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NOTICE

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EXPENSE ANALYSIS IN RATEMAKING AND PRICING

ROGER C. WADE

INTRODUCTION

This paper analyzes the traditional method of including non-loss expenses in property-liability insurance ratemaking processes and makes several suggestions concerning procedures for including expenses in profitability analyses and planning activities.

BACKGROUND

The traditional methods of including expenses in the ratemaking process are of two basic types. First, there is the constant percentage loading approach as used in automobile liability ratemaking. In this approach, a permissible, or expected, loss ratio is calculated by first determining the portion of the premium dollar required for expenses, profits and contingencies on a percentage basis. The complement of this percentage is the permissible loss ratio. This permissible loss ratio, which includes loss adjustment expense, is then used in setting rates for all policies in the state. An example of this calculation is shown in Exhibit I where the commission expense is a budgeted item; general expenses are determined from Insurance Expense Exhibits; taxes, licenses, and fees depend on the state tax structure; and profit and contingencies are set at a flat percentage of written premium. After the pure premium (expected loss and loss expense per unit of exposure) is determined, it is divided by the permissible loss ratio to determine the base rate for the territory. All other rates are then calculated as percentage deviations from this base rate. Thus, if a particular class of business has losses which are 10% higher (lower) than average, its premium rate will also be 10% higher (lower) than the base rate. This means that commissions and expenses are also treated as being 10% higher (lower).

This approach is more fully described in Stern's article on automobile ratemaking.

Exhibit 1
Calculation of Permissible Loss Ratio

Commissions (or production cost)	15%
General Expenses	10
Taxes, Licenses & Fees	3
Underwriting Profit and Contingencies	5
Total Expenses	33%
Permissible Loss Ratio (100% Total Expenses)	67%

Another approach used in current ratemaking procedures involves the use of expense constants. The rationale for this approach is that the cost of certain functions performed in the administration of the insurance process does not vary with the size of the risk. Therefore, in those lines of business (e.g., workmen's compensation) where there is a substantial difference in premium between small and large risks, it is difficult to justify the large risk's paying the same percentage of his premium to cover these administrative costs as does the small risk. The problem is solved in workmen's compensation by the use of premium discount plans and expense constants. The premium discount plans give increasingly larger discounts as premium size increases, while expense constants assign a minimum fixed cost on each policy below a given size. Technically, this treatment of expenses is not part of the ratemaking process as the same rates are used for all sizes of risk, it is the premium that reflects expense gradations through the use of both expense constants and premium discounts. A more detailed example of this approach is given by Morison, within the context of workmen's compensation insurance.[2] Some companies also use premium discount tables in pricing commercial liability insurance, although there is no reflection of their use in the ratemaking process.

Both of these approaches to ratemaking utilize a full absorption basis. That is, all costs within the organization are allocated to an individual line of business. This is to be contrasted with the contribution method which

[1] Stern, P. K., "Ratemaking Procedures for Automobile Liability Insurance", *PCAS*, Vol. LII, p. 139.

[2] Morison, G. D., "The 1965 Study of Expenses by Size of Risk", *PCAS*, Vol. LIII, p. 61.

allocates only those expenses that are closely related to a line of business. Since the full absorption basis allocation is required for completing the Insurance Expense Exhibit filed with the various state insurance departments, it represents a concept which has been sanctioned by the National Association of Insurance Commissioners. This makes it very appealing to use when a rate increase is desired. Also, any variation in the accepted approach requires the maintenance of an additional set of accounting documents. This full absorption method is also intuitively appealing because if all of these allocated costs are covered by the ratemaking process and if the rates for all lines of business are adequate, then it would appear that all expenses will be paid for and profit will be maintained.

Analysis—Constant Expenses

It can be demonstrated that the use of constant expenses to cover those costs which are incurred at a constant level per policy, regardless of premium size, is a dominant pricing strategy as compared to using a constant percentage of premium to cover the expenses for each policy. The strength of this dominance is in direct proportion to the degree of price consciousness exhibited by the consumers for a given line of insurance. The following example is taken from automobile insurance rates in Pennsylvania. A male insured, age 35, with no violations, using the car for pleasure only, and residing in Reading would pay $62 annually for $50,000 single limit liability coverage with $2,000 medical expense and 10/20 uninsured motorists coverage. The same individual, with the same coverage, but living in Philadelphia and driving more than ten miles to work would have an annual premium of $277. Assuming an expense loading of 10% was used in the ratemaking process, the first individual would pay $6.20 for expenses and the second individual would pay $27.70.

If we now use the constant expense approach and assume that one-half of all expenses are not related to premium size (e.g., are related to writing the policy, keeping the policy on file, etc.) and our product mix consisted of the above two policies, then a constant expense of about $8.50 would be required along with a 5% loading for other expenses.

Under this latter mechanism for computing expense, the premium for the insured in Reading would be about $67 and the premium for the Philadelphia resident would be $272.

If we now assume that two companies are competing for these two risks, and that one company uses constant expenses in ratemaking and the other does not, then it is likely that the company using constant expenses will insure the Philadelphia risk for $272 and the other company will insure the Reading risk for $62. The company which does not use constant expenses will obtain $6.20 for expenses. Since this amount is even insufficient to cover the $8.50 of policy writing costs, the firm will be forced to file for a rate increase. If this firm's ratemaking policies do not change, then the rate increase could generate profitable sales in Reading, but its prices in Philadelphia will become even further out of line.

Analysis—Contribution Method

The use of constant expenses is also a complement to a technique for analyzing pricing and marketing operations which is being increasingly used outside of the insurance industry. This technique is called the contribution method. The contribution method is based upon the principles of marginal cost analyses. Its goal is that, after all costs directly associated with selling a particular product are included in the price, the dollars generated to cover overhead expenses are maximized for the product line on the basis of a volume-price tradeoff.

This differs from the use of constant expenses discussed previously in that the constant expense approach assumes that the premium volume is fixed at the level of the previous time period. Thus, the expenses which were not covered by the constant expenses are divided by the premium volume to obtain a percentage loading to be used in the ratemaking process (5% in the example given above). Under the contribution method, the dollars of overhead expense are considered to be independent of the premium volume. If more policies are sold, then the price of each one could be lowered and overhead expenses could still be met. Conversely, if prices are raised, then less volume will be needed to meet overhead expenses. If knowledge concerning the elasticity of demand is available, then the contribution method makes it possible for a company to more nearly maximize its profit. However, the type of knowledge concerning markets and the effect of price differentials on premium volume, which is required for performing marginal analyses, is generally not available in the insurance industry. An example of a profit statement using the contribution method is shown in Exhibit 2.[3]

[3] This is a modification of an example contained in *Techniques of Profitability Analysis* by Sam R. Goodman, Wiley-Interscience, New York, 1970, p. 38.

First the costs of the product, in the form of loss and loss expenses, are subtracted from earnings to identify the variable gross profit. Next, the other variable costs which can be specifically identified with the product are subtracted to obtain the variable profit. This process is sometimes referred to as "direct costing". The variable profit represents the contribution that a particular product makes to a given line of business. Those costs which pertain to the entire line of business are then determined and subtracted from the variable profit to obtain the line of business contribution margin. For example, in the homeowners line of business it might not be possible to split out different marketing costs for Form A and Form B and thus variable profit would not include those marketing costs. When looking at the line of business as a whole, however, it may be possible to specifically identify those marketing costs associated with homeowners.

Exhibit 2
Contribution Method Profit and Loss Statement

Earned Premium			$200
Loss and Loss Expense (variable cost of goods sold)		$130	
Variable Gross Profit			$ 70
Commisions Premium Taxes (other variable costs) Policy Writing		$ 38	
Variable Profit (distribution contribution margin)			$32
Product Promotion Underwriting Marketing Actuarial	(direct line of business costs)	$ 12	
Line of Business Contribution Margin			$ 20
Administration Marketing Management Building and Maintenance	(allocated company overhead)	$ 10	
Line of Business Profit			$ 10

The next group of items to be subtracted represents the allocated company overhead. After completing this subtraction the final result, called line of business profit, is obtained.

This approach can be embellished by adding in considerations for investment income and by ensuring that expenses are assigned to the appropriate policy periods, but these adjustments are outside the scope of this paper.

Under the contribution method, the primary objective of the line of business management is to maximize the line of business contribution margin rather than maximizing the line of business profit.

In addition to permitting management to more realistically assess the impact of changes in volume on profits, this approach also eliminates much of the traditional discussion concerning the appropriate method to be used for allocating overhead costs. Thus, line management is free to devote their energy toward improving the operations under their control rather than toward correcting perceived deficiencies in mechanical allocation procedures.

The value of this approach to analyzing expenses is shown in Exhibits 3 and 4 where two alternative commission structures are being evaluated. In the first case, 10,000 policies are written at a \$100 premium in a particular line of business. In the second case, it was assumed that raising commissions by one point would increase the production to 12,000 policies. In going from Exhibit 3 to Exhibit 4 there is an increase in other variable costs of one dollar per policy to reflect the increase in commission, while the direct line of business costs and the allocated company overhead stay at a fixed total for the line of business. Under the usual full absorption technique of expenses analysis, as seen in Exhibit 3, the overhead (general expense) costs would have been assumed to stay at a constant per policy amount even though the actual expenditures for those items would not increase. Thus the full absorption technique would have shown a line of business profit per policy of \$4, due to the \$1 increase in commission, and a total profit of \$48,000 as compared to \$50,000. This would result in an unfavorable outlook for strategy B, which is the opposite of the indications using the contribution method as shown in Exhibit 4. In effect, increasing the total amount of allocated company overhead would tend to make other lines of business appear more profitable whenever one line increases its sales. The reason for this is that more of the overhead would be allocated to the line of business under examination. This implies that other lines will receive smaller overhead allocation and therefore will appear more profitable even though they have not changed any of their operating procedures.

Exhibit 3
Evaluation of Alternates—Full Absorption Method

Strategy A—10,000 policies	**Per Policy**	**Total**
Earned Premium	$100	$1,000,000
Loss and Loss Expense	65	650,000
Commissions	15	150,000
General Expense	12	120,000
Taxes, Licenses and Fees	3	30,000
Total Underwriting Deductions	$ 95	$ 950,000
Underwriting Profit	$ 5	$ 50,000
Strategy B—12,000 policies		
Earned Premium	$100	$1,200,000
Loss and Loss Expense	65	780,000
Commissions	16	192,000
General Expenses	12	144,000
Taxes, Licenses and Fees	3	36,000
Total Underwriting Deductions	$ 96	$1,152,000
Underwriting Profit	$ 4	$ 48,000

Exhibit 4
Evaluation of Alternatives—Contribution Method

	Per Policy	Total
Strategy A—10,000 policies		
Earned Premium	$100.00	$1,000,000
Loss and Loss Expense (65%)	65.00	650,000
Variable Gross Profit	35.00	350,000
Other Variable Costs (19%)	19.00	190,000
Variable Profit	16.00	160,000
Direct Line of Business Costs ($60,000)	6.00	60,000
Line of Business Contribution Margin	10.00	100,000
Allocated Company Overhead ($50,000)	5.00	50,000
Line of Business Profit	$ 5.00	$ 50,000
Strategy B—12,000 policies		
Earned Premium	$100.00	$1,200,000
Loss and Loss Expense (65%)	65.00	780,000
Variable Gross Profit	35.00	420,000
Other Variable Costs (20%)	20.00	240,000
Variable Profit	15.00	180,000
Direct Line of Business Costs ($60,000)	5.00	60,000
Line of Business Contribution Margin	10.00	120,000
Allocated Company Overhead ($50,000)	4.20	50,000
Line of Business Profit	$ 5.80	$ 70,000

It should be pointed out that the contribution method does not permit growth at any cost. In Strategy B above, if commissions had to be raised three dollars per policy in order to achieve a 20% increase in growth, then the line of business contribution margin would have been $96,000. This would be an undesirable strategy as compared to Strategy A which had a contribution margin of $100,000.

This example may also be used to demonstrate a misuse of the contribution method. The contribution margin per policy should not be used alone as a criterion for performance. For example, under both Strategy A and Strategy B, the contribution margin per policy is $10, even though Strategy B would result in more total profit. The reason for this anomaly is that, on a per policy basis, the cost of generating the increase in premium volume is offset by a decrease in the direct line of business costs. There is also a tendency to express the contribution margin as a percentage of premium. In this case, if two lines of business are compared on the basis of their contribution rates, it is equivalent to allocating all overhead expenses according to premium volume. This defeats the entire purpose of using contribution margins in the first place.

The contribution margins should only be used for marginal analysis problems as large changes in premium volumes will affect the overhead costs (e.g., larger office space). It is therefore inaccurate to use a contribution margin as a measure of performance for an entire line of business.

In summary, the difference between the contribution method of analysis and conventional ratemaking techniques lies in the treatment of fixed costs. In conventional ratemaking, the fact that a variable cost is constant on a per dollar, or per unit, basis simplifies the ratemaking procedure. Thus, commissions, a variable cost which is generally constant per dollar of premium, are readily loaded into the ratemaking process on a percentage basis. Policy record keeping, a variable cost which is constant on a per unit basis, can be treated as a constant expense, as recommended in the first portion of this paper. This same treatment of variable costs can be preserved when using the contribution method. Fixed costs, however, are not constant on a per unit or per dollar basis and thus are difficult to treat in the conventional ratemaking process. The current solution is to treat fixed costs as if they were variable costs similar in nature to commissions. This results in a distorted view of profits, when used for planning purposes, since these fixed costs will not change with small changes in premium volume.

A more detailed discussion of the contribution method and how it can be used for managerial accounting purposes in the insurance industry is given in the book by Schuchardt. [4]

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[4] Schuchardt, Robert A., *Managerial Accounting in the Property and Casualty Insurance Business: A Critical Study*, The National Underwriter Company, Cincinnati, 1969, 209 pp.

Conclusion

It seems clear that the use of constant expenses is a preferred method for ratemaking and that the contribution method offers an opportunity for better decision making. The major obstacle to the implementation of these techniques is the type of data available in most insurance companies. One of the reasons for this lack of appropriate data is the statutory requirement for allocating expenses in the Insurance Expense Exhibit, and the general acceptance by state insurance commissioners of information in such form. This means that expenses must be maintained on two different bases if the contribution method is to be used.

DISCUSSION BY DALE R. COMEY

While the title of Roger Wade's paper, "Expense Analysis in Ratemaking and Pricing", may suggest a narrow discussion of various techniques for developing final premiums from pure premiums, Mr. Wade, explicitly or implicitly, covers a wide range of expense allocation problems with a very few words. Unfortunately, the paper begins with a brief description of the various techniques utilized in the last step of the expense allocation problem, namely the loading of the pure premium. The latter part of the paper presents a discussion of a technique, presumably used in other industries, concerning the problem of allocating (or not allocating) company overhead expenses to line of business. The entire subject of expense allocation, especially its impact on the price individual consumers pay for insurance, might have been put in better perspective had the order of the paper been reversed.

The last half of Mr. Wade's paper is devoted to a discussion of the so-called "contribution method" of analyzing pricing and marketing operations. The explicit suggestion is that this method may serve as a sound basis for line of business decision-making. The implicit suggestion is that the use of this method would generate a more correct basis for projecting line of business operating expense needs which, in turn, would have a significant impact on rate level projections. Although Mr. Wade illustrates the "contribution method" with an example on marketing strategy, it is difficult to understand how this method would be utilized in a long-term pricing environment. The "contribution method" assumes that certain costs may not be affected by the movement of other costs included in the final premium. However, very few "fixed costs" are fixed for very long and some procedure must be devised to move these "fixed costs" over time.

The allocation of company operating expenses to line of business is not, or should not be, performed in a haphazard fashion. The main question is whether the relationship of past expenses to past premium is a reasonable and equitable yardstick for estimating future expense needs as a function of future premiums. The answer to this question must encompass both the socio-economic and political environment in which insurance rates are established as well as the practical result, at the consumer level, of utilizing some other method of expense need projection. It is my belief that, with the exception of atypical variation in the overall line of business pure premium, the projection of future line of business operating expense needs on the basis of historical expense to premium relationships is a reasonable and equitable insurance pricing technique.

While I believe my conclusion with respect to overall line of business expensing for pricing purposes is practical and reasonable, the next step in the pricing procedure, namely allocating expenses by state, territory, classification, etc. presents (or may soon present) a substantial challenge. In the initial sections of his paper, Mr. Wade suggests that equity may be better served if some means other than a "constant percentage" approach is utilized in allocating company operating expenses in the determination of individual risk premiums. Mr. Wade mentions "expense constants" and "expense gradation" which are in use for several lines of insurance. He properly mentions that these techniques are used to adjust expenses on a "total premium basis" when the number of exposure units varies significantly from the average. He then goes on to suggest that, in addition to variation in exposure units per policy, perhaps some expense allocation recognition should be given when the pure premium varies significantly from the average.

While the concept that Roger Wade addresses is not new, as a practicing ratemaker, I have some anxiety concerning the timing of the resurrection of the idea. My anxiety is caused primarily because he is probably correct that some recognition is needed in this era of wide-ranging pure premiums and also because other people are extremely interested in the concept. At the June 1973 meeting of the N.A.I.C., the Rates and Rating Organizations (D1) Subcommittee of the Property and Liability Committee reported on its appointment of a task force to study automobile classifications and rating plans. One of the charges to this task force is to "examine the present system of expense distribution. . .".

Mr. Wade, therefore, has provided a timely paper for inclusion in our *Proceedings*. As the ratemakers in the property and casualty insurance business, we will be called upon to justify our present practices. We will also be called upon to evaluate concepts such as those outlined in Mr. Wade's paper. It is quite possible that some of the age-old support for present practices may no longer be justified. Roger Wade's paper deserves careful reading and the challenges he has presented warrant careful consideration.

DISCUSSION BY ORVAL E. DAHME

Upon first reading Mr. Wade's paper, "Expense Analysis in Ratemaking and Pricing," I was not overly impressed with its content. I felt the paper to be mainly of referential value. The "constant percentage loading" and "use of expense constant" approaches to ratemaking are not unfamiliar to the actuary. Discussion concerning the "full absorption basis" and the "contribution method" seems to be an oversimplified polarization of the expense allocation problem. Though informative, the paper really did not offer anything new or beyond the awareness of the practicing actuary. In fact, in discussing the contribution method, Mr. Wade poses a situation of potentially damaging consequences—at least, to certain segments of the insurance industry.

While not disputing the three types of expenses differentiated by the contribution method of expense allocation: constant per premium dollar, constant per exposure unit and fixed expenses, and how they logically fit into the ratemaking process, I question the propriety of suggesting the insurance industry arbitrarily utilize the method in the manner suggested by Mr. Wade. Even if market and company expense data were available for determining the best mix of insurance writings for the expenditures being experienced, the insurance climate today, especially in the automobile liability field, is such that control of what, where, and how much insurance coverage a company writes is less in the hands of the company itself than under the dictates of the social and regulatory arena. We are called upon to provide for the complete insurance needs of society, but it is society, with the help of regulatory authorities, that is more and more setting the quotas and standards under which we operate.

So much for my initial reaction to Mr. Wade's paper. Not wanting to do Mr. Wade an injustice, I read his paper many additional times and studied what he was saying. It is a good thing I did. Although my initial observations about the paper did not change, I began to realize that Mr. Wade, either by accident or by design, had subtly thrown us a challenge.

The paper made obvious the unfavorable position a company can find itself in when the ratemaking expense formulae do not approximate the actual expense allocation methods followed by the company. It exemplified the importance of accurately formulating company expensing methods in the ratemaking procedures so that predicted results of revisions in rates closely resemble the reported experience that develops from such changes.

What was not so obvious, however, was the subtle questioning of whether companies using their same old traditional methods are allocating expenses in the best manner and whether advantages would accrue to the company that was able to refine its procedures beyond those of its competitors.

The extent to which a company accurately allocates its expenses through successively smaller breakdowns by line of business, kind of business, region, state, territory, coverage, etc. is based on practical considerations of the cost of a finer, accurate breakdown compared with the benefits derived therefrom. Modern computing systems have made it less expensive and easier to accomplish the detailed allocation, but the impetus for doing so has yet to produce tangible results.

Mr. Wade's paper seems to be offering us actuaries a challenge. We must be aware of the shortcomings of our companies' expense allocation practices. We should be the ones to determine the benefits to be gained by narrowing the difference between actual and practiced expensing. We can be the ones to convince our companies of the need for change in this area if such be our conclusion. Our companies are already adhering to the standards set forth for preparing the expense exhibit of the annual statement. They have no reason to go beyond those standards unless we convince them it is to their advantage to do so.

Mr. Wade's paper was refreshingly readable. Though seemingly lacking in technical development, the expense allocation methods practiced by the industry are so varied as to make the reason for such omission obvious. It should be welcomed by the Society for its informative value and as an inducement to others to submit similar type papers. In the challenge Mr. Wade presents to us as actuaries, his paper can be a noteworthy contribution to this Society—much will depend on how we respond to it.

AUTHOR'S REVIEW OF DISCUSSIONS

Rather than attempt to respond to individual points of criticism concerning the paper, although the temptation to do so is sometimes very great, a few basic points concerning the author's motivation for writing a paper on the topic of expense analysis and for selecting the particular format utilized in the paper is perhaps in order.

Selection of Topic

The author firmly believes that a periodic reexamination of all ratemaking procedures is necessary if actuaries are to continue to command the respect of the rest of the insurance industry.

Upon scanning recent *Proceedings* of the Society a dearth of information concerning expense analysis is obvious. The author therefore concluded that an article on expense analysis was necessary and that it should attempt to create an uneasiness in actuaries with respect to the current techniques for including expense provisions in rates.

Selection of Format

A detailed review of procedures currently in use would be of educational value, but, in and of itself, would not necessarily result in actuaries being sufficiently disturbed about those procedures to reexamine them. In addition, in an area as undocumented as expense analysis, there would be many inaccurate details which would draw the attention of reviewers and divert them from the more substantive issues concerning the appropriateness of current techniques.

An alternative approach, which was also rejected, would be to attempt some sort of theoretical justification for an improved system of expense analysis. The difficulty with theoretical proofs is that they are generally either insufficient and readable or sufficient and unreadable. In either event, the author felt that there was little to be gained by attempting theoretical proofs. For the purists who require such proofs it can only be suggested that, while a theoretical proof may be desirable, it was certainly not forthcoming prior to the installation of current procedures and should not be required prior to the installation of improved procedures.

As a result of the above considerations, it was the author's view that the greatest contribution to the Society could be made by giving a cursory review of current techniques and then several provocative examples to illustrate some problems which might result from using "standard" procedures. It was never the intention of the paper to offer a panacea for problems in expense analysis, but rather to point out some current difficulties and perhaps suggest some alternative methods of attack. If an actuary decides to keep his current procedures after examing these alternatives in the light of his organization's data limitations, then the author would indeed be content as it indicates that current procedures are the best available to date. Unfortunately, this will not always be the case.

A SURVEY OF LOSS RESERVING METHODS

DAVID SKURNICK

INTRODUCTION

Proper loss and loss expense reserves are vital for an insurance company, both for financial security and for the production of accurate income statements. The readings for the loss reserving section of the Casualty Actuarial Society examinations describe dozens of different methods for estimating loss and loss expense reserves. Some of these methods are in common use, others are not used at all. Some are in universal use, since they are required in the Annual Statement. Some are quite complex, others are extremely simple. Some are explicitly described in every detail, others are merely outlined. Underlying most of these methods are a few basic principles which have been combined in different ways, sometimes with different terminology.

The primary purpose of this paper is to describe the various loss and loss expense reserving methods using consistent terminology, and to explain the relationships between them. I will also clarify the assumptions underlying the methods. However, I have not compared the methods for accuracy. Such a judgment should be based upon detailed studies of the various methods, covering how well the assumptions are satisfied, the effect of data errors, inflation, large losses, the amount of premiums or losses required for credibility, the expense and difficulty involved in applying the method, and the accuracy of the method as demonstrated by its use with actual data. Perhaps this paper will encourage such studies. In their absence, I have made a few critical comments, confining them to pointing out the inconsistency of certain assumptions with actual data.

Like any statistical analysis, reserving requires grouping the data into appropriate categories, which should be homogeneous but large enough to be credible. More categories also means more labor. The definitions of the categories must therefore vary from company to company. All the loss reserving methods can be applied to any such category, although some methods are recommended as being particularly applicable to certain types of claims. This paper will not cover the proper categorizing of claims by line, class, or geographic regions, but it does examine the categorizing of claims by time units.

Types of Reserves

The total loss reserve for a line of business as of a given date is a liability which should equal the amount of paid loss that will be required to settle all claims which took place prior to the date, not including payments already made. Insurance companies are required by law to carry adequate total loss reserves on the company books. The reserve on the company books will be referred to as the carried total loss reserve.

Of course, it is extremely unlikely that the carried total loss reserve will ever be the precise amount necessary to settle all the claims for which it is meant to provide. We will refer to the precise amount necessary to settle all claims which have taken place as the required total loss reserve. The required total loss reserve, as of a point in time, cannot be known until many years later. A reserving method will produce an estimated total loss reserve. We can estimate current reserves, past reserves, or future reserves. A good reserving method will produce an estimated total loss reserve which is close to the required total loss reserve. An insurance company's carried total loss reserve will generally be set equal to its currently estimated total loss reserve. By definition, the carried total loss reserve as of a given date and the required total loss reserve as of a given date can never change (although the required total loss reserve is generally unknown) but the estimated total loss reserve as of a given date will change with time. Estimates become more accurate (i.e. closer to the required total loss reserves) with the passage of time as more data becomes available.

The total loss reserve provides for payments subsequent to a given date on claims occurring prior to this date. This date is called the reserve date. The evaluation date for a reserve estimate means the date of the most recent accounting or statistical data entering the calculation. Reserve estimates can be categorized as prospective or retrospective. An estimate is retrospective if the evaluation date is later than the reserve date and is prospective if the evaluation date is equal to or earlier than the reserve date.

A reserve test refers to a comparison of an estimated reserve with a carried reserve. The developed reserve is another name for an estimated reserve when the estimate is retrospective.

The total loss reserve can be divided into the reserve for known claims and the incurred but not reported reserve (IBNR). The reserve for known claims represents the amount of paid loss that will be required to settle all reported claims not including payments already made on these claims. The IBNR reserve represents the amount of paid loss that will be required to settle all incurred but not reported claims. Like the total loss reserve, these two reserves can be discussed in terms of required, carried and estimated. The concepts of reserve date, evaluation date, prospective and retrospective estimate, developed reserve, and reserve test all apply to the IBNR and reserve for known claims.

The reserve for known claims is also referred to as the Unpaid Losses Excluding Incurred But Not Reported (Annual Statement Part 3A), the regular reserve, the reserve for claims adjusted or in the process of adjustment and the case reserve. The IBNR is sometimes referred to as the bulk reserve (bulk reserve more commonly refers to any loss or loss expense reserve which is a bulk amount, rather than the sum of individual case reserves). Incurred but not reported claims are referred to as late-reported claims.

The expression "IBNR" is sometimes used to denote a gross IBNR which includes provision for both late reported claims and the deficiency in the reserve for known claims. If the reserve for known claims is redundant, this gross IBNR would denote a reserve for late reported claims minus the redundancy in the reserve for known claims. For example, Bornhuetter and Ferguson recommend this usage.[1] This definition says,

$$\begin{pmatrix}\text{Required gross}\\ \text{IBNR}\end{pmatrix} = \begin{pmatrix}\text{Required total}\\ \text{loss reserve}\end{pmatrix} - \begin{pmatrix}\text{Carried reserve}\\ \text{for known claims}\end{pmatrix}$$

In effect, the error in the total loss reserve becomes fully attributable to the gross IBNR.

In this paper IBNR will always denote the reserve purely for incurred but not reported claims. This usage is consistent with the name of the reserve and, I believe, leads to a clearer exposition.

There is a lag between the date a claim is first reported to an insurance company and the date it is recorded on the company books. It is actually the recorded date which distinguishes whether a claim is provided for in the IBNR or the reserve for known claims. For this reason some people recommend that IBNR be used to abbreviate incurred but not recorded. The incurred but not recorded reserve could be divided into an incurred but not reported reserve and a reported but not recorded reserve. However, under the usual terminology, the word reported is used to mean recorded on the company books. This paper throughout employs the usual terminology.

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[1]R. L. Bornhuetter and R. E. Ferguson, "The Actuary and IBNR", *PCAS*, Vol. LIX, p. 181.

METHODS OF ESTIMATING LOSS RESERVES

Reserve for Known Claims

Individual Case Estimates

An individual case estimate or per case reserve is the value assigned to a specific claim by a field adjuster or home office claims department official based upon an investigation of the claim. This estimated value of the claim, referred to as the gross case reserve, may be revised as more information is discovered. The net case reserve is the gross case reserve less partial payments. It represents the current claims department estimate of the payments remaining to be made on the claim.

The sum of the individual net case reserves for all open claims in a line or other category provides an estimated reserve for known claims for the category. Many companies use this estimate as their carried reserve for known claims; when they do, their carried reserve for known claims is called the case reserve.

The assumption of the individual case estimates method is that the claims department can accurately evaluate a claim. If a large number of claims is included in the given category, over-reserved claims and under-reserved claims will tend to compensate for one another, and the case reserve will be fairly accurate as long as there is no bias in the individual reserves.

Fast Track Reserves

Like the individual case reserve, the fast track reserve is also applied on a claim by claim basis. The fast track reserve is the estimated average value of a claim in the category. If a claim remains open beyond a certain length of time, an individual case reserve will be substituted for this average value.

The use of fast track reserves represents a saving of effort over the use of individual case reserves since many claims will close before they need to have an individual case reserve established for them. Fast track reserves can be accurate if the average value used is accurate. This average value is based upon the average value of similar claims from earlier years. Fast track reserving is appropriate for lines of insurance whose claims, such as auto collision, are similar in size.

Tabular Value

Tabular value reserves are used for certain claims under accident and health or workmen's compensation. Each individual case reserve is taken from a table. For example, a workmen's compensation reserve for benefits to a widow and dependent children arising from a death claim would be based upon tables reflecting the ages of the widow and children, the remarriage probability of the widow, and the benefit level in the state. The tabular reserve is a kind of fast track reserve with an average reserve applied to all claims in a category, where one particular category might be workmen's compensation death claims in New York state with a widow age 46 and dependent children ages 7 and 10. The accuracy of the method depends upon the accuracy of the tables and their applicability to a given company's claims.

Notice-Average Method

This method, as described by Michelbacher and Roos, is an accident year version of the Fast Track Method.[2]

Estimated 12/31/y reserve for known claims, evaluated as of 12/31/y =

$$\sum_{t \leq y}\left[\begin{pmatrix}\text{No. of reported}\\ \text{claims for}\\ \text{accident year t}\end{pmatrix}\times\begin{pmatrix}\text{Estimated average}\\ \text{cost per claim for}\\ \text{accident year t}\end{pmatrix}-\begin{pmatrix}\text{Paid loss to}\\ \text{date on}\\ \text{accident year t}\end{pmatrix}\right]$$

The number of claims used in the formula is not subject to development, since unreported claims are provided for in the IBNR. The method does require an accurate value for the average claim. The value chosen should be based upon average claims from past accident years.

Average Value Method[3]

First estimate the average net case reserve of an open claim, then

$$\begin{pmatrix}\text{Estimated reserve}\\ \text{for known claims}\end{pmatrix}=\begin{pmatrix}\text{No. of open}\\ \text{claims}\end{pmatrix}\times\begin{pmatrix}\text{Estimated average}\\ \text{net case reserve}\end{pmatrix}$$

[2] G. F. Michelbacher and N. R. Roos, *Multiple Line Insurers, Their Nature and Operation* (McGraw-Hill Book Company, Missouri, 1970)

[3] Michelbacher and Roos, *op. cit.*

An estimate of an average value for a particular time grouping of claims should be based on data specifically for that type of time grouping. For example, to estimate the average net case reserve over all claims open at a point in time one should use the average net case reserve over all claims open at an earlier point in time. The average cost of a claim over an accident year as used in the notice-average method is often less than the average net case reserve as used in the average value method. The reason is that for some lines of business, smaller claims close faster than larger ones. The set of open claims at a point in time includes a large share of slow closing larger claims, so the average gross case reserve at a point in time is much larger than the average accident year claim, and even the average net case reserve may be larger than the average accident year claim.

A numerical example will clarify this point. Assume that there are five claims each year, all occurring on June 1. The distribution of sizes and closing times is:

No. of Claims	Closing Time	Average Amount
3	1 month	$100
1	1 year	400
1	2 years	700

Average accident year claim = $1,400 ÷ 5 = $280.

As of any December 31, a $400 claim and a $700 claim from the current accident year will still be open, along with a $700 claim still open from the previous accident year. The average gross case reserve as of December 31 is $600 [($400 + $700 + $700) ÷ 3].

Schedule P, Part 1—Total Suit Liability is an application of the average value method. The number of suits remaining is shown for each accident year and a net reserve amount per suit is specified by the Annual Statement depending upon the age of the accident year. (For 1968 and prior, policy year is used rather than accident year because Schedule P was formerly on a policy year basis.)

Runoff

A runoff is an estimate of a past reserve. The reserve for known claims can be described as the anticipated future payments on known claims. The runoff estimate is

$$\begin{pmatrix}\text{Actual future payments}\\ \text{on known claims}\\ \text{up to a given date}\end{pmatrix} + \begin{pmatrix}\text{Anticipated remaining future payments}\\ \text{on known claims}\\ \text{as of the given date}\end{pmatrix}$$

More precisely, the runoff estimate of the reserve for known claims as of 12/30/70 evaluated as of 6/30/72 is

$$\begin{pmatrix}\text{Paid loss}\\ \text{during the period 1/1/71 through 6/30/72}\\ \text{on all claims}\\ \text{reported prior to 12/31/70}\end{pmatrix} + \begin{pmatrix}\text{Remaining reserve}\\ \text{as of 6/30/72}\\ \text{on all claims}\\ \text{reported prior to 12/31/70}\end{pmatrix}$$

After a sufficient length of time, this runoff becomes fully accurate, either because all claims reported prior to 12/31/70 are settled or because the remaining open claims are accurately reserved. It is not unusual for a partially developed runoff to be an inaccurate indicator. The two common patterns following show actual company data.

RUNOFF OF RESERVE FOR KNOWN CLAIMS 12/31/66

(000,000 omitted)

	Carried Reserve for Known Claims 12/31/66	Number of Months Development							
		3	6	9	12	24	36	48	60
Workmen's Compensation	67.2	66.9	66.3	65.9	66.3	69.2	70.6	70.7	70.7
General Liability BI	58.7	58.9	59.0	59.8	59.9	62.1	65.4	65.6	65.7

The workmen's compensation runoff first moved down and then up. Over-reserved claims tended to settle early, under-reserved claims did not have their reserves increased until somewhat later. The nine-month development made the tested reserve appear redundant, whereas it was actually deficient. The general liability BI runoff consistently moved up. A runoff of 24 months or less would not have shown the full extent of the reserve deficiency.

Schedule G of the Annual Statement is a runoff of the reserve for known claims for fidelity and surety, with the current year end as evaluation date and each of the prior seven year ends as reserve dates.

Correct Case Reserve for Bias

This method, as described by J. A. Scheibl, estimates the reserve for known claims by applying a correction factor to the case reserve. The bias in the prior case reserve is estimated by comparing it with the current estimate of the prior reserve for known claims. The assumption is then made that the same percentage bias exists in the current case reserve.[4] For example,

$$\left(\begin{array}{l}\text{Estimated 12/70}\\ \text{reserve for}\\ \text{known claims}\end{array}\right) = \left(\frac{\text{12/70 estimate of 12/69 reserve for known claims}}{\text{12/69 case reserve}}\right) \times \left(\begin{array}{c}\text{12/70 case}\\ \text{reserve}\end{array}\right)$$

This type of adjustment could alternately be applied to the gross case reserve.

$$\left(\begin{array}{l}\text{Estimated 12/70}\\ \text{reserve for}\\ \text{known claims}\end{array}\right) = \left(\begin{array}{c}\text{Correction}\\ \text{factor}\end{array}\right) \times \left(\begin{array}{c}\text{12/70 gross}\\ \text{case reserve}\end{array}\right) - \left(\begin{array}{c}\text{Paid to date}\\ \text{on open claims}\\ \text{as of 12/70}\end{array}\right)$$

In this case the correction factor is the ratio of the developed 12/69 gross case reserve to the carried 12/69 gross case reserve.

Report Year Loss Development

A report year consists of all claims reported in a given year regardless of accident year or policy year. We can use the report year incurred loss to calculate the reserve for known claims. For example, let us assume that all losses are settled within ten years of being reported. Then

12/70 required reserve for known claims =

Report year 1970 contribution to the reserve for known claims
+ report year 1969 contribution to the reserve for known claims
+ . . . . .
+ report year 1961 contribution to the reserve for known claims.

[4] J. A. Scheibl, "Developments in Formula Reserving Methodology", ***Insurance Accounting and Statistical Association Proceedings,*** 1970.

$$\begin{pmatrix}\text{Report year y} \\ \text{contribution to the} \\ \text{reserve for known claims}\end{pmatrix} = \begin{pmatrix}\text{Report year y} \\ \text{ultimate} \\ \text{incurred loss}\end{pmatrix} - \begin{pmatrix}\text{Report year y} \\ \text{paid loss to date} \\ \text{as of 12/70}\end{pmatrix}$$

This formula is a mathematical indentity. The key is the estimate of the ultimate incurred loss for past report years. Various approaches are recommended by Harnek[5] and Sampson[6] as well as in the Examination of Insurance Companies put out by the New York Insurance Department.[7] These methods will be discussed in the next two sections.

Projection Method

This method, as described by R. F. Harnek, suggests obtaining the estimated report year incurred loss from the paid loss to date, by applying a factor based on the past.[8] The New York Insurance Department Examination of Insurance Companies uses the term "Projection Method" to refer to a paid loss development by "Loss or Report Month, Quarter, or Year, or by Loss Year within policy year or by any other convenient grouping of the 'Time Elements'."[9] Here is a simple example to show how the projection method might be applied.

REPORT YEAR PAID LOSS DEVELOPMENT
(000 omitted)

Based upon Loss through December 31, 1971

Report Year	**Age in Years**			
	1	**2**	**3**	**4**
1968	1,000	1,100	1,210	1,210
1969	1,200	1,320	1,452	
1970	1,400	1,540		
1971	1,600			

For the sake of simplicity, assume that the reserve for known claims at age 3 years is always zero; that is, all claims are closed by the end of the third year of development.

[5] R. F. Harnek, "Formula Loss Reserves", *Insurance Accounting and Statistical Association Proceedings*, 1966.

[6] R. T. Sampson, "Establishing Adequacy of Reserves on Slow Closing Lines—Use of Paid Formulae", *Insurance Accounting and Statistical Association Proceedings*, 1959.

[7] New York (State) Insurance Department, *Examination of Insurance Companies*, Volume 3.

[8] Harnek, *op. cit.*

[9] New York (State) Insurance Department, *op. cit.*

This example was constructed so that the age-to-age development factors are:

One Year Factors		Ultimate Factors	
1 to 2	1.10	1 to 4	1.21
2 to 3	1.10	2 to 4	1.10
3 to 4	1.00	3 to 4	1.00

The report year incurred loss estimates are:

1968	1210 × 1.00 = 1210
1969	1452 × 1.00 = 1452
1970	1540 × 1.10 = 1694
1971	1600 × 1.21 = 1936

and the estimated reserve for known claims as of December 31, 1971 is 490. [(1936 − 1600) + (1694 − 1540) + (1452 − 1452) + (1210 − 1210)].

Also, if we assume that all claims in report years prior to 1968 were closed after three years, then we can obtain an estimated December 31, 1970 reserve for known claims which should be more accurate than the one we obtained using data only through 1970. The estimated reserve for known claims as of December 31, 1970 would then be 426. [(1694 − 1400) + (1452 − 1320) + (1210 − 1210)].

For most lines of insurance a great many years of development would be required for all claims in a report year to be paid. Therefore, it is common practice to carry the paid development to a certain age and use an incurred to paid factor at that age. For an illustration, the previous example can be modified by assuming that the report year incurred loss at age 4 is 1.1 times the report year paid loss at age 4. Then the one-year factors and ultimate factors would become:

One Year Factors		Ultimate Factors	
1 to 2	1.10	1 to ultimate	1.33
2 to 3	1.10	2 to ultimate	1.21
3 to 4	1.00	3 to ultimate	1.10
4 paid to 4 incurred	1.10	4 to ultimate	1.10

Under the revised assumption, there would have to be a contribution to the reserve for known claims as of December 31, 1971 from report years 1967 and prior.

The accuracy of the age-to-age factors chosen will determine the accuracy of the report year paid loss development. This principle also applies to policy year and accident year paid and incurred loss developments. For a given pair of ages, the average age-to-age factor over all report years is often used. In discussing accident year incurred loss development, Bornhuetter and Ferguson recommend a type of average: the sum of three years' losses developed to age n years divided by the sum of the same three years' losses developed to age n-1 years.[10] Trend may be reflected using either a judgment approach or a mathematical approach. When using a trend it is possible to project different one year age-to-age factors for different years. For example, in performing a policy year paid loss development, Balcarek fitted a least squares trend line to the one year age-to-age factors and generated a set of estimated one year age-to-age factors that vary by policy year.[11] The proper choice of age-to-age factors is important for ratemaking as well as reserving.

If all claims closed within a year of being reported, then the Projection Method formula for estimated reserve for known claims as of 12/71 would reduce to a factor multiplied by the report year 1971 paid loss as of 12/71. This is the form in which some authors present the Projection Method.

Payment Development Method

This method, devised by Sampson, is a report year loss development that utilizes the number of claims and their average values in order to estimate the report year incurred loss.[12] An average value method is simpler to apply to a report year than to an accident year or policy year because at the end of the report year all the claims are reported. There is no development in the number of reported claims, so the problem of estimating the reserve for known claims reduces to estimating the average size of claim within a report year.

Sampson uses an inductive method to estimate average size of claim. The inductive process begins with an average claim for an initial report year, which is old enough to be fully developed. The average claim for the report year following the initial year is calculated using the assumption that the percentage increase in ultimate average will be equal to the percentage

[10]Bornhuetter and Ferguson, *op. cit.*

[11] R.J. Balcarek, "Loss Reserving in the Sixties," PCAS Vol. LIX, 1972.

[12] Sampson, *op. cit.*

increase in average paid claim as of the same age. This assumption is successively used to obtain the estimated average claim for all report years up to the current one.

For example, we can estimate the ultimate average claim for report year 1958 with the following data (these figures are from Sampson's paper):

DATA THROUGH DECEMBER 31, 1958

Report year	1952	1953
Average paid claim through 6 years settlements	\$647	\$756
Estimated average claim	655	?

The percentage increase in average paid claim based upon claims closed within 6 years of the beginning of the report year is 17%. [(756 ÷ 647) − 1]. The estimated average claim for 1953 is 117% of \$655 or \$766. The same method is used to estimate the average 1954 claim from the average 1953 claim and so on to the current year.

Another method of estimating the average report year claim would be to assume that the individual case reserves are correct on all open claims. This produces the following formula that can be applied to each report year.

$$\begin{pmatrix}\text{Developed average}\\ \text{cost according to}\\ \text{case reserves}\end{pmatrix} = \frac{\begin{pmatrix}\text{\$ paid loss}\\ \text{to date}\end{pmatrix} + \begin{pmatrix}\text{\$ net case}\\ \text{reserves}\end{pmatrix}}{\text{No. reported}} \text{ or } \frac{\begin{pmatrix}\text{Report year incurred}\\ \text{loss to date}\end{pmatrix}}{\text{Report year No. of claims}}$$

Sampson's paper shows that this method was not as accurate as the Payment Development Method for his company's liability claims.

INCURRED BUT NOT REPORTED RESERVES

Runoff

Like the runoff of the reserve for known claims, this is an estimate of a past reserve. It can be performed at any subsequent date. For example,

$$\begin{pmatrix}\text{IBNR Runoff}\\ \text{as of 12/31/70}\\ \text{based upon}\\ \text{development}\\ \text{through 6/30/72}\end{pmatrix} = \begin{pmatrix}\text{Paid loss during the}\\ \text{period 1/1/71—6/30/72}\\ \text{on all claims incurred}\\ \text{prior to 12/31/70 and}\\ \text{reported subsequent to}\\ \text{12/31/70}\end{pmatrix} + \begin{pmatrix}\text{Remaining reserve}\\ \text{as of 6/30/72}\\ \text{on all claims incurred}\\ \text{prior to 12/31/70 and}\\ \text{reported subsequent to}\\ \text{12/31/70}\end{pmatrix}$$

If the test permits too little time for development, it will definitely underestimate the required IBNR since many late reported claims will still be unreported. The minimum amount of time required for a reasonable development varies from company to company and from line to line. Three months might be enough in auto physical damage, whereas more than five years might be needed for excess general liability. After the passage of sufficient time, all claims incurred prior to 12/31/70 will be reported and settled and the test will become fully accurate.

IBNR Reserve as a Percentage of a Base

The IBNR reserve can be estimated as a certain percentage of a selected base. The base is chosen on the assumption that it is directly proportional to the IBNR. The percentage may come from a retrospective study of past IBNR, from another company, or from judgment. The base used might vary from line to line within a company. The percentage will certainly vary from line to line.

Almost every conceivable base is recommended in the literature.

Premiums in force.
Earned premium.
Written premium.
Incurred loss.
Paid loss.
Late reported incurred loss during a specified brief period after the close of the year.
(Calendar period number of claims) × (average cost per claim), where the average cost is based upon past averages, but the number of claims is the actual number for the calendar period.
Number of claims reported.
Net case reserve.
Gross case reserve.
Number of open claims.

A particular use of this method appears in fidelity and surety, where a special formula fixed by the United States Treasury requires a fidelity IBNR of at least 10% of the premiums in force and a surety IBNR of at least 5% of the premiums in force.

Modify Last Year's IBNR for Growth

This method measures the growth in IBNR by the growth in some indicator. The assumption of this method is that the percentage change in the IBNR will equal the percentage change in the indicator. This assumption is algebraically equivalent to the assumption of the Percent of Base Method, but the operation is a bit different, as the following example will show.

Suppose that the method used in 1969 to set the commercial multiple peril IBNR was to take 10% of earned premium, and the following figures were available at the close of 1970.

1969 earned premium	$10,000,000
1970 earned premium	$12,000,000
Estimated 12/31/69 IBNR based upon runoff or other retrospective test	$ 950,000

Using the Percent of Base Method, we would say that the test confirms that 10% is still a good percentage, so we would recommend a 12/31/70 IBNR of $1,200,000. Using this method, the 10% would never be explicitly mentioned. We would simply recommend a 12/31/70 IBNR of:

$$\frac{12{,}000{,}000}{10{,}000{,}000} \times 950{,}000 = \$1{,}140{,}000.$$

In effect, the Percent of Base Method says:

$$\begin{pmatrix}\text{Current estimate}\\ \text{of IBNR}\end{pmatrix} = \begin{pmatrix}\text{estimated prior IBNR as of}\\ \text{prior evaluation date}\end{pmatrix} \times \begin{pmatrix}\text{growth}\\ \text{factor}\end{pmatrix}$$

$$(\$1{,}200{,}000) = (\$1{,}000{,}000) \times (1.2)$$

(although, the estimated prior IBNR is reviewed using current data).

In contrast to the Percent of Base method, this method says:

$$\begin{pmatrix}\text{Current estimate}\\ \text{of IBNR}\end{pmatrix} = \begin{pmatrix}\text{estimated prior IBNR as of}\\ \text{current evaluation date}\end{pmatrix} \times \begin{pmatrix}\text{Growth}\\ \text{factor}\end{pmatrix}$$

$$(\$1{,}140{,}000) = (\$950{,}000) \times (1.2)$$

The Percent of Base method is more stable and this method is more responsive.

Tarbell Method

Tarbell's method of estimating IBNR also modifies the prior years' reserve for growth.[13] He has two basic formulas:

$$\begin{pmatrix}\text{Estimated IBNR} \\ \text{at the end} \\ \text{of year } y\end{pmatrix} = \frac{N^{y}_{10\text{-}11\text{-}12}}{N^{y-1}_{10\text{-}11\text{-}12}} \times \frac{C^{y}_{10\text{-}11\text{-}12}}{C^{y-1}_{10\text{-}11\text{-}12}} \times I^{y-1}_{(1)\ldots(12)}$$

$$\begin{pmatrix}\text{Estimated IBNR} \\ \text{at the end of month } n \\ \text{of year } y+1\end{pmatrix} = \frac{N^{y+1}_{(n-2)\text{-}(n-1)\text{-}n}}{N^{y}_{10\text{-}11\text{-}12}} \times \frac{C^{y+1}_{(n-2)\text{-}(n-1)\text{-}n}}{C^{y}_{10\text{-}11\text{-}12}} \times I^{y}_{(1)\ldots(n)} \times P_n$$

Where: N = No. of claims

C = Average incurred cost per claim

I = Amount of IBNR runoff

Superscripts designate calendar year

Subscripts designate calendar month

$I^{y}_{(1)\ldots(n)}$ = An n month runoff of the year end y IBNR

$I^{y-1}_{(1)\ldots(12)}$ = A 12 month runoff of the year end (y-1) IBNR

P_n is the factor, based upon experience, necessary to project

$I^{y}_{(1)\ldots(n)}$ to an ultimate basis, since an n month runoff may underestimate the IBNR.

Tarbell starts with the estimated IBNR as of the past year end, as indicated by the runoff, and increases it by the percentage increase in the three-month incurred loss. He points out that the three-month period is arbitrary and recommends varying the length of the period by line.

[13] T. F. Tarbell, "Incurred but not Reported Claim Reserves", *PCAS* Vol. XX, 1933.

Although N × C = incurred loss, Tarbell separates the two factors in order to amend the average cost factor by eliminating abnormal claims where necessary. Since the first formula requires an assumption that a 12-month runoff is fully developed, Tarbell recommends applying a projection factor if necessary.

Total Loss Reserves

Runoff

The runoff of total loss reserves is the sum of the runoffs of the reserve for known claims and IBNR, since it is meant to test the total of the two reserves.

$$\begin{pmatrix}\text{Runoff of total loss}\\ \text{reserve as of 12/31/70}\\ \text{based upon development}\\ \text{through 6/30/72}\end{pmatrix} = \begin{pmatrix}\text{Paid loss during the}\\ \text{period 1/1/71-6/30/72}\\ \text{on all claims incurred}\\ \text{prior to 12/31/70}\end{pmatrix} + \begin{pmatrix}\text{Remaining reserve}\\ \text{as of 6/30/72 on}\\ \text{all claims incurred}\\ \text{prior to 12/31/70}\end{pmatrix}$$

Schedule 0 of the Annual Statement is a runoff of total loss reserves although there is a special treatment of salvage and non-ledger reinsurance (described in Special Topics). Column (16) shows a one year development of the total loss reserve for the previous year end, and column (17) shows a two-year development of the total loss reserve for the second previous year end.

Total Loss Reserves as a Percentage of a Base

Total loss reserves are sometimes tested by representing them as a percentage of a selected base. The magazine *U. S. Investor* annually shows total loss and loss expense reserves as a percentage of earned premium and as a percentage of written premium by major line.

Ruth Salzmann[14] has recommended testing total loss and loss expense reserves by the ratio

$$\frac{\begin{pmatrix}\text{Total loss and loss}\\ \text{expense reserves 12/31/n}\end{pmatrix}}{\begin{pmatrix}\text{Total loss and}\\ \text{loss expense}\\ \text{reserves 12/31/n-1}\end{pmatrix} + \begin{pmatrix}\text{Premiums}\\ \text{earned}\\ \text{calendar year n}\end{pmatrix} - \begin{pmatrix}\text{Paid loss}\\ \text{calendar}\\ \text{year n}\end{pmatrix}}$$

[14] R. Salzmann, "How Adequate Are Loss and Loss Expense Liabilities?", *PCAS* Vol. LIX, 1972.

Although these tests are fallible, they do have the advantage of convenience.

Runoff of Cumulative Incurred Loss

Schedule P, Part 3 tests the total loss reserve by means of a runoff of cumulative incurred loss. It compares the estimated incurred loss for all accident years prior to a certain date (referred to as "Cumulative Total") with later evaluations of the same figure. In this schedule, evaluation date is referred to as "Reserve Date". For an example, look at the 1971 and 1972 Schedule P, Part 3C tables on the attached exhibit.

The 1972 Schedule P, Part 3C shows that the cumulative total through accident year 1971 as of 12/31/71 was \$536,468, but after one year's reserve development the new estimate was \$538,082. This indicates an apparent reserve deficiency of \$1,614. The cumulative 1971 total of \$536,468 can also be obtained from the 1971 Schedule P by adding the last two figures in the column "12/31/71".

This method of reserve development is shown to be equivalent to a runoff of the total loss reserve by subtracting out the cumulative paid loss as of December 31, 1971. The cumulative paid loss as of December 31, 1971 is the sum of the last two numbers in the last column of the 1971 Schedule P—Part 3C: \$413,748 + \$15,001 = \$428,749.

$$\begin{pmatrix}\text{Cumulative incurred loss}\\ \text{as of 12/31/71}\end{pmatrix} - \begin{pmatrix}\text{Cumulative paid loss}\\ \text{as of 12/31/71}\end{pmatrix} = \begin{pmatrix}\text{Carried total loss}\\ \text{reserve as of}\\ \text{12/31/71}\end{pmatrix}$$

$$(\$536{,}468) \quad - \quad (\$428{,}749) \quad = \quad (\$107{,}719)$$

This figure is analogous to 1972 Schedule O, Column (16) "Estimated liability on unpaid losses December 31, 1971 per column 5 Part 3A, 1971."

$$\begin{pmatrix}\text{One year developed}\\ \text{cumulative incurred loss}\\ \text{as of 12/31/71}\end{pmatrix} - \begin{pmatrix}\text{Cumulative}\\ \text{paid loss}\\ \text{as of 12/31/71}\end{pmatrix} = \begin{pmatrix}\text{One year runoff of}\\ \text{total loss reserve}\\ \text{as of 12/31/71}\end{pmatrix}$$

$$(\$538{,}082) \quad - \quad (\$428{,}749) \quad = \quad (\$109{,}333)$$

This one-year runoff of the total loss reserve as of 12/31/71 is analogous to 1972 Schedule O, Column (14) "Total Losses Incurred to December 31 of Current Year on Losses Incurred Prior to 1972". Of course, simple alge-

SCHEDULE P—PART 3C—DEVELOPMENT OF INCURRED COMPENSATION LOSSES SUMS OF COLUMNS (3) AND (10), SCHEDULE P, PART 2

1971

Policy Years	Years In Which Losses Were Incurred	RESERVE DATE 12/31/66	12/31/67	12/31/68	12/31/69	12/31/70	12/31/71	Cumulative Loss Payments As Of December 31, Current Year
	•	•	•	•	•	•	•	•
	•	•	•	•	•	•	•	•
	•	•	•	•	•	•	•	•
	Sub-Total	XXX	XXX	XXX	$430,719	$430,176	$433,995	$383,912
	1970	XXX	XXX	XXX	XXX	54,340	50,998	29,836
	Sub-Total	XXX	XXX	XXX	XXX	484,516	484,993	413,748
	1971	XXX	XXX	XXX	XXX	XXX	51,475	15,001

SCHEDULE P—PART 3C—DEVELOPMENT OF INCURRED COMPENSATION LOSSES
SUMS OF COLUMNS (3) AND (10), SCHEDULE P, PART 2

1972

Policy Years	Years In Which Losses Were Incurred	RESERVE DATE 12/31/67	12/31/68	12/31/69	12/31/70	12/31/71	12/31/72	Cumulative Loss Payments As Of December 31, Current Year
	•	•	•	•	•	•	•	•
	•	•	•	•	•	•	•	•
	•	•	•	•	•	•	•	•
	Cumulative Total	XXX	XXX	$430,719	$430,176	$433,995	$436,298	$392,376
	1970	XXX	XXX	XXX	54,340	50,998	52,814	37,197
	Cumulative Total	XXX	XXX	XXX	484,516	484,993	489,112	429,555
	1971	XXX	XXX	XXX	XXX	51,475	48,970	29,478
	Cumulative Total	XXX	XXX	XXX	XXX	536,468	538,082	459,033
	1972	XXX	XXX	XXX	XXX	XXX	56,296	14,960

bra shows that this runoff of total loss reserve must indicate the same reserve deficiency as the runoff of total incurred loss.

$$\left(\begin{array}{l}\text{One year runoff of}\\ \text{total loss reserve}\\ \text{as of 12/31/71}\end{array}\right) - \left(\begin{array}{l}\text{Carried total loss}\\ \text{reserve}\\ \text{as of 12/31/71}\end{array}\right) = \left(\begin{array}{l}\text{Estimated total loss}\\ \text{reserve deficiency}\\ \text{as of 12/31/71}\end{array}\right)$$

$$(\$109{,}333) \quad - \quad (\$107{,}719) \quad = \quad (\$1{,}614)$$

This figure is analogous to 1972 Schedule O, Column (18) "Change in Such Estimated Liability December 31, 1971".

Accident Year Loss Development

The total loss reserve is related to the accident year incurred loss. For example, assume that all claims are settled within ten years. Then:

12/70 required total loss reserve =

accident year 1961 contribution to the total loss reserve
\+ accident year 1962 contribution to the total loss reserve
\+ . . . . .
\+ accident year 1970 contribution to the total loss reserve.

$$\left(\begin{array}{l}\text{Accident year y}\\ \text{contribution to}\\ \text{the total loss}\\ \text{reserve}\end{array}\right) = \left(\begin{array}{l}\text{Accident year y}\\ \text{ultimate incurred}\\ \text{loss}\end{array}\right) - \left(\begin{array}{l}\text{Accident year y}\\ \text{paid loss to date}\\ \text{as of 12/31/70}\end{array}\right)$$

There is a similar formula using policy year, with the added complication that only half of the final policy year is used.

12/70 required total loss reserve =

policy year 1961 contribution to the total loss reserve
\+ . . . . .
\+ policy year 1969 contribution to the total loss reserve
\+ policy year 1970/accident year 1970 contribution to the total loss reserve.

Both the accident year and policy year formulas are exact. The problem lies in estimating the ultimate incurred loss. A number of methods recommended in the readings are described in the following section.

Loss Ratio Method

The Loss Ratio Method assumes that a line of business will always produce a certain loss ratio. This ratio is multiplied by the policy year earned premium to obtain the estimated policy year incurred loss or is multiplied by the calendar year earned premium to obtain the estimated accident year incurred loss.

Schedule P, Parts 1 and 2 use the Loss Ratio Method to estimate a minimum total loss and loss expense reserve for liability and workmen's compensation. Part 1 assumes a 60% loss and loss expense ratio for liability and Part 2 assumes a 65% loss and loss expense ratio for workmen's compensation, both for the last three accident years.

Accident Year Incurred Loss Development

Accident year incurred loss can be developed by means of loss development factors as in ratemaking. These factors are obtained by observing the rate of development of older accident years and assuming that newer accident years will develop at a similar rate. Note that Schedule P, Part 3 provides the figures necessary to perform such a loss development for liability and workmen's compensation.

The incurred development assumes that accident year losses will be reported and reserved consistently. Rate of payments is irrelevant since the payments do not affect the gross reserves. An inaccurate reserve for known claims will not invalidate the incurred development provided that it is consistent from one accident year to the next.

Bornhuetter and Ferguson recommend an IBNR method which combines the Loss Ratio Method and Accident Year Incurred Loss Development.[15] The accident year incurred loss age-to-age factors are used to obtain an expected losses IBNR factor which is 1.0 − 1.0/ultimate factor, for each accident year.

$$\begin{pmatrix}\text{Accident year y}\\ \text{contribution to}\\ \text{the IBNR}\end{pmatrix} = \begin{pmatrix}\text{Expected Losses}\\ \text{IBNR Factor for}\\ \text{accident year y}\end{pmatrix} \times \begin{pmatrix}\text{Earned premium}\\ \text{calendar}\\ \text{year y}\end{pmatrix} \times \begin{pmatrix}\text{Expected}\\ \text{loss}\\ \text{ratio}\end{pmatrix}$$

It should be noted that this formula estimates a gross IBNR which includes provision for the redundancy in the reserve for known claims.

[15] Bornhuetter and Ferguson, *op. cit.*

The Bornhuetter—Ferguson approach results in a compromise between incurred loss development and the Loss Ratio Method. The relationship can be seen in the following examples which pertain to a partially developed accident year:

		Example A	Example B
1.	Earned premium	2,000	2,000
2.	Expected loss ratio	.50	.50
3.	Expected loss	1,000	1,000
4.	Ultimate factor	1.25	1.25
5.	Expected loss to date (3) ÷ (4)	800	800
6.	Incurred loss to date	800	900
	Estimated subsequent development according to:		
7.	Incurred loss development [(4) − 1.0] × (6)	200	225
8.	Loss Ratio Method (3) − (6)	200	100
9.	Bornhuetter—Ferguson [(4) − 1.0] × (5)	200	200

In Example A the incurred loss to date equals expected loss to date so all three methods agree. In Example B the incurred loss to date is higher than the expected loss to date. The incurred loss development assumes that the increase is due to worsening experience, so the same percentage increase will apply to the subsequent development. The Loss Ratio Method assumes that the increase is due to accelerated loss reporting or strengthened case reserves, so the increase will be offset by an equal dollar reduction in the subsequent development.

