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NOTICE

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NONPROPORTIONAL REINSURANCE AND THE INDEX CLAUSE

RONALD FERGUSON

DISCUSSION BY JOSEPH W. LEVIN

Mr. Ferguson has presented to the Society a paper which has universal appeal to the membership of the Society as well as those in the industry not necessarily technically involved. This paper has an exceptional blend of academic, technical and practical substance.

In his diagnosis and prognosis, Mr. Ferguson presents the problem of inflation from the standpoint of economic theory and from its manifestation in everyday life.

Mr. Ferguson is further to be commended for putting his finger on the more critical points of dealing with inflation. For example, he refers to the "double-barreled inflation effect—the ordinary economic inflation discussed . . . and what might be called social inflation." It is exactly this social inflation which raises doubts in my mind as to the total effectiveness of the index clause to which Mr. Ferguson addresses his paper. One of the biggest challenges facing reinsurers as well as their clients is the precise measurement of the dominant force in society—inflation. Various indices have been in existence for several years, but unfortunately these are subject to the aberrations of statistical methods as well as, I feel, political manipulation.

Mr. Ferguson, in his paper, refers to a study made by Mr. L. H. Roberts entitled "The Impact of Inflation in Reinsurance Costs". The table extracted from Mr. Roberts' study shows that the effect of inflation on layers in excess of given retentions is considerably higher than the overall inflation. For the values shown in this paper, the effect on excess losses is from 2 1/2 times overall inflation at the $10,000 retention level, to over 3 1/2 times at the

$50,000 retention. Extrapolating beyond this, the power becomes more significant. Keeping in mind this tremendous leverage, let us consider the example shown in Table III of Mr. Ferguson's paper. In this table he shows a hypothetical population of 22 losses at various amounts at a 1974 cost level. He then adjusts these to subsequent settlement values into the future, assuming a 10% inflation rate. On the basis of a fixed retention of $50,000, he demonstrates that the indicated reinsurance rate would increase by 15% and 19% respectively, for the two successive years after 1974. This is predicated on an assumed increase in subject premium of 10% a year. He then goes on to demonstrate later in the paper the effect of changing retentions at the same rate of 10% a year. He shows that the relationship of the excess losses above the changing retention to the subject premium remains a constant percentage.

To satisfy my curiosity, I developed a similar table with the assumption that the selected index rises at the given 10% per year, but actual losses increase at the rate of 20% a year, or two times the total rate of inflation. This is not out of line with the leverage mentioned above. The results of this calculation and excerpts from Table III of the paper are summarized below.

Number of Losses	1974 Initial Gross Losses	1974 Accidents Settled at 1978 Value 10% Annual Inflation	20% Annual Inflation
10	$ 30,000	$ 43,923	$ 62,208
5	40,000	58,564	82,944
3	50,000	73,205	103,680
2	60,000	87,846	124,416
1	80,000	117,128	165,888
1	100,000	146,410	207,360
Losses in Excess of $73,205*		146,410	469,380

* $\$50,000 \times 1.10^4 = \$73,205$

What is seen from this exercise is that doubling the rate of inflation on the excess layer of coverage has the effect of more than tripling the costs when the index is tied to the *overall* inflation rate. This is only in the first exposure year!

The rest of the paper deals with some of the mechanics and operations of the index clause, discussion of variations, and in the appendix he shows development of the rate discount for the implementation of such a clause.

Special comment was also directed to the impact of loss reserves. It is essential at this point, to underline his concern over the ramifications on the reserving practices of both ceding and assuming carriers. Since loss reserving is involved with establishing estimates of amounts needed in future transactions, a certain amount of anticipation of future inflation is essential at this point. As has been demonstrated in the past several years, nobody has been able to precisely accomplish that objective. I feel, without firm conviction, that indexing may have a disturbing influence on the loss reserving exercise since underlying history is no longer representative of the present and future. However, is this really different than the shifts in underlying data that we now presently encounter?

Reference was made in the paper to the utilization in other countries of the stabilization clause or index clause as it is called more commonly in the United States. Although there appears to be evidence of success, there still appears reluctance for universal acceptance. It has been observed that the index clause has not become a "standard" clause in most international reinsurance contracts. One of the biggest problems as mentioned in Mr. Ferguson's paper is the problem of multiple claimants or multiple payments over a long period of time. For example, if a claim is paid over several years, the payments must be divided by the indices applicable at the time of payment. The proportion of these adjusted payments in excess of the original retention is applied to the actual total claim payment to determine the amount for which the reinsurer is liable. This problem is made more complex in annuity payments over a long period of time. The European countries have experienced inflation of a more severe degree from a cost standpoint, and are now currently encountering the social inflation or "super-imposed inflation" especially in more current times. We are all concerned with the trend of courts to award substantial damages for other than economic costs. The ballooning of jury awards and settlements well in excess of economic costs have prompted many to take a hard look at the present tort system. I believe this has great impact on excess reinsurance, since one can mentally allocate most of the economic losses to the retention, and proportionately more of the general damages or non-economic losses to the excess portion. This area is highly volatile and is not presently capable of accurate indexation.

Since the underlying theory of index clauses is fairly simple, that is the equitable distribution of the impact of inflation on both the cedent and the assuming reinsurer, I wonder why this concept has not achieved greater acceptance in the United States' market. Perhaps the answer is the natural resistance to changing methods or perhaps the answer lies in the problem itself. What I mean by this is that the uncertainty of inflation may be, in itself, a retardent.

Although ultimately losses are recognized in the rating procedure, there are no other known methods widely employed that achieve the objective of the index clause. One alternative that may have already been tried is to offer a combination of an index clause and a retrospective rating device. As was previously demonstrated, if the rate of inflation affecting the excess losses is more severe than that overall, even the index clause will not achieve the equitable distribution of the impact of inflation. Retrospective rating will help return to the reinsurer some of the additional losses experienced as a result of the leveraged inflation.

Mr. Ferguson has provided the Society a vehicle to further examine this issue and has challenged us all to find a better way to deal with the problem of sharing the impact of inflation in non-proportional reinsurance contracts.

DISCUSSION BY MATTHEW RODERMUND

Mr. Ferguson has done such a thorough job of discussing the effects of inflation on the insurance business, and explaining all the ramifications of the index clause, which is one of the solutions to the inflation problem, that there is little for a reviewer to comment on. Nevertheless, there are practical aspects of the problem that interest this reviewer and that may be useful as a supplement to Mr. Ferguson's paper.

As readers of the paper know, the index clause is a device to distribute between reinsurer and reinsured, on a reasonably equitable basis, the effects of inflation on excess insurance losses. The device consists mainly of applying an index factor to the primary company's retention, so that the company's share of a large loss increases with inflation. Mr. Ferguson has provided a comprehensive treatment of the application of such an index factor.

The index clause, so far, has had relatively little impact in the United States, although American reinsurers have been well aware of it. In Western Europe, however, including the British Isles, the index clause has been used extensively since the mid-sixties. In fact, on the continent it is difficult for a primary insurer to get an excess of loss contract without the index clause.

Why should this difference exist between the U.S. and Europe? For one thing, Europe's problem arrived sooner. In the 1960's, when the U.S. was complacent with an inflation rate of 3% to 6%, England's and Germany's inflation rate was running between 8% and 15%. Thus, the need for the index clause was being felt acutely by European reinsurers.

But even now, when inflation in the U.S. has become painful, American reinsurers have had difficulty peddling the index clause. One of the reasons is that there is greater competition among reinsurers in the U.S. than in Europe. In Europe there is a growing consensus among reinsurers that attaching an index clause is the thing to do. There is no such consensus in this country. American reinsurers have on their books only a relative handful of contracts with index clauses.

The facts of life in the U.S. are that a primary insurer generally will not accept an index clause if he can find a reinsurer who won't insist on it. And he always can.

Why the resistance? Mostly, companies don't like to increase their retentions except of their own free will. Since 1970 this reviewer has been talking about the index clause to groups of insurers around the country. Not infrequently an underwriting executive will comment privately that if he broached this idea to his president and insisted on it, he'd probably get fired.

It can be demonstrated that the final cost of excess loss protection with an index clause is less than the cost without one. But the demonstration assumes the insurer realizes that eventually, and in the long run, he will have to pay for his own excess losses at least at the working level, where the index clause is generally applied. Without an index clause he will pay such losses plus the reinsurer's loading. With an index clause he will retain more losses, but on those he will save the loading.

For example, using the losses suggested by Mr. Ferguson in Tables IV and V of his paper, the rate developed in Table IV, with no index, averaged 4.17% for the three years shown. Loaded by 25% for expenses and contingencies, this rate becomes 5.21%. If it were quoted for 1978, when the expected subject premium would be $14,600,000, the reinsurance premium would be $761,000.

On the other hand, if the 1.46% rate in Table V, using the index, were loaded by 25%, it would become 1.83% and the premium would be only $267,000. But with the index clause the ceding company would retain additional losses which, based on the experience from 1974 to 1976, represent an unloaded rate of 2.71%. This, added to the 1.83% reinsurance rate, produces a total excess loss cost of 4.54%, or 0.67% less than the rate of 5.21% with no index. The saving obviously is 25% (the loading) of the 2.71% rate represented by the additional losses expected to be retained after the index clause is employed. The cost saving is about $98,000.

But all of the foregoing presupposes that the ceding company can't get reinsurance without an index clause for less than 5.21%, or even less than 4.54%. In the real U.S. world he probably can do better than that. There are any number of reinsurance markets which, for the sake of landing a contract, will refuse to concede that losses will develop as badly as current and predicted inflation rates suggest they will.

Even if excess losses do develop as predicted, the primary insurer may be hoping he won't have to pay them back. Maybe he can move from one reinsurer to another fast enough to avoid it. Moreover, with an index clause the increase in retained losses is immediate and certain, whereas without it the pay-back to the reinsurer, plus the loading, might be somewhere in the future.

The reduction in reinsurance premium due to the index clause seems not to be an attraction.

The pity is that loss projections based on the current inflation, and assumptions of the duration of the settlement period, are very likely even less pessimistic than they ought to be, so that rates both with and without the index clause turn out to be too low, and the divergence between them should be greater than is indicated by present trended data. Nevertheless, primary insurers and many reinsurers alike tend to be wishful thinkers.

Thus, in this country at this moment, Mr. Ferguson's exposition is an admirable description of a vital reinsurance device whose day, unfortunately, has not yet come.

I'm afraid that Mr. Ferguson's paper turns into an actuarial exercise he gets into pricing, and into the calculation of the discount from the no-index price to the with-index price. The same may be said for the cogent comments by Mr. Charles F. Cook in his review of Mr. Ferguson's paper.

Mr. Ferguson's and Mr. Cook's algebra, and their logic, are impeccable. But it's hard to imagine any reinsurance underwriter, or actuary, using this algebra in connection with an actual reinsurance quotation. Mr. Ferguson's discount formula is developed in his Appendix II. He sets up an algebraic expression for the price of a contract with an index, and the price of a contract without index, and subtracts the quotient of these from unity. However, the price of the contract with index is tied to the "average excess loss trended and indexed" ($\overline{X}$), and this in turn depends on both the average number of years (t) from occurrence to settlement, and the average number of years (u) from occurrence to mid-point of the new exposure period. The price of the contract without index also depends on t and u. Mr. Cook's improvements on these formulas use the same terms.

This reviewer submits that in a book of excess losses covering three to five accidents years, the size of losses, their frequency, and their settlement periods normally have such great variance that no reinsurance underwriter would ever trust the assumed averages ($\overline{X}$, t, and u) sufficiently to employ them in an actual quotation.

This is not to say that the reinsurance underwriter, using an empirical approach, won't make other equally vulnerable assumptions. He will. (Mr. Ferguson makes this point.) Using the same book of losses, which have little credibility, he will assume that the loss development picture of the past will be repeated in the future a dubious proposition. But typical. Loss rating in the reinsurance business is generally un-actuarial. (For example, one excess loss of $250,000 is given the same rating value as five excess losses of $50,000 each.) A more refined actuarial procedure tends to produce a higher reinsurance rate for good experience than the underwriter's methods will, and a lower rate for bad experience. The customer won't like the former, and the reinsurance underwriter or his president won't like the latter. The point is, in the real world the underwriter is comfortable with an empirical approach, and probably will tolerate Mr. Ferguson's and Mr. Cook's formulas only as material for an actuarial paper.

The foregoing observations notwithstanding, Mr. Ferguson's paper is a valuable one. The Proceedings needs it.

DISCUSSION BY C. K. KHURY

Mr. Ferguson's paper is certainly timely as inflation and its effects have assumed a new prominence in our midst.

It has long been recognized, in literature as well as in practice, that proper accounting for inflationary trends is a necessity in maintaining the actuarial balance of the primary insurer's rate levels. This has also been recognized by the excess writer. Fixed retentions, however, have magnified the effects of inflation on the excess writer. This paper graphically demonstrates the magnification process.

It is of particular note that the problem of the excess writer as respecting fixed retentions is parallel to the primary insurer's problem with deductibles. Both situations translate a given inflation rate into a compound inflation rate on the respective aggregate pure premiums. Even though this problem has existed as long as inflation has, it is now of critical concern in view of the current magnitude of inflation rates. The proposed solution in terms of an indexed retention further suggests that the excess writer has heretofore lived with fixed retentions only through ever increasing [excess] insurance rates. Apparently, the rapidly increasing rates of underlying inflation will produce increases in excess rates of such magnitude that some new alternatives have to be sought. Mr. Ferguson has communicated and demonstrated the stabilizing effect which an indexed retention can produce. This reviewer endorses the concept and the manner in which it is applied. The remainder of this discussion addresses one critical technical aspect of the application of the indexed retention principle.

It would be helpful at first to delineate the ways in which the excess writer is exposed to the ravages of inflation vis-a-vis the primary writer:

- Let X denote a size of loss variable
- Let R denote a fixed retention
- Let i denote a rate of inflation
- Let the losses incurred during a given year of experience be distributed as follows:

Loss Interval	Loss Interval Number
$0 < X \leq R/(1+i)$	I
$R/(1+i) < X \leq R$	II
$R < X$	III

Then the passage of one year's time will generate the following effects on each loss of the various intervals:

Current	**Increase in Each Incurred Loss of**	
Interval	**Primary Insurer**	**Excess Writer**
I	$i(X)$	0
II	$(R - X)$	$(1+i)X - R$
III	0	$i(X)$

By way of added emphasis it should be noted that, under a fixed retention arrangement, losses currently falling in interval II will produce increased frequency for the excess writer, while losses currently falling in interval III will produce greater severity for the excess writer.

The reason for going to these lengths in delineating the nature of the problem is to demonstrate the need to base an indexed retention proposal on the underlying size-of-loss distribution. This would assure an equitable treatment for the primary insurer as well as the excess writer. This is especially true when the [originally] fixed retention is near a cluster point of the underlying size-of-loss distribution. While the percentage impact on losses in excess of R is directly measurable, the frequency impact on the excess writer (and therefore on the primary insurer's excess rate) is ascertainable only in terms of the underlying size-of-loss distribution. This works both ways, and I feel that the point should be carefully noted in understanding the application of indexed retentions. Mr. Ferguson's paper recognized the frequency impact by introducing Δ in Appendix II.

I hope that this paper will spark a parallel treatment in these Proceedings of the corresponding deductible problem. In these days of rampant inflation I am not sure that the day of the indexed deductible if very far away. In the meantime we should be grateful to Mr. Ferguson for a valuable addition to the reinsurance section of the Proceedings.

"REVISING CLASSIFICATION STRUCTURE USING SURVEY DATA"

DAVID SKURNICK, N. ROBERT HEYER AND S. RAY FUNKHOUSER

DISCUSSION BY CHARLES GRUBER

New York State has three crop-raising farm classifications: fruit farms; vegetable and berry farms; farms, not otherwise classified. Before 1965, the basis for assigning a farm to either the fruit or vegetable farm classification was the acreage used for different types of crops. If more than 50% of the farm's acreage was used for fruit or vegetable production, the farm was assigned to the fruit or vegetable classification. After a study in 1965, the New York Compensation Insurance Rating Board felt that the 50% acreage requirement did not properly allocate the farm compensation hazard and changed it to an income requirement: annual income from the sales of fruit or vegetables must constitute more than 50% of the total farm income.

In his 1965 report to the New York State Conference Board of Farm Organizations, Robert S. Smith from the New York State College of Agriculture at Cornell University listed the two major assumptions which underlie the income requirement of the Board's classification structure:

1. The frequency of occurrence of work associated injuries on farm enterprises is directly related to the degree of mechanization of the enterprise, and varies significantly between enterprises or types of farming.
2. Classification by type of farming effectively divides farms by degree of mechanization and therefore by frequency of work associated injuries which can be expected.

A 1974 National Council on Compensation Insurance farm study also stressed the importance of mechanization in determining farm classifications. The National Council created new farm statistical classifications to develop experience.

Both the New York Board and the National Council farm classification studies relied on staff field trips, special farm reports and data developed by state and farm organizations. Messrs. Skurnick, Heyer and Funkhouser have presented an alternate and viable mail survey approach for study and revision of farm classifications structures.

The farm survey results, however, are only applicable to California farms. It would have been most informative if the survey had also asked for cause of loss in order to determine how loss is affected by a farm's degree of mechanization. The survey might also have tried to develop an index of mechanization for each crop; e.g., amount of payroll attributable to machine operations. This crop mechanization index, used as a loss relativity indicator between crops, might then have been useful for classifying farms in other states since farm operations for a particular crop are similar countrywide.

Surveys have previously been used in classification studies, most notably the one utilized by the National Bureau of Casualty Underwriters and the National Automobile Underwriters Association in producing the 1964 Private Passenger Automobile 260 Classification Plan. This survey sampled approximately300,000 automobile risks written by seventeen company groups.

DISCUSSION BY PAUL E. HOUGH

This paper, describing the steps leading to a revised classification scheme for workers' compensation farm risks in California, represents an important contribution to the California farm industry. However, in a more general sense the work is significant as an illustration of how several disciplines can come together to create an improved product. The efforts of the government, the farm industry and the insurance industry, as well as survey specialists, have been merged to create a result greater than each participant's separate contribution.

I was pleased to note the use of talent external to the insurance industry in the development of a new insurance rating structure. Too often, it seems we in the insurance business take the narrow view that only our industry or our company should perform the necessary data gathering and analysis when in fact that may be the most inefficient approach to take. How many times have we burdened our statistical plans with added information requests when that was the most cumbersome and disruptive way we could have satisfied our data needs. In this case, a mail sampling of risk information provided the necessary additional data for a change in an existing rating system.

I was surprised to note that none of the reasons cited for conducting the revision in farming classifications the movement to larger farms, new methods of farming, and new farm machinery were directly reflected in any of the new classifications. This is partially because, and as the paper states, the designers of the study expected that crops would remain as the basis for the revised farm classes. Also there were credibility and sample response concerns that required a limitation on the extent of research into additional classification criteria. With our mandatory workers' compensation experience rating plan for larger risks we can hopefully count on it to respond to those criteria we might have lost by necessarily limiting the study's scope.

I could not help but wonder if we were not looking at an approach that is only feasible in workers' compensation with its centralized and individual risk files of experience. Certainly the general methodology is applicable to other compensation states whose classifications are of sufficient size to justify the expense of this kind of an undertaking and hopefully the paper will spur this kind of activity. It would seem that for other lines, where we must link up individual risk experience with risk characteristics not recorded, we must look to the individual companies to pool their results for the good of an improved industry classification plan. I believe this approach will work in a few remaining lines but with the shift to independence in private passenger automobile, homeowners, and commercial package insurance, workers' compensation remains as one of the few lines of sufficient volume where the use of survey data in conjunction with insurance statistics can be a viable method for the development of the new classification plans.

There are a few concerns that I have noted that taken together would not have changed the resultant California farm classes and their pure premiums with the extensive averaging and judgment involved in their selection. However, they are worthy of comment.

1. The conclusion is drawn that "there is no serious bias in the insurance characteristics of the sample of responding farms, beyond the inherent bias that results from the disproportionate stratified sampling plan" and it is supported by the fact that the difference in pure premiums between the responding and non-responding farms is a mere 3%.

   It is apparent that a greater difference would have been evident had some correction been made for the fact that a higher questionnaire return rate was generated through a telephone follow-up on three of the five farm classes under revision. Their combined average pure premium was nearly double that of the classes where no telephone contact was made and thus the responding farms' pure premiums are correspondingly higher. Had some correction been made for this artificial high frequency of response in the high pure premium classes more of a response bias would have been indicated.

2. The authors note that the disproportionate sampling plan did not create a true cross-section of California farms and though one could have been statistically constructed it was "not essential for comparing the relative hazard among classifications".

   There is an unwarranted assumption that I believe we tend to make in the audited lines, and it is that pure premiums tend to be independent of exposure size. It is observed in the workers' compensation line that results on a standard basis for the smaller risks are relatively poor and although it may be that the non-application of the experience rating plan to these small risks may contribute to this fact, it seems that in the main what we are seeing is a real difference in the risk. Just because one risk has one-tenth the payroll of another within the same classification, it is not necessarily a small scale replica of the large one. My point here is that by not restructuring a true cross-section with each class, we may very well be adding further bias to the study.

3. No mention is made of whether the separate policy year losses of the various hypothetical classification systems were put on a common benefit level. If this were not done, the distortion would only exist in those classes whose exposure is either increasing or decreasing much more rapidly than all the farm classes combined.

In conclusion, I would like to thank the authors for taking time to write up the step-by-step approach taken to respond to the concerns of a growing industry. It is a reminder to all of us to be cognizant of the dynamic society within which the insurance industry plays its part and have us in turn respond with new and imaginative approaches to society's ever-changing needs.

THE CALIFORNIA TABLE L

DAVID SKURNICK

DISCUSSION BY FRANK HARWAYNE

This is both a review of and an alternative to the program described by Mr. Skurnick in *The California Table L* as a generalization of Table M. Table M focuses attention upon risks of a given expected loss size. The aggregate losses of each risk are ordered with the risks producing the least amount of such losses appearing first, the next lowest amount appearing second, etc. From this order, Table M charges or excess pure premium ratios are developed. Simon's[1] methodology generates a family of curves of Table M Values according to expected loss size.

Skurnick's paper carries Simon's program a step further by introducing the accident limitation into the system of excess pure premium ratios. The mathematics are impressive to the point of rivalling some college textbooks. Dropkin's[2] statement on Simon, "It is not to be read casually, commuting to and from work", applies equally here. The theorems and lemmas have been developed and abstracted for general application. Wrestling with them should give the theoretical mathematician or sophisticated actuary some sense of satisfaction. The formulae are sound and useful in developing Table L which sets forth the excess pure premium ratios when claims arising from a single accident are limited for specific amounts.

Application of Skurnick's theorems and lemmas to produce Table L poses a dilemma. If one requires a separate Table L for every accident limit (or excess loss premium factor of which there are 36 in Rhode Island) in each of fifty-two states, one might need as many as 1800 Table L's. Considering that Table M requires 111 printed pages, we could expect to be printing 200,000 pages of Table L, and the more we print, the more difficult is the annual rate approval process required by rate regulation.

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[1] L. J. Simon, "The 1965 Table M," PCAS, LII (1965), p. 1

[2] L. B. Dropkin, "Discussion of 'The 1965 Table M,' L. J. Simon," PCAS, LII (1965) p. 46

In an effort to stem this ruinous tide of paper, I have tried to reevaluate Skurnick's methodology from the practical side. What essentially is Table L? It is Table M on which has been engrafted the charge required for limiting accidents to a specified amount. The difficulty encountered in attempting to combine Table M with excess loss premium factor charges is that Table M is developed from losses on a risk basis and the other is developed from losses on a per accident basis. This means that at certain entry values Table M (which may already contain charges for individual losses in excess of the accident limitations) needs to be coupled with elements that are not normally compiled on a risk basis. It is clear that at entry ratios corresponding to aggregate losses for risks which produce less than the amount equal to the accident limit, Table M contains no overlap problem. It should also be apparent that at the extremely high entry ratios there will be some risks whose losses will consist solely of accidents where claims exceed the accident limitation. In between, there will be some overlap between Table M charges and excess loss premium factors.

If we define the following terms,

r^M_T = attachment point value such that $r^M_i = \dfrac{\text{Accident imit}}{E^M_i}$

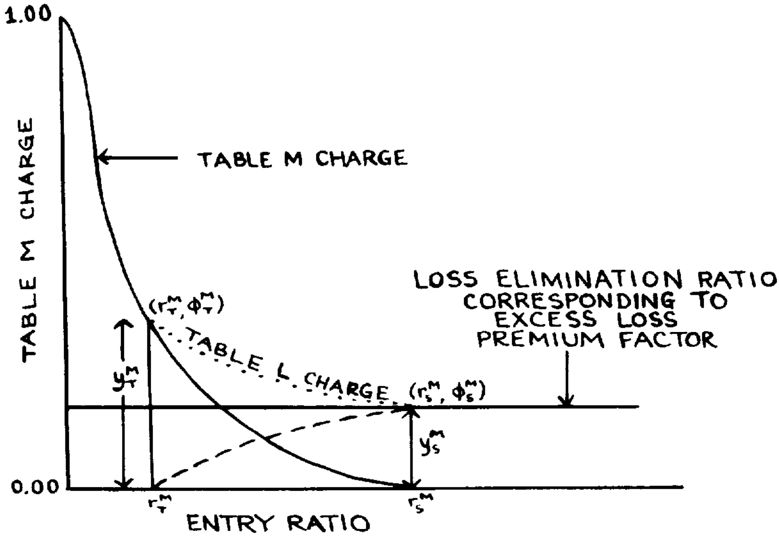
E^M_i = risk expected losses

$Y^M_T = \phi^M_T$ = Table M charge at point r^M_t

r^M_S = asymptotic point at which $\phi^M_i + \Delta\phi(r) = \Delta\phi(r)$

$Y^M_S = \phi^M_S$ Table M Charge at point r^M_S for which the Table L charge is approximately equal to the loss elimination ratio corresponding to the excess loss premium factor.

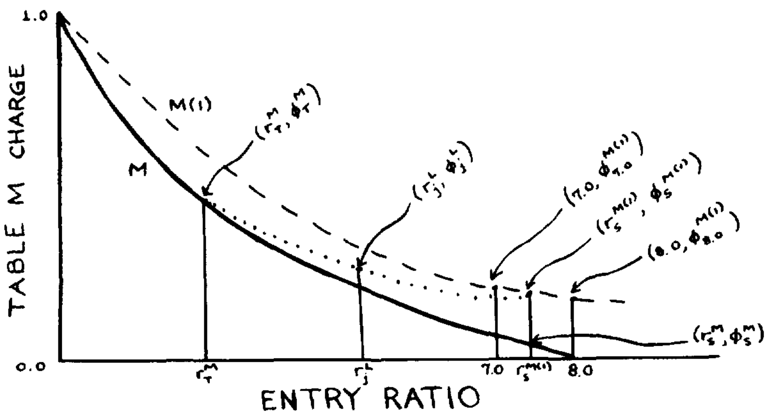
The elements can be graphed as follows:



The broken line represents the loss elimination ratio corresponding to the modified excess loss premium factor (discounted for the overlap with Table M charges). The dotted line connecting the y-values ϕ_T^M and ϕ_S^M represents the net sum of Table M charges and the modified excess loss premium factor. It is the curve of Table L developed by Skurnick. In the form stated here, the degree of overlap of ϕ_i^M and the excess loss premium factor range $r_T^M \leq r_i^M \leq r_S^M$ is not readily expressible as a simple function, or else would entail extensive computation.

The problem may also be looked upon as one of assigning a probability value to the overlap implicit in the Table M values between r_T^M and r_S^M. From this viewpoint, one then asks the question how much should the excess loss premium factor be discounted? Noting that the discount is 100% at point r_T^M and 0% at point r_S^M, and since we are dealing with a continuous function (or nearly so) it is logical that the charge (complement of the discount) be graded in proportion to $r_S^M - r_T^M$. Moreover, in the matter of curve fitting, if we can find another expected loss size M(1) in the same family of curves such that the rate of change at $\phi_T^{M(1)}$ is close to the rate of change at ϕ_T^M and also has the value ϕ_S^M at $r_S^{M(1)}$ with zero or a very small rate of change (i.e. is almost asymptotic), then we will have found two Table M's such that $\phi_j^M \leq \phi_j^L \leq \phi_j^{M(1)}$ for $r_T^M \leq r_j^M \leq r_S^M$.

Graphically, the interpolation and curve fitting can be shown to be as follows:



M is a curve of Table M excess pure premium ratios at the given expected loss size.

M (1) is a curve of Table M excess pure premium ratios that $\phi_{7.0}^{M(1)} \leqq$ excess loss premium factor $\leqq \phi_{8.0}^{M(1)}$.

ϕ_j^L can be estimated to be approximately $\phi_j^M + \Delta\phi(rj)$

$$\text{where } \triangle\phi(rj) = \left[\phi_j^{M(1)} - \phi_j^{M}\right] \times \frac{r_j^M - r_T^M}{r_S^M - r_T^M}$$

$\triangle\phi(rj)$ is the modified excess loss premium factor because $M(1)$ was selected so that $\phi_S^{M(1)} - \phi_S^M$ equals the excess loss premium factor.

It can also be shown that:

$\triangle\phi(rj) + \phi_j^M \geq \phi_S^M$. This means that the adjusted excess loss premium factor plus the table M charge is never less than the undiscounted excess loss premium factor.

The determination of r_S^M for various expected loss size gorups (Table numbers of Table M) forms a new Table of Attenuation Points (see Exhibit I). It is noted that the differences in charges for entry ratios of 7.00 and 8.00 are sufficiently small to meet our requirements regarding asymptotic values. At Table 90 the difference is .065 (less than .00065 for a change of .01 in entry ratio) and at Table 40 the difference is .003 (less than .00003 for a change of .01 in entry ratio). Values for entry ratios less than 7.00 and 8.00 could be selected if less stringent criteria were used. The difference in charge between entry ratio 4.0 and 5.0 is .070 for Table 90 and .025 for Table 40.

Procedurally, the computation of the modified excess loss premium factor is very simple. Entry ratios for the minimum (only if larger than r_T^M), maximum and r_T^M are required. A self-explanatory worksheet (Exhibit II) sets forth the procedure for derivation of the modified loss elimination ratio corresponding to modified excess loss premium factor.

It is possible to construct an equivalent to Table L by adding the modified excess loss premium factor described above to the Table M charge. This was done for Mr. Skurnick's Exhibit 5 for a 25,000 limit and for entry ratios that were above the attachment point ϕ_T^M. Exhibit I was developed from Countrywide Table M charges and these charges for appropriate expected values were used (California Table M would have given somewhat different results). The results are as follows:

		Table M		Table L $25,000 Acc. Limit		
Standard Premium	**Entry Ratio**	**Country-wide**	**Calif.**	**Country-wide***	**Calif.**	**Adjusted#**
$35,000	1.37	.235	.297	.248	.292	.310
50,000	1.32	.215	.268	.243	.268	.296
67,500	1.27	.207	.234	.244	.243	.271
80,000	1.25	.201	.211	.242	.233	.252
254,948	1.08	.157	.187	.214	.219	.244

It should be noted that the differences between countrywide and California Table L values arise from and are smaller than the differences in Table M. It will be seen from the column of Table L, $25,000 Accident Limit, Countrywide Adjusted that the charges for accident limitation are higher than by Skurnick's method and are therefore more conservative.

The alternative suggested here is in no way intended to diminish the logic and insights of Mr. Skurnick's paper. Indeed, his valuable contribution in this area has been the spur for solving a thorny problem in a practical way. Undoubtedly more work in developing refined solutions is to be welcomed.

* Table M plus increment of .1244 developed from application of principles using Exhibits I and II.

Modified excess loss premium factor using countrywide Table M for discounting ELPF's and adding California Table M. charge.

EXHIBIT I

NATIONAL COUNCIL ON COMPENSATION INSURANCE

TABLE OF ATTENUATION POINTS FOR COMPUTATION OF ADJUSTED ELPF'S FROM TABLE M CHARGES

Table	Charge at Entry Ratio		Difference	Table	Charge at Entry Ratio		Difference	Table	Charge at Entry Ratio		Difference
No.	7.00	8.00		No.	7.00	8.00		No.	7.00	8.00	
				74	.255	.211	.044	54	.047	.030	.017
93	.602	.542	.060	73	.246	.202	.044	53	.043	.027	.016
92	.563	.501	.062	72	.237	.194	.043	52	.038	.024	.014
91	.525	.461	.064	71	.229	.187	.042	51	.034	.021	.013
90	.488	.423	.065	70	.220	.178	.042	50	.030	.018	.012
89	.452	.391	.061	69	.211	.170	.041	49	.027	.016	.011
88	.421	.364	.057	68	.202	.162	.040	48	.023	.013	.010
87	.396	.341	.055	67	.193	.154	.039	47	.020	.011	.009
86	.375	.322	.053	66	.184	.145	.039	46	.017	.010	.007
85	.358	.306	.052	65	.175	.137	.038	45	.015	.008	.007
84	.349	.298	.051	64	.165	.118	.047	44	.013	.006	.007
83	.339	.288	.051	63	.142	.100	.042	43	.011	.005	.006
82	.329	.279	.050	62	.121	.085	.036	42	.009	.004	.005
81	.320	.271	.049	61	.103	.072	.031	41	.007	.004	.003
80	.311	.262	.049	60	.088	.061	.027	40	.006	.003	.003
79	.301	.253	.048	59	.074	.050	.024				
78	.292	.245	.047	58	.068	.046	.022				
77	.283	.236	.047	57	.063	.042	.021				
76	.274	.228	.046	56	.057	.038	.019				
75	.264	.219	.045	55	.052	.034	.018				

EXHIBIT II

NATIONAL COUNCIL ON COMPENSATION INSURANCE

Worksheet for Computing Modified
Excess Loss Premium Factor Charges

(1) Loss Elimination Ratio Corresponding to Excess Loss Premium Factor: ____________

(2) Highest Table # where charge at entry ratio of 8.0 is less than (1). Table # ____________

(3) For (2),. Entry Ratio 7.00 Charge ________(a)
8.00 ________(b)
Difference ________(c)

(d) $(r\frac{M}{S}, \phi\frac{M}{S})$ Attenuation Point $= 7.00 + \frac{(3a)-(1)}{(3c)} = 7.00$
$+$ ______ $-$ ______ $=$ ______

(4) Expected Losses (a) ________ and (b) Table # ______

(5) Accident Limit ________

(6) $(r\frac{M}{T}, \phi\frac{M}{T})$ Attachment Point $= (5) \div (4a)$
$=$ ______ $\div$ ______ $=$ ______

(7) Minimum Ratio ______ (use only if larger than (6)).

(8) Maximum Ratio ______

(9) For (7), values of savings for Table #'s (2)–(4),
______ $-$ ______ $=$ ______*

(10) For (8), values of charges for Table #'s (2)–(4),
______ $-$ ______ $=$ ______

(11) [(7)–(6)] ÷ [(3d)–(6)], [______ – ______] ÷ [______ – ______]
$=$ ______

(12) [(8)–(6)] ÷ [(3d)–(6)], [______ – ______] ÷ [______ – ______]
$=$ ______

(13) Incremental Charge for Max. $= (10) \times (12)$
$=$ ____ $\times$ ____ $=$ ____

(14) Incremental Saving for Min. $= (9) \times (11)$
$=$ ____ $\times$ ____ $=$ ____*

(15) Modified Loss Elimination Ratio Corresponding to Excess Loss Premium Factor $= (13)-(14) =$ ____ $-$ ____ $=$ ____

* Enter zero if (7) is smaller than (6).

DISCUSSION BY RICHARD H. SNADER

Mr. Skurnick's thoughtful and thought provoking paper is destined to become required reading for actuarial students and practicing actuaries alike. Growing out of the need to solve a specific, local problem, this fine article accomplishes much more. Clear and compact, it can serve by example as a miniature manual of style for those of us who may feel inclined, in the future, to submit our own ideas for publication.

Apart from the introduction and conclusion, the paper is divided into four distinct sections, each of which serves a specific purpose. In the first two sections we are treated to an elaboration of the purely mathematical qualities of Table L. Section 1, in fact, provides us with an excellent review of the properties of Table M. By making the simple adjustment of mentally dispensing with the asterisks and k's, we have at our disposal a concise and truly rigorous mathematical development of the Table M concept. By the simple readjustment of mentally replacing the asterisks and k's, the transition to Table L is easily made.

In the third section theory is applied, and the continuous form is neatly converted to the discrete situation. This section is a boon to anyone who has ever been perplexed by that mysterious entity known as "Sum 2."

In the fourth section, the numerical properties of Table L are discussed, and the thorough reader is compelled to acquire his own copy of the complete table to supplement the discussion.

It is somewhat disturbing to note that the Table L charge is so close to the corresponding Table M charge over much of the table. It is also mildly disturbing to note the many instances where the Table M charge actually exceeds the Table L charge. There are also numerous instances where the Table L charge for a particular limitation at a given entry ratio exceeds the Table L charge for the next lower limitation.

The author is well aware of these anomalies and discusses them at some length. He correctly reasons that the column of Table M charges is less accurate than the corresponding Table L charges because the Table M data are distorted by large losses. In fact, as the accident limitation increases, each successive column of Table L charges becomes more vulnerable to the distorting effect of large losses. These inconsistencies can be eliminated, suggests the author, by allowing the charges for each premium size group to be developed from the loss elimination ratio pertaining to that group. This procedure will most probably succeed, but the propriety of the measure is somewhat questionable in view of the extreme magnitude of the fluctuations in the k values between premium size groups.

It is possible, of course, that the inconsistencies result from the size of the sample. Perhaps if more data were available, the fluctuations in loss elimination ratios would be less pronounced. In that event, the inconsistencies would tend to eliminate themselves. Perhaps a definite trend in the loss elimination ratios exists but is masked by sparse data. If more data were available, such a trend would become apparent. In that event, it would be necessary for each premium size group to reflect its own loss elimination ratio in the manner suggested by the author.

An alternate approach might also be considered. The problem might be solved by simply requiring that $\phi^* (r_i^L) \geqq \phi (r_i^M)$ within each premium size group. Or, if we may allow the superscript L to become specific by substituting a number for a particular loss limitation (for example, let $\phi^* (r_i^{25})$ denote the Table L charge for the \$25,000 limitation), we can require that $\phi^* (r_i^{25}) \geqq \phi^* (r_i^{30}) \geqq \ldots\ldots\ldots \geqq \phi^* (r_i^{100}) \geqq \phi (r_i^M)$.

The enforcement of this constraint must be embodied in an appropriate graduation procedure. The problem is one of obtaining a smooth surface of Table L charges consistent with the array of tabulated values. The problem is quite similar to the one faced by our life insurance counterparts when graduating a select and ultimate mortality table.

I have chosen to dwell on this particular aspect of the paper because I feel it is an important one with respect to the possible extension of Table L to other states. It is clear that these inconsistencies must be dealt with before the Table L concept can gain acceptance elsewhere.

The consistency problem is by no means the major impediment to universal acceptance. A much more formidable obstacle must be faced in the form of the logistical problem connected with providing a Table L for each state. The number of Table L pages that a home office would be required to maintain would be monstrous if we continued to recognize each state's loss elimination ratio. Perhaps the logistical problem can be minimized by reducing the number of possible loss limitations to a minimum and by grouping states with similar loss distributions by size. Perhaps a formula approach to calculating the incremental charge, which recognizes that the increment must vary with the entry ratio, can be devised. And perhaps this problem is trivial in terms of electronic storage.

It is hoped that the obstacles confining Table L to California can be overcome. It is hard to disagree with the author's contention that from a mathematical point of view, Table L represents an advance over Table M.

AUTHOR'S REVIEW OF DISCUSSIONS

The two reviews suggest alternative approaches to three problems, the incompatibility of California Tables L and M for certain entry values, the multitude of Table L's required for countrywide use, and the difficulty of measuring the incremental charge. Mr. Snader suggests a pragmatic method of graduation to produce a consistent set of tables while maintaining the assumption that the loss elimination ratio is independent of premium size. Mr. Harwayne develops a simple method of estimating the incremental charge for Table M.

This reply includes a previously unpublished method of computing the incremental charge from a risk distribution of losses. The reviews were the stimulus for some further mathematical work, which is also included.

THE "RUINOUS TIDE OF PAPER"

A set of Table L's varying by 52 states, 300 entry ratios, 64 risk sizes, 7 per accident limits, and 4 hazard groups would have 28 million entries filling a hundred thousand pages. To stem this tide, average values are used in place of some of the variables. The California Table L has only 11 size groups and is not subdivided by hazard group. The result is a practical, 66 page table.

The Table L charge $\phi^*(r)$ is the sum of the Table M charge $\phi(r)$ and the incremental charge $\triangle\phi(r)$. Let the per accident charge index $Y(r)$ be the ratio of the incremental charge to the loss elimination ratio. Then

$$\phi^*(r) = \phi(r) + \triangle\phi(r) = \phi(r) + Y(r) \cdot k. \tag{1}$$

A Table L charge is the expected proportion of loss dollars eliminated by excluding the portion of each loss above the accident limit and then excluding the portion of the loss ratio above the entry ratio. These two limiting operations overlap because some loss dollars in excess of the accident limit would have been excluded by the loss ratio limit alone. The greater the overlap, the smaller the per accident charge index.

The amount of overlap depends on the expected loss, the entry ratio, and the accident limit. Harwayne defines the attachment point $r\frac{M}{T}$ by

$$r\frac{M}{T} = (\text{Accident Limit}) \div (\text{Expected Loss}) \tag{2}$$

If the entry ratio is below the attachment point, then for any loss the portion exceeding the accident limit also exceeds the loss ratio limit; the limits entirely overlap, so the per accident charge index is zero. As the entry ratio approaches infinity the overlap disappears and the per accident charge index approaches unity.

