undoubtedly seek to eliminate the marginal risks, thereby increasing his expected return and reducing his variance (risk) for that line.

Thus, if a portfolio selection model is to be developed for property-liability insurance, it must be more complex than the Markowitz model. The expected return for a line of insurance is not single-valued, but is a function of the proportion of the total portfolio committed to that line of insurance and the rate of growth of the total portfolio. Likewise, the variance of return of each line of insurance is not single-valued, but a function of the same variables.

A simple example will illustrate this point. Assume that an insurer has one-half of its portfolio in auto insurance, one quarter in homeowners, and one-quarter in commercial fire. All three lines have an expected return of 3 per cent. The insurer becomes concerned about the future risk in auto insurance, and decides to reduce his auto portfolio to 35 per cent. Homeowners is raised to 35 per cent and commercial fire to 30 per cent.

Under the Ferrari-Markowitz model, the expected return on the portfolio would remain unchanged because the expected return on each line of insurance is single-valued, and not related to the proportion of that line in the total portfolio. In fact, however, the expected returns on auto insurance would undoubtedly rise, and those on homeowners and commercial fire would probably fall if premiums were increased in those lines to maintain total premium writings at a constant level. If the expected return on auto rose to 3.5 per cent, and the return on homeowners and commercial fire both fell to 2.8 per cent, the expected return on the total portfolio would rise above 3 per cent. A similar example relating to the variance (risk) could be cited. Both indicate that the assumption of uniform parameter values for risk and return applying to all portfolio proportions oversimplifies the property-liability insurance model.

Finally, I would suggest that any further work on the Ferrari-Markowitz model might also attempt to incorporate the investment portfolio of an insurer within the model as a means of generalizing its application. Certainly, Mr. Ferrari has written an ingenious and interesting paper, and it merits further exploration and analysis by casualty actuaries.

DISCUSSION BY MATTHEW RODERMUND

Professor Ferrari's paper is scholarly, well-written, interesting, and, not least, courageous. The author is welcomed to the Society as an Associate at the November meeting, but his paper was presented to the Society in May by

invitation, with the understanding that it be subject to the same treatment accorded papers submitted by members of the Society, that is, acceptance or rejection by the Committee on Review of Papers and exposure to critical review.

By stating these ground rules, the reviewer feels less guilty about being critical of a guest. In the reviewer's opinion the paper is irrelevant to the present or future state of the property-liability insurance industry. A company employing the techniques Professor Ferrari describes might have great fun (that is, if computer time can be spared), like an individual pursuing a solution to the twelve-balls problem;* but when the task is completed, has anything useful been accomplished?

The paper aims at providing "an initial report on utilization of portfolio selection techniques to suggest the theoretical optimal diversification of lines of insurance written by property and liability insurance companies." In his portfolio selection analysis the author employs a risk and return concept and assumes that "the expected return of a line of insurance is a function of profitability (as measured by loss and expense ratios) and risk is a function of the variability around the expected return." The technique is based on a study, by Dr. Harry Markowitz, of investment portfolio diversification; in this paper, however, the Markowitz approach is applied to a portfolio of lines of insurance, and from the application is developed the Markowitz E-V criterion, E-V being a handy abbreviation for "expected return on the portfolio and its variance."

Professor Ferrari is aware of the limited usefulness of his approach to portfolio selection. In his concluding paragraph he states:

"The application of portfolio selection techniques to property and liability insurance companies [Professor Ferrari refers to relatively sophisticated techniques like the one he has described] has some *interesting* theoretical possibilities as well as *serious* practical limitations." (The italics are the revewer's, and they are intended to emphasize Professor Ferrari's choice of adjectives: *interesting* possibilities but *serious* limitations.) Had Professor Ferrari pursued the practical limitations with the thoroughness with which he pursues the theoretical possibilties, possibly he might not have written the paper. If, on the other hand, he had not set forth the limitations as completely and objectively as he does, the reviewer might not have had a solid base for this discussion.

