VOLUME LXIV NUMBERS 121 AND 122

PROCEEDINGS

OF THE

Casualty Actuarial Society

ORGANIZED 1914

1977

VOLUME LXIV

Number 121 — May 1977

Number 122 — November 1977

1978 YEAR BOOK

Printed for the Society by
Recording and Statistical Division
Sperry Rand Corporation
Boston, Massachusetts

1977 PROCEEDINGS

CONTENTS OF VOLUME LXIV

Page

1977 PROCEEDINGS

CONTENTS OF VOLUME LXIV (cont.)

NOTICE

The Society is not responsible for statements or opinions expressed in the articles, criticisms, and discussions in these *Proceedings*.

Volume LXIV, Part 1 No. 121

PROCEEDINGS

May 15, 16, 17, 18, 1977

AN ALGORITHM FOR PREMIUM ADJUSTMENT WITH AVAILABLE DATA

RONALD F. WISER

INTRODUCTION

An important part of performing a loss ratio type of rate adequacy study is the ability to restate historical earned premiums at the level implied by the present rate structure. Of course, the most straight-forward and desirable way of accomplishing this restatement is by the extension-of-exposures method. That is, the historical book of business is actually rerated by using today's rate book. With the power of present computers such a procedure is practical if the required exposure information exists in a reliable form. However, the practicing actuary may find that for many lines of insurance reliable exposure information in the required level of detail is not available. Further, the extension-of-exposures procedure requires specialized data processing talents, which may not always be readily available.

Clearly, an approximation method uses realistic assumptions, produces reasonable results, and uses a mathematical simulation of the earnings process is desirable. One approximation method, which has long been popular, is the so-called rectangular method, as explained in Kallop's[1] article on workers' compensation ratemaking. This will be referred to as the traditional method of premium adjustment. A second alternative would be to take the historical written premiums and attempt to approximate the earning process with adjustments for rate level changes. This procedure will be impractical for heavily audited lines where exposure earned premium may be available, but where writings are on a calendar year basis.

[1] Kallop, R., "A Current Look at Worker's Compensation Ratemaking," *PCAS*, LXII (1975), p. 62.

The method introduced in this article attempts to make efficient use of the minimum amount of information, earned premiums and rate change history, that must always be available. Given the available data, the basic concept is to build a straight line approximation to the historical rate of premium writings. This straight line approximating function is determined by the requirements that (1) actual earned premiums are produced by the model, (2) the straight line segments form a continuous curve, and (3) the rate of writings is expressed in terms of the base rate level. Additionally, this algorithm allows the actuary to introduce certain known qualitative information. This is accomplished by designing an "objective function" which is to be minimized, that chooses a "minimal" element from the family of continuous piecewise linear functions that satisfy the above three requirements.

The following discussion shows how the model may be used to obtain better approximations to restated earned premiums at present rates. The model of premium writings may also be useful in quantifying marketing results in terms of measuring an annualized rate of writing at a constant rate level, which is directly proportional to exposure writings. In terms of corporate planning models, if future expected earned premiums are projected and future rate change strategy plotted, this algorithm will produce a required "rate of writings" that allows agents' writings to be monitored month by month to determine if marketing performance is actually fulfilling standards required to meet corporate earned premium projections. With regard to fire insurance, the exposure related premium writings resulting from the algorithm can be modified to reflect increasing amounts of insurance in the final adjusted earned premiums.

BASIC CONSTRUCTION OF THE ALGORITHM

The mathematics of the algorithm can be conveniently developed in terms of linear algebra. For purposes of exposition it is preferable to present a detailed example of the calculations. An object of this demonstration is to familiarize the reader with the idea of choosing a "best" element from a family of approximations as a useful actuarial tool which can easily be modified to meet a particular problem. A mathematical appendix presents the algorithm in terms of matrix algebra, thus making it simple to program the calculations using a mathematical programming language such as APL.

The algorithm exploits analytic expressions for the premium earning process which have recently been made available. For example, if ERCON (y_0, y_1; x_0, x_1; t) represents the contribution to earnings during the time period from x_0 to x_1 of the writings during the time period from y_0 to y_1, Ross[2] presents the following formula:

$$\text{(1) ERCON}\,(y_0, y_1; x_0, x_1; t) = \int_{x_0-t}^{x_0} \frac{(x - x_0 + t)}{t}\, g(x)\,dx + \int_{x_0}^{x_1-t} g(x)\,dx + \int_{x_1-t}^{x_1} \frac{(x_1 - x)}{t}\, g(x)\,dx,$$

where $t =$ term of the policies, and $g(x) = \begin{cases} f(x) \text{ if } y_0 \leq x \leq y_1, \\ 0 \text{ otherwise} \end{cases}$

and f (x) is the exposure related rate of premium writings at time x. This formula assumes that $x_0 \leq x_1 - t$.

Miller and Davis[2] also give formulas for the earning process which will yield the following expression for the earned contributions from a period of writings:

$$\text{(2) ERCON}\,(y_0, y_1; x_0, x_1; t) = \frac{1}{t}\int_{x_0}^{x_1} \int_{a(x)}^{b(x)} f(x - y)\,dy\,dx,$$

where $a(x) = \min(\max(x - y_0, 0), t)$, $b(x) = \max(\min(x - y_1, t), 0)$ and (y_0, y_1) is the period of premium writings, (x_0, x_1) is the earning period, and t is the term of the policies. The proof that these two expressions are actually equivalent is recommended as an exercise for the mathematically inclined reader.

Other formulations for the same process may also be derived. Whichever expression is used, the actuary is always faced with the same problem: he must come up with a rate of exposure (or premium) writings. This rate is a handy theoretical concept which makes the analytic formulas work; unfortunately, it cannot be observed or measured under any practical situations. The best data actuaries can come up with is aggregate writings, i.e., $\int_{y_0}^{y_1} f(y)\,dy$ for some time period (y_0, y_1). Even if such writings are properly related to exposures, we are still faced with the problem of conjuring up the associated rate function f (x) to continue the analysis.

[2] Miller, D. J., and Davis, G. E., "A Refined Model for Premium Adjustment," *PCAS*, LXIII (1976), p. 117.

The approach taken in the design of this algorithm is to start from the assumption that premium writings for a time period can be described by the linear rate of writings function $f(x) = Ax + B$. A different pair of parameters (A, B) is allowed for each writing period, with the continuity condition that the different line segments must meet at common end points of the writing periods. The condition that the model must produce the collected earned premiums makes use of the analytic expressions of the earnings process. With the assumption $f(x) = Ax + B$, we can use an equation such as (2) and calculate:

$$\frac{1}{t}\int_{x_0}^{x_1}\int_{\min(\max(x-y_0,0),t)}^{\max(\min(x-y_1,t),0)} (A(x-y)+B)\,dydx$$
$$= AH(y_0, y_1; x_0, x_1; t) + BG(y_0, y_1; x_0, x_1; t).$$

That is, for each earning and writing period, we obtain numerical coefficients for the unknown parameters A and B of the model. The explicit formulation of H and G is not given because the argument is complicated by the limits of integration. The calculation for any specific (x_0, x_1) and (y_0, y_1) is quite straight-forward. The general formula of H and G is not as easily written and its detailed development adds nothing to the basic demonstration (see Appendix 2). In a practical situation, it is best to program a routine that can handle the necessary logic for limits of integration.

Suppose the data is given as in Table 1, showing earned premiums for the three years 1974, 1975, 1976 and the rate change history for the years 1973 through 1976. We are assuming that the policy term is one year. Table 2 shows the organization of the given data and the results of using formula (1) to calculate the coefficients of the parameters A_i and B_i. Since there are four periods of written premium, there are eight parameters $(A_i, B_i; i = 1,2,3,4)$ to be determined.

TABLE 1

PREMIUM AND RATE CHANGE HISTORY

Accident Year	Earned Premium	Rate Change History	
1974	1600	4/1/73	+15%
1975	1820	7/1/74	+10%
1976	1860	1/1/75	−5%
		5/1/76	+20%

According to Table 2, the contribution of writings from 1/1/74 to 6/30/74 to the earnings of 1974 can be expressed as $.45833A_2 + .375B_2$. To obtain these calculations, consider a time line with 1/1/73 as 0.0 and 1/1/74 as 1.0. The contribution of the writings of one-year policies from 1/1/74 (1.0) to 6/30/74 (1.5) to the earnings of 1974 (the interval 1.0 to 2.0) can be written, using (1):

$$\text{ERCON}(1.0, 1.5; 1.0, 2.0; 1) = \int_{1.0}^{1.5} (A_2x + B_2)(2 - x)\,dx =$$

$$A_2\int_{1.0}^{1.5} (2x - x^2)\,dx + B_2\int_{1.0}^{1.5} (2 - x)\,dx = (.45833)A_2 + (.375)B_2.$$

That is, $H(1.0, 1.5; 1.0, 2.0; 1) = .45833$ and $G(1.0, 1.5; 1.0, 2.0; 1) = .375$.

Once Table 2 has been calculated, one can immediately write down three expressions for the historical earned premiums of the three years. For example, the written premiums generating 1975 earned premiums were written at three different rate levels — 1.150 from 1/1/74 to 6/30/74, 1.265 from 7/1/74 to 12/31/74, and 1.202 from 1/1/75 to 12/31/75. Thus, the total earned premium for 1975 of \$1,820 must satisfy the relationship:

$$1820 = A_2((.16667)(1.15) + (.6667)(1.265)) + B_2((.125)(1.15) + (.375)(1.265)) + A_3(1.16667)(1.202) + B_3(.5)(1.202).$$

Similar expressions can be written for 1974 and 1976 earned premiums yielding the three equations:

$$1600 = .38255A_1 + .57031B_1 + .79062A_2 + .58938B_2,$$

$$1820 = 1.03505A_2 + .61813B_2 + 1.40234A_3 + .601B_3,$$

$$1860 = 1.60266A_3 + .601B_3 + 2.19296A_4 + .65433B_4.$$

In addition to these three equations, we require that our linear approximation must be continuous. This means that the line segments must meet at their end points, i.e.,

$$A_1 \cdot (1) + B_1 = A_2 \cdot (1) + B_2,$$

$$A_2 \cdot (2) + B_2 = A_3 \cdot (2) + B_3,$$

and $A_3 \cdot (3) + B_3 = A_4 \cdot (3) + B_4$.

TABLE 2

CONTRIBUTION OF WRITINGS TO EARNINGS PERIODS — COEFFICIENTS OF PARAMETERS

				Earnings Period					
				1974		1975		1976	
Parameters	Writings Period	Rate Change	Cumulative Change	H	G	H	G	H	G
A_1, B_1	1/1/73 to 3/31/73	1.000	1.000	.00521	.03125	0	0	0	0
	4/1/73 to 12/31/73	1.150	1.150	.328125	.46875	0	0	0	0
A_2, B_2	1/1/74 to 6/30/74	1.000	1.150	.458333	.375	.16667	.125	0	0
	7/1/74 to 12/31/74	1.100	1.265	.208333	.125	.66667	.375	0	0
A_3, B_3	1/1/75 to 12/31/75	.950	1.202	0	0	1.16667	.5	1.3333	.5
A_4, B_4	1/1/76 to 4/31/76	1.000	1.202	0	0	0	0	.876543	.27778
	5/1/76 to 12/31/76	1.200	1.442	0	0	0	0	.790123	.22222

At this point, we have a series of six equations in eight unknown parameters which may be written in the convenient form:

(3)

$$
\begin{aligned}
.3826A_1 + .5703B_1 + .7906A_2 + .5894B_2 &= 1600 \\
1.0351A_2 + .6181B_2 + 1.4023A_3 + .601B_3 &= 1820 \\
1.6027A_3 + .601B_3 &= 1860 - 2\ 1930A_4 - .6543B_4 \\
A_1 + B_1 - A_2 - B_2 &= 0 \\
2A_2 + B_2 - 2A_3 - B_3 &= 0 \\
3A_3 + B_3 &= 3A_4 + B_4
\end{aligned}
$$

This represents a system of six equations in the six unknowns (A_i, B_i; $i = 1, 2, 3$), which can be readily solved; each of (A_i, B_i; $i = 1, 2, 3$) can be written as a linear function of A_4 and B_4. The result should be interpreted as a two parameter family of continuous, piecewise linear functions. That is, for any values of A_4 and B_4, we will obtain values for (A_i, B_i; $i = 1, 2, 3$) that will yield the given earned premiums. The solutions for our problem are as follows in terms of the two parameters A_4, B_4:

$$
(4)\quad
\begin{aligned}
A_1 &= -145{,}170.26 + 318.52A_4 + 98.63B_4 \\
B_1 &= 116{,}462.05 - 252.59A_4 - 78.21B_4 \\
A_2 &= 37{,}994.28 - 82.88A_4 - 25.69B_4 \\
B_2 &= -66{,}702.48 + 148.82A_4 + 46.11B_4 \\
A_3 &= -9{,}286.07 + 19.95A_4 + 6.27B_4 \\
B_3 &= 27{,}858.21 - 56.85A_4 - 17.80B_4
\end{aligned}
$$

Given this description of the family of curves representing the rate of premium writings, it remains for the actuary to choose that particular approximation that seems most appropriate for the situation. Probably the most popular choices, if there is no better information available, would be certain optimal members of the family, such as the "smoothest" or the "flattest". These optimal members can easily be found by methods of ordinary calculus, as the following will show.

Recall that the A_i's of the model are the slopes of the line segments representing the rate of written premiums in each year. Thus, the "smoothest" member of the family can be obtained by minimizing the sum of squares function (which we refer to as the "objective function"):

(5a)

$$S(A_i, B_i; i = 1,\ldots,4) = (A_1 - A_2)^2 + (A_2 - A_3)^2 + (A_3 - A_4)^2.$$

Likewise, the "flattest" member of the family is obtained by minimizing the sum of squares function:

(5b)

$$S(A_i, B_i; i = 1,\ldots,4) = A_1^2 + A_2^2 + A_3^2 + A_4^2.$$

Of course, many other choices are possible for the objective function, including the weighting of its components. For instance, if the actuary has qualitative information that writings for 1973 were relatively flat and a new marketing program started in 1974, he may prefer to design the following objective function:

(5c)

$$S(A_i, B_i; i = 1,\ldots,4) = KA_1^2 + A_2^2 + A_3^2 + A_4^2,$$

where K is chosen as some arbitrary large constant. This procedure will force A_1 to be very small in order to minimize the function.

To continue with the demonstration, assume we have decided the flattest member should be chosen. Then the objective function can be rewritten in terms of the free parameters A_4 and B_4 by using the relationships (4). To minimize the resulting $S(A_4, B_4)$, we take the partial derivatives of $S(A_4, B_4)$ with respect to A_4 and B_4, set the resulting linear equations equal to zero, and solve for A_4 and B_4. The procedure can be conveniently written as follows, by use of the chain rule for differentiation:

$$\frac{\partial}{\partial A_4} S(A_4, B_4) = 2A_1 \frac{\partial A_1}{\partial A_4} + 2A_2 \frac{\partial A_2}{\partial A_4} + 2A_3 \frac{\partial A_3}{\partial A_4} + 2A_4 = 0,$$

$$\frac{\partial}{\partial B_4} S(A_4, B_4) = 2A_1 \frac{\partial A_1}{\partial B_4} + 2A_2 \frac{\partial A_2}{\partial B_4} + 2A_3 \frac{\partial A_3}{\partial B_4} = 0.$$

Substituting for A_i and $\frac{\partial A_i}{\partial A_4}$, $\frac{\partial A_i}{\partial B_4}$, $i = 1, \ldots, 4$ by use of (4), yields the following system of equations in A_4 and B_4 only:

$$-49{,}574{,}336.13 + 108{,}725.20A_4 + 33{,}671.13B_4 = 0,$$

$$-15{,}352{,}848.09 + 33{,}671.13A_4 + 10{,}427.72B_4 = 0.$$

This system can be solved for A_4 and B_4, which in turn will yield values for all the A_i, B_i to produce the flattest writings curve. Thus, solving for A_4 and B_4 yields:

$$A_4 = -27.109,$$

$$B_4 = 1{,}559.846;$$

and for the remaining parameters,

$$A_1 = 48.079, \qquad B_1 = 1{,}309.508,$$

$$A_2 = 172.120, \qquad B_2 = 1{,}185.467,$$

$$A_3 = -51.189, \qquad B_3 = 1{,}632.085.$$

Note that solutions and coefficients have been rounded to three and two decimal places, respectively, so some rounding error will be evident if the reader checks these calculations.

Referring back to Table 2, one sees that the coefficients of the A_i, B_i necessary to produce the earned premiums implied by these writings rates have already been calculated. Hence, the earned premium for 1974 will be \$1,378 ((.00521 + .328125) · (48.079) + (.03125 + .46875) · (1,309.508) + (.45833 + .20833) · (172.120) + (.375 + .125 · (1,185.467)). Likewise, earned premiums for 1975 and 1976 are \$1,492 and \$1,483, respectively. Note that these earned premiums are stated at the premium level in effect at 1/1/73 so they must be restated at the 12/31/76 rate level by multiplying by 1.442. The final cumulative rate level indices to obtain the adjusted earned premiums for this demonstration are shown on Table 3, column (5).

EVALUATION OF RESULTS

Table 3 shows the resulting adjusted earned premiums computed by the algorithm. Note two different minimal elements were considered. Columns (5) and (6) give results for the "flattest" approximating element, while columns (7) and (8) give the "smoothest" approximating element. Various patterns of premium levels were tested to obtain results which may be used to compare the traditional method and the new algorithm. Only the earned premium levels were varied, assuming the same rate level change history. Table 3a details the traditional method of obtaining earned premium adjustment factors from rate change history as explained in Kallop's paper.

Table 3 makes it very evident that the algorithm yields different premium adjustment factors for different patterns of premium volume. This behavior is more realistic than that assumed by the traditional method, which is not affected by premium volume fluctuations. The summary table shows the range of adjustment factors produced by various premium patterns. In most cases, the factors are very close to each other; however, the factors produced by the traditional method for 1976 may be as much as 2% overstated, depending on the actual premium pattern.

Table 4 presents the results of an investigation into the actual accuracy of the algorithm. Briefly, it is assumed that the rate of premium writings is known and can be described by the cubic equation:

$$r(t) = 500t^3 - 1{,}950t^2 + 1{,}150t + 2{,}800,$$

where $0 \leq t \leq 4$. Earned premiums and *actual* earned premium adjustment factors can be calculated for this writing pattern. This is done by means of a table similar in format to Table 2. The same rate history as used in the previous demonstration is assumed. Note that this model presents a fairly complicated writings pattern, as shown by the graph of Figure 1. Comparison of the premium adjustment factors produced by the traditional method shows that they are surprisingly accurate for 1974 and 1975. However, for 1976 premium writings, the rate of writings increases dramatically, resulting in 1976 earned premiums almost double those of 1975 earned premiums. As expected, the traditional adjustment factor for 1976 will overstate premium 4.5%. The algorithm using the smoothest straight line approximation does much better in this extreme case, with only a 1.6% overstatement of premium. Of course, when the rate of exposure writings are known, the adjustment factors can be determined exactly. However, in the absence of any knowledge of the exposure writing history, the algorithm comes up with a very reasonable approximation to writings, as shown in Figure 1, and greatly decreases any distortion to adjusted earned premiums.

TABLE 3

SENSITIVITY OF PREMIUM ADJUSTMENT FACTORS TO PATTERNS IN EARNED PREMIUM

		Adjusted Earned Premiums					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Exposure Year	Actual Earned Premium	Traditional Premium	Traditional Factor	Flattest Premium	Flattest Factor	Smoothest Premium	Smoothest Factor
1974	1600	1989	1.243	1987	1.242	1987	1.242
1975	1820	2152	1.183	2152	1.182	2152	1.182
1976	1860	2158	1.160	2137	1.149	2139	1.149
1974	1600	1989	1.243	1988	1.243	1987	1.242
1975	2000	2364	1.183	2369	1.185	2368	1.184
1976	3000	3480	1.160	3429	1.143	3415	1.138
1974	1600	1989	1.243	1995	1.247	1997	1.248
1975	1200	1418	1.183	1420	1.183	1420	1.183
1976	900	1044	1.160	1037	1.152	1041	1.157
1974	2700	3356	1.243	3370	1.248	3373	1.249
1975	1820	2151	1.183	2154	1.184	2153	1.184
1976	1200	1392	1.160	1385	1.154	1392	1.160
1974	2700	3356	1.243	3374	1.250	3379	1.251
1975	1820	2151	1.183	2160	1.187	2158	1.186
1976	2600	3016	1.160	2966	1.141	2948	1.134
1974	2700	3356	1.243	3372	1.249	3376	1.250
1975	1820	2151	1.183	2157	1.185	2156	1.185
1976	1860	2158	1.160	2130	1.145	2126	1.143

Summary

Empirical Ranges Due to Premium Volume Patterns

Premium Adjustment Factors		% Deviation from (3)
1974 Range	1.242-1.250	−0.1 to +0.6
1975 Range	1.182-1.187	−0.1 to +0.3
1976 Range	1.134-1.160	−2.2 to 0.0

TABLE 3a

PREMIUM ADJUSTMENT FACTOR CALCULATION TRADITIONAL RECTANGULAR METHOD

Year = 1974

(1) Rate Change Date	(2) Manual Change	(3) (2) x (3 Prior) Cumulative Index	(4) Weights*	(5) (3) x (4) Product	Premium Adjustment Factor
1/1/73	Base	1.000	.03125	.03125	
4/1/73	1.150	1.150	.84375	.97031	
7/1/74	1.100	1.265	.1250	.15813	
				1.15969	1.24344

Year = 1975

(1) Rate Change Date	(2) Manual Change	(3) (2) x (3 Prior) Cumulative Index	(4) Weights*	(5) (3) x (4) Product	Premium Adjustment Factor
4/1/73	Base	1.150	.125	.14375	
7/1/74	1.100	1.265	.375	.47438	
1/1/75	.950	1.202	.500	.60100	
				1.21913	1.18281

Year = 1976

(1) Rate Change Date	(2) Manual Change	(3) (2) x (3 Prior) Cumulative Index	(4) Weights*	(5) (3) x (4) Product	Premium Adjustment Factor
1/1/75	Base	1.202	.82986	.99749	
5/1/76	1.20	1.442	.17014	.24534	
				1.24283	1.16026

*Weights are calculated as the fraction of the area of a square of side 1 intersected by 45° lines (angle determined by policy term of 1 year) which originate from point of rate change date. A detailed example of the procedure, with diagrams may be found in Kallop's paper referenced above, Appendix to Section B-2, Exhibit 1-B, "Factor Adjusting Calendar Year Premium to Level of Present Rates."

TABLE 4

TEST OF THE ALGORITHM

	Contribution of Writings to Earnings Period		
Earning Period	1.0 to 2.0	2.0 to 3.0	3.0 to 4.0
Writing Period (&)			
0.0 to 0.25	91.683	0	0
0.25 to 1.0	1304.15	0	0
1.0 to 1.5	826.458	256.51	0
1.5 to 2.0	202.865	564.323	0
2.0 to 3.0	0	654.17	795.833
3.0 to 3.33	0	0	763.323
3.33 to 4.0	0	0	1065.844
Actual Earned Premium	2799	1795	3411
Earned Premium @ 12/31/76 Rates	3497	2127	3785
Actual Premium Adjustment Factor	1.249	1.185	1.110
Traditional Premium Adjustment Factor	1.243	1.183	1.160
% Distortion	–0.5%	–0.2%	+4.5%
Smoothest Algorithm Premium Adjustment Factor	1.253	1.188	1.128
% Distortion	+0.3%	+0.3%	+1.6%

Rate of writings function is $r(x) = 500x^3 - 1950x^2 + 1150x + 2800$.

THE RANGE OF PREMIUM ADJUSTMENT FACTORS

For a given history of rate changes and earned premiums it is often of interest to determine the range of possible values the premium adjustment factors can assume. Such information is of special importance when using this algorithm because the rate change and earned premium history do not determine a unique rate of writings model. Rather, a complete family of such rate of writings functions is obtained, any member of which will produce the historical earned premium numbers. The following discussion will demonstrate the methods involved in obtaining the exact theoretical range of factors obtainable from the family of approximating functions.

The six equations of (4) fully describe all rate of premium writing models which are piecewise linear, continuous, and produce the earned premiums of Table 1. However, the parameters A_4 and B_4 appearing in these equations are not unrestricted; in other words, the rate of premium writings for the final year is not as completely arbitrary as may appear at first glance. The constraints that are put on A_4 and B_4 arise from the requirement that the rate of writings function be positive throughout its domain. Under this condition, the range of premium adjustment factors can be investigated by allowing A_4 and B_4 to vary through their set of admissible values.

The admissible range of the parameters A_4 and B_4 can be determined as follows. For the rate of writings function to be always positive the following four conditions must be satisfied for all t, $0 < t \leq 1$:

$$\begin{aligned}
&A_1 t + B_1 \geq 0 \\
&A_2\,(1+t) + B_2 \geq 0 \\
&A_3\,(2+t) + B_3 \geq 0 \\
&A_4\,(3+t) + B_4 \geq 0.
\end{aligned}$$

Of course, these conditions will be satisfied if and only if they are true for $t = 0$ and $t = 1$. This last observation makes it possible to restate the above conditions in terms of five inequalities:

$$\begin{aligned}
B_1 &\geq 0 \\
A_1 + B_1 &\geq 0 \\
2A_2 + B_2 &\geq 0 \\
3A_3 + B_3 &\geq 0 \\
4A_4 + B_4 &\geq 0.
\end{aligned}$$

Using the equations (4) the above inequality system can be written in terms of A_4 and B_4 alone. The following five inequalities then describe the constraints on the parameters A_4 and B_4:

$$
\begin{aligned}
(6)\quad 252.589\,A_4 + 78.213\,B_4 &\le 116{,}462.054\\
16.950\,A_4 + 5.267\,B_4 &\le 9{,}286.071\\
65.933\,A_4 + 20.421\,B_4 &\ge 28{,}708.206\\
3\,A_4 + B_4 &\ge 0\\
4\,A_4 + B_4 &\ge 0
\end{aligned}
$$

Expressions for the three premiums adjusted to present rates, as well as the premium adjustment factors, can also be written in terms of the parameters A_4 and B_4. By use of the coefficients of the A_i and B_i given in Table 2 and the final cumulative rate level index of 1.442 the adjusted earned premiums can be written as follows:

Accident Year	Adjusted Earned Premium Expression
1974	$1.48067\,A_1 + .721\,B_1 + .96133\,A_2 + .721\,B_2$
1975	$3.36472\,A_2 + .721\,B_2 + 1.68823\,A_3 + .721\,B_3$
1976	$1.92266\,A_3 + .721\,B_3 + 2.40332\,A_4 + .721\,B_4.$

Again by use of the equations (4) the above premium expressions become functions of A_4 and B_4 alone:

Accident Year	Adjusted Earned Premium Expression
(7a) 1974	$2624.1913 - 1.3974A_4 - .4325\,B_4$
(7b) 1975	$2027.2253 + .2729A_4 + .0848\,B_4$
(7c) 1976	$2231.7970 - .2284A_4 - .0642\,B_4.$

At this point the question of the range of the premium adjustment factor for 1974, for example, has been recast as the problem of finding the maximum and minimum of the linear expression (7a) subject to the constraints arising from the system of linear inequalities (6). As stated, the question is almost a linear programming problem but for the fact that neither A_4 or B_4 are constrained to be non-negative. This problem can easily be remedied by writing $A_4 = A_4^+ - A_4^-$ where

$$A_4^+ = \begin{cases} A_4 & \text{if } A_4 \ge 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad A_4^- = \begin{cases} -A_4 & \text{if } A_4 \le 0 \\ 0 & \text{otherwise,} \end{cases}$$

and similarly for B_4. Thus the maximum value of 1974 adjusted earned premium, for example, is obtained by solving the following linear programming problem:

$$\text{Maximize } -1.3974\,A_4^+ + 1.3974\,A_4^- - .4325\,B_4^+ + .4325\,B_4^-$$

subject to the constraints

$$\begin{aligned}
252.589\,A_4^+ - 252.589\,A_4^- + 78.213\,B_4^+ - 78.213\,B_4^- &\leq 116{,}462.054\\
16.950\,A_4^+ - 16.950\,A_4^- + 5.267\,B_4^+ - 5.267\,B_4^- &\leq 9{,}286.071\\
65.933\,A_4^+ - 65.933\,A_4^- + 20.421\,B_4^+ - 20.421\,B_4^- &\geq 28{,}708.206\\
3\,A_4^+ - 3\,A_4^- + B_4^+ - B_4^- &\geq 0\\
4\,A_4^+ - 4\,A_4^- + B_4^+ - B_4^- &\geq 0.
\end{aligned}$$

The six max and min problems of the above type can be solved by using standard computer routines available for solution of linear programming problems. The resulting theoretical ranges of adjusted earned premiums and premium adjustment factors are given in the following table.

Accident Year	Adjusted Earned Premium		Premium Adjustment Factor		
	Minimum	Maximum	Minimum	Maximum	Range
1974	$1976	$2018	1.235	1.261	.026
1975	2139	2156	1.175	1.185	.010
1976	2000	2180	1.075	1.172	.097

The range of 1976 premium adjustment factors (1.075 to 1.172) is wide enough to be disconcerting. However this range should be considered as the uncertainty inherent in the data processed by the algorithm. It is preferable that the analyst be aware of the limits on the information that can be extraced from his data. The alternative use of procedures that present one definite result, when an entire range is possible, can be highly misleading.

INCORPORATING ADDITIONAL DATA

As a final investigation of the algorithm, assume we also know the actual written premiums for the period 1973 to 1976. How would we make use of this additional information? The answer lies in the flexibility afforded by the design of the objective function. The actual premium writings implied by the model of Table 4 are:

1973	\$3169
1974	2216
1975	1743
1976	6482

(For example, $\$3160 = 1.0 \int_{0}^{0.25} r(x)dx + 1.15 \int_{0.25}^{1.0} r(x)dx.)$

For the linear model, the written premium for each year can be written in terms of the parameters A_i and B_i. (For 1974, the writings are calculated as:

$$1.150 \int_{1.0}^{1.5} A_2x + B_3dx + 1.265 \int_{1.5}^{2.0} A_2x + B_2dx.)$$

Hence, the writings are expressed as:

1973	$.57031A_1 + 1.1125B_1$
1974	$1.82563A_2 + 1.2075B_2$
1975	$3.00500A_3 + 1.2020B_3$
1976	$4.78048A_4 + 1.3620B_4$

Clearly, we want to minimize the deviation between the actual written premium and the written premium expressions of the linear model. That is, the proper objective function to be minimized is:

(8)

$$S(A_i, B_i; i = 1, \ldots, 4) = (3169 - .57031A_1 - 1.1125B_1)^2 + (2216 - 1.82563A_2 - 1.2075B_2)^2 + (1743 - 3.005A_3 - 1.202B_3)^2 + (6482 - 4.78048A_4 - 1.362B_4)^2.$$

The matrix techniques developed in the appendix make it a simple matter to find the solution to this particular problem. The correct entries are placed into the objective matrix as defined in appendix 1 that produces the objective function. Solving for A_i, B_i and calculating the corresponding adjusted earned premiums and premium adjustment factors yields:

	Adjusted Earned Premium	Adjustment Factors	% Distortion from Actual
1974	\$3498	1.250	+0.1%
1975	2129	1.186	+0.1%
1976	3773	1.106	−0.3%

This example provides a rather straightforward demonstration of the improvement in results due to the use of more information. Since the traditional method of premium adjustment is not flexible enough to take advantage of all available information, the techniques involved in this new algorithm offer the actuary a more responsive tool to aid in rate adequacy investigations.

SUMMARY AND CONCLUSION

The methods developed in this algorithm treat one source of distortion present in the procedure of restating earned premiums at present rates; the distortion due to fluctuating levels of premium volume. Of course, there may be other sources of distortion such as territorial or classification distributions. These can be treated by refining the data into units small enough to give a reasonable approximation of the effect of distributional shifts. The mathematics of the procedure has been explained to the extent that the reader can modify the individual parts of the algorithm, especially the objective function, to take maximum advantage of all information available.

The results of the algorithm have been compared with a simpler premium adjustment procedure which ignores the effect of premium volume. This is the rectangular method, also referred to as the traditional premium adjustment method. An empirical investigation shows that for the particular rate change history used, the adjusted premium factors can have a 2% range in variation due only to different patterns of yearly earned premiums. An example with severe premium fluctuation is presented in which the traditional premium adjustment method overstates premium by 4.5%. This distortion is significantly reduced by the smoothest straight line approximation to premium writings. In addition, the distortion is virtually eliminated by use of additional data. In this case, the appropriate adjustments to the algorithm's objective function were explained to take into account the exposure related written premiums that were assumed to be available.

The mechanics of obtaining the exact range of premium adjustment factors arising from the family of approximating functions are explained. It is important to realize that even for a fixed rate change history and earned premium pattern there is a range of results rather than a single answer. The choice of factors from within this range is accomplished by minimizing a sum of squares function.

APPENDIX I

RESTATEMENT IN LINEAR ALGEBRA

The specific calculations that we explained for the expository problem in the body of the paper will now be reformulated in terms of matrix manipulations. The economy of notation that is available in the linear algebra formulation is preferable if this procedure is to be used frequently as a computational tool.

Initially, the system of equations (3) can be rewritten as:

$$AX = BY \tag{9}$$

where

$$A = \begin{vmatrix} .3826 & .5703 & .7906 & .5894 & 0 & 0 \\ 0 & 0 & 1.0351 & .6181 & 1.4023 & .601 \\ 0 & 0 & 0 & 0 & 1.6027 & .601 \\ 1.0 & 1.0 & -1.0 & -1.0 & 0 & 0 \\ 0 & 0 & 2.0 & +1.0 & -2.0 & -1.0 \\ 0 & 0 & 0 & 0 & 3.0 & 1.0 \end{vmatrix}$$

$$X = \begin{vmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \\ A_3 \\ B_3 \end{vmatrix}, \quad B = \begin{vmatrix} 1600 & 0 & 0 \\ 1820 & 0 & 0 \\ 1860 & -2.193 & -.6543 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 3.0 & 1.0 \end{vmatrix}, \text{ and } Y = \begin{vmatrix} 1 \\ A_4 \\ B_4 \end{vmatrix}$$

Thus, the solution (4) becomes simply:

$$X = (A^{-1}B)Y \tag{10}$$

For convenience, define the matrix C to be the 6 x 3 matrix $C = A^{-1}B$. Then, it will be helpful in further manipulations to use the augmented matrices:

$$\hat{C} = \begin{vmatrix} 1 & 0 & 0 \\ & C & \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad \text{and } \hat{X} = \begin{vmatrix} 1 \\ X \\ A_4 \\ B_4 \end{vmatrix}$$

In this case,

$$(10a) \qquad \hat{X} = \hat{C}Y.$$

The heart of the calculation lies in the formulation of the objective function as a quadratic form. For instance, in order to produce the "flattest" form of the objective function (5a), we could write:

$$(11) \qquad S(A_i, B_i) = \hat{X}^T(\Theta^T \Theta)\hat{X} \text{ where}$$

$$\Theta = \begin{vmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{vmatrix}$$

The calculation should be viewed as follows:

$$\Theta\hat{X} = (A_1, A_2, A_3, A_4)^T$$

and $(\Theta X)^T(\Theta X) = (A_1, A_2, A_3, A_4) \begin{vmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{vmatrix} = \sum_{i=1}^{4} A_i^2$, exactly the form of (5a).

It is evident that, in the form (11), any of a large class of objective functions can be obtained simply by choosing Θ properly. For example, in order to obtain the objective function selecting the smoothest member of the family of approximating functions, choose Θ to be as follows:

$$\Theta = \begin{vmatrix} 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \end{vmatrix}$$

Clearly, in an interactive computer session (such as APL provides) much information on the range of results can be gained with minimum effort by simply trying different Θ matrices in the calculation program.

Recall that, while $\hat{X}$ contains eight unknown parameters for our demonstration, we can reduce this to two by use of relation (10a). Thus,

$$S(A_i, B_i) = Y^T (\hat{C}^T \Theta^T \Theta \hat{C}) Y = Y^T F Y$$

where the matrix $F = \hat{C}^T \Theta^T \Theta \hat{C}$ is the 3 x 3 matrix of coefficients of the quadratic form. In order to minimize this particular quadratic form, it suffices to set up the system of two linear equations in the unknown parameters A_4 and B_4. This process of taking partial derivatives can also be accomplished by matrix multiplication. If we let

$$E_1 = \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} \quad \text{and } D = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

we can then write the system resulting from setting partial derivatives equal to 0 as:

$$(DFD^T) \begin{vmatrix} A_4 \\ B_4 \end{vmatrix} = -DFE_1.$$

Hence,
$$\begin{vmatrix} A_4 \\ B_4 \end{vmatrix} = (DFD^T)^{-1} (-DFE_1),$$

and once A_4 and B_4 have been determined, the other parameters follow as a result of equation (11).

That is, $(A_1, B_1, A_2, B_2, A_3, B_3)^T = C(1, A_4, B_4)^T$

and, to obtain earned premiums implied by these writings parameters, we form a matrix based on factors used to obtain the A and B matrices of equation (9).

The procedure is as follows. Let P be the vector of adjusted earned premiums. Then,

$$P = KA$$

where $A = (A_1, B_1, A_2, B_2, A_3, B_3, A_4, B_4)^T$

contains the solutions of the parameters $(A_i, B_i; i = 1, \ldots, 4)$, and

$$K = \begin{vmatrix} .333335 & .500 & .6667 & .5 & 0 & 0 & 0 & 0 \\ 0 & 0 & .83337 & .5 & 1.1667 & .5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.3333 & .5 & 1.6667 & .5 \end{vmatrix}$$

Note that K is formed directly from the entries of Table 2. Since the earned premiums P are at the rate level of January 1, 1973, earned premiums at present rates can be obtained by taking (1.442)P.

Finally, for the example analysis of Table 4, the actual written premium is known as well as the earned premium. The object is to force the written premium implied by the model to be as close to actual written premium as possible. If the mathematical tools described in this appendix have been implemented, most likely in the form of a computer program, the solution to this particular problem is easily obtained by merely changing the entries of the Θ matrix. All the matrix calculations remain the same, and the appropriate Θ matrix needed to obtain the objective function (8) is given by:

$$\Theta = \begin{matrix} 3169 & -.57031 & -1.1125 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2216 & 0 & 0 & -1.82563 & -1.2075 & 0 & 0 & 0 & 0 \\ 1743 & 0 & 0 & 0 & 0 & -3.005 & -1.202 & 0 & 0 \\ 6482 & 0 & 0 & 0 & 0 & 0 & 0 & -4.78048 & -1.362 \end{matrix}$$

Of course, the example computations have been developed for a case with policy term of one year and three years of earned premiums. The form of the matrix equations does not change for different policy term or number of years of earned premium. Thus, the same matrix equations will handle all the calculations for a problem with policy term of three years and four years of earned premium data. Note that this problem will involve fourteen unknown parameters, which can be reduced to a parametric family of approximations described by four free parameters. Thus, the objective function can be written in terms of four unknowns and minimized by a procedure of taking four partial derivatives. The problem is reduced to writing a program flexible enough to handle any combination of policy term and years of earned premium.

APPENDIX 2

GENERALIZED EARNINGS CONTRIBUTIONS FORMULA

Let the rate of writing function be given by $f(x) = Ax + B$. Then in order to calculate the contributions to earnings of the period from x_0 to x_1 of the writings from y_0 to y_1, we can use a number of different formulas. This appendix will develop the formula due to Ross[3] where

$$\text{ERCON}(y_0, y_1; x_0; x_1; t) = \int_{x_{0-t}}^{x_0} \frac{x - x_0 + t}{t} g(x)\,dx$$

$$+ \int_{x_0}^{x_{1-t}} g(x)\,dx + \int_{x_{1-t}}^{x_1} \frac{x_1 - x}{t} g(x)\,dx,$$

$$g(x) = \begin{cases} f(x) \text{ if } y_0 \leq x \leq y_1 \\ 0 \text{ otherwise} \end{cases}$$

and t is the policy term. It is assumed that $x_0 \leq x_1 - t$.

