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Volume LXIII, Part 1 No. 119

PROCEEDINGS

May 24, 25, 26, 1976

ACCIDENT LIMITATIONS FOR RETROSPECTIVE RATING

FRANK HARWAYNE

The need for changes in retrospective rating plan accident limitation charges has been apparent for some time.

This paper describes recently adopted recommendations of the National Council on Compensation Insurance for developing such charges. The charges for accident limitations are familiarly known as Excess Loss Premium Factors or ELPF's. They are percentages of standard earned premium which are paid by the policyholder in lieu of his being charged for losses above a selected limit per accident. These charges also vary by industry grouping (hazard groups) to reflect the differences in expected frequency density and size of claim.

About 25 years ago, a system was developed for calculating ELPF's[1]. To Dunbar Uhthoff's great credit, this system has withstood the test of time for most of the 25 years. However, the accelerating impetus of inflation has brought about many qualitative and quantitative changes in insurance. The basic forces at work are:

a. Monetary inflation (the decreasing purchasing power of the accident limitation).

b. Loss development (greater development on more severe injuries).

c. Removal of benefit limitations.

d. Relatively higher medical cost inflation versus indemnity benefit inflation, increasing the spread of the claim size distribution.

[1] D. Uhthoff, "Excess Loss Ratios via Loss Distributions," *PCAS,* Vol. XXXVII, (1950), p. 82ff.

Both frequency of claims and average claim costs have been seriously affected. In particular, claim costs have risen significantly with the dramatic rise in average weekly wages, medical costs and benefits afforded by workers' compensation laws. As an illustration, countrywide average weekly wages during the first six months of 1950 were $52.51 compared with $175.34 for the same period in 1975[2]. Average claim costs for death cases are $97,024 (Illinois) on 1975 benefit levels compared to $3,967 (Illinois) which was used in the 1950 paper.

Since that time, many states which had previously limited the maximum dollar amount payable for such claims have enacted laws which provide lifetime benefits with significantly higher costs. Changes of such magnitude are bound to affect distributions of extreme values. Because charges for accident limitations represent costs which are intended to cover (on the average) amounts in excess of selected limitations, it follows that inflation will shift larger percentages of the total cost over to the higher end of the distribution.

The shift, wherein greater percentages of total cost have been transferred to the higher end of the scale, has been feared and known for some time. Newer tables which reflect the shift were needed. Documentation of the changes in distribution of cost was accomplished by digging into the customary reports used by the National Council on Compensation Insurance for ratemaking. The information was not in readily usable form[3] and revisions of developed individual reports were required for purposes of this study. The need for mature reports is apparent in light of the substantial development of average cost per claim at successive reports. For example, in the state of Connecticut, permanent total average cost per claim for policy year 1969-70[4] was $87,348 at first report, $95,047 at second report and $121,432 at third report.

The program for updating distributions by size of claim called for the use of fourth reports for each of the serious loss categories of fatal, permanent total and major permanent partial injuries. Serious loss categories were used because these are the ones which are likely to result in individual claim costs in excess of the accident limit selected. Distributions as a ratio

[2] NCCI "Call for Wage Data for Injured Employees."

[3] Because normal ratemaking requires aggregate data for developments, such aggregates were captured without taking the time to guarantee that individual claim reports were included in the data base at developed amounts.

[4] Policies issued between August 1, 1969 through July 31, 1970.

of average cost were obtained for the medium or high benefit jurisdictions of Arkansas, Connecticut, District of Columbia, Maryland and Nebraska. By observing each jurisdiction in terms of that portion of cost which represented excess cost per case according to the intervals as a ratio to average, the problem of recognizing different benefit levels and different average costs per case in each of the jurisdictions was minimized. It then became possible to combine the excess cost per case for particular ratios to average. This combination was made based upon the total number of cases for the particular injury type in each of the jurisdictions considered (see Exhibits I-1 and II-1).

EXHIBIT I-1

FATAL—LIMITED

Ratio to Avg.	Excess Ratios Maryland	Nebraska	Arkansas	Average*
.00	1.000	1.000	1.000	1.000
.25	.775	.776	.778	.776
.50	.591	.597	.607	.597
.75	.430	.439	.454	.440
1.00	.289	.300	.339	.308
1.25	.171	.181	.242	.196
1.50	.143	.131	.164	.147
1.75	.123	.124	.102	.116
2.00	.105	.117	.052	.090
2.25	.091	.110	.026	.073
2.50	.078	.103	.017	.063
2.75	.067	.096	.009	.054
3.00	.058	.089	.002	.046
3.25	.047	.082	.000	.039
3.50	.041	.075	.000	.034
Total Number of Claims	85	36	59	xxx

*Average excess ratios weighted by state's total number of claims

The combined results were plotted on semi-logarithmic graph paper and compared with the tables known as "Uhthoff's Tables" (see Exhibits I-3 and II-3). It immediately became apparent that the latter have become seriously out of date at the high end of the scale.

The graphic representation of data for permanent total cases showed a remarkable coincidence with the data for fatal limited cases. For this reason, it was decided to use the fatal limited tables for permanent total as well.

Unlimited fatal cases indicated much lower charges than did the table for limited fatal cases. Due to the paucity of unlimited fatal claims, and in the light of actual results, it was decided to apply the new table for limited fatal cases to the unlimited as well.

Since the values obtained by the averaging method described above were only calculated at each 25% of the average cost per claim, a method was needed to produce a complete table. The method of least squares was used in fitting various equations to the combined results (see Exhibits I-2 and II-2). Each fitted equation was required to produce a value of 1.000 for a zero ratio to average cost. For those selected equations which did not lend themselves to a true least squares analysis, both an approximate least squares method (utilizing logarithms) and a method of collocation were tried. Collocation involves the algebraic solution of the general equation such that the collocation equation thus obtained passes through selected values of the actual data[5]. This is an iterative technique; it was continued until the observed deviations were evenly spread over the entire distribution. The equations, which exhibited the minimum sum of squared differences, were then used to generate complete tables of excess ratios. Exhibits I-4 and II-4 contain the new values developed by the collocation method.

A committee of actuaries reviewed and approved the use of the newly developed tables. The tables will be utilized in conjunction with the current excess loss premium factor calculations[6] until such time as fourth reports of losses by type of injury become available. (See Appendix A for an example of these calculations). At this point, the calculation shall be modified to incorporate the use of estimated actual development by type of injury in lieu of the 1.6 factor which Uhthoff's procedure uses[7]. Detailed comparisons of results under Uhthoff's methods and the new method are described in Appendix A.

[5] For a general description of collocation, see Stephen G. Kellison, *Fundamentals of Numerical Analysis*, Richard D. Irwin, Inc., (1975), p. 20ff.

[6] These are based upon an average of first and second reports, updated by law amendments.

[7] The factor is low in comparison with excess reinsurers' development factors based on actual experience:

EXHIBIT I-2
FATAL—LIMITED

EXCESS RATIOS

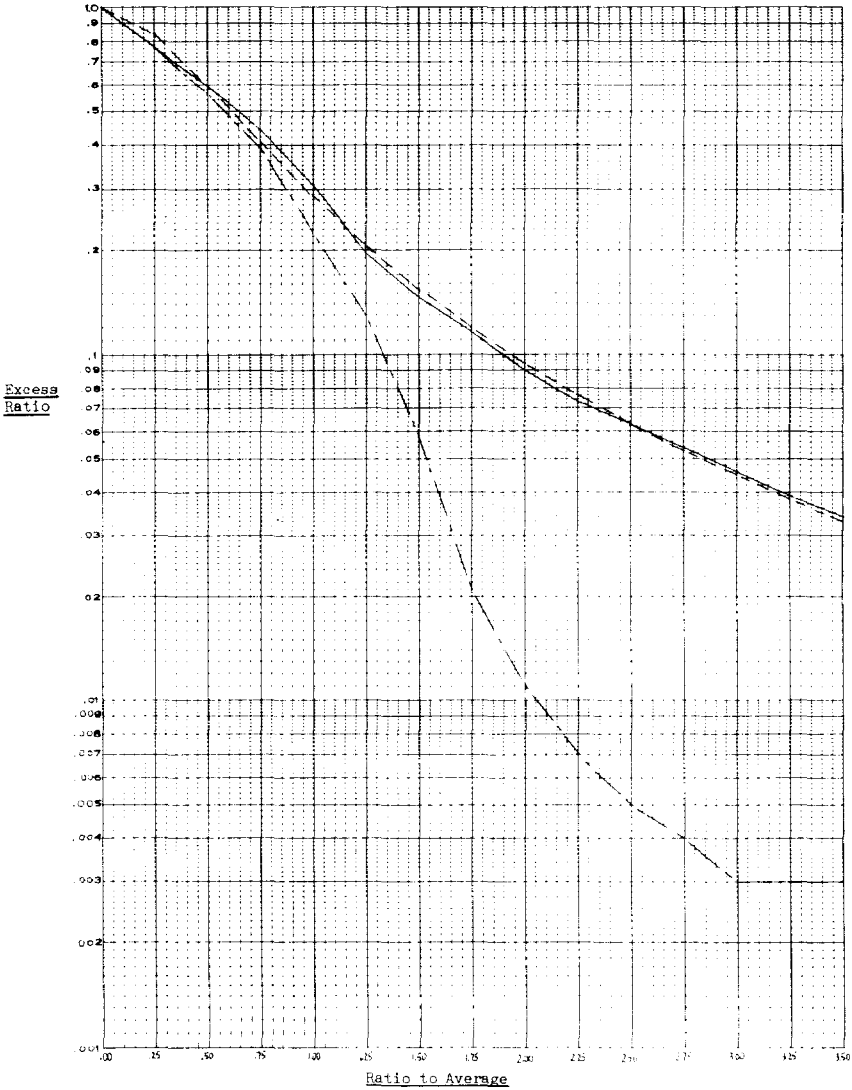
(x) Ratio to Avg.	(y) Actual*	CURVES FIT BY LEAST SQUARES CRITERION $y' = (.342)^x$	$y' = .151x^2 - .779x + 1$	$y' = e^{.127x^2 - 1.417x}$	$y' = \frac{1}{1 + .185x + 2.310x^2}$	$y' = \frac{1}{1 - x + 3.883x^2 - .406x^3}$
.00	1.000	1.000	1.000	1.000	1.000	1.000
.25	.776	.765	.815	.707	.840	1.014
.50	.597	.585	.648	.508	.599	.704
.75	.440	.447	.501	.371	.410	.442
1.00	.308	.342	.372	.275	.286	.288
1.25	.196	.262	.262	.207	.207	.199
1.50	.147	.200	.171	.159	.154	.146
1.75	.116	.153	.099	.124	.119	.112
2.00	.090	.117	.046	.098	.094	.089
2.25	.073	.089	.012	.078	.076	.073
2.50	.063	.068	—.004	.064	.063	.061
2.75	.054	.052	.000	.053	.053	.052
3.00	.046	.040	.022	.045	.045	.045
3.25	.039	.031	.063	.038	.038	.040
3.50	.034	.023	.123	.033	.033	.036
$\frac{\Sigma \mid y - y' \mid}{n}$	xx	.020	.046	.021	.010	.026
$\frac{\Sigma (y - y')^2}{n}$	xx	.00749	.002620	.001264	.000379	.004569

*See Exhibit I-1 for derivation.

EXHIBIT I-3

FATAL - LIMITED

Comparison of Current, Observed and Proposed Excess Ratios



Current — — — — — — —

Observed ————————

Proposed - - - - - - - - - - (Least squares fit of observed values: $y = \frac{1}{1 + .185\, x + 2.310\, x^2}$)

EXHIBIT I-4

FATAL CASES (LIMITED AMOUNT)*

Ratio To Aver.	Excess Per Case	Ratio To Aver.	Excess Per Case	Ratio To Aver.	Excess Per Case
0%	1.000	33	.762	66	.470
1	.998	34	.752	67	.463
2	.995	35	.742	68	.456
3	.992	36	.732	69	.449
4	.989	37	.722	70	.442
5	.985	38	.712	71	.436
6	.981	39	.702	72	.429
7	.976	40	.693	73	.423
8	.971	41	.683	74	.416
9	.966	42	.673	75	.410
10	.960	43	.664	76	.404
11	.954	44	.654	77	.398
12	.947	45	.645	78	.392
13	.941	46	.635	79	.386
14	.934	47	.626	80	.381
15	.926	48	.617	81	.375
16	.918	49	.608	82	.370
17	.911	50	.599	83	.364
18	.902	51	.590	84	.359
19	.894	52	.581	85	.354
20	.885	53	.572	86	.349
21	.877	54	.564	87	.344
22	.868	55	.555	88	.339
23	.859	56	.547	89	.334
24	.849	57	.539	90	.329
25	.840	58	.531	91	.325
26	.830	59	.523	92	.320
27	.821	60	.515	93	.315
28	.811	61	.507	94	.311
29	.801	62	.499	95	.307
30	.792	63	.492	96	.302
31	.782	64	.484	97	.298
32	.772	65	.477	98	.294

$$\text{*Excess per case} = \frac{1}{1 + .185\,(\text{Ratio to Average}) + 2.310\,(\text{Ratio to Average})^2}$$

EXHIBIT I-4 (CONT'D)

Ratio To Aver.	Excess Per Case	Ratio To Aver.	Excess Per Case	Ratio To Aver.	Excess Per Case
99	.290	134	.185	169	.126
100	.286	135	.183	170	.125
101	.282	136	.181	171	.124
102	.278	137	.179	172	.123
103	.275	138	.177	173	.121
104	.271	139	.175	174	.120
105	.267	140	.173	175	.119
106	.264	141	.171	176	.118
107	.260	142	.169	177	.117
108	.257	143	.167	178	.116
109	.253	144	.165	179	.115
110	.250	145	.163	180	.113
111	.247	146	.161	181	.112
112	.244	147	.160	182	.111
113	.240	148	.158	183	.110
114	.237	149	.156	184	.109
115	.234	150	.154	185	.108
116	.231	151	.153	186	.107
117	.228	152	.151	187	.106
118	.225	153	.149	188	.105
119	.223	154	.148	189	.104
120	.220	155	.146	190	.103
121	.217	156	.145	191	.102
122	.214	157	.143	192	.101
123	.212	158	.142	193	.100
124	.209	159	.140	194	.099
125	.207	160	.139	195	.099
126	.204	161	.137	196	.098
127	.202	162	.136	197	.097
128	.199	163	.134	198	.096
129	.197	164	.133	199	.095
130	.194	165	.132	200	.094
131	.192	166	.130	201	.093
132	.190	167	.129	202	.093
133	.188	168	.128	203	.092

EXHIBIT I-4 (CONT'D)

Ratio To Aver.	Excess Per Case	Ratio To Aver.	Excess Per Case	Ratio To Aver.	Excess Per Case
204	.091	239	.068	274	.053
205	.090	240	.068	275	.053
206	.089	241	.067	276	.052
207	.089	242	.067	277	.052
208	.088	243	.066	278	.052
209	.087	244	.066	279	.051
210	.086	245	.065	280	.051
211	.086	246	.065	281	.051
212	.085	247	.064	282	.050
213	.084	248	.064	283	.050
214	.084	249	.063	284	.050
215	.083	250	.063	285	.049
216	.082	251	.062	286	.049
217	.081	252	.062	287	.049
218	.081	253	.062	288	.048
219	.080	254	.061	289	.048
220	.079	255	.061	290	.048
221	.079	256	.060	291	.047
222	.078	257	.060	292	.047
223	.078	258	.059	293	.047
224	.077	259	.059	294	.046
225	.076	260	.058	295	.046
226	.076	261	.058	296	.046
227	.075	262	.058	297	.046
228	.074	263	.057	298	.045
229	.074	264	.057	299	.045
230	.073	265	.056	300	.045
231	.073	266	.056	301	.044
232	.072	267	.056	302	.044
233	.072	268	.055	303	.044
234	.071	269	.055	304	.044
235	.070	270	.055	305	.043
236	.070	271	.054	306	.043
237	.069	272	.054	307	.043
238	.069	273	.053	308	.043

EXHIBIT I-4 (CONT'D)

Ratio To Aver.	Excess Per Case	Ratio To Aver.	Excess Per Case	Ratio To Aver.	Excess Per Case
309	.042	323	.039	337	.036
310	.042	324	.039	338	.036
311	.042	325	.038	339	.035
312	.042	326	.038	340	.035
313	.041	327	.038	341	.035
314	.041	328	.038	342	.035
315	.041	329	.038	343	.035
316	.041	330	.037	344	.035
317	.040	331	.037	345	.034
318	.040	332	.037	346	.034
319	.040	333	.037	347	.034
320	.040	334	.037	348	.034
321	.039	335	.036	349	.034
322	.039	336	.036	350 & Over	.033

A study of experience by hazard group is also under review. The indicated hazard group differentials to average are based upon a review of the experience indications of high, medium and low benefit states as well as the experience of the totals of the three groups of states. The experience is shown in Appendix B (Exhibits B-1 through B-4).

These procedures and tables reflect the situation known today. It is hoped that they may survive periodic review and serve the insurance industry's requirements as long as "Uhthoff's Tables" have.

Accident Years	Range of Factors
1968-1974 (1st Report to Ultimate)	3.10-3.51
1968-1973 (2nd Report to Ultimate)	1.68-2.13
1968-1972 (3rd Report to Ultimate)	1.41-1.67
1968-1971 (4th Report to Ultimate)	1.37-1.46

Reported in March, 1976 issue of *Best's Property/Casualty Review*, pp. 14-18.

EXHIBIT II-1

MAJOR PERMANENT PARTIAL

Ratio to Avg.	Excess Ratios Ark.	Conn.	D.C.	Md.	Neb.	Avg.*
.00	1.000	1.000	1.000	1.000	1.000	1.000
.25	.750	.751	.751	.751	.751	.751
.50	.509	.528	.518	.526	.504	.519
.75	.343	.281	.361	.383	.339	.344
1.00	.248	.194	.253	.285	.242	.249
1.25	.178	.140	.174	.216	.181	.182
1.50	.129	.105	.118	.165	.141	.136
1.75	.096	.082	.081	.127	.113	.103
2.00	.073	.065	.059	.098	.094	.080
2.25	.058	.055	.050	.077	.082	.065
2.50	.046	.045	.042	.063	.075	.053
2.75	.040	.037	.036	.052	.067	.045
3.00	.030	.033	.031	.044	.058	.038
3.25	.025	.029	.028	.038	.052	.033
3.50	.021	.024	.022	.029	.051	.027
3.75	.020	.021	.021	.025	.050	.024
4.00	.017	.018	.018	.021	.049	.021
4.25	.015	.015	.016	.018	.048	.019
4.50	.013	.015	.013	.016	.047	.017
4.75	.012	.012	.010	.013	.046	.015
5.00	.010	.011	.008	.012	.045	.013
5.25	.009	.010	.006	.011	.044	.012
5.50	.008	.009	.005	.010	.043	.011
5.75	.007	.007	.003	.010	.042	.010
6.00	.006	.006	.001	.009	.041	.009
Total Number of Claims	794	666	290	1,022	233	xxx

*Average excess ratios weighted by state's total number of claims

EXHIBIT II-2

MAJOR PERMANENT PARTIAL EXCESS RATIOS

CURVES FIT BY LEAST SQUARES CRITERION

(x) Ratio to Avg.	(y) Actual*	$y' = (.267)^x$	$y' = .072x^2 - .565x + 1$	$y' = e^{.113x^2 - 1.435x}$	$y' = \frac{1}{1 + .555x + 2.655x^2}$	$y' = \frac{1}{1 + .805x + 2.044x^2 + .167x^3}$
.00	1.000	1.000	1.000	1.000	1.000	1.000
.25	.751	.719	.863	.703	.766	.751
.50	.519	.517	.736	.502	.515	.517
.75	.344	.371	.617	.363	.344	.354
1.00	.249	.267	.507	.267	.238	.249
1.25	.182	.192	.406	.198	.171	.181
1.50	.136	.138	.315	.150	.128	.136
1.75	.103	.099	.232	.115	.099	.105
2.00	.080	.071	.158	.089	.079	.082
2.25	.065	.051	.093	.070	.064	.066
2.50	.053	.037	.038	.056	.053	.054
2.75	.045	.026	—.009	.045	.044	.045
3.00	.038	.019	—.047	.037	.038	.038
3.25	.033	.014	—.076	.031	.032	.032
3.50	.027	.010	—.096	.026	.028	.028
3.75	.024	.007	—.106	.023	.025	.024
4.00	.021	.005	—.108	.020	.022	.021

*See Exhibit II-1 for derivation

EXHIBIT II-2 (CONT'D)

MAJOR PERMANENT PARTIAL EXCESS RATIOS

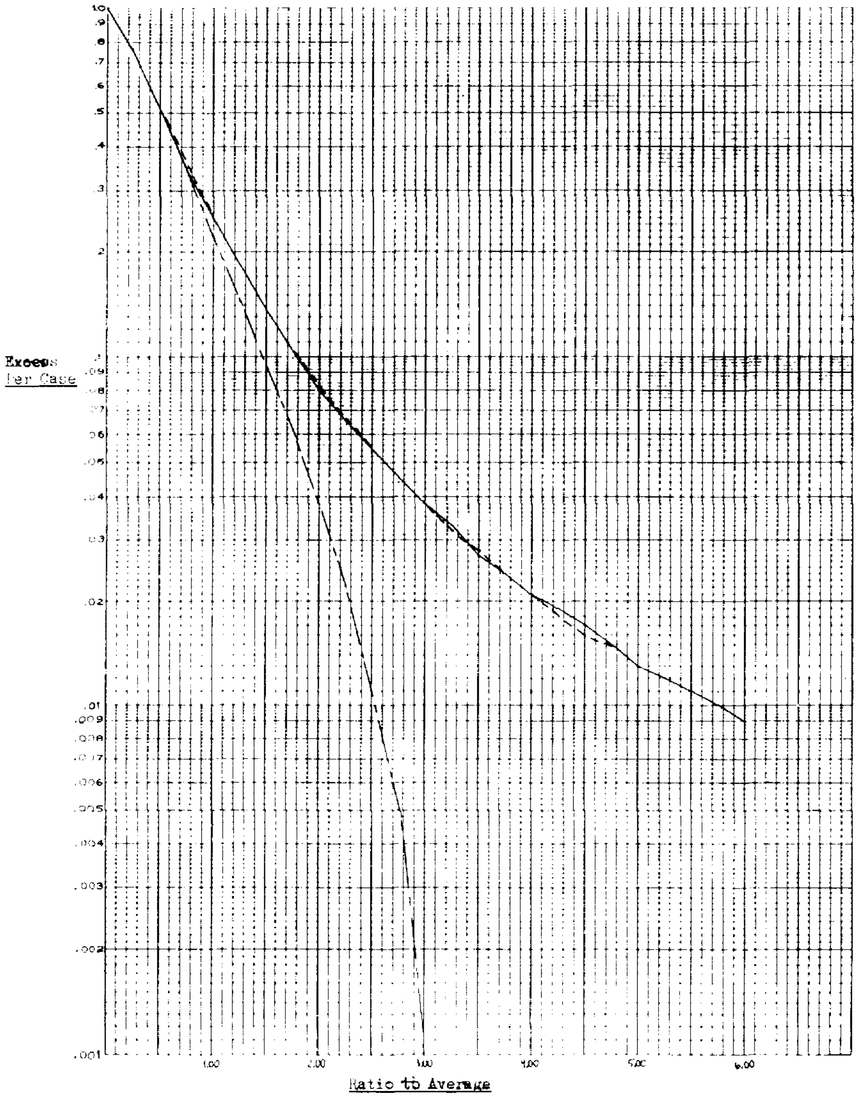
(x) Ratio to Avg.	(y) Actual*	CURVES FIT BY LEAST SQUARES CRITERION				
		$y' = (.267)^x$	$y' = .072x^2 - .565x + 1$	$y' = e^{.113x^2 - 1.435x}$	$y' = \frac{1}{1 + .555x + 2.655x^2}$	$y' = \frac{1}{1 + .805x + 2.044x^2 + .167x^3}$
4.25	.019	.004	—.101	.017	.019	.018
4.50	.017	.003	—.085	.015	.017	.016
4.75	.015	.002	—.059	.014	.016	.015
5.00	.013	.001	—.025	.013	.014	.013
5.25	.012	.001	.018	.012	.013	.012
5.50	.011	.001	.071	.011	.012	.011
5.75	.010	.001	.132	.011	.011	.010
6.00	.009	.000	.202	.011	.010	.009
$\frac{\Sigma \lvert y - y' \rvert}{n}$	xx	.013	.114	.007	.003	.001
$\frac{\Sigma (y - y')^2}{n}$	xx	.000231	.018594	.000160	.000023	.000005

*See Exhibit II-1 for derivation

EXHIBIT II-3

MAJOR PERMANENT PARTIAL

Comparison of Current, Observed and Proposed Excess Ratios



Current — — — — — — —

Observed————————

Proposed- - - - - - - - - - - - (Least squares fit of observed values: $y = \frac{1}{1+.80\text{[illegible]}x+2.044x^2+.167x^3}$)

EXHIBIT II-4
MAJOR PERMANENT PARTIAL CASES*

Ratio To Aver.	Excess Per Case	Ratio To Aver.	Excess Per Case	Ratio To Aver.	Excess Per Case
0%	1.000	33	.669	66	.405
1	.992	34	.659	67	.399
2	.983	35	.650	68	.393
3	.975	36	.640	69	.387
4	.966	37	.630	70	.381
5	.957	38	.621	71	.376
6	.947	39	.612	72	.370
7	.938	40	.603	73	.365
8	.928	41	.593	74	.359
9	.918	42	.584	75	.354
10	.908	43	.576	76	.349
11	.898	44	.567	77	.344
12	.888	45	.558	78	.339
13	.878	46	.550	79	.334
14	.867	47	.541	80	.329
15	.857	48	.533	81	.324
16	.846	49	.525	82	.320
17	.836	50	.517	83	.315
18	.825	51	.509	84	.311
19	.814	52	.501	85	.306
20	.804	53	.494	86	.302
21	.793	54	.486	87	.298
22	.783	55	.479	88	.294
23	.772	56	.471	89	.290
24	.761	57	.464	90	.286
25	.751	58	.457	91	.282
26	.741	59	.450	92	.278
27	.730	60	.443	93	.274
28	.720	61	.437	94	.270
29	.710	62	.430	95	.266
30	.699	63	.424	96	.263
31	.689	64	.417	97	.259
32	.679	65	.411	98	.256

*Excess per case =

$$\frac{4}{1 + .805\,(\text{Ratio to Avg.}) + 2.044\,(\text{Ratio to Avg.})^2 + .167\,(\text{Ratio to Avg.})^3}$$

EXHIBIT II-4 (CONT'D)

Ratio To Aver.	Excess Per Case	Ratio To Aver.	Excess Per Case	Ratio To Aver.	Excess Per Case
99	.252	134	.163	169	.111
100	.249	135	.161	170	.110
101	.246	136	.159	171	.109
102	.242	137	.157	172	.108
103	.239	138	.155	173	.107
104	.236	139	.153	174	.106
105	.233	140	.152	175	.105
106	.230	141	.150	176	.104
107	.227	142	.148	177	.103
108	.224	143	.147	178	.102
109	.221	144	.145	179	.101
110	.218	145	.143	180	.100
111	.216	146	.142	181	.099
112	.213	147	.140	182	.098
113	.210	148	.139	183	.097
114	.207	149	.137	184	.096
115	.205	150	.136	185	.095
116	.202	151	.134	186	.094
117	.200	152	.133	187	.093
118	.197	153	.131	188	.092
119	.195	154	.130	189	.091
120	.192	155	.129	190	.090
121	.190	156	.127	191	.090
122	.188	157	.126	192	.089
123	.185	158	.124	193	.088
124	.183	159	.123	194	.087
125	.181	160	.122	195	.086
126	.179	161	.121	196	.086
127	.177	162	.119	197	.085
128	.175	163	.118	198	.084
129	.172	164	.117	199	.083
130	.170	165	.116	200	.082
131	.168	166	.115	201	.082
132	.166	167	.113	202	.081
133	.164	168	.112	203	.080

EXHIBIT II-4 (CONT'D)

Ratio To Aver.	Excess Per Case	Ratio To Aver.	Excess Per Case	Ratio To Aver.	Excess Per Case
204	.080	239	.059	274	.045
205	.079	240	.059	275	.045
206	.078	241	.058	276	.045
207	.077	242	.058	277	.045
208	.077	243	.057	278	.044
209	.076	244	.057	279	.044
210	.075	245	.057	280	.044
211	.075	246	.056	281	.043
212	.074	247	.056	282	.043
213	.074	248	.055	283	.043
214	.073	249	.055	284	.042
215	.072	250	.054	285	.042
216	.072	251	.054	286	.042
217	.071	252	.054	287	.042
218	.070	253	.053	288	.041
219	.070	254	.053	289	.041
220	.069	255	.052	290	.041
221	.069	256	.052	291	.040
222	.068	257	.052	292	.040
223	.068	258	.051	293	.040
224	.067	259	.051	294	.040
225	.066	260	.050	295	.039
226	.066	261	.050	296	.039
227	.065	262	.050	297	.039
228	.065	263	.049	298	.039
229	.064	264	.049	299	.038
230	.064	265	.049	300	.038
231	.063	266	.048	301	.038
232	.063	267	.048	302	.037
233	.062	268	.047	303	.037
234	.062	269	.047	304	.037
235	.061	270	.047	305	.037
236	.061	271	.046	306	.037
237	.060	272	.046	307	.036
238	.060	273	.046	308	.036

EXHIBIT II-4 (CONT'D)

Ratio To Aver.	Excess Per Case	Ratio To Aver.	Excess Per Case	Ratio To Aver.	Excess Per Case
309	.036	344	.029	379	.024
310	.036	345	.029	380	.023
311	.035	346	.028	381	.023
312	.035	347	.028	382	.023
313	.035	348	.028	383	.023
314	.035	349	.028	384	.023
315	.034	350	.028	385	.023
316	.034	351	.028	386	.023
317	.034	352	.027	387	.023
318	.034	353	.027	388	.022
319	.034	354	.027	389	.022
320	.033	355	.027	390	.022
321	.033	356	.027	391	.022
322	.033	357	.027	392	.022
323	.033	358	.026	393	.022
324	.033	359	.026	394	.022
325	.032	360	.026	395	.022
326	.032	361	.026	396	.021
327	.032	362	.026	397	.021
328	.032	363	.026	398	.021
329	.032	364	.026	399	.021
330	.031	365	.025	400	.021
331	.031	366	.025	401	.021
332	.031	367	.025	402	.021
333	.031	368	.025	403	.021
334	.031	369	.025	404	.021
335	.030	370	.025	405	.020
336	.030	371	.025	406	.020
337	.030	372	.024	407	.020
338	.030	373	.024	408	.020
339	.030	374	.024	409	.020
340	.029	375	.024	410	.020
341	.029	376	.024	411	.020
342	.029	377	.024	412	.020
343	.029	378	.024	413	.020

EXHIBIT II-4 (CONT'D)

Ratio To Aver.	Excess Per Case	Ratio To Aver.	Excess Per Case	Ratio To Aver.	Excess Per Case
414	.020	449	.016	484	.014
415	.019	450	.016	485	.014
416	.019	451	.016	486	.014
417	.019	452	.016	487	.014
418	.019	453	.016	488	.014
419	.019	454	.016	489	.014
420	.019	455	.016	490	.014
421	.019	456	.016	491	.014
422	.019	457	.016	492	.013
423	.019	458	.016	493	.013
424	.019	459	.016	494	.013
425	.018	460	.016	495	.013
426	.018	461	.016	496	.013
427	.018	462	.015	497	.013
428	.018	463	.015	498	.013
429	.018	464	.015	499	.013
430	.018	465	.015	500	.013
431	.018	466	.015	501	.013
432	.018	467	.015	502	.013
433	.018	468	.015	503	.013
434	.018	469	.015	504	.013
435	.018	470	.015	505	.013
436	.017	471	.015	506	.013
437	.017	472	.015	507	.013
438	.017	473	.015	508	.013
439	.017	474	.015	509	.012
440	.017	475	.015	510	.012
441	.017	476	.014	511	.012
442	.017	477	.014	512	.012
443	.017	478	.014	513	.012
444	.017	479	.014	514	.012
445	.017	480	.014	515	.012
446	.017	481	.014	516	.012
447	.017	482	.014	517	.012
448	.016	483	.014	518	.012

EXHIBIT II-4 (CONT'D)

Ratio To Aver.	Excess Per Case	Ratio To Aver.	Excess Per Case	Ratio To Aver.	Excess Per Case
519	.012	547	.011	575	.010
520	.012	548	.011	576	.009
521	.012	549	.011	577	.009
522	.012	550	.011	578	.009
523	.012	551	.010	579	.009
524	.012	552	.010	580	.009
525	.012	553	.010	581	.009
526	.012	554	.010	582	.009
527	.012	555	.010	583	.009
528	.012	556	.010	584	.009
529	.011	557	.010	585	.009
530	.011	558	.010	586	.009
531	.011	559	.010	587	.009
532	.011	560	.010	588	.009
533	.011	561	.010	589	.009
534	.011	562	.010	590	.009
535	.011	563	.010	591	.009
536	.011	564	.010	592	.009
537	.011	565	.010	593	.009
538	.011	566	.010	594	.009
539	.011	567	.010	595	.009
540	.011	568	.010	596	.009
541	.011	569	.010	597	.009
542	.011	570	.010	598	.009
543	.011	571	.010	599	.009
544	.011	572	.010	600 & Over	.009
545	.011	573	.010		
546	.011	574	.010		

APPENDIX A

Three exhibits which follow set forth the calculation of Excess Loss Premium Factors. The first (Appendix A-1) describes the present procedure based on Uhthoff's tables, the second (Appendix A-2) describes the present procedure based on revised tables and the third (Appendix A-3) describes the present procedure (modified) based on revised tables. For convenience, they will be referred to as A-1, A-2 and A-3, respectively

All three exhibits rest upon two policy years of experience; one at a first report and one at a second report. The average claim cost is determined by adjusting the reported incurred losses to reflect law amendment factors and then dividing the result by the number of cases. This is performed by type of injury and is shown in Column 12 of A-1 and A-2. With respect to A-3, not only are the incurred losses adjusted to reflect law amendment factors, they are also modified to reflect loss development by type of injury. The resulting average claim cost is shown in Column 16 of A-3.

