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NOTICE

Volume LIX, Part I No. 111

PROCEEDINGS

May 21, 22, 23, 24, 1972

HOW ADEQUATE ARE LOSS AND LOSS EXPENSE LIABILITIES?

RUTH SALZMANN

This question is continuously asked and it is difficult to answer without making an in-depth study of the company's financial statistics. Yet the quest for a simple yardstick goes on. In this paper I will tackle this formidable and perhaps impossible task.

There have been four approaches thus far, namely:

1. Roger Kenney, Insurance Editor of "U. S. Investor," relates reported loss and loss expense liabilities as of the latest date to the latest calendar year premiums written and premiums earned by major line, but cautions the reader not to draw rash conclusions as to adequacy of reserves solely by reason of the absolute values of these ratios.

2. Schedule P—Part 4 sets forth data for each Schedule P line by accident year which allows the viewer to evaluate loss and loss expense liability levels, as of the latest date (the last diagonal of the date), by comparing the ratios of these liabilities to the respective calendar year premiums earned with updated ratios of prior years at the same stage of development.

3. The present Test 3 in the NAIC Property and Liability Solidity Tests approximates the adequacy of liabilities for each Schedule P line by the excess of reported loss liabilities, as of the latest date, over stipulated percentages of the latest calendar year losses paid plus the excess of reported loss expense liabilities, as of the latest date, over stipulated percentages of the reported loss liabilities for that line.

4. The present Test 4 in the NAIC Tests (a combination of 1 and 2 above) approximates the adequacy of reported loss and loss expense liabilities as of the latest date, by the excess of the ratio of these liabilities to the latest calendar year earned premiums, for all lines combined, over the arithmetic average of similar ratios for the two preceding years, after all of the prior liabilities involved have been adjusted for developments to date.

A BETTER YARDSTICK

The disadvantages and limitations of the above yardsticks are obvious. This paper introduces another yardstick which, though fallible, will, hopefully, be an improvement over those presently in use.

To begin, if one knew, or could predict, the ultimate loss and loss expense ratio, then the calculation of the proper loss and loss expense liability would be simple because one is generated from the other: Incurred equals paid plus ending liabilities less beginning liabilities.

However, if one does not know and cares not to predict a loss and loss expense ratio, then it is necessary to construct a yardstick for ending liabilities that is responsive to the components affecting it.

One can easily accept calendar year premiums earned as an appropriate exposure base for measuring loss and loss expenses incurred. But when ending liabilities are the summation of new and old loss and loss expenses not yet paid, two additional factors become relevant: beginning liabilities and payments during the accounting period. As a result, rather than using premiums earned alone, a better yardstick for measuring liability levels, as of any accounting date, would be the following modification of premiums earned:

$$\text{Liabilities}_{12/31/n-1} + \text{Premiums Earned}_{n} - \text{Losses Paid}_{n}$$

In Exhibit I, loss and loss expense liabilities have been ratioed to this new formula base for thirteen companies (groups). Such ratios include liabilities adjusted for subsequent developments so that prior history will be more meaningful. Because loss expense liabilities were included, and because all financial data was extracted from filed annual statements, the starting point for the analysis became 12/31/68; for it was in 1969 that Schedule P changed, and, among other things, the new Schedule P added a test of loss expense liabilities for the first time. As a result, the data in

Exhibit I includes loss and loss expense liabilities for Schedule P coverages adjusted for developments through 12/31/71, and loss liabilities for Schedule O coverages adjusted up to two years.

Ratios of adjusted liabilities to premiums earned are also calculated in the exhibit for comparison purposes. Note that the range of ratios computed on the new formula base is much narrower than the range of ratios computed on a premiums earned base.

Calendar Year	Range of Ratios of Adjusted Loss & Loss Expense Liabilities To Premiums Earned	To Formula Base
1969	49.6%—128.8%	67.9%—91.2%
1970	45.0%—130.6%	61.1%—85.4%
1971	43.6%—127.9%	59.3%—83.1%

This shrinkage in the range of ratios, plus the fact that the new yardstick will produce ratios which will almost never exceed 100%, will, in and of itself, invite greater acceptance and believability. Likewise, unusual circumstances which involve significant changes in growth or payment of losses will be accommodated more satisfactorily. For these reasons, the new yardstick should be a "better-seller" in addition to being more theoretically sound.

As is true of the other yardsticks, the absolute values of the new ratios cannot be used to rank companies by degree of adequacy. Even though this yardstick should prove to be more effective than the others, because it is more responsive to unusual situations, the new ratios will still vary by company for reasons other than adequacy alone. These reasons are twofold: the unequal influence of premiums earned in the formula base, and the unequal loss potential therein. So to establish a yardstick which answers the question in the title of this paper, further refinement is necessary. Although no simple refinement will disclose the unwavering underlying truth, the margin of error can be significantly reduced by reviewing the past history of the company itself.

Limited history of this kind is set forth in Exhibit I, and two observations can be made:

1. On the new formula base, the 1971 ratios were lower than either of the two prior updated ratios for the same company (group) in

eight out of the thirteen cases, four were within the two-year range, and one exceeded both prior ratios. This fact may, or may not, have significance. The results are based upon only two years of history and we must remember, too, that 1971 was an unusually good year.

2. On the premiums earned base, 1971 ratios were lower than the three prior updated ratios for the same company (group) in five out of thirteen cases, one was within the range, and seven were in excess of all three prior ratios. This situation also may, or may not, have significance, but it comes about with one more year of history. The number of ratios falling above and below the range of prior ratios is, however, noteworthy.

No definite conclusion can be drawn from the results in Exhibit I at this time, but when more history becomes available and further developments occur, a better assessment of the new yardstick can be made. In the meantime I recommend that the new yardstick be introduced into the 1972 NAIC Solidity Tests so that the necessary history can be accumulated. As of 12/31/72, three years of updated history will be available. Though this history is limited, it may prove to be sufficiently representative because the three years include both a good and a poor underwriting year in the industry. In the 12/31/72 test, reported liabilities could be deemed acceptable if the ratio of such liabilities to the new formula base fell within the range of updated ratios in the company's past three-year history. And more specifically, the NAIC Solidity Tests would "accept" 12/31/72 liabilities as reported if the 1972 ratio equalled or exceeded any of the three prior ratios. This accomplishment, backed up by some additional surplus protection, should suffice as a minimum cursory review of loss and loss expense liabilities, the major item of concern in the balance sheet.

On an ultimate basis, I recommend that five years of updated history be used. An illustration of such an analysis is set forth in Exhibit II with data from my own company. In this exhibit, loss and loss expense liabilities for Schedule P coverages have been adjusted for developments through 12/31/71 as if the 1971 Schedule P format and scope had been in existence in prior years, and loss liabilities for Schedule O coverages have been adjusted for developments up to two years. (The difference in data between Exhibits I and II is due entirely to the new testing basis vs. prior methods.) Thus, using the same type of preliminary screening, the 12/31/71 liabilities would be considered neither inadequate nor excessively redundant if the

1971 ratio fell within the prior five-year range of updated ratios, or for Sentry, a ratio between 71.9% and 78.3%, which it does. And for the NAIC Solidity Tests, the 12/31/71 liabilities would be deemed acceptable if the 1971 ratio equalled or exceeded 71.9%, which it does.

PURPOSE OF THE YARDSTICK

A simple yardstick for evaluating the level of loss and loss expense liabilities, as currently reported, is needed by the supervisory authorities to administer their responsibilities regarding the solvency of insurance companies and the early detection of potential insolvencies. Because the yardstick must necessarily be quantified from reported financial data and then reduced to a simple translation, such a yardstick can only produce rough justice in a very difficult and complicated area. This limitation must be recognized and accepted if the primary purpose of the yardstick is to be fulfilled. Hopefully then, any yardstick would err on the conservative side. The yardstick described in this paper has been constructed with this purpose in mind.

CRITICISM AND A FUTURE POSSIBILITY

The criticism in the use of any simple yardstick is in its fallibility and the resulting harm that may be done in its misuse. For that reason, we should take a long, hard look at any quantification and its limitations.

All of the yardsticks introduced to date have encountered no difficulty in identifying the numerator; the problem has been in the composition of the denominator. This is, of course, because, as dictated by its purpose, each yardstick has been balance sheet oriented. Although this emphasis, in and of itself, is not to be criticized, it is very likely that this orientation has misled the user into the belief that an evaluation of the liability can stand completely on its own. You will recall, at the beginning of this paper, the statement was made: "If one knew, or could predict, the ultimate loss and loss expense ratio, then the calculation of the proper loss and loss expense liability would be simple because one is generated by the other." In other words, one cannot evaluate the level of loss and loss expense liabilities without also dictating a resulting loss and loss expense ratio.

It is interesting to note that all of the yardsticks skirted this relationship, but borrowed considerably from it. The components in the true loss and loss expense ratio

$$\frac{\text{Losses Paid}_n - \text{Adjusted Liabilities}_{12/31/n-1} + \text{Adjusted Liabilities}_{12/31/n}}{\text{Premiums Earned}_n}$$

have been used, as follows, in the various yardsticks:

1. Adjusted Liabilities$_{12/31/n}$ ÷ Premiums Earned$_n$
2. Liabilities$_{12/31/n}$ ÷ Losses Paid$_n$
3. And in the yardstick proposed in this paper:

$$\frac{\text{Adjusted Liabilities}_{12/31/n}}{\text{Adjusted Liabilities}_{12/31/n-1} + \text{Premiums Earned}_n - \text{Losses Paid}_n}$$

It is because liabilities continue to be evaluated independently of the loss and loss expense ratio in the new yardstick that it, too, is fallible. The following explains the two areas involved:

1. If the quantity (Losses Paid$_n$ − Adjusted Liabilities$_{12/31/n-1}$) is added to the numerator and denominator, the resulting ratio is the adjusted loss and loss expense ratio for the year(n). Because the addition of an equal amount to both the numerator and denominator of a fraction does not produce a ratio which is arithmetically equivalent, the new yardstick suffers; for when the rate of loss settlement increases, the ratio goes down, or vice versa. (Though a failing, the degree of error is less in this yardstick than the others.)

2. The second area of fallibility is in the inability of the yardstick to compensate for the substantive influence that a lower, or higher, loss and loss expense ratio produces. All else being equal, the lower this ratio, the lower the ratio of liabilities to either the formula reserve base or to premiums earned will be.

From the foregoing, it is relatively clear that a more suitable yardstick would be one that judged the credibility of the adjusted loss and loss expense ratio for the latest year. The recognition that this ratio is the real intangible in any evaluation is most important; for only this understanding will generate the tolerance necessary in the use of simple yardsticks.

When the 1972 statements are filed, adjusted loss and loss expense ratios will be available for three prior years. Three years may not be sufficient, but, in 1974, five years of updated prior history will be available.

At that time, 12/31/74 loss and loss expense liabilities could be evaluated by reviewing the "credentials" of the 1974 adjusted loss and loss expense ratio. Though this approach may have to be subsequently abandoned when experience dictates otherwise, the adjusted loss and loss expense ratio for the latest year should be deemed sufficiently reliable if it falls within the arithmetic average (for the past five years) $\pm$ 2 σ. Likewise, liabilities reported as of the latest date, which satisfy these loss and loss expense ratio parameters, would also be acceptable in any preliminary screening process.

In the case of Sentry, from Exhibit II, the range of liabilities thus generated for 12/31/71 would be $173,617,000 (derived from a loss and loss expense ratio average of 72.98%-5.90%), and $192,814,000 (72.98% + 5.90%). If reported liabilities exceeded the $173,617,000 floor, such liabilities should be deemed adequate in the preliminary review. The surplus requirement for protection against possible optimism in the derivation of reported liabilities or potential adverse deviations on business in force, would simply be the amount that the liabilities at the high end of the range (in this illustration, $192,814,000) exceeded the sum of reported liabilities and excess statutory Schedule P reserves, if any. This difference for Sentry as of 12/31/71, amounts to $13,485,000 ($192,814,000 less $177,660,000 less $1,669,000), or 22.9% of the 12/31/71 reported surplus. So long as this percentage is less than 100%, sufficient surplus protection for the underwriting operation could be assumed to exist.

This approach, though requiring more extensive calculations, is not too complicated. It eliminates many arithmetic pitfalls and provides a wide range of acceptability. But offsetting this greater latitude and tolerance in reported data is the rather stiff provision for surplus protection, which is what solvency is all about.

The problem of quantifying the total surplus needed in the parent company from unconsolidated financial reports is intentionally omitted from this discussion. This problem, though related, should be discussed separately because it is germane to all methods of evaluation. This paper limits itself to the concepts which need to be defined and explored first.

In summary, this paper proposes a new yardstick for implementation in 12/31/72 evaluations. Also, the paper proposes another approach for use beginning with 12/31/74 evaluations, both methods to be used for "fast-track" evaluations only.

From the conclusions in this paper, a much greater appreciation emerges for the use of statutory loss and loss expense reserves in Schedule

P, Parts 1 and 2. This discussion certainly endorses that concept with three major modifications: (1) liabilities for all lines combined are used in the evaluation, (2) the acceptability level for liabilities is individually calculated for each company, and (3) the minimum reserve concept is combined with a quantified amount of surplus back-up. With the third modification, the new approach could serve not only as a basis for computing minimum reserve levels by company, but also as a yardstick for the surplus safety margin needed to support the underwriting operation. This latter quantification would be a great improvement over the arbitrary percentage of a year's premium volume, which is the standard currently in use.

Exhibit I
Page 1

Analysis of Loss and Loss Expense Liability Levels by Company/Group
Liabilities Adjusted for Developments Reported Through 12/31/71

Item	Calendar Year 1968	1969	1970	1971
		Allstate (Group)		
1. Premiums Earned	$1,145,122,103	$1,338,512,756	$1,550,934,266	$1,856,659,863
2. Loss and Loss Expense Paid	—	919,519,399	1,067,384,671	1,222,690,031
3. Adjusted Loss and Loss Expense O/S	760,285,211	894,504,598	977,239,875	1,072,055,715
4. Formula Reserve Base	—	1,179,278,568	1,378,054,193	1,611,209,707
5. % Adjusted O/S to PE: (3) ÷ (1)	66.4%	66.8%	63.0%	57.7%
6. % Adjusted O/S to FRB: (3) ÷ (4)	—	75.9	70.9	66.5
7. Adjusted L. & L. E. Incurred	—	$1,053,738,786	$1,150,119,948	$1,317,505,871
8. Adjusted L. & L. E. Ratio: (7) ÷ (1)	—	78.7%	74.2%	71.0%
		American Mutual (Group)		
1. Premiums Earned	$189,726,768	$201,376,766	$218,315,430	$233,275,926
2. Loss and Loss Expense Paid	—	146,890,494	159,238,756	163,517,887
3. Adjusted Loss and Loss Expense O/S	200,474,013	206,076,850	203,049,973	193,731,205
4. Formula Reserve Base	—	254,960,285	265,153,524	272,808,012
5. % Adjusted O/S to PE: (3) ÷ (1)	105.7%	102.3%	93.0%	83.0%
6. % Adjusted O/S to FRB: (3) ÷ (4)	—	80.8	76.6	71.0
7. Adjusted L. & L. E. Incurred	—	$152,493,331	$156,211,879	$154,199,119
8. Adjusted L. & L. E. Ratio: (7) ÷ (1)	—	75.7%	71.6%	66.1%
		Dairyland Ins. (Co.)		
1. Premiums Earned	$46,119,964	$59,781,811	$72,647,301	$90,760,950
2. Loss and Loss Expense Paid	—	45,256,913	47,911,802	49,116,749
3. Adjusted Loss and Loss Expense O/S	28,636,596	33,091,930	35,334,837	46,328,307
4. Formula Reserve Base	—	43,161,494	57,827,429	76,979,038
5. % Adjusted O/S to PE: (3) ÷ (1)	62.1%	55.4%	48.6%	51.0%
6. % Adjusted O/S to FRB: (3) ÷ (4)	—	76.7	61.1	60.2
7. Adjusted L. & L. E. Incurred	—	$49,712,247	$50,154,709	$60,110,219
8. Adjusted L. & L. E. Ratio: (7) ÷ (1)	—	83.2%	69.0%	66.2%

Exhibit I
Page 2

Analysis of Loss and Loss Expense Liability Levels by Company/Group
Liabilities Adjusted for Developments Reported Through 12/31/71

Item	Calendar Year 1968	1969	1970	1971
		Employers Mutual (Group)		
1. Premiums Earned	$328,964,551	$379,438,388	$431,496,327	$460,561,072
2. Loss and Loss Expense Paid	—	235,048,244	272,364,690	285,952,768
3. Adjusted Loss and Loss Expense O/S	389,753,731	455,691,726	505,000,297	557,702,338
4. Formula Reserve Base	—	534,143,875	614,823,363	679,608,601
5. % Adjusted O/S to PE: (3) ÷ (1)	118.5%	120.1%	117.0%	121.1%
6. % Adjusted O/S to FRB: (3) ÷ (4)	—	85.3	82.1	82.1
7. Adjusted L. & L. E. Incurred	—	$300,986,239	$321,673,261	$338,654,809
8. Adjusted L. & L. E. Ratio: (7) ÷ (1)	—	79.3%	74.6%	73.5%
		Federated Mutual (Co.)		
1. Premiums Earned	$44,808,365	$49,001,259	$52,896,588	$56,534,269
2. Loss and Loss Expense Paid	—	32,802,532	34,193,581	35,219,515
3. Adjusted Loss and Loss Expense O/S	23,840,666	27,189,030	31,213,862	37,069,179
4. Formula Reserve Base	—	40,039,393	45,892,037	52,528,616
5. % Adjusted O/S to PE: (3) ÷ (1)	53.2%	55.5%	59.0%	65.6%
6. % Adjusted O/S to FRB: (3) ÷ (4)	—	67.9	68.0	70.6
7. Adjusted L. & L. E. Incurred	—	$36,150,896	$38,218,413	$41,074,832
8. Adjusted L. & L. E. Ratio: (7) ÷ (1)	—	73.8%	72.3%	72.7%
		INA (Co.)		
1. Premiums Earned	$697,861,857	$764,288,926	$851,666,880	$ 897,730,106
2. Loss and Loss Expense Paid	—	464,060,767	587,449,496	587,878,513
3. Adjusted Loss and Loss Expense O/S	635,553,345	709,633,786	695,348,494	684,499,218
4. Formula Reserve Base	—	935,781,504	973,851,170	1,005,200,087
5. % Adjusted O/S to PE: (3) ÷ (1)	91.1%	92.8%	81.6%	76.2%
6. % Adjusted O/S to FRB: (3) ÷ (4)	—	75.8	71.4	68.1
7. Adjusted L. & L. E. Incurred	—	$538,141,208	$573,164,204	$ 577,029,237
8. Adjusted L. & L. E. Ratio: (7) ÷ (1)	—	70.4%	67.3%	64.3%

Exhibit I
Page 3

Analysis of Loss and Loss Expense Liability Levels by Company/Group
Liabilities Adjusted for Developments Reported Through 12/31/71

Item	Calendar Year 1968	1969	1970	1971
		Liberty (Group)		
1. Premiums Earned	$753,355,852	$ 860,333,794	$ 973,231,567	$ 970,014,612
2. Loss and Loss Expense Paid	—	592,256,622	659,975,558	640,452,612
3. Adjusted Loss and Loss Expense O/S	882,415,976	1,048,837,129	1,163,783,535	1,240,858,314
4. Formula Reserve Base	—	1,150,493,148	1,362,093,138	1,493,345,535
5. % Adjusted O/S to PE: (3) ÷ (1)	117.1%	121.9%	119.6%	127.9%
6. % Adjusted O/S to FRB: (3) ÷ (4)	—	91.2	85.4	83.1
7. Adjusted L. & L. E. Incurred	—	$ 758,677,775	$ 774,921,964	$ 717,527,391
8. Adjusted L. & L. E. Ratio: (7) ÷ (1)	—	88.2%	79.6%	74.0%
		Michigan Mutual Liability (Co.)		
1. Premiums Earned	$68,258,663	$ 73,060,067	$ 73,414,175	$ 77,168,524
2. Loss and Loss Expense Paid	—	48,968,333	49,424,232	52,531,447
3. Adjusted Loss and Loss Expense O/S	90,552,550	94,100,843	95,914,279	97,327,599
4. Formula Reserve Base	—	114,644,284	118,090,786	120,551,356
5. % Adjusted O/S to PE: (3) ÷ (1)	132.7%	128.8%	130.6%	126.1%
6. % Adjusted O/S to FRB: (3) ÷ (4)	—	82.1	81.2	80.7
7. Adjusted L. & L. E. Incurred	—	$ 52,516,626	$ 51,237,668	$53,944,767
8. Adjusted L. & L. E. Ratio: (7) ÷ (1)	—	71.9%	69.8%	69.9%
		Nationwide (Group)		
1. Premiums Earned	$410,502,354	$476,626,663	$543,979,511	$588,438,046
2. Loss and Loss Expense Paid	—	325,387,866	348,763,416	343,840,505
3. Adjusted Loss and Loss Expense O/S	245,054,142	277,785,398	305,455,332	366,477,133
4. Formula Reserve Base	—	396,292,939	473,001,493	550,052,873
5. % Adjusted O/S to PE: (3) ÷ (1)	59.7%	58.3%	56.2%	62.3%
6. % Adjusted O/S to FRB: (3) ÷ (4)	—	70.1	64.6	66.6
7. Adjusted L. & L. E. Incurred	—	$358,119,122	$376,433,350	$404,862,306
8. Adjusted L. & L. E. Ratio: (7) ÷ (1)	—	75.1%	69.2%	68.8%

Exhibit I
Page 4

Analysis of Loss and Loss Expense Liability Levels by Company/Group
Liabilities Adjusted for Developments Reported Through 12/31/71

Item	1968	1969	1970	1971
			Calendar Year	
		St. Paul F & M (Co.)		
1. Premiums Earned	$293,705,900	$339,788,753	$395,287,279	$450,788,769
2. Loss and Loss Expense Paid	—	195,719,878	213,261,249	220,260,615
3. Adjusted Loss and Loss Expense O/S	200,969,023	237,255,963	271,612,945	318,173,721
4. Formula Reserve Base	—	345,037,898	419,281,993	502,141,099
5. % Adjusted O/S to PE: (3) ÷ (1)	68.4%	69.8%	68.7%	70.6%
6. % Adjusted O/S to FRB: (3) ÷ (4)	—	68.8	64.8	63.4
7. Adjusted L. & L. E. Incurred	—	$232,006,818	$247,618,231	$266,821,391
8. Adjusted L. & L. E. Ratio: (7) ÷ (1)	—	68.3%	62.6%	59.2%
		Sentry (Co.)		
1. Premiums Earned	$164,507,568	$182,824,772	$165,343,466	$162,692,755
2. Loss and Loss Expense Paid	—	117,269,007	108,404,089	101,564,479
3. Adjusted Loss and Loss Expense O/S	136,837,334	158,219,282	166,046,689	177,660,196
4. Formula Reserve Base	—	202,393,099	215,158,659	227,174,965
5. % Adjusted O/S to PE: (3) ÷ (1)	83.2%	86.5%	100.4%	109.2%
6. % Adjusted O/S to FRB: (3) ÷ (4)	—	78.2	77.2	78.2
7. Adjusted L. & L. E. Incurred	—	$138,650,955	$116,231,496	$113,177,986
8. Adjusted L. & L. E. Ratio: (7) ÷ (1)	—	75.8%	70.3%	69.6%
		State Farm (Group)		
1. Premiums Earned	$1,384,431,163	$1,634,769,821	$1,876,660,629	$2,143,487,278
2. Loss and Loss Expense Paid	—	1,237,087,125	1,364,645,201	1,412,010,682
3. Adjusted Loss and Loss Expense O/S	687,903,811	810,704,908	843,887,583	934,663,419
4. Formula Reserve Base	—	1,085,586,507	1,322,720,336	1,575,364,179
5. % Adjusted O/S to PE: (3) ÷ (1)	49.7%	49.6%	45.0%	43.6%
6. % Adjusted O/S to FRB: (3) ÷ (4)	—	74.7	63.8	59.3
7. Adjusted L. & L. E. Incurred	—	$1,359,888,222	$1,397,827,876	$1,502,786,518
8. Adjusted L. & L. E. Ratio: (7) ÷ (1)	—	83.2%	74.5%	70.1%

Exhibit I
Page 5

Analysis of Loss and Loss Expense Liability Levels by Company/Group
Liabilities Adjusted for Developments Reported Through 12/31/71

Item	1968	1969	1970	1971
		Travelers (Group)		
1. Premiums Earned	$1,182,593,918	$1,176,824,971	$1,227,408,010	$1,408,016,727
2. Loss and Loss Expense Paid	—	812,302,325	813,253,826	744,510,298
3. Adjusted Loss and Loss Expense O/S	984,591,500	1,030,922,045	1,010,157,646	1,243,445,750
4. Formula Reserve Base	—	1,349,114,146	1,445,076,229	1,673,664,075
5. % Adjusted O/S to PE: (3) ÷ (1)	83.3%	87.6%	82.3%	88.3%
6. % Adjusted O/S to FRB: (3) ÷ (4)	—	76.4	69.9	74.3
7. Adjusted L. & L. E. Incurred	—	$ 858,632,870	$ 792,489,427	$ 977,798,402
8. Adjusted L. & L. E. Ratio: (7) ÷ (1)	—	73.0%	64.6%	69.5%

Calendar Year (columns 1968–1971)

Source: Filed Annual Statements

Definitions

1. Adjusted Loss and Loss Expense O/S includes Schedule P loss and loss expense liabilities adjusted for developments through 12/31/71, and Schedule O loss liabilities adjusted for developments (net as to salvage) through two years, or 12/31/71, whichever date is earlier. For the current year, adjusted O/S is the same as the loss and loss expense O/S reported.
2. Formula Reserve Base is the quantity: adjusted loss and loss expense O/S at the beginning of the period, plus premiums earned for the calendar year, less loss and loss expense paid in the calendar year.
3. Adjusted L. & L. E. Incurred is the calendar year incurred volume of loss and loss expense using adjusted loss and loss expense O/S at the beginning and end of each calendar year in the calculation thereof.

Exhibit II

Analysis of Loss and Loss Expense Liability Levels by Company/Group
Liabilities Adjusted for Developments Through 12/31/71
(had the 1971 Annual Statement been used during this entire period)

Item	Calendar Year 1965	1966	1967	1968	1969	1970	1971
1. Premiums Earned	—	$140,956,960	$151,505,888	$164,507,568	$182,824,772	$165,343,466	$162,692,755
2. Loss and Loss Expense Paid	—	88,111,660	97,753,331	109,722,099	117,269,007	108,404,089	101,564,479
3. Adjusted Loss and Loss Expense O/S	$98,762,903	109,081,179	120,370,620	135,906,422	157,836,921	166,664,844	177,660,196
4. Formula Reserve Base	—	151,608,203	162,833,736	175,156,089	201,462,187	214,776,298	227,793,120
5. % Adjusted O/S to PE (3) ÷ (1)	—	77.4%	79.4%	82.6%	86.3%	100.8%	109.2%
6. % Adjusted O/S to FRB: (3) ÷ (4)	—	71.9	73.9	77.6	78.3	77.6	78.0
7. Adjusted L. & L. E. Incurred	—	$ 98,429,936	$109,042,772	$125,257,901	$139,199,506	$117,232,012	$112,559,831
8. Adjusted L. & L. E. Ratio: (7) ÷ (1)	—	69.8%	72.0%	76.1%	76.1%	70.9%	69.2%

Arithmetic average (1966–1970) = 72.98%
σ = 2.95%

Source: Filed Annual Statements and company records

Definitions:

1. Adjusted Loss and Loss Expense O/S includes Schedule P (including PD) loss and loss expense liabilities adjusted for developments through 12/31/71 and Schedule O loss liabilities adjusted for developments (net as to salvage through two years, or 12/31/71, whichever date is earlier. For the current year, adjusted O/S is the same as the loss and loss expense O/S reported.
2. Formula Reserve Base is the quantity: adjusted loss and loss expense O/S at the beginning of the period, plus premiums earned for the calendar year, less loss and loss expense paid in the calendar year.
3. Adjusted L. & L. E. Incurred is the calendar year incurred volume of loss and loss expense using adjusted loss and loss expense O/S at the beginning and end of each calendar year in the calculation thereof.

DISCUSSION BY MATTHEW RODERMUND

Miss Salzmann has written another good paper. This time she has tackled the elusive subject of the evaluation of current loss and loss expense reserves, and she has proposed not only a yardstick for the measurement of current loss reserves, but also a method of establishing minimum statutory reserves relative to policyholders surplus. Her approach is well thought out, and it seems reasonable that her proposals be tried.

Yet this reviewer has a problem, a problem that relates not to the substance of Miss Salzmann's paper but rather to the mechanics. The problem arises out of the first half of the paper only, which presents the yardstick for current loss reserves.

Miss Salzmann's yardstick is represented by the expression:

$$\frac{\text{Adjusted Liabilities}_{12/31/n}}{\text{Adjusted Liabilities}_{12/31/n-1} + \text{Premiums Earned}_{n} - \text{Losses Paid}_{n}}$$

A study of the denominator of the above fraction reveals that this expression cannot be verbalized, has no meaning, does not exist in the real world. It is a mathematical expression, pure and simple.

This realization was mildly disconcerting to the reviewer. He then noted that the mathematical expression is equal to:

$$\frac{\text{Incurred Losses}_{n} + \text{Liabilities}_{12/31/n-1} - \text{Losses Paid}_{n}}{\text{Premiums Earned}_{n} + \text{Liabilities}_{12/31/n-1} - \text{Losses Paid}_{n}}$$

which is our old friend loss ratio, with a damper provided by adding (or subtracting) identical quantities to the numerator and denominator of the loss ratio fraction.

Unfortunately, this expression cannot be verbalized either, and it would be difficult to explain why dampening a company's loss ratio in this way, which appears to have no inner logic, produces a yardstick by which loss reserves will be evaluated.

The problem is that the yardstick Miss Salzmann has proposed can be described only in mathematical terms. Actuaries need credibility. They stand to lose their credibility if they are not able to talk about their business so that lay people will understand. The worry here is that because this important yardstick cannot be described in words, but must be demonstrated mathematically, it may never be sold and will never be used. The evaluation of loss reserves is not an abstruse subject, like credibility, for instance, or variance. Rather it is an area of our business where almost every informed practitioner has ideas and likes to talk about them. A system that can be explained only mathematically may be rejected by the industry even if it has theoretical merit.

The reviewer hopes that readers of Miss Salzmann's paper and of this review will think seriously about the public presentation of actuarial ideas. Even the concept of credibility can be illustrated for lay people if the analogy of thrown dice, or tossed coins, is used. But the president of an insurance company, if he is not an actuary, is going to question Miss Salzmann's yardstick where his own reserves are concerned, because there is no concept that he can grasp.

It is hoped that Miss Salzmann's rebuttal to this review will also be read and seriously considered, because it is a good one. The subject she has tackled is important to actuaries, but, in the opinion of this reviewer, the communication problem is also important and deserves attention.

AUTHOR'S REVIEW OF DISCUSSION

Mr. Rodermund's review criticizes the yardstick proposed in my paper because of its phantom qualities in that it is a mathematical expression devoid of verbal explanation. This criticism is well deserved and may, as the reviewer points out, seriously detract from both the acceptance and use of the new yardstick.

This lack of verbal identity was a matter of concern to the author when the paper was written, and there are some subtle, and not so subtle, references to this dilemma in the paper. The most obvious reference, of course, is that the author had no better name for the new yardstick than "formula base". Then there was the rather lame argument made in the paper that the results themselves would sell the product.

Mr. Rodermund's review did point out the major stumbling block to any immediate widespread use of the proposed yardstick, but the impact of his criticism diminishes when one considers the effectiveness of the alternatives. Such alternatives are those presently in use which are covered in the paper itself. The other alternatives fall into one general category. These latter approaches, in one way or another, use an expected loss and loss expense ratio to generate expected liability levels with which reported liabilities are then compared. On this basis, unity or greater becomes the rule-of-thumb for redundancy. Such yardsticks would be simple to explain, simple to apply and simple to make comparisons by company. But lost in this shuffle of over-simplification is the fact that all of the answers would be dependent upon the accuracy of the selected expected loss and loss expense ratio.

Because liability levels, and inherent loss and loss expense volumes, are equally difficult to measure, a simplified version of one should not be used to generate a yardstick for the other. Furthermore, any such subtle infusion of assumptions for the sake of simplicity might also tend to underemphasize the tolerance needed in the application of a yardstick to a very complicated and sophisticated actuarial compilation. For this reason alone, such alternatives must never be produced or encouraged by the profession that lays claim to the need for such expertise in this area. As a result, the proposed yardstick incorporates the relevant financial data, as reported in the Annual Statement, unadjusted for any assumptions on that data.

The author recognizes and accepts the criticism made by Mr. Rodermund, but also recognizes the very great need for a better yardstick. This better yardstick has to be one that will produce satisfactory answers while still recognizing the tolerance and judgment needed in its application. This primary principle, in the opinion of the author, should not be compromised to meet other concerns, regardless of their individual merit.

LOSS RESERVING IN THE SIXTIES

R. J. BALCAREK

For some time it has been obvious to the writer that recent loss reserves are not as strong as they were ten or fifteen years ago. This opinion he formed on the basis of loss reserve developments of a handful of selected companies. The steady rise in the bureau development factors used in Private Passenger Auto ratemaking tended to provide additional support for such an opinion.

The purpose of this paper is to analyze some industrywide data to determine more closely what has happened to loss reserve safety margins.

BASIC DATA USED IN THIS STUDY

The basic data were derived from the Supplement to New York Auto Liability Experience published annually by the New York Insurance Department in the Loss and Expense Ratios. This shows developments of policy year loss experience at twelve month intervals up to 84 months. It is the same exhibit used by Frank Harwayne in his paper "Estimating Ultimate Incurred Losses in Auto Liability Insurance" published in PCAS Volume XLV. Attention has to be drawn to the fact that the New York exhibit is not a closed system and therefore we have some slight fluctuations in policy year earned premiums even at a relatively late stage of development. To avoid representing a withdrawal of a company as favorable loss development, the writer decided to follow Mr. Harwayne and relate the incurred, outstanding and paid loss data to earned premiums. These figures are summarized in Exhibit I for policy years 1953 to 1969.

FINDINGS

The most important finding is that there is a progressive increase in the amount of payments at later stages of development. This is shown in Table 1 which relates, by policy year, the cumulative payments at a given stage of development to cumulative payments twelve months earlier. These ratios show a fairly steady rise, and they have been projected for the not yet fully developed policy years by fitting straight lines to the actual figures for the latest eleven years. The ratio of incurred losses at 84 months to paid losses at 84 months also exhibited an increasing trend, and it has been projected in a similar manner to paid losses for the undeveloped policy years.

TABLE 1

AUTO B.I.—N.Y.

Ratio of cumulative paid losses at X months development to paid losses at (X-12) months development

Pol. Yr.	*X=36*	*X=48*	*X=60*	*X=72*	*X=84*	*Ratio of Incurred Losses at 84 months to paid losses at 84 months*
1953	1.6123	1.1845	1.0921	1.0429	1.0211	1.0314
1954	1.6980	1.1829	1.0835	1.0388	1.0182	1.0338
1955	1.6311	1.1818	1.0821	1.0353	1.0205	1.0417
1956	1.6360	1.1938	1.0915	1.0465	1.0262	1.0511
1957	1.6314	1.2066	1.0906	1.0428	1.0336	1.0547
1958	1.6392	1.2116	1.1001	1.0641	1.0314	1.0588
1959	1.6269	1.2195	1.1115	1.0668	1.0343	1.0598
1960	1.6369	1.2177	1.1194	1.0722	1.0489	1.0643
1961	1.6276	1.2504	1.1259	1.0944	1.0300	1.0792
1962	1.6231	1.2620	1.1584	1.0641	1.0512	1.0740
1963	1.6119	1.3052	1.1270	1.0940	1.0621	1.0814
1964	1.6929	1.2313	1.1560	1.1245	1.0730	1.0672
1965	1.6739	1.2890	1.1676	1.1239	1.0677*	1.0850*
1966	1.6896	1.2895	1.1956	1.1264*	1.0725*	1.0891*
1967	1.6843	1.3244	1.1894*	1.1349*	1.0773*	1.0932*
1968	1.7310	1.3202*	1.1991*	1.1434*	1.0820*	1.0973*
1969	1.7130*	1.3310*	1.2088*	1.1519*	1.0868*	1.1014*

*Projected by the use of straight lines fitted to the actual figures for the latest 11 years.

At the end of 1970, policy years 1953 through 1964 have been fully developed to 84 months. For policy years 1965-1969, the incurred losses at 84 months development can be estimated by the use of projected ratios from Table 1. These estimates are shown in the attached Exhibit I.

The figures in Exhibit I indicate that in the fifties the losses, by policy year, were reserved with a margin of safety. This margin at 24 months development ranged from 5.2% for policy year 1954 to 9.5% for policy year 1959. While these safety margins fluctuated, there did not appear to be any particular trends. The real trend began with policy year 1961. Table 2 summarizes a few figures.

TABLE 2

Loss Reserves Margins at 24 Months Development

Policy Years	*% of Outstanding Loss Reserves**
1960	8.65
1961	4.87
1962	0.43
1963	(−)3.55
1964	(−)7.32

* (−) indicates adverse development

The above margins have been determined on the basis of actual development to 84 months, and they indicate a rather sharp change in loss reserving practices. According to the projections, this continued through the sixties and the loss reserves for 1969 policy year at 24 months development appear to be inadequate by 22.4%.

For those who like to observe reserve margin changes by calendar year, the adequacy margins by policy year from Exhibit I can be applied to outstanding loss reserves at the end of a particular calendar year. The results are summarized in Table 3.

TABLE 3
(in 000's)

Policy Years	*Valuation Date*	*Actual Loss Reserve*	*Reserve Margin**	
			Amount	*%*
1955-59	12-31-60	$402,165	$ 56,435	14.0
1956-60	12-31-61	421,569	60,719	14.4
1957-61	12-31-62	428,226	46,838	10.9
1958-62	12-31-63	446,556	33,629	7.5
1959-63	12-31-64	463,996	11,538	2.5
1960-64	12-31-65	499,971	(−) 5,812	(−) 1.2
1961-65	12-31-66	539,237	(−) 6,195	(−) 1.1
1962-66	12-31-67	585,483	(−) 50,373	(−) 8.6
1963-67	12-31-68	633,348	(−) 69,530	(−)11.0
1964-68	12-31-69	671,377	(−)105,403	(−)15.7
1965-69	12-31-70	705,344	(−)132,664	(−)18.8

* (−) indicates loss reserve inadequacy

Thus, during the nine calendar years from 1961 to 1970, the industry appears to have understated their losses to a substantial extent. Based on the reserve margins for the five policy years, this understatement amounted to about $193,000,000, or 3.8% of earned premiums for the nine years.

SOME OBSERVATIONS

It is obvious that during the sixties less adequate loss reserves acquired a fair amount of popularity, if not respectability. This should not be surprising as, in the short run, most of the practical forces affecting the operations of an insurance company make an under-reserved position preferable to an over-reserved position. Examples:

(1) The underwriting department is not unhappy as low loss reserves make their underwriting performance look better than it is.

(2) The field operations and agents are happy because lower loss ratios make for higher contingent commissions and profit sharing. In addition, the lower rates resulting from inadequate loss reserves put them into a more competitive position to secure larger volumes of business.

(3) The operating results are improved and this makes for happy managements and stockholders.

(4) From the point of view of our regulators, the situation also has a few good points: (a) The companies will request fewer and smaller rate increases. (b) Under-reserved losses increase surplus, thus increase capacity.

(5) Inadequate loss reserves are also said to be advantageous from a loss adjusting point of view by supposedly reducing the amount of settlement.

To be sure, there is a disadvantage in indulging in a progressive erosion of your loss reserves. The danger is that in the long run such a company will become insolvent. It is obvious from our study that the long run may be very long indeed, thus it does not go very far in reducing the influence of short-term considerations.

EXHIBIT I

New York Auto Bodily Injury
Development of Loss Experience by Policy Year

Experience as of the End of Calendar Year	*% TO EARNED PREMIUM* Paid Loss	*O/S Loss*	*Incurred Loss*	*Loss Development after the end of Calendar Yr.*** Amount	*% of O/S*
Pol. Yr. 1953					
1954	22.707	31.979	54.686	2.673	8.36
1955	36.611	17.674	54.285	2.272	12.86
1956	43.366	10.038	53.404	1.391	13.86
1957	47.359	5.528	52.887	.874	15.81
1958	49.390	2.936	52.326	.313	10.66
1959	50.430	1.583	52.013		
Pol. Yr. 1954					
1955	23.951	34.873	58.824	1.828	5.24
1956	40.669	18.802	59.471	2.475	13.16
1957	48.107	10.720	58.827	1.831	17.08
1958	52.124	5.701	57.825	.829	14.54
1959	54.149	3.195	57.344	.348	10.89
1960	55.134	1.862	56.996		
Pol. Yr. 1955					
1956	28.588	39.392	67.980	2.350	5.97
1957	46.629	22.373	69.002	3.372	15.07
1958	55.108	12.622	67.730	2.100	16.64
1959	59.634	7.253	66.887	1.257	17.33
1960	61.737	4.378	66.115	.485	11.08
1961	63.001	2.629	65.630		
Pol. Yr. 1956					
1957	28.011	42.958	70.969	3.566	8.30
1958	45.826	25.408	71.234	3.831	15.08
1959	54.708	15.601	70.309	2.906	18.63
1960	59.713	9.622	69.335	1.932	20.08
1961	62.488	5.889	68.377	.974	16.54
1962	64.124	3.279	67.403		

**(−) indicates adverse development

Experience as of the End of Calendar Year	*% TO EARNED PREMIUM* Paid Loss	O/S Loss	Incurred Loss	*Loss Development after the end of Calendar Yr.*** Amount	% of O/S
Pol. Yr. 1957					
1958	29.565	46.801	76.366	4.214	9.00
1959	48.232	28.883	77.115	4.963	17.18
1960	58.199	17.774	75.973	3.821	21.50
1961	63.474	11.432	74.906	2.754	24.09
1962	66.189	6.891	73.080	.928	13.47
1963	68.410	3.742	72.152		
Pol. Yr. 1958					
1959	26.928	45.527	72.455	4.090	8.98
1960	44.140	28.646	72.786	4.421	15.43
1961	53.481	18.359	71.840	3.475	18.93
1962	58.835	11.521	70.356	1.991	17.28
1963	62.606	6.680	69.286	.921	13.79
1964	64.571	3.794	68.365		
Pol. Yr. 1959					
1960	23.244	40.524	63.768	3.834	9.46
1961	37.816	26.376	64.192	4.258	16.14
1962	46.115	17.001	63.116	3.182	18.72
1963	51.256	10.675	61.931	1.997	18.71
1964	54.678	6.044	60.722	.788	13.04
1965	56.554	3.380	59.934		
Pol. Yr. 1960					
1961	19.383	35.446	54.829	3.067	8.65
1962	31.728	22.758	54.486	2.724	11.97
1963	38.635	15.028	53.663	1.901	12.65
1964	43.247	9.358	52.605	.843	9.01
1965	46.368	5.149	51.517	(−) .245	(−) 4.76
1966	48.634	3.128	51.762		
Pol. Yr. 1961					
1962	18.302	34.393	52.695	1.674	4.87
1963	29.788	23.480	53.268	2.247	9.57
1964	37.248	15.356	52.604	1.583	10.31
1965	41.938	10.010	51.948	.927	9.26
1966	45.899	6.146	52.045	1.024	16.66
1967	47.278	3.743	51.021		

**(−) indicates adverse development

Experience as of the End of Calendar Year	*% TO EARNED PREMIUM* *Paid Loss*	*O/S Loss*	*Incurred Loss*	*Loss Development after the end of Calendar Yr.*** *Amount*	*% of O/S*
Pol. Yr. 1962					
1963	18.934	35.191	54.125	.151	0.43
1964	30.732	23.783	54.515	.541	2.27
1965	38.783	16.233	55.016	1.042	6.42
1966	44.927	10.604	55.531	1.557	14.68
1967	47.808	6.434	54.242	.268	4.17
1968	50.255	3.719	53.974		
Pol. Yr. 1963					
1964	18.660	35.670	54.330	(−)1.266	(−) 3.55
1965	30.078	25.541	55.619	.023	0.09
1966	39.258	17.779	57.037	1.441	8.11
1967	44.244	11.616	55.860	.264	2.27
1968	48.404	7.306	55.710	.114	1.56
1969	51.410	4.186	55.596		
Pol. Yr. 1964					
1965	19.426	38.060	57.486	(−)2.785	(−)7.32
1966	32.886	27.325	60.211	(−) .060	(−) .22
1967	40.491	19.439	59.930	(−) .341	(−)1.75
1968	46.806	13.567	60.373	.102	0.75
1969	52.635	7.725	60.360	.089	1.15
1970	56.478	3.793	60.271		
Pol. Yr. 1965					
1966	19.366	39.326	58.692	(−)4.827*	(−)12.27
1967	32.416	28.967	61.383	(−)2.136*	(−) 7.37
1968	41.784	21.261	63.045	(−) .474*	(−) 2.23
1969	48.788	14.217	63.005	(−) .514*	(−) 3.62
1970	54.831	7.931	62.762	(−) .757*	(−) 9.54
1971	58.543*	4.976*	63.519*		
Pol. Yr. 1966					
1967	18.981	39.187	58.168	(−)6.880*	(−)17.56
1968	32.069	28.702	60.771	(−)4.277*	(−)14.90
1969	41.352	20.796	62.148	(−)2.900*	(−)13.94
1970	49.440	12.684	62.124	(−)2.924*	(−)23.05
1971	55.689*				
1972	59.727*	5.321*	65.048*		

**(−) indicates adverse development

Experience as of the End of Calendar Year	*% TO EARNED PREMIUM*			*Loss Development after the end of Calendar Yr.***	
	Paid Loss	*O/S Loss*	*Incurred Loss*	*Amount*	*% of O/S*
Pol. Yr. 1967					
1968	18.927	40.691	59.618	(−)7.398*	(−)18.18
1969	31.879	29.863	61.742	(−)5.274*	(−)17.66
1970	42.221	21.423	63.644	(−)3.372*	(−)15.74
1971	50.218*				
1972	56.992*				
1973	61.398*	5.618*	67.016*		
Pol. Yr. 1968					
1969	18.447	41.251	59.698	(−)8.923*	(−)21.63
1970	31.931	31.526	63.457	(−)5.164*	(−)16.38
1971	42.155*				
1972	50.548*				
1973	57.797*				
1974	62.536*	6.085*	68.621*		
Pol. Yr. 1969	17.537	40.132	57.669	(−)8.974*	(−)22.36
1971	30.041*				
1972	39.984*				
1973	48.333*				
1974	55.675*				
1975	60.508*	6.135*	66.643*		

*Estimated
**(−)indicates adverse development

DISCUSSION BY J. R. BERQUIST

One can hardly read Mr. Balcarek's paper without becoming concerned about the adequacy of the industry's automobile bodily injury reserves in New York, and although there may be reason to expect more upward development of these reserves in New York state than elsewhere, the concern inevitably extends to other states, and eventually to other bodily injury reserves as well.

In view of these implications, it is important that we consider the extent to which the author's results and conclusions may have been influenced by his methodology. The tables which follow will show that, although the paper may exaggerate the magnitude of the trends described therein, the direction is valid and should be a matter of concern for company management and owners alike.

In Table 1 of the paper, the author used a least-squares trend line to project ratios of cumulative paid losses at X months development to paid losses at (X-12) months development. In general, this technique is both sound and practical. However, whenever a trend line is used to extrapolate into the future, as must be done with so many actuarial computations, it must be recognized that even a line developed with sophisticated mathematics is entirely dependent upon the points which have been used to develop that line. Since the selection of the points to be included in the calculation is usually a matter of judgment and convenience, one is at liberty to impose his own judgment upon that selection. In this review, we have purposely slanted those judgment selections toward optimistic results in an effort to answer the question: "At best, how bad can it be?" We have not attempted to answer the other obvious question, or to perform a sophisticated sensitivity analysis.

A careful review of the values in Table 1 of the paper, or preferably a plot of those points, reveals that the trend line could have been over-influenced by unusually large increases in the last few years. Suppose, for example, that we decided to use the same period of time but to eliminate the highest and the lowest value on grounds that each represents spurious deviations. In effect, we are assuming that the values for those two eliminated periods fall on the trend line. The results of a trend line developed in this manner are shown in Table A and Exhibit A.

TABLE A

AUTO B.I.—N.Y.

Ratio of cumulative paid losses at X months development to paid losses at (X-12) months development

Pol. Yr.	*X=36*	*X=48*	*X=60*	*X=72*	*X=84*	*Ratio of Incurred Losses at 84 months to Paid Losses at 84 months*
1965	1.6739	1.2890	1.1676	1.1239	1.0627*	1.0810*
1966	1.6896	1.2895	1.1956	1.1205*	1.0669*	1.0843*
1967	1.6843	1.3244	1.1800*	1.1283*	1.0711*	1.0877*
1968	1.7310	1.3131*	1.1884*	1.1362*	1.0753*	1.0910*
1969	1.7051*	1.3232*	1.1968*	1.1440*	1.0795*	1.0944*

*Projected by the use of straight lines fitted to the actual figures for the latest 11 years but eliminating the highest and the lowest values

In order to get still another "feel" for the range of compound effects of the extrapolated values, a "trend line" was drawn by inspection of past values. The line was selected so that it is the lowest trend line that could reasonably emerge. The results of this effort are shown in Table B and Exhibit B.

TABLE B

AUTO B.I.—N.Y.

Ratio of cumulative paid losses at X months development to paid losses at (X-12) months development

Pol. Yr.	*X=36*	*X=48*	*X=60*	*X=72*	*X=84*	*Ratio of Incurred Losses at 84 months to Paid Losses at 84 months*
1965	1.674	1.289	1.168	1.124	1.058*	1.068*
1966	1.690	1.290	1.196	1.118*	1.059*	1.069*
1967	1.684	1.324	1.172*	1.121*	1.060*	1.071*
1968	1.731	1.320**	1.178*	1.124*	1.061*	1.072*
1969	1.713**	1.331**	1.182*	1.128*	1.062*	1.073*

*"Most Favorable" line
**"Most Favorable" values equivalent to author's

A comparison of the indicated inadequacy levels at the end of the first year of each of the policy years 1965 through 1969, calculated by each of the three methods is as follows:

TABLE C

COMPARISON OF RESULTS

Loss Development After the End of First Year
As a % of Outstanding Reserves

Policy Year	*11 Point Trend Line, per paper (Exhibit I)*	*11 Period but Eliminating Highest and Lowest Points (Exhibit A)*	*Most Favorable Trade Line (Exhibit B)*
1965	(−)12.3	(−)10.9	(−) 8.3
1966	(−)17.6	(−)15.0	(−)12.9
1967	(−)18.2	(−)14.4	(−) 8.2
1968	(−)21.6	(−)16.3	(−) 9.2
1969	(−)22.4	(−)15.8	(−) 7.6

The above comparison shows most of the range within which the actual results are likely to fall. Even acknowledging that the "most favorable" line itself does not have a sound statistical basis, it does, nevertheless, provide an estimate of the lower end of the range of inadequacy. While no effort has been made to develop the upper end of the range by the selection of a most unfavorable line, it is this reviewer's opinion that the author's results are closer to that end of the scale.

The values developed in the above tables and exhibits can be converted to reserve margin indications as the author has shown in Table 3 of the paper. A comparison of these indications is as follows:

TABLE D

COMPARISON OF RESERVE MARGINS

		*Reserve Margin**					
		Table 3		*Exhibit A*		*Exhibit B*	
Policy Years	*Valuation Date*	*Amount*	*%*	*Amount*	*%*	*Amount*	*%*
1961-65	12-31-66	(−)6,195	(−)1.1	(−)3,321	(−)0.6	2,366	0.4
1962-66	12-31-67	(−)50,373	(−)8.6	(−)38,994	(−)6.6	(−)31,267	(−)5.3
1963-67	12-31-68	(−)69,530	(−)11.0	(−)52,104	(−)8.2	(−)13,616	(−)2.1
1964-68	12-31-69	(−)105,403	(−)15.7	(−)74,294	(−)11.1	(−)26,789	(−)4.0
1965-69	12-31-70	(−)132,664	(−)18.8	(−)83,956	(−)11.9	(−)18,451	(−)2.6

*(−) indicates loss reserve inadequacy. Amounts in thousands of dollars while percentages are of outstanding reserves.

Conclusions

Although it appears to this reviewer that the magnitude of the inadequacy is on the high side, it is important to note that the fundamental premise of the paper ("it is obvious that during the sixties less adequate loss reserves acquired a fair amount of popularity, if not respectability") remains unchallenged. Even if the emerging results tend toward the "most favorable" line, they are still unacceptable!

Company management, regulators, and owners certainly must remedy this condition if the companies are to survive over the long pull. One way to do this would be to give more attention to the use of available actuarial and statistical techniques to evaluate the overall levels of their reserves.

It is easy to allow other day-to-day concerns to overshadow this most important task of maintaining adequate reserves. Mr. Balcarek's continuing vigilance, however, has helped to remind us all of our responsibility.

EXHIBIT A

New York Auto B.I.
Development of Loss Experience by Policy Year

Experience as of the End of Calendar Year	*% TO EARNED PREMIUM* Paid Loss	*O/S Loss*	*Incurred Loss*	*Loss Development after the end of Calendar Yr.*** Amount	*% of O/S*
Pol. Yr. 1965					
1966	19.366	39.326	58.692	(−)4.297*	(−)10.93
1967	32.416	28.967	61.383	(−)1.106*	(−) 3.82
1968	41.784	21.261	63.045	.056*	.26
1969	48.788	14.217	63.005	.016*	.11
1970	54.831	7.931	62.762	(−) .227*	(−) 2.86
1971	58.269*	4.720*	62.989*		
Pol. Yr. 1966					
1967	18.981	39.187	58.168	(−)5.918*	(−)15.01
1968	32.069	28.702	60.771	(−)3.315*	(−)11.55
1969	41.352	20.796	62.148	(−)1.938*	(−) 9.32
1970	49.440	12.684	62.124	(−)1.962*	(−)15.47
1971	55.398*				
1972	59.104*	4.982*	64.086*		
Pol. Yr. 1967					
1968	18.927	40.691	59.618	(−)5.872*	(−)14.43
1969	31.879	29.863	61.742	(−)3.748*	(−)12.55
1970	42.221	21.423	63.644	(−)1.846*	(−) 8.62
1971	49.821*				
1972	56.213*				
1973	60.210*	5.280*	65.490*		
Pol. Yr. 1968					
1969	18.447	41.251	59.698	(−)6.719*	(−)16.29
1970	31.931	31.526	63.457	(−)2.960*	(−) 9.39
1971	41.929*				
1972	49.828*				
1973	56.615*				
1974	60.878*	5.539*	66.417*		
Pol. Yr. 1969					
1970	17.537	40.132	57.669	(−)6.331*	(−)15.76
1971	29.902*				
1972	39.567*				
1973	47.354*				
1974	54.172*				
1975	58.479*	5.521*	64.000*		

*Estimated **(−) indicates adverse development

EXHIBIT B

New York Auto B.I.
Development of Loss Experience by Policy Year

Experience as of the End of Calendar Year	*% TO EARNED PREMIUM* Paid Loss	O/S Loss	Incurred Loss	*Loss Development after the end of Calendar Yr.*** Amount	% of O/S
Pol. Yr. 1965					
1966	19.366	39.326	58.692	(−)3.264	(−) 8.30
1967	32.416	28.967	61.383	(−) .573	(−) 1.98
1968	41.784	21.261	63.045	1.089	5.12
1969	48.788	14.217	63.005	1.049	7.38
1970	54.831	7.931	62.762	.806	10.16
1971	58.011*	3.945*	61.956*		
Pol. Yr. 1966					
1967	18.981	34.187	58.168	(−)4.406	(−)12.89
1968	32.069	28.702	60.771	(−)1.803	(−) 6.28
1969	41.352	20.796	62.148	(−) .426	(−) 2.05
1970	49.440	12.684	62.124	(−) .450	(−) 3.55
1971	55.274*				
1972	58.535*	4.039*	62.574*		
Pol. Yr. 1967					
1968	18.927	40.691	59.618	(−)3.355	(−) 8.24
1969	31.879	29.863	61.742	(−)1.231	(−) 4.12
1970	42.221	21.423	63.644	.671	.03
1971	49.483*				
1972	55.470*				
1973	58.799*	4.174*	62.973*		
Pol. Yr. 1968					
1969	18.447	41.251	59.698	(−)3.778	(−) 9.16
1970	31.931	31.526	63.457	(−) .019	(−) .00
1971	42.149*				
1972	49.651*				
1973	55.808*				
1974	59.213*	4.263*	63.476*		
Pol. Yr. 1969					
1970	17.537	40.132	57.669	(−)3.035	(−) 7.56
1971	30.041*				
1972	39.954*				
1973	47.226*				
1974	53.271*				
1975	56.574*	4.130*	60.704*		

*Estimated **(−) indicates adverse development

AUTHOR'S REVIEW OF DISCUSSION

The author is grateful for the detailed discussion by Mr. Berquist. It shows that even after optimistic assumptions made by the reviewer the reserves still look inadequate. The question arises as to how optimistic the reviewer was? Here I have a problem since his most optimistic results are derived by a "trend line" which was drawn, presumably freehand, by "inspection of past values". I do not know how to examine the validity of assumptions underlying such a procedure.

The procedure leading to the less optimistic results can, however, be discussed in more detail. Mr. Berquist reduced the slope of the straight line rather arbitrarily by removing the effect of the lowest and the highest points in the series. This he did not do once, but six times in succession. It would be my opinion that making six optimistic assumptions in succession would tend to result in a rather optimistic result.

