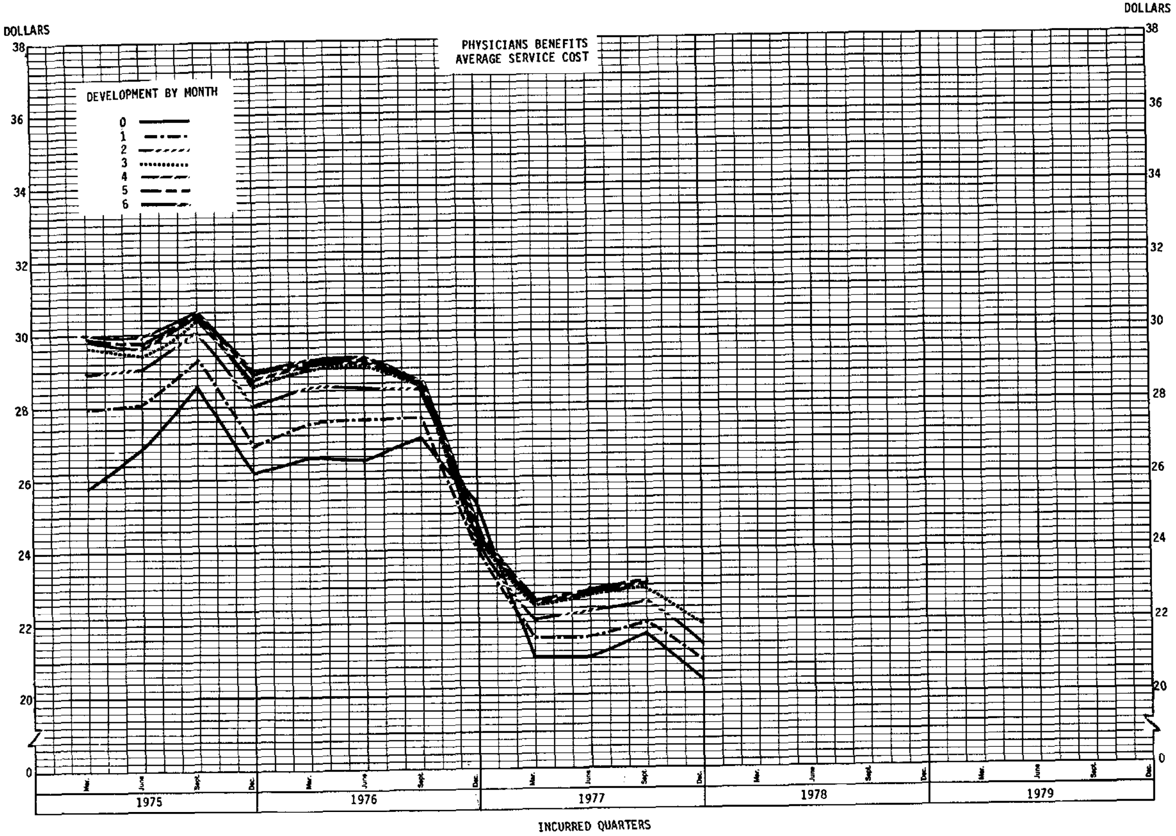

PHYSICIANS BENEFITS
AVERAGE SERVICE COST
DOLLARS
DEVELOPMENT BY MONTH
0
1
2
3
4
5
6
38
36
34
32
30
28
26
24
22
20
0
Mar.
June
Sept.
Dec.
1975
1976
1977
1978
1979
INCURRED QUARTERS

EXHIBIT 9

HOSPITAL BENEFITS
CLAIM REPORT-RECEIPTS

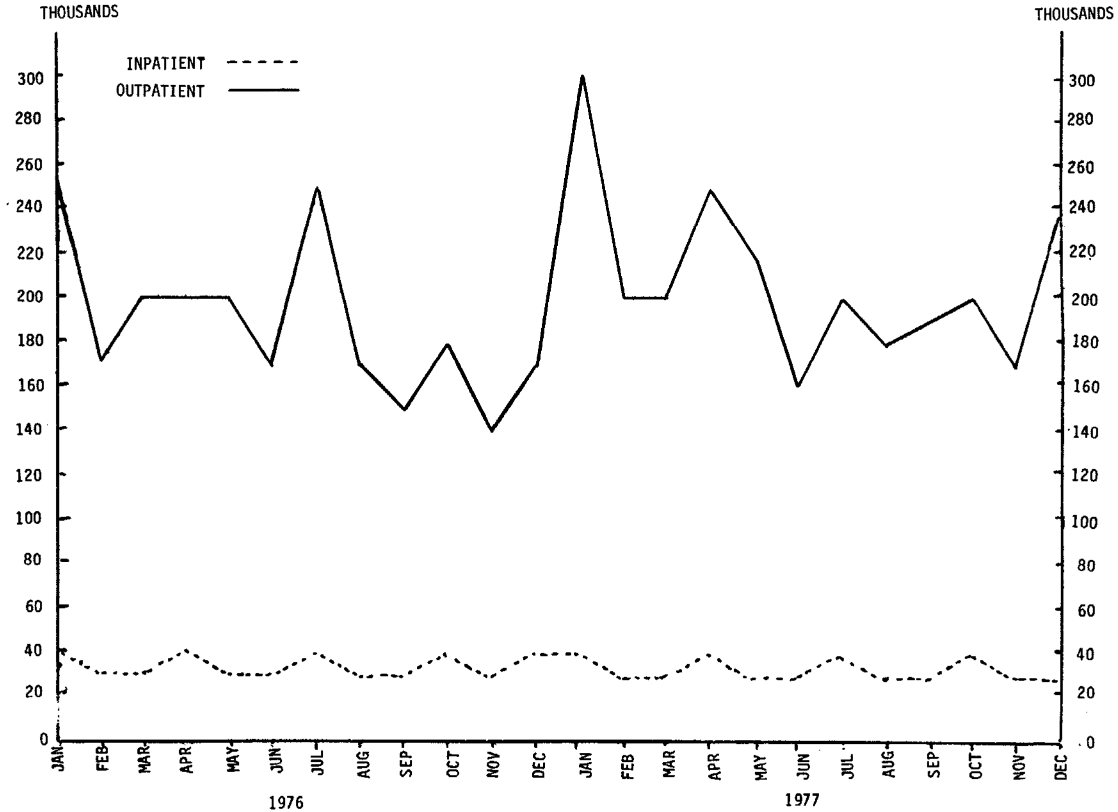


EXHIBIT 10

HOSPITAL BENEFITS
CLAIM REPORT
PAID CLAIMS

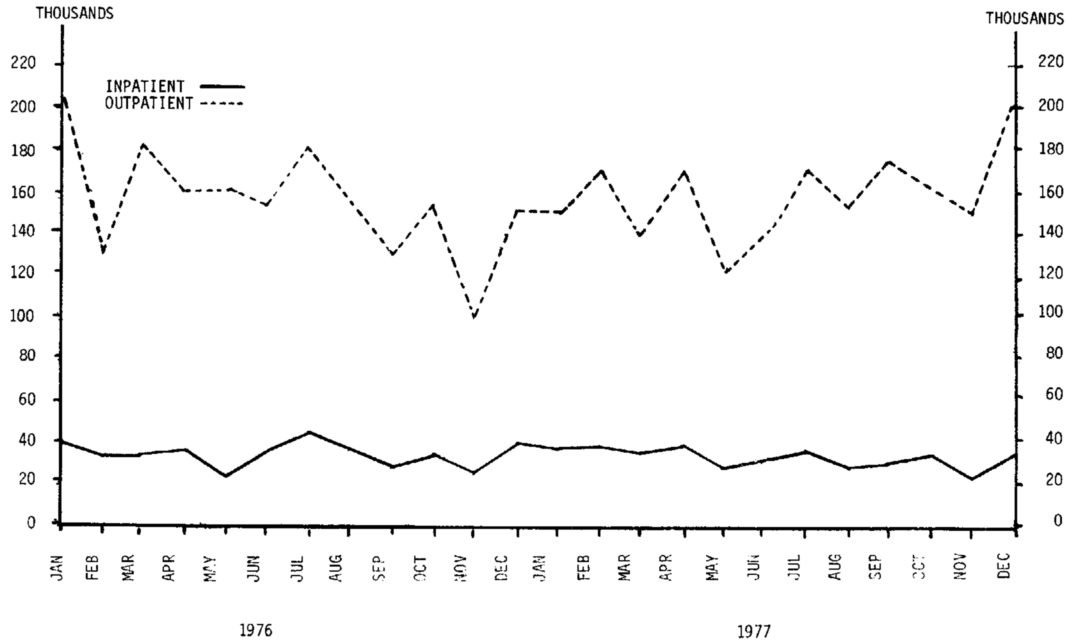


EXHIBIT 11

HOSPITAL BENEFITS
CLAIM REPORT
INVENTORY

EXHIBIT 12

PHYSICIAN BENEFITS
CLAIM RECEIPTS

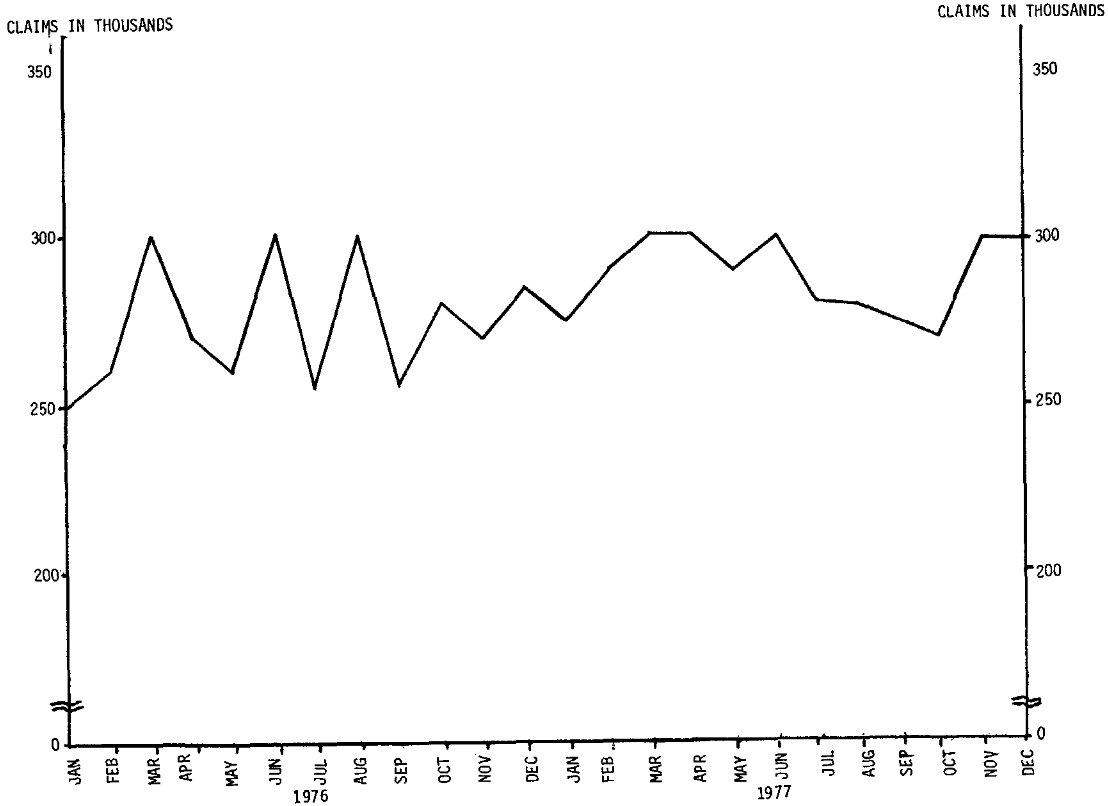


EXHIBIT 13

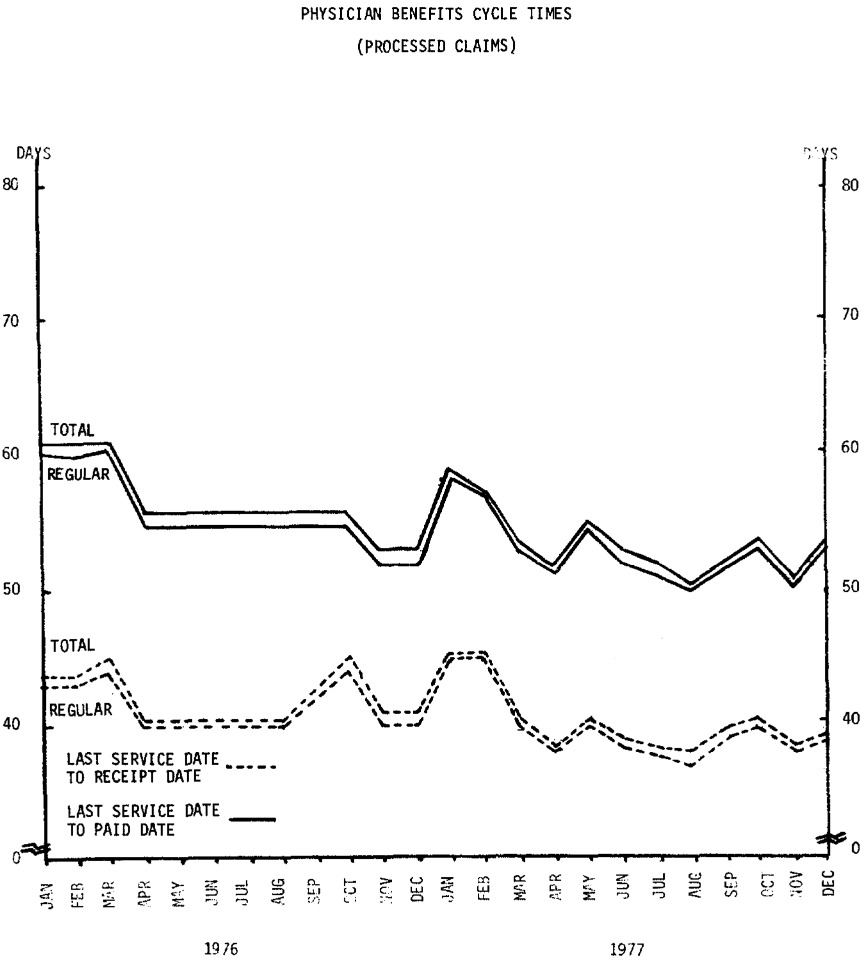

PHYSICIAN BENEFITS CYCLE TIMES
(PROCESSED CLAIMS)
DAYS
80
70
60
50
40
0
TOTAL
REGULAR
TOTAL
REGULAR
LAST SERVICE DATE TO RECEIPT DATE
LAST SERVICE DATE TO PAID DATE
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
1976
1977