The average claim costs are shown on lines 13 (death), 16 (permanent total), and 19 (major) for Exhibits A-1 and A-2. The corresponding lines for Exhibit A-3 are lines 17, 20 and 23. In all three exhibits, the bottom half shows the selected accident limit ranging from $10,000 to $250,000 arranged by columns lettered from (A) through (L). These amounts are expressed as ratios to the average cost for each serious type of claim. These ratios are then used to enter the appropriate table, namely, Uhthoff's or Revised in order to determine the excess ratio contribution by each type of claim. These excess ratios are then weighted in proportion to the contribution to total cost made by each type of claim. The proportion, which is shown on line 22, is derived from the data in Column 11 for A-1 and A-2. These proportions shown on line 26 of A-3 are different from those of A-1 and A-2 because loss development has been included; they are derived from Column 15. The average excess ratio is multiplied by the permissible loss ratio increased by 10% to reflect the conversion of data complied on a per claim basis to a "per accident" basis. It is then increased by flat loadings ranging from .005 to .001 as the accident limit increases. Finally, the indicated Excess Loss Premium Factors are modified by a factor of 1.6 to reflect loss development with respect to the procedures in A-1 and A-2. With respect to A-3, this factor is not necessary since development was included at the beginning; consequently the indicated Excess Loss Premium Factors are the proposed Excess Loss Premium Factors.

The present procedures based on the revised tables tend to produce lower charges for the lower accidents limits and higher charges for the higher accident limits than those based on Uhthoff's tables. This is also true for the present procedure (modified) based on the revised tables wherein loss development by type of injury is included in the calculation of the average claim cost.

It is believed that the revised tables and the modified procedures will effectively generate more appropriate charges since quite frequently the proposed Excess Loss Premium Factors at the lower limits may need to be arbitrarily reduced because they exceed the permissible loss ratio.

APPENDIX A-1

PRESENT PROCEDURE — BASED ON UHTHOFF'S TABLES

		Policy Period (70-71) 2nd				Policy Period (71-72) 1st					
(1) Type Of Injury	(2) No. Of Cases	(3) Indemnity	(4) A.F.	(5) Medical	(6) A.F.	(7) Indemnity	(8) A.F.	(9) Medical	(10) A.F.	(11) Total	(12) Average (11) ÷ (2)
Death	86	2,451,463	1.150	87,228	1.000	2,280,619	1.100	57,109	1.000	5,472,200	63,630
P.T.	19	344,657	1.033	384,440	1.000	1,284,254	1.091	614,261	1.000	2,755,853	145,045
Major	1,271	6,950,644	1.145	2,585,223	1.000	8,763,791	1.086	3,948,947	1.000	24,010,134	18,891
Minor	xx	6,125,412	1.143	2,281,834	1.000	7,238,922	1.089	2,740,605	1.000	19,906,971	xx
T.T.	xx	5,617,151	1.156	3,495,883	1.000	7,883,248	1.097	5,247,349	1.000	23,884,582	xx
Other Med.	xx	xx	xx	2,651,245	1.000	xx	xx	3,584,517	1.000	6,235,762	xx
TOTAL	xx	xx	xx	xx	xx	xx	xx	xx	xx	82,265,502	xx

A.F. = Amendment Factor to current law level.

(Amounts in 1,000's)

	10	15	20	25	30	40	50	75	100	150	200	250
13. Average Death Cost (Incl. Med.)	(A) 63,630	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)
14. Ratio to Average (A), (B), (C), etc. ÷ (13)	.16	.24	.31	.39	.47	.63	.79	1.18	1.57	2.36	3.14	3.93
15. Excess Ratio for Death (from Tables)	.841	.761	.694	.630	.569	.453	.347	.156	.069	.027	.021	.021
16. Average P.T. Cost (Incl. Med.)	145,045											

APPENDIX A-1 (CONT'D)

17. Ratio to Average (A), (B), (C), etc. ÷ (16)	.07	.10	.14	.17	.21	.28	.34	.52	.69	1.03	1.38	1.72
18. Excess Ratio for P.T. (from Tables)	.930	.900	.860	.831	.791	.724	.668	.510	.378	.194	.103	.059
19. Average Major Cost (Incl. Med.)	18,891											
20. Ratio to Average (A), (B), (C), etc. ÷ (19)	.53	.79	1.06	1.32	1.59	2.12	2.65	3.97	5.29	7.94	10.59	13.23
21. Excess Ratio for Major (from Tables)	.496	.324	.200	.127	.081	.031	.007	.001	.001	.001	.001	.001
22. Ratios to Total Cost a. Death	.067											
b. P.T.	.033											
c. Major	.292											
23. Average Excess Ratio	.232	.175	.133	.107	.088	.063	.047	.028	.017	.009	.005	.004
24. Permissible Loss Ratio	.610											
25. (24) × 1.10	.671											
26. (23) × (25)	.156	.117	.089	.072	.059	.042	.032	.019	.011	.006	.003	.003
27. Flat Loadings	.005	.004	.003	.002	.002	.002	.001	.001	.001	.001	.001	.001
28. Indicated ELPF'S (26) + (27)	.161	.121	.092	.074	.061	.044	.033	.020	.012	.007	.004	.004
29. Proposed ELPF'S (28) × 1.6	.258	.194	.147	.118	.098	.070	.053	.032	.019	.011	.006	.006

*23 = [(15) × (22a)] + [(18) × (22b)] + [(21) × (22c)]

APPENDIX A-2

PRESENT PROCEDURE — BASED ON REVISED TABLES

		Policy Period (70-71) 2nd				Policy Period (71-72) 1st					
(1) Type of Injury	(2) No. Of Cases	(3) Indemnity	(4) A.F.	(5) Medical	(6) A.F.	(7) Indemnity	(8) A.F.	(9) Medical	(10) A.F.	(11) Total	(12) Average (11)÷(2)
Death	86	2,451,463	1.150	87,228	1.000	2,280,619	1.100	57,109	1.000	5,472,200	63,630
P.T.	19	344,657	1.033	384,440	1.000	1,284,254	1.091	614,261	1.000	2,755,853	145,045
Major	1,271	6,950,644	1.145	2,585,223	1.000	8,763,791	1.086	3,948,947	1.000	24,010,134	18,891
Minor	xx	6,125,412	1.143	2,281,834	1.000	7,238,922	1.089	2,740,605	1.000	19,906,971	xx
T.T.	xx	5,617,151	1.156	3,495,883	1.000	7,883,248	1.097	5,247,349	1.000	23,884,582	xx
Other Med.	xx	xx	xx	2,651,245	1.000	xx	xx	3,584,517	1.000	6,235,762	xx
TOTAL	xx	xx	xx	xx	xx	xx	xx	xx	xx	82,265,502	xx

A.F. = Amendment Factor to current law level.

	(Amounts in 1,000's) 10	15	20	25	30	40	50	75	100	150	200	250
13. Average Death Cost (Incl. Med.)	(A) 63,630	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)
14. Ratio to Average (A), (B), (C), etc. ÷ (13)	.16	.24	.31	.39	.47	.63	.79	1.18	1.57	2.36	3.14	3.93
15. Excess Ratio for Death (from Tables)	.918	.849	.782	.702	.626	.492	.386	.225	.143	.070	.041	.033
16. Average P.T. Cost (Incl. Med.)	145,045											

APPENDIX A-2 (CONT'D)

17. Ratio to Average (A), (B), (C), etc. ÷ (16)	.07	.10	.14	.17	.21	.28	.34	.52	.69	1.03	1.38	1.72
18. Excess Ratio For P.T. (from Tables)	.976	.960	.934	.911	.877	.811	.752	.581	.449	.275	.177	.123
19. Average Major Cost (Incl. Med.)	18,891											
20. Ratio to Average (A), (B), (C), etc. ÷ (19)	.53	.79	1.06	1.32	1.59	2.12	2.65	3.97	5.29	7.94	10.59	13.23
21. Excess Ratio for Major (from Tables)	.494	.334	.230	.166	.123	.074	.049	.021	.011	.009	.009	.009
22. Ratios to Total Cost a. Death b. P.T. c. Major	.067 .033 .292											
23. Average Excess Ratio	.238	.186	.150	.126	.107	.081	.065	.040	.028	.016	.011	.009
24. Permissible Loss Ratio	.610											
25. (24) × 1.10	.671											
26. (23) × (25)	.160	.125	.101	.085	.072	.054	.044	.027	.019	.011	.007	.006
27. Flat Loadings	.005	.004	.003	.002	.002	.002	.001	.001	.001	.001	.001	.001
28. Indicated ELPF'S (26) + (27)	.165	.129	.104	.087	.074	.056	.045	.028	.020	.012	.008	.007
29. Proposed ELPF'S (28) × 1.6	.264	.206	.166	.139	.118	.090	.072	.045	.032	.019	.013	.011

*23 = [(15) × (22a)] + [(18) × (22b)] + [(21) × (22c)]

APPENDIX A-3

PRESENT PROCEDURE (MODIFIED) — BASED ON REVISED TABLES

(1) Type Of Injury	(2) No. Of Cases†	(3) Indemnity	(4) A.F.	(5) Dev.	(6) Medical	(7) A.F.	(8) Dev.	(9) Indemnity	(10) A.F.	(11) Dev.	(12) Medical	(13) A.F.	(14) Dev.	(15) Total	(16) Average (15) ÷ (2)
Death	110	2,451,463	1.150	1.298	87,228	1.000	1.121	2,280,619	1.100	1.482	57,109	1.000	1.207	7,543,877	68,581
P.T.	28	344,657	1.033	2.329	384,440	1.000	1.121	1,284,254	1.091	2.394	614,261	1.000	1.207	5,355,850	191,280
Major	2,082	6,950,644	1.145	1.385	2,585,223	1.000	1.121	8,763,791	1.086	1.916	3,948,947	1.000	1.207	36,922,405	17,734
Minor	xx	6,125,412	1.143	.971	2,281,834	1.000	1.121	7,238,922	1.089	1.012	2,740,605	1.000	1.207	20,641,937	xx
T.T.	xx	5,617,151	1.156	1.095	3,495,883	1.000	1.121	7,883,248	1.097	1.136	5,247,349	1.000	1.207	27,186,778	xx
Other Med.	xx	xx	xx	xx	2,651,245	1.000	1.121	xx	xx	xx	3,584,517	1.000	1.207	7,298,558	xx
TOTAL	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx	104,949,405	xx

† No. of cases include development factors by type of injury. A.F. = Amendment Factor to current law level.

	Amounts in 1,000's) 10	15	20	25	30	40	50	75	100	150	200	250
17. Average Death Cost (Incl. Med.)	(A) 68,581	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)
18. Ratio to Average (A), (B), (C), etc. ÷ (17)	.15	.22	.29	.36	.44	.58	.73	1.09	1.46	2.19	2.92	3.65
19. Excess Ratio for Death (from Tables)	.926	.868	.801	.732	.654	.531	.423	.253	.161	.080	.047	.033
20. Average P.T. Cost (Incl. Med.)	191,280											
21. Ratio to Average (A), (B), (C), etc. ÷ (20)	.05	.08	.10	.13	.16	.21	.26	.39	.52	.78	1.05	1.31

APPENDIX A-3 (CONT'D)

22. Excess Ratio For P.T. (from Tables)	.985	.971	.960	.941	.918	.877	.830	.702	.581	.392	.267	.192
23. Average Major Cost (Incl. Med.)	17,734											
24. Ratio to Average (A), (B), (C), etc. ÷ (23)	.56	.85	1.13	1.41	1.69	2.26	2.82	4.23	5.64	8.46	11.28	14.10
25. Excess Ratio for Major (from Tables)	.471	.306	.210	.150	.111	.066	.043	.019	.010	.009	.009	.009
26. Ratios to Total Cost a. Death	.072											
b. P.T.	.051											
c. Major	.352											
27. Average Excess Ratio*	.283	.220	.181	.153	.133	.106	.088	.061	.045	.029	.020	.015
28. Permissible Loss Ratio	.610											
29. (28) × 1.10	.671											
30. (27) × (29)	.190	.148	.121	.103	.089	.071	.059	.041	.030	.019	.013	.010
31. Flat Loadings	.005	.004	.003	.002	.002	.002	.001	.001	.001	.001	.001	.001
32. Indicated ELPF's (30) + (31)	.195	.152	.124	.105	.091	.073	.060	.042	.031	.020	.014	.011

* 27 = [(19)×(26a)] + [(22)×(26b)] + [(25)×(26c)]

APPENDIX B

EXHIBIT B-1

Derivation of Indicated Hazard Group Differentials (High Benefit States)†

Hazard Group	(1) All Cases	(2) All Losses	(3) Claims Over 9,999 Cases	(4) Claims Over 9,999 Losses	(5) Claims over 24,999 Cases	(6) Claims over 24,999 Losses
I	106,786	39,062,759	803	16,861,263	191	7,793,946
II	674,620	303,854,815	6,410	141,923,132	1,564	68,176,269
III	267,399	191,174,298	4,329	111,871,561	1,315	65,717,152
IV	39,542	33,592,782	721	21,036,472	274	14,445,293
TOTAL	1,088,347	567,684,654	12,263	291,692,428	3,344	156,132,660

Indicated Hazard Group Relativities: Losses Over 24,999

Hazard Group	(7) Average Excess Ratio [(6) − 25,000 × (5)] ÷ (2)	(8) Indicated Relativities (7) ÷ (7) Total
I	.07728	.60
II	.09569	.75
III	.17179	1.34
IV	.22610	1.77
TOTAL	.12777	

Indicated Hazard Group Relativities: Loss Over 9,999

Hazard Group	(9) Average Excess Ratio [(4) − 10,000 × (3)] ÷ (2)	(10) Indicated Relativities (9) ÷ (9) Total
I	.22608	.76
II	.25612	.86
III	.35874	1.20
IV	.41159	1.38
TOTAL	.29781	

†Includes data from Alaska, Arizona, Connecticut, District of Columbia, Idaho, Illinois, Maine, Michigan, Minnesota, Oregon, and Rhode Island.

APPENDIX B

EXHIBIT B-2

Derivation of Indicated Hazard Group Differentials (Medium Benefit States)†

Hazard Group	(1) All Cases	(2) All Losses	(3) Claims Over 9,999 Cases	(4) Claims Over 9,999 Losses	(5) Claims over 24,999 Cases	(6) Claims over 24,999 Losses
I	94,238	22,064,844	362	6,133,760	46	1,595,904
II	657,109	187,377,649	3,367	64,327,682	563	23,329,029
III	257,383	123,407,829	2,629	58,061,238	606	26,863,892
IV	41,777	25,527,288	596	14,455,738	177	7,812,199
TOTAL	1,050,507	358,377,610	6,954	142,978,418	1,392	59,601,024

Indicated Hazard Group Relativities: Losses Over 24,999

Hazard Group	(7) Average Excess Ratio [(6) − 25,000 × (5)] ÷ (2)	(8) Indicated Relativities (7) ÷ (7) Total
I	.02021	.29
II	.04939	.71
III	.09492	1.37
IV	.13269	1.92
TOTAL	.06920	

Indicated Hazard Group Relativities: Loss Over 9,999

Hazard Group	(9) Average Excess Ratio [(4) − 10,000 × (3)] ÷ (2)	(10) Indicated Relativities (9) ÷ (9) Total
I	.11393	.56
II	.16361	.80
III	.25745	1.26
IV	.33281	1.62
TOTAL	.20492	

†Includes data from Arkansas, Colorado, Hawaii, Indiana, Iowa, Kentucky, Maryland, Missouri, Nebraska, New Hampshire, South Dakota, Tennessee, Vermont and Wisconsin.

APPENDIX B

EXHIBIT B-3

Derivation of Indicated Hazard Group Differentials (Low Benefit States)†

Hazard Group	(1) All Cases	(2) All Losses	(3) Claims Over 9,999 Cases	(4) Claims Over 9,999 Losses	(5) Claims over 24,999 Cases	(6) Claims over 24,999 Losses
I	106,736	28,424,675	486	8,633,223	58	2,396,027
II	693,890	193,922,765	3,853	71,691,448	519	21,896,794
III	334,485	158,990,080	3,750	74,442,781	654	27,209,740
IV	53,826	31,652,377	783	18,042,199	182	8,726,708
TOTAL	1,188,937	412,989,897	8,872	172,809,651	1,413	60,229,269

Indicated Hazard Group Relativities: Losses Over 24,999

Hazard Group	(7) Average Excess Ratio [(6) – 25,000 × (5)] ÷ (2)	(8) Indicated Relativities (7) ÷ (7) Total
I	.03328	.55
II	.04601	.76
III	.06830	1.13
IV	.13196	2.19
TOTAL	.06030	

Indicated Hazard Group Relativities: Loss Over 9,999

Hazard Group	(9) Average Excess Ratio [(4) – 10,000 × (3)] ÷ (2)	(10) Indicated Relativities (9) ÷ (9) Total
I	.13274	.65
II	.17100	.84
III	.23236	1.14
IV	.32264	1.58
TOTAL	.20361	

†Includes data from Alabama, Florida, Georgia, Kansas, Louisiana, Mississippi, Montana, New Mexico, North Carolina, Oklahoma, South Carolina, Utah, and Virginia.

APPENDIX B

EXHIBIT B-4

Derivation of Indicated Hazard Group Differentials (All States)†

Hazard Group	(1) All Cases	(2) All Losses	(3) Claims Over 9,999 Cases	(4) Claims Over 9,999 Losses	(5) Claims over 24,999 Cases	(6) Claims over 24,999 Losses
I	307,760	89,552,278	1,651	31,628,246	295	11,785,877
II	2,025,619	685,155,229	13,630	277,942,262	2,646	113,402,092
III	859,267	473,572,207	10,708	244,375,580	2,575	119,790,784
IV	135,145	90,772,447	2,100	53,534,409	633	30,984,200
TOTAL	3,327,791	1,339,052,161	28,089	607,480,497	6,149	275,962,953

Indicated Hazard Group Relativities: Losses Over 24,999

Hazard Group	(7) Average Excess Ratio [(6) − 25,000 × (5)] ÷ (2)	(8) Indicated Relativities (7) ÷ (7) Total
I	.04925	.54
II	.06897	.76
III	.11702	1.28
IV	.16700	1.83
TOTAL	.09129	

Indicated Hazard Group Relativities: Loss Over 9,999

Hazard Group	(9) Average Excess Ratio [(4) − 10,000 × (3)] ÷ (2)	(10) Indicated Relativities (9) ÷ (9) Total
I	.16882	.69
II	.20673	.85
III	.28991	1.19
IV	.35842	1.47
TOTAL	.24390	

†Includes data from the states listed in exhibits I, II, and III.

ACCIDENT LIMITATIONS FOR RETROSPECTIVE RATING

FRANK HARWAYNE

DISCUSSION BY ROBERT J. FINGER

This paper presents a method for calculating excess loss premium factors (ELPF's). Applying the ELPF to the standard premium determines the premium required to cover losses in excess of a given per accident limitation.

The ELPF is essentially calculated in two phases. First, claim size distributions are required for three types of claims: deaths, permanent total disabilities and major permanent partial disabilities. The claim size distribution gives the percentage of total losses for that injury type which are in excess of a certain accident limitation. Second, the percentage of losses by injury type in excess of the accident limitation are multiplied by the cost of that injury type as a percentage of total premium. Adding the costs for the three injury types yields the ELPF.

The claim size distributions are calculated in a three-step procedure. First, an empirical excess of loss distribution is calculated by state and injury type. This distribution is the percentage of losses in excess of a given amount per claim. The empirical distribution is calculated as a function of ratios to the mean, or average claim size. Second, a composite countrywide distribution is calculated by weighting the state's experience by the number of claims represented. Finally, the empirical distribution is graduated by a function of the form:

$$y = (1 + ax + bx^2 + cx^3)^{-1}$$

where x is the ratio to the mean.

This discussion will explore the applicability of modelling the above claim size distributions by the log-normal probability distribution.

The paper gives empirical data for several states for limited death cases and for major permanent partial cases. The discussion will limit itself to major permanent partial claims, but suitable techniques are applicable to limited death cases.

Table I shows the empirical average excess loss distribution for major permanent partial claims. Also shown are log-normal distributions for coefficient of variations (CV) equal to 0.5, 0.75 and 1.0[1]. It can be seen that the empirical distribution is similar in shape to a log-normal distribution. In fact, it is not too different from a log-normal with a CV of 0.75. Reasons for the discrepancy can be various, but might prove worth exploring. Among the possibilities: (1) the empirical distribution is based in part on case reserves; these reserves may not be entirely accurate: (2) there may be inaccuracies in the data; (3) the data may be distorted by a few abnormal claims or by the weighting by state; (4) limitations in certain states may distort the data; (5) the data may not be log-normally distributed.

It would seem desirable for many reasons to have a generalized model of claim sizes. The log-normal distribution might be a suitable model. Such a model would facilitate making adjustments for particular states, for particular hazard groups or classes, for particular injury types, or for changing claim settlement practices and influences.

TABLE I

SELECTED EXCESS LOSS DISTRIBUTIONS

Ratio To Mean	Log-Normal CV = .5	Empirical Average*	Log-Normal CV = .75	Log-Normal CV = 1.0
.25	75%	75%	75%	75%
.50	51	52	54	56
.75	32	34	38	41
1.0	19	25	26	32
1.5	6	14	13	17
2.0	2	8	7	13
3.0		4	2	6
4.0		2	1	3
5.0		1		2

*Major permanent partial claims; weighted average for five states.

[1] The coefficient of variation is the ratio of the mean to the standard deviation.

ESTIMATING PURE PREMIUMS BY LAYER —AN APPROACH

ROBERT J. FINGER

This paper presents an approach to the estimation of loss costs by layer of insurance coverage. This method uses the log-normal probability distribution as a model for claim sizes. Although the approach has been successfully applied to several different lines of liability insurance, it may not be applicable to property insurance.

The motivation for using a probabilistic model for claim sizes arises largely from the "long-tail" nature of liability insurance. The long tail derives from both the delayed reporting of claims as well as from the lengthy settlement period involved. The long tail makes it difficult to accurately price some liability insurance lines. Since it takes many years to settle claims, the latest year for which a vast majority of claims are settled may well be quite old. Conditions may have changed significantly since that latest mature year. Indeed, average claim costs have increased significantly in most liability lines over the past several years.

In choosing experience data, the ratemaker is thus forced to make a tradeoff between using less mature experience and using more mature (and older) experience with a larger trend factor, to estimate current costs. A method often used to produce more consistent, stable, and mature experience data is to limit individual claims to a certain size, often called "basic limits." A difficulty with this approach is that the value of the basic limits is changing over time. For example, $25,000 in 1963 claim costs was probably quite different than $25,000 in 1973 claim costs. Almost assuredly, the percentage of total limits losses below $25,000 per claim in 1963 was more than the respective percentage below $25,000 in 1973.

When the ratemaker's attention is focused on higher layers of liability, the problems caused by delayed settlements are more significant. The persistent inflation of recent years has pushed both jury verdicts and claim settlements to higher levels. Not only do more claims find their way into higher layers over time, but there is a leverage effect on their amounts; that is, the increase in amount applies only to the highest layer. This paper presents an aid to estimating pure premiums for the higher layers of liability.

The method described in this paper will be applied to two specific problems:

> *Problem No. 1:* A new company, formed to write medical malpractice insurance, wants to purchase excess of loss reinsurance to cover a layer of \$900,000 excess of \$100,000 per claim. How might the premium for this coverage be determined?

> *Problem No. 2:* Experience data is available for medical malpractice claims for policy years 1963 to 1974. The loss data is limited to \$25,000 per claim and premiums are needed for \$100,000 limits. What increased limits factors should be applied to the data to calculate the \$100,000 pure premiums?

THE APPROACH

The approach assumes that the distribution of incurred claim sizes follows a log-normal probability distribution. Knowing two parameters of this distribution, such as the mean and coefficient of variation (CV)[1], one can calculate the percentage of incurred losses by layer. Rather than talking about the losses for a specific layer, it is simpler to talk in terms of the *excess loss distribution.* This distribution is the percentage of total limits losses which are above a certain amount, called the *attachment point,* per claim[2]. Assuming, for example, that the mean of the total limits claim size distribution is \$50,000 and the CV is 3.0, the excess pure premium for an attachment point of \$100,000 is about 40% of the total limits pure premium. For an attachment point of \$250,000 it is about 21%, and for an attachment point of \$1,000,000 it is about 5%. (See Table I.)

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[1] The coefficient of variation is the ratio of the standard deviation to the mean.

[2] Excess losses above a given attachment point are defined as the sum of all claim values larger than the attachment point, less the number of claims above the attachment point multiplied by the value of the attachment point.

TABLE I

EXCESS LOSS DISTRIBUTION (AS A PERCENTAGE OF TOTAL LIMITS LOSSES)

LOG-NORMAL ASSUMPTION

Attachment Point (Times The Mean)	Coefficient Of Variation					
	1.0	1.5	2.0	3.0	4.0	5.0
.5	56%	61%	65%	70%	73%	75%
1.0	32	41	47	55	60	63
1.5	20	30	37	46	52	56
2.0	13	22	30	40	46	50
2.5	9	18	25	35	41	46
3	6	14	21	31	37	42
4	3	9	16	25	32	36
5	2	7	12	21	27	32
10	—	2	5	11	16	21
15	—	1	3	7	12	15
20	—	—	2	5	9	12
25	—	—	1	4	7	10
50	—	—	—	1	3	5
100	—	—	—	—	1	2

The log-normal distribution has appeared previously in the *Proceedings* and other actuarial literature[3]. It is assumed that the natural logarithms of the claim sizes are distributed according to the normal (or Gaussian) probability law. The appendix gives a precise mathematical definition of the log-normal distribution. Exhibit I illustrates the case where the mean is 60 and the CV is 3. The main virtues of the log-normal distribution, from a modelling point of view, are that: (1) it can be a highly skewed distribution[4] and (2) it can be justified on a intuitive basis.

[3] For example, the log-normal distribution is mentioned in: Bickerstaff, D. R. "Automobile Collision Deductibles and Repair Cost Groups: The Lognormal Model" PCAS LIX (1972); Hewitt, C. C. "Credibility for Severity" PCAS LVII (1970); Mayerson, A. L. "A Bayesian View of Credibility" PCAS LI (1964). It is also discussed in Harding, V. "Treatment of IBNR Claims," *IBNR*, Amsterdam: Netherlands Reinsurance Group (1972). A thorough discussion of the log-normal distribution can be found in Aitchison, J. and J. A. C. Brown, *The Lognormal Distribution*, Cambridge University Press (1957).

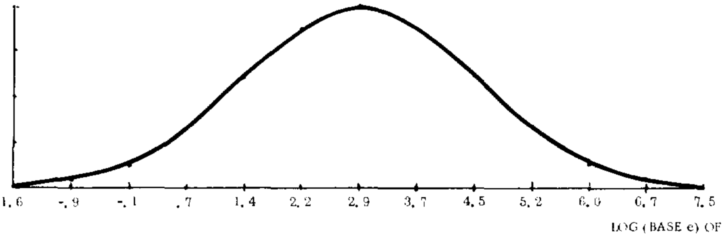
[4] The higher the CV, the more skewed the distribution. This can be seen in Table I.

EXHIBIT I

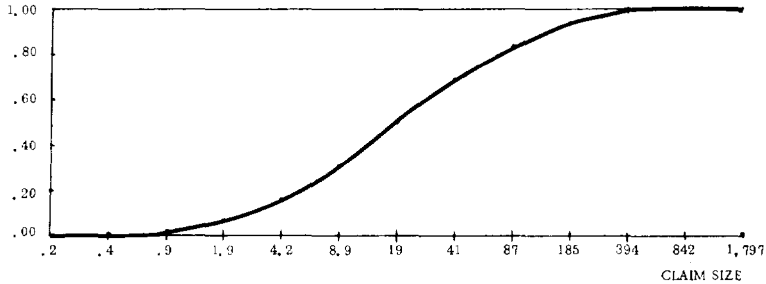
LOG-NORMAL PROBABILITY DISTRIBUTION

MEAN = 60 CV = 3.0

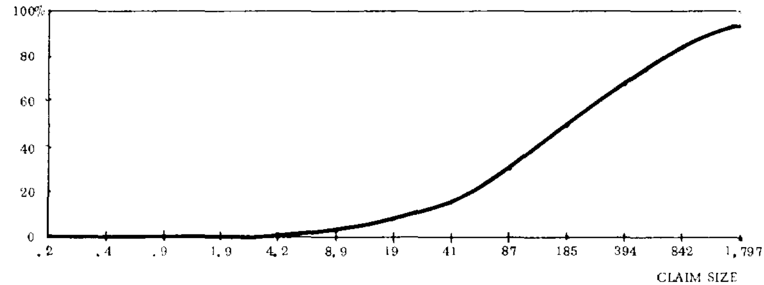
1. PROBABILITY DENSITY FUNCTION



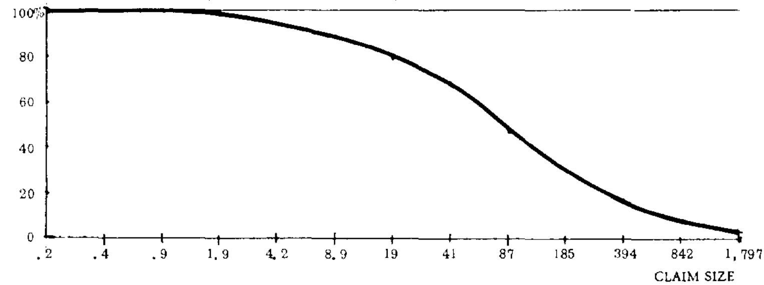
2. CUMULATIVE DISTRIBUTION FUNCTION



3. FIRST MOMENT DISTRIBUTION (AS PERCENTAGE OF MEAN)



4. EXCESS LOSS DISTRIBUTION (AS PERCENTAGE OF MEAN)



Intuitively, the log-normal distribution can be considered appropriate as an analog of the central limit theorem. The central limit theorem states that the average (or sum) of independent random variables will converge to the normal probability distribution. The normal distribution can thus be used as an approximation for the distribution of the sum of a number of independent random variables. If the individual random variables were logarithms, the sum of the logarithms would be approximately normally distributed. The sum of logarithms is analogous to the product of the anti-logarithms.

If we have a number of independent variables, whose product is the observed claim size, we can expect the sum of the logs of these variables to be approximately normally distributed; the claim size would then be approximately log-normally distributed. We might thus expect that any line of business where several independent factors can be multiplied together to determine the claim size will have a log-normal claim size distribution.

Considering an automobile accident, we may theorize that a number of independent factors interact multiplicatively to determine the liability claim size, such as:

- the speed of the vehicles before impact
- the health of the injured party
- the protection (e.g., with seat belts, interior padding), of the victim
- the income of the victim
- the skill of the plaintiff's attorney, and
- the skill of the defendant's claims adjusters.

Regardless of the intuitive justification, the choice of claim size distribution must be sustained in practice. As will be pointed out later, the log-normal distribution seems to provide a good fit for medical malpractice insurance claims.

The log-normal assumption applies to the individual claim sizes (i.e., the claim count). A related distribution is the (first) moment distribution. The moment distribution gives the total amount of losses on claims which are smaller than a given size. Exhibit I, Section 2, illustrates the cumulative distribution function of the claim count distribution. Exhibit I, Section 3, illustrates the cumulative moment distribution, as a percentage of the mean.

The excess loss distribution gives the total amount of losses above a given attachment point per claim. It differs from the complement of the first moment distribution in that the amount of the attachment point is subtracted from every claim greater than the attachment point. Exhibit 1, Section 4, illustrates the excess loss distribution, as a percentage of the mean.

