In this regard Mr. Lange presents us with two basic problems. The explicit one is that there must be developed a technique of rating for areas beyond the so-called basic level. This is a fit technical problem for actuaries to tackle; however, in my opinion it is secondary in importance to his second point — which is implicit in his paper. This is that the Insurance Industry and the Reinsurance Industry must spend much more time and effort in the field of increased limits exposure. This is an area which is already of significant importance and it is growing at an accelerated rate. Both buyers and sellers of insurance and reinsurance must become better acquainted with the price and capacity relationship which is a significant part of the Excess Limits Area. Until this groundwork is laid the acceptance of rating techniques, no matter how elegant or rational, will be difficult to come by.

DISCUSSION BY J. ROBERT HUNTER

THE MISUNDERSTANDING

"Dear Prudence, won't you open up your eyes?"
— John Lennon

The making of rates for increased limits of liability is not, as Mr. Lange points out, given coverage in the *Proceedings* even in proportion to its importance as a premium-producing element in the overall structure of our business. Therefore, not only are executives and underwriters confused by the available experience, but also many actuaries are drawing wrong conclusions. There has been and is much ado about "gravy" in the increased limits factors, but this may well be due to a misunderstanding of long term loss development, different trend, and different credibility criteria (discussed below), as these elements are lost in the unstratified calendar year result. If it does nothing else, Mr. Lange's paper serves as an eye-opener for those in prudent management yet capable of eye-opening. As an aside, this eye-opening process seems to have occurred in the reinsurance area, as evidenced by a contraction, from 1966 to 1968, of 11% in reinsurance company countrywide automobile bodily injury premiums earned, while combined stock and mutual premiums earned increased 18%.[1] Capacity, anyone?

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[1] New York Insurance Department's "1968 Loss and Expense Ratios," page 110. It is recognized that the reinsurance premium split may be misleading, but these data should be indicative of a bad situation.

The author touches on many aspects of difference between basic limits and increased limits. A review of current Mutual Insurance Rating Bureau and Insurance Rating Board combined private passenger car bodily injury data highlights the differences that Mr. Lange notes:

	$10,000/20,000 Limit	Excess of $10,000/20,000
Loss development factor (15 to 63 months)[2]	1.13	1.71
Trend factor (Annual change)[3]	+6%	+15%
Mayerson-Jones-Bower 100% credibility point[4]	5,098 claims	6,972 claims[5]

The rating organizations now reflect claim frequency in their private passenger car ratemaking procedure. This usually results in a downward adjustment in rate level. As Mr. Lange properly points out, inflation adds claims to the excess limits area and this development must be considered in establishing increased limits rates. Additionally, accident frequency data (Motor Vehicle Department generated) in many states trend upward or only slightly downward, while insurance generated claim frequency data trend more sharply downward. Perhaps the impact of the Safe Driver Insurance Plan, and advance payments, come into play in this apparent contradiction. This writer doubts that claim frequency is downward at higher levels of coverage. While the data on the following page are fragmentary, they may be indicative of this situation (particularly when we compare like calendar quarters).

These data are for countrywide private passenger automobiles (bodily injury) for all companies reporting to the Mutual Insurance Rating Bureau and are available for a limited number of quarters only. It is obvious that many more data are needed in this regard and much more work will have to be done in studying the frequency of loss at higher limits.

In discussing frequency trends, consideration should be given to the increasing percentage of insureds carrying higher limits.

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[2] See Exhibit A. Note: Of particular interest is Sheet 4 showing how loss development increases as size of loss increases. Data of this sort might well be used to prove that immaturity of case losses rather than trend is the cause of loss development.

[3] See Exhibit B.

[4] See Exhibit C.

[5] Total limits requirement.

Quarter	All Paid Claims	Over $5,000	Over $10,000	Over $15,000
1st '68	36,281	1,317	498	Not Available
2nd '68	36,948	1,436	504	197
3rd '68	33,684	1,223	314	164
4th '68	35,766	1,445	302	177
1st '69	37,181	1,613	471	244
2nd '69	39,083	1,763	539	267
3rd '69	33,526	1,408	410	192
Average Change of like Calendar Quarters	+2.6%	+20.1%	+10.7%	+26.3%

I will not attempt to quantify the difference in those interconnected elements of risk and reinsurance expense, but it is obvious that Mr. Lange is correct in stating that high limits require more consideration of these elements so that the separate profit and contingency portions of the profit and contingency factor should be increased as the layer of coverage under consideration increases.