The three integrals in this formula can be evaluated for limits of integration a, b and $f(x) = Ax + B$ as follows:

$$\int_a^b \frac{x - x_0 + t}{t} (Ax + B)dx = A \cdot h_1^{(1)}(a, b) + B \cdot h_1^{(2)}(a, b)$$

where

$$h_1^{(1)}(a, b) = \left(\tfrac{1}{6t}\right) (2(b^3 - a^3) + 3\,(b^2 - a^2)(t - x))$$

$$h_1^{(2)}(a, b) = \left(\tfrac{1}{2t}\right) (b^2 - a^2) + 2\,(t - x_0)\,(b - a));$$

$$\int_a^b Ax + Bdx = A \cdot h_2^{(1)}(a, b) + B \cdot h_2^{(2)}(a, b)$$

where

$$h_2^{(1)}(a, b) = (\ 1/2\ )\ (b^2 - a^2)$$

$$h_2^{(2)}(a, b) = b - a;$$

[3] Ross, J. P., "Generalized Premium Formulae," *PCAS*, LXII (1975), p. 50.

$$\int_a^b \frac{(x_1 - x)}{t} \; Ax + Bdx = A \cdot h_3^{(1)}(a, b) + B \cdot h_3^{(2)}(a, b)$$

where

$$h_3^{(1)}(a, b) = \left(\frac{1}{6t}\right)(2(b^3 - a^3) + 3x_1 (b^2 - a^2))$$

$$h_3^{(2)}(a, b) = \left(\frac{1}{2t}\right)((b^2 - a^2) + 2x_1 (b - a)).$$

Define the following logical expression which is a function of the order of four points:

$$j(a, b, c, d) = \begin{cases} 1 \text{ if } a \leq b < c \leq d \\ 0 \text{ otherwise} \end{cases}$$

Using the function j define the following $H_i^{(k)}(a, b, c, d)$ functions for $i = 1, 2, 3$ and $k = 1, 2$:

$$H_i^{(k)}(a,b,c,d) = j(a,c,b,d) \cdot h_i^{(k)}(c, b) + j(a,c,d,b) \cdot h_i^{(k)}(c, d)$$

$$+ j(c,a,b,d) \cdot h_i^{(k)}(a,b) + j(c,a,d,b) \cdot h_i^{(k)}(a,d).$$

At this point we can write

$$\text{ERCON}(y_0, y_1; x_0, x_1; t) = A.H._1(y_0, y_1; x_0; x_1; t)$$

$$+ B.H_2(y_0, y_1; x_0; x_1; t)$$

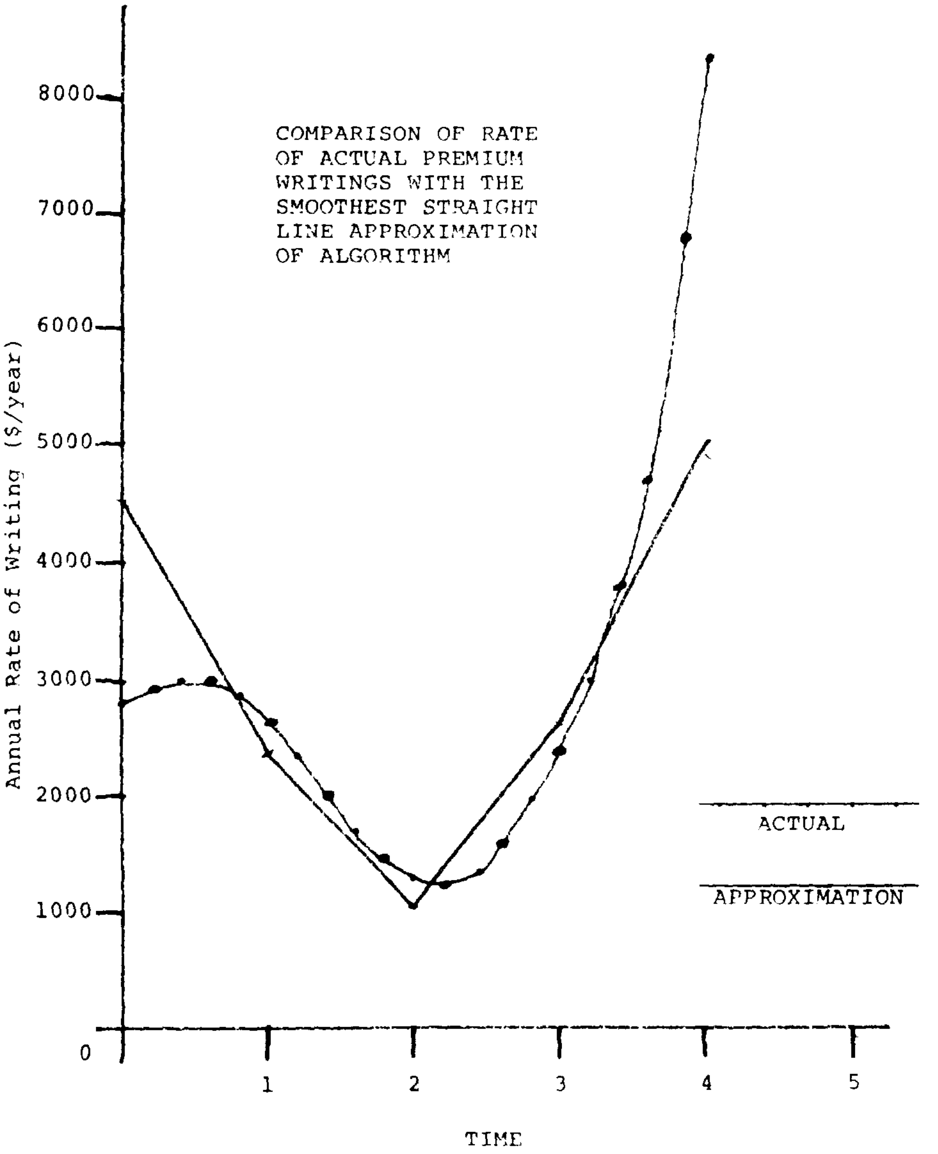
where

$$H_k(y_0, y_1; x_0, x_1; t) = H_1^{(k)}(x_0 - t, x_0, y_0, y_1) +$$

$$H_2^{(k)}(x_0, x_1 - t, y_0, y_1) + H_3^{(k)}(x_1 - t, x_1, y_0, y_1).$$

In this form, the formula can be readily programmed to produce the coefficients of A and B.

FIGURE 1:



Rate of Writing = $500t^3 - 1950t^2 + 1150t + 2800$

ON THE THEORY OF INCREASED LIMITS AND EXCESS OF LOSS PRICING

ROBERT S. MICCOLIS

Since the time of Jeffrey Lange's paper on increased limits[1] in 1969 much has happened to the market for increased limits in the liability lines of insurance. Insureds, particularly commercial insureds, are now interested in purchasing liability coverage with limits in the millions of dollars. This reflects the concern of insureds about exposure to inflation which has greatly increased the magnitude of jury awards and settlements in recent years. The ability of the insurance industry to provide liability insurance for this market greatly depends on sensible pricing.

Currently, in liability insurance there is little experience on losses in excess of $500,000 per occurrence. Indeed the probability that a loss will exceed $500,000 has been quite small. Furthermore, because of the great statistical variation of large losses, there will always be a limit to the credibility of data for making increased limits factors, especially for high limits. Consequently, there will always be a need for judgment in the pricing of high limits and excess of loss coverage.

This paper presents the mathematical foundations of the pricing of increased limits and excess of loss coverage. The paper will attempt to tie together the various aspects of this area of insurance pricing in a logical, straightforward manner by means of a mathematical model. It is hoped that this model will be helpful in making pricing judgments or evaluating such judgments.

Section one presents the mathematical model of expected value pricing by considering frequency and severity separately. An insurance cost function is introduced into the model that should aid greatly in understanding the mathematics of insurance pricing. Such functions are defined for increased limits and excess of loss coverage and are used to derive increased limits factors in concise mathematical terms. A simple formula is found that relates

[1] J. T. Lange, "The Interpretation of Liability Increased Limits Statistics," *PCAS* Vol. LVI (1969), p. 163.

a set of increased limits factors to the loss severity distribution underlying them. From this formula a very useful and convenient test is developed that can identify pricing inconsistencies. The implications of trend in the model are investigated and a commonly used method of adjusting increased limits factors for trend is shown to be undesirable.

Section two considers risk and its relationship to pricing. The variance principle of risk loadings is used to determine a risk charge for increased limits and excess of loss pricing. An analysis of spreading the risk by layering of coverage and reinsurance is also presented and demonstrates a reduction in risk by layering.

Section three describes some ways to treat the many difficulties and practical problems in applying the theory of increased and excess of loss pricing, particularly in regard to obtaining severity distributions.

Section four discusses four related areas of pricing—three in liability insurance and one in property insurance. The mathematics of the leveraged effect of inflation are presented and the consistency test is applied to increased limits factors for aggregate and split limits. Finally, the potential value of the consistency test in other lines of insurance is shown by an application to a similar pricing problem in coinsurance.

The paper will treat only the pure loss element of ratemaking. There are many practical problems concerning expenses, particularly loss adjustment expenses, which cannot be resolved solely by this model.

1. EXPECTED VALUE PRICING

Traditional actuarial ratemaking is predicated on the estimation of expected, mean, or average values. As will be discussed later, these methods can be sufficient for most ratemaking problems. In this section, a general model of expected value pricing is presented and then applied to increased limits. In addition, a test of increased limits factors is developed. Excess of loss coverage is also considered along with an analysis of two different methods of trend adjustment. The next section deals with the determination of a risk loading appropriate for increased limits and excess of loss coverage.

The General Pricing Model

Let us describe the general insurance ratemaking or pricing problem in mathematical terms with the following definitions:

1. Let n be a random variable representing the number of accidents (occurrences) an insured will have over the course of one year (the usual policy period). This is the loss frequency variable.

2. Let x be a random variable representing the dollar amount of damage which the insured incurs given an accident has occurred. This is the loss severity variable.

3. Let g be a function of x representing the dollar amount of coverage[2] provided by the insurer for a loss of size x.[3] This is the insurer's cost function. If $F(x)$ is the cumulative distribution function of x then we can express $E[g(x)]$ in terms of $F(x)$ as follows:

$$E[g(x)] = \int_0^\infty g(x)\, dF(x), \text{ or}$$

$$= \int_0^\infty g(x) \cdot f(x)\, dx, \text{ where } f(x) = \frac{dF(x)}{d(x)}$$

4. Let y be a random variable representing the total dollars of insured losses that an insured will have in one year. This is the pure premium variable.

While y is not easily expressed in terms of n and $g(x)$, we can express the expected value of y, $E[y]$, as

$$E[y] = E[g(x)] \cdot E[n] \tag{1}$$

Equation (1) is merely the mathematical expression for the division of the

[2] The function g represents those coverage provisions which depend only on the size of loss and which treat each loss individually and identically.

[3] As will be mentioned later, the size of a loss may depend on the amount of coverage. The present discussion assumes independence.

average pure premium into the average size of insured loss and the average frequency of loss. The derivation of this equation can be shown as follows:

1. Assume that the distribution of the size of each loss does not depend on how many losses occur during the year under each policy. That is, frequency and severity are independent.
2. Assume also that if more than one loss occurs in a year for a policy, then the size of each loss is independent of the size of any of the other losses.
3. Hence, if n insured losses occur during the year under a given policy, then the expected value of the sum of those losses is equal to n times the expected value of one such loss,

$$E[y \mid n] = n \cdot E[g(x)]$$

4. The expected value of y, the total dollars of insured losses incurred during the year for a given policy, is given by taking the expected value of $E[y \mid n]$ with respect to the random variable n.
5. Therefore,

$$\begin{aligned} E[y] &= E_n\,(E[y \mid n]) \\ &= E_n\,(n \cdot E[g(x)]) \\ &= E[n] \cdot E[g(x)] \end{aligned}$$

Increased Limits Coverage

In liability insurance, a policy generally covers such loss in full up to a specified maximum dollar amount that will be paid on any one loss. If k is such a policy limit then we can express the cost function, $g(x;k)$, for this coverage as

$$g(x;k) = \begin{cases} x, & 0 < x < k,\ k > 0 \\ k, & x \geq k \end{cases} \tag{2}$$

and

$$\begin{aligned} E[g(x;k)] &= \int_0^k x\, dF(x) + k \cdot \int_k^\infty dF(x) \\ &= \int_0^k x\, dF(x) + k \cdot [1 - F(k)] \end{aligned} \tag{3}$$

It is general practice to publish rates for some standard limit called the basic limit, b. Increased limits rates are expressed as a factor, $I(k)$, for a limit k to be applied to the basic limit pure premium rate. The mathematical expression for the increased limits factor is the ratio of expected total losses with k limit coverage to expected total losses with basic limit coverage. Thus, using equation (1) we have

$$I(k) = \frac{E[g(x;k)] \cdot E[n]}{E[g(x;b)] \cdot E[n]} \tag{4}$$
$$= \frac{E[g(x;k)]}{E[g(x;b)]}$$

We can see that the increased limits factor is dependent only on loss severity and the cost function, but not on loss frequency. Note that $E[g(x;b)]$ is simply the average basic limits severity and will be hereafter referred to as ABLS. Consequently, if we know the appropriate loss severity distribution then we can use equations (3) and (4) to determine expected value increased limit factors for various limits.

As will be seen later, the compilation of a loss severity distribution from experience data can be very difficult and in some cases may not be feasible. Consequently, considerable judgment is needed to develop increased limits factors. In many instances it may be easier to make judgments in terms of specific increased limits factors rather than working with loss severity distributions. Therefore, it would be helpful to analyze the loss severity distribution underlying a given set of increased limits factors. The derivation of the necessary mathematical expression is as follows:

$$I(k) = \frac{E[g(x;k)]}{ABLS}, \text{ where ABLS} = E[g(x;b)]$$
$$= \frac{1}{ABLS} \cdot \left(\int_0^k x\, dF(x) + k[1 - F(k)] \right)$$
$$\frac{dI(k)}{dk} = \frac{1}{ABLS} \cdot \left(k \cdot \frac{dF(k)}{dk} + 1 - F(k) - k \cdot \frac{dF(k)}{dk} \right)$$
$$\frac{dI(k)}{dk} = I'(k) = \frac{1 - F(k)}{ABLS} \tag{5}$$

or

$$I'(k) = \frac{G(k)}{ABLS}, \text{ where } G(x) = 1 - F(x) \tag{6}$$

Solving for $F(k)$, the underlying severity distribution, we get

$$F(k) = 1 - ABLS \cdot I'(k) \tag{7}$$

In words, this result shows that the probability that a loss will be greater than k is equal to the product of the average basic limits severity and the rate of change in the increased limits factors at k. Thus we see that there is a loss severity distribution implicitly defined by any set of expected value increased limit factors. Note that the specific distribution is, as we might have suspected, a function of the average basic limits severity. In theory, $I(k)$ and therefore $I'(k)$ exist for all $k > 0$. However, in practice $I(k)$ is defined only for $k > b$ as the term "increased limits" implies. Consequently, any practical application of equation (7) to estimate $F(k)$ from a set of increased limits factors would be limited to sizes of loss greater than the basic limit.

Aside from deriving specific distributions we can also use this relationship to determine general properties of expected value increased limit factors from those of distribution functions.

1. As k approaches ∞, $F(k)$ will approach 1, $I'(k)$ will approach zero, and $I(k)$ will approach some constant. If $I(k)$ becomes constant for all k greater than some value M, then $I'(k) = 0$ and $F(k) = 1$ whenever $k > M$ because there is no probability of a size of loss greater than that value of k. This would imply no additional charge for higher limits.

2. Since $F(k)$ is monotonic increasing, $I'(k)$ will be monotonic decreasing. If $F(k)$ has a point of inflection, then so will $I'(k)$ at the same value of k. The converse of both statements also holds.

3. The probability density function, $f(k)$, can be expressed as follows:

$$f(k) = \frac{dF(k)}{dk} = -\text{ABLS} \cdot \frac{d^2I(k)}{dk^2} \tag{8}$$

Consequently,

$$I''(k) = \frac{d^2I(k)}{dk^2} = \frac{-f(k)}{\text{ABLS}}$$

Note that $I''(k)$ can never be positive since $f(k)$ and ABLS should always be positive. Consequently, to avoid the implication of negative probabilities, $I'(k)$ must be monotonically decreasing and $I(k)$ must be strictly increasing[4]. Also, any modes in $f(k)$ will correspond to inflection points in $I'(k)$.

---

[4] It is permissible for $I(k)$ to reach some limit and stay there for all larger values of k.

The Consistency Test

Using property (3) above, we can construct a "consistency" test for evaluating a given set of increased limits factors. The marginal premium per $1000 of coverage should decrease as the limit of coverage increases. If not, this implies negative probabilities. For example, consider the following set of increased limits factors for per occurrence limits between $25,000 and $10,000,000.

Per Occurrence Limit (in thousands of dollars)	Increased Limits Factor	Marginal Rate[5] per $1000 of Coverage
25	1.000	—
50	1.250	.0100
100	1.425	.0035
200	1.625	.0020
250	1.705	.0016
300	1.775	.0014
350	1.865	.0018*
400	1.915	.0010
500	1.975	.0006
750	2.175	.0008*
1000	2.400	.0009*
1250	2.575	.0007*
1500	2.700	.0005
1750	2.825	.0005
2000	2.950	.0005
2500	3.100	.0003
3000	3.300	.0004*
4000	3.600	.0003
5000	3.800	.0002
7500	4.300	.0002
10000	4.800	.0002

This set of increased limits factors is "inconsistent" at the indicated (*) limits of 350, 750, 1000, 1250, and 3000. These factors are very similar to factors actually in use until 1975.

[5] The Marginal Rate is the difference in increased limits factors between the given limit and the next lower limit, divided by the difference in the limits.

Aside from the mathematical interpretation of this consistency test, it has a very practical meaning. In general, it does not make sense to the insurance buyer to have to pay more for each additional $1000 of coverage since the probability of losses larger than some limit should be less than for a lower limit. Of course there can be anti-selection, that is where the existence of higher limits influences the size of the suit, award or settlement. However, this should not restrict the general applicability of the consistency test. Other applications of the consistency test will be described later in the paper.

Excess of Loss Coverage

In general, an excess of loss contract or non-proportional reinsurance arrangement covers losses greater than a given amount, r, the retention and has a maximum liability of j. Any loss, x, exceeding r is insured for the amount $x - r$, up to the maximum j. We can express the excess of loss cost function, h(x;r,j), as follows:

$$h(x;r,j) = \begin{cases} 0, 0 < x \leq r \\ x - r, r < x < s, s = r + j \\ j, x \geq s \end{cases} \tag{9}$$

and therefore,

$$\begin{aligned} E[h(x;r,j)] &= \int_r^s (x - r)\, dF(x) + j[1 - F(s)] \\ &= \int_r^s x\, dF(x) - r[F(s) - F(r)] + j[1 - F(s)] \\ &= \int_r^s x\, dF(x) + s[1 - F(s)] - r[1 - F(r)] \end{aligned} \tag{10}$$

Consequently,

$$E[h(x;r,j)]^{6} = E[g(x;s)] - E[g(x;r)] \tag{11}$$

Note that the expected number of accidents, E[n], has not changed just because losses less than the retention, r, are not insured under an excess

[6] Equation (11) can also be derived by observing that h(x;r,j) = g(x;s) − v(x;r).

contract. Consequently, the expected value pure premium for the excess contract, is given by

$$E[h(x;r,j)] \cdot E[n]$$

and can be expressed in terms of the basic limits pure premium, ABLS • E[n], as follows,

$$\frac{E[h(x;r,j)] \cdot E[n]}{ABLS \cdot E[n]} = \frac{E[h(x;r,j)]}{ABLS} \tag{12}$$

$$= \frac{E[g(x;s)]}{ABLS} - \frac{E[g(x;r)]}{ABLS}$$

$$= I(s) - I(r)$$

This proves mathematically that the expected value pure premium for an excess contract is equal to the difference in expected value pure premiums of two "first dollar" contracts.

Trend

Inflationary pressures, both economic and social, increase the size of losses over time. Inflationary trends can have substantial effects on pricing increased limits and excess of loss coverages. These effects are difficult to evaluate since the limits and retentions remain fixed while loss severity is shifting[7].

We can investigate the mathematical aspects of trend in terms of a transformation of the loss severity variable. Let's assume that the economic and social values that produce a loss of size x are changing such that a loss of size x' will be produced by the new values after a fixed period of time (one year). This can be described mathematically by equating the probability of a loss size less than or equal to x at a given point in time, with the probability of a loss size less than or equal to x' one year later. If $F(x')$ is the cumulative distribution function of x', then

$$F(x') = F(x) \tag{13}$$

[7] An alternative method of setting retentions is described by Ferguson, R. E., "Non-proportional Reinsurance and the Index Clause," *PCAS* Vol. LVI, (1974), p. 141. The method attempts to reduce the excess carrier's pricing problem caused by inflation.

Let $\alpha(x)$ represent the transformation that describes the relationship between x' and x.

$$x' = \alpha(x) \tag{14}$$

Assuming that $\alpha(x)$ is monotonic, we find

$$F(\alpha(x)) = F(x) \tag{15}$$

Also, since $x = \alpha^{-1}(x')$, we can write

$$F(x') = F(\alpha^{-1}(x')) \tag{16}$$

In the simple case each loss is increased by the same multiplicative factor, a, which is greater than one:[8]

$$\begin{aligned} x' &= \alpha_1(x) \\ &= ax \end{aligned}$$

Here we have $\alpha_1^{-1}(x') = x'/a$, thesefore using equation (16) we find

$$F_1(x') = F(x'/a)$$

Now we would like to know what the trended increased limits factor for policy limit k, $I_1(k)$, should be. Starting from equation (3) using x' and $F_1(x')$,

$$\begin{aligned} E[g(x';k)] &= \int_0^k x'\,dF_1(x') + k[1 - F_1(k)] \\ &= \int_0^k x'\,dF(x'/a) + k[1 - F(k/a)] \end{aligned}$$

Letting $u = x'/a$,

$$\begin{aligned} E[g(x';k)] &= a \cdot \int_0^{k/a} u\,dF(u) + k[1 - F(k/a)] \qquad (17) \\ &= a \cdot E[g(x;k/a)] \end{aligned}$$

[8] This type of trend and its relationship to basic limits trend are studied by Finger, R. J., "A Note on Basic Limits Trend Factors", *PCAS* Vol. LXIII, (1976), p. 106.

Consequently, applying the development of equation (4) to the trended severity we get,

$$I_1(k) = \frac{E[g(x';k)]}{ABLS_1} \tag{18}$$

$$= \frac{a \cdot E[g(x;k/a)]}{a \cdot E[g(x;b/a)]}$$

$$= \frac{I(k/a)}{I(b/a)}$$

For excess of loss coverage,

$$I_1(s) - I_1(r) = \frac{I(s/a) - I(r/a)}{I(b/a)} \tag{19}$$

Also note that differentiating equation (18) gives

$$I_1'(k) = \frac{1}{a} \cdot \frac{I'(k/a)}{I(b/a)} \tag{20}$$

$$= \frac{1}{a} \cdot \frac{G(k/a)}{E[g(x;b/a)]} \text{ (from equation (6))}$$

There is another more commonly used approach to updating increased limits factors for trend. The procedure considers separately:

1) the trend in average severity for basic limits, t_b, and

2) the trend in average severity for increased limits, t_i.

If $ABLS_2$ is the average basic limits severity after one year of inflation, then

$$ABLS_2 = t_b \cdot \text{ABLS}$$

Every "layer or loss" in excess of basic limits is similarly inflated by t_i where such a layer is defined by the excess portion of the increased limits factor, $I(k) - 1$.

For purposes of comparison with the first trend method or other methods, we would like to know the transformation, $\alpha_2(x)$, implied by this second trend procedure.

We can express this procedure as follows:

$$I_2(k) - 1 = t_1 \cdot (I(k) - 1) \tag{21}$$

hence,

$$I_2(k) = 1 + t_1 \cdot (I(k) - 1) \tag{22}$$

also,

$$I'_2(k) = t_1 \cdot I'(k) \tag{23}$$

$$= t_1 \cdot \frac{G(k)}{ABLS} \text{ (from equation (6))}$$

and also from equation (6) we know

$$I'_2(k) = \frac{G_2(k)}{ABLS_2}, \text{ where } G_2(k) = 1 - F_2(k)$$

thus solving for $G_2(k)$ we find

$$G_2(k) = \frac{ABLS_2}{ABLS} \cdot t_1 \cdot G(k) \tag{24}$$

$$= t_b \cdot t_1 \cdot G(k)$$

Now using equation (15) we see that

$$F_2(\alpha_2(x)) = F(x)$$

Therefore,

$$1 - F_2(\alpha_2(x)) = 1 - F(x)$$

$$G_2(\alpha_2(x)) = G(x)$$

$$\alpha_2(x) = G_2^{-1}(G(x))$$

But from equation (24) we find

$$G_2^{-1}(x) = G^{-1}\left(\frac{x}{t_b \cdot t_1}\right)$$

Hence,

$$\alpha_2(x) = G^{-1}\left(\frac{G(x)}{t_b \cdot t_1}\right) \tag{25}$$

We see that $\alpha_2(x)$ is defined in terms of the original severity distribution. In order to see what kind of function $\alpha_2(x)$ is, we can make some assumptions about the severity distribution.

1. If the severity distribution is exponential,

$$G(x) = \exp(-\beta x)$$

$$G^{-1}(x) = \frac{-\ln^x}{\beta}$$

$$\alpha_2(x) = x + \frac{1}{\beta} \cdot \ln(t_b \cdot t_i), \text{ where } \frac{1}{\beta} \cdot \ln(t_b \cdot t_i) \text{ is a constant.}$$

2. If the severity distribution is Weibull,

$$G(x) = \exp(-x^B/A)$$

$$G^{-1}(x) = (-A \cdot \ln x)^{1/B}$$

$$\alpha_2(x) = (x^B + A \cdot \ln(t_b \cdot t_i))^{1/B}$$

3. If the severity distribution is lognormal, a general solution is not available. However, $\alpha_2(x)$ can be computed using numerical approximation techniques.

4. If the only form of the severity distribution is given by a set of increased limits factors represented by $I(x)$, then

$$I'(x) = \frac{G(x)}{ABLS}$$

$$G(x) = ABLS \cdot I'(x)$$

$$G^{-1}(x) = I'^{-1}(x/ABLS)$$

$$\alpha_2(x) = I'^{-1}\left(\frac{I'(x)}{t_b \cdot t_i}\right)$$

Exhibit I gives numerical examples of $\alpha_2(x)$ for the exponential ($\beta = 2.54 \times 10^{-5}$), the Weibull ($A = 42.1898$, $B = .42045$) and the lognormal ($\mu = 8.9146$, $\sigma = 1.7826$) loss severity distributions where $t_b = 1.08$ and $t_i = 1.20$.

EXHIBIT 1

INFLATION BY SIZE OF LOSS UNDER $\alpha_2(x)$

Size of Loss (x)	Exponential $\alpha_2(x)$	Exponential $\alpha_2(x) \div x$	Weibull $\alpha_2(x)$	Weibull $\alpha_2(x) \div x$	Lognormal $\alpha_2(x)$	Lognormal $\alpha_2(x) \div x$
25,000	35,207	1.408	35,207	1.408	35,207	1.408
50,000	60,207	1.204	64,870	1.297	66,241	1.324
100,000	110,207	1.102	121,796	1.217	126,686	1.266
200,000	210,207	1.051	232,102	1.160	244,968	1.224
250,000	260,207	1.040	286,392	1.145	303,432	1.213
300,000	310,207	1.034	340,330	1.134	361,603	1.205
350,000	360,207	1.029	393,997	1.125	419,549	1.198
400,000	410,207	1.025	447,447	1.118	477,303	1.193
500,000	510,207	1.020	553,840	1.107	592,368	1.184
750,000	760,207	1.013	817,784	1.090	878,140	1.170
1,000,000	1,010,207	1.010	1,079,853	1.079	1,162,097	1.162
1,500,000	1,510,207	1.006	1,600,654	1.067	1,726,564	1.151
2,000,000	2,010,207	1.005	2,118,657	1.059	2,288,093	1.144
2,500,000	2,510,207	1.004	2,634,833	1.053	2,847,429	1.138
3,000,000	3,010,207	1.003	3,149,689	1.049	3,405,168	1.135
4,000,000	4,010,207	1.002	4,176,574	1.044	4,517,525	1.129
5,000,000	5,010,207	1.002	5,200,716	1.040	5,626,732	1.125
7,500,000	7,510,207	1.001	7,753,427	1.033	8,388,452	1.118
10,000,000	10,010,207	1.001	10,298,950	1.029	11,144,829	1.114

From the examples, it appears that $\alpha_2(x)$ will generally produce higher trends for small sizes of loss and lower trends for the large losses. Intuitively, it seems that $\alpha_2(x)$ might not be as good a representation of real loss trends as using the same trend for all sizes of loss, represented by $\alpha_1(x)$. In fact, it is more likely that the reverse of $\alpha_2(x)$ is true, i.e., lower trend for small losses, higher trend for large. The reason for these results with $\alpha_2(x)$ stems from the assumption made that *all* excess layers should receive the *same* trend factor. If indeed there is a difference between basic and excess trend, then should not different excess layers be trended differently? This is the contradiction implied by $\alpha_2(x)$.

Consequently, it is preferable to use $\alpha_1(x)$ rather than $\alpha_2(x)$ to adjust for trend.

2. THE CHARGE FOR RISK

A major problem with expected value pricing is that it fails to appropriately charge for the risk of being in the insurance business. Premium rates are usually determined from the expected pure premium, a provision for expenses, and a loading for profit and contingencies. For most lines of insurance, the profit and contingency loading presumably compensates for risk. However, the loading is usually low, and it therefore will only be adequate for relatively low risk lines or coverages. A more volatile line or coverage needs an additional safety loading or risk charge for the added risk in order for it to be on a par with the other lines or coverages. Consequently, while the general provision for profit and contingencies may be sufficient for most lines of insurance, it can be seriously deficient for a high risk line or coverage.

For the purpose of this paper, the meaning of risk will be associated with the degree of uncertainty in the pure premium. It is assumed that one who is averse to risk will desire stability and certainty. Given the choice between insuring ten individuals with $1,000,000 limits each or one insured at $10,000,000, a risk-averse actuary should argue for the ten separate policies in order to reduce the likely variation from the expected losses.[9] However, there should be some risk charge that would make such an actuary indifferent between the two choices based on some rational and objective criteria. Even though attitudes and preferences towards risk can be highly subjective, some measure of risk is desired to establish a reasonable standard for determining such a risk charge. In an article on risk and ratemaking, Lange[10] suggests a measure of risk based on the concept of variance. The discussion that follows will attempt to apply this idea to increased limits and excess of loss pricing.

[9] An additional or alternative criteria in such a situation is to consider the probability of ruin as a basis for actuarial decision making. Utility theory presents yet another decision rule. See Bühlmann, H., *Mathematical Methods in Risk Theory*, Springer-Verlag (1970), p. 85-87, and Freifelder, L. R., *A Decision Theoretic Approach to Insurance Ratemaking*, Irwin (1976), p. 36-56.

[10] J. T. Lange, "Application of a Mathematical Concept of Risk to Property-Liability Insurance Ratemaking," *Journal of Risk and Insurance*, Vol. XXXVI, p. 383.

Sources of Risk

There are two main sources of risk associated with insurance ratemaking. First, variation between actual losses and expected losses can be the result of the stochastic or random nature of the frequency and severity of insurance losses. Freifelder[11] calls this the "process risk". Second, such variation can also result from an inability to estimate expected losses accurately.[12] This is appropriately termed the "parameter risk" by Freifelder. A major cause of difficulty in estimating expected losses in some types of insurance is the occurrence of catastrophes such as hurricanes, tornadoes, earthquakes, etc. Inflationary trends also have a substantial impact in estimating expected losses. For a line of insurance, changes in the mix of business among various classes, coverages and types of insureds can affect expected losses. A small independent insurer is faced with sampling error in estimating expected losses. Incorrect ratemaking data is always a potential problem. Finally, claims practices, underwriting practices, social attitudes, and judicial or legislative climate can undergo drastic and rapid changes which can not always be anticipated to adjust expected losses adequately.

While parameter risk can be substantial, the determination of a risk charge to compensate for this risk is very difficult and is beyond the scope of this paper. In an area such as a catastrophe cover for hurricanes, floods, etc. the parameter risk can be quite large and cannot be ignored. However in many applications the parameter risk should be minimal. This paper will only study the effects of the process risk and develop appropriate risk charges for such risk.

Variance as a Measure of Risk

The source of risk used in this paper for the determination of risk charges for various liability limits is the chance or random variation in the pure premium, i.e. the process risk. As will be shown, this source produces a substantial, measurable difference in risk charge by limit of liability. If we define the measure of this risk as the standard deviation of the pure premium as Lange[13] suggests, we can analyze the properties of risk and risk charges for increased limits and excess of loss coverages. However, the variance of the pure premium is felt to be a more appropriate measure be-

[11] Freifelder, *op. cit.*, p. 70-71.

[12] This second source of risk can also be considered to include errors in estimating any of the moments or parameters which determine the form or shape of the frequency and severity distributions.

[13] Lange, *op. cit.*, p. 386.

cause it satisfies the three basic ratemaking axioms advanced by Freifelder[14] and has some weighty theoretical advantages discussed by Bühlmann[15]. Also, as will be shown, it permits the development of risk adjusted increased limits factors from the severity distribution alone.

The formula for premium determination (excluding expenses) with a safety or contingency loading proportional to risk[16] is

$$\text{Premium} = E[y] + \lambda \cdot \text{Var}[y) \tag{26}$$

Where E[y] is the pure premium and Var[y] is the variance of the pure premium variable. The factor λ must be selected judgmentally. This can be done on the basis of the relative magnitude of Var[y] compared to E[y].

The pure premium variance can be expressed in terms of frequency and severity (assuming independence) as follows:[17]

$$\text{Var}[y] = E[n] \cdot \text{Var}[g(x)] + \text{Var}[n] \cdot E[g(x)]^2 \tag{27}$$

Since

$\text{Var}[g(x)] = E[g(x)^2] - E[g(x)]^2$ we can write

$$\text{Var}[y] = E[n] \cdot E[g(x)^2] + (\text{Var}[n] - E[n] \cdot E[g(x)]^2 \tag{28}$$

In most cases, the frequency variance, Var[n], will be greater than the expected frequency, E[n]. Therefore at a minimum we should have

$$\text{Var}[y] = E[n] \cdot E[g(x)^2] \tag{29}$$

Note that if the frequency of loss distribution is Poisson, equation (29) is exactly right. In addition, the second moment of the severity of insured losses $E[g(x)^2]$, can be many times larger than the square of the first moment, $E[g(x)]^2$, particularly for excess of loss coverage, since the severity distribution has a long tail. Consequently, if we can assume that the ratio of $E[g(x)^2]$ to $E[g(x)]^2$ will be substantially greater than the ratio of $\text{Var}[n] - E[n]$ to $E[n]$, then equation (29) should be adequate for determining risk charges. Further work is needed to test this assumption, how-

14 Freifelder, *op. cit.*, p. 36-56.

15 Bühlmann, *op. cit.*, p. 89-92.

16 This is known as the "variance principle of premium calculation" as discussed by Bühlmann, *op. cit.*, p. 85-87.

17 A. L. Mayerson, D. A. Jones, and N. L. Bowers, Jr., "On the Credibility of the Pure Premium," *PCAS* Vol. LV, (1968), p. 175.

ever it will be used as a first approximation to illustrate the inclusion of risk charges in increased limits factors.

The $E[g(x)^2]$ formulas for the cost functions considered in this paper, $g(x;k)$ and $h(x;r,j)$ as defined in equations (2) and (9) respectively, are given below.

$$E[g(x;k)^2] = \int_0^k x^2\, dF(x) + k^2[1 - F(k)] \tag{30}$$

$$\begin{aligned} E[h(x:r,j)^2] &= \int_r^s (x-r)^2\, dF(x) + (s-r)^2[1 - F(s)],\ s = r + j \\ &= \int_r^s x^2\, dF(x) - 2r \cdot \int_r^s x\, dF(x) + r^2[1 - F(r)] \\ &\quad + (s^2 - 2rs)\,[1 - F(s)] \\ &= \int_r^s x^2\, dF(x) - 2r\left\{\int_r^s x\, dF(x) + s[1 - F(s)] \right. \\ &\quad \left. - r[1 - F(r)]\right\} - r^2[1 - F(r)] + s^2[1 - F(s)] \\ &= \int_r^s x^2\, dF(x) - 2r \cdot E[h(x;r,j)] - r^2[1 - F(r)] \\ &\quad + s^2[1 - F(s)] \\ &= E[g(x;s)^2] - E[g(x;r)^2] - 2r \cdot E\,[h(x;r,j)] \end{aligned} \tag{31}$$

The examples in Exhibit II will demonstrate premium determination including risk charge using equations (26), (1) and (29) for different retentions and policy limits. The assumptions used for Exhibit II are:

1. The expected frequency is the same for each insured. Also, the frequency variance is equal to the expected frequency. The $E[n]$ will be set at 0.10.

2. Insureds are also homogeneous with respect to severity and the severity distribution is given by a lognormal distribution[18] with

[18] The formulas used for approximating the necessary values from the lognormal distribution are given in the Appendix.

parameters $\mu = 8.9146$ and $\sigma = 1.7826$. This distribution has a relatively high coefficient of variation ($\sqrt{\text{Var}[x]/\text{E}[x]}$) and therefore is highly skewed. It should illustrate the potential magnitude of the risk charges.

3. A risk charge of 5% of the expected value pure premium will be assumed adequate for \$25,000 policy limits (zero retention). This produces a λ factor of 2.559×10^{-6}.

The increased limits factors from the same severity distribution as used in Exhibit II are shown in Exhibit III both on an expected value basis and risk adjusted. Note that since equation (29) was used to estimate the pure premium variance, the risk adjusted increased limits factors, $I_r(k)$, do not depend on the frequency of loss.

$$I_r(k) = \frac{\text{Premium for policy limit k}}{\text{Premium for basic limit b}} \tag{32}$$

$$= \frac{E[n] \cdot E[g(x;k)] + \lambda \cdot E[n] \cdot E[g(x;k)^2]}{E[n] \cdot \text{ABLS} + \lambda \cdot E[n] \cdot E[g(x;b)^2]}$$

$$= \frac{E[g(x;k)] + \lambda \cdot E[g(x;k)^2]}{\text{ABLS} + \lambda \cdot E[g(x;b)^2]}$$

It is important to note that it is not appropriate to determine the risk adjustment for excess of loss coverage from the risk adjusted increased limits factors. This will be discussed further in the next section.

Risk Reduction by Layering

The large risk associated with high limits coverage can be significantly reduced by "vertical" layering. This type of layering can be effected by two methods. The first is by insuring through two or more carriers[19], one carrier providing "first-dollar" coverage and the others excess of loss coverage. The second is through the use of non-proportional reinsurance. In this discussion it will be assumed that the insurance coverage is being provided to a large homogeneous group of insureds.

[19] At some point the number of carriers involved in providing coverage for one insured cannot be increased without expense considerations offsetting the risk reduction.

