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PRIVATE PASSENGER AUTOMOBILE INSURANCE RATEMAKING

A CALENDAR YEAR APPROACH

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PREFACE

The primary purpose of this paper is to describe a ratemaking procedure which begins with calendar year data and results in reasonable automobile insurance rates if the proper judgments are exercised.

The format of the paper involves a series of exhibits, supplemented by explanatory narrative. This format was chosen so that the reader can reproduce, step-by-step, all of the ratemaking calculations beginning with the underlying data and ending with the indicated rate level change.

Traditionally the underlying data for automobile insurance ratemaking has been compiled on either an accident year or a policy year basis, whereas the Annual Statement is compiled on a calendar year basis. Occasionally this difference has caused communication problems among the ratemaker, the regulator, and the public. The ratemaking procedure described here begins with Annual Statement type data. The paper explains the adjustments which must be made to the raw calendar year data in order to derive the appropriate rates. As seen in Exhibit II, the first adjustment is to convert the calendar year data into essentially accident year data. This "bridge" allows the ratemaker to reconcile his data directly to the Annual Statement.

In addition to the differences in the underlying data bases, the described ratemaking procedure differs from the traditional ratemaking approaches in several other respects. One of the principal differences is in the treatment of underwriting expenses. A separate-expense trend is calculated rather than the traditional method of "budgeting" expenses. This allows the ratemaker to reflect in the formula an expense trend which may be different from the trend which is applied to the incurred losses.

INTRODUCTION

The rate regulatory laws generally provide that rates shall not be excessive, inadequate, or unfairly discriminatory. Rates meet these criteria if they reasonably reflect the anticipated losses and expenses which will be incurred during the period for which the rates will be in effect.

Emphasis should be placed on the word "anticipated." For it is the anticipated losses and expenses, not the past losses and expenses, upon which the appropriateness of the rates is judged. In formulating this judgment, the past underwriting experience is relevant to the extent that it produces some clue as to what the loss and expense levels will be in the future. The past underwriting experience is also utilized as a starting point to which the judgments concerning the future claim costs, claim frequencies, and underwriting expenses are applied.

Over the years many arguments have been put forward as to whether the starting point (i.e. the underlying data base) for ratemaking should be policy year, calendar year, or calendar/accident year data. To some extent these arguments have been overrated because they tend to place too much emphasis upon the starting point of the ratemaking process. The important consideration is really the ending point (i.e. the anticipated losses and expenses). While it is recognized that circumstances such as availability of data may necessitate the selection of certain data bases over others, if the ratemaker utilizes the same underlying assumptions and applies the correct judgments, all three data bases will produce equally correct rate levels. In the final analysis it is the judgment which the ratemaker exercises, not the underlying data base or mechanical formulas utilized, which determines the reasonableness of the rates promulgated. While a ratemaker may favor one of the three data bases because of clarity or simplicity, clearly none is superior at predicting future rate needs.

As an illustration of the procedure, we will develop the indicated statewide rate level changes for the hypothetical Car Insurance Company. The indicated rate levels will be for the twelve month period beginning July 1, 1978. In addition to developing the indicated rate level changes for all private passenger coverages combined, we will also develop indicated rate level changes individually for the four major private passenger coverages: Bodily Injury/Property Damage Liability, Medical Payments, Comprehensive, and Collision.

In the example the B.I. Liability and P.D. Liability coverages are being treated as a single BI/PD Liability package coverage. This is done primarily because these two coverages are predominately marketed today as a single coverage. In order to avoid getting sidetracked with a discussion of the pros and cons of the package premium approach it is sufficient to say that the calendar year ratemaking procedure is equally applicable if the ratemaker chooses to treat these two coverages separately.

EXHIBIT I

The starting point for calculating the indicated rate changes is the actual calendar year underwriting results as shown in Exhibit I. For this example we have chosen the Car Insurance Company's experience for calendar year 1977 and the first three months of calendar year 1978.

It is necessary to choose an experience period which is both responsive to current conditions and stable so as to avoid large fluctuations in the rates from year to year. Most ratemakers would consider that the Car Insurance Company's volume in State X for the latest year satisfies both requirements.

The inclusion of the most recent available experience through the first three months of 1978 may introduce some seasonal distortion since the fifteen month experience base is heavily weighted with winter months. The significance of this seasonal bias will vary with the state and coverage being considered. In the example any seasonal bias has been ignored. The three months of 1978 experience is included in the example to demonstrate how this particular ratemaking approach can be applied to the most recently available partial year underwriting results.

The calendar year ratemaking approach lends itself to directly utilizing Annual Statement data as the basic source of earned premiums and incurred losses. Unfortunately, there are some problems with directly utilizing Page 14 as the basic source of the ratemaking data in our rating example. These problems are as follows:

1. Page 14 does not provide the necessary detailed breakdown by coverage that we desire to utilize in our example.
2. Page 14 is not available on the monthly basis necessary for the Car Insurance Company to update its rate change indications each month.

3. Page 14 does not provide the experience of the voluntary risks separately from that of those risks insured through the assigned risk pool or other residual market mechanism.
4. Page 14 incurred losses may not include an IBNR amount.

To overcome these problems the Car Insurance Company records its earned premiums and incurred losses monthly for each coverage, with voluntary experience separate from assigned risk experience. This monthly data is compiled in a manner compatible with the Annual Statement data.

Strictly speaking the Exhibit I data does not come directly from the Annual Statement. Rather, the data in Exhibit I, which was recorded on a monthly basis by the Car Insurance Company, is included in the totals on page 14 of the Annual Statement.

Specifically, the BI/PD Liability and Medical Payments earned premiums and incurred losses in Exhibit I are included in the totals shown on Line 19.2, Page 14. The Comprehensive and Collision earned premiums and incurred losses in Exhibit I are included in the totals shown on Line 21.1, Page 14.

The private passenger earned premiums and incurred losses for All Coverages combined in Exhibit I include more than the summation of the four major coverages. The All Coverages data includes all those coverages ordinarily reported on Lines 19.2 and 21.1 of Page 14, excluding all assigned risk earned premiums and incurred losses.

The paid allocated loss adjustment expenses are included with the incurred losses. These expenses are treated in the ratemaking formula as a loss component, rather than an underwriting expense component, because the paid allocated loss adjustment expenses are directly related to the incurred losses. The paid allocated loss adjustment expenses are compiled by the Car Insurance Company on a by coverage, by state basis each month in a manner compatible with the Annual Statement data. These allocated loss adjustment expenses are included in the totals shown in Column 1 of Part 4 of the Annual Statement Underwriting and Investment Exhibit.

Paid rather than incurred allocated loss adjustment expenses are used in the example because the Car Insurance Company does not separately identify allocated loss adjustment expense reserves. It would be appropriate to utilize the incurred allocated loss adjustment expenses if such amounts were available.

The underwriting expenses in Exhibit I include the following incurred expense items.

1) Unallocated loss adjustment.
2) Commissions and brokerage.
3) Other acquisition, field supervision and collection.
4) General.
5) Taxes, licenses and fees.

These expense items are allocated to the lines of business in accordance with New York's Regulation 30.

In the case of the Car Insurance Company, all of the commission and brokerage, other acquisition, field supervision and collection, taxes, licenses and fees are charged directly to State X. Approximately three-fourths of the unallocated loss adjustment and general expenses are charged directly to State X. That portion of the unallocated loss adjustment expenses not charged directly by the Car Insurance Company to State X is allocated based on State X's proportional share of newly reported claims. That portion of the general expenses not charged directly to State X is allocated based on State X's proportional share of policy transactions.

The experience shown for the BI/PD Liability and Medical Payments coverages is total limits. For the Comprehensive and Collision coverages, the experience of all the various deductible options is included.

EXHIBIT II

The incurred losses and incurred claim expenses for any given calendar year include the effect of reserve changes made during that calendar year on prior accident year claims. As a result it is possible that the current calendar year incurred losses do not reflect the current level of claim severity. For instance, claims from prior accident years may have been initially under-reserved and then subsequently increased to the correct amounts during the current calendar year period. Alternately, claims from prior accident years may have been initially over-reserved with the excess amount "washed out" during the current calendar year.

Without an adjustment for the effect of reserve changes on prior accident year claims, the current calendar year incurred losses will not accurately reflect the current level of claim severity. The adjustment necessary to remove this bias from the calendar year experience is summarized in Exhibit II.

As an example of this adjustment, consider an accident year 1976 claim initially reserved for $1,000 in 1976. Assume a payment was made of $500, leaving a

$500 reserve on the claim at the end of 1976. Further, assume that during 1977 the reserve was re-evaluated and increased from $500 to $2,500. In this case, the initial estimate of the total incurred loss of $1,000 was subsequently revised to reflect a total incurred loss estimate of $3,000 (cumulative payments to date of $500 plus current outstanding amount of $2,500). The incurred loss for calendar year 1976 will be $1,000. The incurred loss for calendar year 1977 will be $2,000, reflecting the change in the total incurred loss estimate.

The effect of changes in the total incurred loss estimate for prior accident years does not reflect the current calendar year loss levels. Obviously, if the claims could initially be reserved with absolute accuracy no change in the total incurred loss estimate would be necessary.

In the hypothetical example the effect of the reserve change can be eliminated from the calendar year experience by increasing the actual calendar year 1976 incurred losses by $2,000 and reducing the calendar year 1977 incurred losses by a like amount.

The data necessary to calculate the reserve change adjustments are the outstanding loss amounts and paid loss totals as of the end of each calendar month. The outstanding loss amounts and paid loss totals are recorded separately for each accident year. To maximize the responsiveness of the calendar year ratemaking approach it is desirable to have available these accident year outstanding loss and paid loss amounts at the end of each calendar month.

The calculation of the reserve changes made on accident year y during a given calendar year x is shown by the following general formula.

Let:

P_x = Total accident year y losses paid as of end of calendar year x.

P_{x-1} = Total accident year y losses paid as of beginning of calendar year x.

R_x = Accident year y outstanding losses valued as of end of calendar year x.

R_{x-1} = Accident year y outstanding losses valued as of beginning of calendar year x.

TIL_x = Accident year y total incurred loss estimate valued as of end of calendar year x.

TIL_{x-1} = Accident year y total incurred loss estimate valued as of beginning of calendar year x.

Then:

Reserve changes made on accident year y during calendar year x

$$= [P_x + R_x] - [P_{x-1} + R_{x-1}]$$

$$= TIL_x - TIL_{x-1}$$

Only the Bodily Injury Liability and Property Damage Liability incurred losses have been adjusted for the effect of reserve changes on prior accident year claims. Theoretically the necessity of such adjustment exists for each coverage. From a practical standpoint the reserve adjustment is ordinarily significant only for the slower settling liability coverages.

The calendar year underwriting expenses are also adjusted for reserve changes, reflecting the fact that the Car Insurance Company establishes reserves for unallocated claim expenses as a function of the reserves for the outstanding losses. For those insurers which establish reserves for unallocated claim expenses by some other acceptable method, the ratemaker must be prepared to make the appropriate modifications to the ratemaking formula described in this paper.

By means of the reserve adjustments described in Exhibit II, the calendar year experience from Exhibit I is converted into essentially calendar/accident year data as shown below.

Let: $P_{x/y}$ = Paid claim amount during calendar year x on accidents occurring in year y.

$R_{x/y}$ = Outstanding reserve amount (including IBNR) as of end of calendar year x on accidents occurring in year y.

I_x = Total incurred loss for calendar year x.

Then: $I_x = [P_{x/x} + P_{x/x-1} + P_{x/x-2} + \ldots] + [R_{x/x} + R_{x/x-1} + R_{x/x-2} + \ldots] - [R_{x-1/x} + R_{x-1/x-1} + R_{x-1/x-2} + \ldots]$

Let: $R'_{x-1/y}$ = Outstanding reserve amount (including IBNR) on accidents occurring in year y which should have been carried at the end of calendar year x-1 in light of subsequent developments during calendar year x.

I'_x = Total incurred loss for calendar year x after adjustment for reserve changes.

Then: $I'_x = [P_{x/x} + P_{x/x-1} + P_{x/x-2} + \ldots] + [R_{x/x} + R_{x/x-1} + R_{x/x-2} + \ldots] - [R'_{x-1/x} + R'_{x-1/x-1} + R'_{x-1/x-2} + \ldots]$

Substitute:

$R'_{x\text{-}1/x} = 0$, since the beginning reserve on the current accident year is 0.

$$R'_{x\text{-}1/x\text{-}1} - P_{x/x\text{-}1} = R_{x/x\text{-}1}$$

$$R'_{x\text{-}1/x\text{-}2} - P_{x/x\text{-}2} = R_{x/x\text{-}2}$$

Then after substitution and cancellation:

$$I'_x = P_{x/x} + R_{x/x}$$

The expression $P_{x/x} + R_{x/x}$ is also equivalent to the total incurred loss for accident year x valued at the end of calendar year x.

The use of the "accident year" or "policy year" ratemaking procedure would require that a so-called "loss development" factor be applied at this point in the ratemaking formula. The primary purpose of the loss development factor is to reflect the IBNR which is not ordinarily included in the basic ratemaking data reported to the statistical agents. Since the incurred losses used by the Car Insurance Company include an IBNR amount, the use of a loss development factor is not necessary for that purpose.

In addition to the IBNR consideration, the traditional loss development factors also measure to some extent past inaccuracies in the insurer's reserve amounts. The assumption that the current reserves of the Car Insurance Company are reasonably correct allows the ratemaker to eliminate the use of the loss development factor.

EXHIBIT III

A catastrophe loss is one which should not be assigned exclusively to the year of occurrence because of its unusually large size and infrequent nature. To include such a loss in the basic ratemaking data would produce distorted projections. To penalize insureds with a rate level increase as a result of including the catastrophe loss in the basic ratemaking data would be to ignore the fundamental precept that ratemaking is prospective by nature and not a recoupment process.

Alternately, even if no catastrophe has occurred during the experience period under review, it would be a mistake to assume that the potential for a catastrophe loss is not present. Accordingly, some provision is needed in the rate to reflect the catastrophe hazard.

To properly reflect the catastrophe hazard in the Comprehensive coverage rate, it is appropriate to eliminate the actual catastrophe losses (if any) from the experience period, and then include a catastrophe hazard factor in the loss portion of the premium. Due to the infrequency of the catastrophe loss, the catastrophe hazard factor must be calculated based on a relatively long experience period. Exhibit III shows the calculation of such a factor based on a ten year experience period.

At this point in the ratemaking formula it would be appropriate to adjust the incurred losses for any changes in subrogation or salvage patterns not already reflected in the underlying incurred loss amounts. In the example, the Car Insurance Company's incurred loss amounts are net of subrogation and salvage and there have been neither recent significant changes nor anticipated future changes in the subrogation or salvage procedures which require reflection in the ratemaking data.

Another adjustment to the underlying data base which may be necessary at this point in the formula is the exclusion of any BI/PD Liability incurred losses which arose as the result of a single large claim. These unusually large BI/PD Liability claims are in the category of catastrophe losses and may cause distorted projections if no adjustments are made to the data base. The definition of a large claim will depend upon the judgment of the ratemaker and will vary depending upon the volume of experience in the state. For instance, in State X a single BI/PD Liability incurred loss of $100,000 would have increased the statewide actual loss ratio in 1977 by less than .2%. The inclusion of such a loss in the underlying data base in State X would not cause any significant distortions in the projections. However, the inclusion of a $100,000 incurred loss arising from a single claim in a state with a small volume of experience could have a substantial impact upon the projected losses.

In the example the Car Insurance Company has incurred no single BI/PD Liability claim that is catastrophic in nature during the experience period and which requires special treatment in the ratemaking data.

EXHIBIT IV

Exhibit IV summarizes the underwriting experience after adjustment for reserve changes made on prior accident year BI/PD Liability claims and the inclusion of a Comprehensive catastrophe hazard factor.

Thus far in the ratemaking formula no adjustments have been made to the actual earned premiums. As a result, the earned premiums on Exhibit IV are identical to the earned premiums on Exhibit I.

The BI/PD Liability incurred losses and paid allocated loss adjustment expenses (IL&AE) on Exhibit IV are calculated by adding the loss reserve changes from Exhibit II to the IL&AE amounts in Exhibit I.

The Medical Payments and Collision IL&AE amounts in Exhibit IV are identical to the amounts shown in Exhibit I.

The Comprehensive IL&AE amounts in Exhibit IV are calculated by adding the catastrophe hazard amounts from Exhibit III to the IL&AE amounts shown in Exhibit I.

The All Coverages IL&AE amounts in Exhibit IV are the summation of the IL&AE amounts in Exhibit I, the loss reserve changes from Exhibit II, and the catastrophe hazard amounts from Exhibit III.

The BI/PD Liability and All Coverages underwriting expenses in Exhibit IV equal the underwriting expense amounts in Exhibit I plus the underwriting expense reserve adjustment amounts from Exhibit II.

EXHIBIT V

The calculation of an indicated rate change is a test of the rates currently in effect. It is necessary that the earned premiums utilized in the calculation of the indicated rate change fully reflect the current rate levels. The current level factors set forth in Exhibit V provide this necessary adjustment to the earned premiums.

In the example, the most recent rate change was effective July 1, 1976 Assuming the issuance of only annual policies and that policy renewal dates are spread uniformly throughout the year, one-half of the policies would have been renewed at the new rates by January 1, 1977, the beginning of the experience period. All of the policies would have been renewed at the new rates by July 1, 1977. During the first six months of 1977 an average 75% of the earned premiums would have been earned at the new rates.* During the second six months of 1977 all of the premiums would have been earned at the new rates.

The fact that an average 87.5% of the premiums earned during the entire year of 1977 were earned at the new rate level is reflected in the current level factor calculation by the use of an earned factor of .875.

* Strictly speaking, 87.5% of the earned exposure, not earned premium, is at the new rate level. This is correctly reflected in the calculations on Exhibit V. As pointed out in Mr. Simon's paper "Rate Revision Adjustment Factors," *PCAS* XLV (1958), if the insurer's growth rate is very large some further adjustment in the calculation may be necessary.

Any adjustment to the earned premiums gives rise to the need for adjustments to the underwriting expense dollars because commissions, premium taxes, and some board and bureau assessments are directly related to premiums. Reflecting a commission rate of 10% and a 2½% provision for premium tax and other assessments, a total of 12½% of the premium adjustment is added to the underwriting expenses.

EXHIBIT VI

The experience set forth in Exhibit VI reflects the earned premiums and underwriting expenses adjusted to the current rate level as calculated in Exhibit V.

The IL&AE amounts in Exhibit VI come directly from Exhibit IV.

The experience summarized in Exhibit VI has no particular significance to the ratemaker since no trend factors have as yet been applied to either the losses or the expenses. The Exhibit VI experience summary is set forth in this paper only to provide a recap of all the adjustments made thus far in the ratemaking formula and assist any reader who may attempt to reproduce all the calculations in this paper.

EXHIBIT VII

In developing the projected incurred losses, the ratemaker reviews relevant external and internal statistics in an effort to make the very best prediction possible as to the future frequency and severity of claims. The external data utilized may include general price movements in the economy, the cost of medical and hospital care, new car prices, repair part prices, and garage labor rates. The review of the internal insurance statistics involves a study of the underlying trends in claim severity and claim frequency.

Exhibit VII sets forth the average paid claim costs and the incurred claim frequencies for State X for each of the coverages. The data is calendar year data for the year ending in each calendar quarter as shown. The use of the rolling year ending data eliminates any seasonal bias which might otherwise have an impact on the trend calculation. Each of the average paid claim amounts is calculated by dividing the total amount paid during the year ending in the quarter shown by the total number of paid claims during the same period. Each of the incurred claim frequency amounts is calculated by dividing the total number of incurred claims during the year ending in the quarter shown by the average number of exposures during the same period. The use of incurred claim frequencies eliminates the possibility that any shift in the timing of claim payments could bias the calculation of the underlying frequency trends.

The desire to eliminate the effect of any shift in the timing of claim payments from the claim cost trend would dictate the use of average incurred claim costs, rather than average paid costs. On the other hand, the average incurred claim costs on a calendar year basis will be biased by reserve changes on prior accident year claims as discussed in Exhibit II. In order to circumvent the effect of reserve changes and to facilitate the calculations of the claim cost trend, it is advisable to utilize average paid claim amounts.

If the ratemaker has reason to believe that there has been a recent shift in claim settlement patterns which would bias a cost trend based upon average paid claim costs, then the bias should be measured and the trend adjusted accordingly.

There are many other biases for which the ratemaker should be on the alert and ready to make the appropriate adjustments in the trend data. For instance, a shift in the marketing of deductible physical damage coverages from low deductibles to high deductibles would theoretically decrease the claim frequencies and most likely increase the claim severities. A shift in marketing away from geographical areas or classes of business with high claim frequencies and/or claim severities will also require some adjustment in the trend data. Similarly, catastrophe type losses, such as the 1973 Comprehensive catastrophe loss of $1,000,198 from Exhibit III, should be eliminated from any trend data.

One of the advantages of analyzing the claim frequency and claim severity trends separately, rather than as a combined pure premium trend, is that any distortions in the trend data, such as those listed above, can be more readily recognized. If the ratemaker desires, a good estimate of the underlying pure premium trend can be derived by combining the average paid claim amounts and the incurred frequency amounts, after adjustment for any known biases. Pure premiums so determined will have significance only for the calculation of the trends.

In the example there have been no Comprehensive catastrophe losses (see Exhibit III) during the period covered by the trend. Additionally, the Car Insurance Company has made no changes in its claim settlement procedures or marketing emphasis which would distort the trend. As a result we are able to use the data in Exhibit VII to determine reasonable estimates of the trend without making any adjustments in the underlying claim severity or claim frequency data.

One of the distinct advantages of making rates based on the Car Insurance Company's data, rather than on a combination of data reported by several insurers, is in the determination of the trends. When working with a single insurer's data it is possible to identify and measure changes in claim settlement practices and marketing emphasis. The ratemaker is usually aware of the nature and timing of such changes.

When determining trends based on the combination of data from several companies it is especially difficult to recognize and measure any distortions in the trend data. This is true because each of the reporting insurers tends to manage its portfolio a little differently and generally does not report the nature and timing of its marketing adjustments.

EXHIBIT VIII

There will never exist a single mathematical formula which will produce the correct rates every year. Accordingly, the question as to whether the underlying claim severity and claim frequency trends are best approximated by fitting data to a straight line, exponential curve, or sine curve is a matter that is left to the ratemaker's best expert judgment after weighing all the evidence. Similarly, the length of the experience period over which the trends are calculated is a matter of expert judgment. Such judgment may vary from year to year.

In Exhibit VIII the ratemaker has calculated the trends by fitting a straight line to the data from Exhibit VII utilizing the least squares method. After analyzing the trends from various length experience periods and considering any relevant external trend data, the ratemaker has selected trends, based on his judgment, which will be used to derive the projected incurred losses.

A thorough understanding of the judgment exercised by the ratemaker in selecting the trends in Exhibit VIII would be instructive, but not pertinent to an understanding of the calendar year ratemaking approach, which is the primary purpose of this paper. In order to further the primary purpose of this paper we will assume that the selected trends are reasonable and proceed with a discussion of their application in the formula.

EXHIBIT IX

The selected claim cost and claim frequency trends in Column 2 from Exhibit VIII are applied to the latest available average claim cost and claim frequency data in Column 1 from Exhibit VII in order to derive the projected claim costs and claim frequencies in Column 3.

The average loss level for the calendar year 1977 is represented by the pure premium in Column 4. The average loss level for the first quarter of 1978 is represented by the pure premium in Column 5. It should be noted that the pure premium in Column 5 is for the first quarter of 1978 only, and not for the year ending in the first quarter of 1978. The year ending point would not be representative of the loss levels underlying the incurred losses for the first quarter of 1978.

By comparing the projected pure premiums (Column 3) to the actual pure premiums (Column 4 and Column 5) the loss projection factors in Columns 6 and 7 can be determined for each of the individual coverages. The loss projection factors for the individual coverages are averaged using the distribution of paid losses to derive the All Coverages loss projection factors. The loss projection factors are applied to the 1977 and first quarter 1978 incurred losses and paid allocated loss adjustment expenses from Exhibit VI in order to bring those amounts up to the projected loss levels for the year ending December 31, 1979.

The choice of the correct loss projection date is based on the assumption of an annual policy and regular annual rate revisions. As stated earlier, we are developing rates for the twelve month period following the planned rate change effective date of July 1, 1978. The average loss level for the twelve month calendar year period beginning July 1, 1978 is the midpoint of that period, which is January 1, 1979. If one were to project loss levels to the midpoint of the period, the Car Insurance Company would not achieve the profit level anticipated in the rates because of the lag in earning the rate change. Assuming policy renewals are uniform throughout the year, only one-half of the rate change would be actually earned by the Car Insurance Company during the twelve month period following July 1, 1978. In order to offset this lag in earning rate changes and to realize the anticipated profit, it is necessary to project the loss levels twelve months beyond the effective date.

Perhaps the clearest way to visualize this concept is to consider the problem on a policy year basis. The policies issued during the twelve month period following July 1, 1978, for which the new rates will be effective, will provide coverage for claims during the twenty-four calendar months beginning July 1, 1978. Assuming that policies are written uniformly throughout the policy year and that claims are incurred uniformly over the policy term, it follows that the average loss level for the policy year beginning July 1, 1978 is represented by July 1, 1979, or twelve months beyond the planned effective date.

If rates are calculated by trending twelve months beyond the effective date, then the rates will produce the anticipated profit for each policy year. If the rates produce the anticipated profit for each policy year, it follows that each calendar year will also produce the anticipated profit.

We have projected pure premiums in Exhibit IX Column 3 for the year ending December 31, 1979. These pure premiums represent the average loss level for the calendar year 1979. This is equivalent to projecting loss levels to the average loss level of the policy year beginning July 1, 1978.

EXHIBIT X

The basic data for determining the underlying trend in the average underwriting expenses per policy is the type of data reported on Part 4 of the Annual Statement Underwriting and Investment Exhibit.

The method of calculating the expense trend in Exhibit X recognizes that the commissions, premium taxes, and some board and bureau assessments vary directly with the premium dollar and that the claim, other acquisition, general and other tax expenses are not directly related to premium.

By fitting the data to a straight line, the average annual dollar change per policy is calculated for those expenses not directly related to premiums. In the example the average annual dollar change in the expenses not related to premiums is \$1.106. In relation to the total average expenses per policy, the average annual change of \$1.106 represents an annual trend of +2.1%.

Applying the expense trend to the underwriting expenses from Exhibit VI produces the projected incurred underwriting expenses assuming no change in the current rate level. These projected underwriting expenses reflect only the trend in the expenses not related to premiums. Anticipated changes in the premium related expenses are taken into account in Exhibit XII.

EXHIBIT XI

The experience from Exhibit VI, with incurred losses and paid allocated loss adjustment expenses and underwriting expenses at their projected July 1, 1979 levels, is set forth in Exhibit XI.

The significant items in Exhibit XI are the ratios of IL&AE, Underwriting Expense, and Underwriting Gain or Loss to Earned Premiums. The dollar amounts have no particular significance because there has been no attempt to annualize the amount or estimate policy growth. The ratios reflect the anticipated loss ratio, expense ratio, and resulting per cent of underwriting gain or loss assuming no change in the current rate level.

For instance, based on the first quarter of 1978 experience only, we have predicted an All Coverages underwriting loss of 18.7% for the policy year beginning July 1, 1978 if there is no change in the current rates. Based on the longer 15 month experience period of 1977 and first quarter 1978, we have predicted an All Coverages underwriting loss of 15.2%.

EXHIBIT XII

Before calculating the indicated rate level changes, the proper allowance in the rates for underwriting gain is determined based upon the Car Insurance Company's total financial need. The total financial need of the Car Insurance Company is determined by considering such factors as the expected rate of inflation and the expected real growth (measured by the increase in the number of policies or cars insured). Assuming, for instance, an expected rate of inflation of 6% and real growth of 6%, it will be necessary to increase the Car Insurance Company's surplus by 12% in order to maintain its current financial strength.

Having established quantitatively the total financial need of the Car Insurance Company for the forthcoming year, an amount equivalent to the expected investment income and, if applicable, expected proceeds from the sale of capital stock is deducted. The remainder is the amount of money that must be generated from the Car Insurance Company's underwriting operations.

The determination of the appropriate provision in the rates for underwriting profit has received considerable attention from many authors over the years. While the determination of the Car Insurance Company's total financial need and the resulting indicated provision in the rates for underwriting profit are important subjects, they are not particularly pertinent to an understanding of the mechanics of the calendar year ratemaking approach. Having mentioned the important considerations in determining the total financial need and observing that this figure will vary over time, we will proceed with the assumption that an allowance of 5.0%, before federal taxes, is appropriate for underwriting gain and contingencies in the rates for the Car Insurance Company.

The Loss Ratio Test is defined as the division of the projected loss ratios by the permissible loss ratio.

The permissible loss ratio is dependent upon the projected expense ratio and the desired underwriting profit provision. The projected expense ratio cannot be determined until the indicated rate level is determined because a portion of the underwriting expenses varies directly with premium. The projected expense ratios in

Exhibit XI are correct only if there is no change in the current rates. As a result, the Loss Ratio Test cannot be applied directly to the experience in Exhibit XI.

The way around this problem is to calculate the indicated rate change as a solution to the following algebraic equation. This equation recognizes that 12.5% of the premium change flows into expenses to pay for the commissions (10%) and premium taxes and some board and bureau assessments ($2^1/_2$%), and the remaining 87.5% of the rate change flows into underwriting profit.

Let:	I.C.	= Indicated rate level change.
	g	= Projected underwriting gain or loss as a percent of premium, assuming no rate change.
	G	= Desired underwriting gain ratio = 5.0%.
	C.L.E.P.	= Current level earned premium.
	.125	= Expenses directly variable with premium.
	I.L. & A.E.	= Projected incurred losses and paid allocated loss adjustment expenses.
	U.E.	= Projected underwriting expenses.

Then:

$$\begin{aligned}(\text{C.L.E.P.})(1.000 + \text{I.C.}) = {} & (\text{I.L. \& A.E.} + \text{U.E.}) \\ & + (.125)(\text{C.L.E.P.})(\text{I.C.}) \\ & + (G)(\text{C.L.E.P.})(1.000 + \text{I.C.})\end{aligned}$$

or,

$$1.000 + \text{I.C.} = (1 - g) + (.125)(\text{I.C.}) + (G)(1.000 + \text{I.C.})$$

or,

$$\text{I.C.} = \frac{G - g}{1.000 - .125 - G} = \frac{G - g}{.875 - G}$$

Having determined the indicated overall rate level change, it is possible to calculate a projected expense ratio. The projected expense ratio is the expense ratio which will result if the indicated rate change is implemented. This projected expense ratio can be used in the application of the Loss Ratio Test. The projected expense ratios for the individual coverages are determined based on the historical relationship of the expense ratio for each coverage to the expense ratio for All Coverages combined.

In practice the Car Insurance Company allocates the underwriting expenses of State X to the individual coverages in State X based on the monthly distribution of written premiums by coverage. The one exception is the unallocated claim expense reserves which are allocated to the individual coverages based on the distribution of indemnity reserves.

If the above algebraic formula were applied to each of the individual coverages, the resulting projected expense ratio for each coverage would not be consistent with the expense ratio actually produced by the company's expense allocation formula. As a result, the algebraic formula described above can be applied only to the Car Insurance Company's experience for All Coverages combined. For the individual coverages, the correct rate change indication can be derived utilizing a Loss Ratio Test based upon the projected expense ratio for each coverage from Exhibit XII.

If different accounting procedures were utilized by the Car Insurance Company, then an alternate approach in deriving the indicated rate changes would be dictated. For instance, if the company assumed that all underwriting expenses vary directly with the premium dollar, then our algebraic equation could be used for each individual coverage by substituting the projected expense ratio for the 12.5% factor utilized in our formula.

EXHIBIT XIII

The Loss Ratio Test, applied to the projected loss ratios in Exhibit XI and the projected expense ratios in Exhibit XII, results in the indicated rate level changes for each of the individual coverages and for All Coverages combined. One should note that the indications for All Coverages combined are identical to the All Coverages indication utilizing the algebraic formula in Section I of Exhibit XII (the slight variation in the indication for 1977 is due to rounding).

CONCLUSION

In an effort to restrict this paper to only those matters directly related to the mechanics of a calendar year ratemaking formula, the author has admittedly given only brief reference to some important areas.

The sources of the underlying data and the format and frequency of the necessary internal statistical reports are worthy of greater discussion. The use of total limits BI/PD Liability experience, the rating of BI/PD Liability as a package premium, the precise calculations used in adjusting the loss trends for any distortions due to catastrophe losses or shifts in claim settlement practices, the judgment process used in determining the finally selected trend factors and the specific calculations used in determining the underwriting profit provision are also areas which need to be treated more thoroughly. However, these areas deal with questions that are general and not limited to any particular ratemaking formula. As such, these areas would best be treated as supplements to this paper.

Perhaps the next chapter should carry the ratemaking formula to its next logical step and describe the derivation of the final rates and the preparation of the rate filing. Such a chapter would discuss the allocation of the indicated statewide rate level changes to the various territories, classes of business, limits of coverage, and deductibles.

EXHIBIT I

CAR INSURANCE COMPANY

STATE X

ACTUAL PRIVATE PASSENGER UNDERWRITING EXPERIENCE
(Assigned Risk Experience Excluded)

Calendar Year	Item	BI/PD Liability	% E.P.	Medical Payments	% E.P.	Comprehensive	% E.P.	Collision	% E.P.	All Coverages	% E.P.
1977	E.P.	$ 52,955,922		$ 6,933,324		$ 7,382,934		$ 24,315,485		$ 95,245,692	
	IL & AE	34,103,614	64.4%	4,236,261	61.1%	7,478,912	101.3%	22,856,556	94.0%	72,386,726	76.0%
	U.E.	13,397,848	25.3	1,712,531	24.7	2,141,051	29.0	7,027,175	28.9	26,097,320	27.4
	G or L	5,454,460	10.3	984,532	14.2	-2,237,029	-30.3	-5,568,246	-22.9	-3,238,354	-3.4
1978/3 Mos.	E.P.	$ 15,348,871		$ 1,972,851		$ 2,235,456		$ 7,425,375		$ 27,161,781	
	IL & AE	13,890,728	90.5	1,179,765	59.8	2,273,459	101.7	6,660,561	89.7	22,870,220	84.2
	U.E.	4,174,893	27.2	508,996	25.8	643,811	28.8	2,145,933	28.9	7,686,784	28.3
	G or L	-2,716,750	-17.7	284,090	14.4	-681,814	-30.5	-1,381,119	-18.6	-3,395,223	-12.5
TOTAL	E.P.	$ 68,304,793		$ 8,906,175		$ 9,618,390		$ 31,740,860		$122,407,473	
	IL & AE	47,994,342	70.3	5,416,026	60.8	9,752,371	101.4	29,517,117	93.0	95,256,946	77.8
	U.E.	17,572,741	25.7	2,221,527	24.9	2,784,862	29.0	9,173,108	28.9	33,784,104	27.6
	G or L	2,737,710	4.0	1,268,622	14.2	-2,918,843	-30.3	-6,949,365	-21.9	-6,633,577	-5.4

E.P. = Earned Premiums
IL & AE = Incurred Losses and Paid Allocated Loss Adjustment Expenses
U. E. = Underwriting Expenses
G or L = Underwriting Gain or Loss

EXHIBIT II

CAR INSURANCE COMPANY

STATE X

ADJUSTMENTS TO REFLECT RESERVE CHANGES ON PRIOR ACCIDENT YEAR BODILY INJURY AND PROPERTY DAMAGE LIABILITY CLAIMS

I. Adjustments on Prior Accident Year Bodily Injury Liability Claims

A. Adjustments to Calendar Year 1977 Actual Incurred Losses

Loss Reserve Changes during Calendar Year 1977 on Prior Accident Years:

Accident Years 1974 and Prior	$+ 750,371
Accident Year 1975	$ − 3,300,764
Accident Year 1976	$ + 2,800,452
TOTAL	$+ 250,059

Loss Reserve Changes During 1st Quarter of 1978 on Accident Year 1977	$ + 1,111,000
Net Adjustment to Calendar Year 1977 Incurred Losses ($1,111,000 - $250,059)	$+ 860,941

B. Adjustments to 1st Quarter of Calendar Year 1978 Actual Incurred Losses

Loss Reserve Changes during 1st Quarter of Calendar Year 1978 on Prior Accident Years:

Accident Years 1974 and Prior	$ − 37,257
Accident Year 1975	$ − 211
Accident Year 1976	$ + 373,000
Accident Year 1977	$ + 1,111,000
TOTAL	$ + 1,446,532

Net Adjustment to 1st Quarter of Calendar Year 1978 Incurred Losses	$ − 1,446,532

EXHIBIT II
Continued

ADJUSTMENTS TO REFLECT RESERVE CHANGES ON PRIOR ACCIDENT YEAR BODILY INJURY AND PROPERTY DAMAGE LIABILITY CLAIMS

II. Adjustments on Prior Accident Year Property Damage Liability Claims

A. Adjustments to Calendar Year 1977 Actual Incurred Losses

Loss Reserve Changes during Calendar Year 1977 on Prior Accident Years:

Accident Years 1974 and Prior	$+ 89,452
Accident Year 1975	$ 0
Accident Year 1976	$+ 213,619
TOTAL	$+ 303,071

Loss Reserve Changes During 1st Quarter of 1978 on Accident Year 1977	$+ 287,647
Net Adjustment to Calendar Year 1977 Incurred Losses ($287,647-$303,071)	$− 15,424

B. Adjustments to 1st Quarter of Calendar Year 1978 Actual Incurred Losses

Loss Reserve Changes during 1st Quarter of Calendar Year 1978 on Prior Accident Years:

Accident Years 1974 and Prior	$+ 14,619
Accident Year 1975	$+ 27,342
Accident Year 1976	$ 0
Accident Year 1977	$+ 287,647
TOTAL	$+ 329,608

Net Adjustment to 1st Quarter of Calendar Year 1978 Incurred Losses	$− 329,608

EXHIBIT II
Continued

ADJUSTMENTS TO REFLECT RESERVE CHANGES ON PRIOR ACCIDENT YEAR BODILY INJURY AND PROPERTY DAMAGE LIABILITY CLAIMS

III. Total Reserve Adjustments to Actual Incurred Losses and Underwriting Expenses

A. Adjustments to Incurred Losses

Calendar Year 1977:

Net B.I. Liability Loss Reserve Change	$+ 860,941
Net P.D. Liability Loss Reserve Change	$− 15,424
TOTAL	$+ 845,517

Calendar Year 1978–1st Quarter:

Net B.I. Liability Loss Reserve Change	$− 1,446,532
Net P.D. Liability Loss Reserve Change	$− 329,608
TOTAL	$− 1,776,140

B. Adjustments to Underwriting Expenses*

Calendar Year 1977	$+ 213,693
Calendar Year 1978–1st Quarter	$− 394,594

* Adjustment to underwriting expenses equals 25% of the B.I. Liability loss reserve change plus 10% of the P.D. Liability loss reserve change.

