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NOTICE

"And certainly if it can be demonstrated that you are less apt to have a loss than I, you would be the giver of pure charity and I would be the taker if we pooled our hazards on a fifty-fifty basis. As much as is possible the predisposition to loss is a proper subject for fair discrimination; only the operations of chance are the proper subject for averaging."

— Dudley M. Pruitt

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TREND AND LOSS DEVELOPMENT FACTORS

CHARLES F. COOK

"The mere fact that something has happened a certain number of times causes animals and men to expect that it will happen again."

— Bertrand Russell

During the past year or two it has become apparent that there exist widespread misconceptions about trend and loss development factors. Rather than surface misunderstandings, they appear to result from fundamental confusion between the data base from which the factors are derived and the purpose which they serve. These are essentially laymen's errors, of the kind one might expect to fade away after brief consideration, but they have been surprisingly persistent. Indeed, I have found in private conversations that the overlap fallacy has been uncritically accepted even by many actuaries. The problem may be due to a lack of serious consideration of these difficult concepts outside of the adversary proceedings of disputed rate filings; there has been surprisingly limited treatment of them in the *Proceedings*. The purpose of this paper is to try to clarify a few of the problem areas, and if possible to refute some errors. The crucial importance of avoiding unsound concepts of trend was well illustrated two years before this Society was founded:

"The man who has fed the chicken every day throughout its life at last wrings its neck instead, showing that more refined views as to the uniformity of nature would have been useful to the chicken."

— Bertrand Russell, On Induction (1912)

In order to treat the problems in the use of these factors, it is necessary first to define them. The definitions are general, but for simplicity, we will limit the discussion mainly to automobile — it is the line with the most controversial and best developed practices in this area.

Definitions

There exist in our society dynamic forces — economic and demographic among others — which produce measurable changes in insurance experience as time passes. We call these changes trends. In automobile insurance the three trends most generally considered are those of average claim cost, claim frequency, and classification drift (Physical Damage Age & Symbol Groups). A *trend factor is any index which measures changes over time.* Please note that the changing value itself need not be used; any index which will measure it is acceptable.

The first complete report of an accident year is often compiled as early as three months after the close of the year. For many lines of business, especially the "Schedule P" lines — auto B.I., other liability B.I., and workmen's compensation — claim settlements are often long delayed; even at reports substantially later than three months after the close of the accident year (or policy year or calendar year) the estimated cost of many claims may still be very inaccurate and subject to substantial subsequent revisions. For individual claims (by which loss reports for virtually all lines of insurance are submitted to bureaus) subsequent changes cannot be predicted, but in the aggregate there is a pattern of change from report to report, as more claims are paid and estimates of others are improved. The process of change as an accident year matures is called development. *A calculated past ratio of mature to immature data is called a loss development factor.* This type of factor measures phenomena intrinsic to a specific type of claim, and can only be measured by data completely identical except for age.

The Overlap Fallacy

This idea is probably so persistent because it is so well presented by its proponents. Their arguments are generally logical, well-thought-out, and very attractive. The best summary I have seen is by Commissioner Newton I. Steers, Jr. of Maryland.[1] "I find that inflationary forces in our economy do operate during the period of time between the original estimate of loss

[1] Steers, Newton I. Jr., September 30, 1969 disapproval letter to Mr. Bernard I. Farrell, Manager, Insurance Rating Board Central Atlantic Office: second paragraph, page two.

and the final determination or payment of trial loss. Thus the loss development factor already reflects the trend (inflationary) factor. Since the filing applied these duplicative factors successively and thus compounds them, I find that the IRB has not shown that the combined rates thus derived will not be excessive."

Commissioner Steers has correctly noted that in addition to such obviously proper items as IBNR losses, the loss development factor also includes inflationary effects. Because this inflation appears to take place over approximately the same time period as that which underlies the trend factors, it is deceptively apparent that inflation is counted twice — the two factors "overlap." Q.E.D.

This argument itself is clear and logical, but it is based on a fundamental misunderstanding. *What are rates supposed to do?* Are they intended to provide adequate funds to cover the loss costs which apply at the instant accidents occur? Or are they intended to provide adequate funds to *settle* the claims which result from accidents? If you will accept the latter intent as self-evident, then it immediately follows that an adequate rate must include a provision for any inflation which may occur between the date an accident occurs and the date it is settled. Trend factors, however, are only projected to the average expected accident date. Part of this deficiency is taken care of by evaluating claims three months after the close of the experience period. This is nine months after the average accident, and therefore can include a reasonable estimate of future cost increases. The remaining *unanticipated* inflation will tend to be *precisely* the amount included in a loss development factor.

It may clarify the point to build a model, and consider specific dates in a hypothetical rate review. Let us use accident year 1969 experience, for a revision to be effective January 1, 1971. The average date of accident in the experience period would then be July 1, 1969; the average policy effective date under the revised rates will be July 1, 1971 (assuming annual rate revisions); and the average date of accidents covered by these policies will be January 1, 1972 (assuming one-year policy terms). Thus the total average period of time which will elapse between the actual past accidents on which our rates were based, and the future accidents for which the rates must pay, is 30 months. It is clear that in order to be appropriate to the accidents for which they will pay, the rates must be based on accident year 1969 experience projected forward 30 months by trend factors.

Because the latest available report for accident year 1969 will be as of March 31, 1970, it is also clear that losses will be immature and require development to their estimated ultimate disposition level. One part of the factor used to develop these losses will reflect *unanticipated* increases in cost level between the evaluation date and the ultimate settlement date. To simplify the model, assume that *no* future cost increase is considered at the time reserves are set, and that the average settlement will take three years, to an average date of July 1, 1972. Under these assumptions there should be included in the loss development factor a sizable amount to take care of increasing cost level for the 27-month period from March 31, 1970 to July 1, 1972.

The overlap theory (as I understand it) would suggest that the claim cost trend factor reflects inflationary changes for the 30 months from July 1, 1969 to January 1, 1972, and that the loss development factor reflects inflationary changes for the 27 months from March 31, 1970 to July 1, 1972. There is therefore an *overlap* of 21 months, from March 31, 1970 to January 1, 1972, in which the two factors are at least to some extent reflecting the same inflationary changes.

The error here is in treating dates as if they were *absolutes*, when in fact they are only measuring an *interval of time*. Our model has been set up so that the actual absolute dates can be determined.

Consider three time periods:

A. 7/1/69 to 1/1/72; average experience period accident date to average effective period accident date — inflation is measured by the trend factor.

B. 1/1/72 to 10/1/72; first nine months after average effective period accident date — inflation is measured by a part of the changes in cost estimates during the comparable period (from 7/1/69 to 3/31/70) between the average experience accident date and the accident year evaluation date.

C. 10/1/72 to 1/1/75; remaining 27 months after average effective period accident date, up to the average effective period settlement date (which by our assumptions is three years after the average accident date) — inflation is measured by the loss development factor which is based on a comparable 27-month period, running from 9 to 36 months after an earlier group of accidents.

It can be seen in our example that not two but three measures of inflation are applied successively. However, the actual cost changes we are predicting in ratemaking will also occur successively. The final result is exactly correct; we arrive at the precise cost level needed to *settle* claims arising out of accidents covered by policies written during the effective period of our rates. There is no overlap.

In the real world, of course, claim costs do not generally receive the full impact of inflation after the accident date, because many costs are incurred prior to settlement; neither are reserves established without any consideration of future cost increases. But loss development factors, because they are based on comparable prior developments, tend to measure exactly the things that will probably occur after future accidents. Like any estimate, they are subject to random and/or cyclical errors, but if the factors are based on a period long enough to really approach ultimate cost, they are *valid and unbiased;* on the average over a long period, they are equally likely to be too low or too high.

As a final comment on this subject, it should be conceded that a "real" overlap between trend factors and loss development factors could exist. In an Actuaries' Report on automobile ratemaking procedures, commissioned by the Commonwealth of Virginia, Roberts[2] stated: "Actually, however, [capital gains] are already reflected in that the bureaus do not project loss costs to the anticipated average date of payment. In a generally rising stock market, notwithstanding occasional reversals, capital gains have provided a hedge against the effects of inflation in driving claim costs upward between dates of accident and dates of settlement. It may be in recognition of this factor that the industry has not come forward with arguments for corresponding projection of loss costs."

This lengthened period of trend was not seriously suggested by Roberts for actual use in ratemaking — in fact it appeared in his chapter on investment income — but some profit-starved readers must have been tempted to lift it out of its proper context and apply it in their ratemaking process. Fortunately they did not, for if this were done it would be a perfect example of "true" overlap. One part of this cost trend after the accident would be picked up by foresighted reserving, and the entire remainder by the loss

[2] Roberts, Lewis H. *Actuaries' Report to the State Corporation Commission of Virginia.* Published by Woodward & Fondiller. Distributed by the Virginia Bureau of Insurance (August 17, 1966), page 50.

development process. Thus the *entire* trend beyond the average effective period accident date, up to the settlement date, would be redundant.

Loss Development Factors and Inaccurate Reserves

The most frequent misunderstanding about loss development factors is that they *ought to be* 1.00 or less, because companies are legally required to carry reserves adequate to pay all of their outstanding losses. This incorrect inference results from an invalid analogy between reserves reported to bureaus and reserves shown in the annual statement. They are not generally the same. Most bureau statistical plans require unit loss reserves — individual reserves allocated to specific known cases. A company's annual statement loss reserves, however, are required to be adequate in the aggregate, for all cases whether or not known, and need have no relevance to specific cases. If they are produced on a formula basis which does not utilize the data submitted to the bureaus, occasional substantial differences can obviously be expected; even if the annual statement reserves are built from the same data, however, the minimum difference we can expect is the reserve for incurred-but-not-reported cases, and there may in some instances be other similar differences, such as a reopened case reserve or a special reserve to satisfy the 60% loss ratio minimum reserve requirement from Schedule P.

In many cases these reserves can be the difference between adequacy in the annual statement and inadequacy in bureau data. The problem would be much greater if bureau accident year reserves were reported as of December 31. Fortunately, reserves for the casualty lines (where loss development is most severe) are reported as of March 31, allowing three months for late reported claims and unreserved small claims to "develop" within the company. This is not, however, a sufficient time to eliminate IBNR as a problem. For the General Accident Group, some percentages of accident year 1968 losses which were first reported during the period between March 31, 1969 and September 30, 1969 (the latest report available) were:

Automobile B.I.	4.4%
Automobile P.D.	2.1%
Other liability B.I.	12.5%
Other liability P.D.	6.6%
Workmen's compensation	3.2%

While these amounts *by themselves* do not account for the size of recent loss development factors, they are large enough to show that exactly accurate reserves will still develop upward from a first bureau report.

One could argue that these reserves should be spread over specific claims by a factor, and thus included in bureau reports. Besides the fact that this would in practice simply replace bureau-level loss development factors with individual company loss development factors (derived from smaller statistical bases), there are at least four counter-arguments:

1. It might be considered unsound in principle to slightly over-reserve 90% of the cases to compensate for the 10% which will increase sharply.
2. A blanket distribution may not properly reflect individual state differences, and an individual company may not have sufficient stability for state-by-state allocation.
3. There are some theorists who believe that "loading" individual case reserves encourages higher claim costs, by often permitting adjusters to make over-generous settlements "within the reserve."
4. Sudden changes in reporting practice could result in excessive loss development factors during the changeover period.

Loss Development Factors Compound Reserving Errors

In an earlier section of this paper, it was stated that loss development factors, although valid and unbiased in the long run, "are subject to random and/or cyclical errors." It is the purpose of this section to show that these errors can be most severe and to suggest possible improvements. By now the reader is certainly aware that the author has no reservations about the concept of applying loss development factors. However, the method of calculating and applying the factors is open to criticism.

Loss development factors are generally calculated from the actual development, during the past two-year period, of earlier reports of incurred losses. For purposes of analysis, let us consider a simplified model in which losses reach their ultimate level at 27 months, and a single prior year's development from 15 to 27 months is used as the loss development factor for the current year's 15-month report. This model will have twice as much variance (or 1.4 times as much standard deviation) as the real world, due to random errors, and will have a much shorter period for cyclical errors.

For our model, let the actual losses for each year be $1,100,000. In Case I, reported losses at 15 months are consistently $100,000 less than ultimate. In Case II, the reported losses for two of the years are $50,000 less, and then return to their normal level.

Case I:

Year	*Losses Reported at 15 months*	*Actual Ultimate Losses*	*L.D. Factor (based on Prior Year)*	*Estimated Ultimate Losses*
1	$1,000,000	$1,100,000	—	—
2	1,000,000	1,100,000	1.1000	$1,100,000
3	1,000,000	1,100,000	1.1000	1,100,000
4	1,000,000	1,100,000	1.1000	1,100,000
5	1,000,000	1,100,000	1.1000	1,100,000
Average of years 2 - 5		$1,100,000		$1,100,000

Case II:

1	$1,000,000	$1,100,000	—	—
2	950,000	1,100,000	1.100	$1,045,000
3	950,000	1,100,000	1.158	1,100,100
4	1,000,000	1,100,000	1.158	1,158,000
5	1,000,000	1,100,000	1.100	1,100,000
Average of years 2 - 5		$1,100,000		$1,100,775

On the average, as we had expected, the estimated ultimate losses are very close to the actual ultimate losses. It can also be noted that in years 3 and 5, where the actual development follows the same pattern as the previous year (on which the factor is based), the estimated ultimate losses are accurate. However, in years 2 and 4, when there is a change in pattern, the factors are out of phase and miss badly. Furthermore, the error cannot be detected in advance, because when the lower (or higher) first report for year 2 (or 4) first came in, we could not have known whether the ultimate loss level was changing, or only the adequacy level of the first reports.

The pattern of this model does apply, although smoothed a bit, to the real world. By their nature, the loss development factors we calculate are always the ones that *would have been* right in the past, and they are therefore an accurate measure of the future development of present losses *only* if the present outstanding cases have the same degree of reserve adequacy as did the past ones on which the factors are based.

Furthermore, a significant upward change in loss development factors is likely to indicate an inadequacy of company loss reserves, because the other factors (IBNR, etc.) in loss development should tend to be fairly stable over time. If this is so, company reserve tests will deteriorate and bring about individual company corrective action. From these arguments we can put forward two tentative conclusions which will apply if loss development factors increase sharply:

1. That estimated ultimate incurred losses, *and therefore rates,* were inadequate in the recent past because of inadequate loss development factors.
2. That adequacy of reserves will return to approximately its former level, rendering loss development factors, and therefore estimated ultimate incurred losses, *and therefore rates,* excessive.

This excessiveness will approximately equal the former inadequacy and will thus not *in the long run* unjustly enrich the companies, but it is a form of automatic recoupment of past underwriting losses, which is contrary to traditional ratemaking policy.

If it is agreed that this is an undesirable condition, but it is also agreed that loss development factors are a necessary feature for proper ratemaking, what should be done? I am aware of only one discussion in the literature, set forth by Roberts in the work previously cited.[3] In brief, his proposal was to *test* loss development factors by using *paid* accident year losses, developed to ultimate incurred level on a formula basis. These paid loss development factors would be very large for some lines, of course, but if they were adequate they would produce adequate estimates of ultimate incurred losses. A similar but less sophisticated approach is now used in automobile physical damage, where calendar year paid losses are developed to incurred level by a flat historical factor. The advantage of an approach based on the Roberts test formula would be that the loss development factors could be expected to be more stable, even though very large, because judgment estimates of losses would be eliminated except for a final estimate of cases still open after five or more years. There are two apparent disadvantages:

[3] Roberts, Lewis H. *op. cit.*, pages 40 to 47.

1. Although probably smaller than the random errors in reserves, there will still be random variations in rates of payment. Because the experience base will be small and the factors large, each dollar of random variation in the experience will have a substantially greater effect on rate level. An obvious example of this problem would be unusually poor fourth-quarter experience, which would be reflected only slightly in paid losses through March.
2. In the case of a non-random trend in average speed of payment, the method will be far less responsive than the present approach. The most important example of this would be increasing congestion and delays in courts of law.

The first of these objections, as well as the actual magnitude of the basic advantage of stability, could be objectively analyzed by careful research of proper data. It would be a major project, but just might be very rewarding, and should be encouraged. It is doubtful that this approach could safely be used by itself, but it would provide an *objective* check on reserves, *independent* of company reserving practices, as a supplement to the usual calculation of loss development factors.

There is a second approach to stabilizing loss development, which could be applied either by itself or on top of the paid loss approach. Rather than requiring research, it requires changing a basic axiom of ratemaking. This proposal is simply to stop using data as of 15 months. More mature data, at 21 or 27 months, would be less subject to random error and would require much smaller loss development factors. In the paid loss approach, it would significantly increase the volume of the base. Responsiveness would be reduced, but this does not necessarily lead to inadequate rates, because trend factors would be applied for a longer period to compensate for the older experience. A limited form of this approach would be to go back to the previous year only if the latest year would require a very large loss development factor (e.g. greater than 1.20), but this practice would be biased against the companies, because abnormally low loss development factors would be applied while abnormally large ones would not.

Every decision about ratemaking policy ultimately must wrestle with the dilemma of responsiveness versus stability. Both of these proposals would gain stability at the cost of reduced responsiveness, and it is on this question that they should be considered further or rejected. In making this decision, however, one should avoid a habitual reaction in favor of responsiveness at all costs. Our attitudes in this area are largely based on an earlier day, when responsiveness and adequacy were synonymous. Now that both cost and frequency trend factors are generally applied for the full period from average experience date to average insured event date, this relationship no longer has any necessary validity.

Paid Claim Cost Trend Factors Applied to Incurred Losses

This section is intended to lay to rest another widespread fallacy. Briefly stated, it is the idea that a trend factor based on average paid claim costs can only be properly applied to the paid loss portion of incurred losses, but *not* to the outstanding loss portion. Only a few actuaries have been tempted by this concept, but it is "logical" under casual review and very tempting to laymen. There are several major states where it has recently been the required method of applying trend in automobile liability insurance. Let us consider the apparent alternatives to the application of paid claim cost trend to outstanding losses.

Case I: Average *outstanding* claim cost trend factors should be applied to outstanding losses. The most likely source of error in the ratemaking process as a whole is in setting reserves on outstanding claims. This approach compounds such errors. If we over-reserve, the trend factor will also increase, and we will apply a higher factor to already excessive reserves. Conversely, if we under-reserve, it will result in a lower outstanding claim cost trend factor to be applied to already inadequate reserves. This approach is only accurate to the extent that reserve adequacy does not change. This is parallel to the loss development situation described in the previous section, so it is not necessary to further analyze it here. Suffice to say that outstanding claim cost trend, loss development factors, and the basic incurred loss data will all tend to move together, and thus compound any error three times. Reserving is a major problem in itself; loss development makes it worse. Outstanding claim cost trend factors are a third burden we really do not need, especially when a satisfactory alternative index (average paid claim costs) exists to measure inflation. The situation with regard to loss ratio, average incurred claim cost, or pure premium trend factors is essentially the same.

Case II: No trend factor should be applied to outstanding losses. Some thoughtful consideration will be sufficient for the reader to see that validity for this approach must necessarily imply one of three things:

a. Unpaid claims from *last* year's accidents are valued at the same cost level as that which will ultimately apply to payments made on the accidents which will occur *in the future* and for which we are trying to make rates. This is nonsensical.

b. Loss development factors eliminate the need for trend on outstanding cases; we have previously shown this to be false. No matter how long last year's accidents are developed, no matter how well the reserved anticipated cost increases, next year's claims will still *start out* an average of 30 months later and will probably take just as long to settle, so we will still need to *add trend* for 30 months.

c. Claims incurred next year that are unpaid as of 3 months after the close of the year will finally be settled at last year's cost level; if this were true, none of us would settle claims promptly.

Clearly we can reject the alternatives offered above, but we must still consider the question of whether trend factors based on paid claim costs are valid. Let us again go back to first principles, and be sure that loss development and trend are properly distinguished. A trend factor is not intended to develop *any particular set* of losses to a later date. It is rather an *index* of the rate of inflation of accident costs (or in claim frequency trend, an index of the rate of change of accident frequencies). If there were a proper government index or indexes of such costs, as explored by Masterson,[4] we could use that index just as well. The primary reason to use paid rather than incurred losses is to be objective and avoid the possibility of errors of judgment. Our goal is to predict the level of future costs, based on the assumption that the past rate of cost change will continue. Obviously we must project the entire incurred losses, and anything that measures the past variation is valid for that projection. The problem is to pick the *best* measure. Except for random variations, all valid measures should produce essentially the same result. Our criteria should then be that *subject to the requirement that it measure past claim costs, the best trend index is that index which is most stable.* Average paid claim costs clearly satisfy the first requirement, and *at the present state of the art* it is the most stable index available.

[4] Masterson, Norton E., "Economic Factors in Liability and Property Insurance Claims Costs 1935-1967," *PCAS* Vol. LV, page 61.

Trend and Loss Development Factors for Calendar Year Experience

"How can I tell the signals and the signs?"

—H. W. Longfellow

A loss development factor in the accident year or policy year sense is not calculable for calendar year data. It is appropriate to apply a factor to the raw data in order to reflect the *change* in IBNR and other formula reserves, if they are excluded in the basis data. This serves a purpose similar to that of loss development factors, but a calendar year is not clearly enough defined to permit testing or development in the pure sense. If the reserves, including formula additions, are fully adequate at the beginning and end of the year, calendar year results are adequate and the lack of loss development does no harm. Similarly, if both the beginning and ending loss reserves are *equally* inadequate, *and* there is no growth, the incurred losses are adequate for ratemaking. If, however, there is a change in the adequacy of reserves during the year, or there are consistently inadequate reserves *and* a growth in volume, the calendar year incurred losses are not good enough for ratemaking. Under these conditions a formula additional reserve, which will bring total reserves to an adequate level, must be applied to both the beginning and the ending reserves. If such a factor were only applied to the ending reserve, we would have excessive incurred losses, because a fully adequate ending reserve plus paid losses will then include all incurred losses for the coextensive accident year, *plus* the correction of the previous reserve's inadequacy.

Trend factors are applied to calendar year losses similarly to accident year losses, with one difference: the average date of accident in an accident year can be assumed to be the midpoint of the year, but the average date of accident for calendar year losses is not easily determined. If inadequate reserves exist at the beginning of the calendar year, their development relates to earlier years' accidents. This tends to make the average date of accident earlier, and thus the necessary trend period longer, but the amount of this shift is not readily measurable. The simplest solution to this problem is the formula reserve adjustment referred to in the previous paragraph. After all reserves are raised to the level which the ratemaker believes to be adequate, the following analysis holds:

$$\begin{aligned}\text{Incurred losses} = &- \text{Beginning reserve}\\ &+ \text{Paid losses}\\ &+ \text{Ending reserve}\end{aligned}$$

The right side of this equation can then be subdivided, so that

Incurred losses = − Beginning reserve
+ Paid losses (prior years accidents)
+ Ending reserve (prior years accidents)
+ Paid losses (current year accidents)
+ Ending reserve (current year accidents)

But if the beginning reserve is adequate, then by definition the first three terms exactly cancel, so

Incurred losses = Paid losses (current year accidents)
+ Ending reserve (current year accidents)

This final result is exactly equal to the current accident year incurred losses. Therefore, the midpoint of the year is the appropriate average accident date for a calendar year with fully adequate reserves.

Conclusion

> *"It cannot be that axioms established by argumentation can suffice for the discovery of new works, since the subtlety of nature is greater many times over than the subtlety of argument."*
>
> — Francis Bacon

Loss development and trend factors have tended to increase in recent years, to the point where they account for more than 100% of some rate increases. Despite their magnitude and importance, they have not received adequate treatment in the *Proceedings*. They are not easy concepts to grasp, and their definitions vary, when definitions are given at all. As a result, inadequate knowledge in this area is the typical estate of both laymen and students, and to a lesser degree of many Fellows of this Society. I have expressed a set of positions and opinions in this paper, with which many readers may disagree. Those who can clarify, add to, refute, or support these comments are eagerly invited to join debate. I believe we can all learn quite a bit more about loss development and trend factors, to our mutual benefit.

> *"The history of mankind is an immense sea of errors in which a few obscure truths may here and there be found."*
>
> C. de Beccaria

DISCUSSION BY PAUL J. SCHEEL

Mr. Cook's paper, "Trend and Loss Development Factors," is a welcome addition to the *Proceedings.* His paper treats many problems associated with trend and loss development factors, but I would like to confine my comments to the overlap question.

The concepts of loss development, trend and their relationship are difficult to understand and even more difficult to explain. The fact that some regulators still insist that there exists an overlap between loss development and trend is proof enough that the actuaries have not been able to explain it to the regulator's satisfaction. Perhaps Mr. Cook's paper and the reviews it has stimulated will go a long way in overcoming this past deficiency.

Automobile rate filings, by their very nature, are complicated documents. Thousands of man-hours have gone into thought and discussions of principles and procedures which are inherent in the ratemaking formula contained in the filing. Those who have prepared an automobile rate filing realize that certain techniques utilized are presumed to be accepted by the regulator. This is justified since some ratemaking techniques have survived the test of time. Therefore, one need not fully explain every step in the process each time a filing is prepared. When techniques are presented in the same manner over an extended period of time and go uncontested, it is more difficult to defend those techniques once they are contested. The overlap controversy is a perfect example. The current practice is to first apply the loss development factor to the immature accident year losses and then to apply the trend factor. The factors are successively applied.

The current procedure is a logical order in which to apply the two factors, as it is reasonable to say that immature accident year losses should first be adjusted to maturity before application of any trend factor. When the two factors are applied in this order it does "appear" to create an overlap. This apparent overlap can be easily resolved by reversing the order of application in one's thought process and therefore reversing the order of application in the rate filing.

Because the loss development factor has always been applied first, it is difficult to conceive of it being applied differently. But, isn't it equally as reasonable to say that immature accident year losses should first be placed at the loss level for the period of the new rates? That is, apply the trend factor first. If this is done, the resulting answer is what we expect the losses to be which result from accidents written at the new rates, at the same level of maturity as our loss experience. Since these trended losses are immature we must rely on the past relationship of mature to immature losses to bring these losses to the proper level of maturity. The loss development factor picks up in time where the trend factor stops.

There is, therefore, no overlap.

Mr. Cook's exposition of the overlap question is quite clear and readily understandable to the layman. This section of his paper could be extracted and used as an appendix to an automobile rate filing in those states which have raised the overlap question.

DISCUSSION BY ROBERT W. STURGIS

I intend no disparagement whatsoever of the body of Mr. Cook's paper when I say that one of the most illuminating parts is his introduction. As he points out, there *are* misconceptions, misunderstandings, and confusions; and I can testify to the fact that at least one actuary accepted the trend — development overlap fallacy. In the face of all this, it is indeed surprising that so little has been written on this subject. Hopefully, Mr. Cook's work will be the spur to further scholarly discussion.

Why is this subject so complex? How is it that different clear-thinking professionals can come up with diametrically opposite conclusions? When I finished reading Mr. Cook's arguments I was persuaded that there was no overlap. However, this conviction seemed precarious: I had the unsettling feeling that if I were to read counter arguments, I could be swayed to the other side. I have always waded through logical discourses on trend and development using a time-line visual aid as my guide, but always I wound up worried that I was comparing apples to oranges: effective, expiry, accident, and valuation dates; arising, paid, outstanding, open, and closed claims; inflation acting on past accidents and on future accidents; development of reserves and of number of claims. Of course, it is actuarially unsound to compare apples and oranges, but accepted procedure to relate quarts and liters, feet and meters. The soundness of these relationships, however, makes them no less complex. I was encouraged when I read, "It may clarify the point to build a model." Determined to master the mathematics of the algorithm, I surged ahead, but alas, all I found was the familiar visual aid time-line. I determined to build the model on my own; surprisingly, the model turned out to be simplicity itself.

The question is: To what extent should inflationary trends be reflected in ratemaking? To answer this question we need only consider the effect of inflation on claims; all other variables may be ignored or assumed constant. Consider known claim x, to be used as a value predictor for future claim y. Since we aim to isolate the inflationary factor, we make the following assumption.

Assumption: Claim y occurs m months after claim x, and its final value differs from that of x only by a factor reflecting m months of inflation.

Let: x_s and y_s = final values of claims, and
t = monthly inflation operating on claims x and y, expressed as a decimal.

Now the value of y at settlement, y_s, will be predicted by x_s as follows:

$$(1) \qquad y_s = x_s(1+t)^m$$

In practice we often don't know the settlement value of x. Multiplying the right side of equation (1) by (x_v/x_v),

$$(2) \qquad y_s = x_s(1+t)^m \frac{x_v}{x_v} \quad , \quad \text{or}$$

$$(3) \qquad y_s = x_v(1+t)^m \frac{x_s}{x_v} \quad .$$

Thus, the final value of y will equal the value of x at valuation date, x_v, multiplied by a full trend factor for m months, $(1+t)^m$, and by a full loss development factor, (x_s/x_v). It is also clear that it does not matter if a claim is closed or paid, prior to the valuation date. That would be a special case of the loss development factor where x_v equals x_s, and the factor equals unity. In practice of course, we don't know the actual development of our x claims, and we estimate it by using past x_s/x_v values.

If we had chosen to use a straight line rather than an exponential trend factor, equation (3) would be the same except that the trend factor would be $(1+mt)$.

The suggestion that trend should be applied only to the paid portion of incurred losses has always seemed to me as merely a restatement of the overlap argument. Clearly, this argument has no effect on formula (1), and formulae (2) and (3) are simple mathematical derivations from (1). It is true that inflation will effect x to an extent dependent upon whether x is partially paid, fully paid, or fully outstanding. This fact though, is automatically reflected in the final value of x and thus, in x_s/x_v. If x is fully paid at valuation date, then x_v equals x_s, and x_s/x_v equals 1.00. Similarly, if x is partially paid at valuation date, then x_s/x_v will presumably be smaller than if no payments have been made at valuation date.

Of course, the above exercise in elementary mathematics simply confirms Mr. Cook's conclusion that there is no overlap or duplication in trend and development factors, and all claims that there is have been based on specious reasoning. The chief value of the above model is that the decisive elements in the development of the conclusion are specified. That is, if there is to be a challenge to the conclusion, then that challenge must center on the clearly defined assumption or on the formula (1) representation of it. If ever there was a question that the Society could state an official opinion on, this would seem to be it. Perhaps the overlap fallacy can be finally laid to rest, and the full value of Mr. Cook's contribution realized.

DISCUSSION BY D. R. UHTHOFF

Possibly Mr. Cook's strongest motivation for writing this paper was the increasingly householdish term "overlap." Discussions of loss development factors relative to other type factors intended to project for cost or frequency trends often have been colored by concern and confusion, whether there might be overlap between these. That is, to the extent development factors may at least partially arise from inflationary or otherwise assignable cost trending influences, and these same influences also may be applied as rate level trending factors, there may be duplicative effects. If Mr. Cook were to accomplish nothing other than a clarification of the muddiness of these discussions, which he has done, his paper would be a worthwhile addition to our *Proceedings*; he has, in fact, proceeded further to the examination of quite a few other concepts necessary to intelligent handling of various kinds of experiences and approaches useful for rate level work.

I don't think the reader should anticipate a neat do-it-yourself manual for budding ratemakers by which many things are set forth in ready reference form calculated to quell all future doubts about how to handle variations on the theme of setting up rate level calculation procedures. But the author has provided interestingly readable discussions conducive to logical thinking about the kinds of problems that arise in our modern era of necessity that rate levels must contemplate rapidly changing cost levels. He does not pretend to perfection in all his logic; he may lack complete generality in some model-type explorations. But I suspect he may have deliberately designed his paper that way, so as to excite discussion, to invite probing thoughts. However intended, I think the combination of everything works out well, and we now have some literature in our *Proceedings* concerning this subject about which, Mr. Cook remarks, we have been lacking.

Some of his definitions of principles are well set forth: "a trend factor is any index which measures changes over time;" "a calculated past ratio of mature to immature data is called a loss development factor;" and "by their nature, the loss development factors we calculate are always the ones that *would have been* right in the past, and they are therefore an accurate measure of the future development of present losses only if the present outstanding cases have the same degree of reserve adequacy as did the past ones on which the factors are based."

If the reader may wonder who there may be who doesn't already understand those things, he must nevertheless concede their proper places in a narrative-type analysis such as Mr. Cook has provided. The student particularly will find this paper helpful, as will also the more advanced ratemakers who often can have more fun than anybody kicking some of these questions around.

A feature of the paper is the neat arrangement of time periods in which development factors and trend factors can be said to operate to the complete exclusion of the other, therefore without duplication or overlap. As a general expression of his time period arrangements, I would suggest the following be labeled as *time period number one,* the time span between the center of the experience period upon which the rate level is based, and the center of the forthcoming exposure period dependent upon the intended issue dates of policies to be written under the new rates. *Time period number two* would extend from, or beyond, the central point of occurrence of losses of the future policy effective period to the date necessary for losses to acquire maturity.

Mr. Cook's presentation prefers to apply the cost trending or projection factor to the *period number one,* in effect saying that the cost level of the experience period, before using in rate level, must be transferred to the future cost level of the effective policy issued period. He then would say that the center of loss occurrence of the policy issue period, the *period number two*, is the time in which we must apply the development factor to develop that future accident cost to an ultimate developed cost.

If the periods are handled carefully, and properly coincide with the timing of the trending and development factors, the paper demonstrates that all this holds together well and there is no difficulty with overlapping of factors and timing.

I do feel some concern, though, that assignment of the development step to the period of aging beyond the future accident dates makes the chronological picture seem a bit tenuous, more extended than necessary. The same result might be achieved by this logic: the underdeveloped or immature loss data of the experience basis of rates needs adjustment, by the development factors, to accomplish maturity. This development is, of course, according to the pattern of previous development obtainable from earlier losses of like stages of aging.

Thus, we simply state that immature losses must be developed or made mature before using them in a rate level calculation. They then may be projected or cost trended by appropriate means to a future policy issue loss point.

In this way, we avoid the lengthy visualization of development factors being applied to an aging period, the *time period number two,* of quite a few years in the future, development factors which have been obtained from aging processes of a few years in the past. Actually, we need not assign the development factors to any particular aging future; they simply are required as an elementary step in the process of basing a calculation upon mature losses, whether these have become mature through permission of longer development time or whether they are to be synthetically matured by application of development factors. The only requisite, of course, is that the aging period from which development factors were derived is equivalent, and similar in characteristics, to the aging period contemplated in the process of creating mature from immature losses.

Concerning the possibility that dependence upon development factors can be minimized by using the more mature experience — more mature through the simple process of aging — the paper brings out an interesting point, that there then must be more reliance placed upon cost trending factors. That is, to the extent that responsiveness is not supplied through currency of losses, it must be supplied by factors intended to place these losses on a current basis, in addition to those factors necessary to go from the current to a future basis. In this connection, it is interesting to note that when the accident year system of ratemaking was adopted some years ago, and one of its major advantages was stated to be the improvement in responsiveness, as contrasted to policy year experience, quite a bit of reliance was intended to be placed upon development factors. In theory this was fine, but in practice there has been a retreat, in that too much reliance on a factor of sometimes questionable virtue makes it sensible to use somewhat older experience, despite the need for placing more reliance upon trending factors.

There seems a privilege of reviewers these days to advance at least one of their own pet peeves or inventions. Actually, Mr. Cook has led me quite by the nose to make my offering, which is that we strongly entertain a concept of collecting data by which formula-type incurred losses may be devised. A ratemaker, in applying development factors, is following a method depended upon, at least as an alternate method, by many formula reservers within company shops. And as Mr. Cook mentions the possibility of using paid loss projection factors, again he is referring to another method used by companies for calculating formula incurred losses and, thereupon, reserves.

The obvious question with which the ratemaker must struggle is accuracy of reserves. He is not looked upon as a villain in prudently contemplating inaccuracy possibilities in reserve portions of bureau statistics, especially in these days of rapidly changing cost levels. Thus the ratemaker, after he combines company-by-company experiences in the initial step of accomplishing credible bases, then uses his statistical histories for the development factors by which the accuracy of aggregates may be improved. By this step the ratemaker *has constructed* incurred losses. He has used a simple formula system.

For many lines a development factor system is crude, as compared to more sophisticated systems some companies are using and more fundamentally based upon payment histories, rather than gross aging statistics. I am not suggesting that each company pursue formula methods for bureau submissions. I am suggesting that formula methods be devised for bureau application to aggregates, and that the formula methods so devised then will point to the kinds of statistics each company should submit for implementation, on broad collective bases, of these formula methods.

I am sure many company actuaries would be happy to contribute their perhaps painfully gained expertise in devising their own company formula methods.

When we look at the magnitude of some development factors and the apparent trends in these development factors themselves, and shudder at the possibility that our current rate levels may continue inadequate as these factors must lag with respect to current reserving and with respect to current pressures upon managements by which reserves may be deemed, to say the least, no more than those required for minimum necessity, it seems high time that more sophisticated methods of loss experience valuations be adopted. The individual statistical agencies cannot establish and enforce reserving disciplines within company offices; this would not only be impracticable, but would usurp management and company functions. But isn't it true that the ratemaker assumes something less than his responsibility in not having adequate assurance his loss experiences are as accurate as good actuaries might be able to make them?

Perhaps I would like most of all to applaud Mr. Cook for getting at a troublesome problem in a problem-solving way; he incidentally stirs one to some peripheral thinking too. Shouldn't we, as actuaries, presumably responsible to our function, be vitally concerned with anything and everything about rate levels?

DISCUSSION BY MAVIS A. WALTERS

Charles Cook's paper on trend and loss development factors is a valuable document for any actuary who finds himself or herself in the position of trying to explain ratemaking techniques and procedures to laymen or non-technicians. He defines clearly and concisely the terms "trend" and "loss development," and these definitions help to distinguish the two concepts. The definitions are followed by a statement of the traditional "overlap" fallacy; and in fact, Mr. Cook summarizes the argument much more cogently than some of its chief proponents. He then proceeds to refute the position quite simply and directly by discussing the purpose of the rates, i.e. to provide adequate funds to settle claims which result from accidents. The problems arise from the very simple fact that in the ratemaking procedure the actuary must make adjustments on the experience of the past in anticipation of changes in the future. From a theoretical point of view this paper presents the problem in a manner which is easily understood and then concludes by resolving it quite convincingly.

However, since Mr. Cook's paper is a theoretical presentation, it occurred to me that looking at some real data and setting up a practical example might illustrate even more clearly the conclusion which he has reached. This concrete example also demonstrates that the controversy over a possible overlap between trend and loss development is a significant one in terms of its quantitative effect on rate levels and eventual underwriting results.

In this review we have analyzed the private passenger automobile experience in California for all companies reporting to the Insurance Rating Board in order to test the hypothesis that, in fact, no overlap does exist. California was selected since it is a state where rate revisions are made annually, following the standard IRB formula and usually for relatively small adjustments. If an overlap really did exist, then the IRB ratemaking procedure, which applies loss development and trend successively, would tend to produce slightly more than the profit provision provided for in the rate structure. For bodily injury in particular, an unreasonably high profit might be expected when reviewing past underwriting results, since, presumably, the overlap is most pronounced for this coverage.

The latest available 10/20 B.I. experience in California does not show this to be the case. For the accident year ended June 30, 1969 the actual loss ratio is 76%. Since the break even point for IRB companies is approximately 71% in California, this result indicates that instead of realizing an unusually high profit on bodily injury our companies have suffered an underwriting loss of approximately 5%. These figures would certainly seem to lead to the conclusion that instead of reflecting too much trend, too little trend was actually used in the ratemaking formula. To further verify this conclusion and to determine what the effect on the actual experience would be if an overlap did, in fact, exist, we made some additional calculations.

Since we were reviewing the results for the year ended June 30, 1969, we had to make adjustments on the actual rate levels in effect from July 1, 1967 through June 30, 1969. For the two rate revisions effective during this period (in August, 1967 and September, 1968) we assumed that there was an overlap between trend and loss development and eliminated this overlap by using the model described in Mr. Cook's paper. Although the actual B.I. loss development factors reflect an adjustment to 63 months of maturity, generally 60% to 70% of the development occurs within 39 months. It was assumed for the sake of this analysis that only 39 months of development were used in order that we might proceed along the lines described in the paper.

In the 1967 rate revision, experience for the accident year ended June 30, 1966 was used with a 32 month trend projection. Since fiscal accident year experience is evaluated as of September 30, the loss development factors would presumably adjust for the inflationary changes to December 31, 1968 in this case. The 32 months of trend, similarly, would adjust for inflationary changes from January 1, 1966 to September 1, 1968. In this instance then, an "overlap" of 23 months would be evident: from September 30, 1966 to September 1, 1968. In order to eliminate this overlap a new rate level change was calculated by using only 9 months of trend in lieu of the original 32. In this first revision, i.e., the one effective August 23, 1967, the original indication was for virtually no change for bodily injury. By adjusting the figures to reflect only 9 months of trend we find that this indication is reduced to −2.8%. If this decrease is carried forward and the additional adjustment for trend on the second revision, i.e., the one effective September 4, 1968, is made, the actual indication in that filing of +4.9% is reduced to +0.3%. The actual effect of the two revisions combined was for an increase of 4.3% in bodily injury rates. With the adjustments to eliminate the overlap, the combined effect amounts to a reduction of 2.5% in B.I. rates. Translating these effects into loss ratios, we note that the actual B.I. loss ratio for the accident year ended June 30, 1969 of 76% is increased to 79% by eliminating the presumed overlap. In other words, if in 1967 and 1968 we had believed that there was, in fact, an overlap between trend factors and loss development factors, we would have reduced the B.I. rates by two and a half percent rather than increasing them 4.3%. Furthermore, the actual underwriting loss of 5% would have increased to 8%.

Consequently, from these figures it can be seen quite clearly that eliminating the presumed "overlap" does not have the anticipated result of reducing any immoderately high profit but rather has the effect of producing an even more adverse loss ratio and a greater underwriting loss. These results emphasize the fact that the standard IRB ratemaking formula as used in establishing rates from July 1967 through July 1969 rather than overstating the trend actually resulted in an underestimation of this element. In fact, this example based on California data may very well understate the effects of eliminating the alleged overlap, since the trend factors in this example were relatively small and the actual rate level changes during the period being considered were relatively minor. If another state with higher trend factors and greater indicated rate level changes had been selected, the effect of eliminating the supposed overlap would have been much greater.

In order to verify that the earlier IRB trend procedure was deficient and did not produce excessive profits because of an "overlap" we summarized the latest available 10/20 B.I. experience for a group of 25 states somewhat similar to California in that their rate levels were based substantially on the normal ratemaking procedure and no unusual regulatory delay had been encountered. For these states the actual bodily injury experience was even worse than the California experience. For the group 10/20 bodily injury loss ratio was approximately 79% as compared with the corresponding California figure of 76%. This means that for these states as a whole the underwriting loss was even greater than that suffered in California; and consequently, the conclusions drawn from the further analysis of the California data would appear to be reasonable.

In another section of his paper the author suggests that reserving errors can be compounded by loss development factors and the effect of these errors can be severe. Of course, if we are considering the data for only one company for any one given year this is obviously true as is demonstrated by the example shown in the paper. The loss development procedure in use by the IRB is one in which these possible errors are, in fact, virtually eliminated. Attached to this review is an exhibit setting forth the 15 to 27 month and 27 to 39 month B.I. loss development factors for the latest five years for all companies reporting to the IRB for a representative group of states. There are a couple of observations which can be made on the basis of this exhibit. First, in each instance it is evident that most of the development occurs between the first and second reporting, i.e. between 15 and 27 months. Second, for the larger states as well as the countrywide 10/20 experience the factors from year to year appear to be relatively consistent. The standard procedure at the IRB for the automobile lines is to use an average factor (3 years for the larger states and 5 years for the smaller ones) in order to determine a reasonable approximation of the "true" development to be expected on the latest year's incurred losses. Since an average factor is used for each successive reporting (i.e., in large states the latest three 15 to 27 month factors are averaged, then the latest three 27 to 39 month factors are averaged, etc., and finally for the three latest available years the

51 to 63 month factors are averaged) and these average factors are then multiplied to determine the 15 to 63 month development factor, any reserving errors are corrected in the process. Naturally, the fact that the IRB development factors are based on such a large volume of losses also contributes to the minimizing of any possible reserving errors, and the relative consistency of the factors from year to year.

There were two other valuable sections in Mr. Cook's paper: one discussing the application of paid claim cost trend to incurred losses and the other the application of both trend and loss development to calendar year experience. In the former section one point the author makes is that any index which measures the rate of inflation of accident costs could be used as a trend factor, and a proper government index, if it existed, might be just as valid as paid claim costs. Certainly, further study and exploration in this area should be made.

In summary, Mr. Cook's paper was extremely well-written, and his points well thought out. It is gratifying at last to have a work such as this to refer to when a convincing argument and clear examples are needed.

COMPARISON OF BODILY INJURY LOSS DEVELOPMENT FACTORS

AUTOMOBILE LIABILITY INSURANCE - PRIVATE PASSENGER CARS

All Companies Reporting to I.R.B. — Voluntary and Assigned Risks

15 to 27 Month Factors

Accident Year	Countrywide 10/20 Limits	Calif. 10/20	Ill. 10/20	Ky. 10/20	Miss. 5/10	N.J. 10/20	Pa. 10/20	Wyo. 10/20
1963	1.090	1.107	1.035	1.069	1.002	1.113	1.094	1.052
1964	1.106	1.134	1.089	1.056	1.057	1.178	1.147	1.026
1965	1.124	1.141	1.080	1.103	1.039	1.183	1.136	1.082
1966	1.128	1.140	1.069	1.125	1.028	1.211	1.142	1.097
1967	1.123	1.120	1.068	1.127	1.001	1.155	1.113	.959
Latest 3 Yr. Avg.	1.125	1.134	1.072	1.118	1.023	1.183	1.130	1.046
5 Yr. Avg.	1.114	1.128	1.068	1.096	1.025	1.168	1.126	1.043

27 to 39 Month Factors

Accident Year	Countrywide 10/20 Limits	Calif. 10/20	Ill. 10/20	Ky. 10/20	Miss. 5/10	N.J. 10/20	Pa. 10/20	Wyo. 10/20
1962	1.013	1.008	1.014	1.021	1.016	1.013	1.022	.856
1963	1.027	1.009	1.037	1.026	.990	1.042	1.050	.986
1964	1.027	1.011	1.021	1.016	1.000	1.046	1.030	.951
1965	1.038	1.016	1.029	1.006	.999	1.078	1.043	1.080
1966	1.032	1.025	1.025	.999	1.023	1.046	1.043	.923
Latest 3 Yr. Avg.	1.032	1.017	1.025	1.007	1.007	1.057	1.039	.985
5 Yr. Avg.	1.027	1.014	1.025	1.014	1.006	1.045	1.038	.959

A STOCHASTIC APPROACH TO AUTOMOBILE COMPENSATION

DONALD C. WEBER

1. INTRODUCTION

In recent years various automobile compensation plans have been proposed in response to adverse criticism of the existing automobile liability system. This paper is an effort to present a probability model which conceivably could provide the mathematical framework for some future no-fault insurance system. Before proceeding, it is only fair to warn that utilization of the proposed model far ratemaking purposes would have a far-reaching effect upon members of the Casualty Actuarial Society. It would mean counting accident involvements rather than claims. It would mean calculating involvement costs rather than claim costs. And finally, it would mean insuring an individual driver rather than an automobile.

2. THE MODEL

Suppose the discrete random variable N(t) represents the number of accident involvements experienced by a motorist during a time interval of length $t > O$ and $p(n,t)$ denotes the probability that $N(t) = n$. Let the cost of an accident involvement be represented by the non-negative continuous random variable X having distribution function $G(x)$. We shall assume that X is independent of time and of the costs of prior involvements. Denote by $G_n(x)$ the probability that the cost of n accident involvements is less than or equal to x. Then if $X(t)$ is the total cost of involvements over a time span of t units, the relationship between these random variables is given by the analytic expression:

$$(1) \qquad F(x,t) = \sum_{n=0}^{\infty} p(n,t)G_n(x) \quad , \quad x \geqq 0, t > 0,$$

where:

$$F(x,t) = P_r\{X(t) \leqq x\},$$
$$p(n,t) = P_r\{N(t) = n\},$$
$$G_n(x) = P_r\{X_1 + X_2 + \ldots + X_n \leqq x\} \quad , \quad n > 1 ,$$
$$G_0(x) = 1 \quad , \quad G_1(x) = G(x)$$

With the aid of characteristics functions, it is not difficult to show that the mean and variance of the random variable $X(t)$ are:

$$(2) \qquad E[X(t)] = E[N(t)]\,E(X)\,,$$
$$Var\,[X(t)] = E^2\,(X)\,Var\,[N(t)] + E[N(t)]\,Var\,(X)\,,$$

respectively.

The reader will recognize relation (1) as an adaptation of the basic model employed in collective risk theory as developed by Cramér[1] and others, and discussed by Dropkin[2] in his presentation at the Mathematical Theory of Risk meeting in 1966. The problem now is to obtain realistic and adequate functions for $p(n,t)$ and $G(x)$.

3. THE NEGATIVE BINOMIAL MODEL

In 1920, Greenwood and Yule[3] proposed an accident frequency model which assumes that during a time interval of length t the number of accidents, n, experienced by an individual is a Poisson process with mean and variance λt, i.e.:

$$(3) \qquad p(n,t) = \frac{e^{-\lambda t}(\lambda t)^n}{n!} \quad , \quad n = 0, 1, 2, \ldots, \quad \lambda > 0, t > 0,$$

and that λ is a value of a random variable having a gamma distribution:

$$(4) \qquad u(\lambda) = \frac{(r/m)^r}{\Gamma(r)}\,\lambda^{r-1}e^{-(r/m)\lambda} \quad , \quad \lambda > 0, m > 0, r > 0,$$

where:

$$\Gamma(r) = \int_0^\infty y^{r-1}e^{-y}dy.$$

The resulting unconditional distribution for n accidents in time t is the negative binomial:

$$q(n,t) = \int_0^\infty p(n,t)u(\lambda)d\lambda$$

$$(5) \qquad = \frac{\Gamma(n+r)}{n!\,\Gamma(r)}\left(\frac{r}{r+mt}\right)^r\left(\frac{mt}{r+mt}\right)^n \quad , \quad n = 0, 1, 2, \ldots, \quad r > 0, m > 0, t > 0,$$

with mean mt and variance $mt(1 + mt/r)$.