EXHIBIT II

PREMIUM DETERMINATION INCLUDING RISK CHARGE

Retention	Policy Limits	Expected Value Pure Premium	Pure Premium Variance	Risk Charge ($\lambda \times$ Variance)	Premium (before expenses)
0	25,000	1,113	2.175×10^7	56	1,169
0	50,000	1,579	5.563×10^7	142	1,721
0	100,000	2,083	12.834×10^7	328	2,411
0	300,000	2,811	38.790×10^7	993	3,804
0	500,000	3,074	59.192×10^7	1,515	4,589
0	1,000,000	3,335	95.916×10^7	2,454	5,789
0	1,300,000	3,406	112.144×10^7	2,870	6,276
0	1,500,000	3,439	121.405×10^7	3,107	6,546
0	2,000,000	3,495	140.658×10^7	3,599	7,094
0	3,000,000	3,552	168.506×10^7	4,312	7,864
0	4,000,000	3,581	188.114×10^7	4,814	8,395
300,000	1,000,000	595	37.646×10^7	963	1,558
500,000	1,000,000	365	25.686×10^7	657	1,022
1,000,000	1,000,000	160	12.711×10^7	325	485
2,000,000	1,000,000	57	4.963×10^7	127	184
3,000,000	1,000,000	28	2.561×10^7	66	94

EXHIBIT III

INCREASED LIMITS FACTORS INCLUDING RISK CHARGE

	Increased Limits Factors	
Policy Limit	Expected Value	Risk Adjusted
25,000 (basic limit)	1.000	1.000
50,000	1.419	1.472
100,000	1.872	2.062
300,000	2.526	3.254
500,000	2.762	3.926
1,000,000	2.996	4.952
1,500,000	3.090	5.600
2,000,000	3.140	6.068
3,000,000	3.191	6.727
4,000,000	3.217	7.181

The risk reduction can be demonstrated mathematically by comparing the pure premium variance with and without layering. First consider one insurer providing high limits coverage. With policy limit equal to k, his cost function is given by $g(x;k)$. The variance in the pure premium without layering, $Var[y_0]$, from equation (27) would be:

$$Var[y_0] = E[n] \cdot Var[g(x;k)] + Var[n] \cdot E[g(x;k)]^2 \qquad (33)$$

Next suppose the same coverage is layered between two insurers (or an insurer and reinsurer) where the bottom layer has limit r. The cost functions for the two layers are as follows:

First layer: $g(x;r)$

Second layer: $h(x;r,j)$, where $j = k - r$

Since $g(x;k) = g(x;r) + h(x;r,j)$ we see that the expected value pure premiums for the two layers sum to the non-layered pure premium.

$$E[n] \cdot E[g(x;k)] = E[n] \cdot E[g(x;r)] + E[n] \cdot E[h(x;r,j)]$$

Again using equation (27), the pure premium variances for the two individual carriers[20] are:

[20] The carriers must be entirely separate entities operating from different capital bases. Layering coverage between subsidiaries or affiliates will not produce the desired risk reduction.

First Layer:

$$\mathrm{Var}[y_1] = E[n] \cdot \mathrm{Var}[g(x;r)] + \mathrm{Var}[n] \cdot E[g(x;r)]^2 \tag{34}$$

Second Layer:

$$\mathrm{Var}[y_2] = E[n] \cdot \mathrm{Var}[h(x;r,j)] + \mathrm{Var}[n] \cdot E[h(x;r,j)]^2 \tag{35}$$

For the purpose of comparison, the pure premium variance without layering is needed in terms of the two layers. We can express both $E[g(x;k)]^2$ and $\mathrm{Var}[g(x;k)]$ in terms of $g(x;r)$ and $h(x;r,j)$ as follows:

$$\begin{aligned} E[g(x;k)]^2 &= (E[g(x;r)] + E[h(x;r,j)])^2 \\ &= E[g(x;r)]^2 + 2 \cdot E[g(x;r)] \cdot E[h(x;r,j)] \\ &\quad + E[h(x;r,j)]^2 \end{aligned} \tag{36}$$

and

$$\begin{aligned} \mathrm{Var}[g(x;k)] &= \mathrm{Var}[g(x;r)] + \mathrm{Var}[h(x;r,j)] \\ &\quad + 2 \cdot \mathrm{Cov}[g(x;r),h(x;r,j)] \end{aligned} \tag{37}$$

where

$$\begin{aligned} \mathrm{Cov}[g(x;r),h(x;r,j)] &= E[g(x;r) \cdot h(x;r,j)] \\ &\quad - E[g(x;r)] \cdot E[h(x;r,j)] \end{aligned} \tag{38}$$

However, since

$$\begin{aligned} g(x;r) \cdot h(x;r,j) &= \begin{cases} 0, & 0 \leq x \leq r \\ r(x-r), & r < x < k,\ k = j + r \\ r \cdot j, & k \leq x \end{cases} \\ &= r \cdot h(x;r,j) \end{aligned} \tag{39}$$

we find that

$$\mathrm{Cov}[g(x;r),h(x;r,j)] = (r - E[g(x;r)]) \cdot E[h(x;r,j)] > 0 \tag{40}$$

and therefore, equation (37) becomes

$$\begin{aligned} \mathrm{Var}[g(x;k)] &= \mathrm{Var}[g(x;r)] + \mathrm{Var}[h(x;r,j)] \\ &\quad + 2 \cdot E[h(x;r,j)] \cdot (r - E[g(x;r)]) \end{aligned} \tag{41}$$

Substituting equations (36) and (41) into equation (33), we see that the pure premium variance without layering exceeds the variance with layering by an amount, $R(r,j)$, given by

$$\begin{aligned} R(r,j) &= \mathrm{Var}[y_0] - \mathrm{Var}[y_1] - \mathrm{Var}[y_2] \\ &= 2 \cdot E[n] \cdot E[h(x;r,j)] \cdot (r - E[g(x;r)]) \\ &\quad + 2 \cdot \mathrm{Var}[n] \cdot E[g(x;r)] \cdot E[h(x;r,j)] \\ &= 2 \cdot E[h(x;r,j)] \cdot (r \cdot E[n] + E[g(x;r)] \cdot (\mathrm{Var}[n] - E[n] \end{aligned} \tag{42}$$

If we assume as before that $E[n] \simeq \mathrm{Var}[n]$, then equation (42) simplifies to

$$R(r,j) = 2 \cdot r \cdot E[n] \cdot E[h(x;r,j)] \tag{43}$$

which is just twice the retention times the expected value pure premium of the second layer. This is the reduction in the variance, consequently to get the reduction in the risk charge we multiply by the λ factor[21]. Since there is no reduction in expected value pure premium by layering, the dollar reduction in risk charge is equal to the dollar reduction in premium by layering.

Exhibit IV shows that this reduction by layering can be substantial. The examples in Exhibit IV use the same assumptions as in Exhibit II.

3. APPLICATIONS

The principal applications of the pricing model described in this paper require knowledge of a specific loss severity distribution. The only exception to this is the consistency test. Of course, the development of a severity distribution from experience data is not without difficulties. Special data gathering techniques are required to produce individual losses ranked by size of loss. Loss development also poses certain problems in working with severity distributions. Some approaches to treating these difficulties are outlined below.

One approach to compiling an empirical size of loss distribution is to use all reported claims from a few recent accident (or policy) years. It is very likely that this distribution of immature claim values will change considerably as these claims develop. Some claims with high estimates may be settled for a small amount or adjudicated as no liability. Others which seem unmeritorious initially may ultimately result in very large awards or settlements. Consequently, each open claim has a probability distribution

[21] It is assumed that the same λ factor is appropriate for both carriers.

EXHIBIT IV

RISK REDUCTION BY LAYERING

Total Coverage	First Layer Limit (Second Layer Retention)	Second Layer Expected Value Pure Premium	Premium (before expenses) without layering	Premium Reduction with layering
1,300,000	300,000	595	6,276	914 (14.6%)
1,500,000	500,000	365	6,547	934 (14.3%)
2,000,000	1,000,000	160	7,094	819 (11.5%)
2,000,000	500,000	421	7,094	1,077 (15.2%)
3,000,000	1,000,000	217	7,864	1,111 (14.1%)
3,000,000	2,000,000	57	7,864	583 (7.4%)
4,000,000	1,000,000	246	8,395	1,259 (15.0%)
4,000,000	2,000,000	86	8,395	880 (10.5%)

of its ultimate value. Hachemeister[22] describes a technique for estimating such loss development distributions conditioned on the age of the claim and its estimated value. The method can also be used to estimate a distribution for unreported and reopened claims. The actual procedure for adjusting a severity distribution for loss development using the Hachemeister technique will be left to the interested reader.

There are many other problems in dealing with empirical distributions. Data on individual losses usually come from different policies with different policy limits, causing a bias in the distribution. The credibility of the distribution, especially at the high end, is another area for concern. The use of a theoretical distribution (lognormal, Weibull, etc.) can help considerably in dealing with these problems. One can fit a theoretical distribution to the empirical one and use the fitted distribution for pricing. In a recent paper, Finger[23] fitted a lognormal distribution to medical malpractice data using an empirical procedure based on the particular properties of the lognormal parameters.

In the absence of reliable empirical data it is not unreasonable to assume a theoretical severity distribution to use for pricing. The selection of a particular distribution can be made on the basis of the analytical properties of a distribution such as the mean, variance, coefficient of variation, skewness, etc. Even if the selection of a distribution were based on a subjective evaluation of the resulting increased limits factors, this would be an improvement over selecting factors directly without regard to the loss severity implications.

If a loss severity distribution is available from experience data or by assumption, then the formulas presented in this paper have the following applications.

1) The computation of expected value increased limits factors.

2) The adjustment of the severity distribution and the increased limits factors for trend, where trend is assumed to have the same multiplicative effect on each loss size.

3) The computation of risk charges by limit of liability.

[22] C. A. Hachemeister, "Breaking Down the Loss Reserve Process," presented at the CAS Loss Reserve Symposium (September, 1976).

[23] R. J. Finger, "Estimating Pure Premiums by Layer—An Approach", *PCAS* LXIII (1976), p. 34.

4) The calculation of the reduction in risk charge afforded by "layering" coverage.

5) The computation of the expected value pure premium and risk charge for excess of loss coverage.

If increased limits factors are computed by means other than those described in this paper, it is possible that such factors will produce inconsistencies in the pricing of increased limits and excess of loss coverage. The consistency test described in this paper can be used to evaluate a set of increased limits factors and point out the particular factors that are inconsistent with the rest.

4. RELATED TOPICS

The following are other areas of insurance pricing where the theories developed in this paper, particularly the consistency test, can be applied.

Leveraged Effect of Inflation

The concept of the leveraged effect of inflation is discussed thoroughly by Ferguson[24]. This concept can be expressed analytically in terms of what has been defined in this paper. What we are looking for is the change in the expected value pure premium for excess of loss coverage. Assuming an inflationary trend that has the same multiplicative effect on each size of loss, as defined by $\alpha_1(x) = ax$, the leveraging effect is controlled by the retention. The following formulas can be useful in analyzing the effects of inflation for excess of loss coverage.

1. Average increase in losses with fixed upper limit.

$$\frac{E[g(x';k)]}{E[g(x;k)]} = a \cdot \frac{E[g(x;k/a)]}{E[g(x;k)]}$$

$$= a \cdot \frac{I(k/a)}{I(k)}$$

2. Average increase in excess losses with fixed upper limit.

$$\frac{E[h(x';r,j)]}{E[h(x;r,j)]} = \frac{E[g(x';s)] - E[g(x';r)]}{E[g(x;s)] - E[g(x;r)]},\ s = r + j$$

$$= a \cdot \frac{E[g(x;s/a)] - E[g(x;r/a)]}{E[g(x;s)] - E[g(x;r)]}$$

$$= a \cdot \frac{I(s/a) - I(r/a)}{I(s) - I(r)}$$

[24] Ferguson, *op. cit.*

3. Average increase in excess losses with no upper limit.

$$\frac{E[x'] - E[g(x';r)]}{E[x] - E[g(x;r)]} = a \cdot \frac{E[x] - E[g(x;r/a)]}{E[x] - E[g(x;r)]}$$

The expected value increased limits factors that were computed in the previous examples from the lognormal distribution can be adjusted for inflation using equation (18). Exhibit V shows the effects of inflation for various retentions given an overall inflation of 9% ($a = 1.09$).

The examples in this exhibit indicate somewhat small leveraged effects. This is primarily the result of the specific severity distribution used. Some other distribution could exhibit significantly higher leveraged effects. However, the author has not attempted to study this further. The conclusion from this is that while inflation may cause very serious pricing problems for excess of loss coverage, such problems may not always be as severe as they first appear.

Aggregate Limits

A maximum limitation on the total amount of insured losses for all accidents/occurrences is generally referred to as an aggregate limit. Such a limit usually applies for a one year policy period and can be used in conjunction with a per accident/occurrence limit. Aggregate limits are intended to restrict the exposure to multiple large losses or an excessive frequency of losses. The theoretical pricing structure of aggregate limits and aggregate excess coverage (excess of aggregate limits, also known as stop-loss reinsurance) will not be discussed in this paper. However, the theory does permit the application of a consistency test. The test described previously can be used by analyzing the marginal rate per $1,000 of accident/occurrence limit keeping the aggregate limit constant and also the marginal rate per $1,000 of aggregate limit keeping the accident/occurrence limit constant. Thus, if increased limits factors are displayed in a table where the columns indicate an accident/occurrence limit and the rows indicate an aggregate limit, then each row and each column of increased limits factors should be tested separately in the same manner as a per accident table of factors.

EXHIBIT V

LEVERAGED EFFECT OF INFLATION
(OVERALL INFLATION OF 9%)

Retention	Increased Limits Factors: before inflation adjustment	Increased Limits Factors: after inflation adjustment	Average Increase in Losses Limited to Retention	Leveraged Effect: Average Increase in Losses in Excess of Retention: Limited to $1,000,000	Leveraged Effect: Average Increase in Losses in Excess of Retention: Unlimited
25,000	1.000	1.000	3.8%	10.3%	11.3%
50,000	1.419	1.432	4.8	11.2	12.2
100,000	1.872	1.905	5.7	12.2	13.4
300,000	2.526	2.604	7.1	14.2	15.5
500,000	2.762	2.862	7.6	15.2	16.7
1,000,000	2.996	3.121	8.1	16.7	18.3
2,000,000	3.140	3.282	8.5	18.4	20.1

Per Person, Per Accident Limits

Liability coverage can also be defined by dual or "split" limits. In general, such limits provide for a maximum amount of insured loss for each person injured in an accident in addition to a maximum amount for each accident. To extend the pricing model to this type of coverage would require the introduction of another random variable. This random variable would represent the number of persons injured in an accident. It would also be necessary to change the loss severity variable to a per person basis rather than per accident.

Obviously such changes would complicate the model considerably unless further assumptions are made. It is not clear what advantages split limits have over the single per accident limit other than to further restrict coverage. The elimination of split limits coverage would aid greatly in the pricing of increased limits, both in the evaluation of experience data and in the mathematical model.

The application of the consistency test to evaluate split limits increased limits factors is similar to aggregate limits. Given a table of factors with the columns indicated the per person limit and the rows indicating the per accident limit, the test would be applied to each row and to each column separately.

Property Insurance — Coinsurance Pricing

All aspects of coinsurance including pricing are discussed very thoroughly by Head[25]. However, in discussing the relationships between the rates for different coinsurance requirements Head requires that no premium reversals[26] exist between two coinsurance requirements and that coinsurance rates should decrease at a declining rate with added coverage. The consistency test can be adapted to coinsurance pricing and provide a further check on coinsurance rates.

---

[25] G. L. Head, *Insurance to Value*, Irwin (1971).

[26] *Ibid.*, p. 116.

Consider the following set of factors which relate the rates for various coinsurance requirements to the 80% coinsurance rate. These factors have no premium reversals and produce rates that decrease at a decreasing rate for increasing coinsurance requirements.

Coinsurance Requirement	Coinsurance Factor
100%	.90
90	.94
80	1.00
70	1.07
60	1.15
50	1.28
40	1.50

Next suppose a full value amount of $100,000 and an 80% coinsurance rate of $1.00 per $100 of insurance. The amount of insurance and the premium for the various coinsurance requirements would be:

Coinsurance Requirement			Marginal Premium[28]
Percent	Amount of Insurance	Premium[27]	per $1,000 of Coverage
100%	$100,000	$900	$5.40*
90	90,000	846	4.60
80	80,000	800	5.10
70	70,000	749	5.90
60	60,000	690	5.00*
50	50,000	640	4.00*
40	40,000	600	—

For this pricing to be consistent, the marginal premium per $1,000 of coverage (amount of insurance) should decrease as the coverage increases. This example shows inconsistencies for coinsurance requirements of 100%,

[27] $$\text{Premium} = \text{Coinsurance Percentage} \times \frac{\$100{,}000 \text{ (amount of full value)}}{\$100 \text{ (exposure base)}} \times \text{Coinsurance Factor} \times \$1.00 \text{ (80\% coinsurance rate)}.$$

[28] The Marginal Premium is the premium difference between the given amount of insurance and the next lower amount of insurance, divided by the difference in the amounts of insurance.

60%, and 50% as indicated by *. It is important to note that this result is caused solely by the coinsurance factors, i.e., the same inconsistencies will be indicated regardless of the full value amount or the 80% coinsurance rate. The coinsurance factors used in this example are similar to factors in actual use at the time this paper was written.

5. CONCLUSION

Through the use of a mathematical model, the pricing of increased limits and excess of loss coverage can be analyzed both in theory and in practical application. The model presented in this paper gives a mathematical statement of the pricing problem. The complete solution to this problem requires actual data, judgment and some further study.

The key element to the model is the size of loss distribution. Unfortunately, there is not very extensive knowledge about such distributions, either empirical or theoretical. Techniques must be developed and refined for the collection and evaluation of size of loss data. Moreover, new theoretical distributions must be found that can simulate the many possible types of severity distributions. The treatment of loss adjustment expense is also very important because these expenses are related to the existence and severity of a loss. This relationship must be defined and fit into the model in order to create increased limits factors for actual use.

Other areas where research is needed are a more realistic approach to adjusting for the effects of inflation by size of loss, the detection and implications of anti-selection, the classification of insureds into homogeneous groups with similar severity characteristics, the development of a risk charge for parameter risk, and a pricing model for split and aggregate limits.

Acknowledgements

The lognormal parameters used in the examples were developed by Gary Patrik from medical malpractice closed claim data. Mr. Patrik's work in this area also aided the author in defining the mathematical pricing model. J. Ernest Hansen contributed the original thinking of risk reduction by layering in mathematical terms. The impetus and motivation for the research behind this article was due to Charles Walter Stewart.

APPENDIX

The Lognormal Distribution

For the purpose of this paper, an evaluation of the following integrals is required for various values of x, where f(t) is the lognormal probability density function.

$$\int_0^x f(t)\,dt$$

$$\int_0^x t \cdot f(t)\,dt$$

$$\int_0^x t^2 \cdot f(t)\,dt$$

These three integrals can be evaluated by means of a transformation to values of the normal cumulative distribution function, $\Phi(x)$.

From the definition of the lognormal distribution[29], we know that

$$f(x) = \frac{d}{dx}\,\Phi\left(\frac{\ln x - \mu}{\sigma}\right),\ \Phi(x) = \int_0^x \frac{1}{\sqrt{2\pi}} \cdot \exp(-\tfrac{1}{2}t^2)\,dt$$

$$= \frac{1}{\sqrt{2\pi}\cdot\sigma\cdot x} \cdot \exp\left\{-\tfrac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right\}$$

Consequently,

$$\int_0^x f(t)\,dt = \Phi\left(\frac{\ln x - \mu}{\sigma}\right)$$

The derivation of the formulas for the remaining two integrals follows.

$$\int_0^x t \cdot f(t)\,dt = \int_0^x \frac{1}{\sqrt{2\pi}\cdot\sigma} \exp\left\{-\tfrac{1}{2}\left(\frac{\ln t - \mu}{\sigma}\right)^2\right\} dt$$

$$= \int_{-\infty}^{\frac{\ln x - \mu}{\sigma}} \frac{1}{\sqrt{2\pi}} \exp(\sigma y + \mu) \cdot \exp(-\tfrac{1}{2}y^2)\,dy\,,$$

$$\left\{\begin{aligned} y &= \frac{\ln t - \mu}{\sigma} \\ dy &= \frac{1}{\sigma t}\,dt \\ t &= \exp(\sigma y + \mu) \end{aligned}\right.$$

[29] J. Aitchinson and J. A. C. Brown, *The Lognormal Distribution*, Cambridge University Press (1957).

$$= \frac{1}{\sqrt{2\pi}} \exp(\tfrac{1}{2}\sigma^2 + \mu) \cdot \int_{-\infty}^{\frac{\ln x - \mu}{\sigma}} \exp(-\tfrac{1}{2}(y - \sigma)^2)\, dy$$

$$= \frac{1}{\sqrt{2\pi}} \exp(\tfrac{1}{2}\sigma^2 + \mu) \cdot \int_{-\infty}^{-\sigma + \frac{\ln x - \mu}{\sigma}} \exp(-\tfrac{1}{2}z^2)\, dz , \begin{cases} z = y - \sigma \\ dz = dy \end{cases}$$

$$= \exp(\tfrac{1}{2}\sigma^2 + \mu) \cdot \Phi\left(-\sigma + \frac{\ln x - \mu}{\sigma}\right)$$

$$\int_0^x t^2 \cdot f(t)\, dt = \int_0^x \frac{1}{\sqrt{2\pi} \cdot \sigma} t \cdot \exp\left\{-\tfrac{1}{2}\left(\frac{\ln t - \mu}{\sigma}\right)^2\right\} dt$$

$$= \int_{-\infty}^{\frac{\ln x - \mu}{\sigma}} \frac{1}{\sqrt{2\pi}} \exp(2\sigma y + 2\mu) \cdot \exp(-\tfrac{1}{2}y^2)\, dy ,$$

$$\begin{cases} y = \dfrac{\ln t - \mu}{\sigma} \\ dy = \dfrac{1}{\sigma t}\, dt \\ t = \exp(\sigma y + \mu) \end{cases}$$

$$= \frac{1}{\sqrt{2\pi}} \exp(2\sigma^2 + 2\mu) \cdot \int_{-\infty}^{\frac{\ln x - \mu}{\sigma}} \exp(-\tfrac{1}{2}(y - 2\sigma)^2)\, dy$$

$$= \frac{1}{\sqrt{2\pi}} \exp(2\sigma^2 + 2\mu) \cdot \int_{-\infty}^{-2\sigma + \frac{\ln x - \mu}{\sigma}} \exp(-\tfrac{1}{2}z^2)\, dz ,$$

$$\begin{cases} z = y - 2\sigma \\ dz = dy \end{cases}$$

$$= \exp(2\sigma^2 + 2\mu) \cdot \Phi\left(-2\sigma + \frac{\ln x - \mu}{\sigma}\right)$$

ON THE THEORY OF INCREASED LIMITS AND EXCESS OF LOSS PRICING

ROBERT S. MICCOLIS

DISCUSSION BY SHELDON ROSENBERG

Bob Miccolis has presented a paper which discusses the mathematical theory underlying many aspects of increased limits ratemaking. Committees and staff of Insurance Services Office have put much of this theory into practice in reviewing increased limits loss experience. In so doing, practical problems have arisen and interim solutions developed pending further study. Some of these problems and solutions comprise this discussion.

CONSISTENCY IN INCREASED LIMITS TABLES

Extension to 2 Dimensional Tables

The Miccolis test for consistency is that "the marginal premium per \$1000 of coverage should decrease as the limit of coverage increases." As discussed later in the paper, this consistency test can be extended to two-dimensional tables as well. That is, consider an increased limits table that appears as follows:

Aggregate Limit (in thousands)	Occurrence Limit (in thousands)			
	25	50	100	250
25	1.00			
50	1.50	1.70		
100	1.80	2.03	2.50	
250	2.00	2.25	2.80	3.20

This table must now "pass" the consistency test for each occurrence limit (down each column), as well as for each aggregate limit (across each row). The table passes this test for the \$25,000 occurrence limit because

$\frac{.50}{25} > \frac{.30}{50} > \frac{.20}{150}$. That is, marginal premiums per \$1000 of aggregate coverage are decreasing, as the occurrence limit is held fixed. The table fails to pass this test for the \$250,000 aggregate limit because

$\frac{.25}{25} < \frac{.55}{50}$.

There is yet another consistency test that can be performed on a two-dimensional table. This test has in fact been used in conjunction with the previously discussed test in developing occurrence/aggregate and claim/accident tables at Insurance Services Office.

The test itself can best be described by examining the hypothetical table drawn up above. Consider an insured who is considering switching from a $25,000 occurrence limit to a $50,000 occurrence limit. The question becomes the following: if one assumes that his aggregate limit remains constant, then for which aggregate limit will his decision to change occurrence limits give him the greatest vs. the smallest increases in coverage?

His increase in coverage will be most significant when the aggregate is highest. To see this, consider the extreme situation of having no aggregate at all (i.e. infinite aggregate coverage). In this case, the insured's switching from a $25,000 occurrence limit to a $50,000 occurrence limit will give him a potential increase of $25,000 for each and every occurrence, since no aggregate can ever be applied to stop payments at a certain amount. The other extreme is for an insured who has a $25,000 aggregate and is contemplating a switch from $25,000 to $50,000 in his occurrence coverage. Of course, he gets nothing in additional coverage because the $25,000 aggregate acts as a cap on his occurrence coverage as well.

The pattern thus emerges. For any two occurrence limits in an Increased Limits Table, the differences in the factors must not decrease, as the aggregates grow. This follows from the above discussion because increasing aggregates imply increasing differences in coverage between the given pair of occurrence limits. The way to reflect this in increased limits tables is to make sure that differences between the occurrence limit factors increase (or at least do not decrease) as aggregates increase. Note that our hypothetical table passes this test for the $25,000 and $50,000 occurrence limits because $1.70-1.50 \leq 2.03-1.80 \leq 2.25-2.00$.

This argument can of course be extended to test the differences in any pair of aggregate limits, for all occurrence limits. In comparing the "100" and "250" aggregate limits for the "25", "50", and "100" occurrence limits, the table again passes the test since $2.00-1.80 \leq 2.25-2.03 \leq 2.80-2.50$.

Anti-selection and Consistency

Miccolis notes that while "there can be anti-selection . . . this should not restrict the general applicability of the consistency test".

If one assumes that there is no anti-selection or that its effects are minimal, then one can assume that the inherent underlying severity distribution is identical for all insureds. Thus increased limits factors can be developed using one smooth curve to represent all policy limits. The factors resulting from this curve will then automatically pass the consistency test as described by Miccolis because the curve will produce decreasing severity values by layer. However, if we wish to reflect anti-selection, then each policy limit must have a curve fit to its own loss data.

Increased limits factors for the various policy limits will now result from the various loss curves, each of which is unrelated to any other one. In such a situation, marginal premium per $1000 of additional coverage can increase from one policy limit to the next.

Anti-selection can take two forms. One often encountered is adverse selection, in which purchasing higher limits is associated with adverse loss experience. This can occur for two different reasons. Firstly, insureds who can expect higher loss potential could be more inclined to purchase higher limits. Secondly, liability law suits or settlements may be influenced by the policy limit. Thus the same accident may result in higher losses for the insured that purchased higher policy limits.

The mirror image of "adverse" selection might be labeled "favorable" selection, in which the insureds with the highest policy limits show the best loss experience. There are two reasons why this might occur. Firstly, financially secure insureds may be better risks. Yet since they have more assets to protect, they will be inclined to purchase higher limits. Secondly, insurance companies, knowing that these are the better risks, would be more willing to insure them at higher limits.

While anti-selection can affect increased limits factors, it does not always do so. If anti-selection produces differences by policy limit in the relationships among indemnity severity values, then increased limits factors are affected. However, it may well be that anti-selection produces increasing or decreasing basic limits severities by policy limit, but maintains the same proportionate relationship in severities for all other policy limit cut-offs as well. In this case, increased limits factors would not be affected by anti-selection. Consider the following two examples:

Example A: Anti-selection does affect increased limits factors

Average Indemnity severity resulting from purchasers of:

at "cut-offs" of	POLICY LIMIT $50,000	POLICY LIMIT $100,000
$ 25,000	$ 5,000	$ 7,000
50,000	7,500	8,500
100,000	10,000	11,000

Based on these results, increased limits factors (I.L.F.) could be calculated as follows:

Policy Limit	I.L.F. not reflecting anti-selection	I.L.F. reflecting anti-selection
$ 25,000	1.00	1.00
50,000	1.33 (i.e. $ 8,000/6,000)	1.50 ($ 7,500/5,000)
100,000	1.75 (i.e. $10,500/6,000)	1.57 ($11,000/7,000)

Example B: Anti-selection does not affect increased limits factors

Indemnity severity resulting from purchasers of:

at "cut-offs" of	POLICY LIMIT $50,000	POLICY LIMIT $100,000
$ 25,000	$ 5,000	$ 6,000
50,000	7,500	9,000
100,000	10,000	12,000

Based on these results, increased limits factors could be calculated as follows:

Policy Limit	I.L.F. not reflecting anti-selection	I.L.F. reflecting anti-selection
$ 25,000	1.00	1.00
50,000	1.50 (i.e. $ 8,250/5,500)	1.50 ($ 7,500/5,000)
100,000	2.00 (i.e. $11,000/5,500)	2.00 ($12,000/6,000)

In each example, the basic limit is assumed to be $25,000. Indemnity severities are calculated based on smooth loss curves for each policy limit separately, with the severity values "cut off" (i.e. limited) at various points representing key policy limits in an increased limits table. Note that because loss curves extend for infinite loss values, severity values could be calculated for "cut-offs" higher than the policy limit purchased. Increased limits factors not reflecting anti-selection are calculated by averaging the severity values for both policy limits (this assumes equal weights for both policy limits), and dividing these resulting increased limits average severities by the basic limit average severity. To reflect anti-selection, the severity value at a "cut-off" is equal to the value for that policy limit. This is true for the increased limit in question, as well as the basic limit. Note that in Example A, the increased limits factors differ depending on whether anti-selection was reflected, while in Example B, the factors are identical with or without reflecting anti-selection.

LOSS DEVELOPMENT

As Miccolis states in his paper, "it is very likely that [a] distribution of immature claim values will change considerably as these claims develop". In general, this varying development will exhibit an upward pattern over intervals of claim size. Displayed in Table 1 are the mean and standard deviation for one year of data fitted to a log normal distribution over three different evaluation periods for Physicians, Surgeons, and Hospitals. The increase over time in the mean and standard deviation is evidence of the upward pattern of development. An important reason for this is that IBNR claims, when they eventually get reported, tend to wind up in the higher claim size intervals. The I.S.O. closed claim surveys, for both Malpractice and Product Liability, demonstrate that the longer the time interval between the occurrence of a claim and the reporting of that claim, the higher the average size of the claim.

At Insurance Services Office, loss development factors have been calculated by claim size interval by comparing "theoretical" (the result of smooth loss curves) claim counts per interval as of various evaluation levels. These factors have exhibited the generally upward pattern by claim size interval referred to above.

TABLE 1

POLICY YEAR 1972

Evaluated as of	Physicians		Surgeons		Hospitals	
27 months	Mean	12,421	Mean	13,177	Mean	8,772
	S.D.	58,126	S.D.	53,467	S.D.	42,526
39 months	Mean	16,972	Mean	19,384	Mean	14,364
	S.D.	75,410	S.D.	77,923	S.D.	85,968
51 months	Mean	22,974	Mean	24,784	Mean	19,072
	S.D.	126,314	S.D.	115,847	S.D.	148,045

THE USE OF A THEORETICAL CURVE AS A MODEL FOR LOSS DISTRIBUTION

As Miccolis notes, there are many problems in dealing with empirical distributions.

In addition to those mentioned in his paper two other properties of the claim size distribution tend to bias the results. Firstly, the existence of "cluster points" (intervals where the number of claims drastically rises and immediately drops) magnify the discontinuity between intervals. Usually these cluster points appear at intervals which contain a round number such as $25,000, $50,000 or $100,000. As an example see Table 2 which displays Surgeons data of Companies reporting to ISO for Policy Year 1972 evaluated as of March 31, 1975. This clustering phenomenon may result in a poor fit when any continuous curve is applied to the data. The existence of cluster points should not pre-empt the use of a theoretical distribution, however, since their presence may be artificial for the following reasons:

(1) Policy limits will truncate losses to the limit of an insured's policy. This is not the sole reason for cluster points however, since such points were present for amounts such as $25,000 and $50,000 even when the underlying data corresponded to a policy limit of $100,000. To illustrate this point, consider Table 3 which is Physicians data of companies reporting to ISO for policy year 1974 evaluated as of March 31, 1976. This data contains only those losses which were incurred on policies whose limit was $100,000, yet clusters appear at $5,000, $10,000, $15,000, $20,000, $25,000 and $50,000.

(2) At early evaluation points when the raw data is relatively immature, most losses are still outstanding. These claims are rarely reserved for amounts other than round figures.

A second property of the raw data included in the claim size distribution is that gaps are present for certain intervals where no claims appear. Fitting a curve alleviates this problem because of its smooth nature.

ISO has recognized these weaknesses in the empirical distributions and consequently has chosen to fit a theoretical distribution to the data and use this curve in computing increased limits factor. Initially a log-normal distribution was considered to be an appropriate representation of the data; the fitting problem at cluster points was not considered crucial, as mentioned above. The curve was fit by solving the Maximum Likelihood equations for the parameters μ and σ^2.

One other advantage of using a theoretical distribution to represent the data is that it facilitated the computation of variance at each policy limit. This variance was then used as a basis for risk adjustments.

TREND

When discussing the issue of trending loss distributions, Miccolis first considers the case where trend affects all claims in the same way, i.e. each loss is increased by the same multiplicative factor. This assumption leads to the equation:

$$(1) \qquad \mathcal{F}(x) = F(x/a)$$

where $\mathcal{F}(x)$ is the trended cumulative distribution, "a" is the annual trend factor, and $F(x)$ is the untrended cumulative distribution. If one differentiates both sides of this equation the result is:

$$f(x) = \mathcal{F}'(x) = F'(x/a) - 1/a\, f(x/a)$$

The trended probability distribution function $f(x)$ can thus be defined in terms of $f(x)$.

It is interesting to note that if x is lognormally distributed with parameters μ and σ^2 before trend, then x will also be lognormally distributed with

TABLE 2

INSURANCE SERVICES OFFICE

SURGEONS PROFESSIONAL LIABILITY
CLAIMS SIZE DISTRIBUTION

POLICY YEAR 1972*

Claim Size Intervals	Claims	Loss	Claim Size Intervals	Claims	Loss
0- 250	192	16,321	20,001- 21,000	10	203,022
251- 500	122	50,918	21,001- 22,000	1	22,000
501- 1,000	226	205,580	22,001- 23,000	13	293,007
1,001- 2,000	272	444,820	23,001- 24,000	3	71,290
2,001- 3,000	323	838,154	24,001- 25,000	104	2,548,951
3,001- 4,000	150	545,173	25,001- 30,000	45	1,256,077
4,001- 5,000	425	2,076,934	30,001- 35,000	41	1,388,629
5,001- 6,000	53	303,156	35,001- 40,000	27	1,037,669
6,001- 7,000	60	399,909	40,001- 45,000	15	659,608
7,001- 8,000	116	862,936	45,001- 50,000	67	3,345,275
8,001- 9,000	29	254,676	50,001- 55,000	14	731,821
9,001- 10,000	190	1,875,453	55,001- 60,000	10	600,000
10,001- 11,000	33	345,854	60,001- 65,000	13	822,507
11,001- 12,000	31	368,930	65,001- 70,000	7	486,000
12,001- 13,000	34	427,736	70,001- 75,000	26	1,940,495
13,001- 14,000	9	123,698	75,001- 80,000	9	687,631
14,001- 15,000	130	1,916,898	80,001- 85,000	5	425,000
15,001- 16,000	19	294,270	85,001- 90,000	2	180,000
16,001- 17,000	11	181,967	90,001- 95,000	1	90,540
17,001- 18,000	13	230,802	95,001-100,000	61	6,091,246
18,001- 19,000	7	130,650	100,001-110,000	8	835,605
19,001- 20,000	102	2,039,998	110,001-120,000	2	239,000

TABLE 2 (Cont.)

INSURANCE SERVICES OFFICE
SURGEONS PROFESSIONAL LIABILITY
CLAIMS SIZE DISTRIBUTION
POLICY YEAR 1972*

Claim Size Intervals	Claims	Loss	Claim Size Intervals	Claims	Loss
120,001- 130,000			340,001- 350,000	1	350,000
130,001- 140,000	1	140,000	350,001- 360,000		
140,001- 150,000	5	741,467	360,001- 370,000		
150,001- 160,000			370,001- 380,000		
160,001- 170,000			380,001- 390,000	1	390,000
170,001- 180,000			390,001- 400,000		
180,001- 190,000			400,001- 410,000		
190,001- 200,000	2	400,000	410,001- 420,000		
200,001- 210,000	2	413,750	420,001- 430,000		
210,001- 220,000	1	211,000	430,001- 440,000		
220,001- 230,000			440,001- 450,000		
230,001- 240,000			450,001- 460,000		
240,001- 250,000	2	500,000	460,001- 470,000		
250,001- 260,000			470,001- 480,000		
260,001- 270,000			480,001- 490,000		
270,001- 280,000			490,001- 500,000	1	500,000
280,001- 290,000			500,001- 600,000		
290,001- 300,000	1	300,000	600,001- 700,000		
300,001- 310,000			700,001- 800,000		
310,001- 320,000			800,001- 900,000		
320,001- 330,000			900,001-1,000,000		
330,001- 340,000			1,000,001+		
			TOTAL	3,048	41,836,423

*Evaluated as of 3/31/75

TABLE 3

INSURANCE SERVICES OFFICE

PHYSICIANS PROFESSIONAL LIABILITY
CLAIMS SIZE DISTRIBUTION

POLICY YEAR 1974*
POLICY LIMIT 100/300

Claim Size Intervals	Claims	Loss	Claim Size Intervals	Claims	Loss
0- 250	54	7,324	18,001- 19,000		
251- 500	57	24,980	19,001- 20,000	27	540,000
501- 1,000	123	117,448	20,001- 21,000	1	20,500
1,001- 2,000	90	147,741	21,001- 22,000	2	43,565
2,001- 3,000	108	281,101	22,001- 23,000		
3,001- 4,000	41	149,235	23,001- 24,000	1	24,000
4,001- 5,000	177	876,986	24,001- 25,000	24	600,000
5,001- 6,000	12	70,289	25,001- 30,000	7	207,500
6,001- 7,000	10	67,462	30,001- 35,000	8	280,000
7,001- 8,000	39	295,795	35,001- 40,000	3	118,736
8,001- 9,000	5	43,500	40,001- 45,000	1	45,000
9,001- 10,000	68	680,000	45,001- 50,000	17	850,000
10,001- 11,000	3	31,780	50,001- 55,000		
11,001- 12,000	6	72,000	55,001- 60,000	3	180,000
12,001- 13,000	3	37,500	60,001- 65,000	1	65,000
13,001- 14,000	2	27,333	65,001- 70,000	2	140,000
14,001- 15,000	29	434,620	70,001- 75,000	8	600,000
15,001- 16,000	1	15,804	75,001- 80,000	1	79,020
16,001- 17,000	1	17,000	80,001- 85,000	1	85,000
17,001- 18,000	1	17,500	85,001- 90,000		
			90,001- 95,000		
			95,001-100,000	17	1,700,000
			TOTAL	954	8,993,719

*Evaluated as of 3/31/76

parameters $\mu + \ln a$ and σ^2 after trend. To see that this follows, consider equation (1) and the fact that:

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2}$$

$$\text{Then } f(x) = 1/a\, f(x/a) = \frac{1}{a}\,\frac{1}{(x/a)\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{\ln(x/a) - \mu}{\sigma}\right)^2}$$

$$= \frac{1}{x\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{\ln x - (\mu + \ln a)}{\sigma}\right)^2}$$

i.e. $f(x)$ is a lognormal distribution with parameters $\mu + \ln a$ and σ^x.

Once again it is important to keep in mind that this argument is valid only if one assumes that the effects of inflation are so that each loss is multiplied by the same multiplicative factor. It is clear though, that if this assumption is made and if the lognormal distribution can be assumed to represent the underlying loss severity distribution then trend should not affect the parameter σ^2. Thus if a good fit is achieved via a lognormal distribution and yet the parameter is observed to be changing over time, then this would indicate that trend does not affect all claims equally.

With this point in mind, consider the following values for μ and σ^2 which were computed by fitting a lognormal distribution to the indicated data.

While the σ^2 parameter seems to be changing over time, a hasty conclusion should not be drawn since:

(1) The underlying data is relatively immature (as of 27 months) and thus is largely affected by reserving procedures. To get a proper picture of the effects of trend one should analyze fully developed claims.

(2) As mentioned above, the argument depends on whether or not the data is adequately represented by the lognormal distribution. If the quality of fit changes from year to year, then one cannot analyze the effects of trend by tracing the movement of an artificial parameter.

