formulae (2) and (3) are simple mathematical derivations from (1). It is true that inflation will effect x to an extent dependent upon whether x is partially paid, fully paid, or fully outstanding. This fact though, is automatically reflected in the final value of x and thus, in x_s/x_v. If x is fully paid at valuation date, then x_v equals x_s, and x_s/x_v equals 1.00. Similarly, if x is partially paid at valuation date, then x_s/x_v will presumably be smaller than if no payments have been made at valuation date.

Of course, the above exercise in elementary mathematics simply confirms Mr. Cook's conclusion that there is no overlap or duplication in trend and development factors, and all claims that there is have been based on specious reasoning. The chief value of the above model is that the decisive elements in the development of the conclusion are specified. That is, if there is to be a challenge to the conclusion, then that challenge must center on the clearly defined assumption or on the formula (1) representation of it. If ever there was a question that the Society could state an official opinion on, this would seem to be it. Perhaps the overlap fallacy can be finally laid to rest, and the full value of Mr. Cook's contribution realized.

DISCUSSION BY D. R. UHTHOFF

Possibly Mr. Cook's strongest motivation for writing this paper was the increasingly householdish term "overlap." Discussions of loss development factors relative to other type factors intended to project for cost or frequency trends often have been colored by concern and confusion, whether there might be overlap between these. That is, to the extent development factors may at least partially arise from inflationary or otherwise assignable cost trending influences, and these same influences also may be applied as rate level trending factors, there may be duplicative effects. If Mr. Cook were to accomplish nothing other than a clarification of the muddiness of these discussions, which he has done, his paper would be a worthwhile addition to our *Proceedings*; he has, in fact, proceeded further to the examination of quite a few other concepts necessary to intelligent handling of various kinds of experiences and approaches useful for rate level work.

I don't think the reader should anticipate a neat do-it-yourself manual for budding ratemakers by which many things are set forth in ready reference form calculated to quell all future doubts about how to handle variations on the theme of setting up rate level calculation procedures. But the author has provided interestingly readable discussions conducive to logical

thinking about the kinds of problems that arise in our modern era of necessity that rate levels must contemplate rapidly changing cost levels. He does not pretend to perfection in all his logic; he may lack complete generality in some model-type explorations. But I suspect he may have deliberately designed his paper that way, so as to excite discussion, to invite probing thoughts. However intended, I think the combination of everything works out well, and we now have some literature in our *Proceedings* concerning this subject about which, Mr. Cook remarks, we have been lacking.

Some of his definitions of principles are well set forth: "a trend factor is any index which measures changes over time;" "a calculated past ratio of mature to immature data is called a loss development factor;" and "by their nature, the loss development factors we calculate are always the ones that *would have been* right in the past, and they are therefore an accurate measure of the future development of present losses only if the present outstanding cases have the same degree of reserve adequacy as did the past ones on which the factors are based."

If the reader may wonder who there may be who doesn't already understand those things, he must nevertheless concede their proper places in a narrative-type analysis such as Mr. Cook has provided. The student particularly will find this paper helpful, as will also the more advanced ratemakers who often can have more fun than anybody kicking some of these questions around.

A feature of the paper is the neat arrangement of time periods in which development factors and trend factors can be said to operate to the complete exclusion of the other, therefore without duplication or overlap. As a general expression of his time period arrangements, I would suggest the following be labeled as *time period number one,* the time span between the center of the experience period upon which the rate level is based, and the center of the forthcoming exposure period dependent upon the intended issue dates of policies to be written under the new rates. *Time period number two* would extend from, or beyond, the central point of occurrence of losses of the future policy effective period to the date necessary for losses to acquire maturity.

Mr. Cook's presentation prefers to apply the cost trending or projection factor to the *period number one,* in effect saying that the cost level of the experience period, before using in rate level, must be transferred to the future cost level of the effective policy issued period. He then would say

that the center of loss occurrence of the policy issue period, the *period number two*, is the time in which we must apply the development factor to develop that future accident cost to an ultimate developed cost.

If the periods are handled carefully, and properly coincide with the timing of the trending and development factors, the paper demonstrates that all this holds together well and there is no difficulty with overlapping of factors and timing.

I do feel some concern, though, that assignment of the development step to the period of aging beyond the future accident dates makes the chronological picture seem a bit tenuous, more extended than necessary. The same result might be achieved by this logic: the underdeveloped or immature loss data of the experience basis of rates needs adjustment, by the development factors, to accomplish maturity. This development is, of course, according to the pattern of previous development obtainable from earlier losses of like stages of aging.