[1] Crámer, H., *Collective Risk Theory,* Nordiska bokhandeln, Stockholm, 1955.

[2] Dropkin, L. B., "Loss Distributions of a Single Claim," *PCAS Mathematical Theory of Risk,* 1966.

[3] Greenwood, M., and Yule, G. U., "An inquiry into the Nature of Frequency Distributions Representative of Multiple Happenings with Particular Reference to the Occurrence of Multiple Attacks of Disease or of Repeated Accidents," *Journal of the Royal Statistical Society, 83* pp. 255-279, (1920).

Using different notation, Dropkin[4] introduced this model to the Casualty Actuarial Society and successfully applied it to data obtained in the 1958 California Driver Record Study. With reference to the motor vehicle accident scene, we may interpret the parameter λ to be the theoretical accident rate per unit time associated with an individual driver. We then assume that this parameter varies from individual to individual within a population of drivers according to the probability density $u(\lambda)$ which has mean m and variance m^2/r. It follows that $q(n,t)$ is the distribution of accidents within a population, i.e., $100\ q(n,t)$ gives the percentage of individuals in the population involved in $n = 0,1,2 \cdots$, accidents during a time period of t units.

The California Department of Motor Vehicles kindly provided this writer with data used in their 1964 California Driver Record Study.[5] For this study a random sample constituting about 2% of the licensed drivers in the state was obtained. Of these, data are available on approximately 148,000 motorists over the full observation period of three years, namely 1961-63. These data include information on certain attributes of the individuals in the sample as well as their driving record in terms of traffic offenses and *reportable* accident involvements. In Table 1 we see the fit, using the method of moments, of the negative binomial model (5) to the empirical accident distributions generated by these 148,000 individuals during the specified time intervals. Notice that due to the time lag between the occurrence of an accident and the processing of the resulting accident report, the 1963 period is estimated to represent a 10½-month interval rather than a full year.

The closeness of fit duplicates the results obtained by Dropkin using the earlier California data. However, it is important to observe that the parameters m and r, as shown by their estimates together with the standard deviation of these estimates, do not seem to remain constant over time. This would suggest that a shifting takes place in the underlying distribution (4), $u(\lambda)$, which implies that the parameter λ of relation (3) is a *function of time.*

In Table 2 we find the negative binomial fitted to the California data by sex. As with the combined data, the negative binomial distribution provides

[4] Dropkin, L. B., "Some Considerations on Automobile Rating Systems Utilizing Individual Driving Records," *PCAS* XLVI, p. 165.

[5] California Department of Motor Vehicles, State of, "The 1964 California Driver Record Study Parts 1-9," Sacramento, California 1964 and 1967.

Table 1

Comparison of Actual and Theoretical (Negative Binomial) Accident Distributions
1964 California Driver Record Study

	No. of Accidents	Actual Distribution	Theoretical Distribution
1961 - 1963			
$\hat{m} = 0.0711 \pm 0.0004$	0	122,593	122,638
$\hat{r} = 1.1400 \pm 0.0378$	1	21,350	21,257
$t = 2.875$	2	3,425	3,457
$\chi^2 = 1.61$, 3 d.f.	3	530	550
	4	89	86
	5+	19	18
	Total	148,006	148,006
1961 - 1962			
$\hat{m} = 0.0709 \pm 0.0005$	0	129,524	129,541
$\hat{r} = 1.0773 \pm 0.0473$	1	16,267	16,236
$t = 2$	2	1,966	1,963
$\chi^2 = 4.90$, 3d.f.	3	211	234
	4	31	28
	5+	7	4
	Total	148,006	148,006
1961			
$\hat{m} = 0.0696 \pm 0.0007$	0	138,343	138,353
$\hat{r} = 1.0691 \pm 0.0894$	1	9,072	9,042
$t = 1$	2	547	571
$\chi^2 = 1.47$, 1d.f.	3+	44	40
	Total	148,006	148,006
1962			
$\hat{m} = 0.0722 \pm 0.0007$	0	138,087	138,094
$\hat{r} = 0.8469 \pm 0.0585$	1	9,211	9,191
$t = 1$	2	650	668
$\chi^2 = 1.00$, 1d.f.	3+	58	53
	Total	148,006	148,006
1963			
$\hat{m} = 0.0715 \pm 0.0008$	0	139,326	139,330
$\hat{r} = 0.8712 \pm 0.0701$	1	8,140	8,133
$t = 0.875$	2	505	509
$\chi^2 = 0.07$, 1d.f.	3+	35	34
	Total	148,006	148,006

Table 2

Comparison of Actual and Theoretical Accident Distributions by Sex
1964 California Driver Record Study

	No. of Accidents	Actual Distribution	Theoretical Distribution
Males			
1961			
$\hat{m} = 0.0885$	0	79,595	79,606
$\hat{r} = 1.3420$	1	6,638	6,606
$t = 1$	2	451	479
$\chi^2 = 3.19$, 1d.f.	3+	42	35
		86,726	86,726
1962			
$\hat{m} = 0.0925$	0	79,358	79,365
$\hat{r} = 1.0599$	1	6,775	6,752
$t = 1$	2	538	559
$\chi^2 = 1.37$, 1d.f.	3+	55	50
		86,726	86,726
1963			
$\hat{m} = 0.0901$	0	80,369	80,372
$\hat{r} = 1.0648$	1	5,910	5,902
$t = 0.875$	2	415	420
$\chi^2 = 0.07$, 1d.f.	3+	32	32
		86,726	86,726
Females			
1961			
$\hat{m} = 0.0430$	0	58,748	58,747
$\hat{r} = 1.2423$	1	2,434	2,439
$t = 1$	2	96	91
$\chi^2 = 0.62$, 1d.f.	3+	2	3
		61,280	61.280
1962			
$\hat{m} = 0.0436$	0	58,729	58,726
$\hat{r} = 0.9244$	1	2,436	2,443
$t = 1$	2	112	106
$\chi^2 = 1.16$, 1d.f.	3+	3	5
		61,280	61,280
1963			
$\hat{m} = 0.0451$	0	58,957	58,956
$\hat{r} = 0.9311$	1	2,230	2,232
$t = 0.875$	2	90	88
$\chi^2 = 0.30$, 1d.f.	3+	3	4
		61,280	61,280

a remarkably close fit in every case. Again, the fluctuation in parameter values from one time period to another is apparent. Also, we notice that the distributions of male and female accident involvements are different.

Using the negative binomial model, Arbous and Kerrich,[6] Bates and Neyman[7] and Edwards and Gurland[8] derived various bivariate accident distribution models which differ in certain underlying assumptions. These bivariate models can be used to obtain theoretical distributions of future accidents based upon the number of past accidents. The bivariate negative binomial of Kerrich, which assumes a constant parameter λ (past and future), appeared in the *Proceedings* in a paper by Dropkin[9] and was applied to Canadian data by Hewitt.[10] Actuaries have long recognized, however, that factors other than accident history are related to future automobile accident experience, e.g., age, sex, geographic location, mileage driven, conviction history. In fact, only in recent years has accident experience been incorporated in the ratemaking procedures.

4. ACCIDENT RATE POTENTIAL

For the moment, let us accept the idea that an individual driver's accident frequency over a short period of time is a Poisson process (1). Let us assume that each motorist is characterized by his own particular λ which is a function of accident likelihood variables such as physical, mental and emotional states, attitudes, motor abilities, habits, alertness, environmental driving conditions and amount of driving exposure. In view of the previous section, the parameter λ is a function of time through changing conditions and, therefore, any estimate of this parameter requires frequent updating. However, let us treat λ as a constant over relatively short periods of time in the absence of major changes in the above variables. Thus we may view λ as the result of averaging the individual's accident likelihood variables over

6 Arbous, A. G. and Kerrich, J. E., "Accident Statistics and the Concept of Accident-Proneness," *Biometrics, 7* pp. 340-432 (1951).

7 Bates, G. E. and Neyman, J., "Contributions to the Theory of Accident Proneness," *University of California Publications in Statistics I*, pp. 215-276, (1952).

8 Edwards, C. B., and Gurland, J., "A Class of Distributions Applicable to Accidents," *Journal of the American Statistical Association, 56* pp. 503-517, (1961).

9 Dropkin, L. B., "Automobile Merit Rating and Inverse Probabilities," *PCAS* XLVII, p. 37.

10 Hewitt, Jr., C. C., "The Negative Binomial Applied to the Canadian Merit Rating Plan for Individual Automobile Risks," *PCAS* XLVII, p. 55.

the observation period. This "constant" will hereafter be called the *accident rate potential* associated with the individual driver.

Clearly, most of the accident likelihood variables are not directly measurable. We are therefore confronted with the task of trying to estimate an individual's accident rate potential on the basis of available information, information which at best reflects to an unknown degree the actual accident likelihood of the driver. The information used to estimate λ will be called criteria, or criterion variables.

5. THE MODEL FOR ACCIDENT RATE POTENTIAL

On the basis of the analyses appearing in the 1964 California Driver Record Study, this writer chose as criterion variables: sex, marital status, residence, age, conviction history and accident history. Proceeding on the evidence that the distribution of accidents within a population of drivers is negative binomial, if these criteria are truly effective predictors of λ, they should be able to subdivide the California sample into homogeneous groups with respect to accident rate potential, i.e., into "Poisson groups." In an effort to establish the effectiveness of the criterion variables, the 148,000 individuals in the sample were partitioned into 2,880 groups on the basis

TABLE 3

Criterion Variables used to Partition California Sample

Sex	*Marital Status*	*Residence (Counties)*	
Male	Married	Area 1:	Los Angeles, San Francisco
Female	Single	Area 2:	Alameda, Contra Costa, Marin, Orange, Sacramento, San Mateo, Santa Clara
		Area 3:	Fresno, San Joaquin, Stanislaus, Yolo
		Area 4:	All Other Counties

Age in 1963	*No. of Convictions, 1961-62*	*No. of Accidents, 1961-62*
Less than 21	0	0
21 - 25	1	1
26 - 30	2	2
31 - 40	3	3
41 - 60	4	More than 3
Over 60	More than 4	

of the six chosen criteria. The levels within each variable are presented in Table 3. A computer program printed out the 1963 accident distributions for the 193 groups that contained 100 or more individuals and fitted a Poisson distribution to each such group. In 167 or 86.5% of the cases the hypothesis of a Poisson distribution was acceptable at the .05 level of significance. One may conclude from these results that the six criterion variables did a credible job of classifying the individuals according to negative binomial theory.

On the basis of the above experiment, let us assume that the accident rate potential characterizing an individual is a function of a number of criterion variables, i.e.:

$$\lambda = f(\underline{x} : \underline{\beta}) \tag{6}$$

where $\underline{x}$ represents a vector of criterion variables and $\underline{\beta}$ is a vector of parameters. Our next task is to determine the functional form of f. To do this we turn our attention to the 1964 California Driver Record Study in order to examine the relationship between accident frequency and the selected criterion variables, taken one at a time.

In the partitioning experiment the primary basis used for determining the levels of the area variable was accident rate by drivers residing in a county. During the experience period of the California study, the accident rate per driver is given in Table 4. It reveals that accident rates do indeed

TABLE 4

Accident Rates per driver by Area and County
1964 California Driver Record Study

Area 1 (61,594 cases)	
Los Angeles	0.241
San Francisco	.245
Area 1 Ave.	0.241

Area 2 (37,690 cases)	
Alameda	0.225
Contra Costa	.202
Marin	.201
Orange	.218
Sacramento	.217
San Mateo	.210
Santa Clara	.199
Area 2 Avg.	0.213

Area 3 (7,647 cases)	
Fresno	0.184
San Joaquin	.187
Stanislaus	.172
Yolo	.191
Area 3 Avg.	0.183

Area 4 (40,474 cases)	
All Other	0.147

vary from area to area within the state and, in general, the more populous the area, the higher the accident rate.

To take advantage of the positive correlation between accident rates and population density, it was decided to use the county traffic density index as a criterion variable. This index is defined as the ratio of total registered vehicles in a given county to the total linear miles of roadway in that county. It must be recognized that the use of a countywide index somewhat understates the relationship between accidents and density since the population density within many of the California counties is anything but uniform. In order to fully utilize the predictive power inherent in a traffic density factor with respect to accident frequency an index by geographical area rather than by county lines is needed.

A plot of accident rate versus traffic density index reveals that the mathematical relationship between these two variables is concave downward. The correlation coefficient corresponding to a simple regression analysis of accident rate on the logarithm of traffic density index was 0.85. Accordingly, we will assume that the relationship between mean accident frequency, denoted by y, and the natural logarithm of traffic density index, denoted by x_1, to be:

$$y = a_1 + b_1 x_1$$

where a_1 and b_1 are constants to be estimated.

In Part 5 of the 1964 California Driver Record Study charts are given which visually depict the relationships between accident rates and the personal characteristics (i) sex, (ii) marital status, and (iii) age. These charts again reveal that males and females constitute distinct driving populations, i.e., the relationships are of different character in the two populations. As a consequence, it is necessary to search out a function f in (6) for each of the two sexes.

These charts show that the driving record of married females is better than that of single females at all ages, although the difference is not constant. With the exception of two age groups (under 26 and 56-60), the same statement can be made about male drivers. In order to give recognition to the apparent significant relationship between accident rate, y, and marital status, x_2, we will assume the step function relation:

$$y = a_2 + b_2 x_2$$

where $x_2 = 0$ for a married individual and $x_2 = 1$ for a single person and a_2 and b_2 are constants. Since this assumes a constant difference in mean accident frequencies between marrieds and unmarrieds, the above formula is acknowledged to be an approximation to the actual situation at best.

Using a transformation on age, it is possible to reasonably express the relationship between accident rates, y, and a function of age, x_3, in the linear form:

$$y = a_3 + b_3 x_3$$

where a_3 and b_3 are parameters to be estimated. Applied to the California data, this writer used $x_3 = 5/(age - 13)$ for males and $x_3 = 125/(age - 13)^3$ for females. The estimates for the parameters obtained from weighted regression analyses are given in Tables 5 and 6 together with the fit to the corresponding empirical rates.

TABLE 5

Weighted Regression of Accident Rates on Transformed Ages (Males)
1964 California Driver Record Study (1961-63)

$$y = 0.1823 + 0.3183x_3$$
$$\text{where } x_3 = 5/(\text{age} - 13)$$

Age Class	*Empirical Accident Rate*	*Theoretical Rate y*
Under 21	0.468	0.459
21 - 25	.332	.341
26 - 30	.290	.288
31 - 40	.253	.253
41 - 60	.229	.226
Over 60	.204	.210

In continuing our search for the functional form of f in (6) we next investigate the possibilities of predicting accident involvement using driver record data. Part 4 of the 1964 California Driver Record Study discusses the relationship between accident and conviction frequencies based upon a three-year experience period involving the 148,000 drivers in the sample. At this point a conviction is defined as a traffic conviction which counts toward an individual's negligent operator point total. This includes all violations involving the safe operation of a motor vehicle as defined in Section 12810 of the California Vehicle Code. In this study, the number of convictions understates the actual number of vehicle code violations in that multiple citations relating to a single incident were counted as one. Also,

TABLE 6

Weighted Regression of Accident Rates on Transformed Ages (Females)
1964 California Driver Record Study (1961-63)

$$y = 0.1191 + 0.1365x_3$$
$$\text{where } x_3 = 125/(\text{age} - 13)^3$$

Age Class	*Empirical Accident Rate*	*Theoretical Rate y*
Under 21	0.209	0.209
21 - 25	.138	.136
26 - 30	.118	.124
31 - 40	.121	.121
41 - 60	.120	.120
Over 60	.121	.119

to avoid a "built-in" correlation between accidents and countable convictions, the number of convictions does not include those resulting from an accident investigation. Harwayne's account[11] in the *Proceedings* on the earlier California study revealed the near linear relationship between accident rates and countable convictions. Accordingly, a weighted regression analysis on conviction counts was performed for each sex. The actual and predicted means are given in Table 7 where y is the mean accident frequency and x_4 is the number of conviction counts.

TABLE 7

Weighted Regression of Accident Rates on Number of Countable Convictions
1964 California Driver Record Study (1961-63)

$$\text{Males: } y = 0.1733 + 0.0953x_4$$
$$\text{Females: } y = 0.0999 + 0.0823x_4$$

Number of Convictions (x_4)	*Males*		*Females*	
	Actual	*Theoretical* (y)	*Actual*	*Theoretical* (y)
0	0.17	0.17	0.10	0.10
1	.28	.27	.18	.18
2	.37	.36	.27	.26
3	.45	.46	.37	.35
4	.58	.55	.44	.43
More than 4	.68	.75	.49	.59

[11] Harwayne, F., "Merit Rating in Private Passenger Automobile Liability Insurance and the California Driver Record Study," *PCAS* XLVI, p. 189.

It is necessary to point out explicitly that Table 7 shows a concurrent relationship between accidents and convictions, i.e., the counts for both variables arise from the same experience period. What is of greater interest to us is the predictive nature of past convictions as it concerns future accidents. In this regard Table 8 displays the combined experience of all drivers in the California sample as taken from the tabulation which partitioned the sample into homogeneous groups. There we find that the relationship between 1963 empirical accident rates and 1961-62 conviction counts is dominantly linear by checking the differences in accident rates as we go from one conviction level to the next.

TABLE 8

Observed 1963 Accident Rates by 1961-62 Conviction Counts
1964 California Driver Record Study

No. of Convictions 1961-62	*Empirical Accident Rates 1963*
0	0.0466
1	.0834
2	.1106
3	.1411
More than 3	.1707

Although the relationship in this instance is not as strongly linear as in the concurrent case, let us tacitly assume that the relation:

$$y = a_4 + b_4\, x_4$$

also holds when y is defined as future mean accident frequency and x_4 represents number of convictions as it pertains to the prior time interval.

If we accept the tenet that the negative binomial model is at least an approximation to actual automobile experience, we would expect future accident rates to be linearly related to the incidence of past accident involvements on a theoretical basis. See, for example, Dropkin[12] and Hewitt[13]. To confirm this, iterative weighted regression analyses for:

$$y = a_5 + b_5\, x_5$$

[12] Dropkin, L. B., *op. cit.*
[13] Hewitt, Jr., C. C., *op. cit.*

were performed using a computer. Here y represents 1963 accident rates and x_5 is the number of 1961-62 involvements. Further discussion of the iterative procedure used appears in the next section. The results of these analyses are given in Table 9:

TABLE 9

Weighted Regression of 1963 Accidents on 1961-62 Accident Counts
1964 California Driver Record Study

Males: $y = 0.07234 + 0.03818x_5$
Females: $y = 0.03686 + 0.03090x_5$

No. of Accidents 1961-62 (x_5)	Males		Females	
	Actual Rates	Theoretical (y)	Actual Rates	Theoretical (y)
0	0.0721	0.0723	0.0368	0.0369
1	.1112	.1105	.0677	.0678
2+	.1454	.1529	.1008	.1004

A final candidate for a criterion variable is non countable convictions. A non countable conviction is defined as a traffic conviction which does not involve the safe operation of a motor vehicle, e.g., a conviction in connection with certain non moving offenses. The relationship between accidents and non countable convictions was not given separate analysis in the 1964 California Driver Record Study nor did this writer look into the matter. However, in Part 8 of the California study a significant relationship was observed, at least as it concerns concurrent data. Having no reason to believe that the mathematical form of the relationship between accidents and non countable convictions should be different than that between accidents and countable convictions, let us assume the equation:

$$y = a_6 + b_6 x_6$$

where y is the future accident rate and x_6 is the prior non countable conviction count.

On the basis of the linear relationships between accident rates and the investigated criterion variables, let us hypothesize that, in general, the function of f of (6) is given by:

$$\lambda = f(\underline{x}\,;\underline{\beta}) = \beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k \qquad (7)$$

where the x_i are the criterion variables which functionally determine the value of λ and the β_i are the necessary parameters. Now (7) together with (3) permits us to finalize the form of $p(n,t)$ in (1) in terms of the characteristics of the driver, namely:

$$(8) \qquad p(n,t) = \frac{e^{-t\sum_{i=0}^{k}\beta_i x_i}\left(t\sum_{i=0}^{k}\beta_i x_i\right)^n}{n!} \qquad n = 0, 1, 2, \ldots, \quad \sum_{i=0}^{k}\beta_i x_i > 0,\ t > 0,$$

where; $x_0 = 1$.

Before taking up the problem of estimating λ using the California data, a few comments of the limitations of the data are in order. We have discussed six possible candidates for criterion variables. That does not mean, of course, that these six are the only predictors that have a significant mathematical relationship with accident involvements. For example, miles of driving may be a most significant factor but the California records do not give this information and hence we are unable to *directly* include this variable in our analysis. If at some future date, exposure mileage information by driver were available, it is likely that the relationship between it and accident rate would be found to have a highly significant linear component. Should that be the case, the variable "driving mileage" would take its place as one of the k predictors in relation (7).

At this point it is also appropriate to remind ourselves of other limitations in this study. Recall that our accident count includes only reported accidents, but unreported accidents according to other studies are more numerous than those reported to authorities. Therefore we cannot claim that the relationships derived in this study are applicable when the number of accidents is taken to mean *all* accidents. Also, our estimate of λ in the sections to follow will be based on reported accidents only and so, in terms of all involvements, it will be an understatement. Similarly, our conviction count includes only the incidence of detected violations. Surely, this count is a gross understatement of the number of actual violations and we cannot assume the degree of understatement to be uniform.

6. ESTIMATION OF ACCIDENT RATE POTENTIAL: THEORY

For the sake of simplification but without loss of generality, let $t = 1$ in function (8) in the development that follows. Then the probability that the j^{th} individual in the sample will be involved in n_j accidents during the next unit of time is given by:

$$(9) \qquad p(n_j) = \frac{e^{-\sum_{i=0}^{k} \beta_i x_{ij}} \left(\sum_{i=0}^{k} \beta_i x_{ij} \right)^{n_j}}{n_j!}, \quad n_j = 0, 1, 2, \ldots, \quad \sum_{i=0}^{k} \beta_i x_{ij} > 0.$$

To obtain the maximum likelihood estimates for the parameters we observe that with respect to a sample of size s, the likelihood function is:

$$L = \prod_{j=1}^{s} \frac{e^{-\sum_{i=0}^{k} \beta_i x_{ij}} \left(\sum_{i=0}^{k} \beta_i x_{ij} \right)^{n_j}}{n_j!} .$$

Taking the natural logarithm, we obtain:

$$\ln L = -\sum_{j=1}^{s} \sum_{i=0}^{k} \beta_i x_{ij} + \sum_{j=1}^{s} n_j \ln \left(\sum_{i=0}^{k} \beta_i x_{ij} \right) - \sum_{j=1}^{s} \ln n_j!$$

Differentiating with respect to β_i, $i = 0, 1, 2, \ldots, k$, we get:

$$\frac{\partial \ln L}{\partial \beta_i} = -\sum_{j=1}^{s} x_{ij} + \sum_{j=1}^{s} \frac{n_j x_{ij}}{\left(\sum_{i=0}^{k} \beta_i x_{ij} \right)} .$$

On setting the $k + 1$ partials equal to zero, the system of maximum likelihood normal equations otained is:

$$(10) \qquad \sum_{j=1}^{s} \frac{n_i x_{ij}}{\left(\sum_{i=0}^{k} \hat{\beta} x_{ij} \right)} = \sum_{j=1}^{s} x_{ij} , \quad i = 0, 1, \ldots, k .$$

In a related but slightly different context, Jorgenson[14] showed that a solution to the set of equations (10) can be obtained by using an iterative weighted least squares procedure. If N_j is the random variable having distribution (9), then the parameter λ associated with the j^{th} individual is:

$$\lambda_j = \epsilon(N_j) = \sum_{i=0}^{k} \beta_i x_{ij} = \mathrm{Var}\,(N_j)$$

[14] Jorgenson, D. W., "Multiple Regression Analysis of a Poisson Process," *Journal of the American Statistical Association 56*, pp. 235-245, (1961).

In matrix notation:

$$\underline{\lambda} = \epsilon(\underline{N}) = X\underline{\beta} \quad \text{and} \quad \text{Cov}\,(\underline{N}) = V$$

where the underline of λ and N denote column vectors of dimension s, $\underline{\beta}$ is a vector of $k + 1$ parameters, X is an $s \times (k + 1)$ matrix having the values of the criterion variables as elements and V is an $s \times s$ diagonal matrix with elements $v_j = \sum_{i=0}^{k} \beta_i x_{ij}$. It is well known (e.g., see Goldberger[15] that the minimum variance linear unbiased estimator of $\underline{\beta}$ is:

$$\hat{\underline{\beta}} = (X'V^{-1}X)^{-1}X'V^{-1}\underline{N}$$

with:

$$\text{Cov}\,(\hat{\underline{\beta}}) = (X'V^{-1}X)^{-1}$$

The notation X' denotes the transpose of the matrix X and V^{-1} denotes the inverse of V. According to general linear model theory, if $\underline{x_j}$ is the vector of criterion values corresponding to the j^{th} individual, an unbiased estimator for $\epsilon\,(N_j)$ is $\underline{x_j}\check{\underline{\beta}}$ with the variance of this estimator being $\underline{x'}_j(X'V^{-1}X)^{-1}\underline{x}_j$.

Unfortunately, since $\underline{\beta}$ is unknown, the matrix V is unknown. Our problem then is to obtain an estimate of V which in turn gives us an estimate of $\underline{\beta}$. Following Jorgenson, we let $\hat{V}_m$ denote the estimate of V obtained on the m^{th} iteration and we let the corresponding estimate of $\underline{\beta}$ be:

$$\underline{b}_m = (X'\hat{V}_m^{-1}X)^{-1}X'\hat{V}_m^{-1}\underline{n}\,.$$

Let $\hat{V}^o$ be the $s \times s$ identity matrix and define:

$$\hat{V}_{m+1} = \text{diag}\,[\underline{x}_1'\,\underline{b}_m\,,\,\underline{x}_2'\,\underline{b}_m\,,\,\ldots\,,\,\underline{x}'_s\,\underline{b}_m]$$

where x_j is defined as before. The iterations are continued until convergence is realized, i.e., $\underline{b}_{m+1} = \underline{b}_m$. Denote this equality vector by $\underline{b}$. Then:

(11) $$\underline{b} = (X'\hat{V}^{-1}X)^{-1}X'\hat{V}^{-1}\underline{n}$$

where $\hat{V}$ is the equality matrix $\hat{V}_{m+1} = \hat{V}_m$. As our final estimate of λ_j we may use:

(12) $$\hat{\lambda}_j = \underline{x}_j'\,\underline{b}$$

and as an estimate of the variance of $\hat{\lambda}_j$ we may use:

(13) $$\widehat{\text{Var}}\,(\hat{\lambda}_j) = \underline{x}_j'\,(X'\hat{V}^{-1}X)^{-1}\underline{x}_j\,.$$

[15] Goldberger, A. S., *Econometric Theory* (John Wiley and Sons, Inc., New York, 1964).

Because of having to use $\hat{V}$ instead of V, the estimate (12) is not unbiased and its variance is unknown but Jorgenson[16] points out that it is best asymptotically normal (BA). He also notes that the iterative procedure converges provided that $\check{V}_m$ and $(X'\hat{V}_m^{-1}X)^{-1}$ are positive definite for all m.

Work by Wald[17] provides a theoretical basis for testing:

$$H_o : L\underline{\beta} = \underline{\gamma}$$

where L is a known $l \times (k+1)$ matrix of rank $l \leq k+1$ and $\underline{\gamma}$ is a specified vector of constants. The appropriate test statistic:

$$(L\underline{b} - \underline{\gamma})'[L(X'V^{-1}X)^{-1}L']^{-1}(L\underline{b} - \underline{\gamma})$$

is asymptotically distributed as chi-square with l degree of freedom. This, of course, can be used to test such hypotheses as:

$$H_o : \beta_i = 0 \quad \text{and} \quad H_o : \lambda = \underline{x}'\underline{\beta} = \lambda_o$$

In this study, the vector $\underline{b}_{m+1}$ was calculated using a standard least squares linear regression program after applying a weight of:

$$\left(\sum_{i=0}^{k} b_{i(m)} x_{ij} \right)^{-1/2}$$

to the data. Here $b_{i(m)}$ is the i^{th} element in the vector $\underline{b}_m$. The usual regression program then obtains $\underline{b}_{m+1}$ by solving the system of $k+1$ equations:

$$\sum_{j=1}^{s} \frac{n_j x_{ij} - x_{ij} \sum_{i=0}^{k} b_{i(m+1)} x_{ij}}{\sum_{i=0}^{k} b_{i(m)} x_{ij}} = 0 \quad , \quad i = 0, 1, \ldots, k \, .$$

It is readily seen that this system reduces to (10) when $b_{i(m+1)} = b_{i(m)}$ for all $i = 0, 1, 2 \ldots, k$.

7. ESTIMATION OF ACCIDENT RATE POTENTIAL: EXAMPLES

In this section we illustrate the use of the multiple Poisson regression technique applied to the California data. Recall that in Section 5 we selected

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[16] Jorgenson, D. W., *op. cit.*

[17] Wald, A., "Tests of Statistical Hypotheses Concerning Several Parameters When the Number of Observations Is Large," *Transactions of American Mathematical Society, 54* pp. 426-482, (1943).

six criterion variables to use as accident rate potential predictors. To review, for any given individual in the California sample, these are:

$x_0 \equiv 1$

$x_1 =$ the natural logarithm of the traffic density index of the county in which the driver resides.

$$x_2 = \begin{cases} 0 & \text{, if married,} \\ 1 & \text{, if single.} \end{cases}$$

$$x_3 = \begin{cases} 5/(\text{age} - 13), \text{ if male,} \\ 125/(\text{age} - 13)^3, \text{ if female.} \end{cases}$$

$x_4 =$ the number of countable convictions incurred during years 1961-62.

$x_5 =$ the number of accident involvements incurred during years 1961-62.

$x_6 =$ the number of noncountable convictions incurred during years 1961-62.

Initially, usual least squares analyses were run in order to determine which of the six criterion variables are significant in the presence of the others. The results of these analyses are given in Table 10. Comparing with a critical t value of 1.96 at the .05 significant level, we notice that marital status is a nonsignificant variable in the regression equation for males and two variables, age and noncountable conviction history, are not significant for females. We find that conviction history contributes more to accident prediction than any other variable in both regressions. For males, the degree of contribution to regression by the remaining four significant variables is about equal. The second most significant predictor for females is marital status, while traffic density and accident history provide comparable information in the presence of the other variables.

Table 11 displays the final estimation functions for λ and the estimates of the covariance matrix of the $\underline{\hat{\beta}}$ estimators. In Table 12, the values of $\hat{\lambda}$ and its estimated standard deviation are given for selected values of the criterion variables. Remember that the estimating equations and the estimates of λ found in the tables reflect a time unit of approximately 10½ months rather than 1 year.

8. DISTRIBUTION OF ACCIDENT INVOLVEMENT COSTS

Because of the rarity of the event of an accident in time and the extreme variability in accident costs, a theoretical distribution of accident costs appli-

Table 10

Unweighted Regression of 1963 Accidents on Six Criterion Variables
1964 California Driver Record Study

Males

Analysis of Variance

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F Value
Regression	6	108.4692	18.07820	215.19
Residual	86,463	7,263.9349	0.08401	
Total	86,469	7,372.4041		

i	b_i	s_{b_i}	$t = b_i/s_{b_i}$
0	-0.00211		
1	0.01023	0.00117	8.73
2	-0.00081	0.00265	-0.30
3	0.05746	0.00672	8.55
4	0.01980	0.00097	20.37
5	0.02251	0.00224	10.06
6	0.01768	0.00187	9.47

Females

Analysis of Variance

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F Value
Regression	6	21.7867	3.63112	88.93
Residual	61,118	2,495.6405	0.04083	
Total	61,124	2,517.4272		

i	b_i	s_{b_i}	$t = b_i/s_{b_i}$
0	-0.00857		
1	0.00794	0.00098	8.06
2	0.02099	0.00207	10.16
3	-0.00012	0.00074	-0.17
4	0.01832	0.00141	12.96
5	0.02097	0.00273	7.68
6	0.00356	0.00516	0.69

Table 11

Estimation Function for Accident Rate Potential and Covariance Matrix
1964 California Driver Record Study

Males

$$\hat{\lambda} = 0.00274 + 0.00909x_1 + 0.0532x_3 + 0.0223x_4 + 0.0216x_5 + 0.0169x_6$$

$$(X'\hat{V}^{-1}X)^{-1} = 10^{-4}\begin{bmatrix} 0.1981 & -0.0384 & -0.0754 & 0.0041 & 0.0021 & -0.0024 \\ -0.0384 & 0.0085 & -0.0004 & -0.0012 & -0.0013 & 0.0007 \\ -0.0754 & -0.0004 & 0.4058 & -0.0137 & -0.0057 & -0.0162 \\ 0.0041 & -0.0012 & -0.0137 & 0.0142 & -0.0052 & -0.0050 \\ 0.0021 & -0.0013 & -0.0057 & -0.0052 & 0.0654 & -0.0030 \\ -0.0024 & 0.0007 & -0.0162 & -0.0050 & -0.0030 & 0.0623 \end{bmatrix}$$

Females

$$\hat{\lambda} = -0.00176 + 0.00646x_1 + 0.0209x_2 + 0.0196x_4 + 0.0205x_5$$

$$(X'\hat{V}^{-1}X)^{-1} = 10^{-4}\begin{bmatrix} 0.0991 & -0.0211 & 0.0010 & 0.0016 & 0.0013 \\ -0.0211 & 0.0048 & -0.0015 & -0.0011 & -0.0011 \\ 0.0010 & -0.0015 & 0.0538 & -0.0045 & -0.0044 \\ 0.0016 & -0.0011 & -0.0045 & 0.0355 & -0.0089 \\ 0.0013 & -0.0011 & -0.0044 & -0.0089 & 0.1186 \end{bmatrix}$$

cable to a particular individual cannot be arrived at through the observation of that person's involvement costs over a period of time. Therefore, to gain information about costs applicable to a type of driver it is necessary to look at samples taken from a population of drivers. One such sample is the subject of the study entitled Cost of Motor Vehicle Accidents to Illinois Motorists, 1958,[18] and a subsequent analysis,[19] completed in cooperation with the U.S. Bureau of Public Roads. The passenger car portion is based upon a sample of 2,878 reported and 505 unreported accident involvements. A stratified sampling design was used with the sample size in each stratum determined on the basis of an accuracy level specifying an objective 7% relative error. In terms of a stratum mean $\bar{x}$ and its standard deviation s_x, this implies:

$$s_{\bar{x}}/\bar{x} = 0.07$$

[18] Illinois Department of Public Works and Buildings, State of, "Cost of Motor Vehicle Accidents to Illinois Motorists, 1958," Chicago, 1962.

[19] Billingsley, C. M. and Jorgenson, D. P., "Analyses of Direct Costs and Frequencies of Illinois Motor-Vehicle Accidents, 1958," *Public Roads 32*, pp. 201-213 (1963).

Table 12

Accident Rate Potential Estimates and their Standard Deviations
1964 California Driver Record Study

Sex	Traffic Density	Marital Status	Age	Ct. Conv. History	Accident History	No. Ct. Conv. History	$\hat{\lambda}$	$\sqrt{\widehat{Var}(\hat{\lambda})}$
Male	10	-	60	0	0	0	0.0293	0.0023
Male	50	-	60	1	0	1	.0831	.0027
Male	150	-	60	1	1	1	.1147	.0034
Male	10	-	40	2	0	0	.0781	.0032
Male	50	-	40	0	1	0	.0697	.0027
Male	150	-	40	0	0	0	.0581	.0011
Male	10	-	20	1	1	0	.1056	.0045
Male	50	-	20	0	0	0	.0763	.0035
Male	150	-	20	3	2	1	.2132	.0059
Female	10	Married	-	0	0	-	.0131	.0017
Female	10	Single	-	0	1	-	.0545	.0043
Female	50	Married	-	1	1	-	.0636	.0036
Female	50	Single	-	1	0	-	.0640	.0027
Female	150	Married	-	0	0	-	.0306	.0009
Female	150	Single	-	2	1	-	.1112	.0047

Since the accident data in the 1964 California Driver Record Study, cited earlier, refers to reported involvements, consistency dictates that we confine our attention to the 2,878 reported cases in the Illinois study. These were comprised of 332 fatal injury, 1,730 nonfatal injury and 816 property damage only cases. After appropriate expansion factors were applied, a "population" of 317,051 reported involvement costs was obtained. The distribution of these costs is given graphically in Figure 1.

In the Illinois study, direct costs are defined as "the money value of damages and losses to persons and property resulting directly from accidents, and which might be saved for the motor vehicle owner by the elimination of accidents."[20] Elements of direct costs include damaged property, injuries to persons, value of time lost, loss of use of vehicle, legal and court costs, and damages awarded in excess of costs. Funeral expenses in connection with a motor vehicle accident were not considered a direct cost since such costs are inevitable; an accident merely fixes the time when they are incurred. In evaluating direct costs in multiple car accidents, only those costs associated with the sample car and its occupants were obtained. However, damage to objects other than another motor vehicle, including pedestrians, was obtained.

The Illinois cost study spotlights the two most outstanding characteristics of accident cost distributions:

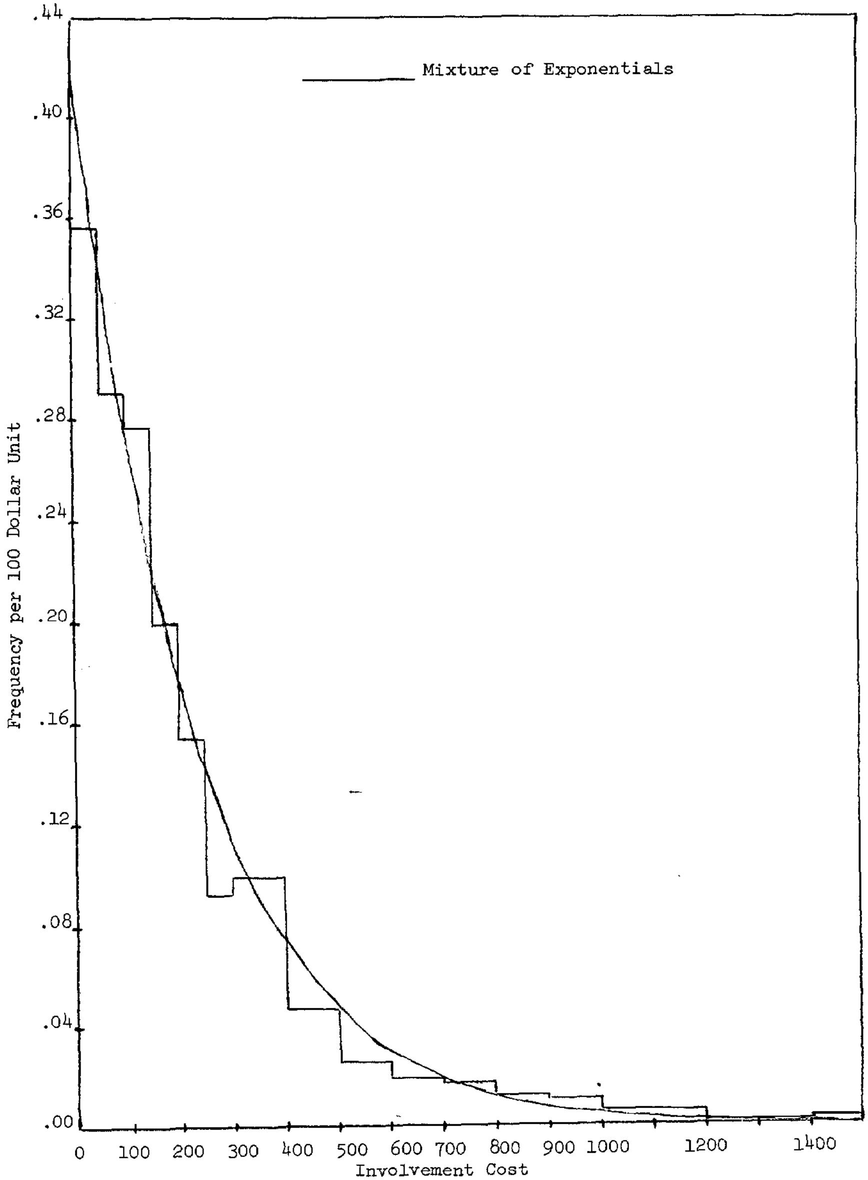
(i) The overall distribution is J-shaped, i.e., low cost accidents are most frequent and high cost accidents least frequent.
(ii) Accident costs depend on where the accident takes place (e.g., urban or rural, divided or undivided highway, intersection or freeway, etc.) and circumstances surrounding the accident (e.g., object struck, number of occupants, speed, etc.).

In an attempt to infer a theoretical distribution of accident costs based on the Illinois data, we must bear in mind that the empirical distribution displayed in Figure 1 is a "dangerous" one and should be taken at somewhat less than "face value." The *constructed* population of involvements is subject to bias through use of incorrect expansion factors in addition to possible large sampling error inherent in such a markedly skewed distribution as manifested by the descriptive statistics: a mean of 471 dollars, a variance of 3,760,963, and a median of 168. Nevertheless, it behooves us

[20] Billingsley, C. M. and Jorgenson, D. P., *ibid.*

Figure 1

Empirical and Theoretical Cost Distributions of a Single Accident Involvement
1958 Illinois Accident Cost Study



to accept the broad characteristics of this constructed distribution as indicative of the true distribution of accident involvement costs in Illinois during the year 1958.

Efforts were made to fit the Illinois data with well-known distributions of non-negative random variables such as the gamma and the lognormal without success. Among the candidates was a mixture of two exponentials:

$$w(x) = \frac{a}{\mu_1} e^{-x/\mu_1} + \frac{(1-a)}{\mu_2} e^{-x/\mu_2} \quad , \quad x \geq 0,\ \mu_1 > 0,\ \mu_2 > 0,\ 0 \leq a \leq 1$$

Using the method of moments described by Rider[21] and used by Dropkin,[22] we concluded that this distribution also seemed to be unsatisfactory. However, if we equate the sample median with the theoretical median in lieu of equating the unstable third moments, the fit appears to be quite reasonable. The derivation of this modified method of moments procedure is found in Weber.[23] Unfortunately, because of the use of a stratified sampling design, no goodness-of-fit test exists which would reveal whether or not the fit is statistically acceptable. The theoretical adaptation to the empirical distribution is shown graphically in Figure 1 and numerically in Table 13.

TABLE 13

Comparison of Empirical and Theoretical Cost Distributions
1958 Illinois Accident Cost Study.

$$W(x) = 1 - 0.9688e^{-x/231.9} - 0.03119e^{-x/7885.2}$$

x	Empirical Cumulative	Theoretical Cumulative
50	0.1783	0.1864
100	.3238	.3384
250	.6395	.6395
500	.8328	.8583
1000	.9229	.9595
2500	.9702	.9772
5000	.9874	.9835

[21] Rider, P. R., "The Method of Moments Applied to a Mixture of Two Exponential Distributions," *Annals of Mathematical Statistics 32*, pp. 143-148 (1961).

[22] Dropkin, L. B., "Loss Distributions of a Single Claim," *PCAS Mathematical Theory of Risk*, 1966.

[23] Weber, D. C., "A Stochastic Model for Automobile Accident Experience," Unpublished Ph.D. Dissertation. North Carolina State University, Raleigh, North Carolina, 1970.

Presumably, if reliable estimation procedures were available, we could improve the fit by increasing the number of exponentials in the mixture, i.e., let:

$$w(x) = \sum_{i=1}^{k} \frac{a_i}{\mu_i} e^{-x/\mu_i} \quad , \quad x \geqq 0, \mu_i > 0, a_i \geqq 0, \quad \sum_{i=1}^{k} a_i = 1 \quad ,$$

for some finite integer k. More generally, we can assume the μ's within the population have a continuous distribution in which case $w(x)$ can be viewed as a mixture of an infinite number of exponential distributions in the same manner that the negative binomial (5) is a mixture of Poisson distributions.

As a consequence, let us assume that the distribution of the cost of an accident involvement for an *individual* driver is exponential, i.e., the probability density function of the random variable X is assumed to be:

(14) $$g(x) = \frac{1}{\mu} e^{-x/\mu} \quad , \quad x \geqq 0, \mu > 0.$$

with mean μ and variance μ^2. The corresponding distribution function is:

(15) $$G(x) = 1 - e^{-x/\mu} \quad , \quad x \geqq 0, \mu > 0.$$

It is well-known that the sum of n independently and identically distributed exponential variables is a gamma variable. (See Feller.[24]) Hence, in terms of our model, the probability density function of $X_1 + X_2 + \cdots + X_n$ is:

(16) $$g_n(x) = \frac{x^{n-1}}{\mu^n (n-1)!} e^{-x/\mu} \quad , \quad x \geqq 0, \mu > 0, \quad n = 1, 2, \ldots ,$$

so that:

(17) $$G_n(x) = \int_0^x g_n(s) ds \quad , \quad x \geqq 0 \quad , \quad n = 1, 2, \ldots ,$$

$$= 1 - e^{-x/\mu} \left[1 + \frac{x/\mu}{1!} + \frac{(x/\mu)^2}{2!} + \ldots + \frac{(x/\mu)^{n-1}}{(n-1)!} \right]$$

and:

$$G_0(x) = \begin{cases} 0 & , \text{ when } x < 0, \\ 1 & , \text{ when } x \geqq 0. \end{cases}$$

[24] Feller, W., *An Introduction to Probability Theory and Its Applications Vol. II.* (John Wiley and Sons, Inc., New York, 1966).

9. AN ESTIMATION PROPOSAL

In this section we will consider the problem of estimating the parameter μ associated with a motorist as a function of his measurable characteristics. Our first inclination on this matter is to construct a model for μ as we did for λ in Section 5. Upon reflection, however, it is not possible to do so, at least with data currently available. Because of the unusually high variability in cost data, sample means have little reliability unless based upon a very large sample. In the face of this variability, the Illinois and similar cost studies are unable to give us concrete information about involvement costs by age and sex of driver, for example. This writer has, therefore, turned to the ratemaking procedures of the casualty insurance industry for a tentative answer to this problem.

As stated in the previous section, if one examines the findings of automobile accident cost studies, it soon becomes apparent that the primary determinant of cost is *location* conditioned by circumstances surrounding the accident. As a case in point, the Illinois study shows that the average involvement cost of an urban accident (one within an incorporate place) to be \$396 as compared to an average of \$931 for one taking place in a rural area. Therefore, to measure the potential cost of an involvement, as it concerns an individual, it is important for us to know where he incurs most of his accidents. Studies indicate, and current ratemaking procedures assume, that generally this is in the immediate vicinity of his residence. Hence, basic to a solution of our estimation problem is the establishment of involvement cost levels by area or territory. The area definitions need to reflect types of highways, population densities, speed limits, geographical and weather conditions, road safety conditions, etc., within a given area. In order to make use of the concept of resident area cost level, we initially assume that all drivers within a given area are characterized by the same μ, say, μ_a. Under this assumption an unbiased estimate for μ_a is given by the statistic $\bar{x}$ with variance μ_a^2/n, where $\bar{x}$ is the mean cost per accident involvement experienced by all drivers residing in the given area and n is the number of involvements upon which $\bar{x}$ is based.

Once an estimate for μ_a is obtained we should be able to assign a μ to each individual driver in that area by applying an appropriate involvement cost index, say, I based upon the characteristics of the motorist. Then the estimate for an individual's μ is given by:

$$\hat{\mu} = \bar{x} I \tag{18}$$

Members of the Casualty Actuarial Society will recognize the index factor *I* as a class differential. As in present automobile ratemaking procedures, the value of *I* applicable to a particular type of driver can be developed statistically on the basic of cost experience.

To illustrate the procedure, a New York Department of Motor Vehicles tabulation[25] classifies 477,101 accident involvements by severity class, age, sex, hour of day and day of week. The percentage distributions of severity class by age and sex found in this bulletin are given in Table 14. Using these distributions it is possible to arrive at an accident cost index by age and sex

Table 14

Distributions of Fatal Injury, Non-Fatal Injury and Property Damage Only Accident Involvements by Sex and Age Group
New York Motor Vehicle Bulletin No. 6 (64)

Males

Age Group	FI	NFI	PDO
Under 21	0.81%	52.60%	46.59%
21 - 24	0.70	56.06	43.24
25 - 29	0.64	57.22	42.14
30 - 39	0.56	57.43	42.01
40 - 49	0.48	55.89	43.63
50 - 59	0.53	54.33	45.14
Over 59	0.65	50.98	48.37
All Ages	0.60%	55.37%	44.03%

Females

Age Group	FI	NFI	PDO
Under 21	0.28%	53.59%	46.13%
21 - 24	0.30	55.56	44.14
25 - 29	0.30	58.34	41.36
30 - 39	0.27	57.32	42.41
40 - 49	0.28	54.48	45.24
50 - 59	0.33	52.40	47.27
Over 59	0.64	47.49	51.87
All Ages	0.31%	54.84%	44.85%

25 New York Department of Motor Vehicles, State of, "Fatal, Non-Fatal, and Property Damage Accidents by Age and Sex, Hour of Day, and Day of Week," *Statistical Bulletin No. 6* (64), Albany, 1964.

if we make assumptions about the relative cost of fatal, nonfatal, and property damage only involvements. Guided by the Illinois cost data, we might use as approximate ratios 30: 6: 1. Applying these weights to the distributions in Table 14 and then converting the results to index form, we find that the patterns displayed in Table 15 emerge.