THE INNOVATION

"Give me the benefit of your convictions, if you have any, but keep your doubts to yourself, for I have enough of my own."
— Goethe

So far, Mr. Lange and I are in complete agreement. I have not yet touched upon the innovation in Mr. Lange's paper, namely, the use of a ratio of increased limits losses for a given policy limit to corresponding basic limits losses for the same limit. Mr. Lange rejects the use of the loss ratio approach because of the difficulty of obtaining accurate premium. Also, he states, "Since increased limits charges are a function of basic limits rates, the isolated fact that increased limits experience is good or bad does not tell the ratemaker whether or not the relationship between increased limits and basic limits rates is correct." The pure premium approach also is rejected, Mr. Lange concluding that there would have to be a great number of breakdowns into class and territory, leading to a credibility problem, and that "the pure premium approach is not particularly convenient for testing

the present rating procedure in which increased limits charges are expressed as a function of basic limits rates."

I do not advocate the pure premium approach either, but fail to see why splits by classification and/or territory would be needed. Certainly any increased limits review would be on a multi-state basis and territory/class differences between basic and increased limits could be reflected by a study of average rates. Nor am I convinced that the problem of testing the present rating procedure couldn't be overcome.

The loss ratio approach was utilized by all rating organizations in establishing increased limits tables (until the Insurance Rating Board utilized a losses to losses approach similar to that suggested by Mr. Lange; M.I.R.B. continues to use a loss ratio approach). The advantage of the loss ratio method (which allows an historical review when adjusted for increased limits table changes) is that it spreads, on a multi-state basis, the impact on excess level of undershooting basic limits rate level needs. Longer periods of review are important, due to the immaturity of loss statistics for excess limits and the credibility problem discussed briefly above.

Regarding Mr. Lange's criticisms of the loss ratio approach, I would make two observations:

1. Premiums can be accurately determined, at least to a degree sufficient for these reviews, through the use of sample distributions.[6]

   These sample premiums can lead to calculations by layer and can, therefore, be used for table-slope testing, as well as rate level requirement determination.

2. Mr. Lange's concern with relationships to basic limits rates can be negated by proper reflection in the loss ratio of the differences discussed above. This is a most curious concern of Mr. Lange, since he applies his loss-to-loss ratios to basic limits charges without regard to adequacy of basic limits rate or how the basic limits rate is determined. (As Mr. Harwayne points out, New York and other states utilize voluntary and assigned risks combined experience,

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[6] I have always felt that attempting to develop rates to apply in the future, with the precision of N.A.S.A.'s landing men on the moon, was an exercise worthy of a Matt Rodermund playlet. Unfortunately, actuaries, perhaps under regulatory pressures, tend to quibble about matters which fail to change rounded rates.

which would require special handling under the losses to losses technique.) The loss ratio for basic limits is available at the time increased limits are reviewed, thereby enabling testing of adequacy of overall basic limits rates as well.

AN APPROACH

"We dance 'round in a ring and suppose
But the Secret sits in the middle and knows."
— Robert Frost

I would suggest that Mr. Lange's loss to loss technique and the loss ratio technique can be used to complement each other since fallacies of each method do not seem to overlap to any significant degree. A sort of "triangulation" toward an optimum answer is necessary, particularly in view of the subjective nature of the reflection of risk and reinsurance expense into these low frequency, high severity areas.

THE AUTHOR

"Still there are some who ask why,
Who want to know, who dare to try.
Every now and then we meet that kind of man"
— Rod McKuen

Mr. Lange has done us all a vital service in bringing this misunderstood area of the ratemaking process into clearer perspective. As always, his thoughts are stimulating and perceptive. He has opened many unexplored areas to our attention. For example, average claim costs vary considerably by state, territory, and classification. I feel intuitively that the closer the average claim cost to the arbitrary basic limits cut-off point, the more likely is an increased limits loss. Should the tables of increased limits factors be reflective of this?

I hope that the many unanswered questions pertaining to this topic of growing importance will prompt further papers by the members of the Casualty Actuarial Society.

AUTOMOBILE LIABILITY INSURANCE - PRIVATE PASSENGER CARS

BASIC LIMITS LOSS DEVELOPMENT FACTORS FOR STATES
WITH 5/10 FINANCIAL RESPONSIBILITY LIMITS

All Companies Reporting
to M.I.R.B. and I.R.B.

