

PROCEEDINGS

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A SURVEY OF RISK CREDIBILITY IN EXPERIENCE
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Anniversaries suggest retrospection. While meditating on the Twentieth Anniversary of our Society, it seemed fitting to select a subject that is peculiar to casualty insurance, that has received serious attention and study from our members, and that has served as the instrumentality through which some real contributions have been made to that body of knowledge which we hope to enlarge and organize so that we may properly call it *Casualty Insurance Actuarial Science*.

A survey of the casualty insurance field will reveal many places where pioneering efforts have resulted in distinctive contributions. Among those of direct interest to actuaries may be cited the development of coverages and premium bases, the devising of statistical systems within the carriers, the organization of central bureaus and boards for collecting and compiling the 'carriers' data, the formulation of methods for reducing these data to uniform (basic) levels, and the development of weighting systems giving credibility on quantitative bases so that the actual experience of individual classifications and risks may receive proper recognition. Each of these might be a fitting subject for this anniversary occasion. I have selected the last because it is almost exclusively actuarial in nature, and because the largest and most distinctive contributions to casualty actuarial knowl-

edge have been made here. It is my intention to confine myself to a particular phase of the broader subject of the credibility of experience by limiting my remarks to the credibility of the experience of the individual risk while making a brief review of the development of credibility in experience rating in compensation insurance. I have selected the compensation field because experience rating was originally developed here and then adapted to other lines and because the only available data for checking results are found here. It is not my purpose to treat the principles and practices of experience rating at length. I intend merely to consider some developments of the past pertaining to the credibility of the individual risk experience and suggest some further studies.

A review of the *Proceedings* will reveal several thoughtful and forward-looking papers concerning experience rating of compensation insurance risks in the early volumes—see Bibliography, Appendix III. The resourcefulness of the writers, their comprehensive treatment of the problem, and their boldness in experiment merit admiration even when reviewed after the lapse of more than a decade and in the light of the information acquired during that time. After these pioneering efforts which shaped the general structure of the experience rating procedure, the subject fell into abeyance so far as our Society records reveal. There are only two papers* since Volume IV devoted wholly to a phase of experience rating. Special phases of experience rating have been treated forcefully in letters, memoranda, and discussions by members of various committees of rating organizations. It is hoped that some of the ideas developed may be added to the permanent records in our *Proceedings* and that interest in both the fundamentals and applications of experience rating may be revived.

DEFINITION AND OBJECT OF EXPERIENCE RATING

The term "experience rating" as now used refers to definitely prescribed procedures for determining individual risk rates depending in whole or in part on the risk's own experience. Risks whose rates have been determined in accordance with some such procedure are said to be experience rated. The compilation of

* Senior, Vol. XI; Kormes, Vol. XX.

definitions, rules, regulations, formulas, and forms necessary to describe and apply the procedure is called the *experience rating plan*.

The object of experience rating is to determine a more equitable rate for the individual risk based in a degree on the evidence presented by its own experience. It is recognized that individual risks within a classification are not alike and that there exist inherent differences due, for example in compensation, to variations in plants and premises, in operating processes, in the materials involved, in the management, in the morale of employees, in claim consciousness, and in the relation to the community. These differences are of such a nature that it is difficult to label them definitely and they cannot be associated with conditions measurable in advance. It is known, however, that variations in experience do exist in a way that definitely precludes ascribing all of them to chance. Experience rating is considered by many as the most practical method yet devised, or even suggested, of giving recognition to variations produced by such factors.

BASIS OF EXPERIENCE RATING

Experience rating is based on the existence of variations in the inherent hazard of the risks which enter into the classification experience. Its object is to measure to a higher degree the hazard of the individual risk by the evidential value of the risk's own experience. This basis needs to be emphasized. If all risks were entirely typical of the classifications, the variation in experience would be purely fortuitous and there would be no place for experience rating; for it would be impossible to reclassify the risks into more homogeneous groups. There are many factors which in different combinations enter into the risk's experience and affect the quality in different degrees. These, at least as yet, can not be classified and recognized so that they may be given individual consideration in rating. They may, however, be reflected to some extent by making use of the effect produced by them as shown in the experience. In the experience rating process, no distinction can be made between similar individual accidents which are fortuitous and those which are indicative of the actual conditions of the risk. The experience of the risk necessarily cannot be divided on such a basis.

APPLICABILITY OF EXPERIENCE RATING

Experience rating is applicable wherever there is a large variation among the risks which make up the classification and where the individual risks are of such nature that they may be expected to develop individual risk experiences of appreciable evidential value. Many lines of casualty insurance have classifications somewhat non-homogeneous, resulting largely from the meager experience available and the present lack of knowledge of the elements which enter into the composition of hazards. Considering only the qualification of having atypical risks within classifications, most casualty lines would be subject to experience rating. The further qualification of having individual risk experiences large enough to be of appreciable evidential value is more restrictive.

Compensation insurance, particularly, is subject to experience rating, for to a considerable degree the losses may be controlled and individuality of management reflected in the experience through the employer's ability to correct defective conditions and to enforce safe practices among employees by his potential power to dismiss or to withhold promotions. There are a few other lines, like employers' liability, workmen's collective, and automobile fleet collision, where the assured has similar power to affect losses. In third party insurance, the assured generally cannot control losses to the same degree, for, notwithstanding that the coverage is for liability of the assured only, the actions of the third party, over whom he has no control, affect the losses. In compensation insurance, risks develop individual risk experiences which in some cases have very high evidential value and, because of the control exerted by the management or other factors, often vary widely even within more homogeneous classifications, relative to occupations covered.