The Bornhuetter—Ferguson approach ignores the incurred loss to date and produces an estimate in between the other two.

Accident Year Paid Loss Development

Accident year paid loss can similarly be developed by means of age-to-age factors. The process is analogous to the report year paid loss development described in the Projection Method for reserving for known claims. Of course, the ultimate value of the accident year paid loss equals the ultimate value of the accident year incurred loss.

Note that Schedule P—Part 4 provides the figures necessary to determine an accident year paid loss and loss expense development for liability and compensation.

The paid development requires that accident year losses be paid at a consistent rate. Accuracy of the reserve for known claims is irrelevant.

Prospective Test of Reserves

Schedule P, Part 4 provides a test of the current total loss and loss expense reserve. It is called a prospective test because the reserve date and evaluation date are equal. The test works by comparing the accident year contributions to the current carried reserve with the accident year contributions to past estimated reserves, relative to paid loss and loss expense to date and relative to the calendar year earned premium. For each accident year we start with loss and loss expense incurred from Schedule P, Parts 1 and 2. Loss and loss expense incurred is the sum of paid loss and loss expense to date plus reserve for known claims plus that accident's year portion of the IBNR and loss expense reserves.

$$\begin{pmatrix}\text{Current estimated loss \&} \\ \text{loss expense reserve} \\ \text{as of a point in time t}\end{pmatrix} = \begin{pmatrix}\text{Loss and} \\ \text{loss expense} \\ \text{incurred}\end{pmatrix} - \begin{pmatrix}\text{Paid loss to} \\ \text{date} \\ \text{as of time t}\end{pmatrix}$$

In order to test the current reserve some assumptions must be made. There are two simple assumptions which enable us to use Schedule P, Part 4.

First, for any number of years of development, there is a fairly constant ratio of accident year total loss and loss expense reserve required to accident year loss and loss expense paid through that number of years. This reserve to paid-to-date ratio does not vary from one accident year to another. This assumption is equivalent to the assumption of consistent accident year paid loss age-to-age factors.

Suppose the upper left-hand portion of Schedule P, Part 4A looks like this:

1971 SCHEDULE P–PART 4A (000 omitted)

	1965	1966	1967	1968	1969	1970	1971
Summary Data from Schedule P—Part 1A							
1. Premiums Earned	25,000	27,000	30,000	33,000	35,000	37,000	42,000
2. Loss & Loss Exp. Inc'd.	15,000	16,500	18,000	19,500	21,000	22,500	26,000
Loss & Loss Expense through 1 year							
3. Paid	10,000	11,000	12,000	13,000	14,000	15,000	16,000
4. Reserve (2) − (3)	5,000	5,500	6,000	6,500	7,000	7,500	10,000

For older accident years, when the estimated reserve at age 1 year is known to be very close to the required reserve, there is a reserve to paid-to-date ratio of 0.5. In fact, this example was constructed so that the 0.5 ratio holds for each accident year except the current one, 1971. Following the first assumption, we suspect that the accident year 1971 portion of the December 1971 loss and loss expense reserve is redundant by $2,000,000. This cannot be a definite conclusion because the increased reserve to paid-to-date ratio might have had causes such as a slowdown in claims settlement or a change in reinsurance or a new bookkeeping method.

Second, for any number of years of development, there is a fairly constant ratio of required accident year loss and loss expense reserve to calendar year earned premium. This reserve to earned premium ratio does not vary from one accident year to another.

Suppose the upper right-hand portion of Schedule P, Part 4A looks like this:

1971 SCHEDULE P–PART 4A
Percentages

		1965	1966	1967	1968	1969	1970	1971
	Summary Data from Schedule P—Part 1A							
1.	Premiums Earned	100.0	100.0	100.0	100.0	100.0	100.0	100.0
2.	Loss & Loss Exp. Inc'd.	64.0	65.0	63.0	62.0	65.0	64.0	57.0
	Loss & Loss Expense through 1 year							
3.	Paid	23.0	24.0	22.0	21.0	24.0	23.0	23.0
4.	Reserve (2)—(3)	41.0	41.0	41.0	41.0	41.0	41.0	34.0

Following the second assumption, we suspect that the accident year 1971 portion of the December 1971 loss and loss expense reserve is deficient by 7% of the 1971 earned premium. This cannot be a definite conclusion because the decreased reserve to earned premium ratio might have had causes such as an improved loss ratio or a speedup of claims settlement.

Lorah Method[16]

This is an accident month loss development based upon separate estimates of number of claims and average claim. For all but the last two accident months, the number of claims is estimated by projecting the number reported to date.

$$\begin{pmatrix}\text{Estimated number}\\ \text{of claims}\end{pmatrix} = \begin{pmatrix}\text{number reported}\\ \text{to date}\end{pmatrix} \times \begin{pmatrix}\text{development}\\ \text{factor}\end{pmatrix}$$

The average cost is based upon claims closed in the most recent 12 calendar months. This is referred to as the claims disposed of (C.D.O.) cost.

$$\begin{pmatrix}\text{C.D.O.}\\ \text{cost}\end{pmatrix} = \frac{\text{gross amount paid on closed claims}}{(\text{number closed with payment}) + (\text{number closed without payment})}$$

[16] J. W. Lorah, "Loss Reserves—Case and Incurred but not Reported, Auto Lines Only", *Insurance Accounting and Statistical Association Proceedings*, 1961.

For the most recent two accident months, an alternate formula is used.

$$\begin{pmatrix}\text{Estimated accident}\\\text{month incurred loss}\end{pmatrix} = \begin{pmatrix}\text{Estimated incurred loss}\\\text{for the accident month}\\\text{one year prior}\end{pmatrix} \times \begin{pmatrix}\text{growth}\\\text{factor}\end{pmatrix}$$

The growth factor can be based upon number of policies or amount of premium.

Lorah splits his accident month estimated incurred loss into reported and unreported portions.

$$\begin{pmatrix}\text{Accident month}\\\text{reported losses}\end{pmatrix} = \begin{pmatrix}\text{No. losses}\\\text{reported}\end{pmatrix} \times \begin{pmatrix}\text{C.D.O.}\\\text{cost}\end{pmatrix}$$

$$\begin{pmatrix}\text{Accident month}\\\text{contribution to IBNR}\end{pmatrix} = \begin{pmatrix}\text{Estimated No. of}\\\text{unreported losses}\end{pmatrix} \times \begin{pmatrix}\text{C.D.O.}\\\text{cost}\end{pmatrix}$$

Summing these contributions provides an estimated reserve for known claims and an estimated IBNR.

This method of reserving requires the usual assumptions of consistency in loss frequency, reporting lag and claims settlement policies. In order to split the reserve into IBNR and reserve for known claims, Lorah assumes that the amount of a claim is independent of the reporting lag. In utilizing the average closed claim to predict an average accident month claim, he assumes that these two averages are equal.

Under certain common circumstances this final assumption cannot be expected to hold. If for a certain line of business large claims settle more slowly than small claims and this line is growing, then the average claim closed in a given calendar year will be less than the average accident year claim. The reason is that the claims closed in a given calendar year will include a proportionately larger number of recent claims than old claims because of growth, and these recent claims are below average in size. An example will clarify this situation.

Assume that all claims in an accident year will close within the first three years of development. There is growth in the number of claims, but there is no change from one accident year to the next in the distribution of claims by size or duration. The closing pattern is indicated in the table.

ACCIDENT YEAR CLOSING PATTERNS

Accident Year	Closing in First Year Number	Closing in First Year Average	Closing in Second Year Number	Closing in Second Year Average	Closing in Third Year Number	Closing in Third Year Average
1968	2	$100	2	$200	2	$300
1969	3	100	3	200	3	300
1970	4	100	4	200	4	300

The average claim for each accident year is \$200. The average claim closed in 1970 is \$178. $\left(\frac{4 \times \$100 + 3 \times \$200 + 2 \times \$300}{4 + 3 + 2}\right)$

Observe also that the average gross case reserve on claims open as of December 31, 1970 is \$264. $\left(\frac{4 \times \$200 + 4 \times \$300 + 3 \times \$300}{4 + 4 + 3}\right)$

Average Value Method

This method, as described by Scheibl, consists of making separate estimates of number of claims and average size of claim by accident year.[17] The count is subject to development. Scheibl recommends an accident year projection of number of claims by means of age-to-age factors.

He mentions four different approaches to estimating the average accident year claim.

a. Estimate the percentage change in average claim for the current accident year based upon the percentage change in average claim for past accident years.

b. Estimate the dollar change in average claim for the current accident year based upon the dollar change in average claim for past accident years.

c. Estimate the second differences in average claim for the current accident year based upon the second differences in average claim for past accident years.

d. Estimate the ultimate average loss for the current year based upon the change in the average paid loss to date over prior years.

[17]Scheibl, *op. cit.*

(The Payment Development Method follows an analogous approach using report year instead of accident year.)

These assumptions can be stated algebraically if we introduce some notation. Let

AI_t^y = average incurred loss for accident year y after t years of development.

AP_t^y = average paid loss for accident year y after t years of development.

Then the four assumptions can be restated as:

a. $\dfrac{AI_{t+1}^y}{AI_t^y}$ is independent of y.

b. $AI_{t+1}^y - AI_t^y$ is independent of y.

c. $(AI_{t+1}^y - AI_t^y) - (AI_t^y - AI_{t-1}^y)$ is independent of y.

d. $\dfrac{AP_{t+1}^y}{AP_t^y}$ is independent of y.

More generally, Scheibl suggests looking for any consistent pattern relating AI_{t+1}^y to AI_t^y or relating AP_{t+1}^y to AP_t^y. An examination of the data at hand should demonstrate which pattern is most consistent.

SPECIAL TOPICS

Reserve for Reopened Claims

A problem in settling claims is that closed cases may be reopened because of developments not foreseen by the claims adjuster. This problem is particularly acute in workmen's compensation. A provision for closed claims which will be reopened must be included within the reserve for known claims. This can be accomplished in several ways.

Estimate Loss Reserves by a Method Which Includes Reopened Claims

Methods which do include a provision for reopened claims are Notice-Average Method, Average Value Method (for reserve for known claims), Case Reserve Runoff, Correct Case Reserve for Bias, Report Year Loss Development, Projection Method, Payment Development Method, Runoff of Total Loss Reserves, Runoff of Cumulative Incurred Loss, Accident Year Loss Development, and Policy Year Loss Development. Methods which do not include a provision for reopened claims are Individual Case Estimate, Fast Track Reserves, Tabular Value, and Average Value Method (for total loss reserves).

Treat Reopened Claims like IBNR

If reopened claims are treated analogously to newly reported claims for the purpose of loss reserve calculations, with the reopened date taken in place of the reported date, then the claim can be treated like a late reported claim. Any of the IBNR methods can be used to calculate a reserve for reopened claims.

Balcarek Method[18]

This is a method for calculating a separate reserve for reopened claims based upon the number of claims closed. There are two steps to this approach. First, estimate the number of closed claims at the end of a particular year which will be reopened at a later date. Claims closed and reopened in the same calendar year are not included. Second, estimate the average incurred cost after reopening.

The number of claims that will reopen is estimated on the basis of the number of claims closed in the last eight years. The formula is the following:

Estimated number of claims that will reopen =

.00460 × no. of claims closed during the present year
+ .00114 × no. of claims closed during the first preceding year
+ .00051 × no. of claims closed during the second preceding year
+ . . .
+ .00002 × no. of claims closed during the eighth preceding year

[18] R. J. Balcarek, "Reserves for Reopened Claims on Workmen's Compensation", *PCAS* Vol. XLVIII, 1961.

We will refer to these reopening probabilities as r_k, so that

$$\begin{pmatrix}\text{Estimated no. of claims that}\\ \text{will reopen as of the}\\ \text{end of year } t\end{pmatrix} = \sum_{k=0}^{8} r_k \times \begin{pmatrix}\text{No. of claims}\\ \text{closed during}\\ \text{year } t\text{-}k\end{pmatrix}$$

The coefficients r_k were calculated by observing the probability of re-opening in a given year based upon past experience and fitting this function to an exponential curve.(See Balcarek Table 2.) The fitted curve was used to find the cumulative probability of reopenings in a given year or later. These cumulative probabilities are used because the reserve for reopened claims covers closed claims which will be reopened in the first subsequent year or second subsequent year, etc. Balcarek's statistics show that only a negligible percentage of claims are reopened after the eighth subsequent year.

In estimating the average incurred cost, Balcarek did not assume that the average reopened claim would equal the average closed claim. Instead, he looked for a factor that would relate the two. He based his average reopened claim upon a developed figure rather than the original estimate. He discovered that for workmen's compensation claims in his company the ratio of average reopened claim to average closed claim was stable at about 4.5 This produced the formula:

Estimated reopened reserve at the end of year t

$$= \sum_{k=0}^{8} \begin{pmatrix}\text{Estimated number of claims closed}\\ \text{in year } (t\text{-}k) \text{ which will reopen}\\ \text{subsequent to year } t\end{pmatrix} \times \begin{pmatrix}\text{Average reopened}\\ \text{value of a claim}\\ \text{closed in year } (t\text{-}k)\end{pmatrix}$$

$$= \sum_{k=0}^{8} r_k \times \begin{pmatrix}\text{Number of claims}\\ \text{closed in year } (t\text{-}k)\end{pmatrix} \times 4.5 \times \begin{pmatrix}\text{Average claim}\\ \text{closed in year } (t\text{-}k)\end{pmatrix}$$

$$= 4.5 \sum_{k=0}^{8} r_k \times \begin{pmatrix}\text{Gross amount of loss on}\\ \text{claims closed in year } (t\text{-}k)\end{pmatrix}$$

Salvage and Subrogation

An insurer will sometimes settle a property loss by agreeing with the insured upon the sound value of the damaged goods, paying him a total loss, and then selling the damaged items for salvage. The amount of the sale is credited to the insurance company. The salvage is booked as a negative paid loss. The doctrine of subrogation gives the insurer whatever rights the insured possessed against responsible third parties. The amount recovered under the right of subrogation is limited by the amount of the loss payment which has been made to the insured. The insurer cannot make a profit by subrogating against the person who caused the loss. The amount realized through subrogation is credited to the insurance company and is booked as a negative paid loss.

At any point in time an insurance company can anticipate receiving a certain amount of salvage and subrogation on claims incurred, be they unreported, in the course of settlement, or closed. This outstanding salvage and subrogation resembles a credit loss reserve. There is a controversy over whether the anticipated salvage and subrogation should be used to reduce reserves. Reserves established with no anticipated salvage and subrogation are said to be Gross of Salvage. Reserves which include anticipated salvage and subrogation (and therefore are lower) are said to be Net of Salvage.

One of the primary functions of loss reserves is to aid in determining the company's financial security. This purpose favors setting reserves conservatively. Therefore statutory insurance accounting requires setting reserves gross of salvage rather than offsetting liabilities with probable but uncertain assets.

The other primary function of loss reserves is to produce accurate income statements for control of underwriting and rates. Net of salvage reserves are the more accurate because an ongoing company will normally collect salvage and subrogation outstanding. Losses and salvage are treated symmetrically by net of salvage reserving. Loss reserves are established on the basis of losses which the company anticipates paying; these reserves are offset by salvage and subrogation which the company anticipates collecting.

Case reserves are always established gross of salvage. The various reserving methods and tests described in this article can be used to set reserves gross of salvage or net of salvage depending upon whether salvage and subrogation are included in the data entering the calculation.

Schedule P of the Annual Statement is net of salvage, as is evident from the fact that it balances to Part-3A. Of course, there is no salvage in liability or workmen's compensation, but there is some subrogation in workmen's compensation.

The Schedule O test of reserves is a compromise between net and gross of salvage. The reserve test as of a year end is gross of salvage on claims paid prior to that year end and net of salvage on claims not paid prior to that year end. The principle followed is this: On claims for which both the loss and salvage are still uncertain, the anticipated salvage may be used to offset the anticipated loss. However, on claims which have already been paid, anticipated salvage may not be used to offset loss reserves since there are no reserves remaining on those claims.

The 1971 Schedule O can be used to illustrate the principle. For the 12/70 runoff, we have column (14) = column (3) + (4) + (11) + (12).

$$\begin{pmatrix}\text{One year runoff of 12/70 total loss reserve net of}\\ \text{salvage \& reinsurance on claims closed in 1971}\end{pmatrix} =$$

$$\begin{pmatrix}\text{Paid loss during 1971}\\ \text{on accident year 1970}\\ \text{net of salvage \& reinsurance}\\ \text{on claims closed in 1971}\end{pmatrix} + \begin{pmatrix}\text{Paid loss during 1971 on}\\ \text{accident year 1969 and prior}\\ \text{net of salvage \& reinsurance}\\ \text{on claims closed in 1971}\end{pmatrix}$$

$$+\begin{pmatrix}\text{Total loss reserve carried}\\ \text{12/71 on accident year 1970}\\ \text{gross of salvage \& reinsurance}\end{pmatrix} + \begin{pmatrix}\text{Total loss reserve carried}\\ \text{12/71 on accident year 1969}\\ \text{\& prior gross of salvage \&}\\ \text{reinsurance}\end{pmatrix}$$

For the 12/69 runoff, we have column (15) = column (4) − (6) + (9) + (12)

$$\begin{pmatrix}\text{Two year runoff of 12/69 total loss reserve net of}\\ \text{salvage \& reinsurance on claims closed in 1970 and 1971}\end{pmatrix} =$$

$$\begin{pmatrix}\text{Paid loss during 1971 on}\\ \text{accident year 1969 and prior}\\ \text{net of salvage \& reinsurance}\\ \text{on claims closed in 1971}\end{pmatrix} - \begin{pmatrix}\text{Salvage \& reinsurance received}\\ \text{during 1971}\\ \text{on accident year 1969 and prior}\\ \text{on claims closed in 1970}\end{pmatrix}$$

$$+\begin{pmatrix}\text{Paid loss during 1970 on}\\ \text{accident year 1969 and prior}\\ \text{net of salvage \& reinsurance}\\ \text{on claims closed in 1971}\end{pmatrix} + \begin{pmatrix}\text{Total loss reserve carried 12/71}\\ \text{on accident year 1969 and prior}\\ \text{gross of salvage \& reinsurance}\end{pmatrix}$$

Relationship between Calendar Year and Accident Year Incurred Loss

This section will demonstrate the fact that the increase in loss reserve redundancy during a year equals the excess of the calendar year incurred loss over the accident year ultimate incurred loss.

To prove this theorem, we first show that

$$\begin{pmatrix}\text{accident year } y\\ \text{ultimate incurred loss}\end{pmatrix} =$$

$$\begin{pmatrix}\text{calendar year } y\\ \text{paid loss}\end{pmatrix} + \begin{pmatrix}12/31/y \text{ required}\\ \text{total loss reserve}\end{pmatrix} - \begin{pmatrix}12/31/(y\text{-}1) \text{ required}\\ \text{total loss reserve}\end{pmatrix}$$

Indeed,

$$\begin{pmatrix}12/31/y \text{ required}\\ \text{total loss reserve}\end{pmatrix} =$$

$$\sum_{t \le y}\left[\begin{pmatrix}\text{accident year t}\\ \text{ultimate incurred loss}\end{pmatrix} - \sum_{s \le y}\begin{pmatrix}\text{accident year t paid loss}\\ \text{during calendar year s}\end{pmatrix}\right].$$

$$\begin{pmatrix}12/31/(y\text{-}1) \text{ required}\\ \text{total loss reserve}\end{pmatrix} =$$

$$\sum_{t \le y-1}\left[\begin{pmatrix}\text{accident year t}\\ \text{ultimate incurred loss}\end{pmatrix} - \sum_{s \le y-1}\begin{pmatrix}\text{accident year t paid loss}\\ \text{during calendar year s}\end{pmatrix}\right].$$

$$\begin{pmatrix}\text{12/31/}y\text{ required}\\ \text{total loss reserve}\end{pmatrix} - \begin{pmatrix}\text{12/31/}y\text{-}1\text{ required}\\ \text{total loss reserve}\end{pmatrix} =$$

$$\begin{pmatrix}\text{accident year } y\\ \text{ultimate incurred loss}\end{pmatrix} - \begin{pmatrix}\text{calendar year } y\\ \text{paid loss}\end{pmatrix}.$$

Using this result, we see that

$$\begin{pmatrix}\text{increase in loss reserve}\\ \text{redundancy during year } y\end{pmatrix} =$$

$$\begin{pmatrix}\text{12/31/}y\text{ carried}\\ \text{total loss reserve}\end{pmatrix} - \begin{pmatrix}\text{12/31/}y\text{ required}\\ \text{total loss reserve}\end{pmatrix} -$$

$$\left[\begin{pmatrix}\text{12/31/}(y\text{-}1)\text{ carried}\\ \text{total loss reserve}\end{pmatrix} - \begin{pmatrix}\text{12/31/}(y\text{-}1)\text{ required}\\ \text{total loss reserve}\end{pmatrix}\right] =$$

$$\begin{pmatrix}\text{calendar year } y\\ \text{paid loss}\end{pmatrix} + \begin{pmatrix}\text{12/31/}y\text{ carried}\\ \text{total loss reserve}\end{pmatrix} - \begin{pmatrix}\text{12/31/}(y\text{-}1)\text{ carried}\\ \text{total loss reserve}\end{pmatrix} -$$

$$\left[\begin{pmatrix}\text{calendar year } y\\ \text{paid loss}\end{pmatrix} + \begin{pmatrix}\text{12/31/}y\text{ required}\\ \text{total loss reserve}\end{pmatrix} - \begin{pmatrix}\text{12/31/}(y\text{-}1)\text{ required}\\ \text{total loss reserve}\end{pmatrix}\right] =$$

$$\begin{pmatrix}\text{calendar year } y\\ \text{incurred loss}\end{pmatrix} - \begin{pmatrix}\text{accident year } y\\ \text{ultimate incurred loss}\end{pmatrix}.$$

As a corollary, a calendar year incurred loss will be the same as an accident year ultimate incurred loss, provided that the beginning and ending carried total loss reserves are at the proper level or provided that these two reserves are inaccurate by equal dollar amounts.

Loss Expense Reserves

Loss expense reserves are established for the purpose of covering all future expenses required to investigate and settle claims already incurred, whether reported or not. Loss expense is also called loss adjustment expense or claim expense. Allocated loss expenses are those which can be allocated to a specific claim, such as legal fees and outside claim adjusters' fees. Unallocated loss expenses are those which cannot be allocated to a specific claim, such as salaries and rent. Different methods are used to set the reserves for allocated and unallocated loss adjustment expense.

Allocated Loss Expense Reserve

Loss Reserve Methods

Since allocated loss expense payments are chargeable to specific claims, individual payments can be recorded in the same detail as the claims themselves. Line, class, accident date, reported date, policy year, state, territory etc. can all be captured. It follows that any method used to establish or test loss reserves can also be used to establish or test allocated loss expense (ALE) reserves. One common method is the establishment of a per case ALE reserve along with the per case loss reserve. Of course, this reserve must be supplemented by a reserve for anticipated ALE on incurred but not reported claims.

Ratio Method

Although the ALE reserve could be based upon premiums, incurred loss, or any of the other bases used for IBNR, the only base recommended in the readings is the total loss reserve.

The simplest formula of this type is

$$\begin{pmatrix}\text{Estimated}\\ \text{ALE reserve}\end{pmatrix} = \left(\text{Factor}\right) \times \begin{pmatrix}\text{Reserve for known}\\ \text{claims plus IBNR}\end{pmatrix}$$

where the factor is the ratio of paid ALE to paid loss for a calendar period. *Examination of Insurance Companies* recommends a factor of

$\dfrac{\text{Paid ALE for 3 calendar years}}{\text{Paid loss for 3 calendar years}}$ for liability claims, provided that the ratio

of calendar year paid ALE to paid loss has remained fairly constant.[19] This simple formula depends upon three assumptions.

a. The loss reserves are accurate.

b. For an individual claim, the ratio of ALE to loss amount is independent of how long it takes to settle the claim.

c. Losses and ALE are paid out at the same rate.

Intuitively, it appears that assumptions (b) and (c) might not hold

[19] New York (State) Insurance Department, *op. cit.*

for legal expense. Slow settling liability claims are more likely to have gone to trial—requiring large amounts of legal expense. Quick settling liability claims are more likely to be settled out of court—requiring little or no legal fees. So, slow closing claims appear to have more legal expense per claim dollar than quick closing claims. Normally, a lawyer submits his bill after the case is settled and since legal expense generally is attached to the slower cases, legal expense would appear to be paid out slower than losses.

Slifka's figures show that assumptions (b) and (c) do not hold for the miscellaneous liability line in his company.[20] His Exhibit II, and Exhibit V show that losses are paid more quickly than ALE. For example, these two exhibits show that 50% of the ultimate loss will be paid within two and one-half years after the start of the accident year, but 50% of the ultimate ALE will not be paid until four years after the start of the accident year. The incremental line in Exhibit III shows that claims which settle later will tend to need more dollars of ALE per dollar of loss than claims which settle earlier. For example, claims settled during the first year have a ratio of ALE to loss which is about 5%. The ALE-to-loss ratio is 10% for claims settled during the second year, 25% for claims settled during the third year, 27% for claims settled during the fourth year, and 40% for claims settled during the fifth year. As Brian says, "The claims paid during a calendar year are heavily weighted by small easy to handle items. It is the severity in the outstanding losses that produce the major portion of the allocated loss expense."[21]

The direction in which these two assumptions fail to hold implies that the Ratio Method will underestimate the required ALE reserve. The longer it takes to settle a claim, the longer it remains in the loss reserves. It follows that the loss reserves include a disproportionately large share of slow settling claims which have a higher than average ratio of ALE to loss. It is also clear that if ALE is paid more slowly than losses, then the ratio of required ALE reserve to required loss reserve will be higher than the ratio of paid ALE to paid loss.

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[20]R. S. Slifka, "Testing of Loss Adjustment (Allocated) Expense Reserve", *Insurance Accounting and Statistical Association Proceedings*, 1968.

[21]R. E. Brian, "Formula Reserving for Loss Expenses", *Insurance Accounting and Statistical Association Proceedings*, 1967.

Brian Method[22]

R. E. Brian recommends a modification of the Ratio Method to correct the inappropriate factor. His formula is also

(ALE reserve) = (Factor) × (Reserve for known claims + IBNR)

but his factor is not a calendar period paid-to-paid factor. Instead, he assumes that there is some appropriate factor which is constant over time and sets out to find it. He determines the factor that would have been appropriate in the past by taking the ratio of ALE runoff to total loss reserve runoff for past year ends. The current ratio is based upon these estimated past ratios with consideration given to historical and trend development. As Brian says, "The above approach follows a complete cycle. The factors are developed on the basis of outstanding losses tc allocated expenses paid and are applied in the reverse manner."

Slifka Method[23]

This is an accident year calculation of ALE Reserve. The formula

$$\begin{pmatrix}\text{Accident year t contribution}\\ \text{to the ALE reserve}\end{pmatrix} = \begin{pmatrix}\text{Accident year t}\\ \text{ultimate ALE}\end{pmatrix} - \begin{pmatrix}\text{Paid ALE}\\ \text{to date}\end{pmatrix}$$

becomes, in Slifka's terminology,

$$\begin{pmatrix}\text{Accident year t}\\ \text{contribution to}\\ \text{the ALE reserve}\end{pmatrix} = \begin{pmatrix}\text{Accident year t}\\ \text{ultimate ALE}\end{pmatrix} - \begin{pmatrix}\text{Accident year t}\\ \text{ALE paid on}\\ \text{closed to date}\end{pmatrix} - \begin{pmatrix}\text{Accident year t}\\ \text{paid on}\\ \text{pending to date}\end{pmatrix}$$

For the most recent four accident years,

$$\begin{pmatrix}\text{Accident year}\\ \text{ultimate ALE}\end{pmatrix} = (\text{Factor}) \times \begin{pmatrix}\text{Accident year ultimate}\\ \text{incurred loss}\end{pmatrix}$$

This factor is based upon the ratio of paid ALE to paid loss for fully developed accident years. The factor is also affected by the paid ALE to date for the given accident year.

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22 Brian, *op. cit.*

23 Slifka, *op. cit.*

For the fifth, sixth, and seventh prior accident years,

$$\begin{pmatrix}\text{Accident year t}\\ \text{contribution to}\\ \text{the ALE reserve}\end{pmatrix} = \begin{pmatrix}\text{No. of}\\ \text{open}\\ \text{claims}\end{pmatrix} \times \begin{pmatrix}\text{Average expected}\\ \text{total ALE payment}\\ \text{on all open claims}\end{pmatrix} = \begin{pmatrix}\text{Paid ALE to}\\ \text{date on}\\ \text{open claims}\end{pmatrix}$$

This formula makes no provision for unreported claims, but relatively few claims are reported more than four years late. The average expected total ALE payment on open claims is actually a weighted average by expected year of closing. That is, first an average ALE per claim is developed based upon the number of years required to close the claim. Then the claims open are assigned a year of closing based upon past patterns. These are multiplied together, summed, and divided by the total number of open claims to obtain the average ALE payment per open claim. For example, Exhibit VIII of Slifka's paper shows that for accident year 1960, 190 claims are open as of 12/66. He projects that 94 of them will close in 1967, 64 in 1968, and 32 in 1969. It is also projected that $1,800 will be the average ALE for those claims closing in 1967, $2,000 for those closing in 1970, and $2,200 for those closing in 1971. The average expected ALE payment for the 190 accident year 1960 claims open as of 12/66 is approximately $2,000 [(94 × $1,800 + 64 × $2,000 + 32 × $2,200) ÷ 190].

Unallocated Loss Expense Reserves

Since unallocated loss expenses are not charged to specific claims, the individual payments cannot be assigned to line, class, accident date or reported date. The total paid unallocated loss expense (ULE) must be allocated to accident year and line based upon a time study or judgment. It is not possible to test the allocation retrospectively. Reserving methods based upon an allocation of ULE can be no more accurate than the allocation itself. For example, Parts 1, 2 and 4 of Schedule P test loss reserves including all loss expense. Schedule P itself cannot be used to determine whether the proper dollars of calendar year ULE were allocated to auto liability, general liability, and workmen's compensation. Nor can it be used to determine whether the allocation of the ULE to accident year was proper. To the degree that either of these allocations is inaccurate the reserve test will be inaccurate.

Ratio Method

$$\begin{pmatrix}\text{ULE}\\ \text{reserve}\end{pmatrix} = (\text{Factor}) \times \left[.50 \times \begin{pmatrix}\text{Reserve for}\\ \text{known claims}\end{pmatrix} + 1.00 \times (\text{IBNR}) \right]$$

where *Examination of Insurance Companies*[24] recommends a factor of $\dfrac{\text{Paid ULE for 3 calendar years}}{\text{Paid loss for 3 calendar years}}$. This formula depends upon the same three assumptions as does the ratio method for ALE reserves, as well as a fourth assumption: that 50% represents a reasonable estimate of the portion of investigation and adjustment already accomplished on open claims.

Projection Method—Accident Year Basis

In 1969 and 1970, Part 3 and 4 of Schedule P prescribed an allocation of ULE to accident year. Although these were dropped from the Statement in 1971, the reserving method which follows from them can still be used.

Part 4 of the 1970 Schedule P distributed the workmen's compensation ULE paid as follows

40% to the current accident year
45% to the first prior accident year
10% to the second prior accident year
5% to the third prior accident year

The table below shows how this distribution can be used to estimate a 12/70 ULE reserve. If we ignore growth, the table can be read as either a calendar year distribution of ULE paid by accident year or as an accident year distribution by calendar year.

DISTRIBUTION OF ULE PAYMENT PERCENTAGES BY CALENDAR YEAR AND ACCIDENT YEAR

Calendar	Accident Year					
Year	1968	1969	1970	1971	1972	1973
1971	5	10	45	40		
1972		5	10	45	40	
1973			5	10	45	40

[24]New York (State) Insurance Department, *op. cit.*

The reserve covers payments to be made 1971 and subsequent on claims occurring in 1970 and prior. If the calendar year paid ULE has been steady, we can take 80% (5% + 10% + 45% + 5% + 10% + 5%) of a typical calendar year's paid ULE as an estimated reserve.

The weakness of this method is that the percentage distribution is only an assumption.

Projection Method—Policy Year Basis

Prior to 1969, Part 3 and Part 4 of Schedule P prescribed an allocation of ULE to policy year. Part 4 of the 1968 Schedule P distributed the paid workmen's compensation ULE as follows:

40% to the current policy year
45% to the first prior policy year
10% to the second prior policy year
5% to the third prior policy year.

Each policy year contains two accident years. The 40% allocated to the current policy year all goes to the first accident year within that policy year, since the second accident year has not yet begun. Therefore, of the remaining 60%, 10% must be allocated to first accident years and 50% to second accident years, assuming that a calendar year's paid ULE is divided equally between first and second accident years within policy years. The reserve calculation will produce the same results regardless of how this ten-fifty split is achieved. For the purpose of illustration we assume here that the allocation percentage are these in the following table.

ASSUMED DISTRIBUTION OF CALENDAR YEAR PAID ULE

	First Accident Year	Second Accident Year	Total
Current policy year	40%	0%	40%
First prior policy year	7	38	45
Second prior policy year	2	8	10
Third prior policy year	1	4	5
	50%	50%	100%

This table can be restated as follows:

DISTRIBUTION OF ULE PAYMENT PERCENTAGES BY POLICY YEAR, ACCIDENT YEAR AND CALENDAR YEAR

Accident Year	Policy Year 1968		1969		1970		1971		1972		1973	
Calendar Year	**68**	**69**	**69**	**70**	**70**	**71**	**71**	**72**	**72**	**73**	**73**	**74**
1971	1	4	2	8	7	38	40	0				
1972			1	4	2	8	7	38	40	0		
1973					1	4	2	8	7	38	40	0

We can take 30% (1% + 4% + 2% + 8% + 7% + 1% + 4% + 2% + 1%) of a typical calendar year's paid ULE as an estimated Reserve.

Dollar Method

Examination of Insurance Companies[25] recommends the application of a percentage to one year's paid ULE. They say, "Such a study requires a cost-accounting analysis of the time and effort spent in a year in servicing claims and distributing the cost to the years of occurrence of these claims." Presumably, the proper percentage is derived as in the Accident Year Projection Method, using the allocation of calendar year payments to accident year revealed by the time study rather than the one previously prescribed by Schedule P.

Brian Method[26]

Brian recommends a method based upon the allocation of the ULE Reserve to five types of loss transactions. The lag in each of these types of transaction is used to calculate the reserve amount.

A simple example will illustrate the basic principle. Suppose that for a certain line of business these are the figures for an average calendar month:

[25] New York (State) Insurance Department, *op. cit.*

[26] Brian, *op. cit.*

Average Calendar Month Loss Transactions

Type of Transaction	Number
Single Payments	60
New Claims	200
Re-openings	70
Closings	270
Outstanding Claims	400
TOTAL	1,000
Amount of unallocated loss expense paid	$10,000

In this illustration we will calculate the portion of the ULE reserve relating to single payments. The average unallocated cost per transaction is $10 ($10,000 ÷ 1,000). The persistence assumption states that if the company were to cancel all business December 31, 1971, the number of single payments in 1972 relating to accidents December 31, 1971 and prior would be

January	60
February	40
March	20
April and subsequent	0
	120

Therefore the single payment portion of the ULE reserve is $1,200 (120 × $10.).

Assuming a persistence pattern is equivalent to assuming a distribution of calendar period payments by accident period. In the case of single payments, the distribution of calendar month payments to accident month which is equivalent to Brian's assumed persistence pattern is

Current accident month	0
First prior accident month	1/3
Second prior accident month	1/3
Third prior accident month	1/3

This assumption permits us to calculate the single payment portion of the ULE reserve using an accident month projection.

DISTRIBUTION OF ULE PAYMENT FRACTIONS BY CALENDAR MONTH AND ACCIDENT MONTH

Calendar Month	Accident Month			
	September	October	November	December
January		1/3	1/3	1/3
February			1/3	1/3
March				1/3

The single payment portion of the ULE reserve is twice the average calendar month ULE paid, or $1,200 (2 × 60 × $10.). Of course, this result is the same as the one we obtained by using the persistence pattern.

The basic assumption of this method is that the persistence pattern derived by allocating all ULE payments among only five transactions in equal amounts per transaction is not too different from the true persistence pattern. This is a reasonable assumption, but the only way to test it is to use another method to estimate the persistence pattern and compare the two estimated patterns. The step that would bring the most improvement to ULE reserve estimation would be to devise a better method for estimating the persistence pattern.

DISCUSSION BY ROBERT A. ANKER

Mr. Skurnick has performed a great service to the students of the Casualty Actuarial Society in the preparation and publication of his paper. Likewise, he has provided the practicing actuary with a consistent and valuable set of definitions for use in the loss reserving field and a comprehensive research and reference source. He has done an excellent job of organizing, distilling, and interrelating a broad spectrum of analytic approaches to loss reserving.

The primary thrust of Mr. Skurnick's paper is educational, and this review will consider it in that light. Thus, a portion of the review will be devoted to comments on specific items in the paper and Mr. Skurnick's interpretation of those items. Because I hope the paper will also serve as a catalyst for further papers in the area of reserving, the remainder of the review is devoted to the description of a broad approach to the selection and application of loss reserving methods.

COMMENTS ON SPECIFIC ITEMS

Fast Track Reserves

In discussing fast track reserves, Mr. Skurnick states this form of reserving is "appropriate for lines of insurance whose claims are similar in size such as auto collision." This statement may be amplified to observe that fast track reserving is suitable for use for particular subsets of claims which exhibit similar size within a given line of insurance. For example, the technique works quite well for workmen's companestion claims whose expected final payment value at the time of indexing is less than a selected limiting value intended to control the size variance of the claims.

Incurred But Not Reported (IBNR)

In discussing runoff tests for IBNR, Mr. Skurnick observes that the time needed for reasonable development varies from company to company and from line to line. It is significant that the variance within a line from company to company is primarily due to different claims administration systems and should be relatively small, normally less than one or two months. However, the variance in development time required between lines is a function of the line or coverage itself and can be substantial. It should also be noted that the time required for reasonable development can vary among states or jurisdictions because of variances in elements of the legal and/or regulatory climates.

Differences in required development time by line and by company exist for all runoff tests, not just IBNR runoff tests.

Total Loss Reserve Runoff

In his discussion of the runoff method for total loss reserves, Mr. Skurnick observes that Schedule O of the annual statement is a runoff method of total reserves, with some minor exceptions. He does not specify the rather significant exception that the IBNR reserves for surety and fidelity are not used in the runoff analysis in Schedule O.

Incurred Loss Development

Mr. Skurnick states that the Bornhuetter-Ferguson method produces an estimate of subsequent development which will lie between the estimates derived from the incurred loss development method and the loss ratio method. This is not true in the circumstances where the redundancy in the known reserve exceeds the expected IBNR as defined by Skurnick, i.e., where the ultimate factor is less than unity. In that case, the incurred loss development method produces the median estimate.

A Broad Approach

Perhaps the greatest service provided by Mr. Skurnick's paper is the demonstration of the multitude of methods available for reserving. The very fact that so many methods exist and are in use is a sufficient demonstration that there is no single "correct" method. Thus, on lines that possess a great potential for variance in reporting rates and severity characteristics, it is imprudent to rely on any single method. Loss reserving here must be a function of a decision process designed to produce an optimum estimate. As many methods as possible within the available time and data limitations should be used, with method selection based on the principle of including methods likely to produce both high and low estimates.

By this means, it is possible to establish ranges for reserve estimation. For each method used, ranges around the calculated value of the method should be established, either statistically or subjectively. If we describe these range limits as probable upper and lower bounds for the method, we can establish the following:

Estimate Range Description	Upper Limit	Lower Limit
Absolute Range	Probable Upper Bound of High Method	Probable Lower Bound of Low Method
Likely Range	Calculated Value of High Method	Calculated Value of Low Method
Best Estimate Range	Probable Upper Bound of Low Method or Lowest Probable Upper Bound of Any Method	Probable Lower Bound of High Method or Highest Probable Lower Bound of Any Method

Under normal circumstances, one would expect the ranges to be successively narrower and the likely range to contain the best estimate range. The likely range will always be contained in the absolute range. When the best estimate range is wider than the likely range, the method selection and application should be reexamined for possible refinement. If it is felt that the method selection and application are proper, then the range of the final reserve estimation should always be the narrowest range determined.

If the high and low method ranges used in establishing the best estimate range do not intersect, in which case the best estimate range does not exist, there is a logic error in the selection or application of methods.

This approach may be expanded to be directly applied to the analysis and evaluation of subsets of data which may have distorting effects on the estimation methods. It allows one to intelligently input information on the probable effects of such things as changes in claim administration, unusual and/or unprecedented claim or risk situations, and current economic and societal conditions.

There is an intrinsic sensibility to the approach which tends to assist in illustrating the need for using multi-source and multi-discipline information in the reserving process. Functional information, both actuarial and other, individual product information, economic information, and managerial and administrative information are all needed. To the extent that the approach can be simply defined and demonstrated, it can assist greatly in gaining the cooperation of individuals outside the actuarial area whose input can be valuable to the reserve valuation process.

I have left the description of this approach intentionally broad so as to not overly limit its use. My comments are not intended as a "how to" guide, any more so than is Mr. Skurnick's paper, but are meant as a further addition to the dialogue on loss reserve methodology that has developed in recent years.

Now that Mr. Skurnick has so admirably organized the existing literature, we must seek the brave soul who can develop a categorization specifying the appropriateness, utility, and limitations of the various methods in application to specific lines and under specific circumstances.

UNDERWRITING INDIVIDUAL DRIVERS: A SEQUENTIAL APPROACH

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AND

LEONARD R. FREIFELDER

Abstract

Adaptive decision models have been used with great success in many fields. This paper shows the value of the adaptive approach in underwriting individual automobile risks. Dropkin's model of the accident process serves as the basis by which adaptive and non-adaptive decisions are compared. The expected value of information about past or future driving experience is explained and developed to illustrate why adaptive and non-adaptive decisions may differ. Further insight into the adaptive model and the underlying accident process is developed by evaluating the value of information from stage *m* and the "true" value of stage *m*. The paper concludes by studying the adaptive model with discounting for the probability that a policy may lapse prematurely.

The insurance underwriter's basic task is risk selection—deciding which risks should be given insurance and which should not. To differentiate between risks, the underwriter must project the future accident experience of a driver and compare these costs against premium revenue. If expected revenues are at least as large as expected costs, the risk is acceptable.

The accuracy of the analysis is extremely important to the insurance company. If the underwriting policy is too restrictive, desirable risks will be overlooked. However, underwriters who are too liberal cost the company money by accepting undesirable business.

For the purposes of this paper, premiums are considered to be the sole source of revenue. On the cost side, expenses and insurer profits are ignored. Only the cost of accidents is considered. A simple decision rule then follows: accept the applicant if

$$\text{Premiums} \geq \text{EV (accident costs)}$$

These assumptions are made to simplify explanation of the model. Adjusting the model to include other sources of revenue and cost is simple and will not hamper its implementation.

The purpose of this paper is to propose a model which can assist the underwriter in selecting risks. The model in no way supplants the need for underwriting expertise, and it requires substantial input from the underwriter to operate properly. Although the paper considers accident frequency in determining the quality of business, other factors are clearly relevant. The effect of lapses, for example, is indicated in the final section of the paper.

It is possible to draw parallels between the underwriting decision and the decision of whether or not to grant credit. Like the underwriter, the credit manager of a company must distinguish between profitable and unprofitable risks. The credit manager must attempt to determine which risks will repay their loans and which will not. By establishing an unnecessarily restrictive policy, good risks are again overlooked. Too liberal a policy incurs unnecessary bad debts expenses.

Bierman and Hausman[1] have studied the problem of granting credit. Their results indicate that the most realistic decisions about extending credit are made when the decision-maker determines his optimal action from a multi-period analysis of the problem. The multi-period framework permits the credit manager to consider both the current and future benefits of granting credit. The model requires that the credit manager make an initial subjective estimate of the customer's probability of repayment. The decision to grant credit is made for one period, but it depends upon the expected value from current and future periods. After one period, the decision is revaluated, based now upon a revised probability of collection. The revised probability value is determined by modifying the prior estimate by the individual's repayment experience in the first period. If the expected monetary value (including costs) is still positive, credit will again be extended. The authors use Bayesian analysis to revise the probability of collection and dynamic programming to permit consideration of the expected returns from current and future periods.

The model presented by Bierman and Hausman is entirely compatible

[1] Harold Bierman, Jr., and Warren H. Hausman, "The Credit Granting Decision", *Management Science*, Vol. 16, No. 8 (1970), pp. B-519—B-532.

with the underwriting problem. Since the insurance company wishes to retain good risks and eliminate bad ones, it is quite natural to consider the underwriting decision in a sequential framework. The underwriter will determine his optimal action, based upon current information, for one period. After observing the accident experience of this period, the underwriter must incorporate the information with the original data and determine his optimal course of action for the next period. Bayesian analysis can be used to revise the underwriter's prior predictions of accident experience.

This paper presents an adaptive model similar to Bierman and Hausman's for use in underwriting individual drivers. Analysis will show that better underwriting decisions are made when the underwriter uses a sequential decision model which considers individual driving records.

A very strong case can be developed for incorporating information about driving records in underwriting decisions. A survey of the insurance literature reveals that individual driving records are often utilized in ratemaking. Studies by Wittick[2] of Canadian driving experience and by Harwayne[3] with California drivers indicate that there is significant and consistent variation in claims experience amongst drivers with different accident and traffic violation histories. Bailey and Simon[4] have established that the accident experience of an individual driver can be given a credibility weight for the purposes of determining his appropriate premium. Dropkin[5] shows that the distribution of the number of accidents for a group of individuals is most accurately described by a negative binomial function. In a subsequent paper, Dropkin[6] describes a method whereby the parameters of the group's negative binomial distribution can be updated by the accident records of individual drivers. The updated distribution serves to indicate future accident experience for each individual.

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[2] Wittick, Herbert E., "The Canadian Merit Rating Plan for Individual Automobile Risks," *PCAS* XLV, pp. 214–220.

[3] Harwayne, Frank, "Merit Rating in Private Passenger Automobile Liability Insurance and the California Driver Record Study," *PCAS* XLVI, pp. 189–195.

[4] Bailey, Robert A. and Simon, Leroy J., "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car," *PCAS* XLVI, pp 159–164.

[5] Dropkin, Lester B., "Some Considerations on Automobile Rating Systems Utilizing Individual Driving Records," *PCAS* XLVI, pp. 165–176.

[6] Dropkin, Lester B., "Automobile Merit Rating and Inverse Probabilities," *PCAS* XLVII, pp. 37–40.

1. A Model of the Accident Process

The negative binomial model is used here as a description of the automobile accident process. The model assumes that:

(1) Each driver generates accident events according to a Poisson process in time with constant rate λ accidents per year. For any time interval of length t, the number of accidents generated by one driver is a random variable having Poisson probability function with parameter λt.

$$P(n|\lambda, t) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}, n = 0, 1, 2, \ldots \qquad (1)$$

The expected value of n is

$$E(n|\lambda, t) = \lambda t$$

(2) The population is heterogeneous; each driver has a different λ value. Prior to observing the experience of a specific driver, the probability density of his λ value is given by

$$g(\lambda|a, b) = \frac{a^b \lambda^{b-1} e^{-a\lambda}}{\Gamma(b)}, \lambda > 0 \qquad (2)$$

The expected value of λ is

$$E(\lambda|a, b) = b/a$$

The marginal distribution of the number of accidents in a time interval of length t, for one driver, may be determined from assumptions (1) and (2).

$$P(n|a, b, t) = \int_0^\infty P(n|\lambda, t) g(\lambda|a, b)\, d\lambda$$

$$P(n|a, b, t) = \binom{n+b-l}{b-l} \left(\frac{a}{t+a}\right)^b \left(\frac{t}{t+a}\right)^n, n = 0, 1, 2, \ldots \qquad (3)$$

The expected value of n in a time period of length t is bt/a and the variance is $tb(a+t) \,/\, a^2$.

In Bierman and Hausman's model the original probabilities of collection (the prior information) are often made subjectively with little or no past information available. People may feel that this fact weakens the results. This criticism can safely be ignored when using the model in underwriting, since in this case more information is available. For any individual, prior information is the probability density of the applicant's value, characterized by a and b. When the underwriter has little or no information about an individual, other than his designated rate class, prior information can be developed without subjective input from the past accident record of the rate class. The accident experience of the class is translated into usable form by fitting a negative binominal distribution to the actual data, and calculating the values of a and b.

2. *The Economic Structure*

Let the underwriting profit from one individual for one year be represented by

$$\pi_1 = P - Cn_1$$

where P represents the pure premium and n_1 represents the actual number of accidents generated by that individual within the time period. The cost of a claim is represented here by its expected value, C. Although claim size is really a random variable its expected value is all that is needed if two conditions are met.

a.) The frequency and severity of accidents are independent random variables. This is commonly held to be true.

b.) The decision criterion utilized is based upon expected value, as is done in this paper. This is reasonable since we are dealing with decisions whose consequences are individually small relative to the overall size of the firm. The more general criterion, maximization of expected utility, would give approximately the same results in this case.

When considering the profit from one individual policy it is necessary to consider the lifetime of the policy. The number of years a policy will remain in force, before it is terminated by the insured, is unknown. Future policy life is of great importance for the decisions to be considered here. It seems most reasonable to express all results as a function of the future duration of the policy. Let m be the number of future years which the individual would renew the policy, if the choice is entirely left to him.

Thus, at the beginning, m is the life of the policy if the company does not terminate it. At any later time, m represents the future life.

Another important economic consideration is the time-value of money. Let

$$\beta = \frac{1}{1+r}$$

be a discount factor for one year, based upon interest rate r representing the best alternative use of money. Thus, consider one dollar to be received one year hence to have present value β.

The present value of profit from one individual policy of m future years duration can be expressed as:

$$\pi_m = \sum_{i=1}^{m} (P - Cn_i)\, \beta^{i-1} \tag{4}$$

Here, n_i represents the number of accidents in year i and all accident costs are treated as if paid at the start of the year.

3. *Non-Adaptive Decision Problem*

Consider the following individual, applying for insurance. The underwriter, considering past records for the applicant's rate class and other factors, suggests that the prior distribution of λ for this person is gamma (equation (2)) with $a = 13.5$ and $b = 1.37$. The underwriting profit function for one year is $\pi_1 = 100 - 1000n$, for the purpose of this example. The expected profit for one year is:

$$E(\pi_1) = \sum_{n=0}^{\infty} (100 - 1000n)\, P\,(n|13.5, 1.37, 1)$$
$$= 100 - 1000\, E(n|13.5, 1.37, 1)$$

By equation (3), $E(n|13.5, 1.37, 1)$ is b/a or .10148 accidents. Therefore,

$$E(\pi_1) = -\$1.48$$

The expected underwriting profit is negative and this applicant is rejected.

Similar results are obtained from considering multi-year policies. Suppose the company can offer a three-year policy which it could not terminate during the three-year period. The expected underwriting profit is

$$E(\pi_3) = E\left\{\sum_{i=1}^{3}(100 - 1000n_i)\,\beta^{i-1}\right.$$

At the time of the decision, n_1, n_2, and n_3 all have the same expected value, .10148 accidents. The result is

$$E(\pi_3) = (-\$1.48)(1 + \beta + \beta^2)$$

This is also negative. If $\beta = 1$, it is just a factor of three times the result for the one-year policy.

Non-adaptive decision-making will result in the rejection of this applicant, regardless of the duration considered for the policy.

It will be shown that an adaptive plan, one which anticipates the utilization of accident experience for decision-making, will give very different results.

4. Information and Its Expected Value

The one piece of information which the underwriter would really like to know is the number of accidents which the applicant will have. Although this information is unattainable, the analysis of its value will still be useful. Suppose, for simplicity, that the policy will last only one year. If future loss experience were known in time to be used in making the underwriting decision, the underwriter would accept the applicant only if the number of accidents, n, is zero. This is easily seen from the profit expression which is negative for all $n > 0$. The future profit from the one-year policy, as a function of n, is 100 if the information is that $n = 0$, and zero if the information is $n > 0$. However, the underwriter must evaluate this information prior to its receipt. The expected value with the information is found by weighting the two possible outcomes by their probabilities. For the applicant having $a = 13.5$ and $b = 1.37$, the probability of no accident in the next year is

$$P(0|13.5, 1.37, 1) = .9067$$

Therefore, the expected value of the process with the information is

$$100(.9067) + 0(.0933) = \$90.67$$

The expected value of perfect information about n, denoted $EVPI_n$, is the above quantity less the expected profit without having the information.

Since it was best to reject the applicant, the latter quantity is zero, and the result is,

$$EVPI_n = \$90.67$$

This quantity is useful as an upper bound on the value of any information to be used in a one-year policy decision. It also serves here to illustrate the concept of expected value of information, which was developed by Raiffa and Schlaifer.[7]

Although the information of the previous type is not available at any price, it is possible that the underwriter can obtain some additional information about the individual's prior distribution from outside sources and use that information in making his decision. The extra information usually involves some cost and it is important to know how much can be paid for it.

The model of the accident process implies that the best information, short of knowing actual accident experience, would be the λ value of the individual applicant. The only parameter of a Poisson probability function is λ, the expected number of claims per year. Knowing λ exactly makes possible exact prediction of the probability of x ($x = 0, 1, 2, \ldots$) accidents.

The best information which is actually available is information about the individual's actual past experience. If many years of past experience are available this information will be almost as valuable as information about λ. However, it is important to realize that even a small amount of this information is quite useful in the decision process.

Suppose that the underwriter can purchase the actual past T years of experience of the applicant and can use this information in his decision of whether to approve a one-year policy. The information will tell him k, the number of accidents the applicant had in the T year period.

Without the information, the expected value of the one-year underwriting profit was found to be

$$E(\pi_1) = E(100 - 1000n) = 100 - 1000\frac{b}{a}$$
$$= -1.48$$

[7] Raiffa and Schlaifer, *Applied Statistical Decision Theory,* The MIT Press, 1961.

However, with the information, the underwriter will modify his prior gamma probability distribution of λ to obtain a posterior gamma probability distribution[8] having parameters

$$a' = a + T$$
$$b' = b + k \qquad (5)$$

which is a blending of the prior knowledge with the sample information. The expected underwriting profit becomes

$$E(\pi_1) = 100 - 1000 \left(\frac{b+k}{a+T}\right)$$

This expression depends upon k. The value of the best decision, as a function of k, is

$$\text{Max} \left\{ 0, 100 - 1000 \left(\frac{b+k}{a+T}\right) \right\}$$

Before k is known, the value of the decision process with the information can be found by taking the expectation of the above expression using the probability function

$$P(k|a, b, T) = \left(\frac{a}{a+T}\right)^b \left(\frac{T}{a+T}\right)^k \binom{b+k-l}{k}$$

This expectation can be represented by

$$\sum_{k=o}^{k^*} \left[100 - 1000\left(\frac{b+k}{a+T}\right)\right] P(k|a, b, T)$$

where k^* is the largest value of k such that

$$\left[100 - 1000\left(\frac{b+k}{a+T}\right)\right] > 0$$

For illustration, let $T = 1$, $a = 13.5$, $b = 1.37$. Then $k^* = 0$, and

$$\left[100 - 1000 \left(\frac{b}{a+1}\right)\right] P(0|a, b, 1) = 5.00$$

[8] See Mayerson, Allen L., "A Bayesian View of Credibility," *PCAS* LI, pp. 85–104.

As already shown, the expected value without the information is zero. Therefore, the expected value of sample information from T years of actual past experience is

$$EVSI(a, b, 1) = \$5.00$$

Summarizing the previous development,

$$EVSI(a, b, T) = \sum_{k=o}^{k^*} \left[100 - 1000 \left(\frac{b+k}{a+T} \right) \right] P(k|a, b, T)$$

$$- \text{Max} \left\{ 0, 100 - 1000 \left(\frac{b}{a} \right) \right\}$$

Information about driving history will appear even more valuable if a multi-period policy is considered. The underwriting profit from a policy which, if granted, will last m years is

$$\pi_m = \sum_{i=1}^{m} (100 - 1000 n_i)\, \beta^{i-1}$$

At the time of the decision all the random variables $n_1, n_2, \ldots, n_m$ have the same expectation; $E(n|a, b, 1)$. Therefore, the expected value of underwriting profit is

$$E(\pi_m) = \left[100 - 1000 \left(\frac{b}{a} \right) \right] \left(\frac{1 - \beta^m}{1 - \beta} \right)$$

Now it can be seen that the expected profit from m years is just a multiple of the expected profit for one year. Therefore, the expected value of sample information to be used in the m year decision is

$$EVSI(a, b, T, m) = \left(\frac{1 - \beta^m}{1 - \beta} \right) EVSI(a, b, T)^{9}$$

It can be proven that $EVSI(a, b, T)$ is a non-negative quantity and is an increasing function of T, increasing at a decreasing rate and approaching an asymptotic value, which would be the expected value of perfect information about the individual's λ value.

The foregoing development establishes the value of information about individual driving experience. Sometimes this experience will be available

[9] As β approaches one, the value of $(1-\beta^m) / (1-\beta)$ approaches m.

from policy records, and the question is whether the information is worth the cost of data processing required to extract it. Even when the information is not available from past records, it is clear that future experience information will be available and the decision-making structure ought to anticipate using the information as it becomes available.

5. The Multi-Period Adaptive Decision

To obtain information about driving experience for use in future periods, the applicant must first be accepted. The underwriter will then receive information which can be used to determine the best future underwriting decisions. A single-decision model ignores the value of such information by basing the underwriting decision solely upon the prior distribution for λ.

A multi-stage dynamic programming model does not have this weakness; it utilizes the information as it becomes available.

Let $V_m(a, b)$ = optimal expected present value of the next m periods when the prior distribution of λ has parameters a, b at the start of the first period.

$\beta = 1/1 + r$ = a factor discounting future returns to consider the time value of money.

The underwriter chooses that action (accept or reject) which results in the optimal expected profit at each stage. The expected profit for rejection at any stage is zero. If the applicant is accepted, the expected profit is the expected first year return plus the discounted returns from future periods. The expected profit from one year, as a function of the parameters a, b of the prior distribution at the start of the year is

$$R(a, b) = E(\pi_1) = P - C\,E(n|a, b, 1)$$

If the experience during the first period is n accidents, and the period is one year, then the prior parameters will be transformed into the posterior parameters $a + 1$ and $b + n$. The value of the remaining $m - 1$ periods will be represented by

$$V_{m-1}(a + 1, b + n)$$

However, n is unknown at the start of the first period and thus the expected value of $V_{m-1}(a+1, b+n)$ must be used. It is[10]

$$E_{n|a,b}\{V_{m-1}(a+1, b+n)\} = \sum_{n=0}^{\infty} P(n|a, b, 1)\, V_{m-1}(a+1, b+n)$$

The dynamic programming equation for V_m, in terms of V_{m-1}, is

$$V_m(a, b) = \text{Max}\,\{0, \overbrace{R(a, b)}^{\text{1st year expected return}} + \beta\, \overbrace{E_{n|a,b}\{V_{m-1}(a+1, b+n)\}}^{\text{expected returns from future periods}}\} \qquad (6)$$

$V_0(a, b)$ is defined as 0 for all a, b.

$V_1(a, b) = \text{Max}\,\{0, P - CE(n|a, b, 1)\}$ and corresponds to a single stage decision.

Example

A simple example has been chosen to illustrate the procedure. The optimal decision for the next three years, $m = 3$, will be determined, assuming a policy period of $t = 1$ year[11] and no discounting ($\beta = 1$). The profit function is $100 - 1000n$ and the prior distribution for λ is $g(\lambda|13.5, 1.37)$. The optimal decision for single stage models, under these conditions, is rejection, as was previously shown.

To calculate $V_3(a, b)$, various values of $V_2(a+1, b+n_1)$ are needed. $V_2(a+1, b+n_1)$ in turn depend upon the values for one stage problems, $V_1(a+2, b+n_1+n_2)$.

The one stage process has the following form for any a, b.

$$\begin{aligned} V_1(a, b) &= \text{Max}\,\{0, 100 - 1000E(n|a, b, 1)\} \\ &= \text{Max}\,\{0, 100 - 1000(b/a)\} \end{aligned}$$

The optimal decision will be to accept if $a > 10b$ and "receive" $100 - 1000(b/a)$; otherwise reject and receive 0.

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[10] The notation $E_{n|a,b}\{\cdot\}$ denotes the same expectation operation previously written $E(\cdot\,|a, b, t)$. It will be used only when $t = 1$ and so the t value can be suppressed.

[11] The optimal policy period is one year. If the company issued a multi-year, non-cancellable contract, the underwriter would be forced to ignore the experience information until the next renewal date.