EXHIBIT III

CAR INSURANCE COMPANY

STATE X

ADJUSTMENTS TO THE COMPREHENSIVE INCURRED LOSSES TO REFLECT THE INCLUSION OF THE CATASTROPHE HAZARD FACTOR

Calendar Year	(1) Comprehensive Incurred Losses and Paid Alloc. Loss Adj. Expenses	(2) Comprehensive Catastrophe Losses	(3) Non-Catastrophe Comprehensive Losses	(4) Column (2) ÷ Column (3)
1968	$3,617,714		$3,617,714	.000
1969	4,630,461		4,630,461	.000
1970	5,796,797		5,796,797	.000
1971	6,363,917		6,363,917	.000
1972	6,557,846		6,557,846	.000
1973	6,361,982	$1,000,198	5,361,784	.187
1974	5,006,674		5,006,674	.000
1975	6,866,817		6,866,817	.000
1976	8,513,478		8,513,478	.000
1977	7,478,912		7,478,912	.000
TOTAL				.187

Catastrophe Hazard Factor = Column (4) total ÷ 10 years = .019

Adjustment to 1977 Comprehensive and All Coverages Incurred Losses to Reflect Catastrophe Hazard

$7,478,912 (Actual 1977 Comprehensive IL & AE) x .019 = $ + 142,099

Adjustment to 1978–1st Quarter Comprehensive and All Coverages Incurred Losses to Reflect Catastrophe Hazard

$2,273,459 (Actual 1978–1st Quarter Comprehensive IL & AE) x .019 = $ + 43,196

EXHIBIT IV

CAR INSURANCE COMPANY

STATE X

PRIVATE PASSENGER UNDERWRITING EXPERIENCE
ADJUSTED FOR RESERVE CHANGES AND THE
COMPREHENSIVE CATASTROPHE HAZARD

Calendar Year	Item	BI/PD Liability	% E.P.	Medical Payments	% E.P.	Comprehensive	% E.P.	Collision	% E.P.	All Coverages	% E.P.
1977	E.P.	$ 52,955,922		$ 6,933,324		$ 7,382,934		$24,315,485		$ 95,245,692	
	IL & AE	34,949,131	66.0%	4,236,261	61.1%	7,621,011	103.2%	22,856,556	94.0%	73,374,342	77.0%
	U.E.	13,611,541	25.7	1,712,531	24.7	2,141,051	29.0	7,027,175	28.9	26,311,013	27.6
	G or L	4,395,250	8.3	984,532	14.2	−2,379,128	−32.2	−5,568,246	−22.9	−4,439,663	−4.7
1978/3 Mos.	E.P.	$ 15,348,871		$ 1,972,851		$ 2,235,456		$ 7,425,375		$ 27,161,781	
	IL & AE	12,114,588	78.9	1,179,765	59.8	2,316,655	103.6	6,660,561	89.7	21,137,276	77.8
	U.E.	3,780,299	24.6	508,996	25.8	643,811	28.8	2,145,933	28.9	7,292,190	26.8
	G or L	−546,016	−3.6	284,090	14.4	−725,010	−32.4	−1,381,119	−18.6	−1,267,685	−4.7
TOTAL	E.P.	$ 68,304,793		$ 8,906,175		$ 9,618,390		$31,740,860		$122,407,473	
	IL & AE	47,063,719	68.9	5,416,026	60.8	9,937,666	103.3	29,517,117	93.0	94,511,618	77.2
	U.E.	17,391,840	25.5	2,221,527	24.9	2,784,862	29.0	9,173,108	28.9	33,603,203	27.5
	G or L	3,849,234	5.6	1,268,622	14.2	−3,104,138	−32.3	−6,949,365	−21.9	−5,707,348	−4.7

E.P. = Earned Premiums
IL & AE = Incurred Losses and Paid Allocated Loss Adjustment Expenses
U. E. = Underwriting Expenses
G or L = Underwriting Gain or Loss

EXHIBIT V

CAR INSURANCE COMPANY

STATE X

ADJUSTMENTS TO REFLECT
EFFECTS OF PAST RATE CHANGES

I. Rate Change effective July 1, 1976:

BI/PD Liability	+ 4.8%
Medical Payments	0.0
Comprehensive	+ 11.0
Collision	+ 15.5
All Coverages	+ 7.2%

II. Current Level Factor $= \dfrac{1 + \text{Rate Change}}{1 + \text{Earned Factor x Rate Change}}$

1977 Current Level Factors:

BI/PD Liability: $\dfrac{1 + .048}{1 + (.875)(.048)} = \dfrac{1.048}{1.042} = 1.006$

Medical Payments: $= 1.000$

Comprehensive: $\dfrac{1 + .110}{1 + (.875)(.110)} = \dfrac{1.110}{1.096} = 1.013$

Collision: $\dfrac{1 + .155}{1 + (.875)(.155)} = \dfrac{1.155}{1.136} = 1.017$

All Coverages: $\dfrac{1 + .072}{1 + (.875)(.072)} = \dfrac{1.072}{1.063} = 1.008$

1978 Current Level Factors are 1.000 for each coverage.

ADJUSTMENTS TO REFLECT
EFFECTS OF PAST RATE CHANGES

EXHIBIT V
Continued

III. Calculation of Current Level Earned Premiums

Coverage	Year	(1) Actual Earned Premiums	(2) Current Level Factor	(3) Current Level Earned Premiums (1) x (2)
BI/PD Liability	1977	$52,955,922	1.006	$53,273,658
	1978/3 mos.	$15,348,871	1.000	$15,348,871
Medical Payments	1977	$ 6,933,324	1.000	$ 6,933,324
	1978/3 mos.	$ 1,972,851	1.000	$ 1,972,851
Comprehensive	1977	$ 7,382,934	1.013	$ 7,478,912
	1978/3 mos.	$ 2,235,456	1.000	$ 2,235,456
Collision	1977	$24,315,485	1.017	$24,728,848
	1978/3 mos.	$ 7,425,375	1.000	$ 7,425,375
All Coverages	1977	$95,245,692	1.008	$96,007,658
	1978/3 mos.	$27,161,781	1.000	$27,161,781

IV. Calculation of Adjusted Underwriting Expenses

Coverage	Year	(1) Underwriting Expenses*	(2) Amount of Premium Change**	(3) Expense Factor Directly Variable with Premium	(4) Adjusted Underwriting Expenses Col(1) + [Col(2)xCol(3)]
BI/PD Liability	1977	$13,611,541	$317,736	.125	$13,651,258
	1978/3 mos.	$ 3,780,299	$ 0	.125	$ 3,780,299
Medical Payments	1977	$ 1,712,531	$ 0	.125	$ 1,712,531
	1978/3 mos.	$ 508,996	$ 0	.125	$ 508,996
Comprehensive	1977	$ 2,141,051	$ 95,978	.125	$ 2,153,048
	1978/3 mos.	$ 643,811	$ 0	.125	$ 643,811
Collision	1977	$ 7,027,175	$413,363	.125	$ 7,078,845
	1978/3 mos.	$ 2,145,933	$ 0	.125	$ 2,145,933
All Coverages	1977	$26,311,013	$761,966	.125	$26,406,259
	1978/3 mos.	$ 7,292,190	$ 0	.125	$ 7,292,190

*Expenses from Exhibit IV.
**Column (3) minus Column (1) from Section III.

EXHIBIT VI

CAR INSURANCE COMPANY

STATE X

CURRENT LEVEL PRIVATE PASSENGER UNDERWRITING EXPERIENCE

Calendar Year	Item	BI/PD Liability	% E.P.	Medical Payments	% E.P.	Comprehensive	% E.P.	Collision	% E.P.	All Coverages	% E.P.
1977	E.P.	$ 53,273,658		$ 6,933,324		$ 7,478,912		$24,728,848		$ 96,007,658	
	IL & AE	34,949,131	65.6%	4,236,261	61.1%	7,621,011	101.9%	22,856,556	92.4%	73,374,342	76.4%
	U.E.	13,651,258	25.6	1,712,531	24.7	2,153,048	28.8	7,078,845	28.6	26,406,259	27.5
	G or L	4,673,269	8.8	984,532	14.2	−2,295,147	−30.7	−5,206,553	−21.1	−3,772,943	−3.9
1978/3 Mos.	E.P.	$ 15,348,871		$ 1,972,851		$ 2,235,456		$ 7,425,375		$ 27,161,781	
	IL & AE	12,114,588	78.9	1,179,765	59.8	2,316,655	103.6	6,660,561	89.7	21,137,276	77.8
	U.E.	3,780,299	24.6	508,996	25.8	643,811	28.8	2,145,933	28.9	7,292,190	26.8
	G or L	−546,016	−3.6	284,090	14.4	−725,010	−32.4	−1,381,119	−18.6	−1,267,685	−4.7
TOTAL	E.P.	$ 68,622,529		$ 8,906,175		$ 9,714,368		$32,154,223		$123,169,439	
	IL & AE	47,063,719	68.6	5,416,026	60.8	9,937,666	102.3	29,517,117	91.8	94,511,618	76.7
	U.E.	17,431,557	25.4	2,221,527	24.9	2,796,859	28.8	9,224,778	28.7	33,698,449	27.4
	G or L	4,127,253	6.0	1,268,622	14.2	−3,020,157	−31.1	−6,587,672	−20.5	−5,040,628	−4.1

E.P. = Earned Premiums
IL & AE = Incurred Losses and Paid Allocated Loss Adjustment Expenses
U. E. = Underwriting Expenses
G or L = Underwriting Gain or Loss

EXHIBIT VII

CAR INSURANCE COMPANY

STATE X

CLAIM COST AND CLAIM FREQUENCY

Year Ended	Average Paid Claim Costs					Incurred Claim Frequency				
	Bodily Injury	Property Damage	Medical Payments	Comprehensive	Collision	Bodily Injury	Property Damage	Medical Payments	Comprehensive	Collision
6/30/75	$2,355.76	$289.79	$410.98	$ 94.01	$386.44	.00949	.05603	.01139	.07791	.06054
9/30/75	2,355.59	298.89	429.30	96.51	396.97	.00916	.05529	.01137	.07732	.06035
12/31/75	2,439.40	303.68	433.76	95.17	404.48	.00904	.05522	.01093	.07781	.06056
3/31/76	2,572.09	308.30	476.72	97.90	398.83	.00919	.05538	.01077	.07715	.06058
6/30/76	2,684.17	312.77	485.82	99.38	402.82	.00899	.05398	.01020	.07534	.05965
9/30/76	2,742.65	317.40	491.62	101.08	407.87	.00929	.05478	.00986	.07456	.06022
12/31/76	2,894.73	324.65	503.33	106.73	425.28	.00932	.05387	.00970	.07457	.05994
3/31/77	2,923.55	331.41	482.00	108.22	448.17	.00930	.05315	.00983	.07517	.06088
6/30/77	2,953.02	339.40	491.17	112.41	456.42	.00916	.05352	.01001	.07741	.06194
9/30/77	2,986.18	342.22	499.22	117.16	464.11	.00909	.05295	.01040	.07963	.06202
12/31/77	3,008.88	347.57	508.55	123.01	464.39	.00910	.05332	.01085	.08234	.06266
3/31/78	2,909.29	355.82	511.59	126.23	477.31	.00893	.05313	.01072	.08471	.06272

EXHIBIT VIII

CAR INSURANCE COMPANY

STATE X

TREND FACTORS

	Claim Cost				Claim Frequency			
	Average Change in Best Fit Line			Change in	Average Change in Best Fit Line			Change in
Coverage	12-Point	8-Point	6-Point	Latest Year*	12-Point	8-Point	6-Point	Latest Year*
Bodily Injury	+8.4%	+ 5.1%	+ 1.4%	− 0.5%	− 0.9%	− 1.2%	− 3.3%	− 4.0%
Property Damage	+6.5	+ 6.9	+ 6.7	+ 7.4	− 2.1	− 1.4	− 0.8	0.0
Medical Payments	+6.3	+ 2.5	+ 2.9	+ 6.1	− 2.9	+ 4.9	+ 9.0	+ 9.1
Comprehensive	+9.7	+12.6	+13.3	+16.6	+ 2.3	+ 7.0	+10.1	+12.7
Collision	+7.3	+ 9.2	+ 7.6	+ 6.5	+ 1.5	+ 3.1	+ 3.5	+ 3.0

*Year Ended 3/31/78 ÷ Year Ended 3/31/77.

	Selected Trends	
Coverage	Claim Cost	Claim Frequency
Bodily Injury	+ 5.0 %	− 2.0%
Property Damage	+ 6.5	− 2.0
Medical Payments	+ 6.0	0.0
Comprehensive	+10.0	+ 5.0
Collision	+ 7.5	+ 3.0

EXHIBIT IX

CAR INSURANCE COMPANY
STATE X
CALCULATION OF PROJECTED INCURRED LOSSES AND ALLOCATED LOSS ADJUSTMENT EXPENSES

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
			Selected	Projected Year			Loss Projection Factors		Paid Loss Distribution	
		Year End	Annual Trend	End 12/31/79 Col(1)x[1 +	Year End	Annualized	1977	1978/3mos.**		1978/3
Coverage	Item	3/31/78	Factor	Col(2)x1.75*]	12/31/77	1978/3 mos.**	Col(3) ÷ Col(4)	Col(3) ÷ Col(5)	1977	mos.
Bodily Injury	Cost	$2,909.29	+ 5.0%	$3,163.85	$3,008.88	$2,363.25				
	Frequency	.00893	− 2.0	.00862	.00910	.00756				
	Cost x Freq.			$ 27.27	$ 27.38	$ 17.87				
Property Damage	Cost	$ 355.82	+ 6.5	$ 396.31	$ 347.57	$ 363.08				
	Frequency	.05313	− 2.0	.05127	.05332	.05348				
	Cost x Freq.			$ 20.32	$ 18.53	$ 19.42				
BI/PD Liability				$ 47.59	$ 45.91	$ 37.29	1.037	1.276	.586	.516
Med. Pay.	Cost	$ 511.59	+ 6.0	$ 565.31	$ 508.55	$ 511.70				
	Frequency	.01072	0.0	.01072	.01085	.01084				
	Cost x Freq.			$ 6.06	$ 5.52	$ 5.55	1.098	1.092	.061	.065
Comprehensive	Cost	$ 126.23	+ 10.0	$ 148.32	$ 123.01	$ 123.33				
	Frequency	.08471	+ 5.0	.09212	.08234	.09964				
	Cost x Freq.			$ 13.66	$ 10.13	$ 12.29	1.348	1.111	.105	.120
Collision	Cost	$ 477.31	+ 7.5	$ 539.98	$ 464.39	$ 492.91				
	Frequency	.06272	+ 3.0	.06601	.06266	.07032				
	Cost x Freq.			$ 35.64	$ 29.10	$ 34.66	1.225	1.028	.248	.299
All Coverages							1.120	1.170		

*Factor of 1.75 extends the annual trend for seven calendar quarters.

**Annualized by multiplying quarterly frequency by four.

EXHIBIT IX
Continued

CAR INSURANCE COMPANY

STATE X

CALCULATION OF PROJECTED INCURRED
LOSSES AND ALLOCATED LOSS ADJUSTMENT EXPENSES

Coverage	Year	(1) Adjusted IL & AE From Exhibit VI	(2) Projection Factors	(3) Projected IL & AE (1) x (2)
BI/PD Liability	1977	$34,949,131	1.037	$36,242,249
	1978-¼	12,114,588	1.276	15,458,214
Medical Payments	1977	4,236,261	1.098	4,651,415
	1978-¼	1,179,765	1.092	1,288,303
Comprehensive	1977	7,621,011	1.348	10,273,123
	1978-¼	2,316,655	1.111	2,573,804
Collision	1977	22,856,556	1.225	27,999,281
	1978-¼	6,660,561	1.028	6,847,057
All Coverages	1977	73,374,342	1.120	82,179,263
	1978-¼	21,137,276	1.170	24,730,613

EXHIBIT X

CAR INSURANCE COMPANY

STATE X

PROJECTED UNDERWRITING EXPENSES

Year	Average Expense Per Policy Not Premium Related*
1966	$19.09
1967	19.04
1968	20.00
1969	20.71
1970	22.36
1971	23.93
1972	25.97
1973	28.52
1974	28.34
1975	28.31
1976	29.66
1977	28.53

Last Point on Line of Best Fit	$30.62
Average Annual $ Change	$1.106
Annual Trend: Expenses Not Premium Related	$1.106 \div 30.62 = +3.6\%$

Average Annual Premium Per Policy = $173.57
Average Premium Related Expense Per Policy = $173.57 x .125 = $21.70

$$\text{Annual Total Expense Trend} = \frac{\$1.106}{\$21.70 + \$30.62} = \frac{\$1.106}{\$52.32} = .021, \text{ or } +2.1\%.$$

*Source: Annual Statement Underwriting and Investment Exhibit Part 4

EXHIBIT X
Continued

CAR INSURANCE COMPANY

STATE X

PROJECTED UNDERWRITING EXPENSES

Coverage	Year	(1) Annual Trend	(2) Time Extension Factor	(3) Trend Factor 1 + [Col(1)xCol(2)]	(4) Adjusted Underwriting Expenses From Exhibit VI	(5) Projected Underwriting Expenses Col(3) x Col(4)
BI/PD Liability	1977	+2.1%	2.000 yrs.	1.042	$13,651,258	$14,224,611
	1978-¼	+2.1	1.375 yrs.	1.029	3,780,299	3,889,928
Medical Payments	1977	+2.1	2.000 yrs.	1.042	1,712,531	1,784,457
	1978-¼	+2.1	1.375 yrs.	1.029	508,996	523,757
Comprehensive	1977	+2.1	2.000 yrs.	1.042	2,153,048	2,243,476
	1978-¼	+2.1	1.375 yrs.	1.029	643,811	662,482
Collision	1977	+2.1	2.000 yrs.	1.042	7,078,845	7,376,156
	1978-¼	+2.1	1.375 yrs.	1.029	2,145,933	2,208,165
All Coverages	1977	+2.1	2.000 yrs.	1.042	26,406,259	27,515,322
	1978-¼	+2.1	1.375 yrs.	1.029	7,292,190	7,503,664

EXHIBIT XI

CAR INSURANCE COMPANY

STATE X

PROJECTED PRIVATE PASSENGER
UNDERWRITING EXPERIENCE

Calendar Year	Item	BI/PD Liability	% E.P.	Medical Payments	% E.P.	Comprehensive	% E.P.	Collision	% E.P.	All Coverages	% E.P.
1977	E.P.	$53,273,658		$6,933,324		$ 7,478,912		$24,728,848		$ 96,007,658	
	IL & AE	36,242,249	68.0%	4,651,415	67.1%	10,273,123	137.4%	27,999,281	113.2%	82,179,263	85.6%
	U.E.	14,224,611	26.7	1,784,457	25.7	2,243,476	30.0	7,376,156	29.8	27,515,322	28.7
	G or L	2,806,798	5.3	497,452	7.2	−5,037,687	−67.4	−10,646,589	−43.1	−13,686,927	−14.3
1978/3 Mos.	E.P.	$15,348,871		$1,972,851		$ 2,235,456		$ 7,425,375		$ 27,161,781	
	IL & AE	15,458,214	100.7	1,288,303	65.3	2,573,804	115.1	6,847,057	92.2	24,730,613	91.0
	U.E.	3,889,928	25.3	523,757	26.5	662,482	29.6	2,208,165	29.7	7,503,664	27.6
	G or L	−3,999,271	−26.1	160,791	8.2	−1,000,830	−44.8	−1,629,847	−21.9	−5,072,496	−18.7
TOTAL	E.P.	$68,622,529		$8,906,175		$ 9,714,368		$32,154,223		$123,169,439	
	IL & AE	51,700,463	75.3	5,939,718	66.7	12,846,927	132.2	34,846,338	108.4	106,909,876	86.8
	U.E.	18,114,539	26.4	2,308,214	25.9	2,905,958	29.9	9,584,321	29.8	35,018,986	28.4
	G or L	−1,192,473	−1.7	658,243	7.4	−6,038,517	−62.2	−12,276,436	−38.2	−18,759,423	−15.2

E.P. = Earned Premiums
IL & AE = Incurred Losses and Allocated Adjustment Expense
U.E. = Underwriting Expenses
G or L = Underwriting Gain or Loss

EXHIBIT XII

CAR INSURANCE COMPANY

STATE X

CALCULATION OF PROJECTED EXPENSE RATIO

I. All Coverages Indicated Rate Change = (G-g) ÷ (.875-G)

$$1977: \quad \frac{5.0\% - (-14.3\%)}{.875 - .050} = \frac{19.3\%}{.825} = +23.4\%$$

$$1978/3\text{ mos}: \quad \frac{5.0\% - (-18.7\%)}{.875 - .050} = \frac{23.7\%}{.825} = +28.7\%$$

II. All Coverages Projected Expense Ratio

Year	(1) Current Level Earned Premium	(2) Indicated Rate Change	(3) Indicated Premium Level [1+Col(2)]×Col(1)	(4) Premium Change Col(3) − Col(1)	(5) 12.5% of Premium Change	(6) Expenses from Exh. XI	(7) Projected Expenses Col(5) + Col(6)	(8) Projected Expense Ratio Col(7) ÷ Col(3)
1977	$96,007,658	+23.4%	$118,473,450	$22,465,792	$2,808,224	$27,515,322	$30,323,546	25.6%
1978/3 mos.	27,161,781	+28.7	34,957,212	7,795,431	974,429	7,503,664	8,478,093	24.3

III. Projected Expense Ratios for Individual Coverages

	1977			1978/3 mos.		
Coverage	(1) Expense Ratio Exh. IV	(2) Relation to All Coverages Expense Ratio Col(1)	(3) Projected Expense Ratio All Cov. × Col(2)	(1) Expense Ratio Exh. IV	(2) Relation to All Coverages Expense Ratio Col(1)	(3) Projected Expense Ratio All Cov. × Col(2)
BI/PD Liability	25.7%	.931	23.8%	24.6%	.918	22.3%
Medical Payments	24.7	.895	22.9	25.8	.963	23.4%
Comprehensive	29.0	1.051	26.9	28.8	1.075	26.1
Collision	28.9	1.047	26.8	28.9	1.078	26.2
All Coverages	27.6	1.000	25.6	26.8	1.000	24.3

EXHIBIT XIII

CAR INSURANCE COMPANY

STATE X

CALCULATION OF INDICATED
RATE CHANGES—LOSS RATIO TEST

	Projected Loss Ratios from Exhibit XI				
Year	BI/PD Liability	Medical Payments	Comprehensive	Collision	All Coverages
1977	68.0%	67.1%	137.4%	113.2%	85.6%
1978/3 mos.	100.7	65.3	115.1	92.2	91.0
TOTAL	75.3%	66.7%	132.2%	108.4%	86.8%

	Projected Expense Ratios from Exhibit XII				
Year	BI/PD Liability	Medical Payments	Comprehensive	Collision	All Coverages
1977	23.8%	22.9%	26.9%	26.8%	25.6%
1978/3 mos.	22.3	23.4	26.1	26.2	24.3
TOTAL	23.5%	23.0%	26.7%	26.7%	25.3%

*The average expense ratio for the total 15 month period is an average of the 1977 and 1978-1st quarter expense ratios calculated by utilizing the current level earned premiums, by coverage, from Exhibit XI as weights.

Desired Profit level = 5.0%, before Federal Income Tax, from Exhibit XII.

Loss Ratio Test: $\dfrac{\text{Projected Loss Ratio}}{1 - \text{Projected Expense Ratio} - \text{Desired Profit Level}} - 1$, expressed as a %.

	Indicated Rate Change—Loss Ratio Test				
Year	BI/PD Liability	Medical Payments	Comprehensive	Collision	All Coverages
1977	− 4.5%	−6.9%	+101.8%	+66.0%	+23.3%
1978/3 mos.	+38.5	−8.8	+ 67.1	+34.0	+28.7
TOTAL	+ 5.3%	−7.4%	+ 93.6%	+58.7%	+24.5%

DISCUSSION BY JOHN J. KOLLAR

Some persons unfamiliar with the different purposes of insurance data have accused insurance companies of keeping "two sets of books." They point out that insurance companies report profits to their shareholders based on Annual Statement data and then file for rate increases based on ratemaking data. They ask, "How can the data be the same when the results are so different?" In his paper Mr. Miller shows that the data are the same by developing accident year data from calendar year data. That is, ratemaking data can be reconciled to Annual Statement data if sufficient detail and flexibility are maintained in a company's data processing system. Unfortunately, this may be beyond the scope of most companies.

While the data underlying calendar year and accident year reports are the same, the methods of compiling them are different because their purposes are different. Calendar year data reflects the past profitability of a company including inaccuracies in reserves established in earlier years. Whatever rate changes are indicated by the ratemaking formula do not change the past profitability, although they are important in anticipating future profitability.

Accident year data provides a matching of premiums with losses and expenses arising from the portions of policies in effect during a twelve month period. For ratemaking purposes these losses and expenses are then projected to future levels to determine what premiums are needed to pay these losses and expenses. Past profitability does not change the indicated rates, although it is an important consideration for a company that is under-capitalized.

Mr. Miller emphasizes the importance in ratemaking of anticipated loss and expense levels as opposed to past loss and expense levels. It is the appropriateness of the revised rates which will determine whether the ratemaker has been successful. While past trends provide a basis for future trends, it is informed judgment which leads to the selection of the appropriate trend factors or trend procedures.

As Mr. Miller indicates, there are several limitations to the use of Annual Statement Page 14 data for ratemaking. To overcome them, his fictional company compiles its data in expanded detail. A company with the necessary data processing capabilities could elect to compile data in additional detail: coverage, basic limits, catastrophe, deductible, territory, class, etc. Such data could then be summarized on either an accident year or calendar year basis, or both. With monthly or even quarterly reserves, this company could increase credibility and avoid the seasonality problem of using a fraction of a year by using the two latest fiscal years of data that are available.

While Mr. Miller's paper does not discuss the use of an IBNR factor, it is of course implicit in calendar year incurred data. This factor, which is used to include reserves on unknown claims and reserve inaccuracies on known claims, is probably the most imprecise part of using an adjusted calendar year for ratemaking. The IBNR factor is comparable to a loss development factor which adjusts an accident year's losses from the twelve month evaluation to their ultimate value. As this factor can be quite large for the liability coverages, particularly bodily injury liability, it is necessary that it be accurately determined. (The IBNR factor is even larger for a fraction of a year.) As with loss development this can probably be best accomplished by considering recent historical patterns in IBNR factors. Much has already been written about establishing IBNR reserves.

Mr. Miller expresses a preference for incurred claim frequency over paid claim frequency because it eliminates the impact of revised claim payment procedures on claim frequency trend. Changes in the procedures for establishing reserves could cause distortions in the incurred claim frequency trend. On the other hand, incurred claim frequencies are more responsive than paid claim frequencies. One can make arguments pro and con for other trend procedures, such as the use of more than one company's data for trend, exponential curve fits, or exponential projections. As Mr. Miller emphasizes in his paper, however, the use of specific ratemaking procedures is not as important as the ratemaker's use of informed judgment in selecting trend factors.

Mr. Miller's application of the selected trend factor is much different from most of today's approaches. (See Exhibit IX.) First, the latest actual trend point (Column 1) is projected (Column 3) for the desired period by the selected annual trend factor (Column 2). This gives very much weight to *one actual* trend point. Second, the loss projection factors (Columns 6 and 7) are used to adjust the 1977 and annualized first quarter 1978 trend points (Columns 4 and 5) to the value of the one projected trend point. This reduces a sample set of two points with different values to a *single* value. That is, if this data were used in determining the indicated rate level changes, the projected incurred loss and allocated loss adjustment expense ratios for 1977 and the first quarter of 1978 would be identical except for average rate differences. Third, however, the loss projection factors based on incurred claim frequencies and paid claim costs are applied to incurred claim frequency and cost data. Although the loss projection factors measure the difference in paid claim costs between two *specific* points in time, they are not necessarily appropriate for measuring the differences in incurred claim costs between the same two *specific* points in time.

A more typical approach to trend would be to extend the selected trend factors for the projection period, combine the cost and frequency factors, and combine the bodily injury and property damage factors. This would result in factors of 1.068 and 1.047 for 1977 and the first quarter of 1978, respectively. (See Appendix for the determination of these factors.) These can be contrasted with Mr. Miller's loss projection factors of 1.037 and 1.276, respectively. Clearly much different rate level indications would result.

As Mr. Miller says in his conclusion, there are many areas of ratemaking on which he comments only briefly. Although I chose to comment on some of these, they are secondary to the purpose of his paper. The key point of Mr. Miller's paper is that financial data and ratemaking data are the same. Mr. Miller proves it by developing accident year data from calendar year data. This is the essence of Mr. Miller's paper and the reason why he has made a valuable contribution to ratemaking theory.

APPENDIX

This section contains an alternate calculation of trend factor with only one difference from Mr. Miller's trend calculation. The selected annual trend factor is extended for the projection period, and then all other calculations are performed in the same fashion.

The proposed effective and trend projection dates are July 1, 1978 and July 1, 1979, respectively. For 1977 the average date of accident is July 1, 1977 yielding a projection period of 2 years. For the first quarter of 1978 the average date of accident is February 15, 1978 yielding a projection period of 1.375 years. For bodily injury the annual trend factors are +5% and −2% for cost and frequency, respectively. For property damage the annual trend factors are +6.5% and −2% for cost and frequency, respectively. The loss weights are assumed to be 60% for bodily injury and 40% for property damage. The loss projection factors are then calculated as follows:

For 1977:

$$.6\{[1+(2x.05)][1+2(-.02)]\} + .4\{[1+(2x.065)][1+2(-.02)]\} = 1.068$$

For the first quarter of 1978:

$$.6\{[1+(1.375\,x\,.05)][1+1.375(-.02)]\}$$
$$+ .4\{[1+(1.375x.065)][1+1.375(-.02)]\} = 1.047$$

ESTIMATION OF THE DISTRIBUTION OF REPORT LAGS BY THE METHOD OF MAXIMUM LIKELIHOOD

EDWARD W. WEISSNER

VOLUME LXV

DISCUSSION BY JERRY A. MICCOLIS

Perhaps the most important contribution the actuarial profession can make to the industry which it serves is the representation of complex insurance phenomena by means of coherent mathematical models. The intelligent formulation of a mathematical model tends to strip away much of the mystery surrounding a given insurance problem. It makes explicit the many assumptions that may be taken for granted in less rigorous approaches. It allows the actuary to make verifiable numerical statements about the most convoluted of insurance problems by building in logical progression upon basic mathematical foundations. Most importantly, though, it aids the actuary in his future research by prompting him to ask the correct questions about the issue under study. For these reasons, Ed Weissner's modelling of the report lag phenomenon is a worthy addition to our *Proceedings*.

This reviewer, after making a few (rather pedestrian) comments on some of the technical aspects of the paper, will concentrate on actual applications of the author's model to real-world situations. The reader is urged, while considering the few minor criticisms which follow, not to lose sight of the overall importance of Mr. Weissner's fine paper.

Reinforcing a Point

It should be stressed that while the data used in the formulation of the model is truncated at various report lags, the parameter that is estimated is not only the parameter of the fitted *truncated* distributions but is also the parameter of the fitted *complete* (untruncated) distribution as well. This is an important point. It is one that the author makes but one that, I feel, bears reinforcement. This technique of fitting complete distributions using incomplete (truncated, censored, etc.) data is a powerful one and has found use in other areas of actuarial work.

The Search for a Maximum

The crucial operation in maximum likelihood estimation is the finding of a maximum of the likelihood (or log-likelihood) function. I must admit to a pet peeve here. Several authors, in their search for a maximum (or minimum) of a given function, set the first derivative of the function equal to zero and automatically assume that the root of this equation is the desired maximum (or minimum). This is, of course, not necessarily so. I am afraid the author is guilty of this assumption in the case of both $g(\theta)$ and $g^*(\theta)$. It would have been a minor task to verify that $\hat{\theta}$, in both cases, provides the maximum.

Interestingly, proof exists for the case of $g(\theta)$ in the Figure in the paper. Note that $g(\theta) = Y_1 - Y_2$. Note also that $Y_1 > Y_2$ for $\theta < \hat{\theta}$ and $Y_1 < Y_2$ for $\theta > \hat{\theta}$. Since $g(\theta)$ is the first derivative of the log-likelihood function, we have a maximum because $g(\theta) = Y_1 - Y_2 > 0$ for $\theta < \hat{\theta}$; $g(\hat{\theta}) = 0$; and $g(\theta) = Y_1 - Y_2 < 0$ for $\theta > \hat{\theta}$.

Domain of Convergence

The author mentions the use of Newton-Raphson iteration. In the examples given, swift convergence to a reasonable result was apparently obtained. In some applications, however, divergence, or convergence to the wrong root, may result. I would have preferred that the author had pointed out these potential covergence problems and shared with us any hints he had on the selection of a proper seed.

Goodness of Fit

Once we have decided upon the form of the theoretical distribution we would like to fit to our data, and estimated the parameters of this distribution (by means of maximum likelihood estimation, for example), we should then test how well the distribution fits our observations. The Kolmogorov-Smirnov (K-S) test is a simple, yet powerful, test for this purpose. A description of the K-S test may be found in [1].

In actual applications, other tests should suggest themselves naturally. For example, using the exponential model in Section 2 of the paper, we are able to compute the estimated number of claims emerging during any calendar month. Comparing this number with the actual number of emerged claims during that calendar month (a diagonal in Table II) provides a good practical test of fit.

Sensitivity

All parameter estimation techniques and tests of fit operate on observed data. One of the uses the author suggests for his model is the estimation of claims incurred but not reported (IBNR). IBNR estimation is one example of *projection* based on the model, i.e., using the model to estimate the future unobserved portion of the data. If one of the reasons for developing a model is to use it for projection, then the testing of the model is incomplete unless it includes some form of sensitivity analysis. Utilizing the IBNR example: if the use of a log-normal distribution, say, over an exponential, results in vastly different IBNR estimates, then much more care in the choice of a distribution function is warranted. Perhaps the most appropriate estimate would be a range of values generated by a family of reasonable distributions.

A Word of Caution

The author mentions in his opening sentence that IBNR estimation is aided by knowledge of the report lag distribution. Indeed, he gives an example of IBNR calculation at the end of Section 1. I believe that it is dangerous to apply the model as it stands to the estimation of IBNR claims. This is my only substantial reservation about the paper.

A crucial assumption made in Section 2 of the paper is that θ, and hence the average report lag, is constant by accident month. Let us assume, alternatively, that the average report lag is increasing by accident month. Let us further assume, as did the author, that the number of (ultimate) occurrences is increasing by accident month. It is clear that these two phenomena will tend to significantly increase the actual IBNR over what would be the case if neither trend were present. It should also be clear that each of these trends will successfully mask the other in the data we have available (i.e., data in the form of Table II). In other words, if October occurrences are greater than April occurrences, we will not notice that fact since they will emerge, on the average, at later lags than did the April occurrences, and will more likely fall in the future unobserved region of Table II. But Table II is all the model has to work with! Hence it cannot distinguish between the "double trend" and "no trend" scenarios above. The model will give accurate results for the "no trend" case but will seriously underestimate IBNR in the "double trend" instance.*

With reasonable effort, the author's model can be generalized to accommodate the assumption of changing report lags and changing number of occurrences by accident month. Hints on how to proceed may be found in [2].

A different phenomenon from continuously changing report lags is the case of an abrupt one-time change in average report lag (due to, say, the implementation of on-line computer claims reporting). This would occur during a particular calendar period and would affect all accident months along a Table II diagonal (doing violence to the implicit independence-by-accident-month assumption necessary to the

* The model will also work in the case of varying occurrences and constant θ, as the author has shown.

formulation of the likelihood function L^*). In this case, I would suggest manual adjustment of all data above the diagonal to put it on the new accelerated-reporting basis rather than adapting the model. This would also be the procedure for other non-recurring type phenomena.

Where from Here?

This model is flexible enough that it may also be used to estimate the lag between claim reporting and claim payment. Both of these lag models, in combination with a model describing claim size amounts by occurrence date and payment date, may then be used to build a complete model of the claim payment process.

Conclusion

This is a significant paper. While the comments above argue against the immediate use of the author's unmodified report lag model as a practical tool, the paper remains important in two respects. First, any responsible attempt, such as this, to mathematically model a complex insurance phenomenon should be heartily welcomed by the actuarial fraternity. Second, the specific fitting technique employed (i.e., estimating a complete distribution function with incomplete, biased data) is extremely useful and has much wider application than the estimation of report lags. The recent technical advances in the field of increased limits pricing owe much to this technique.

This paper should provide a firm foundation for the study of report lags; its techniques should find broad application in other areas of actuarial endeavor; and, in prompting actuaries to "ask the right questions," it should enhance the future state of our science.

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[2] Charles A. Hachemeister, "IBNR Claims Count Estimation with Static Lag Functions," presented to Risk Theory Seminar, American Risk & Insurance Association, April, 1975.

TOTAL RETURN DUE A PROPERTY/CASUALTY INSURANCE COMPANY

SUMMARY OF DISCUSSION PAPERS

JEROME A. SCHEIBL

In his *Treatise on Money*, John Maynard Keynes wrote:

> Thrift may be the handmaid and nurse of enterprise, but equally she may not, for the engine which drives enterprise is not thrift but profit.

Another view of profit in a free enterprise system is that profit is a reward for uncertainty and risk. Certain elements of profit are also viewed as additional premiums for risk bearing, or as compensation for aversion to risk for risky industries.

It is within the framework of these three views of profit that we will consider the total return due a property/casualty insurance company. Indeed, these views of profit apply to all schemes for assuming fortuitous risk in our economy, whether they involve self-insurance, private insurance, state administered funds, or, for that matter, self-insurance plans.

The issue is not whether property/casualty insurance companies are entitled to income, but instead what this income should be, how it should be measured, how the natural forces at work in our economic environment affect income, and what additional artificial stimuli or constraints are necessary to assure that sufficient underwriting capacity is available to meet the needs of our economy and, at the same time, assure that sufficient safeguards are in place to provide the healthy competition so necessary for economic growth.

The response to the Committee on Continuing Education's call for papers and reviews has produced a wealth of material sufficient to encourage spirited discussion that will continue long after we leave this meeting. Thirteen papers and thirteen reviews have been submitted and all participants at this meeting have had an opportunity to wade through the 484 page volume containing these papers. Each person here will have an opportunity to participate in the discussions of approximately half of the papers and reviews. Many of us will leave this meeting more frustrated than satisfied. If that is the case, all is not lost, as academic frustration is the stimulus to creativity.

A number of papers discuss our theme from the viewpoint of economic theory — especially modern concepts of financial theory. Two authors in particular, Lee M. Smith and John S. McGuinness, share the thought that there is not and need not be a special economic theory of risk of insurance or of insurers. Lee M. Smith's primary hypothesis is that a profitability standard developed outside the context of economic theory will have little value. Criteria for profitability which do not ultimately speak to the issue of resultant resource allocation are sterile in that they cannot be demonstrated to be good or bad in their overall effect. Economic theory, then, provides the starting point for discussions of appropriate profit levels for a firm or industry. Guidance may be found in the area of microeconomics. Smith describes the microeconomic factors affecting the equilibrium between supply and demand and optimized resource allocation.

Smith notes that those associated with utility regulation, which he considers to be the most obvious precursor of the movement to regulate rate of return, have found economic principles to be of primary importance in that endeavor. He cautions that, before becoming too enamored with utility regulation, insurance professionals should consider the difficulties and trade-offs involved.

Smith's reviewer, Claus S. Metzner, agrees that the complexities of the profitability issue are so great that caution is called for in using any single approach for measuring profitability. He cautions against automatic adoption of utility regulation as economic fundamentals for analyzing insurance profitability for, as he points out, utilities are regulated because they are a monopoly, while the insurance industry, in contrast, is regulated based on the need to prevent unfettered competition from reducing the price below economic levels and thereby jeopardizing the solvency of insurance companies.

While John S. McGuinness agrees that there is no such thing as a special economic theory of risk, he arrives at his conclusion by a somewhat different route than Smith's. He contends that managerial theory fully and precisely covers the entrepreneurial factor of production. Insurance, as part of security management, has already been fully fitted into managerial theory. He proposes that this theory neatly embraces all types of risk faced by an enterprise, whether it be an insurer or another type.

McGuinness supports his theory generally by describing the elements of managerial theory; he then demonstrates how rates of profit can be conveniently studied by use of an input/output approach.

He suggests that in order to measure the need for profit, one must know the needs for profits. He lists six of these needs:

1. To keep real economic net worth from being reduced by inflation,
2. To meet increasing needs for capacity in a growth economy,

3. To retain existing capital and to attract additional outside capital needed to support other demands for increased capacity,
4. To assure that non-random variation and results will not reduce real underwriting return,
5. To assure that non-random variation and results due to unpredictable outside causes will not reduce the real underwriting return during any year to below zero, and
6. To assure that the value of assets will not decrease by more than a preset percentage during a twelve-month period due to fluctuations in securities prices.

R. Woody Beckman, in his review of McGuinness's paper, suggests that McGuinness's division of profitability needs into six categories is a departure from the thesis that general economic theory is appropriate and no special economic theory is required for the insurance industry. He points out that general economic theory combines the first three of McGuinness's parameters under the caption, "Return on Investment." He also points out that McGuinness's last three needs relate to the protection of earnings from fluctuations in underwriting and investment results and points out that in no other industry is there such a guarantee of profits.

Modern financial concepts such as efficient markets and portfolio theory, which can be used to determine relationships between risk and return for property/casualty insurers, are discussed in Robert P. Butsic's paper. These ideas are introduced as needed and explained in intuitive terms.

A simple deterministic model of a property/casualty insurer is developed to show key accounting relationships, including the fundamental notion of return on surplus as a function of levered underwriting and investment income gains. The model is then extended to treat the basic elements as random variables through which risk is related to premium. The subsequent model illustrates the relationship between systematic and unsystematic risk and explores the problem of finding an optimal balance between asset and underwriting portfolios. The model then applies efficient-market criteria to find the expected underwriting profit margin under equilibrium conditions.

Applications are briefly discussed for the areas of ruin theory, product pricing, marketing, reinsurance, and regulation.

In his review, James N. Stanard points out that the emergence of the discipline of corporate risk management shows that it is anarchistic to view the insurance problem separately from other financial decisions. As actuaries, we must carefully examine the underlying assumptions, the empirical validity, and the resulting implications of applying financial theory to insurance problems.

Whatever one's view on the nature of economic forces guiding the insurance industry's fate, or on the theories underlying the interaction of these forces, one thing is clear: profits must somehow be measured so that they can be related to standards or goals for measuring the effectiveness of resource allocation and regulatory activities. Several of the papers refer to the possible alternative bases for such measurements. Two, in particular, emphasize and evaluate these bases.

Norton E. Masterson's paper emphasizes the "how" of measurement of total return, which is, as he states, a prerequisite to the determination of the "what," or bottom line. He submits that the return-on-assets base is the preferred base for economic analyses and for the appraisal of the production utilization of financial resources. This base draws management's attention to the return on total capital, as compared with the opportunity cost of that capital (that is, what it could earn if invested in other enterprises of equal risk). He observes that, under certain circumstances, the average premium base and the net worth base must be used — premiums for rate-making and net worth for GAAP accounting for parent/holding companies of insurance corporations.

In his review, Robert A. Bailey points out the fact that, since there is a general absence of preferred stock and debt in the insurance industry, the return on total capital is very close to the return on net worth. He states that the assets supplied by policyholders (represented by unearned premiums and unpaid losses) and the corresponding return on these assets, are omitted from a net worth analysis. He argues that this imputed interest, which is also referred to in the McGuinness paper, must be considered in any formula for measuring profitability.

The other paper emphasizing an evaluation of measurement base alternatives is the one presented by Irving Plotkin. Dr. Plotkin first discusses classic economic theory as it might be applied to the business of insurance. He is quick to point out that one cannot predict the magnitude of the rate of return merely by observing the profit margin. He states that this is true for all industries, although he recognizes that the pretax underwriting profit allowance is a useful and even necessary regulatory tool for rate review.

While not ideal for all purposes, the return on total capital is favored by Plotkin as the appropriate yardstick since it has the advantage of minimizing differences in profitability between industries and companies that are due to different equity/debt ratios or financing mixes. He cites three basic reasons for his choice. These are (1) society's view of optimal resource allocation, (2) the underlying source of risk, and (3) marginal investment decisions.

The great profitability debates of the late 60's and early 70's are mentioned frequently by various authors of the discussion papers. Reminiscing about his active participation in these debates, Dr. Plotkin revisits and reaffirms his positions in those debates — most notably his position on imputed interest and his disagreements with Mr. Bailey as to its role in the measurement process.