BODILY INJURY 5/10 BASIS

Accident Year	5/10 Basic Limits Incurred Losses As Of:			Loss Development Factors:		
	15 Months	27 Months	39 Months	15 to 27 Months	27 to 39 Months	15 to 39 Months
1962	$	$ 96,941,033	$ 96,812,668		.999	
1963	94,019,913	100,084,665	100,763,439	1.065	1.007	
1964	98,527,418	104,710,935	106,187,847	1.063	1.014	
1965	75,030,511	81,092,490		1.081		
Average:				1.070	1.007	1.077

	Total Limits Incurred Losses As Of:			Loss Development Factors:		
	39 Months	51 Months	63 Months	39 to 51 Months	51 to 63 Months	39 to 63 Months
1960	$	$129,922,355	$130,184,004		1.002	
1961	113,364,787	113,687,692	113,272,224	1.003	.996	
1962	101,814,598	102,053,766	102,176,432	1.002	1.001	
1963	115,085,286	115,277,011		1.002		
Average:				1.002	1.000	1.002

Loss Development Factors

39 to 63 Months   = 1.002
27 to 63 Months = 1.002 x 1.007 = 1.009
15 to 63 Months = 1.002 x 1.077 = 1.079

Note: Losses Include Allocated Loss Adjustment Expenses.

AUTOMOBILE LIABILITY INSURANCE - PRIVATE PASSENGER CARS

BASIC LIMITS LOSS DEVELOPMENT FACTORS FOR STATES

WITH 10/20 OR HIGHER FINANCIAL RESPONSIBILITY LIMITS

**All Companies Reporting
to M.I.R.B. and I.R.B.**

BODILY INJURY LOSS DEVELOPMENT

	10/20 Basic Limits Incurred Losses As Of:			Loss Development Factors:		
Accident Year	15 Months	27 Months	39 Months	15 to 27 Months	27 to 39 Months	15 to 39 Months
1962	$	$239,594,301	$243,057,453		1.014	
1963	256,073,734	279,709,757	286,742,796	1.092	1.025	
1964	274,468,426	302,931,061	311,433,448	1.104	1.028	
1965	289,992,763	326,196,959		1.125		
Average:				1.107	1.022	1.131

	Total Limits Incurred Losses As Of:			Loss Development Factors:		
	39 Months	51 Months	63 Months	39 to 51 Months	51 to 63 Months	39 to 63 Months
1960	$	$247,318,893	$246,518,246		.997	
1961	270,457,860	269,425,481	268,946,377	.996	.998	
1962	286,318,914	288,032,354	287,904,620	1.006	1.000	
1963	303,436,675	306,173,831		1.009		
Average:				1.004	.998	1.002

Loss Development Factors

39 to 63 Months   = 1.002
27 to 63 Months = 1.002 x 1.022 = 1.024
15 to 63 Months = 1.002 x 1.131 = 1.133

Notes: Losses include allocated loss adjustment expenses.

AUTOMOBILE LIABILITY INSURANCE - PRIVATE PASSENGER CARS

EXCESS LIMITS LOSS DEVELOPMENT FACTORS

BODILY INJURY

All Companies Reporting
to M.I.R.B. and I.R.B.

Accident Year	Excess Losses Over 5/10 Limits As Of:			Loss Development Factors:		
	15 Months	27 Months	39 Months	15 to 27 Months	27 to 39 Months	15 to 39 Months
1962	$	$13,029,499	$14,286,025		1.096	
1963	12,433,974	14,837,950	15,957,400	1.193	1.075	
1964	9,044,834	12,373,901	12,670,490	1.368	1.024	
1965	8,785,610	11,426,655		1.301		
Average:				1.287	1.065	1.371

39 to 63 Months ∅   = 1.002
27 to 63 Months = 1.002 x 1.065 = 1.067
15 to 63 Months = 1.002 x 1.371 = 1.374

Accident Year	Excess Losses Over 10/20 Limits As Of:			Loss Development Factors:		
	15 Months	27 Months	39 Months	15 to 27 Months	27 to 39 Months	15 to 39 Months
1962	$	$24,511,252	$26,303,647		1.073	
1963	16,703,958	24,728,882	27,683,362	1.480	1.119	
1964	19,119,158	28,154,935	32,699,335	1.473	1.161	
1965	22,172,172	35,863,131		1.617		
Average:				1.523	1.118	1.703

39 to 63 Months ∅   = 1.002
27 to 63 Months = 1.002 x 1.118 = 1.120
15 to 63 Months = 1.002 x 1.703 = 1.706

∅ Developments beyond 39 months are on a total limits basis probably
understating the excess limits results.