In this situation $a + 2 = 15.5$, $b = 1.37 + k$, $k = 0, 1, 2, \ldots$

$$\begin{aligned}
V_1(15.5, 1.37) &= \text{Max}\ \{0, 100 - 1000(1.37/15.5)\} = 11.61 \\
V_1(15.5, 2.37) &= \text{Max}\ \{0, 100 - 1000(2.37/15.5)\} = 0 \\
V_1(15.5, 3.37) &= \text{Max}\ \{0, 100 - 1000(3.37/15.5)\} = 0 \\
V_1(15.5, 1.37 + k) &= 0, \text{ for } k = 1, 2, 3, \ldots
\end{aligned}$$

Returning to Stage 2,

$$\begin{aligned}
V_2(a, b) = \text{Max}\ \{0, 100 - 1000(b/a) \\
+ E_{n|a,\,b}\{V_1(a + 1, b + n)\}\}
\end{aligned}$$

For $a = 14.5$, $b = 1.37$,

$$\begin{aligned}
V_2(14.5, 1.37) = \text{Max}\ \{0, 5.52 + V_1(15.5, 1.37)\ P\ (0|14.5, 1.37, 1) \\
+ V_1(15.5, 2.37)\ P\ (1|14.5, 1.37, 1) + \cdots\}
\end{aligned}$$

All terms involving $V_1(15.5, 1.37 + k)$ for $k = 1$ or higher are zero. Therefore

$$V_2(14.5, 1.37) = \text{Max}\ \{0, 5.52 + (11.61)(.91268)\} = \$16.12$$

For $a = 14.5$, $b = 2.37$

$$\begin{aligned}
V_2(14.5, 2.37) = \text{Max}\ \{0, 100 - 1000(2.37/14.5) \\
+ V_1(15.5, 2.37)\ P(0|14.5, 2.37, 1) \\
+ V_1(15.5, 3.37)\ P\ (1|14.5, 2.37, 1) + \cdots\}
\end{aligned}$$

All of the terms for the continued decision are negative or zero. Therefore, $V_2(14.5, 2.37) = 0$. Similarly,

$$V_2(14.5, 1.37 + k) = 0, \text{ for } k = 2, 3, \cdots.$$

Finally, for stage 3,

$$\begin{aligned}
V_3(a, b) = \text{Max}\ \{0, 100 - 1000(b/a) \\
+ E_{n|a,\,b}\{V_2(a + 1, b + n)\}\}
\end{aligned}$$

After exclusion of terms which are zero, this is

$$\begin{aligned}
V_3(13.5, 1.37) &= \text{Max}\ \{0, 100 - 1000(1.37/13.5) \\
&\quad + V_2(14.5, 1.37)\ P\ (0|13.5, 1.37, 1)\} \\
&= \text{Max}\ \{0, -1.48 + (16.12)(.90674)\} \\
&= \$13.14
\end{aligned}$$

The optimal underwriting decision from a sequential analysis is to insure the applicant for the first year. Whether or not further insurance will be granted depends upon the individual's accident experience, but a decision rule has been found. The expected profit is $13.14, as opposed to an expected return of $0 from a single stage analysis (where the optimal action is rejecting the applicant). The increase in expected profit occurs because the underwriter receives and utilizes additional information. The information about individual driving records allows the underwriter to retain good risks and eliminate bad ones. The value of this information is $13.14, the expected profit with the information about driving records less the expected profit without information about driving records.

6. *The Value of Information From Stage m*

At each stage in the sequential process, the underwriter obtains information about the insured's accident rate λ from the individual's accident experience. Since knowledge about driving records enables the underwriter to make better decisions, the information has value.

Define $CVSI_m(n|a, b)$ as the conditional value of the sample information gathered in stage m, which is the first year of an m year process. If the individual is insured during stage m, the underwriter observes the accident experience—n accidents in one year—and has an optimal expected value at stage $m - 1$ of $V_{m-1}(a + 1, b + n)$. If the individual is not insured during stage m, the optimal expected value at stage $m - 1$ is $V_{m-1}(a, b)$.

$$CVSI_m(n|a, b) = \beta\{V_{m-1}(a + 1, b + n) - V_{m-1}(a, b)\}$$

The discounting factor is applied, because the information obtained in stage m can first be used in stage $m - 1$, one year later.

Define $EVSI_m(a, b)$ as the expected value of information from stage m.

$$\begin{aligned} EVSI_m(a, b) &= E_{n|a,\, b}\{CVSI_m(n|a, b)\} \qquad (7) \\ &= \beta\, E_{n|a,\, b}\{V_{m-1}(a + 1, b + n)\} - \beta V_{m-1}(a, b) \end{aligned}$$

The expected value of the information gained during the first year of the three-year period is calculated as an example.

$$\begin{aligned} EVSI_3(13.5, 1.37) &= E_{n|13.5,\, 1.37}\{V_2(14.5, 1.37 + n)\} \\ &\quad - V_2(13.5, 1.37) \\ &= P(0|13.5, 1.37, 1)\, V_2(14.5, 1.37) \\ &\quad - V_2(13.5, 1.37)^{12} \end{aligned}$$

after zero terms are omitted. This reduces to

$$EVSI_3(13.5, 1.37) = 14.62 - 3.52 = 11.10$$

The definition of $EVSI_m(a, b)$ has the important property that it is non-negative. This agrees with the intuitive notion that information is always expected to be beneficial, although on an after-the-fact basis it can have negative value. This is shown in the appendix.

7. *The True Expected Value of Stage m*

It has been shown that an adaptive policy often leads to the acceptance of an individual whose application would be rejected on the basis of a non-adaptive decision. This is the situation when the expected one-year profit, $R(a, b)$, is negative but at the same time, for a horizon of m years, $V_m(a, b)$ is positive. The difference is due to the value of the information to be gained in stage m and utilized thereafter in the $m - 1$ remaining decisions.

It is useful to define $R_m^*(a, b)$ to be the "true" expected value which can be attributed to stage m. If the applicant were not to appear until one year later, then period m would be idle and the present value would be $\beta V_{m-1}(a, b)$. Let the true value of stage m be defined by

$$R_m^*(a, b) = V_m(a, b) - \beta V_{m-1}(a, b),$$

the change in expected value between utilizing and not utilizing stage m. It is shown in the appendix that this is equivalent to

$$R_m^*(a, b) = \begin{cases} R(a, b) + EVSI_m(a, b) & \text{if optimal to continue} \\ 0 & \text{if optimal to terminate or not grant the policy.} \end{cases}$$

[12] $V_2(13.5, 1.37)$ was not calculated before. It requires $V_1(14.5, 1.37 + k)$ for $k = 0, 1, 2$. These also were not calculated previously. The results are $V_1(14.5, 1.37) = 5.52$ and $V_1(14.5, 1.37 + k)$ for $k = 1, 2, 3 \ldots$, are all zero. Finally, $V_2(13.5, 1.37) = 3.52$.

Thus, the true expected value of stage m is the sum of the expected immediate profit of stage m plus the expected value of the information to be received during stage m. Thus, it will be optimal to continue the policy even when $R(a, b)$ is negative, if the value of $EVSI_m(a, b)$ is large enough to make the sum positive.

To illustrate the concept of true expected value, return to the example with $a = 13.5$, $b = 1.37$, $m = 3$, $\beta = 1$. We have found that the

$$EVSI_3(13.5, 1.37) = 11.10$$
$$\text{and } R(13.5, 1.37) = -1.48$$

Therefore

$$R_3^*(a, b) = -1.48 + 11.10 = 9.62$$

and we conclude that the information value far exceeds the small expected loss during the first-year of the adaptive decision. Here the expected loss $R(13.5, 1.37)$ can be viewed as a cost of sampling for information.

8. *The Administration of Adaptive Decision Rules*

The administration of these decision rules begins with the assignment of prior parameter values (a, b) to each new applicant based upon his rate classification and other, possibly subjective, information of use in the underwriting function. The optimum decision rule is calculated to give the accept/reject decision and the decision rule for future periods.

On the anniversary of each policy the prior parameters are updated to include the experience of the past year. The decision to continue or terminate the policy is made according to the decision rule previously calculated.

A useful classification system can be utilized, based upon the calculations shown. All existing policies can be classified into the following states which characterize their current condition:

"Trial" or "Tentative" State—the current values of a, b and m are such that $R(a, b) < 0$ while $R_m^*(a, b) > 0$. Such a policy is being continued only as an experiment which may result in favorable information.

Secure State of Degree n—the current values of a and b are such that $R(a, b) > 0$. Such a policy has positive expected profit for the current year. However, if more than n accidents were to occur during this year, this policy would revert to the trial stage, or even be terminated. In other words, n is the largest number such that $R(a+1, b+n)$ is still positive. The number n is very easy to compute.

The classification of policies into these states emphasizes the various different degrees of security of policies.

Each policy may move, with experience, from one phase to another. Good experience will tend to move a policy into higher states of security while unfavorable experience will move a policy rapidly downward in security level.

9. *The Effect of Lapses*

The expected profit from an applicant depends upon m, the future life of the policy, or the policy horizon. Conditional upon a future life of m years, $V_m(a, b)$ gives the optimal expected profit. The optimal decisions depend strongly upon m; the longer the life of the policy, the more valuable is the adaptive ability. For policies which would always lapse after one year, the adaptive feature is useless. The feature becomes very valuable for small and moderate m. The sensitivity to m is decreased beyond that, however, because of the discounting.

Fortunately, the effect of policy lapse (termination by the insured) can easily be introduced into the dynamic program equations. Let α_i be the probability that the policy will lapse during the i^{th} year given that it has entered the i^{th} year. These "lapse rates" may be constant and equal for all years or they may depend upon the policy age or other policy characteristics. Given the conditional probabilities, the unconditional probability that the policy will remain in force at least n years is

$$\prod_{i=1}^{n} (1-\alpha_i)$$

The unconditional probability that it will lapse first in year n is

$$\prod_{i=1}^{n-1} (1-\alpha_i)\, \alpha_n \quad ^{13}$$

The expected future life is

$$\sum_{n=1}^{\infty} n \prod_{i=1}^{n-1} (1-\alpha_i)\, \alpha_n \text{ which is } \frac{1}{\alpha} \text{ if all } \alpha_i = \alpha$$

[13] $\prod_{i=1}^{n-1} (1-\alpha_i)$ is defined to be equal to one for $n = 1$.

The dynamic programming equation becomes

$$V_m(a, b) = \text{Max}\ \{0, R(a, b) + (1 - \alpha_1)\ \beta \text{E}_{n|a,\,b}\{V_{m-1}(a + 1, b + n)\}\}$$

for the first year of a period of m years duration, and

$$V_{m-i}(a, b) = \text{Max}\ \{0, R(a, b) + (1 - \alpha_{i+1})\ \beta E_{n|a,\,b}\{V_{m-i-1}(a + 1, b + n)\}\}$$

for the successive years.

If the lapse probability for each year is the same, this process amounts to using a larger discount rate $(1 - \alpha)\beta$ in place of β.

The significance of m now is different than previously used. Here it represents the planning horizon of the company rather than the policy horizon. The results will become insensitive to m as long as m is taken to be larger than the expected life of the policy.

To consider the effects of interest and the probability of lapse, the expected returns from each stage must be multiplied by the appropriate discount factor. This discounting factor increases over time. Discounted, the expected returns are very small from stages far in the future. Thus, as m gets large, the discounted value of $V_m(a, b)$ will converge to a constant amount.

Example

Suppose that an applicant having the same description as before is being considered. Again $\beta = 1$. The new feature is that the probability of lapse during the first year is $\alpha_1 = .5$, $\alpha_2 = .5$ for the second year and $\alpha_3 = 1$ for the third year.

Recalling the original results for $a = 13.5, b = 1.37$ without consideration of lapse;

$V_1(13.5, 1.37) = 0$	$V_2(13.5, 1.37) = 3.52$
$V_1(14.5, 1.37) = 5.52$	$V_2(14.5, 1.37) = 16.12$
$V_1(15.5, 1.37) = 11.61$	$V_3(13.5, 1.37) = 13.14$
	$R(13.5, 1.37) = -1.48$

With lapse

(1) $V_1^L(13.5, 1.37) = \text{Max}\{0, R(13.5, 1.37) + (1 - \alpha_3)\beta E_{n|13.5,\,1.37}\{V_0(14.5, 1.37 + n\}\}$
$= 0$

Similarly, $V_1^L(14.5, 1.37) = 5.52$ and $V_1^L(15.5, 1.37) = 11.61$

(2) $V_2^L(13.5, 1.37) = \text{Max}\{0, R(13.5, 1.37) + (1 - \alpha_2)\beta E_{n|13.5,\,1.37}\{V_1^L(14.5, 1.37 + n\}\}$
$= \text{Max}\{0, \text{R}(13.5, 1.37) + [\tfrac{1}{2}\,\text{P}(0|13.5, 1.37)V_1^L(14.5, 1.37) + \tfrac{1}{2}\,\text{P}(1|13.5, 1.37)V_1^L(14.5, 2.37) + \cdots]\}$
$= \text{Max}\{0, -1.48 + \tfrac{1}{2}[(.90674)(5.52) + (.08326)(0)]\}$
$= \$1.02$

A similar calculation will reveal that $V_2^L\ (14.5, 1.37) = 10.82$

(3) $V_3^L(13.5, 1.37) = \text{Max}\{0, R(13.5, 1.37) + (1 - \alpha_1)\beta E_{n|13.5,\,1.37}\{V_2^L(14.5, 1.37 + n\}\}$
$= \text{Max}\{0, R(13.5, 1.37) + [\tfrac{1}{2}P(0|13.5, 1.37)V_2^L(14.5, 1.37) + \tfrac{1}{2}P(1|13.5, 1.37)V_2^L(14.5, 2.37) + \cdots]$
$= \text{Max}\{0, -1.48 + (.90674)(10.82) + (.08326)(0)\}$
$= \$3.43$

The results of this calculation illustrate the importance of adaptive decision-making even when the lapse rate is high. Although the policy has an expected life of less than two years, by making adaptive decisions the underwriter expects to realize a profit for a policy horizon of more than one year.

10. Applications

Applications of the model are not limited to underwriting. With appropriate modifications, the model can also be used as a ratemaking tool.

For risk selection decisions, the decision-maker must be able to formulate a profit function similar to the one shown in section 2. This function can be specified on a net basis by ignoring all sources of revenue and cost except premiums and claims costs. Or, if the appropriate information is available, a profit function including these costs may be derived. The model then requires the decision-maker to specify the applicant's rate class and the loss characteristics of that class. At this point additional information about the applicant will be recognized and appropriate adjustments to the parameters of the prior probability distribution should be made. The best decision follows directly from equation (6) as illustrated in the paper.

For ratemaking applications the model is used somewhat differently. In this case the decision variable is the level of premiums rather than whether to accept or reject the applicant. The decision-maker wishes to find the premium for which $V_m\,(a, b)$ is just equal to zero. This rate represents the minimum amount the company should charge to insure the individual. At any rate lower than the minimum the company would expect to lose money on each person it insures. To determine the minimum rate an underwriting profit function must again be specified and the parameters of the individual's loss distribution must be developed. The model can be a tool for pricing both existing and proposed contracts. One interesting possibility would be to price a non-cancellable, multi-year policy where the premium rate is held constant between renewal dates, irrespective of the insured's accident experience.

Appendix

The Non-negativity of the Expected Value of Information from Stage m

It is to be proved that $EVSI_m(a, b) \geq 0$ for all m. This is equivalent to

$$E_{n|a,\,b}\{V_{m-1}(a+1, b+n)\} \geq V_{m-1}(a, b) \text{ for all } m.$$

This will be done by induction. Letting $m = 2$, by definition,

$$V_1(a+1, b+n) = \text{Max}\,\{0, R(a+1, b+n)\} \text{ for all } n$$

So

$$E_{n|a,\,b}\{V_1(a+1, b+n)\} = E_{n|a,\,b}\{\text{Max}\,\{0, R(a+1, b+n)\}\}$$

Reversing the operations of expectation and maximization, the inequality is found to be

$$E_{n|a,b}\{\text{Max}\,\{0, R(a+1, b+n)\}\} \geq \text{Max}\,\{0, E_{n|a,b}\{R(a+1, b+n)\}\}$$

This right hand side is, by definition, $V_1(a, b)$.

Hence,

$E_{n|a,b}\{V_{m-1}(a+1, b+n)\} \geq V_{m-1}(a, b)$ has been shown for the special case $m = 2$. Now assume the inequality holds for $m - 2$ and consider whether it is true for $m - 1$. A similar argument is used. By definition,

$$E_{n|a,b}\{V_{m-1}(a+1, b+n)\} = E_{n|a,b}\{\text{Max}\,\{0, R(a+1, b+n) + E_{n'|a+1,b+n}\{V_{m-2}(a+2, b+n+n')\}\}\}$$

Now, reversing the operations, the inequality is obtained

$$E_{n|a,b}\{V_{m-1}(a+1, b+n)\} \geq \text{Max}\,\{0, E_{n|a,b}\{R(a+1, b+n)\} + E_{n|a,b}\{E_{n'|a+1,b+n}\{V_{m-2}(a+2, b+n+n')\}\}\}$$

Now it can be shown that

$$E_{n|a,b}\{R(a+1, b+n)\} = R(a, b).$$

The iterated expectation over n and n' can be reversed, using the fact than n and n' are conditionally independent given λ. This reversal gives

$$E_{n|a,b}\{E_{n'|a+1,b+n}\{V_{m-2}(a+2, b+n+n')\}\} = E_{n'|a,b}\{E_{n|a+1,b+n'}\{V_{m-2}(a+2, b+n'+n)\}\}$$

By assumption, the inner term obeys

$$E_{n|a+1,b+n'}\{V_{m-2}(a+2, b+n'+n)\} \geq V_{m-2}(a+1, b+n')$$

Hence, the inequality is obtained

$$E_{n|a,b}\{V_{m-1}(a+1, b+n)\} \geq \text{Max}\,\{0, R(a, b) + E_{n'|a,b}\{V_{m-2}(a+1, b+n')\}\}$$

the right hand side is just $V_{m-1}(a, b)$ and so the result is true for $m - 1$ and, by induction, true for any m.

Appendix

The True Expected Value of Stage m

The true expected value of stage m has been defined as the optimal expected value of an m stage process less that of the $m - 1$ stage process starting from the same state of information but discounted by one period.

$$R_m^*(a, b) = V_m(a, b) - \beta V_{m-1}(a, b)$$

Suppose momentarily that the decision is between continuation of the policy or delaying the renewal one period. This is represented by the dynamic program

$$V_m(a, b) = \text{Max}\ \{\beta V_{m-1}(a, b), \text{R}(a, b) + \beta E_{n|a,b}\{V_{m-1}(a+1, b+n)\}\}$$

The first term in the bracket represents the choice of delaying the decision one period. It will later be shown that this implies that $\beta V_{m-1}(a, b) = 0$ and hence that this dynamic program is equivalent to the one originally discussed.

Equation (7) can be written as

$$\beta E_{n|a,b}\{V_{m-1}(a+1, b+n)\} = EVSI_m(a, b) + \beta V_{m-1}(a, b)$$

Substitution into the above dynamic program gives

$$V_m(a, b) = \text{Max}\ \{\beta V_{m-1}(a, b), R(a, b) + EVSI_m(a, b) + \beta V_{m-1}(a, b)\}$$

subtraction of $\beta V_{m-1}(a, b)$ from all terms gives

$$R_m^*(a, b) = \text{Max}\ \{0, R(a, b) + EVSI_m(a, b)\}$$

Thus the true expected value of stage m is $R(a, b) + EVSI_m(a, b)$ if if it is optimal to continue the policy and zero otherwise.

It remains to show that the above dynamic program implies the original dynamic program. It can be shown from

$$V_m(a, b) = \text{Max}\ \{\beta V_{m-1}(a, b), R(a, b) + \beta E_{n|a,b}\{V_{m-1}(a+1, b+n)\}\}$$

that $V_m(a, b) = \beta V_{m-1}(a, b)$ which implies that

$$V_m(a, b) = V_{m-1}(a, b) = \cdots = V_0(a, b) = 0.$$

This will not be shown here in detail but is based upon the inductive argument that if delay is optimal with m stages remaining it will also be optimal with $m - 1$ stages remaining. By induction it is optimal with zero stages remaining also, and thus has expected value zero.

DISCUSSION BY DONALD A. JONES

Two events that occurred in November 1969 stick in my mind. The University of Michigan football team upset the Ohio State team to become Big Ten champions and the Casualty Actuarial Society and the Society of Actuaries co-sponsored a research seminar based on the decision analysis work of Howard Raiffa and Robert Schlaifer. These concurrent events share the same cell in my memory because my attendance at the latter prevented my witnessing the former!

The Cozzolino and Freifelder approach to underwriting individual drivers is an excellent example of the Raiffa and Schlaifer decision analysis and would have been a highlight of the 1969 research seminar. Some of the seminar presentations were repeated at a 1970 spring meeting of the Society of Actuaries and hence are part of Volume XXII of the *Transactions of the Society.*

When a theoretical model is suggested for an application, the robustness of the model is an important question; i.e., how sensitive are the results to the assumed distribution and its parameter values? To explore this question, I used the authors' gamma-Poisson model with seven pairs of values for the gamma distribution parameters (a, b) chosen so that the mean b/a would equal the authors' 0.10148. These values were $(13.5 \times 10^j, 1.37 \times 10^j)$ for $j = 0$ (the authors' values), ± 1, ± 2, and ± 3. The results for these different parameter values are summarized in Table 1.

Since the standard deviation of the gamma distribution is $\sqrt{b}/a$, the homogeneity of the driver population increases with j. This may be observed in column (4) of Table 1, which shows the standard deviations ranging from 2.742 for $j = -3$ down to 0.00274 for $j = 3$.

As the homogeneity of the driver population increases, we should anticipate the expected value of sample information, *EVSI*, (Column (8), Table 1) and the expected value of sequential decisions over the three year period, $V_3(a, b)$, (Column (11), Table 1) to decrease. Such is the case with these expected values, even being zero for $j = 1$, 2, and 3.

Overall the values in Table 1 show a consistent and monotone pattern that indicates that the gamma-Poisson model reacts well to parameter changes. Perhaps more exploration between $j = 0$ and $j = 1$ would be illuminating.

Let us turn to exploring robustness with respect to the assumed distribution. It would be nice if we could "linearize the problem" to arrive at an analysis that would be distribution-free in the sense of depending on only the first two moments of the underlying distribution. Such is a common approach to credibility theory. Since expected values of truncated random variables, which are the objective of this analysis, depend on more characteristics of the assumed distribution than just the first two moments, linearization is not feasible.

Thus, for a brief exploration of the robustness of the authors' model with respect to the distribution assumption, I calculated the corresponding values under the following assumptions:

(1) The Poisson distribution for the number of accidents was replaced by a Bernoulli distribution:

$$P(n|p,t) = \binom{t}{n} p^n (1-p)^{t-n}, \; n = 0, 1, \cdots, t$$
$$E(n|p,t) = pt$$

(2) The gamma distribution for λ which described the population heterogeneity was replaced by a beta distribution for p:

$$g(p|c,d) = \frac{\Gamma(c+d)}{\Gamma(c)\Gamma(d)} p^{c-1}(1-p)^{d-1} \qquad 0 < p < 1$$

$$E(p|c,d) = \frac{c}{c+d}$$

For these distributions, the number of accidents in "time" t (the parameter in the above binomial distribution), unconditional on p, for one driver is

$$P(n|c,d,t) = \binom{t}{n} \frac{\Gamma(c+d)}{\Gamma(c)\,\Gamma(d)} \; \frac{\Gamma(n+c)\,\Gamma(t+d-n)}{\Gamma(t+c+d)} \qquad n = 0, 1, \cdots\cdot, t$$

$$E(n|t) = tc/(c+d)$$

$$\text{Variance } (n|t) = tcd(t+c+d)/[(c+d+1)(c+d)^2]$$

For the numerical example I set $t = 1$ (the maximum number of accidents per year), $c = 1.37$, and $c + d = 13.5$, which gave the authors' expected value for n. The corresponding calculations for this model are shown in the "Beta-Bernoulli" line of Table 1. You can see that these values are very close to those given by the authors' distribution, which is probably no surprise since both models give nearly the same marginal distribution for n.

I will close with a couple of less quantitative observations. First, the justification for $\pi_1 = P - cn_1$ might be put in familiar analytical form for casualty actuaries. The underwriting profit from one individual for one year is the random variable

$$\pi_1 = P - (X_1 + X_2 + \cdots + X_{n_1}),$$

where the X_i's are the claim amounts. Under the authors' condition (b) that decisions will be based on expected values, we have $E(\pi_1) = P - E(X_1 + X_2 + \cdots + X_{n_1})$. This last term has been evaluated as $E(X_i)E(n)$ under the authors' condition (a) elsewhere in *PCAS* LV, page 179.

As a resident of Michigan, which has the country's newest No-Fault law, I would ask this gathering of actuaries if the authors' fine paper will be the swan song of individual merit rating theory in the *Proceedings?*

TABLE 1

j	a (1)	b (2)	$E[\lambda]$ (3)	$\sigma[\lambda]$ (4)	$R(a, b)$ (5)	$P(0\|a, b, 1)$ (6)	$EVPI$ (7)	$EVSI$ (8)
—3	.0135	.00137	.10148	2.742	—1.48	.99410	99.41	98.07
—2	.135	.0137	.10148	.867	—1.48	.97125	97.12	85.40
—1	1.35	.137	.10148	.274	—1.48	.92687	92.69	38.65
0	13.5	1.37	.10148	.0867	—1.48	.90674	90.67	5.00
1	135.	13.7	.10148	.0274	—1.48	.90384	90.38	0
2	1350.	137.	.10148	.00867	—1.48	.90353	90.35	0
3	13,500.	1370.	.10148	.00274	—1.48	.90350	90.35	0
Beta-Bernouilli			.10148	.0793	—1.48	.89852	89.85	4.96

j	$V_1(a+2, b)$ (9)	$V_2(a+1, b)$ (10)	$V_3(a, b)$ (11)	$V_2(a, b)$ (12)	$EVSI_3$ (13)	$R_3^*(a, b)$ (14)	$V_3^L(a, b)$ (15)
—3	99.32	197.88	195.23	96.58	100.12	98.64	72.21
—2	93.58	180.71	174.03	83.92	91.59	90.11	63.75
—1	59.10	98.00	89.36	37.17	53.67	52.19	30.89
0	11.61	16.12	13.13	3.52	11.09	9.61	3.42
1	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
Beta-Bernoulli	11.61	16.03	12.92	3.48	10.93	9.45	3.36

DISCUSSION BY RICHARD I. FEIN AND JEFFREY T. LANGE

In their paper "Underwriting Individual Drivers: A Sequential Approach," Cozzolino and Freifelder have formulated a model of an aspect of the underwriting process. Like any mathematical model, it is an abstraction of reality and must omit many details of the actual process. This simplification may make some aspects of the model unrealistic; on the other hand, reducing one aspect of the process to the bare essentials does provide insights which would be otherwise obscured. As long as the reader recognizes the limitations inherent in the use of any model, the authors' technique is valuable in assessing risk selection rules and in evaluating merit rating schemes. In order to illustrate the practical value of what may appear to be only a theoretical exercise, we have developed two applications of the authors' model.

Their paper is particularly timely with the changes which are being brought about by No-Fault insurance. First, underwriters must consider changes in their risk selection criteria. Second, some rating techniques, such as the Safe Driver Insurance Plan (SDIP), are being changed or even eliminated by Insurance Departments in No-Fault states. The elimination of such rating tools would imply an adjustment in underwriting standards. If the authors' model can be adapted to describe a company's current underwriting actions (with regard to past driving record), then the parameters of the model may be adjusted to the No-Fault situation (e.g., revised claim frequency) so that alternative underwriting decision rules may be evaluated.

Before accepting the conclusion of a model, the reader must decide whether the underlying assumptions are valid for his situation and whether the mathematics have been correctly carried out. In our opinion, the latter condition is satisfied. The former condition must be examined by each reader before he proceeds to adapt the model to his situation. To aid the reader in gaining an understanding of the model, we have drawn a decision tree depicting the authors' example. (See Figure 1.) The "averaging-out" step, in Raiffa's[1] terminology, is simply the expected value of future returns; the "folding back" is the comparison to zero where the probabilities are from the appropriate negative binomial with the updated parameters.

[1] Raiffa and Schlaifer, *Applied Statistical Decision Theory*, The MIT Press, 1961.

The dotted line in Figure 1 separates the single year decision from the multiperiod adaptive decision, in this case based upon two years. In the former the "insure-don't insure" decision is based solely on the sign of the first year expected return. In this case it was negative and hence the risk is rejected.

In the multiperiod case we encounter a probability step with each branch corresponding to the probability of 0, 1, 2, . . . accidents given the parameters (a, b). At the start of the second year we encounter decision boxes which compare the "do not insure" decision with the one year expected value given the updated parameters based on the experience of the preceding year. In this case the multiperiod decision in the first year is to insure since the expected return in the second year exceeds the expected loss in the first year, thus yielding a positive two year expected value:

$$-1.48 + (.907)(5.52) + (.093)(0) = 3.53 > 0$$

The authors' illustration actually considered the three year adaptive decision although the content of the argument is essentially contained in the two year process.

We may now illustrate how the decision model may be used in a practical application: testing the consistency of underwriting rules with the underlying accident probabilities. We may visualize a kind of underwriting path such that if the tracing of the experience of the risk on the tree leaves the path the resulting decision is not to insure. In other words, the path goes through all of the points of profitable (multiperiod) expectations. This may be easily converted into a set of decision vectors which will contain the admissible experience for continued rating. In the case of the example, there is only one such vector, namely (0, 0), corresponding to no occurrences during each of the first two years. In general, for a horizon m, the set of admissible decision vectors would be of the form

$$\{(K_1, K_2, \ldots, K_{m-1}; K_i \geq 0, i = 1, 2, \ldots m-1\}$$

where

$$V_{m-i}(a+i, b+k, + \ldots + K_{m-i} > 0$$

The same kinds of decision rules could be obtained using $V_m(a,' b')$, that is, using the m year expected return at every step.

Such a decision rule can be thought of as an "underwriting rule" or risk selection criterion. In the simple example cited above, the resulting criterion was to insure if no accidents occur during the experience period but don't insure if there are accidents. (The example does not postulate the existence of a Safe Driver Insurance Plan, as may be the case in some No-Fault states.) Of course, a different choice of parameters would lead to a different, probably more complex, criterion.

As an illustration of a different parameter, one may observe that the ratio of the premium to the average claim cost has an influence on the decision process. In fact, the smaller the ratio, the more sensitive the profit function is to changes in the parameters (a, b). For example—if the average cost is reduced to \$400 (more representative of the physical damage line), it can be shown that it is profitable (from a multiperiod view) to insure the risk even if he has incurred two accidents. Since different parameters lead to significantly different decision (underwriting) rules, one might anticipate that the use of the model would challenge underwriting rules for certain classes.

Since the premium and average claim cost vary by line, the decision process should be evaluated by line and the algebraic sum of the expectations should be the determining criterion when the policy includes several lines. Certainly, a different set of parameter values for each line will be used for the same class of risks, further emphasizing the differences in the underlying processes.

In addition to determining underwriting criteria consistent with accident probabilities, the model may be used to check the logical consistency of premium charges resulting from a merit rating scheme. This is naturally dependent upon the appropriate reevaluation scheme employed. A scheme based on some chosen "marginal profit" defined below is possible. For example, using the well worked illustration and the profit function $\pi = P\text{-}Cn$, we may determine P so that V_3 (13.5, 1.37) = O. We call such a P the Marginal Premium *(MP)*. In this case *MP* is \$95. Suppose the first year passes, and no accidents occur. We may then determine the appropriate *MP* with respect to V_2(14.5, 1.37), which in this case is \$92; if based upon V_3(14.3, 1.37), the three year criterion, the *MP* is \$89. While here the difference is slight, it is conceivable that the difference may border on a competitive disadvantage and so the choice of reevaluation could be significant.

In the case that a single accident occurs, recall that using the "insure—don't insure" scheme, we would not insure the risk. We find the *MP* with respect to $V_3(14.5, 2.37)$ (note the updated parameters) to be \$155, an increase of 68%. However, after only one year of claim free experience, the premium drops to \$145, an increase of 53% over the base. A second year of claim free experience indicates a further reduction to a premium of \$137 or 44% above the base. The *MP* does not, in fact, return to the original value of \$95 after three years (compare SDIP) and it takes ten years of accident free experience to return to that level. Under SDIP, the premium remains constant for the three years after the accident, in contrast to the premium variation indicated by the model.

MP has an additional use—given several classifications, and the availability of negative binomial factors, one may determine the necessary differentials among the classes, based upon the ratios of the associated *MP's*.

Models are particularly useful in situations in which actual data are not available as is the present case with the introduction of No-Fault. However, further refinements of the model may be necessary to provide added realism. Even in its present form, we believe it provides insight into the consistency of certain underwriting criteria. In addition, it could be used to construct a theoretical test of surcharges under a merit rating scheme, such as the Safe Driver Insurance Plan.

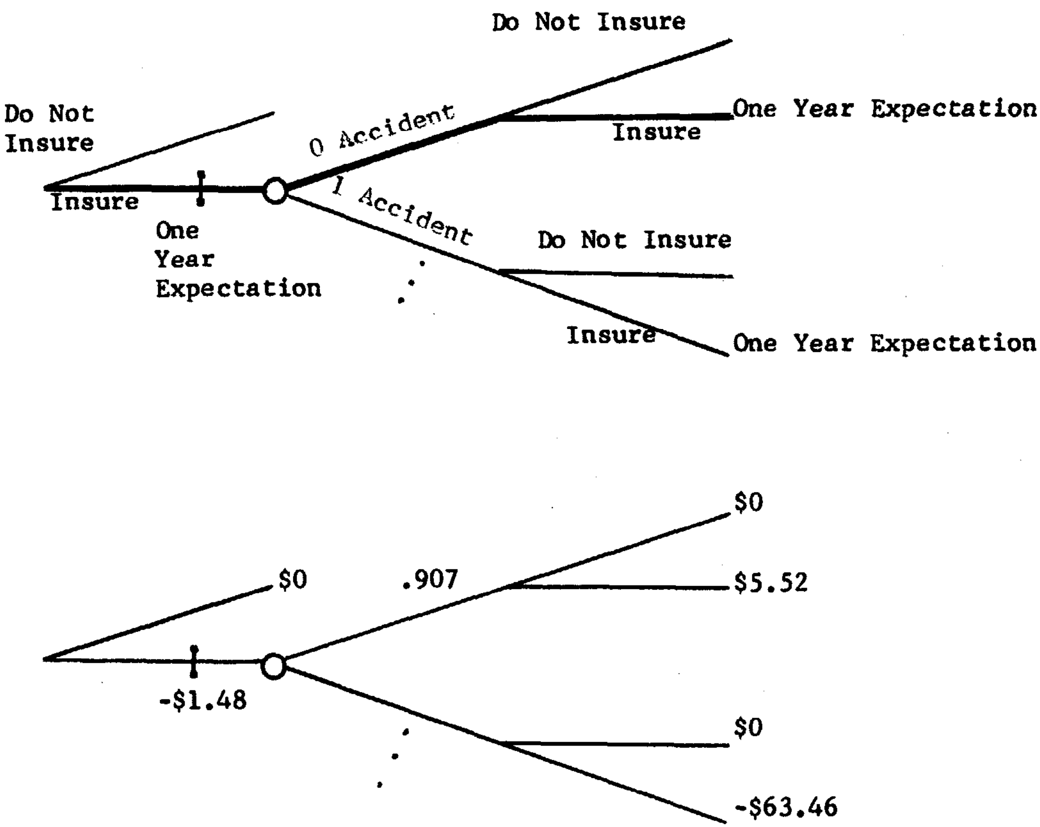
Do Not Insure
Do Not Insure
0 Accident
One Year Expectation
Insure
Insure
One Year Expectation
1 Accident
Do Not Insure
Insure
One Year Expectation
$0
$0
.907
$5.52
-$1.48
$0
-$63.46

FIGURE 1

DISCUSSION BY COSTANDY K. KHURY

Several readings of this paper made it eminently clear that a thorough study of the underlying bibliography is essential for a concise understanding of the proposed concept. In the process of studying Bierman and Hausman's article[1] (B&H) much of the initial gloss and promise which had originally stimulated my interest to thoroughly digest the paper dissolved and moved me to pen this discussion.

At least an assumption, a condition, and a constraint clearly spelled out by B&H are specifically either omitted, revised, or inaccurately reproduced without appropriate, and more importantly, necessary accounting by Cozzolino and Freifelder (C&F). In this manner much of the motivation for the effort as presented by C&F (based on bibliographic equivalence) is open to question. A few examples follow:

A. An Assumption Omitted: The multi-stage dynamic programming model as constructed by B&H specifically assumes that once a default occurs credit will (theoretically) never again be extended. C&F, in adapting the model to the underwriting decision, faithfully reproduce the model but without carrying the implicit assumption through that once an insured incurs a loss, insurance is no longer afforded at subsequent periods during the implied renewal horizon. In other words, given a particular choice of (a, b), there is an immediate implied assumption that there exists a period (horizon) such that if the risk incurs a loss within this period, then insurance will not again be afforded. The ultimate implications of this operating assumption appear to be at odds with the stated objectives of the paper. That is, strict application of this concept will "cause" profitable risks to be overlooked.

B. A Condition Revised: The paper (C&F) recognizes frequency as the sole objective criterion entering the construction of $V_m(a,b)$. B&H, on the other hand, do recognize the amount of credit to be granted in the construction of the model. C&F revised this necessary operating condition by suppressing the severity element by way of utilizing the mean expected loss cost value. The implications of this revision are tantamount to underwriting a risk at renewal time which has incurred a loss and reaching the same

[1] H. Bierman, Jr. and W. H. Hausman, "The Credit Granting Decision," *Management Science*, Vol. 16, No. 8 (1970)

yes/no underwriting decision whether the loss is a \$300 collision loss or a \$10,000 bodily injury liability verdict.

C. A Constraint Inaccurately Reproduced: The time value of money concept along with the lapse problem both tend to effectively diminish the actual derivable value of the proposed approach to evaluating the individual driving record. C&F present these factors as important considerations while the B&H construction, in fact, recognizes these elements only as incidental and that their impact "may be" included. C&F are ultimately strained in effectively incorporating the full impact of these elements in the final constructions.

These examples, I believe, illustrate the degree to which the stated adaptation of C&F departs from B&H vis-a-vis the "entire compatibility" claim as made in the paper.

It was not entirely clear at first, but upon close scrutiny several necessary delineations were missing such that the full intent of the authors is often in doubt, to wit:

A. C&F repeatedly speak of "good" and "bad" risks in an absolute sense. It is axiomatic to the practitioner that any risk may be protected (insured) against any hazard/peril at "some" price . . . not necessarily always affordable. Inasmuch as all the formulas and constructions utilize an implicit pure premium base, it is practically a mandate that the authors should delineate risks by way of a potential profitability standard instead of the phantom absolute good/bad standard.

B. The terms "claim" and "accident" are used interchangeably throughout. Is the reader to assume (accordingly) that culpability is not a factor in the underwriting process . . . or is accident involvement the only criterion?

The repeated lack of specificity does little to help the authors define the precise extent of the idea embraced by their effort.

The basic yes/no criterion, as spelled out in the paper, depends on the prospective validity of:

$$\text{Premium} \geq \text{EV (accident costs)}$$

with appropriate qualifications to both sides of the inequality. In this manner the size of Premium - (EV) is immaterial as long as it is non-negative. In other words, the size of the anticipated multi-period profit as compared to the capital and surplus required to support the corresponding written premium throughout the same multi-period is not a factor. This assumption is not realistic, and, the model [should] can be easily modified to accommodate a risk loading [ε] criterion such that:

$$\text{Premium} \geq \text{EV} + \varepsilon\,(\text{Premium})$$

The investor is more apt to view this approach more seriously to the extent that from his point of view elegance is not presumed to really matter!

The authors' stated objective is aimed at aiding the underwriting process. B&H distilled their conclusion, with full qualification, into a decision table which in turn eliminates the need for computation of the profit expectation for each case separately. Accordingly, perhaps the primary application with respect to insurance should be effected by the agent. In other words, a decision table would be produced for each classification such that the agent could look-up whether the prospect is eligible or not. In this manner administration of the proposed multi-stage model is essentially expense-free. Also, the decision table would naturally reflect the most recent pure premium level, thus preserving the various profitability criteria.

This particular arrangement may be difficult for the authors to accept in view of the particular demonstration outlined in the paper. For example, reviewing Section 3 of the paper, one can spot a "large" gap whereby the authors pass from (emphasis added):

> ". . .records of the <u>applicant's</u> rate <u>class</u> and <u>other factors</u>. . ." to:
>
> "The expected underwriting profit is negative and <u>this applicant</u> is rejected."

and thereby relegate the initial contribution of the driving record to "other factors." Later on (Section 5) the same gap appears where an *(a, b)* combination is produced presumably on the basis of such "other factors." The decision table approach proposed above would force the identification of these mysterious "other factors" and therefore demonstrate how the process initially takes classification attributes and "updates" them to include an individual's driving record. This is a most critical point which neither B&H nor C&F resolve.

Finally, one point which I could not reconcile in my mind. What is the impact of assuming a static pure premium at all branch points of the decision tree? It is not difficult to conjecture that if a risk is "surcharged" after an accident has occurred, then more risks would satisfy the basic yes/no criterion. This is one variation which, if incorporated in the model, will significantly alter the modus operandi and perhaps present yet another opportunity for further research.

DISCUSSIONS OF PAPERS PUBLISHED IN VOLUME LIX

HOW ADEQUATE ARE LOSS AND LOSS EXPENSE LIABILITIES?

RUTH SALZMANN
VOLUME LIX PAGE 1

DISCUSSION BY JOHN A.W. TRIST

Miss Salzmann in this paper has pinpointed an area of need that has been only partially met—that of providing regulatory authorities with a simple yardstick for evaluating the level of loss and loss expense liabilities. She proposes another and admittedly better yardstick but readily concedes its fallibility and notes some of its limitations. Matthew Rodermund has noted another, namely the fact that general acceptance might not be readily forthcoming because the expression

$$\frac{\text{Liabilities}_{12/31/n}}{\text{Adjusted Liabilities}_{12/31/n-1} + \text{Premium Earned}_n - \text{Losses Paid}_n}$$

cannot be verbalized. It can be described only in mathematical terms.

The fact remains, however, that the proposed yardstick does represent a significant improvement over those currently in use. It should be adopted as proposed. The criterion that 12/31 liabilities be accepted only if the ratio of these liabilities to the formula reserve base exceeds the lowest of the corresponding ratios for the most recent five years, might be tempered, however, with some concession to the trend of these ratios. It might be noted for example, that of the thirteen companies for which Miss Salzmann had calculated the adjusted loss and loss expense liability ratios, the ratios were clearly trending downward over the period from 1969 through 1971 for eight of the companies; they were trending upward for one company and indicated no clear pattern for the remaining four. Where a trend is discernable it does warrant recognition.

Having given us one new yardstick Miss Salzmann proceeds to give us another in which both minimum and maximum reserve requirements for current year end are generated from:

(a) A re-estimate of reserves at the beginning of the current calendar year in the light of developments during the year, and

(b) An estimate of the adjusted minimum and maximum loss and loss expense ratios for the current calendar year.

Perhaps more importantly, however, she suggests that the difference between the dollars of reserves generated from the high end of the loss and loss expense ratio range and the dollars of reserves actually carried might be used as a measure of the surplus-safety requirement to support the underwriting operation. This latter proposal appears to represent a significant improvement over the present arbitrary multiple of one year's premium writings.

One wonders though whether both the appropriate minimum and maximum reserve levels really couldn't have been obtained with a sufficient degree of accuracy without the introduction of the direct estimation of current calendar year adjusted loss and loss expense ratios. If for example, we calculate the range (arithmetic average $\pm 2\sigma$) of the percentages of Adjusted O/S to Formula Reserve Base (Item 6. Exhibit 2 in Miss Salzmann's paper) for the five years ending 1970, the range of liabilities thus generated for 12/31/71 would be \$160,002,000 – \$185,606,000 (derived from 75.86% ± 5.62%). The range generated in Miss Salzmann's paper was \$173,617,000 – \$192,814,000. Have we in fact gained very much at all with respect to the maximum reserve requirement by superimposing loss ratio on the initially proposed simple yardstick? Is the difference in this particular case significant? Is the size and direction of the difference in this particular case representative of the result that might be expected from a similar analysis for other companies in the industry? Probably not. It might be noted too that had the range chosen been the arithmetic mean $\pm 3\sigma$ rather than $\pm 2\sigma$, the result would have been \$168,817,000 – \$197,614,000. Which is more appropriate? What might the difference have been had we used other than five years of experience to determine the range or if we had eliminated altogether the experience of the most recent year since it would be the most undeveloped and unreliable?

Miss Salzmann's contributions in the area of loss and loss expense reserve determination, distribution and evaluation are reflected throughout the published records and property/casualty insurance operations of the past two decades. It has always been a pleasurable and rewarding experience to study the output of her incisive mind. This reviewer finds himself readily in agreement with her proposal for the introduction of the new yardstick presented in the early part of the paper. As an initial reaction he subscribes also to the concept of the maximum probable loss and loss expense reserve as a vehicle for determining minimum surplus requirements. He is not convinced, however, that this need be dependent upon the introduction of loss ratio into the estimation process. It is an area that should be explored in greater depth, by type and size of company for example, when a greater and more reliable volume of data becomes available under the currently constituted Schedules O and P.

JOINT UNDERWRITING AS A REINSURANCE PROBLEM

EMIL J. STRUG

VOLUME LIX PAGE 33

DISCUSSION BY WALDO A. STEVENS

Mr. Strug is to be commended for his fine effort in handling a very difficult reinsurance problem. The usual proportional reinsurance arrangement (on a coinsurance basis) is not applicable here due to the criterion that the Dental Plan maintain a contingency reserve, R_E, no less than 10% (W) of its portion of the annual incurred losses on premium accounts, $X \bullet L_p$, in order to preserve a reasonably acceptable (to potential clients and state insurance examiners) financial statement position.

In view of the fact that Mr. Strug's formula for the value of X, the percentage retention of the Dental Plan, is overburdened with numerous variables, the following stubstitutions will prove helpful in delineating the primary variables from those which may be considered, more or less, as parameters:

$$\begin{aligned} D_p &= f^D \bullet P_p & \qquad 0 < f^D < 1 \\ E_p &= f^E \bullet P_p & \qquad 0 < f^E < 1 \\ A_p &= k \bullet D_p = (k \bullet f^D) \bullet P_p & \qquad k > 0 \\ L_p &= Q \bullet P_p & \\ I &= f^R \bullet R_B & \end{aligned}$$

Since the cost of the servicing agreement between the Dental Plan and the Hospital Plan was set as a percent of premium, we may assume the value of f^E to be predetermined at the beginning of each calendar year (this may not be exactly true, since a contract year between insurer and reinsurer is not likely to coincide with a calendar year, but for illustrative purposes our assumption is not unreasonable). Hence, any favorable or unfavorable expense experience associated with E does not affect the value of X.

Because D is a conserative estimate of the direct expenses borne entirely by the Dental Plan, f^D may be considered as a constant for any given calendar year. A, the actual direct expenses incurred by the Dental Plan, is expressed as a percentage, 100k%, of D. Generally k lies somewhere between 0 and 1.00. By its nature, k must be determined at year-end.

Investment income, I, may be represented as a percentage, $100 f^R\%$, of the contingency reserve at the beginning of the calendar year, R_B. In essence, f^R represents an interest rate which must be determined at year-end. Normally, $0 < f^R < 1$, but it may be negative or greater than 1. Q, of course, represents the loss ratio.

The revised formula for X is:

$$X = \left[\frac{1}{Q \bullet (1+W) + f^D + f^E - 1}\right] \bullet \left[\frac{(1+f^R) \bullet R_B + U_C + P_P \bullet f^D \bullet (1-k)}{P_P}\right]$$

The beauty of this formula lies in the fact that it is much easier to isolate the key variables affecting the value of X. For any given calendar year we know in advance the values of W, f^D, f^E and R_B. Hence, the value of X may be expressed as a function of f^R, U_C, P_P, k and Q.

Since we are dealing with one year term insurance, and since f^R and k are reasonably stable from year to year, we may treat these variables as parameters.

An analysis of how fluctuations in the remaining variables affect X reveals that X is most sensitive to Q, moderately sensitive to P_P and generally insensitive to U_C (one wouldn't expect U_C to be very large in relation to P_P since all of U_C must come from favorable expense experience on the cost-plus business). What we find most interesting, though, is that X varies inversely to Q *and* P_P. Hence, if the loss ratio was unexpectedly large in a given calendar year, the retention of the Dental Plan would be less than if the loss ratio had been smaller. assuming all other factors were equal. Although from the viewpoint of the Hospital Service Plan this appears to be a "Heads you win, tails I lose" proposition, one must keep in mind that the loss ratio need not be unexpectedly large. A redundant premium per unit of exposure, though less competitive (and can competition be much of a factor in this market?) is of extreme importance, especially to the Hospital Service Plan.

In contrast, due to the fact that X varies inversely to P_P, the Hospital Plan actually gains if earned premium income increases (either via a rate per unit increase—assuming no significant loss in volume of business—or via a volume increase) and all other factors remain within reasonable range of expected values. This retention pattern helps to compensate, may even overcompensate, for the fact that the Hospital Service Plan stands to gain little and lose much under the first retention pattern (X decreases as Q increases). Thus, the price the Dental Plan pays for their protection is in terms of an exceedingly slow rate of growth.

In due time, as the volume of business increases and as the business on the books matures (loss ratios stabilize), the point may be reached where the Dental Service Plan has a year-end surplus which equals or exceeds 10% (W) of the incurred losses on all of the premium accounts business, in which case the Dental Plan's retention will reach 100%. However, the moment at which total recapture is achieved may be so far in the future that the reinsurance agreement becomes a losing proposition as far as the Dental Service Plan is concerned. As a possible solution, a time limit on the agreement could be resorted to; but, understandably, it would be extremely difficult, if not impossible, to determine an equitable limit based upon actuarial methods. A "gentleman's agreement" may be the best recourse.

AUTHOR'S REVIEW OF DISCUSSION

I concur with Mr. Stevens' comment that the formula as it appears in the paper is somewhat overburdened with numerous variables. Most of these variables were introduced to represent those monetary elements which could affect the operating results of the Dental Plan.

In developing the initial approach to this reinsurance problem, the basic formula contained only those elements which were critical in establishing whether the sharing between the two corporations was equitable and feasible.

The first pass at the formula presumed no income from investment (I) or a gain from any cost plus (U_c) operation. In addition, actual direct expense (A_p) was set equal to expected direct expense (D_p). A further simplification is possible as the Direct Expense (D_p) and Indirect Expense (E_p) allowances in the rates are known. In our example, these were set at $.18\,P_p$. W is also known and was set at .10.

Introducing these simplifications into the formula for X produces the following:

$$X = \frac{R_B}{Lp\,(1.10) + .18P_p - P_p} = \frac{R_B}{1.10\,L_p - .82\,P_p}$$

A further simplification can be accomplished by letting Q equal the loss ratio. The equation becomes

$$X = \frac{R_B}{1.10\,P_pQ - .82\,P_p} = \frac{R_B}{P_p\,(1.10Q - .82)}$$

This simplification is, at this point, essentially that developed by Mr. Stevens and also highlights the sensitivity of the results to the loss ratio and the premium volume as well as the fact that X varies inversely to Q and P_p.

AN ACTUARIAL NOTE ON EXPERIENCE RATING NUCLEAR PROPERTY INSURANCE

RICHARD D. McCLURE

VOLUME LIX PAGE 150

DISCUSSION BY ROBERT L. HURLEY*

There may possibly be some danger that in his impatience to push right into the subject and offer a few thoughts of his own thereon, a reviewer might occasionally be unmindful of his responsibility first to identify the author's argument and evaluate it objectively. With this prime duty, however, once discharged, the reviewer should be free to comment as his conscience dictates and the indulgence of the reader may be expected to reasonably allow.

Our Society is singularly indebted to Mr. McClure for his papers on nuclear energy insurance. His previous paper[1] chronicled for the future student the first decade of the insurance industry's truly remarkable response to the economic and social challenges posed by the peace-time use of nuclear energy. The current McClure paper treats of the insurance industry's plan to relate its prices for nuclear energy property insurance to the developing loss and expense experience.

This "Actuarial Note on Experience Rating Nuclear Property Insurance" presents the background leading to development of the rating plan by the Actuarial Subcommittee of the Nuclear Insurance Rating Bureau; various of the particulars considered by the Subcommittee; the details of the resulting plan and a specific example of its application. Mr. McClure has again contributed a comprehensive and easily readable document and a definitive paper to the CAS *Proceedings*.

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* The relevancy of E. J. Gumbel's *Statistics of Extremes* published in 1958 by Columbia University Press to the actuarial problem of "rare events" was suggested by Thaddeus L. Smith, ISO Mathematical Statistician, but the reviewer reserves as his own any faults in the application thereof.

[1] R. D. McClure, "A review of Nuclear Energy Insurance", *PCAS* Volume LV, (1968) pp. 255-294.

Experience rating of atomic energy losses—would not any such proposal be unthinkable to the earlier generation of casualty actuaries, the Rubinows, the Flynns, the Mobrays? But in all fairness, let us not demean the imaginative skills or the ability of our predecessors to come up with solutions to pressing social and economic problems.

The implications of the horseless carriage, insurance-wise, must have been unfathomable to all but the very few. Who else could have guessed the impact of the social significance of the first workmen's compensation laws under whose shadow, or better promise, our Society was founded and has since flourished? Where otherwise could one have turned to satisfy the needs for aircraft liability and property insurance?

And in stride, the industry has successfully coped with problems presented by hurricanes, earthquakes, flood and social unrest. Why should the atomic energy hazard prove the first instance in which we fail?

With remarkable candor, Mr. McClure draws the reader's attention to the considerations that were given to the practices in other property insurance fields in the process of formulating rating standards for nuclear property insurance. He notes that it was decided to treat the first $5 million of any nuclear property insurance loss as normal experience with the equivalent of 50% weight in the overall rate level, although there was some evidence that losses of such magnitude might reasonably be expected to account for a significantly larger portion of commercial fire losses.

And the selection of the 20 year review period for excess losses was made not without advertence to the then operative critera for extended coverage insurance. The credibility formula, $P/(P+K)$, adopted for excess nuclear property insurance has long been a hallowed tradition for many casualty and property lines stemming from the early researches in our Society.

It has sometimes been observed that no great misfortune would likely stem from giving some modest credibility, say 10%, to an individual account's experience. And in this instance of nuclear property damage insurance, the credibility is not being applied to a single account for a relatively brief time interval, but rather to all accounts, over all states, for a not insignificant period of years.

We need not possess any profound discernment to appreciate the potential vexations of the business assured who has paid considerable premiums, year after year, and witnessed substantial underwriting margins with no resulting reduction in his rates. Such an insurance purchaser recognizes the prime importance of financial stability, but he often expects that continuing favorable underwriting results should ultimately be reflected in rate levels.

It is probably doubtful that the property insurance industry can yet come up with solutions to the credibility problem of rare events that will, at the same time, prove workable for the practical situations confronting us and also satisfy the rigorous standards customarily required by statistical theory. While there has been no flagging in the energy with which scholars have pursued their research, it is not unlikely that Gumbel's 1958 work still serves as benchmark against which all subsequent theoretical contributions are measured on the very difficult questions of the significance to be attached to the occurrence of rate events in the sense of large losses.

In the introductory sections of his magnum opus, which represents a veritable lifetime of intellectual devotion to the subject, Gumbel starts with the Intensity Function from which he develops the Distribution of Repeated Occurrences. The latter is not unrelated to the Pascal or Geometric Distribution which has underscored certain developments of considerable significance to Casualty Actuaries as may be typified by the Dropkin paper[2] concerning the use of the Negative Binomial and other equally distinguished research by the various outstanding CAS pioneers in actuarial theory.

Gumbel presents the Intensity Function as

$$M(x) = \frac{f(x)\,dx}{[1 - F(x)]} \geq f(x)dx$$

where the probability of a value "equal to" or "larger than" x is $[1 - F(x)]$ and $f(x)\,dx$ is the probability of a value between x and $(x + dx)$ and cites as a reference the work of one of our past Presidents, L. H. Longley-Cook.

Gumbel then proceeds to develop the distribution of repeated occurrence of any Large Value x. He starts with q the probability of any occur-

[2] L. B. Dropkin, "Some Considerations on Automobile Rating Systems Utilizing Individual Driving Records", *PCAS* Vol. XLVI, p. 165.

rence smaller than the Large Value x: $q = (1 - p) = F(x)$. Observations are made regularly until the occurrence of a value "equal to" or "larger than" x. The chance that the Large Value x occurs at the vth trial equals pq^{v-1} and decreases as v increases and has a moment generating function $G_v(t) = p/(e^{-t} - q)$ which can be shown as follows:

$$G_v(t) = \sum_1^{\infty} pq^{v-1} \cdot e^{vt} = \frac{p}{q} \sum_1^{\infty} q^v \cdot e^{vt}, \text{ and}$$

$$\sum_{v=1}^{\infty} q^v e^{vt} = (qe^t + q^2e^{2t} + q^3e^{3t} + \cdots)$$

$$= qe^t(1 + qe^t + q^2e^{2t} + \cdots) = qe^t(1 - qe^t)^{-1}$$

$$\text{thus: } G_v(t) = \frac{p}{q}\left(\frac{qe^t}{1 - qe^t}\right)$$

$$= \frac{pe^t}{1 - qe^t} = \frac{p}{(e^{-t} - q)}, \text{ the mean } \bar{v} = \frac{1}{p}$$

which Gumbel identifies as the Return Period $T(x) = \dfrac{1}{[1 - F(x)]} > 1.$

Thus if in any given year a nuclear energy catastrophe equal to or greater than a given magnitude is 2%, the Return Period is 50 years. Thus, Return Period is the reciprocal of the chance of a loss of a given size.

Students of actuarial mathematics, and even occasional dabblers therein like this reviewer, will recognize the similarity of Gumbel's development with Pascal-Fermat "Problem of Points" as detailed in the Probability Chapter in Hall and Knight[3]. This approach also parallels the development of the Geometric Series which, as Feller shows[4], has a variance equal to $\dfrac{q}{p^2}$.

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[3] H. S. Hall and S. R. Knight, *Higher Algebra* (MacMillan and Co., London).

[4] W. Feller, *An Introduction to Probability Theory* (John Wiley & Sons, Inc., New York), Vol. I.

And using Gumbel's relationships above: $q = (1 - p) = F(x)$ and $T(x) = \dfrac{1}{[1 - F(x)]}$ we develop a Standard Deviation σ for the Return Period $\sqrt{T^2 - T}$ as follows:

$$\sigma^2 = \frac{q}{p^2} = \frac{F(x)}{[1 - F(x)]^2} = \left[\frac{T-1}{T}\right] \cdot \left[\frac{1}{T^{-2}}\right] = (T)(T-1)$$

$$\therefore \sigma = \sqrt{T^2 - T}$$

Gumbel notes that the cumulative probability of the event "at" or "before" the *vth* trial is $W(v) = (1 - q^v)$, or in terms of the Return Period, T (i.e. equals $[1 - q]^{-1}$):

$$W(v) = 1 - \left(\frac{T-1}{T}\right)^v = 1 - \left(1 - \frac{1}{T}\right)^v$$

If x is large and p is small, T will be large and $W(v)$ may be written $(1 - e^{-\frac{v}{t}})$

$$\underset{T\to\infty}{\text{limit}}\, W(\mu T) = \left[1 - \underset{T\to\infty}{\text{limit}} \left(1 - \frac{1}{T}\right)^{\mu T}\right] = (1 - e^{-\mu})$$

Hence, the approximation for large T

$$W(T\lambda) - W(T/\lambda) = e^{-1/\lambda} - e^{-\lambda}$$

If we wish to select a $P = 0.9545$ which for the normal curve corresponds to an interval of 2 sigmas about the mean, v would be expected to fall within an interval equal approximately to .05 to 21.5 times the return period. And if the expected annual probability of a $25 million nuclear property damage loss were 2 percent, the range within which such an event would occur would be 2.5 years to 1,075 years.

While the thoughtful reader will likely concede that more than a modicum of genius and scholarship must have been exercised in devising these forecasting techniques, he may wonder how such findings might be used to establish actuarially based rates for a "rare event" insurance commitment. In fairness to these scientists, it must be recognized that their prime interest gravitated toward pure research into mathematical techniques. They had not been charged with the responsibility of developing actuarial based insurance rates. It would be most unfortunate, however, if the above observations were to be construed that such mathematical research might be dismissed cavalierly as of no concern for us. This would be misreading of our intent.