The log-normal distribution has two parameters. In practical usage, the mean and CV can be used as the two parameters. The log-normal distribution has the desirable property that, for a given CV, the distribution can be completely described by a function of a factor times the mean. This means, for example, that the distribution for an attachment point of \$100,000 and a mean of \$50,000 is the same as for a \$200,000 attachment point and \$100,000 mean. In both cases the attachment point is 2.0 times the mean. Tables of the log-normal distribution can thus be prepared (see Table I) as a function of the CV and a factor times the mean. Exhibit II depicts the excess loss distribution graphically as a function of the CV and attachment point, which is defined as a ratio to the mean.

We now tackle the two problems posed earlier:

Problem No. 1: For simplicity we may assume that each primary policy is sold for \$1,000,000 limits. We have concluded from other analysis that \$6,300 is an appropriate pure premium for the coverage. This pure premium is made up of a gross frequency of 26.5%; 50% of the claims are closed without a payment; and the total limits average closed-paid claim will be \$50,000. Based on other evidence, we assume that claim sizes are log-normally distributed with a CV of 3.0. Pure premium by layer can thus be calculated as in Table II. From this table we see that coverage from \$100,000 to \$1,000,000 would cost about \$2,650 —\$330 = \$2,320 in claims per exposure unit. (Pure premium for coverage up to \$1,000,000 is \$6,630 — \$330 = \$6,300.)

TABLE II

EXCESS PURE PREMIUMS BY LAYER

Attachment Point		Excess Losses	
(1000's)	Times Mean	% Of Total	Per Unit
0	0.	100	6,630
25	0.5	70	4,640
50	1.0	55	3,640
100	2.0	40	2,650
250	5.0	21	1,390
500	10.0	11	730
1,000	20.0	5	330
2,500	50.0	1	66

Problem No. 2: For simplicity we may assume that the total limits mean claim size in 1964 is \$10,000; that total limits claim sizes are increasing at 15% annually; and that claim sizes are log-normally distributed with a CV of 3.0. We can then calculate the excess losses for each attachment point for each year. Increased limits factors can be calculated directly from the excess loss distribution. Table III illustrates this problem. It should be noted that the increased limits factors are increasing.

TABLE III
INCREASED LIMITS FACTORS FOR \$100,000 OVER \$25,000

Policy Year	Ratios To Total Limits Mean[a]		Percent Excess Losses[b]		Indicated Increased Limits Factor[c]
	\$25,000	\$100,000	\$25,000	\$100,000	
1963	2.9	11.5	31%	10%	1.32
1964	2.5	10.0	35	11	1.35
1965	2.2	8.7	37	13	1.39
1966	1.9	7.6	41	15	1.44
1967	1.6	6.6	44	17	1.49
1968	1.4	5.7	48	19	1.54
1969	1.2	5.0	50	21	1.59
1970	1.1	4.3	53	24	1.64
1971	.9	3.8	56	26	1.70
1972	.8	3.3	60	29	1.76
1973	.7	2.8	63	32	1.82
1974	.6	2.5	65	35	1.88

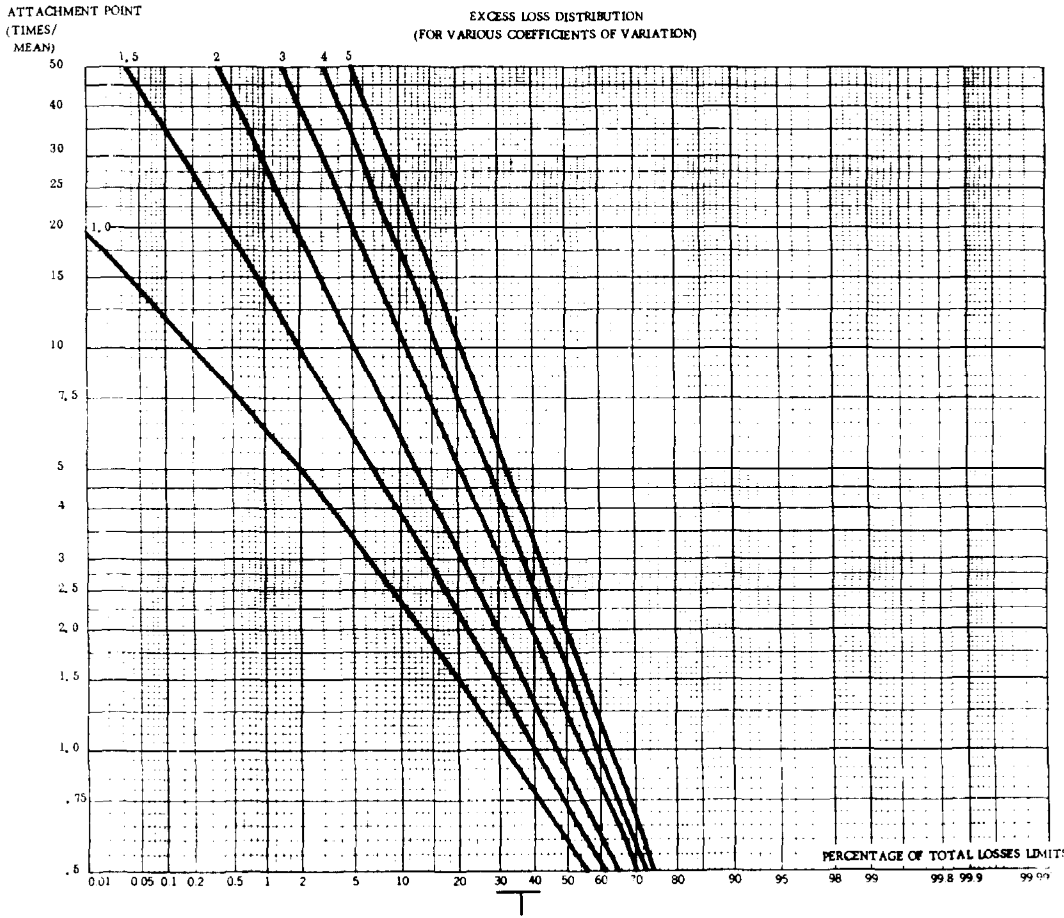
Notes: a. Adjusted for 15% annual inflation.
b. Based on log-normal distribution with CV 3.0.
c. Other columns have been rounded. This is calculated as:

$$\frac{100 - E_{100,000}}{100 - E_{25,000}}$$ where E_x is the percentage of total limits losses above x per claim.

PARAMETER ESTIMATION

To use the approach of this paper, one needs to make assumptions about the total limits mean and CV of the claim size distribution. The basic limits mean is often available from other actuarial analysis. For a given choice of basic limits mean and CV, there is total limits mean. The more difficult parameter to estimate is the CV.

EXHIBIT II


EXCESS LOSS DISTRIBUTION
(FOR VARIOUS COEFFICIENTS OF VARIATION)
ATTACHMENT POINT (TIMES/ MEAN)
PERCENTAGE OF TOTAL LOSSES LIMITS
1.0
1.5
2
3
4
5
50
40
30
25
20
15
10
7.5
5
4
3
2.5
2.0
1.5
1.0
.75
.5
0.01
0.05
0.1
0.2
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
99.8
99.9

A number of practical problems arise in estimating the CV; these include:

- individual claim values are not always known
- claim values tend to cluster at target values, such as $2,500, $5,000 or $10,000
- a large number of nuisance claims are often settled for small amounts, such as $250, $500 or $1,000[5]
- many claims are closed without a payment.

Depending upon the specific situation, the entire claim size distribution may not be log-normally distributed. It is often possible to eliminate some claims from consideration, such as very small claims or claims closed without a payment. The remaining distribution may then closely approximate a log-normal distribution.

This author has found it most convenient to estimate the CV from the observed excess loss distribution. To accomplish this, claims are grouped by interval and the percentage of the total limits losses in excess of a given interval is calculated. There is a unique CV for a given combination of excess percentage and ratio of the attachment point to the mean of the total limits distribution. For example, if excess losses above an attachment point of 2.0 times the mean are 40%, this implies a CV of 3.0. The uniqueness property is illustrated by Exhibit II.

Following the procedure above, the CV is estimated for a number of attachment points. If the estimated CV is the same for each attachment point tested, the distribution can safely be assumed to be log-normally distributed with the observed mean and given CV. If the estimated CV's are randomly distributed about a given value, that value is an appropriate estimate of the CV. If the estimated CV's form a progression (such as 6, 5, 4, 3), the observed data is not log-normally distributed. In the latter case, the data can be truncated, and the remaining data fitted to a log-normal distribution.

[5] Considering that the average allocated expense payment in medical malpractice is over $2,000, there is an incentive to pay a token settlement even when there is no negligence.

This estimation procedure is highly empirical. This may not be a serious drawback since the observed distribution of claim sizes may not be log-normally distributed[6]; one or two large claims, by presence or omission, may distort the observed data. The practical difference resulting from the use of 3.0 versus 2.9, for example, will be small.

An example of the estimation procedure will now be given. The data is shown in Table IV. As might be expected, most claims are relatively small, but a significant amount of the loss dollars are on higher intervals. Estimating the CV from the claim count distribution can be misleading because a majority of the claims are small and the majority of the claim dollars are on a small number of large claims. In the given example, the largest 2.5% of the claims account for 50.9% of the claim dollars. Estimating the CV from the moment distribution can be misleading because of the targeting problem. For example, there may be twenty-five claims for exactly $100,000. Should these claims be considered larger than $100,000 or smaller than $100,000; or are 50% larger and 50% smaller? Using the excess loss distribution largely avoids the targeting problem and it puts the emphasis on the layers where losses have occurred.

If there are a large number of claims closed without a payment, the distribution which includes them is not likely to be log-normally distributed. Table V illustrates this. The basic difference between estimating the CV with and without claims closed without a payment is the indicated mean of the distribution. The data is log-normally distributed if the estimated CV's for different attachment points are the same. If the estimated CV's for higher attachment points exhibit a downward trend, this indicates that the observed mean is too small. In other words, it shows that claims are concentrated too close to the mean. One can raise the observed mean by eliminating claims closed without a payment or by eliminating some of the smaller claims.

Table VI illustrates the estimation process when claims below a given amount (such as $10,000) are excluded from the analysis. The basic difficulty involved in this procedure is in estimating the number and amount of claims which should have appeared below the truncation point. The trun-

[6] The above estimation procedure clearly indicates when the log-normal distribution does not provide a good fit for the data. This occurs when successive estimated CV's form a progression.

TABLE IV

SAMPLE DATA

Interval	Attachment Point	Number Of Claims	Claim Count Distribution		Indemnity On Interval	Moment Distribution	Excess Losses[a]	
			All Claims	Paid Claims	($1000's)		($1000's)	Percent
0	0	2,370	51.4%	0%	$ 0	0%	$50,615	100.0%
1-10,000	10,000	1,496	83.8	66.7	4,500	8.9	38,645	76.4
10,001-25,000	25,000	365	91.7	83.0	6,437	21.6	30,128	59.5
25,001-100,000	100,000	267	97.5	94.9	13,933	49.1	14,245	28.1
100,001-300,000	300,000	99	99.7	99.3	16,488	81.7	4,457	8.8
300,001-1,000,000	1,000,000	15	100.0	100.0	7,207	95.9	1,050	2.1
Over 1,000,000	—	1	100.0	100.0	2,050	100.0	—	—
Total		4,613	(4,613)	(2,243)	$50,615			

Source: AIA (See Table VII)

Notes: a. Excess losses above a given attachment point are the sum of all claims values larger than the attachment point less the number of claims above the attachment point times the value of the attachment point. For example, there are 16 claims larger than $300,000 with an aggregate value of $9,257,000. The excess losses above $300,000 are thus 9,257,000 − 16(300,000) = 4,457,000 or 8.8% of the total limits losses.

TABLE V

ESTIMATING THE CV:
EXCLUSION OF CLAIMS CLOSED WITHOUT A PAYMENT

Case I. Includes all claims. Mean $11,000				Case II. Excludes claims closed without a payment. Mean $22,600		
Attachment Point (1000's)	Attachment Point Times Mean	Excess Percent	Estimated CV[a]	Attachment Point Times Mean	Excess Percent	Estimated CV[a]
10	.9	76 %	[b]	.4	76 %	4.5
25	2.3	60	[b]	1.1	60	4.2
100	9.1	28	6.8	4.3	28	3.6
300	27	8.8	5.4	13	8.8	3.1
1,000	91	2.1	4.8	44	2.1	3.2

[a] Estimated from tables.
[b] More than 10.0.

cation point and the mean of the remaining claims are known. Unfortunately, the relationship between these two items does not specify a unique CV (see Exhibit III). We must therefore pick a provisional CV, calculate the number and amount of claims below the truncation point and then see if the CV estimated from the excess load distribution is the same as our provisional value. Table VI shows that the CV is about 2.4. This result implies that about 38% of the claims should have been truncated or that there should have been about 1,205 claims. Instead the data shows 2,243 claims. We might conclude that there were over 1,000 nuisance claims which cost an average of about $2,300 each.

TABLE VI
ESTIMATING THE CV: TRUNCATION

Case Study:	Truncation Point	$ 10,000
	Remaining Amount	$46,100,000
	Remaining Count	747
	Remaining Mean	$ 61,700
Ratio:	Truncation Point to Remaining Mean	.162

Assumption:	CV 2.0	CV 2.5	CV 3.0
Ratio: Remaining Mean to Complete Mean[a]	1.35	1.58	1.80
Estimated Complete Mean (1000's)	45.7	39.1	34.3
Ratio: Truncation Point to Complete Mean	.22	.26	.29
Percent of Total Amount Truncated[b]	3.5	4.8	5.8
Percent of Total Count Truncated[b]	28.6	39.6	47.7
Estimated Total Amount (1000's)	47,800	48,500	48,900

Attachment Point (1000's)	Excess Losses (1000's)	CV 2.0: Attachment (Times/ Mean)	CV 2.0: Percent Excess	CV 2.0: Estimated CV[b]	CV 2.5: Attachment (Times/ Mean)	CV 2.5: Percent Excess	CV 2.5: Estimated CV[b]	CV 3.0: Attachment (Times/ Mean)	CV 3.0: Percent Excess	CV 3.0: Estimated CV[b]
$ 10	$38,600	.22	81	2.0	.26	80	2.5	.29	79	3.0
25	30,100	.55	63	2.0	.64	62	2.5	.73	62	3.0
100	14,200	2.2	30	2.2	2.6	29	2.5	2.9	29	2.8
300	4,500	6.6	9	2.0	7.7	9	2.3	8.7	9	2.5
1,000	1,000	22	2.1	2.2	26	2.1	2.4	29	2.0	2.5

Notes: a. See Exhibit III
b. From Tables

EXHIBIT III

ESTIMATING THE COMPLETE MEAN FROM A TRUNCATED LOG-NORMAL DISTRIBUTION

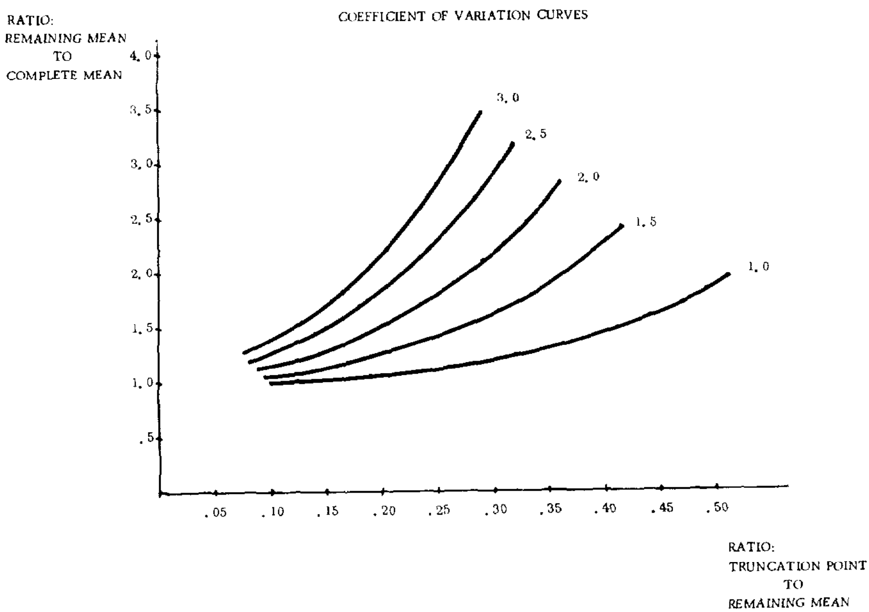


Table VII shows the estimation of the CV for two large groups of countrywide medical malpractice claims. The first group (the AIA study) has already been used in the previous analysis. Calculating the mean from all claims closed with a payment, indicates a CV of about 3.1 to 3.6 for attachment points in excess of $100,000. As previously shown, eliminating nuisance claims indicates a CV of about 2.4 for all attachment points. The second group (NAIC) indicates a higher estimated CV. This is partially due to one more claim in excess of $1,000,000. The higher CV may also be due to the broader group of companies which were included in the study.

SENSITIVITY ANALYSIS

Nuisance claims and other problems tend to distort the estimation process. Nuisance claims may be removed by estimating parameters for a truncated distribution. Because it is somewhat more cumbersome to estimate the CV from a truncated distribution, this section briefly analyzes the magnitude of the errors involved in estimating the CV, while excluding only claims closed without a payment.

Case I: Assume that we fit a log-normal distribution with an actual CV = 2.5 (without nuisance claims) to a distribution with a CV = 3.5. What is the actual error in the postulated excess distribution? Table VIII, Section I, demonstrates that this error is within 2% if the total limits costs. Smaller errors could be obtained by reducing the CV at higher attachments or estimating the CV from a truncated distribution.

Case II: Assume the actual distribution has a CV = 2.5. If about 38% of the claims are nuisance claims, what do we expect the estimated CV's to be for various attachment points? Table VIII, Section II shows that the estimated CV at about 3 times the mean would be 3.9; the estimates are expected to decline to 3.1 at an attachment of 65 times the mean. This is a typical pattern of estimated CV's, which may occur when there are a large number of nuisance claims.

TABLE VII
ESTIMATING THE COEFFICIENT OF VARIATION

I. AIA CLOSED CLAIM STUDY (1974)*

Interval	Number Of Claims	Indemnity On Interval ($1,000's)	Excess Losses ($1,000's)	Excess Losses Percent	Estimated C.V.
1-10,000	1,496	$ 4,500	$38,645	76.4%	4.5
10,001-25,000	365	6,437	30,128	59.5	4.2
25,001-100,000	267	13,933	14,245	28.1	3.6
100,001-300,000	99	16,488	4,457	8.8	3.1
300,001-1,000,000	15	7,207	1,050	2.1	3.2
Over 1,000,000	1	2,050	—	—	—
Total	2,243	$50,615			
Closed No Payment	2,370				

II. NAIC CLOSED CLAIM STUDY (DECEMBER, 1975)**

Interval	Number Of Claims	Indemnity On Interval ($1,000's)	Excess Losses ($1,000's)	Excess Losses Percent	Estimated C.V.
1-10,000	1,124	$ 3,082	$20,800	71.7%	4.0
10,001-50,000	372	7,851	11,029	38.0	3.9
50,001-100,000	83	5,422	6,857	23.6	3.7
100,001-300,000	51	7,607	2,950	10.2	4.0
300,001-1,000,000	5	2,050	1,000	3.4	4.5
Over 1,000,000	2	3,000	—	—	—
Total	1,637	$29,012			
Closed No Payment	2,711				

*Report to the All-Industry Committee Special Malpractice Review: 1974 Closed Claim Survey, Preliminary Analysis of Survey Results. December 1, 1975. Report 9.
**Volume 1, Number 1, December, 1975. Summary 22.

Case III: What if the observed distribution is actually a mixture of two or more distributions which have different means, but the same CV? There might, for example, be different means for different insurers, geographical areas, specialties, or accident years. Table VIII, Section III illustrates the case where half the claims have a mean of 10 and half have a mean of 30; both groups have observed estimated CV = 3.5. The estimated CV for the combined distribution is very close to 4.0. We thus observe what we would have expected, that a mixture of means will increase the coefficient of variation. It should thus be expected that CV's for individual insurers and states should be somewhat below those previously shown in the previous countrywide studies.[7]

[7] This may also explain why the NAIC study, which was based on a broader group of insurers, shows a higher CV than the AIA study.

TABLE VIII

SENSITIVITY ANALYSIS TO PARAMETER ESTIMATION

I. ERROR IN ASSUMING A LARGER COEFFICIENT OF VARIATION

Attachment Point	Theoretical Distribution Mean = 50 CV = 2.5	Assumption Of CV = 3.5 (Fitted At 100) Mean = 35	Assumption Of CV = 3.5 (Fitted At 250) Mean = 31
	Excess Loss Distribution Percentages		
100	35%	35%	33%
250	17	18	17
500	8	10	9
1,000	3	4	4
2,000	1	2	2

II. ERROR IN FITTING COEFFICIENT OF VARIATION

Attachment Point	Theoretical Distribution True Mean = 50 CV = 2.5 Excess Losses	Observed Mean = 31 Estimated Coefficient Of Variation
100	35%	3.9
250	17	3.5
500	8	3.4
1,000	3	3.2
2,000	1	3.1

III. MIXTURE OF DIFFERENT MEANS

Attachment Point	Component Distributions Mean = 10 CV = 3.5	Component Distributions Mean = 30 CV = 3.5	Composite Distribution (Observed)	Fitted Distribution Mean = 20 CV = 4.0
	Excess Loss Distribution Percentages			
5	72%	87%	83%	83%
10	58	78	73	73
20	43	66	60	60
50	24	47	42	41
100	14	32	28	27
200	7	20	16	16
500	2	8	7	7
1,000	1	4	3	3
2,000	—	1	1	1

Case IV: An unanswered and, in many situations, a crucial question is whether or not the coefficient of variation is changing over time. If not, one can estimate the total limits mean at a future date from a trending procedure. This mean and the CV will then completely determine the claim size distribution at the future date.

OTHER APPLICATIONS

Although the data in this paper comes from the medical malpractice line, claim sizes in many other lines appear to be log-normally distributed. Allocated expense payments also seem to be log-normally distributed. It is expected that the log-normal distribution may be appropriate whenever a large number of independent factors contribute multiplicatively to the claim size. Property lines may not provide a proper fit due to: (1) a tangible fixed upper limit on most property claims and (2) widely varying values at risk.

Examples in this paper have stressed excess losses. In many cases the log-normal distribution also yields suitable approximations for deductibles. A potential problem which may call for special handling, however, is nuisance claims.

CONCLUSION

This paper has presented an approach to estimating pure premiums by layer of insurance. It should be helpful to primary carriers for: (1) evaluating the basic limits experience of long-tail lines and (2) evaluating the cost of excess of loss reinsurance. It should be useful to reinsurers, if they have the basic limits experience of their reinsureds; in this case the approach is beneficial because the primary market tends to be more stable and its claims develop more quickly.

The method assumes that claim sizes, except for some nuisance claims, follow the log-normal distribution. In order to apply the method, the actuary needs to know the mean and coefficient of variation of the total limits claim size distribution. The mean is often estimated in the ratemaking process, leaving the coefficient of variation as an unknown. Countrywide data has been presented to estimate the CV for medical malpractice insurance. One sample showed a CV of 2.4, when nuisance claims have been excluded. If nuisance claims are included in the mean, the countrywide CV appears to full in the range from 3.0 to 4.0. For individual carriers or states, the CV should be lower.

APPENDIX

I. THE LOG-NORMAL DISTRIBUTION

The log-normal distribution (with parameters μ and σ^2) is defined as follows:

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma x}\, e^{-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2} \qquad X > 0$$

The mean is $\quad M = e^{\mu + \frac{1}{2}\sigma^2}$

The variance is $\quad e^{2\mu + \sigma^2}\,(e^{\sigma^2} - 1)$

The coefficient of variation is $\quad \beta = (e^{\sigma^2} - 1)^{\frac{1}{2}}$

Let the cumulative distribution function be

$$X1(\alpha \mid \beta) = \int_0^{\alpha M} f(u \mid \beta)\, du$$

where α is a ratio to the mean.

The (first) moment distribution is also log-normally distributed with parameters $\mu + \sigma^2$ and σ^2. This distribution is defined as:

$$X2(\alpha \mid \beta) = \frac{1}{M}\int_0^{\alpha M} u f(u \mid \beta)\, du$$

II. THE EXCESS LOSS DISTRIBUTION

Define $X3(\alpha)$ to be the percentage of total limits losses to be excess of α times the mean of the claim size distribution.

$$X3(\alpha \mid \beta) = (1 - X2(\alpha \mid \beta)) - \alpha\,(1 - X1(\alpha \mid \beta))$$

One property of the log-normal distribution is:

$$X2(\alpha \mid \beta) = X1\left(\frac{\alpha}{1+\beta^2} \mid \beta\right)$$

ESTIMATING PURE PREMIUMS BY LAYER—AN APPROACH

ROBERT J. FINGER

DISCUSSION BY LEE R. STEENECK

Mr. Robert J. Finger in his paper Estimating Pure Premiums By Layer—An Approach suggests that the log-normal probability distribution function can be used as a model for the distribution of a single claim in many instances. Use of the log-normal is based on sound statistical theory and has already been applied to numerous actuarial problems. The "long-tail" evident in liability lines of insurance seems to lead us toward an asymmetrical distribution function like the log-normal.

As a model for claim sizes, in order to be practical, a distribution should have the following desirable characteristics: the estimate of the mean should be efficient and reasonably easy to use; a confidence interval about the mean should be calculable; all moments of the distribution function should exist. The log-normal distribution function has these desirable characteristics. Unfortunately, the log-normal has two annoying qualities, too. One, demonstrated by Mr. Finger, is that there may be fitting problems when there are many small values of the variable under consideration. Making adjustments oftentimes requires a great deal of work. Secondly, from a statistical point of view, the integral in the characteristic function cannot be solved and the convolution cannot be expressed explicitly.[1]

An approximation of the real world severity loss distribution is essential from a reinsurance point of view. On an excess of loss basis the reinsurer is directly involved with the tail of the liability loss distribution. Inflation places the excess of loss reinsurer in a leveraged position where reinsurance claim costs are multiplied significantly with even minor errors in severity or frequency estimation. The cost of error in evaluating these long-tails can produce spectacular underwriting loss as claims develop to ultimate. Reinsurance actuaries have previously realized that distribution functions for claim size would be helpful. Unfortunately, although tools like the log-normal, Pareto, Gamma, and Weibull (to mention a few) have been available for some time now, the estimation of the parameters has been difficult. Few losses exist in these upper layers upon which to make accurate estimates.

[1] Lars-Gunnar Benckert, "The Log-Normal Model For The Distribution Of One Claim," Astin Bulletin, Vol. II (January, 1962) Part I, Pages 2-23.

After detailing the calculation of pure premiums by layer using a log-normal distribution, Mr. Finger applies his approach, for example purposes, to data reported in a Special Malpractice Review.[2] Perhaps some of the problems encountered in fitting a log-normal distribution function to this claim data can be traced directly to the use of survey closed claim data. All the criticisms and caveats implied in using closed claim data will not be repeated here, but suffice it to say that claims included within the survey have accident years dating back into the early 1960's (and claim amounts were not trended). Smaller claims belong to the most recent accident years and are higher in volume relative to the older less frequent severe cases. The poor fit over the entire range of loss values can be attributed to the frequency with which losses close by incurral year. As previously mentioned, the need for an accurate barometer of claim frequency by size is essential. If only we could agree on one.

Several other points deserve comment. To emphasize Mr. Finger's definition of an excess loss distribution—it is defined as "the sum of all claims values larger than the attachment point less the number of claims above the attachment point times the value of the attachment point." Using this definition, Table I represents layers of loss between any two attachment points. This then paves the way for the determination of increased limits factors in Table III. The heading of Table III is a bit misleading. A $100,000 policy increased limit factor is being determined (Basic Limits = $25,000). Coverage is not being rated to $125,000. Perhaps a better title to Table III might be: Indications of $100,000 Policy Increased Limits Factor.

Table VI illustrates an estimation process for determining the CV when claims below a given amount are excluded from the analysis. The problem in dealing with the truncated distribution has also been dealt with in the Benckert article.[3] If the censoring point, c, is such that the excess distribution is greater than 80% (1.0 − L(c)) estimates of the mean and

[2] "Report To The All Industry Committee Special Malpractice Review: 1974 Closed Claim Survey Preliminary Analysis of Survey Results," Prepared by the Insurance Services Office (December, 1975).

[3] Lars-Gunnar Benckert, "The Log-Normal Model For The Distribution Of One Claim," Astin Bulletin, Vol. II (January, 1962) Part I, Pages 2-23.

variance (hence the coefficient of variation is easily calculated) are approximately given by:

$$\sigma^{2*} = \frac{Y_2}{r} - \frac{(Y_1)(\overline{Y_1})}{r} + 0.4\sqrt{\frac{Y_2}{r} - \frac{(Y_1)(\overline{Y_1})\cdot}{r}\;\frac{m}{r}} \bullet (\overline{Y_1} - \log c)$$

$$u^* = \overline{Y_1} - \frac{(m)(\sigma^*)(0.4)}{n}$$

Where $Y_1 = \Sigma \log x_i + m \log c$ for $x_i > c$

$Y_2 = \Sigma \log^2 x_i + m \log^2 c$ for $x_i > c$

$\overline{Y_1} = \frac{Y_1}{n}$

m is the number of claims $\leq c$

r is the number of claims $> c$

$m + r = n$

One final comment regarding the "Other Applications" section of the paper. Although this reviewer has not researched the problem in depth some European actuaries (Benckert [4] and Beard[5]) have suggested the use of the log-normal in connection with fire losses.

I hope this article will spark additional interest in the use of theoretical loss distributions to characterize claim activity. Certainly other functions exist which may provide even better indications for the tail. Insurance data needs to be collected, fitted, analyzed, and published in the testing process of various model distributions. With the sparsity of large claim data, continuous claim size distributions are needed in the rating of high layers of insurance coverage. We are indebted to Mr. Finger for his enlightening exposition on this most flexible rating tool.

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[4] Lars-Gunnar Benckert, "The Premium in Insurance Against Loss of Profit Due to Fire As A Function of the Period of Indemnity," *Transactions of the XVth International Congress of Actuaries*, Vol. II, (1957), Pages 297-305.

[5] R. E. Beard, "Analytic Expressions of the Risks Involved in General Insurance," *Transactions of the XVth International Congress of Actuaries*, Vol. II, (1957), Pages 230-242.

A CURRENT LOOK AT WORKERS' COMPENSATION RATEMAKING

ROY H. KALLOP

DISCUSSION BY CHARLES GRUBER

Mr. Kallop mentions that in the ratemaking procedures utilized by independent bureaus, there are minor variations from the National Council procedures presented in his paper. In New York, there are three differences worth mentioning:

1. Due to the inflationary growth of payroll and therefore the growth of premium without any compensating increase in risk, a wage factor is used to decrease the New York experience-indicated rates. This wage factor measures the increase in the state average wage from the midpoint of the experience period to the midpoint of the policy year for which rates are being changed. There is an offset for the rise in indemnity losses due to increased wages.

2. The New York Compensation Insurance Rating Board uses five policy years of experience for reviewing classifications. For those classifications which develop 100% credibility in less than five years, only the number of years necessary to produce 100% credibility are used. Indicated pure premiums are brought to the level underlying present rates and not, as in the National Council procedure, to current level. In other words, pure premiums are brought to the rate and law benefit levels of the previous filing, not the current filing. The proposed pure premiums are the middle pure premiums of the indicated on level, the formula, and the underlying pure premiums.

3. Proposed classification pure premiums are limited to a 20% change from the underlying. The National Council does not limit pure premiums but limits its proposed rates to a maximum departure from present rates.

In New York, the history of workers' compensation rates has been rather fortunate. From 1950 to 1974, benefits increased by over 100%. Yet, because of favorable experience, rates increased by only about 5%. There has been a sudden, severe change in this favorable experience, however. Calendar year loss ratios have risen from 55% in 1970 to 71% in 1975. This steadily worsening experience makes it imperative that the ratemaking process in New York become more responsive.

The past ratemaking procedure of the New York Rating Board used a 50-50 split between experience indications of one policy year two years before the effective date of the filing, and one calendar year six months before the effective date. The policy year experience was processed from individual unit card data. The experience indications were then modified by the wage factor.

Although this ratemaking procedure was adequate in the past, it is no longer adequate. It seems that past experience has become unrepresentative of current conditions. Even if the experience of the latest calendar year were used, it would still not be an adequate predictor of future experience, without including indicators of change. One problem area is the projected wage factor, which unfortunately measures only the future growth of premiums, without considering future loss conditions. Examples of changing loss conditions are the continuous changes in award liberality and utilization rates of doctors, due to changing economic and social conditions. To get some measure of changes in award liberality, the Rating Board has looked at data on closed compensation cases, provided by the New York State Workmen's Compensation Board. On a common benefit level, the average compensation per case increased from approximately $1,850 in 1970 to $2,090 in 1973, an increase of 13%. It is evident from Exhibit I that most of this increase came from non-scheduled permanent partial cases, where liberality would have the most effect.