The National Council's retrospective rating values come from Table M and from tables of excess loss premium factors. In order to reduce the size of the tables, they averaged over certain variables. Their excess loss premium factors are calculated as if the per accident charge index were always unity; they vary only by state, per accident limit, and hazard group. Their Table M varies only by entry ratio and risk size.

Both reviews point out that it is not feasible to go to a Table L approach and maintain full variation by state and hazard group. A compromise worth considering would be the use of countrywide Table L's varying by hazard group and a choice of about four per accident limits and 20 size groups. This procedure would require the production of 16 separate Table L's, but each would be smaller than the current Table M, and only one would be used to rate a risk. Graduation by size would be easier with fewer size groups. Since the entire Table L charge would vary by hazard group, state, entry ratio, and risk size, this method would be more accurate than the current one. I believe that this increase in accuracy would outweigh the decrease in accuracy from reducing the number of size groups and no longer varying excess loss premium factors by state, particularly because state laws are becoming more uniform as the recommendations of the National Commission on State Workers' Compensation Laws are adopted.

THE HARWAYNE METHOD

Frank Harwayne has discovered a technique by which the National Council can use the Table L method, varying the per accident charge index by entry ratio and premium size, retaining all 73 size groups of the current Table M, and continuing to vary the excess loss premium factor by state and hazard group. The method prevents the paper explosion and uses only currently existing tables, so it can be implemented immediately.

The method is to estimate the Table L charge by interpolating between the Table M charge for the given risk size and the Table M charge for a smaller risk size, which is chosen so that its Table M charge lies above the Table L charge out to a very high entry ratio. The interpolation is performed by filling in a simple worksheet. The resultant approximate incremental charges are reasonably accurate, far surpassing those produced by the current National Council method. (See Exhibit 1.) A disadvantage of the method is that several worksheets have to be filled out each time since a retrospective rating is computed by trial and error. Constructing a set of Table L's would require extra work, but they would be more convenient to use.

For a given risk size the Table L charge $\phi^*(r)$ has certain theoretical properties:

i) The Table L charge equals the Table M charge at entry ratios no greater than the attachment point.

$$\phi^*(r) = \phi(r) \text{ for } r \leq r\frac{M}{T}. \qquad (3)$$

ii) The Table L charge is greater than the Table M charge at entry ratios above the attachment point.

$$\phi^*(r) > \phi(r) \text{ for } r > r\frac{M}{T}. \qquad (4)$$

[1]Proved in a later section.

iii) The Table L charge approaches the loss elimination ratio as the entry ratio becomes large.

$$\lim_{r \to \infty} \phi^*(r) = k. \tag{5}$$

iv) The Table L charge is greater or equal to the loss elimination ratio.

$$\phi^*(r) \geq k. \tag{6}$$

v) The Table L charge is less than or equal to the sum of the Table M charge and the loss elimination ratio.

$$\phi^*(r) \leq \phi(r) + k. \tag{7}$$

vi) The incremental charge is a monotone increasing function of r.[1]

$$\frac{d}{dr} \triangle \phi(r) \geq 0. \tag{8}$$

vii) The Table L charge is a monotone decreasing function of r.

$$\frac{d}{dr} \phi^*(r) \leq 0. \tag{9}$$

viii) The Table L charge is a concave upward function of r.[2]

$$\frac{d^2}{dr^2} \phi^*(r) \geq 0. \tag{10}$$

Exhibit 2 illustrates Harwayne's method. Over most of the range of r the Table L charge $\phi^*(r)$ is close to the Table M charge $\phi(r)$ and far from the reference Table M charge $\phi_1(r)$. This distance is the reason that the curve $\phi_1(r)$ cannot determine $\phi^*(r)$ with perfect accuracy.

The Table L charge produced by Harwayne's formula satisfies the first three theoretical properties and appears to satisfy the fourth, but it need not satisfy the others. The example shown in Exhibit 2 deviates from properties (v) through (viii), although the deviations take place at high entry ratios, which are of little practical importance.

[2]Since $\frac{d}{dr}\left[\int_r^\infty (s-r)\, f^*(s)\,ds + k\right] = -\int_r^\infty f^*(s)\,ds$ and $\frac{d}{dr}\left[-\int_r^\infty f^*(s)\,ds\right]$

$= f^*(r) \geq 0$

Mr. Harwayne has devised a remarkably simple and effective technique for increasing the accuracy of the National Council's retrospective ratings. His method permits an immediate solution to an important practical problem.

MEASURING THE INCREMENTAL CHARGE

The incremental charge can be computed as the Table L charge minus the Table M charge or estimated by choosing a reasonable curve. It can also be measured directly by means of a method devised by the California Inspection Rating Bureau in 1965.

Given a selection of risks numbered 1, . . ., N for a particular size group, with risk n having expected loss E_n, actual loss A_n, and actual limited loss A_n^*, the per accident charge index at entry ratio r can be estimated as

$$\overline{Y}(r) = \frac{\sum_{n=1}^{N} [(Min\ (r, A_n/E_n) - Min\ (r, A_n^*/E_n)]}{\sum_{n=1}^{N} [A_n/E_n - A_n^*/E_n]} . \qquad (11)$$

Let $\cdot\overline{k}$ be the estimated loss elimination ratio for all premium size groups combined. The incremental charge for the particular size group at entry ratio r is then estimated as

$$\triangle\overline{\phi}(r) = \overline{Y}(r) \cdot \overline{K}. \qquad (12)$$

To see why this method works, write k, $\phi(r)$, and $\phi^*(r)$ as

$$k = E\{A/E - A^*/E\} \qquad (13)$$

$$\phi(r) = E\{Max\ [(A/E - r), 0]\} \qquad (14)$$

$$\phi^*(r) = k + E\{Max\ [(A^*/E - r), 0]\} \qquad (15)$$

Then $\triangle\phi(r) = \phi^*(r) - \phi(r)$

$$= E\{A/E - A^*/E + Max\ [A^*/E - r), 0] - Max\ [(A/E - r), 0]\}$$

$$= E\{Min\ [r, A/E] - Min\ [r, A^*/E]\} \qquad (16)$$

Equation (13) shows that $\sum_{n=1}^{N} [A_n/E_n - A_n^*/E_n]/N$ is an estimator for k.

Equation (16) shows that $\sum_{n=1}^{N}$ [*Min* $(r, A_n/E_n)$ — *Min* $(r, A^*_n/E_n)]/N$ *is* an estimator of $\triangle\phi(r)$. Formula (11), the ratio of these two expressions, is an estimator for $Y(r) = \triangle\phi(r)/k$.

Equation (16) can be used to show that $\triangle\phi(r)$ is a monotone increasing function of r.

Theorem: Assume that the loss limitation procedure never increases a loss, that is, $A \geq A^*$. Let r and s be entry ratios with $0 \leq r < s$. Then $\triangle\phi(r) \leq \triangle\phi(s)$.

Proof: Let $X = Min\ [s, A/E] - Min\ [s, A^*/E] - (Min\ [r, A/E] - Min\ [r, A^*/E])$. Then $E\{X\} = \triangle\phi(s) - \triangle\phi(r)$, from equation (16). The value of the random variable X depends on the relative sizes of r, s, A/E, *and* A^*/E:

Condition	Min [s, A/E] — Min [s, A*/E]	Min [r, A/E] — Min [r, A*/E]	X	Sign of X
$A^*/E \leq A/E \leq r < s$	$A/E - A^*/E$	$A/E - A^*/E$	0	0
$A^*/E \leq r < A/E \leq s$	$A/E - A^*/E$	$r - A^*/E$	$A/E - r$	> 0
$A^*/E \leq r < s < A/E$	$s - A^*/E$	$r - A^*/E$	$s - r$	> 0
$r < A^*/E \leq A/E \leq s$	$A/E - A^*/E$	0	$A/E - A^*/E$	≥ 0
$r < A^*/E \leq s < A/E$	$s - A^*/E$	0	$s - A^*/E$	≥ 0
$r < s < A^*/E \leq A/E$	0	0	0	0

Since $X \geq 0$ in all cases, $E\{X\} \geq 0$.

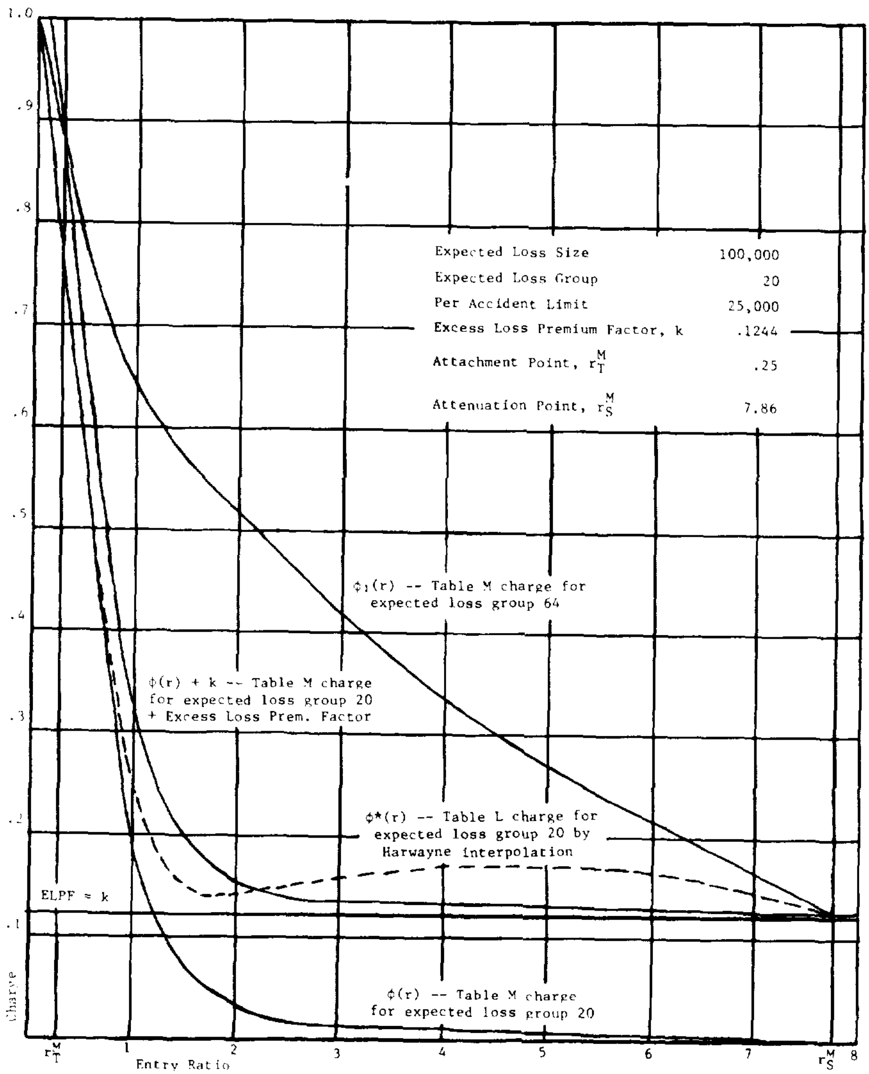
Exhibit 1

TEST OF HARWAYNE INTERPOLATION APPLIED TO CALIFORNIA DATA AT SELECTED ENTRY RATIOS ABOVE THE ATTACHMENT POINT

Expected Loss Size	Per Accident Limit	Entry Ratio	(1) Calif. Table L Charge	(2) Calif. Table M Charge	(3) Calif. Incremental Charge	(4) Incremental Charge by Current National Council Method	(5) Error	(6) Incremental Charge by Harwayne Interpolation	(7) Error
					(1)–(2)		(4)–(3)		(6)–(3)
22,014	25,000	2.0	.193	.188	.005	.124	.119	.037	.032
38,819	25,000	1.5	.207	.194	.013	.124	.111	.044	.031
		2.0	.157	.126	.031	.124	.093	.067	.036
54,781	25,000	1.0	.300	.287	.013	.124	.111	.027	.014
		1.5	.179	.130	.049	.124	.075	.062	.013
		2.0	.139	.061	.078	.124	.046	.090	.012
	50,000	1.0	.290	.287	.003	.050	.047	.003	—
		1.5	.140	.130	.010	.050	.040	.020	.010
		2.0	.082	.061	.021	.050	.029	.034	.013
	100,000	2.0	.070	.061	.009	.027	.018	.004	–.005
161,892	25,000	.5	.533	.529	.004	.124	.120	.011	.007
		1.0	.243	.219	.024	.124	.100	.050	.026
		1.5	.151	.084	.067	.124	.057	.087	.020
		2.0	.130	.039	.091	.124	.033	.109	.018
	50,000	.5	.529	.529	—			.004	.004
		1.0	.222	.219	.003	.050	.047	.027	.024
		1.5	.098	.084	.014	.050	.036	.047	.033
		2.0	.063	.039	.024	.050	.026	.053	.029
	100,000	1.0	.220	.219	.001	.027	.026	.011	.010
		1.5	.088	.084	.004	.027	.023	.024	.020
		2.0	.046	.039	.007	.027	.020	.030	.023

Exhibit 2

HARWAYNE INTERPOLATION APPLIED TO NATIONAL COUNCIL TABLE M



MINUTES OF THE 1975 SPRING MEETING

MAY 18-21, 1975

THE GREENBRIER, WHITE SULPHUR SPRINGS, WEST VIRGINIA

Saturday, May 17, 1975

An informal reception for early arrivals was held in the President's suite from 6:30-7:30 p.m.

Sunday, May 18, 1975

The Board of Directors had a regularly scheduled meeting from 10:00 a.m. to 1:00 p.m.

Registration took place from 4:00-6:00 p.m.

The President's reception for new Fellows and their wives and husbands was from 5:45-6:45 p.m.

An outdoor reception for members and guests was held from 6:30-7:30 p.m.

Monday, May 19, 1975

Registration began at 8:30 a.m.

The Spring meeting formally convened at 9:00 a.m.

Following opening remarks by President M. Stanley Hughey, a welcoming address was given by Honorable Donald W. Brown, Commissioner of Insurance, State of West Virginia.

Plaques were presented to Past Presidents. Those who received their plaques were William Leslie, Jr., Thomas E. Murrin, Harold E. Curry, Charles C. Hewitt, Jr., Paul S. Liscord.

President Hughey then requested the new Associates to stand as he read their names. (Diplomas for the new Associates were distributed at the coffee break after photographs were taken.) After the applause for the new Associates, President Hughey had each new Fellow come forward individually to receive his or her diploma.

NEW ASSOCIATES

David R. Bradley
Fred L. Brewer
Burton Covitz
Charles H. Dangelo
Richard C. Ernst
Sam Gutterman*
David N. Hafling
Urban E. Leimkuhler*
Peter A. Masters
Ronald J. McHugh
Michael F. McManus
Benjamin S. Newville
Richard C. Plunkett
Kenneth R. Rosen
Donna R. Symonds*
Jerome F. Vogel

*Not present.

NEW FELLOWS

Charles H. Berry
Neil A. Bethel
Stephen P. D'Arcy
George H. Dieter, Jr.
John P. Drennan
Janet S. Graves
R. Michael Lamb
Philip D. Miller
Phillip S. Moore
Richard D. Pagnozzi*
Gail E. Tverberg

*Not present.

A review was presented by Frank Harwayne of David Skurnick's paper "The California Table L". This was followed by Richard H. Snader's review of the same paper. An informal Reply to Reviewers of his paper was then given by David Skurnick.

A report on American Academy activities was given by E. J. Moorhead, Past President of the American Academy of Actuaries substituting for Daniel J. McNamara, President of the American Academy. Mr. Moorhead spoke on:

C Conduct
A Authority
S Stewardship

Following a coffee break, there was a panel discussion entitled "Avoiding the Next Capacity Crunch". Participants in this discussion were:

Moderator:	Robert H. McMillen Senior Vice President and Actuary The Travelers Insurance Companies
Panel Members:	Richard J. Doyle Executive Vice President Supervised Investors Services, Inc.
	Robert E. Dineen Consultant National Association of Insurance Commissioners
	Donald Kramer President Kramer Capital Consultants
	Charles L. Niles, Jr. Executive Vice President General Accident Assurance Corporation

Following the panel presentation questions were asked by the audience.

At 12:00 M. the meeting adjourned for luncheon.

At 2:00 p.m. an informal forum entitled "Medical Malpractice" was held covering current developments in the medical malpractice field. Participants in this part of the program were:

Moderator:	Robert W. Sturgis Actuary Aetna Life & Casualty
	Roy R. Anderson Group Vice President Allstate Insurance Company
	J. Haines Boyle Corporate Actuary Signal Insurance Company

Robert G. Oien
Actuary
St. Paul Fire and Marine Insurance Company

The forum concluded at 3:40 p.m.

An outdoor reception for members, their wives and husbands and guests was held from 6:00-7:30 p.m.

Tuesday, May 20, 1975

Mr. Hughey convened the morning session at 9:00 a.m.

Reviews of Ronald E. Ferguson's paper "Nonproportional Reinsurance and the Index Clause" were presented by 1) Joseph W. Levin, 2) Matthew Rodermund, and 3) Costandy K. Khury and were followed by the Author's Reply to Reviewers.

Reviews were presented by 1) Charles Gruber and 2) Paul Hough of "Revising Classification Structure Using Survey Data" authored by David Skurnick, N. Robert Heyer and G. Ray Funkhouser.

A Past President plaque was presented to Mr. LeRoy J. Simon.

The following committee reports were presented:

1. Joint Committee on Independence of the Actuary by P. Adger Williams.
2. E & E Committee by Charles F. Cook and Jeffrey T. Lange.
3. Textbook Committee by W. James MacGinnitie.
4. Financial Reporting by James H. Crowley.
5. Editorial Committee by Luther L. Tarbell, Jr.
6. Nominating Committee by LeRoy J. Simon.
7. Delegate to ASTIN by LeRoy J. Simon. Mr. Simon told the membership that ASTIN will meet this fall in Portugal and the IAA Congress will meet in 1976 in Japan.
8. Joint Committee to Study the Coordination of Activities Among Actuarial Organizations by M. Stanley Hughey.

At 10:00 a.m., Mr. E. J. Moorhead, Past President of the American Academy of Actuaries, spoke on "An Experiment in Statement Making". Mr. Moorhead's talk was well received and enjoyed by all.

Following a coffee break, at 10:45 a.m., a forum entitled "Current Events" was presented. The participants in this forum were:

Moderator: Frederick W. Kilbourne
President
Booz-Allen Consulting Actuaries

Edward H. Budd
Senior Vice President
The Travelers Insurance Companies

Robert Pollack
President
Colonial Penn Insurance Company

Paul S. Liscord
Vice President and Actuary
Insurance Company of North America

Thomas E. Murrin
Senior Vice President and Actuary
Fireman's Fund American Insurance Companies

At noon, the meeting adjourned for luncheon.

From 2:00-5:30 p.m. a choice of six one-hour workshops, each offered twice, were presented:

A. Numerical Analysis Applications

Moderator: David R. Bickerstaff
Consulting Actuary
Milliman & Robertson, Inc.

David J. Grady
Associate Actuary
The Travelers Insurance Companies

B. Computers and the Actuarial Department

Moderator: David G. Hartman
Assistant Vice President and Associate Actuary
Chubb & Son, Incorporated

Richard S. Biondi
Actuarial Supervisor
Insurance Services Office

Dale A. Nelson
Actuary
State Farm Mutual Automobile Insurance Company

C. Loss Reserving Problems

Moderator: Rafal J. Balcarek
Vice President and Actuary
Reliance Insurance Company

James H. Durkin
Actuary
Peat, Marwick, Mitchell & Company

James F. Golz
Assistant Actuary
Employers Insurance of Wausau

Donald E. Trudeau
Assistant Vice President
American Mutual Liability Insurance Co.

D. Current Compensation Problems

Moderator: William C. Aldrich
Assistant Vice President
The Hartford Insurance Group

James A. Hall, III
Assistant Actuary
American Mutual Liability Insurance Company

Frank Harwayne
Vice President and Director of Actuarial Research
National Council on Compensation Insurance

E. Actuarial Applications of Statistical Theory

Moderator: J. Ernest Hansen
Assistant Secretary
The Hartford Insurance Group

David Skurnick
Actuary
California Inspection Rating Bureau

Charles A. Hachemeister
Actuary
Prudential Reinsurance Company

F. General Liability—What's the Future Look Like?

Moderator: Jeffrey T. Lange
Vice President
Royal-Globe Insurance Companies

From 6:30-7:30 p.m. there was a reception on the Colonial Terrace.

Wednesday, May 21, 1975

Wednesday morning was devoted to Corporate Planning with W. James MacGinnitie, Consulting Actuary, Tillinghast & Company serving as Moderator.

8:30-9:30 a.m. *Corporate Planning—An Overview*

W. James MacGinnitie

Planning for the Future Environment

R. Morton Darrow
Vice President
Prudential Insurance Company of America

9:30-10:15 a.m. *Role of the Actuary in Corporate Planning*

Charles C. Hewitt, Jr.
Vice President and Actuary
Metropolitan Property and Liability Insurance Company

Charles R. Rinehart
Assistant Vice President and Associate Actuary
Fireman's Fund American Insurance Companies

Following a coffee break the following Presentations were given:

1) *Planning in the Holding Company Context*

James E. Shaw
Secretary
Hartford Insurance Group

2) *New Venture Planning*

Reginald C. Yoder
Assistant Actuary
Bankers Life Company

3) *Planning for Multi-National Operations*

Neill W. Portermain
Vice President, Corporate Operations
W. R. Berkley Corporation

4) *Modeling for Corporate Planning*

Raymond W. Beckman
Consulting Actuary
Booz-Allen Consulting Actuaries

President Hughey adjourned the meeting at 12:00 noon expressing thanks to James W. Wilson, Chairman of Local Arrangements and to Richard Palczynski, Fred Brewer, John Winkleman, and Michael Blivess who assisted with the registrations.

Registration cards completed by the attendees and filed at the registration desk indicated attendance by 107 Fellows, 71 Associates, 29 guests (including 9 subscribers), and 92 husbands and wives, as follows:

FELLOWS

Adler, Martin
Aldrich, William C.
Alexander, Lee M.
Allen, Edward S.
Anker, Robert A.
Balcarek, Rafal J.
Beckman, Raymond W.
Ben-Zvi, Phillip N.
Berquist, James R.
Berry, Charles H.
Bethel, Neil A.
Bickerstaff, David R.
Bondy, Martin
Bornhuetter, Ronald L.
Boyajian, John H.
Brannigan, James F.
Brian, Robert A.
Budd, Edward H.
Byrne, Harry T.
Connors, John B.
Cook, Charles F.
Crowley, James H.
Curry, Harold E.
D'Arcy, Stephen P.
Dieter, George H.
Drennan, John P.
Dropkin, Lester B.
Ehlert, Darrell W.
Eyers, Robert G.
Ferguson, Ronald E.
Finger, Robert J.
Fitzgibbon, Walter J.
Flaherty, Daniel J.
Flynn, David P.
Fossa, E. Frederick
Foster, Robert B.
Gibson, John A.
Grady, David J.
Graves, Janet S.
Hachemeister, Charles A.
Hall, James A.
Hardy, Howard R.
Hartman, David G.
Harwayne, Frank
Haseltine, Douglas S.
Hewitt, Charles C.
Hoffman, Dennis E.
Hope, Francis J.
Hughey, M. Stanley
Hunter, J. Robert
Inkrott, James G.
Kallop, Roy H.
Kaufman, Allan M.
Khury, Costandy K.
Kilbourne, Frederick W.
Klaassen, Eldon J.
Lamb, Michael R.
Lange, Jeffrey T.
Leslie, William
Levin, Joseph W.
Liscord, Paul S.
MacGinnitie, W. James
Makgill, Stephen S.
McClure, Richard D.
McLean, George E.
Miller, Philip D.
Mills, Richard J.
Moore, Phillip S.
Morison, George D.
Muetterties, John H.
Munro, Richard E.
Murray, Edward R.
Murrin, Thomas E.
Naffziger, Joseph V.
Nelson, Dale A.
Newman, Steven H.
Niles, Charles L.
Oien, Robert G.
Petz, Earl F.
Phillips, Herbert J.
Pollack, Robert
Portermain, Neill W.
Richardson, James F.
Riddlesworth, William
Rinehart, Charles R.
Rodermund, Matthew
Rogers, Daniel J.
Ross, James P.
Ryan, Kevin M.
Scheibl, Jerome A.
Sheppard, Alan R.
Simon, LeRoy J.
Skelding, Albert Z.
Skurnick, David
Snader, Richard H.
Stewart, Charles W.
Sturgis, Robert W.
Switzer, Vernon J.
Tarbell, Luther L.
Thomas, James W.
Toothman, Michael L.
Trudeau, Donald E.
Tverberg, Gail E.
Walsh, Albert J.
Webb, Bernard L.
Williams, P. Adger
Wilson, James C.
Woll, Richard G.

ASSOCIATES

Ashenberg, Wayne R.
Bell, Allan A.
Biondi, Richard S.
Blivess, Michael P.
Bovard, Roger W.
Bradley, David R.

Brewer, Fred L.
Briere, Robert S.
Carbaugh, Albert B.
Carter, Edward J.
Chorpita, Fred M.
Conner, James B.
Covitz, Burton
Daino, Robert A.
Dangelo, Charles H.
DeGarmo, Lyle W.
Donaldson, John P.
Durkin, James H.
Feldman, Martin F.
Fisher, Wayne H.
Foley, Charles D.
Garand, Christopher P.
Gossrow, Robert W.
Grippa, Anthony J.
Gruber, Charles
Hafling, David N.
Head, Thomas F.
Hearn, Vincent W.
Hough, Paul E.
Jaeger, Richard M.
Jensen, James P.
Kaur, Alan F.
Kelly, Anne E.
Klingman, George C.
Krause, Gustave A.
Lindquist, Robert J.
Luneburg, Sandra C.
Masella, Norma M.
Masters, Peter A.
McHugh, Ronald J.
McManus, Michael F.
Mokros, Bertram F.
Moore, Brian C.
Napierski, John D.
Nelson, John K.
Newville, Benjamin S.
Nolan, John D.
Palczynski, Richard W.
Palm, Robert G.
Penniman, Kent T.
Plunkett, Joseph A.
Plunkett, Richard C.
Quirin, Albert J.
Rice, W. Vernon
Riff, Mayer
Rosen, Kenneth R.
Sandler, Robert M.
Sanko, Ronald J.
Singer, Paul E.
Squires, Sanford R.
Staley, Harlow B.
Stephenson, Elton A.
Stergiou, E. James
Torgrimson, Darvin A.
Van Slyke, Oakley E.
Vogel, Jerome F.
Weiner, Joel S.
Winkleman, John J.
Woodworth, James H.
Wulterkens, Paul E.
Yoder, Reginald C.

GUESTS

Anderson, James C.
Anderson, Roy R.
Boyle, J. Haines
Darrow, R. Morton
Dineen, Robert E.
Doyle, Richard J.
Hammond, J. D.
Hansen, J. Ernest
Heiser, John E.
Kramer, Donald
Lyon, Andrew C.
Mack, Dr. Thomas
McMillen, Robert H.
Miller, Robert A.
Moorhead, Ernest J.
Roland, W. Paul
Ruddock, George A.
Shaw, James E.
Stenmark, John A.
Waterfield, Randolph H.

SUBSCRIBERS

Anderson, Ernest V.
Bell, Andrew M.
Dunn, Robert P.
Hoyt, Fred A.
Johnson, John E.
Kaminoff, Harvey
Rais, Arnold M.
White, Bruce R.
Wright, Robert W.

A special program for wives and guests of the members was organized by Elaine Hughey and Cindy Bornhuetter.

Respectfully submitted,

Robert B. Foster
Secretary

Volume LXII, Part II No. 118

PROCEEDINGS

November 16, 17, 18, 1975

PUTTING A PRICE ON THE WHISTLES

PRESIDENTIAL ADDRESS BY M. STANLEY HUGHEY

In today's social and economic climate we are caught up in the turbulence and upheaval of massive currents and cross currents of improving our way of life, providing equal opportunity for all, helping those in need, and building the economy to provide all of these things, both now and in the future. To try and meet these needs, all sorts of new and adapted insurance programs, laws, and social benefits are being proposed and quite a number adopted. These are usually not black or white but some shade of gray, and for the public generally the shade of gray is important. Ben Franklin, 200 years ago, caught the spirit of the problem in his essay on "The Whistle", from which is taken this excerpt:

> "In short I conceived that great part of the miseries of mankind were brought upon them by the false estimates they have made of the value of things, and by their "giving too much for their whistles."

In our bicentennial, as our nation faces up to the social and economic demands of our times, there is a crying need to know and appreciate the price of the many whistles of social and economic improvements we're reaching for. As actuaries, we are uniquely trained, qualified and experienced to "price the whistles." At the same time this pricing represents both an opportunity and a challenge for us to make an important contribution to the public's understanding of the issues and ability to make informed and intelligent decisions.

This kind of social and economic change is certainly not a new phenomenon, and in one form or another has been with us since our cave man ancestors first learned the benefits of combining efforts. However, with the coming of the industrial revolution, our social and economic society shifted into a higher gear and moved with greater speed and turbulence toward a new point of balance.

Without dwelling unduly on past history, it is interesting to note that a sort of balance, reasonably acceptable for those times, was achieved about the turn of the century. The economy was bustling and the first push of industrial growth was being reasonably successfully integrated into the lives and minds of our citizens, new levels of transportation and communication had been established, and there were new frontiers in many directions for the more adventuresome.

For some, however, this turn of the century balance was unsatisfactory, and who can argue that the living conditions, medical facilities and expectancies of the poverty level citizens of that day were sufficient to meet desirable minimum standards of human life, or that the laboring class people of that day were given a reasonable opportunity to share in the proceeds of their labor.

Growing out of this dissatisfaction came general social unrest and the outcry for better working conditions. Of particular interest to us in the insurance industry was the passage of workmens compensation laws in most states in the 1910-14 period, introducing a new concept in caring for workers injured on the job, and almost overnight creating a need for people who understood the insurance mechanism, who had enough of a mathematical background to project probable future costs, and who could construct a set of rates which could reasonably meet those costs within the framework of the insurance mechanism. The Casualty Actuarial Society was founded out of mutual interest in this logical and knowledgeable evaluation of future costs, and the forthright presentation of how best to finance those needs. In a real sense, the work of the CAS members represented a major precedent in evaluating in advance the cost of a new law which would benefit society, albeit in rather modest dimensions at the time.

It is an interesting fact that societal changes come slowly. Stated conversely, human nature is such that it seems to take years and sometimes generations for changes in attitudes and lifestyles to accept and adjust to technological, scientific and social developments. During approximately the 1910-1930 period many changes were taking place—the auto revolutionized our transportation concept, agriculture started its shift from manual to mechanical, and the telephone changed a nation's communication habits. The first World War created new markets for mechanical products, and at least for the U.S. created a boom without a matching terrible accompaniment of loss of lives and permanent family disruption.

For the insurance industry and for the growing casualty actuarial profession, the 1920s provided new challenges in the mushrooming auto insurance field and in the growing health insurance field.

The golden era of the 20s reached a new plateau of economic and social balance. Then came the 30s and the great depression. The economy ground to a halt, the economic problems of the sharecroppers, marginal farmers and the chronic unemployed magnified, and millions of workers and their families found themselves sharing the same soul shaking problems and crises. Under such circumstances, new concepts of social needs were spawned to meet immediate emergency needs, to try to find some way to pull ourselves out of the morass, and to somehow change our structure, approach and planning to try and avoid a recurrence of this kind of serious hardship.

Specific programs included Social Security, Unemployment Insurance and various forms of public aid, along with the real beginnings of private pension plans.

World War II produced an explosive demand cycle, gave a whole generation of people their first look at "one world" and created an entirely new economic and social climate.

Parenthetically, World War II, also marked the take-off point for property and liability premium. It is pertinent to recall that our total industry premium was under five billion dollars in 1945, and will approximate forty eight billion dollars in 1975. Therefore, in thirty years time, our premium has grown ten fold (and more than tripled in the last 15 years.)

All of this is a reflection on the fact that our social and economic circumstances have been building over a long period of time, but they have exploded during the 30 years since World War II. In addition to the explosive economic growth we have built in a whole new set of social values, stemming from our memory of the troubles of the 30s and bolstered by the great expectations which for many were generated by their experience during the war years.

Worth special note is the enactment in 1965 of a new area of public support in the form of medicare and medicaid. Both represented huge new steps into the field of public health care medicare for a large but rather easily defined group, administered by a single agency with cost sharing factors. In contrast, medicaid was introduced in the spirit of social benefit for a very difficult to define group, with wide latitude in the benefits provided, no cost sharing, and administered by a great number of diverse agencies. Currently we are spending something over $11,000,000,000 per year for each of these programs, and medicaid in particular seems destined to go higher unless some means of control can be worked out.

In a sense, all this is just background to the fact that in today's complex economic and social climate we have reached a point where some of us are calling for yet additional programs to ease human problems, while others among us are maintaining that we cannot afford these programs and trying to pay for them can pull the whole nation into deep trouble if not disaster.

Unfortunately, as the current New York City crisis so vividly illustrates, there does seem to be a limit to what we as a society can or perhaps are willing to pay for. Somewhere in the middle is the socially beneficial but affordable balance we should be seeking. To find that balance we need to have a good fix on costs, and the people best qualified to establish these costs are actuaries. Certainly socialogists, economists and even lawyers have amply demonstrated that these costs are elusive.

On given issues, we as professional actuaries should make it our business to be certain that the best estimates of projected costs are available, on which others can depend in making up their minds for or against. Further, if we as private citizens elect to take a position, we should be extra careful that our position on the issue is not allowed to color our objective evaluation of the cost elements.

Having worked into an important principle through the broader aspects of our professional obligations, we should also note that the same principle is one we normally apply in our regular work area—projecting needed rates, setting loss reserves, interpreting experience, and building insurance company models for "what if" kinds of studies. In more recent times we have had other types of challenges in the area of financing future costs—involving new laws and new concepts. Included in this category are the auto no-fault laws, the greatly expanded compensation benefits, residual auto and property markets, limited liability malpractice laws, and all sorts of suggestions for various forms of National Health insurance. In each of these there have been and will undoubtedly be additional calls and opportunities to calculate probable future costs. For the industry and for society generally we can make a worthwile contribution if we can develop properly objective cost estimates based on carefully defined assumptions.

Without in any way arguing the merits for or against any one of these or other developments (and there can be good arguments on both sides) I would encourage, cajole, entreat and implore each of you to contribute all you can to making the best available estimates of future costs available to the "public", who must ultimately decide whether or not these programs should be adopted. We already have a sizeable backlog of unfunded, and in some areas unrecognized, costs in Social Security and pensions to make up, and if we as a society are going to further broaden our public support of major segments of our population it should be done with the full knowledge of the costs involved.

As a professional actuary our first obligation has to be to our employer, setting forth our best judgment of the cost of these programs. Remember also that we have a professional obligation with standards to maintain for the good of all concerned.

Beyond the confines of our regular employment, and out in the public arena, I would call attention to the fact that here also we have an opportunity and an obligation to assure that probable future costs are reasonably calculated and made available to those making decisions. At the local, state and national level there is a continuous stream of proposals on which our cost projections should be more realistic than the ones that are being made.

As always there are problems areas and one in particular warrants special emphasis. In making projections of future costs on these kinds of projects, the actuary is frequently called on to prepare a report which is expressed in layman's language but reflects important actuarial relationships. Further, there are usually a number of assumptions involved, at least some of which are in an area of uncertainty. A good actuarial workman will include basic assumptions in the report but too frequently the end product gets the emphasis and does not include the intermediate assumptions. This sometimes leads to apparent wide differences of calculation which does the profession no good. This is probably not a problem we can ever solve completely but each of us should strive to see that any major assumptions are included as an integral part of the conclusion in any report forecasting costs, and that the assumptions are referred to in any publicity.

Summarizing, our society is reaching for a high level of social benefits, aspiring to the highest humanitarian goals ever achieved in history. Rightly, the insurance industry is in the middle of this effort since insurance is a useful mechanism for achieving many of these goals. However, in our enthusiasm for providing security for all, we must not overlook the need to finance these programs, lest we leave to future generations the framework for disaster.

There must always be someone to ask, "what will it cost?" In the past the answers haven't always been forthcoming, or even worse, some have been misleading or wrong. In today's situation there is an obligation and an opportunity actuaries should not miss to help clarify alternatives and to help others make better informed decisions—all by "putting a price on the whistles."

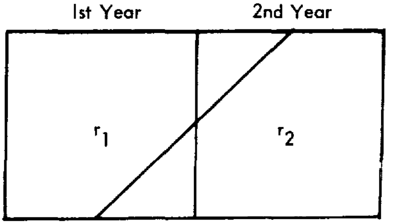
GENERALIZED PREMIUM FORMULAE

JAMES P. ROSS

Socrates: But from which line shall we get it? Try and tell us exactly; and if you would rather not reckon it out, just show what line it is.
Boy: Well, on my word, Socrates, I for one do not know.

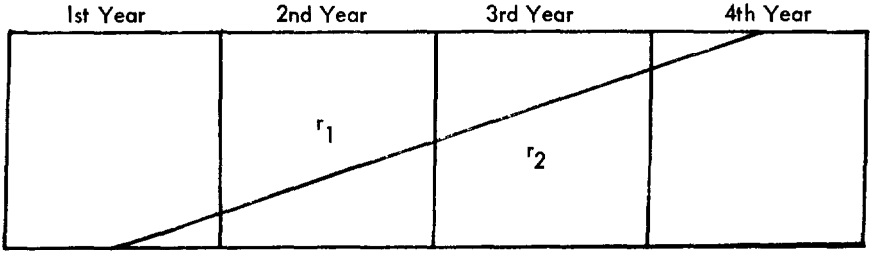
Plato

Fundamental to most ratemaking procedures is the adjustment of historical data to reflect current or anticipated conditions. In ratemaking methods which use premium data it is necessary to adjust the actual premiums to the current rate level. One technique for estimating this adjustment is the parallelogram method, also referred to as the Pro-rata method. This involves drawing a diagram and assigning weights to the different rate levels in proportion to areas on the diagram. In the case where there is an annual policy term the diagram is drawn as follows:



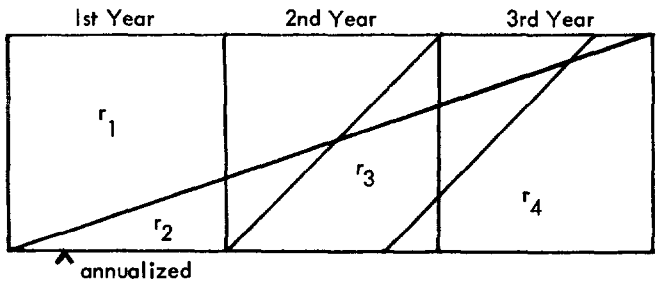
The interpretation of this diagram is that the rate level changed in the middle of the first year from r_1 to r_2. The exposures written at r_1 expire uniformly along the diagonal line and then are renewed at the new rate level r_2. In each year the exposure earned at the new rate level is proportional to the area under the diagonal line. In the first year this is equal to one-eighth of the total area and in the second year is equal to seven-eighths of the total area. Therefore, the average rate level in the first year is given by $7/8r_1 + 1/8r_2$ and the second year is $1/8r_1 + 7/8r_2$. Having determined the average rate level in each year, the factor to adjust to the current rate level is the current rate level divided by the average rate level in that year.

The parallelogram method is also used for policy terms other than annual. In the case of a three year policy term the method is identical except that the slope of the diagonal line becomes one-third rather than one. The diagram becomes:



Again this diagram assumes that a rate change was made in the middle of the first year. The proportion of exposure earned at the new rate is respectively 1/24, 1/3, 2/3, and 23/24 in the first, second, third and fourth years.

The methods described above have been used for many years. This paper had its genesis in a fairly simple problem arising from an application of these methods. Rate level adjustment factors were being calculated from earned premiums and a case was encountered where there had been several rate changes and in addition the policy term had been changed from three years to one year. Using the customary parallelogram approach the diagram looked like this:



In this case the exposure was initially being written on a three year term; the term was changed to annual one-fourth of the way into the first year. Rate changes were made at the beginning of the first and second years and at three fourths of the second year. The problem, of course, is what to do with the crossing lines. Before a solution was found several interesting relationships were discovered and a theoretical framework was developed which may be useful in solving other problems.