TABLE 4

Policy* Year		Physicians	Surgeons	Hospitals
1972	μ	7.8616	8.0562	7.4799
	σ^2	3.1311	2.8601	3.1988
1973	μ	7.9609	8.3099	7.7801
	σ^2	2.5031	2.6183	3.1040
1974	μ	8.1901	8.4578	7.8295
	σ^2	2.3395	2.4240	3.6419

*As of 27 months of maturity.

ALLOCATED LOSS ADJUSTMENT EXPENSES

Miccolis mentions in the final paragraph of his introduction that there are many practical problems concerning loss adjustment expenses which cannot be resolved solely by the mathematical model presented in his paper.

Perhaps the overriding reason for this is that an insurer's legal costs in defending an insured are not bounded by the limit of the insured's policy. The sum of the indemnity portion and the cost of lost adjustment expenses may and in many cases does exceed the limit of the insured's policy. This is where Miccolis' model becomes inoperative since the equation

$$g(x;K) = \begin{cases} x, 0 < x < K, K > 0 \\ K, x \geq K \end{cases}$$

is no longer applicable.

As an alternative to using the model in pricing allocated loss adjustment expenses for layers of coverage above basic limits, ISO has investigated raw data to actually compute the average amount of allocated per claim for each policy limit for which data was available. An increasing pattern in these numbers may be matched by a similar pattern for the average basic limits severity by policy limit. In such a case, a constant percentage charge of the basic limits indemnity will produce an increasing dollar amount for allocated loss adjustment expense by policy limit. Therefore, another test was to compute the ratio of allocated loss adjustment expenses to basic limit losses by policy limit. If this ratio forms an increasing

TABLE 5

INSURANCE SERVICES OFFICE
ALLOCATED LOSS ADJUSTMENT EXPENSE
BY POLICY LIMIT*

PHYSICIANS

(1) Policy Limit	(2) # of Claims	(3) Avg. B/L Ind.	(4) Avg. ALAE	(5) (4) ÷ (3)
100,000	956	$6,713	$3,482	.519
200,000	83	6,120	2,048	.335
250,000	96	7,833	2,635	.336
500,000	57	5,474	2,088	.381
1,000,000	230	8,591	3,130	.364
All LimitsØ Combined	1,601	6,988	3,066	.439

SURGEONS

(1) Policy Limit	(2) # of Claims	(3) Avg. B/L Ind.	(4) Avg. ALAE	(5) (4) ÷ (3)
100,000	1,693	$8,156	$4,115	.505
200,000	260	7,885	2,085	.264
250,000	322	9,161	2,112	.231
500,000	140	9,286	3,157	.340
1,000,000	457	8,565	3,365	.393
All LimitsØ Combined	3,118	8,270	3,496	.423

HOSPITALS

(1) Policy Limit	(2) # of Claims	(3) Avg. B/L Ind.	(4) Avg. ALAE	(5) (4) ÷ (3)
100,000	565	$4,745	$2,508	.529
200,000	178	5,124	2,011	.392
250,000	368	6,307	3,022	.479
300,000	412	6,391	2,697	.422
500,000	818	6,620	2,502	.378
1,000,000	1,242	8,820	3,738	.424
All Policy Limits CombinedØ	4,411	6,397	2,706	.423

*Policy Year 1974 data evaluated as of March 31, 1976.
ØIncludes limits not listed.

progression as policy limits increase then an increasing percentage of the basic limit rate should be charged for allocated loss adjustment expenses.

As the tables for Physicians, Surgeons, and Hospitals below show, however. a progression did not materialize. Therefore, in initial pricing considerations, a constant percentage of the Average Basic Limits Severity was loaded into each policy limit in order to construct the increased limit factors. Of course this decision, based as it was on empirical data, is subject to change as more data become available for analysis.

SUMMARY

The conclusion of Miccolis' paper is that the complete solution to the problem of pricing increased limits coverage requires "actual data, judgment, and 'some' further study." This review, prepared with the assistance of George Burger and Aaron Halpert, has presented data (of companies reporting to ISO) and has indicated the judgment needed to develop such data to a form in which the mathematical models can be applied.

The CAS is indebted to Mr. Miccolis for his contribution to the Proceedings.

USE OF NATIONAL EXPERIENCE INDICATIONS IN WORKERS' COMPENSATION INSURANCE CLASSIFICATION RATEMAKING

FRANK HARWAYNE

The use of national experience indications in workers' compensation insurance classification ratemaking is more familiarly known as small credibility ratemaking. It is a response to a current need and one which is closely akin to processes used during the early days of workers' compensation insurance classification ratemaking.

Historically, classification ratemaking depended to a large extent upon national pure premiums[1], that is, pure premiums were derived from observations of the countrywide classification experience. Differences in pure premium from state to state depended upon measured differences in benefit levels provided by workers' compensation law in each state. Subsequently, this approximation to costs under individual state laws was abandoned as being too crude.

The general movement of state regulation has been in the direction of recognition of each state's own experience. The rates produced as a result of this movement are valid to the extent that the experience within a state is credible. To the extent that the classification experience is not credible, the post World War II techniques used have been those of changing the rates for the non-credible classifications (that is, the non-reviewed classes) only to the extent of general changes in rate level, industry level, or law benefit level. The difficulty with this approach is that the limited experience of the non-reviewed classifications is virtually disregarded as being non-credible. Their rates do not reflect changes in actual costs which take place in non-reviewed classifications. Moreover, this approach tends to produce or perpetuate anomalies with respect to competing manufacturers, processors or distributors who operate in different states within the same industry.

Small credibility ratemaking is a way of continuing to use state experience wherever feasible and to meld the national experience to the extent of its credibility. It is more refined than the prior national pure premium system. This refinement was achieved by adjusting the experience for the

[1] Clarence W. Hobbs, *The National Council on Compensation Insurance*, (Globe Printing Co., New York, circa 1930), pp. 6, 100.

differences between state and average national benefit levels. In its initial stages during the late 1950's and early 1960's, the tentative small credibility ratemaking approach established 50 classifications which have a substantial payroll base and which normally exist in most states. The partial (serious, non-serious and medical) claim frequencies and partial claim costs for these 50 classifications were ascertained on a countrywide basis. A system was devised for obtaining the partial claim frequencies and partial claim costs from the national data base exclusive of the state which would be considered for rate revision. Factors to adjust to state conditions were determined for application to the partial claim costs and partial claim frequencies for all classifications. The program also required that these national partial claim cost and partial claim frequency indications be introduced to a limited degree in the following way. What normally had been the complement of credibility was subdivided into the credible part of the countrywide information (but it could never exceed 50% of the usual complement of the credibility factor) and the balance of the 100% weight was assigned to the state underlying average claim cost or claim frequency of the classification.

Although the process of the tentative approach could work, it appeared to have a number of gaps. Only 50 of the 700 odd classifications were included as the basis for establishing credibilities for all classifications and these were not necessarily large volume classifications. The particular formula[2] which was mathematically correct appeared to require a more so-

[2] The procedure relied upon fifty key codes or manual classifications to adjust experience in different states to a common level. Countrywide weights based on expected losses for the key codes were used to determine key code average frequencies F_i, and key code average severities S_i, for state i. For the state k, for which rates were to be revised, actual A-sheet experience was employed to calculate F_k and S_k: for the remaining states data base records were utilized. Separate averages were calculated for serious, non-serious, and medical losses. The national serious pure premium for classification j, when revising state k, was to be computed as:

$$\text{serious pure premiums} = [S_k \sum_{i \neq k} ({}_iL_j \div S_i)]\,[F_k \div \sum_{i \neq k} ({}_iP_j \times F_i)]$$

where:

S_k = key code average serious severity (cost per case) for state k
${}_iL_j$ = serious losses (from data base) for classification j in state i
S_i = key code average serious severity (cost per case) for state i
F_k = key code average serious frequency (cases per \$100 payroll) for state k
${}_iP_j$ = payroll in hundreds for classification j in state i
F_i = key code average serious frequency (cases per \$100 payroll) for state i

National non-serious and medical pure premiums were similarly derived. The credibility weights assigned to state, national and underlying pure premiums were identical to those finally adopted and described later in the text.

phisticated knowledge than one might reasonably expect of at least some state regulators; explanations of its derivation and operation could not readily be described to the premium paying public and others who were concerned with workers' compensation insurance costs.

A fresh approach to the problem was undertaken. Instead of the tentative approach of using partial claim frequencies and partial claim costs separately, a partial pure premium was utilized and the 50 classifications were replaced by all classes. The experience of other states was modified to permit its inclusion with the state being revised. Separately, the modified national experience[3] serious, non-serious and medical pure premium for each classification was multiplied by the payrolls for that classification code number in the state undergoing revision. The sum of the products for all classifications represents what the dollars of loss would have been if modified national experience were distributed according to the payrolls generated in the state being revised. The difference between these aggregate losses and the actual losses in the state being revised was used to generate a factor to adjust each state's partial pure premium so that it would balance to the average partial pure premium in the state undergoing revision.

With the modified national experience on the level of the state's partial pure premium, the credibility weighting process proceeds. As in the earlier tentative program, the state's own experience is afforded credibility in accordance with customary standards except that credibility intervals of .01 are used in lieu of .10 of the older system. The modified national classification experience is afforded credibility based on number of claims, but is subject to a maximum not to exceed one-half of the complement of the state's credibility for the classification[4]. The remainder of 100% is assigned to the pure premium underlying the present rate for the classification[5]. The process is performed separately for the serious, non-serious and medical pure premiums.

The Appendix contains a technical description, formulae, credibility tables underlying the process described above and an illustrative example.

From an analytical point of view, the new small credibility program looks upon workers' compensation experience at two levels. Primarily, the

[3] The term "modified national experience" used in this paper means the experience of all states except the particular state undergoing a rate revision.

[4] For example: fifty non-serious claims indicates a national credibility of .30. However, if the state non-serious credibility were .60, then the national credibility is limited to one-half the complement of .60.; i.e. .20 in lieu of .30.

[5] Also described as present on rate level pure premium.

first level affords recognition to experience within the state to the extent of the classification credibility. Where the state classification partial pure premium experience is not credible, reliance is placed upon the corresponding partial pure premium for the classification outside the state, with the proviso that the underlying partial pure premium must always be afforded at least half of the complement of the state classification's partial pure premium credibility. In this way, the rate for a manufacturing industry will reflect experience within the state; to the limited extent that no such experience can be relied upon for that industry, reliance is placed upon other states' experience for that manufacturing industry (with appropriate factor adjustments to reflect general state conditions) in combination with the historical record for the class (underlying pure premium) within the state.

This new process is viewed as an improvement in the effort to achieve fair, reasonable, and equitable rates wherein actual experience within and outside the state is expanded substantially.

APPENDIX

SMALL CREDIBILITY RATEMAKING PROCEDURES

The National Council has developed a small credibility ratemaking procedure which is expected to result in refined ratemaking[6]. The procedure involves the use of a data base consisting of individual classification experience on an individual state basis for three policy years. The experience consists of the following records:

1. payroll
2. number of serious cases
3. amount of serious losses
4. number of non-serious cases
5. amount of non-serious losses
6. amount of medical losses
7. policy periods and law level

Proposed partial pure premiums are the sum of (1) the product of the state indicated partial pure premium and state credibility in 1% intervals, (2) the modified national partial pure premium and national credibility in 1% intervals, and (3) the present on rate level partial pure premium and the residual credibility. State credibility is based upon the same 100% standards and the same formula [criterion for credibility value of Z is equal to $Z^{3/2}$ x full credibility standard] as at present except that the formula is evaluated at 1%, in lieu of 10%, intervals. National credibility utilizes the same formula but, for simplicity, is premised on number of cases rather than expected losses and is limited to 50% of the complement of the state credibility. The national serious full credibility standard is 25 serious cases, the national non-serious standard is 300 non-serious cases, and the national medical standard is 300 indemnity (serious and non-serious) cases[7].

The small credibility procedure is premised upon the principles of uniform relative hazard among classes. This principle refers to the hazard for any classification in any state having the same relationship (except for chance variation), after suitable adjustment by indices, to the hazard of any second classification chosen.

---

[6] See Roy H. Kallop, "A Current Look At Workers' Compensation Ratemaking." *P.C.A.S., LXII* (1975).

[7] See Appendix, Exhibit II, "Credibility Criteria for National Experience."

The formula recognizes uniform relative hazard by means of statewide average pure premiums derived from actual experience in each state i and the distribution of payroll among classifications in state k, for which rates are to be revised. For any state i, the state average serious pure premium PP_i is computed as:

$$PP_i = \sum_j ({}_iL_j \div {}_iP_j)\,{}_kP_j \div \sum_j {}_kP_j$$

Values of ${}_kL_j$ and ${}_kP_j$ are taken from A-sheet data[8]; for the remaining states, values of ${}_iL_j$ and ${}_iP_j$ are from the data base. The modified national serious pure premium for classification j when revising state k is:

$$\text{serious pure premium} = \sum_{i \neq k} {}_iL_j\,(PP_k \div PP_i \div \sum_{i \neq k} {}_iP_j$$

where ${}_iL_j$ = serious losses (from data base records) for classification j in state i, and

where ${}_iP_j$ = payroll in hundreds for classification j in state i

Modified national non-serious and medical pure premiums are similarly derived.

[In the case of classifications that would involve division by zero in the formula, modified national pure premiums are defined to be zero and have no credibility.]

The small credibility procedure does not attempt to improve classification ratemaking by the introduction of new credibility standards and/or formulas. Rather, it expands the volume of classification experience by recognizing modified national indications. The result is greater equity among classification rates and no change in overall rate level.

A sample calculation of the process for state k is shown in Exhibit I. For simplicity, it is assumed that states a, b and k comprise the countrywide data base. Within each state, codes 1 and 2 represent all classes.

---

[8] See Roy H. Kallop, "A Current Look At Workers' Compensation Ratemaking" Exhibit II, *P.C.A.S., LXII* (1975).

EXHIBIT 1

STATE k

COMPUTATION OF PROPOSED SERIOUS PURE PREMIUMS FOR ALL CLASSES (CODE 1 AND CODE 2)

	State k		State a		State b	
	Code 1	Code 2	Code 1	Code 2	Code 1	Code 2
1. exposure	10,846,000	8,304,000	7,250,000	110,000,000	3,250,000	210,000,000
2. number of serious cases	15	2	10	10	5	20
3. amount of serious losses	305,100	20,760	220,000	110,000	220,000	440,000
4. state k serious cred.	54%	9%	xx	xx	xx	xx
5. state k present on rate level serious pure premium	2.750	.326	xx	xx	xx	xx

$P_k = 100\,[305{,}100 + 20{,}760] \div [10{,}846{,}000 + 8{,}304{,}000] = 1.702$

$P_a = 100\,[(220{,}000 \div 7{,}250{,}000)(10{,}846{,}000) + (110{,}000 \div 110{,}000{,}000)(8{,}304{,}000)] \div [10{,}846{,}000 + 8{,}304{,}000] = 1.762$

$P_b = 100\,[(220{,}000 \div 3{,}250{,}000)(10{,}846{,}000) + (440{,}000 \div 210{,}000{,}000)(8{,}304{,}000)] \div [10{,}846{,}000 + 8{,}304{,}000] = 3.925$

$N_1 = 100\,(1.702)\,[(220{,}000 \div 1.762) + (220{,}000 \div 3.925)] \div [7{,}250{,}000 + 3{,}250{,}000] = 2.932$

$N_2 = 100\,(1.702)\,[(110{,}000 \div 1.762) + (440{,}000 \div 3.925)] \div [110{,}000{,}000 + 210{,}000{,}000] = .093$

$$\begin{aligned} \text{credibility for } N_1 &= \text{minimum}\left((1 - .54) \div 2;\ ((10 + 5) \div 25)^{2/3}\right) \\ &= \text{minimum}\,(.23;\ .71) \\ &= .23 \end{aligned}$$

$$\begin{aligned} \text{credibility for } N_2 &= \text{minimum}\left((1 - .09) \div 2;\ 1.00\right) \\ &= \text{minimum}\,(.45;\ 1.00) \\ &= .45 \end{aligned}$$

state k indicated serious pure premium for code 1 = 100 (305,100 ÷ 10,846,000) = 2.813

state k indicated serious pure premium for code 2 = 100 (20,760 ÷ 8,304,000) = .250

proposed serious pure premium for code 1 = (2.813)(.54) + (2.932)(.23) + (2.750)(1 − .54 − .23) = 2.826

proposed serious pure premium for code 2 = (.250)(.09) + (.093)(.45) + (.326)(1 − .09 − .45) = .214

EXHIBIT II

CREDIBILITY CRITERIA FOR NATIONAL EXPERIENCE

NATIONAL CREDIBILITY	SERIOUS CRITERION (SERIOUS CASES)	NONSERIOUS CRITERION (NONSERIOUS CASES)	MEDICAL CRITERION (SERIOUS & NONSERIOUS CASES)
1.00	25	300	300
0.99	xx	296	296
0.98	xx	292	292
0.97	24	287	287
0.96	xx	283	283
0.95	xx	278	278
0.94	23	274	274
0.93	xx	270	270
0.92	xx	265	265
0.91	22	261	261
0.90	xx	257	257
0.89	21	252	252
0.88	xx	248	248
0.87	xx	244	244
0.86	20	240	240
0.85	xx	236	236
0.84	xx	231	231
0.83	19	227	227
0.82	xx	223	223
0.81	xx	219	219
0.80	18	215	215
0.79	xx	211	211
0.78	xx	207	207
0.77	17	203	203
0.76	xx	199	199

Exhibit II (Contd.)

CREDIBILITY CRITERIA FOR NATIONAL EXPERIENCE

NATIONAL CREDIBILITY	SERIOUS CRITERION (SERIOUS CASES)	NONSERIOUS CRITERION (NONSERIOUS CASES)	MEDICAL CRITERION (SERIOUS & NONSERIOUS CASES)
0.75	xx	195	195
0.74	16	191	191
0.73	xx	188	188
0.72	xx	184	184
0.71	15	180	180
0.70	xx	176	176
0.69	xx	172	172
0.68	xx	169	169
0.67	14	165	165
0.66	xx	161	161
0.65	xx	158	158
0.64	13	154	154
0.63	xx	151	151
0.62	xx	147	147
0.61	12	143	143
0.60	xx	140	140
0.59	xx	136	136
0.58	xx	133	133
0.57	11	130	130
0.56	xx	126	126
0.55	xx	123	123
0.54	10	120	120
0.53	xx	116	116
0.52	xx	113	113
0.51	xx	110	110

Exhibit II (Contd.)

CREDIBILITY CRITERIA FOR NATIONAL EXPERIENCE

NATIONAL CREDIBILITY	SERIOUS CRITERION (SERIOUS CASES)	NONSERIOUS CRITERION (NONSERIOUS CASES)	MEDICAL CRITERION (SERIOUS & NONSERIOUS CASES)
0.50	9	107	107
0.49	xx	103	103
0.48	xx	100	100
0.47	xx	97	97
0.46	8	94	94
0.45	xx	91	91
0.44	xx	88	88
0.43	xx	85	85
0.42	7	82	82
0.41	xx	79	79
0.40	xx	76	76
0.39	xx	74	74
0.38	6	71	71
0.37	xx	68	68
0.36	xx	65	65
0.35	xx	63	63
0.34	5	60	60
0.33	xx	57	57
0.32	xx	55	55
0.31	xx	52	52
0.30	xx	50	50
0.29	4	47	47
0.28	xx	45	45
0.27	xx	43	43
0.26	xx	40	40

Exhibit II (Contd.)

CREDIBILITY CRITERIA FOR NATIONAL EXPERIENCE

NATIONAL CREDIBILITY	SERIOUS CRITERION (SERIOUS CASES)	NONSERIOUS CRITERION (NONSERIOUS CASES)	MEDICAL CRITERION (SERIOUS & NONSERIOUS CASES)
0.25	xx	38	38
0.24	3	36	36
0.23	xx	34	34
0.22	xx	31	31
0.21	xx	29	29
0.20	xx	27	27
0.19	xx	25	25
0.18	2	23	23
0.17	xx	22	22
0.16	xx	20	20
0.15	xx	18	18
0.14	xx	16	16
0.13	xx	15	15
0.12	xx	13	13
0.11	1	11	11
0.10	xx	10	10
0.09	xx	9	9
0.08	xx	7	7
0.07	xx	6	6
0.06	xx	5	5
0.05	xx	4	4
0.04	xx	3	3
0.03	xx	2	2
0.02	xx	1	1
0.01	xx	xxx	xxx

USE OF NATIONAL EXPERIENCE INDICATIONS IN WORKERS' COMPENSATION CLASSIFICATION RATEMAKING

FRANK HARWAYNE

DISCUSSION BY JAMES F. GOLZ

Frank Harwayne's paper, "Use of National Experience Indications in Workers' Compensation Classification Ratemaking," shows the application of some practical actuarial science to the solution of a lingering problem. The problem: because of low credibility, the rates for some classifications in certain states did not seem to be at, or likely to reach, a reasonable level. The solution: adjust experience from other available states to the exposure distribution and average pure premium level of the state in question and merge it into the classification ratemaking procedure.

A few comments on terminology may be in order. Although the procedure is referred to as "national," it might more properly be termed "multistate" since the data base currently encompasses only those jurisdictions for which the National Council on Compensation Insurance makes rates. Thus, data from about a dozen states (independent bureaus and exclusive funds) is not available. Likewise, although the method is often called a "small credibility" procedure, its use may have some effect on the rate for any classification which does not possess full credibility for all of its partial pure premiums. Indeed, since three years of classification experience are used (as compared to two under the old procedure in most cases) even fully credible classes can end up with a different pure premium from what formerly would have been calculated.

Another item of interest is the subtle change adopted in the calculation of credibility. Credibilities for state experience have been and continue to be based on expected loss dollars. However, credibilities for the national experience are derived from actual claim counts. This slight change signifies, one presumes, no shift in the philosophy underlying credibility, but is merely an adaptation to the data available.

In order to avoid misinterpretations, the National Council has frequently warned against comparing unadjusted classification rates among states. A staff write-up notes that among the factors which cause rates to vary between states are differences in the industries in the state, the definition of exposure (payroll limitation), the benefit level, the administration of the law, the wage levels, the medical facilities, the quality of the labor force, the safety programs in effect, and the degree of attorney involvement. Since the new procedure affects rates by weighting pure premiums (of which rates are a function) between states, it is instructive to observe how these problems are avoided. Any differences in industries are formally adjusted for when the national pure premiums are computed using the exposure distribution in the state under revision. The other factors are not handled individually. Rather, since they all affect costs, they are reflected in the adjustment of each outside state's experience to the subject state's average pure premium level.

One might argue that the benefit level differences could be separately computed by state and the remaining factors then adjusted for in bulk. The technique adopted not only saves this work, but is consistent with a similar treatment employed in the National Council loss ratio trending technique. There, the trending of on-level loss ratios automatically includes all factors which affect costs and avoids the problem of separate identification and measurement of adjustments for items such as wage level changes, medical cost changes, and the host of other items which could be involved.

The procedure described by Mr. Harwayne seems to be based on reasonable actuarial judgment. Although the algebra may momentarily appear complex, the technique is conceptually straightforward. One potential area of concern remains is the vast volume of data involved. The October, 1977, *Scientific American* contains an article on the solution of the classic four-color problem of mathematics; this solution was accomplished by a computer exhaustion of enumerated possible five-color maps. The authors note the reluctance of some to accept their computer proof since it differs so radically from traditional mathematical terseness and verifiability. Similarly here, success in implementing the new technique may depend as much on the ability to demonstrate that accurate data is available and properly adjusted as on any actuarial theory involved.

DISCUSSION BY LESTER B. DROPKIN

Frank Harwayne's paper, which describes the methodology adopted by the National Council on Compensation Insurance with respect to the use of national experience indications, quite properly presupposes a fairly close familiarity with the structure of the Workers' Compensation ratemaking process. For those who have such a familiarity—whether by virtue of having carefully read Roy Kallop's recent paper[1], or by service on one or more of the committees of the National Council, or by other means—the present Harwayne paper will fall naturally into place.

A very valuable and necessary insight into the decisions and methods of the National Council has been provided, and undoubtedly will continue to be provided, for both the membership of the C.A.S. and that wider public readership of the Proceedings by the series of papers devoted to explaining and recording the ideas and concepts that constitute the standard Workers' Compensation ratemaking procedure.

It is, of course, well known that the Workers' Compensation ratemaking procedure has two quite distinct components. The first, concerned with developing the indicated overall rate level change, is today based on aggregate premium and loss experience, i.e. on financial data. The second, which may be referred to as the relativity portion of the rate revision, is concerned with the equitable and reasonable distribution of the otherwise determined overall rate level change to the individual classifications. While the process proceeds in terms of pure premiums—and thus suggests that we are dealing with absolute levels—in fact the process is one of determining the proper *relative* level among the classifications.

Although an actual rate revision proceeds by considering the Serious, Non-Serious and Medical components separately, Mr. Harwayne has found it convenient for illustrative purposes to refer to one component only, the Serious, since the concepts and procedures applying to the Serious component apply to the other components also. This review will also utilize the convenience of referring to only the Serious component.

---

[1] Kallop, Roy H., "A Current Look at Workers' Compensation Ratemaking," P.C.A.S., LXII (1975).

For those classifications with a sufficiently large volume of (Serious) expected losses to receive full (Serious) credibility, the experience or "indicated" pure premium becomes the formula pure premium in accordance with what is meant by full credibility.

It is with respect to those classifications which do not develop the necessary volume of (Serious) expected losses for full (Serious) credibility that the present procedure, utilizing national relativities, differs from the former procedure.

Previously, for those classifications with such lesser amounts of (Serious) expected losses the formula pure premium was determined as the credibility weighted average of the indicated pure premium and the underlying or "present" on (rate) level pure premium. However, since there were many classifications in many National Council jurisdictions which were developing either zero or very modest credibilities, the application of the procedure meant that a large number of classifications were simply taking the overall rate level change or something very close to it. In looking at the classifications in a given state, the state was being viewed as though no other state existed, with a consequent loss of valuable information.

Introduction of the national relativity procedure means that the informational input of the relationships exhibited by the modified national experience will now be utilized as part of the process that determines proper classification relativities. Looking back, we can see in the adoption of the present procedure an almost classic example of Hegelian dialectic with its stages of thesis, antithesis and synthesis:

Thesis — Original, historical use of national experience.
Antithesis — Post Public Law 15 use of state experience.
Synthesis — Present, blended use of both state and national experience.

The new procedure posits the existence of an intrinsic, inherent relativity of hazard among classifications—which, of course, means among employments, operations and businesses—that is independent of state boundaries. To what extent are we willing to accept this premise? This reviewer, for one, has had no difficulty, although the question could be answered more readily perhaps, if we did not have the hundreds of classifications that, in fact, we do have in Workers' Compensation.

Incidentally, it would be interesting to know whether, and if so, how, the National Council adapts the procedure to the case of state special classifications and classifications whose definitions may vary somewhat from one jurisdiction to another.

I have no doubt that the paper should be, and will be, required reading for anyone with an interest in the Workers' Compensation ratemaking process. While the paper sets forth the formulae in a concise mathematical way, it may be useful to present part of the illustrative example in an alternative format which explicitly sets out the logical steps of the process, since the paper will surely also be read and referred to by persons less mathematically oriented than actuaries.

The basic information available to us is restated in Exhibit 1; the underlying logic of the steps used to determine the National Pure Premiums is given in Exhibit 2.

Since the relationships among the various classifications will be expressed, in part, by means of ratios to statewide, overall, all classifications combined pure premiums the first part of the process adjusts the Total Statewide Pure Premiums of states a and b to reflect the distribution by class of state k [Exhibit 2, Col. (6)]. The variation of these Total Statewide Pure Premiums from that of state k [Exhibit 2, Col. (7)] provides factors to be applied to the indicated classification pure premiums of states a and b to produce what may be called Indexed National Pure Premiums [Exhibit 2, Col (10)]. It is these Indexed National Pure Premiums which may be said to constitute the real heart of the process, in the sense that they have preserved the original relativities indicated by experience in states other than state k, yet have been expressed in terms of levels appropriate to state k. This may be seen from the following table:

	Classif. 1	Classif. 2	Ratio
Indicated Pure Premium—State a	3.034	.100	30.34
Indicated Pure Premium—State b	6.769	.210	32.23
Indexed National P. P.—State a	2.9305	.0966	30.34
Indexed National P. P.—State b	2.9350	.0911	32.23

While the illustrative example of the paper of course includes a comparison of the Formula Pure Premiums developed under the two procedures, again it may be useful to exhibit the results in a way which emphasizes the basic concern of this part of a rate revision, viz. the relativities:

	Classif. 1	Classif. 2	Ratio
P. P. Underlying Pres. Rates—State k	2.750	.326	8.44
Indicated Pure Premium—State k	2.813	.250	11.25
National Pure Premium	2.932	.093	31.53
Formula Pure Premium—Prior Procedure	2.782	.326	8.53
—New Procedure	2.826	.214	13.21

Thus we see that while the prior procedure would have changed the present relativity but slightly, the new procedure has allowed a much larger shift, in accordance with the very desirable objective of incorporating the greater body of information provided by the classification experience of other states.

EXHIBIT 1

BASIC INFORMATION

State	Classification	Payroll	Losses	Indicated P.P.	Credibility	Present on Rate Level P.P.
	1	10,846,000	305,100	2.813	.54	2.750
k	2	8,304,000	20,760	.250	.09	.326
	State Total	19,150,000	325,860	1.702		
	1	7,250,000	220,000	3.034		
a	2	110,000,000	110,000	.100		
	State Total	117,250,000	330,000	.281		
	1	3,250,000	220,000	6.769		
b	2	210,000,000	440,000	.210		
	State Total	213,250,000	660,000	.309		

EXHIBIT 2

NATIONAL PURE PREMIUM CALCULATION

(1) State	(2) Classification	(3) Payroll (State k)	(4) Indicated Pure Premium	(5) Incurred Losses (3) × (4)	(6) Adjusted Total State Pure Premium (5) ÷ (3)	(7) Ratio: 1.702 (6)
	1	$10,846,000	3.034	$329,068		
a	2	8,304,000	.100	8,304		
	Total	19,150,000		337,372	1.762	.9659
	1	10,846,000	6.769	734,166		
b	2	8,304,000	.210	17,438		
	Total	$19,150,000		$751,604	3.925	.4336

(8) Classification	(9) State	(10) Indexed National Pure Premium (4) × (7)	(11) Payroll	(12) Incurred Losses (10) × (11)	(13) National Pure Premium (12) ÷ (11)
	a	2.9305	$ 7,250,000	$212,498	
1	b	2.9350	3,250,000	95,388	
	Total		10,500,000	307,886	2.932
	a	.0966	110,000,000	106,700	
2	b	.0911	210,000,000	191,100	
	Total		$320,000,000	$297,800	.093

ACCIDENT LIMITATIONS FOR RETROSPECTIVE RATING

FRANK HARWAYNE

DISCUSSION BY DAVID R. BRADLEY

As Frank Harwayne aptly stated, there have been forces at work in compensation insurance which forced a review of the dollar distribution of losses by size of claim as used to calculate excess loss premium factors for the retrospective rating plan. It should be understandable that inflation alone would not cause the tables developed by Mr. Dunbar Uhthoff to become inaccurate. Mr. Uhthoff carefully shielded his work from this effect by combining state data only after converting them to ratios about the state mean, thereby eliminating dollar amounts. As long as inflation does not cause a change in the shape of the curve describing the distribution by size around the average claim amount, the Uhthoff tables are usable. Even when inflation affects some elements of claim cost (such as medical costs) differently than others (such as wages), basically the overall impact of such a situation is not significant unless the differences in inflation rates affecting the components are both large and of a long-term nature. A simple example of this is the fact that generally changes in hospital and medical fee schedules do not generally cause changes in ELPF's.

However, a revision does become necessary when a change in the *shape* of the size of loss distribution curve occurs. It is logical to expect this phenomenon now because of basic changes that 1) have occurred in the workers' compensation system, and 2) have been incurred by the workers' compensation system. Perhaps the most significant externality is a change in the market for compensation insurance, with concomitant shifts in the distribution of workers and hence payrolls by industry-type. For example, the aerospace industry was virtually nonexistent when Mr. Uhthoff developed his tables. An important internal change is the major aggregate growth in benefits payable under the compensation laws as a result of the recommendations of the National Commission on State Workers' Compensation Laws. While a typical law amendment will cause a change in indicated excess loss premium factors (by raising maximum wage values for benefit computation, etc.), a typical law amendment will generally not cause a major change in claim distribution about the mean. However, when state compensation laws are significantly revised so as to provide livable benefit levels rather than supplementary dollars to claimants, and when, more significantly, workers' compensation laws are revised to allow escalated benefits (benefits that provide the individual claimant with a lifetime increasing annuity), then clearly the distribution of claim sizes will widen. The curve will not merely shift upward but change shape as well under these stimuli. This should not be construed to imply that the industry should feel that significant benefit increases are bad. It does imply that large benefit changes introduce distortions and therefore uncertainty into our loss prediction systems, and one of the areas in which this uncertainty is manifest is our excess loss experience.

Frank Harwayne made two major changes of method from Dunbar Uhthoff's work and they are both improvements. Firstly, Mr. Harwayne fitted his excess loss experience at specific intervals to a curve by the simultaneous application of the methods of collocation and least squares regression. Mr. Uhthoff apparently used linear interpolation to arrive at excess loss values at uneven percentages of the mean. Secondly, Mr. Harwayne plans to use, when it becomes available, fourth and ultimately fifth report unit card data to develop average claim sizes by state by injury type, and to develop the claim size distribution curve. This, as Mr. Harwayne indicates, is an important change made necessary by our seemingly increasing inability to set adequate initial reserves. Development factors in 1950, when Uhthoff did his work, were generally below unity. Development factors today average close to 1.2 for first to ultimate countrywide. Moreover, development factors in some states, mainly those that have experienced major benefit level changes, are running well above 1.3. One must believe that most of the inaccuracy of reserves rests in the setting of large reserves, and this implies that excess loss development is considerably greater.

An additional change in method was caused by data limitations. Mr. Harwayne was forced to use the excess loss experience for limited death benefits as an estimate of excess loss experience in states with unlimited benefits due to insufficient statistics on unlimited death benefit losses. Hopefully, we will be able to improve upon this in the future.

Mr. Harwayne's work represents an important improvement in our measurement of excess losses, and his paper is a valuable, concise, and well-written description of his efforts. However, no compensation actuary should consider the job done. Fifth to ultimate loss development countrywide currently averages 2%, and exceeds 10% in some individual states. The significance of this as it relates to excess losses can be determined by reviewing excess loss development in two states from which the data is available. The New York Compensation Rating Board and the Workers' Compensation Rating and Inspection Bureau of Massachusetts provide their actuarial committees with actual excess loss experience. A recent example of normal and excess (over $50,000) unit statistical plan loss development shows the following:

Development (Latest Three-Year Average)

	New York		Massachusetts	
	On Total Losses	On Excess Losses	On Total Losses	On Excess Losses
1st to 2nd	1.1582	1.6024	1.094	1.5679
2nd to 3rd	1.0835	1.5833	1.048	1.5292
3rd to 4th	1.0506	1.2857	1.029	1.4455
4th to 5th	1.0318	1.2697	1.027	1.1990

Since total loss development from fifth report to "ultimate," based on financial data, is 7.1% for New York and 3.7% in Massachusetts, it seems likely that excess loss development beyond fifth report at least exceeds excess loss development from fourth to fifth reports. Additional support can be derived from converting loss development percentages to dollars. This is more easily achieved using New York data. A comparison of total loss development to excess loss development produces the following:

Development	On Total Losses	On Losses in Excess of $10,000	On Losses in Excess of $25,000
1st to 2nd	$65,145,866	$67,172,648	$29,960,364
2nd to 3rd	38,660,197	54,302,896	31,265,506
3rd to 4th	24,107,566	28,964,362	17,745,885
4th to 5th	10,069,909	23,950,635	14,375,681

(These figures represent total dollars for the three most recent available policy years.)

It seems apparent that, in later reports, virtually all loss development occurs in the adjustment of large claims. This may imply a means of estimating excess loss development beyond the final unit card submission.

To my mind, Frank Harwayne's paper signals not the end but the beginning of the industry's effort to reanalyze its excess loss experience. This is an effort which will ultimately improve not only our retrospective rating plan, but our pro rata resinsurance charges as well.

DISCUSSION BY FRANCIS J. LATTANZIO AND FRANK C. TAYLOR

Mr. Harwayne's paper describes the new procedure for developing Excess Loss Premium Factors (ELPF's). In addition, he mentions two portions of the old method not adopted until after Dunbar Uhthoff[1] wrote his paper. These are: (a) the use of the 1.6 development factor; and (b) the spreading of the average ELPF over the Hazard Groups through the use of Hazard Group differentials. Mr. Harwayne briefly mentions that these differentials were under study. Since this paper was presented, the National Council on Compensation Insurance (NCCI) has filed a revision in the current differentials. The new relativities for all loss limitation sizes are those found in Appendix B, Exhibit B-4, Column (8) of Mr. Harwayne's paper.

The new procedure differs from the most recent procedure in the following ways:

(1) There are now two tables instead of four, with the Fatal Limited tables also being used for Fatal Unlimited and Permanent Total,

(2) The ratios to average have been extended from 3.00 to 3.50 for the Limited Fatal table and from 3.00 to 6.00 for the Major Permanent Partial table, and

(3) The 1.6 development factor will be eliminated when fourth reports of losses by type of injury become available.

The reviewers believe the most important change was the decision to use development by injury type. The remainer of this review will be devoted to a discussion of the necessity of using this type of development and to the presentation of a method for obtaining development factors beyond fifth report.

The ELPF is an important factor not only for ensuring the correctness of the retrospective rating formula, but also as an integral factor in the calculation of excess premiums as used by reinsurers and by the national accounts departments of large insurers. The development of the correct ELPF is essential in all these situations. The question then becomes, how are the proper ELPF's derived?

[1] D. R. Uthoff, "Excess of Loss Ratios via Loss Distributions," *PCAS*, Vol. XXXVII, (1950).

The formula for computing the Excess Loss Ratio (using Snader's notation[2]) is $e^* = w_d e^*_d + w_p e^*_p + w_m e^*_m$. The w's represent the ratios of losses by injury type (death, permanent total, and major permanent partial, respectively) to total losses, while the e*'s represent the excess loss ratios applicable to cases by injury type.

The correctness of the excess ratios by injury type will be accepted for the present. (However, the next revision could be based on data from more states.) The ELPF, then, depends next on the ratios of losses by injury type to total losses, as these are the weights used to combine the individual excess ratios. One assumption of this review is that the development of serious losses is greater than that of total losses and that this difference increases as a direct function of unit report number. Mr. Harwayne indicates that development by injury type will be used as soon as fourth reports of losses become available. This review also assumes that development to fifth report will be used when available. This latter development is currently unavailable and loss development by injury type beyond fifth report is not obtainable under the current unit statistical plan. The absence of data could be especially important in states with large total development beyond fifth report, but it will not be possible to empirically prove this until proper data is available.

If it is assumed that loss development by serious injury type is greater than that for total losses, the problem can be stated as follows: how to measure loss development by injury type beyond fifth report without changing the statistical plan. Even if the statistical plan were changed, the relevance of such factors beyond fifth report is questionable.

There are currently too few development factors by injury type through fifth report for an analysis to be performed. However, loss development factors for losses in excess of certain loss limitation sizes are available. This data is from the New York Compensation Insurance Rating Board and is used in their calculation of ELPF's (a calculation which differs from that of the NCCI). The N.Y.C.I.R.B. displays loss development by unit report from first to fifth for losses in excess of the following loss limitation sizes: \$10,000, \$15,000, \$20,000, \$25,000, and \$50,000.

[2] R. H. Snader, "Fundamentals of Individual Risk Rating and Related Topics," *CAS Study Note*, Part II, Page 16.

This data, together with certain curve fitting techniques, has been used to test not only the New York ELPF charge adequacy in the aggregate, but also the ELPF charge equity by loss limitation size. This technique, while not testing the impact of loss development by injury type beyond fifth report, can be modified to estimate such later development once raw injury type development data is available through fifth report.