LeRoy J. Simon's review also refers to the debate regarding imputed interest. He points out that Plotkin rebuts Bailey's arguments by talking about the use of imputed interest in ratemaking rather than about its validity in the rate-of-return measurement. Simon concludes that inclusion of imputed interest could be a fatal flaw in Plotkin's proposed measurement base because the regulator might force the rate of return to insurers, so calculated, to the same level as in industries which do not have imputed interest. This would result in an insurance investor turning his back on our industry, as there would be no profit in it for him.

The now-famous profitability debates had their roots in the controversy surrounding automobile rate revisions in Massachusetts. Commissioner Stone not only espoused the use of financial analysis for determining a proper underwriting profit loading for ratemaking purposes, but he also developed a procedure for quantitively determining an exact percentage to be used in the ratemaking process. His method determines required profit margins by evaluating the return on equity that the company should receive, given the risk it incurs, and relating return on equity to underwriting profit. The target return on equity is determined by applying the capital asset pricing model.

Jeffrey Brown analyzes the Stone approach and points out that, in his opinion, several types of problems need to be corrected before the system can be used effectively.

Theoretical problems primarily include problems of an incorrect derivation of the profit margin expression and the lack of a consistent time horizon.

Parameter problems are discussed from the standpoint of volatility.

Brown's reviewer, Holmes M. Gwynn, is critical of Brown's observations as well as those of Dr. Fairley, who has written two papers on the subject of profit provisions for automobile ratemaking in Massachusetts.

The capital asset pricing model is of questionable value, in Gwynn's opinion, for determining a profit provision. The problem is in the measurement of systematic risk. The beta coefficients which are supposed to do this job tend to vary within an industry from year to year and, therefore, make the model a difficult tool to use.

Does the traditional ratemaking profit provision ignore investment income, which today is the property/casualty industry's major source of net income? Perhaps some light may be shed on this question by moving from a theoretical discussion of methods and procedures to an actual quantitative analysis of an underwriting profit loading.

Frank Harwayne provides such an evaluation in his paper on workers' compensation ratemaking. He stresses the need for considering investment income on a prospective basis in the ratemaking process, as the statutory standards for rates are prospective in nature.

He describes the National Council approach to estimating expected investment income on a prospective basis as a three-part process:

1. Determination of an appropriate investment yield,
2. Application of this investment yield to unearned premium reserves in order to estimate investment income attributable to unearned premium reserves, and
3. Application of the investment yield to the expected loss reserves in order to estimate investment income attributable to loss reserves.

Harwayne contends that investment income can be given proper prospective consideration without actually being incorporated into the ratemaking mechanics and subjected to the same periodic review as underwriting experience. He discusses a number of reasons which support his contention. He also points out the interrelationship between investment income and contingencies which affect the accuracy of prospective rate level calculations.

David R. Bickerstaff points out in his review that the most cogent sections of the paper are those in which Mr. Harwayne addresses the riskiness of writing workers' compensation insurance with ample documentation of the recent nationwide experience in this line. He recognizes Harwayne's methodology for quantifying the riskiness of this line as being something of a variation on the dual measurements employed in the A. D. Little risk and rate of return studies of a decade ago. Bickerstaff also raises some questions with regard to Harwayne's assumptions in calculating the investment income attributable to loss reserves and unearned premium reserves. He concludes, however, that Harwayne's paper is the most complete and openhanded statement regarding risk and investment income from the insurance industry's viewpoint to be set forth in recent years.

No discussion of required earnings would be complete without considering these earnings or returns from a solvency point of view. Three papers emphasize the analysis from this perspective.

Gary G. Venter addresses his comments to ruin and return implications. He describes the profit and contingency loading as an indivisible unit designed to meet two goals, namely, solvency and profitability. Surplus also contributes to solvency; but, if surplus is too high as a percent of the loading, an insufficient rate of return will result. The problem is to find the set of surplus-loading combinations which give adequate solvency protection *and* a sufficient rate of return. The element in this set with the smallest loading amount is determined; this element gives the surplus-loading combination with the lowest premium consistent with adequate solvency and profitability. The solvency criterion involving the probability that actual losses will exceed the total of expected losses, surplus, and the profit contingency loading amount, is specified as an equation in the two unknowns — loading and surplus — and certain assumed-to-be-known statistics of the portfolio loss distribution.

An expression of the desired return on surplus will also contain the two variables, loading and surplus. Venter points out that, in many cases, the simultaneous solution of these two equations produces the minimum loading sought. He also addresses his comments to distributing the portfolio loading charge by product or contract using what he calls the contract loss distribution function.

Lee R. Steeneck, in his review, expands on the theme that profit and contingency loadings in insurance rates serve two purposes. He notes that the fluctuation-in-loss reserve really protects against both reserve inadequacies and possible prospective rate deficiencies. In part, it also protects against adverse fluctuations in investment results and provides for a return which will be partially paid out in dividends, but mostly retained to allow for increased capacity.

James Stewart's paper reviews two generic approaches to the inclusion of investment income in ratemaking. The first, termed the ownership approach, bases the estimate of investment income upon the expected return that could be earned on unearned premium reserves and loss reserves. In the second, called the cash flow approach, the investment income is an integral part of the total income which accrues to a company from all income sources: return on surplus; underwriting profit; and the discounted cash flow of premiums, losses, and expenses which are associated with a given set of policies.

After reviewing the two approaches, Stewart discusses two computer models, built by the author, designed to simulate each approach. He describes the design, the assumptions made, and the factors used. He then presents the results of the simulations and reaches certain general conclusions about the two approaches.

Stewart suggests that a marked advantage of the cash flow approach is its inherent cohesiveness — it provides a realistic, consistent means for evaluating the impact of a set of policies or a rate change on a company's financial position.

Kenneth R. Frohlich suggests in his review that the two methods are vastly different in their degrees of sophistication. From a theoretical standpoint, the ownership method has little to recommend it. It does have the advantage of using available data, but gives few collateral benefits.

Dr. Frohlich feels that the cash flow method, on the other hand, has both theoretical merit and gives useful management information. He cautions, however, that the cash flow method requires cash flow data, not so readily available for most companies.

The relationship of minimum surplus requirements to probable earnings is described by Robert J. Finger. He describes a model he has developed for calculating minimum surplus requirements. His approach provides a solution for a real situation confronted by many actuaries.

His method is tailored to a monoline insurer, a captive, or a self-insurer. The minimum surplus requirement is defined to be the amount which, when added to the aggregate reserves, equals the 99th percentile of the aggregate reserve distribution; that is, the aggregate reserve is treated as a random variable, the sum of the individual claims.

He describes two types of variations in the aggregate reserve,

1. Statistical fluctuations in the number and size of claims about a given mean value, and

2. Uncertainty in the mean or stated aggregate reserve. Variations about a known mean are calculated by assuming a log-normal distribution for individual claim sizes and independence between the number of claims and the individual claim size distribution. Variations are calculated for various excess-of-loss reinsurance retentions.

Finger combines the uncertainty as to the mean of the aggregate reserve with the fluctuations about a known mean by assuming that both have log-normal distributions. The combined variation has a log-normal distribution since the product of two log-normal variables is also log-normal. The combined variation is then used in a formula to derive the 99th percentile of the aggregate reserve distribution. The result is the minimum surplus requirement, which is expressed as a fraction of the aggregate reserve.

In his review, Robert S. Miccolis points out that the recognition of uncertainty in loss reserve estimates and the measurement of this uncertainty in terms of confidence intervals is an important area for actuarial study. He cautions that the limitations of ruin theory, such as its failure to recognize the potential magnitude of ruin, should also be considered, although he concedes that the effect of these limitations should not be significant.

Papers on planning and inflation, two closely related topics, will round out our discussion.

In his paper illustrating the impact of inflation of insurance company operations, Stephen P. D'Arcy points out that total rates of return for an insurance company from different time periods are not directly comparable. High interest rates are caused by high inflation rates. Since inflation increases premium writings, premium-to-surplus ratios rise with inflation unless surplus is increased by the achievement of a higher rate of return.

D'Arcy constructs a model insurance company to illustrate the impact of inflation. He makes certain assumptions regarding investable assets, then calculates equivalent real total rates of return for various inflation and interest rates. The underwriting profit margin needed to obtain the calculated total rate of return is then determined for each example.

The reviewer, James P. Streff, agrees with D'Arcy that the calculation of the total return due an insurance company and underwriting profit margins cannot be treated independently from the underlying inflation rate. He applauds the author for choosing simplicity, which emphasizes the purpose of the model, over the necessarily complicated model that would have to be developed if the results were to be most accurately applied to the real world.

Another paper on the general subject of underwriting profit and inflation utilizes a simple accounting model to demonstrate how to determine underwriting profits needed to keep pace with increasing growth. In this paper, John H. Muetterties describes historical relationships of annual statement figures for three periods: 1977, 1971, and 1965. Each of these periods represents a different point in an economic cycle: 1977 began with a two-to-one premium-to-surplus ratio and saw a premium growth of 20 percent; 1971 began with a 1.5-to-1.0 ratio and saw a premium growth of 12 percent. Going back to 1965, there was a one-to-one ratio and a 10 percent growth in premiums.

Muetterties points out that, given the 1977 ratio, a before-federal-income-tax underwriting profit of 8.3 percent would have to have been achieved to retain the two-to-one ratio at year-end. Similarly, returns of 7.5 percent in 1971 and 1.4 percent in 1965 would have to have been realized to maintain the beginning premium-to-surplus ratos for each of those years. He also describes a future situation in which the premium-to-surplus ratio could increase to three-to-one and the growth rate to 25 percent. Under these conditions, the necessary underwriting profit would be 4.5 percent.

The discussion is by George E. Davis. He puts Muetterties' conclusions in a somewhat different light. He points out that the 4.5 percent future underwriting margin would result in a 32 percent return on shareholder equity, providing a 7 percent stockholder dividend and a 25 percent growth. He questions whether market forces would long permit such a high return. The influx of investors under these conditions would result in a lower premium-to-surplus ratio and an automatic drop in the return on equity to a more normal level.

A primary objective of financial planning is profit maximization. Randall E. Brubaker uses microeconomic theory to determine the profit-maximizing line mix for a multiline insurance company. Earned premiums of the product lines are the outputs and the capital of the company is the limited input. The company is constrained to line mix alternatives that do not exceed a certain probability of insolvency and impairment. The paper provides a good introduction to microeconomic theory, utilizing a standard model for a firm with one input to production and several outputs.

Brubaker recognizes that actual applications of the methods developed in his paper would require use of some relatively esoteric concepts, such as probability of insolvency or impairment, and means, standard deviations, and correlations of profit among lines of business. He recognizes that these variables do have a very large effect on the efficiency of use of capital.

Michael L. Toothman is the reviewer. He recognizes, as does the author, that the model needs a great deal of additional work and refinement before it can begin to approach reality. He recognizes that the paper is extremely basic, but he concludes that the model is technically sound.

As one reads through these papers and reviews, one can't help but feel that we have entered a new era in the insurance business. Whether or not one agrees with the appropriateness of modern financial economic concepts for analyzing total return due a property/casualty insurance company, one must recognize that many do espouse these theories and will continue to espouse them for some time to come.

Woven through all these papers and reviews is a very subtle debate. Some authors imply that modern financial economic analysis is a field unto itself and therefore beyond the scope of the narrowly defined area of actuarial expertise.

Both Toothman and Simon point out that, since so much of the economic theory can be expressed so simply in mathematical form, the field would seem to be a natural and fertile one for actuaries. Simon reminds us that the scope of actuarial science is very broad. Just as the actuary has needed tools from statistics, mathematics, and the social sciences, so too have we turned to the field of economics when necessary.

To this I might add that, while our profession may make a valid claim to the inclusion of financial economics within the scope of our expertise, we will have to back up our claim with a demonstrated depth of knowledge in this area—as we have done in the fields of statistics, mathematics, and social sciences.

PROFIT, TIME AND CYCLES

RICHARD E. STEWART

The free exchange of ideas may eventually lead to the exchange of professions.

Securities analysts, who ten years ago found truth by dividing reported earnings into stock prices, are now bent over Schedule P. Actuaries, once confined to rates and loss reserves, now discuss total return and the latest inspirations in portfolio theory.

Is the conclusion that both professions have had a rough decade? Perhaps, but a more appealing alternative is that both are trying to understand more about insurance finance.

We are trying to understand how risk-bearing insurance companies work both as parts of the larger economy and as businesses themselves.

Before going further, it is good to emphasize that that is all we are doing — trying to understand better. The casual transfer of what we learn into either short-term management or normative regulation is far more likely to be foolish than the analysis is to be wise.

That caution stated, let us talk generally about insurance profits, although perhaps in an unusual way. It will naturally lead to a look at the reasons for variability in insurance profits and particularly to a look at the underwriting cycle.

Insurance has built up over the years a language of words and numbers which is quite useful for running a company and regulating it on a daily basis. But the language gets in the way of a systematic understanding of the individual firm and of the business as a whole.

That is particularly true with respect to the significance of time. The accepted insurance language began by ignoring time, treating income and outgo as though they were simultaneous and hence keeping underwriting and investments in two different worlds.

The reasons are surely more historical than sinister, but as a result our thinking is imprisoned by a set of concepts which make it very difficult to synthesize what is going on in the whole operation.

The trouble turns up in the perfectly sensible effort to bring time into the picture by attributing investment income to different lines of insurance.

A humorous example is our speaking of investment income on reserves, as though liabilities could be invested at all. Easier to overlook is the unmanageable snarl of cash and accrual accounting, hard and soft numbers, income statement and balance sheet categories, and so forth, with which we try to work.

Finally, our attempts to arrive at total return by line lock us into measurement periods which we sense, correctly, are absurdly short. They lead us into metaphysical disputes about the earning potential, verging on the moral quality, of money one gets to keep forever compared to money one has to pay back.

If we are willing to step away from our inherited insurance concepts, understanding how profits are made gets a lot simpler. Then an insurance company can be explained in terms of just two ideas — earnings on funds invested and the cost of funds.

A representative policy is sold. Premium, net of commission and underwriting costs, is collected and invested. It is probably invested in a security, for simplicity say a bond, which earns interest at a fixed rate. Our accounting conventions and investment habits let us simplify further by ignoring bond market fluctuations, so the investment return is indeed fixed.

From time to time losses and further expenses are paid under the policy.

If we close accounts on the policy after a year, we simply deduct the losses and expenses paid from the premium and interest collected.

If we wait, then after a number of years, the loss payments cease and the books can be toted up. We find that the invested assets and their compounded earnings have been offset by losses and expenses.

If the offset is more than the original investment, then in other financial contexts we would say that we have had to pay for the temporary use of our investable funds. If the offset turns out to be less than the original investment, we have had a negative cost of funds which, instead of being deducted from the investment earnings, is added to them.

In either case, the sum or difference, if still a positive number, would be the total return, which could then be adjusted for time by discounting or some other technique and compared with whatever resource one was measuring return upon.

For an imprecise, but perhaps comforting, invocation of insurance terminology, we are speaking of total return on a fully developed policy year basis, and we are talking about underwriting profit or loss as the cost of investable funds.

Whatever its *dis*advantages, such an approach has three good points.

First, it is extremely simple, makes insurance comparable with other financial businesses, and will account for all the earnings of the enterprise.

Second, it is utterly useless for rate regulation. Much of profit study has been the handmaiden of rate setting in a natural or enforced monopoly or cartel market.

Proper profits are at once easier to determine and more useful where there is only one price. Hence insurance studies borrow heavily from public utility regulation, a proud heritage unless one looks at how well the subject industry performs.

A measure as retrospective as the cost of funds approach would involve so much old data and so much projection that no one would want it for setting rates.

That is good. Just because we can do something does not mean we should. Except in automobile insurance and workers' compensation, where the utility analogy is good or becoming so, insurance prices are finally now set in the main tradition of American economic life — the free market.

The third useful feature is that it makes us acknowledge that underwriting results determine the cost of investable funds.

As in other cases where income on investment is fixed and the cost of funds is variable — as it can be for a banker who borrows short and lends long — the cost of funds can exceed the total earnings on those funds. Then the enterprise loses. Or it can pay something for the funds but not as much as they earn. Finally, in insurance as in few other parts of finance, the cost can itself be a negative number.

So viewed, an insurance company that is writing coverages which contemplate loss payments over a considerable period can only be evaluated over a considerable period. The reason, of course, is that we do not know until the end what the cost of funds has been.

To compare lines, all that is needed is a decision when to close the books. Common sense suggests it should be after the same number of years for all lines and certainly no earlier than the last loss payment in the longest tailed line.

Since we are here on a cash basis, we can skip the vagaries of reserving and proceed directly to ask why the ultimate cash cost of funds is uncertain and variable and what, if anything, can be done about it.

The first place to look is the underwriting cycle. Some of our favorite sayings about the cycle make insurance managements sound suicidal or else the prisoners of events beyond their control. Neither is strictly true.

The insurance industry is cyclical for fundamental economic reasons. The reasons have to do with how expectations about profit affect decisions about supply. When many firms share an expectation and act on it, changes in price or its equivalents follow. Since profits turn on the relation of prices and costs, a change in profit follows too.

Many industries are cyclical because of changes in demand. People suddenly do not want to buy as many cars as Detroit continues to produce. The same goes for pepper grinders and cold rolled steel.

Except when productive capacity has to be added in large increments or not at all, supply remains relatively stable in those industries. Rising demand against stable supply pulls up prices and profit margins. On the way down, the opposite happens and it gets really exciting when discouraged producers and distributors unload inventory.

There are, however, some admittedly cyclical businesses in which the main cause of the cycle is changes in supply.

The classic example is agriculture. The demand for meat, grain or vegetables remains fairly constant and predictable over long periods of time. What is not predictable is how much of those commodities farmers will put in or on the ground.

The farmer has a lot of control over how much wheat or corn he will plant, and he knows his costs pretty well. But he will be selling what he raises some time in the future, and a free market gives him practically no control over what it will sell for. His predicament is making a present commitment to supply based on an anticipation of price many months in the future.

Farmers have similar information and outlooks. It should be no surprise and certainly no disgrace that they would often make similar forecasts as to price. If they do so in a free market, the eventual effect on prices will be just the opposite of the forecast.

Like farmers, insurers meet a fairly constant or predictable demand for what they sell. Even more than farmers, they can vary the amount they sell rather finely and quickly. Later on they may not like what was done with prices, underwriting and so forth — any more than farmers like what happens to their prices when they all plant fencepost to fencepost. But the decision to change supply can be carried out.

In making decisions about supply, meaning sales goals, insurers like farmers tend to look at recent experience. Our elaborate techniques for extrapolation have their counterparts in the barnyard. What they have in common is an inability to call the turns.

But even where we can call the turns, the competitive market prevents the individual firm from taking appropriate action.

For the main lines of insurance and for the industry as a whole, we can call the turns in the underwriting cycle quite reliably two years in advance using a simple equation which compares inflation with insurance price changes, the latter being the difference between written premium growth and the growth of gross national product.

Even when warned, the individual insurer is trapped. He can only lower prices in advance if willing to smooth the cycle by giving up profits before the top. He can only raise prices in advance if willing to give up customers before the bottom. Either one is asking a lot of human nature and even of good business sense.

Both businesses can hedge the cycle. Farmers can use the futures market. Insurers can hedge by retrospective rating, by stop-loss reinsurance, by shifting investments or by executive refinement of loss reserves. But all our known ways of smoothing the cycle are almost surely at the sacrifice of long-term total profitability.

Like most of agriculture, then, most of insurance displays a supply cycle. They are both cyclical because of the basic nature of their businesses, not because of any stupidity or avarice of their managements. Those qualities can add to the thrills, but the essential supply cyclicality is there because of the fundamentals.

The analogy is not perfect, nor does it explain everything. Weather strongly affects both businesses, but in different ways. Again, the insurance supply cycle shows up not directly but in decisions about pricing, coverage and the selection of customers. Demand changes, including substitution in agriculture and new coverages in insurance, affect both industries, though not much compared with supply. Finally, only in courteous agriculture is the product of subordinate creatures referred to as fertilizer.

In most of insurance and most of agriculture the free market dominates in its textbook form — many sellers and many buyers with easy access to each other, undifferentiated products and widespread, current price information. Where that is not true — whether by product differentiation, restricted entry, neglected markets, or pervasive cartel or government control — the whole argument does not hold.

But where the cycle rules, no amount of wisdom in the individual farmer or insurer can beat it. Perhaps it is no accident that those two industries, which defer to no one in the oratory of individualism, have so often been willing to surrender so much of their liberty to government if it would only stabilize their prices.

Both the cost of funds approach to measuring return and the inevitability of underwriting cycles leads us to look at return on a very long term basis. There is one more reason — occasional mad aberrations in profitability.

Since 1910, when the data begins, only during or right after a war have insurance company returns on equity been about as low as they were in 1975 and about as high as they were last year. In each instance there was both a low and a high. It happened with every war in the period—the First and Second World Wars, Korea and the combination of Vietnam and a war in the Middle East in which our economy was part of the issue.

As for the intervals, the insurance business did well in the depression, probably because its price cartel was still working and demand held remarkably steady. It did poorly in the 1960's, probably because the stock market boosted equity so fast that premium leverage was hard to get even with the very aggressive selling which the underwriting results suggest was tried.

Last year was just another postwar peak in insurance company returns on equity. The stock market decline and underwriting losses a few years ago had reduced equity. Rate increases at the absolute bottom of the cycle then restored margins on sales and increased leverage and cash flow. Higher interest rates pushed up the yield on newly investable funds.

The free market, the arrival of new capital and, most important, the nature of the supply cycle will get those returns on equity back down before long. The peak and the trough are real enough, but neither is the stuff for wise judgments.

In summary, fluctuations in the profits of insurers, as of other businesses, follow straightforwardly from the changing relation of their costs and their prices.

The changes can be cyclical, in the natural response of a competitive market, can be secular in the structure and conduct of the business and in the occurrence and cost of the insured event, and can occasionally be in the drama of social and economic dislocations of the magnitude of war.

For all those reasons, we should evaluate insurers only over long periods of time.

For an insurance company seen as an entire financial institution, the fluctuating relation of costs and prices operates by changing the institution's cost of investable funds. We may manage by underwriting first and investing the proceeds later, but we understand the institution best by looking at it the other way around — investments at a known yield made with funds whose cost is eventually determined by underwriting and pricing decisions which are only partly free.

All those conclusions follow from quite elementary economic analysis and from the broadest of looks at the history of our business. It hardly marks the first time we have seen that the business works differently from the way we practitioners sometimes think or hope it works.

Perhaps simple understanding is enough. The ultimate goal of scientific method is hardly a canned precision or a sprawl of concepts. Not just in physics can measurement alter that which is being measured. Not just in Gothic romance are new sciences prone to create monsters.

From here on, we can be more definite only at the sacrifice of more understanding. For there is no proper profit, no perfect rate, no precise reserve, no avoiding the underwriting cycle other than by avoiding competition or the risk-bearing process itself.

As we live it, insurance takes in the risks others cannot bear, and it should be no embarrassment that the commerce of uncertainty is at its heart a bit uncertain.

MINUTES OF THE 1979 SPRING MEETING

May 20-23, 1979

THE BROADMOOR, COLORADO SPRINGS, COLORADO

Sunday, May 20, 1979

The regular quarterly meeting of the Board of Directors took place from 1:00 – 5:00 p.m.

Registration was held from 4:00 – 6:00 p.m.

The President's reception for new Fellows and their spouses was held from 6:00 – 6:45 p.m.

A reception for members and guests was held from 6:30 – 7:30 p.m.

Monday, May 21, 1979

Registration was held from 7:45 – 8:15 a.m.

The Spring Meeting was formally convened at 8:15 a.m. After opening remarks by President Ruth E. Salzmann, a welcoming address was given by J. Richard Barnes, CLU, Commissioner of Insurance of the State of Colorado.

President Salzmann then read the names of the 38 new Associates who rose as their names were called, after which each of the 23 new Fellows was asked to step forward to receive his or her diploma.

FELLOWS

Robert P. Aldorisio	Douglas D. Eland	David J. Oakden
Nolan E. Asch	David N. Hafling	John Pierce
William N. Bartlett	Douglas J. Hoylman	Joseph R. Schumi
Everett G. Bishop	Ronald W. Jean	Edward C. Shoop
James E. Buck, Jr.	Gerald J. Jerabek	Emanuel J. Stergiou
Jerome A. Degerness	Steven G. Lehmann	Frank C. Taylor
Bernard Dorval	Janet R. Nelson	Patricia A. Teufel
Jeanne H. Eddy	Patrick R. Newlin	

ASSOCIATES

J. Paul Austin
William H. Belvin
James E. Biller
James K. Christie
Richard M. Cundy
Susan T. DiBattista
Eric T. Drummond-Hay
Thomas J. Duffy
Claude Dussault
Glenn A. Evans
James M. Foote
Patricia A. Furst
Thomas L. Ghezzi
Eugene E. Harrison
Philip E. Heckman
Barbara J. Higgins
Stephen Jameson
John J. Javaruski
Thomas S. Johnston
Joel M. Kleinman
Gaetane LaFontaine
Richard W. Lo
Edward P. Lotkowski
Howard C. Mahler
Stuart B. Mathewson
Charles W. McConnell, II
William G. McGovern
Evelyn T. Mulder
Francis X. Murphy, Jr.
Curtis M. Parker
Nancy R. Myers
Gary V. Nickerson
Ray E. Niswander, Jr.
John P. Robertson
William J. Rowland
Allan I. Schwartz
Randall J. Wilson
John D. Zicarelli

The first panel discussion was entitled "Inflation and Cost Containment: Two Perspectives." The participants were:

Moderator: Mavis A. Walters
Vice President
Insurance Services Office

Members: M. Stanley Hughey
Executive Vice President
Kemper Insurance Group

H. P. Hudson
President
National Association of Insurance Commissioners

Following the panel, an informal discussion period was held.

Key Note Address: "Profit, Time & Cycles"
Richard E. Stewart
Senior Vice President
Chubb & Son, Inc.

The topic for the call papers was "Total Return Due a Property-Casualty Insurance Company."

Summary of Call Papers: Jerome A. Scheibl
Vice President-Industry Affairs
Employers Insurance of Wausau

At 12:30 p.m. the meeting recessed for a buffet luncheon.

The afternoon was dedicated to concurrent sessions for discussion of the call papers. The four session moderators were:

Session A: Wayne H. Fisher
Vice President and Actuary
Commercial Union Assurance Companies

Session B: Daniel J. Flaherty
Consulting Actuary
Milliman & Robertson, Inc.

Session C: David J. Grady
Secretary & Associate Actuary
North American Reinsurance Corporation

Session D: E. LeRoy Heer
Vice President & Corporate Actuary
W. R. Berkley Corporation

There was an informal discussion and coffee break in midafternoon and the meeting adjourned for the day at 5:00 p.m.

Tuesday, May 22, 1979

The meeting was reconvened at 8:30 a.m.

The morning meeting was a repeat of the concurrent sessions for discussion of the call papers which were held the previous afternoon.

An informal discussion and coffee break took place at mid-morning.

The regular session reconvened at 2:00 p.m. with a workshop program. The workshop subjects and participants were:

Workshop 1 — "Effective Communications"

Moderators: Linda M. Delgadillo
Communications Manager
Society of Actuaries

Frederick D. Hunt, Jr.
Director of Communications and Government Liaison
American Academy of Actuaries

Workshop 2 — "New Paper and Reviews"

Moderator: Ronald F. Wiser
Assistant Vice President and Actuary
Montgomery Ward Insurance Companies

The new paper presented was:

"Private Passenger Automobile Insurance Ratemaking: A Calendar Year Approach" by
Michael J. Miller
Actuary
State Farm Mutual

Reviewers: Neil A. Bethel
Consulting Actuary
Tillinghast, Nelson & Warren, Inc.

John J. Kollar
Associate Actuary & Manager
Insurance Services Office

Workshop 3 — "Statement of Opinion on Loss Reserves"

Moderators: James R. Berquist
Consulting Actuary
Milliman & Robertson, Inc.

Charles A. Hachemeister
Actuary
Prudential Reinsurance Company

Robert F. Lowe
Consulting Actuary
Tillinghast, Nelson & Warren, Inc.

Donald E. Trudeau
Vice President and Controller
American Mutual Liability Insurance Company

Committee meetings were held as scheduled.

At 6:00 p.m. a Western Steak Fry was held at Rotten Log Hollow.

Wednesday, May 23, 1979

The meeting reconvened at 8:30 a.m. with concurrent panel discussions. The three panels, their topics and participants were:

Panel 1 — "Recognition of Anticipated Investment Income"

Moderator: Earl F. Petz
Actuary
Kemper Insurance Group

Panelists: Robert L. Posnak
Partner
Ernst & Ernst

Paul E. Singer
President
Illinois State Medical Insurance Services, Inc.

Panel 2 — "Classifications"

Moderator: E. Frederick Fossa
Senior Vice President & Senior Actuary
Commercial Union Assurance Companies

Panelists: Linda A. Bogue
Financial Industries Consultant
SRI International

Michael Fusco
Vice President and Actuary
Insurance Services Office

James C. Hickman
Professor, Business & Statistics
University of Wisconsin

Panel 3 — "Commercial Package Developments"

Moderator: Vicki S. Keene
Director Field Operations Planning
Aetna Insurance Company

Panelists: Arthur R. Cadorine
Associate Actuary & Manager
Insurance Services Office

Albert J. Quirin
Assistant Secretary
Hartford Insurance Company

John A. Rhoades
Vice President
Central Mutual Insurance

After an informal discussion and coffee break, a business session was held.

During this session, the Michelbacher Prize for 1979 was awarded to Robert P. Butsic for his paper "Risk and Return for Property-Casualty Insurers." This paper was reviewed by James M. Stanard.

The other call papers reviewed were:

"Measurement of Rates of Return for Casualty-Property Companies," by Norton E. Masterson, reviewed by Robert A. Bailey.

"Total Rate of Return and the Regulation of Insurance Profits," by Doctor Irving H. Plotkin, reviewed by LeRoy J. Simon.

"Total Return Pricing in Property-Casualty Insurance: The Massachusetts System," by Jeffrey Brown, reviewed by Holmes M. Gwynn.

"Restatement of the Consideration of Investment Income in Workers' Compensation Insurance Ratemaking," by Frank Harwayne, reviewed by David R. Bickerstaff.

"Insurance Profitability: An Economic Perspective," by Lee M. Smith, reviewed by Claus S. Metzner.

"Basic Economic Theory for an Insurer's Rate of Return and for Its Regulation," by John S. McGuinness, reviewed by R. Woody Beckman.

"Profit/Contingency Loadings and Surplus: Ruin and Return Implications," by Gary G. Venter, reviewed by Lee R. Steeneck.

"Analysis of Return on Surplus under Two Approaches for Including Investment Income in Ratemaking," by James D. Stewart, reviewed by Kenneth R. Frohlich.

"A Model for Calculating Minimum Surplus Requirements," by Robert J. Finger, reviewed by Robert S. Miccolis.

"An Illustration of the Impact of Inflation on Insurance Company Operations," by Stephen P. D'Arcy, reviewed by James P. Streff.

"Underwriting Profits Necessary to Keep Pace with the Increasing Premium Growth for Property-Casualty Companies," by John H. Muetterties, reviewed by George E. Davis.

"A Constrained Profit Maximization Model for a Multi-Line Property/ Liability Company," by Randall E. Brubaker, reviewed by Michael L. Toothman.

The business session was followed by a panel discussion entitled "Prediction of Turning Points."

Moderator: Robert W. Sturgis
Consulting Actuary
Tillinghast, Nelson & Warren, Inc.

Panelists: Robert A. Brian
General Partner
Conning & Company

Sheldon Rosenberg
Associate Actuary & Manager
Insurance Services Office

Stanley Wright
Managing Consultant
Data Resources, Inc.

The closing remarks were made by President Ruth E. Salzmann after which the meeting was adjourned at 12:00 noon.

The meeting was attended by 196 Fellows, 162 Associates, 14 subscribers, 46 guests, 5 students and 197 spouses. The list of attendees follows:

FELLOWS

Aldorisio, R. P.	Anker, R. A.	Bailey, R. A.
Alexander, L. M.	Arata, D. A.	Balcarek, R. J.
Anderson, D. R.	Asch, N. E.	Barnes, G. R.
Angell, C. M.	Atwood, C. R.	Bartlett, W. N.

FELLOWS

Beckman, R. W.
Bell, L. L.
Bellinghausen, G. F.
Bennet, N. J.
Berquist, J. R.
Berry, C. H.
Bethel, N. A.
Beverage, R. M.
Bickerstaff, D. R.
Bill, R. A.
Bishop, E. G.
Bland, W. H.
Bornhuetter, R. L.
Bovard, R. W.
Brannigan, J. F.
Brian, R. A.
Brouillette, Y. J.
Brown, W. W.
Brubaker, R. E.
Buck, J. E.
Carbaugh, A. B.
Carter, E. J.
Cheng, L. W.
Childs, D. M.
Conners, J. B.
Cook, C. F.
Crowley, J. H.
Curley, J. O.
Curry, A. C.
Curry, H. E.
Daino, R. A.
D'Arcy, S. P.
Davis, G. E.
Degerness, J. A.
Dempster, H. V.
Donaldson, J. P.
Dorval, B.
Drennan, J. P.
Dropkin, L. B.
Eddy, J. H.
Eland, D. D.
Even, C. A.
Eyers, R. G.
Faber, J. A.
Fagan, J.
Fallquist, R. J.
Ferguson, R. E.
Fiebrink, M. E.
Finger, R. J.
Fisher, W. H.
Flaherty, D. J.
Flynn, D. P.
Fossa, E. F.
Fowler, T. W.
Fresch, G. W.
Fusco, M.
Garand, C. P.
Gersie, M. H.
Gibson, J. A.
Gillam, W. S.
Gillespie, J. E.
Goddard, D.C.
Gottlieb, L. R.
Grady, D. J.
Graham, T.L.
Grannan, P. J.
Graves, J. S.
Grippa, A. J.
Groot, S. L.
Hachemeister, C. A.
Hafling, D. N.
Hall, J. A.
Hanson, H. D.
Hartman, D. G.
Hartman, G. R.
Harwayne, F.
Haseltine, D. S.
Hazam, W. J.
Heer, E. L.
Hermes, T. M.
Hewitt, C. C.
Hough, P. E.
Hoylman, D. J.
Hughey, M. S.
Inkrott, J. G.
Jean, R. W.
Jerabek, G. J.
Jones, A. G.
Kaliski, A. E.
Kallop, R. H.
Kates, P. B.
Kaufman, A.
Keene, V. S.
Kelly, A. E.
Khury, C. K.
Kilbourne, F. W.
Kollar, J. J.
Krause, G. A.
Kuehn, R. T.
Lamb, R. M.
Lehmann, S. G.
Leonard, G. E.
Leslie, W.
Levin, J. W.
Lino, R. A.
Lowe, R. F.
MacGinnitie, W. J.
Masterson, N. E.
McGuinness, J. S.
McLean, G. E.
McManus, M. F.
Miller, D. L.
Moore, B. C.
Moore, P. S.
Morrison, G. D.
Muetterties, J. H.
Munro, R. E.
Neidermyer, J. R.
Nelson, J. R.
Newlin, P. R.
Oakden, D. J.
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FELLOWS

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Zubulake, T. J.

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Wright, S. H.
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Respectfully submitted,

DAVID P. FLYNN
Secretary

Volume LXVI, Part 2 No. 126

PROCEEDINGS

November 14, 15, 16, 1979

ACCOUNTABILITY: THE ACTUARIAL IMPERATIVE

PRESIDENTIAL ADDRESS BY RUTH E. SALZMANN

"By their fruits ye shall know them."
— Matthew 7:20

The Presidential Address is the "last rite" in the term of office of the president. Last year, the presidents of the Society of Actuaries, the Conference of Actuaries, and the Casualty Actuarial Society chose the subject of professionalism in their addresses. Adger Williams, as you remember, focused our attention on "The Challenge of Being Professional." He stated forthrightly that the professionalism of the casualty actuary is being challenged, and he encouraged us to meet these challenges by communicating a clear identity and by operating so as "to be what we claim to be."

I should like to explore with you the stable-mate of professionalism, which is accountability. "By their fruits ye shall know them" will be more applicable to casualty actuaries in the 1980's than ever before. And how will the harvest be judged?

At the present time, the consumer has developed a new posture of attacking conventional practices. Consumerism is a strong, inner-directed, individualistic movement that says, "I count." The movement has generated skepticism, which has caused an erosion of the credibility of the insurance business and of the credibility of the expert. Though such consumer attitudes may have been influenced by a lack of understanding and by a propensity to generalize from a few incidents, nonetheless they exist and should be addressed.

The ancient advice of Virgil was "Believe an expert." But the best of experts have now and then launched an unsinkable Titanic or built an Edsel. As experts in an inexact science dealing in quantifications affected by future contingent events, we shall certainly miss the mark in many of our measurements. We are not professional soothsayers, and we must make certain that we do not pretend to be. However, with our trained and scientific minds, we should be able to quantify inherent risks and future costs better than any other discipline can, when evaluated over time. For that reason, the members of our profession should be willing to be held accountable over time.

Such a willingness will cause a change in the relationship between actuaries and the lay public. We must in some way convey to the consumer that he is not being unfairly treated; that he is not being placed at the mercy of the "magic of averages" or the "law of large numbers." It should be our mission to endorse and support our methodology without leaving the impression that present methodology is omnipotent. Our responsibility to the public, it seems to me, is to continually examine our methodologies, test the end-results produced therefrom, and be accountable for the success or failure of our work.

In private passenger automobile insurance, a premium for an insured is developed either as the sum or as the product of factors for age, sex, marital status, territory, driving record, etc. Rather than limiting our actuarial studies to refinements in the measurement of the various rating components individually, should we not test the system itself by measuring the reasonableness of each "package" of rating variables? If loss experience, sorted by package of rating variables, proves to be too refined for statistical significance, then a cruder sort, by size of annual policy premiums, would be in order. An analysis of this kind is one that actuaries should be anxious to perform. It would provide a means for determining whether we are measuring what we intend to measure, i.e., the exposure to loss within a risk.

We learned in the property lines that the packaging of coverages produced a new pricing approach. Might the recognition of "packaged rating variables" produce new concepts in the pricing of automobile insurance? Might there be a synergistic effect in a package of rating variables as compared to the independent costing of each? Such a continuous testing program would supply the evidence.

It is the actuary's responsibility to produce a total premium level that is adequate in the aggregate. Also, the actuary is responsible, via classification systems and experience rating techniques, for price distinctions that discriminate fairly among individual insureds. The products emanating from both of these responsibilities must meet the tests of time; and in both cases, the actuary must be willing to be accountable for results. How else can the actuary, or anyone else, determine over time whether the end-result justifies the methodology?

The scientific mind employs deductive reasoning and a systematic approach in a manner that is basically a vertical thought process. This same vertical thought process is used in the continuous updating of methodology and measurements. Such updating is the periodic housecleaning that needs to be done. However, this thought process is not always conducive to innovation. To be innovative, the actuary needs to employ "lateral" thought processes as well. For instance, the actuary should continue to test the appropriateness of each rating component, but the actuary should also establish other tests to determine the reasonableness of the resulting premium relativities by risk. It is the final premium charge to each consumer that matters, not the accuracy of the separate rating components.

"Lateral" thought processes help to introduce external facts and events — even some not directly related — in order that new and broader perspectives can result. "Lateral" thought processes will allow us to deal in concepts, rather than discourses in mathematical derivations. This suggestion does not mean that we should hide our mathematical and statistical techniques in the closet. On the contrary, we must make it clear that the implementations of statistical concepts are dependent on mathematical skills and that such skills are clearly indispensable in the sophisticated conduct of the casualty and fire insurance business. But whereas statistics and logic are our tools, they do not always succeed alone. For instance, might a driver more readily accept the price of gasoline as it passes $1 per gallon, if he were reminded that a gallon of coffee costs $1.75; a gallon of milk, $2; and a gallon of Head and Shoulders shampoo, $28?

This example is more humorous than meaningful. It is included only to make a point. The "lateral" thought process will produce a peripheral view that will encourage us to take a look in at our actuarial practices from the outside. Such continuous examinations are needed to satisfy ourselves that inequities do not occur as by-products of our methodology.

This kind of self-review will help us to build a better relationship with the consumer. But such a relationship should be a two-way street. We should exchange challenges. Let us tell the consumer which aspects of insurance costs are within his control. For instance, we can show him that past driving records are predictive of future loss experience — and to what degree. Violations and "at-fault" accidents are controllable by him, not by the insurance company. The sophisticated consumer must surely recognize that fair discrimination in prices is to his advantage in the long run; for equitable pricing is the best means available to minimize discrimination by selection, a practice that the consumer finds so repugnant.

Then there is the much talked-about problem of affordability. As a first step, might we suggest to the leaders of the consumer movement that they seek an income tax revision that would allow a casualty loss deduction of 50% of the premiums for automobile and homeowners or tenants insurance? The taxpayer now receives a medical deduction of 50% of his medical insurance premiums. In today's world, is not insurance on one's automobile and home equally necessary? It seems to me that the underlying rationale is quite similar.

Whatever the means or subjects selected, we must strive to improve our communications with the consumer. A new "I'm OK; you're OK" relationship should be mutually rewarding.

I would now like to make some comments about our accountability and relationships in another major area of our profession — the quantification of estimated liabilities for loss and loss expense. Harold Schloss stated in his 1968 Presidential Address: "Actuarial science is a 'soft' science and in the realm of loss reserving is perhaps as much art as science." It is now 1979, and the statement is still appropriate.

In recent years, there has been extensive discussion about casualty loss reserves within our Society and within the insurance industry. The proposed Statement of Opinion on casualty loss reserves has intensified these discussions. Arguments are currently centered on the qualifications necessary for the person rendering such an opinion. Considerable controversy has surfaced; and I can see some similarity in these discussions to the old fairy tale about a miraculous looking glass — which always tells the truth.

Mirror, mirror on the wall,
Who's the most qualified of us all?

The mirror's final answer, we hope, will name the casualty actuary.

