ALLOCATING PREMIUM TO LAYER BY THE USE OF INCREASED LIMITS TABLES

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"Most of the literature of insurance to me is cryptic and mystic,
But when I read it I am given pause by a certain actuarial statistic.
Yes, just as some people are fascinated by fisticuffs,
I am fascinated by one group of actuarial statisticuffs . . . . . "

—OGDEN NASH

Since the actuary is expected to know how to use increased limit factors, our literature should contain something on the practical aspects of this subject. It is the intent to provide herein, especially for the student or trainee, a primer on increased limits mathematics. The subject matter is not particularly difficult but is sometimes elusive. The reader is referred to J. T. Lange's excellent paper, "The Interpretation of Liability Increased Limits Statistics," in Volume LVI of the *Proceedings* for a thorough discussion of increased limits ratemaking and related technical problems.

We will approach the subject primarily from the standpoint of a reinsurer, but the same techniques could be used by any company that wished to analyze its experience by layer of coverage. The following table presents a schedule of hypothetical private passenger bodily injury liability reinsurance rates by policy limit. In this example, the reinsurer is providing coverage in excess of $20,000 per person/$20,000 per accident at 100% of the manual increased limits indications. The 100% of manual is used here to simplify the calculations. Normally it would be expected that the reinsurer would pay the ceding company an appropriate "ceding" commission.

In our example there are two layers of coverage. The first layer is described as $20,000/$20,000 excess of $20,000/$20,000 and might be referred to as a working layer (*i.e.*, it is expected that the frequency of claims in this range will be significant, since settlements greater than $20,000 are not uncommon today). The second layer, which might be called the catastrophe layer, has a lower claim frequency and, in this case, would be described as excess of $40,000/$40,000 to the policy limits. The first layer may have enough claim frequency to be experience rated in some fashion, while the terms of the higher layer would not normally be subject to auto-

matic adjustments. It is, of course, possible to have more than two layers but this would not alter the basic premium layering techniques described below.

Policy Limits	*Increased Limits Factors**	*Percent of Total Limits Premium Allocated to Layer:* First Layer $20,000/$20,000 xs $20,000/$20,000	*Percent of Total Limits Premium Allocated to Layer:* Second Layer xs $40,000/$40,000
$ 10,000/ 20,000	1.00	—	—
15,000/ 30,000	1.12	4.46%	—
25,000/ 25,000	1.16	4.31	—
20,000/ 40,000	1.19	6.72	—
25,000/ 50,000	1.23	8.13	1.63%
50,000/100,000	1.35	10.37	7.41
100,000/300,000	1.49	9.40	16.11
Other factors used in this paper			
$ 15,000/ 20,000	1.07		
20,000/ 20,000	1.11		
25,000/ 40,000	1.21		
40,000/ 40,000	1.25		
250,000/500,000	1.59		

*The above factors are from the ISO Automobile Bodily Injury Private Passenger Supplementary Increased Limits Table which became effective in many states on January 1, 1970.

It is apparent that the first layer cannot become involved unless the per person limit or the per accident limit exceeds $20,000. Similarly, it would take an individual claim in excess of $40,000 or a combination of claims with a total in excess of $40,000 on one accident to involve the second layer.

The allocations of total limits premium shown in the table were derived as follows:

1. Since a policy with a limit of \$10,000/\$20,000 cannot penetrate the excess covers, none of the total limits premium is allocated to the layers in excess of \$20,000 per person/\$20,000 per accident. We are here ignoring the problem of an excess of policy limit judgment (bad faith judgment) which may be covered by the reinsurance treaty.

2. Losses occurring when the policy limit is \$15,000/\$30,000 (increased limits factor of 1.12) can penetrate the first excess layer in the case of a multiple-claim accident, for example, two \$15,000 claims. The ceding company is retaining losses up to \$15,000/\$20,000 (increased limits factor of 1.07) with the reinsurer committed for everything over \$20,000 per accident, subject to the policy limit. The reinsurer is thus exposed in the area of nil/\$10,000 excess \$15,000/\$20,000. The reinsurer's premium would be $\frac{1.12-1.07}{1.12}$, or 4.46% of the total limits premium collected by the ceding company on policies with limits of \$15,000/\$30,000.