THE MODEL

The hypothesis set forth here is that loss development factors used in the calculation of ELPF's are a function of time and loss limitation. Hence, the objective is to define a loss development factor function f(T,L) where:

T: T^{th} report to $(T + 1)^{th}$ report, $T \geq 1$

L: Losses in excess of $(L) \times \$5{,}000$ per claim, $L \geq 2$.

For example, f(7,4) is the loss development factor from report 7 to report 8 for the losses in excess of $20,000 per claim.

The starting point in attempting to define f(T,L) is actual loss development data from the N.Y.C.I.R.B., compiled September 19, 1975. The development factors contained therein can be arranged in the form of the following matrix:

Development From (T):	Loss Development Factors by Loss Limitation (L): $10,000	$15,000	$20,000	$25,000	$50,000
1st to 2nd Report	1.610	1.651	1.660	1.665	1.711
2nd to 3rd Report	1.337	1.409	1.474	1.546	1.551
3rd to 4th Report	1.174	1.212	1.247	1.277	1.316
4th to 5th Report	1.109	1.132	1.170	1.166	1.227

Thus, the actual data is defined for: L = 2, 3, 4, 5, 10 and T = 1, 2, 3, 4. The function derived below will calculate loss development during any two adjacent reports for any excess over a given amount per claim ($10,000 or greater), i.e. for all intregal $T \geq 1$, and $L \geq 2$.

In order to arrive at f, the columns and rows of the matrix are analyzed separately. That is, one variable is held fixed, and loss development patterns are examined as the other varies.

Column Analysis

It is assumed that as one proceeds out in time, the loss development from one report to the next report approaches unity as a lower limit. However, each of the equations displayed on Exhibit I, Section A fit data using least squares with an upper limit property. Therefore, the reciprocals of the loss development factors are used as input for the equations. Curve No. II was chosen as most appropriate for the loss development data varying by time after reviewing the indices of determination for the various amounts of excess.

Row Analysis

Exhibit I, Section B lists the curve types to which the loss development data was fit by time T to the independent variable L. Upon review of the indices of determination of the curves, curve type $Y = C + D \ln(L)$ was best overall.

Form of f(T,L)

While it is of interest to extend the original matrix in both directions, i.e., for L greater than 10 as well as for T greater than 4, the primary concern is in the latter direction. Thus, the form of the function $f(T,L)$ will be $f(T,L) = (1 + A_L e^{B_L T})$ where A_L and B_L are functions of L. The constants A and B from curve type No. II (Exhibit I, Section A) are as follows:

Losses in Excess of:	A_L	B_L
\$10,000 (L = 2)	1.07258	–0.58284
15,000 (L = 3)	1.14694	–0.54483
20,000 (L = 4)	1.10248	–0.47205
25,000 (L = 5)	1.20771	–0.48480
50,000 (L = 10)	1.10787	–0.39826

One would hope that there would be some functional relationship between either A_L and L or B_L and L. It has been seen that the loss development factors themselves, for each fixed report, fit very well with the function $Y = C + D \ln(L)$ as L varies. The B_L's do as well, with an index of determination of 0.9501. However, as one can see, the A_L's do not exhibit such a relationship. It becomes necessary to choose a constant A. The function now has the form:

$$f(T,L) = 1 + Ae^{(C + (\ln L)D)T}$$

where the function $C + (\ln L)D$ is substituted for B_L with $C = -0.58785$, $D = 0.08492$. After testing constants in the range of A_L, A is chosen to be 1.10.

Various loss development factors calculated from the function $f(T,L) = 1 + 1.10e^{(0.58785 + (\ln L)(0.08492))T}$ are displayed on Exhibit II. A comparison of Exhibit II with the matrix of actual data reveals an extremely close fit. The ultimate loss development factors as calculated can now be compared with those which would be used by the N.Y.C.I.R.B. in their calculation of ELPF's.

	\$10,000	\$15,000	\$20,000	\$25,000	\$50,000
1. First Report to Ultimate as Calculated by f(T,L)	3.2232	3.8016	4.2740	4.6869	6.4612
2. First Report to Fifth Report (N.Y.C.I.R.B.)	2.8018	3.1917	3.5718	3.8331	4.2844
3. Increase in LDF Obtained ((1) ÷ (2))	1.1504	1.1911	1.1966	1.2227	1.5081

In summary, the reviewers feel that current ELPF charges are inadequate to the extent that they fail to recognize the ultimate development of losses, both in total and by injury type. A method has been presented which estimates this inadequacy. Some improvements in the quality of the derived factors could be achieved if raw loss development factors were available for: (a) no loss limitation, and (b) additional loss limitation sizes.

It is admitted that the data is not the most current and that different data may not fit these curves. However, it is believed that the concept is valid and that curves can be found which will produce a proper fit to more recent and more extensive data.

Finally, the reviewers would like to thank Glenn W. Fresch for his guidance and direction in the preparation of the latter part of this review.

EXHIBIT I

A. LOSS DEVELOPMENT FOR LOSSES IN EXCESS OF ($L \times \$5{,}000$)
INDEX OF DETERMINATION

CURVE TYPE	L=2	L=3	L=4	L=5	L=10
I. Y=(UPPER LIMIT) (A^{BT})	.9983	.9921	.9768	.9324	.9727
II. Y=(UPPER LIMIT)÷($1+Ae^{BT}$)	.9976	.9951	.9814	.9465	.9776

B. LOSS DEVELOPMENT FOR TIME T
INDEX OF DETERMINATION

CURVE TYPE	T=1	T=2	T=3	T=4
1. $Y=C+DL$	.8798	.6449	.8417	.9202
2. $Y=Ce^{DL}$	.8764	.6274	.8313	.9150
3. $Y=CL^{D}$	.9722	.8732	.9811	.9608
4. $Y=C+(D\div L)$	.8734	.9040	.8845	.7830
5. $Y=1\div(C+DL)$	.8728	.6071	.8200	.9094
6. $Y=L\div(CL+D)$	.8803	.9186	.9024	.8001
7. $Y=C+D\ln(L)$	.9722	.8861	.9831	.9574
8. $Y=1\div(C+De^{(-L)})$	.7832	.8983	.8267	.6979
9. $Y=Ce^{(D\div L)}$	.8769	.9119	.8938	.7918

EXHIBIT II

CALCULATED VALUES OF THE FUNCTION f(T,L)

T: Development from:	L: LOSSES IN EXCESS OF $ PER CLAIM: $10,000	$15,000	$20,000	$25,000	$50,000	$75,000	$100,000
Report 1 to 2	1.611	1.648	1.671	1.687	1.736	1.765	1.785
Report 2 to 3	1.339	1.382	1.409	1.430	1.493	1.531	1.560
Report 3 to 4	1.189	1.225	1.249	1.268	1.330	1.369	1.399
Report 4 to 5	1.105	1.133	1.152	1.168	1.221	1.257	1.285
Report 5 to 6	1.058	1.078	1.093	1.105	1.148	1.178	1.203
Report 6 to 7	1.032	1.046	1.057	1.066	1.099	1.124	1.145
Report 7 to 8	1.018	1.027	1.035	1.041	1.066	1.086	1.103
Report 8 to 9	1.010	1.016	1.021	1.026	1.044	1.060	1.074
Report 9 to 10	1.006	1.009	1.013	1.016	1.030	1.042	1.053
Report 10 to 11	1.003	1.006	1.008	1.010	1.020	1.029	1.038
Report 11 to 12	1.002	1.003	1.005	1.006	1.013	1.020	1.027
Report 12 to 13	1.001	1.002	1.003	1.004	1.009	1.014	1.019
Report 13 to 14	1.001	1.001	1.002	1.002	1.006	1.010	1.014
Report 14 to 15	1.000	1.001	1.001	1.002	1.004	1.007	1.010
Report 15 to 16	1.000	1.000	1.001	1.001	1.003	1.005	1.007
Report 16 to 17	1.000	1.000	1.000	1.001	1.002	1.003	1.005
Report 17 to 18	1.000	1.000	1.000	1.000	1.001	1.002	1.004
Report 18 to 19	1.000	1.000	1.000	1.000	1.001	1.002	1.003
Report 19 to 20	1.000	1.000	1.000	1.000	1.001	1.001	1.002
Report 20 to 21	1.000	1.000	1.000	1.000	1.000	1.001	1.001

A REFINED MODEL FOR PREMIUM ADJUSTMENT

BY DAVID MILLER AND GEORGE DAVIS

DISCUSSION BY FRANK KARLINSKI

Many ratemaking problems accompany the expansion of an insurance company into a new line of business. Not the least of these is the problem of accounting for the distortion in on-level factors created by a rapidly expanding exposure base. The authors suggest an interesting extension of the familiar parallelogram method of adjusting the rate level which introduces a third dimension representing the rate at which new exposures are entering the insurer's book of business. Their method is theoretically sound and appears to be rather easy to use as shown by the authors' example. The authors are to be commended for the elegance with which they attacked this rather thorny problem.

One practical consideration which may appear in the application of this three dimensional technique is obscured by the simplicity of the authors' example. One cannot ordinarily expect to find a simple equation that will exactly fit a series of increasing exposures. In a real life situation, development of a suitable equation could well require the use of a skilled technician and a sophisticated computer program. Even with such resources, a perfect fit cannot be expected and the resultant rate level adjustment will be in error to the extent that the equation does not exactly track the data.

A more direct method for accounting for increasing exposures is easily demonstrated using the authors' numbers.

QUARTER	WRITTEN EXPOSURES	EARNED IN 1st YR.	EARNED IN 2nd YR.
1st	125	109.375	15.625
2nd	375	234.375	140.625
3rd	625	234.375	390.625
4th	875	109.375	765.625
Subtotal		687.500	1312.625
5th	1125		984.375
6th	1375		859.375
7th	1625		609.375
8th	1875		234.375
Subtotal			2687.500
GRAND TOTAL			4000.0

Each earned value in the table is found by assuming an earning pattern by quarter of 1/8, 1/4, 1/4, 1/4, 1/8. This discrete method gives a result which is nearly identical to that using the authors' continuous method.

	CONTINUOUS METHOD	DISCRETE METHOD
Exposures at Base Rate Level	33.3%	32.8%
Exposures at 1.200 Rate Level	66.7%	67.2%
Average Rate Level	1.133	1.134
Premium Adjustment Factor	1.059	1.058

The assumption of even writings within each quarter in the discrete method slightly understates the effect of increasing exposures and accounts for the minor differences in the two sets of figures; otherwise, the methods are equivalent. The continuous method produces a slightly more accurate result in this case only because a simple straight line fits the data exactly.

From a theoretical standpoint, I believe the continuous method is superior to the discrete because it enables the actuary to visualize the true nature of increasing exposures and their effect on rate level.

The authors' method could also be applied to lines of insurance in which expected loss levels show significant seasonal variation. Thus, just as the authors adjust for the variation in number of exposures over time, one could also adjust for the fluctuation in the inherent risk of each exposure during the year. The development of a seasonalized exposure curve would enable the actuary to quantify and characterize the seasonal fluctuation and would lend itself to further analysis. It would be enlightening to discover, for example, that loss levels in the auto collision line over time can be approximated by an equation such as

$$y = a + b (\sin x)$$

where y is the seasonal loss level index, x is the month as represented by some appropriate fraction of π, and a and b are constants. It is obvious that to ignore seasonality where it is significant could cause distortion in data of incomplete accident years. It may not be intuitively obvious, but it can also be shown that, under certain conditions, seasonality could also significantly distort the data for complete accident years as well. These conditions include increasing or decreasing premium or exposure levels. (The degree of the distortion depends, of course, on the nature—primarily the skewness—of the seasonality curve.) This suggests the use of a fourth dimension incorporating both exposures and seasonality considerations. In four dimensions, the advantage of the continuous method—that it allows the situation to be visualized—is lost on all but the most imaginative among us and a return to the discrete method is probably advisable.

MINUTES OF THE 1977 SPRING MEETING

MAY 15-18, 1977

THE HYATT REGENCY, WASHINGTON, D.C.

Sunday, May 15, 1977

Early registration took place from 4:00-6:30 p.m.

The CAS registration for ASTIN was held from 6:30-7:30 p.m.

Monday, May 16, 1977

The Board of Directors had their regular quarterly meeting from 8:30 a.m.-12:00 noon.

Registration was held from 11:00 a.m.-1:00 p.m.

The Spring Meeting was formally convened at 1:00 p.m.

After opening remarks by President Morison, an address was given by Maximilian Wallach, Superintendent of Insurance, District of Columbia.

The first panel discussion was entitled "Regulation — What is the Purpose?" Participants were:

Moderator: P. Adger Williams
Vice President and Actuary
The Travelers Insurance Companies

Members: John R. Ingram
North Carolina Insurance Commissioner

J. Robert Hunter, Jr.
Acting Federal Insurance Administrator
and Chief Actuary
Federal Insurance Administration
United States Department of
Housing and Urban Development

John J. Byrne
Chairman of the Board and President
Government Employees Insurance Co.

After a coffee break, President Morison requested the new Associates to rise as he called their names. After a round of applause for the Associates, each new Fellow was asked to step forward and receive his or her diploma. Pictures of new Associates and Fellows were taken at the coffee break which preceded the business session. List of new Associates and Fellows follows.

NEW FELLOWS

Charles M. Angell
Galen R. Barnes
Richard J. Fallquist
Christopher P. Garand
Vicki S. Keene
Floyd R. Radach
Ellen O. Schultz
John J. Schultz, III
James N. Stanard
Mavis A. Walters
Thomas V. Warthen, Jr.
Reginald C. Yoder
R. James Young, Jr.
Theodore J. Zubulake

NEW ASSOCIATES

James E. Buck, Jr.
Daniel A. Crifo
Scott B. Gerlach
Robert A. Giambo*
Warren H. Johnson, Jr.
A. Claude LaFrenaye
Jan A. Lommele
Robert E. Meyer
Mary L. O'Neil
Benny Silberstein
David C. Westerholm*

*Not Present

New papers and reviews of papers were presented.

Three new papers were presented:

1) "An Algorithm for Premium Adjustment with Scarce Data" by Ronald F. Wiser

2) "On the Theory of Increased Limits and Excess of Loss Pricing" by Robert Miccolis

3) "Use of National Experience Indications in Workers' Compensation Insurance Classification Ratemaking" by Frank Harwayne

Three reviews of Frank Harwayne's paper, "Accident Limitations for Retrospective Rating," were made. The reviewers were: 1) Robert Finger, 2) David Bradley, and 3) Frank Taylor presented his and Francis Lattanzio's review.

Robert J. Finger's paper, "A Note on Basic Limits Trend Factors," was reviewed by David Grady.

Two reviews of George Davis and David Miller's paper, "A Refined Model for Premium Adjustment," were made. Theodore Zubulake presented the first review by Frank Karlinski. The second review was made by Jim Ross.

Two reviews of Robert Finger's paper, "Estimating Pure Premiums by Layer — An Approach," were made. The two reviewers were Lee Steeneck and David Grady.

Two reviews of Robert Finger's paper, "Modelling Loss Reserve Developments," were made. The two reviews were made by Charles Hachemeister and Gail Tverberg.

The President's Reception for New Fellows was held at 6:00 p.m.

Tuesday, May 17, 1977

A welcome was given to ASTIN followed by their panel "Actuarial Work Around the World."

Moderator: LeRoy J. Simon
Senior Vice President
Prudential Reinsurance Company

Members: Dott Giovanna Ferrara
Assicurazioni Generale — Italy

Paul Johansen
Director de la Nye Danske Liv
Denmark

A. J. H. Acher
Director Adjoint
Association Generale des Societes
d'Assurances Accidents — Paris

Jurgens Strauss
Handlungsbevallmachtigter
Munchener Ruckversicherungs —
Gesellschaft Germany

Following coffee break a second panel was heard — "The Residual/ Involuntary Market."

Moderator: Stephen L. Groot
Actuarial Research Manager
Allstate Insurance Company

Members: David M. Klein
Actuary
The Hartford Insurance Group

Richard Decker
President
Automobile Insurance Plans
Service Office

James P. Streff
Assistant Vice President
Sentry Insurance Group

At 12:00 noon the meeting recessed for lunch.

From 1:30-5:00 p.m. six workshops were held. Each workshop was held twice during three hourly periods according to the following schedule:

1:30-2:30	A, B, C, D
2:45-3:45	A, B, E, F
4:00-5:00	C, D, E, F

The workshop titles and participants are listed below:

Workshop A — "Follow Up on Certification"

Moderator: Rafal J. Balcarek
Vice President and Actuary
Reliance Insurance Company

Members: Frederick W. Kilbourne
President
Booz-Allen Consulting Actuaries

Alan C. Curry
Vice President and Actuary
State Farm Mutual Automobile
Insurance Company

Workshop B — "No Fault — What Went Wrong Or Did It?"

Moderator: Edward B. Eliason
Actuary
Aetna Life and Casualty

Members: Paul J. Scheel
Vice President and Senior Actuary
United States Fidelity and Guaranty Co.

Russell G. Press, Jr.
Secretary
Travelers Insurance Companies

Workshop C — "Expressing Actuarial Opinion"

Moderator: Ronald L. Bornhuetter
Senior Vice President and Comptroller
General Reinsurance Corporation

Members: James H. Crowley
Assistant Vice President
Aetna Life and Casualty

C. K. Khury
Actuarial Director
Prudential Property and Casualty
Insurance Company

Workshop D — "Education and Examination Discussion"

Moderator: Jeffrey T. Lange
Vice President
Royal-Globe Insurance Companies

Members: Phillip N. Ben-Zvi
Actuary
Royal-Globe Insurance Companies

Robert J. Schuler
Vice President
Blue Cross of Western Pennsylvania

Charles F. Cook
Vice President
American International Underwriters

Workshop E — "Generally Accepted Loss Reserve Methods"

Moderator: James R. Berquist
Consulting Actuary
Milliman and Robertson, Inc.

Members: Martin Bondy
Vice President
Crum & Forster Insurance Companies

Paul M. Otteson
Consultant

Workshop F — "Products Liability — A Problem or a Crisis"

Moderator: James B. Stradtner
Alex Brown & Sons
Manager Insurance Stocks Department

Members: Charles C. Henry
Secretary
The Travelers Insurance Companies

Andre Maisonpierre
Vice President
American Mutual Insurance Alliance

A reception for members and guests was held from 6:30-7:30 p.m.

Dinner and entertainment began at 7:30 p.m. with Matt Rodermund's "How to Succeed as an Actuary."

Wednesday, May 18, 1977

The first panel of the day entitled "What Every Casualty Actuary Should Know About *Life*," began at 9:00 a.m.

Moderator: W. James MacGinnitie
Consulting Actuary
Tillinghast, Nelson & Warren, Inc.

Members: Ardian Gill
Independent Consultant

Roland F. Dorman
Senior Vice President
Connecticut General Life
Insurance Company

Following coffee break was a panel on "Automobile Ratemaking".

Moderator: Charles C. Hewitt, Jr.
Vice President and Actuary
Metropolitan Property and Liability Insurance Company

Members: Paul S. Liscord
Vice President and Actuary
Workers' Compensation Rating and Inspection Bureau of Massachusetts

Michael A. Walters
Vice President-Actuary
Insurance Services Office

Gary Countryman
Assistant Vice President and Director of Corporate Research
Liberty Mutual Insurance Company

President Morison adjourned the meeting at 12:00 noon, expressing thanks to Paul Scheel, Vice President and Senior Actuary, United States Fidelity and Guaranty Company, who ably served as Chairman of Local Arrangements and to Dorothy Zelenko who assisted with the registration. In attendance, as indicated by registration records, were 148 Fellows, 105 Associates, 69 Guests and 93 Spouses.

A list of Fellows, Associates and Guests is shown below.

FELLOWS

Adler, M.
Alexander, L. M.
Angell, C. M.
Anker, R. A.
Arata, D. A.
Bailey, R. A.
Balcarek, R. J.
Balko, K. H.
Barnes, G. R.
Beckman, R. W.
Bennett, N. J.
Ben-Zvi, P. N.
Berquist, J. R.
Bethel, N. A.
Bevan, J. R.
Bickerstaff, D. R.
Bill, R. A.
Bland, W. H.
Blivess, M. P.
Bornhuetter, R. L.
Boyajian, J. H.
Brannigan, J. F.
Brian, R. A.
Brown, W. W.
Bryan, C. A.
Carter, E. J., Jr.
Conners, J. B.
Cook, C. F.
Crowley, J. H.
Curry, A. C.
Curry, H. E.
Davis, G. E.
Dropkin, L. B.

FELLOWS (CONT.)

Ehlert, D. W.
Eide, K. A.
Eliason, E. B.
Eyers, R. G.
Faber, J. A.
Fallquist, R. J.
Farley, J.
Ferguson, R. E.
Finger, R. J.
Fisher, W. H.
Fitzgibbon, W. J., Jr.
Flaherty, D. J.
Flynn, D. P.
Forker, D. C.
Foster, R. B.
Fowler, T. W.
Fresch, G. W.
Fusco, M.
Garand, C. P.
Gibson, J. A., III
Gillam, W. S.
Gottlieb, L. R.
Grady, D. J.
Groot, S. L.
Hachemeister, C. A.
Hall, J. A., III
Hartman, D. G.
Hartman, G. R.
Harwayne, F.
Hazam, W. J.
Heer, E. L.
Hewitt, C. C., Jr.
Hough, P. E.
Inkrott, J. G.
Johe, R. L.
Kates, P. B.
Keene, V. S.
Khury, C. K.
Kilbourne, F. W.
Klein, D. M.
Krause, G. A.
Lamb, R. M.
Lange, J. T.
Latimer, M. W.
Lester, E. P.
Levin, J. W.
Linder, J.
Liscord, P. S.
MacGinnitie, W. J.
Makgill, S. S.
Masterson, N. E.
McClenahan, C. L.
McClure, R. D.
McGuinness, J. S.
McLean, G. E.
Miller, P. D.
Moore, P. S.
Morison, G. D.
Muetterties, J. H.
Naffziger, J. V.
Nelson, D. A.
Newman, S. H.
Palczynski, R. W.
Petz, E. F.
Phillips, H. J.
Pollack, R.
Presley, P. O.
Price, E. E.
Quinlan, J. A.
Richards, H. R.
Richardson, J. F.
Rodermund, M.
Rogers, D. J.
Rosenberg, S.
Ross, J. P.
Roth, R. J.
Ryan, K. M.
Salzmann, R. E.
Scheel, P. J.
Scheibl, J. A.
Scheid, J. E.
Schuler, R. J.
Schultz, E. O.
Schultz, J. J.
Sheppard, A. R.
Simon, L. J.
Smick, J. J.
Smith, E. R.
Spitzer, C. R.
Stanard, J. N.
Steeneck, L. R.
Stephenson, E. A.
Stewart, C. W.
Streff, J. P.
Taht, V.
Tarbell, L. L., Jr.
Tatge, R. L.
Toothman, M. L.
Tverberg, G. E.
Verhage, P. A.
Walters, M. A.
Walters, M. A.
Ward, M. R.
Warthen, T. V.
Webb, B. L.
White, H. G.
Wieder, J. W., Jr.
Williams, P. A.
Wilson, J. C.
Woll, R. G.
Wood, J. O.
Yoder, R. C.
Young, R. J., Jr.
Zelenko, D. A.
Zubulake, T. J.

ASSOCIATES

Alff, G. N.
Applequist, V. H.
Bayley, T. R.
Bell, A. A.
Bell, L. L.
Bradley, D. R.
Buck, J. E., Jr.
Carbaugh, A. B.
Cassity, H. E.
Chorpita, F. M.
Cis, M. M.
Connor, V. P.
Cooper, W. P.
Costello, J. R.
Crifo, D. A.
Davis, R. D.
Donaldson, J. P.
Duperreault, B.
Eddy, J. H.
Eland, D. D.
Faga, D. S.
Feldman, M. F.
Fisher, R. S.
Foley, C. D.
Frohlich, K. R.
Gerlach, S. B.
Gleeson, O. M.
Gossrow, R. W.
Granoff, G.
Gruber, C.
Gutterman, S.
Gwynn, H. M.
Hafling, D. N.
Head, T. F.
Hermes, T. M.
Hobart, G. P.
Hoylman, D. J.
Inderbitzin, P. H.
Jensen, J. P.
Jerabek, G. J.
Johnson, W. H., Jr.
Johnston, D. J.
Jorve, B. M.
Kleinberg, J. J.
Klingman, G. C.
Kolojay, T. M.
Konopa, M. E.
Kozik, T. J.
LaFrenaye, A. C.
Ledbetter, A. R.
Leimkuhler, U. E.
Lindquist, P. L.
Lindquist, R. J.
Livingston, R. P.
Lommele, J. A.
Marks, R. N.
Martin, P. A.
Masella, N. M.
McManus, M. F.
Meyer, R. E.
Miccolis, J. A.
Miccolis, R. S.
Miller, D. L.
Millman, N. L.
Moller, K. G., Jr.
Moore, B. C.
Neidermyer, J. R.
Nelson, J. R.
Neuhauser, F., Jr.
Newlin, P. R.
Newville, B. S.
Oakden, D. J.
O'Neil, M. L.
Patrik, G. S.
Petersen, B. A.
Pflum, R. J.
Piazza, R. N.
Potter, J. A.
Powell, D. S.
Pratt, J. J.
Quirin, A. J.
Radach, F. R.
Reynolds, J. J., III
Rice, W. V.
Riff, M.
Rodgers, B. T.
Roland, W. P.
Rosen, K. R.
Shatoff, L. D.
Shoop, E. C.
Silberstein, B.
Singer, P. E.
Smith, F. A.
Steer, G. D.
Stein, J. B.
Teufel, P. A.
Thompson, P. R.
Torgrimson, D. A.
Wade, R. C.
Weiner, J. S.
Weller, A. O.
Wiser, R. F.
Wooddy, J. C.
Wright, W. C., III
Zatorski, R. T.

GUESTS

*Allen, T. C.
*Anderson, E. V.
Asson, D. C.
*Bednarczyk, F.
*Bell, A. M.
Berger, G.
*Brown, P. S.
Byrne, J. J.
*Chang, C. E.
Christhilf, D. A.
Coover, D.
Countryman, G. L.
Crane, J.
Decker, D.
Dorman, R. F.
*Gamble, R. A.
Gill, A.
*Glaser, L. S., Jr.
Gorman, L.
Gottlieb, J. A.
*Guido, R. N.
*Hager, G. A.
Hartz, M. L.
Havens, C., III
Henry, C.
*Hatfield, B. D.
Hightower, C. V.
Hill, R. J.
*Hoyt, F. A.
Inzerillo, J.
Jewell, W.
Johnson, M. A.
Keatts, G. H.
*Kedrow, W. M.
Kellison, S. G.
Klatt, K.
Klawitter, W. A.
Lachowicz, G.
Linahan, T. J.
Lyon, A. C.
Marlatt, M. H.
McMillen, R. H.
Miller, R. A., III
Murphy, F. X., Jr.
Odell, W. H.
Pelletier, C. A.
*Peterson, T. M.
Press, R. G., Jr.
Rowland, W. J.
Rubino, F.
Saffeir, H. J.
*Schmitt, A. J.
Shayer, N.
*Sheehan-Newick, C.
Simcock, C. E.
Smith, C. L.
Spangler, J. L.
Stark, M.
*Stenmark, J. A.
Stradtner, J. B.
Strickert, C.
Van Amerongen, R. J.
Van Ark, W. R.
Walker, G. M.
Wallach, M.
Walland, B. L.
Williams, J. P.
Yau, M. W.
*Yousri, A.

*Invitational Program

Respectively submitted,

Darrell W. Ehlert
Secretary

Volume LXIV, Part 2 No. 122

PROCEEDINGS

November 20, 21, 22, 1977

PROFESSIONALS' PROGRESS

PRESIDENTIAL ADDRESS BY GEORGE D. MORISON

All goes well with the Casualty Actuarial Society.

Membership continues to grow. More new Fellows were admitted in 1977 than in any year since the founding of the Society. The number of new Associates dipped this year because of the more stringent examination requirements but the climb is expected to resume next year.

Attendance at semi-annual meetings shows a handsome growth pattern for the wearers of each of the several colors of name badges. Some of us maintain that the quality of the programs has as much to do with these turn-outs as does the selection of sites. This particular meeting represents a milestone in the commitment and effort shown by those people who responded to the call for papers by accepting the challenge, handling the difficult deadlines, and sharing their experience and insights in reserving for losses.

I believe we are justified in feeling a sense of satisfaction, even pride, in the fame brought to our Society as its members branch out from the traditional actuarial department assignments into other areas of insurance company operations, into consulting work, reinsurance, risk management, and other endeavors where the value of an actuary's contribution may not have been recognized in the past.

Wherever we look, then, we see remarkable growth and progress for the Casualty Actuarial Society and we are inclined to relish this feeling of self-satisfaction; many folks labored long and hard during the lean years to bring us to this enviable position.

Does a successful present, though, guarantee a bright future? Not necessarily. As a profession which specializes in dealing with future events we can ill afford to enter our own future without some clear idea of what we think it should contain. I would therefore suggest a few specific areas where our attention might appropriately be directed.

The first of these areas is the very planning process itself. For each of the last seven years we have had a group—referred to, simply, as the Planning Committee—comprised of the Vice President and the three newly elected members of the Board whose charge has been to solicit plans for the coming year from the chairman of each committee in the Society. The Planning Committee transmits these responses together with any suggestions of its own in a report to the Board of Directors.

There certainly is merit in obtaining annual plans from each committee; for those which function under less rigid timetables than, say, the Nominating Committee, this survey may occasion a more meaningful program than would otherwise be undertaken. To be sure, asking for plans from representatives on joint committees and, especially, liaison representatives to other associations, becomes somewhat forced. However, it does help in achieving the secondary purpose of the Planning Committee's efforts—familiarizing the members of that group with the activities of all committees, a proven method of introducing the newly elected Directors to one of their responsibilities as members of the Board.

The Planning Committee, then, has provided a satisfactory means of overseeing short term developments. With a few notable exceptions, such as the creation of the Committee on Loss Reserves on recommendation of a recent Planning Committee, long-range plans have not attracted much attention from this group—or any other group.

In today's rapidly changing environment any organization that wants to exercise some degree of control over its future must engage in long-range planning. As recently as eighteen months ago we disbanded a short-lived committee of this type in the mistaken belief that moves toward a restructuring of the profession absolved our Society from the need to plan its own future. How short-sighted that decision was!

It seems ironic that we frequently devote time on our semi-annual meeting program to the topic generally labeled "corporate planning," which turns up as the most popular subject on the interest surveys of the membership, yet we have recently engaged in little or no such activity on behalf of the Society itself. This inertia is even more befuddling when it is recalled that one of the most far-reaching, productive efforts of recent years was that which culminated in the report issued in 1970 by the Committee on the Future Course of the Society—affectionately referred to as the Schloss Committee.

From the many thoughtful recommendations of that committee emerged such Society activities as seminars for high school and college students and faculty. This effort was mounted in the days when we were feverishly trying to attract more candidates into our Society. Extended discussions which led to more demanding examination requirements for Associateship status in lieu of a specified experience period were precipitated by the report of that Future Course Committee. Even the September 1976 one-day seminar on loss reserves grew out of a recommendation of that committee. Its efforts produced significant results and helped achieve the Society's growth and vitality we savor these days.

To ensure continued success, though, we need to embark immediately on an on-going, long-range planning program. A special committee should be appointed for that purpose with a clear charge to draw up 1-year, 2-year, and 5-year plans which would then be updated annually. This committee will, of course, work closely with the people discussing reorganization of the actuarial profession and with other standing committees whose activities are affected by changes in the size and the profile of our membership.

With this type of planning program in place, operating effectively, we will be able to phase into such changes as the inevitable move to a professionally staffed central office, more careful coordination of the Society's various publishing activities, or whatever changes are forecost by the committee—instead of reacting to individual crises as they strike.

Without waiting for suggestions from the long-range planners we should move boldly toward greater involvement by our younger members. The Society has been fairly successful over the years in recognizing the needs and interests of its newer members. Election to the Board of Directors within as little as five years of achieving Fellowship—not uncommon these days—helps to keep our decision-making "youth-oriented." With the continuing growth in membership, though, a more structured program is needed to ensure adequate consideration of the needs of the newer members.

To guarantee that the thoughts, ideas, and aspirations of this increasingly important segment of our Society be properly aired, the Board of Directors should authorize the formation of an Advisory Council to be made up of Fellows who became such within the last five years. The members of this Council might well be selected by each of the local affiliates to enhance the prestige of those organizations as well as to assure reasonable geographical balance on the Council.

The idea of such a vehicle for recognizing the importance of newer members was most recently presented to the Board of Directors by the 1973 Planning Committee. After discussions throughout that year, and part of the next, the idea was abandoned largely due to procedural difficulties and, apparently, some perceived lack of urgency. The time for complacency—if it indeed prevailed—has passed. By 1980, more than half of the members of the Casualty Actuarial Society will have achieved their designation in the decade of the 70's!

In addition to creating this Council for keeping the Society's leadership informed of the concerns of newer members we need to encourage even greater participation by new Fellows in committee and program work. Each new Fellow ought to volunteer to serve on—and be appointed to—the Education & Examination Committee where the individual's recent experience with the examining process can be used to keep the system responsive to the needs of the students. The tour on this committee should be brief to make way for other new Fellows and to allow quick movement into other types of committee work where the special talents and enthusiasm of these new Fellows can be channeled into the various Society activities.

The abundance of thoughtful suggestions available from our newer members was clearly demonstrated in the responses received to a recent request I sent this season's graduates, soliciting their ideas on what the Casualty Actuarial Society ought to be doing differently to respond to members' needs. The answers were gratifying in number and revealing in content. It was suggested, for example, that future meeting programs include such topics as statistical and mathematical techniques, management skills, and the exploration of the actuary's role in maximizing company profits. Several ideas were offered for improving the examination system in such ways as by publishing model answers. In the words of one respondent, this improvement would help to "lift the veil of secrecy" which he sees inhibiting communication between Fellows and candidates. Another very specific suggestion involves a perceived need for this Society to insist that loss reserves be certified by qualified casualty actuaries.

There is no shortage of timely, practical ideas among our newer members. It therefore seems obvious that we must continue to solicit those ideas and implement them as quickly as we can.

At the same time as we embark on a formalized long-range planning program and vigorously pursue a much higher degree of participation by newer members in the affairs of the Society, we should play a major role in the efforts directed toward reorganizing the entire profession.

Those of us who sometimes lose patience with the halting progress of this issue need only recall that it was a mere seven years ago that Dan McNamara, in his Presidential Address, stunned his listeners by suggesting that the CAS begin discussions with the Society of Actuaries on ultimately consolidating the two organizations. This proposal bordered on treason because of the pride most members took in the individuality of the Casualty Actuarial Society and their utter distaste for the thought of being swallowed up by that huge organization of mirthless "life actuaries."

Events since that time, especially those of the last eighteen months when the pace occasionally became frenetic, have been duly chronicled. What does not reveal itself, however, is any especially strong feedback from the membership on this issue, and the leaders of each of the actuarial bodies agree that it is axiomatic that strong support from the membership is needed if any type of reorganization is going to take place.

In weighing the reasons advanced to date for some change in the existing organizational arrangement—as each member of our Society should—I direct attention to the first of the reasons put forth by the Joint Committee on Organizational Coordination—clarification of what an actuary is and what an actuary does. As we succeed in filling the expanding need for those services which actuaries can uniquely provide, we can no longer take refuge in the happy mystery that used to surround our profession. We need to break out of that comfort zone in which we become accustomed to talking primarily to other actuaries, sometimes to underwriters, and less often to other industry people.

Whether we like it or not, actuaries, without qualification as to casualty, life, or pension, are moving into the public eye. When Jack Anderson considers our profession worthy of copy, we have, in one sense at least, arrived! In a syndicated column which was published on various dates around the end of August, this year, Anderson leveled a series of charges about actuaries' arcane tools and obfuscatory language used, in particular, to evaluate defense contractors' pension costs which, to a large extent, are borne by the taxpayer. For those who might be inclined to discount Mr. Anderson's polemics there is an article in the November 1977 issue of *Fortune* magazine presenting observations very similar to those of the syndicated columnist. One particularly incisive passage from the magazine article reads as follows, "Some actuaries are so riled about all the attention unfunded liabilities have been getting that they want to stop using the term altogether. Instead, they prefer calling them 'supplemental present values,' a euphemism worthy of an undertaker."

Pension actuaries, then, are clearly occupying a position in the public spotlight. The lesson to be heeded by casualty actuaries seems obvious. As public awareness of, and interest in, our specialty grows, we need to become more careful of what we say and do; our pronouncements will have to hold up under the glare of public scrutiny. One of the earliest casualties in this arena has been our traditional defense against the use of investment income in ratemaking. As we continued to mouth platitudes about unrealized capital gains, modest profit and contingency allowances, and the like, the outcome on the questions had already been decided by individual rate filers and some regulators. Our steadfastness in a losing cause did little to enhance our public image.

In a similar vein, we should discontinue such practices as attempting to justify the introduction of loss development factors in individual risk ratings in terms of some trumped up benefit to the insured when the real objective is improved cash flow for the insurer. Under public scrutiny we will have to say clearly what we really mean, and what we mean will have to make sense.

Besides acting responsibly to minimize the likelihood of misunderstandings, as a profession, we ought to mount a broad-gauged public relations effort to provide a more meaningful exposition of what an actuary is and what an actuary does. In so doing we might offset the impact of—or even forestall—publicity of the type cited above. Such a concentrated effort requires a strong national body representing all actuaries—however, and by whomever, defined. As we move toward the accomplishment of this goal of clearly defining to the public the role and qualifications of an actuary, we might even reduce the likelihood in the future of outsiders devising their own definitions.

Once a single body earns the right to speak for all actuaries, then other benefits follow beyond clarifying the function of the actuary. Public statements on actuarial questions could more easily be issued by such an organization than by the specialty groups. We have observed a great deal of activity and much success in the recent appearances of representatives of the American Academy of Actuaries before federal committees. As this type of activity expands, I believe we can look for the national body to issue more frequent public expressions of opinion on actuarial matters—a need which has been voiced by many of our members.

Another matter of concern to our members—the intrusion into the casualty/property area by actuaries who have not passed the casualty examinations—could be policed by the strengthening of a strong national body even before it solves the accreditation problem. While the details are not easy to set down, clearly one organization, made up of all actuaries, would be better able to impose strictures on its members' improper activities than would the several actuarial bodies in existence today.

Without discounting the other reasons advanced in favor of some type of reorganization of the actuarial profession, I have chosen to concentrate on the one involving clarification of what an actuary is and what an actuary does because I consider this need urgent. Other people may prefer different reasons, and still others may see no appeal in any of the reasons cited. It is important, though, that our members ponder the questions of reorganization, discuss the issues at every opportunity, and make their views known.

Whatever the outcome on reorganization, the organization which we call the Casualty Actuarial Society will continue to control its future to the extent it plans for that future, and will continue to welcome an increasing number of new members whose properly recognized interests will help give direction to that future. If we are willing to rise to the challenges provided by this continued growth, then those who follow us will be able to take the pride we do in a strong, successful society of professionals.

LOSS RESERVE ADEQUACY TESTING: A COMPREHENSIVE, SYSTEMATIC APPROACH

JAMES R. BERQUIST AND RICHARD E. SHERMAN

While specific guidelines for reserve adequacy testing may be established and specific examples of an actuarial approach to the testing of loss reserves may be offered for particular situations, loss reserving cannot be reduced to a purely mechanical process or to a "cookbook" of rules and methods. The utilization and interpretation of insurance statistics requires an intimate knowledge of the insurance business as well as the actuary's ability to quantify complex phenomena which are not readily measurable. As in the case of ratemaking, while certain general methods are widely accepted, actuarial judgment is required at many critical junctures to assure that reserve projections are neither distorted nor biased. That judgment is specifically required in such decisions as:

1. Ascertaining the optimal combination of the kinds of loss statistics to be used in a reserve analysis,
2. Assessing the impact of changes in company operations and procedures on the loss statistics to be utilized in a reserve analysis,
3. Adjusting the loss data for the influences of known and quantifiable events,
4. Evaluating the strengths and weaknesses of various reserving methods, and
5. Making the final selection of estimates.