TABLE 15

Constructed Involvement Cost Indices, I

Age Groups	*Males*	*Females*
Under 21	.985	.959
21 - 24	1.021	.985
25 - 29	1.032	1.021
30 - 39	1.028	1.005
40 - 49	1.003	.970
50 - 59	.986	.947
Over 59	.953	.907
All Ages	1.005	.977

Before taking a second look at our overall model, a few comments are in order. By keeping the proper statistics on cost experience, through the pooling of data (as is done today), the casualty insurance industry could come up with acceptable estimates for μ_a and I and, in time, test assumption (14). Consideration should be given to including factors other than personal characteristics in constructing the index I, e.g. age and make of the insured's automobile. As with parameter λ, the parameter μ is not constant in time. The cost of having accidents is heavily influenced by prevailing medical, material and wage cost levels. Therefore, it will be necessary to frequently update the estimates of the area μ's, and perhaps employ a trend factor when future costs are involved.

10. THE MODEL REVISITED

We established that the distribution function of $X(t)$, the total cost of accident involvements incurred during a time interval of length t, is given by:

(1) $$F(x,t) = \sum_{n=0}^{\infty} p(n,t)\, G_n(x) \quad , \quad x \geq 0, t > 0.$$

In our development, we have assumed that:

$$(3) \qquad p(n,t) = \frac{e^{-\lambda t}(\lambda t)^n}{n!} \quad , \quad n = 0, 1, 2, \ldots, \quad \lambda > 0, t > 0$$

and:

$$(17) \qquad G_n(x) = 1 - e^{-x/\mu}\left[1 + \frac{x/\mu}{1!} + \frac{(x/\mu)^2}{2!} + \ldots + \frac{(x/\mu)^{n-1}}{(n-1)}\right] \quad , \quad x \geqq 0, \quad n = 1, 2, \ldots,$$

$$G_o(x) = \begin{cases} 0 & , \text{ when } x < 0, \\ 1 & , \text{ when } x \geqq 0. \end{cases}$$

From (2), together with (3) and (14), we obtain the mean and variance of $X(t)$ as:

$$(19) \qquad E(X(t)) = \lambda\mu t \quad \text{and} \quad \text{Var}\,(X(t)) = 2\lambda\mu^2 t.$$

The probability density function of $X(t)$ is given by:

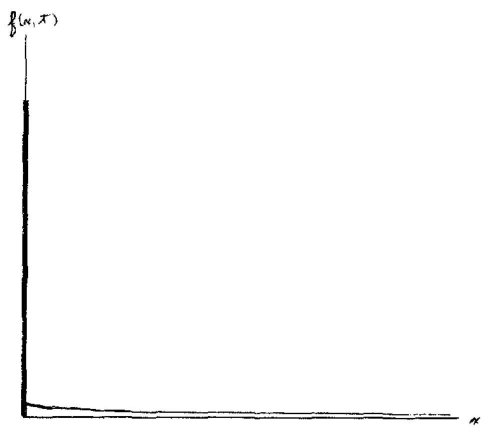
$$(20) \qquad f(x,t) = \begin{cases} \sum_{n=1}^{\infty} p(n,t)\, g_n(x) & , \quad x > 0, t > 0, \\ p(o,t) & , \quad x = 0, t > 0, \end{cases}$$

where $p(n,t)$ has the form (3) and:

$$(16) \qquad g_n(x) = \frac{x^{n-1}}{\mu^n(n-1)!}\, e^{-x/\mu} \quad , \quad x > 0, \mu > 0 \quad n = 1, 2, \ldots.$$

Figure 2 presents a sketch of $f(x,t)$ when t is small.

Figure 2
The Probability Density Function of X(t) for Small t



The graph illustrates that the random variable $X(t)$ is neither discrete nor continuous but rather is mixed. In the case where t is small much of its mass is concentrated at the point $x = 0$ with the remainder spread over the interval $0 < x < \infty$ according to the continuous function:

$$\sum_{n=1}^{\infty} p(n,t)g_n(x) \quad , \quad x > 0.$$

The sketch is exaggerated in that the plot of this continuous function actually is nearer the x-axis than it appears in Figure 2. Again we view the extreme skewness and dispersion that plagues accident researchers. These are the attributes that make it unrealistic to prĕdict the accident experience of an individual driver in anything other than probabilistic terms.

Under the assumption that $p(n,t)$ is a probability function for a Poisson process, the model (1) represents the distribution function of a compound Poisson process. Because of the general applicability of this particular stochastic process, it is discussed in many recent textbooks dealing with the subject of probability and stochastic processes.[26] An important property of the compound Poisson process distribution is its infinite divisibility. It follows that the sum of independent and identically distributed compound Poisson process variables is also a compound Poisson process variable. For us, this implies that the distribution of accumulated costs during one unit of time for k individuals having a common distribution function is the same as that for one of those individuals over a period of k units of time.

Let us consider the sum:

(20) $$S_k(t) = X_1(t) + X_2(t) + \ldots + X_k(t)$$

for a fixed t. If each $X_i(t)$ has $d.f.$ $F(x,t)$ and if the $X_i(t)$ are independent random variables, then the $d.f.$ of $S_k(t)$ is $F(x,kt)$. It follows that:

(21) $$\epsilon[S_k(t)] = k\lambda\mu t \quad \text{and} \quad \text{Var}\,[s_k(t)] = 2k\lambda\mu^2 t.$$

This gives us the capability of studying homogeneous groups of drivers as well as individuals. We note that the average cost per driver represented by the random variable $S_k(t)/k$ has mean and variance:

(22) $$\epsilon[S_k(t)/k] = \lambda\mu t \quad \text{and} \quad \text{Var}\,[s_k(t)/k] = 2\lambda\mu^2 t/k.$$

It can be shown that, for a fixed t, the random variable:

(23) $$\frac{S_k(t)/k - \lambda t\mu}{\sqrt{2\lambda\mu^2 t/k}}$$

[26] Feller, W., *op. cit.*

converges in distribution to that of a normal random variable with mean 0 and variance 1 as $k \longrightarrow \infty$. This, of course, is the central limit theorem in the context of our model.

11. ACCIDENT COST POTENTIAL

With respect to an individual driver, the occurrence of an accident involvement is, in general, an infrequent event. This, together with the variability associated with $X(t)$, means that the empirical average annual involvement costs experienced by a motorist, even if computed over a lifetime, do not adequately reflect the individual's driving skill, exposure in terms of mileage, and environmental driving conditions. Thus, we would expect average annual accident costs generated by two drivers of equal skill and identical exposure to be quite different.

O. Lundberg[27] considered the random variable:

$$Z(t) = X(t)/t$$

He found that:

$$\lim_{t \to \infty} F(zt,t) = \begin{cases} 0 & , \quad \text{when } z < \lambda\mu, \\ 1 & , \quad \text{when } z \geqq \lambda\mu. \end{cases}$$

This result may be interpreted that if we were able to observe a driver under the *same conditions* for many, many years, the distribution of his accident costs per unit of time converges in distribution to a constant, $\lambda\mu$. This "constant," which can be associated with each individual driver may be considered his *accident cost potential* and represents a theoretical cost per unit time. The use of quotation marks in the previous sentence emphasizes the point previously made, namely, that $\lambda\mu$ is not a true constant in that it is a function of the individual and his driving environment and, therefore, is subject to change in time. Although the accident cost potential associated with an individual is indicative of his expected accident costs, it does not uniquely characterize him in the sense that the product $\lambda\mu$ does not specify the individual's $F(x,t)$ uniquely. This is obvious from (21).

12. SOME EXAMPLES

In this section we will look at some probability distributions generated by our model for individual drivers and groups of homogeneous drivers. In Table 16 we find theoretical accident cost distributions related to indi-

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[27] Lundberg, O., *On Random Processes and Their Application to Sickness and Health Statistics* (Almquist and Wiksells, Uppsala, 1940).

viduals over a time span of one year. The tabulated values correspond to $P\{X(1) \leq x\}$ e.g. the probability that an individual characterized by $\lambda = .08$ and $\mu = 500$ will have total accident costs less than or equal to $500 during one year's time is 0.9706. This table demonstrates why it is so difficult to distinguish between "good" and "bad" drivers on the basis of experience over a short interval of time. For example, we expect some of the $\lambda = .04$ individuals to suffer accident loss during a year's time (about 4 percent); yet during that time about 85 percent of the individuals having an accident rate potential four times that of the first group will be cost-free.

Table 16

Evaluation of F(x,1) for Specified Values of the Parameters

$\mu = 500$

Total Costs x	$\lambda = .04$	$\lambda = .08$	$\lambda = .12$	$\lambda = .16$
0	0.9608	0.9231	0.8869	0.8521
50	.9645	.9302	.8971	.8652
100	.9678	.9366	.9063	.8771
250	.9760	.9524	.9294	.9068
500	.9853	.9706	.9559	.9413
1,000	.9945	.9888	.9282	.9767
2,500	.9997	.9993	.9990	.9986
5,000	.9999+	.9999+	.9999+	.9999+
E(X(1))	20	40	60	80
$\sqrt{Var(X(1))}$	141	200	245	283

$\lambda = .12$

Total Costs x	$\mu = 400$	$\mu = 600$	$\mu = 700$	$\mu = 800$
0	0.8869	0.8869	0.8869	0.8869
50	.8995	.8955	.8943	.8934
100	.9106	.9033	.9011	.8995
250	.9372	.9236	.9192	.9157
500	.9652	.9484	.9423	.9372
1,000	.9893	.9765	.9706	.9652
2,500	.9997	.9978	.9961	.9941
5,000	.9999+	.9999+	.9999	.9997
E(X(1))	48	72	84	96
$\sqrt{Var(X(1))}$	196	294	342	392

Table 17

Distribution Functions of $S_{100}(1)$ for Specified Values of the Parameters

$\mu = 500$

Total Costs $S_{100}(1)$	$\lambda = .04$	$\lambda = .08$	$\lambda = .12$	$\lambda = .16$
0	0.0183	0.0003	0.0000+	0.0000+
2,000	.5717	.1535	.0264	.0034
4,000	.9069	.5503	.2162	.0604
5,000	.9629	.7229	.3748	.1390
6,000	.9863	.8444	.5409	.2539
8,000	.9984	.9610	.8033	.5354
10,000	.9998	.9923	.9352	.7739
12,500	.9999+	.9992	.9880	.9323
E(S)	2,000	4,000	6,000	8,000
$\sqrt{Var(S)}$	1,414	2,000	2,449	2,828
E(S/100)	20	40	60	80
$\sqrt{Var(S/100)}$	14.1	20.0	24.5	28.3

$\lambda = .12$

Total Costs $S_{100}(1)$	$\mu = 400$	$\mu = 600$	$\mu = 700$	$\mu = 800$
0	0.0000+	0.0000+	0.0000+	0.0000+
2,000	.0538	.0147	.0090	.0060
4,000	.3748	.1295	.0815	.0538
5,000	.5803	.2407	.1581	.1070
6,000	.7503	.3748	.2589	.1813
8,000	.9352	.6425	.4944	.3748
10,000	.9880	.8337	.7070	.5903
12,500	.9990	.9500	.8790	.7844
E(S)	4,800	7,200	8,400	9,600
$\sqrt{Var(S)}$	1,960	2,939	3,429	3,919
E(S/100)	48	72	84	96
$\sqrt{Var(S/100)}$	19.6	29.4	34.3	39.2

Table 18

Distribution Functions of $S_{1000}(1)$ for Specified Values of the Parameters

Evaluated at $\begin{cases} E(S) - 3\sqrt{Var(S)} \\ E(S) - 2\sqrt{Var(S)} \\ E(S) - \sqrt{Var(S)} \\ E(S) \\ E(S) + \sqrt{Var(S)} \\ E(S) + 2\sqrt{Var(S)} \\ E(S) + 3\sqrt{Var(S)} \end{cases}$

$\lambda = .04,\ \mu = 400$	$S_{1000}(1)$	$F(x,1000)$
	5,267	0.0001
$E(S) = 16{,}000$	8,845	.0129
$\sqrt{Var(S)} = 3{,}578$	12,422	.1578
$E(S/1000) = 16$	16,000	.5223
$\sqrt{Var(S/1000)} = 3.58$	19,578	.8420
	23,155	.9690
	26,733	.9962

$\lambda = .08,\ \mu = 500$	$S_{1000}(1)$	$F(x,1000)$
	21,026	0.0003
$E(S) = 40{,}000$	27,351	.0159
$\sqrt{Var(S)} = 6{,}325$	33,675	.1582
$E(S/100) = 40$	40,000	.5158
$\sqrt{Var(S/1000)} = 6.32$	46,325	.8417
	52,649	.9712
	58,974	.9970

$\lambda = .12,\ \mu = 600$	$S_{1000}(1)$	$F(x,1000)$
	44,114	0.0004
$E(S) = 72{,}000$	53,410	.0172
$\sqrt{Var(S)} = 9{,}295$	62,705	.1584
$E(S/1000) = 72$	72,000	.5129
$\sqrt{Var(S/1000)} = 9.30$	81,295	.8416
	90,590	.9723
	99,885	.9973

One of our impressions about Table 16 might be that differences between successive distributions are trivial. Suppose we find the one year probability distributions associated with groups of 100 motorists where each individual within a group has the same λ and μ. In Table 17 we evaluate $S_{100}(1)$ for various combinations of λ and μ, remembering that the evaluation is the same as that for $X(100)$, the total accident costs acquired by an individual over a period of 100 years (assuming unchanging parameters). No longer do the differences between distributions appear inconsequential, but rather distinct differences in performance between groups of like individuals are apparent. The casualty insurance industry has recognized this, of course, through use of classification plans.

In Table 17 we observe that the standard deviation of mean accident costs, $\sqrt{Var(S/100)}$, is quite large relative to average costs, $E(S/100)$. To show how our predictions about average costs become more reliable as k is increased, in Table 18 we find distribution functions of $S_{1000}(1)$, i.e., for $k = 1000$ and $t = 1$. We also see how the distributions are approaching "normality" as indicated by the asymtotic distribution of the standardized $S_k(t)/k$ random variable displayed as (23).

13. RELEVANCY

At a time when proposals for no-fault automobile accident insurance plans have been introduced in the legislatures of New York and other states, perhaps it is time for the Casualty Actuarial Society to consider new techniques in the event of a universal change in state insurance laws. This writer has described a model which he believes is applicable in a no-fault insurance system.

DISCUSSION BY LESTER B. DROPKIN

Don Weber's paper, "A Stochastic Approach to Automobile Compensation," provides us with a most interesting approach to a subject of considerable current concern. If there were those who thought that the problem of pricing a "no-fault" automobile insurance system was still somewhat academic when the paper was presented last May, more recent events will have quickly brought the realization that the problem is now squarely in the forefront.

Whatever the case may have been at one time, today the unmodified term "no-fault" does not uniquely describe a single system. Rather, the various events which might initiate a claim, and the amounts of such claims, will vary substantially according to the particular specifications of particular systems. However, because the paper is basically presenting an approach — that is, a theoretical model — it is possible to consider the general aspects, which should have a fairly wide range of useful application, separately from the specific numerical facts and results, which may have more limited areas of pertinency.

Indeed, to this reviewer, one of the prime virtues of the model is that it may serve not only in connection with no-fault systems, but also in connection with the traditional third party liability system. Consider, for example, the claim frequency component. If it may be assumed that associated with each insured is a claim event parameter, λ; that λ is constant over the time period of interest; that claims occur according to a Poisson process; and that λ is a linear function of certain criteria variables — then the model is as equally useful in the one system as the other. Whether one is dealing with claim occurrences arising out of an involvement irrespective of fault or a liability claim, or whether he may be using a particular set of criteria variables in one case and another set in the other, in both instances the abstract structure — the model — is the same.

Although the objective of the author is the determination of the distribution of total costs, in fact the paper may be viewed as being composed of three distinct parts. The first part considers the claim frequency component; the second, the claim cost; the third combines the two components to arrive at total cost.

The concept of an insured having an inherent accident rate potential which should be an estimable function of some set of criterion variables, is not, of course, new. However, what the author does do in the present paper is to present, in a very neat blend of theoretical and practical work, a concrete example of a method which results in the setting out of an explicit functional relationship.

Since the reader may not be wholly familiar with the author's least squares procedure which is an extension of the classical least squares situation, it may be of help to the reader to point out two basic points. In classical least squares, a situation of equal variances is assumed. That is, $Var\ (N_j)$ would be equal to $Var\ (N_k)$ for the j^{th} and k^{th} individuals. This assumption no longer holds in the situation of interest here, and it is this absence of equal variances which results in the introduction of the diagonal matrix, V.

The second point is that, since we are dealing with a Poisson process, we know something about these variances along the diagonal of V; viz., that $Var\ (N_j)$ equals $\epsilon(N_j)$, which is, itself, being taken as a linear combination of the criteria variables.

There is one aspect of the treatment of the claim frequency component which this reviewer would have very much liked to have seen in the paper. The paper starts out with the observation that claim involvements follow a negative binomial distribution. This implies a particular distribution of λ in the population. Having subsequently worked out the functional relationship for determining λ, it should be relatively straightforward to compare the distribution of the λ's so determined with the Pearson type III distribution of the λ's underlying the negative binomial.

Many readers will find themselves on somewhat more familiar ground when the subject turns to the question of the distribution of the amount of a claim once the claim initiating event has occurred. Familiar, and yet not quite so. The concept that an observed size of claim distribution for a population may be the result of a mixing of individual exponential distributions is quite intriguing and deserves to be followed up; again, even outside of the immediate concern with "no-fault" systems.

Having separately determined the individual accident rate potential and claim cost distribution, the author proceeds to establish total costs along a well known path. In connection with this last part of the paper, we can note one of the advantages of a paper that combines its theoretical considerations with numeric data, viz., that the reader can get a real feel for what is going on and a real sense of how certain quantities change with changes in the values of the parameters.

This review would not be complete without the following comment: A paper of not inconsiderable size, utilizing mathematics, can be something of a chore to get through. We are therefore all the more appreciative of the fact that Don Weber's paper is well-written and most readable.

This is a paper which undoubtedly will be referred to often. It represents a most welcome addition to the *Proceedings*.

DISTORTION IN IBNR FACTORS

LEROY J. SIMON

The purpose of this actuarial note is to set forth the reasoning and the results involved in a small actuarial problem which occurs from time to time. By establishing a record in our *Proceedings* it is hoped that other actuaries will not have to go through the process of solving the problem independently each time it arises.

When a new line of business is introduced, policies are put on the books in a manner which cumulatively places more and more business in force. We would expect losses to be incurred roughly in proportion to the number of policies in force and, since there are more policies in force at the end of a given accounting period than at the beginning of the period, a factor which measures the incurred but not reported (IBNR) losses will be influenced (that is, distorted) by the relatively heavier weight of policies in force at the end of the accounting period. This same effect also occurs when one changes statistical plans and begins putting business on the books under a new program. It is in this latter vein that these investigations were conducted and this paper is written.

Figure 1 illustrates the build-up of one-year policies from the start of the new program (assumed to be January 1 of a year) until the entire block of business has been converted to the new program at the end of twelve months. The scale "Policies in Force" has been labeled to reach p at the end of the twelve-month period where p represents the proportion of all policies which are on a one-year basis. Figure 2 illustrates the build-up of three-year policies from the start of the new program until the entire block of business has been converted to the new program by the end of thirty-six months. The scale here has been labeled to reach q at the end of the three-year period. We are assuming that only one-year and three-year policies are involved so that $p + q = 1$. Note that it is also assumed that there are a very large number of policies which go on the books in a smooth and regular fashion.

Let us now define the IBNR factor, B, as all incurred losses of a given accident year evaluated at some subsequent date divided by the incurred losses for such accident year reported as of the close of the accounting period. This factor will be subscripted to denote the close of the accounting

FIGURE 1

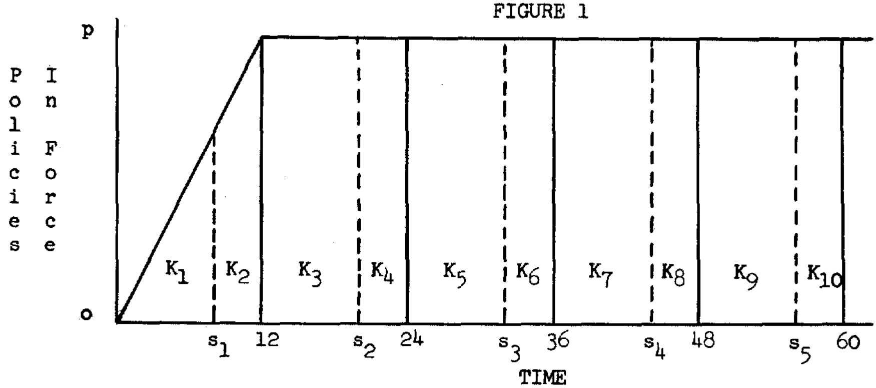
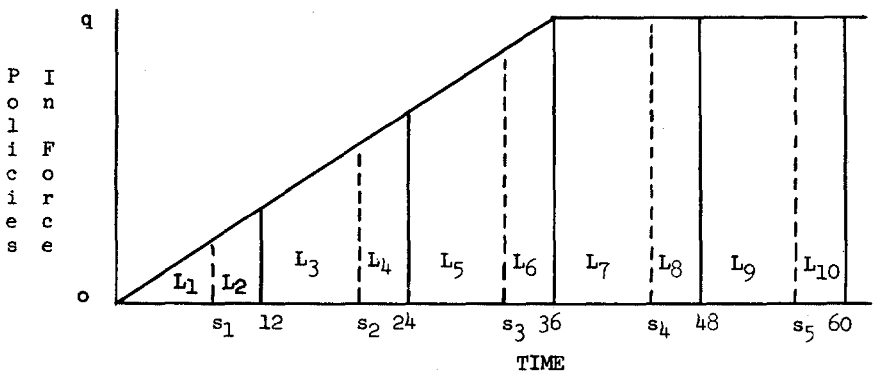


FIGURE 2



period such that B_{12} is the IBNR factor when accounts are closed after the twelfth month.

Our objective is to convert the factor B into an equivalent time period representing the portion of time that is missing from the losses. For example, a factor $B = 1.14$ will represent, under certain conditions, the loss of 1.5 months of incurred losses from the data. We will then assume that during some subsequent accounting period the same conditions will maintain and 1.5 months of losses will be missing from that accounting period also. Note that we are treating the 1.5 figure as a sharp line of demarcation whereas we know from experience that unreported losses will be distributed about this line to some extent. This is not felt to be an inhibiting assumption. These demarcation points are shown along the Time axis of the graphs and are labeled "s." Next we will set forth the formulas which express the area of each of the sections of the two graphs. Following this we will set forth formulas for IBNR factors in terms of these areas.

$$K_1 = ps_1^2/24$$

$$K_1 + K_2 = 6p$$

$$\left.\begin{aligned} K_i &= p[s_{(i+1)/2} - 6(i-1)] \\ K_i + K_{i+1} &= 12p \end{aligned}\right\} \quad i = 3, 5, 7, 9, \ldots.$$

$$L_1 = qs_1^2/72$$

$$L_1 + L_2 = 2q$$

$$L_3 = q(s_2^2 - 144)/72$$

$$L_3 + L_4 = 6q$$

$$L_5 = q(s_3^2 - 576)/72$$

$$L_5 + L_6 = 10q$$

$$\left.\begin{aligned} L_j &= q[s_{(j+1)/2} - 6(j-1)] \\ L_j + L_{j+1} &= 12q \end{aligned}\right\} \quad j = 7, 9, \ldots.$$

It is now possible to calculate the ratio of areas which represent the various IBNR factors and to solve the equations for the points of demarcation, s_i. The first case below is set forth more completely but for the other cases merely the results are stated.

$$\begin{aligned} B_{12} &= (K_1 + K_2 + L_1 + L_2)/(K_1 + L_1) \\ &= (6p + 2q)/(ps_1^2/24 + qs_1^2/72) \\ &= 144/s_1^2 \end{aligned}$$

$$s_1/12 = 1/\sqrt{B_{12}}$$

$$B_{24} = 432(2p + q)/(qs_2^2 + 72ps_2 - 144q - 864p)$$

$$s_2/12 = 1 + 1/B_{24} \quad \text{for } q = 0$$

$$s_2/12 = 3(1 - 1/q) + \sqrt{(-2 + 3/q)^2 + 3(-1 + 2/q)/B_{24}} \text{ otherwise}$$

$$B_{36} = 144(6p + 5q)/(qs_3^2 + 72ps_3 - 576q - 1728p)$$

$$s_3/12 = 2 + 1/B_{36} \quad \text{for } q = 0$$

$$s_3/12 = 3(1 - 1/q) + \sqrt{(-1 + 3/q)^2 + (-1 + 6/q)/B_{36}} \text{ otherwise}$$

$$B_i = (12p + 12q)/\{p[s_{i/12} - (i - 12)] + q[s_{i/12} - (i - 12)]\}$$

$$= 12[s_{i/12} - (i - 12)]$$

$$s_{i/12}/12 = (i - 12)/12 + 1/B_i \quad \text{for } i = 48, 60, \ldots.$$

Now let us define $B_{c:d}$ as the factor which one should apply to data evaluated at time "c" when the latest available IBNR factor is B_d. First we will consider the case where $c - d = 12$. This will be the condition when you have had an opportunity to evaluate the IBNR factor for a given year sometime during the subsequent year and are thus relatively current. (We will later consider the case of an organization which examines its data only once a year and, under the mode of operation, has a twenty-four-month difference between c and d.) The general procedure is illustrated by $B_{24:12}$ where one takes the above equation for B_{24}, puts $s_2 = s_1 + 12$, substitutes for s_1 its value in terms of B_{12} and simplifies. Under these conditions it is found that, given B_{12} one would use at month 24 the following factor:

$$B_{24:12} = 3(2 - q)B_{12}/[q + 2(3 - 2q)\sqrt{B_{12}}]$$

Similarly:

$$B_{36:24} = (6 - q)/[2\sqrt{(3 - 2q)^2 + 3q(2 - q)/(B_{24}} - 2(3 - 2q) + 3(2 - q)/B_{24}]$$

$$B_{48:36} = B_{36} \quad \text{for } q = 0$$

$$B_{48:36} = 1/[(1 - 3/q) + \sqrt{(1 - 3/q)^2 + (-1 + 6/q)/B_{36}}] \text{ otherwise}$$

For higher values of $B_{c:d}$, simply use B_d; that is, no distortion is present thereafter.

Finally, let's consider the case illustrated by a statistical agency which runs its data once a year and has need of the IBNR factor (corrected for distortion) prior to the running of the current year's data. This places $c - d = 24$ and results in the following:

$$B_{36:12} = (6 - q)B_{12}/[(6 - 2q)\sqrt{B_{12}} + q]$$

$$B_{48:24} = B_{24} \quad \text{for } q = 0$$

$$B_{48:24} = 1/[(2 - 3/q) + \sqrt{(2 - 3/q)^2 + 3(-1 + 2/q)/B_{24}}] \quad \text{otherwise}$$

$$B_{60:36} = B_{36} \quad \text{for } q = 0$$

$$B_{60:36} = 1/[(1 - 3/q) + \sqrt{(1 - 3/q)^2 + (-1 + 6/q)/B_{36}}] \quad \text{otherwise}$$

For higher values of $B_{c:d}$, use B_d.

To illustrate the magnitude of the functions and their behavior, we assumed $q = ½$ (which is approximately true for certain personal property lines during 1966-1969) and evaluated $B_{24:12}$, $B_{36:24}$, $B_{48:36}$, $B_{36:12}$, $B_{48:24}$ and $B_{60:36}$ as shown below:

B_d	$B_{24:12}$	$B_{36:24}$	$B_{48:36} = B_{60:36}$	$B_{36:12}$	$B_{48:24}$
1.00	1.0000	1.0000	1.0000	1.0000	1.0000
1.10	1.0543	1.0982	1.0916	1.0533	1.0899
1.20	1.1062	1.1963	1.1831	1.1042	1.1797
1.30	1.1560	1.2943	1.2745	1.1531	1.2694
1.40	1.2039	1.3924	1.3659	1.2001	1.3589

Over this range, these values are nearly linear and could be represented by

$$\hat{B}_{24:12} = .53\, B_{12} + .47$$

$$\hat{B}_{36:24} = .98\, B_{24} + .02$$

$$\hat{B}_{48:36} = \hat{B}_{60:36} = .92\, B_{36} + .08$$

$$\hat{B}_{36:12} = .51\, B_{12} + .49$$

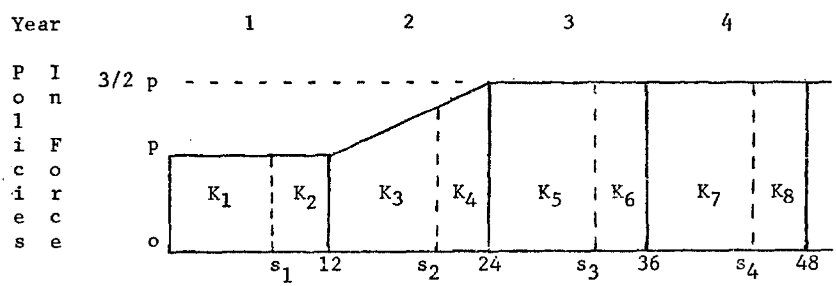
$$\hat{B}_{48:24} = .90\, B_{24} + .10$$

To summarize, we have established the simple formulas $\hat{B}$ above which permit one to remove the distortion caused in IBNR factors computed on an accident year basis when a new line of business or a new statistical program is introduced.

DISCUSSION BY CHARLES F. COOK

This actuarial note lives up to Mr. Simon's reputation for pertinent and handy mathematical models. In addition to providing a workable solution for a specific problem, it clearly illustrates the general technique of projection by geometric areas. This is a common method of modeling a variety of small actuarial problems, but one which is easy to mishandle in practice. Those actuaries who infrequently do this kind of thing should find it beneficial to follow the pattern of mathematical development in this paper, but substitute the precise values and formulas for their own problems as they go. For students, this paper is a "must."

When a new statistical plan or line of business is introduced, individual companies can usually develop proper IBNR reserves by combining data under a new statistical plan with those under the old plan until the new data base is matured. If a new line of business is a package, the same thing can be accomplished by temporarily combining package business with a proper mixture of closely related old lines. In the package case, the same procedure may sometimes be possible for a bureau or statistical agent. Such a treatment, which has long been used, is perfectly reasonable, and Mr. Simon's method is not required in these cases. Combination procedures, however, can only be applied if comparable prior data are available. They generally do exist for an individual company (except in the case of a completely new line of business), but at the bureau level this situation is less likely. Comparable prior data are absent not only for new bureaus, new policies, or new statistical plans, as cited by Mr. Simon, but more generally any time there is a dramatic increase in the volume of data being reported. If, for instance, several new companies began reporting one-year term business on January 1 of year 2, with a volume of policies equal to one-half of the previous total, we could sketch the situation in the notation of this paper as:



Here K_4/K_3 is equal to L_6/L_5 in Figure 2 of Mr. Simon's paper, so $B_{36:24}$ would be equal to his $B_{48:36}$ with $q=0$. The value of $B_{36:12}$, however, would be B_{12}, because $K_2/K_1 = K_6/K_5$, and the formula for $B_{24:12}$ would have to correct for a distortion *opposite* to the one treated in the paper. The point of this little example is to show that this paper has usefulness beyond its declared purpose of solving a single specific problem. A formula for $B_{24:12}$ in the case above, which arises from a different source and has an opposite bias, could easily be determined using Mr. Simon's problem as a guiding model, following his procedure but substituting the appropriate formulas for *K's* and *s's* determined from the drawing of this problem.

Other Types of IBNR Base

I have a strong personal preference for earned premium or premiums in force (rather than reported losses) as an IBNR base, because they have less random variation. My preference has been reinforced by this paper for a new reason. If the IBNR factor B_d' were defined as the ratio of unreported losses as of the close of the accounting period, to either the premiums in force at the end of the period or the premiums earned in the final quarter of the period, reference to the drawings makes it clear that the distortion would be far less. The correction of the distortion would also be easier. Let us assume, for example, that we are trying to evaluate B_{36}' and that we have determined that the unreported losses were U and the reported losses R. Then $s = \sqrt{\dfrac{R}{2(R+U)}}$. If the premiums in force were P_1 at the beginning of the year and P_2 at the end of the year, they averaged

$$[P_1 + P_2 + s(P_2 - P_1)] \div 2$$

during the same period, from s to the end of the year, as the unreported claims were incurred under Mr. Simon's assumptions. Then

$$B' = \frac{2U}{P_1 + P_2 + s(P_2 - P_1)}$$

is an IBNR factor which can be applied to *any* mix of one and three year terms, at *any* point in time, on the base $P_1 + P_2 + s(P_2 - P_1)$ where P_1 and P_2 are defined as above, for the year to which B' is to be applied. This formula will *exactly* reproduce Mr. Simon's results. Now we know that in fact the average date of occurrence for our unreported claims is earlier than $(s + 12) \div 2$, as we assumed above. If we have found (or estimated) it to be A, then $B' = \dfrac{U}{P_1 + A(P_2 - P_1)}$ will be a *better* IBNR factor. This alternative but similar procedure does not take anything away from Mr. Simon's solution, because it is not compatible with a purely pure premium ratemaking method. (It makes total incurred losses partially dependent on the level of premium charged.) However, for calculating reserves at the company level, for a loss-ratio type ratemaking procedure, or for instances where Mr. Simon's assumptions are not valid, it does have some virtues worth considering.

Test of the Procedure

Mathematically, the procedure given by the author is exact to the extent his assumptions hold true. He has avoided critical assumptions very well, only one being significant — the assumption that the IBNR consists of the latest claims in the accident year. Mr. Simon gave a clear discussion of this point, concluding that: "This is not felt to be a limiting assumption." In order to test this assumption independently of the random variations which always occur in real-world applications, I compared the predictions of Mr. Simon's model against "artificial" results built by a model essentially identical to his except that the average actual[1] distributions of accident month by report month for the United Services Automobile Association for 1969 were applied uniformly to each month of the model. The build-up of the in-force followed his model exactly, and the loss distributions were adjusted to eliminate growth. This model was then run through a computer, yielding a set of IBNR factors at 12, 24, 36, and 48 months. Mr. Simon's formulas were then applied to "predict" the later IBNR factors from the earlier ones, and his projections were compared to the "actual" results. Because the loss distributions were constant over time and all other assumptions were identical, random and systematic prediction errors were eliminated. Thus the only source of "error" in this test was the bias resulting from disregarding the actual distribution of IBNR accident dates. The results of three tests for Homeowners for various proportions of three-year business are shown in Tables 1, 2, and 3.

Careful consideration of Mr. Simon's Figures 1 and 2 leads to the conclusion that these results can be generalized beyond the specific data used. He has treated all IBNR losses as occurring in the latest possible time period, when the policies in-force are at their maximum. Because the ratio of

[1] Only claim frequency was considered, for stability. All claims not reported by the twelfth month after the close of the accounting period were assumed to be reported in the eighteenth month.

TABLE 1
Homeowners with q = 0.2

Test Item	Test Value	True Value	Pct. Error
$B_{24:12}$	1.1678	1.1879	−1.69
$B_{36:24}$	1.1874	1.1875	−0.01
$B_{48:36}$	1.1812	1.1824	−0.10
$B_{36:12}$	1.1674	1.1875	−1.69
$B_{48:24}$	1.1812	1.1824	−0.10

TABLE 2
Homeowners with q = 0.5

Test Item	Test Value	True Value	Pct. Error
$B_{24:12}$	1.1801	1.1989	−1.57
$B_{36:24}$	1.1952	1.1959	−0.06
$B_{48:36}$	1.1794	1.1824	−0.25
$B_{36:12}$	1.1767	1.1959	−1.61
$B_{48:24}$	1.1787	1.1824	−0.31

TABLE 3
Homeowners with q = 0.8

Test Item	Test Value	True Value	Pct. Error
$B_{24:12}$	1.1989	1.2159	−1.40
$B_{36:24}$	1.2032	1.2054	−0.18
$B_{48:36}$	1.1775	1.1824	−0.41
$B_{36:12}$	1.1872	1.2054	−1.51
$B_{48:24}$	1.1755	1.1824	−0.58

policies in-force to the 12-months-earlier policies in-force decreases monotonically in the model, this leads to projecting the *minimum* possible increase in IBNR and the *maximum* possible increase in reported cases. Any deviation from this — any IBNR loss which is earlier in time than *any* reported loss — should get a higher "leverage" in its projection to a later date than it gets from Mr. Simon's model. Therefore, although they are reasonably close, his IBNR factors are biased downward. That is, IBNR will tend to be consistently, although slightly, underestimated. In the case of projections

$B_{c:24}$ and $B_{c:36}$, the magnitude of the distortion remaining after correction by the Simon formulas is negligible, but for the factors $B_{c:12}$ it is sufficient to be disturbing. An underestimate in incurred losses of about 1.5% translates into a lot of dollars in rates. It would therefore be reasonable, if permitted, to *add .01 to the formulas for* $B_{24:12}$ *and* $B_{36:12}$. This is sufficiently overjustified by USAA data (15,581 claims), which indicate an adjustment of +.015, so that one can be confident that it would be *at least* .01 for broader-based accident month/report month distributions.

Scope of Application

This paper was oriented specifically to personal property lines, for an accident year valued as of 12 months (immediately at the close of the year). It is interesting to investigate whether the procedure will work acceptably well in other lines or at other valuation dates. I repeated the same test discussed above four more times — for automobile liability (BI and PD combined) and for automobile physical damage valued at 12 months, and for automobile physical damage and homeowners valued at 15 months (considering losses for a calendar-accident year as IBNR only if they were still unreported as of the following March 31). The results of these tests are shown in Tables 4 and 5.

TABLE 4

$B_{24:12}$ (q = 0)

Test Item	Test Value	True Value	Pct. Error
Auto liability	1.1099	1.1360	−2.30
Auto physical damage	1.1043	1.1226	−1.63
Auto physical damage ($B_{27:15}$)	1.0131	1.0177	−0.45

TABLE 5

Homeowners with q = 0.5
Evaluated at 15 months

Test Item	Test Value	True Value	Pct. Error
$B_{24:12}$	1.0257	1.0331	−0.72
$B_{36:24}$	1.0325	1.0328	−0.03
$B_{48:36}$	1.0301	1.0315	−0.14
$B_{36:12}$	1.0252	1.0328	−0.74
$B_{48:24}$	1.0298	1.0315	−0.16

These results confirm the position that Mr. Simon's procedure is sensitive to delays in reporting claims which increase the "spread" of IBNR accident dates. This is especially clear in the 15-month valuation tests. Such later valuations eliminate the vast majority of IBNR claims, which are reported reasonably promptly after occurrence. The remaining IBNR have a much less compact distribution over time and as a result, although the total errors in incurred losses are reduced because there is less total IBNR, the error becomes quite large when compared to the IBNR itself. On this basis the error for automobile physical damage is -26.0% and for $B_{24:12}$ in homeowners it is -22.4%.

Conclusion

Mr. Simon has produced an adequate procedure for controlling the distortion in IBNR factors for personal property insurance during the period following introduction of a new policy, bureau, or statistical plan. He has also provided a set of very simple linear equations to predict IBNR factors under the conditions for the specific problem at hand i.e., a 50-50 mix of one and three-year term policies which eliminate the need for any further significant effort to solve the particular problem.

The results are not perfect. Based on a partially simulated and partially real model, the estimates appear to have a downward bias. In the case of prediction from first-year reports it is about -1.5% which might be considered a significant understatement of losses. The mixture of policy terms makes this particular problem rather complex, however, and his result is certainly far better than the results of other, simpler procedures.

The apparent inability of Mr. Simon's soundly conceived, rather complex procedure to eliminate distortion more completely only highlights the need for better models of the loss development phenomenon. Certainly these test results detract nothing from the paper or the author's workmanship. It is a significant step toward more sophisticated actuarial forecasting of ultimate loss levels.

DISCUSSION OF PAPER PUBLISHED IN VOLUME LV

AN ACTUARIAL NOTE ON ACTUARIAL NOTATION

JEFFREY T. LANGE

VOLUME LV, PAGE 196

DISCUSSION BY LEWIS H. ROBERTS

A paper on actuarial notation in casualty and property insurance is welcome because it forces us to take stock of the basic language of our profession — an important subject that we usually ignore because of the press of more immediate problems. Compact and consistent notation is valuable not only because of its role in communication but also because it assists us in developing our bits and pieces of actuarial knowledge into a coherent and systematic science. In the words of Ernst Mach: "Strange as it may sound, the power of mathematics rests on its evasion of all unnecessary thought and on its wonderful saving of mental operations."[1]

It goes without saying that actuarial notation should be a tool for avoiding confusion, not causing it, yet it has sometimes seemed to this reviewer that the notation used by casualty actuaries worked more in the latter direction. Attempts to read some of the most important theoretical material in our literature are often stymied by difficulty with unique and complex notation. If the authors could have used a familiar, standardized notation, their contributions would have been much more vividly understood and appreciated by their colleagues and by students. In actuarial science, as in other branches of mathematics, "the medium is the message."

Notwithstanding the suggestion by the Committee on Terms, Definitions and Symbols,[2] it may not be the youth of casualty actuarial science that has prevented a stable notation, but at least two other reasons. One is its breadth of scope. It embraces not only the life functions — as in workmen's compensation and accident and health insurance — but mathematical statistics and other fields of applied mathematics. It is natural for notation

[1] E. T. Bell, *Men of Mathematics,* Simon and Schuster, New York, 1961.
[2] *PCAS* Vol. I, p. 76 and Vol. II, pp. 163, 317, and 497.

in casualty actuarial calculations to adopt the conventions of these disciplines but these conventions leave much to individual preference or whim.

A second reason is that few analogues of the complicated standard functions characteristic of life insurance exist in casualty insurance. We do have a variety of elements that can be put together in different ways to form "pure premiums," "loss ratios," etc., but the algebraic need for the compactness and expressiveness of life notation is weak in most routine calculations.

The author's discussion of notational problems in relation to computer programming is timely and well considered. I take one very small exception to his remarks in noting that at least literary Algol admits use of small letters as well as capitals. The only Algol compiler used by our staff, however, permits only capitals.

Adding to what Lange has said, I would urge that any committee appointed to work on notation for programming include, or at least consult with, a professional programmer in order to avoid unnecessary pitfalls. Options that may seem trivial from a formal or mathematical standpoint in a programming language can make a great deal of difference in core memory requirements and running time. To take one of the author's examples, AT5A2 (X, N1, N2) is preferable to AT5A (2, X, N1, N2) because each additional index or argument materially increases computing time. The more information that can be contained in the name itself, the faster a program will run. That this is not a trivial difference can be seen from the following table based on IBM 1130 execution times:

Designation of Variable	Example	Additional Microseconds Relative to an Unsubscripted Variable
No subscripts	A	0
Constant subscript	A(3)	25
One variable subscript	A(K)	280
Two variable subscripts	A(K,M)	390
Three variable subscripts	A(I,J,K)	530

In a one-shot program this is meaningless, of course, but on long production runs such differences can add up to hours.

In conclusion, I suggest that one of the strongest arguments for standardizing notation lies in computer applications, where it could aid materially toward improving the accuracy and speed of programming.

DISCUSSIONS OF PAPERS PUBLISHED IN VOLUME LVI

A REVIEW OF THE LITTLE REPORT ON RATES OF RETURN IN THE PROPERTY AND LIABILITY INSURANCE INDUSTRY

ROBERT A. BAILEY

VOLUME LVI, PAGE 133

DISCUSSION BY RUSSELL P. GODDARD

Anyone familiar with the insurance business must sense intuitively that there is something unreal about the two reports by Arthur D. Little, Inc. on the property and liability insurance industry. The first report, which will be called in this review, for brevity's sake, "Prices and Profits," said that the business was "underearning" and the second report, "Rates of Return," said that the industry's rate of return of 3.634% fell below the average interest rate paid by most savings banks.

Statements or implications like these raise more questions than they answer. How long has this been going on? What caused it? What will cure it? Is the industry in as bad shape as the passenger railroads? Does it need a government subsidy? How long can it go on? How long can an industry starve (i.e., underearn) without starving to death? Will it be only a question of time before the privately employed actuary and underwriter follow the same path as the farrier and the horologer?

Mr. Bailey has accurately put his finger on the cause of the confusion: it is the base to which the rates of return are related. He examines this base, known in the ADL reports as *D2*, and which will be called the "double denominator" in this review, and concludes that the measure involving it "produces a result useless to everyone" and that it is "biased in such a way that it will show the highest rate of return for an insurance company that does no insurance business."

I concur with Mr. Bailey's conclusions with respect to ADL's methods of calculating rates of return, and propose in this review to support these conclusions by another approach, and to attempt to point the way to an answer to one or two of the questions raised by the ADL reports. Since I am primarily interested in determining rates of return to stockholders, I will refrain from comment on those parts of Mr. Bailey's paper which refer to lower premiums and increased loss payments which accrue for the benefit of policyholders.

With respect to the double denominator measure espoused by ADL, the preference for this device was explained in the Prices and Profits report (page 40):

> "Consider a steel mill. It is a tangible asset of bricks and steel. Should it be destroyed, society is less rich by the amount of assets that comprised the mill, no matter how these assets were financed. Should they have been financed by bonds rather than common stock, the loss of the economy would be equally great."

Throughout both reports there are many references to society or to social values. It is difficult to conceive of an arithmetical device which will measure an industry's contribution to society in terms which will be satisfactory to everyone, so possibly the best way to determine the relative usefulness of the new double-denominator measure would be to apply it to an actual case.

A typical insurance company during recent years might be fortunate enough to earn 6% on its invested assets. If it had what ADL refers to as a "50% levered portfolio" (i.e., a one-to-one ratio of reserves to net worth) and sustained a statutory underwriting loss of 1%, it would show a return on net worth of 11%. $(.06 + .06 - .01 = .11)$ The rate of return determined by the ADL method would presumably be .055 since the denominator would consist of 1 for net worth and 1 for premiums.

Now another company with a more conservative investment portfolio might feel able to write a larger volume of premiums in proportion to its net worth. Suppose that it earned only 5% on its invested assets and had a premium volume of three times its net worth. It writes at the same rates as the first company and experiences the same underwriting loss, 1%. Its return on net worth is 17%,

$$\frac{.05 + 3 \times .05 + 3\,(-.01)}{1} = .17$$

but its rate of return under the double-denominator method is .0425,

$$\frac{.05 + 3 \times .05 + 3\,(-.01)}{1 + 3} = .0425$$

As a matter of arithmetic, it will be seen that the double-denominator rate of return is merely the rate of interest earned on invested assets modified by the ratio of underwriting profit to invested assets. It does not tell anyone anything that he wants to know, or in Mr. Bailey's words, "produces a result useless to everyone."

In the illustration above, the second company is obviously providing more revenue to its stockholders, and as for its value to society, it is writing three times as much insurance (and taking three times as much risk) as the first company, at the same rates. And yet on the double-denominator basis, which is invoked in the name of society, it receives a lower mark than the first company!

The ADL reports do not adequately explain why a special measure had to be devised to compare insurance with other industries. For these other industries, the rates of return were computed on net worth in the Prices and Profits report, and on "total assets less current liabilities" in the second report, with average rates of return of about 10% in each case. It would be virtually impossible for a single insurance company, much less an entire industry, to reach a 10% rate of return on the double-denominator basis.

The ADL reports do not answer the question, "How long has this been going on?" since they provide rates of return for only the 13 most recent years. The figures in the Rates of Return report were all taken from Best's Aggregates and Averages and it is a fairly simple job to extract comparable figures for a longer period. The method used in this review (See Exhibit) must be substantially the same as that used by ADL because during the 13 years covered by ADL, the results are very similar. Such differences as do exist may be due to the fact that ADL deducted "current taxes" whereas no attempt has been made to deduct them in the tabulations included here:

STOCK COMPANIES

Year	ADL N4/D1	RPG Total Return on Net Worth	Difference
1967	15.0	15.8	0.8
1966	−5.7	−3.1	2.6
1965	6.7	6.8	0.1
1964	9.9	10.0	0.1
1963	13.3	13.4	0.1
1962	−3.5	−1.8	1.7
1961	20.3	21.0	0.7
1960	5.7	6.6	0.9
1959	9.9	10.5	0.6
1958	21.6	21.5	−0.1
1957	−8.0	−6.0	2.0
1956	4.3	5.0	0.7
1955	15.1	17.1	2.0

For convenience, the subtotals in the Exhibit are summarized here.

Calendar Year	Total Return	Investment	Underwriting
1966-68	8.7	8.9	−0.2
1956-65	8.9	10.1	−1.2
1946-55	14.3	10.8	3.5
1936-45	10.5	7.3	3.2
1926-35	8.1	6.7	1.4

In reviewing these figures, as well as the year-by-year figures in the Exhibit, it is of interest to keep in mind the following dates:

SEUA decision, June 5, 1944
McCarran Act passed March 9, 1941
McCarren Act effective June 30, 1948

The largest underwriting profit of any single year came in 1945, when it was 10.5% of net worth, or 9.5% of earned premiums. The best underwriting period was the eight-year stretch beginning in 1948, although the ten-year period ending in 1945 was almost as profitable. The underwriting results since 1955 should probably be interpreted in the light of the Stanford report, indicating the increased competition from the so-called direct writers, not yet fully met by expense reductions on the part of stock companies.