EXHIBIT 14

DENTAL BENEFITS
DENTAL CLAIMS
PRODUCTION REPORT

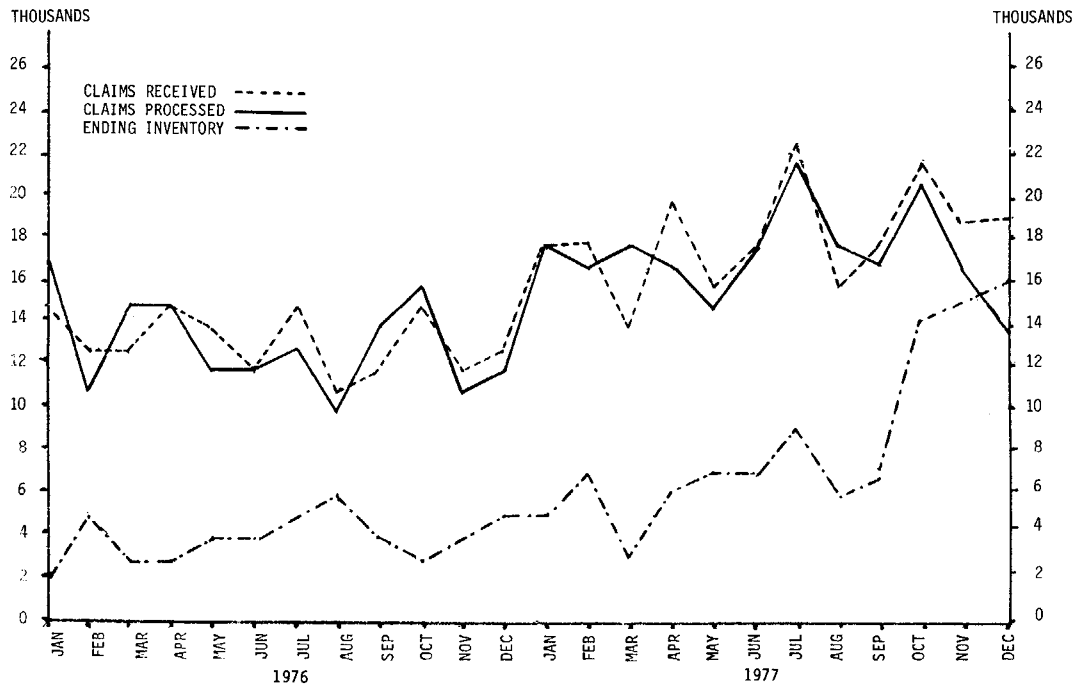


EXHIBIT 15

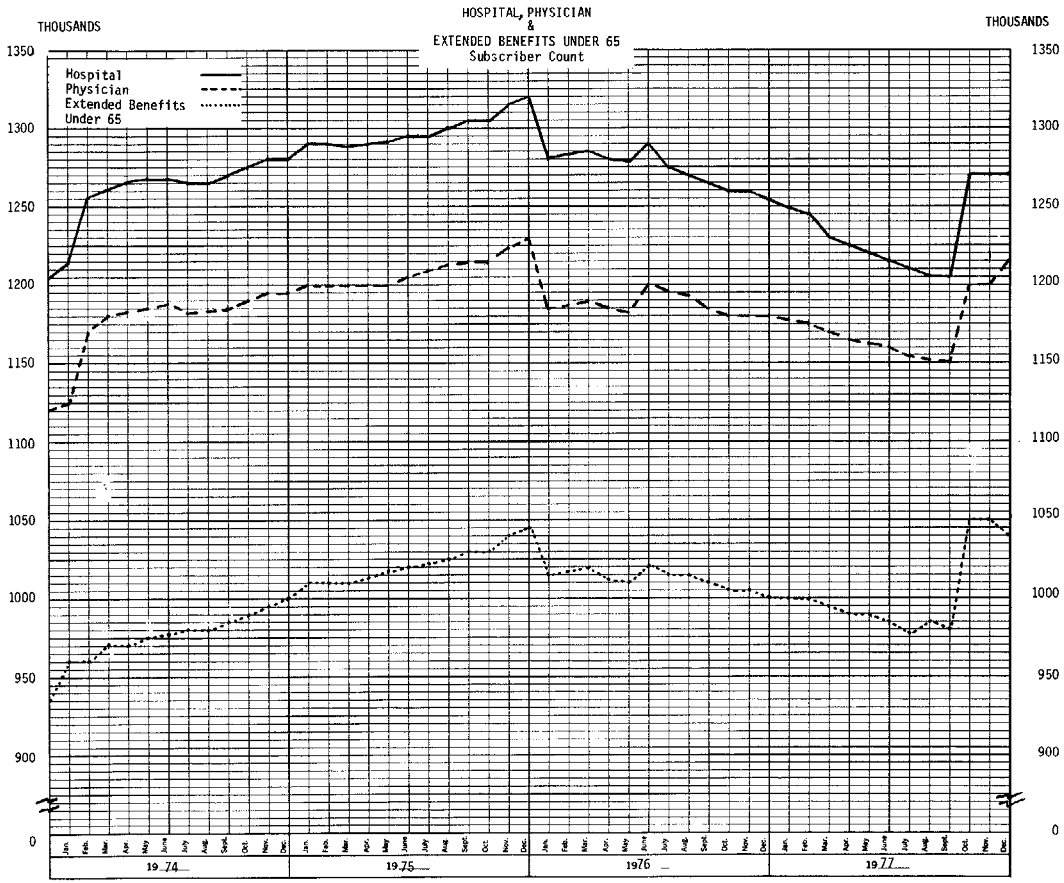

HOSPITAL, PHYSICIAN
&
EXTENDED BENEFITS UNDER 65
Subscriber Count
THOUSANDS
THOUSANDS
Hospital
Physician
Extended Benefits
Under 65
1350
1300
1250
1200
1150
1100
1050
1000
950
900
0
Jan.
Feb.
Mar.
Apr.
May
June
July
Aug.
Sept.
Oct.
Nov.
Dec.
1974
1975
1976
1977

EXHIBIT 15A

EXTENDED BENEFITS 65 AND OVER

Subscriber Count

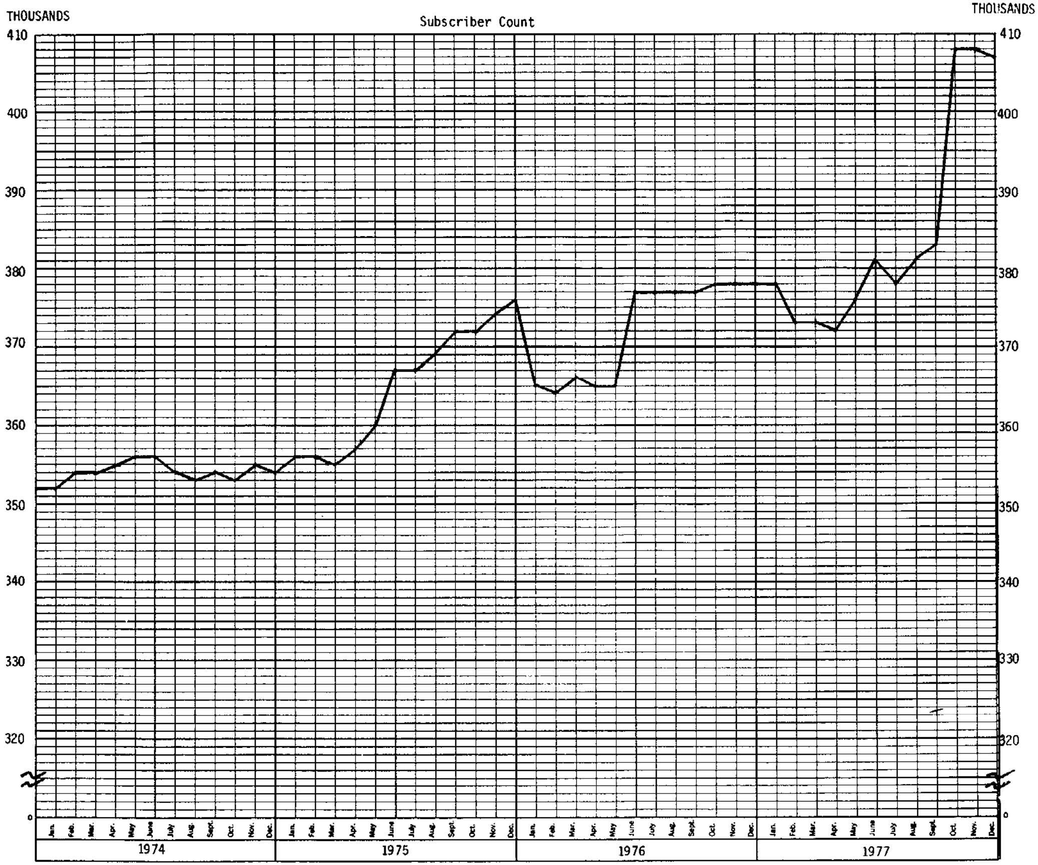
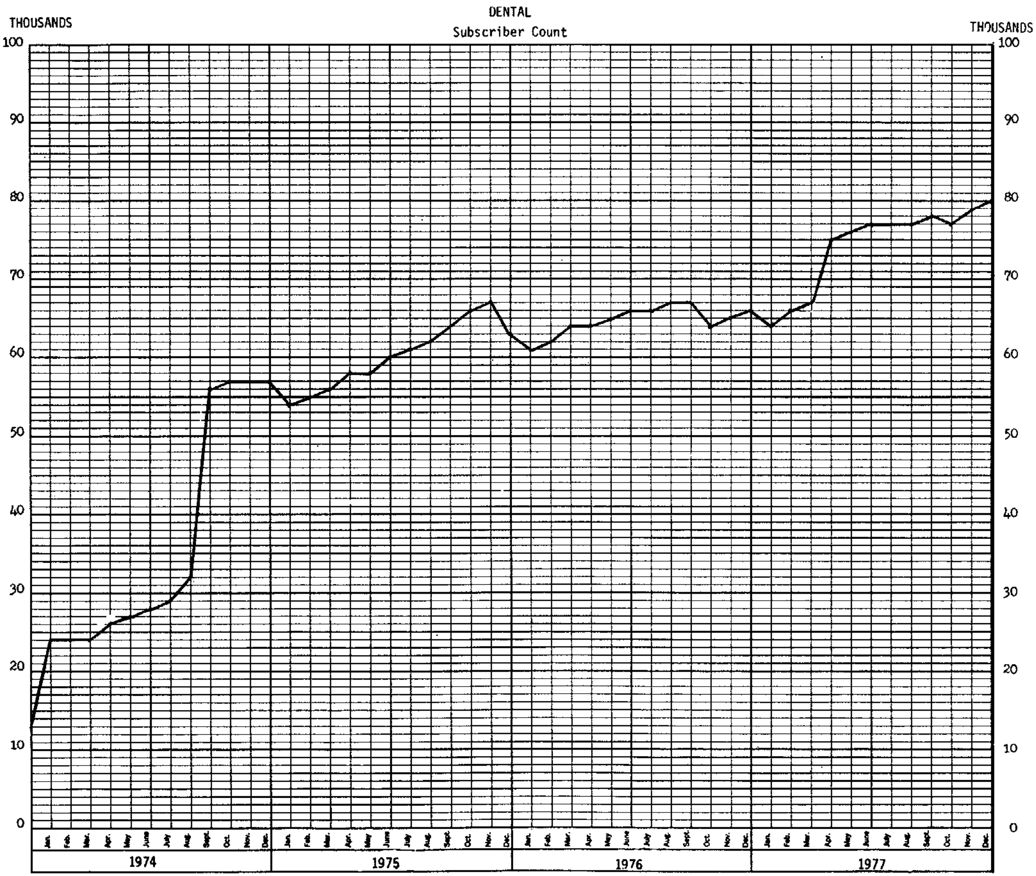


EXHIBIT 15B


DENTAL
Subscriber Count
THOUSANDS
THOUSANDS
100
90
80
70
60
50
40
30
20
10
0
Jan. Feb. Mar. Apr. May June July Aug. Sept. Oct. Nov. Dec.
1974
1975
1976
1977

A METHOD FOR SETTING RETRO RESERVES

CHARLES H. BERRY

OVERVIEW

In a paper presented to the Casualty Actuarial Society in 1965[1], W. J. Fitzgibbon, Jr. explained a method of setting reserves for retrospective premium adjustments. His method is based on the fact that, in general, a group of policies with a low loss ratio will produce a greater retrospective return premium than a group of policies with a high loss ratio. In practice, unfortunately, this relationship is not perfect.

For older groups of policies the actual retrospective adjustments which have already been made provide additional evidence about what the ultimate adjustment will be. This paper describes a systematic method of using this additional information to refine the Fitzgibbon indication and set a more accurate reserve.

THE RETROSPECTIVELY RATED POLICY

A retro policy is an insurance contract which provides for the deposit with the insurer of a *standard premium* at the inception of the policy. Six months after policy expiration and at one-year intervals thereafter, the reported losses arising from the policy are used to determine a *retrospective premium.*

If losses are lower than anticipated, the retro premium will be less than the standard premium, and the difference will be *returned* to the insured. This *retro adjustment* will affect the insurer's books as a *negative* premium. Conversely, losses higher than anticipated will produce an *additional* payment to the insurer, and will have a *positive* impact on the insurer's net premium. This sign convention will be used throughout this paper for actual paid and expected future *deviations* from standard premiums.