One might ask why a professional opinion on casualty loss reserves is needed. Let me provide you with some current data.

In the August 10, 1979, issue of the *Insurance Industry Newsletter*, the following statistics were reported:

> *THERE HAVE BEEN 30 P/L COMPANY INSOLVENCIES* in the last four years, mostly stemming from under-reserving. Out of 240 companies studied by an insurance research firm, 24% were found to be under-reserved by more than one fourth of their end-of-the-year surplus.

Then, in the 1978 Insurance Regulatory Information System (IRIS) reports, the following statistics were shown. The estimated loss and loss expense liabilities reported as of December 31, 1976 were $47.6 billion for the total of all groups and unconsolidated companies. Two years later, as of December 31, 1978, developments in Schedules O and P showed these estimated liabilities to be short by $4.16 billion, or 8.7%.[1] The inadequacy reported after two years of development is not, of course, a final measurement. A final run-off will more than likely show a greater inadequacy.

It should be pointed out that the inadequacy of the December 31, 1976 reserves is not the exception; it is the general rule. With this history of reserve inadequacy and with the insolvency data cited earlier, it is clear that the concern over casualty loss reserve levels by the regulators and the public is real.

The published data on the adequacy of reserves tells us one thing: we know how the score will be kept by the NAIC. The measurement of the adequacy of casualty loss reserves will be based upon the developments for all lines, taken from Schedules O and P (giving credit for all salvage recoveries). Though present measurements exclude loss expense for Schedule O coverages and are limited to developments through two years for Schedule O lines, I believe that extensions will occur in both of these areas in the near future. In any event, we know the ballgame and the rules.

The Casualty Actuarial Society, via its Board of Directors, has supported the position that casualty actuaries should be the professionals to sign Statements of Opinion[2] on casualty loss liabilities. This position is consistent with our Society's

[1] The inadequacy for the aggregate of all companies singly was 7.2%. The difference is difficult to explain, but most probably results from errors in the completion of the individual company or consolidated schedules, or both.

[2] At the time of this writing, whether a Statement of Opinion will be required in the Fire and Casualty Annual Statement, is still to be determined.

objective of promoting the science of measurements affected by future contingent events inherent in fire and casualty insurance. It is axiomatic then that casualty actuaries should be able, over time, to perform better in the area of the measurement of reserves than the representatives of any other discipline. If that axiom is accepted, there should be no hesitation on our part in accepting the accountability that such professional recognition carries with it.

How should we proceed to enhance and follow through on this professional territorial claim? First of all, we should identify and challenge certain myths which lead to serious misconceptions about the preciseness or degree of accuracy obtainable in the quantification of estimated liabilities for loss and loss expense.

The first such myth is the use of the term "reserve strengthening." This phrase frequently appears in print when the underwriting performance for a particular company is being evaluated for a current accounting period. For instance, the statement will be made that the reported results were adversely impacted because of the "reserve strengthening" that occurred. More often than not, an actual dollar figure of such strengthening will be specified. This dollar figure generally represents the amount of the unfavorable run-off of the previous year's reserves, an amount that was absorbed in the current accounting period. Such arithmetic produces only a half-truth; for if last year's reserves were inadequate by $X million and if this year's reserves prove to be inadequate by $X million, then no reserve strengthening has occurred in the current accounting period. The amount of reserve strengthening or weakening that occurs in any calendar year cannot be known until the adequacies of both the beginning and ending reserves are finally determined. If indeed "reserve strengthening" was intended in any accounting period, then any dollar amount should be specified as a "best estimate." Without such a qualification, the commentary implies that the reserves for new claims can be precisely determined. This impression does our science an injustice, because it leads the unwary to believe that a precise measurement is obtainable.

The second such myth is that reserves are the tools by which results are "managed." There is no question that reported profits are affected by a reduction or increase in reserve levels. The problem is that the change in reserve levels cannot be determined until all losses are settled. Statements about managed results, therefore, not only erode confidence but subtly imply that the right amount is known and deviated from. Would that this were so! In any event, the danger in these statements is the misconception conveyed that the right amount is determinable — an injustice to the difficulty and uncertainty involved in the quantification of reserves.

Another myth is the exaggerated role given to IBNR. It is a common belief that case reserves are established by claim departments, and IBNR reserves are established by actuarial or accounting departments. If a Statement of Opinion is to be required in the Fire and Casualty Annual Statement, the Opinion will cover *total* reserves. Adequacy calculations in the IRIS reports are based upon *total* reserves. The financial strength of a company is dependent upon the level of *total* reserves. To make a distinction between case reserves and IBNR is academic; the professional must address the adequacy of the entire valuation. If the professional chooses to use case reserves in his quantification, then case reserves are a component in his methodology — nothing more. The loss reserve actuary should be responsible for the total reserve — both on reported losses and on unreported losses. To continue to isolate our involvement or limit our concern to IBNR reserves is a subterfuge. We must seek the responsibility and accountability for the total reserve, no matter how it is quantified.

These are but a few of the myths; there are more. It is myths such as these that produce confusing impressions to the lay public, damage our image, and understate our role.

Leaving the subject of myths, the next issue that must be met head-on is the matter of "discounting loss reserves." I doubt that there is any quarrel among us with the general principle that recognizes the time value of money; i.e., a dollar that will be paid out in the future is worth less than a dollar today. And actuaries are accustomed to reducing future costs to a present value. But calculating a present value is only a means of preempting or capturing, as of any accounting date, future investment income on cash flow resulting from loss charges not yet disbursed.

By separating the two measurements conceptually, the possibility of a different accounting treatment becomes apparent. Instead of modifying costs to present value, investment income could be modified. In present accounting practice, revenue in each accounting period includes changes in accrued interest income. In like fashion changes in earned but unaccrued interest income could be credited to revenue. The only difference between these two adjustments of revenue is that accrued interest is measurable, while earned but unaccrued interest must be estimated.

Why should we even consider such a nontraditional approach in casualty and fire insurance accounting? Why should we even think of departing from the time-honored techniques employed by our life brethren? The reasons are twofold. First, such an approach would make it abundantly clear that the recognition of the time value of money need not be synonymous with the discounting of loss reserves. The second reason is that loss reserves, at least at present, are tested for adequacy on a full paid-dollar basis. With this background, it is important that we determine which set of adjustments will produce the better basis for evaluating the performance of the loss reserve specialist. The two approaches are accounting equivalents; both succeed in matching costs and revenue; only the accounting entries vary.

Personally, I favor separating the measurements of loss liabilities and earned but unaccrued interest over an integrated measurement on a present-value basis. My reasons are threefold. First, separate accounting is more consistent with the traditional separation of the underwriting and investment functions in fire and casualty financial reporting. Second, due to the fact that casualty and fire losses do not conform to orderly, periodic-payment patterns, it should be simpler to measure earned but unaccrued interest on a crude, lump-sum basis. And third, with separate measurements, the retrospective tests in Schedules O and P could remain intact.

It is apparent from the foregoing that there are many considerations involved in the study of this issue. What is important is that we direct our attention to this matter now, rather than later. If our preference is to adopt the integrated present-value approach, then we must also recommend specific and substantive changes in the measurement of loss reserve adequacy, or the measurement of the loss reserver's performance, or both. If the Society's Committee on Loss Reserves can identify a preference, it should document that preference and publicize it.

The foregoing comments covered items of common interest to us all. There is one additional matter, however, that affects us individually and needs to be addressed as fully as any of the others. This subject is the less-than-full recognition that many of our own company managements concede to the actuary's expertise in the loss reserving area. One cannot be accountable for a responsibility one does not have. Perhaps we need to convince managements that our commitment is real, that our perspective is broad, and that we are willing to accept responsibility with the accountability that goes along with it. It is my belief that managements that do not delegate authority to actuaries in the loss reserving area are hesitant because they recognize that measurements of loss reserves are not entirely mathematical and they fear that actuaries will treat them only as mathematical. I believe it is our practical abilities that managements may be faulting.

Even though in the end it will be our performance that keeps us in the game, perhaps we can get a chance at bat by breaking down some of the barriers to communications. We must make certain that management knows that we know that there is no all-encompassing mathematical formula, or no all-inclusive set of assumptions that will produce a "right" loss reserve amount. Actuaries in the "reserving" business have learned over the years that there is no purity in claim statistics and that there are many imponderables and many not-so-random variables that elude the tools of measurement. To paraphrase Will Rogers, "it don't work as good in practice as it reads in the papers." Informed judgment must always be involved. For the Schedule P lines, the quantification of reserves is the end-result of a continuous series of individual judgments.

It is the judgment area in which management wants to share. As Alexander Pope said, " 'Tis with our judgments as our watches, none go just alike, yet each believes his own." A management always retains the right to superimpose its judgment in any corporate matter. However, in the area of loss reserves, it is difficult to be objective when grading one's own report card. To effectively discourage the need for management to become involved, the loss reserve actuary must earn his credibility. Such credibility is achieved over time by maintaining objectivity, accepting responsibility, and being willing to be held accountable.

The loss reserve actuary is subject to many pressures; but when you became a member of the Casualty Actuarial Society, we "never promised you a rose garden." There will be occasions when you will share Henry Clay's sentiments when he said, "I would rather be right than be president." But on those occasions, be confident of your position. Remember that Speaker Thomas B. Reed answered Congressman Springer's quotation of Clay by saying: "The gentleman need not worry. He will never be either."

It is indeed imperative that accountability go hand-in-hand with loss reserve responsibilities. Anyone who assumes these responsibilities cannot be an individual who succumbs to pressure and prays a lot, for Statements of Opinion will come back to haunt the person who signs them in the name of God and profit.

Though the actuary does not have the inalienable right to the exercise of judgment relating to insurance statistical data, he does have the benefit of a discipline that encourages him to continuously seek new knowledge in quantitative relationships and measurements. Objectivity is an acquired taste, like olives. You have to be habituated to it. But in this inexact science of ours, we must also acquire a sensitivity or, as Dudley Pruitt suggested in his 1958 Presidential Address, a "delicacy of perception." There is a need for us to complement the aloofness conveyed by the mathematical elegance of a logical mind.

A willingness to stand up and be counted will help our image. Though the risks may be high in our inexact science, let us accept those risks. Accountability has no room for an "Oh, pshaw!" attitude or a Mona Lisa smile when hindsight tells us what should have been. Not all work products have the benefit of hindsight. Loss reserving does. It is hindsight that gives 100% credibility to a result after the fact. This knowledge after the fact, however, in no way implies that the actual result was the most likely value at the time of the estimate. Only a review of what was known at the time will enable anyone to make that judgment. Our performance should never be measured by one result, but by many.

For all of these reasons, I urge each of you to look ahead to the next decade with a willingness to accept the accountability that professionalism demands. Remember Harry Truman's desperate cry for a one-armed economist, someone who would not preface every opinion with, "On the other hand . . ." We should be the scientists, not the technicians, in the insurance world of tomorrow. I am confident that our profession will stand the test of time, for "By their fruits ye shall know them."

A STUDY OF RISK ASSESSMENT

RICHARD G. WOLL

Much attention has recently been directed towards the subject of risk assessment in private passenger automobile insurance.

In 1975, SRI International, a research organization, was commissioned to do a major study of insurance classification, or risk assessment. They defined a measure of its efficiency and developed a procedure to utilize this measure for automobile accident frequencies based on the assumption that individual accident experience was Poisson distributed. Based on this analysis they concluded that current pricing and selection practices in automobile insurance did a poor job of creating homogeneous groups of risks [1].

Shortly after the release of the SRI Report, the Massachusetts State Rating Bureau (SRB) addressed the same subject and concluded that current automobile risk assessment practices were not only ineffective but that their use generated side effects that were detrimental to society. They recommended that traditional actuarial rates, based on expected costs, should be modified on the basis of subjective judgments about what was "fair" or what would contribute to the welfare of society [2].

Even more recently, changing social values and arguments like those above helped to create a situation where an NAIC task force condemned present automobile risk assessment practices and concluded that:

> ". . . sex and marital status are seriously lacking in justification and are subject to strong public opposition, and should therefore be prohibited as classification factors." [3]

The fact that such an essential aspect of insurance has come into question indicates a need for more knowledge and a better understanding of how we can measure class homogeneity. It is the contention of this paper that the SRI procedure is based on an oversimplified model of reality and will understate the effectiveness of any risk assessment system because it assumes that no random or stochastic elements affect an individual's exposure to loss. An alternative model of the loss generating process is suggested and a more general measure of class homogeneity is developed which makes use of individual risk experience and the findings of credibility theory.

RISK ASSESSMENT MODELS

The purpose of risk assessment is to partition a risk population into groups whose members have a similar expectation of loss. This requires the assumption that such groups exist and that it is possible to distinguish them.

The best indication that these assumptions are reasonable is the fact that persistent classification differentials do exist and form the basis for present risk assessment systems. This attests to the fact that insureds differ from each other in *consistent* and *predictable* ways.

In order to study risk assessment, it is necessary to focus on the loss generating process. In this paper, the analysis will concentrate on that process as it affects the frequency of automobile accidents.

INDIVIDUAL RISK MODEL

We begin by assuming that the probability of loss for an individual within any period of time is determined by the nature and quality of that individual's driving experience. We will call the expected number of accidents resulting from any set of circumstances, *exposure*, and will use ϕ_i to denote the exposure for an individual i associated with a particular set of circumstances. We consider ϕ_i to be a function of driving environment, amount of driving, and driver characteristics.

More formally, we designate the function:

$$\phi_i = W(E,A,C)$$

where:

(1) E = Driving environment
(2) A = Amount of driving
(3) C = Driver characteristics

Since the value of ϕ_i is determined by individual circumstances which, in turn, are affected by all the uncertainties of daily life, *we consider ϕ_i itself to be the result of a stochastic process which is independent with respect to time.*

We assume further that the actual number of accidents arising from a particular value of ϕ_i is determined by a Poisson process with a parameter equal to ϕ_i [4]. This means that the conditional distribution of claims for the ith individual is:

$$g(X_i | \phi_i) = \frac{(\phi_i)X_i}{X_i!} \quad e^{-\phi_i}$$

where X_i is a random variable denoting the actual number of accidents, given a particular value of ϕ_i, and $g(X_i \mid \phi_i)$ is the conditional probability density function (pdf) of X_i given ϕ_i. If we denote the distribution function of ϕ_i as $V(\phi_i)$, we can write the unconditional pdf of X_i as:

$$g(X_i) = \int_{\phi} \frac{(\phi_i)X_i}{X_i!} e^{-\phi_i} \, dV(\phi_i)$$

It can be seen, given these assumptions, that the actual distribution of accidents for the ith individual is compound-Poisson with moments which can be expressed in terms of the exposure function as follows [5]:

$$E(X_i) = E(\phi_i) \tag{1}$$

$$Var(X_i) = E(\phi_i) + Var(\phi_i) \tag{2}$$

Since the exposure function is independent in time, it is also possible to express the mean and variance of ϕ_i for different time intervals as follows:

$$E_t(\phi_i) = t \times E(\phi_i) \tag{3}$$

$$Var_t(\phi_i) = t \times Var(\phi_i) \tag{4}$$

where t represents the ratio of the time interval of interest to that used to define ϕ_i. The mean and variance of X_i for such a time interval are thus:

$$E_t(X_i) = t \times E(\phi_i) \tag{5}$$

$$Var_t(X_i) = t \times [E(\phi_i) + Var(\phi_i)] \tag{6}$$

GROUP RISK PROCESS

When we consider a group of individuals, we are interested in the unconditional distribution of X which can be thought of as the actual number of accidents happening to an individual selected at random from the group. This requires knowledge about the individual risk process, and the distribution of individual expected losses, the distribution of $E(X_i)$.

We begin by using the random variable M to denote the distribution of expected losses between individuals and define its distribution function as $U(m)$. The function $U(m)$ has been designated the "structure" function and can be thought of as a description of the structure of expected loss differences throughout the given population [6]. It should be evident that the value of M for a particular individual is equal to $E(\phi_i)$, the expectation of the individual's exposure function. That is:

$$M = E(\phi_i)$$

In this context the distribution of X_i, the accident frequency for the i th risk, is conditional on the value of M and we denote its pdf as follows:

$$h(X|m_i) = g(X_i)$$

The unconditional pdf of X is thus:

$$h(X) = \int_m h(X|m)\, dU(m)$$

The mean of this distribution is equal to $E(M)$ and the variance is equal to the variance of the expected accident frequencies, $Var(M)$, plus the expected value of the variance for each individual, $E(Var(X|m_i))$ [7]. Thus:

$$E(X) = E(M) \tag{7}$$

$$Var\,(X) = Var\,(M) + E(Var(X|m_i)) \tag{8}$$

$$= Var\,(M) + E[E(\phi_i) + Var(\phi_i)]$$

$$= Var(M) + E(M) + E[Var(\phi_i)] \tag{9}$$

Thus the unconditional variance of X is equal to the sum of the mean and variance of the structure function plus the average variance of the individual exposure functions.

We can observe the effect of time on the moments of the accident distribution by noting first that it acts as a scaling factor with respect to the moments of the expected loss distribution [8]. That is:

$$E_t\,(M) = t \times E(M) \tag{10}$$

$$Var_t\,(M) = t^2 \times Var(M) \tag{11}$$

When we consider the moments of X, the distribution of actual accident frequencies for different time intervals, we get the following:

$$E_t\,(X) = E_t\,(M) \tag{12}$$

$$Var_t\,(X) = Var_t\,(M) + E[Var_t\,(X|m_i)] \tag{13}$$

$$= t^2 \times Var\,(M) + t \times E[E\,(\phi_i) + Var\,(\phi_i)]$$

$$= t^2 \times Var\,(M) + t \times [E(M) + E(Var\,(\phi_i)] \tag{14}$$

Thus the variance of accident frequencies for a group of individuals is a quadratic function of time with respect to the structure function variance and a linear function of time with respect to the expected variance of individual accident frequencies!

EFFICIENCY STANDARDS

In 1960, R.A. Bailey introduced the idea of evaluating risk assessment systems by comparing the coefficient of variation for classification relativities to the coefficient of variation of the distribution of individual expected losses [9]. L.H. Roberts suggested, in turn, that a ratio of variances resulting in what he called a "coefficient of determination," rather than a ratio of coefficients of variation, might be preferable [10]. Both Bailey and Roberts were interested in what is now termed "class plan efficiency" from the viewpoint of competition.

Sixteen years later, SRI International suggested using the variance measure proposed by Roberts as a way of measuring what percentage of what is ultimately possible has been achieved [11]. It is a measure of how well the system does relative to the ideal situation where the value of M for each individual is known.

It is important to realize that risk assessment represents a partition of the structure function and that the variance of M can be separated into two components related to such a partition:

(a) *Between cell variance* $= BVAR_m$

(b) *Within cell variance* $= WVAR_m$

Thus:

$$Var(M) = BVAR_m + WVAR_m$$

In these terms, the SRI measure can be expressed as:

$$Efficiency = \frac{BVAR_m}{BVAR_m + WVAR_m} = \frac{1}{1 + \dfrac{WVAR_m}{BVAR_m}} \tag{15}$$

SRI International uses the variance produced by the risk assessment system partition to estimate $BVAR_m$. To estimate $Var(M)$, they assume that the distribution of claims for an individual risk, $g(X_i)$ is Poisson and that $U(m)$ is gamma distributed. This in turn leads to the conclusion that:

$$Var(M) = Var(X) - E(X)$$

Thus the SRI procedure consists of measuring classification variance and dividing it by the difference between the estimated mean and variance of the actual claims distribution [12].

Note that the terms $BVAR_m$ and $WVAR_m$ at this point refer to partitions within the structure function. The within class variance term, $WVAR_m$, refers to the average variance of M within the cells of the partition produced by the risk assessment system, while the between class variance term, $BVAR_m$, refers to the variance of expected loss frequencies between the cells. They refer to the variance of expected loss, not actual loss.

The SRI measure is not the only one which can be used for this purpose. Millicent Treloar, a statistical research analyst with the NAII, has noted:

> "If efficiency were expressed as:
>
> $$\frac{BVAR_m}{WVAR_m}$$
>
> we would have a measure which increases as the spread of class relativities and class homogeneity increase. We would also have a quantity of known distribution (an F distribution) by which we could make inferences about the extent of spread of class relativities (and homogeneity). Further, this quantity is that which is employed in classic statistics applications to classification problems dating back to R. Fisher (1936).
>
> "It is most desirable to utilize a measure of efficiency which has a known distribution when one desires to make statements of confidence about a particular value." [13]

MEASURING RISK ASSESSMENT EFFICIENCY-AN EXAMPLE

Before proceeding further with this exposition, a simple example may help to clarify what is meant by risk assessment efficiency. Suppose we have a risk population with the following structure function:

$$U(m) = \begin{cases} 10m & m = .01, .02, .03, \ldots, .10 \\ 0 & \text{otherwise} \end{cases}$$

$E(M)$ in this case is .055 and $Var(M)$ is .000825.

We can illustrate this structure as follows:

	Group									
	1	2	3	4	5	6	7	8	9	10
Accident Frequency:	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10

Suppose that we decide to partition this population into two classes and that our first attempt to do so assigns groups 1, 3, 5, 7, and 9 to the first class and the remainder to the other class. The two classes would look as follows:

	Group									
Class 1	1		3		5		7		9	
Class 2		2		4		6		8		10

This is not a very impressive partition, and the statistics show it:

			Variance		
	Weight	Mean	Within Group	Between Group	Eff.
Class 1	.5	.050	.000800	XX	XX
Class 2	.5	.060	.000800	XX	XX
Total	1.0	.055	.000800	.000025	3%

We learn more about our population, and succeed in producing a better partition:

	Group									
Class 1	1	2			5		7	8		
Class 2			3	4		6			9	10

The statistics for this group verify the fact that it is better:

			Variance		
	Weight	Mean	Within Group	Between Group	Eff.
Class 1	.5	.046	.000744	XX	XX
Class 2	.5	.064	.000744	XX	XX
Total	1.0	.055	.000744	.000081	10%

Continuing our efforts, we come up with a further improvement:

	Group									
Class 1	1	2	3		5		7			
Class 2				4		6		8	9	10

The statistics on this partition are as follows:

	Weight	Mean	Variance Within Group	Variance Between Group	Eff.
Class 1	.5	.036	.000464	XX	XX
Class 2	.5	.074	.000464	XX	XX
Total	1.0	.055	.000464	.000361	44%

Finally, one more plan is produced which divides the risk population as follows:

	Group									
Class 1	1	2	3	4	5					
Class 2						6	7	8	9	10

The statistics for this two partition set are quite impressive:

	Weight	Mean	Variance Within Group	Variance Between Group	Eff.
Class 1	.5	.030	.000200	XX	XX
Class 2	.5	.080	.000200	XX	XX
Total	1.0	.055	.000200	.000625	76%

This set of partitions provides a qualitative idea of what risk assessment efficiency means. It shows that greater efficiency, given the same number of partitions, generally means a greater spread of expected class relativities. This can be seen if one observes the class relativities which result from the partitions just presented:

	Partition 1	2	3	4
Lower Class	.91	.84	.65	.55
Higher Class	1.09	1.16	1.35	1.45

USING RELATIVITIES

In many situations, the variance of expected loss relativities produced by a particular risk assessment partition is more convenient to calculate than the variance of the actual expected loss estimates themselves. It would be convenient to express *Var (M)* in terms of relativities as well, so that direct comparisons can be made. Expected loss relativities are calculated by dividing the value of the random variable M by $E(M)$ and will be denoted by the symbol R. We can express the variance of R in terms of the variance of M as follows:

$$Var(R) = Var\left(\frac{M}{E(M)}\right) = \frac{Var(M)}{(E(M))^2}$$

We can determine the efficiency of any risk assessment partition by calculating the variance of the cell relativities produced by that system and then dividing by $Var(R)$. This, in turn is the same as multiplying by the squared mean of M divided by the variance of M. This quantity will henceforth be designated by the symbol BK such that:

$$BK = \frac{1}{Var(R)} = \frac{(E(M))^2}{Var(M)} = \frac{t^2 \times (E(M))^2}{t^2 \times Var(M)} + \frac{(E_t(M))^2}{Var_t(M)} \qquad (16)$$

Thus BK is independent of time. It can also be seen that:

$$Var(M) = \frac{(E(M))^2}{BK} \qquad (17)$$

It should be noted that BK is the inverse of the normalized variance of the structure function. Since we define homogeneity as the degree of similarity in expected losses for the members of any group, BK is a direct measure of the homogeneity of such a group. A high value of BK indicates a homogeneous group while a low value of BK indicates a relatively heterogeneous group.

It is easy to calculate the efficiency of the different partitions shown in the example above when we know BK. Since we know that $E(M) = .055$ and $Var(M) = .000825$, we have:

$$BK = \frac{(.055)^2}{.000825} = 3.67$$

Since BK is 3.67, we can determine the efficiency of these partitions by calculating the variance of the class relativities that they produce and multiplying the result by

3.67. Shown below are the efficiency estimates for each of these partitions calculated in this manner.

	Partition			
	1	2	3	4
(1) Class Relativities				
Lower Class	.91	.84	.65	.55
Higher Class	1.09	1.16	1.35	1.45
(2) Variance of (1)	.0081	.0268	.1193	.2066
(3) Efficiency				
(2) x 3.67 x 100	3%	10%	44%	76%

The value of *BK* can also be calculated within each class wherein it measures the variance between individuals within that class. In this case, it is a direct measure of class homogeneity!

We can observe the improvements in class homogeneity in the example by calculating the average value of *BK* for each class within each partition. It should be noted that since the average value of *BK* is the inverse of the average of the normalized variance for each class, one first obtains the normalized variance for each class by taking the inverse of *BK*. These values are then averaged and the inverse of the result is then taken. This point becomes more intuitive if one notes that if any single class were perfectly homogeneous, the variance in expected losses for members of that class would be zero and *BK* would be infinite. Clearly, a direct average of *BK* itself could lead to absurd results.

Shown below are the average *BK* values for each partition; these values are calculated in the appropriate manner:

		Partition			
	Population	1	2	3	4
BK Value	3.67	3.69	3.75	4.52	7.89
Efficiency	0%	3%	10%	44%	76%

We can see that more efficient partitions produce more homogeneous class cells.

THE SRI MEASURE OF RISK ASSESSMENT EFFICIENCY

In the example given above, the structure function, *U(m)*, was known. In reality, *U(m)* cannot be observed and must be estimated. *Only the mean and variance of M, however, are necessary for measuring risk assessment efficiency and class homogeneity.*

It is possible to estimate the moments of X, the actual claims distribution, by observing actual data. These estimates are of use in estimating the moments of M. It was shown earlier in formula (12) that:

$$E_t(X) = E_t(M)$$

$$Var_t(X) = E_t(M) + Var_t(M) + t \times E[Var(\phi_i)]$$

Thus:

$$Var_t(M) = Var_t(X) - E_t(X) - t \times E[Var(\phi_i)] \tag{18}$$

Both the SRI Report and the Massachusetts Rating Bureau studies assumed that the distribution of X_i was Poisson and thus $Var(X_i) = E(X_i) = E(\phi_i)$. Since $Var(X_i)$ is also equal to $E(\phi_i) + Var(\phi_i)$, this necessarily implies that $Var(\phi_i)$ is equal to zero in all cases. *In other words, these models assume that there are no elements of chance affecting exposure to loss.* This assumption makes it possible to simplify the formula for $Var(M)$ given above, since $E[Var\ \phi_i)]$ is also equal to zero.

Thus:

$$Var_t(M) = Var_t(X) - E_t(X)$$

We can express this result in terms of BK:

$$Var_t(M) = \frac{(E_t(M))^2}{BK} = Var_t(X) - E_t(X)$$
$$= Var_t(X) - E_t(M)$$

Thus:

$$Var_t(X) = E_t(M) + \frac{(E_t(M))^2}{BK}$$

If we look at the SRI terminology, we see that they write $Var_t(X)$ as follows:

$$Var_t(X) = mt + \frac{(mt)^2}{K}$$

where $mt = E_t(M)$. It can be seen that in this case, BK and K are equal [14].

K can be expressed in all cases (not just the Poisson case) as a function of "excess" variance, the difference between the mean and variance of X, as follows:

$$K = \frac{(E_t(X))^2}{Var_t(X) - E_t(X)} \tag{19}$$

since $E_t(M) = E_t(X)$.

Given these assumptions, *the SRI procedure for assessing the efficiency of a risk assessment system is to calculate the observed mean and variance of the distribution of actual losses for the risk population, $Var_t(X)$, and then to estimate the variance of expected losses, the variance of the structure function, by subtracting the population mean from the population variance:*

$$Var_t(M) = OV - OM$$

where OM and OV are the observed mean and variance of the actual claim distribution.

The SRI method thus "solves" the problem of measuring $Var_t(M)$ by assuming that $E[Var_t(X|m_i)] = E_t(M) = E_t(X)$ and thus that all of the "excess variance" is due to the variance of expected losses. That is:

$$\begin{aligned} Var_t(M) &= Var_t(X) - E[Var_t(X|m_i)] \\ &= Var_t(X) - E_t(X) \end{aligned}$$

If, in fact, $E[Var_t(X|m_i)]$ is not equal to $E_t(X)$ then the SRI method will not work!

MASSACHUSSETTS DEVELOPMENTS

In 1977, the State Rating Bureau disregarded the preliminary nature of the SRI conclusions and made them the basis for a severe indictment of current risk assessment practices. They declared that pricing groups of risks according to their expected loss costs was improper and should be prohibited [15]. In doing so they relied heavily on the SRI conclusions about the efficiency of the risk assessment process:

"current risk assessment schemes in automobile insurance resolve only a small fraction of the uncertainty about individual expected losses." [16]

They pointed out, further, that the SRI study claimed that the "fraction explained" was about 30% and observed that:

"with so much of the difference in expected loss among individuals unresolved, heterogeneous classes are unavoidable." [17]

Finally, they threw cold water on the idea that the way to improve the situation is to improve the class plan. They did this on both practical grounds and because they felt that using certain kinds of information in risk assessment might be socially undesirable [18].

A significant part of the SRB's effort in 1977 was a study of Massachusetts data in order to gain some idea of the expected loss variance in each rating class. Data pertaining to collision coverages was published in a paper on merit rating and is included in this paper as Exhibit I [19].

This data is more suitable than the data used by the SRI to examine the process of risk assessment in insurance because:

(1) It is insurance data.
(2) It represents a complete cross section of insurance business.
(3) It shows differences in homogeneity by class.

MERIT RATING

The intent of the exhibit published by the SRB was to show that class and territorial relativities in Massachussetts should be modified because of the impact of merit rating. In making this point, merit rating data had to be generated through a simulation process since no actual data about individual risk experience existed in Massachussetts at that time. I generated the same data through a computer simulation of the following formula:

$$P(X=x) = \int_m m\ g(x|m)\ dU(m)$$

where $U(m)$ is a gamma distribution function and $g(x|m_i)$ is the Poisson probability of having X claims given the parameter m_i. See the Appendix for a further description of the simulation process.

It is worth noting that the pdf:

$$P(M=m_i|x) = \frac{\int_{m_i} P(X=x|m_i)\; dU(m_i)}{\int_{m} P(X=x|m)\; dU(m)}$$

where x is a discrete number of claims, represents the accident likelihood distribution for risks which have had x claims and is also a gamma distribution. This fact has been pointed out by several commentators including the SRI [20].

It is particularly important to note that the ratio:

$$\frac{E(M|x)}{E(M|0)} = \frac{\int m\; P(M=m|x)\; d(m)}{\int m\; P(M=m|0)\; d(m)}$$

or the ratio of expected loss frequencies for risks who have had x losses and those who have had none, given the assumption that the structure function is gamma, is:

$$\alpha(x)/\alpha(0) = 1 + x/K \tag{20}$$

where $\alpha(x)$ is the expected mean for those risks with x claims. In particular, the ratio of expected means for risks with one claim during this interval of time, and those with none, is:

$$\alpha(1)/\alpha(0) = 1 + 1/K \tag{21}$$

which is dependent only upon the coefficient of variation of expected losses for the subpopulation under consideration.

Exhibit II shows a grid of data by class by merit rating category generated by the simulation process mentioned above. Part 3 of Exhibit II shows the ratio of expected means for risks with X claims divided by the expected mean for risks with 0 claims. The following table reproduces these ratios for risks with one claim:

Class	$\alpha(1)/\alpha(0)$
00	1.98
10	1.57
12	1.46
15	1.48
20	1.54
22	1.54
24	1.58
26	1.37
30	1.35
31	2.76
40	1.37
42	1.29
50	1.44

Using these ratios, it is easy to compute *BK* values for each class using the formula:

$$BK = \frac{\alpha(0)}{\alpha(1) - \alpha(0)} \tag{22}$$

The following table shows the *BK* values estimated in this manner compared to those underlying the simulation:

Class	*BK*	Estimate
00	1.03	1.03
10	1.75	1.75
12	2.15	2.16
15	2.06	2.06
20	1.96	1.91
22	1.95	1.90
24	1.77	1.75
26	2.72	2.72
30	2.83	2.83
31	0.58	0.58
40	2.76	2.74
42	3.51	3.51
50	2.28	2.27

These results show that it is possible to use accident history data rather than the SRI assumptions to estimate class homogeneity.

AN ALTERNATIVE RISK ASSESSMENT MODEL

The SRI method for estimating class plan efficiencies carries with it the implication that the purpose of risk assessment is to determine each risk's exact exposure to loss.

If one considers the nature of the events that determine exposure to loss, it seems more reasonable to assume that exposure is only determinable in a stochastic sense. It was stated earlier that exposure to automobile accidents is determined by the following elements:

(1) Driving environment
(2) Amount of driving
(3) Driver characteristics

Each of these elements is affected by the uncertainties of daily life and should be regarded as random in nature. There are differences in exposure expectations between risks—the success of the current risk assessment system is ample evidence of that—but it seems clear that $Var\ (\phi_i)$, the variance of the exposure function of the individual risk, is likely to be significantly greater than zero and thus the variance of the individual accident distribution, $Var\ (X_i | \phi_i)$, has to be greater than its mean. This follows from formula (2), the formula for the variance of the individual claims distribution given earlier:

$$VAR\,(X_i) \;=\; E(\phi_i) \;+\; Var(\phi_i)$$

In order to estimate the impact that exposure variance might have on the SRI method for estimating risk assessment efficiency, a comparative set of estimates will be calculated, assuming:

(1) $Var(\phi_i) \;=\; 0$
(2) $Var(\phi_i) \;=\; .0625 \;\times\; (E(\phi_i))^2$

For each case, we can calculate the moments of X given these assumptions about $Var\ (\phi_i)$ and the facts about the structure function used in the example given earlier in this paper. In the example, $E(M)$ was .055 and $Var\ (M)$ was .000825.

In the first case, using formula (8):

$$\begin{aligned} Var(X) &= Var\,(M) + E[Var(X|m_i)] \\ &= Var(M) + E(M) + E[Var(\phi_i)] \\ &= .000825 + .055 + 0 \\ &= .055825 \end{aligned}$$

Using the SRI method:

$$\begin{aligned} Var(M) &= Var(X) - E(X) \\ &= .055825 - .055 = .000825 \end{aligned}$$

and:

$$\begin{aligned} BK &= \frac{E(M)^2}{Var(M)} \\ &= .055^2 \div .000825 = 3.67 \end{aligned}$$

We can estimate the efficiency of the various partitions in the example, given the SRI assumptions, by multiplying the variance of the class relativities they produce by 3.67.

In the second case, we have:

$$\begin{aligned} Var(X) &= Var(M) + E[Var(X|m_i)] \\ &= Var(M) + E(M) + E[Var(\phi_i)] \end{aligned}$$

Since $Var(\phi_i) = .0625 \times (E(\phi_i))^2$ and $E(\phi_i) = M$ we have:

$$\begin{aligned} E[Var(\phi_i)] &= .0625 \times E(M^2) \\ &= .0625 \times [Var(M) + (E(M))^2] \\ &= .0625 \times (.000825 + .055^2) \end{aligned}$$

Thus:

$$\begin{aligned} Var(X) &= .000825 + .055 + .000241 \\ &= .056066 \end{aligned}$$

Again applying the SRI method, we calculate *Var (M)* and *BK*:

$$\begin{aligned} Var(M) &= .056066 - .055 = .001066 \\ BK &= .055^2 \div .001066 = 2.84 \end{aligned}$$

Since *Var (M)* is really .000825 and *BK* is really 3.67, it can be seen that the use of the SRI method does not provide an accurate picture of the effectiveness of risk assessment. If we were to use the *BK* estimate of 2.84 to evaluate the efficiency of the partitions used in the example, we would be 23% too low!

The fact is that the SRI method is not really an estimate of risk assessment efficiency at all. It is, in fact, an estimate of the lower bound of that efficiency. If *Var* (ϕ_i) for *any* risk is greater than zero, then the SRI estimate will be too low.

It is not the SRI measure that fails, but the assumption that it is possible to estimate the variance of expected losses, $Var_t\,(M)$, by subtracting the mean of the actual loss distribution, $E_t\,(X)$, from its variance, $Var_t\,(X)$. What is needed is some other method for estimating $Var_t\,(M)$.

Since the structure function itself cannot be directly observed, any inferences that can be made about its characteristics must come from observation of actual claims experience. We know that for any group:

$$Var_t\,(X) \;=\; Var_t\,(M) \;+\; E[Var_t\,(X|m_i)]$$

or, since we have a partition of the risk population achieved by the expected losses for each member:

$$TVAR_x \;=\; BVAR_x \;+\; WVAR_x$$

where $BVAR_x$ is the variance between risks and $WVAR_x$ is the expected value of the within risk variance.

It is particularly important to avoid confusing the concepts of between variance and within variance as used here with their use in the SRI efficiency measure. The total variance term used above refers to the variance of actual losses, $Var_t\,(X)$, while the total variance term used in the SRI measure refers to the variance of expected losses, $Var_t\,(M)$. The within variance term used above refers to the variance of individual losses while the within variance term used in the SRI measure is the variance in expected losses remaining within each partition created by a risk assessment system. It is interesting to note that $BVAR_x$ taken with respect to the distribution of actual losses is identical to $TVAR_m$ taken with respect to the distribution of expected losses. That is:

$$BVAR_x \;=\; TVAR_m \;=\; Var_t\,(M)$$

Since $BVAR_x \;=\; Var_t\,(M)$, it can also be expressed in terms of *BK* by using formula (17) as follows:

$$BVAR_x \;=\; \frac{(E_t\,(M))^2}{BK} \tag{23}$$

and can continue to express the variance of X, or $TVAR_x$, in terms of "excess variance," as follows:

$$TVAR_x = Var_t(X) = E_t(M) + \frac{(E_t(M))^2}{K} \tag{24}$$

See formula (19) for the definition of K as a function of "excess variance."

We know that $Var_t(X_i)$ for any individual is either equal to or greater than $E_t(X_i)$. Thus we also know that $WVAR_x$, or $E[Var_t(X_i)]$, is greater than or equal to $E_t(M)$ since:

$$E[Var_t(X_i)] \geqslant E[E_t(X_i)] = E_t(M)$$

We can, therefore, express $WVAR_x$ in terms of "excess variance" as well, using the quantity WK as the index of the degree to which $WVAR_x$ exceeds $E_t(M)$:

$$WK = \frac{(E_t(M))^2}{WVAR_x - E_t(M)}$$

and thus:

$$WVAR_x = E_t(M) + E[Var_i(\phi_i)] = E_t(M) + \frac{(E_t(M))^2}{WK} \tag{25}$$

We can now write:

$$TVAR_x = WVAR_x + BVAR_x$$

$$E_t(M) + \frac{(E_t(M))^2}{K} = \{ E_t(M) + \frac{(E_t(M))^2}{WK} \} + \{ \frac{(E_t(M))^2}{BK} \}$$

and thus:

$$\frac{1}{K} = \frac{1}{WK} + \frac{1}{BK}$$

$$K = \frac{WK \times BK}{WK + BK}$$

$$WK = \frac{BK \times K}{BK - K}$$

$$BK = \frac{WK \times K}{WK - K}$$

These formulae provide insight into the limits of both BK and WK. We know that K, WK and BK must all be positive (since we have concluded that $Var_t(X_i) \geqslant E_t(X_i)$ and $Var_t(X) \geqslant E_t(X)$) and we see that as either WK or BK approaches K, the other increases without bound. Thus we conclude that K is a lower bound for both variables, and there is no upper bound. It is interesting to note that when WK increases without bound, $WVAR_x$ becomes equal to $E_t(M)$, and $BVAR_x$ to:

$$\frac{(E_t(M))^2}{K}$$

When these conditions obtain, $g_t(X_i)$ becomes a Poisson distribution.

We see, therefore, that the Poisson case is a limiting case of the class of all compound-Poisson individual risk distributions.

Since $WVAR_x$ is at a minimum when the simple Poisson case obtains, $BVAR_x$ is at a maximum, BK is at a minimum, and estimates of risk assessment efficiency are minimized. *When WK possesses a finite value, estimates based on the simple Poisson assumption will invariably be understated.*

A GAMMA-NEGATIVE BINOMIAL SIMULATION

It was found in studying the Massachussetts data under the Poisson assumptions that claims history data gave a good estimate of BK, the index of population or subpopulation homogeneity. A simulation was run under the assumption that $Var(\phi_i)$ was not equal to zero in order to find out whether it was still possible to use the ratio method to get a good estimate of BK and thus of the variance of expected losses. In the simulation, $g_t(X_i)$ was assumed to be negative binomial with a variance equal to:

$$Var_t[X|E(\phi_i)] = \{t \times E(\phi_i)\} + \{\frac{(t \times E(\phi_i))^2}{10}\}$$

Thus:

$$Var_t(\phi_i) = \frac{(t \times E(\phi_i))^2}{10}$$

The results of this simulation are shown in Exhibit III. The value 10 was chosen for the denominator of the second term in the above equation because it seemed to provide results that were reasonably similar to those achieved in the Poisson simulation but which differed enough to provide a reasonable picture of how exposure variance might affect the observable characteristics of the risk population.