On general principles, a person fitting a trend line to a time series has a choice of a type of curve and a choice of a starting point. This provides sufficient scope of making widely different projections, consequently there should not be any need for "doctoring" the actual data underlying the trends. After all, if we are permitted to eliminate the highest and the lowest point in each of the six series, why not eliminate the two highest and two lowest points? Or for that matter, the three highest and three lowest points? Then, if we are at liberty to subtract a few inconvenient points, what is wrong with adding a few more convenient, strategically located points?

The reviewer expresses the opinion that my projections exaggerate the reserve inadequacy. This does not surprise me if he bases his opinion on the results of his projections.

JOINT UNDERWRITING AS A REINSURANCE PROBLEM

EMIL J. STRUG

The reading requirements for the Associateship and Fellowship examinations have included the theory and the functions of reinsurance. Yet in recent years there have been no papers in the *Proceedings* dealing with this aspect as being practically applied by the actuarial profession. In the day to day dealings of the professional reinsurers, undoubtedly their actuaries are faced with unique reinsurance problems but this is probably not the case for the standard property and casualty actuary.

In the study that follows, the author was presented with the problem of developing a joint underwriting arrangement for two non-profit service plans. One organization was a well-established provider of hospital benefits; the other, a new organization incorporated to provide dental benefits.

To those unfamiliar with the term "service plan", a note of explanation is in order. A service plan contracts with the provider of benefits to deliver benefits (services) within the scope of its certificate (contract) to its members. Under this arrangement the service plan reimburses the provider of services directly. This is contrasted to the standard insurance company approach of reimbursing the contract holder who directly pays the provider of services.

A non-profit service corporation differs from a non-profit mutual in that the insured is not a policyholder and owner of the service organization. Generally, service organizations are regulated either under special legislation or interpretation by the state insurance commissioner. The legislation may prescribe that special reserves be established out of its surplus funds for contingencies. These arrangements may also stipulate that services be provided members even though funds may be lacking to pay the provider in full.

With this as a background, let us examine the problems of the newly formed Dental Service Plan. First of all, its initial available capital would not permit rapid growth without endangering its reserve position. To exist in the market place, it must have underwriting capacity yet maintain financial integrity and protect itself and its providers against a run of unexpected losses.

As a new organization, the start up cost to develop a viable organization could be prohibitive. The Dental Service Plan took these problems to the Hospital Service Plan seeking reasonable and realistic solutions. Inasmuch as all the functions necessary to operate a Dental Plan were the same as those already being performed by the Hospital Plan, it was agreed that the Dental Plan would not duplicate these functions but contract for these services to be performed by the Hospital Plan. The cost of this servicing arrangement was ultimately set as a percent of premium. The only expense that the Dental Plan would be directly responsible for would be for such items as legal, boards, bureaus, etc.

The Hospital Plan was agreeable to jointly underwrite business to provide capacity and maintain an adequate financial status. However, the Hospital Plan wanted to limit its loss to a stated amount within a specified time period. At that point, the Dental Plan would have to look to its providers for relief.

If we examine the various problems and limitations of the two service plans, planning to jointly underwrite, we see that they are similar to those commonly handled by a reinsurance company.

Some basic decisions had to be made by the Dental Plan as regards its underwriting policies and its reserve or surplus position. The use of the term reserve is in the sense that the reserve is surplus and not a loss or expense reserve. The Plan decided to write two types of accounts — 1) underwritten or premium, and 2) cost plus. A premium account represents that business underwritten on a guaranteed rate basis. A cost plus account pays as income its claims or losses (with no limit) plus an administrative charge for claims handling and other expenses. Inasmuch as there was no exposure to risk on cost plus business, the reserve requirements would be related to premium business where, of course, an underwriting loss would have to be offset by any accumulated or available reserve.

The two Plans had to mutually agree upon a reserve position, as in the final analysis, this would determine the degree to which each Plan would share in the gains or losses of the jointly underwritten operation. Further agreement had to be reached as to the maximum amount of underwriting loss the Hospital Plan would sustain during the agreed upon term of the arrangement, so as to determine when joint underwriting would terminate.

At the outset, it was established that the reserve should bear a specific ratio to incurred losses and that the joint underwriting agreement would guarantee this ratio up to the point the Hospital Plan reached the maximum underwriting loss it would sustain. All calculations would be based on the gross underwriting results for each calendar year with interim calculations being based upon twelve months ending gross data. The basic data required to develop the formula is contained on page 4 of the Annual Statement Blank.

To facilitate the calculation and provide a somewhat more accurate allocation of the underwritten business to be jointly shared, the income is split between premium business and cost plus business. Expenses are further categorized as to those purchased via a service contract with the Hospital Plan and those directly incurred by the Dental Plan. The nature of the conditions of the joint underwriting agreement precludes any prospective calculation of the pro rata distribution. The crucial point of the agreement is that the proportion ceded is determined retroactively at the end of the year so as to guarantee the predetermined ratio of reserve at the end of the year to losses incurred.

I will now define in general algebraic terms the elements from page 4, as modified, which represent the contributions to reserve for the year and which then can be translated to the desired ratio of reserve to incurred losses from premium accounts. The subscript 'p' will indicate the data from premium business, the subscript 'c' for cost plus and no subscript for the combined results of premium and cost plus business.

W = desired ratio of reserve to incurred losses for premium written business;

X = portion, in terms of percent, that Dental Plan will retain;

P = premium earned;

L = losses incurred;

E = indirect expense via service contract;

D = expected direct expense provided for in premium;

A = actual direct expense;

U = underwriting gain or loss;

I = income from investment and other sources;

R_B = reserve at beginning of period;

Using the above terms, the reserve at the end (R_E) of the period would be expressed as follows:

$$R_E = R_B + P - L - (A + E) + I$$

Using the same basic approach, we can develop a formula from which we can derive X. In the joint underwriting arrangement the Hospital Plan would return to the Dental Plan that portion of the premium representing the expected direct expenses of the Dental Plan.

$$W = \frac{R_B + X(P_p - L_p - E_p) - A_p + (1 - X)D_p + U_c + I}{XL_p}$$

Solving for X: $$X = \frac{R_B - A_p + D_p + U_c + I}{L_p(1 + W) + D_p + E_p - P_p}$$

The calculation of X will only occur when $\frac{R_E}{L_P}$ is less than W. When $\frac{R_E}{L_P} = W$, the equation reduces to unity.

There remains now the development of the maximum loss ratio which the Hospital Plan would sustain to limit its cumulative underwriting loss to the stated maximum.

The joint underwriting in its simplest form becames a form of pro rata reinsurance on a total portfolio of business. The underwriting gain or loss of the Hospital Plan is in direct ratio to that of the Dental Plan. Since X represents the portion of premium business to be underwritten by the Dental Plan then $(1 - X)$ represents the percent to be handled by the Hospital Plan. The underwriting gain or loss incurred by the Hospital Plan for a period would be expressed as $(1 - X)(P_p - L_p - E_p - D_p)$. This sum would be added to any cumulative underwriting results from prior periods. This gives us the basis to determine the maximum allowable loss ratio at which point the joint underwriting arrangement ceases and beyond which the Dental Plan must look to other sources for relief.

If we define Z as the maximum cumulative loss and H_B as the cumulative loss or gain for prior periods we can develop the basic equation from which to develop the maximum loss ratio which we will define as Q.

$$Z = H_B + (1 - X)(P_p - L_p - E_p - D_p)$$

Before substituting for X, let us group certain terms and introduce substitutions and equivalences to simplify the equation.

$$Y = R_B - A_p + D_p + U_c + I$$

$$C = (1 + W)$$

$$L_p = P_p Q$$

$$K = 1 - \frac{(D_p + E_p)}{P_p}$$

$$KP_p = P_p - D_p - E_p$$

Inserting these in the equation for Z produces the following:

$$Z = H_B + \left(1 - \frac{Y}{CP_pQ - KP_p}\right)(KP_p - P_pQ)$$

$$(Z - H_B)(CP_pQ - KP_p) = (CP_pQ - KP_p - Y)(KP_p - P_pQ)$$

$$(Z - H_B)CP_pQ - (Z - H_B)KP_p = CKP_p^2 Q - K^2P_p^2 - KP_pY - CP_p^2 Q^2 + KP_p^2 Q + YP_pQ$$

Dividing by P_p

$$(Z - H_B)CQ - (Z - H_B)K = CKP_pQ - K^2P_p - KY - CP_pQ^2 + KP_pQ + YQ$$

Rearranging the terms and setting the equation to zero produces:

$$CP_pQ^2 + Q[C(Z - H_B) - P_p(CK + K) - Y] + K[KP_p + Y - (Z - H_B)] = 0$$

This is conveniently a quadratic equation which for our use we will define as $aQ^2 + bQ + c = 0$ and further as

$$a = CP_p$$

$$b = C(Z - H_B) - P_p(CK + K) - Y$$

$$c = K[KP_p + Y - (Z - H_B)]$$

$$Q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Logic would suggest and require that Q, the loss ratio, be positive. An analysis of the coefficient 'b' indicates that its sign is always negative thereby guaranteeing the numerator to be positive.

An empirical approach was used to determine that the positive value of the radical must be used to produce results that would occur in the real world. If the negative value is used, X becomes greater than unity and negative underwriting or cessions are developed.

I have worked out an example illustrating how this approach might be applied as a reinsurance vehicle. In the example, when the maximum loss by Reinsurer A is attained, Reinsurer A is no longer considered as the prime reinsurer and the reduction in the amount of loss (incurred losses) necessary to produce the maximum underwriting loss for Reinsurer A is absorbed by the ceding company or another reinsurer.

In the illustration that follows, assumptions were made in regards to the various elements in the formulas:

$$E = .15\text{P}$$

$$D = .03\text{P}$$

$$R_B = \$25{,}000$$

$$W = .10$$

$$Z = \$125{,}000$$

Also for convenience, let H_E equal the reinsurer's cumulative gain or loss at the end of the period under study. If H_E exceeds Z then Q must be calculated. The ending reserve *(H_E)* of a period becomes the beginning reserve *(H_B)* for the next period.

The expenses of the reinsurer (ceded to) are based upon an indirect expense of 15% and direct expense of 3% which would be considered as commission to or a return of the expenses of the direct insurer. The term of the agreement is for a three year period.

In the first year of the program the experience on a direct basis produced an underwriting and operational gain. The ratio of the ending reserve to incurred losses for direct premium business was below the desired ratio of 10% necessitating a cession, the net of which would produce 10%. After the cession the ratio of the net reserve R_E ($105,200) to net loss incurred ($1,052,000), equals 10%. The net reserve becomes the beginning reserve for 1972. At this point, the reinsurer has a gain of $33,700 *($H_E$)* which becomes H_B for 1972.

The second year's experience produced underwriting and operational losses. The reserve ratio dropped below the 10% level requiring a cession. The net ending reserve for the ceding company at this point is $133,820 with the reinsurer's cumulative experience showing a loss of $54,070.

Adverse results for the third year dropped the reserve ratio below 10%. The initial calculation for the cession produced a cumulative loss to the reinsurer in excess of $125,000. Q, the maximum allowable loss ratio to be incurred to limit the loss to $125,000 was calculated. This loss ratio was introduced into the calculation to develop the pro rata amounts to be shared to produce a maximum cumulative loss of $125,000.

The author wishes to acknowledge his indebtedness to Henry D. Jones, President of Massachusetts Blue Cross, Inc. and Walter C. Guralnick, D.M.D. for allowing the use of various material related to the two corporations in this presentation, and George E. McLean, Vice President-Actuary of Massachusetts Blue Cross, Inc., who rendered guidance and assistance in developing the concepts presented.

CALENDAR YEAR 1971

	Direct Business			Premium Accounts	
	Cost Plus($_c$)	Premium ($_p$)	Total	Ceded	Retained
Premium (P)	$1,000,000	$3,000,000	$4,000,000	$1,685,000	$1,315,000
Losses (L)	787,400	2,400,000	3,187,400	1,348,000	1,052,000
Expenses:					
Indirect (E)	157,480	450,000	607,480	252,750	197,250
Direct (A)	26,220	75,000	101,220	50,550	24,450
Total	183,700	525,000	708,700	303,300	221,700
Income:					
Net Underwriting Results (U)	28,900	75,000	103,900	33,700	41,300
Investment Income (I)			10,000	—	38,900*
Total			113,900	33,700	80,200
Reserve at Beginning of Period			25,000[R_B]	— [H_B]	25,000[R_B]
Reserve at End of Period			138,900	33,700[H_E]	105,200[R_E]
Ratio of Ending Reserve to Losses on Premium Business			5.8%	2.5%	10.0%

*Investment Income and Income from Cost Plus

$$\% \text{ to be ceded} = (1.00 - X) = 1.000 - \left\{ \frac{R_B + (I + U_c) - A_p + .03P_p}{1.1L_p - .82\,P_p} \right\}$$

$$= 1.000 - \left\{ \frac{\$25{,}000 + \$38{,}900 - \$75{,}000 + \$90{,}000}{\$2{,}640{,}000 - \$2{,}460{,}000} \right\}$$

$$= 1.000 - \frac{\$\ 78{,}900}{\$180{,}000}$$

$$= 1.000 - .43833333$$

$$= .56166666$$

CALENDAR YEAR 1972

	Direct Business			Premium Accounts	
	Cost Plus($_c$)	Premium ($_p$)	Total	Ceded	Retained
Premium (P)	\$1,500,000	\$4,500,000	\$6,000,000	\$2,925,652	\$1,574,348
Losses (L)	1,181,100	3,825,000	5,006,100	2,486,804	1,338,196
Expenses:					
Indirect (E)	236,220	675,000	911,220	438,848	236,152
Direct (A)	39,330	112,500	151,830	87,770	24,730
Total	275,550	787,500	1,063,050	526,618	260,882
Income:					
Net Underwriting Results (U) ...	43,350	(112,500)	(69,150)	(87,770)	(24,730)
Investment Income (I)			10,000		53,350*
Total			(59,150).	(87,770)	28,620
Reserve at Beginning of Period			105,200[R_B]	33,700[H_B]	105,200[R_B]
Reserve at End of Period			46,050	(54,070)[H_E]	133,820[R_E]
Ratio of Ending Reserve to Losses on Premium Business			1.2%		10.0%

*Investment Income and Income from Cost Plus

$$\% \text{ to be ceded} = (1.00 - X) = 1.000 - \left\{ \frac{R_B + (I + U_c) - Ap + .03P_p}{1.1L_p - .82\,P_p} \right\}$$

$$= 1.000 - \left\{ \frac{\$105{,}200 + \$53{,}350 - \$112{,}500 + \$135{,}000}{\$4{,}207{,}500 - \$3{,}690{,}000} \right\}$$

$$= 1.000 - \frac{\$181{,}050}{\$517{,}500}$$

$$= 1.000 - .3498551$$

$$= .6501449$$

CALENDAR YEAR 1973

	Direct Business			Premium Accounts		Premium Accounts		
	Cost Plus (c)	Premium (p)	Total	Ceded	Retained	First Retention	Ceded	Retained
Premium (P)	$2,500,000	$7,500,000	$10,000,000	$5,295,044	$2,204,956	$7,500,000	$4,909,686	$2,590,314
Losses (L)	1,968,500	6,375,000	8,343,500	4,500,787	1,874,213	6,258,352	4,096,872	2,161,480
Expenses:								
Indirect (E)	393,700	1,125,000	1,518,700	794,257	330,743	1,125,000	736,453	388,547
Direct (A)	65,550	187,500	253,050	158,851	28,649	187,500	147,291	40,209
Total	459,250	1,312,500	1,771,750	953,108	359,392	1,312,500	883,744	428,756
Income:								
Net Underwriting Results (U)	72,250	(187,500)	(115,250)	(158,851)	(28,649)	(70,852)	(70,930)	78
Investment Income (I)			10,000		82,250*			82,250*
Total			(105,250)		53,601			82,328
Reserve at Beginning of Period			133,820[R_B]	(54,070)[H_B]	133,820[R_B]		**(54,070)[H_B]**	**133,820[R_B]**
Reserve at End of Period			28,570	(212,921)[H_E]**	187,421[R_E]		(125,000)[H_E]	216,148[R_E]
Ratio of Ending Reserve to Losses on Premium Business			0.4%		10.0%			10.0%

*Investment Income and Income from Cost Plus
**Exceeds Contractual Limit ($H_E > Z$) Calculate Q

Step #1 % to be ceded = (1.00 − X) =

$$1.00 - \left\{\frac{R_B + (1 + U_c) - Ap + .03P_p}{1.1L_\rho - .82P_\rho}\right\}$$

$$= 1.000 - \left\{\frac{\$133{,}820 + \$82{,}250 - \$187{,}500 + \$225{,}000}{\$7{,}012{,}500 - \$6{,}150{,}000}\right\}$$

$$= 1.000 - \frac{\$253{,}570}{\$862{,}500}$$

$$= 1.000 - .2939942$$

$$= .7060058$$

Step #3 % to be ceded = (1.00 − X) =

$$1.000 - \left\{\frac{\$133{,}820 + \$82{,}250 - \$187{,}500 + \$225{,}000}{\$6{,}884{,}187 - \$6{,}150{,}000}\right\} =$$

$$1.000 - \frac{\$253{,}570}{\$734{,}187} =$$

$$1.000 - .34537522$$

$$.65462478$$

Step #2

$$Q = \frac{\$13{,}246{,}593 + \sqrt{\$272{,}256{,}107{,}649}}{\$16{,}500{,}000} = \frac{\$13{,}768{,}375}{\$16{,}500{,}000} = .83444693$$

ALLOCATING PREMIUM TO LAYER BY THE USE OF INCREASED LIMITS TABLES

RONALD E. FERGUSON

"Most of the literature of insurance to me is cryptic and mystic,
But when I read it I am given pause by a certain actuarial statistic.
Yes, just as some people are fascinated by fisticuffs,
I am fascinated by one group of actuarial statisticuffs . . . . . ."

—OGDEN NASH

Since the actuary is expected to know how to use increased limit factors, our literature should contain something on the practical aspects of this subject. It is the intent to provide herein, especially for the student or trainee, a primer on increased limits mathematics. The subject matter is not particularly difficult but is sometimes elusive. The reader is referred to J. T. Lange's excellent paper, "The Interpretation of Liability Increased Limits Statistics," in Volume LVI of the *Proceedings* for a thorough discussion of increased limits ratemaking and related technical problems.

We will approach the subject primarily from the standpoint of a reinsurer, but the same techniques could be used by any company that wished to analyze its experience by layer of coverage. The following table presents a schedule of hypothetical private passenger bodily injury liability reinsurance rates by policy limit. In this example, the reinsurer is providing coverage in excess of $20,000 per person/$20,000 per accident at 100% of the manual increased limits indications. The 100% of manual is used here to simplify the calculations. Normally it would be expected that the reinsurer would pay the ceding company an appropriate "ceding" commission.

In our example there are two layers of coverage. The first layer is described as $20,000/$20,000 excess of $20,000/$20,000 and might be referred to as a working layer (*i.e.*, it is expected that the frequency of claims in this range will be significant, since settlements greater than $20,000 are not uncommon today). The second layer, which might be called the catastrophe layer, has a lower claim frequency and, in this case, would be described as excess of $40,000/$40,000 to the policy limits. The first layer may have enough claim frequency to be experience rated in some fashion, while the terms of the higher layer would not normally be subject to automatic adjustments. It is, of course, possible to have more than two layers but this would not alter the basic premium layering techniques described below.

		Percent of Total Limits Premium Allocated to Layer:	
Policy Limits	*Increased Limits Factors**	*First Layer $20,000/$20,000 xs $20,000/$20,000*	*Second Layer xs $40,000/$40,000*
$ 10,000/ 20,000	1.00	—	—
15,000/ 30,000	1.12	4.46%	—
25,000/ 25,000	1.16	4.31	—
20,000/ 40,000	1.19	6.72	—
25,000/ 50,000	1.23	8.13	1.63%
50,000/100,000	1.35	10.37	7.41
100,000/300,000	1.49	9.40	16.11

Other factors used in this paper

$ 15,000/ 20,000	1.07
20,000/ 20,000	1.11
25,000/ 40,000	1.21
40,000/ 40,000	1.25
250,000/500,000	1.59

*The above factors are from the ISO Automobile Bodily Injury Private Passenger Supplementary Increased Limits Table which became effective in many states on January 1, 1970.

It is apparent that the first layer cannot become involved unless the per person limit or the per accident limit exceeds $20,000. Similarly, it would take an individual claim in excess of $40,000 or a combination of claims with a total in excess of $40,000 on one accident to involve the second layer.

The allocations of total limits premium shown in the table were derived as follows:

1. Since a policy with a limit of \$10,000/\$20,000 cannot penetrate the excess covers, none of the total limits premium is allocated to the layers in excess of \$20,000 per person/\$20,000 per accident. We are here ignoring the problem of an excess of policy limit judgment (bad faith judgment) which may be covered by the reinsurance treaty.

2. Losses occurring when the policy limit is \$15,000/\$30,000 (increased limits factor of 1.12) can penetrate the first excess layer in the case of a multiple-claim accident, for example, two \$15,000 claims. The ceding company is retaining losses up to \$15,000/\$20,000 (increased limits factor of 1.07) with the reinsurer committed for everything over \$20,000 per accident, subject to the policy limit. The reinsurer is thus exposed in the area of nil/\$10,000 excess \$15,000/\$20,000. The reinsurer's premium would be $\frac{1.12-1.07}{1.12}$, or 4.46% of the total limits premium collected by the ceding company on policies with limits of \$15,000/\$30,000.

3. The next two policy limits can involve only the first layer and are subject to the ceding company's loss retention of \$20,000/\$20,000 (increased limits factor of 1.11).

   a. \$25,000/\$25,000 Policy Limit

   $$\frac{1.16-1.11}{1.16} = 4.31\%$$

   b. \$20,000/\$40,000 Policy Limit

   $$\frac{1.19-1.11}{1.19} = 6.72\%$$

4. A policy with limits of \$25,000/\$50,000 (increased limits factor of 1.23) can penetrate the second layer in the event of a multiple-claim accident. The total ceded premium for both layers would be:

   $$\frac{1.23-1.11}{1.23} = 9.76\%$$

   Since the first layer is exposed in the area of \$5,000/\$20,000 excess of \$20,000/\$20,000, we allocate to that layer the difference between \$20,000/\$20,000 and \$25,000/\$40,000 or

   $$\frac{1.21-1.11}{1.23} = 8.13\%.$$

The second layer's portion is the remainder, 9.76% less 8.13%, or

$$\frac{1.23-1.21}{1.23} = 1.63\%.$$

5. The last two policy limits involve the following calculations:

   a. Policy Limit $50,000/$100,000

   First Layer: $\frac{1.25-1.11}{1.35} = 10.37\%$

   Second Layer: $\frac{1.35-1.25}{1.35} = 7.41\%$

   b. Policy Limit $100,000/$300,000

   First Layer: $\frac{1.25-1.11}{1.49} = 9.40\%$

   Second Layer: $\frac{1.49-1.25}{1.49} = 16.11\%$

In both cases shown in 5 the same number of dollars will be allocated to the first layer since 9.40% × 1.49 = 10.37% × 1.35. This would seem to be logical unless there is some adverse selection involved in the purchasing of increased limits. For example, the above would not be logical if drivers who buy high limits do so, in part, because they expect to have more accidents. Another possible adverse selection factor might be that drivers who buy high limits may have reason to believe that their economic status would predispose them to larger than average claims since people would expect an affluent person to carry high limits and would adjust their claim sights accordingly.

Another example of the application of increased limits factors would be the case where a company writes a primary policy and another company writes excess of the primary policy and the latter company then has reinsurance from a third company. From the third company's point of view this business can be characterized as "excess on excess".

Suppose that a policy with a limit of $250,000/$500,000 (increased limits factor of 1.59) is issued by Company A and that Company A purchases reinsurance in excess of $20,000/$20,000 (increased limits factor of 1.11) from Company B at a rate of $\frac{1.59-1.11}{1.59}$ or 30.19%. Company B in turn purchases reinsurance from Company C, retaining $30,000/$80,000 net for its own account. Company B becomes involved when the direct loss to Company A exceeds \$20,000/\$20,000 while Company C becomes involved when the direct loss to Company A is greater than \$50,000/\$100,000 (since Company B retains the first \$30,000/\$80,000 and Company A retains \$20,000/\$20,000). Company C needs to develop a rate to apply to Company B's premium. This rate would be

$$\frac{1.59\ (\$250{,}000/\$500{,}000) - 1.35\ (\$50{,}000/\$100{,}000)}{1.59\ (\$250{,}000/\$500{,}000) - 1.11\ (\$20{,}000/\$\ 20{,}000)} \text{ or } 50.0\%$$

To further complicate matters the reinsurance which Company B bought from Company C could involve two layers. The first layer being, say, \$50,000/\$200,000 excess \$30,000/\$80,000 and the second layer excess of \$80,000/\$280,000 (keep in mind that Company B's retention of \$30,000/\$80,000 is excess of Company A's \$20,000/\$20,000 retention). The first layer premium would be

$$\frac{1.49\ (\$100{,}000/\$300{,}000) - 1.35\ (\$50{,}000/\$100{,}000)}{1.59\ (\$250{,}000/\$500{,}000) - 1.11\ (\$20{,}000/\$\ 20{,}000)} = 29.17\%,$$

while the second would be

$$\frac{1.59\ (\$250{,}000/\$500{,}000) - 1.49\ (\$100{,}000/\$300{,}000)}{1.59\ (\$250{,}000/\$500{,}000) - 1.11\ (\$\ 20{,}000/\$\ 20{,}000)} = 20.83\%$$

for a total of 50.0%

While the calculations described are rather straightforward, the setting of the ceding commission in a reinsurance transaction is somewhat complex. The commission agreed upon by the ceding company and the reinsurer represents a judgment about the net effect of the many forces that will operate on the expected experience. These forces include the adequacy of the primary rate level; the ceding company's acquisition cost; the ceding company's level of underwriting and claim handling expertise; exposure to excess policy limits judgments and adverse selection; any redundancy or weakness thought to exist in the particular increased limits table utilized.

We have outlined the calculations for those situations which are most likely to be encountered. The possible variations are many but it is hoped that the above examples will provide the reader with the tools he needs to solve similar problems.

DISCUSSION BY JOSEPH A. PLUNKETT

Mr. Ferguson has written an excellent discourse on the use of increased limits factors as a method of excess of loss ratemaking for private passenger automobile bodily injury liability. It must be remembered, as is pointed out in the paper, that this method produces a gross premium which has to be modified to leave the reinsurer with the pure premium plus a margin for profit and contingencies plus a pro-rata portion of the loss adjustment expense for those claims paid by the reinsurer. This modification is accomplished by the use of a commission on the gross increased limits premium.

The manual excess approach to excess of loss reinsurance has a certain appeal in that an equitable premium is being paid for the exposures being assumed, *i.e.* the actual premium collected by the insurer. Calculation of the reinsurance premium written, in force, and unearned can be readily accomplished by tabulating equipment or computer. If the experience is better than anticipated, a share of the excess profit can be returned via contingent commission.

Obviously, it is assumed that the increase limits premium is the correct premium for the exposures covered. Perhaps this is valid over low retentions (10,000/20,000; 15,000/30,000; or 20,000/20,000) but I question the use of this approach to excess of loss rating over higher retentions. The relativities between policy limits related to higher retentions are not the same as with lower ones.

To illustrate this point, let us examine certain relationships which develop from a comparison of the private passenger increased limits tables in effect before and after January 1, 1970. Table I sets forth the percentage increase of increased limits premium for various retentions which will be collected using the new factors which went into effect in most states on January 1, 1970. The column headed "Percentage Increase" reflects the additional premium collected for the increased limits because of the change in factors. The overall increase from the basic limits was 19%. Table II compares the old percentage with the new percentage of premium collected over retentions of $25,000/25,000 and $50,000/50,000. The percentages in the columns headed "Old" and "New" are the percentage relationships between the premium retained by the ceding company and that given to the reinsurer. For example, in Part A, policy limits $50,000/50,000 the

"Old" percentage shown of 10.71% was derived as follows from Table I:

$$\frac{50/50 - 25/25}{25/25} = \frac{1.24 - 1.12}{1.12} = 10.71\%.$$

The reinsurance premium is thus related to the total limits premium. The "Increase" column is the change in percentage of reinsurance premium collected related to premium retained by ceding company. Table II reveals that the change in increased limits factors, which produced an overall increase of 19% from basic limits, does not produce a comparable result for a reinsurer over retentions of $25,000/25,000 or $50,000/50,000.

TABLE I

Policy Limits	Increased Limits Factors Prior to 1/1/70	Subsequent 1/1/70	Percentage Increase
$ 10,000/$ 20,000	1.00	1.00	—
$ 25,000/$ 25,000	1.12	1.16	33.33%
$ 50,000/$ 50,000	1.24	1.29	20.83%
$ 50,000/$100,000	1.30	1.35	16.67%
$100,000/$100,000	1.32	1.37	15.62%
$100,000/$300,000	1.41	1.49	19.51%
$250,000/$500,000	1.50	1.59	18.00%

TABLE II

A. Reinsurance Layer $225,000/475,000 Xs $25,000/25,000

Policy Limits	Old	New	Increase
$ 50,000/$ 50,000	10.71%	11.21%	4.67%
$ 50,000/$100,000	16.07%	16.38%	1.93%
$100,000/$100,000	17.86%	18.10%	1.34%
$100,000/$300,000	25.89%	28.45%	9.89%
$250,000/$500,000	33.93%	37.07%	9.25%

B. Reinsurance Layer $200,000/450,000 Xs $50,000/50,000

Policy Limits	*Old*	*New*	*Increase*
$ 50,000/$100,000	4.84%	4.65%	−3.93%
$100,000/$100,000	6.45%	6.20%	−3.88%
$100,000/$300,000	13.71%	15.50%	13.06%
$250,000/$500,000	20.97%	23.6%	10.92%

There are many other valuable studies pertaining to manual increased limits which can be developed using the techniques described in Mr. Ferguson's paper.

A NOTE ON FULL CREDIBILITY FOR ESTIMATING CLAIM FREQUENCY

J. ERNEST HANSEN*

Introduction

The conventional standards for full credibility are known to be inadequate. This inadequacy has been well treated in the Mayerson, et. al. paper[1] and in the ensuing discussions,[2] where the general problem of estimating pure premium was considered. However, in spite of this previous treatment, that old, familiar number, 1,082, still enjoys widespread patronage.

If, instead of estimating pure premium, we ignore claim severity and estimate only claim frequency, 1,082 claims, with the precision in estimation which it promises, is an acceptable standard, providing we are sampling from a homogeneous risk population and accept the usual assumption of mutual independence among risks having Poisson claim processes. However, we know the insureds are not a homogeneous population. We must provide for a distribution of the Poisson parameter over the population, referred to as the structure function.[3] The structure function introduces additional variation into the claim process which reduces the precision of estimation promised by the conventional credibility standards.

The More General Model

Let m denote the number of claims of an insured selected at random, and let λ denote the parameter of his Poisson claim process for a given interval of time, i.e., the experience period. Then the unconditional probability distribution of m can be represented as:

* J. Ernest Hansen has submitted this paper in response to a presidential invitation. Mr. Hansen is a Research Associate with the Insurance Company of North America.

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1 A. L. Mayerson, D. A. Jones, N. L. Bowers, Jr., "On the Credibility of the Pure Premium", PCAS Vol. LV, p. 175.
2 PCAS Vol. LVI, pp. 63–82.
3 H. Bühlmann, *Mathematical Methods in Risk Theory,* Springer-Verlag, New York, 1970, p. 65.

$$P(m) = \int_{\lambda=0}^{\infty} P(m|\lambda)f(\lambda)d\lambda, \quad m = 0, 1, 2, \cdots$$

where $P(m|\lambda)$ is the Poisson claim process of an individual insured conditional upon λ, and $f(\lambda)$ is the structure function describing the manner in which λ is distributed over a population of insureds.

The gamma distribution is often used as an example of a structure function, where we have:

$$P(m) = \int_{\lambda=0}^{\infty} e^{-\lambda}\frac{\lambda^m}{m!} \cdot \frac{\alpha\beta}{(\beta-1)!} e^{-\alpha\lambda} \lambda^{\beta-1} d\lambda$$

where α and β are the parameters for the gamma distribution. Upon integrating, we have:

$$P(m) = \frac{(m+\beta-1)!}{m!\,(\beta-1)!}\left(\frac{\alpha}{\alpha+1}\right)^{\beta}\left(\frac{1}{\alpha+1}\right)^{m}, \quad m = 0, 1, 2, \cdots$$

The above representation of $P(m)$ is in the form of a negative binomial distribution.

Therefore, if we assume that individual insureds have independent Poisson claim processes and that the Poisson parameter is gamma distributed over the population of insureds, and if we select insureds at random to observe m, then the number of claims from an insured, m has the negative binomial distribution. A number of researchers have found the negative binomial distribution satisfactorily fits automobile claims data[4].

From the general representation of $P(m)$, again assuming a Poisson claim process for individual insureds, and using well known results from conditional probability, we can readily determine:

$$\begin{aligned} E(m) &= E[E(m)|\lambda)] \\ &= E(\lambda) \end{aligned}$$

Also:

$$\begin{aligned} Var(m) &= Var[E(m|\lambda)] + E[Var(m|\lambda)] \\ &= Var(\lambda) + E(\lambda) \end{aligned}$$

[4] H. L. Seal, *Stochastic Theory of a Risk Business*, John Wiley and Sons, Inc., New York, 1969, p. 16.

These results are immediate when we remember $E(m|\lambda) = Var(m|\lambda) = \lambda$ for the Poisson variable m with parameter λ. Therefore, even though the mixed Poisson process $P(m)$ can be mathematically difficult to work with, depending upon what structure function is selected, the mean and variance of m are simply related to those of λ. Considering a random sample of n insureds, we have, by invoking the Central Limit Theorem, the consequence that the sample mean, $\overline{m}$, is approximately normally distributed with a mean of $E(\lambda)$ and a variance of $[Var(\lambda) + E(\lambda)]/n$.

The exponential distribution, with only one parameter, is a convenient choice for $f(\lambda)$ for a numerical example. If $E(\lambda) = .35$, i.e., we expect .35 claims per insured for the experience period, then $Var(\lambda) = E^2(\lambda) = .1225$ for an exponential distribution and $\overline{m}$ is approximately normally distributed with the following parameter values:

$$E(\overline{m}) = E(\lambda) = .35$$
$$Var(\overline{m}) = [Var(\lambda) + E(\lambda)]/n = (.1225 + .35)/n$$
$$= .4725/n$$

If we want the estimator $\overline{m}$ to be within 5% of $E(m)$ with probability .90, we determine n as follows:[5]

$$\text{standard normal deviate, } Z = \frac{\overline{m} - E(\overline{m})}{\sigma_{\overline{m}}}$$

$$1.645 = \frac{.05 \times .35}{\sqrt{\frac{.4725}{n}}}$$

$$n = 4{,}175$$

The number of claims we could expect in a sample of this size would be:

$$n \cdot E(m) = 4{,}175 \times .35 = 1{,}461$$

Therefore, assuming an exponential structure function and an expectation of .35 claims per insured, we find that the standard for full credibility would be 1,461 claims in contrast to the conventional standard of 1,082 claims. The difference is attributable to the additional variation in m introduced by the structure function.

[5] For an exponential structure function, $\overline{m}$ is a maximum likelihood estimator.

However, it is difficult to estimate the shape of the structure function for a particular population of insureds since an insured's risk parameter is not an observable random variable. We can observe the number of claims of a particular insured over time for purposes of estimating this risk parameter, the insured's expected claim frequency, but the true expected claim frequency is never known with certainty.

The conventional standards for full credibility are derived by assuming the structure function is concentrated at a single point,[6] i.e., the risk parameter λ is assumed to be constant over the population of insureds and, therefore, $Var(\lambda) = 0$. If we reconsider the previous numerical example with $E(\lambda) = .35$, but assume the structure function is concentrated at this point, we have:

$$Var(m) = (0 + .35)/n$$

Then:

$$1.645 = \frac{.05 \times .35}{\sqrt{\frac{.35}{n}}}$$

$$n = 3{,}092$$

The number of claims we would expect in a sample of this size would be 3,092 × .35 = 1,082 claims. This is the answer we should have anticipated, the conventional standard for full credibility.

The conventional standard, being adequate only for an extreme and improbable case, violates one of the basic purposes for employing the techniques of statistical inference. This is to establish procedures for estimation which guarantee a level of precision in the estimates, e.g., a probability of at least .90 of being within 5% of the true average claim frequency. The levels of precision associated with conventional standards represent the most precision possible using these standards rather than the least; we have a ceiling on possible precision when what we want is a floor.

Choice of the Structure Function

An ideal choice for a structure function is one that leads to gen-

[6] Simon has used the term "isohazardous" to characterize such a population of insureds in his paper, "The Negative Binomial and Poisson Distributions Compared", PCAS Vol. XLVII, p. 20.

erally conservative standards for estimating claim frequency. Toward this end, we can use the following result from reliability theory: the coefficient of variation for all distributions which have an Increasing Failure Rate is bounded above by that of the exponential distribution.[7] In particular, gamma distributions which may be used as structure functions have increasing failure rates.[8] For such distributions, $Var(\lambda)$ is maximized, and $Var(m) = Var(\lambda) + E(\lambda)$ is maximized by assuming an exponential structure function for a given value of $E(\lambda)$. The maximum variance of m will then be:

$$max.\ Var(m) = E^2(\lambda) + E(\lambda)$$

Credibility standards based on this variance will be adequate for the entire set of structure functions; the standards will be based on the maximum possible rather than the minimum possible variance.

In practice, the actuary is sufficiently familiar with the data he works with to select an upper bound for $E(\lambda)$, the expected claim frequency. Then, using an exponential prior distribution, a more adequate standard for full credibility can be easily computed, as in the previous numerical example. Using the above expression for $max.\ Var(m)$ and letting ϵ denote the tolerance of error as a proportion of $E(\lambda)$, we can rearrange the formula of the example as:

$$n = \frac{1 + E(\lambda)}{E(\lambda)} \cdot \frac{Z^2}{\epsilon^2}$$

[7] R. Barlow and F. Proschan, *Mathematical Theory of Reliability,* John Wiley and Sons, New York, 1965, p. 33. A distribution, $f(x)$, is IFR, has an increasing failure rate, if $f(x)/[1-F(x)]$ increases as x increases. If we restrict a population of insureds to just those for which $\lambda > \lambda_o$, an arbitrary value of λ, then the chance that an insured is close to λ_o is given by the conditional probability $f(\lambda_o)d\lambda/[1-F(\lambda_o)]$. If $f(\lambda)$ is IFR, this conditional probability increases as λ_o increases.

[8] Gamma distributions which are asymptotic to the vertical axis are not intuitively appealing candidates for structure functions.

The following table was constructed using this formula with $Z = 1.645$ and $\varepsilon = .05$:

Full Credibility Standards with a Tolerance of Error of 5% and 90% Confidence

Upper Bound for Claim Frequency	*Sample Size: No. of Exposure Units*	*Expected No. of Claims*
.05	22,731	1,137
.10	11,907	1,191
.15	8,298	1,245
.25	5,412	1,353
.35	4,175	1,461
.50	3,247	1,624
.75	2,526	1,894
1.00	2,165	2,165
1.50	1,804	2,706
2.00	1,624	3,247
3.00	1,443	4,330
5.00	1,299	6,494

Conclusion

The conventional standards for full credibility are known to be minimal for the estimation of claim frequency. They are adequate only when the structure function is concentrated at a point. The exponential distribution appears to present a reasonable bounding case with respect to the additional variance introduced by the structure function. With the assumption of an exponential structure function and the selection of a maximum possible mean value for the Poisson risk parameter, an adequate sample size for estimating claim frequency can be computed.

DISCUSSION BY DAVID J. GRADY

Credibility is the foundation stone of casualty actuarial science. To the theoretician it offers endless opportunities to advance the mathematical basis of our art; to the practioner it provides a means for charting a course between the twin requirements of insurance pricing: stability and responsiveness. To assign too much credibility to an insurer's experience is to court insolvency; to give too little is to risk adverse selection and a declining portfolio. A prime determinant of the appropriateness of a credibility procedure is the level at which full credibility is established.

Mr. Hansen's paper is a concise exploration of the problem of setting the level of full credibility for estimating claim frequency. He traces a clear path through the current difficulties and proposes a rather elegant solution. I would like to make the path which Mr. Hansen has cleared somewhat broader by commenting on the distributions and assumptions employed in his presentation.

Five probability distributions are utilized in the paper. The claim frequencies for individual insureds are assumed to obey independent Poisson processes. The normal distribution is brought into the paper by means of the Central Limit Theorem. The relationships among the means and variances of the exponential, gamma and negative binomial distributions lead directly to Mr. Hansen's choice for a structure function. Since a knowledge of these measures is fundamental to an understanding of this choice, the table presented below may be of some help in following the author's analysis.

Distribution	*Density Function*	*Mean*	*Variance*
Exponential	$f(\lambda) = \alpha e^{-\alpha\lambda}$	$\frac{1}{\alpha}$	$\frac{1}{\alpha^2}$
Gamma	$f(\lambda) = \frac{\alpha^\beta}{\Gamma(\beta)} e^{-\alpha\lambda} \lambda^{\beta-1}$	$\frac{\beta}{\alpha}$	$\frac{\beta}{\alpha^2}$
Negative Binomial	$p(m) = \binom{m+\beta-1}{m}\left(\frac{\alpha}{\alpha+1}\right)^\beta\left(\frac{1}{\alpha+1}\right)^m$	$\frac{\beta}{\alpha}$	$\frac{\beta}{\alpha}+\frac{\beta}{\alpha^2}$

Thus, the means of the gamma and negative binomial distributions are identical. The variance of the negative binomial distribution is equal to the sum of the mean and variance of the gamma distribution.

Since the coefficient of variation is the standard deviation divided by the mean, the coefficient of variation for the exponential distribution equals one. Similarly, the coefficient of variation for the gamma distribution is $\frac{1}{\sqrt{\beta}}$. Since gamma distributions having increasing failure rates require $\beta > 1$, the coefficient of variation for this class of gamma distributions is bounded above by that of the exponential distribution. Chart I shows two members of the class of gamma distributions with increasing failure rates ($\beta = 2$ and $\beta = 10$) and their limiting exponential ($\beta = 1$). A member of the class of gamma distributions with decreasing failure rates ($\beta = 0.5$) is indicated by a dotted line since this class was disqualified by the author. Since the distributions in Chart I were constructed using a fixed mean, the primary purpose of the graph is to provide visual confirmation for Mr. Hansen's statement that an exponential structure function maximizes the variance for a given value of the mean.

The author dismisses the homogeneity assumption underlying current credibility tables as totally unrealistic. In its place he proposes two new assumptions:

a. The class of gamma distributions with increasing failure rate provides a reasonable set of structure functions for the Poisson parameter.

b. The actuary is able to select an appropriate upper bound for expected claim frequency.

The first assumption appears reasonable from two standpoints:

a. The class of gamma distributions under consideration has considerable flexibility.

b. Fairly good results have been obtained in fitting the negative binomial distribution to actual claim data.

The second assumption appears quite innocuous since such knowledge lies at the heart of our profession. However, the key to this problem lies in the closeness of the upper bound to the actual expected claim frequency.

Mr. Hansen's method of determining full credibility for expected claim frequency consists of two basic steps:

1. Maximize the mean.

2. Maximize the variance associated with that mean.

The author provides us with a method for obtaining the least upper bound for the variance, but we are left to our own devices to find a corresponding methodology for obtaining a least upper bound for the mean itself.

Tables I and II and the result of an attempt to investigate the effects of possible errors in estimating expected claim frequency. These tables are merely an expansion of the table in the original paper. The values in the column for the exponential distribution ($\beta = 1$) may be compared with any of the "true" values above them to obtain a measure of the effect of selecting a mean which is too high. The selected value may also be compared with any of the "true" values above and to the right of it in order to determine the compound effect of maximizing both mean and variance.

A graphical analysis of the problem is presented in Charts II and III. The importance of obtaining a least upper bound for expected claim frequency is especially evident in Chart III. This graph points up the fact that the expected number of claims is a linear function of the selected upper bound for claim frequency.

Hence, even using Mr. Hansen's method, the practitioner still is torn between the alternatives of stability versus responsiveness. Although I have pointed up the fact that an overconservative insurer utilizing the author's approach may find the competition running away with rather large chunks of its portfolio, my sympathies actually lie with Mr. Hansen's treatment of the overall problem. In the hierarchy of requirements which an insurance company must meet, solvency must outweigh competitiveness.

Chart I

The Gamma Distribution for a Fixed Mean ($\mu = 0.10$) and Selected Values of β.

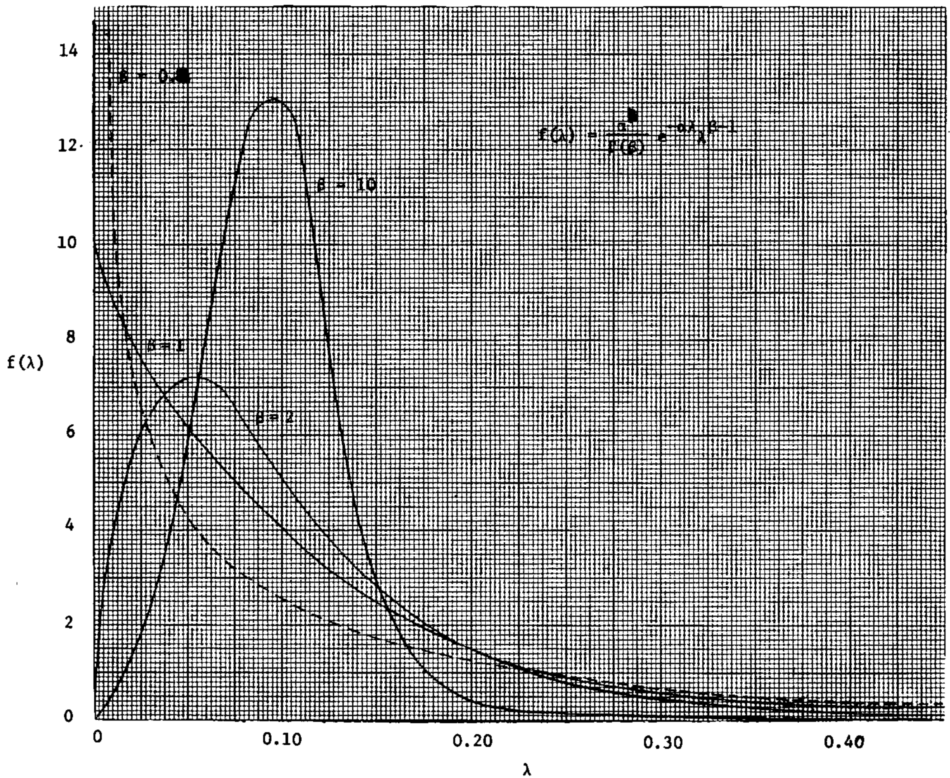


TABLE I

Required Sample Size: Number of Exposure Units
Full Credibility Standards with a Tolerance of Error of 5% and 90% Confidence

Upper Bound for Claim Frequency	*Required Sample Size when β is Equal to:* 0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	3.00	4.00	5.00
0.05	25,978	23,813	23,091	22,731	22,514	22,370	22,267	22,189	22,009	21,919	21,865
0.10	15,154	12,989	12,267	11,907	11,690	11,546	11,443	11,365	11,185	11,095	11,041
0.15	11,545	9,381	8,659	8,298	8,082	7,938	7,835	7,757	7,577	7,487	7,433
0.25	8,659	6,494	5,773	5,412	5,196	5,051	4,948	4,871	4,690	4,600	4,546
0.35	7,422	5,257	4,536	4,175	3,959	3,814	3,711	3,634	3,453	3,363	3,309
0.50	6,494	4,330	3,608	3,247	3,031	2,886	2,783	2,706	2,526	2,435	2,381
0.75	5,773	3,608	2,886	2,526	2,309	2,165	2,062	1,984	1,804	1,714	1,660
1.00	5,412	3,247	2,526	2,165	1,948	1,804	1,701	1,624	1,443	1,353	1,299
1.50	5,051	2,886	2,165	1,804	1,588	1,443	1,340	1,263	1,082	992	938
2.00	4,871	2,706	1,984	1,624	1,407	1,263	1,160	1,082	902	812	758
3.00	4,690	2,526	1,804	1,443	1,227	1,082	979	902	722	631	577
5.00	4,546	2,381	1,660	1,299	1,082	938	835	758	577	487	433

Table II
Expected Number of Claims
Full Credibility Standards with a Tolerance of Error of 5% and 90% Confidence

Upper Bound for Claim Frequency	*Expected Number of Claims when β is Equal to:*										
	0.25	*0.50*	*0.75*	*1.00*	*1.25*	*1.50*	*1.75*	*2.00*	*3.00*	*4.00*	*5.00*
0.05	1,299	1,191	1,155	1,137	1,126	1,119	1,113	1,109	1,100	1,096	1,093
0.10	1,515	1,299	1,227	1,191	1,169	1,155	1,144	1,137	1,119	1,110	1,104
0.15	1,732	1,407	1,299	1,245	1,212	1,191	1,175	1,164	1,137	1,123	1,115
0.25	2,165	1,624	1,443	1,353	1,299	1,263	1,237	1,218	1,173	1,150	1,137
0.35	2,598	1,840	1,588	1,461	1,385	1,335	1,299	1,272	1,209	1,177	1,158
0.50	3,247	2,165	1,804	1,624	1,515	1,443	1,392	1,353	1,263	1,218	1,191
0.75	4,330	2,706	2,165	1,894	1,732	1,624	1,546	1,488	1,353	1,285	1,245
1.00	5,412	3,247	2,526	2,165	1,948	1,804	1,701	1,624	1,443	1,353	1,299
1.50	7,577	4,330	3,247	2,706	2,381	2,165	2,010	1,894	1,624	1,488	1,407
2.00	9,742	5,412	3,969	3,247	2,814	2,526	2,319	2,165	1,804	1,624	1,515
3.00	14,071	7,577	5,412	4,330	3,680	3,247	2,938	2,706	2,165	1,894	1,732
5.00	22,731	11,907	8,298	6,494	5,412	4,690	4,175	3,788	2,886	2,435	2,165

Chart II

Required Sample Size for Selected Values of β

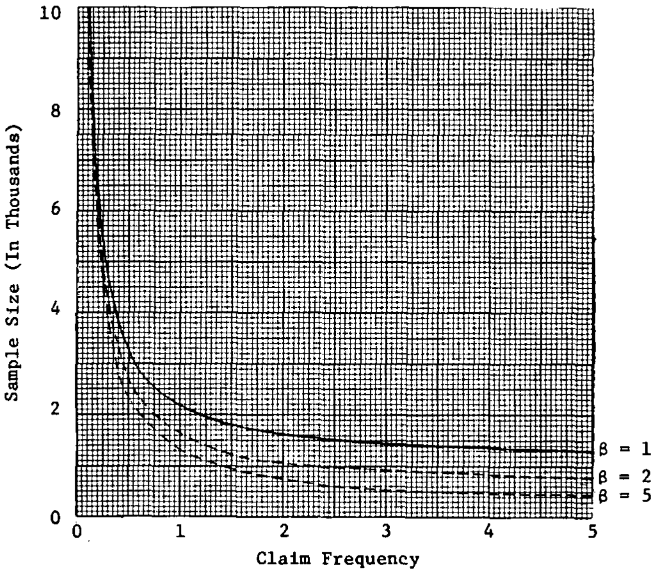
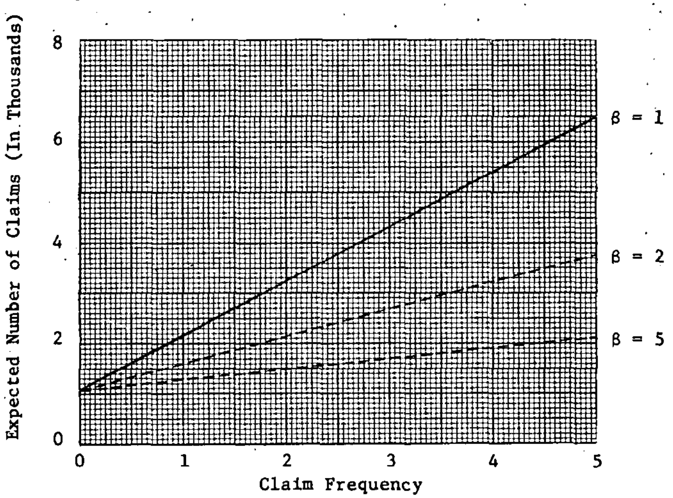


Chart III

Expected Number of Claims for Selected Values of β



DISCUSSION BY ROBERT N. TREMELLING, II

While receiving greater attention in the past few years, credibility standards still lack clarity and structure. This becomes all too apparent when it is realized the standards in common use today are intended for claim frequency only, but are routinely applied to pure premiums. In addition, the assumptions underlying the traditional Poisson claim frequency process are rarely fulfilled. These assumptions include homogeneity of risks and randomness of claims (implying both an "accidental" nature and mutual independence).

Concentrating on claim frequency, Mr. Hansen presents a rationale for the inadequacy of the basic Poisson distribution as a model by attacking the assumption of homogeneity in the risk population. It seems clear that a measure of increased variability must be taken into account if the risks are in fact heterogeneous. The measure described in this paper is a structure function. This, then, becomes the central topic: a standard for full claim frequency credibility through consideration of additional variation inherent in a non-homogeneous population.

The structure function is, of course, of primary importance and should be closely scrutinized. The general form of the gamma function is first developed as a structure function, but later abandoned in favor of the exponential which maximizes the population variability. Mr. Hansen defends the use of the exponential by stating the ideal credibility standards should be "generally conservative". In contrast, I believe the standards should be "generally exact". We do not need a ceiling or a floor, but a correct sample size level. Further, to suggest any one unique structure function for differing lines, covers, deductibles, and territories is rather optimistic.

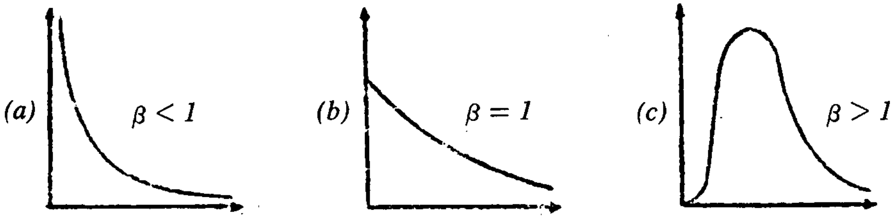
Continuing with the structure function concept, the author states that "gamma distributions which may be used as structure functions have increasing failure rates." In fact, the exponential is a special case of the gamma and has a constant failure rate. The gamma probability distribution function is given by:

$$f(\lambda) = \frac{\alpha^{\beta}}{(\beta - 1)!} e^{-\alpha\lambda} \lambda^{\beta-1} \qquad \begin{array}{l} 0 < \lambda < \infty \\ \alpha > 0 \\ \beta > 0 \end{array}$$

By setting $\beta = 1$, we have the exponential:

$$f(\lambda) = \alpha e^{-\alpha\lambda} \qquad \begin{array}{l} 0 < \lambda < \infty \\ \alpha > 0 \end{array}$$

In general, we may summarize the form of the gamma by the following graphs:



(a) Decreasing failure rate

(b) Constant failure rate (Exponential)

(c) Increasing failure rate (forms similar to the Chi-Square or Log-normal)

As Mr. Hansen notes, those distributions asymtotic to the vertical axis, such as graph (a) above, are not intuitively appealing as structure functions. But graph (c) represents the family of gamma distributions which are all useful as possible structure functions. They are only bounded by the exponential. Perhaps consideration should also be given to another distribution with similar characteristics, the Weibull. The Weibull is widely used for real life systems, and has many forms.[1] In fact, the Weibull distributions also becomes the exponential in the special case where $\beta = 1$.

It should further be noted that an implied characteristic of the constant failure rate is randomness, in contrast to the increasing failure rates which follow a specified pattern. In conclusion, much more work needs to be done on estimation of structure function parameters (especially the shape parameter), although I am sympathetic to the difficulties presented.

[1] See for example, Eugene L. Grant, *Statistical Quality Control,* McGraw-Hill, New York, 1964, pp. 505–507.

Turning to other areas presented in this paper, I offer the following comments:

(1) While dealing with heterogeneity of the risk population, Mr. Hansen accepts "the usual assumption of mutual independence among risks." This cannot be overlooked in a truly comprehensive study. Not only is the claim sample inter-related, there is also the possibility of auto-correlation between samples from different time periods. This auto-correlation becomes even more significant when the ratemaker does not have a large sample. The information can become biased under these conditions, depending largely on the extent to which the same policyholders report claims in different sample periods.

(2) New credibility standards should focus on criteria for pure premiums. Even though Mr. Hansen does present a potentially viable technique for claim frequency, claim severity should also be investigated to provide a more complete answer.

(3) The traditional claim number for full credibility given in this paper is 1,082. However, tables such as those used by the Insurance Services Office show 1,084 as that standard. True, the difference is very minor, especially when the true adequate number is no doubt hundreds of claims higher. But at least this basic number should be consistent. Relying on interpretation of the specific equation used, the required number must be "greater than or equal to" 1,082.4. It seems clear that this number should be the next higher integer value, or 1,083. While the difference in actual sample size is highly insignificant, the lack of agreement is significant. The basic calculation is given below.

$$n \geq \left(\frac{Z_{.05}}{\epsilon}\right)^2 \geq \left(\frac{1.645}{.05}\right)^2 \geq 1{,}082.4 \rightarrow 1{,}083$$

Lastly, I would like to propose the technique of stratified sampling as an alternative to an increase in sample size under simple random sampling from heterogeneous populations. If the risk population of a line is not homogeneous, the derived rates are inequitable. Credits and other rating factors are usually applied to correct this shortcoming, implying a definite partitioning scheme for refining the population into homogeneous subclasses. Stratified sampling is specifically designed for application to non-homogeneous populations. It is also one of the most powerful methods for dealing with skewed distributions (fast becoming a major consideration in credibility studies).