PROSPECTIVE AND RETROSPECTIVE RATING

An experience rating plan in which the experience of the risk is used to determine definite rates for periods in the future is said to be a *prospective* experience rating plan. All plans ever approved for general use have been of this form. A plan in which the experience of a given period is used to determine a final rate to apply to a past period is said to be a *retrospective* plan. Both

of these are entirely legitimate plans and represent definite ways of recognizing variations in the inherent hazards of risks. Both kinds could operate simultaneously, and under an economic system of unrestricted competition probably some carriers would select one form and some the other. The same carrier might even use both forms, applying to some risks one form, and to some the other, or it is even conceivable that both forms might be applied to the same risk. *Retrospective* rating would involve some change in theoretical viewpoint, for experience rating as now applied does not depart from the principle of a known rate fixed in advance.

ESSENTIALS OF EXPERIENCE RATING

The essential operation of experience rating consists of comparing the risk experience and classification experience on a common premium and loss basis, assigning to the risk experience a weight depending on the size of the risk premium and to the classification experience the complementary weight, and deriving a rate therefrom. The adjusted risk rate or experience rate may be looked upon as a weighted average of the rate indicated as necessary by the losses of the risk and the manual rate, that is, the rate indicated by the classification experience. The comparison may be made and has been made in different plans on the basis of indicated losses, pure premiums, or premiums.

In compensation insurance it is required first to "modify" the actual experience of the risk to bring it to the level of current industrial conditions as reflected in the current manual rate level. In the most widely used plan the procedure then is to determine "adjusted losses", the weighted average of the risk's modified losses and the "expected losses" which are indicated by the premium at manual rates*; to derive the ratio of the adjusted losses to the expected losses and apply this ratio to the manual rates* to obtain the final rates. In determining the adjusted losses, the hazard is divided into "normal losses" hazard and "excess losses" hazard. The weight or credibility assigned to the risk's experience is less in determining adjusted excess losses than in determining adjusted normal losses. The large losses occur less frequently than the normal losses and, costing much more individually, their volume in a given risk's experience is less indica-

* Schedule rates are used instead if schedule rating applies.

tive of the real hazard of large losses inherent in the risk than the volume of normal losses is of the real hazard of normal losses.

The technique of each step in the procedure, though worthy of detailed consideration and study, will not be considered here. It has been discussed at times in letters, memoranda, and open discourse in committees of rating bodies. The method of developing loss and payroll modification factors, the use of estimated individual case losses, average value losses whether fixed for all cases or varying with the duration of the case or other conditions, the theoretical and practical advantages and disadvantages of non-split, two-split, or multi-split plans; all these might well receive extended consideration. I propose to consider only risk experience credibility in casualty insurance experience rating, its development, and some criteria of proper credibility, after first mentioning the subject of off-balance produced by experience rating in total premiums because of its inter-relationship with credibility.

OFF-BALANCE OF PLAN.

A phase of the technique of experience rating which has assumed increasing importance is the off-balance of the experience rating plan, that is, the variation of the premium collected on experience rated risks under adjusted rates from that expected at manual rates. There are reasons why one might expect an experience rating plan in which credibility varies with size to be out of balance, when the same elements enter into the modification factors which enter into the manual rate determination. What used to be believed the preponderant, if not the sole cause, an under-reporting of losses on experience rated risks has, it now seems, been over-estimated as to its influence. At least the risk experience so far available from the rather recently established systems of individual risk reports to rating organizations indicate no greater development factor for losses of large risks than they do for losses of small risks which are not subject to experience rating. A factor which is coming more to be recognized as a primary cause of off-balance is the difference in the quality of the experience of large risks and small risks. Generally, the experience of the large risk is more favorable than that of the smaller risk, or of all risks. Necessarily, where the manual rate level is keyed

to the average of all risks and no allowance has been made for this more favorable experience for large risks, it may be expected that an off-balance will be produced from experience rating. Even if recognition is given in the rate level to the more favorable experience for experience rated risks and the experience rating plan keyed to the level of rated risks, there is still left the variation within the experience rated group between the extremely large risks and those risks which just qualify for experience rating. As will be noted from the experience shown for policy year 1931 for New York, Table I, the manual loss ratio for risks in excess of \$10,000 is more than 10% below the average of experience rated risks. These have more favorable experience and by virtue of their size under the experience rating plan receive larger credibility and therefore obtain credits which cannot be expected to be offset by an equal volume of less favorable experience on the smaller experience rated risks whose credibility is less.

DEVELOPMENT OF THE CREDIBILITY FACTOR IN COMPENSATION

National Workmen's Compensation Service Bureau Plans

The part of experience rating plans over which opinion has differed most concerns the reliance placed on the risk's own experience or what is now known as the credibility factor. In this outline of the development of credibility, only the plans of the National Workmen's Compensation Service Bureau and the National Council on Compensation Insurance will be reviewed. In the development of the general principles of experience rating, these may be considered representative. In the first compensation experience rating plans, of which Plan A of 1916 and Plan B of 1917 of the National Workmen's Compensation Service Bureau are typical, there was no general variation in credibility by size of risk. In Plan A there was a limited variation in credibility by size of risk for the schedule rated risks only. In Plan B there was a small variation in credibility by rate size groups but no variation by risk size. The extent of the modification of the risk depended on the amount by which the risk's loss ratio deviated from the average. Soon this failure adequately to consider the size of risk was generally recognized and dealt with in the credi-

bility formulas introduced with Plan D* in 1918. In this plan credibility was determined in two divisions of coverage, from the partial premium corresponding to the death and permanent total disability coverage, and the partial premium for all other coverage. The credibility for each part was obtained from formulas of the form

$$Z = \frac{P}{P + K}, \text{ where } Z \text{ denotes credibility}$$

P denotes partial premium
 K denotes a constant

The Z 's (Z_1 and Z_2 respectively) were taken from separate formulas or curves determined by K values (K_1 and K_2) chosen to give appropriate credibility to the losses in each division, the credibility being less for death and permanent total disability experience than for other losses of the same risks.