The New York Rating Board, in its effort to increase both premium and loss responsiveness, has adopted several ratemaking procedures which the National Council has implemented. The exposure base has been changed from payroll limited to $300 per week to total payroll. In recent filings, the Board has used policy year aggregate totals obtained from financial data reports, i.e., premiums and losses from the latest two policy years evaluated six months before the effective date of the filing. Both premium and losses are developed to an ultimate reporting base. The Board has adopted a new method of adjusting calendar year premium and loss data to the current level. In the past, a geometric method was used; currently, the Board uses policy year contributions to calendar year experience, which more accurately adjusts old claims to the current level.

The Board included a loss ratio trend factor in its most recent filing. This trend factor takes into account New York's wage factor. Loss ratios of the most recent five calendar years are adjusted to current rate, benefit, and wage levels. A least squares trend line is used to project the increase in loss ratios from the midpoint of the experience period to the midpoint of the policy year for which rates are being changed. This procedure is similar to the procedure used to calculate the wage factor. (See Exhibit II for an example of this calculation.)

A basic ratemaking problem lies in discovering accurate predictors of future loss experience, either in insurance data or in outside data. As situations change, existing predictors become inadequate, and additional predictors must be found. Ratemakers continue their efforts despite the sometimes disheartening thought that part of what we are trying to measure may not, in fact, be quantifiable.

EXHIBIT I

ALL DISABILITIES, NON-SCHEDULE PERMANENT PARTIAL, AND TEMPORARY DISABILITIES

COMPENSATED CASES CLOSED, NEW YORK STATE, 1970-1973

Data Provided by the New York State Workmen's Compensation Board

Year of Closing	All Disabilities		Non-Schedule Permanent Partial		Temporary	
	Cases	Compensation	Cases	Compensation	Cases	Compensation
1970	118,537	$188,992,138	3,025	$ 65,243,169	69,649	$30,244,772
1971	123,124	206,526,685	3,011	68,981,730	72,763	33,147,131
1972	122,044	243,907,658	3,687	94,570,672	71,601	36,097,858
1973	117,337	245,524,899	3,549	100,441,054	71,373	36,114,687

Year of Closing	All Disabilities			Permanent Partial Non-Schedule			Temporary		
	Cases	Compensation at 1973 Benefit Level	Compensation Per Case	Cases	Compensation at 1973 Benefit Level	Compensation Per Case	Cases	Compensation at 1973 Benefit Level	Compensation Per Case
1970	118,537	$219,191,240	$1,849	3,025	$ 75,986,347	$25,119	69,649	$35,392,042	$508.15
1971	123,124	217,853,512	1,769	3,011	71,631,181	23,790	72,763	34,610,817	475.67
1972	122,044	251,539,125	2,061	3,687	96,603,197	26,201	71,601	36,999,470	516.75
1973	117,337	245,524,899	2,092	3,549	100,441,054	28,301	70,373	36,114,687	513.19

EXHIBIT II

WORKMEN'S COMPENSATION—NEW YORK

Loss Ratio Trend Factor

Calendar Year	Standard Earned Premium Excluding Expense Constant	Incurred Losses Excluding Interest Adjustment	Loss Ratio	Factor to Bring Premium to 1-1-76 Rate Level	Factor to Bring Losses to 1-1-76 Law Level	Loss Ratio Adjusted to 1-1-76 Rate and Law Level (3) × (5) ÷ (4)	Factor to 1-1-76 Wage Level	Loss Ratio Adjusted to 1-1-76 Rate, Law & Wage Levels	Least Squares Line (.4575 + .0270x)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1970	366,934,084	203,398,073	.5543	1.202	1.276	.5884	.791	.4654	.4575
1971	399,591,276	236,256,628	.5912	1.197	1.210	.5976	.830	.4960	.4845
1972	383,316,891	243,996,414	.6365	1.254	1.187	.6025	.856	.5157	.5115
1973	403,838,132	242,446,219	.5930	1.246	1.158	.5511	.892	.4916	.5385
1974	444,742,065	293,010,752	.6588	1.205	1.092	.5970	.938	.5600	.5655
1975	468,479,146	311,519,805	.7077	1.148	1.021	.6294	.986	.6206	.5925

$$\frac{\text{Calendar Year 1975 Least Squares Loss Ratio (.5925)}}{\text{Policy Years '73 - '74 Least Squares Loss Ratio (.5655)}} = 1.0477$$

$$\frac{\text{1-1-77 Least Squares Loss Ratio (.6330)}}{\text{Calendar Year 1975 Least Squares Loss Ratio (.5925)}} = 1.0684$$

Adjusted Loss Ratio Trend Factor $= .5 \times (1.0000 + 1.0477) \times 1.0684 = 1.0939$

Selected Adjusted Loss Ratio Trend Factor $= 1.0313$

DISCUSSION BY JEROME A. SCHEIBL

The essence of a sound actuarial ratemaking procedure is a balanced intelligent appraisal of all pertinent information leading to a best estimate of future occurrences translated into unit costs. This suggests that a necessary and important element in the ratemaking process is the continuous evaluation of methods and data bases as they relate to the forces affecting losses and expenses. Without such an evaluation, ratemaking becomes a mechanical process of merely measuring past results without proper focus on the accuracy, stability, and responsiveness of rate levels.

Economic, social, technological, and political forces have left their marks on workers' compensation insurance since Ralph Marshall's day. Their dynamic influences continue to be observed along with revolutionary changes in our society's attitudes toward individual rights, the role of government, and the responsibilities of business. As might be expected, therefore, the continual evaluation of the ratemaking process of a line so sensitive to these forces suggests occasional revisions to keep pace with conditions expected to exist during the time rates are to apply.

Mr. Kallop's paper describes the 1975 National Council on Compensation Insurance ratemaking procedure, thereby updating the Marshall paper[1] and filling a void in casualty actuarial literature on workers' compensation ratemaking technique. His presentation serves a second but equally important purpose in that it demonstrates how and why the National Council procedures currently differ from those used years ago. He carefully points out that innovations adopted in the ratemaking process are not suggestive of defects in the older methods but are rather necessary adjustments to develop rates that are responsive to the changing nature of the workers' compensation line and the conditions by which it is affected. Mr. Kallop illustrates the need for flexibility in methodology in arriving at the best estimate of the financial aspects of future occurrences.

[1] Marshall, R. M., *Workmen's Compensation Insurance Ratemaking*, (Revised 1961), Casualty Actuarial Society.

Approximately half of the countrywide workers' compensation premium volume is generated in states where the National Council provides rate calculations from data it has compiled either as a ratemaking organization or on an advisory basis. The other half of the volume is written in states where rates are developed either by an independent rating bureau or by a governmental body.[2]

Ratemaking methods may and do differ among those used by the National Council and by independent state bureaus. This discussion illustrates how one independent bureau has coped with the problem of assuring responsive ratemaking methods through somewhat different approaches than those used by the National Council. Its methods and those of the National Council have the common goal of achieving the best estimate of the financial effects of future occurrences. Therefore, variations in techniques and results should not detract from the actuarial soundness of the rates that are derived therefrom.

Annual premium volume in California is currently about 1½ billion dollars. This represents approximately ⅓ of the business not under the jurisdiction of the National Council or approximately 17% of the countrywide volume for all carriers in non-monopolistic states. Data is gathered and rates are promulgated by the Workers' Compensation Insurance Rating Bureau of California.[3]

California rating practices differ somewhat from those in other states. Rates published in the manual are the minimum rates that must be used by all carriers on all business. Loss and expense constants are not provided for in manual rules which is consistent with the minimum pricing concept. Premium discounts are not permitted and all experience modifications are promulgated on an intra-state basis. Retrospective rating is permitted on a monoline basis only and only through the use of a prescribed tabular plan.

[2] These states are California, Delaware, Massachusetts, New Jersey, New York, Pennsylvania, and Texas.

[3] Formerly the California Inspection Rating Bureau and hereinafter referred to as the California Bureau.

The California Bureau recognized several years ago that the workers' compensation situation in that state was such that rate level adequacy and stability could best be achieved by emphasizing responsiveness to conditions and experience in the ratemaking process. Aggregate policy year data was first used in a 1961 rate revision in lieu of unit report data. This data was used in conjunction with calendar year experience in a manner similar to the National Council procedure except that 60% weight was given to calendar year data rather than the 50% weight used by the National Council. Greater responsiveness was also achieved about the same time through the compilation of calendar quarter data permitting the use of the most recent four quarters in the determination of the rate level adjustment factor.

Development factors through 1970 were based on three-year average incurred policy year loss ratio developments as compared to the National Council practice of developing separate ratios for premium and losses using two-year averages. Losses were assumed to be developed to an ultimate basis at 84 months. In the 1972 revision it was noted that loss development followed a cyclical pattern using incurred data. The use of three-year averages made it difficult to project peaks and troughs of the pattern. It was apparent in 1972 that the incurred loss ratio development pattern was approaching a trough in the cycle. The ratemaking procedure was revised at that time to what was considered to be a more responsive method based on a three-year average paid-to-paid approach. The three-year average incurred-to-incurred approach was readopted in 1975 after it appeared that the trough in the development cycle had been passed.

Subsequent to the presentation of Mr. Kallop's paper, the National Council introduced loss ratio trend into its ratemaking procedure to recognize the imbalance of social and economic inflationary influences on premiums and losses. Although trend factors are derived from twelve-month rolling calendar year loss ratios measured at the end of each half year, such factors are used in conjunction with both the policy year and calendar year data in rate calculations. Observed trends are adjusted for credibility using a Spearman Rank Correlation D-statistic approach.

An on-level loss ratio trend was suspected in California experience as early as 1962. Trend factors were calculated on the basis of twelve-month rolling calendar periods measured at the end of each succeeding quarter. Trend factors were made a part of the formal calculations of the calendar year loss ratio from 1963 through 1968. A change was made in 1970 to base trend calculations on the 16 latest quarterly loss ratios after adjustment for seasonality. In addition, a trend factor other than unity was used only when the data was determined to be significant using the two-sided Spearman Rank Correlation Coefficient and a 95% significance level.

In 1969 it was determined that further responsiveness in the ratemaking process might be achieved if calendar/accident year data were used. Calls for such data have been issued each year since that time and have provided the basis for a major revision in the rate level determination process in 1976.

The new method uses calendar/accident year loss ratios for a number of years adjusted to reflect development to ultimate values and to current premium and benefit levels. This data is trended using a double exponential smoothing method.[4] Since accident year data projected by this method is on an exponential basis giving greatest weight to the latest accident year and progressively diminishing weights to each prior year, trended data can be determined on a cumulative basis. The influence of older years on projected experience diminishes significantly with age. Because of the exponential nature of the curve determined by this method, the loss ratio used in the rate level calculation is derived directly from the extrapolated curve. This is in contrast to the usual linear method of applying the calculated trend to actual experience.

Accident year incurred loss development factors have followed a rather definite upward pattern in recent years. This suggests that the three-year average development used for projection purposes may not be sufficiently responsive for ratemaking purposes. Possible alternatives are the use of trended factors or the factor for the latest year. The later option was selected in a filing made early in 1976.

Mr. Kallop alluded to a new approach under study for developing rates for classes with credibility less than unity. This approach, utilizing countrywide relativities to complement state relativities, may be considered as yet another step toward more rate responsiveness in that it will result in rates more closely corresponding to the peculiarities of each manual class. The California Bureau classification rate calculations also use supplementary

[4] This method is illustrated in the Appendix as it has not been commonly used for ratemaking purposes in the past. It is more fully described in Brown, Robert G., Smoothing, Forecasting & Prediction of Discrete Time Series, 1963. Prentice-Hall.

data when two policy years do not qualify for 100% credibility. Rather than external data, however, the California Bureau achieves responsiveness by adding earlier years until full credibility is reached subject to a maximum of five years.

Both the National Council and California Bureau, each in its own way, have focused much attention and research on the need for responsiveness in ratemaking methods. The fact that techniques may differ is irrelevant as long as each bureau continues to develop what it believes to be its best estimates of future costs under future conditions—the goal of every ratemaker.

APPENDIX

The double exponential smoothing technique may be demonstrated by an example using a filing made by the California Bureau early in 1976.[5] The filing, as applied to new and renewal business, contemplated an effective date of April 1, 1976, with a subsequent revision scheduled for January 1, 1977. Therefore, the midpoint of the exposure period in this illustration is February 15, 1977.

Calendar/accident year loss ratios adjusted to the then current premium and benefit levels as developed to ultimate values are shown in Column (1) of the following table.[6] These values are used to derive the point ($\hat{a}$) and the slope ($\hat{b}$) at the midpoint of accident year 1974. The loss ratio at the midpoint of the exposure period (2.625 years from the midpoint of accident year 1974) is derived by a linear extrapolation from point $\hat{a}_{1974}$ using slope $\hat{b}_{1974}$.

Points and slopes on the exponential curve are defined as:

and $$\hat{a}_t = 2S_t - S_t^{[2]} \qquad (1)$$

$$\hat{b}_t = \frac{\alpha}{1-\alpha}\,(S_t^{[2]}) \qquad (2)$$

[5] Credit is given to David Skurnick, former California Bureau Actuary, who adapted the double exponential smoothing technique to projecting loss ratios for ratemaking purposes.

[6] Accident year loss ratios prior to 1969 are estimated from policy year data. Because of the weighting process inherent in the smoothing technique, the effects of such early year estimates on the projected loss ratio are minimal.

where:

α = a selected weight to be given to the latest X in deriving S_t and to the latest S_t in deriving $S_t^{[2]}$.

$S_t = \alpha X + (1 - \alpha) S_{t-1}$

$S_t^{[2]} = \alpha S_t + (1 - \alpha) S_{t-1}^{[2]}$

X = on level loss ratio

The California Bureau Actuarial Committee used historical policy year data to test various values of α to determine a value that resulted in an optimum balance of rate level adequacy, stability, and responsiveness. After a number of tests, including variations of α by age of data, an α of .2 was selected.

The calculations of $\hat{a}_t$ and $\hat{b}_t$ are straightforward and can be easily traced in the following table. It should be noted that it is not necessary to determine these values for each year—only for the point where extrapolation begins.

Since it is necessary to have a value of S_{t-1} to determine S_t, it is necessary to estimate an initial S_{t-1} using assumed values for $\hat{a}_{t-1}$ and $\hat{b}_{t-1}$. The technique used in this filing was to determine a least squares regression line based on accident year 1966-1970, assume the slope of this line to be b_{t-1} and extrapolate to $t - 1$ to derive the value of $\hat{a}_{t-1}$. Since these estimates are made in rather early years they have a minimal effect on the projected loss ratio.

ACCIDENT YEAR LOSS RATIO PROJECTION BY MEANS OF DOUBLE EXPONENTIAL SMOOTHING

$\alpha = .2$

Midpoint of Accident Year	(1) On Level Loss Ratio	(2) S_t	(3) $S_t^{[2]}$	(4) $\hat{a}_t$	(5) $\hat{b}_t$
		$.2 \times (1) + .8 \times S_{t-1}$	$.2 \times S_t + .8 \times S_{t-1}^{[2]}$	$2S_t - S_t^{[2]}$	$\frac{.2}{.8}(S_t - S_t^{[2]})$
		.5264*	.4828*		
7/1/66	.5879	.5387	.4939		
7/1/67	.5748	.5460	.5043		
7/1/68	.5956	.5559	.5146		
7/1/69	.6510	.5749	.5267		
7/1/70	.6043	.5808	.5376		
7/1/71	.6600	.5966	.5494		
7/1/72	.7391	.6251	.5645		
7/1/73	.7639	.6529	.5822		
7/1/74	.7801	.6783	.6015	.7551	.0192

Projected loss ratio as of:
$2/15/77 = \hat{a}_{1974} + 2.625\,\hat{b}_{1974} = .7551 + 2.625(.0192) = .8055$

*Values obtained by deriving $\hat{a}_{1965}$ and $\hat{b}_{1965}$ from the least squares regression line based on observed loss ratios for accident years 1966-1970 and simultaneously solving the identities in columns (4) and (5) to derive the initial S_t and $S_t^{[2]}$.

GENERALIZED PREMIUM FORMULAE

JAMES P. ROSS

DISCUSSION BY ALAN E. KALISKI

James Ross, in his paper "Generalized Premium Formulae," has mathematically set forth a methodology for determining rate level adjustment factors (i.e., factors to convert actual earned premiums to a present rate level basis) when the earned premiums being put on-level consist of contributions from policies written with different terms. An example of this situation, as posed by Mr. Ross, occurs when: (1) three-year policies are converted to annual policies upon renewal as of a certain date, and (2) the premiums being adjusted to present level consist of earnings from both three-year policies (written prior to date of annualization) and one-year policies (written after date of annualization). In solving this problem, the author has formularized and illustrated many fundamental, yet important relationships among earned exposures, written exposures, rate of exposure writing, and policy term.

With regard to technical ratemaking procedures, this paper is especially relevant at the present time. The Insurance Services Office has recently filed, and received approval of in several states, annualization endorsements for their Special Multi-Peril (SMP) policy form. By attaching these endorsements, SMP policies previously written and rated for three-year terms will now, for the most part, be subject to annual re-rating at first and second anniversaries of policy inception. From a ratemaking standpoint, when faced with the problem of determining rate level adjustment (on-level) factors, the theory developed in the paper under consideration has application to this situation. Some minor modifications are necessary in this case, however, because certain policies—three-year pre-paids and those which develop annual premiums of \$500 or less—are *not* annually re-rated for practical reasons. The following paragraphs discuss the nature of the modifications required in order to make Mr. Ross' paper directly applicable to the ISO annualization of SMP policies.

On page 53, an example is given in which the policy term is changed from three years to one year at time X_0, and the exposure prior to time X_0

had been written at a constant rate K_o. The author then states that f(x) is as follows:

$$f(x) = \begin{cases} \frac{1}{3} K_o, \text{ for } X_o < X \leq X_o + 1 \\ \frac{2}{3} K_o, \text{ for } X_o + 1 < X \leq X_o + 2 \\ K_o, \text{ for } X_o + 2 < X \leq X_o + 3 \\ K_o, \text{ for } X > X_o + 3 \end{cases}$$

In the case of ISO annualization, a modification to the above definition of f(x) is necessary for the following reason: Not all SMP policies are subject to annual re-rating at policy inception anniversary. (More specifically, Deferred Premium Payment (DPP) plan policies where the annual premium is less than \$500 and three-year pre-paid policies are excluded from the effects of annualization.)

Suppose 5% of all SMP policies fall in either of the above two categories and are thus not subject to annualization. Then, under the author's assumptions, f(x) would be defined as follows:

$$f(x) = \begin{cases} .95 \frac{1}{3} K_o + .05 K_o, \text{ for } X_o < X \leq X_o + 1 \\ .95 \frac{2}{3} K_o + .05 K_o, \text{ for } X_o + 1 < X \leq X_o + 2 \\ .95 \frac{3}{3} K_o + .05 K_o, \text{ for } X_o + 2 < X \leq X_o + 3 \\ .95 \frac{3}{3} K_o + .05 K_o, \text{ for } X > X_o + 3 \end{cases}$$

Coincident with the annualization of SMP policies, the ISO is changing the term multiple from 2.7 to 3.0 for all policies and is maintaining its 5% installment surcharge *only* for DPP plan policies whose annual premium is less than \$500. Hence, when determining rate level adjustment factors, the following should be considered as normal rate changes effective as of the date of annualization:

	Rate Change
DPP Plans—premium at least \$500	$\frac{3.0}{2.7\ (1.05)} = 1.058$ (or + 5.8%)
DPP Plans—premium less than \$500	$\frac{3.0\ (1.05)}{2.7\ (1.05)} = 1.111$ (or + 11.1%)
3-Year Pre-Paid Policies	$\frac{3.0}{2.7} = 1.111$ (or + 11.1%)

(The above three rate level effects would be weighted by the respective premiums within each category to obtain the average rate level change effective with annualization.)

Having made the above described modification to the function f(x), and by including the rate level effect of the change in term multiples and installment surcharges, the methodology set forth in Mr. Ross' paper can be applied directly to the ISO situation.

As a final note with respect to the specifics of the paper, there is one place where I believe the author interchanged words. In particular, at the bottom of page 54, an example is given followed by the statement ". . . with an annual term a rate change at the beginning of the year will result in one-half of the premium earned at the old rate and one-half at the new rate." I believe the author meant to say that one-half of the exposures (*not* premium) are earned at the different rate levels, r_1 and r_2.

Although Mr. Ross' paper addresses itself to the solution of a particular problem—the determination of rate level adjustment factors—it is nevertheless recommended to anyone interested in the mathematical formulation of certain fundamental insurance relationships. Also, it can be shown that the rate level adjustment (on-level) factors calculated via the formulae set forth are equivalent to those determined from the traditional parallelogram approach. Hence, this paper can also serve to clear up the "mystique" of the parallelogram approach that might exist among those first introduced to it.

While of a technical nature, "Generalized Premium Formulae" by James Ross touches on a subject of which more needs to be written. In particular, a gap in the literature seems to exist with respect to Commercial Multi-Peril Package policy ratemaking and pricing. This topic would appear to warrant consideration in that CMP has recently become the predominant commercial line. Also, it would be interesting to learn of pricing and packaging approaches used by companies that market their own independent policy forms in addition to the ISO's SMP policy.

MINUTES OF THE 1976 SPRING MEETING

MAY 22-26, 1976

THE BREAKERS, PALM BEACH, FLORIDA

Saturday, May 22, 1976

An informal reception for early arrivals was held in the President's suite from 6:30-7:30 p.m.

Sunday, May 23, 1976

The Board of Directors had their regular quarterly meeting from 1:00-4:00 p.m.

Registration took place from 4:00-6:00 p.m.

The President's reception for new Fellows and their spouses was held from 6:00-6:30 p.m.

A reception for members and guests was held from 6:30-7:30 p.m.

Monday, May 24, 1976

Registration was held from 8:30-9:00 a.m.

The Spring Meeting was formally convened at 9:00 a.m. After opening remarks by President Bornhuetter, an address was given by Philip Ashler, Insurance Commissioner and Treasurer of the State of Florida.

Steven Newman informed the group of the preliminary plans for the ASTIN meeting to be held in Washington, D.C., in the Spring of 1977.

President Bornhuetter then requested the new Associates to rise as he called their names. After a round of applause for the Associates, each new Fellow was asked to step forward and receive his or her diploma. Pictures of new Associates and Fellows were taken at the coffee break which followed the business session. List of new Associates and Fellows follows.

NEW FELLOWS

Dean R. Anderson	Howard H. Kayton
Wayne H. Fisher	Anne E. Kelly
Michael Fusco	C. Robert Spitzer
Leon R. Gottlieb*	Elton A. Stephenson
Anthony J. Grippa	James P. Streff
Steven L. Groot	Robert L. Tatge
Paul E. Hough	Dorothy A. Zelenko
Alan E. Kaliski	

NEW ASSOCIATES

Albert J. Beer	Patrick J. Grannan
Richard M. Beverage	Holmes M. Gwynn
Everett G. Bishop	Walter Haner
Mark M. Cis	Gary P. Hobart
Kenneth R. Frohlich*	Beatrice T. Rogers

*Not Present

S. G. Kellison, Executive Director of the American Academy of Actuaries, reported on Academy activities and the operation of their new office established in Washington, D.C.

Monday, May 24, 1976 (Cont'd.)

The first panel discussion was entitled "Government Regulations— From "Rags to Riches." Participants were:

Moderator: LeRoy J. Simon
Senior Vice President
Prudential Reinsurance Company

Members: George K. Bernstein
Attorney at Law

Don P. McHugh
V.P. & General Counsel
State Farm Mutual Auto Ins. Co.

Benjamin R. Schenck
Shearson Hayden Stone Inc.

Ms. Mavis A. Walters
Assistant Vice President
Insurance Services Office

Following the panelist presentations, questions were asked by the audience.

At 12:00 Noon the meeting recessed for lunch.

The afternoon was devoted to Committee meetings. One Round-Table Discussion entitled "Not For Women Actuaries Only" was conducted by the Committee on Career Enhancement. The introduction was made by Charles C. Hewitt, Vice President and Actuary, Metropolitan Property and Liability Insurance Company. Participants were:

Ms. Carole J. Banfield
Associate Actuary and Manager
Insurance Services Office

Ms. Diana M. Childs
Assistant Actuary
Insurance Company of North America

Ms. Vicki S. Keene
Actuarial Assistant
Aetna Insurance Company

Ms. Mavis A. Walters
Assistant Vice President
Insurance Services Office

A reception was held for members and guests at the Beach Club from 6:30-7:30, followed by a LUAU.

Tuesday, May 25, 1976

The meeting was reconvened at 9:00 a.m.

New papers were presented. The titles and authors were:

"Estimating Pure Premiums by Layer—An Approach" by Robert J. Finger.

"Accident Limitations for Retrospective Rating" by Frank Harwayne.

James P. Ross' paper "Generalized Premium Formulae," presented at the 1975 Fall Meeting was reviewed by Alan E. Kaliski and Richard D. Pagnozzi.

A panel presentation, "The Discounting of Loss Reserves," was given. The participants were:

Moderator: David P. Flynn
Vice President and Actuary
Crum and Forster Insurance Companies

Members: Rafal J. Balcarek
Vice President and Actuary
Reliance Insurance Company

Paul M. Otteson
Senior Vice President and Actuary
Federated Mutual Insurance Company and Federated Life Insurance Company

Ms. Ruth E. Salzmann
Vice President and Actuary
Sentry Insurance Group

Richard H. Snader
Actuary
United States Fidelity and Guaranty Company

After coffee a second panel on the subject "The AICPA Audit Guide" was presented. Participants were:

Moderator: Robert G. Espie
Vice President and Corporate Controller
Aetna Life and Casualty Company

Members: John L. McDonough, Jr.
Partner, Price Waterhouse and Company

Paul E. Singer
President
Illinois Medical Services, Inc.

The meeting was recessed from Noon until 2:00 p.m. for lunch.

Tuesday, May 25, 1976 (Cont'd.)

From 2:00-5:30 p.m. six workshops were held. Each workshop was held twice during three hourly periods according to the following schedule:

2:00-3:00	Workshops A, B, C, D
3:15-4:15	Workshops A, B, E, F
4:30-5:30	Workshops C, D, E, F

The workshop titles and participants are listed below:

Workshop A—Discussion of R. H. Kallop's paper "Workmen's Compensation Ratemaking" presented at the Fall, 1975 Meeting.

Moderator: Roy H. Kallop
Vice President and Actuary
National Council on Compensation Insurance

Members: Charles Gruber
Assistant Actuary
New York Compensation Insurance Rating Board

Stephen S. Makgill
General Manager
Pennsylvania Compensation Rating Bureau

Jerome A. Scheibl
Vice President
Employers Insurance of Wausau

Workshop B—Continuation of the morning panel on "The AICPA Audit Guide" with audience participation. The moderator and members were the same as listed under the above panel.

Workshop C—"Certification By The Actuary" was presented by the following:

Moderator: Dale A. Nelson
Actuary
State Farm Mutual Automobile Ins. Co.

Members: Robert A. Brian
Financial Analyst
Conning and Company

Stephen G. Kellison
Executive Director
American Academy of Actuaries

Tuesday, May 25, 1976 (Cont'd.)

Workshop D—"Trends In Expense Provisions"

Moderator: Edward R. Smith
Vice President and Actuary
The Hartford Insurance Group

Members: Phillip N. Ben-Zvi
Actuary
Royal-Globe Insurance Companies

John B. Conners
Associate Actuary
Liberty Mutual Insurance Company

Frank Harwayne
Vice President and Director of Research
National Council on Compensation Ins.

Workshop E—"Model Building" for solving insurance problems was presented by the following:

Moderator: Charles A. Hachemeister
Actuary
Prudential Reinsurance Company

Members: R. Woody Beckman
Consulting Actuary
Booz-Allen Consulting Actuaries

Richard I. Fein
Woodward and Fondiller

Ernest J. Hansen
Director of Operations Research
Insurance Company of North America

Workshop F—The "Pricing of Long-Term Medical Losses" was the subject of this workshop. Participants were:

Moderator: Joseph W. Levin
Actuary
Employers Reinsurance Corporation

Members:	Martin Adler Vice President and Actuary Government Employees Insurance Co.
	Steven L. Groot Assistant Actuary Allstate Insurance Company
	John Hyland Assistant Vice President General Reinsurance Corporation

A reception for members and guests was held from 6:30 to 7:30 p.m.

Wednesday, May 26, 1976

At 9:00 a.m. an address on econometrics was given by Dr. Allen Sinai of Data Resources Incorporated. A panel on econometrics followed:

Moderator:	Robert Pollack Senior Vice President Colonial Penn Insurance Group
Members:	John B. Gragnola Assistant Vice President Allstate Insurance Company
	Allan M. Groves Senior Research Associate Travelers Insurance Company
	Roger C. Wade Vice President Volkswagen Insurance Company

President Bornhuetter adjourned the meeting at 12:00 Noon, expressing thanks to Phillip B. Kates, President, Independent Fire Insurance Company, who ably served as Chairman of Local Arrangements and to Patrick J. Grannan, John Herzfeld and Diane Young (Mr. Kates' Secretary) who assisted with the registration. In attendance as indicated by registration records were 114 Fellows, 98 Associates, 22 Guests, 12 Subscribers, 6 Students and 105 husbands or wives of members as follows:

FELLOWS

Adler, Martin
Alexander, Lee M.
Anderson, Dean R.
Anker, Robert A.
Balcarek, Rafal J.
Bartik, Robert F.
Beckman, Woody
Bennett, Norman J.
Ben-Zvi, Phillip N.
Bergen, Robert D.
Berquist, James R.
Berry, Charles H.
Bevan, John R.
Bickerstaff, David R.
Bill, Richard A.
Bland, William H.
Bornhuetter, Ronald L.
Brannigan, James F.
Brian, Robert A.
Brown, William W.
Carter, Edward J.
Conners, John B.
Curry, Alan C.
Dahme, Orval E.
Ehlert, Darrell W.
Eliason, Edward B.
Espie, Robert G.
Faber, James A.
Farnam, Walter E.
Ferguson, Ronald E.
Finger, Robert J.
Fisher, Wayne H.
Fitzgibbon, Walter J.
Flynn, David P.
Forker, David C.
Fowler, Thomas W.
Fresch, Glenn
Fusco, Michael
Gibson, John A.
Goddard, Russell P.
Golz, James F.
Grippa, Anthony J.
Groot, Steven L.
Hachemeister, Charles A.
Hartman, David G.
Harwayne, Frank
Hazam, William J.
Hewitt, Charles C.
Honebein, Carlton W.
Hough, Paul E.
Hughey, M. Stanley
Inkrott, James G.
Johe, Richard L.
Jones, Alan G.
Kallop, Roy H.
Kates, Phillip B.
Kayton, Howard H.
Kelly, Anne E.
Khury, Constandy K.
Kilbourne, Frederick W.
Kollar, John J.
Kreuzer, James H.
Lamb, R. Michael
Leslie, William
Levin, Joseph W.
Liscord, Paul S.
Makgill, Steven S.
Masterson, Norton E.
Morison, George D.
Muetterties, John H.
Munro, Richard E.
Murray, Edward R.
Nelson, Dale A.
Newman, Steven H.
Otteson, Paul M.
Perkins, William J.
Petz, Earl F.
Pollack, Robert
Portermain, Neill W.
Riccardo, Joseph F.
Richards, Harry R.
Richardson, James F.
Riddlesworth, William A.
Roberts, Lewis H.
Rodermund, Matthew
Rogers, Daniel V.
Rosenberg, Norman
Ross, James P.
Ryan, Kevin M.
Salzmann, Ruth E.
Scheibl, Jerome A.
Scott, Brian E.
Sheppard, Alan R.
Simon, Leroy J.
Simoneau, Paul W.
Skurnick, David
Smith, Edward R.
Snader, Richard H.
Spitzer, C. Robert
Stephenson, Elton A.
Stewart, Charles W.
Streff, James P.
Strug, Emil J.
Switzer, Vernon J.
Taht, Veljo
Tverberg, Gail E.
Uhthoff, Dunbar R.
Walsh, Albert J.
Walters, Michael A.
Webb, Bernard L.
Williams, P. Adger
Woll, Richard G.
Yount, H. W.
Zelenko, Dorothy A.