3. The next two policy limits can involve only the first layer and are subject to the ceding company's loss retention of \$20,000/\$20,000 (increased limits factor of 1.11).

   a. \$25,000/\$25,000 Policy Limit

   $$\frac{1.16-1.11}{1.16} = 4.31\%$$

   b. \$20,000/\$40,000 Policy Limit

   $$\frac{1.19-1.11}{1.19} = 6.72\%$$

4. A policy with limits of \$25,000/\$50,000 (increased limits factor of 1.23) can penetrate the second layer in the event of a multiple-claim accident. The total ceded premium for both layers would be:

   $$\frac{1.23-1.11}{1.23} = 9.76\%$$

   Since the first layer is exposed in the area of \$5,000/\$20,000 excess of \$20,000/\$20,000, we allocate to that layer the difference between \$20,000/\$20,000 and \$25,000/\$40,000 or

   $$\frac{1.21-1.11}{1.23} = 8.13\%.$$

The second layer's portion is the remainder, 9.76% less 8.13%, or

$$\frac{1.23\text{-}1.21}{1.23} = 1.63\%.$$

5. The last two policy limits involve the following calculations:

   a. Policy Limit $50,000/$100,000

First Layer	Second Layer
$\frac{1.25\text{-}1.11}{1.35} = 10.37\%$	$\frac{1.35\text{-}1.25}{1.35} = 7.41\%$

   b. Policy Limit $100,000/$300,000

First Layer	Second Layer
$\frac{1.25\text{-}1.11}{1.49} = 9.40\%$	$\frac{1.49\text{-}1.25}{1.49} = 16.11\%$

In both cases shown in 5 the same number of dollars will be allocated to the first layer since $9.40\% \times 1.49 = 10.37\% \times 1.35$. This would seem to be logical unless there is some adverse selection involved in the purchasing of increased limits. For example, the above would not be logical if drivers who buy high limits do so, in part, because they expect to have more accidents. Another possible adverse selection factor might be that drivers who buy high limits may have reason to believe that their economic status would predispose them to larger than average claims since people would expect an affluent person to carry high limits and would adjust their claim sights accordingly.

Another example of the application of increased limits factors would be the case where a company writes a primary policy and another company writes excess of the primary policy and the latter company then has reinsurance from a third company. From the third company's point of view this business can be characterized as "excess on excess".

Suppose that a policy with a limit of $250,000/$500,000 (increased limits factor of 1.59) is issued by Company A and that Company A purchases reinsurance in excess of $20,000/$20,000 (increased limits factor of 1.11) from Company B at a rate of $\frac{1.59\text{-}1.11}{1.59}$ or 30.19%. Company B in turn purchases reinsurance from Company C, retaining $30,000/$80,000 net for its own account. Company B becomes involved when the direct loss

to Company A exceeds $20,000/$20,000 while Company C becomes involved when the direct loss to Company A is greater than $50,000/$100,000 (since Company B retains the first $30,000/$80,000 and Company A retains $20,000/$20,000). Company C needs to develop a rate to apply to Company B's premium. This rate would be

$$\frac{1.59\ (\$250{,}000/\$500{,}000) - 1.35\ (\$50{,}000/\$100{,}000)}{1.59\ (\$250{,}000/\$500{,}000) - 1.11\ (\$20{,}000/\$\ 20{,}000)} \text{ or } 50.0\%$$

To further complicate matters the reinsurance which Company B bought from Company C could involve two layers. The first layer being, say, $50,000/$200,000 excess $30,000/$80,000 and the second layer excess of $80,000/$280,000 (keep in mind that Company B's retention of $30,000/$80,000 is excess of Company A's $20,000/$20,000 retention). The first layer premium would be

$$\frac{1.49\ (\$100{,}000/\$300{,}000) - 1.35\ (\$50{,}000/\$100{,}000)}{1.59\ (\$250{,}000/\$500{,}000) - 1.11\ (\$20{,}000/\$\ 20{,}000)} = 29.17\%,$$

while the second would be

$$\frac{1.59\ (\$250{,}000/\$500{,}000) - 1.49\ (\$100{,}000/\$300{,}000)}{1.59\ (\$250{,}000/\$500{,}000) - 1.11\ (\$\ 20{,}000/\$\ 20{,}000)} = 20.83\%$$

for a total of 50.0%

While the calculations described are rather straightforward, the setting of the ceding commission in a reinsurance transaction is somewhat complex. The commission agreed upon by the ceding company and the reinsurer represents a judgment about the net effect of the many forces that will operate on the expected experience. These forces include the adequacy of the primary rate level; the ceding company's acquisition cost; the ceding company's level of underwriting and claim handling expertise; exposure to excess policy limits judgments and adverse selection; any redundancy or weakness thought to exist in the particular increased limits table utilized.

We have outlined the calculations for those situations which are most likely to be encountered. The possible variations are many but it is hoped that the above examples will provide the reader with the tools he needs to solve similar problems.