A revised retrospective premium is calculated annually until at some point the insured and the insurer agree that no further adjustments are needed. The sum of all retro adjustments which took place during this period is called the *ultimate deviation.*

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[1] W. J. Fitzgibbon, Jr., "Reserving for Retrospective Returns," *PCAS* LII, 1965, p. 203.

At any time there may be several policies for which the ultimate deviation has not yet been determined. It is appropriate that the insurer adjust the earned premiums used in stating its underwriting results to reflect any anticipated *remaining* deviations; that is, those deviations which would result if the final retro premium were determined based on the standard premium earned and the losses incurred to date for these policies, including the insurer's provision for losses not yet reported. This adjustment to earned premiums is made through the *retro reserve*. If net remaining deviations are expected to be negative, a *positive* reserve is established and subtracted from earned premiums. However, if positive net remaining deviations are expected, the converse is true.

DATA USED IN CALCULATING THE RETRO RESERVE

The retro reserve could be calculated for all retrospectively rated business combined. However, since greater detail is required for both the Annual Statement and internal underwriting results, it is preferable to calculate a separate reserve for each line of business for which there is a significant volume of retrospectively rated premium.

Some insurers further divide their business into different types of insureds, and it may be appropriate to use different formulas to set retro reserves for these different types. For example, a large account will generally produce a relatively larger retro return than a smaller account with the same loss ratio. This is because the premium discount is returned as part of the retro adjustment, and the large account will have a lower expense ratio.

The method to be described calculates a separate reserve for each *policy year*; that is, for all policies becoming effective during a calendar year. The term of all such policies is one year; three-year agreements must be broken into three pieces. Retro adjustment premiums, audit premiums, late-reported losses, after-closing loss payments, etc., are assigned back to the year of the policy which generated them.

For each cell (i.e., for each line of business by insured type by policy year combination) as of each reserve date, this method requires the paid retro deviations to date, the earned standard premium, and the expected incurred losses which will eventually arise from this earned premium. This loss number should include provision for incurred but not reported (IBNR) losses and for future development of the present estimated values of open claims to their ultimate values.

Since many retro policies provide for limitations on the amount by which any single loss can increase the retro premium, it is also appropriate to remove large individual losses from the incurred loss amount used in calculating the retro reserve. Such large losses should also be removed from the historical data used in calibrating the retro reserve formulas.

BASIC FORMULAS

At any reserve date, for each of the cells described above, the actual deviation paid through that date is a known quantity. If the ultimate deviation can be determined, the retro reserve is easily calculated using the following formula:

$$\text{(Retro Reserve)} = \text{(Paid Deviation)} - \text{(Ultimate Deviation)} \qquad (1)$$

In practice, it is easier to work with quantities which are ratios to earned standard premium (*ESP*). In this way, comparisons of one policy year to another, or of a policy year to itself at different points in time, may be made on a common basis. Therefore, the ultimate deviation is calculated from the ultimate deviation *ratio* (*DRU*) as follows:

$$\text{Ultimate Deviation} = ESP \times DRU \qquad (2)$$

The ultimate deviation ratio used to set the reserve is a weighted average of two indicated deviation ratios ($DR1$ and $DR2$):

$$DRU = [DR1 \times (1 - W2)] + [DR2 \times W2] \qquad (3)$$

$DR2$ is the indication which comes into play as we begin to consider deviations paid to date in estimating the ultimate deviation ratio. It will be discussed later. For "young" policy years (those years which began fewer than 21 months before the date at which the reserve is being set), the weight ($W2$) applied to $DR2$ is 0. During this period, formula (3) above reduces to simply $DRU = DR1$.

THE *DR*1 FORMULA

$DR1$ is a linear function of the incurred loss ratio (*ILR*):

$$DR1 = [(SF \times ILR) + CF] \leq DRM \qquad (4)$$

Note that this is simply Fitzgibbon's formula, with the added restriction that the indicated deviation ratio is capped at a maximum deviation ratio (*DRM*). Ideally, the slope factor (*SF*) and the constant factor (*CF*) of the $DR1$ equation can be determined by a least squares fit of data points representing old policy years[2] for which the ultimate loss ratios and ultimate deviation ratios are fairly accurately known, as in Exhibit I.

Policy year 1971, for example, has a loss ratio of 64.1%. Net retro returns equal to 13.1% of earned standard premium have so far been made. If it happens that a recent policy year, say 1976, presently has a loss ratio of about 64%, we may expect an ultimate deviation ratio of about −13% and establish a retro reserve accordingly. Similarly, policy year 1969 gives an indication that when the aggregate loss ratio for a policy year is as high as 72%, we may expect a net ultimate return of only about 6%.

The least squares fit line provides a method of smoothing out these data points and of interpolating between them. We may also extrapolate to lower and higher loss ratios. Although ultimate loss ratios for large lines of business will tend to vary over only a fairly narrow range, loss ratios for the first few months of new policy years, or even ultimate loss ratios for small volume lines, may be extreme.

Thus, it is appropriate to cap the additional premiums which we expect to collect at, say, 5% of *ESP*, no matter how high the policy year loss ratio is. This is because such policy years probably contain a few policies with extremely high loss ratios which will hit their maximums and produce additional premiums which are too small to offset all the losses. Meanwhile, many other insureds will have low loss ratios and will earn return premiums which may offset most of the additional premium received from the high-loss policies.

Now observe the policy year 1968 point on Exhibit I. This point lies well off the least squares line. If we strictly followed the *DR*1 indication, we would still be looking for an ultimate return of 10% of standard premium despite the fact that returns of only 7.8% have been made so far. Because very few deviations, either positive or negative, are still coming in due to eighth or later adjustments for this old policy year, we clearly should have dropped the indicated retro reserve of 2.2% of standard premium at some previous date.

The difference between the policy year 1972 point and the *DR*1 indication is in the opposite direction. Because this is not as old a year as 1968, a small number of late retro adjustments may still come in. Nevertheless, it is quite

[2] In practice, it is useful to determine the *DR*2 formula values first and then use them to project paid deviations for two or three more recent policy years, thus obtaining additional data points to use in selecting the *DR*1 curve.

unlikely that the total amount of the indicated negative retro reserve is appropriate.

Policy years 1968 and 1972 demonstrate the weakness of a pure *DR*1-type formula. As policy years age, it usually becomes clear that the true ultimate adjustment will be greater than or less than the *DR*1 formula indication. At that point, the reserve must either be changed to zero or must be revised by some amount on a judgment basis. The *DR*2 formula provides a method for making such a revision in a smooth, systematic way.

THE *DR*2 FORMULA

Judgments about the correctness of the *DR*1 indication can be made only after there are significant amounts of actual paid retro adjustments. Consider policy year 1972, for example. The first policies written in this year were effective 1/1/72 and expired 12/31/72. The first retro adjustment was calculated based on losses evaluated six months after policy expiration, or 6/30/73. It probably took two or three months to prepare loss reports, calculate the retrospective premium, and input the adjustments to the accounting system. The first significant paid deviations for policy year 1972 therefore began to appear in August or September of 1973, about 20 or 21 months after the beginning of the policy year. Although a few deviations, probably due to early policy cancellations, were seen before 20 months, it was not until after then that some weight could be given to the *DR*2 indication in formula (3).

At about 33 months, first adjustments for policies effective in December 1972 were completed and second adjustments for January 1972 policies, valued 30 months after policy inception, began to appear. Twelve months later, third adjustments began, and so on.

The consistency of the pattern of paid deviations from one policy year to another can be seen in Exhibit II, which shows cumulative paid deviations as of each month, taken as a ratio to total paid deviations through 60 months. This consistency provides the basis for the second estimate of the ultimate deviation ratio:

$$DR2 = (DPF \times PDR) + (LPF \times ILR) \qquad (5)$$

The first portion of this formula (Deviation Projection Factor times Paid Deviation Ratio) estimates remaining first adjustments. The second portion (Loss Projection Factor times Incurred Loss Ratio) estimates remaining second and later adjustments. In order to understand this formula, refer to Exhibit III to see a typical set of *DR*2 formula values.

Deviation Projection Factor

When setting a retro reserve in any September, the first prior policy year is 21 months old. Observe in Exhibit II that at this age we first begin to see a significant volume of paid deviations coming in. The first part of the *DR*2 formula assumes that first adjustments already paid at any point in time are similar to those first adjustments as yet unpaid. For example, if the early paid deviations are returns equal to 10% of the earned standard premium on the policies producing these deviations, the formula assumes that remaining first adjustment deviations will also be returns equal to 10% of the corresponding premium.

The reciprocal of the 21-month *DPF* of 6.64 represents the portion of all first adjustments which are assumed to have been paid by this age. Multiplying the 21-month paid deviation ratio by this *DPF* thus estimates the ultimate ratio to earned standard premium of deviations due to first retro adjustments. By 36 months it is assumed that all first adjustments have already been processed, and *DPF* decreases to unity, where it remains from that point on.

Loss Projection Factor

In the second part of the *DR*2 formula, the Loss Projection Factor has been so named because it is applied to the expected ultimate incurred loss ratio. Nevertheless, its purpose is to estimate the amount of second and later deviations remaining as of any reserve date.

Note in Exhibit III that from 21 to 31 months, the *LPF* is constant at 0.0500. This is because, no matter how large or small first adjustments are, and no matter whether they are returns or additionals, formula (5) assumes that second and later adjustments will produce an additional premium equal to 5% of incurred losses.

The reason an additional premium is anticipated is that only reported claims, carrying whatever value the Claim Department put on them the last time they were examined, enter the retrospective premium calculation for any policy. Consequently, ''case basis'' losses valued 18 months after policy inception, when the first adjustment is calculated, are, on the average, understated. At later adjustments, previously reported claims may be revalued upward and new claims which were IBNR at previous adjustments may emerge. Thus, it will likely be determined that additional premiums previously paid to the insurer were too small, or that part or all of the returns previously paid to the insured must now flow back to the insurer. In either case, positive deviations resulting from second and later retro adjustments are likely to dominate negative deviations.

Thus, according to the *DR*2 indication, for any policy year which is about 31 months old, there are few first adjustments but many second and later adjustments remaining to be made, and it is appropriate to hold a negative retro reserve. Thereafter, as the anticipated additional premiums flow in, the *LPF* decreases, reducing the size of the indicated negative retro reserve.

The values for *DPF* and *LPF* are readily determined if a monthly history is available which separates deviations into first adjustments, second adjustments, etc. Absent such a history, acceptable answers can be obtained by assuming that all deviations through 33 months are from first adjustments, those from 34 through 45 months are from second adjustments, etc.

COMBINING *DR*1 WITH *DR*2

The last column of Exhibit III sets forth the weight (*W*2) which is applied to the *DR*2 indication in formula (3). As stated previously, no weight is given to *DR*2 during the first 20 months. Then, as paid deviations begin to accumulate, *DR*2 becomes a better and better estimate of the true ultimate deviation ratio, and *W*2 begins increasing linearly.

Inspection of the pattern of actual paid deviations over time reveals that beyond about five years, retro adjustments are likely to be very small, and additional and return premiums are almost equally common. Beyond this point, therefore, it is not worthwhile to attempt to set a retro reserve.[3] The formula is designed so that when a policy year becomes 60 months old, two things happen which cause the retro reserve to disappear:

1. The loss projection factor becomes 0.0000. Because the deviation projection factor is unity at this point, formula (5) simplifies to $DR2 = PDR$
2. At the same time, $W2$ becomes 1.000, and formula (3) simplifies to $DRU = DR2 = PDR$.

That is, for any policy year which is 60 months or more old, the expected ultimate deviation is equal to the current paid deviation, and no retro reserve is

[3] This statement is true only for the particular company studied at this point in time. The decision about how long to hold a retro reserve should be reevaluated periodically.

held. This point has been reached smoothly in the course of 40 months. At each reserve date during this period, the formula has given due consideration to the incurred loss ratio of the policy year as well as to its actual paid retro deviations.

SAMPLE CALCULATION

Exhibit IV shows a history of the retro reserve for a typical policy year, based on the formulas given in Exhibits I and III. Note in particular the following points in time:

A. 1/31/72, age 1 month. The policies written in January 1972 have, by the end of the month, generated an earned premium of $2,074,000. Most of the $1,795,000 of incurred losses is due to an IBNR reserve, as one month is not sufficient time to allow many accidents to occur, be reported, and have estimates of their values put into the system. The indicated loss ratio of 86.55% generates a *DR*1 indication of +6.13% which has been capped at +5.00%. A negative retro reserve has been established in anticipation of a net ultimate additional premium equal to 5% of the standard premium earned so far. No *DR*2 indication has been calculated.

B. 2/29/72, age 2 months. Earned premiums have increased not only because of more policies being written in February, but also because of additional earned premiums generated by January writings. Incurred losses have increased less than premiums, producing a loss ratio of only 84.90% and a *DR*1 indication of +4.71%, which does not need to be capped.

C. 9/30/73, age 21 months. The loss ratio has decreased to 68.52% and *DR*1 now anticipates a return premium of 9.37%. This is the first month for which the *DR*2 indication is considered, and paid deviations have been projected to an ultimate return of 21.47%. This *DR*2 indication predicts that the ultimate policy year 1972 point would be far below the *DR*1 formula line were it graphed on Exhibit I. This *DR*2 indication is too far off the line to be realistic, but it receives a weight of only 2.5%, and we will eventually see that it *does* tend to move the *DR*1 indication in the correct direction.

D. 8/31/74, age 32 months. Almost all first adjustments, but few second adjustments, have been processed. At this point, the maximum return has been reached and remaining retro adjustments will be dominated by additionals. *DR*2, still predicting that ultimate returns will be greater than *DR*1 does, receives a weight of 30%.

E. 11/30/76, age 59 months. *DR*2 has almost 100% weight and the net retro reserve has decreased smoothly despite the large incurred loss decrease since last month. Note that if only *DR*1 were used, this loss decrease would have caused a retro reserve increase of about \$400,000.