ASSOCIATES

Alff, Gregory N.
Alfuth, Terry J.
Banfield, Carole
Barnes, Galen R.
Bassman, Bruce C.
Beer, Albert J.
Bell, Allan A.
Bertles, George G.
Beverage, Richard M.
Biondi, Richard S.
Bishop, Everett G.
Bovard, Roger W.
Bragg, John M.
Brewer, Fred L.
Cadorine, Arthur R.
Carollo, Linda D.
Childs, Diana M.
Chorpita, Fred M.
Chou, Philip S.
Cis, Mark M.
Collins, Douglas J.
Connor, Vincent P.
Covitz, Burton
Daino, Robert A.
Davis, Rex C.
Davis, Rodney D.
Dolan, Michael C.
Durkin, James H.
Eddy, Jeanne H.
Eldridge, Donald J.
Fallquist, Richard J.
Fein, Richard I.
Finkel, Daniel
Gallagher, Thomas L.
Garand, Christopher P.
Gould, Donald E.
Gruber, Charles
Grannan, Patrick
Gwynn, Holmes M.
Hafling, David N.
Hammer, Sidney M.
Haner, Walter
Head, Thomas F.
Herzfeld, John
Hobart, Gary P.
Hoylman, Douglas J.
Inderbitzin, Paul H.
Jensen, James P.
Jerabek, Gerald J.
Johnston, Daniel J.
Jorve, Barry M.
Kaliski, Alan E.
Karlinski, Frank J.
Kaur, Alan F.
Keene, Vicki S.
Kleinberg, James J.
Kolojay, Timothy M.
Lehman, Merlin R.
Lehman, Steven G.
Lindquist, Robert J.
Lino, Richard
Luneburg, Sandra C.
Marino, James F.
Miller, David L.
Moore, Brian C.
Morgan, Stephen T.
Neidermyer, James R.
Nolan, John D.
Ori, Kenneth R.
Penniman, Kent T.
Plunkett, Joseph A.
Plunkett, Richard C.
Potok, Charles M.
Quirin, Albert J.
Renze, David E.
Rice, W. Vernon
Rodgers, Beatrice T.
Rosenberg, Sheldon
Sandler, Robert M.
Schaeffer, Bernard G.
Schulman, Justin
Shoop, Edward C.
Shrum, Roy G.
Singer, Paul E.
Smith, Frances A.
Song, Young B.
Stanard, James N.
Steeneck, Lee R.
Tatge, Robert L.
Torgrimson, Darvin A.
Trees, John S.
Wage, Roger C.
Walters, Mavis A.
Whatley, Michael W.
Whatley, Patrick L.
Woodworth, James H.
Yoder, Reginald C.
Zubulake, Theodore J.

GUESTS

Ashler, Philip	Hansen, Ernest J.	Roland, W. P.
Bernstein, George K.	Hyland, John	Saffeir, Harvey J.
Bowles, Thomas	Kellison, Stephen G.	Sinai, Allen
Davies, Bennett	McCarthy, John F.	Spangler, Joel L.
Forbes, Leon D.	McDonough, John R.	Stenmark, John A.
Gragnola, John B.	McHugh, Don	Trafton, Mark
Groves, Allan M.	McMillen, Robert H.	
Guarini, Leonard T.	Robinson, Michael D.	

SUBSCRIBERS

Anderson, E. V.	Hatfield, Bryan	Ross, Paul Dean
Armstrong, Steven H.	Hinkle, Timothy C.	Schiavo, Michael F.
Bell, Andrew M.	Hoyt, Fred A.	Subeck, Stanton
Dunn, Robert P.	Rinard, Alan V.	Yousri, A.

STUDENTS

Meeks, John M.	Pulis, Ralph S.	Skolnik, Richard S.
Miccolis, Jerry A.	Ragan, Evelyn M.	Tuttle, Jerome E.

A special program for wives and guests of the members was organized by Cindy Bornhuetter. Highlight of this program was a presentation by Mrs. Bobbie Evans, a representative of "The Total Woman, Inc.," outlining the concepts set forth in Marabel Morgan's book *The Total Woman*. A discussion followed.

Respectively submitted,

Darrell W. Ehlert
Secretary

Volume LXIII, Part 2 No. 120

PROCEEDINGS

November 17, 18, 19, 1976

CHALLENGES

PRESIDENTIAL ADDRESS BY RONALD BORNHUETTER

It has been a tradition that the retiring president address the membership, and for many years this occurred twice a year. Fortunately, we now address you only once. In my visits with our sister actuarial organizations at their annual meetings two retiring presidents declined the privilege of making a presidential address—I was tempted; however, that was the easy way out. I was warned by several perceptive past presidents that the beginning of the term of office was none too early to start thinking about this point in time—they were so right.

I will break with tradition somewhat and, instead of focusing on one topic, I would like to put to you several specific challenges involving three general areas; the industry, the actuary, and lastly, the profession.

THE INDUSTRY

We have all heard comments by industry spokesmen covering innumerable financial topics relating to the insurance industry. At this time I would like to put forward one specific challenge to a subject discussed at a workshop during this meeting—the future capacity of our industry.

First, I offer the following question: can the industry, on its own, generate enough capital to fill the needs for the immediate future—under both favorable and unfavorable stock market conditions? Given the current state of affairs, I find it difficult to answer this question in the affirmative.

I would like to list a few "current conditions" that must be taken into consideration.

1. To the best of my knowledge no stock insurance company has in the recent past gone to the equity market to raise substantial additional capital except in a distressed situation. Prospects are not particularly bright in this area; but, I will come back to this.

2. We are about to enter a cycle of investors wanting, and receiving, a greater share of earnings through dividends, thus eroding the amount of new capital that can be generated from within.

3. If one follows "current value accounting" to its ultimate, all securities will be valued at market price. The great cushion that amortized bond values provided in inflationary times will, someday, be gone. This certainly will not help.

4. Few companies have been net buyers of common stock in recent times. Historically this has been an important source of additional capital through appreciation. Several reasons might be cited such as, remembrances of the 1974 "capacity crunch", an economy that seems to be increasingly subject to severe shocks, and a strong need for taxable income to offset sizable tax loss carry-forwards by 1979. These certainly are all valid reasons.

5. Relatively poor and, more importantly, quite erratic underwriting results.

6. Pressure being exerted on current surplus because of rapidly rising price levels. Such pressure will continue as long as we have inflationary forces, both economic and social.

7. A relative distrust or wariness of the property and casualty industry by the investment community. They have been stung badly at least once and are now quite suspicious.

There are probably many more important factors that can be enumerated; however, this gives you the idea.

Obviously, the first and basic requirement to change the situation and attract more capital is strong and relatively consistent earnings performances. One can talk for hours on this aspect alone.

What I would like to call your attention to is a slightly different but related area cited in the first and last conditions on my list: The absence of additional capital being raised through the equity market and the basic distrust by the investment community.

If earnings are to be effective in today's equity market, the investment community must understand and believe in the methodology behind the bottom line numbers and the vast majority do not. This is especially true on the loss reserve side. Admittedly our business is complicated, including two separate but related accounting systems, and those few analysts following our industry have a very very difficult time understanding it. What is worse, most others don't even make the attempt.

We, the industry and, specifically, the actuaries must embark on a program which will make our business more understandable, hence, more marketable to the investing public and the analysts who represent them. Individual companies have made improvements but there is such a long way to go. Unless we can develop statistics that will be understood, and relied upon, we will lose our major market source for future capital.

It is a basic understanding of our numbers that will ultimately lead to a three step result:

1. The credibility of earnings.

2. The achievement of respectable multiples.

3. Ultimately, the attraction of new capital.

We must be able to market ourselves successfully. I leave you with this challenge. The actuarial profession, especially in the area of loss reserves, can be of enormous help. It is essential that we become involved because, if the private sector does not produce adequate capital for future capacity needs, there is always the Man on the Potomac with the printing press who can step in, and will.

THE ACTUARY

I would now like to spend a moment talking about your future, especially the younger members just embarking on their career path. Let me introduce my point with a personal story and two statistical illustrations (every actuarial speech has to have some numbers).

The Society of Actuaries has some fifty-seven hundred members as compared to the Casualty Actuarial Society with seven hundred and fifty. On the other hand, in 1975 United States life and annuity premiums totaled some thirty-nine billion with an additional nineteen billion for accident and health while property and casualty premiums totaled some forty-eight billion. If one reduces the life society membership by roughly thirty percent to reflect Canadian and pension consultants, this leaves some four thousand life actuaries to support fifty-eight billion of premium or, in other words, eighty actuaries for every billion of annual premium (that is, if one includes all the accident and health premiums). The comparable figure for the casualty side is sixteen actuaries per billion of premium volume. I ask you, is life and accident and health so much more complex to require a density of actuaries five times greater?

My second statistical illustration is quite short. According to President Michael O'Brien of the British Institute of Actuaries, ten percent of his one thousand members are employed directly by the securities industry, not the insurance industry, and ten percent more work in investment departments of insurance companies. There are some special reasons for this situation but it is still a startling figure. What percentage of ours compares to this twenty percent? Yet, the investment arena is certainly a logical one for actuarially trained minds.

My third illustration is a brief personal story that occurred about three years ago. I was called into the Chairman's office one afternoon and he explained to me that the Company was being reorganized and they would like me to assume a position in the financial area. Obviously, this was a promotion and yet I was very upset by it all. Because all my life I have strived to achieve the title Actuary and here in ten seconds time the title was being eliminated. I questioned him further as to why the title has been eliminated from my job specification and also from anyone else, as our Company does not now have anyone designated as "The Actuary". He gave me this explanation which has lived on with me and I quote: "The title of 'Actuary' is too limiting on an individual." I know now what was meant three years ago but it took me quite some time to understand this very brief but telling lesson.

The point I'm trying to make is this—for many the actuarial route should be the jumping-off point for other related career paths. Flip through the yearbook of the Society of Actuaries if you really want proof that this can be the case. Not everyone can or will be a chief actuary and, with membership approaching one thousand plus strong, you, as individuals, should be looking at all career paths, and we, as a Society, should provide a better exposure to topics that could assist you in choosing which career path to follow: management, economics, data processing, underwriting, accounting, or investment analysis. All are vital areas where people trained in actuarial science can make significant contributions and be successful.

I'm not advocating the take over of the property and casualty industry by actuaries, but I do challenge you to expand your horizons and challenge the Society to fulfill an obligation using its resources to develop and produce top insurance executives in all disciplines.

THE PROFESSION

During the past twelve months, I have had the opportunity of visiting all the local clubs at least once and several twice; as well as representing you both here and abroad traveling a distance of nearly one hundred thousand miles. During this past year we have hoped to accomplish a few things as well as bring other subjects to the forefront. I would like to take my last few minutes suggesting some challenges on the most important aspect—The Profession.

First, and most important is the change in our growth rate during the past five years, coupled with a tremendous student explosion. As the old expression suggests "we ain't seen nothing yet". Last May we had seventeen hundred ninety-five students taking exams and there were sixteen hundred twenty-seven this November—both numbers are again increases over their respective 1975 figures. More to the point, there were one hundred and seventy individuals who sat for examinations this November who need only one or two examinations to become an Associate, the vast majority of these will be our new nembers in 1977, we are growing up very fast and we had better be ready.

Our average age is declining rapidly and we must be alert that the Society is always responsive to all segments both young and not so young. The consequence for failing to fulfill our obligations will be to lose one of these important segments.

Following through on this point, I wonder if we are not approaching the time when the emphasis of this Society should be shifted somewhat from the national to the regional or local level. To be specific, one of the semi-annual smorgasbord national meetings should be replaced with two or three regional meetings, run by the local clubs which would usually have one underlying theme. I list the following reasons as support for moving in this direction in the near future:

1. One of the important features of our Society is that we are an informal, close knit group—communication has always been easy. Yet, we all comment that the Society is losing this characteristic as we rapidly grow. Certainly regional meetings would preserve some of this for the immediate future.

2. The Society of Actuaries with some fifty-seven hundred members had an attendance of twelve hundred members at their last annual meeting, a twenty-one percent participation. Their regional meeting participation is much higher, almost double ours.

3. Our local club meetings are a huge success with a much higher degree of participation.

4. The workload of the Society could be shared more effectively between local and national staff.

5. Member expenses would be reduced which might encourage greater participation, including advanced students.

6. Done effectively, the Casualty Actuarial Society would increase its outward exposure dramatically.

7. More non-actuarial company executives might be interested in attending a nearby two-day session.

8. Our first one-day, one-topic seminar appeared to be a huge success.

Regionalization would be a dramatic change but one that ultimately must come. Why not sooner than later?

Those in the audience who have had the opportunity to participate in the ASTIN meetings and the International Congresses will understand better what I have to say next. We as a Society have been concentrating all our efforts in the continental United States and to this degree I believe we have been selfish. There is an acute need, call it a search for knowledge, in the property and casualty business worldwide and we are the owners of the biggest bank. Those in foreign countries, practicing in the property and liability area are certainly looked on there as the poor cousins.

This search is one of the many reasons why ASTIN members are coming to Washington next May. These individuals want to see and learn how a large effective casualty actuarial organization functions.

I challenge each of you to make a commitment to go out of your way to make these visitors welcome. Let's start a dialogue, introduce them to your problems and participate in theirs. This is a unique opportunity to reach out, let's not lose it.

One of the goals of this year's officers was to bring the subject of amalgamation to the forefront so that each of you would have an opportunity to review thoroughly all aspects of it. It certainly is time that we all start thinking about it. Amalgamation will be an important and persistent subject in the coming months and years. There isn't much more that needs to be said at this moment, except that the subject deserves your continued careful attention.

Another separate but related subject, certification, was just discussed by three of your Board members. I would point out to you that, since this subject has also come to the forefront in a hurry, I believe we all will find that as this subject unfolds, it will tell us a great deal, very quickly, about whether amalgamation will work or not.

It is absolutely essential that we, the profession, should do the screening for licensing and be the dominant force in the problem area of defining competency. One only needs to look at the fall out from ERISA—enrolled actuaries—federal examinations replacing Society examinations—to know that we should make every effort to handle this ourselves.

Rightfully, your Board of Directors has agreed that the Academy should be the outward vehicle for handling the certification of individuals who will be involved in the certification of property and casualty reserves. The original agreement among the officers of the two organizations, which was accepted in principle by your Board of Directors, is that the qualification procedure should be one of "prior approval" that is, in order for an individual to qualify to be able to certify they must pass prescribed examinations (parts 5, 6, and 7) or be approved by a committee of peers. The result is that the qualifications and the professional code of conduct would be put "up front". Some will argue for a "file-and-use" environment which must rely heavily on the after the fact enforcement of the professional code of conduct. In my personal opinion our Board's position is absolutely essential as the professional code of conduct, on an after the fact basis, is virtually unenforceable.

The law of supply and demand has created and, in the near future, will continue the imbalanced situation now existing among the three major actuarial specialities. Unless we are extremely careful, the standards of professionalism, as represented by our examination procedure, will be eroded as others try to respond to the demand situation.

As the events in the certification arena unfold, we will see if amalgamation can work or whether the certification program through the Academy will be nothing more than a vehicle for unqualified individuals to practice in the property and casualty actuarial field.

Many will criticize my position of having the Casualty Actuarial Society maintain an "exclusive franchise" as parochial or myopic; however, I would respond that we have a good definition for a "competent casualty actuary". To quote a recent comment made by Carroll Nelson to the Conference of Actuaries in Public Practice in introducing me at their annual meeting "Ron represents this newly emerging high demand field of casualty consulting we are all hearing about. You may wish to discuss it with him, but, I warn you his only reply will be 'take the exams!'."

The maintenance of our professional standards is undoubtedly our greatest challenge as the 1970's draw to a close.

* * * * *

My address was somewhat of a montage but I tried to bring forward what we, both individually and collectively, face as immediate challenges that will have a substantial effect on our future, that is:

The Challenge of Marketing our Industry
The Challenge of Developing Future Insurance Executives
The Challenge of Responding to the Society's Growth
The Challenge of Maintaining Participation
The Challenge of Expanding our Horizons Beyond the United States
And lastly, The Challenge of Professionalism

Are you ready?

MODELLING LOSS RESERVE DEVELOPMENTS

ROBERT J. FINGER

The actuarial analysis of loss reserve developments begins by analyzing the patterns in historical claim data. Implicitly this analysis proceeds from a variety of assumptions, which may or may not be acknowledged or tested. By projecting loss reserves from historical data, the analyst is essentially using a mathematical model. This paper presents a general approach aimed at developing and exploring as many alternative models as possible. It is felt that there are indeed some patterns which will continue into the future; at times it may be extremely difficult to uncover these patterns or to even know that they exist. Looking backwards in time, however, it is always possible to describe what has occurred; the historical patterns may be erratic or largely meaningless, but they do exist. Likewise, at some distant future date it should be equally possible to describe the payout on the loss reserves which must now be estimated.

TYPICAL ASSUMPTIONS

Actuarial literature gives many examples of assumptions which are made to create mathematical models of reserve developments. Tarbell[1], for example, assumed that the incurred but not reported (IBNR) liability was proportional to the ratio of incurred losses in the last three months of the last two years. Fisher and Lange[2] assumed that the inflation rate (change in average cost per claim) remains constant for each different age of claim at settlement. Resony[3] assumes that the ratio of paid allocated loss expenses to claims disposed of (change in outstanding reserve by reported year) is constant by age of settlement. A common method of calculating loss development factors for ratemaking purposes assumes that the change in incurred losses from development period to development period will remain constant. On closer scrutiny this single assumption is a composite of several others, such as:

- the reporting pattern of claims by development period will not change and
- the degree of underreserving or overreserving will not change or
- violations of the above assumptions will exactly offset each other.

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[1] Tarbell, T. F. "Incurred But Not Reported Claim Reserves," PCAS XX (1934).
[2] Fisher, W. H., and Lange, J. T., Loss Reserve Testing: A Report Year Approach," PCAS LX (1973).
[3] Resony, A. V. "Allocated Loss Expense Reserves," PCAS LIX (1972).

TERMINOLOGY AND DATA

This paper will refer to a liability as a fixed, though perhaps unknown, amount of money which is owed to others. The term reserve is used to mean an estimate of a liability.

Virtually all loss development data can be put into the characteristic matrix format. This matrix is shown below.

Characteristic Matrix Format

Exposure Period	Development Period 1	2	3	4
1	O_{11}	O_{12}	O_{13}	O_{14}
2	O_{21}	O_{22}	O_{23}	O_{24}
3	O_{31}	O_{32}	O_{33}	
4	O_{41}	O_{42}		
5	O_{51}			

The O_{ij} are observations of some type of reserve data for exposure period i as of development period j. Development periods are successive evaluations of loss development data. Periods are often of twelve-months durations, but can be of one, three, or six-month durations. Exposure periods are groupings of claims. Claims may be grouped by accident years, policy years, report years, or other durations. Exposure periods may also represent groups of claims which were part of a liability (such as the case reserve or IBNR reserve) at a point in time. It is typical, but not necessary, that the durations of the development and exposure periods be the same. This happens, for example, when accident year data is evaluated every 12 months.

Various types of data can fill the characteristic matrix format. The basic variations are: (1) incurred or paid losses and (2) cumulative or incremental developments. The amounts reflected can be aggregate claim amounts, claim counts, or average claim amounts. The amounts can include or exclude allocated loss adjustment expense, subrogation and salvage, ceded and assumed reinsurance, and perhaps deductibles or reinsurance retentions. Claim counts can be defined to include or exclude claims closed without an indemnity payment. Amounts can include several lines of business or coverages.

Various reserving methods utilize the characteristic matrix format in different ways. Among the five most common ways are: (1) using the entire matrix, (2) using one or more diagonals of the matrix, (3) using the ratios of one matrix to another (such as the ratio of paid allocated expense to paid losses), (4) using different matrices for different lines of business (additive combinations), or (5) using a multiplicative combination of matrices (such as those for the number of claims and average claim amounts).

TYPES OF DEVELOPMENT PATTERNS

Assumptions made by a loss reserving method are related to patterns in the characteristic matrix format. The analyst can test the previous accuracy of the assumptions by evaluating the matrix. Further, the analyst can evaluate the potential applicability of other assumptions by reviewing the matrix. In particular, there are a variety of relationships or patterns which are used in different reserving methods.

In analyzing the characteristic matrix format, vertical groups of data represent evaluations of successive groups of claims at the same stage (duration) of development. Horizontal data groups represent successive evaluations of the same group of claims. Diagonal data groups represent developments which occurred during the same calendar period of time.

Many loss reserving methods assume a consistent relationship between two variables, as expressed by the ratio between them. The use of the claim count and the average claim amount is a common example. In this case the average claim amount is actually a ratio of the aggregate losses to the claim count. Ratios may be between two different claim-related variables, between a claim and a premium or exposure variable, or between claims and an external variable. In the first case, an example is the ratio of paid loss expenses to paid losses. In the second case, loss ratios or pure premiums may be evaluated. In the latter case, inflation indices can be used.

Another possible relationship is to model loss developments by a probability distribution. The reporting or payment of claims could, for example, be modeled as a cumulative distribution function in time. A problem that arises with this approach is that time is unbounded, whereas at some point all claims will certainly be reported and all payments will be made. A possible solution would be to fix a certain time period as the ultimate development.

Another possible solution is a different way of looking at reserve developments: claim turnover intervals. Instead of assuming that the development period affects the loss development, it is assumed that the percentage of claims which have been closed affects it. For example, it is assumed that the seventieth to eightieth percentile of closed claims have a constant pattern. (Data is graphically portrayed in this format in Figure 1.) This assumption is useful for lines of business where the claims which remain open a longer period of time close at significantly higher average amounts.

THE BASIC MODEL

All of the previous data formats and relationships can be represented by a generalized loss reserve development model. This model is defined as follows:

(1) $$O_{ij} = C_{ij}F_jS_iK_{i+j} + e_{ij}$$

Where:

- O_{ij} — is the observed values of the process for exposure period i, observed at age j (O_{ij} can be cumulative paid losses, incremental paid losses, or incurred losses).
- C_{ij} — is known items (such as claim counts or inflation indices).
- F_j — is an index of reserve development factors, typically representing the percentage of the ultimate losses paid through j periods. This is estimated from the data.
- S_i — is an index reflecting the relative exposure at exposure period i. This is estimated from the data.
- K_{i+j} — is an index reflecting the relative effect of outside influences during a particular calendar period of time. This is estimated from the data.
- e_{ij} — are the differences between the observations O_{ij} and the estimated values of the process.

Since the C_{ij} are known items, it is possible to divide the O_{ij} by the C_{ij}. For example, C_{ij} may be the number of closed claims and O_{ij} the amount of paid losses. The parameter sets F_j, S_i and K_{i+j} will then effectively be estimating the average closed claim. Assuming that O_{ij} has been divided by C_{ij}, the parameter sets are chosen to model the observations as follows:

Exposure Period	Development Period			
	1	2	3	4
1	$F_1S_1K_1$	$F_2S_1K_2$	$F_3S_1K_3$	$F_4S_1K_4$
2	$F_1S_2K_2$	$F_2S_2K_3$	$F_3S_2K_4$	$F_4S_2K_5$
3	$F_1S_3K_3$	$F_2S_3K_4$	$F_3S_3K_5$	—
4	$F_1S_4K_4$	$F_2S_4K_5$	—	—
5	$F_1S_5K_5$	—	—	—

In a practical situation more than four development periods would be both available and desirable. The data which needs to be estimated to complete the reserve development is:

Exposure Period	Development Period			
	1	2	3	4
1	—	—	—	—
2	—	—	—	—
3	—	—	—	$F_4S_3K_6$
4	—	—	$F_3S_4K_6$	$F_4S_4K_7$
5	—	$F_2S_5K_6$	$F_3S_5K_7$	$F_4S_5K_8$

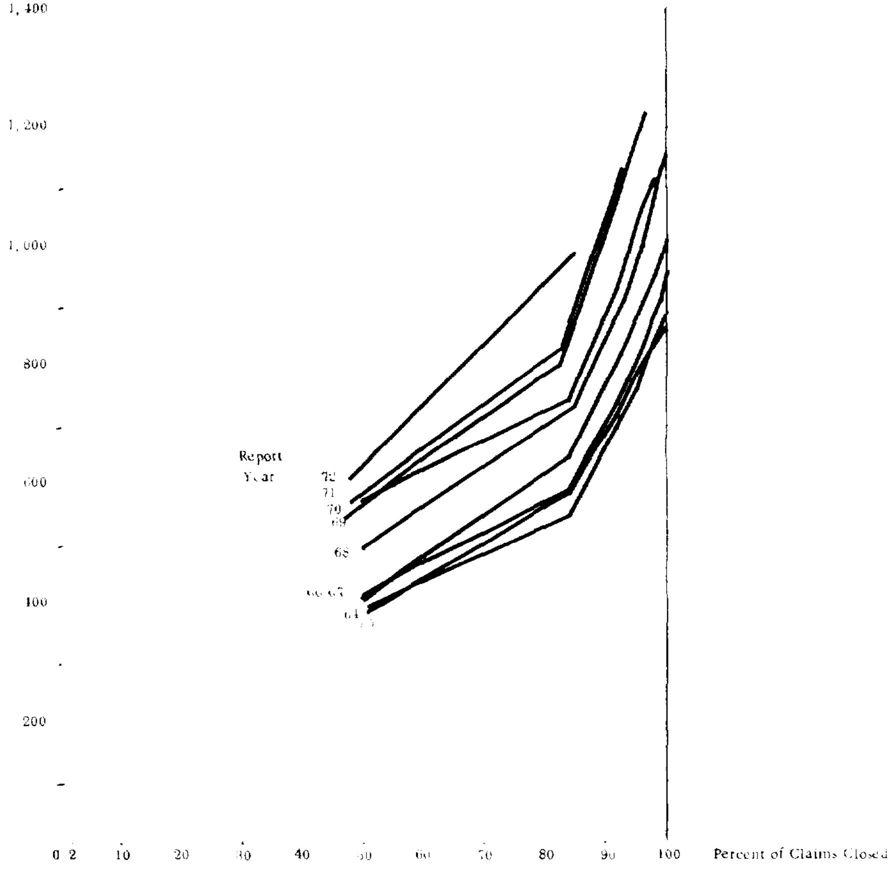
Additionally, some C_{ij} values might need to be estimated.

The F_j, S_i and K_{i+j} are parameter sets which are to be estimated subject to some criteria. They represent things unknown about the loss developments. The C_{ij} represent everything that is known or assumed to affect the developments. The C_{ij} can include measures of exposure, inflation, or claim counts. The C_{ij} could also represent changes in deductible levels or reinsurance retentions.

The F_j, S_i and K_{i+j} sets are all stated in terms of indices. Under certain circumstances they can be eliminated or replaced by functions. F_j has as many independent parameters as the number of development periods. S_i will have as many estimable parameters as the number of exposure periods. K_{i+j} will have as many estimable parameters as the larger of the number of

Figure 1.

CUMULATIVE AVERAGE CLOSED CLAIM



development periods or the number of exposure periods. In addition, to project the reserve developments several additional K_{i+j} terms must be projected.

There is an interesting interpretation of the various models which can be derived from the combination or elimination of F, S, and K parameter sets. Visualize the O_{ij} as incremental payments and ignore C_{ij}. The condition for F_j invariance then determines which parameters will be represented in the model. There are two choices of assumptions: (1) payments are in current or constant dollars, and (2) payments are related to the period of occurrence or the period of payment. Assume, for example, that F_j represents a constant percentage of payments in terms of constant dollars, valued at the occurrence of the claim. Observations will reflect the impact of inflation (current dollars) and a valuation at date of payment. The model thus needs a K_{i+j} term (to convert to constant dollars) and a S_i term (to value the claim at occurrence). The resulting model is thus $O_{ij} = C_{ij}F_jS_iK_{i+j} + e_{ij}$. Other variations are shown in the following table:

Value Of Money	Claim Valuation At	Model, $\hat{O}_{ij} =$
Current Dollars	Occurrence	F_jS_i
Current Dollars	Payment	F_j
Constant Dollars	Occurrence	$F_jS_iK_{i+j}$
Constant Dollars	Payment	F_jK_{i+j}

SOLUTION CRITERIA

There are many possible solution criteria to equation (1). For this paper the chosen criterion is to minimize the sum of squares of the differences between the observations and the estimates. The squares may be weighted by values a_{ij}, which might be chosen to reflect the relative credibility of the various observations. Algebraically the criterion is to minimize Z where:

$$Z = \sum_i \sum_j a_{ij}e_{ij}^2 = \sum_i \sum_j a_{ij}(O_{ij} - C_{ij}F_jS_iK_{i+j})^2.$$

The indices of summation apply to all available items.

A possible alternative, for example, might be Bailey and Simon's[4] minimum chi-square criterion. In the notation of this paper, that would be to minimize Z where:

$$Z = w \sum_i \sum_j a_{ij} \frac{(O_{ij} - C_{ij}F_jS_iK_{i+j})^2}{C_{ij}F_jS_iK_{i+j}}$$

where w is a constant. Bailey and Simon were concerned with rate equity, which can be reflected by the term in the denominator. In a sense, the minimum chi-square criterion attempts to minimize the error relative to the size of the observation. For loss reserving it is possible that the absolute error is more important than the relative error.

BASIC SOLUTION PROCEDURE

In order to derive a solution to the least squares formulation, it is generally necessary to make some simplifying assumptions. For this paper the following assumptions are made:

- The parameter sets F, S and K are independent of each other

- Individual index values within the parameter sets are independent of each other.

The O_{ij} and C_{ij} values are constants. The assumption of independence between different parameter sets is reasonable, since they are constructed to represent the three types of reserve developments (horizontal, vertical, and diagonal). As a practical matter, the S and K sets tend to be redundant. The independence of individual index values (particularly F and K) is in some doubt when the modelled data represents cumulative data.

[4] Bailey, R. A., and Simon, L. J., "Two Studies in Automobile Insurance Ratemaking," PCAS XLVII (1960).

In order to find the values of the parameters which will minimize the criterion function, set the partial derivative of the criterion function with respect to each parameter equal to zero. For the basic model and most variations the solution procedure will be iterative. To obtain a starting solution, one can assume that the F_j are the only parameters in the model.

The solution is thus:

$$\hat{F}_j = \frac{\sum_i a_{ij} O_{ij} C_{ij}}{\sum_i a_{ij} C_{ij}^2}$$

Next assume the model contains only F_j and S_i. Since F_j has been estimated above, it can be used to generate the initial estimates for S_i.

$$\hat{S}_i = \frac{\sum_j a_{ij} O_{ij} C_{ij} F_j}{\sum_j a_{ij} C_{ij}^2 F_j^2}$$

Finally, K_{i+j} can be solved:

$$\hat{K}_{i+j} = \frac{\sum_{i+j=c} a_{ij} O_{ij} C_{ij} F_j S_i}{\sum_{i+j=c} a_{ij} C_{ij}^2 F_j^2 S_i^2}$$

The revised values for F_j and S_i can then be found iteratively using analogous equations:

$$\hat{F}_j = \frac{\sum_i a_{ij} O_{ij} C_{ij} S_i K_{i+j}}{\sum_i a_{ij} C_{ij}^2 S_i^2 K_{i+j}^2}$$

$$\hat{S}_i = \frac{\sum_j a_{ij} O_{ij} C_{ij} F_j K_{i+j}}{\sum_j a_{ij} C_{ij}^2 F_j^2 K_{i+j}^2}$$

The computations proceed iteratively until no improvement in the criterion function can be made.

MODEL VARIATIONS

Some of the possible model variations include the choice of data. Observations can be (1) either paid or incurred losses, or (2) either incremental or cumulative developments. Further, C_{ij} can contain any variables known to affect the loss developments, including claim counts, premium, exposure, or inflation indices. Finally, various simplifications can be made for one of the parameter groups or they can be omitted.

A common assumption might be that inflation is a constant function of exposure period or of the calendar period. In these cases,

$$S_i = (1 + w)^i \text{ or}$$

$$K_{i+j} = (1 + w)^{i+j}$$

Fisher and Lange[5] assume that inflation will be constant for each age at settlement, or:

$$K_{i+j} = (1 + k_j)^i$$

For claim turnover intervals, a substitution is made for F_j, which can be of the general form (recognizing F_j can be bounded by 0 and 1):

$$F_j = 1 - \alpha + \alpha X_{ij}^{\beta}$$

where X_{ij} is the percentage of claims which have been closed.

SOLUTIONS TO MODEL VARIATIONS

The exponent introduced into the model variations makes it difficult to solve directly for the parameters. Newton's Method may be used. To find the minimum of the criterion function, one takes the derivative of the criterion with respect to each parameter and sets the resulting equation to zero. In other words:

$$f(k) = \frac{\partial Z}{\partial k} = 0$$

If k_1 is an initial estimate of k, a better estimate, k_2, can be found by Newton's Method as follows:

$$k_2 \cong k_1 - \frac{f(k_1)}{f'(k_1)}$$

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[5] *Ibid.*

The derivative can also be approximated as:

$$f'(k) \cong \frac{f(k+h) - f(k)}{h}$$

Thus: $$k_2 \cong k_1 - \frac{hf(k_1)}{f(k_1 + h) - f(k_1)}$$

Initial parameter estimates can be obtained as described in an earlier section. The solution procedure will iterate while it successively estimates groups of parameters. When a parameter is estimated by Newton's Method, there will be a sub-iteration. Typical equations are given in the appendix.

