VOLUME LVIII NUMBERS 109 AND 110

PROCEEDINGS

OF THE

Casualty Actuarial Society

ORGANIZED 1914

1971

VOLUME LVIII

Number 109 — May 1971

Number 110 — November 1971

1972 Year Book

Printed for the Society by
Recording and Statistical Division
Sperry Rand Corporation
Boston, Massachusetts

CONTENTS OF VOLUME LVIII

CONTENTS OF VOLUME LVIII (Cont.)

NOTICE

VOLUME LVIII, Part I No. 109

PROCEEDINGS

MAY 16, 17, 18, 19, 1971

FEDERAL INCOME TAXES

R. W. BECKMAN

One of the most important overall financial considerations for any company is Federal Income Taxes. For a property-liability insurance company, taxes are both important and, to a large extent, controllable. Whereas the income of most organizations is fully taxable, insurance companies' income is largely investment income which can be either taxable or tax exempt. This paper explores the subject of Federal Income Taxes and it illustrates how net income can be maximized by minimizing Federal Income Taxes. Because mutuals and life insurance companies fall under different sections of the tax code, they will not be included in this paper.

TAX LAW

The provisions of the Federal Tax Law that apply to insurance companies are essentially those that apply to most corporations. Specifically,

1. *Dividend Credit* — The investment income received from other non-affiliated corporations in the form of dividends is 85% tax free.[1] In

[1] H. Sauvain, *Investment Management,* (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1967) pg. 233.

". . . The purpose of this exclusion is to minimize the triple taxation of corporate earnings that occurs when one company owns stock of another. The triple taxation operates in this way: (a) one company reports earnings and pays the corporate income tax on these earnings; then it pays dividends from the taxed earnings to a second company that owns its stock; the dividends received are part of the earnings of the second company; (b) the second company pays the corporate income tax on its earnings and from the balance of earnings it pays dividends to its stockholders; and (c) the stockholders pay the personal income tax on the dividends. The 85% dividend exclusion of dividends from taxable income of corporations greatly reduces the second application of the tax."

other words, only 15% of dividends is subject to Federal Income Tax. There are a few exceptions to this 85% credit; for example, dividends from certain preferred stocks of public utilities are only allowed a deduction of approximately 60%, but most dividends are eligible for the 85% credit. Dividends from subsidiaries usually fall under another section of the law which exempts inter-corporate dividends from all taxes. The dividend credit is subject to certain limitations which will occasionally cause a loss of part of the dividend credit, but this is a relatively infrequent event and will be discussed later.

2. *Tax Exempt Interest* — The interest received from bonds issued by any state or local government is fully tax exempt. These tax-free bonds are often referred to as municipals or "munis."

3. *Tax-Loss Carry-Over* — When no tax liability exists and total taxable income is negative, a Tax-Loss Carry-Over in the amount of the negative taxable income is established. This carry-over lasts a maximum of eight years — three prior and five succeeding — and must be applied to the earliest year first. A tax refund is generated if the Tax-Loss Carry-Over reduces the prior year's previously computed income tax. Any excess or unused Tax-Loss Carry-Over is then carried to the following year which is also recalculated in a similar fashion.

   A realized capital loss from the selling of investments which is not offset by realized capital gains in a particular year may also be used over an eight year period in the same manner as described above, but only to offset realized capital gains. However, the amount which can be carried back is limited to an amount which does not cause or increase a net operating loss in the carryback year.

4. *Alternate Tax Calculation* — The alternate tax calculation for taxing gains from the sale of assets owned more than six months at a rate less than that applied to other income is also available to all companies. Most companies benefit from this provision when selling buildings and other property while insurance companies benefit when realizing capital gains by selling stocks and bonds.

5. *Tax Rates* — The basic corporate tax rate is effectively 48% and has been for several years. This actually consists of a 22% tax, and a 26% surtax on all taxable income in excess of $25,000. The capital gains tax rate as used in the alternate tax calculations was 25% prior to 1-1-70, was 28% for 1970 and increased to 30% for 1971 and

thereafter. The tax surcharge of 1969 and 1970 was applied to the total tax liability, thus effectively increasing both the basic corporate tax rate and the capital gains tax rate.

Sections 831-832 of the Federal Tax Code relate specifically to Stock Fire and Casualty Insurance Companies. Net earned premium is the revenue base for underwriting operations, with underwriting disbursements including incurred losses (including IBNR), incurred expenses (except that capital items such as automobiles, furniture, fixtures, etc., that are charged directly to expenses in the annual statement must be depreciated over their useful life span for tax purposes), and declared policyholders dividends (not incurred). The primary benefit to insurance companies arising from these sections of the tax law is that expenses (primarily commissions) are charged against income prior to the premium being counted as income. This in effect defers income without deferring the corresponding direct expenses and can amount to a substantial tax benefit, especially for rapidly growing insurance companies. Working in the opposite direction is the handling of policyholders dividends, which are allowed as a tax deduction only when declared.

Determination of Taxes

Federal income taxes must be paid on total income which is the sum of ordinary taxable income and realized capital gains.

The U. S. Corporation Income Tax Return, form 1120, accumulates premiums earned, dividends received, taxable interest income and realized capital gains as total income. Deductions include incurred losses, declared policyholder dividends, salaries, taxes, fees, etc. Gross taxable income is the difference between total income and total deductions. The dividend credit is a special deduction of 85% of dividends received *subject to a maximum of 85% of the gross taxable income* except that this limitation does not apply to a year in which a net operating loss occurs. The net taxable income is the gross taxable income less (1) the dividend credit, and (2) any applicable tax-loss carry-over from prior years.

The federal income tax is the lesser of (1) the net taxable income times the normal tax rate (48%), or (2) the ordinary taxable income (net taxable income less realized capital gains) times the normal tax rate (but not less than zero) plus the realized capital gains times the capital gains tax rate (30%). Step (2) above is the alternate tax calculation and provides for taxing capital gains at a lesser rate. However, whenever ordinary taxable income is negative and capital gains are positive, the net effect of the law is to tax capital gains at a rate between thirty and forty-eight percent. The effective tax rate depends on the relative magnitude of the ordinary taxable income and the capital gains.

When the detail required for the precise calculation is not available, a reasonable approximation to gross taxable income can be achieved by adding (1) statutory underwriting profit, (2) dividends received, (3) taxable interest, and (4) realized capital gains. A number of refinements could be made but they are generally of a minor nature.[2]

An example may help to clarify the calculation of federal taxes. The following assumes tax rates of a) 48% on ordinary income, and b) 30% on realized capital gains, and the following facts about the ABC Insurance Company:

Statutory Underwriting Profit	$-10,000,000
Taxable Investment Income	10,000,000
Tax Exempt Investment Income	10,000,000
Dividends Received	10,000,000
Realized Capital Gains	5,000,000
Net Income Before Taxes	$ 25,000,000

In this situation, ABC has gross taxable income of $15 million, a dividend credit of $8.5 million, net taxable income of $6.5 million, and ordinary taxable income of $1.5 million. The actual tax calculation, including the alternate calculation is shown on Exhibit 1 with the tax liability being $2,220,000.

To illustrate another point, assume the underwriting loss is $15 million. Then the gross taxable income will be $10 million, the dividend credit $8.5 million and the net taxable income $1.5 million. Now the standard calculation indicates a tax of $720,000 (1.5 million dollars @ 48%). The alternate calculation indicates a tax of $1,500,000 and this is greater than the standard formula. Thus the tax liability is $720,000 and the capital gain has been effectively taxed at 48% because (a) the ordinary taxable income loss of $3.5 million (which if there were no

[2] For a detail listing of many of these adjustments see:
W. R. Hammond (ed.), *Insurance Accounting — Fire & Casualty*, (Chilton Company, Philadelphia, 1965), pp. 303-306.

Exhibit I

CALCULATION OF FEDERAL INCOME TAX

Standard Calculation:

Net Income before Taxes	$25,000,000
Less: Tax-Exempt Income	10,000,000
85% of Dividends	8,500,000
Net Taxable Income	$ 6,500,000
Tax @ 48%	$ 3,120,000

Alternate Calculation:

Net Income	$25,000,000
Less: Tax-Exempt Income	10,000,000
85% of Dividends	8,500,000
Capital Gains	5,000,000
Ordinary Taxable Income	$ 1,500,000
Tax @ 48%	$ 720,000
Capital Gains Tax $5 million @ 30%	$ 1,500,000
Total Tax	$ 2,220,000

The tax liability is the lesser of the above two taxes, $2,220,000.

capital gains would have been available as a Tax-Loss Carry-Over to offset ordinary income taxed at 48%) has been used to offset \$3.5 million of realized capital gains, and (b) the remaining \$1.5 million realized capital gain has been taxed at 48% by reason of being included in the Net Taxable Income item.

Any analysis of taxes is hampered by the existence of several interacting variables. Specifically, the major variables are (1) statutory underwriting profit, (2) the split between taxable, tax-exempt and dividend investment income, (3) realized capital gains, and (4) the interest rates on different assets. One of the easier points to illustrate is the effect of varying the investment portfolio. Returning to the ABC Company, if taxable securities are yielding 8% and tax-exempts 6%, then the investment portfolio must consist of \$125 million of taxable bonds and \$166.7 million of tax-exempt bonds. Selling \$10 million of taxable bonds and buying \$10 million of tax-exempt bonds would increase the tax-exempt income by \$600,000 and decrease the taxable income by \$800,000, thus reducing income by \$200,000. However, the federal income tax decreases by \$384,000 so that net after tax income increases by \$184,000. Exhibit II shows the full range of possible investment situations for the Company by increments of \$10 million of assets. This information has been graphed on Exhibit III and will be called the Net Income Curve for the ABC Company.

Exhibit II

THE ABC COMPANY

Taxable Interest	Tax-Exempt Interest	Income Taxes	Net Income
$ 400,000	$17,200,000	$ 0	$22,600,000
1,200,000	16,600,000	0	22,800,000
2,000,000	16,000,000	0	23,000,000
2,800,000	15,400,000	0	23,200,000
3,600,000	14,800,000	619,000	22,780,000
4,400,000	14,200,000	676,000	22,923,000
5,200,000	13,600,000	816,000	22,984,000
6,000,000	13,000,000	1,200,000	22,800,000
6,800,000	12,400,000	1,500,000	22,700,000
7,600,000	11,800,000	1,500,000	22,900,000
8,400,000	11,200,000	1,500,000	23,100,000
9,200,000	10,600,000	1,836,000	22,964,000
10,000,000	10,000,000	2,220,000	22,780,000
10,800,000	9,400,000	2,604,000	22,596,000
11,600,000	8,800,000	2,988,000	22,412,000
12,400,000	8,200,000	3,372,000	22,228,000
13,200,000	7,600,000	3,756,000	22,044,000
14,000,000	7,000,000	4,140,000	21,860,000
14,800,000	6,400,000	4,524,000	21,676,000
15,600,000	5,800,000	4,908,000	21,492,000
16,400,000	5,200,000	5,292,000	21,308,000
17,200,000	4,600,000	5,676,000	21,124,000
18,000,000	4,000,000	6,060,000	20,940,000
18,800,000	3,400,000	6,444,000	20,756,000
19,600,000	2,800,000	6,828,000	20.572,000
20,400,000	2,200,000	7,212,000	20,388,000
21,200,000	1,600,000	7,596,000	20,204,000
22,000,000	1,000,000	7,980,000	20,020,000
22,800,000	400,000	8,364,000	19,836,000

Assumptions: (also on Exhibits IV, V, VI, VII, except as noted)

1. Taxable bond interest rate of 8%.
2. Tax-Exempt bond interest rate of 6%.
3. 1971 tax rates, i.e., ordinary income 48%, capital gains 30%.
4. Statutory underwriting profit of $-10,000,000.
5. Dividends received of $10,000,000.
6. Realized Capital Gains of $5,000,000.

Reviewing the Net Income Curve can help one understand the different aspects of the tax law. Point G represents the ABC Company under our initial assumption of $10 million of taxable investment income and $10 million of tax-exempt investment income, resulting in net income of $22.8 million after taxes. The other points on the curve represent possible situations for ABC resulting from different distributions of the bond portfolio. Explaining the inflection points on the curve should help to clarify the relationship of the Net Income Curve to the tax law:

Point A — This represents zero taxable investment income. In other words, all funds allocated to buying bonds are invested in tax-exempt securities.

Point B — This point is on one side of the only discontinuity of the Net Income Curve. Throughout segment AB the company has negative taxable income. It is assumed that there is no tax refund available from earlier years. If this assumption is invalid, the slope of AB will change but point B will remain fixed. As taxable income increases beyond B, part of the dividend credit will be lost with a corresponding increase in tax liability and decrease in net income.

Point C — This represents the minimum dividend credit possible and segment CD results from reinstating the dividend credit.

Point D — This identifies the point at which the full dividend credit is again received.

Point E — This point identifies the situation where the tax computed from the alternate calculation is identical to the basic tax from the standard formula. The net effect is that the operating loss is being exactly offset by realized capital gains so that the effective capital gains tax rate is 48%. Up to this point all taxes have been obtained from the standard tax calculation formula.

Point F — This is the point at which ordinary taxable income equals zero. The segment EF has negative ordinary income insufficient to offset the capital gains with the resulting tax being the capital gains tax of 30%. The segment FH represents taxes of 48% of ordinary income and 30% of capital gains. Segment FH declines because additional investment income is taxed at 48% bringing the assumed 8% taxable bond yield to an after-tax equivalent of 4.16% which is less than the tax-exempt 6% yield.

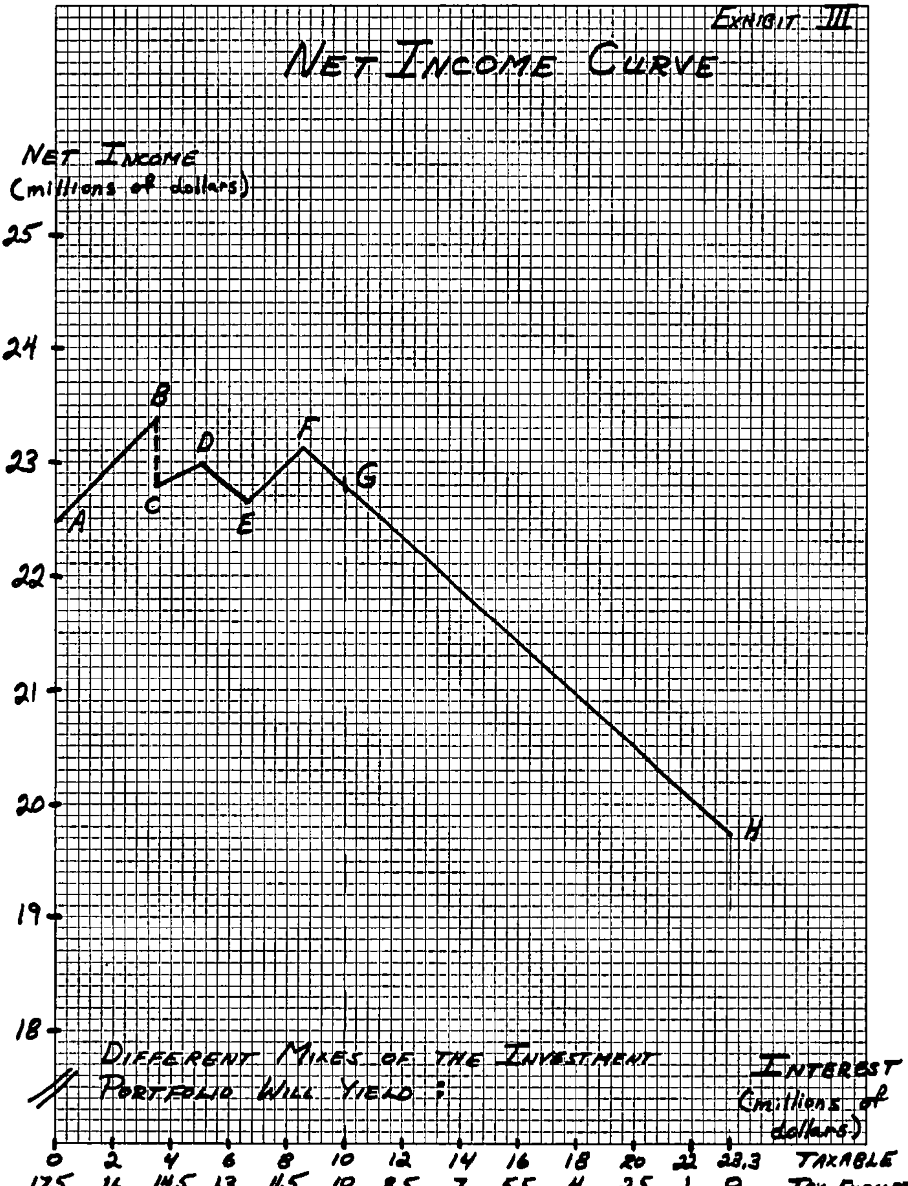


EXHIBIT III
NET INCOME CURVE
NET INCOME
(millions of dollars)
25
24
23
22
21
20
19
18
A
B
C
D
E
F
G
H
DIFFERENT MIXES OF THE INVESTMENT
PORTFOLIO WILL YIELD:
INTEREST
(millions of dollars)
0 2 4 6 8 10 12 14 16 18 20 22 23.3 TAXABLE
17.5 16 14.5 13 11.5 10 8.5 7 5.5 4 2.5 1 0 TAX-EXEMPT

Point G — Identifies the current position of the ABC Company.

Point H — Represents the bond portfolio with no tax-exempt investments.

Thus far we have discussed the calculation of the federal tax liability and have explored the impact of changes in the bond portfolio by using the Net Income Curve. We will now investigate the impact of changes in the other variables again utilizing the Net Income Curve.

Exhibit IV illustrates the effect of different underwriting results. As expected, reducing the underwriting loss increases net income and vice versa as shown by the vertical shift of the curve. However, the entire Net Income Curve is also shifted horizontally by changes in the underwriting loss. Thus the current situation with taxable investment income of $10 million is on line segment FH for underwriting losses of $5 and $10 million, but when the underwriting loss is $15 million the Net Income Curve is intercepted at Point D. In other words, the additional $5 million underwriting loss substantially changes the federal income tax calculation. It is important to note that slopes of all the line segments and the relative position of points B, C, D, E and F remain unaffected by changes in the underwriting results.

Exhibit V illustrates the effect of various capital gains situations. Again as expected, increasing capital gains increases net income and vice versa. However, the structure of the curve changes substantially. Not only is the graph shifted horizontally as with changes in underwriting results, but the length of segments DE and EF is also changed. This occurs because a larger capital gain alters the relative importance of the alternate tax calculation and forces a larger portion of any operating tax loss to be offset against capital gains.

Another important variable is the relative interest rates of taxable and tax-exempt bonds. Exhibit VI illustrates the substantial impact of variations in the tax-exempt interest rate for a fixed taxable investment income yield of 8%. As seen on the graph, Points A through H are unchanged as respects their horizontal separation, but the net income associated with these points changes drastically as does the slope of all the line segments.

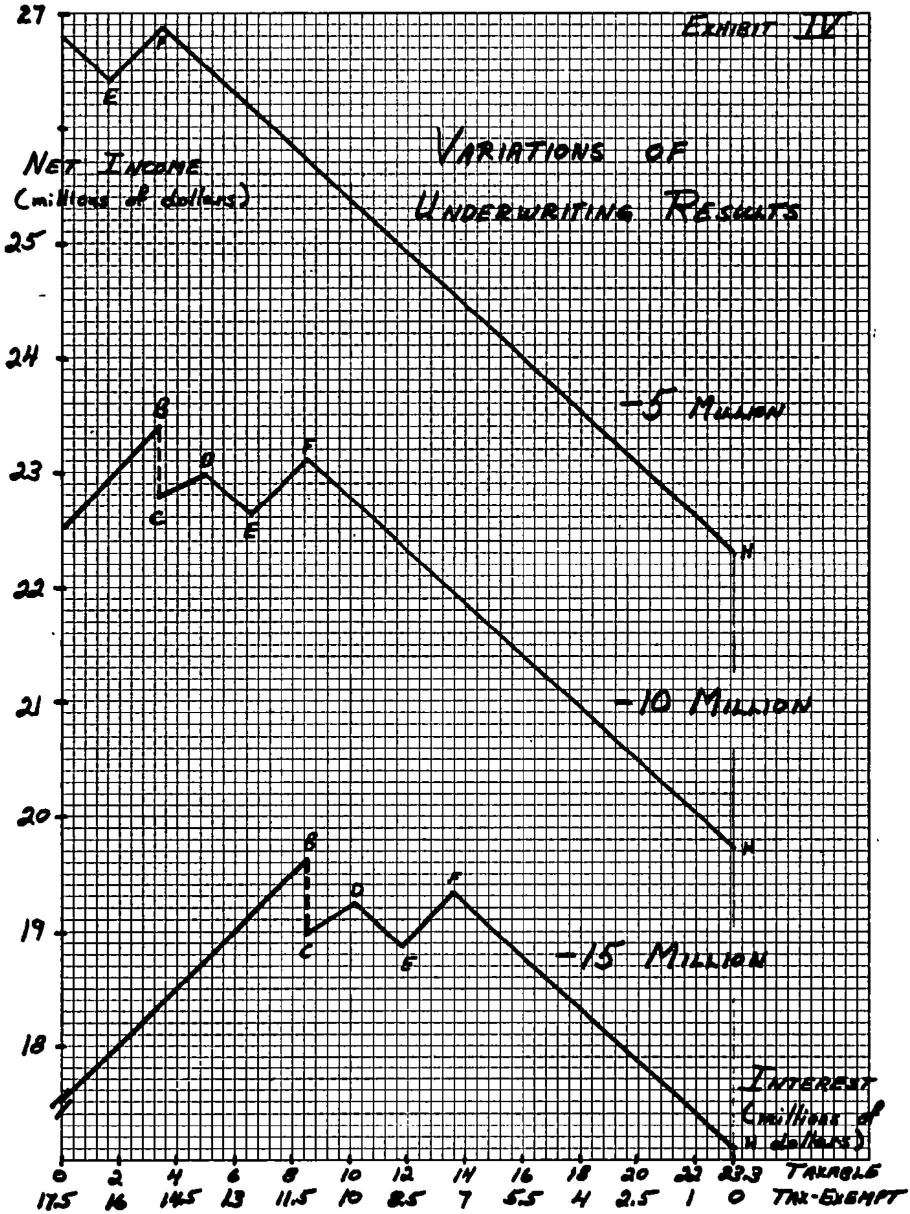


EXHIBIT IV
VARIATIONS OF
UNDERWRITING RESULTS
NET INCOME
(millions of dollars)
27
25
24
23
22
21
20
19
18
-5 MILLION
-10 MILLION
-15 MILLION
B
C
D
E
F
H
INTEREST
(millions of dollars)
0 2 4 6 8 10 12 14 16 18 20 22 23.3 TAXABLE
17.5 16 14.5 13 11.5 10 8.5 7 5.5 4 2.5 1 0 TAX-EXEMPT

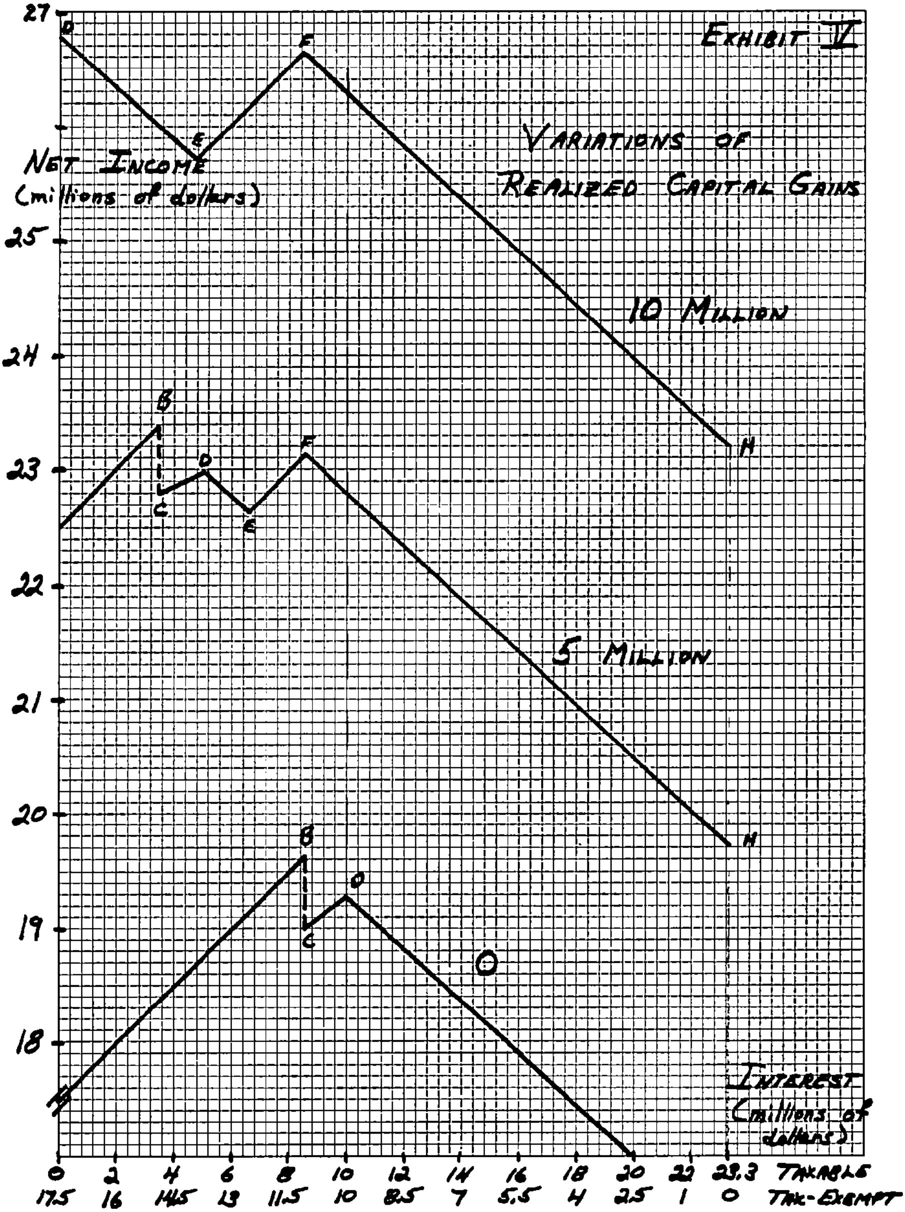


EXHIBIT V
VARIATIONS OF
REALIZED CAPITAL GAINS
NET INCOME
(millions of dollars)
10 MILLION
5 MILLION
0
INTEREST
(millions of dollars)
27
25
24
23
22
21
20
19
18
0 2 4 6 8 10 12 14 16 18 20 22 23.3 TAXABLE
17.5 16 14.5 13 11.5 10 8.5 7 5.5 4 2.5 1 0 TAX-EXEMPT

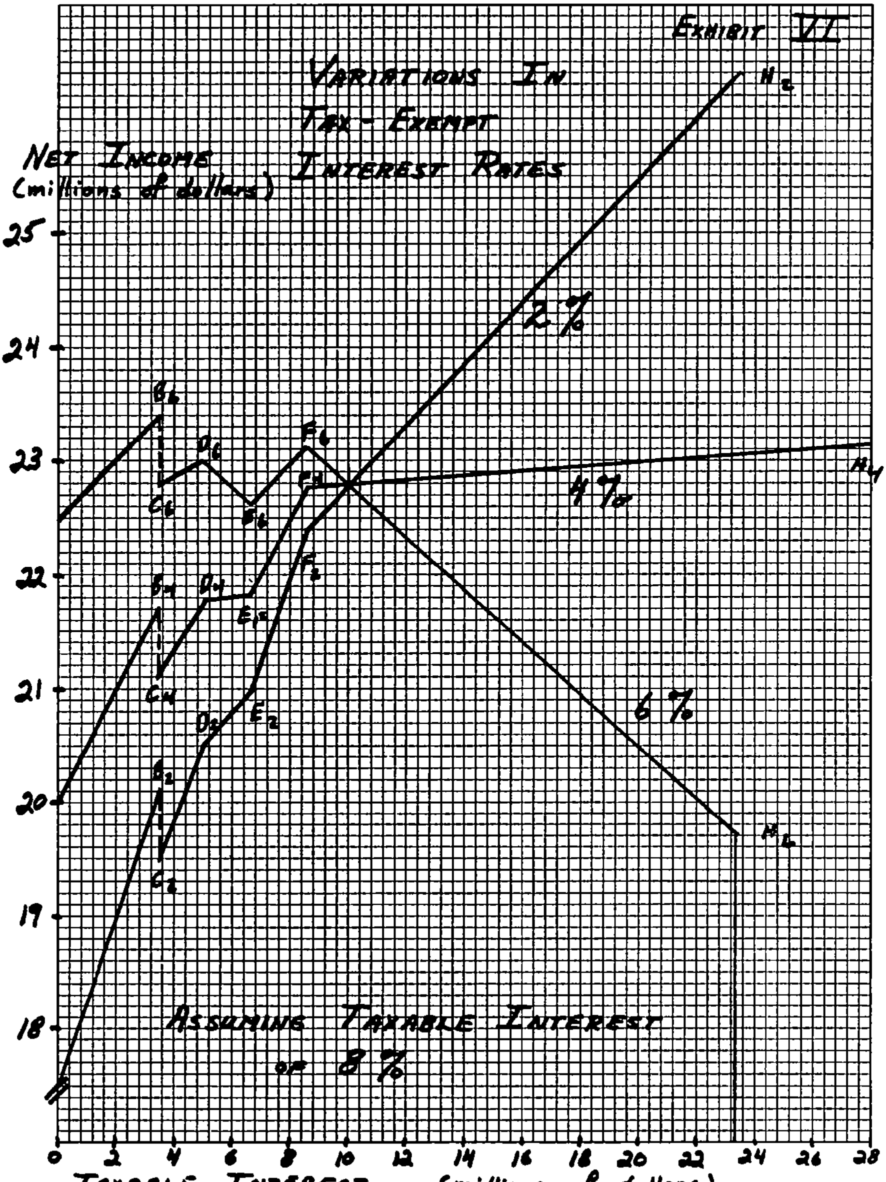


EXHIBIT VI
VARIATIONS IN
TAX - EXEMPT
INTEREST RATES
NET INCOME
(millions of dollars)
25
24
23
22
21
20
19
18
2%
4%
6%
ASSUMING TAXABLE INTEREST
OF 8%
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
TAXABLE INTEREST (millions of dollars)

SUMMARY

This paper has investigated several aspects of federal income taxes including some of the unusual characteristics of the tax law. Several situations resulting in inefficiencies have been noted and the potential for maximizing Net Income has been discussed. Many other factors and problems influence the inter-relationship of taxes and income for an insurance company including:

1. Federal income taxes are based on statutory underwriting results which in effect charge all expenses, including commissions, against earned premium. Consequently, all other things being equal, a company will pay less taxes when it grows faster.

2. Unfortunately, techniques to project underwriting results years in advance have not been perfected. Lacking the ability to accurately foresee results, general investment policies can be pursued to maximize income within ranges of underwriting results, but it is impossible to identify the optimum investment policy in advance.

3. Another important consideration is the impact on the market price of the insurance company's stock caused by variations in overall results. It is possible that some stock analysts would be ill disposed towards a company offsetting realized capital gains with operating losses. Such a philosophy would be based on the theory that poor management is indicated whenever an operating loss that should receive a 48% tax credit is offset by capital gains receiving only a 30% tax credit.

The problems and considerations mentioned above provide a difficult setting for planned taxes. However, if sufficient taxable income from other sources is available to offset any underwriting loss then the optimum investment policy is to invest in tax-exempt securities to the greatest extent possible (assuming the after tax yield from taxable securities is less than the yield on tax-exempt securities as has usually been the case). Returning to the ABC Insurance Company to illustrate this point, the modified Net Income Cu /e on Exhibit VII shows the effect of taxable operating income from external sources sufficiently large to offset any underwriting loss. As can be seen on this exhibit, when taxable investment income is zero, net income is maximized at a level which substantially exceeds that obtained from any other bond portfolio.

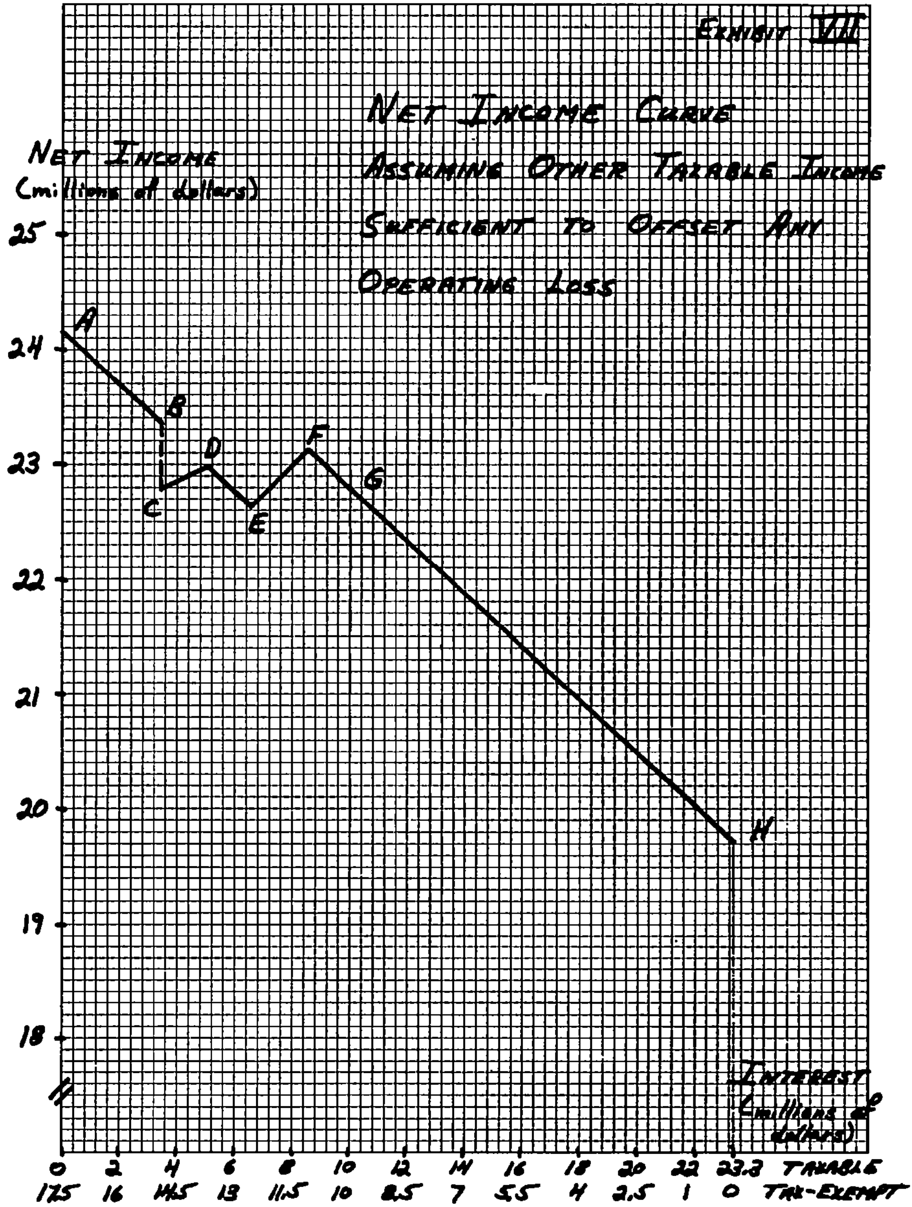


EXHIBIT VII
NET INCOME CURVE
ASSUMING OTHER TAXABLE INCOME
SUFFICIENT TO OFFSET ANY
OPERATING LOSS
NET INCOME
(millions of dollars)
25
24
23
22
21
20
19
18
A
B
C
D
E
F
G
H
INTEREST
(millions of dollars)
0 2 4 6 8 10 12 14 16 18 20 22 23.3 TAXABLE
17.5 16 14.5 13 11.5 10 8.5 7 5.5 4 2.5 1 0 TAX-EXEMPT

This paper has briefly explored the subject of federal income taxes in the hope of stimulating investigations into this important area of insurance company operations. In the final analysis, net income is the sum of underwriting profit, investment income and taxes, and the latter may be the most important *controllable* factor in maximizing income.

DISCUSSION BY M. STANLEY HUGHEY

The very nature of the property and liability insurance industry involves the collection of dollars in the form of advance premium payments, the payment of losses that occur under the insured exposure as they are settled, and the payment of the related expenses of doing business as they fall due. Therefore, at any given time a property and liability insurance company has funds it is holding to make these various payments, as well as the equity funds which its stockholders (or mutual policyholders) have made available to guarantee financial performance.

The wise investment of these funds is an important element in the successful operation of a property and liability insurance company, and there are a number of important considerations to be taken into account:

1. Invested funds should provide security and protect a satisfactory surplus margin. Insurance companies above all else provide security and the investment program must be planned to provide that security.

2. Sufficient cash and liquid funds should be maintained at all times to meet liabilities which are due to be paid in the immediate future. The extremes are reflected in the "liquidation theory" where investments are maintained in such fashion that the company could orderly liquidate all liabilities if it stopped writing business, and the so-called "cash flow" theory where investments are planned on the basis that premium income would continue to flow at about the same level and losses and other expenses could be paid out of current premium receipts. In any given company some of the logic underlying both theories can be helpful in developing the best individual investment program.

Evaluating property and liability insurance company liabilities:

• Unearned premium reserves involve prepaid expenses, expenses falling due over a reasonably short period of time, and potential loss payments and reserves.

- Loss reserves can be thought of in three categories — short term, payable immediately or within a relatively few months; interim, payable in perhaps two to four years; and long term, with very long payout periods.
- Other liabilities are normally short term.

To the extent these reserves call for immediately available funds to meet the short term liabilities falling due, the investment program should provide for such funds in cash or other liquid assets.

3. Protection against inflation should be provided for longer term loss reserves. Particularly in the auto, compensation and liability lines, long tail claims can remain open many years, with some of the settlement factors relating closely to inflation affected elements. In an overall inflationary economy it is desirable to tie some of the longer term loss reserves to investments which will provide some protection against inflation.

4. Underwriting results and surplus position should be recognized in planning investments. A company which is operating with a satisfactory underwriting margin and with a relatively large capital and surplus in relation to exposure can be more aggressive in its investment program than a company which has a thin underwriting margin (or underwriting loss) and a high exposure in relation it its capital and surplus.

5. All legal limitations should be carefully observed. Most state statutes spell out certain restrictions on property and liability insurance company investments, and while the regulations are normally very much in keeping with a sound investment program, it is clearly prudent to make certain that all investments are within whatever legal requirements are applicable.

6. Investment return after taxes should be maximized. Because of the normally higher yields on taxable securities, a property and liability company should strive to balance any underwriting loss with taxable income. Any investment income not needed to offset underwriting loss can normally be more profitably concentrated in tax-exempt and tax-sheltered securities.

All of these considerations can be important in any particular situation and usually they are closely interrelated. Mr. Beckman has concentrated on the last of these, exploring how to maximize investment return after taxes. He is to be commended for his needed and useful summary of the federal tax code as it relates to Stock Fire and Casualty Insurance Companies. His examples of the way net income after tax is affected by the different mixes of taxable investment income, tax-exempt investment income and underwriting loss are both interesting and illuminating.

As Mr. Beckman implies in his conclusions, the investment goal from a tax standpoint is to have taxable income exactly offset any underwriting loss, assuming the current situation where after tax return on tax-exempt and tax-sheltered investments is higher than the after tax return on taxable investments. Recognizing that the tax considerations may be affected by the other considerations commented on above, there is still room for substantial benefits from careful investment planning.

In considering the wide field of investment planning, several points should be emphasized:

- While it is most difficult to predict the key ingredient of the underwriting result for any given year, it is not unreasonable to predict the planned and approximate underwriting result over a period of say five years. With the carryforward and carryback provisions in the tax law, it should be possible to plan for tax-exempt and tax-sheltered investment income which minimizes taxes on the average, and gains the benefit of the tax-free income over several years even though any one year produces an unbalanced result.
- While not completely flexible, the timing of capital gains and losses is subject to some control. Therefore, with careful planning and by making full use of capital gain and loss carry forward and carry back provisions, the use of capital gains to offset ordinary losses can be minimized.
- Similarly, with a minimum of planning, a company operating in a normal range of underwriting results will have enough taxable income to make use of the dividend credit.
- While there are minor variations, mutual property and liability insurance companies are generally subject to the same Federal Income Taxes as stock property and liability companies so that the same general principles apply. However, Federal Income Taxes on both

stock and mutual life insurance companies are very much different and call for a completely different approach to tax planning.

- In Exhibit VI Mr. Beckman has compared the results with differing rates of yield on tax-exempt income. However, as a practical matter investment yield on tax-exempts tends to fluctuate in approximately parallel fashion to the investment yield on taxable securities so that these very sharp variations do not have to be dealt with very often.
- In commenting on tax-exempt investments, Mr. Beckman has concentrated his discussion on municipal bonds which are completely tax free. However, not to be overlooked as tax-sheltered investments are:

1. sound stocks which over a period of years (and assuming that they can be liquidated at the capital gains rate) should yield a net after tax return in the 6-7% range, and
2. real estate investments which due to the depreciation allowance can yield a net after tax return in the 7-8% range (perhaps more in an inflationary economy.)

Mr. Beckman's paper tackles head-on some of the tax ramifications of investment income and suggests some of the possibilities for improving net after tax results. Investment income has not been given the full consideration it deserves by the actuarial profession and it is to be hoped that others will continue to explore the multiple relationships that make up a sound program of investment planning. This is an area which holds promise for a significant professional contribution and, just as importantly, for a significant contribution toward improved company and industry profitability.

DISCUSSION BY J. W. MACGINNITIE

Mr. Beckman has done an excellent job of summarizing the impact of Federal Income Taxes on property/casualty insurance companies and the various factors that influence the total tax liability.

The major variable to which the paper is addressed is the relative amounts of taxable and tax-exempt investment income. In many companies this is the variable over which management has the greatest control and widest latitude of choice. For companies who have both the need and the resources, a more sophisticated approach to tax planning is possible.

Rather than confining the analysis to one variable at a time, several variables can be dealt with simultaneously. These variables would include:

1. Volume and profitability (on a statutory basis) of insurance.
2. Distribution of assets by class, especially tax-exempt bonds, taxable bonds, preferred stocks, and common stocks.
3. Yields by asset class, specified separately for interest, dividends, and capital gains.

It should be noted that there is an often overlooked cost of switching assets between classes (principally commissions) and that capital gains or losses often result when such switches are made. Also, tax loss carryforwards and carrybacks earn interest at a zero rate which is considerably less than the firm's marginal opportunity cost.

It is then possible to introduce frequency distributions for each of the variables listed above and to use a simulation technique to evaluate alternative investment strategies. Additional sophistications can be introduced relating to management of realized capital gains and to a more detailed classification of assets. Investment strategies must also consider the tradeoffs between risk and return, but tax implications must be taken into account. The decision as to whether to buy common stocks with low dividends and high potential appreciation should be tempered by the 30% capital gains rate as compared to a 7.2% tax rate on dividends from a high dividend, low potential appreciation stock. Also, dividends are reportable as ordinary income while capital gains are generally shown separately and only when they are realized.

Insurance companies who file consolidated returns with other parts of a holding company will find their problem of prediction and optimization even more complex. The prediction of profits in some non-insurance businesses is no easier than in property/casualty insurance, and there are likely to be significant differences between reported earnings and taxable earnings for reasons peculiar to each business. In many cases, however, taxable income from non-insurance businesses has less chance with being negative, or at least of being unpredictably negative and this gives the insurance planner more of a cushion when deciding to invest in tax-exempt securities.

A final point which should be mentioned is that the Internal Revenue Service evaluates reserve redundancy by statutory line of business. Returns have been challenged where reserves of an individual line of business have developed redundancies in excess of 15% and deficiencies assessed. Those responsible for establishing reserves by line of business would do well to keep them within this tolerance.

DISCUSSION BY J. A. SCHEIBL

Much has been written in recent years on scientific approaches to management decision-making. Primary factors that have contributed to this surge of literature have been the increasing complexity of the type of decisions necessary in today's increasingly complex world and the development of the electronic computer providing the wherewithal for operations research. A key technique in the new methodology has been the simulation of decision problems through mathematical models.

The ultimate in modeling is the simulation of all operations of a business enterprise and the external forces that affect that enterprise. Through the examination of all likely results from a range of all possible decisions, and, through the repetition of this process as these indicated results lead to more decisions, management teams have at their disposal the means to operate at a high level of efficiency.

Of course, the efficiency attainable by these means depends a great deal on the quality of the corporate model. The model must reflect the action, reaction and interaction of all pertinent factual and assumptive variables. This suggests that an early stage in the construction of a corporate model is to weed out relatively extraneous variables and to trace the actions of only those that are considered pertinent. Mr. Beckman has done a commendable job in demonstrating how this may be done preliminary to constructing the potentially complex income tax phase of a corporate model. He has conveniently and properly ignored many of the minute details of income tax calculations that would detract from his broad illustrations of the actions of the four variables he has chosen to examine. In so doing, he has undoubtedly perpetuated the usefulness of his paper. While, as the saying goes, "there is nothing as certain as death and taxes," we might add by the way of paraphrase, "there is nothing as uncertain as the manner of death or the manner of the tax structure."

The paper does not go into the reaction and interaction of dependent variables and, therefore, stops short of illustrating actual real-life applications which are the next stages in the formation of a corporate model. Obviously, an insurance company does not change its investment portfolio, nor does it experience radical changes in underwriting results without some reaction from other variables affecting profitability. For instance, sales and purchases of bonds usually involve capital gains or losses, as well as changes in the relationship of tax-exempt and taxable income. Also, interest rates on bonds purchased with new money affect net income differently than interest rates on bonds that are approaching maturity. Factors such as these must be taken into consideration when constructing a corporate model simulating real-life conditions.

It should be noted that Mr. Beckman's paper reflects changes in the tax laws made by the 1969 Tax Reform Act and that his calculations are based on the tax rates effective on 1971 business. Anyone reading this paper in the future should be cautioned to determine the tax provisions in effect at the time of the reading to update the illustrations.

Since the scope of Mr. Beckman's paper does not include mutual companies or reciprocal underwriters and inter-insurers, it may be in order here to offer a postscript for the benefit of those who want to apply the concepts of his paper to such companies.

Mutual companies are subject to Sections 821-825 of the Federal Tax Code. Section 826 applies to reciprocals which are taxed as mutuals with minor exceptions. For the most part, mutuals have been taxed exactly the same as stock companies since January 1, 1963. Two notable exceptions are:

1. Mutual companies with gross premium and investment income of $150,000 or less are not subject to income tax. Companies with gross premium and investment income over $150,000, but under $500,000, may be taxed on investment income only, unless they elect to be taxed on total income. A special deduction is allowed companies with gross premium and investment income between $500,000 and $1,100,000 which has the effect of smoothing the transition from an investment income to a total income tax base.

2. Section 824 of the Tax Code requires each mutual company to establish and maintain a Protection Against Loss (PAL) account to be used as a reserve against extraordinary losses, since a mutual company must look to its retained income to meet such emergencies.

Additions to the PAL account are treated as a deduction from underwriting gain for tax purposes.
The code provides specific formulas for establishing and maintaining this special reserve.

It should also be pointed out that, although tax code provisions are similar for mutual and stock companies, the inherent differences in operations and financial structure of the two types of companies may affect the relevancy of some factors in determining maximization of net income after tax.

For one thing, policyholder dividends are more likely to play a significant role in determining the taxable income for mutual companies than for most stock companies. As Mr. Beckman points out, declared policyholders' dividends are a direct deduction from underwriting income. This suggests that company management may look to income from high-yield taxable securities to balance declared dividends in its plan to optimize net income after taxes.

When considering the tax impact of policyholder dividends in corporate planning, it is important to note that only dividends on expired or expiring policies are used to determine underwriting income, although these dividends are a deduction from gross underwriting income earned from all policies in force during and, to some extent, prior to the tax year. This lag may be pertinent in the development of a corporate model concerned with maximizing income after taxes over a span of years.

Another inherent difference between the two types of companies affecting the relevancy of factors in the planning process is in the makeup of investment portfolios — especially with regard to the balance between equity and fixed income securities. As a rule, mutual company investment portfolios lean more heavily toward fixed income securities than do stock company portfolios. Thus, we would expect mutual companies to place less emphasis than stock companies on dividend credits and capital gains or losses from common stocks when planning for maximum after-tax income.

It seems logical to assume that the insurance industry, which is essentially a "numbers" industry, should be an ideal subject for the application of decision theory. Yet, while literature abounds on decision-making and modeling, there is very little published on its application to insurance. Not too many years ago the scope of our Associateship examinations was broadened to include decision theory. More recently, the Insurance Institute of America has prepared a study course in its application. Now, Mr. Beckman has opened the door a bit further by demonstrating how a preliminary stage in the development of a corporate model may be accomplished. More of this needs to be done in other phases of corporate modeling. Then, hopefully, someone, some day, will put all the pieces together.

DISCUSSIONS OF PAPER PUBLISHED IN VOLUME LVII

CREDIBILITY FOR SEVERITY

CHARLES C. HEWITT, JR.

VOLUME LVII, PAGE 148

DISCUSSION BY HANS U. GERBER*

The credibility formulae discussed in this paper may be satisfactory from an *experience rating point of view,* where the premium of a particular risk is only influenced by the *total amount of its claims* experienced in the past. Thus a risk with 10 claims of $1,000 each is rated the same as a risk with just one claim of size $10,000.

However, from a *statistical point of view,* these credibility formulae seem to be oversimplified because they fail to distinguish between the credibility of the claim severity and the credibility of the claim frequency experienced. This simplification may be the reason why Hewitt observes a "reduction in credibility." In the sequel we shall present a credibility formula which is able to distinguish between the credibility of the severity and the one of the frequency.

To establish the terminology, we assume that the claims of each individual risk (described by its two parameters λ, θ) form a compound Poisson process with Poisson parameter λ (expected number of claims per unit time) and distribution $F^{(\theta)}(x)$ of the single claim amounts. With $\mu(\theta)$ and $\sigma^2(\theta)$ we denote the expected value and the variance, respectively for the claim amount of a given risk.

The distributions of λ and θ are supposedly known. However, we need only the values of:

$$m = E[\mu(\theta)] \quad , \quad Var[\mu(\theta)] \quad , \quad E[\sigma^2(\theta)]$$

$$k = E[\lambda] \quad , \quad Var[\lambda]$$

* Mr. Gerber was a guest reviewer of this paper. He is serving currently as Visiting Professor of Statistics in the Department of Mathematics at the University of Michigan, Ann Arbor.

From now on we consider a particular risk (for which we don't know the parameter values). If this risk showed n claims up to time t, let:

$$\overline{m} = \frac{S_1 + S_2 + \ldots S_n}{n}$$

be the average claim size observed and:

$$\overline{k} = \frac{n}{t}$$

the average claim frequency observed.

A credibility formula is an expression which estimates the (Bayesian) conditional expectation:

$$E[\lambda\mu(\theta)\,|\,\overline{k}; S_1, S_2, \ldots, S_n]$$

Let us consider credibility formulas of the form:

$$akm + b\overline{k}m + ck\overline{m} + d\overline{k}\overline{m}$$

(rather than of the form $akm + b\overline{k}\overline{m}$, as Hewitt does). According to Bühlmann's concept, we determine $a,b,c,$ and d in order to minimize the expected squared deviation of the credibility premium from $E[\lambda\mu(\theta)\,|\,\overline{k}; S_1, S_2, \ldots, S_n]$. Assuming that λ and θ are independently distributed, one finds:

$$a = \left(\frac{\dfrac{k}{Var[\lambda]}}{t + \dfrac{k}{Var[\lambda]}}\right) \cdot \left(\frac{\dfrac{E[\sigma^2(\theta)]}{Var[\mu(\theta)]}}{n + \dfrac{E[\sigma^2(\theta)]}{Var[\mu(\theta)]}}\right)$$

$$b = \left(\frac{t}{t + \dfrac{k}{Var[\lambda]}}\right) \cdot \left(\frac{\dfrac{E[\sigma^2(\theta)]}{Var[\mu(\theta)]}}{n + \dfrac{E[\sigma^2(\theta)]}{Var[\mu(\theta)]}}\right)$$

$$c = \left(\frac{\dfrac{k}{Var[\lambda]}}{t + \dfrac{k}{Var[\lambda]}}\right) \cdot \left(\frac{n}{n + \dfrac{E[\sigma^2(\theta)]}{Var[\mu(\theta)]}}\right)$$

$$d = \left(\frac{t}{t + \dfrac{k}{Var[\lambda]}}\right) \cdot \left(\frac{n}{n + \dfrac{E[\sigma^2(\theta)]}{Var[\mu(\theta)]}}\right)$$

Thus the credibility premium may be written as the product:

$$\{(1-z_1)k + z_1\bar{k}\}\ \{(1-z_2)m + z_2\bar{m}\}$$

with:

$$z_1(t) = \frac{t}{t + \dfrac{k}{Var[\lambda]}}$$

$$z_2(n) = \frac{n}{n + \dfrac{E[\sigma^2(\theta)]}{Var[\mu(\theta)]}}.$$

We notice that the two credibilities are properly distinguished now.

For Hewitt's numerical example we find:

$$z_1(t) = \frac{t}{t + 30.1}$$

$$z_2(n) = \frac{n}{n + \dfrac{e^{\sigma^2} - 1}{e^{S^2} - 1} e^{S^2}} = \frac{n}{n + 54.9}$$

Finally, we remark that the assumption of independence between λ and θ is not necessary for the construction of the above described credibility premium. However, in the general case, it will not be possible to write the credibility premium:

$$akm + b\bar{k}m + ck\bar{m} + d\bar{k}\bar{m}$$

in product form (as it was possible in the case of independence).

DISCUSSION BY HANS BÜHLMANN*

This is an inspiring paper very clearly written and well presented. I hope that the point made by Mr. Hewitt comes home, namely that credibility is theoretically justifiable and eminently practical. The main contribution of this paper is the explicit application of general credibility techniques to the

* Dr. Bühlmann was a guest reviewer of this paper. He is currently Professor of Mathematics, E.dg. Technische Hochschule, Zurich.

factorized pure premium, the factors being expected frequency and expected severity. So far, most other applications have been made by either applying the same techniques to the expected frequency alone or directly to the pure premium, without considering its product form.

Mr. Hewitt leads us through two examples in order to illustrate his approach, the discrete die-spinner model and the continuous frequency-severity model for automobile insurance. The first is ingenious from an educational angle, the second leads us right to the center of interest cherished by the casualty actuary. My discussion, therefore, concentrates on this second example.

Mr. Hewitt assumes the frequency of the individual exposure unit to be Poisson distributed with unknown parameter m, the severity of each claim to follow a log normal distribution with unknown "mean of the log" μ but known "variance of the log" σ^2. Assuming the parameters m and μ to be independent and to follow specific distribution functions as well as the usual independence of frequency and severity given these parameters, he arrives at an explicit formula for the constant K in $Z = \frac{n}{n+K}$. Most interesting are his applications of the formula thus obtained to auto merit rating and to a single-split experience rating plan. As shown in the case of Canadian private passenger data, credibility is considerably reduced by taking severity into account. Obviously Mr. Hewitt's approach can be carried through for any *discrete* model with an arbitrary structure (a priori distribution) on the parameter space; and, as he himself points out, the assumption of independence of frequency and severity parameters is not vital then. However, in the continuous case the calculations involved would become very cumbersome if a) the parametric assumptions would differ from the normal (for log of claims) — normal (for mean of log of claims) case and b) independence of frequency and severity parameters is not postulated.

May I say that the Hewitt approach is actually geared to the use of Bayes estimates rather than the use of credibility estimates because he builds up all the machinery needed to compute Bayes estimates. As we know that Bayes estimates are optimal for quadratic loss and that they have many other attractive properties, why not use them? Incidentally, the comparison of Bayes and credibility estimates for the die-spinner-model is very illuminating — and very encouraging for all credibility fans! Credibility estimates can be characterized as closest (in the sense of mean square deviation) approximation to the Bayes estimates provided one knows only *mean* and *variance* of all distributions (including the distributions of the parameters). Let me show how this works by reproducing the Hewitt approach without parametric assumptions (point "a" above) but still assuming independence of frequency and severity parameters (point "b" above),

1. The model (with a slight change of notation)

	random variable	mean	variance	parameter characterizing the distribution
frequency	k	E_k	σ_k^2	$\eta \epsilon \mathrm{H}$
severity (average claim)	$\bar{y}$	$E(\bar{y}\mid k) = E_y$	$Var(\bar{y}\mid k) = \dfrac{\sigma_y^2}{k}$	$\theta \epsilon \Theta$
loss per unit	$x = k\bar{y}$	$E(x) = E_k \,.\, E_y$	$Var(x) = E_k\, \sigma_y^2 + (E_y)^2 \cdot \sigma_k^2$	$(\eta, \theta) \epsilon \mathrm{H}\mathrm{x}\Theta$

2. The parameters and their distribution

The formulae given under 1) are all to be understood for a fixed distribution of frequency and severity (fixed parameter values). We emphasize this by writing:

(1) $E(x \mid \eta,\theta) = E_k(\eta) \cdot E_y(\theta)$

(2) $Var(x \mid \eta, \theta) = E_k(\eta) \cdot \sigma_y^2(\theta) + (E_y(\theta))^2 \cdot \sigma_k^2(\eta)$

In the following the operations E[ ] and Var[ ] (square brackets as opposed to parentheses before !) mean expected value and variance with respect to the structure function (prior distribution) $U(\eta, \theta)$ over $\mathrm{H}\mathrm{x}\Theta$. Let us assume independence of η and θ, an assumption which is equivalent with postulating the "product form" for U, i.e. $U(\eta, \theta) = U_1(\eta) \cdot U_2(\theta)$.

Then we obtain:

(3) $E[E(x \mid \eta, \theta)] = m_k \cdot m_y$

(4) $Var[E(x \mid \eta, \theta)] = w_k w_y + m_k^2 w_y + m_y^2 w_k$

(5) $E[Var(x \mid \eta, \theta)] = m_k v_y + (m_y^2 + w_y) v_k$

with the abbreviations:

$$m_k = E[E_k(\eta)] \qquad m_y = E[E_y(\theta)]$$

$$w_k = Var[E_k(\eta)] \qquad w_y = Var[E_y(\theta)]$$

$$v_k = E[\sigma_k^2(\eta)] \qquad v_y = E[\sigma_y^2(\theta)]$$

3. *Credibility*

We determine the constant K in the credibility formula $Z = \dfrac{n}{n+K}$

$$(6) \quad K = \frac{E[Var(x \mid \eta, \theta)]}{Var[E(x \mid \eta, \theta)]} = \frac{m_k v_y + (m_y^2 + w_y)v_k}{w_k w_y + m_k^2 w_y + m_y^2 w_k}$$

and if the w's are strictly positive:

$$(7) \quad K = \frac{\dfrac{K_y}{m_k} C_k + K_k(C_y + 1)}{C_k + C_y + 1}$$

where:

$$K_k = \frac{v_k}{w_k} \qquad K_y = \frac{v_y}{w_y}$$

$$C_k = \frac{m_k^2}{w_k} \qquad C_y = \frac{m_y^2}{w_y}$$

observe that K_k and K_y are the K-constants in the credibility formulae for the factors alone. Hence the K-constant in the credibility formula of the product appears as:

weighted average of $\dfrac{K_y}{m_k}$ and K_k

(Note the division by m_k which suggests the original assumption

$$Var(y \mid k) = \frac{\sigma_y^2}{k} \text{ !)}$$

We may have assumed from Mr. Hewitt's example that credibility always *decreases* if severity is taken into account. However, this belief now turns out to be incorrect. The correct statement is that credibility *decreases* by taking severity into account exactly if:

$$(8) \qquad K_y > m_k K_k$$

Let me finish my discussion by thanking Mr. Hewitt for his stimulating paper. He has opened a new road of research into *full* credibility (credibility based on severity as well as frequency). For the pure Bayesian the road has already lead to its destination because to him the quantities m, v, w are to be assessed by underwriting judgement. The empirical Bayesian, however, still has a task ahead of him. He must find the appropriate estimates of m, v, w from observations. This can be done also in the non parametric version of the Hewitt approach by applying the method of estimation described in the paper by Mr. Straub and this reviewer.*

* *Editors Note:* This paper appeared in the second edition of ARCH 1972. Copies may be obtained from the Editor, David G. Halmstad, Aviation Reinsurance Unit, Metropolitan Life Insurance Company, 1 Madison Avenue, New York, New York 10010.

DEPARTMENT OF TRANSPORTATION VIEWS ON AUTOMOBILE INSURANCE REFORM

ADDRESS BY CHARLES D. BAKER*

What did the Study show to be wrong with the present system? First, the liability insurance system has limitations in its coverage. Only those who can prove that others were at fault while they were not (or were less at fault) have a legal right to recover their full losses. What does this mean in fact? It means that in more than half the automobile accidents where someone was killed or seriously injured, no benefits were received from the tort liability system. In 10 percent of the cases nothing was received from any system of reparation.

Second, the system looks imbalanced in the way it distributes compensation losses. One would expect that the victim suffering the large economic losses would also have significant intangible losses. One would not anticipate, however, that this type of victim would have a poorer chance of being fully compensated — particularly for his economic losses — than the less seriously injured. Our Study indicates that this is, alas, the case. Only half the total compensable economic losses of seriously or fatally injured victims are compensated — only one third where losses exceed $25,000! Small economic losses fare much better — victims suffering under $500 damage recovered in total through the tort system four-and-one half times their economic loss. You may argue with some of the precise percentages cited here, but unless one is prepared to challenge the conclusions fundamentally, one is forced to the view that compensation is erratic.

And then there is efficiency. Unfortunately it seems that the system has a very high cost/benefit ratio. By our calculations, it costs a dollar to produce a dollar in net victim benefit. Put another way, one premium dollar out of every two does not go to the accident victims. Further not only is the system's cost efficiency in question, it appears that this benefit is poorly timed — it's either too late or too early! Despite commendable

* These excerpts were taken from remarks to the Society by Mr. Charles D. Baker, Assistant Secretary for Policy and International Affairs, Department of Transportation, Washington, D.C. Mr. Baker has kindly consented to have the remarks reproduced in the *Proceedings*.

efforts on the part of the industry to introduce "advance" or partial payment plans, the system looks to be quite slow in providing benefit payments, particularly in terms of when they are needed.

One major problem with this is that there are indications that rehabilitation suffers because slowness of payment discourages early rehabilitative efforts. In fact, the system at times places a premium on deferment of payment beyond the time when rehabilitation could be most effective. Unfortunately, the payment looks to be slowest where the need is greatest — when victims suffer permanent impairment and disfigurement. Nor does the system encourage minimization of very large personal injury losses by the timely use of comprehensive rehabilitation programs for the seriously injured.

It is not just the victim who suffers. As it presently operates, the system places great strains on the insurance industry itself. For many companies, what once were underwriting profits, are turning to underwriting losses, and it's alleged by some that capital may actually be withdrawing from the market. Granted, the threat of capital withdrawal is not a new phenomenon, but actual withdrawal on a large scale would be. I don't think I have to point out to you people what a serious problem this would present, not simply to the industry but to the nation.

But what about the legal profession? The Bar? ALTA? The judiciary? Let me dwell on the latter! The judiciary is feeling the strain! At a time when other demands overburden our legal system, the judiciary handles more than 200,000 auto accident disputes a year — in terms of judge time alone, more than 17 percent of the country's total judiciary resources? Thus we place high demands on our already strained courts. If there is no better alternative — so be it — but, as I'll note in a minute, there is!

But before that, there is another "institutional" issue. Insurer insolvencies have been concentrated among specialty insurers serving the high risk market. This has presented complex problems for consumers, regulators, and the insurance industry in general. And the very complexity of the problem makes them so resistant to solution that they could lead to greater centralization and a loss of local initiative and freedom in insurance regulation.

So, what do we as a nation do? Nothing? I think almost everyone would agree that given the inadequacies of the present system, this is certainly no answer at all. We at DOT think that reform is clearly called for and just as clearly we are convinced that the objective of this reform should be no fault — not just first party — but a contract relationship between insurer and the policyholder which pays benefits when there is loss — regardless of where the fault lies. On this, we and many in and out of the industry — perhaps not all, but many nonetheless — are in accord. But how best to accomplish this? Here is where the going gets complicated! As you are probably aware, there is some difference of opinion about this. Senator Hart and Congressman Moss (among others) have recently proposed legislation that empowers the Federal government to mandate Federal standards for auto insurance and, in effect, also create an insurance "czar" who will execute most insurance regulation. The Administration's approach is different and places responsibility for establishing the principles of change with the Federal government but leaves the detailed implementation as well as regulation to the States. This is the plan that Secretary Volpe presented to Congress last month.

The Department fully endorsed the no-fault approach and urged the Congress to enact a "concurrent resolution" setting forth the basic principles of a reparations system toward which the States should strive, urging them to so move with dispatch!

Why the State approach? In the first place, this Administration is very strongly committed to the belief that the functions of government should be performed and the effective decisions of government made as close to the people as possible (in this case, at the State level). Is this a bad precept? In the face of the clamor for active citizen participation in practically every important issue, I hardly think so! Given the clear call by the electorate for responsibility in the hands of local officials that the electorate can see (and get at), this proposition cannot be dismissed. But some would have us be expedient! "Rise above principle!" Well, I don't think so!

The policy seems clear enough to me! If the States can do the job, then they should. If they cannot, or will not, then Washington has a call for pre-emption, but in my view not until then!

Now, it is our belief that the insurance institution and State regulation have been held at fault for what are really intrinsic inadequacies in the reparations system itself. States regulate now and can continue to do so. Under the present system, various states and regions of the country vary in terms of limits and deductibles. There are clear reasons why this should continue. It would not be fair to impose the standards of New York City, say on Alaska or vice-versa. The States should be allowed to accommodate to their specific situations, given some overall principles for basic uniformity. Is it bad to recognize that Alaska is not New York? Hardly. Alaska is no more New York than Texas is Illinois. Broad similarity? Of course! Special differences? Who can argue that point?

I believe that the States will act, and act quickly. One State has already enacted a no-fault plan and at last count, 27 others had either submitted proposals or were thinking about doing so. And of course, if they don't move, it is certain that in some form the Federal government will. And can we all learn from the movements in the several States? I went to Harvard and yet even so I am not prepared to lay claim to all knowledge! As these fundamental changes come into play we can all learn!

In recent weeks there's been a lot of flak in the media concerning the various parties and proposals involved in the insurance reform controversy. There are those who fear that the predominance of the legal profession in the State legislatures will hinder any effective reform at that level, and others who feel that Federal pre-emption of State regulation of insurance is a clear violation of States' rights. One very vocal critic of the Administration position has had some rather pithy comments about the Department's position. So be it, public airing is good for all of us — even public hot airing!

Gentlemen, we are moving toward no fault! Everyone — the beleaguered legal profession and its courts, the consuming public, and the insurance industry itself — stands to benefit! And I believe that the States can and will step up to the challenge! When the tumult and the shouting dies, the lawyers and the actuaries depart — we'll see a new march forward in the vital industry we know as insurance.

MINUTES OF THE 1971 SPRING MEETING

May 16 - 19, 1971

GREENBRIER HOTEL, WHITE SULPHUR SPRINGS, WEST VIRGINIA

Sunday, May 16, 1971

The Council held its regularly scheduled meeting at the Greenbrier from 2:30 p.m. to 5:30 p.m. Preliminary registration was also held during the afternoon for early arrivals.

A formal reception was held in the early evening for all members and their wives as well as guests. It should also be noted that the Board of Directors of the Academy of Actuaries met at the Greenbrier during the weekend and some members remained to participate in part of the Society activities.

Monday, May 17, 1971

After a brief registration period, the 1971 Spring meeting was formally convened at 9:00 a.m. by President Richard L. Johe who welcomed the gathering and then introduced the Honorable Samuel H. Weese, Insurance Commissioner, State of West Virginia. Commissioner Weese welcomed the Society to West Virginia and presented his thoughts on various timely problems affecting the insurance industry.

At 9:30 a.m. a panel discussion entitled "Consumerism, Insurance and the Actuary" was presented to the entire membership. Participants in this part of the program were as follows:

Moderator: Dunbar R. Uhthoff, Senior Vice President
Employers Insurance of Wausau

Participants: Frank A. Baer, II, Insurance Agent
Commercial Insurance Service, Inc.
Charleston, West Virginia

Samuel R. Boggs, 2nd Vice President
of Communications
Insurance Company of North America

Richard Munro, Actuary
American States Insurance Group

Norman Polovoy, Deputy Attorney General &
Chief of the Divisions of Consumer Protection and Anti-Trust
State of Maryland

This panel discussion was concluded at 11:00 a.m.

Following the coffee break, the membership then participated in a "Delphi" program that had been developed by three of the members. The original questionnaire returned by the membership was discussed and two subsequent ballots were taken following brief discussion periods. An attachment to these minutes sets forth the results of the various questionnaires. The three members responsible for this portion of the program are as follows:

Norman J. Bennett, Secretary & Actuary
Continental Insurance Companies

Charles L. Niles, Jr., Deputy General Manager and Vice President
General Accident Group

Matthew Rodermund, Vice President & Actuary
Munich Reinsurance Company

Following lunch, the afternoon was set aside for individual meetings of the various Society committees.

No formal arrangements were made for the membership at large although a small informal reception was held by the officers for the new Fellows (and their wives) who, at a later time during the meeting, would be presented with their Fellowship diplomas.

Tuesday, May 18, 1971

At 9:00 a.m. President Richard L. Johe reconvened the meeting. The first order of business was the presentation of diplomas to the following new Fellows and Associates:

FELLOWS

Comey, Dale R.
Grady, David J.
Hunter, J. Robert, Jr.
Richardson, James F.
Skurnick, David
Snader, Richard H.
Zory, Peter B.