F. 12/31/76, age 60 months. The reserve for policy year 1972 goes to \$0. Without the *DR*2 formula, we would still have been holding a reserve of:

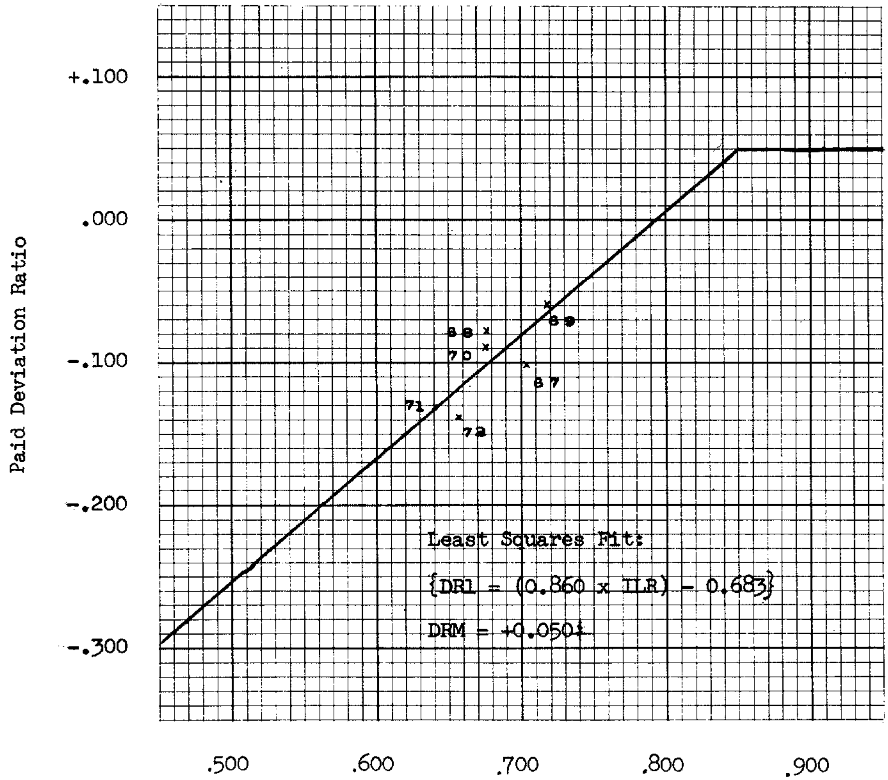
$$-\$10{,}813{,}000 - [\$78{,}128{,}000 \times (-0.1182)] = -\$1{,}578{,}000$$

unless we had dropped part or all of this amount on a judgment basis at some earlier date.

EXHIBIT I

DETERMINATION OF *DR*1 FORMULA
—WORKERS' COMPENSATION

DATA POINTS AS OF 12-31-76



Policy Year	Ratios to Earned Standard Premium	
	Incurred Losses	Paid Deviations
1967	.703	.101−
1968	.677	.078−
1969	.718	.059−
1970	.676	.089−
1971	.641	.131−
1972	.657	.138−

EXHIBIT II

PAID DEVIATIONS—WORKERS' COMPENSATION

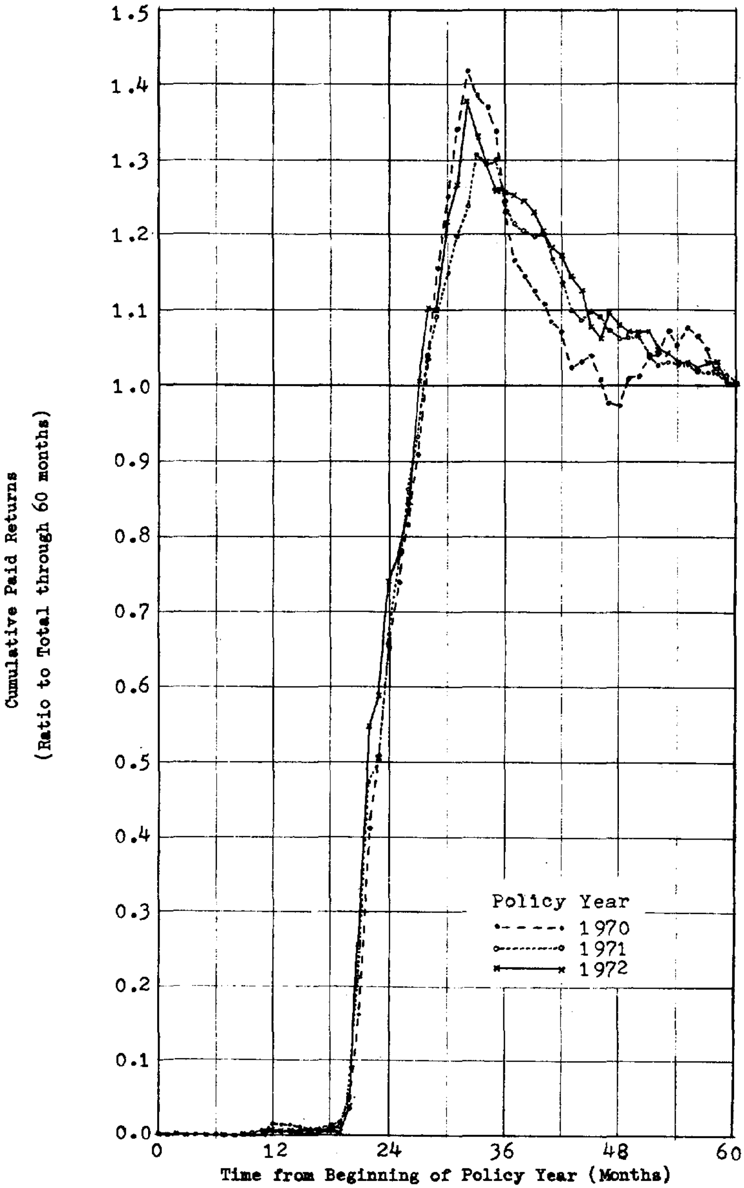


EXHIBIT III

*DR*2 FORMULA VALUES—WORKERS' COMPENSATION

Age of Policy Year (months)	Deviation Projection Factor (*DPF*)	Loss Projection Factor (*LPF*)	Weight for *DR*2 Indication (*W*2)
1 to 20	—	—	.000
21	6.64	0.0500	.025
22	3.96	0.0500	.050
23	3.11	0.0500	.075
24	2.44	0.0500	.100
25	2.14	0.0500	.125
26	1.90	0.0500	.150
27	1.60	0.0500	.175
28	1.43	0.0500	.200
29	1.30	0.0500	.225
30	1.19	0.0500	.250
31	1.11	0.0500	.275
32	1.06	0.0496	.300
33	1.04	0.0444	.325
34	1.02	0.0404	.350
35	1.01	0.0376	.375
36	1.00	0.0340	.400
37	1.00	0.0316	.425
38	1.00	0.0292	.450
39	1.00	0.0252	.475
40	1.00	0.0220	.500
41	1.00	0.0192	.525
42	1.00	0.0164	.550
43	1.00	0.0140	.575
44	1.00	0.0123	.600
45	1.00	0.0102	.625
46	1.00	0.0084	.650
47	1.00	0.0073	.675
48	1.00	0.0060	.700
49	1.00	0.0054	.725
50	1.00	0.0048	.750
51	1.00	0.0038	.775
52	1.00	0.0030	.800
53	1.00	0.0023	.825
54	1.00	0.0016	.850
55	1.00	0.0010	.875
56	1.00	0.0006	.900
57	1.00	0.0004	.925
58	1.00	0.0002	.950
59	1.00	0.0001	.975
60 & up	1.00	0.0000	1.000

$DR2 = (DPF \times PDR) + (LPF \times ILR)$

$DRU = [DR1 \times (1 - W2)] + [DR2 \times W2]$

EXHIBIT IV

Sample Reserve Calculation History
Workers' Compensation—Policy Year 1972

Date	Age	Earned Standard Premium ($000)	Incurred Losses: Dollars ($000)	Incurred Losses: Ratio (4) ÷ (3)	Cumulative Paid Deviations: Dollars ($000)	Cumulative Paid Deviations: Ratio (6) ÷ (3)	Indicated Ultimate Deviation Ratios**: *DR*1	Indicated Ultimate Deviation Ratios**: *DR*2	Indicated Ultimate Deviation Ratios**: *DRU*	Estimated Ultimate Deviation (3) × (10) ($000)	Retro Reserve (6) − (11) ($000)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
(A) 1-72	1	$ 2,074	$ 1,795	.8655	$ 0	.0000	.0613*	—	.0500	$ 104	$ 104−
(B) 2-72	2	5,152	4,374	.8490	0	.0000	.0471	—	.0471	243	243−
3-72	3	8,090	7,272	.8989	0	.0000	.0901*	—	.0500	405	405−
4-72	4	11,260	10,195	.9054	0	.0000	.0956*	—	.0500	563	563−
5-72	5	14,699	13,385	.9106	0	.0000	.1001*	—	.0500	735	735−
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
7-73	19	69,807	48,613	.6964	33−	.0005−	.0841−	—	.0841−	5,871−	5,838
8-73	20	72,450	49,563	.6841	395−	.0055−	.0947−	—	.0947−	6,861−	6,466
(C) 9-73	21	73,531	50,381	.6852	2,755−	.0375−	.0937−	.2147−	.0967−	7,110−	4,355
10-73	22	73,927	51,040	.6904	5,926−	.0802−	.0893−	.2831−	.0990−	7,319−	1,393
11-73	23	74,223	51,620	.6955	6,365−	.0858−	.0849−	.2321−	.0959−	7,118−	753
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
6-74	30	78,284	52,066	.6651	13,155−	.1680−	.1110−	.1667−	.1249−	9,778−	3,377−
7-74	31	78,287	51,992	.6641	13,686−	.1748−	.1119−	.1608−	.1253−	9,809−	3,877−
(D) 8-74	32	78,603	51,868	.6599	14,880−	.1893−	.1155−	.1679−	.1312−	10,313−	4,567−
9-74	33	78,558	51,803	.6594	14,364−	.1828−	.1159−	.1608−	.1305−	10,252−	4,112−
10-74	34	78,509	51,941	.6616	13,997−	.1783−	.1140−	.1551−	.1284−	10,081−	3,916−
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
8-76	56	78,131	51,519	.6594	11,057−	.1415−	.1159−	.1411−	.1386−	10,829−	228−
9-76	57	78,131	51,512	.6593	11,087−	.1419−	.1160−	.1416−	.1397−	10,915−	172−
10-76	58	78,130	51,589	.6603	11,141−	.1426−	.1151−	.1425−	.1411−	11,024−	117−
(E) 11-76	59	78,130	51,128	.6544	10,832−	.1386−	.1202−	.1385−	.1380−	10,782−	50−
(F) 12-76	60	78,128	51,317	.6568	10,813−	.1384−	.1182−	.1384−	.1384−	10,813−	0

* *DR*1 formula indication before capping at +5%.

** *DR*1 = [0.86 × (5)] − 0.683

*DR*2 = [(7) × *DPF*] + [(5) × *LPF*]

DRU = [(8) × (1.0 − *W*2)] + [(9) × *W*2]

MINUTES OF THE 1980 FALL MEETING

November 19–21, 1980

OMNI INTERNATIONAL HOTEL, ATLANTA, GEORGIA

Wednesday, November 19, 1980

The Board of Directors held their regular quarterly meeting from 1:00 p.m. to 5:00 p.m.

Registration took place from 4:00 p.m. to 7:30 p.m.

The President's reception for new Fellows and their spouses was held from 6:00 p.m. to 6:45 p.m.

A reception for members and guests was held from 6:30 p.m. to 7:30 p.m.

Thursday, November 20, 1980

Registration was held from 7:45 a.m. to 8:00 a.m.

The Fall meeting was formally convened at 8:00 a.m. Following his opening remarks, President W. James MacGinnitie introduced the Honorable Johnnie L. Caldwell, Georgia Insurance Commissioner, who welcomed the Society to Atlanta.

Upon thanking Commissioner Caldwell for his remarks, President MacGinnitie then read the names of the new Associates. Each new Associate in attendance rose as his or her name was called. Mr. MacGinnitie then asked each of the new Fellows to step forward to receive his or her diploma.

The names of the twenty-five new Fellows and twenty new Associates follow.

FELLOWS

Debra L. Baer	Patricia A. Furst	Robert S. Miccolis
William H. Belvin	John Herder	Roy K. Morell
David R. Bradley*	Barbara J. Higgins	Richard J. Roth, Jr.
Mark M. Cis	Stephen Jameson*	Bernard G. Schaeffer
Michael D. Covney	Richard W. Lo	Roy G. Shrum
Ronald A. Dahlquist	Edward P. Lotkowski	Alain P. Thibault
Daniel Demers	Stephen P. Lowe	Jerome E. Tuttle
Glenn A. Evans	Michael G. McCarter	Richard T. Zatorski
	Michael A. McMurray	

* Not present.

ASSOCIATES

Robert L. Brown	David D. Hu	Kai-Jaung Pei
Gregory J. Ciezadlo	Martin K. Kelly	Glenn J. Pruiksma
Arthur I. Cohen	Kyleen Knilans	Ollie J. Sherman, Jr.
Eugene C. Connell	Stephen J. Ludwig	Glenn M. Walker
Karl H. Driedger	Ronald R. Miller	Jeffrey C. Warren
Alice H. Gannon	Rebecca A. Moody	Joel D. Yatskowitz
Jonathan B. Hale	William S. Morgan	

Following the admission of the new Fellows and Associates, President MacGinnitie announced the names of those receiving prizes and awards. A joint award of the Dorweiler Prize was given to two authors, C. K. Khury, for his paper, "Loss Reserves: Performance Standards," and Richard G. Woll, for his paper, "A Study of Risk Assessment Using Massachusetts Data."

Following this, President MacGinnitie asked each author to give a short summary of his paper.

At 10:00 a.m. the American Academy of Actuaries Business Session was opened by Mr. Ronald Bornhuetter, President. Mr. E. Boynton then delivered the report of the Nominating Committee. Mr. B. Munson then spoke of the issues of concern to actuaries involved in the area of risk classification and on the present status of various regulatory and legal aspects of the problem.