The Exhibit also gives some clue to the cause of the "apparent riskiness of the insurance industry" mentioned in the second ADL report. It should be noted that the biggest year-to-year fluctuations occur in investments rather than in underwriting. It must be granted that the widest fluctuations probably occur because of the inclusion of unrealized gains and losses and that many would disappear if a two-year moving average were used. Without studying the matter in depth, one may assume that the variations arise primarily from the stock market, rather than from bonds.

We conclude, along with Mr. Bailey, that the measure recommended in the ADL reports, the *N4/D2*, cannot possibly serve any useful purpose either in comparing one insurance company with another, in comparing records of a company at two periods of time, or in comparing the insurance industry with any other industry.

EXHIBIT

Returns on Net Worth*
Including Realized and Unrealized Gains and Losses
Data from Best's Aggregates and Averages
Stock Companies Only

Calendar Year	*Total Return*	*Invest-ment*	*Under-writing*	*Calendar Year*	*Total Return*	*Invest-ment*	*Under-writing*
1968	12.9	14.1	−1.2				
1967	15.8	15.8	0.0				
1966	−3.1	−3.8	0.7				
1966-68	8.7	8.9	−0.2				
1965	6.8	9.6	−2.8	1945	16.9	15.8	1.1
1964	10.0	12.4	−2.4	1944	13.8	11.4	2.4
1963	13.4	15.0	−1.6	1943	18.3	12.5	5.8
1962	−1.8	−1.8	0.0	1942	6.4	3.4	3.0
1961	21.0	20.8	0.2	1941	3.8	1.6	2.2
1960	6.6	6.0	0.6	1940	5.3	2.4	2.9
1959	10.5	9.8	0.7	1939	9.9	6.0	3.9
1958	21.5	22.5	−1.0	1938	15.6	11.0	4.6
1957	−6.0	−1.9	−4.1	1937	−9.3	−13.2	3.9
1956	5.0	6.5	−1.5	1936	20.2	16.9	3.3
1956-65	8.9	10.1	−1.2	1936-45	10.5	7.3	3.2
1955	17.1	14.0	3.1	1935	23.0	18.4	4.6
1954	28.5	22.9	5.6	1934	5.4	1.6	3.8
1953	10.0	4.5	5.5	1933	11.9	7.5	4.4
1952	13.1	9.8	3.3	1932	−0.8	−0.6	−0.2
1951	10.9	10.6	0.3	1931	2.6	3.2	−0.6
1950	17.0	12.9	4.1	1930	−7.8	−6.8	−1.0
1949	23.6	13.1	10.5	1929	5.3	3.9	1.4
1948	9.9	4.3	5.6	1928	17.9	14.6	3.3
1947	1.8	3.2	−1.4	1927	20.4	18.7	1.7
1946	−4.8	−0.3	−4.5	1926	7.9	11.4	−3.5
1946-55	14.3	10.8	3.5	1926-35	8.1	6.7	1.4

*Net worth is the sum of policyholders' surplus plus prepaid commissions and taxes.

DISCUSSION BY RICHARD NORGAARD AND GEORGE SCHICK*

Robert A. Bailey, in his article, "A Review of the Little Report on Rates of Return in the Property and Liability Insurance Industry," has examined the basic A. D. Little (ADL) equation.[1,2] The equation:

$$\text{Return} = \frac{\text{net income}}{\text{net worth} + \text{loss and premium reserves}} \qquad (1)$$

is the basis for their conclusion that the insurance industry is unprofitable. Bailey gives cogent, logical arguments why the profit equation used by ADL has serious shortcomings, and why it substantially understates the actual rate of return.

Although the ADL report is a study of risk and return, risk is a function of return so the return equation is all important. Its importance is enhanced when ADL uses it in the absolute sense for evaluation. For example, ADL compares their return for insurance companies with the return earned on savings deposits, stock market, and among industries. In focusing entirely on return, Bailey has done us a service since other critics have given primary attention to the techniques for measuring risk and sampling.

In examining equation (1), Bailey has forcibly demonstrated the weakness of the ADL conclusions by showing how weak the basic equation is. Many of us have noticed this problem. When we were orginally attempting to find an acceptable method for determining a rate of return, the first thing we examined was the return on investment or ROI.[3] This is the best known

*Professor Norgaard, now of the University of Connecticut, and Professor Schick, of the University of Southern California, were joint guest reviewers of Mr. Bailey's paper. They are best remembered for having launched the first public attack on the ADL Report before the Hart Subcommittee in the U.S. Senate.

[1] Arthur D. Little, Inc., *Prices and Profits in the Property and Liability Insurance Industry,* Report to the American Insurance Association, dated November 1967, but available June 1968.

[2] Arthur D. Little, Inc., *Rates of Return in the Property and Liability Insurance Industry: 1955-1967,* Report to The National Association of Independent Insurers, dated June 1969.

[3] Richard L. Norgaard & G. J. Schick, "Profitability in the Property and Liability Insurance Industry," and "Analysis of Profit Trends in the Property and Liability Insurance Industry," *The Insurance Industry,* Hearings before the Subcommittee on Antitrust & Monopoly, Vol. 14, Washington, U.S. Government Printing Office, July 1968.

profit ratio where:

$$\text{Return} = \text{ROI} = \frac{\text{net income} + \text{fixed charges}}{\text{net worth} + \text{fixed debt}} \tag{2}$$

This is the basis for equation (1). As commonly used ROI is based on book values. We rejected this for, when used in an inter-corporate study, substantial distortion is created because of the different techniques used in accounting by insurance companies compared to other companies. Our solution to the problem was to use market values. The ADL solution as Bailey noticed was to ignore the problem.

Nevertheless, the use of book values in the ROI equation can be an acceptable technique for comparison, if the differences between insurance and non-insurance companies are carefully considered. ADL in their equation have decided to include the loss and premium reserves in their denominator but no imputed earnings. The results, as we know, give insurance companies ridiculously low earnings. This technique so distorts actual results that stocks prove least unprofitable, mutuals more unprofitable, and reciprocals the most unprofitable. Bailey notices these inconsistencies and shows why they come about. The reason is that both loss and premium reserves have implicit income. If the implicit income is ignored, and Bailey thinks it should be because it is unmeasurable, then the reserves themselves should also be ignored. The resulting equation for insurance companies is:

$$\text{Return} = \text{net income}/\text{net worth}. \tag{3}$$

When Bailey readjusts the ADL figures to reflect equation (3), he finds mutuals slightly more profitable than stocks and the overall profit rate slightly less than the average for all industries. In effect Bailey finds that the correct value for the ADL report is approximately the value we have given it in our report.

While we have no criticism of Bailey's approach and conclusions we regret that he has not included three important points:

(1) He has failed to mention the work of others dealing with this problem. For example, both Hofflander and Mason,[4] and Hammond

[4] A. E. Hofflander & R. H. Mason, "Prices and Profits in the Property and Liability Insurance Industry," Review, *Journal of Risk and Insurance*, June 1968.

and Shilling[5] have discussed this problem. Bailey also ignores our discussion of this problem.

(2) While Bailey's approach may be an improvement over ADL's in that it tends to correct some of the ADL understatement of profits, he gives us no ideas whether his adjustments also distort the actual comparison.

(3) Bailey gives no hint as to why ADL has been able to sell its understated income concept so easily to the insurance industry. While we would like Bailey's opinion on this point we can readily understand his reluctance to give it.

DISCUSSION BY J. ROBERT FERRARI

In his paper reviewing the most recent Arthur D. Little (ADL) Report commissioned by the N.A.I.I., Bailey seems to have as his basic objective the development of a rationale for calculating return for property and liability insurance companies as

$$\frac{\text{Net income}}{\text{Net worth}}$$

rather than ADL's preferred approach, which is

$$\frac{\text{Net income}}{\text{Net worth and reserves}}$$

The two ratios produce significantly different returns; the ADL Report shows a return of 8.34% for stock companies with the first formula and only 3.79% with the second formula. Bailey's primary justification for preferring the former ratio and its result is based on certain "returns" to policyholders (discounts on premiums and the time value of deferred loss payments) which he claims exist and which ADL ignored. While I tend to agree with Bailey's choice of a return measure, I have to admit that I did not find his arguments about imputed returns particularly convincing. Furthermore, he failed to discuss the possible relationship of his position with

[5] J. D. Hammond & N. Shilling, "A Review Article: The Little Report on Prices and Profits in the Property and Profits in the Property and Liability Insurance Industry," *Journal of Risk and Insurance*, March 1969.

my recent paper* in the *Proceedings* which set forth certain relationships among various alternative return measures.

In this paper I expressed a relationship between return on net worth, return on assets (or alternatively, return on net worth plus reserves), and return on premiums, in the formula

$$T/S = \frac{I}{A}\left(1 + \frac{R}{S}\right) + \frac{U}{P} \cdot \frac{P}{S}$$

where:

T = Total return

I = Investment profit or loss

U = Underwriting profit or loss

P = Premium income

A = Assets

R = Reserves (excluding equity in unearned premium reserve)

S = Net worth (including equity in unearned premium reserve)

and assuming $T = I + U$ and $A = R + S$.

To illustrate how this formula can be used to compare the two return measures in question, assume that the following data describes stock company performance for the period 1955-1967:

$I/A = 4.11\%$

$R/S = 1.2$

$U/P = -0.70\%$

$P/S = 1.0$

Using this data and the preceding formula, return on net worth (T/S) is 8.34%, which corresponds to ADL's figure when net worth is used as the denominator. ADL's return based on net worth plus reserves in the notation developed here amounts to $\frac{I+U}{R+S}\left(\text{or } \frac{I+U}{A}\right)$ which, based on the illustrative data above, is equal to the figure of 3.79% appearing in the ADL

* J. Robert Ferrari, "The Relationship of Underwriting, Investment, Leverage, and Exposure to Total Return on Owners' Equity," *PCAS*, Vol. LV, pp. 295-302.

report. As was pointed out in the original paper, return on net worth (T/S) is based on an equation which shows clearly the logical relationship of well-known financial variables: leverage, margin, and turnover. However, the ADL preferred approach of return on assets which amounts to $\frac{I}{A} + \frac{U}{A}$ offers little in the way of analytical appeal since the U/A component (i.e., underwriting profit or loss as a percentage of assets) is a relatively meaningless measure of insurance company performance. On this basis, I feel one can argue for the return on net worth measure without resorting to Bailey's rather subtle notions about imputed returns to policyholders. Furtherfore, the "biases" which Bailey contends may result from the return on total assets measure are shown clearly in the T/S formula above through the impact of the R/S and P/S ratios on total financial results. Additionally, actual or expected underwriting profit can be introduced directly with the U/P ratio.

THE INTERPRETATION OF LIABILITY INCREASED LIMITS STATISTICS

JEFFREY T. LANGE

VOLUME LVI, PAGE 163

DISCUSSION BY THOMAS W. FOWLER

In reviewing a paper one must determine at the very outset what the author's primary purpose was in writing the paper. Having ascertained the goal, we are then in a better position to make the determination of whether or not it was attained. It would appear from Mr. Lange's comments that he wishes us to view his paper as a philosophical discussion rather than something definitive and susceptible of rigorous analysis. On reading the paper it is well to keep this fundamental thought in mind since statements are made throughout the paper which implicitly call for further explanation or in themselves raise further questions. In another context this could be a basis for criticism; however, the author's hypothesis gives him a wide latitude in this regard.

Mr. Lange comments that attention in the past, insofar as the papers in the *Proceedings* are concerned, has been almost exclusively limited to rate-making techniques for basic limits coverage since these have been firmly established and widely accepted. In retrospect, it is somewhat unfortunate that this has been the case. Perhaps, inadvertently, the emphasis in this particular area has caused the rate-making process to be associated directly with losses. Another way of saying this is that, although "past experience" is only one of several factors that are considered in the rate-making process, actual practice has given it a much used and unfortunately sometimes abused role. I hope that no one will conclude from my remarks that past loss history, no matter how erratic, should be disregarded, even the situation where there is a total absence of losses. The question becomes one of interpretation of the loss pattern, along with other indications of exposure.

We are painfully aware that a critical area in the rate-making process is that which is involved with low frequency — high severity situations — where, in effect, an absence of losses or a paucity of losses is usual. As a matter of fact, insurance companies in general and, reinsurers in particular, have always been faced with pricing situations where loss patterns are extremely irregular, or even nonexistent, but where potential exposure to severe loss is present. (The relative importance of such situations has increased in recent years because of such things as larger concentrations of value and larger jury awards.)

Even though the paper is admittedly a philosophical one, I was somewhat disappointed that a short explanation of the present method of the Insurance Rating Board in arriving at their excess limits factors was not included. I am not referring to the almost universal procedure of applying factors to basic limit rates, but to the method by which the factors are obtained, especially those at the higher levels. Perhaps this could form the basis for a future supplement to this general topic. Nonetheless, Mr. Lange's suggested use of the ratio of losses, within each increased limits interval to basic limits losses, provides a solid initial approach for further study; however, it raises some pertinent questions as to how such loss statistics should be used. Thus, the loss distributions which are developed arise out of a certain spread of exposures. I think it must be clear to anyone carrying this research further that a functional relationship must be established between loss and exposure before any valid inferences can be drawn from the loss data.

We can follow this line of reasoning further by asking how increased limits intervals are valued (or rated), when the losses within these intervals become thin or become negligible. Mr. Lange alludes to this problem near the end of his paper when he makes the statement that for limits above $100,000 (e.g. $1,000,000), risk is more important than pure premium. He leaves us with the impression that for excess limits intervals possessing a certain undefined level of losses (the lower levels) a procedure would be used using his "ratio of losses" method, to which I previously alluded. We are not told how to proceed when we go beyond these levels; however, one possibility would be to explore the lower distributions as a means of forecasting what will transpire at the higher levels. This is another challenging problem which Mr. Lange gives us to solve.

In my opinion, the greatest contribution of the paper is its timeliness. It is significantly concerned, although indirectly, with one of the major problems of the day — capacity, or more correctly, the lack of capacity. Specifically, Mr. Lange leads us into a rating region where the price for exposure to loss must be measured by means other than the loss itself.

In this regard Mr. Lange presents us with two basic problems. The explicit one is that there must be developed a technique of rating for areas beyond the so-called basic level. This is a fit technical problem for actuaries to tackle; however, in my opinion it is secondary in importance to his second point — which is implicit in his paper. This is that the Insurance Industry and the Reinsurance Industry must spend much more time and effort in the field of increased limits exposure. This is an area which is already of significant importance and it is growing at an accelerated rate. Both buyers and sellers of insurance and reinsurance must become better acquainted with the price and capacity relationship which is a significant part of the Excess Limits Area. Until this groundwork is laid the acceptance of rating techniques, no matter how elegant or rational, will be difficult to come by.

DISCUSSION BY J. ROBERT HUNTER

THE MISUNDERSTANDING

"Dear Prudence, won't you open up your eyes?"
— John Lennon

The making of rates for increased limits of liability is not, as Mr. Lange points out, given coverage in the *Proceedings* even in proportion to its importance as a premium-producing element in the overall structure of our business. Therefore, not only are executives and underwriters confused by the available experience, but also many actuaries are drawing wrong conclusions. There has been and is much ado about "gravy" in the increased limits factors, but this may well be due to a misunderstanding of long term loss development, different trend, and different credibility criteria (discussed below), as these elements are lost in the unstratified calendar year result. If it does nothing else, Mr. Lange's paper serves as an eye-opener for those in prudent management yet capable of eye-opening. As an aside, this eye-opening process seems to have occurred in the reinsurance area, as evidenced by a contraction, from 1966 to 1968, of 11% in reinsurance company countrywide automobile bodily injury premiums earned, while combined stock and mutual premiums earned increased 18%.[1] Capacity, anyone?

[1] New York Insurance Department's "1968 Loss and Expense Ratios," page 110. It is recognized that the reinsurance premium split may be misleading, but these data should be indicative of a bad situation.

The author touches on many aspects of difference between basic limits and increased limits. A review of current Mutual Insurance Rating Bureau and Insurance Rating Board combined private passenger car bodily injury data highlights the differences that Mr. Lange notes:

	$10,000/20,000 Limit	Excess of $10,000/20,000
Loss development factor (15 to 63 months)[2]	1.13	1.71
Trend factor (Annual change)[3]	+ 6%	+ 15%
Mayerson-Jones-Bower 100% credibility point[4]	5,098 claims	6,972 claims[5]

The rating organizations now reflect claim frequency in their private passenger car ratemaking procedure. This usually results in a downward adjustment in rate level. As Mr. Lange properly points out, inflation adds claims to the excess limits area and this development must be considered in establishing increased limits rates. Additionally, accident frequency data (Motor Vehicle Department generated) in many states trend upward or only slightly downward, while insurance generated claim frequency data trend more sharply downward. Perhaps the impact of the Safe Driver Insurance Plan, and advance payments, come into play in this apparent contradiction. This writer doubts that claim frequency is downward at higher levels of coverage. While the data on the following page are fragmentary, they may be indicative of this situation (particularly when we compare like calendar quarters).

These data are for countrywide private passenger automobiles (bodily injury) for all companies reporting to the Mutual Insurance Rating Bureau and are available for a limited number of quarters only. It is obvious that many more data are needed in this regard and much more work will have to be done in studying the frequency of loss at higher limits.

In discussing frequency trends, consideration should be given to the increasing percentage of insureds carrying higher limits.

[2] See Exhibit A. Note: Of particular interest is Sheet 4 showing how loss development increases as size of loss increases. Data of this sort might well be used to prove that immaturity of case losses rather than trend is the cause of loss development.
[3] See Exhibit B.
[4] See Exhibit C.
[5] Total limits requirement.

Quarter	All Paid Claims	Over $5,000	Over $10,000	Over $15,000
1st '68	36,281	1,317	498	Not Available
2nd '68	36,948	1,436	504	197
3rd '68	33,684	1,223	314	164
4th '68	35,766	1,445	302	177
1st '69	37,181	1,613	471	244
2nd '69	39,083	1,763	539	267
3rd '69	33,526	1,408	410	192
Average Change of like Calendar Quarters	+2.6%	+20.1%	+10.7%	+26.3%

I will not attempt to quantify the difference in those interconnected elements of risk and reinsurance expense, but it is obvious that Mr. Lange is correct in stating that high limits require more consideration of these elements so that the separate profit and contingency portions of the profit and contingency factor should be increased as the layer of coverage under consideration increases.

THE INNOVATION

"Give me the benefit of your convictions, if you have any, but keep your doubts to yourself, for I have enough of my own."

— Goethe

So far, Mr. Lange and I are in complete agreement. I have not yet touched upon the innovation in Mr. Lange's paper, namely, the use of a ratio of increased limits losses for a given policy limit to corresponding basic limits losses for the same limit. Mr. Lange rejects the use of the loss ratio approach because of the difficulty of obtaining accurate premium. Also, he states, "Since increased limits charges are a function of basic limits rates, the isolated fact that increased limits experience is good or bad does not tell the ratemaker whether or not the relationship between increased limits and basic limits rates is correct." The pure premium approach also is rejected, Mr. Lange concluding that there would have to be a great number of breakdowns into class and territory, leading to a credibility problem, and that "the pure premium approach is not particularly convenient for testing the present rating procedure in which increased limits charges are expressed as a function of basic limits rates."

I do not advocate the pure premium approach either, but fail to see why splits by classification and/or territory would be needed. Certainly any increased limits review would be on a multi-state basis and territory/class differences between basic and increased limits could be reflected by a study of average rates. Nor am I convinced that the problem of testing the present rating procedure couldn't be overcome.

The loss ratio approach was utilized by all rating organizations in establishing increased limits tables (until the Insurance Rating Board utilized a losses to losses approach similar to that suggested by Mr. Lange; M.I.R.B. continues to use a loss ratio approach). The advantage of the loss ratio method (which allows an historical review when adjusted for increased limits table changes) is that it spreads, on a multi-state basis, the impact on excess level of undershooting basic limits rate level needs. Longer periods of review are important, due to the immaturity of loss statistics for excess limits and the credibility problem discussed briefly above.

Regarding Mr. Lange's criticisms of the loss ratio approach, I would make two observations:

1. Premiums can be accurately determined, at least to a degree sufficient for these reviews, through the use of sample distributions.[6]

   These sample premiums can lead to calculations by layer and can, therefore, be used for table-slope testing, as well as rate level requirement determination.

2. Mr. Lange's concern with relationships to basic limits rates can be negated by proper reflection in the loss ratio of the differences discussed above. This is a most curious concern of Mr. Lange, since he applies his loss-to-loss ratios to basic limits charges without regard to adequacy of basic limits rate or how the basic limits rate is determined. (As Mr. Harwayne points out, New York and other states utilize voluntary and assigned risks combined experience,

[6] I have always felt that attempting to develop rates to apply in the future, with the precision of N.A.S.A.'s landing men on the moon, was an exercise worthy of a Matt Rodermund playlet. Unfortunately, actuaries, perhaps under regulatory pressures, tend to quibble about matters which fail to change rounded rates.

which would require special handling under the losses to losses technique.) The loss ratio for basic limits is available at the time increased limits are reviewed, thereby enabling testing of adequacy of overall basic limits rates as well.

AN APPROACH

"We dance 'round in a ring and suppose
But the Secret sits in the middle and knows."
— Robert Frost

I would suggest that Mr. Lange's loss to loss technique and the loss ratio technique can be used to complement each other since fallacies of each method do not seem to overlap to any significant degree. A sort of "triangulation" toward an optimum answer is necessary, particularly in view of the subjective nature of the reflection of risk and reinsurance expense into these low frequency, high severity areas.

THE AUTHOR

"Still there are some who ask why,
Who want to know, who dare to try.
Every now and then we meet that kind of man"
— Rod McKuen

Mr. Lange has done us all a vital service in bringing this misunderstood area of the ratemaking process into clearer perspective. As always, his thoughts are stimulating and perceptive. He has opened many unexplored areas to our attention. For example, average claim costs vary considerably by state, territory, and classification. I feel intuitively that the closer the average claim cost to the arbitrary basic limits cut-off point, the more likely is an increased limits loss. Should the tables of increased limits factors be reflective of this?

I hope that the many unanswered questions pertaining to this topic of growing importance will prompt further papers by the members of the Casualty Actuarial Society.

Exhibit A
Sheet 1

AUTOMOBILE LIABILITY INSURANCE - PRIVATE PASSENGER CARS

BASIC LIMITS LOSS DEVELOPMENT FACTORS FOR STATES
WITH 5/10 FINANCIAL RESPONSIBILITY LIMITS

All Companies Reporting
to M.I.R.B. and I.R.B.

BODILY INJURY 5/10 BASIS

Accident Year	5/10 Basic Limits Incurred Losses As Of: 15 Months	27 Months	39 Months	Loss Development Factors: 15 to 27 Months	27 to 39 Months	15 to 39 Months
1962	$	$ 96,941,033	$ 96,812,668		.999	
1963	94,019,913	100,084,665	100,763,439	1.065	1.007	
1964	98,527,418	104,710,935	106,187,847	1.063	1.014	
1965	75,030,511	81,092,490		1.081		
Average:				1.070	1.007	1.077

Accident Year	Total Limits Incurred Losses As Of: 39 Months	51 Months	63 Months	Loss Development Factors: 39 to 51 Months	51 to 63 Months	39 to 63 Months
1960	$	$129,922,355	$130,184,004		1.002	
1961	113,364,787	113,687,692	113,272,224	1.003	.996	
1962	101,814,598	102,053,766	102,176,432	1.002	1.001	
1963	115,085,286	115,277,011		1.002		
Average:				1.002	1.000	1.002

Loss Development Factors

39 to 63 Months = 1.002
27 to 63 Months = 1.002 x 1.007 = 1.009
15 to 63 Months = 1.002 x 1.077 = 1.079

Note: Losses Include Allocated Loss Adjustment Expenses.

Exhibit A
Sheet 2

AUTOMOBILE LIABILITY INSURANCE - PRIVATE PASSENGER CARS

BASIC LIMITS LOSS DEVELOPMENT FACTORS FOR STATES

WITH 10/20 OR HIGHER FINANCIAL RESPONSIBILITY LIMITS

All Companies Reporting
to M.I.R.B. and I.R.B.

BODILY INJURY LOSS DEVELOPMENT

Accident Year	10/20 Basic Limits Incurred Losses As Of: 15 Months	27 Months	39 Months	Loss Development Factors: 15 to 27 Months	27 to 39 Months	15 to 39 Months
1962	$	$239,594,301	$243,057,453		1.014	
1963	256,073,734	279,709,757	286,742,796	1.092	1.025	
1964	274,468,426	302,931,061	311,433,448	1.104	1.028	
1965	289,992,763	326,196,959		1.125		
Average:				1.107	1.022	1.131

	Total Limits Incurred Losses As Of: 39 Months	51 Months	63 Months	Loss Development Factors: 39 to 51 Months	51 to 63 Months	39 to 63 Months
1960	$	$247,318,893	$246,518,246		.997	
1961	270,457,860	269,425,481	268,946,377	.996	.998	
1962	286,318,914	288,032,354	287,904,620	1.006	1.000	
1963	303,436,675	306,173,831		1.009		
Average:				1.004	.998	1.002

Loss Development Factors

39 to 63 Months = 1.002
27 to 63 Months = 1.002 x 1.022 = 1.024
15 to 63 Months = 1.002 x 1.131 = 1.133

Notes: Losses include allocated loss adjustment expenses.

Exhibit A
Sheet 3

AUTOMOBILE LIABILITY INSURANCE - PRIVATE PASSENGER CARS

EXCESS LIMITS LOSS DEVELOPMENT FACTORS

BODILY INJURY

All Companies Reporting
to M.I.R.B. and I.R.B.

Accident Year	Excess Losses Over 5/10 Limits As Of: 15 Months	27 Months	39 Months	Loss Development Factors: 15 to 27 Months	27 to 39 Months	15 to 39 Months
1962	$	$13,029,499	$14,286,025		1.096	
1963	12,433,974	14,837,950	15,957,400	1.193	1.075	
1964	9,044,834	12,373,901	12,670,490	1.368	1.024	
1965	8,785,610	11,426,655		1.301		
Average:				1.287	1.065	1.371

39 to 63 Months ϕ = 1.002
27 to 63 Months = 1.002 x 1.065 = 1.067
15 to 63 Months = 1.002 x 1.371 = 1.374

Accident Year	Excess Losses Over 10/20 Limits As Of: 15 Months	27 Months	39 Months	Loss Development Factors: 15 to 27 Months	27 to 39 Months	15 to 39 Months
1962	$	$24,511,252	$26,303,647		1.073	
1963	16,703,958	24,728,882	27,683,362	1.480	1.119	
1964	19,119,158	28,154,935	32,699,335	1.473	1.161	
1965	22,172,172	35,863,131		1.617		
Average:				1.523	1.118	1.703

39 to 63 Months ϕ = 1.002
27 to 63 Months = 1.002 x 1.118 = 1.120
15 to 63 Months = 1.002 x 1.703 = 1.706

ϕ Developments beyond 39 months are on a total limits basis probably understating the excess limits results.

Note: Losses Include Allocated Loss Adjustment Expenses.

Exhibit A
Sheet 4

AUTOMOBILE LIABILITY INSURANCE - PRIVATE PASSENGER CARS

Bodily Injury Excess Limits Loss Development Factors

For States With 10/20 Or Higher Financial Responsibility Limits (Excluding New York)

All Companies Reporting to M.I.R.B.

VOLUNTARY RISKS

Accident Year	Incurred Losses As Of: 15Months	27 Months	39 Months	Loss Development Factors: 15 to 27 Months	27 to 39 Months	15 to 39 Months
Losses Over 0/0 Limits (Total Limits)						
1963	$ --	$31,802,309	$32,497,646	--	1.022	
1964	32,909,330	36,985,051	37,772,132	1.124	1.021	
1965	32,538,171	37,804,817	39,596,501	1.162	1.047	
1966	34,161,919	41,104,051	--	1.203		
Average:				1.163	1.030	1.198
Losses Over 10/20 Limits						
1963	--	3,017,011	3,431,223	--	1.137	
1964	2,242,916	3,265,414	3,644,331	1.456	1.116	
1965	2,219,080	3,507,043	3,993,185	1.580	1.139	
1966	2,744,920	4,275,669	--	1.558		
Average:				1.531	1.131	1.732
Losses Over 25/50 Limits						
1963	--	719,575	996,429	--	1.385	
1964	469,151	787,543	980,992	1.679	1.246	
1965	308,100	718,428	849,044	2.332	1.182	
1966	734,851	1,213,488	--	1.651		
Average:				1.887	1.271	2.398
Losses Over 50/100 Limits						
1963	--	137,500	183,750	--	1.336	
1964	70,000	64,063	124,063	.915	1.937	
1965	10,000	105,000	115,300	10.500	1.098	
1966	179,500	317,941	--	1.771		
Average:				4.395	1.457	6.404

Notes: 1. Losses Exclude Unallocated Adjustment Expenses.

2. Charting these factors is a recommended and enlightening exercise.

Exhibit B

AUTOMOBILE BODILY INJURY LIABILITY INSURANCE

IMPACT OF RECENT CLAIM COST TRENDS ON
$10,000, TOTAL LIMITS AND EXCESS LIMITS

Year and Quarter	Adjusted ∅ Line of Best Fit $10,000 Limit	Adjusted ∅ Line of Best Fit Total Limits	Contribution from Excess of $10,000
1965 3	$ 911.75	$ 981.76	
1965 4	929.55	1,005.20	
1966 1	947.35	1,028.64	
1966 2	965.15	1,052.08	
1966 3	982.95	1,075.52	
1966 4	1,000.75	1,098.96	
1967 1	1,018.55	1,122.40	
1967 2	1,036.35	1,145.84	
1967 3	1,054.15	1,169.28	
1967 4	1,071.95	1,192.72	
1968 1	1,089.75	1,216.16	
1968 2	1,107.55	1,239.60	
1968 3	1,125.35	1,263.04	
1968 4	1,143.15	1.286.48	
1969 1	1,160.95	1,309.92	
1969 2	1,178.75	1,333.36	
Last Point on Line of Best Fit	$1,178.75	$1,333.36	$ 154.61
Annual Increment	$ + 71.20	$ + 93.76	$+ 22.56
Annual Percent Change	+ 6.0%	+ 7.0%	+ 14.6%

∅ Based on Seasonally Adjusted Quarterly Data of All Companies Reporting to the Mutual Insurance Rating Bureau and the Insurance Rating Board.

Exhibit C
Part 1

CREDIBILITY AT VARIOUS CUT-OFF POINTS

\$5,000 B.I., \$10,000 B.I., TOTAL LIMITS B.I.

Utilizing the Mayerson-Jones-Bowers procedure*
as shown in Parts 2 and 3, we find that:

Cutoff Point	Number of Claims Required for Full Credibility (5%k, 90%P)
\$5,000 B.I.	3,931
\$10,000 B.I.	5,098
Total Limits B.I.	6,972†

It is noted that .8% of the claims in the size of claim data for 10/20 states exceed \$10,000. The impact of reflecting their actual value (in lieu of \$10,000 for each such claim) is significant thus indicating an extremely high credibility criteria for these claims.

* Mayerson, A., Jones, D., and Bowers, N., "On The Credibility of the Pure Premium," *PCAS* Vol. LV, Page 175.

† Based on 10/20 states using a calculation identical to that specified in this exhibit.

Exhibit C
Part 2

PRIVATE PASSENGER AUTOMOBILE LIABILITY INSURANCE

Derivation of 100% Credibility Criteria

Based on Countrywide Excluding New York Size of Claim Data of All Companies that Filed with the Mutual Insurance Rating Bureau and the Insurance Rating Board — 1963 Call

Interval	Bodily Injury (All States) No. of Claims f	Losses fx	Average Claim Cost x	x^2
\$ 1 - \$ 24.99	48,686	\$ 657,893	\$ 13.51	183
25 - 49.99	36,911	1,183,711	32.07	1,028
50 - 99.99	42,685	2,783,800	65.22	4,254
100 - 249.99	64,932	9,909,765	152.62	23,293
250 - 499.99	50,312	17,359,394	345.03	119,046
500 - 999.99	56,124	37,990,345	676.90	458,194
1,000 - 1,999.99	37,598	51,229,987	1,362.57	1,856,597
2,000 - 2,999.99	15,832	37,535,893	2,370.89	5,621,119
3,000 - 3,999.99	8,417	28,383,996	3,372.22	11,371,868
4,000 - 4,999.99	4,808	21,025,252	4,372.97	19,122,867
5,000 - 9,999.99	8,770	58,351,870	6,653.58	44,270,127
10,000 (Limit)	3,204	32,040,000	10,000.00	100,000,000
TOTAL	378,329	298,451,911	788.87	

Results:

10/20 States (Bodily Injury)

$\lambda =$ 5,098 * Claims (5% k, 90% P)
= 49,908 Claims (2.5% k, 99% P)

5/10 States (Bodily Injury)

$\lambda =$ 3,931 Claims (5% k, 90% P)
= 38,298 Claims (2.5% k, 99% P)

* See Part 3 for sample derivation of 5,098 claims for full credibility at 90% probability of being within 5% of the expected value.

Exhibit C
Part 3

Derivation of Full Credibility for
\$10,000 Bodily Injury for 5% k, 90% P

(Based on the Paper presented at the Fall, 1968 Meeting of the C.A.S. by Messrs. Mayerson, Jones, and Bowers — "On The Credibility of the Pure Premium," *PCAS* Vol. LV, page 175)

Basic equation (See page 181 of *PCAS* Vol. LV):

$$k\lambda = Z_c\sqrt{\lambda}\sqrt{1+\frac{\mu_2}{\mu^2}+\frac{Z_c^2-1}{6}\cdot\frac{1+3\frac{\mu_2}{\mu^2}+\frac{\mu_3}{\mu^3}}{1+\frac{\mu_2}{\mu^2}}}$$

Where: k = Maximum departure from expected.

λ = Number of claims for 100% credibility.

Z_e = The (100_c) percentile of the standard normal distribution.

μ = Mean = Expected value of x.

μ_2 = Second moment about the mean (μ_2' – about origin).

μ_3 = Third moment about the mean (μ_3' – about origin).

P = Probability of being within k percent of the expected value.

Since, under standard notation:

$$\mu_2 = \mu_2' - \mu^2 \text{, and}$$

$$\mu_3 = \mu_3' - 3\mu\mu_2' + 2\mu^3$$

It follows that:

(A) $1+\frac{\mu_2}{\mu^2}=\frac{\mu_2'}{\mu^2}$, and

(B) $1+3\frac{\mu_2}{\mu^2}+\frac{\mu_3}{\mu^3}=\frac{\mu_3'}{\mu^3}$

Then, for \$10,000 Bodily Injury (see Part 2):

$\mu_2' = 2{,}877{,}265$ $\quad\mu^2 = 622{,}316$

$\mu_3' = 18{,}073{,}982{,}000$ $\quad\mu^3 = 490{,}926{,}423$

Exhibit C
Part 3 (cont.)

By substitution, for *(5% k, 90% P)*

(A) $= 4.623$ and (B) $= 36.816$

Therefore: $$k\lambda = Z_c\sqrt{\lambda}\ \sqrt{4.623} + \frac{Z_c^2 - 1}{6} \cdot \frac{36.816}{4.623}$$

(Let $\sqrt{\lambda} = y$) $$.05y^2 - 1.645y\sqrt{4.623} - .2843 \cdot 7.964 = 0$$

$$5y^2 - 353.7y - 226.42 = 0$$

$$y^2 - 70.7y - 45.28 = 0$$

$$y = 71.4$$

$y^2 = \lambda = 5{,}098$ Claims

IS "PROBABLE MAXIMUM LOSS" (PML) A USEFUL CONCEPT?

JOHN S. McGUINNESS

VOLUME LVI, PAGE 31

AUTHORS REVIEW OF DISCUSSIONS IN VOLUME LVI, PAGES 40-48

The two reviewers have between them raised several points and questions that can be valuable in clarifying the paper and some of the thought underlying it. Mr. Hurley's commentary on the Pareto curve is a very interesting addendum and merits expansion at a later time. His contribution of actual facts is also a positive and helpful addition.

The reviewers' admirably broad range of interests is reflected in their comments. Perhaps it will be an aid to understanding, therefore, first to look at their comments that pertain to the subject of the paper and secondly to look at their other comments. The major points to which the reviewers address themselves seem to be these:

1. the statement in the paper that the concept of PML is "one of the least clear concepts in all insurance"
2. the two-pronged definition of PML
3. how effectively PML now enables underwriters to stabilize their results
4. the fact that the data required for determining PML probabilities are now being collected only for dwellings
5. the significance of Table 1 in the paper
6. whether values at risk can be determined in practice with sufficient accuracy
7. a potential relationship between the confidence level of a set of PML's and the probability of having a large loss
8. whether the probabilities called for by the definition can be measured with sufficient precision (closely related to point 6)
9. the need to balance eagerness for premium volume against the need for stability in underwriting results
10. the usefulness and danger of the PML concept to an insured
11. applicability of the Pareto curve

Point 1: Clarity of the PML Concept. — Reviewer Black goes directly to the heart of the matter in saying ". . . . but I feel [*sic*] strongly that there is a universal meaning as to the end result which all underwriters expect PML to accomplish." He correctly states that an underwriter "feels," but does not "know" about PML. This reviewer refers to an end result for PML to accomplish, not to the meaning of PML itself, and, thereby, reflects the imprecision of thought which the paper aims to overcome.

The author started out some years ago sharing the same feeling, that PML was a clear concept to underwriters. Only when he could not get a clear concept from any underwriter, or the same concept from two or more underwriters, did it occur to him that one clear concept might not exist. This "feeling" needed testing to become a belief, however. So, following Benjamin Rush's example,[1] the author secured the sample of the definitions mentioned in the paper. The collected definitions were omitted from the paper as probably not being of interest to actuaries. They were included in popularized or lay versions of the paper published subsequently elsewhere.[2]

One of the most striking sets of definitions merits repeating here. These came from three property underwriters in the same branch office of a large insurer: (emphasis is supplied by this writer)

> PML is the maximum percentage of the risk *that would be subject to a loss* at one time.
>
> PML is the maximum amount of loss *that can be sustained* within any specifically defined area.
>
> PML is the total amount of loss, expressed in dollars or as a percentage, *expected to be sustained* in the event a fire occurs within a building.

It is remarkable that not one but three definitions come from a single office of an insurer whose underwriting has been outstandingly successful, in relation to that of other companies, over a period of years. Yet here are three

[1] See the fascinating description of Benjamin Rush's painstaking research into chronically unprofitable marine underwriting, and equally painstaking efforts to convince his board of directors of the proper corrective action required, in *Biography of a Business* by Marquis James, New York: Bobbs-Merrill, 1942, pp. 188-200; and *Perils Named and Unnamed* by W. H. A. Carr, New York: McGraw-Hill Book Company, 1967, pp. 82-88. Mr. Rush's example remains a shining beacon to those who would make optimal, soundly based technical and managerial decisions. The present paper obviously covers only the first of Mr. Rush's two steps!

[2] See article of the same title in *Insurance,* New York, 2 August 1969, p. 16; *Assurances,* Montreal, July 1969, p. 83; *Canadian Risk Manager,* Toronto, September/October 1969, p. 15; or *The Review,* London, 31 October 1969, p. 1387.

clearly different concepts of PML! This and the other clear evidence of the lack of clarity in the concepts of PML has in no way been rebutted.[3]

Benjamin Rush realized as well as anyone the need both for full and accurate facts on which to base decisions under uncertainty and also for an effective sales effort to have even the clearest facts and the resulting conclusions accepted by people who are used to thinking along different paths. It is realized that one paper on PML or another subject will not, no matter how factually based, win immediate acceptance from a large number of people whose beliefs and actions it in any manner challenges. But if the presentation of such facts can ultimately win the attention of even one person of influence, communication and acceptance will ultimately be established. Only over a long period, also, will it be possible to demonstrate to a large number of people that actuarial help can be useful in defining and solving problems which are of a quantitative nature or which can be framed in quantitative terms.

It may be that a quotation from Gertrude Stein ("A rose is a rose is a rose.") is more pertinent than the quotation from Shakespeare which was offered by the reviewer. It is easy to get caught in the trap of trying to define something by using one of the words being defined. Mr. Black points up sharply that until the word "probable" is defined in numerical terms as a specific percentage, it is impossible for PML to be clear. And unless we can express in quantitative terms what we are trying to do in this portion of the quantitative part of underwriting, we cannot be sure that any two underwriters, let alone the whole fraternity, will be thinking and acting the same with respect to PML.

Point 2: A Two-Pronged Definition. — Apparently an attempt to make the paper clear has instead resulted in making it unclear. Slightly different forms of the definition were given. Others could also be given for a mortgagee interest or any other insurable or reinsurable interest. The two forms given in the paper are designed to show specifically the elements involved in PML that relate to the property owner and the underwriter. It is felt that a completely generalized definition requires phrasing that may be too abstract to be easily tied by underwriter, actuary, or layman to specific or concrete circumstances.

[3] The popularized articles cited above contrast a sample of several of the conflicting definitions of PML that were collected.

> The PML for a specified financial interest is that proportion of the total value of the interest which will equal or exceed, in a stated proportion of all cases, the amount of any financial loss to the interest from a specified event or group of events.

The reader will have to be the judge of whether *this* feeling is correct.

Mr. Black is absolutely right that a new or standard definition will not change results unless it is used. It is hoped that the definition offered here will soon be used. It will *have* to be used before any material part of the function of determining PML's can be computerized or otherwise meaningfully automated.

Point 3: Effective Use of PML. — One can agree with Mr. Black that current PML concepts and practices can ". . . enable the underwriter to accept maximum lines . . .", but this is not the same as accepting the maximum *safe* lines or *appropriate lines.* The precise concept and measured estimates the paper suggests will by contrast do the latter.

It is also troubling to see mention of "not . . . selecting the maximum *possible* PML in every instance." This reveals a serious logical inconsistency arising from the imprecise concept employed. Not to use the highest PML applicable to any of the covered perils is to defeat the purpose of determining a risk PML in the first place.

The reviewer's expressed opinion (which seems to be the basis for the inconsistency) that the windstorm or tornado PML will almost invariably be greater than the fire PML is open to serious question. Although the hurricane PML, at a 99 per cent confidence level, appears to be far less than 50 per cent for most types of property, it is easy to jump to the conclusion that the tornado PML is 100 per cent (at the same confidence level) for practically all types of risks. As one will see after inspecting the area of damage after any tornado, however, the PML is considerably less than 100 per cent, although higher than for hurricane.

Evidence of inconsistent PML estimating procedures, the facts reported in connection with individual large losses,[4] and studies of tornado and hurricane damage lead the author to the conclusion that at present, because of

[4] *National Fire Protection Association Quarterly,* some rating bureau special hazard reports (prior large losses), and general insurance periodicals such as *The National Underwriter* — Fire and Casualty Edition, report on large fire and allied peril losses in a respectively decreasing degree of detail.

the necessarily crude estimates being made, PML's are most often too high and net retentions are most often too low on the more numerous smaller risks. In a smaller proportion of cases, dangerously, the reverse (on larger risks, which are less numerous) is true. These two types of errors reinforce each other in unstabilizing a portfolio. If PML estimates are too low, the retention tends to be too high and capacity to be over-used; if net retentions are too low, they are apt to be based on faultily high PML estimates, and capacity is under-used. On this basis, an excessive proportion of reinsurance cessions seems more likely to indicate too low retention limits in a company's line sheet. Any adjustments could most practicably and logically be made in the retention schedule rather than through a logically indefensible tinkering with PML estimates.

Point 4: Present Status of Data Collection. — Mr. Black apparently shares, with many other members of the underwriting fraternity with whom the author has communicated, the mistaken belief that the necessary facts to use for determining PML's are presently being collected in the manner required through the statistical plans of the National Insurance Actuarial and Statistical Association. Although amounts of insurance are recorded on premium or exposure cards for both family and business risks, they are recorded only on family or dwelling loss cards under the new NIASA statistical plans. A recommendation to show amounts of insurance on business-risk loss cards was overruled, perhaps on grounds of expense. Since both exposure and loss cards are handled only in bulk, it is impossible under the present plans for the corresponding amounts of insurance and of loss to be put together. This is an important deficiency in the commercial-risk plan which should be corrected. Until it is, underwriters' eager anticipation of facts to support precise PML's will be in vain.

By the same token, the rating bureau reports and analyses of individual loss occurrences are not a satisfactory basis for determining PML's. Just like the reports of all large losses (e.g., those over a certain monetary amount such as $2,500 or $5,000) that in many companies go to supervising underwriters, these rating bureau reports provide only what the actuary or statistician calls a "biased" sample. Study of such material can lead only to biased and inaccurate inferences. Determining the form and manner in which loss data are collected and analyzed is a special field of statistics — design of experiments or design of investigations — in which actuarial expertise is required if accurate inferences are to be drawn by underwriters or others.

Mr. Hurley intimates, and the author agrees, that on a simple class basis the data for any one company will be insufficient to determine PML's with the necessary accuracy for types of risks where they play the most important part viz., the large, not very numerous types. This is the basis of the suggestion in the paper that the data be gathered on an inter-company basis as part of the over-all statistical gathering process.

Because of both an insufficient volume of data and the danger that any available data are being gathered through deficient techniques, any continual review and study that is now going on within companies without actuarial participation is very unlikely to lead to accurate PML estimates.

Point 5: Large Risks v. Small Risks and Table 1. — Mr. Black is fearful that the PML's based on class data would not be sufficiently accurate because the PML percentage is likely to vary significantly among risks of different size. In the absence of facts, one cannot say if this is correct. An opinion that differences in degree of fire resistive compartmentation are more important than differences in size or value might be considered equally valid. In effect, it seems that Mr. Black is saying that while the first of the three stages of accuracy suggested in the paper is meaningful, it can be considerably improved on by refining it to take into consideration such possibly important causes of heterogeneity with variations in size of risk. This seems equivalent to saying that the second or third stages suggested in the paper will produce more accurate results. The author agrees.

Despite differences in size, all the risks in a class can provide useful data for determination of PML's for the class. Homogeneity is a matter of degree rather than a matter of absolutes, or else the classification plan now used has little value. Even though, as Mr. Black suggests, there are many risks of smaller size for which a company with high retentions does not need to determine a PML (because the total value or amount of insurance on each such risk is less than the company's retention limit) it is still necessary to collect the exposure and loss data on smaller risks to provide an adequate picture of the class PML *and of how it may vary with size of risk.* Thus his suggestion for collecting data only from individual losses of at least $25,000, and only for properties valued at $100,000 or more, is inappropriate since it would produce statistically biased results. It would also waste the valuable information and added stability in the statistical results that can be secured from the data on the smaller risks and smaller losses. This is another illustration of the value of, and the need for, a properly designed statistical investigation.

It should also clear up any misunderstanding to point out that Table 1 in the paper applies to all sizes of risks, not just small ones. The table is designed to show how losses under policies with different average clauses should be adjusted to the same basis. It is not designed to serve as a source of PML estimates.

Point 6: Accurate Determination of Values at Risk. — The author did not imply, as Mr. Black infers, "that there is no relation between the Average Clause and the Amount of Insurance purchased . . .", but he is willing to let any facts produced speak for themselves. And while Mr. Black's point that there are bound to be errors in some loss adjustments is quite valid, an assumption that average clause requirements are not enforced in a material proportion of cases raises the question whether inadequate rates or inadequate loss adjustment procedures are responsible for most of the unsatisfactory underwriting results of recent years. The author opts for rate inadequacy.

There will be some inaccuracies in any loss data. The fact that we cannot remove all inaccuracies does not seem good reason for failing to remove those that we can remove. Data from which biases due to different insurance-to-value relationships have been removed or reduced are clearly more accurate than data still containing these biases.

Until we are well into the third stage proposed in the paper, subjective evaluation of risks by seasoned underwriters should be useful in adapting class PML's to individual risks. It is important to realize in this connection, however, that this underwriting activity will resemble much more closely the application of one year's experience twenty times, rather than the application of twenty years' experience, to the extent that it is not continuously improved by the collection of new facts and by the statistically well designed testing of underwriters' theories as they are developed. The cooperative activity of underwriters, who are in the best position to identify actual and potential factors for differentiating risks, and actuaries, who are best equipped to test and measure the pertinence of such factors, is indispensable for progress.

Point 7: Confidence Levels and Probability of Losses. — One must agree with Mr. Hurley that it is easy for an underwriter to confuse the desirable confidence level with the probability of a large loss of some single given size. For example, even if there is only a 95 per cent probability that any loss in a given class of risks will not exceed 50 per cent of value, all losses will not occur to the largest risks. Further, not all of the 5 per cent of losses that exceed 50 per cent of value will occur to the largest risks, and not all of the few total losses in this small group will occur to the largest risks. The probability of total losses to the largest risks in a class is therefore much, much less than 5 per cent (or even than 1 per cent) under such circumstances. It should not be forgotten, however, that no matter what the confidence level used for the PML may be, the underwriter must always be prepared to accept a total loss on any policy he writes.

The PML confidence level for an individual class will be less than the confidence level applying to the stability of a company's complete portfolio, because fluctuations tend to offset from one class to another. Although it would be best to withhold final judgment until a test with actual data can be run, the author believes it not improbable that a 95 per cent confidence level for PML might be satisfactory for all or most classes of risks.

Point 8: Measuring the Required Probabilities. — It is also easy to agree with Mr. Hurley that the suggested definition will have little practical value unless the probabilities to be associated with it can be handled with the statistical assurances required. This is exactly why not only one but three gradually improved methods of obtaining the needed statistical assurances are explained in the paper. A complete and precise methodology for setting retentions — the goal for which PML is simply a tool — has already been provided elsewhere.[5] The missing elements are the needed data to fit the models provided and the conviction of underwriters and executives that existing subjective methods can be improved upon.

Point 9: Balancing Premium Volume and Stability. — Mr. Hurley goes directly to the heart of a dilemma requiring a managerial decision. Mr. Black touches it less directly. Mr. Hurley notes that an underwriter or underwriting manager must at some time make the choice between how much stability he requires in his portfolio and how much potential profit he is willing to forego to achieve it. An underwriter with factually based PML's and also factually based underwriting retentions is of course in a much better position to make this choice than today's underwriter, who has neither.