The formulas represent equilateral hyperbolas which pass through the origin and have as asymptote the line $Z = 1$. This permits one more point arbitrarily to be selected for each curve to determine the curve completely. Originally this point was selected for each division of coverage after experience rating a set of New York risks, both actual and hypothetical, using credibility curves of different degrees of liberality. The members of the committee, after consulting with underwriters, chose those curves which in their opinion produced the best results for the set of risks and thus established the constants K_1 and K_2 and the formulas for New York. The constants for other states were then selected so as to produce approximately the same credibility by parts if the accidents and claims of an average risk had been developed in New York and in each of the other states under their rates and compensation acts.

In determining credibility, the risk premium at latest manual rates was and still is used. This puts all risks on a common basis and eliminates differences that might affect credibility as between risks if actual premiums were used. Such differences might arise from different rate levels in the experience periods used or, and this is more important, from credits and debits in risks previously experienced rated, whereby risks of the same classification

* There was no Plan C for compensation; this letter was used for an employers liability plan.

and of the same size in number of employees and amount of payroll would have different credibility.

Industrial Experience Rating Plan—1920

The National Council's first plan, the Industrial Experience Rating Plan—1920 introduced some modifications in the technique of credibility determination. The credibility formula for death and permanent total disability remained as before, but for all other losses a new constant C was introduced to increase credibility, making the formula

$$Z_2 = \frac{P_2 + C}{P_2 + K_2 + C}$$

A refinement was introduced in the method of dividing the premium between the two coverages, making the division on the basis of the ratios of expected losses in the two divisions in each manual classification. Previously, the classifications had been grouped by size of rate, and average ratios determined, one for each size group. A new feature of the plan was the introduction of "self-rating". This provision was that risks whose subject* premium or whose indicated premium from the losses was \$80,000 or more should have a credibility of unity in each division of coverage. Interpreted graphically, this means that credibility for risks under \$80,000 premium was determined from the Z curves, and for risks of \$80,000 and over the credibility was taken from the line $Z = 1$.

Industrial Experience Rating Plan—1923

When the Industrial Experience Rating Plan—1923 was adopted, the losses were separated into "normal" and "excess" losses for determining credibility, in place of the former two divisions, "Death and Permanent Total", and "All Other" losses.

The credibility formulas were $Z_1 = \frac{P_1}{P_1 + K_1}$, $Z_2 = \frac{P_2}{P_2 + K_2}$, a return to the forms in Plan D. It was agreed to fix the Z curves for each state by selecting K 's so that a single maximum claim on a risk of \$1,000 subject premium having the average

* Subject premium is the premium subject to experience rating, and is obtained by extending the payrolls of the experience period at the manual or schedule modified rate for the effective date of the rating.

state excess ratio would increase the rate by 20% of the manual, 15% of the effect to be on the normal portion and 5% on the excess portion. The self-rating point was set at \$100,000 subject premium or \$60,000 losses for most states. A system of weights applying to both the actual and expected losses which decreased the influence of the older policy years was introduced toward the end of the effective period of this plan.

Industrial Experience Rating Plan—1928

The discontinuities of the credibility curves were removed in the Experience Rating Plan—1928. The formulas remained the same as in the Plan—1923, with K values determined by the same rule as before, but the range of applicability of the formulas was lessened and the discontinuities of the curves removed through the introduction of tangents to the curves from selected self-rating points. Separate normal and excess self-rating points were established. The self-rating point for normal experience was the same as before. Credibility for normal became unity at the point* corresponding to \$100,000 subject premium, and credibility for excess experience was lessened, becoming unity at the point* corresponding to \$200,000 subject premium. In this plan, which is still in effect, the credibility curves have become compound continuous curves, with the first sections arcs of hyperbolas, the second tangents to the hyperbolas, and the last a horizontal line. Tables have been constructed from which the credibility values are taken.

APPRAISING EXPERIENCE RATING

Underwriters and the assured are continually passing judgment on the results for individual risks. Little has been done, however, toward obtaining more systematic or statistical analyses of the results. Various possibilities occur as to the relation of the empirical Z values with what might be regarded as the proper values. The credibility may be everywhere either too high or too low, or it may be too high at one extreme or too low at the other, or the empirical curve may cross the proper value several times. Before commenting on the relation of the credibility

* On an average normal-excess premium split basis.

scale and the results of an experience rating plan, it is necessary to consider again what the experience rating plan is designed to do.

The object of experience rating is to make all experience rated risks within a classification having correct manual rates equally desirable as far as the loss ratio is concerned, or, if all classification rates are assumed correct in their net effect for the total of experience rated risks in the classification, it may be said the object is to make all experience rated risks equally desirable from the loss ratio point of view. In the discussion which follows it will be assumed that the classification rates are correct in their net effect for experience rated risks.

A necessary condition for proper credibility is that the credit risks and debit risks equally reproduce the permissible loss ratio. Also, if the proper credibility has been attained, each sub-group of the credit and debit risks, provided it has adequate volume, should give the permissible loss ratio. While these conditions are necessary for a proper credibility of the experience rating plan, it does not follow that they are also sufficient. For a sufficient condition it would be required to establish that the risks within a group cannot be subdivided on any experience basis so as to give different loss ratios for the subdivisions, assuming the latter have adequate volume.