WRITTEN AND EARNED EXPOSURES

Let the function $f(x)$ stand for the rate of exposure writing at time x. Although the writing of exposures is the result of many discrete transactions we will assume that $f(x)$ is at least piecewise continuous. The written exposure between time x_0 and x_1 may be expressed as:

$$WE(x_0, x_1) = \int_{x_0}^{x_1} f(x)dx \tag{1}$$

The case where exposure is being written at a constant rate is equivalent to $f(x) = K$; the written exposure is given by:

$$WE(x_0, x_1) = \int_{x_0}^{x_1} Kdx = K(x_1 - x_0) \tag{2}$$

To calculate the earned exposure it is necessary to take into account the policy term t. The earned amount between x_0 and x_2 may be derived by partitioning the x-axis into segments $\triangle\, i$; let $x_i \in \triangle\, i$, then the exposure written on the i th partition is approximately equal to $f(x_i) \cdot \triangle\, i$. Assuming that $x_0 \leq x_1 - t$ (identical results are obtained if $x_0 > x_1 - t$, the proof is very similar), then the earned exposure between x_0 and x_1 is approximated by:

a. 0 if $x_i \leq x_0 - t$

b. $((x_i + t - x_0)/t)\, f(x_i) \cdot \triangle\, i$ if $x_0 - t < x_i \leq x_0$

c. $f(x_i) \cdot \triangle\, i$ if $x_0 < x_i \leq x_i - t$

d. $((x_1 - x_i)/t)\, f(x_i) \cdot \triangle\, i$ if $x_1 - t < x_i \leq x_1$

e. 0 if $x_i > x_1$

Summing and taking the appropriate limits the earned exposure is equal to

$$EE(x_0, x_1) = \int_{x_0 - t}^{x_0} \frac{x + t - x_0}{t} f(x)dx + \int_{x_0}^{x_1 - t} f(x)dx + \int_{x_1 - t}^{x_1} \frac{x_1 - x}{t} f(x)dx \tag{3}$$

Similarly the unearned exposure at any time x_0 is given by

$$U(x_0) = \int_{x_0 - t}^{x_0} \frac{x + t - x_0}{t} f(x)dx \qquad (4)$$

Using formulae (1), (3) and (4) it can be shown that

$$EE(x_0, x_1) = U(x_0) + WE(x_0, x_1) - U(x_1) \qquad (5)$$

Three special cases are of interest in that they confirm working formulae.

Special Case I: $f(x) = K$

$$EE(x_0, x_1) = \int_{x_0 - t}^{x_0} \frac{x + t - x_0}{t} Kdx + \int_{x_0}^{x_1 - t} Kdx + \int_{x_1 - t}^{x_1} \frac{x_1 - x}{t} Kdx$$

$$= \tfrac{1}{2} Kt + K(x_1 - t - x_0) + \tfrac{1}{2} Kt$$

$$= K(x_1 - x_0) = WE(x_0, x_1)$$

Thus with a constant rate of writing the earned exposure will equal the written exposure.

Special Case II: $(x_1 - x_0) = t$

$$f(x) = K_1, x_0 - t < x \leq x_0$$

$$f(x) = K_2, x_0 < x \leq x_1$$

$$EE(x_0, x_1) = \tfrac{1}{2} K_1 t + \tfrac{1}{2} K_2 t = \tfrac{1}{2} WE(x_0 - t, x_0) + \tfrac{1}{2} WE(x_0, x_1)$$

This is the earned exposure calculated by the "annual pro-rata" method with annual term.

Special Case III: $(x_1 - x_0) = \tfrac{1}{3} t$

$$f(x) = K_1, x_0 - t < x \leq x_0 - \tfrac{2}{3} t$$

$$f(x) = K_2, x_0 - \tfrac{2}{3} t < x \leq x_0 - \tfrac{1}{3} t$$

$$f(x) = K_3, x_0 - \tfrac{1}{3} t < x \leq x_0$$

$$f(x) = K_4, x_0 < x \leq x_1$$

$$EE(x_0, x_1) = \frac{K_1 t}{18} + \frac{K_2 t}{9} + \frac{K_3 t}{9} + \frac{K_4 t}{18}$$

$$= WE(x_0 - t, x_0 - 2/3\ t)/6 + WE(x_0 - 2/3\ t, x_0 - 1/3\ t)/3$$

$$+ WE(x_0 - 1/3\ t, x_0)/3 + WE(x_0, x_1)/6$$

This is recognizable as a version of the "annual pro-rata" method but with three year term. Other common formulae which are based on the assumption of constant writings over various periods of time may also be derived. It is of interest to note that the various pro-rata formulae also hold true when $f(x) = a + bx$, a and b constants; however, the earned exposure will no longer equal the written exposure.

Another illustrative example is provided in the case where the rate of exposure writing is changing at a uniform rate. In this case $f(x) = Ke^{cx}$, where c is the rate of change. From (1) the written exposure is given by

$$WE(x_0, x_1) = \frac{K}{c}(e^{cx_1} - e^{cx_0})$$

From either equations (3) or (5) it can be shown that

$$EE(x_0, x_1) = WE(x_0, x_1)(1 - e^{-ct})/ct$$

Note that as $C \to O$, $EE(x_0, x_1) \to WE(x_0, x_1)$

For a number of typical values the ratio of earned exposures to written exposure when there is a constant rate of change in writings is as follows:

Ratios of Earned to Written
Annual Rate of Change in Writings[2]

Term	−20%	−10%	+10%	+20%
6 mos	1.0470	1.0242	.9765	.9558
12 mos	1.0970	1.0492	.9538	.9141
36 mos	1.3310	1.1576	.8697	.7702

The concept of importance for what follows is that of earned contribution to the interval (x_0, x_1) from the writings over the interval (y_0, y_1). Roughly this is the portion of $WE(y_0, y_1)$ which is earned between x_0 and x_1. More precisely, first define the function $g(x)$ as follows:

$$g(x) = \begin{cases} O, x \leq y_0 \\ f(x), y_0 < x \leq y_1 \\ O, x > y_1 \end{cases}$$

The earned contribution to the interval (x_0, x_1) from the writings over (y_0, y_1) is equal to

$$EC(y_0, y_1; x_0, x_1) = \int_{x_0 - t}^{x_0} \frac{x + t - x_0}{t} g(x)dx + \int_{x_0}^{x_1 - t} g(x)dx + \int_{x_1 - t}^{x_1} \frac{x_1 - x}{t} g(x)dx \quad (6)$$

In general it can be shown that the following are true:

$$EC(-\infty, x_0; x_0, x_1) + EC(x_0, x_1; x_0, x_1) = EE(x_0, x_1)$$

and when $x_0 < x_1 < x_2$

$$EC(y_0, y_1; x_0, x_1) + EC(y_0, y_1; x_1, x_2) = EC(y_0, y_1; x_0, x_2)$$

[2] The annual rate of change is given by $a = e^c - 1$

and when $y_0 < y_1 < y_2$

$$EC(y_0, y_1; x_0, x_1) + EC(y_1, y_2; x_0, x_1) = EC(y_0, y_2; x_0, x_1)$$

Also of interest are the following:

Let $f(x) = K, \xi > 0$

$EC(-\infty, a; a, a + \xi) = \frac{1}{2} Kt$ when $\xi \geq t$

$EC(a, a + \xi; a, a + \xi) = \frac{1}{2} K \xi^2/t$

That a change in term will have an immediate effect on exposures written is obvious; the same policies are being written but more or less exposure is being booked depending upon whether the term was lengthened or shortened. However, a change in term should not affect exposures earned; this fact allows us to determine the change in $f(x)$ due to a change in t.

If the change from t_0 to t_1 is made at time x_0, then if $f(x) = f_0(x)$ when $x < x_0$ the new function $f(x) = f_1(x)$ for $x \geq x_0$ may be determined by using the following equation with $\xi > 0$

$$EC(-\infty, x_0; x_0, x_0 + \xi) + EC(x_0, x_0 + \xi; x_0, x_0 + \xi) = EE(x_0, x_0 + \xi) \quad (7)$$

The first term on the left side of equation (7) and the term on the right side are calculated using t_0 and $f_0(x)$, while the second term on the left side of the equation contains t_1 and $f_1(x)$. Take the case where $f_0(x) = K_0$ and $\mathfrak{z} < t_1 < t_0$ then we have:

$$\tfrac{1}{2} K_0 t_0 + K_0 (t_0 - \mathfrak{z}) \left(\tfrac{1}{2} (t_0 + \mathfrak{z})/t_0 - 1\right) + \int_{x_0}^{x_0 + \mathfrak{z}} \frac{x_0 + \mathfrak{z} - x}{t_1} f_1(x)\,dx = K_0 \mathfrak{z}$$

simplifying results in the integral equation

$$\int_0^{\mathfrak{z}} (\mathfrak{z} - y)\, f_1(y)\, dy = \tfrac{1}{2} K_0 (t_1/t_0)\, \mathfrak{z}^2$$

which has solution:

$$f_1 = K_0(t_1/t_0)$$

Where f_1 has domain $x_0 < x \leq x_0 + t_1$

The general solution to the problem when $f_0(x) = K_0$ may be obtained by repeated applications of formula (7) with $\mathfrak{z}$ increasing. If N is the largest integer such that $Nt_1 \leq t_0$, it can be shown that $f(x)$ will have the following values:

$$f(x) = K_0(it_1/t_0) \text{ when } x_0 + (i - 1)\, t_1 < x \leq x_0 + it, \quad i = 1, 2 \cdots N$$

and $f(x) = K_0(N + 1)(t_1/t_0)$ when $x_0 + Nt_1 < x < x_0 + t_0$

$$f(x) = K_0\, N(t_1/t_0) \text{ when } x_0 + t_0 \leq x < x_0 + (N + 1)\, t_1$$

and $f(x) = K_0(N + 1)(t_1/t_0)$ when $x_0 + (N + j)\, t_1 \leq x < x_0 + t_0 + jt_1$

$$f(x) = K_0\, N(t_1/t_0) \text{ when } x_0 + t_0 + jt_1 \leq x < x_0 + (N + j + 1)t, \quad j = 1, 2 \cdots$$

A simple example may be helpful at this point: assume at time x_0 the term was changed from three to one; the exposure prior to x_0 had been written at a constant rate of K_0. We then have N equal to three and $f(x)$ is as follows:

$$\begin{aligned} f(x) &= \tfrac{1}{3} K_0, & x_0 &< x \leq x_0 + 1 \\ &= \tfrac{2}{3} K_0, & x_0 + 1 &< x \leq x_0 + 2 \\ &= K_0, & x_0 + 2 &< x \leq x_0 + 3 \\ &= K_0, & x &> x_0 + 3 \end{aligned}$$

This confirms what we would expect, the written exposure drops to one-third the prior rate for one year rises to two-thirds the following year and then as all policies are converted to the one year basis the rate of writing returns to the prior rate.

The change from six month policies to annual policies illustrates another phenomenon; assuming again a constant rate K_0 prior to the change in term, we have N equal to zero and the following:

$$\begin{aligned} f(x) &= K_0\,(N+1)\,(1/0.5) = 2\,K_0\,, && x_0 < x \leq x_0 + 0.5 \\ &= K_0\,N\,(1/.05) = 0 \,, && x_0 + 0.5 < x \leq x_0 + 1 \\ &= K_0\,(N+1)\,(1/0.5) = 2\,K_0, && x_0 + 1 < x \leq x_0 + 1.5 \\ &= K_0\,N\,(1/.05) = 0 \,, && x_0 + 1.5 < x \leq x_0 + 2 \end{aligned}$$

. . . .

As can be seen a permanent distortion in the written exposure has resulted from the change in term. Within six months all policies are on an annual basis and none will be renewed for an additional six months. This is generally true whenever the new term does not evenly divide the old term. For example, a change from five year term to three year term, with the customary assumptions, will have the following effect on written exposures:

$$\begin{aligned} f(x) &= K_0\,(3/5) \,, && x_0 < x \leq x_0 + 3 \\ &= K_0\,(6/5) \,, && x_0 + 3 < x \leq x_0 + 5 \\ &= K_0\,(3/5) \,, && x_0 + 5 < x \leq x_0 + 6 \\ &= K_0\,(6/5) \,, && x_0 + 6 < x \leq x_0 + 8 \\ &= K_0\,(3/5) \,, && x_0 + 8 < x \leq x_0 + 9 \\ &= K_0\,(6/5) \,, && x_0 + 9 < x \leq x_0 + 11 \\ &= K_0\,(3/5) \,, && x_0 + 11 < x \leq x_0 + 12 \end{aligned}$$

Here the pattern of one year writing at K_0 (3/5) followed by two years at K_0 (6/5) continues indefinitely.

EARNED PREMIUMS AND RATE ADJUSTMENT FACTORS

Earned premiums are the result of both earned exposures and rates; by rate we will mean the charge for some fixed amount of exposure, thus a change in term in itself does not result in a change in rate.

With constant rate r the earned premium is given by

$$EP\ (x_0, x_1; r) = rEE\ (x_0, x_1)$$

When there have been different rates $r_1, r_2 \ldots r_n$ which have been in effect on the intervals $(y_0, y_1), (y_1, y_2) \ldots (y_{n-1}, y_n)$ then the earned premium is given by

$$EP\ (x_0, x_1) = \sum_{i=1}^{n} r_i\, EC\ (y_{i-1}, y_i; x_0\, x_1)$$

Example:

$$f(x) = K$$

$$r_1{:}\ f\ x < x_0$$

$$r_2{:}\ f\ x \geq x_0$$

$$x_1 - x_0 = t = 1$$

$$\begin{aligned} EP\ (x_0, x_1) &= r_1\, EC\ (-\infty, x_0; x_0, x_1) + r_2\, EC\ (x_0, x_1; x_0, x_1) \\ &= \tfrac{1}{2}\, r_1\, Kt + \tfrac{1}{2}\, r_2\, Kt = \tfrac{1}{2}\ (r_1 + r_2)\ Kt \end{aligned}$$

Which says that with an annual term a rate change at the beginning of the year will result in one-half of the premium earned at the old rate and one-half at the new rate.

The rate level adjustment factor, which is simply the factor to multiply the actual earned premium by to arrive at what the earned premium would have been if it were all written at a constant rate r_i is given by:

$$AF(x_0, x_1; r_i\mathrm{Q} = r_i\, EE(x_0, x_1)/EP(x_0, x_1)$$

From the example above we have

$$AF(x_0, x_1; r_2) = r_2\, EE(x_0, x_1)/EP(x_0, x_1) = 2\, r_2/(r_1 + r_2)$$

THE ORIGINAL PROBLEM

We now have all the tools necessary to solve the original problem. The problem is to determine the rate level adjustment factors when the following conditions apply:

Rates:

$r_1, x < x_0$

$r_2, x_0 \leq x < x_0 + 1$

$r_3, x_0 + 1 \leq x < x_0 + 7/4$

$r_4, x_0 + 7/4 \leq x$

Terms:

$t = 3, x < x_0 + 1/4$

$t = 1, x \geq x_0 + 1/4$

With the assumption that the exposure was being written at a constant rate K_0 prior to annualization we have $f(x)$ as follows:

$$f(x) = \begin{cases} K_0, x < x_0 + 1/4 \\ 1/3\, K_0, x_0 + 1/4 \leq x < x_0 + 5/4 \\ 2/3\, K_0, x_0 + 5/4 \leq x < x_0 + 9/4 \\ K_0, x \geq x_0 + 9/4 \end{cases}$$

The earned premium at rate level r_4 would have been $r_4 K_0$ in each year. The actual earned premiums are estimated as:

$$\begin{aligned} EP(x_0, x_0 + 1) &= r_1 EC(-\infty, x_0; x_0, x_0 + 1) \\ &+ r_2 EC(x_0, x_0 + 1/4; x_0, x_0 + 1) \\ &+ r_2 EC(x_0 + 1/4, x_0 + 1; x_0, x_0 + 1) \\ &= 5/6\, r_1 K_0 \\ &+ 7/96\, r_2 K_0 \\ &+ 9/96\, r_2 K_0 \end{aligned}$$

$$
\begin{aligned}
EP(x_0+1, x_0+2) &= r_1\, EC(-\infty, x_0; x_0+1, x_0+2) \\
&+ r_2\, EC(x_0, x_0+1/4\,; x_0+1, x_0+2) \\
&+ r_2\, EC(x_0+1/4, x_0+1; x_0+1, x_0+2) \\
&+ r_3\, EC(x_0+1, x_0+5/4; x_0+1, x_0+2) \\
&+ r_3\, EC(x_0+5/4, x_0+7/4; x_0+1, x_0+2) \\
&+ r_4\, EC(x_0+7/4, x_0+2; x_0+1, x_0+2) \\
&= 48/96\, r_1\, K_0 \\
&+ \ \ 8/96\, r_2\, K_0 \\
&+ 15/96\, r_2\, K_0 \\
&+ \ \ 7/96\, r_3\, K_0 \\
&+ 16/96\, r_3\, K_0 \\
&+ \ \ 2/96\, r_4\, K_0
\end{aligned}
$$

$$
\begin{aligned}
EP(x_0+2, x_0+3) &= r_1\, EC(-\infty, x_0; x_0+2, x_0+3) \\
&+ r_2\, EC(x_0, x_0+1/4\,; x_0+2, x_0+3) \\
&+ r_2\, EC(x_0+1/4, x_0+1; x_0+2, x_0+3) \\
&+ r_3\, EC(x_0+1, x_0+5/4; x_0+2, x_0+3) \\
&+ r_3\, EC(x_0+5/4, x_0+7/4; x_0+2, x_0+3) \\
&+ r_4\, EC(x_0+7/4, x_0+9/4; x_0+2, x_0+3) \\
&+ r_4\, EC(x_0+9/4, x_0+3; x_0+2, x_0+3) \\
&= 16/96\, r_1\, K_0 \\
&+ \ \ 8/96\, r_2\, K_0 \\
&+ \ \ 0/96\, r_2\, K_0 \\
&+ \ \ 1/96\, r_3\, K_0 \\
&+ 16/96\, r_3\, K_0 \\
&+ 28/96\, r_4\, K_0 \\
&+ 27/96\, r_4\, K_0
\end{aligned}
$$

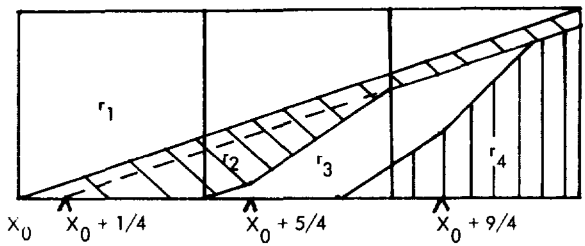
The rate adjustment factors are given by

$$AF(x_0, x_0 + 1; r_4) = r_4 \div [(5/6)\ r_1 + (1/6)\ r_2]$$

$$AF(x_0 + 1, x_0 + 2; r_4)$$
$$= r_4 \div [(1/2)\ r_1 + (23/96\ r_2 + (23/96)\ r_3 + 2/96)\ r_4]$$

$$AF(x_0 + 2, x_0 + 3; r_4)$$
$$= r_4 \div [(1/6)\ r_1 + (8/96)\ r_2 + (17/96)\ r_3 + (55/96)\ r_4]$$

Interestingly, the solution to this problem may be translated into a diagram which would look as follows:



Note that the line separating r_2 and r_3 changes slope at $x_0 + 5/4$ from 1/3 to 2/3; and the line separating r_3 and r_4 changes slope at $x_0 + 9/4$ from 2/3 to 1.

CONCLUSION

When $f(x)$ is other than a very simple formula, many of the equations become quite cumbersome; however, this presents no problem to a computer. More accurate rate level adjustment factors can be determined by making more realistic assumptions regarding the rate of exposure writings.

Throughout this paper it has been assumed that the premiums to be adjusted were calendar year premiums and that the changes in rates or term affected policies as they come due for renewal. In practice, other variations occur; it may be necessary to adjust policy year premiums rather than calendar year premiums. Also the rate or term changes may affect all outstanding policies rather than just renewal policies. These situations require techniques slightly different than those developed in this paper.

Aside from the relatively minor problem with the rate level adjustment factors and possible applications to corporate model building or more general areas where income or costs are deferred, the insight gained in the relationships between term, writings and earnings is of value in itself.

A CURRENT LOOK AT WORKERS' COMPENSATION RATEMAKING

ROY H. KALLOP

INTRODUCTION

The purpose of this paper is to provide a current look at workers' compensation ratemaking procedures employed by the National Council on Compensation Insurance. The paper has been long delayed since notable changes were anticipated both in determining rate level and classification rates. In 1974 the use of policy year aggregates in rate level calculations was introduced. The National Council is also close to completing a new approach in developing rates for classes with small credibility. This paper will describe in detail the rate level calculations and provide the reader with changes made to date in classification relativity since Ralph Marshall's paper was revised in 1961. An additional paper would be appropriate detailing the new classification relativity program when it is implemented.

The workers' compensation pricing program is briefly described in Section A, which covers the pricing of small size insureds including minimum premiums, loss and expense constants, and three year fixed rate policies as well as the pricing of large size insureds including premium discounts and individual risk rating plans.

Section B is a description of manual ratemaking and is divided into the following five parts:

1. Statistical data employed in computing workers' compensation manual rates.
2. Calculation of industry group and overall rate levels.
3. Calculation of manual rates.
4. Ratemaking procedures for classifications having unique conditions.
5. Appendix providing detailed calculations of a rate revision for a typical state.

   The Appendix will include detailed technical steps applicable to each stage of the ratemaking process described in Section B. The paper itself, however, primarily is a narrative description of the techniques employed which, hopefully, will be of interest to those desiring some insight into workers' compensation ratemaking procedures without becoming bogged down in a maze of figures, and it introduces no new concepts in ratemaking. The intention of this paper is to describe current workers' compensation ratemaking procedures.

SECTION A. WORKERS' COMPENSATION PRICING PROGRAM

Overall Pricing in Workers' Compensation Insurance

The first consideration is to determine the classification rate or rates that apply to the particular insured. In workers' compensation there are approximately 700 classifications of various operations for which a separate rate is established. This represents a very refined program to ensure that the price will have a direct tie-in with the occupational hazards involved.

Basis of Determining Premium

The manual rate for each insured is determined according to the business in which the employer is engaged. While workers' compensation insurance basically assigns a single classification descriptive of each employer's business, certain types of employees are generally separately classified and described as standard exceptions. These are clerical office workers, draftsmen, outside salesmen, and drivers, unless specifically included in the scope of the classification. The basic classification approach also contains specific provisions for division of payroll for mercantile operations, construction or erection work, and those businesses which qualify for two or more classifications under the multiple enterprise rule. If two or more classifications, exclusive of standard exceptions, apply to an employer, then the governing classification is the code that carries the largest amount of payroll. The governing classification determines the assignment of the loss constant and the General Inclusion payrolls.

The fundamental concept underlying workers' compensation ratemaking and pricing is that the exposure to risk of each employer is in part a function of the business in which he is engaged. Because it is expected that each employer engaged in the same type of business would have a similar distribution of employees performing comparable functions, it follows that a single all-inclusive classification is the most practical method of determining premium. In this manner, all industries producing the same product or utilizing the same manufacturing process are grouped for rating purposes and pay the same basic premium charge. Consequently, each industry is responsible for its own job-related injuries.

Payroll is the basis of exposure for determining premium. It is readily available and reflects exposure to injury. Manual premium is determined by multiplying the manual rate by the payroll in $100 units. For example, a payroll of $50,000 at a $1.00 manual rate will produce $500 of manual premium. When an employee works overtime, the payroll in excess of straight time pay is excluded. For the last fifteen years or so, payroll used for premium computation purposes had been limited to an average of $300 per week in most states. However, a growing number of workers are earning over $300 per week during the current inflationary period. As a result, these workers were being priced on a head-count basis, with no reflection of hours worked. For example, an employee in the contracting business earning $8 an hour working 40 hours a week is charged the same amount of premium as another worker in the same business earning the same hourly rate working 60 hours a week. There is a greater likelihood that the second employee will suffer an injury than the first since he is on the job for a longer period of time. Hence, it became desirable to utilize the free flow of payroll in order to best reflect exposure to injury. During the past year, the National Council on Compensation Insurance has been filing for the use of total payroll to determine premium, except for executive officers and employees in certain classifications having a relatively high manual rate and wide range of wage-paying practices, such as professional athletes. In these instances, payroll is limited to $500 per week for circuses and athletic teams, and $300 per week for executive officers.

Manual rates apply to all insureds both large and small. The manual rates, however, are only one part of the overall pricing program which also includes various costing programs that bear on the size of the insured's operations.

All members of the National Council including stock carriers, mutual carriers, reciprocals, and competitive state funds use the manual rates published by the Council. There are provisions, however, in many states where a carrier may deviate, such deviation generally falling into the category of a uniform percentage increase or decrease for a period of at least one year.

Pricing of Insureds with Small Premium Volume

For small size insureds where the cost of handling the account and the expectancy of loss represents a much higher percentage of premium than for the large size insureds, programs have been developed toward achieving price equity. This includes minimum premiums, expense constants, and loss constants. In addition, a three year fixed rate policy program is available as a means of writing small businesses for less cost. A description of these programs follows.

Expense Constants

A $15 expense constant is charged per policy to insureds whose annual premium is under $200, and a $10 expense constant is charged per policy for insureds whose annual premium is between $200 and $500. These charges are made because small insureds have a much higher percentage of expense related to manual premium than the large insureds. This comes about since certain fixed costs represent a much greater proportion of a small annual premium than a large annual premium. The expense constant program is a means of distributing expense costs according to need. The expense allowance underlying the manual rates anticipates the collection of expense constant dollars. Hence, manual rates are lower than they would be if no expense constant program were in existence. In other words, the overall premium is the same but more expense dollars are collected from the small insured.

Loss Constants

Another feature of price equity between large and small size insureds is the loss constant program. Loss constants are flat charges which vary by state and by industry group and apply to insureds whose annual premium is less than $500. Normally, there are three industry groups consisting of manufacturing, contracting, and all other classifications. The principle of loss constants is to improve the loss ratios of small insureds. Experience shows that small insureds normally have a loss cost per exposure unit which is greater than the corresponding loss cost for large insureds. Loss constants endeavor to bring the loss ratios of large and small size insureds into closer alignment. Again, as with expense constants, rates are offset in anticipation of collecting loss constant dollars, and consequently the overall premium is unaffected.

Minimum Premiums

The minimum premium is the minimum price for writing a workers' compensation policy. Originally, it was designed to provide premium for one full-time worker. However, the formula was not revised over the years to maintain this level. The concept of a minimum premium is that the carrier must receive a minimum amount to defray the cost of issuing a policy and to provide premium for assuming the hazards being insured.

The minimum premium formula during the past several years assumed an annual wage of $2,500. Today, of course, the average annual wage is over $5,000, and the $2,500 figure is not a true reflection of current conditions.

Small premium policies have always been a problem area in workers' compensation insurance because they do not provide sufficient dollars to cover the cost of policy handling. Also, even a small loss may wipe out premiums of many years. A classification with a $.10 rate in a state with a $15.00 expense constant and a $10.00 loss constant produces a minimum premium of only $28.00 based on an annual wage of $2,500. This would be used almost entirely for the expense of issuing and handling the policy with little or no premium left over for assuming the liability to pay losses.

In order to maintain a market for small size insureds, the following minimum premium program is now being filed in each state:

1. In lieu of an assumed payroll of $2,500, a payroll of $3,500 is used. This means that the minimum premium is calculated by taking 35 times the class rate, plus loss and expense constants.

In addition, it is intended that in future years the minimum premium should be related to the state average annual wage (rounded to the nearest $500) as reported to the National Council semi-annually by the carriers. Thus, instead of 35 times the rate, future revisions would utilize the annual wage rounded to the nearest $500 and establish a multiplier based upon such wage. For instance, if the annual wage should be $5,245, the multiplier would be 50 and the minimum premium would be 50 times the rate, plus loss and expense constants.

Recognizing that utilizing the state average annual wage cannot be accomplished in one step, there will be no proposal to change the minimum premium formula greater than a ten point multiplier in one year. Annually thereafter, the multiplier would change in accordance with any changes that might occur in the state average annual wage.

2. A minimum charge of $35 was established for any insured.

3. A maximum charge of not more than $500 was established for any insured. It was recognized that in those instances of high rated classifications, the minimum premium formula can produce a fairly substantial minimum premium. It was felt that limiting premiums to $500 would be consistent with the principle underlying the establishment of loss and expense constants.

Three Year Fixed Rate Program

The three year fixed rate policy program was established to permit the underwriting of small size insureds at less cost. This is a plan whereby an insured whose annual premium is less than $300 may be written for a period of three years, at the manual rate in effect at the inception date of the policy. This rate will not be changed unless there is an adjustment of outstanding policies in excess of 10% as a result of a law amendment. Law amendments will be described later in this paper.

There is an inducement for the insured to pay his premium in advance. In such instances he would only pay one expense constant for the three year period. If he pays his premium in annual installments, he is charged two expense constants for the three year period.

Pricing of Insureds with Large Premium Volume
Premium Discounts

For insureds whose total annual standard earned premium is in excess of $1,000, premium discounts apply on a mandatory basis. In other words, the amount of discount given to an insured operating in more than one state is based on his total premium for all states where he has operations, not just the premium for one state. Standard earned premium is premium after the application of experience rating which is described in the next section. Premium discounts are afforded since there is a reduction in expenses (as a percentage of premium) incurred by the carrier as the size of the insured increases. There are two schedules of discounts, one for stock carriers and the other for non-stock carriers. The discounts given by non-stock carriers are less than the discounts given by stock carriers because non-stock carriers anticipate granting dividends to policyholders.

Stock carriers may use the non-stock discount table and non-stock carriers may use the stock discount table if they desire. However, in the state or states where they have opted to transfer, they must use the table for a period of at least one year on all of their compensation business in that state. The incidence of companies transferring to the other table is quite low.

For stock carriers, the current discounts are based on the following gradation of expenses:

Standard Premium	(1) Production Expenses	(2) General Expenses	(3) Total of Graduated Items	(4) Amount of Gradation
First $ 1,000	17.5%	8.4%	25.9%	0.0%
Next 4,000	12.5	4.6	17.1	8.8
Next 95,000	7.5	4.6	12.1	13.8
Over 100,000	6.0	4.6	10.6	15.3

The amount of gradation in expense provisions adjusted for profit and taxes determines the percentage of discount allowed. For example, the premium discount for the "Next $4,000" premium interval is derived by dividing 8.8% by the complement of the 2.5% profit allowance and the average countrywide tax allowance of 3.8%. [0.088 ÷ (1.000 − 0.025 − 0.038) = 0.094]. The current premium discount allowances are as follows:

Standard Premium	Premium Discounts
First $ 1,000	0.0%
Next 4,000	9.4
Next 95,000	14.7
Over 100,000	16.3

The expense gradation for non-stock carriers is only available in the aggregate, with the following premium discounts currently in effect:

Standard Premium	Premium Discounts
First $ 1,000	0.0%
Next 4,000	3.0
Next 95,000	6.0
Over 100,000	8.5

Individual Insured Rating Plans

(a) Experience Rating Plan (Prospective)

For those insureds whose annual premium is $750 or more, the manual premium is modified either upward or downward according to the insured's own experience over the past three year period. If the insured develops favorable experience, he receives a reduction (credit) in his manual premium; if the insured develops unfavorable experience, a debit (surcharge) will apply. The experience rating modification will apply to the forthcoming year; hence, the application of the plan is prospective in nature. Since the large size insureds normally have a loss cost per exposure unit which is less than the corresponding loss cost for small insureds, more credits are granted in experience rating than debits.

The experience rating modification is prepared by the rating bureau having jurisdiction and is mandatory regardless of the carrier currently writing the policy.

(b) Experience Rating Plan (Retrospective)

In addition to the mandatory prospective rating plans, there are optional retrospective rating plans available which may be agreed to by the insured and his carrier at the inception of the policy. These plans set forth conditions whereby the premium actually paid depends on the loss experience generated by the insured during the time the policy is in force, subject to a specified maximum and minimum premium. Appropriate net insurance charges offset the effect of the maximum and minimum limitations. The insured and the carrier select the maximum and minimum limitation which best suits the needs of the insured. This can be done from a series of tables (Plans A, B, C, and J) or can be developed from a formula (Plan D). Three year agreements are also available under retrospective rating.

The eligibility requirement for retrospective rating is an annual premium as low as $1,500 for certain plans.

The same expense graduations underlying the Premium Discount Plan are an integral part of all retrospective rating agreements. Hence, if an insured is under a retrospective rating plan, the agreement is in lieu of the Premium Discount Program, not in addition to premium discounts.

SECTION B. DESCRIPTION OF MANUAL RATEMAKING

1. Statistical Data Employed in Computing Workers' Compensation Rates

At the time Ralph Marshall's paper was written, the formula to determine overall rate level was to give equal weight to policy year data and calendar year data. This formula is still in effect today. The only change made is in the source of the policy year data. Until recently, policy year totals from unit statistical reports were used. Now, policy year aggregates from financial data are employed. Unit statistical plan data continues to be used to calculate individual classification rates. The statistical data used is data solely from the state under review. Only the distribution tables used in valuing law amendments which are described later in this paper are developed from countrywide data.

Unit Plan Data (See Appendix, Exhibit II)

Unit plan data is composed of statistical reports which are submitted to the National Council by its members in accordance with the Unit Statistical Plan which has been filed and approved by state regulatory bodies. The Plan provides for the reporting of payroll, manual premium, and incurred loss data by classification code by state for each policyholder. Incurred losses include amounts paid, plus amounts still to be paid. Losses used for ratemaking must represent the total liability of the carrier in discharging its obligation. Losses are valued 18 months after the inception date of the policy, and reports are due to be reported to the National Council two months later. At the time of valuation, there are cases for which the total benefit cost is not yet known. In these instances an estimate is made based upon the facts known at that time. If any losses are still open as of a first valuation date, or are subsequently reopened or reported, a second report is required a year later. A claim is considered to be open if all benefits have not been fully paid. The second valuation could be greater or less than the original estimate depending upon whether the condition of the injured worker has worsened or improved. If any losses are still open as of the loss valuation date of the second report, a third report is required the following year. Similarly, fourth and fifth reports are required if any loss or losses remain open.

Data is submitted by carriers in batches at monthly intervals. The data is keypunched and grouped into 12 month policy periods separately for each state. There is no necessity for such periods to begin on January 1. In order to utilize the most recent experience, policy periods are staggered throughout the year. The policy period is keyed to the anticipated effective date of the proposed rates in the state, allowing sufficient time for preparing the filing.

Losses are identified by type of injury; i.e., fatal, permanent total, permanent partial, and temporary total. Indemnity and medical losses are shown separately. The National Council classifies permanent partial cases into two categories, a major or minor case, according to a critical value which varies by state. These values are adjusted periodically to keep pace with law amendment changes. Losses reported at or in excess of such critical values are classified as major permanent partial claims, and losses reported below such critical values are called minor permanent partial claims. In general, major permanent partial claims involve loss of major members of the body such as a hand, a foot, or a leg, while minor permanent partial claims involve minor members of the body such as a finger, a thumb, or a toe.

Losses reported under the Unit Statistical Plan are limited for use in ratemaking, in order to prevent any one big single claim or multiple claim from having an unduly strong influence on the indicated pure premium. The limitations are as follows:

1. Single claims are limited to 10% of the self-rating point used in experience rating.
2. Multiple claims (an accident where more than one worker is injured) are limited to 20% of the self-rating point used in experience rating.
3. The amount of disease loss that can enter any one class in any one policy year is limited to 25% of the self-rating point used in experience rating.
4. Employers' liability claims are limited to $100,000 exclusive of loss adjustment expense.

The carriers have an option in reporting three year fixed rate policies under the unit statistical plan. They can either submit unit reports for each insured or they may submit data on a Schedule Z basis; i.e., a summary by class by effective year.

For reporting purposes, the experience on three year fixed rate policies is assigned to the year in which the policy became effective, regardless of expiration date. Losses are valued not earlier than March 31 and filed not later than September 1 of the fourth year after the year in which the policy became effective. For example, the experience on three year fixed rate policies becoming effective in 1971 was filed not later than September 1, 1975, with losses valued not later than March 31, 1975. No subsequent reports are made.

Policy Year Aggregates (See Appendix, Exhibit I, Section A)

Over the years, elements in the ratemaking formula have changed as the need required. The workers' compensation ratemaking system has always been under study so that it could keep pace with current conditions. Several years ago it was noted that loss development, that is, the changes in the estimates of the cost of cases over a period of years, were no longer adequately being measured by the use of three consecutive unit statistical reports. It was evident that the character of workers' compensation administrative and benefits programs had changed over the years and the final determination of incurred losses could no longer be considered as available with the use of three subsequent reports. Consequently, the calculation of development factors was changed to use four reports and, later, five reports. Further study indicated that there could be significant development beyond a fifth report. At this point in order to measure such development, it was decided to make use of policy year aggregates valued at calendar year end in lieu of unit statistical reports. This improvement in the process of measuring incurred losses to an ultimate value was made approximately three years ago.

Policy year aggregate data are compilations of loss payments, loss reserve changes, written premium transactions, and unearned premium reserve changes associated with the particular policy year involved. Thus, policy year 1971 would involve all such transactions arising out of policies issued between January 1 and December 31, 1971. Policy year aggregate data would also include the insurance company's judgment as to the amount of incurred but not yet reported claims and the estimated additional cost on closed claims which will be reopened in the future.

In the course of further study of the development problem, it became apparent that the ability to make an adequate determination of losses in the first instance would be improved by use of policy year aggregate data.

Tests showed this to be true. Further, with the dynamic environment in which workers' compensation insurance operates today, it was important to do so to preserve the ability to make rates which are a more current reflection of loss costs. It is expected that the use of policy year aggregates will improve the workers' compensation ratemaking system and make it more responsive to forces affecting costs, both upward and downward. In 1974, the National Council began making filings in which policy year rate levels were based on financial data recorded for first and subsequent reports.

Calendar Year Data (See Appendix, Exhibit I, Section C)

Calendar year experience also is used to determine rate level. The reason for using calendar year experience is to recognize the very latest experience available. These data are obtained from semi-annual calls issued by the National Council to its membership. Standard earned premium and incurred losses are obtained by state. Net earned premium also is obtained in the call requesting data for the full year. Calendar year premiums are determined by adding to the premiums written during the year the unearned premium reserves at the beginning of the year and subtracting the unearned premium reserves at the end of the year. Calendar year losses are determined by adding to the losses paid during the year the loss reserves at the end of the year and subtracting the loss reserves at the beginning of the year.

Calendar year experience is more recent data than policy year experience. Calendar year 1974, for example, includes the incomplete policy year 1974 consisting of all premium and loss transactions on policies effective in 1974 which were recorded in 1974. The complete policy year 1974 aggregates will not be available until the following year. Calendar year experience includes all premiums earned and losses incurred during the calendar year period regardless of the effective date of the policies producing the data. These data reflect all cost factors which affect compensation underwriting results, including not only the most recent changes in wages but also the most recent changes in the frequency and severity of claims.

Financial data is not available on a classification basis. It is statewide data exclusive of excess policies, U.S. Defense Projects Rating Plan risks, and coal mine experience. Carriers are now beginning to exclude experience under the United States Longshoremen's & Harbor Workers' Compensation Act since a separate ratemaking procedure has been established for the classifications falling under this Act—the so-called "F" Classifications. This procedure is described later in the paper. The changes from one year to the next in the policy year aggregates previously mentioned can be summed to reconcile with calendar year data. Since insurers also report calendar year experience to regulatory officials in their Insurance Expense Exhibits annually, such data can be further reconciled to this source. Specifically, the calendar year premiums and losses shown in Part IV of the Insurance Expense Exhibit should agree with the net earned premiums and incurred losses reported to the National Council for calendar year data taking into consideration the aforementioned exclusions. These exclusions are included in the Insurance Expense Exhibit data.

In determining rate level, the amount of loss for a single or multiple accident is limited to 5% of the standard earned premium for the preceding calendar year. The rationale here is that both single claim and multiple claim losses should be included in rate level, except an unusually large catastrophe such as a Texas City disaster. At one time, a much lower limit was applicable for excluding losses from catastrophes. However, there was a one cent loading in the rates for catastrophes. There is no catastrophe charge applicable today.

Distribution Tables—Valuation of Law Amendments
(See Appendix, Exhibit II-B)

The benefits payable to injured workers are adjusted periodically by state legislatures. In these instances it is necessary to determine the percentage increase in cost of the new law to the old law in order to determine what past losses will cost at the new law level. Each state has its own compensation act which prescribes a schedule of benefits for each type of injury. A typical compensation act establishes weekly payments as a percentage of the injured worker's average weekly wage subject to a maximum and a minimum weekly benefit. For example, the injured worker receives a specified percentage, say 66⅔% of his wages earned at the time of injury. A common provision is to set the maximum and minimum benefits as a percentage of the state average weekly wage. If the maximum weekly benefit is established at 100% of the state average weekly wage, and the state average weekly wage is $150, the maximum weekly benefit is $150. A worker earning $300 per week would receive a weekly benefit of two-thirds of $300, limited however to the maximum of $150. Payments usually are made during the entire period of total disability. Most permanent partial disability payments are limited according to a specified schedule. For example, the duration of payments for a dismemberment of an arm might be payable for 200 weeks, a dismemberment of a leg payable for 250 weeks, a loss of a hand payable for 125 weeks, etc. In death cases, benefits generally vary by type of dependency, with widows in many states receiving life pensions providing they do not remarry. In temporary total cases, benefits are payable during disability following a waiting period—usually three days—but payable from day of disability if disability lasts more than a specified duration—usually, one, two, or three weeks.

Whenever benefits change, say the maximum benefits increase, the effect of the law change is determined for each type of injury. This is accomplished by developing monetary costs under the old law, and under the new law, based on (1) the old and new benefit provisions using an accident distribution table in the case of permanent partial cases, (2) a dependency distribution table for fatal cases, (3) a disability table in the case of temporary total cases and (4) a standard wage distribution table to measure the effect of the maximum and minimum weekly limitations in computing the average weekly benefit for each type of injury.[1] The overall cost of the new law is determined by weighting the individual cost effects by type of injury with the latest statewide distribution of losses by type of injury.

Increases in benefits require an adjustment of outstanding policies if the overall increase in benefits results in an adjustment of 1% or more of premium. Such adjustments are made since the carriers are liable for the payment of the higher benefits the day the law goes into effect, and the rates applicable to the policy do not contemplate the higher benefit level.