Possibly a fair appreciation of some of the limitations in our presently developed mathematical forecasting techniques might be obtained from an August, 1954 article in the *Proceedings of the American Society of Civil Engineers,*[5] by H. Alden Foster who has been mentioned as one of the pioneers into mathematical flood forecasting techniques. After admitting that an efficient forecasting service would be helpful in a flood insurance program, Foster stated that he had some doubts as to how much reliance could be placed on present methods. He then proceeded to cite ten different types of floods, some of which would be difficult, or even impossible, to forecast from a probability standpoint.

In the October, 1961 *Journal of the Boston Society of Civil Engineers,* the report of the Committee on Floods cites a number of the statistical formulas which have variously been used for computing the recurrence intervals of floods. While the New England flood records extend back over a respectably long period of years (*i.e.,* over 325 years in some instances), the reader may well get the impression that the predictability of large floods must not be considered amenable to the same statistical precision as in the case of the small flood traceable to less uncommon meteorological conditions.

And it may be somewhat awesome to learn that if we had 10,000 years of records, we could expect 100 large floods, but if these 10,000 years were divided into 100 centuries each, on the average 37 of these centuries would have no such flood, 37 would have one, about 18 of the centuries would experience two floods, 6 would have three, and about 2 of the centuries would have four or more large floods. But it would be impossible to predict into which of these centuries each of these frequencies would occur!

It may seem to the reader that we have reached an impasse. The establishment of any actuarial based system of rates, it has been observed, demands a credible volume of statistics designed for the specific underwriting enterprise with advertence to the possible classification breakdowns for significant differential in loss expectancies.

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[5] Volume 80, Separate No. 483.

It is not always possible to launch a new insurance enterprise with a system of detailed classification rates substantiated by credible statistical experience. But it is a common experience in insurance, as well as in other commercial and industrial enterprises, to set a system of prices (or rates) on a judgement evaluation of all available information, including whatever statistics that may be helpful to the purpose, and then to adjust the rating schedule as the subsequent experience indicates revisions are warranted.

We are indebted to Mr. McClure for keeping us abreast of current developments in nuclear energy insurance and for affording us an insight into the first glimmerings of sound actuarial rating techniques for nuclear energy property losses. Let us hope that the Society will be equally fortunate in that the next generation of actuaries will have an equally competent and experienced chronicler to record the insurance industry's further contributions to sound actuarial rating of nuclear energy property insurance.

THE RELATIONSHIP BETWEEN NET PREMIUM WRITTEN AND POLICYHOLDERS' SURPLUS

RAYMOND W. BECKMAN AND ROBERT N. TREMELLING II

VOLUME LIX PAGE 203

DISCUSSION BY G. L. COUNTRYMAN

Mr. Beckman and Mr. Tremelling conclude from historical evidence that the premium—surplus relationship is unstable and does not show long-term trends. They further conclude that policyholders' surplus has been the volatile element of the ratio primarily because of fluctuations in the stock market.

The conclusion that the relationship has been historically unstable requires some sort of criterion against which stability can be compared. That is, stability—and instability—are relative concepts, and become meaningful only when viewed in the context of some standard or norm. No stability criterion is suggested by the authors.

Characterizing the premium—surplus ratio as unstable may be inappropriate. A review of the historical data presented in Exhibit I of the authors' paper shows that the ratio has varied from about .8 to about 1.7 between 1928 and 1971. In the context of Mr. Thomas Morrill's statement in the June 1970 report on profitability and investment income to the National Association of Insurance Commissioners cited in the authors' study, the variations in the ratio seem quite modest. Mr. Morrill states there is a rule of thumb which sets ". . . $2.00 of premium written for each dollar of surplus as conservative, three or four dollars of premium as safe, but beyond that caution should be observed." Thus, the ratio has fluctuated within very narrow limits when compared to the range of adequacy suggested by Mr. Morrill.

Mr. Morrill's statement raises another issue. If a 2/1 ratio is conservative, why have stock companies historically operated at lower ratios? The authors do not address themselves to this issue.

In controversy with the authors' conclusions, the historical data also suggests a modest long-term trend. It would appear there is a gradual increase in the premium—surplus ratio under way. If the mid-1940's is used as a dividing line, it seems clear that the relationship has drifted upward and may be, on the basis of the most recent evidence, shifting markedly upward.

This tentative conclusion is reinforced by the authors themselves. By forecasting premium writings and policyholders' surplus separately, the authors conclude that by 1976 the relationship may go as high as 2.1 and exceed 2.5 by 1980. If the authors' simple extrapolations are at all valid, it seems that the relationship is expressing a trend that is long-term in character and represents a marked departure from history. The authors fail to identify this experience as a long-term trend and offer very little rationale as to why the relationship is moving towards entirely new levels.

One possible rationale for this upward trend in the premium—surplus relationship may be a more widespread recognition that the Industry is over-capitalized. This rationale leads to what I believe is the more interesting question, which is not what the premium—surplus ratio is, but what the relationship ought to be.

What the relationship ought to be is of interest to many. It is of interest to investors and financial managers because it is a measure of financial leverage and is therefore important in forecasting the level and stability of returns on funds committed to the enterprise. It is of interest to regulators because they are obligated to monitor and assess companies' financial strength. Further, it is of increasing concern to regulators in their attempts to measure profitability. In their report on *Measurement of Profitability—Property and Liability Insurance,* regulators in the New York Insurance Department recognize that premium—surplus ratios play an important role in assessing a firm's profitability.[1] They do not suggest what the relationship should be, but recognize its importance.

The authors do not evaluate what the relationship ought to be. They only attempt a forecast based upon current trends in premium writings and surplus.

Beyond the normal hazards of forecasting from historical evidence, there is the danger that such forecasts will focus on the wrong sources of change. In the authors' case, forecasts are made almost as though premium and surplus levels are influenced only by factors that are beyond the con-

[1] This report was prepared by New York Insurance Department staff and was presented at a hearing in December 1972. Reference is made to the importance of the premium—surplus ratio on pp. 11—14 of their report.

trol of insurance company managements. It is likely over the long term, particularly for the individual company, that premium—surplus relationships are largely within the control of company management. After all, management has the means to loosely control premium growth. Moreover, through pricing, underwriting, investment, and dividend policies, company managements can also, to a degree, control surplus growth. To the extent both premium and surplus levels are within the control of company managements, future premium—surplus relationship will be more responsive to what management perceives the relationship ought to be and less responsive to investment and underwriting uncertainties.

To suggest that management can control the premium—surplus relationship is about the same thing as saying that management can control the firm's capital structure. Theoretically, the optimum capital structure, assuming favorable long-run underwriting results, is a maximum amount of leverage subject to the constraint of adequate surplus necessary to assure solvency. Saying it another way and ignoring the earnings instability arising from leverage, the optimum capital structure will be that which provides the highest premium—surplus ratio but which still assures solvency.[2] What the optimizing ratio ought to be is a difficult question. Certainly it will be different for different companies under various circumstances. A less ambitious question is simply to ask whether or not it is likely stock companies are currently operating at generally optimizing premium—surplus levels.

One way to look at the problem is to asses the contributions to surplus and demands on surplus in a static state under conditions of maximum adversity. During any one annual period there is a nearly certain expectation that contributions will be made to surplus from fixed income securities held and derived from both reserves and surplus. On the other hand demands will be made on surplus arising out of capital losses and unprofitable underwriting performance. Further, premium growth during the period will also make demands on surplus because of gross unearned premium reserving.

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[2] Professor Ferrari, in his paper entitled, "The Relationship of Underwriting, Investment, Leverage, and Exposure to Total Return on Owner's Equity", *PCAS,* Vol. LV, 1968, pp. 295—302, suggests that variability in the earnings effects of leverage maximization may modify this notion of the optimal capital structure. Professor Ferrari points out that stable earnings will be capitalized at a higher value than unstable earnings. Thus, evaluation of optimum capital structures should not only consider leverage but also the earnings instability it imposes.

The amount of surplus needed during one annual period can be given by the simple expression:

$$S = ER(P - P/G) + I_{cl} + U - I_p - I_s$$

Where:

S = policyholders' surplus
E = prepaid expense ratio
R = unearned premium reserve ratio
P = premium
G = premium growth rate
I_{cl} = unrealized capital loss
I_p = net investment income derived from underwriting operations
I_s = net investment income derived from policyholders' surplus
U = statutory underwriting loss

Let's assume some very adverse circumstances:

1. An underwriting loss equal to 15 percentage points of premium.
2. A decline in market value equal to 25 percent of the equity securities held.
3. Premium growth during the annual period of 10 percent.
4. Interest income from fixed interest securities held and arising out of underwriting operations equal to .03 of premium.
5. Interest income arising out of fixed interest securities associated with surplus equal to .025 of surplus.

Since Best's Aggregates and Averages shows that companies' investment in equity securities are about equal to surplus, it is possible to express all of the values in the above equation in terms of either surplus or premium:[3]

[3] Best's Aggregates and Averages shows that during the most recent five-year period the value of equity securities held for all stock companies combined have averaged about 1.08 of policyholders' surplus with very little variation from year to year.

$$
\begin{aligned}
E &= .3 \\
R &= .5 \\
G &= 1.1 \\
I_{cl} &= .25S \\
I_p &= .03P \\
I_s &= .025S \\
U &= .15P
\end{aligned}
$$

By substitution the equation yields:

$$
\begin{aligned}
S &= .172P \\
\text{or} \quad P &= 5.8S
\end{aligned}
$$

If the assumptions are conservative approximations of maximum adversity, it seems possible stock companies are not optimizing their premium—surplus relationships and hence their capital structures.

Of course, the static case has substantial limitations. For example, it is possible that a series of annual periods could collectively produce surplus demands exceeding those illustrated in the static case. To assess this possibility what is needed is an exhaustive analysis of optimal capital structures using a stochastic model of ruin.

The static case is presented here only for purposes of raising serious questions about whether or not the industry is generally optimizing its capital structure. It is not intended to suggest that 5/1 or 6/1 is the right relationship for all companies or for any individual company. I agree with the authors that each company must be considered separately. It would seem, however, that the static case does demonstrate the possibility of over-capitalization.

This possibility should be of interest to regulators, industry management, investors, and policyholders alike. If it is true, financial managers and investors have an opportunity to enhance returns on funds committed to the enterprise. Moreover, regulators and policyholders need not be alarmed if there is a gradual increase in the premium—surplus relationship. In theory, increased leverage will yield higher returns to investors thereby stimulating capacity and causing insurance services to be provided at lower cost.

In the last part of their paper, the authors attempt to show why the premium—surplus relationship ". . . is not completely accepted, cannot be consistently applied, and in several respects is illogical." The authors' arguments may be summarized as follows:

1. Calculations of premium—surplus relationships are inaccurate for an individual company which is a member of a group.
2. The appropriate premium—surplus relationship is dependent upon the nature of the firm's operations and its historical performance.
3. It is difficult to obtain a satisfactory premium—surplus relationship that satisfies all interested parties because stockholders will prefer a high premium—surplus relationship while policyholders and regulators will prefer a low ratio.
4. Different relationships will apply to stock and mutual companies.

I generally agree with these arguments. However, they do not suggest to me that the premium—surplus relationship is not useful. Rather, they only suggest that the relationship should be calculated and applied with care and discretion. I believe the relationship is particularly useful to assess, as I have pointed out, whether or not the Industry is generally over-capitalized.

Finally, I believe the authors have presented an interesting review of the whole subject of premium—surplus relationships and think they have made a valuable contribution in setting the subject into historical perspective. I hope the authors' paper will stimulate actuarial research into this important area of insurance companies' financial structures.

DISCUSSION BY DAVID J. GRADY

Messrs. Beckman and Tremelling have addressed themselves to a question which is of fundamental importance to the insurance industry. The determination of the appropriate relationship between net written premium and policyholders' surplus could provide a key to the problems of pricing, profitability and capacity.

The authors provide a brief summary of the current rules-of-thumb by which regulatory authorities test the adequacy of policyholders' surplus. They point out that the formulas employed today are not the result of recent intensive studies but are merely liberalizations of the two-for-one rule developed nearly forty years ago. These modifications of the original formula assume that the two-for-one rule is soundly based but that in some unspecified way, aside from the fact that it appears to be too stringent, it is inappropriate for current needs. Since this rule is the foundation for the prevailing regulatory procedures, I should like to reinforce the authors' conclusions concerning its applicability with some additional comments.

The two-for-one rule was introduced in the late 1930's by Roger Kenney who was at that time Chief Examiner for the New York Insurance Department. "After making a thorough study of the difficulties into which certain casualty companies had fallen in the early 1930's—difficulties brought on by sharply mounting loss ratios accompanied by an equally sharp decline in the securities markets—he came to the conclusion that in the great majority of cases the venturesome area was entered when a company's premium volume began to exceed $2.00 for every dollar of policyholders' surplus, including any 'free' (or general contingency) reserves."[1]

Although the details of this important study do not appear to be readily available, certain conclusions emerge from Mr. Kenney's statements concerning it.

(1) The approach was a negative one. Apparently, Mr. Kenney's study consisted of examining only insolvent companies, noting a common characteristic of a "great majority of cases", and deducing a law for application to the insurance industry in general. The statistical and logical fallacies of such a procedure are obvious; however, it is sufficient to emphasize that no effort was made to investigate the surplus needs of soundly-managed, solvent companies. In fact, when it was pointed out that a number of reputable companies remained solvent in spite of having been in "violation of the tenets of the Kenney Theory" for decades, Mr. Kenney's response was, "It was more by the grace of God than by any good judgment on their part that they survived at all!"[2]

(2) No attempt was made to relate surplus levels to the standard variables which create the need for surplus, although Mr. Kenney makes frequent appeal to them.

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[1] Roger Kenney, *Fundamentals of Fire and Casualty Insurance Strength* (The Kenney Insurance Studies, Dedham, Massachusetts, 1967), fourth edition, p. 97.

[2] *Ibid.*, page 28.

(3) The study postulated written premium to be the measure of insurer's risk. Although risk may be implicit in the stochastic nature of the pricing process, it is by no means identical with written premium. In essence, written premium is the sum of expected losses (with appropriate loadings for expenses, profit and contingencies) while insurer's risk is generally defined to be adverse deviation from these expected values.

(4) The study did not attempt to distinguish among different types of insurers nor among insurers with vastly differing portfolios. Even superficial analysis reveals that an insurance portfolio consisting of property damage liability coverage on private passenger automobiles should require far less surplus than a group of product liability policies producing the same premium volume.

Since the current premium-to-surplus ratios are merely extensions of this early formula, they have fallen heir to each of the faults of the original study. The major virtue of the variety of premium-to-surplus formulas in use today is that they are simple to apply. Their chief disadvantage is that they are only tenuously related to the actual problem.

I believe that development of the other major conclusion advanced by Messrs. Beckman and Tremelling may be summarized as follows:

(1) Standard and Poor's 500 Stock Index is highly correlated with the total policyholders' surplus for all stock companies combined. Therefore, this stock index is an excellent predictor of future levels of policyholders' surplus.

(2) "The single series, Standard and Poor's 500 Stock year-end closing average, explains 64% of the annual variation (i.e., yearly percent change) in policyholders' surplus."

(3) "Risk (i.e., the variation in rate of return) from insurance operations is minimal when compared to the risk resulting from stockmarket appreciation or depreciation."

(4) The stock market is the major factor affecting policyholders' surplus. Therefore, the major portion of the insurer's risk (investment risk) "could be minimized by eliminating investments in the stock market and investing all assets into bonds."

At first glance this conclusion seems entirely reasonable. However, its possible consequences for the insurance industry are somewhat alarming. These potential effects include far stricter regulation of investments, denial of the right to participate in the stock market, and reduction of the profit and contingencies allowance in the premium dollar to more appropriately reflect the "relatively minimal" underwriting risk. Therefore, I would like to comment briefly on the methodology which the authors use to achieve this conclusion.

(1) The authors indicate that the correlation between Standard and Poor's 500 Stock Index and policyholders' surplus is 98%. Chart I provides a graph of the regression line and a visual confirmation of this apparently remarkable correlation. However, the two sets of data under consideration are actually time series containing distinct autocorrelation. The 98% correlation is somewhat less striking when contrasted with the relatively simple relationship between surplus and the passage of time: the correlation between policyholders' surplus and calendar year is 94%.

(2) The contention that Standard and Poor's 500 Stock Index explains 64% of the annual variation in policyholders' surplus appears to be the result of calculating R^2, the sum of squares due to regression divided by the total sum of squares corrected for the mean. Although the authors have satisfied the book definition of R^2, their casual use of the word "explains" may cause the unwary reader to believe that a deterministic relationship exists between stock market results and policyholders' surplus. Actually, when the word "explains" is used with respect to the fit of a statistical regression model, it is subject to a very narrow technical definition. Regression analysis may reveal association between two variables, but association may be due to a variety of causes. In particular, association may be the result of factors which act jointly on the correlated variables. For example, both the stock market and policyholders' surplus have been subject to considerable growth over time, and they are both influenced adversely by the occurrence of natural or economic catastrophes. In general, simple correlation is not sufficient to prove causation.

An analysis of this data is hindered even further by its heterogeneity. The problem of heterogeneity may be illustrated by an investigation of the following argument:

(a) At least a portion of policyholders' surplus may be considered to be directly invested in the stock market.

(b) The volatility of the market has a direct influence on this portion of policyholders' surplus.

(c) Regression analysis shows that 64% of the annual variation in the total policyholders' surplus may be explained by a leading market indicator.

(d) Therefore, 64% represents the impact which direct investment in the stock market has had on the volatility of total policyholders' surplus.

Apart from the previously indicated weaknesses in this argument, it leads to serious consideration of the portion of policyholders' surplus devoted to common stock. Exhibit I summarizes the commitment of the policyholders' surplus to the stock market by the stock insurance industry over the past 25 years. Exhibit II indicates the percentage of policyholders' surplus invested in common stock of a type similar to that comprising the S&P Index (Industrials, Utilities and Railroads) for the same time period. The exhibits reveal a dramatically increasing commitment of policyholders' surplus to the stock market. This varying participation in the market destroys the homogeneity of the data and introduces a third variable which must be tested. How much has R^2 been strengthened by the increasing commitment of increasing surplus to an increasingly volatile market?

(3) Risk is generally defined to be adverse deviation from expected values. The authors' definition of risk as "variation in rate of return" not only allows them to include several sizeable investment *gains* in their computations but to use the squares of these favorable deviations as contributions to the investment risk.

(4) The industry data used by the authors to support their conclusion contains an unfortunate bias. The law of large numbers tends to act on the pooled results of several hundred stock companies in a manner which gives a somewhat diminished view of underwriting risk.

Although the preceding four points have led me to question whether the stock market is the major factor affecting policyholders' surplus, it is unquestionably a major factor. Certainly, the optimal employment of the assets underlying policyholders' surplus is dependent not only upon risk but upon expected return.

In conclusion I agree with the authors that the myriad of interacting variables makes the problem of policyholders' surplus enormously complex. Unfortunately, premium-to-surplus ratios are easy to condemn but difficult to replace. The authors are to be commended for their pioneering paper on a fascinating and fundamental subject.

CHART I
RELATIONSHIP BETWEEN POLICYHOLDERS' SURPLUS AND THE STANDARD & POOR'S INDEX

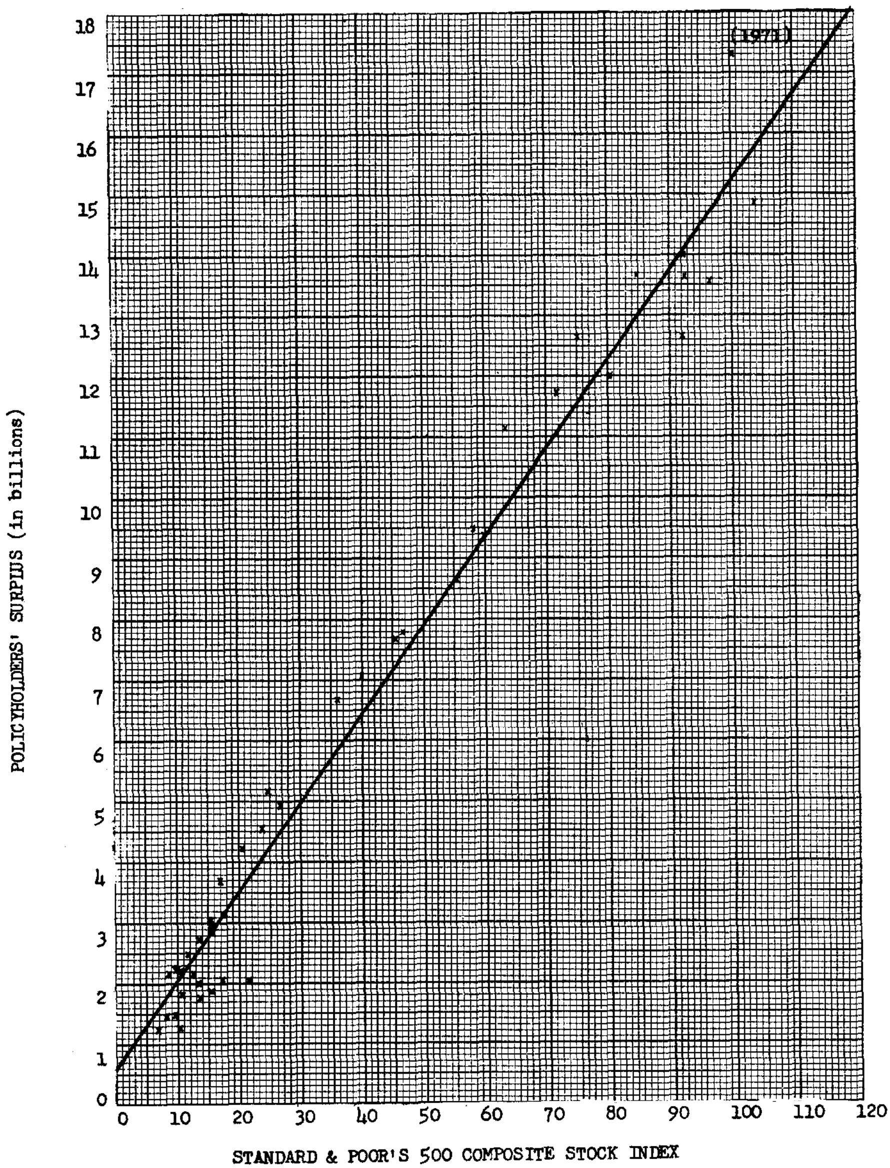


EXHIBIT I
STOCK INSURANCE INDUSTRY PARTICIPATION IN COMMON AND PREFERRED STOCKS
(Amounts in thousands of dollars)

		Admitted Assets in Common Stock		Admitted Assets in Preferred Stock		Admitted Assets in Common and Preferred Stock	
Year	Policyholders' Surplus	Amount	% of Surplus	Amount	% of Surplus	Amount	% of Surplus
1947	2,904,943	1,673,656	57.6	546,606	18.8	2,220,262	76.4
1948	3,066,252	1,718,626	56.0	537,936	17.5	2,256,562	73.6
1949	3,707,539	2,157,148	58.2	591,825	16.0	2,748,973	74.1
1950	4,216,861	2,570,988	61.0	615,454	14.6	3,186,442	75.6
1951	4,542,504	2,919,785	64.3	650,255	14.3	3,570,040	78.6
1952	4,963,904	3,230,418	65.1	715,896	14.4	3,946,314	79.5
1953	5,191,529	3,307,838	63.7	751,154	14.5	4,058,992	78.2
1954	6,697,464	4,589,309	68.5	791,773	11.8	5,381,082	80.3
1955	7,693,594	5,479,911	71.2	791,670	10.3	6,271,581	81.5
1956	7,800,261	5,798,328	74.3	707,684	9.1	6,506,012	83.4
1957	7,073,013	5,257,042	74.3	718,895	10.2	5,975,937	84.5
1958	8,619,370	6,772,003	78.6	709,345	8.2	7,481,348	86.8
1959	9,381,140	7,480,660	79.7	700,091	7.5	8,180,751	87.2
1960	9,494,889	7,631,322	80.4	681,514	7.2	8,312,836	87.6
1961	11,719,406	9,769,815	83.4	661,968	5.6	10,431,783	89.0
1962	11,146,292	9,120,573	81.8	694,770	6.2	9,815,343	88.1
1963	12,642,213	10,709,980	84.7	717,257	5.7	11,427,237	90.4
1964	13,690,544	12,014,739	87.8	733,832	5.4	12,748,571	93.1
1965	13,659,762	12,345,297	90.4	798,523	5.8	13,143,820	96.2
1966	12,006,722	10,952,508	91.2	833,593	6.9	11,786,101	98.2
1967	13,580,010	12,843,063	94.6	936,639	6.9	13,779,702	101.5
1968	14,886,618	14,318,753	96.2	1,050,194	7.1	15,368,947	103.2
1969	12,698,941	13,076,170	103.0	988,968	7.8	14,065,138	110.8
1970	14,014,350	13,653,545	97.4	1,234,258	8.8	14,887,803	106.2
1971	17,308,207	17,188,251	99.3	1,565,616	9.0	18,753,867	108.4
Average % Participation, 1947-1970			77.7		10.0		87.7
Average % Participation, 1947-1971			78.5		10.0		88.5

Source: Best's Aggregates & Averages, 1948-1972

EXHIBIT II
STOCK INSURANCE INDUSTRY PARTICIPATION IN COMMON STOCKS REPRESENTATIVE OF STANDARD & POOR'S 500 COMPOSITE STOCK INDEX
(Amounts in thousands of dollars)

Year	Policyholders' Surplus	Admitted Assets Invested in Common Stock of: Railroads	Utilities	Miscellaneous	Total	Percentage of Policyholders' Surplus
1947	2,904,943	59,502	132,661	767,493	959,656	33.0
1948	3,066,252	61,847	150,247	780,242	992,336	32.4
1949	3,707,539	58,517	250,895	957,894	1,267,306	34.2
1950	4,216,861	72,601	295,039	1,264,723	1,632,363	38.7
1951	4,542,504	67,334	375,423	1,447,398	1,890,155	41.6
1952	4,963,904	86,078	471,799	1,542,193	2,100,070	42.3
1953	5,191,529	77,291	536,383	1,487,964	2,101,638	40.5
1954	6,697,464	108,092	714,941	2,108,676	2,931,709	43.8
1955	7,693,594	118,924	815,782	2,687,060	3,621,766	47.1
1956	7,800,261	111,247	860,353	2,846,089	3,817,689	48.9
1957	7,073,013	76,559	896,324	2,461,387	3,434,270	48.6
1958	8,619,370	92,102	1,230,592	3,332,358	4,655,052	54.0
1959	9,381,140	78,116	1,269,857	3,649,494	4,997,467	53.3
1960	9,494,889	70,343	1,517,477	3,613,260	5,201,080	54.8
1961	11,719,406	76,121	1,923,186	4,504,045	6,503,352	55.5
1962	11,146,292	74,256	1,865,507	3,993,683	5,933,446	53.2
1963	12,642,213	90,469	2,011,528	4,804,167	6,906,164	54.6
1964	13,690,544	97,858	2,270,932	5,575,821	7,944,611	58.0
1965	13,659,762	107,030	2,198,618	6,230,966	8,536,614	62.5
1966	12,006,722	93,540	2,047,510	5,567,071	7,708,121	64.2
1967	13,580,010	103,892	2,037,292	7,124,219	9,265,403	68.2
1968	14,886,618	114,520	2,087,785	7,892,931	10,095,236	67.8
1969	12,698,941	65,753	1,611,400	7,378,017	9,055,170	71.3
1970	14,014,350	66,293	1,643,471	7,106,353	8,816,117	62.9
1971	17,308,207	77,334	1,748,015	9,385,074	11,210,423	64.8

Average Percentage Participation, 1947-1970 51.3

Average Percentage Participation, 1947-1971 51.8

Source: Best's Aggregates & Averages, 1948-1972

DISCUSSION BY ROGER C. WADE

The paper recently presented by Beckman and Tremelling has opened the door on a variety of questions which have not been previously discussed in the *Proceedings*. This review of that paper addresses some of the more interesting questions which were left unanswered. There is no attempt made here to retill the ground covered by Beckman and Tremelling.

The primary thrust of this review is to give some quantitative indications of the amount of surplus needed by a property-liability insurance company.

There are three basic terms which are necessary to discuss this topic.

The first term is surplus which will simply be defined as the excess of assets over liabilities. In statutory accounting this is equivalent to Surplus as Regards Policyholders.[1] If Generally Accepted Accounting Principles (GAAP) are used, then a higher surplus will result on a pretax basis, primarily due to the inclusion of the so called equity in the unearned premium reserve which results from the treatment of prepaid acquisition expense as an asset under GAAP. In this review statutory accounting will be used.

The second term is solvency. The two conditions of solvency are having assets greater than liabilities and being able to meet obligations as they fall due. In insurance operations, the former condition is the one which is most likely to be violated as large amounts of cash usually can be generated to meet obligations.

The third term is solidity. This is a term of comparatively recent vintage[2] that is not readily definable in accounting terms. It refers to the probability of a company remaining solvent over some specified future period of time. This concept has been extensively explored mathematically[3] and

[1] NAIC Fire and Casualty Annual Statement Blank, p. 3, Line 27.

[2] Kimball, Spencer L., "The Purpose of Insurance Regulation", *Minnesota Law Review*, Volume 45 (1961) p. 471.

[3] Seal, Hilary L., *Stochastic Theory of a Risk Business* (John Wiley and Sons, New York, 1969).
Takaes, Lajor, *Combinatorial Methods in the Theory of Stochastic Processes* (John Wiley and Sons, New York, 1967).

is sometimes referred to as the theory of ruin. The theory of ruin is not used in this review because of its complexity and because it was felt to offer little practical advantage over the simpler approach used here.

Calculations for determining the amount of surplus needed by an individual company to establish solidity should take into consideration a variety of factors. The assumptions for the hypothetical firm used here are:

1. Geographical spread is countrywide.
2. Product line mix is the same as the industry average.
3. Reinsurance arrangements are designed such that for the given firm the underwriting results will vary in approximately the same manner as the industry average.
4. The firm's underwriting profitability has been average for a large multi-line insurer.
5. a) The portfolio composition contains the maximum amount of common stock permitted by the New York State law.

   b) The portfolio is an all bond portfolio.

Results are shown for several confidence levels as the amount of surplus should be based on a management decision concerning the amount of risk they are willing to accept as a company. There is no absolute "required" surplus and there is no level of surplus which will guarantee future solvency under all conditions. These confidence levels are based on the assumption that underwriting and investment results are independent and normally distributed. Thus, once a mean and standard deviation have been calculated from historical data it is a simple calculation to determine the probability of any given outcome. The 50% confidence level should be interpreted as the expected outcome. If a −100% change in surplus is indicated for a given confidence level, then it means that insolvency would occur. In Exhibit I, at a 10 to 1 surplus ratio with a common stock portfolio, there is a 1% chance that a greater than 110% decrease in surplus will take place and a 99% chance that less than a 110% decrease in surplus will take place.

Calculations are also shown for varying periods of time. The reason for not showing results beyond a three year period is that the long-term expected outcome for the stock market is positive and thus it is very unlikely that market results will be negative four years in a row. Even if such an outcome were to occur there would be sufficient lead time for management action to counter the adverse trends in surplus. Common stock data was obtained from *Rates of Return on Investment in Common Stock; the Year by Year Record,* 1926-1965 by L. Fisher and J. H. Lorie. Loss Ratio data was used from a large, countrywide, multiple-line insurer.

One of the more interesting results of this table is that if a 2 to 1 surplus ratio is maintained with a common stock portfolio, the apparent risk is greater and the rewards less than with a i0 to 1 surplus ratio and an all bond portfolio.

There are two characteristics of statutory accounting and GAAP which, if altered, could have a significant effect on the results shown here. First, reserves must be maintained which are sufficient to liquidate all outstanding claims on an ultimate value basis. This is in contrast to an approach which permits a present valuing of reserves. Second, bonds are valued on an amortized rather than a market value basis. This eliminates fluctuations in the valuation of bonds. While the propriety of these nethods is open to question, it is assumed that they will both continue to be valid in the future.

The above calculations are merely an indication of the type of work which remains to be done in this area and we can thank Beckman and Tremelling for broaching this subject in the *Proceedings*.

EXHIBIT 1

Maximum Adverse % Change in Surplus in a One Year Period

Surplus and Confidence Level	Portfolio Type: Common Stock	Portfolio Type: All Bond
1 to 1 Surplus Ratio		
50% confidence level	16%	7%
90% confidence level	− 8	4
99% confidence level	− 27	1
99.9% confidence level	− 41	− 1
2 to 1 Surplus Ratio		
50% confidence level	21%	9%
90% confidence level	− 10	3
99% confidence level	− 36	− 2
99.9% confidence level	− 54	− 6
4 to 1 Surplus Ratio		
50% confidence level	30%	13%
90% confidence level	− 16	1
99% confidence level	− 55	− 9
99.9% confidence level	− 82	− 16
10 to 1 Surplus Ratio		
50% confidence level	60%	25%
90% confidence level	− 34	− 5
99% confidence level	−110	− 30
99.9% confidence level	−165	− 47

EXHIBIT 2

Maximum Adverse % Change in Surplus in a Two Year Period

Surplus and Confidence Level	Portfolio Type: Common Stock	Portfolio Type: All Bond
1 to 1 Surplus Ratio		
50% confidence level	32%	14%
90% confidence level	− 2	9
99% confidence level	− 29	6
99.9% confidence level	− 49	3
2 to 1 Surplus Ratio		
50% confidence level	42%	18%
90% confidence level	− 2	9
99% confidence level	− 38	2
99.9% confidence level	− 64	− 3
4 to 1 Surplus Ratio		
50% confidence level	61%	26%
90% confidence level	− 6	9
99% confidence level	− 60	− 5
99.9% confidence level	− 99	− 15
10 to 1 Surplus Ratio		
50% confidence level	119%	50%
90% confidence level	− 13	8
99% confidence level	−121	− 27
99.9% confidence level	−198	− 52

EXHIBIT 3

Maximum Adverse % Change in Surplus in a Three Year Period

Surplus and Confidence Level	Portfolio Types: Common Stock	Portfolio Types: All Bond
1 to 1 Surplus Ratio		
50% confidence level	48%	21%
90% confidence level	7	15
99% confidence level	− 3	11
99.9% confidence level	− 51	8
2 to 1 Surplus Ratio		
50% confidence level	63%	26%
90% confidence level	9	16
99% confidence level	− 35	8
99.9% confidence level	− 67	2
4 to 1 Surplus Ratio		
50% confidence level	91%	39%
90% confidence level	10	18
99% confidence level	− 56	1
99.9% confidence level	−104	− 12
10 to 1 Surplus Ratio		
50% confidence level	179%	75%
90% confidence level	17	23
99% confidence level	−115	− 19
99.9% confidence level	−210	− 50

NO-SPLIT EXPERIENCE RATING PLANS

JOHN P. WELCH

VOLUME LIX PAGE 156

DISCUSSION BY LESTER B. DROPKIN

Although it is clear that Mr. Welch's primary concern is with the experience rating plans in use in the third party lines, he has chosen to use as one main vehicle for his discussion the no-split workmen's compensation experience rating plan of Pennsylvania. This then allows him to compare and contrast the workmen's compensation experience rating plan with the other plans. Having introduced the Pennsylvania workmen's compensation plan, however, he is also led to give some consideration to the more commonly used multi-split workmen's compensation experience rating plans. Accordingly, at one point or another, either explicitly or implicitly, the paper touches on virtually every aspect of experience rating.

It appears to this reviewer that the net result of all of this is that the paper is marked by a certain unevenness of treatment. It is almost as though Mr. Welch could not quite bring the contemplated reader of his paper into focus. Intermingled throughout, we are given some history, some theory, some data, some analysis, some practical considerations and some conclusions.

Yet, there is something else which we are given also; *viz.*, a very properly taken point of view which challenges us to define objectives, and to explicate the criteria by which success in meeting those objectives may be measured.

Let us now look at the author's loss ratio variance test. It is well to remember at the outset, however, that anyone who attempts to analyze the specific results being produced by a given workmen's compensation experience rating plan will be faced with a general problem, whatever his particular approach might be. Because all workmen's compensation experience rating plans contain a provision for combination of entities, the experience modifications are computed on the basis of the risk's combined experience. On the other hand, loss ratios are normally available on a unit report basis only. That is, if the modification applicable to a particular policy also applies to other policies of the risk, ideally one should combine the several policies to form just one risk loss ratio. Although we recognize the practical difficulties involved, if this is not done some distortion will result.

Another possible source of distortion, although probably minor, given broad premium size groupings, arises if the assignment to premium size group is done on the basis of standard premium rather than manual premium. The author does not state the basis for his assignment, but we suspect that it may have been on the basis of standard premium.

While we do not know how great an effect the foregoing might have had on the results shown in Exhibit III-A, and Exhibits A and B of the Appendix, we are nevertheless willing to assume for the purpose of further discussion that the results would have been substantially the same. We do, however, seriously question whether simply comparing the standard deviations of the loss ratio distributions before and after the application of the experience modifications is quite correct,. We would suggest, rather, that a more proper statistic for comparing the loss ratio distributions is the coefficient of variation.

If we compute the coefficients of variation from the data set forth in Exhibits A and B of the Appendix, the following interesting result appears:

Policy Size Group	Coefficient of Variation*		
	(1) Manual	(2) Standard	(3) Ratio (2) ÷ (1)
1	4.42	4.28	.968
2	3.68	3.67	.997
3	3.03	3.01	.993
4	2.29	2.23	.974
5	1.70	1.66	.976
6	.95	.91	.958
7	.69	.66	.957
8	.78	.44	.564
All	1.61	1.43	.888

*Weighted Standard Deviation ÷ Loss Ratio

It is immediately seen that there is a reduction for each of the size groups, as well as in total. Moreover, there is a greater proportional reduction as size increases. Indeed, for the largest policies (size Group 8), the reduction is quite large.

Regardless of whether the experience rating data are analyzed by means of standard deviations or coefficients of variation, it is most important to stress the necessity of taking into account the sampling distribution of individual risk loss ratios over time, particularly with respect to the smaller size risks. While Mr. Welch, of course, is aware of this, and although he refers to it in the final sentences of the section wherein his test is presented, we would suggest that until a detailed analysis is made of the question, and the impact measured, it is really premature to attempt to draw conclusions.

Related to this is the question of whether or not it is correct to set the adjustment of "each risk's loss ratio to the average" as an objective of experience rating. Actually, the question cannot be answered unless it is posed in a much sharper form. Is Mr. Welch suggesting that the objective is a standard premium loss ratio at the average for each risk, each year? We hardly think so. Consider, for example, the smaller risks which typically have no losses for several years running. Such a risk normally has had no losses during the experience period, produced a credit modification, and a zero loss ratio for the year during which the modification applies.

There are apparently two alternatives open to us. If we wish to concentrate on each risk, then the objective has to be set in some such terms as: estimating the expected losses of the risk; "expected losses" being used here in its statistical sense, and, as such, implying an average over time. Alternatively, if we wish to concentrate on a particular year, we would have to set the objective in terms of average results for a group of risks, with the criteria being based on something other than a comparison of the manual and standard loss ratio distributions. Perhaps the only risks for which it might be meaningful to consider an "each risk, each year" approach are the very large ones, where the variance of the sampling distribution over time is presumably minimal. But even in this case one would surely want to investigate actual distributions first.

Before closing this review, I would like to briefly mention another approach to testing how well a given experience rating plan is operating. This consists of making a comparison of the manual premium loss ratios for a given period with the modifications which applied during that period. As the modifications increase, the loss ratios should show a concomitant increase. As an example of this kind of comparison, there is set forth below California experience for policy year 1968, 3rd reports.

Modification Interval	No. of Reports	Average Modification	Manual Premium Loss Ratio
Below .75	666	.608	.3653
.76— .85	6398	.813	.4140
.86— .90	11380	.881	.4719
.91— .95	10667	.929	.5040
.96— .99	5027	.975	.5219
Sub-Total (Credits)	34138	.824	.4453
1.00—1.04	6591	1.018	.6034
1.05—1.14	7710	1.091	.6654
1.15—1.34	5923	1.216	.7620
1.35 and over	1895	1.508	.9453
Sub-Total (Debits)	22119	1.145	.7020

Experience rating is, or should be, a subject of considerable interest to all members of the Society. It is a complex and many-faceted subject. Its many interrelating aspects must be studied from both a theoretical and practical point of view. We therefore welcome the stimulus to further thought provided by Mr. Welch.

ACTUARIAL APPLICATIONS IN CATASTROPHE REINSURANCE

LEROY J. SIMON

VOLUME LIX PAGE 196

DISCUSSION BY DAVID G. HALMSTAD

Actuarial literature is filled with technical studies of varying complexity, but few of these emphasize the importance of consistency in the actuary's work. On the life insurance side, few actuaries would directly recognize that consistency in the results is perhaps the most important reason for using the traditional life insurance actuarial model. All of them should agree that the process of system control afforded by that model—the multi-faceted analysis of expense, mortality, lapse, investment and the role of contractual provision in both the estimation of earnings and the setting of dividends—relies heavily on the use of studies of these variables defined consistently with their use in the model itself. Most life actuaries might even agree that such consistency is a prerequisite for "equity." But I doubt that they would take one additional step with me to hold consistency to be more important than the precise level of any of the individual parameters. Yet I think that this is the case, and that it is exemplified by the case that Mr. Simon has chosen for this paper.

Mr. Simon's case is given by simple illustrations, the sign of a classic paper. In the illustrations used, an insurance is analyzed for which one acceptable price (dependent on a precise definition of the coverage being provided) is given. The problem considered is simply one of finding the consistent costs of alternative definitions of coverage.

To complement Mr. Simon's artful exposition of the relationships between catastrophe coverages of various design, I would like to examine an additional form which I have had to analyze recently. I hope to reinforce Mr. Simon's plea that the actuary can help discern the relative differences amongst alternative designs of such products.

In direct analogy to Mr. Simon's cases, let us consider a catastrophe cover that will cost the insured a pure premium of $\underline{P}$, a lower bound on the premium, if no event occurs, but a maximum premium during the period of $\overline{P}$ ($> \underline{P}$) if the cover has to be paid. This would be a trivial extension if we restricted ourselves to a single possible covered event [equivalent to providing a cover without reinstatement of $L - (\overline{P} - \underline{P})$ when $L > \overline{P}$], but we shall extend the cover "drastically" by providing the cover of L on every such event during the insured period for the maximum total premium of $\overline{P}$.

By making this change, I may well be violating every usual definition of "catastrophe reinsurance," but I do plan to offer my own such definition in due course. Dropping the limitation on the number of events covered may be unsound underwriting in some cases, but there are frequently sound reasons for the actuary to consider the case even if the underwriter imposes such limitations. And in specific cases it will be infeasible to impose such limits on either the number of events or the total possible amount of claims that may be paid.

Allowance for all possible numbers of events during a period involves relatively small changes in premiums for small numbers of expected events. For example, if Mr. Simon had added, as another example, the determination of the premium for an unrestricted cover in his example A, he would have proceeded, with $m = .09553$, to get the pure premium for an unrestricted number of events, $m \times 9 = .85979$, and then to a gross premium (using the same 18% of gross for margins) of $.85979 \div .82 = 1.0485$ (of $1,000,000.) The relative size of making the assumption of coverage for all events in this example, under 5%, is probably less than the uncertainty the underwriter has in the underlying probability of having such an event.

The case I am suggesting for analysis in this review, allowing an unlimited number of events to be covered, with a fixed amount L payable on each event, develops quite naturally from the large claim portion of liability forms in which premiums are defined on a European "swing rate" basis.

When used for non-catastrophe forms, the swing rate structure simply specifies a minimum and maximum premium with a self-insurance role played in those periods when total claims lie between the two extreme premiums. In an American version of swing rates, an expense allowance is paid to the insurer/reinsurer through a guaranteed loss ratio which is less than 1 in the central "self-insurance" band.[1] I shall consider only the simpler and more drastic European form.

[1] In effect, if the year's claims are C and the guaranteed loss ratio is R, the cost of the cover is C ÷ R, subject to minimum and maximum limits of $\underline{P}$ and $\overline{P}$, respectively.

Most coverages offered on such a swing rate basis are on contingencies where the expected losses are believed to be reasonably manageable with the possible exception of rare but spectacular disasters. Thus in the analysis of such coverages, one is faced with the mixture of two components in the risk: one reasonably stable and amenable to standard statistical and actuarial analysis, the second one which may properly be treated as a catastrophe cover. I shall assume hereafter that the premium analysis for the stable portion has been completed and that analysis of the catastrophe portion of the swing rate premium remains.

In such a catastrophe portion, when we limit the claim amount to a single level L, we have (continuing to use Mr. Simon's notation):

$$\text{Expected Losses} = \sum_{c=0}^{\infty} P_c \times cL, \text{ and}$$

$$\text{Expected Premiums} = \sum_{c=0}^{\infty} P_c \times \max(\underline{P}, \min\{\overline{P}, cL\}).$$

After equating these expressions, and dropping the similar central terms from both sides, we find that

$$\sum_{c=0}^{[P \div L]} p_c(\underline{P} - cL) = \sum_{c=1+[\overline{P} \div L]}^{\infty} p_c(cL - \overline{P}), \tag{1}$$

where [•] denotes the lower integer operator (i.e., [x] is the greatest integer less than or equal to x). Expression (1) states the obvious: those cases in which apparent profits occur (left side) must balance those where losses occur (right side).

The question that we are asked in this situation is quite similar to those of consistency explored by Mr. Simon. Since we generally know what probability (and pure premium) should be assigned to the catastrophe element of the cover, we are interested in determining, for a given $\underline{P}$ and $\overline{P}$ given to us, the level equivalent P (or Lm). That is, we wish to find a simple relation between the given set $\underline{P}$, $\overline{P}$ and L, and the logically equivalent P.

The answer is surprisingly simple when $L > \overline{P}$. In this case, which I feel should be made part of the definition of a catastrophe coverage, equation (1) becomes

$$p_0\underline{P} = \sum_{1}^{\infty} p_c(cL - P) = p_0\overline{P} + \sum_{0}^{\infty} p_c(cL - \overline{P})$$
$$= p_0\overline{P} + Lm - \overline{P} = p_0\overline{P} + P - \overline{P}, \text{ or}$$
$$p_0 = \frac{\overline{P} - P}{\overline{P} - \underline{P}} \qquad (2)$$

In the specific case when claims are Poisson, this becomes $e^{-m} = (\overline{P} - P) \div (\overline{P} - \underline{P})$ and, appealing once more to the level claim assumption, we get

$$\text{(Poisson)} \quad e^{-P \div L} = \frac{\overline{P} - P}{\overline{P} - \underline{P}}. \qquad (3)$$

It should be reemphasized that equations (2) and (3) are valid only when $L > \overline{P}$. In the general unit claim swing rate case represented by equation (1), the numerical solution is a bit trickier, but not impossible.

In Appendix A, we illustrate the use of an APL system designed to solve equation (1). Lines beginning in column 7 were typed by the system user; computer responses usually start in column 1. The system responds to inquiries about its status regarding three variables: *TYPE,* the distribution for numbers of events, *MEAN,* the expected number of events, and *SIZE,* the (fixed) claim size for each event, as well as resetting these variables when the user desires and it solves four forms of queries of it:

UPPER FOR	gets $\overline{P}$ for a given $\underline{P}$ (and a global[2] L and $P = mL$),
LOWER FOR	gets $\underline{P}$ for a given $\overline{P}$ (and a global L and $P = mL$),
LEVEL FOR	gets P for a set $\underline{P}$ and $\overline{P}$ (and a global L), and
N FOR	which is essentially the same as *LEVEL FOR.*

A word should be given about the algorithms used to solve equation (1). The applicable programs are given in Appendix B, along with the routines used to handle the Poisson distribution. Lines 7 and 8 (for *LOWER*) and 9 and 10 (for *UPPER*) in the program *<FOR>* solve (1)

[2] A global parameter is one which is essential to the numerical calculation of a function but which is not specifically set in the function definition. Good examples are the specific values for the interest rates and mortality rates in life insurance and annuity functions such as A_x and $\ddot{a}_x$.

for a $\underline{P}$ (or $\overline{P}$) for each possible integer of the limit of c, and take that one which is consistent with $[\underline{P} \div L]$ (or $[\overline{P} \div L]$) for the needed limit.

Solution of (1) for a level P, given $\overline{P}$ and $\underline{P}$ is not as simple in the general case. The expected number of events, m, is not known (the program <*FOR*> ignores *MEAN* in this case) and the solution is accomplished by a Newton approximation (to the root of the line 6 of <*FUNC*>). An equivalent method for expressing the level P is given by *N FOR* which returns the N in

$$P = \underline{P} + (\overline{P} - \underline{P}) \div N. \tag{4}$$

This form is an easier one to communicate to underwriters, partially since, for swing rate coverages, they frequently use the "One Third" rule expressable in (4) by setting $N = 3$. The last request shown in Appendix A is for an N of this form when the swing rate limits are 8 and 14 on a cover of 10 per event, assuming a negative binomial claim number distribution with $k = 1$ (i.e. a geometric distribution).[3] The "One Third" rule is quite accurate in this case.

We have studied the "One Third" rule as it applies to catastrophe coverage on a swing rate basis in the Poisson case. As I indicated previously, I would suggest that a catastrophe coverage be (at least in part) defined as one on which the insurer cannot, from his entire portfolio of business, expect sufficient premium to cover a single claim during the period of insurance. This at least serves as a definition of the conditions under which the use of the "One Third" rule is questionable.

In Appendix C, we present the APL algorithm used to solve the simultaneous equations (3) and (4) for N, given $\underline{P}$ and $\overline{P}$. The algorithm includes the function <*WEW*> which solves the transcendental equation

---

[3] The APL function name for the negative binomial case is NEGBINOMIAL and it is followed, in the Appendix A demonstration by the parameter k which is left free after the mean m has been set. That is, the probability of exactly c claims is, in the NEGBINOMIAL k case,

$$\mathrm{Pc} = \binom{k + c - 1}{k - 1} \left(\frac{m}{k}\right)^{c} \left(1 + \frac{m}{k}\right)^{-(k + c)}$$

$$c = 0, 1, 2, \ldots$$

The system provides for assumption of any of the following distributions for the number of claims: Poisson, binomial, negative binomial, geometric, logarithmic, discrete Pareto, single point and harmonic.

$x = w \times e^w$ for w given a value $x > -e^{-1}$. Using these routines, we have, finally, constructed a contour map for finding a conservative N (giving an understated P) in the Poisson catastrophe reinsurance case. It is presented in Appendix D. The ratio $\underline{P} \div \overline{P}$ is the horizontal axis of this map, and $(\overline{P} \div L) < 1$ the vertical axis. The next integer higher than the true solution for N is shown by odd integers where applicable, with blank bands for even values of N. No values are shown in the area of $N \geq 10$, and it is suggested that $\underline{P}$ be considered as the equivalent value of P in such a case.

I would like to make one final addition that bears directly on the examples presented both by Mr. Simon and myself. All of the "consistency" calculations should be made on a pure (or net) premium basis. Mr. Simon has made this much more explicit than I, but he has neglected to include a marginal ingredient which I believe to be as important as the tax, commission and profit margins: the risk theory fluctuation loading. For catastrophe coverages, this element is both logically includeable and reasonably simple to calculate, especially when the number of possible claims is limited.

APPENDIX A

```
        ⍝ A SAMPLE SESSION WITH THE SWING RATE PACKAGE
        )LOAD SWING
SAVED 22.43.56 06/22/73
             AVIATION REINSURANCE SWING RATES
        STATE
TYPE IS UNDEFINED
MEAN IS  1
SIZE IS  1
PURE EXPECTED CLAIMS = MEAN×SIZE =  1
        UPPER FOR .9
WHAT TYPE OF DISTRIBUTION FOR NUMBERS OF CLAIMS?
⎕:
        POISSON
1.139221119117733
        )DIGITS 6
WAS 16
        LEVEL FOR .9 1.13922
1
        LOWER FOR 1.25
0.82043
        NEGBINOMIAL 2
WAS POISSON
        LEVEL FOR .9 1.13922
0.991011
        MEAN IS .5
WAS  1
        STATE
TYPE IS NEGBINOMIAL  2
MEAN IS  0.5
SIZE IS  1
PURE EXPECTED CLAIMS = MEAN×SIZE =  0.5
        UPPER FOR .3
0.855556
        LOWER FOR .75
0.359375
        NEGBINOMIAL 3
WAS NEGBINOMIAL  2
        LOWER FOR .75
0.353009
        GEOMETRIC
WAS NEGBINOMIAL  3
        LOWER FOR .75
0.375
        NEGBINOMIAL 1
WAS GEOMETRIC
        LOWER FOR .75
0.375
        MEAN IS 10
WAS  0.5
        UPPER FOR 8
13.8794
        LEVEL FOR 8 14
10.04
        N FOR 8 14
2.94111
```

```
      )WSID
SWING
      )GRP SWING
UPPER   LOWER   LEVEL   N       FOR     TYPE    MEAN    SIZE    IS      M       L       TYPE
PTCALC  MLRESET STATE   STATUS  RESET   SWINGAUX        NEWTON  FUNC    POISSON BINOMIAL
NEGBINOMIAL     GEOMETRIC       LOG     DPARETO SINGLE  HARMONIC        ZETA    LNGAM   POIST
BINOMT  NEGBINT GEOMT   LOGT    DPARETOT        HARMONICT       EXPECTED
```

```
      ∇FOR[⎕]∇
    ∇ Z←X FOR Y;PU
[1]   →(0=ρρTYPE)↑OK
[2]   0 0 ρ⎕,0ρ⎕←'WHAT TYPE OF DISTRIBUTION FOR NUMBERS OF
      CLAIMS?'
[3]  OK:→((⌈/,Y÷L)<ρP)/Z←X↓UPPER,LOWER,LEVEL,LEVEL
[4]   T←, 1 0 ↓P←M PTCALC⌈⌈/,Y÷L
[5]   P←, ¯1 0 ↓P
[6]   →Z
[7]  LOWER:Z←(L×+\P×T)+(L×M-+/P×T×Z)-Y×1-+/P×Z←Y≥T×L
[8]   →0,ρZ←(Y≥L×M)×(T=⌊Z÷L)/Z←Z÷+\P
[9]  UPPER:Z←(L×M-+\P×T)-+/P×0⌈Y-T×L
[10]  →(0≠ρZ←(Y≤M×L)×(T=⌊Z÷L)/Z←Z÷(Z≤0)+1-+\P)↑0
[11]  →(M=+/P×T)↑0×ρZ←'IMPOSSIBLE'
[12]  T←, 1 0 ↓P←M PTCALC 2×ρP
[13]  →UPPER,ρP←, ¯1 0 ↓P
[14] LEVEL:→(1=ρZ←,Y)↑FUNC←0
[15]  Z←L× 50 1E¯5 1E¯5 NEWTON PU←¯1↑Y←(>/Y)⌽Y÷L
[16]  →(X≠3)↑0
[17]  Z←(-/Y)÷(1↑Y)-Z÷L
    ∇
```

```
      ∇FUNC[⎕]∇
    ∇ Z←FUNC X;P;T
[1]   →FUNC↓FA,FB
[2]  FB:→((X>0)∧X<1)↓0×Z←1+|X
[3]   →0,ρZ←M+X÷(1-X)×⍟1-X
[4]  FA:T←(P←X PTCALC⌊PU)[1+Z←''ρι1;]
[5]   P←P[Z;]
[6]   Z←X-(PU×1-+/P)++/P×T⌈1↑Y
    ∇
```

```
      ∇POISSON[⎕]∇
    ∇ Z←POISSON
[1]   Z←'WAS',2↓TYPE
[2]   TYPE←0
[3]   M MLRESET L
    ∇
```

```
∇PTCALC[⎕]∇
    ∇ Z←M PTCALC MAX;P;T
[1]   T←(-ι1)+ι1+MAX
[2]   →TYPE↓POISSON,BINOMIAL,NEGBIN,GEOM,LOG,
      DPARETO,SINGLE,HARMONIC
[3]  POISSON:→E,ρP←T POIST M
[4]  BINOMIAL:→E,ρP←T BINOMT(M÷P),P
[5]  NEGBIN:→E,ρP←T NEGBINT K,M
[6]  GEOM:→E,ρP←T GEOMT M
[7]  LOG:→E,ρP←T LOGT M
[8]  DPARETO:→E,ρP←T DPARETOT B
[9]  SINGLE:→E,P←(ρT)↑| 1 0 -P
[10] HARMONIC:→E,P←T HARMONICT B
[11] E:Z←(Z/P),[¯0.5+ι1](Z←P≠0)/T
    ∇
```

```
      ∇POIST[⎕]∇

    ∇ Z←K POIST MEAN;KT
[1]   K←K+8×KT←K<8
[2]   →(Z←~1∊KT)↑OK
[3]   Z←(KT←(ρ,K)ρKT)/,K←(ρK=MEAN)ρK
[4]   Z←(ρK)ρ(~KT)+KT\×⌿(7 6 5 4 3 2 1
      0 ∘.-Z)÷(8,ρZ)ρKT/,MEAN
[5]  OK:KT←(K∘.*,¯2)⊥ 13860 ¯4146 1820
      ¯1287 1716 ¯6006 180180 ÷2162160
[6]   Z←Z×*(K×(1-(MEAN÷K)-⍟MEAN÷K))-(KT÷K)+
      0.5×⍟○2×K
    ∇
```

APPENDIX C

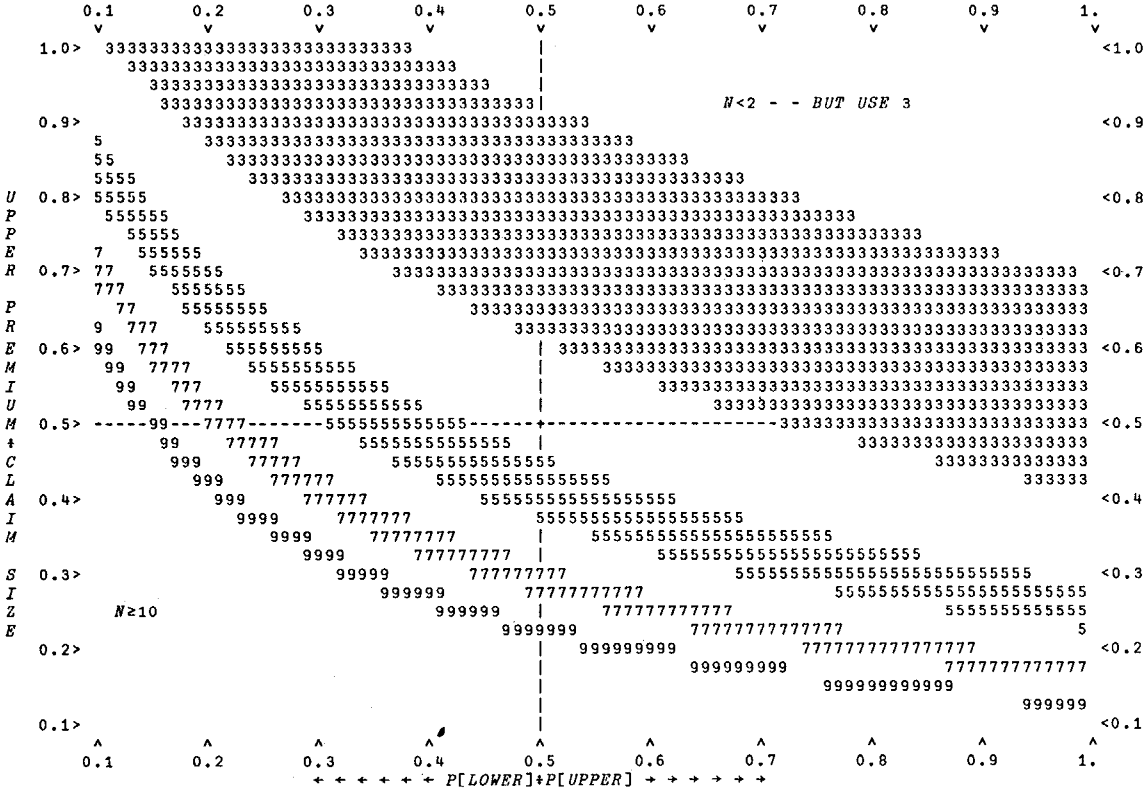
```
      )WSID
SWING
      )GRP POISSONCATASTROPHES
NN      WEW
      ∇NN[⎕]∇
    ∇ Z←PU∆L NN PL∆PU
[1]   Z←÷PU∆L
[2]   Z←1+Z×WEW(PL∆PU-1)÷Z×*÷Z
[3]   Z←(PL∆PU-1)÷PL∆PU-Z
[4]  ⍝ PU∆L IS PU÷L ⋆ PL∆PU IS PL÷PU
    ∇

      ∇WEW[⎕]∇
    ∇ W←WEW X;Z;E;B
[1]   B←(2+1∊X<0)⍴4
[2]   W←(Z←X+(X≤0)÷*1)∘.+,0
[3]   W←(1000⌊X)+(X>0)×(⍟Z)⌈(W⊥ 8 6 0)÷W⊥ 5 14 6
[4]   Z←(⍟X÷W)-W
[5]   W←W×1+E←Z×E÷(1+W)×Z+E←Z-2×(1+W)×1+W+2×Z÷3
[6]   →B←1↓B
[7]   W←W×X≠0
[8]  ⍝ SOLVES X=W×*W FOR W, GIVEN X>-*-1 [¯1<W<0 FOR X<0]
[9]  ⍝ X-W×*W←WEW X IS ¯3E¯17 AT ¯.367<X;
[10] ⍝                  ¯3.5E¯7 AT -*-1.
[11] ⍝ SEE CACM ALGORITHM 443 (1973 FEBRUARY)
    ∇

      .5 NN .1 .2 .3 .4 .5
11.8913  6.81447  5.08915  4.20882  3.66992
      .3 .5 .8 NN .6
5.46871  3.3036  2.14072
      .9 NN .9
1.73106
      .99 NN .99
1.59477
      .999999 NN .999999
1.58198
      .1 NN .1 .05 .02 .01
91.5502  181.553  451.554  901.555
      .01 .02 .05 .1 NN .1
991.505  491.509  191.524  91.5502
      .25 .5 .75 NN .25 .5 .75
13.6179  3.66992  2.08691
```

WEIGHTING 'N' FOR EQUIVALENT LEVEL PREMIUM — *APPENDIX D*

GIVEN AS A FUNCTION OF P[LOWER]÷P[UPPER] AND P[UPPER]÷CLAIM∆SIZE



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DISCUSSION BY JOHN C. WOODDY

Mr. Simon has written an interesting paper which reveals itself to be more a collection of ingenious manipulations of assumed relationships than a set of directions for calculating catastrophe reinsurance premiums.