The necessary and sufficient conditions for the achievement of ideal credibility in an experience rating plan may be illustrated by an analogy to the classification experience. A necessary condition for proper classification rates is that each classification shall reproduce the permissible loss ratio. This condition, however, is not sufficient. A sufficient condition further requires that any subdivision of the classification having adequate volume should reproduce the permissible loss ratio. If two classifications, each of which has its different proper rate, are combined and an average rate established for the combination, the new combined class would reproduce the permissible loss ratio provided the relative volumes in the two original classes remain the same. For this new class, the necessary condition that the new rate reproduce the permissible loss ratio, would have been met. The condition for sufficiency that each sub-group reproduce the permissible loss ratio on the new rate basis would not have been met, for if the new rate were applied to the exposure under each of the original

classifications which entered the combination, the permissible loss ratio would be reproduced for neither.

The necessary and sufficient condition for establishing that the credibility basis of the experience rating plan is correct may be stated as the condition that it is impossible to subdivide the risks on an experience basis differing from the experience rating plan and predict significantly different loss ratios for the subdivisions, providing they have adequate volume to be dependable. When considering the results of any plan, it is impossible to prove that the experience cannot be divided on any other credibility basis to yield better results. The second or sufficient condition is only required to prove that the plan in question is the optimum. The first condition is all that need be considered to test the relative merits of any given plans of experience rating or credibility scales or of a plan of experience rating as compared with no experience rating. The question then is not whether ultimate perfection has been reached but rather whether one plan is better than another, or than no experience rating.

The primary agents in the plan itself, other than the basic data, which affect the experience modification of a risk are the loss modification factors (including the effect of the average value and the payroll factor) and the credibility allowed the risk experience. It may be shown what effect each of these has when the other is assumed to be correct and to remain so. Consider the effect of variation in the loss modification factors on the risks of a premium size group arranged in experience modification groups, as in Table I. If the loss modification factor is too high (produces more modified losses than correspond to the rate level) and if the assumption is made that the rate level and the credibility factor are correct by premium size groups, it may be shown that the loss ratios produced in a given premium size group will have a downward trend as the experience modification increases. Conversely, if the modification factor is too low under the same conditions, the resulting loss ratios will have an upward trend. (See Appendix I.)

TEST OF CREDIBILITY SCALE

In Appendix II the compensation experience of experience rated risks in New York for policy year 1931 has been compiled

in a manner to permit examining the results of the New York Plan in relation to the necessary condition for proper credibility. The risks have been sorted in Table I into premium size groups and then each of these groups has been sorted into experience modification groups of .10 intervals. The sub-groups of the experience rating data resulting from these two sortings will be called "parcels".

It will be noted from Table I and the summary on page 19 that, for the individual parcel, the actual loss ratio is nearer to the permissible loss ratio (.605) than the manual loss ratio is, in 74 of the 97 parcels. For the parcels having credit experience modifications the actual loss ratio is nearer in 45 out of 52, and for the parcels having charge modifications the actual loss ratio is nearer in 29 out of 45.

When the credit parcels within each premium size group are combined, 7 of the 8 combinations show less deviation from the permissible loss ratio for the actual than for the manual loss ratio, the exception being the combined credit parcel for the short-term risks. The same result prevails when all the parcels within a premium size group are combined. When the parcels for the debit experience modifications are combined, 5 of the 8 combinations show less deviation from the permissible loss ratio for the actual than for the manual loss ratio.

When one considers the trends of the straight lines fitted by least squares to the actual loss ratios of the individual premium size groups, it will be noted, page 20, that, in passing from the lower to the higher modifications, of the 8 lines fitted to the credit parcels, 4 have an upward trend and 4 have a downward trend. In the lines fitted to the loss ratios of the debit parcels, the trend in 5 is upward and in 3 downward. When the lines fitted to the loss ratios of all parcels are considered, 6 have upward trends and 2 downward trends.

Interpreting these trend results on the assumption that the loss modifications factors are correct it may be said that they are not unfavorable to the present credibility or "swing" of the plan. It could hardly be expected in view of the limited data that no trends would appear—a condition that would uphold the present credibility. The indicated trends are rather evenly divided between upward and downward trends in the credit and charge experience modification groups. For all groups combined,

which should be the most reliable, the upward trend dominates which would indicate that the present credibility was too restricted. However, when the short-term risks are eliminated, the trend is downward. Too much credence should not be given to the indications, for the data are not only limited but are derived from a single policy year situated in a particular phase of the business cycle. Similar tests should be applied to other policy years in other phases of the cycle and to other experience rating plans, and the results studied before passing final judgment.

I have attempted in these remarks to direct your attention to a problem that is of primary importance in casualty insurance with the hope of stimulating your thought and interest rather than presenting a solution. Compensation insurance was selected for purpose of illustration because the line is well-known; experience rating has been most highly developed in this line, and more extensive data are available for experimental purposes. The interest in the subject should extend to all lines where experience rating is applicable and experience available.

No attempt has been made to give a complete interpretation of the experience presented; this would be hardly justifiable on the basis of one year's experience. The object has been to indicate ways in which tests might be conducted. If a number of our members, either individually or jointly, undertook to analyze the data of experience rated risks for different states and policy periods, possibly along the lines suggested, it is my belief that there would result contributions to both the fundamental principles and applications of experience rating, perhaps comparable to those made in the first decade of our Society.

APPENDIX I

EFFECT OF ERROR IN LOSS MODIFICATION FACTOR

Let the experience of the experience rated risks for a policy year in a given state be sorted into risk premium size groups and effective experience rating modification size groups. Designate these resulting sub-divisions of the experience as "parcels". Assume that the classification rates are correct and that the credibility of the experience rating plan is correct, so that with correct loss modification factors the actual loss ratios for each parcel will be the permissible.

