BOOK NOTES

D. V. Lindley, *Introduction to Probability and Statistics from a Bayesian Viewpoint, Part 1: Probability, Part 2: Inference,* Cambridge University Press, 2 vols., 1965.

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Professor Lindley's two volume work is, professedly, the first attempt to attack the first course in probability and statistics from a Bayesian point of view. This should be of considerable interest to the actuary, since the Bayesian approach to statistics helps the actuary both to explain, in statistical terms, some of the techniques he has been using, and to find applications of some statistical methods to actuarial problems.

The first volume of Lindley's text comprises 4 chapters, 259 tightly reasoned pages, and is concerned with probability. Volume 2, 292 pages, contains chapters 5 to 8, dealing with statistical inference. But let no one assume that these slim volumes are easy reading; the level of mathematical sophistication required is high, and the notation and organization very different from that usually used in American introductory textbooks. Professor Lindley's two books may be an excellent introduction to probability and statistics for the pure mathematician, but they make heavy going for the typical actuarial student and even for many practicing actuaries.

The first chapter of volume I deals with the axioms of probability, the concept of probability as a limit of a relative frequency of successes, and the notions of independence and random trials. Bayes theorem, of course, is given a prominent place, and, throughout the book, the notation is such as to emphasize the conditional nature of many probabilities, with $P(A|B)$ being used in many instances where another author would write $P(A)$. The Bayesian bias of the book is also evident in the statement, which appears after the definition of $P(A|B)$ as the degree of belief in proposition A given that proposition B is true (or that even B has occurred), that: "The main subject matter of statistics is the study of how data (events) change degrees of belief; from prior, by observation of B, to posterior." Lindley also makes clearer than most other authors the relationship between the probability $P(A_i|H)$ where A is an event and H a hypothesis and the likelihood $P(A|H_i)$ and states that: "One speaks of the probability of an event or the likelihood of a hypothesis," depending on which is the variable.

Chapter 2 deals with probability distributions of one variable. It intro-

duces the concept of a random variable, certain theorems dealing with expectations, and, of course, the binomial, Poisson, and normal distributions. Chapter 2 also contains material not often found in introductory texts, such as the simple random walk and the introduction of the characteristic function and the cumulant generating function, in addition to the more familiar moment generating function.

Chapter 3 introduces probability distributions in several variables, and derives joint and conditional distribution functions for Poisson and normal random variables, as well as notions of correlation and regression, in a highly condensed and mathematically elegant fashion. The central limit theorem and both the weak and strong laws of large numbers are given and the chapter also contains an erudite discussion of convergence and ergodicity which are well beyond anything normally found in an introductory statistics textbook. The chapter ends with the use of the Cauchy distribution, "the standard skeleton in the statistician's cupboard" to demonstrate the importance, to the central limit theorem, of the existence of the mean and the variance.

Chapter 4, entitled "Stochastic Processes," is of considerable interest to the actuary, and much of it will be unfamiliar to him. It deals with immigration-emigration and queueing processes, as well as renewal theory and random walks; all of these are related to the risk theory which is becoming so important to the actuary. There is also a 20 page introduction to Markov Chains which covers a good deal of material in a rather limited space.

It is interesting to note that, after the first chapter, the Bayesian ideas of prior distributions and subjective probability are not used until volume 2. Chapters 2-4 of volume 1 are independent of the Bayesian viewpoint which Lindley espouses and which he uses throughout volume 2.

Volume 2 contains chapters 5-8, dealing with statistical inference. Chapter 5 is a masterful exposition of the estimation of parameters from sample data when the random variable has a normal distribution, of course from a Bayesian point of view. Lindley discusses confidence intervals for population means and variances, significance tests, and the meaning and use of sufficient statistics. One would have expected that Lindley's Bayesian orientation would lead him to devote considerable space to decision theory, but he allots it only five pages. The description of confidence intervals and the explanation of the non-uniqueness of the confidence interval prescribed by most elementary texts is valuable and the discussion of the Student-Fisher t distribution and how it differs from the normal is explicit and original. Hypothesis testing is largely ignored: those confusing evil spirits

which jinx many beginning statistics students — Type I and Type II errors — are conspicuous by their absence. As we would expect in this type of book, considerable use is made of the likelihood function, but, to preserve generality, most of Lindley's Bayesian applications assume a uniform prior distribution corresponding to vague prior information.

Chapter 6 extends the theory of statistical inference to multiple samples taken from different normal distributions. Testing the difference between two means, comparing variances using the F test, and an introduction to analysis of variance are among the topics covered.

Chapter 7, entitled "Approximate Methods," discusses the method of maximum likelihood, chi-square goodness of fit tests, and contingency tables. The Bayesian viewpoint and the importance of prior knowledge are often evident, as in a lucid discussion of why the chi-square test is not as good as the usual type of significance test when the form of the distribution is known to be normal.

Chapter 8, which discusses least squares, regression, correlation, and analysis of variance, requires some knowledge of matrix algebra. Lindley's caveat against unthinking use of the correlation coefficient and his warning that it tends to overemphasize the association between two random variables would be useful to many social scientists.

Professor Lindley's two volumes are remarkable. Their unique and unorthodox approach to probability and statistics will be an eye-opener to those unfamiliar with the Bayesian school of statistics. Lindley's unconventional approach to many statistical topics does not make the classical methods obsolete; rather the two approaches complement each other, and reading Lindley will make many facets of statistics, only dimly perceived in the half-light of classical treatment, come alive.

Although not recommended for the neophyte, Lindley's two volume text is very worthwhile reading for the intellectually curious actuary whose mathematics has not grown too rusty. Volume 2 in particular, especially chapter 5, is almost "must" reading for the actuary who, having passed part 2 of the C.A.S. examinations some years ago, has a smug feeling that he knows quite a bit about statistics. A word of warning may not be amiss, however; the book is so compact and the path it follows so different from the furrow plowed so regularly by the great number of elementary statistics textbooks published each year that many readers will find it very difficult at first reading. Do not despair; read it again and you will find it lucid, perceptive, and very educational.