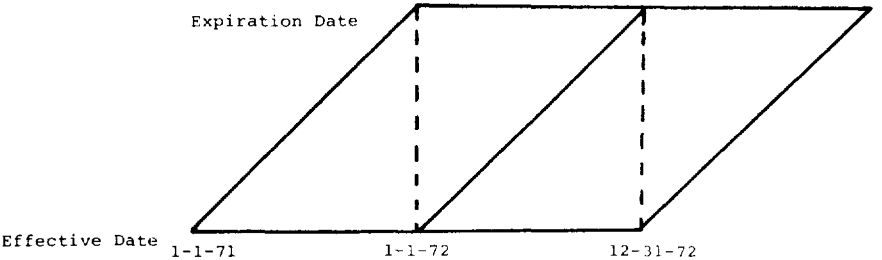
2. *Calculation of Industry Group and Overall Rate Levels (See Appendix, Exhibit I)*

The approach to workers' compensation ratemaking is similar to that used in many lines of insurance whereby premiums and losses of the past are brought up to current conditions, and the resulting loss ratio is compared to an expected loss ratio. If past data is better than expected, a rate level decrease is indicated; and if past data is worse than expected, a rate level increase is indicated. Rates are set prospectively based on past experience at current levels. The rates are designed to produce premium which is adequate to pay for (1) losses which are expected to be incurred, (2)

[1] See Fratello, Barney, 'The "Workmen's Compensation Injury Table" and "Standard Wage Distribution Table"—Their Development and Use in Workmen's Compensation Insurance Ratemaking,' *PCAS* XLII, pp. 110-202.

expenses, and (3) a margin of 2.5% for contingencies and underwriting profit.

The basic data underlying the policy year experience are from financial data records. The two latest complete policy periods are used in the determination of rate level. For example, for policy year aggregates valued as of 12/31/73, policy years 1971 and 1972 would be used. This is illustrated by the following diagram:



Policy year 1971 covers all policies effective in 1971, the last policy expiring as late as 12/31/72; policy year 1972 covers all policies effective in 1972, the last policy expiring as late as 12/31/73.

Policy year data is homogenous data in that the premiums and losses all come from the same set of policies. It is that part of the rate level formula which represents "stability" and is given 50% weight in determining rate level.

Policy year premiums used at the present time are net earned premiums from the Supplementary Call for Policy Year Aggregates, adjusted to a standard earned basis. This adjustment is accomplished by applying the same ratio of standard to net as existed in the two calendar years in which the policy occurred. Carriers now are requested to provide standard earned premiums as well as net earned premiums when submitting policy year aggregates. It is expected that standard earned premiums will be available in the near future.

The standard earned premium thus derived is then brought up to current rate level. This is accomplished through the use of index numbers. When making this adjustment it is assumed that an even distribution of exposure applies throughout the policy period. All rate levels which became effective during or subsequent to the policy period are indexed to a common base. The average rate level for the policy period is determined by depicting rate levels which occur during the policy period according to proportionate areas. This adjustment is computed separately for each of the two policy years used in determining rate level. The indicated premium adjustment for each policy year is the ratio of the current index to the average rate level of the policy year. A further adjustment is made to exclude expense constant premiums. This adjustment is necessary since the premium derived from rates excludes expense constant premium. The permissible loss and loss adjustment ratio referred to later in this section has been adjusted to anticipate this other source of income. Loss constant premium, on the other hand, has not been excluded. Instead, a separate rate reducing factor called the Loss Constant Offset (described later in this paper) is applied to rates to recognize this additional source of income.

Incurred losses also must be brought up to the current benefit levels. This is done in a manner similar to adjusting premium to current rate level; i.e., law amendments which have occurred during or subsequent to the policy period are indexed to a common base. In this instance an even distribution of loss occurrence is assumed throughout the policy period. The average benefit level for the policy period is developed by weighting each benefit level that cuts through the policy period by its proportionate area. The adjustment to apply to each policy year aggregate loss is the ratio of the current index and the average benefit level of the particular policy year.

Tracking losses to their ultimate cost level is necessary if rates are to reflect ultimate liability. The losses used in ratemaking are converted to an ultimate reporting basis through the use of development factors. These factors are determined by tracking the movement of losses for older policy periods to their ultimate level. By applying these factors to the most recent policy periods, it is assumed that the new experience will develop from year to year in the same manner as the older policy year data.

The latest of the two policy years used in rate level is on a first reporting basis; i.e., it represents the first time the complete policy year is available. For example, policy year 1972 valued as of 12/31/73 is considered a first report. The earlier of the two years, policy year 1971 valued as of 12/31/73, is on a second report basis. The development factors are designed to adjust the earlier year from a second to an ultimate reporting basis, and the latest year from a first to an ultimate reporting basis.

The use of the three most recent calls for policy year experience valued at calendar year end provides the movement of premiums and losses by policy year for two consecutive years. Hence, it is possible to get development from a first to a second report basis for the two most recent periods where a second report is available. Similarly, it is possible to determine the movement of premiums and losses from a second to a third reporting basis for the two most recent policy periods where a third report is available. In like manner, the movement from third to fourth reports, fourth to fifth reports, etc., to ultimate can be determined.

The development factors are obtained by multiplying the average development from first to second report (for the two latest policy periods where a second report is available) times the average development from second to third report (for the two most recent years where a third report is available), etc., to an ultmate level. The supplementary call for this information requests carriers to provide policy year aggregates for all prior years. The call for policy year aggregates was a major undertaking and internal recordkeeping procedures in many companies had to be revised. Some companies could not supply such data separately for older policy years. However, they were able to provide this information on a prospective basis. Hence, it is necessary to "match" like companies in each layer of development since all carriers could not provide data for older policy years.

The final adjustment applied to losses is to include loss adjustment expense as a function of losses. Loss adjustment expense includes the cost of investigating cases, representing the employer before claims adjudicating bodies, defending law suits, etc. The allowance includes both allocated and unallocated expense since workers' compensation losses exclude all loss adjustment expenses except allocated loss adjustment expenses for Coverage B claims which are reported as losses. Currently, loss adjustment expense is expressed as 12.5% of expected losses, it having recently been reduced from 13.0% of expected losses. Loss adjustment expense traditionally tracks losses more closely than premiums and, therefore, is more appropriately expressed as a function of loss. The same results can be obtained by setting loss adjustment expense to an equivalent percentage of premium.

To summarize, policy year premiums are adjusted to current rate levels and policy year losses are adjusted to current law levels, converted to an ultimate liability level and further adjusted to include loss adjustment expense. The resulting modified loss and loss adjustment ratio then is divided by the expected loss and loss adjustment ratio to determine policy year overall rate level. The expected loss and loss adjustment ratio, more commonly referred to as the permissible loss and loss adjustment ratio, is the complement of the expense allowance included in manual rates.

Each year expenses are reviewed to determine the expense needs of the carriers for the forthcoming period. This entails a review of past expenses based on data reported in the Insurance Expense Exhibit to observe if any expense item is trending upward or downward. If a trend is apparent, a change in the allowance for such item is made. This could result in either a change in the expense allowance included in the manual rates or a change in the premium discount percentages.

The present allowance for expenses applicable to the first $1,000 of premium is:

1.	Acquisition and Field Supervision	17.5%
2.	General Expenses	8.4
3.	Profit and Contingencies	2.5
	Total for Company Expense and Profit	28.4%
4.	Taxes	Vary by state

The amount of taxes includes state taxes plus a 0.7% allowance for miscellaneous taxes, licenses, and fees. The state taxes include all taxes that are levied as a percentage of premium. Taxes which are levied in the form of assessments based on losses are accounted for in the modification of policy year and calendar year losses to current level. Such assessments become part of the loss modifier in the same manner as development factors and law amendment factors. Assessments based on losses that are limited to certain types of injury such as a sum payable to a Second Injury Fund in a no-dependent death case are included in the experience reported to the National Council and, therefore, no factor is required.

The permissible loss and loss adjustment expense ratio is the complement of the sum of 28.4% and the tax allowance.

A common provision in rate regulatory laws is that due consideration shall be given to a reasonable margin for underwriting profit and contingencies. In workers' compensation insurance, a profit and contingency allowance of 2.5% has been in use for at least 25 years. The 2.5% underwriting profit contemplates additional profits from other sources to realize an adequate rate level.

Earlier in this paper it was mentioned that large size insureds normally have a loss cost per unit of exposure which is less than the corresponding loss cost for small insureds. Hence, in the experience rating plan more credits are given than debits. As a result, the total premium collected after experience rating (i.e., standard earned premium) is less than that premium produced by manual rates. In rate level, since both policy year and calendar year premium are on a standard basis (i.e., after the application of experience rating) and the permissible loss and loss adjustment ratio is a function of standard earned premium, it is not necessary to correct for the off-balance. Under the old rate level method, however, policy year premium generated by extending payrolls times current rates was at manual level (i.e., before the application of experience rating). Therefore, it was necessary to convert this premium to a standard earned basis in order that a proper comparison could be made when the policy year loss ratio was related to the permissible loss ratio. No correction was necessary on the calendar year premium entering rate level because this premium was on a standard earned basis.

The next step is to bring in the effect of the most recent calendar year experience. As mentioned earlier, the rate level formula gives equal weight to policy year and calendar year indications.

The calendar year data used in the rate revision covers all premium and loss transactions during the latest 12 calendar months available. As indicated earlier, calendar year data is obtained from semi-annual calls for experience. Geometrically, calendar year experience can be represented by a square covering 12 months ending June 30, or 12 months ending December 31.

The same procedure for adjusting experience to current level applies to calendar year premium and loss data as was employed with policy year aggregates; i.e., through the use of index numbers, premiums are adjusted from the average rate level of the calendar year to the latest rate level, and the losses are adjusted from the average law level of the calendar year to the latest law level. The same assumption of an even distribution of exposure and loss occurrence are made as were used in adjusting policy year data. It is realized, however, that calendar year incurred losses include changes in reserves of old claims. A new method using policy year contributions to calendar year experience will be implemented shortly in order to more accurately adjust old claims to current level.

The recognition of calendar year experience in rate level is accomplished by the inclusion of a rate level adjustment factor. The rate level adjustment factor expresses the effect of the calendar year data as a multiplier to the policy year indications. The effect of calendar year data expressed as a multiplier is most convenient when developing classification rates described in the next subsection. The calendar year loss ratio at current level and the policy year loss ratio at current level are given equal weight when calculating the rate level adjustment factor. The product of the policy year rate level change and the rate level adjustment factor produces the indicated overall change in rate level.

Rate levels are then determined for three broad industry groups, namely Manufacturing, Contracting, and All Other. This is accomplished by distributing the overall effect according to the relativity indicated by unit statistical plan aggregates. Experience is available by classification under the Unit Statistical Plan and, therefore, can be assembled into Manufacturing, Contracting, and All Other industry groups for this purpose.

Committees of the National Council are constantly reviewing the rate level formula in order to ensure that the existing program responds effectively to current conditions. The recent adverse experience indicates that additional steps may be forthcoming. Several possible areas are being explored, such as the assignment of greater weight to calendar year experience and trend factors.

The use of policy year aggregates from financial data records, in lieu of Unit Statistical Plan data, is felt to be a step forward but not necessarily the final answer.

3. *Calculation of Manual Rates*

After determining the required changes in premium level, the next step in the ratemaking procedure is to distribute these changes among the various industry classifications. The first step is to develop pure premiums for each classification. A pure premium is the amount of loss per $100 of payroll. For example, if the total loss for a classification was $500, and the classification payroll was $50,000, the pure premium would be $500 ÷ [50,000 ÷ 100] = $1.00.

Reviewed Classifications—Pure Premiums (See Appendix, Exhibit II)

The reviewed classifications consist of those classifications whose experience is of sufficient volume to warrant the assignment of some " credibility" or weight to the latest experience for the individual classifications.

Pure premium exhibits are developed which show in detail the experience for each reviewed classification. The data shown in these exhibits are from second reports under the unit statistical plan for the earlier policy year, from first reports under the unit statistical plan for the latest policy year, and from the experience of the three year fixed rate policies for the two latest years. The three year policies are equivalent to a third report for the first 12 months of experience, a second report for the second 12 months of experience, and a first report for the latest 12 months of experience. Losses are at current benefit level and include development factors and loss adjustment expense of 12.5%.

In order to adjust losses to current level, amendment factors are calculated for each type of injury. This is done in the same way that the overall amendment factor was calculated by adjusting the policy year aggregate loss data whereby benefit changes occurring during and subsequent to the policy period are indexed to a common base, and the amendment factor determined by dividing the current index by the average benefit level for the policy period. Again, the average policy year loss level is computed by using proportionate areas of the policy period which is represented geometrically by a parallelogram.

Development factors also are computed in the same manner as development factors for policy year aggregates, by averaging the movement of the premium, indemnity losses, and medical losses for the two latest periods for each reporting from the respective amounts compiled for the preceding report. As indicated earlier in this paper, unit report data is available up to a fifth report. In order to convert policy year unit plan data to an ultimate basis, it is necessary to use the indications of development from fifth report to ultimate from financial data records. A further adjustment is required to develop losses to levels indicated by policy year aggregates used in determining rate level. This is obtained by adjusting losses by the ratio of the policy year earned loss ratio at current level from policy year aggregates and the policy year earned loss ratio at current level from unit plan data.

Losses are combined into serious, non-serious, and medical components. Serious losses consist of death, permanent total, and major permanent partial claims. Non-serious losses consist of minor permanent partial and temporary total claims. Medical losses consist of all medical claims including both compensable and noncompensable cases.

The pure premiums included in these classification exhibits are as follows:

a. *Indicated:* These are the pure premiums indicated by the experience for the classification adjusted to current levels as described above. When a new law amendment develops, and it is known at the time a rate revision is to be prepared, it is included in the amendment factors in both rate level and classification relativity. There are occasions, however, when a law change is not known until the pure premiums have already been prepared. In these instances, the law amendment is applied in the final calculation of rates by parts: serious, nonserious, and medical.

b. *Underlying Present Rates:* These are the pure premiums underlying the rates currently in force. The procedure used to produce these underlying pure premiums involves the following values which are obtained from the previous rate revision:

   Proposed Pure Premiums

   Rate Level Adjustment Factor

   Test Correction Factors (explained later in this section)

   Ratio of Manual Premium to Earned Premium

The calculation is as follows:

The proposed pure premiums from the preceding rate revision for serious, non-serious, and medical are adjusted by applying the Rate Level Adjustment Factor from the preceding rate revision, the corresponding industry group test correction factors from the preceding rate revision, and the present ratio of manual premium to standard earned premium divided by the proposed ratio of manual premium to standard earned premium to each pure premium.

The rationale here is that last year's test correction factor and the rate level adjustment factor were applied after the proposed pure premiums were calculated and must be included as part of this year's underlying pure premiums. Secondly, the pure premium present on the rate level described below includes the policy year rate level change. The rate level change includes any change in the off-balance of the experience rating plan. These changes are reflected in the calculation of rates after the pure premiums are determined and should not affect the pure premiums. The above formula effectively cancels out these changes in the pure premium present on rate level.

The resulting partial pure premiums then are adjusted in instances where there is a law amendment that is included in this year's amendment factors but was not included in last year's rates by applying to them the benefit level change by parts. This adjustment is made in order that this year's proposed pure premiums will include the effect of the law amendment in every instance. As explained later in this paper, in some instances the underlying pure premium is selected as the proposed pure premium. This produces the Partial Pure Premium "Underlying Present Rates".

The total pure premium is obtained by adding the partial pure premiums and rounding the sum to two decimal places.

c. *Present on Rate Level:* These are the pure premiums underlying present rates (see paragraph "b" above) brought to the proposed premium level by the application to the partial pure premiums of factors representing the effect of any proposed changes in policy year premium level. The overall effect of the benefit level change is removed from the policy year premium change before application to the underlying pure premium. The change, exclusive of law, then is applied to the partial pure premiums. The law change has been excluded since it already has been included in the underlying pure premiums.

Whenever there is a change in expenses, such change is reflected in the proposed policy year premium level indication. Therefore, this change must be removed from the pure premium present on rate level because expense changes will be recognized later in the calculation of rates and should not be duplicated in the pure premium exhibits.

d. *Derived by Formula:* The formula pure premium is derived by a mathematical weighting between the indicated and the present on rate level pure premiums. The weight given to the policy year partial indicated pure premium varies from zero percent to 100 percent depending upon the volume of expected losses for serious, non-serious, and medical, respectively, for the classification. Expected losses are derived by multiplying the payroll, in \$100 units, by the partial pure premium underlying the present rates. Expected losses are used in assigning credibility because expected losses represent the normal probability of occurrence. Actual losses, on the other hand, are a matter of chance whereby very favorable experience would produce less credibility than that assigned on the basis of expected losses, and unfavorable experience would produce more credibility than that assigned on the basis of expected losses. The complement of the weight given the indicated pure premium is applied to the present on rate level pure premium. Thus, if 80% credibility is assigned to the indicated, 20% is applied to the present on rate level pure premium. A table of credibilities is used to assign weights to the indications for each of the three industry groups. To the extent a classification grows in volume and attains credibility, the classification makes its own rate. The requirement for full credibility for serious losses is an expected loss amount equal to 25 times the average serious indemnity claim cost; the requirement for full credibility for non-serious losses is an expected loss amount equal to 300 times the average non-serious case. Full credibility for medical is reached if the medical expected losses are equal to or greater than 80% of the expected loss amount to qualify for full credibility for non-serious losses. Partial credibility which is implemented in 10 percentage intervals, is expressed as: Required Expected Losses = (Expected losses required for 100% credibility) x (% credibility)$^{3/2}$. In other words, the percentage of the amount required for full credibility to receive, say, 70% credibility, is determined by the expression $(.70)^{3/2}$ or 58.6%. The exponential expression is used in lieu of a straight line formula in order to produce higher credibilities for partial credibilty.

The rationale behind the development of the formula pure premium is to base such premiums on the indicated pure premiums to the fullest extent that credibility will permit. To the extent that a classification is not credible, the underlying present pure premium is assigned with the assumption that the experience for the classification would change by the same percentage change as the industry group to which the classification belongs; i.e., pure premium present on rate levels.

e. *Proposed:* The proposed pure premiums are the middle ones of the indicated, the formula, and the underlying present rate. Normally, this would be the formula pure premium. However, this selection acts as a stabilizer in those instances where the experience of a class with relatively small credibility moves significantly in one direction while the experience of the industry group under which this class belongs moves significantly in the opposite direction. When the selected pure premium is other than the formula pure premium, the proposed total pure premium is distributed by parts in the same manner as the proposed pure premium.

Non-Reviewed Classifications—Pure Premiums (See Appendix, Exhibit II)

Those classifications whose expected losses are so small that no credibility can be attached to any one of the partial pure premiums (i.e., serious, non-serious, or medical) are called non-reviewed classifications. The expression "non-reviewed" is somewhat of a misnomer in the sense that these classifications are reviewed and have been assigned zero credibility for each partial pure premium. The rate for a non-reviewed classification is determined by modifying the current rate by the change in the industry group rate level into which the classification belongs. Partial pure premiums are maintained for each non-reviewed classification. These partial pure premiums are needed whenever the classification attains sufficient volume to be reviewed. Also, as explained later in this section, whenever a law change occurs, the law amendment is applied by parts to non-reviewed classifications. Further details are provided later in the paper.

Factors to Apply to Proposed Pure Premium to Derive Manual Rates — Reviewed Classes (See Appendix, Exhibit II)

The following items are combined with the proposed pure premium to obtain the final manual rate for a reviewed classification:

a. *Rate Level Adjustment Factor*

The classification experience is compiled excluding the Rate Level Adjustment Factor. It is necessary to bring in this factor when calculating rates as a multiplier to the proposed pure premiums in order to recognize the effect of calendar year experience.

b. *Effect of Legislation*

The partial pure premiums are multiplied by the three part effect of serious, non-serious and medical changes in benefit level that have not already been included in the pure premium exhibits. This may occur, for example, when an experience review is combined with legislation, and the law change is not known until after the pure premium exhibits have been prepared.

c. *Ratio of Manual Premium to Earned Premium*

The ratios of the industry group manual premiums to standard earned premiums are applied to the total pure premium to produce the required level of standard earned premium.

d. *Loss Constant Offsetting Reductions (See Appendix, Exhibit II-E)*

The manual rates include an offsetting reduction for the loss constants so that the premium from such loss constants will not produce premium in excess of the required level. Calculations are made based upon a distribution of size of risk of state experience for the policy year premium level period to produce indicated loss constant offsets each year.

e. *Expense Allowance (See Appendix, Exhibit I-D)*

The expense allowance is introduced into the rate by dividing the product of the proposed pure premiums and the appropriate factors above by the permissible loss and loss adjustment ratio. This operation produces the proposed rate prior to addition of a disease element, if any.

f. *Disease Elements*

The proposed manual rates include specific disease elements for those classes where they apply. The purpose here is to allow the normal occurrence of disease losses to be included in the rate calculations. Abnormally high disease losses are to be excluded. The specific disease elements applicable to those classifications with a high susceptibility to disease exposure provide the carrier with premium for the potential liability which could develop if many diseased workers filed claims at one time. The possibility of an outbreak of claims occurring at one time exists because many workers afflicted with a disease continue working and can at any time file a workers' compensation claim. When workers are reassigned, or long layoffs develop, an emergence of claims might be expected.

Normal disease emergence is an integral part of ratemaking. Typical disease losses include dermatitis, various lung afflictions, lead poisoning, etc. Many diseases have emerged in recent years resulting from the use of new chemical compounds which may involve very high loss potential. Also, the adjudication of disease claims today is much more liberal than was the case years ago. Hence, many cases formerly held non-compensable are now receiving awards.

Those classifications which have a high susceptibility to disease hazard involve exposure to silica dust, rock excavation and quarries, foundries, etc. In these instances, a schedule of specific disease elements which vary by classification applies. The specific disease elements are added to the rates as otherwise calculated to obtain the total manual rate. The elements were established by considering the relative number of employees exposed to the disease hazard, the rate of infection among those employed, and the severity of the resulting disease. If an employer, however, does engage in operations under one or more classifications where a specific disease element applies, and the hazard is not present, manual rules provide that the specific disease element may be removed.

g. *Maximum Departure*

A test is made to make certain that the proposed rates fall within the specified departure from the present rates. Classification rates may not change from one revision to the next by more than the effect of legislation and one-half of the industry group experience change, plus or minus 25%. To illustrate, if a state had an experience change of 1.060 for Manufacturing, with a law change of 10%, the upper swing limit for manufacturing classes would be 38% (i.e., 10% plus 1/2 of 6% = 13%, and 13% plus 25% = 38%). The lower swing limit would be −12% (i.e., 13% −25% = −12%).

h. *Rates — Test Correction Factor*

The payrolls now are extended by the rates presently in effect and by the indicated proposed rates to determine if the required change in manual premium level has been achieved. Since at first this calculation may not yield the required results, an iterative process is initiated which continuously tests the proposed rates including tentative test correction factors until the required change in manual premium level is obtained for each industry group.

Iteratives are necessary because individual class changes are limited. The test correction factors are applied as multipliers to the proposed pure premium.

It is not necessary from a mechanical viewpoint to isolate every factor shown above since there is a balancing out to the indicated rate level. However, it is more meaningful that each item be separately identified.

Factors to Derive Manual Rate — Non-Reviewed Classifications

If the rate revision is a review of experience only, the proposed non-reviewed classification rates are determined by multiplying the present rate excluding the specific disease element by the industry group rate level change and then adding back the specific disease element.

If the rate revision is a review of experience and law amendment combined, the law amendment is applied by parts, serious, non-serious, and medical, to the pure premiums underlying the present rates to derive the current rate modified for law amendments. Then, the industry group rate level change based on experience is applied to the current rate adjusted for the law change to derive the proposed rate.

4. Ratemaking Procedures for Classifications having Unique Conditions

There are certain classifications with characteristics which do not lend themselves readily to the standard ratemaking techniques. In these instances, special procedures are utilized in order to calculate rates.

A. *Per-Capita Classifications*

Per-capita classifications are those classifications comprised of in-servants and out-servants. Payroll is not the ideal basis of exposure for these classifications because in many instances a significant part of the remuneration is in the form of free room and board. Hence, rates for in-servants and out-servants are developed in the same way as any other class, except the number of servants is used in lieu of payroll.

B. *"F" Classifications (See Appendix, Section B-4)*

Prior to November of 1972, employees under the so-called "F" classifications (i.e., stevedores, shipbuilders, tallymen, etc.) received state benefits if they were injured on the dock and were paid benefits under the

United States Longshoremen's & Harbor Workers' Act (USL&HW Act) if they were injured on board ship.

Public Law 92-576, expanded coverage of the USL&HW Act to include dock workers' losses incurred subsequent to November 26, 1972 for the "F" classifications. Hence, stevedores, shipbuilders, tallymen, etc. are, for all practical purposes, completely under the federal act. Also, benefit adjustments under the USL&HW Act will be made annually on October 1. These conditions led to establishing a separate ratemaking program applicable to "F" classifications.

The ratemaking system for "F" classifications is described in detail in the Appendix. Highlights of this program include:

(1) Substituting national "F" classification pure premiums at up to 50% of the credibility that would previously have been assigned to state underlying pure premiums, in instances where the state indicated pure premium is not credible. The rationale here is to give the fullest credibility possible to the actual experience reported for the jurisdiction where rates are being revised. Then, to the extent credibility is not generated, the rate will be based on the national pure premium for the particular classification. However, to avoid any severe swings, the underlying state pure premium is given at least equal weight with the national pure premium.

(2) Since almost all injuries in the "F" classifications are now incurred under one Act, it is expected that rates among the various states would move closer together. Therefore, a range of rates based upon national pure premiums is established. Although not every rate in every state will fall within this range, only movement of rates towards this range is permitted. This technique also recognizes that the experience now available includes some data which is prior to the enactment of Public Law 92-576.

(3) No rate is permitted to increase or decrease by more than 50% from the present rate. This swing limit is more liberal than the limit applicable to other classes because greater fluctuations are anticipated for the next year as a result of the expansion under the Act.

C. *Chemical Classes*

The Chemical and Dyestuff Rating Plan has been established to provide a means of classifying and rating operations for (1) insureds which manufacture chemicals or dyestuffs or (2) insureds where the hazards are of a chemical nature although chemical and dyestuffs are not manufactured by the concern.

The measurement of hazard in terms of basic rates considers first the flammable or explosive nature of substances used or manufactured and second, the hazard created by or during the processes of accomplishing the transformation from raw material to finished product.

The flammable hazard is measured by the flash point. The chemical rates for each state are a grid whereby the abscissae includes four groups with various flash-point ranges and the ordinates indicate the degree of flammability in the processing.

The rates are calculated in the usual manner except that the rates are not permitted to reverse themselves either according to flash point or the degree of processing. When reversals are indicated they are combined with other points on the grid, and a common rate is computed for the group being combined.

D. *Underground Coal Mines*

The rates for underground coal mines are filed under a separate program. The hazards of an underground coal mine are unique because of the high catastrophe hazard present in underground operations. The rates for surface coal mines, auger coal mines, and types of mining other than coal, are developed in the same way as in other classifications except that there is a provision in the rate to cover state and federal black lung claims.

The calculation of the traumatic rate generally is the same as the approach used for calculating non-coal mine rates. However, there are some differences. In most instances, law evaluations have been computed on the basis that the wages received by coal miners will qualify them for maximum benefits. Carriers report calendar year experience and unit plan data for coal mine operations separately. To ensure stability in determining rates, two-thirds weight is given to policy year experience and one-third weight is given to calendar year experience. Expenses included in the manual rate for deep mines are lower than non-coal mine risks, but there are no premium discounts available. There is a catastrophe loading which is a flat charge added to the rate.

The disease rate is a comprehensive rate designed to produce premium to pay for disease claims, primarily black lung, reported under the state act or the federal act. There is an immense loss potential with regard to black lung cases.

The Federal Coal Mine Health and Safety Act of 1969 (FCMHSA), enacted in 1969 and amended in May of 1972, made current coal mine operators and employers who were formerly operators of coal mines liable for the payment of benefits for death or total disability due to pneumoconiosis (black lung) arising out of coal mine employment. The Act also established certain presumptions in the claimant's favor, applicable to black lung determinations:

1. Where a miner with pneumoconiosis has been employed in underground coal mines for 10 years or more, there is a rebuttable presumption that his pneumoconiosis arose out of such employment.
2. Where a deceased miner with 10 years or more of underground coal mine employment died of a respiratory disease, there is a rebuttable presumption that his death was due to pneumoconiosis; and
3. If a miner is suffering from complicated pneumoconiosis, there is an irrebuttable presumption that he is totally disabled due to pneumoconiosis.

As of July 1, 1973 a claimant has the option to file either under the state or federal laws. Black lung benefits payable to a miner or widow are reduced by the amount received under a state program of workers' compensation. This means that those claimants eligible for benefits under the state workers' compensation law will receive the larger of state benefits or federal benefits.

The worker normally would be expected to file under the state act in those jurisdictions where the state benefits exceed the federal benefits and vice versa. Also, there are additional claimants who may file under the state act first, but not qualify for benefits under the state act, and will then file and be eligible for benefits under the federal act.

Although a great many claims have been established and are being compensated under the federal program administered by the Department of Health, Education & Welfare (Part B of the FCMHSA), there is a considerable potential liability remaining to emerge. Some of the features of such liability should be outlined. In the first place, each case of black lung is for all practical purposes a life pension case with an extremely high average cost, currently in the area of $65,000. Thus, the emergence of a number of such cases would be serious indeed. Such an emergence could result from mine shutdowns or from claims by inactive miners or dependents of deceased miners who have not filed claims prior to July 1, 1973. In addition, claims originally filed under the Social Security Administration can be refiled with the Department of Labor to obtain medical benefits which have not previously been available to them. At an estimated amount of $12,000 per claim, application for medical benefits on any significant percentage of the hundreds of thousands of cases filed prior to July 1, 1973 would cost hundreds of millions of dollars.

The rate filed is a complete disease rate anticipating certain claims to be filed under the state law and other claims to be filed under the federal law. With respect to claims filed under the state law, the rate calculations reflect additional amounts that may be payable to the beneficiaries as a result of the federal law. This recognizes that the claimant will receive the federal law's escalated benefits which exceeds state law benefits.

The first step in the derivation of the proposed rate is the estimation of the frequency of successful claims. In this respect various data from reliable sources are interrelated to recognize two principal types of claims: (a) those miners with advanced stages of pneumoconiosis who are disabled, and (b) those miners with mild stages of pneumoconiosis who qualify for benefits under the previously cited presumptions but who refrain from filing a claim until it becomes economically advantageous for them to do so. Coal miners age 62 and over who have filed successful black lung claims receive tax-free income in the form of black lung benefits, social security benefits, and union pensions. The rate computation therefore assumes that the active coal miner age 62 and over will have a successful claim frequency of 25% from July 1, 1973 through June 30, 1974. This frequency is deemed to include those miners age 62 and over with advanced stages of coal miners' pneumoconiosis (progressive massive fibrosis or PMF) and is not in addition to the PMF component of claim frequency.

With respect to those miners under age 62 with progressive massive fibrosis, use is made of a study of 62,876 miners by the National Institute of Occupational Safety & Health (NIOSH) under the provisions of the Federal Coal Mine Health & Safety Act. The study obtained data on active miners who volunteered to be x-rayed. The results of the x-rays then were employed to arrange the miners in distributions according to age, years of service, and stage of pneumoconiosis. Thus, for the age intervals used in the distributions, the ratio of miners with PMF to the total number of miners in the interval can be readily determined. The rate computation assumes that this ratio approximates the true frequency of claims from July 1, 1973 to June 30, 1974 for the age interval. The frequencies for each age interval were applied to an age distribution of coal mine workers to obtain the estimated number of claimants in each age interval. The total number of claimants in all age intervals was then ratioed to the total number of miners to produce the estimated frequency of successful claims. The average age of a claimant was determined by utilizing the estimated numbers of claimants in each age interval as weights against the midpoints of the various intervals.

The average age thus obtained determines the average present value of a claim. There are no temporary total or permanent partial cases eligible for black lung disease benefits under the federal law. Therefore, the evaluation is based upon the present value of life pensions for a miner and his wife. Since most claims are filed by miners of advanced age, the annuity calculations assume there will be a negligible number of cases involving dependents other than wife or widow and relatively few involving a miner alone. It is assumed that when a worker files a claim, his wife, who is approximately two years younger than the miner, will survive him. This assumption is based on the following argument: (1) the mortality rate for miners is expected to be much higher than for non-diseased workers, (2) the mortality rate for men is generally higher than the mortality rate for women, and (3) a miner's wife, on the average, is at least two years younger than the miner.

Benefits payable under the U.S. law are increased automatically whenever the federal pay schedule is revised. When state benefits are initially higher than the corresponding federal benefits, it is assumed that some miners will file successful claims under the state act along with claims under the federal law to protect their interests in receiving supplementary benefits under the federal law in subsequent years when (1) federal benefits have escalated to a level above state benefits or (2) limitations on state benefits apply.

Recognition must also be given to the present value of medical benefits. The rate derivation assumes that average medical costs for black lung disease cases will not differ significantly from the average medical costs for traumatic cases.

The addition of the present value of medical benefits to the average present value of indemnity benefits results in the total average present value of benefits.

The next step toward the proposed rate is to recognize insurance company expenses. An expense allowance of 12.3% plus taxes is included to apply to the disease rate. The traumatic rate will continue to have the full standard expense allowance. A breakdown of the expense allowance is as follows:

Item	Proposed Allowance Applicable to Disease
Taxes	vary by state
Commissions	1.0%
Bureaus	1.0
Profit & Contingencies	2.5
Home Office & Claims	7.8

The present cost (benefits and expenses) per claim is multiplied by the frequency of successful claims to obtain the amount of premium that must be collected per miner to provide the new occupational disease coverage. Division of this per capita charge by the estimated average annual salary in hundreds provides the indicated basic rate.

The basic rate in all states then is increased by 40% to recognize the unknown elements that are not considered in the basic rate. Specifically this includes (1) the so-called junior catastrophes (i.e., closing down of single mines or local layoffs of workers resulting in an acceleration of claims filed), (2) claims filed by inactive miners engaged in other occupations or retired who did not file claims prior to July 1, 1973, and (3) workers who were not eligible for medical payments under the Social Security Administration who would be expected to refile under Part C of the FCMHSA to obtain medical payments on or after January 1, 1974. The loss potential in these areas can be enormous if any significant number of claims occur. For example, 100 claims resulting from a mine closing could easily produce a liability in excess of 65 million dollars.

The above procedure currenty is being reviewed. At the present time only limited data is available on the total liability of claims under policies effective on or after July 1, 1973. However, it is expected that the above procedure will be replaced by a new method using actual data as soon as it becomes available.

E. *Ex-Medical Rates*

Policies may be endorsed to exclude medical coverage. Further, it is necessary that the Board or Bureau having jurisdiction authorizes the writing of this type of policy except where the insured is a hospital. The manual rate for this type of coverage is the manual or authorized rate less 70% of the medical rate. The medical rate is expressed as the medical pure premium divided by the permissible loss ratio. The entire medical rate is not deducted from the full rate to determine the ex-medical rate because (1) the insurance carrier is still liable for the medical loss in case of insolvency by the insured, and (2) the insurance carrier may desire to assume payment of certain medical costs to hasten recovery and enable the injured worker to return to his job as soon as possible. The ex-medical rate is determined by subtracting from the manual rate the product of the manual rate and the ex-medical ratio for the classification involved.

Ex-medical ratios (i.e., 70% times the ratio of the medical pure premiums to the total pure premiums) for the hospital classifications (Codes 8833 and 9040) are printed as footnotes on the state rate pages. Ex-medical ratios for other classes are not printed on the state rate pages but are shown on the exhibits of approved rates and rating values which are distributed to the insurance carriers when an approval notice is released.

CONCLUSION

There exists today some minor variations within National Council states with respect to the procedures described above. This also is true with respect to the ratemaking procedure used by Independent Bureaus. For example, five years of class relativity is used in a few small volume states, and three years are used in a few others. As of this writing, two states are still at the old $100 payroll limitation rule, one state at $200, and some states at $300. Some states have never accepted loss constants. However, despite these variations the same general principles described above underlie the rates in each state.

With the dynamic changes occurring in workers' compensation in recent years, it is a certainty that the workers' compensation ratemaking procedure will be under constant scrutiny to ensure that such procedures effectively respond to these changes.

A few descriptive passages have been taken directly from the filings of the National Council on Compensation Insurance.

Appreciation is hereby extended to the Staff of the National Council for their helpful suggestions.

PREFACE TO APPENDIX

The following exhibits show the step-by-step procedure used to calculate manual rates.

At the time these exhibits were being prepared, loss adjustment expense was included at 13.0% of losses. Subsequently, this allowance was reduced to 12.5% of losses.

Also, the policy year data from unit statistical reports normally consists of two twelve month periods plus two years of data from three year fixed rate policies. However, there are instances when a policy period may be extended or abbreviated to adjust for changes in the normal rate revision effective date. In the attached illustration the earlier of the two policy periods covers ten months of experience, and the most recent period covers twelve months of experience.

EXHIBIT I

Determination of Change in Manual Premium Level

A. *Policy Year Experience—Financial Data*

The data for each policy year are valued as of the year end. Net earned premiums are compiled from the "Supplementary Call for Policy Year Experience Valued at Calendar Year End" and are adjusted to a standard earned premium basis; the calculations underlying such adjustments are found in Exhibit I-A. Premium derived from expense constants is eliminated and all data placed on a current basis (i.e., premiums are on present rate level and losses are on current law level); the calculations of factors to reflect this adjustment are found in Exhibit I-B. Development of both premiums and losses beyond the indicated valuation date is included through factors determined in Exhibit I-C.

	(1)	(2)	(3)	(4)	(5)	(6)
		FACTORS				
	Valued As of 12-31-73	To Current Level	Development	Loss Adjustment Expense	Composite (2)x[(3)x(4)]	Modified Data (1)x(5)
Premiums and Losses of Policies which became effective 1-1-72 through 12-31-72						
Std. Earned Prem.	86,014,777	1.053	1.003	—	1.056	90,831,605
Incurred Losses	48,360,811	1.133	1.118	1.130	1.431	69,204,321
Loss and Loss Adjustment Ratio						.762
Premiums and Losses of Policies which became effective 1-1-71 through 12-31-71						
Std. Earned Prem.	76,583,952	1.022	1.009	—	1.031	78,958,055
Incurred Losses	41,035,648	1.209	1.089	1.130	1.488	61,061,044
Loss and Loss Adjustment Ratio						.773
Total for Policies which became effective 1-1-71 through 12-31-72						
Std. Earned Prem.	xxx	xxx	xxx	xxx	xxx	169,789,660
Incurred Losses	xxx	xxx	xxx	xxx	xxx	130,265,365
Loss and Loss Adjustment Ratio						.767

Exhibit I (Cont.)

B. *Policy Year Indicated Change in Premium Level*

1.	Policy Year Incurred Losses	130,265,365
2.	Policy Year Standard Earned Premium	169,789,660
3.	Policy Year Loss and Loss Adjustment Ratio (1) ÷ (2)	.767
4.	Permissible Loss and Loss Adjustment Ratio (See Exhibit I-D)	.689
5.	Policy Year Change in Premium Level (3) ÷ (4)	1.113

This means that, prior to modification by calendar year results, an average overall increase of 11.3% in premium level is indicated by the policy year experience.

C. *Rate Level Adjustment Factor*

Calendar year premiums are adjusted to present rate level and calendar year losses are adjusted to current law level. The premium derived from the expense constant is eliminated so that the resulting calendar year loss and loss adjustment ratio will be comparable with the policy year loss and loss adjustment ratio. The losses are adjusted to include loss adjustment expense.

The calculation of the Rate Level Adjustment Factor follows:

		Experience of 12 Cal. Mos. End. 6-30-74		
		(a) Actual Basis	(b) Factors to Adj. to Present Law & 10-1-74 Rate Level†	(c) Adjusted Basis (a) × (b)
1.	Standard Earned Premium	106,851,486	1.003	107,172,040
2.	Incurred Losses and Loss Adj. Exp.	80,292,329	1.017	81,657,299
3.	Loss and Loss Adjustment Ratio			.762
4.	Policy Year Loss and Loss Adj. Ratio Based on Earned Prems. (from A)			.767
5.	Mean of (3) and (4)			.7645
6.	Rate Level Adjustment Factor (5) ÷ (4)			.997

†See Exhibit I-B for derivation of these factors.

Exhibit I (Cont.)

D. *Proposed Overall Change in Premium Level*

The product of the policy year indicated change in premium level from B above and the Rate Level Adjustment Factor from C above will produce the required change in premium level. This has the effect of giving equal 50% weightings to the policy year and the calendar year results.

1.	Policy Year Indicated Premium Level Change (from B)	1.113
2.	Rate Level Adjustment Factor (from C)	.997
3.	Overall Change in Premium Level (1) × (2)	1.110

E. *Distribution of Overall Change in Premium by Industry Group*

Since policy year aggregates are not available by industry group, (i.e., Manufacturing, Contracting and All Other), the summaries of Unit Statistical Plan data are used to obtain the distribution by industry group of the overall change in premium level. Exhibit I-E contains such information and, on the basis of the earned premium volume for each industry group the differentials are:

Industry Group	Differential
Manufacturing	.913
Contracting	1.023
All Other	1.036
Overall	1.000

F. *Change in Premium Level by Industry Group*

Applying the industry group differentials from E above produces the following changes in premium level by industry group:

	Industry Groups			
	Mfg.	Cont.	All Other	Total
1. Overall Change in Premium Level (From D)	—	—	—	1.110
2. Industry Group Differentials (From E)	.913	1.023	1.036	1.000
3. Final Change in Premium Level by Industry Group (2) × 1.110	1.013	1.136	1.150	1.110

Exhibit I (Cont.)