Briefly, the population is partitioned by significant differences so as to account for as much of the inherent heterogeneity as possible. Then simple random samples are taken from each subclass and combined by a specific weighting procedure. If the criteria for partitioning is totally without basis, the technique renders the same degree of information as simple random sampling. Effective partitions will bring about a distinct reduction in variability. In other words, a smaller sample size is required under stratified sampling for the same degree of precision as simple random sampling.

In conclusion, Mr. Hansen justly points out that the underlying assumption of risk homogeneity is not generally met. He further provides us with a solution to this shortcoming by introducing a structure function for the risk parameter. Parameter estimation work for the structure function should render this technique viable for claim frequency, although treatment of claim severity is still needed. Rather than increasing the sample size to some specified new level, perhaps serious consideration should be given to variance reduction techniques, such as stratified sampling.

AUTOMOBILE COLLISION DEDUCTIBLES AND REPAIR COST GROUPS:

THE LOGNORMAL MODEL

DAVID R. BICKERSTAFF

Background

Ratemaking methodology in the field of auto physical damage insurance is still in the Stone Age. The peculiarities involved in auto physical damage have really never received the same rigorous scrutiny in actuarial literature that has been given liability ratemaking techniques. The use of driver classification plans, territorial differentials, and other rating factors has been advanced to a remarkable level for physical damage coverages as well as liability. But the real *sine qua non* of a complete physical damage ratemaking procedure—the consideration of the physical characteristics of the vehicle itself—has always been shrouded in actuarial mystery, characterized by ritualistic rules of thumb, crude groupings of cars by left rear-window price stickers, and the absence of an earnest attempt to examine one of the most significant aspects of all physical damage ratemaking: the deductible.

But the absence of significant contributions which get to the real heart of the physical damage problem is not necessarily due to neglect. The simple truth is that at the physical damage table we have been forced to play with less than a full deck of cards. It is probably no secret that the effect of a particular deductible on net loss cost cannot be expected to be the same for automobiles whose expected mean repair costs are different. Unfortunately, an actuarial student's first exposure to the study of deductibles is usually restricted to situations where the underlying expected distribution by size of loss for the particular line is assumed to be identical for all risks and, consequently, the computation of the effect of a deductible (sometimes referred to as "loss elimination ratio") is based on all risks combined. In order to develop the effect of a particular deductible in the case of automobile collision there is the important stipulation that as the

NOTE: The author would like to thank his associates Mr, Thomas J. Patterson, Jr. and Mr. LeRoy P. Kuriger for their assistance in the compilation of the original data and in certain stages of the program used to tabulate net loss cost based on our model.

mean repair cost and underlying distribution by size of loss change from one group of cars to another, then the percentage effect (loss elimination ratio) of the deductible changes as well.

The Lost Card

The distribution by size of loss to which we refer might be defined as the distribution of gross repair costs which would be incurred for a particular automobile when subjected to the full spectrum of possible collisions, weighted by the relative incidence of the particular types of collisions. There is no doubt that mean repair cost by model, together with its underlying size of loss distribution, either measured from actual repair data, or estimated through another measure which is highly correlated to it, is the single most important statistic in physical damage ratemaking. It also happens to be that big trump card which is missing from our deck.

There have been many vain attempts in the past on the part of some companies to compile cost statistics by make and model of vehicle. Of the many reasons such attempts were abortive, the most important one is that, even if the necessary coding quagmire had yielded spotlessly accurate data, it would probably have taken two or three years before any cost studies could have been completely compiled. The value of such knowledge of two year old models would then have been very questionable in estimating the damageability of the corresponding current year models. The physical changes incorporated into the same model from one year to the next are sometimes quite significant. The only answer to this dilemma would be to find a means of estimating the expected average repair costs of a particular model on an *a priori* basis, perhaps even before it leaves the dealer's showroom. Such an exercise would be strictly an engineering probem and well beyond the province of the actuary, so we, along with the industry as a whole, have ignored this problem and have satisfied ourselves that it was necessary to make do with the remaining fifty-one cards.

The Repairability/Damageability Crusades

The only way in which insurers at present distinguish the physical characteristics of insured automobiles is the customary broad list price groups which number from 7 to 10 or so, depending on the company. This grouping assumes, implicitly, that all automobiles falling within a given list price bracket develop about the same average repair costs for both total losses and less than total losses. Recent studies in the last two years which have been given quite widespread publicity have shown that there is a wide disparity in the average repair cost of automobiles which have been placed in the same group by list prices new.[1] Based on the results of these studies, there have been many suggestions by both the automobile manufacturers and the insurance companies in the last two years that physical damage ratemaking formulas be revamped to group cars in some manner by relative repairability and damageability. While these two concepts might be treated as one in some contexts, it might be well to point out that two distinct definitions of the terms are now in the process of evolution. For purposes of this paper, "damageability" will refer to the relative susceptibility to damage given a certain collision situation and "repairability" will refer to the cost of repair parts and labor given a certain degree of damage. As will be brought out later, there are times when the two concepts can be treated as one, but other situations require separate treatment.

Although not all of the repairability/damageability studies which have been completed or are in progress can be thoroughly described in this paper, it might be useful to outline the objectives of these studies and summarize their findings. For a starting point, one industry association undertook a study in 1971 to measure the difference in repair costs of several hundred models from labor and parts prices found in claims adjusters' "flat rate manuals"—basing the "average" repair bill for each model on a predetermined weighted average of parts typically replaced in collision claims.[2] The differences between cars of the same list price group were substantial, ranging towards 80 per cent. Moreover, there was a high degree of overlap in the weighted average repair bill between list price groups. Since the study made no allowance for relative damageability between models, these repairability relativities would not, by themselves, be a completely appropriate "repairability/damageability index"; nevertheless, the study quite graphically illustrated how inequitable the present list price groupings are.

The engineers have decided to take it a step further. Several groups have begun a series of very elaborate tests to determine the relative dam-

[1]See, for example, "Status Report," Insurance Institute for Highway Safety, November 19, 1971, wherein low-speed crash tests of several 1972 models are described and the results outlined.

[2]Mutual Insurance Advisory Association, "Methods of Reflecting Damageability and Repairability Difference in Rating Private Passenger Automobiles", unpublished memorandum of August 25, 1971.

ageability of various models by crashing them into poles and barriers under controlled conditions and then determining the relative damage suffered. These tests have been focused chiefly on the efficiency of various bumper systems. Certainly the most sensational "Naderizing" finding to date was the fact that none of the bumpers tested was able to prevent damage in even a 5 MPH barrier crash. These crashes also showed significant variation between models in susceptibility to damage at many impact speeds.

Even though much useful information has already been derived from these studies and tests, we think it should be safe to assume that the insurance industry will not allow this renaissance movement to come to a close. A completely feasible means to determine a reasonably accurate expected average repair cost which would take into consideration the parts/labor costs and the damageability over the full spectrum of possible collisions for each new automobile model must be developed. The ideas in this paper are predicated on that assumption.

The Need For a Working Mathematical Model of Size-of-Loss Distributions

With the prospect that automobiles can be grouped by expected average repair costs, it follows that half of the basic ratemaking equation will have been supplied to us. For, just as my old physics professor reminded that almost all physics problems can be attacked with the formula F=MA, the pure premium = severity x frequency formula remains equally inviolate. I have inferred from some things I have read concerning the recent repairability/damageability studies that the general feeling is that once there are feasible means of grouping cars by expected severity, the battle will be over and all will be right in the world of physical damage ratemaking. The proper analysis of deductibles, however, requires going a little further.

If the distribution by size of loss for each average repair cost group were available, as well as simply the mean cost itself, then the effect of a particular deductible on loss cost could be analyzed correctly—for the first time. The natural choice would be to find a theoretical distribution function to use as a model for all repair cost groups and to determine the effect of a given deductible based on that model distribution function.

A Likely Candidate—The Lognormal

The usefulness of the Lognormal distribution in representing many kinds of natural phenomena with skewed distributions has been well established. This model has been shown to accurately depict distributions of such things as the number of plankton caught in hauls of a fisherman's net, the distribution of family incomes, the amount of electricity used in middle class American homes, sentence length of various authors, the survival time of insects treated with disinfectant, and the age upon marriage of American spinsters. The use of the lognormal has also been demonstrated in papers in the *Proceedings* of the CAS. Most recently, Hewitt observed that, although a compound form of the lognormal and gamma functions might improve to some degree the closeness of fit, the simple lognormal distribution provided a good working model to represent the distribution by size of loss of auto property damage claims.[3]

Thus, the lognormal is a natural choice as a model for the distribution by size of auto collision claims. It should be emphasized at this point that the distribution we wish to concern ourselves with is the gross cost of claims before the application of a deductible (including those eliminated completely by the deductible), but the only collision data available to us is that from net claim cost, after the deductible. This presents a special problem in estimating the parameters of the lognormal to be fitted to our actual data. How this problem is handled is covered later. First, a brief review of the lognormal mathematics:

Rudiments of the Lognormal Distribution[4]

If $Y = \log X$ is normally distributed with mean μ and variance σ^2, X is said to be lognormally distributed. The probability density function for the lognormal distribution is

$$f(x) = (x \sigma \sqrt{2\pi})^{-1} \exp \left[- \frac{1}{2\sigma^2} (\log_e x - \mu)^2 \right]$$

The standard nomenclature for the lognormal distribution function is

$$\Lambda(x | \mu, \sigma^2) = \int_0^x f(u) du = P [X \leq x]$$

[3]Charles C. Hewitt, Jr., "Credibility for Severity," PCAS, LVII (1970), p. 148.

[4]Most of this brief exposition is adapted from J. Aitchison and J. A. C. Brown, *The Lognormal Distribution*, Cambridge University Press, 1969, Chapter 2. For readers who might have the appetite for a thorough, exhaustive treatment of the lognormal distribution, this book is highly recommended.

and

$$\frac{d}{d_x} \Lambda(x) = f(x).$$

The jth moment about the origin $= \int_O^{\infty} x^j \, d \, \Lambda \, (x)$

$$= e^{j\mu + \frac{1}{2}j^2 \sigma^2}.$$

The mean α and variance β^2 of the distribution, then, are:

$$\alpha = e^{\mu + \frac{1}{2}\sigma^2}$$

$$\beta^2 = e^{2\mu + 2\sigma^2} - e^{2\mu + \sigma^2} = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1)$$

$$= \alpha^2(e^{\sigma^2} - 1) = \alpha^2 \eta^2$$

where $\eta^2 = e^{\sigma^2} - 1$

The quantity η is, therefore, the coefficient of variation of the distribution and the quantity is very useful in our application of the lognormal distribution. Of particular importance is the fact that, as can be seen, if the members of a family of lognormal curves have equal coefficients of variation, they have equal values of the parameter σ^2.

Moment Distributions

A very important theorem concerning the lognormal distribution concerns the concepts of moment distributions. The jth moment distribution function of $\Lambda(x \mid \mu, \sigma^2)$ is defined as

$$\Lambda_j(x \mid \mu, \sigma^2) = \frac{\int_0^x u^j \, d \, \Lambda \, (u \mid \mu, \sigma^2)}{e^{j\mu + \frac{1}{2}j^2\sigma^2}}$$

The theorem states that the jth moment distribution of a lognormal distribution with parameters μ and σ^2 is also a lognormal distribution with parameters $\mu + j\sigma^2$ *and* σ^2, respectively.[5] The first moment distribution Λ_1 would, therefore, be a lognormal distribution with parameters $\mu + \sigma^2$ and σ^2.

[5] Proof in Aitchison and Brown, *op. cit.*, p. 12.

The lognormal distribution and its first moment distribution are the two basic models of this study of distribution of collision losses by size of loss. In studying the effect of a deductible on loss costs in any coverage, two quantities are needed:

1. The ratio of the expected number of claims which exceed the amount of the deductible to the total expected number of claims. Each of these claims would be reduced by the amount of the deductible.

2. The ratio of the aggregate amount of those claims less than the deductible to the total amount of claims over the entire distribution. These claims are eliminated entirely by the application of the deductible.

The reduction in loss costs brought about by the use of a deductible is a combined result of the two quantities above. Using the lognormal model, the first quantity is derived from the lognormal distribution function itself (Λ) and the second quantity from the first moment distribution function (Λ_1).

For the purposes of this paper we will define four functions which will be used to determine the net reduction in loss cost from a deductible, based on the lognormal frequency function $f(x)$ with parameters μ and σ^2:

Function	*Use*
$\Lambda(x) = \int_0^x f(u)\,du$	To determine ratio of number of claims less than amount x to total number of claims. Basic distribution function.
$G(x) = \int_x^\infty f(u)\,du = 1 - \Lambda(x)$	Determine ratio of number of claims in excess of amount x to total number of claims.
$H(x) = \dfrac{\int_0^x uf(u)\,du}{e^{\mu + \frac{1}{2}\sigma^2}} = \Lambda_1(x)$	First moment distribution. To determine ratio of amount of claims less than amount x to total amount of all claims over the distribution.
$J(x) = \dfrac{\int_x^\infty uf(u)\,du}{e^{\mu + \frac{1}{2}\sigma^2}} = 1 - H(x)$	Ratio of amount of claims in excess of the point x to total amount.

Assuming, then, a distribution with mean α (as defined above), the ratio of the reduction in net cost from a deductible D to the "first dollar" cost, with no upper bound, would be expressed as

$$\frac{D \cdot G(D) + \alpha \cdot H(D)}{\alpha}$$

where the two terms of the numerator are, respectively, the very two quantities we set forth earlier as the ones necessary to determine the effect of a particular deductible.

In actual practice of settling collision claims, however, there is, of course, an upper bound to the distribution, represented by the actual cash value of the insured vehicle at the time of the accident. The reduction from loss cost brought about by this upper bound, which we will denote by L, can also be expressed as a ratio to the unlimited, first dollar cost, as follows:

$$\frac{\alpha \cdot J(L) - L \cdot G(L)}{\alpha}$$

The first term in the numerator represents the total amount of claims in excess of the amount L, from the first moment distribution. The second term represents the amount which would actually be paid on those claims, the product of the number of those claims above L and the value of L itself.

Estimating Parameters of the Lognormal Model

We have now assumed that each of several repair cost groups of automobiles have size of loss distributions which, ignoring upper bounds, follow the lognormal model, each group, by definition, having a different value of the parameter μ. Another assumption which would seem to be reasonable is that the value of the parameter σ^2 is equal for all groups. As shown above, this second assumption also means that all groups' distributions have the same coefficient of variation (B/α). Hewitt has shown that, under these circumstances, if all the automobiles from all groups were combined into a single distribution, and the μ's were themselves normally distributed, then the combined distribution would also be lognormal. If the variance of the μ's is S^2, then the variance of the combined distribution would be $S^2 + \sigma^2$, where σ^2 is constant for all individual groups.[6]

[6] Hewitt, *op. cit.*, Appendix A, p. 167.

The only collision data available for estimating the parameters of the lognormal is \$50-deductible collision and any such sample distribution would be a combination of all repair cost groups. Therefore, the quantity $S^2 + \sigma^2$ is estimated directly from the sample and the quantity S^2 is estimated separately by some other means.

The estimation of the parameter $S^2 + \sigma^2$ for a first-dollar collision distribution becomes rather non-elementary when our sample of \$50-deductible collision only gives values greater than or equal to \$50. Such a distribution is known as a truncated lognormal distribution, with truncation at the point of the deductible. Aitchison and Brown have treated the problem rather extensively[7], but it would be beyond the scope of this paper to attempt to summarize this treatment.

The raw data used to estimate the value of $S^2 + \sigma^2$ are shown in Exhibit I. It is based on net \$50-deductible collision claims closed during the calendar year 1971. It was determined that a reasonable working estimate of the quantity $S^2 + \sigma^2$ could be obtained from this truncated distribution by plotting the observed cumulative distribution to normal probability graph paper, using several assumed values of F(50) and then accepting the value which produced the best "fit", i.e., the value which produced the straightest line on the graph. The value so chosen was .05. This graphical process is shown in Exhibit II.

The value of $S^2 + \sigma^2$, then, was found to be approximately 1.08 from the graph. The mean of the μ's from this combined distribution is approximately 5.6, reading from the graph. This mean will be designated N.

The method we will use to estimate the value of S^2, the variance of the μ's, was demonstrated by Hewitt[8]. First assume that the mean repair cost of a repair cost group with a μ which is $2S$ below N is equal to 40% of the average repair cost of the combined distribution of all groups. Thus:

$$e^{N-2S+\sigma^2/2} = .40\, e^{N+(S^2+\sigma^2)/2}$$

and

$$-2S = \log_e 0.40 + S^2/2$$

[7] Aitchison and Brown, *op. cit.*, chapter 9.

[8] Hewitt, *op. cit.*, p. 168.

Solving for S:

$$S = .415$$
$$S^2 = .172$$

then $$\sigma^2 = 1.080 - 0.172 = 0.908$$

The above assumption in solving for S is the equivalent of saying the repair cost groups whose μ's are, respectively, $2S$ below and above N have mean repair costs of about \$185 and \$980. The mean repair cost of the total distribution is about \$455.

But there is still a further correction necessary in our estimate of σ^2. This parameter we are attempting to estimate would be applicable to each repair cost group's distribution with no upper bound. The sample data, even though salvage and subrogation recoveries were excluded, understate the value of σ^2 we are looking for to some degree because the claims shown are net after the application of limits based on list price. Based on some rough calculations, then, our estimate of σ^2 should be raised to about 1.00. Recalling the relationship

$$e^{\sigma^2} = \eta^2 + 1$$

where η = the coefficient of variation, it was decided to round the value of η to 1.3 so that

$$e^{\sigma^2} = 2.69$$
$$\sigma^2 = 0.98954$$
$$\text{and } \sigma = 0.99476$$

Although these assumptions and approximations are admittedly rough, any error shouldn't be large enough to invalidate the general conclusions reached from the model's application.

First conlusions from Model

A family of lognormal distributions with varying μ's and equal values of σ^2 — representing repair cost distributions with no upper bound for various repair cost groups of automobiles — can best be illustrated using normal probability graph paper. Such an illustration is shown in Exhibit III. For the purposes of this paper, a "repair cost group" will be identified with the mean cost, α, of the lognormal distribution representing that group. For example, repair cost group 100 is that group whose distribution has a mean of 100, assuming no upper bound. On the graph each straight line represents a lognormal distribution with means varying from 100 to 900. Since the coefficients of variation are constant — thereby making the value of σ^2 constant — the distributions can be graphed on the normal probability paper as parallel lines.

The first moment distributions corresponding to those distributions in Exhibit III are shown in Exhibit IV. Because of the theorem on moment distributions set forth earlier, the graphs of the first moment distributions can be drawn quite easily by simply "shifting" the basic distributions in Exhibit III to the right a distance equal to σ^2. These two exhibits should now make it rather clear to the reader why the lognormal model was judged to be such a convenient way to represent collision distributions by size of loss.

The effect of a particular deductible on a given repair cost group can be approximated from these two graphs as follows (assuming no upper bound): From Exhibit III the proportion of claims in excess of the deductible can be determined (each of these claims is reduced by the amount of the deductible) and from Exhibit IV the ratio of the amount of claims which are eliminated completely by the deductible to the total amount is determined. Since we can now determine the net cost for a given repair cost group after the effect of a given deductible, what we now need is a graph showing, in continuous form, such net cost for any repair cost group with several different sizes of deductibles. Such a graph is shown in Exhibit V. Several observations and explanations are necessary for this graph:

1. No upper bounds have been imposed upon any of the underlying lognormal distributions, or, the effect of the current market value limitation on a collision claim has been ignored. As will be covered later, the effect of such a limitation is negligible for our purposes in the case of new automobiles, i.e., age group 1. It will be shown later that, if two new automobiles, one costing $3,000 and the other $5,000, both are determined to belong in repair cost group 300, their net costs after certain deductibles would differ only slightly. But, no conclusions should yet be made from Exhibit V concerning older age groups of cars.

2. It is assumed that the absolute frequency is the same for all repair cost groups and, for purposes of illustration in Exhibit V, we have used a common frequency of 1/10. Absolute frequency is defined as the frequency of all claims which would be incurred from first dollar

coverage. Similarly, any time the term net cost per claim is used we will define it to mean the net cost, after a deductible, divided by the total number of claims which would be incurred on a first dollar basis. There is no reason to expect that, all other rating characteristics equal, the absolute frequency of a risk would be any different carrying one deductible than it would carrying another.

3. With an absolute frequency of 1/10, what is actually shown on the graph, then, are relative loss costs of repair cost groups for various deductibles. Any difference in the actual absolute frequency and our estimate of 1/10 would not affect the relative loss costs of the repair cost groups.

4. The net loss cost graph for each deductible approaches asymptotically a line which is parallel to the "first dollar, no deductible" line and a vertical distance below it equal to the product of the absolute frequency times the amount of the deductible. If the value of σ^2 were less than our assumed value the lines would approach their respective asymptotes faster, and vice versa.

5. For all repair cost groups above 200 or 300, the relationship between the net loss costs for $50 and $100-deductible collision is a constant dollar difference, for all practical purposes. For higher deductibles, the relationship between net loss cost by deductible as you go from one cost group to another is not as easy to generalize. But even a cursory glance at the graph leads one to conclude readily that to assume that the various deductibles are related to one another on a constant percentage basis would result in rather substantial error.

6. Using a constant percentage relationship between deductibles as you go from the lower cost groups to the higher ones would undoubtedly lead to understatement of the loss cost for the higher groups, and probably an overstatement in the lower groups. This observation is not only true as you compare two cost groups for a given year, but, perhaps even more important, it is equally true for temporal changes in overall average cost per claim (again, based on absolute frequency, as defined above). For example, if, over all cost groups, the average net loss cost (and, as a result, the gross rate) of $100-deductible collision was 70% of $50-deductible net loss cost in a given year, then as the average claim cost increased from year to year, the $100-deductible rates would become more and more inadequate when related to $50-deductible rates.

If a 70% factor were correct seven years ago, it couldn't be adequate today.

Age Groups, Depreciation Factors, and Trend Factors

The graphs shown in Exhibit V represent net loss costs for age group 1 automobiles only. For automobiles age group 2 and older these additional variables need to be introduced:

1. *Depreciation.* As was mentioned earlier, the list price is of little significance for new automobiles belonging in the same repair group. But as the car depreciates, the reduced value of the upper bound begins to take a larger "slice" out of the top of the distribution (i.e., more "totals") and this factor has to be included in our model. The most feasible way to accomplish this would be to estimate a common depreciation factor d, so that, if the original list price of an automobile in age group 1 is L, the market value for the car when it is age group n can be approximated by Ld^{n-1}. Without making a comprehensive study in this area, the author has chosen, for a rough approximation, .75 as the value of d to use in this study.

2. *Trend Factors.* If the overall absolute frequency were to remain unchanged from one year to the next, the need for trend factors in age group 1 automobiles would be completely obviated by the repair cost grouping — assuming that current repair parts costs and labor charges were used in grouping the new automobiles each year. For age group 2 and older models, to simplify coding, let us stipulate that once a new car is classed into a particular repair cost group, say 300, when new, then it remains in that same class throughout its life. Suppose that it can be determined that the index of repair parts costs and labor charges can be expected to increase at a rate of $1 + r$ per year. Then, if a particular automobile was determined to have a first dollar mean repair cost of α when in age group 1, it could be expected to develop a gross mean repair cost of $\alpha\,(1 + r)^{n-1}$ when it was in age group n. Expected repair costs still would be distributed lognormally with variance σ^2 and with the top of the distribution chopped off by the depreciated market value.

3. *Frequency Decrements by Age Group.* It is a well documented phenomenon that absolute claim frequency decreases as insured vehicles advance from one age group to another. For our model we need to determine what these decrements are, on the average, and to assume that

they would apply to one repair cost group as well as another. To estimate what these decrements should be, we have used the combined frequency experience of all independent companies reporting to the National Association of Independent Insurers. The data, covering the period 1968–1970, are shown in Exhibit VI. We will use the notation C_n to denote the frequency decrement for age group *n,* where C_1 is unity.

The Complete Model for All Age Groups

We can now derive an expression for net loss cost for repair cost group α, age group *n,* with initial list price *L,* depreciation factor *d,* rate of increase in cost of parts and labor $(1 + r)$, with a deductible *D,* as follows:

$$Net\ loss\ cost = AC_n \Big[\alpha(1 + r)^{n-1} - DG(D) - \alpha(1 + r)^{n-1} H(D) - \alpha(1 + r)^{n-1} J(Ld^{n-1}) + Ld^{n-1} G(Ld^{n-1}) \Big]$$

Where the functions *G, H,* and *J* are defined earlier, and the quantity *A* denotes the absolute frequency for age group *1.*

Based on this model formula, we have calculated in tabular form net loss costs (before the frequency decrement is applied) for several combinations of repair cost groups and list price groups for the deductibles customarily in use. For each repair cost group, the tabulation is made for two original list prices, representing the approximate maximum and minimum list price which could be associated with the particular mean repair cost. The trend factor used is 1.05. The computer printouts are shown in Exhibit VII. The reader should again note that the term "net per claim," for the purpose of these calculations, means the net cost after the deductible divided by the total absolute number of claims (including those which are eliminated by the deductible).

By comparing the "net per claim" for a given cost group and deductible with the minimum initial list price with the corresponding net per claim for the maximum list price under the same cost group, the reader should be able to see clearly how the list price is rather insignificant in the first few age groups, but becomes so significant at age 6 and 7 that it would seem that any repair cost rating structure derived from our model might well need to keep some list price distinction, perhaps as a supplement to the repair cost grouping. The number of repair cost groups and/or list price groups to be used in a rating structure would have to be determined primarily from a practical standpoint.

Collision Rates with Built-In Trend Factors

Perhaps the most salient feature of the model ratemaking formula outlined in this study is that, with current year automobiles being grouped in accordance with current expected repair costs and age groups relativities obtained as the product of the net costs shown in Exhibit VII and the frequency decrements in Exhibit VI, the resulting collision rating structure will have a completely built-in trend factor. Theoretically, no base rate change would have to be made unless dictated by a discernible trend in absolute claim frequency, or if the built-in trend factor itself would have to be modified in the formula because of changing economic conditions.

The manner in which the trend factor would be computed for the formula would require special attention. From engineering studies — similar to many that are in progress at the present time — a "typical" average collision could be derived. Such a collision could be defined as a group of weights corresponding to certain automobile parts, the weights being roughly equivalent to the frequency in which the respective parts are in need of repair or replacement. This "average" collision would also include the average number of hours of labor. From such an annual study an overall repairability index could be obtained. From a sequence of these indices over a few years, the trend factor could be obtained.

The trend factor is determined from repairability data only. The damageability factor enters into the picture only in the original placement of a particular model automobile into a particular repair cost group, where the relative susceptibility to damage of a given automobile is quite significant. But the improvement (in the aggregate) of damageability from year to year should play no part in the establishment of a trend factor, since the trend factor would relate only to the cost of repairing a given model (say 1968) in one year (say 1971) compared to the cost in a previous year (e.g., 1969). The damageability of that model should not change appreciably from year to year.

A recent study made in Texas took a sampling of collision claims, originally appraised in years prior to 1970, and reappraised those same claims based on 1970 parts and labor prices.[9] Such a comparison would be closely related to the repairability-oriented trend factor we are suggesting for our model. Based on this study, the average annual increase in parts and labor prices was approximately 8 per cent.[10] Because of the current phase of wage-price controls imposed by the Government — wherein any casualty/property trend factor to be applied beyond the inception date of this phase must be reduced to ⅝ths of the value which otherwise would have been used — the author chose to use a 5 per cent trend factor in all calculations in this study.

Even in such a guise as has been outlined in this paper, automobile collision rates must still be shown to be adequate, not excessive, and not unfairly discriminatory. In the process of making such a determination for rates developed using our model, however, the emphasis is shifted drastically from where it is for current physical damage ratemaking and rate regulation. For example, the two primary items which would have to be subject to serious scrutiny each year would be (1) the actual grouping of new models by repair cost group, based on engineering and other data available, and (2) the determination of the repairability trend factor to be used. Of slightly less importance would be the checking of overall trends in absolute claim frequency and the frequency decrements by age group. Some items which have great significance in present day physical damage ratemaking, such as 2 or 3-year loss ratios at the current rate level, would have little bearing on the ratemaking scheme outlined in this paper.

Conclusion

In using any mathematical model to represent distributions by size of loss of automobile collision claims and making other convenient assumptions, such as we have in this paper, there will always be room for criticism because there will always be many exceptions which don't always abide by our assumptions. But in order to achieve some logic and order to physical damage ratemaking, particularly deductible collision, the use of the model suggested in this paper provides a representation of real-world data which has a proper blend of accuracy and

[9] Grady D. Bruce and Robert E. Witt, *A Survey of Trends in the Consumer Cost of Medical Care and Automobile Repair in Texas*, report to Texas Automobile Insurance Service Office, November, 1970.

[10] *Ibid.*, p. 55.

convenience of handling. More sophisticated models might improve the fit of the actual data a few notches, and other refinements, such as frequency variations between models, might be somewhat more true to life. But the lognormal distribution is a good place to start and to demonstrate that the methods customarily used today in collision ratemaking are inadequate. The prospects of our industry having access to some measure of expected average repair cost by model should provide all the impetus required for us in the physical damage pricing business to finally get our house in order.

EXHIBIT I

DISTRIBUTION BY SIZE OF LOSS OF $50-DEDUCTIBLE COLLISION CLAIMS CLOSED IN 1971

Loss Intervals* (Net after deductible)	Claim Count	Accum. Count (Net)	Loss Amount (Net)	Average Loss	Estimated First Dollar Cum. Count	F(x)
less than deductible		—	—	—	(778)	(.050)
00-25	719	719	13,633.68	18.96	1,497	.096
25-50	856	1,575	31,241.43	36.50	2,353	.151
50-75	904	2,479	57,073.72	63.13	3,257	.209
75-100	921	3,400	80,832.96	87.76	4,178	.269
100-125	898	4,298	100,938.33	112.40	5,076	.326
125-150	905	5,203	124,677.95	137.76	5,981	.385
150-175	712	5,915	115,570.86	162.31	6,693	.430
175-200	703	6,618	131,984.80	187.74	7,396	.476
200-225	557	7,175	118,231.39	212.26	7,953	.511
225-250	555	7,730	132,062.80	237.95	8,508	.547
250-300	909	8,639	249,286.49	274.24	9,417	.605
300-350	684	9,323	221,826.40	324.30	10,101	.649
350-400	610	9,933	229,129.80	374.62	10,711	.689
400-450	507	10,440	215,362.38	424.77	11,218	.721
450-500	461	10,901	219,019.14	475.09	11,679	.751
500-600	670	11,571	367,878.81	549.07	12.349	.794
600-700	552	12,123	358,895.26	650.17	12,901	.829
700-800	431	12,554	323,360.81	750.25	13,332	.857
800-900	372	12,926	315,804.96	848.93	13,704	.881
900-1000	256	13,182	242,234.24	946.22	13,960	.898
1000-1100	218	13,400	228,765.26	1,049.38	14,178	.912
1100-1200	167	13,567	191,713.77	1,147.98	14,345	.922
1200-1300	156	13,723	194,854.35	1,249.06	14,501	.932
1300-1400	119	13,842	161,421.85	1,356.48	14,620	.940
1400-1500	117	13,959	170,597.09	1,458.09	14,737	.948
1500-1600	112	14,071	174,692.77	1,559.75	14,849	.955
1600-1700	93	14,164	153,629.12	1,651.92	14,942	.961
1700-1800	81	14,245	142,029.91	1,753.45	15,023	.966
1800-1900	74	14,319	137,208.47	1,854.16	15,097	.971
1900-2000	42	14,361	82,286.54	1,959.20	15,139	.973
2000-2500	213	14,574	477,137.73	2,240.08	15,352	.987
2500-3000	109	14,683	298,718.15	2,740.53	15,461	.994
3000-4000	72	14,755	244,135.38	3,390.76	15,533	.999
4000-5000	12	14,767	51,857.83	4,321.48	15,545	.999
5000-Up	8	14,775	53,234.96	6,654.37	15,553	1.000
TOTAL	14,775		6,411,329.39	433.93	15,553	

*The top point in each interval used as value of X to compute distribution function.

EXHIBIT II
FITTING OBSERVED DISTRIBUTION OF $50-DEDUCTIBLE COLLISION LOSSES BY SIZE TO LOGNORMAL

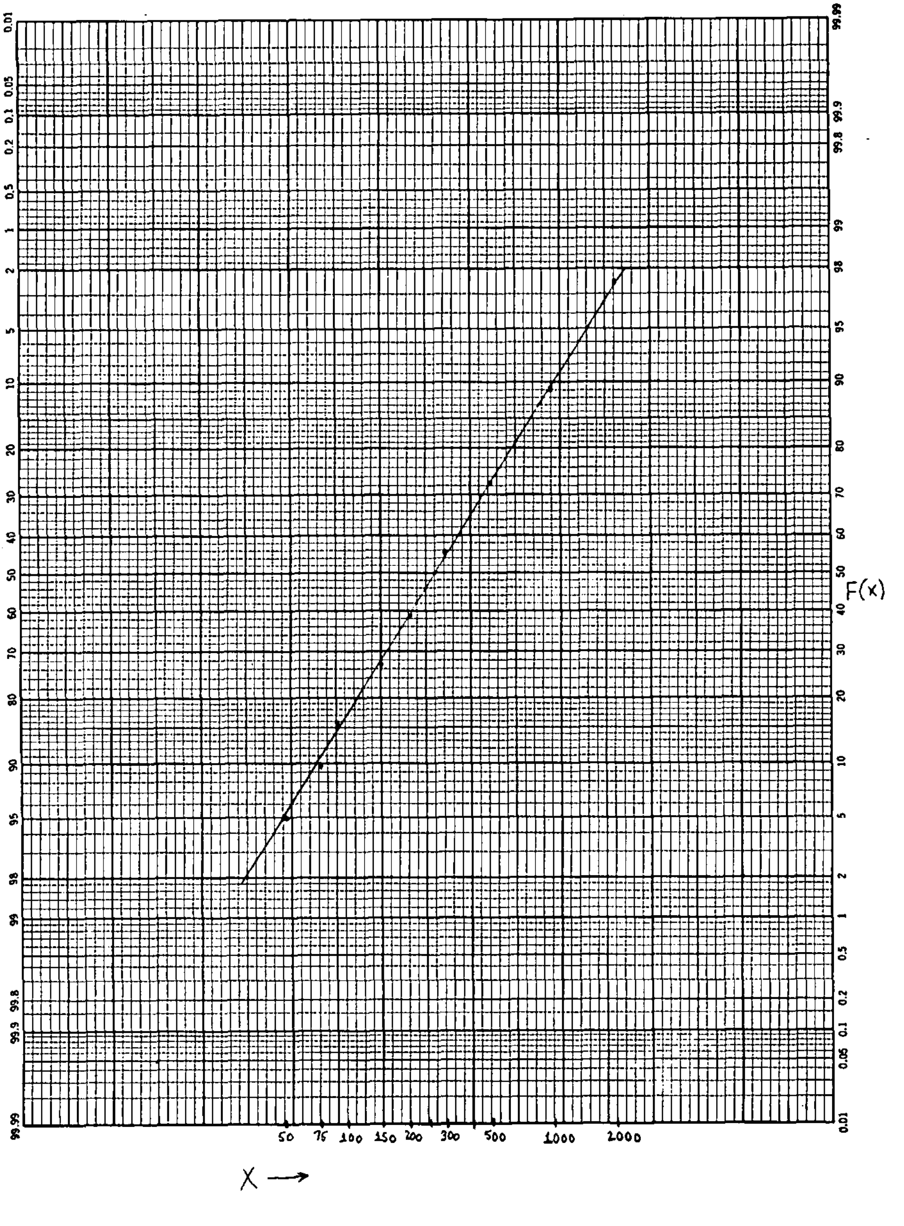


EXHIBIT III
DISTRIBUTION BY SIZE OF LOSS FOR VARIOUS REPAIR COST GROUPS
(LOGNORMAL WITH η = 1.3)

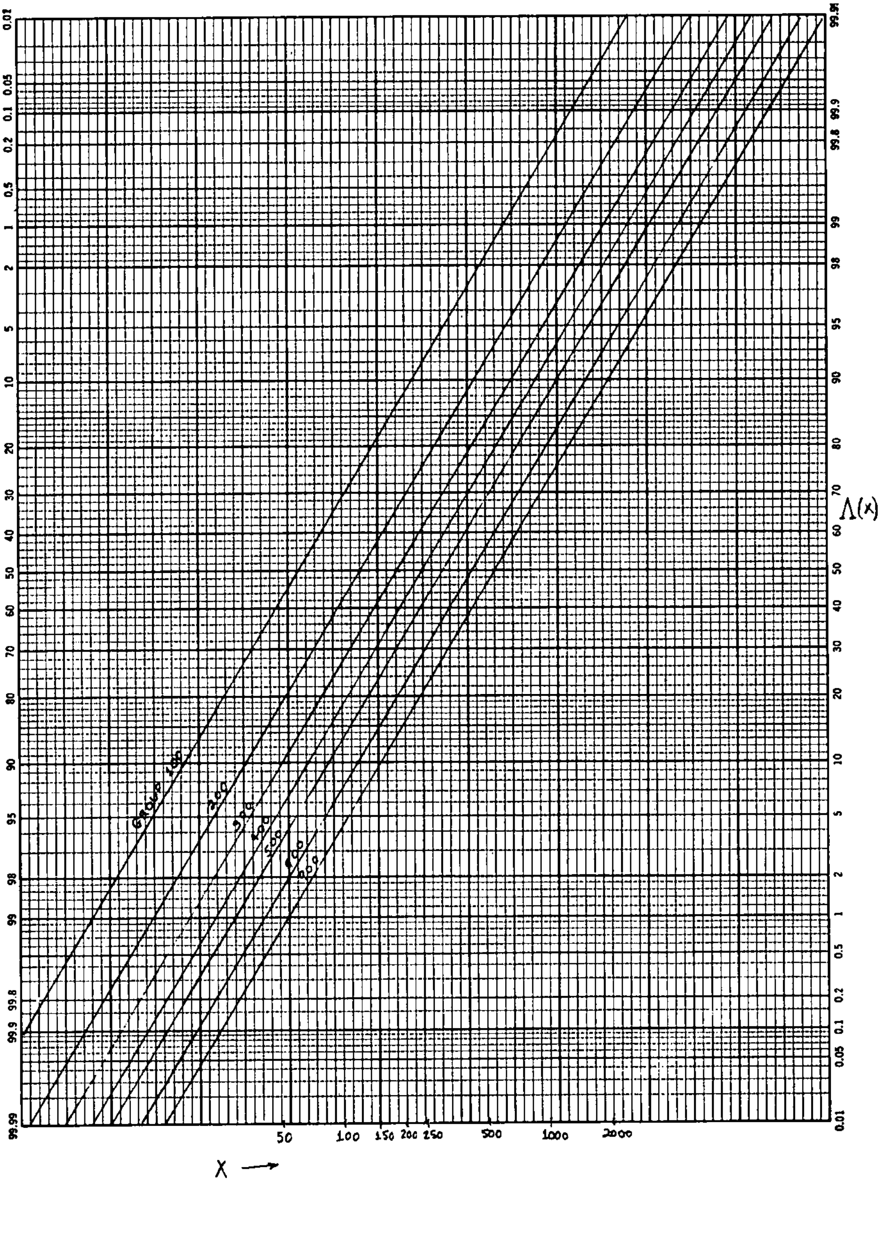


EXHIBIT IV

FIRST MOMENT DISTRIBUTIONS FOR VARIOUS REPAIR COST GROUPS

$\eta = 1.3$

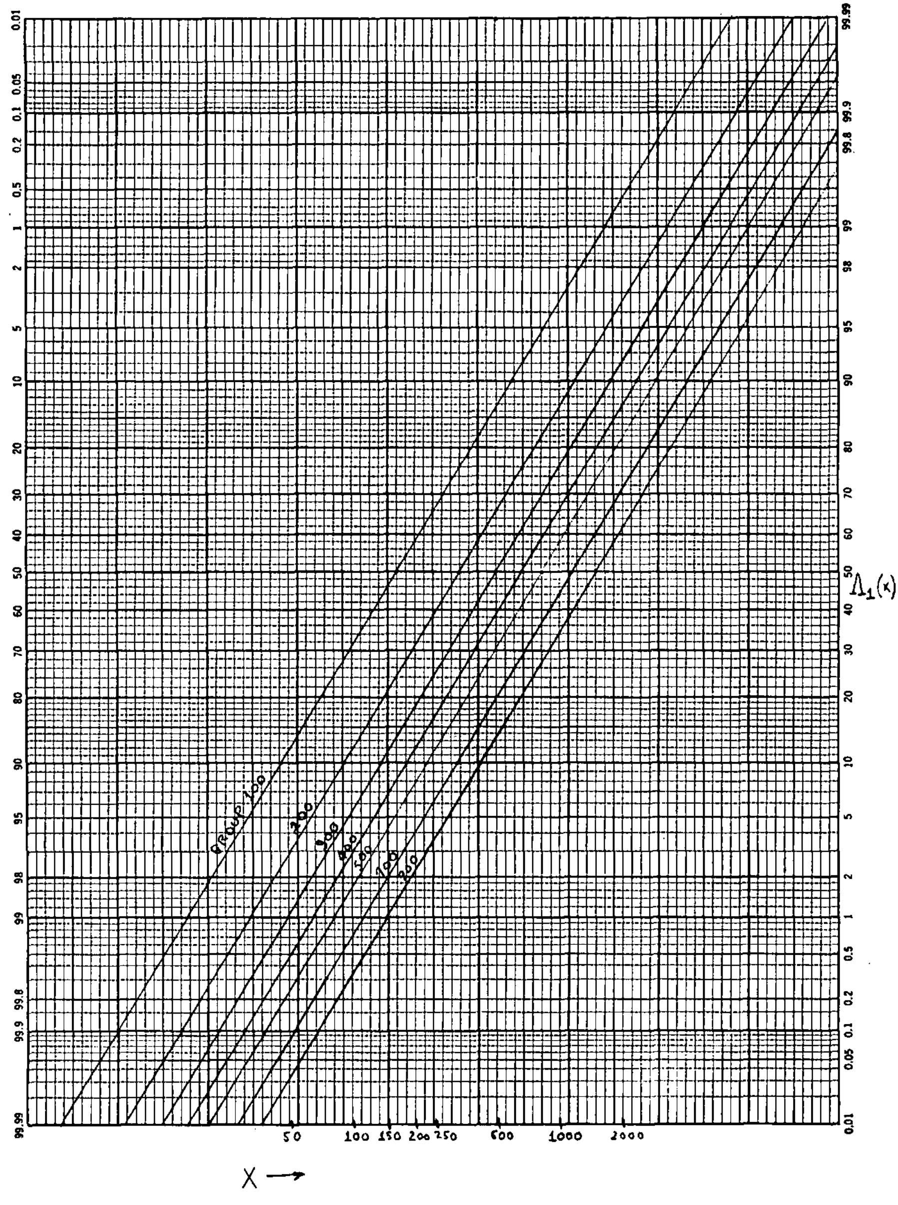


EXHIBIT V

NET LOSS COST BY REPAIR COST GROUP FOR VARIOUS DEDUCTIBLES

ASSUME ABSOLUTE FREQUENCY = 1/10 AND NO UPPER BOUND

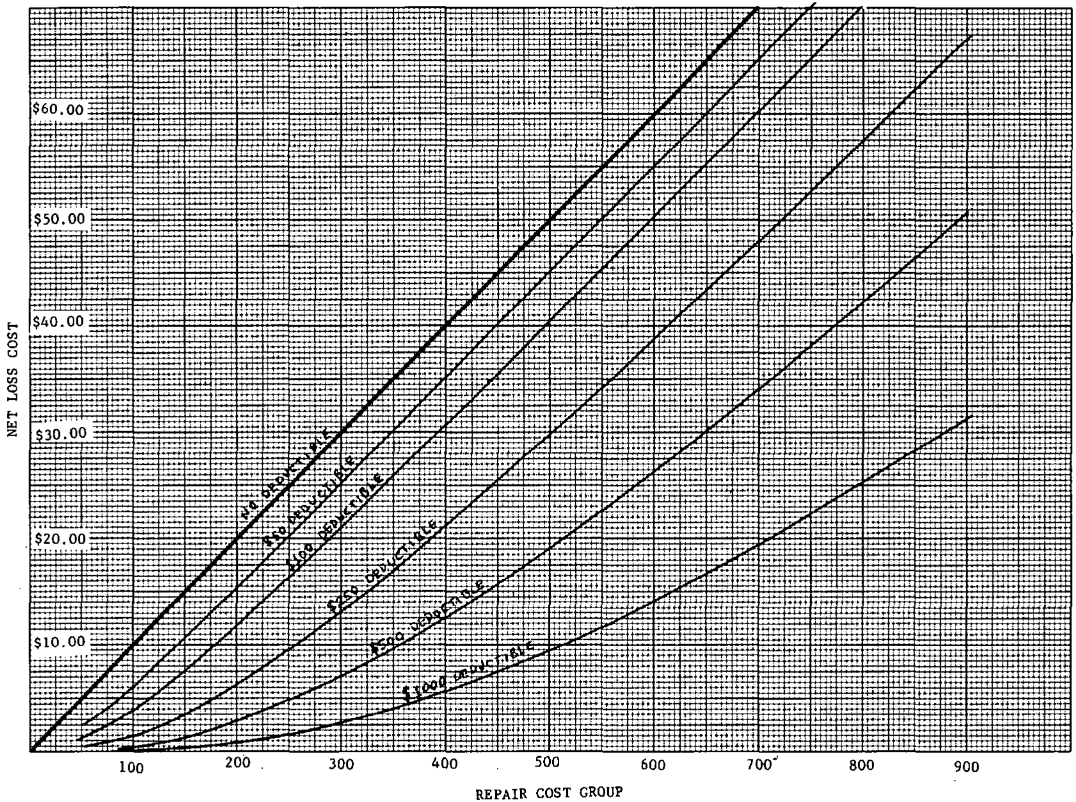


EXHIBIT VI

FREQUENCY DECREMENTS FOR COLLISION BY AGE GROUPS*

Age Group	Freq. per 100 Exposure	Age 4-7 Interpolated**	Relativity To Age 1
1	14.6	14.6	1.000
2	12.8	12.8	.877
3	11.1	11.1	.760
4	9.1	9.6	.658
5	9.1	9.0	.616
6	9.1	8.6	.589
7	9.1	8.4	.575

*Source: National Association of Independent Insurers, Countrywide 1971 Compilation, Private Passenger Non-Fleet, Calendar Years 1968-1970. Used with permission.

**Interpolated frequencies of age groups 4-7 based on rough approximation of relative exposures of these age groups.

EXHIBIT VII—SHEET 1

NET COST BY AGE GROUP FOR REPAIR COST CLASS 300

INITIAL LIST PRICE	2000	DEPRECIATION FACTOR	.75
TREND FACTOR	1.050	COEFFICIENT OF VARIATION	1.3

	AGE 1	AGE 2	AGE 3	AGE 4	AGE 5	AGE 6	AGE 7
MEAN REPAIR COST, NO LIMIT	300.00	315.00	330.75	347.29	364.65	382.88	402.02
LIMIT POINT, L	2000.00	1500.00	1125.00	843.75	632.81	474.61	355.96
G OF L	.0081	.0194	.0420	.0823	.1465	.2379	.3538
G OF L × L	16.20	29.10	47.25	69.44	92.71	112.91	125.94
J OF L	.0793	.1420	.2317	.3464	.4773	.6108	.7323
J OF L × MEAN	23.79	44.73	76.63	120.30	174.05	233.86	294.40
NET REDUCTION FROM LIMIT	7.59	15.63	29.38	50.86	81.34	120.95	168.46
NET PER CLAIM, NO DEDUCTIBLE	292.41	299.37	301.37	296.43	283.31	261.93	233.56
D EQUAL 50							
G OF D	.9038	.9119	.9195	.9266	.9332	.9394	.9450
G OF D × D	45.19	45.60	45.98	46.33	46.66	46.97	47.25
H OF D	.0108	.0095	.0082	.0072	.0063	.0055	.0048
H OF D × MEAN	3.24	2.99	2.71	2.50	2.30	2.11	1.93
NET REDUCTION FROM DEDUCTIBLE	48.43	48.59	48.69	48.83	48.96	49.08	49.18
NET PER CLAIM, 50 DEDUCTIBLE	243.98	250.78	252.68	247.60	234.35	212.85	184.38
D EQUAL 100							
G OF D	.7281	.7441	.7596	.7746	.7891	.8030	.8163
G OF D × D	72.81	74.41	75.96	77.46	78.91	80.30	81.63
H OF D	.0545	.0494	.0446	.0402	.0361	.0324	.0290
H OF D × MEAN	16.35	15.56	14.75	13.96	13.16	12.41	11.66
NET REDUCTION FROM DEDUCTIBLE	89.16	89.97	90.71	91.42	92.07	92.71	93.29
NET PER CLAIM, 100 DEDUCTIBLE	203.25	209.40	210.66	205.01	191.24	169.22	140.27
D EQUAL 250							
G OF D	.3767	.3955	.4145	.4337	.4531	.4726	.4921
G OF D × D	94.18	98.88	103.63	108.43	113.28	118.15	123.03
H OF D	.2480	.2328	.2180	.2039	.1903	.1773	.1648
H OF D × MEAN	74.40	73.33	72.10	70.81	69.39	67.88	66.25
NET REDUCTION FROM DEDUCTIBLE	168.58	172.21	175.73	179.24	182.67	186.03	189.28
NET PER CLAIM, 250 DEDUCTIBLE	123.83	127.16	125.64	117.19	100.64	75.90	44.28
D EQUAL 500							
G OF D	.1560	.1680	.1807	.1939	.2076	.2219	.2368
G OF D × D	78.00	84.00	90.35	96.95	103.80	110.95	118.40
H OF D	.5064	.4869	.4673	.4479	.4285	.4094	.3905
H OF D × MEAN	151.92	153.37	154.56	155.55	156.25	156.75	156.99
NET REDUCTION FROM DEDUCTIBLE	229.92	237.37	244.91	252.50	260.05	267.70	275.39
NET PER CLAIM, 500 DEDUCTIBLE	62.49	62.00	56.46	43.93	23.26	****	****
D EQUAL 1000							
G OF D	.0439	.0486	.0537	.0593	.0653	.0718	.0788
G OF D × D	43.90	48.60	53.70	59.30	65.30	71.80	78.80
H OF D	.7620	.7466	.7307	.7142	.6973	.6800	.6622
H OF D × MEAN	228.60	235.18	241.68	248.03	254.27	260.36	266.22
NET REDUCTION FROM DEDUCTIBLE	272.50	283.78	295.38	307.33	319.57	332.16	345.02
NET PER CLAIM, 1000 DEDUCTIBLE	19.91	15.59	5.99	****	****	****	****

EXHIBIT VII—SHEET 2

NET COST BY AGE GROUP FOR REPAIR COST CLASS 300

INITIAL LIST PRICE	4000	DEPRECIATION FACTOR	.75
TREND FACTOR	1.050	COEFFICIENT OF VARIATION	1.3

	AGE 1	AGE 2	AGE 3	AGE 4	AGE 5	AGE 6	AGE 7
MEAN REPAIR COST, NO LIMIT	300.00	315.00	330.75	347.29	364.65	382.88	402.02
LIMIT POINT, L	4000.00	3000.00	2250.00	1687.50	1265.63	949.22	711.92
G OF L	.0010	.0028	.0077	.0185	.0402	.0792	.1419
G OF L × L	4.00	8.40	17.33	31.22	50.88	75.18	101.02
J OF L	.0176	.0385	.0764	.1375	.2256	.3389	.4693
J OF L × MEAN	5.28	12.13	25.27	47.75	82.27	129.76	188.67
NET REDUCTION FROM LIMIT	1.28	3.73	7.94	16.53	31.39	54.58	87.65
NET PER CLAIM, NO DEDUCTIBLE	298.72	311.27	322.81	330.76	333.26	328.30	314.37
D EQUAL 50							
G OF D	.9038	.9119	.9195	.9266	.9332	.9394	.9450
G OF D × D	45.19	45.60	45.98	46.33	46.66	46.97	47.25
H OF D	.0108	.0095	.0082	.0072	.0063	.0055	.0048
H OF D × MEAN	3.24	2.99	2.71	2.50	2.30	2.11	1.93
NET REDUCTION FROM DEDUCTIBLE	48.43	48.59	48.69	48.83	48.96	49.08	49.18
NET PER CLAIM, 50 DEDUCTIBLE	250.29	262.68	274.12	281.93	284.30	279.22	265.19
D EQUAL 100							
G OF D	.7281	.7441	.7596	.7746	.7891	.8030	.8163
G OF D × D	72.81	74.41	75.96	77.46	78.91	80.30	81.63
H OF D	.0545	.0494	.0446	.0402	.0361	.0324	.0290
H OF D × MEAN	16.35	15.56	14.75	13.96	13.16	12.41	11.66
NET REDUCTION FROM DEDUCTIBLE	89.16	89.97	90.71	91.42	92.07	92.71	93.29
NET PER CLAIM, 100 DEDUCTIBLE	209.56	221.30	232.10	239.34	241.19	235.59	221.08
D EQUAL 250							
G OF D	.3767	.3955	.4145	.4337	.4531	.4726	.4921
G OF D × D	94.18	98.88	103.63	108.43	113.28	118.15	123.03
H OF D	.2480	.2328	.2180	.2039	.1903	.1773	.1648
H OF D × MEAN	74.40	73.33	72.10	70.81	69.39	67.88	66.25
NET REDUCTION FROM DEDUCTIBLE	168.58	172.21	175.73	179.24	182.67	186.03	189.28
NET PER CLAIM, 250 DEDUCTIBLE	130.14	139.06	147.08	151.52	150.59	142.27	125.09
D EQUAL 500							
G OF D	.1560	.1680	.1807	.1939	.2076	.2219	.2368
G OF D × D	78.00	84.00	90.35	96.95	103.80	110.95	118.40
H OF D	.5064	.4869	.4673	.4479	.4285	.4094	.3905
H OF D × MEAN	151.92	153.37	154.56	155.55	156.25	156.75	156.99
NET REDUCTION FROM DEDUCTIBLE	229.92	237.37	244.91	252.50	260.05	267.70	275.39
NET PER CLAIM, 500 DEDUCTIBLE	68.80	73.90	77.90	78.26	73.21	60.60	38.98
D EQUAL 1000							
G OF D	.0439	.0486	.0537	.0593	.0653	.0718	.0788
G OF D × D	43.90	48.60	53.70	59.30	65.30	71.80	78.50
H OF D	.7620	.7466	.7307	.7142	.6973	.6800	.6622
H OF D × MEAN	228.60	235.18	241.68	248.03	254.27	260.36	266.22
NET REDUCTION FROM DEDUCTIBLE	272.50	283.78	295.38	307.33	319.57	332.16	345.02
NET PER CLAIM, 1000 DEDUCTIBLE	26.22	27.49	27.43	23.43	13.69	****	****

EXHIBIT VII—SHEET 3

NET COST BY AGE GROUP FOR REPAIR COST CLASS 400

INITIAL LIST PRICE	3000	DEPRECIATION FACTOR	.75
TREND FACTOR	1.050	COEFFICIENT OF VARIATION	1.3

	AGE 1	AGE 2	AGE 3	AGE 4	AGE 5	AGE 6	AGE 7
MEAN REPAIR COST, NO LIMIT	400.00	420.00	441.00	463.05	486.20	510.51	536.04
LIMIT POINT, L	3000.00	2250.00	1687.50	1265.63	949.22	711.92	533.94
G OF L	.0058	.0145	.0324	.0657	.1210	.2028	.3109
G OF L × L	17.40	32.63	54.68	83.15	114.86	144.38	166.00
J OF L	.0633	.1171	.1972	.3038	.4305	.5648	.6920
J OF L × MEAN	25.32	49.18	86.97	140.67	209.31	288.34	370.94
NET REDUCTION FROM LIMIT	7.92	16.55	32.29	57.52	94.45	143.96	204.94
NET PER CLAIM, NO DEDUCTIBLE	392.08	403.45	408.71	405.53	391.75	366.55	331.10
D EQUAL 50							
G OF D	.9444	.9497	.9546	.9591	.9632	.9670	.9704
G OF D × D	47.22	47.49	47.73	47.96	48.16	48.35	48.52
H OF D	.0048	.0042	.0036	.0032	.0026	.0024	.0020
H OF D × MEAN	1.92	1.76	1.59	1.48	1.26	1.23	1.07
NET REDUCTION FROM DEDUCTIBLE	49.14	49.25	49.32	49.44	49.42	49.58	49.59
NET PER CLAIM, 50 DEDUCTIBLE	342.94	354.20	359.39	356.09	342.33	316.97	281.51
D EQUAL 100							
G OF D	.8149	.8278	.8400	.8516	.8626	.8732	.8831
G OF D × D	81.49	82.78	84.00	85.16	86.26	87.32	88.31
H OF D	.0294	.0262	.0233	.0208	.0184	.0163	.0144
H OF D × MEAN	11.76	11.00	10.28	9.63	8.95	8.32	7.72
NET REDUCTION FROM DEDUCTIBLE	93.25	93.78	94.28	94.79	95.21	95.64	96.03
NET PER CLAIM, 100 DEDUCTIBLE	298.83	309.67	314.43	310.74	296.54	270.91	235.07
D EQUAL 250							
G OF D	.4901	.5097	.5292	.5487	.5680	.5872	.6062
G OF D × D	122.53	127.43	132.30	137.18	142.00	146.80	151.55
H OF D	.1660	.1541	.1427	.1320	.1217	.1122	.1031
H OF D × MEAN	66.40	64.72	62.93	61.12	59.17	57.28	55.27
NET REDUCTION FROM DEDUCTIBLE	188.93	192.15	195.23	198.30	201.17	204.08	206.82
NET PER CLAIM, 250 DEDUCTIBLE	203.15	211.30	213.48	207.23	190.58	162.47	124.28
D EQUAL 500							
G OF D	.2352	.2506	.2664	.2828	.2996	.3168	.3345
G OF D × D	117.60	125.30	133.20	141.40	149.80	158.40	167.25
H OF D	.3924	.3737	.3552	.3372	.3194	.3021	.2852
H OF D × MEAN	156.96	156.95	156.64	156.14	155.29	154.23	152.88
NET REDUCTION FROM DEDUCTIBLE	274.56	282.25	289.84	297.54	305.09	312.63	320.13
NET PER CLAIM, 500 DEDUCTIBLE	117.52	121.20	118.87	107.99	86.66	53.92	10.97
D EQUAL 1000							
G OF D	.0780	.0855	.0934	.1018	.1108	.1203	.1305
G OF D × D	78.00	85.50	93.40	101.80	110.80	120.30	130.50
H OF D	.6641	.6461	.6277	.6090	.5900	.5709	.5515
H OF D × MEAN	265.64	271.36	276.82	282.00	286.86	291.45	295.63
NET REDUCTION FROM DEDUCTIBLE	343.64	356.86	370.22	383.80	397.66	411.75	426.13
NET PER CLAIM, 1000 DEDUCTIBLE	48.44	46.59	38.49	21.73	****	****	****