ASSOCIATES

Engel, Philip L.
Hoffmann, Dennis E.
McClenahan, Charles L.
Miller, Philip D.
Neidermyer, James R.
Rinehart, Charles R.
Thompson, Eugene G.

The next order of business was the presentation of a new paper entitled "Federal Income Taxes" by Raymond W. Beckman.

A review of the paper "Credibility for Severity" by Charles C. Hewitt, Jr., was then presented to the membership by Charles A. Hachemeister. The business session was then adjourned and the membership heard Deputy Superintendent A. E. Fox of the New York Insurance Department present his views on the experience to date under the new open competition rating law in New York.

Following the coffee break, the membership heard a formal address by the Honorable Charles D. Baker, Assistant Secretary for Policy and International Affairs, Department of Transportation. Mr. Baker presented the department's comments concerning the current DOT studies of automobile liability insurance. Mr. Baker's timely comments were then followed by an intensive discussion between various Society members and Mr. Baker. Mr. Alan C. Curry, Vice President and Actuary, State Farm Mutual Automobile Insurance Company, assisted in this portion of the program. The meeting was then adjourned for lunch.

The afternoon session was first devoted to a panel discussion on "Massachusetts No Fault- Status and Prognosis."

Moderator: John R. Bevan, Actuary
Liberty Mutual Insurance Company

Participants: Lee M. Alexander, Actuary
Massachusetts Automobile Rating and Accident Prevention Bureau

Roy Anderson, Vice President
Allstate Insurance Company

Milton G. McDonald, Chief Actuary
Massachusetts Insurance Department

Herbert J. Phillips, Jr., Actuary and Vice President
Employers-Commercial Union Companies

The business session was again reconvened with a joint report by William C. Aldrich, Chairman of the Committee to Review the Constitution, and Matthew Rodermund, Chairman of the Committee on Election Procedures, concerning the activities of the committees leading up to the proposed changes in the Constitution and Bylaws.

After some discussion the Fellows unanimously voted that the proposed changes in the Constitution and Bylaws be adopted.

The afternoon was concluded by the President's report covering various current items of Society activities.

A formal reception was held in the evening for the entire membership with Matthew Rodermund and J. Robert Hunter, Jr. providing some light entertainment for all to enjoy.

Wednesday, May 19, 1971

The meeting was reconvened at 9:00 a.m. by President Richard L. Johe.

The first item of business was a second review of the paper "Credibility for Severity" given by Hans U. Gerber, Visiting Assistant Professor of Statistics, University of Michigan. This guest review was given at the invitation of President Richard L. Johe. The author indicated he reserved the right of rebuttal on both reviews until the November meeting.

The remainder of the morning was devoted to a panel entitled "Investment Income in Ratemaking." Participants were as follows:

Moderator: Jack Moseley, Vice President and
Senior Actuary
United States Fidelity & Guaranty Company

Participants: Paul Benbrook, Executive Vice President
Maryland Casualty Company

Gerald R. Hartman, Associate Professor
Temple University

John H. Muetterties, Actuary
Insurance Services Office

The Spring meeting was then adjourned at 12:00 p.m.

It is noted that the registration cards completed by the attendees and filed at the registration desk indicated, in addition to about 53 wives, attendance by 98 Fellows, 35 Associates, and 28 invited guests as follows:

FELLOWS

Aldrich, W. C.
Alexander, L. M.
Bailey, R. A.
Beckman, R. W., III
Benbrook, P.
Bennett, N. J.
Berquist, J. R.
Bevan, J. R.
Bickerstaff, D. R.
Bornhuetter, R. L.
Boyajian, J. H.
Brannigan, J. F.
Brian R. A.
Comey, D. R.
Cook, C. F.
Crandall, W. H.
Curry, A. C.
Curry, H. E.
Dahme, O. E.
DeMelio, J. J.
Eide, K. A.
Eliason, E. B.
Elliott, G. B.
Farnam, W. E.
Finnegan, J. H.
Fitzgibbon, W. J., Jr.
Flaherty, D. J.
Gibson, J. A., III
Gillespie, J. E.
Grady, D. J.
Graves, C. H.
Hachemeister, C. A.
Hartman, G. R.
Harwayne, F.
Hazam, W. J.
Hewitt, C. C., Jr.
Hillhouse, J. A.
Honebein, C. W.
Hope, F. J.
Hunter, J. R., Jr.
Hurley, R. L.
Jacobs, T. S.
Johe, R. L.
Johnson, R. A.
Kallop, R. H.
Kilbourne, F. W.
Linder, J.
Lino, R.
Liscord, P. S.
MacGinnitie, W. J.
Masterson, N. E.
McClure, R. D.
McGuinness, J. S.
McNamara, D. J.
Meenaghan, J. J.
Menzel, H. W.
Mills, R. J.
Mohnblatt, A. S.
Morison, G. D.
Moseley, J.
Muetterties, J. H.
Munro, R. E.
Murrin, T. E.
Nelson, D. A.
Newman, S. H.
Niles, C. L., Jr.
Oien, R. G.
Petz, E. F.
Phillips, H. J., Jr.
Pollack, R.
Richards, H. R.
Richardson, J. F.
Riddlesworth, W. A.
Roberts, L. H.
Rodermund, M.
Rosenberg, N.
Rowell, J. H.
Ruchlis, E.
Salzmann, R. E.
Scheel, P. J.
Scheibl, J. A.
Schloss, H. W.
Scott, B. E.
Simon, L. J.
Skelding, A. Z.
Skurnick, D.
Snader, R. H.
Strug, E. J.
Switzer, V. J.
Tarbell, L. L., Jr.
Thomas, J. W.
Uhthoff, D. R.
Webb, B. L.
Wieder, J. W., Jr.
Wilcken, C. L.
Wilson, J. C.
Wittick, H. E.
Zory, P. B.

ASSOCIATES

Anker, R. A.
Bell, A. A.
Bergen, R. D.
Bittel, W. H.
Carter, E. J.
Chorpita, F. M.
Cooper, W. P.
Drennan, J. P.
Eyers, R. G.
Ferguson, R. E.
Fossa, E. F.
Gill, J. F.
Greene, T. A.
Hardy, H. R.
Hartman, D. G.

ASSOCIATES

Head, T. F.
Hearn, V. W.
Hoffmann, D. E.
Jensen, J. P.
Krause, G. A.
Levin, J. W.
Linquanti, A. J.
McClenahan, C. L.
McDonald, M. G.
Miller, P. D.
Neidermyer, J. R.
Ratnaswamy, R.
Rinehart, C. R.
Singer, P. E.
Stephenson, E. A.
Thompson, E. G.
Welch, J. P.
Winter, A. E.
Wooddy, J. C.
Young, R. G.

GUESTS

Anderson, R. R.
*Babb, J. A.
Baer, F. A., II
Baker, C. D.
Bitzer, J. F.
Boggs, S. R., II
*Chamberlain, R. H.
*Chan, E.
*Chang, C. I.
*Connolly, C. T.
*Dunn, R. P.
*Eddins, J. M.
Foley, D. J.
Fox, A. E.
Gerber, H. U.
*Griffith, R. W.
*Guarini, L.
*Hayden, R. C.
*Jewell, R. L., Jr.
Kasten, C. W.
*Mingo, G. E.
Pellegrini, P. L.
Polovoy, N.
*Reilly, F. V.
*Smith, D. A.
Watson, C. B.
Weese, S. H.
*White, B. R.

* Invitational Program

Respectfully submitted,

RONALD L. BORNHUETTER
Secretary-Treasurer

DELPHI PROJECT

All in attendance at the Monday morning meeting were invited to participate in a discussion "The Actuary Predicts His Professional Future," which was led by Norman J. Bennett, Secretary and Actuary, Continental Insurance Companies; Charles L. Niles, Jr., Deputy General Manager and Vice President, General Accident Group; and Matthew Rodermund, Vice President and Actuary, Munich Reinsurance Company.

The discussion was triggered by responses to two questions that had been submitted to the entire membership of the Society in March preceding this meeting. In brief, the questions asked the membership to estimate (1) what the percentage of property-liability actuaries would be in the year 2000, given actual percentages of 13% in 1953 and 9% in 1969; and (2) what the percentage of actuaries of all kinds employed by private insurance organizations would be in the year 2000, given actual percentages of 83% in 1953 and 74% in 1969.

The discussion leaders reported that 149 answers had been received and the results on question 1 were:

Year 1953 (as reported) — 13%
Year 1969 (as reported) — 9%
Year 2000 (as estimated) — Median 11%
Inter-quartile range 6% - 15%

The results of question 2 were:

Year 1953 (as reported) — 83%
Year 1969 (as reported) — 74%
Year 2000 (as estimated) — Median 65%
Inter-quartile range 55% - 74%

After open discussion between the leaders and members of the audience on the significance of the results, revealing both pessimistic and optimistic opinions of the future of the property-liability actuary, another vote was taken among those in attendance, and the results were:

Question 1: Median 12%
Inter-quartile range 10% - 15%
Question 2: Median 65%
Inter-quartile range 60% - 70%

The results of the second ballot indicated a narrowing of the inter-quartile range for both questions, but whether this pointed up a herding tendency was uncertain because the sample of voters was smaller and not necessarily possessed of the same biases as on the first ballot.

Further discussions followed, including some criticism of the basic questions and of the balloting system. However, at the end of the morning meeting a third ballot was taken, the results of which (announced Wednesday morning, May 19) were:

Question 1: Median 15%
Inter-quartile range 11% - 19%
Question 2: Median 67%
Inter-quartile range 60% - 75%

VOLUME LVIII, Part II No. 110

PROCEEDINGS

NOVEMBER 14, 15, 16, 1971

A LOOK AHEAD

PRESIDENTIAL ADDRESS BY RICHARD L. JOHE

Introduction

"Nothing endures but change"

— Heraclitus (540-480 B.C.)

Scientific information today has a half-life of less than ten years. This means that in less than ten years, half of today's scientific knowledge will be obsolete. The same is true of business knowledge to a lesser degree. In the insurance industry, ideas, information and attitudes are changing even though insurance is a tradition-bound business, usually slow to innovate and often hampered by uninspired and politically oriented regulation.

Diversification

In a bygone industrial era, an individual human need was met by one, or sometimes several competing specialized American business enterprises. The continuation of each enterprise depended on the degree of efficiency with which it operated but there was no intent that each enterprise should serve all of man's needs. However, today's American business environment emphasizes diversified growth through affiliates, subsidiaries and holding companies to form conglomerates and other large, broadly based business enterprises. We see greater emphasis being placed on serving a broader spectrum of human need.

The trend toward diversification is also affecting the American insurance function which is increasingly being organized around larger and more flexible corporate structures in a position to offer more complete financial services. Forty-seven of the fifty largest property-liability companies have life affiliates, all of the ten largest stock life insurance companies have property-liability affiliates, over three fourths of the largest 100 life insurance companies are developing or currently offering variable annuities, and insurers are sponsoring a substantial number of mutual funds.

It is good business policy to maximize the utilization of financial capacity and of sales and administrative personnel, so it is likely that diversification will continue in the insurance industry.

Environment

American business enterprises are being expected to help solve social problems of ecology, poverty, discrimination and urban sprawl. Employers are being expected to go beyond fulfilling the economic needs of their employees. Today's employees are increasingly looking for job fulfillment; employment which satisfies psychological needs by providing meaning to their lives. Expansion of the role of American business is requiring changes in insurance company management philosophy and behavior.

In addition, American business enterprises are being faced with a force generally called "consumerism"; an increased pressure from consumers who combine forces to obtain influential support in legislatures and in government agencies.

Cooperation with Government

While in the past the insurance industry has used voluntary assigned risk plans and pools to distribute and subsidize marginal and sub-standard risks, growth in the insurance industry has resulted mainly from providing coverage for risks which are generally viewed as insurable.

In recent years, pressure has increased to provide coverage for risks which do not meet the traditional definition of insurable risks. The property-liability insurance industry is cooperating with government in providing coverage for property exposed to flood, as well as crime and fire coverage for property located in extra hazardous urban areas. The insurance industry's Healthcare proposal is an attempt to cooperate with government in meeting the problems of inadequate medical care and the life insurance industry is cooperating with government by investing millions of dollars in high-risk urban areas.

Cooperation with government has taken different forms. In flood insurance, through federal subsidies, the general public will share in the extra cost of insuring highly exposed properties. Under state FAIR plans, private insurance companies are absorbing the extra cost of providing fire coverage for high risk urban properties at inadequate premiums while profits from federal riot reinsurance have been used to fund a federal crime insurance program serviced by and competing with private insurance companies. There is no government subsidy for investment losses which may be sustained by life insurance company's investing in high-risk urban areas. Such investment losses will have to be absorbed by private insurance company stockholders and/or policyholders. It seems likely that in the future, actuaries will need to take into account the extra cost of high-risk cooperation with government.

Future Environment

Just as the present business environment differs from that of a past era, the future business environment will also differ from the present. It seems likely that there will be continued expansion of interaction between government and the insurance industry as our society increasingly looks to the insurance mechanism as a means of tackling social problems of poverty and urban decay, as well as economic problems resulting from automobile accidents and the non-portability of pension funds and other employee fringe benefits.

In the property-liability business, since 1965 we have seen an increasing interest in the mass marketing technique for personal lines coverages i.e. group sales of insurance to employees of a single employer or to members of associations and educational or charitable organizations, usually on the basis of the employee or member paying the total cost. It is likely that in the future, the mass marketing technique will be used to distribute coverage to an increasing share of the personal lines market. At the same time, the American business scene is developing into a service economy with a consequent decline in the relative importance of basic industries such as manufacturing, mining and farming. Service businesses have historically been relatively small firms which do not appear to be ideal markets for the mass marketing technique.

In the life business, companies are expanding into equities and into property-liability insurance in order to increase agent earnings and productivity and to increase underwriting capacity which has been strained by jumbo jets and super tankers. The emergence of a marketable equity-based variable life insurance contract should stimulate interest in permanent life insurance contracts, but could very well result in a significant step toward federal regulation.

The expansion of equity based products introduces a variable investment element and subjects this part of the insurance business to SEC (Securities and Exchange Commission) legislation and regulation. The SEC views variable products as securities and tries to make them fit that mold. This threatens the large front-end load concept which has been so essential to the successful marketing of life insurance personal lines.

One solution would be for Congress to enact legislation taking variable insurance products away from SEC control and transferring jurisdiction to a separate department which would regulate and control these equity based insurance products. Such an exchange of one brand of federal regulation for another would seem to be a step closer to federal regulation of the Canadian type. Whether or not this step is taken, or another means found for increased marketing of variable products, it seems clear to me that actuaries of the future must have a greater knowledge and understanding of the functioning of the entire investment process.

Government Intervention

One issue yet to be determined is the future role of the Social Security system in bringing economic security to Americans. The Social Security role traditionally has been to provide a basic level of protection with occasional liberalizations reflecting increases in the cost of living and our standard of living. However, there are some who envision expanding the Social Security system to provide full economic security to nearly everyone, including the poverty-stricken.

Barring a sustained business recession, it seems clear that the next decade will bring some resolution of the poverty problem, either in the form of a negative income tax or guaranteed income, or in the form of a significance expansion of the Social Security system. Such an expansion would appear to increase wage-related benefits for people in middle and upper income brackets regardless of economic need, as well as those at poverty levels, since an increase in the minimum benefit without a proportionate increase in the maximum would compress the benefit distribution, thus destroying the concept of wage-related benefits. Expansion of the public Social Security system would obviously reduce the private insurance market for income protection.

Another issue yet to be determined concerns the private health insurance field which is being threatened by mounting pressure for the establishment of a compulsory national health insurance system as a partial answer to continuing increases in price levels of medical care. This pressure continues to build even though the vast majority of the American public finances its health care through private health insurance. The Health Insurance Council's 25th Annual Survey found that at the end of 1970, over nine out of every ten Americans below age 65 (about 92 percent) were covered by private hospital expense insurance. Of those covered for some or all of their hospital expenses, 94 percent also had surgical expense protection and 80 percent had non-surgical medical expense coverage. Disability income benefits for non-occupational short-term wage loss provided by insurance companies, and other formal arrangements, protected more than two out of every three in the labor force.

A number of comprehensive proposals designed to solve the nation's health care problems have emerged in recent months. The health insurance business is sponsoring and supporting a cooperative proposal called Healthcare. Healthcare would be a voluntary program with Federal standards for health insurance, tax incentives for employers and individuals to upgrade their private health insurance coverage to meet the Federal standards, federal and state subsidies through state pools of private health insurers to provide comprehensive benefits for the medically indigent (the poor, near poor and those previously uninsurable), modest co-payments by insured persons and progressive expansion of benefits throughout the 1970's to avoid overloading the health care delivery system. Healthcare benefits are designed to dovetail with and supplement Medicare, but to ultimately replace Medicaid as a means of financing medical care for low income families.

At the opposite extreme, in terms of cost and approach to health care problems, is the Health Security Plan sponsored by AFL-CIO and the Committee for National Health Insurance. This proposal would scrap private health insurance plans, finance costs publicly through new and existing federal taxes, scrap co-payments by insured persons and absorb Medicare and Medicaid into the new system. It is likely that the resolution of the Social Security issue and the national health care issue may well have major effects on the role which the federal government will play in the actuary's environment.

The Actuary

"man's yesterday may ne'er be like his morrow"

— Shelley (1792-1822)

Twenty-five years ago, two different companies were required to insure a single private passenger automobile, and most casualty actuaries knew very little about fire insurance. Thanks to multiple line legislation, the horizons of the casualty insurance industry, and of the casualty actuary, have broadened to include all property-liability coverages.

A single document issued by a single company to insure the property-liability exposures found in a single house, or the property-liability exposures of owning or operating a single automobile, are ideas which have become reality. These ideas and dreams of a past generation have been replaced by increasing interest in a family account plan and even a life cycle policy.

A family account plan would enable an insured to make a single monthly payment under a single account billing covering the entire range of financial services (including insurance) required by his family. A life cycle policy, as it is discussed in the life insurance industry, would blend varying amounts and types of life insurance to provide varying mixtures of protection and savings from cradle to grave. Life insurance protection and savings would increase as family responsibilities increased through marriage and child birth, and would decrease as the insured approached retirement age. The life cycle policy concept can easily be broadened to include full family protection through the addition of health insurance and property-liability coverages. Some may argue that the resulting life cycle policy would be a bundle rather than a package, but I submit that today's consumer is not really sympathetic to our internal accounting problems and there is a property-liability precedent in at least one company's Master Insurance Program applicable to commercial risks.

The variable outlay approach to the life cycle policy blends in with the family account plan, either of which would maximize consumer and agent convenience and would increase the probability that a family would look to one company for all insurance and financial service needs. Both variable outlay approaches are consistent with today's trends in American social and business environments. It seems clear to me that actuaries of the future must be much more conversant with all uses of the insurance mechanism.

Traditional thinking can be a Procrustean bed, a perpetuation of the parochial vertical linearity which today argues that a casualty actuary is different, and should continue to be different from a life actuary. It is true that a property-liability company cannot legally write life insurance, and vice-versa, but, the fundamentals of life and casualty actuarial science are the same. We are today divided more by differences in vocabulary and experience, than by any real difference in the common core of actuarial science.

I urge that, jointly with the Society of Actuaries, top priority be given to defining the common core of actuarial science and to completely restructuring our present education and examination requirements for tomorrow's actuary. This redefinition and restructuring must recognize the actuary's function of assisting management in the exercise and building of intelligent and informed fiscal control and long range fiscal plans. Tomorrow's actuary will need a greater knowledge and understanding of all facets of the insurance mechanism including much of the financial and investment knowledge covered by candidates for an MBA (Master's in Business Administration) degree, with emphasis on the mathematics of quantitative methods. Obsolescence must also be avoided by giving top priority to the problems of the continuing education of today's actuary. The continued vitality and relevancy of the actuarial profession depends on our ability to anticipate and adapt to our changing economic and social environment.

ACTUARIAL NOTE ON WORKMEN'S COMPENSATION LOSS RESERVES

RONALD E. FERGUSON

"Not only is there but one way of doing *things rightly, but there is only one way of* seeing *them, and that is, seeing the whole of them."*

— John Ruskin

In the calculation of tabular reserves for long term pension type awards special care must be used when an excess of loss reinsurance coverage is involved. In this situation some or all of the parties interested in the transaction — ceding company, reinsurer, and regulator — frequently do not understand, or sometimes are not even aware of, the proper way to calculate the ceded reserve and, of course, this usually means that the net reserve is also incorrect.

If, for example, a case involving a permanently disabled individual aged 45 with a life pension award of \$7,142 a year, \$137.34 weekly, is presented for reserving, and assuming that the liability is to be discounted for interest (3%) and for mortality using the Survivorship Annuitants' Table of Mortality, and further assuming that a reinsurance contract providing coverage excess of \$50,000 retention is in effect, the reserves are often incorrectly calculated as follows:

Direct Reserve: \$7,142 $\bar{a}_{45}$ or \$129,280, since $\bar{a}_{45} = 18.101$ from the last column of the accompanying table (which column incidentally is found as Table XI in the State of New York Workmen's Compensation Board's Bulletin No. 222). The correct annuity factor is actually $\ddot{a}^{(m)}_{45}$ but since m is fairly large, 52, when weekly payments are involved $\bar{a}_{45}$ is often used since $\lim_{m \to \infty} \ddot{a}^{(m)}_{x} = \bar{a}_x$, and $\bar{a}_x$ is often approximated by $a_x + \frac{1}{2}$ or $\ddot{a}_x - \frac{1}{2}$. In terms of commutation functions this becomes $\dfrac{N_{x+1}}{D_x} + \frac{1}{2}$ which can be written as $\dfrac{N_{x+1} + \frac{1}{2} D_x}{D_x}$ or $\frac{1}{2}\,\dfrac{(N_x + N_{x+1})}{D_x}$ and this approximation appears to be incorporated in the New York Board's Table.

Having calculated the direct reserve of \$129,280 one might then conclude that since a \$50,000 retention is operating on this loss, the net reserve must be \$50,000 and therefore the ceded reserve must be \$79,280 (i.e. \$129,280 — \$50,000).

The direct reserve above is correct but the ceded and net reserves are not. The error arises in part from the fact that assuming a net reserve equal to the retention of \$50,000 ignores the possibility that the annuitant may not survive long enough to exhaust the \$50,000 retention. What must happen to cause the \$50,000 retention to be exhausted? The annuitant must collect for seven years $\frac{\$50{,}000}{\$\ 7{,}142}$ and the probability of his surviving seven years is obviously less than one. Therefore, the expected value of this obligation is less than \$50,000; in fact, it must be \$7,142 $\bar{a}_{45:\overline{7}|}$ or \$43,885. The only time the net incurred can be \$50,000 is when the ceding company has actually paid benefits in the amount of \$50,000. It is interesting to note that when \$7,142 $\bar{a}_{45:\overline{7}|}$ is evaluated at 0% interest (*i.e.*, discounting for mortality only) the reserve becomes \$48,523.

To put it another way, if the ceding company has one hundred similar cases, some annuitants would collect for eight or more years and the ultimate net incurred would be \$50,000 on each of those cases, but some annuitants will survive only one year and have an ultimate incurred of \$7,142, some will survive two years and have an ultimate incurred of \$14,285 *etc.* It then is obvious that the average ultimate net incurred loss must be less than \$50,000.

In addition to failing to take mortality into account, the above reserving method presents a similar problem as respects interest discounting. Even if it were certain the annuitant would survive seven years the present value of this obligation would be less than \$50,000, since the funds set aside as a reserve would be augmented in this case by the assumed 3% investment income. The amount needed would be \$7,142 $\bar{a}_{\overline{7}|}$ or \$45,161.

The correct way to calculate the various reserves is to break the gross or direct reserve into its component pieces. The net reserve must be based on a temporary life annuity, thus taking into account both the mortality and interest discounting discussed above. The ceded reserve is based on a deferred annuity — deferred by the number of years needed to exhaust the ceding company retention.

Direct	Net	Ceded

$$\$7,142\ \bar{a}_{45} = \$7,142\ \bar{a}_{45:\overline{7}|} + \$7,142\ {}_{7|}\bar{a}_{45}$$

or

$$\$7,142 \frac{\overline{N}_{45}}{D_{45}} = \$7,142 \frac{\overline{N}_{45} - \overline{N}_{52}}{D_{45}} + \$7,142 \frac{\overline{N}_{52}}{D_{45}}$$

To calculate the above, the $\overline{N}_x$ and D_x values underlying New York's Table XI are needed. These values are shown in the accompanying table, but the New York Special Bulletin does not show the N_x and D_x values which underlie the $\bar{a}_x$ values, and since this booklet is widely used, the lack of data has no doubt contributed to the confusion that seems to exist concerning proper reserving techniques. The $\overline{N}_x$ and D_x values are derived from the Survivorship Annuitants' Mortality Table for ages 15 through 95. The Survivorship Annuitants' Mortality Table is a "Makehamized" Table (*i.e.*, the l_x values observed were graduated using Makeham's formula $l_x = ks^x g^{c^x}$) for ages 15 through 95 with constants of $log\ s = -.0022402$, $log\ c = .04579609$, $log\ g = -.000093999$, and $log\ k = 5.0226717$, all to the base 10. For these ages, the values shown in the accompanying table are consistent with the $\bar{a}_x$ values shown in Table XI in the State of New York Workmen's Compensation Board's Special Bulletin No. 222.

For ages over 95, the $\bar{a}_x$ values in New York's Bulletin No. 222 are not based on the Survivorship Annuitants' Mortality Table. Accordingly the $\overline{N}_x$ and D_x values shown in the table for ages over 95 were calculated to be consistent with New York's $\bar{a}_x$ values for ages over 95 and with the N_x and D_x values for age 95. For ages under 15 the values were calculated directly from the Makeham formula since neither the New York Workmen's Compensation Board's Bulletin No. 222 nor the Survivorship Annuitants' Mortality Table extends below age 15. It is recognized that the values shown for under 15, and possibly the young adult ages, are not entirely satisfactory since no attempt was made to modify the Makeham formula or adjust the constants. Since the Makeham formula has no minima or maxima or points of inflection, it cannot (without modification) accurately portray some of the peculiarities often observed in the mortality curve below the age of 25.

Following are several examples of correct and incorrect reserves assuming an annual pension of \$7,142 (with weekly payments).

Retention \$50,000

	Correct		Incorrect	
Age	Ceded	Net	Ceded	Net
45	\$85,395	\$43,885	\$79,280	\$50,000
55	60,812	42,969	53,781	50,000

Retention \$100,000

	Correct		Incorrect	
Age	Ceded	Net	Ceded	Net
45	\$52,312	\$76,968	\$29,280	\$100,000
55	30,713	73,068	3,781	100,000

It is clear from the above examples that the difference between the correct and incorrect reserves can be quite significant. Actuaries might well inquire as to how such matters are handled in their own companies.

LIFE AWARDS ON PERMANENT DISABILITY CASES

Present Value of \$1 per annum Payable Until Death, Based on Survivorship Annuitants' Mortality Table and Interest at 3%

(This table does not provide for remarriage factors; but values for ages after 65 can be used for valuing widows' pensions)

Age(x)	D_x	$\overline{N}_x$	$\bar{a}_x$
0	105,336.000	2,781,362.447	26.405
1	101,739.806	2,677,824.544	26.320
2	98,265.624	2,577,821.829	26.233
3	94,909.341	2,481,234.346	26.143
4	91,667.875	2,387,945.738	26.050
5	88,537.303	2,297,843.149	25.953
6	85,513.005	2,210,817.995	25.854
7	82,591.396	2,126,765.794	25.750

Age(x)	D_x	$\overline{N}_x$	$\bar{a}_x$
8	79,769.013	2,045,585.589	25.644
9	77,042.509	1,967,179.828	25.534
10	74,409.391	1,891,453.878	25.420
11	71,865.747	1,818,316.309	25.302
12	69,407.851	1,747,679.510	25.180
13	67,034.212	1,679,458.479	25.054
14	64,740.623	1,613,571.062	24.924
15	62,525.056	1,549,938.223	24.789
16	60,384.253	1,488,483.568	24.650
17	58,316.325	1,429,133.279	24.507
18	56,318.220	1,371,816.006	24.358
19	54,388.179	1,316,462.807	24.205
20	52,522.789	1,263,007.323	24.047
21	50,721.000	1,211,385.428	23.883
22	48,979.089	1,161,535.383	23.715
23	47,296.635	1,113,397.521	23.541
24	45,670.145	1,066,914.131	23.361
25	44,098.755	1,022,029.681	23.176
26	42,579.232	978,690.688	22.985
27	41,111.265	936,845.439	22.788
28	39,692.251	896,443.681	22.585
29	38,320.174	857,437.468	22.376
30	36,993.527	819,780.617	22.160
31	35,710.853	783,428.427	21.938
32	34,470.736	748,337.632	21.709
33	33,271.058	714,466.735	21.474
34	32,110.557	681,775.928	21.232
35	30,988.011	650,226.644	20.983
36	29,901.891	619,781.693	20.727
37	28,850.741	590,405.377	20.464
38	27,833.179	562,063.417	20.194
39	26,847.893	534,722.881	19.917
40	25,893.630	508,352.120	19.632
41	24,969.203	482,920.703	19.341
42	24,073.481	458,399.361	19.042
43	23,205.109	434,760.066	18.736
44	22,363.085	411,975.969	18.422

Age(x)	D_x	$\overline{N}_x$	a_x
45	21,546.459	390,021.197	18.101
46	20,753.554	368,871.190	17.774
47	19,984.072	348,502.377	17.439
48	19,236.243	328,892.220	17.098
49	18,509.384	310,019.407	16.749
50	17,802.389	291,863.520	16.395
51	17,114.453	274,405.099	16.034
52	16,444.609	257,625.568	15.666
53	15,791.751	241,507.388	15.293
54	15,155.069	226.033.978	14.915
55	14,533.617	211,189.635	14.531
56	13,926.531	196,959.561	14.143
57	13,333.206	183,329.692	13.750
58	12,752.366	170,286,906	13.353
59	12,183.735	157,818.855	12.953
60	11,626.207	145,913.884	12.550
61	11,079.286	134,561.137	12.145
62	10,542.363	123,750.312	11.738
63	10,014.735	113,471.763	11.330
64	9,495.9302	103,716.430	10.922
65	8,985.5276	94,475.7015	10.514
66	8,483.1555	85,741.3600	10.107
67	7,988.4857	77,505.5394	9.702
68	7,501.6349	69,760.4791	9.299
69	7,022.3176	62,498.5028	8.900
70	6,550.7915	55,711.9483	8.505
71	6,087.4103	49,392.8474	8.114
72	5,632.4882	43,532.8981	7.729
73	5,186.7672	38,123.2704	7.350
74	4,750.8981	33,154.4377	6.979
75	4,326.2143	28,615.8815	6.615
76	3,913.6716	24,495.9386	6.259
77	3,514.8155	20,781.6951	5.913
78	3,131.1876	17,458.6936	5.576
79	2,764.4087	14.510.8955	5.249
80	2,416.3392	11,920.5216	4.933
81	2,088.7550	9,667.9745	4.629

Age (x)	D_x	$\overline{N}_x$	$\bar{a}_x$
82	1,783.4301	7,731.8820	4.335
83	1,502.0312	6,089.1513	4.054
84	1,245.8651	4,715.2032	3.785
85	1,016.0746	3,584.2333	3.528
86	813.2519	2,669.5700	3.283
87	637.4287	1,944.2297	3.050
88	488.1464	1,381.4422	2.830
89	364.2336	955.2522	2.623
90	264.1173	641.0767	2.427
91	185.5462	416.2450	2.243
92	125.7632	260.5903	2.072
93	81.9761	156.7206	1.912
94	51.1329	90.1661	1.763
95	30.4014	49.3990	1.625
96	17.116	25.640	1.498
97	9.081	12.541	1.381
98	4.512	5.744	1.273
99	2.084	2.446	1.174
100	0.887	0.961	1.084
101	0.345	0.345	1.002
102	0.121	0.112	.928
103	0.038	0.033	.861

Reprints of Papers from Previous Proceedings

The following five papers are being reprinted in this volume of the *Proceedings* in order to make available to the membership and students a few of the pertinent papers published in previous *Proceedings*. Volumes of these *Proceedings* are presently out of print and are difficult to obtain. Since all of these papers are required reading in the examination syllabus and are still applicable to current practices, it is hoped these reprints will supply a readily accessible source for reference.

L. L. Tarbell Jr.,
Editor

NOTES ON EXPOSURE AND PREMIUM BASES

BY

PAUL DORWEILER

When *critical conditions* and *injurable objects* exist in such relationship that accidents may result there is said to be *exposure.* The term *critical conditions* is intended to cover, rather broadly, the presence of or the absence of anything, objective or subjective, generally external to the injurable object, which contributes to the accident frequency and/or the accident severity. It is intended to cover also any part or quality of the injurable object which likewise contributes to accident frequency and/or accident severity. *Injurable objects* will be used to denote any objects, human beings included, which may be damaged or injured, including complete destruction. If the whole insurance field is to be covered, it is necessary to extend the meaning of this term to include non-material things.

As a concrete example it might be conceived that critical conditions consisting of uniform fixed physical obstructions exist in a large but restricted area with inanimate injurable objects all identical in nature, for a period of time. Let it be assumed that the objects move about freely and that when they strike against one of the obstructions they are destroyed, and immediately replaced with others of their kind. Under these simple conditions it may be shown that the hazard varies directly with the product of the three variables: *critical conditions, injurable objects,* and *period of time.* If one of these variables remains constant the hazard varies as the product of the other two and if any two of the variables remain constant the hazard varies directly as the third.

In reality, however, the situation is quite different from this simple case, for the relationship becomes extremely involved. The critical conditions may comprise the presence or absence of material objects, habits, laws, regulations, or yet many other things. They may be external to or form a part of the injurable objects. They generally differ in their contribution to both the accident frequency and the accident severity. The injurable objects also are generally dissimilar. They vary widely in their susceptibility both to the occurrence of accidents and to injury when involved in accidents. Generally the contributions to the hazard occurring from an increase in either the critical conditions or the number of injurable objects, under conditions in which the other variable and the period of time remain constant, cannot be expressed as a linear function of the number of units of the variable. The period of time is the only one of the three variable elements into which the exposure has been divided somewhat arbitrarily, with which the hazard varies directly, the other two variables remaining constant.

Even if the contribution of each of the variables were definitely known and if the hazard underlying the exposure could be expressed as a function of them, such an expression would be too involved for practical purposes. In actual practice the time element is eliminated by considering the aggregate injuries in blocks for selected unit periods of time—usually a year. This procedure brings together injuries of all degrees of severity and it becomes necessary to express the injury aggregate in terms of a common basis. The unit of measure used for this purpose is the economic unit, the dollar. The aggregate of injuries when expressed in the monetary standard of dollars is known as the *losses*.

A new variable, or set of variables, inherent in the *evaluation standard* used is introduced in the process of expressing the aggregate injuries in terms of dollars. This standard for evaluating injuries is the scale of remuneration adopted through a formal law as in compensation, or through custom and precedent as in the courts, or through community opinion as reflected in jury verdicts and private settlements. The effect of this new variable, the evaluation standard, is indicated by the difference in the losses obtained when the same or similar injuries to either human beings or inanimate objects are expressed in monetary units by the use of different evaluation standards.

PREMIUM BASES.

Insurance is the institution devised to transfer the losses arising out of the hazard from the few upon whom they chance to fall to the many exposed by paying the losses from funds called *premiums* which have been specially collected for this purpose. These premium funds are accumulated from charges called the *rate* collected per unit exposure. The exposure medium selected as the basis for the charge of the premium is known as the *premium basis*.

Obviously, the premiums collected are to be proportional to the hazard which is measured by the losses. The medium selected for measuring the exposure is the most important factor in making the premium collections in accordance with the probable loss incidence. The medium most desirable as a premium basis is the one possessing a combination of these two qualifications in the largest degree:

1. *Magnitude of Medium should vary with hazard.*

   It is desirable to have for premium basis an exposure medium whose magnitude varies approximately directly with the hazard when this is measured by the losses. By using a medium which varies directly with the hazard, the total premium may be obtained by multiplying the exposure expressed in units of the premium basis by the rate.

2. *The Medium should be practical and preferably already in use.*

   For measuring the exposure it is desirable to have a medium whose magnitude is readily ascertained and which is already used by the assured for other than insurance purposes. The use of a medium possessing these qualities promotes efficiency, as no additional records are necessary for measuring exposure, and enhances accuracy, as the various existing records may be used as a check.

When one considers the many diverse factors which enter into a hazard and the additional factors which enter into the evaluation of a hazard in terms of losses, one might expect that generally it would be impossible to find a medium whose magnitude varies directly with the losses. The factors underlying the losses, *critical conditions*, *injurable objects*, and *evaluation standard*, are studied as a preliminary to sorting hazards into somewhat homogeneous groups. Divisions made according to the kind of evaluation standard used, the type of injurable object covered, or the origin of critical conditions are known as *lines of insurance*. Divisions within these lines of insurance with regard to the frequency and severity of injuries or for convenience in practical procedure are known as *classifications*. These designations hold only in a very general way and have many exceptions. It is often difficult to distinguish between lines of insurance and classifications as used by different carriers or even by a single carrier.

In this discussion of premium bases it is proposed to review different media that might be used for measuring the exposure. For convenience in outlining the procedure the *injurable objects* will be divided into *human beings* and all other objects. At this time it is intended to consider only premium bases for insurance covering injuries to human beings. The coverages will be treated under four divisions which embrace the more important types. It is not intended to make this an exhaustive analysis including the special cases that may arise.

I. Coverage for injuries to designated persons.
II. Coverage to employers for industrial injuries to their employees.
III. Coverage for liability for injuries to the general public.
IV. Coverage for liability for professional acts.

I. Coverage for Injuries to Designated Persons.

1. Life Insurance.
2. Accident and Health Insurance.

In life, accident, and health insurance, coverage is given to individuals for specified injuries evaluated at specified amounts. These lines differ from the others of the four general divisions in that injuries are appraised at specified values which are prescribed in the contract. Within certain limits these specified values may be selected by the assured when making the contract.

1. Life Insurance.

In life insurance the critical conditions are those conditions external to the assured and also those bodily conditions within the assured which tend to place his life in jeopardy. The injurable object is the assured himself and the injury is the loss of his life. The measure of the injury in dollars by the evaluation standard is the amount named in the policy to be paid in case of death. The underlying assumption is that classifications, when refined with respect to age, occupation, sub-standard conditions, etc., are composed of individuals of equal essential hazard. The losses differ only because different amounts are written in the policies to be paid for the same injury, i.e. loss of life. The amount of insurance specified in the policy is obviously the exposure medium which varies directly with the losses for the very reason that the losses are made a definite function of the amount insured by the specific provision in the policy. A medium consisting of the amount of insurance is quite practical and forms such an ideal premium basis that little thought is given to any other. In group life insurance the situation is not changed although through the use of weighted averages it becomes less obvious.

2. Accident and Health Insurance.

As in life insurance, the critical conditions of accident and health insurance are those conditions external to the assured and those internal which may produce injuries through accidents or ill health. The evaluation standard is more involved than in life insurance. The accident insurance contract generally specifies a maximum loss known as the principal sum, certain lesser losses which have a definite relativity to this maximum, and fixed weekly benefits. In health insurance the losses are evaluated through the weekly benefits written into the contract. The classifications, through sufficient subdivisions according to age, occupation, and other conditions that may affect the hazard, are assumed to be composed of individuals of equal essential hazard. Equivalent injuries sustained differ when evaluated as losses only through the variation in the principal sum and the weekly benefits because, by the terms of the policy, they are definitely related to these items. The principal sum and weekly benefits form ideal premium bases as they are quite practical and vary directly with the losses.

II. Coverage to Employers for Industrial Injuries to Their Employees.

1. Workmen's Compensation Insurance.
2. Employers' Liability Insurance.
3. Workmen's Collective Insurance.

Of the variables underlying the hazard of this group of coverages the *critical conditions* and the *injurable human beings* are substantially the same. The inherent injuries are the same under each of these lines. The difference in the losses, which is due to a change in the *evaluation standard* used, arises out of these sources:

1. In Compensation a wider range of injuries is covered than in Employers' Liability. It does not necessarily follow, however, that the losses are larger under Compensation. In Workmen's Collective the extent of the liability assumed is stated in the contract.
2. The severity of an injury is expressed in monetary units by using different evaluation standards. In Compensation the standard is prescribed in the law. In Employers' Liability it consists in direct settlement mutually acceptable or in a jury verdict under court procedure. In Workmen's Collective the standard is specified in the contract.

Several premium bases have been considered and discussed in different degrees for these lines of insurance. As possible media for use in measuring exposure, these may be considered:

1. Payroll.
2. Restricted Payroll.
3. Man-Year.
4. Value of Product.

1. Payroll. Within the Compensation classifications the total injuries presumably vary directly with the time exposure. The indemnity cost of each injury of given severity varies with the rate of weekly compensation, which depends on the weekly wages. The indemnity losses vary as the product of the time and the weekly wages which product is represented by the payroll. Payroll as a medium does not respond fully to variation in losses to the extent that the losses are legally restricted by the maximum weekly payments and by the maximum amount paid on any case. If there were no limitation on weekly payments or on the maximum cost of a case then payroll would vary directly with the indemnity losses and from this viewpoint form an ideal medium for measuring exposures. Under the earlier Compensation laws these maximum limits were rather low and had an appreciable effect on the losses. Since then the limits have been raised materially by amendments to the law and their effect on the losses has been very much reduced.

Medical losses vary jointly with the accident frequency and severity, and therefore with the time exposure, and with the scale of medical fees. The wages and the medical fee scale supposedly respond to the same general price level and vary with each other. Medical losses thus vary jointly with the time exposure and the wages or as a product of the time and the wages which product is the payroll. The payroll does not reflect either the time or amount limitations on the medical benefits.

The trend in Compensation has been toward raising the limits on the time period and the maximum amount of medical benefits. The present Acts come near to providing unlimited medical benefits thus tending to make the medical losses vary with the payroll.

The exposure in Compensation measured in payroll may be said to vary reasonably with the losses. From the practical viewpoint the payrolls form a desirable medium for measuring exposures. The need of payroll records for internal business administration and for reports for external agencies emphasizes their importance thus serving as an incentive to accuracy.

There is a correlation between payroll and losses in Employers' Liability although this is not so definite as in Compensation. The loss of wages resulting from an accident is a major factor in evaluating injuries whether by direct settlement or through court procedure. Workmen's Collective, in responsiveness of payroll to losses, stands somewhere between Compensation and Employers' Liability, the exact position depending on the limitations on payments written into the Workmen's Collective contract. For both Employers' Liability and Workmen's Collective, payrolls form practical media for measuring exposures.

2. Restricted Payroll. This term is used to denote ordinary payrolls after they have been modified by limiting the maximum weekly wage for any employee to an amount which when multiplied by the weekly percentage compensation rate will equal the maximum weekly payment provided in the law. Under the early, simple Compensation acts the restricted payroll exposure would have varied directly with the indemnity losses aside from the limit as to total amount of the case. At no time would it have accounted for a time limitation on the total amount, for either time or amount limitations on medical, or for the additional hazard involved in overtime work. A further obstacle arose when Compensation laws introduced different weekly limits dependent on the nature of injury or the dependency status. Such restricted payrolls would impose additional records on the assured, would involve more detail in auditing, and as they are not used for other purposes would not provide an external check.

In Employers' Liability Insurance restricted payroll is meaningless as there is no fixed wage that has a definite relation to the award. In Workmen's Collective Insurance the relationship would depend upon the agreement in the contract; probably it would be very much as in Compensation.

3. Man-Year. To measure Compensation exposure in man-year units—the exposure of one man for one year—would not reflect any variation in wages and for that reason would not be expected to vary as constantly with the losses as payroll exposure. In some occupations, e.g. aviation, where wages are so high that in almost every case the maximum weekly payments are made, a man-year exposure medium is more responsive to the indemnity losses than payroll. To account for differences in hazard due to a variation in the number of working days per week or the number of working hours per day, it is necessary to define the man-year unit in terms of man-days or man-hours. This would introduce no special difficulty aside from making the records somewhat more involved. In Employers' Liability there is presumably less correlation between wages and amount of award than in Compensation. For this reason the man-year medium as a basis of premium would be less objectionable here. It could not be expected to vary as constantly with the losses as payroll however. Man-year exposure would probably serve reasonably well in Workmen's Collective insurance, for the weekly payments usually do not vary much and the fixed amounts paid for permanent injuries do not depend on the wages received by the injured.

This exposure medium is the one best adapted for measuring accident frequency or weighted accident severity. Exposure on a man-year basis would be more difficult and costly to obtain as special records would have to be maintained for this purpose. These would not have the general importance of payroll, would probably be less accurate, and would not be subject to check from external sources.

4. Value of Product. The value of product or sales receipts is another medium that has been considered for measuring exposures. It would be difficult to ascertain to what extent the exposure on this basis would vary with the losses. A new system of classification would be required if this medium were used as it would be necessary to recognize the relative degree in which machines enter not on account of the hazard difference between machine and non-machine operations but on account of the additional exposure as measured through this medium, due to the greater production of machines. In these new classifications it would also be necessary to note the degree in which the raw and partially treated materials enter into the process. In insurance for contractors it would be necessary to distinguish between contracts covering all material and cost-plus contracts. The new classifications would have to be on an industrial basis so as to include those employees not engaged in producing a salable product. Some of the present classifications, e.g. public employees, would require some other procedure. In Employers' Liability and Workmen's Collective the same difficulties arise that appear in Compensation. This exposure medium wherever it could be used at all would generally be readily available and subject to check. Measuring exposure on this basis would not require undue effort.

There are certain factors whose existence is now more or less recognized as affecting the losses which are not reflected in any of the media for measuring exposure. The increase in the accident frequency during industrial prosperity and an increase toward malingering during depressions are factors of this nature.

III. Coverage to Assured for His Liability for Injuries to the General Public.

1. Manufacturers' and Contractors'.
2. Owners', Landlords' and Tenants'.
3. Elevator.
4. Teams.
5. Automobile.
6. Airplane.
7. Product.
8. Protective.

The injuries covered under Public Liability Insurance are those sustained by the public while on and/or off the premises from accidents arising out of conditions for which the assured is responsible. The hazards are peculiar to each of the several lines of insurance falling under this division. With respect to one element underlying the hazard, however, these lines of insurance are similar. That element is the evaluation standard used in reducing the severity of the injury to losses. In all Public lines the monetary measure of the injuries is determined by voluntary agreement or by court procedure. The attitude of the community and of the legal profession is an important factor in deciding whether there will be many requests for settlement of trivial, no-liability or even fraudulent cases, whether there will be voluntary settlements, or whether there will be lawsuits. If the last alternative is chosen, the jury selected from the community will determine the money value of the injury and any liberal or conservative viewpoint of the judges construing the law will be reflected in the losses. The attitude of the Community, the Bar, and the Court presumably will be reflected equally in all classifications, if not in all lines. No attempt is made to account for variations from these sources in selecting the exposure medium. These variations in losses are assumed to occur by districts and are provided for by establishing territorial differentials wherever there is a measurable deviation based on adequate data.

There is another factor underlying the evaluation standard which requires different treatment as it is not subject to territorial differentials. This factor arises out of the maximum limits imposed on the amount per injured and the amount per accident. These excess limits will be designated by *a* and *b* respectively. The lines of insurance and also the classifications within lines are divided into a few groups according to the excess hazards. It is assumed that within each group the aggregate losses for any classification when evaluated with a/b limits will have a fixed relativity to the aggregate losses when evaluated under standard (5/10) limits. No effort is made to reflect variations due to different limits in selecting the exposure medium. The rates are quoted for unit exposure on a standard limit evaluation basis. If other limits are desired, the basic rate is modified by applying to the standard limit rate the excess factor corresponding to the desired limits which is taken from a table constructed for each group of excess hazard. The evaluation standard including excess limits has been eliminated in discussing premium bases for these lines of insurance.

1. MANUFACTURERS' AND CONTRACTORS' PUBLIC LIABILITY INSURANCE.

The hazard in this line arises from the contact of the public with the critical conditions of the assured's premises and operations. The problem here is to select an exposure medium which varies with the critical conditions and the number of the public who, by entering and passing, are subjected to the assured's critical conditions. Some of the exposure media that might be given passing consideration are:

1. Number of Public Admissions.
2. Payroll.
3. Man-Year.
4. Area and Frontage.
5. Value of Product.

1. Number of Public Admissions. It would be expected that the number of injuries and their cost would vary with the number of the public who enter or pass the premises or the place of operations and that therefore this number would make a good medium for measuring the exposure. An overpowering objection is that there is generally no record available and that it is quite impractical, if not impossible, to get one.

2. Payroll. Presumably the number of the public desiring admission to the manufacturer's and contractor's premises and place of operations, within a given classification, varies with the size of the plants or operations as roughly measured by the number of employees or the payroll. Payroll exposure responds to the decrease or increase of the aggregate losses brought on by periods of depression and prosperity in industry. It has the practical advantage of being based upon long established records necessary for other purposes, so that it may be readily obtained and checked. The use of payroll records adds to efficiency for, as many risks are insured for Compensation and Public Liability by the same carrier, the same audit may be used for each of these lines.

3. Man-Year. A man-year exposure has the same merits that have just been ascribed to payroll as regards responsiveness to the variation in losses. It is not as practical as payrolls for it is necessary to establish a special record for measuring this exposure. Under present conditions at least this special record could not be used for determining Compensation exposure.

4. Area and Frontage. An area and frontage medium—area of assured's premises and length of premises adjoining public ways—might be used for measuring exposure in Manufacturers' Public Liability Insurance. It would be expected that the number of employees, the payroll, and the size (area and frontage) of the plant would vary in about the same ratio. The area and frontage medium would not respond to variations due to depressions and prosperity. This medium is practical in application as the exposure of manufacturing plants on this basis could be accurately determined with reasonable effort. It would not serve the dual purpose of measuring Compensation exposure at the same time.

The area and frontage basis is poorly adapted to measuring exposure for Contractors' Public Liability Insurance. Area and frontage exposure does not vary and it is fitted to measure only exposures which are continuous and constant. The exposure for a location under Contractors' Public Liability is variable. It begins below average, increases to above average, and then decreases, often tapering off to almost zero.

5. Value of Product. As the value of the product of a given manufacturing establishment reflects the activity it seems not unreasonable to expect that this product value might serve as a medium for measuring Public Liability hazard. This exposure medium, as already stated under Compensation, would require some rearrangement of classifications, it would vary with the total losses under industrial depressions and prosperity, and it could be obtained readily from available records and checked.

For Contractors' Public Liability the equivalent of the value-of-product exposure medium would be the amount of the contract. Within each classification this may be expected to vary roughly with the payroll or with the man-year exposure. A rearrangement of classifications, taking into consideration the extent to which the cost of material is included in the contract, would be required. Exposure measured through this medium could be readily obtained from available records and checked.

2. Owners', Landlords' and Tenants' Public Liability Insurance.

Owners', Landlords' and Tenants' Public Liability Insurance is the term used for public liability insurance on assured's premises, other than Manufacturers' and Contractors', and Elevator Public Liability Insurance. As in Manufacturers' and Contractors' Insurance the hazard arises out of the contact of the public with the critical conditions of the premises. Presumably the classifications have been refined so that critical conditions are similar and uniform within the classification. The variations in conditions from classification to classification are so large, however, that no one exposure medium is adapted to all. The exposure media that will be considered are:

1. Area and Frontage.
2. Number of Admissions.
3. Receipts, Admission Charges.
4. Seat-Year.
5. Sales.
6. Rentals.
7. Payroll.
8. Unit-Year.

1. Area and Frontage. This is a dual basis of premium to account separately for the hazard which may be associated with the area of the premises and that which is related to the frontage along public ways. If the nature of the premises is such that there is no frontage, then the hazard there is zero and the dual exposure becomes a single exposure based on area alone. Presumably the accidents vary with the critical conditions and the number of the public coming in contact with them. Within the same classification the accidents probably vary somewhat directly with the inner area and the linear frontage on public ways. On this assumption, area and frontage exposure may be considered to vary directly with the hazard. This exposure medium is better adapted for hazards that are continuous and uniform, or, if varying by seasons, that average about the same from year to year. It is applied to classifications covering buildings of all kinds, signboards, country estates, cemeteries, etc., where there is little variation in critical conditions or in the number of people exposed year after year. The impossibility of concealing exposure on this basis and the facility with which it may be accurately determined give it an important practical advantage.

2. Number of Public Admissions. The hazard due to the number of the public subjected to the conditions of the assured's premises varies directly with the number admitted to the premises. Considered from this viewpoint this medium is a better measure of the exposure than area. It does not account directly for any outside frontage hazard and is adaptable only where the frontage hazard is negligible or bears a fixed ratio to the area hazard within the classification. It is responsive to changes in hazard due to depressions and periods of prosperity.

The number of admissions can be secured in a practical way in only a few classifications. At present this basis is used only for baseball parks. It might be used in amusement parks, theatres, concert halls, bathing pavilions, restaurants, skating rinks, dance halls, and public museums with turnstiles or admission charges.

3. Receipts, Admission Charges. The receipts vary with the number of admissions and thus with the hazard. Like the number of admissions, this exposure medium reflects depression and prosperity. In classifications where there is no great range in prices this exposure medium might do very well. Where losses do not vary with the admission charge or where they may vary inversely to the charge, the medium is not so well adapted as the number of admissions. This basis is practical for certain classifications where the amount of receipts is more readily ascertained than the number of admissions and where the receipts are subject to check. It is used, at present, in concert halls, stadiums, bathing pavilions, skating rinks, and dance halls. It might be used also in baseball parks.

4. **Seat-Year.** In certain classifications that have a continuous exposure throughout the year, or, if variable, a consistant average hazard from year to year, the number of seats forms a reasonably good measure of exposure. This exposure medium does not respond to a temporary decrease or increase in hazard like the number of admissions or admission receipts. The basis is used for theatres and moving picture houses having regular shows where the number of people exposed during the year bears a reasonably constant ratio to the number of seats. Conceivably it might be used for commercial baseball parks and concert halls but it would not give much responsiveness to losses, for there is a large variation in the number of persons exposed within these classifications, at least as these are constituted at the present time. The exposure on this basis may be readily determined and cannot be concealed for fraudulent purposes.

5. **Sales.** The total receipts from sales might possibly be used as a measure of exposure with some of the O. L. & T. classifications. This medium would require a readjustment of some of the present classifications to make it applicable, and to some it could not be applied at all. In classifications like retail stores of all kinds (when properly subdivided), restaurants, hotels, etc., this premium basis might be used. The public liability hazard would be expected to vary with the number of patrons or purchases and these in turn with the amounts purchased. This exposure is readily ascertained for classifications involving sales.

6. Rentals. As the area is a fair measure of the hazard in connection with buildings, it would seem that the rentals of a building might be used as an exposure medium for buildings where all space is leased. The use of this medium would require some readjustments in those classifications where it is applicable, as the better buildings, which may be expected to have the higher rentals, would have the lesser critical conditions and consequently a smaller hazard. This exposure basis would be practical for only a limited number of building classifications.

7. Payroll. The risks of some of the O. L. & T. classifications have payrolls large enough to be reasonably stable. The payrolls of such risks would vary with the size of the risk when this is measured by other than payroll standards, and might be expected to be responsive to the losses. In classifications like stores, hotels, restaurants, etc., with possibly a few subdivisions, the payrolls would vary reasonably with the number of the public coming in contact with the critical conditions and might be used as an exposure medium.

8. Unit-Year. There are premises that are so nearly identical or that have so small a hazard per unit that for practical purposes all are considered alike. The exposure basis used is the unit-year, which means a flat charge per unit per year. This medium of exposure is simple and practical. It generally applies to things where the total hazard is small. This basis is used at the present time for automatic vending machines, bowling alleys, canoes, tennis courts, dogs, where these are additional hazards to insured premises.

Miscellaneous. There are certain classifications in which the hazard varies so widely within the class that it is impossible to select any practical medium as a reasonable measure of the hazard involved. Items coming under this designation are usually considered individually and a flat charge is given after the factors underlying the hazard have been considered in each individual case. Such flat charges apply to parades, pageants, races, celebrations, etc.

The preceding exposure media for O. L. & T. Liability Insurance may be divided into two divisions according to whether they measure the exposure prospectively or retrospectively. Area and frontage, seats, and unit-years measure the exposure prospectively, while the number of admissions, admission charges, receipts, and rentals measure it retrospectively.

3. Elevator Public Liability Insurance.

The hazard covered in Elevator Public Liability Insurance arises out of the contact of the public with the critical conditions of the elevator. Presumably this hazard varies somewhat jointly with the critical conditions and the number of public passengers. The hazard also varies with the amount of use of the elevator and the efficiency of the operator. The latter, though probably one of the major factors affecting accidents, is not directly considered in selecting the premium basis. Operators are either considered unfit and rejected or considered qualified and accepted without further gradation. The elevators within classifications are graded to some extent through merit rating for special safety devices. Through proper equipment of elevators and selection of operators it is assumed that the critical conditions are approximately the same for individual elevators of a given classification. The possible exposure media are very limited.

1. Number of Passengers. Use of the number of elevator passengers as an exposure medium would give a variation reflecting continuity of use, and to a limited extent congestion, for the hazard in congestion increases in a larger degree than the increase in passengers. Whatever merit the medium may have in responsiveness to hazard is quite offset by the impracticability of getting an accurate measure of the number of passengers in elevators generally.

2. Elevator-Year. The elevator-year exposure medium does not reflect the number of passengers carried, continuity of use, capacity of elevator, average load, congestion, or the efficiency of the operator. It assumes that within a given classification, elevators are equipped approximately equally and average about the same year after year in the passengers carried. This basis is practical and it is in universal use at the present time for measuring the elevator exposure.

It is conceivable that in the modern large building the total elevator hazard might be measured by the factors and conditions used by building engineers to determine the number, the capacity, and the location of the elevators. From these conditions an exposure for the building independent of the number of elevators might be obtained.

4. TEAMS' PUBLIC LIABILITY INSURANCE.

Some of the critical conditions contributing to the hazard covered in Teams' Public Liability Insurance are:

1. Traffic congestion.
2. Nature of the operations.
3. Day or night operations.
4. Accessibility to public.
5. Efficiency of driver.
6. Demeanor of teams.

These are not all independent. The first four are interrelated and some would consider the fifth and sixth as interrelated. Variations in hazard for the first may be accounted for by territorial differentials and for the second and third by classifications. The fourth, fifth and sixth are assumed to be equal for different assureds of the same class. There is only one exposure medium that has been considered practical for application to Teams' generally.

Team-Year. This medium does not respond to any variation of hazard due to continuity of use during the year or the amount of daily use. It assumes that within classifications and territories these average about the same. It does not respond to differences in individual drivers aside from the group differences reflected in classification experience. The exposure medium is simple and its magnitude is readily ascertained.

No other practical medium has been evolved. Mileage, team-day, or team-hour media while responsive to certain variations in hazard are obviously impractical. Driver payrolls might possibly be used in a few classifications where risks have a large number of teams and drivers. Receipts might serve as a basis for risks of a trucking nature. All these media however are impractical for general application to Teams'.

5. Automobile Public Liability Insurance.

Some of the critical conditions that contribute to the hazard covered by Automobile Public Liability Insurance or that cause deviations in this hazard are:

1. The car—age, condition, etc.
2. Highways—road beds, curves, visibility, etc.
3. Traffic density.
4. Laws, regulations, and their enforcement.
5. Efficiency of driver—age, experience, habits, impairments, etc.
6. Mileage.
7. Speed.
8. Weather conditions.
9. Seasonal use of car.
10. Day and/or night use of car.

These are not to be considered a complete list, nor are they to be considered as independent of one another. Too little is known as yet about them to appraise the importance of each. From a casual survey, however, it would appear that (2), (3), and (4) are subject to treatment, if necessary, by territorial differentials. Any appreciable differences in (1) can be corrected through classification of cars. The degree to which (5) affects the hazard is not definitely known. It is probably one of the most important factors enumerated. It is generally recognized that the extremes in age, lack of self-control, and definite impairments disqualify a driver. The effect of the variation in hazard of accepted drivers due to the range of these qualities within accepted limits is not sufficiently known to be considered in determining exposure: The introduction of experience rating is an approach to recognizing these differences. It is generally accepted that hazards would vary approximately with the mileage, other conditions being the same. The extent to which the (7), (8), (9), and (10) contribute to the hazard is unknown.

Among the conceivable exposure media these might be considered:

1. Car-Year.
2. Mileage.
3. Car-Hour.
4. Fuel-Consumption.
5. Payroll.

1. Car-Year. This premium basis does not reflect the continuity of use or the total use of the car. Obviously, other conditions being equal, the hazard will vary with the total mileage of the car. The assumption underlying this basis is that, with proper classification of cars, the differences in the hazard are not large enough to warrant introducing a more involved exposure medium. The merit of this medium is its simplicity and definiteness in measurement which make it difficult to impose fraudulent exposures.

A variation of the car-year unit might be a car-life in which a car would be insured for life at a definitely fixed amount which would be incurred at the beginning, though not necessarily paid in one payment. Conceivably this would serve to promote care and safety as the long use of cars would mean insurance at low cost. It is also possible that such a plan would be adverse to public welfare by keeping old and unsafe insured-for-life cars on the highways. This premium basis probably would not appeal to an installment buying age. It would also fall heavily on car owners who lost their cars early.

2. Mileage. The mileage exposure medium is superior to the car-year medium in yielding an exposure that varies with the hazard, as it responds more to the actual usage of the car. The devices and records necessary for the introduction of this medium make it impractical under present conditions.

3. Car-Hour. A method that would measure exposure by the number of hours the car was operated, i. e., with the motor running, would yield a variation for use of the car, though probably not so responsive as mileage. This medium, however, is even less practical than mileage.

4. Fuel-Consumption. The quantity of fuel consumed as an exposure medium would reflect a variation in the use of the car under similar road conditions. It would, however, penalize the car on country roads as compared with the car on pavements whereas the hazards are just the reverse. Like the two preceding exposure media this would require such an accounting system and other devices that it becomes impractical under present conditions.

5. Payroll. Use of driver payroll as an exposure basis for assureds where several drivers use a variable number of cars responds roughly to the usage of the cars, as wages are paid only for the drivers necessary to keep the cars in use. This basis is somewhat akin to a driver-year basis. There are a few classifications where assureds have several drivers for which this is practical.

This discussion of automobile exposure media has been directed to private passenger and commercial cars. If the passenger hazard of public automobiles is considered, the capacity of the car becomes an important factor. As possible exposure media for the passenger hazard of public automobiles *number of passengers*, *passenger-mile*, and *receipts from fares* should be considered.

The introduction of a mileage, car-hour, or fuel-consumption exposure into rate making would require the prior development of experience on these media. The car-year is the only one of the enumerated media which measures the exposure prospectively, the others require a final adjustment which would be determined retrospectively.

6. Airplane Public Liability Insurance.

In this line of public liability insurance, as in Automobile Public Liability, there exists a natural division of the hazard into passengers and the general public. The hazard of the passengers assumes a greater relative importance than in automobile insurance. Among the more important critical conditions contributing to the hazard are:

1. Plane—type, condition, etc.
2. Use of plane.
3. Capacity of plane.
4. Weather conditions.
5. Topography of country.
6. Efficiency of pilot.

It is hardly to be expected that in this early stage of aviation the available records would be adequate to permit a proper appraisal of these factors. Of these conditions, (1), (2), and possibly (3) may be considered subject to treatment through refined classifications. Conditions (4) and (5) might be recognized to some extent by territorial differentials. The variations in hazard due to them might possibly be somewhat equalized through regulations. The distances covered by planes obviously decreases the effectiveness of territorial differentials. Condition (6), which is probably the most important of all, is not considered after pilots have been approved.

The possible exposure media that will be considered are:

1. Plane-Year. In using this medium for exposure it is assumed that within the classifications the hazard of the planes will average about the same over the period of a year. This medium does not respond to variations in the use of the machine. Accidents presumably vary somewhat with the extent of use of the machine. This medium is simple in application and quite practical.

2. (a) Flying-Hour, (b) Mileage. These media are in some respects similar. Both reflect the use of the machine and probably are more responsive to the losses than the plane-year. They are not as simple in application as the plane-year though they are not as impractical as the corresponding bases for automobile exposure.

3. Number of Flights. Should experience reveal that the hazard connected with the take-off and climb of a flight and the descent and landing is considerably greater than that during the intervening period, then the number of flights might be more responsive to the losses and a better medium for measuring exposure than either of the preceding media. In simplicity this ranks below the plane-year but above either flying-hour or mileage media.

4. (a) Passenger-Hour, (b) Passenger-Mile, (c) Fare Receipts. These media, which are somewhat related, are responsive to the public passenger hazard. They do not respond directly to the hazard of the general public. Although not as simple in application as the plane-year, they are not impracticable, in view of the records available.

5. Number of Passengers. Should the conditions referred to under medium (3) prevail, then the number of passengers carried would be more responsive to the public passenger losses and a better medium for measuring exposure than passenger-hour, passenger-mile, or fare receipts. This medium is not responsive to the hazard of the general public. In simplicity of application it ranks with fare receipts.

7. Product Public Liability Insurance.

Product Public Liability Insurance covers the liability of manufacturers for accidents to the public, arising out of their products. The critical conditions consist in defects in the products, including packing. If the products have been divided into homogenous classifications it may be expected that the critical conditions are somewhat uniformly distributed. These exposure media will be considered:

1. Quantity of product.
2. Units of product.
3. Sales.

1. Quantity of Product. The hazards within a homogeneous class may be considered to vary with the volume on the assumption of a uniform distribution of critical conditions. This quantity exposure medium is probably the best basis in its responsiveness to the hazard. It is not as readily ascertained however as the cost or sales receipts of the products.

2. Units of Product. In responsiveness to hazard this exposure medium stands between quantity of product and sales receipts. It does not reflect variation in hazard due to different sizes of the units within the same classification. The measure of the exposure on this basis for most classifications is not as readily ascertained as that based on the quantity or the value of the product.

3. Sales. An exposure expressed in the medium of receipts from sales would vary approximately with the hazard, for there is a direct relation between sales receipts and volume. If the classifications contained wide variations, the high-priced as compared with low-priced goods would be penalized, for it would be expected that the more costly articles would be the better prepared and the less hazardous. The basis, however, is quite practical, as accurate sales records are essential to sound administration and are found in every line of business.

8. Protective Public Liability.

This coverage is given to owners, landlords, tenants, and contractors for their liability for injuries to the public on premises or operations which have been leased or contracted to others. The critical conditions and injurable objects are generally the same here as under the direct public liability of the lessees or sub-contractors. It is assumed that this secondary liability bears a constant ratio to direct liability and it follows that the exposure media should be the same as under direct liability. This is the procedure followed at present for Landlords' Protective Liability and Tenants' Protective Liability.

In Owners' or Contractors' Protective Public Liability a different exposure medium is used. As the coverage extends to injuries in connection with all material as well as the actual building operations it is believed that the use of the total cost of labor, material, and equipment as exposure medium gives greater responsiveness to losses. It is also recognized that this is in part a defense policy against attack on the owner or contractor in case the financial position of the party assuming direct liability precludes his paying a large verdict. This defense element of the hazard decreases as the financial position of the party assuming direct liability increases, or generally as the size of the contract increases. The ratio of the total hazard under protective liability to the total hazard under primary liability decreases with an increase in the size of the contract. As there is no practical expression which represents such a function, an approximation is made through graded charges, i. e. by charging one rate for a cost up to a fixed amount, then a smaller rate up to another fixed amount, and thereafter a still smaller rate. This is equivalent to decreasing the magnitude of the exposure by a fixed ratio in the second and third intervals. Such graded charges also might be applied to other media, e. g. payroll.

IV. Coverage for Liability for Professional Acts.

1. Physicians and Surgeons, Dentists, Optometrists, and Druggists.

2. Hospitals.

Under this form of insurance the injuries of clients arising out of the professional acts of the assured are covered. The critical conditions consist in defects in the material, errors in treatment, negligence, or lack of ability of the assured. It is obviously most difficult, if not impossible, to get an exposure medium responsive to all these factors. As the hazard is rather small it is not practical to have a complex exposure medium. The following are considered for measuring exposure:

1. PHYSICIANS AND SURGEONS, DENTISTS, OPTOMETRISTS, AND DRUGGISTS.

   Man-Year. This medium for measuring exposure like unit-year in other lines based on unit-years is chosen primarily because of its practicability. The underlying assumption is that professional men within the admitted class do not vary enough from the average to make it advisable to adopt either refined classifications within a profession or to select a more responsive but less practical exposure medium. This basis which is quite practical is used for physicians and surgeons, dentists, optometrists, and druggists. In drug stores there are, in addition to the first charge on the store, supplementary charges for additional employees, making the exposure vary somewhat with the volume of business.

   Other media for measuring exposures that might be considered are: *number of treatments*, *number of patients*, and *professional income*. Each of these media lacks in complete responsiveness to the hazard and requires additional records. In view of the small hazards these media are considered impractical.

2. HOSPITALS.

   Bed-Year. It is apparent that variations in the total hazard between small and large hospitals are too large to be left unrecognized. The bed-year medium for hospitals provides a premium basis which reflects directly the difference in the size of the hospital and indirectly the number of treatments or the number of patients. The magnitude of this exposure is readily obtained, making its use quite practical. There are other conceivable media like *number of patients*, *income* for non-charitable hospitals, *number on staff*, or *payroll* of the hospital. Considering both responsiveness to hazard and practicality, these media just mentioned are deemed inferior to hospital *bed-years*.

INCURRED BUT NOT REPORTED CLAIM RESERVES

BY

THOMAS F. TARBELL

The subject of reserves for incurred but not reported claims has received very scant consideration in our *Proceedings*, nor is there available to the writer's knowledge any written material of consequence on either the theoretical or practical aspects of the subject.

A Committee of the Association of Casualty and Surety Accountants and Statisticians studied the subject in 1927 as respects the fidelity and surety lines and submitted a report recommending that such reserves be determined as a function of premiums in force. Specifically, the Committee recommended the following minimum percentages of in-force premiums: fidelity—10%; surety—3.5%.