Officer reports were given by Mr. S. Kellison, Executive Director; Mr. K. Ryan, Treasurer; and Mr. B. Watson, Secretary.

Mr. Bornhuetter and Mr. Walter Grace, incoming President, then alternately spoke of yesterday's, today's and tomorrow's issues facing the actuary.

After a short break, a panel discussion entitled "Lloyds and the New York Insurance Exchange" was presented. Those participating were:

Moderator: Steven N. Newman
Vice President
American International Group

Panelists: Robin A. G. Jackson
Director
Merrett Dixey Syndicates

Donald E. Reutershan
President
New York Insurance Exchange

At 11:30 a.m. a panel discussion entitled "The Actuary as an Expert Witness" was presented. The participants were:

Moderator: Michael Fusco
Vice President-Actuary
Insurance Services Office

Panelists: Thomas E. Harms
Attorney
Hessian, McKassey & Soderberg

Spencer L. Kimball
Executive Director
American Bar Foundation

Phillip O. Presley
Actuarial Consultant

A luncheon break was held from 12:30 p.m. to 2:00 p.m.

The regular session reconvened at 2:00 p.m. with a workshop program. The workshops were held according to the following schedule:

Workshop A — "Lloyds and the New York Insurance Exchange"
This was a discussion of the morning panel with the panelists.

Workshop B — "The Actuary as Expert Witness"
This was a discussion of the morning panel with the panelists.

Workshop C — "American Academy of Actuaries—Professional Discipline"

Moderator: Dale R. Gustafson
Vice President & Actuary
Northwestern Mutual Life Insurance

Members: William D. Hager
General Counsel
American Academy of Actuaries

Charles C. Hewitt
President
Metropolitan Reinsurance Co.

Workshop D — "American Academy of Actuaries—Discounting Loss Reserves"

Moderator: James R. Berquist
Consulting Actuary
Milliman & Robertson, Inc.

Members: Martin Bondy
Senior Vice President
Crum & Forster, Inc.

James A. Faber
Principal
Peat, Marwick, Mitchell & Co.

Workshop E — "New Paper"
"Implications of Sales as an Exposure Base for Products Liability," by Stephen W. Philbrick, Marsh & McLennan, Inc.

Workshop F — "New Paper"
"General Liability Ratemaking: An Update," by Michael F. McManus, Chubb & Son, Inc; reviewed by Warren H. Johnson, Insurance Services Office.

Workshop G — "New Paper"
"An Analysis of Retrospective Rating," by Glenn G. Meyers, CNA Insurance Companies, reviewed by Mark E. Fiebrink, Wausau Insurance Companies.

Workshop H — "New Paper"
"Estimating Casualty Insurance Loss Amount Distributions," by Gary F. Patrik, Prudential Reinsurance Co.; reviewed by Jerome Jurschak.

Workshop I — "New Paper"
"A Method for Setting Retro Reserves," by Charles H. Berry, Aetna Life & Casualty; reviewed by Roy K. Morell, Liberty Mutual Insurance Company.

Workshop J — "New Paper"
"Determining Ultimate Claim Liabilities for Health Insurance Coverages," by Emil J. Strug, Blue Cross/Blue Shield of Massachusetts; reviewed by Edith E. Price, Kemper Insurance Group.

The day ended with a reception from 7:30 p.m. to 8:00 p.m. at the Omni Terrace.

Friday, November 21, 1980

The business session reconvened at 8:30 a.m., opening with committee reports and reviews of papers.

The Secretary's and Treasurer's Annual Reports were then given.

The election of Officers and Directors followed. Those elected, and their offices, were as follows:

President-Elect	Steven H. Newman
Vice President	Frederick W. Kilbourne
Secretary	David P. Flynn
Treasurer	Michael A. Walters
Editor	C. K. Khury
General Chairman, Education and Examination Committee	Phillip N. Ben-Zvi
Board Members	Wayne H. Fisher
	Anne E. Kelly
	Richard E. Munro

Mr. Fisher was elected on the first ballot; Ms. Kelly and Mr. Munro were elected on the second ballot.

Following the election of Officers and Directors, a panel discussion entitled "Implications of Risk Retention Pools" was presented. Those participating were:

Moderator: Mavis A. Walters
Vice President
Insurance Services Office

Panelists: Dennis R. Connelly
Counsel
American Insurance Association

Warren P. Cooper
Senior Vice President & Actuary
Insurance Company of North America

Robert K. Nelson
Executive Vice President
Insurance Administration Center, Inc.
Consultant
National Association of Wholesaler Distributors

At 10:00 a.m. the Presidential Address was given by Mr. W. James MacGinnitie. This was followed by an informal discussion and coffee break.

At 11:00 a.m. a panel discussion on the topic "The Black Lung Claim Controversy" commenced. Those participating were:

Moderator: Kevin M. Ryan
President
National Council on Compensation Insurance

Panelists: William C. Aldrich
Vice President
Hartford Accident and Indemnity Co.

Robert A. Brian
General Partner
Conning & Company

James De Marce
Executive Assistant to the Director of the Office of
Worker's Compensation Programs
Department of Labor

Richard W. Palczynski
Associate Actuary
Travelers Insurance Co.

A luncheon break was held from 12:00 noon to 1:30 p.m.

The regular session resumed at 1:30 p.m. with a panel discussion entitled "Competitive Rating—Some Proposed Changes." Those participating were:

Moderator: David G. Hartman
Vice President & Actuary
Chubb & Son, Inc.

Panelists: Thomas W. Jenkins
Special Counsel to the Director
Illinois Department of Insurance

Thomas C. Strohmenger
Counsel
Aetna Life & Casualty

Laura P. Sullivan
Senior Assistant Counsel
State Farm Insurance Co.

The closing remarks were made by President-Elect Jerome A. Scheibl after which the Fall Meeting adjourned at 3:00 p.m.

In attendance, as indicated by registration records, were 189 Fellows, 110 Associates, 34 Guests, 11 Subscribers, 12 Students, and 94 spouses. The list follows.

FELLOWS

Adler, M.
Aldrich, W. C.
Anderson, D. R.
Atwood, C. R.
Baer, D. L.
Bailey, R. A.
Bartlett, W. N.
Bassman, B. C.
Bayley, T. R.
Beer, A. J.
Bell, L. L.
Belvin, W. H.
Bennett, N. J.
Ben-Zvi, P. N.
Bergen, R. D.
Berry, C. H.
Bethel, N. A.
Bickerstaff, D. R.
Bill, R. A.
Bishop, E. G.
Bondy, M.
Bornhuetter, R. L.
Bradley, D. R.
Brian, R. A.
Brown, J. W.
Buck, J. E., Jr.
Carter, E. J., Jr.
Cis, M. M.
Collins, D. J.
Conger, R. F.
Curley, J. O.
Dahlquist, R. A.
Daino, R. A.
Davis, G. E.
Degerness, J. A.
Demers, D.
Dempster, H. V., Jr.
Dieter, G. H., Jr.
Donaldson, J. P.
Dorval, B. T.
Drennan, J. P.
Dropkin, L. B.
Ehlert, D. W.
Evans, G. A.
Faber, J. A.
Fiebrink, M. E.
Fisher, W. H.
Flaherty, D. J.
Flynn, D. P.
Ford, E. W.
Forker, D. C.
Fossa, E. F.
Foster, R. B.
Fowler, T. W.
Frohlich, K. R.
Furst, P. A.
Fusco, M.
Garand, C. P.
Gillespie, J. E.
Gleeson, O. M.
Goldberg, S. F.
Grady, D. J.
Graham, T. L.
Grannan, P. J.
Hafling, D. N.
Hall, J. A.
Hartman, D. G.
Harwayne, F.
Hazam, W. J.
Herder, J. M.
Hermes, T. M.
Herzfeld, J.
Hewitt, C. C., Jr.
Higgins, B. J.
Honebein, C. W.
Inkrott, J. G.
Jaeger, R. M.
Jean, R. W.
Jerabek, G. J.
Johe, R. L.
Kaliski, A. E.
Kallop, R. H.
Kates, P. B.
Kaufman, A.
Khury, C. K.
Kilbourne, F. W.
Kist, F. O.
Klaassen, E. J.
Klein, D. M.
Krause, G. A.

FELLOWS

Kuehn, R. T.
Lamb, R. M.
Lattanzio, F. J.
Leonard, G. E.
Leslie, W., Jr.
Levin, J. W.
Liscord, P. S.
Lo, R. W.
Lotkowski, E. P.
Lowe, R. F.
Luneburg, B. C.
MacGinnitie, W. J.
Makgill, S. S.
Masterson, N. E.
McCarter, M. G.
McClenahan, C. L.
McManus, M. F.
McMurray, M. A.
Miccolis, J. A.
Miccolis, R. S.
Miller, D. L.
Moore, B. C.
Morell, R. K.
Morison, G. D.
Muetterties, J. H.
Munro, R. E.
Murray, E. R.
Murrin, T. E.
Nash, R. K.
Nelson, D. A.
Nelson, J. R.
Newman, S. H.
O'Brien, T. M.
Oien, R. G.
Otteson, P. M.
Palczynski, R. W.
Patrik, G. S.
Perkins, W. J.
Petersen, B. A.
Petlick, S.
Philbrick, S. W.
Phillips, H. J.
Pierce, J.
Pinney, A. D.
Presley, P. O.
Price, E. E.
Radach, F. R.
Reichle, K. A.
Richards, H. R.
Rodermund, M.
Roland, W. P.
Roth, R. J.
Roth, R. J., Jr.
Rowland, W. J.
Ryan, K. M.
Salzmann, R. E.
Schaeffer, B. G.
Scheibl, J. A.
Schultz, J. J.
Scott, B. E.
Sheppard, A. R.
Shoop, E. C.
Shrum, R. G.
Skurnick, D.
Snader, R. H.
Spitzer, C. R.
Squires, S. R.
Stanard, J. N.
Steeneck, L. R.
Stephenson, E. A.
Streff, J. P.
Strug, E. J.
Sturgis, R. W.
Swift, J. A.
Tatge, R. L.
Taylor, J. C.
Teufel, P. A.
Thibault, A. P.
Tierney, J. P.
Toothman, M. L.
Trist, J. A. W.
Tuttle, J. E.
Venter, G. G.
Verhage, P. A.
Walters, Ma. A.
Walters, Mi. A.
Webb, B. L.
Weissner, E. W.
White, H. G.
Wilcken, C. L.
Williams, P. A.
Wilson, J. C.
Wiser, R. F.
Wood, J. O.
Wulterkens, P. E.
Young, R. J.
Zatorski, R. T.
Zelenko, D. A.
Zubulake, T. J.

ASSOCIATES

Andler, J. A.
Barrow, B. H.
Bartlett, J. W.
Battaglin, B. H.
Brahmer, J. O.
Chorpita, F.
Ciezadlo, G. J.
Cohen, A. I.
Connell, E. C.
Connor, V. P.
Cooper, W. P.
Corr, F. X.
Crifo, D. A.
Crowe, P. J.
Diamantoukos, C.
Dodd, G. T.
Doepke, M. A.
Driedger, K. H.
Egnasko, G. J.
Einck, N. R.
Feldman, M. F.
Fisher, R. S.
Flack, P. R.
Gaillard, M. B.
Gannon, A. H.
Ghezzi, T. L.
Gottheim, E. F.
Gould, D. E.
Granoff, G.
Hale, J. B.
Hallstrom, R. C.
Hayne, R. M.
Heersink, A. H.
Henkes, J. P.
Hennessy, M. R.
Hu, D. D.
Hurley, J. D.
Jensen, J. P.
Johnson, L. D.
Johnson, M. A.
Johnson, W. H.
Kaur, A. F.
Kelly, M. K.
King, K. K.
Knilans, K.
Kolojay, T. M.
Koski, M. I.
Kucera, J. L.
Larose, J. G.
Leo, C. J.
Linden, O. M.
Livingston, R. P.
Lowe, S. P.
Ludwig, S. J.
Marks, R. N.
Masella, N. M.
McIntosh, K. L.
Meyer, R. E.
Meyers, G. G.
Miyao, S. K.
Morgan, S. T.
Morgan, W. S.
Mulder, E. T.
Myers, N. R.
Neuhauser, F., Jr.
Newville, B. S.
Nickerson, G. V.
Nolan, J. D.
Peacock, W. W.
Pei, K-J.
Philbrick, P. G.
Piersol, K. E.
Potter, J. A.
Powell, D. S.
Pulis, R. S.
Purple, J. M.
Ransom, G. K.
Riff, M.
Ritzenthaler, K. J.
Roach, R. F.
Roman, S. M.
Sandler, R. M.
Schneider, H. N.
Schulman, J.
Schwartz, A.
Seiffertt, B. A.
Shayer, N.
Sherman, O. L., Jr.
Singer, P. E.
Skrodenis, D. P.
Smith, F. A.
Swisher, J. W.
Taranto, J. V.
Thompson, P. R.
Thorne, J. O.
Urschel, F. A.
Vogel, J. F.
Wade, R. C.
Waldman, R. H.
Walker, G. M.
Warren, J. C.
Wasserman, D. L.
Weiner, J. S.
Weller, A. O.
Westerholm, D. C.
Whatley, M. W.
White, F. T.
White, J.
Whitman, M.
Yatskowitz, J. D.

GUESTS—SUBSCRIBERS—STUDENTS

Anderson, C. A.
Bell, A. M.
Belton, E. F.
Benktander, G.
Boyd, L. H.
Caldwell, J. L.
Carpenter, J. G.
Colvin, S. P.
Connolly, D. R.
Costner, J. E.
Coutu, G. R.
Davenport, E.
Davies, R. W.
Dornfeld, J.
Earls, R. R.
Eckley, D. A.
Elisburg, D.
Farmer, D. M.
Fewster, L. B.
Grace, W. L.
Gustafson, D. R.
Hager, G. A.
Hager, W. D.
Harms, T. E.
Hatfield, B. D.
Havens, C. W., III
Heagen, M. G.
Hopkovitz, M. D.
Jackson, R. A. G.
Jenkins, T. W.
Jensen, P. A.
Jurschak, J. F.
Kellison, S. G.
Kimball, S. L.
Knox, F.
Koupf, G.
Larsen, R.
MacKay, B.
Moak, R.
Munson, B. L.
Reade, D. M.
Rech, J. E.
Reott, J.
Reutershan, D. E.
Rushton, I. L.
Schmitt, A. J.
Sharp, C. A.
Smith, J.
Smith, M. J.
Spangler, J. L.
Steinhauser, J.
Stenmark, J. A.
Stevens, E.
Strohmenger, T. C.
Sullivan, L. P.
Whitby, O.
Young, B.