Point 10: The Insured and PML. — In saying that "It seems highly improper to me that the insured should consider anything more than the

[5] J. S. McGuinness, "Controlling the Effects of Catastrophes in Insurance Against Floods and Other Elemental Perils," IV *Transactions of the XVth International Congress of Actuaries,* New York, 1957, pp. 190-203.

total value of his property exposed to *any* peril . . . ," Mr. Black is apparently thinking of an insured who has only a single property that is 100 per cent subject to total loss from a single event. Both the generalized form of the definition given above and the two specific forms in the paper are designed to cover all types of assureds. These forms would include those assureds needing insurance only to the maximum value (per occurrence) represented by a single property of a multiple location account with similar values. The basic definition also includes the person whose other financial resources may equal or exceed the insurable value of his physical properties. An insured and his risk manager need to consider PML in buying insurance as much for pricing as for determining limits of insurance.

The not uncommon practice in marine insurance of securing coverage on hulls only for total losses (because of the Pareto curve involved, only small-percentage losses or total losses are practical possibilities) is one example. The very practical limitation, because of bulk, on the amount of some types of goods that can be burgled at one time makes PML important both for pricing and for determining needed amounts of insurance against open stock burglary. The PML of a protected dwelling in jurisdictions that do not allow rate reductions for inclusion of average clauses in dwelling policies is a very important consideration to the owner or landlord who wants to avoid the extremely excessive premium charges that fire insurance to full value entails. There would be no need for 70, 80, or 90 per cent average clauses (and only 100 per cent average clauses would be needed or in use) if PML was not a practical and necessary consideration for the insured, no matter whether a single property or properties at several locations are involved. Finally, PML estimates of rating bureau engineers in sprinklered risk and special hazard reports must be applied from the insured's point of view. In short, the applicability of the PML concept to the insured and his risk manager is much more complex than the reviewer indicates and is clearly a practical necessity. Modern developments in the theory and practice of risk management would form a valuable subject of study for any underwriter.

Point 11: Applicability of the Pareto Curve. — We are indebted to Mr. Hurley for his erudite discussion of the Pareto curve and some of its history. Since the paper was written, an unpublished doctoral dissertation has been made available to the author.[6] This contains more actual data supporting

[6] G. L. Head, "Insurance to Value," doctoral dissertation submitted to the University of Pennsylvania, Philadelphia, 1968, pp. 115-148.

use of the Pareto curve, some from California from the early 1900's and more recent data from Oregon from the 1960's. The empirical results reported in the dissertation[7] match very nicely the theoretical results of Mandelbrot and others reported in the paper.

Mr. Hurley's mention of the Benktander-Segerdahl paper of 1960 should be supplemented by reference to a later paper by Benktander[8] and one by P. J. H. Green.[9] In the latter, Mr. Green shows that there are other curves that are more dangerous than the Pareto. It should also be noted that "dangerous" as used by these authors refers to the degree of risk that a given excess-of-loss premium would be insufficient if losses are actually distributed according to the curve. It does *not* refer to a risk of being inaccurate, i.e. to any possibility that there may be a more appropriate curve to describe a given loss distribution.

Mr. Hurley should also be thanked for noting the need to point out that the Pareto curve is in usual form asymptotic to the X-axis, and that because property values are finite the tail beyond the 100 per cent of value point on that axis must be cumulated at that point, producing the second leg of the "U."

Summary The reviewers are to be congratulated on bringing out, through the wide range of their remarks, many facets of the paper that needed amplification and clarification. In providing the opportunity for such clarification, not the least of their contributions has been to point up the direct and practical applicability of the paper in demonstrating one path toward improvement of underwriting results. While the paper was not intended to be provocative, it was intended to stimulate action to improve a limited portion of present underwriting techniques. The reviewers' comments, and the opportunity they have provided for amplification, should prove to be of great value toward this end.

[7] *Ibid.*, pp. 143-145.

[8] G. Benktander, "A Note on the Most 'Dangerous' and Skewest Class of Distributions," *Astin Bulletin* Vol. II Part III, April 1963, p. 387.

[9] P. J. H. Green, "Some Skew Distributions," Jubilee Number, *Quarterly Letter*, Algemeene Reinsurance Companies, Amsterdam, July 1964, Vol. II, page 46.

PANEL DISCUSSION

OPEN COMPETITION — FOUR POINTS OF VIEW

MODERATOR

GERALD R. HARTMAN

What does "open competition" really mean? It means what I choose it to mean — neither more nor less. It also means different things to different people. This fact provided the idea for the format of our presentation, i.e., each panelist will discuss what open competition means to him as an insurance department actuary, a bureau and company actuary, a consulting actuary, and a college professor (and consumer) of insurance, respectively. The "open competition" label has been applied to the differently worded laws of several states, and, therefore, the meaning of the label is apt to vary geographically. New York, Illinois, and Minnesota, which are among the "open competition" states, are represented on our panel.

As a prelude to the remarks of our panelists it may be worthwhile to examine the sections of the rating laws, of the foregoing three states, which may justify the label of open competition. Just as one picture may equal a thousand words, in the search for the meaning of a rating law one careful reading thereof may equal a thousand opinions.

In contrast to the All-Industry Committee (AIC) model bills of the 1940's which were not "intended to prohibit or discourage reasonable competition,"[1] these open competition laws are intended to permit and encourage competition between companies on a sound financial basis to the fullest extent possible,"[2] "to encourage, as the most effective way to produce rates that conform to the standards of paragraph (a) [rates shall not be excessive, inadequate or unfairly discriminatory], independent action by and reasonable price competition among insurers,"[3] "to prevent practices that tend to bring about monopoly or to lessen or destroy competition,"[4] "to pro-

[1] Section 1 of All Industry Bills.

[2] Section 472.1, Illustrated Insurance Law.

[3] Section 1, Subdivision 2(b), Minnesota Insurance Law.

[4] *Ibid*, Subdivision 2(d).

hibit price-fixing agreements and other anti-competitive behavior by insurers, to promote price competition among insurers."[5]

Thus, while the wording of the AIC bills did not prohibit competition (but often their administration discouraged it), the avowed purpose of these laws is to encourage competition and to prohibit anti-competitive behavior which flourished for many years under the AIC type of laws. The pendulum has definitely swung in the opposite direction. The laws of all three states prohibit insurer agreements to adhere to rates.

Furthermore, there are teeth in some of these new laws which greatly enhance the chances of their purposes being achieved. For example, in New York "the *superintendent, through the attorney general,* and *any person* injured in his business or property by reason of anything forbidden in subdivision one of this section *may maintain an action to enjoin any violation*" of the law and "any person injured in his business or property by reason of anything forbidden in subdivision one of this section may maintain an action and shall recover *threefold* the *damages* by him sustained."[6] The impact of class actions under these circumstances would be considerable, to say the least. Not only may the superintendent, after a hearing, order premium adjustments when rates are charged that do not comply with the law but also "he shall order the payment of a penalty not to exceed five hundred dollars for each such offense, and if he so finds that such insurer, rating organization or other person knowingly violated this article he shall order the payment of a further penalty not to exceed twenty-five hundred dollars for each such offense. *The issuance, procurement or negotiation of a single policy of insurance shall be* deemed a separate offense."[7]

Another common aspect of the laws in these three states is that generally rates do not have to be filed, and therefore do not have to be approved by the insurance department, *before* they are used. Nor does supporting information have to be filed in all cases. On the other hand records and experience data must be maintained which will enable the commissioner to determine whether there has been compliance with the law, and in some states all rates and supplementary rate information are open to public inspection as soon as the rates become effective. This latter requirement seems especially important since the successful play of competition depends upon knowledgeable buyers and sellers.

[5] Section 175(1) New York Insurance Law.
[6] *Ibid*, Section 177(2)b and c. (author's italics)
[7] *Ibid*, Section 179(3). (author's italics)

Prior to the start of this panel at least one member of the audience quipped about the significance of the word "open" in the term "open competition"; he wanted to know how the term differed from closed competition. The answer may lie in the eleventh definition of "open" given in the unabridged edition of *The Random House Dictionary of the English Language:* "without restrictions as to who may participate: an open competition."[8]

We do not have time today for a thousand other opinions of what open competition really means. Therefore, we shall settle for other opinions from four informed and able men: Kevin Ryan, Steven Newman, Lewis Roberts, and C. Arthur Williams, Jr.

THE REGULATOR

KEVIN M. RYAN

Earlier this year, President Nixon's Council of Economic Advisors stressed the merits of free competition. Their analysis puts it this way: "Traditionally, this nation has accepted the premise that the individual should be as free as possible to decide for himself what goods and services will be best for him and where and how he will exercise his own talents and energies. By and large the resultant system serves us well."

It may not be clear in the non-life-insurance business that the consumer has the opportunity to choose whether or not he will buy the product in the first place. For instance, the purchase of automobile and fire insurance is nearly universal due to social and economic necessity. The consumer *must* buy the product in most instances. There is no effective competition as to whether he will purchase or not, or as to alternative or substitute products. But this is true in other traditionally competitive industries, dealing in the so-called "necessities," e.g., automobile, refrigerators, communication, etc. The circumstance is not a compelling argument against open competition.

Open competition as we refer to it here on this panel is a misnomer. We are not referring to competition but to a pricing process which, for all practical purposes, is the "non-prior approval" pricing process. From the regulator's viewpoint, open competition is a pragmatic realignment of responsibilities with stress supplied by the public and price adjustment from the companies. The open competition which the regulator must look for

[8] Random House, New York 1969, p. 1008.

is a pricing system that benefits the consumer, and not one which merely emphasizes the industry's new freedoms, although the latter consideration is inherently characteristic of this new system. We are viewing a system in which new flexibility is given to the insurance industry for the sole intended result that the consumer will have, in every market, an effective choice among the best possible products. This effective choice will consist of meaningful product variations and price differences.

The result that we seek, if this program is to be deemed successful, is to have, in heretofore neglected areas, active and vital markets where choices are afforded the consumer. The necessary competition will result from an increased capability of a company to move into a market where, having established operations, they can adapt to the conditions there. It has been the feeling of some regulators that the reason for tightly restrictive markets is that companies are fearful of committing themselves to a market to which they cannot adjust. Entering an area and expending start up costs where inadequate rates develop may lead to market restrictions. But these market restrictions become necessary to management if regulators will not allow price adjustments in these new markets. In Illinois, as of January 1, we have given management the flexibility and the power to adjust to a market, without any political or bureaucratic intervention.

It must be kept in mind that this pricing system we speak of is based on the presumption that there is, or can be, the broad type of "open" competition. Without competition, the insurance market has to be regulated. Few argue that, in most endeavors, the existence of competition is by far the best price regulator. A policy of permitting and encouraging competition of all kinds would, if general economic experience is any guide, make the industry more efficient and ultimately benefit the public. But it does depend upon competition. Is there such a thing as effective competition in the insurance market place? Competition broadly has certain characteristics:

1) a large number of competitors that are well informed, act independently, and are sufficiently dispersed in strength and number so that no one of them controls the price by its activity alone;
2) easy entry into and exit from the market;
3) standardized product.

Numbers 2 and 3, easy entry and standardized product, seem to fit the insurance process. On the other hand, within the first requirement, independent action has been, and continues to be, a problem. Despite the fact that there is sufficient independence for reliance upon open competition, companies must act more independently for open competition to fulfill its promise to the satisfaction of all. Furthermore, there is no question but that there is very little competition existing in various areas of the insurance market. Some of those areas are: Medical malpractice, sub-standard automobile insurance in various geographical areas, and low-valued property in ghetto areas. Various causes combine to make very restricted markets in these areas.

Medical malpractice provides a classic example of the companies' inability to properly price a product and establish competitive markets due to changing social conditions that are outstripping price adjustment. Even in the more predictable automobile lines, market restrictions are, at times, a direct result of social upheavals which change the product without changing the price. Due to the inability of the pricing mechanism to respond completely and totally, market restrictions are created.

Obviously, there are areas where, due to very little competition, the open competition rating laws may not be successful. What is the solution to this problem, viz., the problem of trying to regulate rates through competition where there is, in fact, very little competition? An existing, and workable solution is to have the effect of competition artificially created by regulatory agencies. Already the institution of Fair Plans and the expansion of Automobile Assigned Risk Plans have overcome some of the lack of competition. Artificial effects of competition must be developed by means of these plans in order for proper regulation to exist. In this way, the effective and controlled utilization of these plans is not only an effective placement program, but rather, and more importantly to rate regulation in non-competitive areas, it becomes an artificial price regulator.

What happens in effect with Assigned Risk Plans is that a ceiling is put on prices in areas where competition is not strong enough to place a ceiling. It seems reasonable that no carrier in an area affected by the full operations of Assigned Risk Plan can charge, for any large segment of the population of that area, prices higher than the Assigned Risk Plan. Once the new rating system is in full operation, what other items will we be looking for as regulators? What other items are there that will be indicators that the open competition type of rating law is working? We expect competition to increase as the attitudes of price uniformity engendered under the prior approval system recede. By the rate regulator's intervention, originally encouraged by industry and currently imbedded in some industry attitudes, the government not only failed to promote competition, but actually tended to prevent it. One of the concrete changes we expect to encounter is a splintering of the market-place. Why? This is based on the premise that there has been, under the prior approval system, a curtailment of territorial and some classification breakdowns. The state has allowed a restriction of the market-place by creating a need for selective underwriting. Under prior approval it became obvious that, because of pricing restrictions, classifications had clearly definable bad and good risks, i.e., risks that an underwriter knew were better or worse than the stipulated regulated price. Such a situation naturally resulted in market restrictions for those identifiably bad. This unfortunately holds true for territories. By demanding non-selective pricing, the regulator has encouraged selective underwriting and market restriction. We now expect selective pricing and the corresponding lifting of market restriction.

In summary, we expect to see more classifications and more territories, a result that will give the companies the needed flexibility to handle larger portions of the market. In addition, we expect to see greater activities in areas where there has been a definite slow-down of market penetration. We now believe that companies will actively engage in previously restricted areas beacuse they now have the marketing flexibility that prior approval did not give. We look for a modernization of the Illinois Assigned Risk Plans so that they will be more active in areas where *companies have not and will not voluntarily compete.*

Overall we do not look for any abnormal industry-wide price changes, except perhaps in the areas where new competition will be developing. Here, Assigned Risk Plans for property, liability and any other areas where they are necessary, must become the effective regulator. We do not look for great industry-wide upheavals, but expect major marketing changes by companies in those areas where they realize that various Assigned Risk Plans must enter, if they do not. In fact, we are not looking for any dramatic changes which will involve the majority of market places and the majority of the consumers. We are, however, looking for a revitalization of those areas that prior approval restricted. We are looking to those areas where classification and territorial definitions will be amplified and the company's attitude toward its market adaptability will be strengthened for the benefit of those who have suffered because of old attitudes. The regulator has contributed to the problem and must effectively work for the solution.

THE COMPANY ACTUARY AND THE BUREAU ACTUARY

STEVEN H. NEWMAN

Many among those here today are employed by insurance subsidiaries of essentially non-insurance holding companies; many represent subsidiaries of holding companies whose main business is, or has been, insurance; and many represent companies which are about to either succumb to, or reorganize their corporate structure into, one of these two situations.

You may ask, what do these corporate-financial configurations have to do with the subject of this discussion — open competition rating laws and the actuary? The answer is simply that although both subjects involve a great many diverse considerations, the element common to both is profit. Virtually every other business in America lives by the system of competitive private enterprise. Each figures prices on the basis of its estimated costs and then takes the profit or loss, usually without public or governmental interference. Capital can be kept at work in an insurance enterprise only when the risk-return expectation compares favorably with other opportunities. The holding company provides a convenient vehicle through which stockholders can withdraw capital funds from insurance operations and reallocate them more profitably. In 1969 almost $1 billion of such upstream dividends were recorded.

It appears to me, as a not too impartial observer, that open competition rating laws offer insurance managements their last chance to maintain and increase their share of investment capital, and therefore their capacity to underwrite insurance.

So what has all this got to do with the company casualty actuary? Who else but the actuary will be responsible for setting rates at profitable yet competitive levels? Who else is as well qualified by education, training and experience to pinpoint pockets of profit and to price new coverages and packages? And when the crutch of the cumbersome and often unresponsive prior approval regulatory system is removed and open competition laws take effect, upon whom will the ultimate burden fall of explaining to your President why underwriting losses occurred in some sufficiently credible classes of business?

And assuming that we will be able to satisfactorily fulfill these charges, who will then be called upon to maximize corporate earnings from insurance operations? How much business can your company put on its books safely without over-straining surplus and subjecting it to severe depletion. What is a company's capacity? Is it related to its ability to charge a sound rate? Of course! Is it related to the type of coverage sold, the average policy term, the rate of premium collection? How about average size of loss, probable maximum loss, maximum foreseeable loss, claim frequency, and the degree of variance by insured risk for each of these?

In my opinion the company actuary will be called upon in the well-managed company to answer these questions, to simulate the many possible results in connection with the company's corporate model (to which he will substantially assist in developing), and to select the most satisfactory course of action and project his company's profit and growth into the future so that realistic goals can be set up. The implications of such endeavors are enormous, and involve every phase of a company's operations from ratemaking to data processing, underwriting to claims adjusting, personnel requirements to investment of securities.

What does all this mean for the company actuary? At the very least he is saddled with a great deal more responsibility than ever before. In most companies a realignment of responsibility among its various departments will take place (if it has not yet already been accomplished). We can look for a reallocation of manpower, financial resources and internal priorities, all of which will favor the actuarial, data processing and statistical operations. The actuary inevitably will become involved in the mainstream of company operations to a far greater extent than in the past. The stress will shift in the systems and statistical areas from primary emphasis on "statutory reports" and "bottom-line management reports" to actuarial analysis requirements. Information will rival salesmanship and underwriting as the dominant factor in building a growing and profitable insurance portfolio.

The increase in the scope of the actuary's functions and the greater the responsibility he will bear for insurance profits, the larger his staff must become and the more departmentalized and specialized. How else will it be possible to adequately handle responsibilities in ratemaking, research and development, systems and data processing, reinsurance requirements and negotiations, corporate planning and model building, reserve analysis, and management information systems?

And what about the ratemaking and statistical organization actuary? What becomes of his functions under rate laws such as the one currently in effect in New York under which the primary responsibility for establishing and supporting rate levels is transferred to the company actuary? In my opinion the ratemaking roles as they were practiced among actuaries of rating organizations and their member companies will become transposed. Whereas committees of company actuaries actively participated in the development of theoretically proper ratemaking formulas and the necessary refinements, and the bureau staffs guided the adoption of rates to be finally followed on the basis of practical considerations, under open competition laws the roles will be reversed. This outcome reflects the impact of passages such as this one from the present New York insurance statute, section 177 item d, dealing with the prohibition of anti-competitive behavior:

"No insurer or rating organization shall make any agreement with any other insurer, rating organization or other person the effect of which may be substantially to lessen competition in any territory or in any kind, subdivision or class of insurance."

Our lawyers tell us that, in New York, ratemaking in concert via the bureau actuarial committees may be construed as an act that lessens competition through joint establishment of a rate or series of rates. Thus company actuaries will have to set their own rates after independently considering the facts. However, we all recognize the need for establishing a broad data base from which to analyze experience, especially when territory and classification refinement is extensive; and if New York-type rating laws became universal, the company actuary would need, more than ever before, the classified data with theoretically pure adjustments and supplementary projections in order to be in the best possible position to determine his own company's course of action. In my opinion then (and of course, without specifying the many notable exceptions both as to type of regulatory law and rating or statistical organization) whereas in the past the company actuary needed to emphasize the theoretical truths and the bureau actuary the pragmatic considerations, under open competition laws the bureau actuary becomes more the researcher, the technician with unbounded objectivity and more service-oriented in his dealings with affiliated companies, while the company actuary is confronted with the practical considerations of what to charge and how to increase profits without permitting any leakage of profitable business to the competition.

In summary, the advent of open competition rating laws will present challenges in the insurance marketplace that will undoubtedly enhance the value of actuaries to their companies, and at the same time dramatically increase the need for the actuarial services rendered by statistical and advisory ratemaking organizations.

THE CONSULTANT

LEWIS H. ROBERTS

The pricing of insurance is important at several economic levels. At the most parochial level, from the insurance company's point of view, its rates determine the amount and kind of business that it will attract and the profitability of that business. The interest of an actuary in the price of insurance often begins and ends at this level.

From the standpoint of the industry as a whole, the price of insurance determines its profitability and the extent to which it is used as a means of meeting risk, as opposed to other alternatives such as the self-assumption of risk or its elimination through cessation or change in mode of operation. According to economic theory, competition should redound to the benefit of the general public by forcing the price of each coverage to the lowest level consistent with an acceptable profit to the insurer. The aggregate of individual decisions on the amount and kind of insurance purchased at the offered price then determines the extent to which society utilizes insurance as a means of meeting risk. It is in this way that a competitive economic system determines the allocation of economic resources generally.

At the level of the individual company and at the level of the industry as a whole, the price of insurance performs the same economic function as the price of the product of any other business. The insurance industry, however, has a special function in the general economy which transcends the selling of its own product. It determines and assesses from policyholders one of the major costs of carrying on almost every enterprise — the costs of a wide variety of unpredictable contingencies. Inclusion of the insurance cost in the price of a commodity then forces buyers to consider whether they want the product enough to bear the cost of accidental damage and injury to persons that accompanies its production, sale and use. Thus, the insurance industry plays an extremely important role in guiding society to an economically efficient allocation of resources in all industries, not merely its own.

When the government regulates insurance prices, it becomes the arbiter at all of these levels of economic activity. For example, in sanctioning an automobile classification system and rate schedule it not only decides the price of insurance for each individual but also influences the number of cars that will be sold, their price, who will buy them and how much disposable income remains to their purchasers.

It is a basic tenet of a free enterprise society that in the absence of monopoly or unfair competition the best method of pricing commodities and thereby determining the way society allocates its resources is in the market place, through contracts arrived at by free and open bargaining. To be sure, there is ample justification for stringent regulation of the insurance industry in some areas. Unconscionable policy provisions and various undesirable pricing practices, including both unfair discrimination and unfair competition, must be carefully watched for by regulatory offiicals. However, given the present competitive nature of the industry, it is difficult to see any greater need or justification for strict control of insurance prices than for control of prices generally. On the contrary, the absence of a free market has been a bar to the most efficient fulfillment of one of the actuary's most important economic roles — the accurate determination and assessment of that component of a product's or activity's total economic cost due to accidental damage and injury.

Through "prior approval" rate regulation, insurers are straight-jacketed into a "yes" or "no" position on each risk within a classification. The insurer can only accept the risk at the stipulated price or refuse it. That is, he *may* be permitted to refuse it, subject to the limitations on underwriting discretion imposed by Assigned Risk Plans, Fair Plans, and other expressions of public policy. This inflexibility results in a proliferation of such plans and other mandatory rulings which amount to the subsidizing of bad risks by good ones. The insurer cannot play his legitimate pricing role in the economy.

In a competitive environment there should be very few really bad risks. Most risks can be good ones — at the right price. It is squarely up to company actuaries and consulting actuaries to determine that right price by using more flexible and complex techniques than those presently in general use. A company that relies comfortably upon familiar broad classification and territorial systems and depends upon its underwriters to keep it out of trouble will head downhill fast. Other, more enterprising, insurers will attract the best risks from each classification. Otherwise, to break even the company must increase rates, but as it does so the next most desirable layer of risks becomes vulnerable to competitors. We must drive this point home to our companies and to our clients before their competitors do — too late.

Under these conditions the actuary's life is far more stimulating intellectually than under bureaucratically administered pricing regimes. He must be constantly competing to develop better rating techniques. Each improvement should result in his winning some "apples" from his competitors and unloading some "lemons" on them. There ought to be no need for the unloading of lemons to take the form of refusal to insure since that is no longer a rational approach except in extreme cases. It should come about through raising the rate for an unprofitable risk to a more adequate level, thus causing it to go to another company that is still unwittingly pricing it too low. This process will have the economically desirable effect of contiually narrowing the gap between what each policyholder pays for insurance and what he should be paying.

Another gratifying aspect of such competition is that it is unlikely ever to end. Insurance ratemaking, as we know it now, is at once so susceptible to improvement and so potentially complex that there should always be room for improvement — and someone with an idea for accomplishing it. At the not imaginary extreme of refinement, the known probability of loss for some risks becomes so high that insurance is economically not feasible and should be replaced either by direct funding or, much better, by prevention of loss. Safety is furthered through the pressure of rates and the pricing of extreme cases off the road or out of business.

Of the utmost importance to the success of open competition is our willingness and ability to quote a price for risks that many insurers have studiously avoided in the past. Except for those cases where special circumstances exist, such as moral hazard, illegality, or overwhelming catastrophe potential, *the only bad risk is an underpriced one.* We should recognize this and act accordingly.

Open competition should not be looked upon, however, as a panacea whereby uninsurable risks will suddenly become insurable. There will still be coverages that are so potentially catastrophic or so expensive that it will be impossible for the insurer and the risk to reach a meeting of minds in the market place. The insurance industry should not be blamed for this, nor should it be looked upon as a failure of the competitive system. If we price a person out of the market we are telling him that the true economic cost, including insurance, of the enterprise that he is contemplating is more than he is willing or able to pay. Possibly it will be desirable as a matter of public policy that the enterprise be subsidized by other policyholders or by society in general. When that is the case, it can be accomplished by techniques similar to those used in the past. Open competition should alleviate the need for these methods but it will not completely eliminate it.

THE CONSUMER AND THE EDUCATOR

C. ARTHUR WILLIAMS*

"Open-competition" rating laws include all rate regulatory laws that prohibit agreements among insurers (except those under common control) and rating organizations to adhere to certain rates or rules. In early 1970 eleven states had such laws. In varying degrees these laws assign a greater role to competition in the determination of insurance price levels and price structures than other rate regulatory laws and involve the state insurance department less directly in ratemaking.

In five states insurers need not even file their rates — California, Florida, Idaho, Illinois (where filing may be required by regulation), and Montana. Three require filing within a stated period after the rates become effective — Connecticut, New York, and Wisconsin. In the other three — Georgia, Minnesota, and Oregon — insurers must file rates no later than their effective date. In some of these states the commissioner has the authority to impose more severe filing requirements if he finds the existing price competition to be insufficient or irresponsible.

The Consumer Viewpoint

Intelligent consumers will judge open competition rating laws primarily on their ability to provide an adequate supply of insurance at reasonable prices consistent with a "fair" profit for "efficient" insurers. If open competition laws work perfectly, each insured should pay a premium that is reasonable, adequate, and not unfairly discriminatory, in the private equity sense. An adequate supply of insurance should be forthcoming for all insureds at some price. If one insurer is inefficient necessitating high expense charges, earns excessive profits, or overcharges one group of insureds relative to

* Dr. Williams, who was a guest panelist, is Professor of Economics and Insurance in the School of Business Administration, University of Minnesota. He is a Past President of the American Risk and Insurance Association.

others, competition should force this insurer to improve its practices or withdraw from the field. Of course, perfect competition is no more to be expected in insurance than in other areas of business.[1] Consumer knowledge is not perfect; even with complete price information consumer choice would still be extremely difficult because of product and service heterogeneity. The supply of insurance for some insured will be too small because of the risk involved or the lack of insurer interest in certain markets.

Open competition laws will clearly not be acceptable to insureds unless there is active, effective price competition of which the consumer is aware. The public expects government regulators to check constantly on the nature and degree of price competition, to encourage more competition when it is insufficient, and to stop irresponsible practices. The consumer's interest in a continuing review of the status of price competition will be strongest in those states that stipulate different filing and approval requirements, depending upon the regulator's findings. However, it will not be sufficient for the regulator to make such reviews. He must communicate his findings to the public with sufficient documentation so that his conclusions may be evaluated.

Among the types of information to be developed in this review are the following:[2]

1. Number of insurers and premium volume in the state classified by:
   a. Line of insurance
   b. Type of insurer
      1) Domicile
      2) Legal form of organization
      3) Marketing system (independent agency or direct writer)
      4) Pricing system (bureau prices or other prices)
2. Degree of concentration by line
   a. Present status
   b. Trends

[1] For a classic treatise on the problems associated with maintaining price competition, see Arthur R. Burns, *The Decline of Competition* (New York: McGraw-Hill Book Company, Inc., 1936).

[2] See, for example, the section on the competitive structure of the property-liability insurance business in New York in *The Public Interest Now in Property and Liability Insurance Regulation* (New York: State of New York Insurance Department, January 7, 1969), pp. 83-94.

3. Entry and exit
   a. Number of insurers entering and existing during recent period
   b. Growth patterns of new insurers
   c. Insolvencies and their causes

4. Price structures
   a. Role of rating organizations
   b. Frequency distribution of prices
   c. Price elasticity — share of market controlled by low-cost insurers; effect of share of business controlled by individual insurers as their price position changed for selected classes

5. Contract variations and improvements

6. Marketing methods

7. Underwriting practices

8. Non-price competition

9. Insurer loss and expense experience (loss ratios, expense ratios, and profits)
   a. Average and frequency distribution
   b. Latest year and trends

10. Specific market studies — e.g., the competitive characteristics of:
    a. Automobile insurance for young drivers
    b. School property insurance
    c. Property insurance on urban core properties
    d. Malpractice liability insurance
    e. Insurance for farmers

11. Special problems and how they were handled:
    a. Insufficient competition, including an inadequate supply of insurance
    b. Irresponsible competition

Even though the regulator may be able to satisfy himself and the public that price competition is effective under the open competition law, there remains the question whether, from the consumer's point of view, superior results might be achieved under some other approach. Consequently the report on the performance of open competition rating laws should include comparisons with other similarly situated jurisdictions except for their approach to rate regulation.

Consumers expect that regulators will not only review and report what insurers do, but that they will take steps to make price competition more effective. The information on price structures in the report outlined above should explode the still common belief that all insurers charge the same price for all forms of property and liability insurance. A more debatable issue is whether the regulator should distribute charts showing the rates charged by each licensed insurer for selected classes and lines of insurance. As long as (1) the selected classes are changed over time to prevent insurers from paying special attention to selected classes and (2) insureds are alerted to the limitations of the data presented, such charts should improve greatly consumer knowledge and thus make price competition more effective. Georgia has pioneered this approach and its experience deserves further study.[3] Insurers anxious to preserve open competition may themselves take the initiative to improve consumer information. In Great Britain an independent body, the Consumer Council, has recommended that insurers set up local insurance centers where consumers can shop for policies sold by all insurers and obtain comparative price information.[4] Other changes that might improve the ability of consumers to make wise price choices (but which might have offsetting disadvantages) would include the adoption of standard policies, standard rating territories, and other standard rating factors.

Perfect operation of open competition laws, however, will not satisfy another objective of government regulation, the socialization of risk, a goal which is receiving increasing support. From the viewpoint of society, it may be desirable for some consumers to subsidize other consumers; from a broad point of view, this socialization may be in the best interests of the consumers who provide the subsidy.[5] For example, middle-aged drivers may be asked to (and may be willing to) subsidize young drivers in order to increase the proportion of insured young drivers; dwelling-owners in prosperous suburbs may be asked to (and may be willing to) subsidize owners of urban-core property subject to special environmental hazards. Socialization of risk in this way is inconsistent with open competition goals. If one insurer were to

[3] According to the Georgia study, the consumer could save many dollars by shopping around. For example, in 1968 a 45 year old male garaging his car in Atlanta could, depending upon other characteristics, pay a premium for automobile liability insurance (10/20 and 10 limits) ranging from $35 to $83.

[4] "Council Calls for Insurance Markets in U.K.," *Journal of Commerce*, April 14, 1970, pp. 1, 6.

[5] A. F. Whitman and C. A. Williams, Jr., "Environmental Hazards and Rating Urban Core Properties," forthcoming issue of *Journal of Risk and Insurance*.

charge rates based on the socialization of risk, but the others were to price competitively, that insurer would soon be driven out of business. Consequently some modification of open competitions laws is necessary. Depending upon (1) the relative emphasis assigned to socialization of risk as opposed to private equity and (2) the extent of the hazard costs to be socialized, the solution may be the complete abandonment of open competition or some special arrangements for these special hazard costs to be socialized.

If socialization of risk is a primary objective and most hazard costs are to be socialized, the simplest solution is to abolish private insurance and have the government pay losses out of general revenues. A closely related alternative would be a compulsory government insurance program operated by an exclusive government insurer or serviced by private fiscal intermediaries. Another possibility is compulsory insurance written by private insurers, all of whom charge the rates established by a mandatory rating bureau, coupled with a plan to share undesirable business.[6]

If private equity is preferred for most insureds and most hazard costs, but a degree of socialization of some risk is desired, less drastic steps may be satisfactory. At present, some socialization of risk is achieved under open competition laws through assigned risk plans or pools whose members are subsidized by other insureds. In most cases the subsidy is implicit in general rate increases which reflect any underwriting losses on the plan or pool. The subsidy costs are distributed among insureds according to their rate relativities. In property insurance, for which there is a special riot and civil disorder surcharge, the cost allocation formula is more complex.

Consumers may not like either method of allocating environmental hazards or other losses not covered by the plan or pool rates. Insurers may find that when a new risk is to be socialized, unless there is a special loading collectible at the same time, they must bear any excess losses pending a general rate increase. On the other hand, with a special loading, some insurers may collect more than they need and others too little.

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[6] In a thought-provoking paper Professor John Hall has suggested that in "social insurance" lines such as automobile insurance the best approach would be a national mandatory rating bureau that would establish a uniform set of pure premiums, coupled with a plan for "unifying" underwriting experience. This uniform set of pure premiums could favor some insureds over others. Competition would be limited to expense and profit loadings which would differ among insurers. Competition on pure premium structures and underwriting selectivity would cease, but Professor Hall believes that this would be an advantage. See John W. Hall, *The Automobile Insurance Underwriting Problem* (Atlanta: The Center for Insurance Research, Georgia State College, July 1969).

In part insurer reluctance to extend FAIR plans to include theft and vandalism insurance stems from the fact that no satisfactory mechanism has been developed for distributing the cost of non-chargeable environmental hazards, which are substantial with respect to these perils. An alternative approach that might have some merit would be to identify those hazard costs that should be socialized and to seek government subsidies for them. This problem is too important and too complex to be solved by a brief comment in this paper, but the consumer will demand its solution in the near future.

The Educator's Viewpoint

Educators should engage constantly in the "search for truth." Consequently they should be interested in the effects of open competition laws on all types of insurers, consumers, and regulators. In addition they have a special obligation to comment on the overall impact of these laws from the viewpoint of an objective observer.

Of great interest to educators will be the influence of open competition laws on insurance pricing. Insurance literature currently suggests that full-cost pricing has been the rule in insurance. Actuaries have sought the estimated cost of providing protection and then added a profit and contingencies loading to determine the premium. Although in practice consumer demand and the prices charged by other insurers have undoubtedly been considered, they should become more important and more explicit pricing factors in the future. Strict adherence to bureau rates should decline, and company actuaries will have to develop more competence in market research and micro-economies. The development of new price policies should be a fruitful area for research by educators and for case studies in business decision-making. Educators will, of course, also be interested in effect of these laws on the nature and degree of competition in the insurance business and on marketing efficiencies.

Because regulators are much more commonly associated with the overall viewpoint than either insurers or consumers, educators have always been intensely interested in the objectives of regulation, the methods used to achieve these objectives, and the results of this regulation. Because open competition laws affect both the objectives and the methods, a whole new field of inquiry has been opened up. Educators will be interested in how regulators choose to measure the nature and degree of competition, the situations in which they will determine competition to be insufficient or irresponsible, the action they will take in those situations, their experience under the various types of filing requirements, the ways in which they will improve the price information available to insureds, and the resources that must be devoted to the regulatory effort.

Because everyone is a consumer, everyone shares this viewpoint to some degree. In particular educators will study the effect of open competition laws on consumer buying practices and their satisfaction levels. For example, will increased price competition and knowledge of insurance prices cause consumers to shop more before buying? Will it lead to a more desirable emphasis on price or too much emphasis on price? Are the prices for some consumers much higher than they can afford to pay? Is the socialization of risk objective a strong one, and, if so, have some acceptable satisfactory arrangements been made to accomplish this objective?

Finally the educator, having studied the operation of these laws from the viewpoint of all three parties directly involved, should have the understanding and objectivity to assess the laws from society's point of view and to work with the other three groups for an improvement in the rate regulatory process.

The educator's work, in turn, needs to be supported and assessed by the other three groups. Educators often do not understand the total situation as well as others; they may tend to oversimplify problems and their solutions; and unfortunately they are not always objective. Open competition laws, therefore, by opening new areas for study and research, should increase interactions between educators and the other three groups.

MINUTES OF THE 1970 SPRING MEETING

May 24-27, 1970

DIPLOMAT RESORTS and COUNTRY CLUB, HOLLYWOOD, FLORIDA

Sunday, May 24, 1970

Prior to the formal convening of the Spring Meeting the following day, the Council met at the Diplomat from 2:00 p.m. to 5:30 p.m. and the meeting was continued to its conclusion on Monday, May 25, 1970 from 1:00 p.m. to 4:00 p.m.

In the evening the officers sponsored an informal "get acquainted" reception hour for the new Fellows (and their wives) who, later during the Spring Meeting, would be presented with their diplomas.

The membership enjoyed a reception and buffet dinner during the evening hour.

Monday, May 25, 1970

The 1970 Spring Meeting was formally convened at 9:00 a.m. by President Daniel J. McNamara who welcomed the gathering and then introduced the Honorable Broward Williams, Insurance Commissioner, State of Florida.

Commissioner Williams addressed the membership and discussed at some length his views on current problems affecting automobile liability inurance.

Following Commissioner Williams' presentation, President McNamara introduced Henry W. Menzel, General Manager and Actuary, New York Compensation Insurance Rating Board who moderated a panel "Profitability and Pricing." Mr. Menzel then introduced the following participants:

Dr. Irving H. Plotkin
Senior Economist
Arthur D. Little, Inc.

Alan C. Curry
Senior Actuary
State Farm Mutual Automobile Insurance Company

Jeffrey T. Lange
Secretary and Associate Actuary
Royal-Globe Insurance Companies

As part of this portion of the program, reviews of Robert A. Bailey's paper "A Review of the Little Report on Rates of Return in the Property and Liability Insurance Industry" were presented by the following:

Russell P. Goddard
Actuary
Bowles, Andrews and Towne, Inc.

Richard L. Norgaard
Head, Department of Finance
University of Connecticut

J. Robert Ferrari
Associate Professor of Insurance
University of Pennsylvania

Goddard's review was read by Bernard L. Webb, Norgaard's by Jack Moseley, and Ferrari's by George D. Morison. The panel discussion continued until 11:00 a.m.

Following the panel discussion the membership had their choice of attending one of two concurrent workshop seminars as follows:

Seminar #1 — "The Changing Annual Statement"

Moderator: Phillip B. Kates
President
Independent Fire Insurance Co.

Participants: Matthew Rodermund
Vice President and Actuary
Munich Reinsurance Company

Ruth E. Salzmann
Vice President and Actuary
Sentry Insurance Group

William H. Thompson
Assistant Vice President
Hartford Insurance Group

Seminar #2 — "Decision Theory and the Actuary"

Moderator: Charles F. Cook
Chief Actuary
United Services Automobile Association

Participants: Darrell W. Ehlert
Director of Actuarial Research
Allstate Insurance Company

Dale A. Nelson
Actuary
State Farm Automobile Insurance Company

Warren P. Cooper
Assistant Vice President
Chubb and Son, Incorporated

After luncheon, various Society committees as well as the Council met to consider their assigned subjects.

Tuesday, May 26, 1970

The meeting was reconvened at 9:00 a.m. by Vice President LeRoy J. Simon.

Seminar moderators Phillip B. Kates, Jr. and Charles F. Cook reported to the membership a brief summary of their seminars held on Monday morning.

Following these summaries the following committee chairmen reported current activities of their respective committees.

Advisory Committee to the Department of Transportation	— Paul S. Liscord
Education and Examination Committee	— M. Stanley Hughey
ASTIN	— Charles C. Hewitt, Jr.
Public Relations Committee	— James R. Berquist
Committee on Professional Conduct	— Thomas E. Murrin

After these reports the membership split to attend two concurrent workshop seminars as follows:

Seminar #1 — "What Does Open Competition Really Mean?"

Moderator:	Gerald R. Hartman Associate Professor Department of Insurance, Temple University
Participants:	Lewis H. Roberts Vice President and Manager Woodward and Fondiller Inc.
	Steven H. Newman Vice President and Casualty Actuary American International Underwriters Corporation
	Kevin M. Ryan Deputy Director Department of Insurance, State of Illinois
	C. Arthur Williams, Jr. Professor of Economics and Insurance School of Business Administration University of Minnesota

Seminar #2 — "Necessary Crime Coverages — Are They Available?"

Moderator:	Augustin J. Cima Actuary Allstate Insurance Company
Participants:	Phillip N. Ben-Zvi Assistant Actuary Royal-Globe Insurance Companies
	Richard H. Snader Senior Actuarial Assistant United States Fidelity and Guaranty Co.
	A. W. Hanington, Jr. Assistant Vice President Maryland Casualty Company

After completion of these seminars the meeting was recessed for luncheon.

Vice President Simon reconvened the meeting at 2:00 p.m.

President McNamara then presented diplomas to the following new Associates and Fellows:

ASSOCIATES

Bill, Richard A.	Napierski, John D.	Skurnick, David
Head, Thomas F.	Sandler, Robert M.	Stephenson, Elton A.

FELLOWS

Beckman, Raymond W.	Kilbourne, Frederick W.	Scheel, Paul J.
Jacobs, Terry S.	Munro, Richard E.	White, William D.

The next item was the presentation of three new papers as follows:

1. "Trend and Loss Development Factors" by Charles F. Cook, Chief Actuary, United Services Automobile Association
2. "A Stochastic Approach to Automobile Compensation" by Donald C. Weber, Lecturer, Miami University (Oxford, Ohio). Professor Weber's paper was presented by Robert A. Bailey.
3. "Distortion in IBNR Factors" by LeRoy J. Simon.

The next order of business was the presentation of a panel on "Open Competition Laws: Are They Working?" This panel was moderated by Harold E. Curry, Senior Vice President, State Farm Mutual Automobile Insurance Company, who introduced his panelists as follows:

George K. Bernstein
Federal Insurance Administrator
U.S. Department of Housing and Urban Development

Benjamin R. Schenck
First Deputy Superintendent
New York Insurance Department

P. C. Gallagher
President
Florida State Agents Association

Thomas E. Murrin
Senior Vice President and Actuary
Fireman's Fund American Insurance Companies

This session was recessed at 5:00 p.m.

During the evening hour a reception was held at the hotel for members and their guests.

Wednesday, May 27, 1970

The meeting was reconvened at 9:00 a.m. by Vice President Richard L. Johe.

The business session was continued with reports of Tuesday's seminars by the two moderators Gerald R. Hartman and Augustin J. Cima.

Following these reports, a review of Jeffrey T. Lange's paper, "The Interpretation of Liability Increased Limits Statistics," was presented by Thomas W. Fowler, Actuary, North American Reinsurance Corporation. L. H. Roberts, Vice President and Manager, Woodward and Fondiller, Inc., presented a review of an earlier paper by Mr. Lange, "An Actuarial Note on Actuarial Notation."

An author's rebuttal to prior reviews of his paper "Is Probable Maximum Loss (PML) A Useful Concept?" was presented by John S. McGuinness, President, John S. McGuinness Associates.

Following the conclusion of the business session, a presentation was made to the membership, including movies and slides, on "Highway Loss Reduction; Structuring The Problem and Examples of Payoff." The presentation was made by Dr. William Haddon, Jr., President, Insurance Institute for Highway Safety.

At the conclusion of this presentation President McNamara adjourned the Spring Meeting at 12:30 p.m.

It is noted that the registration cards completed by the attendees and filed at the registration desk indicate, in addition to about 25 wives, attendance by 92 Fellows, 47 Associates, and 35 invited guests as follows:

FELLOWS

Adler, M.	Bevan, J. R.	Cook, C. F.
Aldrich, W. C.	Bickerstaff, D. R.	Curry, A. C.
Alexander, L. M.	Bland, W. H.	Curry, H. E.
Allen, E. S.	Bondy, M.	Dahme, O. E.
Bailey, R. A.	Bornhuetter, R. L.	Dorf, S. A.
Balcarek, R. J.	Boyajian, J. H.	Ehlert, D. W.
Beckman, R. W.	Brown, W. W., Jr.	Eliason, E. B.
Bennett, N. J.	Byrne, H. T.	Elliott, G. B.
Ben-Zvi, P. N.	Carlson, E. A.	Farnam, W. E.
Berquist, J. R.	Cima, A. J.	Finnegan, J. H.

FELLOWS

Fowler, T. W.
Gibson, J. A., III
Gillam, W. S.
Graham, C. M.
Graves, C. H.
Hartman, G. R.
Harwayne, F.
Hazam, W. J.
Hillhouse, J. A.
Hope, F. J.
Hughey, M. S.
Hurley, R. L.
Jacobs, T. S.
Johe, R. L.
Kallop, R. H.
Kates, P. B.
Kilbourne, F. W.
Kormes, M.
Lange, J. T.
Leslie, W., Jr.
Linder, J.
Lino, R.
Liscord, P. S.
MacGinnitie, W. J.
Makgill, S. S.
Masterson, N. E.
McGuinness, J. S.
McNamara, D. J.
Meenaghan, J. J.
Menzel, H. W.
Mohnblatt, A. S.
Morison, G. D.
Moseley, J.
Muetterties, J. H.
Munro, R. E.
Murrin, T. E.
Naffziger, J. V.
Nelson, D. A.
Newman, S. H.
Petz, E. F.
Phillips, H. J., Jr.
Pollack, R.
Riddlesworth, W. A.
Roberts, L. H.
Rodermund, M.
Rosenberg, N.
Roth, R. J.
Ryan, K. M.
Salzmann, R. E.
Scheel, P. J.
Scheibl, J. A.
Simon, L. J.
Smick, J. J.
Smith, E. R.
Sturgis, R. W.
Tarbell, L. L., Jr.
Uhthoff, D. R.
Verhage, P. A.
Walsh, A. J.
Webb, B. L.
White, W. D.
Wilcken, C. L.

ASSOCIATES

Bergen, R. D.
Bill, R. A.
Chorpita, F. M.
Comey, D. R.
Cooper, W. P.
Davis, R. C.
Feldman, M. F.
Ferguson, R. E.
Flynn, D. P.
Franklin, N. M.
Gill, J. F.
Gossrow, R. W.
Gould, D. E.
Grady, D. J.
Greene, T. A.
Hammer, S. M.
Hardy, H. R.
Head, T. F.
Hunter, J. R., Jr.
Jensen, J. P.
Jones, A. G.
Jones, D. R.
Jones, N. F.
Jorve, B. M.
Klingman, G. C.
Levin, J. W.
McDonald, M. G.
McIntosh, K. L.
Napierski, J. D.
Peel, J. P.
Plunkett, J. A.
Price, E. E.
Raid, G. A.
Ratnaswamy, R.
Richardson, J. F.
Royer, A. F.
Sandler, R. M.
Scammon, L. W.
Schneiker, H. C.
Singer, P. E.
Skurnick, D.
Snader, R. H.
Stephenson, E. A.
Stewart, C. W.
Strug, E. J.
Walters, M. A. (Miss)
Walters, M. A.

GUESTS

*Babb, J. A.
*Banfield, C. J.
*Battaglin, B. H.
Bernstein, G. K.
Buzby, E. K.
Chamberlain, R. H.
*Dunn, R. P.
*Eddins, J. M.
Farr, D. G.
Gallagher, P. C.
*Griffith, R. W.
Guarini, L.
Haddon, W., Jr.
Hall, J. W.
Hanington, A. W., Jr.
*Hart, J. F.
*Hayden, R. C.
*Katzman, I.
*Kedrow, W. M.
Knox, F.
*McClenahan, C. L.
*Miller, P. V.
Olsen, W. G.
*O'Shea, H. J.
Pellegrini, P. L.
Plotkin, I. H.
Reid, J. N.
*Reilly, F. V.
Reinbolt, J. B.
Schenck, B. R.
Thompson, W. H.
Trafton, M.
*White, B. R.
Williams, B.
Williams, C. A., Jr.

* Invitational Program

Respectfully submitted,

RONALD L. BORNHUETTER
Secretary-Treasurer

VOLUME LVII, Part II | No. 108

PROCEEDINGS

NOVEMBER 15, 16, 17, 1970

ECUMENICISM

PRESIDENTIAL ADDRESS BY DANIEL J. McNAMARA

Traditional adherence to the literal provisions of the bylaws of the Casualty Actuarial Society requires the retiring president, at each annual meeting, to share with his colleagues some of his personal views — with perhaps a sprinkling of previews — about the business we serve and the profession we enjoy. I shall not disturb this tradition although I briefly explored methods to circumvent it. I have, however, satisfied myself that previous presidential addresses to our Society have provided a sense of continuity and perspective as well as a periodic restoration of the spirit. Thus, with a renewed appreciation of the innovative thinking advanced in the addresses of my predecessors, I have decided not to be the one to break tradition. But enough for introduction! Where are we today?

The old Arabic adage, "I wept because I had no shoes until I saw a man who had no feet," seems properly to set the stage if rephrased to say "I wept because I had no profit until I saw a man who had no business."

It has been our fortune — or misfortune, depending upon your particular point of view — to have just come through a careening decade in the underwriting, pricing, marketing, and regulation of insurance. It has been a bewildering decade for individual insurers and, beset with underwriting losses and new forms of governmental pressure, many have begun to diversify their efforts and to change — or to have changed for them — their corporate structures.