NUMERICAL EXAMPLE

To compare the results of a variety of models, data from the Fisher-Lange paper are presented. Exhibit I shows the cumulative payments by reported year and development year; 84 months is considered the ultimate incurred loss. Also shown are the incremental average payments and the cumulative closed claim count. Complete data was not available in the original paper on the number of claims; it is therefore assumed that there are 1,000 claims per year.

Various models can be fitted to this data. For comparison, Exhibit II shows the estimated reserves for a variety of models. In each case the C matrix was taken as the number of closed claims. The K vectors were extrapolated based on a least-squares fit of the data points which could be estimated directly. Estimates are shown for both cumulative and incremental payments. The claim turnover approach can be shown graphically by Figure 1, where the cumulative average closed claim cost is shown as a function of the percentage of claims which have been closed. Equations for solving some of these models are given in the Appendix.

The example depicted in Exhibit II portrays some general results in the use of these models. First, models using K_{i+j} parameters are more difficult to use, since the parameters must be projected for future calendar year periods. In addition, closed claim counts must also be projected for future periods; with claim turnover intervals, however, closed claim count projections have no impact on the estimated reserve. Using cumulative data probably gives too much weight to early developments; thus, incremental data can lead to significantly different results. Estimating too many parameters yields arbitrary parameter values; for example, the inflation factors often add no explanatory value to that already provided by the S_i or K_{i+j} parameters; in addition, diagonal inflation can lead to the same result as exposure-period inflation.

Exhibit III shows the estimated parameters and projected developments for the F_jS_i model. Since the C_{ij} matrix is the cumulative closed claim count, the F_j vector is interpreted as the relative average claim value and the S_i vector is the average incurred claim cost for reported year i.

CONCLUSION

This paper presents a general approach to the modelling of loss reserve developments. All reserving methods are essentially mathematical models; all essentially assume that certain past events will be repeated in the future. This paper presents a methodology for understanding the assumptions made in any given model. In addition, it provides a means to generate a large number of alternative models. In particular, it stresses the use of all available information. This includes using the entire characteristic matrix format, instead of one observation or one diagonal; this also includes the use of endless types of collateral information, such as changes in deductible levels and external economic data. A general mathematical formulation is presented which allows the incorporation of all this data.

Most reserving methods are dependent upon certain fundamental assumptions, which may not be valid. How can one evaluate situations where: case reserving is inconsistent? the speed of claim settlements (payments) is changing? reinsurance retentions have changed? inflation is known to affect the data? Possible solutions to these questions will be briefly examined.

If case reserving is inconsistent, it may be best to evaluate only claim payments. If the rate of claim settlements are changing, the approach of claim turnover intervals is applicable. If reinsurance retentions have changed, adjustments can be incorporated into the C matrix. Inflation can be handled in a variety of ways. It can be assumed that inflation impacts either claim payments or claim occurrences. External economic functions or industrywide data can be used to model inflation. Finally, the inflation can be estimated from the claim data itself.

The value of the claim liability depends upon events which will occur in the future. A means of projecting the consequences of these events is to explore the various patterns which may continue with the future.

EXHIBIT I

INPUT DATA

I. Cumulative Payments

REPORTED YEAR	AGE 12	24	36	48	60	72	84
64	202184	465254	636658	726568	798577	829441	860385
65	197679	487722	660090	750090	807950	851498	891980
66	204848	489428	681620	795544	864044	899372	958682
67	224220	545194	760171	870281	930568	970908	1013188
68	247500	621480	823834	964447	1062748	1120069	1154607
69	286769	626641	852976	1026736	1153576	0	0
70	256695	658941	976191	1179090	0	0	0
71	275229	688579	1051977	0	0	0	0
72	291924	829946	0	0	0	0	0
73	350396	0	0	0	0	0	0

II. Incremental Average Payment

REPORTED YEAR	AGE 12	24	36	48	60	72	84
64	398	790	2348	2430	3429	2572	1934
65	393	871	2128	2500	2630	3629	3114
66	413	837	2288	2998	3425	2944	5931
67	444	961	2471	3146	3173	4034	4228
68	495	1084	2438	4261	4681	5211	4934
69	577	988	2865	4344	5285	0	0
70	545	1146	3375	4317	0	0	0
71	577	1181	3598	0	0	0	0
72	612	1466	0	0	0	0	0
73	698	0	0	0	0	0	0

EXHIBIT I (CONT'D)

III. Cumulative Closed Claim Count

REPORTED YEAR	AGE 12	24	36	48	60	72	84
64	508	841	914	951	972	984	1000
65	503	836	917	953	975	987	1000
66	496	836	920	958	978	990	1000
67	505	839	926	961	980	990	1000
68	500	845	928	961	982	993	1000
69	497	841	920	960	984	0	0
70	471	822	916	963	0	0	0
71	477	827	928	0	0	0	0
72	477	844	0	0	0	0	0
73	502	0	0	0	0	0	0

EXHIBIT II

COMPARATIVE SOLUTIONS FOR DIFFERENT MODELS (WHERE C_{ij} IS CLOSED CLAIM COUNT)

Model $O_{ij} = C_{ji}(\ \bullet\ )$	Number of Parameters	Estimated Reserve ($ Million) Cumulative Payments	Incremental Payments	Standard Error* (Cumulative)
F_jS_i	17	2.83	2.83	17.5
$F_j(1+w)^i$	9	2.77	2.81	25.6
$F_j(1+w)^{i+j}$	9	2.77	2.81	25.6
F_jK_{i+j}	17	2.83	2.83	17.8
$F_j(1+w_j)^i$	20	2.67	3.48	37.7
$X_{ij}(1-\alpha(1-X_{ij})^{\beta})S_i$	12	2.86		13.7

*Standard error calculated as square root of (sum of squares of differences between observations and projections divided by the number of observations less the number of parameters estimated) (shown only for cumulative payments model).

EXHIBIT III

MODEL OUTPUTS

I. Cumulative Payments

REPORTED YEAR	AGE 12	24	36	48	60	72	84
64	187943	453242	628325	732777	797809	829871	870945
65	191205	462923	647703	754488	822253	855266	894868
66	198851	488230	685346	799908	869871	904762	943787
67	218747	529400	745309	866965	941772	977548	1019713
68	244424	601730	842941	978420	1065012	1106561	1150804
69	256735	632844	883065	1032830	1127701	1165782	1216066
70	271106	689227	979693	1154446	1251454	1298994	1355024
71	284461	718426	1028321	1192352	1296584	1345838	1403889
72	317981	819590	1139583	1332852	1449366	1504425	1569315
73	350247	853733	1192710	1394989	1516935	1574560	1642476

II. Incremental Average Payment

REPORTED YEAR	AGE 12	24	36	48	60	72	84
64	370	797	2398	2823	3097	2672	2567
65	380	816	2281	2966	3080	2751	3046
66	401	851	2347	3015	3498	2908	3903
67	433	930	2482	3476	3937	3578	4216
68	489	1036	2906	4105	4123	3777	6320
69	517	1093	3167	3744	3953	6347	5028
70	576	1191	3090	3718	5706	4754	5603
71	596	1240	3068	5126	5212	4925	5805
72	667	1367	4210	4832	5826	5506	6489
73	698	1490	4237	5057	6097	5763	6792

III. Model Parameters

F_1	F_2	F_3	F_4	F_5	F_6	F_7
.425	.618	.789	.885	.942	.968	1.0

S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
871	895	944	1,020	1,151	1,216	1,355	1,404	1,569	1,642

APPENDIX

The following gives examples of models which can be solved by Newton's Method.

I. Model: $\hat{O}_{ij} = C_{ij}F_j(1 + w)^i$

Criterion: Minimize Z where $Z = \sum_i \sum_j a_{ij}(O_{ij} - \hat{O}_{ij})^2$

$$\frac{\partial Z}{\partial w} = 2 \sum_i \sum_j a_{ij}(O_{ij} - C_{ij}F_j(1 + w)^i)C_{ij}F_j(i - 1)(1 + w)^{i-1}$$

II. Model: $\hat{O}_{ij} = C_{ij}X_{ij}(1 - \alpha\,(1 - X_{ij}{}^{\beta}))S_i$

where X_{ij} is the cumulative fraction of exposure period i claims closed as of development period j.

Criterion: Minimize Z where $Z = \sum_i \sum_j a_{ij}(O_{ij} - O_{ij})^2$

$$\frac{\partial Z}{\partial \beta} = -2 \sum_i \sum_j a_{ij}[O_{ij} - C_{ij}S_iX_{ij}(1 - \alpha\,(1 - X_{ij}^{\beta}))]$$

$$C_{ij}S_iX_{ij}\,\alpha\,X_{ij}{}^{\beta}\ln X_{ij}$$

A NOTE ON BASIC LIMITS TREND FACTORS

ROBERT J. FINGER

It is widely accepted that excess layers of insurance suffer an inflationary impact greater than that attributable to the overall growth in claim costs. A necessary corollary of this thesis, and perhaps one not often acknowledged, is that the primary layer (basic limits) suffers a lesser impact than the overall rate. In other words, one may assume that aggregate claim costs are increasing at a certain annual rate. The trend in basic limits costs will be less than this overall rate. The trend in excess layer costs will be more than this rate. This paper will discuss the relationship between basic limit trends and the overall increase in claim costs. A method is presented for estimating the basic limit trend when the overall trend is known.

TERMINOLOGY

The term "claim costs" can have different meanings. Claim costs can change in several ways, for many different reasons. Fundamental changes in costs are due to changes in claim <u>frequency</u> (the number of claims per exposure unit) and claim <u>severity</u> (the average claim size). Claim severity is impacted by these forces, as a minimum: changes in the overall price level in the economy, changes in claim settlement practices and changes in social forces.

This paper is not concerned with changes in claim frequency. If it is assumed that such changes do not affect the claim size distribution, the conclusions of this paper will apply to any level of claim frequency.

This paper does not differentiate between the various sources causing changes in claim severity. It is assumed that these different causes can be suitably combined and that changes in their relative impact over time does not change the claim size distribution. <u>Trend</u> is defined as the change in claim severity.

Liability insurance ratemaking methods usually define certain limits as the <u>basic limits</u>. For example, this could be $25,000 per claim and $75,000 for all claims occurring within the 12-month policy period. In most cases, no insurance policy is sold for limits of less than this amount. In this paper it is assumed that there is a single basic limit per policy (e.g., the $25,000 above). The total amount of insured losses will be referred to as unlimited. The average claim size of the unlimited losses will be referred to as the mean of the claim size distribution.

For a given overall trend in claim costs, the trend in basic limit costs will generally depend upon the relationship between the basic limit value and the mean. The shape of the claim size distribution is also of some importance. If the basic limit is much higher than the mean, relatively few claims are affected by the basic limit; consequently most of the overall trend is felt within the basic limit. If on the other hand, the basic limit is close to the mean, relatively many claims are necessarily above the basic limit. The trend on claims above the basic limit is obviously not reflected in the basic limit cost. The relative trend is defined as the ratio of the basic limit trend to the overall trend. The relative trend varies as the relationship between the basic limit and the mean changes. As the mean gets larger, the ratio of the basic limit to the mean becomes smaller. The average relative trend is the average of the instantaneous relative trends over a period of time.

METHODOLOGY

The basic assumption made in this paper is as follows. When there is a trend in claim costs, the claim size distribution itself does not change, but the value of money does. In effect, this is equivalent to assuming that if overall costs increase 25%, each individual claim increases 25%. Finding the average relative trend is analogous to the following situation. Suppose a Mexican insurance company writes a policy limit of 100,000 pesos on risks located in the United States. When the peso is devalued, what is the increase in claim costs? The ratio of the change in claim costs to the revaluation of the dollar is analogous to the average relative trend, for a basic limit of 100,000 pesos.

Assume that the claim size distribution is known. For a given basic limit A, the unlimited losses, T, can be divided into basic limit losses, B, and excess limit losses, E:

$$T(M) = B(A/M) + E(A/M)$$

where: M is the (unlimited) mean claim size
T(M) is the total amount of losses
B(A/M) is the total amount of losses limited to A per claim
E(A/M) is the total amount of losses in excess of A per claim.

The basic limit losses are defined as:

$$B(A/M) = CMX2(A/M) + CA[1 - X1(A/M)]$$

where: C is the number of claims
X2(A/M) is the percentage of the total amount of losses (moment distribution) on claims which are less than A
X1(A/M) is the percentage of the total number of claims which are less than A.

The average relative trend, ART, is a function of the (beginning) basic limit value and the unlimited trend. In other words, the unlimited losses will be increased by certain trend, i. At the same time the basic limit losses will be increased by a lesser amount. The average relative trend is the percentage increase in basic limit losses as a fraction of the percentage increase in total limit losses. Thus:

$$ART(A, i) = \frac{\dfrac{B(A/(1+i)M) - B(A/M)}{B(A/M)}}{\dfrac{T((1+i)M) - (T(M)}{T(M)}}$$

To derive usable results, two assumptions are made. It is assumed that unlimited losses are proportional to the unlimited mean. Symbolically:

$$\frac{T(M')}{T(M)} = \frac{M'}{M}$$

It is also assumed that the percentage distributions X1 (claim count) and X2 (moment) are a function of the ratio of the basic limit to the unlimited mean. This assumption holds, for example, for the log-normal and Pareto distributions.

By the second assumption:

$$B(A/M) = CM(X2(R) + R[1 - X1(R)]) = CMX(R)$$

where: $R = \dfrac{A}{M}$ and X(R) is the percentage of the total amount of losses which are below a basic limit value of R times the unlimited mean, per claim

This leads to the redefinition of ART as:

$$\mathrm{ART}(R, i) = \frac{\dfrac{B\left(\dfrac{R}{1+i}\right) - B(R)}{B(R)}}{(1+i) - 1}$$

$$\mathrm{ART}(R, i) = \frac{1}{i} \; \frac{(1+i)X\left(\dfrac{R}{1+i}\right) - X(R)}{X(R)}$$

In plain English, these equations state that there exists a distribution, X(R), which represents the percentage of unlimited losses which are less than R per claim, where R is defined as a ratio to the unlimited mean. Assume there is a trend in overall claim costs of fraction i during a period; only basic limit losses which were previously less than $\frac{R}{1+i}$ will now be included within the new basic limit. The entire distribution, however, will be $(1 + i)$ times larger. In other words, assume the initial basic limit is \$25,000 and inflation is 25%. Under the new circumstances only the basic limit losses under the previous \$20,000 basic limit will be below the new basic limit. The entire loss distribution, however, is 25% larger. Algebraically:

$$\mathrm{ART} = \frac{1}{.25} \; \frac{1.25X(20) - X(25)}{X(25)}$$

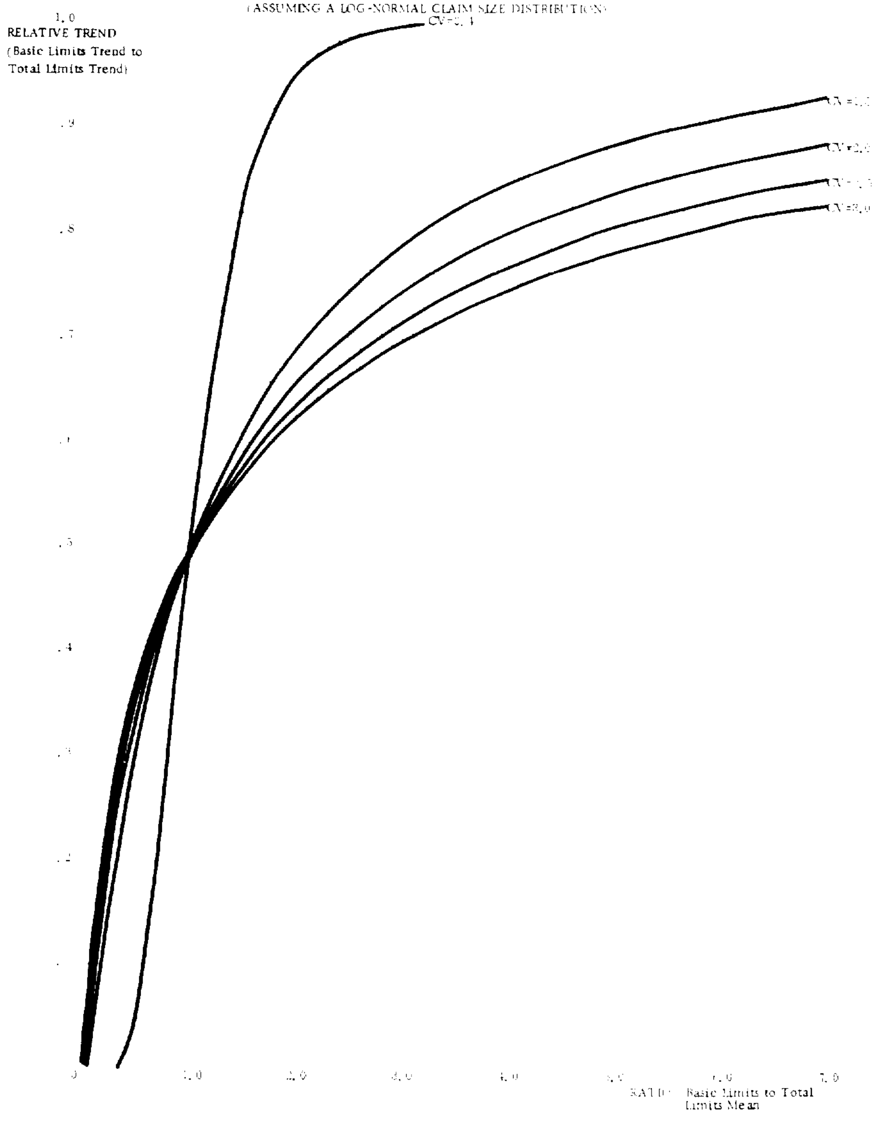
where: X(x) is the percentage of the total amount of losses below x per claim.

For this paper, it is assumed that the claim size distribution follows the log-normal probability law[1]. Results for this law can be produced in terms of two parameters: the coefficient of variation (CV), and the ratio to the unlimited mean. The second parameter can be used to represent the basic limit. Results vary somewhat as a function of CV, but this parameter is not as crucial as the basic limit value. Exhibit I illustrates the relative trend for several choices of CV. A method for calculation of the average relative trend is described in the appendix.

[1] For a discussion of this distribution, see Finger, R. J., "Estimating Pure Premiums By Layer—An Approach", PCAS LXII (1976).

EXHIBIT I

RELATIVE TREND FOR VARIOUS COEFFICIENTS OF VARIATION
(ASSUMING A LOG-NORMAL CLAIM SIZE DISTRIBUTION)



EXAMPLE NO. 1—LIABILITY INSURANCE

Basic limits rates are being prepared for liability insurance. For this purpose two policy years are used which are 3.5 and 4.5 years removed from the average effective date for the new rates. The basic limit trend is measured at 10% per annum based on claims occurring an average of 10 years prior to those expected under the new rates. For claims entering the trend calculation, the basic limit is about 3.8 times the observed unlimited mean.

Looking at Exhibit I, and assuming a CV of 3, the relative trend is about .74 at 3.8 times the mean. This implies an unlimited trend of about $\frac{10\%}{.74} = 13.5\%$. If the CV is 2, the relative trend is .79 and the unlimited trend is 12.7%. Assuming the CV is 3, the basic limit is expected to be $3.8 \times 1.135^{(4.5-10)} = 1.89$ and 1.67 times the mean for the above policy years and 1.07 times the mean in the policy year for the new rates. The average relative trend from 1.89 to 1.07 is .56 and from 1.67 to 1.07 is .55. Thus the average basic limit trend should be $(.56)13.5\% = 7.6\%$ and $(.55)13.5\% = 7.4\%$ for the two policy years. The basic limit trend factors should be $(1.076)^{4.5} = 1.39$ rather than $(1.10)^{4.5} = 1.54$ and $(1.074)^{3.5} = 1.28$ instead of $(1.10)^{3.5} = 1.40$.

Assuming the CV is 2, the basic limit would be 1.97 and 1.75 times the mean for the given policy years and 1.15 in the new policy year. The average relative trends would be .59 and .58. The basic limit trends would be 7.5% and 7.4%. The basic limits trend factors would be 1.38 and 1.28.

This example points out some general conclusions:

- The choice of CV has relatively little impact on the results.

- The use of a basic limit trend factor based solely on previous experience may overstate the projected basic limit losses; in the given example it was by about 10%.

EXAMPLE NO. 2—WORKERS' COMPENSATION PAYROLL OFFSET

The same general approach can be taken to evaluate the effect of increasing wages on collectible premiums in workers' compensation insurance. A few states have a payroll limitation which acts much like a basic limit to curb the growth of subject payroll. The main practical difference between a payroll limitation and basic limits is that the subject distribution is much less skewed for workers' compensation payroll. Table I compares the Standard Wage Distribution Table with a log-normal distribution with CV of 0.4. These tables are based on claimant data and may not represent the same distribution as that for exposed workers.

TABLE I
COMPARISON OF STANDARD WAGE DISTRIBUTION TABLE AND LOG-NORMAL DISTRIBUTION

Ratio to Mean	Standard Wage Table[2]		Log-Normal (CV = 0.4)	
	% Workers	% Wages	% Workers	% Wages
.1	.1	—	—	—
.2	.5	.1	—	—
.3	1.3	.3	.2	—
.4	2.9	.8	1.4	.5
.5	6.3	2.4	5.4	2.3
.6	12.7	8.9	12.9	6.4
.7	22.1	12.0	23.2	13.2
.8	33.2	20.4	35.0	22.0
.9	44.9	30.2	46.8	32.1
1.0	56.5	41.2	57.6	42.4
1.1	66.4	51.6	67.0	52.2
1.2	74.4	60.8	74.7	61.1
1.3	80.5	68.4	80.9	68.7
1.4	85.4	75.0	85.7	75.2
1.5	89.0	80.2	89.3	80.5
1.6	91.6	84.3	92.1	84.8
1.7	94.1	88.4	94.2	88.2
1.8	95.7	91.1	95.7	90.9
1.9	97.0	93.5	96.8	93.0
2.0	98.0	95.5	97.7	94.6

[2] Source: Fratello, Barney, "The 'Workmens Compensation Injury Table' and 'Standard Wage Distribution Table,' " PCAS XLII (1955).

Assume the statewide average wage is $200 and the payroll limitation is $300. If total wages can be expected to grow by 7%, subject premium will only grow by 5.6%. That is, the payroll limitation changes from 1.5 to 1.4 times the mean and the relative trend is about .8. Currently used ratemaking methods consider many other factors and may indirectly adjust for this shortfall in collectible premium.

SUMMARY

This paper has explored the problem of estimating the basic limits trend once the overall trend has been determined. Although the log-normal, has been used for numerical examples, it can be expected that the general conclusions hold for most actual claim size distributions.

Generally speaking, the relative trend (that is, the basic limit trend relative to the unlimited trend) is less than 1.0 and decreasing as the ratio of the basic limit to the unlimited mean is decreasing.

Practical applications of the relative trend concept are not limited to basic limits ratemaking. An example is presented to show what the increase in subject wages will be for workers' compensation insurance, given a fixed dollar payroll limitation.

APPENDIX

FINDING THE AVERAGE RELATIVE TREND

The relative trend varies as the relationship between the basic limits value and the mean changes. To measure the average relative trend over a period of time, one must take into account the changes in that relationship.

The relative trend, f(x), is defined at the particular instant when the ratio of the basic limit to the unlimited mean is x. This function can be defined as a limiting distribution of ART, or:

$$f(x) = \lim_{i \to o} \frac{1}{i} \frac{(1+i)X\left(\frac{x}{1+i}\right) - X(x)}{X(x)}$$

The relationship between a fixed basic limit value, A, and the mean of the unlimited distribution is not changing as a linear function of time. For example, after one time period of inflation i, the new unlimited mean is $\frac{A}{1+i}$ where A was the original mean. After two time periods the mean is $\frac{A}{(1+i)^2}$. For fractional time periods, t, we can use the function $e^{-\delta t} = (1+i)^{-t}$ to represent the changing value of the mean. Thus $\frac{A}{(1+i)^t} = Ae^{-\delta t}$.

The arguments of ART will be revised to represent the beginning and ending ratios of the basic limit to the unlimited mean. If A is the beginning ratio and there is an annual trend of i for T years, the ending ratio will be $Ae^{-\delta T}$.

Assume:
1. The total limits annual trend is i; or $1 + i = e^{\delta}$
2. The beginning value of the basic limit is A times the mean
3. $f(x)$ is the relative trend as a function of x, the ratio of the basic limit to the mean
4. The time period under study is T years.

The average relative trend, ART, can be written as

$$\text{ART}(A, Ae^{-\delta T}) = \frac{1}{T}\int_{0}^{T} f(Ae^{-\delta t})dt$$

Substituting $y = Ae^{-\delta t}$

$$\text{ART} = \frac{1}{\delta T}\int_{Ae^{-\delta t}}^{A} \frac{1}{y} f(y)dy$$

Substituting $z = \ln y$

$$\text{ART} = \frac{1}{\delta T}\int_{\ln (A - \delta t)}^{\ln A} f(z)dz$$

Table II shows the tabulation of $\int_{o}^{\ln A} f(z)dz$ for various values of A and several choices of CV. From this table $\int_{\ln (A - \delta T)}^{\ln A} f(z)dz$ can be obtained by one subtraction. The quantity δT is the difference between the natural logarithms of the initial and ending ratios of the basic limits to the mean. This quantity can also be obtained by one subtraction.

Example. Given:
1. i is 15% per annum.
2. A is 5.0 times the mean.
3. T is 5 years.
4. The CV is 3.0.

Solution: $Ae^{-\delta T}$ is about 2.5 times the mean.

From Table II we have:

$$\text{ART}(5.0, 2.5) = \frac{1.811 - 1.309}{1.609 - .916} = .72$$

TABLE II

CALCULATION VALUES FOR AVERAGE RELATIVE TREND

RATIO A: Basic Limits to Total Limits Mean	Ln A	$\int_0^{\text{Ln A}} f(z)dz$ where z is Ln[ratio A]			
		CV = 0.4	CV = 2.0	CV = 3.0	CV = 4.0
.1	−2.303	0	.044	.100	.148
.2	−1.609	0	.121	.215	.285
.3	−1.204	0	.199	.315	.396
.4	− .916	.002	.274	.402	.489
.5	− .693	.008	.343	.479	.571
.6	− .511	.021	.408	.549	.644
.7	− .357	.044	.469	.613	.709
.8	− .223	.077	.526	.673	.770
.9	− .105	.118	.580	.728	.825
1.0	0	.165	.632	.779	.877
1.1	.095	.217	.680	.827	.924
1.2	.182	.270	.725	.871	.969
1.3	.262	.325	.768	.914	1.011
1.4	.336	.381	.810	.955	1.051
1.5	.405	.437	.850	.993	1.089
1.6	.470	.492	.888	1.030	1.125
1.7	.531	.545	.925	1.066	1.160
1.8	.588	.597	.961	1.100	1.194
1.9	.642	.647	.996	1.133	1.226
2.0	.693	.696	1.029	1.165	1.257
2.5	.916	.913	1.181	1.309	1.396
3.0	1.099	1.094	1.313	1.433	1.517
3.5	1.253	1.248	1.430	1.543	1.622
4.0	1.386	1.381	1.535	1.641	1.717
4.5	1.504	1.499	1.630	1.730	1.802
5.0	1.609	1.604	1.718	1.811	1.880
6.0	1.792	1.786	1.873	1.956	2.019
7.0	1.946	1.940	2.008	2.082	2.140
8.0	2.079	2.074	2.127	2.194	2.247
9.0	2.197	2.192	2.234	2.294	2.344
10.0	2.303	2.297	2.331	2.385	2.431

A REFINED MODEL FOR PREMIUM ADJUSTMENT

DAVID L. MILLER AND GEORGE E. DAVIS

INTRODUCTION

Loss ratio ratemaking is an important actuarial technique, especially for actuaries working with less sophisticated data than is available to a rating bureau; and despite the movement toward pure premium ratemaking, loss ratio ratemaking is still essential for several lines of business.

The premium adjustment factors used in the loss ratio method are familiar to most actuaries at least to the extent of Ralph Marshall's[1] or Roy Kallop's[2] descriptions in their papers on Workers' Compensation ratemaking. These papers contain an adequate discussion of the mechanics for calculating premium adjustment factors, but the conceptual background is sketchily drawn and the method used in those papers assumes a constant level of exposures.

Jim Ross, in *Generalized Premium Formulae*[3], presents mathematical expressions which fit the parallelogram approach in a variety of situations. He introduces to the Proceedings a description of the mathematical theory underlying the use of the parallelogram approach. Unlike the previously cited authors, he allows for changing levels of exposures but he does not address their impact on rate level indications or modify the geometrical model to accommodate their representation.

After reviewing the geometry of the traditional two-dimensional model, this paper will introduce a third dimension which will allow for the geometrical representation of levels of exposures; the mathematics fitting this model will be explored. Finally, an example will illustrate the practical application of this model while examing the impact of changing levels of exposure on rate level indications.

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[1] Marshall, R. M., *Workmen's Compensation Insurance Ratemaking*, (Revised, 1961), Casualty Actuarial Society

[2] Kallop, R., *A Current Look at Workers' Compensation Ratemaking*, PCAS LXII (1975)

[3] Ross, J. P., *Generalized Premium Formulae*, PCAS LXII (1975)

TRADITIONAL MODEL

In the usual technique a square is drawn to represent each calendar year's earned exposure (Figure 1). The horizontal or x-axis is always identified as time but the vertical or y-axis is not described. A little reflection will show that the second dimension represents the portion of the policy term expired, ranging from zero to one.

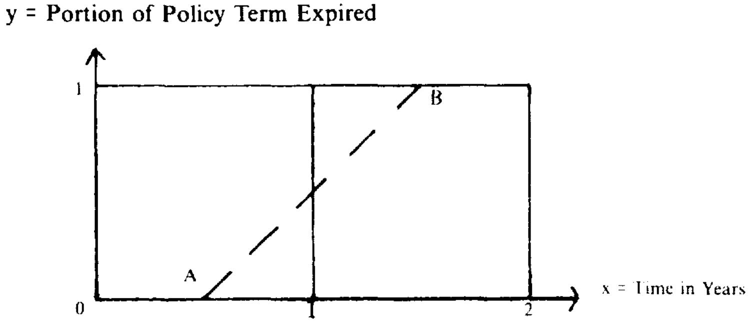


Figure 1

In terms of this model any point (x_t, o) represents the writing of exposures at time x_t because, with $y = o$, exposures are completely unearned. As we move forward in time the exposures written at time x_t are uniformly earned until at time $x_t + k$ (where k is the term of the policy) the exposures are fully earned. This pattern is shown in the geometrical configuration by a diagonal line connecting (x_t, o) and $(x_t + k, 1)$. For example, the earning of exposures on annual policies written at time ½ would be described by the line AB in Figure 1. All other exposures on annual policies, regardless of the time written, will follow a pattern of earning described by lines parallel to AB.

By assuming that exposures are written continuously over time, each square is viewed as being covered by a collection of diagonal lines. It is important to note, for use later in the paper, that any point (x_t, y_t) can be traced to the end point $(x_t - ky_t, o)$ of the diagonal line on which it lies.

In the application of the parallelogram method, the particular diagonal lines drawn mark the boundaries of areas of earned exposures where different rate levels are in effect. The various areas, taken as a percent of the total, are used as weights applied against the various rate levels to produce an average rate level for that period's earned exposures. The ratio of the current rate level to this average rate level is used to modify the period's earned premium. (For an example see pages 76 and 104 of Roy Kallop's paper.)

The method has been presented with an example using only annual term policies. In general where policies of several terms are involved we find it easiest to handle the adjustments for each term of policy separately. Once the separate adjustments are made, simply adding the individual results gives the total adjusted earned premium.

THREE-DIMENSIONAL MODEL

We have seen that the two dimensional model deals with time and the portion of policy term earned. The possibility of varying levels of exposures was addressed by Jim Ross who introduced a function, $f(x)$, representing the rate of exposure writing at time x. In the two dimensional model $f(x)$ cannot be shown. By introducing a third dimension we can account for changing levels of exposure. In the three dimensional model the x-axis and y-axis are defined as before; the z-axis will be defined as the level of exposures.

In order to make the model and the mathematics compatible we will let $z = g(x, y)$ define the level of exposures. Thus each value of z is a function of time and the portion of the policy term earned. It should be clear that $g(x, o)$ is the rate of exposure writing at time x and thus $g(x, o) = f(x)$. Using the relationship noted in the two-dimensional model that any point in the plane (x_t, y_t) can be traced to the point $(x_t - ky_t, o)$ we establish that $g(x_t, y_t) = g(x_t - ky_t, o)$ with the condition that all policies are held to full term. (The assumption of no cancellations will generally be acceptable. If the rate of cancellations is significant in a particular situation the relationship of $g(x, y)$ to $f(x)$ can be appropriately modified. For example, if on the average 10% of written exposures are cancelled during the term of the policy, we can approximate this situation by letting $g(x, y) = (1 - .1y)\, f(x - ky)$.)