EXHIBIT VII—SHEET 4

NET COST BY AGE GROUP FOR REPAIR COST CLASS 400

INITIAL LIST PRICE 6000 DEPRECIATION FACTOR .75
TREND FACTOR 1.050 COEFFICIENT OF VARIATION 1.3

	AGE 1	AGE 2	AGE 3	AGE 4	AGE 5	AGE 6	AGE 7
MEAN REPAIR COST, NO LIMIT	400.00	420.00	441.00	463.05	486.20	510.51	536.04
LIMIT POINT, L	6000.00	4500.00	3375.00	2531.25	1898.44	1423.83	1067.87
G OF L	.0006	.0020	.0055	.0137	.0310	.0632	.1170
G OF L × L	3.60	9.00	18.56	34.68	58.85	89.99	124.94
J OF L	.0130	.0296	.0607	.1131	.1916	.2967	.4225
J OF L × MEAN	5.20	12.43	26.77	52.37	93.16	151.47	226.48
NET REDUCTION FROM LIMIT	1.60	3.43	8.21	17.69	34.31	61.48	101.54
NET PER CLAIM, NO DEDUCTIBLE	398.40	416.57	432.79	445.36	451.89	449.03	434.50
D EQUAL 50							
G OF D	.9444	.9497	.9546	.9591	.9632	.9670	.9704
G OF D × D	47.22	47.49	47.73	47.96	48.16	48.35	48.52
H OF D	.0048	.0042	.0036	.0032	.0026	.0024	.0020
H OF D × MEAN	1.92	1.76	1.59	1.48	1.26	1.23	1.07
NET REDUCTION FROM DEDUCTIBLE	49.14	49.25	49.32	49.44	49.42	49.58	49.59
NET PER CLAIM, 50 DEDUCTIBLE	349.26	367.32	383.47	395.92	402.47	399.45	384.91
D EQUAL 100							
G OF D	.8149	.8278	.8400	.8516	.8626	.8732	.8831
G OF D × D	81.49	82.78	84.00	85.16	86.26	87.32	88.31
H OF D	.0294	.0262	.0233	.0208	.0184	.0163	.0144
H OF D × MEAN	11.76	11.00	10.28	9.63	8.95	8.32	7.72
NET REDUCTION FROM DEDUCTIBLE	93.25	93.78	94.28	94.79	95.21	95.64	96.03
NET PER CLAIM, 100 DEDUCTIBLE	305.15	322.79	338.51	350.57	356.68	353.39	338.47
D EQUAL 250							
G OF D	.4901	.5097	.5292	.5487	.5680	.5872	.6062
G OF D × D	122.53	127.43	132.30	137.18	142.00	146.80	151.55
H OF D	.1660	.1541	.1427	.1320	.1217	.1122	.1031
H OF D × MEAN	66.40	64.72	62.93	61.12	59.17	57.28	55.27
NET REDUCTION FROM DEDUCTIBLE	188.93	192.15	195.23	198.30	201.17	204.08	206.82
NET PER CLAIM, 250 DEDUCTIBLE	209.47	224.42	237.56	247.06	250.72	244.95	227.68
D EQUAL 500							
G OF D	.2352	.2506	.2664	.2828	.2996	.3168	.3345
G OF D × D	117.60	125.30	133.20	141.40	149.80	158.40	167.25
H OF D	.3924	.3737	.3552	.3372	.3194	.3021	.2852
H OF D × MEAN	156.96	156.95	156.64	156.14	155.29	154.23	152.88
NET REDUCTION FROM DEDUCTIBLE	274.56	282.25	289.84	297.54	305.09	312.63	320.13
NET PER CLAIM, 500 DEDUCTIBLE	123.84	134.32	142.95	147.82	146.80	136.40	114.37
D EQUAL 1000							
G OF D	.0780	.0855	.0934	.1018	.1108	.1203	.1305
G OF D × D	78.00	85.50	93.40	101.80	110.80	120.30	130.50
H OF D	.6641	.6461	.6277	.6090	.5900	.5709	.5515
H OF D × MEAN	265.64	271.36	276.82	282.00	286.86	291.45	295.63
NET REDUCTION FROM DEDUCTIBLE	343.64	356.86	370.22	383.80	397.66	411.75	426.13
NET PER CLAIM, 1000 DEDUCTIBLE	54.76	59.71	62.57	61.56	54.23	37.28	8.37

EXHIBIT VII—SHEET 5

NET COST BY AGE GROUP FOR REPAIR COST CLASS 500

INITIAL LIST PRICE	3000	DEPRECIATION FACTOR	.75
TREND FACTOR	1.050	COEFFICIENT OF VARIATION	1.3

	AGE 1	AGE 2	AGE 3	AGE 4	AGE 5	AGE 6	AGE 7
MEAN REPAIR COST, NO LIMIT	500.00	525.00	551.25	578.81	607.75	638.14	670.05
LIMIT POINT, L	3000.00	2250.00	1687.50	1265.63	949.22	711.92	533.94
G OF L	.0107	.0250	.0523	.0997	.1722	.2718	.3939
G OF L × L	32.10	56.25	88.26	126.18	163.46	193.50	210.32
J OF L	.0962	.1671	.2652	.3862	.5196	.6508	.7660
J OF L × MEAN	48.10	87.73	146.19	223.54	315.79	415.30	513.26
NET REDUCTION FROM LIMIT	16.00	31.48	57.93	97.36	152.33	221.80	302.94
NET PER CLAIM, NO DEDUCTIBLE	484.00	493.52	493.32	481.45	455.42	416.34	367.11
D EQUAL 50							
G OF D	.9654	.9690	.9723	.9753	.9779	.9804	.9827
G OF D × D	48.27	48.45	48.62	48.77	48.90	49.02	49.14
H OF D	.0024	.0021	.0018	.0015	.0013	.0012	.0009
H OF D × MEAN	1.20	1.10	.99	.87	.79	.77	.60
NET REDUCTION FROM DEDUCTIBLE	49.47	49.55	49.61	49.64	49.69	49.79	49.74
NET PER CLAIM, 50 DEDUCTIBLE	434.53	443.97	443.71	431.81	405.73	366.55	317.37
D EQUAL 100							
G OF D	.8687	.8789	.8885	.8975	.9060	.9140	.9214
G OF D × D	86.87	87.89	88.85	89.75	90.60	91.40	92.14
H OF D	.0172	.0152	.0135	.0119	.0103	.0091	.0080
H OF D × MEAN	8.60	7.98	7.44	6.89	6.26	5.81	5.36
NET REDUCTION FROM DEDUCTIBLE	95.47	95.87	96.29	96.64	96.86	97.21	97.50
NET PER CLAIM, 100 DEDUCTIBLE	388.53	397.65	397.03	384.81	358.56	319.13	269.61
D EQUAL 250							
G OF D	.5790	.5981	.6170	.6356	.6538	.6717	.6892
G OF D × D	144.75	149.53	154.25	158.90	163.45	167.93	172.30
H OF D	.1162	.1069	.0981	.0899	.0822	.0750	.0683
H OF D × MEAN	58.10	56.12	54.08	52.04	49.96	47.86	45.76
NET REDUCTION FROM DEDUCTIBLE	202.85	205.65	208.33	210.94	213.41	215.79	218.06
NET PER CLAIM, 250 DEDUCTIBLE	281.15	287.87	284.99	270.51	242.01	200.55	149.05
D EQUAL 500							
G OF D	.3094	.3269	.3448	.3631	.3816	.4005	.4195
G OF D × D	154.70	163.45	172.40	181.55	190.80	200.25	209.75
H OF D	.3094	.2924	.2758	.2596	.2440	.2289	.2143
H OF D × MEAN	154.70	153.51	152.03	150.26	148.29	146.07	143.59
NET REDUCTION FROM DEDUCTIBLE	309.40	316.96	324.43	331.81	339.09	346.32	353.34
NET PER CLAIM, 500 DEDUCTIBLE	174.60	176.56	168.89	149.64	116.33	70.02	13.77
D EQUAL 1000							
G OF D	.1162	.1261	.1365	.1475	.1591	.1714	.1841
G OF D × D	116.20	126.10	136.50	147.50	159.10	171.40	184.10
H OF D	.5790	.5598	.5403	.5209	.5013	.4817	.4622
H OF D × MEAN	289.50	293.90	297.84	301.50	304.67	307.39	309.70
NET REDUCTION FROM DEDUCTIBLE	405.70	420.00	434.34	449.00	463.77	478.79	493.80
NET PER CLAIM, 1000 DEDUCTIBLE	78.30	73.52	58.98	32.45	****	****	****

EXHIBIT VII—SHEET 6

NET COST BY AGE GROUP FOR REPAIR COST CLASS 500

INITIAL LIST PRICE	6000	DEPRECIATION FACTOR	.75
TREND FACTOR	1.050	COEFFICIENT OF VARIATION	1.3

	AGE 1	AGE 2	AGE 3	AGE 4	AGE 5	AGE 6	AGE 7
MEAN REPAIR COST, NO LIMIT	500.00	525.00	551.25	578.81	607.75	638.14	670.05
LIMIT POINT, L	6000.00	4500.00	3375.00	2531.25	1898.44	1423.83	1067.87
G OF L	.0013	.0040	.0102	.0238	.0503	.0961	.1670
G OF L × L	7.80	18.00	34.43	60.24	95.49	136.83	178.33
J OF L	.0227	.0482	.0927	.1621	.2586	.3785	.5115
J OF L × MEAN	11.35	25.31	51.10	93.83	157.16	241.54	342.73
NET REDUCTION FROM LIMIT	3.55	7.31	16.67	33.59	61.67	104.71	164.40
NET PER CLAIM, NO DEDUCTIBLE	496.45	517.69	534.58	545.22	546.08	533.43	505.65
D EQUAL 50							
G OF D	.9654	.9690	.9723	.9753	.9779	.9804	.9827
G OF D × D	48.27	48.45	48.62	48.77	48.90	49.02	49.14
H OF D	.0024	.0021	.0018	.0015	.0013	.0012	.0009
H OF D × MEAN	1.20	1.10	.99	.87	.79	.77	.60
NET REDUCTION FROM DEDUCTIBLE	49.47	49.55	49.61	49.64	49.69	49.79	49.74
NET PER CLAIM, 50 DEDUCTIBLE	446.98	468.14	484.97	495.58	496.39	483.64	455.91
D EQUAL 100							
G OF D	.8687	.8789	.8885	.8975	.9060	.9140	.9214
G OF D × D	86.87	87.89	88.85	89.75	90.60	91.40	92.14
H OF D	.0172	.0152	.0135	.0119	.0103	.0091	.0080
H OF D × MEAN	8.60	7.98	7.44	6.89	6.26	5.81	5.36
NET REDUCTION FROM DEDUCTIBLE	95.47	95.87	96.29	96.64	96.86	97.21	97.50
NET PER CLAIM, 100 DEDUCTIBLE	400.98	421.82	438.29	448.58	449.22	436.22	408.15
D EQUAL 250							
G OF D	.5790	.5981	.6170	.6356	.6538	.6717	.6892
G OF D × D	144.75	149.53	154.25	158.90	163.45	167.93	172.30
H OF D	.1162	.1069	.0981	.0899	.0822	.0750	.0683
H OF D × MEAN	58.10	56.12	54.08	52.04	49.96	47.86	45.76
NET REDUCTION FROM DEDUCTIBLE	202.85	205.65	208.33	210.94	213.41	215.79	218.06
NET PER CLAIM, 250 DEDUCTIBLE	293.60	312.04	326.25	334.28	332.67	317.64	287.59
D EQUAL 500							
G OF D	.3094	.3269	.3448	.3631	.3816	.4005	.4195
G OF D × D	154.70	163.45	172.40	181.55	190.80	200.25	209.75
H OF D	.3094	.2924	.2758	.2596	.2440	.2289	.2143
H OF D × MEAN	154.70	153.51	152.03	150.26	148.29	146.07	143.59
NET REDUCTION FROM DEDUCTIBLE	309.40	316.96	324.43	331.81	339.09	346.32	353.34
NET PER CLAIM, 500 DEDUCTIBLE	187.05	200.73	210.15	213.41	206.99	187.11	152.31
D EQUAL 1000							
G OF D	.1162	.1261	.1365	.1475	.1591	.1714	.1841
G OF D × D	116.20	126.10	136.50	147.50	159.10	171.40	184.10
H OF D	.5790	.5598	.5403	.5209	.5013	.4817	.4622
H OF D × MEAN	289.50	293.90	297.84	301.50	304.67	307.39	309.70
NET REDUCTION FROM DEDUCTIBLE	405.70	420.00	434.34	449.00	463.77	478.79	493.80
NET PER CLAIM, 1000 DEDUCTIBLE	90.75	97.69	100.24	96.22	82.31	54.64	11.85

EXHIBIT VII—SHEET 7

NET COST BY AGE GROUP FOR REPAIR COST CLASS 600

INITIAL LIST PRICE	4000	DEPRECIATION FACTOR	.75
TREND FACTOR	1.050	COEFFICIENT OF VARIATION	1.3

	AGE 1	AGE 2	AGE 3	AGE 4	AGE 5	AGE 6	AGE 7
MEAN REPAIR COST, NO LIMIT	600.00	630.00	661.50	694.58	729.31	765.78	804.07
LIMIT POINT, L	4000.00	3000.00	2250.00	1687.50	1265.63	949.22	711.92
G OF L	.0080	.0194	.0420	.0823	.1465	.2379	.3538
G OF L × L	32.00	58.20	94.50	138.88	185.41	225.82	251.88
J OF L	.0793	.1420	.2317	.3464	.4774	.6108	.7323
J OF L × MEAN	47.58	89.46	153.27	240.60	348.17	467.74	588.82
NET REDUCTION FROM LIMIT	15.58	31.26	58.77	101.72	162.76	241.92	336.94
NET PER CLAIM, NO DEDUCTIBLE	584.42	598.74	602.73	592.86	566.55	523.86	467.13
D EQUAL 50							
G OF D	.9773	.9798	.9821	.9841	.9860	.9876	.9891
G OF D × D	48.87	48.99	49.11	49.21	49.30	49.38	49.46
H OF D	.0013	.0012	.0010	.0008	.0007	.0006	.0006
H OF D × MEAN	.78	.76	.66	.56	.51	.46	.48
NET REDUCTION FROM DEDUCTIBLE	49.65	49.75	49.77	49.77	49.81	49.84	49.94
NET PER CLAIM, 50 DEDUCTIBLE	534.77	548.99	552.96	543.09	516.74	474.02	417.19
D EQUAL 100							
G OF D	.9038	.9119	.9195	.9266	.9332	.9394	.9450
G OF D × D	90.38	91.19	91.95	92.66	93.32	93.94	94.50
H OF D	.0108	.0095	.0082	.0072	.0063	.0055	.0048
H OF D × MEAN	6.48	5.99	5.42	5.00	4.59	4.21	3.86
NET REDUCTION FROM DEDUCTIBLE	96.86	97.18	97.37	97.66	97.91	98.15	98.36
NET PER CLAIM, 100 DEDUCTIBLE	487.56	501.56	505.36	495.20	468.64	425.71	368.77
D EQUAL 250							
G OF D	.6490	.6671	.6847	.7019	.7187	.7350	.7508
G OF D × D	162.25	166.78	171.18	175.48	179.68	183.75	187.70
H OF D	.0841	.0768	.0700	.0637	.0578	.0523	.0473
H OF D × MEAN	50.46	48.38	46.31	44.24	42.15	40.05	38.03
NET REDUCTION FROM DEDUCTIBLE	212.71	215.16	217.49	219.72	221.83	223.80	225.73
NET PER CLAIM, 250 DEDUCTIBLE	371.71	383.58	385.24	373.14	344.72	300.06	241.40
D EQUAL 500							
G OF D	.3767	.3955	.4145	.4337	.4531	.4726	.4921
G OF D × D	188.35	197.75	207.25	216.85	226.55	236.30	246.05
H OF D	.2480	.2328	.2180	.2039	.1903	.1773	.1648
H OF D × MEAN	148.80	146.66	144.21	141.62	138.79	135.77	132.51
NET REDUCTION FROM DEDUCTIBLE	337.15	344.41	351.46	358.47	365.34	372.07	378.56
NET PER CLAIM, 500 DEDUCTIBLE	247.27	254.33	251.27	234.39	201.21	151.79	88.57
D EQUAL 1000							
G OF D	.1560	.1680	.1807	.1939	.2076	.2219	.2368
G OF D × D	156.00	168.00	180.70	193.90	207.60	221.90	236.80
H OF D	.5064	.4869	.4673	.4479	.4285	.4094	.3905
H OF D × MEAN	303.84	306.75	309.12	311.10	312.51	313.51	313.99
NET REDUCTION FROM DEDUCTIBLE	459.84	474.75	489.82	505.00	520.11	535.41	550.79
NET PER CLAIM, 1000 DEDUCTIBLE	124.58	123.99	112.91	87.86	46.44	****	****

EXHIBIT VII—SHEET 8

NET COST BY AGE GROUP FOR REPAIR COST CLASS 600

INITIAL LIST PRICE	7000	DEPRECIATION FACTOR	.75
TREND FACTOR	1.050	COEFFICIENT OF VARIATION	1.3

	AGE 1	AGE 2	AGE 3	AGE 4	AGE 5	AGE 6	AGE 7
MEAN REPAIR COST, NO LIMIT	600.00	630.00	661.50	694.58	729.31	765.78	804.07
LIMIT POINT, L	7000.00	5250.00	3937.50	2953.13	2214.85	1661.14	1245.86
G OF L	.0014	.0043	.0110	.0255	.0532	.1010	.1742
G OF L × L	9.80	22.58	43.31	75.30	117.83	167.78	217.03
J OF L	.0243	.0511	.0975	.1691	.2679	.3893	.5228
J OF L × MEAN	14.58	32.19	64.50	117.45	195.38	298.12	420.37
NET REDUCTION FROM LIMIT	4.78	9.61	21.19	42.15	77.55	130.34	203.34
NET PER CLAIM, NO DEDUCTIBLE	595.22	620.39	640.31	652.43	651.76	635.44	600.73
D EQUAL 50							
G OF D	.9773	.9798	.9821	.9841	.9860	.9876	.9891
G OF D × D	48.87	48.99	49.11	49.21	49.30	49.38	49.46
H OF D	.0013	.0012	.0010	.0008	.0007	.0006	.0006
H OF D × MEAN	.78	.76	.66	.56	.51	.46	.48
NET REDUCTION FROM DEDUCTIBLE	49.65	49.75	49.77	49.77	49.81	49.84	49.94
NET PER CLAIM, 50 DEDUCTIBLE	545.57	570.64	590.54	602.66	601.95	585.60	550.79
D EQUAL 100							
G OF D	.9038	.9119	.9195	.9266	.9332	.9394	.9450
G OF D × D	90.38	91.19	91.95	92.66	93.32	93.94	94.50
H OF D	.0108	.0095	.0082	.0072	.0063	.0055	.0048
H OF D × MEAN	6.48	5.99	5.42	5.00	4.59	4.21	3.86
NET REDUCTION FROM DEDUCTIBLE	96.86	97.18	97.37	97.66	97.91	98.15	98.36
NET PER CLAIM, 100 DEDUCTIBLE	498.36	523.21	542.94	554.77	553.85	537.29	502.37
D EQUAL 250							
G OF D	.6490	.6671	.6847	.7019	.7187	.7350	.7508
G OF D × D	162.25	166.78	171.18	175.48	179.68	183.75	187.70
H OF D	.0841	.0768	.0700	.0637	.0578	.0523	.0473
H OF D × MEAN	50.46	48.38	46.31	44.24	42.15	40.05	38.03
NET REDUCTION FROM DEDUCTIBLE	212.71	215.16	217.49	219.72	221.83	223.80	225.73
NET PER CLAIM, 250 DEDUCTIBLE	382.51	405.23	422.82	432.71	429.93	411.64	375.00
D EQUAL 500							
G OF D	.3767	.3955	.4145	.4337	.4531	.4726	.4921
G OF D × D	188.35	197.75	207.25	216.85	226.55	236.30	246.05
H OF D	.2480	.2328	.2180	.2039	.1903	.1773	.1648
H OF D × MEAN	148.80	146.66	144.21	141.62	138.79	135.77	132.51
NET REDUCTION FROM DEDUCTIBLE	337.15	344.41	351.46	358.47	365.34	372.07	378.56
NET PER CLAIM, 500 DEDUCTIBLE	258.07	275.98	288.85	293.96	286.42	263.37	222.17
D EQUAL 1000							
G OF D	.1560	.1680	.1807	.1939	.2076	.2219	.2368
G OF D × D	156.00	168.00	180.70	193.90	207.60	221.90	236.80
H OF D	.5064	.4869	.4673	.4479	.4285	.4094	.3905
H OF D × MEAN	303.84	306.75	309.12	311.10	312.51	313.51	313.99
NET REDUCTION FROM DEDUCTIBLE	459.84	474.75	489.82	505.00	520.11	535.41	550.79
NET PER CLAIM, 1000 DEDUCTIBLE	135.38	145.64	150.49	147.43	131.65	100.03	49.94

EXHIBIT VII—SHEET 9

NET COST BY AGE GROUP FOR REPAIR COST CLASS 700

INITIAL LIST PRICE	5000	DEPRECIATION FACTOR	.75
TREND FACTOR	1.050	COEFFICIENT OF VARIATION	1.3

	AGE 1	AGE 2	AGE 3	AGE 4	AGE 5	AGE 6	AGE 7
MEAN REPAIR COST, NO LIMIT	700.00	735.00	771.75	810.34	850.86	893.40	938.07
LIMIT POINT, L	5000.00	3750.00	2812.50	2109.38	1582.04	1186.53	889.90
G OF L	.0067	.0163	.0362	.0723	.1312	.2170	.3284
G OF L × L	33.50	61.13	101.81	152.51	207.56	257.48	292.24
J OF L	.0695	.1270	.2111	.3212	.4498	.5840	.7090
J OF L × MEAN	48.65	93.35	162.92	260.28	382.72	521.75	665.09
NET REDUCTION FROM LIMIT	15.15	32.22	61.11	107.77	175.16	264.27	372.85
NET PER CLAIM, NO DEDUCTIBLE	684.85	702.78	710.64	702.57	675.70	629.13	565.22
D EQUAL 50							
G OF D	.9845	.9862	.9879	.9893	.9906	.9919	.9929
G OF D × D	49.23	49.31	49.40	49.47	49.53	49.60	49.65
H OF D	.0008	.0007	.0006	.0005	.0004	.0003	.0002
H OF D × MEAN	.56	.51	.46	.41	.34	.27	.19
NET REDUCTION FROM DEDUCTIBLE	49.79	49.82	49.86	49.88	49.87	49.87	49.84
NET PER CLAIM, 50 DEDUCTIBLE	635.06	652.96	660.78	652.69	625.83	579.26	515.38
D EQUAL 100							
G OF D	.9277	.9342	.9402	.9459	.9510	.9558	.9602
G OF D × D	92.77	93.42	94.02	94.59	95.10	95.58	96.02
H OF D	.0070	.0062	.0054	.0047	.0040	.0035	.0030
H OF D × MEAN	4.90	4.56	4.17	3.81	3.40	3.13	2.81
NET REDUCTION FROM DEDUCTIBLE	97.67	97.98	98.19	98.40	98.50	98.71	98.83
NET PER CLAIM, 100 DEDUCTIBLE	587.18	604.80	612.45	604.17	577.20	530.42	466.39
D EQUAL 250							
G OF D	.7046	.7213	.7376	.7533	.7685	.7832	.7973
G OF D × D	176.15	180.33	184.40	188.33	192.13	195.80	199.33
H OF D	.0628	.0569	.0515	.0465	.0419	.0377	.0339
H OF D × MEAN	43.96	41.82	39.75	37.68	35.65	33.68	31.80
NET REDUCTION FROM DEDUCTIBLE	220.11	222.15	224.15	226.01	227.78	229.48	231.13
NET PER CLAIM, 250 DEDUCTIBLE	464.74	480.63	486.49	476.56	447.92	399.65	334.09
D EQUAL 500							
G OF D	.4368	.4562	.4757	.4952	.5148	.5343	.5538
G OF D × D	218.40	228.10	237.85	247.60	257.40	267.15	276.90
H OF D	.2016	.1882	.1752	.1629	.1511	.1399	.1293
H OF D × MEAN	141.12	138.33	135.21	132.00	128.56	124.99	121.29
NET REDUCTION FROM DEDUCTIBLE	359.52	366.43	373.06	379.60	385.96	392.14	398.19
NET PER CLAIM, 500 DEDUCTIBLE	325.33	336.35	337.58	322.97	289.74	236.99	167.03
D EQUAL 1000							
G OF D	.1960	.2099	.2243	.2392	.2548	.2707	.2872
G OF D × D	196.00	209.90	224.30	239.20	254.80	270.70	287.20
H OF D	.4448	.4255	.4064	.3875	.3688	.3504	.3324
H OF D × MEAN	311.36	312.74	313.64	314.01	313.80	313.05	311.81
NET REDUCTION FROM DEDUCTIBLE	507.36	522.64	537.94	553.21	568.60	583.75	599.01
NET PER CLAIM, 1000 DEDUCTIBLE	177.49	180.14	172.70	149.36	107.10	45.38	****

EXHIBIT VII—SHEET 10

NET COST BY AGE GROUP FOR REPAIR COST CLASS 700

INITIAL LIST PRICE	8000	DEPRECIATION FACTOR	.75
TREND FACTOR	1.050	COEFFICIENT OF VARIATION	1.3

	AGE 1	AGE 2	AGE 3	AGE 4	AGE 5	AGE 6	AGE 7
MEAN REPAIR COST, NO LIMIT	700.00	735.00	771.75	810.34	850.86	893.40	938.07
LIMIT POINT, L	8000.00	6000.00	4500.00	3375.00	2531.25	1898.44	1423.83
G OF L	.0016	.0046	.0116	.0267	.0556	.1047	.1796
G OF L × L	12.80	27.60	52.20	90.11	140.74	198.77	255.72
J OF L	.0255	.0534	.1011	.1744	.2747	.3973	.5310
J OF L × MEAN	17.85	39.25	78.02	141.32	233.73	354.95	498.12
NET REDUCTION FROM LIMIT	5.05	11.65	25.82	51.21	92.99	156.18	242.40
NET PER CLAIM, NO DEDUCTIBLE	694.95	723.35	745.93	759.13	757.87	737.22	695.67
D EQUAL 50							
G OF D	.9845	.9862	.9879	.9893	.9906	.9919	.9929
G OF D × D	49.23	49.31	49.40	49.47	49.53	49.60	49.65
H OF D	.0008	.0007	.0006	.0005	.0004	.0003	.0002
H OF D × MEAN	.56	.51	.46	.41	.34	.27	.19
NET REDUCTION FROM DEDUCTIBLE	49.79	49.82	49.86	49.88	49.87	49.87	49.84
NET PER CLAIM, 50 DEDUCTIBLE	645.16	673.53	696.07	709.25	708.00	687.35	645.83
D EQUAL 100							
G OF D	.9277	.9342	.9402	.9459	.9510	.9558	.9602
G OF D × D	92.77	93.42	94.02	94.59	95.10	95.58	96.02
H OF D	.0070	.0062	.0054	.0047	.0040	.0035	.0030
H OF D × MEAN	4.90	4.56	4.17	3.81	3.40	3.13	2.81
NET REDUCTION FROM DEDUCTIBLE	97.67	97.98	98.19	98.40	98.50	98.71	98.83
NET PER CLAIM, 100 DEDUCTIBLE	597.28	625.37	647.74	660.73	659.37	638.51	596.84
D EQUAL 250							
G OF D	.7046	.7213	.7376	.7533	.7685	.7832	.7973
G OF D × D	176.15	180.33	184.40	188.33	192.13	195.80	199.33
H OF D	.0628	.0569	.0515	.0465	.0419	.0377	.0339
H OF D × MEAN	43.96	41.82	39.75	37.68	35.65	33.68	31.80
NET REDUCTION FROM DEDUCTIBLE	220.11	222.15	224.15	226.01	227.78	229.48	231.13
NET PER CLAIM, 250 DEDUCTIBLE	474.84	501.20	521.78	533.12	530.09	507.74	464.54
D EQUAL 500							
G OF D	.4368	.4562	.4757	.4952	.5148	.5343	.5538
G OF D × D	218.40	228.10	237.85	247.60	257.40	267.15	276.90
H OF D	.2016	.1882	.1752	.1629	.1511	.1399	.1293
H OF D × MEAN	141.12	138.33	135.21	132.00	128.56	124.99	121.29
NET REDUCTION FROM DEDUCTIBLE	359.52	366.43	373.06	379.60	385.96	392.14	398.19
NET PER CLAIM, 500 DEDUCTIBLE	335.43	356.92	372.87	379.53	371.91	345.08	297.48
D EQUAL 1000							
G OF D	.1960	.2099	.2243	.2392	.2548	.2707	.2872
G OF D × D	196.00	209.90	224.30	239.20	254.80	270.70	287.20
H OF D	.4448	.4255	.4064	.3875	.3688	.3504	.3324
H OF D × MEAN	311.36	312.74	313.64	314.01	313.80	313.05	311.81
NET REDUCTION FROM DEDUCTIBLE	507.36	522.64	537.94	553.21	568.60	583.75	599.01
NET PER CLAIM, 1000 DEDUCTIBLE	187.59	200.71	207.99	205.92	189.27	153.47	96.66

EXHIBIT VII—SHEET 11

NET COST BY AGE GROUP FOR REPAIR COST CLASS 800

INITIAL LIST PRICE	6000	DEPRECIATION FACTOR	.75
TREND FACTOR	1.050	COEFFICIENT OF VARIATION	1.3

	AGE 1	AGE 2	AGE 3	AGE 4	AGE 5	AGE 6	AGE 7
MEAN REPAIR COST, NO LIMIT	800.00	840.00	882.00	926.10	972.41	1021.03	1072.08
LIMIT POINT, L	6000.00	4500.00	3375.00	2531.25	1898.44	1423.83	1067.87
G OF L	.0058	.0145	.0324	.0657	.1210	.2028	.3109
G OF L × L	34.80	65.25	109.35	166.30	229.71	288.75	332.00
J OF L	.0633	.1171	.1972	.3038	.4305	.5648	.6920
J OF L × MEAN	50.64	98.36	173.93	281.35	418.62	576.68	741.88
NET REDUCTION FROM LIMIT	15.84	33.11	64.58	115.05	188.91	287.93	409.88
NET PER CLAIM, NO DEDUCTIBLE	784.16	806.89	817.42	811.05	783.50	733.10	662.20
D EQUAL 50							
G OF D	.9890	.9903	.9915	.9927	.9936	.9944	.9951
G OF D × D	49.45	49.52	49.58	49.64	49.68	49.72	49.76
H OF D	.0005	.0004	.0004	.0002	.0002	.0003	.0002
H OF D × MEAN	.40	.34	.35	.19	.19	.31	.21
NET REDUCTION FROM DEDUCTIBLE	49.85	49.86	49.93	49.83	49.87	50.03	49.97
NET PER CLAIM, 50 DEDUCTIBLE	734.31	757.03	767.49	761.22	733.63	683.07	612.23
D EQUAL 100							
G OF D	.9444	.9497	.9546	.9591	.9632	.9670	.9704
G OF D × D	94.44	94.97	95.46	95.91	96.32	96.70	97.04
H OF D	.0048	.0042	.0036	.0032	.0026	.0024	.0020
H OF D × MEAN	3.84	3.53	3.18	2.96	2.53	2.45	2.14
NET REDUCTION FROM DEDUCTIBLE	98.28	98.50	98.64	98.87	98.85	99.15	99.18
NET PER CLAIM, 100 DEDUCTIBLE	685.88	708.39	718.78	712.18	684.65	633.95	563.02
D EQUAL 250							
G OF D	.7491	.7645	.7793	.7936	.8073	.8205	.8330
G OF D × D	187.28	191.13	194.83	198.40	201.83	205.13	208.25
H OF D	.0478	.0431	.0387	.0348	.0312	.0279	.0249
H OF D × MEAN	38.24	36.20	34.13	32.23	30.34	28.49	26.69
NET REDUCTION FROM DEDUCTIBLE	225.52	227.33	228.96	230.63	232.17	233.62	234.94
NET PER CLAIM, 250 DEDUCTIBLE	558.64	579.56	588.46	580.42	551.33	499.48	427.26
D EQUAL 500							
G OF D	.4901	.5097	.5292	.5487	.5680	.5872	.6062
G OF D × D	245.05	254.85	264.60	274.35	284.00	293.60	303.10
H OF D	.1660	.1541	.1427	.1320	.1217	.1122	.1031
H OF D × MEAN	132.80	129.44	125.86	122.25	118.34	114.56	110.53
NET REDUCTION FROM DEDUCTIBLE	377.85	384.29	390.46	396.60	402.34	408.16	413.63
NET PER CLAIM, 500 DEDUCTIBLE	406.31	422.60	426.96	414.45	381.16	324.94	248.57
D EQUAL 1000							
G OF D	.2352	.2506	.2664	.2828	.2996	.3168	.3345
G OF D × D	235.20	250.60	266.40	282.80	299.60	316.80	334.50
H OF D	.3924	.3737	.3552	.3372	.3194	.3021	.2852
H OF D × MEAN	313.92	313.91	313.29	312.28	310.59	308.45	305.76
NET REDUCTION FROM DEDUCTIBLE	549.12	564.51	579.69	595.08	610.19	625.25	640.26
NET PER CLAIM, 1000 DEDUCTIBLE	235.04	242.38	237.73	215.97	173.31	107.85	21.94

EXHIBIT VII—SHEET 12

NET COST BY AGE GROUP FOR REPAIR COST CLASS 800

INITIAL LIST PRICE	9000	DEPRECIATION FACTOR	.75
TREND FACTOR	1.050	COEFFICIENT OF VARIATION	1.3

	AGE 1	AGE 2	AGE 3	AGE 4	AGE 5	AGE 6	AGE 7
MEAN REPAIR COST, NO LIMIT	800.00	840.00	882.00	926.10	972.41	1021.03	1072.08
LIMIT POINT, L	9000.00	6750.00	5062.50	3796.88	2847.66	2135.75	1601.81
G OF L	.0017	.0048	.0121	.0277	.0574	.1076	.1838
G OF L × L	15.30	32.40	61.26	105.17	163.46	229.81	294.41
J OF L	.0265	.0551	.1040	.1785	.2801	.4034	.5374
J OF L × MEAN	21.20	46.28	91.73	165.31	272.37	411.88	576.14
NET REDUCTION FROM LIMIT	5.90	13.88	30.47	60.14	108.91	182.07	281.73
NET PER CLAIM, NO DEDUCTIBLE	794.10	826.12	851.53	865.96	863.50	838.96	790.35
D EQUAL 50							
G OF D	.9890	.9903	.9915	.9927	.9936	.9944	.9951
G OF D × D	49.45	49.52	49.58	49.64	49.68	49.72	49.76
H OF D	.0005	.0004	.0004	.0002	.0002	.0003	.0002
H OF D × MEAN	.40	.34	.35	.19	.19	.31	.21
NET REDUCTION FROM DEDUCTIBLE	49.85	49.86	49.93	49.83	49.87	50.03	49.97
NET PER CLAIM, 50 DEDUCTIBLE	744.25	776.26	801.60	816.13	813.63	788.93	740.38
D EQUAL 100							
G OF D	.9444	.9497	.9546	.9591	.9632	.9670	.9704
G OF D × D	94.44	94.97	95.46	95.91	96.32	96.70	97.04
H OF D	.0048	.0042	.0036	.0032	.0026	.0024	.0020
H OF D × MEAN	3.84	3.53	3.18	2.96	2.53	2.45	2.14
NET REDUCTION FROM DEDUCTIBLE	98.28	98.50	98.64	98.87	98.85	99.15	99.18
NET PER CLAIM, 100 DEDUCTIBLE	695.82	727.62	752.89	767.09	764.65	739.81	691.17
D EQUAL 250							
G OF D	.7491	.7645	.7793	.7936	.8073	.8205	.8330
G OF D × D	187.28	191.13	194.83	198.40	201.83	205.13	208.25
H OF D	.0478	.0431	.0387	.0348	.0312	.0279	.0249
H OF D × MEAN	38.24	36.20	34.13	32.23	30.34	28.49	26.69
NET REDUCTION FROM DEDUCTIBLE	225.52	227.33	228.96	230.63	232.17	233.62	234.94
NET PER CLAIM, 250 DEDUCTIBLE	568.58	598.79	622.57	635.33	631.33	605.34	555.41
D EQUAL 500							
G OF D	.4901	.5097	.5292	.5487	.5680	.5872	.6062
G OF D × D	245.05	254.85	264.60	274.35	284.00	293.60	303.10
H OF D	.1660	.1541	.1427	.1320	.1217	.1122	.1031
H OF D × MEAN	132.80	129.44	125.86	122.25	118.34	114.56	110.53
NET REDUCTION FROM DEDUCTIBLE	377.85	384.29	390.46	396.60	402.34	408.16	413.63
NET PER CLAIM, 500 DEDUCTIBLE	416.25	441.83	461.07	469.36	461.16	430.80	376.72
D EQUAL 1000							
G OF D	.2352	.2506	.2664	.2828	.2996	.3168	.3345
G OF D × D	235.20	250.60	266.40	282.80	299.60	316.80	334.50
H OF D	.3924	.3737	.3552	.3372	.3194	.3021	.2852
H OF D × MEAN	313.92	313.91	313.29	312.28	310.59	308.45	305.76
NET REDUCTION FROM DEDUCTIBLE	549.12	564.51	579.69	595.08	610.19	625.25	640.26
NET PER CLAIM, 1000 DEDUCTIBLE	244.98	261.61	271.84	270.88	253.31	213.71	150.09

DISCUSSION BY WILLIAM S. GILLAM AND DANIEL P. FRAME

In this paper Mr. Bickerstaff presents a proposed ratemaking procedure for automobile collision insurance which incorporates two concepts not currently reflected in the standard ratemaking techniques on which we will comment in this review. The first is the reflection of the physical characteristics of vehicles in terms of their "damageability" and "repairability". That is, their relative susceptibility to damage given a certain collsion situation and their relative cost of repair parts and labor given a certain degree of damage. The second is the use of the lognormal distribution as a model for the distribution by size of automobile collision claims and the use of this model to determine the relationships between the rates for different deductibles.

As far as damageability and repairability are concerned, Mr. Bickerstaff in his paper recognizes the practical problems involved in collecting claim cost statistics by make and model of vehicle within a time frame that would permit their use in prospective ratemaking. He concludes that the answer would be to find a means of estimating the expected average repair costs of particular models on an *a priori* basis by means of engineering analyses. He briefly summarizes several of the studies that are being made by the industry and optimistically concludes that it would be safe to assume that the insurance industry will not stop "until there is developed completely a feasible means to determine for each new automobile model a reasonably accurate expected average repair cost which would take into consideration the parts/labor costs and the damageability over the full spectrum of possible collisions." He further states that "The ideas in this paper are predicated on that assumption."

In his summary of studies under way in this area he does not mention the continuing study of collision loss statistics that has been initiated by the Insurance Institute for Highway Safety in cooperation with a relatively small number of automobile insurers that, nevertheless, account for a very sizeable share of the automobile insurance market. For a summary of this project see the report submitted by Dr. William Haddon, Jr., President of the Institute, to the Automobile Insurance Problems (D3) Subcommittee of the NAIC at its meeting in December of 1971.

We can only echo Mr. Bickerstaff's hope that the industry will be able to develop a feasible way to measure differences in expected average repair costs due to differences in the physical characteristics of vehicles so as to make possible their reflection in collision insurance rating. The progress made to date in this regard does not lead us to be as optimistic as Mr. Bickerstaff that this will be accomplished in the near future.

As far as the use of the lognormal distribution is concerned, we will leave to others more mathematically inclined to comment on the appropriateness of this distribution as a model for the distribution by size of automobile collision claims. They may also comment on his solutions to the problem of estimating the parameters of the lognormal to be fitted to actual data that are reported net as to a deductible. The problems created by the fact that there is in the real world an upper bound to the distribution, represented by the actual cash value of the insured vehicle at the time of the accident, need to be commented upon also.

As to the application of the lognormal model in the determination of the relationships between the rates for different deductibles, Mr. Bickerstaff concludes that the relationship between the net loss costs for $50 and $100 deductible collision is, for all practical purposes, a constant dollar difference. He feels that using a constant percentage relationship as one goes from the lower cost groups to the higher ones would "undoubtedly lead to an understatement of the loss costs for the higher groups and an overstatement in the lower groups."

However, in developing his net loss cost relationships by repair cost group for various deductibles, he "assumed that the absolute frequency (the frequency of all claims if no deductible were purchased) is the same for all repair cost groups, and that there is no reason to expect that, all other rating characteristics being equal, the absolute frequency of a risk would be any different carrying one deductible than it would carrying another."

We suggest that the absolute frequency should be expected to vary according to various characteristics of the risk such as the age, sex, and marital status of the operators. The uses to which the car is to be put, the income of the owner, and the expected annual mileage should also have some effect on the absolute frequency. We also believe that these considerations will affect the choice of the type of car owned, and hence its repair cost group, and ultimately the selection of a deductible.

In other words, the one thing we know for certain is that the people who purchase $50 deductible coverage are a completely different set of people than those who purchase $100 deductible coverage.

Therefore, rather than to switch to the use of a mathematical model based upon such assumptions, why not continue to develop rates for the two basic collision coverages, $50 and $100 deductible. More than 95% of the private passenger automobile insureds purchase these coverages, so why not use their actual experience? In the standard automobile physical damage ratemaking procedure the overall statewide rate levels and the base premiums for each territory are determined separately for $50 deductible and $100 deductible coverages based on the actual experience. This reflects all the characteristics of the two groups of risks that will affect their losses. The relationships between the rates for the current symbol groups are determined based upon analysis of experience separately for $50 deductible and $100 deductible coverages.

If the industry is successful in developing a feasible way to measure differences in the expected average repair costs of the different models of cars and if these lead to the development of classifications based upon such expected average repair costs, either in place of, or in addition to, the current symbol classifications based upon the value of the car—new, or as Mr. Bickerstaff expresses it, "crude groupings of cars by left rear window price stickers," there is no reason why the appropriate relationships cannot be determined from the real world data separately for $50 deductible and $100 deductible.

For deductibles higher than $100, where the volume of experience is not sufficient to permit analysis by state and territory and by symbol classification, the use of the lognormal to determine appropriate relationships warrants, in our judgment, further study.

Based upon the use of the lognormal and the expected average repair costs of each year's new car production by make and model, Mr. Bickerstaff constructs a mathematical model for complete collision rating which also incorporates a depreciation factor, trends in repair costs and frequency decrements by age group. Evaluation of this model is beyond the scope of our review. Suffice to say that the ideas underlying it should serve to stimulate those concerned with automobile collision insurance ratemaking to re-examine current procedures and to test Mr. Bickerstaff's assumptions against the real world collision experience for automobiles of Age Group 2 and older.

DISCUSSION BY J. S. SAWYER

Mr. Bickerstaff has presented an excellent paper on the rating of Automobile Collision coverage. As he points out, there is a great need for a review of our Automobile Physical Damage ratemaking techniques. Unlike the Liability coverages which have been treated rather exhaustively in respect to the characteristics contributing to loss, our Physical Damage ratemaking techniques have largely ignored the loss characteristics inherent in the automobile itself.

His use of the lognormal model provides a fine example of the use of a mathematical tool in the solution of a practical problem. As with any model there are practical problems which must be solved prior to its actual use in determining rates.

Among the problems that I foresee are the following:

1. It will be difficult to secure reliable information regarding the damageability and repairability of a given model of automobile prior to its introduction to the public.

2. The model assumes that given a certain vehicle with an expected severity of loss, one can calculate the appropriate deductible credits to be applied. From past experience, we know that such things as geographical location of the risk and type of driver can contribute heavily to the loss severity. For example, let us assume that all cars in a given cost group are used in an urban area where the impact speed of any collision is assumed to be relatively low and damage is largely confined to bumpers and fenders. In this particular instance, the value of the deductible would be greater than if we took the identical group of cars and used them in a rural area where impact speeds of 60-100 m.p.h. might be common and therefore severity much worse. Thus, it would seem that a grouping by vehicle repairability characteristics alone would not be sufficient to calculate the value of the deductible. From this, one might argue that the basic territorial rate would remove any distortion along these lines and that automobiles of various cost groups would tend to stay in some relation to each other. However, from the physical characteristics of some of the crash tests that have been conducted recently, it is not clear to me that two automobiles which sustain substantially different damage at

some given impact speed will not have nearly the same amount of damage at some other impact speed. This would necessitate extensive tests at various impact speeds to determine if an automobile of one model was properly grouped with an automobile of some other model.

3. The assumption was made in Mr. Bickerstaff's paper that the absolute frequency was the same for all repair cost groups. This seems to me to be a dangerous assumption since it is perhaps more likely for a youthful operator to operate an automobile in one of the lower repair cost groups. Admittedly, I have not seen which automobiles might be in the groups contemplated by Mr. Bickerstaff, but assuming that a Chevrolet is not to be grouped with a Cadillac, it might well turn out that the absolute frequency for the two groups is substantially different simply because of the mix of operator characteristics involved. Geographical variation of frequency may also significantly affect the charge for the proper deductible and may also tend to nullify the assumption of constant frequency.
4. The use of the truncated lognormal distribution is valid only when applied to raw data which conforms to the lognormal distribution above the deductible amount. In actual practice, the problem of "padding" deductibles may alter the true deductible Collision data by throwing claims under the deductible into the distribution and thus make it unreliable in estimating the first dollar Collision claim distribution. This would be especially true if the data to be considered were for $100 deductible claims rather than the $50 deductible used. Since the type of deductible chosen by the insured seems to vary in proportion to his Collision cost, it may well be that the only data available in certain areas would be the $100 deductible experience in which case this "padding" would be most prevalent.

The foregoing is not meant to criticize the intent of the paper presented, but rather to point out the practical problems which I foresee will need to be overcome before a workable model can be established. I am in wholehearted agreement that we need to put our house in order in regard to Physical Damage ratemaking and the steps outlined in Mr. Bickerstaff's paper present a thought provoking and valuable contribution to such a change. The reader should also consider the extension of this valuable tool to other ratemaking disciplines such as the pricing of deductibles in the various amount of insurance categories in the Homeowners line. The path which Mr. Bickerstaff has started us on may well prove to be one of the most valuable practical tools in ratemaking.

DISCUSSION BY DALE NELSON

This paper, in the reviewer's mind, is one of the finest to have been presented to the Society in recent years. Not only for its subject matter, but also for the author's approach. It is a superb example of an actuarial attack on an important problem. Most papers tend to concentrate on either the practical or the theoretical side of the problem; this one encompasses a very readable blending of both sides.

In view of the widespread publicity in recent years concerning the damageability and repairability characteristics of today's cars and the rash of studies underway to isolate and quantify those characteristics, Mr. Bickerstaff's paper is certainly opportune. On the assumption these studies will be successful in developing some appropriate damageability indices or expected average repair costs, he advances a ratemaking approach for automobile physical damage which would directly take this new information into account. In the process, he develops a fairly complete model of the expected loss profile for the physical damage coverages, particularly Collision, including the effects of deductibles and Actual Cash Values. This in itself is a much needed, and long overdue analysis.

Also incorporated in this model is a concept which may prove to be the most useful of the whole paper—the built-in trend (or inflationary) factor. It is conceivable that, in the not-too-distant future, rate structures for both automobile liability and physical damage will incorporate such built-in automatic adjustment factors based on standardized indices. Something similar is already in use in the Homeowner's area—through Insurance to Value programs—and, of course, Mr. Masterson's recent *Proceedings* papers fall right down this alley. In this manner, many of the rate filings for routine (and, nowadays, expected) upward adjustments could be avoided—thus saving considerable time and effort for both the regulators and the insurers.

Getting back to the paper at hand, one might quibble with the author's statement that "The peculiarities involved in auto physical damage have never received the same rigorous scrutiny . . . that has been given liability ratemaking techniques" on the grounds that auto liability has not been confronted with a problem of quite the same maginitude. (No-Fault, however, is now correcting that situation.) I suppose the lack of attention to the physical characteristics of the automobile stems largely from the somewhat incongruous dichotomy which used to pervade the respective ratemaking approaches for the casualty and the property lines. Oversimplified, this dictated that in casualty ratemaking, once the classifications were established, one relied solely on past experience, almost to the complete exclusion of other considerations (including engineering), while in property ratemaking just the opposite was emphasized. One could speculate that the problem at hand would have been solved long ago had auto physical damage been kept in the property area.

Neither extreme is the answer, though. Both are important, and both should be taken into account. Thus, while the casualty actuary might tend to express a rating relativity (R) as

$$R_{new} = Z \cdot A + (1\text{-}Z) \cdot R_{old}$$

where A is the actual (projected) experience indication and Z is a suitable weight or credibility factor, and the property actuary might express it as

$$R_{new} = E,$$

where E is an indicator based on engineering or theoretical considerations, perhaps what is really needed is

$$R_{new} = Z \cdot A + (1\text{-}Z) \cdot E$$

In this light, Mr. Bickerstaff has provided us with a suitable candidate for E in the case of automobile physical damage—at least for repairability/ deductible elements; namely, the lengthy expression given in his paper for the Net Loss Cost for repair group α, age group n, with initial list price L, depreciation factor d, inflationary rate r, and deductible D. Actually, he is suggesting that E stand on its own, but I think in practice a more suitable approach would involve a weighting of the experience with the model's indications. Once a new automobile model has been introduced, and some actual crash experience is available, it would be foolhardy to rely completely on the prior estimates of the expected repair costs.

As partial confirmation that this model is in the right direction, it might be noted that the experience for the present symbol groups substantiates the author's finding that the relationships between loss costs by deductible tend toward constant dollar differences rather than uniform percentage differences.

As noted earlier, the paper is concerned primarily with the Collision coverages and doesn't touch on the closely related problem of the Comprehensive coverage. But it is evident that the same approach lends itself to handling that coverage also. It would be probably be necessary to segment the coverage by cause of loss (fire, theft, windstorm, etc.) or in some fashion "gimmick" the size of loss distribution in order to recognize the disproportionate number of total losses. Also, since the comprehensive perils, particularly theft and windstorm, vary considerably by geographical area, some care would be necessary in piecing the parts back together. This latter type problem doesn't affect Collision quite so much, although a case could be made for taking into account the variation in the number of single car crashes, which tend to involve greater damage on average.

Finally, turning to the technical portion of the paper, some further work undoubtedly needs to be done in the area of parameter estimation for the size of loss distributions. Specifically, the method used by the author to eliminate the "spread" parameter S^2 is a little weak. It appears that he has simply assumed an arbitrary value for S, which is independent of the underlying data. Initially I thought that a handle could be gotten from the first moment distribution; but, as it turned out, this only provides another estimate of $S^2 + \sigma^2$. In view of the truncation problem we may find that the graphical estimate for the total variance is, in fact, a good, practical estimate of σ^2.

AUTHOR'S REVIEW OF DISCUSSIONS

I guess it would be impossible to consider any ratemaking scheme which is at odds, to any degree, with current methods without many practical transitional problems surfacing early in the game. Each of the reviewers has done an excellent job of considering the practical difficulties which would accompany the scheme outlined in my paper and I am indebted to each of them for their thorough, incisive discussions. I am in general agreement with the reviewers' comments and, moreover, the only point of disagreement I have with them may be due mostly to a poorly worded definition of an important concept in the original paper. My comments are as follows:

Absolute Frequency and Risk Characteristics

Some poor wording in the original paper led two of the reviewers to interpret that the efficacy of the entire ratemaking formula depended on the "absolute frequency" being constant for all repair cost groups. The reviewers correctly observed that such an asumption is not likely to be true because of the fact that frequency-related risk characteristics of one cost group might well be different than another. However, the workability of the model does not depend on the truth of such an assumption and, furthermore, it isn't the assumption the author intended to make in the first place.

The assumption on which the model is based is that a given risk within a particular class/territory/age group subset would develop about the same absolute frequency regardless of the cost group his car would fall in and whatever deductible he might choose. Of course, it is true that "the group of people who purchase $50-deductible collision is a completely different set of people than those who purchase $100-deductible," as Mr. Frame and Mr. Gillam have reminded us. Indeed, I have seen recent statewide statistical reports which show that the average annual premium (and probably statewide pure premium, as well) for $100-deductible collision is actually higher than the average for $50-deductible, even though for one individual risk the rate for $100-deductible is from 70 to 85 per cent of the corresponding $50-deductible rate. But, again, this heterogeneity, caused by a marked selection of the higher deductible as age, territory, and symbol classifications tend to the higher side of the rating scale, has no bearing on the validity of our presumed relationship between the various deductibles.

To further clarify the absolute frequency/risk characteristic assumption, perhaps the full expression for net loss cost needs some work. The formula, as originally set forth, would apply as a "base" absolute frequency corresponding to a particular class/territory combination, both of whose absolute frequency relativities are unity. For other classes and territories, appropriate absolute frequency relativities would have to be determined and included as an additional factor in the full formula. Just how class and territory relativities would be superimposed on our repair cost grouping would be of utmost importance in the actual implementation of our ratemaking scheme. This facet of the overall problem was virtually ignored in the original paper, and, by implication, the reviewers wisely advised that the class and territory considerations require much further study.

Severity Differences by Territory

Can class and territory relativities be based on frequency differences only? I suppose there is an implicit assumption in the original paper that they can. Mr. Sawyer has pointed out that automobiles in an urban environment might be expected to develop a lower average repair cost than rural autos because of lower average impact speed. This would cause a slightly different relationship between deductibles in urban areas compared to rural territories. In our original paper we defined the average repair cost underlying a given repair cost group as the average cost of repair and labor charges for a given group of automobiles when subjected to the full spectrum of possible collisions, weighted by the relative incidence of the particular types of collisions. It was implied that this "full spectrum" would embrace all geographical conditions, urban and rural. It is probably true, as Mr. Sawyer has observed, that urban areas would be weighted slightly to the lower side of the overall mean expected repair cost, and vice versa. Also implicit in the original thinking was that the necessary territorial distinctions in the final gross rate would be a function of measurable differences in absolute frequency only. However, in going through the process of self-assessment, particularly after the review of Mr. Sawyer, I find that the ignoring of severity differences by territory and its effect on the overall ratemaking scheme is that portion of my paper which requires much further study.

One possible solution would be to divide all territories into two groups—urban and rural—and vary the formula for the two groups. Instead of simply determining the average expected repair cost over the full spectrum of collision situations, using weights which reflect the overall incidence in both rural and urban environments, perhaps the weighted mean expected cost could be done separately for urban and rural situations. For example, let us say that a group of cars developed an expected average repair cost of $400, weighted proportionately for both urban and rural situations. But using weights characteristic of urban accidents alone, let us say the average was $380, and the "rural" average was $425. Perhaps some reasonable generalization could be made that the "urban" average would be X% lower than the overall average and the "rural" average would be Y% higher than the overall—for any cost group.

From a practical standpoint, most any block of data you look at these days seems to bear out the fact that most urban-rural distinctions, as far as rating characteristics are concerned, are beginning to fade somewhat. More and more these days, automobile owners who can be classified as rural drive into the urban areas rather frequently instead of once a week (Saturday morning, maybe). One indication of this change in rural driving habits is the gradual diminution of the justifiable "farm discount" on private passenger automobiles over the past ten years. It would seem to follow that the expected severity differences within one overall cost group between those garaged in urban areas and those in rural areas would tend to diminish in like fashion. Perhaps before two separate average expected repair costs are computed for urban and rural territories, an effort should be made to determine exactly how much difference there would be and if it would necessitate a distinction from a practical standpoint.

How "Real" is Real World Data?