The writer, in conjunction with his office associates, has given considerable study to this subject during recent years and has maintained numerous records designed to aid in the calculation of this particular reserve liability. While no claim is made that an entirely complete solution has been reached, the results of our methods have been so generally satisfactory as to encourage a discussion of the subject before this Society.

For the purposes of this paper an incurred but not reported claim is defined as a claim arising out of an event or accident which occurred on, or prior to, a certain date, but notice of which was not received by the home office of the company until after such date. The date we usually associate with this definition is December 31, since this date is of particular significance from the annual statement viewpoint. Unless otherwise stated, the subject will be considered from the standpoint of this date.

The definition submitted is inclusive and specific and covers all situations and practices, in that the governing condition is the fact of notice of the claim being received or not received on or before the particular date. It is assumed that all notices received as of the particular date will be recorded as of such date, although the actual physical recording may take place at a subsequent date —i.e., that notices received up to and including December 31 will be recorded as December notices, although the actual recording may not be completed until the first day or two of January.

It is the opinion of the writer that the problem of incurred but not reported claim reserves is essentially actuarial or statistical. Of course a certain part of the reserve can be determined from notices of prior accidents received after the close of the year up to and including the date of closing the annual statement records, but for most lines of business this period is ordinarily too short to produce more than a small part of the reserve and the amounts so determined must be supplemented by additional amounts determined from experience or judgment.

The fundamental principles underlying the establishment of reserves for incurred but not reported claims involve the use of the experience of the immediate past, modified to reflect the effect of current conditions or trends upon such experience. By the experience of the immediate past is meant the amount of incurred but not reported claims of the preceding year developed down to the end of (or for the first eleven months of) the current year, modified, if necessary, by a factor to project such claims to an ultimate basis, and it is assumed that such a record is available. It is not material how this record is maintained. The basic data may be obtained by keeping an itemized record of all such paid cases plus reserve values of all such outstanding cases brought down to the end of November or December, or in the aggregate by recording a symbol on the paid punch cards to indicate an incurred but not reported case and a similar ear marking of outstanding cases whether or not these are recorded on punch cards.

For the more important lines of business the method followed by the company with which the writer is associated is to keep such record on an aggregate incurred loss basis. A card is punched for each notice of loss or accident and the cards for those cases with date of accident December 31 or prior and reported subsequently carry both the accident year and the report year. The card shows the original estimate. For every subsequent change two additional cards are punched, one charging up the changed estimate (or final amount paid) and the other crediting the last previous estimate. The record is maintained on this basis for compensation, automobile liability and liability other than auto. For the other casualty lines only the original estimates are recorded; changes are disregarded. It has been found that for such lines the original estimates produce a satisfactory reserve—in the aggregate slightly redundant. The record of the incurred but not reported claims is tabulated monthly on an accumulative basis.

Current factors affecting past experience are:

(1) Comparative volume of exposure
(2) Comparative accident frequency
(3) Comparative average notice or claim costs

The Committee of the Association of Casualty and Surety Accountants and Statisticians attempted to reflect the effect of current conditions by basing the reserve for incurred but not reported fidelity and surety claims on the volume of business in force. The method should produce satisfactory results provided the percentages reflect previous experience and there is no change in accident (or claim) frequency, or in average claim cost, but these factors do not remain constant for many of the casualty lines over any considerable period.

It has also been contended that the incurred but not reported reserve may be determined as a function of the reserve for known cases. This is more or less correct for lines of business where the average claim is small and varies within rather narrow limits and if, further, claims are liquidated rather speedily—such for example as automobile property damage and plate glass—but does not apply to the major casualty lines, compensation, auto liability and other liability, where there is a lag in the liquidation of claims. For the major lines the reserve will be too low if the volume of business is increasing and conversely, if the volume of business is decreasing the reserve will be too high.

It has, therefore, been our theory that having determined the amount of reserve for incurred but not reported claims for the previous year, the reserve for the current year may be determined by modifying such amount by those factors which most nearly reflect the modifications required in the light of current conditions—change in volume of business, change in accident frequency and change in average notice (or claim) cost. The comparative number of notices reflects not only change in volume of business, but change in accident frequency. The trend in claim cost or claim severity is reflected in the average notice cost.

The general method may be expressed in formula form, as follows:

$$\text{Reserve} = \frac{N^{y}_{10-11-12}}{N^{y-1}_{10-11-12}} \times \frac{C^{y}_{10-11-12}}{C^{y-1}_{10-11-12}} \times I^{y-1}_{(1)-(12)}$$

Where N = number of notices

C = Average incurred cost per notice

I = Amount of incurred but not reported claims

y designates the current calendar year

$y - 1$ designates the previous calendar year

Subscripts designate calendar months

It will be noted that the comparative number of notices and average notice costs are based upon statistical data for the last three months of each calendar year. This is purely arbitrary and should be varied according to the volume of the particular line of business being dealt with. The period should be such as to include sufficient statistical data to produce dependable results. Obviously, the theory of credibility may be applied to the problem. The basic factor—the amount of incurred but not reported claims at the end of year $(y-1)$ as disclosed by developments during year (y)—embraces a full year's development of claims modified to an ultimate basis, if necessary, in order to produce as accurate an ultimate incurred amount as possible.

The formula as stated contains three factors. It is quite obvious that it can be reduced to two factors:

$$\text{Reserve} = \frac{A^{y}_{10-11-12}}{A^{y-1}_{10-11-12}} \times I^{y-1}_{(1)-(12)}$$

Since $N \times C = A$ (The amount of incurred losses)

It is desirable, however, as will be brought out later, to provide for the determination of both the N and C factors.

It is at once apparent to anyone who has dealt with the practical aspects of the problem that the foregoing is not an inflexible formula to be applied without modification to each casualty line. It should rather be considered as a formula which furnishes an approach to the desired result rather than the result itself. The formula is not applicable to lines having a low accident frequency and a large factor of variation in average claim costs, such for example as death and dismemberment claims under personal accident policies, burglary, boiler and machinery; but for most other lines and coverages, provided the volume of business is substantial enough, the formula can be used without material modification. As a rule the formula can be used without much modification for accident and health indemnity claims, compensation, plate glass, auto property damage, auto collision and miscellaneous property damage and collision.

For auto liability, other liability, fidelity and surety it will frequently be desirable to amend the average cost factors by eliminating any abnormal claims. No fixed rule can be given as the limitation depends upon volume of business and the effect of one or more large claims upon the average costs. Notice averages should be developed on a net retention basis and if a company reinsures liability losses in excess of standard limits and its net retention on fidelity and surety business is comparatively low, no modification of averages will, in general, be necessary.

In the case of death and dismemberment claims under accident policies, burglary, boiler and machinery, there is probably no better method of determining the incurred but not reported reserve than that of accumulating the amounts or estimates on such claims reported during the period immediately following the close of the year with the addition of such amount as a factor of safety as past experience indicates to be necessary.

While the above formula is designed primarily for the determination of the reserve at the end of the year, it may be used with certain changes for the monthly reserve during the following year. It is frequently desirable to make such modification so that in event of changes in volume of exposure, accident frequency and accident severity, any change from one year-end to the next may be reflected gradually rather than abruptly.

Since the formula at the end of the next calendar year will be

$$\text{Reserve} = \frac{N^{y+1}_{10-11-12}}{N^{y}_{10-11-12}} \times \frac{C^{y+1}_{10-11-12}}{C^{y}_{10-11-12}} \times I^{y}_{(1)\text{---}(12)}$$

it follows that the formula for the end of any month of year $(y+1)$ will be

$$\frac{N^{y+1}_{(n-2)-(n-1)-n}}{N^{y}_{10-11-12}} \times \frac{C^{y+1}_{(n-2)-(n-1)-n}}{C^{y}_{10-11-12}} \times I^{y}_{(1)\text{---}(n)} \times P_n$$

Where n designates the calendar month of reserve,

$$I^{y}_{(1)\text{---}(n)}$$

the incurred but not reported claims reported to end of month n, and P_n the factor based upon experience necessary to project $I^y_{(1)\ldots(n)}$ to an ultimate basis.

The foregoing formula reduces to

$$\frac{A^{y+1}_{(n-2)-(n-1)-n}}{A^y_{10-11-12}} \times I^y_{(1)\ldots(n)} \times P_n$$

It is obvious that considerable judgment must be exercised in using the results obtained by this formula, since the factor $I^y_{(1)\ldots(n)} \times P_n$ is not subject to accurate statistical determination. In fact it has been found in practice that for the first three months of the year it is better to substitute for this factor the factor $I^{y-1}_{(1)\ldots(12)}$.

For such lines of business as automobile property damage and plate glass where the average claim costs do not vary materially during a twelve months period, the variation in the monthly reserve for incurred but not reported claims will be satisfactorily reflected for practical purposes by disregarding the change in average notice cost; that is, by use of the formula —

$$\frac{N^{y+1}_{(n-2)-(n-1)-n}}{N^y_{10-11-12}} \times I^y_{(1)\ldots(n)} \times P_n$$

In conclusion the writer wishes to emphasize the fact that the formulae presented and discussed are not put forward as furnishing a complete solution of the problems under consideration, but it is believed, as a result of experience, that they may contribute to at least a partial and in many cases a satisfactory solution of a difficult problem which admittedly is not susceptible of accurate solution.

VOL. XXI, PART I

No. 43

PROCEEDINGS

NOVEMBER 22, 1934

A SURVEY OF RISK CREDIBILITY IN EXPERIENCE RATING

PRESIDENTIAL ADDRESS AT TWENTIETH ANNIVERSARY,

PAUL DORWEILER

Anniversaries suggest retrospection. While meditating on the Twentieth Anniversary of our Society, it seemed fitting to select a subject that is peculiar to casualty insurance, that has received serious attention and study from our members, and that has served as the instrumentality through which some real contributions have been made to that body of knowledge which we hope to enlarge and organize so that we may properly call it *Casualty Insurance Actuarial Science.*

A survey of the casualty insurance field will reveal many places where pioneering efforts have resulted in distinctive contributions. Among those of direct interest to actuaries may be cited the development of coverages and premium bases, the devising of statistical systems within the carriers, the organization of central bureaus and boards for collecting and compiling the carriers' data, the formulation of methods for reducing these data to uniform (basic) levels, and the development of weighting systems giving credibility on quantitative bases so that the actual experience of individual classifications and risks may receive proper recognition. Each of these might be a fitting subject for this anniversary occasion. I have selected the last because it is almost exclusively actuarial in nature, and because the largest and most distinctive contributions to casualty actuarial knowledge have been made here. It is my intention to confine myself to a particular phase of the broader subject of the credibility of experience by limiting my remarks to the credibility of the experience of the individual risk while making a brief review of the development of credibility in experience rating in compensation insurance. I have selected the compensation field because experience rating was originally developed here and then adapted to other lines and because the only available data for checking results are found here. It is not my purpose to treat the principles and practices of experience rating at length. I intend merely to consider some developments of the past pertaining to the credibility of the individual risk experience and suggest some further studies.

A review of the *Proceedings* will reveal several thoughtful and forward-looking papers concerning experience rating of compensation insurance risks in the early volumes—see Bibliography, Appendix III. The resourcefulness of the writers, their comprehensive treatment of the problem, and their boldness in experiment merit admiration even when reviewed after the lapse of more than a decade and in the light of the information acquired during that time. After these pioneering efforts which shaped the general structure of the experience rating procedure, the subject fell into abeyance so far as our Society records reveal. There are only two papers* since Volume IV devoted wholly to a phase of experience rating. Special phases of experience rating have been treated forcefully in letters, memoranda, and discussions by members of various committees of rating organizations. It is hoped that some of the ideas developed may be added to the permanent records in our *Proceedings* and that interest in both the fundamentals and applications of experience rating may be revived.

Definition and Object of Experience Rating

The term "experience rating" as now used refers to definitely prescribed procedures for determining individual risk rates depending in whole or in part on the risk's own experience. Risks whose rates have been determined in accordance with some such procedure are said to be experience rated. The compilation of

* Senior, Vol. XI; Kormes, Vol. XX.

definitions, rules, regulations, formulas, and forms necessary to describe and apply the procedure is called the *experience rating plan.*

The object of experience rating is to determine a more equitable rate for the individual risk based in a degree on the evidence presented by its own experience. It is recognized that individual risks within a classification are not alike and that there exist inherent differences due, for example in compensation, to variations in plants and premises, in operating processes, in the materials involved, in the management, in the morale of employees, in claim consciousness, and in the relation to the community. These differences are of such a nature that it is difficult to label them definitely and they cannot be associated with conditions measurable in advance. It is known, however, that variations in experience do exist in a way that definitely precludes ascribing all of them to chance. Experience rating is considered by many as the most practical method yet devised, or even suggested, of giving recognition to variations produced by such factors.

Basis of Experience Rating

Experience rating is based on the existence of variations in the inherent hazard of the risks which enter into the classification experience. Its object is to measure to a higher degree the hazard of the individual risk by the evidential value of the risk's own experience. This basis needs to be emphasized. If all risks were entirely typical of the classifications, the variation in experience would be purely fortuitous and there would be no place for experience rating; for it would be impossible to reclassify the risks into more homogeneous groups. There are many factors which in different combinations enter into the risk's experience and affect the quality in different degrees. These, at least as yet, can not be classified and recognized so that they may be given individual consideration in rating. They may, however, be reflected to some extent by making use of the effect produced by them as shown in the experience. In the experience rating process, no distinction can be made between similar individual accidents which are fortuitous and those which are indicative of the actual conditions of the risk. The experience of the risk necessarily cannot be divided on such a basis.

Applicability of Experience Rating

Experience rating is applicable wherever there is a large variation among the risks which make up the classification and where the individual risks are of such nature that they may be expected to develop individual risk experiences of appreciable evidential value. Many lines of casualty insurance have classifications somewhat non-homogeneous, resulting largely from the meager experience available and the present lack of knowledge of the elements which enter into the composition of hazards. Considering only the qualification of having atypical risks within classifications, most casualty lines would be subject to experience rating. The further qualification of having individual risk experiences large enough to be of appreciable evidential value is more restrictive.

Compensation insurance, particularly, is subject to experience rating, for to a considerable degree the losses may be controlled and individuality of management reflected in the experience through the employer's ability to correct defective conditions and to enforce safe practices among employees by his potential power to dismiss or to withhold promotions. There are a few other lines, like employers' liability, workmen's collective, and automobile fleet collision, where the assured has similar power to affect losses. In third party insurance, the assured generally cannot control losses to the same degree, for, notwithstanding that the coverage is for liability of the assured only, the actions of the third party, over whom he has no control, affect the losses. In compensation insurance, risks develop individual risk experiences which in some cases have very high evidential value and, because of the control exerted by the management or other factors, often vary widely even within more homogeneous classifications, relative to occupations covered.

Prospective and Retrospective Rating

An experience rating plan in which the experience of the risk is used to determine definite rates for periods in the future is said to be a *prospective* experience rating plan. All plans ever approved for general use have been of this form. A plan in which the experience of a given period is used to determine a final rate to apply to a past period is said to be a *retrospective* plan. Both of these are entirely legitimate plans and represent definite ways of recognizing variations in the inherent hazards of risks. Both kinds could operate simultaneously, and under an economic system of unrestricted competition probably some carriers would select one form and some the other. The same carrier might even use both forms, applying to some risks one form, and to some the other, or it is even conceivable that both forms might be applied to the same risk. *Retrospective* rating would involve some change in theoretical viewpoint, for experience rating as now applied does not depart from the principle of a known rate fixed in advance.

Essentials of Experience Rating

The essential operation of experience rating consists of comparing the risk experience and classification experience on a common premium and loss basis, assigning to the risk experience a weight depending on the size of the risk premium and to the classification experience the complementary weight, and deriving a rate therefrom. The adjusted risk rate or experience rate may be looked upon as a weighted average of the rate indicated as necessary by the losses of the risk and the manual rate, that is, the rate indicated by the classification experience. The comparison may be made and has been made in different plans on the basis of indicated losses, pure premiums, or premiums.

In compensation insurance it is required first to "modify" the actual experience of the risk to bring it to the level of current industrial conditions as reflected in the current manual rate level. In the most widely used plan the procedure then is to determine "adjusted losses", the weighted average of the risk's modified losses and the "expected losses" which are indicated by the premium at manual rates*; to derive the ratio of the adjusted losses to the expected losses and apply this ratio to the manual rates* to obtain the final rates. In determining the adjusted losses, the hazard is divided into "normal losses" hazard and "excess losses" hazard. The weight or credibility assigned to the risk's experience is less in determining adjusted excess losses than in determining adjusted normal losses. The large losses occur less frequently than the normal losses and, costing much more individually, their volume in a given risk's experience is less indica-

* Schedule rates are used instead if schedule rating applies.

tive of the real hazard of large losses inherent in the risk than the volume of normal losses is of the real hazard of normal losses.

The technique of each step in the procedure, though worthy of detailed consideration and study, will not be considered here. It has been discussed at times in letters, memoranda, and open discourse in committees of rating bodies. The method of developing loss and payroll modification factors, the use of estimated individual case losses, average value losses whether fixed for all cases or varying with the duration of the case or other conditions, the theoretical and practical advantages and disadvantages of non-split, two-split, or multi-split plans; all these might well receive extended consideration. I propose to consider only risk experience credibility in casualty insurance experience rating, its development, and some criteria of proper credibility, after first mentioning the subject of off-balance produced by experience rating in total premiums because of its inter-relationship with credibility.

Off-Balance of Plan.

A phase of the technique of experience rating which has assumed increasing importance is the off-balance of the experience rating plan, that is, the variation of the premium collected on experience rated risks under adjusted rates from that expected at manual rates. There are reasons why one might expect an experience rating plan in which credibility varies with size to be out of balance, when the same elements enter into the modification factors which enter into the manual rate determination. What used to be believed the preponderant, if not the sole cause, an underreporting of losses on experience rated risks has, it now seems, been over-estimated as to its influence. At least the risk experience so far available from the rather recently established systems of individual risk reports to rating organizations indicate no greater development factor for losses of large risks than they do for losses of small risks which are not subject to experience rating. A factor which is coming more to be recognized as a primary cause of off-balance is the difference in the quality of the experience of large risks and small risks. Generally, the experience of the large risk is more favorable than that of the smaller risk, or of all risks. Necessarily, where the manual rate level is keyed to the average of all risks and no allowance has been made for this more favorable experience for large risks, it may be expected that an off-balance will be produced from experience rating. Even if recognition is given in the rate level to the more favorable experience for experience rated risks and the experience rating plan keyed to the level of rated risks, there is still left the variation within the experience rated group between the extremely large risks and those risks which just qualify for experience rating. As will be noted from the experience shown for policy year 1931 for New York, Table I, the manual loss ratio for risks in excess of $10,000 is more than 10% below the average of experience rated risks. These have more favorable experience and by virtue of their size under the experience rating plan receive larger credibility and therefore obtain credits which cannot be expected to be offset by an equal volume of less favorable experience on the smaller experience rated risks whose credibility is less.

Development of the Credibility Factor in Compensation

National Workmen's Compensation Service Bureau Plans

The part of experience rating plans over which opinion has differed most concerns the reliance placed on the risk's own experience or what is now known as the credibility factor. In this outline of the development of credibility, only the plans of the National Workmen's Compensation Service Bureau and the National Council on Compensation Insurance will be reviewed. In the development of the general principles of experience rating, these may be considered representative. In the first compensation experience rating plans, of which Plan A of 1916 and Plan B of 1917 of the National Workmen's Compensation Service Bureau are typical, there was no general variation in credibility by size of risk. In Plan A there was a limited variation in credibility by size of risk for the schedule rated risks only. In Plan B there was a small variation in credibility by rate size groups but no variation by risk size. The extent of the modification of the risk depended on the amount by which the risk's loss ratio deviated from the average. Soon this failure adequately to consider the size of risk was generally recognized and dealt with in the credibility formulas introduced with Plan D* in 1918. In this plan credibility was determined in two divisions of coverage, from the partial premium corresponding to the death and permanent total disability coverage, and the partial premium for all other coverage. The credibility for each part was obtained from formulas of the form

$$Z = \frac{P}{P+K}, \text{ where } Z \text{ denotes credibility}$$

P denotes partial premium

K denotes a constant

The Z's (Z_1 and Z_2 respectively) were taken from separate formulas or curves determined by K values (K_1 and K_2) chosen to give appropriate credibility to the losses in each division, the credibility being less for death and permanent total disability experience than for other losses of the same risks.

The formulas represent equilateral hyperbolas which pass through the origin and have as asymptote the line $Z = 1$. This permits one more point arbitrarily to be selected for each curve to determine the curve completely. Originally this point was selected for each division of coverage after experience rating a set of New York risks, both actual and hypothetical, using credibility curves of different degrees of liberality. The members of the committee, after consulting with underwriters, chose those curves which in their opinion produced the best results for the set of risks and thus established the constants K_1 and K_2 and the formulas for New York. The constants for other states were then selected so as to produce approximately the same credibility by parts if the accidents and claims of an average risk had been developed in New York and in each of the other states under their rates and compensation acts.

In determining credibility, the risk premium at latest manual rates was and still is used. This puts all risks on a common basis and eliminates differences that might affect credibility as between risks if actual premiums were used. Such differences might arise from different rate levels in the experience periods used or, and this is more important, from credits and debits in risks previously experienced rated, whereby risks of the same classification

* There was no Plan C for compensation; this letter was used for an employers liability plan.

and of the same size in number of employees and amount of payroll would have different credibility.

Industrial Experience Rating Plan—1920

The National Council's first plan, the Industrial Experience Rating Plan—1920 introduced some modifications in the technique of credibility determination. The credibility formula for death and permanent total disability remained as before, but for all other losses a new constant C was introduced to increase credibility, making the formula

$$Z_2 = \frac{P_2 + C}{P_2 + K_2 + C}$$

A refinement was introduced in the method of dividing the premium between the two coverages, making the division on the basis of the ratios of expected losses in the two divisions in each manual classification. Previously, the classifications had been grouped by size of rate, and average ratios determined, one for each size group. A new feature of the plan was the introduction of "self-rating". This provision was that risks whose subject* premium or whose indicated premium from the losses was $80,000 or more should have a credibility of unity in each division of coverage. Interpreted graphically, this means that credibility for risks under $80,000 premium was determined from the Z curves, and for risks of $80,000 and over the credibility was taken from the line $Z = 1$.

Industrial Experience Rating Plan—1923

When the Industrial Experience Rating Plan—1923 was adopted, the losses were separated into "normal" and "excess" losses for determining credibility, in place of the former two divisions, "Death and Permanent Total", and "All Other" losses. The credibility formulas were $Z_1 = \dfrac{P_1}{P_1 + K_1}$, $Z_2 = \dfrac{P_2}{P_2 + K_2}$, a return to the forms in Plan D. It was agreed to fix the Z curves for each state by selecting K's so that a single maximum claim on a risk of $1,000 subject premium having the average

* Subject premium is the premium subject to experience rating, and is obtained by extending the payrolls of the experience period at the manual or schedule modified rate for the effective date of the rating.

state excess ratio would increase the rate by 20% of the manual, 15% of the effect to be on the normal portion and 5% on the excess portion. The self-rating point was set at $100,000 subject premium or $60,000 losses for most states. A system of weights applying to both the actual and expected losses which decreased the influence of the older policy years was introduced toward the end of the effective period of this plan.

Industrial Experience Rating Plan—1928

The discontinuities of the credibility curves were removed in the Experience Rating Plan—1928. The formulas remained the same as in the Plan—1923, with K values determined by the same rule as before, but the range of applicability of the formulas was lessened and the discontinuities of the curves removed through the introduction of tangents to the curves from selected self-rating points. Separate normal and excess self-rating points were established. The self-rating point for normal experience was the same as before. Credibility for normal became unity at the point* corresponding to $100,000 subject premium, and credibility for excess experience was lessened, becoming unity at the point* corresponding to $200,000 subject premium. In this plan, which is still in effect, the credibility curves have become compound continuous curves, with the first sections arcs of hyperbolas, the second tangents to the hyperbolas, and the last a horizontal line. Tables have been constructed from which the credibility values are taken.

Appraising Experience Rating

Underwriters and the assured are continually passing judgment on the results for individual risks. Little has been done, however, toward obtaining more systematic or statistical analyses of the results. Various possibilities occur as to the relation of the empirical Z values with what might be regarded as the proper values. The credibility may be everywhere either too high or too low, or it may be too high at one extreme or too low at the other, or the empirical curve may cross the proper value several times. Before commenting on the relation of the credibility

* On an average normal-excess premium split basis.

scale and the results of an experience rating plan, it is necessary to consider again what the experience rating plan is designed to do.

The object of experience rating is to make all experience rated risks within a classification having correct manual rates equally desirable as far as the loss ratio is concerned, or, if all classification rates are assumed correct in their net effect for the total of experience rated risks in the classification, it may be said the object is to make all experience rated risks equally desirable from the loss ratio point of view. In the discussion which follows it will be assumed that the classification rates are correct in their net effect for experience rated risks.

A necessary condition for proper credibility is that the credit risks and debit risks equally reproduce the permissible loss ratio. Also, if the proper credibility has been attained, each sub-group of the credit and debit risks, provided it has adequate volume, should give the permissible loss ratio. While these conditions are necessary for a proper credibility of the experience rating plan, it does not follow that they are also sufficient. For a sufficient condition it would be required to establish that the risks within a group cannot be subdivided on any experience basis so as to give different loss ratios for the subdivisions, assuming the latter have adequate volume.

The necessary and sufficient conditions for the achievement of ideal credibility in an experience rating plan may be illustrated by an analogy to the classification experience. A necessary condition for proper classification rates is that each classification shall reproduce the permissible loss ratio. This condition, however, is not sufficient. A sufficient condition further requires that any subdivision of the classification having adequate volume should reproduce the permissible loss ratio. If two classifications, each of which has its different proper rate, are combined and an average rate established for the combination, the new combined class would reproduce the permissible loss ratio provided the relative volumes in the two original classes remain the same. For this new class, the necessary condition that the new rate reproduce the permissible loss ratio, would have been met. The condition for sufficiency that each sub-group reproduce the permissible loss ratio on the new rate basis would not have been met, for if the new rate were applied to the exposure under each of the original classifications which entered the combination, the permissible loss ratio would be reproduced for neither.

The necessary and sufficient condition for establishing that the credibility basis of the experience rating plan is correct may be stated as the condition that it is impossible to subdivide the risks on an experience basis differing from the experience rating plan and predict significantly different loss ratios for the subdivisions, providing they have adequate volume to be dependable. When considering the results of any plan, it is impossible to prove that the experience cannot be divided on any other credibility basis to yield better results. The second or sufficient condition is only required to prove that the plan in question is the optimum. The first condition is all that need be considered to test the relative merits of any given plans of experience rating or credibility scales or of a plan of experience rating as compared with no experience rating. The question then is not whether ultimate perfection has been reached but rather whether one plan is better than another, or than no experience rating.

The primary agents in the plan itself, other than the basic data, which affect the experience modification of a risk are the loss modification factors (including the effect of the average value and the payroll factor) and the credibility allowed the risk experience. It may be shown what effect each of these has when the other is assumed to be correct and to remain so. Consider the effect of variation in the loss modification factors on the risks of a premium size group arranged in experience modification groups, as in Table I. If the loss modification factor is too high (produces more modified losses than correspond to the rate level) and if the assumption is made that the rate level and the credibility factor are correct by premium size groups, it may be shown that the loss ratios produced in a given premium size group will have a downward trend as the experience modification increases. Conversely, if the modification factor is too low under the same conditions, the resulting loss ratios will have an upward trend. (See Appendix I.)

Test of Credibility Scale

In Appendix II the compensation experience of experience rated risks in New York for policy year 1931 has been compiled in a manner to permit examining the results of the New York Plan in relation to the necessary condition for proper credibility. The risks have been sorted in Table I into premium size groups and then each of these groups has been sorted into experience modification groups of .10 intervals. The sub-groups of the experience rating data resulting from these two sortings will be called "parcels".

It will be noted from Table I and the summary on page 19 that, for the individual parcel, the actual loss ratio is nearer to the permissible loss ratio (.605) than the manual loss ratio is, in 74 of the 97 parcels. For the parcels having credit experience modifications the actual loss ratio is nearer in 45 out of 52, and for the parcels having charge modifications the actual loss ratio is nearer in 29 out of 45.

When the credit parcels within each premium size group are combined, 7 of the 8 combinations show less deviation from the permissible loss ratio for the actual than for the manual loss ratio, the exception being the combined credit parcel for the short-term risks. The same result prevails when all the parcels within a premium size group are combined. When the parcels for the debit experience modifications are combined, 5 of the 8 combinations show less deviation from the permissible loss ratio for the actual than for the manual loss ratio.

When one considers the trends of the straight lines fitted by least squares to the actual loss ratios of the individual premium size groups, it will be noted, page 20, that, in passing from the lower to the higher modifications, of the 8 lines fitted to the credit parcels, 4 have an upward trend and 4 have a downward trend. In the lines fitted to the loss ratios of the debit parcels, the trend in 5 is upward and in 3 downward. When the lines fitted to the loss ratios of all parcels are considered, 6 have upward trends and 2 downward trends.

Interpreting these trend results on the assumption that the loss modifications factors are correct it may be said that they are not unfavorable to the present credibility or "swing" of the plan. It could hardly be expected in view of the limited data that no trends would appear—a condition that would uphold the present credibility. The indicated trends are rather evenly divided between upward and downward trends in the credit and charge experience modification groups. For all groups combined, which should be the most reliable, the upward trend dominates which would indicate that the present credibility was too restricted. However, when the short-term risks are eliminated, the trend is downward. Too much credence should not be given to the indications, for the data are not only limited but are derived from a single policy year situated in a particular phase of the business cycle. Similar tests should be applied to other policy years in other phases of the cycle and to other experience rating plans, and the results studied before passing final judgment.

I have attempted in these remarks to direct your attention to a problem that is of primary importance in casualty insurance with the hope of stimulating your thought and interest rather than presenting a solution. Compensation insurance was selected for purpose of illustration because the line is well-known; experience rating has been most highly developed in this line, and more extensive data are available for experimental purposes. The interest in the subject should extend to all lines where experience rating is applicable and experience available.

No attempt has been made to give a complete interpretation of the experience presented; this would be hardly justifiable on the basis of one year's experience. The object has been to indicate ways in which tests might be conducted. If a number of our members, either individually or jointly, undertook to analyze the data of experience rated risks for different states and policy periods, possibly along the lines suggested, it is my belief that there would result contributions to both the fundamental principles and applications of experience rating, perhaps comparable to those made in the first decade of our Society.

APPENDIX I

Effect of Error in Loss Modification Factor

Let the experience of the experience rated risks for a policy year in a given state be sorted into risk premium size groups and effective experience rating modification size groups. Designate these resulting sub-divisions of the experience as "parcels". Assume that the classification rates are correct and that the credibility of the experience rating plan is correct, so that with correct loss modification factors the actual loss ratios for each parcel will be the permissible.

It is proposed to determine the effect produced by an error in the loss modification factor on the trend of the actual loss ratios of the parcels in a given risk premium size group when the parcels are arrayed in increasing experience modification order. The loss modification factor herein will be understood to embrace the combined effect of the present modification factors for losses and payrolls and the effect of using average values. The modification factor will be considered correct when the losses of a given year are brought to the loss level underlying the manual rates.

Let F denote correct loss modification factor

F' denote actual loss modification factor

L_s denote actual losses of the experience period of parcel s

P_s denote subject premium of parcel s

E denote expected loss ratio

M_s denote correct experience modification of the risks in parcel s (derived by using the correct modification factor F)

M'_s denote actual experience modification (derived by using the actual modification factor F')

Z denote credibility of risks in premium size group

x denote difference between F' and F, or correction in F'

Then

$F' = F + x$

$EP_s =$ expected losses of parcel s

$FL_s =$ modified losses of parcel s, using correct loss modification factor

$(F + x)\,L_s =$ modified losses of parcel s, using actual loss modification factor

$$M_s = \frac{ZFL_s + EP_s(1-Z)}{EP_s}$$

$$M'_s = \frac{Z(F+x)L_s + EP_s(1-Z)}{EP_s}$$

$$= M_s + \frac{Z\,x\,L_s}{EP_s}$$

Consider the loss ratio of the experience developed in parcel s during the effective periods of the ratings based on F'

${}_fL_s$ denotes actual losses in parcel s during effective (future) period

${}_fP_s$ denotes manual premium in parcel s during effective (future) period

${}_fr_s$ denote loss ratio of parcel s during effective (future) period with ratings based on F

${}_fr'_s$ denote loss ratio of parcel s during effective (future) period with ratings based on F'

Then

$$ {}_fr'_s = \frac{{}_fL_s}{{}_fP_s\ M'_s} = \frac{{}_fL_s}{{}_fP_s\left(M_s + \dfrac{Z\,x\,L_s}{EP_s}\right)} $$

$$ = \frac{{}_fL_s}{{}_fP_s\ M_s} \cdot \frac{1}{1 + \dfrac{Z\,x\,L_s}{EP_s\ M_s}} $$

$$ = \frac{{}_fL_s}{{}_fP_s\ M_s} \frac{1}{1 + \dfrac{Z\,x\,L_s}{ZFL_s + EP_s(1-Z)}}, \text{ since } EP_s\ M_s = ZFL_s + EP_s(1-Z) $$

$$ = {}_fr_s \cdot \frac{1}{1 + \dfrac{x}{F + \dfrac{EP_s}{L_s} \cdot \dfrac{1-Z}{Z}}}, \text{ where } {}_fr_s,\ x,\ F, \text{ and } \frac{1-Z}{Z} \text{ are constant and } 0 < Z < 1 $$

As M_s increases these relations hold

	Case I $x > 0$	Case II $x < 0$
$\dfrac{{}^*EF}{L_s}$	decreases	decreases
$\dfrac{EP_s}{L_s} \cdot \dfrac{1-Z}{Z}$	decreases	decreases
$F + \dfrac{EP_s}{L_s} \cdot \dfrac{1-Z}{Z}$	decreases	decreases
$\dfrac{x}{F + \dfrac{EP_s}{L_s} \cdot \dfrac{1-Z}{Z}}$	increases	decreases

$1+\dfrac{x}{F+\dfrac{EP_s}{L_s}\cdot\dfrac{1-Z}{Z}}$	increases	decreases
$\dfrac{1}{1+\dfrac{x}{F+\dfrac{EP_s}{L_s}\cdot\dfrac{1-Z}{Z}}}$	decreases	increases
${}_fr'_s$	decreases	increases

This shows that if the actual modification factors are greater than the correct modification factors there will result a downward trend in the loss ratios and if the actual factors are less than the correct factors there will result an upward trend in the loss ratios, assuming that the rest of the experience rating plan is correct.

$^*\dfrac{EP_s}{L_s}$ is independent of x and decreases with an increase in M_s as the parcels are assumed to be arrayed in that order.

APPENDIX II

Results of New York Plan

Table I is an exhibit showing data of experience rated risks in New York for policy year 1931 compiled by the Compensation Insurance Rating Board of New York. In this exhibit the risks have been separated into full term and short term risks. The full term risks have been further separated into seven premium size groups based on actual annual premiums. The short term risks have been shown separately as it was impracticable to make a size division on an annual premium basis. There is also one group for all full term risks combined and another group for the total of all risks. The risks within each premium size group have been divided according to the experience rating modification factor underlying the risk rate in effect for policy year 1931.

A separation into size groups on the basis of manual rate annual premiums would have been preferable as reflecting more nearly the relative size of exposure. To place the experience on this basis, however, would require the determination of the manual premium for individual risks. The manual loss ratios given were derived by the Compensation Insurance Rating Board from manual premiums calculated for groups of risks at intervals of .01 of experience modification by division of the group actual premiums by their experience rating modifications.

For each experience modification division in each of the premium size groups there are shown the number of risks, the actual premiums expressed in $1,000 units, the actual loss ratio, and the manual loss ratio. The totals of these items for all credit risks, all charge risks and all risks are also shown. The object is to test the effect of experience rating on the loss ratios of the individual parcels into which the policy year experience has been sorted by the division into premium size groups and experience modification groups. If the experience rating procedure produces rates more equitable than the manual rates which they supersede, then, assuming adequate exposure, the deviations from the permissible loss ratio should be less for the actual loss ratios than for the manual loss ratios of the individual parcels.

In Table IA for each experience modification group a "1" has been placed in the proper column and line for each parcel to indicate whether the actual loss ratio or the manual loss ratio was nearer to 60.5%, the permissible loss ratio. The columns also have been summed for all credit modification groups, all charge modification groups, and all modification groups. At the bottom of Table IA, on the last three lines, it has been indicated in a similar manner whether the actual loss ratio or the manual loss ratio was nearer the permissible for all credit risks combined, for all charge risks combined, and for all experience rated risks combined. The results for the individual parcels of premium size groups and the whole premium size groups in Table IA when summarized are as follows:

Experience Modification Group	Number of Parcels where the Permissible Loss Ratios are nearer to		Number of Premium Size Groups where the Permissible Loss Ratios are nearer to	
	Actual L. R.	Manual L. R.	Actual L. R.	Manual L. R.
Credit Groups	45	7	7	1
Charge Groups	29	16	5	3
All Groups	74	23	7	1

In these tabulations the short-term risks were considered as one premium size group. This short-term group is responsible for the entries in the first and last line of the last column denoting that the permissible loss ratio is nearer to the manual than to the actual.

If the manual rates for the classifications were quite correct for every premium size group, if the experience rating plan were perfect, and if the volume of experience under each partition were adequate, the actual loss ratio in each partition should equal the permissible. Under these ideal conditions the deviations from the permissible loss ratio would be purely fortuitous and be plus and minus with equal frequency. Then straight lines fitted by least squares to the actual loss ratios of the parcels in any direction should have no trend, and a plane fitted to the whole field should be level.

In Table IB are shown loss ratios lying on straight lines fitted to the actual loss ratios of experience modification groups of each premium size group in Table I by the method of least squares, using the actual premiums in thousands as weights. In the column headed "All", the loss ratios derived from the fitted straight lines are given. In the other columns, under "Cr", the loss ratios on straight lines fitted to the credit modification groups only are given, and, under "Dr", the loss ratios on straight lines fitted to the charge modification groups only are given.

An effort has been made to gain in this way some knowledge as to the effect of the credibility factor or the "swing" of the plan. If all the conditions were correct, a line showing an upward trend in loss ratios with increasing experience modification groups would indicate that the swing of the plan is too restricted, for a wider swing would increase the credits and charges which would result in higher loss ratios for credit risks and lower loss ratios for charge risks. The change, if sufficient, could be made to overcome the trend so that, generally, the actual loss ratios for the charge risks would be no higher than those for credit risks.

It will be observed from Table I, that for all premium size groups, except the highest two, the actual loss ratios for the charge risks exceed those of the credit risks. In the "$10,000-$49,999" premium size group, the predicted charge risks had a manual loss ratio of 58.9% as compared with 48.8% for the whole group. The application of the charges from experience rating produced an actual loss ratio for this group of 50.2% as compared with 54.7% for the whole premium size group. If these limited data were accepted as fully reliable, this would indicate a swing which is too large, or a credibility factor which is too high, in the experience rating plan. In the "$50,000 up" premium size group, the debit risks are even more out of line. The predicted debit risks actually have a trifle better manual loss ratio than the group as a whole and, with the charges imposed, the actual loss ratio becomes very much better than that of the whole group.

The results, aside from these two high groups, are rather favorable to the present credibility of the experience rating plan. It is possible that the self-rating points established arbitrarily may have an influence on these large premium size groups. However, the complete reversal of form of the eleven predicted charge risks in the highest group, assuming they were correctly reported and rated, cannot be explained by any change in credibility factor. The actual explanation would require a detailed examination of the underlying losses which enter into the rating procedure and the conditions prevailing in the risks during policy year 1931 and the preceding years when the experience underlying the experience rating procedure was developed.

In Table IB, the loss ratios on straight lines fitted to the data in Table I show trends as the experience modification increases. These trends for the loss ratios on the lines fitted to the credit groups, the lines fitted to the charge groups, and to all groups for the seven full term premium groups, and the one group including all short-term risks combined, may be summarized as follows:

Experience Modification Groups	Straight Line Loss Ratio Trends as Experience Modification Increases	
	Trend Upward	Trend Downward
Credit Groups	4	4
Charge Groups	5	3
All Groups	6	2

The results for the credit groups are evenly divided between upward and downward trends. The results for the charge groups and all groups show an upward trend, though not a very decisive one.

TABLE I—COMPARISON OF ACTUAL LOSS RATIOS AND MANUAL LOSS RATIOS

EXPERIENCE RATED COMPENSATION RISKS IN NEW YORK POLICY YEAR 1931
DATA OF COMPENSATION INSURANCE RATING BOARD

Risks Grouped According to Size of Experience Modification: Full Term Risks Subdivided into Premium Size Groups

Experience Modification in Per Cent	No. of Risks	Act. Prem. in Thou.	Act. L. R.	Man. L. R.	No. of Risks	Act. Prem. in Thou.	Act. L. R.	Man. L. R.	No. of Risks	Act. Prem. in Thou.	Act. L. R.	Man. L. R.	No. of Risks	Act. Prem. in Thou.	Act. L. R.	Man. L. R.	No. of Risks	Act. Prem. in Thou.	Act. L. R.	Man. L. R.
	UNDER $500				$500—999				$1000—2499				$2500—4999				$5000—9999			
0– 30%	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
30– 39	....	....	....	....	2	2	364.1	132.7	....	....	....	....	....	....	....	....	2	17	90.6	29.7
40– 49	5	2	31.3	14.4	4	3	28.3	13.1	6	11	14.0	6.7	6	22	58.6	26.4	2	15	23.6	10.8
50– 59	16	2	165.3	91.9	9	7	48.4	26.3	22	35	59.9	33.0	18	66	48.4	26.5	14	96	36.9	20.7
60– 69	52	12	32.8	21.6	34	26	43.6	28.4	102	171	49.9	32.7	62	217	60.7	39.7	30	225	66.7	43.4
70– 79	211	58	82.6	62.5	196	147	48.3	38.0	273	432	46.6	35.2	139	474	50.4	37.9	58	384	62.5	47.0
80– 89	1,018	298	54.1	46.8	973	708	45.3	38.8	776	1,161	51.9	44.5	237	796	61.5	52.2	97	671	55.7	47.2
90–100	3,524	1,090	53.8	50.6	1,939	1,316	55.9	52.6	893	1,345	57.2	53.5	243	806	50.1	47.0	112	797	50.5	47.6
Credits	4,826	1,462	55.0	50.0	3,157	2,209	51.9	46.4	2,072	3,155	53.3	45.6	705	2,381	55.0	44.8	315	2,205	55.4	44.4
100–109	1,003	305	66.8	69.3	832	591	62.5	64.8	609	943	51.7	53.9	173	591	66.6	69.4	77	515	55.4	58.1
110–119	426	129	54.1	61.6	416	296	65.6	74.9	371	570	55.5	63.3	113	398	56.2	64.1	48	342	60.6	69.0
120–129	171	51	66.3	82.4	177	126	59.4	73.6	169	272	57.6	71.3	79	276	50.6	62.8	38	266	85.1	106.4
130–139	67	20	117.8	156.8	85	59	52.2	69.9	92	147	67.7	90.2	41	150	88.4	118.7	34	232	59.6	80.0
140–149	35	11	42.8	60.4	36	25	71.1	102.5	52	81	58.3	82.7	27	92	59.2	85.2	13	84	38.4	55.1
150 Up	27	7	48.3	76.6	28	20	132.4	211.8	57	99	43.2	71.4	24	80	42.0	70.9	17	114	47.8	82.4
Charges	1,729	523	64.8	71.3	1,574	1,117	63.9	71.1	1,350	2,112	54.4	62.1	457	1,587	61.6	71.7	227	1,553	60.8	72.5
TOTAL	6,555	1,985	57.6	54.9	4,731	3,326	56.0	53.5	3,422	5,267	53.8	51.1	1,162	3,968	57.7	53.2	542	3,758	57.6	53.4

Experience Modification in Per Cent	No. of Risks	Act. Prem. in Thou.	Act. L. R.	Man. L. R.	No. of Risks	Act. Prem. in Thou.	Act. L. R.	Man. L. R.	No. of Risks	Act. Prem. in Thou.	Act. L. R.	Man. L. R.	No. of Risks	Act. Prem. in Thou.	Act. L. R.	Man. L. R.	No. of Risks	Act. Prem. in Thou.	Act. L. R.	Man. L. R.
	$10,000—49,999				$50,000 and Over				Full Term—All Sizes				Short Term—All Sizes				All Risks			
0– 30%	1	50	120.1	34.9	....	....	....	....	1	50	120.1	34.9	....	....	....	....	1	50	120.1	34.9
30– 39	2	34	35.6	11.8	....	....	....	....	6	53	62.5	20.6	2	4	4.0	1.4	8	57	58.7	19.5
40– 49	4	102	79.7	36.2	....	....	....	....	27	155	65.0	29.6	1	6	75.3	30.9	28	161	65.4	29.7
50– 59	6	169	77.7	42.5	1	75	25.9	14.2	86	450	54.8	30.2	16	19	9.9	5.4	102	469	52.9	29.1
60– 69	17	322	50.7	33.1	5	440	59.1	36.9	302	1,413	57.0	36.8	53	49	108.6	72.2	355	1,462	58.7	37.9
70– 79	41	749	52.6	39.6	2	141	66.8	49.2	920	2,385	53.9	40.7	164	145	62.8	47.2	1,084	2,530	54.4	41.0
80– 89	69	1,389	57.8	49.3	7	620	47.2	39.4	3,177	5,643	53.9	46.0	451	382	89.5	76.4	3,628	6,025	56.2	47.8
90–100	55	895	56.7	53.2	6	660	57.5	54.6	6,772	6,909	54.8	51.5	964	575	69.7	65.4	7,736	7,484	55.9	52.6
Credits	195	3,710	58.1	44.1	21	1,936	54.0	42.3	11,291	17,059	54.9	45.0	1,651	1,180	75.8	64.4	12,942	18,238	56.2	46.2
100–109	58	1,073	47.7	49.6	4	371	38.0	40.6	2,756	4,389	54.5	56.9	451	281	84.7	88.8	3,207	4,670	56.4	58.8
110–119	36	625	48.1	54.9	4	288	41.1	46.4	1,414	2,648	53.9	61.5	304	343	72.0	82.4	1,718	2,991	56.1	63.9
120–129	15	292	60.9	76.6	3	192	30.1	36.9	652	1,475	58.7	73.1	211	210	88.1	109.6	863	1,685	62.4	77.7
130–139	15	226	56.3	75.4	....	....	....	....	334	834	66.1	88.6	85	72	110.6	148.9	419	906	69.8	93.5
140–149	10	206	60.3	86.0	....	....	....	....	173	499	56.2	80.4	51	74	63.7	91.4	224	573	57.2	81.8
150 Up	13	248	39.5	66.2	....	....	....	....	166	568	45.6	76.5	78	116	83.5	133.2	244	684	52.0	86.5
Charges	147	2,670	50.2	58.9	11	851	37.3	41.8	5,495	10,412	55.5	64.1	1,180	1,096	81.6	97.7	6,675	11,509	57.9	67.1
TOTAL	342	6,380	54.7	48.8	32	2,787	48.9	42.2	16,786	27,471	55.2	50.7	2,831	2,276	78.6	77.6	19,617	29,747	57.0	52.6

TABLE IA—FROM DATA OF TABLE I

COMPARISON OF ACTUAL LOSS RATIOS (ALR) AND MANUAL LOSS RATIOS (MLR) OF TABLE I WITH PERMISSIBLE LOSS RATIOS

A "1" in Columns "ALR" or "MLR" Indicates Respectively whether the Actual Loss Ratio or the Manual Loss Ratio is nearer to the Permissible Loss Ratio

Experience Modification in Per Cent	FULL TERM RISKS—PREMIUM SIZE GROUPS																All Short Term Risks		All Risks	
	Under $500		$500–999		$1,000–2,499		$2,500–4,999		$5,000–9,999		$10,000–49,999		$50,000 and Over		All Groups					
	ALR	MLR	ALR	MLR	ALR	MLR	ALR	MLR	ALR	MLR	ALR	MLR	ALR	MLR	ALR	MLR	ALR	MLR	ALR	MLR
0– 30%	..	..	..	..	..	..	..	..	..	.	..	1	..	..	..	1	..	..	..	1
30– 39	..	..	..	1	..	..	..	..	1	..	1	..	..	..	1	..	1	..	1	..
40– 49	1	..	1	..	1	..	1	..	1	..	1	..	..	..	1	..	1	..	1	..
50– 59	..	1	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..
60– 69	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..	..	1	1	..
70– 79	..	1	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..
80– 89	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..	..	1	1	..
90–100	1	..	1	.	1	..	1	..	1	..	1	..	1	..	1	..	..	1	1	..
Credits	4	2	6	1	6	..	6	..	7	..	7	1	5	..	7	1	4	3	7	1
100–109	1	..	1	..	.	1	1	..	..	1	..	1	..	1	..	1	1	..	..	1
110–119	..	1	1	.	..	1	..	1	1	..	..	1	..	1	..	1	1	..	..	1
120–129	1	..	1	..	1	..	..	1	1	..	1	..	..	1	1	..	1	..	1	..
130–139	1	..	1	..	1	..	1	..	1	..	1	..	..	..	1	..	1	..	1	..
140–149	..	1	1	..	1	..	1	..	..	1	1	..	..	..	1	..	1	..	1	..
150 Up.	1	..	1	..	..	1	..	1	1	..	..	1	..	..	1	..	1	..	1	..
Charges	4	2	6	..	3	3	3	3	4	2	3	3	..	3	4	2	6	..	4	2
TOTAL	8	4	12	1	9	3	9	3	11	2	10	4	5	3	11	3	10	3	11	3
Credit Group	1	..	1	..	1	..	1	..	1	..	1	.	1	..	1	..	..	1	1	..
Charge Group	1	..	1	..	..	1	1	..	1	..	..	1	..	1	..	1	1	..	1	..
All Risks	1	..	1	..	1	..	1	..	1	..	1	..	1	..	1	..	..	1	1	..

Table IB—From Data of Table I

Showing Trends of Loss Ratios with Risks Grouped by Size of Experience Modification Within Premium Size Groups

The Values given lie on Straight Lines Fitted by Least Squares to the Actual Loss Ratios in Table I, Weighted according to Actual Premiums

In the Columns headed "All" the Lines were Fitted to the Loss Ratios of All Modification Groups. In the Other Columns, under "Cr." the Lines were fitted to the Credit Modification Groups only, and under "Dr." to the Charge Modification Groups

Experience Modification in Per Cent	FULL TERM RISKS—PREMIUM SIZE GROUPS																All Short Term Risks		All Risks	
	Under $500		$500–999		$1,000–2,499		$2,500–4,999		$5,000–9,999		$10,000–49,999		$50,000 and Over		All Groups					
	Cr.	All	Cr.	All	Cr	All	Cr.	All	Cr.	All	Cr.	All	Cr.	All	Cr.	All	Cr.	All	Cr.	All
0– 30%	....	....	....	.. .	....	....	....	....	....	.. .	77.0	67.2	....	....	62.6	56.2	....	. . .	62.2	56.2
30– 39	....	....	37.2	28.9	...	....	....	....	66.9	55.7	73.5	65.2	....	....	61.3	56.0	74.1	70.5	61.2	56.5
40– 49	77.9	48 0	39.9	33.2	38.2	48.9	59 0	54.1	64.5	56 0	70.1	63.8	.	. . .	60 0	55.9	74 4	71 7	60.2	56.7
50– 59	73.0	49.8	42.6	37.6	41.9	49 8	58.0	54.8	62.1	56.3	66.6	62.0	52.3	49.1	58.7	55.7	74 7	72.8	59.2	56 9
60– 69	68.1	51.6	45.3	41.9	45 5	50.7	56 9	55 4	59 7	56 6	63 2	60.3	53.0	48.8	57 4	55.6	75.0	74.0	58.2	57.1
70– 79	63 2	53.5	48.0	46.2	49.1	51.6	55 9	56.1	57.3	57 0	59 8	58.6	53.6	48.4	56.1	55.4	75.3	75.1	57.2	57.4
80– 89	58 3	55.3	50.8	50 5	52.8	52.5	54.8	56.8	54.9	57 3	56.3	56.8	54.3	48.1	54.8	55 3	75 7	76.3	56.2	57.6
90–100	53 4	57.2	53.5	54.8	56.8	53 4	53.7	57.4	52.6	57.6	52.9	55.1	54.9	47.8	53 5	55.1	76.0	77.5	55 2	57.8
	Dr.		Dr.		Dr.		Dr.		Dr.		Dr.		Dr.		Dr.		Dr.		Dr.	
100–109	64.5	59.0	60.7	59.2	53.8	54.3	63.9	58.1	62.4	57.9	49 9	53.4	39 8	47.5	55.5	55 0	80.6	78.6	57.4	58.1
110–119	64.9	60.8	64.5	63.5	54.4	55.2	62 2	58 8	61 5	58.2	50.1	51.7	36 6	47.1	55.5	54.8	81.2	79.8	57.9	58 3
120–129	65 3	62.7	68 2	67.8	54.9	56.1	60.6	59 4	60.7	58 6	50.3	49.9	33.5	46.8	55.5	54 7	81.7	80 9	58.4	58.5
130–139	65.8	64.5	72 0	72.1	55.5	57.0	57.0	60.1	59.8	58 9	50.4	48.2	....	....	55.5	54.5	82.2	82 1	58 8	58.8
140–149	66.2	66.4	75.8	76.4	56.0	57.9	57 4	60.8	58 9	59 2	50 6	46.5	....	....	55.4	54.4	82.7	83 3	59 3	59.0
150 Up	67.0	70.0	83.4	85.1	57.1	59.7	54.2	62.1	57.2	59 8	50.9	43.0	.. .	...	55.4	54.0	83.8	85.6	60 2	59.4

APPENDIX III

Bibliography of Experience Rating

Proceedings of Casualty Actuarial Society

I, 227-240, Senior—"The Effect of Schedule and Experience Rating on Workmen's Compensation Risks in New York."

II, 347-355, Greene—"Should the Compensation Premium Reflect the Experience of the Individual Risk?"
III, 54-75, Discussion.

II, 356-369, Woodward—"The Experience Rating of Workmen's Compensation Risks."
III, 54-75, Discussion.

III, 43-48, Fisher—"Note on Application of Bayes' Rule in the Classification of Hazards in Experience Rating."
III, 268-273, Discussion.

IV, 274-292, Whitney—"The Theory of Experience Rating."
V, 133-159, Discussion.

IV, 293-324, Michelbacher—"The Practice of Experience Rating."
V, 133-159, Discussion.

VIII, 345, Current Notes—Experience Rating.

IX, 157-159, Current Notes—Burglary Experience Rating.

IX, 370-373, Current Notes—Experience Rating.

XI, 173-175, Current Notes—"Automobile Insurance Merit Rating of Individual Drivers."

XI, 211-217, Senior—"Experience Rating In Rem and In Personam."
XII, 117-128, Discussion.

XII, 161, Current Notes—"Compensation Experience Rating in Wisconsin."

XII, 162-165, Actuarial and Statistical Notes, "Credibility Tables for Experience Rating."

XIII, 338-342, Current Notes—Garrison—"Experience Rating for Plate Glass Insurance."

XV, 106-107, Current Notes—"National Council Plan for Adjustment of Large and Small Risk Rates."

XV, 223-235, Roeber—"Recent Developments in Workmen's Compensation Insurance Rate Making."

XVI, 218, Current Notes—"Results of Merit Rating Plan are Being Studied" (Automobile).

XVI, 391, Current Notes—"Merit Rating of Public Liability."

XIX, 195, Current Notes—"Activities of the National Council during 1932, Merit Rating."

XX, 68-81, Kormes—"Correction of Certain Deficiencies in the Experience Rating Plan by the So-called Account Current Method."
XX, 346-352, Discussion.

Record of American Institute of Actuaries

VII, 37-44, Greene—"Certain Actuarial Aspects of Workmen's Compensation Insurance."

Transactions of Actuarial Society of America

XXX, 130-139, Keffer—"An Experience Rating Formula."

Transactions of Eighth International Congress of Actuaries
I, 324-335, Mowbray—"Experience Rating of Risks for Workmen's Compensation in the United States."

Proceedings of National Convention of Insurance Commissioners
Year 1924, pp. 272-282, Hobbs—"Experience Rating in Compensation Insurance."

Transactions of National Safety Council (called Proceedings before 1926)
Year 1921, pp. 207-210, Hall—"Experience Rating of Contractors' Risks."
Same in Economic World, 11/5/1921, 573-574.
Year 1933, pp. 105-106, Richardson—"Relation of Safety Equipment to Experience Rating."

Safety Engineering
February, 1927, pp. 67-72, Hall—"Experience Rating for Compensation Insurance Risks."

Economic World
April 15, 1916, Beyer—"Experience or Morale Rating in Workmen's Compensation Insurance."
November 17, 1917, Michelbacher—"Theory of Experience Rating in Workmen's Compensation Insurance."
(Same in Insurance Commercial Magazine, November, 1917.)
November 5, 1921, Hall—"Experience Rating of Contractors' Risks."
(Same in Proceedings of National Safety Council for 1921.)

Journal of Commerce
In re Experience Rating
Theo. E. Gaty, February 21, 1916.
R. J. Sullivan, February 24, 1916.
Edmund Dwight, February 25, 1916.
Dr. R. S. Keelor, February 28, 1916.
L. S. Senior, March 1 and April 15, 1916.
A. C. Hendrick, March 3, 1916.

United States Bureau of Labor Statistics
Bulletin No. 212, 1917, pp. 249-260, Beyer—"Experience Rating versus Schedule Rating."
Discussion, pp. 261-266.
Bulletin No. 333, 1923, pp. 252-260, Senior—"Schedule Rating versus Experience Rating: A Critical Review of Systems for Rating Workmen's Compensation Risks."
Discussion, pp. 260-267.
Monthly Labor Review, January, 1919, pp. 276-278—"Massachusetts Workmen's Compensation—Revised Experience Rating Plan."

Treatises on Insurance
Blanchard—Liability and Compensation Insurance, pp. 253-264. (1917)
Kulp—Casualty Insurance, pp. 262-264, 281-295, 359-360, 455. (1928)
Michelbacher and Nial—Workmen's Compensation Insurance, pp. 308-314, 469-474. (1925)
Michelbacher—Casualty Insurance Principles, pp. 223-226, 229-239. (1930)

DEDUCTIBLE AND EXCESS COVERAGES LIABILITY AND PROPERTY DAMAGE LINES, OTHER THAN AUTOMOBILE

BY

JAMES M. CAHILL

Relatively few risks under the various Liability and Property Damage lines, other than Automobile, have been written in the past on either a deductible or an excess coverage basis. There is, however, a growing trend toward writing certain types of risks under these lines of insurance on a deductible basis. The reason why these forms of coverage have been given such scant consideration as underwriting tools is undoubtedly that most casualty insurance men are unfamiliar with them as applied to the miscellaneous Liability and Property Damage lines. The advantages of writing deductible or excess coverage in certain cases remain unappreciated because of a lack of knowledge of the mathematical derivation of the discounts, the method of applying the discounts to the basic rates, the method of experience rating such risks, etc. The purpose of this paper is to assemble the available data which may be published in order that there may be a more general understanding of the rate structure for deductible and excess coverages.

First, it would be well to define the coverage provided by policies written on a deductible or on an excess basis.

Deductible Coverage

The insurance company investigates, defends and settles all claims, paying total first aid medical, total allocated claim adjustment expense, and any indemnity in excess of the assured's retention of liability, subject to the limits of the policy.

The assured pays all indemnity up to the amount of his retention of liability per claim or per accident. In actual practice, the insurance company usually pays the total loss and subsequently secures reimbursement from the assured for his portion of the indemnity loss.

Excess Coverage

The assured investigates, defends and settles all claims not in excess of his retention of liability per claim or per accident.

The insurance company cooperates in the investigation, defense and settlement of such claims only as are necessary for the protection of its interests. The insurance company pays any allocated claim adjustment expense thus incurred by itself and any indemnity in excess of the assured's retention of liability, subject to the limits of the policy.

Deductible coverage is usually written for relatively small amounts of assured's retention of liability in connection with risks which experience a high claim frequency. This gives the assured a direct interest in controlling accidents and tends to make desirable risks which might be uninsurable on a full-coverage basis. On the other hand, excess coverage is usually written for high amounts of assured's retention of liability for risks which desire to self-insure all except the more costly claims or catastrophe losses. In Part I of this paper, deductible coverage will be discussed. Excess coverage will be treated in Part II.

PART I — DEDUCTIBLE COVERAGE

Distribution of Losses by Size of Claim

In order to calculate rates for deductible coverage, it is necessary to compile a distribution of incurred losses by size of claim, $1-$10, $11-$25, etc. The discounts currently in use were calculated from the following compilations of such data by line of insurance for claims settled in calendar years 1925 and 1926:

Line of Insurance (1)	Claims Settled in Calendar Years: (2)	Territorial Subdivisions (3)	Classification Groups (4)
Elevator P. L.	1925-26	Countrywide	Total
Elevator P. D.	1925-26	Countrywide	Total
Mfrs.' & Contrs.' P. L.	1925-26	Countrywide	(a) Manufacturing (b) Contracting (c) Public Utilities (d) All Other
Mfrs.' & Contrs.' P. D.	1925-26	Countrywide	Total
O. L. & T. P. L.	1925-26	Countrywide	Total
O. L. & T. P. D.	1925-26	Countrywide	Total
Teams' P. L.	1925-26	Countrywide	Total
Teams' P. D.	1925-26	Countrywide	Total

In 1935, the National Bureau of Casualty and Surety Underwriters called upon its member companies to file more recent data for the important Liability and Property Damage lines, other than Automobile, to serve as the basis for the calculation of revised discounts for these lines. The recent calls which have been compiled are as follows:

Line of Insurance (1)	Claims Settled in Calendar Years: (2)	Territorial Subdivisions (3)	Classification Groups (4)
Elevator P. L.	1934	(1) New York State	Total
		(2) Remainder of Country	Total
Mfrs.' & Contrs.' P. L.	1933	Countrywide	Total
Mfrs.' & Contrs.' P. D.	1933	Countrywide	Total
O. L. & T. P. L.	1934	(1) New York City	Apartments & Tenements
		(2) New York State	(a) Area & Frontage Classes excluding New York City Apartments & Tenements Classes.
			(b) Miscellaneous Classes.
		(3) Remainder of Country	(a) Area & Frontage Classes.
			(b) Miscellaneous Classes.
Product P. L.	1934	(1) New York State	(a) Bakeries.
			(b) All Other Foodstuffs—Stores & Mfg. Classes.
			(c) All Other Classes.
		(2) Remainder of Country	(a) Bakeries.
			(b) All Other Foodstuffs—Stores & Mfg. Classes.
			(c) All Other Classes.

It will be noted that these recent calls provide for a subdivision of the data by classification groups in certain instances and also between the state of New York and the remainder of the country for certain lines. The National Bureau has recognized the probability that a rather wide variation in the distribution of claims by size exists within classification groups and it is for this reason that the recent calls have included more subdivisions than the previous calls.

In these calls, the size of a claim was determined by the amount of incurred indemnity and medical combined, excluding allocated claim adjustment expense. The total allocated claim adjustment expense was recorded for all size groups combined. It might be pointed out that the medical losses should theoretically be handled in the same manner as the allocated claim expense rather than to be combined with the indemnity in determining the size of claim, since the insurance carrier is liable for both the medical and the allocated claim adjustment expense under deductible coverage. This is not a serious error, since for the various Liability lines, other than Automobile and Employers' Liability, the ratio of medical losses to total losses including allocated claim expense is less than 1%. It is recommended, however, that future calls provide for the determination of size of claim by the amount of indemnity alone, excluding all medical and allocated claim adjustment expense.

Rating Making Method

The method currently employed in determining the discounts for deductible coverage is as follows. The portion of the indemnity losses eliminated by the deductible feature is calculated from the distribution of incurred losses by size of claim. This percentage is deducted from 100% in order to determine the percentage of the indemnity losses which will be incurred by the insurance company. The product of this residual percentage and the permissible loss ratio excluding the provision for allocated claim expense determines the percentage of full-coverage rates which the insurance company may expect to incur in indemnity losses under the deductible form. To this percentage are added the provision for allocated claim expense and the full loadings in the manual rates for unallocated claim expense, Home Office administration, payroll audit and inspection. This total in terms of manual rates is then divided by .70 in order to load percentagewise for acquisition (25%), taxes (2½%) and profit (2½%). This calculation determines the indicated percentage of full-coverage rates which is necessary to give the proper allowances for losses and expenses under the deductible form. The indicated discount is calculated by deducting this percentage from 100%. In order to provide a safety margin, the indicated discount is multiplied by .90 and this discount is then rounded to the lower .025 interval.

The details of the calculation of the discount for $250 deductible coverage for the O. L. & T. Public Liability line are given in the following exhibit:

O. L. & T. PUBLIC LIABILITY
Calculation of Discount for $250 Deductible

(1)	Incurred indemnity losses under $250 per claim	$3,874,396
(2)	Number of claims over $250 per claim	7,312
(3)	First $250 of loss on claims over $250 per claim (2) × $250	$1,828,000
(4)	Total first $250 of loss (1) + (3)	$5,702,396
(5)	Total indemnity losses (5/10 limits)	$8,689,185
(6)	Portion of indemnity losses eliminated by $250 deductible (4) ÷ (5)	.656

	Full Coverage	$250 Deductible
Losses (excl. allocated claim expense)	.473	.473 × (1.000 — .656) = .163
Allocated claim expense	.037	.037
Unallocated claim expense	.080	.080
Administration	.075	.075
Inspection	.035	.035
Sub-Total	.700	.390
Acquisition, Taxes and Profit	.300	$.30 \times \frac{.390}{.70} = .167$
Total	1.000	.557
Indicated discount for $250 deductible		1.000 — .557 = .443
Safety factor applied		.443 × .90 = .399
Discount rounded to lower .025 interval		.375

The actual calculation of the discounts for the various deductible amounts is simplified by the use of formulas. The rate for deductible coverage is calculated from the manual rate for full-coverage as follows:

$$R_d = R\ (1.00 - \text{Discount})$$

The formulas for calculating the discount for each of the important Liability and Property Damage lines, other than Automobile, are as follows, where k is the percentage reduction in indemnity losses by reason of the deductible feature:

Mfrs.' & Contrs.' P. L. & P. D.
O. L. & T. P. L. & P. D.
Product P. L. & P. D.
Theatre P. L. & P. D.

$$\text{Discount} = \frac{.90 \times k \times (.510 - .037)}{1.000 - (.250 + .025 + .025)}$$
$$= .6081k \text{ (Rounded to lower .025 interval)}$$

Teams' P. L. & P. D.

$$\text{Discount} = \frac{.90 \times k \times (.520 - .037)}{1.000 - (.250 + .025 + .025)}$$
$$= .6210k \text{ (Rounded to lower .025 interval)}$$

Elevator P. L.

$$\text{Discount} = \frac{.90 \times k \times (.545 - .037 - \text{Inspection cost ratio})}{1.000 - (.250 + .025 + .025)}$$

(NOTE: The inspection cost ratio is the inspection pure premium divided by the manual rate. This ratio varies by type of elevator and by territory.)

Elevator P. D.

$$\text{Discount} = \frac{.90 \times k \times (.245 - .037)}{1.000 - (.250 + .025 + .025)}$$
$$= .2674k \text{ (Rounded to lower .025 interval)}$$

Employers' Liability

The following table of discounts (taken from page 17 of the September, 1923 edition of the manual of Employers' Liability Insurance) is used in the calculation of rates for deductible per claim coverage for the respective amounts of assured's retention of liability shown. These discounts are applicable only to the indemnity portion of the rate.

Assured's Retention of Liability	Per cent Discount Deductible per Claim
$ 100	5.0%
150	10.0
250	20.0
500	25.0
1,000	30.0
2,000	45.0
2,500	50.0
3,000	52.5
3,500	55.0
4,000	57.5
4,500	60.0

In calculating the rate for deductible coverage for a policy written on an ex-medical basis, the full-coverage rate is first multiplied by unity minus the ex-medical ratio in order to obtain the ex-medical rate, and then the deductible discount specified in the table is applied to this ex-medical rate.

To obtain the rate for deductible coverage for a policy written on a full-medical basis, the discount specified in the table is applied to the ex-medical rate and to this result is added the medical portion of the rate in order to determine the final deductible rate.

Rate Filing with New York Insurance Department

The National Bureau's filing with the New York Insurance Department does not consist of a definite, complete schedule of discounts for the various amounts of assured's retention of liability for each line of insurance. The filing consists of the formulas previously given in this paper to be used in calculating the discounts for the smaller amounts of assured's retention, together with an explanation of a modification of these formulas to provide for the graduation of the discounts for the amounts of assured's retention above $1,000 per claim for the Public Liability lines and above $250 per accident for the Property Damage lines. The discounts for the higher amounts of assured's retention are established by judgment in order to graduate to a discount of .80 for $5,000 deductible coverage on a per claim basis on a standard limits Public Liability policy, or for an assured's retention of $1,000 per accident on a standard limits Property Damage policy.

The Product P. L. and P. D. lines are considered to be on an "*a*" rated basis for deductible coverage; that is, discounts are quoted which fit the characteristics of each risk.

Under the present filing, it would theoretically be possible to use the distribution of losses by size for a group of classifications or for an industry group rather than the totals for a line of insurance in establishing the proper discount for a given risk, if it were considered that this procedure would establish a more accurate rate for the risk.

If the assured's retention of liability is in excess of standard limits, the rate is determined by applying the following multiplier to the manual rate:

$$M - .80N$$

Where $M =$ Table multiplier for limits desired

$N =$ Table multiplier for limits of assured's retention

Comments on Present Deductible Rate Making Method

Under the present method of determining the discounts for deductible coverage, the provision for allocated claim expense is .037 of the full coverage rate for all lines of insurance. This ratio was derived from the claim expense data compiled in the Supplement to the 1928 New York Casualty Experience Exhibit for the Owners', Landlords' and Tenants', the Manufacturers' and Contractors', the Elevator, and the Teams' Public Liability lines combined. The ratio of allocated and unallocated claim expenses combined to earned premiums was .117. Since the loading in the manual rates for unallocated claim expense is .080, the difference between .117 and .080, or .037, was assumed to represent the ratio of allocated claim expense to earned premium.