It is proposed to determine the effect produced by an error in the loss modification factor on the trend of the actual loss ratios of the parcels in a given risk premium size group when the parcels are arrayed in increasing experience modification order. The loss modification factor herein will be understood to embrace the combined effect of the present modification factors for losses and payrolls and the effect of using average values. The modification factor will be considered correct when the losses of a given year are brought to the loss level underlying the manual rates.

Let F denote correct loss modification factor

F' denote actual loss modification factor

L_s denote actual losses of the experience period of parcel s

P_s denote subject premium of parcel s

E denote expected loss ratio

M_s denote correct experience modification of the risks in parcel s
(derived by using the correct modification factor F)

M'_s denote actual experience modification
(derived by using the actual modification factor F')

Z denote credibility of risks in premium size group

x denote difference between F' and F , or correction in F'

Then

$$F' = F + x$$

EP_s = expected losses of parcel s

FL_s = modified losses of parcel s , using correct loss modification factor

$(F + x)L_s$ = modified losses of parcel s , using actual loss modification factor

$$M_s = \frac{ZFL_s + EP_s(1-Z)}{EP_s}$$

$$M'_s = \frac{Z(F+x)L_s + EP_s(1-Z)}{EP_s}$$

$$= M_s + \frac{ZxL_s}{EP_s}$$

Consider the loss ratio of the experience developed in parcel s during the effective periods of the ratings based on F'

${}_sL_s$ denotes actual losses in parcel s during effective (future) period

${}_sP_s$ denotes manual premium in parcel s during effective (future) period

${}_sr_s$ denote loss ratio of parcel s during effective (future) period with ratings based on F

${}_sr'_s$ denote loss ratio of parcel s during effective (future) period with ratings based on F'

Then

$$\begin{aligned} {}_sr'_s &= \frac{{}_sL_s}{{}_sP_s M_s} = \frac{{}_sL_s}{{}_sP_s \left(M_s + \frac{Z x L_s}{EP_s} \right)} \\ &= \frac{{}_sL_s}{{}_sP_s M_s} \cdot \frac{1}{1 + \frac{Z x L_s}{EP_s M_s}} \\ &= \frac{{}_sL_s}{{}_sP_s M_s} \frac{1}{1 + \frac{Z x L_s}{ZFL_s + EP_s(1-Z)}}, \text{ since } EP_s M_s = ZFL_s + EP_s(1-Z) \\ &= {}_sr_s \cdot \frac{1}{1 + \frac{x}{F + \frac{EP_s}{L_s} \cdot \frac{1-Z}{Z}}}, \text{ where } {}_sr_s, x, F, \text{ and } \frac{1-Z}{Z} \text{ are constant and } 0 < Z < 1 \end{aligned}$$

As M_s increases these relations hold

	Case I	Case II
	$x > 0$	$x < 0$
$\frac{{}^*EF}{L_s}$	decreases	decreases
$\frac{EP_s}{L_s} \cdot \frac{1-Z}{Z}$	decreases	decreases
$F + \frac{EP_s}{L_s} \cdot \frac{1-Z}{Z}$	decreases	decreases
$\frac{x}{F + \frac{EP_s}{L_s} \cdot \frac{1-Z}{Z}}$	increases	decreases

$$1 + \frac{x}{F + \frac{EP_s}{L_s} \cdot \frac{1-Z}{Z}} \quad \text{increases} \quad \text{decreases}$$

$$\frac{1}{1 + \frac{x}{F + \frac{EP_s}{L_s} \cdot \frac{1-Z}{Z}}} \quad \text{decreases} \quad \text{increases}$$

$$r'_s \quad \text{decreases} \quad \text{increases}$$

This shows that if the actual modification factors are greater than the correct modification factors there will result a downward trend in the loss ratios and if the actual factors are less than the correct factors there will result an upward trend in the loss ratios, assuming that the rest of the experience rating plan is correct.

$$\frac{*EP_s}{L_s} \text{ is independent of } x \text{ and decreases with an increase in } M_s$$

as the parcels are assumed to be arrayed in that order.

APPENDIX II

RESULTS OF NEW YORK PLAN

Table I is an exhibit showing data of experience rated risks in New York for policy year 1931 compiled by the Compensation Insurance Rating Board of New York. In this exhibit the risks have been separated into full term and short term risks. The full term risks have been further separated into seven premium size groups based on actual annual premiums. The short term risks have been shown separately as it was impracticable to make a size division on an annual premium basis. There is also one group for all full term risks combined and another group for the total of all risks. The risks within each premium size group have been divided according to the experience rating modification factor underlying the risk rate in effect for policy year 1931.

A separation into size groups on the basis of manual rate annual premiums would have been preferable as reflecting more nearly the relative size of exposure. To place the experience on this basis, however, would require the determination of the manual premium for individual risks. The manual loss ratios given were derived by the Compensation Insurance Rating Board from manual premiums calculated for groups of risks at intervals of .01 of experience modification by division of the group actual premiums by their experience rating modifications.

For each experience modification division in each of the premium size groups there are shown the number of risks, the actual premiums expressed in \$1,000 units, the actual loss ratio, and the manual loss ratio. The totals of these items for all credit risks, all charge risks and all risks are also shown. The object is to test the effect of experience rating on the loss ratios of the individual parcels into which the policy year experience has been sorted by the division into premium size groups and experience modification groups. If the experience rating procedure produces rates more equitable than the manual rates which they supersede, then, assuming adequate exposure, the deviations from the permissible loss ratio should be less for the actual loss ratios than for the manual loss ratios of the individual parcels.

