For each time period, the mean error and variance of each underwriter could be compared with the over-all company mean and variance, or with the over-all mean and variance of underwriters handling the same types of risks. Separate consideration of results with family risks and with business risks would be the minimum split needed if underwriters are specialized on that basis in the company. A review and analysis of the largest percentage errors from each underwriter's results could lay the foundation for better results in succeeding periods. A comparison of the mean errors and variances over time, both for individuals and for the company as a whole, could keep management abreast of whether the desirable downward trend was present in each case and of which underwriters needed help in improving their results.

DISCUSSION BY ROBERT L. HURLEY

There is much that the reader may find remarkable in the paper, "Is Probable Maximum Loss (PML) a Useful Concept?" The term, itself, is believed one of those esoteric symbols of the underwriting fraternity whose members must, in turn, sometimes find certain actuarial arcana a bit mystifying. It is not possible that PML can convey to the actuary the associations (not necessarily all pleasant) that these letters can suggest to the experienced fire underwriter. Presented with the McGuinness warnings on large fire losses, an underwriter may well reflect that there have been fire catastrophies before McCormick place, which he, incidentally, might not regard as likely destined to be the last of such disasters. Nevertheless, a lifelong schooling not to hazard, needlessly, an undue portion of his company's assets in a single occurrence would typically dissuade the underwriter from placing any significant reliance upon a purely fatalist approach to risk evaluation. Moreover, he could not help being at least a bit curious about any such approach as Dr. McGuinness's which might be construed as showing the underwriter how much he could safely write on the risks offered to him. The actuary, too, would have more than a passing interest in any such demonstration, although, understandably, the underwriter would be the most immediate beneficiary of any such mathematical solution to the age old problem of determining PML.

But before attempting to evaluate the McGuinness proposal, it may be helpful to identify his mathematical sources since they stem more from the economics and sociological than from the actuarial literature. About the

turn of the present century Vilfredo Pareto, who had recently assumed the Chair of Economics at Lausanne previously graced by the distinguished economist Leon Walras, published a two-volume tone on economic theory buttressed, if not somewhat laden, with mathematics. Probably the feature which, at the time, caught the fancy, not only of the professional economist, but also of the reading public, was the Pareto law which claimed that with an ascent in the income scale, while the number of recipients thereof declined sharply, the relative percentage of the total income absorbed by the dwindling number did not decline at the same rate. Pareto expressed his law as $N = k\,x^{-a}$ where N is the number receiving incomes of x greater than k, a threshold value. Not satisfied with his slightly meteoric thrust into notoriety, Pareto pushed along into the wider fields of sociology and philosophy.

Time has relegated Pareto's economic law to a respectable, but maybe nonetheless deserved, neglect. To cite just one teacher who has long been in the vanguard of economic theory, Paul Samuelson noted:

> "According to the Pareto law, there is an inevitable tendency for income to be distributed according to a logarithmic curve whereon the upper tail of the income data of many different countries and many different times fell along straight lines of almost the same slopes. He came to believe this as a fundamental law, regardless of social and political institutions, and regardless of taxation. In the past 50 years, more careful studies have refuted the universality of Pareto's law as well as its inevitability."

Pareto's sociological writings won for him only the opprobrium (and this probably not at all deserved) as one of the philosophical fathers of 20th century fascism. Moreover, the earlier disciples of his mathematical theories may have escaped only a somewhat lesser disenchantment faced with the charge that Pareto's work was solely a trivial extension of the somewhat "outdated" system of densities introduced by Karl Pearson in 1894. And even in the current revival of Pareto mathematics, some may believe the contributions to be of more heuristic than corroborative value.

However, this reviewer believes that the CAS is not responsible for the partialities with which the accolades may be distributed in other learned disciplines, and is concerned only with the possible significance of the findings in the allied professions to actuarial problems. And; in this regard, we are indebted to Dr. McGuinness for directing our attention to the research cur-

rently being conducted by European actuaries on the Pareto curve. To the McGuinness list of references one might add the paper in the 16th International Congress at Brussels in 1960 by Benktander and Segerdahl pointing out "the Pareto distribution is essentially the most 'dangerous' analytical expression that can be used to describe a claim distribution, notwithstanding the values of the parameters involved."

While not unappreciative of the almost disingenuous shifts to which even scholars may sometimes resort who are moved by an uncritical reverence for an author, it is believed still incumbent on us not to dismiss summarily the use being made of the Pareto curves in Europe, but to research, such as Dr. McGuinness has suggested, possible applications to U. S. insurance problems. Solely as an addendum to this commentary on the McGuinness proposals, there are offered some fire (excl. dwellings) loss distributions related to the actual value of the properties, taken from the public records of various fire rating bureau large deductible filings in the middle 1960's. It is suggested that these might be viewed as not unrelated to the Pareto equation with some modifications therein.

Now the McGuinness paper proposes three objectives in order to show how PML can be made a useful and valuable tool, by suggesting:

(1) a precise definition of PML,

(2) how the accuracy of PML estimates is related to the stability of a portfolio of risks,

(3) methods of measurable accuracy for determining PML of a risk.

1. The definition of PML

Dr. McGuinness noted that a four-year investigation among company underwriting executives revealed a singular lack of unanimity on the meaning of Probable Maximum Loss. One of my former underwriting associates had a favorite jingle pointing up the shades of meaning which underwriters attach to PML. He was, however, once somewhat taken aback when an underwriting trainee who, on being questioned as to the PML on a particular acceptance, responded that since the policy authorized $100,000 which was the full value of the risk, he judged that the PML should not likely be more than that figure.