A review of the allocated claim expense ratios reported in the 1935 Casualty Experience Exhibit indicates that this ratio of .037 is only approximately half the average allocated claim expense ratio actually being incurred in connection with the Liability lines, other than Automobile. There is also considerable variation in the indicated allocated claim expense ratio by line of insurance. For all stock companies combined, the allocated claim expense ratios shown in the Supplement to the 1935 Casualty Experience Exhibit are as follows:

CASUALTY EXPERIENCE EXHIBIT—CALENDAR YEAR 1935

Line of Insurance	Allocated Claim Expense Ratio
Elevator P. L.	2.3%
Mfrs.' & Contrs.' P. L.	7.9
O. L. & T. P. L.	7.3
Teams' P. L.	8.4
Employers' Liability	6.2
Product P. L.	8.4
All other Liability lines, other than Auto	11.7
Total Liability other than Automobile	7.2

It is quite likely that the allocated claim expense ratio incurred on risks written on a deductible basis is higher on the average than that incurred on risks written on a full coverage basis because assured's whose coverage is on a deductible basis frequently endeavor to influence the insurance company to contest more cases than normal. Giving consideration to this point and also to the fact that the ratio of .037 is seriously out of line with the indications of the latest data on actual allocated claim expense ratios, it is the opinion of the writer that the present rate making method for deductible coverage does not include an adequate provision for allocated claim expense.

The foregoing table indicates that the provision for allocated claim expense on deductible risks should vary by line of insurance. It is the writer's recommendation that the allocated claim expense ratio to be used in the determination of the discounts for deductible coverage be determined in the following manner. In conjunction with the loss data reported by size of claim, the allocated claim expense incurred on the claims included in the report is shown in total as a separate amount. The ratio of the total allocated claim expense to the total of the standard limits indemnity losses and the allocated claim expense combined could be determined. Applying this ratio to the permissible loss ratio for the line of insurance would develop the indicated necessary provision for allocated claim expense on the basis of the assumption that the total loss experience incurred for the line of insurance would equal the permissible.

If it should be considered undesirable to use the data reported in connection with the call for experience by size of claim as the basis for this calculation, the data reported in the regular call for loss ratio experience by line of insurance could be substituted.

It would be preferable to determine the allocated claim expense ratio by the recommended method rather than to adopt a ratio based on the indications of the Casualty Experience Exhibit. The latter ratios are apt to be unreliable for some of the less important lines of insurance and, furthermore, the actual allocated claim expense ratio varies considerably with the character of the general loss experience, reflecting the effect of a favorable or an unfavorable loss ratio.

The present method of graduating the discounts for the higher amounts of assured's retention to produce a discount of .800 for

$5,000 deductible coverage apparently does not give an adequate provision for the expenses incurred in servicing such risks. The breakdown of the .200 of the full coverage rate which is charged for servicing a $5,000 deductible risk may be assumed to be as follows:

PRESENT METHOD

Expense Item	Ratio to Full Coverage Rate
Acquisition, Taxes and Profit (30% × .200)	.060
Unallocated Claim Expense	.080
Allocated Claim Expense	.037
Available for H. O. Admin., Insp., and Payroll Audit	.023
Total	.200

It will be noted that even with a provision of only .037 for allocated claim expense, the residue available for Home Office administration, inspection and payroll audit is .023 as compared with the provision of .110 in the manual rates for the important lines of insurance. If the provision for allocated claim expense indicated by the tabulation previously given were allowed, there would be nothing specifically available for Home Office administration, inspection and payroll audit. The above analysis assumes, of course, that the same number of claims would be incurred under deductible coverage as under full coverage. It seems quite likely, however, that some beneficial effect on the number of claims would normally result from writing the coverage on a deductible basis rather than on a full coverage basis, similar to that which has actually been experienced when Workmen's Compensation risks have been written under the Retrospective Rating Plan instead of on a guaranteed cost basis. Such a tendency for deductible coverage to reduce the number of claims would offset, to some extent, the apparent inadequacy in the expense provision.

If consideration is given to the theory underlying deductible coverage, it is apparent that there should be the same provision for company expenses in the deductible rate that there is in the full coverage rate. If the discount for $5,000 deductible coverage is calculated in accordance with this theory, the discount indicated for the important lines of insurance is .676 as compared with the discount of .800 allowed at present. The discount of .676 is calculated as follows:

PROPOSED METHOD

Expense Item	Ratio to Full Coverage Rate
Acquisition, Taxes and Profit (30% × .324)	.097
Unallocated Claim Expense	.080
Allocated Claim Expense	.037
H. O. Admin., Insp., and Payroll Audit	.110
Total	.324

If the indicated necessary provision for allocated claim expense were included, the discount calculated would be somewhat less than .676.

Under the present rate making method, it is questionable whether an insurance company could actually afford to insure a risk on a deductible basis with the assured's retention of liability approximating $5,000 per claim because of the apparently inadequate expense allowance which would be received. Consideration should be given to the desirability of revising the present method of graduating the discounts for the higher amounts of assured's retention so that a larger expense allowance will be provided. In the writer's opinion, the discount allowed for an assured's retention of $5,000 per claim should be considerably less than .800 as at present.

Per Claim vs. Per Accident Deductible Coverage

The formulas given for the Public Liability lines apply only when the deductible coverage is written on a per claim basis. No statistics of the distribution of losses by size on a per accident basis are available. It would be very difficult for the insurance companies to respond to a call for the distribution of losses by size on a per accident basis because of the manner in which their statistical records are maintained. When deductible coverage on a Public Liability policy is written on a per accident basis, the discount allowed is .05 less than the discount calculated on a per claim basis for all lines except Elevator P. L., for which .025 is deducted from the discount applicable on a per claim basis.

For the Property Damage lines, the formulas given are for a per accident basis since Property Damage deductible coverage is always written on a per accident basis and never on a per claim basis. This procedure is necessary in view of the difficulty of defining a claim under Property Damage coverage. This difficulty is not experienced with the Public Liability lines since the number of claims is a function of the number of persons injured in each accident.

Minimum Premiums

The deductible discounts are also applicable to the minimum premium for individual locations or operations on specific risks where the minimum premium is the controlling premium. In no event, however, may the deductible discount operate to reduce the premium charge per policy below the minimum premium charge (if not in excess of $10.00) which would apply if the policy were canceled by the assured.

Excess Limits

When excess limits coverage is provided on a policy written on a deductible basis with an assured's retention of less than standard limits, the premium charge for the excess limits portion of the coverage must be the same as would be made on a risk written on a full-coverage basis. The liability of the insurance company with regard to the excess limits portion of the coverage is not affected by the deductible provision applicable to the standard limits portion of the coverage. For example, if a $6,000 indemnity loss were incurred on a policy written for 50/100 limits and on a $250 deductible basis, the assured would be liable for $250 and the insurance company for $4,750 under the standard limits portion of the coverage and for $1,000 under the excess limits portion of the coverage. Under a full-coverage policy, the portion of the loss chargeable against the excess limits coverage would likewise be $1,000.

To illustrate the manner in which the final rate is calculated for a risk written on a deductible basis, with excess limits coverage, the following example is included:

Example—O. L. & T. P. L. risk subject to Table B
50/100 limits factor = 1.320
\$250 deductible discount = .375

Factor applicable to 5/10 manual rate:
$1.00 \times (1.000 - .375) = .625$
$.32 \times 1.000 = .320$

Total........ .945

If the 5/10 manual rate were \$.50, the rate for 50/100 limits, \$250 deductible, would be .945 × \$.50, which equals \$.473.

If experience rating modifications are applicable, the final adjusted rate for the above example would be calculated as follows. Assume a standard limits experience modification of .700 and an excess limits experience modification of .800.

$.625 \times .700 = .4375$
$.320 \times .800 = .2560$

Total .6935

.6935 × \$.50 = \$.347 Final adjusted rate

Aggregate Limits

For certain lines of insurance, an aggregate limit as well as the usual per person and per accident limits applies. All of the specified limits of liability—whether per person, per accident or the aggregate liability under the policy—apply to the gross indemnity cost of the claims incurred regardless of the portion of such costs which may be retained by the policyholder under the deductible form of coverage. It is therefore necessary that the insurance company maintain a record of the gross indemnity cost of all claims on each policy written on a deductible basis under those lines which are subject to an aggregate limit, in order to determine when the aggregate policy limit has been exhausted.

Classification Experience

The experience of risks written on a deductible basis is excluded from the classification experience reported for rate making. The experience of all risks written on a deductible basis is reported in total under a specified code number for each line of insurance.

No attempt is made to compile a record by deductible amount, because the volume of business which has been written to date on a deductible basis has not been sufficiently large to be of any value for rate making purposes.

Experience Rating

The Public Liability Experience Rating Plan is applicable on an intra-state basis in three states: Minnesota, New York and Wisconsin. A Public Liability risk written on a deductible basis qualifies for experience rating if it has developed an exposure during either the latest year or the latest two years of the experience period such that the application thereto of the manual rates for full coverage (standard limits only) produces a premium of the same amount as required for a full coverage risk to qualify for experience rating.

The experience rating of Public Liability risks is in accordance with the coverage to be provided on renewal. Full coverage experience is adjusted to the deductible basis if the risk is to be written on the deductible form on renewal and, vice versa, any deductible experience is built up to a full coverage basis before using in the experience rating calculation if the risk is to be afforded full coverage on renewal. In conformance with the rule that there should be only one experience rating modification outstanding for a risk at one time, it would be desirable to provide that if a portion of the coverage is to be written on a full coverage basis and the remainder on a deductible basis on renewal, the experience rating calculation should be based on the combined data compiled accordingly. For a risk written in such a manner, it is the writer's opinion that there should not be separate experience rating calculations based in the one case with all of the experience adjusted to a deductible basis and in the other case with all of the experience built up to a full coverage basis.

In developing the experience rating modification for a risk which is to be written on a deductible basis on renewal, the following changes in the Public Liability experience rating plan are necessary:

Actual Losses

The actual losses experienced under full coverage are reduced to an equivalent deductible amount by subtracting the deductible amount from the indemnity payments. Allocated loss expense and medical losses are included in full. The adjusted indemnity loss is combined with the allocated loss expense and the medical losses before separating any loss into normal and excess. In dividing actual losses between normal and excess, the deductible amount is first subtracted from the normal loss amount of Table A and the remainder is used as the normal amount for the deductible coverage.

Expected Losses

(1) The total expected losses on the deductible basis are obtained by multiplying the full coverage premium subject at standard limits by the ratio given below for each line of insurance, where r is the ratio of the manual rate for the deductible coverage to the manual rate for full coverage $\left(r = \frac{R_d}{R} \right)$:

Line of Insurance	Ratio Applicable to Full Coverage Premium Subject
General Formula	$.70\,r$ — (Unallocated Cl. Exp. + H. O. Admin. + Insp. + P. A.)
Mfrs.' & Contrs.' P. L. O. L. & T. P. L. Product P. L. Theatre P. L.	$.70\,r - .19$
Teams' P. L.	$.70\,r - .18$
Elevator P. L.	$.70\,r$ — (.155 + Inspection cost ratio)

For example, for the O. L. & T. Public Liability line, the total expected losses for a risk written on a \$250 deductible basis for which the discount is .375 would be equal to .2475 times the full coverage premium subject ($.70 \times .625 - .19 = .2475$).

(2) Under *any* of the following conditions, the total standard limits expected losses (deductible basis) shall be considered to be composed entirely of excess standard limits expected losses (deductible basis) and in such cases it will not be necessary to split either the expected losses or the actual losses into the usual normal and excess divisions:

(a) When the deductible amount is equal to or greater than the normal loss amount of Table A.

(b) When the ratio of the manual rate for the deductible coverage to the manual rate for full coverage is equal to or less than the ratio given for each line of insurance in the following table:

Line of Insurance	Ratio of Deductible Rate to Full Coverage Rate
General Formula	$.70r -$ (Unallocated Cl. Exp. + H. O. Admin. + Insp. + P. A.) $= .40$*(Full Cov. Perm. L. R.)
Mfrs.' & Contrs.' P. L. O. L. & T. P. L. Product P. L. Theatre P. L.	$.70r - .19 = .40 \times .51$ $r = .56$ or less
Teams' P. L.	$.70r - .18 = .40 \times .52$ $r = .55$ or less
Elevator P. L.	$.70r -$ (.155 + Insp. Cost Ratio) $= .40 \times$ (.545 − Insp. Cost Ratio) $r = .53 + .86$ Insp. Cost Ratio, or less.

(c) When the normal credibility in all other cases calculated as provided for in Rule (5) below is less than the excess credibility determined in accordance with Rule (4).

(3) In cases other than those described under Rules (2a) and (2b), the normal and excess expected losses are determined by the following formulas:

(a) The normal expected losses (deductible basis) are equal to the product of the ratio given in the following table and the premium subject (full coverage).

Line of Insurance	Ratio Applicable to Full Coverage Premium Subject
General Formula	$.70r -$ (.40 × Full Cov. Perm. L. R. + Unalloc. Cl. Exp. + H. O. Admin. + Insp. + P. A.)
Mfrs.' & Contrs.' P. L. O. L. & T. P. L. Product P. L. Theatre P. L.	$.70r - (.40 \times .51 + .19)$ $= .70r - .394$
Teams' P. L.	$.70r - (.40 \times .52 + .18)$ $= .70r - .388$
Elevator P. L.	$.70r -$ [.40 (.545 − Insp. Cost Ratio) + .155 + Insp. Cost Ratio] $= .70r - .373 - .60$ Insp. Cost Ratio

(b) The excess expected losses (deductible basis) are obtained by applying the ratio shown in the following table to the premium subject (full coverage).

Line of Insurance	Ratio Applicable to Full Coverage Premium Subject
General Formula	.40 (Full Cov. Permissible L. R.)
Mfrs.' & Contrs.' P. L. O. L. & T. P. L. Product P. L. Theatre P. L.	$.40 \times .51 = .204$
Teams' P. L.	$.40 \times .52 = .208$
Elevator P. L.	.40 (.545 − Insp. Cost Ratio) = .218 − .40 Insp. Cost Ratio.

*Note: In the Public Liability Experience Rating Plan, the excess standard limits premium subject is equal to .40 of the total standard limits premium subject.

Credibility

(4) In all cases the excess credibility factor shall be the same as for full coverage and, therefore, shall be read from Table B using excess expected losses calculated in accordance with rule (3b).

(5) The normal credibility factor shall be determined from Table B by using the normal expected losses (deductible basis) as calculated in accordance with Rule (3a). In the event that the normal credibility factor so determined is less than the excess credibility factor as determined by Rule (4), the excess credibility factor shall be substituted and used for normal.

The derivation of the various ratios specified to be used in experience rating Public Liability risks written on a deductible basis can be reproduced by referring to the Public Liability Experience Rating Plan and to the data given in this paper showing the methods employed in calculating the discounts for deductible coverage.

Underwriting Considerations

From an underwriting standpoint, the risks which it is preferable to write on a deductible rather than on a full coverage basis are those with high accident frequency. Through writing such risks on a deductible basis, the assured is directly impressed with the necessity for introducing accident prevention measures in order to reduce his own share of the incurred losses. Many risks of this nature which would produce very unfavorable experience for the insurance company if written on a full coverage basis prove to be satisfactory when written on a deductible coverage basis. Deductible coverage for an assured's retention of such amounts as \$100 or \$250 is most frequently written on Product Public Liability risks, department stores for O. L. & T. Public Liability coverage, and Theatre Public Liability risks. Many risks of these types would be almost uninsurable on a full coverage basis but the loss experience can be controlled when the risks are written on a deductible basis because of the cooperation which is received from the assured through his realization of the monetary loss which he will directly suffer if accidents occur.

As a sales argument, it might be well to recommend deductible rather than full coverage for any fairly large risks with a tendency to produce almost no losses. Concerns of this type should be willing to carry their risk up to a nominal amount provided that the insurance company continues to furnish the necessary service and protection against severe losses. On risks of this type, a review of the past experience will indicate whether it is likely that the discount received by reason of the deductible coverage will more than offset the assured's share of the probable incurred losses.

It should be emphasized that the insurance company must retain control over the settlement of all losses, regardless of amount, and not obligate itself to consult an assured as to whether a claim should be settled or contested. Some assureds with their coverage written on a deductible basis would want every claim, regardless of merit, fought in order to avoid payment under their retention of liability, if possible. Whereas the insurance company might decide that certain claims should be settled in order to avoid the legal expense of court actions, the assured might object to making any payments under his retention unless forced to through legal judgments. Unless the insurance company retains full control of the settlement of all claims, it will be found that the cost of allocated claim expense will be increased substantially over the average experienced on risks written on a full coverage basis. In addition to incurring unusually high allocated claim expense through permitting the adoption of a policy of contesting all claims, the insurance company might find its portion of the indemnity losses increased because of substantial judgments in the case of certain claims which would have been settled out of court if the decision had been entirely in the hands of the insurance company and had not been affected by the assured's judgment.

In the settlement of losses incurred under a deductible policy, it is customary, as previously stated, for the insurance company to pay each loss in full and then to secure reimbursement from the assured for the portion of the loss for which he is liable because of his retention. The usual procedure for securing reimbursement is to bill the assured for his portion of each claim immediately after the loss is paid. Since some of the losses on a Public Liability policy may not be paid until several years after the policy has expired, the claim adjuster should always be certain that it will be possible to secure the reimbursement from the assured if the loss is paid in full. Otherwise, the insurance company should pay only its share of the incurred loss. This problem should not arise in the case of any risk for which the insurance company is still writing the current coverage.

Recommended Alternative Method of Writing Deductible Coverage

Sometimes, the criticism is expressed by risks written on a deductible basis that the insurance company is settling too many cases, regardless of liability, and that a considerable portion of the indemnity payments made must be borne by the assured because of the deductible coverage feature. In these cases, the assured undoubtedly feels that the insurance company is paying out his money in order to decrease the possibility of loss under the insurance coverage. In order to meet this criticism, the suggestion is advanced that deductible coverage might be more satisfactory and salable if it were written to provide that the insurance company and the assured would share equally the portion of any loss lower than a specified amount. For instance, instead of writing $250 deductible coverage on a particular risk, it could be provided that the insurance company and the assured would share equally the first $500 of any indemnity loss and the insurance company would pay in full the portion of any loss in excess of $500, subject to the policy limits. The maximum amount of loss which the assured would have to pay on any one claim not exceeding the policy limits would still be $250. Since the insurance company would be obligated to pay at least an equal amount with the assured in the settlement of every claim, it could no longer be accused of needlessly settling claims for amounts within the assured's retention in order to avoid incurring any loss under its portion of the coverage.

The discount for this co-insurance coverage would be 50% of the usual discount for deductible coverage equal to the total amount of loss for which the insurance company and the assured are jointly liable. For purposes of comparison, the discount for $250 deductible coverage for O. L. & T. Public Liability insurance is 37.5% whereas one-half the discount for $500 deductible coverage would be 23.8%.

It is the writer's opinion that this suggestion of writing co-insurance coverage instead of deductible coverage under certain circumstances possesses sufficient merit to justify thorough study of this proposal on the part of the committees which deal with the rate making problems for deductible coverage. It may be found that this form of coverage contains sufficient advantages to warrant its addition to the plans which are now available on an optional basis.

PART II — EXCESS COVERAGE

Rate Making Method

In calculating the rates for excess coverage when the assured's retention is less than standard limits, the same distribution of incurred losses by size of claim is employed as in calculating the rates for deductible coverage. The expense loading is treated differently, however, reflecting the difference in the degree of service which the insurance company gives under these two forms of coverage. Under excess coverage, only the provision for payroll audit expense and two-thirds of the provision for Home Office administration expense are treated as fixed. Unallocated claim expense, inspection, acquisition, taxes, profit, and one-third of the Home Office administration expense vary with the premium. Reflecting the manner in which losses are adjusted and defended under this coverage, the allocated claim expense is necessarily treated in the same manner as the indemnity cost.

The rate for excess coverage is calculated from the rate for full-coverage in this manner:

$$R_e = R\ (1.00 - \text{Discount})$$

The discount for each line of insurance is calculated by means of the following formula, where k is the percentage reduction in indemnity losses by reason of writing the coverage on an excess basis:

$$\text{Discount} = \frac{.90 \times k \times (\text{Indemnity} + \text{Allocated Claim Expense})}{1.00 - \begin{array}{l}(\text{Acquisition} + \text{Taxes} + \text{Profit} + \text{Inspection} \\ \quad + \text{Unallocated Claim Expense} + \tfrac{1}{3}\ \text{H. O. Admin.})\end{array}}$$

The formulas employed in calculating the discounts for excess coverage for the important lines of insurance are:

Mfrs.' & Contrs.' P. L. & P. D.

$$\text{Discount} = \frac{.90 \times k \times .510}{1.00 - (.25 + .025 + .025 + .015 + .08 + .025)}$$
$$= .7914k \text{ (Rounded to lower .025 interval)}$$

O. L. & T. P. L. & P. D.
Theatre P. L. & P. D.

$$\text{Discount} = \frac{.90 \times k \times .510}{1.00 - (.25 + .025 + .025 + .035 + .08 + .025)}$$
$$= .8196k \text{ (Rounded to lower .025 interval)}$$

Teams' P. L. & P. D.

$$\text{Discount} = \frac{.90 \times k \times .520}{1.00 - (.25 + .025 + .025 + .005 + .08 + .025)}$$
$$= .7932k \text{ (Rounded to lower .025 interval)}$$

The formulas for calculating the discounts for excess coverage for the Product P. L. & P. D. and the Elevator P. L. & P. D. lines are on an "*a*" rated basis.

Rate Filing with New York Insurance Department

The National Bureau's filing with the New York Insurance Department for excess coverage is similar to that for deductible coverage. The filing does not consist of a complete schedule of discounts for the various amounts of assured's retention for each line of insurance but only of the formulas to be used in calculating the discounts for the lower amounts of assured's retention, together with an explanation of a modification of these formulas to provide for the graduation of the discounts for the higher amounts of assured's retention. The discounts for the higher amounts of assured's retention are graduated by judgment to produce a discount of 100% for a \$5,000 retention per claim on a standard limits Public Liability policy, or for an assured's retention of \$1,000 per accident on a standard limits Property Damage policy. The graduation applies to the amounts of assured's retention above \$1,000 per claim for the Public Liability lines and above \$250 per accident for the Property Damage lines.

When the assured's retention is in excess of standard limits, as is frequently the situation, the rate is determined by taking the difference between the excess limits table multipliers for the upper limits desired and for the limits of the assured's retention.

Suggested Rate Making Formula

In the present rate making formula for excess coverage, it is considered that the provisions for inspection, unallocated claim expense, and one-third of Home Office administration should vary with the premium. It is the writer's suggestion that this portion of the company expenses be considered instead to vary with the losses rather than with the premium. If this adjustment is made, the formula for calculating the discount for excess coverage would be as follows:

$$\text{Discount} = \frac{.90 \times k \times (\text{Ind.} + \text{Alloc. Cl. Exp.} + \text{Insp.} + \text{Unalloc. Cl. Exp.} + \tfrac{1}{3}\text{ H. O. Admin.})}{1.00 - (\text{Acq.} + \text{Taxes} + \text{Profit})}$$

This suggestion is made because the present formula for calculating the discount for excess coverage does not allow credit for the proportion of company expenses contemplated because the denominator used in this formula is higher than the corresponding denominator employed in calculating manual rates. The recommended formula would produce results more in line with those intended by the theory underlying the application of the expense loadings in the calculation of excess rates. The discounts produced by the suggested formula would be somewhat larger than those developed by the present formula.

Per Claim vs. Per Accident Excess Coverage

The formulas given in this paper produce the indicated discounts for excess coverage on a per claim basis for the Public Liability lines and on a per accident basis for the Property Damage lines. When excess coverage on a Public Liability policy is written on a per accident basis, the discount allowed is .05 less than the discount calculated on a per claim basis for all lines except Elevator Public Liability, for which .025 is deducted from the discount applicable on a per claim basis. For the Property Damage lines, excess coverage is always written on a per accident basis and never on a per claim basis.

Minimum Premiums

The discounts for excess coverage also apply to the minimum premium for individual locations or operations on specific risks where the minimum premium is the controlling premium. In no event, however, may the discount for excess coverage operate to reduce the premium charge per policy below the minimum premium charge (if not in excess of $10.00) which would apply if the policy were canceled by the assured.

Excess Limits

The charge for excess limits coverage on a policy written on an excess basis is the same as that which would be made for the corresponding excess limits portion of the coverage on a risk written on a full coverage basis. To illustrate the manner in which the final rate is calculated for a risk written on an excess coverage basis with excess limits coverage the following example is given:

Example—O. L. & T. P. L. risk subject to Table B

50/100 limits factor = 1.320
$250 excess discount = .525

Factor applicable to 5/10 manual rate:

$$1.00 \times (1.000 - .525) = .475$$
$$.32 \times 1.000 = .320$$

Total .795

If the 5/10 manual rate were $.50, the rate for 50/100 limits, $250 excess coverage, would be .795 × $.50, which equals $.398.

Aggregate Limits

Theoretically, the aggregate limit specified for certain lines of insurance should apply on the basis of the gross amount of incurred indemnity losses, including those incurred by the assured under his retention. As a practical matter, however, it would be impossible to treat the policy limits in this manner where the insurance is written on an excess coverage basis. The insurance company would obviously not be able to maintain a record of the losses settled within the assured's retention and, for this reason, it would be necessary to provide that the aggregate limit would apply instead on the basis of the net amount of losses incurred by the insurance company under the excess coverage. The premium charge for excess coverage should reflect the extension of coverage, of course, where the aggregate liability under the insurance policy applies on the basis of the net incurred losses rather than on the basis of the gross incurred losses.

For example, the calculation of the factor applicable to the standard limits manual rate for Contractors' P. D. where the assured's retention is $5,000 per accident and the insurance company is assuming liability in excess of this amount to the extent of $25,000 per accident, with an aggregate limit of $100,000 applying on the basis of the gross amount of losses, would be as follows:

Example—Contractors' P. D. risk subject to Table II

$30,000/100,000 factor	= 1.68
$ 5,000/100,000 factor	= 1.42
Factor for $25,000 per accident coverage in excess of $5,000 per accident, with an aggregate limit of $100,000 applying on the basis of the gross incurred losses	.26

When, recognizing the impracticability of treating the aggregate policy limit in this manner, it is specified that the aggregate limit will apply instead on the basis of the net losses incurred by the insurance company, the factor calculated in the above manner should be increased somewhat to reflect the extension of coverage. This adjustment would be similar in character to that which is made in modifying the discounts calculated for deductible coverage on a per claim basis to reflect the increased insurance protection afforded when the deductible provision is to apply on a per accident basis instead.

Few risks under the lines of insurance involving aggregate limits have been written to date on an excess coverage basis. Because the whole question of aggregate limits for the casualty lines of insurance is still in the experimental stage, no definite procedure for determining the proper premium charge for risks written on an excess coverage basis has been worked out, but the method outlined above appears to offer a reasonable solution of the problem.

Classification Experience

The experience of risks written on an excess coverage basis is excluded from the classification experience employed in deriving manual rates. The experience of all risks written on an excess coverage basis is reported in total under a specified code number for each line of insurance. No attempt is made to compile a record by excess amount, because the volume of experience developed is too limited to justify such refinement for statistical purposes.

Experience Rating

The Public Liability Experience Rating Plan is applicable on an intra-state basis in three states: Minnesota, New York and Wisconsin. A Public Liability risk written on an excess coverage basis qualifies for experience rating if it has developed an exposure during either the latest year or the latest two years of the experience period such that the application thereto of the manual rates for full coverage (standard limits only) produces a premium of the same amount as required for a full coverage risk to qualify for experience rating.

The experience rating of Public Liability risks is in accordance with the coverage to be provided on renewal. Full coverage experience is adjusted to an excess coverage basis if the risk is to be written on the latter basis on renewal. Conversely, any experience developed on an excess coverage basis should theoretically be built up to a full coverage basis before using in the experience rating calculation if the risk is to be afforded full coverage on renewal. As a practical matter, this latter adjustment would be very difficult, if not impossible, because the insurance company would not have a record of the losses incurred by the assured under his retention.

In developing the experience rating modification for a risk which is to be written on an excess coverage basis on renewal, the following changes in the Public Liability experience rating plan are necessary:

Actual Losses

The actual losses experienced under full coverage are reduced to an equivalent excess coverage amount by subtracting the assured's retention from the indemnity payments. Allocated loss expense and medical losses are excluded, except where the allocated loss expense was incurred with the insurance actually written on an excess coverage basis. In dividing the adjusted actual losses between normal and excess, the assured's retention is first subtracted from the normal loss amount of Table A and the remainder is used as the normal amount for the excess coverage.

Expected Losses

(1) The total expected losses on the excess coverage basis are obtained by multiplying the full coverage premium subject at standard limits by the ratio given below for each line of insurance, where r is the ratio of the manual rate for excess coverage to the manual rate for full coverage $\left(r = \frac{R_e}{R}\right)$:

Line of Insurance	Ratio Applicable to Full Coverage Premium Subject
General Formula	r — [Full Coverage Expense Loading — (1.000—r) × Expense Loading in Excess Discount]
Mfrs.' & Contrs.' P. L.	$r - [.490 - (1.000 - r)\ .420]$ $= .58r - .07$
O. L. & T. P. L. / Theatre P. L.	$r - [.490 - (1.000 - r)\ .440]$ $= .56r - .05$
Teams' P. L.	$r - [.480 - (1.000 - r)\ .410]$ $= .59r - .07$

For example, for the O. L. & T. Public Liability line, the total expected losses for a risk written on a $250 excess coverage basis for which the discount is .525 would be equal to .216 times the full coverage premium subject ($.56 \times .475 - .050 = .216$).

(2) Under *any* of the following conditions, the total standard limits expected losses (excess coverage basis) shall be considered to be composed entirely of excess standard limits expected losses (excess coverage basis) and in such cases it will not be necessary to split either the expected losses or the actual losses into the usual normal and excess divisions:

(a) When the assured's retention is equal to or greater than the normal loss amount of Table A.

(b) When the ratio of the manual rate for the excess coverage to the manual rate for full coverage is equal to or less than the ratio given for each line of insurance in the following table:

Line of Insurance	Ratio of Excess Rate to Full Coverage Rate
General Formula	r — [Full Cov. Expense Loading — (1.000 — r) × Expense Loading in Excess Discount] = .40 (Full Cov. Perm. L. R.)
Mfrs.' & Contrs.' P. L.	$r - [.490 - (1.000 - r)\ .420] = .40 \times .51$ $.58r - .070 = .204$ $r = .47$ or less
O. L. & T. P. L. / Theatre P. L.	$r - [.490 - (1.000 - r)\ .440] = .40 \times .51$ $.56r - .050 = .204$ $r = .45$ or less
Teams' P. L.	$r - [.480 - (1.000 - r)\ .410] = .40 \times .52$ $.59r - .070 = .208$ $r = .47$ or less

(c) When the normal credibility in all other cases calculated as provided for in Rule (5) below is less than the excess credibility determined in accordance with Rule (4).

(3) In cases other than those described under Rules (2a) and (2b), the normal and excess expected losses are determined by the following formulas:

(a) The normal expected losses (excess coverage basis) are equal to the product of the ratio given in the following table and the premium subject (full coverage):

Line of Insurance	Ratio Applicable to Full Coverage Premium Subject
General Formula	$r -$ [Full Cov. Expense Loading $- (1.000 - r)$ $\times$ Expense Loading in Excess Discount $+ .40$ (Full Cov. Perm. L. R.)]
Mfrs.' & Contrs.' P. L.	$r - [.490 - (1.000 - r)\,.420 + .40 \times .51] = .58r - .274$
O. L. & T. P. L. Theatre P. L.	$r - [.490 - (1.000 - r)\,.440 + .40 \times .51] = .56r - .254$
Teams' P. L.	$r - [.480 - (1.000 - r)\,.410 + .40 \times .52] = .59r - .278$

(b) The excess expected losses (excess coverage basis) are obtained by applying the ratio shown in the following table to the premium subject (full coverage):

Line of Insurance	Ratio Applicable to Full Coverage Premium Subject
General Formula	.40 (Full Cov. Perm. L. R.)
Mfrs.' & Contrs.' P. L.	$.40 \times .51 = .204$
O. L. & T. P. L. Theatre P. L.	$.40 \times .51 = .204$
Teams' P. L.	$.40 \times .52 = .208$

Credibility

(4) In all cases the excess credibility factor shall be the same as for full coverage and, therefore, shall be read from Table B using excess expected losses calculated in accordance with Rule (3b).

(5) The normal credibility factor shall be determined from Table B by using the normal expected losses (excess coverage basis) as calculated in accordance with Rule (3a). In the event that the normal credibility factor so determined is less than the excess credibility factor as determined by Rule (4), the excess credibility factor will be substituted and used for normal.

The derivation of the various ratios specified to be used in experience rating Public Liability risks written on an excess coverage basis can be reproduced by referring to the Public Liability Experience Rating Plan and to the data given in this paper showing the methods employed in calculating the discounts for excess coverage.

Underwriting Considerations

Excess insurance is mainly written on very large risks which self-insure the smaller amounts of loss but wish to purchase insurance protection against an unusual or catastrophic loss, and on those risks which insure the primary portion of their coverage in one company and purchase the higher limits coverage from another. A large part of this excess coverage is written by London Lloyds, undoubtedly because the premium charge is less than that determined by the rating methods which have been established for this coverage by the Bureau companies.

Except for coverage which involves a severe catastrophe hazard, such as on oil refining operations or on theatres, it may be considered that business written on an excess coverage basis is desirable provided that the assured's retention is a fairly large amount.

Summary

As stated previously, the main purpose of this paper was to assemble the available data on rate making, etc. which may be published for deductible and excess coverages. In addition, certain observations and suggestions have been advanced by the writer with regard to the rating methods and insurance practices for these coverages. The information included in this paper may prove to be of help to underwriters and others in the writing of business under either of these forms of coverage. Undoubtedly, some important points may have been omitted unintentionally by the writer, but it is likely that any such matters will be treated in the written discussions of this paper.

EXPERIENCE RATING PLAN CREDIBILITIES

BY

FRANCIS S. PERRYMAN

For some time past certain criticisms have been made of the Compensation Experience Rating Plan. These have touched on various aspects of the Plan; some of them have been directed to the way in which the Plan works in particular instances. Other criticisms of the Plan have been in respect of some of the more debatable questions such as the period of experience to be used and the swing of the plan. This is the old question of Stability *vs.* Responsiveness and some of the critics have shown a surprising tendency to ignore the essential conflict between these two qualities. With these criticisms, those responsible for setting up and administering the Plan can doubtless deal. It is not in any way my intention to do more than mention them here as leading up to the subject of this paper. The Experience Rating Plan has recently been the subject of intensive studies by the responsible committees with the objects of seeing what there is of merit in the criticisms and of endeavoring to revise the Plan to make it better adapted to present-day conditions. The lessons gained from the, on the whole, successful working of the Plan over a large number of years are, of course, the principal guides in such studies.

One of the ideas being thus investigated is to see whether the Plan could not be simplified, particularly in the actual day-to-day process of rating, which is largely done by clerical help not particularly well trained in actuarial science, and scrutinized by agents, brokers, field men and assureds who, again, are not generally experts in casualty rate-making. One specific suggestion is that considerable simplicity would be obtained if, in respect of the small and medium-sized risks which are a great majority of the total number of rated risks, the large or excess loss experience were not rated. This idea has a lot of merit and the main purpose of this paper is to help it along by working out, systematically, the way in which the credibilities should be handled under such a plan. In effect under it the excess credibility will be zero unless the size of the risk is large, and considerable research and testing has to be done to be sure that such a plan will give consistent results and that the excess experience can be worked in satisfactorily for large risks.

In order to present a logical account of this investigation it is necessary first to give a fairly full account of the treatment of credibility under the present form of the Plan and this is done in the first two parts of the paper. The remaining parts are devoted, first, (since it seemed desirable to discuss some definite plan) to a brief description of a concrete plan, the multi-split plan,* which gives no excess credibility except for large risks. The balance of the paper is given up to a full discussion, with examples, of the determination of credibilities under this Plan.

While the paper discusses a particular Compensation Experience Rating Plan, I have tried to treat the question in such a way as to bring out the principles that should be used with the thought that these principles will be applicable to any similar experience rating plan, whether for Compensation or for any other kind of insurance, for which experience rating is suitable.

Part I

Credibilities in No Split Plans

1. *Analysis of Modification for Simplest Case—No Split Plan.*

First of all we will deal with the case of an experience rating plan with no splits, that is, where all losses (loss costs) are used with equal weight. In this case the ordinary formula for the modification (that is, the multiplier to be applied to manual rates) is

$$\frac{ZA + (1 - Z)E}{E} \tag{1}$$

where A denotes the actual losses
E denotes the expected losses
and Z is the credibility assigned to the risk.

In this paper I will not deal with questions of loss or payroll modification factors, or the number of years experience used, and

* I want to make it clear that no implication is intended that I was the originator of the multi-split plan. I wish I had been.

will assume that these are all incorporated in the "actual" and "expected" losses.

This modification can be put in the form (which I shall often have occasion to use later)

$$1 - Z + Z\frac{A}{E} \tag{2}$$

Note that this expression is in three parts:—

(i) unity, corresponding to no change from manual rates, as, for instance, if $Z = 0$

(ii) $-Z$, being the credit for clear experience, that is, if $A = 0$

and (iii) $+Z\frac{A}{E}$ being the charge for the actual losses of A.

2. *K Formula for the Credibility.*

The values to be given to Z in this modification are usually determined from the formula

$$Z = \frac{E}{E + K} \tag{3}$$

where K is a constant, i.e., does not vary with E.

Substituting this in (1) we get

$$\frac{A + K}{E + K} \tag{4}$$

In practice we can obtain the modifications either from (1) or from (4). If we use (1) we must have a reference table of Z from which to get the value to be substituted in (1). If we use (4) we need only to know the value of K. It is therefore somewhat easier to use (4) in this simple case but, as we shall see, when we come to use a split plan with provision for self rating for large risks, it is then easier to use a formula analogous to (1).

The value of K is determined from consideration of the "swing" it is desired to give the plan. K is usually fixed so as to give for a certain sized risk a definite credit (e.g., 10%) for clear experience or a definite charge (say 25%) for a single maximum loss.

The expression (3) gives for Z a value between 0 and 1, continually increasing as E increases but never quite reaching unity. In fact if Z is plotted as a function of E, Z moves along a branch of a hyperbola which has $Z = 1$ as an asymptote. (See Fig. I).

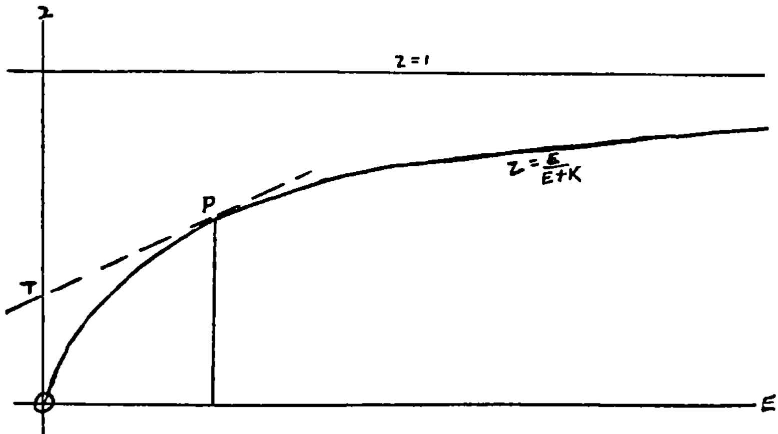


Fig. I.

3. *Conditions to which Z must be Subject.*

At this point it is advantageous to set down some conditions that the credibility Z should satisfy. These are general conditions derived from *a priori* considerations, and are applicable to the more complicated rating formulas we shall consider later.

(i) The credibility should be not less than zero and not greater than unity.

(ii) The credibility should increase (or more strictly speaking not decrease) as the size of the risk increases.

(iii) As the size of the risk increases the percentage charge for any loss of given size should decrease.

(i) and (ii) are obvious requirements; (iii) is perhaps not quite as evident at first, but a little thought will show it is desirable that, given two risks with differing expected losses, then if both have a single actual loss of the same amount the addition to the modification on account of the single loss should be less for the larger risk.

For instance, if we have two risks, the first with expected losses of 1,000 and the second with expected losses of 10,000: if each have a loss of 5,000, then on account of this loss

(a) by (i) above the addition to the premium in each case is positive and not greater than the equivalent of the 5,000 loss (that is if the expected loss ratio is 60%, the addition is not more than 8,333);

(b) by (ii) the addition is greater for the second risk than for the first; and

(c) by (iii) the addition is a *smaller* percentage of the (manual) premium for the second risk than for the first.

If we consider large self rated risks the reasons for (iii) becomes perhaps clearer: For these risks the addition to the premium is the same for a given loss of say 5,000, whatever the size of risk (for example the addition is 8,333 if the expected loss ratio is 60%) but the percentage addition gets smaller as the risk gets bigger.

The conditions mentioned can be expressed mathematically as

$$\left.\begin{array}{rl} \text{(i)} & 0 \leq Z \leq 1 \\ \text{(ii)} & Z' \text{ is not negative} \\ \text{(iii)} & (Z/E)' \text{ is negative} \end{array}\right\} \qquad (5)$$

where to economize space and to facilitate printing we have employed the common notation of Z' for $\frac{dZ}{dE}$: similarly we write W' for $\frac{dW}{dE}$, M' for $\frac{dM}{dE}$ and so on where W, M, etc. are functions of E. All differentiations are to be understood to be with respect to E. We have also written above Z/E for the constantly occurring expression $\frac{Z}{E}$ and we shall often employ this notation. $(Z/E)'$ means of course $\frac{d}{dE}\frac{Z}{E}$. We shall also often say "Z increases" or "Z/E decreases" meaning "Z increases as E increases" or "Z/E decreases as E increases" as will be clear from the context.

It is easily seen that Z as determined by (3) fulfills these conditions: for as E is positive (and K also) Z is > 0 and < 1: also $Z' = K/(E+K)^2$ and is positive, while $(Z/E)' = -1/(E+K)^2$ which is negative.

A useful geometrical interpretation of the conditions is as follows:

Plotting Z as a function of E (as in Fig. I which shows the curve $Z = E/(E + K)$)

(i) means the curve must be bounded by the E axis $Z = 0$ and by the straight line $Z = 1$ parallel to it;

(ii) means that as E increases the curve must always rise from $Z = 0$ towards $Z = 1$ or at most be parallel to the E axis or in other words the tangent at P must slope upwards from left to right or at most be parallel to the E axis;

(iii) means that the tangent must pass above the origin O and cut the Z axis above O; for the tangent at P cuts the Z axis at T where $OT = Z - EZ'$, (where E, Z are the co-ordinates of P), and the condition $(Z/E)' = (EZ' - Z)/E^2$ is negative means that $Z - EZ'$ is positive.

4. *Self Rating.*

In paragraph 2 we have seen that formula (3) for Z gives values that continually approach unity as E increases but never reach that value.

For practical reasons it is often desirable that for risks over a certain size the credibility Z be exactly unity. This certain size is called the self rating point and risks with credibilities equal to unity are called self-rated risks. We will denote the value of E at the self rating point by S. So for $E \geq S$, Z must be unity.

The question now arises as to the proper way to modify formula (3) so as to reach unity at S. Originally all that was done was to draw a straight line from some arbitrary point $(Q_1, Q_1/(Q_1 + K))$ to the self rating point $(S, 1)$ (see Fig. II) and use for Z between Q_1 and S the values given by this line. This however gives discontinuity to the values of Z at Q_1 and at S. So instead of using an arbitrary point Q_1, a tangent was drawn from the point $(S, 1)$ touching the curve $Z = E/(E + K)$ at $E = Q_2$. This is the present practice and does away with the discontinuity at Q_2 but leaves that at S. It would have been better, while making the change to have drawn a curve (e.g., a second degree parabola) touching the line $Z = 1$ at $E = S$ and also touching the curve $Z = E/(E + K)$ at $E = Q$. (See Fig. II).

(*Note*: We shall use Q generally to denote the value of E at the point of departure from the original credibility curve.)

Let us work out the equations of the tangent $s\ q_2$ and the touching parabola $s\ q$.

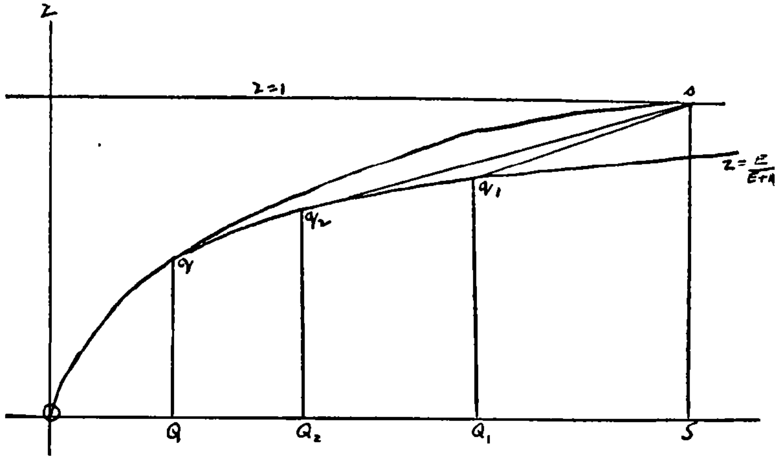


Fig. II.

The tangent to the (hyperbola) $Z = E/(E + K)$ at the point $E = Q_2$, $Z = Q_2/(Q_2 + K)$ is

$$Z = \frac{E\,K + Q_2{}^2}{(Q_2 + K)^2}$$

and this passes through $E = S$, $Z = 1$ if

$$\left.\begin{aligned} Q_2 &= \frac{S - K}{2} \\ \text{The tangent is then } Z &= 1 - \frac{4\,K}{(S + \mathrm{K})^2}\ (S - E) \end{aligned}\right\} \qquad (6)$$

A simple parabola of the m-th degree, $Z = 1 - H\ (S - E)^m$, where H is a constant and m is $\nless 1$ will touch $Z = 1$ at $E = S$. It will also touch $Z = E/(E + K)$ at $E = Q$ if

$$\frac{K}{Q + K} = H\ (S - Q)^m \text{ for each must equal } 1 - Z$$

$$\frac{K}{(Q + K)^2} = H\ m\ (S - Q)^{m-1} \text{ for each must equal } Z'$$

from which

$$\left.\begin{aligned} m &= \frac{S - Q}{Q + K} \\ H &= \frac{K\,m}{(S - Q)^{m+1}} \end{aligned}\right\} \qquad (7a)$$

Thus we can either (i) choose m (greater than one) and

$$\left.\begin{aligned} \text{then } Q &= \frac{S - mK}{m+1} \\ \text{and } H &= \frac{K\,(m+1)^{m+1}}{m^m\,(S+K)^{m+1}} \end{aligned}\right\} \tag{7b}$$

or (ii) choose Q (which must be less than $Q_2 = (S - K)/2$) then m and H can be calculated from equations (7a). If Q is taken as zero $m = S/K, H = S^{-\frac{S}{K}}$.

Thus by taking $m > 1$ and $\leq S/K$, or $Q \geq 0$ and $< (S - K)/2$ we can obtain the equation of a simple parabola (not usually a second degree parabola) which touches the credibility curve $Z = E/(E + K)$ at Q and touches the line $Z = 1$ at S. The credibility to be used will be that given by $Z = E/(E + K)$ from 0 to Q, that given by $Z = 1 - H\,(S - E)^m$ from Q to S, and $Z = 1$ for $E > S$.

To determine which parabola (or which value of Q to use, which is the same thing) other considerations (such as the credibilities to be given for various values of E) have to be invoked. Probably for most purposes the second degree parabola obtained by putting $m = 2$ will be satisfactory. For this

$$\left.\begin{aligned} m = 2 \qquad\qquad Q &= \frac{S - 2K}{3} \\ Z &= 1 - \frac{27\,K\,(S - E)^2}{4\,(S + K)^3} \end{aligned}\right\} \tag{8}$$

This is (in an unfamiliar guise or disguise) the familiar "square root" formula used elsewhere in casualty actuarial science as a credibility formula.

Note that the case of the tangent can be deduced by putting $m = 1$.

Note also that if Q is made equal to zero we use the parabola all the way from 0 to S and the original credibility curve has apparently been dropped entirely. Its influence, however, is still present in determining the slope of the parabola at $E = 0$. This case can of course be treated separately as the use of a family of curves:—

$$Z = 1 - \left(\frac{S - E}{S}\right)^m \tag{9}$$

where the parameter m has to be settled from other considerations such as the swing to be given to the plan. It will probably be found in many cases that a credibility curve of this type will rise too fast, or in other words if it gives satisfactory values for small values of E it will give too large values for intermediate values. For example this would usually be so if we took $m = 2$ to get the "square root" formula.

It is important to note that as all the parabolas suggested are concave to the E axis the conditions (5) of paragraph 3 are complied with. Z is between 0 and 1, Z' is positive and so is $Z - EZ'$. This is also true of the straight line tangent.

In applying credibilities as thus adjusted to rise to unity at the self rating point it would be very complicated to use the formula in each case, as suggested for the second alternative method in paragraph 2. It is apparently better to use the first alternative there mentioned and have a table of Z values to which reference may be made to get the proper value for a given E; in other words to use as a working formula (1) as opposed to a modified (4).

5. *Another Method of Reaching Self Rating.*

The last sentence represents the general view in the past. However, we can retain most of the advantages of using a formula like (4) by proceeding as follows:—For values of E greater than Q calculate K_E from

$$Z = \frac{E}{E + K_E}$$

where Z is the credibility value from the parabola: thus $K_E = E\,(1 - Z)/Z$. Construct a table for K_E for all values of E, putting $K_E = K$ for $E < Q$. Then apply formula (4) thus

$$\text{modification} = \frac{A + K_E}{E + K_E}. \tag{10}$$

By this method the great majority of risks will be rated by the simple formula (4) with a constant K and for large risks all that is necessary is to ascertain the value of K_E and use the same simple formula. In practice, however, the complications introduced by the present method of splitting into normal and excess would preclude the adoption of this scheme.

This suggests, nevertheless, another method of attaining self rating, namely, by using (4) and gradually reducing the constant

K as E goes from Q to S. Thus if we were to construct values of K_E so that, at Q, $K_E = K$ and $K'_E = 0$ and, at S, $K_E = 0$ and $K'_E = 0$ we would get credibility values which would join smoothly with those given by $Z = E/(E + K)$ at Q and with $Z = 1$ at S.

We will not at present pursue this further, but as will be seen later this idea is used in the more complicated questions of split plans and multi-split plans.

6. *Justification for Departing from Usual Credibility Formula.*

At this point it would seem desirable to see what theoretical objections there may be to departing from the usual or standard credibility formula $Z = E/(E + K)$ or, to put it the other way, whether we can justify departures such as dealt with above. The first thing to be remembered here is that the standard credibility formula itself does not give an exact measure of the proper credibility that shall be given to the risk experience. It is an approximation to an approximation of an expression for the credibility that was based on some necessarily rather arbitrary assumptions as will be seen from the classic papers of Messrs. Whitney and Michelbacher, (P.C.A.S., Vol. IV), describing the genesis of the present form of experience rating. I do not mean to be understood to be attacking the general validity of the usual formula or to be advocating its abandonment. The formula is a very satisfactory, practical instrument that gives credibility values conforming in a reasonable manner to what we would expect and it is because of this that it has stood the test of time. I do mean to state, however, that any not too violent departures from the formula arising out of the self-rating adjustments given in the preceding paragraph cannot be condemned merely for the reason that they are departures. If—as they do—these departures give values that also are reasonable in the light of our a priori judgment and that conform to the criteria of paragraph 3, then our system of credibility values is just as defensible as those given by the unadulterated standard credibility formula.

To anticipate a little so as to collect together all the remarks on departure from the standard formula, similar considerations apply to the usual form of split plan dealt with in Part II. As for the multi-split plan dealt with in the remainder of the paper, the question there arises as to the validity of the method used of handling the excess credibility. This is kept at zero for small and medium-sized risks and for large risks is brought up to unity at the self-rating point. If the excess portion is considered by itself there is little theoretical justification for this procedure but excess experience *is* excess and always arises in connection with the corresponding normal experience and never by itself, so we must consider the normal and excess parts together. Then whether we look at the risk's average or over-all credibility or whether we look at the effect of any reasonable combination of normal and excess experience we will find that the credibilities by the multi-split plan are not unreasonable.

PART II

CREDIBILITIES IN SPLIT PLANS

7. *Application to "Split" Plans.*

So far we have dealt with a no-split plan as explained in paragraph 1. We now shall consider the necessary modifications of the preceding theory so as to apply it to a split plan. It is not my intention to deal with the history of experience rating (for which see Mr. Kormes' recent papers, P.C.A.S., Vols. XXI and XXII) and so I will merely state here that almost invariably losses (both Actual and Expected) are divided into "normal" and "excess," that is to say the risk is considered in two parts; first, the experience on losses limited to a certain amount per case (say $1,000 indemnity and $100 medical), this being the "normal" part; and second, the experience on the loss cost in excess of this certain amount, this being the "excess" part. The expected losses are divided in the same way (from the available statistics) and the final rate for the risk is the sum of the adjusted rates for each of the two parts.

Less credibility is given to the excess losses since they are more unusual. The reason for making the split is fairly obvious. Without a split a single loss of, say, 3,000 gets as much weight as six losses of 500 each and it is both theoretically and practically desirable to give the six losses much more weight.

The rating formula is as follows where E_n, A_n, Z_n denote the normal expected losses, actual losses and credibility respectively and E_e, A_e, Z_e are the same for the excess part, (note that $E_n + E_e = E$ and $A_n + A_e = A$).

$$\text{Modification} = \frac{E_n}{E} \frac{Z_n A_n + (1 - Z_n) E_n}{E_n} + \frac{E_e}{E} \frac{Z_e A_e + (1 - Z_e) E_e}{E_e}$$

$$= \frac{Z_n A_n + (1 - Z_n) E_n + Z_e A_e + (1 - Z_e) E_e}{E} \qquad (11)$$

If as usual we use

$$\frac{E_n}{E_n + K_n} \text{ for } Z_n \text{ and } \frac{E_e}{E_e + K_e} \text{ for } Z_e$$

(where by making K_e much larger than K_n we give much less credibility to the excess losses) we get for the modification

$$\frac{E_n}{E} \frac{A_n + K_n}{E_n + K_n} + \frac{E_e}{E} \frac{A_e + K_e}{E_e + K_e}$$

which is not subject to much simplification for working purposes. In fact, it is easier to read Z_n and Z_e out of a prepared table and apply (11) particularly as (i) the normal and excess ratios E_n/E and E_e/E vary for risks according to the classifications involved and (ii) by using (11) it is easy to modify Z_n and Z_e (in accordance with the principles set out in Part I) to attain self-rating at S_n and S_e respectively (these self-rating points usually differ). Z_n and Z_e are usually brought to self-rating by means of tangents as shown in paragraph 4, equations (6), although I think it would be better to use a second degree parabola as per equations (8).

It is to be noted that since both Z_n and Z_e comply with the conditions (5) of paragraph 3, so does also the combination of the two in (11) whatever be the proportions of the normal and the excess portions.

8. *Analysis of Split Plan Modification.*

It is useful to note (for it will be needed later) the following analysis of (11).

$$\left.\begin{array}{l} 1 - Z_n \dfrac{E_n}{E} + Z_n \dfrac{A_n}{E} - Z_e \dfrac{E_e}{E} + Z_e \dfrac{A_e}{E} \\[2ex] \text{or } 1 + \dfrac{E_n}{E}\left\{- Z_n + Z_n \dfrac{A_n}{E_n}\right\} + \dfrac{E_e}{E}\left\{- Z_e + Z_e \dfrac{A_e}{E_e}\right\} \end{array}\right\} \qquad (12)$$

This is analogous to the analysis in paragraph 1 of expression (1) into (2): here the parts are:

(i) unity $\left(\text{equal to } \dfrac{E_n + E_e}{E}\right)$.

(ii) (a) $-Z_n\, E_n/E$ the credit for clear normal experience.

(b) $-Z_e\, E_e/E$ the credit for clear excess experience.

(iii) (a) $+Z_n \dfrac{A_n}{E}$ or $Z_n \dfrac{A_n}{E_n} \cdot \dfrac{E_n}{E}$ the charge for the actual normal losses of A_n.

(b) $+Z_e \dfrac{A_e}{E}$ or $Z_e \dfrac{A_e}{E_e} \cdot \dfrac{E_e}{E}$ the charge for the actual excess losses of A_e.

PART III

THE MULTI-SPLIT PLAN—DERIVATION OF FORMULAS

9. *The Multi-Split Plan.*

The present state of the experience rating plan (as far as the scope of this paper is concerned) is practically as described in Part II. Recently, however, studies have been made with a view to improve the plan and the remainder of this paper arose out of considering some aspects of suggestions which took the form of (i) advocating the so-called multi-split plan and (ii) endeavoring to reduce the working formula to as simple a form as possible, the aim being something like (4).

The so-called multi-split plan consists of a different way of dividing the total losses into "normal" and "excess", or rather as originally proposed, it reduced all losses to normal losses leaving out of account the remainder (or excess) losses, which are not so great as under the ordinary plan. The principle invoked is to take the first (say) 500 of each loss at its face value, the next 500 at (say) two-thirds of its actual value or at a reduction of one-third, the next 500 at another one-third reduction, namely, four-ninths of its actual value, and so on. Thus a very large loss could not be taken at more than 1,500 (using the above values which are illustrative only). The reduction is achieved by means of a table of discounted values showing the discounted value to be used for each size of loss exceeding 500. For losses not greater than 500 the full value is to be used. Thus a loss of 1,000 would have a discounted value of 833 (equal to 500 plus two-thirds of 500), a loss of 1,500 a discounted value of 1,055 (equal to 833 plus two-thirds of two-thirds of 500) and so on. Intermediate values (e.g. for a loss of 800) would be shown in the tables, calculated from the formula:—

$$\text{Discounted value for loss of } x \ (x > 500) = 1{,}500\left\{1 - (2/3)^{\frac{x}{500}}\right\}$$

or if a is the starting point (corresponding to the 500 above) and ρ (< 1) is the discounting ratio (corresponding to the $2/3$ above)

$$\text{Discounted value for loss of } x \ (x > a) = a\,\frac{1-\rho^{\frac{x}{a}}}{1-\rho} \qquad (13)$$

The maximum discounted value is obviously $a/(1-\rho)$.

From the risk's experience the discounted losses A_n would be determined (it being necessary to enter the table of discounted values only for losses $> a$) and from collective statistics the corresponding expected discounted losses E_n would be determined.

From A_n and E_n by a simple credibility formula (several suggestions as to this are given below) the risk's modification would be calculated. For the great majority of risks, no attention would be paid to the "remainder" losses $A - A_n$ (or excess losses) the experience on these being brought in only above a certain size of risk (i.e., after a certain Q point) to attain ultimate self-rating (at a certain S point).

It is not my purpose here to go into the details or to discuss the soundness or otherwise, or the merits and demerits of the multi-split plan except to say that I believe the idea to be a good one (better than the current split-plan) and that the discounted values given by the exponential curve (13) seem, from tests and from theoretical considerations, to give a good approximation to the relative weight that should be given to losses of various sizes. I hope to give a fuller account of these tests, theoretical and practical, at another time. In this paragraph I have given the above brief account of the plan so as to render intelligible the ideas of the remainder of this paper which is concerned with the credibility formulas to be used in connection with the multi-split plan *or any other plan* where the excess credibility used is zero up to a certain (Q) point and then is gradually brought up to unity at a self-rating (S) point as is in effect done in the multi-split plan. In any case it is not desirable to pass judgment on the multi-split plan until an exploration has been made of how to manage the credibilities this plan is to grant. It is the main purpose of this paper to do some of this exploring.

10. *First Formula for the Modification.*

The first formula we shall consider for the modification to be used in the multi-split plan is arrived at in this way.

If in (11) we put $Z_e = 0$ we get

$$\frac{Z_n A_n + (1 - Z_n) E_n + E_e}{E}$$

and now if, for simplicity, we put $Z_n = E/(E + K)$ (instead of the usual $E_n/(E_n + K_n)$) we get

$$\frac{A_n + E_e + K}{E + K}$$

and we take this for the modification when $E \leq Q$, when $Z_e = 0$.

Now we can get self-rating by adding $A - (A_n + E_e + K)$ or $A_e - E_e - K$ to the numerator of this expression and subtracting $(E + K) - E$ or K from the denominator: we accordingly use for the modification for $E > Q$

$$\frac{A_n + E_e + K + W\,(A_e - E_e - K)}{E + K - W\,K}$$

where W is to be zero for $E \leq Q$ and unity for $E \geq S$, and in between zero and unity for E between Q and S.

Thus:

$$\left.\begin{aligned} \text{Modification} &= \frac{A_n + E_e + K}{E + K} \\ &\qquad \text{for } E \leq Q \\ \text{and} &= \frac{A_n + E_e + K + W\,(A_e - E_e - K)}{E + K\,(1 - W)} \\ &\qquad \text{for } E > Q \text{ and } \leq S \end{aligned}\right\} \qquad (14)$$

where W is a function of E (to be determined), equal to zero for $E = Q$ and rising from 0 to 1 as E goes from Q to S.

This is perhaps not quite as simple as a formula (see (31)) to be considered later but I deal with it first because of the greater ease of handling the theoretical work.

It will be observed that if $A_n = E_n$ (and $A_e = E_e$ if $E > Q$) the modification equals unity as it should.

Now (14) can be analyzed into:

$$\left.\begin{array}{ll} 1 - \dfrac{E_n}{E+K} + \dfrac{A_n}{E+K} & \text{for } E < Q \\ \text{and } 1 - \dfrac{E_n}{E+K(1-W)} + \dfrac{A_n}{E+K(1-W)} & \\ \quad - \dfrac{W E_e}{E+K(1-W)} + \dfrac{W A_e}{E+K(1-W)} & \text{for } E > Q \end{array}\right\} \quad (15)$$

whence by a comparison with (12)

$$\left.\begin{array}{ll} Z_n = \dfrac{E}{E+K}, \; Z_e = 0 & \text{for } E < Q \\ Z_n = \dfrac{E}{E+K(1-W)}, \; Z_e = \dfrac{W E}{E+K(1-W)} & \text{for } E > Q \end{array}\right\} \quad (16)$$

We see that $Z_n = 0$ for $E = 0$

and $Z_n = Z_e = 1$ for $E = S$ where $W = 1$

also $Z_n > Z_e$ for $E < S$ (except for $E = 0$)

It will be noted that here, and this is true generally of the multi-split plan as we shall discuss it, that there is only one self-rating point, not one for normal losses and one for excess as in the case of the present plan. This is deliberately done as one means of simplification, and is justifiable if the self-rating point is not too low.

11. *Conditions for W to fulfill.*

Before proceeding to the determination of W, it is necessary to consider how this function must behave. We see at once that as well as $W = 0$ for $E = Q$ and $W = 1$ for $E = S$, we must have $W' = 0$ for $E = Q$ and for $E = S$ in order that we have smooth junctions with $Z_n = E/(E+K)$ and $Z_e = 0$ at $E = Q$ and with $Z_n = Z_e = 1$ at $E = S$.

Furthermore we must also determine W in such a manner that the credibilities comply with the conditions (5), paragraph 3. For $E < Q$, Z_n obviously complies with these (as has been shown above) and so does Z_e and therefore so does any combination of Z_n and Z_e.

For $E > Q$ both Z_n and Z_e comply with (5i) but, on the other hand, Z_e *cannot* comply with (5iii) as will readily be seen from the geometrical interpretations of this condition given in paragraph 3. As Z_e has to rise from zero at $E = Q$ to unity at $E = S$ the tangent to the curve $Z_e =$ function of E must, at any rate for the first part of the range $E = Q$ to $E = S$ cut the Z_e axis below the origin (see Fig. III). This of course applies to all varieties of plan where $Z_e = 0$ up to a point $E = Q$ and then rises to unity at a point $E = S$, in such a manner that there is a smooth junction at Q.

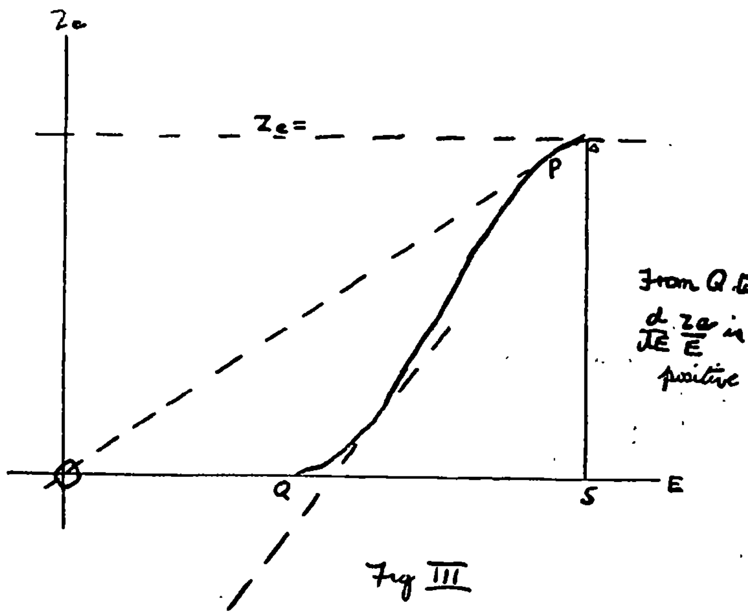


Fig III

Let us consider, however, any single loss and let the ratio of the excess portion of this to the normal be θ.

Since we can have a "normal" loss with no excess portion but cannot have an "excess" loss without a corresponding "normal" portion, it follows that θ can range from 0 to some maximum value which we will call α. To take the illustration given in paragraph 9 where $a = 500$ and $\rho = 2/3$, if the maximum possible actual loss is 7,500, it is easily seen that α will be very nearly equal to 4 for the maximum normal loss is 1,500. (The actual value of α in this case is 4.01).

Then it is $Z_n + \theta Z_e$ which must comply with the conditions (ii) and (iii) of (5) and for all possible values of θ. Since θ can be zero, Z_n must certainly comply with these conditions; and then $Z_n + \theta Z_e$ will also comply for all values of θ if it complies for the maximum value of θ regardless of whether Z_e complies or not, for the conditions in question are linear in Z_n and Z_e. Thus we must have Z_n and $Z_n + \alpha Z_e$ (which we will call ζ) both complying with (ii) and (iii). As regards condition (ii) it is desirable (but not necessary) that Z_e also comply (and this can be arranged.)

We observe that at Q $\zeta/E = 1/(Q + K)$ and at S it equals $(1 + \alpha)/S$ so that for ζ/E to decrease from Q to S as required by conditions (iii) we must have

$$S > (1 + \alpha)\,(Q + K) \tag{17}$$

This is of course a condition limiting the choice of S when Q has been chosen and vice versa.

12. *Examination of Conditions*

We see from (16) and from

$$\zeta = Z_n + \alpha Z_e = \frac{E\,(1 + \alpha W)}{E + K\,(1 - W)} \tag{18}$$

that we can either determine W directly or first settle on ζ from which we can get W and the other functions. Before deciding which we will do we shall first collect together and "boil down" the requirements that must be fulfilled.

A. Terminal Conditions

(i) W must be 0 at $E = Q$ and 1 at $E = S$
W' must be 0 at Q and at S

(ii) Z_n must be $Q/(Q + K)$ at $E = Q$ and 1 at $E = S$
Z'_n must be $K/(Q + K)^2$ at $E = Q$ and 0 at $E = S$

(iii) ζ must be $= Z_n$ at $E = Q$ and $= 1 + \alpha$ at $E = S$
ζ' must be $= Z'_n$ at $E = Q$ and $= 0$ at $E = S$

(iv) Z_e must be 0 at $E = Q$ and $= 1$ at $E = S$
Z'_e must be 0 at $E = Q$ and at $E = S$

It is easily seen that any one of the sets of conditions (i) to (iv) is equivalent to the other three, e.g., if (iii) holds then (i), (ii) and (iv) must.

B. Conditions for $E > Q$ and $< S$

As E increases

(i) Z_n should increase

(ii) Z_n/E should decrease

(iii) ζ should increase

(iv) ζ/E should decrease

It is also desirable but not mandatory that in addition

(v) Z_e should increase

(vi) W should increase

(The solutions given will comply with (v) and (vi))

Let us see if all the B conditions are independent and if not let us reduce them to the fewest possible.

First expressing Z_n in terms of ζ by eliminating W from (18) and the expression for Z_n in (16) we get

$$Z_n = \frac{\alpha E + \zeta K}{\alpha E + (\alpha + 1) K} \tag{19}$$

Differentiating* this

$$\{\alpha E + (\alpha+1) K\}^2 Z'_n = \alpha K (\alpha + 1 - \zeta) + \{\alpha E + (\alpha+1) K\} K \zeta'$$

and as $\alpha + 1 - \zeta$ is positive, we find that Z'_n is if ζ' is. So B (iii) includes B (i).

Also

$$\frac{Z_n}{E} = \frac{\alpha + K \zeta/E}{\alpha E + (\alpha + 1) K}$$

and it is obvious, without differentiating, that if ζ/E decreases as E increases, so does Z_n/E. Thus B (iv) includes B (ii).