The key sentence is in Section 2. It reads; "Attention will be focused on situations where it will be appropriate to assume that any loss which hits the cover will run all the way through it, that is, all losses will be total losses." This assumption is not commonly used in catastrophe reinsurance of life portfolios, and I understand that it is not the European practice in non-life catastrophe reinsurance. However, the assumption does have the consequence that the same mathematical results can be obtained by considering one death, two deaths, etc., among a group of lives, each having the same probability of death and the same amount of life insurance. This allows the usual risk theory assumptions of independence, stationarity and exclusion of multiple events to be made, so as to permit use of the Poisson distribution to specify the probabilities of a given number of deaths.

Using the expected number of claims of examples A and C in the paper, namely .09553, the sum of the probabilities of 0, 1, 2, 3, and 4 claims is equal to .999999, that is, practically all of the probability. As indicated in the paper, the pure premium to cover one payment of $9 million, in the event that at least one claim occurs in the year, is $819,981. Furthermore, the pure premium payable in advance to provide for payment of $9 million on the first claim plus another $9 million if there is at least one more claim during the year, is $858,528. However, the pure premium payable in advance to provide a payment of $9 million on each claim without limit on the number of claims is only $859,770.

I think that, psychologically, we are prepared to accept results like this when dealing with four lives each having a probability of death of about 2.4% where we know that the life either fails or does not fail. I do not feel equally comfortable with only a little more than $1,000 difference between the unlimited cover and the two-event maximum discussed above when catastrophe claims are, in fact, characterized by severity as well as frequency. I guess the question that I am really raising is whether catastrophe reinsurance priced under the assumption of "all losses will be total losses" provides a good framework for illustrating consistency of pricing.

Another point which calls for mention in the examples in the paper is the reference to provision for "overhead and profit". Of course, we may take it that the provision for profit includes the security loading, or provision for the risk of adverse fluctuations, but I must say that I would have expected a larger percentage of the gross premium than those suggested in the examples. I would expect a security loading to be a function of the standard deviation and/or the variance of the distribution of catastrophe claims.

Mr. Simon mentions that if the assumption that "all losses will be total losses" causes difficulty, "it may be necessary to apply this model to narrow sublayers of a given treaty". A paper entitled: "Rückversicherung des Kumulrisikos in der Lebensversicherung", by Paul Strickler, in the 1960 *Transactions of the International Congress of Actuaries (TICA)* presented in Brussels gives a procedure for calculating the net premium for a catastrophe cover applicable to a life insurance portfolio. An English translation appeared as item 3 in the first issue of *Actuarial Research Clearing House*.[1] The formulae in the paper require determination of a function $A(y)$, the annual number of deaths per million of general population, from all accidents causing the deaths of y or more persons. Also required is the frequency function for the amount z (limited by the warranted maximum risk on any one life) payable on account of one death in a catastrophic accident, given that such a death has occurred. Since the $A(y)$ function, as used in the premium formula, gives rise to a summation of the costs of one death, two deaths, three deaths, etc., in excess of the number of deaths retained by the ceding company, the total layer covered by the catastrophe reinsurance treaty is, in effect, subdivided into the separate layers defined by the successive numbers of deaths.

Of course, a catastrophe treaty usually specifies the deductible or ceding company retention and the reinsurance limit in dollars rather than numbers of lives. The conversion factor between the two is usually the average amount at risk on a life.

[1] Formula (11) is correct in *Actuarial Research Clearing House* but erroneous in the *TICA*.

DISCUSSION BY CHARLES R. RINEHART

First, let me commend Mr. Simon on a thought provoking paper in an area which has received very little attention in our *Proceedings*.

When first asked to review this paper, I was something less than enthusiastic. I had initially expected the paper to present a methodology for pricing catastrophe reinsurance treaties and, in this context, it did not seem particularly remarkable that Mr. Simon was able to calculate the probability of a claim in a particular year given the premium, a fixed severity and an assumption regarding the frequency distribution for a particular reinsurance treaty. It was only after rereading the paper that I was able to appreciate the benefits of a technique for testing the logical consistency of the assumptions underlying the pricing of catastrophe treaties.

This is, actually, similar to the technique used by the Bayesians to test the internal consistency of the underlying assumptions in a decision process. The key to any such approach is in surfacing the critical assumptions implied by any decision strategy. For the catastrophe reinsurance treaties described in the paper, the premium is the equivalent of the decision strategy while the principal factor implied by the premium is the potential for a claim in a particular year. Mr. Simon has provided a technique whereby the claim probability implied by a particular treaty premium can be computed. Ostensibly, such probabilities can be compared for similar treaties, or variations on the same treaty, to determine if the probability assumptions underlying the quoted premiums are logically consistent.

This does not imply that logical consistency is a necessity or even always desirable. In addition to the basic claim considerations, other factors such as competitive pressures or a reinsurer's relationship with a particular company will undoubtedly have a bearing on the ultimate treaty premium. The advantage to this technique, however, is that it will point out when the underlying assumptions are inconsistent and thereby necessitate that the individual pricing the treaties, at least, rationalize the deviation.

In addition to the test for logical consistency, this procedure further lends itself to a general test for reasonableness of the claims probability implied by a particular premium. As an illustration: in Mr. Simon's Example A, the gross premium of $1,000,000 implied that the probability of hitting the cover within a one year period was approximately 10%. If this were a wind treaty, say in excess of a $5,000,000 retention for a moderate sized company, the 10% probability would appear reasonable. However, if this were the top layer of an earthquake treaty, with the underlying layers and company retentions amounting to, say, $50,000,000, the probability would not be reasonable and the treaty premium possibly too high. While this is an extreme case, the reinsurance expert should be capable of far more refined analyses. Certainly, he is continually using his own experience to review his current prices, but, because the dynamics of the market place are continually changing his frame of reference, it should be beneficial to occasionally review the underlying assumptions implied by his prices to insure their continued reasonableness.

Finally, it should be noted, that the technique of testing for logical consistency has many other applications in addition to reinsurance. While there has never been sufficient data to test the myriad of individual rating factors used in the fire schedules, it might be very enlightening to test the internal consistency of the effects of each of the rating factors on the ultimately developed fire rate. The many rating factors present in a No Fault Class Plan would similarly avail themselves to tests for relative consistency. In the latter case, there are some data available and, to the extent reliable, they could be incorporated into the analysis, further enhancing the results.

In this review, I have not attempted to address any of the assumptions utilized by Mr. Simon in the actual application of his technique. Two areas which probably require further analysis are:

1. Is the Poisson distribution the appropriate mathematical model for the occurrence of claims?

2. Is the assumption that all losses are total losses realistic?

Regardless of how these questions might be resolved, Mr. Simon has provided a valuable contribution in the overall concept of testing the logical consistency of the assumptions underlying a ratemaking process.

DISCUSSION BY COSTANDY K. KHURY

Once again Mr. Simon favors the *Proceedings* with a new direction that should receive significant attention in the future. The importance of maintaining logical consistency among various reinsurance alternatives can hardly escape either the reinsurer of the reinsured. Perhaps the accent which this paper has placed on the concept will serve to underscore the degree of care that every primary insurer must exercise in assuring logical consistency among its own various plans—whether they concern deductible options, excess coverage plans, etc.

There are three assumptions underlying the treatment of the problem as posed by Mr. Simon:

(1) The Poisson distribution is the appropriate mathematical model for the occurrence of claims.

(2) The subject treaties are unbalanced, that is, they attach at a high limit such that the pure premium is small in relation to the size of the cover. (Emphasis added)

(3) All covered losses are total losses, that is, any loss which penetrates the cover will run all the way through it.

Assumption (1) presents no problem in that the principles espoused in the paper are fundamentally independent of the Poisson distribution—as specifically implied by the last sentence of the paper. Also, there is considerable literature in support of this particular choice. The use of another distribution would impact only upon the methodology and would leave the basic principles intact.

Assumption (2) presents some interpretation problems. First, what constitutes a "high" point of attachment is certainly different for different reinsureds. Also, what constitutes a "small" pure premium in relation to the size of the cover is a matter on which different observers could easily disagree. With each of these two characterizations open to debate—in the absence of specific definition criteria—the combination thereof is in turn open to compound interpretation problems inasmuch as they are not independent of each other. Close examination of the paper failed to yield the specific point(s) at which this assumption became operative. It appears that this assumption provides essentially a restatement of what a catastrophe is interpreted to be in relation to assumption (3) and [therefore] to the subsequent methodology. In fact, if the paper is re-read without benefit of this assumption, then the methodology is unaffected provided assumption (3) is accepted without further qualification. The matter of including or excluding this assumption fundamentally rests, at first blush, with the degree of aesthetic elegance sought in bridging from assumptions to conclusions. In this same regard, a proposition is strongest only when the set of assumptions has been reduced to an absolute minimum. More on this assumption later.

Due to the impact that assumption (3) has on molding the main body of the paper, it is in fact the focal point of this discussion. The implications of assuming all losses to be total losses (in most casualty insurance situations) are numerous. Among them:

(1) If consecutive reinsurers, at all layers, adopted the same philosophy, then once a loss penetrates the first layer it must be unlimited. A loss must stop somewhere, *a priori.*

(2) If a cover of size L excess of a specified retention requires a pure premium P; then a cover of size kL must imply a pure premium of kP for all $k > 0$. This linear movement of the pure premium could cause the cost of reinsurance to become prohibitive for $k > 1$ and inadequate for $k < 1$. A competing reinsurer may be able to capitalize on this feature by making appropriate adjustments for successive layers of equal thickness, etc.

These conditions, as well as others not mentioned above, can be eliminated at once provided a size-of-loss distribution is incorporated in the various formulations in the paper. An illustration of how such a distribution could be utilized is set forth below in terms of an arbitrary but fixed size-of-loss distribution.

Given a reinsurance treaty with a limit C excess over a specified retention c. The cedent's recovery is at a ratio r such that a loss of size x with $x > c$ generates reinsurance recoverable of $r(x - c)$ subject to the maximum rC. Let the size-of-loss random variable be x with a probability density function $f(x)$. In this discussion x may be assumed to be continuous without loss of generality. Let $g(n)$ be the distribution representing the model for the occurrence of exactly n claims during a specified time interval Δt and each of which is in excess of c. It should be noted here that $g(n)$ and $f(x)$ are independent. Finally, let $R(x)$ be the reinsurance recoverable by the cedent given the occurrence of a loss of size x:

$$R(x) = \begin{cases} 0 & \text{if} \quad c \geq x \geq 0 \\ r(x-c) & \text{if } C+c \geq x > c \\ rC & \text{if} \quad x > C+c \end{cases}$$

At first the development will track a "no-reinstatement" assumption during a time interval Δt. Two key probabilities of occurrence of claims are:

$$g(0) \text{ and } g(n \geq 1)$$

The reinsurer's expectation of loss (the pure premium $\overline{P}_1$)[1] is given by:

$$\overline{P}_1 = 0 \cdot g(0) + * \cdot g(n \geq 1)$$

with * yet to be determined.

Given that a loss has occurred, then $f(x)$ would yield the following probabilities:

$$Pr(C + c \geq x > c) = \int_c^{C+c} f(x)dx) \text{and}$$

$$Pr(x > C + c) = \int_{C+c}^{\infty} f(x)dx$$

These, in turn, give immediate rise to the following conditional probabilities:

$$Pr(C + c \geq x > c \mid x > c) = \int_c^{C+c} f(x)dx \Big/ \int_c^{\infty} f(x)dx$$
$$= Q(c, C + c, f(x)), \text{ and}$$

$$Pr(x > C + c \mid x > c) = \int_{C+c}^{\infty} f(x)dx \Big/ \int_c^{\infty} f(x)dx$$
$$= Q(C + c, \infty, f(x))$$

[1] This assumed equivalence reflects Mr. Simon's choice of not including a separate risk loading.

Accordingly, the reinsurer's conditional expectation of loss is given by:

$$E(R(x) \mid C + c \geq x > c) = \int_c^{C+c} r(x-c) f(x) dx \Big/ \int_c^{C+c} f(x) dx$$

$$E(R(x) \mid x > C + c) = rC$$

Hence, * is given by:

$$* = [Q(c, C+c, f(x)) . \int_c^{C+c} r(x-c)\, f(x) dx \Big/ \int_c^{C+c} f(x) dx]$$
$$+ Q(C+c, \infty, f(x)) . rC$$

And the pure premium $\overline{P}_1$, resolves to:

$$\overline{P}_1 = * \cdot g(n \geq 1)$$

If the expression obtained by Mr. Simon in (3.1) is recast into this new format (where rC, the maximum possible single loss reinsurance recoverable under the contract, is substituted for the expected average loss derived via the size-of-loss distribution when the loss is less than total), it would appear as follows:

$$P_1 = \{Q(c, C+c, f(x)) \cdot rC + Q(C+c, \infty, f(x)) \cdot rC\} \cdot g(n \geq 1)$$

Thus, the additional (built-in) pure premium due to assumption (3) is given by:

$$(P_1 - \overline{P}_1) = (rC - [\int_c^{C+c} r(x-c) f(x) dx \Big/ \int_c^{C+c} f(x) dx]) \cdot$$
$$Q(c, C+c, f(x)) \cdot g(n \geq 1)$$

Trivially, this expression is always non-negative.

It should be noted here that the expression given above for $\overline{P}_1$, given the same circumstances of example A, is equally usable as a vehicle for extracting the Poisson parameter by iteration since the introduction of a specific size-of-loss distribution did not produce any additional unknowns.

Also, since $(P_1 - \overline{P_1}) \geq 0$, it follows immediately that the Poisson parameter m, as implied by example A, is at its natural maximum only if the size-of-loss distribution ultimately implies $(P_1 - \overline{P_1}) = 0$ and is overstated whenever $(P_1 - \overline{P_1}) > 0$.

Now turning to example B, the probabilities of occurrences are:

$$g(0), g(1), \text{ and } g(n \geq 2)$$

and the pure premium $\overline{P_2}$ is given by:

$$\overline{P_2} = 0 \cdot g(0) + * \cdot g(1) + 2 \cdot * \cdot g(n \geq 2)$$

$$= * \cdot [g(1) + 2\, g(n \geq 2)] \text{ with } * \text{ as defined above.}$$

Once again, recasting Mr. Simon's definition expression for P_2 in the above format would produce:

$$P_2 = \{Q(c, C + c, f(x)) \cdot rC + Q(C + c, \infty, f(x)) \cdot rC\} \cdot [g(1) + 2g\,(n \geq 2)]$$

Thus the additional (built-in) pure premium due to assumption (3) is given by:

$$(P_2 - \overline{P_2}) = (rC - [\int_c^{C+c} r(x - c) f(x) dx \Big/ \int_c^{C+c} f(x) dx]) \cdot Q(c, C + c, f(x)) \cdot [g(1) + 2g(n \geq 2)]$$

and again this expression is trivially non-negative. Note that $(P_2 - \overline{P_2})$ compounds the original difference term $(P_1 - \overline{P_1})$. As was true for example A, the expression for $\overline{P_2}$ is equally usable for the determination of the Poisson parameter since the introduction of the size-of-loss distribution did not produce any additional unknowns. Finally, $(P_2 - \overline{P_2}) \geq 0$ implies that m as derived from $\overline{P_2}$ will never exceed m as derived from P_2.

Now turning to the critical relationships of $\overline{P_2}$ to $\overline{P_1}$ and P_2 to P_1:

$$\overline{P_2} \,/\, \overline{P_1} = [* \cdot g(1) + 2 \cdot * \cdot g(n \geq 2)] \,/\, * \cdot g(n \geq 1)$$

$$= [g(1) + 2\, g(n \geq 2)] \,/\, g(n \geq 1)$$

$$= [g(1) + g(n \geq 2) + g(n \geq 2)] \,/\, [g(1) + g(n \geq 2)]$$

$$= 1 + [g(n \geq 2) \,/\, g(n \geq 1)]$$

Using Mr. Simon's definition expression for P_1 and P_2 (with L replaced by C), we have:

$$P_2 / P_1 = [Cp_1 + 2C \sum_{i=2}^{\infty} p_i] \Big/ C \sum_{i=1}^{\infty} p_i$$

$$= [p_1 + 2 \sum_{i=2}^{\infty} p_i] \Big/ \sum_{i=1}^{\infty} p_i$$

$$= [p_1 + \sum_{i=2}^{\infty} p_i + \sum_{i=2}^{\infty} p_i] \Big/ [p_1 + \sum_{i=2}^{\infty} p_i]$$

$$= 1 + [\sum_{i=2}^{\infty} p_i \Big/ \sum_{i=1}^{\infty} p_i]$$

which is the same expression as derived above as $1 + [g(n \geq 2) / g(n \geq 1)]$. Therefore, the relationships $\overline{P_2} / \overline{P_1}$ and P_2 / P_1 are independent of the size of the cover and dependent only upon the all important point of attachment of the treaty and therefore, on the particular selection of the distribution for the occurrence of claims in excess of the retention. Perhaps the preceding discussion in connection with assumption (2) as to what constitutes a "high" point of attachment could now be viewed as requiring a definition of the term "high" inasmuch as it turns out to be the controlling feature in comparing the various reinstatement alternatives.

As a generalization of the preceding findings, any two proposals involving reinstatements could be compared. In this manner the examples given in Mr. Simon's paper (e.g., A and C) can be treated as special cases of the larger class of problems they represent.

If P_{i+1} *and* P_{j+1} represent two options with i and j reinstatements respectively, then:

$$P_{i+1} / P_{j+1} = [\sum_{u=0}^{i} u\, g(u) + (i+1)\, g(n \geq i+1)] \, /$$

$$[\sum_{u=0}^{j} u\, g(u) + (j+1)\, g(n \geq j+1)]$$

One other interesting generalization can be obtained in connection with the reinsured wishing unlimited reinstatements. That is, P_∞ at the inception of the time interval Δt is given by:

$$P_\infty = * \cdot \sum_{u=o}^{\infty} u\, g(u)$$

and in case of the Poisson distribution, this expression is reduced to the compact, and perhaps obvious, form:

$$P_\infty = * \cdot \sum_{u=o}^{\infty} u\, g(u) = * \cdot \sum_{u=o}^{\infty} u \cdot [m^u \ e^{-m} / u\,!] = * \cdot m$$

In conclusion, it should be acknowledged that it is the author's exclusive privilege to select his underlying assumptions and proceed within the guidelines provided thereby. In this discussion it is hoped that additional light has been shed on this problem in terms of reducing the set of working assumptions, a generalization of the problem and its solution, and finally a more direct approach to the comparison of reinstatement options. Once again, we should be grateful to Mr. Simon for opening the doors to this extremely exciting and glamorous area of actuarial research.

ALLOCATED LOSS EXPENSE RESERVES

ALLIE V. RESONY

VOLUME LIX PAGE 141

DISCUSSION BY EARL F. PETZ

Mr. Resony has presented an interesting method for determining the allocated loss expense reserve and his work displays a good deal of ingenuity. Furthermore, I suspect that, at least for him, the method[1] works. There are some parts of his formula which, I think, fall short of the ideal and which might be considered for modification or, as suggested later in this review, for which an alternative approach might have merit. The less than ideal elements[2] would include the following:

1. The F ratio method requires the use of "created year". This is information which is not generally required and carrying it forward, in addition to accounting year, accident year and policy year, just adds one more thing which can go wrong. In the case of our company, we do not carry created year through our statistical routines and, therefore, were unable to make a direct test of Mr. Resony's methods.

   Could accident year be substituted? As a measure of age of claim, accident year might even be superior. The theory is that the ratio of allocated loss expense[3] to loss increases with age of claim. In this respect, isn't a claim from "x" accident year "created" in year "x + 2" more likely to resemble other claims from accident year "x" than to resemble other more recent claims?

2. The F ratio is an abstract quantity. The ratio of paid ALE during a calendar year to the change in loss reserve outstanding during the same calendar year is a ratio which has no meaning in and of itself. Presumably, these ratios have worked empirically but their use makes it difficult to explain the method to Insurance Department Examiners or to management.

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[1] Subsequently referred to as the "F ratio" method.

[2] Most of these items were mentioned by the author in his paper and are repeated in this review primarily as a basis for suggesting possible variation to the F ratio method.

[3] Subsequently abbreviated "ALE".

Would it be possible, instead, to use the ratio of ALE paid to losses paid in a calendar year? These would be split by created year or accident year, of course. For those years in which there has been no change in the estimated ultimate losses, the F ratio and the ratio of paid ALE to paid loss would be the same. For those years where there has been a change in the estimate of the ultimate total incurred losses, the ratio would be different, but it is not clear that the F ratio would be superior to the paid ratio. During any one calendar year, there is no necessary relationship between paid ALE and either paid losses or losses "disposed of".

3. The "disposed of" ratios are artificial and may be unstable. The amount of loss disposed of is equal to the change in loss outstanding or paid losses, plus or minus the change in the estimated ultimate total incurred loss which has taken place during the year. While the ratios given in the paper show reasonable stability, this would not necessarily be the case, especially for a line like general liability. For this line, our company actually developed some "disposed of" ratios which were greater than 1.000 (for accident years 1969 and 1970 valued successively 12-31-71 and 12-31-72.)

4. The F ratio method is independent of redundancy of loss reserves if and only if the degree of redundancy or inadequacy does not change. While a stable condition of redundancy or inadequacy would be ideal, it is unlikely to be realized under actual conditions. When there is a change in degree of redundancy over a period of time, this method is, probably, no more stable than one which sets the ALE reserve as a percentage of the loss reserve and which, consequently, will be over, or under, as the loss reserve itself is over or under. For example, if there was a loss reserve redundancy in the years which were used to determine the F ratio and none in the year to which it was being applied, the ALE reserve would be low. This is because the amount of loss "disposed of" (the denominator of the F ratio) would be artificially large when disposing of claims which contain a redundancy. Such an F ratio produces a proper answer only when applied to a year with a like amount of redundancy.

5. The method does not recognize special situations. This is perhaps more a failing of any formula reserving system than it is of the F

ratio system in particular, but, in developing experience by risk or by agent or in any other limited category, the formula is unable to recognize exceptional situations where, for example, an unusually large ALE reserve is required on a particular claim or where there is variation in the legal activity from territory to territory. For this kind of use, there appears to be no substitute for case basis reserves.

As a possible alternative to the F ratio method, or perhaps as a supplementary check thereto, the following system is suggested for consideration:

(a) Determine the ratio of paid ALE to paid loss, cumulatively, by accident year at successive valuations.

(b) From this array, select ultimate ratios by accident year.

(c) Apply the selected ultimate ratio to losses incurred for the accident year.

(d) The allocated loss expense reserve is the difference between the ultimate figure calculated in step (c) and the paid ALE to date, by accident year.

The following exhibit shows an array of the type referred to in steps (a), (b) and (c) above. The approach to selecting the ultimate values might well be the subject of considerably more study, but in a surprisingly high proportion of the cases reasonable values will be pretty well self-evident.

RATIO OF ALLOCATED LOSS EXPENSE PAID TO LOSS PAID
GENERAL LIABILITY BI

Accident Year	Valued As Of (Months) 12	24	36	48	60	72	84	96	108	120	132	Selected Ultimate
1962	.03	.09	.19	.26	.31	.33	.33	.37	.38	.36	.36	.36
1963	.04	.12	.23	.28	.33	.34	.34	.37	.36	.38		.38
1964	.03	.12	.22	.30	.29	.33	.36	.36	.38			.38
1965	.04	.13	.27	.33	.38	.41	.40	.39				.40
1966	.03	.13	.26	.36	.41	.43	.44					.45
1967	.03	.15	.41	.31	.36	.35						.39
1968	.10	.12	.20	.31	.34							.38
1969	.04	.13	.26	.36								.40
1970	.03	.13	.25									.39
1971	.03	.17										.39
1972	.07											.39

This suggested method has the advantage of using ratios of paid ALE to paid loss which, though perhaps no more meaningful in themselves than the F ratios, are more likely to be acceptable to Insurance Department Examiners and management as appearing to be reasonable. The use of accident year, rather than created year, avoids the necessity for carrying additional statistical information since accident year is required for many other purposes anyway. This system has the drawback of being dependent upon the adequacy of the loss reserves. If they are over, or under, the ALE reserve will also be over, or under.

If this system is used as a pure formula reserve, it has the same deficiency as number 5. for the F ratio method and suggests another way of using this approach, which is to use case basis reserves established by the claim department, the adequacy of which is tested by this method, with feedback to the claim department to assist them in establishing the reserves at an adequate level.

The major advantage of the proposed alternate system is that it is simpler. It requires no separate treatment for IBNR, nor adjustment for other than year-end dates, yet it may very well produce ALE reserves which are as accurate as those produced by the F ratio method. To produce an accurate reserve under either method, one needs to know what is happening in the way of both internal and external influences. With that information, a good reserve can be produced by either method and, without that information, neither method will produce a good reserve.

THE ACTUARY AND IBNR

RONALD L. BORNHUETTER AND RONALD E. FERGUSON

VOLUME LIX PAGE 181

DISCUSSION BY WARREN P. COOPER

This will not be a very spicy review. I have been unable to find the one mandatory flaw which, despite all succeeding praise, puts the logical flow of the paper in question. To the contrary, I find the authors' suggested treatment of Incurred But Not Reported Losses altogether congenial. So much so that I shall simply give four varyingly garrulous comments and pose one question. Hopefully, they will serve to amplify the authors' position, not just expose my own prejudices.

1. The idea of using losses to predict losses is a particularly happy one. A genuinely unscientific telephone survey has convinced me that a minimum of 80% of our industry's total IBNR is established by referencing premiums in one way or another, my own company not excepted. As the authors point out, premiums in force now seem to be in vogue, although I'm not sure I understand all the critical points of the rationalization. In the "longest-tail" lines, such as products and professional liability, the bulk of IBNR claims arise from expired policies, not those in force, and while such claims may not predominate in the usual third-party unreported reserves, they add up to be a substantial portion. Earned premiums, according to some of our sages, do the trick, particularly calendar-year earnings used to predict accident-year IBNR. Certainly all the now-classical arguments in favor of this accounting marriage are convincing, unless we face up to one usually unexpressed assumption: premiums are a proper measure of exposure. Wouldn't that solve a bunch of problems if it were true? We could even stop compromising our definitions of exposure, where we can define it, and cease looking for definitions where we can't. But, alas, the assumption is not necessarily true for many reasons: regulations, individual risk rating, etc. Consider the lively marketplace of 1973, where our favorable underwriting results for the previous two years are generating strong pressures to reduce rates. Better experience is expected to bring rate decreases, but unfortunately, the illogical, non-actuarial work does not always match the individual cost reduction with a cognate reduction in loss exposure. Competition is more generally felt. The same book of, say, commercial multiple peril busi-

ness in 1973 could produce a substantially lower premium flow than it did in 1972. While the severity, perhaps even the frequency, of IBNR it brings along to the insurers should also drop, I doubt it it will go as far. In a highly competitive arena prices tend to over-react. This is just another face of the cyclical swings we find in insurance profits. Alternate speed-ups and lags in pricing are one of the causes of these swings. Do we really want to tie our reserves to the pendulum?

For those lines where earned exposures are realistic and measurable, we should be able to use them to predict a stable, adequate IBNR. I hope someone will devote some time to the relationship and share their findings with the rest of us. But for now, the procedures that Messrs. Bornhuetter and Ferguson have described allow us to calculate reserves that are responsive, as the outside world demands, and efficiently consistent across lines, as our comptrollers demand.

The authors of the paper point out that expected loss ratios are best considered in certain cases and there's no faulting their logic, as most of us involved in "long-tailed" business will testify. "Expected loss ratios" is a not very well hidden reference to premiums and my remarks above caution against premiums in any disguise. However, my arguments do not solve the problems that loss ratio budgeting does in slender-data circumstances. There, they must be used. In other cases, the authors note, the results of calculating IBNR by expected loss ratios on the one hand and extending losses on the other should put our reserves in just about the same condition. Ideally this is so, but realistically it is probably not so. More bears on the real situation than the Central Limit Theorem.

2. I suspect the authors' lumping of classical IBNR and development on known cases together will rouse uneasy feelings in some of our colleagues. There are reasonable arguments for putting the "development-on-known" portion in either the IBNR or the outstanding column. Pragmatically, does it matter as long as the liabilities are fully accounted for? At the very least, the authors have given us a vehicle to get where we want to go and where the law wants us to be. However, I can see reason to separate the two pieces for control purposes, i.e., in order to interpret distortions in development patterns. Under the paper's definition, unreported reserves cover four situations: "true" unreported, reopenings, known but unrecorded items and known case development. In the third-party lines, we find, this last is less accurately predictable than the sum of the first three. Hence the excellent papers we have in the *Proceedings* on

problems with changing reserve margins. Fluctuation in margins is not altogether random; the management of our claims departments has considerable influence upon it. In a primary company, we should know what unusual actions the claims people have taken: extra periodic reviews, wholesale markdowns, special analyses, etc., and what policy changes they may institute. Armed with these bits of information, we can understand strange patterns and make corrective adjustments. For a reinsurer, the case is not so clear, since he is dealing with many claims staffs most of whom are probably strangers to him. The primary carrier might consider two routines, one like that the authors present and another similar one that ignores known case development. The difference is, of course, the latter. As an expression of our belief in the control value of the second routine, we sort accident-year losses by known and unreported to watch their separate development over several years. We suspect the margin change patterns are dissimilar, but we do not know definitely as yet.

3. The crucial step in the authors' IBNR calculation is the selection of the year-to-year development factors, those extrapolated from their Exhibit A and developed on Exhibit B. In the paper, a three-year average, weighted by the earlier years' incurred claims value, is used. Like all averaging procedures, the calculation smoothes out underterminable yearly variations. It also smoothes out known aberrations. I am certain that the authors do not advise slavish use of their formula to derive the factors; in the text they suggest curve fitting, trending and judgment adjustments. I would like to underline their suggestion. One of the elements bunched into the IBNR is what we might call incurred but not recorded (perhaps abbreviated IBNR' or a $_{I}$IBNR), the load of claims that for one reason or another are clogged in the processing pipeline and don't make it to the drain by the time of the end of period cut-off. This is the kind of imponderable we actuaries like to believe is a constant and can, therefore, be conveniently ignored. To our misfortune, it is not, and it is probably impossible to derive an algorithm to account for it. Let me give you an example. A few years ago our company shifted to an entirely new common master file system for all losses in the house. We expected conversion drags and a strong jump in entry rejects; we got disruption in profusion. The situation stabilized after two or three quarters, but, one of the unstable quarter-ends was also a year-end when we had more than double the expected processing lag. We made exceptional surveys to account for this aberration in the Annual Statement. We couldn't correct entry date, so, on an accident-year argument, the rejects flowed into our reports as late

reportings. Two years later, an analysis very similar to that in the paper was made of our third-party experience. If we had believed the three-year average factor, our IBNR would have been overstated by several millions of dollars; a plus, perhaps, on the solvency side, but a definite minus to the IRS. If there is a moral to this tale, it is to emphasize the need for inspection of the data, isolation of strange patterns and judgment adjustments. After all, we actuaries are clinicians, aren't we? Mathematical procedures offer extensive help, but, of necessity, they are impersonal and the data sluggish. Together they might leave you with an IBNR substantially above, or below, the necessary amount. Experienced diagnosis and prescription is a critical step in selecting development factors.

4. Throughout the paper, the authors refer to varying ways their systematic approach may be refined. This flexibility is a prime virtue and should be explicitly stated. Their concept, after all, is a model for predicting IBNR as defined. Like any model, it may be contracted, or expanded, as the case warrants. The warranting case might be the peculiarity of a particular line (witness the choice between using expected loss ratios or extending incurred losses); or it might be some unique company characteristic. The model can be more, or less, rigorous mathematically, depending on how the company views the value of rigor. As rehearsed above, it certainly can incorporate the clinical actuary's diligent insights. Not the least impressive aspect of the system's flexibility is its ability to monitor the reserve once established and allow for interim adjustments. The authors develop some alternatives in this area, while most of our IBNR computational schemes do not serve in this regard at all.

5. Finally the question. Why do the authors express surprise that IBNR flow in prior accident years is skewed? If the most recent year's development reflects a constantly diminishing rate, why not the other years, at least in frequency? Severity, especially in the later years of development, will probably be random. Both factors should make the 25% per quarter distinction unlikely.

DISCUSSION BY HUGH G. WHITE

I was very impressed by the quality of this paper by two men whose company's fortunes turn so heavily on the accuracy of the measurement of the Incurred But Not Reported Claim Reserve.

The authors define the IBNR Reserve in such a way as to embrace all changes in the reported incurred loss figures for an accident period from the date of current valuation to ultimate. Mr. Tarbell in his 1934 paper "Incurred But Not Reported Claim Reserve", *PCAS* Vol. XX 1934 considers as IBNR all unreported losses as of a given valuation date, including their development from first notice to ultimate. Presumably, although he does not seem to mention it in the paper, he would consider development on known cases to be a separate consideration.

My own personal preference is to segregate the Bornhuetter-Ferguson definition mentally into three segments: IBNR (unreported to the company at the branch level), "in transit", and supplemental (loss development) and to separate it practically into two parts, the first consisting of IBNR and "in transit" to the first notice level, and the second of supplemental on recorded claims whether they are reported as of the reserve date or not. In other words, development on an unreported claim from the first notice level is considered part of the same reserve component as future development on known cases as of the reserve date. This is an attempt to segregate the causes of non-instantaneous reporting of claim producing events, and their effects on reserve levels, from the causes of inaccurate first notices and their effects.

Complete combination into one component has psychological advantages since, when variations appear from time to time in the supplemental or IBNR "in transit" segments, it helps to spread it over a wider base. Nevertheless, it is helpful when a potential perturbation rears its head in the external world to know what portion and what proportion of your development factor is likely to be affected. Let us use the authors' example of a mail strike which, of course, affects primarily the "in transit" portions of the total bulk reserve. Suppose you are fortunate enough to write a line of insurance where your aggregate development factor on outstanding claims from any valuation date to ultimate is unity because you are closing without payment a reported claim for every one late reported. It would be difficult to try to account for the effects of three weeks without mail if you have not segregated your development factor into the two components of late reported and claims closed with no incurred amount.

Another advantage of the separation of the bulk reserve into at least two components is the availability of different bases for calculation of the reserve components. Premiums in force are a reasonable basis for calculation of IBNR and "in transit" claims, at least to the first notice level, and yet what could be more appropriate as a calculation basis for loss development, from the first notice level to ultimate, in a major line than the latest information on what is subject to it (i.e. case basis outstanding)?

What follows is in essence a series of marginal notes on specific points raised throughout the paper. As they begin the discussion of the loss development approach, the authors indicate that it is necessary that all loss and allocated loss expense data carry both accident date and original notification date. It would seem that the attachment of the notification date to the claim record would only be necessary if one expected different development patterns after first report on two groups of losses, one group reported three months after date of loss and another group reported four months after date of loss, and have a method of taking this difference into account in reserve calculation. It would seem sufficient, at least for primary insurers, to allow the data to establish its own notification date (and, of course, accident date), separately for a first report, or subsequent development on a claim as it is recorded on the company's records. If this is done monthly, it should adequately establish the pattern of first report and development of an accident month's incurred losses.

I certainly agree with the authors that expected loss should be used as a test of reasonableness of a reserve, but feel that, provided sufficient individual attention can be given to the results in a small or volatile line, its use to actually set the reserve for the line should be avoided as much as possible, since, by doing otherwise, you are specifically ignoring most of the information on which the reserve would normally be based. To illustrate some of the difficulties associated with expected losses, at least for myself, I offer the following problem. You are trying to establish the reserve for commercial automobile bodily injury and the reported proportion of expected losses as of statement date for the current accident year period is 8% higher than it should be. Do you:

1. Reduce the bulk reserve a corresponding amount (because you sense an acceleration in the rate of report);

2. Leave the bulk reserve at the same percentage level of expected losses (because you sense a random fluctuation such as a large

loss); or

3. Increase the bulk reserve in proportion to the increase of actual reported over expected reported (because you don't have 100% confidence in your "expected losses")?

Obviously, none of the three suggested "answers" is satisfactory without further extensive investigation, and yet, all are reasonable. While it is a gross over-simplification of the question the reserve actuary will face, it still illustrates the limitations of the effectiveness of expected losses.

An additional thorny problem raised in conjunction with the use of expected losses is the decision, which must be made separately for each company and for each line of insurance, as to what constitutes an "exceptionally large loss"; a portion of which should be excluded from the determination of the incurred but not reported reserve. Inclusion of the full amount of such an item may cause you to over-reserve, unless you are using the expected loss method, in which case it may cause you to under-reserve, but, of course, the exclusion of too much or too many will do the reverse.

In the section on "Interim Reserving Techniques", the authors point out the skewedness of the distribution of development, by quarter, throughout the year, both for the most recent accident year and for prior accident years. The figures indicated are 40%, 70%, 85% and 100%, by quarter, for the most recent accident year and 33%, 60%, 80% and 100% for prior accident years. Perhaps, as they point out, because the experience they are quoting is excess of loss and would have a much greater average time lag in reporting, their distributions are much less skewed than the distributions arising out of our company's experience in Canada. The distribution, by quarter, that we get for the most recent accident year is 80%, 90%, 95% and 100% and for prior accident years, 36%, 62%, 83% and 100%.

The authors mention the possible snowballing effect of an observed deterioration in a prior accident year which is allowed not only to affect the reserve for that year, but also to cause the recalculation of all subsequent accident year's reserves, based on the revised developmental indications for that year. One must be careful, in observing such a deterioration, to distinguish between a random deterioration and a definite indication of a basic flaw in the reserve structure. One does not want to be too light on one's feet in responding to an indication of either deterioration or retarded development since, as the authors point out in one of their examples, the adjustment of one accident year's supplemental and IBNR reserve by $200,000 could carry with it the implication of an adjustment of over five times that amount.

Another suggestion, in the section on "Interim Reserve Techniques", is that the required alteration in a budgeted growth in the IBNR reserve, because of an increase in expected losses, could be achieved by multiplying the increment in expected losses by the current year's IBNR factor. If one admits that IBNR is an attempt to account for time lag in reporting, it would not seem sufficiently conservative to use the IBNR factor for a group of losses (reported or unreported), which are, on the average, six months old, to produce an incurred but not reported reserve increment by multiplying by a group of expected losses (reported and unreported), which could be as little as one and one half months old, if one uses quarterly interim reserves.

In their section on the fiscal-accident year approach, the authors point out that the suggested method is more expensive and more time consuming than the rougher estimates which are available without reference to both month and year of loss in all claim data. But, I believe, that there would be no question today that the cost associated with it is worthwhile. I believe, also, that there are no major primary insurers who do not use some variation on a fiscal-accident year approach for at least their major lines of insurance.

The amount of information available to most actuaries who are responsible for the establishment of supplemental and IBNR reserves is enormous and the computational techniques are myriad and sound. Nevertheless, enough judgment situations arise month by month in arriving at numbers which directly affect operating results that it is of extreme importance for the actuary to keep in as close touch as possible with all aspects of his company's operation in both branch and head offices, and particularly in the claim and data processing departments. While it is the duty of actuarial science to substitute some facts for some impressions, it is the duty of an actuary to prepare himself in such a way that he receives the correct impression from a set of facts. I believe that nowhere in the wide range of actuarial responsibilities is informed judgment called into greater play than in the establishment of supplemental and IBNR reserves. The authors of this paper deserve congratulations for pointing their colleagues toward sources of the necessary information.

MINUTES OF THE 1973 SPRING MEETING

May 20—23, 1973

NEVELE COUNTRY CLUB, ELLENVILLE, NEW YORK

Sunday, May 20

The Board of Directors held its regularly scheduled meeting at the Nevele Country Club from 2:00—6:00 p.m.

Advance registration was held from 5:00—6:00 p.m. for early arrivals.

A reception was held 6:30—7:30 p.m.

Monday, May 21

The registration period began at 8:30 a.m. The 1973 Spring meeting was formally convened at 9:00 a.m. with opening remarks by President Hewitt who then introduced Superintendent of Insurance Benjamin R. Schenck of the State of New York. Superintendent Schenck welcomed the Society to New York and spoke on the purposes of insurance to provide for contingencies, as a means of financing public purposes and as a contributor to loss prevention or loss reduction.

The following reviews of papers were presented:

Reviewer	Paper	Author
John A. W. Trist Associate Actuary Insurance Company of North America (presented by James A. Hall)	"How Adequate are Loss and Loss Expense Liabilities?"	Ruth E. Salzmann Vice President & Actuary Sentry Insurance Group
Waldo A. Stevens Senior Vice President Blue Cross Association (presented by Robert B. Foster)	"Joint Underwriting as a Reinsurance Problem"	Emil J. Strug Assistant Vice President Massachusetts Blue Cross, Inc. Massachusetts Blue Shield, Inc.
Robert L. Hurley Associate Actuary Insurance Services Office	"An Actuarial Note on Experience Rating Nuclear Property Insurance"	Richard D. McClure Assistant Actuary Kemper Insurance Group

Following a coffee break, there was a panel discussion entitled "GAAM— A CAS Position for the Future". Participants in this part of the program were as follows:

Moderator:	Harold W. Schloss, Senior Vice President Royal-Globe Insurance Companies
Panel:	Stanley C. DuRose, Jr., Commissioner of Insurance State of Wisconsin
	John S. McGuinness, President John S. McGuinness Associates
	Robert H. McMillen, Vice President & Actuary The Travelers Insurance Companies
	Paul M. Otteson, Vice President and Actuary Federated Mutual Insurance Company
	C. L. Breslin, Partner Peat, Marwick, Mitchell & Co.

The panel discussion was concluded at 12:00 noon.

The afternoon program convened at 1:30 p.m. with a panel discussion "Redefinitions Under No-Fault". Participating on the panel were:

Moderator:	Paul J. Scheel, Vice President & Senior Actuary United States Fidelity & Guaranty Company
Panel:	Rex C. Davis, Assistant Vice President Allstate Insurance Company
	Robert A. Bailey, Director, Insurance Bureau State of Michigan
	Walker Richardson, Assistant Vice President Liberty Mutual Insurance Company
	Robert D. Bergen, Associate Actuary Insurance Services Office

The panel discussion was concluded at 2:45 p.m.

After a fifteen minute coffee break the meeting convened at 3:00 p.m. with two concurrent sessions:

A. Continuation of the Non-Fault subject on an informal basis.

B. Panel Discussion: "The Actuary and Marketing Research".

The participants in the panel discussion on marketing research were:

Moderator: Robert Pollack, President
Colonial Penn Insurance Company

Panel: Morris Gottlieb, Vice President
Market Facts, New York

John Gragnola, Director of Market Research
Allstate Insurance Company

Wayne Sorenson, Director of Research
State Farm Mutual Automobile Insurance Company

A. E. Billings, Jr., Director, Office of Consumer Information
The Travelers Insurance Companies

At 4:00 p.m. after a short break there were two more concurrent sessions as follows:

A. Panel Discussion: "Ratemaking and Reserve Problems in Professional Liability".

Moderator: William J. Hazam, Vice President and Actuary
American Mutual Liability Insurance Company

Panel: Robert A. Anker, Assistant Actuary
Employers Insurance of Wausau

Robert G. Oien, Staff Actuary
St. Paul Fire and Marine Insurance Company

David G. Hartman, Associate Actuary
Chubb & Son, Inc.

B. Continuation of the market research discussion on an informal basis.

These sessions were concluded at 5:00 p.m.

The President's reception for new Fellows and their wives was held from 6:00—7:00 p.m.

Tuesday, May 22

President Hewitt convened the business session at 9:00 a.m.

President Hewitt presented diplomas to the following new Fellows and Associates:

FELLOWS

Robert G. Eyers	Dennis E. Hoffmann	James P. Ross
E. Frederick Fossa	Costandy K. Khury	

ASSOCIATES

Roger W. Bovard	Robert A. Daino	Raymond S. Lis, Jr.

By vote, the membership ratified the election of Ronald L. Bornhuetter to the Board of Directors to fill the vacancy created by the election of George D. Morison as General Chairman of the Education and Examination Committee.

It was announced that the Nominating Committee for the November 1973 election consists of Daniel J. McNamara, Chairman, William J. Hazam, Richard L. Johe, Harold W. Schloss, and LeRoy J. Simon and that a preferential ballot would be distributed to Fellows in July.

By invitation, Gary L. Countryman, Research Associate, Liberty Mutual Insurance Company, presented his review of Raymond W. Beckman and Robert N. Tremelling II's paper, "The Relationship Between Net Premiums Written and Policyholders' Surplus". Reviews of the same paper were presented by David J. Grady, Assistant Actuary, The Travelers Insurance Company and Roger C. Wade, Chief Actuary, Utica Mutual Insurance Company.

Lester B. Dropkin, General Manager, California Inspection Rating Bureau, presented a review of John P. Welch's paper "No-Split Experience Rating Plans".

David G. Halmsted, Metropolitan Life Insurance Company, presented by invitation a review of LeRoy J. Simon's paper "Actuarial Applications in Catastrophe Reinsurance".

Following a coffee break the meeting reconvened at 10:15 a.m. at which time President Hewitt introduced the featured speaker Mr. J. Albert Burgoyne, President, Metropolitan Property and Liability Insurance Company. Mr. Burgoyne spoke about the challenge the insurance industry faces to meet the new demands of the consumer.

At 11:00 a.m. the Committee on Experience Requirement presented the pros and cons of an experience requirement. Ronald L. Bornhuetter, Chairman of the committee, provided background information. LeRoy J. Simon presented arguments in favor of an experience requirement. John H. Muetterties substituting for James MacGinnitie described arguments against an experience requirement. Differing opinions were advanced by several members of the Society after the presentations had been made.

At 11:30 a.m. there was a one hour presentation by the Committee on Government Statistics, Phillip N. Ben-Zvi, Chairman, on the subject "Outside Statistics and Their Impact Upon Insurance". Presentations were made by Harold E. Curry, Darrell W. Ehlert, J. Robert Hunter, Jr. and Michael A. Walters.

After lunch, which was served at 12:30 p.m., various Committee meetings were held starting at 1:30 p.m.

A reception was held at 6:00 p.m. for members, wives (or husbands) and guests followed by a banquet at 7:00 p.m.

Wednesday, May 23

The meeting convened at 9:00 a.m. The following papers were presented:

> "Expense Analysis in Ratemaking & Pricing" by Roger C. Wade, Chief Actuary, Utica Mutual Insurance Company.
>
> "A Survey of Loss Reserving Methods" by David Skurnick, Assistant Actuary, Insurance Company of North America.
>
> "Underwriting Individual Drivers: A Sequential Approach" by Professor John Cozzolino and Leonard R. Freifelder, both of the University of Pennsylvania. This paper was submitted by invitation and was presented by Leonard R. Freifelder.

The following reviews of papers were presented:

> John C. Wooddy, Senior Vice President, North American Reassur-

ance Company, Charlès R. Rinehart, Assistant Actuary, Fireman's Fund American Insurance Companies, and Costandy Khury, Associate Director, Prudential Property and Casualty Insurance Company presented reviews of LeRoy J. Simon's paper "Actuarial Applications in Catastrophe Reinsurance".

A reply to the reviews was made by the author, LeRoy J. Simon, Vice President, Prudential Property and Casualty Insurance Company.

Earl F. Petz, Actuary, Kemper Insurance Group reviewed the paper "Allocated Loss Expense Reserves" by Allie V. Resony, Associate Actuary, The Hartford Insurance Group.

The author's reply was presented by George C. Munterich.

Reviews of "The Actuary and IBNR", jointly authored by Ronald L. Bornhuetter, Vice President and Actuary, General Reinsurance Corporation and Ronald E. Ferguson, Assistant Vice President, General Reinsurance Corporation, prepared by Warren Cooper, Vice President & Actuary, Chubb & Son, Inc. and Hugh G. White, Assistant Actuary, The Travelers Indemnity Company of Canada, were presented by David G. Hartman and David J. Grady respectively.

The business session ended at 10:30 a.m. and was followed by a fifteen minute coffee break.

At 10:45 a.m. there was a panel discussion "Group Marketing—Ratemaking and Problems". Participating in this panel discussion were:

Moderator:	LeRoy J. Simon, Vice President Prudential Property and Casualty Insurance Company
Panel:	John W. Carleton, Senior Vice President Liberty Mutual Insurance Company
	Robert G. Young Robert G. Young and Associates
	John H. Rowell, Vice President Marsh & McLennan, Inc.
	Richard E. Munro, Actuary California Casualty Group

The panel discussion was concluded at 12:15 p.m.

President Hewitt then adjourned the meeting.

It is noted that the registration cards completed by the attendees and filed at the registration desk indicated attendance by 104 Fellows, 47 Associates, 27 invited guests and in addition, 57 wives.

FELLOWS

Aldrich, W. C.
Alexander, L. A.
Anker, R. A.
Bailey, R. A.
Balcarek, R. J.
Beckman, R. W.
Ben-Zvi, P. N.
Bergen, R. D.
Bevan, J. R.
Bickerstaff, D. R.
Bill, R. A.
Bondy, M.
Bornhuetter, R. L.
Boyajian, J. H.
Boyle, J. I.
Brannigan, J. F.
Brian, R. A.
Carleton, J. W.
Comey, D. R.
Cook, C. F.
Crowley, J. H.
Curry, A. C.
Curry, H. E.
Dahme, O. E.
DeMelio, J. J.
Dropkin, L. B.
Ehlert, D. W.
Eide, K. A.
Eliason, E. B.
Eyers, R. G.
Farnam, W. E.
Ferguson, R. E.
Fossa, E. F.
Foster, R. B.
Fowler, T. W.
Gibson, J. A.
Gillam, W. S.
Gillespie, J. E.
Grady, D. J.
Graves, C. H.
Hachemeister, C. A.
Hartman, D. G.
Hartman, G. R.
Harwayne, F.
Hazam, W. J.
Heer, E. L.
Hewitt, C. C., Jr.
Hoffmann, D. E.
Honebein, C. W.
Hughey, M. S.
Hunter, J. R., Jr.
Hurley, R. L.
Jacobs, T. S.
Kallop, R. H.
Khury, C. K.
Leslie, W., Jr.
Levin, J. W.
Linden, J. R.
Linder, J.
Lino, R.
Liscord, P. S.
McLean, G. E.
McGuinness, J. S.
Menzel, H. W.
Mohnblatt, A. S.
Morison, G. D.
Muetterties, J. H.
Munro, R. E.
Munterich, G. C.
Nelson, D. A.
Newman, S. H.
Niles, C. L., Jr.
Oien, R. G.
Otteson, P. M.
Petz, E. F.
Phillips, H. J.
Pinney, A. D.
Pollack, R.
Portermain, N. W.
Richards, H. R.
Richardson, J. F.
Rinehart, C. R.
Roberts, L. H.
Rodermund, M.
Rosenberg, N.
Ross, J. P.
Rowell, J. H.
Ruchlis, E.
Salzmann, R. E.
Scheel, P. J.
Scheibl, J. A.
Schloss, H. W.
Simon, L. J.

FELLOWS

Skelding, A. Z.
Skurnick, D.
Snader, R. H.
Stewart, C. W.
Tarbell, L. L., Jr.
Walters, M. A.
Ward, M. R.
Wilcken, C. L.
Wilson, J. C.
Wittick, H. E.
Zory, P. B.

ASSOCIATES

Balko, K. H.
Bovard, R. W.
Chorpita, F. M.
Cohen, H. S.
Conner, J. B.
Daino, R. A.
Davis, R. C.
Drennan, J. P.
Durkin, J. H.
DuRose, S. C.
Golz, J. F.
Gould, D. E.
Hall, J. A.
Hardy, H. R.
Haseltine, D. S.
Head, T. F.
Jensen, J. P.
Kolodziej, T. M.
Krause, G. A.
Lindquist, R. J.
Lis, R. S., Jr.
McClenahan, C. L.
Miller, M. J.
Miller, P. D.
Moore, P. S.
Napierski, J. T.
Nelson, J. K.
Ori, K. R.
Powell, D. S.
Plunkett, J. A.
Rice, W. V.
Richardson, H. F.
Schultz, J. J.
Sheppard, A. R.
Singer, P. E.
Stephenson, E. A.
Thompson, E. G.
Torgrimson, D. A.
Trees, J. S.
Tverberg, G. E.
Wade, R. C.
Woll, R. G.
Wilson, O. T.
Winkleman, J. J., Jr.
Wood, J. O.
Wooddy, J. C.
Young, R. G.

GUESTS

* Anderson, E. V.
* Bell, A. M.
Billings, A. E., Jr.
Breslin, C. L.
Burgoyne, J. A.
* Butcher, R. W.
Countryman, G. L.
* Dunn, R. P.
* Eddins, J. M.
* Farmer, R. J.
Freifelder, L.
Gottlieb, M.
Gragnola, J. B.
Guaschi, F.
Halmstad, D. G.
* Hatfield, B. D.
* Hoyt, F. A.
Jones, T. L.
* Kaminoff, H.
* Kedrow, W. M.
Knox, F.
McMillen, R. H.
Richardson, W.
Schenck, B. R.
* Stevens, V. I.
Sorenson, W. W.
* Yoder, R.

*Invitational Program

Respectfully submitted,

Robert B. Foster
Secretary-Treasurer

Volume LX, Part II No. 114

PROCEEDINGS

November 11, 12, 13 1973

TOMORROW IS WHERE YOU LIVE

PRESIDENTIAL ADDRESS BY CHARLES C. HEWITT, JR.

". . . perfection is of this instant;
tomorrow is a new time,
and *tomorrow is where you live.*"[1]

The choice of topic for this Presidential Address will probably come as a disappointment to some of you who might have hoped that I would have dealt with some controversial actuarial matter; I have the feeling that in some quarters I have been regarded as a controversial member of the profession. There are many direct actuarial issues upon which I could focus my attention. However, it happens that I feel rather strongly that the most important thing I could contribute to the Casualty Actuarial Society at the present time is to put ourselves and our culture in a proper perspective. So, please do not search for direct applicability to our business or our profession in what I am about to say.

Most of us may be a little tired of hearing that we live in changing times or of hearing mind-boggling statements such as "the total knowledge of mankind has doubled in the last ten years." Nevertheless, I doubt if I will find many arguments from my audience today if I make the statement that rapid change is the chief character of our age.

If this be an acceptable premise, then it is vitally important that we

[1]Garet Garrett (well-known newspaperman of the early twentieth century) in a letter to Adolph Ochs (publisher of *The New York Times)* in 1921- as quoted in *The Kingdom and the Power* by Gay Talese (The World Publishing Company).

place ourselves in a logical perspective. I propose to do this, first, by looking at our instant culture and society and, then, to put this culture and society into an historical perspective. From such a perspective we should be able to see much more clearly where we have been and, more importantly, where we are most likely to be going. In a sense I will be asking you to climb with me for a few moments out of the valley of your current problems and thoughts and climb to the top of a mountain from which we can view with some detachment longer range problems and from which we can share longer range thoughts.

For a good fundamental understanding of our present culture and society and of ourselves as individuals I turn to the work of the Harvard psychologist, Abraham Maslow, of which many of you have heard and with which some of you are more familiar than I. Maslow postulates that human values relate directly to human needs and that, in fact, the two words "needs" and "values" may be used interchangeably. Thus, in referring to groups of human beings we can use the hyphenated phrase "needs-values" in discussing both the needs and the values of the group.

In today's society, says Maslow, there exists a hierarchy of "needs-values" with five distinct stratifications:

1. Survival
2. Security
3. Belonging
4. Esteem

---

5. Self-actualization

Let us, for convenience, regard each of these categories as mutually exclusive and let us examine each layer of society briefly in order to put this hierarchy in perspective.

At the very lowest level of our culture is that group of individuals whose needs or values center around mere survival—in other words they simply concentrate their efforts upon staying alive. Their need is for food, drink, sleep, warmth, etc. Individuals in this level of society are closely tied to their physical enviroment and their problems relate to conquering this environment rather than achieving a permenent place in society.

SURVIVAL
(10%)

The next step upward in the sociological scale from the mere need to survive is the need for protection from threats in the future—threats which

SECURITY (15%)

may come from Nature or from Society itself. Individuals at this level have found the wherewithal to survive, but desire to protect this wherewithal both for the present and for the future.

BELONGING (42%)

The third rung on this hierarchical ladder finds that group of individduals which has graduated from merely caring about their own welfare and now finds a need to be a part of something bigger than themselves—something such as a family, a clan, a lodge, a veteran's organization, an ethnic group or a nation. Persons in this needs-values group desire to love and be loved—to fit in with other people.

ESTEEM (30%)

The fourth rank, and, in one sense, the highest rank, is the next step upward from merely belonging to something or some group. It contains those individuals who want others to think well of them—they want to stand out in the group. Persons in this group want self-respect, but more than that, confidence, status and importance. In our society this can be achieved in many ways—power, wealth and professional attainment are just a few of the bases for Esteem.

Before discussing the fifth, and final, group of needs-values it is important to underscore the hierarchical nature of the first four levels. Security is attained only after Survival is assured. Belonging is only valuable if Security has been obtained and Esteem is clearly founded upon the sense of Belonging to some group whose Esteem is desired or needed.

SELF-ACTUALIZATION (3%)

However, the fifth needs-values group is different—and in an essential way which we'll come back to later. For lack of a better term this level is called Self-actualization and is possible at any of the other needs levels, but is more often found when all of the other needs have been satisfied. It may best be described as the need for the psychologically mature individual to express, to the fullest extent, his own interests and talents and, in his own way, "to do his thing." Persons in this group desire to achieve an inner potential rather than an outer display. Self-satisfaction is more important than fame.

A better understanding of these needs-values groups may be obtained if we also view them as hierarchical steps in the personality development of each individual. At birth the baby cries for food, drink, and warmth—for its Survival. Next we witness the baby sucking its thumb, holding on to a blanket or crying when about to be separated from a parent—a need for Security. Next I suggest you watch the behavior of a well-adjusted five or six-year-old in a family gathering. The child is most anxious to see that everyone, including the child itself, feels that he or she is part of the whole group—experiences togetherness. Then, somewhere during the school years, if it is going to be evidenced at all, there comes a desire to shine—academically, athletically, socially, or politically. And, finally, in psychologically mature individuals, comes the inner satisfaction of knowing when one has done well, or done the right thing—a sense of satisfaction which does not require that all others acknowledge this achievement.

You will have noticed during the exposition of the Maslovian needs hierarchy that I have placed numbers against each level. These numbers represent reasonable estimates of that proportion of our population in this country today which falls within each level. Clearly then the "needs-values" of Belonging and Esteem, with 72% of the people, dominate our present culture. Clearly, also, the number of individuals in the Belonging class has been substantially augmented during the past generation. Aren't these individuals what Nixon has chosen to call "the silent majority"?

It is most important to recognize that we have a society with at least five distinct sets of needs and values—each level demanding in its own way to be listened to and to be satisfied. We have a pluralistic society; we each understand and respect that there are other groups of individuals with needs and values different from our own—each with a right to fulfillment of these needs. I have already emphasized that change is the essence of our era, but it is almost as important to recognize that a Polaroid picture of our present society would emphasize the variability rather than the singeleness of purpose within our culture.

Lest I lead you into the trap of thinking that "Belonging" is the dominant force in our culture, merely because a plurality of our citizens are in this group, I must point out that simply counting heads does not measure the force and effect of various needs and values upon society. For a true measure a "power profile" rather than a "numbers profile" is essential. For it is the needs and values of those in power which most accurately determine the tone of a society. The United States today is clearly an Esteem-oriented society because that is the orientation of those in power. Attributes most sought after are financial resources, political influence, business or professional position, anything which is scientific and modern.

Up until this point in my address I hope I have had the indulgence of those members of my audience who heard me give a similar discourse at a meeting of the Midwestern Actuarial Forum several years ago. I hope their forbearance is justified because, in our effort to understand our tomorrow, I now intend to put the future in a long range perspective.