* The reviewer will be delighted to describe the twelve-balls problem to any reader who has never heard of it.

Professor Ferrari indicates his own doubts frequently. But he does not explore their implications. When he first introduces the subject of the constraints that had to be recognized in his study, he says:

"In every portfolio selection application, the combined influence of regulation, managerial policy and practical considerations places constraints on the freedom of action." In the same paragraph: "The obstacles to such [freedom of] action result primarily from the maintenance of agency relationships, the insurance consumption pattern of insureds, and competition among insurers." Such obstacles are not inconsiderable; their influence is frequently decisive.

According to Professor Ferrari, "There are two . . . . areas of difficulty that . . . . limit the . . . . application of the Markowitz technique to property-liability company insurance portfolios. The first pertains to the nature of the input assumptions of the model. The second is the uncertain relationship between the Markowitz E-V criterion and the objectives and behavior of non-life insurance companies."

With respect to input assumptions, the author admits that the historical method described by Markowitz, which can be used for quantifying expectations of future return, "is deficient to the extent that it ignores the dynamic aspects of the insurance business. For example, the relative adequacy of future rate levels may differ from that evident in the historical data." Then he makes the surprising suggestion that "this and other similar difficulties can be alleviated by introducing expectations into historical parameters by adjustments based on *subjective judgment.*" (Reviewer's italics.) Thus seat-of-pants wisdom, a traditional tool of the underwriter, is introduced to the computer!

Another input problem: "Even if a property-liability insurer could significantly alter its insurance portfolio, this action could so seriously affect loss and expense ratios as to destroy the assumptions on which the reallocation was based." The reviewer agrees.

As to the relationship of the Markowitz E-V criterion to company objectives and behavior, it is here that Professor Ferrari strikes the most telling blow against the feasibility of his theoretical portfolio selection technique:

"The structure of the insurance business is such that non-life insurance companies can attain great diversification by lines of insurance without conscious marginal risk-return decisions. A large company can be expected to establish variety in its lines of insurance simply because of the nature of the marketing channels, the sheer size of the portfolios, and the complementarity

of certain lines of insurance, for example, auto bodily injury liability, auto property damage liability, and auto physical damage."

Three comments:

(1) The large company, as Professor Ferrari implies, hardly needs a theoretical portfolio selection approach. The medium size company and the small company have so many practical considerations to resolve—such as the insurance needs of the area in which they operate, the number and quality of employees capable of servicing these needs, the available capital and surplus, the nature of production arrangements, and the strength of the competition—that a theoretical study of profitability becomes too expensive in terms of the limited influence that the results of the study might have on decision-making.

(2) Professor Ferrari refers to the complementarity of certain lines of insurance. Clearly auto bodily injury liability and auto property damage liability will continue to be offered in a package regardless of the profitability of either coverage with respect to the other. Similarly fire and extended coverage in the dwelling field. Complementarity also has its broader aspects. A company writing all lines but specializing throughout its history in personal lines is not likely either to alter the composition of its personal lines portfolio or to move more strongly into commercial lines as the result of a sophisticated profitability study. Nor, if it specializes in commercial lines, will it adjust the distribution of those lines or move into personal lines in a big way. A company writing a good volume of workmen's compensation business, and jealous of its markets, will not alternately increase and decrease its general liability business, auto fleet liability business, or even commercial fire business, *on the basis of profitability studies.* It may move in or out of certain classes, set new capacity limits, or adjust its agency set-up, but generally it will try to hold its markets. And when it does make portfolio adjustments, considerations other than profitability are apt to be the major factors.

Professor Ferrari undoubtedly knows that the package concept is becoming increasingly prevalent in property-liability insurance, but he may not have realized that a company's portfolio can no longer be pictured adequately by a percentage distribution of the lines of business designated in the annual statement. In his Table 2, Professor Ferrari illustrates the percentage composition of the portfolio of an anonymous company by using annual statement lines of business, and applies the Markowitz E-V criterion to that portfolio. Application of the Markowitz technique produces results (as

shown in Tables 3, 4, and 5) that he concedes in some respects are not acceptable. He might have fared better by finding out from his anonymous company the dozen or so main types of policies issued by the company, obtaining the premium distribution for those types of policies, and applying his technique to the rearranged portfolio distribution. The limitations of the technique would not have been obviated, but the results might have been less unacceptable.