Further, differentiating (18) we get

$$\{E + K (1 - W)\}^2 \zeta' = K (1 - W)(1 + \alpha W) + \{\alpha E + (\alpha+1) K\} E W'$$

---

* We shall frequently have occasion to differentiate an expression of the form $Z = \frac{X}{Y}$ where X, Y and Z are functions of E. To save space we will usually not write the result in the form $Z' = \frac{Y X' - X Y'}{Y^2}$ but instead will put it in the form

$$Y^2 Z' = Y X' - X Y'$$

which shows that if W' is positive so is ζ'. Thus B (~~iv~~ vi) includes B (iii) and therefore also B (i).

Also, as $Z_e = W Z_n$, if W' is positive and therefore Z'_n is, so is Z'_e. Thus B (vi) includes B (v).

The B conditions therefore can be reduced to:

B (iii) ζ should increase

B (iv) ζ/E should decrease

which are mandatory, or to the following which comprises all the mandatory and desirable conditions:

B (iv) ζ/E should decrease

B (vi) W should increase

We could now proceed for example to make Z_n go from its value $Q/(Q+K)$ at $E=Q$, to 1 at $E=S$ (using the methods of paragraph 4) and see whether the resulting Z_n values gave W and ζ values which complied with B (iv) and B (iii) or B (vi), but this is an indirect way of working. It is better to determine one of the functions so that the conditions are directly complied with. It appears that the most suitable function to operate on is either ζ or W for these are the functions appearing in the conditions B (iv), B (iii) and B (vi).

I have found that ζ is somewhat preferable. I construct a formula for it so as to satisfy B (iii) and B (iv) and then find it also satisfies B (vi).

The alternative of constructing W itself so as to comply with B (vi) and B (iv) is a little more complicated but (as shown in Appendix III) leads to identically the same results as by the method I have used, namely, constructing ζ first.

13. *Construction of* ζ.

We have then to construct ζ so that (i) at $E=Q$, ζ equals $Q/(Q+K)$ and $\zeta' = K/(Q+K)^2$; (ii) at $E=S$, ζ equals $1+\alpha$ and $\zeta'=0$; (iii) ζ' must be always positive, and (iv) $(\zeta/E)'$ must be always negative. It is understood (17) that $S > (1+\alpha)(Q+K)$.

We could try drawing a simple parabola of the m-th degree as in paragraph 4 from $(S, 1+\alpha)$ touching the curve $E/(E+K)$ at $E=Q$, but this is possible only if the tangent at $E=Q$ to the curve $E/(E+K)$ cuts the line $\zeta = \alpha + 1$ at $E = S_1$ where $S_1 < S$. It is easily found that

$$K S_1 = (Q+K)^2 (\alpha + 1) - Q^2$$

while S_2 the minimum value of S from (17) is given by

$$K S_2 = K (Q+K) (\alpha+1)$$

and therefore

$$K (S_1 - S_2) = Q \{\alpha Q + (\alpha+1) K\} \text{ and so } S_1 > S_2.$$

So if S lies between S_1 and S_2, no such parabola can be drawn. (What the above proves is that if S is between S_1 and S_2, the curve for ζ must contain a point of inflexion between Q and S which is evident if a diagram is drawn.)

We could use in some cases a non-simple cubic parabola of the form

$$\zeta = a_1 (S-E)^3 + a_2 (S-E)^2 + a_3 (S-E) + (1+\alpha)$$

but this again would not work for all combinations of Q, K and S and in any event if we used such a parabola we would have to investigate to see that the necessary requirements for ζ and W were met, and this would lead to many restrictions. As we are looking for a universal construction we must try something else.

14. *Construction of ζ by Method Finally Used.*

I have accordingly devised a method of constructing an expression for ζ which will give the required values to ζ and its first differential coefficient at both $E = Q$ and $E = S$ and for which ζ continually increases and ζ/E continually decreases as E increases. In order not to burden the body of the paper unduly with mathematics, I have relegated the details of this construction to Appendix I. However, in order to preserve continuity I have numbered the equations in that appendix just as though the appendix were placed here; thus equations (20) to (27i) inclusive are to be found in Appendix I.

The construction is given in detail but it will be seen that all the calculation of the constants is contained in the equations (27b) to (27g). Then from (27h) and (27i) ζ is readily obtainable for all required values of E from Q to S.

15. *This Construction Fulfills Required Conditions.*

From ζ as thus determined W is found from (18) which gives

$$W = \frac{(\zeta - 1) E + \zeta K}{\alpha E + \zeta K} \tag{28}$$

from which W is readily calculated for values of E.

If our object is to calculate W as quickly as possible, we can eliminate the step of calculating ζ from Y—see equation (27i)—and use instead

$$W = \frac{E + K - Y}{\alpha\, Y + K} \tag{28a}$$

We also have for E from Q to S,

$$\left.\begin{aligned} Z_n &= \frac{\alpha\, E + \zeta\, K}{\alpha\, E + (\alpha + 1)\, K} \\ Z_e &= \frac{(\zeta - 1)\, E + \zeta\, K}{\alpha\, E + (\alpha + 1)\, K} \end{aligned}\right\} \tag{29}$$

These of course give the proper values to Z_n, Z_e, Z'_n and Z'_e at Q and at S. Also of course W, Z_n, Z_e are all between 0 and 1 and $Z_n > Z_e$ (because $\zeta < \alpha + 1$).

We also know from paragraph 12, that as ζ' is positive and $(\zeta/E)'$ is negative Z'_n is also positive and $(Z_n/E)'$ is negative.

We can prove that W (and therefore also Z_e) increases with E for our construction. The proof will be found in Appendix II.

This completes, for the moment, the discussion of formula (14) for the modification. Let us note, however, that the construction for W does not depend upon the value of the excess ratio E_e/E or r.

16. *Second Formula for the Modification.*

We will now consider another formula that has been suggested for the modification for the multi-split plan on the ground that is rather simpler than (14) in practical application.

This formula was derived as follows: For $E < Q$ use the normal modification as the modification for the risk: For $E > Q$ amplify the formula so as to equal A/E at $E = S$ just as was done for the previous formula (14). The result is

$$\left.\begin{aligned} &\frac{A_n + K}{E_n + K} && \text{for } E < Q \\ \text{and} \\ &\frac{A_n + K + W\,(A_e - K)}{E_n + K + W\,(E_e - K)} && \text{for } E > Q \end{aligned}\right\} \tag{30}$$

but if we analyze this as per (12) we get

$$Z_n = \frac{E}{E_n + K + W\,(E_e - K)},\ Z_e = W\, Z_n.$$

Now if $E_c > K$, Z_n is greater than unity, contravening condition (5) (i) of paragraph 3. This means that if $E_c > K$ (whether E is less or greater than Q and whatever W is—except unity) the charge for a normal loss will be greater than the premium equivalent. However, we can adjust (14) so as to overcome this, as follows:—First of all we must lay down the condition that K must be greater than E_c for $E = Q$; then instead of the constant, K, in (30) we put a function of E, which we will call K_E, such that this is equal to the constant K for $E \leq Q$ but increases as E increases above Q so that K_E is always greater than E_c and also so that $K'_E = 0$ for $E = Q$ (this insures a continuous join of K and K_E at Q.)

We thus have for the modification

$$\left.\begin{array}{ll} \dfrac{A_n + K}{E_n + K} & \text{for } E < Q \\[2ex] \dfrac{A_n + K_E + W\,(A_c - K_E)}{E_n + K_E + W\,(E_c - K_E)} & \text{for } E > Q \text{ and } \leq S \end{array}\right\} \qquad (31)$$

Leaving the determination of K_E aside for the moment and putting $M = K_E - E_c$ where M is of course a function of E we have

$$\left.\begin{array}{l} \text{for } E < Q \quad Z_n = \dfrac{E}{E_n + K} = \dfrac{E}{E + M} \\[2ex] \text{for } E > Q \quad Z_n = \dfrac{E}{E_n + K_E - W\,(K_E - E_c)} = \dfrac{E}{E + M\,(1 - W)} \\[2ex] \qquad Z_c = W\,Z_n = \dfrac{W\,E}{E + M\,(1 - W)} \end{array}\right\} (32)$$

Now M is positive and so Z_n is > 0 and < 1 until $W = 1$ when $Z_n = 1$: $Z_c = 0$ while $W = 0$ and then as W rises from 0 to 1, Z_c is > 0 and < 1 until $W = 1$ when $Z_c = 1$. Also $Z_n > Z_c$.

17. *Construction of W for Formula (31).*

We now determine $\zeta = Z_n + a\,Z_c$ in a manner similar to that used for formula (14).

Put M_Q for the value of M at Q. We have

$M' = K'_E - E'_c = K'_E - E_c/E$, and $M - E\,M' = K_E - E\,K'_E$ and

$$\frac{d}{d\,E}\,\frac{E}{E + M} = \frac{(E+M) - E\,(1+M')}{(E + M)^2} = \frac{M - E\,M'}{(E+M)^2} = \frac{K_E - E\,K'_E}{(E + M)^2}$$

Now at $E = Q$, $K'_E = 0$ and so at that point

$$\frac{d}{dE}\frac{E}{E+M} = \frac{K}{(Q+M_Q)^2}$$

So we must have

$$\text{at } Q \qquad \zeta = \frac{Q}{Q+M_Q} \quad \zeta' = \frac{K}{(Q+M_Q)^2}$$

$$\text{at } S \qquad \zeta = \alpha + 1 \qquad \zeta' = 0.$$

Now if we denote E_e/E, the excess ratio, by r and put

$$\zeta = \frac{E}{Y(1-r)} \tag{33}$$

we must have (compare with the method used in Appendix I)

$$\text{at } Q \qquad Y = (Q+M_Q)\,/(1-r) = Q + K/(1-r),\ Y' = 1$$

$$\text{at } S \qquad Y = \frac{S}{(\alpha+1)(1-r)} \qquad Y' = \frac{1}{(\alpha+1)(1-r)}$$

Now if (i) $(\alpha+1)(1-r)$ is greater than unity, which it will be for r is small, say less than ½, while α is greater than one,

$$\text{and if (ii)} \qquad S > (\alpha+1)\{Q(1-r)+K\} \tag{34}$$

(this corresponds to the condition (17) and means that ζ/E must be less at S than at Q), we can proceed to determine Y just as previously (see after equation (20)—Appendix I)

u will in this case be $1/(1+\alpha)\ (1-r)$ and w will be

$$\frac{S-(\alpha+1)\{Q(1-r)+K\}}{(S-Q)(1+\alpha)(1-r)}$$

Note that w is positive and $u - w$ is positive, by (34).
Thus $0 < w < u < 1$.
Thus we determine λ, p, k, h, j, t as before and we get:

$$Y = \left\{\frac{h}{k} - \frac{t}{j}\right\}\left(S-Q\right) + \left(Q + \frac{K}{1-r}\right) - \frac{h(S-Q)^2}{E-Q+(S-Q)k} + \frac{t(S-Q)^2}{j(S-Q)+Q-E} \tag{35a}$$

$$\zeta = \frac{E}{Y(1-r)} \tag{35b}$$

Note that because K_E increases with E, $E + M$, which is the same as $E_n + K_E$, increases as E does and faster than $E_n + K$ so that $(1 - r)\, Y$, which is less than $E_n + K$, is *a fortiori* less than $E_n + K_E$: thus ζ is greater than $E/(E_n + K_E)$ or $E/(E + M)$.

18. *This Construction Fulfills Requirements.*

Now as for formula (14) we have

$$W = \frac{(\zeta - 1)\, E + \zeta\, M}{a\, E + \zeta\, M} \tag{36}$$

As before we can express W in terms of Y namely

$$W = \frac{E + M - Y\,(1 - r)}{a\, Y\,(1 - r) + M} \tag{36a}$$

Also

$$\left.\begin{aligned} Z_n &= \frac{a\, E + \zeta\, M}{a\, E + (a + 1)\, M} \\ Z_e &= \frac{(\zeta - 1)\, E + \zeta\, M}{a\, E + (a + 1)\, M} \end{aligned}\right\} \tag{37}$$

These of course give the proper values to Z_n, Z_e, W and their first derivatives at $E = Q$ and at $E = S$.

Also, since $\zeta > E/(E + M)$, W, Z_n and Z_e are all between 0 and 1, and Z_n is greater than Z_e (except at S).

Examining now Z'_n we find

$$\{a\, E + (a+1)\, M\}^2\, Z'_n = a\,(a + 1 - \zeta)\,(M - E\, M') \\ + \{a\, E + (a+1)\, M\}\, M\, \zeta'$$

and Z_n will certainly be positive if $M - E\, M'$ is. Now, as shown above, this last expression is the same as $K_E - E\, K'_E$: this means Z'_n will certainly be positive if $(K_E/E)'$ is negative and we will so construct K_E.

Now to examine $(Z_n/E)'$

$$\frac{Z_n}{E} = \frac{a + M\,\zeta/E}{a\, E + (a + 1)\, M}$$

Now the denominator of this equals

$$E\,\{(1 + a)(1 - r) - 1\} + (a + 1)\, K_E$$

which, as $(1+a)(1-r)-1$ is positive, increases with E. As for the numerator, ζ/E decreases as E increases and if M does also then the whole numerator does, and so if M decreases, Z_n/E will also unquestionably decrease. On the other hand if M increases, we find by differentiation that

$$\{a\,E+(a+1)\,M\}^2\,(Z_n/E)'=\{a\,E+(a+1)\,M\}\,M\,(\zeta/E)' \\ -a\,(a+M\,\zeta/E)-a\,(a+1-\zeta)\,M'$$

and the right hand side is certainly negative if M' is positive for $(\zeta/E)'$ is negative. Thus whether M increases or decreases, Z_n/E decreases. (Note, the construction we adopt, in paragraph 19, makes M' negative for the first part of the range Q to S and positive for the latter part).

We can also show that W (and therefore Z_e) increases with E, for our construction. As in the case of the corresponding proof for the formula (14) construction we have put this proof in Appendix II.

19. *Determination of K_E.*

We now come to the determination of K_E. We must have

(a) $K_E = K$ for $E = Q$.
(b) $K'_E = 0$ at Q and positive for $E > Q$.
(c) $(K_E/E)'$ negative.
(d) $K_E > E_e$.

We first note that (d) is the only condition involving E_e (or in other words r) and if $K_E > E_e$ for the maximum value of r it will be so for all values of r: so we will make $K_E > E_e$ for the maximum value of r and then we can use the same series of values of K_E for all values of r. Let this maximum value of r be g; note that as K must be greater than E_e for $E = Q$ we must have $K > Q\,g$.

For $E > Q$ we will let K_E be given by the hyperbola

$$(K_E - g\,E)(E + a_1) = a_2$$

which is asymptotic to $K_E = g\,E$ (see Fig. IV). We will determine the constants a_1 and a_2 so that the curve touches $K_E = K$ at $E = Q$. We have

$$K_E = \frac{a_2}{E + a_1} + g\,E \qquad \text{so that } K = \frac{a_2}{Q + a_1} + \overset{g}{\cancel{k}}\,Q$$

$$K'_E = g - \frac{a_2}{(E + a_1)^2} \qquad \text{so that } g = \frac{a_2}{(Q + a_1)^2}$$

whence $a_1 = \dfrac{K - 2\,Q\,g}{g}$ $\qquad a_2 = \dfrac{(K - Q\,g)^2}{g}$

and thus:

$$K_E = \frac{(K - Q\,g)^2}{g\,E + (K - 2\,Q\,g)} + g\,E \tag{38}$$

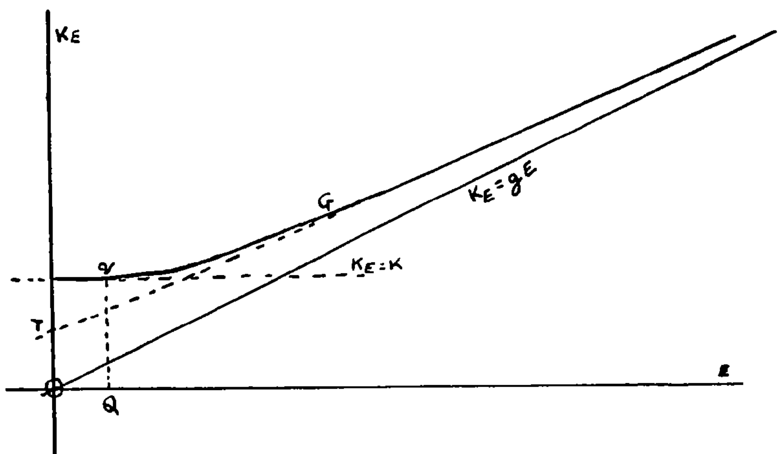


Fig. IV.

The curve is $q\,G$ in Fig. IV. The tangent at any point G cuts the K_E axis at T above 0 showing that $(K_E/E)'$ is negative. Thus all the conditions (a) to (d) are complied with.

For Appendix II it is necessary to note that the maximum value of $O\,T$ occurs for $E = Q$, that is, the maximum value of $K_E - E\,K'_E$, which equals $M - E\,M'$, is K.

We have now completed for the moment the discussion of formula (31). We will return later to consider how to deal with the different values of r that arise. The construction given above for W depends on the value of r used; note, however, that the formula (38) for K_E is useable for all values of r.

20. *Third Formula for the Modification.*

Let us now see what we must do if we apply the ordinary modification formula (11) to the multi-split plan. Making $Z_e = 0$ we have for the modification for $E < Q$

$$\frac{E_n}{E}\ \frac{A_n + K_n}{E_n + K_r} + \frac{E_e}{E}$$

which we can write as

$$\frac{K + \frac{E_n}{E}\,(A_n + E_e)}{E_n + K} \quad \text{for } E < Q \qquad (39a)$$

For $E > Q$ and $\leq S$ we can put $E_n + K\,(1 - W)$ for the denominator and we must add to the numerator

$$W\,\{(A - A_n - E_e)\,E_n/E - K\}$$

and we get the rather cumbersome formula

$$\frac{K + (A_n + E_e)\,\frac{E_n}{E} + W\left\{(A_e - E_e)\,\frac{E_n}{E} - K\right\}}{E_n + K\,(1 - W)} \quad \text{for } E > Q\ (39b)$$

for which $Z_n = \dfrac{E_n}{E_n + K\,(1 - W)}$, $Z_e = W\,Z_n$

It will be seen that

$$Z_n = \frac{E}{E + \dfrac{(1 - W)\,K}{1 - r}}$$

which is of the same form as Z_n in (16) with $K/(1 - r)$ for the K there; and indeed if we multiply the top and bottom of (39b) by E/E_n and put ${}^rK$ for $K/(1 - r)$ we get

$$\frac{{}^rK + A_n + E_e + W\,(A_e - E_e - {}^rK)}{E + {}^rK\,(1 - W)} \qquad (40)$$

which is of the same form as (14) with ${}^rK$ for K.

So we can determine W just as for (14) but using ${}^rK$ for K. We note, however, that as for formula (31) the values of W depend on the value of r.

21. *Value of Excess Ratio to be Used.*

Now let us consider this question of the value of r that enters into the determination of W. We have discussed three formulas for modifications, namely, (14), (31) and (39). For the first W does *not* depend on r but for the last two it does. It is obviously impractical to calculate a series of values of W for each separate possible value of r and we will therefore see if we cannot use, for all values of r, the values of W calculated for one particular r, say the average value or the maximum or the minimum value. Let us take (39) first, and suppose we have calculated values of W for a certain excess ratio r and use them for risks with a different excess ratio x. Then, since $W = 0$ at Q and $= 1$ at S and $W' = 0$ at both Q and S, Z_n will join smoothly at Q with the values below Q, and will be tangential to $Z_n = 1$ at S; also Z_e will $= 0$ at Q and 1 at S and Z'_e will $= 0$ at both Q and S.

Now since $\zeta = E\,(1 + a\,W)/\{E + {}^xK\,(1 - W)\}$

$$\{E + {}^xK\,(1 - W)\}^2\,\zeta' = {}^xK\,[(1 - W)(1 + a\,W) + (1 + a)\,E\,W'] + a\,E^2\,W'$$

which will be positive for all values of x since W' is positive. We also see that Z'_n is positive by putting $a = 0$ in the above, when ζ becomes Z_n. Also Z'_e is positive, for $Z_e = W\,Z_n$.

Now to consider Z_n/E and ζ/E. We easily find by differentiation that

$$\{E + {}^xK\,(1 - W)\}^2\,(\zeta/E)' = {}^xK\,(1 + a)\,W' - \{(1 + a\,W) - a\,E\,W'\}$$

and by considering that this expression is negative if r is put for x, we see it remains negative if ${}^rK > {}^xK$: we see similarly $(Z_n/E)'$ is certainly negative if ${}^rK > {}^xK$ (put $a = 0$ in the above expression).

So if ${}^rK > {}^xK$ or $r > x$ we can certainly use with safety for the case of an excess ratio x the W's derived for the ratio r. On the other hand there is some margin in the fulfillment of the conditions by the W's derived for ratio r (except perhaps in a borderline case where S is only a little greater than $(1 + a)(Q + {}^rK)$—see (17)) and if x is not much greater than r we probably will still have Z_n/E and ζ/E decreasing.

We note that the condition $r > x$ is what we would expect; for if $r > x$, then ${}^rK > {}^xK$ and Z_n for $E = Q$ will be greater for x than for r. Thus at Q, Z_n/E and ζ/E, which are equal at Q, will be greater for x than for r. On the other hand at S, Z_n/E and ζ/E are equal for all values of excess ratio being equal to $1/S$ and $(1 + a)/S$ respectively. So for x the ratio Z_n/E or ζ/E has further to decrease as E goes from Q to S than it has for r and we should not be surprised therefore that the W values calculated for r will work satisfactorily for a smaller ratio x.

To come now to formula (31) we first note that we have taken care of K_E by using the maximum excess ratio in fixing it. As far as Z_n and ζ are concerned, we easily find that if the excess ratio is x

$$\{E + M\,(1-W)\}^2\,\zeta' = (M - E\,M')(1-W)(1+a\,W) + \{a\,E + (a+1)\,M\}\,E\,W'$$

Now $M - E\,M' = K_E - E\,K'_E$ which is positive and so the right hand side is positive whatever the value of x. If we put $a = 0$ in the above equation, ζ becomes Z_n and the right hand side is of course still positive. Thus Z'_n and ζ' are positive for all values of x. The question, however, is not so simple when we come to consider Z_n/E and ζ/E.

We have

$$\{E + M\,(1-W)\}^2\,(\zeta/E)' = M\,(1+a)\,W' - \{(1+a\,W) - a\,E\,W'\} - M'\,(1-W)(1+a\,W)$$

Now in this M refers to an excess ratio x and if we write, temporarily, $\overline{M}$ for the M for the ratio r, we have

$$M = \overline{M} - (x - r)\,E \qquad\qquad M' = \overline{M}' - (x - r)$$

and the right hand side of the above equation becomes

$$\overline{M}\,(1+a)\,W' - \{(1+a\,W) - a\,E\,W'\} - \overline{M}'\,(1-W)(1+a\,W) + (x-r)\,\{(1-W)(1+a\,W) - (1+a)\,E\,W'\}$$

which we will call $X + (x - r)\,\mu$.

Now X we know is negative for it is what the above right hand side becomes if $x = r$. As for μ, this $= 1$ for $E = Q$ and $= 0$ for $E = S$, but as we shall see as E goes from Q to S μ rapidly becomes negative and remains negative till E reaches S. If we write, for the moment, V for $W - E\,W'$, V is the distance above the origin that the tangent to the curve for W (as a function of E) cuts the W axis $E = 0$. μ becomes $(1-2\,W-a\,W^2) + (1 + a)\,V$. The first term in this equals 1 for $W = 0$ $(E = Q)$, equals 0 for $W = \{-1 + \sqrt{1 + a}\}/a$, equals $-(1 + a)$ for $W = 1$ $(E = S)$ and decreases continually from $W = 0$ to $W = 1$. As for the second term, V equals 0 at $E = Q$ and equals 1 at $E = S$. As will be seen from the examples given below V is negative from $E = Q$ until E is well advanced towards S. Thus we find μ starting from

1 at Q rapidly becomes negative, reaches a minimum and then rises to 0 at S. Now if $x > r$ and μ is negative $(\zeta/E)'$ will be negative, but if μ is positive $(\zeta/E)'$ will be negative only if $(x - r)\ \mu$ is not greater than $-X$. Thus if $x > r$, $(\zeta/E)'$ will be certainly negative over the greater part of the range from Q to S and the only region it can be positive is in the earlier part of the range and then only if there is not much "margin," i.e., only if the relationship of Q and S is such that there is not much drop in ζ/E from Q to S. Further, if in any particular case where there is not much margin and where, therefore, ζ/E does not decrease continuously in the earlier part of the range Q to S, we can improve the situation by using a higher value of η in calculating the W values. It will readily be seen on examination of the construction of Y in Appendix I that a higher value of η will give higher values of Y and lower values of W and ζ/E. Thus increasing η should tend to eliminate the up and down behaviour of ζ/E in the early part of Q to S in borderline cases.

On the other hand, if $r > x$, (ζ/E) will certainly decrease in the first part of Q to S but in the latter part there is danger of an increase and the only thing to prevent this is the "margin" (in the sense used above): but here we must note that in the case of formula (31) if $r > x$, Z_n for Q is *less* for x than for r and therefore ζ/E for Q is less for x than for r and so (as at S ζ/E is the same for x and for r) there is less drop in ζ/E from Q to S for x than for r so it will be easier for ζ/E to increase. The opposite is, of course, the case if $r < x$: there will be a bigger drop in ζ/E from Q to S for x than for r.

The conclusion is that x should be greater than r for formula (31). This is borne out by the examples given below—where it will be seen that $x < r$ gives quite unsatisfactory results, while $x > r$ gives usually quite good ones though not in all borderline cases. An example is given of how increasing the value of η improves a borderline case.

In the above discussion we have dealt with ζ/E. A similar analysis can be made of Z_n/E but it is fairly plain that if we get proper results for ζ/E we will also get them for Z_n/E.

Thus in the case of formula (39), to calculate the W values we should use a value of r at or nearly at the maximum of its range while for formula (31) we should use r near the minimum.

A word about the minimum value for S. In respect of formula (39) we must have

$$S > (1 + \alpha)\,\{Q + K/(1 - r)\} \tag{41a}$$

and we should see that S complies with this for the maximum value of r. (Some margin of compliance is desirable.)

In respect of formula (31) we must have

$$S > (1 + \alpha)\,\{Q\,(1 - r) + K\} \tag{41b}$$

and in this case we should see that S complies for the minimum value of r. (The values of K will, of course, probably be quite different for the two cases). We see that the necessity here of using, for r, the maximum value for formula (39) and the minimum for formula (31) agrees with the requirements for the W values.

In respect of formula (14) no question of r arises and we must simply have

$$S > (1 + \alpha)(Q + K) \tag{41c}$$

22. *Other Formulas for the Modification.*

I have now given three different formulas, (14), (31) and (39), for the multi-split plan modification and it is clear that many more could be devised, but the three given are sufficient to illustrate the principles involved. It will be observed that the procedure consists of

(a) Choosing a formula for the modification for $E < Q$. This is the most important step since the greater number of risks fall in this range, and in addition the credibilities for risks where $E > Q$ are settled, to a large extent, by the "swing" below Q.

(b) Adjusting the modification formula for $E > Q$ by the addition of terms involving a parameter W so that the credibilities join smoothly at Q to those below Q and reach unity tangentially at S.

(c) Calculating the values of W so as to fulfill these conditions and the conditions set out in paragraph 3. The technique developed above consists in calculating ζ so that it and its first derivative ζ' take the required values at Q and at S and so that ζ increases and ζ/E decreases. Then it is necessary to check that these values when used in conjunction with the modification formula give values of Z_n, Z_e and W that increase and values of Z_n/E that decrease.

It is of interest to note that when the modification formula for $E < Q$ is settled, it is possible to choose more than one formula for $E > Q$ and that the calculation of the ζ values is independent of the choice of the modification formula for E greater than Q. For instance, instead of formula (14) for E greater than Q we could have

$$\frac{A_n + E_e + K(1-W)}{E + K(1-W)} + W\frac{A_e - E_e}{E} \tag{14A}$$

which gives $Z_n = \dfrac{E}{E + K(1-W)}$, $Z_e = W$.

The same ζ's as determined for (14) are applicable here and it will be found that the resulting values for W, Z_n, and Z_e are satisfactory. However, to calculate W from ζ requires the solution of a quadratic equation and all-in-all (14A) is not as simple to work with as is (14).

Another, and easily worked, variation of 14 is

$$\frac{A_n + E_e + K}{E + K}(1-W) + \frac{A}{E}W \tag{14B}$$

which gives

$$Z_n = \frac{E + WK}{E + K},\ Z_e = W.$$

Here again the ζ's are the same as for (14) and it will be found that

$$Z_n = \frac{aE + \zeta K}{aE + (a+1)K} \qquad Z_e = \frac{(\zeta - 1)E + \zeta K}{aE + (a+1)K}$$

These are the same as for (14) showing that (14B) gives the *same* values of Z_n and Z_e as does (14). (The W values are different, of course.) Thus (14B) could be used in place of (14) if it gives a better "working formula" and if it is felt that it is easier of explanation, to the layman, than is (14).

However, I will not pursue further this discussion of alternative formulas but will proceed to consider some practical aspects of the three original formulas.

Part IV

Multi-Split Plan—Practical Considerations

23. *Comparison of the Three Formulas.*

We will now examine some of the characteristics of the three formulas (14), (31) and (39), we are discussing. We will pay particular attention to the credibilities given for low values of E, that is those below Q.

For $E \not> Q$ Z_e is zero and Z_n is equal to:—

$$\frac{E}{E+K} \text{ by formula (14)}$$

$$\frac{E}{E_n+K} \text{ by formula (31)}$$

$$\frac{E_n}{E_n+K} \text{ by formula (39)}$$

(The K's will not necessarily be the same).

Therefore (a) for a fixed value of E, i.e. for a fixed total premium the (normal) credibility for varying normal ratios E_n/E, i.e. for varying amounts of normal premiums contained in the fixed total premiums, will

(i) not vary, for formula (14)
(ii) increase as the amount of normal premiums *decreases*, and vice versa, for formula (31)
(iii) increase as the amount of normal premium *increases*, and vice versa, for formula (39)

and (b) for a fixed value of E_n, i.e. for a fixed normal premium, the (normal) credibility for varying normal ratios, i.e. for varying amounts of total premium, will

(i) increase as the amount of total premium *increases*, and vice versa, for formula (14)
(ii) increase as the amount of total premium *increases*, and vice versa, for formula (31)
(iii) not vary for formula (39)

For formula (39) this behavior is, of course, in accordance with our accepted notions (as the formula is, of course, the ordinary one) but for formula (31) the behavior in particular in respect of (a) (ii) is rather strange.

Formula (14) comes in between the other two and its characteristics are quite defensible. Nevertheless, as the excess ratios are low for the multi-split plan, the disadvantages of (31) are not as serious as they otherwise would be and the working scheme for this formula is very simple.

Now let us look at another aspect of the three credibilities. If as is customary we fix K by its effect for a low or minimum value of E (either by way of the charge for a maximum loss or the credit for clear experience) we find the formulas give different results for larger values of E say in the neighborhood of Q. Since in thus fixing K it is customary to use an average value of the excess ratio, formulas (14) and (39) will give the same credibilities (for the average value of r) at higher values of E if the K's are chosen so as to give the same effect at a low value of E. (The K's will differ—if r is the average excess ratio used, K by formula (39) will be $(1-r)$ times the K by formula (14)). On the other hand the credibilities at higher values of E given by (31) will be considerably greater than those given by formula (14) or (39) with the same effect at a low value of E. This will be an advantage of formula (31) if we desire to give a wider swing to the plan for medium values of E without opening up the swing too much for small sizes of E, and it has been suggested that there would be considerable merit in doing this since no credibility is given to the excess experience as long as E is less than Q.

24. *Working formulas.*

We come now to the question of the form in which the "working formula" should be put.

First we call attention to the point that both for formulas (14) and (31) if in either the numerator or the denominator we take the sum of the coefficient of W and of the remaining terms we get A in the case of the numerator and E in the case of the denominator. For formula (39) we get $A(1-r)$ and E_n respectively but if we put this formula in the alternative form (40) we again get A and E respectively. This, of course, is the same as saying that we get self-rating for $W=1$.

Thus we can arrange our working formulas as follows:

I. Formula (14)

We give two alternatives

(i) We require a table of W for values of $E > Q$ and $< S$. We arrange our work sheet to give (a) ballasted actual discounted (normal) losses plus unrated expected excess losses, namely, $A_n + E_e + K$ where K is the "ballast" (b) ballasted expected losses, $E + K$. Then if $E \not> Q$ the modification is $\frac{(a)}{(b)} = \frac{A_n + E_e + K}{E + K}$

but if $E > Q$ we subtract from the top (c) the proportionate surplus of ballasted actual losses being W times the difference between (a) and the total actual losses, namely, $W\{(A_n + E_e + K) - A\}$, and we subtract from the bottom (d) the proportionate surplus of ballasted expected losses, being W times the difference between (b) and the actual expected losses or $W\{(E + K) - E\}$ and the modification is $\frac{(a) - (c)}{(b) - (d)}$

or (ii) We require a table of W as before and also a table of ballasts B equal to $K(1 - W)$. For $E < Q$, $B = K$. We arrange our work sheets to give (a) actual discounted (normal) losses plus unrated expected excess losses $A_n + E_e$ (b) the total expected losses. Then if $E < Q$ the modification is $\frac{(a) + \text{ballast}}{(b) + \text{ballast}} = \frac{A_n + E_e + K}{E + K}$

If $E > Q$ to the top we add (c) the proportionate remainder losses being W times the difference between the total actual losses and (a) or $W\{A - (A_n + E_e)\}$. Then the modification is

$$\frac{(a) + (c) + \text{ballast}}{(b) + \text{ballast}}$$

where the ballast is B from the table.

The second alternative seems to me to be the preferable.

II. Formula (31)

As before we give alternatives

(i) We require a table of W for $E > Q$ and of K_E the ballast ($= K$ for $E < Q$). Then we get (a) ballasted actual discounted losses, $A_n + K_E$ and (b) ballasted expected discounted (normal) losses. If $E < Q$ the modification is

$$\frac{(a)}{(b)} = \frac{A_n + K}{E_n + K}$$

but if $E > Q$ we subtract from the top (c) the proportionate surplus ballasted discounted losses being W times the difference between (a) and the total actual losses or $W\{(A_n + K_e) - A\}$, and from the bottom we subtract (d) the proportionate surplus expected discounted losses being W times the difference between (b) and the total expected losses; then the modification is

$$\frac{(a) - (c)}{(b) - (d)}$$

or (ii) We require a table of W as before and also a table of ballasts B equal to $K_E(1 - W)$. For $E < Q, B = K$. We get (a) actual discounted (normal) losses (b) expected discounted losses and if $E < Q$ the modification is

$$\frac{(a) + \text{ballast}}{(b) + \text{ballast}} = \frac{A_n + K}{E_n + K}$$

but if $E > Q$ we add to the top (c) the proportionate remainder actual losses being W times the difference between the total actual losses and (a), and to the bottom we add (d) the proportionate remainder expected losses being W times the difference between the total expected losses and (b). Then the modification is

$$\frac{(a) + (c) + \text{ballast}}{(b) + (d) + \text{ballast}}$$

where the ballast is B from the table.

Again the second alternative seems to be the preferable.

III. Formula (39)

In the form (39) this formula is not very suitable for easy working. It would be best to put it in the form (40) and then proceed as for formula (14) but in all cases dividing the ballast—whether K or B—by $(1 - r)$ before using so as to give ${}^rK$ or ${}^rB$ as the case may be. This makes the application of this formula a little more complicated than (14) which again, at any rate for $E < Q$, is neither quite as simple as (31) nor perhaps as attractive when explained to the layman. For (31) the layman is told, we get the modification by dividing the ballasted discounted actual losses by the ballasted (discounted) expected losses, while for (14) he is told we get the modification by dividing the ballasted discounted actual loss plus the (unrated) expected excess losses by the ballasted (undiscounted) expected losses.

25. *The Basic Constants.*

The fundamental quantities entering into all the calculation in connection with the multi-split plan credibilities as set out above are S, Q, and K and the auxiliary quantities are r (except in the case of (14)) and α. A few observations on these are offered.

Taking α first, we see that no particular harm is done by choosing it on the high side and therefore it seems possible and desirable to choose a value for it which can be the same for all states and need not be changed for every rate revision. This will simplify our calculations by eliminating one source of variation. As for the value to be assigned, if we use actual values in respect of death and more particularly permanent total cases, we shall obtain very high values but if as seems desirable we use, as at present, average values for these types of losses α will come out at a moderate value. In the examples given below I have used the value 4. This is possibly on the small side for universal use.

As for the excess ratio r, this does not enter into (14) at all (except incidentally into the determination of K). It enters into the calculations for (39) (apart from its use in fixing K) so that theoretically we should have different sets of W values for each r. If we use a fixed value of r, preferably near the maximum value we should get satisfactory results (see paragraph 21). There is not yet much information available as to the range of r except that it seems probable it will be fairly small (e.g. with a maximum of perhaps 40% and an average of 15% to 20%) for the values of a and ρ likely to be used in practice for discounting (see paragraph 7). In formula (31) the ratio r enters first into the determination of K_E and as shown in paragraph 20, a maximum value g should be used here. In the examples given below, I have used $g = .333$ which is possibly too low. As for the value of r to be used for formula (31) in determining the W values, the investigation in paragraph 21 shows that a low value should be used but it is not certain in respect of this formula (31) that a single value of r will work satisfactorily in all cases—particularly if the inequality (41b) is complied with by only a small margin. As in the case of α it would be a great simplification in practice if a universal value could be adopted for the fixed value of r to be used in determining the W's but until more is known about the actual values r can take, it cannot be decided if this is possible for formula (31).

Coming now to K, we have mentioned above the usual procedure for the fixing of this constant. As for Q and S these also must be settled on in some more or less arbitrary manner. Suggestions have been made to take S as a certain multiple (say twenty) of the average D. and P. T. value and Q as a fixed percentage of S. (Care must be taken, of course, that S and Q together with the K value chosen satisfy the condition (41) (a), (b) or (c) as the case may be). The taking of Q as a fixed proportion of S would greatly simplify the calculation of the W's.

If α (and the value of r if any to be used) are fixed then the determination of y depends solely on one parameter, namely, the value of w, which can vary, in accordance with the choice of K in relation to S and Q, from 0 to u. This assumes we take η equal to a fixed value say ½ in (27a). So it would be easy to compile a standard table of y. Now if in addition Q/S is a fixed ratio q then Y/S (which equals $y(1-q)+(q+K/S)$ for (14) for example) will also depend solely on a single parameter fixed by the relationship of K and S and therefore so will ζ/S and there also W expressed in terms of E/S. Thus if q is fixed W depends only on the relationship of K and S (and if this were fixed one table of W would do!)

The task of preparing a table of W for any state can thus be made much easier by deciding on fixed values for α, r, g and q, although as a matter of fact it is not burdensome to calculate W *ab initio*. We first calculate u and w: the expressions for these quantities are in Appendix I for formula (14) and in paragraph 17 for formula (31); for formula (39) use the same expressions as for formula (14) but with ${}^rK$ in place of K.

Then by equations (27b) to (27h) we get the expressions for Y (for formula (31) use equation (35a) instead of (27h)). From Y we get W by using equation (28a) for formula (14), (36a) for formula (31) and (28a) with ${}^rK$ for K for formula (39). For formula (31) we must in addition calculate K_E and M.

26. *Which Formula should be used?*

As to which of the three formulas should be used, the final determination of this question will rest on practical grounds, regard being had principally to the ease of explanation and facility of operation of the plan. This seems to rule out the rather more complicated (39) and give a slight preference to (31), or in other words the order of preference is likely to be (31), (14) and (39), the exact reverse of the order of theoretical desirability. However, if theoretical soundness is given enough weight then the "middle of the road" (14) might be chosen—and the mathematics of derivation and calculation will be considerably simplified. Of course (see paragraph 22) many other formulas are possible and it may well be that one far better may be devised.

My personal preference so far is with (14) but I have tried to present the alternatives impartially.

Part V

Illustrations of Multi-Split Plan Credibilities

27. At the end of the paper will be found some tables giving examples of W values and credibilities for the multi-split plan. These have been calculated in accordance with the foregoing and with basic values similar to those that might be expected to be used in practice.

The examples are chosen so as to be applicable to

I. New York State—with high benefits
II. Massachusetts—with medium benefits
III. Georgia with low benefits

In all cases the S values has been taken as approximately twenty times the average D. and P. T. value and the Q value is 10% of the S value (so that the q of paragraph 25 is 0.1). The actual S and Q values used were

New York	$S = 140000$	$Q = 14000$
Massachusetts	90000	9000
Georgia	42000	4200

(Note that as everywhere else in this paper these are in terms of *expected losses* so that the subject premiums would be about two-thirds greater).

In all cases the value of α used is 4, and the value of η is $\frac{1}{2}$.

In all the tables the various values are given for specimen values of E/S so as to facilitate comparisons from one state and one table to another. The at first sight odd percentages between Q and S were chosen as to give round percentages of the interval between Q and S: thus $E/S = 55\%$ represents a point half way between Q and S. This scheme of specimen values is possible because Q/S is constant.

28. In table I are given values worked out on the assumption that formula (14) is used for the modification.

The value of r is accordingly immaterial, except in fixing K where an average value of one-sixth was used. The values used for K are New York 6900, Massachusetts 5520, Georgia 4140; these were chosen so as to give a charge of 20% for a maximum loss and a credit of 6⅔% for clear experience for expected losses of 600 for New York, 480 for Massachusetts, and 360 for Georgia, the maximum losses used being 1500 for New York, 1200 for Massachusetts, and 900 for Georgia. (These are discounted values of course).

In table II are given values on the assumption that formula (31) is to be used. The excess ratio used in calculating K_B (that is the g of the paragraph 20) is in all cases one-third. For each of the three states three sets of values are given—with $r = 333$, $r = .167$ and $r = 0$ respectively. (Of course the value $r = 0$ cannot arise in practice but the values are given for this to show how the formulas behave when r is very small). The values of K used are New York 7000, Massachusetts 5000, Georgia 4200, which as before, were chosen so as to give the same charge for a maximum loss and the same credit for clear experience for the same expected losses (with the same average value of one-sixth for r) as for Table I.

The values shown in Tables I and II, for each selected value of E/S are E, K_B (Table II only), W, B, Z_n, Z_e, $S\,Z_n/E$ and $S\,\zeta/E$. The last two functions are given to show the way in which they decrease with E, or in other words to illustrate the negativeness of $(Z_n/E)'$ and $(\zeta/E)'$.

The values of u and w involved in the example in Tables I and II are

	u	w		
	All States	New York	Massachusetts	Georgia
Formula (14) Table I	.2	.0563	.0430	.00159
Formula (31) Table II				
$r = .333$	.3	.1388	.1184	.0555
$r = .167$	.24	.0890	.0727	.0223
$r = 0$	.2	.0550	.0420	0

The fact that $w = 0$ for Table II, Georgia, $r = 0$, shows that for this example S is equal to instead of being greater than $(1 + a) \{Q (1 - r) + K\}$. So in this case, $y = 0$ for all values of x and therefore Y is also constant. Thus W is linear and equal to $(E - Q)/(S - Q)$ and there is no smooth junction for any of W, Z_n and Z_e at Q or at S. This is, of course, the limiting case and as observed above $r = 0$ does not arise in practice. If w were equal to (or less than) zero for a possible value of r, then S, or Q or K would have to be changed.

I have given no examples of the application of formula (39) for this is a simple modification of (14). In fact, Table I gives the values for formula (39) for K values equal to the K's of that table multiplied by $(1 - r)$ whatever r may be. There is little to comment on in these Tables I and II. The functions behave of course as they should in the light of the foregoing theory.

29. To illustrate the discussion in paragraph 21, in respect of formula (31), of the effect of using values of W, derived from a fixed value of the excess ratio, for the case of a different, varying, value of the ratio, I show in Table III values of Z_n, Z_e, $S\, Z_n/E$ and $S\,\zeta/E$ that occur with a variable excess ratio x if W values are used calculated for a fixed value r. These are shown for the same values of E/S as before, for each of the three States, for all combinations of r and x equal to .333, .167 and 0. The values for $r = x$ are not given as they are in Table II. (Here again I must mention that the results shown for r or $x = 0$ are merely illustrative of the limit of the effect of a low excess ratio.)

Chart I (shown at the end of the Tables) has been included to show graphically and a little more fully the behavior of ζ/E if r does not equal x. It shows for each of the nine combinations of the three States and the three x values how ζ/E behaves in going from Q to S when r equals each of the three values we have selected (including the case of $r = x$).

It will be seen that in accordance with the theory given in paragraph 21

(a) if $r = x$ the function ζ/E decreases satisfactorily (for Georgia, $r = x = 0$, ζ/E follows a horizontal straight line which at Q and S is not tangential to the curves for $E < Q$ and $> S$—but this is a limiting case);

(b) if $r > x$ ζ/E does not behave satisfactorily: it decreases, then rises and then falls again.

(c) if $r < x$ ζ/E decreases satisfactorily, except in the case of the Georgia values: there, for $r = .167$, $x = .333$, the behavior is bad for the early part of the interval Q to S (but not bad as, say, for $r = .333$ $x = .167$). In any case this is quite close to a borderline case. For $r = 0$ Georgia, the values of ζ/E are of course even worse.

In paragraph 21 it was suggested that in a borderline case such as Georgia $r = .167$, $x = .333$ where ζ/E, instead of continually decreasing, first decreases then increases and then decreases again, improvement would result if we increased the value of η used to calculate the W's. To show how this works out in this particular case I give on Chart II a graph of ζ/E for Georgia $r = .167$ $x = .333$ both for $\eta = 1/2$ (the value used in Chart I and Table III) and for $\eta = 1$, the highest possible value. It will be seen that the up and down behavior of ζ/E is eliminated when $\eta = 1$.

30. Finally, I give Table IV to illustrate the remarks in paragraph 23 regarding the different effects of the three formulas with respect to the credibilities given at higher value of E if the K values are chosen so as to give the same effect at a certain low value of E. In the table IV the K values used for formulas (14) and (31) are the same as in the previous tables and the K values used for formula (39) were chosen so as to give the same effects as the other formulas at minimum values of E. In Table IV are shown for selected E values the Z_n values and also the average credibilities (i.e. the credit for clear experience) taking into account the (zero) excess credibility.

. . . .

Appendix I

Construction of ζ for formula (14).

The construction referred to in paragraph 14 is as follows:

$$\text{Put } \zeta = \frac{E}{Y} \tag{20}$$

We will construct Y and derive ζ from it.

Y must be such that

(i) at $E = Q$, Y must equal $Q + K$ and be tangent to the line $Y = E + K$ i.e. Y' must equal 1;

(ii) at $E = S$, Y must equal $S/(1+\alpha)$ and be tangent to the line $Y = E/(1+\alpha)$ i.e. Y' must equal $1/(1+\alpha)$;

(iii) $Y' = (E/\zeta)'$ must be always positive;

(iv) $(Y/E)' = (1/\zeta)'$ must be always negative.

Thus (see Fig. V) we must make Y go from q to s and be tangent at q to $L\,q$ and at s to O_s, so that Y continually rises and its tangent cuts $0\,Y$ above 0.

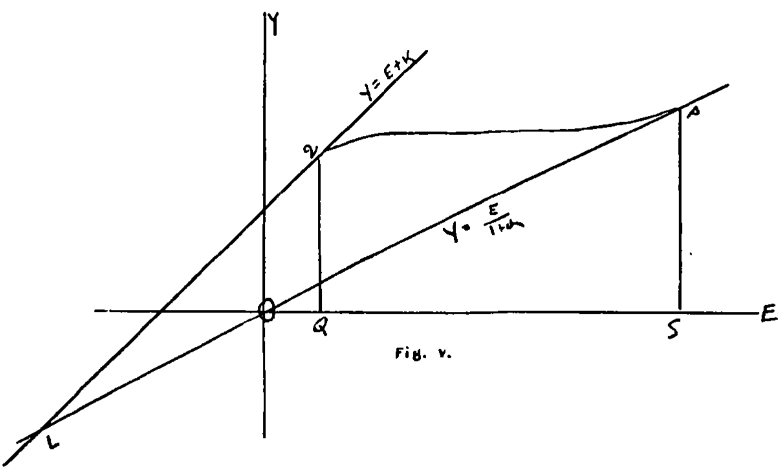


Fig. V.

q must be lower than s which is, of course, the same as the necessary condition (17).

We now put Y equal to the sum of the ordinates of two hyperbolas

$$Y = A_1 - \frac{B_1}{C_1 + E} \quad \text{and } Y = \frac{B_2}{C_2 - E} - A_2$$

where the A's, B's and C's are constants that will be determined so that the sum of these partial curves will meet the necessary conditions, namely that the combined curve touches $L\,q$ at q and O_s at s. B_1 and B_2 are to be positive and $C_1 > -Q, C_2 > S$: then the vertical asymptotes of the two hyperbolas are to the left of Q and the right of S respectively. In both hyperbolas Y' is positive (between Q and S) for in both Y increases from $E = Q$ to $E = S$, therefore, for the combined curve Y' is positive. Again the first partial curve is continually concave to the E axis from Q to S and so Y'' is always negative but it increases continually (that is, gets less negative) from Q to S; also the second curve is continuously convex to the E axis from Q to S, and so Y'' is always positive and it increases continually from E to S: so the sum of the two Y'''s which commences by being negative at q and ends by being positive at S can change sign only once between Q and S: in other words there is one and only one point of inflexion between Q and S and the tangent to the combined curve, starting from $L\,q$ at $E = Q$ and ending at $L\,s$ at $E = S$ can never cut $O\,Y$ below O as an examination of Fig. V will show. In other words, for the combined curve $(Y/E)'$ will always be negative, as required. (The tangent not only always cuts $O\,Y$ above O but also always cuts $L\,q$ above L: this fact will be needed in Appendix II).

To determine the constants we will simplify the calculations by transferring the origin to q and making $S - Q$ the unit i.e. we put

$$y = \frac{Y - Q - K}{S - Q} \qquad\qquad x = \frac{E - Q}{S - Q} \tag{21}$$

then the required curve will be

$$y = \frac{h}{k} - \frac{h}{x+k} + \frac{t}{j-x} - \frac{t}{j} \tag{22}$$

where h, k, t must be > 0 and $j > 1$.

In addition we must have

(i) for $x = 0$ $y = 0$ (this is taken care of the form of (22))

(ii) for $x = 0$ $y' = 1$

(iii) for $x = 1$ $y = \{S/(1+\alpha) - (Q+K)\}/(S-Q)$ or w (say)

(iv) for $x = 1$ $y' = 1/(1+\alpha)$ or u (say) (ii), (iii) and (iv) give us

$$\left.\begin{aligned}\frac{h}{k^2} + \frac{t}{j^2} &= 1 \\ \frac{h}{k\,(k+1)} + \frac{t}{j\,(j-1)} &= w \\ \frac{h}{(k+1)^2} + \frac{t}{(j-1)^2} &= u\end{aligned}\right\} \tag{23}$$

Note that $w = \dfrac{S-(1+\alpha)(Q+K)}{(S-Q)(1+\alpha)}$ which is positive and that $u - w = \dfrac{\alpha Q + (\alpha+1)K}{(S-Q)(1+\alpha)}$ which is also positive and so

$$0 < w < u < 1 \tag{24}$$

To solve (23) I put

$$p = \frac{h}{k\,(k+1)} \qquad \lambda = \frac{h}{k^2} \tag{25}$$

Then

$$w - p = \frac{t}{j\,(j-1)} \qquad 1 - \lambda = \frac{t}{j^2}$$

from which we get

$$\frac{p^2}{\lambda} = \frac{h}{(k+1)^2} \qquad \frac{(w-p)^2}{1-\lambda} = \frac{t}{(j-1)^2}$$

so we must have

$$\frac{p^2}{\lambda} + \frac{(w-p)^2}{1-\lambda} = u \tag{26}$$

and if we can find values of p and λ that satisfy this and such that $w - p > 1 - \lambda > 0$ and $\lambda > p > 0$ then these values will give a solution of (23).

Now (26) can be written

$$(p - w\,\lambda)^2 = \lambda\,(1-\lambda)(u - w^2)$$

and as $w < u < 1$ therefore $u > w^2$ so put $u - w^2 = \sigma$ which is positive and we have

$$(p - w\,\lambda)^2 + (\lambda - \tfrac{1}{2})^2\,\sigma = \sigma/4$$

which is an ellipse in p and λ (see Fig. VI) with center $\lambda = \frac{1}{2}$, $p = w/2$, passing through the origin $(0, 0)$ and touching the p axis there, also passing through $(w, 1)$ and touching $\lambda = 1$ there. It cuts the λ axis at $\lambda = 0$ and $\lambda = \sigma/u$ and also cuts $p = w$ at $\lambda = 1$ and $\lambda = w^2/u$.

Also the line $w - p = 1 - \lambda$ passes through $(w, 1)$ and cuts the λ axis at $\lambda = 1 - w$. Also since w is less than unity the line $p = \lambda$ which is parallel to $w - p = 1 - \lambda$ passes through the origin and lies to the left of $w - p = 1 - \lambda$.

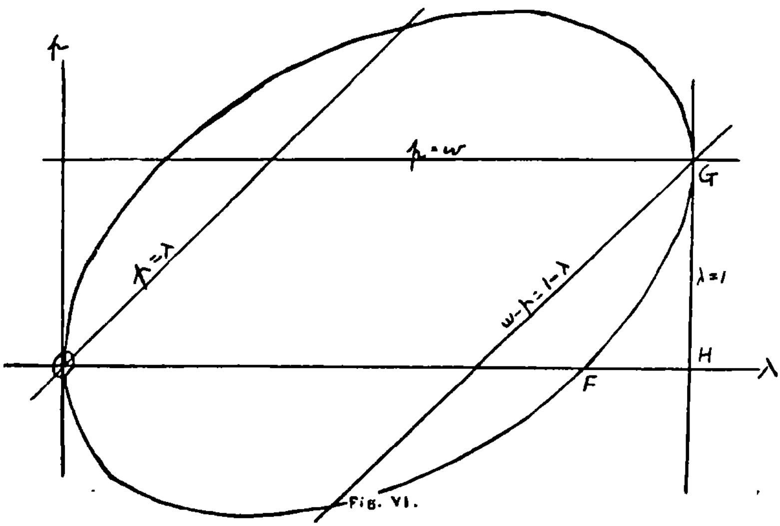


Fig. VI.

Thus all the solutions are given by the arc of the ellipse from $p = 0$, $\lambda = \sigma/u$ to $p = w$, $\lambda = 1$ or (in Fig. VI) from F to G.

There is one "degree of freedom" in this solution as there is one more constant in (22) than there are conditions to be fulfilled.

This is expressed by the possibility of choosing any point on the arc $F\,G$ to give values of p and λ. As $F\,H = w^2/u$ and is usually small compared with $O\,H$ which equals one, a good set of values for p and λ is usually obtained by putting $\eta = ½$ in $p/\lambda = (1 - \eta)\,w$, the equation which gives all the solutions by varying η from 0 to 1.

The solution is thus:

$$\text{Put } \frac{p}{\lambda} = (1 - \eta)\,w \qquad 0 < \eta < 1 \tag{27a}$$

Then solving (26) for λ

$$\lambda = \frac{u - w^2}{u - w^2\,(1 - \eta^2)} \tag{27b}$$

$$p = (1 - \eta)\,w\,\lambda. \tag{27c}$$

Then from (25)

$$k = \frac{p}{\lambda - p} \tag{27d}$$

$$h = \lambda k^2 \tag{27e}$$

$$j = \frac{w - p}{(w - p) - (1 - \lambda)} \tag{27f}$$

$$t = (1 - \lambda) j^2 \tag{27g}$$

Then from $y = \frac{h}{k} - \frac{t}{j} - \frac{h}{k + x} + \frac{t}{j - x}$

$$Y = \left(\frac{h}{k} - \frac{t}{j}\right)(S-Q) + (Q+K) - \frac{h\,(S-Q)^2}{E-Q+(S-Q)\,k} + \frac{t\,(S-Q)^2}{j\,(S-Q)+Q-E} \tag{27h}$$

$$\zeta = \frac{E}{Y} \tag{27i}$$

If η is taken as 0, $p = 0$ and $\lambda = \sigma/u$, the partial curve $y = \frac{h}{k} - \frac{h}{k + x}$ degenerates to $y = 0$ and the curve for Y is not a proper tangent at q: similarly if $\eta = 1$, $p = w$ and $\lambda = 1$, the partial curve $y = \frac{t}{j - x} - \frac{t}{j}$ degenerates to $y = 0$ and the curve for Y is not a proper tangent at S. η should therefore be taken between 0 and 1 say at ½ as suggested above.

The equation for Y is of the form

$$Y = A_1 - \frac{B_1}{C_1 + E} + \frac{B_2}{C_2 - E} = \frac{C_3 + B_3 E - A_1 E^2}{(C_1 + E)(C_2 - E)}$$

and so the equation for ζ is of the form

$$\zeta = \frac{E\,(C_1 + E)(C_2 - E)}{C_3 + B_3 E - A_1 E^2}$$

a cubic equation. (All the A's, B's and C's are constants).

Appendix II

Proof that W (and therefore Z_e) increases with E.

We wish to show that for our construction of W for formula (14)—and for formula (31)—W increases with E. An algebraical proof is given below but first it is constructive to examine the question geometrically and in terms of Y and E as shown in Fig. V.

Taking equation (28a)

$$(\alpha Y + K)\, W = E - Y + K$$

we can regard this as the equation of a family of curves in Y and E with W as the parameter. The equation can be written as

$$(\alpha W + 1)\, Y = E + K\,(1 - W)$$

showing this represents a family of straight lines. Each one passes through the point L of Figure V, the intersection of $Y = E + K$ and $Y = E/(1 + \alpha)$, the coordinates of which are

$$E_L = -\frac{1+\alpha}{\alpha}\,K, \quad Y_L = -\frac{K}{\alpha}$$

For $W = 0$ the line is $Y = E + K$ or the line $L\,q$, and for $W = 1$ the line is $(\alpha + 1)\, Y = E$ or the line $L\,O\,s$: and as W goes from 0 to 1 the line rotates round L from $L\,q$ to $L\,s$. Now drawing

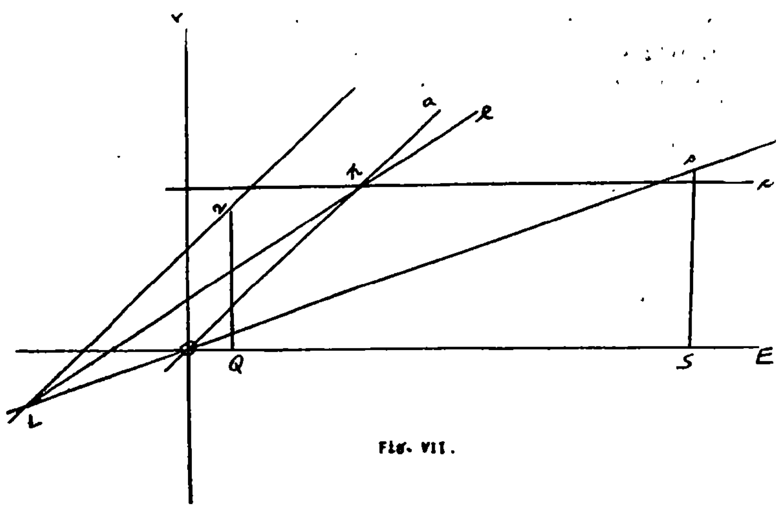


FIG. VII.

Fig. VII we see that if, at any point p of the curve $q\,s$ we are constructing for Y, W is to decrease, the tangent to the curve at p

must fall in the angle $a\,p\,l$ where a is on $O\,p$ extended and l is on $L\,p$ extended.

Now the conditions to which the curve $q\,p\,s$ is subject are that the tangent is in the angle $a\,p\,c$ where $p\,c$ is parallel to the E axis $O\,Q\,S$ and since $a\,p\,l$ falls inside $a\,p\,c$ it is possible to construct the curve so that W decreases: but we observe that our construction does not permit of this: for as noted in Appendix I the tangent always cuts $L\,q$ above L and thus the tangent always falls in the angle $l\,p\,c$. Thus W cannot decrease for our construction.

We can now give an algebraic proof of the increasing of W with E. To do this we obtain the inequality expressing the fact noted above that the tangent to $q\,s$ cuts $L\,q$ above L. If the coordinates of the intersection of $L\,q$ and the tangent are E_T and L_T we have

$$Y_T = Y'\,(E_T - E) + Y = E_T + K$$

whence $E_T = \dfrac{Y - K - E\,Y'}{1 - Y'}$

therefore $\dfrac{Y - K - E\,Y'}{1 - Y'} > E_L > -\dfrac{\alpha + 1}{\alpha}\,K$

$$\text{or } \alpha\,Y + K > Y'\,\{\alpha\,E + (\alpha + 1)\,K\}.$$

Translating this back into terms of ζ we put $Y = E/\zeta$ and $Y' = (\zeta - E\,\zeta')/\zeta^2$ and get

$$E\,\zeta'\,\{\alpha\,E + (\alpha + 1)\,K\} > \zeta\,K\,(\alpha + 1 - \zeta).$$

Now differentiating (28)

$$(\alpha\,E + \zeta\,K)^2\,W' = E\,\zeta'\,\{\alpha\,E + (\alpha + 1)\,K\} - \zeta\,K\,(\alpha + 1 - \zeta)$$

which is positive by the inequality just proved.

Thus W' is positive.

We will now give a proof in the case of the construction given for formula (31): the geometrical proof is considerably complicated by the variability of M and we will not give it. We can, however, readily extend the algebraic proof as follows:

Proceeding as in the proof for formula (14) we have E_L given by

$$\frac{E_L}{(1 + \alpha)(1 - r)} = E_L + \frac{K}{1 - r} \quad \text{or} \quad E_L = -K\,\frac{\alpha + 1}{\alpha\,(1 - r) - r}$$

and E_T is given by $Y'\,(E_T - E) + Y = E_T + \dfrac{K}{1 - r}$

so $E_T = \dfrac{Y - E\,Y' - K/(1 - r)}{1 - Y'}$ which is greater than E_L

Thus

$$\{a\,(1-r)-r\}\ Y+K/(1-r)>Y'\ [\{a\,(1-r)-r\}\ E+(a+1)\ K].$$

Now putting $Y=\dfrac{E}{\zeta\,(1-r)}$ $\qquad Y'=\dfrac{\zeta-E\,\zeta'}{\zeta^2\,(1-r)}$

we get $E\,\zeta'\ [\{a\,(1-r)-r\}\ E+(a+1)\ K]>\zeta\,K\ (a+1-\zeta)$.

Now $K_E \nless K$ so we can put K_E for K in the left hand side of this inequality which then becomes $E\,\zeta'\ \{a\,E+(a+1)\ M\}$.

Also the maximum value of $M-E\,M'$ is, as we have seen in paragraph 19, equal to K: so we can put $M-E\,M'$ for K in the right hand side. So we have

$$\zeta'\,E\,\{a\,E+(a+1)\ M\}>\zeta\,(M-E\,M')\,(a+1-\zeta).$$

Differentiating (36) we get

$$(a\,E+\zeta\,M)^2\,w'=\zeta\,E\ \{a\,E+(a+1)\ M\}-\zeta\,(M-E\,M')\,(a+1-\zeta)$$

and by the inequality just proved the right hand side is positive and so W (and therefore Z_e) increases with E.

Appendix III

Direct Construction of W for Formula (14).

At the end of paragraph (12) I had to choose between

(a) constructing ζ so that ζ' is positive and $(\zeta/E)'$ negative and then seeing if W' is positive; or

(b) constructing W so that W' is positive and $(\zeta/E)'$ negative.

I chose (a) but stated that (b) would lead to identical values of W.

In this Appendix we will work out (b).

We must first express in terms of W the condition that $(\zeta/E)'$ must be negative. Dividing (18) through by E and differentiating we get

$$\{E+K\ (1-W)\}^2\ (\zeta/E)'=W'\ \{a\,E+(a+1)\ K\}-(1+a\,W)$$

and the right hand side multiplied by a is equal to

$$\{a\,E+(a+1)\ K\}^2$$

times the derivative with respect to E of

$$\frac{1+a\,W}{a\,E+(a+1)\,K}$$

So the condition that ζ/E must decrease is equivalent to the condition that $(1+a\,W)/\{a\,E+(a+1)\ K\}$ must decrease.

Now if we put

$$a\,\omega = 1 + a\,W \qquad a\,\epsilon = a\,E + (a+1)\,K$$

or in other words change the origin from $E = 0$, $W = 0$ to

$$E = -\frac{a+1}{a}\,K \qquad W = -\frac{1}{a}$$

the conditions W' is to be positive and $(\zeta/E)'$ is to be negative become ω' is to be positive and $(\omega/\epsilon)'$ is to be negative (where the differentiations are here with respect to ϵ). These are very similar to the conditions under which we constructed ζ. We have the terminal conditions that

(i) when $\epsilon = Q + \dfrac{a+1}{a}\,K \qquad \omega = \dfrac{1}{a},\ \omega' = 0$

(ii) when $\epsilon = S + \dfrac{a+1}{a}\,K \qquad \omega = \dfrac{1+a}{a},\ \omega' = 0.$

Now if we put $\omega = \dfrac{\epsilon}{a\,V}$ we have to go

from $\epsilon = Q + \dfrac{a+1}{a} K \qquad V = Q + \dfrac{a+1}{a}\,K$ with $V' = 1$

to $\quad \epsilon = S + \dfrac{a+1}{a} K \qquad V = \dfrac{S}{1+a} + \dfrac{K}{a}$ with $V' = \dfrac{1}{1+a}$

so that V' is positive and $(V/\epsilon)'$ is negative.

These conditions are very similar to those for Y in Appendix I. In fact if we refer to Fig. V in Appendix I we see that if we change the origin from O (or $E = 0$, $Y = 0$) to L

$$(E = -K\,(a+1)/a,\ Y = -K/a)$$

by putting

$$a\,\epsilon_1 = a\,E + (a+1)\,K \qquad a\,V_1 = a\,Y + K$$

the conditions to which V_1 is subject become exactly those to which V is subject—except that the condition Y/E must decrease does not become the condition V_1/ϵ_1 must decrease. In other words the ϵ_1 and V_1 which we get this way, by transfering E and Y are exactly the ϵ and the V we have just derived from E and W: for it is easily seen that the two ϵ's are the same and as for the two V's the V_1 derived from Y equals

$$\frac{a\,Y + K}{a}$$

which is the same as $\dfrac{\alpha E/\zeta + K}{\alpha}$

$$\text{or} \quad \frac{\alpha E\{E + K(1 - W)\} + K E(\alpha W + 1)}{\alpha E(\alpha W + 1)} \quad \text{by (18)}$$

$$\text{or} \quad \frac{\alpha E + (\alpha + 1) K}{\alpha(\alpha W + 1)}$$

which equals $\epsilon/\alpha\,\omega$ or V derived from E and W.

Thus the only difference between the conditions for V and for Y are that for the former V/ϵ must decrease and for the latter Y/E. These represent the difference between the conditions with which we started. In constructing Y in Appendix I we required that this should make ζ' positive and in setting up V we required that this should make W' positive.

Now if Y/E is to decrease the tangent to the curve $q\,s$ must cut $O\,Y$ above O: and if V/ϵ is to decrease the tangent must pass above L or, as it can be put must cut $L\,q$ above L. It will be recalled that our construction actually fulfills *both* these conditions (or rather as it fulfills the harder condition that the tangent should pass above L it also fulfills the easier condition that it should pass above O) and it was because of this that W' proved to be positive as well as ζ'.

So if we finish the construction of V by

(i) transferring the origin $\epsilon = 0$ $V = 0$ from L to q at the same time making the unit $S - Q$ (just as we did in Appendix I for Y) and denoting the transformed ϵ by x and the transformed V by y; and

(ii) constructing y in terms of x just as in Appendix I

then we get the same values of y as in Appendix I and these give values of V in terms of ϵ that give the same values of W in terms of E as we get from the values of Y as obtained in Appendix I. Thus we see that if we set out to construct W direct so as to make W' positive and $(\zeta/E)'$ negative we arrive at exactly the same W values as we do by constructing ζ first as in Appendix I.