Respectfully submitted,

DAVID P. FLYNN,
Secretary

REPORT OF THE SECRETARY

The purpose of this report is to bring the membership up to date on the activities of the Board of Directors and its various committees during the past year.

In reviewing the minutes of the four Board meetings held since my last report to you, it became apparent that our principal activities were more inner-rected than external. I will briefly review some of the major items on which e Board took action.

(1) *Nomination and Election Procedures*

A Task Force was established in November, 1979 under the direction of Mr. Robert Foster to review the existing nomination and election procedures for CAS offices. These procedures were last formally reviewed in 1969. The report of the Task Force was presented to the Board at its September, 1980 meeting. Among other items, the Task Force recommended a sequence of preferential ballots followed by mail balloting conducted approximately 30 days prior to the Annual Meeting. The most recent *Actuarial Review* carried a full article on the report. The Board would appreciate hearing comments from the members before a decision is made in March 1981.

(2) *Review of Papers*

Acting on comments received from the committee, authors, and potential authors, the Board authorized the expansion of the Committee on Review of Papers from five to six members, with the chair now acting as a non-voting administrator. In the future, authors will deal directly with the chairman. This, we believe, will speed up work and eliminate some of the written correspondence which is now necessary.

(3) *Review of Anti-Trust Exposure*

In the event that the McCarren-Ferguson Act were repealed, the Board commissioned a review of potential CAS exposure to the federal anti-trust laws arising from the routine conduct of Society affairs. Mr. Michael Fusco carried out this review and concluded that our present meeting formats and committee structures are consistent with those of a learned society and, of themselves, should not subject the CAS to anti-trust action. Members interested in more information about this area should refer to Mr. Fusco or to the booklet developed by Mr. William Hager of the American Academy of Actuaries.

(4) *CAS Textbook*

Acting on a report from a review committee, the Board decided that the textbook should not be published in its present form. The present draft was sent to the Education Committee for consideration of the use of the individual chapters as separate study notes. A working group was established to report to the Board on the need, intended readership, contents, and methods of production of a future CAS Textbook.

In addition to these areas, the Board reviewed the activities of all of its committees and our regional affiliates. Action is proceeding in many areas: Career Enhancement, Public Relations, Risk Classification, Continuing Education, Professional Conduct, Reserves, Long Range Planning, and Editorial. These committees and others are working diligently at their tasks.

On the administrative side, the Board appointed Mr. Phillip N. Ben-Zvi to the Board to fill the unexpired term of Mr. C. K. Khury who resigned to serve as Editor. Mr. David Forker resigned as Editor because of business demands.

The good news on membership fees is that 1981 membership dues will not be increased. The bad news on 1981 examination fees is that:

1. Examination fees for Parts 1 through 4 will be increased $5.00 per part;
2. Examination fees for Parts 5 through 10 will be increased $10.00 per part; and
3. Administrative fees for exam transfers and refunds will be increased to $10.00 from $5.00.

During 1980, we welcomed 58 new Associates and 38 new Fellows to our ranks. Total membership now stands at approximately 890. Registrations were received for 1147 examinations for Parts 5 through 10.

Total cumulative contributions to the Joint Solicitation for Minority Student Scholarships amount to $31,310.

The Fraternal Actuarial Association announced its dissolution effective September, 1980.

Finally, let me again extend my thanks to all of you who aided in the administrative work of the Society during the past year. Many additional duties were added to the shoulders of Edith Morabito and Carole Olszewski of our New York office and to my own secretary, Pamela Sawas. Their work has again been invaluable and deeply appreciated.

Respectfully submitted,

DAVID P. FLYNN
Secretary

REPORT OF THE TREASURER

The audited financial statement for the fiscal year ended September 30, 1980 shows a decline in assets of $16,126.60 but an even greater decline in liabilities of $33,420.00, for an overall increase in Members' Equity of $17,293.40.

The net increase from interest income for the Michelbacher and Dorweiler Funds as well as donations to the newly established CAS Trust amounted to $2,489.36, while the net gain from CAS regular operations was $14,804.04. This came about principally because of a $5,000 increase in the sale of Readings over the amount budgeted, a savings of almost $4,000 from anticipated printing expenses, and an increase in interest received.

The primary reason for the reduction in liabilities was the faster payout for CAS *Proceedings*. The corresponding reduction in assets invested was offset by a switch to higher yield assets, specifically an interest-bearing checking account and a money market fund.

Next year's budget, which is based upon no increase in dues, is expected to yield another small increase in surplus, barring any major unforseen contingencies which have affected some of the other actuarial bodies, such as expanded legal and office expenses. The CAS surplus in the past year would have covered about seven months of incurred expenses, while next year it is estimated to be equivalent to about 7.4 months worth of expenses.

The CAS office expenses continue to be efficiently handled through a cooperative agreement with the National Council, which has projected only a modest increase in its expenses billable to the CAS for 1981. Offsetting that and some of the increase in printing expenses is an increase in exam fees for 1981, up $10 per CAS exam, paralleling the increase in the exams jointly sponsored with the Society of Actuaries.

One other point worth noting is the clarification that in the future all funds payable to the CAS should be remitted in U.S. currency, to avoid the inequity and bookkeeping inconvenience of dealing with exchange rates for foreign currencies. Inequities could occur, for example, by the payment of fees to the CAS in Canadian currency, while refunds are transmitted in U.S. funds.

Respectfully submitted,

MICHAEL A. WALTERS
Treasurer

FINANCIAL REPORT
Fiscal Year Ended 9/30/80

Income		Disbursements	
Dues	$ 55,437.00	Printing & stationery	$ 53,002.18
Exam fees	35,334.97	Office expenses	51,407.76
Meetings	78,070.45	Examination expenses	1,250.16
Sale of *Proceedings*	13,522.52	Meeting expenses	82,357.02
Sale of Readings	10,718.83	Library	817.50
Invitational program	3,125.00	Math. Assoc. of America	1,500.00
Interest	14,178.37	Insurance	4,223.00
Actuarial Review	254.86	Dues overpayment	100.00
Miscellaneous	−728.04	Miscellaneous	452.30
Total	$ 209,913.96	Total	$195,109.92

Income	$ 209,913.96
Disbursements	195,109.92
Change in Surplus	$+14,804.04

ACCOUNTING STATEMENT

Assets	9/30/79	9/30/80	Change
Bank accounts	$ 86,328.17	$ 28,818.51	$−57,509.66
Money market fund	—	65,064.06	+65,064.06
U.S. Treasury Bonds	4,325.00	—	−4,325.00
U.S. Treasury Notes	124,535.00	99,535.00	−25,000.00
Accrued income	11,396,00	17,040.00	+5,644.00
Total	$226,584.17	$210,457.57	$−16,126.60
Liabilities			
Office services	$ 22,872.00	$ 12,612.00	$−10,260.00
Printing expenses	49,342.00	32,542.00	−16,800.00
Examination expenses	0	300.00	+300.00
Minority Education fund	7,150.00	0	−7,150.00
Other	10.00	500.00	+490.00
Total	$ 79,374.00	$ 45,954.00	$−33,420.00
Members' Equity			
Michelbacher fund	$ 34,131.43	$ 36,266.88	$ +2,135.45
Dorweiler fund	7,681.49	7,757.60	+76.11
CAS trust	0	277.80	+277.80
Surplus	105,397.25	120,201.29	+14,804.04
Total	$147,210.17	$164,503.57	$+17,293.40

Michael A. Walters
Treasurer

This is to certify that the assets and accounts shown in the above financial statement have been audited and found to be correct.

Finance Committee
Walter J. Fitzgibbon, Jr., Chairman
Glenn W. Fresch
David M. Klein
James W. Thomas

1980 EXAMINATIONS—SUCCESSFUL CANDIDATES

Examinations for Parts 5, 7 and 9 of the Casualty Actuarial Society syllabus were held on November 1 and 2, 1979 and on November 17 and 18, 1980. Examinations for Parts 6, 8 and 10 were held on May 7 and 8, 1980.

Parts 1, 2, 3 and 4 are jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries. These exams were given in November of 1979, May of 1980 and November of 1980. Candidates who passed these exams were listed in the joint releases of the two Societies.

The Casualty Actuarial Society and the Society of Actuaries jointly awarded prizes to the undergraduates ranking highest on the General Mathematics examination. For the November, 1979 examination the $200 prize was awarded to Gregory J. Pastino. The additional $100 prize winners were Timothy P. Hesterbert, Stephen R. Hilding, Chun-Nip Lee and David C. Scheinerman. For the May, 1980 examination, the $200 prize was awarded to Miller S. Puckette. The additional $100 prize winners were Paul M. Green, David S. Laster, Denis Latulippe and Harlan Messinger. For the November, 1980 examination, the $200 prize was awarded to Robert L. Zako. The additional $100 prize winners were James R. Braue, Chiao S. Chung, Manuel V. Hidalgo, Bud Chiv Kwan and Eric L. Taillefer.

The following candidates were admitted as Associates and Fellows at the May, 1980 meeting as a result of their successful completion of the Society requirements in the November, 1979 examinations:

FELLOWS

Bass, Irene K.
Beer, Albert J.
DiBattista, Susan T.
Dolan, Michael C.
Faga, Doreen S.
Giambo, Robert A.
Lattanzio, Francis J.
O'Neil, Mary L.
Rodgers, Beatrice T.
Rowland, William J.
Van Slyke, Oakley E.
Weissner, Edward W.
Wisecarver, Timothy L.

ASSOCIATES

Berens, Regina M.
Brown, Nicholas M., Jr.
Burger, George
Campbell, Catherine J.
Clark, David G.
Dean, Curtis G.
Dodd, George T.
Easton, Richard D.
Edie, Grover M.
Engles, David
Goldfarb, Irwin H.
Gorman, Deborah A.
Hayne, Roger M.
Horowitz, Bertram A.
John, Russell T.
Johnson, Judy A.
Jones, Bruce R.
Koch, Leon W.
Larsen, Michael R.
Leo, Carl J.
Leong, Winsome
Linden, Orin M.
Lombardo, John S.
McDaniel, Gail P.
Mealy, Dennis C.
Meyers, Glenn G.
Pinto, Emanuel
Seguin, Louis G.
Surrago, James
Thompson, Kevin B.
Walker, Roger D.
Weidman, Thomas A.
Weiland, William T.
Woods, Patrick B.
Youngerman, Hank

The following is the list of successful candidates in examinations held in November, 1979:

Part 5

Addie, Barbara J.
Allin, Larry V.
Alpert, Bradley K.
Balling, Glenn R.
Baum, Edward J.
Belden, Stephen A.
Biscoglia, Terry J.
Blanchard, Ralph S., III
Boone, James P.
Braithwaite, Paul
Brown, Nicholas M., Jr.
Bujaucius, Gary S.
Bursley, Kevin H.
Callahan, James J.
Carpenter, Thomas S.
Carponter, John D.
Chuck, Allan
Colgren, Karl D.
Conlon, Aileen M.
Dembiec, Linda A.
Diss, Gordon F.
Egnasko, Valere M.
Ehrlich, Warren S.
Ellefson, Thomas J.
Faltas, Bill
Frost, Stanley R., Jr.
Fueston, Loyd L., Jr.
Gerard, Felix R.
Gillespie, Bryan C.
Goldberg, Terry L.
Guarini, Leonard T.
Hanover, Richard F.
Hofmann, Richard A.
Holmberg, Randall D.
Horowitz, Bertram A.
Josephson, Gary R.
Josephson, Philip K.
Klawitter, Warren A.
Kollmar, Richard
Kurtinaitis, Charles R.
LeClair, Peter T.
Liuzzi, Joseph R.
McDaniel, Gail P.
McIntosh, Karol A.
Miner, Neil B.
Moore, Gregory A.
Moy, Kenneth W.
Musante, Donald R.
Muza, James J.
Nelson, Cheryl L.
O'Connor, Michael P.
Pearce, Leesa I.
Port, Rhonda D.
Rapoport, Andrew J.
Robbins, Kevin B.
Rodby, Craig R.
Sanders, Robert L.
Somers, Edward C.
Soul, Harry W.
Townsend, Christopher J.
Tucker, Warren B.
Walker, Glenn M.
Watson, Lois A.
Wilson, Ronald L.
Winkelstein, Jerome
Yonkunas, John P.
Young, Bryan G.

Part 7

Behan, Donald F.
Berens, Regina M.
Boley, Russell A.
Brown, Robert L.
Burger, George
Campbell, Catherine J.
Campbell, Kenrick A.
Chernick, David R.
Ciezadlo, Gregory J.
Clark, David G.
Dean, Curtis G.
Dodd, George T.
Doellman, John L.
Easton, Richard D.
Edie, Grover M.
Edwalds, Thomas P.
Engles, David
Faga, Doreen S.
Fahrenbach, John J., Jr.
Friedberg, Bruce F.
Gannon, Alice H.
Godbold, Nathan T.
Goldfarb, Irwin H.
Gorman, Deborah A.
Hale, Jonathan B.
Harrison, David C.
Hayne, Roger M.
Henzler, Paul J.
Ingco, Aguedo M.
Jacobus, Jay A.
Jensen, Patricia A.
John, Russell T.
Johnson, Judy A.
Johnson, Warren H., Jr.
Jones, Bruce R.
Kelly, Martin K.
Koch, Leon W.
Larsen, Michael R.
Lattanzio, Francis J.
Leo, Carl J.
Leong, Winsome
Linden, Orin M.
Lombardo, John S.
Ludwig, Stephen J.
McCollum, Richard C.
Mealy, Dennis C.
Meyers, Glenn G.
Mill, Ralph A.
Miller, Allen H.
Miller, Ronald R.
Morgan, William S.
Murr, Rebecca A.
Newville, Benjamin S.
Pinto, Emanuel
Pruiksma, Glenn J.
Rapp, Jerry W.
Seguin, Louis G.
Steinhauser, John W.
Surrago, James
Thompson, Kevin B.
Tom, Darlene P.
Wainscott, Robert H.
Walker, Roger D.
Warren, Jeffrey C.
Weidman, Thomas A.
Weiland, William T.
Woods, Patrick B.
Youngerman, Hank
Yunque, Mark A.