The five needs-values levels of the Maslovian hierarchy are the products of the total evolution of mankind over a period of approximately four million years—give or take a few hundred thousand years either way. To understand this four-million year development we will make use of a thesis[2] that there are basically four stages in the historical evolution of our society:

1. Primitive Survival Society
2. Primitive Identity Society
3. Civilized Survival Society
4. Civilized Identity Society

Without corrupting the pattern of the author of this thesis I would like to suggest that the transition from Survival to Identity in each instance involves going through half-phases that might be labeled Primitive Security and Civilized Security Societies.

In the earliest stages of his evolution in an anthropoid form mankind had one simple goal—to survive in a rigorous and often hostile environment. This fight for survival so pre-occupied him that he had little time for other matters. The question may logically be asked, "Why did man survive in such a hostile environment when many other forms of animal life—for example, the dinosaur—disappeared from the face of the earth?" The principal answer seems to be that he learned to cooperate with his fellow human beings. Those individuals who did not learn to cooperate did not survive as frequently, hence did not procreate as frequently. The process of natural selection favored those forms of human life which found—in cooperation— the easiest means to survive.

PRIMITIVE SURVIVAL & SECURITY SOCIETY (3,500,000 yrs.)

The relic of this Primitive Survival Society in our modern world is the lowest needs-value level of the Maslow hierarchy. For practical purposes there are no examples left of whole societies of this type, although we come

[2] *The Identity Society* by William Glasser (Harper & Row)

pretty close to such an existence among an African people known as the Ik tribe[3]. Colin Turnbull, an anthropologist who lived with these people for three years during the last decade, reports that the Iks had been brought to a level of near-starvation because the government of Uganda had forbidden them to hunt—their normal method of procuring food—and instead had ordered them to farm on land that was semi-desert in character. In fairness to the government of Uganda it should be mentioned that there was a desire to protect the wild animals of the region, but unfortunately this decision meant saving the lives of animals at the expense of the lives of people.

In the Ik language the word for "goodness" is simply "food" and a "good" man is a man with a full stomach. There are many chilling stories told. Perhaps the most frightening concerns a mother who was delighted when a leopard made off with her own child because it meant that she was relieved of the responsibility of carrying the child about and of providing food for it. Because we cannot possibly understand what living in a Primitive Survival Society must be like, this experience of Turnbull's is important to review as a grim lesson for people in an affluent society.

Toward the end of the three and a half million year period covering the Primitive Survival Society, people began to achieve a better than survival level. Mankind made a transition from a Primitive Survival Society to what might be referred to as a Primitive Security Society. First, as man learned to cooperate with other men, he found increasing freedom from the stress of survival efforts, greater security and hence more free time for rest periods. During the rest periods he found that he could engage in activities which actually gave him pleasure. Among those things that gave him pleasure was the company of other human beings. Because he enjoyed the company of others he became involved in their affairs. Ultimately this led to the establishment of individual identities—each person recognizing his own identity, his own peculiarities, his own pleasures and desires and, more importantly, the same inclination among other people who might even have different peculiarities, pleasures and desires.

Thus mankind arrived at what we will call, for convenience, a Primitive Identity Society. Man wanted to become involved in matters which concerned his fellow human beings. He wanted to create an identity.

[3]*The Mountain People* by Colin Turnbull (Simon & Schuster)

PRIMITIVE IDENTITY SOCIETY (500,000 yrs.)

Depending upon the level of such identity, each man attained status. A principal means for identity and status were kinship systems—particularly tribal groups with their concomitant rituals, ceremonies and dances. Examples of Primitive Identity Societies are found today and in the recent past. In the twentieth century we have the Bushmen of the Kalahari Desert or the recently discovered Tasaday Tribe in the Southern Philippine Islands. In the nineteenth century we had the Cheyenne Indians of our own Great Plains.

Developments within the Primitive Identity Society plus the introduction of agriculture (as opposed to hunting and gathering of food) contributed in a major way to the next major evolution of society— the development of the Civilized Survival Society. An important circumstance within the Primitive Identity Society which led to this change was the beginning of large families—large families required more room and more room meant territorial expansion and impingement upon other Primitive Identity Societies. At the same time it is impossible to over-estimate the effect of the introduction of agriculture. Agriculture required a fixed base for farm activity and made land a valuable commodity; whereas in the earlier hunting and gathering activities, land was a disposable commodity.

Although there were undoubtedly tribal conflicts among Primitive Identity Societies, these conflicts were basically "status things" and did not represent in any sense major warfare. However, with the increase in the number of human beings and the importance of fixed territories for their living and for the growing of food, tribal conflicts turned into wars for geographical boundaries. In this activity was born the third principal state of the evolution of our society—the Civilized Survival Society. Very simply stated, the goal of this Civilized Survival Society is to survive in an environment of conflict with one's fellow human beings. The means for this survival involved aggression—man sought to overpower, destroy or enslave other men who might threaten his survival. In the Primitive Survival Society man sought to get the best of his physical environment. In the Civilized Survival Society, man sought to get the best of his fellow man.

CIVILIZED SURVIVAL & SECURITY SOCIETY (10,000 yrs.)

Examples of aggressions within the Civilized Survival Society are the many wars that have taken place since we first began to record history. Perhaps the least palatable examples of these aggressions are the interactions with more primitive societies—for example the manner in which European colonists and the American frontiersmen seized land from the American Indians. Another example of aggression is the enslavement of Africans by their fellow Africans who then sold them as slaves to ship captains for delivery into the New World.

Acts of aggression in a Civilized Survival Society have been rationalized as being necessary for survival. Even in today's highly developed culture it is accepted that an individual will do things otherwise unacceptable, justifying such action on the basis of surviving. "I was only doing my job" is a commonplace excuse for many things in our culture.

An inevitable result of any struggle for survival is that a large number of individuals become subservient to the smaller group who are leaders. Those who are subservient are expected to support all group efforts and to work toward the goals established by the leadership group. Inevitably a hierarchy of control by one man over another comes into being.

The followers become conditioned to "doing their duty." A good illustration of the philosophy of doing one's duty (often without really knowing why) is found in the oft-quoted nineteenth century poem *The Charge of the Light Brigade* by Tennyson. Another nineteenth century example is found in the following quotation from our own Ralph Waldo Emerson—

"So nigh is grandeur to our dust
So near is God to man
When duty whispers low 'Thou must'
The youth replies 'I can' "

While it is easy to imagine these lines being used by high-school valedictorians in the early years of this century, I wonder if you would expect to hear them on many college campuses today?

The Darwinian process of natural selection in a Civilized Survival Society favors the procreation of the types who are either aggressors or the types who are willing to be subservient and tends to discourage the existence of individuals (in between these two levels) who might wish to establish and preserve an identity other than that of aggressor or follower. Thus it is extremely difficult for individuals to establish and preserve their identity in a Civilized Survival Society. Instead this Society forces them to move toward goals and norms established by those in power. Individual expression which is not normative to the goals of society is actively discouraged by both the leaders and the followers.

Ultimately, the stronger Civilized Survival Societies developed a certain security in much the same sense as the later stages of Primitive Survival Societies developed security. Just as primitive mankind found rest periods from which he could relax in the struggle against his environment the civilized man found rest periods, when conflicts or wars were not necessary for his survival and, during these periods (Civilized Security Societies), mankind has begun to assert the question of individual identity within civilized society. A good example of this assertion is our own Declaration of Independence and our Constitution with its consequent Bill of Rights. Other civilized Security Societies, many in (or deriving from) Western Europe, developed along similar lines. Examples would include Canada, Switzerland, Sweden, Australia, and New Zealand, but there would also be many others.

CIVILIZED IDENTITY SOCIETY

(20 yrs.)

Sometime after the end of World War II the United States and other countries had achieved a level of affluence and a diminution in the quest for survival to the extent that many individuals in these societies (particularly the younger people) no longer felt the tremendous stress of the conflict for survival. They no longer felt the tremendous concern for Security, Belonging and Esteem which are the end products of the sum of our cultures. Instead these individuals substituted a desire to establish their individual identities, which identities might be independent of the goals and norms of the society which considered them as members. Thus began, sometime within the period since the end of World War II, the fourth major stage of society which can appropriately be referred to as a Civilized Identity Society.

Evidence of the need to establish individual identities within our civilized culture are all around us. Most of them are found among our younger people. The desire to dress differently, to have a different life style, wear long hair, beards, and so forth, is clear evidence, albeit superficial, of individuals or groups of individuals saying to the other members of society that they don't necessarily subscribe to the goals of that society; that they feel it is more important to discover first what their identity really is.

Clearly these are the individuals to whom Maslow refers as Self-actualizers. So we see that the level of Self-actualization is not a fifth level of the needs-values hierarchy of a Civilized Survival Society (and its predecessor societies) but rather the first wave of an entirely new process in the evolution of mankind.

Some of you in my audience are remnants of a Civilized Survival Society. Some of you are charter members in a Civilized Identity Society. But I suspect most of us are, unwittingly, trying to bridge the gap between the two. Those of you who are still deeply committed to the goals established by the Civilized Survival Society may not have established an identity of your own; you certainly have difficulty understanding the new generation. On the other hand the new generation, born in affluence and steeped in security, clearly has great difficulty in understanding the problems and hence the goals of a Survival Society.

So at this moment in history we do not have just a generation gap in the old historical sense in which each successive generation claims that it failed to understand "what the younger generation was coming to." Rather —and this is most important to all of us to understand—we have superimposed upon a normal generation gap a genuine cultural gap between the last stages of a Civilized Survival Society and the embryonic stages of a Civilized Identity Society. However, unlike earlier transition periods between major cultural eras, we are not afforded the luxury of time—a luxury in earlier eras of millions or hundreds of thousands of years. We must solve the problems created by our cultural gap in a matter of tens of years.

A great deal of what I have said could be aligned with similar thinking which has appeared in such books as *Future Shock* by Alvin Toffler and *The Age of Discontinuity* by Peter Drucker. However, Toffler concentrates rather heavily on technological change in the near future and Drucker concentrates of discontinuities in our economic and political systems. If I have made myself clear, you should now understand that I am talking about change in human behavioral patterns. The whole point of this Presidential Address is that these future changes in behavioral patterns will dominate either technological, economic or political changes.

In a Civilized Survival Society, where most individuals have values and goals associated with Belonging or Esteem, it is essential that the individuals adapt their behavioral patterns to technological change, to economic change or to political change. The title of my address is "Tomorrow

Is Where you Live." It is becoming clearer all the time that the tomorrow in which you and I will live will find technology, economics and politics adapting to change in human behavior.

Two great movements in this country today are perfect illustrations. Consumerism has at its very core the principal that economic change and technological change must adapt to the behavioral patterns and values of the people in a society. Similarly, Environmentalism has at its core that technologies must change and adapt to the needs and values of the society of today and tomorrow.

What can we expect from the balance of this century in which we live? We can expect a shift away from Esteem orientation. As an illustration, today's college students often view the goals of money and influence as anachronisms. It is possible that overt displays of wealth will be replaced by more deeply meaningful personal goals. Authoritarian use of power (already under severe attack) will be replaced by persuasion based upon giving people insight and understanding of their own needs and values as well as the needs and values of society as a whole.

By the end of this century it is probable that more than half the population of the United States will be in either the fourth or fifth levels of the Maslovian needs hierarchy. We can therefore expect a full flowering of individual expression instead of a uniform society where everyone is expected to have the same set of values. It is a virtual certainty that we are moving into an era of pluralism where the multiplicity of needs and values of individuals is recognized and tolerated. I think we are already well into the period when a single purpose, a single goal or a single answer no longer satisfies the diversified interests of the individuals in the higher level needs-values groups. The "American Way" which has been a rallying point for our entire society since World War I is already being replaced by subcultures and this change will continue to evolve. Finally, and perhaps restating the obvious, the emphasis on material need which is at the root of many of today's social values will give way to emphasis on the ability to make a personal choice. Measures of value which are internal to the individual will become more important than the measures of value which are external.

In our lifetime it seems to me that the greatest need will be for understanding—I don't think that mere tolerance will be sufficient. We need a generation of transitionalists who will understand that tomorrow's society, where there is a pluralism of needs, values and activities, rests upon the foundations of security and the ability to survive. On the other hand transitionalists must realize that to project the future merely as a straight line extension of past progress made during the evolution of a Civilized Survival Society is impossible.

So the challenge that I give you today is first to understand where we are in the perspective of the development of our culture, to understand the pluralism of the needs and values of modern society, and to work toward giving others who are more strongly committed to either a Survival Society or an Identity Society the same understanding that we have of the importance of bridging the gap between these two. We cannot rest upon past successes. We must prepare for the future by understanding what is in store for us.

Lest I be accused of decamping completely from a Civilized Survival Society, I will conclude with a poem by its archetype, John Wooden:*

"Remember this your lifetime through
Tomorrow, there will be more to do
And failure waits for all who stay
With some success made yesterday"

*Basketball coach at UCLA

LOSS RESERVE TESTING: A REPORT YEAR APPROACH

WAYNE H. FISHER AND JEFFREY T. LANGE

"Two roads diverged in a wood, and I—
I took the one less traveled by,
And that has made all the difference."
—Robert Frost

During the nineteenth century, the functions of the actuary were twofold: the calculation of premium rates and the setting of reserves. While reserves for life insurance may be set by actuarial formulae, casualty reserves are more frequently established by claims adjusters on an individual case basis. One of the tasks of the casualty actuary is to test the adequacy of reserves. Such tests are generally carried out on a statistical basis, reviewing a whole portfolio of case reserves at one time. As a result of the actuary's test, it may be necessary to increase or decrease existing reserves, add special bulk reserves, or issue new instructions redirecting the claims adjusters in the setting of reserves. The significance of these steps for the financial solvency of the company cannot be overestimated. As Balcarek has clearly shown,[1] changes in the degree of reserve adequacy have a very substantial impact on the earnings of casualty insurance companies. Thus, reserve tests should be, and usually are, the concern of senior management.

The most familiar reserve tests are those incorporated in the annual statement. Currently, Schedule P is organized on an accident year basis and provides a means of performing a reserve test. However, it can be argued that Schedule P alone cannot be used to determine the amount of overstatement or understatement of the current reserves for a given line. Skurnick[2] includes more elaborate accident year tests in his survey of reserve calculations and, no doubt, the accident year approach is favored by many actuaries for reserve evaluation.

This paper will discuss a reserve test which is based on an alternative scheme for organization of the data. Fundamental to this approach is the tabulation of claims (both reserves and payments) by report year. The

[1]R. J. Balcarek, "Effect of Loss Reserve Margins in Calendar Year Results", *PCAS* Vol. LIII, p. 5.
[2]D. Skurnick, "A Survey of Loss Reserving Methods", *PCAS* Vol. LX.

latter is defined to be the year in which the claim was reported to the company, regardless of the accident date. In his survey, Skurnick mentions two methods of using report year data. The first is a projection, or loss development, approach dealing with total report year losses at various points of development. The second is a payment development method in which it is assumed that the percentage increase in the ultimate incurred value will be the same as the increase in average paid claim costs for claims of a similar age. In this paper, we will describe a third, more complex method utilizing report year data, and will then proceed to show how this approach is used in practice.

The report year approach is designed to test the adequacy of a portfolio of reserves for known cases. By known cases, we mean cases that have been reported to the company and for which the claim department has established a reserve. Since the method is a test, it is possible to apply it even in situations where the claim department employs an average reserving or similar system in setting some reserves. The test is usually applied to the entire known case reserve for a given line, such as auto bodily injury liability; however, in some cases it has been applied to subdivisions of a line, when the subdivision itself constitutes a unique reserving problem. The test is customarily applied only to third party lines. It is designed to reveal whether the reserve is adequate, and to measure: (1) the extent of any redundancy or inadequacy, (2) the slippage or strengthening of the equity position of the reserve since the last evaluation, and (3) the contribution of various report years to the overall reserve position. The first two results are significant in the financial position of the company in that one deals with solvency while the second deals with possible distortions in the income statement. The third result is of value in the administration of the claim department in that it tells us whether any redundancy is due to old or new cases. In other words, it indicates where corrective action is needed and, in later evaluations, monitors that corrective action. While the first two results are, of course, attainable under an accident year approach, the third result is not available in exactly the same format unless one uses a report year approach. It is believed that, from the point of view of administering a claim department, the report date is more significant than the accident date. Claim department policy would deal with claims reported during some time frame. That policy, or changes in it, can be tested by examining appropriate report (not accident) years.

There are three key features in the report year methodology. As previously mentioned, the first is that the data are organized by report year. This implies that the number of cases for any report year to be tested is fixed at the close of the year. Such a method is superior to an accident year approach in which the number of cases is subject to change at each successive evaluation. Thus, the report year approach substitutes a known quantity for an estimate. The second key characteristic is that virtually all parameters for the reserve test may be estimated from paid loss data. This is an advantage over those techniques employing loss development factors in which reserves are included in the calculation of the parameters of the test. Hence, the report year approach provides a truly independent check on the reserves. A third characteristic of the methodology is that it can be readily adjusted to reflect management's views on the change in the way in which claims will be disposed of by the claim department and on the change in the future rate of inflation. It is possible to show the impact of different rates of inflation on the adequacy of the reserves.

In the next section, we explain the report year methodology using a detailed example. First, the organization of the data base is outlined. Next, the two fundamental calculations are described: the estimation of claim costs and the computation of the disposal rate of claims. Then, these results are combined and the equity position of the reserve is determined. Since one goal of the paper is to instruct actuarial students who are unfamiliar with loss reserving techniques, a complete step-by-step description is provided. While actual data have not been used in this exercise, it has been constructed so that it is representative of third party lines.

This report year test first requires that the various outstanding claims in a given reserve be divided into groups (report year reserves) depending on the year in which the case was initially reported to the company. For example, assume the December 31, 1973 reserve for known cases in a particular line is $55.0 million on 13,761 cases; of these 13,761 cases, 8,372 cases were reported in 1973, 2,764 cases in 1972, 1,416 in 1971, 787 in 1970, and the remainder reported in prior years. Each group of cases, of course, has associated with it an aggregate dollar value—the sum of the individual estimates established for those cases by the claim department.

In order to develop the individual report year reserves, claim data must be available which separate and track the development of claims reported in a given year. The following data, for example, would be maintained on those claims reported in 1970:

TABLE 1

1970 Report Year Statistics on Paid, Outstanding and Incurred Bases

Calendar Year	Cumulative Paid/Closed Number	Cumulative Paid/Closed Dollars (000)	Outstanding Number	Outstanding Dollars (000)	Incurred Number	Incurred Dollars (000)	Incurred Average
1970	10015	$ 5458	11248	$18304	21263	$23762	$1118
1971	17478	14011	3785	12916	21263	26927	1266
1972	19477	20758	1786	8762	21263	29520	1388
1973	20476	25071	787	5154	21263	30225	1421

The report year data above are assembled in this format to facilitate the comparisons and calculations described later. Several items on the above table should be noted now, however. First, the number of cases incurred for the report year (21,263) is determined by adding the number of 1970 report year cases actually paid in 1970 (10,015) to those 1970 report year cases still outstanding at the end of 1970 (11,248). This figure for the number incurred is then "frozen" for this report year in the subsequent years of development (i.e., a claim closed without payment after the initial year does not decrease the number of claims incurred for the report year). The values in the first column are a combination of claims paid and claims closed—with or without payment. For the initial year, in this case 1970, the figure represents the number of 1970 report year cases actually paid during 1970 (10,015). For the subsequent years, the number in this column is the sum of the cases paid during the initial year and those cases closed, with or without payment, in the following years. The number of cases closed is the difference between the number outstanding at the beginning of the period and those still outstanding at the end of the period.

Arranging the data in the above format allows one to notice several characteristics which are fundamental to the report year methodology. During 1970, roughly half of the total number of claims incurred for the report year were settled, but less than one-fourth of the total dollars incurred were paid on these cases; thereby implying a relatively low average claim cost. In the subsequent years, fewer claims were actually settled, but they were settled at relatively higher averages. In their report year approaches, Harnek and Sampson[3] did not utilize this difference in the average claim costs by age; however, this difference is an essential feature of the approach presented in this paper.

The data in the above table are presented on a cumulative basis; reassembling these data on a segmented basis, year by year, enables one to observe the pattern in average claim costs more readily. For example, the paid data for the 1970 report year would be displayed as follows:

TABLE 2

1970 Report Year

Calendar Year of Closing	Time Since Beginning of Report Year	Number Paid/Closed	Dollars Paid (000)	Average Claim Cost
1970	(0-12 months)	10015	$5458	$ 545
1971	(13-24 months)	7463	8553	1146
1972	(25-36 months)	1999	6747	3375
1973	(37-48 months)	999	4313	4317

With the report year data in this format, one notices a pattern of increased average claim costs as the time of settlement moves farther away from the year in which the claim was reported. This pattern is typical of that seen for other report years, and implies that the claims settled three years after the year in which they were reported are substantially different from the claims settled, say, in the first year.

In addition, this methodology incorporates the assumption that the claims settled in the same time period (i.e., 13 to 24 months after the beginning of the report year) are essentially similar type claims, and can be compared with claims from other report years which were settled in the same relative time period. In the following table, the data for the 1970 report year are combined with data for other report years, grouping together, for comparison, those averages relating to claims settled in the same time frame.

[3] R. F. Harnek, "Formula Loss Reserves", *Insurance Accounting and Statistical Association Proceedings,* 1966 and R. T. Sampson, "Establishing Adequacy of Reserves on Slow Closing Lines—Use of Paid Formulae", *Insurance Accounting and Statistical Association Proceedings,* 1959.

TABLE 3

Average Claim Cost for Claims Settled in Interval Indicated

Age of Claim Measured in Number of Months from Beginning of Report Year to Settling of Claim	Report Year									
	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
0—12	398	393	413	444	495	577	545	577	612	698
13—24	790	871	837	961	1084	988	1146	1181	1466	
25—36	2348	2128	2288	2471	2438	2865	3375	3598		
37—48	2430	2500	2998	3146	4261	4344	4317			
49—60	3429	2630	3425	3173	4681	5285				
61—72	2572	3629	2944	4034	5211					
73—Ultimate*	1934	3114	5931	4228	4934					

*These averages include the current claim department estimate for any claims still outstanding.

This composite exhibit allows one to observe the effect of inflationary factors on the average claim costs from year to year. It is important to note that the claim costs are increasing at difference rates for the various age-of-claim groups. For example, the claim costs in the 49 to 60 month group are increasing nearly twice as fast as the claims in the 0 to 12 month category. The actual increases are shown later, and at this point only the fact that the percentage increases are different, or at least can be different, is important. This methodology incorporates these different trends by projecting average claim costs independently for each age-of-claim group by utilizing the historical trend for that group. The approaches described by Harnek and Sampson do not reflect this phenomenon.

The requirement for projecting these claim costs can be seen in Table 3. For the 1970 report year, we know that the cases settled in the initial year were settled at an average of $545. Similarly, those settled in the next year were settled at an average of $1,146; in the next year, $3,375; and so forth. We do not know, however, what the average will be for those cases that will be settled in 1974 (49 to 60 months), or in the years after that. Assuming for the moment that we know the number of cases that will be settled in each age-of-claim group for this report year and that we can project the necessary future average claim costs, then we can obtain a weighted average incurred claim cost for the entire report year. This weighted average incurred claim cost can then be compared to the actual average incurred claim cost, based on the claim department estimates, to determine the current reserve adequacy for the particular report year. An average incurred claim cost, rather than an average outstanding claim cost, is utilized for clarity in presentation. This will be explained later.

Projecting the necessary average claim costs can be done in many ways; however, certain techniques work well with this methodology. Table 4 displays, in parentheses, projected averages based on a least squares fit of an exponential curve to the available data for that age group. The exponential curve was utilized as it implies a constant percentage increase in inflation, and this was felt to be most indicative of the situation today. In addition, projections based on a weighted exponential least squares fit are also valuable, as they give added weight to the more recent experience. A linear projection could also be utilized; however, this is unrealistic due to its implied decreasing percentage trend.

Table 4 also displays the percentage increases underlying the individual exponential projections. These percentages can then be weighted

TABLE 4

Average Claim Cost for Claims Settled in Interval Indicated

	Report Year										Average % increase in Claim Cost
Age of Claim	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973	(exponential)
0—12	398	393	413	444	495	577	545	577	612	698	6.6
13—24	790	871	837	961	1084	988	1146	1181	1466	(1426)	7.0
25—36	2348	2128	2288	2471	2438	2865	3375	3598	(3639)	(3906)	7.4
37—48	2430	2500	2998	3146	4261	4344	4317	(5251)	(5883)	(6591)	12.0
49—60	3429	2630	3425	3173	4681	5285	(5368)	(5986)	(6676)	(7445)	11.5
61—72	2572	3629	2944	4034	5211	(5624)	(6546)	(7620)	(8869)	(10322)	16.4
73—Ult.*	1934	3114	5931	4228	4934	(7216)	(8973)	(11158)	(13874)	(17252)	24.3
											9.1%**

NOTE: Numbers in parentheses are projected values.

*These averages include the current claim department estimate for any claims still outstanding.

**Weighted average of percentage increases by age of claim, with weights proportional to the product of the appropriate claim costs (above) and disposal rates (from Table 6) for the latest report year (1973).

to obtain the overall percentage increase inherent in the estimate, in this case 9.1%. If this increase is felt to be either excessive or inadequate (presumably because of information from some external source), the projected averages can be modified to reflect the anticipated rate of increase.

After obtaining projected average claim costs, the second step is to determine the number of claims which will be settled for each age group. From Table I we observe that, for the 1970 report year, 47.1% of the total cases incurred were settled in the first year, 35.1% in the second year, 9.4% in the third, and so forth. Combining these percentages, which we will refer to as disposal rates, with similar data for the remaining report years, we notice definite patterns from report year to report year. Table 5 displays these disposal rates in the same format as the claim cost data in Table 3.

Examining the disposal rates in this format allows one the opportunity to observe any trends in the pattern of claim settlement. For example, these data show a lessening in recent years in the time required to settle claims. Settling 50.2% of the 1973 report year claims in the first year (as compared to 47.7% in this time frame for each of the two prior report years), 36.7% of the 1972 report year claims in the 13-24 month group (as compared to 35.0% and 35.1%), and 10.1% for the 1971 report year (as compared to 9.4% and 7.9%) bear out this observation. It should be noted that these percentages—50.2%, 36.7%, and 10.1%—along with the remaining values on that diagonal, pertain to those claims settled in the latest calendar year (1973).

The speed-up in claim settlement noted above may be the result of a deliberate plan by the claim department; in this example we assumed that it was planned and would extend into the future. Accordingly, the anticipated disposal rates shown in parentheses in Table 6 were selected with this in mind. The selected values may appear to be low for certain intervals; however, this is the result of settling more claims in the earlier periods, thereby leaving fewer claims to be settled later.

Selecting disposal rates can be done in a variety of ways. If one feels the recent pattern is representative of current claim settlement practices, the disposal rates from the most recent calendar year can be utilized exclusively; or a weighted average of the last few years can be employed if such is felt to be more in line with anticipated trends. Disposal rates from

TABLE 5

Percentage of Report Year Total Claims Incurred Settled in Interval Indicated

Age of Claim Measured in Number of Months from Beginning of Report Year to Settling of Claim	Report Year									
	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
0—12	.510	.503	.496	.505	.500	.497	.471	.477	.477	.502
13—24	.333	.333	.340	.334	.345	.344	.351	.350	.367	
25—36	.073	.081	.084	.087	.083	.079	.094	.101		
37—48	.037	.036	.038	.035	.033	.040	.047			
49—60	.021	.022	.020	.019	.021	.024				
61—72	.012	.012	.012	.010	.011					
73—Ultimate	.016	.013	.010	.010	.007					

TABLE 6

Percentage of Report Year Total Claims Incurred Settled in Interval Indicated

Age of Claim Measured in Number of Months from Beginning of Report Year to Settling of Claim	Report Year									
	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
0 - 12	.508	.503	.496	.505	.500	.497	.471	.477	.477	.502
13 - 24	.333	.333	.340	.334	.345	.344	.351	.350	.367	(.349)
25 - 36	.073	.081	.084	.087	.083	.079	.094	.101	(.091)	(.087)
37 - 48	.037	.036	.038	.035	.033	.040	.047	(.040)	(.036)	(.035)
49 - 60	.021	.022	.020	.019	.021	.024	(.022)	(.019)	(.017)	(.016)
61 - 72	.012	.012	.012	.010	.011	(.010)	(.009)	(.008)	(.007)	(.007)
73 - Ultimate	.016	.013	.010	.010	.007	(.006)	(.006)	(.005)	(.005)	(.004)

the most recent year were used in this example. Specifically, the 1973 report year value for the 13-24 month age group was determined as follows:

$$\frac{\text{(1972 report year claims settled 13-24 mos.)}}{\text{(1972 report year claims outstanding at 13 mos.)}} = \frac{.367}{1.000 - .477} = .702$$

$$.702 \times \text{(1973 report year claims outstanding at 13 mos.)} = .702 \times (1.000 - .502) = .349$$

Values for the remaining disposal rates were determined in a similar manner, utilizing the latest data for percentage of claims settled in each age group. A more mathematical projection technique, similar to that used in estimating claim costs, could have been used in determining disposal rates instead of that described above.

After determining the projected average claim costs and corresponding disposal rates, the estimated average incurred claim cost for a report year can be calculated. For example, the 1970 report year average incurred claim cost is determined by weighting the 1970 report year average claim costs in their respective age groups from Table 4 by the corresponding 1970 disposal rates from Table 6. Note that the first four products in the weighted average are actual observed values whereas the latter three are projections. The resulting estimated average incurred claim cost of \$1,410 may be compared to the actual average claim cost incorporating the current claim department reserves (\$1,421, as shown in Table 1). This difference (\$1,421 − \$1,410 = \$11) is then multiplied by the total number of incurred claims (21,263 from Table 1) to determine the dollar redundancy for this reserve (\$11 × 21,263 = \$234,000). In this case, the actual average incurred claim cost exceeds the estimated value and the reserve is redundant by \$234,000. If the estimated value had exceeded the actual average, the reserve would have been deficient. This redundancy, or deficiency, is the reserve equity. In the following table, the report year equity positions for the remaining years are calculated.

TABLE 7

Calculation of Report Year Reserve Positions

	Report Year				
	1969	1970	1971	1972	1973
Estimated Average Incurred	$ 1,253	$ 1,410	$ 1,493	$ 1,618	$ 1,679
Actual Average Incurred (at 12/73)	$ 1,259	$ 1,421	$ 1,508	$ 1,613	$ 1,637
Margin per Claim Incurred	+$6	+$11	+$15	−$5	−$42
Number of Claims Incurred	20,462	21,263	22,613	23,124	23,716
Report Year Researve Position	+$123,000	+$234,000	+$339,000	−$116,000	−$996,000

The current, overall equity position for this reserve can now be determined by adding the individual results (i.e., the last line in the table) obtained for the various report years. Accordingly, this reserve appears to be deficient by $416,000. It is interesting to note, however, that the equity positions vary greatly between the various report years, with the older claims being over-reserved and the more current claims being under-reserved. Information of this type may be useful in giving guidance to the claim department.

This same picture of slippage in the recent claims can be seen in a comparison of the estimated and actual average incurred claim costs set forth in Table 7. The estimated average (i.e., the first line of data in the table) for the 1973 report year is up 3.8% over the 1972 report year average. A greater increase would have been expected based on the 9.1% rate of inflation inherent in the estimate; however, the substantial acceleration in disposal rates serves to lower the estimated averages and the percentage increases from year to year. This 3.8% increase in the estimated average exceeds the 1.5% increase in the actual average (the second data line in the table), thereby indicating that the claim department may be establishing too low an average on the recent cases. Similarly, the 1972 report year estimated average shows an 8.4% increase over 1971, while the actual average increased only 7.0%. Graphing these estimated and actual average incurred claim costs, or the percentage increases from report year to report year, can be helpful in presenting the results of the report year test.

We have now determined the equity position of the December 31, 1973 reserve for this particular line. In order to obtain the strengthening, or slippage, in this reserve during 1973 we must first calculate the equity position of this reserve at December 31, 1972. An increase in reserve equity at December 31, 1973 indicates a strengthening during 1973, while an increase in the reserve deficiency (or decrease in savings) indicates a slippage during the year.

Table 8 sets forth equity positions for the 1971, 1972, and 1973 year-end reserves. These equity estimates are obtained by adding the savings already emerged for the report year to the current estimate of the savings to emerge in the future. The savings already emerged would be determined from report year information similar to that set forth in Table 1. Specifically, subtracting the 1970 report year dollars incurred as of December 31, 1973 ($30.2 million) from the dollars incurred as of December 31, 1971 ($26.9 million) yields the emerged savings on the December 31, 1971 reserve (−$3.3 million).[4]

Examining Table 8, we notice a substantial strengthening ($3.6 million) for this reserve during 1973 (going from a deficit of $4.0 million at December 31, 1972 to a current deficit of $0.4 million) following a slippage ($0.7 million) during 1972 (from a deficit of $3.3 million to a deficit of $4.0 million). These movements in the level of reserve equity have a direct effect on underwriting results, and accordingly could be used to restate such results on a more accurate basis.

It must be noted that this methodology, as described in the example, contains two potential sources of distortion: reopened claims and partial claim payments.

Reopened claims, if included in the report year data, will distort both the disposal rates and the average claim costs. For those lines with a substantial volume of reopened cases, such as workmen's compensation, the approach described in the example can be appropriately modified. However, since reserves are frequently maintained and tested separately for reopened cases, excluding these cases from the report year data developed for this test is probably the best solution.

[4] Equity positions are not calculated for report years prior to 1969. This methodology is not sensitive to report years which are already at 72, or more, months of development. Equity positions for these report years could be determined either by increasing the number of age groups—73 to 84 months, 85 to 96 months, etc.—or by projecting an estimated average incurred loss by applying a development factor to the average paid loss to date. In practice, the method is extended for certain slow closing lines. For this example, we will assume these early report years to be correctly reserved (no inadequacy or redundancy).

TABLE 8

Reserve Equity Position

($000)

December 31, 1971 Reserve

Report Year	Outstanding Reserve (at 12/71)	Emerged Savings (as of 12/73)	Current Position	Reserve Position
1968 and Prior	7,576	+ 110	—	+ 110
1969	8,724	+ 400	+123	+ 523
1970	12,916	−3,299	+234	−3,065
1971	18,432	−1,213	+339	− 874
TOTAL	47,648	−4,002	+696	−3,306

December 31, 1972 Reserve

Report Year	Outstanding Reserve (at 12/72)	Emerged Savings (as of 12/73)	Current Position	Reserve Position
1968 and Prior	3,566	− 360	—	− 360
1969	5,201	− 300	+123	− 177
1970	8,762	−1.522	+234	−1,288
1971	14,671	−1,843	+339	−1,504
1972	20,472	− 567	−116	− 683
TOTAL	52,672	−4,592	+580	−4,012

December 31, 1973 Reserve

Report Year	Outstanding Reserve (at 12/73)	Emerged Savings (as of 12/73)	Current Position	Reserve Position
1968 and Prior	904	—	—	—
1969	1,832	—	+123	+123
1970	5,154	—	+234	+234
1971	8,246	—	+339	+339
1972	15,125	—	−116	−116
1973	23,761	—	−996	−996
TOTAL	55,022	—	−416	−416

Partial claim payments also can be handled by modifications in the reserve test. In practice, this is done only for workmen's compensation, where partial payments are prevalent, although similar modifications may become necessary in auto bodily injury as No-Fault begins to exert greater influence. Partial payments involving allocated loss adjustment expense must also be considered in lines where allocated expenses are a major factor. Performing the test on pure indemnity data would, of course, remove this problem.

By summarizing algebraically the report year methodology described above, an alternative method of estimating the average claim costs can be derived. The procedure described thus far makes independent estimates of the average claim costs for each age of claim. The alternative method, on the other hand, estimates these average claim costs simultaneously for all ages of cases.

In making this summary, we will refer to cases settled within the first twelve months of the beginning of the report year as being in age group 1, cases settled within the thirteenth to twenty-fourth month after the beginning of the report year as being age group 2, and so on.[5] In the following notation, the age group will be denoted by the first subscript $_i$, while the particular report year (e.g., report year 1972) will be denoted by the second subscript $_t$.

n_{it} = number of cases closed in *i-th* age group in report year t

g_{it} = observed disposal rate for *i-th* group in report year t

$$= \frac{n_{it}}{\sum_{j=1}^{h} n_{jt}}$$

where h simply represents the number of age groups into which the report year data has been divided.

X_{it} = paid claim cost for *i-th* age group in report year t

$\overline{X}_{\cdot t}$ = ultimate average paid claim cost for report year t

$$= \sum_{i=1}^{h} g_{it} X_{it}$$

[5]This section will assume that only annual subdivisions of the data are available, although quarterly subdivisions are used in practice.

Of course, X_{it} is unknown[6] for $i > 1974\text{-}t$ in our problem where our latest complete report year is 1973. Thus, estimates of X_{it} must be used in computing $\overline{X}_{\cdot t}$. This latter quantity can be compared to the actual incurred average claim cost, $\overline{W}_t$, carried on the company books, where

$$\overline{W}_t = \frac{\text{Total incurred losses for report year } t}{m_t}$$

m_t = number of cases reported in year t, and

$m_t\,(\overline{W}_t - \overline{X}_{\cdot t})$ = future runoff savings (or deficit if minus) for the report year t reserved at the end of the current year.

Note that, for the current report year, the future savings equal the reserve position for the current reserve. In reevaluating reserves for prior calendar years, the emerged savings to date for that reserve must be considered:

e_c = emerged runoff savings (deficit if minus) on reserve for calendar year c at the end of the current year.

d_c = reserve position for reserve for calendar year c

$$= e_c + \sum_{t=T}^{c} m_t\,(\overline{W}_t - \overline{X}_{\cdot t}),$$ where T is the first year of data included in the analysis.

s_c = strengthening (slippage if minus) of the reserves during calendar year c

$$= d_c - d_{c-1}$$

The goal of our test was to compute d_c and s_c. The first quantity tells us about the equity position of the reserves and is related to the company's financial solvency. The second quantity gives us the dollar impact of reserve movements (change in reserving policy) on the income account for the year. Specifically, by dividing s_c by the earned premium for the line, the impact of a change in reserve policy may be expressed in terms of points of loss ratio.

We now have summarized the methodology as explained in the pre-

[6] The disposal rates, g_{tt}, are also unknown. However, in this section, it is assumed that the g_{tt} are relatively stable and, when necessary, may be estimated using the method described in the prior section.

vious section and can return to the problem of estimating the unknown X_{it}. We shall use a different weighting and reorganize the data so that the known (as opposed to the estimated) X_{it} can be combined into a calendar year average claim cost, $\overline{U}_{\cdot c}$ where

$$\overline{U}_{\cdot c} = \frac{\sum_{i=1}^{h} n_{i,\,c-i+1} X_{i,\,c-i+1}}{\sum_{i=1}^{h} n_{i,\,c-i+1}}$$

As defined above, the $\overline{U}_{\cdot c}$ are known quantities since the specific X_{it} is selected so that $i \leq 1974\text{-}t$. We now may consider the relative cost, r_{ic} where:

$$r_{ic} = \frac{X_{i,\,c-i+1}}{\overline{U}_{\cdot c}}$$

It may be noted that these r's reveal the relationship of age of claim to the relative cost of settling the claim. This is in accordance with the assumption mentioned earlier that claims settled in a particular age group (e.g., 13 to 24 months) are similar-type claims and can be compared from report year to report year.

Having reorganized the data, we shall now proceed to use the restructed data to estimate the future X_{it}'s. We will do this by decomposing the known average claim costs, X_{it}, into three components: the impact of inflation, the effect of age of claim, and the general level of costs for the line.

Specifically, to measure inflation let us assume that there exists for each year some underlying rate of increase in claim costs, y_c, which is expressed as an index with the latest calendar year indexed as unity. Furthermore, assume that over any span of a few years, the effect of age of claim, the r_{ic}'s are dependent only on age group and not on the calendar year of observation; hence we will replace r_{ic} by r_i in subsequent equations.

Finally, let us define a scalar B representing the average paid claim cost for the latest calendar year so that the r's and y's appear as indices. This allows us to obtain an estimate $\widetilde{X}_{it}$ of the actual average claim cost for a particular age group and report year (X_{it}, where $i \leq 1974 - t$ and $t \leq 1973$):

$$\widetilde{X}_{it} = Br_i\, y_{t+i-1} \quad \text{and} \quad X_{it} = \widetilde{X}_{it} + \xi$$

where ξ is an error term for the difference between the observed value X_{it} and the computed value $\widetilde{X}_{it}$. Given five years of data, an h by 5 matrix is formed in which the X's are the entry values. Using the iterative techniques described by Bailey and Simon,[7] we may now solve for the best set of r's and y's (denoted $\hat{r}$'s and $\hat{y}$'s) to minimize the sum of the squares of the error term ξ.

An average annual increase in claim costs, q, can be determined from the $\hat{y}$'s. In practice, we use least squares to fit the $\hat{y}$'s to an exponential curve, although other functional relationships could be utilized.[8]

Using this value for q, and the $\hat{r}$'s determined above, projected average claim costs $\hat{X}_{it}$ can be determined for those claims which will be settled in the future:

$$\hat{X}_{it} = B\hat{r}_i\, q^{t+i-1974}$$

These projected values can then be used along with the actual claim costs, X_{it} where $i \leq 1974 - t$ and $t < 1973$, to determine the ultimate average paid claim cost $\overline{X}_{\cdot t}$ as described earlier. Using the projected values $\hat{X}_{it}$ from this technique, one develops a second estimate of the reserve position.

There are two advantages to this approach. First, all of the data are used simultaneously in computing the projection of the average costs instead of subdividing our data by age and making h separate projections. Second, a number of different values of q can arbitrarily be used to compute the projected $\hat{X}_{it}$ so that the sensitivity of d and s to changes in q can be observed. This is relatively simple to do in practice since, after computing B and r, all values other than q are known; hence, d and s may be expressed as polynomial functions of q of degree h, then numerically evaluated.

It must be emphasized that this particular report year approach is but one way of testing reserve adequacy and no single test is completely reliable. Actuaries would wish to use more than one testing procedure to assess the position of the reserves. In our opinion, the report year method has much to recommend itself as one such procedure.

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[7] The difference between the observed claim cost and the product of the age relativity and the calendar year index and the constant B was minimized using the "minimum Chi-Square" technique described in Bailey, R. A. and Simon, L. J., "Two Studies in Automobile Insurance Ratemaking" *PCAS* Vol. XLVII, pp. 11 and 12.

[8] In practice, for certain lines the problem must be divided into two parts (e.g., cases less than 3 years old and cases more than 3 years old) to compute two distinct q's in order to obtain a reasonably good fit to the actual data.

COMMERCIAL FIRE INSURANCE RATEMAKING PROCEDURES

BY

ROBERT L. HURLEY

FOREWORD

It is difficult to imagine that anyone with a just appreciation for historical facts is likely to be much influenced by those who would summarily dismiss all that has gone before in the fire insurance field as the workings of an industry, hide-bound in its conservatism, devoted to the past, and inflicted with a never-ending infancy. Hopefully, we can help to dispel any such notion in the few introductory sections of this paper which, with a due regard for historical precedence, will attempt to explain the actuarial procedures currently used to evaluate commercial fire insurance rate levels and to determine classification adjustments needed to implement such requirements. And, maybe, it will not be taken amiss that one who has never hidden his partialities (well founded, we believe) for the fire insurance ways, must warn of the changes even now upon us. Things which seemed impossible just a short time ago are now taking place with the dynamic changes in industry thinking and responses. But this is a story for another occasion, and, possibly, a more adventuresome pen.

CONTENTS

1. Some Background in the Development of Fire Insurance Rates

Although the history of the fire insurance industry in the United States reaches back to, and is even intimately connected with, the lives of some of our founding fathers, fire insurance rates did not quickly, or easily, win the acceptance and authority with which they began to be regarded shortly after the turn of the century. The early days, the archives suggest, were marked with frequent mistakes and many failures.

It must be remembered that, at the start of the 19th century, the Alleghenies were the nation's frontier, and as the pioneers began to push west, the insurance companies, mainly located on the Atlantic Coast, found it necessary to appoint special agents to travel the new territories and investigate the type of risks being written for them by the local business community. It, undoubtedly, must be difficult today to appreciate the initial frustrations of these companies attempting to introduce some order and stability into the many burgeoning local communities wherein, it seemed, periodic catastrophes would hardly interrupt the almost incessant rate wars. It might not be wrong to picture the temper of those times as an environment of mutual recriminations by outraged insurance practioners; each expostulating with the rate-shaving practices of his confreres, unmindful of his own failings on the same score.

It has been said that chaos seldom happens by mere chance. On the contrary, it is often the result of developments which, in retrospect, can usually be isolated and, with some study, evaluated reasonably well. In this particular instance, the initial calamities were understandable, and correctable. The hamlets, villages and towns were new-built with little regard to planning for safety. These first fire insurance men, for most part, were novices with little or no knowledge of construction and fire protection techniques. There was no standardized fire insurance policy, no real financial or accounting requirements, and no statistics.

It was a situation in which things just had to get better.

Many of the difficulties of those early days have been corrected. For example, we have had for many years a standard fire policy. There is no longer any lack of financial and accounting requirements imposed on the industry. But some of the other problems have not afforded ready solutions, and it is interesting to read that Horace Binney, in April 1852, singled out the festive occasion of the Centennial Anniversary of his Philadelphia Contributionship to complain of: "the defective state of our knowledge in regard to the facts that ought to govern us in our estimating the risk of loss by fire, and the proper rates of premiums of fire insurance".

It took almost another half century before any significant advancements were made in fire insurance rating practices. The subsequent landmarks would certainly embrace the introduction, in 1893, of F.C. Moore's Universal Mercantile Schedule which, one authority notes: "No system was set up for maintaining the schedule as originally conceived—and it has tended to be adapted and modified—to fit the particular needs of each state—however, variations of this schedule are the basis of schedules in use today in almost half the states".

Probably no other single work has shared the prestige accorded A.F. Dean's Analytic System for the Measurement of Relative Fire Hazard which, from early pamphlets in the 1880's, Dean successively expanded and modified until the October 1904 publication, as titled above. The "Analytic System" has been copyrighted, maintained and periodically revised. It has been noted that the Dean Schedule was based on the fundamental concept of "relativities of hazards", whereby the hazard relativities, once established, could be adapted to differences in loss costs, or burning rates, between one territory and another, or between one period of time, and another. As E.R. Hardy notes in his book, *The Making of the Fire Insurance Rate*[1], "There are few men who have contributed more to the solution of the rating problem than A.F. Dean".

It is hoped that these brief preliminary paragraphs may indicate well enough that the early fire insurance instrumentalities and practices did not, like the legendary Athene, spring full-grown from the forehead of Zeus, but rather evolved gradually, and at times falteringly, from modest beginnings. Moreover, with recognition that our present knowledge of the fire insurance business is, in itself, not absolute and immutable, but that it too must be periodically reassessed in terms of the emerging challenges. Let us proceed along with our investigation into the developments within modern times.

2. *The Changing Nature and Function of the Fire Insurance Rate in Modern Times*

The desideratum for the "Making of Rates" under item 3(a)2 of the January 24, 1947 All Industry Fire, Marine, Inland Marine Model Rate

[1]E.R. Hardy, *The Making of the Fire Insurance Rate* (The Spectator Co., Philadelphia, 1926).

Regulatory Bill was succinctly proclaimed as: "Rates shall not be excessive, inadequate, or unfairly discriminatory". Maybe with some apprehension that such a ringing phrase might appear to indict the authors of a greater concern for "sound" than "substance", an article 3(a)3 was added, enumerating various factors to which consideration might properly be afforded in the implementation of the ratemaking responsibility. It is interesting to note, for possible future reference, that this particular article 3 carries the specific provision that: "in the case of fire insurance rates, consideration shall be given to the experence of the fire insurance business during a period of not less than the most recent five years".

It would not be difficult, but obviously unfair and a misreading of the intent and of the times, to accuse the proponents of the model rate regulatory bills of a purely legalistic approach, with little or no concern for the philosophical and economical realities underlying fire rate level determinations. It must be remembered that these particular men were charged with the responsibility of suggesting some model language that would be acceptable and would unite the leaders of the industry (who represented different and, on some items, widely divergent points-of-view) in support of legislation that was of vital concern to the industry in one of the most trying periods of its history. It was not a time, nor proper occasion, for a philosophical and economic treatise on fire insurance ratemaking.

Somewhat later, in the early 1950's, Professor Clarence Kulp[2], in a contribution to the Duke University's symposium on Regulation of Insurance, addressed himself, in the course of his remarks on ratemaking, to the underlying problem of the nature and the function of the insurance rate. While admitting freely, at the outset, the importance both of the "non-excessive" and "non-discriminatory" aspects of the insurance rate, Professor Kulp strongly advocated that these criteria, desirable as they may be, would be significantly less important than the need for "rate adequacy".

He cautioned his listeners that the insurance rate should produce total funds to cover the insurer's obligations. If the rate is insufficient over the years, there can be no insurance business, since the accumulation of reserve funds for future contingencies requires capital, and capital costs money as every city and state, and even the Federal Government, finds out each time it competes for funds in the capital markets against the securities issued by

[2]*Law and Contemporary Problems, Regulation of Insurance,* School of Law, Duke University Vol. 15, No. 4, Autumn 1950

all other borrowers. Of course, at the end position, a Government might requisition all the funds it needs without turning to the capital markets; but the U.S. Government has not been founded on this principle.

In his analysis of the standards of "equitable" and "not unfairly discriminatory" rates, Professor Kulp noted that these criteria were a much later technical development. Making a "fair" rate is, technically, a far more complicated process than making one that is simply "adequate". It should be patently clear that procedure wherein the losses for a single year or multiple years are divided by the corresponding exposures and then loaded for expenses cannot be expected to produce, necessarily, "equitable" and "not unfairly discriminatory" rates. There are the inherently technical problems of classification of risks by coverage, territory, composite characteristics, the adjustment of the premiums and losses to current levels, and the, always overriding, tests of the credibility of the experience.

3. Maintenance of Fire Insurance Rates in The Transition Period

There is probably little need to trace, in detail, the development of the fire rating bureaus; created in response to the early rate regulatory laws, and functioning, almost, as an adjunct to the state regulatory authority. As one author noted, "The state was the watchdog against excessive rates, and the rating system (i.e. operated by the bureaus) was the device by which property owners were treated uniformly and without unfair discrimination".[3]

Within its operational capabilities, the bureau system of fire ratemaking guaranteed, as surely as any human institution can, "equitable" and "not unfairly discriminatory" rates. The rate was developed for each individual risk based on an analysis of its own particular fire hazards by an engineer trained in the profession and completely impartial as to which insurance company might be writing the risk. There was, however, some feeling that if the rigid bureau system had any significant limitation it was, most likely, in the area of delayed rate level adequacy and an inherent reluctance to visualize risks from other than the aspect of the physical fire hazards presented. The industry's response, in recent years, to this challenge will be highlighted in some detail in subsequent sections.

While the bureau's tariff, or minimum, rates may have, at one time, set the rate for fire coverage on dwelling property, there is no comparable

[3]Kent H. Parker, ***Property & Liability Insurance Handbook*** (Long & Gregg, Richard D. Irwin, Illinois, 1965), Chapter 13.

limitation in the case of the fire schedule rate used on practically all non-dwelling properties. The individual fire schedule rate is used to compute blanket rates for fire policies covering more than one piece of property. It is the basis of the Multiple Location (Fire, ECE, & Allied Lines) Rating Plans. It has also been used to figure rates for Installation Risk, Garment Contractor, Jewelers Block, Dies and Patents, and other Inland Marine and floater type policies. And as is well known, the fire schedule rate on the individual commercial risk serves as the starting point of the SMP policy and of many of the independently filed package policies.

However, the schedule fire rate has not often maintained its original identity. Normally, it is adjusted (usually downward) as it is fed into its various subsidiary applications. Progressively, it has become more and more difficult to determine what portion of the final policy rate is attributable to the fire hazard, and many believe that such a disclosure, were it possible, would be futile. Nonetheless, the pervasiveness of the fire schedule rate, even if incognito throughout the somewhat amorphous package policy development, would seem to be of some potential significance.

With all that has happened since the "SEUA Decision", it is difficult to imagine that one might be tempted to maintain that there need not be any connection between fire rates and the evolving loss experience. But, maybe, he would be guilty of no less a temerity were he to insist that the fire rates be tied to a prescribed body of statistical experience without a critical investigation into the appropriateness of the particular statistics. Those of a scholarly persuation would, likely, hope that there might be some middle ground, albeit a little breathing space, for discussion between these two extreme positions.

Insurance rates are established, as is generally recognized, on a prospective, not a retrospective, basis. Consequently the subsequent experience, whether it happens to be favorable, unfavorable, or just what was expected, is not necessary proof that the rate levels were either "right" or "wrong". The progressively evolving experience is used successively only to gauge, once again, what the next year's prospective average overall rate level should be. Sometimes the subsequent loss ratios will be higher, sometimes lower, but seldom ever exactly the same as the balance point loss ratio. We can suspect that our rating system has an inherent defect of some serious consequence, only if the experience is persistently, over the years, either almost always above—or conversely, almost always below—the balance point loss ratio.

Students of the business are aware that for fire insurance (except for dwellings) the chance of other than "bagatelle" losses is of a relatively low order of probability. For example, it has been estimated that, on commercial and industrial properties, 92% of the losses, by number, account for only some 15% of the total payments. The following abstract from the National Board of Fire Underwriters' 1964 tabulation of adjusters' reports is indicative. Since the adjusters' reports did not commonly include losses under $250, these have been provided from research associated with, and subsequent to, an earlier paper.

Fire Classes—Other Than Dwelling

Loss Size	Frequency No. Losses	%	Severity Pd. Loss	%
Under $5,000	191,035	92.1	$ 81,328,000	16.6
$5,000 + Over	16,429	7.9	410,001,000	83.4
Total	207,464	100.0	$491,329,000	100.0

4. Introduction Of The Actuarial Discipline Into Fire Insurance Rating

Maybe, 1958 should be singled out as the transition year. Prior to the date, the industry's fortunes had been pledged to the many local rating bureaus whose response to a nationwide commitment would naturally have been conditioned by the necessary concern for purely parochial interests. The creation of the Inter-Regional Insurance Conference (IRIC) was inspired, it might be viewed in retrospect, as some sort of an early ecumenical movement in the commercial field of insurance to encourage a business community understanding of the industry's substantive and nonsubstantive needs and responsibilities.

The challenge was great. The response, even with some understandable but regretable disaffections, may have been more than should have been expected. Certainly, the general willingness to share set the sights for what had to be done.

In 1958, IRIC came out with its first Recommended Procedure for Fire Rate Level Adjustments, subsequently revised in later years. In hindsight, it was not a very ambitious undertaking. To suggest to the local fire rating bureaus that they should use "earned" rather than "written" experience, that they should adjust prior collected earned premiums to current rate levels, that they should give more weight to the most recent year's experience, may now seem needless and even somewhat trivial. But, believe it or not, the selling job was not always easy.

No useful purpose would now be served to retrace the anxieties, the misunderstandings, the disappointments that eventually lead to the IRIC demise. It would be much better to record that the IRIC established the first actuarial committee which was composed, almost exclusively, of professionals with membership in the Casualty Actuarial Society and which was charged with the continuing responsibility for introducing sound rating principles into the property insurance field. The honor roll of this particular membership would include so many actuaries, who have also contributed to the affairs of the CAS, that one, instinctively, hesitates to attempt the list lest he, inadvertently, might fail to record even one, among so many, who served with such little concern for personal aggrandizement.

The subsequent, and final, section of this paper will detail the present procedures used in establishing commercial fire statewide rate levels and classification relativities as they have been developed from the continuing research of the industry's actuarial committees which succeeded IRIC. It is believed, however, that any account purporting to record, even if only "en passant", the IRIC actuarial contributions would, most surely, have to cite two of its responses to, perhaps, subsidiary, but certainly somewhat related, challenges.

The first of these was the rating plan to provide for the recognition of of windstorm hazard in the Extended Coverage Endorsement, as requested of the industry by the National Association of Insurance Commissioners (NAIC).

The NAIC, due to the severe catastrophe along the Atlantic Seaboard and the Gulf States during the 1950's, was concerned with the effect of such occurrences on rate level gyrations and/or market availability of windstorm coverage. Consequently, it appointed a committee, representative of the various segments of the industry, to study the problem and, if possible, recommend a feasible solution thereto. This Special Windstorm Catastrophe Rating Subcommittee's findings were reported by the NAIC at its June 1962 convention in Montreal.

Subsequently, based on this principle, the IRIC Actuarial Committee recommended to the local fire rating bureaus an Extended Coverage Rate Level Review Procedure incorporating these criteria. This prototype

EC rating plan, encompassing the windstorm catastrophe element, went through a number of editions and was subsequently adapted to the needs of the present monoline commercial EC requirements and the Homeowners rating procedures. A number of the actuaries who developed this original IRIC Catastrophe Windstorm Rating Plan (while possibly no longer so active in primarily technical responses to such challenges) are, for most part, still in the forefront of industry developments.

While this windstorm catastrophe element has been prescribed reading for CAS examination candidates for a number of years, the property insurance industry's joint report to the 1962 NAIC convention has not generally been readily available, and for this reason it is included as Memorabilia A to this paper for future students of the industry.

The second IRIC actuarial study of possible interest to future students of the commercial fire insurance business would, conceivably, be the 1961-1962 deductible investigations. Previously, it had not been possible for the industry to make actuarially credible determinations of the probable savings under various deductible contracts because of the almost universal practice of fire insurance companies to share large commercial lines and the limited significance that could be accorded, even to any one large company's full-line writings, because of the underlying credibility requirements.

With the cooperation of the General Adjustment Bureau and the Factory Insurance Association, data on some 80,000 losses totaling some $190 million in loss payments were analyzed, and deductible rating plans were recommended by the IRIC and Fire Insurance Research and Actuarial Association, FIRAA (i.e. a successor to IRIC), to the local fire rating bureaus as supporting materials for rate filings to be made on behalf of their member and subscriber companies. Again, to record this IRIC initial property actuarial research for present and future scholars of the business, there is enclosed as Memorabilia B the deductible rating plans, and supporting materials thereto, recommended to the local fire rating bureaus. Experience has indicated that the materials contained therein are self-contained and fully comprehensible, even to knowledgeable persons with no special training in actuarial theory

5. *Present Actuarial Procedures For Evaluating Commercial Monoline Fire Statewide Rate Levels And Classification Relativities Thereon*

The concluding section will treat separately in some, but hopefully not exhausting, detail present rating practices. A few prefatory reminders may be in order. The fire insurance business is now much different than when Horace Binney issued his 1852 clarion call for rating reforms. Frederick Moore and Alfred Dean, important as their contributions at the "turn of the century" were, would hardly be equipped to cope with the industry's present rating problems.

Even those industry leaders who successfully steered the troubled ship "Insurance" through the uncertain waters of Public Law 15 and the All-Industry Rating Laws might well discover it difficult to relate to the social, economic, and moral changes that have dominated our post World War II era. The Insurance Industry has had to re-examine its position, reassess its capabilities, and reallocate certain of its resources.

Consequently, there is described in the following paragraphs solely the present rating practices, without any implication that further changes therein may not shortly be required. An understanding, however, of the current procedures is a requisite to determine what changes may be needed to adapt our rating methods to the future challenges.

A. Evaluation of Statewide Rate Levels

Exhibit I presents, on a single page, an actual rate level workout for a filing with an effective date of June 30, 1973. The top section shows the calendar year collected earned premiums and incurred losses of the companies whose experience is used to evaluate rate levels in the particular state. No actuarial adjustments are made directly on these actual dollar premium and loss figures, which are no longer needed once the "Unadjusted Loss Ratios" in Column (1) of the section "Rate Level Calculation" have been computed. In this particular exhibit, the 1971 Unadjusted Loss Ratio of 55.3% is obtained, by dividing incurred losses of $23,835,327 by earned premiums of $43,132,303.

I. Adjustment of Losses

In Columns (2) and (3) of the section "Rate Level Calculation", the loss ratios for each of the calendar years are brought up to the current cost levels using the latest available Current Cost Factors (CCF). The loss ratios are then adjusted to the prospective cost levels 12 months beyond the anticipated effective date of filing, using the Trended Cost Factor (TCF) in Column (4).

The CCF is calculated from a weighted average of the U.S. Bureau of Labor Statistics Consumer Price Index and the U.S. Department of Commerce Composite Construction Cost Index, as shown in Exhibit I(a). The weights used give recognition to the relative volume of contents and building expected losses, respectively.

The Trended Cost Factor is calculated from the projection of the fitted (i.e. linear least-squares) Composite Current Cost Index (CCCI) to the point 12 months beyond the anticipated effective date of the rate filing, as shown in Exhibit I(b).