Mr. Frame and Mr. Gillam suggest that, even if the industry is successful in developing repair cost groups, there would be "no reason why the appropriate relationships (between deductibles) cannot be determined from the real world data for $50-deductible and $100-deductible." I would assume that this means, as is customary in present ratemaking procedures for ratemaking organizations and for many large companies, that countrywide loss ratios by symbol (or repair cost) groups would be analyzed with premiums adjusted to the base symbol group, and symbol or repair cost group relativities calculated therefrom. There is a very big reason why "the appropriate relationships" cannot be determined accurately from this type of "real world" data—unless it can be assumed that within each symbol/repair cost group each subdivision by state, territory, class, age group, etc., produced the same loss ratio as the whole group—a very unlikely event. Otherwise, such an analysis almost inevitability leads to distorted, biased results, particularly when using companywide statistics. There are ways to eliminate much of the bias when all elements of a multi-dimensional rating structure (symbol, age, territory, etc.) are analyzed simultaneously[1] but, apparently because of the complexities of such analysis, such an effort to avoid the bias has not customarily been made. Indeed, it is the author's strong recollection that, in the most recent adjustment made by the rating organizations of symbol relativities, the revised relativities were identical for $50-deductible and $100-deductible collision, perhaps with a view towards convenience at the expense of accuracy.

Mr. Frame and Mr. Gillam also state that, "in the standard ratemaking procedure, the overall statewide rate levels and the base premiums for each territory are determined separately for $50-deductible and $100-deductible based on actual experience, which reflects all the characteristics of the two groups of risks that will affect their losses." Again, because of the same inherent bias which is present in what I presume they are calling the "standard ratemaking procedure", some interesting situations can result. Suppose that in a given state the base rate for $50-deductible collision in territory 1 is 125% of the remainder-of-state $50-deductible rate, but the $100-deductible base rate for the same territory is only 105% of the $100-deductible remainder-of-state rate (such a situation, I think the reader will find, is not uncommon). If one policyholder driving the same car moved from the remainder-of-state territory to territory 1 his rate would go up 25% if he carried $50-deductible but only 5% if he carried $100-deductible. I submit that this is absurd. The different "characteristics of the two (symbol) groups which will affect their losses" should be provided for in class plan, age group, etc., relativities to such a large degree that different territorial relativities for different deductibles are, in my opinion, indefensible.

Actual Crash Experience vs. Prior Estimates

Despite the above critique of present day "real world" analytical techniques as far as symbol groups are concerned, I agree wholeheartedly with Mr. Nelson's comment that "once a new automobile model has been in-

[1]See, for example, R. A. Bailey and L. J. Simon, "Two Studies in Automobile Insurance Ratemaking," *PCAS XLVII* and Bailey, "Insurance Rates With Minimum Bias, *PCAS L.*

troduced, and some actual crash experience is available, it would be foolhardy to rely completely on the prior estimates of the expected repair cost." Based on some pilot studies being undertaken at the present time to measure actual repair cost differences by model from actual crashes[2] it seems that the earliest possible date that any credible, real world crash data would be available would be after the model year is over half-way expired. But once this actual crash data is available, I agree that the revised repair cost grouping should be a weighted average of the original expected repair cost and the actual crash data.

Technical Notes Concerning the Model

Mr. Nelson is correct in his observation that the method I used to eliminate the "spread" parameter S^2 is a little on the rough side. Actually, the value I assumed for S is not as arbitrary as would appear at first glance. The estimate for S is based roughly on a countrywide exposure distribution by symbol groups and a rough correlation between symbol groups and average repair cost. Mr. Sawyer correctly observed that in the process of evaluating a truncated distribution as we did in the original paper, with the point of truncation being the deductible amount, we have run the risk of what he terms deductible "padding". The author was aware of such a hazard and, moreover, in attempting to fit the truncated distribution to a straight line on the probability graph paper there was an unnatural "hump" in the data around the point of deductible which simply couldn't be smoothed out no matter what value of F(50) was used in the process of fitting the data. The "padding" problem is something we simply had to live with. It is my feeling, however, that the error produced by the rough estimation of S and the "padding" problem would not be beyond the tolerance we had originally set up. Our final estimate of the coefficient of variation based on the original data was 1.3. Even allowing for the possible errors Mr. Sawyer and Mr. Nelson have called to our attention, we believe that the estimate of 1.3 should be correct within the tolerance of, say, plus or minus 0.1.

Application to Other Coverages

I believe that the type of deductible analysis illustrated in the original paper would also lend itself to other coverages. Mr. Nelson suggested that

[2]For example, the tests currently in process in conjunction with the Insurance Institute for Highway Safety.

automobile comprehensive could be analyzed the same way once an allowance is made for the disproportionate share of total losses. Mr. Sawyer suggested this type of analysis for Homeowners. One other coverage I would like to add to the list which would be perhaps even a more likely candidate for such an analysis is Crop Hail. This coverage is written with various types of deductibles—some straight deductibles and some disappearing deductible forms. It is my understanding that, in the present rating structure, the relationship between one deductible and another is a constant percentage relationship. I wouldn't be a bit surprised to find that, after an analysis of the losses by size in crop hail, the proper relationship between these various deductibles would be more of a constant dollar relationship as was found to be the case in automobile collision.

Again, I would like to thank each of the reviewers for their very helpful comments concerning the paper—particularly those comments which called attention to areas in the original paper which required substantial clarification.

DISCUSSIONS OF PAPER PUBLISHED IN VOLUME LVIII

ACTUARIAL NOTE ON WORKMEN'S COMPENSATION LOSS RESERVES

RONALD E. FERGUSON

VOLUME LVIII PAGE 51

DISCUSSION BY EDWARD M. SMITH

In this presentation Mr. Ferguson has noted an error in reserving which he believes to be common practice where there is excess of loss reinsurance on long term disability losses. Having recognized the error he also presents a means of correctly reserving the greater portion of these cases.

The reserving error which he discusses involves the calculation of the net retention on such cases. This net value is often set at the contractual retention level rather than at the present value of the retention amount recognizing interest and mortality over the term of disability. The result is an overstatement of the direct writer's portion of the loss and an understatement of the reinsurer's portion of the loss reserve by a like amount.

I think that there is little question as to the validity of his estimate that many companies are making these calculations incorrectly. The procedure recommended for properly establishing the net retention is correct for most cases and represents an improvement in all cases. For most companies this error in reserve is probably of little significance. My company is fairly typical of large writers of Workmen's Compensation and we have only a handful of Workmen's Compensation cases involving reinsurance. These are all old cases involving substantial amounts of continuing medical expenses.

Mr. Ferguson has suggested the lack of N_x and D_x values has been responsible for failure to use correct reserving techniques. This is a kindness, at least in my instance. I found that we did have the ℓ_x values underlying the Survivorship Annuitants Table of Mortality and was able to spot check some of the D_x values in the table presented in the paper. With a little effort we could have developed a complete table of values.

Until now we had not considered the problem. In our case the impact of changing our reserves on these losses would not be sufficient to make the establishment of new procedures worthwhile. For those companies which find that this reserve area is one they must correct, Mr. Ferguson's presentation will provide the basis for a good start.

There will remain the problem of handling the reserve calculations for permanent disability losses in states which do not use Table XI in Bulletin 222, and some decision must be made in regard to the proper method to be used in establishing present value reserves on those large losses involving continuing medical care. Some thought must be given as to whether or not such cases, involving permanent hospital care, can be expected to incur mortality rates similar to those used in creating the valuation tables presently in use.

It seems to me that few companies will decide to change their reserving methods concerning net retention in these cases, for the effect on their total loss reserve will be insignificant. However, it may well be a problem for reinsurers covering substantial amounts of Workmen's Compensation. If this is a problem to them it probably will only be solved through the establishment of precautionary reserves on their part. I am fairly certain that this will not be the only area where such reserves are needed.

DISCUSSION BY JAMES F. GOLZ

Mr. Ferguson has given us an admirably clear and concise case for the use of temporary annuities in the calculation of net reserves under excess of loss reinsurance. Mr. Ferguson points out that one should not simply take the primary retention as the net reserve whenever the direct reserve exceeds the retention. In order to properly reflect the benefits to the insurer of interest earnings and mortality experience, the net reserve should equal the present value of a temporary annuity of period sufficient to accrue benefits equal to the retention. The ceded reserve is then the present value of an annuity deferred for this same period. The sum of the two will, of course, equal the original direct reserve. Mr. Ferguson illustrates the situation with examples based upon reinsurance excess of \$50,000 and \$100,000 retentions.

However, a major insurer, if it has excess of loss reinsurance, is likely to operate under a retention considerably greater than those used by Mr. Ferguson. If for example the retention were \$250,000, then even with a pension of \$10,000 a year, the direct reserve would exceed the primary retention only for a annuitant under fourteen years of age if we use the table accompanying Mr. Ferguson's paper (Survivorship Annuitants' Mortality Table at 3%). And might not inaccuracies in non-pension portions of the reserve outweigh the difference between the "correct" and "incorrect" methods of splitting the direct reserve?

There is the additional problem of catastrophes. When the reserve is composed of more than one pension, one should calculate the net reserve as the sum of present values of each possible primary insurer payment pattern weighted by its probability of occurrence. For example, if annuitants of ages 45 and 55 each receive \$10,000 a year paid continuously under reinsurance excess of \$250,000 retention, then the net reserve would be

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} ({}_{m-1|}q_{45} \cdot {}_{n-1|}q_{55}) (\bar{a}_{\overline{mm}|} + \bar{a}_{\overline{nn}|})$$

where mm and nn, the payment years to the younger and older annuitants respectively, are appropriately limited by the primary retention. Such a calculation seems unduly complex.

Mr. Ferguson hypothesizes that the lack of availability of N_x *and* D_x commutation columns may have contributed to the use of incorrect methods. Although it may not be possible to find the values used in the construction of an annuity table, developing columns consistent with the annuity values is a relatively straightforward process. Having the annuity values $\ddot{a}_x$, one need merely set D_0 equal to some convenient constant. Then $N_0 = \ddot{a}_0 \cdot D_0$ and the columns may be completed by backing off using

$$N_x = N_{x-1} - D_{x-1}$$

$$D_x = \frac{N_x}{\ddot{a}_x}$$

If it is desired, one may conveniently alter the interest assumption of the table by computing

$$D'_x = D_x \cdot \left(\frac{1+i}{1+i'}\right)^x \quad D_x \cdot \left(\frac{1+i}{1+i'}\right)^x$$

As with any such procedure, one must be careful not to carry the results beyond the significance of the input data.

My conclusion, then, is that the problem Mr. Ferguson has examined is one which occurs so rarely (at least in its simple form) that the benefits of the theoretically correct procedure are outweighed by the efforts of implementing it. Areas such as this do provide an opportunity for fruitful cooperation between insurer and reinsurer. Since reinsurers may suffer more from reserve inaccuracies, they have a legitimate interest in the techniques utilized. Perhaps it is time for reinsurers to help develop and implement reserving methods which serve their needs as well as those of primary insurers. In this respect, Mr. Ferguson's article forms an excellent first step.

AUTHOR'S REVIEW OF DISCUSSIONS

I am grateful to Mr. Golz for an interesting review of my paper. Mr. Golz accomplished at least three things in his review: he presented his opinion that the reserving technique is probably not worthwhile since the basic problem does not occur frequently; he pointed to a significant gap in my paper, as respects catastrophes; and he provided us with a technique for determining working values of N_x and D_x given only $\ddot{a}_x$.

I believe our differences of opinion on the value of implementing the reserving techniques described in my paper result from our different perspectives. Mr. Golz, for example, states that with retentions in the neighborhood of $250,000 one would only infrequently encounter cases where the recommended reserving technique would matter. While the retention of $250,000 may be used by a large compensation writer, such as the one for which Mr. Golz works, there remain hundreds of companies with much lower retentions, sometimes as low as $10,000. Clearly, for these companies, the difference between the correct and incorrect reserving technique can be significant.

The reviewer suggests that the "non-pension portions of the reserve outweigh the difference between correct and incorrect methods of splitting the direct reserve." While it is true that the non-pension portions of a loss do complicate the issue, they do not present an insurmountable problem. Very often their present values can be calculated as is done for pension benefits. Frequently, for example, medical care costs can be expressed as $x per year and then handled as an annuity.

The ink was scarcely dry on my paper when I realized that I had failed to cover the problem of catastrophes (multiple person accidents). Generally speaking, the ceding company's retention applies on a per accident basis rather than on a per claim basis, and Mr. Golz is quite correct in pointing out that this condition will complicate the task of computing the correct reserve. Fortunately, multiple person accidents do not seem to be as common as one might guess and, as Mr. Golz points out, the theory for calculating the correct reserve does exist. With modern day computing machinery such complicated reserving practices may not be as formidable as they seem. In any event, I resist the notion that because it is difficult to calculate correct reserves in a multiple person accident, we should fail to make an attempt to calculate the correct reserve when it is feasible.

Finally, Mr. Golz is to be congratulated for developing an algorithm for determining the N_x and D_x values consistent with $\ddot{a}_x$ values. The reader will recall that the N_x and D_x values were not published in the New York Tables (Bulletin 222).

While not agreeing with all of Mr. Golz's observations and conclusions, I am, nevertheless, indebted to him for a good review of my paper.

MINUTES OF THE 1972 SPRING MEETING

May 21-24, 1972

LAKE LAWN LODGE, DELAVAN, WISCONSIN

Sunday, May 21, 1972

The Board of Directors held its regularly scheduled meeting at Lake Lawn Lodge, from 2:00 p.m. to 6:00 p.m. Preliminary registration was also held during the late afternoon for early arrivals.

An informal reception was held in the early evening for all members and their wives as well as guests. It should be noted that the Board of Directors of the American Academy of Actuaries met at Lake Lawn during the weekend and some members remained to participate in part of the Society's activities.

Monday, May 22, 1972

After a brief registration period, the 1972 Spring meeting was formally convened at 9:00 a.m. by President LeRoy J. Simon who welcomed the gathering and then introduced the Honorable Stanley C. DuRose, Jr., Commissioner of Insurance, State of Wisconsin and also a member of the Society.

Commissioner DuRose welcomed the Society to Wisconsin and presented his thoughts on various timely problems affecting the insurance industry. Following his remarks Commissioner DuRose answered questions from the floor.

As time permitted, the following new papers were presented to the membership by the authors:

> "How Adequate are Loss and Loss Expense Liabilities" by Ruth E. Salzmann, Vice President and Actuary, Sentry Insurance Group.
>
> "Loss Reserving in the Sixties" by Rafal J. Balcarek, Vice President and Actuary, Reliance Insurance Co.

Following a coffee break, a panel discussion entitled "Measurements of Profitability" was presented to the entire membership. Participants in this part of the program were as follows:

Moderator: Seymour E. Smith
Senior Vice President and Actuary
The Travelers Insurance Companies

Participants: Robert A. Bailey
Insurance Bureau
State of Michigan

Philip L. Defliese
Lybrand, Ross Brothers & Montgomery

Robert Dineen
National Association of Insurance Commissioners

Leandro S. Galban, Jr.
Wood, Struthers & Winthrop

William J. Pugh, Jr.
Associate General Counsel
Insurance Company of North America

A brief question and answer period followed the formal presentation and the panel discussion was concluded at 12:00 noon.

Following lunch, the afternoon program convened at 1:30 p.m. with a panel presentation to the entire membership entitled "Car Design, Damageability and Repairability". Participants in this part of the program were as follows:

Moderator: John S. Trees
Pricing Director and Actuary
Allstate Insurance Company

Participants: John P. Drennan
Associate Actuary
Allstate Insurance Company

P. K. Sturgeon
Assistant Vice President
Lumbermen's Mutual Casualty Co.

J. C. Purcell
Director Consumer Services
General Motors Corporation

Following this presentation, the membership was split into two groups for the purpose of attending one of two concurrent sessions. The first session was a continuation of the subject "Car Design, Damageability and Repairability" on an informal basis consisting of questions, answers and discussions on the prior presentation.

The second concurrent session was the formal panel presentation of "Corporate Modeling".

Participants in this discussion were as follows:

Moderator: W. James MacGinnitie
Consulting Actuary
Milliman and Robertson

Participants: Raymond W. Beckman
Assistant Actuary
Fireman's Fund American Insurance Co.

Martin Albaum
Director of Research
Prudential Property and Casualty Ins. Co.

Gail Cullen
Controller
Allstate Enterprises Operations

At 4:00 p.m. the membership again had its choice of concurrent sessions. Those interested in pursuing the subject of "Corporate Modeling" could attend the continuation of this subject on an informal basis while others could attend the second session which was a group discussion entitled "Actuarial Smorgasbord" moderated by Dunbar R. Uhthoff, Senior Vice President, Employers of Wausau.

The afternoon activities recessed at 5:00 p.m.

No formal evening arrangements were made for the membership at large. A small reception was held for the new Fellows and their wives who, at a later time during the meeting, would be presented with their Fellowship diplomas.

Tuesday, May 23, 1972

President Simon reconvened the meeting at 9:00 a.m. The first order of business was the presentation of diplomas to the following new Fellows and Associates:

FELLOWS

Robert A. Anker	Edward R. Murray
Robert D. Bergen	Charles R. Rinehart
David G. Hartman	Charles W. Stewart

ASSOCIATES

Howard V. Dempster	Ronald C. Retterath
James B. Reinbolt	Daniel J. Rogers

The next portion of the business meeting was devoted to three Committee reports. Earl Petz discussed the new examination Syllabus as part of the report of the activities of the Education Committee. Vernon Switzer followed with a brief summary of the report of the Committee on the Constitution and By-Laws which was approved by the Board of Directors at its meeting on Sunday, May 21. The last report was a brief outline by Charles Hachemeister of the forthcoming Actuarial Research Conference to be held at the University of Waterloo, Waterloo, Ontario.

The next portion of the business meeting was the presentation of another new paper entitled:

> "Joint Underwriting as a Reinsurance Problem" by Emil Strug, Assistant Vice President and Associate Actuary, Massachusetts Blue Cross/Blue Shield Inc.

Following the coffee break, President Simon introduced our featured speaker, Dean Spencer L. Kimball, University of Wisconsin Law School. Dean Kimball's topic was "The Future of Insurance Regulation" and his address was followed by a short question and answer period.

At 11:30 a.m. a group discussion was held as a continuation of the subject "The Future Course of the Casualty Actuarial Society" which was previously discussed at the November 1971 meeting. Participants in this part of the program were as follows:

Moderator: Richard L. Johe
Vice President and Actuary
United States Fidelity & Guaranty Co.

Participants: "Levels of Certification"
John H. Muetterties
Actuary
Insurance Services Office

"Report of the Special Task Force to Enlist Candidates"
James R. Berquist
Consulting Actuary
Milliman & Robertson

"Areas of Cooperation with Other Actuarial Bodies"
Paul S. Liscord
Vice President
Insurance Company of North America

Following lunch the afternoon was set aside for individual meetings of various committees.

A reception was held at 6:00 p.m. for the entire membership and was followed by a banquet.

Wednesday, May 24, 1972

At 9:00 a.m. President Simon reconvened the business meeting with the presentation of the following three new papers:

"Allocating Premium to Layer by the Use of Increased Limits Tables" by Ronald E. Ferguson, Assistant Secretary, General Reinsurance Corporation.

"A Note on Full Credibility for Estimating Claim Frequency" by J. Ernest Hansen, Research Associate, Insurance Company of North America.

"Automobile Collision Deductibles and Repair Cost Groups: the Lognormal Model" by David Bickerstaff, Actuary, Southern Farm Bureau Casualty Insurance Company.

The business session was concluded with the presentation of two reviews of a paper previously presented.

"Actuarial Note on Workmen's Compensation Loss Reserves" by Ronald E. Ferguson.

Reviews by:

James Golz
Actuarial Assistant
Employers Insurance of Wausau

Edward M. Smith
Actuary
The Travelers Insurance Company

(Mr. Smith's review was read by Robert Foster)

The author offered brief comments on both reviews.

Following the coffee break, the membership was presented a panel discussion entitled "Computerization within Actuarial Departments" (non data processing applications).

Participants in the panel were as follows:

Moderator: Dale A. Nelson
Actuary
State Farm Mutual Automobile Insurance Co.

Participants: Warren P. Cooper
Vice President and Actuary
Chubb and Son Inc.

Charles A. Hachemeister
Associate Actuary
Allstate Insurance Company

Charles W. Stewart
Research Associate
Insurance Company of North America

The last item of the agenda was a panel discussion entitled "No Fault Revisited".

Participants in this panel were as follows:

Moderator:	Robert A. Brian Associate Actuary Aetna Life and Casualty
Participants:	Massachusetts George C. Klingman Assistant Director The Travelers Insurance Cos.
	Delaware Howard V. Dempster, Jr. Assistant Secretary Insurance Company of North America
	Florida Dale R. Comey Assistant Actuary The Hartford Insurance Group
	Oregon David P. Flynn Assistant Actuary Fireman's Fund American Insurance Co.

At the conclusion of this panel President Simon adjourned the meeting at 12:00 p.m.

It is noted that the registration cards completed by the attendees and filed at the registration desk indicated, in addition to approximately 25 wives, attendance by 84 Fellows and 47 Associates, and 29 invited guests as follows:

FELLOWS

Adler, M.
Alexander, L. M.
Atwood, C. R.
Bailey, R. A.
Balcarek, R. J.
Beckman, R. W.
Bennett, N. J.
Ben-Zvi, P. N.
Bergen, R. D.
Berquist, J. R.
Bevan, J. R.
Bickerstaff, D. R.
Bornhuetter, R. L.
Boyajian, J. H.
Brian, R. A.
Brown, W. W., Jr.
Byrne, H. T.
Comey, D. R.
Cook, C. F.
Curry, A. C.
Curry, H. E.
Eliason, E. B.
Ferguson, R. E.
Flynn, D. P.
Foster, R. B.
Gibson, J. A.
Gillam, W. S.
Gillespie, J. E.
Graves, C. H.
Hachemeister, C. A.
Hartman, D. G.
Hazam, W. J.
Hewitt, C. C., Jr.
Hillhouse, J. A.
Honebein, C. W.
Hughey, M. S.
Jacobs, T. S.
Johe, R. L.
Kilbourne, F. W.
Klaassen, E. J.
Liscord, P. S.
MacGinnitie, W. J.
Masterson, N. E.
McClure, R. D.
Meenaghan, J. J.
Menzel, H. W.
Mills, R. J.
Mohnblatt, A. S.
Morison, G. D.
Muetterties, J. H.
Monro, R. E.
Murray, E. R.
Naffziger, J. V.
Nelson, D. A.
Otteson, P. M.
Perreault, S. L.
Petz, E. F.
Phillips, H. J.
Portermain, N. W.
Richards, H. R.
Richardson, J. F.
Rinehart, C. R.
Roberts, L. H.
Rodermund, M.
Ryan, K. M.
Salzmann, R. E.
Scheibl, J. A.
Scheid, J. E.
Schloss, H. W.
Simon, L. J.
Smith, E. R.
Smith, S. E.
Snader, R. H.
Stankus, L. M.
Stewart, C. W.
Strug, E. J.
Switzer, V. J.
Tarbell, L. L., Jr.
Uhthoff, D. R.
Verhage, P. A.
Webb, B. L.
Wilcken, C. L.
Wilson, J. C.
Zory, P. B.

ASSOCIATES

Anker, R. A.
Balko, K. H.
Bell, A. A.
Bill, R. A.
Cadorine, A. R.
Carter, E. J., Jr.
Chorpita, F. M.
Cooper, W. P.
Davis, R. C.
Dempster, H. V., Jr.
Drennan, J. P.
DuRose, S. C.
Gill, J. F.
Golz, J. F.
Head, F. F.
Hearn, V. W.
Hoffmann, D. E.
Jensen, J. P.
Khury, C. K.
Klingman, G. C.
Krause, G. A.
Lindquist, R. J., Jr.
Margolis, D. R.
McClenahan, C. L.
Millman, N. L.
Moore, P. S.
Napierski, J. D.
Neidermyer, J. R.
Plunkett, J. A.
Price, E. E.
Retterath, R. C.
Rogers, D. J.
Ross, J. P.
Sandler, R. M.
Schaeffer, B. G.
Schneiker, H. C.
Singer, P. E.
Stephenson, E. A.
Swaziek, R. R.
Thompson, E. G.
Torgrimson, D. A.
Trees, J. S.
VanCleave, M. E.
Welch, J. P.
Young, D. M.
Young, E. W.
Young, R. G.

GUESTS

Albaum, M.
*Babb, J. A.
*Bell, A. M.
*Bell, G. W.
*Chang, C. I.
Cullen, G. F.
Defliese, P. L.
Dineen, R.
*Dunn, R. P.
*Eddins, J. M.
*Galban, L. S., Jr.
*Griffith, R. W.
Hansen, J. E.
*Hatfield, B. D.
Hoyt, F. A.
Jewell, R. L., Jr.
*Katzman, I.
*Kedrow, W. M.
Kimball, S. L.
Knox, F.
Peterson, R. P.
Pugh, W. J., Jr.
Purcell, J. C.
*Smith, D. A.
Sturgeon, P. K.
*Taylor, J. R.
Watson, C. B. H.
*White, B. R.
White, J.

*Invitational Program

Respectfully submitted,

RONALD L. BORNHUETTER
Secretary-Treasurer

Volume LIX, Part II No. 112

PROCEEDINGS

November 8, 9, 10, 1972

KNOW THYSELF, ACTUARY

PRESIDENTIAL ADDRESS BY LeROY J. SIMON

The title of this year's presidential address will be the reference point for each of the subjects developed. I will discuss how we can better know ourselves by knowing the intruders into our profession, by recognizing other educational areas which we must develop, by viewing other spheres of insurance operations which are in great need of actuarial assistance, by questioning whether we really understand the effects of our pricing activities and by challenging all those who profess to call themselves colleagues: "Know thyself, actuary".

It has been my privilege to work for a number of years on behalf of the Casualty Actuarial Society in positions on committees and on the Board of Directors. One of the more enjoyable parts of casualty actuarial work is that one has an opportunity to express oneself in a field which has ahead of it virtually unlimited horizons of theoretical and practical development. In working on some of the fascinating actuarial problems which have been encountered over the years, we have each had the opportunity to influence the course of actuarial developments and the direction in which our business moves. Over the years I have become extremely proud of our profession and very pleased to be a part of it. Thus, I firmly plant my feet and say, in a paraphrase, "I am a casualty actuary". Out of this pride and devotion grows a sense of concern and uneasiness caused by some developments observed over the recent past. Let me explain.

Over the course of the last few years, your Board of Directors and officers have been increasing the degree of coordination and cooperation with the other actuarial societies in the United States. Society members were active in the formation of the American Academy of Actuaries, and we continue to have active participation by many of our members. We have established a number of joint activities in the field of risk theory, education and research, and we have also engaged in the exchange of liaison people with committees of other actuarial societies. All of these activities are progressive and positive in and of themselves. However, if these activities lead some actuaries to feel that if they are qualified and experienced in one actuarial specialty, they become casualty actuaries automatically, they have erred—and perhaps we have erred by allowing such a presumption to arise. This most often comes to the surface when actuaries are employed as special consultants for particular jobs. It also surfaces when actuaries either as consultants or company employees, are asked to testify before insurance departments or special committees of legislatures and are expected to be experts in the field to which they are testifying. Is it possible for a person who has worked in the pension field to evaluate a proposed automobile no-fault statute? Conversely, is it possible for an actuary experienced in pricing automobile liability insurance to evaluate a pension plan? Can a group insurance actuary establish liability for workmen's compensation insurance? Is it proper for a retrospective rating expert to establish dividend plans for group life? We can list 20 subjects in which actuaries from different disciplines are acknowledged experts and ask ourselves whether the permutations of these 20 items taken two at a time are all acceptable. I think we would readily admit that expert qualifications in one field does not mean expert qualification in another field. But, do our actuarial colleagues who are not qualified casualty actuaries really believe this and conduct themselves in this way? Recent evidence suggests not.

All actuaries will recognize that there are certain generalized actuarial principles which can be applied across all lines. We will certainly recognize that basic mathematical, statistical and probability concepts are quite universally applicable. Concepts of present values, pure loss cost and expense loadings are certainly basic actuarial tools that we all use. However, I object and I object very strongly to a person who has qualified himself through experience to apply these tools in an informed and intelligent way to one particular area, feeling that this ability automatically qualifies him as an expert in the application of these tools to another area of insurance. An actuary who understands how to make life insurance rates can certainly apply the basic principles and concepts more quickly and will be able to acquire the knowledge and experience helpful in making fire insurance rates in a shorter period of time than one without any rate-making background. However, we should seriously question the professional status of anyone claiming to be an expert in a field in which he has not served his apprenticeship and the test of time.

Fundamentally, we believe that poor advice will eventually lead to ruin or at least the malfeasor will be found out. But, will he really? Times change and conditions change, and it is difficult to trace back exactly what advice led to what consequences. If we sincerely believe that the recommendations and advice which we give are of serious impact to the recipients, then we must be prepared to face up to the importance of the actions which people take as the result of such advice. It may not be sufficient to look back and evaluate what the patient died of when the death was brought about by the overzealous advice of an unqualified individual.

How do you obtain experience we might be asked if you can't become an expert until you have had it and you can't get the experience without being an expert. The answer, of course, is obvious in that one must serve an appropriate apprenticeship whether he be 20 years old and starting into the business or whether he be 50 years old and deciding that he would like to expand his activities into an additional sphere of operation. It does not suffice to say I am a member of one industry actuarial association and, therefore, I am qualified to handle any industry actuarial problem. When the come-latelies and the diletantes have served that apprenticeship and been seasoned in the crucible of experience, then and only then, will they be recognized as qualified experts and not as mere opportunists.

Let me cite another area in which this erosion of our profession might be taking place. We can be justly proud of the fact that the Casualty Actuarial Society has a continuous history as a single organization dating back to 1914. We have never been involved in mergers with other semi- or quasi-actuarial bodies, but have maintained the standards of entrance to our profession within our own control over all the years. Thus, no one has entered our society through a club or similar group which was then merged in and its members given full status. Yet, it does seem confusing and perhaps somewhat misleading when one reads about a society of actuaries which is not a society of all actuaries, but yet it does not properly delineate its sphere of operation. Although it might have been thought of as something that happened long ago which would not occur today, consider the newsletter of recent vintage which purports to speak for the actuary, but yet represents but one segment of the field. Such a situation certainly must be quite misleading to some people—particularly those who are not actuaries. I call on all actuaries to be sufficiently precise in their terminology so as to avoid misleading others—or themselves.

Other examples could also be cited, but the point has been sufficiently made, I believe. Where does this lead us? Can casualty actuaries feel that the spirit of cooperation and joint effort so actively engaged in over the past few years is soundly based and that the spheres of operation in which we each operate are mutually respected by all actuaries? As of this date, I fear the answer is less than satisfactory. Actuaries in other disciplines that I know personally are quite the opposite of the types who cause or contribute to the difficulties referred to. Therefore, I would like to see a wider range of contacts over a broad spectrum of subjects with particular emphasis on joint efforts aimed at increasing the understanding between the various actuarial specialties. Next, we must have a stern approach to those who do not recognize restraint in their interpretation of who is a qualifed actuary in a particular field. Finally, we must take steps to make it clear that the American Academy of Actuaries is the only actuarial body that represents all actuarial specialties. Let us hopefully see this situation turn around—and do so promptly so that the fine early stages of cooperation and coordination started by my predecessors and developed during my term as President can proceed further. But, let us also recognize that if we cannot reach a proper working relationship with other branches of the total profession, the future is not of grave concern to us—the next 58 years look every bit as promising and challenging as the last. We are growing rapidly, the demand for services is very high and, after all, this is where the action is.

If we are to better know ourselves as casualty actuaries and the special type of actuarial work which we do, we will have to be sure that our educational process is aimed at covering the proper areas of education. One particular area where we need considerably more effort in the educational process is economics. Many things have been done in the past in this field which affect insurance, and changes are being made today which have an influence on our actuarial profession. We should study this field and know it well. Today's actuary should be able to discuss the economic concepts of price elasticity, cost-push inflation and the value added tax as readily as he can discuss pure premium concepts, the negative binominal distribution and stochastic financial models. These economic concepts have an impact on our pricing that is just as great as the actuarial concepts which we know so well. We are going to have to understand economics and its effect on our business if we are to properly carry out our jobs as actuaries. Today's Syllabus of Examinations is a step in the right direction, but we must go further. And, to the actuary who qualified in the past I must ask, "Are we, today's actuaries, prepared to use the tools of today's profession"? This is a dynamic profession, and continuing education must rank high in our priorities.

Just exactly what are our functions as actuaries? We will readily admit that we are the pricing experts and recognize the financial evaluation aspects of our work. However, actuaries are now finding rich new fields of endeavor for which their actuarial training has made them particularly valuable. Actuaries are increasingly coordinating the responsibilities for both actuarial and underwriting operations within their companies. What could be more natural than to view the design of the product, its pricing and the selection of insureds as a comprehensive whole. This will be an uneasy position for many actuaries who were brought up in an era where actuaries were strictly staff people. The line responsibility of having to be fully accountable for the success of their operations is something which has to be learned by doing rather than learned by the educational process. It will be a chilling experience for some to have bottom line profit responsibility for their actions. The actuary of the future who is involved in pricing but has not spent a significant segment of his early years as an underwriter in his company will be hard put to meet the full responsibilities of his future development. The man who best knows how to price a product is going to be the man who has actually applied those prices to individual risk situations as an underwriter. Conversely, the underwriter who has had actuarial training will be better able to operate by viewing the prospective aspects of matching selection and price. The underwriter who can only look backwards to see what havoc the pricing and selection process has wrought upon his book of business and his company will be as extinct as the running boards, coal burning stoves and Hollerith cards of a bygone era.

Another area that must receive additional attention from actuaries is that of taxation. Our *Proceedings* have recently contained writings on this subject and it must be pursued further. The long range planning and the complex, but yet mathematical, inter-relationships among the taxation variables are areas in which persons who have had actuarial training will have a distinct advantage.

The involvement of actuaries in investment operations also seems to be a very natural outgrowth of current developments. As we increasingly weave the functions of investment into our insurance and reinsurance pricing, the actuary will, of necessity, have to become more deeply involved in the investment function. Certainly his analytical training will stand him in good stead, but more importantly, the aspects of long range planning and short range flexibility make the field particularly appealing to the casualty actuary.

Finally, I would like to draw attention to an area of insurance operations which has received relatively little attention from actuaries if one measures the quantity of work effort expended in proportion to the magnitude of the problems which exist. This is a field of reinsurance where I am convinced that an extraordinary opportunity for actuaries exists on both sides of the reinsurance transaction. The normal reaction of the veteran reinsurance man to such a statement is to throw up his hands in horror because he thinks that actuaries can operate only in areas having massive sets of statistics on homogeneous risks. We must be quick to recognize that the pricing of many reinsurance agreements is much more of an art than an actuarial science. One of the important contributions that the actuary can make to the reinsurance field is the maintenance of logical consistency among the various alternatives that may be considered at different stages of the negotiation process. It is quite common for modifications in terms to be discussed such as altering the retention, changing the thickness of the layer and subdividing the layer into two or more strata. Although it may not be possible to claim that the various alternatives are actuarially equivalent in the strict sense, the actuary can help assure that they are at least logically consistent with each other. Other positive contributions from the actuary in the reinsurance field would include pricing estimates themselves, determining incurred but not reported and developmental reserves, assessing inflationary impact and evaluating financial aspects of his own company or of prospective reinsureds. A great deal of sound thinking, together with innovation and the open acceptance of new ideas, is required in the reinsurance field. An actuary's training is very helpful in developing the type of individual needed.

As President of the Society principally devoted to pricing casualty and property insurance products, I must comment on the resounding success we have produced as evidenced by the profit margins actually realized during 1971 and reported this far in 1972. We should all be rather proud of ourselves, I suppose, but yet I can't help but ask you, weren't you really a little surprised? Yes, I think everyone in the industry was not just a little surprised; they were absolutely amazed to find rather suddenly that we were making money in the property and casualty insurance business after a decade of mountainous underwriting losses requiring carefully detailed explanations or, lacking those, just bare excuses. And what kinds of answers did we have for those who said "What caused the sudden change"? We didn't really have very good answers at all, did we? We fumbled around and tried to see of we could conjure up a set of circumstances that would lead to the facts which we were observing. Perhaps we would have done as well if we had dug out the reports to our stockholders or policyholders over the recent past and simply prepared a list of logical conjugates for the explanations and excuses advanced therein. If our ratemaking process is such a lumbering giant that we don't know what really causes a change when it occurs, we must ask ourselves whether we truly understand our jobs as actuaries.

Now, after 20 years of failing to meet our avowed profit goals, how do we react after perhaps 20 months of success? We are doing what the industry has done before, and I fear will repeat again, and that is to dance in the euphoria of success and dance and dance in ever increasing circles, trying to gobble up more and more premium volume until we shall. again, drive the prices down and the cost of operation up to the point where we can go back to the good old days—the days of mountainous underwriting losses, carefully detailed explanations and bare excuses. Do you know your job as an actuary in times of profitable underwriting results as well as you learned it in times of difficulty? A rate reduction aimed at producing an additional profit through increased sales is obviously a wise move—just as wise as a rate increase designed to produce an additional profit despite the risk of reduced sales. However, how many of us actually follow through and try to discover if insurance pricing, particularly in the personal lines area, really has the high price elasticity which the sales representatives claim. Maybe it is necessary at times to refuse to fall into this trap and simply hold the line on the number of units sold or renewed until a more propitious time to adjust rates. Does a rate cut really produce addition business or help us hold the business we have? Will the lowest rate in a territory produce significantly more business than a rate slightly above the general market. Hypotheses such as these do not have to go untested. We have enormously helpful tools in our computer machinery which would allow actual tests to be made. For example, the excellent paper by P. D.

Johnson and G. B. Hey entitled "Statistical Review of a Motor Insurance Portfolio" in the May, 1972 ASTIN Bulletin gives the details of a system in actual operation in Great Britain which is capable of putting these theories under the bright spotlight of detailed testing.

To further this effort of trying to better understand ourselves, consider the wide range of estimates in the cost of no-fault bills. Three different organizations come up with three different estimates of the impact of a given legislative proposal. Why can't the actuaries agree? First, let's make sure that the basic determinations are made by qualified casualty actuaries. Then, let's make sure that the estimates are in fact actuarial estimates and not just wishful flights of fancy taken by someone who has used the actuary's basic work but has modified it to suit his own purpose. Next, let's make sure the actuary who makes these studies is willing to stand up and be counted when the time comes. Basic professional integrity should require the actuary to be open to questioning and discussion of his methods if his results are used in an advocacy proceeding. I know of nothing that will improve professionalism any more than making individuals accountable for the statements attributed to them. Finally, let's make clear that while the future is more accurately predicted by the qualified actuary, point estimates are subject to uncertainty. Hence, even if two actuaries used the same underlying data, they would, undoubtedly, treat the judgmental areas of uncertainty in a different manner. If you think this range of uncertainty is large in no-fault, have you seen some of the recent quotes being put out on commercial risks? Let's recognize this uncertainty and stop trying to sell a specific cost saving figure as a benefit under no-fault. Stop legislating rate levels. Let the normal forces of competition and the profit motive as we know it in American free enterprise force the price and product benefits to their appropriate level. Let the regulator be ever wary of self-destructive rate cutting, but let competition provide the benefits in price and product that the public deserves.

One of the criticisms that we like to make of ourselves and which others freely make of us is that we must be practical men. A "Practical man" was once defined as a person who practiced the errors of his forefathers. Let's not be seduced by the pleas of those who say that we must be able to reduce the complex concepts with which we work to a base level and make sure laymen can understand them. Perhaps we try to be too practical and to explain too clearly ideas and concepts which really cannot be put in a language within the understanding of everyone. There are times when we must simply say that certain matters are technical in nature and if an employer, regulator or legislator wants to understand them, he must make the investment in time and training to learn how to do so. Everyone cannot be an actuary and everyone cannot understand all the actuarial concepts. Perhaps, in our attempt to simplify automobile insurance ratemaking and to try to explain it to everyone, we are simply making it appear to be much simpler than it really is, and we are thus deceiving others into thinking that once they have grasped the practical explanation, they understand the entire field. I fear we have fallen into a trap best illustrated by the immortal words of Pogo—"We have met the enemy, and they are us."

We have now covered some of my views on who is a qualified casualty actuary, how we must better understand related fields, what additional facets of the insurance operations are benefiting from actuarial advice and direction, and certain areas in which we must ask how well we really understand ourselves. These are the kinds of gauntlets we actuaries like to have thrown down. These are the kinds of challenges to business and to professionalism that we enjoy meeting. In closing, I would like to give you some words of cheer as you are confronted with these and the many other problems which we have come to expect actuaries to tackle and solve. It was Casey Stengel who said "You know, they say it can't be done. But, sometimes that doesn't always work."

ALLOCATED LOSS EXPENSE RESERVES

ALLIE V. RESONY

Little, if any, reference to the problems and methodology of reserving for loss expense has appeared in the *Proceedings* of the Casualty Actuarial Society. In addition to initiating the publication of some information on the subject, this paper offers a device for the development of formula type allocated loss expense reserves for use in ratemaking. Directed primarily to those casualty lines of business which traditionally require the submission of allocated loss expense with loss in the ratemaking procedure, the theory presented is equally applicable to the determination of statement allocated loss expense reserves for these, as well as other, casualty lines of business where data is of sufficient volume to be credible.

Examination of accident year loss and allocated loss expense development in ratemaking statistics indicates that a significant contribution to the upward development appearing in recent years has been due to inadequate allocated loss expense reserves, which in most cases are probably the sum of the individual estimates of claim examiners.

This analysis not only offers a procedure for establishing more adequate allocated loss expense reserves on a formula basis for ratemaking, but also relieves claim examiners of the task of making individual estimates[1]. Aside from the savings in time and expense, claim men will admit that this is a most difficult area to make accurate estimates, especially at the time of the initial claim reserve creation.

The procedure uses past experience to determine a set of factors which, when applied to the individually estimated loss reserve, produces the corresponding allocated loss expense reserve. In addition, a similar single factor is derived for application to the Incurred But Not Reported (IBNR) loss reserve to provide the IBNR allocated loss expense reserve for statement purposes.

In order to initiate the analysis, the term "loss outstanding" (O/S) is defined as the individually estimated (known) loss reserves, and a "created year" is defined as the year in which the initial individual claim reserve

[1] It is still necessary to make individual estimates of allocated loss expense reserves on certain risks where formula type reserves could be criticized on an individual claim basis; i.e., Retrospective Rating where allocated loss expense incurred on a claim-by-claim basis is included in the rating and other large risks where individual claim experience is subject to inspection by the insured. These estimates are ignored in the compilation of the ratemaking data.

is recorded. "As of" dates are reference points in time with respect to the created year. A created year as of 12 months is as of its 12-31 date; i.e., created year 1970 as of 12 months is as of 12-31-70. Similarly, a created year as of 24 months is as of the 12-31 date of the following year; i.e., created year 1970 as of 24 months is as of 12-31-71, and so on for as of 36, 48, and 60 months. It should be noted that once a created year reaches its 12 months date, a certain block of claims is identified, and future as of dates have reference to that particular block of claims only.

Observation of past experience of the amount of loss still O/S through annual points in time for each of a series of created years is used to predict the future progression in time of the amount of loss O/S, for each created year of the current total loss O/S.

This prediction is shown below in Exhibit I, as it might appear for Automobile Bodily Injury at 12-31-71, for example. Past experience of the amount of loss still O/S by created year at the indicated points in time is shown to the left of the diagonal line. Ratios of the amounts of loss O/S are calculated through the progression in time, and an average of the latest three such ratios is used to predict the future progression in time of the

EXHIBIT I
AUTOMOBILE BODILY INJURY
(000's OMITTED)

	PROGRESSION OF LOSS O/S BY CREATED YEAR						
As Of	1965	1966	1967	1968	1969	1970	1971
12 Months	$50,000	$58,000	$66,700	$76,000	$70,000	$76,000	$80,000
Ratio 24/12	.630	.609	.610	.609	.611	.633	(.618)
24 Months	31,500	35,300	40,700	46,300	42,800	48,100	(49,440)
Ratio 36/24	.619	.575	.590	.633	.605	(.609)	(.609)
36 Months	19,500	20,300	24,000	29,300	25,900	(29,293)	(30,109)
Ratio 48/36	.559	.542	.567	.597	(.569)	(.569)	(.569)
48 Months	10,900	11,000	13,600	17,500	(14,737)	(16,668)	(17,132)
Ratio 60/48	.491	.491	.515	(.499)	(.499)	(.499)	(.499)
60 Months	5,350	5,400	7,000	(8,733)	(7,354)	(8,317)	(8,549)

Total Loss O/S at 12-31-71
$184,500*

*Includes $6,000 loss O/S for created years 1966 and prior current loss O/S for each created year[2]. This is shown to the right of the diagonal line and appears in parentheses.

The difference in the amount of loss O/S between two successive as of dates is considered as the amount of loss O/S disposed of in that particular 12-month period[3]. Thus, referring to Exhibit I, for created year 1970 the amount of loss O/S disposed of between as of dates 12 and 24 months is $76,000—$48,100 or $27,900. Similarly, the prediction of the future amount of loss O/S to be disposed of between as of dates 12 and 24 months for created year 1971 is $80,000—$49,440 or $30,560, etc.

Utilizing this concept, the future disposition of the loss O/S at 12-31-71 may be arranged in the following format:

EXHIBIT II
AUTOMOBILE BODILY INJURY
(000's OMITTED)

FUTURE DISPOSITION OF LOSS O/S AT 12-31-71

Disposal Interval	Created Year					
	67 & Prior	68	69	70	71	Total
12 to 24 Months	$ xx	$ xx	$ xx	$ xx	$30,560	$ 30,560
24 to 36 Months	xx	xx	xx	18,807	19,331	38,138
36 to 48 Months	xx	xx	11,163	12,625	12,977	36,765
48 to 60 Months	xx	8,767	7,383	8,351	8,583	33,084
Over 60 Months	13,000	8,733	7,354	8,317	8,549	45,953
Total	$13,000	$17,500	$25,900	$48,100	$80,000	$184,500

The disposal intervals shown are a measure of the age (since initial claim reserve creation) of the particular claims at disposal. It might be expected that the amount of allocated loss expense paid per unit of loss O/S disposed would vary with the age at disposal.

In order to determine these relationships, past experience in the form of the allocated loss expense paid by created year is obtained for each of the last three calendar years[4]. By rearranging the amounts of loss O/S

[2]Throughout this analysis, simple averages are used so as not co complicate the text. More sophisticated trending or weighted average procedures may be substituted.

[3]Actually, the amount of loss O/S taken down at settlement or cancellation and the net of changes in estimate.

[4]These relationships do not actually represent the cost in allocated loss expense per unit of loss O/S disposed, except as "disposed" is defined in footnote #3. Since a portion of the allocated expense paid is on loss still O/S at the end of the calendar period, the "F" ratios, as subsequently derived, are only an artificial device in the mechanics of the formula.

disposed of from past experience (left of diagonal line in Exhibit I) and relating the corresponding allocated loss expense paid, a ratio of allocated loss expense paid to the amount of loss O/S disposed, for each of the disposal intervals, can be developed as follows:

EXHIBIT III
AUTOMOBILE BODILY INJURY
(000'S OMITTED)

Calendar Year 1969

Created Year	Disposal Interval	Loss O/S Disposed	Allocated Exp. Paid	Ratio
68	12-24 Mo.	$29,700	$2,000	.0673
67	24-36 Mo.	16,700	2,300	.1377
66	36-48 Mo.	9,300	1,400	.1505
65	48-60 Mo.	5,550	990	.1784
64 & Pr.	Over 60 Mo.	5,400*	1,100	.2037

Calendar Year 1970

Created Year	Disposal Interval	Loss O/S Disposed	Allocated Exp. Paid	Ratio
69	12-24 Mo.	$27,200	$1,800	.0662
68	24-36 Mo.	17,000	2,150	.1265
67	36-48 Mo.	10,400	1,600	.1538
66	48-60 Mo.	5,600	950	.1696
65 & Pr.	Over 60 Mo.	5,050*	1,000	.1980

Calendar Year 1971

Created Year	Disposal Interval	Loss O/S Disposed	Allocated Exp. Paid	Ratio
70	12-24 Mo.	$27,900	$1,800	.0645
69	24-36 Mo.	16,900	2,050	.1213
68	36-48 Mo.	11,800	1,800	.1525
67	48-60 Mo.	6,600	1,150	.1742
66 & Pr.	Over 60 Mo.	5,700*	1,200	.2105

*Not obtainable from Exhibit I, but derived from extension of same data.

Designating an average of the above ratios as "F", the following ratios of allocated loss expense paid to loss O/S disposed are determined for each disposal interval:

EXHIBIT IV
AUTOMOBILE BODILY INJURY

Disposal Interval	F
12 to 24 Months	.0660
24 to 36 Months	.1285
36 to 48 Months	.1523
48 to 60 Months	.1741
Over 60 Months	.2041

Recalling Exhibit II, which predicts the future disposition of the current (12-31-71) loss O/S, and applying the appropriate F ratios, the prediction of the future allocated loss expense to be paid (in effect, the allocated loss expense O/S) for each development year is made as follows:

EXHIBIT V
AUTOMOBILE BODILY INJURY
(000's OMITTED)

		Future Disposition of Loss O/S at 12-31-71					
		Created Year					
Disposal Interval	F	67 & Prior	68	69	70	71	All
12 to 24 Months	.0660	$ xx	$ xx	$ xx	$ xx	$30,560	$ 30,560
24 to 36 Months	.1285	xx	xx	xx	18,807	19,331	38,138
36 to 48 Months	.1523	xx	xx	11,163	12,625	12,977	36,765
48 to 60 Months	.1741	xx	8,767	7,383	8,351	8,583	33,084
Over 60 Months	.2041	13,000	8,733	7,354	8,317	8,549	45,953
Total Loss O/S		$13,000	$17,500	$25,900	$48,100	$80,000	$184,500
Allocated Loss Expense O/S		2,653	3,309	4,486	7,491	9,717	27,656
Allocated Loss Expense O/S ÷ Loss O/S		.2041	.1891	.1732	.1557	.1215	.1499

The ratios of allocated loss expense O/S to loss O/S so determined for each created year are designated as "Allocated Loss Expense Reserve Factors." In the compilation of the ratemaking data, each claim's loss O/S is examined with respect to its created year. Application of the proper Allocated Loss Expense Reserve Factor to the loss O/S provides the corresponding allocated loss expense O/S. In the example shown, the procedure relates to the ratemaking data reported at 12-31-71.

Since ratemaking data is normally required at the end of each quarter, it becomes necessary to determine the factors at these interim dates.

Until such time as an additional calendar year's experience (1972 in this example) becomes available, it is reasonable to assume that the factors as derived above at 12-31-71 would be repeated at 12-31-72, offset one created year. That is, the factor for created year 1972 at 12-31-72 would be the same as that of created year 1971 at 12-31-71; the factor for created year 1971 at 12-31-72 would be the same as that of created year 1970 at 12-31-71, etc.

Making this assumption, and using straight line interpolation for the quarterly points (with the exception of the 1972 created year), the factors would appear as follows:

EXHIBIT VI
AUTOMOBILE BODILY INJURY
ALLOCATED LOSS EXPENSE RESERVE FACTOR

	Created Year						
At	67 & Pr.	68	69	70	71	72	All
12-31-71	.2041	.1891	.1732	.1557	.1215	xx	.1499
3-31-72	.2041	.1929	.1772	.1601	.1301	.1144	.1499
6-30-72	.2041	.1966	.1811	.1644	.1386	.1164	.1499
9-30-72	.2041	.2004	.1851	.1688	.1472	.1188	.1499
12-31-72	.2041	.2041	.1891	.1732	.1557	.1215	.1499

The quarterly factors shown for created year 1972 can be determined by awaiting the actual distribution of the loss O/S by created year in 1972, and calculating the factors based on the assumption that the factor for all created years remains constant throughout the year. Alternatively, these factors can be determined in advance in the same manner by observations of past distribution of the loss O/S by created year at the quarterly points.

As indicated in the opening paragraph of this paper, the theory presented in the preceding analysis may also be used in the determination of statement reserves for the casualty lines of business where sufficient data is available[5]. Thus, in the example as shown in Exhibit V, the application of the All created years' factor of .1499 to the total loss O/S at 12-31-71 produces the corresponding total allocated loss expense O/S of $27,656.

Determination of a similar factor applicable to IBNR loss reserves requires consideration of the manner in which IBNR loss development emerges.

From past experience, observations of IBNR loss payments in the 12-month period immediately following the reserve date, i.e., IBNR loss payments in 1971 on claims incurred but not reported at 12-31-70, may be related to the corresponding IBNR allocated loss expense payments.

Continuing with the example established above, these relationships might appear for the latest three years for Automobile Bodily Injury as follows:

EXHIBIT VII
AUTOMOBILE BODILY INJURY
(000's Omitted)

IBNR With Respect To	Subsequent 12 Months' Payments		
	Loss	Allocated Loss Exp.	Ratio
12-31-68	$5,500	$205	.0373
12-31-69	5,200	200	.0385
12-31-70	5,700	210	.0368

[5] It may be necessary to treat certain pools, associations and large risks individually.

It is reasonable to assume that an average of these ratios, when applied to that portion of the current 12-31-71 IBNR loss reserve which can be assigned to IBNR paid losses in the immediately following 12 months, produces the corresponding allocated loss expense reserve. For this example, the average is .0375.

The remaining portion of the current IBNR loss reserve is assigned to the amount of loss O/S 12 months from the reserve date on IBNR claim reserve creations during that period, plus the amount of IBNR still unreported at that time.

As indicated, all of this IBNR loss O/S arises in the created year following the reserve date. For this example, it is created year 1972 which will be as of 12 months at that time. It is noted from Exhibit VI, after making the indicated assumption, that the Allocated Loss Expense Reserve Factor for this created year as of 12 months is .1215.

Making the assumption that the still unreported IBNR loss 12 months after the reserve date bears the same factor[6], it is now necessary to determine from past experience the proportional part of the total ultimate IBNR loss development which arises from payments during the first 12 months subsequent to the reserve date. Proper weighting, by proportional parts, of the two factors (.0375 and .1215) as determined above, then produces a single factor for application to the current IBNR loss reserve to obtain the corresponding allocated loss expense reserve.

Continuing the example, suppose it is determined from past experience that the first 12 months' IBNR loss payments subsequent to the reserve date make up 30% of the ultimate IBNR loss development. Then, making use of the proportional parts, as indicated above, the single factor of .0963 (.30 × .0375 + .70 × .1215) is produced for application to the current IBNR loss reserve to determine the IBNR allocated loss expense reserve.

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[6] The inclusion of the still unreported with the IBNR loss O/S deserves comment. The continuity of the analysis could be maintained by determining the same relationships and proportional parts for IBNR loss and allocated loss expense arising in subsequent 12-month periods; i.e., 12 to 24 months, 24 to 36 months, etc., after the reserve date. Experience indicates that the relationship of allocated loss expense paid to loss paid on claims arising in these periods approximates the factor (.1215 in the example) applicable to the IBNR loss O/S at 12 months. Since the IBNR loss O/S at the end of each of these periods (arising during the periods) would bear the same factor, it is felt that it is permissible to include the still unreported with the IBNR loss O/S at the end of the first 12 months, as indicated in the text.

Although the example presented in the text was made relatively stable for ease of illustration, annual redetermination of the elements of the analysis may reveal trends or dislocations which reflect alteration of practices affecting the continuity of the basic data. For instance, although the formula will produce proper allocated loss expense reserves for a line of business that has been consistently overreserved with respect to loss, a sharp correction of the problem would lead to a surge in the amount of loss O/S disposed in the year of correction. This, in turn, would distort the pattern used to predict future disposal of the current loss O/S and also the most current F ratios. At this point, the success of the procedure would depend upon a knowledgeable adjustment of the past experience to the corrected loss O/S level. This is an extreme example but serves to illustrate the fact that the procedure requires continual monitoring.

In this connection, it should be emphasized that any mathematical system used in the determination of reserves based upon past experience is always subject to variations in both internal and external influences, and should, in the last analysis, be dependent upon judgment based on the currently available knowledge of these factors.

Annual determinations will also allow measurement of the adequacy of previous year-end allocated loss expense reserves. For instance, Exhibit V shows the current still O/S allocated loss expense with respect to 12-31-68 as $3,309 plus $2,653 or $5,962. This O/S, coupled with the proper allocated loss expense paid of $17,640 from Exhibit III, is a current measure of the adequacy of the 12-31-68 reserve.

Although the theories presented are illustrated for Automobile Bodily Injury, they are, as previously indicated, equally applicable to other lines of casualty insurance and, in addition, to sublines and geographical divisions where data is available and in sufficient volume to be credible. The indicated subdivisions are particularly desirable in the ratemaking application.

AN ACTUARIAL NOTE ON EXPERIENCE RATING NUCLEAR PROPERTY INSURANCE

RICHARD D. McCLURE

By the end of 1971, the insurance pools writing nuclear property insurance (Nuclear Energy Property Insurance Association and Mutual Atomic Energy Reinsurance Pool) had since inception in 1959 earned about $47 million in premiums and had incurred loss and loss expense of nearly $15 million. After an accounting for all expenses, it was clear that a substantial profit had been made. However, it is recognized that this is a highly catastrophic coverage. The pools are now writing many policies with combined limits of $100 million. A nuclear accident might wipe out, in literally a few seconds, more than twice all premiums earned during the thirteen-year life of the pools. Since the level of earned premiums on all risks at the end of 1971 was only $11.5 million, the insurers were opposed to any reduction in rates, in spite of the good record.

Nevertheless, the insurers felt that an in-depth study of rating procedures was in order, and late in 1971 asked the Nuclear Insurance Rating Bureau to make one. NIRB appointed a subcommittee of five actuaries to undertake such a study and to make whatever recommendations it felt advisable. The subcommittee was also given an immediate assignment of developing a plan to make nuclear property rates more responsive to experience.