In Table IA for each experience modification group a "1" has been placed in the proper column and line for each parcel to indicate whether the actual loss ratio or the manual loss ratio was nearer to 60.5%, the permissible loss ratio. The columns also have been summed for all credit modification groups, all charge modification groups, and all modification groups. At the bottom of Table IA, on the last three lines, it has been indicated in a similar manner whether the actual loss ratio or the manual loss ratio was nearer the permissible for all credit risks combined, for all charge risks combined, and for all experience rated risks combined. The results for the individual parcels of premium size groups and the whole premium size groups in Table IA when summarized are as follows:

Experience Modification Group	Number of Parcels where the Permissible Loss Ratios are nearer to		Number of Premium Size Groups where the Permissible Loss Ratios are nearer to	
	Actual L. R.	Manual L. R.	Actual L. R.	Manual L. R.
Credit Groups	45	7	7	1
Charge Groups	29	16	5	3
All Groups	74	23	7	1

In these tabulations the short-term risks were considered as one premium size group. This short-term group is responsible for the entries in the first and last line of the last column denoting that the permissible loss ratio is nearer to the manual than to the actual.

If the manual rates for the classifications were quite correct for every premium size group, if the experience rating plan were perfect, and if the volume of experience under each partition were adequate, the actual loss ratio in each partition should equal the permissible. Under these ideal conditions the deviations from the permissible loss ratio would be purely fortuitous and be plus and minus with equal frequency. Then straight lines fitted by least squares to the actual loss ratios of the parcels in any direction should have no trend, and a plane fitted to the whole field should be level.

In Table IB are shown loss ratios lying on straight lines fitted to the actual loss ratios of experience modification groups of each premium size group in Table I by the method of least squares, using the actual premiums in thousands as weights. In the column headed "All", the loss ratios derived from the fitted straight lines are given. In the other columns, under "Cr", the loss ratios on straight lines fitted to the credit modification groups only are given, and, under "Dr", the loss ratios on straight lines fitted to the charge modification groups only are given.

An effort has been made to gain in this way some knowledge as to the effect of the credibility factor or the "swing" of the plan. If all the conditions were correct, a line showing an upward trend in loss ratios with increasing experience modification groups would indicate that the swing of the plan is too restricted, for a wider swing would increase the credits and charges which would result in higher loss ratios for credit risks and lower loss ratios for charge risks. The change, if sufficient, could be made to overcome the trend so that, generally, the actual loss ratios for the charge risks would be no higher than those for credit risks.

It will be observed from Table I, that for all premium size groups, except the highest two, the actual loss ratios for the charge risks exceed those of the credit risks. In the "\$10,000-\$49,999" premium size group, the predicted charge risks had a manual loss ratio of 58.9% as compared with 48.8% for the whole group. The application of the charges from experience rating produced an actual loss ratio for this group of 50.2% as compared with 54.7% for the whole premium size group. If these limited data were accepted as fully reliable, this would indicate a swing which is too large, or a credibility factor which is too

high, in the experience rating plan. In the "\$50,000 up" premium size group, the debit risks are even more out of line. The predicted debit risks actually have a trifle better manual loss ratio than the group-as a whole and, with the charges imposed, the actual loss ratio becomes very much better than that of the whole group.

The results, aside from these two high groups, are rather favorable to the present credibility of the experience rating plan. It is possible that the self-rating points established arbitrarily may have an influence on these large premium size groups. However, the complete reversal of form of the eleven predicted charge risks in the highest group, assuming they were correctly reported and rated, cannot be explained by any change in credibility factor. The actual explanation would require a detailed examination of the underlying losses which enter into the rating procedure and the conditions prevailing in the risks during policy year 1931 and the preceding years when the experience underlying the experience rating procedure was developed.

In Table IB, the loss ratios on straight lines fitted to the data in Table I show trends as the experience modification increases. These trends for the loss ratios on the lines fitted to the credit groups, the lines fitted to the charge groups, and to all groups for the seven full term premium groups, and the one group including all short-term risks combined, may be summarized as follows:

Experience Modification Groups	Straight Line Loss Ratio Trends as Experience Modification Increases	
	Trend Upward	Trend Downward
Credit Groups	4	4
Charge Groups	5	3
All Groups	6	2

The results for the credit groups are evenly divided between upward and downward trends. The results for the charge groups and all groups show an upward trend, though not a very decisive one.

TABLE I—COMPARISON OF ACTUAL LOSS RATIOS AND MANUAL LOSS RATIOS
EXPERIENCE RATED COMPENSATION RISKS IN NEW YORK POLICY YEAR 1931
DATA OF COMPENSATION INSURANCE RATING BOARD

Risks Grouped According to Size of Experience Modification: Full Term Risks Subdivided into Premium Size Groups