The results of this simulation compared with the gamma-Poisson case are as follows:

(1) *The number of risks with 0, 1, 2, . . . claims in a three year period is virtually the same in both instances!* Part 2 for both Exhibits II and III shows this distribution within each class for both cases. Shown below are the statewide claims distributions for each case along with negative binomial distributions possessing the same mean and variance.

	Compound Distributions			
	Gamma/Poisson		Gamma/Neg. Bin.	
Number of Claims	Actual	Neg. Binomial	Actual	Neg. Binomial
0	.652	.652	.653	.652
1	.247	.246	.249	.246
2	.074	.074	.073	.074
3	.020	.020	.020	.020
4	.005	.005	.005	.005
5	.001	.001	.001	.001

(2) *The ratio of expected losses for groups having x accidents in a three year period, $\alpha(x)$, to those having none, $\alpha(0)$, is substantially lower in the negative binomial case than it is in the Poisson case.* Part 1 of Exhibits II and III shows the values of $\alpha(x)$ within each class for the two simulations, while Part 3 shows their relativity to the 0 accident class. Part 4 shows the relativities to the class mean frequency. It is interesting to note that in both cases the frequency of claims in classes 20 and 22 is so high that even risks with one claim are better than the average for the class and should be charged a rate below the class average!

Shown below are the statewide expected frequencies and their relativities to the expected frequency of the group with zero accidents:

	Compound Distributions			
Number of	Gamma/Poisson		Gamma/Neg. Bin.	
Claims	Frequency	Relativity	Frequency	Relativity
X	$\alpha(X)$	$\alpha(X)/\alpha(0)$	$\alpha(X)$	$\alpha(X)/\alpha(0)$
0	.126	1.00	.135	1.00
1	.199	1.58	.191	1.42
2	.274	2.16	.247	1.83
3	.351	2.78	.304	2.26
4	.433	3.43	.363	2.69
5	.522	4.13	.425	3.15

These results can be explained by the fact that the expected loss distribution underlying the negative binomial case has less variance than that underlying the Poisson case. This is consistent with a model which assumes that more of the total population variance is explained by the variance of the individual risk processes, and less by the variance between risks. This is also evident in the K and BK values resulting from each case. Column 1 from Part 1 of Exhibits II and III shows the BK values underlying the class expected loss distributions in each case, while column 2 shows the K value underlying the actual distribution of claim frequencies. These two columns should be identical in Exhibit II, the Poisson case, but the limitations of the simulation process resulted in slight differences.

In Exhibit III, the value BK of the expected loss distribution over the entire state is 2.22 while the value K of the claim frequency distribution is 1.68. Thus it can be seen that if the negative binomial assumption used in the example is a better picture of reality than the Poisson assumption, a given class plan will actually be 32% more efficient than the SRI methodology would indicate. This difference in class plan efficiency estimates can be observed when we test the efficiency of rates based on the claim frequencies shown in the SRB exhibit (Exhibit I). Shown below are the class relativities and their variance:

Class	Relativity (R_i)	Distribution ($Prob\,(R_i)$)	Variance
00	1.067	3.5%	XX
10	0.938	58.0	XX
12	0.889	10.2	XX
15	0.726	9.3	XX
20	2.213	0.1	XX
22	2.192	0.2	XX
24	1.514	1.0	XX
26	1.313	7.4	XX
30	1.067	2.8	XX
31	0.807	1.3	XX
40	1.621	1.9	XX
42	1.800	2.9	XX
50	1.319	1.3	XX
Total	1.000	100.0%	.053

From the formula given earlier for estimating the efficiency of a risk assessment system:

$$Efficiency = BK \times \Sigma\ [(R_i - 1)^2 Prob\,(R_i)]$$

we see that this class plan would be 8.9% efficient if *BK* were equal to 1.68, (the Poisson case), and 11.8% efficient if *BK* were equal to 2.22 (the negative-binomial case).

(3) *The efficiency of a merit rating plan is reduced in the negative binomial case, compared with the Poisson case.* The total efficiency of rates based on the indicated frequencies shown in Exhibit III Part 1 is 26.4%, while it would be 28.9% if rates were based on Exhibit II Part 1, generated from the gamma-Poisson model. This is all the more surprising since the class plan by itself (without claims history) is more effective in the negative binomial case. This effect is due, of course, to the reduced variance underlying the accident likelihood distribution shown in Exhibit III. The efficiency contribution of the claims history portion of such a rating plan is 14.6% in the negative binomial case and 20.0% in the Poisson case!

(4) *The ratio of expected frequencies for risks with one claim in three years to those with none is still a good indicator of class homogeneity, but not quite as good as in the Poisson case.* Shown below are the ratios for each class, the indicators of class *BK* values based on them, and the actual *BK* values underlying the simulation:

Class	$\alpha(1)/\alpha(0)$	*BK* (Est)	*BK* (Actual)
00	1.770	1.30	1.25
10	1.410	2.44	2.33
12	1.317	3.15	3.02
15	1.338	2.96	2.85
20	1.378	2.65	2.48
22	1.380	2.63	2.47
24	1.412	2.43	2.30
26	1.230	4.35	4.09
30	1.219	4.57	4.34
31	2.462	0.68	0.66
40	1.227	4.40	4.12
42	1.158	6.34	5.88
50	1.294	3.41	3.22
Total	1.419	2.39	2.22

These ratios give a reasonably good estimate of the *BK* values underlying the accident likelihood distribution, *but are definitely biased.*

CREDIBILITY THEORY AND RISK ASSESSMENT

It seems evident that dividing risks into groups according to the number of claims they have experienced over a particular period of time and then observing the results over a subsequent period can provide insight into class homogeneity and the efficiency of risk assessment.

There is a need, however, for a better understanding of the way that individual experience and expected loss distributions relate to each other.

It has long been recognized that in many instances greater rate accuracy can be gained by utilizing both group information and individual risk experience. Credibility theory was developed, in part, as a tool for combining these two sources of information.

In *Mathematical Models in Risk Theory* H. Bühlmann discussed Bayesian methods for estimating the expected losses for an individual risk given its actual losses. He pointed out that most such methods require knowledge of the parametric distributions of the individual risk processes and of the structure function. Since such knowledge is lacking in most practical applications, Bühlmann suggested the use of formulae based on linear approximations of the theoretically correct quantities. In effect, he suggested that the theoretically correct quantities could be approximated by a straight line fitted to the regression of expected losses over actual losses, using the method of least squares.

We can represent such a line as follows:

$$E\,(M|x) = a + bx$$

where the linear expression on the right side of the equation represents the line of best fit of the regression of expected losses over actual losses [21].

It is well known that the slope of such an equation is equal to the covariance of the dependent and independent variables divided by the variance of the independent variable [22].

Thus:

$$b = \frac{Cov\,(M,\,X)}{Var\,(X)}$$

In turn:

$$\begin{aligned} Cov\,(M,\,X) &= E\,(M,\,X) - E\,(M)\,E\,(X) \\ &= E\,(M,\,X) - (E(M))^2 \end{aligned} \tag{26}$$

since $E(X) = E\,(M)$. (See formula (7).) Furthermore:

$$\begin{aligned} E\,(M,\,X) &= \int_0^\infty \sum_{j=0}^\infty M\,X_j\,P(M,\,X_j)\,dM \\ &= \int_0^\infty M \sum_{j=0}^\infty X_j\,P(M)\,P(X_j|M)\,dM \\ &= \int_0^\infty M\,P(M)\,E(X|M)\,dM \\ &= \int_0^\infty M^2\,P(M)\,dM \\ &= E(M^2) \end{aligned} \tag{27}$$

since $E(X|m) = m$. (Note that $m = E(\phi_i)$, and see formula (1).) Thus:

$$Cov\,(M, X) = E(M^2) - (E(M))^2 = Var\,(M) \tag{28}$$

and the slope of the credibility equation is:

$$b = \frac{Var\,(M)}{Var\,(X)} \tag{29}$$

Since the constant in a least squares regression line is equal to the mean of the dependent variable minus the slope of the line times the mean of the independent variable, we can express the constant in this case as (see note [22]):

$$\begin{aligned} a &= E(X) - b \times E(X) \\ &= E(X) - E(X) \times \frac{Var\,(M)}{Var\,(X)} \\ &= E(X) \times \{ 1 - \frac{Var\,(M)}{Var\,(X)} \} \end{aligned} \tag{30}$$

Thus the linear Bayesian formula for estimating expected losses for an individual risk, EL, given its actual experience, X, is the familiar credibility equation:

$$EL = E(X) \times (1 - Z) + X \times Z \tag{31}$$

where X is the observed experience for the risk and:

$$Z = \frac{Var\,(M)}{Var\,(X)} \tag{32}$$

If we interchange the order of integration and summation in formula (27), we can express $E\,(M, X)$ as follows:

$$\begin{aligned} E\,(M, X) &= \sum_{j=0}^{\infty} \int_0^{\infty} X_j\, M\, P(X_j, M)\, dM \\ &= \sum_{j=0}^{\infty} X_j\, P(X_j) \int_0^{\infty} M\, P(M|X_j)\, dM \\ &= \sum_{j=0}^{\infty} X_j\, P(X_j)\, E(M|X_j) \end{aligned} \tag{33}$$

The importance of this last expression lies in the fact that all of its components can be estimated from observable data. The quantity $P(X_j)$ can be estimated from the number of risks having X_j losses during any given observation period, while $E(M|X_j)$ can be estimated by observing those risks with X_j losses during a subsequent observation period. It should also be noted that since $Cov(M, X) = Var(M)$:

$$\begin{aligned} Var(M) &= E(M,X) - E(M)E(X) \\ &= \{\sum_{j=0}^{\infty} X_j\, P(X_j)\, E(M|X_j)\} - (E(X))^2 \end{aligned} \tag{34}$$

Thus Var (M) can be estimated by making two observations of a risk population, estimating $P(X_j)$ for all j from the first observation, $E(M|X_j)$ from the second, summing over all j, and then subtracting the square of the population mean. It should be noted that since the second observation is being used to estimate conditions prevailing during the first period, adjustments should be made to reflect any changes in conditions between the first and second periods, such as differences in the underlying population mean [23].

At this point we will define new terms which are helpful in estimating $Var(M)$ using the covariance method:

$$\alpha(X_j) = E(M|X_j)$$

and:

$$r(\alpha(X)_j)) = P(X_j)$$

We further define the term t as the adjustment factor reflecting those differences between the observation periods which affect the group as a whole.

Using these identities, we can estimate $Var(M)$ as follows:

$$\begin{aligned} Var(M) &= E(M,X) - E(M)E(X) \\ &= \{\sum_{j=0}^{\infty} X_j\, r(\alpha(X_j))\,(a(X_m) \div t)\} - (E(X))^2 \end{aligned} \tag{35}$$

where $\alpha(X_j)$ is calculated from a subsequent observation period and is adjusted to conditions prevailing during the first. Note that:

$$E(X) = \sum_{j=0}^{\infty} X_j\, r(\alpha(X_j))$$

$$Var(X) = \{\sum_{j=0}^{\infty} X_j^2\, r(\alpha(X_j))\} - (E(X))^2$$

C. Hewitt provided a useful example of a loss generating process and its relationship to Bayesian credibility theory which will be used to illustrate the relationships just discussed [24]. Mr. Hewitt's example used a die and spinner to create a population with four loss processes, all equally represented in the population. The following matrix shows the joint probability of each process and its outcome [25]:

	Outcome		
State	0	2	14
A_1B_1	.83333	.13889	.02778
A_1B_2	.83333	.08333	.08333
A_2B_1	.50000	.41667	.08333
A_2B_2	.50000	.25000	.25000

Suppose that we have been able to observe this population for three repetitions of this process (three "years") and wish to estimate the variance of expected losses by comparing the last repetition to the first two. We obtain the following matrix of joint probabilities:

Joint Probabilities of Loss—$(P(X_1X_2))$

3rd Year	2 Year Losses (X_1)					
Losses (X_2)	0	2	4	14	16	28
0	.35185	.16049	.03498	.08025	.02881	.01029
2	.08025	.06996	.02281	.02881	.01560	.00480
14	.04012	.02881	.00780	.02058	.00960	.00420
$r(\alpha(X_j))$	.47222	.25926	.06559	.12963	.05401	.01929
$\alpha(X_j)$	1.5294	2.0953	2.3608	2.6667	3.0667	3.5467

where the subscripts refer to observations made from the first and second periods respectively.

We can use this information to compute the mean, variance, and covariance of these outcomes and can estimate $Var\ (M)$ by recognizing that the mean for the group during the second observation period will be only half of that during the first, since only one period of time is utilized for the second observation while two are utilized for the first. Thus t, in this case, will be .50 and using formula (35) we have:

1. $E(X)$	4.000
2. $Var(X)$	40.444
3. t	.500
4. $E(M,X)$	22.222
5. $Var(M)$	6.222
6. Z	.154
7. BK	2.571

We can compare the estimates of $E(M|X_j)$ generated by formula (31), the credibility equation,

$$E(M|X_j) = \{4.000 \times (1 - .154)\} + X_j \times .154$$

to the results obtained using Bayes theorem (since we have the necessary information in this simulation).

	$E(M\|X_j)$		
X_j	Bayesian	Credibility	Difference
0	3.0588	3.3846	0.3258
2	4.1906	3.6922	−0.4984
4	4.7216	4.0000	−0.7216
14	5.3334	5.5384	0.2050
16	6.1334	5.8460	−0.2874
28	7.0934	7.6922	0.5988
Total	4.0000	4.0000	0.0000

Hewitt made several observations about the nature of the credibility estimate compared to the true, or Bayesian, estimate, which are particulary cogent at this point. He observed that:

1. Credibility does not (necessarily) produce the optimum estimate while the Bayesian estimate is optimum.
2. Credibility does produce the "least-squares" fit to the optimum (Bayesian) estimates for all possible outcomes weighted by the respective probabilities of those outcomes.
3. Both estimates—credibility and Bayesian—are "in-balance" for all possible outcomes [26].

It can be seen from this example how these points apply. The credibility estimates are quite biased in most cases and thus are not optimum. They are in balance, however, since the expectation of the credibility estimates is equal to $E(X)$!

It is clear, therefore, that Var (M) can be estimated using observable data as long as at least two observations of the risk population can be made. These estimates are unbiased and do not require any assumptions about the nature of the loss processes for individual risks, or about the distribution of expected losses!

RATIO ESTIMATES

It was pointed out earlier in the paper that a reasonably accurate estimate of BK, and thus $Var\ (M)$, was obtained by the simple ratio of merit rating frequencies for risks with one accident to that for risks who were claim free. That is (see formula (22)):

$$BK = \frac{\alpha(0)}{\alpha(1) - \alpha(0)}$$

The credibility estimate for $\alpha(n)$ is:

$$\alpha(n) = \{ E(X) \times (1-Z) \} + \{ Z \times n \}$$

Thus:

$$\frac{\alpha(1)}{\alpha(0)} = \frac{\{ E(X) \times (1-Z) \} + \{ Z \}}{E(X) \times (1-Z)} = 1 + \{ \frac{Z}{1-Z} \times \frac{1}{E(X)} \}$$

and:

$$\frac{\alpha(1) - \alpha(0)}{\alpha(0)} = \frac{Var\ (M)}{E(X) \times \{E(X) + E[Var\ (\phi_i)]\}}$$

$$= \frac{Var\ (M)}{(E(X))^2 + E(X) \times E[Var\ (\phi_i)]}$$

$$= \frac{E(X)}{BK \times \{ E(X) + E[Var\ (\phi_i)] \}}$$

Thus:

$$\frac{\alpha(1)}{\alpha(0)} = 1 + \{ \frac{1}{BK} \times \frac{E(X)}{E(X) + E[Var\ (\phi_i)]} \}$$

Since the Poisson assumption is only valid when $E[Var\,(\phi_i)]$ is equal to zero, this simplifies to:

$$\frac{\alpha(1)}{\alpha(0)} = 1 + \frac{1}{BK}$$

It can be seen that this ratio test does produce unbiased estimates of BK when the Poisson assumptions hold. It is interesting to note that the ratio test is exact in the gamma-Poisson case, since it was shown earlier (see formula (20)) that the ratio of $\alpha(1)$ to $\alpha(0)$ was equal to one plus the inverse of BK (since in the Poisson case, K is equal to BK).

If the individual risk process is not Poisson, the ratio test will be biased by the amount:

$$\frac{E(X) + E[Var\,(\phi_i)]}{E(X)}$$

This explains why the results were biased when this test was applied to the negative binomial simulation where $E[Var\,(\phi_i)]$ was greater than zero.

CLAIM FREE DISCOUNT

If the regression of expected losses over actual losses is reasonably linear, which it usually is when only accident frequencies are involved, there is another convenient way to estimate $Var\,(M)$ using merit rating data.

To begin with, we note that:

$$\alpha(0) = E(X) \times (1 - Z)$$

Therefore:

$$E(X) - \alpha(0) = \frac{E(X) \times Var\,(M)}{Var\,(X)}$$

and:

$$Var\,(M) = Var\,(X) \times \left\{ 1 - \frac{\alpha(0)}{E(X)} \right\}$$

The quantity $\alpha(0) \div E(X)$ represents the ratio of expected losses for risks with claim free experience to the ratio of losses for all risks and thus the quantity in braces represents the claim free discount. We can see, therefore, that the variance of expected losses can be estimated by multiplying the variance of actual losses, $Var\,(X)$, by the claim free discount!

NORTH CAROLINA EXPERIENCE

The methods outlined above can be applied to actual data by assuming that the observed frequency of the events, X_1 and X_2 are unbiased estimators of the true joint probabilities of these events. The following table shows the experience of North Carolina drivers over a four year period, split between the first three years and the fourth year [27].

Second Period (X_2)	Number of Losses First Period (X_1) 0	1	2	3	4	5	6	7
0	2002577	295414	45203	7666	1441	300	82	25
1	104048	26776	6255	1577	375	83	20	4
2	5931	2362	811	247	80	30	13	7
3	438	231	102	34	11	10	0	3
4	30	16	12	2	3	1	0	1
5	5	9	3	2	0	0	0	0
$r(\alpha(X_j))$	.8445	.1298	.0209	.0038	.0008	.0002	.0000	.0000
$\alpha(X_j)$	.0555	.0994	.1574	.2300	.3037	.4175	.4000	.7750

We note the following facts:

1. First period mean		.1874
2. Second period mean		.0643
3. *Var(X)*		.2316
4. *t*	{Quotient of means for two periods}	.3432
5. *E(M,X)*		.0688
6. *Var(M)*		.0337
7. Claim Free Discount	{1.0 − (.0555 ÷ (2))}	.1369
8. *Var(M)* from Claim Free Discount	{(7) × (3)}	.0317
9. *Z*	{(6) ÷ (3)}	.1455
10. *BK*	{from covariance formula}	1.0421
11. *K*		.8656

We see therefore, that we have been able to estimate the homogeneity of the North Carolina driving population without having to make any estimates about a gamma-Poisson process. We note further that there is a significant difference between the two estimates of *BK* (since *K* is the SRI estimate of *BK*) and thus there is a clear indication that the SRI method does not accurately measure the homogeneity of the North Carolina population!

The following table shows merit rating relativities from actual experience, the credibility indicated relativities, and the relativities indicated by the Poisson model (using the four year K value of .8656 [28]):

		Merit Rating Relativities		
X_j	$r(\alpha(X_j))$	Actual	Credibility	Poisson
0	.845	.864	.855	.822
1	.130	1.546	1.630	1.772
2	.021	2.448	2.406	2.722
3	.004	3.576	3.182	3.672
4	.001	4.722	3.958	4.622
5	.000	6.492	4.733	5.571
6	.000	6.220	5.509	6.521
Total	1.000	1.000	1.000	1.000

It can be seen that a merit rating procedure based on the Poisson assumptions would undercharge the 84.5% of the population who were claim free and would substantially overcharge the 15.1% with one or two claims.

The actual data also shows a noticeable departure from linearity for those groups with three or more claims, which suggests that the gamma distribution may not be an appropriate description of the structure of the distribution of expected losses for the North Carolina driving population!

MEASURING HETEROGENEITY

Various ways of estimating the variance of expected losses within an insurance population or subpopulation have been explored in this paper. In all cases, attention has been focused on estimating the variance of the structure function, *Var (M)*, since it is the measure of how much heterogeneity there actually is in the population. If one can measure *Var (M)* for any given group, one has a direct measure of the homogeneity of that group.

The first measure explored was that used by the SRI study which consisted of estimating *Var (M)* by substracting the mean of the actual loss experience, *E(X)*, from its variance. This measure can be thought of as the "excess variance" method. It has been shown that the use of this method requires the assumption that there are no random or stochastic elements affecting exposure to loss, ϕ_i. *If, in fact, this assumption is invalid then any conclusions about the effectiveness of current risk assessment practices based on this measure are not appropriate.*

The second measure consisted of estimating BK (and thus $Var\ (M)$ indirectly) by calculating the ratio of merit rating experience for risks with one accident and zero accidents respectively. This method was termed the "ratio method" and proved reasonably effective even when the Poisson assumption was not made. It was shown, however, that it would be biased by the ratio of the average within risk variance to the population mean:

$$\frac{E(X) + E[Var(\phi_i)]}{E(X)} = \frac{E[E(\phi_i)] + E[Var(\phi_i)]}{E[E(\phi_i)]}$$

A third method for estimating $Var\ (M)$ is to multiply the indicated claim free discount by the variance of the claims experience. That is:

$$Var\ (M) = CFD \times Var(X)$$

where CFD is the claim free discount. This measure was shown to be independent of the Poisson assumption, but it is dependent on the linearity of the regression of expected loss over actual loss. It gives reasonable results if the departure from linearity is not too great but can give poor results as in the Hewitt example where the difference between the actual and linear estimate of the claim free discount is approximately 10%. This situation is likely to exist in most pure premium applications.

The fourth method uses the relationship

$$E(M, X) = \sum_{j=0}^{\infty} X\ P(X_j)\ E(M|X_j)$$

and the fact that the expression on the right can be estimated from observable data taken over to successive periods of time to estimate $Var\ (M)$.

This measure is unbiased, is not affected by the linearity of the regression of expected losses over actual losses, and requires no assumptions about the distribution of losses for individual risks or the distribution of expected loss between members of the risk population. It is, however, subject to sampling variance and the possibility that the characteristics of groups selected on the basis of their loss experience may change with respect to the rest of the population from one period to the next. This would occur, for instance, if individual risk experience were not independent over time.

This method also provides a measure of heterogeneity of the distribution of expected losses over the entire period observed. If each observation period is two years, then the measure estimates $Var\ (M)$ where $M = E\ (\phi_i)$ is the expected loss for the ith risk over the entire four year period.

CONCLUSIONS

This paper has explored questions of risk assessment efficiency and class homogeneity. It has been shown that:

1. The SRI efficiency measure itself is an intuitively reasonable way to gain an overall idea of the effect of risk assessment on the variance within and between classes.

2. Since it is impossible to observe the structure function directly, it is necessary to make inferences about its nature using data which can be observed.

3. SRI International and the SRB "solve" the problem of estimating *Var (M)* by making use of the following relationship:

   $$Var_t\,(M) \;=\; Var_t\,(X) \;-\; E_t\,(X) \;-\; t \times E[Var\,(\phi_i)]$$

   (see formula (18)). They assume that there are no random, or stochastic, elements affecting exposure to loss and thus conclude that $E[Var\,(\phi_i)]$ is equal to zero. This conclusion makes it possible to use the "excess variance" method of determining *Var (M)* which consists of subtracting the observed mean of the actual loss experience from the variance. That is:

   $$Var\,(M) \;=\; OV \;-\; OM$$

   If there are, in fact, random elements associated with exposure, estimates using the "excess variance" method will be biased and misleading.

4. It is possible to estimate *Var (M)* without making arbitrary assumptions about the variance of exposure, ϕ_i, or the nature of the loss process and the shape of the structure function by observing actual experience over more than one period of time and utilizing the fact that:

   $$Var\,(M) \;=\; \{\sum_{j=0}^{\infty} X_j\, r(\alpha(X))\, (\alpha(X_j) \div t)\,\} \;-\; (E(X))^2$$

   where t represents the ratio of the average loss frequency for the first observation to that of the second observation, $r(\alpha(X_j))$ represents the probability that a risk will have X_j losses, and $\alpha(X_j)$ represents the expected losses of that group as estimated from a second observation period.

The purpose of risk assessment is to create homogeneous groups of insureds. The covariance method provides a readily available tool to measure group homogeneity directly, as long as credible subgroups are the object of measurement, and thus provides a way of measuring and monitoring the effectiveness of risk assessment. It provides an objective methodology for defining partitions of the insurance population and also builds in a mechanism for responding to changes in circumstances which might indicate a need for a different type of partitioning system.

The consequences of a lack of class homogeneity were pointed out by the Massachusetts State Rating Bureau:

> "If . . . classes are homogeneous, then each such class average is indeed typical of the expected loss associated with all policies in that class.
>
> "But when classes are heterogeneous, the mean expected loss for each class—however accurately it is estimated—is not at all typical of what each policy is expected to cost."[29]

In the future, actuaries will no longer be allowed to focus their attention exclusively on mean class rates without explicit concern about the types of classes that they are defining and working with. There is valid public concern about the possibility that "good" risks may be paying more than they should for their insurance, while "bad" risks are paying less. There is no evidence whatsoever that this is taking place, but our past inability to demonstrate that our classes are relatively homogeneous has troubled many reasonable people. Actuaries can hope to provide this reassurance only by developing objective measures and standards for class homogeneity. The methods and analyses presented in this paper should provide the basis for such objective measures and standards, and it is up to practicing actuaries to determine how they may be developed and applied.

APPENDIX
SIMULATING PRIVATE PASSENGER AUTOMOBILE EXPERIENCE RATING FREQUENCIES

The purpose of these simulations was to produce annual expected loss frequencies for groups of risks partitioned on the basis of the number of losses experienced during the prior three years. This can also be thought of as a way of generating the actual loss frequency expected during a fourth year, in which case it represents a simulation of the results of two observations of the population of interest.

The simulation procedure consists of the following steps:

1. Create a discretized structure function for the group or subgroup being analyzed.
   In this paper, gamma distributions were generated on the computer for each class shown on Exhibit I. The means of these gamma distributions were set equal to the means of the various classes. In the Poisson simulation, the variance of the gamma distributions was set equal to:

   $$\frac{m_i^2}{K}$$

   where the subscript refers to the class. In the negative-binomial simulation, the variance of X for each class was set equal to the variance of X in the Poisson simulation, so the variance of the gamma structure function was set equal to the following:

   $$\frac{m_i^2}{K} \times \frac{(10 - K)}{11}$$

   This adjustment reflects the fact that for each class:

   $$Var(\phi_i) = \frac{(t \times E(\phi_i))^2}{10}$$

   as shown on page 103 of the text.
   The result of this procedure for each simulation was a 62 by 13 matrix. The rows represent a partition of the domain of the structure function and the columns represent the 13 classes. Exhibit IV shows selected values from these matrices for each simulation. It can be seen, for example, that

that members of Class 10 had a 4.7% probability of having expected losses, $E\ (\phi_i)$, between .05 and .06. Note that in terms of the distributions discussed in the paper, each column of the matrix represents the discrete density function of the structure function, $U(m_i)$.

2. Calculate for each discrete value of m_i, the Poisson and negative binomial conditional probabilities of X losses given $t \times m_i$. This results in a 62 by 6 matrix where the rows represent the discrete values of m_i, as before, and the columns represent the values of X, $X = 0, 1, \ldots, 5$. The values in this matrix represent the probability of X accidents in three years, given an annual frequency rate, $E\ (\phi_i)$, equal to m_i. These values are shown in Exhibit V. If one refers to Exhibit V, Part 2, it can be seen that the negative binomial probability of being claim free for three years, given an annual expected frequency of .055, is 84.9%, while the Poisson probability for the same event shown on Part 1 of Exhibit V is 84.8%.
3. Calculate the matrix of $r(\alpha(X))$ values as follows (using matrix notation):

   $$r\ (\alpha(X)) = U' \times H$$

   where H is the matrix of conditional probabilities of X accidents given m_i and $h(X|\ m_i)$, and U' is the transpose of the structure function matrix, U.
4. Calculate the matrix of $\alpha(X)$ values by first defining the matrix W as being the product of the ith row of U and the scalar $m_i \div r(\alpha(X_i))$ and then taking the matrix product:

   $$\alpha(X) = W' \times H$$

   Clearly $\alpha(X)$ represents the following expectation:

   $$\alpha(X) = E(m_i|X)$$

   where X represents the accident experience during the prior three years.

Part 1 of Exhibits II and III shows the matrix of $\alpha(X)$ values while Part 2 shows the matrix of values of $r(\alpha(X))$. Shown below for illustrative purposes is data which can be used to generate values of $\alpha(X)$ and $r(\alpha(X))$ for the structure function provided in the partitioning example used in the paper. It will be recalled that the only possible values of m_i were .01, .02, .03, . . . ,.10 and that the third partition was 44% efficient. The probability of any particular value of m_i within each partition was .2. Thus we have the matrix U reflecting the structure function within each class and within the overall population, as follows:

	Structure Function: (U)		
	Class		Population
m_i	1	2	
.01	.2	.0	.1
.02	.2	.0	.1
.03	.2	.0	.1
.04	.0	.2	.1
.05	.2	.0	.1
.06	.0	.2	.1
.07	.2	.0	.1
.08	.0	.2	.1
.09	.0	.2	.1
.10	.0	.2	.1
Total	1.0	1.0	1.0

Assuming that $g(x_i)$ is negative binomial with an exposure variance equal to:

$$.0625 \times (E(\phi_i))^2$$

as discussed earlier (see page 99), the conditional probability matrix, *H*, assuming an initial three year observation period, is as follows:

Probability of X claims given m_i: (H)

	Number of Claims (X)					
m_i	0	1	2	3	4	5
.01	.97045	.02910	.00045	.00000	.00000	.00000
.02	.94180	.05644	.00173	.00004	.00000	.00000
.03	.91401	.08211	.00376	.00012	.00000	.00000
.04	.88705	.10618	.00649	.00027	.00001	.00000
.05	.86091	.12873	.00983	.00051	.00002	.00000
.06	.83555	.14984	.01371	.00085	.00004	.00000
.07	.81096	.16956	.01810	.00131	.00007	.00000
.08	.78710	.18796	.02291	.00190	.00012	.00001
.09	.76396	.20511	.02811	.00262	.00019	.00001
.10	.74151	.22107	.03364	.00348	.00028	.00002

With this information, we begin by calculating the matrix of values of $\alpha(X)$:

	$\alpha(X)$					
Class	0	1	2	3	4	5
1	.03463	.04762	.05641	.06144	.06433	.06610
2	.07260	.07911	.08401	.08745	.08985	.09156
Total	.05254	.06813	.07727	.08281	.08647	.08905

Next we calculate the matrix of values of $r(\alpha(X))$:

	$r(\alpha(X))$					
Class	0	1	2	3	4	5
1	.89963	.09319	.00677	.00040	.00002	.00000
2	.80303	.17403	.02097	.00183	.00013	.00001
Total	.85133	.13361	.01387	.00111	.00007	.00000

From these two matrices it is now possible to make the following series of calculations:

	Class		
	1	2	Total
1. $E(X_1)$	.10800	.22200	.16500
2. $E(X_2)$	.03600	.07400	.05500
3. t	.33333	.33333	.33333
4. $E(M,X)$	.00528	.01782	.01155
5. $Cov(M,X)$	.00139	.00139	.00247
6. $Var(M)$	.00046	.00046	.00082
7. $Var_t\,(X)$	.11250	.22728	.17314
8. $E[Var(\phi_i)]$	.00011	.00037	.00024

The value of $E[Var\,(\phi_i)]$ is calculated using the following formula:

$$E[Var\,(\phi_i)] = \{\, Var_t\,(X) - [E_t(X) + Var_t\,(M)]\,\}\,/\,t$$

In the above table, $E(X_1)$ is equal to $E_t\,(X)$, and the three year variance of the structure function, $Var_t\,(M)$, is nine times the one year variance (see pages 85—88). The details on how the values of $\alpha(X)$ and $r(\alpha(X))$ are put to use can be found on pages 107—116.

NOTES AND REFERENCES

[1] SRI International, *The Role of Risk Classification in Property and Casualty Insurance: A Study of the Risk Assessment Process*. (1976). (Note that there were three volumes issued as a part of this project: an Executive Summary, the Final Report, and a Supplement to the Final Report.)

The most important discussion of the SRI Report findings on class homogeneity and risk assessment efficiency is found on pp. 81–82. of the Final Report: "within each group there remains a wide range of accident likelihoods. The risk assessment process is still imprecise for individual insureds . . . "

[2] The attitude of the Massachusetts Division of Insurance, of which the State Rating Bureau was a part, was expressed most strongly in "Insurance Rates and Social Policy." This paper was presented at the 1977 hearings on 1978 Massachusetts Auto Rates conducted by Commissioner Stone.

The SRB recommendations about departing from actuarial rates are found in another paper presented at the 1977 hearings: "Identifying Equitable Insurance Premiums for Risk Classes: An Alternative to the Classical Approach" by Dr. J. Ferreira, Jr. found as Chapter IV of *Automobile Insurance Risk Classification: Equity and Accuracy* issued in 1978 by the Massachusetts Divison of Insurance.

[3] "Report of the Rates and Rating Procedures Task Force of the Automobile Insurance (D3) Subcommittee," November 1978, p. 6.

[4] A brief description of the assumptions underlying the use of the Poisson process is provided on pp. 175–176 and 212–213 of the Supplement to the SRI Report. Another discussion of the importance of the compound-Poisson process in risk theory is given in *Mathematical Methods in Risk Theory* by H. Bühlmann (Springer-Verlag, 1970). Dr. Bühlmann discusses what he calls "infinitely divisible" probability distributions and makes the statement that *for distributions defined on the non-negative integers, every infinitely divisible characteristic function is compound-Poisson!* (See pp. 69–73)

Intuitive support for the Poisson assumption can be derived by consideration of the fact that the limit of a binomial process taken over shorter and shorter time intervals is Poisson.

[5] The moments of the compound-Poisson process can be derived from the fact that the unconditional expectation of a random variable can be expressed in terms of conditional expectations. If we let μ_i represent the conditional mean of the ith state, we can express the unconditional variance of such a random variable, X, as:

$$\begin{aligned} Var\,(X) &= E\,\{\,E(X^2|\mu_i)\,\} \;-\; \{\,E[E\,(X|\mu_i)]\,\}^2 \\ &= E\,\{\,Var\,(X|\mu_i) \;+\; [\,E\,(X|\mu_i)]^2\,\} \;-\; \{\,E[E\,(X|\mu_i))]\,\}^2 \\ &= E\,[Var\,(X|\mu_i)] \;+\; E(\mu_i)^2 \;-\; (E(\mu_i))^2 \\ &= E\,[Var\,(X|\mu_i)] \;+\; Var\,(\mu_i) \end{aligned}$$

since $E(X|\mu_i) = \mu_i$. In the compound-Poisson distribution, the mean and variance of the conditional Poisson process are both equal to the parameter of the process, ϕ_i, and thus:

$$Var\,(X) = E(\phi_i) + Var\,(\phi_i)$$

[6] Bühlmann, p. 65.

[7] See note [5]. In this case the conditional mean is m_i.

[8] If m_i represents the average losses for the ith risk during a single period of time, then $t \times m_i$ will represent the average losses for t units of time. If $Var\,(M)$ represents the variance of the structure function for a single period of time, then $Var\,(t \times M)$ will represent that variance for t units time. Thus:

$$Var_t\,(M) = t^2 \times Var\,(M)$$

[9] R.A. Bailey, "Any Room Left for Skimming the Cream?" *PCAS* XLVII (1960), p. 30.

[10] See discussion by L.H. Roberts of Bailey, op. cit. p. 213.

[11] SRI Final Report pp. 46–55, and Supplement pp. 200–203.

[12] **The SRI procedure is discussed further on pages 93—95 of this paper.**

[13] Private letter from M. Treloar to R.G. Woll, June 1978.

[14] The mean and variance of the accident distribution are defined in the SRI Supplement, p. 177.

[15] Ferreira, p. 110. Dr. Ferreira states: ". . . it is recommended that the factors other than class means be considered in setting 1978 auto insurance rates in Massachusetts. *It is further recommended that consideration be given to the homogeneity of such classes* and that either the method incorporated in this paper or an approach incorporating its basic principles be used *in place of the traditional actuarial method for determining class and territorial differentials*." (Emphasis added)

[16] Ibid. p. 85.

[17] Ibid.

[18] Ibid. p. 86.

[19] J. Ferreira, Jr. "Merit Rating and Automobile Insurance," *Automobile Insurance Classification: Equity and Accuracy*, Chapter III, p. 69.

[20] SRI Supplement, pp. 205–206.

[21] Bühlmann, pp. 100–103.

[22] For example, see Hoel, Port, and Stone, *Introduction to Statistical Theory*, (Houghton Mifflin, 1971), p. 115. They show that if we write the regression equation as:

$$Y = a + b(X - \bar{X})$$

then:

$$a = \bar{Y}$$

and:

$$b = \rho \frac{s_y}{s_x}$$

where s_Y and s_X are the standard deviations of Y and X respectively, and ρ is the correlation coefficient of Y and X. Thus:

$$b = \frac{Cov\ (Y, X)}{\sqrt{Var\ (Y)\ Var\ (X)}} \times \frac{\sqrt{Var\ (Y)}}{\sqrt{Var\ (X)}}$$

$$= \frac{Cov(Y,X)}{Var\ (X)}$$

Note that the constant term becomes:

$$a = \bar{Y} - \frac{Cov\ (Y, X)}{Var\ (X)} \times \bar{X}$$

In other words, the constant term is equal to the mean of the dependent variable minus the mean of the independent variable times the slope of the regression line.

[23] I am indebted to Dr. D. Rosenfield of Arthur D. Little (ADL) who helped me realize how the covariance of M and X could be utilized to estimate $Var\ (M)$.

[24] C. C. Hewitt, Jr., "Credibility for Severity," *PCAS* LVII (1968), pp. 148–171.

[25] Ibid. p. 150.

[26] Ibid. p. 152. The probabilities shown in the table are taken from the description of the die and spinner probabilities assuming independence.

[27] R. Stewart and R.J. Campbell, "The Statistical Association between Past and Future Accidents and Violations," (1970). This study is very useful for analyzing the concepts discussed in this paper, since it contains the data used in this paper along with other combinations of observation periods and driver groups. It is not insurance data, and it is hard to know how indicative results based on such data might be of actual insurance results.

[28] This value of K was calculated by estimating the values of $r(\alpha(X_i))$ from four year North Carolina data. First, values of $E(X)$ and $Var\ (X)$ were calculated:

$$E(X) = \sum_{j=0}^{\infty} X_j\, r\,(\alpha(X_j))$$

$$Var\ (X) = \{\sum_{j=0}^{\infty} X_j^2\, r(\alpha(X_j))\ \} - (E(X))^2$$

and then K was set equal to the following:

$$K = \frac{(E(X))^2}{Var\ (X) - E(X)}$$

$$= \frac{.2517^2}{.3249 - .2517}$$

$$= .8656$$

[29] J. Ferreira, Jr. "Identifying Equitable Insurance Premiums for Risk Classes: An Alternative to the Classical Approach," p. 82.

EXHIBIT I

EFFECT OF MERIT RATING ON CLASS RELATIVITIES*

Driver Class	1975 Exposure %	Observed 1975 Average Claim Frequency (× 100)	k** Value	Observed 1975 Driver-Class Relativity (before)	Predicted Driver-Class Relativity (after 3 years of Merit Rating)	Percent Change (6) − (5) × 100
(1)	(2)	(3)	(4)	(5)	(6)	(7)
00	3.55	17.31	1.025	1.067	1.016	− 4.8%
10	58.11	15.21	1.752	.938	.952	+ 1.5%
12	10.27	14.42	2.160	.889	.924	+ 3.9%
15	9.34	11.78	2.066	.726	.780	+ 7.4%
20	.07	35.90	1.902	2.213	1.770	−20.0%
22	.24	35.56	1.894	2.192	1.759	−19.8%
24	1.00	24.55	1.747	1.514	1.362	−10.0%
26	7.37	21.30	2.720	1.313	1.272	− 3.1%
30	2.77	17.31	2.842	1.067	1.085	+ 1.7%
31	1.28	13.09	0.570	.807	.745	− 7.7%
40	1.92	26.30	2.739	1.621	1.480	− 8.3%
42	2.94	29.19	3.493	1.800	1.650	− 8.3%
50	1.33	21.40	2.272	1.319	1.262	− 4.3%
State -wide Average	100.0	16.22	1.969***	1.00	1.00	0.0%

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**The value of k is an estimate of class homogeneity. (The square root of the reciprocal of k is the coefficient of variation of the claim frequency distribution underlying the class.)

