factor first. If this is done, the resulting answer is what we expect the losses to be which result from accidents written at the new rates, at the same level of maturity as our loss experience. Since these trended losses are immature we must rely on the past relationship of mature to immature losses to bring these losses to the proper level of maturity. The loss development factor picks up in time where the trend factor stops.

There is, therefore, no overlap.

Mr. Cook's exposition of the overlap question is quite clear and readily understandable to the layman. This section of his paper could be extracted and used as an appendix to an automobile rate filing in those states which have raised the overlap question.

DISCUSSION BY ROBERT W. STURGIS

I intend no disparagement whatsoever of the body of Mr. Cook's paper when I say that one of the most illuminating parts is his introduction. As he points out, there *are* misconceptions, misunderstandings, and confusions; and I can testify to the fact that at least one actuary accepted the trend — development overlap fallacy. In the face of all this, it is indeed surprising that so little has been written on this subject. Hopefully, Mr. Cook's work will be the spur to further scholarly discussion.

Why is this subject so complex? How is it that different clear-thinking professionals can come up with diametrically opposite conclusions? When I finished reading Mr. Cook's arguments I was persuaded that there was no overlap. However, this conviction seemed precarious: I had the unsettling feeling that if I were to read counter arguments, I could be swayed to the other side. I have always waded through logical discourses on trend and development using a time-line visual aid as my guide, but always I wound up worried that I was comparing apples to oranges: effective, expiry, accident, and valuation dates; arising, paid, outstanding, open, and closed claims; inflation acting on past accidents and on future accidents; development of reserves and of number of claims. Of course, it is actuarially unsound to compare apples and oranges, but accepted procedure to relate quarts and liters, feet and meters. The soundness of these relationships, however, makes them no less complex. I was encouraged when I read, "It may clarify the point to build a model." Determined to master the mathematics of the algorithm, I surged ahead, but alas, all I found was the familiar visual aid

time-line. I determined to build the model on my own; surprisingly, the model turned out to be simplicity itself.

The question is: To what extent should inflationary trends be reflected in ratemaking? To answer this question we need only consider the effect of inflation on claims; all other variables may be ignored or assumed constant. Consider known claim x, to be used as a value predictor for future claim y. Since we aim to isolate the inflationary factor, we make the following assumption.

Assumption: Claim y occurs m months after claim x, and its final value differs from that of x only by a factor reflecting m months of inflation.

Let: x_s and $y_s =$ final values of claims, and

$t =$ monthly inflation operating on claims x and y, expressed as a decimal.

Now the value of y at settlement, y_s, will be predicted by x_s as follows:

$$(1) \qquad y_s = x_s(1 + t)^m$$

In practice we often don't know the settlement value of x. Multiplying the right side of equation (1) by (x_v/x_v),

$$(2) \qquad y_s = x_s(1 + t)^m \, \frac{x_v}{x_v} \quad , \quad \text{or}$$

$$(3) \qquad y_s = x_v(1 + t)^m \, \frac{x_s}{x_v} \quad .$$

Thus, the final value of y will equal the value of x at valuation date, x_v, multiplied by a full trend factor for m months, $(1 + t)^m$, and by a full loss development factor, (x_s/x_v). It is also clear that it does not matter if a claim is closed or paid, prior to the valuation date. That would be a special case of the loss development factor where x_v equals x_s, and the factor equals unity. In practice of course, we don't know the actual development of our x claims, and we estimate it by using past x_s/x_v values.

If we had chosen to use a straight line rather than an exponential trend factor, equation (3) would be the same except that the trend factor would be $(1 + mt)$.

The suggestion that trend should be applied only to the paid portion of incurred losses has always seemed to me as merely a restatement of the overlap argument. Clearly, this argument has no effect on formula (1), and

formulae (2) and (3) are simple mathematical derivations from (1). It is true that inflation will effect x to an extent dependent upon whether x is partially paid, fully paid, or fully outstanding. This fact though, is automatically reflected in the final value of x and thus, in x_s/x_v. If x is fully paid at valuation date, then x_v equals x_s, and x_s/x_v equals 1.00. Similarly, if x is partially paid at valuation date, then x_s/x_v will presumably be smaller than if no payments have been made at valuation date.

Of course, the above exercise in elementary mathematics simply confirms Mr. Cook's conclusion that there is no overlap or duplication in trend and development factors, and all claims that there is have been based on specious reasoning. The chief value of the above model is that the decisive elements in the development of the conclusion are specified. That is, if there is to be a challenge to the conclusion, then that challenge must center on the clearly defined assumption or on the formula (1) representation of it. If ever there was a question that the Society could state an official opinion on, this would seem to be it. Perhaps the overlap fallacy can be finally laid to rest, and the full value of Mr. Cook's contribution realized.

DISCUSSION BY D. R. UHTHOFF

Possibly Mr. Cook's strongest motivation for writing this paper was the increasingly householdish term "overlap." Discussions of loss development factors relative to other type factors intended to project for cost or frequency trends often have been colored by concern and confusion, whether there might be overlap between these. That is, to the extent development factors may at least partially arise from inflationary or otherwise assignable cost trending influences, and these same influences also may be applied as rate level trending factors, there may be duplicative effects. If Mr. Cook were to accomplish nothing other than a clarification of the muddiness of these discussions, which he has done, his paper would be a worthwhile addition to our *Proceedings*; he has, in fact, proceeded further to the examination of quite a few other concepts necessary to intelligent handling of various kinds of experiences and approaches useful for rate level work.

I don't think the reader should anticipate a neat do-it-yourself manual for budding ratemakers by which many things are set forth in ready reference form calculated to quell all future doubts about how to handle variations on the theme of setting up rate level calculation procedures. But the author has provided interestingly readable discussions conducive to logical