Throughout the entire process of testing the adequacy of loss reserves, the actuary's expertise must be called upon in tailoring the methodology to the characteristics of the insurer's book of business, the specific data available, and to recent changes in company operations and procedures.

The purpose of this paper is to present what we believe to be some essential guidelines for any comprehensive and systematic approach to testing the adequacy of loss reserves. In this paper, these guidelines will often be illustrated by a specific example of an actuarial approach (among many) to a particular problem. Within the framework of these guidelines, however, much latitude exists for the development of a wide variety of actuarially sound approaches to loss reserving. These guidelines, and the sections devoted to discussion of them, are as follows:

1. A thorough understanding of the existing data base and the *trends* and *changes* underlying that data base is a prerequisite to the application of actuarially sound reserving methods. Familiarity with the underwriting, claims, data processing and accounting operations within a company, and knowledge of *changes* in the operations and procedures of these departments which have occurred during the experience period, are essential to the accurate interpretation and evaluation of various reserving methods. Comprehension of key developments and trends in the legal, regulatory and socio-economic environment in which an insurer operates is also a prerequisite to the formulation of accurate reserve estimates. (Section I).

2. Where possible, loss data which has been relatively unaffected by changes in company procedures and operations should be utilized in testing loss reserves. The possibility of subdividing or combining the data in order to increase its homogeneity or to minimize the distorting effects of underlying or procedural changes on the data should be fully explored. The quality and reliability of the various kinds of available data should also influence the choice of the forms of data to be analyzed. (Section II).

3. Whenever the impact of changes in company procedures or operations on loss data can be isolated or reasonably quantified, adjustment of the data may be advisable before applying various reserving methods. Whenever possible, the underlying assumptions of each method should be tested statistically. It may be possible to adjust the historical data so that the underlying assumptions of a method are more nearly satisfied. New projections may then be computed. (Section III).

4. No single reserving method can possibly produce the best estimates in all situations. Every reserving method is based on certain underlying assumptions, which may or may not be satisfied in a given situation.

Thus, several methods should be applied. Where possible, these should include:[1]

A. Projections of incurred losses,

B. Projections of paid losses,

C. Projections of ultimate reported claims and ultimate losses per ultimate reported claim,

D. Estimates of the number and average amount of outstanding losses, and

E. Loss ratio estimates.

Wherever appropriate, the concepts of credibility, regression analysis and data smoothing should be incorporated into the actuarial methods utilized. The methods applied should range from those which are highly stable (i.e., representative of the average of experience over several years) to those which are highly responsive to trends and to more recent experience. The actuary must then decide which methods provide the appropriate balance between stability and responsiveness in accordance with the credibility of the data and whether or not past trends may be expected to continue into the future. (Section IV).

5. In determining which methods are believed to be the best in a given situation, the following procedures should be implemented: (Section V).

   A. Whenever regression analysis has been incorporated into a method, some measure of goodness of fit (such as the coefficient of determination)[2] should be noted in evaluating the appropriateness of that method's projections. Additionally, the possibility that seasonal variations or cycles have been mistaken for trends should be carefully explored.

[1] Ruth Salzmann, "Estimated Liabilities for Losses and Loss Adjustment Expenses," Chapter 3, *Property-Liability Insurance Accounting*, ed. Robert W. Strain, The Merritt Company, Santa Monica, California, 1974; and David Skurnick, "A Survey of Loss Reserving Methods," PCAS, Vol. LX (1973), p. 16.

[2] G. G. C. Parker and E. L. Segura, "How to Get a Better Forecast," *Harvard Business Review*, March-April 1971, p. 99; and D. L. McLagan, "A Non-Econometrician's Guide to Econometrics," *Business Economics*, Vol. VIII, No. 3, May 1973, p. 38.

B. Whenever sufficient loss history is available, each method should be tested retrospectively to determine its historical record of accuracy and freedom from bias in projecting future paid losses. The projections of each method should then be adjusted for any detectable bias.

C. Significant differences between the projections of the various methods should be explained, where possible, in terms of changes in company procedures and operations. The convergence of the projections of several methods after the data has been adjusted for changes or trends in company procedures and operations (see Section III) may serve to considerably narrow the range of reasonable reserve estimates.

D. In making the final selections, the actuary must attach judgmental credibilities to basic as well as sophisticated methods as applied to both unadjusted and adjusted data. These judgmental credibilities should be based upon an evaluation of the relative strengths and weaknesses of each method in the context of the data to which it is applied.

E. A final check which should be applied to the selected estimates for the most recent accident years or quarters is a review of the loss ratios, pure premiums, frequencies and severities by accident period which result from those selections. The reasonableness of such statistics, when compared with those of immediately prior accident periods, may increase confidence in the reserve estimation process or raise questions which must be more thoroughly investigated before a conclusion is reached.

I. GATHERING DATA AND SEARCHING FOR PROBLEM AREAS

The first part of gathering information is the review of all available sources of data which may be reasonably utilized in a reserve analysis. It is, of course, unlikely (and unnecessary) that any given company will have all the data in the detail prescribed in Appendix A.

The reconciliation of the most important source documents and tabulations for a reserve analysis with the Annual Statement or with other public documents or audited data is a necessary and often instructive exercise.

Frequently an inability to reconcile reveals an unsuspected missing piece of the book of business which will also require analysis. Additionally, a review of the available data, with an eye to spotting significant shifts, changes, and seeming irregularities, can raise many questions. When such questions are directed to top management as well as underwriters, claims and data processing personnel, and accountants, they can yield invaluable insights into the interpretation of the history of losses which often could never have been obtained through the most sophisticated statistical analyses.

Another integral part of a reserve analysis is the development of a deeper understanding of changes in company operations which have occurred during the experience period. Such changes frequently result in distortions in the loss history that the actuary is analyzing in his attempts to forecast future developments and trends. The actuary must concern himself seriously with the task of determining the nature of such changes and the extent to which such changes have affected the data under analysis. To do this, the actuary should engage in discussions with the most knowledgeable members of management within the underwriting, claims, data processing, and accounting departments and with the actuaries specializing in ratemaking.

Appendix B provides a sampling of the kinds of questions which can be directed to the management of the various departments in an effort to pinpoint problem areas and to more accurately interpret the loss data and reserve projections. Throughout the course of these discussions, the unquestioning acceptance of opinions should naturally be avoided. Wherever possible, supplementary data should be sought to support and to quantify (or to counter) the opinions expressed.

II. TREATING PROBLEM AREAS THROUGH DATA SELECTION AND REARRANGEMENT

Appendix C provides a sampling of the types of problems which can seriously affect the consistency of loss data or cause subsequent losses to develop in ways markedly different from past patterns. To consistently and effectively deal with such problems, a systematic and analytic approach is often helpful. The following questions provide an outline of one such approach:

1. What type of event or trend could potentially cause data problems?

2. What is the expected impact of this problem on each of the available forms of data? On each of the proposed reserve methods?

3. Will this problem result in shifts in the loss data between *successive* accident years? Calendar years? Report years? Policy years? Years of development? Are such shifts observable in the loss data?

4. Is the problem serious enough to warrant further attention?

5. Will the problem be so serious as to render past history irrelevant in predicting future developments?

6. What forms of data and actuarial methods will be substantially unaffected by this problem? How can these be used in a reserve analysis? Can the available data be subdivided or reorganized to isolate the problem?

7. Does there exist supplementary data which accurately quantifies the magnitude of the impact of the problem?

Essentially, there are two stages in this analysis. In the first, the nature of the problem is defined. Its impact is estimated and, whenever possible, accurately quantified. In the second stage, the search for solutions, one of two general approaches is followed:

1. Utilization of data and actuarial methods which are relatively unaffected by the problem.

2. Accurate quantification of the impact of the problem and the application of adjustments to the data before utilizing the various reserve methods.

The first approach will be discussed in this section and the second in Section III.

Two primary means may be employed in obtaining data which is relatively unaffected by a given problem. The first is the selection of substitute types or forms of data. Examples of this would include the following:

1. Utilization of earned exposures in place of claim counts when count data is of questionable accuracy or there has been a major change in the definition of claim count.

2. Substitution of policy year data for accident year data when there has been a significant change in policy limits or deductibles between successive policy years.

3. Substitution of report year data for accident year data when there has been a dramatic shift in the social or legal climate which causes claim severity to more closely correlate with the report date than with the accident date.

4. Substitution of accident quarter for accident year data when the rate of growth of earned exposures changes markedly, causing distortions in development factors due to significant shifts in the average accident date within each exposure period.

The second means of obtaining relatively unaffected data is that of subdividing the loss experience into more homogeneous groups of exposures and/or types of claims. This is particularly desirable whenever there have been major changes in the composition of business by state, subline, class, territory or size of risk. However, it may not be advisable if it results in a marked decline in the credibility of each new block of experience.

The subdivision of loss experience into more homogeneous types of claims is particularly important whenever the types of claims in the experience are widely heterogeneous or a particular procedural change impacts only a few types of claims. While it may be possible to recompile loss experience based on types of claims (e.g., property versus liability losses under multi-peril policies), it is sometimes more expedient to use certain characteristics of various types of claims to segregate their loss experience fairly effectively. Such characteristics include the lag between accident date and settlement date or adjuster's estimates of incurred losses. For example, in homeowners multi-peril, claims closed within the first two years of development are primarily property claims while those closed after the first two years are primarily liability claims. This observation suggests that claims closing after the first two years of development should be analyzed separately from those which closed within the first two development years.

Another effective means of accomplishing the segregation of claims into more homogeneous groups is the analysis of loss experience by separate size of loss categories or separate layers of loss.[3] Examples of this technique which have long been used in ratemaking are the separation of basic limits from total limits experience[4] and the determination of catastrophe loadings.[5] Similar procedures should also be employed in reserve analyses whenever large claims comprise a significant portion of total losses. An important refinement which should be a part of any size of loss analysis is that the definitions of each category should be adjusted for inflation over each successive accident year, as shown below.

Size of loss or layer of loss	Accident year	Months of Development 12	24	36
$ 1– 99	1974	x	x	x
1– 109	1975	x	x	
1– 120	1976	x		
$ 100– 999	1974	x	x	x
110– 1,099	1975	x	x	
121– 1,209	1976	x		
$ 1,000– 9,999	1974	x	x	x
1,100–10,999	1975	x	x	
1,210–12,099	1976	x		
$10,000 & over	1974	x	x	x
11,000 & over	1975	x	x	
12,100 & over	1976	x		

One problem which is susceptible to the size of loss approach is that of shifts in emphasis by the claims department on priorities in settling large versus small claims. Such a shift can cause major distortions in the loss

[3] Ruth Salzmann, "Rating by Layer of Insurance," PCAS, Vol. L (1963), p. 15; David R. Bickerstaff, "Automobile Collision Deductibles and Repair Cost Groups: The Lognormal Model," PCAS, Vol. LIX (1972), p. 68; Robert J. Finger, "Estimating Pure Premiums By Layer—An Approach," PCAS, Vol. LXIII (1976), p. 34; and Charles A. Hachemeister, "Breaking Down the Loss Reserving Process."

[4] Jeffrey T. Lange, "The Interpretation of Liability Increased Limits Statistics," PCAS, Vol. LVI (1969), p. 163.

[5] Michael A. Walters, "Homeowners Insurance Ratemaking," PCAS, Vol. LXI (1974), p. 15.

projections of nearly all reserving methods. This problem may be adequately dealt with by analyzing loss history separately by size of loss category. Within each size of loss category, paid losses should be examined at equal percentiles of claims closed. (See Section III).

The analysis of loss experience by size of loss categories may also be quite effective in handling the problem of changes in the claims procedures for very small or trivial claims. For example, when guidelines for the establishment of a claim file for very small claims are changed, such a change may result in noticeable distortions in claim count data. These distortions may adversely affect frequency and severity projections for either ratemaking or reserving purposes. By defining several size of loss categories so that the experience for the very small claims is isolated, such distortions in count data can be adequately treated.

III. TREATING PROBLEM AREAS THROUGH DATA ADJUSTMENT

Whenever reformulations of the format of the data base will not yield satisfactory solutions to problems such as those enumerated in Appendix C, the primary alternative is the accurate quantification of the extent of the problem and the application of adjustments to the loss experience before utilizing it to estimate reserves. The existence of supplementary data which can accurately quantify the magnitude of the change should be fully explored in communications with other departments. In general, the nature of the problem and the kind of supplementary data available will often suggest the types of data adjustments to be made. The two most common problems encountered in reserve analyses are treated specifically in the remainder of this section.

Detecting Changes in the Adequacy Level of Case Reserves and Reducing the Impact of Such Changes on Incurred Loss Projections

The sensitivity of projections of ultimate losses based on incurred loss development factors to changes in the adequacy level of case reserves increases significantly for the long-tail lines. To illustrate this sensitivity and to indicate a general method for significantly reducing the distortions created by changing case reserve adequacy, an example from medical malpractice will be explained in this subsection.

The development of incurred losses for the eight most recent accident years and projections of ultimate losses based on average development factors is displayed in Exhibit A. Before utilizing the incurred projections derived in Exhibit A for reserving purposes, the primary underlying assumption of the incurred loss development method should be tested. Has the adequacy level of case reserves remained relatively constant during the experience period[6]? Several approaches may be taken in testing this hypothesis, but only one will be discussed here. In this approach, severity trends derived from changes in case reserves per open claim (Exhibit B) for each separate year of development are compared with severity trends in paid losses per closed claim (Exhibit C) for each separate year of development as well as over successive calendar years. The severity trends obtained from the fitting of exponential curves to the case reserves per open claim from Exhibit B range from + 27.6% to 34.2%, with the exception of such averages at 12 months of development. In contrast, the severity trends derived from the array of paid losses per closed claim in Exhibit C range from + 6.7% to 14.3%. Furthermore, the traditional approach of estimating the severity trend from the fitting of an exponential curve to calendar year paid losses per closed claim produces a trend of + 15.0% (with a coefficient of determination of .9793).

In the above example, no evidence was found which supported the notion that the severity trend for paid losses was inaccurate, and the indicated severity trend of + 15.0% was close to that experienced by many malpractice carriers. Thus, severity trends on the order of + 30% derived from changes in case reserves per open claim were rejected as unreasonable and the + 15% severity trend was selected as being representative of the underlying trend. The + 15% severity trend was thus used as the basis for adjusting the magnitude of case reserves in past years to their approximate value under the assumption that they are at the same relative adequacy level as the case reserves as of December 31, 1976. Working separately within each column of the array shown in Exhibit B, the value of case reserves per open claim as of December 31, 1976, was selected as the basis for readjusting the case reserves for past years. The year-end 1976 average case reserve was reduced by 15% per year for each year of development separately to obtain estimates of adjusted case reserves per open claim. Each adjusted average reserve estimate was then multiplied by the corresponding number of open claims (Exhibit D) to obtain an estimate of case reserves for some past year which is on approximately the same adequacy level as the year-end

[6] W. H. Fisher and E. P. Lester, "Loss Reserve Testing in a Changing Environment," PCAS, Vol. LXII (1975).

1976 case reserves. Each recomputed reserve was then added to the corresponding amount of cumulative paid losses (Exhibit E) to obtain a hypothetical history of incurred losses (Exhibit F) based on a relatively constant level of adequacy. The incurred projections obtained by again accepting the arithmetic mean of the development factors for each respective column are shown in the last column of Exhibit F.

In this example, the historical values of cumulative paid losses were adjusted to reduce the impact of increases in the rate of settlement of claims. Exhibit G provides a comparison of the reserve estimates derived from the incurred and the paid projections, both before and after the above adjustments. The aggregate difference between the paid and incurred estimates of loss reserves was reduced by 80% by applying the above adjustments, and apparent overstatements in those estimates were markedly reduced by these adjustments.

Detecting Changes in the Rate of Settlement of Claims and Adjusting Paid Losses for Such Changes

The importance of recognizing the impact of shifts in the rate of settlement of claims upon historical paid loss data has received previous attention in the Proceedings.[7] In this section a specific numerical method for making adjustments for changes in settlement rates will be described in detail. Exhibit H displays the accident year history of cumulative paid losses for automobile B.I. liability which will be adjusted for changing settlement rates. Exhibits I and J show the corresponding history of cumulative closed and cumulative reported claims. For each accident year, the ultimate claims disposed ratios contained in Exhibit K were derived by dividing the cumulative closed claims in Exhibit I by the projected ultimate number of reported claims in Exhibit J. Close examination of each column of claims disposed ratios for trends should reveal any persistent shifts in settlement rates. Caution should be exercised in this analysis and the impact of any procedural changes within the company should be particularly noted in terms of their influence on the claim count data from which these ratios were derived. In general, however, the absence of trend within the columns of Exhibit K indicates that no adjustment to the paid loss history in Exhibit

[7] David Skurnick, Discussion of "Loss Reserve Testing: A Report Year Approach" (W. H. Fisher and J. T. Lange, PCAS, Vol. LX (1973), p. 189), PCAS, Vol. LXI (1974), p. 73.

H would be recommended before analysis of such data by various actuarial methods.

Skurnick[8] has described a general approach to be taken in making adjustments for changing settlement rates. However, data in the format that Skurnick prescribes is frequently not available from many companies. A few minor modifications of Skurnick's approach, however, yields a more general method which can be applied to loss data maintained by most companies.

The first step in this process is the identification of a mathematical curve which closely approximates the relationship between the cumulative number of closed claims (X) and cumulative paid losses (Y). In the case of the automobile B.I. data in Exhibits H and I, a curve of the form $Y = ae^{bX}$ fits exceptionally well. As Exhibit L indicates, the coefficient of determination of this curve, when fitted to the loss data from Exhibits H and I for accident year 1969, is .99573. This coefficient increases to .99821 when the first point is dropped. Of course, a different curve may be required for a different company or line of business and it may be that no simple mathematical function reasonably describes the above relationship. In that event, generalized numerical methods, such as Lagrange's formula[9], may be applied in the interpolation process.

Since the exponential curve ($Y = ae^{bX}$) very closely approximates the relationship between cumulative closed claims and cumulative paid losses in our example, it may be used as the basis for exponential interpolation in applying adjustments for shifting claims disposed ratios. First, a representative claims disposed ratio was selected for each year of development. Selection of the claims disposed ratios for the latest calendar year of the experience (1976) possesses some key advantages. First, it eliminates the need for extrapolation into the future in making adjustments, and second, it leaves the most recent values of cumulative paid losses for each accident year unadjusted. However, some adjustments may be necessary in the event that these selected ratios do not progress upward in a smooth fashion from lower to higher years of development.

The claims disposed ratios for calendar year 1976 appear as the column headings in Exhibit M. These ratios are then applied to the projected

[8] *Ibid*, p. 83.

[9] Stephen G. Kellison, *Fundamentals of Numerical Analysis*, Richard D. Irwin, Inc., Homewood, Illinois, 1975, pp. 100-102.

ultimate number of reported claims for each accident year to obtain the number of cumulative closed claims which would be equivalent to the indicated claims disposed ratio for that year of development and accident year. For example, for accident year 1969, a selected claims disposed ratio of 88.55% for 36 months of development is equivalent to 6,926 cumulative closed claims. Since the coefficient of determination (.99821) of the exponential curve is exceptionally high, interpolation by means of only a two point curve fit seems appropriate. In order to approximate the value of cumulative paid losses which corresponds to 6,926 cumulative closed claims, the exponential curve ($Y = ae^{bX}$) is fitted to the two points (6,616, $5,398,000) and (7,192, $7,496,000) for accident year 1969 (from Exhibits H and I). The resultant approximation of $6,441,000, as well as other similarly derived estimates, are shown in Exhibit N. These adjusted estimates of cumulative paid losses may then be analyzed by the methods described in Section IV (or by other suitable mathematical procedures) to derive a set of reserve estimates.

IV. APPLYING A VARIETY OF RESERVING METHODS

In this section, some of the methods of projection frequently utilized in reserve analyses will be described. The specific methods presented in this section are representative of those which we are currently utilizing and serve only as examples of what we believe are acceptable procedures. These methods will, of course, undergo refinement as continuing advances are made in actuarial science.

In this example, the data analyzed by these methods is the unadjusted automobile B.I. data introduced in Section III. This data is in the form of paid losses per ultimate reported claim. Projections of the ultimate number of reported claims were first derived from an analysis of the historical development of cumulative reported claims contained in Exhibit J. These estimates are shown in the last column of Exhibit J by accident year. For each individual accident year, cumulative paid losses at the end of each year of development (Exhibit H) were then divided by the projected ultimate number of reported claims. The resultant averages are shown in the upper portion of Exhibit O. For each accident year, two sets of averages are shown above the diagonal. The first is that of development year paid losses per ultimate claim, while the second is that of cumulative paid losses per ultimate claim. This array of averages was then analyzed by six projection methods and the resultant projections are shown in the lower triangle of

Exhibit O. These projections are displayed in clusters of six for ease of comparison, with the Method I estimate at the top, the Method II estimate next, and so forth. Usually an estimate is selected for each cluster of estimates or a particular method is selected and its estimates are totalled.

The methods described in this section are not limited in application to accident period data such as paid losses per ultimate claim. They may also be applied to report or policy period loss history (either paid or incurred) which is in a triangular form. While Methods I, II and V do not require that the loss data be divided by claim counts or exposures, Methods III, IV and VI do require this (unless the volume of business over the experience period has been changing at a constant rate)[10]. Each of these methods may also be applied to accident period arrays of reported claims or reported claims per earned exposure.

For the purpose of describing these methods, mathematical notation will be introduced in order to shorten the narrative. The following matrices will frequently be mentioned in this section:

A — Paid losses per ultimate claim

C — Cumulative paid losses per ultimate claim

D — Development factors of cumulative paid losses

A and C are (m) x (n) matrices ($m \geq n$) while D is ($m - 1$) x ($n - 1$). The A and C matrices represent loss data for m exposure periods over n periods of development:

PAID LOSS MATRIX

Exposure Period	Development Period					
	1	2				n
1	$a_{1,1}$	$a_{1,2}$	•	•	•	$a_{1,n}$
2	$a_{2,1}$	$a_{2,2}$	•			$a_{2,n}$
•	•	•		•		•
•	•	•			•	•
•	•	•				•
m-1	$a_{m-1,1}$	$a_{m-1,2}$	•	•	•	$a_{m-1,n}$
m	$a_{m,1}$	$a_{m,2}$	•	•	•	$a_{m,n}$

where $a_{i,j} = 0$ if $i + j \geq m + 2$.

[10] Methods III, IV and VI are primarily based on the application of estimated trend factors to loss statistics which have been divided by some measure of the volume of business or of claims. Such statistics would include claim frequency or severity, pure premium or paid losses per ultimate claim, but not incurred losses or paid losses.

The matrix D of development factors of cumulative paid losses is identical to that of development factors of the matrix C since a constant divisor is used for each accident year in deriving C from the array of cumulative paid losses.

In general terms, the six methods differ in terms of the data from which trend factors are estimated, the statistical technique utilized in estimating the trend factors and the data to which the estimated trend factors are applied in making projections. The following table summarizes these differences:

Method	Data from which Trend Factors are Estimated	Technique for Estimating Trend Factors	Data to which Trend Factors are Applied
	Projections of Paid Loss Development Factors		
I	D	Linear regression	D
II	D	Weighted average	D
V	D-1	Adjusted exponential	D-1
	Estimates of Claim Cost Growth Rates		
III	C	Exponential curve fit	A
IV	C	Adjusted exponential	A
VI	A	Adjusted exponential	A

As this table indicates, Methods I, II and V are based upon projections of paid loss development factors. These methods differ only with respect to the statistical technique which is applied to the paid loss development factors (Exhibit P) in order to project the factors shown in Exhibit Q. In Method I, linear regression projections are determined separately for each year of development. In Method II, a weighted average of the development factors is computed for each column (development year) of Exhibit P. As can be seen from Exhibit Q, this weighted average is assumed to be constant for each year of development. In Method V, an adjusted exponential projection technique is applied. An exponential growth rate (trend factor) is first determined for each column of the array D-1 (1.0 is subtracted from each factor shown in Exhibit P). A weighted average of these growth rates is then obtained for the entire matrix. This weighted average is then "credibility weighted" with the initially determined growth rate of each column to determine the adjusted growth rate for that column. This adjusted growth rate is then utilized in projecting the development factors shown in the Method V section of Exhibit Q. For each of Methods I, II, and V, the development factors shown in Exhibit Q are applied successively to the corresponding average of cumulative paid losses per ultimate claim to estimate such cumulative averages for each future development period. These cumulative averages are then de-cumulated to obtain the estimates shown in Exhibit O.

The computations utilized in the development of estimates by Methods III, IV and VI all involve the following steps:

1. Estimation of claim cost growth rates for each year of development.

2. Utilization of the estimated growth rates to increase historical values of paid losses per ultimate claim during a given year of development to the estimated calendar year 1977 claim cost level.

3. Estimation of paid losses per ultimate claim during calendar year 1977 for the given year of development by computing a weighted average of past paid losses per ultimate claim on the estimated 1977 claim cost level.

4. Estimation of paid losses per ultimate claim during calendar years beyond 1977 by successive applications of the estimated claim cost growth rate to the estimate for calendar year 1977.

The calculations required by the second and third steps above are shown in Exhibit R for Methods III, IV and VI. The calculations shown are only those required to compute the estimates of paid losses per ultimate claim for accident year 1974 during calendar year 1977 (the fourth year of development). These calculations differ only in terms of the trend factors in Column 2 which are used to increase past paid losses per ultimate claim to the 1977 cost level. In this example, square weights are utilized but many other weighting procedures, as deemed appropriate, may be used here.

The calculations required by the fourth step of Methods III, IV and VI are illustrated in Exhibit S. For each method and year of development, a constant growth rate is successively applied to the estimate of that method for calendar year 1977.

In the above discussion, a broad framework within which the various methods may be viewed has been described. The remainder of this section contains a more precise, mathematical description of those methods.

Method I—Linear Regression Estimates of Paid Loss Development Factor

In this method, a linear least squares trend line is fitted to each column of D which contains three or more development factors. These fitted regression lines are then used to project down each column to provide estimates of the future development factors. Let D denote the array that includes all the projections, then

$$D = \begin{vmatrix} d_{1,1} & d_{1,2} & \cdot & \cdot & \cdot & d_{1,n-1} \\ d_{2,1} & d_{2,2} & \cdot & \cdot & \cdot & d_{2,n-1} \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ d_{m-1,1} & \hat{d}_{m-1,2} & \cdot & \cdot & \cdot & \hat{d}_{m-1,n-1} \\ d_{m,1} & \hat{d}_{m,2} & \cdot & \cdot & \cdot & \hat{d}_{m,n-1} \end{vmatrix}$$

where each $\hat{d}_{i,j}$ denotes a linear least squares estimate (if $m \geq n + 2$).

If $m = n$, the last two columns are taken to be

$\hat{d}_{i,n-1} = d_{1,n-1}, i = 2, 3, \cdots, m$ and

$\hat{d}_{i,n-2} = (d_{1,n-2} + d_{2,n-2})/2, i = 3, 4, \cdots, m.$

If $m = n + 1$, the last column is defined by

$\hat{d}_{i,n-1} = (d_{1,n-1} + d_{2,n-1})/2, i = 3, 4, \cdots, m.$

In both of these special cases, the remaining $\hat{d}_{i,j}$'s are the linear least squares estimates.

The Method I estimates of average paid losses per ultimate claim can then be calculated as follows, for $1 \leq k \leq m$, $1 \leq j \leq n$:

$$\hat{c}_{k,j} = \begin{cases} c_{k,j} & \text{if } k + j < m + 2 \\ c_{k,m+1-k} \times \prod_{l=m+1-k}^{j-1} \hat{d}_{k,l} & \text{if } k + j \geq m + 2 \end{cases}$$

$$\text{and} \quad \hat{a}_{i,j} = \begin{cases} \hat{c}_{i,j} - \hat{c}_{i,j-1} & i + j \geq m + 2 \\ a_{i,j} & i + j < m + 2 \end{cases}$$

$\hat{A}_I$, the array of Method I estimates is then:

$$\hat{A}_I = \left| \begin{array}{cccccc} a_{1,1} & a_{1,2} & \bullet & \bullet & \bullet & a_{1,n} \\ a_{2,1} & a_{2,2} & \bullet & \bullet & \bullet & a_{2,n} \\ \bullet & \bullet & \bullet & & & \bullet \\ \bullet & \bullet & & \bullet & & \bullet \\ \bullet & \bullet & & & \bullet & \bullet \\ a_{m-1,1} & a_{m-1,2} & \bullet & \bullet & \bullet & \hat{a}_{m-1,n} \\ a_{m,1} & \hat{a}_{m,2} & \bullet & \bullet & \bullet & \hat{a}_{m,n} \end{array} \right|$$

Method II—Weighted Average Estimates of Paid Loss Development Factors

A set of weights, $W = \{w_1, w_2, \cdots, w_{m-1}\}$, is first selected. These selections may be based upon such factors as the credibility of the data and/or a general assessment of the comparative relevance of newer (versus older) experience to future developments. In general, with data of low credibility and large random variations, weights of relatively equal magnitude should be used. With data of full credibility greater weight should generally be assigned to more recent experience simply on the grounds that it is likely to be more relevant to future developments.

For the jth column of D, the weighted average estimate is

$$\hat{d}_{i,j} = \frac{\sum_{k=1}^{m-j} d_{k,j} w_{j+k-1}}{\sum_{k=1}^{m-j} w_{j+k-1}}, i+j > m.$$

Thus, all of the projected development factors for any given column (i.e., period of development) are identical. The notational representation for D is the same as that for Method I. The $\hat{c}_{i,j}$'s and hence $\hat{A}_{II}$ are then calculated according to the same equations as those for Method I.

Method III—Exponential Curve Fit Estimates of Claim Cost Growth Rates

In Method III, a growth rate β_j, for the jth development period is computed by fitting an exponential curve $y = \alpha_j e^{\beta_j x}$ to the jth column of the matrix C. Once all β_j's have been estimated, then each element of A is adjusted to current cost level by applying the appropriate power of $\gamma_j = e^{\beta_j}$ to $a_{i,j}$:

$$A' = \left| \begin{array}{cccccc} \gamma_1^{m-1} \cdot a_{1,1} & \gamma_2^{m-2} \cdot a_{1,2} & \cdot & \cdot & \cdot & \gamma_n^{m-n} \cdot a_{1,n} \\ \gamma_1^{m-2} \cdot a_{2,1} & \gamma_2^{m-3} \cdot a_{2,2} & \cdot & \cdot & \cdot & \gamma_n^{m-n-1} \cdot a_{2,n} \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ \gamma_1^2 \cdot a_{m-2,1} & \gamma_2 \cdot a_{m-2,2} & \cdot & \cdot & \cdot & 0 \\ \gamma_1 \cdot a_{m-1,1} & a_{m-1,2} & \cdot & \cdot & \cdot & 0 \\ a_{m,1} & 0 & \cdot & \cdot & \cdot & 0 \end{array} \right|$$

A weighted average $\hat{a}_j$, of the adjusted $a_{i,j}$'s, $\{\gamma_j^{m-i-j+1} \cdot a_{i,j};\ i = 1, \cdots, m - j + 1\}$ is then computed for each column:

$$\hat{a}_j = \frac{\sum_{i=1}^{m-j+1} w_i \gamma_j^{m-i-j+1} a_{i,j}}{\sum_{i=1}^{m-j+1} w_i}$$

Each $\hat{a}_j$ is then projected into the future by applying the appropriate power of γ_j to increase $\hat{a}_j$ from current cost level to expected future cost levels:

$$\hat{A}_{III} = \left| \begin{array}{ccccccc} a_{1,1} & a_{1,2} & a_{1,3} & \cdot & \cdot & \cdot & a_{1,n} \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdot & \cdot & \cdot & a_{2,n} \\ \cdot & \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & & \cdot & \cdot \\ a_{m-1,1} & a_{m-1,2} & \gamma_3 \cdot \hat{a}_3 & \cdot & \cdot & \cdot & \gamma_n^{n-2} \cdot \hat{a}_n \\ a_{m,1} & \gamma_2 \cdot \hat{a}_2 & \gamma_3^2 \cdot \hat{a}_3 & \cdot & \cdot & \cdot & \gamma_n^{n-1} \cdot \hat{a}_n \end{array} \right|$$

Method IV—Adjusted Exponential Estimates of Claim Cost Growth Rates

Method IV is a modification of Method III. In Method IV, each γ_j derived in Method III is adjusted to γ_j' before it is applied in computing $\hat{a}_j$ and $\hat{a}_{i,j}$. These adjustments are made by the credibility-type formula

$$\gamma_j' = [w_j\gamma_j + (w_1 - w_j)\gamma]/w_1$$

where $\gamma = \sum_{j=1}^{n} w_j\gamma_j \Big/ \sum_{j=1}^{n} w_j \cdot$

and w_j is taken to be $(m + 1 - j)^2$, the square of the number of historical averages in the jth column of A. The determination of $\{\gamma_j', j = 1, \cdots, n\}$

from $\{\gamma_j, j = 1, \cdots, n\}$ is illustrated in Exhibit T. In that exhibit, each γ_j is referred to as an initial growth rate, $\gamma(1.0797)$ is the overall growth rate and each γ_j' is an adjusted growth rate. Each γ_j' may thus be viewed as the credibility weighted average of the initial growth rate, γ_j, and the overall growth rate, γ. In terms of Exhibit T, this equation becomes

$$\gamma_j' = \frac{\{(\text{Col. } 2 \times \text{Col. } 3) + (\Sigma\text{Col. } 4) \times ((64/203) - \text{Col. } 3)\}}{(64/203)}$$

In Method IV estimates are then calculated from the adjusted $a_{i,j}$'s, $\{(\gamma_j')^{m-i-j+1} \cdot a_{i,j};\ i = 1, \cdots, m - j + 1\}$ in the same way as for Method III.

Method V—Adjusted Exponential Estimates of Paid Loss Development Factors

The adjusted exponential projection technique utilized in Method V is completely analogous to that described for Method IV. The only differences between these methods are summarized in the first table of this section. In Method IV, $\{\gamma_j', j = 1, \cdots, n\}$ is derived from the matrix C, but in Method V it is derived from D-1. In Method V, $\{\gamma_j', j = 1, \cdots, n\}$ is applied to D-1, instead of to A (Method IV). The matrix D-1 is obtained by subtracting 1.0 from each nonzero element of D.

Method VI—Adjusted Exponential Estimates of Claim Cost Growth Rates

Method VI differs from Method IV only with respect to the matrix from which the set of initial growth rates, $\{\gamma_j\}$, is derived. For Method IV, the γ_j's are derived from the matrix C, while for Method VI, they are derived from A. Both of these methods are applied to the matrix A in determining their estimates. Thus, Method VI is more responsive to the growth rates of paid losses per ultimate claim for higher years of development.

****

The six methods described in this section provide an example of a group of methods which comprises a range of varying degrees of stability versus responsiveness. If these methods were ranked from the most stable to the most responsive, they would probably appear in this order: II, IV, III, I, VI, and V. Additionally, three of these methods are based on the development factor hypothesis, while the other three are based on a growth rate hypothesis. Stated in another way, Methods I, II and V are based upon projections across each row of the triangular array of loss data by means of estimated development factors. On the other hand, Methods III, IV and VI are based on projections down each column of the triangular array by means of estimated claims cost growth rates.

V. ANALYZING THE PROJECTIONS OF THE VARIOUS METHODS AND DETERMINING SELECTIONS

Every method of estimating loss reserves is based on the general assumption that the future can in some way be extrapolated from the past. Each method is also based on various specific assumptions such as the consistent relative adequacy of case reserves (incurred projections) or consistent settlement rates (paid projections). To the extent that the underlying assumptions of a method are violated in a systematic and non-random manner, the projections of that method will likewise be systematically distorted. Thus, an evaluation of the extent to which the underlying assumptions of a method are violated should become a vital part of the process of making actuarial projections. When the actuary applies a variety of projection techniques and thereby obtains a range of reserve estimates, he is then faced with the task of making value judgments on the relative appropriateness of each method. The mere taking of an average of the initial estimates may not be a satisfactory approach, although this procedure has more merit than the blind acceptance of the projections of only one method. As the example of the medical malpractice estimates in Section III indicates, it may well happen that the actual range of reasonable (i.e., adjusted) estimates lies entirely outside the range of the initial estimates (see Exhibit G).

In terms of the automobile B.I. experience shown in Exhibits H through T, an examination of the primary underlying assumptions for projections of incurred and paid losses yielded the following observations:

First, the relative adequacy level of case reserves has increased significantly in the last four years, indicating that projections of incurred losses will most likely overestimate reserves.

Second, the rate of settlement of open claims has generally declined over the last eight years and has undergone a major drop between 1975 and 1976. Because of this decline, it may be expected that projections of paid losses will tend to underestimate loss reserves.

Application of the adjustments described in Section III to the automobile B.I. data results in a reduction of the incurred estimate of total loss reserves from $43.5 to $40.6 million and an increase in the paid projections from $35.3 to $42.5 million. These adjustments have thus resulted in reducing the difference between the incurred and the paid projections by 76%. A comparison of the initial and the adjusted loss reserve estimates is provided in Exhibit U.

Retrospective Tests for Bias

Retrospective tests for bias and accuracy can provide much information regarding the appropriateness of various methods in testing loss reserves. Such tests are, however, not infallible, for it may happen that the underlying changes in the data during the experience period (which caused a particular method to under or overestimate) may not continue to occur in the future. The actuary must therefore exercise judgment as to the validity of these tests as a measure of the accuracy and bias in future projections.

A method of estimating the accuracy and bias of each of the six methods described in Section IV is exemplified in Exhibit V. Method II projections are used in this illustration. The top section of Exhibit V displays some of the historical values of paid losses per ultimate claim from Exhibit O. For each of the averages contained in the top section, estimates of that average were developed by each of the six methods—based entirely on data from Exhibit O for calendar years prior to that of the given average. The estimates thereby developed by Method II are shown in the middle section of Exhibit V. The percentage deviations for Method II, are shown in the bottom section of Exhibit V. The average and median of these percentage deviations were then computed. The results for each method were as follows:

Method	Average Deviation	Median Deviation
I	− 0.48%	− 0.48%
II	− 4.44	− 4.95
III	− 6.14	− 7.16
IV	− 6.64	− 7.76
V	+ 2.16	+ 2.34
VI	− 4.25	− 4.89

Both the average and median deviations for each method (except V) are negative. This indicates that each of these methods has historically underestimated the actual values of paid losses per ultimate claim for the first calendar year subsequent to the known data. By way of comparison, the method of utilizing the arithmetic mean of the prior development factors would produce an average deviation of -6.28% and a median deviation of -8.02%. Thus, each of the above methods, with the possible exception of Method IV, would appear to be preferable to simply employing the mean of the development factors.

Under the hypothesis that the future projections of each method will have percentage deviations equal to the median deviation shown above, the projections of each method shown in Exhibit O may be adjusted for their expected bias. The median was selected instead of the mean since the latter can easily be distorted by extreme values. Since the formula for each percentage deviation is:

$$\text{Percentage Deviation} = \frac{100 \times (\text{Estimated Value} - \text{Actual Value})}{\text{Actual Value}}$$

a rearrangement of this equation becomes:

$$\text{Actual Value} = \frac{100}{100 + \text{Percentage Deviation}} \times \text{Estimated Value}$$

Thus, the adjustment factor for a projection of one year was taken to be the quantity, $\{100/(100 + \text{Percentage Deviation})\}$. As an approximation to the adjustment factor for a projection of N years, the quantity, $\{100/(100 + \text{Percentage Deviation})\}^N$, was used.

The coefficient of variation of the retrospectively adjusted estimates of the total reserve was 71% less than that of the initial estimates, indicating that the range between the various adjusted estimates is noticeably less than the range between the initial estimates. This observation tends to lend support to the appropriateness of the adjusted estimates.

Selection of Estimates

For each accident year, the selected estimate in Exhibit U is a weighted average of the various projections. The weights were selected on the basis of a judgmental assessment of the relative strengths and weaknesses of each method.