G. *Effect of the 1-1-75 Benefit Changes*

The calculations up to this point have been carried through on the July 1, 1974 law level. A benefit change was enacted 1-1-75 and is applied as a final step as shown below.

The change in manual premium level by industry group determined in Section F must be further modified by the effect of the benefit change as follows:

	Change in Manual Premium Level (From Sect. F)	Effect of 1-1-75 Benefit Change	Final Change in Manual Premium Level
Manufacturing	1.013	1.014	1.027
Contracting	1.136	1.014	1.152
All Other	1.150	1.014	1.166
Total	1.110	1.014	1.126

The final change in premium level, therefore, is a 12.6% overall increase.

Manufacturing	2.7% increase
Contracting	15.2% increase
All Other	16.6% increase
Overall	12.6% increase

EXHIBIT I-A

Conversion of Net Earned Premium to Standard Earned Premium

A. *Conversion of 1971 Policy Year Net Earned Premium to Standard Earned Premium*

Assuming an even distribution of business, one-half of Policy Year 1971 falls in Calendar Year 1971, and one-half falls in Calendar Year 1972. Therefore, to derive standard earned premium for Policy Year 1971, equal weight is given to the ratio of standard to net premium for Calendar Years 1971 and 1972 to derive Policy Year 1971 net earned premium.

(1) Calendar Period	(2) Standard Earned Premium	(3) Net Earned Premium	(4) Conversion Factor (2) ÷ (3)
1-1-71/12-31-71	77,246,171	72,221,796	1.070
1-1-72/12-31-72	84,370,151	77,238,092	1.092
			1.081

	(5) Net Earned Premium	(6) Conversion Factor	(7) Standard Earned Premium (5) × (6)
Policy Year 1971 as of 12-31-73	70,845,469	1.081	76,583,952

B. *Conversion of 1972 Policy Year Net Earned Premium to Standard Earned Premium*

(1) Calendar Period	(2) Standard Earned Premium	(3) Net Earned Premium	(4) Conversion Factor (2) ÷ (3)
1-1-72/12-31-72	84,370,151	77,238,092	1.092
1-1-73/12-31-73	96,734,165	88,410,138	1.094
			1.093

	(5) Net Earned Premium	(6) Conversion Factor	(7) Standard Earned Premium (5) × (6)
Policy Year 1972 as of 12-31-73	78,696,045	1.093	86,014,777

EXHIBIT I-B

Factor Adjusting 1972 Policy Year Premium to Level of Present Rates

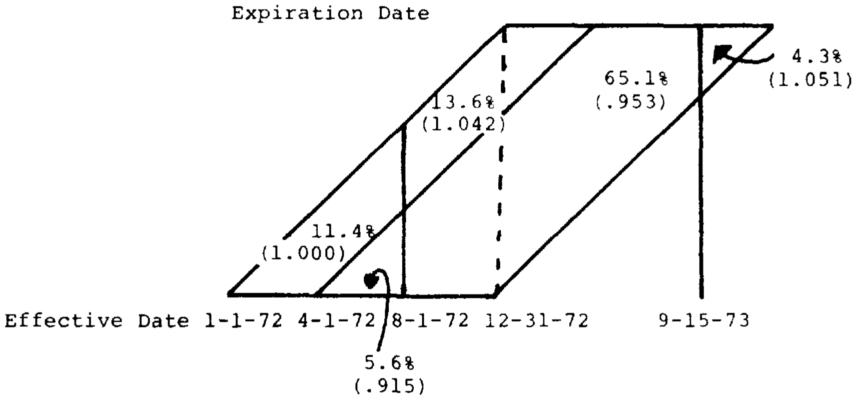
There are two types of rate adjustments. One is applicable to new and renewal business. This type of change can be represented geometrically by a diagonal line. For example, the new and renewal rate level change effective 4-1-72 is shown in the diagram below as a diagonal line. It indicates an average reduction in rate level of 8.5% applicable to all new and renewal policies effective on and after 4-1-72. The other type of change occurs when there is a law amendment or a medical fee change which requires an adjustment to outstanding policies. The 8-1-72 change shown below increased new, renewal, and outstanding policies by 4.2%. This type of change can be represented geometrically as a vertical line since it affects all policies in force on and after a specified date.

The 8-1-72 outstanding adjustment affected policies written under the 2-1-71 rates as well as policies written under the 4-1-72 rates. The new and renewal change effective 9-15-73 consisted of a review of experience and a benefit increase. The experience indications were somewhat favorable and, combined with the benefit adjustment, produced a net change of 7%. The outstanding policies were adjusted by a flat 10.3% for the unexpired portion of the policies to recognize the law change. The benefit increase was actually 10.6% but was reduced because of restrictions imposed by the Economic Stabilization Program. The outstanding adjustment cut across the tail end of Policy Year 1972 as shown below.

The rate level changes are indexed to a common base as shown in column (2) below. By computing proportionate areas to each rate level appearing in Policy Year 1972, the weights in column (3) are determined. These weights are then applied to the rate level indices in column (2) to determine the average policy year rate level index of .972 in column (4). The factor to bring the policy year data to current rate level is the ratio of the current index (1.044) in column (2) divided by the average policy year rate level (.972) to produce a factor of 1.074 in column (5). Following the removal of expense constant premium, the factor is reduced to 1.053 in column (7).

Exhibit I-B (Cont.)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Premium Level Changes		Weight		Adj. Factor	Adj. For	Prem. Adj.
Date	Manual Change	Cumulative Index	(See diagram)	Product (2) × (3)	Pres. Index ÷ Sum. Col. (4)	Exp. Const. Removal	Factor (5) × (6)
2-1-71	Base	1.000	.114	.114	1.074	.980	1.053
4-1-72	.915	.915	.056	.051			
8-1-72(AO)	1.042	1.042	.136	.142			
8-1-72(NR)	1.042	.953	.651	.620			
9-15-73(AO)	1.103	1.051	.043	.045			
9-15-73(NR)	1.070	1.020	—	—			
10-1-74	1.024*	1.044	—	—			
				.972			



* Applicable to "all outstanding" as well as new and renewal.
AO = All Outstanding.
NR = New and Renewal Business Only.

Factor Adjusting 1972 Pol. Year Losses to Level of Present Law

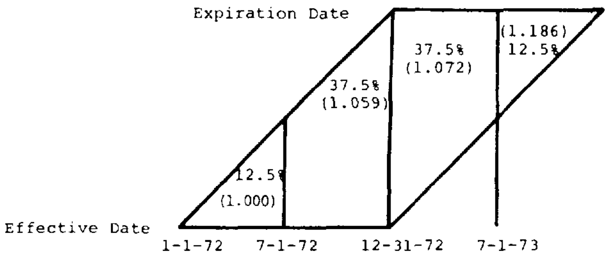
Benefit changes resulting from legislative enactments, medical fees, and hospital changes are represented geometrically by a vertical line since they are applicable to all new claims regardless of policy effective dates.

Set forth below are the benefit changes which have occurred during or subsequent to the policy period and indexed to the level of benefits applicable on 1-1-72. Using proportionate areas 12.5% of losses are at the 1-1-72 level; 37.5% are at the 7-1-72 level; 37.5% are at the 1-1-73 level; and 12.5% are at the 7-1-73 level. The weighted average law level for the policy year (using index numbers shown in column (9) is 1.072 in column (11)). The current index of 1.215 divided by 1.072 is the factor to ad-

Exhibit I-B (Cont.)

just the policy year losses to current law level, namely 1.133 as shown in column (12).

Date	(8) Benefit Change	(9) Cumulative Index	(10) Weight (See Diagram)	(11) Product (9) × (10)	(12) Adj. Factor Pres. Index ÷ Sum. Col. (11)
1-1-72	Base	1.000	.125	.125	1.133
7-1-72	1.059	1.059	.375	.397	
1-1-73	1.012	1.072	.375	.402	
7-1-73	1.106	1.186	.125	.148	
1-1-74	1.014	1.203	—	—	
7-1-74	1.010	1.215	—	—	
				1.072	



The procedures to adjust policy year 1971 premiums and losses to current levels are performed in a similar manner as shown below.

Factor Adjusting 1971 Pol. Year Premium to Level of Present Rates

Date	(1) Premium Level Changes Manual Change	(2) Cumulative Index	(3) Weight (See diagram)	(4) Product (2) × (3)	(5) Adj. Factor Pres. Index ÷ Sum. Col. (4)	(6) Adj. For Exp. Const. Removal	(7) Prem. Adj. Factor (5) × (6)
8-15-70	Base	1.000	.003	.003	1.043	.980	1.022
2-1-71	1.041*	1.041	.910	.947			
4-1-72	.915*	.953	—	—			
8-1-72 (AO)	1.042	1.085	.087	.094			
8-1-72 (NR)	1.042	.993	—	—			
9-15-73 (AO)	1.103	1.095	—	—			
9-15-73 (NR)	1.070	1.063	—	—			
10-1-74	1.024*	1.089	—	—			
				1.044			

* Applicable to "all outstanding" as well as new and renewal.
AO = All Outstanding. NR = New and Renewal Business Only.

Exhibit I-B (Cont.)

Factor Adjusting 1971 Pol. Year Losses to Level of Present Law

Date	(8) Benefit Change	(9) Cumulative Index	(10) Weight (See Diagram)	(11) Product (9) × (10)	(12) Adj. Factor Pres. Index ÷ Sum. Col. (11)
1-1-71	Base	1.000	.125	.125	1.209
7-1-71	1.001	1.001	.375	.375	
1-1-72	1.006	1.007	.375	.378	
7-1-72	1.059	1.066	.125	.133	
1-1-73	1.012	1.079	—	—	
7-1-73	1.106	1.193	—	—	
1-1-74	1.014	1.210	—	—	
7-1-74	1.010	1.222	—	—	
				1.011	

DIAGRAM FOR PREMIUM ADJUSTMENT

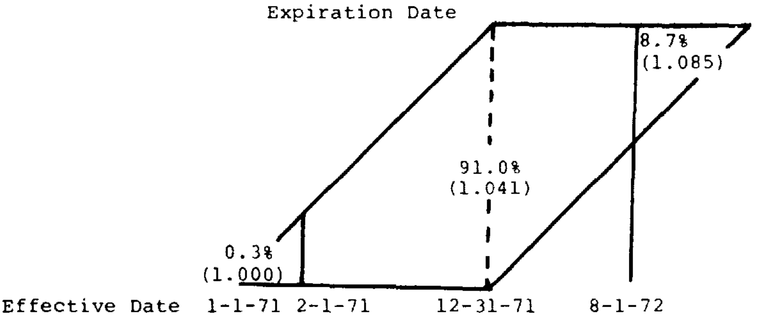


DIAGRAM FOR LOSS ADJUSTMENT

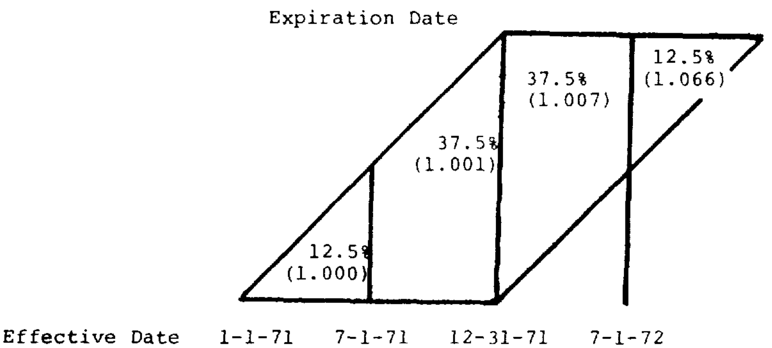


Exhibit I-B (Cont.)

Factor Adjusting Calendar Year Premium to Level of Present Rates

The same procedure is used to adjust calendar year premiums and losses to current levels as was used to adjust policy year premiums and losses to current levels.

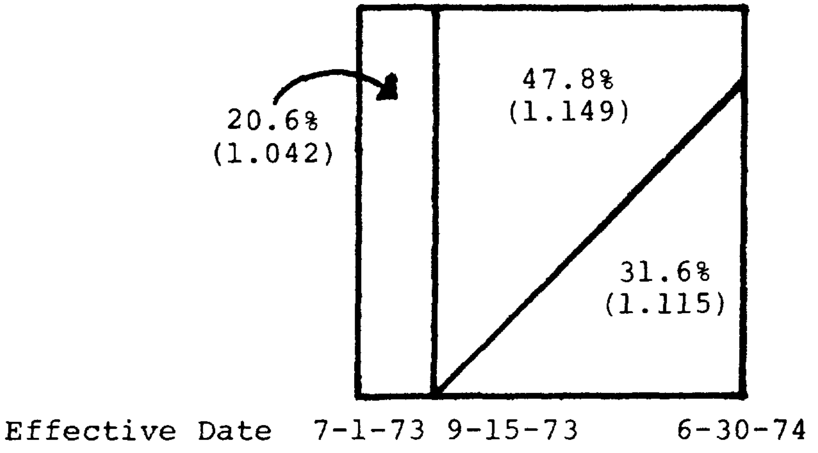
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Premium Level Changes		Weight		Adj. Factor	Adj. For	Prem. Adj.
Date	Manual Change	Cumulative Index	(See diagram)	Product (2) × (3)	Pres. Index ÷ Sum. Col. (4)	Exp. Const. Removal	Factor (5) × (6)
4-1-72	Base	1.000	—	—	1.023	.980	1.003
8-1-72	1.042*	1.042	.206	.215			
9-15-73 (AO)	1.103	1.149	.478	.549			
9-15-73 (MR)	1.070	1.115	.316	.352			
10-1-74	1.024*	1.142	—	—			
				1.116			

Factor Adjusting Calendar Year Losses to Law Level Underlying Present Manual Rates

	(1)	(2)	(3)	(4)	(5)
	Benefit Changes				Adj. Factor
Date	Change	Cumulative Index	Weight (See Diagram)	Product (2) × (3)	Pres. Index ÷ Sum. Col. (4)
7-1-73	Base	1.000	.500	.500	1.017
1-1-74	1.014	1.014	.500	.507	
7-1-74	1.010	1.024	—	—	
				1.007	

Exhibit I-B (Cont.)

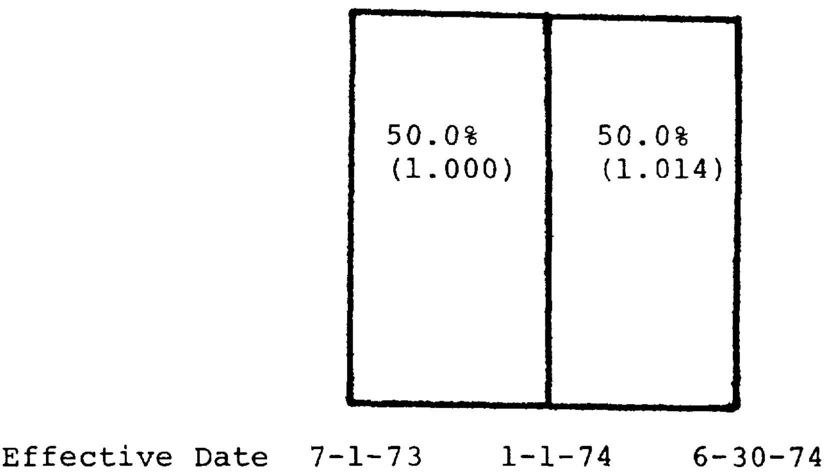
DIAGRAM FOR PREMIUM ADJUSTMENT



BASIC DATA

1.	Standard Earned Premium	106,851,486
2.	Incurred Losses Ex. Loss Adj.	71,055,158
3.	Incurred losses, (2) × 1.130	80,292,329

DIAGRAM FOR LOSS ADJUSTMENT



* Applicable to "all outstanding" as well as new and renewal.

EXHIBIT I-C

CALCULATION OF DEVELOPMENT FACTORS (1st to 5th)

The calculation of development factors from second report to ultimate and from first report to ultimate follows. In computing development from a first report to a second report the aggregate figures of all carriers that submitted reports from first report to second report are used; in computing development from a second report to third report the aggregate figures of all carriers that submitted reports from a second to a third report are used etc. In other words, in computing development from one report to the next the aggregates must represent the same carriers.

Premium development is not carried beyond a fifth report since no significant development is expected beyond that point.

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		NET EARNED PREMIUM AND TOTAL INCURRED LOSSES FOR MATCHING COS.								
		1st Report	2nd Report	3rd Report	4th Report	5th Report	1st/2nd	2nd/3rd	3rd/4th	4th/5th
1967	Prem.	xxx	xxx	xxx	43,085,575	43,101,142	xxx	xxx	xxx	1.000
	Losses	xxx	xxx	xxx	25,468,539	25,517,526	xxx	xxx	xxx	1.002
1968	Prem.	xxx	xxx	xxx	44,457,862	44,344,785	xxx	xxx	xxx	.997
	Losses	xxx	xxx	xxx	27,048,083	27,731,066	xxx	xxx	xxx	1.025
1968	Prem.	xxx	xxx	44,030,869	44,158,317	xxx	xxx	xxx	1.003	xxx
	Losses	xxx	xxx	26,593,494	26,947,988	xxx	xxx	xxx	1.013	xxx
1969	Prem.	xxx	xxx	53,075,479	53,283,244	xxx	xxx	xxx	1.004	xxx
	Losses	xxx	xxx	30,938,657	31,701,046	xxx	xxx	xxx	1.025	xxx
1969	Prem.	xxx	52,982,736	52,695,898	xxx	xxx	xxx	.995	xxx	xxx
	Losses	xxx	29,938,634	30,755,330	xxx	xxx	xxx	1.027	xxx	xxx
1970	Prem.	xxx	58,706,720	59,675,421	xxx	xxx	xxx	1.017	xxx	xxx
	Losses	xxx	35,681,348	36,602,354	xxx	xxx	xxx	1.026	xxx	xxx
1970	Prem.	57,769,741	58,141,229	xxx	xxx	xxx	1.006	xxx	xxx	xxx
	Losses	34,186,877	35,061,430	xxx	xxx	xxx	1.026	xxx	xxx	xxx
1971	Prem.	67,140,830	65,837,749	xxx	xxx	xxx	.981	xxx	xxx	xxx
	Losses	37,588,806	38,630,481	xxx	xxx	xxx	1.028	xxx	xxx	xxx
	Unweighted Average									
	Prem.						.994	1.006	1.004	.999
	Losses						1.027	1.027	1.019	1.014
	Dev. Factors: 2nd to 5th Report (7)x(8)x(9)									
1971	Prem.	1.009								
	Losses	1.062								
	Dev. Factors: 1st to 5th Report (6)x(7)x(8)x(9)									
	Prem.	1.003								
	Losses	1.091								

Exhibit I-C (Cont.)

CALCULATION OF DEVELOPMENT FACTORS (5th to ultimate)

Policies Becoming Effective During Period	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	TOTAL INCURRED LOSSES FOR CORRESPONDING COMPANIES AS PER				DEVELOPMENT FACTORS				
	5th Report	6th Report	7th Report	8th Report	5th to 6th (2) ÷ (1)	6th to 7th (3) ÷ (2)	7th to 8th (4) ÷ (3)	5th to 7th (5) × (6)	5th to 8th (8) × (7)
1964	xxx	xxx	19,800,947	19,782,024	xxx	xxx	.999	xxx	xxx
1965	xxx	xxx	21,430,394	21,671,573	xxx	xxx	1.011	xxx	xxx
1965	xxx	21,239,964	21,350,885	xxx	xxx	1.005	xxx	xxx	xxx
1966	xxx	24,029,594	24,166,650	xxx	xxx	1.006	xxx	xxx	xxx
1966	23,562,465	23,783,049	xxx	xxx	1.009	xxx	xxx	xxx	xxx
1967	25,611,420	25,608,236	xxx	xxx	1.000	xxx	xxx	xxx	xxx
			Unweighted Average:		1.005	1.006	1.005	1.011	1.016

(10) Change in losses from 12-31-71 to 12-31-72 for All Policy Years Prior to 1964	68,575
(11) Total incurred losses for corresponding companies for Policy Year 1963 valued as of 12-31-71	18,504,166
(12) Development Factors from 8th Report to Ultimate Development for All Policy Years Prior to 1964 [(10)+(11)]÷(11)	1.004
(13) Change in losses from 12-31-72 to 12-31-73 for All Policy Years Prior to 1965	251,029
(14) Total incurred losses for corresponding companies for Policy Year 1964 valued as of 12-31-72	19,822,402
(15) Development Factors from 8th Report to Ultimate Development for All Policy Years Prior to 1965 [(13)+(14)]÷(14)	1.013
(16) Unweighted Average of 8th to Ultimate Development Factors [(12)+(15)]÷2	1.009
(17) Development Factors from 5th Report to Ultimate Development (9)×(16)	1.025

	2nd to 5th	1st to 5th	5th to Ultimate	2nd to Ultimate	1st to Ultimate
2nd	1.062	xxx	1.025	1.089	xxx
1st	xxx	1.091	1.025	xxx	1.118

Note: The development of losses beyond an eighth report are lumped together and related to the policy period on an eighth reporting basis to obtain development from eighth to ultimate. The two latest developments from eighth to ultimate determines the development factor from eighth to ultimate.

EXHIBIT I-D

Allowances for Expenses, Taxes, Profit and Contingencies

Underlying the proposed rates are allowances of 25.9% of standard premium for company expenses, 2.5% of standard premium for profit and contingencies, 2.7% of standard premium for taxes, coupled with 13.0% of expected losses for loss adjustment expenses, plus an expense constant on premiums under $500.

The items comprising the expense allowance are as follows:

	Item	
(1)	Acquisition and Field Supervision	17.5%
(2)	General Expenses	8.4
(3)	Total for Company Expenses (1) + (2)	25.9%
(4)	Taxes, Licenses and Fees other than Federal Income Tax	2.7
(5)	Profit and Contingencies	2.5
(6)	Total for Company Expenses, Taxes and Profit and Contingencies (3) + (4) + (5)	31.1%
(7)	Permissible Loss and Loss Adjustment Ratio	68.9
	Loss Adjustment Expense:	
(8)	Related to Premium	7.9
(9)	Related to Losses	13.0
(10)	Total Expense Allowance Related to Premium (6) + (8)	39.0%
(11)	Expense Constant	
	Risks Under $200 Premium	$15.00
	Risks Between $200 and $500 Premium	$10.00

As a matter of information, the following paragraphs develop the allowance of the net rate; i.e., the manual rate after premium discounts have been applied.

It should be borne in mind that the allowances shown above apply only to the first $1,000 of premium. For risks with premium over $1,000 which in this state represent about 27.6% of the total number of risks and about 90.4% of the total premium, manual rules provide for a reduction of rates through application of premium discounts (or their equivalents included in the Retrospective Rating Plan Values). Premium discounts result from the reduction of expense requirements for Acquisition and General

Exhibit I-D (Cont.)

Administration with increasing premium size. The premium discounts are as follows:

Division of Standard Premium		Stock Co. Discount*	Non-Stock Co. Discount
First	$ 1,000	—	—
Next	4,000	9.4%	3.0%
Next	95,000	14.7	6.0
Over	100,000	16.3	8.5

*To be used by all carriers for policies issued under an assigned risk plan.

A tabulation of the state experience by risk size for the latest available policy period shows that for stock carriers the proposed discounts would produce a net discount of 10.3%. This figure undoubtedly is on the conservative side because in actual practice the discounts, which increase by risk size, are based on the total risk premium, including premium developed by operations in all states.

The tables below indicate for the stock carriers, the proposed expense, taxes, and profit and contingencies allowances on two bases. Column (1) lists the net allowances after reduction for the proposed premium discounts, such allowances being expressed as a percentage of standard premium. Column (2) expresses these allowances as a percentage of the net premium resulting from premium discounts.

Item	(1) Net Allowance (% of Standard Premium)	(2) Net Allowance (% of Net Prem.) (Col. (1) ÷ .897)
Acquisition and Field Supervision	10.8%	12.0%
General Expenses	5.4	6.0
Total for Company Expenses	16.2%	18.0%
Taxes, Licenses, and Fees other than Federal Income Taxes	2.4	2.7
Profit and Contingencies	2.2	2.5
Loss Adjustment Expense—Related Premium	7.9	8.8
Losses	61.0	68.0
Total	89.7%	100.0%
Premium Discounts	10.3	xxx
Total	100.0%	100.0%

EXHIBIT 1-E

CALCULATION OF INDUSTRY GROUP DIFFERENTIALS

Industry group totals compiled under the Unit Statistical Plan are used to establish industry group relativities. These relativities are adjusted to unity on an overall basis and then applied to the proposed overall rate level.

Policies Becoming Effective During Period	(1) Premiums At 10-1-74 Manual Rates**	(2) Losses and Loss Adjustment Expense On 7-1-74 Law Level***
Manufacturing Group—Schedules 5-25 Inclusive††		
7-1-70 to 4-30-71	19,289,641	10,797,825
5-1-71 to 4-30-72*	20,707,220	11,937,564
1968†	61,959	14,932
1969†	35,250	7,773
TOTAL	40,094,070	22,758,094
Contracting Group—Schedules 26 and 27††		
7-1-70 to 4-30-71	21,732,247	12,574,145
5-1-71 to 4-30-72*	26,303,258	15,976,434
1968†	62,556	120,741
1969†	54,379	6,164
TOTAL	48,152,440	28,677,484
All Other Group—Other Schedules Except Schedule 29††		
7-1-70 to 4-30-71	30,663,550	19,536,945
5-1-71 to 4-30-72*	39,166,320	23,833,808
1968†	601,007	477,552
1969†	455,039	359,499
TOTAL	70,885,916	44,207,804
All Industry Groups		
7-1-70 to 4-30-71	71,685,438	42,908,915
5-1-71 to 4-30-72*	86,176,798	51,747,806
1968†	725,522	613,225
1969†	544,668	373,436
TOTAL	159,132,426	95,643,382

* Last policy expired April 30, 1973.

† Three Year Fixed Rate Policies, last policy expired December 31, 1972.

†† Schedules are those set forth in Classifications Code Book issued by National Council.

** Derived by extending policy year payrolls by current rates exclusive of off-balance factor (manual to earned) and the loss constant offset.

*** Losses developed to an ultimate reporting level, adjusted to current benefit level, and further adjusted to include loss adjustment expense.

Exhibit I-E (Cont.)

In order to obtain the rate level by industry group the overall rate level change must be distributed by industry group using policy year differentials.

The Expected Losses—column (2)—are calculated by multiplying the premium at current manual rates by the ratio of earned premium to manual premium to recognize the Experience Rating Plan and by the permissible loss and loss adjustment ratio. The indicated losses are the losses and loss adjustment expense on the current law level brought up to the proposed rate level.

(1) Industry	(2) Expected Losses	(3) Indicated Losses	(4) Ratio (3) ÷ (2)	(5) Group Differentials (4) ÷ 1.112
Manufacturing	26,464,572	26,854,551	1.015	.913
Contracting	29,726,620	33,839,431	1.138	1.023
All Other	45,275,047	52,165,209	1.152	1.036
Overall	101,466,239	112,859,191	1.112	1.000

EXHIBIT II

Calculation of Rates for Reviewed Classifications

Indicated Pure Premium

Losses for each classification must be adjusted to current levels in the same manner as the policy year aggregates which were used to determine rate level. The factors are different, however, because the time period is different. Unit statistical report compilations are submitted monthly and, therefore, it is not necessary that the latest twelve month policy period commence on January 1.

The step by step development of the proposed pure premium for Classification Code 2003—"Bakeries" is as follows:

The indicated pure premium for Code 2003 is determined by first taking the losses as reported under the unit statistical plan and modifying them as indicated above (see Exhibit II-A). The losses (including loss adjustment expenses) on current level are related to payrolls in $100 units to determine the indicated pure premium.

	(1) Losses and Loss Adjustment Expense†	(2) Payroll	(3) Payroll In Units of $100 (2) ÷ (100)	(4) Indicated Pure Prem. (1) ÷ (3)
Serious	130,652	xx	xx	.207
Non-Ser.	461,337	xx	xx	.730
Medical	265,010	xx	xx	.419
TOTAL	856,999	63,231,980	632,319.80	1.36

† See Exhibit II-A.

Exhibit II (Cont.)

Pure Premium Underlying Present Rates

	Ser.	Non-Ser.	Med.	Total
1. Proposed Pure Premiums (Previous Revision)	.300	.524	.311	1.14
2. Product of RLAF and Test Correction Factor (Previous Revision)	1.067	1.067	1.067	xx
3. Adjusted Pure Premiums (Previous Revision) (1) × (2)	.320	.559	.332	1.21
4. Effect of Legislation 7-1-74	1.036	1.033	1.000	xx
5. Adjusted Pure Premium Including Law Change (Previous Revision) (3) × (4)	.332	.577	.332	1.24
6. Ratio of Manual To Earned (Mfg. Grp.) (Prev. Rev.)	1.062	1.062	1.062	xx
7. Ratio of Manual To Earned (Mfg Grp.) (Current Rev.)	1.044	1.044	1.044	xx
8. Factor to Adjust Underlying Pure Premium from Previous Revision (6) ÷ (7)	1.017	1.017	1.017	xx
9. Pure Premiums Underlying Present Rates (Current Revision) (5) × (8)	.338	.587	.338	1.26

Note: If there was a law amendment which was included in this year's pure premium exhibits which is not included in the present rates, the effect of the law amendment is applied by parts to the pure premiums shown in line (9).

Present on Rate Level Pure Premium

These are the pure premiums underlying present rates brought to the proposed premium level by the application to the partial pure premiums of factors representing the effect of the changes in policy year premium level. The overall effect of the benefit level change is removed from the policy year premium change before application to the underlying pure premium. The derivation of the policy year change in premium level for the Manufacturing Group, exclusive of benefit change, and of the present on rate level pure premium for Code 2003 follows:

1. Proposed Change in Prem. Level—Mfg. Group	1.027
2. Rate Level Adjustment Factor	.997
3. Policy Year Change in Premium Level (1) ÷ (2)	1.030
4. Effect of 1-1-75 Benefit Change	1.014
5. Policy Year Change Excl. 1-1-75 law—Mfg. Group (3) ÷ (4)	1.016

	Ser.	Non-Ser.	Med.	Total
6. Underlying Pure Premiums	.338	.587	338	1.26
7. Present on Rate Level Pure Prems. (5) × (6)	.343	.596	.343	1.28

Exhibit II (Cont.)

Determination of Credibility

The expected loss credibility criteria for assigning 100% credibility to an indicated partial pure premium are determined as follows:

	Serious	Non-Ser.	Medical	Total
1. No. of Cases—All Classes	1,375	30,388	xx	xx
2. Modified Losses—All Classes	29,730,836	37,763,181	28,139,265	95,643,382
3. Average Cost per Case (2) ÷ (1)	21,630	1,243	xx	xx
4. Basis for 100% Credibility—No. of Cases	25	300	*	xx
5. 100% Cred. Criteria on Actual Losses (3) × (4)	540,750	372,900	298,320*	xx
6. Expected Losses Based on Underlying Pure Premium—All Classes**	34,069,966	38,599,777	28,857,479	101,527,222
7. Factor to Adjust from Actual to Underlying (6) ÷ (2)	xx	xx	xx	1.062
8. Expected Losses Required for 100% Credibility (5) × (7)	574,277	396,020	316,816	xx

* 100% Credibility Criterion for Medical equals 80% of Non-Serious Criterion.

** Expected losses in line (6) are the sum of the product of the total payroll in $100 units times the underlying pure premiums for all classes. The expected losses for Code 2003 are as follows:

	Serious	Non-Ser.	Medical	Total
1. Payroll in Units of $100 (Code 2003)	xx	xx	xx	632,319.80
2. Underlying Pure Premiums (Prev. Rev.)	.332	.577	.332	xx
3. Expected Losses (1) × (2)	209,930	364,849	209,930	xx

The formula to determine partial credibility, which is implemented in 10 percentage point intervals is:

(100% criteria) × (% credibility) = Required expected losses

The credibility table for "State X" is shown in Exhibit II-D.

Exhibit II (Cont.)

The serious, non-serious, and medical expected losses of $209,930, $364,849, and $209,930 derived above, when compared to the credibility table (Exh. II-D), results in serious, non-serious, and medical credibility assignments of 50%, 90%, and 70%, respectively.

Formula Pure Premium

These pure premiums are determined by adding the product of the indicated pure premium times its credibility and the product of the present on rate level pure premium times the unassigned credibility. Shown below is this calculation for Code 2003:

	Serious	Non-Serious	Medical	Total
1. Indicated Pure Premium	.207	.730	.419	1.36
2. Credibility	50%	90%	70%	xx
3. Present on Rate Level Pure Premium	.343	.596	.343	1.28
4. Unassigned Credibility [100% − (2)]	50%	10%	30%	xx
5. Formula Pure Premium [(1) × (2)] + [(3) × (4)]	.275	.717	.396	1.39

Proposed Pure Premium

The proposed pure premiums are derived based on selection of the middle of the total pure premiums for indicated, underlying, and formula. The total pure premiums for Code 2003 are:

Indicated	1.36
Formula	1.39
Underlying	1.26

Since for 2003 the indicated total pure premium is the middle of the three, the indicated pure premium is selected as the proposed pure premium (serious, non-serious, and medical). If either the indicated or the underlying total pure premium is selected as the middle pure premium, the partial pure premiums for the selected are adjusted so as to be in the same relativity as the formula partial pure premiums, while still summing to the selected total pure premium. Redistributing the partial indicated pure premiums produces the partial proposed pure premiums as follows:

	Serious	Non-Serious	Medical	Total
Proposed Pure Premium	.269	.702	.387	1.36

Computation of Manual Rate

The purpose of selecting the middle of the three pure premiums to be the proposed pure premium is to prevent the rates for classes which are not fully credible from moving significantly away from the industry group indications and to add an additional force for maintaininng the stability of rates from year to year.

The following items are combined with the proposed pure premium to obtain the final manual rate for a reviewed classification:

(1) Rate Level Adjustment Factor

See Exhibit I for Derivation of this Factor

(2) Effect of Legisation

The partial pure premiums are multiplied by the three part effect of the January 1, 1975 legislation change in benefit level, namely:

Serious	1.017
Non-Ser.	1.023
Medical	1.000

(3) Ratios of Manual Premiums to Earned Premiums

The ratios of manual premiums to earned premiums by industry group have also been excluded from the classification experience, and it is necessary to apply these factors to the proposed pure premiums. These factors

Exhibit II (Cont.)

Computation of Manual Rate

are determined by dividing the manual premium by the earned premium for the two policy periods combined. These premiums are the actual reported earned and manual premiums at policy year level. The factors for "State X" were as follows:

Industry Group	Rates of Manual Premium to Earned Premium
Manufacturing	1.044
Contracting	1.116
All Other	1.079

(4) Loss Constant Offsetting Reductions

The present manual rates include an offsetting reduction for the loss constants so that the premium from such loss constants will not produce premium in excess of requirements. This proposal contemplates the continuance of existing loss constants. Calculations based upon a distribution of size of risk of the state experience for the policy year premium level period used in this filing indicate revised offsetting reductions as follows:

Industry Group	Loss Constants	Offsetting Reduction in Manual Rate	
		Present	Proposed*
Manufacturing	$15.00	.999	.999
Contracting	8.00	.999	.999
All Other	5.00	.997	.998

The product of these factors referred to in (3) and (4) above are as follows:

	(1)	(2)	(3)
Industry Group	Ratio Of Man. Prem. To Earned Prem.	Loss Const.	Product (1) × (2)
Manufacturing	1.044	.999	1.0430
Contracting	1.116	.999	1.1149
All Other	1.079	.998	1.0768

* For Derivation of these factors see attached Exhibit II-E.

Computation of Manual Rate

(5) Expense Allowance

The expense allowance is introduced into the rate by dividing the product of the proposed pure premium and the appropriate factors above by the permissible loss and loss adjustment expense ratio.

(6) Disease Elements

The proposed manual rates include specific disease elements for those classifications where they apply. There is no specific disease element for Code 2003.

(7) Rates—Test Correction Factor

The payrolls are now extended by the rates presently in effect and by the indicated proposed rates to determine if the required change in manual premium level has been achieved. Since at first this calculation may not yield the required results, an iterative process is initiated which continuously tests the proposed rates including tentative test correction factors until the required change in manual premium level is obtained. The iterative process also adjusts for the effect of limited classes indicated in the next paragraph.

In the computer program the factors are then rearranged in the order indicated in the illustration that follows. In this way, next year's underlying pure premium can be identified and stored.

The factors referred to in (1) and (5) above as as follows:

Industry Group	(1) Test Correction Factor	(2) Rate Level Adjust. Factor	(3) Product (1) × (2)
Manufacturing	.993	.997	.990
Contracting	.989	.997	.986
All Other	1.027	.997	1.024

A test is made to make certain that the proposed rates fall within the following departures from the present rates:

Manufacturing	from 27% above to 23% below
Contracting	from 33% above to 17% below
All other	from 34% above to 16% below

Exhibit II (Cont.)

Computation of Manual Rate

These limits have been calculated in accordance with the following formula:

Max. Deviation = Effect of Law Amendment plus ½ (% Change (+ or −) in Premium Level Excluding Law Amendment) plus or minus 25% rounded to the nearest 1%.

For example, the upper limit for the All Other group is:

+ 1.4% + ½ (15.0%) + 25% =

1.4% + 7.5% + 25% = 33.9% = 34% (rounded)

The changes in manual premium level used are those derived in Exhibit I, Section G.

Exhibit II (Cont.)

CALCULATION OF PROPOSED RATE CODE 2003—MANUFACTURING GROUP

A. REVIEWED CLASSIFICATIONS

	Serious	Non-Ser.	Medical	Total
1. Proposed pure premiums (Exhibit II-A)	.269	.702	.387	1.36
2. Product of RLAF and test correction factor	.990	.990	.990	xx
3. Adjusted pure premiums, unrounded (1) × (2)	.26631	.69498	.38313	xx
4. Effect of benefit change eff. 1-1-75	1.017	1.023	1.000	xx
5. Proposed pure premiums (3) × (4)	.271	.711	.383	1.37
6. Ratio of manual to earned premium and loss constant offsets				1.0430
7. Permissible loss and loss adjustment ratio				.689
8. Proposed manual rate [(5) × (6) ÷ (7)]				2.07

Calculation of Rates for Non-Reviewed Classifications

The proposed rates for the non-reviewed classifications are obtained as follows:

(1) The current rates are adjusted by removing the specific disease element, if any. The rate exclusive of disease is then modified by the changes in manual premium level excluding the effect of the January 1, 1975 legislation. These changes are calculated as follows:

	(1)	(2)	(3)
Industry Group	Final Change In Manual Premium Level Incl. Law Amendment	Effect of Legislation	Change in Man. Premium Level Excl. Effect Of 1975 Legislation Col. (1) ÷ Col. (2)
Mfg.	1.027	1.014	1.013
Cont.	1.152	1.014	1.136
A.O.	1.166	1.014	1.150

(2) The rates resulting from above are increased by applying the effect of the January 1, 1975 legislation to three parts (Serious 1.017, Non-Serious 1.023, Medical 1.000) to the corresponding pure premiums underlying those rates.

(3) The addition of the proposed specific disease element, if any, produces the final manual rate.

EXHIBIT II-A

Code 2003—"Bakeries"

Policy Period 7-1-70 to 4-30-71

Type of Injury	(1) Incurred Losses	(2) Amendment Factor*	(3) Development Factor**	(4) Loss Adj. Expense	(5) (2) × [(3) × (4)] Composite	(6) (1) × (5) Modified Loss Loss Adjustment Expense
Death	—	3.075	1.123	1.130	3.902	—
Permanent Total	-	2.192	1.123	1.130	2.782	—
Major Perm. Partial	63,929	1.066	1.123	1.130	1.353	86,496
Serious	63,929					86,496
Minor Perm. Partial	57,893	1.157	1.123	1.130	1.468	84,987
Temporary Total	66,669	1.426	1.123	1.130	1.810	120,671
Non-Serious	124,562					205,658
Medical	101,393	1.000	1.119	1.130	1.264	128,161
Policy Period 5-1-71 to 4-30-72						
Death	—	3.033	1.211	1.130	4.149	-
Permanent Total	—	2.179	1.211	1.130	2.981	—
Major Perm. Partial	30,600	1.055	1.211	1.130	1.443	44,156
Serious	30,600					44,156
Minor Perm. Partial	87,161	1.118	1.211	1.130	1.529	133,269
Temporary Total	69,158	1.294	1.211	1.130	1.770	122,410
Non-Serious	156,319					255,679
Medical	106,865	1.000	1.131	1.130	1.278	136,573
Three Year Fixed Rate Policies						
Policy Period 1968						
Death		3.103	1.145	1.130	4.015	—
Permanent Total	-	2.429	1.145	1.130	3.143	—
Major Perm. Partial	-	1.280	1.145	1.130	1.656	-
Serious	-					
Minor Perm. Partial	—	1.417	1.145	1.130	1.834	-
Temporary Total	-	1.451	1.145	1.130	1.878	—
Non-Serious	—					
Medical	187	1.167	1.120	1.130	1.477	276
Policy Period 1969						
Death	—	3.077	1.145	1.130	3.982	-
Permanent Total	—	2.217	1.145	1.130	2.869	—
Major Perm. Partial	—	1.160	1.145	1.130	1.501	
Serious	—					
Minor Perm. Partial	-	1.267	1.145	1.130	1.639	-
Temporary Total	—	1.419	1.145	1.130	1.836	-
Non-Serious	—					
Medical	—	1.083	1.120	1.130	1.371	-

* See Exhibit II-B
** See Exhibit II-C

EXHIBIT II-B

Calculation of Amendment Factors

A separate amendment factor is calculated for each type of injury for each policy period. Each factor is calculated in the same manner as amendment factors used in the rate level; i.e., by use of index numbers the latest benefit level is related to the average benefit level during the policy period to determine the amendment factor.

