In all probability the sampling mean, as an unbiased estimator, will have zeroed in upon true expectation by the end of all eternity, but classification rates normally rest upon at most only five or six years of experience. When at most only five or six observations are to be used in a given rate calculation, a probability of 0.55 or more that a single observation will fall below expectation, and a probability of 0.5 that it will fall below 95% of expectation, would seem significant even though credibility will be low in such cases. Where the rating formula ultimately rests upon truncated distribution,[3] the effects of skewness will be minimized, and perhaps may be reduced to negligible proportions. Nevertheless, the matter seems worth investigation.

Entirely apart from the present application specifically to the credibility problem, the Cornish-Fisher expansion seems to offer a simple technique whereby empirical distributions of loss may be developed readily, either when a theoretical distribution cannot be fitted, or when a theoretical distribution, if fitted, is too complex for routine practical calculation. Although the estimation of annuity costs as such may be of little interest to most casualty actuaries, as an example of techniques readily applicable to casualty problems, Mr. Bowers' paper[4] cited by the present authors will repay study by anyone interested in actuarial methods.

It is to be hoped that Messrs. Mayerson, Jones, and Bowers will not rest with their present significant contribution, and that additional data will become available to permit practical application of their results.

DISCUSSION BY DALE NELSON

In their paper, the authors present a distribution-free approach to the problem of evaluating the full credibility standard for a specific block of business, after having briefly reviewed the customary approach. Their motivation stems from two principal concerns:

(i) the usual derivation is based on the distribution of the number of claims and, generally, ignores the distribution of claim amounts

[3] E.g., when, as in private passenger automobile, basic limits experience, rather than total limits experience, is used.

[4] Bowers, Newton L., Jr., "An Approximation to the Distribution of Annuity Costs," *TSA* Vol. XIX (1967), p. 295.

even though the results are applied to such loss statistics as pure premiums and loss ratios, in addition to claim frequencies; and

(ii) the third and higher moments of these distributions are usually glossed over and simply accounted for by means of a normal approximation.

The results derived in the paper reaffirm the fact that the effect of (i) is very substantial, and establish that the importance of (ii) is relatively minor.

Most of my comments are technical in nature, but I would like to remark first on a curious situation. Despite the fact that the factor $A = 1 + \frac{S^2}{M^2}$ needed to compensate for (i) has been known for over 35 years, that it is fairly easy to rationalize, and that it is rather large in size (most calculations yield values ranging from 2 to 5), the actuarial community has been almost united in their indifference to its use. Part of the reason for this undoubtedly lies in the difficulty encountered in evaluating S, given the form of most insurance data. But estimates have been made for most lines, at one time or another, and in view of the conservative nature of most actuarial techniques it is surprising that some convenient, arbitrary value of S (say $2M$), has not been used in place of the implicit value $S = 0$.

The following are a few technical notes pertaining to the authors' paper which may be of interest.

(1) The authors have used the first two terms of the Cornish-Fisher expansion to approximate the $100e$ percentile, t_e, of the distribution of $\frac{T - E(T)}{\sigma_T}$ in terms of the corresponding percentile, Z_e, for the standard normal distribution:

$$t_e \sim Z_e + \frac{1}{6}(Z_e^2 - 1). \frac{E[T - E(T)]^3}{\sigma_T^3}$$

They have not commented on the accuracy of this expansion; but about all that can be said, in general, of this particular two term approximation is that the error term goes to zero with n^{-1}, where n is the number of exposure units.

In checking this approximation formula against known distributions a fair degree of accuracy was found to exist, particularly in the tails of these distributions. With the Gamma distribution, for example, the relative error was less than 5% for the cases tested.

(2) By using this approximation and the usual definition of full credibility, the authors derive the full credibility standard, Λ, by setting

$$\frac{kE(T)}{\sigma_T} = t_e, \text{ with } e = \frac{1+P}{2}$$

This, as the authors admit, produces a somewhat conservative standard. A more correct formulation would have been to determine e and e' such that:

$$t_e = \frac{kE(T)}{\sigma_T} = -t_{e'}$$

$$\text{and } Pr\,[Z_{e'} \leq Z \leq Z_e] = P$$

Although the arithmetic can get rather burdensome, it is possible to determine Λ in this fashion. For instance, in the authors' first example, this procedure yields $\Lambda = 4{,}573$ (compared to the authors' *4,713*).

This raises a couple of interesting questions since the authors derived a slightly higher standard, $\Lambda = 4{,}577$, by ignoring the third moment. Specifically,

(a) Does the presence of positive skewness in the distributions yield a lower standard of credibility than the symmetric case? or

(b) Is this phenomenon spurious and only a reflection of the error in the Cornish-Fisher approximation?

It should be noted that similar results are also obtained for the other examples. Intuitively, it doesn't seem that (a) should be true. This would lead one to conclude, then, that perhaps the third moment effect is of the same order of magnitude as the error term in the formula used to measure it and that the usual normal approximation is entirely satisfactory for most purposes.

(3) The authors' approach, and other published results, take the normal approximation as their point of departure. This produces, in effect, an increasing sequence of lower bounds to the full credibility standard as the conditions on the higher moments are relaxed. It is intellectually, if not practically, interesting to approach the problem from the other side — i.e. to devise upper bounds to the standard.

Thus, Chebyshev's theorem states that:

$$Pr\left[\,|t| \leq \frac{kE(T)}{\sigma_T}\,\right] \geq 1 - \frac{\sigma_T{}^2}{k^2E(T)^2}$$

regardless of the form of the distributions. Resolving this yields the following standard for full credibility:

$$\Lambda = \frac{A}{k^2(1-P)}$$

This estimate is much more conservative; for example, in comparison to the usual standard of 1,084 claims, the same parameters produce the value $\Lambda = 4{,}000$ for claim frequency and $4{,}000A$ for the pure premium. There are more elaborate Chebyshev-type relations, involving higher moments, which could be used to reduce this upper bound. From a practical standpoint, however, these are not useful since the required moments are not available.

DISCUSSION BY LEROY J. SIMON

This fine paper is providing a new stimulus to the thinking of actuaries on the important subject of credibility. A primary purpose of this review is to place additional information before the Society relating to another line of business, namely fire.

The Actuarial Bureau of the National Board of Fire Underwriters and, more recently, the National Insurance Actuarial and Statistical Association have assembled, under the direction of Dr. J. H. Finnegan, statistical data on fire losses in the United States. The latest compiled information was for 1964 and the results are shown in the accompanying table. The data were derived from "Adjusters' Loss Reports" which are forms completed by adjusters upon the settlement of each claim. The reports reflect the payment made to all involved companies as a combined total. Thus, if a claim were split among ten companies the entry would be made as one entry for the full amount and not as ten separate reports for shares of the amount. For our purposes, the method of compilation in the accompanying table is much better than the usual compilation of data in the fire field where split losses would be reported separately and never pulled together into a single combined total.

In many instances an adjuster's report represents the total damage sustained in a fire, but if the insured had one group of policies on his building