It was an interesting challenge. How do you recognize in a rating formula the real possibility of very severe losses? No such losses have yet occurred (the largest to date was one for about $3.5 million) but the potential is ever-present. A complete melt-down of the core of a large power reactor would probably cost the pools $30 million, and this assumes that no radioactivity escapes the reactor vessel. Contamination of the primary or secondary loops, or of the turbine equipment, would quickly run the loss very much higher.

The subcommittee, after consultations with engineers and underwriters, approached the problem by dividing the loss portion of the premium dollar into two equal parts: normal loss and excess loss. The subcommittee also assigned to the excess loss portion in the rating formula limited credibility, until the volume of premium has grown much greater.

Normal losses were arbitrarily defined as the first $5 million part of every loss, and excess losses as those parts of any losses in excess of $5 million. The split of total expected losses between normal losses and excess losses was necessarily arrived at on a judgment basis. There is no body of large nuclear property losses. Distributions of large fire losses were examined, and according to these indications, 50% is too much to assign to losses between $5 million and $100 million. While the standard should be considerably less, the engineers and the underwriters felt that the peril of nuclear contamination greatly enhances the probability of catastrophe loss and that a substantial part of the loss money must be set aside for such an event.

Based on these concepts, a complete rating guide was developed and submitted to the Nuclear Insurance Rating Bureau, which adopted it to apply to policies effective on or after March 1, 1972.

Industry Experience Rating Guide

The experience rating guide is traditional in form, in that it seeks to compare expected losses with actual losses, modified by various credibility factors, and thus to establish credits or debits prospectively. It is unique in the following respects:

1. It applies equally to all domestic policies issued by the pools, regardless of the loss history of any particular risk, and thus provides an element of stability and uniformity to insurers and insureds alike. To proceed otherwise would either mean very wide swings in the rates for particular insureds or a relatively insensitive formula.

2. The experience rating period is 10 years for normal losses and 20 years for excess losses. Normal and excess losses are defined as above. The bulk of the total losses is expected to be normal losses, and a 10-year period is felt to be sufficient to give them full credibility. The 20-year period for excess losses, for which there is precedent in the making of windstorm rates in many states, is needed to provide stability and continuity in the nuclear structure. Until such time as 20 years of experience is available, the total experience since inception will be used. Limited credibility is given to excess losses.

Rating Formula

The formula for the rate modification is:

$$\frac{A_n + A_e Z + E_e (1.000 - Z)}{E_t}$$

Where A_n is actual Normal Limits loss ratio,
A_e is actual Excess Limits loss ratio
Z is the credibility factor
E_e is expected Excess Limits loss ratio, and
E_t is total expected loss ratio

"Loss" includes loss expense.
The expected loss ratio is unity minus the expense ratio.

The Normal Limits loss ratio is the normal losses incurred in the latest 10 years divided by the total premiums earned in the 10 years. The Excess Limits loss ratio is the excess losses incurred in the latest 20 years divided by the total premiums earned in the 20 years. All earned premiums are modified to eliminate the effect of the experience rating modifications produced by this guide.

The experience periods end November 30 of the year prior to the rating year for which experience rates are to be calculated. This coincides with the fiscal year of the pools. The rating year commences the subsequent March 1, an arrangement which permits sufficient time to collect and consolidate the experience and to make the necessary rating calculations. The experience rating modification applies to all policies effective on and after that date, for a period of one year, and policies may not be cancelled and rewritten to take advantage of, or to avoid, a change in such factor. All nuclear property policies are written for a period of one year.

Expenses

The expenses since inception were carefully reviewed and current expense levels were determined. These include commissions, pool administration, inspections (both fire and boiler and machinery), taxes, company overhead and a loading for profit and contingencies.

The total expense factor for the first year of the rating guide is .336. Thus, the total expected loss ratio is .664, and is divided equally between expected normal and expected excess loss ratios, or .332 each.

The expense factors are to be reviewed periodically and updated to reflect actual current costs.

Credibility

The Normal Limits loss ratio receives 100% credibility. The Excess Limits loss ratio receives up to 50% credibility, depending on the total earned premium during the 20 year period, as follows:

Total Earned Premium in Millions of Dollars	Credibility Factor
0-12	.00
13-40	.05
41-71	.10
72-106	.15
107-145	.20
146-189	.25
190-240	.30
241-300	.35
301-369	.40
370-452	.45
453-552	.50
over 552	to be determined

In this context also, the earned premium is adjusted to eliminate the effect of experience rating factors of this guide.

The table is based on the formula:

$$\frac{P}{P + K}$$

where P is the 20-year earned premium and K is a constant. The subcommittee wanted a substantial amount of premium built up before applying a credibility factor as high as 50%. From reports of power reactors under construction and in planning, it is estimated that at the end of the next ten years, or about 1982, the pools will have earned \$500 million premium since inception. Thus, it appeared reasonable to set K at \$500 million and to construct the table. It is not intended to interpolate the credibility factors, and on this basis the premium intervals were calculated.

The course of action to be taken when earned premium over a 20-year period exceed $552 million was deferred for a future decision.

COMPUTATION OF A SAMPLE MODIFICATION

1.	Normal Limits losses For 10 year rating period	$15,000,000
2.	Total earned premium for 10 year rating period	45,000,000
3.	Actual Normal Limits loss ratio (1) ÷ (2)	.333
4.	Excess Limits losses for 20 year rating period	$ 0
5.	Total earned premium for 20 year rating period	$48,000,000
6.	Actual excess Limits Loss Ratio (4) ÷ (5)	.000
7.	Total Expected loss ratio	.664
8.	Excess Limits expected loss ratio .5 × (7)	.332
9.	Credibility factor based on (5)	.100
10.	Modification: $\dfrac{(3) + (6)(9) + (8)[1.00 - (9)]}{(7)}$	.951
11.	Credit	4.9%

COMMENTS

The actual modification produced for 1972 was a rate credit of 7.7%. This is a modest credit, and it is certainly hoped that such a situation will continue indefinitely. Tests have shown that if there are no excess losses and if the normal limits loss ratio continues to hover around 30%, credits will gradually build up to about 16% in 1975 and over 30% in 1979.

On the other hand, a $25 million loss in 1972, a thoroughly catastrophic event to the pools, would produce a modification of .998, or a credit of 0.2% on the rates. This calculation assumes that the 1972 earned premium, unmodified, is $15 million and that the normal loss ratio is the expected, .332.

Under the same conditions a $50 million loss in 1972 would produce a debit of 6.0%.

If there were $12 million losses in 1972, with no single loss over $5 million, the resulting debit would be 12.7%.

Thus it is clear that the rating guide, as constructed, prevents wild rate swings from year to year, yet appears to produce a reasonable and balanced response to actual experience.

In conclusion, it is hoped that there will be a frequent review of this guide, and of the many assumptions underlying it, in the light of actual experience. Perhaps the $5 million excess loss definition could be set at $7.5 million, or $10 million, with an increasing emphasis on normal loss experience. Even without such change, consideration may be given to the allocation of more than 50% of the loss portion of the premium to expected normal losses, with less to expected excess losses. The expense portion, of course, should be updated constantly to reflect actual costs. The ultimate objective is to produce the most equitable results to insureds and insurers alike.

NO-SPLIT EXPERIENCE RATING PLANS

JOHN P. WELCH

As experience rating evolved over the years, a great deal of effort went into the refinement of the multi-split experience rating plan of Workmen's Compensation. Perhaps because of this emphasis, attention has been directed away from no-split experience rating plans. The purposes of this paper are: to draw together some ideas on no-split plans by comparing a no-split compensation plan with the multi-split compensation plan, to measure these two plans as to their ability to respond to theoretical requirements and to discuss these implications on currently used no-split plans in lines of business other than Workmen's Compensation.

WHAT IS EXPERIENCE RATING?

Experience rating may be described as a process which prospectively alters the premium of each member of a class, based on each member's recent past experience. The process attempts to balance the indications of a risk's classification rate and the risk's own experience. A review of the early writings[1] on the subject reveals that experience rating is an attempt to measure four critical input items: exposure, hazard, credibility of the manual rate and dispersion of risks within a class. Present experience rating plans reflect the first characteristic, exposure, by assigning varying credibility by size of risk. For the very large risk, the rate for the risk will be based solely on the risk's own experience; whereas, in the case of the small risk, very little credence can be given to the risk's own experience and his rate will depend largely on the experience of the class to which that risk belongs. The varying hazard (frequency of loss) of the risk is treated in experience rating in much the same manner as exposure is treated. For a very hazardous risk, one which may, because of its hazard, develop many losses for similar exposure, the credibility will be high. It is therefore easier to identify the average loss experience of that risk.

A large hazard will affect the class rate as well as the individual risk's experience. It is not difficult to imagine examples of large risks in relatively small states where the size of the risk heavily influences the manual rate. In these instances, the risk's experience serves a dual rating purpose and the net effect of this varying credibility is not immediately clear.

[1]Whitney, A. C., "The Theory of Experience Rating", *P.C.A.S. IV*.

To be theoretically precise, an experience rating plan should account for the variation in the credibility of the manual rate. Again, we can picture the single state situation where manual rates are often constructed for classes with small volumes of experience. If the manual rate is stable, more weight can be given to it and, hence, less weight can be given to the individual risk (all other things being equal). For the unstable rate, greater relative credibility can be given to the risk's experience.

It is obvious that every risk in a class is not typical of the class. The classification system does a better job in some cases than in others. The problem is to measure the dispersion of risks within a class. The frequency distribution of the loss ratios of risks in a class, like "bakeries", will differ sharply from a class like "roofers". (I am assuming that "bakeries" is a class with many small and medium losses, while "roofers" is a class with relatively more larger losses.) For a class with a concentrated distribution about the mean, we can surmise that a risk's experience that departs from the average of the class can be accounted for as due to chance rather than an inherent difference in the degree of hazard for the risk. On the other hand, if risks are diverse it is likely that a risk's experience that departs from the average will be accounted for by a real difference in hazard. To be theoretically more precise, an experience rating plan should have a credibility weighting that varies by class (as well as size).

Early experience rating plans grappled with these problems. In spite of the obvious difficulties, plans were developed which have stood the test of time, at least in their basic theoretical construction. Perhaps their strength lies in the fact that they satisfy the primary functions of individual risk rating plans[2]:

1. To achieve greater equity in rating of insurance
2. To stimulate loss prevention control
3. To abet competition.

No-Split Plans

A no-split plan has been defined as one in which no attempt is made to divide losses into primary and excess elements. Examples of no-split plans are: the Pennsylvania Workmen's Compensation Experience Rating Plan,

[2]Kulp, C. A. and Hall, J. W., *Casualty Insurance* (The Ronald Press Company, New York, 1968), 4th edition, Chapter 22.

the Automobile Liability Experience Rating Plan, and the Liability other than Auto, or as we will refer to it, the General Liability Experience Rating Plan. Though the Auto and General Liability plans may differ by company, I will refer to the plans of the Insurance Services Office for uniformity. In all of these plans the usual formula for experience rating applies (with variations), *viz.*:

$$M = \frac{AZ + (1\text{-}Z)E}{E}, \text{ where,}$$

M = Modification

A = The actual losses

Z = Credibility

E = The expected losses

Comparison Of A Workmen's Compensation No-Split Plan With A Workmen's Compensation Multi-Split Plan

To begin our understanding of no-split plans, let us compare the Pennsylvania Workmen's Compensation no-split experience rating plan with a multi-split experience rating plan. In Workmen's Compensation, a given risk begins with the same manual rate and the identical loss experience as it enters the experience rating process regardless of the company that writes the risk. Each company arrives at the same price for the risk prior to the application of premium discount, dividends or retrospective rating plans. An example of the experience rating plan for Pennsylvania is given on Exhibit I-A.

In the no-split formula for Pennsylvania, $M = \frac{AZ + (1\text{-}Z)E}{E}$, the entire value of each loss enters the rating unless the loss exceeds the "Maximum Value of One Accident" figure that is shown in Exhibit I-A. In the multi-split plan, each loss enters the rating in two pieces: the primary portion of the loss and the excess portion of the loss. The primary value is determined by the formula, primary loss value $= \frac{\text{actual loss}}{\text{actual loss} + 3000} \times 3750$; therefore the excess loss value equals actual loss value minus primary loss value. The multi-split formula can be written as,

$$M = \frac{ApZp + (1\text{-}Zp)Ep + AeZe + (1\text{-}Ze)Ee}{E}, \text{ where,}$$

Ap = Primary actual losses
Ep = Primary expected losses
Zp = Primary credibility
Ae = Excess actual losses
Ee = Excess expected losses
Ze = Excess credibility
E = Ep + Ee.

With this background, let us compare these plans in four areas: off-balance, expected loss, the ability to reflect differences in hazard and the ability to respond to the theory of minimizing the variance of the loss ratio distribution.

OFF-BALANCE

In no-split plans the column entitled "Maximum Value of One Accident" (Exhibit I-A) implies that, when losses of a certain size occur, the actual amounts of losses entering the rating are diminished to the tabular value. Total actual losses, therefore, must be less than the total expected losses of risks entering experience rating in the aggregate. The fact that actual losses used in experience rating (in the aggregate) are less than expected losses (in the aggregate) can be termed "off-balance". In addition to the off-balance created by limiting individual losses that enter rating, there are other reasons for off-balance. It has been noted that larger risks tend to have loss ratios less than smaller risks. A distribution of loss ratios by size of risk (see Exhibit II) would, therefore, indicate that risks that do not qualify for rating, that is those that are too small to be rated, would have loss ratios in excess of the expected loss ratios built into rates. Risks that are subject to experience rating tend to have loss ratios (in the aggregate) less than the expected loss ratios built into rates. The fact that these actual losses for risks subject to rating are, in the aggregate, less than the expected losses anticipated by rates is another contributing factor to the off-balance of experience rating.

Multi-split experience rating plans also have off-balance. To the extent that loss ratios vary by size of risk and primary and excess actual loss is less than primary and excess expected loss, there will be off-balance. Also, individual losses are limited to the state's accident limitation under multi-split plans. These limitations also reduce the dollars of actual loss and, therefore, contribute to off-balance. The off-balance which may exist in plans for lines other than Workmen's Compensation is not recognized in rate level calculations as is the off-balance in Compensation plans. This will be discussed later.

Expected Losses—Hazard

Expected losses and the ability of an experience rating plan to reflect hazard are two concepts which should be discussed jointly. Expected loss refers to the column of expected losses or expected loss rates as shown in Exhibit I-B. The expected loss rate times the payroll provides the dollars of expected loss for the individual risk for the experience period under consideration in experience rating. Expected losses in multi-split experience rating plans are similar to those calculated in the Pennsylvania Workmen's Compensation no-split plan, in that an expected loss rate is published for each class in both plans. The multi-split plans further subdivide the expected losses into the expected primary losses and expected excess losses. To determine the expected primary losses, the expected losses are multiplied by the classification "D" ratio. The "D" ratio is a direct reflection of varying hazard. A high "D" ratio indicates that a larger percentage of total losses are coming from smaller type losses. We would expect the class "bakeries" to have a high "D" ratio and "roofers" to have a low "D" ratio.

The calculation of an expected loss rate is shown in Marshall's paper on Workmen's Compensation ratemaking[3]. The purpose of calculating an expected loss rate is to arrive at a value of expected loss which is comparable to the actual losses that are used in experience rating. Instead of adjusting the actual losses (which may be two or three years old) to current benefit levels, the expected losses are adjusted to the benefit levels at the time the actual losses were incurred. This will be important later in our comparison of no-split plans for third party lines.

[3] Marshall, R. M., "Workmen's Compensation Ratemaking", *P.C.A.S. XLI.*

Measuring the Plan's Ability to Respond

As indicated earlier, an experience rating plan has many functions. In the absence of experience rating, we would expect that a group of risks would distribute themselves about an average loss ratio. The experience rating process attempts to more tightly distribute the same group of risks about the average. By increasing the premium on poor risks and lowering the premium on good risks, the plan attempts to adjust each risk's loss ratio to the average. If we accept this as one of the standards that an experience rating plan should meet, we are then left with the problem of measuring a plan's ability against this standard.

To measure this ability, I suggest that we calculate the standard deviation of the loss ratio distribution of a group of risks without experience rating (the manual premium loss ratio distribution) and compare it to the standard deviation of the loss ratio distribution of the same group of risks after experience rating (the standard premium loss ratio distribution). This information is available for the Pennsylvania Experience Rating Plan. The experience is included in Exhibits III-A and III-B.

All experience rated risks for Pennsylvania for policy years 1966 and 1967 were analyzed. The risks were divided by arbitrary size groupings in order to see if the plan worked better for the larger sizes of risks. One year (1966) was at first report, while the other (1967) was at second report. I wanted to see if the later reporting of losses affected the distributions. One conclusion that I drew was that the additional development of losses to second report has little or no effect on the pattern of results. On the first report basis, the standard deviations of the two loss ratio distributions are not markedly different. On the second report basis, there is no apparent improvement in this pattern. We will be safe if we concentrate on Exhibit III-A for further analysis.

We note some peculiarities on Exhibit III-A:

1. Though the standard deviations for the two loss ratio distributions are similar, our size groupings are made up of broad ranges of risks and this may be biasing our result. We note that the loss ratio on a manual premium basis begins to depart significantly from the mean loss ratio as size of risk increases (weighted mean versus unweighted mean).

2. There is a significant difference in the overall premium developed on a manual basis as opposed to a standard basis.

In answer to the first point, I have attached an appendix which shows the same data carried through a weighting procedure. This should dispel any fears that the standard deviations are being affected by either the size groupings of the risks or the disparity in loss ratio. The weighted standard deviation of the manual premium loss ratio distribution is not reduced by the experience rating process. Or, to put it another way, the weighted standard deviations of the standard premium distributions are not significantly less than the weighted standard deviations of the manual premium distributions.

Now that we know that the data on Exhibit III-A are unbiased, we can turn to the second point: the reduction of premium. The application of the Pennsylvania Experience Rating Plan generates roughly a 20% reduction in overall premium each year. Let us assume for the moment that we have no experience rating plan. Wouldn't we want to credit the class "large risks" because they have better than average experience? Let us assume further that we give large risks (those subject to experience rating) a 20% flat credit. If we now look at the standard deviations of this loss ratio distribution and compare it with the standard deviation of the loss ratio distribution of experience rated risks (Exhibit III-C), we see that experience rating does tighten the loss ratio distribution. When measured against a plan that generates only flat credits, we find that the Pennsylvania no-split plan is superior.

This same type of analysis was performed on a sample of risks that were rated under the multi-split experience rating plan. The sample contained one policy year of "intra-state only" risks. The risks were at first report. The standard deviations, by size of premium groupings similar to Pennsylvania, were very close over all sizes of risk on both the standard and manual premium bases. The aggregate premium on the standard premium basis was very close to the aggregate premium on a manual premium basis. For this body of experience there was no evidence that the loss ratio distribution was tightening after experience rating. Perhaps a more interesting study would be one where the manual premium loss ratio distribution is measured against a standard premium loss ratio distribution when the standard premium loss ratio covers a longer period than one year. This should reduce the variation in the one-year test of standard premium loss ratios considerably, and it should conclusively prove that the experience rating plan does indeed indentify the "better than average" and "worse than average" risks.

Conclusion on Compensation Plans

Before any testing of the loss ratio distribution after experience rating, why should we expect the multi-split plan to be superior to the Pennsylvania Workmen's Compensation Plan? I think the one big advantage of the multi-split plan is the "D" ratio concept. For the majority of risks, those below the Q point, there is no excess credibility. For these risks,

$$M = \frac{ZpAp + (1\text{-}Zp)Ep + Ee}{E}.$$

Here, in the simple case, we can see the importance of the *Ep* term. For any two risks with the same primary credibility, the "D" ratio *(Ep)* distinguishes between the inherent hazard of risks (bakers vs. roofers). A high "D" ratio reflects the high incidence of small loss. The claim-free "baker" will get a larger credit modification than the claim-free "roofer". Exhibit IV shows the relationship of the "D" ratio and the modification for the claim-free risk. For a given primary credibility, a larger credit is given to the claim-free risk with the highest "D" ratio.

The "D" ratio was non-existent in the earliest experience rating plans. The earliest plan calculated two partial premiums: one for the death and permanent total loss portion and the other for the remaining indemnity and medical losses. The splitting of losses, therefore, is an ingenious way to incorporate, within the workings of the plan, the catastrophe type of loss (death or permanent total injury) with the run-of-the-mill loss. It satisfies a fundamental principle of experience rating stated by Michelbacher, ". . . experience rating . . . should not excessively penalize an assured for the occurrence of an accident which, as regards the individual risk, may be considered fortuitous."[4]

Both the no-split and the multi-split plans create off-balance. Both types of plans retain overall manual rate level by adjusting class rates for this off-balance in Workmen's Compensation. The no-split plan would

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[4] Michelbacher, G. F., "The Practice of Experience Rating", P.C.A.S. IV

seem to be easier to implement because the calculations of the modification are easier. Perhaps the ultimate test would be to apply both plans to the same group of risks to see if the increased calculations of the multi-split plan warrant its use in favor of the no-split plan.

Comparison of Workmen's Compensation Plans With Third Party Line Plans

The Insurance Services Office promulgates a number of individual risk rating plans. States have responded differently to these plans. In those states that permit maximum rating flexibility, the ISO has filed its Experience and Schedule Rating Plan for Auto Liability and General Liability. These states are referred to as "open states". Only four states are currently referred to as "closed states" (North Carolina, Texas, Louisiana and Virginia). These states do not permit schedule rating in the aforementioned lines of business, so the ISO files its Experience Rating Plan in these states. New York was a member of this group, but the Open Competition Rating Law in New York now permits the filing of a schedule rating plan. Exhibit V shows a sample page of the North Carolina Automobile Liability Experience Rating Plan (a closed state plan). Exhibit VI shows a sample page of an open state's plan: the Maryland General Liability Experience and Schedule Rating Plan.

Closed States' Plan

Though there are only four closed states, it is worth identifying features of this type of plan because the construction is different from other types of no-split plans. The closed states' plan uses a "D" ratio concept and an excess limits credibility for risks developing more than $30,000 of basic limits premium. The "D" ratio that is used in this plan is the measure of the off-balance created by limiting losses to the maximum single loss values. To the extent that it represents the ratio of small losses to total losses, it is similar to the "D" ratio of the Compensation Plan. That "D" ratio is defined as the ratio of primary losses to total losses. Besides the "D" ratio, it is also worth noting that the closed states' plan uses a modification formula with loss ratios in lieu of dollars of actual and expected loss. All General Liability risks, for example, would use the same expected loss ratio. For risks developing less than $30,000 of basic limits premium the formula is:

$$M = \frac{\text{Actual Basic Limits Ratio} \times Z + (1\text{-}Z) \text{ Expected Loss Ratio} \times \text{"D" Ratio}}{\text{Expected Loss Ratio} \times \text{"D" Ratio}}$$

The use of loss ratio, as opposed to dollars, does not distort the result. It was noted earlier, however, that the expected loss rates in the Compensation Plan were used to adjust the expected losses to the same benefit levels as actual losses. In the closed states' plan, no attempt is made to put the actual and expected loss ratios on the same economic level, i.e., the actual loss ratio is comprised of losses which are valued at some date approximately three months prior to the rating date and premiums which are adjusted to current levels. Losses, at this time, are basic limits paid losses and outstanding reserves on reported cases. For basic limits losses, it is safe to assume that most of the actual loss dollars are known at this time, although some losses will be unreported. To the extent that losses were paid in the earlier part of the experience period, some adjustment of these losses should be made to bring these losses to expected cost levels. If a company's liability reserves indicated an upward loss development pattern, it would be safe to assume that this, too, would cause an understatement of actual loss dollars at the time of calculation of the modification.

Open States Plan

Exhibit V gives us an idea of the way that the open states' plan of the ISO is constructed. This plan is very similar in construction to the Pennsylvania Workmen's Compensation Experience Rating Plan. Neither of these plans employs a "D" ratio. The General Liability Plan and Auto Liability Plan are used in conjunction with schedule rating. The maximum credibility assigned in the liability plans is .75; the Pennsylvania Compensation Plan identifies the self-rating value as $208,567 of expected loss (25 times the average death and permanent total loss for Pennsylvania).

There are some major differences in these two plans. Let us segment these differences as follows:

A. Historical developments
B. Effects on rate level.

A. *Historical Developments*

Workmen's Compensation rates and rating plans have received a great deal of scrutiny over the years. I think the evidence is clear that the ratemaking process develops rates which are more appropriate, risk by risk, than can be developed in any other commercial line of business. As the political climate has changed with respect to compensation over the years, the ratemaking process has reflected these changes. As benefit levels and wage levels have increased, rates have responded with minimum delay. Since rates are reviewed annually, large swings in manual rates are kept at a minimum. State regulators, employers and the insurance companies tend to view the rating process as responsive to their needs.

Perhaps, as we look back into the Fifties and early Sixties, General Liability also responded well to the rating needs of employers and state regulators. However, in the late Sixties, as the rate of inflation increased and attitudes of the public were reflected in changes in the interpretation of the law, premium levels for this line of business came under great stress. We have seen malpractice and products liability claims increasing in frequency and severity in reflection of this change in attitude.

The rating plans in this line of business are geared to give the underwriter maximum flexibility. In many instances, the underwriter needs this flexibility. In other instances, the lack of appreciation of the workings of individual risk rating plans have surely caused problems. In a line of business where significant loss development is continuing three years after the close of a policy year, responsiveness of rates can be a serious problem.

B. *Effects on Rate Levels*

I had previously indicated that Compensation maintains overall rate level by adjusting all class rates for the off-balance created by the experience rating plan. In General Liability and Commercial Automobile there is no formal procedure to adjust rate levels for off-balance. In the closed states' plan, the "D" ratio helps to reduce off-balance, but in the open states' plan, off-balance is assured by the limitation on individual loss amounts. We have already identified other factors that contribute to off-balance in the Liability Plans, such as unreported losses, possible upward loss development and losses which have not been adjusted for economic changes. This off-balance is not restored to class rates in General Liability and Commercial Automobile.

There are explanations for ignoring this off-balance. One is that the plan is not mandatory. Another might be that schedule rating can offset the off-balance created by experience rating. To the first argument, I would reply that the plan is probably applied as if it were mandatory. Credit risks will demand to be rated. Debit risks will be glad to pass up the opportunity. Of course, passing them up increases the off-balance. To the second argument, there is no clear answer. We might argue that to schedule debit risks that have proven themselves to be better than average, under experience rating, is discriminatory.

Schedule rating, like experience rating, is unmeasured. Current statistical plans do not require the recording of either experience or schedule modifications for individual risks. Perhaps the Commercial Risk Statistical Plan will start a precedent in that it requires the coding of the "Percentage Premium Modification".

REVISION OF A PLAN

What options are open to us in revising an experience rating plan of the "open states' type"? In a relatively stable insurance pricing mechanism, the revision of the experience rating plan is not necessarily a problem. Recently, however, because of our rising economy, we have seen the necessity to modify certain of the factors inherent in the ISO experience rating plans and in the Pennsylvania Workmen's Compensation Plan. As values of individual losses increase due to increased costs in medical expenses, etc., the values will more often exceed the maximum single loss value as published in our present plans. Increased frequency of large loss tends to accentuate the off-balance in the present experience rating plans. As loss experience in the third party lines continues to deteriorate, companies are taking a more realistic view of the third party experience rating plan. The loss experience on which experience modifications are calculated in the third party lines is the risk's own experience as produced on individual account loss runs. These loss runs are usually subject to some deficiencies such as losses which have been incurred that are not yet reported. This inadequacy of total loss dollars, of course, is built into the calculation of the experience modification. In attempting to get more dollars of loss into rating, some revision of the experience rating plan has taken place. Some changes have been made in the swing of the experience rating plan, in self-rating point or in the credibility formulas, i.e., the "K" value in the credibility formula. Since, in the third party line experience rating plan, the dollars of actual losses are very difficult to adjust to a final fully developed value, and since it is very difficult to identify all losses which have been incurred but are not yet reported, there is a tendency to look at the dollars of expected losses with anticipation of adjusting these instead of trying to adjust the actual losses. Those plans with a current off-balance due to maximum single loss (or the other dollars of missing loss) cannot necessarily be adjusted to full value as far as the actual losses are concerned. It is possible, however, to measure the total dollars of off-balance in the plan and to bring the plan back into balance, *i.e.*, to strengthen the plan by adjusting the dollars of expected loss to a truer expected loss value than that anticipated in overall rate level. In other words, the expected loss ratio should be adjusted to some lesser value, as was done in the closed states' plan with the "D" ratio. However, we need not restrict the role of the "D" ratio to the adjustment for maximum single loss only.

SUMMARY

The currently used no-split experience rating plans affect large premium volumes. These plans have not been revised frequently. Since many of these plans have not been formally tested, there should be concern as to the values used in the plans, such as: the credibility curve, the self-rating point and the swing of the plan. Perhaps we should also be concerned with the way that the relative hazard of a risk is reflected in these plans. The "D" ratio of the multi-split plan segregates hazard. In the liability no-split plans, any two risks of equal premium size are treated the same, though one may be a risk with heavy OL&T exposure, and the other may have heavy products exposure. Off-balance is a measure of difference from established manual rate level. Underwriters must recognize this fact. With the advent of detailed Commercial Multiple Line experience from the Commercial Risk Statistical Plan, developments of experience rating plans for Commercial Package risks can be anticipated.

Experience rating plans are very useful tools in fostering competition and safety. As shown previously, some compensation plans, as presently constructed, do not minimize the loss ratio variance, *i.e.*, they are not necessarily distributing costs equitably. To the extent that this may be judged important, factors in the plans should be adjusted. An additional implication is that the third party line plans may also fail on this point. They too should be tested. I hope the items discussed in this paper will be beneficial to those who periodically come in contact with these plans.

Exhibit I-A

Pennsylvania Experience Rating Plan

Expected Losses (1)	Credibility "C" (2)	Maximum Value of One Accident (3)
1420 or less	.050	6750
1421-1564	.055	6786
1565-1709	.060	6825
1710-1856	.065	6862
1857-2004	.070	6896
.	.	.
.	.	.
.	.	.
5347-5534	.175	7773
5535-5725	.180	7821
5726-5918	.185	7869
5919-6114	.190	7917
6115-6312	.195	7967
.	.	.
.	.	.
.	.	.
12777-13066	.335	9643
13061-13360	.340	9716
13367-13659	.345	9791
13660-13963	.350	9866
13964-14271	.355	9942
.	.	.
.	.	.
.	.	.
94078-95816	.815	29125
95817-97574	.820	29480
97575-99362	.825	29839
99363-101176	.830	30202
101177-103012	.835	30567
.	.	.
.	.	.
.	.	.
173932-178666	.975	45205
178667-183964	.980	46254
183965-190077	.985	47468
190078-197615	.990	48951
197616-208566	.995	51028
208567 & over	1.000	55873

Exhibit I-B

PENNSYLVANIA EXPERIENCE RATING PLAN

Manual Rates and Expected Loss Rates

Code No.	Manual Rate	Exp. Loss Rate	Code No.	Manual Rate	Exp. Loss Rate	Code No.	Manual Rate	Exp. Loss Rate
005	3.50	1.99	251	.63	.35	505	1.85	1.05
007	3.55	2.02	255	1.10	.62	506	.88	.50
009	7.70	4.38	257	1.25	.71	507	1.75	.99
025	4.00	2.27	281	.51	.29	508	2.35	1.33
028	1.60	.91	305	1.95	1.11	509	.85	.48
050	3.50	1.99	323	1.05	.59	510	4.25	2.42
051	3.20	1.82	401	1.70	.96	512	1.90	1.08
053	3.00	1.70	402	3.30	1.88	513	.91	.51
055	2.35	1.33	404	1.40	.79	533	2.15	1.22
101	1.75	.99	406	1.35	.76	535	.63	.35
103	.37	.21	407	1.30	.74	551	1.30	.74
105	1.00	.56	408	1.30	.74	553	.70	.39
107	.90	.51	409	.80	.45	555	.48	.27
108	1.30	.74	411	2.55	1.45	557	1.30	.74
109	1.30	.74	413	1.45	.82	563	1.00	.56
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
163	.44	.25	458	.35	.19	651	1.55	.88
165	.93	.52	459	.30	.17	653	1.50	.85
167	.67	.38	461	.87	.49	654	2.00	1.13
201	1.55	.88	463	1.65	.93	655	5.95	3.39
204	.47	.26	467	.54	.30	656	3.05	1.73
205	.70	.39	473	.54	.30	658	2.00	1.13
221	1.30	.74	475	1.15	.65	661	.94	.53
222	1.10	.62	483	.29	.16	662	.66	.37
225	1.20	.68	487	.28	.15	663	.99	.56
227	.79	.44	501	.60	.34	665	2.50	1.42
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.

Exhibit II

WORKMEN'S COMPENSATION

Size of Risk Premium Range	Total Standard Premium	Loss Ratio
$ 1,678—$ 6,623	$265,776,009	60.4
$ 6,624—$13,687	$149,737,975	59.3
$13,688—$47,340	$222,551,145	57.6
$47,341 and over	$217,213,861	56.6

Source: Simon, LeRoy J., "The 1965 Table M", *P.C.A.S.*, LII

This exhibit does not show the loss ratio for risks not subject to experience rating but it does demonstrate that loss ratios diminish as risk size increases.

Exhibit III-A

PENNSYLVANIA WORKMEN'S COMPENSATION
Policy Year 1966 (1st Report)

Size of Policy	Number of Policies	Incurred Losses	Standard Premium	Standard Loss Ratio	Standard Mean	Standard Std. Dev.	Manual Premium	Manual Loss Ratio	Manual Mean	Manual Std. Dev.
*1(Under— 500)	390	$ 150,643.	$ 103,494.	145.56	115.55	581.94	$ 107,643.	139.95.	115.45	589.60
2(500— 999)	1,207	509,966.	966,714.	52.75	54.38	208.40	997,198.	51.14	53.16	202.15
3(1,000— 2,499)	1,968	1,617,372.	3,332,414.	48.53	47.90	144.03	3,441,004.	47.00.	47.30	142.01
4(2,500— 4,999)	2,463	4,536,414.	8,661,421.	52.37	51.95	116.47	9,022,956.	50.28.	51.26	118.12
5(5,000— 9,999)	1,921	7,206,209.	13,299,927.	54.18	54.40	91.83	14,220,821.	50.67	52.98	91.07
6(10,000—49,999)	1,389	13,953,419.	26,214,228.	53.23	52.38	50.72	29,670,060.	47.03	49.04	48.59
7(50,000—99,999)	161	6,265,718.	11,265,167.	55.62	56.34	38.25	15,848,688.	39.53.	44.70	31.29
8(100,000—Over)	71	7,807,874.	14,685,593.	53.17	55.46	25.86	25,176,272.	31.01	43.98	29.00
ALL SIZES	9,570	42,047,615.	78,528,958.	53.54	54.67	171.03	98,481,645.	42.70	53.16	170.93

*Size group #1 is included for completeness. Modifications were calculated for these risks before it became evident that the premium was too small to qualify the risks for experience rating.

Exhibit III-B

PENNSYLVANIA WORKMEN'S COMPENSATION
Policy Year 1967 (2nd Report)

Size of Policy	Number of Policies	Incurred Losses	Standard				Manual			
			Premium	Loss Ratio	Mean	Std. Dev.	Premium	Loss Ratio	Mean	Std. Dev.
*1(Under— 500)	343	$ 119,473.	$ 64,045.	186.55	145.59	977.21	$ 69,149.	172.78	140.99	804.10
2(500— 999)	839	410,045.	670,690.	61.14	61.37	445.07	696,672.	58.86	67.99	587.68
3(1,000— 2,499)	1,779	1,554,003.	3,086,803.	50.34	50.46	193.84	3,176,823.	48.92	49.58	185.32
4(2,500— 4,999)	2,656	4,625,417.	9,406,756.	49.17	48.91	125.81	9,822,967.	47.09	47.99	125.04
5(5,000— 9,999)	2,205	8,076,203.	15,409,702.	52.41	52.11	128.33	16,422,798.	49.18	50.73	127.17
6(10,000—49,999)	1,702	17,793,889.	32,899,856.	54.09	55.78	80.91	37,033,594.	48.05	53.21	85.46
7(50,000—99,999)	217	8,205,736.	14,951,246.	54.88	54.93	42.90	19,325,803.	42.46	47.62	40.42
8(100,000—Over)	124	12,843,856.	24,146,269.	53.19	52.75	31.37	36,756,781.	34.94	42.17	25.77
ALL SIZES	9,865	53,628,622.	100,635,366.	53.29	52.21	256.60	123,300,911.	43.49	51.17	259.05

*Size group #1 is included for completeness. Modifications were calculated for these risks before it became evident that the premium was too small to qualify the risks for experience rating.

Exhibit III-C

PENNSYLVANIA WORKMEN'S COMPENSATION
LOSS RATIO BY EXPERIENCE MODIFICATION
MEAN AND STANDARD DEVIATION OF LOSS RATIOS
MANUAL YEAR 1966

Size of Policy	Number of Policies	Incurred Losses	Standard				Manual (Adjusted)			
			Premium	Loss Ratio	Mean	Std. Dev.	Premium	Loss Ratio	Mean	Std. Dev.
1(Under— 500)	390	$ 150,643.	$ 103,494.	145.56	115.55	581.94	$ 86,115.	174.93	144.31	737.00
2(500— 999)	1,207	509,966.	966,714.	52.75	54.38	208.40	797,758.	63.92	66.45	252.68
3(1,000— 2,499)	1,968	1,617,372.	3,332,414.	48.53	47.90	144.03	2,752,801.	58.75	59.13	177.51
4(2,500— 4,999)	2,463	4,536,414.	8,661,421.	52.37	51.95	116.47	7,218,361.	62.85	64.07	147.65
5(5,000— 9,999)	1,921	7,206,209.	13,299,927.	54.18	54.40	91.83	11,376,644.	63.34	66.23	113.84
6(10,000—49,999)	1,389	13,953,419.	26,214,228.	53.23	52.38	50.72	23,736,023.	58.79	61.30	60.73
7(50,000—99,999)	161	6,265,718.	11,265,167.	55.62	56.34	38.25	12,678,948.	49.42	55.88	39.11
8(100,000—Over)	71	7,807,874.	14,685,593.	53.17	55.46	25.86	20,141,017.	38.77	54.97	36.25
ALL SIZES	9,570	42,047,615.	78,528,958.	53.54	54.67	171.03	78,785,067.	53.37	66.45	213.66

Exhibit IV

WORKMEN'S COMPENSATION

Let

E	=	Expected Losses	M	=	Modification
Ze	=	Excess Credibility	Ap	=	Primary Actual Loss
Zp	=	Primary Credibility	Ep	=	Primary Expected Loss
Q	=	Q Point	Ee	=	Excess Expected Loss
			E	=	$Ep + Ee$

For $E < Q$, $Ze = 0$

$$M = \frac{ZpAp + (1-Zp)Ep + Ee}{E}$$

If $Ap = 0$, $M = \frac{Ep(1-Zp) + Ee}{E}$

But $Ep = \text{"D"} \times E$ and $Ee = E - Ep$

Therefore, $Ee = E - \text{"D"}E$

Now $M = \frac{\text{"D"}E(1-Zp) + E - \text{"D"}E}{E}$

$$M = \text{"D"}(1-Zp) + 1.0 - \text{"D"}$$

$$M = 1.0 - \text{"D"}Zp$$

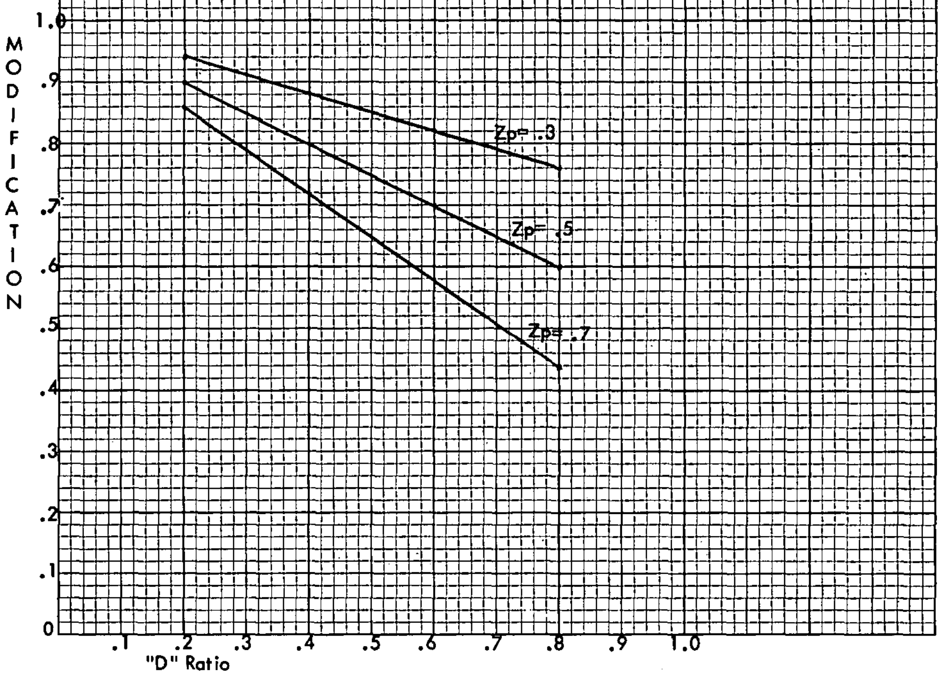


Exhibit V

INSURANCE SERVICES OFFICE

Automobile Liability Experience Rating Plan—North Carolina Credibility Table

Premium	Basic Limits Credibility Z	"D" Ratio	Basic Limits Maximum Loss Including Allocated Claim Expense: All Other	Taxis	Garages	Excess Limits Cred. Ze
24,944- 25,977	.56	.899	7,400	7,700	7,100	—
25,978- 27,058	.57	.902	7,600	7,900	7,300	—
27,059- 28,192	.58	.904	7,800	8,100	7,500	—
28,193- 29,382	.59	.906	8,000	8,400	7,700	—
29,383- 30,632	.60	.909	8,200	8,600	7,900	.10
30,633- 31,948	.61	.911	8,400	8,800	8,100	.10
31,949- 33,333	.62	.913	8,700	9,100	8,400	.11
33,334- 34,794	.63	.915	8,900	9,400	8,600	.11
34,795- 36,338	.64	.918	9,200	9,600	8,900	.11
36,339- 37,971	.65	.920	9,500	9,900	9,200	.12
.	.	.	.	.	.	.
82,565- 88,108	.81	.956	18,100	19,000	17,500	.24
88,109- 94,285	.82	.958	19,200	20,100	18,500	.25
94,286-100,503	.83	.961	20,300	21,300	19,600	.26
100,504-106,721	.84	.963	21,400	22,400	20,700	.27
106,722-112,939	.85	.965	22,400	23,600	21,700	.29
.	.	.	.	.	.	.
207,457-216,071	1.00	.990	30,000	30,000	30,000	.44
216,072-225,000	1.00	.990	30,000	30,000	30,000	.45
225,001-234,259	1.00	.990	30,000	30,000	30,000	.46
234,260-243,867	1.00	.990	30,000	30,000	30,000	.47
243,868-253,846	1.00	.990	30,000	30,000	30,000	.48
253,847-274,999	1.00	.990	30,000	30,000	30,000	.49
275,000 and over	1.00	.990	30,000	30,000	30,000	.50

Exhibit VI

INSURANCE SERVICES OFFICE

Maryland
General Liability Experience Rating
Credibility and Maximum Single Loss Table

Premium	Credibility	Maximum Single Loss	Premium	Credibility	Maximum Single Loss
1- 304	.01	2,850	12,521-13,057	.39	4,600
305- 512	.02	2,900	13,058-13,613	.40	4,700
513- 725	.03	2,900	13,614-14,188	.41	4,800
726- 942	.04	2,950	14,189-14,782	.42	4,850
943- 1,164	.05	2,950	14,783-15,393	.43	4,950
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
3,669- 3,952	.16	3,350	23,011-23,956	.54	6,100
3,953- 4,242	.17	3,400	23,957-24,943	.55	6,300
4,243- 4,539	.18	3,450	24,944-25,977	.56	6,400
4,540- 4,844	.19	3,500	25,978-27,058	.57	6,600
4,845- 5,157	.20	3,550	27,059-28,192	.58	6,700
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
8,777- 9,197	.31	4,100	43,493-45,573	.69	9,100
9,198- 9,629	.32	4,150	45,574-47,796	.70	9,400
9,630-10,075	.33	4,200	47,797-50,175	.71	9,700
10,076-10,534	.34	4,250	50,176-52,727	.72	10,100
10,535-11,007	.35	4,350	52,728-55,471	.73	10,400
11,008-11,496	.36	4,400	55,472-58,431	.74	10,900
11,497-12,000	.37	4,500	58,432 and over	.75	.188 x Prem.
12,001-12,520	.38	4,550			

APPENDIX

It was noted that the results on Exhibit III-A may be heavily biased by the method of grouping the risks. If we are given a set of numbers $X_1, X_2, \ldots X_N$, the standard deviation would be defined as:

$$S = \sqrt{\frac{\sum_{j=1}^{N} (X_j - \overline{X})^2}{N}} \,.$$

In our case, each X is a loss ratio. If $X_1, X_2, \ldots X_N$ occur with frequencies $f_1, f_2, \ldots f_N$, then the standard deviation would be defined as:

$$S = \sqrt{\frac{\sum_{j=1}^{N} f_j (X_j - \overline{X})^2}{N}} \,.$$

If we want to remove any doubt concerning the effects of grouping the risks in this study, we can consider the premium as a weighting factor similar to frequency. On Exhibits A and B of this appendix are shown the weighted standard deviations for each size group and for all risks combined. For each size group the difference in the weighted standard deviations from Exhibit A (standard premium) to Exhibit B (manual premium) is very small. We can conclude that the grouping of risks on Exhibit III-A has not biased our result.

Exhibit A

PENNSYLVANIA WORKMEN'S COMPENSATION

LOSS RATIO BY EXPERIENCE MODIFICATION

MEAN AND STANDARD DEVIATION OF LOSS RATIOS
POLICY YEAR 1966
STANDARD (UNADJUSTED) PREMIUMS

SIZE OF POLICY	NUMBER OF POLICIES	INCURRED LOSSES	STANDARD PREMIUM	LOSS RATIO	MEAN	STANDARD DEVIATION	WEIGHTED STD. DEV.
1(Under — 500)	390	$ 150,643.	$ 103,494.	145.56	115.55	581.94	622.51
2(500— 999)	1,207	509,966.	966,714.	52.75	54.38	208.40	193.38
3(1,000— 2,499)	1,968	1,617,372.	3,332,414.	48.53	47.90	144.03	145.98
4(2,500— 4,999)	2,463	4,536,414.	8,661,421.	52.37	51.95	116.47	116.67
5(5,000— 9,999)	1,921	7,206,209.	13,299,927.	54.18	54.40	91.83	89.80
6(10,000—49,999)	1,389	13,953,419.	26,214,228.	53.23	52.38	50.72	48.63
7(50,000—99,999)	161	6,265,718.	11,265,167.	55.62	56.34	38.25	36.82
8(100,000—Over)	71	7,807,874.	14,685,593.	53.17	55.46	25.86	23.13
ALL SIZES	9,570	42,047,615.	78,528,958.	53.54	54.67	171.03	76.41

Exhibit B

PENNSYLVANIA WORKMEN'S COMPENSATION

LOSS RATIO BY EXPERIENCE MODIFICATION

MEAN AND STANDARD DEVIATION OF LOSS RATIOS
POLICY YEAR 1966
MANUAL (READJUSTED) PREMIUMS

SIZE OF POLICY	NUMBER OF POLICIES	INCURRED LOSSES	STANDARD PREMIUM	LOSS RATIO	MEAN	STANDARD DEVIATION	WEIGHTED STD. DEV.
1(Under — 500)	390	$ 150,643.	$ 107,643.	139.95	115.45	589.60	618.93
2(500— 999)	1,207	509,966.	997,198.	51.14	53.16	202.15	188.08
3(1,000— 2,499)	1,968	1,617,372.	3,441,004.	47.00	47.30	142.01	142.47
4(2,500— 4,999)	2,463	4,536,414.	9,022,956.	50.28	51.26	118.12	114.89
5(5,000— 9,999)	1,921	7,206,209.	14,220,821.	50.67	52.98	91.07	86.17
6(10,000—49,999)	1,389	13,953,419.	29,670,060.	47.03	49.04	48.59	44.85
7(50,000—99,999)	161	6,265,718.	15,848,688.	39.53	44.70	31.29	27.15
8(100,000—Over)	71	7,807,874.	25,176,272.	31.01	43.98	29.00	24.31
ALL SIZES	9,570	42,047,615.	98,481,645.	42.70	53.16	170.93	68.64

THE ACTUARY AND I B N R

R. L. BORNHUETTER AND R. E. FERGUSON

Incurred But Not Reported loss reserves (hereinafter referred to as IBNR reserves) represent vast sums of money, exceeding $100,000,000 for a number of U.S. property and liability insurers. Nevertheless, the subject has had little attention in the literature of insurance, especially the *Proceedings of the Casualty Actuarial Society*. Although the situation is changing in recent times, the lack of articles, discussions and other related means of presenting the theory and practice of IBNR reserves leads one to conclude that the subject has suffered from neglect over the years and companies have not been allocating sufficient time and talent to this subject.

In an attempt to generate some actuarial interest in this important subject, the authors will describe some theories concerning IBNR reserves which have evolved over the years of handling one of the largest such reserve structures in the United States. Interestingly enough, the theories discussed in this paper have the added advantage of being adaptable to small or medium size insurers and can also apply to areas where credible statistics are unavailable, areas such as reinsurance, casualty umbrella business, etc.

DEFINITION OF IBNR

IBNR reserves represent an important cog in the insurance accounting machinery, especially where a substantial amount of casualty insurance is written. Obviously, inaccurate IBNR reserves will lead to non-optimal management decisions. It is not only prudent accounting practice to have proper IBNR provisions, but it is required by law. It is, for example, stated in Article 72 of the New York Insurance law that: "every insurer shall maintain reserves in an amount estimated in the aggregate to provide for the payment of all losses or claims incurred on or prior to the date of settlement whether reported or unreported which are unpaid as of such date and for which such insurer may be liable, and also reserves in an amount estimated to provide for the expenses of adjustments or settlement of such claims".

Many companies appear to take the narrow view that an IBNR reserve is intended to provide only for liability which is presently unknown because the claims have not yet been reported. Perhaps the first point that should be emphasized is that there is no rational basis for this practice. The words which stand behind the expression IBNR, *Incurred But Not Reported,* do not require the restricted definition which has been traditionally accorded them. It is our contention that liability which is unknown at present but which will eventuate as a result of adverse developments on reported claims is *Incurred But Not Reported,* just as surely as liability on unreported cases.

It is recognized that the developments on reported cases could be favorable and such developments should be contemplated in the overall IBNR reserve structure. As will be developed later in the text, it is quite practical to provide for both types of liability when developing the IBNR reserve. While there is nothing to be gained by splitting the two types of development as far as the establishment of reserves is concerned, it is recognized that it is desirable to be able to discern the two types of development in a management information system. For example, this separate distinction will allow the use of adverse, or favorable, developments on previously reported cases as a form of "report card" on the Claims Department.

For many property-oriented companies, IBNR reserves mainly serve the purpose of providing for the lag in booking November and December losses. By three months beyond the closing date most loss developments have been booked and a particular IBNR reserve is needed for only a short period of time. Companies are finding that the same is not true for casualty business. There is still a year-end lag; however, there is also a further tail for which substantial IBNR reserves must be carried. For those companies in the casualty excess or reinsurance area, IBNR reserves have become enormous in size.

1934 Tarbell Paper

As mentioned previously, the *Proceedings* are relatively barren of papers on the subject of IBNR reserves; however, a paper written by Mr. Thomas F. Tarbell in 1934 is an excellent treatise on the subject.

If one were to survey today's IBNR computational techniques and then reread Mr. Tarbell's 1934 paper, one might conclude that it must still be 1934. On the one hand this is a great tribute to Mr. Tarbell, who very ably and concisely articulated some basic IBNR concepts. Yet, on the other hand, it is also a serious indictment against the actuarial profession that those particular skills have not been sharpened in almost 40 years.

Mr. Tarbell's basic formula was [1] :

$$\frac{N^{y}\ 10\text{-}11\text{-}12 \times C^{y}\ 10\text{-}11\text{-}12}{N^{y-1} 10\text{-}11\text{-}12 \times C^{y-1}\ 10\text{-}11\text{-}12} \times I^{y-1}\ (1) \ldots (12)$$

Where: N = Number of notices
C = Average incurred per notice
I = Amount of IBNR
y = Designates current year
y-1 = Designates previous year
Subscripts denote month

In other words, the actual IBNR (narrow definition) as realized in a given period of time was related to some base, and the resulting factor was then applied to the current base. Any number of things can be used as a base including earned premiums, case incurred, outstanding case reserves, or premiums in force, as long as the selected base is responsive to changes in IBNR exposure. Mr. Tarbell used case incurred losses while today the premiums in force appears to be the favorite base, although reasonable arguments can be marshalled for other bases.

One of the many things which should be considered when selecting an exposure base is the potential distortion in the IBNR reserve which can occur if the book of business is growing rapidly (especially a new book of business). This problem arises because: "we would expect losses to be incurred roughly in proportion to the number of policies in force and, since there are more policies in force at the end of a given accounting period than at the beginning of the period, a factor which measures the incurred but not reported (IBNR) losses will be influenced (that is distorted) by the relatively heavier weight of policies in force at the end of the accounting period".[2] The various solutions to this problem, arrived at by means of geometric modeling, are discussed in detail by Mr. Simon and his reviewer, Charles F. Cook, and will not be discussed here.

1 Tarbell, Thomas F. "Incurred But Not Reported Claim Reserves," *Proceedings of the Casualty Actuarial Society,* Vol. XX, 1934, p. 275

2 Simon, LeRoy J., "Distortion in IBNR Factors," *Proceedings of the Casualty Actuarial Society,* Vol. LVII, 1970, p. 64

This distortion can be a very real problem, and is partly a function of the base used, the term of the business, and the tail of the business (i.e. loss distribution over time). Fortunately, the distortion is relatively less significant on the heavy IBNR lines since they are normally short term policies of 6 months or 1 year, and these lines have a long tail. Mr. Cook points out, in his review, that the choice of the base can also help to minimize the possible distortions and that, for this and other reasons, he favors premiums in force or earned premium as a base.

If one is establishing reserves for a fast closing line, such as the property lines where there is practically no development after 24 months (e.g. 1972 accident year incurred losses as known at 12/31/73) there is absolutely nothing wrong with a Tarbell type of approach. On the other hand, if development is expected after 24 months, as is probable for the Schedule P lines, a more rigorous actuarial approach is indicated.

A one year run-off method, such as Tarbell's, will lead to a woefully inadequate reserve structure if loss development patterns are deteriorating (almost a certainty in the face of modern inflation) and especially if the volume of business is increasing.

For example:

$$\frac{\text{1969 IBNR Observed}}{\text{1969 Premium In Force}} = \frac{\$\ 1{,}000{,}000}{\$10{,}000{,}000} = .10$$

$$\frac{\text{1970 IBNR Observed}}{\text{1970 Premium In Force}} = \frac{1{,}500{,}000}{13{,}000{,}000} = .1154$$

$$\begin{aligned}\text{1971 IBNR Reserve} &= 11.54\% \times \text{1971 Premium In Force}\\ &= 11.54\% \times \$16{,}900{,}000\\ &= \$1{,}950{,}000\end{aligned}$$

We would expect, based on 1969 experience, that the numerator for the 1970 calculation would be $1,300,000, an increase of $300,000 and in line with the increased exposure. But an "extra" $200,000 of development has surfaced. If the extra $200,000 of development is due to a deterioration in the 3rd to 4th report and exposure has been increasing 30% per year, the 1971 IBNR reserve should be $3,039,000 rather than $1,950,000.

The snowballing effect of a deterioration in loss development patterns can best be appreciated by viewing the IBNR reserve structure as it really is, a function of past and present exposures, and not as it seems to be in terms of a Tarbell type formula. The Tarbell formula would lead us to believe that IBNR is totally a function of the in force or incurred, at a given point in time. In reality, the IBNR at time t is partially a function of the claims or exposure relevant to accident year t, and partially a function of claims or exposure relevant to accident year $t-1$, $t-2$, etc. If circumstances change so that more development is expected on year $t-4$, there will also, *ceteris paribus*, be an effect on accident years $t-3$, $t-2$, $t-1$ and t as they ultimately play out.

Loss Development Approach

Determination of Loss Development Factors

The starting point in the establishment of a complete IBNR reserve is the study of developments on total incurred losses on an accident year basis. To facilitate this type of review, it is necessary that all loss and allocated loss expense data carry both accident date and (original) notification date. The authors have found that month and year serve as adequate identification, and that the month identification is essential for the more sophisticated reserving techniques discussed in a later section.

Having studied the emerging loss development data, and resulting loss development factors for the various reporting intervals (first to second report, second to third report, etc.), it is a relatively simple matter to construct an IBNR reserve. Normally, the individual loss development factors would be those indicated by averaging the development patterns for several years, and possibly making judgment adjustments to reflect trends apparent in the data. It is, of course, possible to attempt to select loss development factors in a more objective manner by smoothing the resulting IBNR factors (i.e. unity less the reciprocal of the loss development factor to ultimate) with a fitted curve. Thomas W. Fowler found that a modified exponential curve fitted the IBNR factors reasonably well for the data he was reviewing.[3]

On the attached exhibits we display a purely hypothetical IBNR reserve computation procedure. Exhibit A portrays the actual case loss developments along with the individual loss development factors by interval. In the left column, as a point of reference, the earned premiums net of

3 Fowler, Thomas W., *IBNR*, "Liability IBNR Reserves" p. 35 and Exhibit C, published by Nederlandse Reassurantie Groupe, N.V. Amsterdam, 1972.

commission for the calendar year associated with each accident year are displayed. This is the basic tool with which the IBNR reserve is developed.