As an illustration, the calculation of the amendment factor to bring death cases incurred under policy period 1970 – 71 to current level is as follows:

Effective Date of Benefit Changes	Effect of Amendment	Cumulative Index	Weight	Product	Adj. Factor Pres. Index ÷ Sum. Col. (1)
7-1-70	Base	1.000	.583	.583	3.075
7-1-71	1.018	1.018	.350	.356	
1-1-72	1.005	1.023	.067	.069	
1-1-73	1.003	1.026		1.008	
7-1-73	2.881	2.956			
1-1-74	1.029	3.042			
7-1-74	1.019	3.100			

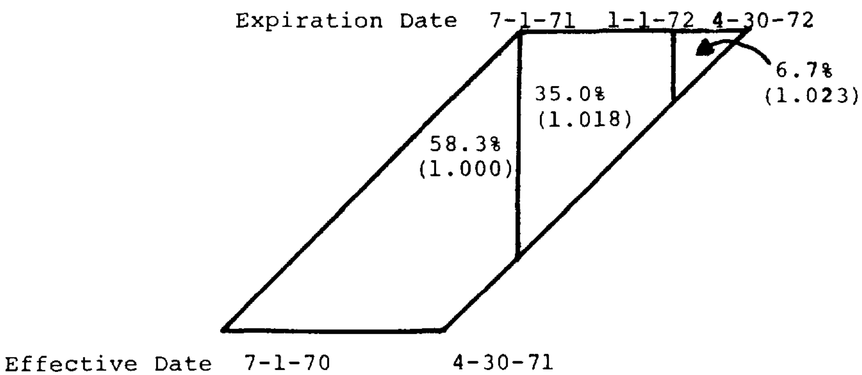


EXHIBIT II-C

Calculation of Development Factors—Unit Plan Data

Policies Becoming Effective During Period	Item	(1) 1st Report	(2) 2nd Report	(3) 3rd Report	(4) 4th Report	(5) 5th Report
		AMOUNT AS PER				
7-1-66 – 6-30-67	Prem.	xxx	xxx	xxx	49,010,952	49,010,952
	Indem.	xxx	xxx	xxx	17,146,758	16,933,673
	Med.	xxx	xxx	xxx	8,177,953	8,162,219
7-1-67 – 6-30-68	Prem.	xxx	xxx	49,735,163	49,742,733	49,742,733
	Indem.	xxx	xxx	17,028,459	17,140,724	17,059,899
	Med.	xxx	xxx	8,166,397	8,138,786	8,188,438
7-1-68 – 6-30-69	Prem.	xxx	55,356,751	55,356,751	55,356,751	xxx
	Indem.	xxx	18,211,826	18,208,402	18,104,070	xxx
	Med.	xxx	9,113,940	9,112,897	9,147,308	xxx
7-1-69 – 6-30-70	Prem.	65,650,651	65,658,595	65,658,595	xxx	xxx
	Indem.	21,305,115	22,776,486	23,764,284	xxx	xxx
	Med.	11,058,639	11,168,837	11,330,799	xxx	xxx
7-1-70 – 4-30-71	Prem.	67,976,290	67,998,360	xxx	xxx	xxx
	Indem.	19,648,378	21,341,197	xxx	xxx	xxx
	Med.	10,579,794	10,675,280	xxx	xxx	xxx

	Prem.		(a)
	Indem.		(b)
	Med.		(c)
	Indem.	(b)÷(a)	
	Med.	(c)÷(a)	
7-1-69 – 6-30-70 3rd	Indem.	(b)÷(a)	
	Med.	(c)÷(a)	
7-1-70 – 4-30-71 2nd	Indem.	(b)÷(a)	
	Med.	(c)÷(a)	
5-1-71 – 4-30-71 1st	Indem.	(b)÷(a)	
	Med.	(c)÷(a)	

EXHIBIT II-C

Calculation of Development Factors—Unit Plan Data

(6)	(7)	(8)	(9)	(10)	(11)	(12)
DEVELOPMENT FACTORS						
1st to 2nd (2)÷(1)	2nd to 3rd (3)÷(2)	3rd to 4th (4)÷(3)	4th to 5th (5)÷(4)	3rd to 5th (8)×(9)	2nd to 5th (7)× [(8)×(9)]	1st to 5th (6)× [(7)×(8)×(9)]
xxx	xxx	xxx	1.000	xx	xx	xx
xxx	xxx	xxx	.988	xx	xx	xx
xxx	xxx	xxx	.998	xx	xx	xx
xxx	xxx	1.000	1.000	xx	xx	xx
xxx	xxx	1.007	.995	xx	xx	xx
xxx	xxx	.997	1.006	xx	xx	xx
xxx	1.000	1.000	xx	xx	xx	xx
xxx	1.000	.994	xx	xx	xx	xx
xxx	1.000	1.004	xx	xx	xx	xx
1.000	1.000	xx	xx	xx	xx	xx
1.069	1.043	xx	xx	xx	xx	xx
1.010	1.015	xx	xx	xx	xx	xx
1.000	xx	xx	xx	xx	xx	xx
1.086	xx	xx	xx	xx	xx	xx
1.009	xx	xx	xx	xx	xx	xx
Unweighted Factors						
1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.078	1.022	1.001	.992	.993	1.015	1.094
1.010	1.008	1.001	1.002	1.003	1.011	1.021
Combined Factors						
xx	xx	xx	xx	.993	xx	xx
xx	xx	xx	xx	1.003	xx	xx
xx	xx	xx	xx	xx	1.015	xx
xx	xx	xx	xx	xx	1.011	xx
xx	xx	xx	xx	xx	xx	1.094
xx	xx	xx	xx	xx	xx	1.021

1968 & 1969 3 Year Fixed Rate Policies

	Indemnity	Medical
3rd to 5th	.993	1.003
2nd to 5th	1.015	1.011
1st to 5th	1.094	1.021
	3/3.102	3/3.035
	1.034	1.012

EXHIBIT II-C (Contd.)

CALCULATION OF DEVELOPMENT FACTORS (5th to Ultimate)

Policies Becoming Effective During Period	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	TOTAL INCURRED LOSSES FOR CORRESPONDING COMPANIES AS PER				DEVELOPMENT FACTORS				
	5th Report	6th Report	7th Report	8th Report	5th to 6th (2) ÷ (1)	6th to 7th (3) ÷ (2)	7th to 8th (4) ÷ (3)	5th to 7th (5) × (6)	5th to 8th (8) × (7)
1964	xxx	xxx	19,800,947	19,782,024	xxx	xxx	.999	xxx	xxx
1965	xxx	xxx	21,430,394	21,671,573	xxx	xxx	1.011	xxx	xxx
1965	xxx	21,239,964	21,350,885	xxx	xxx	1.005	xxx	xxx	xxx
1966	xxx	24,029,594	24,166,650	xxx	xxx	1.006	xxx	xxx	xxx
1966	23,562,465	23,783,049	xxx	xxx	1.009	xxx	xxx	xxx	xxx
1967	25,611,420	25,608,236	xxx	xxx	1.000	xxx	xxx	xxx	xxx
			Unweighted Average:		1.005	1.006	1.005	1.011	1.016

(10) Change in losses from 12-31-71 to 12-31-72 for All Policy Years Prior to 1964	68,575
(11) Total incurred losses for corresponding companies for Policy Year 1963 valued as of 12-31-71	18,504,166
(12) Development Factors from 9th Report to Ultimate Development For All Policy Years Prior to 1964 [(10)+(11)] ÷ (11)	1.004
(13) Change in losses from 12-31-72 to 12-31-73 for All Policy Years Prior to 1965	251,029
(14) Total incurred losses for corresponding companies for Policy Year 1964 valued as of 12-31-72	19,822,402
(15) Development Factors from 9th Report to Ultimate Development For All Policy Years Prior to 1965 [(13)+(14)]÷(14)	1.013
(16) Unweighted Average of 8th to Ultimate Development Factors [(12)+(15)]÷2	1.009
(17) Development Factors from 5th Report to Ultimate Development (9)×(16)	1.025

	2nd to 5th		1st to 5th		5th to Ultimate	7/1/70 to 4/30/71 2nd to Ultimate		5/1/71 to 4/30/72 1st to Ultimate	
	Indemnity	Medical	Indemnity	Medical		Indemnity	Medical	Indemnity	Medical
2nd	1.015	1.011	xxx	xxx	1.025	1.040	1.036	xxx	xxx
1st	xxx	xxx	1.094	1.021	1.025	xxx	xxx	1.121	1.047

1968 & 1969 3 Year Fixed Rate Policies (Indemnity 1.034 × 1.025 = 1.060) (Medical 1.021 × 1.025 = 1.037)

The above factors are further developed to the level underlying financial data. A factor of 1.080 computed by relating policy year earned loss ratios based on financial data to unit statistical plan data is applied to the above factors.

EXHIBIT II-D

Credibility Group	$E = \sqrt{W3}$	Credibility Criteria On Assignment Level Serious (9a)×(12)	Non-Serious (9b)×(12)	Medical (9c)×(12)
100	1.000	574,277	396,020	316,816
90	.854	490,433	338,201	270,561
80	.716	411,182	283,550	226,840
70	.586	336,526	232,068	185,654
60	.465	267,039	184,149	147,319
50	.354	203,294	140,191	112,153
40	.253	145,292	100,193	80,154
30	.164	94,181	64,947	51,958
20	.089	51,111	35,246	28,197
10	.032	18,377	12,673	10,138
0	.000	xx	xx	xx

For example, the determination of the required expected losses for at least 60% credibility is as follows:

	Serious	Non-Serious	Medical
1. 100% Criteria	574,277	396,020	316,816
2. $\sqrt{(.60)^3}$	.465	.465	.465
3. 60% Criteria (1)×(2)	267,039	184,149	147,319

EXHIBIT II-E

DETERMINATION OF LOSS CONSTANT OFFSETS

Industry Group	Mfg.	Cont.	A.O.
1. Prem. at Man. Rates	40,053,976	48,104,288	70,673,258
2. Loss Constant Offsets Underlying Manual Rates	.999	.999	.997
3. Prem.—No Loss Constant Program (1)÷(2)	40,094,070	48,152,440	70,885,916
4. Prop. Change in Manual Rate Level	1.027	1.152	1.166
5. Prem. at Proposed Level (No Loss Constant Program) (3)×(4)	41,176,610	55,471,611	82,652,978
6. Number of Risks Under $500 (All Experience Periods)	1997	6598	29069
7. Amount of Loss Constants	15	8	5
8. Amount of Premium Expected from Application of Loss Constant (6)×(7)	29955	52784	145345
9. Prop. Loss Constant Offsets [(5)−(8)]÷(5)	.999	.999	.998
10. Change in Loss Constant Offsets (9)÷(2)	1.000	1.000	1.001

RATEMAKING PROCEDURE — "F" CLASSIFICATIONS

A description of the features of the ratemaking program which are different from the standard ratemaking program follows:

Under the ratemaking program for "F" classifications, rates are based on unit statistical plan data. Policy year aggregate and calendar year data are not currently available for "F" classes separate from other classes. Carriers now are developing programs in order that this information be available in future years.

A. *Pure Premium Calculation*

1. *Indicated Pure Premiums* are derived by compiling past losses reported under the state act and converting them to the current federal law level and adding to such losses past federal losses converted to the current federal benefit level. As a result of the expansion of the Longshoremen's Act in 1972, it is expected that practically all losses previously incurred under the state acts will now be paid under the U.S. law. The losses are developed to ultimate level by using the state development factors. Loss adjustment expenses are also included. The average indicated pure premiums are determined by giving 60% weight to the experience of the latest policy period and 40% weight to the earlier year. The program for giving more weight to the most recent year will apply during the time when the policy periods used for ratemaking contain some experience prior to the 1972 law change. It is believed that experience under the new expanded law is more indicative of future loss level than prior data.

2. *Underlying Present Rates:* These are the pure premiums underlying the "F" classification rates currently in force. The procedure used to determine these underlying pure premiums is the same as has been used for determining underlying pure premiums in general revisions of workers' compensation rates.

3. *Indicated by National Pure Premium:* The losses used to determine the national pure premiums are the sum of the losses for each state as described above. The payrolls for each state are converted to a total payroll rule basis and then summed to determine national payroll. The national payroll is then converted to the state's

payroll limitation rule prior to dividing into the national losses in order to obtain the national pure premium.

4. *Derived by Formula:* The formula pure premium is derived by weighting among the indicated, underlying, and national pure premiums. The weight given to the indicated pure premium varies from zero to 100 percent, depending upon the volume of the expected losses. If the indicated pure premium receives less than 100% credibility, the national pure premium is assigned its national credibility, limited as follows: the national pure premium may not be assigned a credibility greater than one-half of (100% – state credibility). Thus, if a state indicated pure premium is 40% credible and the pure premium indicated by the national figures has 90% credibility, the national pure premium is assigned a weight of 30%, [(100% – 40%) ÷ 2], and the underlying pure premium is also assigned 30%, (100% – 40% – 30%).

B. *Calculation of Proposed Rates*

The following factors are applied to the formula pure premiums to derive rates.

1. *Ratio of Manual Premiums to Earned Premiums*
   Ratios of manual premiums to earned premiums have been calculated on a national basis separately for three groups of "F" classes. The three groups are "Shipbuilding and Repairs", "Stevedoring", and "Non-Appropriated Fund Instrumentalities".

2. *Law Amendments*
   Law amendments not included in the pure premium exhibits are applied by parts (serious, non-serious, and medical).

3. *Expense Allowance*
   The expense allowance, which is the same as in the general rate revision for the state, is included in the rate by dividing the product of the proposed pure premiums and the appropriate factors from (1) and (2) above by the permissible loss and loss adjustment ratio. This operation produces an indicated rate which then is subject to limitations as described in the next paragraph.

C. *Limitation of Rate Change*

It is recognized that a portion of the ratemaking experience now available is prior to the November 26, 1972 amendment to the Longshoremen's Act and, therefore, is influenced by cost-related conditions as they apply in varying degrees state by state. Therefore, the manual rate is limited according to the following program.

Establish a range as being 10% below to 10% above the national indicated rate for each classification. The following conditions apply:

1. If the present rate and the indicated rate both fall in the range, the indicated rate is proposed without further adjustment.
2. If the present rate falls inside the range and the indicated manual rate falls outside the range, the proposed rate will be limited to the rate establishing the boundary of the range.
3. If the present rate is outside the range and the indicated rate falls inside the range, the indicated rate is used without further adjustment.
4. If both the present and the indicated rate fall outside the range (on the same side), the present rate is retained if the indicated rate is further away from the range; if the indicated rate is closer to the range, then the proposed rate is the indicated rate without adjustment.
5. If both the present and the indicated rate fall outside the range (on opposite sides), then the range boundary nearest the indicated rate is the propsed rate.

Finally, each proposed rate is limited to a change of not more than 50% (up or down) from the present manual rate in order to prevent any drastic rate change from occurring.

CALCULATION OF AN EX-MEDICAL RATE[2]

$$\text{Ex-Medical Rate} = \text{Statutory medical rate} - 70\% \text{ of Medical Rate}$$

$$= \text{Standard Rate} - \frac{.70 \text{ Medical Pure Premiums}}{\text{Permissible Loss Ratio}}$$

$$\text{The Ex-Medical Ratio} = 1.0 - \frac{\text{Ex-Med. Rate}}{\text{Standard Rate}}$$

$$= 1.0 - \left(\frac{\text{Standard Rate} - \dfrac{.70 \text{ Med. P.P.}}{\text{Perm. L. R.}}}{\text{Standard Rate}} \right)$$

$$= 1.0 - \left(1.0 - \frac{.70 \text{ Med. P. P.}}{\text{Perm. L. R.}} \times \frac{1}{\text{Standard Rate}} \right)$$

But Perm. L. R. $\times$ Standard Rate = Total Pure Premium.

$$\text{Therefore Ex-Medical Ratio} = 1.0 - \left(1.0 - .70 \frac{\text{Med. P. P.}}{\text{Total P. P.}} \right)$$

$$= \quad .70 \frac{\text{Med. P. P.}}{\text{Total P. P.}}$$

[2] Reprinted from Marshall, Ralph, "Workmen's Compensation Insurance Ratemaking," Casualty Actuarial Society, 1961

A MATHEMATICAL MODEL FOR LOSS RESERVE ANALYSIS

CHARLES L. McCLENAHAN

"Contrariwise," continued Tweedledee, "if it was so, it might be; and if it were so, it would be; but as it isn't, it ain't. That's logic."

— *Lewis Carroll*

It has long been recognized that loss reserving is, or should be, within the domain of the Casualty Actuary; but in no other area have we applied our expertise with as little success. We have devised classification systems which generate unique automobile insurance rates for single female farmers living in Manhattan and we have developed so many formulae for partial credibility that we are in danger of losing ours. In our sixty year history we have truly put the "science" in "actuarial science." But, as a review of the experience of the past few years points out, we still have difficulty establishing accurate loss reserves.

One reason for this difficulty is the dearth of analytical tools with which to quantify the effects of changes in payment patterns, inflation, frequency and other factors upon reserve adequacy. Where a line of business has a "long tail" we must go back several years in order to examine a relatively complete development pattern — and the intervening years may have brought changes which should be taken into account in establishing current loss reserves.

Over the years, actuaries and others have developed several mathematical models to deal with the projection problem. These models range from the rather simple deterministic model underlying the calculation of loss development factors to the sometimes quite complex models of incurred losses which have been built into probabilistic planning models.

More recently, attention has been turned to the use of mathematical models in the analysis of loss reserves.[1] Not only in the area of adequacy determination but also in the area of financial planning it is becoming more

[1] See, for example, Simon, "Distortion in IBNR Factors", *P.C.A.S. LVII, p. 64.*

and more important that paid losses and loss reserves be treated separately rather than being dealt with, on a combined basis, as incurred losses. As cash flow begins to rival profitability as the key area for analysis by management, investors and regulators, the need for reserving models based upon paid losses becomes more intense. This paper presents one such model.

Any loss payment model which is proposed for use in the analysis of loss reserves must meet certain minimum requirements. First, the cumulative paid losses for a given incurred period must obviously converge to the ultimate incurred losses. Second, the model should allow for the varying of frequency and severity assumptions separately. Finally, the model should provide a reasonable approximation of reality. Where the model is designed to serve as a component of a larger corporate model it is also desirable that the model be simple — especially if the macro model is probabilistic.

The model described herein represents the results, to date, of the formulation and testing (mostly on a trial-and-error basis) of several paid loss development models.

THE MODEL

Assume that, where severity — that is the pure loss cost resulting from the average claim — is constant over time, losses of 1 incurred during a given (accident) month m are paid during subsequent months $m+n$ in amounts equal to pq^{n-d} where $0 < p < 1$, $q = 1-p$, $n \geq d$, and where d is the average delay in months between loss occurrence and loss reporting. In other words, assume that no payments are made for the first d months and then monthly payments are made at the rate of 100p% of the unpaid losses at the beginning of each subsequent month.[2]

If we let x represent the uniform monthly rate of change in severity, and y the uniform monthly rate of change in accident month incurred losses due to claim frequency and exposure volume increases or decreases, we are able to develop certain relationships between paid losses, incurred losses and loss reserves. It is necessary that assumptions x and y be treated separately because, while x impacts the amount of loss through the date of payment, the effect of y is felt only through the incurred date.

[2] This, of course, assumes that all losses occur on the first day of the month and represents an average delay of d-½ months assuming a uniform distribution of loss occurrence.

Let ${}_nP_m$ represent the amount paid during month $m+n$ ($n \geq 0$) on losses incurred during accident month m. If $n < d$ *then*, ${}_nP_m = 0$. If $n \geq d$ *then*, ${}_nP_m$ can be expressed as:

$$ {}_nP_m = cpq^{n-d}\,(1+x)^{m+n}\,(1+y)^m \tag{1}$$

where c represents the constant-dollar losses (i.e. incurred losses where $x=0$) for some base accident month ($m=0$).

It will be helpful at this point to define three additional values:

$$z = x + y + xy$$

$$r = q(1+x)$$

$$b = cp(1+x)^d$$

Note that z represents the combined effects of x and y (i.e. $1+z = (1+x)\ (1+y)$); r is a combination of the effects of severity increases $(1+x)$ during a month and the unpaid loss factor (q); and b represents the payments during month d on losses of c incurred during month $m=0$. Substituting into (1):

$$ {}_nP_m = br^{n-d}(1+z)^m \quad , \qquad n \geqslant d \geqslant 0 \tag{2}$$

Formula (2) is the basis for the model described in this paper. All of the subsequent formulae and relationships follow directly from (2).

Defining I_m as the losses incurred in month m:[3, 4]

$$ I_m = \sum_{k=d}^{\infty} {}_kP_m = \frac{b(1+z)^m}{1-r} \tag{3}$$

And, defining ${}_nU_m$ as the losses incurred in month m which remain unpaid at the end of month $m+n$:

$$ {}_nU_m = \sum_{k=n+1}^{\infty} {}_kP_m = \frac{br^{n-d+1}(1+z)^m}{1-r}\ , n \geqslant d \geqslant 0 \tag{4}$$

---

[3] The derivations of the formulae in this section will be found in Technical Appendix 1.

[4] In this and all subsequent formulae it is assumed that $-1 < x < p/q$ (i.e. $0 < r < 1$). Note that if $\geq p/q$ ultimate incurred losses are infinite as severity is increasing the value of unpaid losses faster than the losses are being settled.

Now, if we let R_m represent the total required reserve at the end of month m:

$$R_m = \sum_{k=0}^{d-1} I_{m-k} + \sum_{k=d}^{\infty} {}_kU_{m-k}$$

$$= \frac{b(1+z)^{m-d+1}}{1-r}\left[\frac{(1+z)^d-1}{z} + \frac{r}{1+z-r}\right], d \geqslant 0 \quad (5)$$

One final definition is necessary. Let ${}_mP_{tot}$ be the total losses paid during month m. Then:

$${}_mP_{tot} = \sum_{k=d}^{\infty} {}_kP_{m-k} = \frac{b(1+z)^{m-d+1}}{1+z-r}, d \geqslant 0 \quad (6)$$

ACCIDENT YEAR MODEL

We can now examine the paid loss model in the accident year mode.

Let: AI_t = incurred losses for accident year t;

${}_nAP_t$ = accident year t losses paid during the year $t+n$ $(n \geqslant 0)$;

${}_nAR_t$ = required reserve for accident year t at the end of the year $t+n$ $(n \geqslant 0)$.

The accident year incurred formula is fairly straightforward. Since the payment model is predicated upon losses incurred in a given month, the accident year incurred losses are simply the sum of twelve months of incurred losses:[5]

$$AI_t = \sum_{k=12t}^{12t+11} I_k = \frac{b(1+z)^{12t}}{1-r}\left[\frac{(1+z)^{12}-1}{z}\right] \quad (7)$$

The accident year payment formulae present a more difficult computational task. Even if there were no delay between incurred date and reported date, a separate formula would be required for the payments made during the accident year. Where d exceeds 1 additional formulae are re-

[5] The derivations of the formulae in this section will be found in Technical Appendix 2.

quired. In developing the following it has been assumed that d is at least 1 month but does not exceed 12 months. It is obvious that d can never be zero as that would require the average loss to be reported 1/2 month prior to its occurrence.

${}_0AP_t$ represents the payments on accident year t incurred losses made during year t. Thus:

$$ {}_0AP_t = \sum_{k=12t}^{12t+11-d} \sum_{j=d}^{12t+11-k} {}_jP_k $$

$$ = \frac{b(1+z)^{12t}}{1-r} \left[\frac{(1+z)^{12-d}-1}{z} - r \left[\frac{(1+z)^{12-d}-r^{12-d}}{1+z-r} \right] \right], $$

$$ 1 \leqslant d \leqslant 12 \qquad (8) $$

${}_1AP_t$ represents the payments on accident year t incurred losses during the first subsequent year. Where $2 \leqslant d \leqslant 12$ payments on the last $d-1$ months of the accident year will not be made until the full d month delay has elapsed. For this reason the formula for ${}_1AP_t$ requires two double summations as follows:

$$ {}_1AP_t = \left[\sum_{k=12t}^{12t+12-d} \sum_{j=12t+12-k}^{12t+23-k} {}_jP_k \right] + \left[\sum_{k=12t+13-d}^{12t+11} \sum_{j=d}^{12t+23-k} {}_jP_k \right] $$

$$ = \frac{b(1+z)^{12t}(1-r^{12})}{1-r} \left[\frac{(1+z)^{13-d}-r^{13-d}}{1+z-r} \right] $$

$$ + \frac{b(1+z)^{12t+13-d}}{1-r} \left[\left[\frac{(1+z)^{d-1}-1}{z} \right] - (r^{13-d}) \left[\frac{(1+z)^{d-1}-r^{d-1}}{1+z-r} \right] \right], 2 \leqslant d \leqslant 12 \qquad (9) $$

Where $n \geqslant \frac{d+11}{12}$, each accident month has payments during each of the twelve months of the year $t+n$. Therefore, a single formula will serve for all such years as follows:

$$ {}_nAP_t = \sum_{k=12t}^{12t+11} \; \sum_{j=12(n+t)-k}^{12(n+t)-k+11} {}_jP_k $$

$$ = \frac{b(1-r^{12})(1+z)^{12t}(r^{12n-11-d})}{1-r} \left[\frac{(1+z)^{12}-r^{12}}{1+z-r}\right], \; n \geqslant \frac{d+11}{12} \tag{10} $$

The accident year reserve formulae present similar problems to those encountered with the accident year paid. Again the formulae assume $1 \leqslant d \leqslant 12$.

The reserve at the end of the accident year is the difference between the accident year incurred and the amount paid during the accident year:

$$ {}_0AR_t = AI_t - {}_0AP_t $$

$$ = \frac{b(1+z)^{12t}}{1-r} \left[\left[\frac{(1+z)^{12}-(1+z)^{12-d}}{z}\right] + r \left[\frac{(1+z)^{12-d}-r^{12-d}}{1+z-r}\right] \right], \quad 1 \leqslant d \leqslant 12 \tag{11} $$

The reserve for accident year t at the end of the year $t+n$ $(n \geqslant 1)$ can be expressed as follows:

$$ {}_nAR_t = \sum_{k=12t}^{12t+11} {}_{12(t+n)-k+11}U_k $$

$$ = \frac{br^{12n-d+1}(1+z)^{12t}}{1-r} \left[\frac{(1+z)^{12}-r^{12}}{1+z-r}\right], \; n \geqslant \frac{d}{12} \tag{12} $$

COMPARISON OF MODEL WITH ACTUAL ACCIDENT YEAR DATA

In order to test the model, the automobile bodily injury loss data for accident year 1968 of five large writers of automobile insurance was compiled. This data was as follows (000 omitted):

Paid through 12/31/68	$112,528
Paid through 12/31/69	328,420
Paid through 12/31/70	453,371
Paid through 12/31/71	528,505
Paid through 12/31/72	575,449
Paid through 12/31/73	597,843
Paid through 12/31/74	608,204
Reserve at 12/31/74	9,931

The accident year model was then applied using the following values:

$$p = .0498 \qquad x = .004 \qquad c = 44{,}414$$
$$d = 2 \qquad y = .007 \qquad t = 0$$

The following tables detail the relationship between the actual and theoretical data. It should be noted that no attempt was made to obtain the best fit between theoretical and actual data. p was estimated from the actual data and d, x, and y were selected as representative of the line and the conditions extant in 1968.[6]

TABLE I
DISTRIBUTION OF ACCIDENT YEAR 1968 LOSSES

	Theoretical	Actual	Difference	Percent of Actual
Paid during 1968	110,947	112,528	− 1,581	− 1.4
Paid during 1969	217,453	215,892	+ 1,561	+ 0.7
Paid during 1970	125,078	124,951	+ 127	+ 0.1
Paid during 1971	71,082	75,134	− 4,052	− 5.4
Paid during 1972	40,396	46,944	− 6,548	− 13.9
Paid during 1973	22,957	22,394	+ 563	+ 2.5
Paid during 1974	13,047	10,361	+ 2,686	+ 25.9
Reserve 12/31/74	17,175	9,931	+ 7,244	+ 72.9
Total Incurred	618,135	618,135		

[6] See Technical Appendix 3 for an explanation of this application.

TABLE II
ACCIDENT YEAR 1968 REQUIRED RESERVES

Reserve Date	Theoretical	Actual	Difference	Percent of Actual
12/31/68	507,188	505,607	+ 1,581	+ 0.3
12/31/69	289,735	289,715	+ 20	+ 0.0
12/31/70	164,657	164,764	− 107	− 0.1
12/31/71	93,575	89,630	+ 3,945	+ 4.4
12/31/72	53,179	42,686	+ 10,493	+ 24.6
12/31/73	30,222	20,292	+ 9,930	+ 48.9
12/31/74	17,175	9,931	+ 7,244	+ 72.9
Average	165,104	160,375	+ 4,729	+ 2.9

TABLE III
ACCUMULATIVE PERCENTAGE OF 1968
ACCIDENT YEAR LOSSES PAID

Date	Theoretical	Actual
12/31/68	17.9%	18.2%
12/31/69	53.1	53.1
12/31/70	73.4	73.3
12/31/71	84.9	85.5
12/31/72	91.4	93.1
12/31/73	95.1	96.7
12/31/74	97.2	98.4

The above tables indicate that, while the model does not provide as close a fit to the sample data as might be desirable (especially in the later years of development), the fit is sufficiently close to allow us to use the model to advantage—particularly where total reserves (as opposed to reserves for a specific accident year) are the subject of study. For example, the model can be quite useful in the analysis of the effects upon reserve adequacy of changes in various exogenous variables and in the testing of the established loss reserves on a prospective basis.

APPLICATIONS TO LOSS RESERVE ANALYSIS

The remainder of this paper is devoted to the analysis of loss reserves through the use of certain theoretical relationships developed from the model.

Effect of "discounting" loss reserves

If, instead of reserving a full dollar for each dollar to be paid t months hence, we "discount" the t month-deferred dollar by v^t, where $v = (1+i)^{-1}$ and i represents the assumed monthly yield on our invested reserves, the result is the present value of the loss reserve.

The model may be used to approximate the effect of such "discounting" of the loss reserves. Defining DR_m as the present value of R_m:[7]

$$DR_m = \left[\sum_{k=0}^{d-1} \sum_{j=d}^{\infty} (v^{j-k})\ {}_jP_{m-k}\right] + \left[\sum_{k=d}^{\infty} \sum_{j=k+1}^{\infty} (v^{j-k})\ {}_jP_{m-k}\right]$$

$$= \frac{bv(1+z)^{m-d+1}}{1-vr}\left[\frac{v^d(1+z)^d-1}{v(1+z)-1}+\frac{r}{1+z-r}\right]$$

And the ratio of DR_m to R_m can be expressed as:

$$\frac{DR_m}{R_m} = \frac{vz(v-r)}{(1-vr)\,[v(1+z)-1]}$$
$$\left[\frac{v^d(1+z)^{d-1}(1+z-r)-(1-vr)}{(1+z)^{d-1}(1+z-r)-(1-r)}\right]$$

Effect upon reserve adequacy of a change in x

The same approach may be taken in determining the effect of a change at the end of month m in the monthly severity increase rate x. If the "new" rate is denoted by x', and defininig $r' = q(1+x')$ and $z' = x' + y + x'y$, then the required reserve at m, adjusted for the change from x to x', can be expressed as follows simply by replacing v in the expression for DR_m by $\frac{1+x'}{1+x}$:

$$R'_m = \frac{b(1+x')\,(1+z)^{m-d+1}}{(1+x)\,(1-r')}\left[\frac{(1+z')^d-1}{z'}+\frac{r}{1+z-r}\right]$$

And:

$$\frac{R'_m}{R_m} = \frac{z(1+x')\,(1-r)}{z'(1+x)\,(1-r')}$$
$$\left[\frac{(1+z')^d\,(1+z-r)-(1+z)\,(1-r')}{(1+z)^d\,(1+z-r)-(1+z)\,(1-r)}\right]$$

[7] The derivations of the relationships described in this section will be found in Technical Appendix 4.

Required reserve relative to current monthly payment rate

Where a line of business has been written for a sufficient period to have closed substantialy all of the losses incurred during the first year of writing, an approximation based upon the model may be used in testing the adequacy of current loss reserves.

The test consists of determining the ratio of R_m to ${}_mP_{tot}$ from the model, and multiplying that ratio by the current average monthly loss payment rate. The resultant product is an approximation of the current required reserve. The ratio of R_m to ${}_mP_{tot}$ can be expressed as:

$$\frac{R_m}{{}_mP_{tot}} = \frac{(1+z)^d - (1+z)}{z} + \frac{(1+z)^d}{1-r}$$

CONCLUSION

The model described in this paper is but one of many models of loss payment patterns which can be developed and successfully applied to reserving problems. The applications described herein likewise represent but a few of the potential applications of such a model. It is this author's hope and expectation that the next few years will see additional actuarial papers presented on this and related subjects.

TECHNICAL APPENDIX 1

Development of formula (3):

$$I_m = \sum_{k=d}^{\infty} {}_kP_m = \sum_{k=d}^{\infty} br^{k-d}(1+z)^m = br^{-d}(1+z)^m \sum_{k=d}^{\infty} r^k$$

$$= \frac{b(1+z)^m}{1-r}, \quad 0 < r < 1$$

Development of formula (4):

$$ {}_nU_m = \begin{cases} \displaystyle\sum_{k=n+1}^{\infty} {}_kP_m = br^{-d}(1+z)^m \sum_{k=n+1}^{\infty} r^k = \frac{br^{n-d+1}(1+z)^m}{1-r}, & n \geqslant d \\ I_m, & n < d \end{cases}$$

Development of formula (5):

$$R_m = \sum_{k=0}^{d-1} I_{m-k} + \sum_{k=d}^{\infty} {}_kU_{m-k} = \sum_{k=0}^{d-1} \frac{b(1+z)^{m-k}}{1-r}$$

$$+ \sum_{k=d}^{\infty} \frac{br^{k-d+1}(1+z)^{m-k}}{1-r}$$

$$= \frac{b(1+z)^m}{1-r} \left\{ \sum_{k=0}^{d-1} (1+z)^{-k} + r^{1-d} \sum_{k=d}^{\infty} \left(\frac{r}{1+z}\right)^k \right\}$$

$$= \frac{b(1+z)^m}{1-r} \left\{ \frac{(1+z)^d-1}{z(1+z)^{d-1}} + r^{1-d} \left[\frac{r^d}{(1+z)^{d-1}(1+z-r)}\right] \right\}$$

$$= \frac{b(1+z)^{m-d+1}}{1-r} \left[\frac{(1+z)^d-1}{z} + \frac{r}{1+z-r}\right]$$

Development of formula (6):

$$ {}_{m}P_{tot} = \sum_{k=d}^{\infty} {}_{k}P_{m-k} = \sum_{k=d}^{\infty} br^{k-d}(1+z)^{m-k} $$

$$ = br^{-d}(1+z)^{m} \sum_{k=d}^{\infty} \left(\frac{r}{1+z}\right)^{k} $$

$$ = \frac{b(1+z)^{m-d+1}}{1+z-r} $$

TECHNICAL APPENDIX 2

Development of formula (7):

$$AI_t = \sum_{k=12t}^{12t+11} I_k = \frac{b}{1-r} \sum_{k=12t}^{12t+11} (1+z)^k$$

$$= \frac{b(1+z)^{12t}}{1-r} \left[\frac{(1+z)^{12}-1}{z} \right]$$

Development of formula (8):

$$_0AP_t = \sum_{k=12t}^{12t+11-d} \sum_{j=d}^{12t+11-k} {}_jP_k = b \sum_{k=12t}^{12t+11-d} (1+z)^k \sum_{j=d}^{12t+11-k} r^{j-d}$$

$$= b \sum_{k=12t}^{12t+11-d} (1+z)^k \left(\frac{1-r^{12t+12-d-k}}{1-r} \right)$$

$$= \frac{b}{1-r} \left[\sum_{k=12t}^{12t+11-d} (1+z)^k - r^{12t+12-d} \sum_{k=12t}^{12t+11-d} \left(\frac{1+z}{r} \right)^k \right]$$

$$= \frac{b}{1-r} \left[(1+z)^{12t} \left[\frac{(1+z)^{12-d}-1}{z} \right] - r^{12-d}(1+z)^{12t} \left[\frac{(1+z)^{12-d}-r^{12-d}}{r^{11-d}(1+z-r)} \right] \right]$$

$$= \frac{b(1+z)^{12t}}{1-r} \left[\frac{(1+z)^{12-d}-1}{z} - r \left[\frac{(1+z)^{12-d}-r^{12-d}}{1+z-r} \right] \right],$$

$$1 \leqslant d \leqslant 12$$

Development of formula (9):

$$
{}_1AP_t = \sum_{k=12t}^{12t+12-d} \; \sum_{j=12t+12-k}^{12t+23-k} {}_jP_k
+ \sum_{k=12t+13-d}^{12t+11} \; \sum_{j=d}^{12t+23-k} {}_jP_k
$$

$$
= b \sum_{k=12t}^{12t+12-d} (1+z)^k \sum_{j=12t+12-k}^{12t+23-k} r^{j-d}
+ b \sum_{k=12t+13-d}^{12t+11} (1+z)^k \sum_{j=d}^{12t+23-k} r^{j-d}
$$

$$
= b \sum_{k=12t}^{12t+12-d} (1+z^k r^{12t+12-k-d} \left(\frac{1-r^{12}}{1-r}\right)
+ b \sum_{k=12t+13-d}^{12t+11} (1+z)^k \left(\frac{1-r^{12t+24-k-d}}{1-r}\right)
$$

$$
= \frac{b}{1-r}\Bigg[(1-r^{12}) r^{12t+12-d} \sum_{k=12t}^{12t+12-d} \left(\frac{1+z}{r}\right)^k
+ \sum_{k=12t+13-d}^{12t+11} (1+z)^k - r^{12t+24-d} \sum_{k=12t+13-d}^{12t+11} \left(\frac{1+z}{r}\right)^k \Bigg]
$$

$$
= \frac{b}{1-r}\Bigg[(1-r^{12}) r^{12t+12-d} \left(\frac{1+z}{r}\right)^{12t} \left[\frac{(1+z)^{13-d} - r^{13-d}}{r^{12-d}(1+z-r)}\right]
+ (1+z)^{12t+13-d} \left[\frac{(1+z)^{d-1} - 1}{z}\right]
- r^{12t+24-d} \left(\frac{1+z}{r}\right)^{12t+13-d} \left[\frac{(1+z)^{d-1} - r^{d-1}}{r^{d-2}(1+z-r)}\right] \Bigg]
$$

$$= \frac{b(1+z)^{12t}(1-r^{12})}{1-r} \left[\frac{(1+z)^{13-d}-r^{13-d}}{1+z-r}\right]$$

$$+ \frac{b(1+z)^{12t+13-d}}{1-r}\left[\left[\frac{(1+z)^{d-1}-1}{z}\right] - (r^{13-d})\right.$$

$$\left.\left[\frac{(1+z)^{d-1}-r^{d-1}}{1+z-r}\right]\right], 2 \leqslant d \leqslant 12$$

Development of formula (10):

$${}_nAP_t = \sum_{k=12t}^{12t+11} \sum_{j=12(n+t)-k}^{12(n+t)-k+11} {}_jP_k = b \sum_{k=12t}^{12t+11} (1+z)^k$$

$$\sum_{j=12(n+t)-k}^{12(n+t)-k+11} r^{j-d}$$

$$= b \sum_{k=12t}^{12t+11} (1+z)^k [r^{12(n+t)-k-d}] \left(\frac{1-r^{12}}{1-r}\right)$$

$$= \frac{b(1-r^{12})r^{12(n+t)-d}}{1-r} \sum_{k=12t}^{12t+11} \left(\frac{1+z}{r}\right)^k$$

$$= \frac{b(1-r^{12})\ r^{12(n+t)-d}}{1-r} \left[\frac{(1+z)^{12t}}{r^{12t+11}}\right] \left[\frac{(1+z)^{12}-r^{12}}{1+z-r}\right]$$

$$= \frac{b(1-r^{12})(1+z)^{12t}(r^{12n-11-d})}{1-r} \left[\frac{(1+z)^{12}-r^{12}}{1+z-r}\right],$$

$$n \geqslant \frac{d+11}{12}$$

Development of formula (11):

$$
{}_0AR_t = AI_t - {}_0AP_t
$$

$$
= \frac{b(1+z)^{12t}}{1-r}\left[\left[\frac{(1+z)^{12}-1}{z}\right]-\left[\frac{(1+z)^{12-d}-1}{z}\right] + r\left[\frac{(1+z)^{12-d}-r^{12-d}}{1+z-r}\right]\right]
$$

$$
= \frac{b(1+z)^{12t}}{1-r}\left[\frac{(1+z)^{12}-(1+z)^{12-d}}{z} + r\left[\frac{(1+z)^{12-d}-r^{12-d}}{1+z-r}\right]\right],
$$

$$
1 \leqslant d \leqslant 12
$$

Development of formula (12):

$$
{}_nAR_t = \sum_{k=12t}^{12t+11} 12(t+n)-k+11^{U}k
$$

$$
= \frac{br^{12(t+n)+12-d}}{1-r}\sum_{k=12t}^{12t+11}\left(\frac{1+z}{r}\right)^k
$$

$$
= \frac{br^{12n-d+1}(1+z)^{12t}}{1-r}\left[\frac{(1+z)^{12}-r^{12}}{1+z-r}\right], \qquad n \geqslant \frac{d}{12}
$$

TECHNICAL APPENDIX 3

3.1 Estimation of x and y

For the five companies and groups included in the study, automobile bodily injury net written premium for 1967 and 1968 was used to determine the total premium growth rate as follows:

Net written premium 1968	$1,212,517,000
Net written premium 1967	$1,062,432,000
Growth factor 1968/1967	1.1413

A review of automobile bodily injury rate filings made during 1968 and 1969 indicated that the average annual trend factors being used were approximately:

Severity	+ 5.0% per year
Frequency	− 1.0% per year

x and y were then determined as follows:

1) Premium growth factor	.1413
2) Severity growth factor	.0500
3) Frequency growth factor	− .0100
4) Pure premium growth factor	
$(1.0500)(.9900) - 1$	.0395
5) Volume growth rate	
$(1.1413/1.0395) - 1$	.0979
6) Frequency and volume combined	
$(.9900)(1.0979) - 1$	.0869
7) $x = (1.0500)^{1/12} - 1 =$	.004
8) $y = (1.0869)^{1/12} - 1 =$	.007

3.2 Selection of d

2 was selected as the value of d based upon the author's experience with automobile bodily injury loss reporting patterns.