The sequence of the computations may be summarized as follows:

Adjustment to current cost levels

The loss ratio for each calendar year is multiplied by its appropriate Current Cost Factor (CCF), which is developed by dividing the latest available average quarterly CCCI reading by the average CCCI reading for the particular calendar year, and is shown in Column (2) of Exhibit I.

Determination of trend

The trend in the CCCI is computed from the linear "least-squares" line fitted to the twelve (12) latest available average quarterly CCCI readings (i.e. quarters ended March 31, June 30, September 30, and December 31). The statistical calculations are shown in Exhibit I(b).

Development of Trended Cost Factor (TCF) will then involve the following mathematical computations:

a. Count the number of months between the latest CCCI readings used above (midpoint of quarter) and the date 12 months beyond the anticipated effective date of the filing and divide by 12.

b. Multiply by the latest available annual rate of change in the fitted value of the CCCI as determined in Exhibit I(b).

c. Add unity, the resulting sum is the TCF which appears in the heading of Column (4) of Exhibit I.

In Column (5) of the section "Rate Level Calculations", the calendar year loss ratios are adjusted for changes in coverage. In this particular example, a small deductible was introduced in the early years of the experience review period and, consequently, the losses prior to 1969 had to be reduced slightly for the LER (Loss Elimination Ratio), which is the complement of the savings in losses under the new deductible coverage.

II. Adjustment of Premiums

In Columns (6) and (7), the calendar year loss ratios are adjusted to current rate levels by reflecting the effect of prior rate changes through the "Premium Conversion Factors" (PCF). These PCF's are computed, for this particular example, in Exhibit I(c).

This Exhibit I(c) is divided into three sections of which the top gives both a history of prior average rate changes for all commercial classifications, in the particular year, and a series of index numbers of these rate changes.

The middle section of Exhibit I(c) presents, in parallel columns for each year, the average index and the rate modification factor. For example, the May 8, 1968 rate increase of 3.8% produces an average calendar year index of 1.391 for the calendar year 1968 since the prior rate level of 1.358 was in effect through May 15 or 9/24ths of the year and the then new rate level of 1.410 became effective for 15/24ths of 1968, calculated as follows:

$$[\tfrac{9}{24} \times 1.358] + [\tfrac{15}{24} \times 1.410] = 1.391$$

In the same section, the second column, "Rate Modification Factors" (RMF), gives the factors to be applied to the segments of each year's earned premium contributed from the policy premiums written in prior years. These RMF's are all expressed in terms of current rate level index by dividing each year's index into the current index. For example, the RMF for 1968 of 1.088 is obtained with the division of the 1968 average calendar year index of 1.391 into the index which is in effect as of the date when the rate level evaluation is being calculated (i.e. 1.513).

In the bottom section of Exhibit I(c), the PCF's are finally computed. It will be noted that, of the calendar year 1971 earned premiums 47.5% came from policy premiums written in 1971, 41.5% came from premiums written in 1970, 7.0% from 1969 writings, etc. Consequently, the PCF for calendar year 1971 is obtained by taking: 47.5% of the RMF for 1971 (i.e. 1.000), plus 41.5% of the RMF for the year 1970 (i.e. 1.035), plus 7.0% of the RMF for the year 1969 (i.e. 1.073), and 3.8% of the RMF for 1968 (i.e. 1.088), and similarly for 1967 and 1966. The sum of these products gives 1.024, the PCF for calendar year 1971.

Returning to Exhibit I, we are now ready to determine the rate level indication by extending the rate level loss ratios in Column (7) by the series of weights, which attach greater significance to the more recent experience. The sum of these weighted loss ratio factors, in Column (9) of 64.0%, is multiplied by 1.065 to include the loss adjustment expense. The series of calendar year weights have been established on an underwriting judgment basis. The 1.065 loss adjustment expense factor is developed by an analysis of the ratio of the loss adjustment expense to incurred losses for the three latest calendar years.

Normally, this rate level loss ratio of 68.2% (including loss adjustment expense) would be divided by the "Balance Point" loss ratio to determine the overall commercial fire rate level indication for the particular state. The "Balance Point" loss ratio equals unity less the sum of the profit provision, tax elements, and all expenses except loss adjustment expense, or 56.5% as detailed in the following. Since this example is based on the actuarial calculations underlying an actual rate filing, it was subject to the, then operative, Phase II Federal Price Guidelines which provided that the Company General Expense (i.e. 9.5%), the Other Acquisition Expense (i.e. 3.5%) and the Profit Provision (i.e. 5.0%) be limited to a 2.5% increase, while the budgetary provision of 21.5% for Production Cost (excluding Other Acquisition), Taxes (3.0%) and the Loss Contingency or Catastrophe Allowance of 1.0% might be treated as a direct function of premium.

It is believed important to observe that it is intended that the Federal Economic Stabilization Program would be continued solely for the emergency period and that the industry would return thereafter to the normal Balance Point loss ratio procedures.

The resulting rate level change of (+) 16.3% is finally modified to reflect the Civil Disorder (C-D) Element (1.3% in this particular instance). The C-D factor has been computed from an analysis of each state's distribution of population between metropolitan and rural areas and subsequently adjusted for the developing experience.

B. Determination of Classification Rate Level Relativities

Before attempting to outline the general guidelines for classification rate adjustments, we believe it may be helpful to retrace certain fundamentals.

Basically, fire insurance rates must be geared to expected losses. It is not improbable that from its earliest beginnings, the fire insurance rate has been visualized, at least implicitly, in such terms. In the days when a rate of, say, $1.00 was charged for a relatively broad spectrum of risks, there must have been the expectation that the resulting funds would be adequate for the losses and expenses that would ensue. With the subsequent advances in fire protection engineering, schedules were developed to measure, with progressively more detailed treatment, the hazards presented by individual risks.

However, over the years, supervisory authorities have been increasingly interested in correlating proposed fire rate changes to the classification loss ratios. This trend, it is believed, does some violence to basic concepts in that it weakens the original visualization of the rate, as a measure of "expected losses", by attempting to "true-up" rates with the vagaries of class loss experience. And, on occasions, it introduces certain elements of rigidity, which work against the realization of a proper overall rate level. The situation wherein the rate level inadequacy can be traced to a limited number of classes, and the needed rate increase cannot, practically, be realized just from these few occupancy—protection—contruction groupings is an example of this.

Consequently, once the statewide rate level has been determined, the procedure described hereinafter rests on the cardinal principle that each subsidiary classification adjustment should depart from this statewide norm only to the extent that there are actuarial indications for such a differential.

Throughout the earlier years, fire classification rate level adjustments had generally been made on an informal, semi-statistical basis, with the result, sometimes, being that the sum of the individual revisions did not equal the indicated overall statewide rate level need. The objective of the present procedure, therefore, is to provide a systematic means of determining classification rate level adjustments with a greater degree of actuarial precision than possible heretofore, and at the same time to facilitate the achievement of overall statewide rate level indications.

State individual classification experience often does not, in itself, provide a sufficiently credible basis for determining classification rate level indications. The presence, or absence, of large losses during the review period, plus the low order of fire frequencies, can produce intolerable fluctuations in the classification experience and, therefore, normally requires that consideration be given to a broader base of experience. This procedure contemplates review of classification experience in conjunction with statewide major industry results and with regional classification data—all on the basis of incurred losses and earned premiums adjusted to current rate levels.

The underlying credibility tables recognize premium volume and a broad judgment of inherent classification hazard. Each of the major classifications and industry groupings was reviewed, both from its underwriting and engineering aspects (i.e. inherent physical hazards) and from its actuarial aspects (i.e. the average annual loss ratio variation). Each classification was then assigned to one of three credibility tables. Credibility Table A was established for use on classes with low hazard risks and with expectation of extremely stable loss ratios. Table B is to apply to medium hazard risks with expectation of average loss ratio stability. Table C is to apply to high hazard risks with the expectation of poor loss ratio stability. Each credibility table was graded by premium volume utilizing a curve, $Z = P/(P + K)$, where Z is credibility and P is the premium volume for the latest 6 years. As indicated in the footnote to Exhibit II, Page 1, the K values are respectively \$500,000 for Table A, \$2,500,000 for Table B and \$10,000,000 for Table C.

Exhibit II presents the procedure used in developing the indicated classification rate level adjustments:

1. The statewide overall commercial fire rate level indication is calculated as outlined in the previous section.

2. The indicated rate level change for each major industry group is then determined:

   a. The state major group loss ratio (${}_sL_g$) is given the credibility value (${}_sZ_g$) corresponding to its earned premium at present rates and the credibility table to which it has been assigned. In the summary portion of Exhibit II these credibility values are shown in column (4).

   The complement of this credibility ($1-{}_sZ_g$) is assigned to the regional loss ratio (${}_rL_g$) for the same major group. In the summary section, these complements are not shown because column (8) is used in the other sections to show the actual regional credibility values as discussed below.

b. A credibility weighted state and regional major industry group loss ratio (M_g) is calculated by multiplying the state group loss ratio ($_sL_g$) by its credibility factor ($_sZ_g$) and adding to this result the product of the regional group loss ratio ($_rL_g$) and the complement of the state group loss ratio credibility ($1-{_sZ_g}$). The result of this calculation is shown in column (9) of the summary section.

Example: Major Industry Group II, Mercantile

State major group loss ratio ($_sL_g$) $= 71.9\%$

State major group credibility ($_sZ_g$) $= .97$

Regional major group loss ratio ($_rL_g$) $= 64.9\%$

Regional major group credibility ($1-{_sZ_g}$) $= .03$

$$M_g = [{_sL_g} \cdot {_sZ_g}] + [{_rL_g}(1-{_sZ_g})]$$

$$M_g = [(71.9)(0.97) + (64.9)(0.03)] = 71.7\%$$

The total commercial weighted mean loss ratio of 62.6% is obtained by summing the weighted means for each major group by its state relative earned premiums as given in column (1).

The relativity index in column (10) for each major classification group results from dividing its own weighted mean loss ratio in column (9) by the total commercial weighted mean loss ratio in the same column. Specifically, the relativity for Group 02 "Mercantile" of 1.145 comes from dividing 71.7% by 62.6%.

The final column (11), the indicated rate adjustment for Group 02 "Mercantile" of +34.9%, results from extending the overall rate level requirement of 1.178 by its appropriate relativity of 1.145 given in the previous paragraph.

3. The individual class rate level adjustments are then determined as follows:

   a. The state class loss ratio is assigned its appropriate credibility value ($_sZ_c$) and the regional class loss ratio its appropriate credibility ($_rZ_c$). If the sum of these two credibilities

$({}_sZ_c + {}_rZ_c)$ equals or exceeds 100% (i.e., 1.00), then the regional class credibility is assigned the complement of the state class credibility.

For instance: if the state class credibility is 40% and the regional class credibility is 90% the sum is 130%. Thus, the regional class loss ratio is assigned 60% (rather than 90%) credibility, which is the complement of the state class credibility (i.e., 100% less 40% equals 60%). However, if the sum of the state class credibility $({}_sZ_c)$ and the regional class credibility $({}_rZ_c)$ is less than 100%, then the regional major industry group experience is used to make up the balance of the needed 100% credibility.

b. A credibility weighted classification loss ratio (M_c) in column (9) is then calculated.

Example: Rating group 37 Laundries in Manufacturing major group.

State class loss ratio $({}_sL_c) = 45.2\%$

State class credibility $({}_sZ_c) = 0.16$

Regional class loss ratio $({}_rL_c) = 52.7\%$

Regional class credibility $({}_rZ_c) = 0.41$

Regional major industry group loss ratio $({}_rL_g) = 68.4\%$

$$M_c = ({}_sL_c \cdot {}_sZ_c) + ({}_rL_c \cdot {}_rZ_c) + {}_rL_g(1 - [{}_sZ_c + {}_rZ_c])$$

$$M_c = (45.2 \times 0.16) + (52.7 \times 0.41) + (68.4\,[1.00 - (0.16 + 0.41)])$$

$$M_c = 58.3\%$$

c. The rate level relativity .988 in column (10) is determined by dividing the particular class weighted mean loss ratio (i.e. 037 Laundries at 58.3%) by the 62.6% weighted mean loss ratio for all groups. However, the class loss ratio of 58.3% is first adjusted to the Manufacturing rate level loss ratio by the factor (69.8 ÷ 65.8), therefore:

$$\left\{ \left[58.3 \times \left(\frac{69.8}{65.8} \right) \right] \div 62.6 \right\} = .988$$

d. Finally, the relativity for the specific class (i.e. Laundries) of .988 is multiplied by the overall rate level indicated change of 1.178 to give a +16.4% increase (i.e., $1.178 \times .988 = 1.164$).

It will be noted that the overall commercial fire rate adjustment indicated for the state is distributed to each of the Major Classification Groupings. And then each of these Major Classification Grouping rate adjustments is distributed to each of the individual classes which make up the Major Classification Group. Consequently, the indicated adjustment for each of the individual classifications is keyed back into particular states overall commercial fire rate level requirements.

SOME AFTER-THOUGHTS

Certain aspects of the rate level procedures may seem to have been treated too cursorily herein. Others, the reader may feel, were barely mentioned, or even totally neglected. The subject of the loss trending techniques may serve as an example of the former and the extended coverage, which parallels fire at many points, an instance of the latter. Our sole defense, but no plea for exculpation, may lie in the consideration that no single paper could reasonably cover all aspects of the subject exhaustively and that emphasis is often a matter of personal preference.

As a partial amends for any failure on this score, we should like to conclude with a catalogue of CAS papers concerned with commercial fire insurance which, it is believed, may be read with profit by all and possibly with special delight by the studiously inclined:

1. Some Random thoughts concerning Fire Insurance—Is a Statistical Basis for Rating Possible?

   E. R. Hardy, *PCAS* Volume X

2. A Casualty Man Looks at Fire Insurance Rate Making

   M. H. McConnell, *PCAS* Volume XXXVIII

3. Problems of Fire Insurance Rate Making

   L. H. Longley-Cook, *PCAS* Volume XXXVIII

4. A Statistical Study of Large Fire Losses

   L. H. Longley-Cook, *PCAS* Volume XXXIX

5. The Uniform Statistical Plans for Fire and Allied Lines

C. H. Graves, *PCAS* Volume XL

6. Statistics of the National Board of Fire Underwriters

J. H. Finnegan, *PCAS* Volume XLIII

7. Ratemaking for Fire Insurance

J. J. Magrath, *PCAS* Volume XLV

8. Notes on Some Actuarial Problems of Property Insurance

L. H. Longley-Cook, *PCAS* Volume XLVI

9. Mathematical Limits to the Judgement Factor in Fire Schedule Rating

K. L. McIntosh, *PCAS* Volume XLVIII

10. An Introduction to Credibility Theory

L. H. Longley-Cook, *PCAS* Volume XLIX

11. Commercial Package Policies—Rating and Statistics

R. A. Bailey, E. J. Hobbs, F. J. Hunt, R. E. Salzmann, *PCAS* Volume L

12. A Mathematical Approach to Fire Classification Rates

K. L. McIntosh, *PCAS* Volume LII

13. Implications of Sampling Theory for Package Policy Ratemaking

J. T. Lange, *PCAS* Volume LIII

14. Underwriting Profit in Fire Bureau Rates

L. H. Longley-Cook, *PCAS* Volume LIII

15. Is Probable Maximum Loss (PML) a Useful Concept?

J. S. McGuinness, *PCAS* Volume LVI

INSURANCE SERVICES OFFICE Exhibit I

INDICATED COMMERCIAL FIRE RATE LEVEL CHANGE

YEAR	EARNED PREMIUMS	INCURRED LOSSES	LOSS RATIO
1966	24,840,666	13,811,578	0.556
1967	25,115,097	13,412,369	0.534
1968	25,122,692	15,655,167	0.623
1969	27,075,311	10,831,257	0.400
1970	32,883,716	20,520,943	0.624
1971	43,132,303	23,835,327	0.553

RATE LEVEL CALCULATION

YEAR	(1) UNADJ. LOSS RATIOS	(2) CURRENT COST FACTORS THROUGH 6/72 (See Exhibit Ia)	(3) L/R AT CURR. COSTS (1) × (2)	(4) L/R AT 6/74 COSTS (3) × 1.070 (See Exhibit Ib)
1966	.556	1.367	.760	.813
1967	.534	1.319	.704	.754
1968	.623	1.253	.781	.835
1969	.400	1.175	.470	.503
1970	.624	1.102	.688	.736
1971	.553	1.038	.574	.614

YEAR	(4) L/R AT 6/74 COSTS (3) × 1.070	(5) LER ADJ. FACT.	(6) RATE LEVEL ADJ. FACT. PCF (See Exhibit Ic)	(7) RATE LEVEL LOSS RATIO (4) × (5) / (6)	(8) WGTS.	(9) LOSS RATIO FACTOR (7) × (8)
1966	.813	.997	1.262	.642	.10	.064
1967	.754	.998	1.201	.627	.10	.063
1968	.835	.999	1.131	.738	.10	.074
1969	.503	1.000	1.061	.474	.15	.071
1970	.736	1.000	1.019	.722	.25	.181
1971	.614	1.000	0.986	.623	.30	.187
					SUM =	.640

Loss Ratio (Including Loss Adjustment Expense):

As Filed Under Phase II = (.640 × 1.065) = 0.682

Under Standard Procedure = 0.682 (1.111 ÷ 1.070) = 0.708

Balance Point Loss Ratio = 0.565

Indicated Rate Level Change (Ex. Civil Disorder)

$$\text{As Filed Under Phase II} = \frac{0.682 + 1.025\,(.035 + .095 + .050)}{1.000 - (.215 + .010 + .030)} = 1.163$$

Under Standard Procedure = (0.708 ÷ 0.565) = 1.253

Filed Rate Level Change (Incl. Civil Disorder) = (1.163 × 1.013) = 1.178

11/21/72 INSURANCE SERVICES OFFICE Exhibit I(a)

DEVELOPMENT OF CURRENT COST FACTORS (CCF) AND TRENDED COST FACTOR (TCF)

Quarter Ending June 30, 1972

Part A: Establishment of Monthly Composite Current Cost Index (CCCI), With:
40% Weight to Consumer Price Index (CPI) U. S. Dept. of Labor (BLS), And
60% Weight to Composite Construction Cost Index—U. S. Dept. of Commerce (DC)
(BLS Base: 1967 = 100 DC Base: 1967 = 100)

	1969				1970				1971			
MO	BLS	DC	CCCI	3 MOS AVE	BLS	DC	CCCI	3 MOS AVE	BLS	DC	CCCI	3 MOS AVE
7	110.2	115	113.1	112.3	116.7	122	119.9	119.5	121.8	132	127.9	127.1
8	110.7	116	113.9	113.1	116.9	123	120.6	120.1	122.1	133	128.6	127.9
9	111.2	116	114.1	113.7	117.5	123	120.8	120.4	122.2	133	128.7	128.4
10	111.6	117	114.8	114.3	118.1	124	121.6	121.0	122.4	133	128.8	128.7
11	112.2	118	115.7	114.9	118.5	125	122.4	121.6	122.6	133	128.8	128.8
12	112.9	118	116.0	115.5	119.1	125	122.6	122.2	123.1	134	129.6	129.1

	1970				1971				1972			
MO	BLS	DC	CCCI	3 MOS AVE	BLS	DC	CCCI	3 MOS AVE	BLS	DC	CCCI	3 MOS AVE
1	113.3	118	116.1	115.9	119.2	125	122.7	122.6	123.2	135	130.3	129.6
2	113.9	118	116.4	116.2	119.4	125	122.8	122.7	123.8	135	130.5	130.1
3	114.5	118	116.6	116.4	119.8	127	124.1	123.2	124.0	136	131.2	130.7
4	115.2	120	118.1	117.0	120.2	129	125.5	124.1	124.3	136	131.3	131.0
5	115.7	121	118.9	117.9	120.8	130	126.3	125.3	124.7	137	132.1	131.5
6	116.3	122	119.7	118.9	121.5	131	127.2	126.3	125.0	137	132.2	131.9

Part B: Use of Average Annual CCCI To Calculate Current Cost Factors (CCF)

	CALENDAR YEAR AVERAGE CCCI			CURRENT COST FACTORS BASED ON AVERAGE CCCI VALUE FOR QUARTER ENDING JUNE 30, 1972 = 131.9
YEAR	BLS	DC	CCCI	
1965	94.5	93	93.6	131.9 / 93.6 = 1.409 *
1966	97.2	96	96.5	131.9 / 96.5 = 1.367
1967	100.0	100	100.0	131.9 / 100.0 = 1.319
1968	104.2	106	105.3	131.9 / 105.3 = 1.253
1969	109.8	114	112.3	131.9 / 112.3 = 1.175
1970	116.3	122	119.7	131.9 / 119.7 = 1.102
1971	121.3	131	127.1	131.9 / 127.1 = 1.038

* To Be Used For First 4 Years of EC 10 Year Non-Catastrophe Period, If Using Data Through 1970.

11/21/72 Exhibit I(b)

DEVELOPMENT OF CURRENT COST FACTORS (CCF) AND TRENDED COST FACTOR (TCF)

Part C: Computation of Trended Cost Factor (TCF)

CAL. YR.	QUARTER ENDING	TIME (2X)	AVERAGE CCCI (Y)	(2XY)	($4X^2$)
1969	SEP 30	−11	113.7	−1250.7	121
1969	DEC 31	−9	115.5	−1039.5	81
1970	MAR 31	−7	116.4	−814.8	49
1970	JUN 30	−5	118.9	−594.5	25
1970	SEP 30	−3	120.4	−361.2	9
1970	DEC 31	−1	122.2	−122.2	1
1971	MAR 31	1	123.2	123.2	1
1971	JUN 30	3	126.3	378.9	9
1971	SEP 30	5	128.4	642.0	25
1971	DEC 31	7	129.1	903.7	49
1972	MAR 31	9	130.7	1176.3	81
1972	JUN 30	11	131.9	1450.9	121
		0	1476.7	492.1	572

Equations: $Y = A + BX$

$SY = NA + BSX$

$SXY = ASX + BSX^2$

Where A = Mean of fitted line

B = Average quarterly increment

S = Summation

N = Number of observations

$S2XY = 492.1$ or $SXY = 246.05$ $\quad S4X^2 = 572$ or $SX^2 = 143$

A (mean of fitted line) = 1476.7/12 = 123.06

B (ave. quarterly increment) = 246.05/143 = 1.721

Ave. annual increment = 4 × 1.721 = 6.88

Fitted CCCI trend at midpoint of qtr. ending Jun 30, 1972 =
123.06 + (5.5 × 1.721) = 132.53

Latest Annual Rate of Change = 6.88/132.53 = 5.2%

Trended Cost Factor* = 1.000 + (.052 × 25.5**/12) = 1.111

*Phase II Price Limitation (5/8s of indicated TCF) = 1.07 (See Column 4 of Exhibit I)

**For a filing with an anticipated effective date of June 30, 1973, i.e., the time interval would be 25.5 months between the midpoint of the latest quarter (May 15, 1972) and June 30, 1974.

INSURANCE SERVICES OFFICE Exhibit I (c)
MASSACHUSETTS 2/14/73
STATISTICAL IMPLEMENTATION—PART A
COMPUTATION OF PREMIUM CONVERSION FACTORS
(COMMERCIAL FIRE)

I. Record of Rate Changes and Rate Level Indices:

EFFECTIVE DATE	ADJ. EFF. DATE	RATE CHANGE (%)	RATE LEVEL INDEX
4/ 1/62	4/15/62	2.3	1.023
9/ 5/62	9/15/62	7.3	1.098
1/20/64	1/31/64	2.5	1.125
5/14/65	5/15/65	3.3	1.162
5/27/66	5/31/66	8.0	1.255
11/ 6/67	11/15/67	8.2	1.358
5/ 8/68	5/15/68	3.8	1.410
6/22/70	6/30/70	7.3	1.513

II. Calculation of Rate Modification Factors:

YEAR	AVERAGE CALENDAR YEAR INDEX	RATE MODIFICATION FACTORS
1961	1.000	1.513
1962	1.038	1.458
1963	1.098	1.378
1964	1.123	1.347
1965	1.148	1.318
1966	1.216	1.244
1967	1.268	1.193
1968	1.391	1.088
1969	1.410	1.073
1970	1.462	1.035
1971	1.513	1.000

III. Development of Premium Conversion Factors:
Distribution of Calendar Year Earned Premium by Year Written

YR. WRITT	1966	1967	1968	1969	1970	1971
N-5	.026	.022	.022	.016	.005	.001
N-4	.043	.036	.036	.010	.003	.003
N-3	.071	.069	.069	.048	.038	.038
N-2	.103	.102	.102	.096	.091	.070
N-1	.394	.366	.366	.434	.411	.415
N	.363	.405	.405	.416	.468	.475
PCF	1.310	1.247	1.174	1.102	1.058	1.024
CORRECTED FOR C-D *	1.262	1.201	1.131	1.061	1.019	0.986

* On state's population distribution, the Civil Disorder change averaged 3.8% which produced a 0.963 PCF correction factor: [1.000 ÷ (1.000 + .038)] = 0.963.

Exhibit II

Page 1

2/16/73

INSURANCE SERVICES OFFICE

COMMERCIAL FIRE-CLASS RATE LEVEL INDICATIONS

MASSACHUSETTS EAST COAST REGION

OVERALL INDICATION = 17.8%

SUMMARY—MAJOR INDUSTRY GROUPINGS—KEYED TO STATE OVERALL RATE LEVEL INDICATION

			STATE EXPERIENCE				REGIONAL EXPERIENCE						
			(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Rtg. Grp.	Description	Cred. Table	Earned Premiums	Incurred Losses	Loss Ratio (%)	Credi-bility	Earned Premiums	Incurred Losses	Loss Ratio (%)	Credi-bility	Wtd. Mean	Relati-vity	Ind. Adj. (%)
01	I Residential (Ex. Dwg.)	A	20,436,230	12,710,208	62.2	.98	93,482,243	64,549,521	69.1	.99	62.3	.995	+17.2
02	II Mercantile	B	70,207,996	50,448,197	71.9	.97	390,337,072	253,356,892	64.9	.99	71.7	1.145	+34.9
03	III Non-Mfg. (Ex. Whses.)	B	40,006,359	24,562,073	61.4	.94	220,995,097	140,834,395	63.7	.99	61.5	.982	+15.7
04	IV Manufacturing	C	19,331.406	13,631,852	70.5	.66	159,815,749	109,322,286	68.4	.94	69.8	1.115	+31.3
05	V Sprinklered Risks	B	35,665,108	14,719,104	41.3	.93	119,905,724	61,672,554	51.4	.98	42.0	.671	−21.0
06	Total Commercial		185,647,099	116,071,434	62.5		984,535,885	629,735,648	64.0		62.6	1.000	+17.8

Experience is at Current Rate Level.

Credibility Computations used (P = Adjusted Earned Premium):

A: Credibility = P/ (P + 500,000)
B: Credibility = P/ (P + 2,500,000)
C: Credibility = P/ (P + 10,000,000)

* Indicates Rating Group Excluding those subgroups shown separately.

***** Indicates loss ratio exceeding 999.9.

Exhibit II

Page 2

2/16/73

INSURANCE SERVICES OFFICE
COMMERCIAL FIRE-CLASS RATE LEVEL INDICATIONS

MASSACHUSETTS — EAST COAST REGION

OVERALL INDICATION = 17.8%

Rtg. Grp.	Description	Cred. Table	STATE EXPERIENCE (1) Earned Premiums	(2) Incurred Losses	(3) Loss Ratio (%)	(4) Credibility	REGIONAL EXPERIENCE (5) Earned Premiums	(6) Incurred Losses	(7) Loss Ratio (%)	(8) Credibility	(9) Wtd. Mean	(10) Relativity	(11) Ind. Adj. (%)
	I RESIDENTIAL (EX. DWG)												
07	Bdg. & Rmg. Ho. Etc.	B	1,878,366	1,219,973	64.9	.43	6,459,122	4,415,017	68.4	.72	66.9	1.037	+22.2
08	Motels, Camps, Etc.	B	2,032,289	920,504	45.3	.45	12,265,583	6,070,572	49.5	.83	47.6	.738	−13.1
09	Farm Property	A	3,221,272	2,320,323	72.0	.87	33,349,312	21,459,892	64.3	.99	71.0	1.100	+29.6
10	Apartment Buildings	B	13,304,303	8.249,408	62.0	.84	41.408,226	32,604,040	78.7	.94	64.7	1.003	+18.2
01	Total	A	20,436,230	12,710,208	62.2		93,482,243	64,549,521	69.1		64.2	.995	+17.2
	II MERCANTILE												
11A	Stores and Dwellings	A	7,523,592	5,370,088	71.4	.94	45,201,516	29,878,727	66.1	.99	71.1	1.158	+36.4
11B	Merc. Bldgs.—Class Rated	A	0	−11,037	0.0	.00	6,476,506	5,151,618	79.5	.93	78.5	1.279	+50.7
11C	Merc. Bldgs.—Sched. Rated	A	28,259,612	22,221,496	78.6	.98	126,626,500	90,090,902	71.1	1.00	78.4	1.277	+50.4
11D	HHG. in Merc. Bldgs.	A	671,276	293,907	43.8	.57	3,873,120	1,824,349	47.1	.89	45.2	.736	−13.3
11E	Heavy Stocks Incl. Mach.	B	795,601	654,577	82.3	.24	5,361,667	3,189,681	59.5	.68	65.4	1.065	+25.5
11F	Wearing Apparel, Textiles	B	2,600,604	1,617,564	62.2	.51	17,723,485	9,347,462	52.7	.88	57.5	.937	+10.4
11G	Food Prod. & Beverages	B	4,297,561	2,955,634	68.8	.63	23,562,357	15,492,234	65.7	.90	67.7	1.103	+29.9
11H	Restaurants and Bars	B	8.037,091	5,337,599	66.4	.76	40,780,242	21,413,928	52.5	.94	63.1	1.028	+21.1
11I	Light Merchandise	B	9,493,001	5,943,405	62.6	.79	63,220,835	40,302,214	63.7	.96	62.8	1.023	+20.5
11J	Extra Hazardous Stocks	B	1,043,739	450,108	43.1	.29	6,274,060	3,641,929	58.0	.72	53.7	.875	+ 3.1
12	Whses.—misc.—Mercantile	C	5,252,376	3,960,502	75.4	.34	40,312,715	26,469,767	65.7	.80	69.0	1.124	+32.4
13	Lumber, Coal & Wood Yds.	C	2,233,543	1,654,354	74.1	.18	10,924,069	6,554,081	60.0	.52	64.0	1.042	+22.7
02	Total	B	70,207,996	50,448,197	71.9		390,337,072	253,356,892	64.9		70.3	1.145	+34.9

Exhibit II

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INSURANCE SERVICES OFFICE

COMMERCIAL FIRE-CLASS RATE LEVEL INDICATIONS

MASSACHUSETTS — EAST COAST REGION

OVERALL INDICATION = 17.8%

			STATE EXPERIENCE				REGIONAL EXPERIENCE						
			(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Rtg. Grp.	Description	Cred. Table	Earned Premiums	Incurred Losses	Loss Ratio (%)	Credibility	Earned Premiums	Incurred Losses	Loss Ratio (%)	Credibility	Wtd. Mean	Relativity	Ind. Adj. (%)
	III NON-MFG. (EX. WHSES.)												
14	Offices & Banks, Etc.	A	7,360,350	3,905,096	53.1	.94	36,843,358	21,837,298	59.3	.99	53.5	.847	− .2
15	Hotels, Comm. Bdg. Ho. Clubs	B	4,977,908	2,784,648	55.9	.67	31,422,001	19,331,173	61.5	.93	57.7	.914	+ 7.7
16	Theatres & Auditoriums	B	2,647,369	2,038,787	77.0	.51	10,106,921	7,105,525	70.3	.80	73.7	1.167	+37.5
17	Amusement Properties	B	3,778,393	1,514,904	40.1	.60	23,948,791	11,049,900	46.1	.91	42.5	.673	−20.7
18	Hospitals, Etc.	B	1,367,086	1,529,757	111.9	.35	6,614,112	3,500,061	52.9	.73	73.5	1.164	+37.1
19	Churches, Etc.	B	1,823,127	1,157,654	63.5	.42	14,524,584	13,883,889	95.6	.85	82.1	1.300	+53.1
20	Auto Garages, Filling Sta.	A	5,058,336	3,126,010	61.8	.91	38,702,391	22,528,587	58.2	.99	61.5	.974	+14.7
21	Airplane Hangars	C	121,732	65,936	54.2	.01	1,172,350	669,407	57.1	.10	62.9	.996	+17.3
22	Penal Institutions	C	147,873	340,537	230.3	.01	1,818,683	2,342,499	128.8	.15	75.1	1.189	+40.1
23	Educational Institutions	C	8,482,075	5,328,579	62.8	.46	35,070,334	24,856,188	70.9	.78	67.2	1.064	+25.3
24	Bridges, Piers, Wharves	C	322,787	242,871	75.2	.03	2,199,521	2,160,056	98.2	.18	70.3	1.113	+31.1
25	Builders' Risks	B	3,361,015	2,480,652	73.8	.57	14,899,787	10,409,423	69.9	.86	72.1	1.142	+34.5
26	Police, Fire, Waterworks	C	558,308	46,642	8.4	.05	3,672,264	1,160,389	31.6	.27	52.3	.828	− 2.5
03	Total	B	40,006,359	24,562,073	61.4		220,995,097	140,834,395	63.7		62.0	.982	+15.7

Exhibit II

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2/16/73

INSURANCE SERVICES OFFICE
COMMERCIAL FIRE-CLASS RATE LEVEL INDICATIONS

MASSACHUSETTS EAST COAST REGION

OVERALL INDICATION = 17.8%

			STATE EXPERIENCE				REGIONAL EXPERIENCE						
			(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Rtg. Grp.	Description	Cred. Table	Earned Premiums	Incurred Losses	Loss Ratio (%)	Credi-bility	Earned Premiums	Incurred Losses	Loss Ratio (%)	Credi-bility	Wtd. Mean	Relati-vity	Ind. Adj. (%)
	IV MANUFACTURING												
27	Whses.—Misc.—Manufacturing	C	444,825	438,671	98.6	.04	4,026,538	3,515,868	87.3	.29	75.1	1.273	+50.0
28	Grain Elev.-Term. & Ctry.	C	69,016	44,503	64.5	.01	491,726	940,032	191.2	.05	74.5	1.262	+48.7
29	Food & Kindred Products	C	2,162,881	2,690,626	124.4	.18	15,000,002	11,896,457	79.3	.60	85.0	1.440	+69.6
30	Textiles, Furs, Leather	C	1,836,858	1,047,460	57.0	.16	17,150,138	12,579,306	73.3	.63	69.7	1.181	+39.1
31	Wood Products	C	1,870,064	1,011,969	54.1	.16	14,137,581	8,251,580	58.4	.59	60.2	1.020	+20.2
32	Paper & Pulp Products	C	351,879	1,444,838	410.6	.03	3,151,732	4,467,249	141.7	.24	96.3	1.632	+92.2
33	Printing & Lithographing	C	601,427	308,541	51.3	.06	4,873,875	3,497,880	71.8	.33	68.5	1.161	+36.8
34	Chemicals, Plastic, Rubber	C	2,003,651	1,895,848	94.6	.17	27.027,750	16,699,203	61.8	.73	68.0	1.152	+35.7
35	Stone, Clay, Glass Plts.	C	744,514	495,671	66.6	.07	11,358,974	7,322,282	64.5	.53	66.2	1.122	+32.2
36	Metalworkers	C	6,457,781	3,059,264	47.4	.39	48,830,961	32,652,063	66.9	.83	59.3	1.005	+18.4
37	Laundries & Dry Cleaning	C	1,975,912	892,561	45.2	.16	7,084,210	3,731,309	52.7	.41	58.3	.988	+16.4
38	Oil Distributing Sta.	C	812,598	301,900	37.2	.08	6,682,262	3,769,057	56.4	.40	61.1	1.035	+21.9
04	Total	C	19,331,406	13,631,852	70.5		159,815,749	109,322,286	68.4		65.8	1.115	+31.3

Exhibit II

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INSURANCE SERVICES OFFICE

COMMERCIAL FIRE-CLASS RATE LEVEL INDICATIONS

MASSACHUSETTS EAST COAST REGION

OVERALL INDICATION = 17.8%

			STATE EXPERIENCE				REGIONAL EXPERIENCE						
			(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Rtg. Grp.	Description	Cred. Table	Earned Premiums	Incurred Losses	Loss Ratio (%)	Credi-bility	Earned Premiums	Incurred Losses	Loss Ratio (%)	Credi-bility	Wtd. Mean	Relati-vity	Ind. Adj. (%)
	V SPRINKLERED RISKS												
39	Sprinklered-Non Mfg.—Total	C	16,605,640	9,611,998	57.9	.62	52.780,798	33,409,969	63.3	.84	60.0	.932	+ 9.8
40	Sprinklered-Mfg.—Total	B	19,059,468	5,107,106	26.8	.88	67,124,926	28,262,585	42.1	.96	28.6	.444	−47.7
05	Total	B	35,665,108	14,719,104	41.3		119,905,724	61,672,554	51.4		43.2	.671	−21.0

MEMORIBILIA A

REPORT OF INDUSTRY COMMITTEE TO NATIONAL ASSOCIATION OF INSURANCE COMMISSIONERS' SUBCOMMITTEE—TO STUDY THE CATASTROPHE FACTOR AND ITS USE IN EXTENDED COVERAGE FORMULA OF RATES AND RATING ORGANIZATIONS COMMITTEE

In response to the invitation of the Honorable Rufus D. Hayes, Chairman of the NAIC Subcommittee to Study the Catastrophe Factor and its use in Extended Coverage Formula, the following report is addressed seriatim to the Subcommittee's June 5, 1961 comprehensive outline of the problems.

Careful reflection on the complexities with which the Subcommittee is confronted has imposed an obligation of reassessing the fundamentals of present EC rate level adjustment methods. It was deemed advisable to recommend the continuance of those precedures which have worked successfully. At the same time there has been no reluctance to recommend changes in those methods which are not consonant with the facts indicated by developing catastrophe experience.

The EC rating problem and its solution are of great practical consequence to the property insurance business as well as to the supervisory authorities. Admittedly the EC coverage has not been a satisfactory underwriting venture in its 20-year history. Our approach to EC underwriting experience evaluation methods must be consistent with the underlying catastrophe loss expectancies. Neither the policyholder, nor the supervisory authorities, nor the companies really benefits from an inadequate, a redundant or a wildly gyrating rate structure.

It is thought that answers to the Subcommittee's June 5, 1961 questions will indicate the general principles on which the EC rate level adjustment procedure should be based.

The first problem—the definition of an EC catastrophe.

As a result of our studies, it is recommended that a catastrophe be defined as any occurrence under any one of the EC perils which results in aggregate losses in excess of whichever of the following amounts is the greater:

(a) $1,000,000 or

(b) 50% of the earned EC premium of the State at the current rate level as applied to the year of occurrence of the loss.

Our tests have suggested that on such a definition, even a major storm should not typically result in a subsequent rate adjustment of more than reasonable proportions. Moreover, under this conservative definition of a catastrophe, the thousands of ordinary losses each year can be accommodated according to the established rating procedures which have been employed in the property insurance field.

The second problem—the territorial basis for application of Catastrophe definition.

On the basis of our studies it is recommended that the catastrophe definition be applied to each State individually.

It should be noted that the State can serve as a reasonable basis for application of an EC rate level review formula if the following three interrelated conditions are fulfilled:

(a) an appropriate definition of a catastrophe,

(b) the selection of a compatible catastrophe experience review period of a sufficient span of years, and

(c) a provision for a minimum catastrophe factor in all States whether or not the particular State has suffered a catastrophe loss.

Moreover, the State constitutes a practical geographical basis for rates in that it is consistent with the pattern of rate regulation in the United States which delegates to each of the several States individually the direct responsibility for the supervision of the insurance business, including the review of rates. This is so even though it is recognized the major perils under EC, i.e. Wind and Hail, transcend State lines.

The third problem—the period of time over which the Catastrophe experience should be related to rates.

As a result of our studies it is recommended that the non-catastrophe losses be reviewed over the latest available six calendar year experience period. It is proposed that the catastrophe factor (except the minimum factor for which provision is otherwise made) be reviewed over not less than the latest available fifteen calendar year experience period with the additional suggestion that consideration should be given to a longer period as the developing experience may indicate.

The fourth problem—the conversion of the Catastrophe experience into a percentage factor or element to be added to the normal EC rate.

As a result of our studies it is recommended that the overall EC rate level by State reflect both the non-catastrophe portion and the long-term catastrophe factor or element. The non-catastrophe portion and the catastrophe element would each be computed individually. However, it is proposed that, while the non-catastrophe portion be reviewed annually, the catastrophe factor would be subject to further modification, upward or downward, only at long-term intervals as the developing catastrophe experience may warrant.

However, there should be a minimum catastrophe factor (i.e. 1%) for any State regardless of whether or not the particular State had suffered an EC Catastrophe.

The following general principles, based on the answers to the above questions, are proposed as a guide in the determination of overall Extended Coverage rate level adjustments, and are supplementary to the general principles established for property insurance overall rate level adjustments:

General Principles—Extended Coverage Rate Level Adjustments

1. Extended Coverage rates shall contain a separate element (1% minimum) for catastrophe losses. This element shall be determined from all available pertinent data including, if possible, the loss experience of not less than the most recent 15 years. It is contemplated that this separate catastrophe element shall be subject to revision, upward or downward, only at long-term intervals as the developing catastrophe experience may warrant.

2. A catastrophe is defined as any occurrence (under any one of the ECE perils) which results in an aggregate loss in excess of whichever of the following amounts is the greater: (a) $1,000,000 or (b) 50% of the State ECE earned premiums in the year of occurrence.

3. Non-Catastrophe ECE experience shall be defined as experience excluding the catastrophe element of the premiums and the portion of losses of

any catastrophe within the State which is in excess of the aggregate loss amount used to define catastrophe. Non-Catastrophe ECE experience shall be reviewed for the latest six-year period including that of the immediate past year, with earned premiums adjusted to reflect current tariff rate levels.

Clyde H. Graves, Assistant Manager,
Mutual Insurance Advisory Association

Walter L. Hays, President,
American Fire and Casualty Company

Ambrose B. Kelly, General Counsel,
Associated Factory Mutual Fire Insurance Companies

Kent H. Parker, General Manager,
Inter-Regional Insurance Conference

April 18, 1962

MEMORIBILIA B

INTER-REGIONAL INSURANCE CONFERENCE
125 MAIDEN LANE, NEW YORK 38, N. Y.

K. H. PARKER, GENERAL MANAGER
R. M. BECKWITH, ASS'T GENERAL MANAGER
C. P. BUTLER, GEN'L COUNSEL

MANAGERS
—
H. F. PERLET
J. T. SORENSEN

July 15, 1963

To Members of
Inter-Regional Insurance Conference

Gentlemen:

Disappearing Deductible Clause ($5,000 Maximum)
Memorandum on Rating Plan for Deductible Insurance
Fire and Allied Line Perils

Following a long period of study under the direction of our Committee on Rate Level Adjustments, the Executive Committee has approved for country-wide recommendation the attached Rating Plan for Disappearing Deductible Insurance contemplating various disappearing deductibles with a $5,000 maximum. The Executive Committee deferred taking action at this time with respect to a recommendation as to larger deductibles involving straight deductibles of $10,000 or more.

The basic study and the method of development of deductible credits is outlined in the attached memorandum which, likewise, has been furnished to the rating organizations and which is believed sufficiently explanatory to permit development of needed information in support of the Plan and a full understanding of the methods followed. While the present recommendation to the rating organizations is confined to the disappearing deductible involving amounts not exceeding $5,000, the memorandum covers the study in condensed form as to both straight and disappearing deductibles.

This information is sent to member companies to fully acquaint you with the current status of studies and recommendations of the Conference.

Yours very truly,

K. H. Parker
General Manager

Enclosure

July, 1963.

INTER-REGIONAL INSURANCE CONFERENCE
NEW YORK, N. Y.

MEMORANDUM ON RATING PLAN FOR DEDUCTIBLE INSURANCE FIRE AND ALLIED LINE PERILS

1. *Introduction*

It is thought that the attached research findings and analysis will be helpful as a reference frame within which to review the recommended plan of credits from tariff rates for deductible insurance on fire and allied perils.

At the outset, a few words ought to be said on both the source and the scope of the data. With the approval of its Executive Committee, the Inter-Regional Insurance Conference inaugurated in 1962 a research study on fire and allied peril deductible insurance. While the study was in response to the expressed interest of its membership, it was generally recognized that the entire industry would benefit from a detailed research into a volume of data substantially greater than the statistical sources used in early studies in this field.

The IRIC was most fortunate in, and is appreciative of, the co-operation of the General Adjustment Bureau, Inc., and the Factory Insurance Association in the development of the basic loss statistics. These organizations conducted a nine month (January-September, 1962) survey of countrywide case losses. Detailed information as indicated in the Loss Report Form (c.f. Exhibit I) was collected on 80,150 individual losses, encompassing all classifications of fire and allied peril business—excluding only dwellings and farms. These 80,150 losses represented total loss payments to policyholders of $189,182,341.

Traditionally, fire insurance rates have been developed to provide funds whereby the general public may be indemnified for the whole loss. For other than residential properties, the fire rates reflect an analysis of each individual risk according to an engineering schedule of the hazards presented. They were not developed to fraction off the rate for the probabilities of various loss sizes for the individual risk locations.

There are good reasons to believe that the evaluation of such probabilities cannot be arranged on a risk by risk engineering evaluation. Actually they can at best only be approximated. And maybe the easiest approach is through an analysis of the totality of available statistics. Consequently, the 80,000 losses totaling almost $190 million in payments were used as a basis for the deductible rate credits developed herein.

2. *Fundamental Principles Used in Study*

In order to determine the appropriate discount in tariff rates for deductible coverage, it is necessary and sufficient:

a. to establish what portion of the tariff premium dollar must be reserved for losses and what portion is available for expenses, taxes and profit.

b. to develop a schedule of the savings in losses (hereinafter referred to as LER or Loss Elimination Ratios) for the various deductible classification groupings.

c. to determine the extent to which the expense, tax and profit provisions represented in the initial tariff rate should be reduced by reason of the expected loss savings.

a. *Allocation of Tariff Rate to various Functional Components*

It is well known that there are no such standard allocations recognized countrywide for the type of business which is eligible for deductible coverage. In the absence of such criteria, it was deemed advisable to use the Stock Company countrywide actual experience indications with solely those modifications which would be appropriate for the classes of business eligible for deductible coverage.

A fairly close reading of loss and expense distributions with a due regard to the most currently available data suggested that the tariff premium rate for eligible deductible business could be broken down into the following functional components:

Normal Losses & Loss Adjustment Expense	$ 0.54
Expenses Varying with Written Premiums	
Taxes and Bureau	0.04
Production Costs	0.20
Other Company Expense	0.16
Allowance for Underwriting Profit & Contingencies	0.06
Total Tariff Premium Dollar	$ 1.00

b. *Savings in Loss Cost or Loss Elimination Ratios (LER)*

The Loss Elimination Ratios are the ratios to total losses of all loss payments excluded by the deductible provisions. For example, if the total losses were a $1,000,000. and $100,000. were avoided by the Company (in other words, paid by the policyholders because of the deductible provision) the LER (Loss Elimination Ratio) in this instance would be 10%.

From "a priori" considerations the LER's must be a joint function of the insurable value and the amount to be deducted from each loss. Restating this principle in more specific terms, we would observe that one should normally expect to eliminate a larger proportion of the losses occurring on a $100,000. insurable value with a $25,000. deductible than with a $1,000. deductible. Likewise, one should normally expect to eliminate with a $1,000. deductible a larger percentage of the losses on a $10,000. than on a $1,000,000. insurable value.

Consequently, we would expect that the LER's should increase directly with the size of the deductible and inversely with the size of the insurable value involved. As the deductible amount approaches close to the insurable value, the LER's approach unity or 100%. Conversely, for a specific deductible amount, the LER's approach zero as the insurable value approaches an infinitely large number.

However, the LER's do not vary in direct proportion with the amount of the deductible. Nor do they decrease step by step with the increase in the insurable value involved. In mathematical terms, the functional relationships are not linear. They do not vary in a straight line.

To obtain from the statistics the best possible estimates of the Loss Elimination Ratios (LER's) the some 80,000 losses involving almost $190 million in payments were tabulated by line size and loss size. The LER's were computed separately for Fire and ECE initially on a straight deductible basis as shown in Exhibit 2. Subsidiary analyses were made of possible variations in the LER's by construction, protection and occupancy classification groupings, but these details are still under review and are not included in this recommended filing.

The Loss Elimination Ratios (LER's) were plotted on graph as shown in Exhibit 3. These points on the graph are observed to adhere to a pattern. It will be noted that the family of curves conforms to this pattern and is consistent with the "a priori" considerations outlined above; namely that the LER's should decrease with the increase in the insurable value and approach gradually to the value zero as the insurable value increases towards infinity.

The Disappearing Deductible which is confined to the smaller deductible amounts (i.e., 5,000. or less) was afforded a slightly different treatment at certain points in the analysis and some specific comment is required. At the risk of belaboring the obvious, the $1,000. Deductible disappearing at a $10,000. loss means that the policyholder:

a) pays himself for all losses under $1,000.

b) receives full payment on all losses of $10,000. or more.

c) receives 111% of the difference between the actual loss and $1,000. on all losses within the range of $1,000. to $10,000.

    1) the 111% is the vehicle for making the deductible disappear and comes from dividing $10,000. by ($10,000. less $1,000. or $9,000.).

Consequently the same loss data used for the Straight Deductibles is again reworked for the Disappearing Deductible. Rather than applying the arithmetic in steps "a", "b" and "c" just above to each and every loss, it is possible to come up with the identical answer by applying the following algebraic formula to the summary data available from the tabulations:

$$S = (1 + x)\,n\,d - x\,L$$

Where

S = the dollar loss payments eliminated in the "Disappearing Deductible" range.

n = the number of losses in the "Disappearing Deductible" range.

d = the deductible amount.

L = the total loss incurred in the "Disappearing Deductible" range before the application of the deductible.

(1 + x) = the multiplicative loss factor (i.e., the 111% as given above).

x = the multiplicative factor less unity (i.e., 111% less 100% or 11%).

The derivation of this formula and a test of it against an arithmetical example is presented in Exhibit 4.

c. *Relation of LER's (Loss Elimination Ratios) to the Credits in Tariff Rates*

Even if all the losses were eliminated through the application of the deductible feature, the insurance company would still incur certain underwriting, servicing, engineering, etc. expenses for which a charge must logically be made to the assured.

On the other hand, there are loss adjustment expenses which are intimately connected with losses and for which no charge need be made on losses escaped by the deductible. Still other items of outgo such as taxes, bureau assessments, production costs and profit margins are a function of the finally adjusted premium . . . although they are usually a constant percentage of the indicated premium.

Therefore, the final premium rate must provide for the sum of the residual normal losses after the application of the deductible clause plus the loss adjustment expenses on these losses, plus the Company expense, all loaded for the items of functional outgo. The formula for the appropriate discount in tariff rates is as follows:

$$C = 1 - \left[\frac{L(1-d) + F}{1 - (S+T+P)}\right] \quad \text{where:}$$

Symbols

C = credit in tariff premium rates

L = provision for losses and loss adjustment expense in tariff premium rates

d = loss elimination ratios

F = provision in tariff rates for Company expense (not otherwise provided for)

S = percent of final adjusted premium for production cost

T = percent of final adjusted premium for taxes and bureau assessments

P = percent of final adjusted premiums for underwriting profit and contingency allowance.

3. *Development of the Indicated Credits from Tariff Rates*

From the previous section of this memorandum, it was indicated that the final credits from tariff rates must be some percentage, not exceeding 100%, of the expected LER's (loss elimination ratios or relative savings in losses under the deductible coverage selected).

Given the allocations of the tariff rate to its various functional components as set forth in Section 2 (a) and the formula relationship as in Section 2 (c), the appropriate credit in tariff rates will equal 77.1% of the indicated LER's in the previously identified notation:

$$C = 1 - \left[\frac{L(1-d) + F}{1 - (S + T + P)}\right] \text{ or}$$

$$C = 1.00 - \left[\frac{.54(1-d) + .16}{1.00 - (.20 + .04 + .06)}\right]$$

$$C = 1.00 - \left[\frac{.70 - .54d}{.70}\right]$$

$$C = \frac{(.70 - .70 + .54d)}{.70}$$

$C = 0.771d$ or 77.1% of the LER's.

With the development of the formula to convert LER's to indicated rate credits, it remains only to select the best estimates of the LER's for the various deductible provisions contemplated under the respective plans.

The study findings indicate a definite pattern in the relationship between the LER's and the corresponding insurable values over the entire scale for which readings are available. However, it was decided to confine the Deductible Plan to those ranges within which such coverage might be responsive to the ordinary requirements of the policyholder.

For example, it was difficult to imagine that there would be an appreciable demand for as high as a $1,000. deductible on a $5,000. insurable value, or for a $10,000. deductible on, say, a $25,000. insurable value. Consequently, it was decided to develop a schedule of indicated credits from tariff rates for selected ranges of percent deductible to insurable value—up to 10% of the insurable value.

In the development of the plan of credits, as previously noted, the LER's (Loss Elimination Ratios) were computed directly from statistics for the various observation points. Then these averages were plotted on a graph, fitted to a curve, and the corresponding LER's taken from the equation of the curve.

After applying the expense adjustment factor of 77.1% (as derived above) to the LER's, the indicated rate credits both for the unmodified arithmetical indications and the comparable readings from the mathematical curve were reviewed from the point of view of reasonableness and consistency. Wherever possible, the unmodified arithmetical reading was used for the rate credit. However, in some instances, the unmodified arithmetical indication was obviously out of line with the general pattern of the indicated rate credits for neighboring values.

On such an occasion, either the reading from the mathematical curve was used or some adjoining point on the basis of underwriting judgment. However, the departures were seldom of any magnitude. Actually the modifications that were made must be evaluated for significance in terms of the knowledge that:

1. *As far as the averages from the pure arithmetic are concerned,* while the overall sample was very large, the specific values for certain of the detailed line sizes by coverage could be vulnerable to the play of chance fluctuations, and

2. *As far as the readings from the mathematical curves are concerned,* while the mathematical curves observe the pattern suggested by the "a priori" considerations and generally fit the actual observations reasonably well according to established statistical tests, their merit is more likely one of utility and economy than it is the possession of some guarantee of "super-precision."

Exhibit 5 presents the basic data and the computation work sheets which underlie the credits from tariff rates for the recommended Disappearing Deductible Plan.

4. *Conclusions*

The presentations herein have involved the full scope of the Inter-Regional study of deductible insurance for Fire and ECE. It would have been possible to restrict the material solely to the statistics required in support of the specific recommended filing—namely, a Disappearing Deductible Plan with a maximum $5,000. deductible per loss. However, the knowledge of the basic relationships involved over the entire range of deductible coverage on fire and allied lines insurance should be of additional interest and help in evaluating the plan as presently recommended.

In recapitulation, this memorandum has set forth:

1. An explanation of the source, nature and significance of the statistics,
2. The approach used to determine appropriate credits from tariff rates for deductible coverage—both the straight and the disappearing type,
3. A detailed account of basic relationships existing between:
   a) Loss Elimination Ratios (LER's) and the distribution of losses by size and the insurable value involved, and
   b) The LER's and the credits from tariff rates,
4. The basic data and the computations of LER's for straight deductible coverage,
5. The basic data and the computation work sheets showing both the LER's and the indicated credits from tariff rates underlying the recommended Disappearing Deductible Plan.

Exhibit I

GENERAL ADJUSTMENT BUREAU, INC.—DIRECT PROPERTY DAMAGE LOSS SURVEY FOR INTER-REGIONAL INSURANCE CONFERENCE

FIRE POLICIES WITH OR WITHOUT EXTENDED COVERAGE (INCLUDING VANDALISM & MALICIOUS MISCHIEF), OTHER THAN DWELLING (MAXIMUM FOUR FAMILIES) AND FARM PROPERTY

No.	Principal Occupancy of Premises Involved in Loss		Loss To		Public Fire Protection		Predominant Construction			Peril—Cause of Loss				Actual Cash Value of Property Involved In Loss	Whole Loss or Damage
	Code	Type of Business (Use Only for Code 5 Explanation)	Build-ing	Con-tents	Yes	No	Fire Resistant	Non-Comb. All Steel	All Other	Fire or Lightning	Wind or Hail	Explosion	All Other ECE Perils		
1														$	$
2															
3															
4															
5															
6															
7															
8															
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22															
23															
24															
25															

Code	
1	Apartment (5 or More—Family)
2	Mercantile
3	Non-Mfg. Whse. & Service
4	Institutional
5	Manufacturing (Explained Above)

Branch ________________ State ________________ Date ________________ Sheet No. ________

No.	Remarks

LOSS ELIMINATION RATIOS—STRAIGHT DEDUCTIBLE

Exhibit 2
Page 1

Insurable Value	Paid ($1000) Less than Deductible			No. Losses Above Deduct.			Savings in Loss ($1000)			Total Loss ($1000)			Loss Elmination Ratios		
	Fire	ECE	Total	Fire	ECE	Total	Fire	ECE	Total	Fire	ECE	Total	Fire	ECE	Total
$250 DEDUCTIBLE															
0 — 9,999	$ 461	$ 737	$ 1,198	3,885	1,739	5,624	$ 1,432	$ 1,172	$ 2,604	$ 8,439	$ 2,072	$10,511	17.0%	56.6%	24.8%
10,000— 24,999	369	777	1,146	4,146	2,507	6,653	1,406	1,404	2,810	19,068	3,066	22,134	7.4	45.8	12.7
25,000— 49,999	229	457	686	3,187	2,046	5,233	1,026	969	1,995	25,083	2,756	27,839	4.1	35.2	7.2
50,000— 99,999	170	300	470	2,544	1,709	4,253	806	727	1,533	29,180	2,839	32,019	2.8	25.6	4.8
100,000—249,999	137	211	348	2,166	1,552	3,718	678	599	1,277	30,466	3,601	34,067	2.2	16.6	3.7
250,000—499,999	52	89	141	958	713	1,671	292	267	559	17,664	1,439	19,103	1.7	18.6	2.9
500,000 & Over	109	136	245	2,130	1,416	3,546	641	490	1,131	33,519	5,687	39,206	1.9	8.6	2.9
Total	1,527	2,707	4,234	19.016	11,682	30,698	6,281	5,628	11,909	163,419	21,460	184,879	—	—	—
Average													3.8	26.2	6.4
$500 DEDUCTIBLE															
0 — 9,999	792	1,098	1,890	2,953	681	3,634	2.267	1,438	3,705	8,439	2.072	10,511	26.9	69.4	35.2
10,000— 24,999	698	1,311	2,009	3,209	960	4,169	2,302	1,791	4,093	19,068	3,066	22,134	12.1	58.4	18.5
25,000— 49,999	479	830	1,309	2,483	978	3,461	1,721	1,319	3,040	25,083	2,756	27,839	6.9	47.9	10.9
50,000— 99,999	371	579	950	1,978	914	2,892	1,360	1,036	2,396	29,180	2,839	32,015	4.7	36.5	7.5
100,000—249,999	324	436	760	1,647	904	2,551	1,148	888	2,036	30,466	3,601	34,067	3.8	24.7	6.0
250,000—499,999	132	186	318	735	445	1,180	500	409	909	17,664	1,439	19,103	2.8	28.4	4.8
500,000 & Over	289	310	599	1,639	923	2,562	1,109	772	1,881	33,519	5,687	39,206	3.3	13.6	4.8
Total	3,085	4.750	7,835	14,644	5,085	20,499	10,407	7,653	18,060	163,419	21,460	184,379	—	—	—
Average													6.4	35.7	9.8

LOSS ELIMINATION RATIOS—STRAIGHT DEDUCTIBLE

Exhibit 2
Page 2

Insurable Value	Paid ($1000) Less than Deductible			No. Losses Above Deduct.			Savings in Loss ($1000)			Total Loss ($1000)			Loss Elmination Ratios		
	Fire	ECE	Total	Fire	ECE	Total	Fire	ECE	Total	Fire	ECE	Total	Fire	ECE	Total
$1,000 DEDUCTIBLE															
0 — 9,999	$ 1,323	$ 1,360	$ 2,683	2,200	291	2,491	$3,523	$1,651	$ 5,174	$ 8,439	$ 2.072	$10,511	41.7 %	79.7 %	49.2 %
10,000— 24,999	1,140	1,710	2,850	2,578	375	2,953	3,718	2,085	5,803	19,068	3,066	22,134	19.5	68.0	26.2
25,000— 49,999	834	1,216	2,050	1,985	411	2,396	2,819	1,627	4,446	25,083	2,756	27,839	11.2	59.0	16.0
50,000— 99,999	696	919	1,615	1,517	429	1,946	2,213	1,348	3,561	29,180	2,839	32,019	7.6	47.5	11.1
100,000—249,999	617	736	1,353	1,232	475	1,707	1,849	1,211	3,060	30,466	3,601	34,067	6.1	33.6	9.0
250,000—499,999	297	313	610	508	264	772	805	577	1,382	17,664	1,439	19,103	4.6	40.1	7.2
500,000 & Over	633	546	1,179	1,160	584	1,744	1,793	1,130	2,923	33,519	5,687	39,206	5.4	19.9	7.5
Total	5,540	6,800	12,340	11,180	2,829	14,009	16,720	9,629	26,349	163,419	21,460	184,879	—	—	—
Average													10.2	44.9	14.3
$5,000 DEDUCTIBLE															
0 — 9,999	5,557	1,884	7,441	437	29	466	7,742	2,029	9,771	8,439	2,072	$10,511	91.8	97.9	93.0
10,000— 24,999	4,240	2,261	6,501	1,362	73	1,440	11,050	2,651	13,701	19,068	3,066	22,134	58.0	86.5	61.9
25,000— 49,999	2,774	1,877	4,651	1,203	73	1,276	8,789	2,242	11,031	25,083	2,756	27,839	35.0	81.3	39.6
50,000— 99,999	2,226	1,596	3,822	884	80	964	6,646	1,996	8,642	29,180	2,839	32,019	22.8	70.3	27.0
100,000—249,999	1,956	1,522	3,478	656	104	760	5,236	2,042	7.278	30,466	3,601	34,067	17.2	56.7	21.4
250,000—499,999	873	747	1,620	241	47	288	2,078	982	3,060	17,664	1,439	19,103	11.8	68.3	16.0
500,000 & Over	2,164	1,476	3,640	465	159	624	4,489	2,271	6,760	33,519	5,687	39,206	13.4	39.9	17.2
Total	19,790	11,363	31,153	5,248	570	5,818	46,030	14,213	60,243	163,419	21,460	184,879	—	—	—
Average													28.2	66.2	32.6

* excludes Losses not assigned an insurable value.