Actually, McGuinness offered two definitions of PML and seemed to favor the following modification of the second:

Fire (Ex. Dwellings) - Ratio of First $1000 of Loss to Total Loss – Correlated to Insurable Value of Risk

Value of Risk (Mean of Range) (x)	Losses Paid* Total	Losses Paid* Segment Under $1000	% Loss Under $1000 (y)	Log y (z)	(Log x)	Log Log x (w)	(w^2)	(wz)
$ 7500	8439	3523	41.7%	1.62014	3.87506	0.58827	0.34606	0.95308
17,500	19 068	3718	19.5%	1.29003	4.24304	0.62768	0.39398	0.80913
37,500	25,083	2819	11.2%	1.04922	4.57403	0.66036	0.43608	0.69286
75,000	29,180	2213	7.6%	0.88081	4.87506	0.68802	0.47337	0.60601
175,000	30,466	1849	6.1%	0.78533	5.24304	0.71958	0.51780	0.56511
375,000	17,664	805	4.6%	0.66276	5.57403	0.74617	0.55677	0.49453
Sub total	129,900"	14,927"		6.28829"		4.03008"	2.72406"	4.12132"
Open End	33,519	1793						
Total	163,419"	16,720"						

$\Sigma z = na + b \Sigma w$ $\quad 6.28829 = 6a + 4.03008\,b$

$\Sigma wz = a \Sigma w + b \Sigma w^2$ $\quad 4.12132 = 4.03008\,a + 2.72406\,b \times \left(\frac{6}{4.03008}\right) = 6.13584 = 6a + 4.05555\,b$

$0.15245 = (-)\ 0.02551\,b$

$b = (-)\ 5.9761$

$a = +\ 5.0621$

Least Squares Equation $z = 5.0621 - 5.9761\,w$

$\log y = 5.0621 - 5.9761 \log \log x$

$y = 10^{5.0621}(\log x)^{-5.9761}$

$y = \frac{10^{5.0621}}{(\log x)^{5.9761}} = \frac{a}{t^b}$ or $y = a \cdot t^{-b}$ where $t = \log x$

This is a common form of the Pareto Distribution

* In $1000

PML

"The probable maximum loss under a given insurance contract is that portion of *100 (m + k) %* of the limit of liability which, with probability '*p*,' is greater than, or equal to, any loss covered by the contract."

I am reasonably convinced that my former underwriting associate would not be at all inclined to take exception to this definition, as being much less meaningful than the others with which he was familiar, once the terms had been explained to him. It is likely, however, that he would have a number of searching questions as to the basis of the m and k and particularly the p values. It is not likely that he would be much impressed by a 5% or 1% confidence limit, in the feeling that he could not afford to accept, albeit, such a small probability, in view of the even smaller probability of any large fire loss.

However, this reviewer is inclined to regard the McGuinness definition as being more compact and certainly more mathematically precise, once the parameters of his test have been set. Nevertheless, there is still the lurking suspicion that there may be no substantial gain in understanding, via any such mathematical definition, if the probabilites to be assocated with it cannot be handled with the statistical assurances required.

2. *How accuracy of PML estimates is related to the stability of a portfolio of risks*

It is difficult within a given framework to disagree with the McGuinness proposition that the immediate purpose of PML is to select the maximum amount of insurance that an underwriter should retain on the risk for his own account — at least, to the extent that this observation may be tautological. Nor can one easily take exception to the McGuinness formula $Ca - Ce = k$ where Ca is the total, Ce the expected claims, and k is a constant.

It is noted, however, that an underwriter might arrange his risk selections so that his annual loss ratio variation would be minimal by writing relatively small lines on acceptable risks. Conversely, it is possible for the same underwriter, while allowing for a greater variation in his annual loss ratio expectancy, to increase his company's long-term profit by writing large lines on super-choice risks.

3. *Methods for measuring PML*

It is believed that Dr. McGuinness is correct that the statistics needed to determine PML, as defined, are not now collected (except possibly for

dwelling risks) on any formal industry program. The McGuinness proposal is believed to involve the collection of losses related to insurable value on initially a simple class basis. He would then determine the maximum percentage of loss involved in, for instance, 90% of all claims in each category.

This procedure is seemingly the reverse of the typical deductible analysis. It is believed that one will readily appreciate the considerably more difficult task of making reliable estimates of the appropriate charge in risk rates for losses in excess of, say, 90% of insurable value than determining the expected savings under a 1% valued deductible. Incidentally, the percent deductible savings is a function of risk size which, also, would not likely prove a negligible factor in the McGuinness proposal.

It is possible that some companies are now collecting, for their own use, data on the percent loss to insurable value, and such statistics may well be helpful in setting company line sheets and underwriting risk gradings. It is thought that many underwriters are not unaware of the danger involved in projecting top line loss experience in view of the relatively small likelihood of loss in these upper regions, and are guided accordingly in the PML evaluations.

In summary, this reviewer believes that Dr. McGuinness is to be commended for an interesting and thought provoking article of particular value to the CAS membership as a reminder of the work by European actuaries on the Pareto curves.

DISCUSSION BY EDWARD B. BLACK*

The author's treatment of the Probable Maximum Loss concept is both interesting and thought-provoking from an underwriter's viewpoint. It is a subject of great importance because a clear understanding of PML and its application can spell the difference between profit or loss, success or disaster, in the property insurance line. Mr. McGuinness aptly establishes this fact in his reference to the large losses at the oil refinery in Louisiana and the exhibition building in Chicago, Illinois. No one can debate the serious outcome of the reported deficiencies in the PML factors in such instances and I suggest these two examples could be multiplied many times in any year

* Mr. Black was a guest reviewer of this paper. He is Secretary-Underwriting at the Insurance Company of North America and is in charge of that company's commercial fire and allied lines underwriting.