Comparison of Values

Year Ended	Average Paid Claim Cost	Line of Best Fit	Graduation No. 1 $z=2, a=2$	Graduation No. 2 $z=3, a=2$	
3/31/60	624	600.00	604.84	611.56	
6/30/60	602	609.56	610.58	610.53	
9/30/60	603	619.12	617.39	614.29	
12/31/60	620	628.68	625.85	622.30	
3/31/61	624	638.24	635.74	633.25	
6/30/61	661	647.80	646.54	645.42	
9/30/61	669	657.36	657.05	657.14	
12/31/61	672	666.92	666.90	667.61	
3/31/62	678	676.48	676.34	677.09	
6/30/62	670	686.04	685.96	686.50	
9/30/62	690	695.60	696.39	696.82	
12/31/62	718	705.16	707.42	708.48	
3/31/63		714.72	718.44	721.49	
6/30/63		724.28	729.46	735.84	
9/30/63		733.84	740.49	751.53	Extrapolated
12/31/63		743.40	751.51	768.56	Values
3/30/64		752.96	762.53	786.93	
6/30/64		762.52	773.56	806.65	
Projection Factor		1.081	1.093	1.139	

AUTHOR'S REVIEW OF DISCUSSIONS

Mostly I have only to thank Messrs. Nelson and Snader for their kind reviews.

Mr. Nelson recalls reading my remarks of twenty-five years ago on the subject of Whittaker-Henderson formulas, incidental to a paper on tables of risks inferred from the then rather new "excess ratio" tables. He says excess ratios were his main concern and the passing remarks on Hendersonian graduations got but passing attention from him. That was the emphasis intended. I wonder if he missed, as I find others have, the graphs which were for some reason printed on pages preceding the paper.

Graphical representations are so useful. I have read that the great Karl Pearson stressed graphical treatment. Therefore I appreciate Mr. Nelson's

word that when one plots the actual and the smoothed data, the measure of relevancy in graduations is "seen."

There had been no references to Whittaker-Henderson formulas in the Proceedings before 1942. We were investigating the implied distributions of risks by loss ratio, floundering in trial and error, when Dr. Franklin Satterthwaite, who was then in our companies' group insurance operations and active in this Society, suggested that Formula A, as found in C. A. Spoerl's paper, was the tool to use, and so it proved. This was my introduction to Formula A. One fixed impression as to the formula I expressed at that time: "The biggest difficulty in a Whittaker-Henderson Formula A graduation is to get the right start."

In the course of using the formula now and then over the years, not ungrateful for Spoerl's corrections for unsatisfactory starts given in his paper, the impression remained. The right start was still the stumbling block. The "involved methods," of which the Society of Actuaries' Monograph speaks, seemed not in keeping with the relative simplicity of the operating formulas. Therefore, a year or two ago, I was happy to discover that Henderson's auxiliary u''' column for deriving initial values could be lengthened out by the up-and-down iteration to any desired accuracy of initial values.

Mr. Snader tries out the fourth- and sixth-order difference equation graduations on P. K. Stern's average paid claim costs (Snader's Graduation No. 1 and Graduation No. 2). He used $a = 2$. The smaller the a, the weaker the graduating effect. I would prefer $a \geqq 3$ for Graduation No. 2 because the stronger graduating effect when minimizing the higher order of differences seems desirable. It is interesting that Graduation No. 1 produces practically the same projection factor as fitting the line to the logs of the average claim values instead of to the values themselves. This substitutes a least squares pro rate increase for a least squares absolute increase.

A comment on the time-consuming aspect may be in order. It is my experience that a person reasonably conversant with the processes could complete any of the graduations mentioned, the one in the Notes and the several in the Reviews, in an hour or two with an office desk calculator.

Both the paper and the reviews have referred to laws underlying the data. I trust this does not commit any of us to any rigid views about the nature of phenomena.