Incidentally, the author errs in including treaty reinsurance in the portfolio items subject to his computer techniques. Treaty reinsurance is a conglomeration of all the other lines of business (or types of policies) in the portfolio, and therefore the decision as to expansion or contraction of the reinsurance portfolio runs up against countless variables not at all analogous to those involved in decisions affecting primary lines of business.

(3) Finally, with respect to the relationship of the Markowitz E-V criterion to company objectives and behavior, it seems to the reviewer that a company's principal objectives are first and foremost to provide markets and to grow, and its behavior is based on these objectives. Profitability of the business and the degree of variability of operating results are not to be ignored; indeed, without attention to these the company will cease to exist. But there is an old axiom in the insurance business that any risk (subject to certain qualifications inherent in the business) can be insured; the problem is to obtain the proper premium. Thus, a company's first decisions on its portfolio generally relate to what markets it feels itself prepared to provide, and profitability then becomes a function of the premiums it needs to provide the required market. Obtaining a proper premium, under the present regulatory system, is not always easy; but the solution to *that* problem is hardly furnished by a theoretical portfolio selection technique based on profitability. If there is a ready market for a type of insurance coverage, the responsible company is not likely to de-emphasize that coverage because its returns do not meet expectations. More likely the responsible company will devote its energies to servicing that market more efficiently and at the same time working for higher premiums.

The technical aspects of Professor Ferrari's paper are presented in quite understandable terms, intelligible to the lay actuarial reader. Professor Ferrari emphasizes that profitability is not solely a function of combined loss and expense ratios, but also a function of the variability of such ratios around the expected returns. He further indicates that the risk of a given portfolio is not simply the weighted sum of the variances of the individual lines, but a function of both the risk of each individual line and the cor-

relation of returns between each pair of lines. The rationale is well presented and the technique is neat.

The reviewer has prided himself on a progressive attitude toward the introduction of refined actuarial techniques in the management of our business. Now he seems to be rejecting an interesting, forward-looking technique, and saying nonsense, it's not relevant to our business, it's not practical, it will never get off the ground. Undoubtedly the Markowitz E-V criterion has its uses, and the investment portfolio may be a fruitful area for its employment. But the underwriting portfolio, because of the profound practical considerations that Professor Ferrari lists but does not sufficiently evaluate, is not, in the reviewer's opinion, a proper field for effective use of the technique described.

DISCUSSION BY LEROY J. SIMON

The Ferrari paper is one of the most significant papers we have had in the Casualty Actuarial Society *Proceedings*. It will stand as a landmark to be referred to many, many times in the future by researchers and actuaries alike. The paper touches me in a personal way because for at least six years I have carried a note to myself to attempt to develop a "balanced book approach combining profit with stability." This paper is the first significant step in that direction.

The author is a very strict critic of his own work because he never hesitates to point out the areas in which caution must be exercised. He does not offer his paper as a panacea for management or as a computerized substitute for decision making. He does, however, give us an insight into a very powerful tool and shows how it would operate. Particularly impressive is the fact that he has actually applied the technique in a concrete situation and presents the results for the reader to review. As one would suspect, the results do not say "do this" or "don't do that" but rather point in directions where the company would benefit if they would place additional emphasis or impose some restraints. This may give direction to field force efforts, channel advertising themes, or suggest areas for agency contests which the company may wish to pursue. It is rather doubtful that a manager would examine these results and cut out a given line of business merely because of the indications. As the author points out, there are many more factors to be considered other than the results of a statistical analysis. However, management now has an additional signpost pointing in the proper direction which should be a helpful guide in their decision making process.

We must all keep in mind that many procedures and techniques in the