TABLE I

Examples of Results Produced by Formula (14)

$a = 4 \qquad \eta = \frac{1}{2}$

		E/S										
		.01	.05	.10	.145	.19	.28	.37	.55	.73	.91	1.00
				Q								S
New York	E	1400	7000	14000	20300	26600	39200	51800	77000	102200	127400	140000
	W	..	..	.000	.040	.095	.211	.328	.558	.776	.958	1.000
$S = 140000$	B	6900	6900	6900	6624	6245	5444	4637	3050	1546	290	0
$Q = 14000$	Z_n	.169	.504	.670	.754	.810	.878	.918	.962	.985	.998	1.000
$K = 6900$	Z_e	.000	.000	.000	.030	.077	.185	.301	.536	.763	.956	1.000
	$S\,Z_n/E$	16.90	10.08	6.70	5.20	4.26	3.14	2.48	1.75	1.35	1.10	1.00
	$S\ \zeta/E$	16.90	10.08	6.70	6.03	5.88	5.78	5.74	5.65	5.53	5.30	5.00
Massachusetts	E	900	4500	9000	13050	17100	25200	33300	49500	65700	81900	90000
	W	..	..	.000	.041	.095	.208	.320	.543	.760	.950	1.000
$S = 90000$	B	5520	5520	5520	5294	4996	4372	3754	2523	1325	276	0
$Q = 9000$	Z_n	.140	.449	.620	.711	.774	.852	.899	.952	.980	.997	1.000
$K = 5520$	Z_e	.000	.000	.000	.029	.074	.177	.288	.517	.745	.947	1.000
	$S\,Z_n/E$	14.00	8.98	6.20	4.90	3.07	3.02	2.43	1.73	1.34	1.10	1.00
	$S\ \zeta/E$	14.00	8.98	6.20	5.71	5.62	5.57	5.54	5.49	5.42	5.26	5.00
Georgia	E	420	2100	4200	6090	7980	11760	15540	23100	30660	38220	42000
	W	..	..	.000	.050	.100	.200	.301	.502	.703	.904	1.000
$S = 42000$	B	4140	4140	4140	3933	3726	3312	2894	2062	1230	397	0
$Q = 4200$	Z_n	.092	.337	.504	.607	.682	.780	.843	.918	.961	.990	1.000
$K = 4140$	Z_e	.000	.000	.000	.030	.068	.158	.253	.460	.675	.894	1.000
	$S\,Z_n/E$	9.20	6.74	5.04	4.19	3.59	2.79	2.28	1.67	1.32	1.09	1.00
	$S\ \zeta/E$	9.20	6.74	5.04	5.02	5.02	5.02	5.02	5.02	5.02	5.01	5.00

TABLE II

Examples of Results Produced by Formula (31)—when excess ratio of risk is the same as that for which the W's are calculated

$a = 4$ $\eta = ½$ $g = .333$

			E/S										
			.01	.05	.10	.145	.19	.28	.37	.55	.73	.91	1.00
					Q								S
New York		E	1400	7000	14000	20300	26600	39200	51800	77000	102200	127400	140000
		K_B	7000	7000	7000	8000	9700	13580	17630	25900	34240	42610	46790
$S = 140000$	$r = .333$	W	..	..	.000	.034	.092	.218	.346	.594	.813	.971	1.000
$Q = 14000$		B	7000	7000	7000	7728	8808	10620	11530	10515	6403	1236	0
$K = 7000$		Z_n	.176	.600	.857	.946	.972	.990	.995	.999	1.000	1.000	1.000
		Z_e	.000	.000	.000	.032	.089	.216	.344	.593	.813	.971	1.000
		$S\,Z_n/E$	17.60	12.00	8.57	6.52	5.11	3.54	2.69	1.82	1.37	1.09	1.00
		$S\ \zeta/E$	17.60	12.00	8.57	7.41	6.85	6.62	.641	6.13	5.82	5.36	5.00
	$r = .167$	W	..	..	.000	.041	.105	.238	.369	.611	.819	.970	1.000
		B	7000	7000	7000	7672	8682	10348	11125	10075	6197	1278	0
		Z_n	.171	.546	.750	.821	.850	.880	.901	.938	.970	.995	1.000
		Z_e	.000	.000	.000	.034	.089	.210	.333	.573	.794	.965	1.000
		$S\,Z_n/E$	17.10	10.92	7.50	5.66	4.47	3.14	2.44	1.71	1.33	1.09	1.00
		$S\ \zeta/E$	17.10	10.92	7.50	6.60	6.35	6.14	6.04	5.87	5.68	5.34	5.00
	$r = 0$	W	..	..	.000	.049	.118	.258	.389	.623	.820	.968	1.000
		B	7000	7000	7000	7608	8555	10076	10772	9764	6163	1364	0
		Z_n	.167	.500	.667	.727	.757	.795	.828	.888	.943	.989	1.000
		Z_e	.000	.000	.000	.036	.090	.205	.322	.553	.773	.957	1.000
		$S\,Z_n/E$	16.70	10.00	6.66	5.01	3.99	2.84	2.24	1.61	1.29	1.09	1.00
		$S\ \zeta/E$	16.70	10.00	6.67	6.01	5.87	5.77	5.72	5.64	5.53	5.30	5.00

TABLE II — *Continued*

Examples of Results Produced by Formula (31)—when excess ratio of risk is the same as that for which the W's are calculated

$a = 4$ $\quad \eta = 1/2$ $\quad g = .333$

			E/S .01	.05	.10	.145	.19	.28	.37	.55	.73	.91	1.00
					Q								S
Massachusetts		E	900	4500	9000	13050	17100	25200	33300	49500	65700	81900	90000
		K_B	5600	5600	5600	6060	6980	9250	11730	16920	22210	27550	30230
$S = 90000$	$r = .333$	W	..	..	.000	.030	.083	.202	.326	.572	.796	.967	1.000
$Q = 9000$		B	5600	5600	5600	5878	6401	7382	7906	7242	4531	909	0
$K = 5600$		Z_n	.145	5.23	.776	.887	.935	.973	.987	.996	.999	1.000	1.000
		Z_e	.000	.000	.000	.027	.077	.197	.322	.569	.795	.967	1.000
		$S\,Z_n/E$	14.50	10.46	7.76	6.12	4.92	3.48	2.67	1.81	1.37	1.10	1.00
		$S\,\zeta/E$	14.50	10.46	7.76	6.86	6.55	6.29	6.15	5.95	5.72	5.35	5.00
	$r = .167$	W	..	..	.000	.038	.098	.225	.352	.591	.802	.965	1.000
		B	5600	5600	5600	5830	6296	7169	7601	6920	4398	964	0
		Z_n	.142	.481	.687	.778	.821	.866	.890	.933	.967	.994	1.000
		Z_e	.000	.000	.000	.030	.080	.195	.314	.551	.775	.959	1.000
		$S\,Z_n/E$	14.20	9.62	6.87	5.37	4.32	3.09	2.41	1.70	1.32	1.09	1.00
		$S\,\zeta/E$	14.20	9.62	6.87	6.18	6.01	5.88	5.81	5.71	5.57	5.31	5.00
	$r = 0$	W	..	..	.000	.048	.113	.247	.374	.605	.804	.961	1.000
		B	5600	5600	5600	5769	6191	6965	7343	6683	4353	1074	0
		Z_n	.138	.446	.616	.693	.734	.783	.819	.881	.938	.987	1.000
		Z_e	.000	.000	.000	.033	.083	.193	.306	.553	.754	.949	1.000
		$S\,Z_n/E$	13.80	8.92	6.16	4.78	3.86	2.80	2.21	1.60	1.28	1.08	1.00
		$S\,\zeta/E$	13.80	8.92	6.16	5.69	5.61	5.56	5.53	5.48	5.42	4.69	5.00

TABLE II — *Continued*

Examples of Results Produced by Formula (31)—when excess ratio of risk is the same as that for which the W's are calculated

$a = 4$ $\eta = \frac{1}{2}$ $g = .333$

			E/S									
			.01	.05	.10	.145	.19	.28	.37	.55	.73	.91	1.00
					Q								S
Georgia		E	420	2100	4200	6090	7980	11760	15540	23100	30660	38220	42000
		K_E	4200	4200	4200	4320	4590	5390	6370	8560	10890	13290	14510
$S = 42000$	$r = .333$	W	..	..	.000	.028	.072	.174	.283	.510	.737	.942	1.000
$Q = 4200$		B	4200	4200	4200	4199	4260	4452	4567	4194	2973	771	0
$K = 4200$		Z_n	.094	.375	.600	.732	.817	.906	.948	.982	.994	.999	1.000
		Z_e	.000	.000	.000	.021	.059	.158	.168	.500	.733	.941	1.000
		$S\ Z_n/E$	9.40	7.50	6.00	5.05	4.30	3.23	2.56	1.79	1.36	1.10	1.00
		$S\ \zeta/E$	9.40	7.50	6.00	5.63	5.54	5.49	5.46	5.42	5.38	5.23	5.00
	$r = .167$	W	..	..	.000	.040	.091	.201	.314	.535	.745	.936	1.000
		B	4200	4200	4200	4147	4172	4307	4370	3980	2777	850	0
		Z_n	.092	.353	.545	.658	.729	.811	.857	.914	.954	.989	1.000
		Z_e	.000	.000	.000	.026	.067	.163	.269	.489	.711	.925	1.000
		$S\ Z_n/E$	9.20	7.06	5.45	4.54	3.84	2.90	2.31	1.66	1.46	1.13	1.00
		$S\ \zeta/E$	9.20	7.06	5.45	5.25	5.24	5.23	5.22	5.22	5.20	5.15	5.00
	$r = 0$	W	..	..	.000	.053	.109	.224	.338	.552	.745	.919	1.000
		B	4200	4200	4200	4091	4090	4183	4217	3835	2777	1076	0
		Z_n	.091	.333	.500	.598	.661	.738	.787	.858	.917	.973	1.000
		Z_e	.000	.000	.000	.032	.072	.166	.266	.473	.683	.894	1.000
		$S\ Z_n/E$	9.10	6.66	5.00	4.12	3.48	2.64	2.13	1.56	1.26	1.07	1.00
		$S\ \zeta/E$	9.10	6.66	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00

TABLE III

Examples of Results Produced by Formula (31)—when excess ratio of risk (x) is different from that (r) for which the W's are calculated

$a = 4$ $\eta = \frac{1}{2}$ $g = .333$

r	x		E/S: .01	.05	.10	.145	.19	.28	.37	.55	.73	.91	1.00
					Q								S
New York		E	1400	7000	14000	20300	26600	39200	51800	77000	102200	127400	140000
.333	.167	Z_n	.171	.546	.750	.820	.848	.877	.898	.936	.970	.995	1.000
		Z_e	.000	.000	.000	.028	.078	.191	.311	.556	.789	.966	1.000
		$S\,Z_n/E$	17.10	10.92	7.50	5.66	4.47	3.14	2.42	1.71	1.33	1.09	1.00
		$S\ \zeta/E$	17.10	10.92	7.50	6.43	6.10	5.87	5.80	5.74	5.66	5.33	5.00
	0	Z_n	.167	.500	.667	.725	.751	.787	.818	.880	.941	.990	1.000
		Z_e	.000	.000	.000	.025	.069	.172	.283	.523	.765	.961	1.000
		$S\,Z_n/E$	16.70	10.00	6.67	5.00	3.95	2.81	2.21	1.60	1.43	1.09	1.00
		$S\ \zeta/E$	16.70	10.00	6.67	5.68	5.40	5.26	5.26	5.40	5.47	5.31	5.00
.167	.333	Z_n	.177	.600	.857	.945	.973	.990	.996	.999	1.000	1.000	1.000
		Z_e	.000	.000	.000	.039	.102	.236	.368	.610	.819	.970	1.000
		$S\,Z_n/E$	17.70	12.00	8.57	6.52	5.12	3.54	2.70	1.82	1.37	1.09	1.00
		$S\ \zeta/E$	17.70	12.00	8.57	7.59	7.27	6.90	6.66	6.26	5.85	5.36	5.00
	0	Z_n	.167	.500	.667	.726	.754	.792	.823	.884	.943	.990	1.000
		Z_e	.000	.000	.000	.030	.079	.188	.304	.540	.772	.960	1.000
		$S\,Z_n/E$	16.70	10.00	6.67	5.01	3.97	2.83	2.22	1.67	1.29	1.09	1.00
		$S\ \zeta/E$	16.70	10.00	6.67	5.84	5.63	5.52	5.52	5.53	5.52	5.31	5.00

TABLE III — *Continued*

Examples of Results Produced by Formula (31)—when excess ratio of risk (x) is different from that (r) for which the W's are calculated

$a = 4$ $\eta = \frac{1}{2}$ $g = .333$

			E/S										
			.01	.05	.10	.145	.19	.28	.37	.55	.73	.91	1.00
r	x				Q								S
New York (Cont'd)													
0	.333	Z_n	.176	.600	.857	.947	.973	.990	.996	.999	1.000	1.000	1.000
		Z_e	.000	.000	.000	.046	.115	.255	.387	.622	.820	.961	1.000
		$S\,Z_n/E$	17.60	12.00	8.57	6.54	5.12	3.54	2.69	1.82	1.37	1.09	1.00
		$S\ \zeta/E$	17.60	12.00	8.57	7.80	7.54	7.18	6.87	6.34	5.87	5.35	5.00
	.167	Z_n	.171	.546	.750	.823	.851	.882	.904	.940	.971	.995	1.000
		Z_e	.000	.000	.000	.040	.100	.228	.352	.586	.796	.963	1.000
		$S\,Z_n/E$	17.09	10.92	7.50	5.08	4.48	3.15	2.45	1.71	1.33	1.09	1.000
		$S\ \zeta/E$	17.09	10.92	7.50	6.78	6.58	6.41	6.24	5.96	5.70	5.32	5.00
Massachusetts		E	900	4500	9000	13050	17100	25200	33300	49500	65700	81900	90000
.333	.167	Z_n	.142	.481	.687	.776	.819	.862	.889	.930	.966	.994	1.000
		Z_e	.000	.000	.000	.023	.068	.174	.290	.532	.769	.961	1.000
		$S\,Z_n/E$	14.20	9.62	6.87	5.36	4.31	3.08	2.40	1.69	1.32	1.09	1.00
		$S\ \zeta/E$	14.20	9.62	6.87	5.99	5.74	5.56	5.54	5.56	5.54	5.32	5.00
	0	Z_n	.138	.446	.616	.689	.728	.773	.808	.872	.935	.989	1.000
		Z_e	.000	.000	.000	.021	.060	.156	.263	.499	.744	.956	1.000
		$S\,Z_n/E$	13.80	8.92	6.16	4.75	3.84	2.76	2.19	1.58	1.28	1.09	1.00
		$S\ \zeta/E$	13.80	8.92	6.16	5.33	5.09	4.99	5.03	5.21	5.36	5.29	5.00

TABLE III — *Continued*

Examples of Results Produced by Formula (31)—when excess ratio of risk (x) is different from that (r) for which the W's are calculated

$a = 4$ $\eta = \frac{1}{2}$ $g = .333$

Massachusetts (Cont'd)			E/S										
			.01	.05	.10	.145	.19	.28	.37	.55	.73	.91	1.00
r	x				Q								S
.167	.333	Z_n	.145	.523	.776	.888	.936	.974	.988	.996	.999	1.000	1.000
		Z_e	.000	.000	.000	.034	.092	.219	.348	.589	.801	.965	1.000
		$S\,Z_n/E$	14.50	10.46	7.76	6.12	4.92	3.48	2.67	1.81	1.38	1.10	1.00
		$S\;\zeta/E$	14.50	10.46	7.76	7.07	6.87	6.61	6.44	6.09	5.76	5.34	5.00
	0	Z_n	.138	.446	.616	.691	.731	.778	.814	.877	.937	.988	1.000
		Z_e	.000	.000	.000	.026	.072	.175	.287	.518	.751	.953	1.000
		$S\,Z_n/E$	13.80	8.92	6.16	4.77	3.84	2.78	2.20	1.59	1.29	1.08	1.00
		$S\;\zeta/E$	13.80	8.92	6.16	5.48	5.36	5.28	5.30	5.36	5.40	5.27	5.00
0	.333	Z_n	.145	.523	.776	.881	.938	.975	.988	.997	.999	1.000	1.000
		Z_e	.000	.000	.000	.042	.106	.241	.370	.603	.803	.961	1.000
		$S\,Z_n/E$	14.50	10.46	7.76	6.07	4.93	3.48	2.67	1.81	1.37	1.10	1.00
		$S\;\zeta/E$	14.50	10.46	7.76	7.24	7.16	6.92	6.67	6.20	5.77	5.32	5.00
	.167	Z_n	.142	.481	.687	.780	.824	.869	.896	.935	.968	.993	1.000
		Z_e	.000	.000	.000	.037	.093	.215	.335	.566	.778	.954	1.000
		$S\,Z_n/E$	14.20	9.62	6.87	5.38	4.34	3.11	2.42	1.70	1.32	1.09	1.00
		$S\;\zeta/E$	14.20	9.62	6.87	6.40	6.29	6.17	6.04	5.81	5.59	5.28	5.00

TABLE III — *Continued*

Examples of Results Produced by Formula (31)—when excess ratio of risk (x) is different from that (r) for which the W's are calculated

$a = 4 \qquad \eta = \frac{1}{2} \qquad g = .333$

			E/S	.01	.05	.10	.145	.19	.28	.37	.55	.73	.91	1.00
	r	x				Q								S
Georgia			E	420	2100	4200	6090	7980	11760	15540	23100	30660	38220	42000
	.333	.167	Z_n	.092	.353	.545	.655	.725	.806	.852	.909	.953	.990	1.000
			Z_e	.000	.000	.000	.018	.052	.140	.241	.464	.702	.933	1.000
			$S\,Z_n/E$	9.20	7.06	5.45	4.52	3.82	2.88	2.30	1.65	1.31	1.09	1.00
			$S\ \zeta/E$	9.20	7.06	5.45	5.01	4.91	4.88	4.91	5.03	5.15	5.19	5.00
		0	Z_n	.091	.333	.500	.592	.652	.725	.773	.846	.915	.980	1.000
			Z_e	.000	.000	.000	.017	.047	.126	.219	.431	.674	.923	1.000
			$S\,Z_n/E$	9.10	6.66	5.00	4.08	3.43	2.59	2.09	1.54	1.25	1.08	1.00
			$S\ \zeta/E$	9.10	6.66	5.00	4.55	4.42	4.39	4.46	4.67	4.95	5.13	5.00
	.167	.333	Z_n	.094	.375	.600	.735	.820	.910	.950	.983	.994	.999	1.000
			Z_e	.000	.000	.000	.029	.075	.183	.298	.526	.741	.935	1.000
			$S\,Z_n/E$	9.40	7.50	6.00	5.07	4.32	3.25	2.57	1.79	1.36	1.10	1.00
			$S\ \zeta/E$	9.40	7.50	6.00	5.87	5.90	5.86	5.79	5.61	5.42	5.21	5.00
		0	Z_n	.091	.333	.500	.595	.657	.732	.781	.853	.917	.978	1.000
			Z_e	.000	.000	.000	.024	.060	.147	.245	.456	.683	.915	1.000
			$S\,Z_n/E$	9.10	6.66	5.00	4.10	3.46	2.61	2.11	1.55	1.26	1.07	1.00
			$S\ \zeta/E$	9.10	6.66	5.00	4.77	4.72	4.71	4.76	4.87	5.00	5.10	5.00

TABLE III — *Continued*

Examples of Results Produced by Formula (31)—when excess ratio of risk (x) is different from that (r) for which the W's are calculated

$a = 4$ $\eta = 1/2$ $g = .333$

Georgia (Cont'd)

r	x		.01	.05	.10	.145	.19	.28	.37	.55	.73	.91	1.00
					Q			E/S					S
0	.333	Z_n	.094	.375	.600	.737	.824	.912	.952	.983	.994	.999	1.000
		Z_e	.000	.000	.000	.039	.090	.204	.322	.543	.741	.918	1.000
		$S\,Z_n/E$	9.40	7.50	6.00	5.08	4.34	3.26	2.57	1.79	1.36	1.10	1.00
		$S\ \zeta/E$	9.40	7.50	6.00	6.16	6.23	6.17	6.05	5.74	5.42	5.13	5.00
	.167	Z_n	.092	.353	.545	.660	.733	.816	.861	.916	.954	.985	1.000
		Z_e	.000	.000	.000	.035	.080	.183	.291	.506	.711	.905	1.000
		$S\,Z_n/E$	9.20	7.06	5.45	4.55	3.86	2.91	2.33	1.67	1.31	1.08	1.00
		$S\ \zeta/E$	9.20	7.06	5.45	5.52	5.54	5.53	5.47	5.35	5.20	5.06	5.00

TABLE IV

Credibilities given for selected value of E if K is chosen to give credit of $6\frac{2}{3}\%$ for clear experience at qualification point

				Formula (14)		Formula (31)		Formula (39)	
	r	E	E_n	Z_n	Average Credibility	Z_n	Average Credibility	Z_n	Average Credibility
New York									
	.333	14000	9333	.670	.447	.857	.571	.619	.413
Qualification	.167	14000	11667	.670	.558	.750	.625	.670	.558
point $E = 600$	0	14000	14000	.670	.670	.667	.667	.709	.709
	.333	14000	9333	.670	.447	.857	.571	.619	.413
	.167	11200	9333	.619	.516	.686	.571	.619	.516
	0	9333	9333	.575	.575	.571	.571	.619	.619
Massachusetts									
	.333	9000	6000	.620	.413	.776	.517	.566	.377
Qualification	.167	9000	7500	.620	.517	.687	.573	.620	.517
point $E = 480$	0	9000	9000	.620	.620	.616	.616	.662	.662
	.333	9000	6000	.620	.413	.776	.517	.566	.377
	.167	7200	6000	.566	.472	.621	.517	.566	.472
	0	6000	6000	.521	.521	.517	.517	.566	.566
Georgia									
	.333	4200	2800	.504	.336	.600	.400	.448	.299
Qualification	.167	4200	3500	.504	.420	.546	.456	.504	.420
point $E = 360$	0	4200	4200	.504	.504	.500	.500	.549	.549
	.333	4200	2800	.504	.336	.600	.400	.448	.299
	.167	3360	2800	.448	.373	.480	.400	.448	.373
	0	2800	2800	.404	.404	.400	.400	.448	.448

CHART I

Values of $\frac{\zeta}{E}$ Formula (31) $\alpha = 4$, $\eta = \frac{1}{2}$

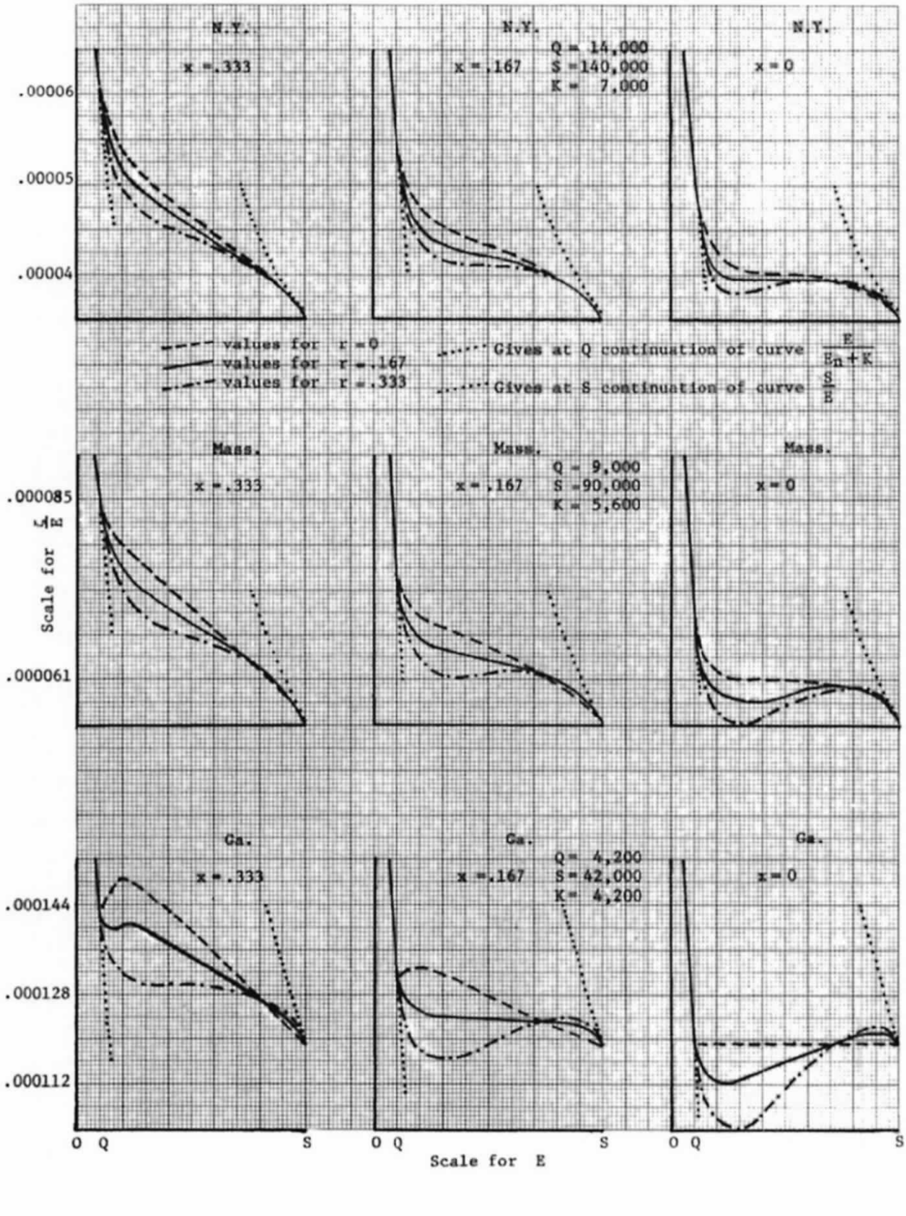
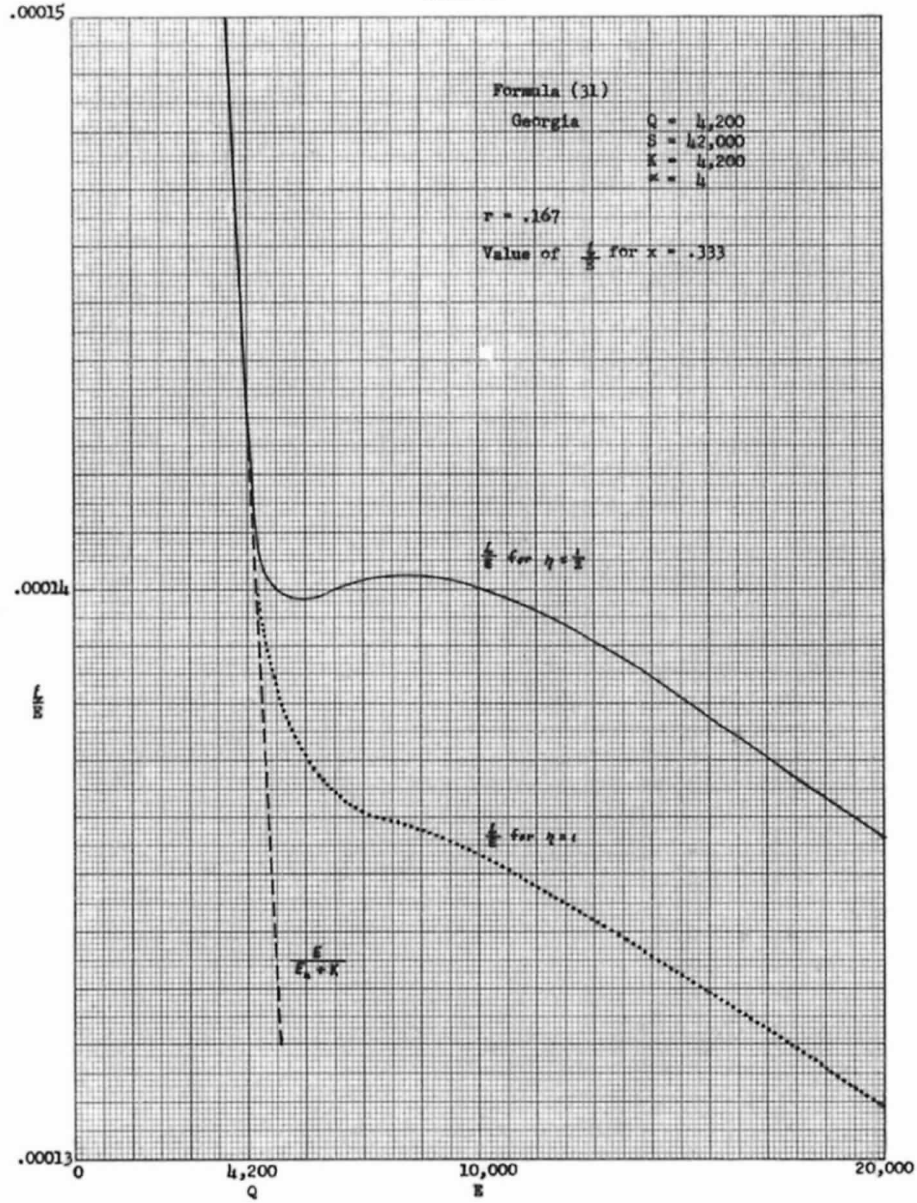


CHART II


Formula (31)
Georgia
Q = 4,200
S = 42,000
K = 4,200
κ = 4
r = .167
Value of $\frac{L}{E}$ for x = .333
.00015
.00014
.00013
$\frac{L}{E}$
0
4,200
Q
10,000
E
20,000

MINUTES OF THE 1971 ANNUAL MEETING

November 14 - 16, 1971

CHERRY HILL INN, CHERRY HILL, NEW JERSEY

Sunday, November 14, 1971

Prior to the formal convening of the Annual Meeting on the following day, the Board of Directors met at the Cherry Hill Inn from 2:30 p.m. to 5:30 p.m.

During the evening a small reception was held for the new Fellows (and their wives) who, later during the Annual Meeting, were presented with their Fellowship diplomas. At 6:30 p.m. an informal reception was held for the entire membership present.

Monday, November 15, 1971

The 1971 Annual Meeting was formally convened at 9:00 a.m. by President Richard L. Johe who welcomed the gathering and introduced the Honorable Robert L. Clifford, Commissioner of Insurance, State of New Jersey. Commissioner Clifford welcomed the gathering to New Jersey and presented his views on several areas of casualty insurance as it affects the New Jersey Department as well as the insurance buying public within the state.

Following Commissioner Clifford's address, President Johe presented diplomas to the following new Associates and Fellows:

ASSOCIATES

John B. Conners	Kenneth R. Ori	Lee M. Smith
Dorothy K. Dropick	Willard W. Peacock	Raymond R. Swaziek
James F. Golz	James P. Ross	Oliver T. Wilson
Douglas S. Haseltine	Harwood Rosser	John J. Winkleman, Jr.
Robert J. Lindquist	Bernard G. Schaeffer	Danny M. Young
Michael J. Miller	Edward C. Shoop	Edward W. Young
Neil L. Millman	Martin M. Simons	

FELLOWS

Clarence R. Atwood	Glenn W. Fresch
Ronald E. Ferguson	Alan G. Jones

The entire membership then observed a moment of silence in memory of the passing of the following individuals during the past year:

Augustin J. Cima	Hartwell L. Hall	Walter F. Sullivan
Frank A. Fleming	Allen L. Mayerson	Donald M. Wood

The next item was the Presidential address, a copy of which is printed in the *Proceedings*.

Ronald E. Ferguson then presented his new paper "Actuarial Note on Workmen's Compensation Loss Reserves."

At 10:15 a.m. a panel discussion was presented wherein members of the insurance press were interviewed by four members of the Society. This was the main feature of the morning program. Participants in the program were as follows:

Moderator: Luther L. Tarbell, Jr.
Second Vice President and Actuary
The Travelers Insurance Companies

Panel Members: John C. Burridge, Managing Editor
A. M. Best Company

Robert J. Cole, Insurance Editor
New York Times

Emanuel Levy, Editor
Insurance Advocate

David R. Bickerstaff, Actuary
Southern Farm Bureau Casualty Ins. Co.

Frederick W. Kilbourne, Consulting Actuary
Milliman and Robertson, Incorporated

Henry W. Menzel, Vice President
Insurance Services Office

Mavis A. Walters, Assistant Actuary
Insurance Services Office

The panel discussion was concluded at 11:30 a.m.

At 11:30 a.m. the following reviews of papers were presented to the membership:

> "Credibility for Severity" by Charles C. Hewitt, Jr. Guest review by Hans Bühlmann, which was read by Lester P. Dropkin.
>
> "Federal Income Taxes" by Raymond W. Beckman. Review by James W. MacGinnitie.

At 12:00 noon a formal luncheon was held after which an address was presented by Kenneth C. Foster, President, The Prudential Insurance Company of America. Mr. Foster discussed the various reasons the Prudential had for entering the property and casualty insurance business.

From 2:00 p.m. to 3:00 p.m. a panel discussion was held on National Health Insurance. Participants in this discussion were:

Moderator:	Paul E. Singer, Vice President and Actuary CNA/Insurance
Panel Members:	Leslie P. Hemry, President Health Insurance Association of America
	Roger A. Johnson, Actuary Blue Cross of Greater Philadelphia
	Robert Pollack, Executive Vice President Colonial Penn Insurance Company
	Vernon J. Switzer, Health Actuary State Farm Mutual Automobile Ins. Co.

After a brief break, a discussion of the report of the Committee on the Future Course of the Society was held. Leaders in this discussion were as follows:

Moderator:	Charles C. Hewitt, Jr., Actuary Allstate Insurance Company
Panel Members:	Rafal J. Balcarek, Vice President and Actuary Reliance Insurance Company
	James R. Berquist, Consulting Actuary Milliman and Robertson, Incorporated

William J. Hazam, Vice President and Actuary
American Mutual Liability Ins. Co.

M. Stanley Hughey, Executive V.P.
Lumbermen's Mutual Casualty Company

Paul J. Scheel, Associate Actuary
United States Fidelity and Guaranty Co.

Luther L. Tarbell, Jr.
Second Vice President and Actuary
The Travelers Insurance Companies

P. Adger Williams, Vice President
The Travelers Insurance Companies

The meeting was recessed at 5:00 p.m.

Following a reception for the entire membership from 6:00 to 7:00 p.m. a group banquet was held for the membership. After dinner, a musical program by Miss Laurie Barron, a music major at Temple University, was presented to the members and guests.

Tuesday, November 16, 1971

The meeting was reconvened at 9:00 a.m. with the election of Officers as the first item of business. The results of the election were as follows:

President	LeRoy J. Simon
President-Elect	Charles C. Hewitt, Jr.
Vice President	Paul S. Liscord
Secretary-Treasurer	Ronald L. Bornhuetter
Editor	Luther L. Tarbell, Jr.
Chairman, Education and Examination Committee	M. Stanley Hughey
Directors	Charles F. Cook George D. Morison John H. Muetterties
Director to fill unexpired term of Allen L. Mayerson	Dunbar R. Uhthoff

The next item of business was the presentation of the Financial Report for the completed year as well as the Secretary-Treasurer's Report of the activities of the Board of Directors. These reports are printed in the *Proceedings*.

Following the conclusion of the business meeting, three additional reviews were presented as follows:

> "Federal Income Taxes" by Raymond W. Beckman. Review by M. Stanley Hughey.
>
> "Federal Income Taxes" by Raymond W. Beckman. Guest review by Clyde Fulton, Travelers Insurance Companies which was read by David C. Forker.
>
> "Federal Income Taxes" by Raymond W. Beckman. Review by Jerome A. Scheibl.

Following the coffee break a panel discussion entitled "Working with Washington" was presented to the membership. The panel covered past, present and future relationships with the Federal Government.

Participants in this portion of the program were:

Moderator:	James J. Meenaghan Vice President and Actuary Fireman's Fund American Ins. Cos.
Panel Members:	J. Robert Hunter, Jr., Chief Actuary Federal Insurance Administration (HUD)
	Robertson Mackay, Secretary Aetna Life and Casualty
	Donald P. McHugh Vice President and General Counsel State Farm Fire and Casualty Company

At 11:15 a.m. the final portion of the program, a panel discussion presented by members of the academic staff of Temple University, was held.

The subject of the discussion was "Medical Malpractice — Can the Disease be cured?". The participants were as follows:

Moderator:	Gerald R. Hartman Associate Professor Temple University
Panel Members:	Charles P. Hall, Department Chairman Health Administration Temple University
	Samuel Polsky Professor of Law Temple University
	Robert Tyson, M.D. Medical School Temple University

The Annual Meeting was adjourned at 12:15 p.m. It is noted that the registration cards completed at the registration desk indicate, in addition to about 10 wives, attendance by 103 Fellows, 57 Associates and 12 invited guests, as follows:

FELLOWS

Allen, E. S.
Atwood, C. R.
Balcarek, R. J.
Barker, L. M.
Ben-Zvi, P. N.
Berquist, J. R.
Bevan, J. R.
Bickerstaff, D. R.
Bland, W. H.
Bornhuetter, R. L.
Boyajian, J. H.
Brown, W. W., Jr.
Carlson, E. A.
Comey, D. R.
Cook, C. F.
Crowley, J. H.
Curry, A. C.
Curry, H. E.
Dahme, O. E.
DeMelio, J. J.
Dickerson, O. D.
Drobisch, M. R.
Dropkin, L. B.
Eide, K. A.
Elliott, G. B.
Faber, J. A.
Ferguson, R. E.
Flaherty, D. J.
Forker, D. C.
Foster, R. B.
Fowler, T. W.
Fresch, G. W.
Gibson, J. A., III
Gillespie, J. E.
Gowdy, R. C.
Grady, D. J.
Hartman, G. R.
Hazam, W. J.
Hewitt, C. C., Jr.
Hughey, M. S.
Hunt, F. J., Jr.
Jacobs, T. S.
Johe, R. L.
Johnson, R. A.
Jones, A. G.
Kilbourne, F. W.
Klaassen, E. J.
Kormes, M.
Leslie, W., Jr.
Linder, J.
Liscord, P. S.
Longley-Cook, L. H.
Lowe, R. F.
MacGinnitie, W. J.
Makgill, S. S.
Masterson, N. E.
McClure, R. D.
McGuinness, J. S.
McLean, G. E.
McNamara, D. J.

Meenaghan, J. J.
Menzel, H. W.
Miller, P. V.
Mills, R. J.
Mohnblatt, A. S.
Morison, G. D.
Muetterties, J. H.
Munro, R. E.
Naffziger, J. V.
Newman, S. H.
Niles, C. L., Jr.
Oien, P. G.
Otteson, P. M.
Perkins, W. J.
Phillips, H. J., Jr.
Pollack, R.
Portermain, N. W.
Presley, P. O.
Quinlan, J. A.
Richards, H. R.
Riddlesworth, W. A.
Rodermund, M.
Ruchlis, E.
Salzmann, R. E.
Scheel, P. J.
Scheibl, J. A.
Scheid, J. E.
Simon, L. J.
Skelding, A. Z.
Skurnick, D.
Smick, J. J.
Smith, E. R.
Switzer, V. J.
Tarbell, L. L., Jr.
Uhthoff, D. R.
Verhage, P. A.
Walsh, A. J.
Ward, M. R.
Webb, B. L.
White, H. G.
Williams, D. G.
Williams, P. A.
Wilson, J. C.

ASSOCIATES

Balko, K. H.
Bergen, R. D.
Cadorine, A. R.
Carson, D. E. A.
Chorpita, F. M.
Coates, W. D.
Conners, J. B.
Cooper, W. P.
Copestakes, A. D.
Davis, R. C.
Dropick, D. K.
Fossa, E. F.
French, J. T.
Golz, J. F.
Gossrow, R. W.
Hardy, H. R.
Hartman, D. G.
Haseltine, D. S.
Head, T. F.
Jensen, J. P.
Klingman, G. C.
Levin, J. W.
Lindquist, R. J.
Linquanti, A. J.
Margolis, D. R.
Miller, M. J.
Millman, N. L.
Mokros, B. F.
Moore, P. S.
Napierski, J. D.
Neidermyer, J. R.
Ori, K. R.
Peacock, W. W.
Pilon, A.
Richardson, H. F.
Ross, J. P.
Rosser, H.
Sandler, R. M.
Sawyer, J. S., III
Scammon, L. W.
Schaeffer, B. G.
Shoop, E. C.
Simons, M. M.
Singer, P. E.
Smith, L. M.
Spitzer, C. R.
Spooner, F. A.
Stern, P. K.
Swaziek, R. R.
Thompson, E. G.
Trees, J. S.
Walters, M. A.
Welch, J. P.
Wilson, O. T.
Winkleman, J. J., Jr.
Young, D. M.
Young, E. W.

GUESTS

Anderson, E. V.
Blanc, R.
Burridge, J. C.
Clifford, R. L.
Cole, R. J.
Foster, K. C.
Hall, J. W.
Hemry, L. P.
*Kedrow, W. M.
Knox, F. J.
Levy, E.
McHugh, D. P.

* Invitational Program

Respectfully submitted,

RONALD L. BORNHUETTER
Secretary-Treasurer

REPORT OF THE SECRETARY-TREASURER

The Board of Directors met during the year on the following dates:

December 11, 1970
March 19, 1971
May 16, 1971
October 8, 1971
November 14 and 15, 1971

In addition, mail votes were conducted on several items during the year.

The highlights of the actions taken by the Board at these meetings are best summarized into categories as follows:

A. *Future Course of the Casualty Actuarial Society*

Special committees with specific assignments were established and reports considered during the year were in the following areas:

1. *Interest Areas*
   The final report of this Committee, which summarized the results of an extensive questionnaire, was accepted.
2. *Levels of Certification*
   The final report of this Committee has been received by the Board and will be acted upon during the next year.
3. *Recruitment of New Candidates*
   Interim reports have been furnished to the Board during the year with a final report expected to be considered by the Board in 1972.
4. *Forms of Amalgamation*
   The report of this Committee has been received by the Board and will be considered in depth during 1972.
5. *Education and Examination*
   Activities in this area are still in progress.
6. *Editorial Committee*
   The Board accepted the recommendation that the *Proceedings* continue to be printed in hard cover form.

B. *Professional Conduct*

The Board adopted two opinions (CAS-1 and CAS-2). Only the Guides will be printed in the *Year Book* while the members will be sent copies of the two opinions.

C. *Bylaws and Constitution*

This Committee's main assignment was completed with the adoption of a new Constitution and Bylaws at the May 1971 membership meeting. Several additional assignments remain to be presented to the Board of Directors in 1972.

D. *Examinations*

A new program of furnishing grades to failing candidates was introduced in 1971 and is to be continued. The Board also inaugurated an experimental program of bringing exam graders together for one or two day sessions in order to expedite the completion of the exam results. This program met with such a high degree of success that it has been adopted on a permanent basis.

E. *High School Mathematical Contest*

The Board approved the CAS's being a co-sponsor of the High School Mathematics Contest through a contribution of $1,500.00.

F. *Nominating Committee Guideline*

The Board adopted new nominating committee guidelines in line with the changes in the Constitution and Bylaws. In addition, the Board instructed the Nominating Committee to be guided by the Preferential Ballot in determining the number of candidates to be offered for the office of Vice President.

G. *Finance Committee*

The Board adopted a new insurance program for the Society. In addition, the Financial Statement, as certified to by the Finance Committee, was also approved.

The Board also approved for 1972 an increase in the dues for a Fellow and an Associate (more than 5 years membership) from $50.00 to $60.00 and all other Associates from $25.00 to $40.00.

The exam fee for Part 3 was increased from $9.00 to $10.00 and for Parts 4-9 from $15.00 to $20.00.

The Budget for 1972 was also approved by the Board.

H. *New Committees and Delegates*

The Board approved the establishment of the following new Committees or individual assignments:

a) Committee on Government Statistics
b) Alternate Delegate to ASTIN
c) Liaison Representative to Joint Actuarial Committee on Financial Reporting
d) Assistant to Secretary-Treasurer

I. *Sites*

The Board approved the switch of the November 1972 meeting from Ann Arbor, Michigan to the West Coast area. The meeting is be held in San Francisco, California.

The Board also agreed with the Site Committee's recommendation that the Greenbrier, White Sulphur Springs, West Virginia be investigated for the 1975 Spring meeting and Atlanta or Williamsburg for the 1976 Fall meeting.

J. *Secretary-Treasurer's Office*

The Board adopted the recommendation of a Special Committee that the office of the Secretary-Treasurer remain at the National Council for the foreseeable future. It would be uneconomical for the CAS to establish its own office.

K. *Joint Sponsorship of Examinations*

The Board has extended an invitation to other actuarial bodies to jointly sponsor any parts of the CAS examinations which count toward membership in that body. In addition, these organizations would be invited to appoint a liaison representative to work with the CAS Education and Examination Committee.

Copies of the detailed 1970-1971 Financial Statement of the Society were available at the November meeting. During the year total income amounted to $54,274.78; expenses were $47,285.67 resulting in an increase in assets of $6,989.11. As of September 30, 1971 the assets of the CAS amounted to $74,904.00.

Respectfully submitted,

RONALD L. BORNHUETTER
Secretary-Treasurer

FINANCIAL REPORT

Income and Disbursements
(from October 1, 1970 through September 30, 1971)

Income		Disbursements	
Dues	$16,880 00	Printing and stationery	$20,938 82
Examination fees	12,267 10	Secretary-Treasurer's office	2,400 00
Meetings	11,610 05	Examination expense	4,739 55
Sale of *Proceedings*	3,734.00	Meeting expense	11,387 11
Sale of Readings	911 23	Library	716 24
Invitational program	1,500 00	Insurance	262 00
Michelbacher Fund	820 57	Meeting refunds	347 50
Interest	5,251 83	Examination refunds	270 50
Registration-ACNY	1,300 00	ACNY	1,300 00
		Investment	20 00
		Georgia State	2,500 00
		Mathematical Association	1,500 00
		Miscellaneous	903 95
Total	$54,274 78		$47,285 67

Assets

As of 10/1/70		As of 9/30/71		Change
Checking Account	$ 858.26	Checking Account	$ 3,596 34	$ 2,738 08
Savings Account	12,000.69	Savings Account	44,637 62	32,636.93
Investments	55,055 94	Investments	26,670.04	—28,385 90
	$67,914 89		$74,904 00	$ 6,989.11

Investments

	Cost
U S A Treasury Bond #1673 Due 11/15/74	$ 1,000 00
U S A Treasury Bond #1674 Due 11/15/74	1,000 00
U.S A Treasury Bond #299 Due 2/15/75	4,981 25
U S A Treasury Bond #5263 Due 2/15/80	4,325 00
U S A Treasury Bond #21733 Due 11/15/71	
U S A Treasury Bond #7478 Due 11/15/71	15,363 79*
*Cost price includes $265.35 accrued interest	$26,670 04

This is to certify that we have audited the accounts and the assets shown above and find same to be correct

Finance Committee
JOHN H BOYAJIAN
THOMAS W FOWLER
ALBERT J WALSH
HENRY W MENZEL, Chairman

1971 EXAMINATIONS — SUCCESSFUL CANDIDATES

Examinations for Parts 3, 5, 7 and 9 of the Casualty Actuarial Society syllabus were held May 12 and 14, 1971 and examinations for Parts 4, 6 and 8 were held November 9 and 10, 1971. Parts 1 and 2, jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries, were given May 13 and November 4. Those who passed Parts 1 and 2 were listed in the joint releases of the two Societies dated June 28, 1971 and December 21, 1971.

The following candidates successfully completed the requirements for Fellowship and Associateship in the November 1970 examinations and were awarded their diplomas at the May 1971 meeting:

FELLOWS

Comey, Dale R.	Richardson, James F.	Snader, Richard H.
Grady, David J.	Skurnick, David	Zory, Peter B.
Hunter, J. Robert, Jr.		

ASSOCIATES

Engel, Philip L.	Miller, Philip D.	Rinehart, Charles R.
Hoffmann, Dennis E.	Neidermyer, James R.	Thompson, Eugene G.
McClenahan, Charles L.		

MAY 1971 EXAMINATIONS

Following is the list of successful candidates in the examinations held in May, 1971:

FELLOWSHIP EXAMINATIONS

Part 7

Anker, Robert A.	Hoffmann, Dennis E.	Spitzer, Charles R.
Bell, Allan A.	Khury, Constandy K.	Stephenson, Elton A.
Bill, Richard A.	Krause, Gustave A.	Swaziek, Raymond R.
Engel, Philip G.	Moore, James E.	Tatge, Robert L.
Eyers, Robert G.	Neidermyer, James R.	Trees, John S.
Fossa, E. Frederick	Plunkett, Joseph A.	Walters, Michael A.
Hardy, Howard R.	Rinehart, Charles R.	Welch, John P.
Head, Thomas F.	Sandler, Robert M.	

Part 9

Anker, Robert A.	Ferguson, Ronald E.	Murray, Edward R.
Atwood, Clarence R.	Fresch, Glenn W.	Rinehart, Charles R.
Bartik, Robert F.	Hartman, David G.	Stewart, Charles W.
Conner, James B.	Jones, Alan G.	Walters, Mavis A.

NEW FELLOWS ADMITTED MAY 1971: Left to right: David J. Grady, David Skurnick, J. Robert Hunter, Jr., President Richard L. Johe, Peter B. Zory, James F. Richardson, Richard H. Snader, and Dale R. Comey.

NEW ASSOCIATES ADMITTED MAY 1971: Left to right: Eugene G. Thompson, Dennis E. Hoffmann, Philip D. Miller, President Richard L. Johe, James R. Neidermyer, Charles R. Rinehart, and Charles L. McClenahan. Missing from the picture was new Associate Philip L. Engel.

ASSOCIATESHIP EXAMINATIONS

Part 3

Alexander, Stephen A.
Alfuth, Terry J.
Andler, James A.
Biondi, Richard S.
Chesney, Richard A.
Chou, Philip S.
Davidson, David A.
Degerness, Jerome A.
Demers, Daniel
Edwards, John S.
Evans, Dale M.
Fallquist, Richard J.
Forman, Ben J.
Fusco, Michael
Golz, James F.
Graves, Janet S.
Harper, Pamela C.
Henry, Dennis R.
Hough, Paul E.
Inkrott, James G.
Jewell, William S.
Jones, James D.
Kenney, Roger K.
Kline, Douglas F.
Kochanski, Nancy M.
Kreuzer, James H.
Lis, Raymond S., Jr.
Marks, Rosemary N.
Mohl, Frederic J.
Penniman, Kent T.
Radach, Floyd R.
Schaeffer, Bernard G.
Sheppard, Alan R.
Simons, Martin M.
Smith, Lee M.
Streff, James P.
Strickland, Michael E.
Vincenzo, James J.
Winkleman, John J., Jr.
Woll, Richard G.
Wood, James O.
Young, Danny M.

Part 5

Brouillette, Yves J.
Conners, John B.
Dropick, Dorothy K.
Gardner, John E.
Golz, James F.
Grippa, Anthony J.
Gruber, Charles
Hall, James A.
Haseltine, Douglas S.
Kelly, Ann E.
Klein, David M.
Lester, Edward P.
Lieberman, Judy L.
Lindquist, Robert J.
Masella, Norma M.
Miller, Michael J.
Millman, Neil L.
Murphy, Edward J., Jr.
Ori, Kenneth R.
Pagnozzi, Richard D.
Peacock, Willard W.
Reinbolt, James B.
Retterath, Ronald C.
Rice, W. Vernon
Rogers, Daniel J.
Ross, James P.
Rosser, Harwood
Shoop, Edward C.
Smith, Lee M.
Swaziek, Raymond R.
Tverberg, Gail E.
Wilson, Oliver T.
Winkleman, John J., Jr.
Woll, Richard G.
Young, Danny M.
Young, Edward W.

As a result of the above examinations, four new Fellows and twenty new Associates were admitted at the Annual Meeting, November 15, 1971.

FELLOWS

Atwood, Clarence R.
Ferguson Ronald E.
Fresch, Glenn W.
Jones, Alan G.

ASSOCIATES

Conners, John B.
Dropick, Dorothy K.
Golz, James F.
Haseltine, Douglas S.
Lindquist, Robert J.
Miller, Michael J.
Millman, Neil L.
Ori, Kenneth R.
Peacock, Willard W.
Ross, James P.
Rosser, Harwood
Schaeffer, Bernard G.
Shoop, Edward C.
Simons, Martin M.
Smith, Lee M.
Swaziek, Raymond R
Wilson, Oliver T.
Winkelman, John J., Jr.
Young, Danny M.
Young, Edward W.

NEW FELLOWS ADMITTED NOVEMBER 1971: Left to right: Clarence R. Atwood, Ronald E. Ferguson, Glenn W. Fresch, outgoing President Richard L. Johe and Alan G. Jones.

NEW ASSOCIATES ADMITTED NOVEMBER 1971: Left to right, standing: Raymond R. Swaziek, Edward W. Young, Michael J. Miller, Edward C. Shoop, Lee M. Smith, James F. Golz, Danny M. Young, James P. Ross, Oliver T. Wilson, Bernard G. Schaeffer, Martin M. Simons, and John J. Winkleman, Jr.; seated left to right: John B. Conners, Neil L. Millman, Willard W. Peacock, outgoing President Richard L. Johe, Harwood Rosser, Dorothy K. Dropick, and Douglas S. Haseltine.

NOVEMBER 1971 EXAMINATIONS

The successful candidates in the November 1971 examinations were:

FELLOWSHIP EXAMINATIONS

Part 6

Bradshaw, John G., Jr.
Bryan, Charles A.
Conners, John B.
Dickson, Jeffrey J.
Dieter, George H., Jr.
Fossa, E. Frederick
Hall, James A.
Hartman, David G.
Haseltine, Douglas S.
Hearn, Vincent W.
Klein, David M.
Moore, Phillip S.
Murray, Edward R.
Pagnozzi, Richard D.
Rinehart, Charles R.
Ross, James P.
Sanko, Ronald J.
Smith, Lee M.
Stewart, C. Walter
Tatge, Robert L.
Winkleman, John J., Jr.

Part 8

Anker, Robert A.
Bartik, Robert F.
Bergen, Robert D.
Bill, Richard A.
Cadorine, Arthur R.
Drennan, John P.
Eyers, Robert G.
Golz, James F.
Hoffmann, Dennis E.
Kolodziej, Timothy M.
McDonald, Charles
Miller, Philip D.
Penniman, Kent T.
Rinehart, Charles R.
Smith, Lee M.
Song, Young B.
Stephenson, Elton A.
Walters, Michael A.
Zarrella, Edward G.

ASSOCIATESHIP EXAMINATIONS

Part 4 (b)

Rogers, Daniel J.

Part 4

Alexander, Stephen A.
Alff, Gregory N.
Andler, James A.
Bailey, Michael W.
Berry, Charles H.
Bertles, George G.
Biondi, Richard S.
Brouillette, Yves J.
Chou, Philip S.
Creswell, David L., Jr.
Curley, James O.
Davis, Rodney D.
Degerness, Jerome A.
Demers, Daniel
Dempster, Howard V., Jr.
Evans, Dale M.
Fallquist, Richard J.
Fisher, Wayne H.
Fusco, Michael
Gardner, John E.
Glover, William D.
Godbold, Nathan T.
Graves, Janet S
Griswold, Gerald W.
Grogan, Shirley M.
Gwynn, Holmes M.
Hough, Paul E.
Inderbitzin, Paul H.
Inkrott, James G.
Irwin Larry L.
Jones, James D.
Kaufman, Allan M.
Kayton, Howard H.
Kelly, Anne E.
Kline, Douglas F.
Kochanski, Nancy M.
Kollar, John J.
Kramer, Lawrence D.
Kuehn, Ronald T.
Lamb, John A.
Lamb, Michael R.
Lester, Edward P.
Marino, James F.
Marks, Rosemary N.
Mohl, Frederic J.
Nolan, John D.
Olson, Arthur J.
Petrelli, Joseph L.
Rapp, Jerry W.
Reinbolt, James B.
Retterath, Ronald C.
Roman, Spencer M.
Sargent, Dennis J.
Schultz, John J.
Sheppard, Alan R.
Stanard, James N.
Stergiou, Emanuel J.
Streff, James P.
Szczepanski, Gerald R.
Tinkler, William P.
Toothman, Michael L.
Watford, James D.
Wood, James O.

Six candidates for Fellowship and three candidates for Associateship completed their requirements in the above examinations and will be admitted at the Spring Meeting in 1972:

NEW FELLOWS

Anker, Robert A.	Hartman, David G.	Rinehart, Charles R.
Bergen, Robert D.	Murray, Edward R.	Stewart, C. Walter

NEW ASSOCIATES

Dempster, Howard V., Jr.	Reinbolt, James B.	Rogers, Daniel J.

OBITUARIES

AUGUSTIN J. CIMA

1932 — 1971

Augustin J. Cima, a Fellow of the Casualty Actuarial Society since November, 1966, died March 30, 1971, as a result of injuries suffered in an automobile accident.

Guy Cima was born in Latrobe, Pennsylvania, and attended Lehigh University from 1950 to 1952. His academic career was interrupted by service in the Army from September, 1952 to August, 1954. He was graduated from the University of Chicago in 1958 and did graduate work at the Illinois Institute of Technology.

Guy began his insurance career with Allstate Insurance Company in October, 1959. He was promoted to Associate Actuary in November, 1965, to Actuary in March of 1969 and most recently, to Pricing Director in 1971.

While Guy's actuarial career was all too brief, it was one of active involvement. He served on the Society's examination committee. He was a member of the FAIR Plan Procedural Advisory Committee — Accounting Subcommittee. He was chairman of the Florida Windstorm Underwriting Association Accounting Committee and the Alabama Insurance Underwriting Association Accounting Committee in addition to serving as a member of other accounting committees, namely that of the New York FAIR Plan, the New Jersey FAIR Plan and the Florida Sinkhole Reinsurance Association.

He was also a member of the American Academy of Actuaries and the Midwestern Actuarial Forum.

The tragic death of Guy Cima was shocking to his friends and fellow workers. He will best be remembered as one who truly cared for all humanity and who worked diligently to better the lives of those less fortunate.

He is survived by his wife, Mrs. Nancy Lyn Cima, by four sons; Christopher, 14, Stephen, 12, Thomas, 10 and Jeffrey, 5 and by two daughters; Kathryn, 8 and Rebecca, 3.

FRANK A. FLEMING

1893 — 1971

Frank A. Fleming was admitted to the Society as an Associate on November 16, 1923. His death on February 12, 1971 at the age of 78 ended a long career devoted to the service of mutual fire and casualty insurance. Although Mr. Fleming started his career with a life insurance company, he soon became interested and involved in the fire and casualty business. He was the manager of the New York Office of the American Mutual Insurance Alliance when the Mutual Insurance Rating Bureau was organized on September 27, 1929 and was named the first general manager. At the time of its organization the Mutual Bureau was a one state — one line rating organization (automobile liability insurance in New York). Under the direction of Mr. Fleming, the Mutual Bureau became a multi-line rating organization in all states.

On October 29, 1947, the Mutual Insurance Advisory Association was formed as an advisory organization under the rating laws of the various states. Mr. Fleming became the first general manager of the advisory organization and served in that capacity, as well as general manager of the Mutual Bureau, until retirement from both organizations on December 31, 1957. Under his direction, the interests of mutual companies relative to rating matters in workmen's compensation and fire and allied lines, in addition to the casualty lines of insurance under the jurisdiction of the Mutual Bureau, were well served.

After retirement, Frank Fleming left New York City where he had lived for nearly 40 years and spent a few years in traveling. He lived for a while in Mexico and later in Spain. Returning to this country Mr. Fleming spent the remainder of his years in San Francisco, California and Phoenix, Arizona.

Frank never married and at the time of his death in Phoenix, his only survivors were a sister and an older brother.

The high esteem in which Mr. Fleming was held was founded upon his personal qualities as a man, as well as upon his lifetime of important services to mutual insurance.

HARTWELL LEON HALL

1890 — 1971

Hartwell Leon Hall, former Chief Examiner of the State of Connecticut Insurance Department, died February 15, 1971 at his home in West Hartford, Connecticut, at the age of 80.

He was born in Ground Point, New York, and was a graduate of Cornell University.

He became Assistant Actuary of the Connecticut Insurance Department in 1925 and later Chief Examiner, serving in the latter capacity until his retirement in 1958.

As Chief Examiner during the Depression of the 1930's he won wide praise for his sound judgment, foresight and fairness in dealing with companies experiencing difficult times, thus helping them to weather the economic storm.

On the occasion of his retirement he received many messages of good wishes. Charles J. Zimmerman, then President and now Chairman of the Connecticut Mutual Life Insurance Company, wrote: "My associates . . . express the highest regard for you both as an examiner and as an individual. Man after man has commented on your fairness, on your friendliness, and on your ability to get the job done efficiently while at the same time winning friends for yourself and for your associates."

He was a member of the Casualty Actuarial Society and was active for many years on the Committee on Blanks of the National Association of Insurance Commissioners.

He served in the Army in World War I and was a member and former officer of Rau-Locke Post, American Legion. He was a member of the Church of the Redeemer (Universalist) of West Hartford and was a 32nd Degree Mason for fifty years.

Surviving Mr. Hall are his widow, Grace Lombard Hall, and a son, Robert, of West Hartford.

ALLEN L. MAYERSON

1925 — 1971

Allen L. Mayerson, Professor of Insurance and Actuarial Mathematics at the University of Michigan, died on September 11, 1971 at the age of forty-six. Two years ago he was diagnosed to have cancer. With characteristic fortitude, he concealed this fact from all but a few of his close relatives and friends, and maintained an active life almost up to the day of his death.

Born in Brooklyn, New York, he early exhibited the intellect and drive which soon won him recognition. After attending rapid advance schools in his native city, he graduated as a mathematics major from the University of Michigan at the age of nineteen; being elected to Phi Beta Kappa in his senior year. He was commissioned in the Navy upon graduation in 1944 and served until 1946. He then returned to Michigan to earn a Master's degree in actuarial mathematics.

He was an associate in statistics and research at the Institute of Life Insurance from 1947 to 1949 and served as actuary of the National Surety Corporation from 1949 to 1951. At that time, upon becoming a Fellow of the Society of Actuaries, he was appointed as Principal Life Actuary of the New York Insurance Department, where he served until 1956. During this period he was on leave for one year to accept a Fulbright Scholarship at the University of Paris.

In 1956, he returned to the University of Michigan with a dual appointment as Assistant Professor of Insurance in the Graduate School of Business Administration and of Mathematics in the College of Literature, Science and Arts. Here he continued until the time of his death, except for an interlude (1963-1966) as Insurance Commissioner of the State of Michigan, probably the only Commissioner who has held fellowships in both the life and casualty actuarial bodies. As Commissioner of Insurance he applied himself vigorously and achieved a number of improvements in the Department's organization.

Professor Mayerson's interests were far reaching. In addition to his technical competence, he was an accomplished linguist, being sufficiently fluent in French and Spanish to deliver papers in those languages to European audiences. He made frequent trips to Europe and was a member of the actuarial organizations of Great Britain, France, Spain and Switzerland, in addition to his memberships in this country. He served on the Board of Governors of the Society of Actuaries, also as President of the Michigan Actuarial Society, as a Vice-President of the American Academy of Actuaries and on the Council of the Casualty Actuarial Society. In recent years he provided technical assistance to insurance cooperatives in Peru and Chile and participated in designing an actuarial program for the Hebrew University of Jerusalem and served as a Visiting Professor there.

In spite of his demanding academic and professional activities, he somehow found time to pursue his interests in music, theater, art and archeology and to engage in his athletic hobbies of sailing and tennis. In all of this he was joined by his wife, the former Dorli Baenninger of Zurich, Switzerland, who gloried in her husband's distinguished career and thoroughly enjoyed her role of travelling companion and hostess. His colleagues in the University and in his profession, not to mention numerous students with whom he continued contacts, suffered a grievous loss in his untimely death in the prime of life.

WALTER F. SULLIVAN

1906 — 1971

Walter F. Sullivan, an Associate of the Casualty Actuarial Society since 1930, died in his home in San Francisco, California, January 7, 1971.

Mr. Sullivan, a native of Bethlehem, Pennsylvania, graduated from Pennsylvania State Forestry School (now a part of Penn State University) and then received a M.A. degree from Iowa State University in forestry. He then came to California to work for the forestry service but soon left it to become the Statistician for the Associated Indemnity Company in San Francisco in 1929. He remained with the Associated Indemnity Company (now a part of the Fireman's Fund American Insurance Companies) until 1941 at which time he joined the State Compensation Insurance Fund of California as a Statistician. In 1942, he was promoted to Assistant Actuary and became Actuary in 1953. During this period, he served upon the Actuarial Committee of the California Inspection Rating Bureau and his constructive advice upon matters pertaining to merit rating plans and overall rate level changes was always sought by the other members of the Committee. Mr. Sullivan was also a member of the San Francisco Actuarial Club.

Mr. Sullivan retired from the State Compensation Insurance Fund in May of 1970 to devote his full time to his lifelong hobby of photography. As a photographer, he was a member of the Photochrome Club of San Francisco and many of his photographs won awards in local competition, and several of his favorite photographs were exhibited in national competition. On several occasions he was asked to judge photographs at regional showings.

Mr. Sullivan is survived by his mother, Mrs. Frank Sullivan of Bethlehem; three sisters, Mrs. John Schimmel of Bethlehem, Mrs. Gertrude Bannann of New Jersey, and Mrs. Edith McCormick of Coopersburg, Pennsylvania; and a brother, Arthur of Evanston, Illinois.

DONALD M. WOOD

1882 — 1971

Donald M. Wood, an Associate of the Casualty Actuarial Society since 1915, died on September 6, 1971 at the age of 89.

He was a partner in the insurance agency of Childs & Wood for 63 years and was actively engaged in the affairs of the office up to the time of his death. He had been in the investment business with W. H. Calvin & Co.

before becoming a partner in Childs, Young & Wood in 1908. He was a past vice president of the Union League Club, a past president of the Glen View Club and a former director of Kroger Co. He put into operation and directed the affairs of an insurance company for the Kroger Co. — the Selective Insurance Co. of Cincinnati, and was a former chairman of the executive committee of the Manufacturers and Merchants Indemnity Co.

He is survived by his son Donald M., Jr.; three grandchildren, and three great-grandchildren.

INDEX TO VOLUME LVIII

INDEX TO VOLUME LVIII (Cont.)

INDEX TO VOLUME LVIII (Cont.)

CASUALTY ACTUARIAL SOCIETY

ORGANIZED 1914

1972 YEAR BOOK

Corrected to January 1, 1972

FOREWORD

The Casualty Actuarial Society was organized in 1914 as the Casualty Actuarial and Statistical Society of America, with 97 charter members of the grade of Fellow; the Society adopted its present name on May 14, 1921.

Actuarial science originated in England in 1792, in the early days of life insurance. Due to the technical nature of the business, the first actuaries were mathematicians; eventually their numerical growth resulted in the formation of the Institute of Actuaries in England in 1848. The Faculty of Actuaries was founded in Scotland in 1856, followed in the United States by the Actuarial Society of America in 1889 and the American Institute of Actuaries in 1909. In 1949 the two American organizations were merged into the Society of Actuaries.

In the United States problems requiring actuarial treatment were emerging in sickness, disability, and casualty insurance—particularly workmen's compensation, introduced in 1911 The differences between the new problems and those of life insurance led to the organization of the Casualty Actuarial Society in 1914 Dr. I M. Rubinow, who was responsible for its formation, became the Society's first president. Since the problems of workmen's compensation were the most urgent, many members played a leading part in developing the present scientific basis for that line of insurance. The object of the Society was, and is, the promotion of actuarial and statistical science as applied to the problems of insurance other than life insurance by means of personal communication, presentation and discussion of appropriate papers, collection of a library, and by other desirable means.

From its beginning the Society has grown constantly in membership, scope of interests, and scientific and related contributions to the non-life field. These contributions are found in original papers prepared by members of the Society and published in the annual *Proceedings* The presidential addresses constitute a valuable record of actuarial problems, some of them still unsolved, that have faced the insurance industry over the years.

In November 1950 the Constitution and Bylaws were amended to enlarge the scope of the Society to include all lines of insurance other than life insurance (specifically, fire and allied lines) in recognition of the multiple line powers granted by many states to both casualty and fire companies.

The membership of the Society includes actuaries employed by insurance companies, ratemaking organizations, and state insurance departments, and as independent consultants. The Society has two grades of membership, Fellowship and Associateship. Examinations for the two grades are held in May and November in various cities in the United States and Canada.

On the inside front cover of the *Year Book* are listed the *Proceedings* and other publications of the Society and their respective prices. The *Year Book* is published annually *Recommendations for Study* is a pamphlet outlining the course of study recommended for examination. The two booklets may be obtained free upon request to the Secretary-Treasurer, Casualty Actuarial Society, 200 E. 42nd Street, New York, N Y. 10017

TABLE OF CONTENTS

"The society exists for the benefit of its members; not the members for the benefit of the society."

—Herbert Spencer

THE BOARD OF DIRECTORS

January 1, 1972

**Officers:*

LeRoy J. Simon	*President*
Charles C. Hewitt, Jr.	*President-Elect*
Paul S. Liscord	*Vice President*
Ronald L. Bornhuetter	*Secretary-Treasurer*
Luther L. Tarbell, Jr.	*Editor*
M. Stanley Hughey	*General Chairman, Education and Examination Commitee*

†*Ex-Presidents:*

Daniel J. McNamara	1972
Richard L. Johe	1973

†*Elected:*

Norman J. Bennett	1972
Henry W. Menzel	1972
Dunbar R. Uhthoff (a)	1972
Alan C. Curry	1973
W. James MacGinnitie	1973
James J. Meenaghan	1973
Charles F. Cook	1974
George D. Morison	1974
John H. Muetterties	1974

*Terms expire at the 1972 Annual Meeting except for the Editor whose term expires May 1, 1973.

† Terms expire at the Annual Meeting of the year given.

(a) Appointed by the Board to fill the unexpired term of Allen L. Mayerson.

COMMITTEES

The functions of all committees are subject to the policy determination and overall direction of the Board of Directors.

PROGRAM COMMITTEE

This committee plans programs for Society meetings.

RONALD L. BORNHUETTER *(ex officio)*
CHARLES C. HEWITT, JR. *(ex officio)*
PAUL S. LISCORD *(ex officio)*
LEROY J SIMON *(ex officio)*

EDUCATION AND EXAMINATION COMMITTEE

The Education and Examination Committee determines the scope and content of the syllabus and course of reading for the examinations, and is responsible for the preparation of educational material. The committee is also responsible for the organization, management, and administration of the examinations, and for determining the standards to be achieved by successful candidates.

M. STANLEY HUGHEY, GENERAL CHAIRMAN
ALAN C. CURRY, EDUCATION CHAIRMAN
EARL F. PETZ, EDUCATION VICE CHAIRMAN

DAVID R. BICKERSTAFF
HARRY T BYRNE
RONALD E. FERGUSON
WILLIAM S. GILLAM
GERALD R. HARTMAN
EDWARD J. HOBBS
HUGH G. WHITE
FREDERIC J. HUNT, JR.
ROBERT L. HURLEY
JAMES J. MEENAGHAN
HARRY R. RICHARDS
JEROME A. SCHEIBL
ROBERT J. SCHULER

JOSEPH J. DEMELIO, EDUCATION VICE CHAIRMAN

RAYMOND W. BECKMAN
CHARLES A. HACHEMEISTER
BERNARD L. WEBB
DALE A. NELSON
PAUL A. VERHAGE

GEORGE D. MORISON, EXAMINATION CHAIRMAN
DANIEL J. FLAHERTY, EXAMINATION VICE CHAIRMAN
STEVEN H. NEWMAN, EXAMINATION VICE CHAIRMAN

MARTIN ADLER
PHILLIP N. BEN-ZVI
DALE R. COMEY
ORVAL E. DAHME
WALTER E. FARNAM
DAVID J. GRADY
CARLTON W. HONEBEIN
TERRY S. JACOBS
FREDERICK W. KILBOURNE
ROBERT F. LOWE
WILLIAM D. WHITE
RICHARD E. MUNRO
DALE A. NELSON
ROBERT S. OIEN
PHILLIP O. PRESLEY
JOHN A. QUINLAN
KEVIN M. RYAN
DAVID SKURNICK
RICHARD H. SNADER
MICHAEL R. WARD
BERNARD L. WEBB

Examination Consultants

NORMAN J. BENNETT
CHARLES F. COOK
WILLIAM S. GILLAM
RUTH E. SALZMANN
FRANK HARWAYNE
PAUL S. LISCORD
JACK MOSELEY

NOMINATING COMMITTEE

The Nominating Committee nominates candidates for offices of the Society and for Board membership and submits its selections to Fellows of the Society prior to the annual elections. The committee also makes recommendations to the Board to fill interim vacancies in Society offices or in Board membership.