***The actual statewide value of k is 1.685 (See Exhibit II, Part 1).

EXHIBIT II

MASSACHUSETTS: POISSON SIMULATION

PART 1

Expected Claim Frequencies: $\alpha(X)$

				$\alpha(X)$					
Class	*BK*	*K*	*E(X)*	0	1	2	3	4	5
00	1.022	1.025	.173	.115	.227	.340	.451	.563	.675
10	1.753	1.752	.152	.121	.190	.258	.327	.396	.464
12	2.162	2.160	.144	.120	.176	.231	.287	.343	.398
15	2.068	2.066	.118	.101	.149	.198	.247	.296	.345
20	1.872	1.904	.359	.227	.349	.471	.592	.714	.837
22	1.865	1.895	.356	.225	.347	.468	.589	.711	.833
24	1.739	1.747	.245	.172	.272	.371	.469	.568	.667
26	2.722	2.720	.213	.173	.236	.299	.363	.426	.488
30	2.845	2.842	.173	.146	.198	.249	.301	.352	.404
31	0.568	0.570	.131	.077	.214	.350	.486	.622	.758
40	2.734	2.739	.263	.204	.279	.354	.428	.502	.576
42	3.488	3.493	.292	.233	.300	.367	.434	.500	.567
50	2.271	2.272	.214	.167	.240	.314	.387	.460	.533
Total	1.684	1.685	.162	.126	.199	.274	.351	.433	.522

PART 2

Distribution within Class and Merit Rating Category: $r(\alpha(X))$

	$r(\alpha(X))$						
Class	0	1	2	3	4	5	Weight
00	.657	.226	.077	.026	.009	.003	.035
10	.667	.241	.069	.018	.004	.001	.580
12	.674	.243	.064	.015	.003	.001	.102
15	.722	.218	.049	.010	.002	.000	.093
20	.428	.292	.153	.072	.032	.014	.001
22	.431	.291	.152	.071	.031	.013	.002
24	.542	.280	.114	.042	.015	.005	.010
26	.563	.292	.103	.031	.008	.002	.074
30	.621	.273	.081	.028	.005	.001	.028
31	.742	.172	.055	.019	.007	.003	.013
40	.500	.306	.128	.045	.015	.004	.019
42	.458	.320	.144	.053	.017	.005	.029
50	.568	.284	.103	.032	.009	.003	.013
Total	.652	.247	.074	.020	.005	.001	1.000

EXHIBIT II

MASSACHUSETTS: POISSON SIMULATION

PART 3

Merit Rating Relativities to Claim Free Rate: $\alpha(X) \div \alpha(0)$

	$\alpha(X) \div \alpha(0)$					
Class	0	1	2	3	4	5
00	1.000	1.982	2.963	3.936	4.908	5.887
10	1.000	1.571	2.141	2.711	3.279	3.844
12	1.000	1.462	1.925	2.388	2.850	3.309
15	1.000	1.483	1.966	2.451	2.937	3.425
20	1.000	1.540	2.076	2.610	3.147	3.691
22	1.000	1.542	2.080	2.616	3.156	3.701
24	1.000	1.579	2.154	2.726	3.297	3.873
26	1.000	1.368	1.735	2.102	2.467	2.831
30	1.000	1.351	1.702	2.054	2.406	2.756
31	1.000	2.758	4.526	6.278	8.028	9.793
40	1.000	1.368	1.734	2.098	2.460	2.823
42	1.000	1.288	1.575	1.859	2.143	2.430
50	1.000	1.441	1.882	2.320	2.756	3.193
Total	1.000	1.576	2.164	2.777	3.425	4.125

PART 4

Merit Rating Relativities to Class Mean: $\alpha(X) \div E(X)$

	$\alpha(X) \div E(X)$					
Class	0	1	2	3	4	5
00	0.663	1.313	1.963	2.608	3.252	3.901
10	0.793	1.246	1.699	2.151	2.601	3.050
12	0.833	1.219	1.604	1.990	2.375	2.757
15	0.854	1.267	1.679	2.093	2.508	2.925
20	0.634	0.976	1.315	1.654	1.995	2.339
22	0.635	0.979	1.321	1.661	2.004	2.350
24	0.702	1.108	1.512	1.913	2.314	2.719
26	0.810	1.108	1.405	1.702	1.997	2.293
30	0.846	1.143	1.440	1.737	2.035	2.331
31	0.591	1.631	2.676	3.712	4.747	5.791
40	0.776	1.061	1.345	1.627	1.908	2.189
42	0.799	1.029	1.258	1.486	1.713	1.941
50	0.779	1.123	1.467	1.809	2.148	2.489
Total	0.779	1.228	1.686	2.164	2.669	3.214

EXHIBIT III

MASSACHUSETTS: NEGATIVE BINOMIAL SIMULATION

PART 1

Expected Claim Frequencies: $\alpha(X)$

Class	BK	K	$E(X)$	$\alpha(X)$ 0	1	2	3	4	5
00	1.245	1.017	.173	.124	.220	.312	.400	.485	.568
10	2.331	1.748	.152	.128	.181	.232	.283	.332	.380
12	3.019	2.153	.144	.127	.167	.207	.245	.284	.321
15	2.853	2.059	.118	.105	.141	.176	.210	.244	.277
20	2.473	1.836	.359	.257	.354	.447	.536	.622	.707
22	2.462	1.830	.356	.255	.351	.444	.533	.619	.704
24	2.299	1.729	.245	.189	.267	.342	.415	.485	.553
26	4.094	2.712	.213	.186	.229	.271	.312	.353	.393
30	4.344	2.831	.173	.156	.190	.223	.256	.289	.322
31	0.658	0.565	.131	.084	.207	.325	.437	.545	.650
40	4.125	2.727	.263	.224	.274	.324	.372	.420	.465
42	5.888	3.487	.292	.257	.298	.337	.376	.415	.452
50	3.221	2.265	.214	.180	.233	.285	.336	.385	.433
Total	2.219	1.679	.162	.135	.191	.247	.304	.363	.425

PART 2

Distribution within Class and Merit Rating Category: $r(\alpha(X))$

Class	$r(\alpha(X))$ 0	1	2	3	4	5	Weight
00	.656	.229	.076	.025	.009	.003	.035
10	.666	.243	.068	.017	.004	.001	.580
12	.674	.244	.063	.015	.003	.001	.102
15	.721	.219	.048	.010	.002	.000	.093
20	.426	.297	.153	.070	.031	.031	.001
22	.429	.296	.152	.070	.030	.013	.002
24	.540	.283	.113	.041	.015	.005	.010
26	.562	.294	.102	.030	.008	.002	.074
30	.620	.274	.080	.020	.005	.001	.028
31	.740	.175	.055	.019	.007	.003	.013
40	.498	.309	.127	.045	.014	.004	.019
42	.456	.323	.144	.052	.017	.005	.029
50	.567	.287	.102	.032	.009	.003	.013
Total	.651	.249	.073	.020	.005	.001	1.000

EXHIBIT III

MASSACHUSETTS: NEGATIVE BINOMIAL SIMULATION

PART 3

Merit Rating Relativities to Claim Free Rate: $\alpha(X) \div \alpha(0)$

	$\alpha(X) \div \alpha(0)$					
Class	0	1	2	3	4	5
00	1.000	1.770	2.510	3.221	3.906	4.569
10	1.000	1.410	1.811	2.203	2.586	2.959
12	1.000	1.317	1.628	1.934	2.235	2.531
15	1.000	1.338	1.669	1.996	2.317	2.635
20	1.000	1.378	1.740	2.087	2.425	2.756
22	1.000	1.380	1.743	2.092	2.432	2.764
24	1.000	1.412	1.809	2.193	2.563	2.923
26	1.000	1.230	1.456	1.678	1.896	2.112
30	1.000	1.219	1.434	1.646	1.858	2.070
31	1.000	2.462	3.868	5.209	6.497	7.744
40	1.000	1.227	1.449	1.666	1.876	2.081
42	1.000	1.158	1.313	1.464	1.613	1.757
50	1.000	1.294	1.580	1.860	2.133	2.398
Total	1.000	1.419	1.834	2.256	2.691	3.150

PART 4

Merit Rating Relativities to Class Mean: $\alpha(X) \div E(X)$

	$\alpha(X) \div E(X)$					
Class	0	1	2	3	4	5
00	0.718	1.272	1.803	2.314	2.806	3.283
10	0.843	1.189	1.527	1.858	2.181	2.495
12	0.880	1.159	1.433	1.702	1.967	2.227
15	0.894	1.196	1.492	1.784	2.071	2.355
20	0.718	0.989	1.249	1.499	1.741	1.979
22	0.719	0.992	1.253	1.504	1.748	1.987
24	0.771	1.088	1.395	1.690	1.976	2.253
26	0.873	1.073	1.270	1.464	1.655	1.843
30	0.898	1.095	1.288	1.479	1.669	1.860
31	0.640	1.574	2.474	3.332	4.156	4.954
40	0.850	1.043	1.231	1.415	1.594	1.768
42	0.880	1.018	1.155	1.288	1.419	1.546
50	0.843	1.090	1.332	1.567	1.797	2.021
Total	0.831	1.179	1.523	1.873	2.235	2.616

EXHIBIT IV
PART 1A

MASSACHUSETTS: POISSON SIMULATION

Gamma Structure Function Probabilities: $U(m_i)$

	Class					
m_i	00	10	12	15	20	22
.00005	.00052	.00000	.00000	.00000	.00000	.00000
.00055	.00483	.00024	.00005	.00011	.00003	.00004
.00550	.04873	.01276	.00629	.01121	.00228	.00237
.01500	.05215	.02783	.01937	.03109	.00573	.00591
.02500	.04960	.03663	.03031	.04525	.00842	.00864
.03500	.04702	.04213	.03866	.05454	.01064	.01089
.04500	.04452	.04542	.04463	.05995	.01250	.01278
.05500	.04212	.04711	.04856	.06239	.01408	.01436
.06500	.03983	.04763	.05080	.06262	.01541	.01570
.07500	.03764	.04729	.05167	.06125	.01652	.01681
.08500	.03557	.04632	.05146	.05876	.01745	.01773
.09500	.03360	.04489	.05043	.05554	.01821	.01849
.10500	.03174	.04314	.04877	.05187	.01883	.01910
.11500	.02997	.04118	.04667	.04796	.01932	.01957
.12500	.02830	.03908	.04427	.04399	.01969	.01993
.13500	.02672	.03690	.04168	.04007	.01997	.02019
.14500	.02523	.03471	.03898	.03629	.02015	.02036
.15500	.02381	.03252	.03626	.03269	.02025	.02045
.16500	.02248	.03038	.03356	.02932	.02028	.02046
.17500	.02122	.02830	.03093	.02619	.02025	.02042
.18500	.02003	.02630	.02840	.02332	.02016	.02031
.19500	.01890	.02438	.02599	.02069	.02002	.02016
.20500	.01784	.02256	.02371	.01831	.01984	.01997
.21500	.01683	.02084	.02157	.01616	.01962	.01973
.22500	.01588	.01922	.01957	.01423	.01937	.01947
.23500	.01499	.01769	.01772	.01251	.01909	.01917
.24500	.01414	.01627	.01600	.01097	.01878	.01886
.25500	.01335	.01494	.01443	.00960	.01846	.01852
.26500	.01259	.01371	.01299	.00839	.01811	.01816
.27500	.01188	.01256	.01167	.00732	.01775	.01779
.28500	.01121	.01150	.01047	.00638	.01738	.01741
.29500	.01058	.01051	.00938	.00555	.01700	.01701
.30500	.00998	.00961	.00839	.00483	.01661	.01662
.31500	.00942	.00877	.00750	.00419	.01621	.01621
.32500	.00888	.00800	.00669	.00363	.01581	.01580
.33500	.00838	.00729	.00596	.00315	.01541	.01539
.34500	.00791	.00665	.00531	.00273	.01500	.01498
.35500	.00746	.00605	.00472	.00236	.01460	.01457
.36500	.00704	.00551	.00420	.00204	.01420	.01416
.74000	.00754	.00117	.00032	.00005	.03628	.03561
.84000	.00420	.00041	.00008	.00002	.02413	.02359
.94000	.00234	.00014	.00003	.00000	.01586	.01545

EXHIBIT IV
PART 1B

MASSACHUSETTS: POISSON SIMULATION

Gamma Structure Function Probabilities: $U(m_i)$

	Class						
m_i	24	26	30	31	40	42	50
.00005	.00000	.00000	.00000	.01429	.00000	.00000	.00000
.00055	.00012	.00000	.00000	.03789	.00000	.00000	.00001
.00550	.00605	.00078	.00103	.13601	.00045	.00005	.00219
.01500	.01339	.00393	.00558	.08564	.00231	.00045	.00765
.02500	.01817	.00827	.01204	.06508	.00497	.00138	.01315
.03500	.02166	.01297	.01902	.05369	.00798	.00279	.01812
.04500	.02426	.01760	.02568	.04604	.01108	.00460	.02243
.05500	.02617	.02188	.03159	.04038	.01411	.00670	.02603
.06500	.02756	.02569	.03652	.03595	.01696	.00899	.02894
.07500	.02851	.02893	.04039	.03234	.01957	.01137	.03122
.08500	.02911	.03160	.04320	.02933	.02189	.01376	.03292
.09500	.02942	.03368	.04503	.02676	.02391	.01609	.03410
.10500	.02949	.03523	.04597	.02453	.02561	.01831	.03482
.11500	.02937	.03627	.04615	.02258	.02701	.02036	.03516
.12500	.02909	.03685	.04569	.02086	.02812	.02222	.03515
.13500	.02868	.03703	.04469	.01932	.02895	.02387	.03486
.14500	.02816	.03686	.04327	.01793	.02952	.02529	.03433
.15500	.02755	.03639	.04152	.01669	.02986	.02649	.03360
.16500	.02687	.03567	.03954	.01555	.02998	.02745	.03272
.17500	.02614	.03474	.03740	.01452	.02992	.02819	.03171
.18500	.02537	.03364	.03516	.01357	.02969	.02872	.03061
.19500	.02457	.03242	.03287	.01271	.02931	.02905	.02943
.20500	.02375	.03110	.03058	.01191	.02881	.02919	.02820
.21500	.02292	.02971	.02833	.01117	.02819	.02916	.02695
.22500	.02208	.02827	.02614	.01049	.02749	.02897	.02568
.23500	.02124	.02682	.02403	.00986	.02672	.02864	.02440
.24500	.02041	.02535	.02201	.00927	.02588	.02819	.02314
.25500	.01958	.02390	.02010	.00873	.02500	.02763	.02190
.26500	.01877	.02248	.01831	.00822	.02409	.02698	.02068
.27500	.01797	.02108	.01663	.00775	.02315	.02625	.01950
.28500	.01719	.01973	.01507	.00731	.02220	.02546	.01835
.29500	.01643	.01843	.01362	.00690	.02124	.02462	.01724
.30500	.01569	.01717	.01229	.00651	.02029	.02374	.01618
.31500	.01497	.01598	.01106	.00615	.01934	.02282	.01516
.32500	.01428	.01484	.00994	.00581	.01840	.02189	.01418
.33500	.01360	.01376	.00892	.00549	.01748	.02095	.01326
.34500	.01295	.01273	.00799	.00519	.01658	.02000	.01238
.35500	.01233	.01177	.00714	.00491	.01570	.01906	.01154
.36500	.01172	.01087	.00638	.00465	.01485	.01812	.01075
.74000	.01337	.00285	.00046	.00655	.00979	.01131	.00467
.84000	.00725	.00099	.00010	.00403	.00432	.00470	.00190
.94000	.00389	.00033	.00005	.00250	.00186	.00189	.00076

EXHIBIT IV
PART 2A

MASSACHUSETTS: NEGATIVE BINOMIAL SIMULATION

Gamma Structure Function Probabilities: $U(m_i)$

	Class					
m_i	00	10	12	15	20	22
.00005	.00011	.00000	.00000	.00000	.00000	.00000
.00055	.00182	.00002	.00000	.00000	.00000	.00000
.00550	.03052	.00404	.00122	.00289	.00042	.00044
.01500	.04128	.01436	.00730	.01472	.00177	.00184
.02500	.04356	.02435	.01642	.02958	.00345	.00358
.03500	.04400	.03275	.02625	.04334	.00525	.00542
.04500	.04352	.03931	.03539	.05426	.00706	.00727
.05500	.04252	.04409	.04309	.06184	.00882	.00907
.06500	.04120	.04728	.04902	.06621	.01050	.01078
.07500	.03970	.04910	.05312	.06780	.01208	.01237
.08500	.03808	.04979	.05551	.06715	.01353	.01383
.09500	.03641	.04955	.05639	.06480	.01485	.01516
.10500	.03472	.04858	.05601	.06125	.01604	.01636
.11500	.03303	.04705	.05461	.05692	.01710	.01741
.12500	.03137	.04511	.05243	.05215	.01803	.01833
.13500	.02975	.04289	.04968	.04721	.01883	.01913
.14500	.02817	.04047	.04656	.04231	.01951	.01979
.15500	.02664	.03795	.04321	.03757	.02007	.02034
.16500	.02518	.03538	.03977	.03311	.02052	.02079
.17500	.02377	.03283	.03633	.02898	.02088	.02112
.18500	.02242	.03033	.03296	.02522	.02114	.02136
.19500	.02114	.02791	.02973	.02182	.02131	.02152
.20500	.01991	.02559	.02668	.01879	.02140	.02159
.21500	.01875	.02339	.02382	.01610	.02141	.02159
.22500	.01765	.02132	.02118	.01375	.02136	.02152
.23500	.01660	.01938	.01875	.01169	.02125	.02139
.24500	.01561	.01757	.01654	.00991	.02108	.02120
.25500	.01467	.01590	.01454	.00838	.02087	.02097
.26500	.01378	.01435	.01275	.00706	.02061	.02069
.27500	.01294	.01294	.01114	.00593	.02031	.02038
.28500	.01215	.01164	.00971	.00498	.01997	.02003
.29500	.01140	.01045	.00844	.00416	.01961	.01965
.30500	.01070	.00937	.00732	.00347	.01922	.01925
.31500	.01004	.00839	.00634	.00289	.01881	.01882
.32500	.00941	.00751	.00547	.00241	.01838	.01838
.33500	.00883	.00670	.00472	.00200	.01794	.01793
.34500	.00827	.00598	.00406	.00165	.01748	.01746
.35500	.00776	.00533	.00349	.00137	.01702	.01698
.36500	.00727	.00474	.00299	.00113	.01655	.01650
.74000	.00558	.00036	.00007	.00000	.03403	.03323
.84000	.00281	.00009	.00000	.00000	.02060	.02002
.94000	.00141	.00004	.00000	.00000	.01222	.01181

EXHIBIT IV
PART 2B

MASSACHUSETTS: NEGATIVE BINOMIAL SIMULATION

Gamma Structure Function Probabilities: $U(m_i)$

	Class						
m_i	24	26	30	31	40	42	50
.00005	.00000	.00000	.00000	.00773	.00000	.00000	.00000
.00055	.00001	.00000	.00000	.02699	.00000	.00000	.00000
.00550	.00154	.00003	.00005	.11876	.00001	.00000	.00025
.01500	.00554	.00048	.00075	.08325	.00021	.00001	.00182
.02500	.00974	.00182	.00302	.06593	.00083	.00005	.00477
.03500	.01369	.00422	.00716	.05572	.00199	.00019	.00862
.04500	.01724	.00756	.01287	.04855	.00372	.00051	.01295
.05500	.02035	.01162	.01960	.04307	.00593	.00111	.01740
.06500	.02300	.01610	.02670	.03866	.00854	.00205	.02170
.07500	.02520	.02071	.03356	.03500	.01141	.00337	.02567
.08500	.02698	.02521	.03974	.03188	.01442	.00507	.02917
.09500	.02837	.02938	.04490	.02918	.01744	.00714	.03214
.10500	.02941	.03308	.04886	.02682	.02039	.00952	.03455
.11500	.03012	.03620	.05158	.02472	.02316	.01215	.03639
.12500	.03056	.03870	.05307	.02285	.02569	.01493	.03768
.13500	.03074	.04055	.05344	.02117	.02794	.01779	.03847
.14500	.03071	.04176	.05283	.01965	.02986	.02063	.03880
.15500	.03049	.04238	.05139	.01826	.03144	.02337	.03871
.16500	.03010	.04245	.04929	.01700	.03268	.02594	.03827
.17500	.02958	.04203	.04670	.01585	.03358	.02828	.03752
.18500	.02895	.04120	.04376	.01479	.03416	.03035	.03653
.19500	.02823	.04002	.04060	.01382	.03442	.03209	.03532
.20500	.02743	.03856	.03734	.01292	.03441	.03351	.03396
.21500	.02657	.03687	.03407	.01209	.03414	.03458	.03248
.22500	.02566	.03502	.03085	.01133	.03364	.03531	.03091
.23500	.02472	.03306	.02776	.01061	.03295	.03570	.02929
.24500	.02377	.03103	.02482	.00995	.03208	.03578	.02764
.25500	.02279	.02898	.02207	.00934	.03108	.03557	.02599
.26500	.02182	.02693	.01952	.00877	.02996	.03509	.02435
.27500	.02085	.02492	.01719	.00824	.02876	.03438	.02275
.28500	.01989	.02296	.01506	.00774	.02749	.03347	.02118
.29500	.01894	.02108	.01315	.00728	.02618	.03238	.01967
.30500	.01801	.01928	.01143	.00684	.02484	.03116	.01823
.31500	.01710	.01757	.00990	.00644	.02348	.02982	.01684
.32500	.01622	.01597	.00855	.00606	.02214	.02840	.01553
.33500	.01537	.01447	.00736	.00571	.02080	.02692	.01429
.34500	.01454	.01307	.00631	.00537	.01950	.02540	.01312
.35500	.01374	.01178	.00540	.00506	.01822	.02388	.01203
.36500	.01297	.01059	.00461	.00477	.01699	.02235	.01101
.74000	.00925	.00063	.00018	.00553	.00406	.00342	.00175
.84000	.00428	.00003	.00000	.00322	.00125	.00078	.00051
.94000	.00195	.00016	.00000	.00188	.00036	.00032	.00013

EXHIBIT V
PART 1

MASSACHUSETTS: POISSON SIMULATION

Probability of X Claims Given m_i: *(H)*

	Number of Claims: *(X)*					
m_i	0	1	2	3	4	5
.00005	.99985	.00015	.00000	.00000	.00000	.00000
.00055	.99835	.00165	.00000	.00000	.00000	.00000
.00550	.98364	.01623	.00013	.00000	.00000	.00000
.01500	.95600	.04302	.00097	.00001	.00000	.00000
.02500	.92774	.06958	.00261	.00007	.00000	.00000
.03500	.90032	.09453	.00496	.00017	.00000	.00000
.04500	.87372	.11795	.00796	.00036	.00001	.00000
.05500	.84789	.13990	.01154	.00063	.00003	.00000
.06500	.82283	.16045	.01564	.00102	.00005	.00000
.07500	.79852	.17967	.02021	.00152	.00009	.00000
.08500	.77492	.19760	.02519	.00214	.00014	.00001
.09500	.75201	.21432	.03054	.00290	.00021	.00001
.10500	.72979	.22988	.03621	.00380	.00030	.00002
.11500	.70822	.24434	.04215	.00485	.00042	.00003
.12500	.68729	.25773	.04833	.00604	.00057	.00004
.13500	.66698	.27013	.05470	.00738	.00075	.00006
.14500	.64726	.28156	.06124	.00888	.00097	.00008
.15500	.62814	.29208	.06791	.01053	.00122	.00011
.16500	.60957	.30174	.07468	.01232	.00152	.00015
.17500	.59156	.31057	.08152	.01427	.00187	.00020
.18500	.57407	.31861	.08841	.01636	.00227	.00025
.19500	.55711	.32591	.09533	.01859	.00272	.00032
.20500	.54064	.33249	.10224	.02096	.00322	.00040
.21500	.52466	.33841	.10914	.02346	.00378	.00049
.22500	.50916	.34368	.11599	.02610	.00440	.00059
.23500	.49411	.34835	.12279	.02886	.00509	.00072
.24500	.47951	.35244	.12952	.03173	.00583	.00086
.25500	.46533	.35598	.13616	.03472	.00664	.00102
.26500	.45158	.35901	.14271	.03782	.00752	.00120
.27500	.43823	.36154	.14914	.04101	.00846	.00140
.28500	.42528	.36362	.15545	.04430	.00947	.00162
.29500	.41271	.36525	.16162	.04768	.01055	.00187
.30500	.40052	.36647	.16766	.05114	.01170	.00214
.31500	.38868	.36730	.17355	.05467	.01292	.00244
.32500	.37719	.36776	.17928	.05827	.01420	.00277
.33500	.36604	.36787	.18486	.06193	.01556	.00313
.34500	.35523	.36766	.19026	.06564	.01698	.00352
.35500	.34473	.36714	.19550	.06940	.01848	.00394
.36500	.33454	.36632	.20056	.07320	.02004	.00439
.74000	.10861	.24111	.26763	.19805	.10992	.04880
.84000	.08046	.20276	.25548	.21460	.13520	.06814
.94000	.05961	.16809	.23701	.22278	.15706	.08858

EXHIBIT V
PART 2

MASSACHUSETTS: NEGATIVE BINOMIAL SIMULATION

Probability of X Claims Given m_i: (H)

	Number of Claims: (X)					
m_i	0	1	2	3	4	5
.00005	.99985	.00015	.00000	.00000	.00000	.00000
.00055	.99835	.00165	.00000	.00000	.00000	.00000
.00550	.98365	.01620	.00015	.00000	.00000	.00000
.01500	.95609	.04283	.00106	.00002	.00000	.00000
.02500	.92800	.06908	.00283	.00008	.00000	.00000
.03500	.90082	.09360	.00535	.00022	.00001	.00000
.04500	.87451	.11649	.00853	.00045	.00002	.00000
.05500	.84904	.13782	.01230	.00080	.00004	.00000
.06500	.82438	.15768	.01659	.00127	.00008	.00000
.07500	.80051	.17615	.02132	.00188	.00013	.00001
.08500	.77740	.19331	.02644	.00263	.00021	.00001
.09500	.75502	.20922	.03189	.00353	.00032	.00002
.10500	.73334	.22395	.03761	.00459	.00046	.00004
.11500	.71235	.23757	.04357	.00581	.00063	.00006
.12500	.69202	.25013	.04972	.00719	.00084	.00009
.13500	.67232	.26169	.05602	.00872	.00110	.00012
.14500	.65324	.27232	.06244	.01041	.00141	.00016
.15500	.63476	.28205	.06893	.01225	.00177	.00022
.16500	.61684	.29094	.07547	.01424	.00218	.00029
.17500	.59949	.29903	.08204	.01637	.00265	.00037
.18500	.58266	.30637	.08860	.01864	.00318	.00047
.19500	.56636	.31301	.09514	.02103	.00378	.00058
.20500	.55055	.31897	.10164	.02356	.00444	.00072
.21500	.53523	.32431	.10808	.02619	.00516	.00088
.22500	.52038	.32905	.11443	.02894	.00595	.00105
.23500	.50598	.33322	.12070	.03180	.00681	.00125
.24500	.49202	.33687	.12686	.03474	.00773	.00148
.25500	.47848	.34002	.13290	.03778	.00872	.00174
.26500	.46534	.34270	.13881	.04089	.00979	.00202
.27500	.45261	.34494	.14459	.04408	.01092	.00233
.28500	.44025	.34677	.15022	.04733	.01212	.00267
.29500	.42827	.34820	.15571	.05064	.01338	.00305
.30500	.41664	.34927	.16103	.05400	.01471	.00345
.31500	.40536	.34999	.16620	.05740	.01611	.00389
.32500	.39442	.35039	.17121	.06084	.01757	.00437
.33500	.38380	.35049	.17604	.06431	.01909	.00488
.34500	.37349	.35030	.18071	.06780	.02067	.00543
.35500	.36348	.34985	.18520	.07130	.02230	.00601
.36500	.35377	.34915	.18952	.07482	.02400	.00663
.74000	.13468	.24466	.24446	.17765	.10489	.05335
.84000	.10567	.21269	.23546	.18957	.12401	.06989
.94000	.08339	.18343	.22192	.19527	.13960	.08598

METHODS FOR FITTING DISTRIBUTIONS TO INSURANCE LOSS DATA

CHARLES C. HEWITT, JR. AND BENJAMIN LEFKOWITZ

SUMMARY

The methods described in this paper can be used to fit five types of distribution to loss data: gamma, log-gamma, log-normal, gamma + log-gamma, and gamma + log-normal. The paper also discusses applications of the fitted distributions to estimation problems; e.g., computing the effects of inflation on the loss portion of deductible credits and increased limits charges, and determining changes to claim frequencies and severities brought about by changes in deductibles and limits. A computer program carries out all the calculations.

INTRODUCTION

Casualty actuaries frequently wish to extract information from insurance loss data. Generally, an actuary will group individual losses by size of loss and then fit a continuous positive distribution to the aggregated data. In this way, he can characterize the universe from which the sample was selected. For example, a distribution fitted to one month's losses could be used to characterize the distribution of annual losses.

Bickerstaff and Dropkin have shown that the log-normal distribution closely approximates certain types of homogeneous loss data [1], [2]. Hewitt showed that two other positive distributions, the gamma and the log-gamma, also give good fits [3], [4]. Used alone, each of these distributions assumes that the observed losses are generated by a single underlying process. This may not always be the case. For example, a sample of observed losses may contain some that involved litigation and others that did not. In this situation, a single distribution may not fit the aggregate data as well as a combination of two (or more) distributions added together.[1] Herein, such combinations are called compound distributions. This paper describes algorithms for fitting two particular compound distributions, gamma + log-gamma, and gamma + log-normal, and three simple distributions: gamma, log-gamma and log-normal.

[1]Each component distribution has its own form, i.e., gamma, log-gamma or log-normal, and its own parameters, e.g., mean, variance. If the proportion of losses (either claim count or amount) in one distribution is P, then the proportion in the second distribution is 1-P.

The authors do not claim that all insurance loss data can be fitted by the methods described below or, in fact, by any analytical methods. However, after many years' experience, we are convinced that these methods will produce useful results for most practical problems.

ALGORITHMS FOR FITTING DISTRIBUTIONS

The usual method for fitting a distribution to observations involves estimating the distribution's parameters or moments from a sample of actual loss frequencies, and then using those parameters to compute the distribution's densities, i.e., its theoretical loss frequencies. The normal distribution's parameters, for example, are the mean and variance. Given their values, one can obtain loss frequencies by consulting tables of the normal distribution.

This method cannot be applied to a compound distribution because its parameters are not directly computable from the sample observations. Instead, an iterative procedure must be used to approximate them. The procedure, described in Appendix A, repeats the following steps:

1. Split the data between the two distributions.
2. Estimate each distribution's parameters.
3. Fit the distributions.
4. Compare the computed frequencies to the actual frequencies.

Each iteration attempts to adjust the data split and distribution fits so as to improve the correspondence between actual and theoretical frequencies. There is no guarantee that the correspondence will improve each iteration, or that the best fit will be obtained after a finite number of trials. Generally, it takes fewer than ten iterations to reach stability, by which we mean that the mean of the fitted compound distribution changes little from one iteration to the next.

A problem common to fitting any distribution—single or compound—to aggregate loss data is the location of the "mass-point" of each loss interval.[2] A single value must represent all observations within an interval; very often, the interval midpoint is used for this purpose. The choice of mass-point influences the distribution's parameters and hence the quality of the resulting fit. In most distributions arising in casualty insurance applications, losses are skewed toward the upper boundary of their intervals. However, in the normal distribution and distributions like it, losses are skewed toward the lower boundary

[2] I.e., the first two moments, loss amount and loss amount squared weighted by frequency.

in intervals lying to the left of the mode, and towards the upper boundary in intervals lying to the right of the mode.

The algorithms described in Appendix A include calculations that correct the possible bias introduced by the exclusive use of the interval midpoint. The mass-points of the amount-of-loss and the square of the amount-of-loss must be adjusted at each successive iteration, because they are used to compute the moments needed to estimate the parameters of the individual distributions. In most instances, the correction substantially affects only the uppermost and lowermost intervals.

EXAMPLE

The quality of compound distribution fits can be illustrated by an example. Table 1 contains automobile bodily injury loss data along with log-normal and gamma + log-normal fits to the data.[3]

As can be seen, the compound distribution, gamma + log-gamma, is superior to the log-normal alone because it better approximates the frequency of low value and high value losses.

The goodness-of-fit can be measured by the Chi-square statistic, χ^2 [5]. The difference between actual and log-normal distribution has a $\chi^2 = 28.7$ with 15 degrees of freedom. This means that there is about a 2.5% chance the log-normal explains the data. The difference between actual and the gamma + log-gamma distribution has $\chi^2 = 3.5$ with 12 degrees of freedom. There is only about a 1% chance that the agreement could arise by chance alone.

DISTRIBUTION TABLE

A distribution fitted to the number of losses can be used to compute the cumulative dollars of loss and the deductible credit or "buy back." A deductible credit is the proportional loss reduction caused by imposing a deductible.[4] Readers may be more familiar with the term "loss elimination ratio" [6]. Notice that the limits in Table 2 are not the same as the limits in Table 1.

[3]The data is from a 1969 Department of Transportation study of automobile injuries. It shows general damages on serious injury cases in California.

[4]Purchasers of automobile collision insurance understand that they pay something less than full cost when they are willing to pay the first $50, $100, etc. of any loss.

To illustrate the use of the distribution table, refer to the limit of \$10,000. The table shows that losses of \$10,000 or less account for 82.15% of the losses by number but only 12.64% of the loss dollars. These loss dollars plus the first \$10,000 on the 17.85% of the losses which exceed \$10,000 account for 25.88% of all loss dollars. Put another way, for a policy with no limit but with a \$10,000 deductible, the table indicates a (loss dollar) credit of 25.88%.

The distribution table also can be used to determine the loss portion of charges for increased limits. The formula is:

$$Charge = \frac{\begin{array}{c}(deductible\ credit\ for\ the\ increased\ limit \\ -\ deductible\ credit\ for\ the\ basic\ limit)\end{array}}{deductible\ credit\ for\ the\ basic\ limit}$$

For example, to determine the loss portion of the charge for a layer of coverage between \$10,000 and \$100,000, compute

$$\frac{.6029 - .2588}{.2588} = 1.33$$

i.e., the loss portion of the increased limits charge is 133% of basic limits losses. (The increased limits factor is, of course, 2.33 if expenses remain proportionate.)

EFFECT OF INFLATION

It is possible to construct distribution tables that take into account the effect of inflation on loss settlements, thereby allowing actuaries to answer questions of the following type: What would happen to loss costs if future losses were distributed in a manner similar to past losses, but settlement costs were 100% higher?

The answer is found in Table 3 which uses data from Table 2 but assumes a 100% inflation rate.[5] Note the contrast between the two tables. Only 71.09% of the inflated losses (Table 3) are less than \$10,000 and they account for only 6.83% of the loss dollars, whereas 82.15% of the uninflated losses (Table 2) are less than \$10,000 and they account for 12.64% of the loss dollars. Similarly, under 100% inflation, the deductible credit decreases from 25.88% to 17.54%. Inflation causes the loss portion of the increased limits charge for \$100,000 limits to rise from 133% to 184%.

[5] I.e., a 100% increase in the value of each loss from the settlement date to the date for which losses are being used.

The algorithm for measuring the effect of inflation is shown in Appendix B.

FITTING TRUNCATED DISTRIBUTIONS

Because insureds use deductibles or retentions in many lines of casualty insurance, data collected for use by the actuary may be incomplete in that nothing is available for losses below some fixed dollar amount (such as $100). The flexibility of the gamma, log-gamma and log-normal distributions is such that their moments (where they exist) also are distributed respectively as gamma, log-gamma and log-normal [1], [3], [4]. The missing portion of a truncated distribution may contain many losses, but often the missing loss dollars do not amount to a great deal. Therefore, it is suggested that the actuary initially use the dollar amount of losses to fit the distribution. The number of omitted losses resulting from the use of the deductible can then be estimated and used to fit the distribution of the number of cases.

The method for obtaining the parameters of the distribution of cases after estimating the parameters of the distribution of loss dollars is shown in Appendix C.

For large losses, the data collected by the actuary may be inaccurate because of policy loss limits. Here there are no missing cases, but the arbitrary limit obscures the true (unbounded) value of these larger losses.

This problem can be solved by calculating the "true" value[6] for the mass-point of the uppermost interval in a manner that is independent of the reported values of the larger losses. The actuary selects the lower limit of the uppermost interval for the data to be fitted so that all cases which may have been arbitrarily valued fall into the uppermost interval. Then, by fitting the number of cases and not their dollar value, the effect of policy limits is ignored. The method for calculating an interval mass-point is explained in Appendix A.

Example: Suppose the raw data in Table 3 contains no losses under $100, because of the existence of a $100 deductible. One could fit a distribution to this data under the assumption that the interval $0-100 is empty. (This is equivalent to assuming that the cumulative frequency of loss dollars up to the $250 limit equals the frequency of loss dollars in the interval $100-250.) The error introduced by this assumption (0 cf. .0002), is less than the error intro-

[6]As opposed to some arbitrary, a priori assumption, guess or inaccuracy in the raw data itself.

duced by postulating that there are no claims in the \$0-100 loss interval (0 cf. .1433).

After fitting a distribution to raw loss dollars, one can deduce the parameters of the distribution of claim counts as shown in Appendix C. These latter parameters can be used to calculate the hypothetical proportion of claims under \$100. That proportion can be used to fit the augmented claim count distribution.

RELATIVE FREQUENCIES AND SEVERITIES

Changes in claim frequencies and severities can be determined when deductibles (or retentions) and limits are changed. Assume an insured has a retention of \$1,000 per claim, then what are the relative frequencies when the retention is increased to \$5,000 per claim? From Table 2,

$$\frac{1 - .7109}{1 - .3862} \doteq .471$$

the new frequency is 47.1% of the old frequency.

The relative severities under unlimited coverage are:

$$\frac{\dfrac{1 - .1754}{1 - .7109}}{\dfrac{1 - .0545}{1 - .3862}} = \frac{2.852}{1.540} = 1.852$$

or the new severity is 185.2% of the old severity.

Suppose limits are increased from \$10,000 per claim to \$100,000 per claim. What happens to the relative severities? From Table 2,

$$\frac{.6029}{.2588} = 2.330$$

that is, the new severity will be 233.0% of the old severity.

Suppose a reinsurer has data collected on the basis of a retention of \$10,000 and a limit of \$100,000, and suppose loss costs have increased 100% since the period for which the data was collected. How will the relative frequencies and severities change? From Tables 2 and 3, the relative frequencies are:

$$\frac{1 - .7109}{1 - .8215} = 1.620$$

that is, there will be 62.0% more claims at the same retention because of inflation. The relative severities are:

$$\frac{\dfrac{.4981 - .1754}{1 - .7109}}{\dfrac{.6029 - .2588}{1 - .8215}} = \frac{1.116}{1.928} = .579$$

The new severity is 57.9% of the old severity.

PROGRAM HEWITZ

All computations described in this paper were performed with a computer program called HEWITZ.[7] This program fits five distributions to input data: gamma, log-gamma, log-normal, gamma + log-gamma and gamma + log-normal.

HEWITZ has the following characteristics and capabilities:

1. The user can select different intervals for the input data and the output distribution table.
2. The user can halt the iterative algorithms in one of several ways, but usually by specifying the maximum number of iterations.
3. The user can create a wholly new distribution by presetting any distribution's parameters.
4. The program computes the Chi-square goodness-of-fit statistic.

[7]Program HEWITZ is written in G-Level Fortran IV, and has been implemented on an IBM 370/158 computer. The program occupies about 100k bytes of core. The program took ten seconds to fit all five distributions to the loss data described earlier.