Experience Modification in Per Cent	No. of Risks	Act. Prem. in Thou.	Act. L. R.	Man. L. R.	No. of Risks	Act. Prem. in Thou.	Act. L. R.	Man. L. R.	No. of Risks	Act. Prem. in Thou.	Act. L. R.	Man. L. R.	No. of Risks	Act. Prem. in Thou.	Act. L. R.	Man. L. R.	No. of Risks	Act. Prem. in Thou.	Act. L. R.	Man. L. R.	
UNDER \$500					\$500—999				\$1000—2499				\$2500—4999				\$5000—9999				
0- 30%					2	2	364.1	132.7									2	17	90.6	29.7	
30- 39					4	3	28.3	13.1	6	11	14.0	6.7	6	22	58.6	26.4	2	15	23.6	10.8	
40- 49	5	2	31.3	14.4	9	7	48.4	26.3	22	35	59.9	33.0	18	66	48.4	26.5	14	96	36.9	20.7	
50- 59	16	2	165.3	91.9	34	26	43.6	28.4	102	171	49.9	32.7	62	217	60.7	39.7	30	225	66.7	43.4	
60- 69	52	12	32.8	21.6	196	147	48.3	38.0	273	432	46.6	35.2	139	474	50.4	37.9	58	384	62.5	47.0	
70- 79	211	58	82.6	62.5	973	708	45.3	38.8	776	1,161	51.9	44.5	237	796	61.5	52.2	97	671	55.7	47.2	
80- 89	1,018	298	54.1	46.8	1,939	1,316	55.9	52.6	893	1,345	57.2	53.5	243	806	50.1	47.0	112	797	50.5	47.6	
90-100	3,524	1,090	53.8	50.6																	
Credits	4,826	1,462	55.0	50.0	3,157	2,209	51.9	46.4	2,072	3,155	53.3	45.6	705	2,381	55.0	44.8	315	2,205	55.4	44.4	
100-109	1,003	305	66.8	69.3	832	591	62.5	64.8	609	943	51.7	53.9	173	591	66.6	69.4	77	515	55.4	58.1	
110-119	426	129	54.1	61.6	416	296	65.6	74.9	371	570	55.5	63.3	113	398	56.2	64.1	48	342	60.6	69.0	
120-129	171	51	68.3	82.4	177	126	59.4	73.6	169	272	57.6	71.3	79	278	50.6	62.8	38	266	85.1	106.4	
130-139	67	20	117.8	156.8	85	59	52.2	69.9	92	147	67.7	90.2	41	150	88.4	118.7	34	232	59.6	80.0	
140-149	35	11	42.8	60.4	36	25	71.1	102.5	52	81	58.3	82.7	27	92	59.2	85.2	13	84	38.4	55.1	
150 Up	27	7	48.3	76.6	28	20	132.4	211.8	57	99	43.2	71.4	24	80	42.0	70.9	17	114	47.8	82.4	
Charges	1,729	523	64.8	71.3	1,574	1,117	63.9	71.1	1,350	2,112	54.4	62.1	457	1,587	61.6	71.7	227	1,553	60.8	72.5	
TOTAL	6,555	1,985	57.6	54.9	4,731	3,326	56.0	53.5	3,422	5,267	53.8	51.1	1,162	3,968	57.7	53.2	542	3,758	57.6	53.4	
\$10,000—49,999					\$50,000 and Over				Full Term—All Sizes				Short Term—All Sizes				All Risks				
0- 30%	1	50	120.1	34.9					1	50	120.1	34.9		2	4	4.0	1.4	1	50	120.1	34.9
30- 39	2	34	35.6	11.8					6	53	62.5	20.6		8	57	58.7	19.5	8	57	58.7	19.5
40- 49	4	102	79.7	36.2					27	155	65.0	29.6		28	161	65.4	29.7	28	161	65.4	29.7
50- 59	6	169	77.7	42.5	1	75	25.9	14.2	86	450	54.8	30.2	16	19	9.9	5.4	102	469	52.9	29.1	
60- 69	17	322	50.7	33.1	5	440	59.1	36.9	302	1,413	57.0	36.8	53	49	108.6	72.2	355	1,462	58.7	37.9	
70- 79	41	749	52.6	39.6	2	141	66.8	49.2	920	2,385	53.9	40.7	164	145	62.8	47.2	1,084	2,530	54.4	41.0	
80- 89	69	1,389	57.8	49.3	7	620	47.2	39.4	3,177	5,643	53.9	46.0	451	382	89.5	76.4	3,628	6,025	56.2	47.8	
90-100	55	895	56.7	53.2	6	660	57.5	54.6	6,772	6,909	54.8	51.5	964	575	69.7	65.4	7,736	7,484	55.9	52.6	
Credits	195	3,710	58.1	44.1	21	1,936	54.0	42.3	11,291	17,059	54.9	45.0	1,651	1,180	75.8	64.4	12,942	18,238	56.2	46.2	
100-109	58	1,073	47.7	49.6	4	371	38.0	40.6	2,756	4,389	54.5	56.9	451	281	84.7	88.8	3,207	4,670	56.4	58.8	
110-119	36	625	48.1	54.9	4	288	41.1	46.4	1,414	2,648	53.9	61.5	304	343	72.0	82.4	1,718	2,991	56.1	63.9	
120-129	15	292	60.9	76.6	3	192	30.1	36.9	652	1,475	58.7	73.1	211	210	88.1	109.6	863	1,685	62.4	77.7	
130-139	15	226	56.3	75.4					334	1,334	66.1	88.6	85	72	110.6	148.9	419	1,906	69.8	93.5	
140-149	10	206	60.3	86.0					173	499	56.2	80.4	51	74	63.7	91.4	224	573	57.2	81.8	
150 Up	13	248	39.5	66.2					166	568	45.6	76.5	78	116	83.5	133.2	244	684	52.0	86.5	
Charges	147	2,670	50.2	56.9	11	851	37.3	41.8	5,495	10,412	55.5	64.1	1,180	1,096	81.6	97.7	6,675	11,509	57.9	67.1	
TOTAL	342	6,380	54.7	48.8	32	2,787	48.9	42.2	16,786	27,471	55.2	50.7	2,831	2,276	78.6	77.6	19,617	29,747	57.0	52.6	

TABLE IA—FROM DATA OF TABLE I

COMPARISON OF ACTUAL LOSS RATIOS (ALR) AND MANUAL LOSS RATIOS (MLR)
OF TABLE I WITH PERMISSIBLE LOSS RATIOSA "1" in Columns "ALR" or "MLR" Indicates Respectively whether the Actual Loss Ratio or the
Manual Loss Ratio is nearer to the Permissible Loss Ratio