Figure 2 shows, in three dimensions, the standard assumption of level exposures. The plane ABCD is comparable to line AB in figure 1 except that in three dimensions we are able to show the level of exposures. Because the value of z is the same throughout, the volumes will be proportional to the areas in the traditional model and the same weights will be obtained. Note that no dimensions are placed on the z-axis. In practice we can graduate the z-axis to absolute amounts of exposure or we can index the exposures to the level of exposures at any convenient time.

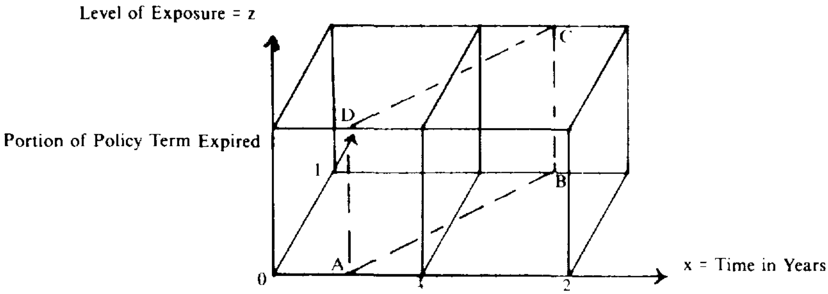


Figure 2

MATHEMATICAL DEVELOPMENT

We have defined $f(x)$ as the rate of exposure writing at time x and $g(x, y)$ as $f(x - ky)$ where k is the policy term in years. The range of y, the portion of policy earned, is such that $0 \leq y \leq 1$ and we require always $g(x, y) \geq 0$.

Since $g(x, y)$ is a density function, its integral describes an amount of exposures. For example, the written exposures between time x_0 and time x_1 may be expressed as:

$$\int_{x_0}^{x_1} g(x, o)dx = \int_{x_0}^{x_1} f(x)dx \tag{1}$$

By integrating over all values of y at a fixed time x_t, we can evaluate the rate of exposures being earned per unit of time. Thus,

$$\int_0^1 g(x_t, y)dy = \int_0^1 f(x_t - ky)dy \tag{2}$$

This differs from the usual notion of "in-force" which is not consistently on an annual basis; e.g., a three year policy would be tallied as three annual exposures in a usual accounting of in-force although only one annual exposure is being earned at any time during the policy period. The traditional concept of in-force would be obtained within this three dimensional model by introducing a factor for the policy term as shown in formula (3).

$$\text{In-force} = k \int_0^1 g(x_t, y)\,dy \tag{3}$$

Since formula (2) gives the rate of exposures being earned at time x_t, its integral over time describes earned exposures. The expression for earned exposures between time x_0 and time x_1 is shown as formula (4).

$$\int_{x_0}^{x_1} \int_0^1 g(x, y)\,dydx = \int_{x_0}^{x_1} \int_0^1 f(x - ky)\,dydx \tag{4}$$

Before proceeding with an example showing the practical application of these formulas we will set forth two additional relationships that can be seen within the three-dimensional framework.

First, noting that $y = 1$ indicates points at which exposures are expiring, the integral of z at $y = 1$ over a time interval x_0 to x_1 will give the level of exposures expiring in that interval. The mathematical notation for this integral is as follows:

$$\int_{x_0}^{x_1} g(x, 1)\,dx = \int_{x_0}^{x_1} f(x - k)\,dx \tag{5}$$

The final formula presented will develop the value for unearned exposures at time x_t. We can view the unearned exposures as the amount of exposures earned following time x_t from all policies written prior to time x_t. Since the last policy contributing to this earned will expire at time $x_t + k$, we will be integrating between time x_t and time $x_t + k$. This will be similar to formula (4) except the lower limit for y will be the diagonal line connecting $(x_t, 0)$ and $(x_t + k, 1)$ instead of $y = 0$. The equation for this line is $y = (x - x_t)/k$, hence the integral for unearned exposures becomes:

$$\int_{x_t}^{x_t + k} \int_{\frac{x - x_t}{k}}^{1} g(x, y)\,dydx =$$

$$\int_{x_t}^{x_t + k} \int_{\frac{x - x_t}{k}}^{1} f(x - ky)\,dydx \tag{6}$$

APPLICATIONS

In developing rate indications for an established line it is unlikely that a bureau will see a significant change in written exposures in one year, but for new lines or declining lines there would be significant changes. On an individual company basis new lines, new program deviations and new market penetrations will in some cases cause changes in exposure large enough to have an impact on rate level indications. In these cases the change in exposure can have an effect on rate level indications which can be measured using the three dimensional model.

As an example of increasing exposures consider the following table of data which might represent the pattern for a new line of business.

Time	Written Exposures
1st Quarter Year	125
2nd Quarter Year	375
3rd Quarter Year	625
4th Quarter Year	875
5th Quarter Year	1125
6th Quarter Year	1375
7th Quarter Year	1625
8th Quarter Year	1875

With $f(x)$ defined as the annual rate of written exposures it is necessary to convert the actual quarterly exposures to an equivalent annual rate in developing the equation for $f(x)$.

Time	Written Exposures	Annual Rate
$0 \leq x \leq .25$	125	500
$.25 \leq x \leq .50$	375	1500
$.50 \leq x \leq .75$	625	2500
$.75 \leq x \leq 1.00$	875	3500
$1.00 \leq x \leq 1.25$	1125	4500
$1.25 \leq x \leq 1.50$	1375	5500
$1.50 \leq x \leq 1.75$	1625	6500
$1.75 \leq x \leq 2.00$	1875	7500

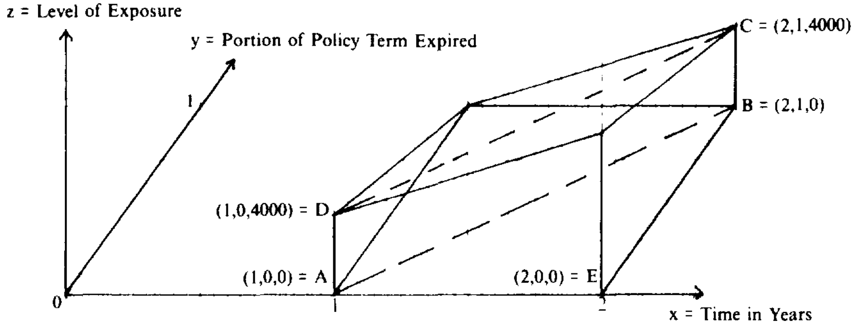


Figure 3

This data indicates a linear pattern since the growth between quarters is constant. By fitting each value to the midpoint of the time interval the equation f(x) = 4000x is developed. Figure 3 describes this situation for policies with one-year term. From our prior development we have g(x,y) = f(x-y) = 4000 (x-y).

We have assumed a rate level increase of 20% at the end of the first year. In figure 3 the diagonal plane ABCD separates the exposures at different rate levels. In our example the earned exposures at the higher rate level in year two are represented by the volume above triangle ABE. To evaluate this volume we apply formula (4) except that the upper limit for y will be the diagonal line AB whose equation is y = x-1. The expression to evaluate this volume is then:

$$\int_1^2 \int_0^{x-1} 4000\,(x-y)dydx = 2667$$

The proportion this volume is of the full year's earned exposure is obtained from dividing the above result by the total earned exposure volume for the year. This latter value is developed directly from formula (4) as:

$$\int_1^2 \int_0^1 4000\,(x-y)\,dydx = 4000$$

The following table compares the results of this method with those of the traditional method.

	Increasing Exposures	Level Exposures
Exposures at Base Rate Level	33%	50%
Exposures at 1.200 Rate Level	67%	50%
Average Rate Level	1.133	1.100
Premium Adjustment Factor	1.059	1.091

If the actual and expected loss ratios were 70% and 60% respectively the traditional method would yield a rate level indication of +6.9% whereas recognizing increasing exposures yields + 10.2%[4]. The above example shows that when rates have increased during a time when exposures are steadily increasing the traditional approach underestimates the average rate level for earned exposure. As a result the necessary premium adjustment is overestimated and the adjusted loss ratio is too low, leading to an inadequate rate indication.

In the case of declining exposures during a period of rising rate levels the traditional method overestimates rate level indications. This situation has been encountered recently in developing rates for a diminishing book of monoline fire business.

In addition to growth situations, irregular exposure patterns may also occur in a stable line where policy writing is heavily weighted towards specific effective dates (e.g., January 1, July 1). Whenever a non-level pattern of exposures is evident, it would be appropriate to look further for the actual exposure pattern. Certainly monthly exposure data would be optimal, but it may be more practical to rely on premium figures. This may be in the form of internal data such as quarterly production reports, monthly bureau transmittals, etc. This data will allow one to judge the value of a refined calculation in the particular instance. If a calculation is warranted, techniques from numerical analysis can be used to fit the data to an integrable function.

[4] $\frac{.70}{1.091} \div .60 = 1.069; \quad \frac{.70}{1.059} \div .60 = 1.102$

A MATHEMATICAL MODEL FOR LOSS RESERVE ANALYSIS

CHARLES L. McCLENAHAN

DISCUSSION BY DAVID SKURNICK

Actuaries generally predict the ultimate cost of a partially paid accident year from the pattern of earlier accident years' payments. But this procedure ignores the development pattern of the current year itself. McClenahan's paper utilizes each year's development pattern by means of certain assumptions concerning the rates of payment, growth, and inflation; the result is a well-defined mathematical model which can serve several useful functions.

The fundamental assumption is that for a given accident month there is a delay of d months before the loss payments begin. Under constant severity these payments then would decrease at a geometric rate. Severity depends upon payment month, and it changes at a uniform geometric rate. Frequency depends upon accident month, and it also changes at a uniform geometric rate. For a given accident month, the combined effects of the decreasing payment rate and the change in severity produce a geometric decline in which each month's payments are r times the prior month's payments.

The assumptions lead to the development of a variety of formulas relating to paid and unpaid losses by accident month and by accident year. The formulas can be used for both cash flow and reserve analyses. The model allows one to measure the effect of a change in frequency, severity, or payment rate. The author also uses the model to evaluate the amount by which loss reserves can be reduced if the payments are discounted. Although the formulas are complicated, the presentation is clear and easy to follow.

Properly estimating a model's parameters is as important as constructing the model. A sensitivity test of the model will show how much accuracy is required for each parameter. For a paid loss development on a casualty line, the indicated reserve is highly sensitive to the rate of payment, particularly at the later ages. For example, under stable conditions, a change of .01 in the 120 month to 132 month age to age factor will produce 1% more loss development for each of the ten most recent years. Thus, it will change the indicated loss reserve by 10% of a year's incurred loss. The age to age factors for the last portion of the development influence the most accident years. Unfortunately, these factors are based on the oldest data; thus they are the least reliable.

The geometric distribution is a special case of the negative binomial. Three methods of estimating the negative binomial's parameters are described by Johnson and Kotz.[1] In the case of a geometric where each term is r times the preceding term all three methods estimate $1/(1-r)$ as the sample mean, which in this case is the average length of time to pay a dollar of loss. The reviewer applied this method to fit geometric distributions separately to each of three accident years using paid workers' compensation claims. For accident year 0 with actual paid loss amounts $A_0, A_1, \ldots, A_N$ during years $0, 1, \ldots, N$ respectively the geometric rate of decline was estimated from the formula

$$1/(1-\hat{r}) = \sum_{n=1}^{N} nA_n \Big/ \sum_{n=1}^{N} A_n \qquad (1)$$

Note that year 0 was omitted because the initial reporting delay prevents the geometric pattern from beginning until year 1.

As shown on Exhibit 1, the fit is only fair. The fitted curve substantially underestimates actual paid loss at later years. By comparison, in the automobile bodily injury example in the paper the model overestimated paid loss at later years. Probably these results reflect the different characteristics of the two lines of business.

There is a bias in this estimation procedure. It underestimates $1/(1-r)$ since it represents the mean of a truncated series of payments. Some adjustment should be made because the observations stop at year N, the latest year for which data is available, if substantial amounts of claims remain unpaid at that time.

Probably the best application of McClenahan's results lies in sensitivity analysis. His formulas directly show the effect of changing the discount rate, the growth rate, or the payment rate. Many readers of this paper will want to experiment to see whether his formulas provide more accurate reserve estimates than the usual methods. This thoroughly developed model is a significant addition to the actuarial literature.

[1] Norman L. Johnson and Samuel Kotz, *Discrete Distributions*, Houghton, Mifflin Company 1969, distributed by John Wiley & Sons, Inc., Salt Lake City, Utah, p. 131-137.

Exhibit 1

GEOMETRIC DISTRIBUTION OF WORKERS' COMPENSATION ACCIDENT YEAR PAID LOSSES*

n	Paid During the Year	(1) Actual Paid	(2) Theoretical Paid Kr^{n}**	(3) Difference (2) − (1)	(4) Percent of Actual (3) ÷ (1)
			Accident Year 1968		
0	1968	11,790,455	11,790,455	—	—
1	1969	10,402,479	10,985,242	582,763	5.6%
2	1970	6,370,883	5,976,239	−394,644	−6.2
3	1971	2,847,065	3,251,220	404,155	14.2
4	1972	1,896,985	1,768,743	−128,242	−6.8
5	1973	1,082,910	962,239	−120,671	−11.1
6	1974	658,942	523,482	−135,460	−20.6
7	1975	492,688	284,787	−207,901	−42.2
		35,542,407	35,542,407		
			Accident Year 1969		
0	1969	13,378,723	13,378,723	—	—
1	1970	14,277,955	15,171,631	893,676	6.3%
2	1971	8,027,259	7,927,389	−99,870	−1.2
3	1972	4,029,497	4,142,172	112,675	2.8
4	1973	2,282,755	2,164,343	−118,412	−5.2
5	1974	1,421,190	1,130,899	−290,291	−20.4
6	1975	1,088,689	590,911	−497,778	−45.7
		44,506,068	44,506,068		
			Accident Year 1970		
0	1970	16,816,141	16,816,141	—	—
1	1971	17,593,975	18,995,304	1,401,329	8.0%
2	1972	9,238,517	9,373,211	134,694	1.5
3	1973	4,571,356	4,625,201	53,845	1.2
4	1974	2,914,044	2,282,300	−631,744	−21.7
5	1975	2,084,322	1,126,198	−958,124	−46.0
		53,218,355	53,218,355		

*For each accident year, year 0 was excluded from the distribution

**Accident year values for r and K

	r	K
1968	.544024371	20,192,555
1969	.522513997	29,035,836
1970	.493448876	38,494,978

MINUTES OF THE 1976 FALL MEETING

NOVEMBER 17-19, 1976

SHERATON HARBOR HOTEL, SAN DIEGO, CALIFORNIA

Wednesday, November 17, 1976

The Board of Directors had their regular quarterly meeting from 1:00 p.m. to 5:00 p.m.

Registration was held from 4:00 p.m. until 6:00 p.m.

The President's reception for New Fellows and their spouses was held from 5:45-6:30 p.m.

A reception for members and guests was held from 6:30-8:00 p.m.

Thursday, November 18, 1976

Registration was held from 8:00-8:30 a.m. President Bornhuetter opened the Annual Meeting at 8:30 with a short welcoming address.

Wesley J. Kinder, California Commissioner of Insurance, then presented his welcoming address. Mr. Kinder quoted from several previous addresses of former C.A.S. presidents and related these addresses to current problems in the Property and Casualty Insurance Business. President Bornhuetter then read the names of the 63 New Associates who rose as their names were called and received the applause of the assembly. A short biography of each New Fellow was then read as each came forward to receive his diploma. All New Associates and Fellows were asked to assemble for group pictures at the coffee break. The New Associates and New Fellows are:

NEW FELLOWS

David A. Arata
Michael P. Blivess
George E. Davis
Howard V. Dempster, Jr.
Richard M. Jaeger
Gustave A. Krause
Richard W. Palczynski
Robert G. Palm
Sheldon Rosenberg
Sanford R. Squires
Lee R. Steeneck
John J. Winkleman, Jr.
Paul E. Wulterkens

NEW ASSOCIATES

Virgil H. Applequist
Betty H. Barrow
Irene K. Bass
Thomas R. Bayley
Donald A. Bealer
John O. Brahmer
H. Earl Cassity
Joseph S. Cheng
Laurence W. Cheng
Christopher Diamantoukos
Nancy R. Einck
Doreen S. Faga
Janet L. Fagan
Russell S. Fisher
Bernard J. Galiley
Timothy L. Graham
Gary Grant
Gary Granoff
Steven C. Herman
Cecily A. Hine
Robert P. Irvan
Larry D. Johnson
James A. Kenney
Kerry K. King
Frederick O. Kist
Thomas J. Kozik
John A. Lamb
Francis J. Lattanzio
Stephen P. Lattanzio
Alan R. Ledbetter
Roy P. Livingston
Anne B. Matson
John M. Meeks
Jerry A. Miccolis
Stanley K. Miyao
Roy K. Morell
Richard F. Murphy
Frank Neuhauser
David J. Oakden
Terrence M. O'Brien
John A. Pagliaccio
David M. Patterson
Joseph L. Petrelli
Richard N. Piazza
John Pierce
John A. Potter
C. Ronald Riley
William P. Roland
Spencer M. Roman
Martin Rosenberg
Harold N. Schneider
Barbara A. Seiffertt
Larry D. Shatoff
Richard S. Skolnik
Richard A. Stroud
Joseph O. Thorne
John P. Tierney
Robert H. Waldman
Forrest Wasserman
Alfred O. Weller
James D. Wickwire, Jr.
Ronald F. Wiser
Richard T. Zatorski

The election of Officers and Board Members followed. Those elected, and their offices were as follows:

President-elect	— P. Adger Williams
Vice President	— Ruth E. Salzmann
Secretary	— Darrell W. Ehlert
Treasurer	— Walter J. Fitzgibbon, Jr.
Editor	— David C. Forker
General Chairman Education and Examination Committee	— Charles C. Cook
Board Members	— Norman J. Bennett
	— Charles A. Hachemeister
	— Frederick W. Kilbourne

The amendments to Article III of the Constitution and Article VI of the By-Laws were approved as submitted to the membership on October 8, 1976. The reading of the Minutes of the May 22-26, 1976 meeting was waived. The Secretary's and the Treasurer's Annual Reports were read. A moment of silence was observed for the C.A.S. members who had died since the last Annual Meeting. (See Obituaries)

The Woodward and Fondiller Prize for 1976 was awarded to Robert J. Finger for his paper "Estimating Pure Premium by Layer—An Approach" by Lewis H. Roberts of Woodward and Fondiller.

President Ronald L. Bornhuetter then delivered the report of Harold W. Schloss on the activities of Organization Coordination and Proposed Reorganization of the Actuarial Bodies in North America. Mr. Schloss was unavoidably absent and sent his deepest regrets.

Robert J. Finger then presented two of his papers which had been accepted by the Committee on Review of Papers:

"A Note on Basic Limits Trend Factors," and
"Modelling Loss Reserve Developments".

After a short break, a panel discussion entitled "Commercial Lines—A Liability?" was presented. Daniel J. McNamara, President of the Insurance Services Office, was the moderator. Participants were:

John W. Carleton, Senior Vice President, Liberty Mutual Insurance Company,
James J. Meenaghan, Vice President and Operations Executive, Fireman's Fund American Insurance Company,
Jack Moseley, Executive Vice President, United States Fidelity and Guaranty Company, and
LeRoy J. Simon, Senior Vice President, Prudential Reinsurance Company.

A formal luncheon was held at noon and the Honorable Alan Cranston, Senior Senator of the State of California was the speaker.

The regular session reconvened at 2:00 p.m. with a workshop program. Six workshops were held, each twice, and four at a time from 2:00 to 5:20 p.m. according to the following schedule:

2:00—3:00 Workshops A, B, C, D
3:10—4:10 Workshops A, B, E, F
4:20—5:20 Workshops C, D, E, F

The workshop subjects and participants are listed below:

Workshop A. *Specific Applications of Econometrics*

Moderator —Stephen S. Makgill, General Manager, Pennsylvania Compensation Rating Bureau

Participants —Allerton Cushman, Jr., Morgan Stanley & Co., Incorporated

Workshop B. *Cumulative Injury—An Industry Trauma*

Moderator —James A. Hall, III, Vice President and Actuary, California Casualty Group

Participants —William B. Whiting, M.D., State Compensation Insurance Fund of California

—Melvin S. Witt, Chairman of Workers' Compensation Appeals Board of California

Workshop C. *Homeowners—A Loss Leader?*

Moderator —Peter B. Zory, Actuary, The Travelers Insurance Companies

Participants —David A. Arata, Actuarial Analyst, Fireman's Fund American Insurance Companies

—John D. Napierski, Assistant Vice President-Actuary, State Farm Fire & Casualty Company

Workshop D. *Early Warning Systems*

Moderator —Ruth E. Salzmann, Vice-President & Actuary, Sentry Insurance Group

Participant —Charles L. McClenahan, Associate Actuary, CNA Insurance

Workshop E. *Capacity*

Moderator —Norman J. Bennett, Vice-President & Actuary, Continental Insurance Companies

Participants —James R. Berquist, Consulting Actuary, Milliman & Robertson, Incorporated

—Robert C. Gowdy, Vice-President, Industrial Indemnity Company

Workshop F. *Catastrophes*

Moderator —Richard J. Roth, Vice-President & Actuary, CNA Insurance

Participant —Don G. Friedman, Director of Corporate Planning & Research, The Travelers Insurance Companies

Members and Guests were invited on a sightseeing boat tour of San Diego Harbor at 6:00 p.m. followed by a Western Barbeque at the convention hotel. During the dinner, President Bornhuetter thanked the Arrangements Committee which was directed by Albert J. Walsh, Vice-President and General Manager, Interinsurance Exchange of the Automobile Club of Southern California. The Southern California Casualty Actuaries Club assisted Mr. Walsh in arranging the meeting and other activities.

The regular meeting resumed at 8:30 a.m. on November 19 with President-elect George D. Morison presiding.

A paper entitled "A Refined Model for Premium Adjustment" written by George Davis and David Miller was presented by Mr. Davis.

Several reviews of papers previously presented were given.

Frank Harwayne's paper—"Accident Limitations for Retrospective Rating" was reviewed by Frank Taylor and Francis Lattanzio, with the former presenting the review. Two other reviews of this paper were presented by David Bradley and Robert J. Finger.

Robert J. Finger's paper—"Estimating Pure Premiums by Layer —An Approach" received two reviews by Lee Steeneck and David Grady.

A panel discussion on "Certification of Annual Statements by the Casualty Actuary" followed.

Moderator —Dale A. Nelson, Actuary, State Farm Mutual Automobile Insurance Company.

Participants —Rafal J. Balcarek, Vice-President & Actuary, Reliance Insurance Company

—David R. Bickerstaff, Consulting Actuary, Milliman & Robertson, Incorporated

After a short coffee break, President Ronald L. Bornhuetter delivered his presidential address. President-elect Morison then presented Mr. Bornhuetter with a plaque commemorating his year as President and thanked Mr. Bornhuetter for his years of service to the Society. The membership responded with a hearty round of applause.

The second panel of the morning—"The Stanford Research Institute Study on Automobile Classification" was then presented.

Moderator —Frederick W. Kilbourne, President, Booz-Allen Consulting Actuaries

Participants —Charles A. Hachemeister, Actuary, Prudential Reinsurance Company

—Frank Riley—Assistant Administrator of the Federal Insurance Administration

—Carl L. Spetzler, M.D., Director, Stanford Research Institute.

A list of the attendees follows:

FELLOWS

Anderson, D. R.
Anker, R. A.
Arata, D. A.
Atwood, C. R.
Balcarek, R. J.
Barker, L. M.
Beckman, R. W.
Bennett, N. J.
Berquist, J. R.
Berry, C. H.
Bethel, N. A.
Bickerstaff, D. R.
Bill, R. A.
Blivess, M. P.
Bondy, M.
Bornhuetter, R. L.
Boyajian, J. H.
Brannigan, J. F.
Carleton, J. W.
Carter, E. J.
Cook, C. F.
Crowley, J. H.
Curry, A. C.
D'Arcy, S. P.
Davis, G. E.
Dempster, H. V., Jr.
Drennan, J. P.
Drobisch, M. R.
Dropkin, L. B.
Ehlert, D. W.
Faber, J. A.
Ferguson, R. E.
Finger, R. J.
Fisher, W. H.
Fitzgibbon, W. J., Jr.
Flynn, D. P.
Forker, D. C.
Fossa, E. F.
Foster, R. B.
Fowler, T. W.
Fusco, M.
Gibson, J. A., III
Gillam, W. S.
Gillespie, J. E.
Ginsburgh, H. J.
Golz, J. F.
Gottlieb, L. R.
Gowdy, R. C.
Grady, D. J.
Graves, J. S.
Grippa, A. J.
Hachemeister, C. A.
Hall, J. A., III
Hartman, D. G.
Harwayne, F.
Haseltine, D. S.
Hazam, W. J.
Heer, E. L.
Hewitt, C. C., Jr.
Hillhouse, J. A.
Inkrott, J. G.
Jaeger, R. M.
Kaliski, A. E.
Kates, P. B.
Kaufman, A. M.
Kayton, H. H.
Kelly, A. E.
Khury, C. K.
Kilbourne, F. W.
Klaassen, E. J.
Kline, D. F.
Kollar, J. J.
Krause, G. A.
Kreuzer, J. H.
Lamb, R. M.
Leslie, W., Jr.
Levin, J. W.
Liscord, P. S.
Lowe, R. F.
MacGinnitie, W. J.
Makgill, S. S.
Masterson, N. E.
McClenahan, C. L.
McClure, R. D.
McLean, G. E.
McNamara, D. J.
Mohl, F. J.
Moore, P. S.
Morison, G. D.
Munro, R. E.
Nelson, D. A.
Newman, S. H.
Oien, R. G.
Pagnozzi, R. D.
Palczynski, R. W.
Palm, R. G.
Perkins, W. J.
Phillips, H. J.
Price, E. E.
Quinlan, J. A.
Retterath, R. C.
Richards, H. R.
Riddlesworth, W. A.
Rinehart, C. R.
Roberts, L. H.
Rodermund, M.
Rosenberg, N.
Rosenberg, S.
Ross, J. P.
Roth, R. J.
Ryan, K. M.

FELLOWS (Cont'd)

Salzmann, R. E.
Scheibl, J. A.
Scheid, J. E.
Scott, B. E.
Sheppard, A. R.
Simon, L. J.
Squires, S. R.
Steeneck, L. R.
Stephenson, E. A.
Sturgis, R. W.
Switzer, V. J.
Tarbell, L. L., Jr.
Tatge, R. L.
Toothman, M. L.
Trudeau, D. E.
Walsh, A. J.
White, H. G.
Williams, P. A.
Winkleman, J. J., Jr.
Woll, R. G.
Wood, J. O.
Wulterkens, P. E.
Zory, P. B.

ASSOCIATES

Andler, J. A.
Angell, C. M.
Applequist, V. H.
Barnes, G. R.
Barrette, R.
Bartlett, W. N.
Bass, I. K.
Bayley, T. R.
Bealer, D. A.
Biondi, R. S.
Bovard, R. W.
Bradley, D. R.
Brahmer, J. O.
Briere, R. S.
Brubaker, R. E.
Cassity, H. E.
Cheng, J. S.
Cheng, L. W.
Childs, D. M.
Chorpita, F. M.
Christiansen, S. L.
Cohen, H. S.
Connor, V. P.
Cooper, W. P.
Covney, M. D.
Crowe, P. J.
Curley, J. O.
DeGarmo, L. W.
Degerness, J. A.
Diamantoukos, C.
Donaldson, J. P.
Einck, N. R.
Eland, D. D.
Evans, D. M.
Faga, D. S.
Fagan, J. L.
Fallquist, R. J.
Fiebrink, M. E.
Fisher, R. S.
Flack, P. R.
Galiley, B. J.
Garand, C. P.
Gleeson, O. M.
Godbold, M. E.
Godbold, N. T.
Goddard, D. C.
Goldberg, S. F.
Graham, T. L.
Granoff, G.
Grant, G.
Greene, T. A.
Gruber, C.
Head, T. F.
Henkes, J. P.
Hermes, T. M.
Herzfeld, J.
Hine, C. A.
Hoylman, D. J.
Inderbitzin, P. H.
Isaac, D. H.
Jensen, J. P.
Johnson, L. D.
Johnston, D. J.
Judd, S.W.
Kenney, J. A.
King, K. K.
Kist, F. O.
Kitzrow, E. W.
Kozik, T. J.
Lamb, J. A.
Lattanzio, F. J.
Lattanzio, S. P.
Ledbetter, A. R.
Lindquist, R. J.
Lino, R. A.
Livingston, R. P.
Marks, R. N.
Matson, A. B.

ASSOCIATES (Cont'd)

McManus, M. F.
Meeks, J. M.
Miccolis, J. A.
Miccolis, R. S.
Miyao, S. K.
Mokros, B. F.
Moller, K. G., Jr.
Moore, B. C.
Moore, B. D.
Morell, R. K.
Murphy, R. F.
Napierski, J. D.
Neis, A. R.
Neuhauser, F., Jr.
Newville, B. S.
Nishio, J. A.
Oakden, D. J.
O'Brien, T. M.
Patrik, G. S.
Patterson, D. M.
Pearl, M. B.
Petit, C. I.
Petlick, S. A.
Piazza, R. N.
Pierce, J.
Plunkett, R. C.
Potter, J. A.
Pratt, J. J.
Ratnaswamy, R.
Reynolds, J. D.
Riley, C. R.
Roach, R. F.
Roland, W. P.
Roman, S. M.
Rosenberg, M.
Sandler, R. M.
Schneider, H. N.
Schultz, E. O.
Schultz, J. J., III
Schumi, J. R.
Seiffertt, B. A.
Shatoff, L. D.
Sherman, R. E.
Shoop, E. C.
Singer, P. E.
Skolnik, R. S.
Stergiou, E. J.
Stroud, R. A.
Swisher, J. W.
Taylor, F. C.
Taylor, J. C.
Thorne, J. O.
Tierney, J. P.
Torgrimson, D. A.
Van Slyke, O. E.
Vogel, J. F.
Wade, R. C.
Waldman, R. H.
Warthen, T. V., Jr.
Wasserman, F.
Weiner, J. S.
Weller, A. O.
Wilson, O. T.
Wiser, R. F.
Young, E. W.
Young, R. J., Jr.
Zatorski, R. T.

GUESTS

Belton, E. F.
Benktander, G.
Carpenter, J. G.
Clause, R. E.
*Cotter, M.
Cushman, A., Jr.
Davidson, D. A.
*Forbes, L. D.
Friedman, D. G.
Guaschi, F. E.
*Hatfield, B. D.
Heller, D. M.
*Hoyt, F. A.
*Johnson, J. E.
Kellison, S. G.
Lyon, A. C.
McCarthy, R.
McMillen, R. H.
*O'Shea, H. J.
Reilly, F. V.
*Rinard, A. V.
*Smith, D. A.
Spangler, J. L.
*Stenmark, J. A.
Tepper, D.
Whiting, W. B.
Wilkinson, M. E.

*Invitational Program

The closing remarks were made by President-elect Morison, who again thanked the committee in charge of arrangements, all those participating in the meeting and the membership who attended. All were invited to reconvene in Washington, D.C. in May of 1977.

The meeting adjourned at 1:10 p.m., P.S.T.

Respectively submitted,

Darrell W. Ehlert,
Secretary

REPORT OF THE SECRETARY

As our society increases in size, so also have the activities of the Officers, Board of Directors and committees increased in scope and intensity. It is expected that the pressures on our business, our profession and the community at large will continue to intensify in the years ahead, and create additional opportunities for our membership to participate in active service to our profession. For example, there are 178 positions on committees that were filled for the 1976-'77 year.

The following list of highlights of the past year will give you some idea of the broadening areas of our concerns and responsibilities.

A special meeting of the Board of Directors was held in January to review the American Institute of Certified Public Accountants' "Discussion Paper" on the audit guide for property and casualty companies. The C.A.S. Committee on Financial Reporting has studied the issues and the Board spent the better part of a day debating these issues.

The Education and Examination Committee expanded their membership during the year and has taken great strides in defining the areas of knowledge expected of our students and in updating the Syllabus. More professional help is being used to increase the quality of the exams. Much effort has been put into developing procedures for handling accusations of cheating on exams.

The Committee on Review of Papers is revising the Guides for Submission of Papers and sees an increasing workload as our new (and older) members increase the volume of papers being submitted for the *Proceedings*.

Increased contacts and cooperation with other Actuarial "Learned Societies" have progressed on several fronts.

1. The Joint Committee on the Independence of the Actuary is attempting to codify disclosure procedures and other ethical standards.