TABLE 1

Automobile Bodily Injury Loss Data

Loss Amount ($)	Number of Cases		
	Actual	Log-Normal	Gamma + Log-Gamma
1– 50	27	18	27
51– 100	4	10	4
101– 150	1	8	2
151– 200	2	6	2
201– 250	3	5	3
251– 300	4	4	3
301– 400	5	7	6
401– 500	6	6	5
501– 750	13	12	12
751–1,000	8	8	10
1,001–1,500	16	12	15
1,501–2,000	8	9	11
2,001–2,500	11	7	9
2,501–3,000	6	5	7
3,001–4,000	12	8	11
4,001–5,000	9	6	8
5,001–7,500	14	10	13
Over 7,500	40	48	41
TOTAL	189	189	189

TABLE 2

Distribution Table

Upper Limit of Loss Amount ($)	Cumulative Frequency of Cases	Cumulative Frequency of Dollars	Deductible Credit
100	.1623	.0003	.0065
250	.2008	.0008	.0156
500	.2724	.0028	.0298
750	.3344	.0057	.0427
1,000	.3862	.0090	.0545
2,000	.5271	.0242	.0943
2,500	.5739	.0320	.1109
5,000	.7109	.0683	.1754
7,500	.7795	.0995	.2221
10,000	.8215	.1264	.2588
20,000	.8992	.2075	.3570
25,000	.9176	.2380	.3908
50,000	.9582	.3432	.4981
100,000	.9803	.4570	.6029
250,000	.9934	.6047	.7263
500,000	.9973	.7040	.8028
1,000,000	.9990	.7873	.8633
Unlimited	1.0000	1.0000	1.0000

TABLE 3

Distribution Table with 100% Inflation

Upper Limit of Loss Amount ($)	Cumulative Frequency of Cases	Cumulative Frequency of Dollars	Deductible Credit
100	.1433	.0002	.0034
250	.1676	.0004	.0081
500	.2008	.0008	.0156
750	.2373	.0017	.0229
1,000	.2724	.0028	.0298
2,000	.3862	.0090	.0545
2,500	.4298	.0126	.0655
5,000	.5739	.0320	.1109
7,500	.6564	.0508	.1463
10,000	.7109	.0683	.1754
20,000	.8215	.1264	.2588
25,000	.8501	.1501	.2891
50,000	.9176	.2380	.3908
100,000	.9582	.3432	.4981
250,000	.9848	.4939	.6350
500,000	.9934	.6047	.7263
1,000,000	.9973	.7040	.8028
Unlimited	1.0000	1.0000	1.0000

APPENDIX A

The following symbols are used:

Symbol	Meaning
$a+1$	Scale parameter of the log-gamma distribution
A	Scale parameter of the gamma distribution
c_i	Actual number of cases in the i-th loss (claim) interval
C_i	Computed number of cases in the i-th loss (claim) interval
DC_j	Cumulative proportion of cases in the first j intervals of the distribution table
DD_j	Cumulative proportion of loss dollars in the first j intervals of the distribution table
DE_j	Deductible credit for the first j intervals of the distribution table
$E_G(X)$	Mean of X, gamma
$E_G(X^2)$	Mean of X^2, gamma
$E_L(x)$	Mean of x, log-gamma
$E_L(x^2)$	Mean of x^2, log-gamma
$\bar{E}_L(x)$	Estimate of $E_L(x)$ used in the first iteration of the gamma + log-gamma algorithm
$E(X)$	Mean of the compound distribution
$E_L(X)$	Mean of X, log-gamma. Equals $\left(\frac{a+1}{a}\right)^{p+1}$
$E_N(x)$	Mean of x, log-normal
$E_N(x^2)$	Mean of x^2, log-normal
$E_N(X)$	Mean of X, log-normal. Equals $exp\,(E_N(x) + \sigma^2_N/2)$
f_i	Relative frequency of cases in the i-th loss (claim) interval
F_i	Cumulative of f_i
FG_i	Cumulative frequency of gamma distribution in the i-th interval
$H(j)$	Proportion of claims in j-th loss interval after allowing for inflation
$I(y,w)$	Value of the incomplete gamma function ratio for the variable y and the parameter w. This is the cumulative density of the ratio up to and including y
N	Index of last loss (claim) interval
P	Proportion of total claims in log-gamma or log-normal distribution
$p+1$	Shape parameter of the log-gamma distribution
Q	Proportion of total claims in gamma distribution. Equals $1 - P$
R	Shape parameter of the gamma distribution

x_i	$Log_e\ X_i$
xh_i	$Log_e\ XH_i$
X	Value of loss
X_i	Midpoint of the i-th loss (claim) interval
XH_i	Upper boundary of the i-th loss (claim) interval
XL_i	Lower boundary of the i-th loss (claim) interval
y_j	$Log_e\ Y_j$
Y_j	Upper boundary of the j-th distribution table interval
λ	Inflation factor. Equals one plus the rate of inflation expressed as a fraction
$\Phi(z_i)$	Normal curve cumulative density from $-\infty$ to z_i
$\sigma^2{}_G$	Variance of X in the gamma
$\sigma^2{}_L$	Variance of X in log-gamma
$\tilde{\sigma}_L$	Estimate of σ_L used in the first iteration of the gamma + log-gamma algorithm
$\sigma^2{}_N$	Variance of x in log-normal

1) *Gamma Distribution*

The gamma distribution, actually the incomplete gamma function ratio, is the cumulative density function:

$$I\,(v_i,\ R-1) = \begin{cases} 0\ , & i=0 \\ \dfrac{1}{\Gamma(R)} \displaystyle\int_0^{v_i\sqrt{R}} y^{(R-1)}\, e^{-y}\, dy, & 0<i<N \\ 1\ , & i=N \end{cases}$$

where

$$v_i = A \bullet XH_{i_i} / \sqrt{R}$$

In the k-th iteration, the distribution parameters

A - the scale parameter

R - the shape parameter

are estimated as follows:

$$A = E_G(X) / \sigma^2_G, \quad R = A \bullet E_G(X)$$

The k-th iteration values of $E_G(X)$ and σ_G^2 are:

$$E_G(X) = \Sigma f_i X_i \ , \ \sigma_G^2 = \Sigma f_i X_i^2 - \overline{E_G(X)}^2$$

Initially, $X_i = \frac{1}{2}(XH_i + XL_i)$. After the k-th iteration, the repaired interval midpoints are:

$$X_i = \frac{g_i' E_G(X)}{g_i^*}$$

where $E_G(X)$ is the mean of the gamma computed in the $(k\text{-}1)$st iteration, and g_i^* is the proportion of cases in the i-th interval computed using the gamma fitted in the k-th iteration.

$$g_i^* = I(v_i, R-1) - I(v_{i-1}, R-1)$$

The quantity g_i' is the proportion of dollar loss in the i-th interval computed using the gamma fitted in the k-th iteration

$$g_i' = I(v_i', R) - I(v_{i-1}', R)$$
$$v_i' = A \bullet XH_i / \sqrt{R+1}$$

In the k-th iteration, the repaired values of X_i^2

$$X_i^2 = \frac{g_i'' \bullet E_G(X^2)}{g_i^*}$$

where $E_G(X^2)$ is the average of the squared midpoints computed in the $(k\text{-}1)$ st iteration, and g_i'' is the proportion of the X^2-value in the i-th interval computed using the gamma fitted in the k-th iteration

$$g_i'' = I(v_i'', R-1) - I(v_{i-1}'', R+1)$$
$$v_i'' = A \bullet XH_i / \sqrt{R+2}$$

The number of claims in the i-th interval computed using the fitted gamma distribution is:

$$C_i = [C^* \{ I(v_i, R-1) - I(v_{i-1}, R-1)\} + .5]$$
$$C^* = \Sigma c_i$$

The square brackets represent the greatest integer function.

2) Log-gamma Distribution

The log-gamma distribution, actually the incomplete gamma function ratio applied to the logarithms of loss data, is the cumulative density function:

$$I(u_i, p) = \begin{cases} 0\ , \ i=0 \\ \dfrac{1}{\Gamma(p+1)} \displaystyle\int_0^{u_i\sqrt{p+1}} y^p\, e^{-y} dy, & 0<i<N \\ 1\ , \ i=N \end{cases}$$

where

$$u_i = (a+1) \bullet xh_i / \sqrt{p+1}$$

In the k-th iteration, the distribution parameters

$a + 1$ — the scale parameter

$p + 1$ — the shape parameter

are estimated as follows:

$$a+1 = E_L(x) / \sigma_L^2, \quad p+1 = max\{0, (a+1) \bullet E_L(x)\}$$

The k-th iteration values of $E_L(x)$ and σ_L^2 are:

$$E_L(x) = \Sigma f_i x_i\ , \quad \sigma_L^2 = \Sigma f_i x_i^2 - \overline{E_L(x)}^2$$

Initially $x_i = log_e \{1/2 (XH_i + XL_i)\}$. After the k-th iteration, the repaired interval midpoints are:

$$x_i = E_L(x) \bullet f_i' / f_i^*\ ,$$

where $E_L(x)$ is the mean of X in the log-gamma computed in the $(k-1)$st iteration, and f_i^* is the proportion of cases in the i-th interval computed using the log-gamma fitted in the k-th iteration.

$$f_i^* = I(u_i, p) - I(u_{i-1}, p).$$

The quantity f_i' is the proportion of x in the i-th interval

$$f_i' = I(u'_i, p+1) - I(u'_{i-1}, p+1)$$
$$u''_i = (a+1) \bullet xh_i / \sqrt{p+2}$$

In the k-th iteration, the repaired values of x_i^2 are:

$$x_i^2 = \frac{f_i'' \bullet E_L(x^2)}{f_i^*}$$

where $E_L(x^2)$ is the average squared log of the midpoints computed in the $(k\text{-}1)$st iteration, and f_i'' is the proportion of the x-value in the i-th interval computed using the log-gamma fitted in the k-th iteration.

$$f_i'' = I(u_i'', p+2) - I(u_{i-1}'', p+2)$$
$$u_i'' = (a+1) \bullet xh_i / \sqrt{p+3}$$

The number of claims in the i-th interval computed using the fitted log-gamma distribution is

$$C_i = [C^* \{I(u_i,p) - I(u_{i-1},p)\} + .5]$$
$$C^* = \Sigma c_i$$

The square brackets represent the greatest integer function.

3) Log-normal Distribution

The cumulative frequency of the log-normal distribution is:

$$\Phi(z_i) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_i} e^{-\frac{1}{2} z_i^2} dz_i$$

where

$$z_i = \{ xh_i - E_N(x) \} / \sigma_N$$

In the k-th iteration the distribution parameters

$E_N(x)$ — the mean of the log-normal
σ_N^2 — the variance of the log-normal

are estimated as follows:

$$E_N(x) = \Sigma f_i x_i$$
$$\sigma_N^2 = \Sigma f_i x_i^2 - \overline{E_N(x)}^2$$

Initially $x_i = \log_e \{ \frac{1}{2} (XH_i + XL_i) \}$. After the k-th iteration, the repaired interval midpoints are:

$$x_i = f_i' / f_i^*$$

where f_i^* is the proportion of cases in the i-th interval.

$$f_i^* = \Phi(z_i) - \Phi(z_{i-1}),\ \Phi(z_0) = 0,\ \Phi(z_N) = 1$$

$$f_i' = \{E_N(x)\,\Phi(z_i) - \frac{\sigma_N}{\sqrt{2\pi}} e^{-\frac{1}{2} z_i^2}\} - \{E_N(x)\,\Phi(z_{i-1}) - \frac{\sigma_N}{\sqrt{2\pi}} e^{-\frac{1}{2} z_{i-1}^2}\}$$

In the k-th iteration, the repaired values of x_i^2 are:

$$x_i^2 = f_i''/f_i^*$$

where

$$f_i'' = \{(\overline{E_N(x)}^2 + \sigma_N^2)\,\Phi(z_i) - (E_N(x) + x_i) \bullet \frac{\sigma_N}{\sqrt{2\pi}} e^{-\frac{1}{2} z_i^2}\}$$
$$- \{(\overline{E_N(x)}^2 + \sigma_N^2)\,\Phi(z_{i-1}) - (E_N(x) + x_{i-1}) \bullet \frac{\sigma_N}{\sqrt{2\pi}} e^{-\frac{1}{2} z_{i-1}^2}\}$$

The number of claims in the i-th interval computed using the fitted log-normal distribution is:

$$C_i = [C^* \{\Phi(z_i) - \Phi(z_{i-1})\} + .5]$$
$$C^* = \Sigma\, c_i$$

The square brackets represent the greatest integer function.

4) Gamma + Log-gamma

Fitting a compound distribution is a trial-and-error process. Initially, the log-gamma distribution is fitted to the data. Generally this will result in fewer computed claimants in the lower loss intervals than are actually there. The gamma distribution is fitted to the excess claimants. These calculations "split" the data between the gamma and log-gamma distributions. The compound distribution is the weighted sum of the two distributions, the weights being:

P — the proportion of total claims in the log-gamma

$Q = (1\text{-}P)$ — the proportion of total claims in the gamma.

As before, the interval midpoints must be repaired to recognize intra-interval skewness.

The gamma plus log-gamma (*GPLG*) distribution is the cumulative density function:

$$GPLG_i = \begin{cases} 0 \ , \ i=0 \\ Q \bullet I(v_i, R-1) + P \bullet I(u_i, p) \ , \ 0 < i < N \\ 1 \ , \ i=N \end{cases}$$

where

$$v_i = A \bullet XH_i / \sqrt{R}$$
$$u_i = (a+1) \bullet xh_i / \sqrt{p+1}$$

Two methods are used to estimate the log-gamma distribution parameters. One applies to the first iteration only, the other to the remaining iterations. On the first iteration:

$$a = \frac{\tilde{E}_L(x)}{\tilde{\sigma}_L^{\,2}} \qquad p+1 = a \bullet \tilde{E}_L(x)$$

where

$$\tilde{E}_L(x) = \frac{\Sigma f_i X_i \bullet x_i}{\Sigma f_i X_i}$$

$$\tilde{\sigma}_L^{\,2} = \frac{\Sigma f_i X_i \bullet x_i^2}{\Sigma f_i X_i} - \{\tilde{E}_L(x)\}^2$$

The mean of the compound distribution is:

$$E(X) = P \bullet E_L(X) + Q \bullet E_G(X)$$

where

$$E_L(X) = \left(\tfrac{a+1}{a}\right)^{(p+1)} \text{ and } E_G(X) = \frac{R}{A}$$

The formulas for repairing x_i, x_i^2, X_i and X_i^2 are shown earlier.

The number of claims in the *i*-th interval is:

$$C_i = [C^* \{GPLG_i - GPLG_{i-1}\} + .5]$$

$$C^* = \Sigma c_i$$

The square brackets represent the greatest integer function.

5) Gamma + Log-normal

Fitting the gamma + log-normal (*GPLN*) distribution is analogous to fitting the gamma + log-gamma distribution.

The *GPLN* distribution is the cumulative density function:

$$GPLN_i = \begin{cases} 0 \;,\; i=0 \\ \\ Q \bullet \mathrm{I}(v_i, R-1) \;+\; P \bullet \Phi(z_i) \;,\; 0<i<N \\ \\ 1 \;,\; i = N \end{cases}$$

$$v_i = A \bullet XH_i \,/\, \sqrt{R} \;,\; z_i = \{xh_i - E_N(x)\} \,/\, \sigma_N$$

The formulas for estimating the parameters of the gamma and log-normal, A, R, $E_N(x)$ and σ_N^2, are the ones used to fit the gamma and the long-normal separately. Similarly, the procedure used to split the total loss data between the two distributions is the one used in fitting the gamma + log-gamma, but with the log-normal distribution substituted for the log-gamma distribution where appropriate.

In all other iterations:

$$a+1 = \frac{E_L(x)}{\sigma_L^2} \;,\; p+1 = max\,\{0, (a+1) \bullet E_L(x)\}$$

The log-gamma is fitted to the intervals, yielding theoretical cumulative frequencies:

$$DL_i = I(u_i, p)$$

The next step in the calculation splits the total distribution between the gamma and the log-gamma, and estimates the proportion of total claims in each distribution. The calculation consists of the following steps:

Determine whether the data can be split

1. Set the split proportion estimate $P' = 1$, and the interval index $j = 0$
2. Compute proportion of total claims in first interval of gamma $G_1 = F_1 - DL_1$
3. If $G_1 \leq 0$, then the gamma distribution cannot be fitted to the data. This can happen when small valued losses go unreported.

Split the data

4. Increase interval index j by one

5. Compute approximate proportion of total claims in the log-gamma and in the gamma

$$H_j = P' \cdot DL_j$$
$$G_j = F_j - H_j$$

6. Compute estimate of P:

$$P = 1 - G_{j+1} \text{ (Initially } G_2 = G_1 \text{ ; see Step 2)}$$

7. Compute proportion of total claims in the j-th and $(j+1)$st intervals of the log-gamma, and the gamma

$$H_j = P \cdot DL_j$$
$$G_j = F_j - H_j$$
$$H_{j+1} = P \cdot DL_{j+1}$$
$$G_{j+1} = F_{j+1} - H_{j+1}$$

8. Compute the difference of successive intervals of the gamma

$$\Delta = G_{j+1} - G_j$$

9. If $\Delta < 0$, go to Step 10, otherwise set $P' = P$ and return to Step 4.

Compute Frequencies of the Gamma

10.

$$G_j = \begin{cases} F_i - P \cdot DL_i \text{ , } & i < j \\ 1 \text{ , } & i \geq j \end{cases}$$

After the data has been split, the parameters of the gamma distribution are estimated as follows:

$$A = E_G(X) / \sigma_G^2 \quad , \quad R = A \bullet E_G(X)$$

where

$$E_G(X) = \Sigma\, G_i \bullet X_i \ , \quad \sigma_G^2 = \Sigma\, G_i \bullet X_i^2 - \overline{E_G(X)}^2$$

The formulas for repairing x_i, $x^2{}_i$, X_i^2 *are shown earlier.*

The number of claims in the i-th interval is

$$C_i = [C^* \{GPLN_i - GPLN_{i-1}\} + .5]$$

$$C^* = \Sigma\, c_i$$

The square brackets represent the greatest integer function.

The mean of the compound distribution is:

$$E(X) = P \bullet E_N(X) + Q \bullet E_G(X)$$

where

$$E_N(X) = exp\ (E_N(x) + \frac{\sigma_N^2}{2}) \text{ and } E_G\,(X) = \frac{R}{A}$$

APPENDIX B

EFFECT OF INFLATION

In the formulas given below the subscripts 1, 2 and 3 refer to the gamma, log-gamma and log-normal distributions respectively, and the index j runs over the Distribution Table loss intervals. The parameter λ, is one plus the rate of inflation expressed as a fraction.

$$F_1(j) = I(v_j, R - 1)\ , \quad v_j = \frac{Y_j}{\lambda} \cdot \frac{A}{\sqrt{R}}$$

$$F_2(j) = I(u_j, p) \quad , \quad u_j = \frac{a + 1}{\sqrt{p + 1}}\ log\ \frac{Y_j}{\lambda}$$

$$F_3(j) = \Phi(z_j) \quad , \quad z_j = log\ \frac{Y_j}{\lambda} - E_N(x) / \sigma_N$$

$$G_1(j) = I(v_j^*, R) \quad , \quad v_j^* = \frac{Y_j}{\lambda} \cdot \frac{A}{\sqrt{R+1}}$$

$$G_2(j) = I(u^*, p) \quad , \quad u_j^* = \frac{a}{\sqrt{p+1}} \cdot \log \frac{Y_j}{\lambda}$$

$$G_3(j) = \Phi(z_j^*) \quad , \quad z_j^* = z_j - \sigma_N$$

$$H_1(j) = G_1(j) + \frac{Y_j}{\lambda} \{1 - F_1(j)\} / E_G(X)$$

$$H_2(j) = G_2(j) + \log \frac{Y_j}{\lambda} \{1 - F_2(j)\} / E_L(x)$$

$$H_3(j) = G_3(j) + \log \frac{Y_j}{\lambda} \{1 - F_3(j)\} / E_N(x)$$

For the two compound distributions, we get

Gamma + log-gamma

$$F(j) = Q \cdot F_1(j) + P \cdot F_2(j)$$

$G(j) = (1 - S) \cdot G_1(j) + S \cdot G_2(j)$, where

$$S = \frac{P \cdot E_L(X)}{E(X)} \quad \text{and} \quad E(X) = P \cdot E_L(X) + Q \cdot E_G(X)$$

$$H(j) = G(j) + \frac{Y_j}{\lambda} \{1 - F(j)\} / E(X)$$

Gamma + log-normal

$$F(j) = Q \cdot F_1(j) + P \cdot F_3(j)$$

$G(j) = (1 - S) \cdot G_1(j) + S \cdot G_3(j)$, where

$$S = \frac{P \cdot E_L(X)}{E(X)} \quad \text{and} \quad E(X) = P \cdot E_N(X) + Q \cdot E_G(X)$$

$$H(j) = G(j) + \frac{Y_j}{\lambda} \{1 - F(j)\} / E(X)$$

APPENDIX C

Parameter	Fit on \$	Fit on #
	Gamma	
Shape	$R + 1$	R
Scale	A	A
	Log-Gamma	
Shape	$p + 1$	$p + 1$
Scale	a	$a + 1$
	Log-Normal	
Mean	$E_N(X)$	$E_N(X) - \sigma_N^2$
Variance	σ_N^2	σ_N^2

Example of the use of this Appendix: If a gamma distribution with parameters R and A is fitted to numbers of claim counts (#), then the parameters of the distribution of loss amounts (\$) are $R + 1$ and A respectively.

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AN OVERVIEW OF THE PROPERTY/CASUALTY FINANCIAL CONDITION

ROBERT J. SCHRAEDER

The element which quite possibly presents the most danger to the insurance industry today is the widespread tendency to think in terms of status quo. Status quo thinking is endemic to almost all enterprises. We successfully weathered the trials thrust upon us in the past and, therefore, will do so again. The misconception is that today's trials are not the same. There is something new; external change has accelerated to such an extent that status quo thinking can cause the industry to fall so far behind that the only way it can be brought into line with the perceived needs of society is for it to be accomplished by outside forces.

The concept of insurance as a means of ameliorating catastrophic loss has changed over the past thirty to forty years to the point where insurance is now considered a mechanism of reparation for escaping almost all consequences of unfortuitous economic loss. The manner in which this has come about is no longer important, because the inescapable fact is that the users of insurance now view their policies in an entirely different light than they did some years ago. The insurance industry, while participating in bringing about these changed expectations, has not paid much attention to, or to any appreciable extent understood, the meaning or consequences of this change in attitude. However, if insurance people fail to appreciate the distinction between what they think they are selling and what the policyholders think they are buying, then the outcome will probably be disastrous for the insurance industry, for it is considerably outnumbered. Where public demands have not been met by the industry, their demands have been met through regulatory decrees and/or by the legislative process.

Thus the tendency by the industry to view the problem simply as the public's distorted conception of insurance, in turn calls for a program to educate the public about the business. This is worse than shoveling sand against the sea because not only is it virtually impossible to get any significant percentage of the public interested in the purposes and problems of insurance, it is effectively putting a halt to further thinking on the part of those who guide the destiny of the business and believe they have dealt with the problem.

This basic psychology is not limited solely to the external industry problems, but also permeates to the very vital core of underwriting results. Those who discuss the offset of underwriting losses through investment gains establish a very poor philosophical base for doing business. The necessity of an underwriting profit as a fundamental in the company's conduct of its business is inescapable. Underwriting is the very foundation of the source of money for investments. If profits cannot consistently be produced from underwriting, then ultimately the company will be leveraged into an impossible position. Planning that concedes an underwriting loss runs contrary to the very purpose of engaging in the insurance business and contributes to the erosion of the basic undertaking, as well as the fundamental concepts, of the business.

The remarkable underwriting profits recorded in 1976, 1977 and 1978 have been eroded by the accelerating change in social and judicial attitudes which have been responsible for substantially higher rates of inflation than have existed during the past quarter century. Not only must these danger signals be reckoned with, but the industry must simultaneously refrain from indulging in competition for premium dollars which might push rates to dangerously low levels. The results of such unfortuitous action are more than evident from past history when more, not less, money was needed to cover an ever expanding risk potential. It is to be hoped that the folly of ignoring the necessity for underwriting profits as reflected by the memory of 1974 and 1975 has infused a lasting acknowledgment of the contribution of adequate rates to an underwriting profit.

Adequate rate levels, moreover, demand the establishment of sound loss reserves. Such funds today, more than at any other time in the history of the insurance business, represent the most volatile item on the balance sheet. They are of prime importance to future solvency.

Behind any calculation of proper rate levels in the casualty insurance business is the existence of sound loss reserves. Such funds today, more than ever, are subject to uncertainties over an extended time span that makes doubtful any real confidence in their accuracy. Long term loss reserves, largely a consequence of casualty insurance underwriting, have grown with the expansion of the casualty business. That expansion, since the advent of multiple line underwriting in the mid-1950's, has been dramatic. Twenty-five years ago the major lines of property/casualty insurance were divided 65% property and 35% casualty; by the end of 1978 the division was 29% property and 71% casualty, or more than a complete reversal. This change occurred gradually over the 25 years and, we think, was rather more unnoticed than planned by carrier managements that were oriented to property insurance and had not concerned themselves with the liability side with its concomitant loss reserves and their leveraging effect. Perhaps not enough attention was paid to the significance of the change in the makeup of the underwriting portfolio. One of the ironies of it is that as new forms of insurance protection were needed, they inevitably seemed to fall into the casualty category. This in turn resulted in more and more casualty premium volume accompanied by claims calling for greater and greater loss reserves.

This increase in the size of loss reserves is equalled by their growing importance as a factor in insurer solvency. Since multiple line underwriting began, insurance industry loss reserves have grown by nearly 1,600%, but overall premium volume is up less than half of that — 770%. Unearned premium reserves have advanced 430%, and policyholders' surplus has risen by 528%.

Companies heavily involved in third party lines have had to make substantial additions to their reserves in recent years, to such an extent that the rate of growth in long-tail casualty loss reserves has outstripped the annual advances in earned premiums. By the end of 1978, loss and loss adjustment expense reserves on outstanding claims in workers compensation alone totalled $16.3 billion. Earned premiums for that line during 1978 were $11 billion. If the workers compensation reserves were 10% deficient (as they were over the previous five accident year periods ending 1977), the impact on the workers compensation operating ratio would be 14.8 points. Even at its current high level — approximately 10% of earned premiums—investment income on investable funds for this class of business would not be adequate to offset such a reserve deficiency. When one considers that many carriers discount their workers compensation reserves, then the leverage problem is compounded further in that future investment income is already dedicated to the offset of reserve requirements.

The relationship between loss reserves and earned premiums for general liability and medical malpractice insurance reveals a potential impact on combined ratios even more severe than that in workers compensation. General liability loss reserves were $11.5 billion at year-end 1978, compared with earned premiums of $6.2 billion; medical malpractice loss reserves of $3.2 billion were more than double the earned premium volume of $1.2 billion. If loss reserves in these lines were 10% deficient, the impact on the operating ratio would be 18.5 points and 26.7 points, respectively.

The other casualty lines covered in Schedule P of the Annual Convention Statement — automobile liability and multiple peril coverages — do not reveal such dramatic potentially adverse impact arising from a reserve deficiency. Automobile liability loss and loss adjustment expense reserves of $19.6 billion at the end of 1978 were slightly less than the earned premiums that year of $20.0 billion, while multiple peril coverage loss reserves of $6.9 billion were significantly less than the $15.0 billion of 1978 earned premium volume. Compared with the considerably longer time required to close cases in workers compensation, medical malpractice, products liability and general liability, the automobile and multiple peril reserving practices exhibit trends closer to actual experience. Deficiencies in these lines over the five accident years ending 1977 were moderate, and the hypothesis of a 10% deficiency would impact the operating ratio of automobile at 9.8 points and for multiple peril, 4.6 points.

The establishment of loss reserves for the final adjudication of claims at some undetermined time in the future is a major challenge to the actuarial sciences. Such funds are subject to the limitations of information available to those who must attempt to determine a realistic figure; and once set, reserves are subject to the vicissitudes of economic fluctuation, social and judicial change, and liberalizing legislation — to say nothing of the second thoughts of those who put up the reserves in the first place. We stress that loss reserves are today, more than at any other time in the history of the insurance business, the most explosive item on the balance sheet; they are of prime importance to future solvency.

To relate loss reserves to policyholders' surplus provides a fairly simple test of the leveraged position of insurers. It is a test which we believe tells a good deal more about the extent of underwriting exposure than does the premium-to-surplus ratio.

The leverage of reserves to surplus in a time of high inflation is a critical matter. A company with $100 million in reserves and $100 million in surplus pays dollar for dollar from surplus for a deficiency in loss reserves; a company that has $200 million in reserves and $100 million in surplus and is 10% under-reserved will lose nearly 20% of its surplus making up the 10% deficiency. When the leverage exceeds two-and-a-half times reserves to surplus, deficiencies in the loss reserves become very serious and even more so if there is the added volatility of the securities market. The severity of this also is affected by the size of the stock portfolio in relation to surplus.

One final thought on loss reserving practices. Many companies report bringing loss reserves up to the required levels as though it were a voluntary act designed to strengthen the company's financial position. In fact, it is more often a reflection of the company's having fallen out of phase with the actual loss situation. Strengthening loss reserves is a euphemism for reducing surplus.

Each class of business has its own loss development characteristics, which are mirrored in the size of the loss reserves that line of business requires concomitant with the time lag in the final settlement of claims. This premise recognizes the importance of loss reserves to the solvency of a company. Furthermore, the premise establishes a base for the use of loss reserves as a monitor of growth in premium production and as a major factor in the determination of investment philosophy.

Experience has established that new business, carefully underwritten, develops poorer overall results than that which has been reunderwritten, and the latter produces poorer results than that recorded by a seasoned or older book of underwriting risks. The length of time demanded for the seasoning process relates directly to the type of insurance, the class of risk and the underwriting standards established by the carrier. With this in mind, each carrier must determine how these factors apply in its case, so that the proper relationship of loss reserves to policyholders' surplus can be determined. These factors also will provide the basis for construction of a model which can indicate acceptable growth levels for each line and class of business consistent with the company's financial position. This in turn establishes the base for the determination of loss reserve requirements in connection with a given rate of growth for each class of business.

The model can be programmed to develop the proper mix of business in accordance with a predetermined overall rate of growth for a given year or to establish the maximum growth rate for one or more lines that management desires to stress. These growth patterns are predicated on relationships set by management between loss reserves and policyholders' surplus which, in turn, relates to the volatility that the investment portfolio can have. As the ratio of loss reserves to surplus approaches and/or exceeds two-to-one, management must become wary of its exposure to the impact on surplus that can arise out of adverse trends in the securities market.

Over the past quarter century the marked change in underwriting mix has been accompanied by a dramatic change in the sources of investable funds: unearned premium and loss reserves, and policyholder's surplus. These funds responsible for investment income changed from a base of money to be retained (i.e., unearned premiums and policyholders' surplus) to a base of funds that are to be paid out (i.e., loss reserves). At the start of the period 74% of investable funds were represented by unearned premiums (38%) and policyholders' surplus (36%) and the remainder (26%) by loss reserves. As of the 1978 year the distribution of investable funds was sharply different with 51% of such monies represented by loss reserves and the remainder by unearned premiums (23%) and policyholders' surplus (26%). This shift in the base of investment income and its implications has gone largely unnoticed and by many outside the industry is viewed as profits achieved to the detriment of policyholders and claimants. The need for investment income and the role it plays in company stability and growth is not understood or is misunderstood by nearly everyone not in the business and probably by too many within it.

The marvel of investment income today is that it is becoming the dominant factor in determining operating results. Thus the ability to face another period of underwriting hardship with composure arises primarily from the new perspective insurance managements and most analysts of the industry have adopted in their view of underwriting losses. This perspective allows them to regard a deteriorating underwriting situation calmly because there exists sizable investment income which can obscure a poor underwriting showing and produce a bottom line gain that a few years ago could have been achieved only with the help of a combined ratio well below 100%. The strength of the market for property/casualty insurance stocks despite widely predicted underwriting reverses demonstrates that investors are ignoring underwriting in favor of the bottom line. The danger in this is that insurer managements also tend to think in those terms.

Just since 1970, investment income has shot ahead in dollars by 3.6 times; more importantly, as a percentage of earned premiums it has in the same time span risen by 2.6 points to 9.2% as of the end of 1978. By the end of this year, investment income will be close to 10% of the industry's earned premiums. That can cover a lot of underwriting loss. Insurance company investment income is rising faster than premium income, faster than losses, faster even than inflation.

Attention must be called to the other side of the investment income coin and to suggest that there may be something here in need of thoughtful consideration. There are problems inherent in this increasing fund of money, problems of sufficient magnitude that it would be well to be aware of the possibility that the rapid growth in this component of insurance operations could be a curse in disguise.

The first bad effect of high investment income is the already-noted opportunity it presents managements to cover up a poor underwriting operation. Poor underwriting begins with competitive excesses which can be fostered by the protective blanket of a secondary source of earnings.

Already there have been announcements by insurance regulators to the effect that rate increases requested against the background of a successful operating report (in which the underwriting outcome in fact may have been miserable) simply are not in order. This is a not-too-subtle means of incorporating investment income into rate making. The next step is to formalize this ready-made medium for holding down costs to the consumer.

Current levels of investment income are a product of inflation which is reflected by the volatility of claims that demand the establishment of ever larger loss reserves, the major base of investible assets. It cannot be expected that interest rates will remain in their present range even for any extended period of time, and when they decline insurers must face the realities. The greater the extent to which a company relies on income from a peripheral operation (investments) to maintain its overall soundness and growth, the greater the difficulty in maintaining balance when income from that peripheral operation declines. This situation would be compounded if by then there existed a legislated formula for incorporating investment income into rate making.

Indications for 1979 are for poor underwriting results with the outlook for 1980 also not encouraging. If investment income in 1979 is 10% of earned premiums and the combined loss and expense ratio is 102%, there remains 8% in additional surplus to support growth. But if inflation continues at around 12% to 13%, more than 60% of the surplus gain in 1979 will be required to cover the increase in losses and higher rates occasioned by inflation. That leaves a 3% increase in surplus to fund net growth, which at a 2.5-to-1 premium to surplus ratio, would come to something like 7.5%. Remember that the 5% in added surplus needed to fund inflation will be called upon regardless of whether or not rates rise — the claims will be there at inflation-level costs. If rates are held down, the losses still must be paid. Thus even with a huge investment income, inflation — which causes investment income to rise and simultaneously creates the need for so much added surplus — sees to it that the real growth of insurance companies is modest if not negligible unless it is supported by funding from profitable underwriting.

MINUTES OF THE 1979 FALL MEETING

November 14–16, 1979

ORLANDO HYATT HOTEL, KISSIMMEE, FLORIDA

Wednesday, November 14, 1979

The Board of Directors held their regular quarterly meeting from 1:00–5:00 p.m.

Registration took place from 4:00–7:30 p.m.

The President's reception for new Fellows and their spouses was held from 6:00–6:45 p.m.

A reception for members and guests was held from 6:30–7:30 p.m.

Thursday, November 15, 1979

Registration was held from 8:00–8:45 a.m.

The Fall Meeting was formally convened at 8:45 a.m. After her opening remarks, President Ruth E. Salzmann read the names of the 30 new Associates. Each new Associate in attendance rose as his or her name was called. Ms. Salzmann then asked each of the 24 new Fellows to step forward to receive his or her diploma.

FELLOWS

Terry J. Alfuth
Thomas R. Bayley
Richard S. Biondi
Joseph W. Brown, Jr.
Robert F. Conger
Edward W. Ford
Steven F. Goldberg
Robert J. Hemstead
John Herzfeld
Frederick O. Kist
Steven P. Lattanzio
Stuart N. Lerwick
Sandra C. Luneburg
Charles W. McConnell II
John M. Meeks
Jerry A. Miccolis
Bruce D. Moore
Russell K. Nash
Stephen W. Philbrick
Kurt A. Reichle
William P. Roland
Grant D. Steer
John P. Tierney
Forrest Wasserman

ASSOCIATES

Donald T. Bashline
Howard L. Cohen
John Dawson
Terrence A. Flanagan
Eric F. Gottheim
Robert C. Hallstrom
Agnes H. Heersink
Mary E. Hennessy
John M. Herder
William J. Hibberd
Mikhael I. Koski
Jeffrey L. Kucera
J. Gary LaRose
Charles M. Lederman
John J. Limpert
Robert A. Miller III
Raymond S. Nichols
Gerald H. Pastor
Polly G. Philbrick
Kim E. Piersol
Andre R. Racine
Gary K. Random
Domenico Rosa
John P. Ryan
Michael Sansevero, Jr.
Mark J. Sobel
Andrea M. Sweeny
William R. Van Ark
David L. Wasserman
Clifford Wess

The election of Officers and Directors followed. Those elected, and their offices, were as follows:

President-Elect	Jerome A. Scheibl
Vice President	Steven H. Newman
Secretary	David P. Flynn
Treasurer	Michael A. Walters
Editor	C. K. Khury
General Chairman, Education and Examination Committee	Jeffrey T. Lange
Board Members	Carlton W. Honebein
	Charles L. McClenahan
	Donald E. Trudeau

The Secretary's and Treasurer's Annual Reports were then given.

There were no awards for either the Woodward and Fondiller Prize or the Dorweiller Prize.

There were no reviews of papers during the business session.

After a short break, a panel discussion entitled "Principles of Risk Classification" was presented.

Moderator: P. Adger Williams
Vice President and Actuary
The Travelers Insurance Companies

Panelists: Charles C. Hewitt, Jr.
President
Metropolitan Reinsurance Company

Dale A. Nelson
Assistant Vice President & Actuary
State Farm Mutual Automobile Insurance Company

Sanford R. Squires
Vice President & Actuary
Commercial Union Assurance Companies

Mavis A. Walters
Vice President, Government and Industry Relations
Insurance Services Office

During a formal luncheon, Robert J. Schraeder, Vice President, A. M. Best Company, gave a presentation entitled "An Overview of the Property/Casualty Financial Condition."

The regular session reconvened with a workshop program.

Workshop A— "Discussion of Luncheon Speech"

Moderator: Robert J. Schraeder
Vice President
A.M. Best Company

Workshop B— "Insuring Nuclear Risks"

Moderator: Richard D. McClure
Consulting Actuary

Members: Robert B. Foster
Actuary
The Travelers Insurance Companies

Richard W. Newcomb, Sr.
Senior Vice President
Arkwright-Boston Insurance Company

Workshop C— "Actuarial Problems of Captive Insurers"

Moderator: Howard V. Dempster
Assistant Vice President & Actuary
Insurance Company of North America

Members: Robert S. Miccolis
Vice President & Consulting Actuary
Carroon & Black Corporation

Kevin M. Ryan
Consulting Actuary
Milliman and Robertson, Inc.

Workshop D— "New Papers"

"A Study of Risk Assessment," by Richard G. Woll, reviewed by Natalie Shayer.

"Methods For Fitting Distributions to Insurance Loss Data," by Charles C. Hewitt, Jr. and Benjamin Lefkowitz, reviewed by Lee R. Steeneck.

Workshop E— "Principles of Risk Classification"

This was a discussion of the morning panel with the panelists.

Workshop F— "Education Policy of the CAS"

Moderator: Jeffrey T. Lange
Senior Vice President
Royal-Glove Insurance Companies

Members: Phillip N. Ben-Zvi
Vice President and Actuary
Royal-Globe Insurance Companies

David C. Hartman
Vice President and Actuary
Chubb & Son, Inc.

Barbara J. Lautzenheiser
Vice President and Actuary
Bankers Life Insurance Company of Nebraska

The day ended with a reception at 7:00 p.m. followed by dinner at 8:00 p.m. with "Kaleidoscope."

During this program a momento of appreciation was given to Matthew Rodermund in recognition of his long and devoted service to the Society. The presentation was made by Charles C. Hewitt, Jr.

Friday, November 16, 1979

The regular meeting resumed at 8:15 a.m. with a panel discussion entitled "Living with Open Competition." The participants were:

Moderator: Frederick W. Kilbourne
Consulting Actuary

Members: Stewart W. Kemp
Special Counsel to the Senate Judiciary Subcommittee on Antitrust Monopoly and Business Rights

Harry J. Solberg
Manager, Insurance Industries Program
SRI International

At 9:30 a.m. the Presidential Address was given by Ruth E. Salzmann. This was followed by an informal discussion and coffee break.

At 10:15 a.m. there was a panel discussion on the topic "The Florida Workers Compensation Scene." The participants were:

Moderator: Anthony J. Grippa
Actuary
National Council on Compensation Insurance

Members: William G. McCue, Jr.
Director of Legislation
Florida Association of Insurance Agents

Kenneth H. MacKay
Senator, 6th District
State of Florida

Mary Ann Stiles
Vice President and General Counsel
Associated Industries of Florida

A formal luncheon was held at 12:00 noon, during which Robert F. Frohlke, President, Health Insurance Association of America gave a presentation entitled "Let's Keep Health Care Healthy."

The General Session resumed at 1:30 p.m. with a panel discussion entitled "The Casualty Actuarial Society, A Look Ahead." The participants were:

Moderator: George D. Morison
President
Workers Compensation Rating and Inspection Bureau of Massachusetts

Members: Carlton W. Honebein
Senior Vice President and Actuary
Fireman's Fund Insurance Companies

Edward R. Smith
Vice President and Actuary
The Hartford Insurance Group

Hugh G. White
Actuary
Travelers of Canada

The closing remarks were made by President James MacGinnitie after which the Fall Meeting adjourned at 3:00 p.m.

In attendance as indicated by registration records were 167 Fellows, 127 Associates, 19 guests, 14 subscribers, 4 students and 148 spouses. The list follows:

FELLOWS

Adler, M.
Alfuth, T. J.
Anker, R. A.
Ashenberg, W. R.
Bailey, R. A.
Balko, K. H.
Barrette, R.
Bartlett, W. N.
Bassman, B. C.
Bayley, T. R.
Beckman, R. W.
Bell, L. L.
Bennett, N. J.
Ben-Zvi, P. N.
Bergen, R. D.
Berquist, J. R.
Bertiles, G. G.
Bethel, N. A.
Biondi, R. S.
Bishop, E. G.
Bondy, M.
Bornhuetter, R. L.
Brown, J. W.
Brown, W. W., Jr.
Bryan, C. A.
Conger, R. F.
Conners, J. B.
Cook, C. F.
Curry, H. E.
Daino, R. A.

FELLOWS

Dempster, H. V.
Ehlert, D. W.
Eldridge, D. J.
Eyers, R. G.
Faber, J. A.
Fagan, J. L.
Farnam, W. E.
Ferguson, R. E.
Fisher, W. H.
Fitzgibbon, W. J., Jr.
Flaherty, D. J.
Flynn, D. P.
Ford, E. W.
Forker, D. C.
Foster, R. B.
Fowler, T. W.
Fresch, G. W.
Fusco, M.
Gallagher, T. L.
Gibson, J. A., III
Goldberg, S. F.
Grady, D. J.
Grannan, P. J.
Grippa, A. J.
Hachemeister, C. A.
Hafling, D. N.
Hall, J. A., III
Hanson, H. D.
Hartman, D. G.
Harwayne, F.
Haseltine, D. S.
Hazam, W. J.
Heer, E. L.
Hermes, T. M.
Herzfeld, J.
Hewitt, C. C., Jr.
Honebein, C. W.
Hope, F. J.
Inkrott, J. G.
Irvan, R. P.
Jaeger, R. M.
Johe, R. L.
Kallop, R. H.
Kates, P. B.
Kaufman, A.
Khury, C. K.
Kilbourne, F. W.
Kist, F. O.
Kollar, J. J.
Krause, G. A.
Kreuzer, J. H.
Lange, J. T.
Lattanzio, S. P.
Leimkuhler, U. E.
Leslie, W., Jr.
Levin, J. W.
Lino, R. A.
Luneburg, B. C.
MacGinnite, W. J.
Makgill, S. S.
Marker, J. O.
Masterson, N. E.
McClenahan, C. L.
McLure, R. D.
McConnell, C. W., II
McLean, G. E.
McManus, M. F.
Meeks, J. M.
Miccolis, J. A.
Miller, D. L.
Mills, R. J.
Mohl, F. J.
Moore, B. C.
Moore, B. D.
Morison, G. D.
Muetterties, J. H.
Nash, R. K.
Neidermyer, J. R.
Nelson, D. A.
Newman, S. H.
O'Brien, T. M.
Oien, R. G.
Otteson, P. M.
Pagnozzi, R. D.
Palczynski, R. W.
Patrik, G. S.
Perkins, W. J.
Petersen, B. A.
Philbrick, S. W.
Phillips, H. J.
Pierce, J.
Pollack, R.
Presley, P. O.
Radach, F. R.
Reichle, K. A.
Retterath, R. C.
Richardson, J. F.
Rodermund, M.
Roland, W. P.
Rosenberg, S.
Roth, R. J.
Ryan, K. M.
Salzmann, R. E.
Scheibl, J. A.
Schultz, J. J., III
Scott, B. E.
Sheppard, A. R.
Sherman, R. E.
Shoop, E. C.
Smith, E. R.
Squires, S. R.
Steeneck, L. R.
Steer, G. D.
Stephenson, E. A.

