AN ACTUARIAL NOTE ON ACTUARIAL NOTATION

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DISCUSSION BY JOHN C. WOODDY

Jeffrey Lange has sketched some of the values and characteristics of a system of notation. Without mentioning it explicitly, he discusses the matter of the extent of acceptance of a particular system, pointing out that the canon of *life* actuarial notation has been fixed by an International Congress of Actuaries. It might also be noted that many symbols, as for instance the plus sign and the integral sign, enjoy even wider understanding without having been decreed by any official body.

This brings out the fact that in order for any system of notation to be practical and effective it must appeal to a sufficiently large body of practitioners. In preparing an elementary text on the mathematical theory of risk I laid out a set of symbols, drawn largely from earlier works, which would be internally self-consistent and which would provide for most of the concepts in the field. I went so far as to gather together in an appendix all the symbols and formulae developed in the text. I do not really expect, however, that this notation will be widely used; there are just not enough people doing work in the field of risk theory. Another example of an attempt to establish a system of notation for a particular purpose occurs in the article on exposed-to-risk formulae by E. W. Marshall in Volume XLVI of the *Transactions of the Actuarial Society*. The symbols he used for new entrants, survivors, deaths, withdrawals, and existing policyholders remain, but the *system* of angles, dots, brackets, subscripts, superscripts, etc., has been scrapped in favor of a *verbal* description of the specifications, such as mean age, age last birthday, etc., for each element in a given exposure formula. This scrapping of the system came after an attempt over some ten years to enshrine it by including it in the examination syllabus of the Society of Actuaries.

A good system of notation will be succinct, precise, and consistent. A given symbol will always mean the same thing. When the definitions of two

different symbols are only slightly different, the reader will realize that the concepts *must* be different because different symbols are used. A good system of notation facilitates the making of distinctions between closely related entities. For instance, in the mathematical theory of risk, the density function usually written as $p(z)$ is the distribution of the relative probabilties of the amount of one claim given that a claim occurs, but it is not the conditional distribution of the aggregate amount of claims given that exactly one claim occurs, which would be symbolized by $F(x,t|y=1)$.

A good notational system also reveals relationships which may be obscured by purely verbal descriptions or by ad hoc schemes of notation. For example, the present life actuarial notation makes it clear that an endowment insurance is the same as level term insurance plus a pure endowment. Interestingly, it does *not* readily reveal that such an insurance with its cash values (or reserves) is equivalent to a decreasing term insurance plus a savings fund.

Where, then, do we find ourselves when considering how to make computers do actuarial calculations? In the first place, any involved manipulation of notational symbols is presumably performed manually by the actuary. When he has solved his problem conceptually and is prepared to feed some numbers into the computer and get some other numbers out, verbal labels would seem to be the most flexible for the purpose. In order to permit future modification of the computer program, of course, the job record must contain reasonably complete notes of both the actuary's algebra and the programmer's formulation.

In my own observation, which I must confess is limited and incomplete, most of the jobs involving only those symbols defined in the International Notation have already been programmed: reserves, premiums, asset shares. The sorts of things actuaries are now investigating require the use of symbols defined specifically for the problem in hand.

I do think that there is a need for a sort of "Guide to the Selection of Symbols" to be used by anyone writing a mathematical work. I am thinking of something analogous to Strunk & White's "The Elements of Style," which might be described as a collection of the "hard" information needed by any writer of English. Certainly there are varying degrees of clarity in various writers' private notations. One intriguing example is Cramer's "Mathematical Methods of Statistics," which uses symbols drawn not only from the English and Greek alphabets, which most of us can make shift to

recognize and pronounce, but also from the Gothic alphabet, the names of whose letters are unknown to me and probably to most readers in this country. How can one think about a particular function which one cannot put a name to? This reinforces the proposition that every mathematics book should have a glossary which names and defines all symbols used.

To return to the specifics of Jeff Lange's paper, I should point out that the two dots used over a letter, as $\ddot{a}$, are a diaeresis, not an umlaut. (The umlaut is a substitute for the letter *e* following the letter so decorated.) Also, the distinction between a and $\ddot{a}$ is not between "permanent and temporary annuities" but between annuities with payments beginning at the end and the beginning, respectively, of the initial period.

By this time you will have noticed that I have refrained from revealing my ignorance by attempting to comment on Jeff's suggestions with respect to a standard notation for casualty and property actuarial work. Actually, my reference to a manual of style for notation is pertinent here, although such a manual should have a broader sphere of applicability than the purely actuarial. Perhaps the ideal body to develop such a manual is a well-organized group of highly qualified professionals, such as our Society, with no vested interest in an existing code, and having expertise in the general field of mathematics. Such an endeavor could be undertaken with full regard for the idiosyncrasies of computers but without imposing limitations which may inhibit all generations up to the present and yet be of no consequence to machines of the near future. Do you remember the first color television sets with the mechanical color wheel?

DISCUSSION BY R. GUSTAVE OIEN

In his note, Mr. Lange has demonstrated diligent research on the problem of standardized notation for actuarial work. He has conveyed a sense of the history of the development of the notation used by life insurance actuaries, a sense of the utility derived from the standardization of that notation, and a sense of the problems which still exist in that area. The author develops the inter-relationship of the problems of standardizing notation for working purposes with those of standardizing expressions for use in computer language systems and those of reasonable notation for printing purposes.