Thus, we simply state that immature losses must be developed or made mature before using them in a rate level calculation. They then may be projected or cost trended by appropriate means to a future policy issue loss point.

In this way, we avoid the lengthy visualization of development factors being applied to an aging period, the *time period number two,* of quite a few years in the future, development factors which have been obtained from aging processes of a few years in the past. Actually, we need not assign the development factors to any particular aging future; they simply are required as an elementary step in the process of basing a calculation upon mature losses, whether these have become mature through permission of longer development time or whether they are to be synthetically matured by application of development factors. The only requisite, of course, is that the aging period from which development factors were derived is equivalent, and similar in characteristics, to the aging period contemplated in the process of creating mature from immature losses.

Concerning the possibility that dependence upon development factors can be minimized by using the more mature experience — more mature through the simple process of aging — the paper brings out an interesting point, that there then must be more reliance placed upon cost trending factors. That is, to the extent that responsiveness is not supplied through cur-

rency of losses, it must be supplied by factors intended to place these losses on a current basis, in addition to those factors necessary to go from the current to a future basis. In this connection, it is interesting to note that when the accident year system of ratemaking was adopted some years ago, and one of its major advantages was stated to be the improvement in responsiveness, as contrasted to policy year experience, quite a bit of reliance was intended to be placed upon development factors. In theory this was fine, but in practice there has been a retreat, in that too much reliance on a factor of sometimes questionable virtue makes it sensible to use somewhat older experience, despite the need for placing more reliance upon trending factors.

There seems a privilege of reviewers these days to advance at least one of their own pet peeves or inventions. Actually, Mr. Cook has led me quite by the nose to make my offering, which is that we strongly entertain a concept of collecting data by which formula-type incurred losses may be devised. A ratemaker, in applying development factors, is following a method depended upon, at least as an alternate method, by many formula reservers within company shops. And as Mr. Cook mentions the possibility of using paid loss projection factors, again he is referring to another method used by companies for calculating formula incurred losses and, thereupon, reserves.

The obvious question with which the ratemaker must struggle is accuracy of reserves. He is not looked upon as a villain in prudently contemplating inaccuracy possibilities in reserve portions of bureau statistics, especially in these days of rapidly changing cost levels. Thus the ratemaker, after he combines company-by-company experiences in the initial step of accomplishing credible bases, then uses his statistical histories for the development factors by which the accuracy of aggregates may be improved. By this step the ratemaker *has constructed* incurred losses. He has used a simple formula system.

For many lines a development factor system is crude, as compared to more sophisticated systems some companies are using and more fundamentally based upon payment histories, rather than gross aging statistics. I am not suggesting that each company pursue formula methods for bureau submissions. I am suggesting that formula methods be devised for bureau application to aggregates, and that the formula methods so devised then will point to the kinds of statistics each company should submit for implementation, on broad collective bases, of these formula methods.

I am sure many company actuaries would be happy to contribute their perhaps painfully gained expertise in devising their own company formula methods.

When we look at the magnitude of some development factors and the apparent trends in these development factors themselves, and shudder at the possibility that our current rate levels may continue inadequate as these factors must lag with respect to current reserving and with respect to current pressures upon managements by which reserves may be deemed, to say the least, no more than those required for minimum necessity, it seems high time that more sophisticated methods of loss experience valuations be adopted. The individual statistical agencies cannot establish and enforce reserving disciplines within company offices; this would not only be impracticable, but would usurp management and company functions. But isn't it true that the ratemaker assumes something less than his responsibility in not having adequate assurance his loss experiences are as accurate as good actuaries might be able to make them?

Perhaps I would like most of all to applaud Mr. Cook for getting at a troublesome problem in a problem-solving way; he incidentally stirs one to some peripheral thinking too. Shouldn't we, as actuaries, presumably responsible to our function, be vitally concerned with anything and everything about rate levels?

DISCUSSION BY MAVIS A. WALTERS

Charles Cook's paper on trend and loss development factors is a valuable document for any actuary who finds himself or herself in the position of trying to explain ratemaking techniques and procedures to laymen or nontechnicians. He defines clearly and concisely the terms "trend" and "loss development," and these definitions help to distinguish the two concepts. The definitions are followed by a statement of the traditional "overlap" fallacy; and in fact, Mr. Cook summarizes the argument much more cogently than some of its chief proponents. He then proceeds to refute the position quite simply and directly by discussing the purpose of the rates, i.e. to provide adequate funds to settle claims which result from accidents. The problems arise from the very simple fact that in the ratemaking procedure the actuary must make adjustments on the experience of the past in anticipation of changes in the future. From a theoretical point of view this paper pre-