FELLOWS

Stergiou, E. J.
Streff, J. P.
Strug, E. J.
Swift, J. A.
Thomas, J. W.
Tierney, J. P.
Trudeau, D. E.
Venter, G. G.
Walters, Mavis
Walters, Michael
Ward, M. R.
Warthen, T. V.
Wasserman, F.
Webb, B. L.
White, H. G.
White, W. D.
Wilcken, C. L.
Williams, P. A.
Wilson, J. C.
Wiser, R. F.
Woll, R. G.
Wright, W. C., III
Zory, P. B.

ASSOCIATES

Applequist, V. H.
Austin, J. P.
Barrow, B. H.
Bashline, D. T.
Bass, I. K.
Bradley, D. R.
Cadorine, A. R.
Carson, D. E. A.
Cheng, J. S.
Chorpita, F. M.
Chou, P. S.
Christie, J. K.
Cohen, H. L.
Cohen, H. S.
Connor, V. P.
Copestakes, A. D.
Covney, M. D.
Currie, R. A.
Dahlquist, R. A.
Davis, L. S.
Davis, R. D.
Dawson, J.
Drummond-Hay, E. T.
Fisher, R. S.
Flanagan, T. A.
Gerlach, S. B.
Gottheim, E. F.
Gould, D. E.
Granoff, G.
Grant, G.
Gruber, C.
Gwynn, H. M.
Hallstrom, R. C.
Haner, W. J.
Head, T. F.
Hearn, V. W.
Heersink, A. H.
Hennessy, M. R.
Herder, J. M.
Herman, S. C.
Hibberd, W. J.
Hurley, J. D.
Jensen, J. P.
Judd, S. W.
Kaur, A. F.
King, K. K.
Kleinman, J. M.
Klingman, G. C.
Kolojay, T. M.
Konopa, M. E.
Koski, M. I.
Kucera, J. L.
Larose, J. G.
Ledbetter, A. R.
Lederman, C. M.
Lehman, M. R.
Limpert, J. J.
Lowe, S. P.
Marino, J. F.
Mathewson, S. B.
McHugh, R. J.
Meyer, R. E.
Miccolis, R. S.
Miller, R. A., III
Millman, N. L.
Moore, J. E.
Morell, R. K.
Mulder, E. T.
Napierski, J. D.
Neuhauser, F. Jr.
Nichols, R. S.
Nolan, J. D.
Peacock, W. W.
Penniman, K. T.
Perry, L. A.
Pflum, R. J.
Philbrick, P. G.
Piazza, R. N.
Piersol, K. E.
Plunkett, R. C.
Potter, J. A.
Pulis, R. S.
Purple, J. M.
Racine, A. R.
Ratnaswamy, R.
Riley, C. R.
Ritzenthaler, K. J.

ASSOCIATES

Rodgers, B. T.
Rosenberg, M.
Sandler, R. M.
Sansevero, M., Jr.
Schwartz, A. I.
Shayer, N.
Silberstein, B.
Singer, P. E.
Skrodenis, D. P.
Stein, J. B.
Sweeny, A. M.
Swisher, J. W.
Symonds, D. R.
Taranto, J. V.
Thorne, J. O.
Tuttle, J. E.
Van Ark, W. R.
Wade, R. C.
Weiner, J. S.
Weissner, E. W.
Weller, A. O.
Wess, C.
White, F. T.
White, J.
Whitman, M.
Wickwire, J. D., Jr.
Young, R. G.
Zatorski, R. T.
Zicarelli, J. D.
Ziock, R. W.

GUESTS—SUBSCRIBERS—STUDENTS

Bell, A. M.
Belton, T.
Benson, D. W.
Blazer, B.
Brown, A.
Campbell, C. J.
Canfield, P. A.
Carpenter, J. G.
Clowes, W. M.
Costner, J. E.
Dallaire, M. J.
Edie, G. M.
Froehlke, R. F.
Hager, G. A.
Hanna, J.
Hatfield, B. D.
Hinkle, T. C.
Hoyt, F. A.
Johnson, J. E.
Katz, A.
Knilans, K.
Koupf, G. I.
Kraysler, S. F.
Lautzenheiser, B. J.
McCue, W. G., Jr.
Murr, R. A.
Newcomb, R. W.
Pope, D. W.
Rowland, V. T.
Rubino, F.
Schraeder, R. J.
Smith, D. A.
Solberg, H. J.
Spangler, J. L.
Stenmark, J. A.
Stiles, M. A.

Respectfully submitted,

DAVID P. FLYNN
Secretary

REPORT OF THE SECRETARY

The purpose of the Secretary's Report is to provide the membership with information on the activities of the Board of Directors since the last Annual Meeting.

The Board met five times in 1979. The meetings took place in Chicago on January 9th; in Tarpon Springs, Florida on March 15–16; in Colorado Springs, Colorado on May 20th; in Chicago on September 9–10; and in Orlando, Florida on November 14th. Our activities can be categorized in three broad areas: external, organizational and administrative.

The year was particularly busy with activity having to do with factors external, but nonetheless important, to the Society. Foremost among these is the question of certification of loss reserves. President Salzmann convened a special meeting of the Board on January 9th on this particular matter to consider the Society's course of action with respect to the then-significant possibility that all CPAs would automatically be qualified as loss reserve specialists by the NAIC. The Board resolved at that meeting that we support instructions on the certification of Fire and Casualty Blanks that are essentially the same as the instructions that exist for the Life and Accident and Health Blanks. The President was directed to present this resolution to the next meeting of the NAIC Blanks Subcommittee.

Much has transpired since that time. The question of CPA certification appears to have been satisfactorily resolved, although the final answer will not be available until at least the December 1979 NAIC Meeting. In the interest of brevity, I will not relate the entire situation as it exists today. However, it appears that in the future, accountants will audit and loss reserve specialists will certify. Persons will qualify as loss reserve specialists in a number of ways, one of which may be the successful completion of an examination or series of examinations sponsored by the Casualty Actuarial Society. To this end, your Board, at its September 9–10 meeting, expressed its willingness to identify pertinent parts of Exams 5, 6 and 7, and to administer either separate or a single exam on this material.

In closing my report on this item, it is appropriate and important to note the invaluable work done by the Committees on Financial Reporting of the Society and of the American Academy. Ronald Bornhuetter, President-Elect of the

Academy and former President of the Society, was particularly active in these tasks.

There were also other external matters which should receive your attention:

1. The Society has now received tax-exempt status for its Trust Fund from the Internal Revenue Service. We are indebted to Ms. Joan R. Good of the Travelers for her work in securing this approval.
2. Action was taken by the Board on the Committee on Committees Report. This was a major activity of the Council of Presidents to bring more harmony to the structures and relationships of the various actuarial bodies. More about this activity later.

Activities related to the internal organization of the Society were both numerous and wide ranging.

1. The findings and recommendations of a Task Force on Publications was adopted by the Board at its September Meeting. In brief, the Board's action expands the responsibility of the Editorial Committee to include all publications of the Society with Editors and Editorial Committees appointed for each of our major publications. Specific target dates were set for each publication.
2. The Board established an Education Policy Committee which will recommend education policy and goals to it. This new committee is a result of the recommendation of the Committee on Committees.
3. The Board disbanded the Committees on Financial Reporting and on Professional Conduct and Discipline. Most areas formerly covered by these committees will now be handled by appropriate committees within the Academy of Actuaries. Guides to Professional Conduct will continue to be subject to Board approval. Discipline of members remains within our Society to be handled on an ad hoc basis. Our Society will also be well represented on the Academy's Committee on Financial Reporting.
4. The Education and Examination Committee has again been very active this year. More than eighty members of the Society are involved in this important area. Difficult liaison and restructuring tasks for Parts 3 and 4 were completed. This year the Committee administered approximately twelve hundred exams for six hundred candidates in more than sixty examination centers.

5. During the year the Board also authorized work, presently in progress, on a review of our anti-trust exposure, on new booklets for minority recruiting and on the eligibility standards for new papers.
6. The Continuing Education Committee conducted a very successful Call Paper Program in May and laid plans for a similar program for our May 1980 Meeting.

In the administrative area, there is both good news and bad. On the happy side, our Society continues to grow. During 1979 we added 55 new Associates and 47 new Fellows. Our total membership has grown to 874, comprised of 440 Associates and 434 Fellows.

It may be of interest to briefly review the growth in our Society as we prepare to enter the next decade. It took forty years for our membership to reach three hundred members, another twenty-one years to add another three hundred members, and it is likely that it will take only six years to add another three hundred. At the beginning of this decade, we had 451 members and now we are nearly double that size. Most of that increase came in the last six years, during which our membership increased by 330. Exam enrollment indicates that our growth will continue into the forseeable future.

On the less positive side, the Board has found it necessary to increase both dues and exam fees. Beginning in 1980, the dues will be $80 for Fellows, $60 for Associates and $80 for Associates of over five years standing. Invitation Program fees have been increased to $100. Examination fees have been increased to $30 for each exam administered by the CAS.

This report would not be complete without some mention of the activity of our regional affiliates. They are each alive, well, and providing important forums for discussion of problems and the presentation of new ideas. Unfortunately, even a brief review of their activities would extend me far beyond my allotted time.

Finally, it is incumbent upon me to again extend my thanks for the professional job being done by Edith Morabito and Carole Olshefski in our New York office, and to my own secretary, Janet Seiler. They each have been truly invaluable to the operation and continued well being of our Society.

Respectfully submitted,

DAVID P. FLYNN
Secretary

REPORT OF THE TREASURER

The audited financial statement for the fiscal year ended September 30, 1979 showed assets of $226,584, up $3,698, and liabilities of $79,374, up $13,693. The major liabilities are printing expenses for the 1977 & 1978 Proceedings and secretarial service expenses owed to the National Council on Compensation Insurance.

Although we had budgeted to break even, there was a loss on operations of $12,430 caused by the following items:

1. Meetings produced a loss of $7,000 overall with a $12,000 loss in New York and a profit of $7,000 realized in Colorado. Expenses for Board and Committee meetings were $2,000.
2. Printing and stationery expenses were $6,000 higher than expected.
3. Exam fees were $3,000 lower than budgeted while invitational program charges and interest earned were $2,000 ahead of budget.

The Michelbacher and Dorweiler Funds were increased during the year by investment income. The Michelbacher Fund now stands at $34,131, up $1,913, and the Dorweiler Fund has a balance of $7,681, up $523.

Overall, with the loss on operations, and the increase in the special funds, Members' equity was reduced $9,995 for the year.

There were no new investments made during the year. We continue to be invested in U.S. Treasury Bonds and Notes.

The operating budget for next year has once again been set at the break even level.

Membership dues will be increased $10.00 for the coming year. Fellowship dues are now $80.00; Associateship dues are $60.00 for the first five years and $80.00 thereafter. Residents outside the U.S. and Canada will pay $60.00. The fee for the invitational program will be raised from $75.00 to $100.00 for next year.

In anticipation of the election of Mike Walters as Treasurer, the CAS financial records have been moved to New York. As Assistant Treasurer, Mike has been conducting the Society's financial affairs in recent months and I'm confident that he will do an excellent job as our new Treasurer.

Respectfully submitted,

WALTER J. FITZGIBBON, JR.
Treasurer

FINANCIAL REPORT
Fiscal Year Ended September 30, 1979

Income		Disbursements	
Dues	$ 43,757.81	Printing & stationery	$ 49,034.24
Exam fees	26,077.61	Secretary's office	45,744.00
Meetings & registration fees	73,385.79	Examination expenses	3,777.11
Sale of *Proceedings*	4,827.20	Meeting expenses	80,599.74
Sale of Readings	8,648.02	Library	193.97
Invitational program	5,400.00	Math. Assoc. of America	1,500.00
Interest	10,198.66	Insurance	2,012.00
Actuarial Review	276.90	Miscellaneous	1,942.66
Miscellaneous	−198.73		
Total	$172,373.26	Total	$184,803,72

Income	$172,373.26
Disbursements	184,803.72
Change in Surplus	$−12,430.46

ACCRUAL BASIS ACCOUNTING STATEMENT
AS OF SEPTEMBER 30, 1979

ASSETS

Assets	9/30/78	9/30/79	Change
Bank accounts	$ 89,277.71	$ 86,328.17	$−2,949.54
U. S. Treasury Bonds	4,325.00	4,325.00	0
U. S. Treasury Notes	124,535.00	124,535.00	0
Accrued income	4,748.43	11,396.00	6,647.57
Total	$222,886.14	$226,584.17	$ 3,698.03

LIABILITIES, SURPLUS AND OTHER FUNDS

Liabilities			
Secretarial services	$10,371.70	$22,872.00	$12,500.30
Printing expenses	28,500.00	49,342.00	20,842.00
Examination expenses	18,735.00	0	−18,735.00
Meeting expenses	974.58	0	−974.58
Minority Education Fund	6,900.00	7,150.00	250.00
Other	200.00	10.00	−190.00
Sub-Total	$65,681.28	$79,374.00	$13,692.72
Members' Equity			
Michelbacher Fund	$ 32,218.45	$ 34,131.43	$ 1,912.98
Dorweiler Fund	7,158.70	7,681.49	522.79
Surplus	117,827.71	105,397.25	−12,430.46
Sub-Total	$157,204.86	$147,210.17	$−9,994.69
Total	$222,886.14	$226,584.17	$3,698.03

Walter J. Fitzgibbon, Jr.
Treasurer

This is to certify that the assets and accounts shown in the above financial statement have been audited and found to be correct.

Finance Committee
Robert B. Foster, Chairman
Harold E. Curry
David M. Klein
Michael A. Walters

1979 EXAMINATIONS—SUCCESSFUL CANDIDATES

Examinations for Parts 5, 7 and 9 of the Casualty Actuarial Sociaty syllabus were held November 2 and 3, 1978. Examinations for Parts 4, 6, 8 and 10 were held May 9 and 10, 1979. Parts 1, 2 and 3 jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries were given in November of 1978 and May of 1979. Those who passed Parts 1, 2 and 3 were listed in the joint releases of the two Societies sent out in January and July 1979.

The Casualty Actuarial Society and the Society of Actuaries jointly awarded prizes to the undergraduates ranking highest on the General Mathematics examination.

In January 1979, the $200 prize was awarded to Joshua D. Bernoff. The additional $100 prize winners were Howard J. Karloff, Dennis J. Monaco, Denise M. Ridolfi and Timothy J. Steger. In July, the $200 prize was awarded to Mark P. Kleiman. The additional $100 prize winners were Robert A. Brady, Brenda J. Fahey, Kevin J. Kelley and Sasedis Parashis.

The following candidates were admitted as Associates and Fellows at the May 1979 meeting as a result of their successful completion of the Society requirements in the November 1978 examinations:

FELLOWS

Aldorisio, Robert P.
Asch, Nolan E.
Bartlett, William N.
Bishop, Everett G.
Buck, James E., Jr.
Degerness, Jerome A.
Dorval, Bernard
Eddy, Jeanne H.
Eland, Douglas D.
Hafling, David N.
Hoylman, Douglas J.
Jean, Ronald W.
Jerabek, Gerald J.
Lehmann, Steven G.
Nelson, Janet R.
Newlin, Patrick R.
Oakden, David J.
Pierce, John
Schumi, Joseph R.
Shoop, Edward C.
Stergiou, Emanuel J.
Taylor, Frank C.
Teufel, Patricia A.

ASSOCIATES

Austin, J. Paul
Belvin, William H.
Biller, James E.
Christie, James K.
Cundy, Richard M.
DiBattista, Susan T.
Drummond-Hay, Eric T.
Duffy, Thomas J.
Dussault, Claude
Evans, Glenn A.
Foote, James M.
Furst, Patricia A.
Ghezzi, Thomas L.
Harrison, Eugene E.
Heckman, Philip E.
Higgins, Barbara J.
Jameson, Stephen
Javaruski, John J.
Johnston, Thomas S.
Kleinman, Joel M.
Lafontaine, Gaetane
Lo, Richard W.
Lotkowski, Edward P.
Mahler, Howard C.
Mathewson, Stuart B.
McConnell, Charles W., II
McGovern, William G.
Mulder, Evelyn T.
Murphy, Francis X., Jr.
Myers, Nancy R.
Nickerson, Gary V.
Niswander, Ray E., Jr.
Robertson, John P.
Rowland, William J.
Schwartz, Allan I.
Wilson, Randall J.
Zicarelli, John D.

The following is the list of successful candidates in examinations held in November, 1978:

Part 5

Amundson, Richard B.
Austin, J. Paul
Berens, Regina M.
Bertrand, Francois
Boison, Leroy A., Jr.
Brown, Robert L.
Camp, Jeanne H.
Campbell, Catherine J.
Chernick, David R.
Chou, Li-Chuan
Ciezadlo, Gregory J.
Cimini, Edward D., Jr.
Clark, David G.
Clinton, R. Kevin
Colvin, Samuel P.
Dean, Curtis G.
DeLiberato, Robert V.
Doellman, John L.
Doran, Phyllis A.
Douglas, Frank H.
Doyle, Michael J.
Easton, Richard D.
Edwalds, Thomas P.
Engles, David
Erie, Steven L.
Fahrenbach, Jack
Faix, Paul J.
Fallon, Patricia D.
Fiebrink, Dianne C.
Fitz, Loy W.
Fitzpatrick, Kathleen M.
Friedberg, Bruce F.
Friedman, Howard H.
Gannon, Alice A.
Gogol, Daniel F.
Gorman, Deborah A.
Greco, Ronald E.
Hale, Jonathan B.
Halpern, Nina S.
Hayne, Roger M.
Heller, David M.
Howard, C. Douglas
Huber, Debra S. R.
Johnson, Judy A.
Kelly, Martin K.
Lally, Mary-Ellen
Lange, Dennis L.
Larsen, Michael R.
Lee, Stephen
Leo, Carl J.
Leong, Winsome
Leung, Kung L.
Lobosco, Virginia R.
Ludwig, Stephen J.
Lynch, John J.
Malik, Sudershan K.
Mealy, Dennis C.
Mellia, Joanne C.
Miller, Ronald R.
Moeller, Victoria L.
Montigney, Brian A.
Moody, Andrew W.
Muleski, Robert T.
Munt, Donna S.
Murphy, Edward J., Jr.
Murphy, William F.
Murr, Rebecca A.
Newton, Brian R.
Nichols, Richard W.
Nikstad, James R.
Ostergren, Gregory V.
Pachyn, Karen A.
Pelletier, Bernard A.
Pence, Clifford A., Jr.
Priester, David C.
Pruiksma, Glenn J.
Remis, David E.
Ryan, John P.
Scott, Diane D.
Seguin, Louis G.
Sherwood, Douglas L.
Silverman, Mark J.
Stiefel, Stanley M.
Suchoff, Stuart B.
Tom, Darlene P.
Visner, Steven M.
Vitale, Lawrence A.
Wade, John E.
Walker, David G.
Walker, Leigh M.
Washburn, Monty J.
Weidman, Thomas A.
Withers, David A.
Yunque, Mark A.
Zolnowski, Raymond M.

Part 7

Bashline, Donald T.
Beer, Albert J.
Belvin, William H.
Biller, James E.
Boyd, Lawrence H.
Brown, Nicholas M.
Burg, David R.
Christiansen, Stephan L.
Christie, James K.
Cohen, Howard L.
Connell, Eugene C.
Cundy, Richard M.
Dawson, John
DeConti, Michael A.
DeGarmo, Lyle W.
Demers, Daniel
DiBattista, Susan T.
Driedger, Karl H.
Drummond-Hay, Eric T.
Duffy, Thomas J.
Dussault, Claude
Eddy, Jeanne H.
Evans, Glenn A.
Flanagan, Terrence A.
Foote, James M.
Furst, Patricia A.
Ghezzi, Thomas L.
Giambo, Robert A.
Gottheim, Eric F.
Hallstrom, Robert C.
Harrison, Eugene E.
Heckman, Philip E.
Heersink, Agnes H.
Hennessy, Mary E.
Herder, John M.
Herzfeld, John
Hibberd, William J.
Higgins, Barbara J.
Horowitz, Bertram A.
Hu, David D.
Jameson, Stephen
Javaruski, John J.
Jerner, Donald C.
Johnson, Larry D.
Johnston, Thomas S.
Judd, Steven W.
Kleinman, Joel M.
Knilans, Kyleen
Kozik, Thomas J.
Lafontaine, Gaetane
LaRose, J. Gary
Lederman, Charles M.
Lee, Young S.
Limpert, John J.
Lo, Richard W.
Lotkowski, Edward P.
Mahler, Howard C.
Mathewson, Stuart B.
McConnell, Charles W., II
McDaniel, Gail P.
McGovern, William G.
Mulder, Evelyn T.
Murphy, Francis X., Jr.
Nickerson, Gary V.
Niswander, Ray E., Jr.
O'Neil, Mary L.
Parker, Curtis M.
Pastor, Gerald H.
Pei, Kai-Jaung
Piersol, Kim E.
Racine, Andre R.
Ransom, Gary K.
Robertson, John P.
Roman, Spencer M.
Rosa, Domenico
Rosenberg, Martin
Rowland, William J.
Ryan, John P.
Sansevero, Michael, Jr.
Schott, Barbara
Schwartz, Allan I.
Sherman, Ollie L., Jr.
Sobel, Mark J.
Taylor, Frank C.
Varca, John J.
Waldman, Robert H.
Walker, Glenn M.
Wasserman, David L.
Wess, Clifford
Westerholm, Sharon W.
Wilson, Randall J.
Wolf, Philip M.
Yatskowitz, Joel D.

Part 9

Aldorisio, Robert P.
Asch, Nolan E.
Baer, Debra L.
Bartlett, William N.
Bayley, Thomas R.
Bishop, Everett G.
Brown, Joseph W., Jr.
Buck, James E., Jr.
Cheng, Joseph S.
Cis, Mark M.
Conger, Robert F.
Covney, Michael D.
Dahlquist, Ronald A.
Degerness, Jerome A.
Dorval, Bernard
Egnasko, Gary J.
Eland, Douglas D.
Faga, Doreen S.
Ford, Edward W.
Hafling, David N.
Henry, Dennis R.
Hoylman, Douglas J.
Jean, Ronald W.
Jerabek, Gerald J.
Kist, Frederick O.
Lattanzio, Stephen P.
Lehmann, Steven G.
Lerwick, Stuart N.
Lowe, Stephen P.
Meeks, John M.
Miccolis, Jerry A.
Miccolis, Robert S.
Moore, Bruce D.
Morgan, Stephen T.
Nash, Russell K.
Nelson, Janet R.
Newlin, Patrick R.
Oakden, David J.
Philbrick, Stephen W.
Pierce, John
Schumi, Joseph R.
Shoop, Edward C.
Shrum, Roy G.
Stergiou, Emanuel J.
Teufel, Patricia A.
Thibault, Alain P.
Tierney, John P.
Torgrimson, Darvin A.
Tuttle, Jerome E.

The following candidates were admitted as Associates and Fellows at the November, 1979 meeting as a result of their successful completion of the Society requirements in the May 1979 examinations:

FELLOWS

Alfuth, Terry J.
Bayley, Thomas R.
Biondi, Richard S.
Brown, Joseph W., Jr.
Conger, Robert F.
Ford, Edward W.
Goldberg, Steven F.
Hemstead, Robert J.
Herzfeld, John
Kist, Frederick O.
Lattanzio, Stephen P.
Lerwick, Stuart N.
Luneburg, Sandra C.
McConnell, Charles W., II
Meeks, John M.
Miccolis, Jerry A.
Moore, Bruce D.
Nash, Russell K.
Philbrick, Stephen W.
Reichle, Kurt A.
Roland, William P.
Steer, Grant D.
Tierney, John P.
Wasserman, Forrest

ASSOCIATES

Bashline, Donald T.
Cohen, Howard L.
Dawson, John
Flanagan, Terrence A.
Gottheim, Eric F.
Hallstrom, Robert C.
Heersink, Agnes H.
Hennessy, Mary E.
Herder, John M.
Hibberd, William J.
Koski, Mikhael I.
Kucera, Jeffrey L.
LaRose, J. Gary
Lederman, Charles M.
Limpert, John J.
Miller, Robert A., III
Nichols, Raymond S.
Pastor, Gerald H.
Philbrick, Polly G.
Piersol, Kim E.
Racine, Andre R.
Ransom, Gary K.
Rosa, Domenico
Sansevero, Michael, Jr.
Sobel, Mark J.
Sweeny, Andrea M.
Van Ark, William R.
Wasserman, David L.
Wess, Clifford

The following is the list of successful candidates in examinations held in May, 1979:

Part 4

Amundson, Richard B.
Balling, Glenn R.
Baum, Edward J.
Belden, Stephen A.
Biscoglia, Terry J.
Blanchard, Ralph S., III
Boone, J. Parker
Braithwaite, Paul
Burks, Michael L.
Campbell, Kenrick A.
Canetta, John A.
Carlton, Kenneth E.
Carpenter, Thomas S.
Carponter, John D.
Cassuto, Irene A.
Choi, Louise Y.
Chou, Li-Chuan L.
Ciezadlo, Gregory J.
Clinton, R. Kevin
Colin, Barbara
Colin, Steven L.
Conlon, Aileen M.
Davenport, Elena S.
Davidson, Shelley T.
Dembiec, Linda A.
Djordjevic, Nancy G.
Dodd, George T.
Driedger, Karl H.
Gannon, Alice H.
Gapp, Steven A.
Gerald, Felix R.
Gillam, William R.
Gillespie, Bryan C.
Gogol, Daniel F.
Goldberg, Terry L.
Groves, Jeffrey, A.
Henzler, Paul J.
Hershkowitz, Steven
Hofmann, Richard A.
Holmberg, Randall D.
Hsu, Ho Wan M.
Huber, Debra S. R.
Jones, Bruce R.
Katz, Aaron J.
Keatts, Glenn H.
Knilans, Kyleen
Kuo, Chung-Kuo
Lacefield, David W.
LaRose, J. Gary
Laurin, Pierre G.
Lebrun, Richard
Lehman, Layne B.
Leiner, William W., Jr.
Levine, Alexander J.
Lew, Elizabeth L.
Limpert, John J.
Liuzzi, Joseph R.
Llewellyn, Barry I.
Lo, Eddy L.
Ludwig, Stephen J.
Lynam, Paula
Lynch, John J.
Malik, Sudershan K.
Mason, Karol A.
Matthews, Robert
Miller, David L.
Murphy, William F.
Murr, Rebecca A.
Muza, James J.
Noback, Jodee B.
Pearce, Leesa I.
Pelletier, Bernard A.
Pence, Clifford A., Jr.
Philbrick, Polly G.
Port, Rhonda D.
Potts, Cynthia M.
Ramanujam, Srinivasa
Raws, Alfred, III
Rosenberg, Deborah M.
Ryan, John P.
Sanders, Robert L.
Schwartzman, Joy A.
Sherman, Harvey A.
Siewert, Jerome J.
Silverman, Janet K.
Silverman, Mark J.
Sirkin, Jeffrey S.
Somers, Edward C.
Steinhorst, Gail L.
Sweeny, Andrea M.
Umansky, Steven D.
Vogel, Charles D.
Wade, John E.
Walker, Leigh M.
Warren, Jeffrey C.
Watson, Lois A.
Weissman, Michael
Whiting, David R.
Wick, Peter G.
Williams, Lincoln B.
Windwehr, Debra R.
Withers, David A.
Yee, William
Yen, Chung-Ye
Young, Bryan G.

Part 6

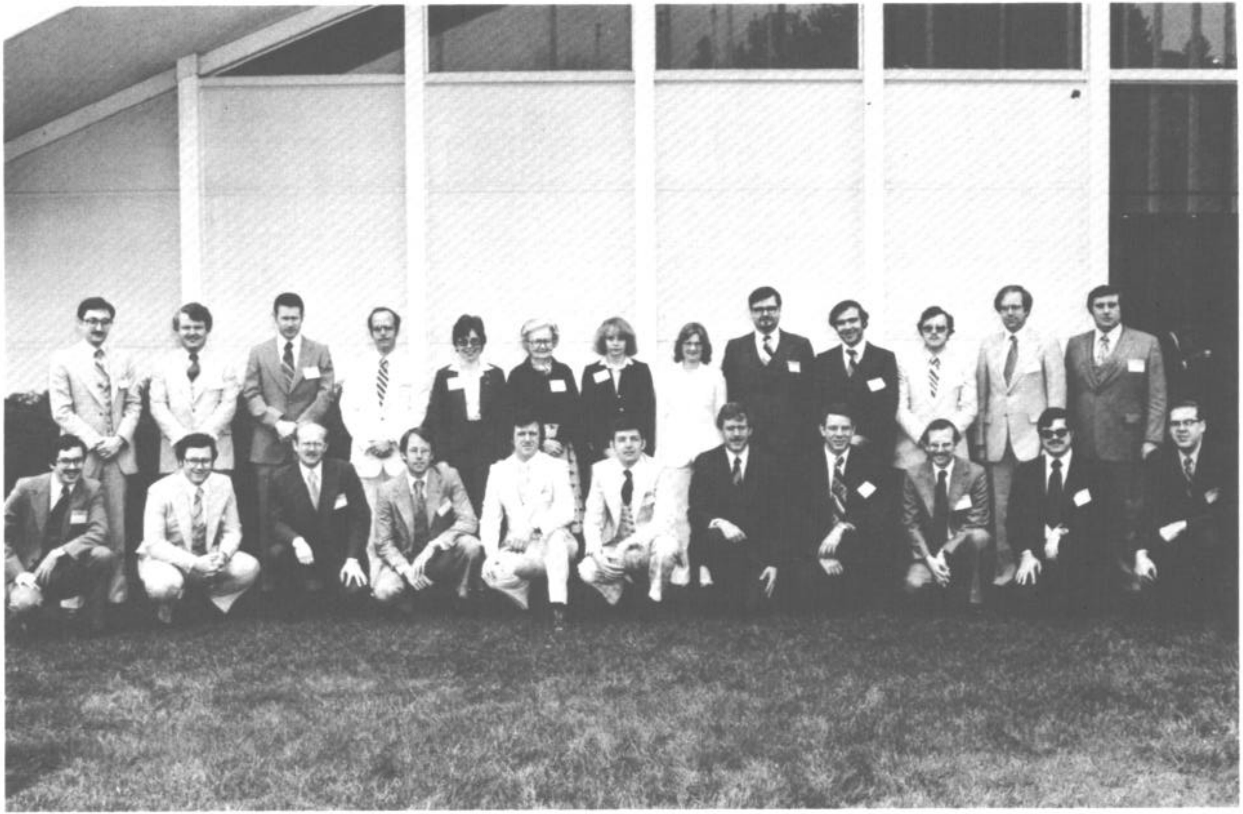
Abramson, Gary R.
Barlow, Pamela J.
Bashline, Donald T.
Berens, Regina M.
Campbell, Catherine J.
Cohen, Howard L.
Dawson, John
Dean, Curtis G.
Dineen, D. Kevin
Doellman, John L.
Douglas, Frank H.
Eagelfeld, Howard M.
Easton, Richard D.
Ehrlich, Warren S.
Engles, David
Flanagan, Terrence A.
Friedberg, Bruce F.
Gluck, Spencer M.
Goldfarb, Irwin H.
Gorman, Deborah A.
Gottheim, Eric F.
Hallstrom Robert C.
Halpert, Aaron
Hayne, Roger M.
Heersink, Agnes, H.
Hennessy, Mary E.
Herder, John M.
Hibberd, William J.
Johnson, Judy A.
Jones, Bruce R.
Keates, Katharine L.
Kelly, Martin K.
Koch, Leon W.
Kolk, Stephen L.
Killmar, Richard
Koski, Michael I.
Koupf, Gary I.
Kucera, Jeffrey L.
Lange, Dennis L.
Larsen, Michael R.
Lederman, Charles M.
Leo, Carl J.
Leong, Winsome
Linden, Orin M.
Lobosco, Virginia R.
Malloy, Linda M.
Mealy, Dennis C.
Miller, Robert A., III
Montigcuy, Brian A.
Munt, Donna S.
Nichols, Raymond S.
Nichols, Richard W.
Pachyn, Karen A.
Pastor, Gerald H.
Piersol, Kim E.
Pinto, Emanuel
Racine, Andre R.
Ransom, Gary K.
Reutershan, John T.
Rosa, Domenico
Ryan, John P.
Sansevero, Michael, Jr.
Seguin, Louis G.
Sobel, Mark J.
Stanco, Edward J.
Strange, Deborah L.
Suchoff, Stuart B.
Van Ark, William R.
Wasserman, David L.
Webster, Patricia J.
Weidman, Thomas A.
Weiland, William T.
Wess, Clifford
Wilson, Ronald L.
Youngerman, Hank

Part 8

Applequist, Virgil H.
Baer, Debra L.
Barrow, Betty H.
Biondi, Richard S.
Boison, LeRoy A., Jr.
Brahmer, John O.
Brooks, Dale L.
Brown, Joseph W., Jr.
Brown, Nicholas M., Jr.
Cassity, H. Earl
Cheng, Joseph S.
Christie, James K.
Clark, David G.
Corr, Francis X.
Davis, Lawrence S.
DeGarmo, Lyle W.
Edie, Grover M.
Evans, Glenn A.
Furst, Patricia A.
Godbold, Mary Jo E.
Godbold, Nathan T.
Henry, Dennis R.
Herman, Steven C.
Higgins, Barbara J.
Hurley, James D.
Jameson, Stephen
Javaruski, John J.
John, Russell T.
Johnson, Warren H., Jr.
Kleinberg, James J.
Lattanzio, Francis J.
Lee, Yoong S.
Lerwick, Stuart N.
Lo, Richard W.
Lotkowski, Edward P.
Mathewson, Stuart B.
McMurray, Michael A.
Miccolis, Jerry A.
Myers, Nancy R.
Niswander, Ray E., Jr.
Parker, Curtis M.
Patterson, David M.
Petrelli, Joseph L.
Schaeffer, Bernard G.
Taranto, Joseph V.
Thibault, Alain P.
Truttmann, Everett J.
Tuttle, Jerome E.
Walker, Roger D.
Whatley, Michael W.
Wilson, Randall J.
Woods, Patrick B.
Zatorski, Richard T.
Zicarelli, John D.

Part 10

Alfuth, Terry J.
Bayley, Thomas R.
Brown, Joseph W., Jr.
Conger, Robert F.
Demers, Daniel
DiBattista, Susan T.
Faga, Doreen S.
Ford, Edward W.
Giambo, Robert A.
Goldberg, Steven F.
Hemstead, Robert J.
Herzfeld, John
Judd, Steven W.
Kist, Frederick O.
Lattanzio, Stephen P.
Lerwick, Stuart N.
Lombardo, John S.
Lommele, Jan A.
Luneburg, Sandra C.
McConnell, Charles W.
Meeks, John M.
Miccolis, Jerry A.
Moore, Bruce D.
Nash, Russell K.
O'Neill, Mary Lou
Philbrick, Stephen W.
Rapp, Jerry W.
Reichle, Kurt A.
Rodgers, Beatrice T.
Roland, William P.
Rowland, William J.
Steer, Grant D.
Tierney, John P.
Wasserman, Forrest
Weissner, Edward W.
Westerholm, David C.
Wickwire, James D., Jr.
Wisecarver, Timothy L.
Young, Edward W.



NEW FELLOWS ADMITTED MAY, 1979: The twenty-three new Fellows admitted at Colorado Springs are shown with President Salzmann.

NEW ASSOCIATES ADMITTED MAY, 1979: Thirty-three of the thirty-eight new Associates admitted at Colorado Springs are shown with President Salzmann.

NEW FELLOWS ADMITTED NOVEMBER, 1979: The twenty-four new Fellows admitted at Orlando are shown with President Salzmann.

NEW ASSOCIATES ADMITTED NOVEMBER, 1979: Twenty-three of the thirty new Associates admitted at Orlando are shown with President Salzmann.

OBITUARIES

N. MATTHEW FRANKLIN

HAROLD W. SCHLOSS

JACK J. SMICK

HIRAM O. VAN TUYL

N. MATTHEW FRANKLIN
1923–1979

N. Matthew Franklin, an Associate of the Casualty Actuarial Society since 1952 and a Member of the American Academy of Actuaries, died on April 10, 1979 in Long Island, New York at the age of 56 after a lengthy illness.

Born January 2, 1923, Matt graduated with a Bachelor of Arts degree in Mathematics from the University of Connecticut in 1943. He was a Captain in the United States Army from 1944 to 1946, where he was awarded the Silver Star. Upon discharge from the Army, he received a Masters Degree from the University of Michigan in Mathematical Statistics.

Matt began his insurance career with the Surety Association of America in July of 1948 as Actuary. After 18 years with the Surety Association of America, he joined the National Insurance Actuarial and Statistical Association in 1966 and served that organization, as Actuary, until it merged with several others to form Insurance Services Office in 1971. Matt served as an Associate Actuary with ISO until the time of his illness.

Matt is survived by his wife, Betty and his two children, Robert and Pamela.

HAROLD W. SCHLOSS
1919–1979

Harold W. Schloss, a Fellow and past president of the Casualty Actuarial Society, died suddenly November 12, 1979. At his death he was Executive Vice President of the Royal-Globe Insurance Companies.

Mr. Schloss joined Royal-Globe's actuarial department in 1946. He was appointed Secretary in 1958, receiving the additional title of Actuary in 1960, Vice President and Actuary in 1965, and Senior Vice President in 1969. He was named Executive Vice President in 1974.

Prior to joining Royal-Globe, Mr. Schloss served as commanding officer with the rank of lieutenant aboard a missile-firing vessel in the amphibious forces of the Pacific Fleet during World War II. He was awarded the Navy Commendation Ribbon, the Navy Unit Citation, and two Bronze Stars.

Mr. Schloss was born in Brooklyn June 11, 1919. After graduating from Brooklyn College, he did postgraduate work in actuarial science at the University of Iowa. He became a Fellow of the Casualty Actuarial Society in 1948, was elected Editor of the Society in 1961, Vice President in 1965, and President in 1967. Harold Schloss was the first one-term president since 1940, the Society having decided in 1967 that its membership growth was producing so many presidential possibilities that the practice of keeping its presidents for two terms was denying many talented members a chance at the office.

Mr. Schloss's presidency was distinguished by the graceful and informal, yet efficient, manner in which he presided over Society and Board meetings. He was one of the few presidents who felt free to comment on speeches and panel discussions he had just heard, and his comments were invariably pertinent and good-humored.

At the 1968 annual meeting Mr. Schloss found time to take the role of Narrator in a tongue-in-cheek dramatic skit presented after the traditional banquet.

He was the author of a notable paper, "Valuation of the Death Benefits Provided by the Workmen's Compensation Law of New York," *PCAS* XXXV (1948), which became required reading for many years thereafter for students taking the Society examinations.

Harold Schloss was a member and former Vice President of the American Academy of Actuaries. He was also a member of the International Association of Actuaries. He held the Chartered Property Casualty Underwriter (CPCU) designation and served on committees appointed by the National Association of Insurance Commissioners. Mr. Schloss was also a member of the Executive Committee and Board of Directors of Underwriters Salvage Company of Chicago.

Mr. Schloss resided with his family in Brooklyn, New York, and is survived by his widow Frances A., a son Robert J., a daughter Linda D., and his mother, Mrs. Max Schloss of West Palm Beach, Florida.

JACK J. SMICK
1906–1979

Jack J. Smick, a Fellow of the Casualty Actuarial Society since 1932, died on March 9, 1979 in Westport, Connecticut at the age of 73. Jack was also a Fellow of the Conference of Actuaries in Public Practice and a Member of the American Academy of Actuaries.

Born February 22, 1906, Jack graduated with a Bachelor of Philosophy degree from Yale University in 1928.

From 1929 to 1946 Jack was connected with the National Council on Compensation Insurance and supervised much of its work in connection with compensation rates and rating problems. During this time period, he was awarded the Woodward-Fondiller prize for original thinking and research in a paper submitted to the Casualty Actuarial Society.

For the next twelve years, Jack worked as an independent consultant, where his services were employed by the War Department, Atomic Energy Commission and many insurance companies and private organizations.

From 1961 to 1963 he was a partner in Smick and Steinhaus Consulting Actuaries.

In 1964, he founded Smick and Co., where he continued to work with his son, Robert L. Smick, until the time of his death.

His wife Sonia passed away in 1976. Jack is survived by his sons Edward and Robert.

HIRAM ORVIS VAN TUYL
1886–1979

Hiram Orvis Van Tuyl, a Fellow of the Casualty Actuarial Society since 1919, was born on a farm in Steuben County, N.Y. on April 1, 1886 and died in St. Petersburg, Florida on January 10, 1979. He was a direct descendant of the Jan Otto Van Tuyl who emigrated from Holland to New Amsterdam in 1663.

On graduation from Albany Business College in 1905 he taught in that school for three years and then entered civil service. For sixteen years he was an examiner in the New York Insurance Department. When assigned to the newly established Rating Bureau of that department he saw the need for a more detailed breakdown of underwriting expenses and accordingly drew up the "Casualty Experience Exhibit" which has become a required filing ever since.

In 1926 he resigned from the Insurance Department and accepted a position as actuary of the Constitution Indemnity Co., a casualty insurance company newly organized as an affiliate of the Fire Association in Philadelphia. The new company did not outlast the depression and in 1933 he accepted a position as superintendent of the accounts department of the London Guarantee and Accident Company in New York City. A year later all companies of the Phoenix-London Group were placed under a single management and he was placed in charge of premium collections of the seven fire and casualty insurance companies. Later, he became internal auditor for the group, which position he held at the time of his retirement in 1952.

Living in White Plains for 25 years, he was an active layman in the First Baptist Church, serving as treasurer for seven years. His wife died in April, 1970. He is survived by three daughters and eight grandchildren.

INDEX TO VOLUME LXVI

INDEX TO VOLUME (CONT.)

Page