WILLIAM J. HAZAM, CHAIRMAN

HAROLD E. CURRY
RICHARD L. JOHE
DANIEL J. MCNAMARA
HAROLD W. SCHLOSS

EDITORIAL COMMITTEE

The Editorial Committee is responsible for publication of the *Proceedings* and the *Year Book* The committee's functions include negotiation with the printer, planning and organization of the publications, preparation of copy for the printer, and correction of galley proofs and page proofs.

LUTHER L. TARBELL, JR., CHAIRMAN

DAVID C. FORKER
ROBERT L. HURLEY
MATTHEW RODERMUND
GEORGE D. MORISON
NEIL W. PORTERMAIN

COMMITTEE ON REVIEW OF PAPERS

The Committee on Review of Papers reviews and approves, or rejects, papers, reviews, and replies (other than guest papers invited by the President) submitted for publication in the *Proceedings*, applying the standards and rules set forth in the Guides for the Submission of Papers The committee also awards the Woodward-Fondiller Prize and the Dorweiler Prize.

ROBERT A. BAILEY, CHAIRMAN

CHARLES F. COOK
JAMES J. MEENAGHAN
JEROME A. SCHEIBL
LUTHER L. TARBELL, JR. *(ex officio)*

FINANCE COMMITTEE

The Finance Committee audits the financial records to insure that all transactions conform to the Bylaws. This function includes verification of the assets and certification that the financial statements prepared by the Secretary-Treasurer accurately reflect the Society's true financial position.

The committee also makes recommendations to the Board in all aspects of financial structure, including—but not limited to—preparation of the budget, establishment of dues and examination fees, investment of funds, fund-raising measures, administration of funds given or bequeathed to the Society, and major expenditures.

HENRY W. MENZEL, CHAIRMAN

WILLIAM H. CRANDALL
RICHARD D. MCCLURE
STEVEN H NEWMAN

PUBLIC RELATIONS AND PUBLICITY COMMITTEE

The Public Relations and Publicity Committee comprises separate subcommittees for the public relations and publicity functions. The public relations subcommittee has as its objective the development of ways and means of giving the public, and especially high school and college students, a better understanding of the work of the casualty actuary. The subcommittee also strives to promote interest in the actuarial profession among students. To those ends it cooperates with corresponding committees of other actuarial societies.

The publicity subcommittee publicizes, locally and nationally, Society activities such as meetings, panel discussions, significant committee reports, and admission of new Fellows and Associates. The primary purpose is to create knowledge and appreciation of the casualty actuarial function.

PAUL J. SCHEEL, CHAIRMAN

FREDERICK W. KILBOURNE, PUBLIC RELATIONS VICE CHAIRMAN

DALE R. COMEY
JAMES P. JENSEN
RONALD E. FERGUSON
W. JAMES MACGINNITIE
MAVIS A. WALTERS

ARNOLD S. MOHNBLATT, PUBLICITY VICE CHAIRMAN

ROBERT F. LOWE
R. GUSTAVE OIEN
JAMES B. M. MURRAY
STEPHEN L. PERREAULT

COMMITTEE ON SITES

The Committee on Sites investigates and suggests to the Board appropriate sites and dates for Society meetings. Upon Board approval, and on behalf of the Society, it enters into agreement with the chosen facility. The Committee also recommends to the President the membership of a local arrangements committee. Under the direction of the Program Committee, it helps the local committee with details of the meetings. Finally, it establishes and maintains guidelines on Society meetings to assist the local committee and the facility.

RICHARD LINO, CHAIRMAN

JOHN H. BOYAJIAN
VERNON J. SWITZER
EDWARD R. SMITH
JOHN P. WELCH
JAMES C. WILSON

COMMITTEE ON PROFESSIONAL CONDUCT

The Committee on Professional Conduct maintains continuous supervision of the guides to professional conduct, recommending, when necessary, revision or repeal of existing guides, or the adoption of new ones. The committee answers inquiries on professional conduct, both general inquiries and those relating to particular situations, but not those involving named members, consulting with the President if inquiries of the latter type are not referred by him. The committee assists the President and the Board in reviewing and evaluating any problem of professional conduct. It maintains liaison with other actuarial organizations in regard to their corresponding guides.

WILLIAM J. HAZAM, CHAIRMAN

HAROLD E. CURRY
FRANK HARWAYNE
DANIEL J. MCNAMARA

ANNUAL STATEMENT COMMITTEE

The Annual Statement Committee prepares any studies relating to financial accounting by insurance companies that may be requested by the Board. In addition, the committee informs the membership of current developments on items of interest. It also initiates projects and conducts the necessary research in any area of financial accounting of particular concern to its members.

RUTH E. SALZMANN, CHAIRMAN

ROBERT A. BAILEY
JAMES R. BERQUIST
JOHN W. CARLETON
JAMES H. CROWLEY
JAMES F. GILL
PHILLIP B. KATES
RICHARD E. MUNRO
PAUL M. OTTESON
MATTHEW RODERMUND

LIAISON REPRESENTATIVE JOINT ACTUARIAL COMMITTEE ON FINANCIAL REPORTING

This member represents the Society on the Joint Committee which is composed of twelve other representatives, three each from the American Academy of Actuaries, the Canadian Institute of Actuaries, the Conference of Actuaries in Public Practice, and the Society of Actuaries. The committee will consider the professional role of the actuary in financial reporting, the basic principles involved in financial reporting for life insurance companies, specialized problems such as deferred income taxes and special reinsurance agreements, and related problems in other lines of insurance.

RUTH E. SALZMANN

REPRESENTATIVES ON THE JOINT COMMITTEE ON REVIEW OF EDUCATION AND EXAMINATIONS

The joint committee is composed of three representatives from each of the six recognized professional actuarial organizations in the United States and Canada: the American Academy of Actuaries, the Canadian Institute of Actuaries, the Casualty Actuarial Society, the Conference of Actuaries in Public Practice, the Fraternal Actuarial Association, and the Society of Actuaries. The function of the committee is to conduct a continuing review of policy matters relating to the education and examination of actuaries and, after studying such matters, to make appropriate recommendations to the governing bodies of the organizations represented. The Society representatives are:

M. STANLEY HUGHEY
RICHARD L. JOHE
W. JAMES MACGINNITIE

DELEGATE TO ASTIN

ASTIN is the non-life insurance section of the International Actuarial Association. The delegate to ASTIN represents the Society on the ASTIN Committee and at ASTIN functions and informs Society members of ASTIN activities. He is the Society's primary link with actuaries and actuarial associations outside the United States and Canada.

CHARLES C. HEWITT, JR.
LEROY J. SIMON (Alternate)

RISK THEORY LIAISON REPRESENTATIVE

This member represents the Society in joint ventures with other actuarial organizations in the field of risk theory and related subjects.

CHARLES A. HACHEMEISTER

ADDITIONAL COMMITTEES

COMMITTEE ON GOVERNMENT STATISTICS

PHILLIP N. BEN-ZVI, CHAIRMAN

JAMES R BERQUIST
HAROLD E. CURRY
STANLEY A DORF
DARRELL W. EHLERT
J ROBERT HUNTER, JR.
RICHARD J. MILLS
NORTON E. MASTERSON
MICHAEL A WALTERS

TOPICS COMMITTEE

RAFAL J. BALCAREK, CHAIRMAN

ROBERT A. BRIAN
TERRY S JACOBS

PLANNING COMMITTEE

PAUL S. LISCORD, CHAIRMAN

CHARLES F COOK
JOHN H. MUETTERTIES
GEORGE D. MORISON

LIAISON REPRESENTATIVE TO THE COMMITTEE OF PRESIDENTS OF STATISTICAL SOCIETIES

DARRELL W. EHLERT

LIAISON REPRESENTATIVE TO MATHEMATICAL ASSOCIATION OF AMERICA

PAUL J. SCHEEL

ASSISTANT TO SECRETARY-TREASURER

ROBERT B. FOSTER

SPECIAL COMMITTEES

TEXTBOOK COMMITTEE

The Textbook Committee is arranging for and assisting in the preparation and publication of a textbook on casualty insurance mathematics in cooperation with the actuarial faculty of Georgia State University. The Committee acts as a liaison between Georgia State University and the Board.

RICHARD L. JOHE, CHAIRMAN

NORMAN J. BENNETT
CHARLES A. HACHEMEISTER
WILLIAM J HAZAM
CHARLES C. HEWITT, JR.
M. STANLEY HUGHEY
DANIEL J. MCNAMARA

COMMITTEE TO REVIEW THE CONSTITUTION AND BYLAWS

In the light of recent changes in election procedures voted by the membership of the Society, and in consideration of newly developing attitudes toward the legal responsibilities of the actuarial profession, this committee is charged with revising the Constitution and Bylaws to make them consistent with and responsive to the long-term requirements and goals of the Society.

WILLIAM C. ALDRICH, CHAIRMAN

DANIEL J. MCNAMARA
ROBERT POLLACK
MATTHEW RODERMUND
VERNON J. SWITZER

SPECIAL TASK FORCE TO STUDY RECRUITMENT OF NEW CANDIDATES TO THE PROFESSION

JAMES R. BERQUIST, CHAIRMAN

PAUL J. SCHEEL, VICE CHAIRMAN

RONALD L. FERGUSON
DAVID G. HARTMAN
GERALD R. HARTMAN
FREDERICK W. KILBOURNE
W. JAMES MACGINNITIE
NEIL W. PORTERMAIN
HARRY R. RICHARDS
MAVIS A. WALTERS
DEWEY G. WILLIAMS

COMMITTEE ON LEVELS OF CERTIFICATION

WILLIAM J. HAZAM, CHAIRMAN

NORMAN J. BENNETT
JOHN W. WIEDER, JR.

COMMITTEE TO IDENTIFY INTEREST AREAS

RAFAL J. BALCAREK, CHAIRMAN

WALTER J. FITZGIBBON, JR.
PAUL A. VERHAGE

COMMITTEE TO STUDY FORMS OF AMALGAMATION

P. ADGER WILLIAMS, CHAIRMAN

JOHN R. BEVAN
DARRELL W EHLERT
PAUL E. SINGER
JOHN C. WOODDY
MATTHEW RODERMUND

SCHEDULE OF MEMBERSHIP
NOVEMBER 16, 1971

	Fellows	Associates	Total
Membership, November 17, 1970	249	212	461
Increase by:			
Election	....	....	....
Reinstatement	...	....	....
Examination	11	27	38
	260	239	499
Decrease by:			
Death	3	4	7
Withdrawal	....	1	1
Transfer from Associate to Fellow ..	....	11	11
	257	223	480

ANALYSIS OF MEMBERSHIP BY
TYPE OF EMPLOYMENT

Type of Employment	Fellows			Associates		
	Nov 1955	Nov. 1965	Nov. 1971	Nov. 1955	Nov. 1965	Nov. 1971
Insurance company:						
Property-liability	70	103	145	39	69	103
Life and A. & H.	17	16	13	45	37	26
Bureaus and						
Associations	21	21	17	12	14	13
Consultants	18	21	25	15	15	16
Government	5	9	5	10	11	12
Academic	4	5	6	3	5	3
Other	3	6	7	3	7	14
Retired	31	37	39	15	32	36
Total	169	218	257	142	190	223

FELLOWS OF THE SOCIETY
NOVEMBER 16, 1971

Those Marked (†) were Charter Members at date of organization, November 7, 1914

Admitted	
Nov. 17, 1969	ADLER, MARTIN, Assistant Vice President and Associate Actuary, Crum & Forster Insurance Companies, Madison Avenue at Canfield Road, Morristown, New Jersey 07960
Nov. 21, 1930	AINLEY, JOHN W. (Retired), 33 Paxton Road, West Hartford, Connecticut 06107
Nov. 20, 1964	ALDRICH, WILLIAM C., Associate General Counsel, The Hartford Insurance Group, Hartford Plaza, Hartford, Connecticut 06115
Nov. 20, 1964	ALEXANDER, LEE M., Actuary, Massachusetts Workmen's Compensation Rating & Inspection Bureau, Massachusetts Automobile Rating and Accident Prevention Bureau, and Massachusetts Motor Vehicle Insurance Plan, 89 Broad Street, Boston, Massachusetts 02110
Nov. 14, 1947	ALLEN, EDWARD S., Associate Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 15, 1971	ATWOOD, CLARENCE R., Assistant Actuary, Great American Insurance Company, 6310 San Vicente Boulevard, Los Angeles, California 90030
Nov. 18, 1955	BAILEY, ROBERT A., Director, Insurance and Actuarial Section, Insurance Bureau, State of Michigan, 111 N. Hosmer Street, Lansing, Michigan 48913
Nov. 15, 1962	BALCAREK, RAFAL J., Vice President and Actuary, Reliance Insurance Company, 4 Penn Center Plaza, Philadelphia, Pennsylvania 19103
Nov. 20, 1924	BARBER, HARMON T. (Retired), 18 Ridgewood Road, Windsor, Connecticut 06095
Nov. 19, 1954	BARKER, GORDON M., Gulf Insurance Company, P. O. Box 1771, Dallas, Texas 75221
Nov. 14, 1947	BARKER, LORING M., Assistant Vice President and Actuary, Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, California 94120
Nov. 20, 1942	BART, ROBERT D., Vice President-Services & Employee Relations The West Bend Company, 400 Washington Street, West Bend, Wisconsin 53095
Nov. 18, 1932	BARTER, JOHN L. (Retired), 1028 Farmington Avenue, Apartment 2F, West Hartford, Connecticut 06107
Nov. 13, 1931	BATHO, ELGIN R., Route 49, Pittsfield, Massachusetts 01201
May 26, 1970	BECKMAN, RAYMOND W., Assistant Actuary, Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, California 94120
Nov. 14, 1958	BENBROOK, PAUL, Executive Vice President, Maryland Casualty Company, Box 1228, Baltimore, Maryland 21203
Nov. 16, 1956	BENNETT, NORMAN J., Secretary and Actuary, Continental Insurance Companies, 80 Maiden Lane, New York, New York 10038

Admitted	
Nov. 19, 1968	BEN-ZVI, PHILLIP N., Secretary and Associate Actuary, Royal-Globe Insurance Companies, 150 William Street, New York, New York 10038
Nov. 22, 1957	BERQUIST, JAMES R., Consulting Actuary, Milliman & Robertson, Inc., 80 South Lake Avenue, Pasadena, California 91101
Nov. 19, 1953	BEVAN, JOHN R., Actuary, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Massachusetts 02117
Nov. 17, 1969	BICKERSTAFF, DAVID R., Actuary, Southern Farm Bureau Casualty Insurance Company, 515 East Amite, P. O. Box 78, Jackson, Mississippi 39205
Apr. 20, 1917	BLANCHARD, RALPH H., Professor Emeritus of Insurance, Columbia University, Plympton, Massachusetts 02367
Nov. 19, 1968	BLAND, WILLIAM H., Senior Actuarial Analyst, Aetna Life & Casualty, Hartford, Connecticut 06115
Nov. 19, 1959	BLODGET, HUGH R., Assistant Vice President, Data Processing Development Department, Aetna Life & Casualty, Hartford, Connecticut 06115
Nov. 16, 1956	BONDY, MARTIN, Vice President and Actuary, Crum & Forster Insurance Companies, Madison Avenue at Canfield Road, Morristown, New Jersey 07960
Nov. 22, 1957	BORNHUETTER, RONALD L., Vice President and Actuary, General Reinsurance Corporation, 400 Park Avenue, New York, New York 10022
Nov. 16, 1956	BOYAJIAN, JOHN H., Actuary, New Jersey Manufacturers Insurance Company, Sullivan Way, Trenton, New Jersey 08607
Nov. 19, 1959	BOYLE, JAMES I., Associate Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 16, 1961	BRANNIGAN, JAMES F., Assistant Vice President & Associate Actuary, Great American Insurance Company, 9310 San Vicente Boulevard, Los Angeles, California 90048
Nov. 17, 1970	BRIAN, ROBERT A., Assistant Actuary, Aetna Life & Casualty, Hartford, Connecticut 06115
Nov. 21, 1952	BRINDISE, RALPH S., Risk Consultant, Standard Oil Company (Indiana), Box 5910-A, Chicago, Illinois 60680
Oct. 22, 1915	BROWN, HERBERT D. (Retired), Glenora-on-Lake Seneca, Dundee, New York 14837
Nov. 17, 1969	BROWN, WILLIAM W., JR., Assistant Actuary, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Massachusetts 02117
Nov. 16, 1961	BUDD, EDWARD H., Vice President, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 23, 1928	BURLING, WILLIAM H. (Retired), 31 Woodland Street, Hartford, Connecticut 06105
Nov. 19, 1959	BYRNE, HARRY T., Associate Actuary, Aetna Life & Casualty, Hartford, Connecticut 06115
Nov. 19, 1929	CAHILL, JAMES M. (Retired), 6 Balfour Lane, Ramsey, New Jersey 07446

Admitted	
Nov. 18, 1932	CAMERON, FREELAND R. (Retired), 2415 East Club Drive N. E., Atlanta, Georgia 30319
Nov. 17, 1938	CARLETON, JOHN W., Senior Vice President, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Massachusetts 02117
Nov. 13, 1967	CARLSON, EDWIN A., Assistant Systems Director, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov 18, 1949	CLARKE, JOHN W., President, Hartford Life Insurance Company, Hartford Plaza, Hartford, Connecticut 06115
Nov. 15, 1918	COATES, BARRETT N (Retired), 1007 Cragmont Avenue, Berkeley, California 94708
Nov 17, 1922	COATES, CLARENCE S. (Retired), 1730 Washington Avenue, Wilmette, Illinois 60091
Feb. 19, 1915	COLLINS, HENRY (Retired), Employers-Commercial Union Companies, 110 Milk Street, Boston, Massachusetts 02107
May 18, 1971	COMEY, DALE R., Associate Actuary, Hartford Insurance Group, Hartford Plaza, Hartford, Connecticut 06115
Nov. 18, 1966	COOK, CHARLES F., Chief Actuary, United Services Automobile Association, 4119 Broadway, San Antonio, Texas 78715
Nov. 22, 1934	COOK, EDWIN A., President and General Manager, Interboro Mutual Indemnity Insurance Company, 155 Mineola Boulevard, Mineola, New York 11501
Nov 18, 1925	CORCORAN, WILLIAM M. (Retired), 9 Parkview Drive, Bronxville, New York 10708
Nov. 18, 1966	CRANDALL, WILLIAM H., Vice President-Actuary, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pennsylvania 19101
Nov. 19, 1926	CRANE, HOWARD G., Vice President and Consultant, General Reinsurance Corporation, 400 Park Avenue, New York, New York 10022
Nov. 21, 1952	CRITCHLEY, DOUGLAS, E. B. Savory, Milln & Company, Basildon House, Moorgate, London E. C. 2, England
Nov. 22, 1946	CROUSE, CHARLES W., Assistant Professor of Mathematics, The Cleveland State University, 19602 Purnell Avenue, Rocky River, Ohio 44116
Nov. 18, 1960	CROWLEY, JAMES H., Assistant Vice President, Comptroller's Department, Aetna Life & Casualty, Hartford, Connecticut 06115
Nov. 16, 1965	CURRY, ALAN C., Vice President and Actuary, State Farm Mutual Automobile Insurance Company, 112 East Washington Street, Bloomington, Illinois 61701
Nov. 19, 1953	CURRY, HAROLD E. (Retired), R. R. 1, Carlock, Illinois 61725
Nov. 18, 1966	DAHME, ORVAL E., Senior Associate Actuary, State Farm Mutual Automobile Insurance Company, 112 East Washington Street, Bloomington, Illinois 61701
Nov. 18, 1927	DAVIS, EVELYN M., Partner, Woodward, Ryan, Sharp & Davis, 26 Broadway, New York, New York 10004
Nov. 16, 1965	DEMELIO, JOSEPH J., Vice President and Actuary, Home Insurance Company, 59 Maiden Lane, New York, New York 10038

Admitted	
Nov. 18, 1960	DICKERSON, O. D., Professor, Risk and Insurance, Florida State University, Tallahassee, Florida 32306
Nov. 16, 1965	DORF, STANLEY A., Supervising Actuary, New York Insurance Dept., 123 William Street, New York, New York 10038
Nov. 22, 1957	DROBISCH, MILES R., Assistant Actuary, California Inspection Rating Bureau, 1453 Mission Street, San Francisco, California 94103
Nov. 14, 1958	DROPKIN, LESTER B., Assistant General Manager and Actuary, California Inspection Rating Bureau, 1453 Mission Street, San Francisco, California 94103
Nov. 24, 1933	EDWARDS, JOHN (Retired), P. O Box 148, Hastings, Ontario, Canada
Nov. 16, 1965	EHLERT, DARRELL W., Director of Actuarial Research, Allstate Insurance Company, 321 Middlefield Road, Menlo Park, California 94025
Nov 19, 1959	EIDE, K. ARNE, Assistant Vice President-Insurance Relations, Metropolitan Life Insurance Company, One Madison Avenue, New York, New York 10010
Nov. 13, 1967	ELIASON, EDWARD B., Associate Actuary, Aetna Life & Casualty, Hartford, Connecticut 06115
Nov. 15, 1940	ELLIOTT, GEORGE B., General Manager, Pennsylvania Compensation Rating Bureau, 1819 John F. Kennedy Boulevard, Philadelphia, Pennsylvania 19103
Nov. 17, 1922	ELSTON, JAMES S (Retired), 1640 Palmer Avenue, Winter Park, Florida 32789
Nov. 15, 1935	EPPINK, WALTER T., 1st Vice President, Treasurer & Actuary, Merchants Mutual Insurance Company, 250 Main Street, Buffalo, New York 14240
Nov. 14, 1958	ESPIE, ROBERT G., Vice President & Corporate Comptroller, Aetna Life & Casualty, 151 Farmington Avenue, Hartford, Connecticut 06115
Nov. 18, 1966	EVEN, CHARLES A., JR., Associate Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 17, 1969	FABER, JAMES A., Assistant Secretary, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pennsylvania 19101
Nov. 18, 1955	FAIRBANKS, ALFRED V., Assistant Vice President and Actuary, Monarch Life Insurance Company, 1250 State Street, Springfield, Massachusetts 01101
†	FALLOW, EVERETT S. (Retired), 28 Sunset Terrace, West Hartford, Connecticut 06107
Nov. 15, 1940	FARLEY, JARVIS, Chairman of the Board and Chief Executive Officer, Massachusetts Indemnity and Life Insurance Company, 100 William Street, Wellesley, Massachusetts 02181
Nov. 17, 1969	FARNAM, WALTER E., Associate Actuary, Aetna Life & Casualty, 151 Farmington Avenue, Hartford, Connecticut 06115
†	FARRER, HENRY (Retired), R. D 3, Box 439 A, Fleetwood, Pennsylvania 19522
Nov. 18, 1960	FAUST, J. EDWARD, JR., Consulting Actuary, R. R. #1, West Gray Road, Zionsville, Indiana 46077

Admitted	
Nov. 15, 1971	FERGUSON, RONALD E., Assistant Secretary, General Reinsurance Corporation, 400 Park Avenue, New York, New York 10022
May 25, 1956	FINNEGAN, JOSEPH H., Assistant to the Actuary, Insurance Services Office, 160 Water Street, New York, New York 10038
Nov. 16, 1961	FITZGIBBON, WALTER J., JR., Actuary, Aetna Life & Casualty, 151 Farmington Avenue, Hartford, Connecticut 06115
Nov. 15, 1935	FITZHUGH, GILBERT W., Chairman of the Board, Metropolitan Life Insurance Company, One Madison Avenue, New York, New York 10010
Nov. 18, 1966	FLAHERTY, DANIEL J., Actuary, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pennsylvania 19101
Nov. 17, 1970	FLYNN, DAVID P., Associate Actuary, Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, California 94120
Nov. 18, 1966	FORKER, DAVID C., Associate Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 18, 1955	FOSTER, ROBERT B., Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 18, 1955	FOWLER, THOMAS W., Actuary, North American Reinsurance Corporation, 245 Park Avenue, New York, New York 10017
Nov. 15, 1971	FRESCH, GLENN W., Assistant Actuary, Aetna Life & Casualty, 151 Farmington Avenue, Hartford, Connecticut 06115
Nov. 22, 1934	FULLER, GARDNER V (Retired), Conover, Wisconsin 54519
Nov. 17, 1970	GERUNDO, LOUIS P., JR., Assistant Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 13, 1967	GIBSON, JOHN A., III, Vice President & Actuary, Colonial Penn Insurance Company, 5 Penn Center Plaza, Philadelphia, Pennsylvania 19103
Nov. 22, 1957	GILLAM, WILLIAM S., Manager, Research Division, Insurance Services Office, 160 Water Street, New York, New York 10038
Nov. 20, 1964	GILLESPIE, JAMES E., Assistant Actuary, CNA/insurance, 310 South Michigan Avenue, Chicago, Illinois 60604
Nov. 20, 1924	GINSBURGH, HAROLD J. (Retired), P. O. Box 283, Brookline, Massachusetts 02146
Nov. 21, 1930	GLENN, JOSEPH BRYAN, Actuarial Consultant, Department of Defense, Washington, D. C.
Nov. 13, 1931	GODDARD, RUSSELL P., Chief Actuary, South Carolina Department of Insurance, 1401 Hampton Street, Columbia, South Carolina 29201
Nov. 17, 1969	GOWDY, ROBERT C., Manager, Employee Benefit Underwriting, Industrial Indemnity Company, 255 California Street, San Francisco, California 94120
May 18, 1971	GRADY, DAVID J., Assistant Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115

Admitted	
Nov. 19, 1926	GRAHAM, CHARLES M., Consulting Actuary, 13760-104th Terrace, North, Largo, Florida 33540
Nov. 19, 1953	GRAVES, CLYDE H., Vice President and Actuary, American Mutual Insurance Alliance, 20 North Wacker Drive, Chicago, Illinois 60606
Nov. 19, 1968	HACHEMEISTER, CHARLES A., Associate Actuary, Allstate Insurance Company, 321 Middlefield Road, Menlo Park, California 94025
Nov. 19, 1953	HALEY, JAMES B., JR., Consulting Actuary, Four Country Club Plaza, Orinda, California 94563
Nov. 16, 1956	HART, W. VAN BUREN, JR., Senior Actuarial Assistant, Aetna Insurance Company, 55 Elm Street, Hartford, Connecticut 06115
May 27, 1969	HARTMAN, GERALD R., Director, Program In Actuarial Science, Temple University, Philadelphia, Pennsylvania 19122
Nov. 17, 1950	HARWAYNE, FRANK, Consulting Actuary, 3 Stuyvesant Oval, New York, New York 10009
Nov. 19, 1926	HAUGH, CHARLES J. (Retired), 25 Le May Street, West Hartford, Connecticut 06107
Nov. 17, 1950	HAZAM, WILLIAM J., Vice President and Actuary, American Mutual Liability Insurance Company, Quannapowitt Parkway, Wakefield, Massachusetts 01880
Nov. 17, 1969	HEER, LEROY E., Associate Actuary, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pennsylvania 19101
Nov. 16, 1951	HEWITT, CHARLES C., JR., Actuary, Allstate Insurance Company, Allstate Plaza, Northbrook, Illinois 60062
Nov. 18, 1966	HILLHOUSE, JERRY A., Actuary, State Farm Mutual Automobile Insurance Company, 112 East Washington Street, Bloomington, Illinois 61701
Nov. 16, 1961	HOBBS, EDWARD J., Senior Vice President, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pennsylvania 19101
Nov. 17, 1970	HOLT, WILLIAM T., Assistant Actuary, Mutual & United of Omaha, 33rd and Dodge, Omaha, Nebraska 68131
Nov. 17, 1969	HONEBEIN, CARLTON W., Assistant Actuary, Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, California 94120
Nov. 22, 1934	HOOKER, RUSSELL O., Consulting Actuary, Hooker & Holcombe, Inc., 100 Constitution Plaza, Hartford, Connecticut 06103
Nov. 17, 1950	HOPE, FRANCIS J., Actuary, Hartford Insurance Group, Hartford Plaza, Hartford, Connecticut 06115
Nov. 14, 1947	HUGHEY, M. STANLEY, Executive Vice President, Lumbermens Mutual Casualty Company, Kemper Insurance—B-4, Long Grove, Illinois 60049
Nov. 19, 1959	HUNT, FREDERIC J., JR., Secretary-Underwriting, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pennsylvania 19101
May 18, 1971	HUNTER, J. ROBERT, JR., Chief Actuary, Federal Insurance Administration, United States Department of Housing and Urban Development, 451 7th Street South West, Washington, D.C 20410

Admitted	
Nov. 18, 1955	HURLEY, ROBERT L., Associate Actuary, Insurance Services Office, 160 Water Street, New York, New York 10038
May 26, 1970	JACOBS, TERRY S., Personal Lines Actuary, Prudential Property and Casualty Insurance Company, Prudential Plaza, Newark, New Jersey 07101
Nov. 19, 1954	JOHE, RICHARD L., Vice President and Actuary, United States Fidelity and Guaranty Company, Calvert and Redwood Streets, Baltimore, Maryland 21203
Nov. 14, 1941	JOHNSON, ROGER A., Actuary, Blue Cross of Greater Philadelphia, 1333 Chestnut Street, Philadelphia, Pennsylvania 19107
Nov. 15, 1971	JONES, ALAN G., Actuarial Assistant, Aetna Insurance Company, 55 Elm Street, Hartford, Connecticut 06115
Nov. 16, 1939	JONES, HAROLD M., Group Statistician, John Hancock Mutual Life Insurance Company, 200 Berkeley Street, Boston, Massachusetts 02117
Nov. 16, 1956	KALLOP, ROY H., Actuary, National Council on Compensation Insurance, 200 East 42 Street, New York, New York 10017
Nov. 22, 1957	KATES, PHILLIP B., President, Independent Fire Insurance Company, P. O. Box 629, Jacksonville, Florida 32201
Nov. 19, 1926	KELTON, WILLIAM H. (Retired), 122 Arundel Avenue, West Hartford, Connecticut 06107
May 26, 1970	KILBOURNE, FREDERICK W., Consulting Actuary, Milliman & Robertson, Inc., 80 South Lake Avenue, Pasadena, California 91101
Nov. 19, 1959	KLAASSEN, ELDON J., Associate Actuary, CNA/insurance, 310 South Michigan Avenue, Chicago, Illinois 60604
Nov. 14, 1941	KOLE, MORRIS B., Actuary, The State Insurance Fund, 199 Church Street, New York, New York 10007
Nov. 24, 1933	KORMES, MARK, President, Actuarial Associates, Inc., 415 Lexington Avenue, New York, New York 10017
Nov. 19, 1953	KUENKLER, ARTHUR S., Consultant, Route 7, Box 35, West 7th Street Extended, Frederick, Maryland 21701
Nov. 18, 1949	LACROIX, HAROLD F., Executive Vice President, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 20, 1964	LANGE, JEFFREY T., Assistant Vice President & Actuary, Royal-Globe Insurance Companies, 150 William Street, New York, New York 10038
May 5, 1961	LATIMER, MURRAY W., Murray W. Latimer, Actuaries, 1625 K Street, N. W., Washington, D. C. 20006
Nov. 17, 1950	LESLIE, WILLIAM, JR., Executive Vice President, The INSCO Systems Corporation, 2901 State Highway #66, Neptune, New Jersey 07753
Nov. 16, 1961	LINDEN, JOHN R., Associate Actuary, Aetna Life & Casualty, Hartford, Connecticut 06115
Nov. 20, 1924	LINDER, JOSEPH, 25 Roosevelt Terrace, Bayonne, New Jersey 07002
Nov. 16, 1956	LINO, RICHARD, Associate Actuary, Insurance Services Office, 160 Water Street, New York, New York 10038
Nov. 18, 1955	LISCORD, PAUL S., Vice President and Actuary, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pennsylvania 19101

Admitted	
Nov. 17, 1950	LIVINGSTON, GILBERT R., Casualty Actuary, Connecticut Insurance Department, State Office Bldg., Hartford, Connecticut 06115
Nov. 16, 1951	LONGLEY-COOK, LAURENCE H., Consultant, Special Lecturer and Research Consultant, Department of Insurance, Georgia State University, 33 Gilmer Street S. E., Atlanta, Georgia 30303
Nov. 17, 1969	LOWE, ROBERT F., Assistant Actuary, Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, California 94120
Nov. 13, 1936	LYONS, DANIEL J., President, Associated Actuaries Incorporated, 120 Sanhican Drive, Trenton, New Jersey 08618
Nov. 1, 1963	MACGINNITIE, W. JAMES, Vice President, CNA Financial Corporation, 310 South Michigan Avenue, Chicago, Illinois 60604
Nov. 18, 1958	MAGRATH, JOSEPH J. (Retired), 3100 South Ocean Boulevard, Delray Beach, Florida 33444
Nov. 22, 1957	MAKGILL, STEPHEN S., Systems Director, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 23, 1928	MARSHALL, RALPH M. (Retired), Catts Corner, Worton, Kent County, Maryland 21678
Nov. 18, 1927	MASTERSON, NORTON E., Consulting Actuary, 1516 Clark Street, Stevens Point, Wisconsin 54481
Nov. 19, 1926	MATTHEWS, ARTHUR N., 475 Poquonock Avenue, Windsor, Connecticut 06095
May 19, 1915	MAYCRINK, EMMA C. (Retired), 32 Chittenden Avenue, Crestwood, New York 10707
Nov 1, 1963	MCCLURE, RICHARD D., Assistant Actuary, Kemper Insurance Group, Long Grove, Illinois 60049
Nov. 15, 1935	MCCONNELL, MATTHEW H, Superintendent, Compensation and Liability Dept., General Accident Group, 414 Walnut Street, Philadelphia, Pennsylvania 19106
Nov. 18, 1960	MCGUINNESS, JOHN S, President, John S. McGuinness Associates, Consultants in Actuarial Science and Management, 15 Kevin Rd., Scotch Plains, New Jersey 07076
Nov. 20, 1964	MCLEAN, GEORGE E., Vice President—Actuary, Massachusetts Blue Cross Incorporated, Massachusetts Blue Shield Incorporated, 133 Federal Street, Boston, Massachusetts 02106
Nov. 15, 1962	MCNAMARA, DANIEL J, President, Insurance Services Office, 160 Water Street, New York, New York 10038
Nov. 15, 1962	MEENAGHAN, JAMES J., Vice President and Actuary, Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, California 94904
Nov. 18, 1955	MENZEL, HENRY W., Vice President, Insurance Services Office, 160 Water Street, New York, New York 10038
†	MICHELBACHER, GUSTAV F. (Retired), 15201 Quito Road, Saratoga, California 95070
Nov. 17, 1938	MILLER, JOHN HAYNES, Actuarial Consultant, North American Reassurance Company, 451 Russell Avenue, Suffield, Connecticut 06078
Nov. 1, 1963	MILLER, NICHOLAS F., JR., Secretary-Executive Department, Aetna Life & Casualty, Hartford, Connecticut 06115

Admitted	
Nov. 18, 1937	MILLS, JOHN A. (Retired), Point Placid 141R4, Reeds Springs, Missouri 65737
Nov. 22, 1957	MILLS, RICHARD J., Assistant Actuary, Kemper Insurance Group, Long Grove, Illinois 60049
Nov. 13, 1967	MOHNBLATT, ARNOLD S., Associate Actuary, Crum & Forster Insurance Companies, Madison Avenue at Canfield Road, Morristown, New Jersey 07960
Nov. 15, 1962	MORISON, GEORGE D., President, New York Compensation Insurance Rating Board, 200 East 42nd Street, New York, New York 10017
Nov. 16, 1961	MOSELEY, JACK, Vice President and Senior Actuary, United States Fidelity and Guaranty Company, Calvert and Redwood Streets, Baltimore, Maryland 21203
Nov. 17, 1920	MUELLER, LOUIS H., 2845 Lake Street, San Francisco, California 94121
Nov. 16, 1956	MUETTERTIES, JOHN H., Actuary, Insurance Services Office, 160 Water Street, New York, New York 10038
May 26, 1970	MUNRO, RICHARD E., Actuary, California Casualty Group, 1900 Alameda de las Pulgas, San Mateo, California 94402
Nov. 17, 1950	MUNTERICH, GEORGE C., Associate Actuary, The Hartford Insurance Group, Hartford Plaza, Hartford, Connecticut 06115
Nov. 19, 1954	MURRIN, THOMAS E., Senior Vice President and Actuary, Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, California 94120
Nov. 19, 1959	MYERS, ROBERT J., Professor of Actuarial Science, Temple University, 9610 Wire Avenue, Silver Springs, Maryland 20901
Nov. 19, 1968	NAFFZIGER, JOSEPH V., Actuary, State Farm Mutual Automobile Insurance Company, 112 East Washington Street, Bloomington, Illinois 61701
Nov. 16, 1965	NELSON, DALE A., Actuary, State Farm Mutual Automobile Insurance Company, 112 East Washington Street, Bloomington, Illinois 61701
Nov. 13, 1967	NEWMAN, STEVEN H. Vice President & Casualty Actuary, American International Group, 102 Maiden Lane, New York, New York 10005
Nov. 14, 1958	NILES, CHARLES L., JR., Deputy General Manager and Vice President, General Accident Group, 414 Walnut Street, Philadelphia, Pennsylvania 19105
Nov. 16, 1965	OIEN, ROBERT G., Staff Actuary, St. Paul-Fire and Marine Insurance Company, 385 Washington Street, St. Paul, Minnesota 55108
Nov. 22, 1957	OTTESON, PAUL M , Vice President and Actuary, Federated Mutual Insurance Company and Federated Life Insurance Company, 129 East Broadway, Owatonna, Minnesota 55060
Nov. 21, 1919	OUTWATER, OLIVE E. (Retired), 2404 Loring Street, San Diego, California 92109
Nov. 15, 1962	PARLIN, R. W., Actuary, Neckura Insurance, 6000 Frankfurt/Main, Adickesallee 67, Germany

Admitted	
Nov. 18, 1960	PENNYCOOK, ROD B., Executive Officer, Policyholder Service, Great-West Life, 60 Osborne Street North, Winnipeg, Manitoba, Canada R3C 3A5
Nov. 22, 1957	PERKINS, WILLIAM J., Assistant Group Actuary, London Life Insurance Company, 255 Dufferin Avenue, London, Ontario, Canada
Nov. 17, 1969	PERREAULT, STEPHEN L., Secretary, The Hartford Insurance Group, Hartford Plaza, Hartford, Connecticut 06115
Nov. 14, 1941	PETERS, STEFAN, Consultant, Arthur D. Little, Inc., 35 Acorn Park, Cambridge, Massachusetts 02140
Nov. 21, 1952	PETZ, EARL F., Actuary, Kemper Insurance Group, Long Grove, Illinois 60049
Nov. 19, 1959	PHILLIPS, HERBERT J., JR., Actuary and Vice President, Employers-Commercial Union Companies, 110 Milk Street, Boston, Massachusetts 02107
Nov. 24, 1933	PICKETT, SAMUEL C. (Retired), Connecticut Rating Supervisor, State of Connecticut, Hartford, Connecticut 06115
Nov. 22, 1957	PINNEY, ALLEN D., Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 17, 1922	PINNEY, SYDNEY D. (Retired), 290 Wolcott Hill Road, Wethersfield, Connecticut 06109
Nov. 19, 1959	POLLACK, ROBERT, Executive Vice President, Colonial Penn Insurance Company, 5 Penn Center Plaza, Philadelphia, Pennsylvania 19103
Nov. 16, 1965	PORTERMAIN, NEILL W., Actuarial Consultant, P.O. Box 265, Concord, New Hampshire 03301
Nov. 13, 1967	PRESLEY, PHILIP O., Assistant Vice President and Associate Actuary, American Mutual Liability Insurance Company, Quannapowitt Parkway, Wakefield, Massachusetts 01880
Nov. 17, 1969	QUINLAND, JOHN A., Associate Actuary, The Hartford Insurance Group, Hartford Plaza, Hartford, Connecticut 06115
Nov. 18, 1955	RESONY, ALLIE V., Assistant Secretary, The Hartford Insurance Group, Hartford Plaza, Hartford, Connecticut 06115
Nov 18, 1949	RESONY, JOHN A., Senior Vice President, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 18, 1966	RICCARDO, JOSEPH F., JR., Director-Financial Statements, Corporate Accounting Department, Aetna Life & Casualty, Hartford, Connecticut 06115
Nov. 1, 1963	RICHARDS, HARRY R., Associate Actuary, National Council on Compensation Insurance, 1099 Wall Street West. Lyndhurst, New Jersey 07071
May 18, 1971	RICHARDSON, JAMES F, Actuary, The Hanover Insurance Company, 440 Lincoln Street, Worcester, Massachusetts 01605
Nov. 1, 1963	RIDDLESWORTH, WILLIAM A., Associate Actuary, Aetna Life & Casualty, Hartford, Connecticut 06115
Nov. 14, 1958	ROBERTS, LEWIS H., Vice President and Manager, Woodward and Fondiller, Inc., 730 Fifth Avenue, New York, New York 10019
Nov. 14, 1947	RODERMUND, MATTHEW, Vice President-Actuary, Munich Reinsurance Company, 410 Park Avenue, New York, New York 10022

Admitted	
Nov. 14, 1947	ROSENBERG, NORMAN, Assistant Vice President-Actuary, Farmers Insurance Group, 4680 Wilshire Boulevard, Los Angeles, California 90054
Nov. 18, 1966	ROTH, RICHARD J., Senior Vice President & Actuary, Great American Insurance Companies, 6310 San Vicente Boulevard, Los Angeles, California 90030
Nov. 14, 1947	ROWELL, JOHN H., Vice President, Marsh & McLennan, Inc., 231 South LaSalle Street, Chicago, Illinois 60604
Nov. 17, 1938	RUCHLIS, ELSIE, Assistant Actuary, Insurance Services Office, 160 Water Street, New York, New York 10038
Nov. 19, 1968	RYAN, KEVIN M., Regional Vice President, Insurance Services Office, Sixth and Chestnut Streets, Philadelphia, Pennsylvania 19106
Nov. 14, 1947	SALZMANN, RUTH E., Vice President and Actuary, Sentry Insurance Group, 1421 Strongs Avenue, Stevens Point, Wisconsin 54481
Nov. 1, 1963	SARASON, HARRY M., Editor, BICAT, 1246 (A) Chelsea, Santa Monica, California 90404
May 26, 1970	SCHEEL, PAUL J., Associate Actuary, United States Fidelity & Guaranty Company, Calvert & Redwood Streets, Baltimore, Maryland 21203
Nov. 18, 1966	SCHEIBL, JEROME A., Associate Actuary, Employers Insurance of Wausau, 2000 Westwood Drive, Wausau, Wisconsin 54401
Nov. 17, 1969	SCHEID, JAMES E., Associate Actuary, The Hartford Insurance Group, Hartford Plaza, Hartford, Connecticut 06115
Nov. 19, 1948	SCHLOSS, HAROLD W., Senior Vice President, Royal-Globe Insurance Companies, 150 William Street, New York, New York 10038
Nov. 13, 1967	SCHULER, ROBERT J., Vice President, Blue Cross of Western Pennsylvania, One Smithfield Street, Pittsburgh, Pennsylvania 15222
Nov. 18, 1966	SCOTT, BRIAN E., Systems Director, Data Processing Development Department, Aetna Life & Casualty, 151 Farmington Avenue, Hartford, Connecticut 06115
Nov. 18, 1937	SHAPIRO, GEORGE I., 934 East 9 Street, Brooklyn, New York 11230
Nov. 13, 1931	SILVERMAN, DAVID, Consulting Actuary, Peat, Marwick, Mitchell & Co., 345 Park Avenue, New York, New York 10022
Nov. 19, 1954	SIMON, LEROY J., Vice President, Prudential Property and Casualty Insurance Company, Prudential Plaza, Newark, New Jersey 07101
Nov. 18, 1960	SIMONEAU, PAUL W., Actuary, Aetna Life & Casualty, Hartford, Connecticut 06115
Nov. 19, 1929	SKELDING, ALBERT Z., 162 Hamilton Road, Hempstead, New York 11550
May 18, 1971	SKURNICK, DAVID, Assistant Actuary, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pennsylvania 19101
Nov. 18, 1932	SMICK, J. J., Consulting Actuary, Smick & Co., Inc., 300 E. 46th Street, New York, New York 10017
Nov. 14, 1958	SMITH, EDWARD M., Actuary, The Travelers Insurance Companies One Tower Square, Hartford, Connecticut 06115

Admitted	
Nov. 18, 1966	SMITH, EDWARD R., Assistant Vice President and Actuary, Hartford Insurance Group, Hartford Plaza, Hartford, Connecticut 06115
Nov. 15, 1940	SMITH, SEYMOUR E., Senior Vice President and Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
May 18, 1971	SNADER, RICHARD H., Assistant Actuary, United States Fidelity and Guaranty Company, Calvert and Redwood Streets, Baltimore, Maryland 21203
Nov. 15, 1962	STANKUS, LEO M., Director of Executive Information, Allstate Insurance Company, Allstate Plaza, Northbrook, Illinois 60062
Nov. 17, 1970	STRUG, EMIL J., Assistant Vice President & Associate Actuary, Massachusetts Blue Cross, Incorporated, Massachusetts Blue Shield, Incorporated, 133 Federal Street, Boston, Massachusetts 02106
Nov. 19, 1968	STURGIS, ROBERT W., Associate Actuary, Aetna Life & Casualty, Hartford, Connecticut 06115
Nov. 18, 1966	SWITZER, VERNON J., Health Actuary, State Farm Mutual Automobile Insurance Co., 112 E. Washington Street, Bloomington, Illinois 61701
May 25, 1956	TAPLEY, DAVID A., President, Transamerica Insurance Company, 1150 South Olive Street, Suite 2100, Los Angeles, California 90015
Nov. 14, 1958	TARBELL, LUTHER L., JR., Second Vice President and Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 16, 1956	THOMAS, JAMES W., Associate Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
†	THOMPSON, JOHN S., Newark Athletic Club, Newark, New Jersey 07102
Nov. 19, 1953	TRIST, JOHN A. W., Associate Actuary, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pennsylvania 19101
Nov. 15, 1962	TRUDEAU, DONALD E., Vice President of Finance/Treasurer, Medallion Insurance Group, 1907 Grand Avenue, Kansas City, Missouri 64108
Nov. 14, 1947	UHTHOFF, DUNBAR R., Senior Vice President, Employers Insurance of Wausau, 2000 Westwood Drive, Wausau, Wisconsin 54401
Nov. 23, 1928	VALERIUS, NELS M. (Retired), 94 Maple Hill Avenue, Newington, Connecticut 06111
Nov. 21, 1919	VAN TUYL, HIRAM O. (Retired), 125 56th Avenue, South, St. Petersburg, Florida 33705
Nov. 16, 1965	VERHAGE, PAUL A., Actuary, Sentry Insurance Group, 1421 Strongs Avenue, Stevens Point, Wisconsin 54481
Nov. 16, 1951	VINCENT, LEWIS A. (Retired), Carter Road, Post Office Box 9, New London, New Hampshire 03257
Nov 19, 1962	WALSH, ALBERT J., Vice President and General Manager, Interinsurance Exchange of the Automobile Club of Southern California, 2601 South Figueroa Street, Los Angeles, California 90054

Admitted	
Nov. 17, 1970	WARD, MICHAEL R., Assistant Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 16, 1965	WEBB, BERNARD L., Associate Professor of Actuarial Science and Insurance, Georgia State University, 33 Gilmer Street S E., Atlanta, Georgia 30303
Nov. 17, 1970	WHITE, HUGH G., Assistant Actuary, The Travelers Insurance Company of Canada, 400 University Avenue, Toronto 100, Ontario, Canada
May 26, 1970	WHITE, WILLIAM D., Actuary, Woodward and Fondiller, 730 Fifth Avenue, New York, New York 10019
Nov. 14, 1947	WIEDER, JOHN W., JR., Vice President and Actuary, Aetna Life & Casualty, Hartford, Connecticut 06115
Nov. 18, 1960	WILCKEN, CARL L., Actuary, Insurance Bureau of Canada, 170 University Avenue, Toronto 1, Ontario, Canada
Nov. 1, 1963	WILLIAMS, DEWEY G., Vice President, Actuary, Texas Employers' Insurance Association, Employers Casualty Company, 423 So. Akard Street, P. O. Box 2759, Dallas, Texas 75221
Nov. 15, 1935	WILLIAMS, HARRY V., Chairman of the Boards and President, The Hartford Insurance Group, Hartford Plaza, Hartford, Connecticut 06115
Nov. 22, 1957	WILLIAMS, P. ADGER, Vice President, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 14, 1941	WILLIAMSON, W. RULON, Self-employed, 2400 Fairhill Drive, Suitland, Maryland 20023
Nov. 18, 1960	WILLSEY, LYNN W., Second Vice President and Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 16, 1961	WILSON, JAMES C., Vice President, Actuary, INTEGON General Insurance Corporation, 420 North Spruce Street, Winston-Salem, North Carolina 27102
Nov. 13, 1931	WITTICK, HERBERT E. (Retired), 34 Old Bridle Path, Toronto 7, Ontario, Canada
Nov. 14, 1958	WRIGHT, BYRON (Retired), Post Office Box 177, Arendtsville, Pennsylvania 17303
Nov. 19, 1953	YOUNT, HUBERT W. (Retired), Box 489, Amherst, Massachusetts 01002
May 18, 1971	ZORY, PETER B., Associate Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115

ASSOCIATES OF THE SOCIETY

NOVEMBER 16, 1971

Admitted	
Nov. 15, 1918	ACKERMAN, SAUL B., 405 Lexington Avenue, New York, New York 10017
Nov. 16, 1939	AIN, SAMUEL N., Consulting Actuary, 120 Broadway, New York, New York 10005
Nov. 15, 1962	AMLIE, WILLIAM P., Associate Actuary, Employers-Commercial Union Companies, 110 Milk Street, Boston, Massachusetts 02107
Nov. 18, 1955	ANDREWS, EDWARD C. (Retired), 19 Avalon Road, West Hartford, Connecticut 06119
Nov. 17, 1970	ANKER, ROBERT A., Assistant Actuary, Employers Insurance of Wausau, 2000 Westwood Drive, Wausau, Wisconsin 54401
Nov. 21, 1930	ARCHIBALD, A. EDWARD, 200 Richardson Street, Lookout Mountain, Tennessee 37350
Nov. 17, 1970	BALKO, KAREN H., Actuarial Assistant, The Hartford Insurance Group, Hartford Plaza, Hartford, Connecticut 06115
Nov. 19, 1959	BANNISTER, DAN W, President, Horace Mann Educators, 216 East Monroe Street, Springfield, Illinois 62701
Nov. 19, 1968	BARTIK, ROBERT F., Assistant Actuary, Kemper Insurance Group, Long Grove, Illinois 60049
Nov. 23, 1928	BATEMAN, ARTHUR E., Pine Grove Rest Home, Marlboro, Massachusetts 01752
Nov. 15, 1940	BATHO, BRUCE W., Executive Vice President-Administration, Life Insurance Company of Georgia, Life of Georgia Tower, Atlanta, Georgia 30308
Nov. 17, 1970	BATTAGLIN, BERNARD H., Manager-Homeowners Division, Insurance Services Office, 160 Water Street, New York, New York 10038
Nov. 16, 1965	BELL, ALLAN A., Senior Assistant Actuary, State Farm Fire & Casualty Company, 112 East Washington Street, Bloomington, Illinois 61701
Nov. 19, 1968	BELL, LINDA L., Assistant Actuary, Crum & Forster Insurance Companies, Madison Avenue at Canfield Road, Morristown, New Jersey 07960
Nov. 16, 1956	BERG, ROY A., JR., Assistant Actuary, Old Republic Life Insurance Company, 307 North Michigan Avenue, Chicago, Illinois 60601
Nov. 19, 1968	BERGEN, ROBERT D., Associate Actuary, Insurance Services Office, 160 Water Street, New York, New York 10038
Nov. 14, 1958	BERNAT, LEO A., Executive Director, Minnesota Research Associates, 204 Franklin Avenue West, Minneapolis, Minnesota 55404
May 26, 1970	BILL, RICHARD A., Actuary, Country Mutual Insurance Company, Post Office Box 565, Bloomington, Illinois 61701
Nov. 18, 1925	BITTEL, W. HAROLD, Chief Actuary, Department of Insurance, State of New Jersey, Trenton, New Jersey 08618

Admitted	
Nov. 14, 1958	BLUMENFELD, M. EUGENE, Assistant Actuary, Bankers Life and Casualty Company, 4444 W. Lawrence Avenue, Chicago, Illinois 60630
Nov. 22, 1934	BOMSE, EDWARD L., Manager-Commercial Lines Liability, Royal-Globe Insurance Companies, 150 William Street, New York, New York 10038
May 27, 1969	BRADSHAW, JOHN G., JR., Actuarial Assistant, The Hartford Insurance Group, Hartford Plaza, Hartford, Connecticut 06115
Nov. 22, 1957	BRAGG, JOHN M., Vice President and Chief Actuary, Life Insurance Company of Georgia, Life of Georgia Tower, Atlanta, Georgia 30308
Nov. 15, 1962	BUFFINTON, PHILIP G., Vice President, State Farm Fire and Casualty Company, 112 East Washington Street, Bloomington, Illinois 61701
Nov. 20, 1924	BUGBEE, JAMES M. (Retired), 115 Hawthorn Road, Baltimore, Maryland 21210
Mar. 31, 1920	BURT, MARGARET A., Office of George B. Buck, Consulting Actuary, 60 Worth Street, New York, New York 10013
Nov. 19, 1959	BUTLER, RICHARD H. (Retired), Newgate Road, East Granby, Connecticut 06026
Nov. 17, 1969	CADORINE, ARTHUR R., Assistant Actuary, Insurance Services Office, 160 Water Street, New York, New York 10038
Nov. 15, 1962	CARSON, DAVID E. A., Vice President and Actuary, The Hartford Insurance Group, Hartford Plaza, Hartford, Connecticut 06115
Nov. 13, 1967	CARTER, EDWARD J., JR., Actuary, United Services Automobile Association, 4119 Broadway, San Antonio, Texas 78215
Nov. 18, 1927	CHEN, S. T., Consulting Actuary, The Wing On Life Assurance Company Ltd., Wing On Life Bldg., 22 Des Voeux Road, Central, Hong Kong
Nov. 16, 1961	CHERLIN, GEORGE, Vice President and Actuary, National Health and Welfare Retirement Association, Inc., 360 Park Avenue South, New York, New York 10010
Nov. 13, 1967	CHORPITA, FRED M., Assistant Actuary, National Council on Compensation Insurance, 200 East 42nd Street, New York, New York 10017
Nov. 22, 1957	CHURCH, HARRY M., Coates, Herfurth & England, 301 East Colorado Boulevard, Pasadena, California 91101
Nov. 18, 1955	COATES, WILLIAM D., Vice President, National-Ben Franklin Life Insurance Corp., 360 W. Jackson Boulevard, Chicago, Illinois 60606
Nov. 18, 1966	CONNER, JAMES B., Assistant Director, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06033
Nov. 15, 1971	CONNERS, JOHN B., Actuarial Analyst, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Massachusetts 02117
Nov. 19, 1953	CONTE, JOSEPH P., Assistant to the President, Bermans Motor Express, Incorporated, P. O. Box 1566, Binghamton, New York 13902

Admitted	
May 27, 1969	COOPER, WARREN P., Vice President and Actuary, Chubb & Son, Incorporated, 51 John F. Kennedy Parkway, Short Hills, New Jersey 07078
Nov. 19, 1959	COPESTAKES, A. D., Assistant Vice President-Reports, American Mutual Liability Insurance Company, Wakefield, Massachusetts 01880
Nov. 24, 1933	CRAWFORD, WILLIAM H., Financial Consultant, Industrial Indemnity Company, 155 Sansome Street, San Francisco, California 94104
Nov. 19, 1953	CROFTS, GEOFFREY, Dean and Director, Graduate School of Actuarial Science, Northeastern University, 360 Huntington Avenue, Boston, Massachusetts 02115
Nov. 21, 1952	DANIEL, CHARLES M., IBM, 2116 Grand Avenue, Des Moines, Iowa 50312
Nov. 13, 1967	DAVIS, REX C., Pricing Director and Actuary, Allstate Insurance Company, Allstate Plaza, Northbrook, Illinois 60062
Nov. 16, 1965	DICKSON, CAROL D (MRS), 34 Brookline Drive, West Hartford, Connecticut 06107
Nov. 17, 1970	DRENNAN, JOHN P, Associate Actuary, Allstate Insurance Company, Allstate Plaza, Northbrook, Illinois 60062
Nov. 15, 1971	DROPICK, DOROTHY K., Actuarial Assistant, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 1, 1963	DURKIN, JAMES H., Actuary, Peat, Marwick, Mitchell & Company, 345 Park Avenue, New York, New York 10022
Nov. 14, 1958	DUROSE, STANLEY C., JR., Commissioner of Insurance, State of Wisconsin, 212 North Bassett Street, Madison, Wisconsin 53703
Nov. 19, 1954	EATON, KARL F, Vice President and Controller, National Fidelity Life Insurance Company, 1002 Walnut, Kansas City, Missouri 64106
June 5, 1925	EGER, FRANK A. (Retired), 1119 Prospect Ridge Boulevard, Haddon Heights, New Jersey 08035
May 18, 1971	ENGEL, PHILIP L., Assistant Actuary, CNA/insurance, 310 South Michigan Avenue, Chicago, Illinois 60604
Nov 19, 1968	EYERS, ROBERT G, Actuarial Assistant, Aetna Insurance Company, 55 Elm Street, Hartford, Connecticut 06115
Nov. 22, 1957	FELDMAN, MARTIN F., Associate Actuary, New York State Insurance Department, 123 William Street, New York, New York 10038
Nov. 16, 1961	FERDEN, STEIN, Undelstadlia 8, Asker, Norway
Nov. 13, 1967	FERRARI, J. ROBERT, Department of Economic and Investment Research, Prudential Life Insurance Company, Prudential Plaza, Newark, New Jersey 07101
Nov. 15, 1962	FINKEL, DANIEL, Associate Actuary, The State Insurance Fund, 199 Church Street, New York, New York 10007
Nov. 16, 1956	FLACK, PAUL R., Reinsurance-Actuary, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pennsylvania 19101
Nov. 19, 1968	FOSSA, E. FREDERICK, Associate Actuary, Employers-Commercial Union Companies, 110 Milk Street, Boston, Massachusetts 02107

Admitted	
Nov. 21, 1952	FRANKLIN N MATTHEW, Associate Actuary, Insurance Services Office, 160 Water Street, New York, New York 10038
Nov. 19, 1968	FRENCH, JAMES T., Assistant Vice President, CNA/insurance, 310 South Michigan Avenue, Chicago, Illinois 60604
Nov. 18, 1966	FULTON, CLYDE B., JR., Director, Tax Administration, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 19, 1954	GAINES, NATHANIEL, Associate Actuary, George B Buck Consulting Actuaries, Incorporated, Two Pennsylvania Plaza, New York, New York 10001
Nov. 18, 1932	GETMAN, RICHARD A., Assistant Actuary, Life Dept., The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 17, 1922	GIBSON, JOSEPH P., JR. (Retired), 2970 Lorain Road, San Marino, California 91108
Nov. 16, 1923	GILDEA, JAMES F. (Retired), 236 Nott Street, Wethersfield, Connecticut 06109
Nov. 1, 1963	GILL, JAMES F., Vice President and Actuary, National Association of Independent Insurers, 30 West Monroe St., Chicago, Illinois 60603
Nov. 14, 1947	GINGERY, STANLEY W., Vice President and Actuary, Prudential Insurance Company, Prudential Plaza, Newark, New Jersey 07101
Nov. 15, 1971	GOLZ, JAMES F., Actuarial Assistant, Employers Insurance of Wausau, 2000 Westwood Drive, Wausau, Wisconsin 54401
Nov. 13, 1967	GOSSROW, ROBERT W., Associate Actuary, Allstate Insurance Company, Allstate Plaza, Northbrook, Illinois 60062
Nov. 16, 1961	GOULD, DONALD, Associate Actuary, Woodward & Fondiller, Inc , 730 Fifth Avenue, New York, New York 10019
Nov. 18, 1927	GREEN, WALTER C. (Retired), 923 South 2300 East, Salt Lake City, Utah 84108
Nov. 16, 1961	GREENE, THOMAS A., Vice President, General Reinsurance Corporation, 400 Park Avenue, New York, New York, 10022
Nov. 15, 1940	GROSSMAN, ELI A., Senior Vice President, Security-Connecticut Life Insurance Company, 1000 Asylum Avenue, Hartford, Connecticut 06101
Nov. 15, 1935	GUERTIN, ALFRED N., Actuarial Consultant, 2 Pennsylvania Plaza, New York, New York 10001
Nov. 16, 1939	HAGEN, OLAF E., Senior Assistant Actuarial Supervisor, Metropolitan Life Insurance Company, One Madison Avenue, New York, New York 10010
Nov. 13, 1936	HAM, HUGH P. (Retired), Apt. 901 "A", 1141 Royal York Road, Islington, Toronto, Ontario, Canada
Nov. 1, 1963	HAMMER, SIDNEY M., Assistant Manager, Actuarial Department, The Home Insurance Company, 59 Maiden Lane, New York, New York 10008
Nov 16, 1965	HANSON, H. DONALD, Director, Planning & Analysis, CNA Financial Corporation, 310 South Michigan Avenue, Chicago, Illinois 60604

Admitted	
Nov. 19, 1953	HARACK, JOHN, Senior Vice President-Actuary, Health Service, Inc. Medical Indemnity of America, Inc., 200 North Michigan Avenue, Chicago, Illinois 60601
Nov. 19, 1968	HARDY, HOWARD R., Assistant Actuary, Great American Insurance Companies, 6310 San Vicente Boulevard, P. O. Box 30172, Los Angeles, California 90030
Mar. 24, 1932	HARRIS, SCOTT, Vice Chairman, Joseph Froggatt & Company, Inc., 74 Trinity Place, New York, New York 10006
Mar. 25, 1924	HART, WARD VAN B., 49 Robbins Drive, Wethersfield, Connecticut 06109
Nov. 19, 1968	HARTMAN, DAVID G., Assistant Actuary, Chubb & Son, Incorporated, 51 John F. Kennedy Parkway, Short Hills, New Jersey 07078
Nov. 15, 1971	HASELTINE, DOUGLAS S., Actuarial Assistant, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 19, 1953	HEAD, GLENN O., President, First Investors Life Insurance Company, 120 Wall Street, New York, New York 10005
May 26, 1970	HEAD, THOMAS F., Assistant Actuary, Nationwide Mutual Insurance Company, 246 North High Street, Columbus, Ohio 43216
Nov. 17, 1970	HEARN, VINCENT W., Actuarial Assistant, The Home Insurance Company, 59 Maiden Lane, New York, New York 10008
Nov. 19, 1959	HICKMAN, JAMES C., Professor, Department of Statistics, University of Iowa, Iowa City, Iowa 52240
May 18, 1971	HOFFMANN, DENNIS E., Actuarial Assistant, The Hartford Insurance Group, Hartford Plaza, Hartford, Connecticut 06115
Nov 16, 1961	HOROWITZ, MILTON, Principal Actuary, The State Insurance Fund, 199 Church Street, New York, New York 10007
Nov. 19, 1929	JACOBS, CARL N (Retired), 1909 Plover Street, Stevens Point, Wisconsin 54481
Nov. 15, 1962	JENSEN, JAMES P., Assistant Actuary, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Massachusetts 02117
Nov. 19, 1968	JONES, DEL R., Assistant Director, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 21, 1919	JONES, LORING D. (Retired), 64 Raymond Avenue, Rockville Centre, New York 11570
Nov. 21, 1952	JONES, NATHAN F., Vice President and Associate Actuary, Prudential Insurance Company, Prudential Plaza, Newark, New Jersey 07101
Nov. 19, 1968	JORVE, BARRY M., Secretary, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 13, 1967	KAUR, ALAN F., Account Executive, Hornblower Weeks, Hemphill-Noyes, 134 South LaSalle, Chicago, Illinois 60603
Nov. 20, 1964	KHURY, COSTANDY K., Actuary, Utica Mutual Insurance Company, Box 530, Utica, New York 13503
Nov. 15, 1935	KITZROW, ERWIN W. (Retired), 1042 E. Dolores Dr., Altadena, California 91001

Admitted	
Nov. 19, 1968	KLINGMAN, GEORGE C., Assistant Director, Rating, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 17, 1970	KRAUSE, GUSTAVE A., Senior Actuarial Assistant, CNA/insurance, 310 South Michigan Avenue, Chicago, Illinois 60604
Nov. 19, 1959	KROEKER, JOHN, Actuary, James E. Coughlin and Associates, Ltd., 904 Lady Ellen Place, Ottawa 3, Ontario, Canada
Nov. 19, 1959	LEIGHT, ARTHUR S., Assistant Actuary, Guardian Life Insurance Co., 201 Park Avenue South, New York, New York 10003
May 27, 1969	LEVIN, JOSEPH W., Actuary-Property & Liability Branch, State of Illinois-Department of Insurance, 525 West Jefferson, Springfield, Illinois 62706
Nov. 15, 1971	LINDQUIST, ROBERT J., Manager-Actuarial Department, Transamerica Insurance Company, 1150 South Olive Street, Los Angeles, California 90015
Nov. 19, 1968	LINQUANTI, AUGUST J., Assistant Actuary, Royal-Globe Insurance Companies, 150 William Street, New York, New York 10038
Nov. 18, 1925	MALMUTH, JACOB, Chief-Rating Bureau, New York Insurance Department, 123 William Street, New York, New York 10038
Nov. 16, 1961	MARGOLIS, DONALD R., Assistant to President, Union Fidelity Corporation, 1515 Locust Street, Philadelphia, Pennsylvania 19102
Nov. 20, 1964	MARKELL, ANDREW S., Director of Finance, League Insurance Group, Post Office Box 5010, Detroit, Michigan 48235
Nov. 16, 1956	MATHWICK, LLOYD F., Manager, Group Health, Life and Pension Products, Employers Insurance of Wausau, 2000 Westwood Drive, Wausau, Wisconsin 54401
Nov. 13, 1936	MAYER, WILLIAM H., JR., Manager, Group Contract Bureau, Metropolitan Life Insurance Company, One Madison Avenue, New York, New York 10010
Nov. 15, 1971	McCLENAHAN, CHARLES L., Actuarial Analyst, Government Employees Insurance Company, 1705 L Street North West, Washington, District of Columbia 20036
Nov. 13, 1967	McDONALD, CHARLES, Manager, Actuarial Department, Employers Casualty Company, P. O Box 2759, Dallas, Texas 75221
May 26, 1955	McDONALD, MILTON G., Chief Actuary, Massachusetts Insurance Department, 100 Cambridge Street, Boston, Massachusetts 02202
Nov. 16, 1961	McINTOSH, KENNETH L., Property and Casualty Actuary, Arkansas Insurance Department, University Towers, Little Rock, Arkansas 72204
Nov. 13, 1931	MILLER, HENRY C. (Retired), 35 Lower Crescent, Sausalito, California 94965
Nov. 15, 1971	MILLER, MICHAEL J., Assistant Actuary, State Farm Mutual Automobile Insurance Company, 112 East Washington Street, Bloomington, Illinois 61701
May 18, 1971	MILLER, PHILIP D., Assistant Actuary, Insurance Services Office, 160 Water Street, New York, New York 10038

Admitted	
Nov. 15, 1971	MILLMAN, NEIL L., Assistant Director-Actuarial Department, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pennsylvania 19101
Nov. 20, 1964	MOKROS, BERTRAM F., Underwriting Research Manager, Allstate Insurance Company, 321 Middlefield Road, Menlo Park, California 94025
Nov. 17, 1922	MONTGOMERY, JOHN C. (Retired), 165 Westervelt Avenue, Tenafly, New Jersey 07670
Nov 19, 1968	MOORE, JAMES E., Assistant Director, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
May 25, 1923	MOORE, JOSEPH P., 115 St. Catherine Road, Outremont, Quebec, Canada
Nov 17, 1970	MOORE, PHILLIP S., Assistant Actuary, Employers Insurance of Wausau, 2000 Westwood Drive, Wausau, Wisconsin 54401
Nov. 16, 1961	MOSS, ROBERT G., Vice President and Actuary, Marsh & McLennan, Inc., 515 Olive Street, St. Louis, Missouri 63101
Nov. 22, 1957	MUIR, JOSEPH M. (Retired), 591 McCulloch Place, Haworth, New Jersey 07641
Nov. 1, 1963	MUNIZ, ROBERT M , Mutual Insurance Advisory Association, 733 Third Avenue, New York, New York 10017
Nov. 18, 1966	MURRAY, EDWARD R., Actuarial Assistant, Royal-Globe Insurance Companies, 150 William Street, New York, New York 10038
Nov. 18, 1966	MURRAY, JAMES B. M , Casualty Superintendent, Prudential Assurance Co., Ltd of England, 635 Dorchester Boulevard West, Montreal, Quebec, Canada
May 26, 1970	NAPIERSKI, JOHN D., Senior Associate Actuary, State Farm Fire and Casualty Company, 112 East Washington Street, Bloomington, Illinois 61701
May 18, 1971	NEIDERMYER, JAMES R., Assistant Actuary, Reliance Insurance Company, 4 Penn Center Plaza, Philadelphia, Pennsylvania 19103
Nov. 19, 1968	NELSON, JOHN K., Senior Associate Actuary, State Farm Mutual Automobile Insurance Company, 112 East Washington Street, Bloomington, Illinois 61701
Nov. 16, 1961	NELSON, ROLAND E , Manager for Canada-Life, Accident and Health, The Travelers Insurance Companies, 101 Richmond Street W , Toronto, Ontario, Canada
Oct. 27, 1916	NEWELL, WILLIAM (Retired), 1225 Park Avenue, New York, New York 10028
Nov 18, 1925	NICHOLSON, EARL H., Actuary and Deputy Insurance Commissioner, Nevada Insurance Division, Nye Building, Carson City, Nevada 89701
Nov. 15, 1971	ORI, KENNETH R., Assistant Actuary, State Farm Mutual Automobile Insurance Company, 112 East Washington Street, Bloomington, Illinois 61701
May 23, 1919	OTTO, WALTER E., Consultant and Member of the Board of Directors, Michigan Mutual Liability Company, 28 West Adams Avenue, Detroit, Michigan 48226
Nov. 19, 1926	OVERHOLSER, DONALD M., 30 Fairlawn Street, Ho-ho-kus, New Jersey 07423