Note: Losses Include Allocated Loss Adjustment Expenses.

AUTOMOBILE LIABILITY INSURANCE - PRIVATE PASSENGER CARS

Bodily Injury Excess Limits Loss Development Factors

For States With 10/20 Or Higher Financial Responsibility Limits (Excluding New York)

All Companies Reporting
to M.I.R.B.   VOLUNTARY RISKS

| Accident Year | Incurred Losses As Of: | | | Loss Development Factors: | | |
	15Months	27 Months	39 Months	15 to 27 Months	27 to 39 Months	15 to 39 Months
Losses Over 0/0 Limits (Total Limits)						
1963	$ --	$31,802,309	$32,497,646	--	1.022	
1964	32,909,330	36,985,051	37,772,132	1.124	1.021	
1965	32,538,171	37,804,817	39,596,501	1.162	1.047	
1966	34,161,919	41,104,051	--	1.203		
Average:				1.163	1.030	1.198
Losses Over 10/20 Limits						
1963	--	3,017,011	3,431,223	--	1.137	
1964	2,242,916	3,265,414	3,644,331	1.456	1.116	
1965	2,219,080	3,507,043	3,993,185	1.580	1.139	
1966	2,744,920	4,275,669	--	1.558		
Average:				1.531	1.131	1.732
Losses Over 25/50 Limits						
1963	--	719,575	996,429	--	1.385	
1964	469,151	787,543	980,992	1.679	1.246	
1965	308,100	718,428	849,044	2.332	1.182	
1966	734,851	1,213,488	--	1.651		
Average:				1.887	1.271	2.398
Losses Over 50/100 Limits						
1963	--	137,500	183,750	--	1.336	
1964	70,000	64,063	124,063	.915	1.937	
1965	10,000	105,000	115,300	10.500	1.098	
1966	179,500	317,941	--	1.771		
Average:				4.395	1.457	6.404

Notes: 1. Losses Exclude Unallocated Adjustment Expenses.

2. Charting these factors is a recommended and enlightening exercise.

AUTOMOBILE BODILY INJURY LIABILITY INSURANCE

IMPACT OF RECENT CLAIM COST TRENDS ON

$10,000, TOTAL LIMITS AND EXCESS LIMITS

Year and Quarter	Adjusted ∮ Line of Best Fit $10,000 Limit	Adjusted ∮ Line of Best Fit Total Limits	Contribution from Excess of $10,000
1965 3	$ 911.75	$ 981.76	
1965 4	929.55	1,005.20	
1966 1	947.35	1,028.64	
1966 2	965.15	1,052.08	
1966 3	982.95	1,075.52	
1966 4	1,000.75	1,098.96	
1967 1	1,018.55	1,122.40	
1967 2	1,036.35	1,145.84	
1967 3	1,054.15	1,169.28	
1967 4	1,071.95	1,192.72	
1968 1	1,089.75	1,216.16	
1968 2	1,107.55	1,239.60	
1968 3	1,125.35	1,263.04	
1968 4	1,143.15	1.286.48	
1969 1	1,160.95	1,309.92	
1969 2	1,178.75	1,333.36	
Last Point on Line of Best Fit	$1,178.75	$1,333.36	$ 154.61
Annual Increment	$ + 71.20	$ + 93.76	$+ 22.56
Annual Percent Change	+ 6.0%	+ 7.0%	+ 14.6%

∮ Based on Seasonally Adjusted Quarterly Data of All Companies
   Reporting to the Mutual Insurance Rating Bureau and the
   Insurance Rating Board.

<div align="right">

Exhibit C

Part 1

</div>

CREDIBILITY AT VARIOUS CUT-OFF POINTS

$5,000 B.I., $10,000 B.I., TOTAL LIMITS B.I.

Utilizing the Mayerson-Jones-Bowers procedure*
as shown in Parts 2 and 3, we find that:

Cutoff Point	Number of Claims Required for Full Credibility (5%k, 90%P)
$5,000 B.I.	3,931
$10,000 B.I.	5,098
Total Limits B.I.	6,972†

It is noted that .8% of the claims in the size of claim data for 10/20 states exceed $10,000. The impact of reflecting their actual value (in lieu of $10,000 for each such claim) is significant thus indicating an extremely high credibility criteria for these claims.

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* Mayerson, A., Jones, D., and Bowers, N., "On The Credibility of the Pure Premium," *PCAS* Vol. LV, Page 175.
† Based on 10/20 states using a calculation identical to that specified in this exhibit.