2. The reorganization of the actuarial bodies in North America has been thoroughly explored and much discussed.

3. The definition of and the role of the "enrolled actuary" required in E.R.I.S.A. has received much attention.
4. Contemplated legislation requiring actuarial certification of property and casualty statements occupied many hours of committee work by our Board, individual members and the Special Committee on Certification.
5. The Actuarial Education and Research Foundation is operative and may soon embark on independent actuarial research projects.
6. Common rules and procedures for examinations are being discussed with the Society of Actuaries, as well as additional joint exams.

The Committee on Career Enhancement is taking positive steps to increase recruiting among minority groups and women.

The Ad Hoc Committee on Actuarial Communications made extensive recommendations for improving communications within the society and with the public. These recommendations were assigned to standing committees for implementation.

The five regional affiliates of the society continue to grow and to provide additional opportunities for continuing education of members.

The new Committee on Loss Reserves conducted an all day Symposium on loss reserve techniques in September which was attended by 227 members and guests. Other such Symposiums may be scheduled in the future because of the success of this first one. This is the first meeting to be recorded in full, and transcripts are being prepared for the attendees.

The Astin Organizing Committee is busily preparing for the Washington, D.C. meeting of this international actuarial organization next Spring.

A new recruiting booklet—"The Casualty Actuary" was produced and published by the Public Relations Committee.

The Textbook Committee is nearing completion of a property and casualty actuarial textbook. Publication may be possible in 1977 or 1978.

The Finance Committee has been active in an unsuccessful attempt to change our tax status. They have also recommended changes in the By-Laws to update the "Waiver of Dues" provisions. They did *not* recommend a dues increase.

The Committee on Professional Conduct was involved in a major project of preparing a report regarding an Opinion on Advertising in conjunction with the Joint Committee on Professional Conduct. A Supreme Court Decision striking down Bar Association restrictions on advertising by members aborted this effort.

The Program Committee has met almost monthly in order to provide quality programs and participants at our meetings.

The Board of Directors met five times since last November. Meetings were held in Chicago in January; Jacksonville, Florida in March; Palm Beach, Florida in May; Atlanta, Georgia in September, and here in San Diego. Attendance by Board Members exceeded 90% for these five meetings.

Our society continues to grow. The exams last November and May brought in 74 new Associates and 28 new Fellows, both records.

These records are expected to be broken each year for the next several years as the number of students applying for exams is still increasing, although at a decreasing rate. Students signed up to take 3,422 exams in 1976, versus 3,182 in 1975.

With the death of Everett Fallow this year, John S. Thompson becomes the last surviving charter member of the Casualty Actuarial Society. Perhaps Mr. Thompson would enjoy a personal expression of appreciation from individual members.

In closing, I wish to express my personal thanks to the Officers and Board Members and Committee Members who have helped me this past year and to Bob Foster who has allowed me to draw on his secretarial experience. As you all know, Edith Morabito, who supervises the C.A.S. Office in New York, is the sustaining force in the Office of Secretary. Without her help, I could not survive. My own secretary, Randy Pietroski, has also been of great help to me and the Society this past year.

Respectfully submitted,

Darrell W. Ehlert
Secretary

REPORT OF THE TREASURER

The audited financial statement for the fiscal year ended September 30, 1976 showed cash and invested assets of $167,469.34, an increase of $30,710.20 for the year. Most of the increase was due to more prompt recording of examination fees, underspending the printing budget and showing a profit on the loss reserves symposium. Some of this gain is temporary since examination fees received for jointly administered exams will be paid over to the Society of Actuaries.

Your finance committee and Board of Directors have recognized that cash basis statements do not present the financial position of our Society very clearly. Therefore, the financial statement for September 30, 1976 shows both the continuation of the cash accounting basis and the conversion to an accrual basis. This conversion recognizes two major liabilities. One for printing the 1975 Proceedings and the other to reflect November, 1976 examination fees for jointly administered parts soon to be paid to the Society of Actuaries. The accrual statement shows membership equity of $133,022.44. This equity includes two funds which have been established using amounts received from past presidents of our Society, Gustav F. Michelbacher and Paul Dorweiler. The Michelbacher fund of $26,137.97 represents royalties received by the Society over the years from Mr. Michelbacher's books. The Dorweiler fund of $6,375.13 is based on a legacy received by the Casualty Actuarial Society. No specific purposes have yet been established by the Board of Directors for these funds.

In February, 1976, a $100,000 U.S. Treasury bill paying 5.3% interest which had been purchased three months earlier matured and the proceeds were reinvested in a U.S. Treasury note maturing in May, 1981 and paying 7.5% interest. In May, $25,000 was placed in a 1 year time savings account paying 6½% interest.

The operating budget approved for the coming year contains some increases in postage and printing expenses. Increased mailings and the December 28, 1975 increase in postal rates along with printing of study materials and copies of the syllabus have caused the increases. Interest income will be reduced somewhat since earnings on the newly established Michelbacher and Dorweiler Funds will accrue to these funds and will not be used to support current operations.

The level of membership dues will be unchanged. Fellowship dues are $70.00. Associateship dues are $50.00 for the first five years and $70.00 thereafter. Residents outside the United States and Canada will pay $50.00 dues.

The amendment to the by-laws approved today will permit the Board of Directors to waive partially or fully the dues of any member who considers that payment of these dues constitutes a financial hardship. This will include, but not be limited to, members on maternity leave or raising families who, in the past, have been required to continue payment of dues in order to retain Society membership.

A petition to the IRS to change the Casualty Actuarial Society's tax exempt status from Section 501(c)(6) to Section 501(c)(3) was denied. If approved, the change would have allowed tax deductions to those making donations to the Casualty Actuarial Society. The legal fees of $2,130.02 shown in the financial statement were incurred in connection with the IRS petition.

The limit on the Society's surety bond was increased early in the year from $150,000 to $175,000. Our general liability policy was endorsed to provide coverage for all those involved in administering the examination program of the Society. This policy provides $300,000 single limit coverage.

Respectfully submitted,

W. J. Fitzgibbon, Jr.
Treasurer

Presented to membership on
November 18, 1976

FINANCIAL REPORT

FOR FISCAL YEAR ENDED SEPTEMBER 30, 1976

INCOME

Dues	$ 36,010.00
Examination Fees	71,209.00
Meetings & Registration Fees	33,022.78
Sale of Proceedings	7,145.15
Sale of Readings	1,855.50
Invitational Program	3,600.00
Michelbacher Royalties	1,177.05
Interest	9,308.62
Actuarial Review	220.00
Misc.	—24.59
Total	$163,523.51

DISBURSEMENTS

Printing & Stationery	$ 31,594.25
Secretary's Office	31,777.00
Examination Expenses	37,938.39
Meeting Expenses	26,438.12
Library	330.10
Math. Assoc. of America	1,500.00
Insurance	629.00
Dorweiler Prize	200.00
Legal Fees	2,130.02
Misc.	276.43
Total	$132,813.31

Increase in cash and invested assets	$ 30,710.20
Cash & invested assets 9/30/75	136,759.14
Cash & invested assets 9/30/76	167,469.34

ACCRUAL BASIS ACCOUNTING STATEMENT AS OF 9/30/76

ASSETS

Bank Accounts	$ 63,609.34
U.S. Treasury Bond	4,325.00
U.S. Treasury Note	99,535.00
Accrued interest—Savings	627.74
Accrued interest—Investments	2,788.36
Total	$170,885.44

LIABILITIES, SURPLUS & OTHER FUNDS

LIABILITIES

Printing PCAS 1975	$ 20,000.00
Examination Expense	16,163.00
Actuarial Review	1,700.00
Sub-Total	$ 37,863.00

MEMBERS EQUITY

Michelbacher Fund	$ 26,137.97
Dorweiler Fund	6,375.13
Surplus	100,509.34
Sub-Total	$133,022.44
Total	$170,885.44

W. J. Fitzgibbon, Jr.
Treasurer

* * * * * *

This is to certify that the assets and accounts shown in the above financial statement have been audited and found to be correct.

Robert B. Foster
Chairman of Finance Committee

1976 EXAMINATIONS—SUCCESSFUL CANDIDATES

Examinations for Parts 4, 6, 8 and 10 of the Casualty Actuarial Society Syllabus were held May 6 and 7, 1976 and examinations for Parts 5, 7 and 9 were held November 10 and 12, 1976. Parts 1, 2 and 3, jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries were given in May and November. Those who passed Parts 1, 2 and 3 were listed in the joint release of the two Societies dated July 23, 1976 and January 21, 1977.

The following candidates successfully completed the requirements for Fellowship and Associateship in the May 1976 Examinations.

NEW FELLOWS

Arata, David A.	Krause, Gustave A.	Squires, Sanford R.
Blivess, Michael P.	Palczynski, Richard W.	Steeneck, Lee R.
Davis, George E.	Palm, Robert G.	Winkleman, John J., Jr.
Dempster, Howard V., Jr.	Rosenberg, Sheldon	Wulterkens, Paul E.
Jaeger, Richard M.		

NEW ASSOCIATES

Appelquist, Virgil H.	Johnson, Larry D.	Petrelli, Joseph L.
Barrow, Betty H.	Kenney, James A.	Piazza, Richard N.
Bass, Irene K.	King, Kerry K.	Pierce, John
Bayley, Thomas R.	Kist, Frederick O.	Potter, John A.
Bealer, Donald A.	Kozik, Thomas J.	Riley, C. Ronald
Brahmer, John O.	Lamb, John A.	Roland, William P.
Cassity, H. Earl	Lattanzio, Francis J.	Roman, Spencer M.
Cheng, Joseph S.	Lattanzio, Stephen P.	Rosenberg, Martin
Cheng, Laurence W.	Ledbetter, Alan R.	Schneider, Harold N.
Diamantoukos, Christopher	Livingston, Roy P.	Seiffertt, Barbara A.
Einck, Nancy R.	Matson, Anne B.	Shatoff, Larry D.
Faga, Doreen S.	Meeks, John M.	Skolnik, Richard S.
Fagan, Janet L.	Miccolis, Jerry A.	Stroud, Richard A.
Fisher, Russell S.	Miyao, Stanley K.	Thorne, Joseph O.
Galiley, Bernard J.	Morell, Roy K.	Tierney, John P.
Graham, Timothy L.	Murphy, Richard F.	Waldman, Robert H.
Granoff, Gary	Neuhauser, Frank, Jr.	Wasserman, Forrest
Grant, Gary	Oakden, David J.	Weller, Alfred O.
Herman, Steven C.	O'Brien, Terrence M.	Wickwire, James D., Jr.
Hine, Cecily A.	Pagliaccio, John A.	Wiser, Ronald F.
Irvan, Robert P.	Patterson, David M.	Zatorski, Richard T.

MAY 1976 EXAMINATIONS

Following is a list of successful candidates in the examinations held in May 1976:

FELLOWSHIP EXAMINATIONS

Part 8

Arata, David A.
Childs, Diana M.
Daino, Robert A.
Dangelo, Charles H.
Dempster, Howard V., Jr.
Dolan, Michael C.
Dorval, Bernard
Duperreault, Brian
Eddy, Jeanne H.
Eldridge, Donald J.
Engel, Philip L.
Ernst, Richard C.
Evans, Dale M.
Fiebrink, Mark E.
Frohlich, Kenneth R.
Gersie, Michael H.
Gleeson, Owen M.
Grannan, Patrick J.
Jaeger, Richard M.
Jean, Ronald W.
Jerabek, Gerald J.
Judd, Steven W.
Krause, Gustave A.
Lehmann, Steven G.
Lindquist, Peter L.
McManus, Michael F.
Miccolis, Robert S.
Moller, Karl G., Jr.
Moore, Bruce D.
Newlin, Patrick R.
Patrik, Gary S.
Potok, Charles M.
Rapp, Jerry W.
Reynolds, John D.
Rice, Walter V.
Riff, Mayer
Rodgers, Beatrice T.
Smith, Frances A.
Squires, Sanford R.
Taylor, Frank C.
Teufel, Patricia A.
Wood, Charles P., Jr.

Part 10

Angell, Charles M.
Arata, David A.
Ashenberg, Wayne R.
Barnes, Galen R.
Bassman, Bruce C.
Bellinghausen, Gary F.
Blivess, Michael P.
Brubaker, Randall E.
Carbaugh, Albert B.
Davis, George E.
Eland, Douglas D.
Hermes, Thomas M.
Johnston, Daniel J.
Keene, Vicki S.
Leimkuhler, Urban E.
Marker, Joseph O.
Palczynski, Richard W.
Palm, Robert G.
Quirin, Albert J.
Roach, Robert F.
Rosenberg, Sheldon
Schultz, Ellen O.
Shoop, Edward C.
Squires, Sanford R.
Steeneck, Lee R.
Taylor, Jane C.
Van Slyke, Oakley E.
Venter, Gary G.
Vogel, Jerome F.
Warthen, Thomas V.
Winkleman, John J., Jr.
Wright, Walter C., III
Wulterkens, Paul E.
Yoder, Reginald C.
Young, R. James, Jr.
Zubulake, Theodore J.

ASSOCIATESHIP EXAMINATIONS

Part 4(a)

Graves, George G.
Johnson, Warren H., Jr.
McCarter, Michael G.
Nickerson, Gary V.
Noceti, Stephen A.

Part 4(b)

Almer, Monte
Andrus, William R.
Bartlett, John W.
Bayley, Thomas R.
Carpenter, James G.
Christhilf, David A.
Cloutier, Guy
Gaillard, Mary B.
Hanover, Richard F.
Heitt, Maurice
Hine, Cecily A.
Irvan, Robert P.
LaFrenaye, A. Claude
Leo, Carl J.
Livingston, Roy P.
Lommele, Jan A.
Lowe, Stephen P.
McDaniel, Gail P.
Mellia, Joanne C.
Miller, Robert A., III
Racine, Andre R.
Ragan, Evelyn T. M.
Riley, C. Ronald
Rowland, Vincent T.
Rush, Mary L.
Strickoff, Carol L.
Thibault, Alain P.
Van Ark, William R.
Wach, Michael M.
Winter, John C., III
Wisecarver, Timothy L.

Part 4

Arzberger, Peter W.
Belvin, William H.
Buck, James E., Jr.
Callahan, James J.
Cheng, Laurence W.
Conger, Robert F.
Doepke, Mark A.
Duffy, Thomas J.
Ford, Edward W.
Giambo, Robert A.
Granoff, Gary
Harrison, Eugene E.
Holdredge, Wayne D.
Johnson, Marvin A.
Johnston, Thomas S.
Kucera, Jeffrey L.
Lerwick, Stuart N.
Leung, Gilbert K.
Levine, Michael M.
Mair, Sharon A.
McConnell, Charles W., II
McGovern, William G.
Myers, Nancy R.
Nash, Russell K.
Orlowicz, Charles P.
Reichle, Kurt A.
Roach, William L.
Robertson, John P.
Rosa, Domenico
Schmidt, Lowell D.
Tremblay, Monique
Tuttle, Jerome E.
Weissner, Edward W.
White, Jonathan
Whitman, Mark
Wilson, Randall J.
Youngner, Ruth E.

Part 6

Appelquist, Virgil H.
Balchunas, Anthony J.
Barrow, Betty H.
Bass, Irene K.
Bealer, Donald A.
Biller, James E.
Brahmer, John O.
Brown, Joseph W.
Buck, James E., Jr.
Cassity, H. Earl
Cheng, Joseph S.
Cheng, Laurence W.
Clark, David G.
Diamantoukos, Christopher
Drummond-Hay, Eric T.
Egnasko, Gary J.
Einck, Nancy R.
Faga, Doreen S.
Fagan, Janet L.
Fisher, Russell S.
Galiley, Bernard J.
Giambo, Robert A.
Gnazzo, Polly R.
Graham, Timothy L.
Granoff, Gary
Grant, Gary
Herman, Steven C.
Hine, Cecily A.
Irva, Robert P.
Johnson, Larry D.
Kenney, James A.
King, Kerry K.
Kist, Frederick O.
Klein, Richard C.
Kozik, Thomas J.
LaFrenaye, A. Claude
Lamb, John A.
Lattanzio, Francis J.
Lattanzio, Stephen P.
Ledbetter, Alan R.
Matson, Anne B.
Meeks, John M.
Meyer, Robert E.
Miccolis, Jerry A.
Miyao, Stanley K.
Morell, Roy K.
Murphy, Richard F.
Neuhauser, Frank, Jr.
Oakden, David J.
O'Brien, Terrence M.
Pagliaccio, John A.
Parker, Curtis M.
Patterson, David M.
Petrelli, Joseph L.
Philbrick, Stephen W.
Piazza, Richard N.
Pierce, John
Potter, John A.
Roland, William P.
Roman, Spencer M.
Rosenberg, Martin
Rush, Mary L.
Ryan, John F.
Schneider, Harold N.
Seiffertt, Barbara A.
Shatoff, Larry D.
Shayer, Natalie
Skolnik, Richard S.
Stroud, Richard A.
Surrago, James
Thorne, Joseph O.
Tierney, John P.
Waldman, Robert H.
Wasserman, Forrest
Weller, Alfred O.
Westerholm, David C.
Wickwire, James D., Jr.
Wiser, Ronald F.
Zatorski, Richard T.

The following candidates successfully completed the requirements for Fellowship and Associateship in the November 1976 examinations.

NEW FELLOWS

Angell, Charles M.
Barnes, Galen R.
Fallquist, Richard J.
Garand, Christopher P.
Keene, Vicki S.
Radach, Floyd R.
Schultz, Ellen O.
Schultz, John J., III
Stanard, James N.
Walters, Mavis A.
Warthen, Thomas V., Jr.
Yoder, Reginald C.
Young, R. James, Jr.
Zubulake, Theodore J.

NEW ASSOCIATES

Buck, James E., Jr.
Crifo, Daniel A.
Gerlach, Scott B.
Giambo, Robert A.
Johnson, Warren H., Jr.
LaFrenaye, A. Claude
Lommele, Jan A.
Meyer, Robert E.
O'Neil, Mary L.
Silberstein, Benny
Westerholm, David C.

NOVEMBER 1976 EXAMINATIONS

Following is a list of successful candidates in the examinations held in November 1976:

FELLOWSHIP EXAMINATIONS

Part 7

Aldorisio, Robert P.
Bass, Irene K.
Bayley, Thomas R.
Bealer, Donald A.
Bell, Linda C.
Beverage, Richard M.
Bishop, Everett G.
Bradley, David R.
Brooks, Dale L.
Brown, Joseph W.
Cheng, Joseph S.
Cheng, Laurence W.
Cis, Mark M.
Cohen, Arthur I.
Collins, Douglas J.
Connor, Vincent P.
Curley, James O.
Currie, Ross A.
Dahlquist, Ronald A.
Dangelo, Charles H.
Dorval, Bernard
Duperreault, Brian
Eland, Douglas D.
Fagan, Janet L.
Frohlich, Kenneth R.
Gaillard, Mary B.
Gnazzo, Polly R.
Godbold, Mary Jo E.
Goddard, Daniel C.
Grant, Gary
Henkes, Joseph P.
Henry, Dennis R.
Hobart, Gary P.
Irvan, Robert P.
Jerabek, Gerald J.
Johnson, Marvin A.
Karlinski, Frank J.
Kist, Frederick O.
Lattanzio, Stephen P.
Ledbetter, Alan R.
Lehman, Merlin R.
Lehmann, Steven G.
Linquist, Peter L.
Livingston, Roy P.
Lowe, Stephen P.
McManus, Michael F.
Meeks, John M.
Metzner, Claus S.
Miccolis, Robert S.
Miyao, Stanley K.
Moller, Karl G., Jr.,
Morell, Roy K.
Murad, John A.
Neis, Allan R.
Neuhauser, Frank Jr.
Newlin, Patrick R.
Nichols, Raymond S.
Oakden, David J.
Patrik, Gary S.
Philbrick, Stephen W.
Piazza, Richard N.
Quirin, Albert J.
Renze, David E.
Reynolds, John D.
Reynolds, John J., III
Rodgers, Beatrice T.
Roland, William P.
Roth, Richard J., Jr.
Schaeffer, Bernard G.
Schultz, Ellen O.
Shatoff, Larry D.
Shayer, Natalie
Stanard, James N.
Sweeny, Andrea M.
Teufel, Patricia A.
Tierney, John P.
Van Ark, William R.
Wiser, Ronald F.
Wood, Charles P., Jr.
Zatorski, Richard T.

Part 9(a)

Hanson, H. Donald
Millman, Neil L.
Radach, Floyd R.
Young, R. James, Jr.

Part 9(b)

Stanard, James N.
Walters, Mavis A.

Part 9

Angell, Charles M.
Barnes, Galen R.
Childs, Diana M.
Fallquist, Richard J.
Fiebrink, Mark E.
Garand, Christopher P.
Gersie, Michael H.
Gleeson, Owen M.
Grannan, Patrick J.
Gruber, Charles
Herzfeld, John
Keene, Vicki S.
Lino, Richard A.
Schultz, John J., III
Sherman, Richard E.
Taylor, Frank C.
Venter, Gary G.
Warthen, Thomas V., Jr.
Wasserman, Forrest
Yoder, Reginald C.
Zubulake, Theodore J.

ASSOCIATESHIP EXAMINATIONS

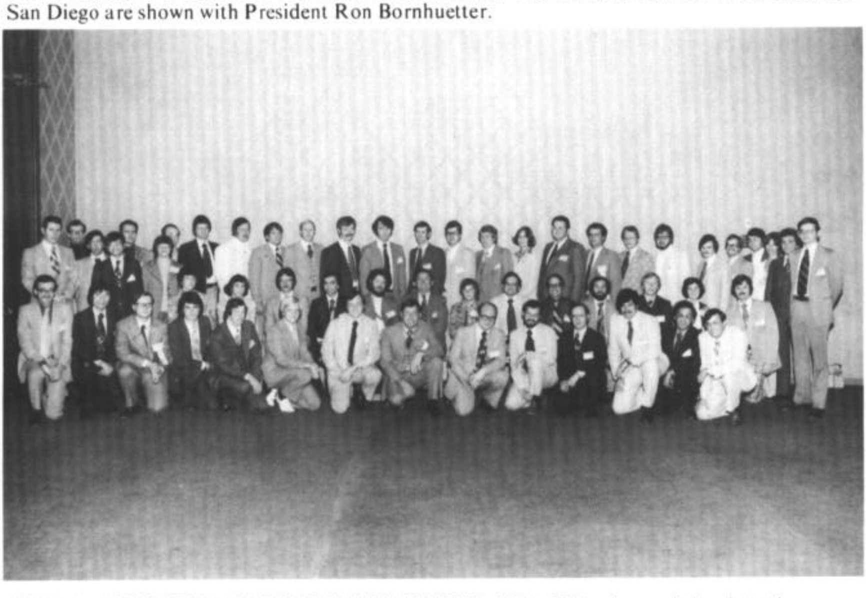
Part 5

Abramson, Gary R.
Antolino, Michael R., Jr.
Baer, Debra L.
Belvin, William H.
Beversdorf, William R.
Booher, John P.
Buck, James E., Jr.
Burg, David R.
Cloutier, Guy
Cohen, Howard L.
Cola-Luca, Suzanne E.
Conger, Robert F.
Davis, Lawrence S.
DiBattista, Susan T.
Dodd, George T.
Doepke, Mark A.
Dornfeld, James L.
Duffy, Thomas J.
Edie, Grover M.
Foote, James M.
Ford, Edward W.
Furst, Patricia A.
Gerlach, Scott B.
Ghezzi, Thomas L.
Giambo, Robert A.
Harrison, Eugene E.
Hart, John A.
Hartz, Melvin L.
Heckman, Philip E.
Heersink, Agnes H.
Higgins, Barbara J.
Holdredge, Wayne D.
Javaruski, John J.
Johnston, Thomas S.
Knilans, Kyleen
Koski, Mikhael I.
Kucera, Jeffrey L.
LaFontaine, Gaetane
LaFrenaye, A. Claude
LaMonica, Michael A.
Lee, Yoong S.
Lerwick, Stuart N.
Lo, Richard W.
Lombardo, John S.
Lommele, Jan A.
Merves, Brian B.
Meyer, Robert E.
Miller, Allen H.
Miller, Robert A., III
Morgan, William S.
Nash, Russell K.
Neeson, Charles P.
Niswander, Ray E., Jr.
O'Neil, Mary L.
Orlowicz, Charles P.
Perry, Loren A.
Racine, Andre R.
Rosa, Domenico
Rowland, Vincent T., Jr.
Rowland, William J.
Silberstein, Benny
Skrodenis, Donald P.
Smith, Mary Jane
Swallow, James R.
Taranto, Joseph V.
Taylor, Thomas F.
Thibault, Alain P.
Thompson, Kevin B.
Truttmann, Everett J.
Weissner, Edward W.
Westerholm, David C.
White, Frank T.
Wilson, Randall J.
Wilson, William F.
Wisecarver, Timothy L.
Yatskowitz, Joel D.
Yuan, Hui-Lin
Zicarelli, John D.

NEW FELLOWS ADMITTED MAY 1976: Thirteen of the fifteen new fellows admitted at the Breakers in Palm Beach are shown with President Ron Bornhuetter.

NEW ASSOCIATES ADMITTED MAY 1976: Eight of the ten new associates admitted at the Breakers in Palm Beach are shown with President Ron Bornhuetter.

NEW FELLOWS ADMITTED NOVEMBER 1976: The thirteen new fellows admitted at San Diego are shown with President Ron Bornhuetter.



NEW ASSOCIATES ADMITTED NOVEMBER 1976: Fifty-three of the sixty-three new associates admitted at San Diego are shown with President Ron Bornhuetter.

OBITUARIES

BARRETT N. COATES

1893-1976

Barrett N. Coates, a fellow of the Casualty Actuarial Society since 1918 died on September 16, 1976 in Berkeley, California.

When Mr. Coates became a Fellow, he was employed by the Fraternal Brotherhood in Los Angeles. In 1921 he became Assistant Secretary and Actuary of the Western States Life Insurance Company of San Francisco. He went into consulting actuarial work in San Francisco in 1924, and in 1928 became a partner in the consulting actuarial firm of Coates and Herfurth, also in San Francisco. He stayed there until he retired in 1954. Until his death he lived in Berkeley, California.

As a member of the CAS, Mr. Coates served on the Committee on Book Reviews. He was a Fellow of the Society of Actuaries, class of 1921, and a Charter Member of the American Academy of Actuaries.

EVERETT S. FALLOW

1885-1976

Everett S. Fallow, a charter member of the Casualty Actuarial Society died on February 19, 1976 at home in West Hartford, Connecticut.

Born in East Hartford, he lived in the Hartford area all his life. He worked as an actuary for the Travelers Company for 45 years before retiring in 1950. From 1921 until his retirement, he was the Actuary in the Accident Department of the Travelers.

Mr. Fallow served twice as a Member of the Council (now the Board of Directors), from 1922 to 1924 and from 1927 to 1930. He also served on the Examination Committee and the Committee on Book Reviews. In addition to being a charter member of the Casualty Actuarial Society, he was also a charter member of the American Academy of Actuaries.

JOSEPH H. FINNEGAN

1904-1976

Joseph H. Finnegan, a Fellow of the Casualty Actuarial Society since 1956 died on July 25, 1976.

Born in 1904, Mr. Finnegan did undergraduate and graduate work at the New York University School of Commerce, receiving his Doctorate from them in 1944.

Mr. Finnegan joined the National Board of Fire underwriters in 1946. He stayed with the NBFU and its successor organizations until his retirement in 1971. Prior to his retirement he was a Manager in their Property Division.

HAROLD F. La CROIX
1924-1976

Harold F. La Croix, a Fellow of the Casualty Actuarial Society since 1949 died August 25, 1976 at his home in West Hartford, Connecticut.

Born in Quincy, Massachusetts, he was graduated Magna Cum Laude in 1943 from Harvard University with an A.B. in Mathematics. He served as a Lt. Commander in the U. S. Navy during World War II.

Mr. La Croix joined the Travelers in 1946 as an actuarial student. He was named Assistant Actuary in 1950, Associate Actuary in 1956 and Secretary in the Group Department in 1959. He was appointed Second Vice President in 1961, Vice President in 1965, and became Senior Vice President and Actuary in 1967 in charge of the Corporate Actuarial and Control Department. In July, 1968 he was appointed to head the Casualty-Property Department, and in 1971 was appointed Executive Vice President.

He resigned that post in 1973 for health reasons, but remained at the Travelers as a consultant until his death. He was a Charter Member of the American Academy of Actuaries and in 1949 presented a paper on Group Accident & Health Insurance.

GILBERT R. LIVINGSTON
1902-1976

Gilbert R. Livingston, a Fellow of the Casualty Actuarial Society since 1950 died February 4, 1976 at his home in Nutley, New Jersey.

A lifelong resident of Nutley, Mr. Livingston was a graduate of the Loomis School in Windsor, Connecticut and Union College in Schenectady, New York.

Mr. Livingston worked for the National Bureau of Casualty Underwriters, now the Insurance Services Office from 1925 to 1958. Following his retirement, he became an actuary for the Connecticut Insurance Department, retiring from this position in 1975. He maintained his residence in Nutley, commuting to Hartford two or three times a week during this period.

Mr. Livingston spent more than 50 years in the insurance business. During this time he was chairman of the Actuarial Committee of the National Bureau of Casualty Underwriters, a member of the National Association of Insurance Commissioners Workers' Compensation Ratemaking Committee and was librarian of the Casualty Actuarial Society. He was also a charter member of the American Academy of Actuaries.

LOUIS H. MUELLER

1896-1975

Louis H. Mueller, a Fellow of the Casualty Actuarial Society since 1920 died November 4, 1975.

Upon graduation from the University of California in 1917 he became a test pilot for the United States Army Air Force and aviation from that time on had been one of his main interests.

When he received his Fellowship, Mr. Mueller was Statistician of the California State Compensation Insurance Fund. In 1922 he became Actuary-Statistician of the Associated Industrial Insurance Corporation. He became Vice President and Treasurer in 1928. In 1929 Mr. Mueller accepted the position of President of Varney Air Lines, and in 1932 he became Resident Executive of United Air Lines.

In 1935 Mr. Mueller returned to the insurance industry as Director of the Associated Insurance Fund. He was made President in 1938, succeeding Claude W. Fellows a former Fellow of this Society. He remained at Associated, now part of the "Firemen's Fund" until his retirement in 1947. He continued to live in the San Francisco area until his death. He was charter member of the American Academy of Actuaries.

Mr. Mueller is survived by his widow, Mrs. Mueller, who lives in San Francisco.

WALTER E. OTTO

1889-1976

Walter E. Otto, an Associate of the Casualty Actuarial Society since 1919 died April 6, 1976 at a Convalescent Home in Bloomfield, Michigan.

Mr. Otto, former president, board chairman and director emeritus of the Michigan Mutual Insurance Company was a prominent figure in the insurance industry for more than 60 years. He began his career in 1906 as an actuarial clerk at Michigan Mutual Life. The following year he was employed by the Michigan Insurance Department as an actuary and was later named Deputy Commissioner of Insurance.

In 1918 he joined Michigan Mutual Liability Insurance Company as treasurer. In 1922 he was elected to the board of directors. He was promoted to secretary-treasurer in 1924 and in 1936 was elected president. He served in that position until 1958 when he was elevated to chairman of the board. He retired from active participation in Michigan Mutual management in 1966, but continued as a member of the board until 1972.

In the insurance industry he was active in the American Mutual Insurance Alliance and was president of the National Association of Mutual Casualty companies and the National Association of Automobile Mutual Insurance companies. He was a member of the insurance committee of the United States Chamber of Commerce and was chairman of the Rehabilitation Institute.

ARMAND SOMMER

1898 1976

Armand Sommer, an Associate of the Casualty Actuarial Society, died on June 12, 1976 in Chicago, Illinois at the age of 78.

Born in Salt Lake City, Utah on January 15, 1898, he was a graduate of the University of California. After ten years experience with other companies, Mr. Sommer joined Continental Casualty Company in 1932. He was named executive assistant vice president in 1952 and vice president in 1956. After his retirement in 1967, he served as an executive consultant to the Accident and Health Department. At the time of his death he was a director of Old Equity Life Insurance Company.

Mr. Sommer was the author of two books: "Manual of Accident & Health Insurance" and "Your Future in Insurance". He was the founder and first president of the Chicago Health Insurance Association, which later became part of the International Association of Health Underwriters. He was the recipient of many health insurance honors, including the IAHU award as Health Insurance Man of the Year in 1966.

Mr. Sommer is survived by his wife, Leah; two daughters, Mrs. Jane S. Mason and Mrs. S. Holmes; and five grandchildren.

M. ELIZABETH UHL

1976

M. Elizabeth Uhl, an associate of the Casualty Actuarial Society died June 4, 1976 in New York.

Ms. Uhl received her undergraduate degree from the University of California. She earned a masters degree from the University of Michigan in 1920. She joined the National Board of Casualty Underwriters in 1921 and remained with them until 1964 when she retired from her position in charge of rate filings.

After her retirement, Ms. Uhl continued to live in New York City until her death.

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