As a check of the reasonableness of the selected estimates in Exhibit U for the most recent accident years, the projections of ultimate losses corresponding to the selected reserve estimates, as well as the projections of the ultimate number of reported claims (Exhibit J), were translated into the resultant loss ratios, frequencies, severities and pure premiums:

	Accident Year			Percentage Change	
	1974	1975	1976	AY 1975 / AY 1974	AY 1976 / AY 1975
Loss Ratio	78.0%	74.9%	76.3%	−4.0%	+1.9%
Frequency	.0197	.0195	.0201	−1.0	+3.1
Severity	$2,214	$2,545	$2,805	+15.0	+10.2
Pure Premium	$43.65	$49.57	$56.39	+13.6	+13.8

Since the degree of variability possible in the above statistics for accident year 1974 is much smaller than for 1975 or 1976, it was chosen as the basis for comparison of the loss statistics of 1975 and 1976. The reasonableness of the percentage changes in the above statistics between accident year 1974 and 1975, and 1975 and 1976, serve to verify the accuracy of the selected loss reserve estimates for accident years 1975 and 1976. If, on the other hand, the selected estimates result in apparently unreasonable loss statistics, this should not lead to the modification of the selections unless further investigation provides sufficient justification.

Concluding Remarks

We have gone to great lengths to explain *one* actuarial approach to the estimation of ultimate loss costs and the outstanding reserve associated with those costs. Although it has already been stated, we probably have not emphasized enough, that many times we do not have the luxury of obtaining *all* the data described herein. We have found, however, that by building a system which is designed to utilize such detail, we seem to obtain considerably more input data than one might expect.

Throughout this paper, we have attempted to emphasize that the successful reserving system must merge a great deal of basic information derived from "field oriented" executives *with* sophisticated actuarial methods.

Finally, it should be emphasized that however much the process of testing reserve adequacy may be refined and improved, reserve estimates will always be subject to a considerable degree of variability. The forecasting of future events is inherent in the act of estimating loss reserves. No matter how closely past events may be examined and analyzed, precise predictions of future events will never be obtainable. While this fact should serve to prevent us from becoming overconfident in our estimates, it should not, however, dissuade our profession from seeking to develop the best possible projections based on as much data and information as may be obtained at a reasonable cost. The importance of reserve analysis in safeguarding solvency and assisting the ratemaking actuary in the task of projecting ultimate losses for recent experience, should thoroughly convince us of the need for continuing advancements in this branch of actuarial science.

EXHIBIT A

MEDICAL MALPRACTICE

INCURRED LOSSES

(000's omitted)

Accident Year	MONTHS OF DEVELOPMENT 12	24	36	48	60	72	84	96	Projected Ultimate
1969	$ 2,897	$ 5,160	$10,714	$15,228	$16,661	$20,899	$22,892	$23,506	$ 23,506
1970	4,828	10,707	16,907	22,840	26,211	31,970	32,216		33,086
1971	5,455	11,941	20,733	30,928	42,395	48,377			52,247
1972	8,732	18,633	32,143	57,196	61,163				79,634
1973	11,228	19,967	50,143	73,733					112,443
1974	8,706	33,459	63,477						145,426
1975	12,928	48,904							215,275
1976	15,791								175,991

AVERAGE INCURRED LOSS DEVELOPMENT FACTORS

12-24	24-36	36-48	48-60	60-72	72-84	84-96
2.532	1.921	1.503	1.171	1.205	1.052	1.027

EXHIBIT B

CASE RESERVE PER OPEN CLAIM

Accident Year	MONTHS OF DEVELOPMENT 12	24	36	48	60	72	84	96
1969	$ 3,817	$ 5,660	$ 9,262	$10,151	$11,793	$16,627	$19,238	$21,423
1970	7,250	10,635	12,960	14,221	17,067	23,411	24,551	
1971	5,877	8,122	10,613	14,373	21,706	29,044		
1972	8,324	11,433	15,499	25,040	28,019			
1973	10,124	13,785	30,223	33,266				
1974	8,261	22,477	34,402					
1975	11,176	32,160						
1976	13,028							
Severity Trend	+15.3%	+29.5%	+31.1%	+34.2%	+32.8%	+32.2%	+27.6%	

EXHIBIT C

PAID LOSSES PER CLOSED CLAIM

Accident Year	MONTHS OF DEVELOPMENT							
	0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96
1969	$402	$ 539	$2,971	$ 8,620	$ 9,199	$12,669	$17,084	$16,634
1970	110	919	5,487	9,129	12,403	18,452	19,533	
1971	706	1,115	5,644	4,928	12,994	14,948		
1972	161	862	5,782	9,477	14,085			
1973	724	541	4,003	11,709				
1974	518	1,394	7,635					
1975	517	1,494						
1976	525							
Severity Trend	+12.9%	+12.0%	+11.5%	+6.7%	+14.2%	+8.6%	+14.3%	

EXHIBIT D

MEDICAL MALPRACTICE

OPEN CLAIMS

Accident Year	MONTHS OF DEVELOPMENT							
	12	24	36	48	60	72	84	96
1969	749	840	1,001	1,206	1,034	765	533	359
1970	660	957	1,149	1,350	1,095	755	539	
1971	878	1,329	1,720	1,799	1,428	1,056		
1972	1,043	1,561	1,828	1,894	1,522			
1973	1,088	1,388	1,540	1,877				
1974	1,033	1,418	1,663					
1975	1,138	1,472						
1976	1,196							

EXHIBIT E

MEDICAL MALPRACTICE

CUMULATIVE PAID LOSSES

(000's omitted)

Accident Year	MONTHS OF DEVELOPMENT 12	24	36	48	60	72	84	96	Projected Ultimate
1969	$ 125	$ 406	$ 1,443	$ 2,986	$ 4,467	$ 8,179	$12,638	$15,815	$ 23,506
1970	43	529	2,016	3,641	7,523	14,295	18,983		35,289
1971	295	1,147	2,479	5,071	11,399	17,707			46,322
1972	50	786	3,810	9,771	18,518				83,220
1973	213	833	3,599	11,292					99,042
1974	172	1,587	6,267						134,954
1975	210	1,565							124,997
1976	209								112,042

EXHIBIT F

ADJUSTED INCURRED LOSSES

(000's omitted)

Accident Year	MONTHS OF DEVELOPMENT 12	24	36	48	60	72	84	96	Projected Ultimate
1969	$ 3,707	$12,085	$18,564	$25,924	$23,516	$24,979	$24,017	$23,506	$ 23,506
1970	3,760	15,830	24,616	33,170	30,722	33,363	32,216		31,539
1971	5,982	25,585	41,385	50,323	46,191	48,377			45,668
1972	7,819	33,795	51,362	64,559	61,163				61,346
1973	9,533	34,586	49,668	73,733					68,719
1974	10,348	41,241	63,477						78,965
1975	13,102	48,904							92,918
1976	15,791								117,248

EXHIBIT G

COMPARISON OF RESERVE ESTIMATES

(000's omitted)

Accident Year	Before Adjustment: Incurred Projection	Before Adjustment: Paid Projection	Before Adjustment: Difference	After Adjustment: Incurred Projection	After Adjustment: Paid Projection	After Adjustment: Difference
1969	$ 7,691	$ 7,691	$ 0	$ 7,691	$ 7,691	$ 0
1970	14,103	16,306	−2,203	12,556	14,967	−2,411
1971	34,540	28,615	+5,925	27,961	25,607	+2,354
1972	61,116	64,702	−3,586	42,828	49,072	−6,244
1973	101,151	87,750	+13,401	57,427	79,665	−22,238
1974	139,159	128,687	+10,472	72,698	77,943	−5,245
1975	213,710	123,432	+90,278	91,353	88,931	+2,422
1976	175,782	111,833	+63,949	117,039	122,094	−5,055
	$747,252	$569,016	$+178,236	$429,553	$465,970	$−36,417

EXHIBIT H

AUTOMOBILE BODILY INJURY LIABILITY

CUMULATIVE PAID LOSSES

Accident Year	MONTHS OF DEVELOPMENT 12	24	36	48	60	72	84	96
1969	$1,904	$5,398	$ 7,496	$ 8,882	$ 9,712	$10,071	$10,199	$10,256
1970	2,235	6,261	8,691	10,443	11,346	11,754	12,031	
1971	2,441	7,348	10,662	12,655	13,748	14,235		
1972	2,503	8,173	11,810	14,176	15,383			
1973	2,838	8,712	12,728	15,278				
1974	2,405	7,858	11,771					
1975	2,759	9,182						
1976	2,801							

EXHIBIT I

CUMULATIVE CLOSED CLAIMS

Accident Year	MONTHS OF DEVELOPMENT 12	24	36	48	60	72	84	96
1969	4,079	6,616	7,192	7,494	7,670	7,749	7,792	7,806
1970	4,429	7,230	7,899	8,291	8,494	8,606	8,647	
1971	4,914	8,174	9,068	9,518	9,761	9,855		
1972	4,497	7,842	8,747	9,254	9,469			
1973	4,419	7,665	8,659	9,093				
1974	3,486	6,214	6,916					
1975	3,516	6,226						
1976	3,230							

EXHIBIT J

CUMULATIVE REPORTED CLAIMS

Accident Year	MONTHS OF DEVELOPMENT 12	24	36	48	60	72	84	96	Projected Ultimate
1969	6,553	7,696	7,770	7,799	7,814	7,819	7,820	7,821	7,822
1970	7,277	8,537	8,615	8,661	8,675	8,679	8,682		8,674
1971	8,259	9,765	9,884	9,926	9,940	9,945			9,950
1972	7,858	9,474	9,615	9,664	9,680				9,690
1973	7,808	9,376	9,513	9,562					9,590
1974	6,278	7,614	7,741						7,810
1975	6,446	7,884							8,092
1976	6,115								7,594

EXHIBIT K

AUTOMOBILE BODILY INJURY LIABILITY

ULTIMATE CLAIMS DISPOSED RATIOS

Accident Year	MONTHS OF DEVELOPMENT 12	24	36	48	60	72	84	96
1969	.52148	.84582	.91946	.95807	.98057	.99067	.99616	.99795
1970	.51002	.83257	.90960	.95474	.97812	.99102	.99574	
1971	.49387	.82151	.91136	.95658	.98101	.99045		
1972	.46409	.80929	.90268	.95501	.97719			
1973	.46079	.79927	.90292	.94818				
1974	.44635	.79565	.88553					
1975	.43450	.76940						
1976	.42534							

EXHIBIT L

UTILIZATION OF AN EXPONENTIAL CURVE TO ESTIMATE

CUMULATIVE PAID LOSSES

Accident Year 1969

Months of Development	X Cumulative Closed Claims	Y Cumulative Paid Losses	Predicted Y Value ($Y = ae^{bX}$) (8 Points)	Predicted Y Value ($Y = ae^{bX}$) (7 Points)
12	4,079	$ 1,904	$ 1,850	
24	6,616	5,398	5,885	$ 5,443
36	7,192	7,496	7,653	7,439
48	7,494	8,882	8,783	8,762
60	7,670	9,712	9,518	9,639
72	7,749	10,071	9,867	10,061
84	7,792	10,199	10,062	10,299
96	7,806	10,256	10,127	10,377
	Coefficient of Determination		.99573	.99821
		a	$287.741	$150.625
		b	.000456	.000542

EXHIBIT M

AUTOMOBILE BODILY INJURY LIABILITY

CUMULATIVE CLOSED CLAIMS

At Equal Percentiles of Ultimate Claims Closed

Accident	PERCENTAGE OF ULTIMATE CLAIMS CLOSED							
Year	42.53%	76.94%	88.55%	94.82%	97.72%	99.05%	99.57%	99.80%
1969	3,327	6,018	6,926	7,417	7,644	7,748	7,788	7,806
1970	3,693	6,681	7,690	8,234	8,486	8,602	8,647	
1971	4,232	7,656	8,811	9,435	9,723	9,855		
1972	4,121	7,455	8,580	9,188	9,469			
1973	4,079	7,379	8,492	9,093				
1974	3,322	6,009	6,916					
1975	3,442	6,226						
1976	3,230							

EXHIBIT N

CUMULATIVE PAID LOSSES

At Equal Percentiles of Ultimate Claims Closed

Accident	PERCENTAGE OF ULTIMATE CLAIMS CLOSED							
Year	42.53%	76.94%	88.55%	94.82%	97.72%	99.05%	99.57%	99.80%
1969	1,398	4,222	6,441	8,506	9,585	10,066	10,187	10,256
1970	1,705	5,116	7,845	10,160	11,309	11,739	12,031	
1971	1,938	6,168	9,580	12,261	13,571	14,235		
1972	2,191	7,127	11,034	13,843	15,383			
1973	2,523	7,892	11,943	15,278				
1974	2,240	7,189	11,771					
1975	2,670	9,182						
1976	2,801							

EXHIBIT O

AUTOMOBILE BODILY INJURY LIABILITY

A COMPARISON OF THE ESTIMATES OF METHODS I-VI PAID LOSSES PER ULTIMATE CLAIM

Accident Year	MONTHS OF DEVELOPMENT 12	24	36	48	60	72	84	96
1969	$244	$ 447	$ 268	$ 177	$ 106	$ 46	$ 16	$ 7
	244	690	958	1,136	1,242	1,287	1,304	1,311
1970	257	464	280	202	104	47	32	8
	257	721	1,001	1,203	1,307	1,353	1,385	8
								8
								8
								8
								8
1971	245	493	333	200	110	49	26	8
	245	738	1,072	1,272	1,382	1,431	27	8
							27	9
							28	9
							31	9
							31	9

EXHIBIT O (Cont'd.)

AUTOMOBILE BODILY INJURY LIABILITY

A COMPARISON OF THE ESTIMATES OF METHODS I-VI PAID LOSSES PER ULTIMATE CLAIM

Accident Year	MONTHS OF DEVELOPMENT 12	24	36	48	60	72	84	96
1972	258	585	375	244	124	55	30	10
	258	843	1,219	1,463	1,587	57	31	10
						52	29	9
						54	30	9
						61	40	11
						56	36	10
1973	296	612	419	266	130	58	32	10
	296	908	1,327	1,593	138	62	34	10
					134	55	31	10
					133	58	32	10
					146	69	49	13
					137	61	41	11
1974	308	698	501	307	143	65	37	12
	308	1,006	1,507	297	157	70	39	12
				283	145	58	33	11
				279	144	63	35	11
				320	172	83	64	16
				292	150	67	48	12

EXHIBIT O (Cont'd.)

AUTOMOBILE BODILY INJURY LIABILITY

A COMPARISON OF THE ESTIMATES OF METHODS I-VI PAID LOSSES PER ULTIMATE CLAIM

Accident	MONTHS OF DEVELOPMENT							
Year	12	24	36	48	60	72	84	96
1975	341	794	584	355	159	72	42	13
	341	1,135	526	327	173	78	42	13
			511	309	157	61	35	12
			503	303	156	68	38	12
			590	378	203	99	82	19
			539	323	164	74	55	14
1976	369	898	679	408	174	80	47	15
	369	819	550	342	181	81	44	14
		826	561	337	170	64	37	12
		822	548	328	168	73	41	13
		907	697	447	239	119	106	23
		855	606	357	179	81	64	15

EXHIBIT P

AUTOMOBILE BODILY INJURY LIABILITY

PAID LOSS DEVELOPMENT FACTORS

Accident Year	MONTHS OF DEVELOPMENT 24	36	48	60	72	84	96
1969	2.8341	1.3886	1.1849	1.0934	1.0369	1.0127	1.0057
1970	2.8005	1.3881	1.2016	1.0865	1.0359	1.0236	
1971	3.0090	1.4511	1.1868	1.0865	1.0354		
1972	3.2644	1.4450	1.2003	1.0851			
1973	3.0687	1.4611	1.2003				
1974	3.2664	1.4978					
1975	3.3272						
Average	3.0815	1.4386	1.1948	1.0879	1.0361	1.0181	1.0057
Average Latest 4	3.2317	1.4637	1.1973	1.0879	1.0361	1.0181	1.0057
Weighted	3.2192	1.4632	1.1969	1.0868	1.0359	1.0190	1.0057
Weighted Latest 4	3.2724	1.4782	1.1986	1.0860	1.0358	1.0197	1.0057

EXHIBIT Q

AUTOMOBILE BODILY INJURY LIABILITY

THE PROJECTED DEVELOPMENT FACTORS UTILIZED IN DETERMINING THE ESTIMATES OF METHODS I, II AND V

Accident Year	MONTHS OF DEVELOPMENT 12-24	24-36	36-48	48-60	60-72	72-84	84-Ult.
			Method I				
1970							1.0142
1971						1.0181	1.0142
1972					1.0346	1.0181	1.0142
1973				1.0816	1.0339	1.0181	1.0142
1974			1.2037	1.0791	1.0332	1.0181	1.0142
1975		1.5145	1.2066	1.0765	1.0325	1.0181	1.0142
1976	3.4344	1.5362	1.2096	1.0740	1.0317	1.0181	1.0142
Goodness of Fit	.7969	.8883	.3271	.7364	.9548	—	—

EXHIBIT Q (Cont'd.)

AUTOMOBILE BODILY INJURY LIABILITY

THE PROJECTED DEVELOPMENT FACTORS UTILIZED IN DETERMINING THE ESTIMATES OF METHODS I, II AND V

Accident Year	MONTHS OF DEVELOPMENT 12-24	24-36	36-48	48-60	60-72	72-84	84-Ult.
			Method II				
1970							1.0142
1971						1.0190	1.0142
1972					1.0359	1.0190	1.0142
1973				1.0868	1.0359	1.0190	1.0142
1974			1.1969	1.0868	1.0359	1.0190	1.0142
1975		1.4632	1.1969	1.0868	1.0359	1.0190	1.0142
1976	3.2192	1.4632	1.1969	1.0868	1.0359	1.0190	1.0142
			Method V				
1970							1.0145
1971						1.0219	1.0148
1972					1.0384	1.0245	1.0152
1973				1.0916	1.0399	1.0273	1.0155
1974			1.2120	1.0939	1.0414	1.0305	1.0159
1975		1.5201	1.2191	1.0963	1.0430	1.0341	1.0163
1976	3.4574	1.5467	1.2264	1.0988	1.0447	1.0381	1.0167
Goodness of Fit	.7966	.8831	.3304	.7417	.9567	1.0000	—

EXHIBIT R

AUTOMOBILE BODILY INJURY LIABILITY

DETERMINATION OF THE ESTIMATES OF PAID LOSSES PER ULTIMATE CLAIM BY METHODS III, IV AND VI FOR ACCIDENT YEAR 1974 DURING CALENDAR YEAR 1977

Accident Year	(1) Average Paid Losses Calendar Year AY+3	(2) Factor to Adjust Claim Costs to Calendar Year 1977 Level	(3) Estimated Average Paid Losses at Calendar Year 1976 Cost Level	(4) Weights	(5) Weighted Average of Column (3)
			Method III		
1969	$177.17	1.0912^5	$274.15	9/135	$ 18.28
1970	201.73	1.0912^4	286.06	16/135	33.90
1971	200.18	1.0912^3	260.13	25/135	48.17
1972	244.14	1.0912^2	290.72	36/135	77.53
1973	265.87	1.0912^1	290.13	49/135	105.31
			Method III Estimate =		$283.19

EXHIBIT R (Cont'd.)

AUTOMOBILE BODILY INJURY LIABILITY

DETERMINATION OF THE ESTIMATES OF PAID LOSSES PER ULTIMATE CLAIM BY METHODS III, IV AND VI FOR ACCIDENT YEAR 1974 DURING CALENDAR YEAR 1977

Accident Year	(1) Average Paid Losses Calendar Year AY+3	(2) Factor to Adjust Claim Costs to Calendar Year 1977 Level	(3) Estimated Average Paid Losses at Calendar Year 1976 Cost Level	(4) Weights	(5) Weighted Average of Column (3)
		Method IV			
1969	177.17	1.0842^5	265.44	9/135	17.70
1970	201.73	1.0842^4	278.76	16/135	33.04
1971	200.18	1.0842^3	255.13	25/135	47.25
1972	244.14	1.0842^2	286.99	36/135	76.53
1973	265.87	1.0842^1	288.26	49/135	104.63
		Method IV Estimate =			$279.15

EXHIBIT R (Cont'd.)

AUTOMOBILE BODILY INJURY LIABILITY

DETERMINATION OF THE ESTIMATES OF PAID LOSSES PER ULTIMATE CLAIM BY METHODS III, IV AND VI FOR ACCIDENT YEAR 1974 DURING CALENDAR YEAR 1977

	(1)	(2)	(3)	(4)	(5)
Accident Year	Average Paid Losses Calendar Year AY+3	Factor to Adjust Claim Costs to Calendar Year 1977 Level	Estimated Average Paid Losses at Calendar Year 1976 Cost Level	Weights	Weighted Average of Column (3)
			Method VI		
1969	177.17	1.1061^5	293.30	9/135	19.55
1970	201.73	1.1061^4	301.93	16/135	35.78
1971	200.18	1.1061^3	270.88	25/135	50.16
1972	244.14	1.1061^2	298.68	36/135	79.65
1973	265.87	1.1061^1	294.07	49/135	106.74
			Method VI Estimate =		$291.88

NOTE:
1. Factors shown in Column (2) are rounded. The actual factors used in the calculations are 1.091241 (Method III), 1.084217 (Method IV) and 1.106077 (Method VI).

EXHIBIT S

AUTOMOBILE BODILY INJURY LIABILITY

FINAL CALCULATIONS IN THE DETERMINATION OF THE ESTIMATES OF METHODS III, IV AND VI

Accident Year	Calendar Year AY + 1	AY + 2	AY + 3	AY + 4	AY +5	AY + 6	AY + 7
				Method III			
1970							11.84
1971						27.36	12.47
1972					52.21	29.07	14.26
1973				133.87	55.04	30.89	15.49
1974			283.19	144.92	58.02	32.82	17.31
1975		511.04	309.02	156.88	61.16	34.87	18.86
1976	825.61	560.59	337.22	169.82	64.47	37.06	20.20
Growth Rate	+8.82%	+9.70%	+9.12%	+8.25%	+5.42%	+6.25%	+7.76%
Goodness of Fit	.9668	.9764	.9695	.9337	.9991	1.0000	—

EXHIBIT S (Cont'd.)

AUTOMOBILE BODILY INJURY LIABILITY

FINAL CALCULATIONS IN THE DETERMINATION OF THE ESTIMATES OF METHODS III, IV AND VI

Accident Year	Calendar Year AY + 1	AY + 2	AY + 3	AY + 4	AY + 5	AY + 6	AY + 7
				Method IV			
1970							11.84
1971						27.89	12.47
1972					54.18	30.09	14.29
1973				133.33	58.31	32.45	15.53
1974			279.14	144.05	62.74	35.01	17.33
1975		503.06	302.65	155.63	67.52	37.76	18.81
1976	822.09	548.04	328.14	168.15	72.66	40.73	20.07
Growth Rate	+8.62%	+8.94%	+8.42%	+8.04%	+7.61%	+7.86%	+7.97%
Goodness of Fit	.9668	.9764	.9695	.9337	.9991	1.0000	—

EXHIBIT S (Cont'd.)

AUTOMOBILE BODILY INJURY LIABILITY

FINAL CALCULATIONS IN THE DETERMINATION OF THE ESTIMATES OF METHODS III, IV AND VI

Accident Year	Calendar Year						
	AY + 1	AY + 2	AY + 3	AY + 4	AY + 5	AY + 6	AY + 7
				Method VI			
1970							11.84
1971						30.62	12.50
1972					56.04	35.50	14.36
1973				136.74	61.43	41.15	15.67
1974			291.89	149.53	67.35	47.69	17.64
1975		539.36	322.85	163.52	73.83	55.28	19.58
1976	854.84	605.64	357.10	178.82	80.94	64.08	21.47
Growth Rate	+10.43%	+12.29%	+10.61%	+9.36%	+9.63%	+15.91%	+10.65%
Goodness of Fit	.9734	.9827	.9321	.7330	.9723	1.0000	—

EXHIBIT T

AUTOMOBILE BODILY INJURY LIABILITY

ESTIMATION OF ADJUSTED GROWTH RATES FOR

METHOD IV

Months of Development	(1) Slope of Trend Line	(2) Initial Growth Rate EXP (1)	(3) Weights	(4) Overall Growth Rate	(5) Adjusted Growth Rate
12	.0611	1.0630	64/203	.3351	1.0630
24	.0845	1.0882	49/203	.2627	1.0862
36	.0926	1.0970	36/203	.1946	1.0894
48	.0873	1.0912	25/203	.1344	1.0842
60	.0793	1.0825	16/203	.0853	1.0804
72	.0528	1.0542	9/203	.0467	1.0761
84	.0606	1.0625	4/203	.0209	1.0786
				1.0797	

EXHIBIT U

AUTOMOBILE BODILY INJURY LIABILITY

COMPARISON OF LOSS RESERVE ESTIMATES

(000's omitted)

	Initial Estimates			Adjusted Estimates			
				Mean of Methods I-VI			
					Incurred Loss Development		
Accident Year	Mean of Methods I-VI	Incurred Loss Develop-ment	Hindsight Average O/S Losses	Data Adjusted for Shifts in Settle-ment Rates	Projections Adjusted for Bias	Constant Adequacy Level	Selected Estimate
1969	$ 87	$ 87	$ 87	$ 87	$ 87	$ 87	$ 87
1970	173	192	192	187	174	192	192
1971	493	549	514	522	494	520	517
1972	1,092	1,256	1,231	1,282	1,147	1,240	1,233
1973	2,496	2,908	2,994	3,092	2,650	2,961	2,953
1974	4,592	6,161	5,688	5,950	4,904	5,714	5,576
1975	9,716	12,697	11,677	12,198	10,446	11,589	11,406
1976	16,653	19,678	18,704	19,196	18,044	18,267	18,618
1969-76	$35,301	$43,526	$41,087	$42,512	$37,945	$40,570	$40,582

EXHIBIT V

AUTOMOBILE BODILY INJURY LIABILITY

A RETROSPECTIVE TEST OF METHOD II

Accident Year	Calendar Year AY + 1	AY + 2	AY + 3	AY + 4	AY + 5
			Historical Averages		
1971	493.02	333.13	200.18	109.94	48.94
1972	585.08	375.30	244.14	124.45	
1973	612.34	418.83	265.87		
1974	698.12	500.83			
1975	793.75				
			Estimates Derived From Prior Calendar Years		
1971	443.52	286.66	209.55	113.41	50.12
1972	500.48	356.72	233.01	128.61	
1973	625.02	393.45	258.80		
1974	644.43	447.04			
1975	737.08				
			Percentage Deviations		
1971	−10.04%	−13.95%	+4.68%	+3.16%	+2.41%
1972	−14.46	−4.95	−4.56	+3.34	
1973	+2.07	−6.06	−2.66		
1974	−7.69	−10.74			
1975	−7.14				

APPENDIX A

RELEVANT DATA FOR A RESERVE ANALYSIS

I. Incurred, Paid and Outstanding Losses

This data may be provided by accident year, report year, policy year or calendar year (in descending order of preference) by year of development for the latest five to twenty years. The number of years of experience should be great enough to assure that any further development in reported counts or incurred losses will be negligible for the oldest years. Accident quarters, report months, etc., and quarters or months of development may be used in place of years. The loss history may include or exclude allocated loss adjustment expenses or may provide a separate history of such expenses in the same detail as losses. Paid losses should either include partial payment or paid losses on closed claims and partial payments should be shown separately and in the same detail. The losses may be direct, gross or net with respect to reinsurance and gross or net of salvage and subrogation recoveries.

The loss history should be provided separately for each line of business and, if possible, for major blocks of business within each line which represent more homogeneous groupings of risks or types of claims. These may include loss experience by subline, state, underwriting or claims office, size of risk, policy limit or deductible amount. They may also include separate loss experience for personal versus commercial risks, voluntary versus assigned risks and prospectively versus retrospectively rated risks. The loss history should also provide separate detail by size of loss or layer of loss, although this information is usually available only in terms of a listing of large claims or catastrophic losses. If possible, the loss history should be provided separately by kind of claim (fast track versus regular, medical versus indemnity for workers' compensation, or by status of the law suit). Where the book of business consists of a large number of small risks and a few large risks, it may be necessary to review the loss experience for the large risks separately.

II. Reported, Closed, Reopened and Outstanding Counts

This data should be provided in the same detail as the history of incurred, paid and outstanding losses. Closed counts may also be broken down into claims closed with payment and without payment.

III. Earned or Written Premium and Earned or Written Exposures

Earned or written premium may be provided at collected levels or at current rates. Premiums or exposures may be provided by year, quarter or month to match the detail of the loss history (but not its periods of development). Premiums or exposures should be provided separately for each line of business, for each significant subline, state, underwriting office, policy limit or deductible, and for each significant subdivision of business (e.g., personal versus commercial, voluntary versus assigned risk and prospectively versus retrospectively rated risks) even if a history of losses is not available in the same detail. Where a few large risks are underwritten, separate premium data for such risks may be useful.

IV. Miscellaneous Documents

A history of reinsurance treaties, the latest NAIC examination report, and annual statements as well as quarterly and annual reports to stockholders and/or policyhoders for the most recent three or four years are examples of documents which may provide useful additional information and insight into company operations.

V. Industrywide Frequency and Severity Data and External Indices

Such information is often available by line of business and may prove useful where company experience is not fully credible or marked changes in company operations or procedures have significantly altered or distorted frequency or severity trends. This type of data may be helpful even where reserves from specialty underwriting (such as substandard business) are being analyzed.

APPENDIX B

SAMPLE QUESTIONS FOR DEPARTMENT EXECUTIVES

Questions for a Claims Executive:

1. What specific objectives and guidelines does your department have in setting case reserves? Are case reserves established on the basis of what it would cost to settle the case today, or has a provision for inflation between now and the estimated time of settlement of the claim been included in the case reserve?
2. Have there been any significant changes in the guidelines for setting and reviewing case reserves during the last five years?
3. Have there been any changes in the definitions of or rules for establishing bulk or formula reserves for reported claims in the last five years?
4. Are any special procedures or guidelines applied in the reserving of large or catastrophic claims? If so, please describe.
5. Has the size of the caseload of the average claims adjuster changed significantly in the past several years?
6. When, in the sequence of events, is a claim file established?
7. Is a claim file established for each claimant or for each accident? What procedures are followed when there are multiple claimants from the same accident? Is a claim file established for each coverage or for all coverages combined?
8. What procedures are followed in recording reopened claims? Are such claims coded to the report date of the original claim or to the date of reopening? How will the reopening of a claim affect aggregate data for paid, open or reported claims and paid, outstanding or incurred losses?
9. Have there been any noticeable shifts in the reporting or nonreporting of very small or trivial claims? In the procedures for the recording of such?
10. Has there been any shift in emphasis in settling large versus small

claims? In the relative proportion of such claims? In attitudes in adjusting such claims?

11. Have there been any changes in the guidelines on when to close a claim? For example, is a P.D. claim kept open until the associated B.I. claim is closed, or only until the P.D. portion is settled?

12. Have there been any noticeable changes in the rate of settlement of claims recently?

13. Has there been any shift from the employment of company adjusters to independent adjusters? Or vice versa? If so, how has this affected the operations of the claims department?

14. Has there been any change in the timing of the payment of allocated loss adjustment expenses? For example, are such payments made as these expenses are accrued (or incurred) or when the claim is closed?

15. Has there been any change in the definition and limit for one-shot or fast-track claims in recent years? What is that limit?

16. What safeguards against fraudulent claims are now employed? Are any special procedures followed in the event of the filing of apparently questionable or non-meritorious claims? Have these safeguards changed in recent years?

17. Have there been any shifts toward (or away from) the more vigorous defense of suits in recent years?

18. Could you provide copies of all bulletins to the field issued in the last five years in which details of the changes in claims procedures are provided?

Questions for an Underwriting Executive:

1. What significant changes have occurred in your company's book of business and mix of business in the past five to seven years? How are the risks insured today different from those of the past?

2. Do you underwrite any large risks which are not characteristic of your general book of business?

3. Have any significant changes occurred in your underwriting guidelines in recent years?

4. Has the proportion of business attributable to excess coverages for self-insurers changed in recent years? Can a distribution of such business be obtained by line, retention limit, class, etc.? Is a record of self-insured losses and claims available?

Questions for a Data Processing or Accounting Executive:

1. Has there been any change in the date on which the books are closed for the quarter? the year?

2. How are loss payments handled for claims which have already been paid, but which have not yet been processed to the point where they can be allocated to accident quarter? Are they excluded from the loss history until they are allocated to accident quarter or are they loaded into an arbitrary quarter?

3. Have new data processing systems been implemented in recent years? Have they had a significant impact on the rate of processing claims or on the length of time required from the reporting to the recording of a claim?

4. To what extent have each of the data sources supplied (see Appendix A) been crosschecked and audited for accuracy and for balancing to overall company statistics? Comment on the degree of accuracy with which each kind of statistic has been properly allocated to accident quarter, to line of business, to size of loss, etc.

5. Have there been any changes in coding procedures which would affect the data supplied?

6. Would it be possible for partial payments to exceed the case reserve on a claim? In such an event, what adjustments are made? Are case reserves taken down by the amount of partial payments?

Questions for Actuaries Specializing in Ratemaking:

1. Have there been any changes in company operations or procedures which have caused you to depart from standard ratemaking procedures? If so, please describe those changes and how they were treated.

2. What data which is currently used for ratemaking purposes could also be used in testing loss reserves?

3. Have you noted any significant shifts in the composition of business by type of risk or type of claim within the past several years?

4. Do you have any of the following sources of information which may be of value in reserve testing:

   a. External economic indices,

   b. Combined loss data for several companies (e.g., data obtainable from bureau rate filings),

   c. Special rating bureau studies,

   d. Changes in state laws or regulations, and

   e. Size of loss or cause of loss studies?

APPENDIX C

PROBLEMS AFFECTING THE UNDERLYING ASSUMPTIONS OF LOSS RESERVE METHODS

Significant internal changes in company operations:

A. Claims procedures (See Questions for a Claims Executive in Appendix B)

B. Data processing and recording procedures (See Questions for a Data Processing or Accounting Executive in Appendix B)

C. Loss experience by heterogeneous groups of exposures or types of claims:

   1. Utilization of total limits experience when large claims comprise a significant portion of losses.

   2. Inclusion of catastrophic losses in the loss experience.

   3. Loss experience for the multi-peril coverages (a mixture of property and liability claims).

   4. Major changes in the rate of growth of earned exposures — causing a shift in the average accident date within a given accident period and producing distortions in development factors.

   5. Utilization of combined data for two or more types of risks when each type of risk comprises a significant portion of the experience.

      Examples would include combined data for:

      A. Large versus small risks.

      B. Personal versus commercial risks.

      C. Voluntary versus assigned risks.

      D. Direct business versus pooled risks.

      E. Prospectively versus retrospectively rated risks.

      F. Primary versus excess or umbrella business

      G. Different states, sublines, classes, territories, policy limits or deductibles.

D. Changes in the mix of business — utilization of combined loss data when there have been major changes in the composition of business by type of risk or type of claim.

External changes:

A. Legislation or court decisions significantly modifying claimant's legal rights.

B. High rates of inflation or wide fluctuations in inflation rates.

C. Changes in the social climate producing shifts in claims consciousness.

D. Impact of publicity of any kind regarding an insurer (e.g., lack of solidity or withdrawal from a given state).

E. Seasonal or cyclical fluctuations in loss experience.

F. Changes in the liberality of juries in granting awards.

G. Changes in the incidence of fraudulent claims and in the insurer's safeguards against such claims.

H. Changes in state regulations affecting company practices.

MINUTES OF THE 1977 FALL MEETING

NOVEMBER 20-22, 1977

BOCA RATON HOTEL AND CLUB, BOCA RATON, FLORIDA

Sunday, November 20, 1977

The Board of Directors Meeting began at 8:30 a.m. in the Madrid Room, with adjournment at 12:15 p.m.

At 11:00 a.m. registration was held in the Cloister Lobby.

The Annual Meeting was called to order at 1:00 p.m. in the Great Hall by President George Morison, followed by a few brief remarks by President Morison. A summary of call papers was then presented by C. K. Khury, Actuarial Director, Prudential Property and Casualty Insurance Company.

Presentations and reviews of papers 1, 2, and 3 of the Concurrent Session A began at 1:45 p.m. in the Granada Room. Charles A. Hachemeister, Actuary, Prudential Reinsurance Company, moderated these three papers entitled:

1. "The Tabular Approach to Claim Reserves and Liabilities," authored by John M. Bragg, President and Chief Executive Officer, Life Insurance Company of Georgia. Reviewed by James T. French, Assistant Vice President and Assistant Actuary, Mutual of Omaha.

2. "A Theory of Loss Development," authored by James A. Kenney, Chief Programmer, Coates and Crawford, Inc. Reviewed by John S. McGuinness, President, John S. McGuinness Associates, Consultants in Actuarial Science and Management.

3. "Statistical Testing of a Nonlife Insurance Runoff Model," authored by Dr. Gregory C. Taylor, Consulting Actuary, E. S. Knight & Co. Due to his absence, Charles A. Hachemeister presented the paper. The review was made by Richard I. Fein, Associate Actuary, Woodward & Fondiller, Division of Martin E. Segal Co.

Presentations and reviews of papers 4, 5, and 6 of the Concurrent Session B were held in the Barcelona Room. Richard H. Snader, Actuary, United States Fidelity & Guaranty Company, moderated these three papers entitled:

4. "Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach," authored by James R. Berquist, Consulting Actuary, Milliman & Robertson, Incorporated and Richard E. Sherman, Assistant Actuary, Milliman & Robertson, Incorporated. Reviewed by James A. Faber, Manager, Peat, Marwick, Mitchell and Company.

5. "Loss Reserves: Performance Standards," authored by C. K. Khury, Actuarial Director, Prudential Property and Casualty Insurance Company. Reviewed by E. LeRoy Heer, Assistant Vice President and Actuary, United Services Automobile Association.

6. "Determining Unpaid Claim Liabilities For Health Insurance Coverages," authored by Emil J. Strug, Assistant Vice President and Associate Actuary, Blue Cross of Massachusetts. Reviewed by Earl F. Petz, Actuary, Kemper Insurance Group.

Presentations and reviews of papers 7, 8, 9, and 10 of the Concurrent Session C were held in the Great Hall. M. Stanley Hughey, Executive Vice President, Kemper Insurance Group, moderated these four papers entitled:

7. "Reserves and the Emergence of Surplus," authored by Sidney Benjamin of Bacon and Woodrow. No review was made.

8. "Taking Down Reserves for Retrospective Adjustments," authored by Charles H. Berry, Associate Actuary, Aetna Life & Casualty. Reviewed by John R. Bevan, Vice President and Actuary, Liberty Mutual Insurance Company but was presented by Roy K. Morell.

9. "An Objective Standard for Testing Loss and Loss Expense Reserves," authored by M. Stanley Hughey, Executive Vice President, Kemper Insurance Group.

10. "An Integrated View of Insurance Company Results," authored by Richard G. Woll, Actuary, The Hartford Insurance Group. Reviewed by Marc B. Pearl, Senior Actuarial Associate, Royal-Globe Insurance Companies.

The President's Reception for New Fellows was held at 6:00 p.m. in the Presidential Suite.

A reception and dinner followed at 6:30 p.m. in the Camino Hall.

Monday, November 21, 1977

The Business Session, held in the Great Hall, was called to order at 8:15 a.m. by President Morison.

The Report of the Secretary was then presented by Darrell W. Ehlert. The Report of the Treasurer was given by Walter J. Fitzgibbon.

President Morison then requested the new Associates to rise as he called their names. After a round of applause for the Associates, each new Fellow was asked to step forward and receive his or her diploma. Richard A. Lino's diploma was presented by his father, Richard Lino. Pictures of new Associates and Fellows were taken at the coffee break following the Business Session. List of new Associates and Fellows follows:

NEW FELLOWS

Barrette, Raymond
Brubaker, Randall E.
Childs, Diana M.
Collins, Douglas J.
Curley, James O.
Dangelo, Charles H.
Donaldson, John P.
Fiebrink, Mark E.
Gersie, Michael H.
Goddard, Daniel C.
Hanson, H. Donald
Karlinski, Frank J.
Lino, Richard A.
McManus, Michael F.
Moore, Brian C.