Part 9

Bass, Irene K.
Bealer, Donald A.
Beer, Albert J.
Belvin, William H.
Biller, James E.
Connell, Eugene C.
Dawson, John
Demers, Daniel
DiBattista, Susan T.
Doepke, Mark A.
Dolan, Michael C.
Driedger, Karl H.
Dussault, Claude
Evans, Glenn A.
Furst, Patricia A.
Gaillard, Mary B.
Ghezzi, Thomas L.
Giambo, Robert A.
Gottheim, Eric F.
Hallstrom, Robert C.
Heersink, Agnes H.
Hennessy, Mary E.
Herder, John M.
Hibberd, William J.
Higgins, Barbara J.
Ingco, Aguedo M.
Isaac, David H.
Jameson, Stephen
Johnson, Larry D.
Koski, Mikhael I.
Lederman, Charles M.
Lo, Richard W.
Lotkowski, Edward P.
Mahler, Howard C.
McCarter, Michael G.
McMurray, Michael A.
Miyao, Stanley K.
Murad, John A.
Myers, Nancy R.

Part 9

O'Neil, Mary L.
Piersol, Kim E.
Purple, John M.
Racine, Andre R.
Ransom, Gary K.
Rodgers, Beatrice T.
Rowland, William J.
Schwartz, Allan I.
Sobel, Mark J.
Van Slyke, Oakley E.
Weissner, Edward W.
White, Jonathan
Wisecarver, Timothy L.
Zatorski, Richard T.
Zicarelli, John D.

The following candidates were admitted as Associates and Fellows at the November, 1980 meeting as a result of their successful completion of the Society requirements in the May, 1980 examinations:

FELLOWS

Baer, Debra L.
Belvin, William H.
Bradley, David R.
Cis, Mark M.
Covney, Michael D.
Dahlquist, Ronald A.
Demers, Daniel
Evans, Glenn A.
Furst, Patricia A.
Herder, John M.
Higgins, Barbara J.
Jameson, Stephen
Lo, Richard W.
Lotkowski, Edward P.
Lowe, Stephen P.
McCarter, Michael G.
McMurray, Michael A.
Miccolis, Robert S
Morell, Roy K.
Roth, Richard J., Jr.
Schaeffer, Bernard G.
Shrum, Roy G.
Thibault, Alain P.
Tuttle, Jerome E.
Zatorski, Richard T.

ASSOCIATES

Brown, Robert L.
Ciezadlo, Gregory J.
Cohen, Arthur I.
Connell, Eugene C.
Driedger, Karl H.
Gannon, Alice H.
Hale, Jonathan B.
Hu, David D.
Kelly, Martin K.
Knilans, Kyleen
Ludwig, Stephen J.
Miller, Ronald R.
Moody, Rebecca A.
Morgan, William S.
Pei, Kai-Jaung
Pruiksma, Glenn J.
Sherman, Ollie L., Jr.
Walker, Glenn M.
Warren, Jeffrey C.
Yatskowitz, Joel D.

The following is the list of successful candidates in examinations held in May, 1980:

Part 6

Addie, Barbara J.
Allin, Larry V.
Almer, Monte
Anson, Donald W.
Bear, Robert A.
Belden, Stephen A.
Bertrand, Francois
Biscoglia, Terry J.
Blanchard, Ralph, S., III
Boone, James P.
Braithwaite, Paul
Brown, Robert L.
Camp, Jeanne H.
Carpenter, Thomas S.
Carponter, John D.
Chernick, David R.
Chou, Li-Chaun L.
Chuck, Allan
Ciezadlo, Gregory J.
Cimini, Edward D., Jr.
Clinton, R. Kevin
Cohen, Arthur I.
Colin, Barbara
Conlon, Aileen M.
Connell, Eugene C.
Curran, Kathleen F.
Davidson, Shelley T.
Dembiec, Linda A.
Driedger, Karl H.
Dudick, Alicia J.
Dupuis, Camille
Faltas, Bill
Fleming, Kirk G.
Fortunato, Stephen J.
Friedman, Howard H.
Fueston, Loyd L., Jr.
Gannon, Alice H.
Gillespie, Bryan C.
Goldberg, Terry L.
Gorman, Linda A.
Hale, Jonathan B.
Holmberg, Randall D.
Hu, David D.
Josephson, Gary R.
Katz, Aaron J.
Keatts, Glenn H.
Knilans, Kyleen
Lacefield, David W.
Leung, Kung L.
Licht, Peter M.
Llewellyn, Barry I.
Loucks, William D., Jr.
Ludwig, Stephen J.
Martin, Paul C.
Matthews, Robert W.
McAllister, Kevin C.
McIntosh, Karol A.
Miller, Ronald R.
Moody, Andrew W.
Moody, Rebecca A.
Moore, Gregory A.
Morgan, William S.
Morris, Barbara W.
Muza, James J.
Onufer, Layne B.
Pearce, Leesa I.
Pei, Kai-Jaung
Pelletier, Bernard A.
Port, Rhonda D.
Prill, Donna A.
Pruiksma, Glenn J.
Rodby, Craig R.
Rosenberg, Deborah M.
Sakowitz, Reina M.
Sanders, Robert L.
Sarosi, Joseph F.
Scholl, David C.
Schwartzman, Joy A.
Sherman, Harvey A.
Sherman, Ollie L., Jr.
Silverman, Mark J.
Somers, Edward C.
Tucker, Warren B.
Vaillancourt, Jean
Vaughan, Robert C.
Vitale, Lawrence A.
Walker, Glenn M.
Warren, Jeffrey C.
Watkin, Mark
Wilkinson, Margaret E.
Wiseman, Michael L.
Yatskowitz, Joel D.
Yingling, Mark E.
Yonkunas, John P.

Part 8

Berens, Regina M.
Campbell, Catherine J.
Cloutier, Guy
Cohen, Howard L.
Dean, Curtis G.
Demers, Daniel
Doellman, John L.
Doepke, Mark A.
Drummond-Hay, Eric T.
Duffy, Thomas J.
Dussault, Claude
Easton, Richard D.
Egnasko, Gary J.
Engles, David
Fasking, Dennis D.
Gaillard, Mary B.
Goldfarb, Irwin H.
Gottheim, Eric F.
Heersink, Agnes H.
Hennessy, Mary E.
Herder, John M.
Hibberd, William J.
Horowitz, Bertram A.
Johnson, Marvin A.
Kleinman, Joel M.
Koch, Leon W.
Kucera, Jeffrey L.
Lafontaine, Gaetane
Lange, Dennis L.
LaRose, J. Gary
Larsen, Michael R.
Lederman, Charles M.
Linden, Orin M.
Mellia, Joanne C.
Pachyn, Karen A.
Pastor, Gerald H.
Piazza, Richard N.
Piersol, Kim E.
Pinto, Emanuel
Racine, Andre R.
Ransom, Gary K.
Sansevero, Michael, Jr.
Sweeny, Andrea M.
Wasserman, David L.
Weiland, William T.
White, Jonathan
Wilson, Ronald L.
Wilson, William F.

Part 10

Baer, Debra L.
Belvin, William H.
Boison, LeRoy A.
Bradley, David R.
Brown, Nicholas M., Jr.
Christiansen, Stephan L.
Cis, Mark M.
Covney, Michael D.
Dahlquist, Ronald A.
Evans, Glenn A.
Fisher, Russell S.
Furst, Patricia A.
Grant, Gary
Haner, Walter J.
Herder, John M.
Herman, Steven C.
Higgins, Barbara J.
Jameson, Stephen
John, Russell T.
Johnston, Thomas S.
Lo, Richard W.
Lotkowski, Edward P.
Lowe, Stephen P.
McCarter, Michael G.
McMurray, Michael A.
Miccolis, Robert S.
Morell, Roy K.
Neis, Allan R.
Newville, Benjamin S.
Niswander, Ray E., Jr.
Roth, Richard J., Jr.
Schaeffer, Bernard G.
Shrum, Roy G.
Sobel, Mark J.
Taranto, Joseph V.
Thibault, Alain P.
Truttmann, Everett J.
Tuttle, Jerome E.
Van Ark, William R.
Woods, Patrick B.
Zatorski, Richard T.

The following candidates will be admitted as Associates and Fellows at the May, 1981 meeting as a result of their successful completion of the Society requirements in the November, 1980 examinations:

FELLOWS

Brown, Nicholas M., Jr.
John, Russell T.
Ledbetter, Alan R.
Lehman, Merlin R.
Lombardo, John S.
Miller, Michael J.
Niswander, Ray E., Jr.
Rapp, Jerry W.
Rosenberg, Martin
Wickwire, James D., Jr.
Woods, Patrick B.

ASSOCIATES

Abramson, Gary R.
Bertrand, Francois
Blanchard, Ralph S., III
Boison, LeRoy A., Jr.
Boone, James P.
Camp, Jeanne H.
Carponter, John D.
Chernick, David R.
Chuck, Allan
Clinton, R. Kevin
Colin, Barbara
Davidson, Shelley T.
Douglas, Frank H.
Ehrlich, Warren S.
Gluck, Spencer M.
Holmberg, Randall D.
Josephson, Gary R.
Keatts, Glenn H.
Lange, Dennis L.
Liuzzi, Joseph R.
Lobosco, Virginia R.
McAllister, Kevin C.
Montigney, Brian A.
Mueller, Conrad P.
Munt, Donna S.
Muza, James J.
Orlowicz, Charles P.
Pachyn, Karen A.
Pearce, Leesa I.
Pelletier, Bernard A.
Ryan, John F.
Sanders, Robert L.
Silverman, Mark J.
Suchoff, Stuart B.
Tom, Darlene P.
Truttmann, Everett J.
Wilkinson, Margaret E.
Wiseman, Michael L.
Yonkunas, John P.

The following is the list of successful candidates in examinations held in November, 1980:

Part 5

Abell, Ralph L.
Allaben, Mark S.
Barclay, David L.
Barlow, Pamela J.
Bear, Robert A.
Bensimon, Abbe
Bhagavatula, Raja R.
Boccitto, Bonnie L.
Boley, Russell A.
Boulanger, Francois
Bouska, Amy S.
Bowen, David S.
Brockmeier, Donald R.
Burks, Michael L.
Cantin, Claudette
Carlton, Kenneth E.
Coffin, John D.
Colin, Barbara
Costner, James E.
Cutler, Janice Z.
DeConti, Michael A.
Deutsch, Robert V.
Dominiak, Lynn A.
Downer, Robert B.
Duffy, Brian
Edmondson, Alice H.
Epstein, Michael
Farwell, Randall A.
Fleming, Kirk G.
Forney, John R., Jr.

Part 5

Gillam, William R.
Gilles, Joseph A.
Gorman, Linda A.
Halpert, Aaron
Harwood, Catherine B.
Haskell, Gayle E.
Henzler, Paul J.
Hoppe, Kenneth J.
Howald, Ruth A.
Human, Joyce K.
Johnson, Andrew P.
Keatts, Glenn H.
Kelley, Kevin J.
Kolk, Stephen L.
Kostka, Thomas C.
Koupf, Gary I.
Krakowski, Israel
Lacefield, David W.
Lacko, Paul E.
Lee, Diana
Levine, George M.
Lonergan, Kevin F.
Lonergan, Thomas X.
Loucks, William D., Jr.
Martin, Paul C.
Mashitz, Isaac
Mayer, Jeffrey H.
Mendelssohn, Gail A.
Mittal, Madan L.
Murdza, Peter J., Jr.
O'Connell, Paul G.
Odell, W. H.
Pelly, Brian G.
Potts, Cynthia M.
Rau, Frank J., Jr.
Raws, Alfred, III
Rosenberg, Deborah M.
Sarosi, Joseph F.
Scholl, David C.
Schwartzman, Joy A.
Sherman, Harvey A.
Siewert, Jerome J.
Smith, Byron W.
Smith, Richard A.
Splitt, Daniel L.
Strange, Deborah L.
Tresco, Frank J.
Varca, John J.
Wacek, Michael G.
Wainscott, Robert H.
Wallace, Thomas A.
Webster, Patricia J.
White, Charles S.
Whiting, David R.
Wick, Peter G.
Wilkinson, Margaret E.
Windwehr, Debra R.
Woomer, Roy T., III
Yau, Michael W.
Yingling, Mark E.

Part 7

Abramson, Gary R.
Alpert, Bradley K.
Amundson, Richard B.
Baum, Edward J.
Bell, Charles T.
Bertrand, Francois
Blanchard, Ralph S., III
Boison, LeRoy A., Jr.
Boone, James P.
Brahmer, John O.
Bursley, Kevin H.
Camp, Jeanne H.
Carponter, John D.
Chuck, Allan
Clinton, R. Kevin
Davidson, Shelley T.
Dornfeld, James L.
Douglas, Frank H.
Ehrlich, Warren S.
Epstein, Michael
Esposito, David L.
Fasking, Dennis D.
Fiebrink, Dianne C.
Gapp, Steven A.
Gluck, Spencer M.
Haner, Walter J.
Hapke, Alan J.
Holmberg, Randall D.
Jaso, Robert J.
Josephson, Gary R.
Keatts, Glenn H.
Lange, Dennis L.
LeClair, Peter T.
Leiner, William W., Jr.
Liuzzi, Joseph R.
Lobosco, Virginia R.
McAllister, Kevin C.
McGuan, Jane A.
Miller, David L.
Montigney, Brian A.
Mueller, Conrad P.
Muleski, Robert T.
Munt, Donna S.
Muza, James J.
Nikstad, James R.
Odell, W. H.
Ogden, Dale F.
Orlowicz, Charles P.