As we enter the last thirty years of the Twentieth Century, any remaining isolationism of the property-liability insurance business from the economy of the country is a historic shadow. The business of insurance is inextricably involved in the social and the economic pressures of our time. We have only to glance at or hear the headlines of the communication media to realize that society's problems are our problems. The breakdown of law and order, inflation, urban decay, and the proliferation of natural and man-made disasters have made our business most sensitive to its environment.

In this highly-charged atmosphere, I submit that leaders in both our business and our Government have some rather basic thinking to do and some significant steps to take in the months and years to come if the insurance business is to continue to meet human needs in this era of rapid social change. This, after all, is the only reason our business exists.

Similar reflection, I would further submit, is equally appropriate for the Casualty Actuarial Society, where the last ten years have blurred forever many of the distinctions that have traditionally identified the separate life and non-life actuarial professions. I will offer my observations on this point later.

For now, let me suggest that an ecumenical spirit — of the kind which has deeply affected the personal lives of so many of us — should prevail in our reflections of how best can be blended the needs of the business we serve and the persons whose needs, to a significant degree, we help to fulfill. This type of spirit, which implies a subordination of differences within broad areas of agreement — and let me emphasize subordination, not a surrender of rights — could be enhanced by recognition of two basic precepts:

> First, that every right implies a corresponding responsibility, and second, that the individual good is, indeed, subordinate to the common good.

Observers in great numbers and in stentorian tone are predicting with confidence that we are beginning a decade which, while it will be a period of major technological and economic progress, will be markedly different from any that heretofore has been experienced by the business of insurance.

This progress, coupled with control of, or diminution of, other influences on life and society, will, in my opinion, permit us to direct more of our resources toward peacetime efforts, toward a calming of urban unrest and toward social justice for all Americans.

In the final analysis, every enterprise exists by using its capital and its other resources to meet human needs, and the way people perceive some of these needs is changing as rapidly as social values are changing.

The importance that people attach to their perceived needs and the concern they show when such needs are inadequately met carry a significant message to the managers of any corporation. Private enterprise can hope to succeed only to the degree that it recognizes changes in social values, identifies the resulting new or changed human needs, and proceeds to serve those needs in a manner satisfactory to the public.

Along with others, I have concluded that we are in the midst of a period of more than soaring material expectations. There is a great heightening, I believe, in the moral expectations of more and more Americans. It is this latter type of expectation that has helped to spur the consumer movement, to point up and advance the civil rights movement, and to enlarge and solidify the forces supporting the peace movement.

How can the goals of these and similar movements be met? The usual approach is to look to Government, and certainly that is a part of the answer. But, the potential power of Government alone is not sufficient to respond fully to the demands of today's society. All kinds of businesses, as well as you and I as individuals, must combine our talents and resources with those of Government to meet these new and pressing demands.

Society today is more mobile, more industrialized, more knowledgeable, and more demanding. Society as a whole is insisting upon greater accountability from all institutions — public and private — that lay claim on its money through capital investment, through charitable donation, through taxation, or through outright purchase.

It is no revelation to this audience that society seeks to have its business and personal possessions insured and its injuries compensated at the least possible cost consistent with good service. But there is ample evidence that today our people and our corporations are much more vocal and are more determined to accomplish this goal.

It seems clear to me that providing a full range of protection through insurance for all of our society no longer can be considered the singular function of the private insurance mechanism. Increasingly, filling the needs for insurance protection is requiring cooperative action between insurance entrepreneurs and Government. We cannot stick our head in the sand and ignore the expanding role of Government in finding ways to meet public demand that protection be readily available, irrespective of the problems involved in trying to provide it through the traditional insurance mechanism.

I would like then to look briefly at two principles that should embrace the future partnership between the insurance enterprise and Government, keeping in mind the precepts concerning rights and responsibilities and the common good.

First, our business has every right, as private enterprise, to demand and to receive from Government the sanction to develop and price our products and services without regulatory interference. But in insisting upon this right, we must be ready to accept the corresponding responsibility thereafter to meet the insurance needs of society in a truly competitive manner.

The mainspring of the private enterprise system is the free-price mechanism. It is clear that Government must continue to move positively to eliminate the artificial pricing restrictions that the model All-Industry Rate Regulatory Laws imposed on our business in a bygone era.

The quid pro quo is that our business must respond to this freedom by making insurance more readily available and, especially for the personal lines of insurance, by moving toward greater price diversity among individual insurers. We all know that there is no one right rate for most insurers for most insurance lines and in this new climate our actions should reflect this fact. Lest I be misunderstood, I am not suggesting that we abandon the necessary and sound practice of pooling risk data. But once these data have been translated into appropriate measurements — and future measurements are an art, incorporating judgment, not a pure actuarial science — individual insurers should move ahead on their own to provide products and services at a price which is suitable and proper for the individual company and its policyholders. Our business has adapted well to the creative procedures developed to handle and rate large commercial enterprises which presented, in another era, problems of both coverage variation and individually unique risk characteristics. The cake of custom is slowly crumbling and I am confident that, despite the corporate and producer dislocations, we can also adapt to these new directions in the mass market lines of insurance without undue selectivity. The cream-skimming of risks that characterized our failure in the 1960's must be arrested under these new-type laws or we invite further Government involvement.

Secondly, Governmental insistence that all members of society be able to obtain insurance carries with it a mutual responsibility to create conditions that permit this goal to be accomplished on an economically sound basis. This Government demand does not change the inherent loss potential of a risk — even though the form of the guarantee against loss may be changed or transferred from private insurance to Government.

Insistence that a risk be assumed by insurance companies may solve a political problem but at the same time it may create an intolerable economic problem. When Government decides that some method of idemnification must be developed to meet society's needs at a price society thinks it can afford to pay — be it for losses associated with the automobile, crime, flood, mud slide, sink holes, or sickness and injury — there must be an appreciation that total reliance upon the private insurance mechanism may not be the most feasible nor the most equitable way to spread the cost of these losses.

I would set forth the proposition that Government and business must solve this problem ecumenically or our current frustrations will be magnified in the years to come to the detriment of the public we both serve.

In any appraisal of the nation's future insurance needs — personal, commercial, and group — the partnership of Government and business must be balanced. We have now in many lines of insurance a gradual but growing trend toward the sharing of responsibility between Government and private enterprise. The winds of change seem to be blowing in the direction of programs which for certain kinds of risks will provide more publicly financed basic limits of protection, supplemented by private programs under which individuals and groups can obtain at their own expense the kinds of additional protection they want.

Nevertheless, in any future environment, it seems clear to me that the private insurance mechanism will continue to have a major role in providing basic coverages as well as the sole role in providing the supplemental coverages. But in each case, the role of our business can be expected to undergo substantial change, and I would hope and trust that the resulting procedures and performance would be more satisfactory both for the public and the insurance companies.

The ramifications of this trend were described recently by Superintendent Stewart of New York in this way: "Thus the present national debates over what we now call health insurance and crime insurance, and other foreseeable debates over protection from economic loss, are far less debates over regulation than debates over program. They are far less debates over insurance than debates over other economic and social problems in which insurance is seen, and dealt with, as only a part of the problem or an implement for its solution. Once the program decisions are made, the national government's relationship with the insurance business is most likely to be that of planner, partner, employer, supplier, customer, competitor or successor."

In this changing pricing and regulatory environment, actuaries, as individuals, certainly will have an increasingly expanded role. The accomplishments to date of our Society, as published in our *Proceedings*, and the successful management careers of many of our members, speak for themselves and it is a justifiably proud history. We have earned the right not only to be heard, but to be listened to, and we must preserve and improve upon this right by becoming even stronger and more useful.

These changes in direction present distinct challenges to all of us to reevaluate the conventional approaches to our problems. We must find the new approaches that will help insurers move toward solutions to many of today's problems and toward proper profitability. Product simplification and group approaches in a computer, consumer-oriented society, as well as coverage packaging with broader deductible application in an inflationary economy, are but a few of the approaches that must be studied to satisfy legitimate demands.

Some reform of the legal system as it applies to automobile insurance is inevitable for it is only through such reform that the insurance business can provide the service the public deserves at a cost it is willing to pay — and we, as actuaries, must be deeply involved in suggesting the forms this reform should take.

We must also find ways to simplify the structure of our business, which over the years has too often adopted parochial practices and procedures that were designed to meet vocal demands made locally rather than substantive need. Lest again I be misunderstood, variations in pricing procedures by geographical area or by individual company must be encouraged, but such variations are useful and efficient only when they are based on sound reason and meet significant need in the national multiple line operations of many insurers.

And, we must help Government develop new pricing and rating reforms, if not tax reforms, to make possible the goal of a fully insured public without continued subsidization by our business. This fact of inadequate return, which can only cause further decapitalization at a time when more capital is needed to meet increasing demands for insurance, must be reversed if we are to have a viable private insurance business.

Turning again to our profession itself, I believe that it will undergo changes that if not in magnitude, at least in direction, will match the changes affecting our business. The rights and privileges we now enjoy as actuaries carry with them, I believe, a responsibility to multiply our number and to enhance — not merely maintain — our professional standards of competence and conduct.

The cold, hard fact is that our Society has not met the increasing demand for actuarial expertise and creativity. Our growth rate over the last three decades in comparison with the growth of the business and our counterpart, the Society of Actuaries, has been poor, and I would urge that we place top priority on the need to attract bright young people to our profession. This critical need can only be met, in my opinion, by a complete re-evaluation and alteration of our present educational and examination requirements, both procedurally and substantively, and present Society efforts in this area must be aggressively and dramatically increased.

Irrespective of the fruit of such efforts, I nevertheless believe that many of the distinctions that have carefully insulated the life and non-life actuarial professions are gone forever as more and more companies move into the business of providing a broad range of financial services.

It is inevitable, in my judgment, that there will be one professional actuarial society with a number of specialities, and I conclude these personal remarks by publicly suggesting what already has been suggested in some private quarters, that the Casualty Actuarial Society take the first giant step by commencing discussions with the Society of Actuaries with the purpose of ultimately consolidating these two organizations. The property-liability actuarial profession, although small in number, has created a significant impression on our entire business and, in any possible consolidation, I believe, in the words of the late Franklin Delano Roosevelt, "We have nothing to fear but fear itself."

CREDIBILITY FOR SEVERITY

CHARLES C. HEWITT, JR.

"THE UNSOLVED PROBLEM

"In casualty insurance, the inherent hazard of an insured, or of a classification of insureds, is the product of an inherent frequency of loss occurrence and an inherent average amount of loss, and it is the value of this product for which an estimate is desired. Such an estimate must be expressed in terms of the amounts of the individual losses which have occurred and the a priori *knowledge as to average frequencies, average amounts of losses, the distribution of frequencies and loss amounts about such averages and* a priori *knowledge as to the correlation between frequencies of loss and average loss amounts.*

"The expected value, or estimate, of such a product would, no doubt, be more complicated in form than the results obtained for the simpler cases studied herein. The form such an estimate should take would be very desirable information for the actuary to have, even though, at the present time, there is little or no knowledge as to the correlation between frequencies of loss and average loss amounts in casualty insurance. It is the hope of the writer that someone with a knowledge of the statistical behavior of products will undertake the development of the appropriate procedure."

— A. L. Bailey [1]

"Most credibility formulas in use today measure the credibility of a given number of claims. What is really needed, however, is the credibility of the pure premium, which depends on claim severity as well as claim frequency."

— Allen L. Mayerson [9]

This paper accepts the challenge laid down by these two distinguished actuaries.

Credibility Defined

The literature of this Society amply reflects the widespread use of the formula:

$$Z = \frac{n}{n + K}$$

where:

n equals the number of trials or exposure units
K equals a constant (whose derivation will be the principal subject of this discourse)
z equals *credibility* (with range *0 to 1*).

Credibility is applied in the following manner:

1) there is an hypothesis concerning the mean expectation from some observations,
2) for n trials or exposure units a mean value of the observations is established,
3) with two values to choose from, the question is asked, "To what extent do we believe the expectation and to what extent do we believe the observations?",
4) the degree of belief in the observations is expressed as a measure z, and the degree of belief in the hypothesis as the complement, $1 - z$,
5) or more formally in the linear relationship:

$$C = zR + (1 - z)H$$

where:

R equals the mean of observation (*R*esult)
H equals the mean of the *H*ypotheses, and
C equals the value to be used as a *C*ompromise estimate

Thus, *credibility is a linear estimate of the true* (or inherent) *expectation derived as the result of a compromise between hypothesis and observation.*

Bühlmann [3] has demonstrated that:

$$K \text{ equals } \frac{\text{Expected value of the process variance}}{\text{Variance of the hypothetical means}}$$

The implications of this derivation of K are discussed more fully in [8].

Credibility and Bayesian Estimation

Before proceeding further it will be helpful to underscore the relationship between credibility and *a posteriori* (or Bayesian) estimation and to weigh their relative advantage and disadvantages.

Credibility does not (necessarily) produce the optimum estimate.

Bayesian analysis produces the optimum estimate.

Credibility does produce the "least-squares" fit to the optimum (Bayesian) estimates for all possible outcomes weighted by the respective probabilities of those outcomes.

Both estimates — credibility and Bayesian — are "in-balance" for all possible outcomes.

The estimate resulting from the application of credibility always falls on or between the hypothetical mean and the observed result. The Bayesian resultant frequently does not satisfy this condition. Hence, the use of credibility often produces results more easily explained to the layman, whether he be a customer or an underwriter.

On the other hand the Bayesian resultant can never fall outside the range of hypotheses, whereas the credibility-produced estimate can fall outside the "realm of possibility," although such a happening is unlikely.

Determination of the Bayesian estimate can be extremely complex, even on one trial, and is predictably too complex to handle for more than a few trials (or exposure units). The theoretical part of using credibility is encompassed in the fixing of the value for K; once K is determined, credibility may be applied by a clerk and understood by virtually anyone concerned.

The Risk Process

The basic process in risk and insurance is the compounding of a number of events or occurrences (labelled claims) with a value assigned to each separate event or occurrence (called the amount of the claim). The value of the number of occurrences is discrete: 0, 1, 2, 3, ..., k, ... The amount assigned to each occurrence may be constant, or it may vary over a wide range, often considered to be a continuum for convenience of analysis. The compound process is discussed and expressions for moments of the compound process are derived in [10]. For purposes of this section it will be sufficient to draw the following expressions adapted from [10]:

$$_{x}E = {_k}E \cdot {_{x_1}}E$$

and $$_{x}\sigma^2 = {_k}E \cdot {_{x_1}}\sigma^2 + {_k}\sigma^2 \cdot {_{x_1}}E^2$$

Where the prefatory subscripts have the following significance:

x — the compound process (or distribution)

k — the discrete process of determining the number of occurrences

x_1 — the distribution of the values of a single claim

Thus $_kE$ is the mean number of occurrences, $_{x_1}\sigma^2$ is the variance of the amounts of a single claim, etc. Of most importance are $_xE$ and $_x\sigma^2$ which are the mean and variance of the compound process.

The inhibitions placed upon the use of the expression for the mean and variance of the compound process are:

(a) the value attributed to each separate occurrence is independent of all other values so attributed,
(b) the values so attributed are drawn from the same probability distribution,
(c) the number of occurrences is statistically independent of the values attributed to the occurrences.

There are circumstances in practice under which these inhibitions are breached — non-reporting of smaller claims as opposed to the more likely reporting of larger claims is clearly in this category. However, there are many situations in insurance for which these inhibitions are not violated to any important degree. Judicious selection of the proper event-producing process and/or what properly constitutes a single event (or claim) will generally provide adequate reassurance that the expression for the compound process variance is quite satisfactory.

It is important, nay vital, to realize that these inhibitions which qualify the "process variance" do not, in any way, affect the "variance of the hypothetical means," which depends only upon defining the possible states and quantifying their *a priori* probabilities.

Two Illustrative Examples

In order to bridge the gap between the theoretical derivation by Bühlmann [3] of the credibility "K" and actual application thereof, two examples will be used to explain how the "variance of the hypothetical means" and the "expected value of the process variance" may be derived. In so doing the theoretically correct method for inclusion of the severity component will also be illustrated — for the first time as far as this author is aware.

A Discrete Example

A die is selected at random from a pair of "honest" dice. It is known that one die has one marked face and five unmarked faces and that the other die has three marked and three unmarked faces.

For reference we will define:

- A_1 as the state of having drawn the die with one marked and five unmarked faces
- A_2 as the state of having drawn the die with three marked and three unmarked faces

A spinner is selected at random from a pair of spinners.

It is known that one spinner has six equally-likely sectors five of which are marked *two* and one of which is marked *fourteen,* and that the other spinner has six equally-likely sectors three of which are marked *two* and three of which are marked *fourteen.*

For reference we will define:

- B_1 as the state of having selected the spinner with five *twos* and one *fourteen*
- B_2 as the state of having selected the spinner with three *twos* and three *fourteens.*

Initially it will be specified that the selection of the die and the spinner are completely independent.

Thus there are four *equally-likely* compound states:

$$A_1 \cap B_1$$

$$A_1 \cap B_2$$

$$A_2 \cap B_1$$

$$A_2 \cap B_2$$

The state once determined will remain the same throughout, but will be *unknown* to the participants.

First, the die which was drawn will be rolled. If a marked face appears uppermost this constitutes a claim; if not, there is no claim. If there is a claim, the selected spinner will be spun to determine the amount of the claim.

The process of rolling the die, once drawn, is assumed to be binomial, i.e either marked or unmarked face appears.

A Continuous Example

A private passenger automobile insurance risk will be chosen from a class of such risks. Each risk in the class has its own inherent measure of hazard which can never be exactly known to an insurer.

For frequency of occurrence of a claim we will define:

M as the state of having chosen a risk with the inherent frequency of claims in one exposure unit (one car-year) m (see [4], [5] and [6] for an earlier treatment of this situation)

The *a priori* probability of having chosen M is given by the gamma distribution:

$$T(m)dm = \frac{a^r}{\Gamma(r)} m^{r-1} e^{-am} dm$$

where m varies between *zero* and (positive) infinity.

The process by which the number of claims is given is assumed to be Poisson for the particular risk, M. Thus the probability of n claims in one car-year is given by:

$$Pr(n) = \frac{m^n}{n!} e^{-m}$$

where n is any non-negative integer.

It is important to distinguish between the frequency process for the individual risk (Poisson as already stated) and the frequency process for the class of risks from which it is drawn. The latter process will be negative binomial as developed in [4], [5] and [6].

For severity of an individual claim, use will be made of a distribution of some auto property damage losses by size which are fitted quite precisely by a compounding of the log-normal and gamma distributions. (A paper describing the method of fit and the accuracy thereof is in preparation.) For simplicity here it will be assumed that the distribution of a single property damage claim (X) follows the log-normal pattern.

We will define:

θ as the state of having chosen a risk with the inherent severity (average amount of a single claim):

$$E(X) = e^{\mu + \frac{\sigma^2}{2}}$$

where the loss amounts for the risk θ are distributed log-normally:

$$\Lambda(X; \mu, \sigma)dX = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$x = \log_e X$ and varies from negative infinity to positive infinity. The *a priori* probability of having chosen μ is given by the normal distribution:

$$N(\mu; N, S)d\mu = \frac{1}{S\sqrt{2\pi}} e^{-\frac{(\mu-N)^2}{2S^2}} d\mu$$

where μ varies from negative infinity to positive infinity and it is assumed that σ does not vary from risk to risk in this particular class of risks.

(Appendix A demonstrates that the class distribution of amounts of a single claim (Y) is also log-normal:

$$\Lambda(Y; N, S, \sigma)dY = \frac{1}{\sqrt{2\pi(S^2+\sigma^2)}} e^{-\frac{(y-N)^2}{2(S^2+\sigma^2)}} dy$$

for the whole class.)

$$y = \log_e Y$$

Thus amounts of a single claim are distributed log-normally for both individual risks and the whole class. It is important, however, to distinguish among the parameters of the respective log-normal distributions.

For simplicity at this point it will be assumed that the inherent risk parameters m and μ are completely independent.

Thus there is an infinitude of compound states:

$$M \cap \theta$$

whose likelihood is given by the distribution:

$$T(m) \cdot N(\mu; N, S)d\mu dm$$

The inherent hazard of the risk will be assumed to remain the same throughout.

Exposure Units

Discrete Example — One roll plus, if necessary, one spin to determine the amount of the claim.

Continuous Example — One car-year.

Moments of Frequencies and Severities

Discrete Example

(Frequency) The probability of a claim for state A_1 is $1/6$ and for state A_2 is $3/6$. Remembering that the variance in the binomial process is given by *npq* and that *n* equals *one* for one for one exposure unit, the respective variances are:

A_1: $1 \cdot (1/6) \cdot (5/6) = 5/36$

A_2: $1 \cdot (3/6) \cdot (3/6) = 9/36 = 1/4$

Summarizing in the special notation adopted for the risk process:

State	${}_kE$	${}_k\sigma^2$
A_1	1/6	5/36
A_2	3/6 (or 1/2)	1/4

(Severity) When a claim occurs the amount thereof is determined by use of the spinner. If B_1 has been selected, then the key moments of a single claim may be calculated as follows:

Amount of Claim (X)	Probability of Amount $f(X)$	$X f(X)$	$X^2 f(X)$
2	5/6	5/3	10/3
14	1/6	7/3	98/3
Total	1	4	36

Thus the mean for state B_1 is *4* and the variance (mean of the squares less the square of the mean) is *20*.

In a similar manner it can be shown that the mean and variance of the amount of a single claim for state B_2 are *8* and *36* respectively. Summarizing in the special notation adopted for the risk process:

State	${}_{x_1}E$	${}_{x_1}\sigma^2$
B_1	4	20
B_2	8	36

Continuous Example

(Frequency) The frequency of property damage claims in one car-year for a risk with state M is m and since the process for an individual risk is Poisson the variance is also m.

Summarizing:

State	${}_kE$	${}_k\sigma^2$
M	m	m

(Severity) The severity (average amount of a single claim) for a risk with state θ is:

$$e^{\mu+\frac{\sigma^2}{2}}$$

And, for the log-normal distribution, the variance of a single claim (for state θ) is:

$$e^{2\mu+\sigma^2}(e^{\sigma^2}-1)$$

Summarizing:

State	${}_{x_1}E$	${}_{x_1}\sigma^2$
θ	$e^{\mu+\frac{\sigma^2}{2}}$	$e^{2\mu+\sigma^2}(e^{\sigma^2}-1)$

(The reader will surely recognize that the "sigmas" in the risk process notation are *not* the same as the "sigmas" in the log-normal distribution. There is a need to compromise between distinguishing symbols and still using familiar notation.)

Pure Premium

The product of frequency and severity (for one exposure unit) is commonly referred to as the pure premium (Π). When multiplied by the number of exposure units the pure premium indicates the charge necessary to cover expected losses. For the risk process the pure premium is given by:

$$\Pi = {}_xE = {}_kE \cdot {}_{x_1}E$$

Discrete Example

Table 1

Mean and Variance
of the
Pure Premium

State	Probability of State	Frequency	Severity	Pure Premium	Square of Pure Premium
$A_i \cap B_j$	$f(A_i \cap B_j)$	${}_kE$	${}_{x_1}E$	Π	Π^2
(1)	(2) 1/4*	(3) (As calculated)	(4)	(5) (3) x (4)	(6) (5) squared
$A_1 \cap B_1$	1/4	1/6	4	2/3	4/9
$A_1 \cap B_2$	1/4	1/6	8	4/3	16/9
$A_2 \cap B_1$	1/4	1/2	4	2	4
$A_2 \cap B_2$	1/4	1/2	8	4	16
Total	1	Weighted — by (2) — Total		2	50/9

* Remember that each state in the dice-spinner example is equally likely.

Thus, if one knew that the true state was $A_2 \cap B_2$, one would charge a pure premium of *4*. Since one never knows (in this example) which state one is dealing with, one would start *a priori* by charging the mean (of the hypotheses) pure premium of *2*.

The variance of the hypothetical means (mean of the squares less the square of the mean) is (14/9).

Continuous Example

Recalling that ${}_kE = m$ and ${}_{x_1}E = e^{\mu + \frac{\sigma^2}{2}}$

the pure premium for the state $M \cap \theta$ is:

$$\Pi = m \cdot e^{\mu + \frac{\sigma^2}{2}}$$

Also recalling that the probability of state $M \cap \theta$ is:

$$T(m) \cdot N(\mu; N, S)\, d\mu dm$$

the mean (class) pure premium is given by:

$$E(\Pi) = \int_0^\infty mT(m) \left[\int_{-\infty}^{\infty} e^{\mu + \frac{\sigma^2}{2}} N(\mu; N, S)\, d\mu \right] dm$$

$$E(\Pi) = \frac{r}{a} e^{N + \frac{S^2+\sigma^2}{2}}$$

In a similar manner the mean of the pure-premium-squared is given by:

$$E(\Pi^2) = \int_0^\infty m^2 T(m) \left[\int_{-\infty}^{\infty} e^{2\mu+\sigma^2} N(\mu; N, S) d\mu \right] dm$$

$$E(\Pi^2) = \frac{(r+1)r}{a^2} e^{2N+2S^2+\sigma^2}$$

Thus the *variance of the hypothetical means* is:

$$\sigma^2(\Pi) = \frac{r}{a^2} e^{2N+S^2+\sigma^2}[(r+1)e^{S^2} - r]$$

Expected Value of the Process Variance

In the risk process each separate state has its own process. Recall that the variance of the compound process is given by:

$${}_x\sigma^2 = {}_kE \cdot {}_{x_1}\sigma^2 + {}_k\sigma^2 \cdot {}_{x_1}E^2$$

Discrete Example

Table 2

Process Variance

State	Probability of State	Frequency: Mean	Frequency: Variance	Severity: Variance	Severity: Mean-Squared	Process Variance
$A_i \cap B_j$	$f(A_i \cap B_j)$	${}_kE$	${}_k\sigma^2$	${}_{x_1}\sigma^2$	${}_{x_1}E^2$	${}_x\sigma^2$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
	1/4	(As calculated)		(As calculated)		[(3) × (5)] + [(4) × (6)]
$A_1 \cap B_1$	1/4	1/6	5/36	20	16	50/9
$A_1 \cap B_2$	1/4	1/6	5/36	36	64	134/9
$A_2 \cap B_1$	1/4	1/2	1/4	20	16	14
$A_2 \cap B_2$	1/4	1/2	1/4	36	64	34
Total	1	Weighted — by (2) — Total				154/9

It is pertinent to note at this point that the *process variance* of the individual states does not change, but, as actual experience is obtained, the probabilities of the individual states may change. New probabilities, after experience is obtained, are *a posteriori.* The mean of the process variances using the *a priori* probabilities is the *expected value of the process variance* — in this example $(154/9)$

Continuous Example

Recalling that $\quad {}_kE = {}_k\sigma^2 = m \quad$ and that

$$ {}_{x_1}\sigma^2 = e^{2\mu+\sigma^2}(e^{\sigma^2} - 1), \quad \text{and} \quad {}_{x_1}E^2 = e^{2\mu+\sigma^2} $$

$$ {}_x\sigma^2 = m[e^{2\mu+\sigma^2}(e^{\sigma^2} - 1) + e^{2\mu+\sigma^2}] $$

$$ {}_x\sigma^2 = m \cdot e^{2(\mu+\sigma^2)} $$

This is the process variance for each individual risk with state $M \cap \theta$. Weighting each such variance by the *a priori* probability of the respective states and integrating over m and μ is:

$$ E({}_x\sigma^2) = \int_0^\infty mT(m)\left[\int_{-\infty}^{\infty} e^{2(\mu+\sigma^2)}N(\mu; N, S)d\mu\right]dm $$

$$ E({}_x\sigma^2) = \frac{r}{a}\, e^{2(N+S^2+\sigma^2)} $$

(Expected value of the process variance)

Credibility "K"

From the definition of credibility (Bühlmann)

$$ K = \frac{\text{Expected value of the process variance}}{\text{Variance of the hypothetical means}} $$

Discrete Example

$$ K = \frac{(154/9)}{(14/9)} = 11, $$

Now

$$ Z = \frac{n}{n+K} $$

so that credibility for one exposure unit (one roll — and spin, if required)

$$ Z = \frac{1}{1+11} = \frac{1}{12} $$

and for two rolls — 2/13, three rolls — 3/14, etc.

Continuous Example

$$K = \frac{\frac{r}{a} e^{2(N+S^2+\sigma^2)}}{\frac{r}{a^2} e^{2N+S^2+\sigma^2}[(r+1)e^{S^2} - r]} = \frac{ae^{S^2+\sigma^2}}{(r+1)e^{S^2} - r}$$

If the individual risks in the class being investigated were assumed to have the same severity, then S would be zero and K would reduce to:

$$ae^{\sigma^2}$$

Furthermore, if severities were ignored and the same amount used for each claim, then σ would be zero and K would be further simplified to a, which is the same value determined in [6] and [8] when severities had not yet been introduced into credibility formulas.

It should not be inferred that theoretical loss distributions are necessary for this method to work. First and second moments of raw data may also be used as estimators.

Recapitulation

This concludes the major thesis of this paper:

To demonstrate how to determine theoretically correct credibilities for the pure premium by making use of the Bühlmann definition of credibility and the formula for the variance of the compound process.

The steps involved are summarized below:

To Determine the Variance of the Hypothetical Means

(1) Enumerate all of the possible states,

(2) Assign an *a priori* probability to each state,

(3) For *each state separately* assign a mean of the number of occurrences or events (labelled claims) per exposure unit,

(4) For *each state separately,* assign a mean value for one occurrence of an event (labelled a claim),

(5) The product of the values assigned in (3) and (4) for *each state separately* is weighted by the *a priori* probability of that state,

(6) The sum of the weighted products in (5) is the mean of the hypothetical means, elsewhere referred to as H, the mean of the hypotheses,

(7) The products described in (5) are squared for *each state separately* and weighted by the *a priori* probability of that state,

(8) The sum of the weighted products in (7) less the square of the mean of the hypothetical means is the *variance of the hypothetical means*, as used in the expression for K in the credibility formula.

To Determine the Expected Value of the Process Variance

Utilize the values obtained in Steps (1) through (4) in determining the variance of the hypothetical means.

(5) Square the value obtained in (4) for *each state separately,*

(6) Obtain the variance of the mean number of occurrences, per exposure unit, and the variance of the amount of a single claim for *each state separately,*

(7) Obtain the product of the value in (3) with the variance of the amount of a single claim for *each state separately,*

(8) Similarly, obtain the product of the value in (5) with the variance of the mean number of occurrences, per exposure unit, for *each state separately,*

(9) The sum of the products for *each state separately* obtained in (7) and (8) is the process variance for each state respectively,

(10) The sum of the process variances weighted by the *a priori* probability of each respective state is the *expected value of the process variance.*

The balance of the paper is devoted to applications of this new approach and to a comparison of credibility results with the results of Bayesian estimation.

Credibility vis-a-vis Bayesian Estimation

While the discrete example of the dice and the spinners is fresh, it is instructive to compare credibility with the results of Bayesian analysis. To start, the probabilities of obtaining all possible outcomes under all possible states are set forth in Table 3.

Table 3
Probabilities of States and Outcomes

State	Outcome for One Trial in 144ths: 0	2	14	Marginal Total
$A_1 \cap B_1$	30	5	1	36
$A_1 \cap B_2$	30	3	3	36
$A_2 \cap B_1$	18	15	3	36
$A_2 \cap B_2$	18	9	9	36
Total	96	32	16	144

To illustrate the method of determining the values in the above table:

Probability of state $A_2 \cap B_1$ equals 1/4

Probability of a claim given this state equals 3/6

Probability of claim amount "2" given a claim and given this state equals 5/6

Therefore, Probability $(2 \cap A_2 \cap B_1)$ equals $1/4 \times 3/6 \times 5/6$ equals 15/144

The inverse, or Bayesian, probabilities derived from the above table given the outcome of one trial but not knowing the true state, are obtained by dividing the individual cell probabilities by the probability of the outcome (column) in which the cell falls. The results, plus a refresher on the pure premiums of each individual state, are given in Table 4.

Table 4
Inverse Probabilities

State	Given Outcome: 0	2	14	Pure Premium [Table 1, Column (5)]
$A_1 \cap B_1$	5/16	5/32	1/16	2/3
$A_1 \cap B_2$	5/16	3/32	3/16	4/3
$A_2 \cap B_1$	3/16	15/32	3/16	2
$A_2 \cap B_2$	3/16	9/32	9/16	4
Total	1	1	1	

The probabilities in each of the "Given Outcome" columns represent the *a posteriori* probabilities of the respective individual states if the outcome is as given. They are, then, a revision of the *a priori* state probabilities found in Column (2) of Tables 1 and 2. The pure premiums in the last column above weighted by the *a posteriori* probabilities in each respective "Given Outcome" column are the Bayesian estimates, e.g. for the outcome "2":

$$\text{Pure Premium/Given "2"} = \frac{(5 \times 2) + (3 \times 4) + (15 \times 6) + (9 \times 12)}{96}$$

$$= \frac{55}{24} = 2\tfrac{7}{24}$$

In a similar manner the Bayesian estimates for the outcomes "0" and "14" are $1\frac{3}{4}$ and $2\frac{11}{12}$ respectively.

It is interesting to note that credibility has a stochastic aspect. For one could use the *a posteriori* probabilities of the individual states to recalculate the *variance of the hypothetical means* and the *expected value of the process variance* and hence "K." The new "least squares" line so determined would, of necessity, pass through the new Bayesian estimate of the pure premium ($2\frac{7}{24}$ instead of the *a priori* 2, if the outcome had been a "2"), and would no longer necessarily produce the same estimate after a second trial as the original "K" would for *two* trials. In practice it is doubtful if the stochastic approach would be used.

However, the purpose of this section is to compare the credibility-produced estimates with the Bayesian pure premiums. This is done in Table 5.

Table 5
Pure Premiums
Bayesian vs. Credibility-Produced

Outcome	Credibility-Produced Estimate	Bayesian Estimate	Square of the Differences
(1)	(2)	(3)	(4)
	[(1/12) × (1)] + [(11/12) × 2]	See Narrative	[(2) − (3)] squared
0	11/6	7/4	1/144
2	2	55/24	49/576
14	3	35/12	1/144

The reader who has persisted with this discrete example this far is encouraged to weight the "difference-squares" in Column (4) above by the probabilities of the outcomes from Table 3: 2/3, 2/9, and 1/9 respectively — take the sum and verify for himself by trying alternatives that the credibility of 1/12 does, in fact, produce a "least squares" fit to the Bayesian pure premiums in Column (3).

Finally, it should be observed in this example that, as was pointed out much earlier, the Bayesian estimate (55/24) does not fall between the observed result (2) and the hypothetical mean (also 2). While this is easy enough for the probabilist to understand, it is awfully difficult to interpret for a layman.

Interdependence of Frequency and Severity

In the discrete example it was originally specified that the selection of the die (determining frequency) and the spinner (determining severity) are completely independent. Such a specification is not necessary in order to obtain credibility "K" using the method described herein.

In fact, assume that state B_1 can only be associated with state A_1 and that state B_2 can only be associated with state A_2 — total interdependence of frequency and severity. It is only necessary to make a change in the state probabilities in the second column of Tables 1 and 2. ($A_1 \cap B_1$ becomes 1/2 as does $A_2 \cap B_2$, while $A_1 \cap B_2$ and $A_2 \cap B_1$ become zero.) Greater familiarity with the method could be obtained by the reader by making this substitution and carrying out the balance of the steps. (For the record, the new pure premium is $2\frac{1}{3}$ and K equals 7.12.)

The important point is that hypothetical frequencies and severities may be interrelated without vitiating this method of determining the credibility of the pure premium.

Auto Merit Rating — Application of Method (See Continuous Example)

In private passenger automobile insurance the theory with respect to merit rating ([5] and [6]) is pretty well established, if severity is ignored entirely. The connection between merit rating and credibility has been pointed out ([2] and [7]), but for frequency of occurrence only.

When severity is ignored, it has been shown in [6] and [8] that credibility for Canadian private passenger data — Class 1 (Adult-Pleasure Use) — is determinable from the parameters:

$$r = 2.62$$

$$(K =) \quad a = 30.1$$

$$\frac{r}{a} = .08704 \quad \text{(frequency-class)}$$

$$Z = \frac{1}{1 + 30.1} = .032$$

Bringing in the additional dimension of severity, for para-realistic automobile property damage data (log-normally distributed-Appendix A):

$$N = 5.289$$

$$S^2 + \sigma^2 = 0.738$$

$$S^2 = 0.01932$$

$$e^{N + \frac{S^2 + \sigma^2}{2}} = \$286.60 \quad \text{(severity-class)}$$

$$E(\Pi) = \frac{r}{a} \, e^{N + \frac{S^2 + \sigma^2}{2}} = \$24.95 \quad \text{(pure premium-class)}$$

From the continuous example:

$$K = \frac{a e^{S^2 + \sigma^2}}{(r + 1) e^{S^2} - r} = \frac{(30.1) e^{0.738}}{(3.62) e^{.01932} - (2.62)} = 58.8$$

$$Z = \frac{1}{1 + 58.8} = .017$$

So the second dimension (severity) has the effect of *halving* credibility *in this instance.* For a coverage with wider dispersion of loss values (of a single claim), say bodily injury, there would have been an even greater reduction in credibility.

Rating Plans With Normal/Excess Loss Splits

Workmen's Compensation Insurance has a multi-split experience rating plan and many forms of commercial insurance have single-split experience rating plans. As a change-of-pace from symbols, the results for a single-split experience rating plan have been calculated using the same data as in the previous section to illustrate how a plan with (complete) credibility, i.e. for amounts as well as occurrences, might work. (See Appendix B for complete details.)

To illustrate the effect of splitting losses upon credibility values it was assumed that all risks would have the same size-of-loss distribution. It would be possible with a computer and Monte Carlo methods to determine K for variable size-of-loss distributions with splitting, but this is beyond the needs of this paper.

How would experience rating work for the values thus obtained? Table 6 below sets forth the credibilities and experience-rated pure premiums for *no losses*, and for a *single loss* in the amount of $100, $500, $1,000 and $3,000 respectively:

Table 6
Class Pure Premium $24.95
Experience-Rated Pure Premium

Amount of (one) Loss	Loss Split-Point $50	$100	$250	$500	$1,000	None
(z-normal)	.03184	.03095	.02739	.02298	.01901	.01557
(z-excess)	.01088	.00757	.00289	.00083	.00014	0
$ 0	$24.59	$24.57	$24.50	$24.47	$24.50	$24.56
100	26.73	27.67	27.23	26.77	26.40	26.12
500	31.08	30.70	32.07	35.96	34.00	32.35
1,000	36.52	34.49	33.51	36.37	43.51	40.13
3,000	58.28	49.63	39.29	38.03	43.79	71.27

This table tells only a small part of a much larger story. For example, if the amounts of loss used above were divided among two or more separate occurrences, the resulting experience-rated pure premium would be different. However, it is clear that the individual risk suffers most (relative to the amount of loss) when the single loss is right at the split point. Also the inconsistency in values, read horizontally for a particular amount of loss, illustrates the fact that credibility does not necessarily produce an optimum estimate but rather is a "least squares" value fitted to a series of optimum estimates. It also shows the inconsistency of subsuming credibility into *normal* and *excess*, thus implying that there is no interrelationship between the empirical number and amount of losses in each category.

Conclusion

Credibility is theoretically justifiable and eminently practicable when amount of loss is considered in addition to frequency of occurrence. The results produced by so using credibility are "least squares" approximations to Bayesian estimates.

The necessary "tools" are:

1) *A priori* probabilities of all possible states (frequency and severity may or may not be interdependent),
2) The *first* and *second moments* for each state of the —
   a) Discrete process which determines the number of occurrences, and of the —
   b) Amount of a single claim.

APPENDIX A

Log-Normal (Class) Distributions by Size-of-Loss

Given X = amount of claim:
$x = \log_e X$

then the p. d. f. of X is:

$$\Lambda(X;\mu,\sigma)dX = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx$$

Also given that μ varies from risk to risk according to:

$$N(\mu;N,S)d\mu = \frac{1}{S\sqrt{2\pi}}e^{-\frac{(\mu-N)^2}{2\sigma^2}}dx$$

While σ does not vary from risk to risk.

$$Pr(X) = \sum_{\mu} Pr(X \mid \mu)\cdot Pr(\mu)$$

$$= \int_{-\infty}^{\infty} \Lambda(X;\mu,\sigma)N(\mu;N,S)d\mu$$

$$= \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \frac{1}{S\sqrt{2\pi}} e^{-\frac{(\mu-N)^2}{2S^2}}d\mu$$

Combining exponents and completing the square produces:

$$Pr(X) = \frac{e^{-\frac{(x-N)^2}{2(S^2+\sigma^2)}}}{\sqrt{2\pi(S^2+\sigma^2)}}\int_{-\infty}^{\infty} \frac{1}{\frac{\sigma S}{\sqrt{S^2+\sigma^2}}\sqrt{2\pi}} e^{-\frac{\left(\mu-\frac{S^2x+\sigma^2N}{S^2+\sigma^2}\right)^2}{2\frac{\sigma^2S^2}{S^2+\sigma^2}}}d\mu$$

$$= \frac{1}{\sqrt{2\pi(S^2+\sigma^2)}}e^{-\frac{(\mu-N)^2}{2(S^2+\sigma^2)}} \quad \text{(also log-normal)}$$

where S^2 is the variance of the individual μ's about N, the mean of μ's.

As mentioned earlier some special-purpose auto property damage data, available to the writer will be used for illustrative purposes. For the log-normal fitted to this data the parameters are:

$$N = 5.289 \quad ; \quad (S^2 + \sigma^2) = 0.738$$

For purposes of illustrating method arbitrarily assume that the severity of an individual risk, with a μ which is $2S$ below N, is equal to 75% of the average severity of the class. Then:

$$e^{N-2S+\frac{\sigma^2}{2}} = 75\% \; e^{N+\frac{S^2+\sigma^2}{2}}$$

and: $$-2S = \log_e 0.75 + \frac{S^2}{2}$$

and solving for S: $S = 0.139$

$$S^2 = 0.01932$$

APPENDIX B

A Numerical Illustration for Normal/Excess Split Plans

From Appendix A assume, for simplicity, that all risks in a class have the same size-of-loss distribution. Then $S = 0$ and:

$$N = 5.289$$

$$\sigma^2 = 0.738$$

And from the paper itself:

$$r = 2.62$$

$$a = 30.1$$

making for the class of risks:

$$\text{frequency} = \frac{r}{a} = .08704$$

$$\text{severity} = e^{N+\frac{\sigma^2}{2}} = \$286.60$$

$$\text{pure premium} = \frac{r}{a} e^{N+\frac{\sigma^2}{2}} = \$24.95$$

$$K = ae^{\sigma^2} = 63.0 \quad \text{(no split)}$$

Table B-1 and the explanation which follows show how credibility should be calculated for a single split plan.

TABLE B-1
KEY PARAMETERS

Split Point (1) X	Cumulative Frequency (2) $F(X)$	$_{x_1}E$ (3) from Log-Normal*	$_{x_1}E^2$ (4) $(3)^2$	$_{x_1}\sigma^2$ (5) from Log-Normal*	$_x\sigma^2$ (6) $(.087)\times[(4)+(5)]$	$\sigma^2(\pi)$ (7) $[(.00288)\times(4)]$	K (8) $\frac{(6)}{(7)}$
				Normal (Below Split-Point)			
$ 50	.05447	$ 49.26	2,427	14.32	212.40	6.99	30.4
100	.2131	92.72	8,597	297.40	773.80	24.76	31.3
250	.6064	178.05	31,702	5,577.00	3,243.00	91.30	35.5
500	.8595	238.40	56,835	23,142.00	6,958.00	163.68	42.5
1,000	.97029	272.60	74,311	52,678.00	11,048.00	214.00	51.6
No Limit	1.000	286.60	82,140	89,640.00	14,945.00	236.60	63.0
				Excess (Above Split-Point)			
	$1-(2)_N$				$[(.087)\times(2)]$ $[(4)+(5)]$	$(.00288)\times$ $(2)^2\times(4)$	
$ 0	1.000	286.60	82,140	89,640	14,945	236.60	63.0
50	.94553	251.00	63,001	116,093	14,739	162.22	90.9
100	.7869	246.40	60,713	146,284	14,179	108.25	131.0
250	.3936	275.80	76,066	265,654	11,687	33.94	344.3
500	.1405	343.10	117,718	535,686	7,985	6.692	1,193
1,000	.02971	471.20	222,029	1,285,577	3,890	0.5644	6,892

* Calculation not reproduced here

The numbers in Table B-1 have very little value per se, but the method of computing *K* for split losses is illustrated. Some explanation is required, particularly for anyone unfamiliar with rating plans in use in the United States on commercial insurance. "Normal losses" for a split-point of, say, $500, would be the total amount of any claim below $500 and the first $500 on any claim above that point. "Excess losses" would be that portion of any loss over $500 in excess of $500.

Table B-1, Column (2) indicates that 85.95% of all losses will be $500 or less and consequently that only 14.05% (see under Excess) of all losses will be excess losses. Thus the frequency of excess losses is obtained by multiplying the regular frequency of (.087) by the probability that, given a loss, it is an excess loss. It has been stated by Verbeek [11] that excess losses taken from a Poisson process also follow a Poisson process. Thus the variance in Column (7) must be multiplied by the square of the probability of an excess loss in Column (2).

Column (3) indicates that the average value of a loss limited to $500 is $238.40 and also that the 14.05% of losses which are excess have an average (excess) value of $343.10. Columns (4) and (5) are self-explanatory. Column (6) illustrates the calculation of the *expected value of the process variance*. Since all risks have the same severity the variance of the hypothetical means can be obtained (Column (7)) by multiplying the variance of the hypothetical frequencies by the square of the average severity. By now Column (8) should be self-explanatory also.

In summary, if claim amounts are disregarded, *K* has the value *a* of 30.1. For low split-points — $50 and $100 — the effect is not much different from just counting claims as far as normal losses are concerned. But as the split-point is increased *K* increases and credibility given to normal losses would therefore decrease, until with no limit on the split-point *K* equals the previously calculated value of *63.0* for a no-split rating plan.

On excess losses, *K* starts out at *63.0* with the split-point at $0, as might have been expected. However, when the split-point increases the credibility for excess loss approaches the vanishing point (*K* equals 1,193 for excess of $500 losses).

In summary, credibility is greatest when severity is ignored entirely (as has been the case in the past); when severity is introduced, credibility can be retained by limiting the value for which a loss enters the rating, but credibility decreases as more and more of the value of the individual claim enters the rating until it reaches a fixed value when all loss amounts are subject to inclusion. If losses above a certain value (excess losses) are rated, credibility has a maximum at the same fixed value applying to the rating of all losses and then decreases to zero as the excess point moves upward toward infinity.

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THE ACTUARIAL PROFESSION

DAVID G. SCOTT

As a preface to the rather rambling dissertation which I am about to make, I have been an officer of the Society of Actuaries in the past, and was active in Society affairs, however I no longer hold any position in the Society, and therefore the opinions expressed are strictly my own. I feel it only fair to give you a little background material so that you will be able to make sufficient allowance for the personal bias in what I am about to say. My actuarial experience has been solely grounded in life insurance, and my acquaintance with the casualty field is based on observation only. Since I was associated with a company whose affiliate was Continental Casualty Company this may appear to be odd, but it is nevertheless true. As a matter of fact, it is doubtful that I have had much of an actuarial connection at all for the past two years, since I have been working for CNA Financial Corporation, a non-operating holding company. Most of my time has been spent in areas in which there is little or no actuarial content to my work. However, I think that this is an advantage for the kind of talk I am about to give, because it will probably be a good deal more objective than it would have been if I had made it a few years ago. Just as distance lends enchantment, it also introduces a little objectivity.

There have been a number of occasions in the past, when, considering the contrast between the life insurance actuary and the casualty actuary, I have congratulated myself on being fortunate enough to have started my business career in a life insurance company. Few other professions have had so many forces working together to enhance the value of its services. It is difficult to remember a time when there was a surplus of life actuaries on the market, in spite of the considerable effort to increase the numbers of practitioners for many years. I believe the profession has been the beneficiary of some extremely fortuitous events, each of which substantially increased the demands for the peculiar kinds of knowledge at the disposal of the life insurance actuary.

Each decade of the last thirty years has produced a new wave of such

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Editor's Note: Mr. Scott, Executive Vice President of the CNA Financial Corporation, presented this address at the Society's luncheon, November 16, 1970.

events. The forties brought us changes in Federal Income Tax Laws and Regulations, and with it the growing need for pension actuaries to act as consultants.

The 1950's saw the introduction of the computer into the operations of the life insurance company. Actuaries of life insurance companies were quick to apply electronics to the calculation of a myriad of values for dividends, cash values, and reserves, as well as to processing of daily routines, and in the process, expanded their influence into other areas of company operations, creating for themselves in the process a whole new range of positions.

The next wave in the series was stirred up by the 1959 Federal Income Tax Act. The implications and long range effects of this piece of legislation were so obscure that they were originally little understood by other than actuaries. This was a natural consequence, since actuaries were heavily represented on the industry committees that provided the background information that led to the law.

Now, just as this vein seems to be playing out, we seem to have struck another, in the forthcoming changes required by the American Institute of Certified Public Accountants in reporting life insurance earnings on an adjusted basis. The demand for the actuary's services seems to be secure for the decade of the seventies, at least.

During that span of thirty years, these and other changes have taken place affecting the actuary's work. In the aggregate, they seem to have made the position of the life actuary more secure. At least, the demands for his services is greater than ever and the number of persons professionally qualified has multiplied. To all appearances, his field of practice seems to have become a genuine profession.