IBNR Reserve Computation

The attached Exhibit B sets forth the actual IBNR reserve computation based upon the data in Exhibit A. After determining the loss development factors to ultimate (column 4), one could simply apply these factors to the appropriate case incurred losses, although, in some cases, working backwards from expected losses to an IBNR reserve is the recommended procedure. This is accomplished by subtracting the reciprocal of the appropriate loss development factor from unity and applying the resulting factors to the accident year expected losses.

The decision as to whether to develop the reserve as a direct function of case incurred losses or as a function of expected losses turns on the expected volatility of the data. If the data are extremely thin, the presence or absence of several large losses will impact greatly on the IBNR reserves if the reserve is a function of the case incurred. A strictly fortuitous event such as an exceptionally large loss should not be allowed to distort the IBNR reserves. Then too, if one is working with an unusual line with a long tail, or perhaps simply a new line, it might be desirable to derive the IBNR as function of expected losses. It can be argued that the most prudent course is, when in doubt, to use expected losses, inasmuch as it is certainly indicated for volatile lines, and in the case of a stable line, the expected loss ratio should be predictable enough so that both techniques produce the same result.

A more comprehensive explanation of Exhibit B follows:

> To produce average loss development factors for each interval (one report to the next) the latest three years of data were used. Columns (1) and (2) are the basic data from the loss development triangle (Exhibit A). For example, the sum of the first reports for accident years 1968, 1969 and 1970 is $10,250,000 while $14,500,000 is the sum of the incurred losses as of the second report for those same three accident years. The division of these figures produces the average loss development factor of 1.415 shown in column (3).

Column (4) is simply the upward accumulation of the column (3) loss development factors which will then produce the total loss development factor to project a given accident year to its estimated ultimate result. The factor from third report to ultimate is obtained by multiplying 1.000 x 1.032, while the factor 1.166 from second to ultimate, is the product of 1.000 x 1.032 x 1.130.

The expected losses for each accident year are set forth in column (6). In the example, they are obtained by applying an expected loss ratio of 95% to the premiums earned net of commissions from Exhibit A. The 95% reflects a 5% adjustment to eliminate estimated overhead costs. The selection of an expected loss ratio also is affected by the stability of the data. If for example, a company demonstrates that it can consistently produce an ultimate loss ratio of 60%, then that, to be sure, is the ratio to be used in the IBNR calculations. On the other hand, if the expected loss ratio cannot be selected with much accuracy, a high ratio should be used on the assumption that it is better to err on the conservative side (but not so conservative as to run afoul of the Internal Revenue Service). In rare instances, one might deem that an expected loss ratio exceeding 100% would be appropriate.

Column (7) sets forth the conversion of the loss development factors in column (4) to a basis appropriate for use with expected losses. The factor of .394 for accident year 1971 indicates that 60.6% of the total losses for that accident year have been reported as of the first report (12/31/71), and thus that 39.4% of the losses are yet to emerge. The factor of .394 is determined as follows:

IBNR factor = 1.000 - 1.000/loss development factor to ultimate
= 1.000 - 1.000/1.650
= .394

The IBNR reserves are set forth in column (8) and are the product of the IBNR factors in column (7) and the expected losses in column (6). Thus $2,994,000 is the IBNR reserve assigned to accident year 1971, while $1,012,000 is allocated to accident year 1970, and $206,000 for accident year 1969, resulting in a total reserve of $4,212,000 at 12/31/71.

The technique for developing the IBNR reserve as a function of case incurred losses, Column (9), Exhibit B, involves applying the loss development factor, Column (4), Exhibit B, less unity to the proper case incurred losses (e.g. $4,250,000 x .65; $4,800,000 x .166 etc.). A refinement, results shown in Column (10), on the above method, which is sometimes used to "smooth out" the data, is to average the case incurred for the same report, say for two years; adjust that average case incurred figure to reflect the relative difference between the average exposure for the two years studied and the current year; times the IBNR factor e.g.:

$$\frac{(4{,}250{,}000 + 3{,}300{,}000)}{2} \times \frac{8{,}000{,}000}{\frac{(7{,}500{,}000 + 8{,}000{,}000)}{2}} \times .65 = \$2{,}533{,}000$$

$$\frac{(4{,}800{,}000 + 5{,}200{,}000)}{2} \times \frac{7{,}500{,}000}{\frac{(7{,}000{,}000 + 7{,}500.000)}{2}} \times .166 = \$859{,}000$$

In our example, each method produced a different IBNR reserve. The first method (IBNR reserve as a function of expected losses) could be brought more into line with the other two methods if it was felt that a lower expected loss ratio could be justified. If the ultimate loss ratio to premiums earned net of commission was completely predictable, all three methods would produce the same reserve. Of course, if loss ratios were that predictable, the determination of IBNR reserves would be a trivial matter. One would simply subtract the incurred losses to date by accident year from the expected ultimate incurred, the remainder being the needed IBNR reserve.

Homogenous Data

It makes sense to perform the loss development reviews and IBNR calculations independently for types of business which are known or thought to be different. For example, automobile liability loss development patterns are different than those observed for workmen's compensation. Commercial business may very well have different loss development characteristics than personal business. Umbrella business will clearly have a different loss development pattern than general liability. On the other hand, combining B.I. and P.D. might make sense, especially in light of the recent Annual Statement changes, especially since the B.I., P.D. mix is not likely to change significantly over short periods of time.

The product mix can be an important factor, not so much because two somewhat dissimilar items are combined, but because they may have different rates of growth. For example, a company may have personal and commercial automobile loss development experience combined over the years although, if it were looked at separately, commercial business would require higher loss development factors. As long as the relative exposure between the two categories remains constant there is no problem; however, picture the situation if personal automobile increased at a 5% annual rate while commercial automobile, although relatively small, is growing at a 25% annual rate. The reader may wish to construct a model along these lines and he will be surprised with the results.

Of course, the volume of data is an important factor in determining what kind of breakdowns of the data are feasible. If the data are subdivided so finely that most groups have only a small volume of data, the subdivisions may accomplish nothing useful. Or to quote Mr. Longley-Cook's delightful analogy, "We may liken our statistics to a large crumbly loaf cake, which we may cut in slices to obtain easily edible helpings. The method of slicing may be chosen in different ways—across the cake, lengthwise, down the cake, or even in horizontal slices, but only one method of slicing may be used at a time. If we try to slice the cake more than one way at a time, we shall be left with a useless collection of crumbs" [4]

Interim Reserving Techniques

Between the annual calculations of an IBNR reserve structure it is necessary to periodically review the reserve from two angles. At the close of each accounting period it must be determined if changes in the amount of the reserve are necessitated by changing exposures. Secondly, the reserves established at the prior year-end must be continuously monitored to see if the loss developments observed are what were contemplated when the reserve was established. To the extent that the actual development is different from the expected and credible, the reserve structure should be "fine-tuned".

It is a relatively easy matter to determine the amount of development

4 Longley-Cook, Lawrence, H., "An Introduction to Credibility Theory," *Proceedings of the Casualty Acturial Society*, Vol. XLIX, 1962, p. 194.

expected in the year following the establishment of the reserve. The IBNR reserve, for a given accident year multiplied by a factor, which is developed by dividing the loss development factor for the period (e.g. second to third) minus unity by the ultimate loss development factor (e.g. second report to ultimate) minus unity, will produce the amount of development expected during the ensuing calendar year (for that accident year). In Exhibit B, the expected development in 1972 on the 1971 accident year would be derived as follows:

$$\frac{1.415 - 1.000}{1.650 - 1.000} \times \$2{,}994{,}000 = \$1{,}911{,}669$$

A summation of similar calculations over all accident years produces the total expected development for the calendar year.

If an IBNR computation tied to expected losses rather than actual losses is employed, the determination of the expected development is a little easier. The expected losses for a given accident year multiplied by the difference between that year's and the previous year's IBNR factor will produce the expected development for the next calendar year. Referring again to Exhibit B, the 1971 accident year development would be: $7,600,000 × (.394 − .142) = $1,915,000. Of course, algebraically, we are accomplishing the same thing and it is only rounding errors that cause the resulting numbers to be slightly different.

Having determined the expected development for the year, the next step is to allocate the expected development to quarters. Here one can make judgments or rely on empirical studies. In the absence of data, it might be reasonable to assume that the cummulative distribution of development by quarter for the most recent accident year is skewed say 40% at three months, 70% at six months, 85% at nine months, 100% at 12 months, and that the distribution for prior accident years is uniform: 25%, 50%, 75% 100%. Upon further study the authors were somewhat surprised to find that their data revealed prior year's development were also skewed; approximate distribution: 33%, 60%, 80%, 100%. The data reviewed were excess of loss and it is recognized that distributions observed may not be typical of ordinary business.

If the quarterly (or perhaps semi-quarterly) monitoring indicates a deterioration, it is necessary to pay attention to where, in terms of accident years, the deterioration exists. If the deterioration occurs in an old accident year one must consider the possible snowballing effect alluded to in the first part of the paper.

Changes in exposure and the concomitant changes in reserves can be handled in a variety of ways. The simplest and most treacherous is to relate the entire reserve to the current year's net earned premium and then apply the resulting factor to the increase or decrease in exposure (at 6/30, this would be the difference between 12 months earned premium at 12/31 and 12 months earned premium as of 6/30; this, of course, simplifies to the difference between earned premium for the first six months of each year). This technique will be quite satisfactory as long as the exposure is increasing or decreasing at a uniform rate (see Exhibit C) but will be considerably wide of the mark if the growth rate varies.

A more refined approach to reserve increases due to exposure changes would take into account varying growth rates. One such approach would be to estimate what the IBNR reserve would be at the end of next year (assuming no change in loss development patterns) if next year's exposure is the same as this year's exposure. In Exhibit A, such a calculation would show that a reserve of $4,295,000 would be necessary. The increment could be budgeted by quarter and then if the exposure as of the interim date appeared to be up, the increment in expected losses would be multiplied by the current years IBNR factor (.394 in the example). This latter approach should result in a fairly orderly change in the reserve over the year.

FISCAL/ACCIDENT YEAR APPROACH

Perhaps the ultimate answer for establishing interim point INBR reserves is a fiscal/accident year system. Working from a fiscal/accident year data base it is possible to create at the close of each quarter a completely new IBNR reserve structure. This procedure will automatically take into account any credible changes in loss development patterns and it will all be keyed to current exposure levels.

Admittedly, creating a fiscal/accident year data base could be expensive and, obviously, the more involved computation procedures will be more time consuming than the rough hewn interim procedures described in the preceding section. Unquestionably, a better product will be produced, although it is recognized that unless the reserves involved are substantial in relation to surplus it may be difficult to justify the effort.

A fiscal/accident year embraces losses which occur in the twelve months running (for example) from 4/1 to 3/30. These losses would be evaluated as of second report (12 months later), third report (24 months later), etc. Loss development triangles similar to Exhibit A would then be assembled. Expected losses, as a function of earned premium, are developed on a fiscal year basis and IBNR calculations would proceed as in Exhibit B.

CONCLUSION

As in so many areas of actuarial endeavor, the setting of IBNR reserves is far from an exact science. As was made clear in the above presentation, there are numerous judgments which have to be made by the individual responsible for the reserves. The methods available range from the crude techniques discussed in the first section to the relatively sophisticated fiscal/accident year approach described in the last section.

No matter what approach is taken, one must be ever mindful of the fact that the forces which operate on the IBNR liabilities are dynamic and frequently will beyond the control of the company. A list of the many factors, internal and external, which will affect IBNR includes: inflation, claims adjusting philosophy, processing lags, no-fault programs, reinsurance arrangements, court back logs, product mix, etc. The list can be very long and, as an illustration of the unusual situations which can occur, a special IBNR reserve at March 31, 1970 was established in our own company to anticipate the effects of the mail strike in effect at that time.

It is hoped that more casualty actuaries will involve themselves in this important area. IBNR reserves deserve more than just a clerical or cursory treatment and we believe, as did Mr. Tarbell, that "the problem of incurred but not reported claim reserves is essentially actuarial or statistical". [5] Perhaps in today's environment the quotation would be even more relevant if it stated that the problem " . . . is more actuarial than statistical".

5 Tarbell, *op. cit.*, p. 276.

HYPOTHETICAL LOSS DEVELOPMENT DATA

Exhibit A

Earned Premiums Net of Commission	Accident Year	First Report	Second Report	Third Report	Fourth Report	Fifth Report	Sixth Report
		Incurred Case Losses (including Allocated Claim Expense)					
$5,000,000	1966	$2,500,000	$3,650,000	$4,200,000	$4,325,000	$4,335,000	$4,330,000
		1.460	1.151	1.030	1.002	.999	
5,500,000	1967	2,150,000	3,225,000	3,775,000	3,965,000	3,960,000	
		1.500	1.171	1.050	.999		
6,000,000	1968	3,250,000	4,500,000	5,050,000	5,150,000		
		1.385	1.122	1.020			
7,000,000	1969	3,700,000	5,200,000	5,775,000			
		1.405	1.111				
7,500,000	1970	3,300,000	4,800,000				
		1.455					
8,000,000	1971	4,250,000					

Exhibit B

HYPOTHETICAL IBNR RESERVE COMPUTATION

Dev. Period	(1) Three Year Data: Beginning	(2) Three Year Data: Ending	IBNR Computation as of Dec. 31, 1971: (3) Loss Dev. Factor: Indicated	(4) Loss Dev. Factor: To Ult.	(5) Accident Year	(6) Expected Losses ϕ	(7) Expected Losses IBNR Factor#	(8) Indicated IBNR	(9) Loss Method IBNR	(10) Adjusted Loss Method IBNR
1st-2nd	$10,250,000	$14,500,000	1.415	1.650	1971	$7,600,000	.394	$2,994,000	$2,763,000	$2,533,000
2nd-3rd	12,925,000	14,600,000	1.130	1.166	1970	7,125,000	.142	1,012,000	797,000	859,000
3rd-4th	13,025,000	13,440,000	1.032	1.032	1969	6,650,000	.031	206,000	185,000	186,000
4th-Ult.	8,290,000*	8,290,000*	1.000	1.000	1968	5,700,000	—	-0-	-0-	-0-
								$4,212,000	$3,745,000	$3,583,000

*Only two years of data
ϕEarned Premium (net of commission) × .95
#Factor = 1.000 − 1.000/ultimate loss development factor

Exhibit C

HYPOTHETICAL IBNR COMPUTATION

	As Of 12/31/71			As Of 12/31/72		
	Expected Losses	IBNR Factor	IBNR Reserve	Expected Losses	IBNR Factor	IBNR Reserve
1966	$1,000,000	−.062	$ −62,000	$ —	—	—
1967	1,200,000	−.014	−17,000	1,200,000	−.062	$ 74,000
1968	1,440,000	.078	112,000	1,440,000	−.014	−20,000
1969	1,728,000	.232	401,000	1,728,000	.078	135,000
1970	2,074,000	.535	1,110,000	2,074,000	.232	481,000
1971	2,488,000	.814	2,025,000	2,488,000	.535	1,331,000
1972				2,986,000	.814	2,431,000
Total	$9,930,000		$3,569,000	$11,916,000		$4,284,000

Increase in the current year's exposure is $498,000 ($2,986,000 − $2,488,000). Increase in total IBNR is $715,000 ($4,284,000 − $3,569,000).

The increase in IBNR reserve can be developed by applying the ratio of the 12/31/71 IBNR to the 1971 premiums to the increase in expected losses. For example, $3,569,000/$2,488,000 = 1.435 and 1.435 × $498,000 = $715,000.

At first glance it seems odd that the increase in the IBNR reserve ($715,000) would be greater than the increase in exposure ($498,000). This phenomenon results from the fact that it is not only the current year exposure that is increased but each of the prior years is also increased (e.g. the 1st prior year to which a factor of .535 applies has increased from $2,074,000 to $2,488,000). To put it in another way, while 1972 exposure has increased by $498,000 over 1971, the total exposure subject to an IBNR factor has increased by $1,986,000 ($11,916,000 − $9,930,000).

Assumptions: Premium growth rate of 20%, expected losses are a direct function of premiums earned, IBNR factors unchanged. (Factors shown above are much higher than the factors found in Exhibit B and are typical of what can be expected on excess of loss business).

ACTUARIAL APPLICATIONS IN CATASTROPHE REINSURANCE

LEROY J. SIMON

1. The pricing of catastrophe reinsurance treaties is much more of an art than an actuarial science. The parties involved are usually well informed and are free to bargain in the spirit of free and open competition. Reinsurance has also been favored with a high degree of integrity on the part of all participants.

One of the important contributions that the actuary can make to the reinsurance field is the maintenance of logical consistency among the various alternatives that may be considered at different stages of the negotiation process. It is quite common for modifications in terms to be discussed, such as altering the retention; changing the thickness of the layer; subdividing the layer into two or more strata. Although it may not be possible to claim that the various alternatives are actuarially equivalent in the strict sense, the actuary can help assure that they are at least logically consistent with each other. I would hasten to add that other positive contributions from the actuary in the reinsurance field would include: pricing estimates themselves; determining incurred but not reported and developmental reserves; assessing inflationary impact and evaluating financial aspects of his own company or of prospective reinsureds.

A great deal of sound thinking, together with innovation and the open acceptance of new ideas, is required in the reinsurance field. An actuary's training is very helpful in developing the type of individual needed. In reinsurance, very few pricing situations lend themselves to statistical or rating manual analysis. However, the maintenance of logical consistency within various reinsurance quotations is greatly aided by a mathematical model and appropriate study of the implied actuarial relationships. The purpose of this note is to study some of the relationships in the catastrophe reinsurance area.

2. Let there be a reinsurance treaty with an exposure to the reinsurer* L excess over a specified retention at a pure premium of P. Attention

* The author is most indebted to Matthew Rodermund for suggesting an improvement in the paper that led to this definition of L. Instead of considering the vertical slice of a layer which represents the amount at risk to the reinsurer, an alternative definition of L as the complete 100% thickness of the layer could also be used. The premiums thus determined would be on a 100% basis and would have to be modified to fit the terms of the specific cover and the reinsurer's portion.

will be focused on situations where it will be appropriate to assume that any loss which hits the cover will run all the way through it, that is, all losses will be total losses. While this assumption is not strictly true, the reinsurance company normally assesses treaties on this basis, and it is very nearly the true situation. If this assumption causes difficulty, it may be necessary to apply this model to narrow sub-layers of a given treaty.

Further, we shall consider here treaties which are unbalanced, that is, they attach at a high level such that the pure premium will be small with respect to the limit L. It will be further assumed that the Poisson distribution (with parameter m) is the appropriate mathematical model for the occurrence of claims, and we shall designate p_c as the probability of having exactly c claims.

Some of the functions of interest are:

(2.1) $p_c = m^c e^{-m}/c!$

And, in particular:

(2.2) $p_0 = e^{-m}$

(2.3) $p_1 = m e^{-m}$

(2.4) $\sum\limits_0^\infty p_c = 1$

(2.5) $\sum\limits_1^\infty p_c = 1 - e^{-m}$

(2.6) $\sum\limits_2^\infty p_c = 1 - e^{-m} - me^{-m}$

The last two functions are the probability of having one or more claims and two or more claims respectively. Also in general:

(2.7) Expected Losses = Expected Pure Premium

3. Let us first take the simple case of a treaty designed to cover one catastrophe event in the year with no agreement to reinstate the coverage if the insured utilizes it during the one year period. In this case we have:

$$P_1 = 0.p_0 + L \sum_1^\infty p_c$$

or:

(3.1) $P_1 = L(1 - e^{-m})$

It follows that:

(3.2) $e^{-m} = 1 - P_1/L$

and:

(3.3) $m = -ln(1 - P_1/L)$

where *ln* is the natural logarithm.

Example A: Analyze a 90% of \$10 million cover which sells for a gross premium of \$1 million with no automatic reinstatement provision. It will be assumed that commisssion to the broker and the reinsurer's overhead and profit provisions total 18%. Therefore, $P_1 = 1(.82) = .82$ and $L = .90(10) = 9.0$. We can now note that the expected number of times the cover will be hit in a one year period by (3.3), is .09553. The probability of running claim free for a year is, by (2.2) and (3.2), .90889. From equation (2.6) we calculate the probability of hitting the cover two or more times to be .00428.

4. A more common case in reinsurance is for a catastrophe treaty to have an automatic reinstatement provision. In this case the insured will receive a reinstatement of the amount of cover he has utilized in hitting it, up to an amount *L*, so that the cover is immediately available again for a second event. In exchange for this reinstatement he pays a gross premium (and hence a pure premium) equal to the proportion of the cover which is reinstated times the proportion of time left to run in the contract year times the original gross premium (and hence times the original pure premium *P*). To develop the mathematics of this case, let *f* designate the mean portion of the year from the start of the contract to the occurrence of the first claim.

We may then write the following equations:

$$\text{Expected Losses} = 0.p_0 + Lp_1 + 2L\sum_{2}^{\infty} p_c$$

$$= Lme^{-m} + 2L(1 - e^{-m} - me^{-m})$$

or:

(4.1) $\text{Expected Losses} = 2L - 2Le^{-m} - Lme^{-m}$

(4.2) $\text{Expected Pure Premium} = P_2 + P_2\,(1 - f)$

Hence, by (2.7):

(4.3) $2P_2 - P_2 f = 2L - 2Le^{-m} - Lme^{-m}$

This can be rearranged in many ways. To solve for m when P and L are given, the following form is helpful:
(4.4) $e^{-m}(2L + Lm) - 2(L - P_2) - P_2 f = 0$

To obtain an expression for f, we note that for zero claims, $f = 1$; for one claim, $f = 1/2$; for two claims, $f = 1/3$; for three claims, $f = 1/4$ and so forth (we are assuming that a claim is equally likely to occur at any time during the year). Hence:

$$f = (1)e^{-m} + (1/2)me^{-m} + (1/3)m^2e^{-m}/2! + (1/4)m^3e^{-m}/3! + \ldots$$
$$= e^{-m}(1 + m/2! + m^2/3! + m^3/4! + \ldots)$$

(4.5) $f = (1 - e^{-m})/m$

Substituting this value in (4.4):

(4.6) $e^{-m}(2L + Lm + P_2/m) - P_2/m - 2(L - P_2) = 0$

Thus, the case of reinstatement requires an iterative process whereby m is approximated, substituted in (4.6) and the value improved through repeated trials of values of m.

Example B: Analyze a treaty for 95% of \$5 million with a gross premium of \$1 million with one automatic reinstatement. It will be assumed that there is no brokerage commission and that profit and overhead are 12%. We thus have $L = .95(5) = 4.75$ and $P_2 = 1(.88) = 0.88$. Substituting these in (4.6) we write $e^{-m}(9.50 + 4.75m + .88/m) - .88/m - 7.74 = V$. As a first approximation, $m = -ln(1 - .88/4.75) = -ln(.81474) = .205$ by (3.3). Since the formula used for the approximation does not include consideration of the reinstatement coverage, it can be assumed that this approximation of m is a little too high so we shall start off with an initial value, $m_1 = .20$. This value is substituted in the equation for V to produce a value V_1. A second estimate designated m_2 is then made and the process repeated. Thereafter, improved estimates of m may be obtained by linear interpolation or extrapolation of the values of V until we reach a value of m where V equals zero.

In this example, the following values are obtained:

$m_1 = .20$ $\qquad V_1 = +.01815$

$m_2 = .204$ $\qquad V_2 = +.00103$

$m_3 = .204267$ $\qquad V_3 = -.00011$

$m_4 = .204244$ $\qquad V_4 = -.00001$

The process has converged quite rapidly indicating that the function V must be very nearly a straight line in the vicinity of m. We terminate after obtaining m_4 and conclude that the conditions of this treaty imply $m = .20424$.

5. Suppose now that we are interested in determining the relationship between the pure premium P_1 for a one event, no reinstatement cover and the pure premium P_2 for a similar cover involving one automatic reinstatement at pro-rata of the gross premium (and hence the pure premium).

From (3.1):

(5.1) $P_1 = L(1 - e^{-m})$

From (4.6):

(5.2) $P_2 = (mL)(2 - 2e^{-m} - me^{-m})/(e^{-m} + 2m - 1)$

To express P_2 in terms of P_1 and L, which will be given, use is made of (5.1), (3.2) and (3.3) in rewriting (5.2) as:

(5.3) $P_2 = L(ln(1 - P_1/L))[2 - (1 - P_1/L)(2 - ln(1 - P_1/L))]/[P_1/L + 2\, ln(1 - P_1/L)]$

Example C: If the reinsured in Example A now asked for an automatic reinstatement at pro-rata of premium, what is the new pure premium? $P_1 = .82$, $L = 9.0$. Thus by (5.3, $P_2 = .82057$.

It is interesting to note that the ratio

$$P_2/P_1 = .82057/.82$$
$$= 1.0007/1.000$$

Example D: In the Example B case, we observed that $L = 4.75$ and $P_2 = 0.88$ resulting in $m = .20424$. What is that treaty worth without the automatic reinstatement? By (5.1):

$$P_1 = 4.75(1 - e^{-.20424})$$
$$= .87748$$

Also note that:

$$P_2/P_1 = .88/.87748$$
$$= 1.0029/1.000$$

As a corollary, it is of interest to calculate the expected pure premium income under Example B. This would be the mean amount one would expect to actually collect over the long run. By (4.2), (4.5), (3.3) and (3.1) one would expect to collect $P_2[2 - (P_1/L)/(- \ln(1 - P_1/L))] = .96405$

Naturally, this same result could be obtained from (4.1) since the identity of (4.1) and (4.2) was used in determining m.

6. As an extension of the theory, consider the quotation of a premium for a third event cover when a company has hit its original cover once. The original cover has one automatic reinstatement and the company desires the additional protection of the third event cover for the remainder of the year. One would simply apply the appropriate equation to the conditions of the original cover to determine the implied m and then apply it to the future cover.

Example E: What would the pure premium be for a third event cover to be effective for the last half of the year when the reinsured in Example B has hit the cover in the middle of the policy period and desires the additional coverage for the last six months? We know from the solution of Example B that $m = .20424$. On pure theory, m for the last half of the year is $.10212$.*

To evaluate the pure premium, we have:

$$P = 0.p_0 + 0.p_1 + L \sum_{2}^{\infty} p_i$$
$$= L(1 - e^{-m} - me^{-m})$$
$$= 4.75\,(1 - e^{-.10212} - .10212e^{-.10212})$$
$$= .02314$$

* Practical considerations such as the effective and expiration dates of the treaty with respect to the hurricane season would affect this "pure theory". Also, since the reinsured has recently hit the cover, his new premium would undoubtedly be based on a higher value for m. A possible approach might be to re-price the original cover in light of the current experience and thus determine a new value for m.

In dollars, then, the insured would pay a pure premium of $23,140. Note how this would represent a mimimum which the reinsurer could use but which would undoubtedly be far below the final bargaining premium. It is apparent that if the reinsured makes similar actuarial calculations, he will be in a much improved bargaining position when the negotiations begin.

7. Many other interesting applications can be made of these simple concepts in order to keep the pricing of covers in a rational relationship to one another. Perhaps future theoretical developments will enable us to use more sophisticated models than the simple Poisson — for example the Pareto or the negative binominal.

THE RELATIONSHIP BETWEEN NET PREMIUM WRITTEN and POLICYHOLDERS' SURPLUS

Raymond W. Beckman
&
Robert N. Tremelling II

Nothing is so firmly believed as what we least know.

—Michel DeMontaigne

Introduction

Property-liability insurance company financial strength is important to the purchasers of the insurance product as well as to company stockholders, for this strength protects insureds against any possible company defaults. Because of this unusual buyer *caveat,* there are advantages to developing some generally applicable measures of insurance company strength. To meet this need, Roger Kenney[1] developed two of these measures approximately forty years ago. At that time insurance law did not permit multiple line companies, and consequently Kenney devised two measures, one for fire insurers and another for casualty insurers. For fire insurers the standard test was a one-to-one relationship between the unearned premium reserve and policyholders' surplus, i.e., total assets less total liabilities (this is the statutory definition of policyholders' surplus). For casualty insurers the standard test was a two-to-one relationship between net premium written and policyholders' surplus. When insurers were allowed to write both property and casualty business, the Kenney tests were maintained and applied separately after an allocation (based on premium) of surplus to distinct property and casualty "surplus accounts."

Today, the relationship of unearned premium to surplus has almost been forgotten, and the relationship between net premium written and policyholders' surplus is of prime interest, although there is no generally

[1] Kenney, R., *Fundamentals of Fire and Casualty Insurance Strength*, The Kennedy Insurance Studies, Dedham, Massachusetts, 1967, fourth ed., pg. 19ff, 97ff.

accepted standard. The June 1970 report of the National Association of Insurance Commissioners, ***Measurement of Profitability and Treatment of Investment Income in Property and Liability Insurance,*** discussed "The Amount of Needed Capital and Surplus." Included in the report was a quote from Mr. Thomas Morrill, President of State Farm Mutual Automobile Insurance Company.[2]

> "It is a well-established insurance management principle that premium volume should be kept within a reasonable relationship to surplus (or capital and surplus in a stock company), although there is no consensus as to what that relationship should be. The purpose of surplus is first for solvency, second for solidity. Surplus must absorb the ebb and flow of losses from both underwriting and investments. While the element of risk present in both the underwriting and investment portfolios affects the need for surplus, there is a rule of thumb which sets $2.00 of premiums written for each dollar of surplus as conservative, $3.00 or $4.00 of premium as safe, but beyond that caution should be observed."

The report concludes, " . . . the current measures of solidity relating premium to surplus are little more than rules of thumb. But until advanced computer and mathematical techniques can be applied to the 'needed' surplus question, they are the best standards we have".

More recently, (February 3, 1972) the Insurance Commissioner of New Jersey rendered a decision concerning ratemaking. That decision included an evaluation that the required capital (net worth) is one-half of the premium. In other words, a premium-net worth ratio of two-to-one is considered sound by the New Jersey Insurance Commissioner.

There have been other devlopments recently in the regulatory aspects of the insurance business. Eleven financial tests are completed on all insurance companies' Annual Statements, beginning with the 1971 statement. One of these tests, Number 7, examines the net premium written-net worth (statutory policyholders' surplus plus 20% of the unearned premium reserve) ratio. If this ratio exceeds 3.00, the company will fall into the "bad range".

It is apparent that the premium-surplus ratio is undergoing extensive revaluation. This paper explores the entire net premium written-policy-

[2] National Association of Insurance Commissioners, *Measurement of Profitability and Treatment of Investment Income in Property and Liability Insurance,* June, 1970, p. 125.

holders' surplus relationship, first by providing insight with a historical perspective followed by the short term future outlook, then ending with the significance of this relationship and conclusions. "Industry" data are for all stock companies combined.

Review of Stock Insurance Industry Premium-Surplus Ratio, 1928-1970

The net premium written-policyholders' surplus ratio is the primary method of quickly measuring insurance company strength, largely because of a lack of other useful and meaningful measures of insurers' strength. Given the large importance ascribed to this ratio, it would seem reasonable to expect a relatively stable premium to surplus relationship, perhaps showing long term trends. This is not the case. The following chart, Exhibit I, shows the premium-surplus ratio since 1929 and illustrates that there have been substantial fluctuations, both from year to year and over the long term. To understand the behavior of this series over the last forty years, it is helpful to review the individual components—net premium written and policyholders' surplus.

The net premium written of all stock companies, as depicted on Exhibit II, has shown fairly stable growth, not patently dependent on short term economic conditions but generally following long term economic growth. It is true that during the last several years the rate of growth in premium volume has increased substantially, but it is premature to predict the beginning of a new era rather than a slight statistical fluctuation. Given the stability of the premium volume and the fluctuations in the premium-surplus ratio, we are led to the conclusion that policyholders' surplus has been the volatile element.

In an effort to explain the year to year fluctuations in policyholders' surplus, we performed statistical regression analyses and conclude that the major cause of fluctuations in policyholders' surplus is changes in the stockmarket. The single series, Standard and Poor's 500 Stock year end closing average, explains 64% of the annual variation (i.e., yearly percent change) in policyholders' surplus. The following chart, Exhibit III, further illustrates how successfully this stockmarket index can be mathematically used to explain the policyholders' surplus of the insurance industry, although the residuals are not random. In this case, there is a 98% correlation between the two series. Additional statistical analysis might indicate that underwriting results or changes in leverage help explain the remaining variation in policyholders' surplus but, for our purposes, the

EXHIBIT I

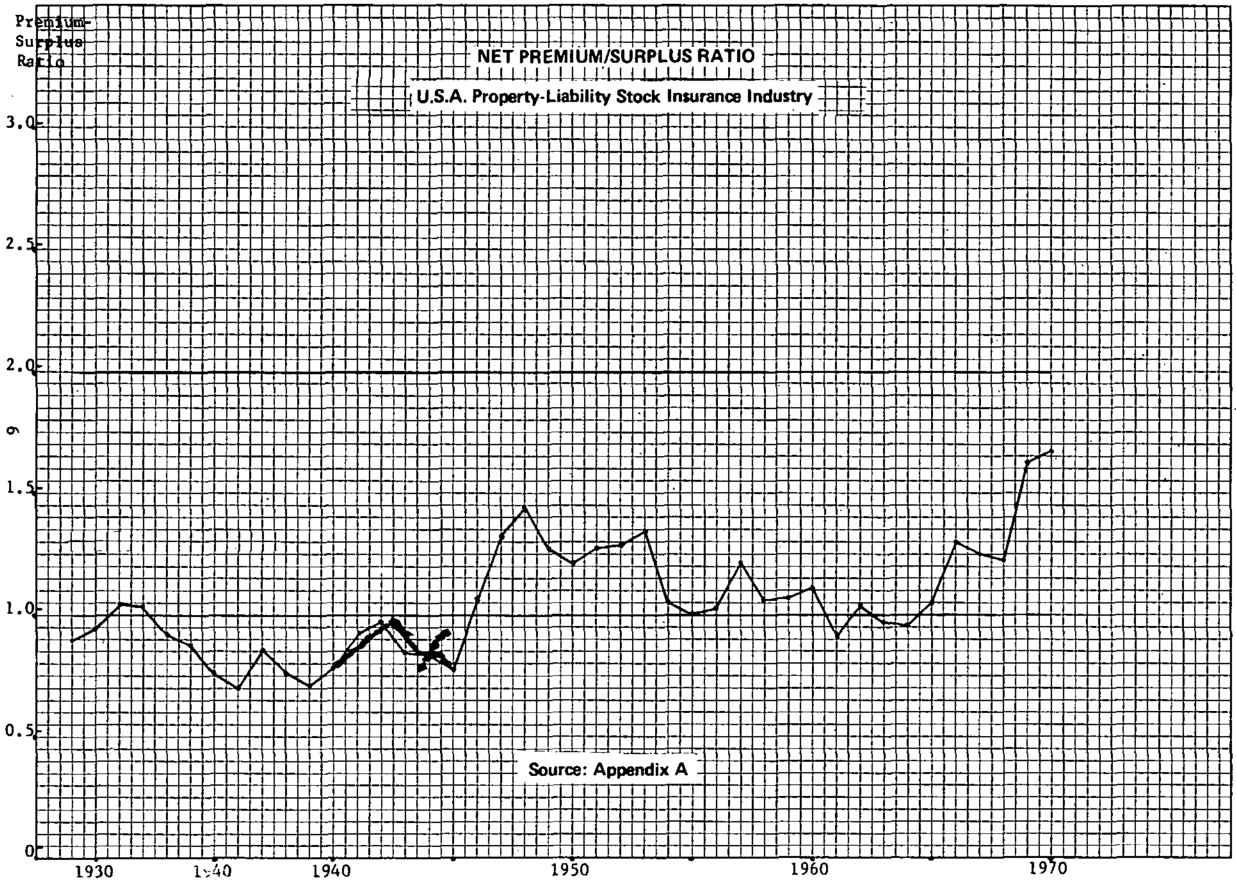
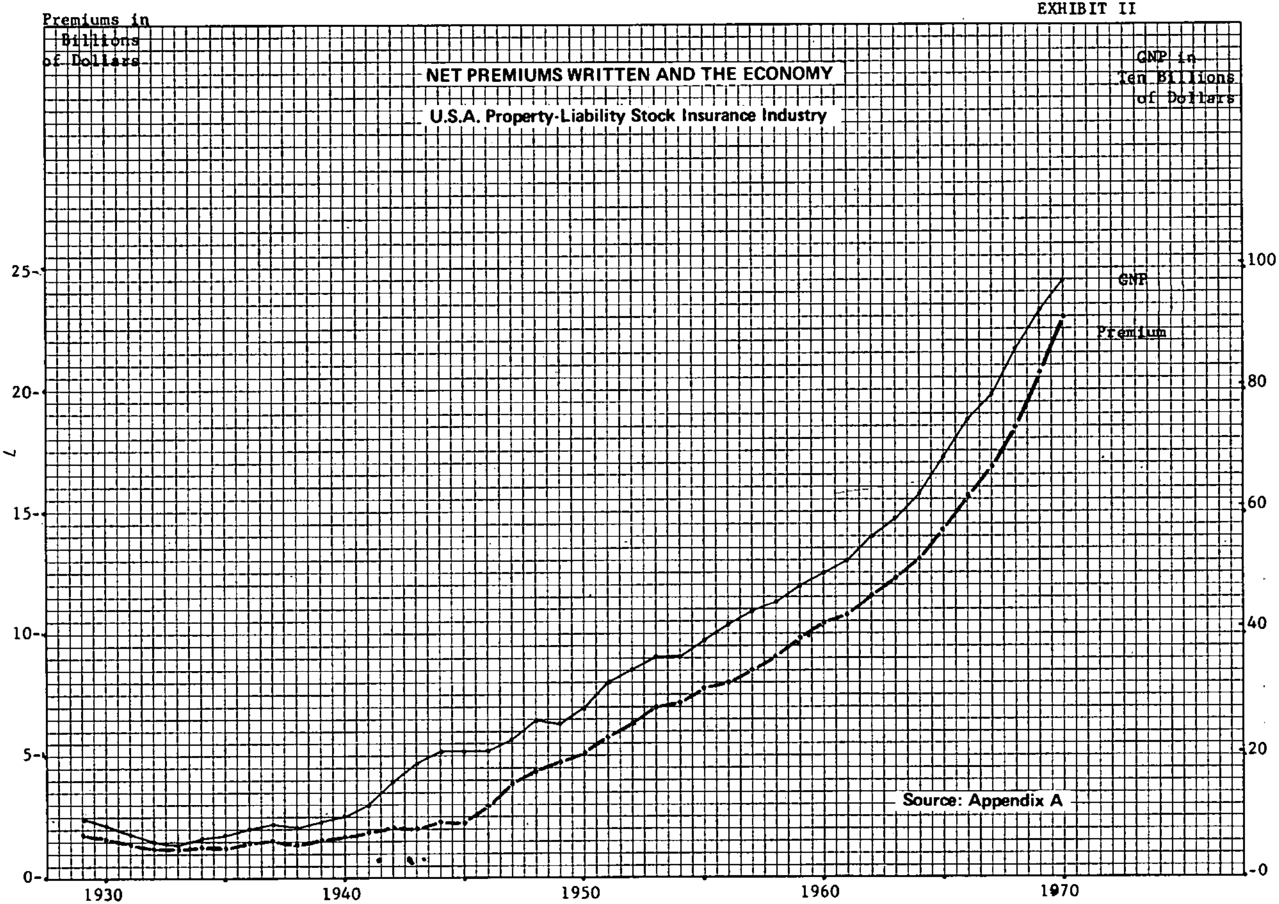
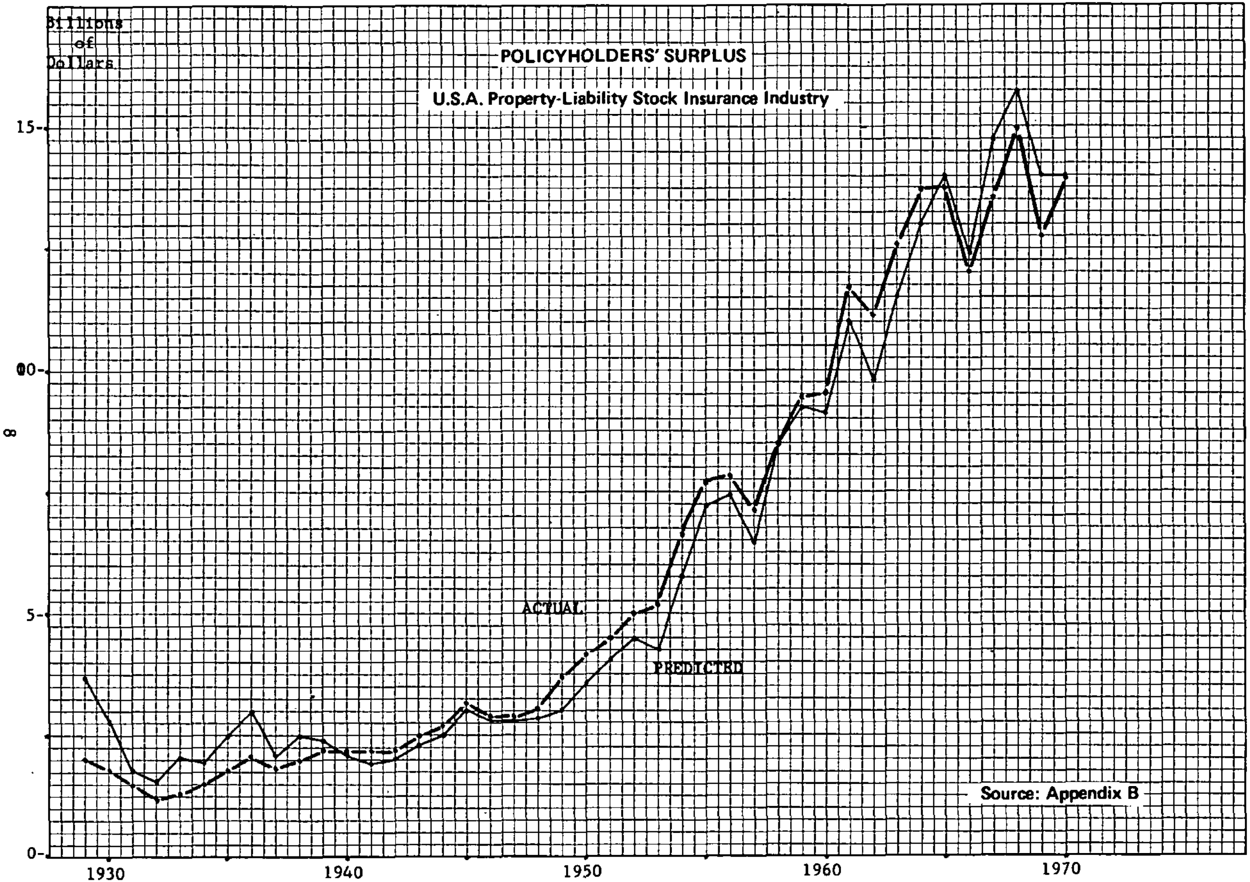

Premium-
Surplus
Ratio
NET PREMIUM/SURPLUS RATIO
U.S.A. Property-Liability Stock Insurance Industry
3.0
2.5
2.0
1.5
1.0
0.5
0
1930
1940
1950
1960
1970
Source: Appendix A

EXHIBIT II



Premiums in Billions of Dollars
NET PREMIUMS WRITTEN AND THE ECONOMY
U.S.A. Property-Liability Stock Insurance Industry
GNP in Ten Billions of Dollars
GNP
Premium
Source: Appendix A
0
5
10
15
20
25
0
20
40
60
80
100
1930
1940
1950
1960
1970

EXHIBIT III


Billions of Dollars
POLICYHOLDERS' SURPLUS
U.S.A. Property-Liability Stock Insurance Industry
15
10
5
0
ACTUAL
PREDICTED
Source: Appendix B
1930
1940
1950
1960
1970

Standard and Poor's 500 Stock Index is adequate. The premium volume, surplus, and stockmarket index are all shown on Exhibit IV.

There are additional factors which have not been reflected but which would influence the changes in the premium-surplus ratio. They are:

(1) The statistics used are Best's Aggregates and Averages which, for policyholders' surplus, is the sum of the surplus of all companies, and not a true indication of the policyholders' surplus in the insurance industry obtained by consolidating all insurance groups; this factor tends to overstate the surplus in the insurance business. Furthermore, the disability business of some insurers has been ceded to life company affiliates in recent years, causing an inconsistency in net premium written (we have adjusted for the only transaction of this type identified in Best's[3]).

(2) The flight of "surplus surplus" from the insurance business to non-insurance parent corporations has also had a substantial impact on policyholders' surplus in recent years and tends to increase the premium-surplus ratio.

(3) Life insurance company affiliates are included in a property-liability insurance company's assets at book value whereas, in reality, the market value may be substantially greater. This method of accounting for affiliates can result in an underestimation of policyholders' surplus.

Property-Liability Insurance Industry: Premium-Surplus Expectations

In this section we predict what will happen to the premium-surplus ratio in the next five years and we explore the significance of this forecast.

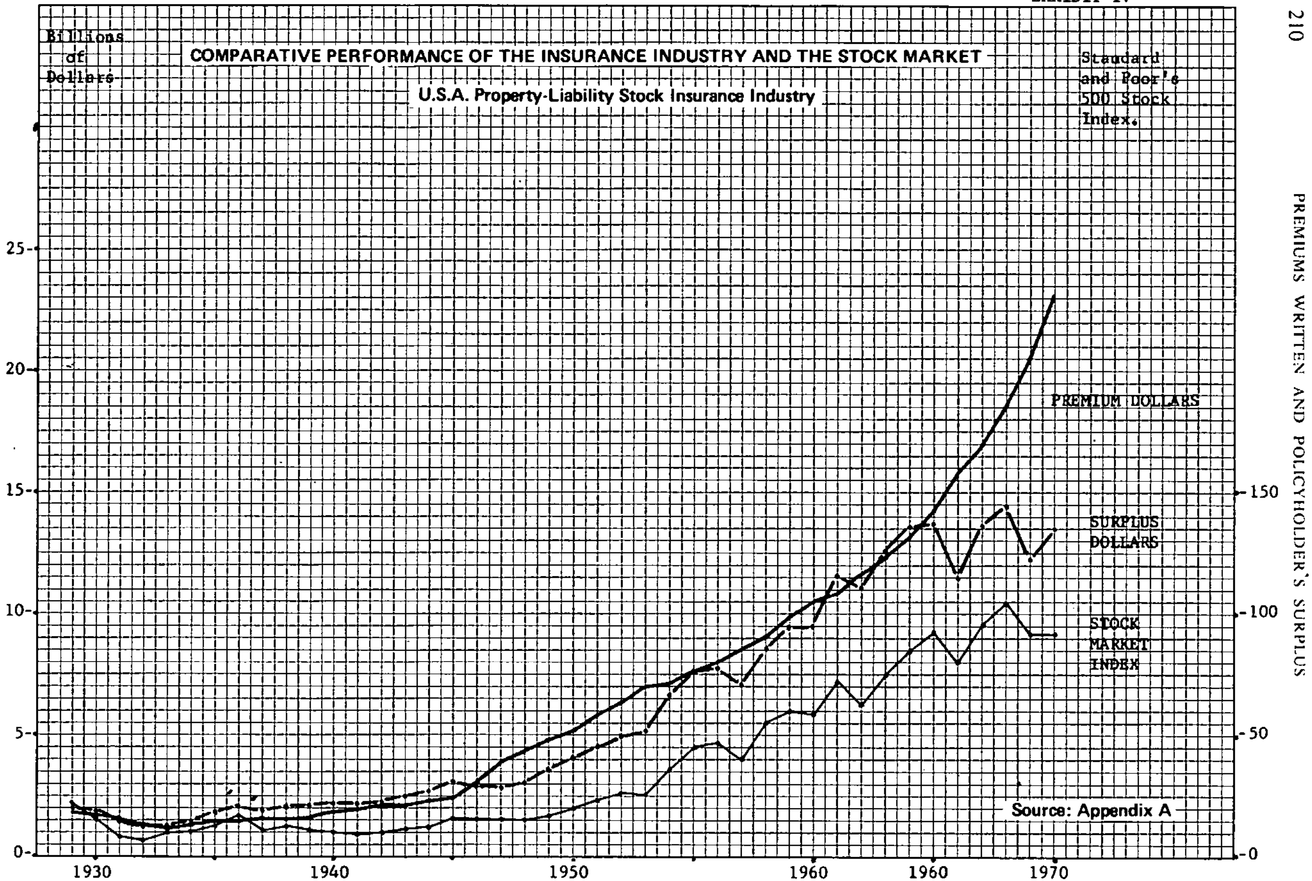
Over the last forty years, industry premium volume has increased at an average annual rate of 7%. However, during the last five years, premiums have grown at a 10% annual rate. Consequently, projecting future growth is extremely difficult.

There are several factors which determine premium growth, chief among which are:

(1) Rate Changes—Rate changes reflect changes in both frequency and severity. Although it is difficult to predict changes in frequency, we

[3]Premiums for 1963-1970 have been adjusted to include Traveler's A & H.

EXHIBIT IV


Billions of Dollars
COMPARATIVE PERFORMANCE OF THE INSURANCE INDUSTRY AND THE STOCK MARKET
U.S.A. Property-Liability Stock Insurance Industry
Standard and Poor's 500 Stock Index.
PREMIUM DOLLARS
SURPLUS DOLLARS
STOCK MARKET INDEX
Source: Appendix A
0
5
10
15
20
25
0
50
100
150
1930
1940
1950
1960
1960
1970
PREMIUMS WRITTEN AND POLICYHOLDER'S SURPLUS

can expect severity (average claim size) to increase because of inflation.

(2) Insurance Coverage—Inflation has a second impact on premium volume because as inflation erodes the value of the dollar, more coverage will be needed in order that property be insured to full value. Also, the use of deductibles is increasing and coverages are changing.

(3) Insurance Buying Population—Defining this class as persons age 20 and over, the insurance buying population is as follows:

Year	Population Age 20 and Over	Average Annual Increase
1964	116.4 Million	—
1970	127.3 Million	1-1/2%
1975	138.2 Million	1-3/4%
1980	150.2 Million	1-3/4%

Source: U. S. Department of Commerce (Series D projections).

From the above we conclude that growth in the insurance buying population will be greater in the next ten years than it was during the last six years.

In addition to the above factors, there are numerous other influences which cannot be quantified but are also important. These include: the adoption of no-fault auto insurance, better built and safer autos, safer working conditions, the greater proportion of low exposure service industries which reduces workmen's compensation premium volume, the higher average number of vehicles per person, greater insurance awareness of the general population, and mandatory automobile insurance coverages. The possibility of the federal government selling property-liability insurance coverages must not be overlooked, but this analysis is based on the assumption that the federal government will not intervene.

The impact of the foregoing on the stock company insurance industry net premium written is difficult to quantify, but an annual growth of 8 to 10% appears reasonable. This would indicate a 1976 premium volume of approximately $38 billion for all stock companies combined.

Projecting the 1976 policyholders' surplus in the insurance business is subject to substantial error. Assuming no significant in or out flow of capital, the major problem is to estimate the December 31, 1976, Standard and Poor's 500 Stock Index. The average annual appreciation in this stock index over the last five years, 1967 to 1971, is 5.5%; and this growth rate will be used for the next five years. Based on the relationship of annual growth in stocks to annual growth in surplus, the projected 1976 policyholders' surplus for the stock insurance industry is $18.4 billion (refer to Appendix B).

The premium-surplus ratio for the stock insurance industry might therefore be expected to increase from the 1970 level of 1.7 (the highest in the entire 40-year time period reviewed) to a new high of 2.1 in 1976. Furthermore, if the same patterns continue through 1980, the premium-surplus ratio will exceed 2.5. These simple extrapolations indicate that the entire industry will exceed the two-to-one boundary within the next decade. If the entire industry is at a 2.5 ratio, one would expect that many companies' premium-surplus ratio would be substantially higher. The significance of these higher ratios will be explored in the third section of this report.

Significance Of Premium-Surplus Ratio

In light of the volatility of the premium-surplus ratio, it is quite logical to ask why so much importance is given to the premium and surplus relationship. As mentioned earlier, this standard was devised in an attempt to measure the strength of insurance companies. Because of the publicity received, the use of this ratio has developed into a widely held "rule of thumb" for what is acceptable in the way of an operating ratio for insurance companies. The New York Insurance code that was revised in the late 1930's contained a provision that "a stock company cannot pay dividends of more than ten per cent of the capital stock unless (a) surplus to policyholders is twenty-five per cent of its unearned premium liability, or, (b) surplus above capital equals fifty per cent of the minimum capital required, whichever is greater."[4] This provision in the New York code is not particularly restrictive, limiting stockholder dividends only when the company is not sufficiently strong, and it says nothing about a desirable premium-surplus relationship. Nevertheless, the two-to-one rule is much

[4]Kenney, *Op. Cit.*, p. 98.

more widely held and discussed than any other measure of insurance company strength.

It is quite interesting that, in Mr. Kenney's "Fundamentals of Fire and Casualty Insurance Strenght," it is acknowledged that there is disagreement on the two-to-one rule and that variations from this rule are acceptable under certain circumstances.[5] However, we would go one step further and state that the application of this premium-surplus ratio is illogical for many reasons, including:

(1) The premium-surplus ratio is applied by individual company and not by group, thus causing a distortion that produces a lower ratio for the individual companies than is actually true for the group in the aggregate.

(2) The importance of the premium volume is dependent upon the geographical spread of business that the company writes, the mix of business, reinsurance arrangements, the general profit margin included in the lines of business, the long term profit record of the company, and the size of the company. Thus each company should be independently analyzed.

(3) Policyholders' surplus is a function of the total amount of paid-in capital and surplus, the rate of growth of the insurance company, the underwriting profit margin achieved, the adequacy of the loss reserves, and to a large extent, the amount of money invested in stocks and year-end level of the stock market.

(4) The rationale for using policyholders' surplus rather than stockholders' equity is ". . . the much vaunted equity is 'locked up' even when conditions become critical within a company. Which is to say that come a catastrophe or an economic upheaval, you can't release that equity without selling the birthright of a company through panicky reinsurance of the entire insurance portfolio."[6] This is a nice statement made by Mr. Kenney, but it tends to cloud the issue, being more a concern of the stockholder than of the policyholder. The premium-surplus ratio measure of strength is a tool for the policyholder and is only a measure of leverage in the eyes of the stockholder. In other words, a stockholder should prefer a higher premium-surplus ratio, while a policyholder might prefer a lower ratio.

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[5] *Ibid*, p. 99.

[6] *Ibid*, p. 102.

(5) Any good measure of insurance company strength should apply to all types of companies, and the two-to-one rule is probably not applicable to mutual insurance companies and certainly not appropriate for many specialty insurers. For example, specialty insurers might be more or less risky depending on the individual company and the specialty lines written.

In summary, the premium-surplus ratio is not completely accepted, cannot be consistently applied, and, in several respects, is illogical.

In view of the questions raised on the premium-surplus relationship, it might be well to consider the objectives of this ratio. As mentioned previously, the use of the premium-surplus ratio is an attempt to measure the strength of an insurance company. Strength of a company means the ability to withstand the risks of insurance. Although generally not delineated, there are two types of risks inherent in the insurance business:

(1) Underwriting risks, which is the exposure of surplus from normal insurance underwriting operations of an insurance company, and

(2) Investment risk, which is encountered because most insurance companies invest in the stock market.

Given that there are two different types of risks for insurance companies, the logical question concerns the relative risks to the net value or worth of an insurance company by reason of underwriting operations versus investment operations. For purposes of this analysis we have defined net worth simply as capital and surplus plus the equity in the unearned premium reserve (assumed to be 20%). During any calendar year the underwriting risk to net worth is equal to the underwriting profit or loss during the year plus the amount of investment income derived from policyholder-supplied funds. (It is not unreasonable to assume that policyholder-supplied funds produce approximately one-half of net investment income, excluding capital gains.) The investment risk during the calendar year is represented by realized and unrealized capital gains plus the remaining one-half of net investment income. Federal income taxes have been ignored.

The following table sets forth, for the period 1940-1970, the impact on net worth from underwriting risk and investment risk (as defined above) in each calendar year. The source of the unadjusted data is Best's Aggregates and Averages.

U.S.A. Property-Liability Stock Insurance Industry
(millions of dollars)

Year	Net Worth At Jan. 1*	Insurance Profit or Loss+	Investment Profit or Loss	Increase or Decrease In Net Worth From: Insurance	Investment
1971	$16,262	NA	NA	NA	NA
1970	14,785	$566	$ 530	3.83%	3.58%
1969	16,805	223	−1,111	1.33	−6.61
1968	15,380	349	1,729	2.27	11.30
1967	13,711	503	1,809	3.67	13.19
1966	15,265	551	−1,000	3.61	−6.55
1965	15,206	1	1,040	0.0	6.84
1964	14,009	43	1,430	0.31	10.20
1963	12,558	141	1,657	1.12	13.20
1962	13,068	339	−566	2.59	−4.33
1961	10,829	340	2,206	3.14	20.37
1960	10,670	362	359	3.39	3.37
1959	9,820	338	754	3.44	7.68
1958	8,227	151	1,830	1.84	22.24
1957	8,891	−131	−396	−1.47	−4.46
1956	8,740	79	365	0.90	4.18
1955	7,687	452	950	5.88	12.36
1954	6,143	566	1,402	9.21	22.83
1953	5,849	496	104	8.48	1.78
1952	5,344	332	402	6.21	7.52
1951	4,465	149	409	3.34	9.16
1950	3,934	317	474	8.06	12.05
1949	3,270	528	421	16.15	12.88
1948	3,077	294	58	9.56	1.89
1947	3,019	17	43	0.57	1.42
1946	3,285	−75	−89	−2.28	−2.71
1945	2,829	106	441	3.75	15.59
1944	2,584	142	261	5.50	10.11
1943	2,312	219	266	9.47	11.51
1942	2,248	135	23	6.01	1.03
1941	2,284	119	−25	5.21	−1.01
1940	2,250	131	−3	—	—
			MEAN:	4.17	7.02
			STANDARD DEVIATION:	3.87	8.10
			VARIANCE:	15.00	65.69

*Capital and surplus plus equity (20%) in unearned premium reserve

+Underwriting profit or loss plus 50% of investment profit or loss

The foregoing table clearly demonstrates that risk (i.e., the variation in rate of return) from insurance operations is minimal when compared to the risk resulting from stock market fluctuations.