NEW ASSOCIATES

Brown, Joseph W.
Dahlquist, Ronald A.
Lowe, Stephen P.
McCarter, Michael G.*
McConnell, D. Michael
Reichle, Kurt A.
Roth, Richard J., Jr.*
Shayer, Natalie

*Not Present

A moments silence was taken for William M. Corcoran, a Fellow of the Society, who died April 13, 1977.

The report of the Nominating Committee was given by Charles C. Hewitt, Jr. The Nominating Slates were as follows:

NOMINATIONS FOR OFFICERS

President-Elect	—	Ruth Salzmann
Vice President	—	W. J. MacGuinnitie
Secretary	—	Darrell W. Ehlert
Treasurer	—	Walter J. Fitzgibbon
Editor	—	David G. Forker
General Chairman Education and Examination Com.	—	Jeffrey T. Lange

A motion was made, seconded and carried that the nominations be closed, and the Secretary cast one unanimous ballet for the Nominating Committee Slate of Officers.

NOMINATIONS FOR DIRECTORS
(3 to be elected)

Robert A. Bailey
David G. Hartman
C. K. Khury
Joseph W. Levin
James F. Richardson
Michael A. Walters

The election of directors proceeded with Charles F. Cook as Chief Teller and Jerome A. Scheibl, Martin Bondy, and Dale A. Nelson as Tellers.

Those elected were Robert A. Bailey, David G. Hartman, and C. K. Khury.

The proposed change in the By-Laws (VI. Dues) carried.

There were no new papers presented, but there were reviews of the following three papers:

1. "Use of National Experience Indications in Workers' Compensation Insurance Classification Ratemaking," authored by Frank Harwayne, Vice President and Director of Actuarial Research, National Council on Compensation Insurance. Two reviews of

this paper were made—one by James F. Golz, Associate Actuary, Employers Insurance of Wausau and one by Les Dropkin, Vice President and Chief Actuary, Industrial Indemnity Company presented by Daniel C. Goddard.

2. "An Algorith for Premium Adjustment with Scarce Data," authored by Ronald F. Wiser, Senior Actuarial Analyst, CNA Insurance Companies. Two reviews were made—one by Bruce D. Moore, Consulting Actuary, Tillinghast, Nelson & Warren, Inc. and one by James E. Buck, Jr., Actuarial Department, Prudential Property and Casualty Insurance Company, presented by Frank J. Karlinski.

3. "On the Theory of Increased Limits and Excess of Loss Pricing," authored by Robert S. Miccolis. Two reviews were made—one by James A. Hall, III, Vice President and Actuary, California Casualty Group and one by Sheldon Rosenberg, Assistant Actuary, Insurance Services Office. Mr. Miccolis responded to the reviews.

Lew Roberts presented the Woodward and Fondiller Prize to Robert Miccolis, author of "On the Theory of Increased Limits and Excess of Loss Pricing."

No Dorweiler Prize was given.

The Call Paper Prize Award Committee consisted of members from the Loss Reserve Committee and the Committee on Review of Papers and was chaired by Charles F. Cook. President Morison presented the $200 prize to James R. Berquist, Consulting Actuary, and Richard E. Sherman, Assistant Actuary, Milliman & Robertson, Incorporated, for their paper entitled "Loss Reserve Adequacy Testing: A Comprehensive Systematic Approach."

John S. Trees, Vice President, Allstate Insurance Company, reported on activities of the All Industry Research Advisory Committee.

David G. Hartman, Vice President and Actuary, Chubb & Son, Inc., reported on the Joint Special Interest Meeting of April 1978 to be held at the New York Hilton. This meeting will be jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries.

The meeting recessed from 10:15 a.m. until 12:30 p.m. during which time the American Academy of Actuaries Annual Meeting was held in the Great Hall.

After a 12:30 p.m. break for lunch, the Concurrent Sessions presented on Sunday, November 20, 1977 were repeated.

At 6:30 p.m. a reception was held in the Camino Hall. Dinner followed at 7:30 p.m.

Tuesday, November 22, 1977

At 8:30 a.m. in the Great Hall, the panel entitled "Reorganization—Yes, No, or Maybe" was moderated by Jerome A. Scheibl, Vice President, Employers Insurance of Wausau. The panel members were: 1) Robert A. Bailey, Actuary, Director of NAIC Data Base, National Association of Insurance Commissioners, and 2) Charles A. Hachemeister, Actuary, Prudential Reinsurance Company.

At 9:30 a.m. the Presidential Address was given.

Following coffee break in the Camino Hall, the second panel entitled "Automobile Classifications—Looking Ahead" was moderated by William S. Gillam, Assistant to the President and Associate Actuary, Insurance Services Office. The panel members were: 1) John A. Gibson, III, Vice President and Actuary, Colonial Penn Insurance Company, 2) Ann E. Kelly, Supervising Actuary, New York State Insurance Department, 3) W. James MacGinnitie, Consulting Actuary, Tillinghast, Nelson & Warren, Inc. and 4) Richard G. Woll, Actuary, The Hartford Insurance Group.

Closing remarks were heard in the Great Hall at 11:45 a.m. The meeting formally adjourned at noon.

In attendance, as indicated by registration records, were 145 Fellows, 118 Associates, 53 Guests, and 123 Spouses. A list of Fellows, Associates and Guests is attached.

FELLOWS

Adler, M.	Barnes, G. R.	Ben-Zvi, P. N.
Angell, C. M.	Barrette, R.	Bergen, R. D.
Bailey, R. A.	Batho, E. R.	Berquist, J. R.
Balcarek, R. J.	Beckman, R. W.	Berry, C. H.
Balko, K. H.	Bennett, N. J.	Bickerstaff, D. R.

FELLOWS (CONT'D.)

Bill, R. A.
Bondy, M.
Bornhuetter, R. L.
Boyajian, J. H.
Brian, R. A.
Brouillette, Y. J.
Brubaker, R. E.
Carter, E. J.
Childs, D. M.
Collins, D. J.
Cook, C. F.
Curley, J. O.
Curry, A. C.
Dangelo, C. H.
D'Arcy, S. P.
Davis, G. E.
Dempster, H. V., Jr.
Donaldson, J. P.
Drobisch, M. R.
Ehlert, D. W.
Even, C. A., Jr.
Eyers, R. G.
Faber, J. A.
Ferguson, R. E.
Fiebrink, M. E.
Fisher, W. H.
Fitzgibbon, W. J., Jr.
Flaherty, D. J.
Flynn, D. P.
Fossa, E. F.
Fowler, T. W.
Fresch, G. W.
Gersie, M. H.
Gillam, W. S.
Goddard, D. C.
Golz, J. F.
Grady, D. J.
Graves, C. H.
Graves, J. S.
Grippa, A. J.
Hachemeister, C. A.
Hall, J. A., III
Hardy, H. R.
Hartman, D. G.
Hartman, G. R.
Harwayne, F.
Haseltine, D. S.
Hazam, W. J.
Heer, E. L.
Hewitt, C. C., Jr.
Hough, P. E.
Hughey, M. S.
Hunt, F. J., Jr.
Inkrott, J. G.
Jaeger, R. M.
Johe, R. L.
Kates, P. B.
Kaufman, A.
Kelly, A. E.
Khury, C. K.
Kilbourne, F. W.
Kline, D. F.
Kollar, J. J.
Kreuzer, J. H.
Kuehn, R. T.
Lange, J. T.
Levin, J. W.
Linder, J.
Lino, R.
Lino, R. A.
Liscord, P. S.
Lowe, R. F.
MacGinnitie, W. J.
Makgill, S. S.
McClure, R. D.
McGuinness, J. S.
McLean, G. E.
McManus, M. F.
McNamara, D. J.
Miller, P. D.
Mohl, F. J.
Moore, B. C.
Moore, P. S.
Morison, G. D.
Muetterties, J. H.
Munro, R. E.
Murray, E. R.
Nelson, D. A.
Newman, S. H.
Oien, R. G.
Otteson, P. M.
Pagnozzi, R. D.
Palczynski, R. W.
Perkins, W. J.
Phillips, H. J.
Pollack, R.
Presley, P. O.
Retterath, R. C.
Richards, H. R.
Richardson, J. F.
Riddlesworth, W. A.
Roberts, L. H.
Rodermund, M.
Rogers, D. J.
Rosenberg, S.
Ryan, K. M.
Salzmann, R. E.
Scheibl, J. A.
Sheppard, A. R.
Skurnick, D.
Smick, J. J.
Smith, E. R.
Snader, R. H.
Spitzer, C. R.

FELLOWS (CONT'D.)

Strug, E. J.
Taht, V.
Tarbell, L. L., Jr.
Tatge, R. L.
Trudeau, D. E.
Tverberg, G. E.
Uhthoff, D. R.
Walsh, A. J.
Walters, M. A.
Walters, M. A.
Webb, B. L.
Wieder, J. W., Jr.
Williams, P. A.
Winkleman, J. J., Jr.
Woll, R. G.
Wood, J. O.

ASSOCIATES

Anderson, R. C.
Andler, J. A.
Applequist, V. H.
Asch, N. E.
Banfield, C. J.
Bartlett, W. N.
Battaglin, B. H.
Bell, L. L.
Bellinghausen, G. F.
Bertles, G. G.
Beverage, R. M.
Bishop, E. G.
Bovard, R. W.
Bragg, J. M.
Brahmer, J. O.
Brewer, F. L.
Briere, R. S.
Brown, J. W., Jr.
Cadorine, A. R.
Cheng, J. S.
Cheng, L.
Chorpita, F. M.
Christiansen, S. L.
Conner, J. B.
Connor, V. P.
Crowe, P. J.
Dahlquist, R. A.
Davis, R. D.
DeGarmo, L. W.
Dolan, M. C.
Duperreault, B.
Durkin, J. H.
Eddy, J. H.
Einck, N. R.
Eldridge, D. J.
Faga, D. S.
Fagan, J.
Fein, R. I.
Fisher, R. S.
Flack, P. R.
Foley, C. D.
French, J. T.
Giambo, R. A.
Gleeson, O. M.
Goldberg, S. F.
Grannan, P. J.
Granoff, G.
Gruber, C.
Gwynn, H. M.
Hammer, S. M.
Haner, W. J.
Hanson, H. D.
Head, T. F.
Ingco, A. M.
Irvan, R. P.
Jean, R. W.
Jensen, J. P.
Johnston, D. J.
Kenney, J. A.
King, K. K.
Klingman, G. C.
Kolojay, T. M.
Lattanzio, F. J.
Lehman, M. R.
Lowe, S. P.
Luneburg, S. C.
Marker, J. O.
McHugh, R. J.
Meyer, R. E.
Miccolis, R. S.
Millman, N. L.
Mokros, B. F.
Moore, B. D.
Napierski, J. D.
Neidermyer, J. R.
Neis, A. R.
Neuhauser, F., Jr.
Nishio, J. A.
Nolan, J. D.
Oakden, D. J.
Patrik, G. S.
Patterson, D. M.
Peacock, W. W.
Pearl, M. B.
Petlick, S.
Pflum, R. J.
Plunkett, R. C.
Potok, C. M.
Pratt, J. J.
Quirin, A. J.

ASSOCIATES (CONT'D.)

Reichle, K. A.
Riff, M.
Roach, R. F.
Roman, S. M.
Rudduck, G. A.
Sandler, R. M.
Schneiker, H. C.
Shayer, N.
Sherman, R. E.
Shoop, E. C.
Shrum, R. G.
Singer, P. E.
Skolnick, R. S.
Smith, F. A.
Stergiou, E. J.
Taylor, F. C.
Thompson, E. G.
Thompson, P. R.
Thorne, J. O.
Torgrimson, D. A.
Trees, J. S.
Van Slyke, O. E.
Wade, R. C.
Weiner, J. S.
Weller, A. O.
Westerholm, D. C.
Young, E. W.
Young, R. G.

GUESTS

Altschuler, M. C.
Angle, J. C.
Bartlett, D. K., III
Belton, E. F.
Benjamin, S.
Biller, J. E.
Bowles, T. P., Jr.
Butsic, R. P.
*Canfield, P. A.
Carlson, D. L.
Carpenter, J. G.
Casner, T. R.
Chen, H.
Cohen, A. I.
*Dinstein, M. P.
Dunn, R. P.
Edwards, R. E.
Furst, P. A.
*Glaser, L. S., Jr.
Gorman, L. A.
Gustafson, D. R.
*Hager, G. A.
Hany, D.
*Hatfield, B. D.
Haughey, T. D.
Hill, R. J.
Hogg, J. J.
Jacobus, J. A.
*Johnson, J. E.
Kellison, S. G.
Knox, F. J.
*Kraysler, S. F.
Lo, R. W.
Loynes, J.
Maguire, R. D.
McConnell, D. M.
McMillen, R. H.
Miller, R. A., III
Odell, W. H.
Parker, C. M.
*Peterson, T. M.
*Pope, D. W.
*Posnak, R. L.
Ryan, J. P.
Saffeir, H. J.
*Smith, D. A.
*Spangler, J. L.
*Stenmark, J. A.
*Subeck, S. I.
Wilson, R. J.
Winters, R. C.
Wright, R. W.
Young, D. W.

*Invitational Program

Respectfully submitted,

Darrell W. Ehlert
Secretary

REPORT OF THE SECRETARY

In preparation for this report, I reviewed the activities of the Board and Committees for the past year and the Report of the Secretary for the last few years. With some exceptions, I find that many of the challenges facing the CAS have not changed significantly, but that progress to meet these challenges has occurred. Although our more action oriented members and our outside critics may think we are moving too slowly towards resolution of some of these issues, I believe it is a credit to our Society to have a Board of Directors and Committees that thoroughly discuss the ramifications of our corporate actions in terms of impact on our business and our current and future membership before making final decisions.

Any volunteer organization must rely on the dedication of its members to get the jobs done. To those of you who are not involved, I would like to say that your representatives on the Board and the Committees are dedicated people who represent the whole membership. Many people devote long hours to the business of the CAS—without pay and, in some cases, with little thanks. We should not overlook their contributions, nor the contributions of their employers which bear the costs of this participation.

Some highlights of 1977:

The ASTIN Meeting in conjunction with the Washington CAS Meeting was a huge success. The ASTIN Organizing Committee (all CAS members) raised $30,000 from 30 companies to host the meeting for our 170 foreign member guests and 50 CAS members of ASTIN. A highlight of this meeting was Matt Rodermund's "How to Succeed as an Actuary" performed by members and friends.

Attendance at the Spring Meeting in Washington and the Fall Meeting here in Boca Raton continued to set records with total registrations of 415 and 490, including guests.

The Sites Committee had a busy time after the cancellation of the Bermuda Meeting because of a new tax law. The attendance here attests to their good judgement and the flexibility of our membership. Although some complaints have been voiced about our "exotic sites," attendance indicates that the Sites Committee is doing something right.

A Special Committee on Reorganization was formed to study the various proposals of other actuarial bodies for reorganizing the actuarial profession in North America. The Board approved the recommendations of this committee to discuss reorganization with the other bodies without endorsing any of the presently existing proposals.

The CAS is working with the Society of Actuaries on a Joint Special Interest Meeting to be held in New York in April of 1978. A program of subjects of interest to both societies is being planned. The Committee on Continuing Education is handling the details.

This meeting in Boca Raton is a first in that the emphasis is on the discussion of Call Papers on Loss Reserving. The Committee on Continuing Education and the Committee on Review of Papers worked long and hard on this project. The number of papers submitted, and approved, the cooperation of members acting as reviewers and the attendance at this meeting attest to the success of the efforts of these committees.

The Committee on Review of Papers also proposed new Guides for Submission of Papers which were approved by the Board for inclusion in the 1978 *Yearbook.*

The Education and Examination Committee also had a busy year. An independent investigation of the 1976 Part 9 exam and grading was conducted. The committee was expanded again to handle the increased workload and the grading and publishing of exam results were completed in a timely fashion. Work is continuing on updating the Syllabus to help the students in their studies. In this regard, the CAS regional affiliates are collecting comments from students to aid in constructing better exams.

Exam fees for Parts 1, 2 and 3 must be raised to $20 in 1978 to cover the increased costs of administration. The number of students taking exams reached a new record of 3,443 in 1977, but only slightly above the previous record of 3,422 in 1976. Applicants for Parts 1, 2 and 3 decreased somewhat, but the increase in the number taking other exams was enough to increase the totals.

The number of new members entering the society decreased over prior years with only 20 persons receiving their Associateship, probably due to the increase to seven exams for this designation. The number of members attaining Fellowship was 29.

The CAS will provide financial support to the Actuarial Education and Research Foundation on a project to produce a book on loss distributions. The ASTIN Organizing Committee also donated money as have some companies. It appears that this CAS sponsored project will be the first for AERF.

The Financial Reporting Committee and the Loss Reserve Committee, along with the Board, studied proposals for certification of casualty statements by actuaries made by the National Association of Insurance Commissioners. This subject will get additional attention in 1978.

A donation of $2,000 was made to the Joint Committee Minority Scholarship Fund at the recommendation of the Committee on Career Enhancement.

The Editorial Committee struggled with printer apathy in attempting to get current on the publishing of the *Proceedings* and the *Yearbook*.

The Finance Committee submitted a budget for 1978 and was again able to hold dues to present levels. However, fiscal 1978 expenses are expected to exceed income by $27,800.

Our five regional affiliates are continuing programs of education for members on the local level.

The Program Committee met almost monthly to plan the programs.

The CAS Office in New York will be moved shortly to One Penn Plaza, 250 West 34th Street, New York, New York 10001.

The Board of Directors had four meetings in 1977: Panama City, Florida in February; Washington, D.C. in May; Lake Geneva, Wisconsin in September and at Boca Raton, Florida in November.

In closing, I would like to offer a personal "thank you" to Paul Scheel and Arthur Cadorine and their helpers for handling the arrangements for the Washington and Boca Raton Meetings. Also, thanks to Edie Morabito and the crew at the CAS Office in New York and to my own secretary, Randy Pietroski. Their help has been necessary, outstanding and greatly appreciated.

Respectfully submitted,

Darrell W. Ehlert
Secretary

REPORT OF THE TREASURER

The audited financial statement for the fiscal year ended September 30, 1977 showed assets of $207,963.61 up $37,078.17 for the year. Liabilities increased $30,437.00 to $68,300.00. The liabilities include printing costs for the 1975 and 1976 Proceedings as well as examination fees for jointly administered examinations soon to be paid to the Society of Actuaries.

Membership equity increased $6,641.17 to $139,663.61. This is comprised of the Michelbacher Fund of $28,889.86, up $2,751.89 for the year, the Dorweiler Fund of $6,845.30 up $470.17 and surplus of $103,928.45 up $3,419.11.

No new investments were made during the year and none are contemplated for the near future. Our principal investments are a $100,000 U.S. Treasury Note maturing in May, 1981, yielding 7.5% and a 1 year time savings account of $26,702.90 paying 6½% maturing in May, 1978. The time savings account was an automatic extension of the account opened one year earlier.

Our operations had been budgeted at the breakeven level for the past year. Three items account for most of the $3,419 surplus growth experienced. Exam fees were higher than expected and the Board of Directors in September approved two contributions: Two thousand dollars was approved for a Joint Minority Scholarship Program contribution which had not been budgeted and $5,000 was approved for the Actuarial Education and Research Fund. The budget contained only $2,000 for this item. The $5,000 will be used for a project involving preparation of a textbook on loss distributions.

The Michelbacher and Dorweiler Funds grew as a result of interest credited on fund balances and $982.52 received as royalties on Mr. Michelbacher's books.

The operating budget approved for the coming year shows only slight changes overall from last year. Printing and stationery will increase $4,500 or 13% and the charge for Secretarial Services provided by the National Council will increase $2,750 or 7.5%. These increases are offset by a reduction in the contributions budget. The operating budget has been established at the breakeven level on an accrual basis. On a cash basis, a deficit will be produced since printing bills should be paid for two issues of the Proceedings.

The level of membership dues will be unchanged. Fellowship dues are \$70.00. Associateship dues are \$50.00 for the first five years and \$70.00 thereafter. Residents outside the United States and Canada will pay \$50.00 dues.

Last year it was reported that our petition to the IRS to change the Casualty Actuarial Society's tax exempt status from Section 501(c)(6) to Section 501(c)(3) was denied. If approved, the change would have allowed tax deductions to those making donations to the Casualty Actuarial Society. Since the denial, the IRS informed us that recently enacted law provides for judicial review of any adverse IRS rulings and determinations made after 1-1-76 relating to exemption qualification. We were told we could request reissuance of the adverse ruling and thus keep the issue alive. The request was made and the IRS has yet to reissue their ruling. We have not yet decided whether this matter is worth pursuing further.

Respectfully submitted,

Walter J. Fitzgibbon, Jr.
Treasurer

FINANCIAL REPORT

FOR FISCAL YEAR ENDED SEPTEMBER 30, 1977

INCOME

Dues	$ 39,230.00
Exam Fees	69,782.25
Meetings & Registration Fees	58,910.87
Sale of Proceedings	7,442.00
Sale of Readings	1,245.00
Invitational Program	2,781.00
Interest	9,391.54
Actuarial Review	303.50
Misc.	—122.64
Total	$188,963.52

DISBURSEMENTS

Printing & Stationery	$ 33,971.06
Secretary's Office	33,739.00
Examination Expenses	42,790.31
Meeting Expenses	62,035.84
Library	24.03
Math. Assoc. of America	1,500.00
Insurance	656.00
Actuarial Educ. & Res. Fund	5,500.00
Joint Minority Scholarship Prog.	2,000.00
Misc.	328.17
Total	$185,544.41

Increase in Surplus	$ 3,419.11
Cash & Invested Assets 9/30/76	167,469.34
Cash & Invested Assets 9/30/77	170,888.45

ACCRUAL BASIS ACCOUNTING STATEMENT
AS OF SEPTEMBER 30, 1977

ASSETS	9/30/76	9/30/77
Bank Accounts	$ 63,609.34	$ 97,025.61
U.S. Treasury Bond	4,325.00	4,325.00
U.S. Treasury Note	99,535.00	99,535.00
Accrued Income	3,416.10	7,078.00
Total	$170,885.44	$207,963.61
LIABILITIES, SURPLUS AND OTHER FUNDS		
LIABILITIES		
Printing Expenses	$ 20,000.00	$ 43,835.00
Examination Expenses	17,863.00	17,265.00
Actuarial Educ. & Research Fund	0	5,000.00
Joint Minority Scholarship Program	0	2,000.00
Other	0	200.00
Sub-Total	$ 37,863.00	$ 68,300.00
MEMBERS' EQUITY		
Michelbacher Fund	$ 26,137.97	$ 28,889.86
Dorweiler Fund	6,375.13	6,845.30
Surplus	100,509.34	103,928.45
Sub-Total	$133,022.44	$139,663.61
Total	$170,885.44	$207,963.61

Walter J. Fitzgibbon, Jr.
Treasurer

* * * * * *

This is to certify that the assets and accounts shown in the above financial statement have been audited and found to be correct.

Finance Committee
R. B. Foster, *Chairman*
H. E. Curry
S. L. Perreault
P. A. Verhage

1977 EXAMINATIONS—SUCCESSFUL CANDIDATES

Examinations for Parts 4, 6, 8 and 10 of the Casualty Actuarial Society Syllabus were held May 4 and 5, 1977 and examinations for Parts 5, 7 and 9 were held November 14 and 15, 1977. Parts 1, 2 and 3, jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries were given in May and November. Those who passed Parts 1, 2 and 3 were listed in the joint release of the two Societies dated July 27, 1977 and January 31, 1978.

The Casualty Actuarial Society and the Society of Actuaries jointly award prizes to the undergraduates ranking highest on the General Mathematics examination.

The winner of the $200 for the May 1977 examination was Ernest Simon Davis. $100 prizes were awarded to Rajiv Gupta, Alan Seth Minkoff, David John Rusin and Stephen Ngok Tsun. The $200 prize was awarded to Richard A. Pérusse for the November 1977 examination. The additional $100 prize winners were Maria F. DiPaoli, Jonathan B. Loring, Hal M. Switkay and Patrick A. Walker.

The following candidates successfully completed the requirements for Fellowship and Associateship in the May 1977 Examinations.

NEW FELLOWS

Barrette, Raymond	Dangelo, Charles H.	Hanson, H. Donald
Brubaker, Randall E.	Donaldson, John P.	Karlinski, Frank J.
Childs, Diana M.	Fiebrink, Mark E.	Lino, Richard A.
Collins, Douglas J.	Gersie, Michael H.	McManus, Michael F.
Curley, James O.	Goddard, Daniel C.	Moore, Brian C.

NEW ASSOCIATES

Brown, Joseph W.	McCarter, Michael G.	Roth, Richard J. Jr.
Dahlquist, Ronald A.	McConnell, D. Michael	Shayer, Natalie
Lowe, Stephen P.	Reichle, Kurt A.	

MAY 1977 EXAMINATIONS

Following is a list of successful candidates in the examinations held in May 1977:

Part 4(a)

Boison, LeRoy A., Jr.
Dahlquist, Ronald A.
Mill, Ralph A.
Murphy, Thomas M.
Purple, John M.

Part 4(b)

Abrams, Paul T.
Amling, Michael A.
Bell, Charles T.
Clark, David G.
Corr, Francis X.
Dupuis, Camille
Dussault, Claude
Egnasko, Gary J.
Flanagan, Terrence A.
Heckman, Philip E.
Henry, Dennis R.
Javaruski, John J.
Lake, Gary D.
McMurray, Michael A.
Polagye, Karen C.
Shayer, Natalie
Urschel, Frederick A.
Van Domelen, James P.
Wengertsman, John F.
Wenner, Richard M.

Part 4

Antolino, Michael R., Jr.
Austin, John P.
Baer, Debra L.
Brooks, Dale L.
Brown, Joseph W.
Brown, Nicholas, Jr.
Burg, David R.
Byrne, Patrick J.
Chiang, Chia-Chi
Christie, James K.
Costner, James E.
Cundy, Richard M.
Davis, Lawrence S.
DiBattista, Susan T.
Diss, Gordon F.
Drummond-Hay, Eric T.
Dudick, Alicia J.
Edie, Grover M.
Esposito, David L.
Evans, Glenn A.
Foote, James M.
Ghezzi, Thomas L.
Golin, Thomas
Gorman, Linda A.
Heersink, Agnes H.
Higgins, Barbara J.
Horowitz, Bertram A.
Howard, C. Douglas
Iverson, Randall L.
Jameson, Stephen
John, Russell T.
Kleinman, Joel M.
Koski, Mikhael I.
LaMonica, Michael A.
Lo, Richard W.
Lombardo, John S.
Lotkowski, Edward P.
Mathewson, Stuart B.
McGowan, John S.
Miller, Allen H.
Mueller, Conrad P.
Nichols, Raymond S.
Nichols, Richard W.
Niswander, Ray E.
Perry, Loren A.
Piersol, Kim E.
Reutershan, John T.
Russell, Charles B.
Russell, David M.
Sce, Michael A.
Schwartz, Allan I.
Sobel, Mark J.
Strange, Deborah L.
Taranto, Joseph V.
Thompson, Kevin B.
Truttmann, Everett J.
Undis, David J.
Weiland, William T.
Woods, Patrick B.
Yatskowitz, Joel D.
Zolnowski, Raymond M.

Part 6

Anderson, Bruce C.
Andrus, William R.
Antolino, Michael R., Jr.
Bartlett, John W.
Belvin, William H.
Beversdorf, William R.
Boyd, Lawrence H.
Brown, Andrew F., Jr.
Chernicky, Celeste G.
Cloutier, Guy
Conger, Robert F.
DiBattista, Susan T.
Doepke, Mark A.
Elia, Dominick A.
Ford, Edward W.
Godin, Gene W.
Harrison, Eugene E.
Heckman, Philip E.
Hurley, James D.
Jensen, Patricia A.
Johnston, Thomas S.
Kadison, Jeffrey P.
LaFontaine, Gaetane
LaMonica, Michael A.
Lerwick, Stuart N.
Lombardo, John S.
Lowe, Stephen P.
McCarter, Michael G.
McConnell, Charles W.
McConnell, D. Michael
McDaniel, Gail P.
McGovern, William G.
Mellia, Joanne C.
Meyers, Glenn G.
Murphy, Francis X., Jr.
Myers, Nancy R.
Nash, Russell K.
Nickerson, Gary V.
Orlowicz, Charles P.
Reichle, Kurt A.
Roth, Richard J., Jr.
Rowland, Vincent T., Jr.
Rowland, William J.
Schwartz, Allan I.
Skrodenis, Donald P.
Smith, Byron W.
Sweeney, Andrea M.
Thibault, Alain P.
Tom, Darlene P.
Tuttle, Jerome E.
Walker, Roger D.
Weissner, Edward W.
White, Frank T.
White, Jonathan
Whitman, Mark
Wilson, William F.
Wisecarver, Timothy L.
Yuan, Hui-Lin
Zicarelli, John D.

Part 8

Aldorisio, Robert P.
Asch, Nolan E.
Barrette, Raymond
Bass, Irene K.
Beer, Albert J.
Bell, Linda C.
Bishop, Everett G.
Bovard, Roger W.
Brubaker, Randall E.
Buck, James E., Jr.
Cheng, Laurence W.
Cis, Mark M.
Covney, Michael D.
Crifo, Daniel A.
Degerness, Jerome A.
Faga, Doreen S.
Fagan, Janet L.
Fisher, Russell S.
Giambo, Robert A.
Goddard, Daniel C.
Gutterman, Sam
Henkes, Joseph P.
Hobart, Gary P.
Irvan, Robert P.
Kist, Frederick O.
Lattanzio, Stephen P.
Ledbetter, Alan R.
Livingston, Roy P.
Lommele, Jan A.
Luneberg, Sandra C.
Newville, Benjamin S.
Oakden, David J.
O'Brien, Terrence M.
O'Neil, Mary L.
Pearl, Marc B.
Petlick, Steven
Pierce, John
Roland, William P.
Rosenberg, Martin
Sherman, Richard E.
Shrum, Roy G.
Steer, Grant D.
Swift, John A.
Symonds, Donna R.
Tierney, John P.
Vaughan, Robert C.
Wasserman, Forrest
Weller, Alfred O.
Whatley, Patrick L.
Wickwire, James D., Jr.
Wiser, Ronald F.

Part 10

Barrette, Raymond
Bartlett, William N.
Bertles, George G.
Childs, Diana M.
Collins, Douglas J.
Connor, Vincent P.
Curley, James O.
Dangelo, Charles H.
Donaldson, John P.
Dorval, Bernard
Duperreault, Brian
Eldridge, Donald J.
Ernst, Richard C.
Fiebrink, Mark E.
Frohlich, Kenneth R.
Gallagher, Thomas L.
Gersie, Michael H.
Gleeson, Owen M.
Goddard, Daniel C.
Graham, Timothy L.
Grannan, Patrick J.
Gutterman, Sam
Hafling, David N.
Hanson, H. Donald
Hoylman, Douglas J.
Jean, Ronald W.
Jerabek, Gerald J.
Karlinski, Frank J.
Lehman, Merlin R.
Lehmann, Steven G.
Lindquist, Peter L.
Lino, Richard A.
McManus, Michael F.
Miller, David L.
Moore, Brian C.
Neidermyer, James R.
Nelson, Janet R.
Newlin, Patrick R.
Petersen, Bruce A.
Petlick, Steven
Renze, David E.
Schumi, Joseph R.

The following candidates successfully completed the requirements for Fellowship and Associateship in the May 1977 Examinations.

NEW FELLOWS

Ashenberg, Wayne R.
Bassman, Bruce C.
Bertles, George G.
Bovard, Roger W.
Carbaugh, Albert B.
Daino, Robert A.
Eldridge, Donald J.
Engel, Philip L.
Ernst, Richard C.
Fein, Richard I.
Frohlich, Kenneth R.
Gallagher, Thomas L.
Gleeson, Owen M.
Graham, Timothy L.
Grannan, Patrick J.
Hermes, Thomas M.
Leimkuhler, Urban E.
Lindquist, Peter L.
Marker, Joseph O.
Miller, David L.
Petersen, Bruce A.
Petlick, Steven
Quirin, Albert J.
Renze, David E.
Taylor, Jane C.
Wright, Walter C., III

NEW ASSOCIATES

Andrus, William R.
Antolino, Michael R., Jr.
Bartlett, John W.
Cloutier, Guy
Conger, Robert F.
Doepke, Mark A.
Egnasko, Gary J.
Ford, Edward W.
Hurley, James D.
LaMonica, Michael A.
Lerwick, Stuart N.
McMurray, Michael A.
Nash, Russell K.
Purple, John M.
Skrodenis, Donald P.
Thibault, Alain P.
Tuttle, Jerome E.
Weissner, Edward W.
White, Frank T.
White, Jonathan
Whitman, Mark
Wilson, William F.
Wisecarver, Timothy L.

Part 5

Balchunas, Anthony J.
Bashline, Donald T.
Bell, Charles T.
Biller, James E.
Boudakian, Armen M.
Brutto, Richard S.
Byrne, Patrick J.
Canetta, John A.
Christhilf, David A.
Christie, James K.
Cohen, Elliot J.
Connell, Eugene C.
Cundy, Richard M.
Davidson, Shelly T.
Dawson, John
Djordjevic, Nancy G.
Doak, Kenneth A.
Driedger, Karl H.
Drummond-Hay, Eric T.
Dupuis, Camille
Dussault, Claude
Esposito, David L.
Evans, Glenn A.
Gluck, Spencer M.
Goldfarb, Irwin H.
Gottheim, Eric F.
Griffith, Roger E.
Gutman, Ewa
Hallstrom, Robert C.
Harrison, David C.
Hennessy, Mary E.
Herder, John M.
Hibberd, William J.
Hu, David D.
Jameson, Stephen
Jaso, Robert J.
John Russell T.
Johnson, Richard E.
Jones, Bruce R.
Kidd, Joseph L.
Kleinman, Joel M.
Koch, Leon W.
Koerber, Alan J.
Kramer, Neal M.
Landwehr, James B.
LaRose, J. Gary
Lederman, Charles M.
Levine, Alex J.
Limpert, John J.
Linden, Orin M.
Liu, Robert T. T.
Lotkowski, Edward P.
Maguire, Richard J.
Mahler, Howard C.
Mair, Sharon A.
Mathewson, Stuart B.
McGovern, William G.
Meyers, Glenn G.
Milligan, Alfred W.
Mueller, Conrad P.
Parker, Curtis M.
Pastor, Gerald H.
Pei, Kai-Jaung
Pelletier, Charles A.
Philbrick, Polly G.
Philbrick, Stephen W.
Piersol, Kim E.
Pinto, Emanuel
Prill, Donna A.
Ransom, Gary K.
Reutershan, John T.
Robertson, John P.
Rouillard, Marc L.
Sansevero, Michael, Jr.
Schmidt, Lowell D.
Schwartz, Allan I.
Seligman, Edward J.
Sherman, Ollie L., Jr.
Silverman, Janet K.
Sobel, Mark J.
Stadler-Hrbacek, Elisabeth
Surrago, James
Sweeny, Andrea M.
Van Ark, William R.
Van Domelen, James P.
Warren, Jeffrey C.
Wasserman, David L.
Weiland, William T.
Wenner, Richard M.
Wess, Clifford
Westerholm, Sharon
Wiseman, Michael L.
Wolf, Philip M.
Woods, Patrick B.
Youngerman, Howard
Youngner, Ruth E.

Part 7

Andrus, William R.
Antolino, Michael R., Jr.
Applequist, Virgil H.
Austin, John P.
Baer, Debra L.
Bartlett, John W.
Brewer, Fred L.
Buck, James E., Jr.
Cloutier, Guy
Conger, Robert F.
Corr, Francis X.
Covitz, Burton
Davis, Lawrence S.
Degerness, Jerome A.
Dineen, David K.
Doepke, Mark A.
Egnasko, Gary J.
Einck, Nancy R.
Eramo, Robert P.
Fisher, Russell S.
Ford, Edward W.
Gleeson, Owen M.
Grannan, Patrick J.
Graves, George G.
Herman, Steven C.
Hine, Cecily A.
Hurley, James D.
Jean, Ronald W.
Johnson, Richard W.
Koski, Mikhael I.
Kucera, Jeffrey L.
LaMonica, Michael A.
Lerwick, Stuart N.
McMurray, Michael A.
Miccolis, Jerry A.
Miller, Robert A., III
Myers, Nancy R.
Nash, Russell K.
Nishio, Jo Anne
Perry, Loren A.
Petlick, Steven
Potok, Charles M.
Pratt, Joseph J.
Pulis, Ralph S.
Purple, John M.
Schneider, Harold N.
Skrodenis, Donald P.
Smith, Frances A.
Subeck, Stanton I.
Swisher, John W., Jr.
Taranto, Joseph V.
Taylor, Jane C.
Thibault, Alain P.
Tuttle, Jerome E.
Urschel, Frederick A.
Van Slyke, Oakley E.
Wasserman, Forrest
Weissner, Edward W.
Westerholm, David C.
Whatley, Michael W.
Whatley, Patrick L.
White, Frank T.
White, Jonathan
Whitman, Mark
Wickwire, James D., Jr.
Wilson, William F.
Wisecarver, Timothy L.
Zicarelli, John D.

Part 9

Ashenberg, Wayne R.
Barrow, Betty H.
Bassman, Bruce C.
Bell, Linda C.
Bellinghausen, Gary F.
Bertles, George G.
Beverage, Richard M.
Bovard, Roger W.
Bradley, David R.
Carbaugh, Albert B.
Cheng, Laurence W.
Daino, Robert A.
Eldridge, Donald J.
Engel, Philip L.
Ernst, Richard C.
Fagan, Janet L.
Fein, Richard I.
Frohlich, Kenneth R.
Gallagher, Thomas L.
Goldberg, Steven F.
Graham, Timothy L.
Heckman, Philip E.
Hemstead, Robert J.
Hermes, Thomas M.
Irvan, Robert P.
Lattanzio, Francis J.
Leimkuhler, Urban E.
Lindquist, Peter L.
Luneburg, Sandra C.
Marker, Joseph O.
McConnell, Charles W., II
Miller, David L.
Morell, Roy K.
Nichols, Raymond S.
O'Brien, Terrence M.
Patrik, Gary S.
Petersen, Bruce A.
Quirin, Albert J.
Reichle, Kurt A.
Renze, David E.
Reynolds, John D.
Roland, W. Paul
Roth, Richard J., Jr.
Steer, Grant D.
Swift, John A.
Thorne, Joseph O.
Wiser, Ronald F.
Wood, Charles P., Jr.
Wright, Walter C., III

WILLIAM M. CORCORAN
1898-1977

William M. Corcoran a Fellow of the Casualty Actuarial Society died on April 13, 1977 at the age of 79. Mr. Corcoran was born at New London, Connecticut. He graduated from Yale in 1920 and upon graduation went to work for the Massachusetts Insurance Department. He was admitted as a Fellow to the Casualty Actuarial Society on Nevember 18, 1925. He was also a Fellow of the Society of Actuaries and The Conference of Actuaries in Public Practice and was a member of the Fraternal Actuarial Society and the American Academy of Actuaries. He was a registered public accountant in New York and was co-author of a book entitled, "Inheritance Tax Calculations".

Mr. Corcoran was assistant actuary with the Massachusetts Insurance Department from 1920 to 1923. From 1923 to 1928 he was the Actuary for the Connecticut Insurance Department. In 1928 he became partner in the Consulting Actuarial firm of Wolfe, Corcoran & Linder. He was a senior partner in this firm until January 1, 1965 when the firm merged with Peat, Marwick, Mitchell & Co. After the merger he became a special consultant until his retirement in 1966.

NEW ASSOCIATES ADMITTED MAY 1977: Nine of the eleven new associates admitted at the Hyatt Regency in Washington, D.C. are shown with President George Morison.

NEW FELLOWS ADMITTED MAY 1977: The fourteen new fellows admitted at the Hyatt Regency in Washington, D.C. are shown with President George Morison.

NEW ASSOCIATES ADMITTED NOVEMBER 1977: Five of the eight new associates admitted at Boca Raton, Florida are shown with President George Morison.

NEW FELLOWS ADMITTED NOVEMBER 1977: The fifteen new fellows admitted at Boca Raton, Florida are shown with President George Morison.

INDEX TO VOLUME LXIV

INDEX TO VOLUME LXIV (Cont.)

Brian Scott
78 Brookmoor Rd
Avon CT