3.3 Selection of r

.0498 was selected as the value for r by setting the theoretical ratio of losses paid through 12/31/69 to total incurred losses equal to the actual ratio and solving for r:

$$\frac{{}_1AP_0}{AI_0} = \frac{328{,}420 - 112{,}528}{618{,}135} = .3493$$

3.4 Determination of b and c

b was determined to be 2229.56 by solving $AI_0 = 618{,}135$ for b.

c was determined to be 44,414 by solving $b = cp(1 + x)^d$ for c.

TECHNICAL APPENDIX 4

$$DR_m = \sum_{k=0}^{d-1} \sum_{j=d}^{\infty} (v^{j-k})_j P_{m-k} + \sum_{k=d}^{\infty} \sum_{j=k+1}^{\infty} (v^{j-k})_j P_{m-k}$$

$$= \sum_{k=0}^{d-1} \sum_{j=d}^{\infty} (v^{j-k}) b r^{j-d} (1+z)^{m-k}$$

$$+ \sum_{k=d}^{\infty} \sum_{j=k+1}^{\infty} (v^{j-k}) b r^{j-d} (1+z)^{m-k}$$

$$= b r^{-d} (1+z)^m \left[\sum_{k=0}^{d-1} v^{-k} (1+z)^{-k} \sum_{j=d}^{\infty} (vr)^j \right.$$

$$\left. + \sum_{k=d}^{\infty} v^{-k} (1+z)^{-k} \sum_{j=k+1}^{\infty} (vr)^j \right]$$

$$= \frac{b r^{-d} (1+z)^m}{1 - vr} \left[\sum_{k=0}^{d-1} v^{-k} (1+z)^{-k} (vr)^d \right.$$

$$\left. + \sum_{k=d}^{\infty} v^{-k} (1+z)^{-k} (vr)^{k+1} \right]$$

$$= \frac{b r^{-d} (1+z)^m}{1 - vr} \left[(vr)^d \sum_{k=0}^{d-1} v^{-k} (1+z)^{-k} + vr \sum_{k=d}^{\infty} \left(\frac{r}{1+z} \right)^k \right]$$

$$= \frac{b r^{-d} (1+z)^m}{1 - vr} \left[(vr)^d \left(\frac{v^d (1+z)^d - 1}{v^{d-1} (1+z)^{d-1} [v(1+z) - 1]} \right) \right.$$

$$\left. + \frac{vr(1+z)}{1+z-r} \left(\frac{r}{1+z} \right)^d \right]$$

$$= \frac{bv(1+z)^{m-d+1}}{1 - vr}\left[\frac{v^d(1+z)^d-1}{v(1+z)-1} + \frac{r}{1+z-r}\right]$$

$$\frac{DR_m}{R_m} = \frac{\dfrac{bv(1+z)^{m-d+1}}{1 - vr}\left[\dfrac{v^d(1+z)^d-1}{v(1+z)-1} + \dfrac{r}{1+z-r}\right]}{\dfrac{b(1+z)^{m-d+1}}{1 - r}\left[\dfrac{(1+z)^d-1}{z} + \dfrac{r}{1+z-r}\right]}$$

$$= \frac{\dfrac{v}{1-vr}\left[\dfrac{v^d(1+z)^d-1}{v(1+z)-1} + \dfrac{r}{1+z-r}\right]}{\dfrac{1}{1-r}\left[\dfrac{(1+z)^d-1}{z} + \dfrac{r}{1+z-r}\right]}$$

$$= \frac{v(1-r)\left[\dfrac{v^d(1+z)^d-1}{v(1+z)-1} + \dfrac{r}{1+z-r}\right]}{(1-vr)\left[\dfrac{(1+z)^d-1}{z} + \dfrac{r}{1+z-r}\right]}$$

$$= \frac{vz(1-r)}{(1-vr)[v(1+z)-1]}\left[\frac{v^d(1+z)^{d-1}(1+z-r) - (1-vr)}{(1+z)^{d-1}(1+z-r) - (1-r)}\right]$$

$$\frac{R_m}{{}_mP_{tot}} = \frac{\dfrac{b(1+z)^{m-d+1}}{1 - r}\left[\dfrac{(1+z)^d-1}{z} + \dfrac{r}{1+z-r}\right]}{\dfrac{b(1+z)^{m-d+1}}{1 + z - r}}$$

$$= \frac{(1+z-r)\left[\dfrac{(1+z)^d-1}{z} + \dfrac{r}{1+z-r}\right]}{(1 - r)}$$

$$= \frac{(1+z)^d-1}{z} + \frac{(1+z)^d-1+r}{(1-r)}$$

$$= \frac{(1+z)^d-(1+z)}{z} + \frac{(1+z)^d}{1-r}$$

LOSS RESERVE TESTING IN A CHANGING ENVIRONMENT

WAYNE H. FISHER AND EDWARD P. LESTER

Determining accurate loss reserves is one of the most challenging tasks facing the actuary, and through the years numerous approaches have been devised to assist in developing reasonable estimates. Many of these are outlined in Skurnick's "A Survey of Loss Reserving Methods."[1]

An actuary, in testing reserve adequacy, frequently employs several different methodologies before determining a final estimate. The approach which produced the most accurate result in previous evaluations might again be heavily relied upon. In a stable environment, an approach which has given reasonable results over time may be expected to do so again. However, in a rapidly changing environment, a previously accurate approach may no longer be appropriate; in fact, the estimate produced might be extremely inaccurate since different methodologies react in varying degrees to changes in underlying experience.[2]

The purpose of this paper is to demonstrate the importance of determining any underlying changes in the claim environment in selecting a reserve test. Basically, this paper examines and compares how several different reserve methodologies react to changes in two "variables": the calendar/accident year loss ratio and the adequacy of the reserves for reported claims. All other factors which normally may change over time are assumed to remain constant.

Other factors, of course, do change and influence reserve tests. The introduction of no-fault insurance, for example, certainly precludes the rate application of a previously acceptable model. In this paper, however, we have chosen to concentrate on the impact of the two items mentioned above as typical of the problems that can develop. It should be recognized that any other factor which causes patterns in the underlying data to be unrepresentative of the current situation can produce a somewhat similar distortion.

[1] D. Skurnick, "A Survey of Loss Reserving Methods," PCAS, LX, 1973.

[2] This situation has been noted previously. For example, see Skurnick, *Ibid.*, p. 37.

[3] R. L. Bornhuetter and R. E. Ferguson, "The Actuary and IBNR," PCAS, LIX, 1972, p. 182.

The paper examines the overall adequacy of the reserves; hence, IBNR is defined as the difference between the ultimate liability and the reported loss reserves (i.e., IBNR includes both development on reported claims and the emergence of unreported claims). This IBNR definition was used by Bornhuetter-Ferguson[3], although, as they note, a more restricted definition is sometimes more appropriate.

Three methodoligies are analyzed, all of which are based on incurred losses and could be said to belong to the "development" family. They are as follows:

(i) *Loss Development* — This is essentially the standard loss development approach used in manual ratemaking. Ultimate loss development factors are determined for each accident year based on recent emergence patterns of incurred losses. (In this paper, for all three methodologies, development factors are based on the latest three points, where available.) These factors are used to estimate the ultimate loss liability for each accident year. The required IBNR is then the sum over the individual accident years of the differences between the estimated ultimate loss liability and the corresponding emerged losses.

(ii) *Expected Loss Approach* — As described in Bornhuetter-Ferguson[4], this approach is based on the ultimate loss development factors determined as in (i). The IBNR need for each accident year is estimated as the product of the accident year expected losses (based on the expected loss ratio) and $1 - \left[\frac{1}{\text{U.L.D.}}\right]$, where U.L.D. is the appropriate ultimate loss development factor. The total IBNR need is the sum over the accident years of these products.

(iii) *Percentage of Premium Method* — For each accident year, IBNR factors are computed from historical emerged losses as a percentage of premium. The IBNR estimate is the sum over the accident years of the product of the appropriate IBNR factor and the corresponding earned premium.

For the purpose of this analysis, the three development methodologies are applied to several different situations and the resulting IBNR estimates are compared. It is assumed that the real underlying situation is known, including both the ultimate loss ratio for each accident year and the ade-

[4] *Ibid.*, p. 186.

quacy of the reserves for reported claims. Clearly, the actuary trying to estimate the IBNR need would not be aware of this information. However, by assuming these items known, one is able to compare the answers produced by the different methodologies, both with each other and with the actual need in the various situations. In a real-life reserve test, part of the task is to make an informed judgment as to the underlying situation so as to choose the most appropriate technique.

THE STATIC SITUATION

When using a development approach to determine the IBNR need, the best results are obtained in a static environment. In such a static situation, one can expect that all three development methodologies would produce the same IBNR estimates. For example, consider the situation depicted in Exhibit I. The ultimate loss ratio for each accident year is constant; in addition, the loss emergence patterns do not change, i.e., the percentage development in incurred losses is the same for each accident year at common valuation dates. As seen below, all three methods yield the same (and the correct) IBNR need in this situation:[5]

Loss Development Method

Accident Year	(1) Ultimate Loss Development Factor[6]	(2) Acc. Year Losses at Current Valuation	(3) Estimated IBNR Required (Col. (1)—1) x Col. (2)
8	2.000	720,000	720,000
7	1.333	990,000	329,670
6	1.159	1,035,000	164,565
5	1.054	1,024,650	55,331
4	1.023	938,124	21,577
3	1.009	832,351	7,491
2	1.003	717,724	2,153
1	1.000	600,000	0
			1,300,787

[5] The minor differences in the Estimated IBNR Required result from rounding the various factors.

[6] Details of the underlying calculations are shown in Exhibit V.

Expected Loss Method

Accident Year	(1) IBNR Factor[7] $1 - \frac{1}{\text{Ult. Loss Dev. Factor}}$	(2) Expected Losses	(3) Estimated IBNR Required Col. (1) x Col. (2)
8	.500	1,440,000	720,000
7	.250	1,320,000	330,000
6	.137	1,200,000	164,400
5	.051	1,080,000	55,080
4	.022	960,000	21,120
3	.009	840,000	7,560
2	.003	720,000	2,160
1	.000	600,000	0
			1,300,320

Percentage of Premium Method

Accident Year	(1) IBNR Factor[8]	(2) Earned Premium	(3) Estimated IBNR Required Col. (1) x Col. (2)
8	.301	2,400,000	722,400
7	.151	2,200,000	332,200
6	.083	2,000,000	166,000
5	.031	1,800,000	55,800
4	.014	1,600,000	22,400
3	.006	1,400,000	8,400
2	.002	1,200,000	2,400
1	.000	1,000,000	0
			1,309,600

Note that the actual IBNR need, as can be determined from Exhibit I, is $1,302,000 which agrees with the estimates produced by all three of the development methodologies. If the results for a particular line of insurance are static over a period of several accident years, both as to ultimate loss ratio and loss emergence patterns, the choice of a particular methodology from the development family is not an issue as all three will yield the same result.

[7] See Exhibit V.

[8] See Exhibit V.

DETERIORATING LOSS RATIO WITH NO RESERVE STRENGTHENING

Unfortunately for actuaries, the static situation described in the last paragraph is rarely observed in real life. The large underwriting losses experienced by the industry in 1974 and (most likely) in 1975 after profitable years in 1972 and 1973 exhibit a changing environment that is more the rule than the exception. Exhibit II, whch shows a deteriorating loss ratio, while retaining a constant loss emergence pattern as in Exhibit I, would be more typical of the situation one might encounter. The rote application of the three methods produces strikingly different IBNR estimates, in light of the fact that the loss ratio deterioration on Exhibit II is not unusual. (Note that a $361,000 difference in IBNR need produces a loss ratio distortion for the year of 15 points.)

Loss Development Method

Accident Year	(1) Ultimate Loss Development Factor	(2) Acc. Year Losses at Current Valuation	(3) Estimated IBNR Required (Col. (1)—1) x Col. (2)
8	2.000	960,000	960,000
7	1.333	1,237,500	412,088
6	1.159	1,207,500	191,993
5	1.054	1,195,425	64,553
4	1.023	1,016,301	23,375
3	1.009	832,351	7,491
2	1.003	717,724	2,153
1	1.000	600,000	0
			1,661,653

Expected Loss Method

Accident Year	(1) IBNR Factor $1 - \frac{1}{\text{Ult. Loss Dev. Factor}}$	(2) Expected Losses	(3) Estimated IBNR Required Col. (1) x Col. (2)
8	.500	1,440,000	720,000
7	.250	1,320,000	330,000
6	.137	1,200,000	164,400
5	.051	1,080,000	55,080
4	.022	960,000	21,120
3	.009	840,000	7,560
2	.003	720,000	2,160
1	.000	600,000	0
			1,300,320

Percentage of Premium Method

Accident Year	(1) IBNR Factor	(2) Earned Premium	(3) Estimated IBNR Required Col. (1) x Col. (2)
8	.344	2,400,000	825,600
7	.165	2,200,000	363,000
6	.088	2,000,000	176,000
5	.032	1,800,000	57,600
4	.014	1,600,000	22,400
3	.006	1,400,000	8,400
2	.002	1,200,000	2,400
1	.000	1,000,000	0
			1,455,400

The actual IBNR need determined from Exhibit II is $1,663,000; since the loss emergence patterns are consistent, it is clear that the loss development methodology produces the correct need.

The Expected Loss Method yields exactly the same IBNR estimate as in the static situation in the prior section. This is not surprising since the Expected Loss Method depends only on the loss development factors and the expected loss rato, neither of which has changed. Clearly, this is a serious weakness in this approach. If the actuary does not change the expected loss ratio, the method will underestimate the required reserve in such a situation.

Given the situation displayed in Exhibit II, one would probably be motivated to modify the expected loss ratio for the more recent accident years. However, although a few of the accident years already are showing a loss ratio a little above 60 based on the losses already emerged, for the two most recent accident years (which produce the bulk of the IBNR need), the emerged loss ratios are below 60. Consequently, the higher-than-expected emergence could be deemed to be attributable to reserve strengthening. In addition, one purpose of the paper is to examine the sensitivity of the method to varying situations and this example stresses the fact that without a further judgmental descision, the Expected Loss Method will in this case produce too low an IBNR estimate.

For the situation shown in Exhibit II, the Percentage of Premium Method produces an IBNR need between the estimates produced by the Loss Development and Expected Loss Methods. The loss development patterns are consistent; hence, as the loss ratio deteriorates, an increasing percentage of the premium will emerge as IBNR. However, this approach fails to determine the true need, since the IBNR factors are directly related to the loss ratios in the prior accident years, which are below the current ultimate loss ratio. (Of course, if one can adjust current premiums to offset any rate inadequacy, this problem is eliminated.)

Two further points should be mentioned. First, the Percentage of Premium Method tends to be "self-correcting" in this situation since a deteriorating loss ratio will create increased IBNR factors (percentage of premium emerged) which produces an increased IBNR estimate. In fact, if the loss ratio then stabilizes for several years, the Percentage of Premium Method will eventually produce the correct IBNR need. This is in contrast to the Expected Loss Method where the IBNR estimate will not change unless there is a judgmental decision made to revise the expected loss ratio. In addition, it should be noted that, as with the Expected Loss Method, the emerged losses of the most recent accident year are not used at all in the computation, and in certain situations, this is a weakness of the method.

Given the changing situation depicted in Exhibit II, the actuary must make a choice among the three methodologies since each yields a different answer. In this particular case, only the choice of the Loss Development

Method would yield the correct IBNR need although the Percentage of Premium Method produces a more accurate answer than the Expected Loss Method. Similar comments would apply when the ultimate loss ratio is improving and the loss emergence patterns remain consistent.

CONSISTENT LOSS RATIO WITH RESERVE STRENGTHENING

The static situation set forth in Exhibit I consists of a long-term stable loss ratio along with an equally stable incurred loss development pattern. The development pattern is dependent on both the adequacy of the reported claim reserves at various stages of maturity and the actual rate of emergence of late-reported claims. In this section, the data underlying the stable situation of Exhibit I is modified to reflect a changing level of reserve adequacy for reported claims.

First, the paid losses underlying the experience were selected. A stable payment rate was assumed, viz., at the twelve-month valuation date 15% of the accident year ultimate incurred losses were paid, at 24 months 45%, at 36 months 65%, at 48 months 75%, at 60 months 85%, at 72 months 90%, at 84 months 95%, and 100% at 96 months.

Second, the outstanding losses were modified to reflect reserve adequacy levels different than the consistent, long-term levels underlying the static situation. The long-term levels are those shown below for accident years 1 and 2; the remaining ones are the result of an assumed slippage followed by an abrupt strengthening and return to the historical reserve adequacy levels.

Assumed Levels of Reserve Adequacy

Valuation	Accident Year							
Date	1	2	3	4	5	6	7	8
12 Months	85%	85%	85%	85%	80%	75%	70%	85%
24 Months	90	90	90	90	85	85	90	
36 Months	95	95	95	90	90	95		
48 Months	100	100	95	95	100			
60 Months to Ult.	100	100	100	100				

The resulting incurred losses, together with a sample calculation of the adjustment, are displayed in Exhibit III. As the calculations below show, all three methods overstate the required IBNR in this situation with the Loss

Development Method nearly 15% over the mark, which is $1,302,000 as in the static situation. The Loss Development Method is the most susceptible to distortions from changes in the adequacy level of the reserves for reported claims. This is, of course, largely because the bulk of the IBNR required is attributable to the current accident year and by incorporating the actual emerged losses the relative adequacy of the current reserve exerts considerable leverage.

All three methods, however, contain the same type of distortion which stems from the relationship between the composition of the development factor and the current incurred losses. In a time of unexpected rapid inflation, the reserves carried one or more years ago are almost certain to have been somewhat inadequate at that time and raising them now to current values adds an additional increment to the development factor. The current reserves, however, are being set in the midst of an environment of higher inflation and assuming we do not experience another significant jump in the rate of inflation, these reserves should not develop as adversely as anticipated in the loss development factor. Used together without modification, the estimated reserve will be overstated. In this specific example, a rote application of the Loss Development Method would incorrectly add nearly 10 points to the current year's loss ratio.

The Expected Loss and Percentage of Premium Methods are distorted to a lesser extent because they are not subject to a leverage impact from the emerged incurred losses. However, in both cases, the factors utilized do assume a certain amount of future development which incorrectly includes a provision for the extra reserve strengthening.

Loss Development Method

Accident Year	(1) Ultimate Loss Development Factor	(2) Acc. Year Losses at Current Valuation	(3) Estimated IBNR Required (Col. (1)—1) x Col. (2)
8	2.182	720,000	851,040
7	1.353	990,000	349,470
6	1.169	1,035,000	174,915
5	1.061	1,024,650	62,504
4	1.023	938,124	21,577
3	1.009	832,351	7,491
2	1.003	717,724	2,153
1	1.000	600,000	0
			1,469,150

Expected Loss Method

Year Accident	(1) IBNR Factor 1 — 1 / Ult. Loss Dev. Factor	(2) Expected Losses	(3) Estimated IBNR Required Col. (1) x Col. (2)
8	.542	1,440,000	780,480
7	.261	1,320,000	344,520
6	.145	1,200,000	174,000
5	.057	1,080,000	61,560
4	.022	960,000	21,120
3	.009	840,000	7,560
2	.003	720,000	2,160
1	.000	600,000	0
			1,391,400

Percentage of Premium Method

Accident Year	(1) IBNR Factor	(2) Earned Premium	(3) Estimated IBNR Required Col. (1) x Col. (2)
8	.325	2,400,000	780,000
7	.157	2,200,000	345,400
6	.087	2,000,000	174,000
5	.035	1,800,000	63,000
4	.014	1,600,000	22,400
3	.006	1,400,000	8,400
2	.002	1,200,000	2,400
1	.000	1,000,000	0
			1,395,600

Numerous methods are available to test the reserves for reported cases ranging from relatively sophisticated procedures to simple run-off tests. These tests have the advantage of dealing with a group of claims about which some information is known. They do not directly test the adequacy of the overall reserves required. Tests of reported cases are beyond the scope of this paper; however, this example, which assumes a relatively modest shift in reserve adequacy, shows their importance in completing the overall tests.

The impact on pricing can also be significant if one does not consider movement in the level of adequacy in the reported claim reserves. In this example, the use of the unadjusted loss development factors would overstate the ultimate incurred losses for the last two accident years by nearly 15%, thereby possibly causing one to raise rates excessively and be placed in an uncompetitive position.

It is interesting at this point to note the almost opposite reaction of the tests to the two basic situations described. The Loss Development Method yields the only correct answer in the deteriorating loss ratio situation (unless one can estimate fairly well the ultimate loss ratio) but is the most vulnerable to distortion from reserve strengthening. The tests emphasizing expected losses (or premiums) are less influenced by reserve shifts but react extremely slowly to a deteriorating loss ratio and can mask the underlying severity of the situation by artificially lowering the calendar year loss ratios.

LOSS RATIO DETERIORATION AND RESERVE STRENGTHING

This example is simply a composite of the two changing situations described previously. The incurred losses utilized in the deteriorating loss ratio example are adjusted to reflect the assumed reserve adequacy levels (and payment patterns) underlying the strengthening example. The resulting incurred losses are shown in Exhibit IV.

As might be expected, the three tests produce substantially different estimates of the required IBNR reserve. In fact, the variance in the range is roughly equal to 20 points of the current year's loss ratio. The individual estimates react as one might expect from the previous examples. The Loss Development Method overstates the required IBNR reserve by $219,000 as it correctly interprets the loss ratio deterioration but does not adjust the loss development factors so as not to double-up on the reserve strengthening. The Expected Loss Method produces an estimate $272,000 too low as it reacts the slowest to the deteriorating loss ratio situation and the overstatement from the reserve strengthening is fairly small. The Percentage of Premium Method is only $101,000 short in this example, as this estimate reacts faster than the Expected Loss Method to the loss ratio deterioration.

Clearly, selection and modification of the most appropriate test is vital, and would depend on the "mix" of loss ratio and reserve adequacy changes in the data being analyzed.

Loss Development Method

Accident Year	(1) Ultimate Loss Development Factor	(2) Acc. Year Losses at Current Valuation	(3) Estimated IBNR Required (Col. (1)—1) x Col. (2)
8	2.182	960,000	1,134,720
7	1.353	1,237,500	436,838
6	1.169	1,207,500	204,068
5	1.061	1,195,425	72,921
4	1.023	1,016,301	23,375
3	1.009	832,351	7,491
2	1.003	717,724	2,153
1	1.000	600,000	0
			1,881,566

Expected Loss Method

Accident Year	(1) IBNR Factor $1 - \frac{1}{\text{Ult. Loss Dev. Factor}}$	(2) Expected Losses	(3) Estimated IBNR Required Col. (1) x Col. (2)
8	.542	1,440,000	780,480
7	.261	1,320,000	344,520
6	.145	1,200,000	174,000
5	.057	1,080,000	61,560
4	.022	960,000	21,120
3	.009	840,000	7,560
2	.003	720,000	2,160
1	.000	600,000	0
			1,391,400

Percentage of Premium Method

Accident Year	(1) IBNR Factor	(2) Earned Premium	(3) Estimated IBNR Required Col. (1) x Col. (2)
8	.374	2,400,000	897,600
7	.173	2,200,000	380,600
6	.093	2,000,000	186,000
5	.036	1,800,000	64,800
4	.014	1,600,000	22,400
3	.006	1,400,000	8,400
2	.002	1,200,000	2,400
1	.000	1,000,000	0
			1,562,200

This paper has considered only reserve tests incorporating incurred losses. Such tests, whether they use incurred losses directly or employ claim counts and average incurred (or outstanding) claim costs, are impacted by changes in reserve adequacy. Methods projecting incurred losses from paid losses, for example R. E. Salzmann's "Extrapolation from Accumulated Paid Losses,"[9] would produce the correct result in each of the examples given. However, these methods are limited to "coverages where payment patterns and claim durations are relatively stable,"[10] and, although we assume these patterns to remain constant in the paper, in practice they may not do so for many slow-settling lines.

CONCLUSION

While the paper has concentrated on one family of reserve tests and on two elements which may vary from year to year, the main point is that every reserve test can be severely distorted by changing conditions and that different tests react in varying ways. In addition to changing loss ratios and reserve levels, the results can be influenced by changes in disposal rates of claims, claims handling practices, legal costs, general social conditions, etc. It is therefore important that the actuary carefully examine the reserve testing methodologies he utilizes and attempt to identify which of these factors may influence the various procedures.

"Man's yesterday may ne'er be like his morrow,
Nought may endure but Mutability."

Shelley
"Mutability"

[9] R. E. Salzmann, "Estimated Liabilities for Losses and Loss Adjustment Expenses," Chapter 3, *Property-Liability Insurance Accounting*, Robert W. Strain, Editor (California, The Merritt Company, 1974), p. 36.

[10] *Ibid.*, p. 36.

EXHIBIT I

THE STATIC SITUATION

ACCIDENT YEAR

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Earned Premium	1,000,000	1,200,000	1,400,000	1,600,000	1,800,000	2,000,000	2,200,000	2,400,000
Ult. Loss Ratio	.60	.60	.60	.60	.60	.60	.60	.60
Expected Loss Ratio	.60	.60	.60	.60	.60	.60	.60	.60
Incurred Losses Valued at:								
12 months	300,000	360,000	420,000	480,000	540,000	600,000	660,000	720,000
24 months	450,000	540,000	630,000	720,000	810,000	900,000	990,000	
36 months	517,500	621,000	724,500	828,000	931,500	1,035,000		
48 months	596,250	683,100	796,950	910,800	1,024,650			
60 months	586,328	703,593	820,859	938,124				
72 months	594,537	713,443	832,351					
84 months	598,104	717,724						
96 months	600,000							

EXHIBIT II

LOSS RATIO DETERIORATION WITH NO RESERVE STRENGTHENING

ACCIDENT YEAR

	1	2	3	4	5	6	7	8
Earned Premium	1,000,000	1,200,000	1,400,000	1,600,000	1,800,000	2,000,000	2,200,000	2,400,000
Ult. Loss Ratio	60	.60	.60	.65	.70	.70	.75	.80
Expected Loss Ratio	60	.60	.60	.60	.60	.60	.60	.60
Incurred Losses Valued at:								
12 months	300,000	360,000	420,000	520,000	630,000	700,000	825,000	960,000
24 months	450,000	540,000	630,000	780,000	945,000	1,050,000	1,237,500	
36 months	517,500	621,000	724,500	897,000	1,086,750	1,207,500		
48 months	569,520	683,100	796,500	986,700	1,195,425			
60 months	586,328	703,593	820,859	1,016,301				
72 months	594,537	713,443	832,351					
84 months	598,104	717,724						
96 months	600,000							

EXHIBIT III

CONSISTENT LOSS RATIO WITH RESERVE STRENGTHENING

ACCIDENT YEAR

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Earned Premium	1,000,000	1,200,000	1,400,000	1,600,000	1,800,000	2,000,000	2,200,000	2,400,000
Ult. Loss Ratio	.60	.60	.60	.60	.60	.60	.60	.60
Expected Loss Ratio	.60	.60	.60	.60	.60	.60	.60	.60
Incurred Losses Valued at:								
12 months	300,000	360,000	420,000	480,000	517,765*	550,588	578,471	720,000
24 months	450,000	540,000	630,000	720,000	792,000	880,000	990,000	
36 months	517,500	621,000	724,500	817,623	919,421	1,035,000		
48 months	569,250	683,100	788,603	901,260	1,024,650			
60 months	586,328	703,593	820,859	938,124				
72 months	594,537	713,443	832,351					
84 months	598,104	717,724						
96 months	600,000							

*The comparable incurred losses from Exhibit I are $540,000. Paid losses are assumed to be 15% of the ultimate incurred losses of $1,080,000 or $162,000. This results in a reserve of $378,000 which is at the historical adequacy level for reserves at a valuation date of 12 months. An 80% adequacy level is obtained by multiplying the reserve by 80/85 yielding $355,765. Adding to this amount the paid losses of $162,000, one obtains $517,765.

EXHIBIT IV

LOSS RATIO DETERIORATION AND RESERVE STRENGTHENING

ACCIDENT YEAR

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Earned Premium	1,000,000	1,200,000	1,400,000	1,600,000	1,800,000	2,000,000	2,200,000	2,400,000
Ult. Loss Ratio	.60	.60	.60	.65	.70	.70	.75	.80
Expected Loss Ratio	.60	.60	.60	.60	.60	.60	.60	.60
Incurred Losses Valued at:								
12 months	300,000	360,000	420,000	520,000	604,059	642,353	723,088	960,000
24 months	450,000	540,000	630,000	780,000	924,000	1,026,667	1,237,500	
36 months	517,500	621,000	724,500	885,368	1,072,658	1,207,500		
48 months	569,250	683,100	788,603	976,365	1,195,425			
60 months	586,328	703,593	820,859	1,016,301				
72 months	594,537	713,443	832,351					
84 months	598,104	717,724						
96 months	600,000							

EXHIBIT V

CALCULATION OF ULTIMATE LOSS DEVELOPMENT AND IBNR FACTORS (UTILIZING DATA FROM EXHIBIT I)

Loss Development Factors

Development Period	(1) Most Recent Observation	(2) First Prior Observation	(3) Second Prior Observation	(4) Three-Year Mean
12 to 24 Mos.	1.500[a]	1.500	1.500	1.500
24 to 36 Mos.	1.150	1.150	1.150	1.150
36 to 48 Mos.	1.100	1.100	1.100	1.100
48 to 60 Mos.	1.030	1.030	1.030	1.030
60 to 72 Mos.	1.014	1.014	1.014	1.014
72 to 84 Mos.	1.006	1.006	N/A	1.006
84 to 96 Mos.	1.003	N/A	N/A	1.003

Percentage of Premium Factors

Development Period	(5) Most Recent Observation	(6) First Prior Observation	(7) Second Prior Observation	(8) Three-Year Mean
12 to 24 Mos.	0.150[b]	0.150	0.150	0.150
24 to 36 Mos.	0.068	0.068	0.068	0.068
36 to 48 Mos.	0.052	0.052	0.052	0.052
48 to 60 Mos.	0.017	0.017	0.017	0.017
60 to 72 Mos.	0.008	0.008	0.008	0.008
72 to 84 Mos.	0.004	0.004	N/A	0.004
84 to 96 Mos.	0.002	N/A	N/A	0.002

Ultimate Loss Development and IBNR Factors

Development Period	(9) Loss Development Method Ultimate Loss Dev. Factor[c]	(10) Expected Loss Method IBNR Factor $(1 - \frac{1}{\text{Col. (9)}})$	(11) Percentage of Premium Method IBNR Factor[d]
12 to Ult.	2.000	0.500	0.301
24 to Ult.	1.333	0.250	0.151
36 to Ult.	1.159	0.137	0.083
48 to Ult.	1.054	0.051	0.031
60 to Ult.	1.023	0.022	0.014
72 to Ult.	1.009	0.009	0.006
84 to Ult.	1.003	0.003	0.002
96 to Ult.	1.000	0.000	0.000

(a) 990,000/660,000 = 1.500

(b) (990,000-660,000)/2,200,000 = 0.150

(c) Upward multiplicative accumulation of Column (4)

(d) Upward additive accumulation of Column (8)

AN ECONOMETRIC MODEL OF WORKERS' COMPENSATION

JAN A. LOMMELE AND ROBERT W. STURGIS

DISCUSSION BY ROBERT A. BRIAN

Essentially, this paper describes a model for using regression analysis in the forecasting of Workers' Compensation underwriting results and in evaluating past underwriting results.

In my line of work, which is in the research department of an insurance-oriented stock brokerage firm, we are continually forecasting results for property and casualty insurance companies.

One of the most surprising developments during the present underwriting cycle has been the deterioration in Workers' Compensation insurance results. We had always thought that Workers' Compensation ratemaking was the ultimate in ratemaking methodology and that this would protect Workers' Compensation results from adverse developments. However, this has turned out not to be the case.

We have learned some lessons from this development. The first is that we should not get too comfortable with a ratemaking method just because it has served us well in the past. The second is that we should step back from the ratemaking scene and do some independent forecasting of results using the lastest social and economic factors in our forecasting. At that time, we can then ask ourselves whether or not we believe that the present ratemaking formulas will deliver the rate needed to produce a profit under the developing social and economic scene.

We believe that the value of the Sturgis-Lommele paper is that it presents a *technique* for forecasting Workers' Compensation results. The models presented are just a beginning, but they represent valuable steps in the direction of doing independent forecasting.

If we were to be critical of the models, we would caution against using too much ratemaking data in the forecasting models. If the same assumptions are used in the models that are used in ratemaking, then any inadequacies in the ratemaking formula may be equally present in the forecasting model. To provide an independent forecast of results, it seems that the models should use assumptions and data that are independent of those used in the ratemaking area.

Overall, this paper represents a valuable step in the direction of independent forecasting of Workers' Compensation results. In speaking with the authors, I have learned that they have since revised some of the models and they expect that the models will be continually revised and improved upon.

DISCUSSION BY DAVID SKURNICK

This paper tells how regression analysis was used to relate Workers' Compensation premiums and losses to various economic variables. Messrs. Lommele and Sturgis conducted a lengthy and complex study combining statistical and actuarial techniques and finally arrived at three models that they considered acceptable.

Model I says annual countrywide Workers' Compensation written premium equals 0.6% of total adjusted wages disbursed, plus a constant of $289 million. The wage adjustments take into account rate level changes, the percentage of the work force covered, and the effect of payroll limitations. This model meets the author's statistical criteria, and it tends to confirm a reasonable relationship: an increase in wages, in rate level, or in percentage of work force covered, or a decrease in the effect of payroll limitation is associated with an increase in premiums.

Perhaps the constant term should have been omitted. Its presence implies that a certain percentage increase in adjusted wages is associated with a smaller percentage increase in premium. At first glance it appears that these rates of increase should be identical, but rising wages may have been associated with increasing self-insurance. In any event, the constant term is not too significant for current years. It is only about 6% of the 1973 premium, although it constitutes nearly 40% of the 1948 premium.

Model II says that a year's incurred loss equals 58% of the prior year's incurred loss, plus 0.2% of adjusted wages, plus $11.5 million times the unemployment in percent, minus a small constant of $18 million. This model is important because it indicates that a percentage point increase in the unemployment rate is associated with an $11.5 million increase in incurred losses, and this relationship is shown to be statistically significant.

In one respect Model II is unappealing to the intuition. It says that a percentage point increase in unemployment is associated with a fixed dollar increase in losses regardless of the magnitude of the year's losses. The incurred loss was $426 million in 1948 and $3.6 billion in 1973, so the model implies that in 1948 a percentage point increase in unemployment would have raised losses 2.6%, but in 1973 a percentage point increase in unemployment would have raised losses only 0.3%. This reviewer believes that the variables are related in a multiplicative fashion. For example, Model II might be replaced by:

$$\text{Model II}': \text{LOSS}_i = A\,(\text{PRODUCT}_i)^{B_1}\,(\text{UNEMP}_{(i-1)})^{B_2}\,(\text{LOSS}_{(i-1)})^{B_3}\,(\text{error}),$$

or its equivalent:

$$\text{Model II}'': \log \text{LOSS}_i = A + B_1 \log \text{PRODUCT}_i + B_2 \log \text{UNEMP}_{(i-1)} + B_3 \log \text{LOSS}_{(i-1)} + \text{error}.$$

Linear regression can be used to estimate the parameters of Model II″. Incidentally, readers should be warned that Model II in the paper measures current year's loss in thousands of dollars but measures prior year's loss in dollars.

Actuaries commonly estimate a year's earned premium as the average of the current and prior years' written premium. Model III shows that this approximation is not accurate for Workers' Compensation; in fact, the premium earned in a year depends almost entirely on the premium written that year, and only slightly on premium written the year before. As the authors state, this situation results from the general practice of collecting substantial additional audit premiums. Also, a log in entering premium onto the company books will cause several months worth of premium to be earned the month the written premium is entered.

Over the years there have been periods when Workers' Compensation loss ratios increased and periods when they decreased. Recently, sharp increases have led to unprofitable underwriting experience in most jurisdictions. A primary motive for the Lommele-Sturgis study was to explain Compensation's recent unprofitability.

It is disappointing to report that the authors failed to solve this problem. The only clue they discovered is that an increase in unemployment seems to cause a small increase in losses. The recent high loss ratios may result from social factors that cannot be quantified. However, before we conclude that regression analysis cannot be used to find an explanation for the rising loss ratio, we should try out many other reasonable models. In particular models should be tested in which the dependent variable is loss ratio at current rate and benefit levels. Such models would measure changes in profitability more directly than the ones here, which estimate premiums and losses separately.

Thanks to the authors' hard work, this paper demonstrates high technical quality. The paper is itself a model a model of how a study ought to be conducted. The authors sought out many external sources of economic data. They graphically tested the variables for a linear relationship before adopting a linear model. Each proposed model was tested and evaluated on the basis of seven criteria. The authors measured the error in each model, so one can estimate how accurate its predictions would be. They also discuss autocorrelation and how to adjust for it a most serious concern in the study of time series.

Regression analysis can be a useful tool for the actuary, and this paper will encourage and help other actuaries to work with linear regression. The Society owes a debt of gratitude to the authors for their efforts in this important field of research.

MINUTES OF THE 1975 FALL MEETING

NOVEMBER 16-18, 1975

LE CHATEAU CHAMPLAIN, MONTREAL, QUEBEC, CANADA

Sunday, November 16

The Board of Directors held its regularly scheduled meeting at Le Chateau Champlain from 1:00-5:00 p.m.

Registration was held from 4:00-6:00 p.m.

The President's reception for new Fellows and their wives took place from 6:00-7:00 p.m.

A reception for members and guests was held from 6:30-7:30 p.m.

Monday, November 17

Registration was held from 8:00-9:00 a.m.

The Fall meeting formally convened at 9:00 a.m.

Following opening remarks by President M. Stanley Hughey, a welcoming address was given by Richard Humphrys, F.S.A., F.C.I.A., Superintendent of Insurance, Department of Insurance, Ottawa, Canada. Mr. Humphrys spoke on the status of the Canadian insurance business.

The business session began at 9:30 a.m. Diplomas were presented to the new Fellows who were present. New Associates were asked to stand as their names were read. Applause was then given to each group of ten new Associates.

NEW ASSOCIATES

Aldorisio, Robert P.
Anderson, Robert C., Sr.
Asch, Nolan E.
Carlin, James G.*
Carollo, Linda D.
Christiansen, Stephan L.
Connor, Vincent P.
Costello, Jeanette R.
Dolan, Michael C.
Dorval, Bernard
Duperreault, Brian
Eddy, Jeanne H.
Eland, Douglas D.
Eldridge, Donald J.

Fiebrink, Mark E.
Gersie, Michael H.*
Gleeson, Owen M.
Goddard, Daniel C.
Henkes, Joseph P.
Hermes, Thomas M.
Lino, Richard A.
Mansur, Joseph M.*
Marker, Joseph O.
Miccolis, Robert S.
Moore, Bruce D.*
Morgan, Stephen T.*
Murad, John Aram
Nelson, Janet R.
Newlin, Patrick R.
Nishio, Jo Anne
Patrik, Gary S.
Petersen, Bruce A.
Pflum, Roberta J.
Pratt, Joseph J.
Reynolds, John J., III
Ritzenthaler, Kenneth J.
Roach, Robert F.
Ruddock, George A.
Sherman, Richard E.
Shrum, Roy G.
Smith, Frances A.
Steer, Grant D.
Taylor, Frank C.
Teufel, Patricia A.
Tobing, Diane W.*
Venter, Gary G.
Whatley, Michael W.*
Whatley, Patrick L.*
Wiegert, Paul M.
Wood, Charles P., Jr.

*Not present.

NEW FELLOWS

Brouillette, Yves J.
Carter, Edward J., Jr.
Kollar, John J.
Kreuzer, James H.
Kuehn, Ronald T.
Leonard, Gregory E.
Mohl, F. James
Taht, Veljo
Wood, James O.*

*Not present.

The following Officers and Directors were elected:

President-Elect	George D. Morison
Vice President	P. Adger Williams
Secretary	Darrell W. Ehlert
Treasurer	Walter J. Fitzgibbon, Jr.
Editor	David C. Forker (as of May 1, 1976)
General Chairman, Education & Examination Committee	Charles F. Cook

Directors	Rafal J. Balcarek David R. Bickerstaff Ronald E. Ferguson

The Secretary's report was presented by Robert B. Foster.