The underwriting risk cannot be avoided as long as a company continues doing insurance business, although the risk is minimized by reinsurance, diversification, and the stabilizing aspects of large size. On the other hand, given the current practice of carrying bonds at amortized values, the investment risk could be minimized by eliminating investments in the stock market and investing all assets in bonds. Such an investment policy would have additional advantages:

(1) Net investment income and net operating income, the reported earnings of the company, would increase.

(2) Fluctuations in policyholders' surplus and the resulting fluctuations in the premium-surplus ratio would be minimized.

(3) It would avoid any possible problem of the accounting for marketable securities, currently being investigated by accountants. However, if the evaluation of bonds is changed to market value, the stabilizing aspect of bond investments would be at least partially lost.

The long term impact on policyholders' surplus should undergo further investigation. Although net operating income, and consequently increases in policyholders' surplus, would be greater if all investments were in bonds, it is true that the past growth in surplus resulted largely from the appreciation of stock market investments. The significance of this point, however, is contingent upon an evaluation of the long term prospects of investments in the stock market and should not hinge solely on the historical appreciation realized by stock investments.

In summary, the fluctuations of the net worth of insurance companies arise primarily from investment operations, particularly capital gains and losses. On the other hand, the substantial growth of policyholders' surplus and net worth over the last forty years can, to a large extent, be attributed to investment appreciation.

CONCLUSIONS

The foregoing historical analysis and projections lead to several basic conclusions:

(1) The stock market is the major factor affecting policyholders' surplus and the premium-surplus ratio.

(2) The premium-surplus ratio measures the leverage of an insurance company and consequently the stockholders should prefer a higher ratio. From the policyholder's viewpoint, this ratio is an indication of the strength of the insurer, and thus a lower ratio indicates a more heavily capitalized and "stronger" insurer.

(3) The net premium written-policyholders' surplus ratio, as shown previously, is distorted because policyholders' surplus has been overstated. The following table shows industry data, as presented on Exhibit I, and includes a premium-surplus ratio based on consolidated insurance industry surplus[7] (consolidated data are not available for earlier years):

Net Premium Written-Policyholder's Surplus Ratio

	Stock Insurance Industry	
Year	Aggregate	Consolidated
1970	1.66	2.20
1969	1.63	1.87
1968	1.24	1.41
1967	1.25	1.42
1966	1.31	1.50
1965	1.05	1.23
1964	.96	1.13
1963	.97	1.16
1962	1.04	1.22
1961	.92	1.10
1960	1.11	1.31
Average	1.19	1.41

The above presentation indicates that the actual premium-surplus ratio (reflecting consolidated policyholders' surplus after eliminating the double counting of surplus of subsidiary insurance companies) is approximately 20% higher than the previous analysis would indicate.

[7]Annual Statements are consolidated with interownership eliminated through holdings shown in Schedules D1 and D2. *Report of the Advisory Committee on Use of Investment Income in Making Rates for Automobile Insurance to the State Board of Insurance of Texas*, October 19 and 20, 1970, p. 87.

(4) Carrying bonds at amortized values is a major factor for property-liability insurance companies. This practice eliminates short and medium term swings in surplus and contributes stability to the policyholders' surplus account. Any proposal to change this method of accounting should be closely scrutinized to eliminate undesired consequences. Conversely, the practice of carrying all stock investments at market value contributes substantially to short and medium term swings in surplus and contributes instability to the policyholders' surplus account. The discussions on accounting for marketable securities should be closely followed for both stock and bond investments.

This paper was not intended to be the final commentary on the net premium written-policyholders' surplus ratio, but rather a review of the subject. Hopefully this paper will stimulate actuarial research in other financial areas which heretofore have been largely overlooked.

APPENDIX A—STOCK INSURANCE INDUSTRY DATA

Year	Standard & Poor's	% Change	GNP	% Change	Surplus	% Change	Premium	% Change	P/S Ratio
1929	21.45	−11.9	1044	6.31	2037	11.8	1826	5.31	.896
1930	15.34	−28.5	911	−12.74	1824	−10.5	1700	− 6.90	.932
1931	8.12	−47.1	763	−16.25	1466	−19.6	1532	− 9.88	1.045
1932	6.89	−15.1	585	−23.33	1243	−15.2	1288	−15.93	1.036
1933	10.10	46.6	560	− 4.27	1288	3.6	1182	− 8.23	.918
1934	9.50	− 5.9	650	16.07	1472	14.3	1282	8.46	.871
1935	13.43	41.4	725	11.54	1784	21.2	1332	3.90	.747
1936	17.18	27.9	827	14.07	2079	16.5	1445	8.48	.695
1937	10.55	−38.6	908	9.79	1828	−12.1	1579	9.27	.864
1938	13.21	25.2	852	− 6.17	1972	7.9	1508	− 4.50	.765
1939	12.49	− 5.5	911	6.92	2179	10.5	1571	4.18	.721
1940	10.58	−15.3	1006	10.43	2209	1.4	1730	10.12	.783
1941	8.69	−17.9	1258	25.05	2164	− 2.0	1989	14.97	.919
1942	9.77	12.4	1591	26.47	2222	2.7	2165	8.85	.974
1943	11.67	19.4	1925	20.99	2494	12.2	2091	− 3.42	.838
1944	13.28	13.8	2114	9.82	2729	9.4	2258	7.99	.827
1945	17.36	30.7	2136	1.04	3151	15.5	2425	7.40	.770
1946	15.30	−11.9	2107	− 1.36	2879	− 8.6	3063	26.31	1.064
1947	15.30	0.0	2343	11.20	2905	0.9	3862	26.09	1.329
1948	15.20	− 0.7	2594	10.71	3066	5.5	4403	14.01	1.436
1949	16.76	10.3	2581	− 0.50	3708	20.9	4760	8.11	1.284
1950	20.41	21.8	2848	10.34	4217	13.7	5138	7.94	1.218
1951	23.77	16.5	3284	15.31	4543	7.7	5759	12.09	1.268
1952	26.57	11.8	3455	5.21	4964	9.3	6411	11.32	1.291
1953	24.81	− 6.6	3646	5.53	5192	4.6	7000	9.19	1.348
1954	35.98	45.0	3648	0.05	6697	29.0	7144	2.06	1.067
1955	45.48	26.4	3980	9.10	7694	14.9	7662	7.25	.996
1956	46.67	2.6	4192	5.33	7800	1.4	7991	4.29	1.024
1957	39.99	−14.3	4411	5.22	7073	− 9.3	8640	8.12	1.222
1958	55.21	38.1	4473	1.41	8619	21.9	9077	5.06	1.053
1959	59.89	8.5	4837	8.14	9381	8.8	9931	9.41	1.059
1960	58.11	− 3.0	5037	4.13	9495	1.2	10527	6.00	1.109
1961	71.55	23.1	5201	3.26	11719	23.4	10783	2.43	.920
1962	63.10	−11.8	5603	7.73	11146	− 4.9	11599	7.57	1.041
1963	75.02	18.9	5905	5.39	12642	13.4	12296	6.01	.973
1964	84.75	13.0	6324	7.10	13691	8.3	13090	6.46	.956
1965	92.43	9.1	6849	8.30	13660	− 0.2	14339	9.54	1.050
1966	80.33	−13.1	7499	9.49	12007	−12.1	15728	9.69	1.310
1967	96.47	20.1	7939	5.87	13580	13.1	16915	7.55	1.246
1968	103.86	7.7	8650	8.96	14887	9.6	18457	9.12	1.240
1969	92.06	−11.4	9314	7.68	12669	−14.9	20668	11.98	1.631
1970	92.15	0.1	9765	4.84	14014	10.6	23215	12.32	1.657

Notes:
1. Standard & Poor's 500 Stock Index at December 31.
2. Premium and Surplus from A. M. Best's Aggregates & Averages.
3. Premiums for 1963-1970 have been adjusted to include Travelers' A & H.
4. Percentage change from previous year.
5. Premiums and Surplus given in millions of dollars.

APPENDIX B
STOCK INSURANCE INDUSTRY
POLICYHOLDERS' SURPLUS
(MILLIONS OF DOLLARS)

Year	Actual	Predicted	Error
1929	$ 2037	$ 3714	$-1677
1930	1824	2822	- 998
1931	1466	1768	- 302
1932	1243	1588	- 345
1933	1288	2057	- 769
1934	1472	1969	- 497
1935	1784	2543	- 759
1936	2079	3091	-1012
1937	1828	2123	- 295
1938	1972	2511	- 539
1939	2179	2406	- 227
1940	2209	2127	81
1941	2164	1851	312
1942	2222	2009	212
1943	2494	2286	207
1944	2729	2521	207
1945	3151	2117	33
1946	2879	2816	62
1947	2905	2816	88
1948	3066	2802	263
1949	3708	3030	677
1950	4217	3563	653
1951	4543	4053	489
1952	4964	4462	501
1953	5192	4205	986
1954	6697	5836	860
1955	7694	7224	469
1956	7800	7397	402
1957	7073	6422	650
1958	8619	8644	- 25
1959	9381	9328	52
1960	9495	9068	426
1961	11719	11031	687
1962	11146	9797	1348
1963	12642	11537	1104
1964	13691	12958	732
1965	13660	14080	- 420
1966	12007	12313	- 306
1967	13580	14670	-1090
1968	14887	15749	- 862
1969	12669	14026	-1357
1970	14014	14039	- 25

Note: The above predictions are based on the equation:
Surplus $ = (Stock Index) ($146) + $582.6

For example, the Standard and Poor's 500 Stock Index was 92.15 on December 31, 1970. Therefore the predicted Surplus (in millions) is 92.15 times $146 plus $582.6, or $14,039 million.

MINUTES OF THE 1972 ANNUAL MEETING

November 8 - 10, 1972

HOTEL ST. FRANCIS, SAN FRANCISCO, CALIFORNIA

Wednesday, November 8, 1972

The Board of Directors held its regularly scheduled meeting at the Hotel St. Francis from 1:30 p.m. to 5:30 p.m. Preliminary registration was also held during the late afternoon for early arrivals.

An informal reception, sponsored by the Fireman's Fund American Insurance Companies, was held in the early evening for all members and their wives as well as guests. Also during the evening the President's reception for the new Fellows and their wives was held.

Thursday, November 9, 1972

After a brief registration period, the 1972 Annual meeting was formally convened at 9:00 a.m. by President Simon who welcomed the gathering and then introduced the Honorable Gleeson L. Payne, Commissioner of Insurance, State of California.

Commissioner Payne welcomed the Society to California and presented his thoughts on various timely problems affecting the Insurance Industry.

President Simon then presented diplomas to the following new Fellows and Associates:

FELLOWS

William P. Amlie
Robert F. Bartik
Richard A. Bill
Joseph W. Levin
Lee M. Smith
Michael A. Walters
John P. Welch

ASSOCIATES

Richard S. Biondi
Yves J. Brouillette
Howard S. Cohen
Jerome Degerness
Jeffrey J. Dickson
Dale M. Evans

Richard J. Fallquist
Michael Fusco
Janet S. Graves
James A. Hall
Paul E. Hough
James G. Inkrott
Joseph R. Jersey
Allan M. Kaufman
Howard H. Kayton
Douglas F. Kline
John J. Kollar
Timothy M. Kolodziej
Ronald T. Kuehn
R. Michael Lamb
Edward P. Lester
Rosemary N. Marks
Norma M. Masella
F. James Mohl
Richard D. Pagnozzi
Kent T. Penniman
David S. Powell
Floyd R. Radach
Jerry W. Rapp
Walter V. Rice
John J. Schultz
Alan R. Sheppard
James P. Streff
Michael L. Toothman
Gail E. Tverberg
Richard G. Woll
James O. Wood

Following a coffee break, a panel discussion entitled "Rate Making Under Open Competition" was presented to the entire membership. Participants in this part of the program were as follows:

Moderator:
Richard J. Roth
Senior Vice President and Actuary
Great American Insurance Companies

Panel Members:
Lawrence C. Baker, Jr.
Chief Deputy Insurance Commissioner
California Insurance Department

Charles F. Cook
Chief Actuary
United Services Automobile Association

John E. Riley
Assistant Vice President
Safeco Insurance Companies

Richard S. L. Roddis, Dean
School of Law
University of Washington

A brief question and answer period followed the presentation and the panel discussion was concluded at 12:00 noon.

A formal luncheon was then held with Louis William Niggeman, Chairman of the Board, Fireman's Fund American Insurance Companies, addressing the gathering.

The afternoon program convened at 2:00 p.m. with the presentation of reviews of papers previously presented. Although several of the reviews were presented the following day, the complete list of reviews presented during the meeting is as follows:

(1) "Loss Reserving in the Sixties" by Rafal J. Balcarek
Review by — James R. Berquist, Consulting Actuary
Millman and Robertson

(2) "Automobile Collision Deductibles and Repair Cost Groups: The Lognormal Model" by David R. Bickerstaff

Reviews by — (a) William S. Gillam
Associate Actuary
Insurance Services Office

Daniel P. Frame
Assistant Actuary
Insurance Services Office

(This review was read
by Robert Bergen)

(b) J. Stewart Sawyer, III
Assistant Actuary
Fireman's Fund American Ins. Cos.

(c) Dale A. Nelson
Actuary
State Farm Mutual Insurance Co.

(Mr. Nelson's review was read by
Jerome Hillhouse)

(3) "Allocating Premium to Layer by the Use of Increased Limits Tables" by Ronald E. Ferguson

Reviews by — (a) Jeffrey T. Lange
Assistant V.P. and Actuary
Royal-Globe Insurance Cos.

(b) Joseph A. Plunkett
Assistant Vice President
American Reinsurance Co.

(4) "A Note on Full Credibility for Estimating Claim Frequency" by J. E. Hansen

Reviews by — (a) David J. Grady
Assistant Actuary
The Travelers Insursance Companies

(b) Robert N. Tremelling
Actuarial Analyst
Fireman's Fund American Ins. Cos.

(5) "How Adequate are Loss and Loss Expense Liabilities?" by Ruth E. Salzmann

Review by — Matthew Rodermund
Vice President-Actuary
Munich Reinsurance Company

At 2:30 p.m. a panel discussion entitled "Catastrophe Insurance and Reinsurance Programs" was presented to the entire membership. Participants in this part of the program were as follows:

Moderator: Ronald L. Bornhuetter
Vice President and Actuary
General Reinsurance Corporation

Participants: Philip G. Buffinton
Vice President
State Farm Fire and Casualty Co.

Honorable John A. Durkin
Insurance Commissioner
State of New Hampshire

Shigetoni Hashimoto
Nippon Fire & Marine Ins. Co., Ltd.

Henry T. Kramer
President
North American Reinsurance Corporation

Carl Steinbrugge
Manager, Earthquake Department
Insurance Services Office

A brief question and answer period followed the presentation with the panel discussion being concluded at 4:00 p.m.

The next part of the afternoon program was two simultaneous sessions between which members could choose. They were as follows:

Session A.	Workshop continuation of the subject "Catastrophe Programs" with emphasis on specific accounting and actuarial problems (catastrophe reserves, internal reinsurance pools, etc.)
Moderator:	James H. Crowley Assistant Vice President Aetna Life and Casualty
Participant:	Walter J. Fitzgibbon, Jr. Actuary Aetna Life and Casualty
Session B. Panel:	Workmen's Compensation—Dividend Plans and Retrospective Rating
Moderator:	Harry R. Richards Associate Actuary National Council on Compensation Ins.
Panel Members:	Del R. Jones Assistant Director The Travelers Insurance Co.
	Jerome A. Scheibl Actuary Employers Insurance of Wausau

William C. Aldrich
Associate General Counsel
The Hartford Insurance Group

James R. Berquist
Consulting Actuary
Millman and Robertson

Phillip O. Presley
Actuary
New Hamphsire Insurance Dept.

The afternoon activities concluded at 5:15 p.m.

Although no formal dinner arrangements were held, members, their wives and guests attended an evening reception at the Hotel.

Friday, November 10, 1972

President Simon convened the business session at 9:00 a.m. and a summary of events taking place are as follows:

1. President Simon delivered his Presidential address entitled "Know Thyself, Actuary" to the membership.
2. The reading of the minutes of the last meeting was waived.
3. The Secretary and Treasurer reports were presented to the membership.
4. A brief moment of silence was held in rememberance of members and friends who died during the past year.

   Henry Farrer
   Roger A. Johnson
   William H. Kelton
   Frederick H. Trench
   Frances Abel Roberts

5. The following new papers were presented to the membership:
   (1) "Allocated Loss Expense Reserves" by Allie V. Resony, Assistant Secretary, the Hartford Insurance Group
   (2) "An Actuarial Note on Experience Rating Nuclear Property Insurance" by Richard D. McClure, Assistant Actuary, Kemper Insurance Group

(3) "No-Split Experience Rating Plans" by John P. Welch, Vice President, Argonaut Insurance Company

(4) "The Actuary and IBNR" by Ronald L. Bornhuetter, Vice President and Actuary, General Reinsurance Corporation and Ronald E. Ferguson, Assistant Vice President, General Reinsurance Corporation

(5) "Actuarial Applications in Catastrophe Reinsurance" by LeRoy J. Simon, Vice President, Prudential Property and Casualty Insurance Companies

(6) "The Relationship Between Net Premium Written and Policyholder's Surplus" by Raymond W. Beckman, Assistant Actuary and Director of Operations Research, Fireman's Fund American Insurance Companies and Robert N. Tremelling, Actuarial Analyst, Fireman's Fund American Insurance Companies.

6. The following Officers and Directors were then elected by the membership:

President-Elect	Paul Liscord
Vice President	M. Stanley Hughey
Secretary-Treasurer	Robert B. Foster
Editor	Luther L. Tarbell
Chairman—Education & Examination Committee	George Morison
Directors (Class of '75)	William C. Aldrich
	Jeffrey T. Lange
	Paul J. Scheel

7. The Woodward-Fondiller prize was awarded to David R. Bickerstaff for his paper "Automobile Collision Deductibles and Repair Cost Groups: The Lognormal Model". The Dorweiler prize was omitted this year.

Following the coffee break, a workshop entitled "Discussion of IBNR and Other Formula Reserves" was presented to the membership. Participants in this workshop were as follows:

Moderator	Charles L. Niles, Jr. Deputy General Manager and Vice President General Accident Group
Participants	Rafal J. Balcarek Vice President and Actuary Reliance Insurance Company
	Loring M. Barker Assistant Vice President and Actuary Fireman's Fund American Insurance Cos.
	Ronald E. Ferguson Assistant Vice President General Reinsurance Corporation
	Warren P. Cooper Vice President and Actuary Chubb and Son, Incorporated
	Orval E. Dahme Senior Associate Actuary State Farm Mutual Automobile Insurance Co.
	Jeffrey T. Lange Assistant Vice President and Actuary Royal-Globe Insurance Companies
	Earl F. Petz Actuary Kemper Insurance Group
	William A. Riddlesworth Associate Actuary Aetna Life and Casualty
	Dunbar R. Uhthoff Senior Vice President Employers Insurance of Wausau

The workshop session was recessed for lunch and was then continued in the afternoon until 3:30 p.m. at which time the workshop was concluded. President Simon then adjourned the meeting.

It is noted that the registration cards completed by the attendees and filed at the registration desk indicated, in addition to approximately 90 wives, attendance by 118 Fellows and 65 Associates, and 26 invited guests as follows:

FELLOWS

Adler, M.
Aldrich, W. C.
Alexander, L. M.
Amlie, W. P.
Anker, R. A.
Atwood, C. R.
Balcarek, R. J.
Barker, L. M.
Bartik, R. F.
Beckman, R. W.
Bennett, N. J.
Bergen, R. D.
Berquist, J. R.
Bickerstaff, D. R.
Bill, R. A.
Bland, W. H.
Blodget, H. R.
Bornhuetter, R. L.
Boyajian, J. H.
Boyle, J. I.
Brannigan, J. F.
Brian, R. A.
Brown, W. W., Jr.
Burling, W. H.
Cook, C. F.
Crowley, J. H.
Curry, A. C.
Dahme, O. E.
Dorf, S. A.
Drobisch, M. R.
Dropkin, L. B.
Elliott, G. B.
Faber, J. A.
Ferguson, R. E.
Fitzgibbon, W. J., Jr.
Flaherty, D. J.
Forker, D. C.
Foster, R. B.
Gibson, J. A.
Gillespie, J. E.
Goddard, R. P.
Gowdy, R. C.
Grady, D. J.
Graves, C. H.
Hachemeister, C. A.
Hartman, D. G.
Hartman, G. R.
Harwayne, F.
Hazam, W. J.
Heer, E. L.
Hewitt, C. C., Jr.
Hillhouse, J. A.
Honebein, C. W.
Hughey, M. S.
Hunt, F. J., Jr.
Jacobs, T. S.
Johe, R. L.
Kallop, R. H.
Klaassen, E. J.
Lange, J. T.
Levin, J. W.
Linder, J.
Lino, R.
Liscord, P. S.
Lowe, R. F.
MacGinnitie, W. J.
Masterson, N. E.
McClure, R. D.
Meenaghan, J. J.
Mohnblatt, A. S.
Morison, G. D.
Muetterties, J. H.
Munro, R. E.
Murray, E. R.
Murrin, T. E.
Newman, S. H.
Niles, C. L., Jr.
Oien, R. G.
Otteson, P. M.
Petz, E. F.
Phillips, H. J.
Pollack, R.
Presley, P. O.
Quinlan, J. A.
Resony, A. V.
Richards, H. R.
Richardson, J. F.
Riddlesworth, W. A.
Rinehart, C. R.
Rodermund, M.
Rosenberg, N.
Roth, R. J.
Ryan, K. M.
Salzmann, R. E.
Scheel, P. J.
Scheibl, J. A.
Scheid, J. E.
Schloss, H. W.
Schuler, R. J.
Scott, B. E.
Simon, L. J.
Skurnick, D.
Smick, J. J.
Smith, E. R.
Smith, L. M.
Snader, R. H.
Stewart, C. W.
Sturgis, R. W.
Switzer, V. J.
Tarbell, L. L., Jr.
Uhthoff, D. R.
Verhage, P. A.
Walsh, A. J.
Walters, M. A.
Webb, B. L.
Welch, J. P.
White, H. G.
White, W. D.

ASSOCIATES

Biondi, R. S.
Buffinton, P. G.
Chorpita, F. M.
Cohen, H. S.
Cooper, W. P.
Copestakes, A. D.
Crawford, W. H.
Davis, R. C.
Degerness, J. A.
Evans, D. M.
Fallquist, R. J.
Finkel, D.
Franklin, N. M.
Fusco, M.
Gill, J. F.
Gossrow, R. W.
Graves, J. S.
Hall, J. A., III
Hammer, S. M.
Head, T. F.
Inkrott, J. G.
Jensen, J. P.
Jersey, J. R.
Jones, D. R.
Kaufman, A. M.
Kayton, H. H.
Kollar, J. J.
Kolodziej, T. M.
Krause, G. A.
Kuehn, R. T.
Lamb, R. M.
Lester, E. P.
Lindquist, R. J.
Margolis, D. R.
Marks, R. N.
Masella, N. M.
McDonald, M. G.
Miller, H. C.
Mohl, F. J.
Mokros, B. F.
Neidermyer, J. R.
Nelson, J. K.
Pagnozzi, R. D.
Plunkett, J. A.
Raddach, F. R.
Raid, G. A.
Rapp, J. W.
Ratnaswamy, R.
Reinbolt, J. B.
Sandler, R. M.
Sawyer, J. S., III
Schneiker, H. C.
Schultz, J. J.
Sheppard, A. R.
Singer, P. E.
Stein, J. B.
Streff, J. P.
Toothman, M. L.
Tverberg, G. E.
Wade, R. C.
Walters, M. A.
Wilson, O. T.
Woll, R. G.
Woodworth, J. H.
Young, E. W.

GUESTS

Baker, L. C., Jr.
*Bell, A. M.
Buscemi, J.
*Chamberlain, R. H.
*Chan, E.
*Chang, C. I.
Cook, R. H.
Durkin, J. A.
*Eddins, J. M.
*Galban, L. S., Jr.
Gamble, R. A.
*Griffith, R. W.
Hall, M. L.
Hashimoto, S.
*Hatfield, B. D.
Hoyt, F. A.
Karr, R.
Kramer, H. T.
Payne, G. L.
*Peterson, R. P.
Riley, J. E.
Roddis, R. S. L.
*Smith, D. A.
Steinbrugge, C.
Varney, B. K.
Wohlner, E. A.

*Invitational Program

Respectfully submitted,

Ronald L. Bornhuetter
Secretary-Treasurer

REPORT OF THE SECRETARY

Again, as in the past two years, I am pleased to report to you the activities of your Board of Directors for the past year. The President's Newsletters have kept you informed on many activities; however, I will try to bring you up to date through the Board of Directors meeting held last Wednesday, November 8th.

Your Board of Directors met four times during 1972. Two day meetings were held on February 7-8th and September 14-15th and one day meetings were held on May 21st and November 8th. Your Directors certainly earned their salary this year.

I will try to summarize the year's activities by Committee in order to provide some continuity between the various meetings throughout the year.

Special Committees

As you know, as a result of the report of the Committee on the Future Course of the Society, many Special Committees were established with specific assignments—some of which carried over to this year. These Committees are as follows:

Special Committee on Amalgamation

The final report of this Committee was accepted by the Board and the Committee was discharged. The Board then asked the Planning Committee to enumerate areas of co-operation between Societies both present and future. In addition, the membership was informed that the general subject was being discussed within the Board through both the Newsletter and at the May 1972 meeting. At the present time this general subject is not before the Board although, as is true for any subject, it may be brought up again.

Special Committee on Levels of Certification

The final report of this Committee was accepted and the Committee was discharged. A committee of the Board was instructed to pursue the subject further, and at the September meeting, the Board agreed in principle that, in addition to examination requirements, an experience requirement should be established for membership in the Casualty Actuarial Society. A special Committee of three Directors is now drafting specific requirements that the Board will consider at its next meeting. It is also understood that any specific program agreed upon will be presented to the membership at the May 1973 meeting.

Special Task Force to Enlist Candidates

An extensive report with many recommendations was reviewed by the Board and specific action was taken on each recommendation, with many being referred to regular Society Committees for further review and recommendations. Although this Committee has been discharged, its work is going on in many areas within the Society.

One recommendation has already resulted in the completion of an extensive survey of actuarial student programs within the membership which has been distributed to members who might find the report informative.

A second survey concerning meeting attendance was also accomplished and the Officers were asked to attempt to have senior members involved in future meeting programs in hopes of encouraging better attendance from this segment of the Society's membership.

Regular Committees

As to regular Committees, their activites were as follows:

Education and Examination Committee

A complete reprint of the Syllabus was completed and distributed this year and the Committee continues to work on an overall revision in this area. The Committee is also discussing with the Society of Actuaries the possibility of another joint examination, Part III, covering numerical analysis and/or other subjects of joint interest. Maximum joint sponsorship with other actuarial organizations has also been approved. In addition, the Committee distributed an informative letter and copies of the Syllabus and Essential Executive to twelve hundred heads of Mathematics Departments in colleges throughout the country.

Text Book Committee

Work on the joint Textbook project with the staff of Georgia State University is proceeding although progress has been slower than expected.

Editorial Committee

As you know the 1971 *Proceedings* included the reprint of five old papers still being used in the Syllabus. The Board instructed the Committee to investigate the possibility of reprinting other old papers for use by students. This Committee will also be sponsoring a Logo contest among the membership.

Committee to Review Constitution and By-Laws

The work of this Committee has been completed with the adoption of changes in the Guides for the Submission of Papers and a model constitution for affiliated organizations. In this connection, the Board also approved the application of Casualty Actuaries of New York and the Southern California Actuaries Club as affiliated organizations.

Finance Committee

The Board is continuing an extensive evaluation of the Invitational Subscriber program prepared by this Committee. For the present the program will continue as a vehicle for interested individuals to attend Society meetings.

On the financial side, the Committee has audited the Society books which, as of September 1972, show a net worth of $97,104, an increase of $22,200 from last year. In addition, the Board approved the Committee's recommended budget of $73,500 for 1973.

Committee on Review of Papers

Although this small Committee works quietly behind the scenes, no report would be complete without mentioning their outstanding contribution to the Society as well as the contribution of the authors who are now writing papers.

New Committees

The Board also established a new Committee on Financial Reporting, which replaced the Committee on the Annual Statement. In addition, a Joint Committee on Risk Theory has been established with the Society of Actuaries.

Perhaps it might be of interest to all if I could point out a few items that have come to mind during my three year tenure as your Secretary-Treasurer which indicate to me just how strong a Society we have going for us right now.

(1) From purely a dollar and cents point of view, during the three year period the net worth of the CAS increased from $52,540 to $97,104, an increase of 84.8%. We have now built a base from which the Society can move in many directions.

(2) Our membership roll increased from 451 members as of November 30, 1969 to 518 members as of today. This is an increase of 14.9% which, in its absolute value, is not an impressive figure for a three year period except when you consider that the prior three year growth was somewhat less than 7%.

(3) As to future members, I might point out to you that in November 1969 we had 232 students taking exams. Next week there will be 752 students scratching their heads over the exams. This is an increase of 520 students in the relatively short period of time. For additional comparison purposes, there were 469 students taking exams last year.

(4) As you know from yesterday morning, 1972 produced a record number of new Associate members for any one given year. This is only half the story when you realize a few supplementary facts:

a) 33 students need only one exam to become an Associate.

b) 72 students need only two exams to become an Associate.

c) 107 students have two exams behind them and are well on their way to becoming Associates.

November 1972 will not be an oddity as to our future membership enrollment.

You should be proud of your efforts in all areas of the Society. We are now a growing, viable Society that will be able to respond to the future needs of our growing industry.

No report for the three year period would be complete if I did not acknowledge here and in the *Proceedings* the invaluable assistance to the Secretary-Treasurer rendered by several hard working individuals who have made the office a success.

Arthur Cadorine whose work in handling the Library needs of the students was done in an efficient manner.

Robert Foster who assisted this year and has earned the full responsibility of the office for the next few years.

Ronald Ferguson, an associate of mine, who filled in many gaps and was always there to give a hand when needed.

Edith Morabito who really is the heart behind our whole operation and has given exceptional service to the Society in many ways throughout the many years she has been working with us. I can't say enough for what she has meant to the success of the Society.

Lastly I would like to thank you, the membership, for all your help in making the three years a success to me and, I hope, to you.

Respectfully submitted,

RONALD L. BORNHUETTER
Secretary-Treasurer

RESOLUTION

RESOLVED, that the Board of Directors of the Casualty Actuarial Society does hereby acknowledge with profound gratitude the wisdom and devotion which

GLADYS SORENSEN FAWCETT

brought to bear upon the affairs of this Society during her seventeen years of service to it and expresses regret about her decision to retire from the National Council on Compensation Insurance on December 1, 1972. The energy and enthusiasm with which she helped meet the challenges of a growing Society contributed greatly to the performance of the Secretary-Treasurer's office over the years. The most sincere best wishes of her friends in the Society are extended to her.

(2) *Study Notes*

An expense item of $1,500 has been budgeted for in order to provide the Education and Examination Committee the facility to pay outside authors for the development of study notes for use in the examination program.

(3) *Secretary's Office*

The National Council on Compensation Insurance has increased the CAS 1973 assessment to $19,175 from $10,000 last year. This reflects the increased activity in the examination area.

In total the 1973 budget anticipates revenues of $73,500 and expenditures at $71,000 which will increase the net worth of the Society at September 30, 1973 to approximately $100,000.

Respectfully submitted,

RONALD L. BORNHUETTER
Secretary-Treasurer

REPORT OF THE TREASURER

The complete financial statement, as well as the supporting records, have been audited and certified to by the Finance Committee at their meeting on October 26, 1972.

In summary, the net worth of the Society now stands at $97,103.90 as of September 30, 1972—an increase of $22,199.90 during the past twelve months. This unusual increase in surplus was caused by the combination of several favorable factors as follows:

(1) *Dues*

The increased dues for 1972, as well as the increase in the number of members, created additional revenue.

(2) *Examination Fees*

An increase of $9,008 in examination fees was realized while expenses increased slightly during the year. This will be a onetime gain as our 1973 assessment from the National Council will reflect the increased activity in this area.

(3) *Proceedings*

Due to the lack of papers the cost of the *Proceedings* was substantially less than budgeted for.

(4) *M.A.A. Contest*

Due to a lag in receiving the CAS assessment for this year's contest, no monies were expended in 1972. Again, this is a one-time item.

Your Board of Directors adopted a 1973 budget which anticipates no increase in dues or examination fees for the ensuing year. Specific expenses budgeted for next year that may be of interest are as follows:

(1) *Georgia State*

It is anticipated that the joint textbook project with Georgia State University will be completed and the second half of the grant ($2,500) has been budgeted for.

FINANCIAL REPORT

Income and Disbursements
(from October 1, 1971 through September 30, 1972)

Income		Disbursements	
Dues	$21,130.00	Printing and Stationary	$16,049.51
Examination Fees	21,661.00	Secretary's Office	10,332.70
Meetings & Registration Fees	13,254.83	Examination Expense	5,424.15
Sale of Proceedings	4,350.95	Meeting Expense	9,644.66
Sale of Readings	478.97	Library	892.28
Invitational Program	1,740.00	Insurance	478.00
Michelbacher Fund	609.75	Meeting Refunds	764.00
Interest	3,675.86	Examination Refunds	655.50
Registration-ACNY	1,300.00	ACNY	1,300.00
Miscellaneous	11.03	Miscellaneous	471.69
Total	$68,212.39		$46,012.49

Assets

As of 9/30/71		As of 9/30/72		Change
Checking Account	$ 3,596.34	Checking Account	$ 645,83	$− 2,950.51
Savings Account	44,637.62	Savings Account	85,151.82	40,514.20
Investments	26,670.04	Investments	11,306.25	−15,363.79
	$74,904.00		$97,103.90	$ 22,199.90

Investments

	Cost
U. S. A. Treasury Bond #1673 Due 11/15/74	$ 1,000.00
U. S. A. Treasury Bond #1674 Due 11/15/74	1,000.00
U. S. A. Treasury Bond # 299 Due 2/15/75	4,981.25
U. S. A. Treasury Bond #5263 Due 2/15/80	4,325.00
	$11,306.25

This is to certify that we have audited the accounts and the assets shown above and find same to be correct.

Finance Committee
WILLIAM H. CRANDALL
RICHARD D. MCCLURE
STEVEN H. NEWMAN
HENRY W. MENZEL, Chairman

1972 EXAMINATIONS—SUCCESSFUL CANDIDATES

Examinations for Parts 3, 5, 7 and 9 of the Casualty Actuarial Society syllabus were held May 15 and 16, 1972 and examinations for Parts 4, 6 and 8 were held November 16 and 17, 1972. Parts 1 and 2, jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries, were given May 11 and November 9. Those who passed Parts 1 and 2 were listed in the joint releases of the two Societies dated June 23, 1972 and December 27, 1972.

The following candidates successfully completed the requirements for Fellowship and Associateship in the November 1971 examinations and were awarded their diplomas at the May 1972 meeting:

NEW FELLOWS

Anker, Robert A.
Bergen, Robert D.
Hartman, David G.
Murray, Edward R.
Rinehart, Charles R.
Stewart, C. Walter

NEW ASSOCIATES

Dempster, Howard V., Jr.
Reinbolt, James B.
Rogers, Daniel J.
Retterath, Ronald C.

MAY 1972 EXAMINATIONS

Following is the list of successful candidates in the examinations held in May, 1972:

FELLOWSHIP EXAMINATIONS

Part 7

Balko, Karen H.
Bartik, Robert F.
Bradshaw, John G., Jr.
Conners, John B.
Dempster, Howard V., Jr.
Haseltine, Douglas S.
Kline, Douglas F.
Miller, Philip D.
Millman, Neil L.
Moore, Phillip S.
Pagnozzi, Richard D.
Peacock, Willard W.
Retterath, Ronald C.
Rogers, Daniel J.
Ross, James P.
Smith, Lee M.
Torgrimson, Darvin A.
Winkleman, John J., Jr.

Part 9

Amlie, William P.
Bill, Richard A.
Eyers, Robert G.
Fossa, E. Frederick
Hoffmann, Dennis E.
Klein, David M.
Krause, Gustave A.
Levin, Joseph W.
McClenahan, Charles L.
Ross, James P.
Sandler, Robert M.
Sawyer, J. Stewart, III
Smith, Lee M.
Walters, Michael A.
Welch, John P.

ASSOCIATESHIP EXAMINATIONS

Part 3

Alff, Gregory N.
Allaire, Andre
Armstrong, Donald L.
Ashenberg, Wayne R.
Bak, Harris N.
Bartlett, William N.
Bassman, Bruce C.
Berry, Charles H.
Bertles, George G.
Blivess, Michael P.
Brewer, Fred L.
Briere, Robert S.
Brouillette, Yves J.
Bryan, Charles A.
Costello, Jeanette R.
Covney, Michael D.
Creasey, Frank C., Jr.
Daino, Robert A.
Davis, Rodney D.
Dee, Vicki S.
DeGarmo, Lyle W.
Desjardins, Jean-Pierre
Dickson, Jeffrey J.
Eklof, Cary B.
Fasking, Dennis D.
Finger, Robert J.
Foley, Charles D.
Gaudreault, Andre
Genest, Francois J.
Gleeson, Owen M.
Godbold, Mary Jo E.
Godbold, Nathan T.
Goldberg, Steven F.
Gutman, Ewa
Hall, James A.
Inderbitzin, Paul H.
Isaac, David H.
Jaeger, Richard M.
Kaufman, Allan M.
Klein, David M.
Kollar, John J.
Kuehn, Ronald T.
Lafrenaye, A. Claude
Lamb, R. Michael
Lariviere, Normand
Lau, Geegym
Lehman, Merlin R.
Leonard, Gregory E.
Lester, Edward P.
Longchamps, Renaud E.
Masella, Norma M.
Miller, David L.
Neis, Allan R.
O'Donnell, Joanne R.
Pagnozzi, Richard D.
Palczynski, Richard W.
Pfaff, Edwin W.
Powell, David S.
Pye, Tommie L.
Racine, Andre R.
Ragnes, Andrew P.
Renze, David E.
Reynolds, John D.
Riff, Mayer
Salato, Susan D.
Sanko, Ronald J.
Savard, Claude
Schultz, John J.
Shapiro, Barry R.
Silberstein, Benny
Steeneck, Lee R.
Stergiou, Emanuel J.
Swift, John A.
Symonds, Donna R.
Toothman, Michael L.
Turcotte, Gerard J.
Tverberg, Gail E.
Wald, Allan R.
Warthen, Thomas V., Jr.
Zeitz, Claudia

Part 5

Alfuth, Terry J.
Bailey, Michael W.
Barrette, Raymond
Biondi, Richard S.
Bovard, Roger W.
Chou, Philip S.
Cohen, Howard S.
Cox, Allan M., Jr.
Curley, James O.
Daino, Robert A.
Degerness, Jerome A.
Dickson, Jeffrey J.
Donaldson, John P.
Evans, Dale M.
Fallquist, Richard J.
Fisher, Wayne H.
Fusco, Michael
Graves, Janet S.
Hough, Paul E.
Inkrott, James G.
Jersey, Joseph R.
Kaufman, Allan M.
Kayton, Howard H.
Kline, Douglas F.
Kochanski, Nancy M.
Kollar, John J.
Kolodziej, Timothy M.
Kuehn, Ronald T.
Lamb, R. Michael
Lis, Raymond S., Jr.
Marks, Rosemary N.
Mohl, Frederic J.
Moore, Brian C.
Nolan, John D.
Penniman, Kent T.
Petit, Charles I.
Radach, Floyd R.
Rapp, Jerry W.
Schultz, John J.
Sheppard, Alan R.
Stanard, James N.
Streff, James P.
Toothman, Michael L.
Wood, James O.

As a result of the above examinations seven new Fellows and 37 new Associates were admitted at the Annual Meeting, November 9, 1972.

NEW FELLOWS

Amlie, William P.
Bartik, Robert F.
Bill, Richard A.
Levin, Joseph W.
Smith, Lee M.
Walters, Michael A.
Welch, John P.

NEW ASSOCIATES

Biondi, Richard S.
Brouillette, Yves J.
Cohen, Howard S.
Degerness, Jerome A.
Dickson, Jeffrey J.
Evans, Dale M.
Fallquist, Richard J.
Fusco, Michael
Graves, Janet S.
Hall, James A.
Hough, Paul E.
Inkrott, James G.
Jersey, Joseph R.
Kaufman, Allan M.
Kayton, Howard H.
Kline, Douglas F.
Kollar, John J.
Kolodziej, Timothy M.
Kuehn, Ronald T.
Lamb, R. Michael
Lester, Edward P.
Marks, Rosemary N.
Masella, Norma M.
Mohl, Frederic J.
Pagnozzi, Richard D.
Penniman, Kent T.
Powell, David S.
Radach, Floyd R.
Rapp, Jerry W.
Rice, Walter V.
Schultz, John J.
Sheppard, Alan R.
Streff, James P.
Toothman, Michael L.
Tverberg, Gail E.
Woll, Richard G.
Wood, James O.

NOVEMBER 1972 EXAMINATIONS

The successful candidates in the November 1972 examinations were:

FELLOWSHIP EXAMINATIONS

Part 6

Balko, Karen H.
Berry, Charles H.
Brouillette, Yves J.
Carter, Edward J., Jr.
Curley, James O.
Dropick, Dorothy E.
Eyers, Robert G.
Finger, Robert J.
Gruber, Charles
Hardy, Howard R.
Hoffmann, Dennis E.
Hough, Paul E.
Kaufman, Allan M.
Kayton, Howard H.
Kline, Douglas F.
McClenahan, Charles L.
Retterath, Ronald C.
Stanard, James N.
Streff, James P.
Toothman, Michael L.
Tverberg, Gail E.
Woll, Richard G.

Part 8

Biondi, Richard S.
Bryan, Charles A.
Creasey, Frank C.
Dempster, Howard V., Jr.
Fossa, E. Frederick
Hall, James A.
Haseltine, Douglas S.
Khury, Costandy K.
Klein, David M.
Kline, Douglas F.
Lester, Edward P.
Marino, James F.
Neidermyer, James R.
Nolan, John D.
Rogers, Daniel J.
Ross, James P.
Sawyer, J. Stewart, III
Schaeffer, Bernard G.
Sheppard, Alan R.
Toothman, Michael L.
Wood, James O.

ASSOCIATESHIP EXAMINATIONS

Part 4

Anderson, Dean R.
Ashenberg, Wayne R.
Barden, John P.
Barnes, Galen R.
Bassman, Bruce C.
Bellinghausen, Gary F.
Bethel, Neil A.
Blivess, Michael P.
Bovard, Roger W.
Brown, Andrew F., Jr.
Brubaker, Randall E.
Carbaugh, Albert B.
Connor, Vincent P.
Crowe, Patrick J.
Daino, Robert A.
D'Arcy, Stephen P.
Davis, George E.
Eddy, Jeanne M.
Fein, Richard
Finger, Robert J.
Forman, Ben J.
Gallagher, Thomas L.
Garand, Christopher P.
Godbold, Mary Jo
Hemstead, Robert J.
Hoover, Gary A.
Hoylman, Douglas J.
Ingco, Aguedo M.
Isaac, David H.
Jaeger, Richard M.
Jaffe, Jay M.
Jerabek, Gerald J.
Kaliski, Alan E.
Karlinski, Frank J.
Kass, Sheldon
Knaus, Charles B.
Lehman, Merlin R.
Leimkuhler, Urban E.
Leonard, Gregory E.
Lis, Raymond S., Jr.
Miller, David L.
Morgan, Stephen T.
Newville, Benjamin S.
Ostrowski, Ellen M.
Palczynski, Richard W.
Penton, Ann M.
Petit, Charles I.
Pfaff, Edwin W.
Quirin, Albert J.
Reynolds, John J., III
Riff, Mayer
Roach, Robert F.
Rogan, Timothy J.
Seiffertt, Barbara A.
Shapiro, Barry R.
Skolnik, Richard S.
Steeneck, Lee R.
Stone, James M.
Swift, John A.
Swisher, John W., Jr.
Taht, Veljo
Taylor, Frank C.
Taylor, Jane C.
Taylor, John R.
Van Slyke, Oakley E.
Weiner, Joel S.
Weller, Alfred O.
Wulterkens, Paul E.
Young, Robert J., Jr.
Zeitz, Claudia
Zelenko, Dorothy A.

Five candidates for Fellowship and three candidates for Associateship completed their requirements in the above examinations and will be admitted at the Spring Meeting in 1973:

NEW FELLOWS

Eyers, Robert G.
Fossa, E. Frederick
Hoffmann, Dennis E.
Khury, Costandy K.
Ross, James P.

NEW ASSOCIATES

Bovard, Roger W.
Daino, Robert A.
Lis, Raymond S., Jr.

NEW ASSOCIATES ADMITTED MAY 1972: Seated left to right: Daniel J. Rogers, President LeRoy J. Simon. Standing left to right: Ronald C. Retterath, Howard V. Dempster, James B. Reinbolt.

NEW FELLOWS ADMITTED MAY 1972: Seated left to right: David G. Hartman, President LeRoy J. Simon. Standing left to right: Edward R. Murray, Charles W. Stewart, Charles R. Rinehart, Robert D. Bergen, Robert A. Anker.

NEW ASSOCIATES ADMITTED NOVEMBER 1972: Seated left to right: Floyd R. Radach, Alan R. Sheppard, Howard H. Kayton, Jerome Degerness, Janet S. Graves, outgoing President LeRoy J. Simon, Richard S. Biondi, Howard S. Cohen, Allan M. Kaufman, Norma M. Masella, Michael L. Toothman, Edward P. Lester. Standing left to right: Richard J. Fallquist, Dale M. Evans, F. James Mohl, Timothy M. Kolodziej, Rosemary N. Marks, Paul E. Hough, James A. Hall, Gail E. Tverberg, James P. Streff, Joseph R. Jersey, R. Michael Lamb, Richard D. Pagnozzi, Jerry W. Rapp, Michael Fusco, James G. Inkrott, Richard G. Woll, John J. Kollar, John J. Schultz, Ronald T. Kuehn. Missing from the picture were new Associates Yves J. Brouillette, Jeffrey J. Dickson, Douglas F. Kline, Kent T. Penniman, David S. Powell, Walter V. Rice and James O. Wood.

NEW FELLOWS ADMITTED NOVEMBER 1972: Left to right: Richard A. Bill, Joseph W. Levin, John P. Welch, outgoing President LeRoy J. Simon, Robert F. Bartik, Michael A. Walters, William P. Amlie, Lee M. Smith.

BOOK NOTES

Spencer L. Kimball and Herbert S. Denenberg, *Insurance, Government, and Social Policy: Studies in Insurance Regulation,* 487 pages, R. D. Irwin, Homewood, Illinois, 1969.

REVIEWED BY RONALD E. FERGUSON

Although the book is not new, having been first printed in July of 1969, it was thought that it should be reviewed for the membership since we on the Education Committee have recently taken notice of it. As you probably know, the Education Committee recently updated the Syllabus and eight chapters of this book are now included in the Part Six readings and one chapter is included in the Part Nine readings. These nine chapters represent in bulk approximately one-third of the book. The reader will recognize the editors as being two of the most prominent insurance academicians. The genesis of the book was work that the two editors did in the revision and recodification of the insurance laws of the state of Wisconsin. Related to their work they developed a series of lectures which were presented at the University of Pennsylvania and these lectures form the core of the book.

I am happy to report that the quality of the work indeed lives up to the reputation of the two editors. I think the book would be a valuable addition to the library of any serious student of the insurance business. Most of the chapters are good and a few (which will be discussed below) are excellent, perhaps even "must" reading for anyone who wants to understand the contemporary insurance scene. On the negative side, the book suffers, as most collections of this type do, by being somewhat disjointed in places. Also a couple of the chapters are perhaps of questionable relevancy to what I take to be the main theme of the book.

Messrs. Kimble and Spencer individually or jointly authored approximately 20% of the book with the remaining portions of the book contributed by 14 well known academicians and industry people. In fact, two members of our own society made significant contributions to the book. Mr. LeRoy Simon authored Chapter Thirteen which very nicely puts into perspective the roll of statistical agencies, while the late Allan Mayerson wrote Chapter Nine which covers the concept of regulating for the solvency or solidity of property and liability companies.

A substantial portion of the book is devoted to what might be loosely called the federal interest in insurance regulation problem. This subject should be of great current interest because of the numerous insurance topics now on the national horizon, such as National No-Fault, National Health Insurance and disaster coverages. It is not possible to discuss each and every chapter of the book; however, with the general observation that perhaps, with only one or two exceptions, all of the chapters are at least worthwhile, I will single out those sections that appear to be especially good.

Part I of the book encompassing the first four chapters develops a more or less philosophical discussion of the purposes and objectives of insurance regulation and is recommended reading. Part III, "Financial Structure, Reserves, and Other Methods of Ensuring the Solidity of the Insurance Enterprise" includes Chapters 6-9 and is highly recommended.

Chapter Six, "Capital and Surplus Requirements", which also includes an excerpt dealing with Surplus Surplus from the report of the Special Committee on Insurance Holding Companies, was written by Alfred E. Hofflander. In this essay Prof. Hofflander brings into play some elementary risk theory concepts with an eye towards determining the proper surplus requirements of a company. Chapter Seven, "Life Insurance Reserves", by Joseph M. Belth, is alone worth the price of the book. Professor Belth's essay is the most lucid explanation of life loss reserves as they affect net worth and the incidence of earnings that I have come across. The author presents us with easy to understand explanations (with examples in some cases) of the various possible valuation approaches—net premium, modified net premium, gross premium, refined gross premium including natural reserves, and asset shares. The author also casts some doubt on the validity of the "Rule of Ten" (*i.e.* a 1% change in rate of interest, changes the reserve by 10%) used by the IRS and most stock analysts.

The last two chapters of Part III, "Regulation of Investment" by Kimball and Denenberg and "Solidity of Property and Liability Insurers" by Mayerson are both done very well. All in all, I was impressed with the book although more impressed with the first one half or so than with the last half.

OBITUARIES

FRANK A. EGER

1894-1972

Frank A. Eger, retired Secretary-Comptroller for the Insurance Company of North America, died October 29, 1972 at the age of 78. He had been an Associate of the Casualty Actuarial Society since 1925.

Born on August 11, 1894, Mr. Eger spent his insurance career with the Insurance Company of North America completing over 38 years of service prior to his retirement in 1959. He was an army veteran of World War I and a member of the Haddon Heights Post 149, American Legion. Mr. Eger served as the commissioner of the Medford Lakes community in New Jersey and was a past president of the Medford Lakes Colony Club. He also was a past master of Haddon Heights Masonic Lodge No. 191.

He is survived by a son, Frank Jr. of Kailua, Hawaii; a daughter, Mrs. Whitney Carleton of Cherry Hill, New Jersey; a sister and six grandchildren.

HENRY FARRER

1882-1972

Henry Farrer, retired Chief Accountant for the Insurance Company of North America, died August 1, 1972 at the age of 90. He was one of the charter members of the Casualty Actuarial Society and served on the Council from 1926 through 1929.

Born in 1882, Mr. Farrer was serving as Chief Statistician with the Hartford Accident and Indemnity when the Casualty Actuarial Society was formed in 1914. He was later appointed Actuary in the same company prior to his leaving in 1922 to take a position as Assistant Secretary with the Independence Indemnity Company in Philadelphia. He joined the Indemnity Insurance Company of North America in 1930 and was Chief Accountant for the New York office of the Insurance Company of North America from 1932 until his retirement in 1949.

He is survived by two sons and a daughter.

ROGER A. JOHNSON

1914-1972

Roger A. Johnson, actuary of the Blue Cross of Greater Philadelphia, died suddenly at his home on April 20, 1972.

Born in Cleveland, Ohio on May 25, 1914, he was the son of Roger Arthur Johnson, who later headed the mathematics department at Brooklyn College. He attended high school in Elmhurst, New York and received his bachelor's degree from Columbia University in 1935.

He was actuary for the New York Compensation Insurance Rating Board from 1935 until 1948, when he joined the Utica Mutual Insurance Company, serving as actuary for that company until January of 1960. In February of that year he became actuary for Philadelphia Blue Cross, holding that position until the time of his death.

An outstanding member of his profession, the six-feet four, mild-mannered actuary was a Fellow of the Casualty Actuarial Society (1941); a member of the American Academy of Actuaries and of the Casualty Actuaries Club of Philadelphia.

Mr. Johnson was a philatelist and among his other hobbies were bowling and curling. He was a past treasurer of the Philadelphia Curling Club.

He is survived by his wife, the former Martha Triggs; a son, Roger F.; a daughter, Mrs. John Van Noy; two brothers, Hayden B. and Duncan G.; and one grandchild.

WILLIAM H. KELTON

1894-1972

William Henry Kelton, a Fellow of both the Casualty Actuarial Society and the Society of Actuaries, died June 28, 1972. Mr. Kelton was born in Manchester, Vermont on August 16, 1894 and attended Troy Conference Academy at Poultney, Vermont. He graduated from Williams College with a B.A. degree, magna cum laude in Mathematics and Physics and was elected to Phi Beta Kappa.

Following service with the field artillery in World War I, he joined the Travelers Insurance Company in the life actuarial department. In 1923 he became a Fellow of the Actuarial Society of America, which amalgamated with the American Institute of Actuaries (of which he was also a Fellow), to form the Society of Actuaries. He became a Fellow of the Casualty Actuarial Society in 1926.

Mr. Kelton was appointed Assistant Actuary in 1925, was promoted to Associate Actuary in 1947 and to Actuary in 1957. His work embraced several fields, but he specialized in compiling and analyzing financial statements and various statements associated with them. He will be remembered by his many friends for his integrity and friendly cooperation with his associates.

Prior to his retirement in 1960, he purchased a "home away from home" on the eastern shore of Lake Memphremagog in Newport, Vermont, about a half mile from the Canadian border where he and his family enjoyed many happy months.

He was a member of the Immanuel Congregational Church in Hartford.

He is survived by his wife, Irene S. Kelton; a son, a daughter and seven grandchildren.

EMMA C. MAYCRINK

1879—1972

Emma C. Maycrink, a Fellow of the Society for over fifty-seven years, died at her home in Crestwood, New York, on December 7, 1972. She was the first woman member of the Society, being admitted on May 19, 1915, a few months after the charter organization of the Society in 1914.

Born on July 31, 1879 she spent her early girlhood in Manhattan and following graduation from college became a teacher of mathematics. She entered her business career as an auditor for the Compensation Rating Bureau and in 1923 became an Examiner for the New York Insurance Department. In 1949 she was elected Secretary-Treasurer of the Association of New York State Mutual Casualty Companies, holding this position until her retirement in 1957.

Supplementing her long business activity, were many years of service to the Casualty Actuarial Society. In 1919 she became a member of the Committee on Book Reviews. For the *Proceedings* she submitted two book reviews, seven discussions of papers, and a paper entitled, "Procedures in the Examination of Casualty Companies by Insurance Departments." She served on the Examination Committee in 1923 and 1924 and on the Education Committee for fourteen years from 1925. For ten years from 1944 she was on the Committee on Publications and was Editor of the *Proceedings.* In 1938 she was elected to a term on the Council.

She had a sense of humor that enlivened the informal meetings and social gatherings of the members. As a woman she was a pioneer in establishing the distaff participation in development of actuarial methods for casualty insurance.

OLIVE E. OUTWATER

1885-1972

Olive E. Outwater, Actuary for the Benefit Association of Railway Employees prior to her retirement, died on December 3, 1972 at the age of 87. Miss Outwater was one of the first female members of the Casualty Actuarial Society being admitted as a Fellow by examination in 1919. She was very active in Society affairs, serving as Editor in 1922 and 1923 and as a member of the Council from 1924 to 1926 in addition to contributing a number of papers, discussions and book reviews for the *Proceedings*. In addition, Miss Outwater served on the Examination Committee, the Committee on Papers and the Educational Committee.

Born in Portland, Michigan on February 11, 1885, Miss Outwater attended the University of Michigan and took a great interest in university affairs and was very active in the alumni association.

Her early insurance career was spent with the National Workmen's Compensation Service Bureau and the National Bureau of Casualty and Surety Underwriters in New York City. In 1924 she became the actuary for the Maccabees in Michigan leaving in 1928 to join the Benefit Association of Railway Employees in Chicago where she worked until retirement in 1951.

She is survived by her brother, Mr. Kenneth R. Outwater of Encino, California.

FREDERICK H. TRENCH

1882-1972

Frederick H. Trench, an Associate of the Casualty Actuarial Society since November 21, 1919 died January 9, 1972 in Utica, New York at the age of 89. He was one of the early members of the Society admitted to Associateship through examination.

Born in New York City on September 8, 1882, Mr. Trench spent most of his business career associated with the Utica Mutual Insurance Company. He joined the company in 1919 as Underwriting Manager and was named Budget Director in 1954. In 1959 he was named Treasurer and was elected to the Board of Directors where he served until his retirement in 1964.

He leaves his wife, Alice and a son, Edward of New York City.

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