The actuary has always had a problem in defining the nature and limits of his professional field of activity. When we depended largely upon manual calculations we had a natural reluctance to limit our profession to that as a field. We wanted to be regarded, not as experts in the computation of numerical values, but as experts in the field of life insurance. We used definitions of an actuary such as, "Mathematician with a business sense." When trying to attract young men into the profession, the best companies, in their recruiting policies, attempted to find persons with the combination of abilities that would enable them to pass the actuarial exams and also become good managers. The career path of the young actuary, as described by the recruiters, ended up inevitably in some form of management, and the technical skills of our profession stressed in the examinations were regarded by most of us as a stepping stone to our real career as a manager in a life insurance company. This attitude seems to reflect a somewhat apologetic attitude on the part of the life insurance actuary towards the purely technical side of his profession.

The world, on the other hand, accepts with difficulty the viewpoint that we are part businessman and only part technician. The popular view is that the actuary is possessed of a secret body of knowledge, which he applies with intricate techniques, to produce precise answers to problems otherwise insoluble.

I recently received what to me was a startling piece of evidence confirming this. I was present at a luncheon where a world-famous demographer gave a talk on population movements. When I was introduced to him as an actuary, he said, "Oh, you are only interested in everything to the right of the decimal point, while I am interested in everything to the left." Even though I have known for a long time the way the world at large regards the actuary, it came to me as a great surprise that a demographer, who is certainly a scientist, would regard me as being interested only in the extreme exactness of a calculation, while he was more interested in the broad view.

Nevertheless, it is true that life insurance actuaries have risen to be important members of the management team, certainly to a far greater degree than have casualty actuaries. It might be well to examine some of the reasons why they have done so. It is perfectly obvious that the lengthy duration of the life insurance or annuity contract requires a different technique than the short term contracts which prevail in the casualty insurance industry. Life contingencies vary for each year of age. Premiums for long term contracts require the recognition of interest earnings. Factors such as these require more complicated formulae than the simple probability of a loss, multiplied by the average claim cost.

Add to this the necessity for building adequate actuarial reserves over a long period of time to ensure solvency, and you have an obvious need for a technician. In earlier times the value of technical mathematical proficiency was highly valued because of the limited ability to make lengthy computations and life actuaries had learned to cope with the problem through graduation formulae and commutation functions. Remember, too, that earnings statements of a life insurance company tend to conceal or distort its true financial picture. They need interpretation by an actuary, or at least assurance from him that earnings are presumably greater than those shown in the income statement. In short, the actuary was an absolute necessity in any well run life insurance company.

Because the concepts of level premium life insurance and the need for actuarial reserves were not always understood by laymen, the actuary was regarded as much a necessity to a well run company as an engineer was to a builder of aircraft. It was his good fortune that he operated during a long period of declining mortality rates, and his reputation was enhanced by the underlying profitability in the premiums which he had calculated.

Let us contrast this picture of one professional with his fellow professional in the casualty business. There the risks to be measured are of short duration and, therefore, seem less complicated to the layman. Nevertheless, the variables used to determine casualty premiums are not always so readily obtainable, either because of a lack of relevant statistics, or because the variables themselves are affected by economic and social forces. The number of variables to be considered are frequently greater than for computing a single mortality rate. In addition, the casualty actuary has to deal with a greater dispersion in the probability of loss among the risks covered, due to the practice in the casualty field of underwriting against a price, rather than determining premiums for a group of risks with an identical probability of loss, as in life insurance. In many ways the casualty actuary has the more complex problem, but because the concepts are superficially simple and more understandable to the layman, the casualty actuary fails to make the impression that he is indispensable to the good management of a casualty insurance company.

Of course, life insurance actuaries have made mistakes from time to time. For example, they underestimated the effect that the depression of the 1930's would have on the incidence of disability claims. In the same era, they miscalculated the future course of interest rates. Mortality rates used in setting annuity rates were no more accurate than those used to determine life insurance premiums, but with the opposite financial effect. However, the losses in that era were more than off-set by the mortality gains in life insurance.

This would seem to indicate that while life insurance actuaries are human and can make mistakes, these mistakes have been largely forgotten.

Yes, the life insurance actuary can make mistakes, but usually there is a considerable lapse of time between the determination of the premium and the discovery of an error in one of its factors. Usually the error has been off-set by a favorable fluctuation in the interest rate, or better yet, the perpetrator has long since been promoted to management. The mistakes of the casualty actuary, on the other hand, stand revealed within a short time. One additional factor which has worked to his disadvantage is the continuing inflation which has produced higher claim costs than he could have contemplated. No one loves the bearer of bad news and the casualty actuary has had little news that has been good for some time.

Even after having painted this happy picture of the life insurance actuary, and contrasted it with his often unhappy fellow casualty actuary, every sign indicates that the two professions are drawing more closely together. I do not say that simply because we now have an American Academy of Actuaries, to which we both belong, or because we have some examinations in common. I say it because we are being driven to use the same kinds of techniques to solve similar problems in our different industries. In accident and health insurance, actuaries of whatever persuasion have to use both rates of disability, which vary by age, and interest assumptions. On the other hand, life insurance actuaries are becoming more and more familiar with risk and ruin problems, long regarded as the proper field for a casualty actuary only, and papers on these subjects are appearing in the *Transactions*. There is no doubt in my mind that the separation of our two professions is not the result of any fundamental difference, but is entirely due to the fact that we were applying different techniques to problems that once looked dissimilar and now appear to be very much the same.

One of the important influences on this is the fact that life insurance companies and casualty insurance companies are now often associated as affiliates. Sometimes these companies are held by a holding company which has its origin in the insurance industry, but not always. In either event, the management of a holding company eventually will require common standards of performance in its management process, so as to provide a fair basis of comparison between the diverse industries it holds. When a holding company has subsidiaries in the field of real estate development, leasing, and finance business, as well as the casualty and life insurance field, one has to weigh carefully the many opportunities for investment in new developments. There exists a more rigorous requirement to make the best selection than when the new developments were limited to one industry only. Whether in the casualty or the life insurance industry or out of it, discounted cash flows are being used to measure performance and determine the profitability of an investment to be made in the new product. Fortunately, actuaries have at their command the skills which will enable them to accurately develop these measures.

Actuaries of all kinds have a sense of foreboding about their present status. We question ourselves as to whether we really are a profession. It has even been said, half seriously, that the real body of knowledge which inherently belongs to the actuarial profession is whatever is contained in the actuarial examinations. This would seem to indicate that the core of our professional knowledge changes with the times, and that we have no fundamental body of knowledge and techniques exclusive to us.

In a recent article in the *Journal of Risk and Insurance*, the statement is made that, "Actuarial science, despite its prestige position in the insurance world, represents only one branch of applied mathematics that may have neither the range nor the power to solve the problems facing the insurance industry and the insurance consumer."

With the development of computer techniques and their application in operations research, and with these techniques sometimes being used in the construction of models, we find both in the life insurance and casualty industry that the actuary's domain is being encroached upon by people with different and perhaps superior techniques.

Actuaries were, of course, early users of the model company concept to demonstrate the incidence of statutory earnings, and asset shares are, of course, a much used tool in the actuarial bag of tricks, but the flexibility of the new techniques and the mystery which surrounds the field of knowledge of the operations research people have placed us on the defensive.

Where does the future of the company actuary lie? As you may have observed, most of what I said excludes the pension consultant, who is a very different animal. Perhaps our future as company actuaries lies in the common core of knowledge of both our professions. All actuaries should be viewed as measurers of risk. All actuaries should possess the essential skills required to determine price after having measured the risk. However, the arithmetic techniques which were our stock in trade in the past do not provide a solid enough foundation on which to build a lasting profession. These techniques, when usable under modern conditions, will be adopted by others outside the profession.

All mathematics is in the public domain. What has happened is that we have failed to identify and apply knowledge gained in other fields to our own. We have tended to limit the scope of our profession because of our devotion to our own mathematical literature.

The common task of both casualty and life insurance actuaries continues to be the evaluation of risks. If we are to survive as a profession it must be because of our skill in assessing all the factors that affect the future of life and casualty insurance. Our professional work gives us a perspective that enables us to evaluate opportunities in the insurance business, analyze its problems and suggest solutions, based upon our knowledge of history and our analytical ability. Long range planning, the development of new products, the discovery of new markets, and intelligent pricing, require skills which are not readily found outside of the range of our knowledge.

If we can combine this particular skill with a theoretical background based upon all the available mathematical techniques, whether developed within or without our own discipline, we will continue to be an indispensable part of the insurance industry.

PANEL DISCUSSION

EXAMINATIONS — SHOULD THEY BE THE ONLY WAY TO SOCIETY MEMBERSHIP?

INTRODUCTORY REMARKS

H. RAYMOND STRONG*

Before trying to set the stage for a discussion of the system of educating and certifying actuaries, I want to introduce the other members of the panel.

Charles B. H. Watson, a Fellow of the Society of Actuaries, is the Executive Director of both the Society of Actuaries and the American Academy of Actuaries. He is also Secretary of the Joint Committee on Review of Education and Examinations. Because of these positions, he has a unique overview of nearly everything that is happening in the actuarial profession today.

Dr. James C. Hickman, Fellow of the Society of Actuaries, and Associate of the Casualty Actuarial Society, whose permanent place of business is the Department of Statistics of the University of Iowa, is presently a visiting professor of Statistics and Business at the University of Wisconsin. He is an Academy Advisor of the ad hoc Committee of the Academy on the Alternate Route and has done much of the work in the preparation of a sample comprehensive examination proposed for use with the Alternate Route. He is also one of the Academic Advisors to the Committee to Study Further Membership Requirements of the Conference of Actuaries in Public Practice.

Stephen G. Kellison, a Fellow of the Society of Actuaries, occupies the Chair of Actuarial Science at the University of Nebraska. Professor Kellison has also been an Academic Advisor to both the Academy Committee on the Alternate Route and the Conference Committee to Study Further Membership Requirements. Both Professors Hickman and Kellison are members of the American Academy of Actuaries.

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* Mr. Strong is the President of the American Academy of Actuaries. A consulting actuary with the firm of Strong & Thompson of Dallas, Texas, he is a Fellow of the Conference of Actuaries in Public Practice.

Briefly, the "Alternate Route" is the proposed acceptance of a degree in actuarial science from a "recognized" school with passage of a comprehensive examination, as the equivalent of some number (usually five) of the present Society of Actuaries' examinations. This same concept could be applied to some number of the Casualty Actuarial Society's examinations.

There are indications that the present system does not, and probably will not, supply enough qualified actuaries to meet the demand.

The professional actuarial organizations in England, Scotland, and Australia have all recently made changes in their examination systems somewhat similar to that proposed as the Alternate Route on this continent.

BACKGROUND OF THE PROBLEM

CHARLES B. H. WATSON

The purpose of these remarks is to give the factual background behind a number of proposals about education and examinations being considered in the actuarial profession today.

First is the Alternate Route. Originally proposed by the Conference of Actuaries in Public Practice, this concept has been developed by an Academy Committee into a recommendation that credit for the first five actuarial examinations* be given to a graduate from an accredited actuarial program including specified subjects, who passes a comprehensive examination covering the subject matter of those five examinations. There has been and will be considerable debate of this proposition, and possible variations on it, at national and local actuarial meetings; those favoring it emphasize the desirability of promoting greater diversity and experimentation in actuarial education, while its opponents stress the need to maintain high and consistent standards. There is at present no consensus of opinion on this topic.

The second major proposal under consideration is Joint Sponsorship. Under this, the various actuarial organizations in the United States and Canada would become joint sponsors of those of the existing actuarial examinations (administered now by the Casualty Actuarial Society and the Society of Actuaries) which pertained to their own membership require-

* *Editor's Note:* Here referring to the Society of Actuaries. For the Casualty Actuarial Society the number of examinations would represent a comparable portion of its syllabus.

ments. As the Boards of all six actuarial organizations have acted to approve this proposition, although the practical details of its implementation (e.g. sharing of responsibilities, work loads, and expenses) remain to be worked out, it is likely that steps will be taken in this direction in the near future. The recommendations of the Joint Committee on Review of Education and Examinations, with members from all six bodies, and of the Education and Examination Committees of the Casualty Actuarial Society and the Society of Actuaries, will be very important in achieving a working system of Joint Sponsorship.

Other matters are in the wind. The Joint Committee on Review is exploring the nature of the common core of knowledge essential to the education of an actuary, in the thought that the number and content of the common examinations could be expanded. This, combined with a potential parallelism of structure and topic on the later examinations, could go far to establishing the educational foundation for a broader concept of the actuarial profession.

THE ALTERNATE ROUTE — AN INCREASED ROLE FOR UNIVERSITIES IN ACTUARIAL EDUCATION

THE CASE FOR

JAMES C. HICKMAN

Each generation has the right and obligation to examine the institutions that have served earlier generations and to judge whether they are adequate for the current age. Those of us who are middle-aged and conservative may sometimes wish that the process of examination and judgment would proceed in a more leisurely and orderly fashion, but we cannot deny the necessity for a continuing critical review of the organizations and programs that serve society.

The professional actuarial organizations have provided for a continuing review of the system for educating and certifying actuaries. It will be my thesis that there are factors in the current situation that make this review of extraordinary importance. Indeed, I shall claim that there are persuasive reasons for introducing more flexibility into the actuarial education system and that the revised system should provide for making greater use of our universities.

Barry Watson and Ray Strong have already described some of the proposals for reforming the actuarial education system. They have also told a little about the committees that are developing and reviewing these proposals.

The alternate route proposed, developed by a committee of the American Academy of Actuaries, calls for accrediting high quality collegiate programs of actuarial science. To earn accredition a program would have to meet certain academic standards and regularly offer courses ón the fundamentals of actuarial science. This plan provides for graduates of accredited programs to be eligibile to take a comprehensive examination which, if they are successful, would entitle them to entry into the actuarial profession at the Associateship level.

Although I favor the alternate route, the arguments that I will present will not be directed to supporting this particular proposal. I have elected to present more general arguments because there is no specific propòsal before the Casualty Actuarial Society and because many members of this Society have not had an opportunity to be participants in the current review of actuarial education and may not be acquainted with the basic arguments.

The traditional Anglo-American actuarial education system has stressed self-study and apprenticeship training structured by a sequence of difficult examinations on sharply defined topics. This system has been justified by tradition, the relatively small size of the profession, and the assumed unique nature of actuarial science. With the exception of tradition, which must be given some weight but which is hardly sufficient justification for any action, the force of the other justifications for the current system has declined.

To document the fact that the actuarial profession is no longer small I suggest that one examine four pieces of evidence: (1) the number of students who are taking preliminary actuarial examinations at each of the two examination dates each year (1,201 took Part 1 in November, 1970), (2) the number of new Fellows admitted to the principal actuarial societies each year (156 new Fellows of the Society of Actuaries in 1969), (3) the membership of the American Academy of Actuaries (2,608 in November 1969), and (4) the projections of the demand for actuaries.

In addition to being measured by a census of its membership, a profession may be measured by the scope of the problems that it solves for society and the depth of the body of the theory that it applies in its practice. By these measures also the actuarial profession has expanded. As society, in its quest for security, has called for the creation of new insurance systems in both the public and private sectors, the complexity of the risk measurement and management problems handled by actuaries has grown. At the same time the body of theory which actuaries apply has also been growing, as may be sensed by reviewing the world's principal actuarial journals. There are many implications of this numerical and intellectual growth, but within the context of the present discussion I would suggest that they include the idea that the existing system of actuarial education may not be flexible enough to supply a sufficient number of new actuaries, equipped with an innovative spirit, to properly serve society. I also suggest that the profession is now large enough to justify the creation of a few high quality university centers of actuarial education and research such as serve the other professions.

However it is the third justification for a continuation of the present actuarial education system that I believe is the most open to question. At its birth actuarial science was probably unique as a discipline which applied mathematical models to the management of a business operation. At that time mathematics, the queen of the sciences, was applied, if at all, in the natural sciences; mathematical ideas seemed irrelevant to the management of business and political affairs. Particularly in the years since World War II, mathematical models, and the computers that are used to manipulate them, have become almost indispensable to the management of private and public enterprises. A host of disciplines with rather hazy boundaries (operations research, systems analysis, management science, etc.) have been created to describe and systematize this mathematical modeling activity. In a real sense we are seeing the rest of business management adopt actuarial methods.

Perhaps two rather concrete examples will help to establish this point. Collective risk theory is one of the most sophisticated models created within actuarial science. Yet it is, for all practical purposes, identical to the queueing model which was originally developed to study congestion in telephone switch boards. The queueing model has become highly developd and is applied to a variety of practical problems by practitioners of operations research. It is a regrettable fact that these two important models were developed without interaction. However, all of this is in the past. The implication for the future is that collective risk results may be useful to those studying queues, and actuaries may be able to learn a great deal from those studying applied stochastic processes under different labels.

A second example may be found in finance. From the beginning of the development of a theory of finance it was reoganized that the management of an investment portfolio involves deciding on some balance between the expected rate of return and the volatility of the rate of return. However, only in the past score of years have students of finance quantified these concepts and developed programs for managing portfolios to achieve articulated investment goals. Once again we see the development and application of actuarial methods outside the mainstream of traditional actuarial science.

The potential loss in isolating actuarial education from developments in related areas of quantitative management is large. The emphasis within the present system on mastering what are supposedly unique actuarial methods may frustrate talented students and deprive actuarial science of needed new ideas.

To embark on an actuarial education program such as the Alternate

Route obviously creates some risks. However, I claim that by the use of the twin controls of accreditation standards and the comprehensive examination, this risk may be bounded, and this risk is more than offset by the potential gains. In our efforts at building a revised education system we can utilize the experience of other professions such as engineering, law, medicine, and dentistry that have worked with problems of university accreditation and professional certification for many years. The experience of other professional actuarial groups, such as in Australia, may also serve as a guide.

THE ALTERNATE ROUTE

THE CASE AGAINST

STEPHEN G. KELLISON

The purpose of our panel today is to reexamine the procedures by which new entrants into the actuarial profession gain professional qualification. Currently new entrants into both the Casualty Actuarial Society and the Society of Actuaries gain professional standing by passing a series of professional examinations. As most of you know, the so-called "Alternate Route" is a proposal which would grant greater recognition to course work taken at accredited actuarial universities.

I would like to discuss with you today some of the severe problems and disadvantages that I see associated with the adoption of the Alternate Route. I am afraid that this may be a rather unusual and unexpected role for an actuarial professor to play. However, I can assure you that I am quite pleased to be able to participate in this seminar in this capacity.

Let me preface my remarks with the observation that my background in the actuarial profession stems from my membership in the Society of Actuaries. Although a portion of my remarks may be more applicable to the Society of Actuaries than to the Casualty Actuarial Society, most of my remarks are germane to the entire profession.

I have broken down my arguments against the Alternate Route into seven broad areas.

The first argument involves the question of professional standards. Over the years the actuarial profession has developed an examination system for admitting new members which ranks highly in terms of maintenance of standards, objectivity of grading procedures, impartiality of passing standards, and quality of questions. I am concerned that the Alternate Route would result in an erosion of these standards, which are both *high* and *uniform.*

On the question of high standards, there is no doubt that standards on actuarial examinations are significantly higher than standards in college course work. Certainly, passing an actuarial examination is a much higher level of performance than passing a college course.

The proponents of the Alternate Route intend that the comprehensive examination would not require the same high level of performance as the regular examinations, but instead would be only a mild screening device with a high passing percentage designed to weed out only the poorest students. Thus, it is easy to conceive of students qualifying for entry into the actuarial profession via the Alternate Route, who would be unable to pass the regular examinations.

Most university administrators and even faculties are not as dedicated to the attainment and maintenance of high professional standards as most of you would presume. There are many factors, e.g. budgetary considerations, which are not always in harmony with maintaining standards. The trend toward pass-fail grading at many universities is quite disturbing in this connection.

Related to the question of high standards is the question of uniform standards. At the present time the actuarial profession is blessed with standards which do not vary from state to state or university to university. Many other professions plagued with varying standards would be happy to trade places with us. For example, the legal profession is giving consideration to uniform standards for accrediting lawyers at the present time.

The Alternate Route would result in an erosion of these uniform standards, since the background of the students would vary signfiicantly depend-upon which universities were attended. The inevitable result is that many students would seek out those universities providing the softest program.

The proponents of the Alternate Route have used as an argument that actuarial organizations in other countries, in particular England and Australia, have substituted college courses for certain actuarial examinations. However, it should be noted that the higher educational systems in these countries are much more uniform and homogeneous than is the case in the United States with its pluralistic educational system. Thus, I doubt that these examples from abroad can serve as valid precedents.

The application of two different standards within the same university could also create major problems. A mediocre student might pass the comprehensive examination, while a better student has somewhat less success with the regular examinations. Probably the clever student will hedge his bets by taking the regular examinations during the course of his schooling, figuring that if he doesn't get his Associateship that way, he can still take the comprehensive examination. It is entirely possible that a student with a succession of failures on the regular examinations could nevertheless gain his Associateship by passing the comprehensive examination. This would seem to be a very undesirable situation.

A study of recent graduates of actuarial programs has been made by the American Academy of Actuaries Committee on the Alternate Route. This study indicates that, on the average, graduates of actuarial universities do slightly better on examinations than the rest of our students. I am not surprised that this is true; in fact, if it is not true, we professors should find something else to do other than teach. However, this should not be interpreted, as it has been by some of the proponents, that the Alternate Route would not result in a lowering of standards. Extrapolating the past into the future may easily be invalid, when there is such a fundamental change in the ground rules for qualifying as an actuary. For example, most of my students indicate to me that they study harder for actuarial courses in order to pass actuarial examinations than they would in the absence of the regular examinations. This additional effort over and above the requirements of the courses, resulting in a deeper understanding of the subject material, would very likely diminish under the Alternate Route. The backstop of the regular examinations has always been of real value to me in setting appropriate levels of performance in my classes. There would very likely be pressures to lower standards, if the necessity for passing the regular examinations were removed.

Finally, it has been my experience that standards maintained by the present system are not unattractive to students. Many of my students have informed me that one of the reasons they were originally attracted to the actuarial profession was the challenge of the examinations, coupled with uniform, objective standards.

The second argument relates to one of the points raised by the proponents of the Alternate Route: namely, that it would help free our universities from the rigidity of the present system and would introduce greater flexibility into actuarial education.

Here I must admit to a fair degree of ignorance concerning the syllabus of the Casualty Actuarial Society. However, I do not think this charge stands up to examination in the case of the Society of Actuaries. The current Education and Examination Committee makes every attempt to keep our syllabus up to date.

For example, in 1968 a committee was formed to review the mathematical content of the examinations. The work of this committee was greatly influenced by comments solicited from actuarial professors and directly led to several improvements in the associateship examinations.

In those cases in which the syllabus has appeared to lag, a key factor has frequently been the lack of good study material. When appropriate study material has been available, improvements have usually been quickly adopted. There is nothing in the Alternate Route proposal that would improve this situation. Either better study material is available or it is not. If a university has access to better study material, the profession should also.

The problem of rigidity may not be as serious as it is purported to be. A professor can always introduce material into a course which he feels is important and which is not on the syllabus. Even more significantly, most actuarial programs have seminar courses in which a wide variety of topics, many not directly on the syllabus, can be discussed.

Finally, if the Alternate Route is adopted, there will be an incentive for actuarial programs to gear themselves to the material to be covered on the comprehensive examination. Thus, in effect, we will wind up substituting a new rigidity for an old rigidity.

The third argument is that the Alternate Route may not appeal to any significant number of students. In discussing the Alternate Route with the students in my classes I have found few, if any, who would prefer the Alternate Route to the present system. Most of them recoiled in horror at the thought of writing an examination at one point in time that covered as much material as the present Associateship examinations. Most students showed a marked preference for writing examinations one at a time, just after completing course work related to that examination, rather than waiting until they had completed their entire program, and then taking an all-or-nothing gamble on the comprehensive examination. Incidentally, my colleagues in the Psychology Department informed me that in many respects a series of examinations at periodic intervals is superior to one giant examination at the end of a college career. Among the students I interviewed, the Alternate Route would generate substantial interest only if the comprehensive examination were substantially easier to pass than the present Associateship examinations.

The fourth argument against the Alternate Route is that there is an extreme shortage of actuarial programs and professors. I question whether we have enough educational facilities at universities to effectively handle any significant transfer of the education and examination effort from the profession to the universities, either at the present time or in the near future. This is true for members of the Society of Actuaries and the shortage is even more acute for members of the Casualty Actuarial Society.

Any lack of innovative approaches to actuarial science at universities, as alleged by the proponents of the Alternate Route, is more attributable to this extreme shortage of qualified actuarial programs and professors than it is to our present examination system for admitting new entrants. Furthermore, the Alternate Route will have little effect in encouraging more qualified actuaries to enter teaching.

In most actuarial programs one or two individual professors can often largely determine who does or does not get the necessary marks in course work and the degree required for the Alternate Route. This is placing a significant amount of power in the hands of one or two persons, since the result is going to be used for something as important as a student's qualifying for professional standing.

Moreover, it appears to be assumed in most of the discussions to date that these same professors would be intimately involved in the setting and grading of questions on the comprehensive examination. I question whether this is desirable in view of our intent to maintain the comprehensive examination for objective, uniform standards. The actuarial profession has always recognized this problem and not allowed professors to serve on an examination committee which covered material they were teaching. I believe this has been a sound policy.

In teaching I naturally develop a close rapport with my students. Frankly, I would question my own ability to be completely objective in giving grades which would affect a student's qualifying as an actuary. Moreover, I feel that adoption of the Alternate Route would impair my effectiveness as a teacher because of the pressure created by the importance of these grades. I would much prefer to have students qualify on the basis of examinations taken anonymously as is currently done. I would not like to see the actuarial profession create the possibility of a moral problem for its professors.

The fifth argument involves the commitment to education and examinations that would be required by the profession. The actuarial profession already devotes thousands of hours per year of volunteer labor in operating the present system. The actuarial profession as a profession undoubtedly devotes a considerably higher percentage of its time and talent to the training of new members than any profession.

Yet the Alternate Route would not result in any significant reduction in the commitment to the present system, since the majority of students would continue to qualify as in the past. Moreover, the Alternate Route would significantly increase the manpower commitment required for education and examinations. This would arise from staffing the comprehensive examination, performing the accreditation of acceptable universities, and coordinating the two routes.

The sixth area of my concern involves the impact that the Alternate Route would have on the universities. The proponents of the Alternate Route state that it would strengthen university programs, and this is probably true for the larger, well-established programs. However, it could easily prove to have adverse effects on the newer or smaller programs.

One such effect would be to discourage the formation of new actuarial programs. It would be difficult to start a new program on a small scale and then attempt to develop it. Being unaccredited for several years would be a major handicap. In fact, accreditation might also jeopardize certain programs already in existence. Universities do not like unaccredited programs, since it gives the entire institution a black eye. Faced with the choice between an unaccredited program and no program, many university administrators pressed for funds might opt for no program.

It is not always a simple matter to get an unaccredited program accredited. It may take dollars. However, the dollars may not be easy to come by. Because of widespread taxpayer dissatisfaction with increasing taxloads and the unrest on our campuses characterized by demonstrations and student strikes, universities are fast entering a period of much more stringent budgeting than in the immediate past.

I am less optimistic than the proponents that we will be able to effectively accredit actuarial programs. Accreditation can shed some light on the background of the faculty, library facilities, computing facilities, and so forth; but it sheds very little light on what is happening in the classroom and in the students' minds. The problems involved in accrediting actuarial programs are particularly severe because of the extremely small size of the faculties.

If we have stringent accreditation guidelines, we will run the risk of adversely affecting our newer and smaller programs. If we have lax accreditation guidelines, we will run the risk of a substantial lowering of standards. I suggest that we might better spend our time and effort improving our procedures for evaluating new entrants into our profession on an individual basis, rather than attempting to evaluate university programs. In other words, we should evaluate the result instead of the process.

The seventh and final argument is a counterargument to the concept that the Alternate Route is a natural evolution which would have the actuarial profession follow the same pattern other professions have followed. It is true that most other professions do provide for training and qualification through the university system.

However, in considering this argument more deeply there are several factors which should be kept in mind. First of all, most other professions do not have an *alternate* route; university training is the *only* route. This splintering of ways to enter a profession, leading to duplication of effort and a double standard, is not typical.

Secondly, most other professions have never had an examination system outside the universities which would even approach in caliber the present system for actuarial science. Thus, the need to utilize the universities for other professions was more pressing.

Thirdly, as previously mentioned, the number of actuaries actively engaged in teaching is very small. This creates unique problems for our profession which are not present for the larger professions with more extensive university facilities. Not only does it create a problem for individual professors in objectively evaluating a student's performance, but it creates a problem in evaluating a university's accreditation every time there is a change of even one person in the faculty.

Fourthly, it is important to note that in some other professions, such as law, a significant transfer to the universities was effected *after* the development of relatively large university facilities, and not *before*.

To finish my remarks, I would like to say that I sympathize with much of the motivation lying behind the Alternate Route as it relates to actuarial education. I think we would all agree that the goal of actuarial education is not to create people who can pass actuarial examinations but is to create people who can solve real-world problems.

However, in the real world examinations are a necessity. Every profession needs to make a determination concerning which individuals it will admit to membership and which it will not. No system of doing this is perfect. Although not perfect, I feel that a system of objective, uniform examinations which are graded anonymously is an effective means of making this determination for a profession such as ours, characterized by small number and high entrance requirements.

There are several possibilities for improving actuarial education and examinations without creating the severe problems associated with the Alternate Route.

One of the most promising of these ventures is joint sponsorship of examinations, which is currently being considered by the Joint Committee on Review of Education and Examinations. This is a very important development for the profession since it will reflect the inherent unity in our discipline. It has several advantages in comparison with the Alternate Route, since it involves a consolidation of effect rather than a duplication of effort and since it involves a move toward more uniform standards rather than a splintering of standards.

Another proposal of substantial merit is to permit members of the actuarial academic community to serve as consultants to our Education and Examination Committee. Although it would be very unwise to involve the professors in the setting and grading of questions on the examinations, academic advisors could play a valuable role in keeping the syllabus up to date with new developments, in preparing text material where needed, and in providing an educational philosophy to the committee.

Still another avenue for greater involvement of the academic community is in the area of continuing education. The Society of Actuaries is just beginning to undertake a rather massive effort in attempting to keep all of its members as up to date with new developments as possible. Actuarial professors and university facilities promise to play a major role in this endeavor.

The Alternate Route raises a number of serious questions and asks us to reexamine the method of selecting new entrants into our profession. Since this is a very important issue for all actuaries, I would encourage all of you to give this issue your thoughtful consideration in the months ahead.

MINUTES OF THE 1970 ANNUAL MEETING
NOVEMBER 15 - 17, 1970

PALMER HOUSE, CHICAGO, ILLINOIS

Sunday, November 15

Prior to the formal convening of the Annual Meeting on the following day, the Council met at the Palmer House from 2:00 p.m. to 5:30 p.m.

In the evening, the Illinois based companies and trade associations sponsored a welcoming reception for all members of the Society. Concurrently a separate reception was held for new Fellows (and their wives) who, later during the Annual Meeting, would be presented with Fellowship diplomas.

Monday, November 16

The 1970 Annual Meeting was formally convened at 9:00 a.m. by President Daniel J. McNamara, who welcomed the gathering and then introduced the Honorable James R. Baylor, Director, Department of Insurance, State of Ilinois. Director Baylor welcomed the gathering to Chicago and presented his views on various problems affecting the insurance industry.

At 9:30 a panel discussion pertaining to current developments of automobile insurance systems was presented to the entire membership. Participants in this program were as follows:

Moderator: Paul S. Liscord, Consultant
Chairman, Advisory Committee to the Department of Transportation
Casualty Actuarial Society

Participants: Harold S. Baile
President
General Accident Group

M. Stanley Hughey
Executive Vice President
Lumbermens Mutual Casualty

Donald L. Schaffer
Vice President
Allstate Insurance Company

Edward B. Rust
President
State Farm Mutual Automobile Insurance Company

This panel discussion was concluded at 11:30 a.m

The morning session was then recessed in order that the American Academy of Actuaries could hold their Annual Meeting, which included the election of their new Board of Directors. Members of the CAS who were also members of the Academy participated in this meeting.

A ticket luncheon was then held for the entire membership at which time the Society was addressed by Mr. David G. Scott, Executive Vice President, CNA Financial Corporation.

The afternoon program consisted of two consecutive seminars as follows:

2 p.m. - 3:45 p.m.
How Do You Regulate for Solvency?

Moderator: Stanley C. DuRose, Jr.
Commissioner of Insurance
State of Wisconsin

Participants: James R. Baylor, Director
Department of Insurance
State of Illinois

Allen L. Mayerson
Professor of Insurance and Actuarial Mathematics
University of Michigan

James H. Crowley
Assistant Vice President
Aetna Life and Casualty

4 p.m. - 5:30 p.m.
Examinations — Should They Be the Only Way to Society Membership?

Moderator: H. Raymond Strong
President-Elect
American Academy of Actuaries

Participants: Charles B. H. Watson
Executive Director
Society of Actuaries

Stephen G. Kellison
Chairman of Actuarial Science
University of Nebraska

James C. Hickman
Visiting Professor of Statistics and Business
University of Wisconsin

A reception and banquet was held during the evening. A delightful skit under the authorship of John Muetterties was presented to the membership for their enjoyment.

Tuesday, November 17

The meeting was reconvened at 9:00 a.m. by President McNamara.

After the appropriate motion was passed, the calling of the roll and the reading of the minutes of the previous meeting were dispensed with. The next order of business was the formal report of the Secretary-Treasurer covering the activities of the Society in addition to a summary of the financial status as of the close of the year. Messrs. Harwayne and Rodermund offered several comments concerning the portion of the Secretary-Treasurer's report pertaining to Council action on the scope and duties of the Committee on Review of Papers.

President McNamara then presented diplomas to the following new Associates and Fellows:

ASSOCIATES

Anker, Robert A.	Drennan, John P.	Moore, Phillip S.
Balko, Karen H.	Hearn, Vincent W.	Spooner, F. Allen
Battaglin, Bernard H.	Krause, Gustave A.	Tatge, Robert L.

FELLOWS

Brian, Robert A.	Holt, William T.	Ward, Michael R.
Flynn, David P.	Strug, Emil J.	White, Hugh G.
Gerundo, Louis P., Jr.		

The entire membership observed a moment of silence in memory of the passing of the following individuals during the past year:

Ernest T. Berkeley	Dr. Robert Riegel	James B. Donovan
George F. Haydon	John B. St. John	

Although Mr. Donovan was not a member of the Casualty Actuarial Society he was honored because of the frequent and highly expert legal services he gave to the Society during his brilliant career.

The next order of business was the election of the President, two Vice Presidents, Secretary-Treasurer, and three members of the Council. The following were elected:

President	Richard L. Johe
Vice Presidents	LeRoy J. Simon
	Charles C. Hewitt, Jr.
Secretary-Treasurer	Ronald L. Bornhuetter
Members of Council	James J. Meenaghan
	Allen C. Curry
	W. James MacGinnitie

The membership, acting under the provision of Article V of the Constitution, voted to ratify the following elections made by the Council:

Editor	Luther L. Tarbell, Jr.
Librarian	William S. Gillam
General Chairman, Education and Examination Committee	M. Stanley Hughey

President McNamara then presented the Woodward-Fondiller prize to Jeffrey T. Lange, Secretary and Associate Actuary, Royal-Globe Insurance Companies, for his paper, "The Interpretation of Liability Increased Limits Statistics."

The business session was temporarily adjourned for the membership to hear a presentation by Philo Smith of Philo Smith, Landstreet and Company, Inc. on the subject "The Investment Analyst Looks at the Insurance Business."

At 11:15 the business session was again reconvened with an announcement by W. James MacGinnitie of a public relations seminar for students, which was to be held in the afternoon.

President McNamara then delivered his presidential address, which appears in the present *Proceedings* of the Society.

The business session was concluded with the reading of the following new papers and reviews:

New Paper

"Credibility for Severity" by Charles C. Hewitt, Jr.

Reviews

"The Interpretation of Liability Increased Limits Statistics" by Jeffrey T. Lange, review by J. Robert Hunter, Jr.

"Trend and Loss Development Factors" by Charles F. Cook, reviews by Paul J. Scheel, Robert W. Sturgis, Dunbar R. Uhthoff, and Mavis A. Walters.

"A Stochastic Approach to Automobile Compensation" by Donald C. Weber, review by Lester B. Dropkin.

"Distortion in IBNR Factors" by LeRoy J. Simon, review by Charles F. Cook.

President McNamara adjourned the meeting at 12:00.

It is noted that registration cards completed by the attendees and filed at the registration desk indicate, in addition to about 20 wives, attendance by 94 Fellows, 55 Associates and 30 invited guests, as follows:

FELLOWS

Adler, M.
Alexander, L. M.
Allen, E. S.
Bailey, R. A.
Balcarek, R. J.
Barker, L. M.
Bennett, N. J.
Berquist, J. R.
Bevan, J. R.
Bickerstaff, D. R.
Blodget, H. R.
Bornhuetter, R. L.
Boyajian, J. H.
Boyle, J. I.
Cahill, J. M.
Cima, A. J.
Cook, C. F.
Crowley, J. H.
Curry, A. C.
Curry, H. E.
Drobisch, M. R.
Dropkin, L. B.
Elliott, G. B.
Fairbanks, A. V.
Finnegan, J. H.
Flynn, D. P.
Gerundo, L. P., Jr.

FELLOWS

Gibson, J. A., III
Gillam, W. S.
Gillespie, J. E.
Gowdy, R. C.
Graham, C. M.
Graves, C. H.
Hachemeister, C. A.
Harwayne, F.
Hazam, W. J.
Heer, E. L.
Hewitt, C. C., Jr.
Hillhouse, J. A.
Hobbs, E. J.
Holt, W. T.
Honebein, C. W.
Hughey, M. S.
Hunt, F. J., Jr.
Hurley, R. L.
Johe, R. L.
Johnson, R. A.
Klaassen, E. J.
Linden, J. R.
Linder, J.
Liscord, P. S.
Longley-Cook, L. H.
Lowe, R. F.
MacGinnitie, W. J.
Makgill, S. S.
Masterson, N. E.
Mayerson, A. L.
McClure, R. D.
McNamara, D. J.
Menzel, H. W.
Mills, R. J.
Mohnblatt, A. S.
Morison, G. D.
Muetterties, J. H.
Munro, R. E.
Myers, R. J.
Naffziger, J. V.
Nelson, D. A.
Niles, C. L., Jr.
Oien, R. G.
Otteson, P. M.
Perreault, S. L.
Petz, E. F.
Phillips, H. J.
Rodermund, M.
Rowell, J. H.
Ryan, K. M.
Salzmann, R. E.
Scheibl, J. A.
Scheid, J. E.
Schloss, H. W.
Scott, B. E.
Simon, L. J.
Smick, J. J.
Strug, E. J.
Switzer, V. J.
Tarbell, L. L., Jr.
Trudeau, D. E.
Uhthoff, D. R.
Verhage, P. A.
Ward, M. R.
Webb, B. L.
White, H. G.
Wilson, J. C.

ASSOCIATES

Anker, R. A.
Atwood, C. R.
Balko, K. H.
Battaglin, B. H.
Bell, L. L.
Berg, R. A., Jr.
Bill, R. A.
Bittel, W. H.
Bradshaw, J. G.
Brian, R. A.
Buffinton, P. G.
Cadorine, A. R.
Carter, E. J.
Chorpita, F. M.
Coates, W. D.
Conner, J. B.
Cooper, W. P.
Copestakes, A. D.
Davis, R. C.
Drennan, J. P.
Durkin, J. H.
DuRose, S. C.
Ferguson, R. E.
Fossa, E. F.
Franklin, N. M.
French, J. T.
Gossrow, R. W.
Greene, T. A.
Hanson, H. D.
Hartman, D. G.
Head, T. F.
Hearn, V. W.
Hickman, J. C.
Gill, J. F.
Hunter, J. R., Jr.
Khury, C. K.
Krause, G. A.
Levin, J. W.
Mokros, B. F.
Moore, P. S.
Murray, E. R.
Plunkett, J. A.
Price, E. E.
Ratnaswamy, R.
Royer, A. F.
Sandler, R. M.
Scammon, L. W.
Schneiker, H. C.
Singer, P. E.
Stephenson, E. A.
Tatge, R. L.
Trees, J. S.
Van Cleave, M. E.
Walters, M. A.
Weber, D. C.

GUESTS

*Babb, J. A.
Baile, H. S.
*Banfield, C. J.
Baylor, J. R.
Benson, C. R.
*Blanc, R.
Bowles, T. P.
Chamberlain, R. H.
*Daniels, E.
*Griffith, R. W.
Hall, J. W.
Halvorson, W. A.
*Hayden, R. C.
*Katzman, I.
Kellison, S. G.
*McClenahan, C. L.
*Reiner, J. G.
Rugland, W. L.
Rust, E. B.
Schaffer, D. L.
Scott, D. G.
Sloan, W. K.
*Smith, D. A.
Smith, P.
Strong, H. R.
Taylor, J. R.
Walton, W. G.
Watson, C. B. H.
*Wickard, W. G.
Williams, D. R.

* Invitational Program

Respectfully submitted,

R. L. BORNHUETTER,
Secretary-Treasurer

REPORT OF THE SECRETARY-TREASURER

This report will be brief, as the two issues of the Newsletter released in 1970 discussed most of the details on important actions taken by the Council.

The Council met during the year on the following dates:

February 9, 1970
May 24, 1970
May 25, 1970
September 18, 1970
November 15, 1970

In addition, mail votes were conducted on several items during the year.

The highlights of the actions taken by the Council at these meetings are summarized into categories as follows:

A. *Sites*

1) The fall 1972 site in Ann Arbor, Michigan, was switched from the Sheraton to the Campus Inn.

2) The following site selections were approved:
El Conquistador Hotel, San Juan, Puerto Rico — May 1974
Marriott Hotel, New Orleans — November 1974
Chateau Champlain, Montreal, Canada — November 1975

Sites have now been selected through November 1975 except for Spring 1975.

B. *Committee on Professional Conduct*

Adopted revised Guides to Professional Conduct and two related opinions. These Guides and Opinions were developed jointly with the Academy and Society of Actuaries. The Guides will be printed in the 1971 Year Book.

C. *Committee on Election Procedures*

The following changes in election procedures were approved:

1) Council
Normally 5 or 6 candidates will be nominated for the three slots,

but not more than two and preferably fewer than two will have previously served on the Council. This change was implemented in elections at the 1970 Annual Meeting.

2) Officers
President — One nominee, the President-Elect
President-Elect — One nominee, normally the Vice President
Vice President — normally one nominee

3) Change the name of the Council to Board of Directors.

The latter two actions were turned over to a Committee to Review the Constitution and Bylaws of the Society. The membership will consider the proposed Constitutional changes at the Spring 1971 meeting for implementation in the November 1971 elections.

D. *Public Relations and Publicity Committee*

The following action applied to this Committee:

1) Investigate possible co-sponsorship of high school mathematics test with the Society of Actuaries.

2) Establish eight regional contact men who would handle inquiries from students concerning the actuarial professsion. The Secretary's office will work with the Committee on this program.

3) Increase the scope and function of this Committee to encompass cooperation with other Actuarial Societies.

E. *Textbook Committee*

Actions in this area are as follows:

1) Approved a grant of $5,000 to Georgia State University towards the cost of the preparation of a text book on actuarial mathematics.

2) Funds are to come from the Paul Dorweiler bequest to the CAS. There will be an appropriate dedication in the text to Paul Dorweiler.

3) The CAS will receive 50% of the royalties from the sale of the text and the monies will replenish the Dorweiler Fund.

4) Approved the formal contract to be entered into with Georgia State University.

F. *Dorweiler Legacy Committee*

1) Textbook project (see action under Textbook Committee).

2) Established a $200 annual prize to be awarded for best eligible papers submitted by an Associate or Fellow (not eligible for Wood-

ward-Fondiller Prize). The prize will come from funds in the bequest.

3) The appropriate rules will be printed in the 1971 Year Book.

G. *Finance Committee*

The Council approved the budget for 1971 as recommended by this Committee. Anticipated income will be $48,600.00 and anticipated expense will be $38,000.00.

H. *Advisory Committee to Department of Transportation*

The Council action is as follows:

1) Accepted the report of this Committee. Agreed that a copy can be furnished to the DOT as an opinion of the Committee only, not the Council nor the Society membership as a whole.

2) The Committee should consider the feasibility of future CAS research on the data in the closed claim survey.

3) The Committee should consider the possibility of future CAS Committee on Government Statistics to work with various departments of the Federal Government.

I. *Committee on Review of Papers*

1) At the February 9, 1970, meeting the Council reconfirmed the present scope, functions, and methods of operation of this Committee as outlined in the Year Book.

2) At the May 24, 1970 meeting the Council heard a Society member speak on the issue involved in the previous action and voted as follows:

a) Reconfirmed action at previous meeting in which it confirmed the present scope, functions, and methods of operation of the Committee on Review of Papers as outlined in the Year Book.

b) Approved the following clarification to the Guides for Submission of Papers:
"An author may appeal to the President in case of rejection, and the President will make such inquiries as he deems appropriate and will make recommendations to the Council."

c) Asked the Committee on Review of Papers to review current administrative and mechanical procedures followed by the Committee. Such review has now been completed.

J. *Education and Examination Committee*

The Council acted in the following areas:

1) Consistent with the Society of Actuaries action, fees for Parts 1, 2 and 3 are increased from $7.50 to $9.00 effective with the May 1971 exams.

2) Approved the furnishing of numerical grades for those who fail Parts 3 to 9. Successful candidates will continue to receive a "Pass" grade only.

3) Encouraged the exploration of the concepts of joint sponsorship of education and examinations, and alternate route to membership.

4) Authorized CAS representatives in the Joint Committee on Review of Education and Examinations to represent the CAS in discussions of both subjects (item 3) with the appropriate committee of other actuarial organizations.

5) Instructed Education section of Education and Examination Committee to explore and develop possible alternative routes to Society membership.

6) In the area of joint sponsorship:

a) Indicate the CAS willingness to accept joint sponsorship with Society of Actuaries on their Parts 3 and 4.

b) Extend invitation to American Academy, Canadian Institute, and Conference of Actuaries in Public Practice to co-sponsor CAS Parts 3, 4, 5 and 6. This will also include Part 7 at a later date.

c) The CAS will be co-sponsor for Parts 1 and 2 of the Society of Actuaries exams.

Copies of the detailed 1969-1970 financial report of the Society were available at the November meeting. During the year total income amounted to $48,934.56, expenses were $33,559.60 and the $15,374.96 increase in assets produces a total net asset figure of $67,914.89 as of September 30, 1970.

RONALD L. BORNHUETTER
Secretary-Treasurer

FINANCIAL REPORT

Income and Disbursements

(from October 1, 1969 through September 30, 1970)

Income		Disbursements	
Dues	$16,400.00	Printing and stationery	$16,450.29
Examination fees	8,977.58	Secretary's office	2,509.67
Meetings	4,365.08	Examination expense	2,861.38
Registration fees	4,785.80	Meeting expense	8,692.93
Sale of *Proceedings*	6,199.80	Library	589.74
Sale of Readings	352.45	Insurance	193.00
Invitational program	2,150.00	Meeting refunds	92.50
Michelbacher Fund	715.28	Examination refunds	215.60
Interest	3,845.72	Registration refunds	90.00
Registration-ACNY Review Courses	1,050.00	*Proceedings* refund	20.00
Miscellaneous	92.85	ACNY	1,050.00
		Investment expense	115.00
		Miscellaneous	679.49
Total	$48,934.56	Total	$33,559.60

Assets

As of 10/1/69		As of 10/1/70		GAIN
Checking account	$ 5,461.06	Checking account	$ 858.26	$—4,602.80
Savings account	17,060.09	Savings account	12,000.69	—5,059.40
Investments	30,018.78	Investments	55,055.94	25,037.16
	$52,539.93		$67,914.89	$ 15,374.96

Investments

	Cost
U.S.A. Treasury Bond #1673 Due 11/15/74	$ 1,000.00
U.S.A. Treasury Bond #1674 Due 11/15/74	1,000.00
U.S.A. Treasury Bond #299 Due 2/15/75	4,981.25
U.S.A. Treasury Bond #5263 Due 2/15/80	4,325.00
U.S.A. Treasury Bill #4738731 Due 11/19/70	9,646.30
U.S.A. Treasury Bill #5726487/88 Due 9/30/71	18,739.60
U.S.A. Treasury Note #21733 Due 11/15/71 U.S.A. Treasury Note #7478 Due 11/15/71	15,363.79*
	$55,055.94

*Cost price includes $265.35 accrued interest.

This is to certify that we have audited the accounts and the assets shown above and find same to be correct.

Finance Committee
JOHN H. BOYAJIAN
THOMAS W. FOWLER
ALBERT J. WALSH
HENRY W. MENZEL, Chairman

1970 EXAMINATIONS — SUCCESSFUL CANDIDATES

Examinations for Parts 3, 5, 7, and 9 of the Casualty Actuarial Society syllabus were held May 20 and 21, 1970, and examinations for Parts 4, 6, and 8 were held November 12 and 13. Parts 1 and 2, jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries, were given May 13 and November 4. Those who passed Parts 1 and 2 were listed in the joint releases of the two Societies dated June 30, 1970 and December 24, 1970.

The following candidates successfully completed the requirements for Fellowship and Associateship in the November 1969 examinations and were awarded their diplomas at the May 1970 meeting:

NEW FELLOWS

Beckman, Raymond W.	Kilbourne, Frederick W.	Scheel, Paul J.
Jacobs, Terry S.	Munro, Richard E.	White, William D.

NEW ASSOCIATES

Bill, Richard A.	Napierski, John D.	Skurnick, David
Head, Thomas F.	Sandler, Robert M.	Stephenson, Elton A.