Experience Modification in Per Cent	FULL TERM RISKS—PREMIUM SIZE GROUPS																All Short Term Risks		All Risks	
	Under \$500		\$500-999		\$1,000-2,499		\$2,500-4,999		\$5,000-9,999		\$10,000-49,999		\$50,000 and Over		All Groups					
	ALR	MLR	ALR	MLR	ALR	MLR	ALR	MLR	ALR	MLR	ALR	MLR	ALR	MLR	ALR	MLR	ALR	MLR	ALR	MLR
0-30%	..	..	..	1	..	..	..	..	1	..	1	1	..	..	1	1	..	..	1	1
30-39	..	..	1	..	1	..	1	..	1	..	1	..	..	..	1	..	1	..	1	..
40-49	1	..	1	..	1	..	1	..	1	..	1	..	..	..	1	..	1	..	1	..
50-59	..	1	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..
60-69	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..
70-79	..	1	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..
80-89	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..
90-100	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..
Credits	4	2	6	1	6	..	6	..	7	..	7	1	5	..	7	1	4	3	7	1
100-109	1	..	1	..	1	1	1	..	1	..	1	..	1	..	1	1	1	..	1	1
110-119	..	1	1	..	1	..	1	..	1	..	1	..	1	..	1	1	1	..	1	..
120-129	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	1	1	..	1	..
130-139	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	1	1	..	1	..
140-149	..	1	1	..	1	..	1	..	1	..	1	..	1	..	1	1	1	..	1	..
150 Up.	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	1	1	..	1	..
Charges	4	2	6	..	3	3	3	3	4	2	3	3	..	3	4	2	6	..	4	2
TOTAL	8	4	12	1	9	3	9	3	11	2	10	4	5	3	11	3	10	3	11	3
Credit Group	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..	..	1	1	..
Charge Group	1	..	1	..	..	1	1	..	1	..	..	1	..	1	..	1	1	..	1	..
All Risks	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..	..	1	1	..

TABLE IB—FROM DATA OF TABLE I

SHOWING TRENDS OF LOSS RATIOS WITH RISKS GROUPED BY SIZE OF EXPERIENCE MODIFICATION
WITHIN PREMIUM SIZE GROUPS

The Values given lie on Straight Lines Fitted by Least Squares to the Actual Loss Ratios in Table I,
Weighted according to Actual Premiums

In the Columns headed "All" the Lines were Fitted to the Loss Ratios of All Modification Groups. In the Other
Columns, under "Cr." the Lines were fitted to the Credit Modification Groups only, and under "Dr." to the
Charge Modification Groups

Experience Modification in Per Cent	FULL TERM RISKS—PREMIUM SIZE GROUPS																All Short Term Risks		All Risks	
	Under \$500		\$500-999		\$1,000-2,499		\$2,500-4,999		\$5,000-9,999		\$10,000-49,999		\$50,000 and Over		All Groups					
	Cr.	All	Cr.	All	Cr.	All	Cr.	All	Cr.	All	Cr.	All	Cr.	All	Cr.	All	Cr.	All		
0-30%			37.2	28.9					66.9	55.7	77.0	67.2			62.6	56.2			62.2	56.2
30-39			37.2	28.9					66.9	55.7	73.5	65.2			61.3	56.0	74.1	70.5	61.2	56.5
40-49	77.9	48.0	39.9	33.2	38.2	48.9	59.0	54.1	64.5	56.0	70.1	63.8			60.0	55.9	74.4	71.7	60.2	56.7
50-59	73.0	49.8	42.6	37.6	41.9	49.8	58.0	54.8	62.1	56.3	66.6	62.0	52.3	49.1	58.7	55.7	74.7	72.8	59.2	56.9
60-69	68.1	51.6	45.3	41.9	45.5	50.7	56.9	55.4	59.7	56.6	63.2	60.3	53.0	48.8	57.4	55.6	75.0	74.0	58.2	57.1
70-79	63.2	53.5	48.0	46.2	49.1	51.6	55.9	56.1	57.3	57.0	59.8	58.6	53.6	48.4	56.1	55.4	75.3	75.1	57.2	57.4
80-89	58.3	55.3	50.8	50.5	52.8	52.5	54.8	56.8	54.9	57.3	56.3	56.8	54.3	48.1	54.8	55.3	75.7	76.3	56.2	57.6
90-100	53.4	57.2	53.5	54.8	56.8	53.4	53.7	57.4	52.6	57.6	52.9	55.1	54.9	47.8	53.5	55.1	76.0	77.5	55.2	57.8
100-109	Dr.	59.0	Dr.	59.2	Dr.	53.8	Dr.	58.1	Dr.	57.9	Dr.	53.4	Dr.	47.5	Dr.	55.0	Dr.	78.6	Dr.	58.1
110-119	64.9	60.8	64.5	63.5	54.4	55.2	82.2	58.8	61.5	58.2	50.1	51.7	36.6	47.1	55.5	54.8	81.2	79.8	57.9	58.3
120-129	65.3	62.7	68.2	67.8	54.9	56.1	80.6	59.4	60.7	58.6	50.3	49.9	33.5	46.8	55.5	54.7	81.7	80.9	58.4	58.5
130-139	65.8	64.5	72.0	72.1	55.5	57.0	57.0	60.1	59.8	58.9	50.4	48.2			55.5	54.5	82.2	82.1	58.8	58.8
140-149	66.2	66.4	75.8	76.4	56.0	57.9	57.4	60.8	58.9	59.2	50.6	46.5			55.4	54.4	82.7	83.3	59.3	59.0
150 Up	67.0	70.0	83.4	85.1	57.1	59.7	54.2	62.1	57.2	59.8	50.9	43.0			55.4	54.0	83.8	85.6	60.2	59.4

APPENDIX III

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