Admitted	
Nov. 15, 1971	PEACOCK, WILLARD W., Actuarial Assistant, United States Fidelity and Guaranty Company, Calvert and Redwood Streets, Baltimore, Maryland 21203
Nov. 16, 1961	PEEL, JERALD P., Vice President Reinsurance, Security Mutual Casualty Company, 222 South Riverside Plaza, Chicago, Illinois 60606
Nov. 20, 1924	PENNOCK, RICHARD M. (Retired), Foxcroft Apartments, A-8, 6851 Roswell Road, North East, Atlanta, Georgia 30328
Nov. 14, 1947	PERRY, ROBERT C., Executive Vice President, State Farm Life Insurance Company, 112 East Washington Street, Bloomington, Illinois 61701
Nov. 19, 1929	PHILLIPS, JOHN H. (Retired), 915 Steuben Street, Wausau, Wisconsin 54401
Nov. 17, 1920	PIKE, MORRIS (Retired), 19 Old Mamaroneck Road, Apt. 2G, White Plains, New York 10605
Nov. 17, 1969	PILON, ANDRE, Manager, Quebec Division, Laurentian Group, 2065 Brulart, Quebec 6e, Canada
Nov. 13, 1967	PLUNKETT, JOSEPH A., Assistant Vice President, American Re-Insurance Company, 99 John Street, New York, New York 10038
Nov. 17, 1922	POORMAN, WILLIAM F. (Retired), 4915 Country Club Boulevard, Des Moines, Iowa 50312
Nov. 13, 1936	POTOFSKY, SYLVIA (Retired), 175 West 12th Street, New York, New York 10011
Nov. 13, 1967	PRICE, EDITH E., Senior Actuarial Assistant, Kemper Insurance Group, Long Grove, Illinois 60049
Nov. 20, 1964	RAID, GARY A., Actuary, Unigard Insurance Group, 217 Pine Street, Seattle, Washington 98101
Nov. 16, 1965	RATNASWAMY, RAJ, Staff Actuary, St. Paul Insurance Companies, 385 Washington Street, St. Paul, Minnesota 55102
Nov. 15, 1918	RAYWID, JOSEPH (Retired), 322 West 72nd Street, New York, New York 10023
Nov. 19, 1932	RICHARDSON, HARRY F. (Retired), 170-C Rossmoor Drive, Jamesburg, New Jersey 08831
Nov. 19, 1953	RICHMOND, OWEN D., Accounting Vice President, Business Men's Assurance Company, Post Office Box 458, Kansas City, Missouri 64141
Nov. 18, 1960	RIPANDELLI, JOHN S., Actuary and Pension Consultant, P. O Box 3552, Tallahassee, Florida 32303
May 18, 1971	RINEHART, CHARLES R., Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, California 94120
Nov. 18, 1932	ROBERTS, JAMES A., Actuarial Statistician, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 15, 1962	ROOD, HENRY F., Honorary Chairman, Lincoln National Life Insurance Company, 1300 South Harrison Street, Fort Wayne, Indiana 46801
Nov. 15, 1971	ROSS, JAMES P., Actuarial Assistant, Aetna Life & Casualty, Hartford, Connecticut 06115
Nov. 15, 1971	ROSSER, HARWOOD, Chief Actuary, Insurance Department, Commonwealth of Pennsylvania, Finance Building, Harrisburg, Pennsylvania 17120

Admitted	
Nov. 19, 1959	ROYER, ALAN F., Actuary, Multi-Line Insurance Rating Bureau, 160 Water Street, New York, New York 10038
May 26, 1970	SANDLER, ROBERT M., Associate Actuary, American International Group, 102 Maiden Lane, New York, New York 10005
Nov. 14, 1958	SARNOFF, PAUL E., Assistant Actuary, The Prudential Insurance Company of America, Prudential Plaza, Newark, New Jersey 07101
Nov. 16, 1923	SAWYER, ARTHUR (Retired), 13751 St. Andrews Drive, Leisure World, Ap. 1-I, Seal Beach, California 90740
Nov 17, 1969	SAWYER, J. STEWART, III, Assistant Actuary, Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, California 94120
Nov. 14, 1947	SCAMMON, LAWRENCE W. (Retired), 172 Green Street, Stoneham, Massachusetts 02180
Nov. 15, 1971	SCHAEFFER, BERNARD G., Assistant Actuary, Insurance Services Office, 160 Water Street, New York, New York 10038
Nov. 14, 1958	SCHLENZ, JOHN W., Senior Vice President and Actuary, Federal Life and Casualty Company, 78 West Michigan Avenue, Battle Creek, Michigan 49016
Nov. 22, 1957	SCHNEIKER, HENRY C., Secretary, The Home Insurance Company, 59 Maiden Lane, New York, New York 10038
Nov. 19, 1954	SCHULMAN, JUSTIN, Group Leader, Mathematical Analysis, Programming, Kollsman Instruments Corporation, 575 Underhill Boulevard, Syosset, New York 11791
Nov. 14, 1947	SCHWARTZ, MAX J., Chief, Accident & Health Rating Section, New York State Insurance Department, 324 State Street, Albany, New York 12210
Nov. 20, 1930	SEVILLA, EXEQUIEL S., President, National Life Insurance Company, Post Office Box 2056, Manila, Philippines
Nov. 20, 1924	SHEPPARD, NORRIS E., Actuary, Ontario Teachers' Superannuation, 789 Don Mills Road, Don Mills, Ontario, Ontario, Canada
Nov. 15, 1971	SHOOP, EDWARD C., Actuarial Assistant, Aetna Life & Casualty, Hartford, Connecticut 06115
Nov. 15, 1971	SIMONS, MARTIN M., Assistant Actuary, Unigard Insurance Group, 217 Pine Street, Seattle, Washington 98101
Nov. 1, 1963	SINGER, PAUL E., Vice President and Actuary, CNA/insurance, 310 South Michigan Avenue, Chicago, Illinois 60604
Nov. 15, 1971	SMITH, LEE M., Aetna Insurance Company, 55 Elm Street, Hartford, Connecticut 06115
Nov. 18, 1925	SOMMER, ARMAND, Vice President, CNA/insurance, 310 South Michigan Avenue, Chicago, Illinois 60604
Nov. 19, 1968	SPITZER, C. ROBERT, Actuarial Assistant, Employers-Commercial Union Companies, 110 Milk Street, Boston, Massachusetts 02107
Nov 17, 1970	SPOONER, F. ALLEN, Associate Actuary, Mutual of New York, 1740 Broadway, New York, New York 10019
Nov. 1, 1963	STALEY, HARLOW B., Consulting Actuary, Taylor, Ballard & Company, 1025 Ashworth Road, Suite 426, West Des Moines, Iowa 50265

Admitted	
Nov. 19, 1959	STEIN, JOAN BERKMAN, Assistant Actuary, Insurance Rating Board, 125 Maiden Lane, New York, New York 10038
Nov. 20, 1924	STELLWAGEN, HERBERT P., Director, Insurance Company of North America, 721 Mount Pleasant Road, Bryn Mawr, Pennsylvania 19010
May 26, 1970	STEPHENSON, ELTON A., Assistant Actuary, Allstate Insurance Company, Allstate Plaza, Northbrook, Illinois 60062
Nov. 15, 1956	STERN, PHILIPP K., Actuary, Property-Liability Insurance, Department of Insurance, State of New Jersey, Trenton, New Jersey 08618
Nov. 19, 1959	STEVENS, WALDO A., Senior Vice President, Administrative Services, Blue Cross Association, 840 North Lake Shore Drive, Chicago, Illinois 60611
Nov. 17, 1969	STEWART, CHARLES W., Research Associate, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pennsylvania 19101
Nov. 16, 1923	STOKE, KENDRICK (Retired), 11052 McKinney, Detroit, Michigan 48224
Nov. 15, 1971	SWAZIEK, RAYMOND R., Assistant Vice President, Guy Carpenter and Company, Inc., 110 William Street, New York, New York 10038
Nov. 17, 1970	TATGE, ROBERT L., Actuary, Farm Bureau Mutual Insurance Company, 507 Tenth Street, Des Moines, Iowa 50307
May 18, 1971	THOMPSON, EUGENE G., Actuarial Assistant, General Accident Group, 414 Walnut Street, Philadelphia, Pennsylvania 19105
Nov. 1, 1963	THOMPSON, PHILIP R., Statistician, Federated Mutual Insurance Company, 129 East Broadway, Owatonna, Minnesota 55060
Nov. 18, 1966	TOREN, CHESTER J., Secretary, Zurich-American Insurance Companies, 111 West Jackson Boulevard, Chicago, Illinois 60604
Nov. 13, 1967	TORGRIMSON, DARVIN A., American States Insurance Company, American States Plaza, 500 North Meridian Street, Indianapolis, Indiana 46206
Nov. 18, 1966	TREES, JOHN S., Assistant Vice President, Allstate Insurance Company, Allstate Plaza, Northbrook, Illinois 60062
Nov. 21, 1919	TRENCH, FREDERICK H. (Retired), 1629 Genesee Street, Apt. B-4, Utica, New York 13501
Nov. 20, 1924	UHL, M. ELIZABETH (Retired), 320 East 53rd Street, New York, New York 10022
Nov. 14, 1958	VAN CLEAVE, MARVIN E., Assistant Deputy Commissioner, Office of the Commissioner of Insurance, State of Wisconsin, 212 North Bassett Street, Madison, Wisconsin 53703
Nov 20, 1964	VANDERHOOF, IRWIN T , Senior Vice President and Chief Actuary, Standard Security Life Insurance Co. of New York, 111 Fifth Avenue, New York, New York 10003
Nov. 17, 1969	WADE, ROGER C., Research Associate, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pennsylvania 19101

Admitted	
Nov. 18, 1966	WALTERS, MAVIS A., Assistant Actuary, Insurance Services Office, 160 Water Street, New York, New York 10038
Nov. 13, 1967	WALTERS, MICHAEL A., Associate Actuary, Insurance Services Office, 160 Water Street, New York, New York 10038
Nov. 19, 1959	WEBER, DONALD C., Assistant Professor, Miami University, Department of Mathematics, Oxford, Ohio 45056
Nov. 18, 1932	WEINSTEIN, MAX S., Consulting Actuary, 29 Elk Street, Albany, New York 12207
Nov. 18, 1966	WELCH, JOHN P., Actuary, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pennsylvania 19101
Nov. 18, 1925	WELLMAN, ALEX C., 638 Ridge Road, Roebuck Springs, Birmingham, Alabama 35206
Nov. 21, 1930	WELLS, WALTER I. (Retired), 7 Pinewood Drive, West Boylston, Massachusetts 01583
Nov. 18, 1927	WHITBREAD, FRANK G., Second Vice President, The Lincoln National Life Insurance Company, 1301 South Harrison Street, Fort Wayne, Indiana 46801
Nov. 19, 1948	WHITE, AUBREY, Manager, Peat, Marwick, Mitchell & Company, 1500 Walnut Street, Philadelphia, Pennsylvania 19102
Nov. 13, 1967	WILLIAMS, W. THOMAS, Consultant, The Wyatt Company, 1900 Republic National Bank Tower, Dallas, Texas 75201
Nov. 15, 1971	WILSON, OLIVER T., Actuarial Analyst, Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, California 94120
Nov. 15, 1971	WINKLEMAN, JOHN J., JR., Actuarial Assistant, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 13, 1967	WINTER, ARTHUR E., Assistant Director, The Travelers Insurance Companies, One Tower Square, Hartford, Connecticut 06115
Nov. 16, 1939	WITTLAKE, J. CLARKE, Executive Vice President, Business Men's Assurance Company, P. O. Box 458, Kansas City, Missouri 64141
Nov. 18, 1937	WOOD, DONALD M., JR., Partner, Childs & Wood, 175 West Jackson Boulevard, Chicago, Illinois 60604
Nov. 17, 1950	WOODDY, JOHN C., Senior Vice President, North American Reassurance Company, P. O. Box 2888, New York, New York 10017
Nov. 22, 1934	WOODWARD, BARBARA H. (Retired), Edge Lea, South Lyme, Connecticut 06376
Nov. 16, 1956	WOODWORTH, JAMES H., Assistant Secretary, The Hartford Insurance Group, Hartford Plaza, Hartford, Connecticut 06115
Nov. 18, 1925	WOOLERY, JAMES M., Consultant, 3207 Sussex Road, Raleigh, North Carolina 27607
Nov. 15, 1971	YOUNG, DANNY M., Actuarial Assistant, The Hartford Insurance Group, Hartford Plaza, Hartford, Connecticut 06115
Nov. 15, 1971	YOUNG, EDWARD W., Associate Actuary, Allstate Insurance Company, Allstate Plaza, Northbrook, Illinois 60062
May 5, 1961	YOUNG, ROBERT G., Robert G. Young and Associates, 1313 Catalpa, Royal Oak, Michigan 48067

DECEASED FELLOWS

The (†) denotes charter members at date of organization, November 7, 1914.

Admitted		Died
Nov. 13, 1931	Gilbert E. Ault	Apr. 13, 1965
Nov. 19, 1948	Arthur L. Bailey	Aug. 12, 1954
May 23, 1924	William B. Bailey	Jan. 10, 1952
†	Roland Benjamin	July 2, 1949
Nov. 22, 1934	Ernest T. Berkeley	Dec. 26, 1969
†	S. Bruce Black	Dec. 7, 1968
May 24, 1921	Edward J. Bond	Nov. 12, 1941
May 19, 1915	Thomas Bradshaw	Nov. 10, 1939
†	William Breiby	Aug. 5, 1968
June 5, 1925	William Brosmith	Aug. 22, 1937
Nov. 18, 1927	F. Stuart Brown	Oct. 21, 1967
†	George B. Buck, Sr.	Apr. 12, 1961
†	William A. Budlong	June 4, 1934
Nov. 18, 1932	Charles H. Burhans	June 15, 1942
Apr. 20, 1917	William H. Burhop	Oct. 11, 1963
Feb. 19, 1915	F. Highlands Burns	Mar. 30, 1935
†	Edmund E. Cammack	Dec. 17, 1958
Nov. 21, 1930	Thomas O. Carlson	July 15, 1964
†	Raymond V. Carpenter	Mar. 11, 1947
Feb. 19, 1915	Gorden Case	Feb. 4, 1920
Nov. 18, 1966	Augustin J. Cima	Mar. 29, 1971
Oct. 27, 1916	Edmund S. Cogswell	Apr. 25, 1957
Nov. 23, 1928	Walter P. Comstock	May 11, 1951
Nov. 22, 1934	William J. Constable	Apr. 19, 1959
†	Charles T. Conway	July 23, 1921
†	John A. Copeland	June 12, 1953
†	Walter G. Cowles	May 30, 1942
†	James D. Craig	May 27, 1940
†	James McIntosh Craig	Jan. 20, 1922
Nov. 20, 1964	Robert A. Craig	Feb. 8, 1965
May 26, 1916	Frederick S. Crum	Sept. 2, 1921
Nov. 18, 1932	E. Alfred Davies	Jan. 14, 1967
†	Alfred Burnett Dawson	June 21, 1931
†	Miles Menander Dawson	Mar. 27, 1942
May 25, 1956	Elden W. Day	June 9, 1969
†	Elmer H. Dearth	Mar. 26, 1947
†	Eckford C. DeKay	July 31, 1951
May 19, 1915	Samuel Deutschberger	Jan. 18, 1929
Nov. 17, 1920	Paul Dorweiler	May 17, 1968
†	Ezekiel Hinton Downey	July 9, 1922
May 19, 1915	Earl O. Dunlap	July 5, 1944
†	David Parks Fackler	Oct. 30, 1924
†	Edward B. Fackler	Jan. 8, 1952

DECEASED FELLOWS

Admitted		Died
Feb. 19, 1915	Claude W. Fellows	July 15, 1938
†	Benedict D. Flynn	Aug. 22, 1944
Feb. 19, 1915	Richard Fondiller	Apr. 29, 1962
†	Charles S. Forbes	Oct. 2, 1943
May 26, 1916	Lee K. Frankel	July 25, 1931
†	Charles H. Franklin	May 1951
Nov. 18, 1927	C. H. Fredrickson	Jan. 12, 1969
Feb. 25, 1916	Joseph Froggatt	Sept. 28, 1940
†	Harry Furze	Dec. 26, 1945
Feb. 19, 1915	Fred S. Garrison	Nov. 14, 1949
†	Theodore E. Gaty	Aug. 22, 1925
May 19, 1915	James W. Glover	July 15, 1941
†	Edward S. Goodwin	Jan. 27, 1966
†	William H. Gould	Oct. 28, 1936
Oct. 22, 1915	George Graham	Apr. 15, 1937
Oct. 22, 1915	Thompson B. Graham	July 24, 1946
†	William J. Graham	Feb. 11, 1963
May 25, 1923	William A. Granville	Feb 4, 1943
†	Winfield W. Greene	Mar. 26, 1965
†	Robert Cowen Lees Hamilton	Nov. 15, 1941
†	H. Pierson Hammond	Apr. 10, 1963
Oct. 27, 1916	Edward R. Hardy	June 29, 1951
Oct. 22, 1915	Leonard W. Hatch	Nov. 23, 1958
Nov. 21, 1919	Robert Henderson	Feb. 16, 1942
†	Robert J. Hillas	May 17, 1940
Nov. 15, 1918	Frank Webster Hinsdale	Mar. 18, 1932
May 23, 1924	Clarence W. Hobbs	July 21, 1944
Nov. 19, 1926	Charles E. Hodges	Jan. 22, 1937
Oct. 22, 1915	Lemuel G. Hodgkins	Dec. 26, 1951
†	Frederick L. Hoffman	Feb. 23, 1946
Oct. 22, 1915	Charles H. Holland	Dec. 28, 1951
Nov. 21, 1919	Carl Hookstadt	Mar. 10, 1924
Nov. 18, 1932	Solomon S. Huebner	July 17, 1964
†	Charles Hughes	Aug. 27, 1948
Nov. 19, 1929	Robert S. Hull	Nov. 30, 1947
†	Burritt A. Hunt	Sept. 3, 1943
†	Arthur Hunter	Jan. 27, 1964
Nov. 28, 1921	William Anderson Hutcheson	Nov. 19, 1942
Feb. 25, 1916	Charles William Jackson	Sept. 21, 1959
Nov. 19, 1929	Henry Hollister Jackson	May 27, 1955
May 19, 1915	William C. Johnson	Oct. 7, 1943
Nov. 23, 1928	F. Robertson Jones	Dec. 26, 1941
Nov. 18, 1921	Thomas P. Kearney	Feb. 11, 1928
Nov. 19, 1926	Gregory Cook Kelly	Sept. 11, 1948
Oct. 22, 1915	Virgil Morrison Kime	Oct. 15, 1918
†	Edwin W. Kopf	Aug. 3, 1933
Nov. 23, 1928	Clarence Arthur Kulp	Aug. 20, 1957
Feb. 17, 1915	John M. Laird	June 20, 1942
Nov. 13, 1931	Stewart M. LaMont	Aug. 22, 1960

DECEASED FELLOWS

Admitted		Died
Feb. 19, 1915	Abb Landis	Dec. 9, 1937
Nov. 24, 1933	John Robert Lange	Apr. 12, 1957
Nov. 17, 1922	Arnette Roy Lawrence	Dec. 1, 1942
†	James R. Leal, Sr.	Dec. 26, 1957
†	William Leslie	Dec. 12, 1962
Nov. 18, 1921	James Fulton Little	Aug. 11, 1938
Nov. 23, 1928	Edward C. Lunt	Jan. 13, 1941
Feb. 19, 1915	Harry Lubin	Dec. 20, 1920
Nov. 19, 1954	Harold E. MacKeen	July 14, 1970
†	William N. Magoun	Dec. 11, 1954
Nov. 14, 1958	Allen L. Mayerson	Sept. 11, 1971
Nov. 16, 1923	D. Ralph McClurg	Apr. 27, 1947
May 23, 1919	Alfred McDougald	July 28, 1944
Oct. 31, 1917	Robert J. McManus	Aug. 15, 1960
Feb. 15, 1915	Franklin B. Mead	Nov. 29, 1933
Apr. 20, 1917	Marcus Meltzer	Mar. 27, 1931
†	David W. Miller	Jan. 18, 1936
†	Samuel Milligan	Aug. 8, 1965
†	James F. Mitchell	Feb. 9, 1941
†	Henry Moir	June 8, 1937
Nov. 18, 1921	Victor Montgomery	May 2, 1960
Feb. 19, 1915	William J. Montgomery	Aug. 20, 1915
Nov. 19, 1926	William L. Mooney	Oct. 21, 1948
†	George D. Moore	Mar. 11, 1959
May 19, 1915	Edward Bontecou Morris	Dec. 19, 1929
†	Albert H. Mowbray	Jan. 7, 1949
†	Frank Mullaney	Jan. 22, 1953
May 28, 1920	Ray D. Murphy	Feb. 24, 1964
Nov. 1, 1963	S. Tyler Nelson	Aug. 9, 1969
†	Lewis A. Nicholas	Apr. 21, 1940
†	Edward Olifiers	May 13, 1962
†	Robert K. Orr	Oct. 5, 1967
†	Stanley L. Otis	Oct. 12, 1937
Nov. 13, 1926	Bertrand A. Page	July 30, 1941
Nov. 18, 1921	Sanford B. Perkins	Sept. 16, 1945
Nov. 15, 1918	William Thomas Perry	Oct. 25, 1940
Nov. 21, 1930	Francis S. Perryman	Nov. 30, 1959
†	Edward B. Phelps	July 24, 1915
Nov. 19, 1926	Jesse S. Phillips	Nov. 6, 1954
Nov. 13, 1931	Dudley M. Pruitt	June 27, 1967
†	Charles Grant Reiter	July 30, 1937
†	Charles H. Remington	Mar. 21, 1938
Nov. 16, 1951	Homer D. Rice	May 12, 1967
May 23, 1919	Frederick Richardson	July 22, 1955
Nov. 19, 1926	Otto C. Richter	Feb. 17, 1962
May 24, 1921	Robert Riegel	Mar. 12, 1970
Nov. 16, 1923	William F. Roeber	Mar. 21, 1960
Nov. 17, 1943	Samuel M. Ross	July 24, 1951
†	Isaac M. Rubinow	Sept. 1, 1936

DECEASED FELLOWS

Admitted		Died
†	Harwood Eldridge Ryan	Nov. 2, 1930
†	Arthur F. Saxton	Feb. 26, 1927
†	Emil Scheitlin	May 2, 1946
†	Leon S. Senior	Feb. 3, 1940
Nov. 24, 1933	Robert V. Sinnott	Dec. 15, 1952
Apr. 20, 1917	Charles Gordon Smith	June 22, 1938
Nov. 24, 1933	John B. St. John	Nov. 22, 1970
Nov. 18, 1927	Edward C. Stone	June 6, 1964
Feb. 19, 1915	John T. Stone	May 9, 1920
Feb. 25, 1916	Wendell Melville Strong	Mar. 30, 1942
Oct. 22, 1915	William R. Strong	Jan. 10, 1946
†	Robert J. Sullivan	July 19, 1934
Nov. 17, 1920	Thomas F. Tarbell	July 2, 1958
Nov. 22, 1934	Walter H. Thompson	May 25, 1935
Nov. 18, 1921	Guido Toja	Feb. 28, 1933
†	John L. Train	June 12, 1958
Nov. 17, 1922	Antonio Thomas Traversi	Apr. 20, 1961
Nov. 19, 1948	Paul A. Turner	Jan. 30, 1961
Nov. 17, 1920	Alan W. Waite	Aug. 17, 1969
Nov. 15, 1935	Harry V. Waite	Aug. 14, 1951
Nov. 18, 1925	Lloyd A. H. Warren	Sept. 30, 1949
May 23, 1919	Archibald A. Welch	May 8, 1945
Nov. 19, 1926	Roy A. Wheeler	Aug. 26, 1932
†	Albert W. Whitney	July 27, 1943
†	Lee J. Wolfe	Apr. 28, 1949
†	S. Herbert Wolfe	Dec. 31, 1927
Nov. 18, 1949	Richard J. Wolfrum	Oct. 31, 1967
May 24, 1921	Arthur B. Wood	June 14, 1952
†	Joseph H. Woodward	May 15, 1928
†	William Young	Oct. 23, 1927

Admitted		Died
May 23, 1924	Milton Acker	Aug. 16, 1956
Apr. 5, 1928	Austin F. Allen	Oct. 8, 1969
Nov. 15, 1918	Robert E. Ankers	Mar. 1, 1964
Oct. 22, 1915	Don A. Baxter	Feb. 10, 1920
Nov. 17, 1920	Nellas C. Black	Dec. 24, 1962
Nov. 15, 1940	John M. Blackhall	Nov. 14, 1957
Nov. 15, 1918	Helmuth G. Brunnquell	June 3, 1958
Oct. 22, 1915	Louis Buffler	July 19, 1963
Nov. 17, 1922	Leo D. Cavanaugh	July 18, 1965
Nov. 18, 1925	Malvin E. Davis	Aug. 26, 1966
Nov. 14, 1941	William F. Dowling	June 29, 1968
May 25, 1923	Harilaus E. Economidy	Apr. 13, 1948
Nov. 16, 1923	Frank A. Fleming	Feb. 12, 1971
Nov. 20, 1924	John Froberg	Oct. 11, 1949
Nov. 19, 1929	Maurice L. Furnivall	June 16, 1962
Nov. 22, 1934	John J. Gately	Nov. 3, 1943
Nov. 14, 1947	Harold J. George	Apr. 1, 1952
Nov. 19, 1929	Harold R. Gordon	July 8, 1948
Nov. 18, 1921	Robert E. Haggard	July 26, 1958
Nov. 17, 1922	Hartwell L. Hall	Feb. 15, 1971
Nov. 20, 1924	Leslie LeVant Hall	Mar. 8, 1931
Nov. 21, 1919	George F. Haydon	Sept. 7, 1970
Nov. 17, 1927	Grady Hayne Hipp	June 25, 1965
Oct. 31, 1917	Edward T. Jackson	May 8, 1939
Nov. 18, 1921	Edward S. Jensen	Sept. 2, 1966
Mar. 24, 1927	Charles V. R. Marsh	Sept. 12, 1967
Nov. 17, 1922	Rosswel A. McIver	Apr. 1, 1959
Nov. 21, 1919	Rolland V. Mothersill	July 25, 1949
Nov. 19, 1929	Fritz Muller	Apr. 27, 1945
Nov. 23, 1928	Karl Newhall	Oct. 24, 1944
Nov. 22, 1957	C. Otis Shaver	June 15, 1966
Nov. 15, 1918	John L. Sibley	Mar. 10, 1957
Nov. 18, 1921	Arthur G. Smith	May 2, 1956
Nov. 19, 1926	William F. Somerville	Nov. 12, 1965
Nov. 18, 1927	Alexander A. Speers	June 25, 1941
Nov. 15, 1918	Harold S. Spencer	Mar. 18, 1968
Nov. 19, 1959	Henry W. Steinhaus	Aug. 8, 1966
Nov. 21, 1930	Walter F. Sullivan	Jan. 7, 1971
Mar. 23, 1921	Arthur E. Thompson	Jan. 17, 1944
Nov. 21, 1919	Walter G. Voogt	May 8, 1937
May 23, 1919	Charles S. Warren	May 1, 1952
Nov. 18, 1925	James H. Washburn	Aug. 19, 1946
Nov. 17, 1920	James J. Watson	Feb. 23, 1937
Nov. 18, 1921	Eugene R. Welch	Jan. 17, 1945
Nov. 16, 1951	Michael T. Wermel	Feb. 6, 1962
Mar. 21, 1929	Charles A. Wheeler	July 2, 1956
Nov. 15, 1918	Albert Edward Wilkinson	June 11, 1930
Oct. 22, 1915	Donald M. Wood	Sept. 6, 1971
Oct. 22, 1915	Charles E. Woodman	Dec. 16, 1955

PRESIDENTS AND VICE PRESIDENTS

Elected	*President*	*Vice Presidents*	
1914-1915	*Isaac M. Rubinow	*Albert H. Mowbray	*Benedict D. Flynn
1916-1917	*James D. Craig	*Joseph H. Woodward	*Harwood E. Ryan
1918	*Joseph H. Woodward	*Benedict D. Flynn	*George D. Moore
1919	*Benedict D. Flynn	*George D. Moore	*William Leslie
1920	*Albert H. Mowbray	*William Leslie	*Leon S Senior
1921	*Albert H. Mowbray	*Leon S. Senior	*Harwood E. Ryan
1922	*Harwood E. Ryan	Gustav F Michelbacher	*Edmund E. Cammack
1923	*William Leslie	Gustav F. Michelbacher	*Edmund E. Cammack
1924-1925	Gustav F. Michelbacher	*Sanford B. Perkins	Ralph H. Blanchard
1926-1927	*Sanford B. Perkins	*George D. Moore	*Thomas F. Tarbell
1928-1929	*George D. Moore	Sydney D. Pinney	*Paul Dorweiler
1930-1931	*Thomas F. Tarbell	*Roy A. Wheeler	*Winfield W. Greene
1932-1933	*Paul Dorweiler	*William F. Roeber	*Leon S. Senior
1934-1935	*Winfield W. Greene	Ralph H. Blanchard	Charles J. Haugh
1936-1937	*Leon S. Senior	Sydney D. Pinney	*Francis S. Perryman
1938-1939	*Francis S. Perryman	Harmon T. Barber	*William J. Constable
1940	Sydney D. Pinney	Harold J. Ginsburgh	James M. Cahill
1941	Ralph H. Blanchard	Harold J. Ginsburgh	James M. Cahill
1942	Ralph H. Blanchard	Albert Z. Skelding	Charles J. Haugh
1943-1944	Harold J. Ginsburgh	Albert Z. Skelding	Charles J. Haugh
1945-1946	Charles J. Haugh	James M. Cahill	Harry V. Williams
1947-1948	James M. Cahill	Harmon T. Barber	Russell P Goddard
1949-1950	Harmon T. Barber	*Thomas O Carlson	Norton E. Masterson
1951-1952	*Thomas O. Carlson	Joseph Linder	Seymour E Smith
1953-1954	Seymour E. Smith	*Dudley M. Pruitt	John A. Mills
1955-1956	Norton E. Masterson	*Clarence A. Kulp	Arthur N Matthews
1957-1958	*Dudley M. Pruitt	John W. Carleton	William Leslie, Jr.
1959-1960	William Leslie, Jr.	*Ernest T. Berkeley	Laurence H. Longley-Cook
1961-1962	L H. Longley-Cook	Thomas E. Murrin	*Richard J Wolfrum
1963-1964	Thomas E Murrin	Harold E. Curry	William J. Hazam
1965-1966	Harold E. Curry	Charles C Hewitt, Jr.	Harold W. Schloss
1967	Harold W. Schloss	William J. Hazam	Daniel J. McNamara
1968	William J. Hazam	Richard L. Johe	Daniel J. McNamara
1969	Daniel J. McNamara	Richard L. Johe	LeRoy J. Simon
1970	Richard L. Johe	Charles C. Hewitt, Jr.	LeRoy J. Simon
		President-Elect	*Vice President*
1971	LeRoy J. Simon	Charles C. Hewitt, Jr.	Paul S. Liscord

**Deceased*

Elected	Secretary-Treasurer
1914-1917	*C. E. Scattergood
1918-1952	*R. Fondiller
1953-1968	A. Z Skelding
1969-1971	R. L. Bornhuetter

Editor

1914	*W. W Greene
1915-1917	*R. Fondiller
1918	*W. W Greene
1919-1921	G. F. Michelbacher
1922-1923	O. E. Outwater
1924-1932	*R. J. McManus
1933-1943	*C. W. Hobbs
1944-1954	E. C. Maycrink
1955-1958	E. S. Allen
1959-1960	R. P. Goddard
1961-1964	H W. Schloss
1965-1969	M. Rodermund
1970-1971	L. L. Tarbell, Jr.

Librarian

1914	*W. W. Greene
1915	*R. Fondiller
1916-1921	L. I. Dublin
1922-1924	*E. R. Hardy
1925-1936	*W Breiby
1937-1947	*T. O. Carlson
1948-1950	*S. M. Ross
1951-1957	G. R. Livingston
1958-1969	R. Lino
1970	W. S. Gillam

General Chairman Examination Committee

1949-1951	R. A. Johnson
1952-1956	J. W. Wieder, Jr.
1957-1961	W. J. Hazam
1962-1968	N. J. Bennett

General Chairman Education and Examination Committee

1969-1971	M. S. Hughey

**Deceased.*

CONSTITUTION

(AS AMENDED MAY 18, 1971)

ARTICLE I.—*Name*

This organization shall be called the CASUALTY ACTUARIAL SOCIETY.

ARTICLE II —*Objects*

The objects of the Society shall be to advance the knowledge of actuarial science as applied to the problems of insurance, other than life insurance, and to promote and maintain high standards of conduct and competence within the actuarial profession. The Society shall further these ends by holding meetings, by personal communication, by the presentation, discussion and publication of appropriate papers, by promoting educational activities in the actuarial sciences for its students and members, and by such other means as may be found desirable.

ARTICLE III.—*Membership*

The membership of the Casualty Actuarial Society shall be composed of two classes, Fellows and Associates. Fellows only shall be eligible to hold office, make nominations, or have the right to vote.

The Fellows of the Society shall be the present Fellows and those who may be duly admitted to Fellowship as hereinafter provided. The Associates shall be the present Associates and those who may be duly admitted to Associateship as hereinafter provided.

Any applicant shall be enrolled as an Associate at a meeting of the Society provided that:

(i) the applicant passes the examinations prescribed by the Board of Directors for Associateship and compiles with any further requirements the Board may prescribe;

(ii) the applicant, upon fulfilling all the requirements outlined in (i), is approved by a majority vote of the Board of Directors.

An Associate shall be enrolled as a Fellow of the Society at the Society meeting following the successful completion of the examinations prescribed by the Board of Directors for Fellowship, subject to any further requirements the Board may prescribe.

Otherwise no one shall be admitted as an Associate or a Fellow unless recommended at a duly called meeting of the Board of Directors with not more than two negative votes followed by an affirmative vote in a secret ballot of at least three-fourths of the Fellows present and voting at a meeting of the Society

The Board of Directors may waive, subject to such other requirements as it may prescribe, any examination of the Casualty Actuarial Society if the applicant has passed an examination required by another recognized actuarial organization that the Board of Directors deems equivalent to such examination of the Casualty Actuarial Society.

ARTICLE IV.—*Officers and Board of Directors*

The Officers of the Society, all of whom shall be Fellows, shall consist of a President, a President-Elect, a Vice President, a Secretary-Treasurer, an Editor, and a General Chairman of the Education and Examination Committee. The Board of Directors shall consist of the Officers, nine other Fellows and, for the two years following the expiration of their terms of office, the ex-Presidents.

The Board of Directors may fill vacancies occasioned by death or resignation of any

Officer or other elected member of the Board. An appointed Officer shall serve until the next annual meeting. Any other member so appointed by the Board of Directors shall serve, subject to ratification by the Fellows at the next meeting of the Society, until the expiration of the term of office of the Officer or Board member being replaced.

ARTICLE V.—*Election of Officers and Board of Directors*

At the 1971 annual meeting the Officers shall be elected by a majority vote in a secret ballot of the Fellows present and voting for the term of one year or until their qualified successors shall be duly elected. Thereafter, at the annual meeting the President-Elect shall assume the office of the Presidency and the Fellows shall, in the manner described above, elect a President-Elect, a Vice President, a Secretary-Treasurer, an Editor and a General Chairman of the Education and Examination Committee for the term of one year or until their qualified successors shall be duly elected. Three members of the Board of Directors shall, in a similar manner, be annually elected to serve from the close of the annual meeting for the term of three years. Any retiring elected member of the Board of Directors shall not be eligible for re-election at the same meeting.

A majority of the votes cast shall be required for election as an elected member except that, in the event of a second or subsequent ballot, Fellows receiving the greatest number of votes shall be elected, provided the number of votes received is not less than one-third of those cast.

The terms of all Officers except the Editor shall begin at the close of the annual meeting at which they are elected. The term of the Editor shall begin on May 1 of the calendar year following the annual meeting at which he is elected.

ARTICLE VI.—*Duties of Officers and Board of Directors*

The duties of the Officers shall be such as are customarily incident to their respective offices and such other duties as specified in the Bylaws. The duties of the Board of Directors shall be to pass upon candidates for membership, to decide upon the publication of papers presented at meetings of the Society, to supervise the examination of candidates and prescribe fees for such examinations, to call meetings, to ratify such committees as may be appointed by the President, and, in general, to manage the affairs of the Society, and, for the latter purpose, shall determine all questions arising with respect to the interpretation or administration of this Constitution and the Society's Bylaws not inconsistent therewith.

ARTICLE VII.—*Meetings*

There shall be an annual meeting of the Society on such date in the last quarter of each calendar year as may be fixed by the Board of Directors, but other Society meetings may be called by the Board from time to time and shall be called by the President at any time upon the written request of twenty Fellows. At least two weeks notice of all Society meetings shall be given to the members by the Secretary-Treasurer.

ARTICLE VIII.—*Quorum*

Ten members of the Board of Directors shall constitute a quorum. Forty Fellows of the Society shall constitute a quorum at every meeting of the Society.

ARTICLE IX.—*Public Expression of Professional Opinion*

No opinion with respect to questions of public interest shall be publicly expressed by, or on behalf of, the Casualty Actuarial Society, the Board of Directors, or any committee except on matters within the special professional competence of actuaries and then only in accordance with authority given and procedures determined in each instance by the Board and in accordance with the following conditions:

(i) An opinion of the Casualty Actuarial Society shall require advance approval by an affirmative vote of at least ninety percent of the Fellows who vote in a mail ballot.

(ii) An opinion of the Board of Directors or a committee authorized by the Board to express an opinion shall indicate that it does not purport to represent the views of the Casualty Actuarial Society, but only of the Board of Directors or the committee, as the case may be.

ARTICLE X —*Resignation and Discipline of Members*

Any member who is not in default in payment of dues, and against whom no complaints or charges are pending, may at any time file his resignation in writing with the Secretary-Treasurer. Notwithstanding the foregoing, the Board of Directors may, in its discretion, permit the resignation of a member against whom a complaint or charge is pending. The Board, on written application of any member who has resigned while in good standing, may reinstate such member subject to such conditions as it may prescribe.

No member of the Society shall be disciplined, suspended, or expelled except upon action of the Board of Directors and the membership as provided for in the Bylaws of the Society

ARTICLE XI.—*Amendments*

This Constitution may be amended by an affirmative vote of two-thirds of the Fellows present at any meeting held at least one month after notice of such proposed amendment shall have been sent to each Fellow by the Secretary-Treasurer.

BYLAWS

(As amended May 18, 1971)

Article I.—*Order of Business*

At a meeting of the Society the order of business shall be in accordance with an agenda sent to the members prior to the meeting, but at the annual meeting shall include:

1. Address or remarks by the President
2. Minutes of the last meeting
3. Report by the Board of Directors on business transacted by it since the last annual meeting of the Society
4. Enrollment of new Fellows and Associates
5. Reports of Officers and committees
6. Election of Officers and Board members
7. Unfinished business
8. New business
9. Reading of papers
10. Discussion of papers

Article II.—*Meetings of the Board of Directors*

Meetings of the Board of Directors shall be called whenever the President or three members of the Board so request, but not without sending notice to each member of the Board seven or more days before the time appointed. Such notice shall state the objects intended to be brought before the meeting, and should other matter be passed upon, any member of the Board shall have the right to reopen the question at the next meeting.

Article III.—*Duties of Officers*

The President, or, in his absence, the President-Elect, or, in the absence of both, the Vice President, shall preside at meetings of the Society and of the Board of Directors. At the Society meetings, the presiding officer shall vote only in case of a tie, but at the Board meetings he may vote in all cases. The President shall appoint all committees and shall perform all duties customarily incident to the office of President and such other duties as may be prescribed by the Board of Directors from time to time. The President-Elect and the Vice President shall have such duties as may be assigned to them by the President or by the Board of Directors.

The Secretary-Treasurer shall keep a full and accurate record of the proceedings at the meetings of the Society and of the Board of Directors, and send out notices for such meetings. Subject to the direction of the Board, he shall have immediate charge of the office and archives of the Society, and shall have charge of the books, pamphlets, manuscripts, and other literary or scientific material collected by the Society.

The Secretary-Treasurer shall collect the annual dues of members, pay all bills for ordinary expenditures incurred by the Society and any other bills as authorized by the Board of Directors, keep a detailed record of all receipts and expenditures, and present an accounting of the same at the annual meetings, after it has been audited by a committee appointed by the President The Secretary-Treasurer shall perform all duties customarily incident to the office of Secretary-Treasurer and such other duties as may be assigned to him from time to time by the President or by the Board of Directors.

The Editor shall, under the general supervision of the Board of Directors, have charge of all matters connected with editing and printing the Society's publications The *Pro-*

ceedings shall contain only the proceedings of the meetings and the original papers, reviews or discussions on said papers by members that may be expressly authorized by the Board to appear in such *Proceedings*. The *Proceedings* may also contain any other matter expressly authorized by the Board.

The General Chairman of the Education and Examination Committee shall, under the general supervision of the Board of Directors, have charge of the education and examination system and of the examinations held by the Society for admission to the grades of Associate and Fellow.

ARTICLE IV.—*Discipline of Members*

The Board of Directors shall have the power to consider and take action, as herein provided, with respect to all questions which may arise as to the conduct of a member of the Casualty Actuarial Society in his relations to the Society or its members, or in his profession, or in the practice thereof, or affecting the interests of the actuarial profession. The Board may, on its own initiative, investigate and take action with respect to any such question, and may also receive and hear any complaint relating to the conduct of a member preferred in writing and subscribed to by a member. In the course of dealing with questions and complaints relating to the conduct of members, the Board may appoint, from among the Fellows of the Casualty Actuarial Society, committees and boards vested with the powers specified herein:

(a) Investigating committees empowered to investigate questions and complaints and to prefer charges against a member;

(b) Prosecuting committees empowered to prosecute charges against a member at hearings before the Board of Directors or a disciplinary board;

(c) Disciplinary boards empowered to hear evidence relating to questions and complaints and to make findings with respect to such evidence.

The procedures for such committees and boards shall be prescribed by the Board of Directors. The Board of Directors may retain counsel for the assistance of the Board of Directors and of committees and boards appointed by it

In any hearing before the Board of Directors or a disciplinary board, a member proceeded against shall have the right to appear personally and by counsel, to be informed of the nature and content of the question or complaint, to examine the evidence presented, to examine adverse witnesses, and to present witnesses and evidence in his behalf. Any member preferring a complaint may appear personally and by counsel. Witnesses called in the course of hearings involving conduct shall vouch for the truth of their statements on their word of honor.

In all proceedings under this Article, the Board of Directors shall decide, directly or upon review of the findings of a body appointed by it, whether or not misconduct has occurred If the Board finds that misconduct has occurred, it may warn, admonish, reprimand, suspend, or expel the member, provided that no order reprimanding, suspending, or expelling a member shall be issued except after a hearing before the Board of Directors or a disciplinary board.

A member against whom an order of suspension or expulsion has been rendered shall, upon application to the Board of Directors within thirty days thereafter, be entitled to appeal to the Fellows attending a meeting of the Casualty Actuarial Society upon the following conditions:

(a) All rights and privileges of membership shall be suspended during the pendency of the appeal, and

(b) The notice of appeal shall be in writing and shall stipulate that the appealing member consents to the mailing to the Fellows of a transcript of the evidence and copies of exhibits in the form approved by a majority of the Board of Directors, and

(c) The appealing member shall, within ten days after an invoice of the amount due is sent to him, deposit with the Secretary-Treasurer the cost of transcribing and printing the transcript of the evidence and copies of any and all exhibits. In the event the decision of the Board shall be set aside, the Secretary-Treasurer shall return to the appealing member the amount of the deposit. Otherwise, the deposit shall be retained by the Casualty Actuarial Society

In the event of an appeal to the Fellows, the decision of the Board of Directors may be affirmed, modified, or set aside by the vote of a majority of the Fellows present and voting at a meeting of the Casualty Actuarial Society

The Board of Directors may, in its discretion, reinstate to membership at any time a member suspended or expelled under this Article, provided in the event the suspension or expulsion has been affirmed by the Fellows, the reinstatement shall not take effect unless and until confirmed at a meeting of the Casualty Actuarial Society by a vote of a majority of the Fellows present and voting.

Except as otherwise provided, all proceedings under this section shall be deemed confidential and kept secret The Board of Directors, however, shall notify the members of its action in all instances in which the Board orders the suspension or expulsion of a member Such notification shall not be given until the time to appeal has expired or, in the event of an appeal, until a majority of the Fellows present at a meeting of the Society have voted in favor of suspension or expulsion. At the same time notification is given to the members, the Board may also give notice of such suspension or expulsion to such newspapers or journals as it may select.

In the event of subsequent reinstatement of the member, the Board of Directors shall give notice of such action to the members of the Society and to any newspapers or journals previously advised by the Board of the member's suspension or expulsion

ARTICLE V.—*Indemnification of Officers, Members of the Board of Directors, and Committee Members*

Each person who at any time shall serve, or shall have served, as an Officer, member of the Board of Directors, committee member, or member of any disciplinary board of the Society (and his heirs, executors, administrators, and personal representatives) shall be indemnified by the Society against all costs and expenses (including but not limited to legal fees, amounts of judgments paid, and amounts paid in settlements) reasonably incurred in connection with the defense of any claim, action, suit, or proceeding, whether civil, criminal, administrative, or other, in which he or they may be involved by virtue of such person being or having been an Officer, member of the Board of Directors, committee member, or member of any disciplinary board of the Society, or in connection with any appeal therein; provided, however, that in the event of a settlement the indemnification herein provided shall apply only when the Board of Directors approves such settlement; and provided further that such indemnity shall not be operative with respect to any matter as to which such person shall have been finally adjudged liable in such claim, action, suit, or proceeding on account of his own wilful misconduct.

The rights accruing to any person under this Article shall be without prejudice to any rights or benefits given by the Board of Directors inconsistent therewith in special cases and shall not exclude any other rights or benefits to which he may be lawfully entitled.

make it difficult for him to act independently. Even if there is no question as to his ability to act independently, he will not act unless there has been a full disclosure of the situation to all parties involved and the parties have expressly agreed to his performance of the service.

4. *Calculations and Recommendations*

A. The member will customarily include in any report or certificate quoting actuarial costs, reserves, or liabilities a statement or reference describing or clearly identifying the data and the actuarial methods and assumptions employed

B. The member will exercise his best judgment to ensure that any calculations or recommendations made by him or under his direction are based on sufficient and reliable data, that any assumptions made are adequate and appropriate, and that the methods employed are consistent with the sound principles established by precedents or common usage within the profession.

C. If, nevertheless, a client or employer requests the member to prepare a study which in his opinion deviates from this practice, any resulting report, recommendation, or certificate submitted by him will include an appropriate and explicit qualification of his findings.

5. *Advertising and Relations with Other Members*

A. The member will neither engage in nor condone any advertising or other activity which can reasonably be regarded as being likely to attract professional work unfairly, or where the tone, form and content are not strictly professional.

B. The member will conduct his professional activities on a high plane. He will avoid unjustifiable or improper criticism of others and will not attempt to injure maliciously the professional reputation of any other actuary He will recognize that there is substantial room for honest differences of opinion on many matters

6. *Remuneration*

The member will make full and timely disclosure to a client as to all direct and indirect compensation that he or his firm may receive from all sources in relation to any assignment the member or his firm undertakes for the client.

7. *Titles*

The member will use a designation dependent upon elective or appointive qualifications within the Society, such as "President," "Member of the Board," or "Member of the Educational and Examination Committee," only when he is acting in such capacity on behalf of the Society.

(Copies of interpretative opinions for these Guides may be obtained from the Secretary-Treasurer of the Society.)

GUIDES FOR THE SUBMISSION OF PAPERS

(AS AMENDED NOVEMBER 15, 1970)

A. *Method of Review.* All papers and reviews of papers are reviewed by the Committee on Review of Papers. The Committee consists of members appointed by the President, plus, ex officio, the Editor of the *Proceedings* Unanimous vote of the regular Committee is necessary for acceptance of a paper or a review, except that if there is only one vote for rejection, the paper or review will be submitted to the Editor for acceptance or rejection.

B. *Scope and Standards*—1. Broad latitude will be allowed in the choice of a subject, provided it is a subject of interest to property and casualty actuaries. However, it must be clearly suitable for inclusion in the *Proceedings.*

2. The paper must contain original ideas or new material of reasonable value, unless it has a definite educational value for other reasons.

3. When a paper includes material that the Committee finds it is not qualified to review, the Committee will seek advice or opinion from other members of the Society or from recognized experts outside of the Society.

4. Disagreement by the Committee with opinions of the author or reviewer of a paper will not be a bar to acceptance of an otherwise suitable paper or review. Where, however, the Committee believes a paper or review to be fallacious in logic or misleading in matters of fact, the Committee may reject it. An author may appeal to the President in case of rejection, and the President will make such inquiries as he deems appropriate and will make recommendations to the Board.

5. Reviews of papers are expected to be free of criticism of a personal nature Opportunity will be given to the authors of papers to respond to reviews. Authors' replies will also be reviewed by the Committee and will be treated in the same manner as reviews

6. The paper or review should show care in preparation. A reasonable minimum standard will be required as to form, clarity, and literary quality. When a paper or review, otherwise acceptable, does not meet these standards, the Committee may return it to the author or reviewer and invite resubmission after editing or rewriting. The Committee may also make suggestions to the author as to possible improvements in an accepted paper

7. Papers and reviews should be kept within the general limits of length indicated by past acceptances, ordinarily about twenty printed pages for papers and two or three pages for reviews.

C. *Procedures and Regulations.*—1. Papers may be submitted only by Fellows or Associates of the Casualty Actuarial Society, except that papers may be submitted by non-members of the Society upon invitation of the President. A member may collaborate in joint authorship with a non-member who possesses particular qualifications in respect to the subject of a paper.

2. Papers should be submitted in quintuplicate to the Secretary-Treasurer of the Society. The name of the author should not appear on the copies of the paper submitted to the Secretary-Treasurer, but should be included in the covering letter. The Secretary-Treasurer is authorized to return to the author copies of a paper that in his opinion are not legible.

3. Reviews of papers and authors' replies to reviews should be submitted in quintuplicate to the Chairman of the Committee on Review of Papers. Names of reviewers should

be identified on the copies of their reviews The Chairman will return to the reviewer or to the author copies of a review or of an author's reply that in his opinion are not legible.

4 In submitting a paper, the author must answer the following questions on a separate sheet attached to each of the five copies of the paper.

(a) Name of paper.

(b) Has the paper been published elsewhere, in whole or in part, in identical or similar form?

(c) Is the paper being simultaneously submitted elsewhere, or will it be so submitted before decision by the Committee on Review of Papers?

(d) In the case of co-authorship with a non-member, to what extent has the Society member contributed?

(e) If the paper has been requested by the President or General Chairman of the Education and Examination Committee, attach the letter of request, removing any reference which would identify the author

(f) If the paper contains factual data from some organization, has the organization given the author permission to publish it?

5. Papers and reviews should be typed double-spaced on letter-size stationery, on one side of each sheet. The first line of each paragraph should be indented. Tables and footnotes may be single-spaced. Pages should be numbered. Footnotes should be numbered consecutively throughout the paper.

6. Major captions should be centered and typed in capitals; subcaptions should appear in the left-hand margin in italics (single underscore). In technical papers paragraphs may be numbered to simplify reference; in non-technical papers paragraphs should not be numbered.

7. So far as possible, tables should be arranged so that they can be printed on a single page of the *Proceedings* without undue reduction in size of type Column headings must be clear and concise.

8. Mathematical formulas and symbols may be handwritten in ink rather than typewritten. They must be legible especially as to subscripts and superscripts. There must be no possibility of confusion between, for instance, dx and d_x; × (the sign for multiplication) and x; a and α (alpha). The lower case L (l) should not be used as a mathematical symbol. The exclamation point (!) should be used to indicate factorials in binominal expansions Where necessary, instructions to the printer may be inserted in pencil on the manuscript. The Committee strongly recommends that authors of mathematical papers refer to the ***Style Manual of the American Institute of Physics*** for precise information on preparation of a manuscript. A copy of the ***Style Manual*** may be borrowed from the Editor of the ***Proceedings*** or it may be purchased from the Editor for one dollar When life contingency symbols are applicable the International Actuarial Notation should be used This code is described in the *Proceedings*, Vol. XXXVI, page 123.

9. References to books and periodicals and to proceedings of professional societies should be sufficiently complete to permit obtaining a copy of the source without additional research.

10. If the manuscript has been prepared carefully in accordance with the foregoing suggestions, there should be only a few minor corrections necessary. The paper as originally submitted should not be considered simply as a draft to which extensive alterations can be made.

11. Authors will be notified of the acceptance or rejection of their papers by the

Secretary-Treasurer. If a paper is rejected, original and copies will be returned. The Committee does not promise a decision on a paper submitted fewer than sixty days prior to the meeting for which the paper has been prepared A review of a paper will be considered to have been accepted by the Committee unless the reviewer is otherwise notified.

12. Authors of accepted papers are requested to notify the Secretary-Treasurer whether or not they can supply additional copies for use at meetings or for further distribution prior to publication. (Photographic reproduction is less expensive than printing and insures accuracy.)

13. After acceptance of a paper and before its reproduction, the author should have the following statement typed at the bottom of the first page. "Presented at the (date) meeting of the Casualty Actuarial Society at (city and state). Reproduction in whole or in part without acknowledgment to the Casualty Actuarial Society is specifically prohibited."

14. Presentation of papers and reviews will be within time limits of the meeting schedule and subject to the discretion of the presiding officer. Generally, an author may expect an allowance of ten minutes to present a summary of his paper stating its purpose and conclusions. A typewritten copy of this summary should be forwarded to the Publicity Committee Chairman well in advance of the meeting. Reviewers may expect an allowance of five minutes for their presentation A reviewer should send a typewritten copy of his review to the author and any other persons known to be reviewing the same paper, and to the Chairman of the Committee on Review of Papers as required by paragraph C-3. Authors and reviewers are cautioned that in a verbal presentation mathematical formulae and statistical data are difficult for the audience to grasp.

15. Reviews may be presented at the meeting which are not intended for publication in the *Proceedings* Such reviews will not be reviewed by the Committee Reviews which are submitted for publication may be presented at the meeting prior to submission to or action by the Committee.

16. The Editor of the *Proceedings*, in consultation with the author or reviewer, may edit the paper or review prior to publication.

WOODWARD-FONDILLER PRIZE

This award, made in commemoration of Joseph H. Woodward and Richard Fondiller, is intended to stimulate original thinking and research and will be made to the best eligible paper each year submitted by an Associate or Fellow who has attained his designation within the last five years. To be eligible the paper must show evidence of ability for original research and the solution of advanced insurance problems. If no paper is considered eligible in a given year, the award shall not be made Papers previously submitted to the Society or elsewhere shall not be eligible.

The amount of the prize will be $200 and the papers will be judged by the Society's Committee on Review of Papers, whose decision will be final.

The announcement of the award will be made at the November meeting each year, based on papers submitted to the Society at the previous November and May meetings.

DORWEILER PRIZE

This award, made in commemoration of Paul Dorweiler, is subject to the same conditions as those specified for the Woodward-Fondiller Prize, *except* that the Dorweiler Prize will be awarded to the best eligible paper each year submitted by an Associate or Fellow who has *not* attained his designation within the last five years

The amount of the Dorweiler Prize also is $200.

RULES REGARDING EXAMINATIONS FOR ADMISSION

(Effective with 1970 Examinations)

1. Dates of Examinations

Examinations for Parts 1 and 2 will be given twice yearly, in May and November. Parts 3, 5, 7, and 9 will be given once a year, in May; Parts 4, 6, and 8 will be given once a year, in November.

The schedule of dates on which the examinations will be given appears on the last page of this publication. It is customary to administer the examinations in such cities as will be convenient. Candidates will be advised by the Secretary-Treasurer as to the times and places of these examinations.

2. Filing of Application

A candidate who wishes to take Part 1 or Part 2, or both, must make application on the Society's application form, which may be obtained from the Secretary-Treasurer.

A candidate who has previously submitted his application on the Society's application form, and who wishes to take one or more examinations other than Parts 1 and 2, need not again make use of the Society's application form, but may simply write to the Secretary-Treasurer, stating the part or parts for which he is applying.

Each application must be accompanied by the appropriate examination fee, in check, draft, or money order payable to the Casualty Actuarial Society.

Applications must be received by the Secretary-Treasurer by April 1 for the Spring examinations and by October 1 for the Fall examinations.

3. Associateship and Fellowship Examinations

There are five examinations which the candidate must pass in order to become an Associate of the Casualty Actuarial Society. Part 1, the General Mathematics examination, and Part 2, the Probability and Statistics examination, are jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries. Successful candidates will be given credit for these examinations by both Societies regardless of the Society through which the candidate registers.

A candidate may write any one or more of the five examinations and will receive credit for those passed, except that Parts 1 and 2 must be taken in numerical order.

There are four examinations which a candidate must also pass to become a Fellow of the Casualty Actuarial Society. A candidate may present himself for one or more of the Fellowship examinations either if he has previously passed the Associateship examinations or if he concurrently presents himself for and submits papers for all unpassed Associateship examinations given during that examination period. Subject to the foregoing requirements, a candidate will be given credit for any examination which he may pass.

4. Fees

The examination fee schedule is as follows:

Parts 1 & 2	$ 9 for each Part
Part 3	$10
Parts 4-9	$20 for each Part

Examination fees are payable each time the candidate presents himself Check, draft, or money order payable to the order of the Casualty Actuarial Society must be received by the Secretary-Treasurer before April 1 for the Spring examinations, or before October 1 for the Fall examinations.

5. Prize Awards

The Casualty Actuarial Society and the Society of Actuaries will jointly ward one $200 and four $100 prizes to the five successful undergraduates ranking highest in the General Mathematics examination. These prize awards will be granted for both the Spring and Fall examinations.

6. Credit for Examination Parts under Former Syllabus

A candidate who has passed, or been credited with, one or more of the Associateship or Fellowship examinations under the 1967 Syllabus will receive credit for the corresponding examinations of the 1969 Syllabus in accordance with the following table:

Parts Passed or Credited under 1967 Syllabus	Parts Credited under 1969 Syllabus
Associateship, Part 1	Associateship, Part 1
Associateship, Part 2	Associateship, Part 2
Associateship, Part 3(a)	Associateship, Part 3
Associateship, Parts 3(b) and 4	Associateship, Parts 4 and 5
Fellowship, Part 5	Fellowship, Part 6
Fellowship, Part 6	Fellowship, Part 7
Fellowship, Part 7	Fellowship, Part 8
Fellowship. Part 8	Fellowship, Part 9

7. Waiver of Examinations for Associateship

Waiver of certain Associateship examinations will be allowed for a candidate who has passed or been credited with corresponding examinations of the Society of Actuaries, in accordance with the following:

Casualty Actuarial Society	Society of Actuaries
Part 1	Part 1, General Mathematics, passed prior to 1963 (before joint sponsorship)
Part 2	Part 2, Probability and Statistics, passed prior to 1966 (before joint sponsorship)
Part 3	Part 4, Life Contingencies, passed prior to 1969
Part 3	Parts 3 and 4 both, if Part 4 is passed after 1968

Candidates who take the Advanced Mathematics test of the Graduate Record examinations may apply for credit for Part 1. Credit will be granted if the candidate's score on the Graduate Record Advanced Mathematics test is equivalent, as determined by the Casualty Actuarial Society, to the passing score on Part 1. An application to the Casualty Actuarial Society for such credit may be completed either in advance of taking the Graduate Record Advanced Mathematics test or within two years after taking it. The necessary application form may be secured from the Secretary-Treasurer of the Casualty Actuarial Society.

The Board may waive, subject to such other requirements as it may prescribe, any examinations of the Casualty Actuarial Society which it deems equivalent to examinations required by another recognized actuarial organization which have been passed by an applicant while not a resident of the United States or Canada, or during his first year of temporary or permanent residence in the United States or Canada.

LIBRARY

All candidates registered for the examinations of the Casualty Actuarial Society and all members of the Casualty Actuarial Society have access to all the library facilities of the Insurance Society of New York, the Casualty Actuarial Society, and the Society of Actuaries. These libraries, with combined operations, are located at 150 William Street, New York, New York 10038.

Registered candidates may have access to the library by receiving from the Society's Secretary-Treasurer the necessary credentials. Books and manuals may be withdrawn from the library for a period of one month without charge. In general, not more than two references may be in the hands of a borrower at one time. The Insurance Society is responsible for postage and insurance charges for sending books to out-of-town borrowers, and borrowers are responsible for the safe return of the books.

Address requests for books to:

Ronald L. Bornhuetter, Secretary-Treasurer
Casualty Actuarial Society
200 East 42nd Street
New York, New York 10017

SYLLABUS OF EXAMINATIONS

ASSOCIATESHIP

Part	*Time Allowed*	*Subject*
1	3 hours	General Mathematics (jointly sponsored with the Society of Actuaries)
2	3 hours	Probability and Statistics (jointly sponsored with the Society of Actuaries)
3	2 hours	Compound Interest and Life Contingencies
4	3 hours	(a) Principles of Economics: Theory of Risk and Insurance (b) Insurance Coverages and Policy Forms
5	3 hours	(a) Principles of Ratemaking (b) Insurance Statistics and Data Processing

FELLOWSHIP

6	3 hours	(a) Insurance Law; Supervision, Regulation, and Taxation (b) Statutory Insurances
7	3 hours	(a) Insurance Accounting and Expense Analysis (b) Premium, Loss, and Expense Reserves
8	2 hours	Individual Risk Rating
9	3 hours	Advanced Insurance Problems

EXAMINATION DATES

1972 EXAMINATIONS

Parts 1 and 2	May 11, November 9
Part 3	May 15
Part 4	November 16
Part 5	May 16
Part 6	November 17
Part 7	May 15
Part 8	November 16
Part 9	May 16

1973 EXAMINATIONS

Parts 1 and 2	May 17, November 15
Part 3	May 10
Part 4	November 8
Part 5	May 11
Part 6	November 9
Part 7	May 10
Part 8	November 8
Part 9	May 11

1974 EXAMINATIONS

Parts 1 and 2	May 16, November 14
Part 3	May 9
Part 4	November 7
Part 5	May 10
Part 6	November 8
Part 7	May 9
Part 8	November 7
Part 9	May 10

AMERICAN ACADEMY OF ACTUARIES

The American Academy of Actuaries was organized October 25, 1965 as the culmination of efforts on the part of the four actuarial bodies in the United States—the Casualty Actuarial Society, the Conference of Actuaries in Public Practice, the Fraternal Actuarial Association, and the Society of Actuaries. A principal purpose of the Academy is to secure state and federal legal recognition of actuaries. Fellows of the Casualty Actuarial Society are eligible for membership if they have had five years of experience in responsible actuarial work, as defined in the Bylaws, at date of application. Beginning January 1, 1971, Associates of the Casualty Actuarial Society will be eligible for membership if, at the date of application, they have had seven years of experience in responsible actuarial work, as defined in the Bylaws, and if they have passed a comprehensive examination in subject categories defined in the Bylaws. Application blanks, and a copy of the Year Book of the American Academy, may be obtained from the Secretary, 208 South LaSalle Street, Chicago, Illinois 60604

BOARD OF DIRECTORS 1971-1972

Officers:

President	ROBERT J. MYERS
President-Elect	MORTON D MILLER
Vice President—Term Expires 1972	HAROLD E. CURRY
	ERNEST J. MOORHEAD
Term Expires 1973	ROBERT E. BRUCE
	JULIUS VOGEL
Secretary	WILLIAM A. HALVORSON
Treasurer	DALE R. GUSTAFSON

Past Presidents:

WALTER L. RUGLAND	1972*
H. RAYMOND STRONG	1973*

Elected Directors

1974*	1973*	1972*
EDWIN F BOYNTON	JOHN H. BIGGS	REUBEN I JACOBSON
M. STANLEY HUGHEY	W. HAROLD BITTEL	MENO T. LAKE
KENNETH H. ROSS	THOMAS P. BOWLES, JR.	EDWIN B. LANCASTER
HENRY F. SCHEIG	LOUIS GARFIN	DANIEL J. MCNAMARA
CHARLES L. TROWBRIDGE	RICHARD L. JOHE	JOSEPH MUSHER
ROBERT C. WINTERS	FREDERICK P. SLOAT	ROBERT H. TAYLOR

*For terms expiring at annual meeting of the year given.

The 1972 annual meeting will be held in Bal Harbour, Florida, on October 23, 1972 at the Americana Hotel.

INTERNATIONAL ACTUARIAL ASSOCIATION

The first International Congress of Actuaries was held in 1895 in Brussels under the auspices of the Permanent Committee for International Congresses (Comite Permanent des Congres Internationaux d'Actuaires). This organization provided continuity of arrangements for successive International Congresses, the last of which was held in Munich in June 1968. The 19th International Congress will be held in Oslo, Norway, June 19-24, 1972 and the next Congress is scheduled for 1976 in Japan.

The name of the organization was changed to the International Actuarial Association in 1968. The Association cooperates with special Organizing Committees of the host nations to prepare the work of International Congresses and assists in the publication of the proceedings of such Congresses. It also issues each year a bulletin which lists the membership of the Association and brings together selected information on actuarial organizations, actuarial publications, and highlights of insurance developments in various countries

Individual actuaries in North America can support the work of the International Actuarial Association by joining the United States or Canadian sections of the Association Membership in the Association is one of the prerequisites for membership in and attendance at International Actuarial Congresses. Currently the annual dues for membership are 150 Belgian francs; a sum of $4.50 should be remitted to the order of the Society of Actuaries in payment of these dues and incidental minor expenses. A notice about these dues, which are payable by July 1, will be mailed each spring. A late fee of $1.50 will be charged all those who pay their dues on or after July 1.

Inquiries regarding the International Actuarial Association should be directed to *either*

Secretary for United States Section
International Actuarial Association
Edward A. Lew
Metropolitan Life Insurance Company
One Madison Avenue
New York, New York 10010

Secretary for Canadian Section
International Actuarial Association
Archie R. McCracken
Vice President and Chief Actuary
North American Life Assurance Company
105 Adelaide Street, West
Toronto 1, Ontario, Canada

ASTIN SECTION

ASTIN (Actuarial Studies in Non-Life Insurance) is the first section of the International Actuarial Association to be formed under the revised regulations adopted in 1957 at the XVth International Congress in New York. It was established to study applications of modern statistical and mathematical methods in the field of non-life insurance.

It has for its aims the promotion of actuarial research in general insurance and the maintenance of contacts between actuaries, groups of actuaries, and others interested in this field.

Membership in ASTIN is open to all members of the International Actuarial Association upon application and payment of annual dues of 250 Belgian francs. Arrangements have been made for these dues, amounting to $6.00, to be paid with the dues for membership in the International Actuarial Association.

ASTIN publishes a ***Bulletin*** periodically as well as occasional papers on topics related to its interests. These are made available only to members

At annual or biennial intervals colloquia are conducted on topics of special interest, and these are hosted by national actuarial organizations Inquiries regarding ASTIN should be directed to P J. H. Green, Secretary, ASTIN, 130 Fenchurch Street, London, E C. 3., England There will be no Colloquium in 1972 because of the International Congress in that year.

The members of the Committee of ASTIN are:

Chairman	Jan Jung—Sweden
Vice President	Hans Bühlmann—Switzerland
Secretary	Peter J. H Green—Great Britain
Treasurer	Paul Thyrion—Belgium
Editor	Henry G. Verbeek—Netherlands
Other Members	Paul Johansen—Denmark
	Jean Sousselier—France
	Giuseppe Ottaviani—Italy
	Charles C. Hewitt, Jr.—U. S A.

FUTURE MEETINGS OF THE CASUALTY ACTUARIAL SOCIETY

1972 Spring Meeting	May 21, 22, 23, 24 Lake Lawn Lodge Delavan, Wisconsin
1972 Annual Meeting	November 9, 10 Hotel St. Francis San Francisco, California
1973 Spring Meeting	May 20, 21, 22, 23 Nevele Country Club Ellenville, New York
1973 Annual Meeting	November 11, 12, 13 Sheraton-Boston Hotel Boston, Massachusetts
1974 Spring Meeting	May 19, 20, 21, 22 El Conquistador Hotel & Club Fajardo, Puerto Rico
1974 Annual Meeting	November 17, 18, 19 Marriott Motor Hotel New Orleans, Louisiana
1975 Spring Meeting	May 18, 19, 20, 21 The Greenbrier White Sulphur Springs, West Virginia
1975 Annual Meeting	November 16, 17, 18 Le Chateau Champlain Montreal, Canada