The proposed change in the Constitution to add a new section providing for the disposition of assets in the event the Society dissolved was approved by vote of the Fellows.

The Treasurer's report was presented by Walter J. Fitzgibbon, Jr.

A brief moment of silence was held in remembrance of members who died during the past year:

Freeland R. Cameron
James F. Gildea
Scott Harris
Arthur S. Kuenkler
Jacob Malmuth

President Hughey delivered his Presidential address entitled "Putting a Price on the Whistles".

Following a coffee break, a panel discussion entitled "Classifications—Too Many or Too Few?" was presented to the membership from 10:30 a.m. until noon. Participants in this discussion were as follows:

Moderator:	Charles C. Hewitt, Jr. Vice President and Actuary Metropolitan Property and Liability Insurance Company
Panel Members:	James R. Berquist Consulting Actuary Milliman & Robertson, Incorporated
	Edward B. Eliason Actuary Aetna Life & Casualty

Philipp K. Stern
Actuary, Property-Liability Insurance
Department of Insurance
State of New Jersey

Rex C. Davis
Assistant Vice President and Actuary
Allstate Insurance Company

A formal luncheon was held from 12:30 p.m. until 2:00 p.m. in the Ballroom. The guest speaker, who was introduced by Mr. Hughey, was Mr. Jarvis Farley, F.C.A.S., M.A.A.A., General Chairman of American Academy of Actuaries General Committee on Financial Reporting Principles and former Chairman of the Board of the Massachusetts Indemnity and Life Insurance Company.

Starting at 2:00 p.m. the following concurrent workshops were presented:

A. Commercial Property Ratemaking in a Recession

Robert L. Hurley, Moderator
Associate Actuary
Insurance Services Office

David C. Forker
Commercial Actuary
Allstate Insurance Company

Albert J. Quirin
Senior Actuarial Assistant
The Hartford Insurance Group

Michael R. Ward
Associate Actuary
The Travelers Insurance Companies

B. Reinsurance—Umbrella Policies

David P. Flynn, Moderator
Vice President and Actuary
Crum & Forster Insurance Companies

Thomas W. Fowler
Actuary
North American Reinsurance Corporation

Frederick J. Knox
Vice President—Actuarial
St. Paul Fire & Marine Insurance Company

Joseph A. Plunkett
Vice President
American Re-insurance Company

C. Living Under A Corporate Plan

Neill W. Portermain, Moderator
Vice President, Corporate Operations
W. R. Berkley Corporation

Carlton W. Honebein
Vice President and Actuary
Fireman's Fund American Insurance Companies

James E. Moore
Associate Director
The Travelers Insurance Companies

D. Current Status of Auto No-Fault Pricing

Richard M. Jaeger, Moderator
Assistant Actuary
Insurance Services Office

David S. Powell
Associate Actuary
Insurance Company of North America

Jerry W. Rapp
Assistant Actuary
State Farm Mutual Automobile Insurance Company

E. The Expanding Role of the Casualty Actuarial Consultant

James A. Faber, Moderator
Manager
Peat, Marwick, Mitchell and Company

Janet S. Graves
Assistant Actuary
Milliman & Robertson, Incorporated

Robert F. Lowe
Consulting Actuary
Nelson & Warren, Inc.

John S. McGuiness
President
John S. McGuinness Associates, Consultants in Actuarial Science and Management

F. The Use of Trend Factors in Ratemaking

Michael A. Walters, Moderator
Vice President—Actuary
Insurance Services Office

Frank Harwayne
Vice President and Director of Actuarial Research
National Council on Compensation Insurance

Edith E. Price
Actuarial Associate
Kemper Insurance Group

Lewis H. Roberts
Vice President and Manager
Woodward and Fondiller, Division of Martin E. Segal Company

Workshops A, B, C, and D were given from 2:00-3:00 p.m., Workshops A, B, E, and F were given from 3:15-4:15 p.m., and Workshops C, D, E, and F were given from 4:30-5:30 p.m.

From 4:15-4:30 p.m. there was a coffee break.

There was a reception for members, their wives and husbands and guests from 7:00 p.m. to 8:00 p.m.

Tuesday, November 18

President Hughey convened the morning session at 8:30 a.m. and introduced Jacques Dallaire, Chief Actuary, Quebec Insurance Department, who spoke briefly.

The following papers and reviews were presented:

Papers

1. "Loss Reserve Testing in a Changing Environment" by Wayne H. Fisher and Edward P. Lester, presented by Mr. Lester.
2. "Generalized Premium Formulae" by James P. Ross.
3. "A Current Look at Worker's Compensation Ratemaking" by Roy H. Kallop.
4. "A Mathematical Model for Loss Reserve Analysis" by Charles L. McClenahan, presented by Gustave A. Krause.

Reviews

1. Robert A. Brian presented a review of Robert W. Sturgis' and Jan A. Lommele's paper "An Econometric Model of Workmen's Compensation".
2. David Skurnick also presented a review of the Sturgis and Lommele paper.

Author's Reply to Reviews

David Skurnick presented a reply to reviewers of his paper "California Table L". The Woodward-Fondiller prize was awarded to David Skurnick for his paper "California Table L".

The Dorweiler prize was awarded to Frank Harwayne for his review of the paper "California Table L".

Plaques were presented to Past Presidents Norton E. Masterson, William J. Hazam, Harold W. Schloss and Daniel J. McNamara by Mr. Hughey.

Mr. Morison presented a summary of the Planning Committee report.

Mr. Hughey expressed thanks to Andre Pilon, Chairman of the Local Committee on Arrangements, and to Yves Brouillette, Daniel Demers, and Bernard Dorval, who served on this committee.

Following a coffee break, at 10:30 a.m. a panel discussion "A Look at Canadian Insurance Today and Tomorrow" was presented. The participants were as follows:

Moderator: Carl L. Wilcken
General Manager
Insurance Bureau of Canada

Panel Members: Yves J. Brouillette
Actuary
The Commerce Group

Veljo Taht
Actuary
Insurance Bureau of Canada

Hugh G. White
Assistant Actuary
The Travelers Indemnity Company of Canada

At 11:45 a.m. a panel discussion "Malpractice: Why Did The Actuary Fail?" was presented. Participants were as follows:

Moderator: Warren P. Cooper
Vice President and Actuary
Chubb & Son, Incorporated

Panel Members: Richard B. Buckley
Associate Professor of Law
Syracuse University

John E. Linster
Senior Vice President
Employers Insurance of Wausau

Roger O. Egeberg, M.D.
Special Assistant to the Secretary
Department of Health, Education and Welfare

Mr. Bornhuetter presented a Past President's plaque to M. Stanley Hughey and adjourned the meeting at 1:15 p.m.

Registration cards completed by the attendees and filed at the registration desk indicated a record attendance of 387, consisting of 129 Fellows, 120 Associates, 23 guests (including 7 subscribers), 16 four or more exam students, and 99 accompanying persons, as follows:

FELLOWS

Anker, Robert A.
Balcarek, Rafal J.
Balko, Karen H.
Barker, Loring M.
Beckman, Woody
Ben-Zvi, Phillip N.
Berquist, James R.
Bethel, Neil A.
Bevan, John R.
Bickerstaff, David R.
Bill, Richard A.
Bondy, Martin
Bornhuetter, Ronald L.
Boyajian, John H.
Boyle, James I.
Brian, Robert A.
Brouillette, Yves J.
Brown, William W.
Carter, Edward J.
Conners, John B.
Cook, Charles F.
Crowley, James H.
Dahme, Orval E.
D'Arcy, Stephen P.
Drennan, John P.
Drobisch, Miles R.
Dropkin, Lester B.
Ehlert, Darrell W.
Eliason, Edward B.
Faber, James A.
Farley, Jarvis
Ferguson, Ronald E.
Finger, Robert J.
Fitzgibbon, Walter J.
Flynn, David P.
Forker, David C.
Fossa, E. Frederick
Foster, Robert B.
Fowler, Thomas W.
Fresch, Glenn W.
Gillam, William S.
Gibson, John A.
Gillespie, James E.
Golz, James F.
Grady, David J.
Graves, Janet S.
Hachemeister, Charles A.
Hall, James A.
Hardy, Howard R.
Hartman, David G.
Harwayne, Frank
Hazam, William J.
Hewitt, Charles C.
Honebein, Carlton W.
Hughey, M. Stanley
Hurley, Robert L.
Jones, Alan G.
Kallop, Roy H.
Kates, Phillip B.
Kaufman, Allan
Khury, Costandy K.
Kilbourne, Frederick W.
Klaassen, Eldon J.
Klein, David M.
Kollar, John J.
Kormes, Mark
Kreuzer, James H.
Kuehn, Ronald T.
Lamb, Michael R.
Lange, Jeffrey T.
Leonard, Gregory E.
Lester, Edward P.
Levin, Joseph W.
Linden, John R.
Linder, Joseph
Lino, Richard
Liscord, Paul S.
Lowe, Robert F.

Makgill, Stephen S.
Masterson, Norton E.
McClure, Richard D.
McGuinness, John S.
McLean, George E.
McNamara, Daniel J.
Meenaghan, James J.
Mohl, F. James
Moore, Phillip S.
Morison, George D.
Muetterties, John H.
Munro, Richard E.
Newman, Steven H.
Otteson, Paul M.
Pagnozzi, Richard D.
Perkins, William J.
Petz, Earl F.
Phillips, Herbert J.
Pollack, Robert
Portermain, Neill W.
Price, Edith E.
Quinlan, John A.
Retterath, Ronald C.
Richardson, James F.
Richards, Harry R.
Roberts, Lewis H.
Rodermund, Matthew
Ross, James P.
Rosenberg, Norman
Roth, Richard J.
Ryan, Kevin M.
Salzmann, Ruth E.
Sarason, Harry M.
Scheel, Paul J.
Scheibl, Jerome A.
Scheid, James E.
Schloss, Harold W.
Simon, Leroy J.
Simoneau, Paul W.
Skurnick, David
Smith, Lee M.
Stankus, Leo M.
Taht, Veljo
Tarbell, Luther L.
Toothman, Michael L.
Walters, Michael A.
Ward, Michael R.
Webb, Bernard L.
White, Hugh G.
Wilcken, Carl L.
Williams, P. Adger
Woll, Richard G.

ASSOCIATES

Aldorisio, Robert
Anderson, Robert C.
Andler, James A.
Asch, Nolan
Banfield, Carole J.
Barnes, Galen R.
Barrette, Raymond
Bartlett, William N.
Bell, Allan A.
Bertles, George G.
Briere, Robert S.
Cadorine, Arthur R.
Carollo, Linda D.
Carson, David E.
Childs, Diana
Chorpita, Fred M.
Christiansen, Stephan L.
Connor, Vincent P.
Cooper, Warren A.
Costello, Jeannette R.
Crowe, Patrick J.
Dangelo, Charles H.
Davis, Rex C.
Davis, Rodney D.
Demers, Daniel
Dolan, Michael C.
Dorval, Bernard
Duperreault, Brian
Durkin, James H.
Eddy, Jeanne H.
Eland, Douglas D.
Eldridge, Donald J.
Fasking, Dennis D.
Fein, Richard I.
Fiebrink, Mark E.
Fisher, Wayne H.
Gallagher, Thomas L.
Garand, Christopher P.
Gleeson, Owen M.
Godbold, Mary Jo
Godbold, Nathan T.
Goddard, Daniel C.
Groot, Steven L.
Gruber, Charles
Harack, John
Head, Thomas F.
Henkes, Joseph P.
Hermes, Thomas
Isaac, David H.
Jaeger, Richard M.
Jean, Ronald W.
Jensen, James P.
Johnston, Daniel J.
Jones, Del R.
Jorve, Barry M.
Judd, Steven W.
Kaliski, Alan E.

Karlinski, Frank J.
Keene, Vicki S.
Kolodziej, Timothy M.
Konopa, Milan E.
Krause, Gustave A.
Leimkuhler, Urban E.
Lindquist, Peter L.
Lino, Richard A.
Luneberg, Sandra C.
Markell, Andrew S.
Marker, Joseph O.
Marks, Rosemary N.
Masella, Norma M.
Miccolis, Robert S.
Millman, Neil L.
Mokros, Bertram F.
Moore, James E.
Murad, John A.
Neidermyer, James R.
Nelson, Janet R.
Newlin, Patrick R.
Nishio, Joanne
Palczynski, Richard W.
Patrik, Gary
Petersen, Bruce A.
Petit, Charles I.
Pflum, Roberta J.
Plunkett, Joseph A.
Plunkett, Richard
Potok, Charles M.
Potvin, Robert
Powell, David S.
Pratt, Joseph J.
Quirin, Albert J.
Rapp, Jerry W.
Reynolds, John
Riff, Mayer
Ritzenthaler, Kenneth J.
Roach, Robert F.
Ruddock, George A.
Sherman, Richard E.
Shoop, Edward C.
Shrum, Roy G.
Singer, Paul E.
Smith, Frances A.
Stanard, James N.
Steer, Grant D.
Steeneck, Lee R.
Stern, Philipp K.
Swift, John A.
Swisher, John W.
Tatge, Robert L.
Taylor, Frank C.
Teufel, Patricia A.
Thompson, Eugene G.
Toren, Chester J.
Venter, Gary
Wade, Roger C.
Wiegert, Paul M.
Winter, Arthur E.
Wood, Charles P.
Zeitz, Claudia
Zubulake, Theodore J.

GUESTS

Buckley, Richard B.
Chang, Dr. Lena
Clark, Kenneth T.
Dallaire, Jacques
Egeberg, Roger O., M.D.
Erickson, Arthur E.
Guaschi, Francis E.
Humphrys, Richard
Knox, Frederick J.
Lauer, Henry
Linster, Jack E.
Savage, George A.
Sorensen, Thomas B.
Spangler, Joel L.
Wise, Paul S.

SUBSCRIBERS

Bell, Gerald W.
Chang, Ching I.
Guido, Robert N.
Johnson, John E.
Smith, Duane A.
Subeck, Stanton
Trescott, Harold C.

STUDENTS

Barrow, Betty H.
Beversdorf, William R.
Fisher, Russell S.
Guarini, Leonard T.

Respectfully submitted,

Robert B. Foster
Secretary

REPORT OF THE SECRETARY

With this report I conclude three years as Secretary-Treasurer and Secretary, not counting my year's apprenticeship as Assistant to the Secretary-Treasurer. They have been busy years and exciting ones.

Since November of 1971, our Society has grown from a membership of 480 to a membership of about 680, including the 57 members who have received their Associateship designation at this meeting. The number of persons who signed up for exams this Fall was 1,560, an increase of 256 over last year and an increase of 1,091 over those who signed up in the Fall of 1971. The increase in number of students promises substantial growth in membership over the next few years.

Highlights of the year:

> Application has been made with the IRS to change the status of the CAS so as to permit donations to the Society to be recognized as charitable contributions eligible for tax deduction.
>
> Recognition of past Presidents was initiated through distribution to living Presidents of a special plaque.
>
> An invitation to attend one of our meetings was extended to students who have passed four or more exams. For the first time we have sixteen students with us wearing a green "Student" badge.
>
> We began to use the newly authorized CAS logo.
>
> A new high in attendance at a meeting with 387 here in Montreal. (The previous high was set a year ago in New Orleans).
>
> Our meeting at The Greenbrier and our meeting here in Montreal both have to be considered in any list of highlights.
>
> We have joined with other actuarial organizations in the formation of a Joint Actuarial Education and Research Foundation. The Foundation may sponsor and finance actuarial research in such topics as: size of loss distribution, residual markets, unlimited lifetime medical expense benefits, loss reserving, and econometric analysis. The CAS budget for next year contains the sum of $2,500 earmarked for research in one of these areas.

The past year has been one of much activity by the Officers, Directors and those working on the many CAS committees.

The Board of Directors met on the following dates in 1975:

March 7-8 at the Sheraton Tobacco Valley Inn, Windsor, Connecticut

May 18 at The Greenbrier, White Sulphur Springs, West Virginia
September 11-12 at Arlington Park Hilton Towers, Arlington Heights, Illinois

November 16 at Le Chateau Champlain, Montreal, Canada

Highlights of actions taken by the Board of Directors include the following:

1. Approved a new Committee on Career Enhancement to assist "in recruiting qualified persons for the actuarial profession in the following categories:

   Minority Group Members, Women, and Physically Handicapped and Otherwise Disadvantaged Persons". In addition to recruiting, the Committee will engage in "result-oriented procedures for prompt recognition and full utilization of the abilities and talents for such qualified persons".

2. Approved seventy-three candidates for membership.

3. Approved an exam fee increase to $15 for each of Parts 1, 2, and 3 effective in 1976.

4. Made the following decisions in connection with future meetings:

   Rescinded a previous rule that limited Fall meetings to 1½ days so we can look forward to a two-day meeting in the Fall of 1976 in San Diego and in the Fall of 1977 in Bermuda.

   Approved the Hyatt Regency in Washington, D.C. for the May 1977 meeting.

   Approved Camelback, Scottsdale, Arizona for the Spring of 1978 and New York City for the Fall of 1978.

5. Approved a revision to the "Guides for Submission of Papers" to clarify responsibility of the Secretary and the Committee on Review of Papers.

6. Approved a resolution advising committee chairmen of procedures to be used in bringing proposed public statements to the attention of the Board so that the Board can give direction to the committee.

In closing I wish to thank those who have shared in handling the work addressed to the Secretary. This includes Edith Morabito, Carol Olzewski, and Frank Kugel in our New York office, my secretary, Harriet Massicotte, and the Assistant to the Secretary, Darrell Ehlert. I am confident your new Secretary will find his work as stimulating and rewarding as I have. Mes amis, j'ai fini.

Respectfully submitted.

Robert B. Foster
Secretary

REPORT OF THE TREASURER

The audited financial statement for the fiscal year ended September 30, 1975 showed assets of $136,759.14, an increase of $3,229.28 for the year. Individual income and disbursement items were all close to expected levels.

A $100,000 US Treasury Bill was purchased in November, 1974 with a 6 month maturity paying an interest rate of 7.6%. When this amount was reinvested in May, 1975, again for a 6 month term, interest rates had fallen to 5.5% and are now at this low level. While we continue to believe that U. S. Government obligations are the most appropriate investment type for our Society, the Finance Committee is now considering alternatives to Treasury Bills which would offer higher yields but would require longer maturities.

The budget proposed by the Finance Committee and approved by the Board calls for changes in the level of membership dues. This is the first change in the dues scales in four years. Fellowship dues have increased from $60 to $70. Associates for the first five years will now pay $50 instead of $40. Associates after five years will continue to pay the same dues as Fellows and, thus, will be increased from $60 to $70. Residents outside of the United States and Canada will have dues increased from $30 to $50.

The budget reflects some increase in printing and stationery expenses and a modest increase in costs associated with the secretary's office. Because the number of students taking examinations beyond Part 3 is expected to drop slightly in 1976, the income expected from this source in the coming year is reduced. Interest income which had risen 40% in 1975 over 1974 is not expected to increase during the coming year.

The Society's insurance program was modified effective October 4, 1974 to add Director's and Officer's liability coverage for a three year period with a policy limit of $1 million. The limit on the Society's Surety Bond is being increased from $150,000 to $175,000. This increase is being made even though assets are not expected to increase in 1976 because the assets would otherwise exceed the bond limit for the first three quarters of the year.

Respectfully submitted,

W. J. Fitzgibbon, Jr.
Treasurer

FINANCIAL REPORT

FOR FISCAL YEAR ENDED SEPTEMBER 30, 1975

INCOME	
Dues	$ 27,287.00
Examination Fees	46,943.06
Meetings & Registration Fees	19,453.79
Sale of Proceedings	6,900.00
Sale of Readings	1,386.45
Invitational Program	2,400.00
Michelbacher Royalties	982.24
Interest	9,898.81
Misc.	262.56
TOTAL	$115,513.91
DISBURSEMENTS	
Printing & Stationery	$ 32,901.46
Secretary's Office	29,906.00
Examination Expense	24,921.55
Meeting Expense	20,453.27
Library	433.45
Math Assoc. of America	1,500.00
Insurance	1,558.00
Dorweiler Prize	200.00
Misc.	410.90
TOTAL	$112,284.63
Change in Assets	+$ 3,229.28
Assets 9/30/74	133,529.86
Assets 9/30/75	136,759.14

ANALYSIS OF ASSETS

	9/30/74	9/30/75
Bank Accounts	$ 39,984.96	$ 35,170.14
U.S. Treasury Bonds	11,306.25	4,325.00
U.S. Treasury Bills	82,238.65	97,264.00
TOTAL	$133,529.86	$136,759.14

Walter J. Fitzgibbon, Jr.
Treasurer

* * * * * *

This is to certify that the assets and accounts shown in the above financial statement have been audited and found to be correct.

Steven H. Newman
Chairman of Finance Committee

1975 EXAMINATIONS—SUCCESSFUL CANDIDATES

Examinations for Parts 4, 6, 7, 8, 9 and 10 of the Casualty Actuarial Society Syllabus were held May 7, 8 and 9, 1975 and examinations for Parts 5, 7 and 9 were held November 6 and 7, 1975. Parts 1, 2 and 3, jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries were given May 13 and 15 and November 12 and 13. Those who passed Parts 1, 2 and 3 were listed in the joint release of the two Societies dated July 11, 1975 and January 16, 1976.

The following candidates successfully completed the requirements for Fellowship and Associateship in the November 1974 examinations and were awarded their diplomas at the May 1975 meeting:

NEW FELLOWS

Berry, Charles H.
Bethel, Neil A.
D'Arcy, Stephen P.
Dieter, George H., Jr.
Drennan, John P.
Graves, Janet S.
Lamb, R. Michael
Miller, Philip D.
Moore, Phillip S.
Pagnozzi, Richard D.
Tverberg, Gail E.

NEW ASSOCIATES

Bradley, David H.
Brewer, Fred L.
Covitz, Burton
Dangelo, Charles H.
Ernest, Richard C.
Gutterman, Sam
Hafling, David N.
Leimkuhler, Urban E.
Masters, Peter A.
McHugh, Ronald J.
McManus, Michael F.
Newville, Benjamin S.
Plunkett, Richard C.
Rosen, Kenneth R.
Symonds, Donna R.
Vogel, Jerome F.

MAY 1975 EXAMINATIONS

Following is a list of successful candidates in the examinations held in May 1975:

FELLOWSHIP EXAMINATIONS

Part 7

Alff, Gregory N.
Angell, Charles M.
Ashenberg, Wayne R.
Bassman, Bruce C.
Biondi, Richard S.
Brouillette, Yves J.
Brubaker, Randall E.
Childs, Diana M.
Daino, Robert A.
Davis, George E.
Donaldson, John P.
Gallagher, Thomas L.

Gerlach, Scott B.
Goldberg, Steven F.
Kaur, Alan F.
Keene, Vicki S.
Leimkuhler, Urban E.
Marino, James F.
Palczynski, Richard W.
Palm, Robert G.
Rosenberg, Sheldon
Squires, Sanford R.
Steeneck, Lee R.
Stergiou, Emanuel J.
Taht, Veljo
Thompson, Eugene G.
Vogel, Jerome F.
Wood, James O.
Wulterkens, Paul E.
Yoder, Reginald C.
Zelenko, Dorothy A.
Zubulake, Theodore J.

Part 8

Bartlett, William N.
Bertles, George G.
Blivess, Michael P.
Chou, Philip S.
Collins, Douglas J.
Graham, Timothy L.
Hafling, David N.
Hermes, Thomas M.
Hoylman, Douglas J.
Kollar, John J.
Kreuzer, James H.
Kuehn, Ronald T.
Lis, Raymond S., Jr.
Martin, Pamela A.
Moore, Brian C.
Neis, Allan R.
Palm, Robert G.
Petit, Charles I.
Quirin, Albert J.
Renze, David E.
Shoop, Edward C.
Warthen, Thomas V., Jr.
Wright, Walter C., III
Young, Robert G.

Part 9 (a)

Arata, David A.
Brubaker, Randall E.
Donaldson, John P.
Kaliski, Alan E.
Kelly, Anne E.
Leonard, Gregory E.
Steeneck, Lee R.
Weller, Alfred O.
Winkleman, John J., Jr.

Part 9 (b)

Brouillette, Yves J.
Carter, Edward J., Jr.
Mohl, F. James

Part 9

Alfuth, Terry J.
Ostrowski, Ellen M.
Pearl, Marc B.
Potvin, Robert
Weiner, Joel S.

Part 10

Daino, Robert A.
Dickson, Jeffrey J.
Fallquist, Richard J.
Fein, Richard I.
Fisher, Wayne H.
Garand, Christopher P.
Gottlieb, Leon R.
Groot, Steven L.
Gruber, Charles
Kaliski, Alan E.
Kelly, Anne E.
Leonard, Gregory E.
Radach, Floyd R.
Schultz, John J., III
Stephenson, Elton A.
Stergiou, Emanuel J.
Streff, James P.

ASSOCIATESHIP EXAMINATIONS

Part 4 (a)

Casey, Doreen S.
Fagan, Janet L.
Kozik, Thomas J.
Lattanzio, Stephen P.
Ledbetter, Alan R.
Pierce, John
Ryan, John F.
Tierney, John P.
Wickwire, James D., Jr.
Wiser, Ronald F.

Part 4 (b)

Applequist, Virgil H.
Balchunas, Anthony J.
Bealer, Donald A.
Beer, Albert J.
Beverage, Richard M.
Beversdorf, William R.
Bishop, Everett G.
Chung, Karl Keh-Shen
Cohen, Arthur I.
Currie, Ross A.
Fisher, Russell S.
Lafontaine, Gaetane
Frohlich, Kenneth R.
Furst, Patricia A.
Gidos, Peter M.
Glasser, Mark S.
Grannan, Patrick J.
Hartz, Melvin L.
Heller, David M.
Herman, Steven C.
Johnson, Larry D.
Kist, Frederick O.
Koerber, Alan J.
Lau, Geegym
Merves, Brian B.
Meyer, Robert E.
Miccolis, Jerry A.
Oakden, David J.
Pulis, Ralph S.
Rowland, William J.
Schneider, Harold N.
Sczech, James R.
Skrodenis, Donald C.
Smith, Byron W.
Surrago, James
Valenti, Anthony T.
Waldman, Robert H.
Westerholm, David C.
White, Frank T.
Wilson, Doris S.
Wilson, William F.
Zatorski, Richard T.

Part 4

Cheng, Joseph S.
Lyons, Joseph P.
Meyers, Glenn G.
Parker, Curtis M.

Part 6

Aldorisio, Robert P.
Anderson, Robert C.
Asch, Nolan E.
Bayley, Thomas R.
Beer, Albert J.
Beverage, Richard M.
Bishop, Everett G.
Burger, George
Carlin, James G.
Carollo, Linda D.
Christiansen, Stephan L.
Connor, Vincent P.
Dahlquist, Ronald A.
Dolan, Michael C.
Dorval, Bernard
Duperreault, Brian
Eddy, Jeanne H.
Eland, Douglas D.
Eldridge, Donald J.
Fiebrink, Mark E.
Flaherty, Morgan P.
Frohlich, Kenneth R.
Gersie, Michael H.
Gleeson, Owen M.
Goddard, Daniel C.
Grannan, Patrick J.
Graves, George G.
Heller, David M.
Henkes, Joseph P.
Hobart, Gary P.

Javaruski, John J.
Johnston, Daniel J.
Judd, Steven W.
Karlinski, Frank J.
Kleinberg, James J.
Konopa, Milan E.
Lehmann, Steven G.
Lindquist, Peter L.
Lino, Richard A.
Livingston, Roy P.
Lommele, Jan A.
Mansur, Joseph M.
Marker, Joseph O.
McMurray, Michael A.
Miccolis, Robert S.
Moore, Bruce D.
Morgan, Stephen T.
Murad, Aram
Nelson, Janet R.
Newlin, Patrick R.
Nishio, Jo Anne
Patrik, Gary S.
Pflum, Roberta J.
Pratt, Joseph J.
Purple, John M.
Reynolds, John J., III
Riley, C. Ronald
Ritzenthaler, Kenneth
Roach, Robert F.
Rodgers, Beatrice T.
Petersen, Bruce A.
Rudduck, George A.
Sherman, Richard E.
Shrum, Roy G.
Smith, Frances A.
Steer, Grant D.
Taylor, Frank C.
Teufel, Patricia A.
Thompson, Kevin B.
Tobing, Diane
Venter, Gary G.
Wengertsman, John F.
Whatley, Michael W.
Whatley, Patrick L.
Wickman, Alan E.
Wiegert, Paul M.
Wood, Charles, P., Jr.

As a result of the above examinations 9 new Fellows and 57 new Associates were admitted at the Annual Meeting November 1975:

NEW FELLOWS

Brouillette, Yves J.
Carter, Edward J., Jr.
Kollar, John J.
Kreuzer, James H.
Kuehn, Ronald T.
Leonard, Gregory E.
Mohl, F. James
Taht, Veljo
Wood, James O.

NEW ASSOCIATES

Aldorisio, Robert P.
Anderson, Robert C.
Asch, Nolan E.
Carlin, James G.
Carollo, Linda D.
Christiansen, Stephan L.
Connor, Vincent P.
Costello, Jeanette R.
Dolan, Michael C.
Dorval, Bernard
Duperrault, Brian
Eddy, Jeanne H.
Eland, Douglas D.
Eldridge, Donald J.
Fiebrink, Mark E.
Gersie, Michael H.
Gleeson, Owen M.
Goddard, Daniel C.
Henkes, Joseph P.
Hermes, Thomas M.
Johnston, Daniel J.
Judd, Steven W.
Karlinski, Frank J.
Kleinberg, James J.
Konopa, Milan E.
Lehmann, Steven G.
Lindquist, Peter L.
Lino, Richard A.
Mansur, Joseph M.
Marker, Joseph O.
Miccolis, Robert S.
Moore, Bruce D.
Morgan, Stephen T.
Murad, Aram
Nelson, Janet R.
Newlin, Patrick R.
Nishio, Jo Anne
Patrik, Gary S.
Petersen, Bruce A.

Pflum, Roberta J.
Pratt, Joseph J.
Reynolds, John J., III
Ritzenthaler, Kenneth
Roach, Robert F.
Rudduck, George A.
Sherman, Richard E.
Shrum, Roy G.
Smith, Frances A.
Steer, Grant D.
Taylor, Frank C.
Teufel, Patricia A.
Tobing, Diane
Venter, Gary G.
Whatley, Michael W.
Whatley, Patrick L.
Wiegert, Paul M.
Wood, Charles P., Jr.

NOVEMBER 1975 EXAMINATIONS

The successful candidates in the November 1975 examinations were:

FELLOWSHIP EXAMINATIONS

Part 7

Anderson, Dean R.
Anderson, Robert C.
Andler, James A.
Arata, David A.
Asch, Nolan E.
Barnes, Galen R.
Barrow, Betty H.
Bertles, George G.
Carbaugh, Albert B.
Carlin, James G.
Carollo, Linda D.
Covney, Michael D.
Crowe, Patrick J.
Dean, Charles E., Jr.
Dolan, Michael C.
Eldridge, Donald J.
Ernst, Richard C.
Fein, Richard I.
Fiebrink, Mark E.
Garand, Christopher P.
Gersic, Michael H.
Graham, Timothy L.
Grippa, Anthony J.
Gutterman, Sam
Hafling, David N.
Hemstead, Robert J.
Hermes, Thomas M.
Hough, Paul E.
Hoylman, Douglas J.
Inderbitzin, Paul H.
Johnston, Daniel J.
Kaliski, Alan E.
Kelly, Anne E.
Kleinberg, James J.
Lino, Richard A.
Luneberg, Sandra C.
Marker, Joseph O.
McCarter, Michael G.
McConnell, D. Michael
Miller, David L.
Moore, Bruce D.
Nelson, Janet R.
O'Brien, Terrence M.
Patterson, David M.
Pearl, Marc B.
Petersen, Bruce A.
Pierce, John
Plunkett, Richard C.
Reichle, Kurt A.
Rice, W. Vernon
Roach, Robert F.
Schumi, Joseph R.
Sherman, Richard E.
Shoop, Edward C.
Shrum, Roy G.
Steer, Grant D.
Swift, John A.
Thorne, Joseph O.
Venter, Gary G.
Warthen, Thomas V., Jr.
Weller, Alfred O.
Wright, Walter C., III
Young, R. James, Jr.

Part 9 (a)

Crowe, Patrick J.
Curley, James O.
Davis, George E.
Eddy, Jeanne H.
Fisher, Wayne H.
Neidermyer, James R.
Palczynski, Richard W.
Reynolds, John J., III
Schaeffer, Bernard G.
Stephenson, Elton A.

Part 9 (b)

Fusco, Michael
Jaeger, Richard M.
Kayton, Howard H.
Spitzer, C. Robert
Tatge, Robert L.
Zelenko, Dorothy A.

Part 9

Barrette, Raymond
Blivess, Michael P.
Collins, Douglas J.
Dangelo, Charles H.
Goddard, Daniel C.
Gottlieb, Leon R.
Groot, Steven L.
Karlinski, Frank J.
McManus, Michael F.
Moore, Brian C.
Palm, Robert G.
Petlick, Steven
Rosenberg, Sheldon
Squires, Sanford R.
Streff, James P.
Taylor, Jane C.
Wulterkens, Paul E.

ASSOCIATESHIP EXAMINATIONS

Part 5

Almer, Monte
Andrus, William R.
Applequist, Virgil H.
Bartlett, John W.
Bayley, Thomas R.
Bealer, Donald A.
Beer, Albert J.
Beverage, Richard M.
Bishop, Everett G.
Brooks, Dale L.
Brown, Joseph W.
Burger, George
Cheng, Joseph S.
Cheng, Lawrence W.
Cis, Mark M.
Cohen, Arthur I.
Corr, Francis X.
Currie, Ross A.
Egnasko, Gary J.
Eramo, Robert P.
Fisher, Russell S.
Flaherty, Morgan P.
Flanagan, Terrence A.
Friedberg, Thomas H.
Frohlich, Kenneth R.
Gaillard, Mary B.
Gidos, Peter M.
Glenn, John L.
Grannan, Patrick J.
Granoff, Gary
Haner, Walter J.
Henry, Dennis R.
Herman, Steven C.
Hess, David M.
Hine, Cecily A.
Irvan, Robert P.
Johnson, Larry D.
Johnson, Marvin A.
Kist, Frederick O.
Livingston, Roy P.
Lowe, Stephen P.
Maier, Robert V.
McAllister, Kevin C.
McConnell, Charles W., II
McGovern, Eugene
McMurray, Michael A.
Miccolis, Jerry A.
Myers, Nancy R.
Nichols, Raymond S.
Noceti, Stephen A.
Oakden, David J.
Pulis, R. Stephen
Ragan, Evelyn T. M.
Reichle, Kurt A.
Riley, C. Ronald
Schneider, Harold N.
Shayer, Natalie
Tuttle, Jerome E.
Urschel, Frederick A.
Waldman, Robert H.
White, Jonathan
Whitman, Mark
Wickman, Alan E.
Zatorski, Richard T.

Fifteen candidates for Fellowship and ten candidates for Associateship completed their requirements in the above examinations and will, upon approval of the Board of Directors, be admitted at the 1975 Spring Meeting:

NEW FELLOWS

Anderson, Dean R.	Groot, Steven L.	Spitzer, C. Robert
Fisher, Wayne H.	Hough, Paul E.	Stephenson, Elton A.
Fusco, Michael	Kaliski, Alan E.	Streff, James P.
Gottlieb, Leon R.	Kayton, Howard H.	Tatge, Robert L.
Grippa, Anthony J.	Kelly, Anne E.	Zenlenko, Dorothy A.

NEW ASSOCIATES

Beer, Albert J.	Cis, Mark M.	Gwynn, Holmes M.
Beverage, Richard M.	Frohlich, Kenneth R.	Haner, Walter J.
Bishop, Everett G.	Grannan, Patrick J.	Hobart, Gary P.
		Rogers, Beatrice T.

NEW FELLOWS ADMITTED MAY 1975: Ten of the eleven new fellows admitted at the Greenbrier are shown with President Stan Hughey.

NEW ASSOCIATES ADMITTED MAY 1975: Twelve of the sixteen new associates admitted at the Greenbrier are shown with President Stan Hughey.

NEW FELLOWS ADMITTED NOVEMBER 1975: Eight of the nine new fellows admitted at Montreal are shown with President Stan Hughey.

NEW ASSOCIATES ADMITTED NOVEMBER 1975: 46 of the 57 new associates admitted at Montreal are shown with President Stan Hughey.

OBITUARIES

JAMES F. GILDEA

SCOTT HARRIS

ARTHUR S. KUENKLER

JAMES A. ROBERTS

BARBARA WOODWARD

JAMES F. GILDEA

1892-1975

James F. Gildea, an Associate Member of the Society since November 16, 1923 and a retired Assistant Actuary in the Casualty Fire Actuarial Department of The Travelers Insurance Companies died February 11, 1975 in Rocky Hill, Connecticut. He had been with The Travelers forty years retiring in August 1957.

Mr. Gildea was born in Danbury, Connecticut, July 22, 1892; educated at Saint Thomas Seminary, Bloomfield, Connecticut, and the University of Louvain in Belgium; and was a U.S. Army veteran of World War I. A widower for many years he had lived in the Hartford, Connecticut area since the end of World War I.

SCOTT HARRIS

1896-1975

Scott Harris, former 1st Vice Chairman of the Board of Directors at Joseph Froggatt and Company Inc. died at the age of 78 on January 20, 1975.

Scott was born on September 13, 1896 in Ridgewood, N.J.

Scott joined Joseph Froggatt and Company Inc. in 1921 as a junior accountant. Mr. Harris served in various capacities in the New York office over the years, and in 1940 assumed the responsibilities of Executive Vice President. He worked at Joseph Froggatt for 51 years.

He attended Cornell University and received an honorary LL.D. degree from Panzer College, East Orange, in 1958. He was for many years Chairman of the Board of Trustees of Panzer College of Physical Education and Hygiene.

He served in the United States Navy, on a submarine chaser in World War I. He was Chairman of Civilian Defense in his home city of East Orange, New Jersey during World War II.

Mr. Harris held C.P.A. certificates in California, Connecticut, Illinois, Michigan, Missouri, New Jersey, and New York. He was a member of the Conference of Actuaries in Public Practice, National Association of Casualty and Surety Executives, American Academy of Actuaries, and an associate Member of the Casualty Actuarial Society since March 24, 1932. He served two terms as Director of Research of the Insurance Accounting and Statistical Association. He also was a Member of Theta Chi Fraternity, Sons of the American Revolution and the Masonic Order.

Reflecting his interest in education, he has served many years on the Board of Education in East Orange and was Vice Chairman of the Board of Directors of Bloomfield (N.J.) College in addition to his Panzer College chairmanship.

Scott retired in 1972 and moved to Sarasota, Florida with his wife.

He is survived by his wife, Mrs. Dorothea Harris; two daughters, Patricia and Elizabeth; nine grandchildren.

ARTHUR STEFAN KUENKLER
1907-1975

Arthur Stefan Kuenkler, a Fellow of the Casualty Actuarial Society and a member of the American Academy of Actuaries, died May 26, 1975 at the age of 68.

Born in Biersdorf, Germany, and emigrating to the United States as a boy with his family. He graduated from the University of Wisconsin in 1930. Following graduation he joined the actuarial department of the National Council on Compensation Insurance, beginning an outstanding professional career and service in a number of organizations.

He was appointed to the position of statistician in the National Council on Compensation Insurance in 1931, and in 1934 joined the Wisconsin

Compensation Rating and Inspection Bureau as actuary and assistant manager. Entering the Army in 1942 he served in the insurance branch of the office of the Undersecretary of War, becoming a lieutenant colonel in 1944 and chief of the branch in 1945.

Following the war he joined the United States Fidelity and Guaranty Co. in Baltimore, becoming vice president and actuary of the company. In 1958 he was appointed executive vice president of the Security Insurance Group where he served until retiring in 1967. He later became a consultant on insurance matters for the International Telephone and Telegraph Corp. in both Germany and England.

During his career he was active in a wide variety of insurance activities, where his keen mind and unfailing sense of humor contributed substantially to his own organization, to the Casualty Actuarial Society, and to the industry as a whole.

He is survived by his wife, Grace K. Kuenkler; two sons, the Rev. Richard Kuenkler and Stephen Kuenkler; a brother, a sister and a grandchild.

JAMES A. ROBERTS

1901-1973

Mr. James A. Roberts, an Associate of the Casualty Actuarial Society since November 1932 died December 21, 1973, after a long illness. He was born in Dover, New Hampshire and after he graduated from the University of New Hampshire in 1923, he did post graduate work in Danvers, Massachusetts and taught mathematics in a private school in Stonington, Connecticut.

In 1927 he joined the Life Actuarial Department in the Travelers Insurance Company, subsequently becoming a statistician in the Group Actuarial Department. In addition to being an Associate of our Society, he was also an Associate of the Society of Actuaries.

He was quiet and reserved, a much respected family man and over the years was active in church work. He is survived by a daughter, son and six grandchildren.

BARBARA H. WOODARD

1908-1975

Barbara H. Woodard, an Associate of the Casualty Actuarial Society since November 1934 died November 5, 1975. She graduated from Fordham Law School in 1939 with a LL.B. degree.

From 1928 to 1934 she was an Actuarial Clerk with the National Bureau of Casualty and Surety Underwriters. From 1934 to 1942 she was an Examiner in the New York Insurance Department. In 1942 she joined the law firm of Hughs, Hubbard and Ewing where she remained until she joined Ruben H. Donnelley Corporation in 1950. In 1968 she retired from Ruben H. Donnelley and at that time was their Assistant General Counsel.

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INDEX TO VOLUME LXII (Cont.)