MAY 1970 EXAMINATIONS

Following is the list of successful candidates in the examinations held in May, 1970:

FELLOWSHIP EXAMINATIONS

Part 7

Hunter, J. Robert, Jr.	McClenahan, Charles L.	Stewart, Charles W.
Jones, Alan G.	Price, Edith E.	White, Hugh G.
Levin, Joseph W.	Skurnick, David	

Part 9

Brian, Robert A.	Grady, David J.	Skurnick, David
Comey, Dale R.	Holt, William T.	Strug, Emil J.
Flynn, David P.	Khury, Costandy K.	Ward, Michael R.
Gerundo, Louis P., Jr.	Price, Edith E.	White, Hugh G.
Gilmartin, Leo J.		

NEW FELLOWS AND ASSOCIATES ADMITTED MAY 1970: Standing, left to right: Paul J. Scheel, Fellow; David Skurnick, Associate; William D. White, Fellow; Terry S. Jacobs, Fellow; Richard E. Munro, Fellow; Raymond W. Beckman, Fellow; Elton A. Stephenson, Associate; and Thomas F. Head, Associate; seated, left to right: Richard A. Bill, Associate; John D. Napierski, Associate; President Daniel J. McNamara; Robert M. Sandler, Associate; and Frederick W. Kilbourne, Fellow.

ASSOCIATESHIP EXAMINATIONS

Part 3

Bremer, John P.
Cohen, Howard S.
Conners, John B.
Dropick, Dorothy E.
Grippa, Anthony J.
Hess, Albert L.
Johnson, Allan W.
Kelly, Juan N.
Kolodziej, Timothy M.
Krause, Gustave A.
Marino, James F.
Miller, Michael J.
Millman, Neil L.
Moore, Phillip S.
Ori, Ken R.
Peacock, Willard W.
Priester, David C.
Rapp, Jerry W.
Retterath, Ronald C.
Rice, Walter V.
Rinehart, Charles R.
Ross, James P.
Thompson, Eugene G.
Weiner, Joel S.

Part 5

Anker, Robert A.
Battaglin, Bernard H.
Crescio, Joseph P.
Dempster, Howard V., Jr.
Drennan, John P.
Engel, Philip L.
Gwynn, Holmes M.
Hearn, Vincent W.
Hoffman, Dennis E.
Kass, Sheldon
Kramer, Lawrence D.
Krause, Gustave A.
McClenahan, Charles L.
Miller, Philip D.
Moore, Phillip S.
Neidermyer, James R.
Powell, David S.
Rinehart, Charles R.
Schaeffer, Bernard G.
Siegfried, Michael C.
Simons, Martin M.
Spooner, F. Allen
Sullivan, Jerry J.
Tatge, Robert L.
Thompson, Eugene G.
Vogel, Jerome F.

NEW FELLOWS ADMITTED NOVEMBER 1970: Left to right, seated: William T. Holt, David P. Flynn, Louis P. Gerundo Jr., and Robert A. Brian; standing, left to right: Hugh G. White, outgoing President Daniel J. McNamara, Michael R. Ward, and Emil J. Strug.

As a result of the above examinations seven new Fellows and nine new Associates were admitted at the Annual Meeting, November 17, 1970:

NEW FELLOWS

Brian, Robert A.	Holt, William T.	Ward, Michael R.
Flynn, David P.	Strug, Emil J.	White, Hugh G.
Gerundo, Louis P., Jr.		

NEW ASSOCIATES

Anker, Robert A.	Drennan, John P.	Moore, Phillip S.
Balko, Karen H.	Hearn, Vincent W.	Spooner, F. Allen
Battaglin, Bernard H.	Krause, Gustave A.	Tatge, Robert L.

NEW ASSOCIATES ADMITTED NOVEMBER 1970: Left to right, seated: John P. Drennan, Karen H. Balko, Vincent W. Hearn, and Bernard H. Battaglin; standing left to right: Phillip S. Moore, Robert A. Anker, outgoing President Daniel J. McNamara, Gustave A. Krause, and Robert L. Tatge.

NOVEMBER 1970 EXAMINATIONS

The successful candidates in the November 1970 examinations were:

FELLOWSHIP EXAMINATIONS

Part 6

Anker, Robert A.
Bartik, Robert F.
Battaglin, Bernard H.
Bergen, Robert D.
Bill, Richard A.
Comey, Dale R.
Ferguson, Ronald E.
Grady, David J.
Ori, Kenneth R.
Powell, David S.
Simons, Martin M.
Skurnick, David
Snader, Richard H.
Swaziek, Raymond R.
Vogel, Jerome F.

Part 8

Bradshaw, John G., Jr.
Fresch, Glenn W.
Grippa, Anthony J.
Hardy, Howard R.
Hough, Paul E.
Hunter, J. Robert, Jr.
Jones, Alan G.
Krause, Gustave A.
Levin, Joseph W.
Richardson, James F.
Skurnick, David
Stewart, Charles W.
Zory, Peter B.

ASSOCIATESHIP EXAMINATIONS

Part 4 (b)

Haseltine, Douglas S.
McClenahan, Charles L.
Obermeyer, Charles T.

Part 4

Bryan, Charles A.
Cohen, Howard S.
Davidson, David A.
Dickson, Jeffrey J.
Dieter, George H., Jr.
Donaldson, John P.
Dropick, Dorothy E.
Eckblom, Carl W.
Engel, Philip L.
Eubanks, Lowell T.
Golz, James F.
Gruber, Charles
Hall, James A., III
Hoffmann, Dennis E.
Klein, David M.
Kolodziej, Timothy M.
Kreuzer, James H.
Masella, Norma M.
Miller, Philip D.
Millman, Neil L.
Neidermyer, James R.
Pagnozzi, Richard D.
Peacock, Willard W.
Penniman, Kent T.
Pillay, Thanu A.
Radach, Floyd R.
Rice, Walter V.
Rinehart, Charles R.
Sanko, Ronald J.
Shoop, Edward C.
Smith, Lee M.
Thompson, Eugene G.
Tverberg, Gail E.
Warthen, Thomas V.
Winkleman, John J., Jr.
Woll, Richard G.

Seven candidates for Fellowship and seven candidates for Associateship completed their requirements in the above examinations and will be admitted at the Spring Meeting in May 1971:

NEW FELLOWS

Comey, Dale R.
Grady, David J.
Hunter, J. Robert, Jr.
Richardson, James F.
Skurnick, David
Snader, Richard H.
Zory, Peter B.

NEW ASSOCIATES

Engel, Philip L.
Hoffmann, Dennis E.
McClenahan, Charles L.
Miller, Philip D.
Neidermyer, James R.
Rinehart, Charles R.
Thompson, Eugene G.

BOOK NOTES

Hans Bühlmann, *Mathematical Methods in Risk Theory*, 210 pages, Springer-Verlag, 1970.

Reviewed by JAMES C. HICKMAN

Actuarial *theory* and the mathematical theory of risk are practically identical. There are, of course, many aspects of actuarial science and actuarial practice that are effectively independent of risk theory. Yet if one attempts to define the singular body of theory on which actuarial science is built, he comes up with a statement that satisfactorily describes risk theory. Unfortunately, despite the fundamental importance of the subject to actuarial science, twentieth century developments in risk theory have not, in any large measure, been incorporated into the mainstream of North American actuarial education. One can easily conjecture several explanations for this regrettable lag, but one which seems very plausible has been the lack of an English language textbook on the subject.

Since 1968 this impediment to the incorporation of new risk theory ideas into North American actuarial education and practice has been removed by the publication of four important books. In 1968 *The Economics of Uncertainty* by Karl Borch, Norwegian economist and actuary, was published by Princeton University Press. Borch provides his readers with a panoramic view of a collection of ideas for introducing probability components into economic models. This is not a book on risk theory; yet because so many of the examples are drawn from insurance, this book belongs on any list of volumes contributing to the propagation of risk theory ideas among English speaking actuaries.

In 1969 the book *Risk Theory* by Beard, Pentikäinen, and Pesonen was published by Metheun. It is fashionable for specialists to criticize this slim volume for oversights and misprints. Yet within this book the main ideas of collective risk theory are succinctly developed, several of the most promising methods for making approximate probability statements using the collective risk model are discussed, and a collection of miscellaneous topics including ruin probability approximations and the use of risk models in business planning are summarized. All of this is done at a mathematical level which is not inconsistent with that required of North American actuarial students.

Also in 1969 the book *Stochastic Theory of a Risk Business* by H. L. Seal was published by John Wiley. With his usual thoroughness Seal surveys and orders the scattered literature of risk theory and provides an invaluable guide to the relationship between risk theory and its mathematical parent, the theory of stochastic processes. Now in 1970, with the publication of Bühlmann's monograph, the diligent reader of English may work his way very close to the frontier of risk theory.

A reviewer must start by complimenting Bühlmann on his honesty. In the preface he makes it quite clear that he is concerned only with the development of mathematical models appropriate for risk processes. True enough; except for a very few examples that illustrate the application of some of the models, the practical problems involved in estimating the parameters of the models receive little attention in this book. Basically this volume is a research monograph, not a textbook. Yet Bühlmann claims that it may be read by a person "who is moderately familiar with [probability] theory on an intermediate level (without the use of measure theory)." Once again he is right. By prudently avoiding proofs that involve the direct use of measure theory, he has made it possible for a reader with the background he prescribes to appreciate his main ideas.

Perhaps the best way to indicate the scope of Bühlmann's book is to conduct a brief tour of the chapters. Chapter one contains a review of the fundamentals of probability theory required in the remainder of the book. Since most of the technical mathematics problems in probability relate to integration, Bühlmann has thoughtfully provided an appendix in which some of the principal definitions and theorems concerning the generalized Riemann-Stieltjes integral are summarized. Chapter one is amazingly self contained. However, it is practically impossible to encompass within 34 pages all of the basic probability ideas that one needs to advance to the frontier of modern risk theory. For example, chapter one says relatively little about techniques for establishing limiting distribution results. Yet the exercises for chapter one require the reader to employ the unstated and rather deep "continuity theorem" for characteristic functions to solve a limiting distribution problem.

In chapter two the fundamental risk model is developed. Those expecting a restatement of the classical collective risk model will be in for a surprise. Bühlmann does not redevelop the Poisson distribution for the number of claims, but rather he makes the weaker assumption that the number of claims is a Markov process (that is, the probability of a claim in a period is permitted to depend on the number of past claims). Some of the interesting results when contagion is present in the form of a claims number intensity function that is a linear function of past claims, are developed. Bühlmann's general formulation of the distribution of total claims payments does not appear until page 55 and the classical collective risk model then turns up as a special case on page 58.

Although in chapter two the basic risk model is presented in greater generality than is customary, the ideas used in this chapter are in the mainstream of the study of general stochastic processes. However, in chapter three, The Risk in the Collective, Bühlmann commenced to introduce ideas that are directly motivated by practical insurance problems and have special actuarial interpretations. The key idea is that the parameters of a risk process, defined by claims number and claims amount distributions, are not known. These parameters may be thought of as belonging to a set of possible parameters called a collective. A distribution defined on the collective, or some subset of it, which provides information on the probability of the realization of a particular risk process is called a structure function. Bühlmann does not stress the estimation of the structure function. He admits that statistical investigations may cast some light on the structure distribution; but the possibility that it might be estimated by a blend of statistical and ancillary prior information, as could be permitted if one adopts a subjective view of probability, is not mentioned. Nor is the idea that the role of underwriting is, in this model, to reduce the dispersion of the structure distribution discussed.

The remainder of chapter three is taken up with embedding the previously developed general risk model into the still more general model that permits selection, according to the structure distribution, of the risk characteristics. Bühlmann devotes the final section of this chapter to discussing properties of portfolios of risks selected according to the structure distribution. The principal results pertain to the convergence in probability of the average claims payment in a portfolio of risk selected from the collective in the case where the number of risks increases (convergence obtains) and where the time period increases (convergence fails unless the risks are homogeneous).

Chapter four is spent on premium calculations and starts the second part of the book which is devoted to the consequences of the theoretical model developed in the first part. Four alternative principles (expected claims loaded proportionately, expected claims loaded by a constant times the standard deviation of claims, expected claims loaded by a constant times the variance of claims, and expected claims plus a loading which will make the insurer's expected utility upon entering the insurance contract equal to its current utility) are discussed. The principal point in the chapter is that the risk premium, associated with a particular risk from the collective, remains essentially unknown because changing risk conditions prevent the collection of enough data to permit a precise estimate of the risk premium. Statistics are useful primarily for the estimation of the collective premium, the expected premium taken with respect to structure distribution over the set of risk premiums.

Bühlmann defines a credibility premium for a risk, using the variance loading principle, which he proves in Section 4.3.5 has the desired property that as the number of experience periods increase the credibility premium approaches the risk premium. In the remainder of chapter four the practical problem of estimating successive credibility premiums, starting with the collective premium, is discussed. In this project Bühlmann applies the ancient idea of approximating a function, in this case certain conditional expectations, with least squares lines. With some ingenious approximations and manipulations he develops a linearized approximate credibility premium which can be computed using claims statistics from the collective and which, as is shown in an exercise, retains the property that it converges to the risk premium.

Chapter five is spent on the important risk theory problem of making rational reinsurance decisions. Only individual risk reinsurance contracts of the proportional and excess of loss types are considered. Two problems are formulated. The first is to determine the optimal reinsurance agreement for each risk, in the sense of minimizing the variance of retained profits given a fixed profit expectation, for proportional and excess of loss reinsurance contracts. The second problem is to solve the absolute retention problem. In this problem the objective is no longer to spread the reinsurance coverage over individual risks, but it is that of determining the company's global reinsurance policy. The first problem is solved, but the second requires the risk carrier to formulate its overall business objective.

This requirement, to specify the firm's objective in order to solve the absolute retention problem, provides a bridge to chapter six in which insurance carrier stability criteria are discussed. Thus chapter six provides a brief introduction to the intersection between risk theory and modern applied economics. The decision variables which are assumed to be under management control are the premium loadings, the retention level, and the amount of initial free reserve funds. It is management's objective to adjust these variables in order to achieve stability as measured by the probability of avoiding ruin, the present expected value of future dividends, or the achievement of the maximization of the expected utility associated with the future profit stream. The classical ruin probability results of collective risk theory are developed and alternative novel proofs are presented in some cases. The survival probability objective is used to solve the absolute retention problem which had been left in limbo in chapter five. In recent years, led by de Finetti and Borch, many actuaries have become interested in expected dividends as a business objective. Because new results due to Gerber (a doctoral student of Bühlmann's at Zurich), are presented for the classical collective risk model, the development of the stability criterion is of special interest. Finally, not only does Bühlmann discuss the maximization of expected utility as a business objective, but he makes his discussion almost self contained by providing a proof of the existence of a utility function over the class of risk processes by reasoning from basic axioms concerning preference orderings.

This is an important book. Most of the criticisms that a reviewer might make follow from the limitations on the scope of the book that Bühlmann imposed on himself. One yearns for more ideas on methods for solving the estimation problems in risk process, yet Bühlmann warned us that he would stick to model building. The outline that Bühlmann apparently set for himself is incomplete in places, such as in the case of dividend and utility stabilization criteria. However once again Bühlmann usually points out the missing steps in the outline and indicates the reason for the incompleteness. Several aspects of the models that he introduces are only partially exploited. For example, the idea of the collective and the structure distribution provide models for studying sales and underwriting operations. Although properties of posterior distributions play a vital role in proving the properties of Bühlmann's credibility premium, he avoids any direct Bayesian interpretation of the credibility premium. In summary, this is a challenging book. Those actuaries who read it will be carried to the working frontier of risk theory and will be rewarded with a stock of stimulating ideas which they can develop, expand and apply.

R. E. Beard, T. Pentikäinen, and E. Pesonen, *Risk Theory*, 191 pages, Methuen, London, 1969.

Reviewed by CHARLES C. HEWITT, JR.

This little book is recommended without reservation to all American actuaries who have even a passing interest in questions which involve the European theory of risk. As a by-product one obtains a logical exposition of the processes which generate claims and of the distributions of claims by size of loss. The discussion of distributions by size of loss moves logically from those involving a single claim to those involving mutliple claims. Whether or not one is concerned with the somewhat artificial problems of European risk theory, there is much meat in this book for everyone.

This is the first opportunity for American actuaries to study this subject in English and from the ground up. The method of exposition is superior. Most chapters contain relatively simple examples which illustrate the method and answers to the examples are furnished at the end of the principal portion of the text.

Without in any way detracting from the effort of these authors, the American actuary must be cautioned that those problems considered fundamental by European actuaries are not those with which American actuaries must come to grips.

The two principal problems of European risk theory may be summarized in the following questions:

1) What will be the result of a risk business as of some point in time, t?
2) What is the probability of ruin at some point between time zero and time t; and then, by allowing t to increase to infinity, what is the probability of ruin at any time in the future?

At about the same time as the release of *Risk Theory*, we have the welcome addition to the literature of Hilary Seal's book, *Stochastic Theory of a Risk Business* and Hans Bühlmann's book, *Mathematical Methods in Risk Theory*. Each of these texts is also an excellent source of material on European risk theory but without the purposeful simplicity of the Beard, Pentikäinen, and Pesonen effort. Most advanced students of the literature may pass over *Risk Theory* rather quickly and devote themselves to the Seal and Bühlmann books. Neophytes should definitely begin with the book being reviewed here. The early chapters discuss theory of risk in a general way, then proceed to expain and discuss simple stochastic processes, proceeding from the simple Poisson process to the generalized Poisson distribution. Then there are three chapters on approximations for the generalized risk distribution function, followed by a discussion of the simulation of the generalized risk function using Monte Carlo methods.

There is then the inevitable discussion of applications to reinsurance portfolios, followed by a chapter on varying basic probabilities which includes the concept of the compound Poisson process and the Polya process. The antepenultimate and penultimate chapters delve into the question of ruin probabilities — finite and infinite. The final chapter makes a preliminary bow in the direction of Utility Theory as expounded by the Norwegian, Karl Borch, who is well known in the United States. If one is permitted to forecast, it is quite probable that in future editions of *Risk Theory* the space devoted to the applications of Utility Theory in assisting business decisions and business planning will increase and the space devoted to ruin theory and related topics will decrease.

Once again this book is a little gem, if for no other reason than because of the simplicity of approach and the clarity of exposition. In working the numerical examples which go with the text, this reviewer has found one error which should be called to the attention of prospective readers. Exercise 2.5.1 on page 14 should refer to a friendly society with 1,000 members rather than with 18,000 members. This is most certainly a typographical error.

Hilary L. Seal, *Stochastic Theory of a Risk Business*, 210 pages, Wiley, New York, 1969.

Reviewed by CHARLES A. HACHEMEISTER

In his preface, Dr. Seal clearly indicates that, "This monograph is the result of an attempt to survey all the literature relating to the mathematical foundations of risk taking as a business," and that his "concern has been with mathematical and statistical theory; not with its application in practice." Accordingly, the casual reader should be aware of the author's intent and not be disappointed if he finds that proper understanding of its content requires a more-than-casual commitment on his part. Regardless, this reviewer is extremely appreciative of the great amount of work that has gone into the reviewing of virtually all references in English and other main

European languages that are the backbone of this text. The bringing together in an orderly fashion of virtually every important paper relating to the theoretical aspects of actual mathematics makes this book an invaluable reference source. To further increase the value of this work, the author has extracted what he believes are the "principal contributions of each of the articles." It should be noted, however, that the amount of emphasis placed on any piece of subject matter is directly proportional to the amount of theoretical work done already in print concerning it. To emphasize this, the lack of any work related toward the "dynamic" aspects of casualty insurance (e.g. trending and changes in mix of business, etc.) should be pointed out.

After the introductory chapter, the main body of the text is divided into five chapters:

2. The Distribution of Aggregate Claims
3. Calculation of "Fair" Net Premiums
4. The Probability of Ruin of a Risk Business
5. Premium Loading and Reinsurance
6. Utility Theory and Its Application to Reinsurance and Profit Prospects

The first of these, The Distribution of Aggregate Claims, contains many references of attempts to fit empirical aggregate loss data to theoretical distributions. Among these, Dr. Seal includes a discussion of the "excess pure premium ratio" and, implicitly, methods of fitting Table M"; however, he is in error when he states that Hewitt (1967) "proposes a gamma distribution with unit mean for the excess pure premium ratio." The purpose of Mr. Hewitt's 1967 paper, "Loss Ratio Distribution: A Model," *PCAS* LIV, was to fit a gamma distribution to loss ratios. The excess pure premium ratio would then be calculated from it.

After outlining the many attempts at fitting aggregate loss distributions to simple theoretical models, Dr. Seal skillfully produced for the reader the concepts involved in developing the "convolution-mixed distribution" by first introducing the distribution of individual claims (frequency, in casualty terminology) and then the distribution of an individual claim (severity). A good bit of work related to the distribution of frequency is based upon the assumption that some compound Poisson distribution adequately describes the distribution of number of claims. We are, for example, well familiar with the use of the negative binomial as a model for the distribution of the number of claims. Most of these models arise from the assumption that the number of claims of an individual insured in unit time is distributed according to a Poisson distribution with the expected number of claims λ unknown to the insurer and that λ itself varies over the whole population of insureds such that the proportion of insureds with λ less than a certain value can be described by some theoretical distribution function such as the gamma distribution. This is the accident proneness concept.

The description of this work does not include the discussion of merit rating at this point, but leaves it to the following chapter. Although this may be necessary within the structure of the text, it is unfortunate in that the number of articles describing the accident proneness concept far outnumber those delving into the merit rating aspects of it. For example, Dr. Seal includes discussions of articles describing bivariate accident proneness without commenting on the ratio available solution to the grouped contract rating problem. For example, to what extent does the Homeowners' claim affect the insured automobile rate and vice versa?

The fitting of distributions of individual claim size is commented on very quickly before leading into a discussion of the convolution-mixed distribution which is produced by combining frequency and severity distributions into an overall model. Most of the published work in this area has been associated with the Scandinavian studies relating to ruin theory.

The chapter entitled Calculation of "Fair" Net Premiums should be of greatest interest to a casualty actuary. The subject matter includes here, as in other chapters, the high points of available work produced within a mathematical framework. Unfortunately, since a good piece of casualty actuarial work in practice has not been documented from the theoretical point-of-view as well as it could have been, the material discussed does not drive home the subjects dearest to the casualty actuary's heart. The explicit references to credibility towards the end of the chapter are only a fraction of the implicit uses of credibility in other sections of the chapter. Within the explicit discussion, this review does agree enthusiastically with the author's concern as to "whether elaborate investigations into suitable mathematical forms for Z have not tended to obscure the fact that [proper credibility weights] could be estimated from actual data." Although not explicitly stated, the credibility concept is also included in the development on the top of page 52, on page 70 relating to Simberg's work, and again on page 71 in the discussion of Franckx' article. The merit rating concept is contained in the section entitled Experience Rating the Premium of an Individual Contract Holder. A very important relationship not shown, to my knowledge, elsewhere in actuarial literature, is given within this section. That is equation 3.38 on page 65 which shows a formula from which one can easily calculate the proper merit rate from empirical data with virtually no assumptions on the distribution frequency.

Chapter 4, The Probability of Ruin of a Risk Business, is indeed mathematical, as is conceded by Dr. Seal in his preface. To properly read this chapter, one needs a substantial background in probability theory. The mathematical content of this chapter, however, should not lead us to believe that the application of ruin theory to problems of surplus allocation and risk reserves is difficult in practice. Most of the work given is directed towards producing mathematical solutions of ruin probabilities. If, however, one is willing to adopt a discrete time model and has a computor available, the determination of ruin probabilities by simulation is relatively straight-forward.

The remaining two chapters of the text deal with reinsurance — a subject the reviewer suggests he cannot consider himself well qualified to comment upon. Some of my colleagues, however, have pointed out that here Dr. Seal has apparently been directing himself to life reinsurance terminology.

Mention should be given to the Appendices, the first of which gives a short resumé of the basic elements of renewal processes. Apparently, Dr. Seal's reason for including the appendix is related to the usefulness of the renewal process model in the determination of ruin probabilities. We of the casualty profession in the United States have not, in the reviewer's mind, recognized the power of this model.

The second appendix describes procedures for the empirical inversion of Laplace transforms and includes an actual numerical example. To point out the value of these procedures, one should note that if he is attempting to estimate the probability density function of a continuous random variable from a sample of observations directly by using the empirical distribution functions, the estimate is not continuous. However, if one estimates the characteristic function on the basis of sample, the characteristic function is continuous and can be inverted to produce a continuous estimate of the probability density.

In conclusion, the reviewer would like to sincerely thank Dr. Seal for his tremendous effort. If, in any place, the reviewer has been critical, this has only been made possible by Dr. Seal's putting this material together in the first place. For the most part, most of this material was unknown or not readily available to the reviewer; and he thanks Dr. Seal for the introduction to many valuable pieces of technical risk literature. It is, indeed, gratifying to realize that the serious student of casualty actuarial procedures no longer needs survey literature for himself but has here a strong basis from which to begin further work. The reviewer hopes that the whole of the casualty actuarial profession will join him in thanking Dr. Seal for carrying out this monumental task.

Robert Clinton, *Investment Return, Cash Flow and Loss Reserves,* 54 pages, printed by Robert Clinton, Nahant, Mass. 1970.

Reviewed by PHILLIP N. BEN-ZVI

Mr. Clinton's writings constitute a brief monograph, rather than a full-fledged book, in which he presents several elements which he believes should be included as useful tools in a company's Management Information System.

The monograph is divided into three basic areas. Chapters 1 and 2, and the two appendices, discuss the subject of the investment income that is developed from the insurance operation, and its relationship to actual underwriting experience. Mr. Clinton presents a method of calculating the magnitude of this investment income for a particular line of business, and of calculating the "breakeven" loss ratio at which the underwriting loss is exactly offset by the investment earnings. Finally, he displays comparative investment earnings for several lines of business which can then be utilized by management for testing and setting optimum distributions of business under existing business conditions.

In Chapter 3, Mr. Clinton discusses the importance of cash flow as a planning tool for management. He proposes the development of two additional financial exhibits as part of the MIS, one of which he calls a "Flow of Funds Statement," and the second, a "Cash Flow Statement." The first half of the Flow of Funds Statement involves a rearrangement of the "Reconciliation of Ledger Assets," on Page 12 of the Convention Annual Statement Blank, including a breakdown into the flow from underwriting items and the flow from investment and miscellaneous items. The second part of the statement then removes all non-earning assets so that the final totals are the book values of the earnings assets and the changes from year to year. This statement is then further refined in the Cash Flow to Earning Asset Statement by elimination of the effect of bookkeeping adjustment to arrive at the net cash flow itself. Mr. Clinton suggests that "such forecasting of the flow of cash and other funds will greatly aid in forward planning and budgeting and do much to improve the efficient use of cash when it is received."

In his last chapter, Mr. Clinton discusses one means of testing the adequacy of loss reserves. He goes into great detail about the "paid ratio" method, in which one takes past patterns of the loss payments between successive valuation points and uses these to project existing partially developed accident or policy years to their ultimate settlement values. These ultimate values, less the payments already made, can then be used to test the adequacy of reserves being carried by the company.

It is this pattern of loss payments that constitutes the unifying element among the three areas of the monograph. In addition to its use in testing loss reserves, Mr. Clinton singles out this factor as being the most critical to future cash flow, and he discusses the data available from the annual statement which can be used as a basis for future projections.

With respect to the investment income calculations, it is the relative speed of loss payments that is the only element used by Mr. Clinton to produce differing investment returns for the various lines he exhibits. Thus, for auto physical damage with its quick payout rates followed by some recoveries from salvage and subrogation, the investment return is virtually zero, while at the other extreme the long delays in reporting and settling professional liability claims results in a calculated return of 28% of premiums earned. One can therefore conclude that auto physical damage experience must produce underwriting profits to make it desirable business, while on the other hand the writing of professional liability policies would remain advantageous to the company overall, even at a moderately large underwriting loss.

Personally, I found the discussions of the effect of investment income from underwriting operations to be the most interesting (and timely in view of the current uproar on this subject). The method he uses is akin to one I have employed in recent months. I regret, however, that his method chooses to concentrate only on the loss payout rate while ignoring the other parts of the profit or loss equation, namely, the premium income received and underwriting and loss adjustment expenses disbursed. Variations in these elements between lines, between policy terms, and between company operational procedures can have a material effect on the result. One might also suggest the additional refinement of analyzing the payment rate in more detail during the first and second years, in view of the high percentage paid in many lines during this period.

I must also admit to being somewhat disappointed with the discussions of cash flow and loss reserves in that they seemed overly concerned with the accounting details and adjustments required on annual statement figures. While a discussion of these adjustment is useful, I felt that they represented too great a proportion of the monograph, and thus served to distract and detract from the main subject matter. I would also have hoped for a broader discussion of loss reserve testing techniques, and certainly felt that a more complete evaluation of the advantages and shortcomings of the paid ratio method was in order.

In general, I found Mr. Clinton's monograph both timely and well worth reading. After having read and reread it, however, I found that it whetted my appetite, but did not satisfy it.

J. B. M. Murray, *Automobile and Casualty Insurance Ratemaking in Canada*, 212 pages, The Insurance Institute of Canada, 1969.

Reviewed by WILLIAM S. GILLAM

The title of this book by our Canadian associate may be somewhat misleading to most of us here in the States in that it is more an elementary textbook on ratemaking using the Canadian procedures as illustrations than it is an exposition of the Canadian ratemaking procedures. The first half of the book covers such things as elementary probability and statistics, the method of least-squares, the loss ratio and pure premiums methods of ratemaking and credibility. The third quarter of the book explains briefly the ratemaking procedures used in Canada and the last quarter contains questions and answers that might be used as tests for each chapter.

One way of describing the first half of the book would be to say that it covers the material included in Parts 1, 2, 4(a), and 5 of the Syllabus in 100 pages. Obviously this can be done only on a very elementary level.

I believe that this book would be valuable for underwriters and students of the business who are interested in learning the elements of ratemaking so that they can better understand what actuaries are doing and talking about. In addition, it might be used as the very first exposure to property and liability insurance ratemaking for those entering the actuarial field with little prior knowledge of our business.

U. S. DEPARTMENT OF TRANSPORTATION — AUTOMOBILE AND COMPENSATION STUDY, Superintendent of Documents, U. S. Government Printing Office, Washington, D.C.

REVIEWS OF FOUR REPORTS

Public Attitudes toward Auto Insurance, a report of the Survey Research Center Institute for Social Research of the University of Michigan, 266 pages, March 1970; and Supplement, 45 pages, September 1970.

Reviewed by DALE A. NELSON

This is the first of several volumes that have been published as part of the Department of Transportation's Automobile Insurance and Compensation Study. It contains two separate reports dealing with public attitudes: one is on the opinion survey conducted for the DOT by the Survey Research Center, Institute of Social Research, University of Michigan; the other is a compilation of selected complaint letters received by the DOT. The latter consists of excerpts from some 83 complaint letters on various automobile insurance problems — cancellations, non-renewals, premium charges, claim handling, etc. — along with the insurance company's explanation for its particular action on each case. As is typical with such complaints, the majority seem to stem from misunderstandings or the lack of sufficient information on the part of one or both parties involved. Most of the attention given to this volume, though, has been focused on the opinion survey. The Survey Research Center has had considerable experience in this area, and its surveys have formed the basis for several widely used "indices" of consumer intentions.

The Center's report on public attitudes toward automobile insurance is based on a national cross-section of households, excluding Alaska and Hawaii. A standard, area probability sampling technique was used to select the households, and the survey results are based on personal interviews conducted during 1969 with 3,075 respondents. Actually, most of the attitudes concerning auto insurance are based on the interviews for the 2,534 car-owning households. This, together with the fact that most of the respondents were heads-of-household, makes it somewhat questionable whether or not the survey really reflects a true cross-section of *public* opinion.

Attitude surveys always present problems in interpretation, largely because of the many subjective factors influencing people's opinion. This is especially true of this study, and as noted at the beginning of the report, ". . . some of the attitudes presented in this report must be assumed to represent *ad hoc* attitudes rather than attitudes resulting from careful consideration or deliberation. No doubt some respondents had not given any thought to the matters about which they were asked, and they replied according to how they felt at the moment."

Unfortunately, this word of caution was generally ignored in the early press releases on this study — many of which alleged that public opinion was shifting towards a no-fault system. This "finding" appears to be based on the responses to the following questions:

> A65 In most states, this is how automobile liability insurance is set up now: If you are involved in an accident, you have a claim against another person (or his insurance company) only if you can prove that the other person *alone* is at fault. Would you say that this is a good system, a bad system, or what?
>
> A67 Suppose auto insurance were made similar to fire or hospital insurance. Then, in case of an accident your losses — including damage to your car, hospital or doctor bills, and loss of wages — would be paid by your own insurance company, no matter whether you or the other driver were at fault. Would you be in favor of or opposed to such a system?

On the first question, 56% felt the present system was good and 28% bad. The responses to the second are seemingly contradictory, since 57% would be in favor of the proposed system and 30% opposed. If anything, though, it probably indicates only that a great number of people would be satisfied with either system.

A large amount of other, more factual, information was also developed in the survey, relating to such items as the kinds of insurance coverage carried by the household, their claim experience, etc. In reviewing this material one needs to keep in mind that the responses are for households, not individuals. For example, the survey indicates that 93% carry liability insurance, which seems high in comparison with the usual figure of 80-85%

for the percentage of insured automobiles. However, it apparently means that 93% of the households carry liability insurance on at least one of their cars.

Many of the tables contained in this study include cross-tabulations of the attitudinal responses with these more factual data — such as age, driving record, income, etc. By-and-large, the responses follow a predictable pattern. For instance, persons who have had recent accidents or problems with finding insurance are less satisfied with the present system. This portion of the study has been further extended in a more recent DOT publication, *Public Attitudes Supplement to The Economic Consequences of Automobile Accident Injuries,* which summarizes the attitudes expressed by the respondents in the DOT's serious injury study.

Economic Consequences of Automobile Accident Injuries, Vol. I, Part 1, Summary and Analysis; Vol. I, Part 2, Reference Tables, 383 pages, Vol. II, Part 3, Appendices, 294 pages, April 1970.

Reviewed by JEFFREY T. LANGE

The Department of Transportation has issued more than twenty volumes as part of its Automobile Insurance and Compensation Study. Many of these studies have been academic treatises and only a few volumes have been based upon statistical investigations. This particular study, prepared for the Department by Westat Research, Inc., with the assistance of the U. S. Bureau of the Census, falls into the latter category and is a major contribution to the literature on the workings of the automobile accident reparations system. It presents a detailed statistical picture of the economic losses sustained and the compensation received by individuals seriously injured in automobile accidents.

The report consists of less than sixty pages of text followed by several hundred pages of statistical tables and appendices. The text provides an overview of the report and the major conclusions which were drawn from the data. The tables are in sufficient detail so that the reader may use them not only to check the conclusions, but also for other research work on automobile accidents. To most actuaries, the tables will be the most interesting part of the report since they present useful auto accident data, some of which was not previously available. Finally, the appendices provided a discussion of methodology involved in the study.

The data was collected in a scientifically drawn sample of 1,376 individuals who were seriously injured in auto accidents. The population injured was identified from police and court records and an initial sample of reported injured was drawn. These individuals were sent a screening questionnaire to determine whether they were seriously injured. They were considered seriously injured if they had been hospitalized for two weeks or more, *or* had $500 or more of medical costs (other than hospital costs), *or* had missed three weeks of work, *or* had missed six weeks of normal activities. A subsample of 1,435 was selected for personal interviews, and 1,376 responses were ultimately obtained for tabulation. From an insurance standpoint, the study is especially interesting since it was selected from non-insurance records and was based on detailed personal interviews conducted well after the accident date. Most insurance studies, on the other hand, have been made from insurance records and have been conducted as part of the claim settlement process.

The report examines both the amount of loss sustained by the injured person and his degree of compensation. It measures not only the normal components of economic loss — medical expenses, wage loss, property damage, miscellaneous out-of-pocket costs — but also the future wage loss sustained by the disabled and by the families of those killed in auto accidents. It contrasts these losses with the recovery from various insurance sources: auto liability, medical payments, collision, life insurance, wage replacement, and hospitalization. Data for these variables are summarized by size of loss, size of recovery, and degree of recovery. Separate analyses are made of recoveries for persons receiving tort settlement (vs. those with no tort settlement) and of the legal costs involved. The data displays are in sufficient detail to permit an analysis of the interaction of the several variables. It should be a very valuable reference work for those interested in the subject of automobile insurance reparations systems.

Perhaps the most striking conclusion of the study was that only about half of the total personal and family economic loss was recovered in the case of a serious injury in an automobile accident. In cases where the economic loss was less than $500, the injured party on the average recovered more than twice his loss. For cases over $5,000, the average recovery was less than the average loss, with the lowest proportion of the loss being recovered in cases where the loss was over $25,000. Of the recoveries, about 54 percent came from the automobile insurance system, with the remainder coming largely from other forms of insurance. Some have concluded from the study that the existing automobile insurance system did an inadequate job in compensating those seriously injured, although this is not explicitly stated in the report itself.

Price Variability in the Automobile Insurance Market, a report of the Division of Industry Analysis, Bureau of Economics, Federal Trade Commission, 270 pages, August 1970.

Reviewed by ROBERT A. BAILEY

The purpose of this study is to measure the objective basis for subjective dissatisfaction with the demand-supply relationship in the automobile insurance market.

It concludes that the average price of automobile liability insurance is not too high, but that the price for about 2% of the risks is much higher than average. It concludes that the high prices for high risk insured and the lack of availability of insurance for atypical and extreme high-cost buyers are the natural result of a competitive market, and of the tort liability principle of reparation which attempts to place the cost of accidents on those who cause them. It concludes that regulatory interference with economic pricing can aggravate the availability problem, but that free competitive pricing cannot solve the availability problem. It concludes that the best way to reduce the high-risk problem is to reduce the number of high-risk drivers. It also concludes that a pooling or subsidy program, either voluntary or enforced, is necessary to meet the needs of the atypical and high-risk buyers which the open competitive market does not accommodate.

The study was made during the summer of 1969 and is based on information which was readily available at that time from existing data compilations and publications. It includes statistical data obtained from several unnamed leading nationwide insurers, the Insurance Rating Board, and the Massachusetts Automobile Rating and Accident Prevention Bureau. The statistical data shows exposure distributions for various classification plans and price groupings. It shows rate relativities, pure premiums, claim frequencies, claim severities, etc., for various rating criteria.

The study describes in 270 pages how the pricing and underwriting mechanisms work under the present system and explains why a certain relatively small segment of insureds has difficulties with high prices and availability of automobile liability insurance.

Quantitative Models for Automobile Accidents and Insurance, a monograph by Joseph Ferreira, Jr., MIT, 184 pages, September 1970.

Reviewed by PHILIP O. PRESLEY

Mr. Joseph Ferreira, Jr. has presented us with an excellent example of the use of operations research techniques in this report. While his work was directed at evaluating various strategies which could be adopted by a state licensing or regulatory agency in reducing accident involvement, applications of the approach, and of this particular model itself, to insurance problems are readily apparent. Mr. Ferreira suggests that his model could be used in evaluating automobile merit rating plans. Other conceivable areas would include underwriting risk selection and renewal strategies and evaluation of safety engineering efforts. In addition, he has included the various Fortran routines which he developed in the course of his work and which should be helpful to individuals attempting to adapt or extend his model.

In constructing his mathematical model, Mr. Ferreira used the Poisson distribution to describe the probability of a motorist being involved in a given number of accidents over a period of time. He then postulates that the likelihood of being involved in accidents, the parameter of the Poisson distribution, varies from driver to driver, and assumes that the probability of an arbitrarily selected driver having a particular accident involvement "likelihood" is described by the Gamma distribution. Integrating the product of these distributions over all possible accident involvement "likelihoods" yields the Negative Binomial distribution. This model is, of course, familiar to actuaries and has appeared several times in our *Proceedings*.[1]

Using Bayes' Theorem, this model enables one to predict future accident involvement of drivers selected according to their past records. By measuring differences in future accident involvement when remedial programs have been used from those predicted by the model without any such program, one can gauge the efficiency and cost effectiveness of these programs. The best program can then be selected. It should be cautioned that such comparisons and selections can be made only if sufficient information is given about the likely effects of any remedial programs. A model of this

[1] In addition to the two references from our *Proceedings* found in the bibliography to this report, see "Negative Binomial Rationale," by Thomas O. Carlson, *PCAS* XLIX, p. 177, and "A Bayesian View of Credibility," by Allen L. Mayerson, *PCAS* LI, p. 85.

sort does not provide us with the programs per se. The programs themselves, and their likely effects, are inputs, not outputs. To develop such programs, and to estimate their effectiveness initially (assuming they are not in actual operation anywhere), we must rely on talented individuals from many disciplines: educators, social scientists, psychologists, lawyers, and so forth. This is characteristic of the multi-disciplinary effort normally required in the successful use of operations research.

This model also does not provide us with much information as to actions which could be taken outside the licensing area. Thus, any solutions it indicates must be further evaluated in the broader context of the entire "automobile accident and compensation system." Obvious examples of other solution areas which come to mind include better highway construction, more crash resistant vehicles, and more intensive use of mass transportation. It is interesting to note that this last example points out that answers may even be found outside the "automobile accident and compensation system" itself. It has also been suggested that making negligent operation of an automobile punishable by fines and imprisonment would have a significant deterrent effect. All of this is not meant to disparage Mr. Ferreira's work; it only points out that mathematical modeling is only one aspect of the total effort needed to find a solution to the automobile accident problem. Mr. Ferreira recognizes this, and I do not believe he would propose that his work provided any absolute answers, only techniques (note that he never uses an actual remedial driver training program, only hypothetical effects).

In discussing his use of the Poisson distribution, Mr. Ferreira makes a homogeneity assumption that the accident involvement likelihood for a particular doctor must remain constant from one time period to the next. He points to several factors which would cause variations in this value, but dismisses them as having only second-order effect. I find this somewhat disturbing in that I feel there can be significant changes over a period of years, especially in the case of newly licensed drivers. Admittedly these latter drivers are of less interest to Mr. Ferreira in that his underlying driver population, by definition, had several years of driving experience. However, new drivers must be a concern to any licensing agency considering alternative programs.

One answer to this problem lies in the fact that the Poisson distribution is flexible, and can apply to situations where the probability of the occurrence varies over time. It is not difficult to show that if $r(t)$ is the continuous density function of events (in this case, accidents) per unit time at any instant t, then the probability of exactly n events occurring in a time interval is described by a Poisson distribution with parameter m, where m is the integral of $r(t)$ over that time interval.[2] Thus, the probability of any given number of accidents for a particular insured can still be represented by the Poisson distribution, even though we relax the homogeneity assumption. It becomes necessary, however, to assume that the overall distribution of accident "likelihoods" remains constant, or homogeneous over time. The effect on the conditional probability of future accidents given a particular past driving record becomes somewhat less clear, and would be an interesting subject for investigation.

Mr. Ferreira also suggests a Lognormal distribution might be substituted for the Gamma distribution in order to allow a more skewed accident distribution to be approximated. I agree that this should be investigated. At the Spring 1967 meeting of the Operations Research Society of America held in New York, Dr. William Horvath of the University of Michigan presented a paper entitled "A New Type of Compound Poisson Process with Applications to Medicine and Biology." In this he fitted a Poisson/Lognormal conjugate distribution to, I believe, hospital admissions data. This resulted in markedly improved fits, especially in the "tail" of the distribution. It may well be that similar improvements could be made in the present instance. The problem, of course, with using the Lognormal distribution is that one must resort to computers, although this is less of a problem than it once was.

In conclusion, I feel that Mr. Ferreira has made a valuable contribution to the study of the automobile accident problem. Moreover, his paper provides a good example of the use of operations research techniques in seeking answers to problems of interest to actuaries. I recommend it to students of our Society.

[2] For those interested in a rather simple proof, see Wadsworth, George P., and Bryan, Joseph G., *Introduction to Probability and Random Variables*, McGraw-Hill, 1960, pp. 69-70.

OBITUARIES

GEORGE F. HAYDON

HAROLD E. MACKEEN

ROBERT RIEGEL

JOHN B. ST. JOHN

GEORGE F. HAYDON

1882 — 1970

George F. Haydon, General Manager of the Wisconsin Compensation Rating Bureau from 1917 when it was organized until his retirement in 1950, died September 7, 1970. He had been an Associate of the Casualty Actuarial Society since 1919.

Mr. Haydon was born in Liverpool, England, went to sea in 1901, and served as a marine engineer until 1911 when he migrated to the United States. As an engineer he was employed by the Ocean Insurance Company, the Prudential Casualty Company, and the Chicago Bonding and Insurance Company, before he was selected to manage the Wisconsin Compensation Rating and Inspection Bureau.

After his retirement he wintered in Florida and summered in Milwaukee until 1962 when Mrs. Haydon died. Thereafter his year-round home was in Florida. In 1963 he married his late wife's best friend, who survives.

At the age of 84 he wrote an account of his experiences and philosophies which was published in 1967 under the title, *The Trial of Edgar Ransome.*

While he worked in Milwaukee he was active in a number of civic organizations, including the Whitefish Bay (a suburb) Civic Association of which he was first president, and the Whitefish Bay Village Board of Appeals, of which he was chairman.

In addition his wife Phyllis, he is survived by a daughter, Mrs. Alexander Hazelwood of Minocqua, Wisconsin; two sons, George and Charles of Milwaukee and New Canaan, Connecticut, respectively; and granddaughters in Dallas, Texas, and Frisco, Colorado. He is also survived by a brother, William.

HAROLD E. MacKEEN

1904 — 1970

Harold E. MacKeen, with The Travelers Insurance Company for 43 years until his retirement in 1969, died on July 14, 1970 at the age of 65.

Mr. MacKeen was born October 14, 1904. He was graduated from Yale in 1926 and joined The Travelers the same year. He became an Associate of the Casualty Actuarial Society in 1931 and spent a number of years in the Life, Accident and Health Actuarial Department before returning to the casualty and fire insurance lines. He then completed the Fellowship examinations and was admitted as a Fellow of the Casualty Actuarial Society in 1954. In December of 1953 he was appointed an Assistant Actuary in the Fire Actuarial Department of the Travelers.

Mr. MacKeen served as Chairman of the Board of the Central Baptist Church in Hartford, Treasurer of the New England Ashram Associates, and member of the Board of the Connecticut Valley Girl Scout Council.

He is survived by his wife, the former Elvira Schlatter; a son, George D. MacKeen of Avon, Connecticut; two daughters, Mrs. Charles G. Young of Storrs, Connecticut, and Mrs. Philip Giramonti of Guam; and eleven grandchildren.

ROBERT RIEGEL

1890 — 1970

Robert Riegel, professor emeritus of insurance and statistics at the State University of Buffalo, died suddenly at his home March 12, 1970. Active to the date of his death, Professor Riegel was a prolific and versatile teacher and writer.

Born in Philadelphia on January 31, 1890, Dr. Riegel earned a B.A., M.A., and Ph.D. at the Wharton School of the University of Pennsylvania. As a faculty member in the insurance department at Pennsylvania, he had as colleagues such well-known teachers as Professors Huebner, Blanchard, Kulp, and Loman.

During World War I he was a civilian instructor for the Navy and the Merchant Marine. Based on this experience he wrote a book on merchant vessels. In 1921 he became a Fellow of the Casualty Actuarial Society. In the same year he and Professor Harry J. Loman published the first edition of *Insurance Principles and Practices*, a pioneering introductory text covering all lines of insurance in detail.

At an early date Dr. Riegel sensed the growing importance of statistical tools and machines that would make calculations easier and faster. In 1924 he published one of the nation's first textbooks on statistics.

In 1929 Professor Riegel moved to the newly formed School of Business Administration of the University of Buffalo, then a private college. There he had an extremely active career as a teacher of both insurance and statistics, chairman of his department, member and chairman of several key school and University committees, coach of the University tennis team, and director of the University's Bureau of Business Research. In 1939 he was president of the American Risk and Insurance Association and in 1951-52 served as president of the University's chapter of the American Association of University Professors.

Although retired officially in 1960, Dr. Riegel continued on the University staff as a consultant on insurance matters and maintained his interest in general University affairs, visiting his office almost every day. His interest in computers actually increased after his retirement and he became an accomplished programmer. In 1966 he co-authored with Jerome S. Miller the fifth edition of *Insurance Principles and Practices.*

A slightly-built person, he used his mind to compensate for any lack of physical strength in sports (especially tennis) and other activities. A man of rare wit and many interests, he was widely sought as a toastmaster at various banquets. An able, close observer of the current scene, he was an active critic of what he considered to be injustices in all areas of life. A friendly counselor to many students and colleagues, he affected significantly the careers of all those who came in close contact with him.

JOHN B. ST. JOHN

1904 — 1970

John B. St. John, a Fellow of the Casualty Actuarial Society, died suddenly on November 22, 1970 at his home in Penllyn, Pennsylvania.

Mr. St. John was born in Tientsin, China, of missionary parents. He was educated in the United States and received degrees from Northwestern University and Columbia University. He was also a Fellow of the Society of Actuaries and the Canadian Institute of Actuaries and a member of the American Academy of Actuaries.

Mr. St. John began his actuarial career with the Metropolitan Life Insurance Company. Subsequently, he was appointed Chief of the Actuarial Section of the Bureau of Old Age and Survivors Insurance of the Social Security Board. In 1944 Mr. St. John entered the consulting actuarial field with Towers, Perrin, Forster & Crosby. In 1949 he established his own consulting firm. In 1967 Mr. St. John's firm was merged with The Wyatt Company and this relationship continued until his death.

Mr. St. John was one of the pioneers in the development of variable annuities and enjoyed an outstanding reputation as a consulting actuary. His ability to grasp the essentials of a problem and his unfailing good humor and understanding will be missed by his associates and his many friends.

Mr. St. John is survived by his widow, Elizabeth, and five daughters.

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