Part 7

Pachyn, Karen A.
Pearce, Leesa I.
Pelletier, Bernard A.
Petrelli, Joseph L.
Robbins, Kevin B.
Rudduck, George A.
Ryan, John F.
Sanders, Robert L.
Silverman, Janet K.
Silverman, Mark J.
Soul, Harry W.
Suchoff, Stuart B.
Truttmann, Everett J.
Watford, James D.
Wilkinson, Margaret E.
Wilson, Ronald L.
Wiseman, Michael L.
Yonkunas, John P.
Young, Bryan G.

Part 9

Briere, Robert S.
Brown, Nicholas M., Jr.
Campbell, Catherine J.
Cloutier, Guy
Cohen, Howard L.
Corr, Francis X.
Cundy, Richard M.
Dean, Curtis G.
Doellman, John L.
Drummond-Hay, Eric T.
Easton, Richard D.
Engles, David
Goldfarb, Irwin H.
Grant, Gary
Hayne, Roger M.
John, Russell T.
Johnson, Judy A.
Johnson, Marvin A.
Kleinman, Joel M.
Lafontaine, Gaetane
LaMonica, Michael A.
LaRose, J. Gary
Ledbetter, Alan R.
Lee, Yoong S.
Lehman, Merlin R.
Leong, Winsome
Linden, Orin M.
Lombardo, John S.
Lommele, Jan A.
Ludwig, Stephen J.
Mathewson, Stuart B.
McGovern, William G.
Mealy, Dennis C.
Miller, Michael J.
Miller, Ronald R.
Moody, Rebecca A.
Niswander, Ray E., Jr.
Parker, Curtis M.
Pratt, Joseph J.
Rapp, Jerry W.
Robertson, John P.
Rosenberg, Martin
Schneider, Harold N.
Seguin, Louis G.
Smith, Frances A.
Walker, Roger D.
Warren, Jeffrey C.
Wasserman, David L.
Weidman, Thomas A.
Wess, Clifford
Whitman, Mark
Wickwire, James D., Jr.
Woods, Patrick B.

NEW FELLOWS ADMITTED MAY, 1980: Twelve of the thirteen new Fellows admitted at San Juan are shown with President MacGinnitie.

NEW ASSOCIATES ADMITTED MAY, 1980: Thirty-three of the thirty-eight new Associates admitted at San Juan are shown with President MacGinnitie.

NEW FELLOWS ADMITTED NOVEMBER, 1980: Twenty-three of the twenty-five new Fellows admitted at Atlanta are shown with President MacGinnitie.

NEW ASSOCIATES ADMITTED NOVEMBER, 1980: Fifteen of the twenty new Associates admitted at Atlanta are shown with President MacGinnitie.

OBITUARIES

JOHN W. AINLEY
–1980

John W. Ainley, a fellow of the Casualty Actuarial Society since 1930, died on February 6, 1980 in West Hartford, Connecticut.

A native of New Britain, Connecticut, Mr. Ainley was an honors graduate of Trinity College, Hartford.

His actuarial career was spent with the Travelers Insurance Companies, where he was an actuarial officer.

He is survived by his wife, Muriel.

JOHN L. BARTER
1896–1980

John L. (Jerry) Barter, a Fellow of the Casualty Actuarial Society since 1932, died on December 22, 1980 at age 84.

Jerry was very well liked by all who knew him and is considered the father of the casualty actuarial profession at the Hartford. A graduate of the University of California, Jerry joined the San Francisco office of the Hartford Accident and Indemnity Company in 1921 as an automobile underwriter. He later moved to the compensation and liability area of which he became superintendent in 1933.

Jerry anticipated the value of the actuarial function to the company and privately pursued his Fellowship while working as an underwriter. Jerry was called to the Home Office in 1934 to establish the Rating and Research Department. He formed the department around the late Robert Sinnott and Harry Williams, who subsequently rose to become company President and Chairman of the Board.

In 1936, Jerry was elected Assistant Secretary; three years later he was elected Secretary with responsibility for Casualty Underwriting as well as Rating and Research. In 1945 Jerry became Vice President. He retired in 1961 after 40 years of distinguished service with the Hartford.

He is survived by two sons, Christie C. Barter and John H. Barter, and a daughter, Mary Armstrong.

WILLIAM H. CRAWFORD
1902–1979

William H. Crawford, an Associate of the Casualty Actuarial Society since 1933, died on September 27, 1979 at age 76.

Bill, whose insurance career spanned 55 years, retired from the Industrial Indemnity Company in 1972, having served that company in various financial positions since 1951. Prior to joining Industrial Indemnity, Bill served for 24 years in executive financial and actuarial positions with the Loyalty Group of Insurance Companies (now Continental of New York) in New York City, Chicago and San Francisco.

Bill was also a Member of the American Academy of Actuaries and held membership in the Financial Executives Institute and the Society of Insurance Accountants. He was active as a director of the San Mateo County Heart Association.

He is survived by a son and a daughter.

JAMES S. ELSTON
1890–1980

James S. Elston, a Fellow of the Casualty Actuary Society since 1922, died April 14, 1980 in Winter Park, Florida at the age of 90.

Mr. Elston was born in Martensdale, Montana in 1890.

He joined the Travelers Insurance Company in 1913. He was named Assistant Actuary of the life department in 1919 and was named Associate Actuary in 1950 before his retirement in 1955.

He was a former Vice President and member of the Board of Governors of the American Institute of Actuaries, a former member of the council of the Actuarial Society of America, a Fellow of the Society of Actuaries and a Member of the American Academy of Actuaries. He is survived by his sister, Helen Readie; and a grandson, Paul Fortin.

CARL N. JACOBS
1895–1980

Carl N. Jacobs, an Associate of the Casualty Actuarial Society since 1929, died April 21, 1980.

After Wisconsin's enactment in 1911 of the first Workmen's Compensation Law in the United States, Mr. Jacobs became the first employee of the newly formed Wisconsin Limited Mutual Liability Insurance Company, organized in 1913 to write this new line of insurance. This company was a predecessor to Hardware Mutual Casualty Company and Sentry Insurance.

In his early career he was active in accounting, statistical and rating bureau matters. In 1925 he engaged Woodward, Fondiller and Ryan, consulting actuaries, to make the financial audit and to make a special annual certification of the adequacy of loss reserves to his Board of Directors.

During his 54 year insurance career he advanced to President and Chairman of the Board. He was active in many organizations, including the U.S. Chamber of Commerce, American Mutual Alliance, and was a Trustee of the American Institute for Property and Liability Underwriters. In his honor in 1966, the Carl N. Jacobs Lecture Series in Mathematics was established at the University of Wisconsin—Stevens Point.

ARTHUR SAWYER
1888–1980

Arthur Sawyer, an Associate of the Casualty Actuarial Society since 1923, died June 1, 1980 in California at the age of 91.

Mr. Sawyer was born October 18, 1888 in Hazelwood, Indiana.

He joined the Indianapolis Life Insurance Company in Indianapolis, Indiana in 1914. He left in 1919 to join the London Guarantee Accident Company in New York City. He served that company for nine years, joining the Royal Liverpool Group, a predecessor of Royal Globe, in 1928. During the following twenty-five years, he served successively in the Statistical, Treasurer's and Actuarial Departments and retired October 1, 1953.

He is survived by a daughter, JoAnn Hager of California.

DAVID SILVERMAN
1905–1980

David Silverman, a Fellow of the Casualty Actuarial Society since 1931, died on April 21, 1980 at the age of 75.

Mr. Silverman was born in Ontario, Canada in 1905. He was a graduate of McGill University, where he received a degree in mathematics.

Mr. Silverman joined the consulting firm of Wolfe, Corcoran and Linder in 1934 and was named a partner to the firm in 1945. He continued his consulting work with Peat, Marwick, Mitchell and Co. when it merged with the Corcoran firm in 1965.

Mr. Silverman was a Member of the Conference of Actuaries in Public Practice and the American Academy of Actuaries.

SEYMOUR E. SMITH
1913–1980

Seymour E. Smith, a Fellow and past President of the Casualty Actuarial Society, died November 10, 1980 in Hartford, Connecticut at the age of 67.

Born in New York City, Seymour Smith lived most of his life in the Hartford area. He went to work in the Supply Department of The Travelers Insurance Companies in 1934, after graduation from Trinity College. In 1937, he transferred to the Actuarial Department where he served until 1943 when he was granted a military leave of absence.

During World War II, he served in the Navy until 1946 when he returned to Travelers and became Assistant Secretary in the Underwriting Department. In 1950, he was appointed Secretary and in 1955, Vice President and Actuary. He was named Senior Vice President in 1965, a title he held until retirement.

Mr. Smith became a Fellow of the Casualty Actuarial Society in 1940, served as Vice President in 1951 and 1952 and President in 1953 and 1954. His continuing interest in education was highlighted in his presidential addresses and was further exhibited by his election as a life trustee of Trinity College where he was serving as Secretary of the Board at the time of his death.

Besides being a leader in his community, his company, and his profession, Seymour Smith was a leader in the insurance industry. He served as Chairman of the American Insurance Association, President of the National Association of Casualty and Surety Executives, and Chairman of the National Insurance Actuarial and Statistical Association.

Mr. Smith lived in Wethersfield, Connecticut and is survived by his wife Margaret Maslin Smith; two sons, Seymour Smith and Malcolm Smith; and a daughter Constance Christian.

JOHN S. THOMPSON
1884–1979

John S. Thompson died on October 27, 1979 at the age of 95. Mr. Thompson was the last surviving Charter Member of the Casualty Actuarial Society; one of the original 97 people who founded the Society in 1914.

Mr. Thompson was born in 1884, a native of Prince Edward County, Ontario, Canada. He attended the University of Toronto where he received the B.A. and M.A. degrees.

He became a Fellow of the Actuarial Society of America (which eventually became the Society of Actuaries) in 1909, and a Fellow of the British Institute of Actuaries in 1911. Mr. Thompson served a very active role in the Society of Actuaries, culminating in his term as President from 1932 to 1934.

Mr. Thompson began his insurance career with the Mutual Life Insurance Company of New York in 1905. In 1926, he joined the Mutual Benefit Life

Insurance Company of Newark where he worked until his retirement in 1953. He served as President of the company from 1946 to 1953.

Due to his extensive services in the formative years of the Medical-Surgical Plan of New Jersey (Blue Shield), he was voted an Honorary Member of the Medical Society of New Jersey.

Mr. Thompson was very active in community services in New Jersey, and received several awards in recognition of his numerous contributions.

He is survived by three children, nine grandchildren and three great-grandchildren.

FRANK G. WHITBREAD
1904–1979

Frank G. Whitbread, an Associate of the Casualty Actuarial Society since 1927, died in Fort Wayne, Indiana on December 8, 1979.

Mr. Whitbread was born in England in 1904 and graduated in 1924 from the University of Manitoba. He became an Associate Actuary with the Great-West Life Assurance Company in Winnipeg, later Vice President of the Reliance Insurance Company of Pittsburgh, and then Second Vice President of the Lincoln National Life Insurance Company of Fort Wayne, Indiana until his retirement in 1969.

Mr. Whitbread was a member of the First Presbyterian Church of Fort Wayne and was well known in insurance circles throughout the United States and Canada. He was a Fellow of the Society of Actuaries and was a Member of the American Academy of Actuaries, the Home Office Underwriters Association and the Institute of Home Office Underwriters.

He is survived by his wife, Ruth; a son, E. Peter; a daughter, Frances; a sister, Lily; and two brothers, Robert and John.

W. RULON WILLIAMSON
1889–1980

W. Rulon Williamson, a Fellow of the Casualty Actuarial Society since 1941, and a Fellow of the Society of Actuaries, died on July 26, 1980 in Windsor, Connecticut at the age of 91.

Mr. Williamson was born in Wales, New York, on July 1, 1891. Immediately after graduation in 1910 he joined the life actuarial department of the Travelers Insurance Company. In 1934, he was one of two actuaries who were loaned by the private sector to the Committee on Economic Security, established by President Franklin D. Roosevelt to develop the Social Security program. Mr. Williamson's responsibility in this connection was in the field of unemployment insurance. After this assignment, Mr. Williamson returned to the Travelers, but in 1936 he accepted an appointment by the Social Security Board to be its Actuarial Consultant (chief actuarial officer).

In 1947, Mr. Williamson left the government and became president of the Wyatt Company when its founder died at a relatively young age. After a few years, Mr. Williamson left that company and devoted all of his time to research and writing in the field of Society Security.

All persons who knew him had a very high regard for his sincerity and efforts to have the Social Security program serve what he considered an appropriate role. He was always very kind and considerate to those with whom he disagreed, and invariably people respected and loved him.

Besides his intellectual and academic pursuits, he was a great devotee of the outdoor life, and was an ardent practicing member of the Potomac Appalachian Trail Club. In addition, he was a faithful member of the Episcopalian Church. He is survived by two sons, William R. Williamson, Jr. (ASA), and Addison H. Williamson.

J. CLARKE WITTLAKE
1910–1980

J. Clarke Wittlake, an Associate of the Casualty Actuarial Society since 1939, died October 11, 1980 in Kansas City, Missouri at the age of 70.

A native of McCook, Nebraska, Mr. Wittlake received his bachelor of science degree from the University of Nebraska and a masters degree from the University of Iowa.

He served in the U.S. Army in World War II, attaining the rank of major.

Mr. Wittlake joined the actuarial department of Business Men's Assurance Company in 1936, and served as Actuarial Supervisor, Assistant Actuary, and Assistant to the President.

Mr. Wittlake served as Director and Executive Vice President of home office operations before being elected President in 1973.

After retiring in 1977, he continued with the company as a member of the Board of Directors.

In addition to his membership in the Casualty Actuarial Society, Mr. Wittlake was a Member of the American Academy of Actuaries, Beta Gamma Sigma and Pi Mu Epsilon. He was a former member of the board of directors of the Greater Kansas City Sports Commission, a former board member and Vice President of the Heart of America United Way and Chairman of the corporate division of the United Way Campaign.

Mr. Wittlake is survived by his wife, Jamie; a daughter, Linda Lewis; and a grandson.

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