Exhibit C
Part 2

PRIVATE PASSENGER AUTOMOBILE LIABILITY INSURANCE

Derivation of 100% Credibility Criteria

Based on Countrywide Excluding New York Size of Claim Data
of All Companies that Filed with the Mutual Insurance Rating Bureau
and the Insurance Rating Board — 1963 Call

| Interval | Bodily Injury (All States) | | | |
	No. of Claims f	Losses fx	Average Claim Cost x	x^2
$ 1 - $ 24.99	48,686	$ 657,893	$ 13.51	183
25 - 49.99	36,911	1,183,711	32.07	1,028
50 - 99.99	42,685	2,783,800	65.22	4,254
100 - 249.99	64,932	9,909,765	152.62	23,293
250 - 499.99	50,312	17,359,394	345.03	119,046
500 - 999.99	56,124	37,990,345	676.90	458,194
1,000 - 1,999.99	37,598	51,229,987	1,362.57	1,856,597
2,000 - 2,999.99	15,832	37,535,893	2,370.89	5,621,119
3,000 - 3,999.99	8,417	28,383,996	3,372.22	11,371,868
4,000 - 4,999.99	4,808	21,025,252	4,372.97	19,122,867
5,000 - 9,999.99	8,770	58,351,870	6,653.58	44,270,127
10,000 (Limit)	3,204	32,040,000	10,000.00	100,000,000
TOTAL	378,329	298,451,911	788.87	

Results: *10/20 States (Bodily Injury)*
$\lambda = $ 5,098 * Claims (5% k, 90% P)
$ = $ 49,908   Claims (2.5% k, 99% P)

5/10 States (Bodily Injury)
$\lambda = $ 3,931 Claims (5% k, 90% P)
$ = $ 38,298 Claims (2.5% k, 99% P)

* See Part 3 for sample derivation of 5,098 claims for full credibility at 90% probability of being within 5% of the expected value.

Exhibit C
Part 3

Derivation of Full Credibility for
$10,000 Bodily Injury for 5% k, 90% P

(Based on the Paper presented at the Fall, 1968 Meeting of the C.A.S. by
Messrs. Mayerson, Jones, and Bowers — "On The Credibility of the Pure
Premium," *PCAS* Vol. LV, page 175)

Basic equation (See page 181 of *PCAS* Vol. LV):

$$k\lambda = Z_c\sqrt{\lambda}\,\sqrt{1+\frac{\mu_2}{\mu^2}+\frac{Z_c^2-1}{6}\cdot\frac{1+3\frac{\mu_2}{\mu^2}+\frac{\mu_3}{\mu^3}}{1+\frac{\mu_2}{\mu^2}}}$$

Where: k = Maximum departure from expected.

λ = Number of claims for 100% credibility.

Z_e = The (100_c) percentile of the standard normal distribution.

μ = Mean = Expected value of x.

μ_2 = Second moment about the mean (μ_2' — about origin).

μ_3 = Third moment about the mean (μ_3' — about origin).

P = Probability of being within k percent of the expected value.

Since, under standard notation:

$\mu_2 = \mu_2' - \mu^2$, and

$\mu_3 = \mu_3' - 3\mu\mu_2' + 2\mu^3$

It follows that:

(A) $1+\frac{\mu_2}{\mu^2} = \frac{\mu_2'}{\mu^2}$, and

(B) $1+3\frac{\mu_2}{\mu^2}+\frac{\mu_3}{\mu^3} = \frac{\mu_3'}{\mu^3}$

Then, for $10,000 Bodily Injury (see Part 2):

$\mu_2' = 2,877,265$ $\mu^2 = 622,316$

$\mu_3' = 18,073,982,000$ $\mu^3 = 490,926,423$

Exhibit C
Part 3 (cont.)

By substitution, for *(5% k, 90% P)*

$$(A) = 4.623 \quad \text{and} \quad (B) = 36.816$$

Therefore: $$k\lambda = Z_c\sqrt{\lambda} \; \sqrt{4.623} + \frac{Z_c^2 - 1}{6} \cdot \frac{36.816}{4.623}$$

(Let $\sqrt{\lambda} = y$) $\quad .05y^2 - 1.645y\sqrt{4.623} - .2843 \cdot 7.964 \doteq 0$

$$5y^2 - 353.7y - 226.42 = 0$$

$$y^2 - 70.7y - 45.28 = 0$$

$$y = 71.4$$

$$y^2 = \lambda = 5,098 \text{ Claims}$$