LOSS ELIMINATION RATIOS—STRAIGHT DEDUCTIBLE

Exhibit 2
Page 3

Insurable Value	Paid ($1000) Less than Deductible			No. Losses Above Deduct.			Savings in Loss ($1000)			Total Loss ($1000)			Loss Elmination Ratios		
	Fire	ECE	Total	Fire	ECE	Total	Fire	ECE	Total	Fire	ECE	Total	Fire	ECE	Total
							$10,000 DEDUCTIBLE								
10,000— 24,999	$ 8,871	$ 2,567	$11,438	719	34	753	$16,061	$ 2,907	$18,968	$19,068	$ 3,066	$22,134	84.3 %	94.8%	85.7%
25,000— 49,999	5,498	2,165	7,663	826	33	859	13,758	2,495	16,253	25,083	2,756	27,839	54.8	90.5	58.4
50,000— 99,999	3,872	1,932	5,804	656	33	689	10,432	2,262	12,694	29,180	2,839	32,019	35.7	79.7	39.6
100,000—249,999	3,151	1,908	5,059	487	48	535	8,021	2,388	10,409	30,466	3,601	34,067	26.3	66.3	30.6
250,000—499,999	1,280	938	2,218	184	20	204	3,120	1,138	4,258	17,664	1,439	19,103	17.7	79.1	22.3
500,000 & Over	3,347	2,049	5,396	293	74	367	6,277	2,789	9,066	33,519	5,687	39,206	18.7	49.0	23.1
Subtotal**	26,019	11,559	37,578	3,165	242	3,407	57,669	13,979	71,648	154,980	19,388	174,368	—	—	—
							$25,000 DEDUCTIBLE								
25,000— 49,999	13,618	2,626	16,244	346	4	350	22,268	2,726	24,994	25,083	2,756	27,839	88.8	98.9	89.8
50,000— 99,999	7,779	2,269	10,048	421	12	433	18,304	2,569	20,873	29,180	2,839	32,019	62.7	90.5	65.2
100,000—249,999	5,668	2,320	7,988	328	21	349	13,868	2,845	16,713	30,466	3,601	34,067	45.5	79.0	49.1
250,000—499,999	2,307	1,184	3,491	121	5	126	5,332	1,309	6,641	17,664	1,439	19,103	30.2	91.0	34.8
500,000 & Over	5,430	2,690	8,120	161	29	190	9,455	3,415	12,870	33,519	5,687	39,206	28.2	60.0	32.8
Subtotal**	34,802	11,089	45,891	1,377	71	1,448	69,227	12,864	82,091	135,912	16,322	152,234	—	—	—

** For checking purposes only . . . does not include losses on Insurable Values less than the Deductible.

EXHIBIT #3

IRIC DEDUCTIBLE STUDY – STRAIGHT DEDUCTIBLE
FIRE LOSS ELIMINATION RATIOS – ALL CLASSES

NOTE: MARKS (● × O □) ARE THE OBSERVATION POINTS ON WHICH THE GRADUATION CURVES ARE COMPUTED

Loss Elimination Ratios (Y)

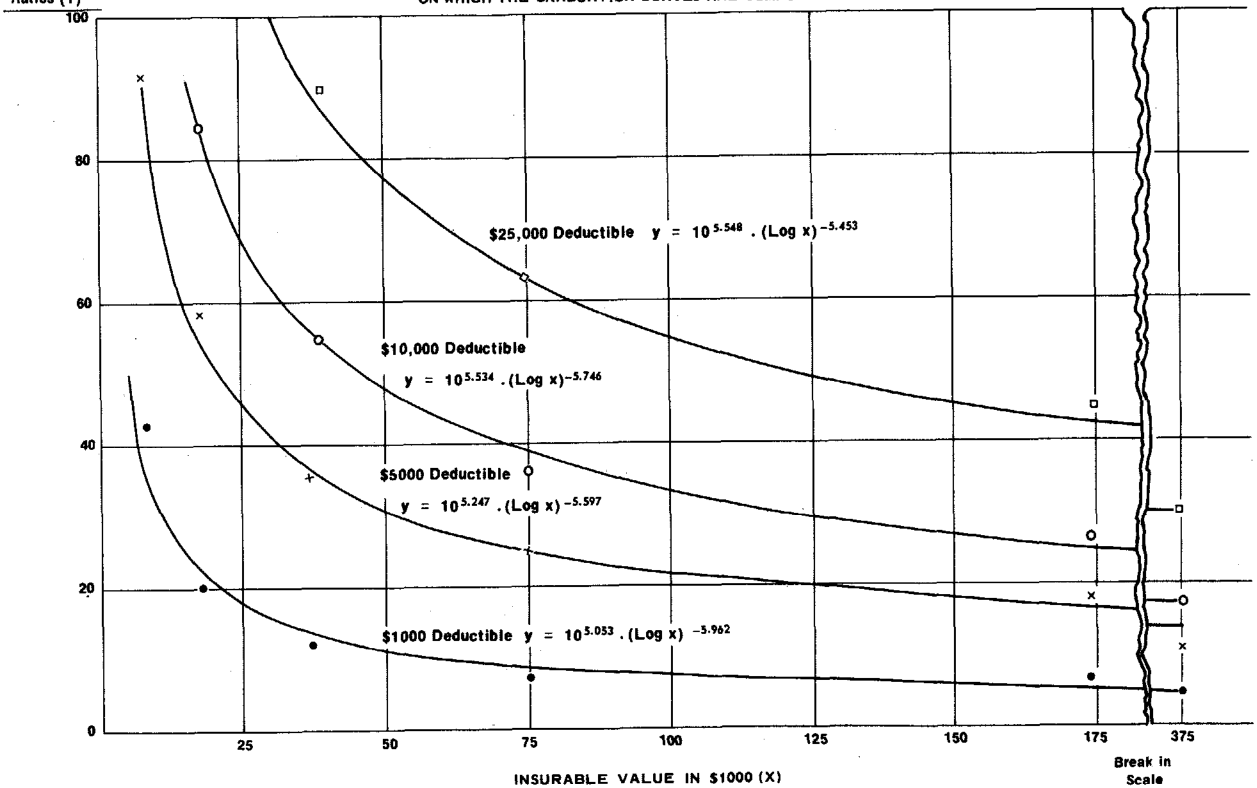

Exhibit 4

DISAPPEARING DEDUCTIBLE FORMULA

The formula used in the calculation of the loss savings within the disappearing deductible range is.

$$S = (1 + x)\,nd - xL$$

Where:

S = Savings in loss payments by the Insurance Company
$(1 + x)$ = the multiplication factor applied to the loss net of the deductible
n = the number of losses in the Disappearing Deductible range
d = the amount of the initial deductible
x = the multiplication factor less unity
L = the sum of the losses in the disappearing deductible range

This formula is developed, in effect, from the following typical example involving three losses (A – B and C) all within the disappearing deductible range: (1000-5000)

Loss	Total Amount of Loss	Deductible Disappearing @ $5000	Net Loss	Paid to * Policyholder	Savings to Insurance Co.
A	2000	1000	1000	1250	750
B	3000	1000	2000	2500	500
C	4000	1000	3000	3750	250
Total	9000	3000	6000	7500	1500

Now by the Formula given above, the Savings (S) equals:

$$S = (1 + x)\,nd - xL$$
$$S = (1.25)(3)(1000) - (0.25)(9000)$$
$$S = 3750 - 2250$$
$$S = 1500$$

It will be noted that this $1500 produced by the formula is the same figure obtained by applying the arithmetical approach to each of the three losses individually.

* With a $1000 deductible disappearing at $5000 the loss multiplication factor becomes $\dfrac{5000}{5000 - 1000} = 1.25$

NOTE

In addition to these savings resulting from the losses in the disappearing deductible range (i.e., in this example $1000 to $5000), we must add the total dollars involved in all losses for less than the deductible (i.e., in this example less than $1000)—in order to develop the total dollar loss savings.

Exhibit 5 Page 1

IRIC DEDUCTIBLE STUDY—LOSS ELIMINATION RATIOS & RATE CREDITS

$500 DEDUCTIBLE DISAPPEARING AT $5,000

Insurable Value	Losses Less than Deductible Amt. Pd ($1000)	Losses in Disappearing Range No.	Losses in Disappearing Range Amt. ($1000)	Losses in Disappearing Range Savings* in $1000	Total Eliminated Loss ($1000)	Total Loss in $1000 Under Review	LER's	Indicated Credit in Tarrif Rates
FIRE								
$ 0— 9,999	793	2516	4765	870	1663	8439	19.7	15.2
10— 24,999	698	1847	3541	636	1334	19068	7.0	5.4
25— 49,999	479	1280	2296	456	935	25083	3.7	2.9
50— 99,999	371	1094	1855	401	772	29180	2.6	2.0
100—249,999	324	991	1633	369	693	30466	2.3	1.8
250—499,999	132	494	741	192	324	17664	1.8	1.4
500,000 & over	289	1174	1873	446	734	33519	2.2	1.7
Total Fire	3086	9396	16704	3370	6455	163419	3.9	3.0
ECE								
$ 0— 9,999	1099	652	786	274	1373	2072	66.3	51.1
10— 24,999	1311	882	951	384	1695	3066	55.3	42.6
25— 49,999	830	905	1047	386	1216	2756	44.1	34.0
50— 99,999	579	834	1017	350	929	2839	32.7	25.2
100— 249,999	436	800	1085	324	760	3601	21.1	16.3
250—499,999	185	398	562	159	344	1439	24.0	18.5
500,000 & over	310	764	1167	295	605	5687	10.6	8.2
Total ECE	4750	5235	6615	2172	6922	21460	32.3	24.9

* $S = [(1 + x)nd - xL]$ where: S = Savings in losses; n= number of losses; d = deductible amount
$(1 + x)$ = multiplication factor, and x = multiplication factor less unity.

Exhibit 5 Page 2

IRIC DEDUCTIBLE STUDY—LOSS ELIMINATION RATIOS & RATE CREDITS

$1000 DEDUCTIBLE DISAPPEARING AT $10,000

Insurable Value	Losses Less than Deductible Amt. Pd ($1000)	Losses in Disappearing Range			Total Eliminated Loss ($1000)	Total Loss in $1000 Under Review	LER's	Indicated Credit in Tarrif Rates
		No.	Amt. ($1000)	Savings* in $1000				
FIRE								
$ 0— 9,999	1323	2200	7116	1654	2977	8439	35.3	27.2
10— 24,999	1140	1859	7734	1207	2347	19068	12.3	9.5
25— 49,999	834	1159	4663	770	1604	25083	6.4	4.9
50— 99,999	696	861	3176	604	1300	29180	4.5	3.5
100—249,999	617	745	2534	547	1164	30466	3.8	2.9
250—499,999	297	324	983	251	548	17664	3.1	2.4
500,000 & over	633	867	2713	662	1295	33519	3.9	3.0
Total Fire	5540	8015	28919	5695	11235	163419	6.9	5.3
ECE								
$ 0— 9,999	1360	291	712	244	1604	2072	77.4	59.7
10— 24,999	1710	341	857	284	1994	3066	65.0	50.1
25— 49,999	1214	378	948	315	1529	2756	55.5	42.8
50— 99,999	919	396	1013	328	1247	2839	43.9	33.8
100—249,999	736	427	1171	344	1080	3601	30.0	23.1
250—499,999	313	244	626	202	515	1439	35.8	27.6
500,000 & over	548	510	1504	400	948	5687	16.7	12.9
Total ECE	6800	2587	6831	2117	8917	21460	41.6	32.1

* $S = [(1 + x)nd - xL]$ where: S = Savings in losses; n = number of losses; d = deductible amount
$(1 + x)$ = multiplication factor, and x = multiplication factor less unity.

IRIC DEDUCTIBLE STUDY—LOSS ELIMINATION RATIOS & RATE CREDITS

$5000 DEDUCTIBLE DISAPPEARING AT $25,000

Insurable Value	Losses Less than Deductible Amt. Pd ($1000)	Losses in Disappearing Range: No.	Losses in Disappearing Range: Amt. ($1000)	Losses in Disappearing Range: Savings* in $1000	Total Eliminated Loss ($1000)	Total Loss in $1000 Under Review	LER's	Indicated Credit in Tarrif Rates
FIRE								
$ 0— 9,999	5559	437	2880	2010	7569	8439	89.7	69.2
10— 24,999	4240	1362	14828	4805	9045	19068	47.4	36.5
25— 49,999	2774	857	10843	2645	5419	25083	21.6	16.7
50— 99,999	2226	463	5554	1505	3731	29180	12.8	9.9
100—249,999	1956	328	3712	1122	3078	30466	10.1	7.8
250—499,999	873	120	1435	391	1264	17664	7.2	5.6
500,000 & over	2162	304	3266	1087	3249	33519	9.7	7.5
Total Fire	19790	3871	42518	13565	33355	163419	20.4	15.7
ECE								
$ 0— 9,999	1884	29	188	134	2018	2072	97.5	75.2
10— 24,999	2261	78	805	287	2548	3066	83.1	64.1
25— 49,999	1877	69	748	245	2122	2756	77.0	59.4
50— 99,999	1596	68	673	257	1853	2839	65.3	50.3
100—249,999	1522	83	798	319	1841	3601	51.1	39.4
250—499,999	747	42	437	154	901	1439	62.6	48.3
500,000 & over	1478	130	1214	507	1985	5687	34.9	26.9
Total ECE	11365	499	4863	1903	13268	21460	61.8	47.6

* $S = [(1 + x)nd - xL]$ where: S = Savings in losses; n = number of losses; d = deductible amount
$(1 + x)$ = multiplication factor, and x = multiplication factor less unity.

MINUTES OF THE 1973 ANNUAL MEETING

November 11-13, 1973

SHERATON-BOSTON HOTEL, BOSTON, MASSACHUSETTS

Sunday, November 11

The Board of Directors held its regularly scheduled meeting at the Sheraton-Boston Hotel from 2:00-6:00 p.m.

Advance registration was held from 5:00-6:00 p.m. for early arrivals.

The President's reception for new Fellows and their wives was held from 6:00-7:00 p.m.

A reception sponsored by the American Mutual, Commercial Union and Liberty Mutual Insurance Companies was held from 6:30-7:00 p.m. for members and guests and members of the Board of Directors of the American Academy of Actuaries.

Monday, November 12

The registration period began at 8:30 a.m. The 1973 Fall meeting formally convened at 9:00 a.m. with opening remarks by President Hewitt. Mr. Hewitt then introduced Commissioner of Insurance for the Commonwealth of Massachusetts, John G. Ryan, who welcomed the Society to Massachusetts.

The business session convened at 9:30 a.m. with the admission of new Fellows and Associates. President Hewitt presented diplomas to the following:

ASSOCIATES

Andler, James A.	Chou, Philip S.	Issac, David H.
Ashenberg, Wayne R.	D'Arcy, Stephen P.	Jaeger, Richard M.
Banfield, Carole J.	Demers, Daniel	Kelly, Anne E.
Bassam, Bruce C.	Dieter, George H., Jr.	Kochanski, Nancy M.
Berry, Charles H.	Donaldson, John P.	Kreuzer, James H.
Bertles, George G.	Finger, Robert J.	Leonard, Gregory E.
Bethel, Neil A.	Fisher, Wayne H.	Luneburg, Sandra C.
Blivess, Michael P.	Forman, Ben J.	Marino, James F.
Bryan, Charles A.	Hemstead, Robert J.	Nolan, John D.
Carbaugh, Albert B.	Hoylman, Douglas J.	Palczynski, Richard W

Potvin, Robert	Stergiou, Emanuel J.	Waulterkens, Paul E.
Riff, Mayer	Swift, John A.	Zeitz, Claudia
Sanko, Ronald J.	Taht, Veljo	

FELLOWS

Hall, James A.	Kline, Douglas F.	Rogers, Daniel J.
	Toothman, Michael L.	

The following Officers and Directors were then elected:

President-Elect	- M. Stanley Hughey
Vice President	- Ronald L. Bornhuetter
Secretary-Tresurer	- Robert B. Foster
Editor	- Luther L. Tarbell, Jr.
Chairman, Education & Examination Committee	- George D. Morison
Directors (Terms to Expire in 1975)	- Steven H. Newman Earl F. Petz Kevin M. Ryan

Following a coffee break a panel discussion was held entitled "No Fault and the Blues". Participants in this discussion were:

Moderator:

Charles F. Cook
Chief Actuary
United Services
Automobile Association

Panel Members:

Hon. John G. Ryan
Commissioner of Insurance
Commonwealth of Massachusetts

George E. McLean
Vice President-Actuary
Massachusetts Blue Cross, Inc.

Robert S. Seiler
Assistant Vice President & Counsel
Allstate Insurance Company

Eldon J. Klaassen
Associate Actuary
CNA/Insurance

At 11:30 the American Academy Annual Meeting convened with members of the Casualty Actuarial Society attending by invitation.

The afternoon session convened at 2:00 p.m. with the presentation of reviews of papers previously presented.

Paper—"Expense Analysis in Ratemaking and Pricing" by Roger C. Wade, Chief Actuary, Utica Mutual Insurance Company.

Reviews by: 1) Dale R. Comey, Actuary, Hartford Insurance Group.

2) Orval E. Dahme, Senior Associate Actuary, State Farm Mutual Automobile Insurance Company. (Presented by Joseph V. Naffziger)

3) C. K. Khury, Actuary, Utica Mutual Insurance Company.

A reply to the reviews was made by the author, Roger C. Wade.

Paper—"Underwriting Individual Drivers: A Sequential Approach" by Dr. John M. Cozzolino and Leonard Freifelder, University of Pennsylvania.

Review by: 1) Donald A. Jones, Professor, University of Michigan. (Presented by Robert A. Bailey)

After the presentation of reviews of papers a panel discussion was held entitled "Property and Casualty Insurance Profitability—Past, Present and Future". The participants in this panel discussion were as follows:

Moderator: Daniel J. McNamara
President
Insurance Services Office

Panel Members: Roy R. Anderson
Group Vice President
Allstate Insurance Company

David E. A. Carson
Senior Vice President
The Hartford Insurance Group

Jack Moseley
Executive Vice President
United States Fidelity and
Guaranty Company

Starting at 3:45 p.m. there were three workshop sessions:

A. "No Fault Ratemaking—Statistical Plans"

Moderator:

Clyde H. Graves
Vice President and Actuary
American Mutual Insurance
Alliance

Discussion Leaders:

David S. Powell
Assistant Actuary
Insurance Company of
North America

David R. Bickerstaff
Vice President-Actuary
Southern Farm Bureau
Casualty Insurance Company

Dennis E. Hoffman
Associate Actuary
Hartford Insurance Group

B. "Actuarial Aspects of Health Maintenance Organizations"

Moderator:

Lloyd F. Mathwick
Manager, Group Health, Life
and Pension Products
Employers Insurance of Wausau

Discussion Leaders:

James T. French
Assistant Vice President
CNA/Insurance

Robert J. Schuler
Vice President
Blue Cross of Western
Pennsylvania

C. "Ratemaking for Black Lung Coverage"

Moderator:	Stephen S. Makgill General Manager Pennsylvania Compensation Rating Bureau
Discussion Leaders:	George F. Reall General Manager National Council on Workmen's Compensation Insurance
	David J. Grady Assistant Actuary The Travelers Insurance Companies

At 6:00-7:00 p.m. a reception was held followed by a Banquet. "How to Succeed as an Actuary" by Matthew Rodermund, Vice President-Actuary, Munich Reinsurance Company was presented by John Muetterties, J. Robert Hunter, Paul Liscord, Virginia Hunter, Robert Foster, Norman Bennett, Luther Tarbell, Charles Cook, Barry Jorve, Matthew Rodermund, Adger Williams, Barbara Cook, Nancy Kochanski, Ann Phillips, and Sharon Faber. The performance was well received by the audience.

Tuesday, November 13

President Hewitt convened the business session at 9:00 a.m.

The Secretary and Treasurer reports were presented to the membership.

A moment of silence was held in remembrance of members whose death the Society had been apprised of since the last Annual Meeting:

Ralph H. Blanchard	Ward Van Buren Hart, Sr.
Henry Collins	Emma C. Maycrink
Frank A. Eger	Olive E. Outwater
	Sidney D. Pinney

President Hewitt delivered his Presidential address entitled "Tomorrow Is Where You Live".

The reading of the minutes of the last two meetings was waived.

The following committee reports were presented:

Examination and Education Committee by Mr. Morison, Chairman. It was noted that 1164 students had signed up for the November exams.

Editorial Committee by Mr. Tarbell, Chairman. Orval Dahme was named winner of the Logo Contest.

Liaison Representative to the Society of Actuaries Research Committee, Charles A. Hachemeister, told of plans for a jointly sponsored conference on Credibility and Experience Rating scheduled for September 19-21, 1974 at Berkeley, California.

Mr. Hewitt reported that the next International Congress of Actuaries would be held in Japan in 1976.

The Woodward-Fondiller Prize was presented to C. K. Khury for his review of LeRoy Simon's paper "Actuarial Applications in Catastrophe Reinsurance".

The Dorweiler Prize was jointly awarded to LeRoy Simon for his paper "Actuarial Applications in Catastrophe Reinsurance" and to Ronald L. Bornhuetter and Ronald E. Ferguson for their paper "The Actuary and IBNR".

The following new papers were presented:

"Loss Reserve Testing—A Report Year Approach" by Jeffrey T. Lange, Vice President and Wayne Fisher of the Royal-Globe Insurance Companies. (Presented by Wayne Fisher)

"Commercial Fire Ratemaking" by Robert L. Hurley, Associate Actuary, Insurance Services Office.

After a fifteen minute coffee break the following workshop sessions were held:

A. "Homeowners Ratemaking Procedures and Problems"

Moderator: Michael A. Walters
Associate Actuary
Insurance Services Office

Discussion Leaders:

William H. Crandall
Vice President
Insurance Company of North America

Richard H. Snader
Associate Actuary
United States Fidelity and Guaranty Company

B. "Current General Liability Ratemaking Procedures and Problems"

Moderator:

Edward H. Budd
Vice President
The Travelers Insurance Companies

Discussion Leaders:

Martin Bondy
Vice President and Actuary
Crum & Forster Insurance Companies

Carlton W. Honebein
Assistant Vice President and Associate Actuary
Fireman's Fund American Insurance Companies

A formal luncheon was held with R. Earl Roberson, President, American Mutual Liability Insurance Company addressing the gathering.

The afternoon program convened at 2:00 p.m. with the presentation of reviews of papers previously presented:

Paper—"Underwriting Individual Drivers: A Sequential Approach" by Dr, John M. Cozzolino and Leonard Freifelder, University of Pennsylvania.

Reviews by: 1) Jeffrey T. Lange, Vice President, and Richard Fein, both of Royal-Globe Insurance Companies, presented by Mr. Lange.

2) C. K. Khury, Actuary, Utica Mutual Insurance Company.

Paper—"A Survey of Loss Reserving Methods" by David Skurnick, Actuary, California Inspection Rating Bureau.

Review by: 1) Robert A. Anker, Associate Actuary, Employers Insurance of Wausau.

At 2:30 p.m. a panel discussion entitled "Developing and Interpreting Key Experience Reports" was presented. Participating were:

Moderator:

P. Adger Williams
Vice President and Actuary
The Travelers Insurance Companies

Panel Members:

Leo M. Stankus
Commercial Development Director
Allstate Insurance Company

Charles L. Niles, Jr.
Deputy General Manager and Vice President
General Accident Group

Richard McClure
Assistant Actuary
Kemper Insurance Group

From 2:00-4:00 a seminar entitled "The Casualty Actuarial Profession" was held for college students, mathematics teachers, business and finance teachers and guidance counselors from the Boston area high schools and New England area colleges. Participants in this seminar were as follows:

Coordinator:

James P. Jensen
Associate Actuary
Liberty Mutual Insurance Company

Panel Members:

John B. Conners
Actuarial Analyst
Liberty Mutual Insurance Company

E. Frederick Fossa
Associate Actuary
Commercial Union Assurance Companies

Norma M. Masella
Actuarial Supervisor
Insurance Services Office

James F. Richardson
Actuary
The Hanover Insurance Company

President Hewitt turned the meeting over to the new President, Paul S. Liscord, who made a few brief remarks before adjourning the meeting at 3:15 p.m.

Registration cards completed by the attendees and filed at the registration desk indicated attendance by 124 Fellows, 98 Associates, 32 invited guests and 59 wives.

FELLOWS

Alder, M.
Alexander, L. M.
Allen, E. S.
Amlie, W. P.
Anker, R. A.
Bailey, R. A.
Barker, L. M.
Bennett, N. J.
Bergen, R. D.
Berquist, J. R.
Bevan, J. R.
Bickerstaff, D. R.
Bill, R. A.
Bondy, M.
Bornhuetter, R. L.
Boyajian, J. H.
Boyle, J. I.
Brian, R. A.
Brown, W. W., Jr.
Budd, E. H.
Comey, D. R.
Cook, C. F.
Crandall, W. H.
Crowley, J. H.
Curry, H. E.
DeMelio, J. J.
Drobisch, M. B.
Dropkin, L. B.
Faber, J. A.
Ferguson, R. E.
Fitzgibbon, W. J.
Flaherty, D. J.
Foster, R. B.
Fossa, E. F.
Gibson, J. A., III
Gillam, W. S.
Gillespie, J. E.
Goddard, R. P.
Grady, D. J.
Graves, C. H.
Hachemeister, C. A.
Hall, J. A., III
Hartman, D. G.
Hartman, G. R.
Harwayne, F.
Hazam, W. J.
Hearn, V. W.
Hewitt, C. C., Jr.
Hoffmann, D. E.
Honebein, C. W.
Hughey, M. S.
Hunt, F. J., Jr.
Hunter, J. R.
Hurley, R. L.
Jacobs, T. S.
Jones, A. G.
Kallop, R. H.

Khury, C. K.
Kilbourne, F. W.
Klaassen, E. J.
Kline, D. F.
Kormes, M.
Lange, J. T.
Levin, J. W.
Linden, J. R.
Linder, J.
Lino, R.
Liscord, P. S.
Lowe, R. F.
McClure, R. D.
McGuinness, J. S.
McLean, G. E.
McNamara, D. J.
Makgill, S. S.
Masterson, N. E.
Menzel, H. W.
Morison, G. D.
Moseley, J.
Muetterties, J. H.
Munro, R. E.
Murray, E. R.
Myers, R. J.
Naffziger, J. V.
Newman, S. H.
Niles, C. L., Jr.
Oien, R. G.
Otteson, P. M.
Perkins, W. J.
Phillips, H. J.
Portermain, N. W.
Presley, P. O.
Richards, H. R.
Richardson, J. F.
Rodermund, M.
Rogers, D. J.
Ross, J. P.
Ryan, K. M.
Salzmann, R. E.
Sarason, H. M.
Scheibl, J. A.
Scheid, J. E.
Schloss, H. W.
Schuler, R. J.
Simon, L. J.
Skurnick, D.
Smick, J. J.
Smith, E. R.
Snader, R. H.
Stankus, L. M.
Stewart, C. W.
Strug, E. J.
Switzer, V. J.
Tarbell, L. L., Jr.
Toothman, M. L.
Trudeau, D. E.
Verhage, P. A.
Walsh, A. J.
Walters, M. A.
Webb, B. L.
Welch, J. P.
White, H. G.
Williams, D. G.
Williams, P. A.
Wilson, J. C.

ASSOCIATES

Andler, J. A.
Ashenberg, W. R.
Balko, K. H.
Banfield, C. J.
Bassman, B. C.
Bell, A. A.
Berry, C. H.
Bertles, G. C.
Bethel, N. A.
Blivess, M. P.
Bradshaw, J. G., Jr.
Bragg, J. M.
Brouillette, Y. J.
Bryan, C. A.
Cadorine, A. R.
Carbaugh, A. B.
Carson, D. E. A.
Carter, E. J.
Chorpita, F. M.
Chou, P. S.
Cohen, H. S.
Conners, J. B.
Cooper, W. P.
Copestakes, A. D.
Crofts, G.
D'Arcy, S. P.
Davis, R. C.
Demers, D.
Donaldson, J. P.
Drennan, J. P.
Evans, D. M.
Fallquist, R. J.
Finger, R. J.
Fisher, W. H.
Forman, B. J.
French, J. T.
Gill, J. F.
Graves, J. S.
Greene, T. A.
Hardy, H. R.
Hough, P. E.
Hoylman, D. J.

Inkrott, J. G.
Isaac, D. H.
Jaeger, R. M.
Jensen, J. P.
Jersey, J. R.
Jorve, B. M.
Kaufman, A. M.
Kaur, A. F.
Kayton, H. H.
Kelly, A. E.
Klingman, G. C.
Kochanski, N. M.
Kolodziej, T. M.
Krause, G. A.
Kreuzer, J. H.
Kuehn, R. T.
Lamb, R. M.
Leonard, G. E.
Lindquist, R. J.
Linquanti, A. J.
Luneburg, S. C.
McClenahan, C. L.
McDonald, M. G.
Marino, J. F.
Marks, R. N.
Masella, N. M.
Mathwick, L. F.
Millman, N. L.
Mokros, B. F.
Napierski, J. D.
Nelson, J. K.
Nolan, J. D.
Palczynski, R. W.
Pilon, A.
Plunkett, J. A.
Potvin, R.
Powell, D. S.
Retterath, R. C.
Rice, W. V.
Riff, M.
Sanko, R. J.
Simons, M. M.
Singer, P. E.
Spitzer, C. R.
Stergiou, E. J.
Swift, J. A.
Taht, V.
Tatge, R. L.
Thompson, E. G.
Torgrimson, D. A.
Tverberg, G. E.
Wade, R. C.
Woll, R. G.
Wulterkens, P. E.
Young, E. W.
Zeitz, C.

GUESTS

*Anderson, E. V.
Anderson, R. R.
Balfour, E. H.
*Bell, A. M.
Cohn, H. T.
Cox, A. M.
Devers, L. H.
DiSanto, W. R.
*Dunn, R.
Eubanks, L.
*Galban, L. S., Jr.
*Griffith, R. W.
Halmstad, D. G.
Harrington, T. M., Jr.
*Hatfield, B. D.
*Hoyt, F. A.
Johnson, J. E.
Lamb, J. A.
Lofgren, P. G.
McKenna, K. A.
Mullaney, P.
Musher, J.
Reall, G. F.
Roeser, K.
Rugland, W. S.
Seiler, R. S.
Trafton, M.
Trowbridge, C. L.
*VanSlyke, O. E.
West, A. B.
*Wright, R. W.
*Yoder, R. C.

*Invitational Program

Respectfully submitted,

Robert B. Foster
Secretary-Treasurer

REPORT OF THE SECRETARY

I will try to briefly bring you up-to-date on the activities of the Board of Directors and the various committees over the past twelve months. During this time the Casualty Actuarial Society membership increased by 33 members to 546. Suggesting even more growth ahead, 1,164 persons signed up for the November 1973 exams, a substantial increase over the 755 who signed up a year ago. In addition, numerous inquiries have been received by the Society from people interested in pursuing a career as a casualty actuary, not only from this country but also from England, Pakistan, Australia, West Germany and Nigeria.

The Board of Directors met during the year on the following dates:

March 1-2, 1973 at the Tobacco Valley Inn, Windsor, Connecticut
May 20, 1973 at the Nevele, Ellenville, New York
September 27-28, 1973 at the O'Hare International Tower, Chicago, Illinois
November 11, 1973 at the Sheraton-Boston Hotel, Boston, Massachusetts

Actions taken by the Board of Directors reflect the hundreds of man-hours spent on behalf of the Society by your Officers, Directors and Committee members.

1. A new procedure for handling Board of Directors mail votes was ratified and approved.
2. Enrollment of 41 new Associates was approved.
3. Committee descriptions were up-dated.
4. The Editor was authorized to have reprints made of *Proceedings* papers and reviews to be grouped by examination part.
5. On behalf of the Casualty Actuarial Society a wine cooler was presented to the Institute of Actuaries on its 125th Anniversary.
6. A Logo Contest was held. Selection of one or more winning entries is in the hands of the Editorial Committee. The Board was unable to agree on any one entry for adoption. (Later at the Boston meeting it was announced by the Editorial Committee that Orval Dahme was the contest winner.)

7. Palm Beach and San Diego were approved as meeting sites in 1976.

8. Committee appointments for fiscal year 1973-1974 have been approved.

9. Applications have been received for waiver of examinations for those who have passed the examinations of other actuarial organizations. The Board has approved waiver when it has been satisfied that the applicant has passed the equivalent of our examinations. In no case has waiver gone beyond Parts 1, 2 and 3.

10. A special committee appointed to study the CAS Library operation made an extensive report to the Board in September. The Library will be moved from the Insurance Society of New York to the CAS Office at 200 East 42nd Street, New York by January 1, 1974. The committee will continue to seek ways of improving operations in the coming year.

11. An increase of $1.00 in the examination fee for Parts 1 and 2 for 1974 (from $9 to $10) was approved. The $1 increase will offset the cost of providing sample exam questions.

12. A Task Force was appointed to explore restructure of the Syllabus as an alternative approach to an experience requirement. This approach has received support from the Board.

13. Because progress on the Georgia State Textbook has been slow, consideration is being given to printing individual chapters that are finished for the benefit of students.

14. To provide newer members with a forum, the formation of a Junior Advisory Board, on a trial basis, is being considered. The Board would be asked to submit suggestions for modifying the Society's operations and procedures.

15. A new Joint Committee of Actuarial Societies with CAS representation has been formed to share knowledge relating to the formation of tax exempt foundations.

16. The Board has approved Opinion CAS-3 submitted by the Committee on Professional Conduct. Copies will be distributed to the membership.

17. Foreseeing an increase in the workload of the Secretary-Treas-

urer's office the Finance Committee recommended separate offices of Secretary and Treasurer. A revision of the Constitution and By-Laws will be considered at the next Board meeting and, if approved, will be sent to Fellows for approval at the May 1974 meeting. The change would be implemented with the 1974 elections in November.

18. $1,500 has been appropriated for the Mathematical Association of America for the annual high school mathematics contest. This is the same amount given in previous years.

19. A new committee has been appointed to "Articulate a Member-Guest Policy". The committee reported at the November Board meeting but action has been deferred to the next board meeting.

I cannot close this report without expressing my thanks to Arthur Cadorine who, in his capacity as Librarian Assistant, has carried the burden of the Library operation, Walter Fitzgibbon who has recently taken on the handling of our many financial transactions, Edith Morabito and her assistant, Carol Olszewski, who keep things humming at 200 East 42nd Street and my own secretary, Harriet Massicotte. She's done a great Job.

Finally, it has been very helpful to be able to turn to my predecessor, Ronald Bornhuetter, for guidance as I have done on many occasions.

Respectfully submitted,

Robert B. Foster
Secretary

REPORT OF THE TREASURER

The Finance Committee met on October 26, 1973, audited the financial accounts and verified the assets of the CAS as of September 30, 1973.

The net worth of the Society now stands at $107,673—an increase of $10,569 during the past twelve months, Recognizing this increase the fidelity bond limit has been increased to $125,000.

The budget approved by the Board for fiscal year 10/1/73 to 9/30/74 calls for a budget of $85,500 with no increase in net worth. The most noteworthy change in expenses for 1974 will be the cost of the Secretary's office. Over the past few years the Society has begun to pay its own way. This trend is being extended to cover the library operation which will be centralized at 200 East 42nd Street by January 1, 1974. A payment to Georgia State of $2,500 provided for by the Dorweiler Fund and included in the budget for next year will only be made if the long awaited textbook is completed by September 30, 1974.

The cost of reprinted papers and study notes budgeted for next year represents an investment that will produce income over the next several years.

Dues for the current year will continue at $40 for Associates for less than five years, $60 for Fellows and Associates of five or more years and $75 for Subscribers to the Invitational Program.

I appreciate the support of the Board and Officers in providing for assistance in this function with the designation of Walter J. Fitzgibbon, Jr., as Assistant to the Secretary-Treasurer and look forward to approval by the membership of the establishment of the separate office of Treasurer effective with our annual meeting in 1974.

Respectfully submitted,

Robert B. Foster
Treasurer

FINANCIAL REPORT

Income and Disbursements
(from October 1, 1972 through September 30, 1973)

Income		Disbursements	
Dues	$22,160.00	Printing & Stationery	$22,679.48
Examination Fees	27,301.23	Secretary's Office	19,175.00
Meetings & Registration Fees	10,918.92	Examination Expense	10,768.34
Sale of Proceedings	3,853.00	Meeting expense	7,860.86
Sale of Readings	1,677.55	Library	726.85
Invitational Program	1,860.00	Insurance	430.00
Michelbacher Royalties	833.86	Meeting Refunds	392.00
Interest	7,283.40	Exam Refunds	819.00
Misc.	60.00	Dues Overpayment	40.00
		Math. Assoc. of America	1,500.00
		Misc.	987.80
TOTAL	$75,947.96	TOTAL	$65,379.33

Assets

As of 9/30/72		As of 9/30/73		Change
Checking Accounts	$ 645.83	Checking Accounts	$ 521.09	−124.74
Savings Accounts	85,151.82	Savings Accounts	95,845.19	10,693.37
Investments	11,306.25	Investments	11,306.25	0.00
	$97,103.90		$107,672.53	$10,568.63

Investments

	Cost
U.S.A. Treasury Bond No. 1673 Due 11/15/74	$ 1,000.00
U.S.A. Treasury Bond No. 1674 Due 11/15/74	1,000.00
U.S.A. Treasury Bond No. 299 Due 2/15/75	4,981.25
U.S.A. Treasury Bond No. 5263 Due 2/15/80	4,325.00
	$11,306.25

Dorweiler Fund (included in assets)

Balance as of 9/30/73 $6,051.72

* * * * * *

This statement reflects $16,408.46 for printing bill not actually paid as of 9/30/73.

This is to certify that we have audited the accounts and the assets shown above and find same to be correct.

Finance Committee:

Steven H. Newman, Chairman
William H. Crandall
Richard D. McClure
David G. Hartman

1973 EXAMINATIONS—SUCCESSFUL CANDIDATES

Examinations for Parts 3, 5, 7 and 9 of the Casualty Actuarial Society syllabus were held May 10 and 11, 1973 and examinations for Parts 4, 6 and 8 were held November 8 and 9, 1973. Parts 1 and 2, jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries were given May 17 and November 15. Those who passed Parts 1 and 2 were listed in the joint releases of the two Societies dated July 6, 1973 and January 10, 1974.

The following candidates successfully completed the requirements for Fellowship and Associateship in the November 1972 examinations and were awarded their diplomas at the May 1973 meeting:

NEW FELLOWS

Eyers, Robert G.
Fossa, E. Frederick
Hoffmann, Dennis E.
Khury, Costandy K.
Ross, James P.

NEW ASSOCIATES

Bovard, Roger W.
Daino, Robert A.
Lis, Raymond S., Jr.

MAY 1973 EXAMINATIONS

Following is a list of successful candidates in the examinations held in May 1973:

FELLOWSHIP EXAMINATIONS

Part 7

Alfuth, Terry J.
Berry, Charles H.
Carter, Edward J.
Drennan, John P.
Finger, Robert J.
Golz, James F.
Graves, Janet S.
Hall, James A.
Inkrott, James G.
Kaufman, Allan M.
Kayton, Howard H.
Klein, David M.
Kuehn, Ronald T.
Lamb, R. Michael
Lester, Edward P.
Miller, Michael J.
Mohl, Frederic J.
Moore, Brian C.
Penniman, Kent T.
Sheppard, Alan R.
Streff, James P.
Toothman, Michael L.
Wade, Roger C.
Woll, Richard G.
Young, Edward W.

Part 9

Conners, John B.
Dempster, Howard V., Jr.
Hall, James A.
Head, Thomas F.
Hough, Paul E.
Inkrott, James G.
Kaufman, Allan M.
Kline, Douglas F.
Miller, Philip D.
Retterath, Ronald C.
Rogers, Daniel J.
Toothman, Michael L.
Woll, Richard G.

ASSOCIATESHIP EXAMINATIONS

Part 3

Anderson, Dean R.
Anderson, Robert C.
Angell, Charles M.
Arata, David A.
Arends, Herman J.
Barden, John P.
Barrette, Raymond
Bellinghausen, Gary F.
Bertronski, Anthony J.
Bethel, Neil A.
Carbaugh, Albert B.
Carraway, Delores J.
Chhabra, Jatindar P.
Cis, Mark M.
Cohen, Alan J.
Collins, Douglas J.
Crowe, Patrick J.
Dangelo, Charles H.
D'Arcy, Stephen P.
Davis, George E.
De Angelis, Linda J.
Dieter, George H., Jr.
Donaldson, John P.
Dupuis, Camille
Eddy, Jeanne M.
Fein, Richard
Fisher, Wayne H.
Frohlich, Kenneth R.
Furst, Patricia A.
Gallagher, Thomas L.
Gerlach, Scott B.
Gottlieb, Leon R.
Grant, Gordon D.
Groot, Steven L.
Hafling, David N.
Haner, Walter J.
Harland, Henri
Hermes, Thomas M.
Hoylman, Douglas J.
Husarsky, Harry
Ingco, Aguedo M.
Jago, Richard J.
Jean, Ronald W.
Jerabek, Gerald J.
Kaliski, Alan E.
Karlinski, Frank J.
Kelly, Anne E.
King, Kerry K.
Lamb, John A.
Lamonde, Claude
Lo, Fu-Ching
Make, Ronald P.
Mansur, Joseph M.
Martin, Pamela A.
Moore, Brian C.
Morency, Bernard
Nolan, John D.
Pagliaccio, John A.
Palm, Robert G.
Paquette, Donald R.
Pearl, Marc B.
Penton, Ann M.
Petlick, Steven
Plunkett, Richard C.
Potok, Charles M.
Pouliot, Roch M.
Pratt, Joseph J.
Quirin, Albert J.
Ritzenthaler, Kenneth J.
Rosen, Kenneth R.
Rosenberg, Sheldon
Saucier, Alain
Schiavo, Michael F.
Shrum, Roy G.
Simkiss, John A., Jr.
Skolnik, Richard S.
Skrodenis, Donald P.
Stenmark, John A.
Tardif, Jean-Marc
Taylor, Frank C.
Taylor, Jane C.
Teufel, Patricia A.
Tinkler, William P.
Tu, Nancy Y.
Walker, Roger D.
Watford, James D.
Weigert, Paul M.
Weller, Alfred O.
Wengertsman, John F.
Westerholm, David C.
Whatley, Michael W.
Whatley, Patrick L.
Wright, Walter C., III
Wulterkens, Paul E.
Zelenko, Dorothy A.
Zubulake, Theodore J.

Part 5

Adams, Galen H.
Andler, James A.
Ashenberg, Wayne R.
Bartlett, William N.
Bassman, Bruce C.
Berry, Charles H.
Bertles, George G.
Bethel, Neil A.
Blivess, Michael P.
Brubaker, Randall E.
Bryan, Charles A.
Carbaugh, Albert B.
Creasey, Frank C., Jr.
D'Arcy, Stephen P.
Dee, Vicki S.
DeGarmo, Lyle W.
Demers, Daniel
Dieter, George H., Jr.
Ernst, Richard C.
Finger, Robert J.
Foley, Charles D.
Forman, Ben J.
Goldberg, Steven F.
Gottlieb, Leon R.
Groot, Steven L.
Guarini, Leonard
Hemstead, Robert J.
Hoylman, Douglas J.
Isaac, David H.
Jaeger, Richard M.
Knaus, Charles B.
Kreuzer, James H.
Leonard, Gregory E.
Luneburg, Sandra C.
Marino, James F.
McHugh, Ronald J.
McManus, Michael F.
Noceti, Stephen A.
Palczynski, Richard W.
Potvin, Robert
Riff, Mayer
Sanko, Ronald J.
Silberstein, Benny
Stergiou, Emanuel J.
Swift, John A.
Swisher, John W., Jr.
Taht, Veljo
Winser, Denise C.
Wulterkens, Paul E.
Zeitz, Claudia
Ziock, Richard W.

As a result of the above examinations four new Fellows and 38 new Associates were admitted at the Annual Meeting, November 12, 1973:

NEW FELLOWS

Hall, James A.
Kline, Douglas F.
Toothman, Michael L.
Rogers, Daniel J.

NEW ASSOCIATES

Andler, James A.
Ashenberg, Wayne R.
Banfield, Carole J.
Bassman, Bruce C.
Berry, Charles H.
Bertles, George G.
Bethel, Neil A.
Blivess, Michael P.
Bryan, Charles A.
Carbaugh, Albert B.
Chou, Philip S.
D'Arcy, Stephen P.
Demers, Daniel
Dieter, George H., Jr.
Donaldson, John P.
Finger, Robert J.
Fisher, Wayne H.
Forman, Ben J.
Hemstead, Robert J.
Hoylman, Douglas J.
Isaac, David H.
Jaeger, Richard M.
Kelly, Anne E.
Kochanski, Nancy M.
Kreuzer, James H.
Leonard, Gregory E.
Luneburg, Sandra C.
Marino, James F.
Nolan, John D.
Palczynski, Richard W.
Potvin, Robert
Riff, Mayer
Sanko, Ronald J.
Stergiou, Emanuel J.
Swift, John A.
Taht, Veljo
Wulterkens, Paul E.
Zeitz, Claudia

NOVEMBER 1973 EXAMINATIONS

The successful candidates in the November 1973 examinations were:

FELLOWSHIP EXAMINATIONS

Part 6

Alff, Gregory N.
Bassman, Bruce C.
Bellinghausen, Gary F.
Connor, Vincent P.
D'Arcy, Stephen P.
Davis, George E.
Donaldson, John P.
Drennan, John P.
Fallquist, Richard J.
Fusco, Michael
Gallagher, Thomas L.
Garand, Christopher P.
Golz, James F.
Graves, Janet S.
Head, Thomas F.
Inkrott, James G.
Kolodziej, Timothy M.
Lehman, Merlin R.
Leimkuhler, Urban E.
Leonard, Gregory E.
Lester, Edward P.
Miller, Michael J.
Mohl, Frederic J.
Ostrowski, Ellen M.
Radach, Floyd R.
Roach, Robert F.
Schultz, John J.
Sheppard, Alan R.
Stephenson, Elton A.
Stergiou, Emanuel J.
Taylor, Jane C.
Torgrimson, Darvin A.
Van Slyke, Oakley E.
Weiner, Joel S.
Wulterkens, Paul E.
Zelenko, Dorothy A.

Part 8

Anderson, Dean R.
Ashenberg, Wayne R.
Balko, Karen H.
Bell, Allan A.
Bethel, Neil A.
Brubaker, Randall E.
Conners, John B.
Degerness, Jerome A.
Dieter, George H.
Fein, Richard I.
Finger, Robert J.
Fisher, Wayne H.
Inderbitzin, Paul H.
Inkrott, James G.
Isaac, David H.
Kaliski, Alan E.
Kaufman, Allan M.
Kelly, Anne E.
Kollar, John J.
Kreuzer, James H.
Kuehn, Ronald T.
Lamb, R. Michael
Luneburg, Sandra C.
McClenahan, Charles L.
Millman, Neil L.
Moore, Phillip S.
Palczynski, Richard W.
Price, Edith E.
Retterath, Ronald C.
Reynolds, John J., III
Sanko, Ronald J.
Steeneck, Lee R.
Taht, Veljo
Tverberg, Gail E.
Winkleman, John J., Jr.
Woll, Richard G.
Young, R. James, Jr.

ASSOCIATESHIP EXAMINATION

Part 4

Aldorisio, Robert P.
Alfuth, Terry J.
Angell, Charles M.
Arata, David A.
Asch, Nolan E.
Barrette, Raymond
Bartlett, William N.
Bradley, David R.
Brickel, Stephen G.
Briere, Robert S.
Carlin, James G.
Cassity, Howard E.
Childs, Diana M.
Collins, Douglas J.
Costello, Jeanette R.
Covney, Michael D.
Creasy, Frank C., Jr.
Dean, Charles E., Jr.
DeGarmo, Lyle W.
Eagelfeld, Howard M.
Einck, Nancy R.
Eland, Douglas D.
Eldridge, Donald J.
Fasking, Dennis D.
Foley, Charles D.
Galiley, Bernard J.
Gleeson, Owen M.
Goddard, Daniel C.
Goldberg, Steven F.
Gottlieb, Leon R.
Graham, Timothy L.
Groot, Steven L.
Hafling, David N.
Hansen, Charles E.
Hermes, Thomas M.
Herzfeld, John
Jean, Ronald W.
Jenkins, Elmore, Jr.
Johnson, Marvin F.
Johnston, Daniel J.
Judd, Steven W.
Keene, Vicki S.
Levine, Kenneth M.
Lindquist, Peter L.
Lino, Richard A.
Make, Ronald P.
Marker, Joseph O.
Martin, Pamela A.
Masters, Peter A.
Matson, Anne B.
McHugh, Ronald J.
Meeks, John M.
Meteer, James W., Jr.
Miccolis, Robert S.
Mill, Ralph A.
Miyao, Stanley K.
Moller, Karl G., Jr.
Moore, Brian C.
Morell, Roy K.
Neis, Allan R.
Nelson, Janet R.
Neuhauser, Frank, Jr.
Nishio, Jo Anne
Palm, Robert G.
Patterson, David M.
Pearl, Marc B.
Petersen, Bruce A.
Petlick, Steven
Piazza, Richard N.
Potok, Charles M.
Potter, John A.
Pratt, Joseph J.
Renze, David E.
Reynolds, John D.
Ritzenthaler, Kenneth J.
Rosenberg, Sheldon
Rudduck, George A.
Schumi, Joseph R.
Sherman, Richard E.
Simkiss, John A., Jr.
Spinella, Joseph J.
Squires, Sanford R.
Steer, Grant D.
Stroud, Richard A.
Teufel, Patricia A.
Tobing, Diana
Venter, Gary G.
Walker, Roger D.
Wetherell, William H.
Whatley, Michael W.
Whatley, Patrick L.
Wittnebel, Marjory A.
Wood, Charles P., Jr.
Wright, Walter C., III
Yoder, Reginald C.
Ziock, Richard W.
Zubulake, Theodore J.

Eight candidates for Fellowship and 13 candidates for Associateship completed their requirements in the above examination and will be admitted at the Spring Meeting in 1974:

NEW FELLOWS

Conners, John B.
Inkrott, James G.
Kaufman, Allan M.
Klein, David M.
McClenahan, Charles L.
Price, Edith E.
Retterath, Ronald C.
Woll, Richard G.

NEW ASSOCIATES

Alfuth, Terry J.
Barrette, Raymond
Bartlett, William N.
Creasey, Frank C., Jr.
DeGarmo, Lyle W.
Foley, Charles D.
Goldberg, Steven F.
Gottlieb, Leon R.
Groot, Steven L.
Keene, Vicki S.
Klein, David M.
Moore, Brian C.
Ziock, Richard W.

NEW ASSOCIATES ADMITTED MAY 1973: Left to right: Robert A. Daino, Raymond S. Lis, Jr., Roger W. Bovard, President Charles C. Hewitt, Jr.

NEW FELLOWS ADMITTED MAY 1973: Left to right: James P. Ross, E. Frederick Fossa, Robert G. Eyers, Dennis E. Hoffmann, Costandy K. Khury, President Charles C. Hewitt, Jr.

NEW ASSOCIATES ADMITTED NOVEMBER 1973: Seated left to right: John A. Swift, Michael P. Blivess, Daniel Demers, Claudia Zeitz, Carole J. Banfield, Sandra C. Luneberg, outgoing President Charles C. Hewitt, Jr., Nancy M. Kochanski, Anne E. Kelly, Mayer Riff, Emanuel J. Stergiou, Richard W. Palczynski, James H. Kreuzer.

Standing left to right, center row:

George G. Bertles, Stephen P. D'Arcy, David H. Isaac, Veljo Taht, Robert Potvin, Paul E. Wulterkens, John P. Donaldson, James A. Andler, Richard M. Jaeger, Ben J. Forman, Robert J. Finger, Charles H. Berry, Ronald J. Sanko.

Standing left to right, top row:

Philip S. Chou, Wayne R. Ashenberg, Gregory E. Leonard, Charles A. Bryan, Douglas J. Hoylman, Neil A. Bethel, Bruce C. Bassman, Wayne H. Fisher, John D. Nolan, James F. Marino, Albert B. Carbaugh.

NEW FELLOWS ADMITTED NOVEMBER 1973: Left to right: Michael L. Toothman, James A. Hall, Douglas F. Kline, Daniel J. Rogers, outgoing President Charles C. Hewitt, Jr.

OBITUARIES

RALPH H. BLANCHARD

1891—1973

Ralph H. Blanchard, age 82, died in the Silver Lake Nursing Home, Plympton, Massachusetts on September 28, 1973 after an attach of pneumonia which had weakened him seriously. Born in Plympton, Massachusetts, he was graduated from Dartmouth College in 1911 and received his Doctorate at the University of Pennsylvania where he taught until 1917. He then joined the faculty of Columbia University where he was appointed professor in 1927 and remained until his retirement in 1957.

He was president of the Casualty Actuarial Society in 1941 and 1942 and also served as president of the American Association of University Teachers of Insurance, as vice president and manager of the American Managers Association, as director of the Insurance Society of New York and was a member of several other professional organizations.

Doctor Blanchard was the author of numerous publications, and instituted and edited the McGraw-Hill Insurance Series, of which twenty volumes appeared before his retirement and which has served as a standard for the insurance profession. He also produced the "Dictionary of Insurance Terms" in 1949.

Doctor Blanchard was an advisor to the industry and was also enlisted as a consultant by the Defense and Treasury departments of the federal government and the Social Security Board.

In 1958 he was elected to the Insurance Hall of Fame established by the Griffith Foundation and Ohio State University.

He never married and leaves no direct surviving relatives.

HENRY COLLINS

1883—1972

Mr. Henry Collins, Fellow of the Casualty Actuarial Society, died on June 18, 1972, in Windermere, Florida at the age of 90.

Having completed 44 years of service with the Commercial Union Companies Mr. Collins retired in 1947. He was born in 1883 in England and began his insurance career there in 1900, moved to Canada in 1903, and joined the Metropolitan, New York office of the now Commercial Union Companies in 1907.

During these many years of service dedicated to the insurance industry, Henry progressed from Assistant Manager in 1914 to Deputy Manager in 1929 and finally United States Manager of the Ocean Accident and Guarantee Corporation, Limited in 1930. In that same year he was also elected president of the affiliated Columbia Casualty Company.

In 1938 Mr. Collins was appointed Deputy United States Manager of the Commercial Union, the position he held until his retirement in 1947.

Mr. Collins became a Fellow of the Society on February 19, 1915, a short three months after the date of organization.

WARD VAN BUREN HART

1893—1973

Ward Van Buren Hart, an Associate of the Casualty Actuarial Society, died on September 5, 1973. He was born in Yorktown, New York on October 16, 1893.

He attended Yale University and graduated in 1914 with a Philosophical Orations honor which today would be summa cum laude. He was a member of Zeta Psi fraternity and was elected to membership in Phi Beta Kappa in his junior year.

Following graduation from Yale, Mr. Hart began his lifelong career with the Connecticut General Life Insurance Company. He was named assistant actuary in 1924 and associate actuary in 1949. Following his retirement in 1958 he served as a consultant to the Connecticut General on actuarial matters.

Mr. Hart became a Fellow in the Society of Actuaries in 1924 and an Associate in the Casualty Actuarial Society in the same year. He was very active in both Societies contributing a number of articles to their *Proceedings* and serving on discussion panels. His paper "Recent Developments in Commercial Accident and Health Insurance" published in the *Proceedings of the Casualty Actuarial Society,* Volume XXI was in the syllabus of the examinations of the Society until the early 1950's. He also wrote a number of discussions of papers and a book review.

He was a member of Immanuel Congregation Church in Hartford, where he served on the Prudential Committee for 13 years, and was a member of the University Club of Hartford and the Old Guard of West Hartford.

He leaves two sons, Ward Van Buren Hart, Jr. of Newington, Connecticut and Gilbert W. Hart of Yonkers, New York; a brother, Morgan D. Hart of Pluckemin, New Jersey, and five grandchildren.

SAMUEL C. PICKETT

1890—1970

Samuel C. Pickett, a Fellow of the Casualty Actuarial Society since 1933 and the retired Supervisor of the casualty-property rating division of the state of Connecticut insurance department, died August 21, 1970 at the age of 80.

Born in Escondido, California, Mr. Pickett was a graduate of Whittier College, California and Haverford College, Pennsylvania. He served in the Yankee Infantry Division during World War I and received the Purple Heart and the Croix de Guerre.

Mr. Pickett began his career with the Connecticut insurance department as an examiner. With the advent of prior approval regulations, he established the rating division for the state and was chief of the division for eight years prior to his retirement in 1951.

He was an active member of the Trinity United Methodist Church in Windsor, Connecticut, serving as secretary of the church's board for 16 years and as treasurer of the board of trustees.

He leaves two sons, Samuel E. Pickett of Windsor, Connecticut, and Kilbourne C. Pickett of Bayshore, New York; a brother, E. S. Pickett of Whittier, California; two sisters, Mrs. William J. Blount and Mrs. Alonzo White, both of Long Beach, California; six grandchildren and six great-grandchildren.

SYDNEY D. PINNEY

1897—1973

Sydney Dillingham Pinney, a former president of this Society, died April 24, 1973 at the age of 76. He is survived by his wife, the former Louisa Griswold Wells; by a son, Sydney, Jr. of Hartford, Connecticut; a daughter, Mrs. Robert W. Brewer, of Center Sandwich, N. H.; four grandchildren and a brother, Robert E. Pinney, of Bloomfield, Conn.

Born in Windsor Locks, Conn., he was a life-long resident of the Hartford area. He attended the public schools of Hartford and entered Trinity College in 1914. His college education was interrupted when he enlisted in the 101st Machine Gun Battalion, 26th Division, U. S. Army and served in France during World War I. He returned to graduate from Trinity with a B. S. degree in 1920.

Following his graduation, he joined The Travelers Insurance Company as one of several original trainees in the newly formed casualty insurance actuarial unit. During the next decade his work embraced a broad experience with various casualty lines, although workmen's compensation rate-making and rating plans occupied the center of his attention. One subject of major interest to him was retrospective rating and the relevant study of the dispersion of risk loss ratio by premium size.

He was admitted as a Fellow in the Casualty Actuarial Society in 1922 and assumed an active role in Society affairs by contributing several papers to the *Proceedings* and serving as a member of various standing committees. He was elected a vice president for two non-consecutive terms and was elected president of the Society in 1940.

During his active business life his wit, sense of humor and likeable personality helped immeasurably to gain acceptance for the role of the casualty actuary in dealings with underwriting executives and state regulatory authorities.

In 1943, he was obliged to resign from active employment because of ill-health. An attack of chronic arthritis resulted in various disabilities including the gradual and complete loss of vision first of one eye and ultimately of both eyes. Although the telephone and typwriter were indispensible implements in his work, by far the greatest assistance came from a devoted wife who attended to his personal discomforts, read to him the words he could not see, escorted him to meetings and led him by the hand in unfamiliar surroundings. To her belongs solely the deep satisfaction of sharing in an eventful career which was truly great in all the finest connotations of the term.

Many members of our Society point to the attainment of Fellowship status as one of the outstanding events of their careers. In this one instance it seems that the direction of acclaim should be reversed and the Society may well be proud of its former affiliation with Sydney Pinney.

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